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A STUDY OF THE GREEK EPIGRAM BEFORE 300 B. C.

By Florence Alden Gragg.

A STUDY OF THE GREEK EPIGRAM BEFORE 300 B. C.[1]

By Florence Alden Gragg.

Presented by H. Weir Smyth, February 9, 1910. Received May 10, 1910.

Although there exist several collections of Greek epigrams and many treatises on individual epigrammatists, such as Anacreon, Simonides, and Plato, no one has as yet collected and arranged in chronological order the epigrams which have been preserved both on stone and in MSS. and examined them in detail with the purpose of throwing light

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[1] Selected Bibliography together with a List of Abbreviations by which the works are cited:

AEMO = Archaeologische epigraphische Mittheilungen aus Oesterreich.

AM = Mittheilungen des Deutschen Archaeologischen Instituts, athenische Abtheilung.

AP = Anthologia Palatina.
I–IX, 563, ed. H. Stadtmüller. Leipzig, 1894–99.
IX–end, with appendix, ed. F. Jacobs. Leipzig, 1814.

A Pl = Anthologia Planudea, ed. F. Jacobs. Leipzig, 1884.

Allen = F. D. Allen, On Greek Versification in Inscriptions. Papers of the American School of Classical Studies at Athens, vol. 4, pp. 37–204. Boston, 1888.

B, Bergk, or PLG = Th. Bergk, Poetae Lyrici Graeci[4]. Leipzig, 1892.

BCH = Bulletin de Correspondence Hellénique.

Boas = M. Boas, De Epigrammatis Simonideis. Groningen, 1905.

CIA = Corpus Inscriptionum Atticarum.
Vol. I, ed. Kirchhoff. Berlin, 1873.
Vol. II, ed. Koehler. Berlin, 1877–88.
Vol. III, ed. Dittenberger. Berlin, 1878–82.
Vol. IV, ed. Koehler. Berlin, 1877–91.

CIG = Corpus Inscriptionum Graecarum.
Vol. I, ed. Boeckh. Berlin, 1828.
Vol. II, ed. Boeckh. Berlin, 1843.
Vol. III, ed. Franz. Berlin, 1853.
Vol. IV, ed. Curtius & Kirchhoff. Berlin, 1877.

CRAI = Comptes Rendus de l'Académie d'Inscriptions et de Belles-Lettres.

Fava = D. Fava, Gli Epigrammi di Platone. Milan, 1901.

Haenel, J. De Epigrammatis Graeci Historia. Breslau, 1852.

H = E. Hoffmann, Sylloge Epigrammatum Graecorum quae ante medium saeculum a. Chr. n. tertium incisa ad nos pervenerunt. Halle, 1893.

Hauvette = A. Hauvette, De l'Authenticité des Epigrammes de Simonide. Paris, 1896.

Henkenrath, R. Studien zu den griechischen Grabschriften. Feldkirch, 1896.

on the development of this branch of Greek literature. Historians of Greek letters[2] have touched but lightly on the epigram. Haenel's book, published in 1852,[3] is now out of date and, even at the time of its publication, did not do justice to its subject. Mackail, in a work as useful as it is charming,[4] has included only such epigrams as are

Hiller = E. Hiller, Zu den Simonideischen Epigrammen, Philologus, 48, 229–247.
IA = Inscriptiones Argolidis, ed. M. Fraenkel. Berlin, 1902.
IGA = Inscriptiones Graecae Antiquissimae, ed. Roehl. Berlin, 1882.
IGS = Inscriptiones Graeciae Septentrionalis, ed. Dittenberger. Berlin, 1892.
IIS = Inscriptiones Italiae et Siciliae, ed. Kaibel. Berlin, 1890.
IP = Die Inschriften von Pergamon, ed. M. Fraenkel. Berlin, 1890–95.
JHS = Journal of Hellenic Studies.
JOAI = Jahresheft des Oesterreichischen Archaeologischen Instituts zu Wien.
Junghahn, A. A. De Simonidis Cei Epigrammatis Quaestiones. Berlin, 1869.
K = G. Kaibel, Epigrammata Graeca ex Lapidibus Conlecta. Berlin, 1878.
K(RM) = G. Kaibel, Supplementum Epigrammatum Graecorum, Rheinisches Museum, 34 (1879), 181 ff.
Kaibel, G. Quaestiones Simonideae, R. M. 28 (1873), 436 ff.
Kirchhoff, A. Zur Geschichte der attischen Epigrammen, Hermes 5 (1871), 48 ff.
Mackail, J. W. Select Epigrams from the Greek Anthology. London and New York, 1906.
Von Mess, A. Quaestiones de Epigrammate Attico et Tragoedia Antiquiore Dialecticae. Bonn, 1898.
Olympia = Die Inschriften von Olympia, ed. Dittenberger & Purgold. Berlin, 1896.
Pr = Th. Preger, Inscriptiones Graecae Metricae ex Scriptoribus praeter Anthologiam Collectae. Leipzig, 1891.
REG = Révue des Études Grecques.
RM = Rheinisches Museum für Philologie.
Reitzenstein = R. Reitzenstein, Epigramm und Skolion. Giessen, 1893.
Reitzenstein, R., in Pauly-Wissowa, Real-Encyclopaedie, s. v. Epigramm.
Roberts = E. S. Roberts, An Introduction to Greek Epigraphy, vol. I. Cambridge, 1887.
Smyth, H. W. Sounds and Inflections of the Greek Dialects; Ionic. Oxford, 1894.
Wagner, R. Quaestiones de Epigrammatis Graecis. Leipzig, 1883.
Weber = L. Weber, Anacreontea. Göttingen, 1895.
Wilamowitz-Moellendorff, Ein Altattisches Epigramm, Hermes, 20 (1886), 62 ff.
Simonides der Epigrammatiker, Göttinger Nachrichten, 1897, 306 ff.
Wilhelm, A. Simonideische Gedichte, JOAI 2 (1899), 221 ff.

The lyric poets are cited by Bergk's numbers; Aeschylus by the edition of Sidgwick (Oxford, 1902); Sophocles by that of Jebb (Cambridge, 1906); Euripides by that of Prinz-Wecklein (Leipzig, 1898–1902); the fragments of the tragic poets by the numbers of Nauck, Tragicorum Graecorum Fragmenta², Leipzig, 1889.

[2] I have consulted the works of Bergk, Berlin, 1872–87; Bernhardy, Halle, 1880; K. O. Muller (ed. Heitz), Stuttgart, 1882; Sittl, Munich, 1884–7; Bender, Leipzig, 1886; Croiset, Paris, 1887–99; Christ, Munich, 1905; Flach, Ges. d. Gr. Lyrik nach d. Quellen dargestellt, Tubingen, 1883–4.

[3] De Epigrammatis Graeci Historia, Breslau, 1852.

[4] Select Epigrams from the Greek Anthology, London and New York, 1906.

remarkable for beauty of thought or execution. Consequently he has admitted few early epigrams and his arrangement by subject precludes any attempt at chronological order. Reitzenstein, in "Epigramm und Skolion," discussed brilliantly the nature and history of the epigram,[5] but his interest was chiefly for the work of the Alexandrian period and he treated the early verses only as they could be made to support his original theory about those of later date. A recent article by the same scholar[6] is by far the most satisfactory presentation of the subject known to me, but the necessity of discussing the whole history of the epigram in a few pages has prevented him from giving much space to the early period or considering individual epigrams to any extent. Moreover, since the publication of the collections of Kaibel, Allen, Preger, and Hoffmann many new epigrams have come to light. These, together with numerous suggestions of various scholars, are scattered in footnotes and separate dissertations, where they easily escape the notice of the general reader.

For these reasons it seemed to me a profitable task to collect the early[7] epigrams and, so far as I could, to trace the changes which gradually took place in the nature of the epigram and its relation to other branches of literature.

Appended to this paper is a list of epigrams earlier than 300 B. C.[8] To the material already at hand in the various thesauri I have added such epigrams as I have myself gathered from the chief classical journals published since the appearance of Hoffmann's book in 1893. I trust that no epigram has been omitted for lack of care or diligence on my part, but, even so, I cannot hope that the list will seem complete to every reader. We possess, in the Palatine Anthology and elsewhere, epigrams which give us absolutely no clue to their age, though certain of them may seem to individual scholars to be early.[9] If any of these are missed, it is because I did not feel warranted in inserting in a list from which historical conclusions were to be drawn, any epigrams which are assigned to an early date merely by the "literary feeling" of this or that scholar. On purely literary grounds it is often possible to say with comparative certainty that an epigram is later than the fifth century; it is practically never possible to say that it is earlier than the third, for the distinctive characteristics of the epigrams composed after 400 B. C. do not make their appearance earlier, while the austere and

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[5] pp. 87 ff.

[6] Pauly-Wissowa, Real-Encyclopaedie, s. v. Epigramm.

[7] I. e., those composed before 300 B. C.

[8] pp. 45 ff.

[9] E. g. Pr 64, 65, 206. Cf. PLG 2. 377 ff.

simple epigram belongs — in greater or less degree — to all periods. I have also thought it safer to omit epigrams which, though they are extant only in inscriptions of a late period, are thought by some to reproduce early inscriptions.[10]

At the end of the list[11] I have enumerated the epigrams which, in my opinion, are wrongly attributed to early poets.

When Haenel said that we ought to call no poem an epigram unless we know when it was composed[12] he meant to draw a distinction between epigram, as we commonly use the word, and *ἐπίγραμμα* as used by the Greeks of the sixth and fifth centuries B. C. A Greek epigram[13] is to most of us any short poem — irrespective of the sentiment expressed — complete in itself and composed in the elegiac metre. Such a poem, Haenel says, would not have been an epigram at all to the Greeks of the sixth century B. C. In this he is of course quite right, for it is clear that the early Greeks would have assented to the definition we find in Suidas — *πάντα τὰ ἐπιγραφόμενά τισι, κἂν μὴ ἐν μέτροις εἰρημένα, ἐπιγράμματα λέγεται.*[14] This continued to be the meaning of the word for a long time, for there is no proof that in Herodotus the word is "on the point of acquiring its literary sense,"[15] if by "literary sense" is meant any sort of poem as distinguished from prose, and that Demosthenes could still apply the term to prose is abundantly evident from his orations.[16] It is not until 94 A. D. that we have actual proof that the word signified a poem in elegiacs. In an inscription of that date discovered near Rome[17] we find the word *ἐπιγράμματα* prefixed to elegiacs to distinguish them from the hexameters which precede. Even in the Palatine Anthology the word appears but twice[18] and the two verses in question — both very late — merely prove that the authors understood epigrams to be poems; they are not in themselves positive evidence that the term included poems which were not inscribed.

Still, that the idea of epigram actually had changed long before 94 A. D., all agree. Collectors of *ἐπιγράμματα* in the third century B. C.

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[10] E. g. CIG 1050, 1051. See PLG 2. 238.

[11] pp. 55 ff.

[12] p. 18.

[13] We are not concerned here with the somewhat different coloring of "epigram" as applied to poems in Latin and other languages.

[14] S. v. *ἐπίγραμμα.*

[15] Mackail, p. 1.

[16] E. g. Or. 22. 72; 24. 180.

[17] K 618.

[18] AP 9. 342: *Φημὶ πολυστιχίην ἐπιγράμματος οὐ κατὰ Μούσας εἶναι· μὴ ζητεῖτ' ἐν σταδίῳ δόλιχον.*

AP 9. 369: *Πάγκαλόν ἐστ' ἐπίγραμμα τὸ δίστιχον· ἢν δὲ παρέλθῃς τοὺς τρεῖς, ῥαψῳδεῖς κοὐκ ἐπίγραμμα λέγεις.*

included in their lists many verses that were never meant to appear on stones. Philochorus [19] and Polemo,[20] indeed, seem to have started with the intention of gathering only inscriptions, but Polemo himself admitted at least one poem of a different sort;[21] and for his contemporaries "ἐπιγράμματα" were no longer "inscriptiones." We learn this from Athenaeus, for when he quotes ἐπιγράμματα it is probable that he quotes them by the titles which their authors gave them. In Hedylus,[22] Nicaenetus,[23] Posidippus [24] we find the name given to convivial poems, and the meaning which the word had assumed in the time of Athenaeus himself is clear from many passages.[25] In short, among the Greeks epigram came to have an even broader meaning than it has with us.

Under these circumstances, if we should try to trace the history of the form of literature which the Greeks from age to age called epigram, we should be met by almost insurmountable difficulties, since neither the times or the causes of the changes in meaning can be determined with any degree of accuracy. Epigram, then, in this paper will have its later meaning, a short, complete elegiac poem. For if we kept the earliest meaning, we should have to exclude from our consideration all verses except those on stone. This would be most unfortunate, for we are searching for the origin of a particular kind of poem, not of a name, and it is the purpose of this investigation to learn whence the later epigram had its source rather than to discover what finally developed out of the early inscription.

Therefore, for purposes of literary history, it is absurd to deny to the following verses of Theognis [26]

Ἄφρονες ἄνθρωποι καὶ νήπιοι οἵ τε θανόντας
κλαίουσ', οὐδ' ἥβης ἄνθος ἀπολλύμενον,

the name which we give to these lines from the Anthology [27]

Τοὺς καταλείψαντας γλυκερὸν φάος οὐκέτι θρηνῶ,
τοὺς δ' ἐπὶ προσδοκίῃ ζῶντας ἀεὶ θανάτου.

merely because the former date from the sixth or fifth century while the latter are some centuries younger. The same is the case with many other distichs, e. g.

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[19] Suidas, s. v. Φιλόχορος.
[20] Athen. 10. 436 d.
[21] Athen. 10. 442 e: Ἦλις καὶ μεθύει καὶ ψεύδεται· οἷος ἑκάστου οἶκος, τοιαύτη καὶ συνάπασα πόλις.
[22] Athen. 11. 473 a.
[23] Id. 15. 673 b.
[24] Id. 10. 415 b.
[25] E. g. Athen. 2. 39 c; 3. 125 c; 4. 162 a; 13. 604 f.
[26] 1069, 1070.
[27] AP 11. 282.

Ἥβα μοι, φίλε θυμέ · τάχ' ἄν τινες ἄλλοι ἔσονται
ἄνδρες · ἐγὼ δὲ θανὼν γαῖα μέλαιν' ἔσομαι,[28]

or

Αἲ γὰρ ἄτερ νούσων τε καὶ ἀργαλέων μελεδωνῶν
ἑξηκονταέτη μοῖρα κίχοι θανάτου.[29]

All these we should call epigrams, if only we could be sure that they are complete in themselves.

By far the greater number of the extant early epigrams were inscribed so that it becomes necessary to examine all early metrical inscriptions, whether elegiac or not — all elegiacs, whether inscribed or not — that we may learn as accurately as possible what causes went to the making of the later epigram.

It is easy to distinguish the inscribed epigrams (if I may be allowed the apparent pleonasm) of the fifth century and earlier, since the forms of the letters testify to their age. It is harder to be sure of those belonging to the fourth century,[30] but usually the style or the content comes to help out any doubtful epigraphical evidence.[31] When, however, we come to the epigrams which are preserved only in MSS. the case is quite different. In the first place the works of the Greek lyric poets are extant in so fragmentary a condition that very often we are unable to say whether a given distich is a complete poem — i. e. an epigram — or a shred torn from a longer elegy. The difficulty will be obvious if we compare the following verses.

Soph. O. C. 1224–8.

μὴ φῦναι μὲν ἅπαντα νι-
κᾷ λόγον · τὸ δ' ἐπεὶ φανῇ
βῆναι κεῖθεν, ὅθενπερ ἥκει
πολὺ δεύτερον ὡς τάχιστα

Theog. 425–428

πάντων μὴ φῦναι ἐπιχθονίοισιν ἄριστον
μηδ' ἐσιδεῖν αὐγὰς ὀξέος ἠελίου ·

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28 Theog. 877, 878.

29 Mimnermus, 6.

30 In the period before 300 B. C. no evidence as to the date is furnished by the arrangement of verses in inscriptions. As early as the sixth century each verse may begin a new line and throughout the period we find epigrams where the verses are not so separated. The latter method is more common in the sixth century, the former in the fifth and fourth. There is no example of an indented pentameter, unless it be 221, v. 2, which in CIA, II, 3. 2339 and in Kumanudes, Insc. Sepulch. Att. 858, is represented as indented one letter. I have not seen the stone.

31 From the collections of Kaibel and Allen I have taken no epigrams which those editors do not definitely assign to a date earlier than 300 B. C.

φύντα δ' ὅπως ὤκιστα πύλας Ἀίδαο περῆσαι
καὶ κεῖσθαι πολλὴν γῆν ἐπαμησάμενον.

AP 9. 359.

Ποίην τις βιότοιο τάμῃ τρίβον; . . .
. . . .
Ἦν ἄρα τοῖνδε δυοῖν ἑνὸς αἵρεσις, ἢ τὸ γενέσθαι
μηδέποτ' ἢ τὸ θανεῖν αὐτίκα τικτόμενον.

Upon examining these passages we can say without hesitation that the first is not an epigram and that the third is, — about the second we are quite at a loss. Sometimes the presence of a particle, as δέ or γάρ, prevents our including such fragments among epigrams, for in the whole Anthology such a particle introduces only one epigram which is not manifestly corrupt or lacking its original beginning.[32] In many cases, however, we are left in doubt, although, even so, it is only in name that they differ from true epigrams. If we should discover for a certainty that they were parts of longer elegies, they would still be of use for historical purposes, since the epigram itself is but a species of elegy.

Epigrams which have been handed down to us in MSS. seem, at first sight, to furnish three kinds of evidence by which we may determine their age. In some famous persons or events are celebrated; some are attributed to known poets; in others we have only the diction or the sentiment to help us. This testimony is not, however, so valuable as it at first appears. It is easy to see how uncertain the first test is, for it merely supplies us with a terminus post quem and, as a matter of fact, of the numerous epitaphs purporting to be those of men of the sixth century not one can with any probability be assigned to a date earlier than the third century.[33] The second test is somewhat surer. Still, when an epigram attributed to Anacreon [34] is found inscribed in letters considerably later than the age of that poet, and when poems in praise of the works of Myron [35] are assigned to the same author, it is easy to see how blind is the trail we follow. More than this, recent discoveries have proved that certain epigrams of four verses [36] preserved

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32 AP 9. 547, where δ' is evidently inserted for the sake of including all the letters of the alphabet in one and the same verse.

33 The epitaphs Pr 238–47, which celebrate early sages, are an excellent example of a set of epigrams composed probably by one author at a late period. Cf. also Preger's note, p. 199.

34 124. (Arabic numerals refer to epigrams on pp. 46 ff.)

35 Anac. 115, 116.

36 83 (= Sim. 96) and 125 (= Sim. 150).

to us in MSS. originally (i. e. in the fifth century) consisted of only two verses. The joinings had escaped the notice of critics for centuries and it is impossible to say how many more such pieces of patchwork there may be in the Anthology and elsewhere.

And yet to admit all this is not necessarily to believe with many scholars [37] that no confidence is to be placed in those MSS. which assign epigrams to definite authors. For, though the tests of authorship which we can apply are most uncertain, still, unless we can bring forward at least highly probable arguments to the contrary, we are bound to give the benefit of the doubt to the only evidence we have. However weak may be the authority of the Palatine Anthology, it is not for us to make it actually testify against itself. Therefore Reitzenstein seems to go too far when he says,[38] "Es ist meines Erachtens unmethodisch bei dieser Art Pseudo-tradition auch nur den Beweis der Unechtheit zu verlangen."

There are two reasons why scholars incline to reject the testimony of the Anthology. In the first place they are reluctant to believe that the early poets wrote epigrams at all — a reluctance which has no evidence to support it. When this art was so generally cultivated were the famous poets the ones to neglect it? Because very few epigrams of these poets have come down to us, are we to reject even those that we have? This is to let individual conjecture weigh against probability and, indeed, against some actual evidence. In the second place many seem convinced that the scribes of the Anthology were possessed by a desire to assign every poem to too early a date. This is certainly, however, not the case, for the discovery of a number of epigrams inscribed on stones [39] has proved that, even if they are not the work of the particular poets whose names they bear in the Anthology, these names point at least to the approximate dates. In some cases epigrams are actually assigned to too late a period. For example 217 — found inscribed in letters of the fourth century — bears in the Anthology the name of Gaetulicus, a poet, indeed, little known to us, but, if we may judge from the other epigrams attributed to him, much later than the fourth century. Again 123, a poem certainly inscribed in the fifth

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37 Wilamowitz (Goett. Nachr. 1897, p. 320): Für uns ist die Consequenz unvermeidlich, dass wir die Autorität selbst der alexandrinischen Sammlung da auch sehr niedrig schätzen, wo die Gedichte selbst keinen unmittelbaren Anstoss geben.

Kaibel (RM 28, p. 441): Cavendum utique igitur est ne citius unquam huic testi (AP) credamus; mendax est ac fallax ut non magis alter.

A. A. Junghahn, De Simonidis Cei Epigrammatis Quaestiones (Berlin, 1869), p. 30: Paene nulla in his rebus fides est Anthologiae.

38 Pauly-Wissowa, p. 80.

39 E. g. 75 and 125.

century, is assigned to no poet at all, so that we may conclude that famous names were not sprinkled over the contents of the Anthology quite so profusely and indiscriminately as some would have us think. Indeed, in all probability, mistakes in authorship come not so much from the perversity of scribes as from confusion and changes in arrangement. Finally, we ought always to bear in mind what gaps there are in our knowledge of Greek literature — gaps which are nowhere wider than in our knowledge of the lyric poets. Under these conditions we ought to give the MSS. at least an unprejudiced hearing.

The third test — that of style — helps us less than one might at first expect. Since the approximate date of the inscribed epigrams can usually be determined with certainty, we should naturally look to them for the standard by which to judge the epigrams preserved only in MSS. But the standard they set is hardly adequate, for while the inscriptions are the work of men widely different in rank, education, and ability, many of the epigrams preserved in MSS. may be the work of famous poets and it would be unfair to deny to a great master the authorship of a given epigram merely because it exhibits more charming sentiment, more graceful diction, more brilliant genius than do the inscriptions composed by ordinary men. Some assistance is given us by the fact that certain formulas seem to leap into favor at certain periods, but any such evidence must be used with caution, for it may be that the original use of a phrase by a great poet gave that phrase its popularity with a later generation.[40]

For our purpose it will be sufficient to determine the age of the epigrams without considering their authorship, but from what has been said above it is evident that even this is difficult enough. It is, therefore, with diffidence that I have approached the task, especially when I remember that certain epigrams which recent discoveries have shown to belong to the fifth century were pronounced late by very excellent scholars.[41] If 217 and 224 had come down to us only in MSS. I venture to think there would be no lack of critics to assign them to a far

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[40] E. g. 19 (= Sappho, 119). Cf. Table III.

[41] Some have even denied that they were inscribed at all. So Kaibel (RM 28, p. 455): Ante omnia Simonidi abiudicanda cum Bergkio aliis [*sic*] epp. 95 et 96, manifesto demonstrativa.

Hauvette, p. 94 (Sim. 108): A notre avis l'inscription ne peut être ni de Simonide ni même d'un poète du 5e siècle qui l'aurait composée dans les premières années de la guerre du Peloponnèse. Cf. Kaibel, RM 28, p. 456.

Hauvette, p. 133 (Sim. 150): Par sa forme, par les idées qu'elle exprime, et par son style cette pièce ne saurait . . . passer pour une inscription réelle, gravée au début du 5e siècle sur une statue dans l'académie . . . une telle formule ne convient qu'à une pièce composée après coup à l'occasion d'une offrande.

later date than the fourth century to which they actually belong. I have therefore tried to err on the side of accepting rather than of rejecting too much. In a case where, so far as internal evidence goes, an epigram might be late or early, the burden of proof rests with those who would assign it to a date later than that indicated by ancient tradition. It is not enough for them to show that it may be late; they must also show that in all probability it is not early. I have not rejected without specific reasons any epigram which any ancient authority assigns to a date earlier than 300 B. C.[42]

Because of the greater number of hexameter inscriptions of the sixth century, it is necessary to devote more attention to them than to later hexameters.[43] The great majority, consisting as they do of a single verse, do not exhibit that difference in feeling which distinguishes the later series of hexameters from the elegy.

In the beginning the Greeks used inscriptions merely as a means of informing the reader as briefly and easily as possible of the reason for setting up the stones on which they were inscribed. Iliad H. 17[44] is familiar evidence that some sort of epitaph was usual. Doubtless in earliest times merely the names of the dedicator and the divinity or of the dead man and his father were cut upon the stones — a practice which survived in combination with the later custom. Cf. IGA 149 (= H 58) Καλλία Αἰγίθ(θ)οιο· τὺ δ' εὖ πρᾶσ(σ' ὦ) παροδῶτα.

Thus, although among the Greeks poetry precedes prose as a literary form, it must have been itself preceded by a ruder form of expression. The use of metre testifies to a certain degree of conscious art and therefore we cannot wonder if in the earliest epigrams which we possess some attempt is made to adorn the bare record of facts. The earliest Greek metrical inscriptions ever composed must have represented, not the first attempts to convey certain information in writing, but the first attempts to convey that information in artistic form. Without doubt long before the time from which our earliest inscriptions date poets had composed songs in memory of the dead and had celebrated offerings to the gods.[45] When the custom arose of inscribing such poems upon stone, those who could not or would not employ the services of professional poets, turned

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[42] See pp. 55 ff. [43] See p. 8.

[44] καί ποτέ τις εἴπῃσι καὶ ὀψιγόνων ἀνθρώπων
. . . .
ἀνδρὸς μὲν τόδε σῆμα πάλαι κατατεθνεῶτος
ὅν ποτ' ἀριστεύοντα κατέκτανε φαίδιμος Ἕκτωρ.

The vocabulary of this passage is echoed by the early inscriptions. Cf. 8, 88, 89.

[45] It is worth while to remember that our earliest Attic inscription is metrical. See AM 6 (1881), p. 107.

poets themselves — they wrote indocti doctique — and the results are what might be expected. Hence the crudeness of some of the verses, which is to be attributed to the particular author, not to antiquity in general. Afterward, when they had become more accustomed to composing and had more models before their eyes, even ordinary men with no greater inspiration than their predecessors acquired greater ease of style and produced fewer rude epigrams. Again the extreme simplicity of many verses is the result of the restraint, not of the lack of skill of the authors, since the Greek of early times felt that only simplicity could be in place in approaching his gods or his dead. So it happens that an epigram very probably written by Anacreon shows the same characteristics as the epigrams on the stones of the Dipylon.[46]

In the simplest epigrams a few common and familiar words fill out the metre, often merely forming a complete sentence of words that in the earliest times had been disconnected. So

Λυσέᾳ ἐνθάδε σῆμα πατὴρ Σήμων ἐπέθηκεν [47]

adds nothing to the meaning of the earlier form Λυσέα Σήμωνος. In

Στάλα ΞενϜάρεος τοῦ Μ*h*είξιος εἴμ' ἐπὶ τύμβῳ [48]

only the words ἐπὶ τύμβῳ are added to the primitive formula.

The same is the case with dedicatory inscriptions, e. g.

'Α]λκίβιος ἀνέθηκεν κιθαρῳδὸς νησιώτης.[49]

Μνῆμ'] ἀνέθηκε Τύχανδρος ἀπαρχὴν τἀθηναίᾳ.[50]

IGA 410 (= K 1098) perhaps shows most clearly the metrical inscription in the making.

'Αλξήνω[ρ ἐ]ποίησεν ὁ Νάξιος· ἀλλ' ἐσίδεσ[θε.

It is the desire to conform to the fashion of the time that has led the artist to this naïve expression of pride in his work. Epithets of the gods, too, suggest a convenient method of filling out the verse, especially since the poet found them adapted to dactylic measure and ready to his hand. Examples are

Δ]εινανό[ρ]ης μ' ἀνέθηκεν ἑκηβόλῳ 'Απόλλωνι.[51]
'Αθηναίᾳ πολιούχω[ι [52]

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[46] Cf. 7 and 20.
[47] CIA I. 468 (= K 5).
[48] IGA 344 (= K 181).
[49] CIA I. 357 (= Roberts 48).
[50] CIA IV. 373[78] p. 86 (= H 240). Cf. IGA 466 (= H 286) and CIA I. 344 (= H 216).
[51] IGA 408 (= H 300).
[52] CIA IV. 373[95], p. 89.

ἐκηβόλῳ ἰσχεαίρῃ [53]
Ποτ]ειδάϝωνι ϝάνακτι [54]

From the very nature of the metre early elegiac inscriptions tend to be more diffuse than hexameters, but in them, too, the addition to the original bare formula may be only the name of the father or some word describing the gift (e. g. *ἄγαλμα*) or some phrase which had come to be a familiar part of the pentameter, e. g. *εὐξάμενος δεκάτην* (32, 36), *παιδὶ χαριζομένη* (15), *παιδὶ Διὸς μεγάλου* (31, 32, 36). Epp. 26 and 32 are two of the many concise distichs in which every word except the epithet gives the reader some definite information.

26. *Παλλ]άδι μ' ἐγρεμάχᾳ Διονύσιο[ς τό]δ' ἄγαλμα*
στῆσε Κολοίου *παῖς [εὐξά]μενος δεκάτην.* [55]

In epigrams such as this the art appears in the adoption of a metrical form not, barring the epithet, in the choice of words or the method of expressing the sentiment. Thus 8 is a poem rough and without charm, written with more effort than success,

8. *Σῆμ]α τόδ' (ἐ)γ(γ)ὺς ὁδοῦ Θεοσήμου [στῆσα- . . .*
ἀν]δρὸς ἐμοῦ (γ)ε φίλου κἀγαθοῦ ἀνφ[ότερον.

On the other hand 5, though scarcely more ornate, shows a style somewhat easier and freer.

5. *Τῇ]δε φί[λην ἄλοχον ὁ δεῖνα] κατέθηκε θανοῦσαν*
Λ[αμπι]τὼ αἰδοίην, γῆς ἀπὸ πατρωίης.

Often we find the prayer, which was indeed always in the mind of a Greek, that the divinity may be graciously pleased to return an equivalent for the gift offered.[56] Now it is good report that is desired —

42. *δὸς δε ϝ' ἰν ἀνθρώποις δόξαν ἔχειν ἀγαθ(ά)ν,* [57]

now gain —

38. *πότνια τῶν ἀγαθῶν τῷ σὺ δὸς ἀφθονίαν,*

now we find expressed that craving to be remembered among men,[58] to which, indeed, the very existence of the stones bears witness.

Of a different sort are epp. 1, 2, and 11.

2. *Ἄνθρωπε ὃς (σ)τείχε[ι]ς καθ' ὁδὸν φρασὶν ἄλ(λ)α μενοινῶν*
στῆθι καὶ οἴκτιρον σῆμα Θράσωνος ἰδών.

11. *Σῆμα Φρασικλείας. κούρη κεκλήσομαι αἰεί*
ἀντὶ γάμου παρὰ θεῶν τοῦτο λαχοῖσ' ὄνομα.

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[53] IGA 107 (= H 299). [54] IGA 20, 12 (= H 293).

[55] Cf. 10, 22, 10.

[56] In a late epigram (AP 6. 42) we read even *δὸς πλέον ὧν ἔλαβες.*

[57] Cf. 27, 13. [58] 21.

It is not only that these verses are far more charming than any yet quoted, but we can see in them the beginnings of that principle which characterizes the elegy in contrast to the epic. For the epic, with all its simplicity and directness of construction, depends for much of its effect on sonorous and splendid words, while the charm of the elegy is in familiar, even intimate sentiment never overshadowed by mere magnificence of vocabulary. Even in these poems, bare and brief as they are, it is the personal feeling of the writer that is expressed, and expressed with pathos all the more touching because of the simple means employed. In hexameters, even those which express grief, the writer is telling a story, he is objective; in the elegiacs he is subjective. Cf. the following poems.

Πραξιτέλει τόδε μνᾶμα Fίσων ποίFησε θανό[ντι·
τ]οῦτο δ' ἑταῖροι σᾶμα χέαν βα[ρ]έα στενάχοντες,
Fέργων ἀντ' ἀγ[α]θῶν, κήπάμερον ἐξετέλεσ(σ)αν.[59]

Παι[δὸς ἀπο]φθιμένοιο Κ[λεοί]του τοῦ Μενεσαίχμου
μνῆμ' ἐσορῶν οἴκτιρ' ὡς καλὸς ὢν ἔθανε.[60]

Passion as well as pathos is expressed in elegiacs and the author of 38 went so far as to threaten the enemies of the dedicator with human or divine anger, for the general sense is plain, though the last verse is mutilated.

οἵ τε λέγ[ου]σι λόγους ἀδίκ[ους] ψευδᾶς κα[τ'] ἐκ[είνου.

In 8 the writer even comes forward in the first person.

These examples are enough to show that as early as the sixth century men entrusted to the stones their thoughts and griefs and desires.[61] Compressed and restrained though most of the epigrams are, there is in them the personal element, the lyric quality, which comes out more freely in the work of the fourth century and later.

The first traces of poetic color come less, perhaps, from deliberate art than from almost unconscious imitation. In 6 — *ὃν θάνατο[ς δακρυ]όεις καθ[έ]χει* — and in IGA 15 (= K 463a. add.), *τὸν ὤλεσε πόντος ἀναι[δής*, the well-known epic vocabulary shows the absence of originality in the writer. The words or phrases were ready to his hand as familiar to his readers as to himself, and he is a poet because he chose to use them in his verses, not because he made them or used them in any

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59 IA 800.

60 7. Cf. 1 and 2.

61 Cf. also CIA IV. p. 118 (= K 19), the only hexameter inscription which belongs to this class.

'Ενθάδ' ἀνὴρ ὤμο(σ)εν τὰ hόρκια παιδὸς ἐρα[σ]θ[ε]ίς
νείκεα συγμε[ί]σ[γειν] πόλεμόν θ' ἅμα δακρυόεντα, κ. τ. λ.

way peculiarly his own. On the other hand, 11, 17, 23, 43, 46 differ from the verses just quoted in that the writers have somehow managed to make their own the familiar expressions. 43 will illustrate —

Δέξο Ϝάναξ Κρονίδα Ζεῦ 'Ολύνπιε καλὸν ἄγαλμα
ἱλήϜῳ θυμῷ τῷ Λακεδαιμονίῳ.

But in spite of the simplicity of these early epigrams, their variety is remarkable. The same ideas are expressed, the same words used, in a number of constructions, e. g. the name of the dead and of the divinity appear each in four cases,[62] that of the dedicator in three. Now it is the tomb or statue that speaks — now the buried man or the dedicator; now the god, now the passer-by is addressed.

We possess a few early epigrams which show greater poetic power, poems where art and elegance seemed to the authors as important as utility. An example is the well-known 25 —

Ἔθνεα Βοιωτῶν καὶ Χαλκιδέων δαμάσαντες
παῖδες 'Αθηναίων, ἔργμασιν ἐν πολέμου,
δεσμῷ ἐν ἀχνυόεντι σιδερέῳ ἔσβεσαν ὕβριν·
τῶν ἵππους δεκάτην Παλλάδι τάσδ' ἔθεσαν —

an epigram which approaches more nearly those of the next century, because phrases like ἔθνεα δαμάσαντες, ἔργμασιν ἐν πολέμου, ἔσβεσαν ὕβριν give to the whole poem a poetic coloring.

Appended to this paper are tables showing the elements which appear more or less constantly in the inscriptional epigrams. It is remarkable how definitely they speak, how consistently they keep the reason for their existence before our eyes. In the sepulchral inscriptions we find always the name of the dead (but it is in the verses themselves, never *extra metrum*);[63] always some word meaning "tomb," except in the

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[62] See Tables I, II.

[63] Koehler thinks that in 10 the name of the dead was omitted in the verses and inscribed above them. His restoration, however, is by no means certain and it seems to me more reasonable to suppose it faulty than to accept on conjecture a reading which would make the inscription an exception, not only in the sixth but in the fifth century, at least as far as we can tell from the evidence at our command. The only inscription that could possibly support Koehler's view is 12.

. . . . . ἦν γ]ὰρ ἀπάσης
νοῦν τε καὶ ἀνο[ρέ]αν ἔξοχος ἡλικίας
'Επ]ιστήμων τόδ' ἐπόει 'Ιπ(π)οσ[τράτ]ου σῆμα.

This is, however, not a parallel, for the name almost certainly appeared in the missing part of the hexameter and it appears below, not as part of the epitaph, repeating *extra metrum* information already given in verse, but as part of a second and separate inscription with quite a different function.

very early epigram 1, where, however, the language indicates plainly that the verses were inscribed on a tomb. Almost always we find some word meaning "dead" (e. g. *θανών*, *φθίμενος*). In dedicatory inscriptions we may expect to find the name of the dedicator,[64] the name of a divinity, a verb of dedication. In 24 the last element is lacking, but it must have been sufficiently evident from the place where the stone was set up that it was a dedicatory offering.

> Ἀστῶν θαλ(λ)όντων, πολιήοχε πότνι' Ἀθάνα,
> Σμίκρου καὶ παίδων μνῆμ' ἔχοι ἥδε πόλις.

For these reasons, then, we are justified in refusing to assign to this early period any epigrams preserved in MSS. only, which would require for the explanation or completion of their meaning any words on the stone *extra metrum*. In the case of dedicatory epigrams the information given in the verses may be supplemented by inferences drawn from the places where the stones were set up. So we sometimes miss the verb of dedicating, as in 24. This is especially likely to be the case when the dedicatory offering takes the form of an honorary statue.[65] The epigrams of the fifth century show that the verb of dedicating was regularly omitted in inscriptions for such statues.

Of the epigrams preserved only in MSS. the great majority were intended to be inscribed. We observe in them the same stages of development as in the inscriptions, although in neither case does fuller development necessarily indicate later date.[66] 53 is as severely simple as any verse carved on stone.

> Πραξαγόρας τάδε δῶρα θεοῖς ἀνέθηκε Λυκαίου
> υἱός· ἐποίησεν δ' ἔργον Ἀναξαγόρας.

19 expresses with greater elaboration, but with no greater charm, the same sentiment as 11. The very fact that of the epigrams attributed to illustrious poets some are as brief and severe as the inscriptions, while others are more elaborate, may serve as an indication at least that they are correctly attributed. 49, Archilochus's *unico fiore, nato di due petali soli*, could hardly be simpler.

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[64] In 42 the words ὁ κεραμεύς take the place of the name of the dedicator. The appearance of the name itself below is due to the fact that the same man was both dedicator and potter. In other words Νικόμαχος does not repeat the name of the dedicator, but adds, in a separate inscription, the name of the potter. Cf. also n. 63.

[65] 45. Cf. 61, 62. Cf. p. 22.

[66] For example 1, probably the oldest elegiac inscription, is one of the longest and most elaborate.

Ἀλκιβίη πλοκάμων ἱερὴν ἀνέθηκε καλύπτρην
Ἥρῃ, κουριδίων εὖτ' ἐκύρησε γάμων.[67]

In 66 we have a bare formula clothed in poetic language.

Ποικιλομήχαν' Ἔρως, σοὶ τόνδ' ἱδρύσατο πρῶτος
Χάρμος ἐπὶ σκιεροῖς τέρμασι γυμνασίου.[68]

In 50, as in 25, the poet has taken pleasure in merely exhibiting his skill in composing. 56, which describes a painting or a relief, is important as an early example of a style of epigram very common later.[69] 59 and 60 are by far the most ornate of the early epigrams, but even here the ornament is applied to quite common and familiar phrases.[70] In 63 and 64 — the well-known Hipparchus epigrams — we may fancy we see the influence of Solon or some other worthy, or we may agree with Professor Gildersleeve that the "moralizing is national. No Greek lets us off from that."[71]

Nearly all conform to the requirements laid down above.[72] In 49 and 55 we may perhaps miss τῇδε or some such word, but we miss it equally in 24. The dedicatory inscriptions, because usually set up in temples or on sacred ground, are often less definite in the information they give.[73] So in 65 — as in 24 — the word of dedicating is lacking; no one, however, could doubt that it was meant to be inscribed. 21 breaks entirely with the established form, but its contents are such that even if we admit that it was actually inscribed,[74] we cannot expect to judge it by the same tests as the other epitaphs.

Εἰμὶ νεκρός, νεκρὸς δὲ κόπρος, γῆ δ' ἡ κόπρος ἐστίν·
εἰ δ' ἡ γῆ θεός ἐστ', οὐ νεκρὸς ἀλλὰ θεός.

68–73 are manifestly neither sepulchral nor dedicatory. They are not inscriptional at all. But the fact that they are not inscriptions is no

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[67] Cf. 51, 52, 54, 57, 58, 61.

[68] Cf. 55, 62.

[69] We have no evidence as to whether this epigram was actually inscribed. The only inscription of this nature (47) is too mutilated to serve as a model and the omission of any indication in the verses themselves that they were inscribed is not more strange than the same omission in the verses that accompanied honorary statues. See p. 17. Cf. also p. 17, n. 169. For later epigrams of this kind cf. APl 200, 207.

[70] With 59 cf. 66; with 60 cf. 34, 42.

[71] Pindar, Olympian and Pythian Odes, p. 129.

[72] pp. 16 ff.

[73] Cf. p. 17.

[74] If we do not regard this as a possible sentiment for an actual epitaph, we must suppose the verses later than the 6th century. See pp. 19 ff.; cf., however, the Latin inscription cited by M. Haupt (Opusc. 2. 190).

Cinis sum, cinis terra est, terra dea est;
Ergo ego mortua non sum.

reason for rejecting them as epigrams in our sense of the word. They ought to be all the more carefully examined because they are few and treasured as the seeds from which the later epigram sprang. So in 68–70 we have early examples of satiric epigram.

E. g. 69.

> Καὶ τόδε Δημοδόκου· Χῖοι κακοί· οὐχ ὁ μέν, ὃς δ' οὔ,
> πάντες πλὴν Προκλέους· καὶ Προκλέης δὲ Χῖος.

Reitzenstein[75] claims that the later satiric epigram grew out of jests at banquets; it is at least equally probable that it merely continued such epigrams as these, which give no indication that they were convivial witticisms, though they may have been. 72 is one of the ancestors of the later narrative epigram.

There remains ep. 74, a poem which I cannot think was ever inscribed in the sixth century, because it contains no word for "tomb" or any other indication that the verses are an epitaph.

> Ἀβδήρων προθανόντα τὸν αἰνοβίην Ἀγάθωνα
> πᾶσ' ἐπὶ πυρκαΐης ἤδ' ἐβόησε πόλις·
> οὔτινα γὰρ τοιόνδε νέων ὁ φιλαίματος Ἄρης
> ἠνάρισεν στυγερῆς ἐν στροφάλιγγι μάχης.

For the same reason I cannot regard it as an early epideictic epitaph. Such poems, i. e. epigrams not meant to be inscribed themselves, but imitating inscriptions, are, I take it, of two sorts. They may be accurate imitations of real inscriptions — exercises, as it were, in writing epitaphs or dedications. In this case they are composed merely to display the author's skill, which would be hardly worth displaying if he tripped in a matter so simple as an essential word or formula. Or (the second possibility) the aim of such a poem may be, not the accurate imitation of an inscription, but the use in a general way of the inscriptional form as a vehicle for jest or satire — a parody rather than an imitation of an inscription. In this case it is not the difference in form but the difference in content that marks the verses as epideictic. Moreover, so long as men considered primarily the utility of the epigram they were not likely to compose epideictic epigrams. For these reasons, if 74 was intended as an epitaph, real or imitative, it cannot belong to the sixth century, since it omits an element found in all actual epitaphs of that century and yet gives no further evidence that it is of an epideictic character. It is, however, quite possible that the poem has no reference to a tomb at all. If this is so, there is no reason

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[75] Epigramm u. Skolion, p. 92.

why it cannot belong to an early date, since it would be a short elegy. That these existed in early times we cannot reasonably deny, especially when we reflect that many fragments of Theognis may be separate poems.[76]

The vocabulary of the sixth century epigram is drawn largely from words of every day, but many words are borrowed from the epic, e. g. δακρυόεις (6), ἔξοχος (12), κάρτερος (20), πολιήοχε (24), ἐγρεμάχα (26), κρατερόφρονι (39), κουριδίων (49), εὐρυχόροιο (51). Cf. also 1 — εἴτε ξένος ἄλλοθεν ἐλθών with Od. η. 33 — ὅς κ' ἄλλοθεν ἔλθῃ; Od. η. 52 — εἰ καί ποθεν ἄλλοθεν ἔλθοι; Od. ρ. 382 — ἄλλοθεν αὐτὸς ἐπελθών and cf. 8 — ἀνδρὸς ἐμοῦ γε φίλου κἀγαθοῦ ἀμφότερον — with Il. Γ. 179. ἀμφότερον βασιλεύς τ' ἀγαθὸς κρατερός τ' αἰχμητής.[77]

There are also some words which occur not in epic but in lyric poetry. These are ποντομέδων (23), χρυσοτρίαιν' (23), ἀμενφές (46), κλεινός (15), εὐκλέισον (50).

Sometimes, as it is quite natural, the words or sentiments of the elegy are echoed by the epigram. Cf.

42	δόξαν ἔχειν ἀγαθάν.	Solon 13. 4	δόξαν ἔχειν ἀγαθήν.
15	παιδὶ χαριζομένη.	Theog. 774	παιδὶ χαριζόμενος.
K. 742	ἔχοι κλέος ἄπθιτον αἰϜεί.	" 867	μέγα κλέος οὔποτ' ὀλεῖται.
50	εὐκλέισον γενεάν	Tyrt. 12. 24	πατέρ' εὐκλείσας.

Words which are ἅπαξ λελεγμένα are enumerated below.[78]

Even from the small number of seventh and sixth century epigrams which we possess we see that certain combinations of words had already crystallized into recognized formulas[79] — an indication of the great popularity of the inscribed poem.

The epigrams of the fifth century still contain nearly all the information that the reader needs. In the epitaphs we have always the name of the dead, always some indication that the verses are inscribed, but actual synonyms for "dead" and for "tomb" are often missing, though the idea may be expressed in different form. E. g. οἶδε (81), κεῖται (78), ἐνθάδ' (79), and less clearly 75, 83, 86. In some cases it would be difficult to tell from the contents alone whether the verses were intended for a tomb or for an honorary statue,[80] but the phraseology never leaves any doubt that they were inscribed somewhere. These considerations make it, in my judgment, impossible to accept as genuine Sim

76 Cf. pp. 7 ff.

77 Cf. also Pindar, Olymp. VI. 17, ἀμφότερον μάντιν τ' ἀγαθὸν καὶ δουρὶ μάρνασθαι. For epic *forms* see pp. 40 ff.

78 p. 36.

79 See Table III.

80 E. g. 75, 81, 86 and among the epigrams in MSS. 96, 102, 104, 109, 113–115.

101 and 114. In 101 there is not the slightest indication that the verses are an inscription.

Παῖδες Ἀθηναίων Περσῶν στρατὸν ἐξελάσαντες
ἤρκεσαν ἀργαλέην πατρίδι δουλοσύνην.

Cf. with this ep. 96 (= Sim. 91) where τῇδε supplies just what is lacking in Sim. 101.[81]

Μυριάσιν ποτὲ τῇδε τριακοσίαις ἐμάχοντο
ἐκ Πελοποννάσου χιλιάδες τέτορες.

In Sim. 114 the name of the dead man does not appear at all and I cannot feel with Mackail[82] that its place is adequately supplied by ὁ μέν.

Ἠερίη Γεράνεια, κακὸν λέπας, ὤφελεν Ἴστρον
τῆλε καὶ ἐς Σκυθέων μακρὸν ὁρᾶν Τάναϊν,
μηδὲ πέλας ναίειν Σκειρωνικὸν οἶδμα θαλάσσης
ἀγέα μαινομένης ἀμφὶ Μολουριάδα.
νῦν δ' ὁ μὲν ἐν πόντῳ κρυερὸς νέκυς, οἱ δὲ βαρεῖαν
ναυτιλίην κενεοὶ τῇδε βοῶσι τάφοι.[83]

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[81] Not every epigram which contains τῇδε, οἵδε, or similar words is necessarily an epitaph, real or epideictic, since these words might occur in a merely reflective poem and indicate that the author composed it with the tomb before his eyes; but every epitaph must contain some such indication of place.

[82] p. 364.

[83] Mackail (l. c.) quotes K 89 in support of his theory and thinks that "τόνδε in that epigram is like the ὁ μέν of Simonides here":

Σῆμα τόδ' ἐν κενεῇ κεῖται χθονὶ [σῶμα δ' ἐπ' ἀγροῦ
Ὠρείου κρύπτει πυρκαϊὴ φθιμένου.
Τόνδ' ἔτι παπταίνοντ' ἐπὶ γούνασι πατρὸς μάρψας
Ἀίδης οἱ σκοτίας ἀμφέβαλεν πτέρυγας, κτλ.

A comparison of the two poems makes it evident at once that they are not parallel and, even if they were, K 89 is later than Simonides. In inscriptions the name is never omitted till the fourth century and very rarely then (five times. See Table I.) For arguments against Bergk's theory that the opening of the epigram, which contained the name, is missing, see Mackail (l. c.).

Wilamowitz (Goett. Nachr. 1897, pp. 306 ff.) thinks that this is proved genuine by an imitation by Callimachus (Call. 17 = AP 7. 271).

Ὤφελε μήδ' ἐγένοντο θοαὶ νέες· οὐ γὰρ ἂν ἡμεῖς
παῖδα Διοκλείδου Σώπολιν ἐστένομεν.
νῦν δ' ὁ μὲν εἰν ἁλί που φέρεται νέκυς, ἀντὶ δ' ἐκείνου
οὔνομα καὶ κενεὸν σῆμα παρερχόμεθα.

But the case is more likely to be reversed, and we have in Callimachus the model which the nameless poet of Simonides 114 copied. See also Hauvette, pp. 102 ff.

In the dedicatory inscriptions of the fifth century we find always the name of the dedicator, always some verb of dedicating. The name of the divinity is, however, frequently omitted, since the site of the stone made it sufficiently clear to whom the offering was made. In the epigrams preserved in MSS. we sometimes miss the verb of dedicating also. In 158 *δώροισι* may be said to take its place, but in 144, 167, 169, 170, 181 there is no such equivalent. These epigrams are all, however, quite as much honorary as dedicatory and conform to the type of such verses in the sixth century.[84]

Thus far the epigrams of the fifth century do not differ much from the earlier ones in the elements which they contain. There is, however, one important point of difference, — the name of the dead or of the dedicator is sometimes repeated *extra metrum*.[85] This means that the epigram is no longer primarily a means of giving necessary information, but an ornament. Reitzenstein indeed says,[86] "Dennoch ist noch bis über die Mitte des vierten Jahrhunderts hinaus das Epigramm keine anerkannte Form der Kunstdichtung," but he gives no proof of his statement. On the contrary, the moment the name was added *extra metrum* the epigram must have been regarded as a poem rather than as a poetic label. 81 shows this clearly. The names of the fallen warriors and the place where they fell were inscribed first. There was no need of anything further. A poem, however, was added giving the same information in verse, because thus the monument was made more splendid. The names of the individual dead are necessarily omitted in these lines, but in private monuments, as stated above,[87] the name is never omitted in the epigram. To be sure, occasionally information not contained in the epigram is added *extra metrum*, but it is never information essential to the interpretation of the epigram.[88] An interesting example is 138, where the name of the author was inscribed on the stone, — a peculiarity which does not occur again till the second century B. C.

But in spite of this difference between the epigrams of the fifth and those of the sixth century, we must not suppose that the poems of the two periods are sharply divided by an impassable barrier. The simple epigram continues to exist, e. g.

78. Ἐνθάδ' ἀνὴρ ἀγαθὸς κεῖται Καλλιστράτου ὑός,
Καλλίμαχος φυλ(ῆ)ς Κεκροπί(δ)ος Μελιτεύς.

131. Ε]ὐχὰν ἐκτελέσαντι Διονύσῳ Νεομήδης,
ἔργων ἀντ' ἀγαθῶν μνᾶμ' ἀνέθηκε τόδε.[89]

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[84] See p. 17. Cf. ep. 24.
[85] 78, 79, 84, 135.
[86] Ep. u. Sk., p. 121.
[87] p. 20.
[83] Cf. 84, 135.
[89] Cf. 80, 82, 122, 123.

Such epigrams are common to all periods. Cf. K 139, which dates from the Roman period:

Στήλλην Παρθ[ε]νόπης ἴδιος γαμέτης ἐπόησεν
Δαΐνης ἀλόχ[ω] τοῦτο χαριζόμενος.[90]

Thus 89 is hardly more elaborate than 17, and 126, 127, 133, 135 are exceedingly simple. A new feature, however, is the tendency of poets to make the epigram longer by simply spreading out their material over a larger space.

In the earlier epigrams something, no matter how brief, was *added* to fill out the distich; now the amplifying process is often adopted. See, for example, 128, of which the pentameter, πατρὶ δὲ τῷ τήνου Δαμοφόων ὄνομα, might be expressed in one word, Δαμοφόοντος. In the pentameter τόνδε θε(ᾶ)ι τῆδε, ἣ τόδ' (ἔ)χει τέμενος (127), ἣ τόδ' ἔχει τέμενος merely repeats τῆδε.

Epigrams of four verses begin to be very frequent, and the four verses are filled in various ways. Sometimes the material here is diluted, as in the examples cited above. Cf. 93, where the author has spread over four verses a sentiment which is expressed with perfect ease in two verses by the author of 5. Sometimes the material is not diluted but elaborated, e. g. 75, 90, 138. Sometimes new material is added, as in 77 b, c, 81, 86, 132.

But it is not only the vocabulary and the sentiments that have come down from the earlier time. The poetic color which distinguishes the work of this century appeared earlier in poems like 25, and the development of the epigram is thus unbroken, though without doubt the achievements and the glory of the Persian wars were an inspiration to the poets of that age. For with the beginning of the fifth century a new spirit was breathed into all Greece. To say that men suddenly woke to the realization that the individual was but a part of one great nation, and recognized that the liberty bequeathed to them by their fathers was a national possession, a κτῆμα ἐς ἀεί, in defence of which every man must cast aside personal considerations — this is to repeat what has been said again and again. But nowhere, except in the "Persians" of the warrior poet, can we trace more clearly the fierce valor, the burning patriotism, the indomitable pride than in the epigrams of the time. They form a little group set apart from other epigrams, for in them we miss the individualistic tone which otherwise characterizes the epigram from the beginning to the end of its history. Before this men involuntarily and almost unconsciously had laid stress on the individual and his thoughts and feelings; later they were to do

[90] Cf. also K 134, 791.

the same thing with more self-conscious art and set purpose; but the men of this age carried their disregard of the individual as compared with the state even into that form of poetry which had been most individualistic. Even the sepulchral epigrams are no exception. When grief is expressed it is the grief of the state, rarely of individuals.[91] It is sometimes said that Simonides brought in a new kind of epigram.[92] Whether Simonides wrote all the epigrams attributed to him is a matter for dispute, but it cannot be disputed that not Simonides but the spirit and purpose of the age furnished the material for those epigrams. Eloquence and grandeur of expression he or some other poet may have contributed, but the spirit was the spirit of all Greece.[93]

The epigram at this period reached the height of its splendor. 83, one of the noblest that have come down to us, is indeed strikingly simple:

Ὦ ξένε, εὔυδρόν ποκ' ἐναίομες ἄστυ Ϙορίνθου
νῦν δ' ἁμὲ Αἴαντος νᾶσος ἔχει Σαλαμίς,

but this simplicity is of a different kind from that of 17 and 20. With set purpose the pride of the dead is, as it were, imitated in the verses, and thus the very simplicity becomes the most perfect art. That this is characteristic of the fifth century is shown by the fact that two epigrams attributed to Simonides, originally consisting each of one simple distich, were lengthened by later writers, and thus lost much of their magnificence.[94]

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91 Cf. 77 c, 79, 86, 87, 88 with 1, 2, 6, 7.

92 On the epigrams of Simonides, see Kaibel, Junghahn, Hiller, Hauvette, Bergk, Boas, Wilamowitz, Wilhelm (see n. 1).

93 Cf. Reitzenstein, p. 106: Der Versuch aus der Geschichte des Epigramms die entscheidende Persönlichkeit des Simonides zu streichen, indem man ihm nur lässt, was der dürren und dürftigen Form aus dem sechsten Jahrhundert von namenlosen Privatleuten gesetzten Inschriften entspricht, weist die überraschende Fortbildung des Epigramms und die Bildung der neuen auf Jahrhunderte hinaus wirksamen Formen nur nicht dem grossen Dichter, welchen hierfür das Altertum kennt, sondern namenlosen, wenig jüngeren Zeitgenossen desselben zu.

Cf. Wilamowitz (Goett. Nachr.), 1897, p. 320: Der damals ziemlich allerorten in Hellas für die metrischen Aufschriften geltende Stil verdient das hohe Lob, das bisher der Person des Simonides gezollt ward.

94 83 and 125. To these Wilhelm (JOAI 2, pp. 221 ff.) would add (with great probability) 99 and 193, and (less likely) 101, 105, 109. See also Wilamowitz (Goett. Nachr., 1897, pp. 306 ff.). Boas (p. 109) has shown that the later addition to 83 was imitated from 99, and the spurious part of 99 from the genuine lines of 83. The motive which caused the lengthening of 83 and 125 was evidently the desire to give in the epigram certain definite and important facts which were often found in inscriptions. The additions (?) in 99 and 193 could be accounted for in the same way, but such is not the case with 101, 105, 109, where the last verses do not add facts at all.

More ornate are 77 b, c, 81, 86, 132, 138.

In this century tragedy and rhetoric were coming more and more to the front, and we can begin to trace their influence in the epigram. This appears at first more in general sentiment and style of composition than in particular words or phrases. The tragic poets have had their effect on such epitaphs as 77 c, 79–81, 86. Cf. any one of these with 90, — an epigram quite untouched by this influence and carrying on the somewhat rude simplicity together with the phraseology of earlier inscriptions. Tragedy and rhetoric alike have helped to give us the antitheses and the metaphors in 77 b, *Αἰθὴρ μὲμ ψυχάς . . . σώματα δὲ χθών*;[95] 77 c, *Ἄνδρας μὲμ . . . ψυχὰς δέ, ψυχὰς δ' ἀντίρροπα θέντες ἠ[λλ]άξαντ' ἀρετήν.*[96] That this influence should appear first in epitaphs is natural enough, since the emotions of pity, grief, and affection are capable of more poetic treatment than the somewhat cool gratitude of the dedicatory inscriptions. However, in a few of the dedications preserved in MSS. we find poetic touches.[97]

In the inscriptions of this century the same formulas appear repeatedly, and not only those that are simple and almost essential, but often those that are more elaborate and original.[98]

The epigrams which are preserved in MSS. show the same tendencies as the inscriptions. A few are very simple. So —

110. *Σῆμα Θεόγνιδός εἰμι Σινωπέος, ᾧ μ' ἐπέθηκεν*
Γλαῦκος ἑταιρείης ἀντὶ πολυχρονίου.

153. *Ταῦτ' ἀπὸ δυσμενέων Μήδων ναῦται Διοδώρου*
ὅπλ' ἀνέθεν Λατοῖ μνάματα ναυμαχίας.[99]

Some are diluted, e. g. 175.[100] Some are made more ornate, e. g. 145.[101] In some, as in 148, new material is added.[102] Elaborate and

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but merely pad what has gone before. It is, moreover, impossible to base upon the evidence of two poems (which alone are *proved* to have been lengthened) any conclusions that will enable us to detect with certainty similar pieces of patchwork among the epigrams preserved only in MSS., especially as we possess inscriptions of the fifth century which contain genuine lines of much the same character as the spurious lines of 83 and 125 (e. g. 75 and 79). Without the proof, which discovery of the actual stones alone can give, we have only the merest conjecture to go upon.

95 Cf. Eur. Suppl. 543, *πνεῦμα μὲν πρὸς αἰθέρα, τὸ σῶμα δ' ἐς γῆν.* The Suppliants was produced twelve years after the battle commemorated in ep. 77, but the sentiment of the inscription is tragic. See G. H. Macurdy, Classical Weekly, March 6, 1909, p. 139.

96 Cf. Aesch. Ag. 438: *ὁ χρυσαμοιβὸς δ' Ἄρης σωμάτων*
καὶ ταλαντοῦχος ἐν μάχῃ δορός.

97 E. g. 145, 148, 164.

98 See Table III.

99 Cf. 111, 149, 150.

100 Cf. 109, 157, 168.

101 Cf. 95, 102, 103, 105, 108, 112, 152.

102 Cf. 102, 164, 167, 169.

sometimes artificial diction, which becomes more and more common, may add beauty to the verses, but at times the imitator falls far short of the splendor of his master, and produces a poem mediocre in comparison with his noble model. Cf. for example 103 and 104:

103. Ἄσβεστον κλέος οἵδε φίλῃ περὶ πατρίδι θέντες
κυάνεον θανάτου ἀμφεβάλοντο νέφος·
οὐδὲ τεθνᾶσι θανόντες, ἐπεί σφ' ἀρετὴ καθύπερθεν
κυδαίνουσ' ἀνάγει δώματος ἐξ Ἀίδεω.

104. Κυανέη καὶ τούςδε μενέγχεας ὤλεσαν ἄνδρας
μοῖρα, πολύρρηνον πατρίδα ῥυομένους,
ζωὸν δὲ φθιμένων πέλεται κλέος, οἵ ποτε γυίοις
τλήμονες Ὀσσαίαν ἀμφιέσαντο κόνιν.

The two epigrams employ the same figures and almost the same phraseology, both show the strong influence of rhetoric; and yet the superiority of the former is as remarkable as it is apparent.[103] Cf. also 81 and 109, which show that it is one thing to ornament and elaborate a given theme and quite another to dilute it.

So in 102 and 103 rhetoric has lent a certain grandeur to the verses; in 106 the fine lines are marred by the almost frigid beginning. In 102, 103, 148 the elaboration is vivid and virile, in 105 it is labored if not actually inept.

In 158, as in 125, by the device of directly addressing the dedicator a certain life and eloquence is given to the epigram, although the words themselves are plain enough.

Εὔχεό τοι δώροισι, Κύτων, θεὸν ὧδε χαρῆναι
Λητοΐην ἀγορῆς καλλιχόρου πρύτανιν,
ὥσπερ ὑπὲρ ξείνων τε καὶ οἳ ναίουσι Κόρινθον
αἶνον ἔχεις χαρίτων, δέσποτα, τοῖς στεφανοῖς.

The noble simplicity of 83 is rivalled by the proud humility of 94:

Ὦ ξεῖν' ἀγγέλλειν Λακεδαιμονίοις ὅτι τῇδε
κείμεθα, τοῖς κείνων ῥήμασι πειθόμενοι.

There are also epigrams which approach more nearly the style of many verses of the fourth century, where the uninspired author has tried to make up for his limitations by filling with added details the space he could not fill with poetic charm and color, e. g.

166. Ἀρτέμιδος τόδ' ἄγαλμα· διηκόσιαι γὰρ ὁ μισθὸς
δραχμαὶ ταὶ Πάριαι, τῶν ἐπίσημα τράγος —
ἀσκητῶς ἐποίησεν Ἀθηναίης παλάμῃσιν
Ἄξιος Ἀρκεσίλα υἱὸς Ἀριστοδίκου.

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[103] See p. 49, n. 176.

So 171 and 174, in spite of a few happy touches, for the most part drag hopelessly.

In epigrams earlier than 300 we have almost nothing of that half-reverential, half-intimate affection for nature which is part of the charm of so much of the later work. We find in the fifth century but one metaphor drawn from nature, — that of the harvest of war (81). When special localities are mentioned, now and then a picturesque word or phrase accompanies them; e. g. ὑπὸ κροτάφοις Ἑλικῶνος (98), Δίρφυος ὑπὸ πτυχί (101), σκοπέλοισιν Ἄθω (163). 191 is the first epigram which frankly enlarges on the beauty of nature, and 164 first expresses the simple confidence of man in natural forces:

164. Εὔδημος τὸν νηὸν ἐπ' ἄγρου τόνδ' ἀνέθηκεν
τῷ πάντων ἀνέμων πιοτάτῳ Ζεφύρῳ,
εὐξαμένῳ γάρ οἱ ἦλθε βοαθόος, ὄφρα τάχιστα
λικμήσῃ πεπόνων καρπὸν ἀπ' ἀσταχύων.

The general tone of this epigram is so strikingly like that of many later verses [104] that it is tempting to assign it to the third century, especially as even the fourth century offers no parallels to it. There is in it, however, nothing which we can fairly say *could* not have been written in the fifth century, and Jebb [105] decides that there is nothing to prevent its having been written by Bacchylides himself.

We possess, too, many epigrams attributed to the fifth century which were never meant to be inscribed. These are of various sorts. The well-known 188 is a true παίγνιον:

Ἄκρον ἰητρὸν Ἄκρων, Ἀκραγαντῖνον πατρὸς ἄκρου,
κρύπτει κρημνὸς ἄκρος πατρίδος ἀκροτάτης —

but there are no good grounds for rejecting it, especially when we remember that Lasus is said to have indulged in a similar *tour de force* in the shape of a poem which did not contain the letter σ.[106]

189 is the first epitaph that we have a right to call epideictic:

πολλὰ φαγὼν καὶ πολλὰ πιὼν καὶ πολλὰ κάκ' εἰπὼν
ἀνθρώπους, κεῖμαι Τιμοκρέων Ῥόδιος.

Anyone can see that this was not meant to be inscribed, but it imitates an inscription so far as the form is concerned. Of the same character is 190, which obviously parodies 189. Such poems could be composed when the sepulchral epigram was considered an ornament

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104 Cf. Mackail's section II.

105 In his edition of Bacchylides.

106 See Christ, Gr. Lit.-Ges.[4] p. 193.

rather than a necessity, and not before. Then poets began to cultivate this branch of literature as they would any other. We are not forced to the conclusion that such poems were always "παίγνια beim Gelage."[107] The παίγνιον of course sometimes imitated the form of an inscription; the epigram quoted by Reitzenstein (Ep. u. Sk., p. 99) is obviously a παίγνιον which has usurped the form of an inscription, not an inscription perverted into a παίγνιον. Short elegies came finally to be called epigrams, not because παίγνια usually imitated inscriptions, nor because most inscribed verses were elegiac (for in the fourth century other metres again came to the front), but because by far the greater number of short elegies were actually inscribed — which is a very different matter. I am unable to see why Reitzenstein holds[108] that epideictic epigrams could not be composed till inscriptions were collected in book form. Surely it is conceivable that verses on stone pleased men and suggested imitation just as readily as did verses written on parchment; nor did the love of parody make its first appearance in the fourth century.

The poems just discussed (189, 190) show by their content, not by their form, that they are epideictic. They conform, therefore, to the principle laid down above.[109] On the other hand, 191 and 192, which differ from actual epitaphs only in form, i. e. by the omission of any indication that the verses were inscribed, cannot have been intended by the authors even as imitative inscriptions. For surely if they had wished to imitate the established form of an epitaph they could have done it more cleverly than this! The influence of inscriptions is without doubt to be seen in these verses, but epitaphs, whether real or epideictic, they are not.

Among these epigrams I have ventured to insert some fragments (?) of Theognis.[110] Together with 196–205, they include almost every kind of epigram, — gnomic, satiric, epideictic, erotic, convivial, narrative. To exclude such poems from the list of early epigrams in order to make the history of Greek literature conform to a scheme which we have arbitrarily and, it may very well be, falsely mapped out, appears to me to be arguing in a circle.

The diction of fifth-century epigrams is largely epic. The following epic words are but a small part of those which occur:

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107 Reitzenstein (p. 96): "Sie (i. e. the epigrams of Asclepiades and his contemporaries) ergeben ein einheitliches Bild sobald wir sie als Lieder beim Gelage auffassen." He has, however, failed to show the necessity for arriving at "ein einheitliches Bild."

108 p. 104 (l. c.).

109 p. 19.

110 See p. 7.

κλινάμενοι (76)	πολύρρηνον (104)	πολυμήλῳ (136)
ἱππόβοτον (79)	παράκοιτις (93)	πυροφόροιο (108)
φθιμένου (79, 97, etc.)	εὐρυχόρου (102)	ἀγέρωχος (120)
μοῖρα κίχη θανάτου (92)	ἀταλασθίην (107)	ἀμφιρύτου (138)

Indeed, many of the poets might take as their motto the words of a late epigrammatist: ἵν' εἴπωμεν καθ' Ὅμηρον.[111] Cf. especially 145, where, within four verses, we read, πτολέμοιο δακρυόεντος, ὑπωρόφια, στονόεντα κατὰ κλόνον, ἐν δαΐ φωτῶν.

A curious circumstance at this time is the dearth of epithets of the gods. Against the earlier

χρυσοτρίαινος (23)	κρατερόφρων (39)	αργυρότοξος (58)
ποντομέδων (23)	χρύσαιγις (47)	ποικιλομήχανος (66)
πολιήοχος (24)	ὀβριμοπάτρη (47)	φιλαίματος (74)
ἐγρεμάχα (26)	φιλοστέφανος (54)	

we can set only ἀγέρωχος (120), Τριτογενής (123), καλλίκομος (125), αἰγίοχος (142), πολυώνυμος (167). The deficiency is, however, more than balanced by the frequency with which we find epithets applied to πατρίς, γαῖα, and similar words. In the sixth century we find only εὐρύχορος (51). In the fifth century the following occur:

ἀμφίρυτος (138)	εὐρύχορος (102, 139, 143, 150, 157, 193)	
ἀπόρθητος (138)	ἱππόβοτος (79)	πολύμηλος (136)
εὔδοξος (134)	καλλίχορος (75, 138)	πολύρρηνος (104)
εὔυδρος (83)	πολύανδρος (139)	πυροφόρος (108)

Some phrases recall the elegy. Cf.

81. εὐκλέισαμ πατρίδα[112]	Tyrt. 12. 24, ἄστυ τε καὶ λαοὺς καὶ πατέρ' εὐκλείσας
94. ῥήμασι πειθόμενοι =	Theog. 1262 and 1239 b (cf. 194 and 380; Solon 4. 6 and 13. 12)
103. οὐδὲ τεθνᾶσι θανόντες	Theog. 245, οὐδὲ τότ' οὐδὲ θανὼν ἀπολεῖς κλέος, ἀλλὰ μελήσεις | ἄφθιτον ἀνθρώποις αἰὲν ἔχων ὄνομα

A few passages directly recall the tragic poets. With the figure in 81, ὥστ' ἐχθροὺς στενάχειμ πολέμου θέρος ἐκκομίσαντας, cf. Aesch. Suppl. 636, Ἄρη τὸν ἀρότοις θερίζοντα βροτοὺς ἐν ἄλλοις; Pers. 822, ἄτης, ὅθεν πάγκλαυτον ἐξαμᾷ (ὕβρις) θέρος. With 174, μέγα δ' ἔστενε Ἀσὶς ὑπ' αὐτῶν πληγεῖσ', cf. Pers. 548, στένει γαῖ' Ἀσίς; 61, οὓς πέρι πᾶσα χθὼν Ἀσιᾶτις στένεται.[113] These few passages are the forerunners of many in the fourth century which show the profound influence of tragedy on the epigram.

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[111] AP 6. 61. For Homeric *forms* see pp. 41 ff.

[112] Cf. 77 c, 175.

[113] Cf. also p. 25 and notes.

Although the tone of the epigrams of the fifth century often differs from that of the earlier work, the difference between the poems of the fifth and of the fourth centuries is even more marked. In the fifth century, not only during the Persian wars, but even at the end of the century,[114] patriotism and the good of the state were foremost in men's minds. But with the fourth century the individualistic tendency returned, and it appears in other fields besides the epigram. A striking testimony to the passing of the great period in Greek national life is the sudden drop in tone of the epigrams of the fourth century. The tendency to individualism was fostered by philosophy, particularly by the Peripatetic school, and flattered by rhetoric, but the individual was as yet a somewhat unsatisfactory and uninspiring subject, and the details with which the verses now begin to be loaded are for the most part dry facts expressed in curt or rambling style.[115] A man's country, his age, his trade or profession,[116] his whole genealogical tree,[117] — these are not the touching and intimate details which give to the later epigram its charm, — details that move our sympathy for the dead as for a friend,[118] and make us feel the gracious, kindly presence of the gods.[119]

While the earlier epigrams select for glorification valor in arms or success in the great games, now we find men praised also for learning and for excellence in the fine arts, — qualities which belong to men as individuals, not as members of a state. In the fifth century it was the glory of a noble death that appealed to the poet; in the fourth, as in the sixth, it was the sadness that affected him — again a change that corresponds with the shifting of interest from the state to the individual. But though this feeling is expressed in the sixth century, it is expressed with restraint, sometimes hardly more than hinted; in the fourth century it is revealed more freely. More and more men brooded over the idea of death and deplored the power of envious fate which could snatch men from the pleasures and opportunities of life. In almost

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114 Cf. 81.

115 Cf. 265 and 247.

116 See Tables I, II.

117 E. g. 265, 274, 275, 286.

118 Cf. 245 with AP 7. 502:

Ἠρίον εἰμὶ Βίτωνος, ὁδοῖπορε· εἰ δὲ Τορώνην
λείπων εἰς αὐτὴν ἔρχεαι Ἀμφίπολιν,
εἰπεῖν Νικαγόρᾳ παίδων ὅτι τὸν μόνον αὐτῷ
Στρυμονίης Ἐρίφων ὤλεσε πανδυσίη.

Nowhere do we find the touch, so light and yet so sure, of AP 7. 453

Δωδεκετῆ τὸν παῖδα πατὴρ ἀπέθηκε Φίλιππος
ἐνθάδε, τὴν πολλὴν ἐλπίδα, Νικοτέλην.

119 Cf. any of the dedicatory epigrams of this century with Mackail, § 2. 9–12.

every sepulchral inscription we find some such word as *ποθεινός*, *πένθος*, *πῆμα*, and such expressions as *κοινοταφὴς θάλαμος* (230), *τὸν ἀνάνκης κοινὸν Φερσεφόνης πᾶσιν θάλαμον* (220 a), *ὤλεσεν ἡ φθονερὰ τοῖς ἀγαθοῖσι τύχη* (249), *μοῖραν, τῆς μέτα πᾶσι βροτοῖς* (253) show the spirit of an age for which *Τύχη* was a dread and too powerful goddess.[120] To these detached phrases it may be worth while to add one complete poem :

228. Οὐ γάμος οὐδ' ὑμέναιος ἔμειν[ε] μ[ε] τλήμονι μοίρᾳ
ἀ]λλά με ἔλεν [κ]άμ[α]τος λυγρὸς ἀεικέλιον
πρὶν πολιο[κ]ρ[οτ]άφοιο χρόνου γῆρας προσικέσθαι
κ[α]ὶ λ[εί]πω πένθος πᾶσιν ἐμοῖσι φίλοις·
Ὅσ[σ]α δ' [ἐ]χρῆν φύσεως θνητῆς παιδεῖα ἐφικέσθαι
π[ά]ντα μ' ἔχοντα εἷλεν μοῖρα Τύχης δυνάμει.

In only one epitaph of this century (245) is death a rest from toil — *Ἀτώτας | ἧς γαίας τηλοῦ σῶμ' ἀνέπαυσε πόνων.* Elsewhere it is the horror of dying that is uppermost — *τὸν Ἀίδαν γὰρ οὐδὲ γῆρας οἶδε φιλεῖν.*[121]

Yet as it was with the sixth and fifth centuries, so we find it with the fourth. The general sentiment and point of view may change, but the same varieties of epigram are handed on from age to age. Now, as always, the short and simple epigram holds its own :

226. Μνῆμα δικαιοσύνης καὶ σωφροσύνης ἀρετῆς τε
Σωσίνου ἔστησαν παῖδες ἀποφθιμένου.

266. Ἀντ' ἀγαθῶν ἔργω[ν] Ἀ[σκ]λαπιέ, τούσδ' ἀνέθηκ[ε]
αὐτοῦ καὶ παίδων δῶρα τά[δ'] Ἀντίφιλος.[122]

Others just as brief show greater elaboration :

238. Ἐνθάδε τὴν πάσης ἀρετ(ῆ)ς ἐπὶ τέρμα μολοῦσαν
Φαναγόραν κατέχει Φερσεφόνης θάλαμος.[123]

The subject-matter is diluted far more than in the fifth century. Cf. 221, 235, 265, 273.

The influence of rhetoric is very apparent. It appears most often, as in the fifth century, in the use of antithesis ; but this figure, which lends the earlier poems grandeur and loftiness, now becomes too often frigid and lifeless. The phrases *σῶμα μὲν . . . ψυχὴ δέ*, *θνητὸς ἀθάνατον* have become catch phrases, appearing in even the shortest epigrams :

231. Ἥ]δ' ἔθανεν προλιποῦσα πόσιν καὶ μητ[έρα σεμνὴν
κ]αὶ κλέος ἀθάνατον σωφροσύνης [ἔλαβεν.

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[120] Cf. 281 : ἀλλὰ τύχη κρείσσων ἐλπίδος [ἐξ]εφάνη.

[121] Soph. fr. 275.

[122] Cf. 225, 227, 234, 251, 252, 268, 270, 271, 284.

[123] Cf. 230, 231, 239, 243.

240. Εὐρυμάχου ψυχὴν καὶ ὑπερφιάλους διανοίας
αἰθὴρ ὑγρὸς ἔχει. σῶμα δὲ τύμβος ὅδε.

227 is an excellent example to show the change in taste since the fifth century; the verses are half rhetoric, half jest, wholly frigid:

Οὗτος ὃς ἐνθάδε κεῖται ἔχει μὲν τοὔνομα κριοῦ
φωτὸς [δὲ] ψυχὴν ἔσχε δικαιοτάτου.

The fact that the name is sometimes omitted altogether[124] from the epigram shows that the epigram has come by this time to be regarded as pure ornament. Unfortunately it is an ornament which does not always adorn, and at times differs from the most prosaic prose only by being metrical. So, for example, 276 a–f are verses of an unbroken mediocrity; it would be difficult to find a group of epigrams which displays less charm and eloquence.

That the utility of the epigram had fallen into the background is well shown by 222:

a) Ψυχὴ μὲν προλιποῦσα τὸ σὸν [Δημήτριε σῶμα
οἴχεται εἰς Ἔρεβος· σωφροσύνης δὲ χάρις
θάλλει ἀγήρατος· τύμβῳ δέ σ[ε θῆκε θανόντα
Ἔρξις ἴσον στέρξασ' οἷσι τέκ]εσσιν ὁμοῦ.

b) Ἄφθονον εὐλογίας πηγὴν Δημ[ήτριε λείπεις
ἀσκήσας κόσμον σωφροσύνη[ν τε τρόπων
ὧν σε χάριν στέρξασ' Ἔρξις τεκ]έεσσιν ὁμοίως
μνημεῖον φιλίας τεῦξε τάφ[ον φθιμένῳ.

Here we have two complete epigrams expressing the same ideas in almost the same language.[125] The poet, or rather versifier, wished to exhibit his skill. If we find such exercises actually inscribed, it is no proof of spuriousness that two epigrams, which seem to be intended for the same tombstone, should be ascribed to Plato.[126]

Numerous epigrams of this century show that the authors had failed to attain the grandeur of the earlier masters, and had not as yet acquired that polished elegance which is the charm of much of the Alexandrian work. But among many empty and affected verses a few stand out superior to their surroundings.

233 has something of the early dignity:

a) Εἰ] τοιῶνδε ἀνδρῶν εἴη πόλις, οὔποτ' ἂν αὐτ[ῆς
ἐ]χθροὶ στήσαιεν Ζηνὶ τρόπαιον ἔδος.

b) Κ]τ[ώμ]ενον εὔκλεια[ν δ]ορὶ καὶ χερὶ τόνδε πρὸς ἀ[ν]δρὸς
ἐχθροῦ Ἀρι]σ[τ]όκρ[ιτ]ον ὤλεσε θοῦρος Ἄρης

124 See Table I.

125 Cf. 220. 241.

126 See p. 34.

217 has a grave beauty of its own, though it cannot rival Sim. 92, which it recalls.

> 217. Ὦ Χρόνε, παντοίων θνητοῖς πανεπίσκοπε δαίμων,
> ἄγγελος ἡμετέρων πᾶσι γενοῦ παθέων.
> ὡς ἱερὰν σῴζειν πειρώμενοι Ἑλλάδα χώραν
> Βοιωτῶν κλεινοῖς θνήισκομεν ἐν δαπέδοις.[127]

Another epigram, which, like that just quoted, shows how these later poets adapted to their own times the earlier sentiments, is 237 :

> Τοὺς ἀγαθοὺς ἔστερξαν Ἄρης, ἐφίλησε δ' ἔπαινος
> καὶ γήρᾳ νεότης οὐ παρέδωχ' ὑβρίσαι·
> ὧγ καὶ Γ(λ)αυκιάδας δήους ἀπὸ πατρίδος ἔργω[ν
> ἦλθ' ἐπ[ὶ] πάνδεκτον Φερσεφόνης θάλαμον.[128]

Splendid severity had passed away, and in its place were coming charm and grace.

Although the name of the dead is occasionally omitted from the epigram,[129] most epigrams give the reader all necessary details. The name of the dedicator and the verb of dedicating are always present in dedicatory inscriptions; in epitaphs, while a synonym for "dead" or "tomb" may be lacking (as in the fifth century), the fact that the verses are epitaphs is usually indicated by expressions like ἐνθάδε, κεῖται, etc. In a very few cases there is no indication that the verses were inscribed, — a peculiarity which appears now in sepulchral inscriptions for the first time ; e. g.

> 239. Οὐ σπάνις ἐστὶ γυναικὶ ἐσθλὴν καὶ σώφρονα φῦναι
> τὴν αὐτὴν δοκίμως, τοῦδε τύχεγ Γλυκέρα.[130]

In this century we find inscriptions written neither in elegiacs nor hexameters, but in irregular combinations of hexameters and pentameters. These verses bear witness to the growing passion for novelty, but otherwise they add nothing to our knowledge of the epigram, and are interesting chiefly because the commonplace authors of some of them have evidently attempted to imitate earlier and better poems.[131]

The epigrams preserved in MSS. differ little from the inscriptions, though among them there are a greater number that are excellent. Almost all show the signs of the times. The epitaph in honor of Plato shows a common formula slightly modified :

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127 Cf. 224, 292. 128 Cf. 20. 129 See p. 32.
130 Cf. 230.
131 See pp. 36 ff. Cf. K. F. Smith, Some Irregular Forms of the Elegiac Distich, AJP 22, pp. 165 ff.

256. Σῶμα μὲν ἐν κόλποις κατέχει τόδε γαῖα Πλάτωνος
ψυχὴ δ' ἰσοθέων τάξιν ἔχει μακάρων.

The writers are usually self-conscious, fond of rhetoric, given to praising learning as readily as valor. Whereas in early days one verse or a few words sufficed for the name of the artist, now an entire epigram is devoted to his name and his boasts; e. g. 312.[132] 254, although different from the rest in contents, is not alien to the taste of this century, whether we regard it as a real or an imitated epitaph:

Τοὔνομα θῆτα ῥῶ ἄλφα σὰν ὖ μῦ ἄλφα χῖ οὖ σάν.
πατρὶς Χαλκηδών· ἡ δὲ τέχνη σοφίη.

By far the most original epigram in our collection is 313:

Μνᾶμα μὲν Ἑλλὰς ἅπασ' Εὐριπίδου. ὀστέα δ' ἴσχει
γῆ Μακεδών. ἡ γὰρ δέξατο τέρμα βίου.
πατρὶς δ' Ἑλλάδος Ἑλλάς, Ἀθῆναι. πλεῖστα δὲ μούσαις
τέρψας, ἐκ πολλῶν καὶ τὸν ἔπαινον ἔχει.

The expression Ἑλλάδος Ἑλλάς seems to some critics too rhetorical for the fourth century. That is a question that can hardly be answered with any degree of certainty, but to me it seems not inconsistent with the style of Euripides himself. At any rate, the poem makes use of formulas which are characteristic of this period, and I am unwilling to reject it without further arguments against its genuineness.

There remain the epigrams attributed to Plato, with regard to which I have been and still am doubtful. Fava[133] denies that any of them belong to Plato or to the fourth century at all. Bergk accepts only 322. It seems to me, however, that we have no right to reject them all because some are surely or very probably late. In 318 and 319 we find early formulas combined with the freedom of expression which belongs to the fourth century. We cannot reject them on the ground that they are written for one and the same stone, since we know that two such epigrams might actually be inscribed side by side.[134] As for the rest that I have included, they may, of course, be late.[135] Still it is at least probable that in the fourth century there were prototypes of the epigrams so frequent in the third,[136] and every one, I suppose, would be willing to date these epigrams assigned to Plato as early as

[132] Cf. 302, 310, 311, 312 a.
[133] Gli Epigrammi di Platone. Cf. Reitzenstein, Ep. u. Sk., pp. 181 ff.
[134] See p. 32. It is impossible to tell whether they are real or epideictic epitaphs.
[135] Cf. pp. 11 ff.
[136] Cf. 314 with AP 5. 171; 320 with AP 5. 82, 83.

the third century. The spirit of 315 and 316, indeed, is not unlike that of 213–215, verses of Theognis. 321 and 322 are among the most famous epigrams which have come down to us, and would not shame the philosopher himself:

321. Ἀστὴρ πρὶν μὲν ἔλαμπες ἐνὶ ζωοῖσιν Ἑῷος,
νῦν δὲ θανὼν λάμπεις Ἕσπερος ἐν φθιμένοις.

322. Αἱ χάριτες τέμενός τι λαβεῖν ὅπερ οὐχὶ πεσεῖται
ζητοῦσαι, ψυχὴν εὗρον Ἀριστοφάνους.

In the fourth century we find the following epithets of the gods:

ἀκερσεκόμας (297)	πανεπίσκοπος (217)	τριτογένεια (271)
μεγαλόφρων (224)	τριτογενής (261)	ὑψιμέδων (298)
πολιοῦχος (268)		

The following epithets are applied to a country or city:

ἀγχίαλος (308)	εὐρύχορος (289)	κλεινότατος (221)
ἀνίκητος (259)	ἠγάθεος (234)	κλυτός (318)
αὐτόνομος (304)	ἱερός (274, 280, 304)	μεγάλαυχος (259)
εὔολβος (269)	κλεινός (217, 243, 259)	περίκλυστος (265)

As to the diction of the fourth century, epic words are gradually disappearing, displaced largely by the language of tragedy. This is clear from the following parallel passages.[137] I have tried to include only such words or phrases as are found solely or chiefly or first in the tragic poets.

237. Τοὺς ἀγαθοὺς ἔστερξεν Ἄρης.	Aesch. fr. 100. ἀλλ' Ἄρης φιλεῖ ἀεὶ τὰ λῷστα πάντα τἀνθρώπων στρατοῦ. Soph. fr. 657. τοὺς εὐγενεῖς γὰρ κἀγαθούς, ὦ παῖ, φιλεῖ Ἄρης ἐναίρειν. Phil. 436, 7. πόλεμος οὐδέν' ἄνδρ' ἑκὼν αἱρεῖ πονηρόν, ἀλλὰ τοὺς χρηστοὺς ἀεί.
289. ὑψηλὸμ πύργων ἀμφέθετο στέφανον.	Soph. Ant. 122. στεφάνωμα πύργων.
217. Χρόνε παντοίων . . . πανεπίσκοπε δαίμων.	O. T. 1213. ὁ πάνθ' ὁρῶν χρόνος. fr. 280. ὁ πάνθ' ὁρῶν καὶ πάντ' ἀκούων πάντ' ἀναπτύσσει Χρόνος.
220 a. κοινὸν Φερσεφόνης θάλαμον.	Soph. Aj. 1192. τὸν πολύκοινον Ἅιδαν. Eur. Suppl. 797. κοινὸν ἐς Ἅιδην καταβᾶσα.

137 The list does not pretend to be exhaustive.

230. κοινοταφὴς θάλαμος.	Soph. Ant. 804. παγκοίταν θάλαμον (cf. 810).
237. πάνδεκτον θάλαμον.[138]	Eur. Suppl. 1022. Φερσεφονείας θαλάμους.
281. τέρμα βίου.	Aesch. fr. 362. 2. εἰ μὴ τέρμα συντρέχοι βίου (cf. Prom. 259).
	Soph. O. T. 1530. πρὶν ἂν τέρμα τοῦ βίου περάσῃ.
	Eur. Alc. 643. ἐπὶ τέρμ' ἥκων βίου.
220 b. λέλοιπας πένθος.[139]	Soph. O. C. 1708. οὐδὲ πένθος ἔλιπ' ἄκλαυτον.

The following rare words are also found in tragedy :
ἐκγενέται (274) in Eur. Andr. 128 ; Bacch. 1155.
νυμφίδιος (244) in Eur. Alc. 885 ; Hipp. 1140.
ἁβροδίαιτος (311) in Aesch. Pers. 41.
ποθεινός, a word very frequent in epigrams of the fourth century, is a favorite of the tragic poets, especially Euripides.[140]

The following words are ἅπαξ λελεγμένα :

ἄγρωσσα (112)	δεκατόσπορος (309)	ὀστρειογραφής (301)
αἰνοβίης (74)	ἰθυμαχος (154) ?	πάνδεκτος (237)
ἀχνυόεις (25)	κοινοταφής (230)	ποικιλομήχανος (66)
βαθυχαιτήεις (108)	μενέγχης (104)	χρυσελεφαντήλεκτρος (301)

Finally I add a list of parallel passages from the early epigrams to show how well known these verses must have been, whether on stone or in MSS., and how freely they were imitated.

1. οἰκτίρας ἄνδρ' ἀγαθὸν παρίτω.	90. κα]τοικτίρας ἄνδρα ἀγαθὸν παρίτω.
83. εὔυδρόν ποκ' ἐναίομες ἄστυ Ϙορίνθου.	98. τοί ποτ' ἔναιον ὑπὸ κροτάφοις Ἑλικῶνος.
100. ἐλευθερίης ἀμφέθετο στέφανον.	289. πύργων ἀμφέθετο στέφανον (cf. 106).
104. ἀμφιέσαντο κόνιν.	250. ἀμφιέσαντο κόνιν.
	196. Πιερίην γῆν ἐπιεσσαμένη.
79. ἵκετ' ἄχος φθιμένου.	86. ἕξει ἄχος φθιμένου.
77 b. Ποτειδαίας δ' ἀμφὶ πύλας ἔδ[αμεν.	115. Σελινοῦντος ἀμφὶ πύλας ἔθανον.
106. εἰ τὸ καλῶς θνήσκειν ἀρετῆς μέρος ἐστὶ μέγιστον, ἡμῖν . . . τοῦτ' ἀπένειμε τύχη.	Allen XXVI (S IV). εἰ τὸ καλῶς ἐστι θανεῖν κἀμοὶ τοῦτ' ἀπένειμε τύχη. Cf. K 65 (S IV–II), εἰ τὰ θεῶν τιμᾶν . . . δόξα ἀρετή τε βροτοῖς.

138 See Table III.

139 Cf. Table III.

140 E. g. Hel. 623, I. T. 515, Phoen. 1737.

220 a. ηὔρηται δὲ ἄφθονος εὐλογία.	222 b. ἄφθονον εὐλογίας πηγήν.
106. ἀγηράντῳ χρώμενοι εὐλογίῃ	276 *f.* εὐδόξῳ χρώμενος εὐλογίῃ.
81. ἀθάνατον μνῆμα ἀρετῆς ἔθεσαν.	109. κάλλιστον δ' ἀρετῆς μνῆμ' ἔλιπον φθίμενοι.
258. μνῆμα . . . ἀθάνατον θνητοῖς.	273. ἀθάνατον θνητῷ πατρὶ γέρας τελέσας.
230. παῖδας παίδων ἐπιδοῦσαν.	K 44 (S IV/III). παῖδας παίδων ἐπιδοῦσα (cf. H 76).
84. Χαίρετε οἱ παριόντες ἐγὼ . . . κεῖμαι τῇδε θανὼν πατρίδα γῆν προλιπών.	K 23 (S IV). Χαίρετε δ' οἱ παριόντες, ἐ[γ]ὼ δὲ λιπὼν πατρίδα ἐνθάδε κεῖμαι.
170. Ἑλλήνων προμαχοῦντες Ἀθηναῖοι Μαραθῶνι.	REG 1904, 2. Ἑλλήνων προμαχοῦντες, Ἀθηναίων πολεμητάς.
174. Ἐξ οὗ γ' Εὐρώπην.	K 768 (S V/IV). Ἐ[ξ] οὗ τ' Εὐρώπην.
	K 844 (S IV). Ἐξ οὗ Κέκροπα λαός.
83. Ὦ ξεῖν' . . .	94. Ὦ ξεῖν' . . .
235. Ἐνθάδε τὸν πάσης ἀρετῆς ἐπὶ τέρμα μολόντα.	238. Ἐνθάδε τὴν πάσης ἀρετῆς ἐπὶ τέρμα μολοῦσαν.
59. Εὔχεο Τιμώνακτι . . .	158. Εὔχεό τοι δώροισι . . .
149. ἐπεὶ στρατὸν ὤλεσε Μήδων.	156. ἐπεὶ στρατὸς ὤλετο Μήδων.
217. Βοιωτῶν κλεινοῖς θνήσκομεν ἐν δαπέδοις.	259. κεῖται δ' ἐν κλεινοῖς Θετταλικοῖς πεδίοις.
243. ἔθανον κλειναῖς ἐν Ἀθήναις.	K 90 (S IV). κε[ῖ]μα[ι] κλειναῖς ἐν Ἀθήναις.
219. ἡνιόχος τέχνης τραγικῆς.	323. κιθάρας δεξιὸν ἡνιόχον.
177. παλαισμοσύνας δεξιὸν ἡνιόχον.	
239. οὗ σπάνις ἐστὶ τυχεῖν.	K 52 (S IV). οὗ σπάνις ἀνδρὶ τυχεῖν.
255. καὶ φθόνον οὐ φέρεται.	302. ᾧ φθόνος οὐχ ἕπεται.

Cf. 103 and 104 (see p. 26); 81 and 109; and see Table III.

The facts, then, which are clear from this investigation are briefly as follows:

The epigrams of the sixth century are characterized by a severe simplicity, which, however, cannot quite conceal the personal tone. Simple devices are usually employed to fill out the metre, though occasionally we find traces of more elaborate poetic treatment. In this century nothing except the name of the artist or stonecutter is inscribed on the stone *extra metrum*, and the verses themselves, with the exception of those meant for honorary statues, must indicate that they are inscriptions. We find no purely imitative inscriptions, but epigrams of satire or lament have already made their appearance. The chief influence from other literature is from the epic.

The fifth century is the period when splendor of expression is combined with simplicity and nobility of thought. Now the individual sinks out of prominence for a time. The name of the dead or of the dedicator may be repeated and certain other information added *extra metrum*, but the verses, still with the exception of epigrams for honorary statues, must indicate that they were inscribed. Dedicatory inscriptions often omit the name of the divinity. We find for the first time epideictic epitaphs and dedications; satiric, convivial, gnomic, and love epigrams are developing rapidly. The chief literary influence is still that of the epic, though we begin to see the effect of tragedy and rhetoric.

The work of the fourth century is marked by individualism and by a drop in tone. The great majority of the epigrams are commonplace, loaded with details, and utterly without charm. In the best work grace has taken the place of splendor, and we have an intimation of what the later epigram is to be. For the first time now art and letters are mentioned as titles to fame. The purely ornamental character of the inscribed epigram is shown by the fact that the name is sometimes omitted in the verses, which often give no indication that they were inscribed. In this century epic influence is dying out, and it is replaced by the influence of tragedy and rhetoric.

The frequent repetition of formulas and the imitation of one epigram by another show that the epigram was early considered a distinct branch of literature. Finally we have found examples of practically every variety of epigram in the early period. There is therefore no reason to suppose that the epideictic epigram of the third century sprang into being full-grown, as it were, nor that it developed from a perverted use of inscriptions.

The main part of this study ends here, but it may not be out of place to add a few remarks on the metre and the dialect of the epigrams.

The metre of inscriptions is discussed fully by Allen, and there is little to add to his investigations, since the epigrams preserved in MSS. observe the same principles as the inscriptions. I add, however, a few instances of the occurrence of the more unusual phenomena.[141]

Caesura in the third foot of the hexameter is lacking in 183. 1 (MS.) and 276 a. 1.

Elision in the third foot of the hexameter after the caesura occurs in 83. 1 and 138. 3 (Allen, p. 50, cites two examples, K 43 (S IV/III) and Allen XCVII 20 (S III)).

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[141] See also J. Mesk, Satz u. Vers in eleg. Distichon d. Griechen, Brünn, 1900; A. Langen, De Disticho Graecorum Elegiaco, Breslau, 1868.

Spondaic lines are 129. 1, 171. 3 (MS.), 210. 3 (MS.), 275. 1, 301. 1 (MS.).

Hiatus between the cola of the pentameter occurs in 103. 2 (MS.), 127.2 (Allen, p. 63, cites but one example, Allen XXVI (S IV)).

Syllaba anceps ends the first colon of the pentameter in 48. 2, 67. 4 (MS.), 127. 2, 190. 2 (MS.) (Allen, p. 63, cites but one example, K 24 = 233 b. 2).

A *monosyllable* ends the pentameter in 67.4 (MS.), 160 a. 2 (MS.), 267. 2, 270. 2 (Allen, p. 65, cites but one example, K 519).

I append also a table showing the structure of the distich.

	Hexameter and Pentameter distinct.		Pentameter grammatically dependent on Hexameter, but expressing Complete Thought.		Pause in Hexameter but none between Verses.		Pause in Pentameter but none between Verses.		No Pause.		
	Insc.	*Ms.*	*Insc.*	*Ms.*	*Insc.*	*Ms.*	*Insc.*	*Ms.*	*Insc.*	*Ms.*	
Seventh and sixth centuries	6	4	12	4	1	8	3	2	10	20	= 70
Fifth century	7	18	13	28	4	15	2	6	18	96	= 207
Fourth century	24	9	24	10	9	9	4	3	56	33	= 181

If we reduce the above figures to per cents, we get the following table :

	Per cent of Distichs where there is a Pause between Hexameter and Pentameter.	
	Isnc.	*Ms.*
Seventh and sixth centuries	$56\frac{1}{4}$	$21\frac{1}{19}$
Fifth century	$45\frac{5}{11}$	$28\frac{36}{163}$
Fourth century	$41\frac{3}{117}$	$29\frac{11}{16}$

It is evident that the tendency to separate the verses is much greater in the inscriptions than in the poems preserved in MSS. only. This tendency, however, grows steadily less in the case of inscriptions, while it increases slightly in the case of the epigrams preserved only in MSS.

The following table shows the structure of epigrams of two distichs :

	Pause in Sense between Distichs.		Pause in v. 2, not between Distichs.		Pause in v. 3, not between Distichs.		Pause after v. 3, not between Distichs.		No Pause.		
	Insc.	*Ms.*	*Insc.*	*Ms.*	*Insc.*	*Ms.*	*Insc.*	*Ms.*	*Insc.*	*Ms.*	
Seventh and sixth centuries		6				1	2			1	= 10
Fifth century	3	18	1		2	3	1	1	5	18	= 52
Fourth century	11	13			4	2	2	1	11	7	= 51

The evidence here is somewhat scanty, but it is surprising that there is a greater tendency to separate the distichs sharply in the epigrams preserved in MSS. than in the inscriptions.

The epigrams composed of more than two distichs are so few that it is not worth while to examine their structure.

The dialect of the epigram has long been a subject for discussion, and a somewhat unsatisfactory subject, since the only sure testimony is that of the inscriptions, and the material, except possibly for the Attic epigram, is extremely meagre. Among the various views which scholars have held, Fick and v. Mess represent the two extremes. Fick [142] claims that we find in inscriptions only such forms as belong to the language of the author or of those for whom the verses were composed. Kirchhoff [143] agrees with him so far as the Attic epigram is concerned. v. Mess,[144] on the other hand, holds that forms drawn from various dialects may, for different reasons, appear in the same epigram. Other scholars have taken various positions between these two extremes. Wilamowitz [145] thinks that the dialect is either epichoric or "die internationale Form des elegisch-epischen Dialectes." Wagner [146] admits the appearance of words of various dialects in one epigram, and thinks it is explained by the adoption of words or phrases taken bodily from different styles of literature and by the fact that the author and those for whom the poem was composed spoke different dialects. He holds also that when an author wished to write in a dialect not his own, he was satisfied to adopt only the more striking features of that dialect. Reitzenstein [147] contents himself with the following general statement: "Der Dialect ist im wesentlichen epichorisch; wenn der Tote im Ausland begraben ist, der seiner Heimat. Doch hat die Einwirkung des Epos oder der Lyrik ab und an auch die dialektische Form beeinflusst." [148]

Let us first consider the epigrams found in Attica. The following forms are not Attic:

SEVENTH AND SIXTH CENTURIES.

σαοφροσύνης (3)	*πατρωίης* (5)	*'Αίδαο* (9)
αἰδοίην (5)	*ἀπο]φθιμένοιο* (7)	*κούρῃι* (11, 28, 31)

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142 Die Homerische Ilias, Goettingen, 1886, pp. v ff.

143 Hermes 5, pp. 56 ff.

144 Quaestiones de Epigrammate Attico et Tragoedia Antiquiore Dialecticae, Bonn, 1898.

145 Hermes 20, pp. 69 ff.

146 Quaestiones de Epigrammatis Graecis, Leipzig, 1883.

147 Pauly-Wissowa, s. v. Epigramm, p. 78.

148 See also H. W. Smyth, Ionic, p. 61.

πολιήοχε (24)	ἀνορέαν (12)	Ἀθάνα (24)
Ἀθήνηι (29)	Ἀθαναίαι (22)	φρασμοσύναι(39)
φρασίν (2)	ἠδέ (3)	ἐγρεμάχαι (26)

In this list there are no Ionic words which are not also epic. That Ἀθήνη is epic, not Ionic, has been shown by v. Mess,[149] since it appears only in epic poets, and in one inscription from Naxos,[150] probably in hexameter. Except in two words, which will be discussed a little later, *a purum* is not represented by η. All the epic forms are adopted bodily from the epic, and in every case where they appear the Attic form would be metrically impossible in that position in the verse. These same words are found in the Attic form in other epigrams. Cf. κούρη with κόρη (23), ἀποφθιμένοιο with φθιμένου (79).

That the Doric forms are taken from lyric poets and show their influence on the vocabulary of poetry is generally admitted.[151] They are always side by side with Attic forms, so that they cannot be regarded as evidence of the native region of the author.[152] In order to explain the form φρασίν, a form found elsewhere only in Pindar, v. Mess[153] conjectures that the phrase φρασὶν ἄλλα μενοινῶν was common in Boeotia and thus crept into the inscriptions of the adjoining country. It is, however, unlikely that the contracted form μενοινῶν was common in Boeotia in the sixth century,[154] and it is more probable that we have in φρασίν another evidence of the influence of lyric poetry, unless, indeed, it may be also Attic.

FIFTH CENTURY.

ἀριστῆες (75)	εἰργάσσατο (123)	ἱπποσύναι (75)
κοῦροι (75)	Κυδωνιήτας (123)	Ἑλλάνων (75)
ξεῖν' (83)	πόληας (126)	ποκ' (83)
Ἀθηναίηι (117)	Πυθαγόρην (79)	ἐναίομες (83)
Ἄρεος (117)	Σαλυβρίαν (79)	
φιλοξενίης (117)	βαρνάμενοι (81)	

We find here Ionic, or rather epic, forms as in the preceding centuries. Again two words show -ιη. If such forms were used by natives of Attica, the statement made above, that epic forms were used only when the Attic form was metrically impossible, is incorrect. But if

[149] p. 12.
[150] Collitz u. Bechtel, Sammlung d. gr. Dialekt-Inschriften, 5418.
[151] Cf. Wilamowitz and Wagner (ll. cc.), and Wilhelm, JOAI 2. 244.
[152] Even κόρα appears, JHS 13. 126, n. 18; and CIA IV, 373^{109}, p. 91.
[153] p. 21.
[154] See Ahrens, De Graecae Linguae Dialectis, Goettingen, 1839, I, 201.

we examine the epigrams in which these words appear (5, 117), we see that both were written in honor of foreigners. Therefore the forms in -ιη may be really Ionic and not epic. Other inscriptions which are certainly in honor of foreigners are 79 and 83. 79 is, indeed, composed in the Attic dialect because it was the Athenian state which set up the stone, and the official language would naturally be used; but 83 is in the dialect of the Corinthians who lay beneath the stone. From these facts we may reasonably conclude that 5 and 17 were also composed in the dialects of the dead, and we may still hold that Attic forms are replaced by epic only for metrical convenience. This position is strengthened if we examine the following words, ἡλικίας (12), νεαράν (1), ξενίαν (16), ἄγρας (23), βίαι (76), σφετέραν (81), γενεᾶς (80), all of which have retained the Attic form, although at least two of them appear in the epic as frequently as πατρωίης and αἰδοίην.

'Αθηναίηι (117) is explained by v. Mess in the same way as 'Αθήνηι (see above), but here the case is somewhat different, since the Attic form 'Αθηναίαι would have suited the metre equally well. Therefore, while 'Αθήνηι might appear in an inscription in Attic, 'Αθηναίηι, in my opinion, could not.[155]

Since only words which appear in epic or lyric poetry are found in the epigrams in epic or Doric forms, it seems fair to conclude that words not found in epic or lyric poetry could not be given epic or Doric forms merely to add a poetic tone to the verses. Hence πρᾶγμ' (1) is the only possible form for that epigram, and in the much-discussed ep. 171,[156] πράγμασι must have been the form which appeared on the stone, if, as we now suppose, πρᾶγμα was a word not used in the epic.

There remains a difficulty which no one has as yet been able to solve, — the form Πυθαγόρην (79). The inscription, as was to be expected, is almost wholly Attic, even to the genitive Πυθαγόρου which precedes the verses. The Doric form Σαλυβρίαν is easily explained, since the name of a Doric town would be likely to remain unchanged in any surroundings. Of the form Πυθαγόρην v. Mess writes as follows:[157] "Formam vero Πυθαγόρην in carmine nostro non ex Iade vulgari sed ex poetica moris epici imitatione ortam esse apparet, praesertim cum in titulo suprascripto eiusdem nominis forma Attica, Πυθαγόρου, legeretur. Forma Πυθαγόρης fortasse inde explicatur, quod eis fere temporibus

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155 v. Mess cites Hoffmann 258 as an example of an Attic epigram containing Ἀθηναίηι. But this epigram may also have been written by or for a foreigner. The name Σμ]ικύθη (which is partly conjectural) is not found in Attica alone, and it is not certain that the form ἑαυτῆς could not be employed by an Ionic poet.

156 See Wilamowitz, Hermes 20, pp. 69 ff.

157 p. 14.

Pythagorae philosophi nomen in poesi dactylica et philosophorum et poetarum saepissime laudabatur." His first suggested explanation cannot be correct if we have been right in concluding that epic forms were borrowed or transferred, not imitated; but his second suggestion may be the answer to the question. The genitive Πυθαγόρου may be due to the fact that it was not in the original copy given to the stonecutter, who supplied it in his native Attic.

Editors have wished to change Κυδωνιήτας (123) to Κυδωνιήτης or Κυδωνιάτας. v. Mess keeps the form as inscribed on the stone and calls it Dorico-epic.[158] I cannot think that the stonecutter purposely "epicized" his own name, especially as he kept the ending -τας. It is more probable that η is a mistake for α.

In the fifth century, too, we find Doric forms. Ἑλλάνων (75) is not preserved on the stone, but appears in the MS., which has, however, the incorrect form ἱπποσύνηι. On βαρνάμενοι see v. Mess,[159] who cites two early inscriptions.[160]

FOURTH CENTURY.

γαῖα (218, 220 b, 234, 245)	ἀρετᾶς (242)
ἐών (229)	Ἀθάναι (246)
Ἀχιλλῆος (245)	(cf. Ἀ]θην . . . v. 1.)
Πυλαμένεος (245)	

It is noticeable how very few non-Attic forms appear. In this century Attic forms were not only crowding out epic and Doric-lyric forms in Attica itself, but they were displacing the epichoric forms in other regions. In 218, an epitaph of Corcyreans, and 225, the epitaph of a Cytherean woman, the Attic dialect may indeed be explained by the fact that the former was inscribed by the Athenian state and the latter by the master of the dead woman, himself probably an Athenian; her name, however, which appears *extra metrum*, retains the Doric form, Μαλίχα. The monument of a Corinthian woman for which 231 was written may likewise have been set up by Athenians, but the widespread use of the Attic dialect at this period makes it impossible to speak with certainty.

In regard to the dialect of epigrams of the sixth and fifth centuries from other regions than Attica we have very little information. Not many inscriptions come from any one place, and in the case of numerous dedications found at Olympia and Delphi we have no means of determining the nationality of the authors or dedicators. So far as we can tell, epigrams seem to be in the epichoric dialect; cf. forms

158 p. 15. 159 p. 20. 160 Hoffmann, 47, 51.

like *Fἰν* (42), *Fάναξ* (43), *ἐνπίδες* (85), *ἀρισστεύων* (88), *τήνου*, *ὄνυμα* (128), *Fοι ℎεσλός* (136). On the other hand, epigrams like 88 and 92 refute Fick's statement and seem to support Wagner's views.[161] 84 and 86, epitaphs of foreigners, are written in the dialect of the dead, not of the people among whom they died.

Epic forms are exceedingly rare. We find only *ξείνοισι* (88), *ὁδοῖο* (89), *κίχηι* (92), *εἵνεκα* (132). In 42 the form *ἰλήFωι*, in a Doric setting, is still a stumbling-block to commentators.

In the fourth century, as stated above, the Attic dialect spread rapidly over other districts. In Doric countries we still find the Doric *α* as a rule: *ὀλόμαν* (248), Μ]*ατρ*[*όπολ*]*ις*, *γᾶ*, *στᾶσαν* (265), Ἀ[*σκ*]*λαπιέ* (266); in Ionic regions the Ionic *η*: *ἡλικίην* (252), Ἀ*θηναίηι* (268), but these are almost the only distinguishing marks of dialect which we find, and even these are not invariable. In 267 we read *νεώ*, *ἵλαος*, *γενεήν*; in 269 *κῆρυξ*, *πτανοῖς*; in 274 *νύμφα*, *λαός*, *κοῦρος*, *μνῆμ'*; in 247 (from Euboea) *θεραπείαι*; in 268 (from Erythrae) *πολιούχωι*. Epic forms have almost entirely disappeared. I am unable to see why Preger[162] says that in the fourth century the epic-Ionic dialect began to prevail. Surely for "epic-Ionic" we should read "Attic."

The only epic *forms* which are found to any extent in this century are words ending in *-οιο*. This termination persists because of its metrical convenience, as we see from the fact that such forms tend to occupy fixed places in the verse. The same is true of the endings *-οισι* and *-αισι*.

Ending.	Total No. of Occurrences.	Number of Occurrences in Hexameter.					Number of Occurrences in Pentameter.			
		1st & 2d ft.	2d and 3d ft.	3d ft.	5th ft.	6th ft.	2d ft.	2d & 3d ft.	4th ft.	5th ft.
-οιο	17			8	4	4				1
-οισι	31 (-οις, 78)	1	1	8	7	6	1	3	1	3
-αισι -ησι	5 (-αις, 26)				4	1				
Total		1	1	16	15	11	1	3	1	4

[161] See p. 40.

[162] p. xviii.

The above table covers the occurrence of these terminations in all epigrams of the early period, whether inscribed or not.

It is evident from this table that the favorite positions for such terminations are just before the feminine caesura of the third foot and in the fifth and sixth feet of the hexameter. The number of cases where they stand in other positions is so small that we can hardly say that one position is preferred over another.

The chief result of the preceding investigation has been to show how little we can really state positively from the material at our command. We know enough about Attic epigrams to be able to say with comparative certainty when we may expect to find Ionic or epic forms, but in all other cases our knowledge is so limited that we can draw no conclusions which would enable us to make corrections or even to justify MS. tradition in those epigrams for which we have not the testimony of the stones. Reitzenstein's general statement[163] is the best we can do till we have more material to examine.

Appendix

The following epigrams are grouped by centuries: in each century the sepulchral epigrams stand first, followed by the dedicatory and epideictic. Within each group the inscriptions precede the epigrams which are preserved in MSS. only. The inscriptions are arranged according to their provenience to facilitate comparison of dialect. It is quite possible that some epigrams written at the beginning of a century have been wrongly grouped among those belonging to the end of the preceding century and *vice versa*,[164] but I trust that no such mistake has been made in any case where it would affect the historical conclusions which have been drawn. I have made no attempt to give all the sources for an epigram nor to indicate all the collections where it may be found, but when an epigram is included in the collections of Kaibel, Preger, Hoffmann, or Bergk, and when it is cited by an author earlier than 300 B. C., I have so indicated. In the case of inscriptions the first reference is to a facsimile and the restoration adopted is by the editor of the facsimile unless it is otherwise stated. Where the reputed author of an epigram is mentioned it is merely for purposes of citation and is not meant as an assertion of the correctness of the attribution.

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[163] See p. 40.

[164] This is especially likely in the case of Simonides. Doubtless some of the epigrams attributed to him belong to the sixth century, but there is no way of distinguishing them from those of the fifth century, and it has seemed simpler to group them all together except such as are obviously later than the fifth century.

An asterisk* indicates that an epigram is preserved both on stone and in MSS.

Square brackets [—] indicate that the epigram is usually cited as an elegy or part of an elegy.

Epigrams

Seventh and Sixth Centuries

1. CIA I, 463 (= K 1 = H 2).
2. CIA IV, 477^{h}, p. 112 (= K(RM) 2 a = H 13).
3. CIA IV, 477^{b}, p. 48 (= K 2 = H 9).
4. CIA I, 482 (= K 17 = H 255).
5. CIA I, 477 (= K 13 = H 11).
6. CIA I, 479 (= K 15 = H 27).
7. CIA IV, 477^{c}, p. 48 (= K, add. 1 a = H 22).
8. CIA I, 473 (= K 10 = H 23).
9. CIA I, 481 (= K 16 = H 28).
10. CIA IV, 477^{p}, p. 188 (= H 1).[165]
11. CIA I, 469 (= K 6 = H 7)
12. CIA I, 471 (= K 8 = H 6).[165]
13. CIA I, 487 (= K 18 = H 19).
14. AEMO 11 (1887), p. 187 (= H 53). Paros.
15. IGA 495 (= K(RM) 229 a = H 61). Erythrae.
16. IGS 3501 (= H 57). Provenience unknown. Restored by Dittenberger.
17. Hermes 20 (1885), p. 158 (= H 54). Thessaly.
18. AP 7. 304 (= Pr 25 = PLG 2, p. 24).
19. AP 7. 489 (= Sappho 119).
20. AP 7. 160 (= Anac. 101).[166]
21. Pr 49 (= PLG 2, p. 239).
22. CIA IV, 373^{105}, p. 90 (= H 214).
23. CIA IV, $373^{54.68.17.9}$, p. 179 (= H 256). Restored by Lolling.
24. CIA IV, 373^{106}, p. 91 (= H 242).

*25. CIA IV, 334 a, p. 78 (= K 748 = H 249 = Pr 72 : Hdt. 5.77).

26. CIA IV, 373^{216}, p. 102 (= H 226).
27. CIA IV, 373 x, p. 128 (= H 230).
28. CIA IV, $373^{28.36.211}$, p. 180 (= H 221).

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[165] See n. 33.

[166] Reitzenstein (Ep. u. Sk., p. 135, n. 1) thinks v. 2 imitates Aeschylus, fr. 100. There is, however, nothing in the epigram inconsistent with an early date, and 237 shows how a later poet imitated the fragment of Aeschylus.

On the epigrams of Anacreon, see L. Weber, Anacreontea, Goettingen, 1895.

29. CIA IV, 373^{107}, p. 91 (= H 243).
30. CIA IV, 373^{85}, p. 87 (= H 218). Restored by Allen, VIII.
31. CIA IV, $373^{a5.58.29.53}$, p. 163 (= H 211) Restored by Kirchhoff.
32. CIA IV, 373^{202}, p. 100 (= H 224).
33. CIA IV, 373^{218}, p. 102 (= H 246).
34. CIA IV, 373^{188}, p. 98 (= H 222). Restored by Kirchhoff.
35. CIA IV, 373^{201}, p. 100 (= H 223). Restored by Kirchhoff and Hoffmann.
36. CIA IV, 373^{208}, p. 183 (= H 220).
*37. CIA IV, 373^{e}, p. 41 (= K 743 a, pref. = H 238 = Pr 71 : Thuc. 6. 54).
38. CIA IV, 422^{13}, p. 185 (= H 253).
39. CIA IV, 373^{99}, p. 89 (= H 251).
40. CIA IV, 373^{79}, p. 86 (= H 232).
41. IGA 62^{a}, p. 174 (= H 307). Sellasia.
42. IIS 652 (= H 305). Metapontum.
*43. Olymp. 252 (= K 743 = H 311 = Pr 57 : Paus. 5. 24. 3).
44. Olymp. 157 (= H 375). Restored by Kirchhoff.
45. Olymp. 154 (= H 371).
46. IGA 412 (= K 740 = H 290). Melos.
47. IGA 393 (= H 321). Ceos. Restored by Kirchhoff.
48. BCH 29 (1905), p. 214. Delos.
49. AP 6. 133 (= Archil. 18).
50. AP 6. 269 (= Sappho 118).[167]
51. AP 6. 135 (= Anac. 102).
52. AP 6. 142 (= Anac. 103).
53. AP 6. 139 (= Anac. 105).
54. AP 6. 140 (= Anac. 106).[168]
55. AP 6. 141 (= Anac. 107).
56. AP 6. 134 (= Anac. 108).[169]
57. AP 6. 136 (= Anac. 109).
58. AP 6. 137 (= Anac. 110).
59. AP 6. 143 (= Anac. 111).

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[167] Although this epigram is longer than most early inscriptions, it is no longer than the early Corcyrean inscription, K 179. The sentiment is simple and frequent on stones of the sixth century.

[168] Wilamowitz (quoted by Weber, p. 34) holds that the name 'Αρηιφίλου proves this epigram late. For Homeric epithets as proper names see Thuc. 7. 34 (Δίφιλος), Thuc. 8. 64 (Διιτρέφης), Xen. Hell. 1. 3. 13 (Θεογένης). These examples, though not so early as this epigram, are certainly not "late."

[169] Weber's objection that these verses describe a painting and are therefore late, does not hold, since there is no indication that they describe a picture rather than a relief. Cf. 47 and Paus. 5. 17. 6.

60. AP 6. 346 (= Anac. 112).
61. Pr 106 (Paus. 5. 10. 3).
62. Pr 123 (Paus. 6. 13. 10).
63. Pr 197ª (Plato, Hipp. 229 A).[170]
64. Pr 197ᵇ (Plato. Hipp. 229 A).[170]
65. Pr 53 (Phot. Lex. s. v. Κυψελίδων).
66. Pr 70 (Ath. 13. 609d).
67. AP 6, 341 (= Pr 109 : Hdt. 4. 88).
68. PLG Demod. 1 (Arist. Nic. Eth. 7. 9).
69. AP 11, 235 (= PLG Demod. 2).
70. PLG Phocyl. 3 (Strabo 10. 487).
[71. PLG Archil. 2 (Ath. 1. 30 f).]
[72. PLG Archil. 6 (Plut. Mor. 239 B).]
[73. Theog. 877, 878.]
74. AP 7. 226 (= Anac. 100).

Fifth Century

*75. JOAI 2 (1899), p. 221 (= AP 7. 254 = Sim. 108).
76. CIA I, 333 (= K 749 = H 266).[171]
77. CIA I, 442 (= K 21 = H 34) vv. 1–4 restored by Boeckh and Kaibel.
78. CIA II, 3. 2338 (= H 38).
79. CIA IV, 491^{12}, p. 115 (= K 36 = H 32).
80. CIA IV, 477^{e}, p. 49 (= H 20). Restored by Boeckh and Kirchhoff.
81. CIA IV, 446^{a}, p. 108 (= H 36)
82. CIA IV, 491^{8}, p. 114 (= K 73 = H 33).
*83. AM 22 (1897) p. 53 and tab. 9 (= Sim. 96). Salamis.
84. IGA 368 (= K 22 = H 66). Aegina.
85. AM 31 (1906), p. 89 and tab. 13. Megara.[172]
86. IGS 2531 (= K 488 = H 171). Tanagra ?
87. IGA 146 (= K 486 = H 56). Thespii.
88. IGA 167 (= K 487 = H 59). Thisbe.
89. IGA 329 (= K 182 = H 51). Acarnania.
90. IGA 325 (= H 55). Pharsalus. Restored by Cauer.
91. BCH 24 (1900), p. 267. Thasos.
92. BCH 24 (1900), p. 266. Thasos.
93. IGA 382 (= H 63). Chios.

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170 See CIA I, 522, and cf. K. F. Smith. AJP 22, pp. 165 ff.
171 Cf. Bormann, JAOI 6 (1903), pp. 241 ff.
172 Cf. Solmsen, AM 31 (1906), p. 342.

94. AP 7. 249 (= Sim. 92 = Pr 21 : Hdt. 7. 228).
95. AP 7. 677 (= Sim. 94 = Pr 20 : Hdt. 7. 228).
96. AP 7. 248 (= Sim. 91 = Pr 200 : Hdt. 7. 228).
97. PLG Sim. 93 (= Pr 22).
98. Pr 23 (PLG 3, p. 428).
99. AP 7. 250 (= Sim. 97 = Pr 5).[173]
100. AP 7. 347 (= Sim. 98 = Pr 4).
101. PLG Sim. 89 (= A Pl 26).[174]
102. AP 7. 512 (= Sim. 102).
103. AP 7. 251 (= Sim. 99).[175]
104. AP 7. 255 (= PLG Aesch. 3).[176]
105. AP 7. 508 (= PLG 2. p. 260 = Pr 40).
106. AP 7. 253 (= Sim. 100 = Pr 8).
107. PLG Sim. 111 (= Pr 31 : Thuc. 6. 59).
108. PLG Aesch. 4 (= Pr 39).
109. AP 7. 258 (= Sim. 105).[177]
110. AP 7. 509 (= Sim. 118).
111. AP 7. 507b (= Sim. 124 B).
112. PLG Sim. 130 (= Pr 51).
113. AP 7. 270 (= Sim. 109).
114. PLG Eurip. 1 (= Pr 9).
115. Pr 41 (PLG 3, p. 517 : Plut. Mor. 217 F).
116. AP 10. 105 (= Sim. 122).
117. CIA I, 374 (= K 752).
118. CIA I, 418 (= K 763). Restored by Kaibel.
119. CIA I, 382 (= K 754).
120. CIA I, 353 (= K 765).
121. CIA I, 397 (= K 753).
122. CIA I, 349 (= K 756 = H 236).
*123. CIA I, 403 (= K 751 = AP 13. 13). Restored by Meineke.
*124. CIA I, 381 (= K 758 = Anac. 104 = AP 6. 138).

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173 Vv. 1 and 2 form a complete epigram (see Kaibel, RM 28. 444), and vv. 3–6 are not quoted by any author before Aristides. This makes it probable that 3–6 are a later addition (see Wilhelm, JOAI 2, p. 244, and Wilamowitz, Goett. Nachr., 1897, p. 308). Kaibel (l. c.) says of vv. 1 and 2: "Nihil habet quod reprehendas praeterquam quod nec mortuos homines neque hostes devictos quinam fuerint significat; unde antiquo sepulcro insculptum fuisse nequit." But cf. 81. See also p. 24, n. 94.

174 Wilhelm (l. c.) thinks vv. 3, 4 a later addition.

175 Cf. AP 7. 242. With v. 2 cf. Sim. 37. 8 and Iliad II. 66.

176 Evidently an imitation of 103. It is very likely epideictic. The *καί* looks like an imitation of an epigram like 171 b, where the *καί* is in place, as it is not here.

177 Wilhelm (l. c.) thinks only vv. 1, 2 are original. Cf. note on 101.

*125. JAOI 2 (1899), p. 230 (= Sim. 150 = AP 6. 144). Attica.
126. CIA IV, 373[268], p. 204 (= K 759 = H 322).
127. CIA IV, 422[16], p. 185 (= H 271).
128. IGA 354 (= K 761 = H 315). Aegina.
129. IA 1207. Epidaurus.
130. IGA 7 (= H 324). Euboea.
131. IGS 1794 (= K 757a pref. = H 317). Thisbe.
*132. Olymp. 253 (= H 312 = Pr 59).
133. Olymp. 147 (= K(RM) 940 b = H 376).
134. Olymp. 149 (= K(RM) 941 c = H 377).
135. Olymp. 144 (= K(RM) 940 a = H 378).
136. Olymp. 266 (= K 744 = H 309).
137. Olymp. 150 (= IGA 355).
138. CRAI 1901, p. 681. Delphi.[178]
139. AM 31 (1906), p. 530. Delphi.
140. AM 11 (1886), p. 450 (= H 318). Larissa.
141. IGA 402 (= K 750 = H 301). Paros.
142. IGA 401 (= K 750 a, add. = H 302). Paros.
143. Olymp. 630 (= H 401).
144. PLG Sim. 131 (= Pr 152).
145. AP 6. 2 (= Sim. 143).
146. Pr 73 (Phil. ap. Harpocrat. s. v. πρὸς τῇ πυλίδι 'Ερμῆς).
147. Pr 74 (Arist. 'Αθην. πολ. 7).
148. AP 6. 213 (= Sim. 145).
149. PLG Sim. 138 (= Pr 84 : Thuc. 1. 132).
150. PLG Sim. 139 (= Pr 85).
151. PLG Sim. 141 (= Pr 83. vv. 1. 2. 5. 6 in scholia, Pind. P. 1. 155; vv. 1–4 = AP 6. 214).[179]
152. Pr 86 (PLG 3, p. 516 : Diod. 11. 14).
153. PLG Sim. 134 (= Pr 67 : Plut. Mor. 870 F).
154. PLG Sim. 137 (= Pr 68 : Ath. 13. 573 d).
155. AP 6. 50 (= Sim. 140 = Pr 78).[180]
156. Pr 103 (= Sim. 135 : Plut. Them. 8).
157. Pr 100 (PLG 3, p. 516 : Ath. 12. 536 b).
158. AP 6. 212 (= Sim. 164).
159. Pr 125 (Paus. 6. 10. 7).
160. Pr 126. 176 (Paus. 8. 42. 9, 10).

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178 This is the only epigram earlier than the second century B. C. where the name of the poet is mentioned. See Boas, p. 45, n. 16.

179 See Hauvette, no. 65. Wilamowitz (Goett. Nachr., 1897, p. 313) rejects vv. 3–6.

180 Wilamowitz (l. c.) rejects v. 2.

161. Pr 58. 175 (Paus. 5. 25. 10).
162. Pr 56 (Paus. 5. 27. 12).
163. Pr 82 (Diod. 13. 41).
164. AP 6. 53 (= PLG Bacchyl. 49).
165. Pr 108 (Porphyr. Vit. Pythag. 3).
166. PLG Sim. 157 (= Pr 105 : Di. La. 4. 45).
167. AP 6. 313 (= PLG Bacchyl. 48).
168. PLG Philox. 15 (= AP 9. 319).
169. PLG Sim. 147 (= Pr 136).
170. PLG Sim. 90 (= Pr 199 : Lycurg. Leocr. 109).[181]
171. Pr 153 (Aeschin. 3. 184, 185).
172. Pr 154 (Aeschin. 3. 190).
173. Pr 178 (Paus. 6. 20. 14).
174. AP 7. 296 (= Pr 269 = Sim. 142).[182]
175. Pr 140 (Ath. 1. 19 b).
176. Pr 127 (Paus. 7. 17. 7).
177. PLG Sim. 149 (= A Pl 2).
178. PLG Sim. 152 (= Pr 124 ; Paus. 6. 9. 9).
179. PLG Sim. 153 (= A Pl 3).
180. PLG Sim. 163 (= Pr. 144 ; Arist. Rhet. 1. 7. 32).
181. Pr 142 (Schol. Aristoph. Acharn. 214).
182. Pr 180 (Ath. 2. 48 b).
183. Pr 174 (Paus. 6. 10. 5).
184. PLG Sim. 161 (= AP 9. 757).
185. PLG Sim. 160 (= Pr 179; Paus. 10. 27. 4).
186. Pr 209 (Arist. Nic. Eth. 1. 9).[183]
187. Pr 207 (Porphyr. de Abstin. 2. 19).
188. PLG 2, p. 260 (Di. La. 8. 65).
189. PLG Sim. 169 (= Pr 253 ; Ath. 10. 415 f).
190. AP 7. 349.
191. AP 7. 22 (PLG 2, p. 314).[184]
192. AP 7. 515 (= Sim. 117).
193. AP 7. 301 (= Sim. 95).[185]

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[181] The word Μαραθῶνι seems more suited to an honorary statue than to an actual tomb on the battle field.

[182] See Boas, p. 205.

[183] Cf. Theog. 255; Soph. fr. 329.

[184] The absence of τῇδε or any similar indication that this is an inscription has led me to place it among the epideictic epitaphs.

[185] If this is an epitaph, it would be hard to tell whether it was for Leonidas or for the Spartans (see Hauvette); this uncertainty seems to me impossible in an epitaph, and the verses are probably an occasional poem. τῇδε may be explained by supposing that the author had the tomb of Leonidas before him.

194. AP 7. 514 (= Sim. 120).[184]
195. AP 7. 302 (= Sim. 121).
196. PLG Sim. 167 (Ath. 3. 125 c).
197. PLG Soph. 5 (Plut. Mor. 785 B).
198. PLG Eurip. 2 (Ath. 2. 61 b).
199. PLG Soph. 4 (Ath. 13. 604 f).
200. PLG Sim. 175 (Stob. Ecl. Phys. 1. 8. 15).
201. PLG 2, p. 268 (Stob. Ecl. Phys. 1. 8. 16).
[202. PLG Euenus 2 (= AP 11. 49).]
[203. PLG Euenus 3.]
[204. PLG Euenus 1 (Ath. 9. 367 e).]
[205. PLG Euenus 4 (Stob. 51. 17).]
[206. Theog. 1069, 1070.][186]
[207. Theog. 425–8.]
[208. Theog. 797, 8.]
[209. Theog. 257–260.]
[210. Theog. 993–6.]
[211. Theog. 351–4.]
[212. Theog. 649–652.]
[213. Theog. 567–570.]
[214. Theog. 1341–4.]
[215. Theog. 1299–1304.]
[216. Theog. 1329–34.]

Fourth Century

*217. CIA II, 3. 1680 (= K 27 = H 106 = AP 7. 245).
218. CIA II, 3. 1678 (= K 37 = H 70).
219. CIA II, 3. 2263 (= K 39 = H 117).
220. CIA II, 3. 3620 (= K 35 = H 90).
221. CIA II, 3. 2892 (= K 71 = H 83).
222. CIA II, 3. 3602 (= K 35 a. add. = H 101). Vv. 2, 8, 9 restored by Kaibel.
223. K 38 (I have found no facsimile). Athens.
224. CIA II, 3. 2339 (= K 34 = H 74).
225. CIA II, 3. 3111 (= K 47 = H 87)
226. CIA II, 3. 2867 (= K 54 = H 82)
227. CIA II, 3. 3880 (= K 63 = H 94).
228. AM 19 (1894), p. 140. Athens.

[186] Some of these verses attributed to Theognis may belong to the sixth century. Since, however, some are surely of the fifth century and all are probably earlier than 400 B. C. (see Reitzenstein, p. 81), they are grouped together here for convenience.

229. CIA II, 3. 1995 (= K 62 = H 73). V. 2 restored by Kumanudes.
230. CIA II, 3. 3903 (= H 105).
231. CIA II. 3. 3086 (= K (RM) 58 a = H 86). Restored by Kaibel.
232. CIA II, 3. 2646 (= K 25 = H 78). Restored by Kaibel.
233. CIA II, 3. 2717 (= K 24 = H 79). Restored by Kaibel after Boeckh.
234. CIA II, 3. 2453 (= H 69).
235. CIA II, 3. 1994 (= K 49 = H 129).
236. CIA II, 3. 2496 (= K 64 = H 75).
237. CIA II, 3. 2718 (= H 80).
238. CIA II, 3. 1774 (= K 50 = H 72).
239. CIA II, 3. 3577 (= K 53 = H 89).
240. CIA II, 3. 3720 (= K 41 = H 92).
241. BCH 17 (1893), p. 194. Piraeus.
242. CIA II, 3. 2643 (= K 69 = H 77).
243. CIA II, 3. 3673 (= K 91 = H 91).
244. CIA II, 3. 4054 (= H 96).
245. CIA II, 3. 3260 b, p. 355 (= H 109).
246. CIA II, 3. 1376 (= K 43 = H 351). Restored by Kaibel.
247. K 209 (I have found no facsimile). Euboea.
248. AM 1 (1876), p. 233 (= K 471 a, add. = H 177). Sparta.
249. IGS 2536 (= K 489 = H 181). Thebes. V. 1 restored by Wilamowitz; vv. 3–7 by Keil and Kaibel.
250. BCH 25 (1901) p. 271. Tegea.[187]
251. CIG II (add.), 2254 v (= K 219 = H 172). Amorgos.
252. Ann. d. Instit., 1864, p. 103 (= K 220 = H 173). Amorgos.
253. CIG II, 3648 (= K 86 = H 174). Provenience unknown.
254. Pr 260 (Ath. 10. 454 f).
255. AP 7. 60 (= Pr 11).
256. Pr 12 (PLG 2, p. 329 = A Pl 31).[188]
257. Pr 13 (Steph. Byz. s. v. Φασηλίς).
258. AP 7. 300 (= Sim. 123).[189]
259. Pr 24 (Ath. 13. 589 b).
260. CIA II, 2. 1078 (= K 1043 = H 416). Restored by Boeckh.

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187 See Wilhelm, AM 29 (1904), p. 108, and Loring, JHS, 1895, p. 90.

188 Cf. 'Εφ. 'Αρχ., 1839, p. 227, n. 264, a fragment which may be metrical. For Bergk's reading of it see PLG 2, p. 330.

189 Although this epigram bears the name of Simonides, there are in it indications that it belongs to the fourth century — κέκευθε γαῖα, ἐρατῆς ἥβης τέλος ἄκρον, ἀθάνατον θνητοῖς.

261. CIA II, 3. 1427 (= K 770 = H 273).
262. CIA II, 3. 1425 (= K 771 = H 350). Restored by Kaibel.
263. AM 30 (1905), p. 298 (= H. 281). Athens.
264. CIA II, 3. 1441 (= K(RM) 773 a = H 274). Restored by Kaibel.
265. IA 583 (= K 846 = H 358). Argos.
266. IA 1099. Epidaurus.
267. 'Εφ. 'Αρχ. 1884, pp. 49/50. Epidaurus.
268. Lebas, Voyage Arch. en Grèce et en Asie Mineure 5. 38 (= K 769 = H 325). Erythrae.
269. IGS 530 (= K 938 = H 390). Tanagra.
270. BCH 24 (1900), p. 235, n. 2. Crete.
271. IP I, 2, p. 2 (= H 331).
272. AM 8 (1883), p. 23 (= H 328). Larissa.
273. CIG II, 2104 (= K 773). Pantacapaeum.
274. AM 14 (1889), p. 17 (= H 326). Delphi. Vv. 1, 3 restored by Kaibel.
275. BCH 21 (1897), p. 598. Delphi.
276. BCH 21 (1897), pp. 592 ff, n. 2–7.[190] Delphi.
277. BCH 6 (1882), p. 446 (= H 383). Delphi.
278. BCH 24 (1900), p. 171. Delphi.[191]
279. CIA II, 3. 1302 (= K 940 = H 366).
280. CIA II, 3. 1311 (= H 369).
281. IGS 2532 (= K 492 = H 179). Thebes.
282. IGS 2533 (= K 492 b, pref.) Thebes.
283. Olymp. 166 (= H 382).
284. Olymp. 164 (= K(RM) 942 a).
285. Olymp. 161. Restored by H. Förster.
286. Olymp. 293 (= K 875 a, add. = H 357).
287. IGS 2470 (= K 938 a, pref. = H 386). Thebes.
288. IGS 2462 (= K 768 a, pref. = H 356). Thebes.
289. JHS 9 (1888), p. 239. Paphos.
290. Pr 156 (Plut. Mor. 838 D).
291. Pr 157 (PLG 2, p. 329: Plut. Mor. 839 B).
292. Pr 75 (Philodemus). Restored by Boeckh.
293. Pr 143 (Pollux 4. 92).
294. Pr 88 (Stob. Flor. 1. 49. 52).
295. Pr 164 (Strabo. 10, p. 463).

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190 Although n. 7 is irregular in form, I have included it for the sake of keeping complete the series of which it is a part. See p. 33.

191 Ulrichs (Reise über Delphi, p. 43, n. 5) gives the whole inscription, which was complete when he saw it.

296. Pr 141 (Ath. 14, 629 a).
297. Pr 60 (Paus. 5. 22. 3).
298. Pr 146 (Paus. 6. 3. 14).
299. Pr 214 (= AP 9. 684).
300. Pr 99 (= AP 9. 786).
301. Pr 115 (PLG 2, p. 325 : Plut. Timol. 31).
302. Pr 158 (PLG 2, p. 326 : Suidas s. v. σαυτὴν ἐπαινεῖς).
303. PLG. Sim. 186 (= AP app. 77).
304. Pr 161 (Paus. 9. 15. 6).
305. Pr 162 (= A Pl 33).
306. Pr 163 (= PLG Arist. 4 : Di. La. 5. 7).
307. PLG Sim. 188 (= Pr 129 : Hephaest. p. 116, ed. Gaisford).
308. Pr 130 (Paus 6. 4. 6).
309. Pr 147 (Strabo 10, p. 463).
310. Pr 184 (Aristid. 2. 521).
311. Pr 181 (PLG 2, p. 320 : Ath. 12. 543 d).
312. Pr 182 (PLG 2, p. 321 : Ath. 12. 543 e).
312 a. Pr 183 (PLG 2, p. 321 : Ath. 12. 544 a).
313. AP 7. 45 (= Pr 259 : PLG 2, p. 267).
314. AP 5. 77 (= PLG Plato 1).
315. AP 5. 78 (= Plato 2).
316. AP 5. 79 (= Plato 3).
317. AP 7. 100 (= Plato 8).
318. AP 7. 256 (= Plato 9).
319. AP 7. 259 (= Plato 10).
320. AP 7. 669 (= Plato 14).
321. AP 7. 670 (= Plato 15).
322. PLG Plato 29 (Th. Magist. Vit. Aristoph., p. 160, ed. Westermann).
323. Pr 10 (Steph. Byz. s. v. Μίλητος).
324. Plut. Mor. 603 C.

The following epigrams, cited by Bergk among the poems of ancient authors, seem later than the dates to which they are attributed. In the case of some it has been enough to refer to the work of scholars whose arguments against them seem to me just; in the case of others I add some considerations which have occurred to me.

Aesop. PLG 2, p. 164. This epigram is so very unlike all early epigrams and so like later ones (e. g. AP 9. 359, 360) that it can hardly be assigned to the time of Aesop.

Archil. 17. Although formulas like γαῖ' ὑπένερθεν ἔχεις do occur in Homer[192] and Theognis,[193] yet they do not appear in epigrams till

[192] Od. O. 31, πρὶν καί τινα γαῖα καθέξει Il. II. 629, πάρος τινὰ γαῖα καθέξει.
[193] 973: ὃν πότν' ἐπὶ γαῖα καλύψῃ.

the fourth century, when they become very frequent.[194] Hence I have hesitated to assign the epigram to Archilochus. Moreover, the words ὑψηλοὺς κιόνας savor of rhetoric, and the address ὦ . . . γαῖ' suggests a later period.

Archil. 19. I have rejected this epigram because it is obviously so incomplete that it is impossible to say what was the original character of the poem to which it belonged. The content is not a reason for rejecting it.

Sappho 120. Reitzenstein (p. 107) says "μνήματα κακοζοΐας stehen nicht auf Marmor sondern im Buch." Although I do not feel certain of the truth of this, I have rejected the epigram because it gives no indication that it was intended for a tomb. See pp. 16 ff.

Demod. 3, 4. See PLG.

Anac. 113. The absence of any indication that these verses were inscribed shows that they cannot have been written at the time of Anacreon as an inscription. The style appears too ornate and elaborate for an early epideictic epigram. (Cf. PLG.) Kaibel's[195] argument that the words πατρίδος αἴης appear only here in an epigram of the sixth century does not appear to me conclusive, since the phrase is not common in any age, and there is nothing in it inconsistent with the style of the sixth century.

Anac. 115, 116. The name of Myron shows that these verses are later than the time of Anacreon, and their striking resemblance to the sort of epigram so common in the Alexandrian age[196] makes it improbable that they are contemporary with Myron himself.

Erinna 3–5. The question as to how many poets by the name of Erinna there were in early times, and the precise periods when they flourished, may be passed over here. These three epigrams at any rate were not composed before the third century. Cf. with 4 AP 9. 736 and A Pl 248, poems of the Alexandrian era. The style and the content alike are inconsistent with an early date.

Sim. 101. See Hiller, Phil. 48 (1889), p. 231, and p. 21 of this article.

Sim. 103. See Boas, p. 216, and Kaibel, RM 28, 457.

Sim. 104. See Boas, pp. 92 ff.

Sim. 106. See Hauvette and Boas, pp. 213 ff.

Sim. 107. See PLG.

Sim. 110. See Boas, p. 137, n. 103, and Hauvette.

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194 See Table III.

195 Stil u. Text d. Ἀθην. Πολ., p. 131, n. 3 (Berlin, 1893).

196 Cf. A Pl 146, 248; AP 713, 714.

Sim. 113. The vagueness of the epigram as to the identity of Kallias makes it most improbable that this epigram is fifth-century work, and the phrase εὖτ' ἂν ἴδωμαι also points to a later period.

Sim. 114. See Hauvette and p. 21 of this article.

Sim. 115, 116.[197] The style is obviously Alexandrian.

Sim. 119. See Hauvette.

Sim. 124ª. See PLG.

Sim. 127–129. See Hauvette.

Sim. 133. See PLG and Hauvette.

Sim. 136. See Boas, pp. 73, 86, and Wilamowitz, Goett. Nachr., 1897, p. 311.

Sim. 144. Bergk thinks this genuine because he considers that it was imitated by Mnasalcas (AP 6. 125, 128). Cf., however, AP 6. 124, by Hegesippus, — a poem far simpler and better than this. Moreover, we miss the name of the dedicator, an omission never found in the fifth or even in the fourth century.[198] If the writers of the later epigrams mentioned above had any model before their eyes, it may have been ep. 55, which is recalled by the epigram of Hegesippus.

Sim. 146. See Hauvette.

Sim. 154. The form of this epigram shows its late date. Cf. AP 7. 64, 79, 163, 470, 552.

Sim. 156. See Hauvette.

Plato 4–7, 11–13, 16–28, 30, 31. See PLG and Fava.

With regard to other epigrams attributed to the fifth and fourth centuries and not included in the preceding lists, see Bergk's notes. In every case the attribution rests on very uncertain evidence, or the poems themselves show plainly their late date.

Smith College
January 15, 1910.

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[197] Cf. R. Weisshäupl, Die Grabgedichte d. gr. Anthologie (Vienna, 1889), p. 102.

[198] In ep. 145 there is no dedicator's name, but the language makes it clear that the offering was made by the whole people.

TABLE I.

INFORMATION GIVEN BY INSCRIBED EPITAPHS.

Century.	Name of the dead.			Name of parent.				Age of the dead.			
	In the verses.	Outside the verses.	Omitted in the verses.	In the verses.	Outside the verses.	Omitted.	Birthplace of the dead.	Expressed by ηβη or the like.	Expressed in years.	Profession or office of the dead.	Cause of death.
VII VI	*Gen.* 2, 6, 7, 8, 11, 17 *Dat.* 3, 15?, 16 *Acc.* 1. 5, 9 *Voc.* 13?	*Gen.* 12		*Nom.* 3, 6?, 10?, 15 *Gen.* 7		1, 2, 11, 14?, 17		1, 9, 11, 14?			*War*, 1, 17
V	*Nom.* 78, 84, 88? *Gen.* 80, 82, 89, 93 *Dat.* 87, 90, 91 *Acc.* 79, 85 *Voc.* 86	*Nom.* 78, 81, 82, 84 *Gen.* 79	81[1] 83[1]	*Nom.* 87, 90 *Gen.* 78, 79, 84, 65, 86	*Nom.* 78	82, 89, 91, 93	75, 77c, 78, 79, 83, 84, 86	75, 81			*War*, 75, 77 b. c. 81, 88, 89
IV	*Nom.* 221, 224, 234, 237, 239, 241a 243, 244, 245, 249? *Gen.* 226, 227, 240, 251, 252, *Dat.* 253 *Acc.* 218, 230, 233, 235, 236,	*Nom.* 219, 221, 222, 225, 226, 227, 228, 229, 231, 232? 235, 236, 239, 241, 242, 243, 245, 253	217,[1] 225 228, 231 242 247	*Nom.* 247, 253 *Gen.* 221 224 241 249 252	*Gen.* 228, 229 239 242 253	218, 219, 220, 222, 223, 225, 226, 227, 230, 231, 233, 234, 235, 236 237, 238, 240, 243, 245, 251	218, 221, 224, 225 226 (outside 228 verses) 229 " 231 " 234 235 (outside 242 verses) 245 248	219, 221, 228, 230, 237 252	244? 246	219, 223, 225, 226, 245	*War*, 217, 233, 237 *Other causes*, 228 241 251

	238, *Voc.* 219, 220 a, b, 222, 223, 229, 241 b										

TABLE I. — (*continued*).

Century.	Praise of the dead.					Word for tomb.			Speaker.			
	For valor.	For friendship.	For other virtues.	For excellence in the arts.	For various reasons.	σῆμα or the like.	Omitted.	Site of tomb.	The dead.	The tomb.	Expression of grief.	Name of artist in the verses.
VII VI	3, 12, 17	6, 8, 10	1, 3, 8, 12, 13, 16		7, 16	2, 3, 6, 7, 8, 12, 14, 15, 17	1	5, 8	11	14, 17	9?	10
V	75, 77 a, c, 79, 81, 88	82, 86 88	78, 79, 80, 90, 93		80	82, 87, 89, 91, 93	75, 77, 78, 79, 81, 83, 84, 86	83, 84, 89, 93	83, 84	77 a, 87	77 c, 79, 85? 86, 87, 90, 92	
IV	224 229 232 233 237 247	220 b, 235	219, 220a, b, 222, 224, 225, 226, 227, 231, 235, 236, 238, 239, 241, 242, 245, 247	219, 223, 236, 245		222 a, b, 222, 235, 240, 242, 247, 251, 252, 253	217, 218, 220, 221, 223, 225, 227, 228, 229, 230, 231, 233, 234, 236, 237, 238, 239, 241, 243, 245	234 235 243 245	217 228 232 242 243 245 248		220 221 223 228 229 235 236 243 251	

[1] Name omitted because of the large number of the dead.

TABLE II.

INFORMATION GIVEN BY INSCRIBED DEDICATIONS.

Century.	Name of dedicator.			Name of father.			Birthplace of dedicator.	Profession or office of dedicator.	Name of deity.		
	In the verses.	Outside the verses.	Omitted.	In the verses.	Outside the verses.	Omitted.			In the verses.	Outside the verses.	Omitted.
VII VI	*Nom.* 22, 23, 25, 26, 27, 28?, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 44, 45, 47, 48 *Gen.* 24 *Dat.* 43 46	*Nom.* 42		*Gen.* 26, 31, 33, 37, 44		22, 23, 24, 27, 28, 32, 38?, 40, 41, 42, 46, 48		42	*Nom.* 23 *Gen.* 33, 37, 47, 48 *Dat.* 22, 25, 26, 27, 28, 29, 30, 32, 36, 38, 39, 40, 41 *Voc.* 24, 42, 43, 46		
V	*Nom.* 117, 118, 120?, 121, 122, 123, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 140, 142 *Voc.* 125	*Nom.* 135		*Nom.* 128, 134 *Gen.* 117, 121, 123, 125, 126, 127, 129, 133, 135, 136	*Gen.* 135	131, 138, 140 142	121, 126, 129, 133, 134?, 135, 138		*Dat.* 117, 119, 122, 123, 125, 126, 127, 129, 131, 140, 142 *Voc.* 121, 141		124, 128, 133, 134, 135, 138

IV	*Nom.* 261, 264?, 265, 266, 267, 268, 269, 270, 271?, 272, 273, 274, 275, 276 f, 278	*Nom.* 261		*Gen.* 268 269 270 271 272 273 *Nom.* 275	*Gen.* 261	267 276 f	261 265 273 274 276 f	276 f (outside the verses)	*Dat.* 261, 265, 268, 270, 272, 273, 275, 276 f *Voc* 262? 263, 264, 266, 267, 271, 274		260 269 278

TABLE II. — (*continued*).

Century.	Site of the offering.	Reasons for dedication.				Offering.		Offering speaks.	Recompense asked.	Verb of dedicating omitted.	Artist's name in the verses.
		Victory in battle.	Victory in games.	Fulfilment of vow.	Various.	Described.	Not described.				
VII VI	37	25	40 44	26, 30, 31, 32, 36, 48	23, 24, 35, 37, 41, 42, 46	23, 25, 26, 27, 31, 32, 36, 40, 43, 46, 48	22, 24, 33, 37, 38, 39, 42	30, 33, 34, 38, 39, 41, 42, 44	29, 38, 42	24, 43, 46	
V	142	132 138	133 134 135	120, 121, 122, 123, 130, 131, 140, 142	117, 126, 129, 136	125, 126, 132, 135, 138, 140, 141, 142	117, 121, 122, 123, 124, 127, 129, 131, 133, 134	117, 122, 129, 130, 140	121, 124, 142		
IV	260 264	274 278	265 269	275	260, 261, 264, 266, 273, 276 f	260, 261, 265, 267, 269, 270, 273, 275	268 272 276 f	260? 265	262, 264, 267, 272		268

TABLE III.

FORMULAS WHICH OCCUR MOST FREQUENTLY IN INSCRIPTIONAL EPIGRAMS.[1]

EPITAPHS.

	Σῆμ' ἐπέθηκε ὁ δεῖνα.	τοῦ δεῖνα τόδε σῆμα.	ἐνθάδε κεῖται or κεῖμαι.	γαῖα δέξατο or κατέχει.	ἦλθ' ἐπὶ Περσεφόνης θάλαμον.	ὤλεσαν ἀγλαὸν ἥβην	αἰθὴρ μὲν ψυχὰς ὑπεδέξατο, σώματα δὲ χθών.	ἐπὶ τέρμα μολόντα	ἐν προμάχοις ἔθανον	πένθος θῆκε or πέλε	πατρίδ' εὐκλέισαν
VII VI	3, 6 10, 15 16	6, 7, 8, 11, 12, 17				1				9?	
V	87, 90 91, 93	89, 93	78, 84			75, 81, 101, 109	77 b		77 c, 88, 102, 109	87, 92	77 c, 81
IV	226 252 253	251	221, 222, 227, 241, 242	218, 220 b. 225, 234, 235, 236	220 a, 229, 230, 237, 238		220 b, 222, 240	235, 238, 249		220 b, 221, 228, 251	

DEDICATIONS.

	τοῦ δεινά εἰμι	χάριν ἀντιδίδου	εὐξάμενος δεκάτην or ἀπαρχήν	ἐστεφάνωσε πολιν				ἐπὶ τέρμα μολόντα			
VII VI	33	29, 34	26, 28, 31, 32, 36, 48								
V		121	120, 121, 122, 123, 131, 142	138, 139							
S IV				269, 280, 277				281			

[1] The formulas show slight variations.

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THE REACTIONS OF EARTHWORMS TO ACIDS

By S. H. Hurwitz.

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THE REACTIONS OF EARTHWORMS TO ACIDS.

By S. H. Hurwitz.

Presented by E. L. Mark, May 11, 1910. Received May 18, 1910.

The present investigation was made to ascertain the influence of inorganic and organic acids on the responses of the common manure worm, Allolobophora foetida (Sav.). Apart from its specific application, this problem bears upon the more general one of the relation of electrolytes — acids, bases, and salts — to physiological processes. For the suggestion of this problem and for much valuable advice as to method and interpretation, I am greatly indebted to Professor G. H. Parker.

The sense of taste even in man is not well understood. What its nature in an invertebrate may be, is only a matter of conjecture. Experiments designed to determine the relation between the chemical nature of solutions of acids to the sensations of taste in man, and especially to ascertain how closely the sensation corresponds to the degree of dissociation were made by Kahlenberg ('98) and by Richards ('98). It was my purpose to ascertain to what extent the conclusions reached in a study of human taste were applicable to a typical invertebrate like the earthworm.

The modern theories of solutions lead to the conclusion that the taste of a solution of an electrolyte must depend upon the taste of its ions, its undissociated molecules, or both; furthermore, the taste of a solution in which ionization is practically complete should be due simply to its ions. At dilutions of $\frac{N}{100}$ such as were used in the present investigation, strong acids, at least, would be almost completely dissociated. And their sour taste or stimulating value has been shown both by Kahlenberg and by Richards to depend upon hydrogen ion. It was further shown by Richards that the degree to which such acids stimulate is directly determined by the number of hydrogen ions.

For the purpose of the present study, three inorganic and one organic acid were used; they were hydrochloric, nitric, sulphuric, and acetic acids. The degree of dissociation of these acids diminishes in the order named, hydrochloric acid being most and acetic acid least dissociated.

The strength of these solutions was so chosen that they should give distinct and measurable reactions, not sufficiently strong, however, to produce in any case lasting injurious effects upon the worm. Preliminary trial showed that an $\frac{N}{400}$ solution was serviceable for this purpose, since at higher concentrations the acids were so stimulating that the worms withdrew within two-tenths of a second, and, consequently, it was impossible to state whether at these concentrations one acid was a more effective stimulus than another.

The method of handling the worm was, with few modifications, like that used by Parker and Metcalf (:06) in their study of the reactions of earthworms to salts. The worms were thoroughly rinsed in tap-water till they were cleaned externally of foreign matter, and each worm was suspended by a silk thread passed through the posterior tip of its body and loosely tied. The worms thus prepared and numbered were kept singly in small, open, glass vessels lined with moistened filter paper, upon which the worms were allowed to crawl. The method of keeping them in tap-water was found undesirable because the water seemed to excite the secretions of mucus with the result that the mucus thus formed prevented the easy penetration of the acid to the skin of the worm and made the reaction-times slow and unreliable. The filter paper, on the other hand, served to remove all excess of mucus from the worms.

The apparatus with which the tests were made consisted of a base with an upright wooden post to which a pivoted arm was fixed. One of the ends of the arm was notched so that the silk thread carrying a worm could be inserted into it, and the other end was used as a handle by which the arm could be moved so as to raise the worm or lower it into the solution contained in a glass vessel on the base. When a test was to be made, a worm was taken up by its thread and attached to the arm of the apparatus. The superfluous water was drained from it, and, after it had lengthened fully, it was lowered gently, but quickly, into the solution to the depth of the anterior edge of the clitellum. As the tip of the worm cut the surface of the solution, a stop watch was started, and when as a result of the contraction of the worm, the tip withdrew from the solution, the watch was stopped. The interval of time thus recorded to fifths of a second was taken as the reaction-time for that particular experiment. If the worm failed to react after two minutes, the experiment was discontinued, and the worm was taken out of the solution. Such extremely long reactions were common when the worms were kept in tap-water, but seldom occurred when they were allowed to crawl on the filter paper.

After the worm under ordinary circumstances had withdrawn from the solution, it was rinsed in tap-water, returned to its glass, and allowed to rest about five minutes before another trial with it was made. To rule out the possible disturbing factor of fatigue, it was decided to use no worm for more than a given, arbitrarily determined, number of reactions (twelve).

The acids were first experimented with in pairs; each acid being compared with every other one; hydrochloric acid, for instance, being compared with nitric, sulphuric, and acetic acids successively. Although it is not known that the particular sequence in which the acids are used makes any material difference, nevertheless to avoid any error which might possibly creep in by passing from a more to a less stimulating acid, the solutions were used alternately; for it is highly probable that when a strong solution is first applied, the effect of it is apt to be so vigorous as to obscure a subsequent response to weaker solutions. Where all the acids were compared at once the procedure adopted was to give each acid first place with at least one worm of those tested.

In tables I to XVI will be found the reaction-times of the earthworm, Allolobophora foetida, for the four different acids used.

Tables I to XVI. — Reaction Times, in Seconds, of Allolobophora foetida to Solutions of Nitric and Hydrochloric, Sulphuric and Hydrochloric, Sulphuric and Nitric, and Hydrochloric and Acetic Acids.

TABLE I.

$\frac{N}{400}$ HNO_3.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	0.4	25.0	38.2	1.2	12.6	32.4	18.3
II.	25.2	9.4	20.0	7.6	3.2	...	13.0
III.	18.8	11.2	29.8	11.0	10.8	...	16.3
IV.	0.2	0.6	0.8	0.4	0.4	7.2	1.6
V.	4.0	2.0	15.4	28.8	1.0	13.6	10.8
	General average						12.0

TABLE II.

$\frac{N}{400}$ HCl.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	0.6	5.0	8.0	17.6	22.8	17.8	11 9
II.	7.2	11.0	27.4	11.6	0.6	. . .	11.5
III.	20.0	5.4	14.0	12.0	2.4	. . .	10.7
IV.	0.4	0.6	0.4	0.4	0.4	1.0	0.5
V.	6.4	25.0	19.0	8.0	5.0	5.2	11.4
	General average						9.0

TABLE III.

$\frac{N}{400}$ HNO_3.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	0.4	14.4	14.0	1.0	1.0	1.0	5.1
II.	0.6	0.6	0.8	0.6	0.8	33.0	6.0
III.	0.4	24.0	7.0	20.0	21.0	11.2	13.9
IV.	0.2	0.2	0.6	0.6	0.6	35.2	6.2
V.	0.6	0.4	0.6	0.2	0.6	. . .	0.5
	General average						6.3

TABLE IV.

$\frac{N}{400}$ HCl.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	0.8	0.6	1.0	1.0	0.6	...	0.8
II.	0.8	0.8	0.8	1.0	1.0	1.2	0.9
III.	24.4	21.4	30.6	16.4	21.0	23.0	22.8
IV.	0.4	0.2	0.2	0.2	0.4	1.0	0.4
V.	0.4	0.6	0.6	0.8	...	...	0.5
	General average						5.0

TABLE V.

$\frac{N}{400}$ H_2SO_4.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	33.2	23.8	13.8	29.0	30.0	34.0	27.3
II.[1]	0.8	36.0	71.0	25.0	61.6	43.0	...
III.	0.8	1.2	14.0	1.0	22.4	20.8	10.0
IV.	0.2	0.6	1.0	0.6	1.0	2.0	0.9
V.	0.2	39.4	20.6	2.6	26.2	15.0	17.3
	General average						11.1

[1] Owing to the great variation in the readings obtained in Tables V and VI with worm number II, it was decided to discard these readings in making the general average.

TABLE VI.

$\frac{N}{400}$ HCl.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	26.2	0.2	1.2	31.0	40.0	51.6	25.0
II.[1]	54.0	62.8	65.2	34.0	0.4	31.0	...
III.	1.0	0.4	1.0	0.2	0.4	0.2	0.5
IV.	0.2	0.4	0.4	0.2	0.8	0.2	0.3
V.	1.0	0.2	0.4	1.0	0.6	16.0	3.2
	General average						5.8

TABLE VII.

$\frac{N}{400}$ H_2SO_4.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	7.0	30.8	0.6	22.4	26.4	8.2	15.9
II.	0.4	4.0	3.2	1.0	1.8	25.0	5.9
III.	0.4	0.6	0.4	1.0	2.0	8.4	2.1
IV.	25.4	0.6	22.8	5.0	0.4	0.8	9.1
V.	18.6	17.6	22.2	10.2	11.0	14.6	15.7
	General average						9.7

TABLE VIII.

$\frac{N}{400}$ HCl.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	14.6	0.4	0.6	8.4	19.0	17.0	10.0
II.	0.2	1.0	0.6	1.0	4.8	40.4 [1]	1.5
III.	0.4	0.4	0.2	0.4	0.8	2.2	0.7
IV.	0.4	22.8	15.0	2.4	23.2	22.0	14.3
V.	0.2	0.2	17.6	8.0	16.6	33.6 [1]	8.5
	General average						7.0

[1] The readings marked [1] show a great variation from the readings recorded in the other five tests. In the general average they were not included.

TABLE IX.

$\frac{N}{400}$ H_2SO_4.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	1.6	19.4	46.2	52.4	30.2	39.0	31.4
II.	21.0	8.2	20.0	16.2	8.0	32.0	17.5
III.	14.8	3.2	16.0	17.2	9.0	33.0	15.0
IV.	13.0	18.0	1.0	12.0	2.0	0.6	7.7
V.	15.2	18.2	16.2	0.2	0.2	. . .	10.0
	General average						16.4

TABLE X.

$\frac{N}{400}$ HNO_3.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	1.2	30.0	54.0	39.6	36.0	67.6	38.0
II.	19.2	22.0	31.0	19.2	13.0	40.0	24.0
III.	22.6	4.0	9.0	21.0	18.2	21.0	15.9
IV.	2.4	0.6	0.4	1.4	2.0	11.2	3.0
V.	0.2	17.0	11.4	0.4	0.4	. . .	5.9
	General average						17.3

TABLE XI.

$\frac{N}{400}$ H_2SO_4.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	17.0	12.0	8.2	11.6	33.8	11.6	15.7
II.	20.0	4.2	5.2	12.0	7.0	18.8	11.2
III.	19.0	15.4	12.0	0.6	1.0	. . .	9.6
IV.	12.0	27.4	48.2	20.4	11.0	47.0	27.6
V.	11.6	15.0	13.8	7.6	0.2	. . .	10.2
	General average						14.8

TABLE XII.

$\frac{N}{400}$ HNO_3.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	11.0	2.2	16.0	7.0	14.0	23.0	12.2
II.	20.8	7.0	1.2	5.2	11.0	2.0	7.8
III.	15.2	0.4	15.8	0.6	0.4	...	6.4
IV.	20.6	11.4	17.0	22.6	49.0	42.0	27.1
V.	19.4	0.8	15.0	11.0	4.6	...	10.1
	General average						12.7

TABLE XIII.

$\frac{N}{400}$ HCl.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	0.6	1.0	0.6	0.4	4.0	0.2	1.1
II.	22.0	57.6	66.0	47.2	70.0	66.2	54.8
III.	0.2	0.8	0.2	0.8	2.0	1.0	0.8
IV.	0.6	4.0	0.6	1.2	21.2	13.0	6.7
V.	0.6	0.4	0.6	0.2	0.6	...	0.5
	General average						12.8

TABLE XIV.

$\frac{N}{400}$ $C_2H_4O_2$.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	19.4	15.0	22.4	23.6	9.0	3.6	15.5
II.	62.4	78.8	75.6	55.0	29.6	71.6	62.1
III.	0.8	3.0	27.6	2.6	26.4	5.4	10.9
IV.	33.0	10.6	25.0	11.8	7.4	31.0	19.8
V.	20.8	1.0	8.4	5.4	0.8	...	7.3
	General average						23.1

TABLE XV.

$\frac{N}{400}$ HCl.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	0.4	0.4	0.4	0.4	0.2	0.4	0.4
II.	1.0	30.6	10.8	0.6	0.2	...	8.6
III.	0.6	0.4	21.4	0.4	1.0	19.4	7.2
IV.	0.4	1.0	11.0	28.4	...	...	10.2
V.	1.2	0.8	0.4	0.6	0.6	3.2	1.1
	General average						5.5

TABLE XVI.

$\frac{N}{400}$ $C_2H_4O_2$.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	4.0	1.0	5.2	5.0	1.6	57.8	12.4
II.	18.0	9.0	14.4	1.2	0.6	...	8.6
III.	44.4	16.8	22.0	2.0	...	29.8	23.0
IV.	1.4	17.2	5.6	...	...	...	8.0
V.	38.8	13.0	13.6	29.6	20.0	22.4	22.9
	General average						14.9

It will be seen by inspecting the tables that each solution was as a rule used on each one of the five worms six times. The worms were tested in such sequence that when the reaction-times for each acid were averaged the factor of exhaustion was so distributed as to be non-cumulative for any class.

Notwithstanding the care exercised in keeping the experimental conditions as nearly uniform as possible, it will be seen that the various worms present striking differences; some exhibiting characteristically slow, others quick reactions.

The slow reaction was in the main found associated with two modes of behavior, unlike each other. In not a few of the cases recorded, the portion of the worm anterior to the clitellum would rapidly contract as soon as it cut the surface of the solution, thus exposing a smaller receptive area to the action of the acid than would otherwise have been done. In such cases the worm was probably less stimulated and therefore would naturally be slower in withdrawing from the stimulating solution. In many of the slow reactions, the behavior was directly opposite of that just described. After cutting the surface of the solution, the anterior end of the worm would greatly elongate and squirm about, as if testing its environment in all directions. Such random movements executed in an endeavor to escape the stimulus are described by Jennings (:06) for the earthworm. A worm crawling upon a flat surface might escape an irritating stimulus with which it came in contact anteriorly, either by forward movement in a direction different from that in which it had been moving, or by a backward contraction. When in a suspended condition, however, escape from a stimulating solution can be effected only by a contraction. It is highly probable, however, that the anterior elongations and squirming of the worm observed in some slow reactions corresponds to the effort of a crawling worm to escape an irritation by moving forward in a new direction. This ineffective forward reaction most likely delayed the backward one and so lengthened the reaction time. Although the quick reaction is the more characteristic one, the general average of slow and quick reactions may be taken as a fair measure of the reaction-time.

Assuming that these reaction-times are indicative of the degree of stimulation, it is clear from the figures in Tables I to XVI that hydrochloric acid is more stimulating than nitric, sulphuric, or acetic acids, but beyond this it is difficult to go. This method was therefore abandoned for one that gave through a greater number of worms a more immediate comparison of the different acids.

In this second set of observations, twenty-four instead of five worms were tested in all four acids and their relative reaction times were recorded in the manner previously described. The tests were so made that each acid had first place with some worm. Thus the possibility of error due to always passing from a stronger to a weaker acid with a given worm was eliminated. The results are recorded in the following table.

By inspecting this table, it will be seen that the order of acids arranged according to their stimulating value is hydrochloric, nitric, sulphuric, and acetic. Furthermore, that the reaction times for hydrochloric and nitric are practically identical, while that for sulphuric is nearly twice that for hydrochloric or nitric acids. The stimulating value of acetic acid, according to these results, is about half that of sulphuric and scarcely a third of that of hydrochloric and nitric acids.

These last conclusions differ from those of Kahlenberg ('98) for sensations of taste in man, in that at $\frac{N}{400}$ he was unable to distinguish between hydrochloric, nitric, and sulphuric acids. As Table XVII shows, the earthworm, on the contrary, distinguishes between these acids; and since in each solution the hydrogen ions are the stimulating elements, it must follow that the difference in reaction is due to a difference in concentration of these ions.

As compared with other acids, solutions of acetic acid are peculiar in that their sour taste is more intense than would be expected from their degree of dissociation. Kahlenberg, for instance, found that a $\frac{N}{200}$ solution of acetic acid, being dissociated only about 6 per cent, has a sour taste about four times as strong as it would be expected to have, assuming that the taste is due simply to the hydrogen ions momentarily present. Richards obtained a similar result, in that he found that the acetic acid was about one-third as sour as an equivalent solution of hydrochloric acid, though the acetic acid was only dissociated to the extent of one-fourteenth as much as the hydrochloric acid was.

To determine whether this discrepancy existed also for the earthworm, the following test was made. Two solutions were prepared, an $\frac{N}{400}$ solution of acetic acid and a weaker solution of hydrochloric acid. Both solutions, however, contained equivalent numbers of hydrogen ions. Worms were now tested in these two solutions and the results are recorded in Tables XVIII and XIX.

It is clear from these tables that the earthworm, like man, is stimulated by acetic acid more vigorously than should be expected in accordance with the dissociation hypothesis. The explanation for this peculiarity cannot be given. It is not unlikely, however, that it may be ascribed to the undissociated molecules of acetic acid which in a way serve as a reverse for hydrogen ions.

TABLE XVII.

Reaction-Times, in Seconds, of Twenty-four Worms to Solutions of Hydrochloric, Nitric, Sulphuric, and Acetic Acids.

	No. of Worm.																								Average.
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	
HCl	.4	.6	51.0	.2	1.8	8.0	.4	.6	.4	.4	.2	19.6	.2	12.6	.2	.6	.2	.2	1.0	.2	.8	11.6	4.2	.4	4.7
HNO_3	.4	.4	32.4	.8	.4	11.4	. .	13.4	3.2	.4	.4	12.4	2.8	.2	.2	22.0	.2	.8	1.2	.4	.4	1.4	1.2	.2	4.8
H_2SO_4	20.4	.6	22.4	8.4	5.2	2.8	.2	15.4	7.0	.6	.4	. .	3.4	13.2	.6	19.0	.4	15.0	.6	.6	17.2	11.2	6.4	.4	7.5
$C_2H_4O_2$	30.0	18.0	18.2	20.4	21.0	8.0	19.2	31.2	7.4	2.2	5.4	18.6	16.8	. .	10.0	3.2	25.0	3.0	.8	14.0	16.0	25.0	13.6	.6	14.3

TABLE XVIII.

Reactions of Allolobophora foetida to Solutions of Hydrochloric and Acetic Acids containing Equivalent Numbers of Hydrogen ions.

$HCl \rightleftharpoons \frac{N}{400}\ C_2H_4O_2$.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	7.0	23.6	28.0	19.0	21.2	18.0	19.4
II.	17.2	15.0	19.0	18.0	8.8	18.2	16.0
III.	16.4	27.0	14.0	19.4	13.0	10.0	16.6
IV.	12.0	19.6	16.0	19.4	15.0	12.4	15.7
V.	12.6	11.2	11.0	7.4	16.6	13.0	11.9
	General average						15.9

TABLE XIX.

$\frac{N}{400}\ C_2H_4O_2$.

No. of Worm.	Trials.						Averages.
	1	2	3	4	5	6	
I.	23.2	17.0	27.0	11.0	18.2	15.0	18.5
II.	1.8	1.0	1.8	1.8	11.4	28.0	7.6
III.	16.2	6.2	9.0	10.0	17.0	19.4	12.9
IV.	5.6	7.6	4.4	19.4	13.0	15.0	10.8
V.	7.0	12.2	11.4	2.2	8.0	10.0	8.4
	General average						11.6

Summary.

1. The responses of the earthworm Allolobophora foetida to solutions of acids may be ascribed to the hydrogen ions that they contain.

2. The reaction-time of the earthworm depends upon the number of hydrogen ions present in the solution of the acid.

3. Using the reaction-time as a basis, the earthworm was found to discriminate more certainly than man between solutions of acids at a concentration of $\frac{N}{400}$.

4. The response of the earthworm to solutions of acetic acid was more active than would have been anticipated from the degree of dissociation of this acid, and in this respect the earthworm's reactions are in agreement with human sensations as worked out by Kahlenberg, and by Richards.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE MAGNITUDE OF AN ERROR WHICH SOMETIMES AFFECTS THE RESULTS OF MAGNETIC TESTS UPON IRON AND STEEL RINGS.

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THE MAGNITUDE OF AN ERROR WHICH SOMETIMES AFFECTS THE RESULTS OF MAGNETIC TESTS UPON IRON AND STEEL RINGS.

By B. Osgood Peirce.

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The theory of the magnetic properties of a homogeneous ring of iron or steel uniformly wound about by turns of insulated wire through which a steady current of electricity can be made to pass, was first investigated by Kirchhoff, who showed that the intensity of the magnetic field in the metal which thus forms the core of a ring solenoid, must be inversely proportional to the distance from the axis of revolution of the ring. He computed the mean value of the field in a ring of rectangular cross section,[1] and pointed out the advantages which rings offer for measurements of the magnetic permeabilities of the metals of which they are made. The next year, Stoletow, working under the advice of Kirchhoff, took up the subject practically in the Physical Laboratory of the University of Heidelberg and in 1872 published the results of a long series of experiments upon a ring forged from a wrought iron rod. In 1873 appeared an account of the important work of Rowland, begun three years before, on rings (toroids) of circular cross section, made of various kinds of iron and steel, and since that time countless measurements of permeability have been made by many observers[2] upon iron and steel rings; and when these rings

[1] Gustav Kirchhoff, Pogg. Ann. Ergzbd. **5**, 1 (1870). Gesammelte Abhandlungen, 223; A. Stoletow, Pogg. Ann. **146**, 442 (1872); H. Rowland, Phil. Mag. **46**, 140 (1873); **48**, 321 (1874); C. Bauer, Wied. Ann. **11**, 349 (1880).

[2] J. A. Ewing, Magnetic Induction in Iron and other metals; G. vom Hofe, Wied. Ann. **37**, 482 (1889); H. Lehmann, Wied. Ann. **48**, 406 (1893); A. von Ettingshausen, Wied. Ann. **8**, 554 (1879); H. du Bois, Magnetische Kreise, **110** *et seq.* Berlin, 1894; L. Boltzmann, Anz. d. Wiener Akademie, 203 (1878); J. Sauter, Wied. Ann. **62**, 85 (1897); L. Mues, Inaug. Diss. Greifswald (1893); I. Schuetz, Journal f. Mathematik, **113**, 161 (1894); Carl Neumann, Ueber Ring Potentiale, Halle (1864).

have been turned out of masses of solid metal, and not forged up and welded from bars, the results have usually been satisfactory. The value of the mean intensity of the magnetic force within the mass of a ring of circular cross section was given without proof by Bauer in 1880; a proof was printed by Lehmann in 1893, and an interesting diagram based on the formulas of Kirchhoff and Bauer, and showing the ratio of the mean magnetizing force to the value of the force at the mean radius for rings and toroids of different relative dimensions, was given by Morton[3] in the Bulletin of the Bureau of Standards for February, 1909.

In determining the permeability of an iron ring it is usual to demagnetize the metal as thoroughly as possible at the outset, and then, either by the "Method of Ascending Reversals" or the "Step-by-step Method" to determine for each of a number of values of the magnetomotive force, the whole flux of magnetic induction through the ring. The ratio (B') of this flux to the area of the cross section of the ring is then plotted against the mean value of the magnetic force in the metal to get an HB diagram for the given magnetic journey of the iron. It is clear, however, as the earliest workers in this field saw, that the process here described is only approximately exact, for the induction often has very different values at the points of the ring nearest the axis of revolution and at those farthest away from it. Indeed, in a ring of soft iron of the dimensions of the specimens employed in a well-known form of commercial testing apparatus used in Europe, the value of B at points on the inner edge of the ring when the average value of the force in the metal is unity, may be as high as 2,000, while the value at points at the outer surface is only 700. In this case there is a considerable difference between the average value (B') of the flux in the metal and the real value (B'') of B at points of the ring where H has the average value. For relatively slender rings and fairly high excitations the discrepancy is not so great, and various attempts have been made to estimate its amount beforehand for materials of different kinds. It sometimes happens, however, that one has at command only a small piece of the iron to be tested, and it becomes necessary to make the measurements upon a relatively stout ring not much larger than a finger ring, as Dr. A. Campbell of the National

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[3] J. A. Moellinger, Elect. Zs., **22**, 379 (1901); B. Soschinski, Elect. Zs., **24**, 292 (1903); J. W. Esterline, Proc. Am. Soc. for Testing Materials, **3**, 288 (1903); R. Richter, Elect. Zs., **24**, 710 (1903); R. Edler, Mitt. Techn. Gewerbe-Museums, Vienna, **16**, 67 (1906); M. G. Lloyd, Bulletin of the Bureau of Standards, Feb. (1909).

Physical Laboratory, Teddington, Middlesex, England, has so successfully done. If in such a case great accuracy is required, the work has to be carried out with considerable care and some attention has to be paid to the fact that there is a real, if usually small, difference between the value of B corresponding to the mean H, and the mean value of B.

I have had occasion of late to determine the permeability of a small ring of extremely pure soft iron, and have found it helpful to compute by the aid of accurate HB diagrams, previously made for two or three different kinds of iron and steel in the form of long rods, what the discrepancy $(B' - B'')$ would be for these materials at different excitations, if they were made into rings of the dimensions of the one I was compelled to use. This paper gives some results which seem instructive, for a very soft kind of Norway wrought iron and for a specimen of Bessemer steel fairly typical of what one meets with in practice.

The straight rods used were magnetized and demagnetized in a uniform solenoid about five meters long, consisting of 20904 turns of well-insulated wire wound on a stout, solid-drawn brass tube through which a stream of tap water could be kept running about the rod to prevent any sensible rise of temperature. The axis of the solenoid was horizontal and perpendicular to the meridian. The flux of induction in the rods was measured by means of a test coil of fine wire wound on the rod at its centre. This coil was protected by rubber tape and its leads were insulated from the water by rubber tubes of fine bore slipped over them. The ballistic galvanometer employed had a period so long[4] that no detectable error was introduced into the readings by the fact that a measurable time was needed to make the magnetic changes incident to a reversal of current in the solenoid. The rods were demagnetized by means of a long series of currents in the solenoid, alternating in direction and gradually decreasing in intensity; and the fact that this process was successful showed that the rods were practically homogeneous throughout. The rods were so long that the corrections for the ends, as given by du Bois or by Shuddemagen,[5] were very small.

Tables I and II give the results of determinations[6] of corresponding values of H and B made by the method of ascending reversals by Mr. John Coulson and myself. A number of diagrams were obtained for each rod to make sure that the rather elaborate apparatus for demagnetizing the specimens was effective and that the metal was practically

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[4] Peirce, These Proceedings, **44**, 283 (1909).

[5] C. L. B. Shuddemagen, These Proceedings, **43**, 183 (1907).

[6] Peirce, American Journal of Science, **27**, 273 (1909).

homogeneous throughout, and although the larger values of B are given in the tables rather more exactly than the observations warrant, the slow-moving ballistic galvanometers employed permitted of very accurate measurements of the flux changes in the testing coil.

After a good HB diagram, accurately drawn on a large scale, has been obtained for a given kind of iron or steel, it is possible to find out how nearly the mean value of the magnetic induction in a given ring

TABLE I.

Annealed Norway Iron Rod 1.25 *cms. in Diameter, magnetized in a Uniform Solenoid about five Meters long. Results obtained by the Method of Reversals.*

H.	*B.*	*H.*	*B.*	*H.*	*B.*
0	0	4.0	11280	16.	15700
0.2	90	4.5	11980	18.	15900
0.4	260	5.0	12560	20.	16040
0.5	395	5.5	13000	25.	16320
0.6	570	6.0	13400	30.	16520
0.8	1125	6.5	13700	35.	16740
1.0	2150	7.0	13900	40.	16920
1.2	3160	7.5	14100	45.	17100
1.4	4140	8.0	14300	50.	17220
1.5	4600	8.5	14490	60.	17450
1.6	5200	9.0	14660	70.	17630
1.8	5825	9.5	14800	80.	17820
2.0	6600	10.	14940	90.	18020
2.5	8240	11.	15100	100.	18210
3.0	9480	12.	15360	105.	18300
3.5	10460	14.	15540		

made of this material would differ from the real induction corresponding to the mean value of the field in the metal, for any given excitation. Suppose, for example, that the ring is to be a toroid and that the radius of the circular cross section is to be a, while the centre of the section is distant c cms. from the axis, OY, of revolution of the ring. Suppose that the excitation is to be such as to make the value of H, at points distant c from OY, H_c, then the value of H at a point P (Figure 1) is $H_c \cdot c/(\mathrm{OP})$. Let the numerical value of this quantity be computed for say $n + 1$ points evenly dividing the space WV, and let the numerical values of B corresponding to these values of H be read with the help of a lens from the HB diagram. Let P represent one of the points of division, let y represent the product of the value of B corresponding to the value of H at P, and the width, ST, of the ring at P, and let a curve be drawn with the y's as ordinates and the OP's as abscissas. The ratio of the area under this curve, — obtained by the help of a good Amsler's planimeter, — to πa^2, gives the mean value (B') of the magnetic induction in the ring. The average value of the field (H) is $\frac{2cH_c}{a^2}(c - \sqrt{c^2 - a^2})$ and the value (B'') of the induction corresponding to this value of H can be found from the HB diagram.

TABLE II.

Rod of Bessemer Steel 1.25 cm. in Diameter, magnetized in a Uniform Solenoid about five Meters long. Results obtained by the Method of Reversals.

H.	*B.*	*H.*	*B.*	*H.*	*B.*
0	0	4.5	4830	40.	15200
0.2	40	5.0	5700	50.	15720
0.4	90	5.5	6410	60.	16120
0.5	115	6.0	7060	70.	16460
0.6	145	6.5	7650	80.	16750
0.8	210	7.0	8130	90.	17000
1.0	280	7.5	8600	100.	17220
1.2	354	8.0	9040	120.	17640
1.4	426	8.5	9420	160.	18240
1.5	465	9.0	9760	200.	18800
1.6	505	9.5	10070	240.	19200
1.8	610	10.0	10390	270.	19620
2.0	730	10.5	10650	300.	19800
2.5	1180	11.	10890	350.	20240
3.0	1935	15.	12400	400.	20660
3.5	2800	20.	13360		
4.0	3760	30.	14480		

An illustration may help to make the details of the process more intelligible. Consider a toroid of the Norway iron, the circular cross section of which has a radius of one centimeter and the mean radius (c) of which is 7 centimeters. If the excitation is to be such that the value of the magnetic field at C is unity, the values of H at points distant 6, 6.1, 6.2, 6.4, 6.6, 6.8, 7.0, 7.2, 7.4, 7.6, 7.8, 8 cms. respectively from the axis of revolution of the ring, are 1.167, 1.147, 1.129, 1.094, 1.061, 1.030, 1.000, 0.972, 0.946, 0.921, 0.897, 0.875, and the values of B which correspond to these as determined from the HB diagram for the iron, are 2995, 2895, 2810, 2635, 2460, 2310, 2150, 2005, 1870, 1745, 1620, 1505. If 2895 be multiplied by the thickness of the ring at a distance of 6.1 cms. from the axis (OY), 2810 by the thickness at a distance of 6.2 cms. from OY, etc., a curve of the form KNQ shown in Figure 1 will be obtained. This curve was actually laid down on a large scale by the help of a needle point on a sheet of good coördinate

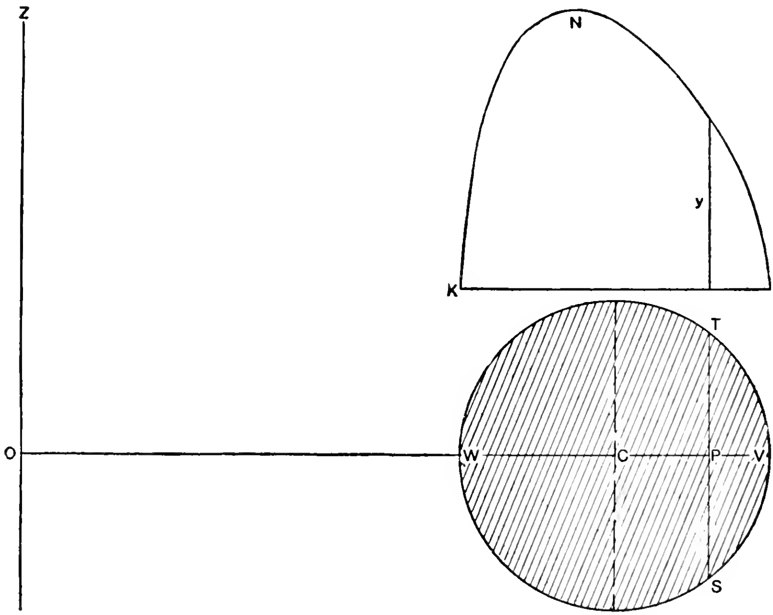

FIGURE 1.

paper, and the area under it was determined to be 6816, though the last significant figure is not determined. This divided by πa^2 gives 2170 as the mean value (B') of B in the ring. The mean value of H in the ring is 1.0052 and the value (B'') of B which corresponds to this is 2176. Although these results have been obtained with great care, they cannot of course be assumed to be quite correct; but it appears to be true that the error in this case is not very large.

The corresponding process in the case of a ring with rectangular cross section is much simpler and the results are more trustworthy, for the ring has a uniform thickness and the curve which bounds the nearly trapezoidal area to be measured often has so slight a curvature that the application of some form of Simpson's Rule may be made to yield a result much more accurate than a planimeter can be expected to furnish.

In the case of such a ring, as appears from the last two columns of Tables III, IV, and V, B' is usually a trifle larger than B'', for very small values of the mean H in the iron, but is equal to it for a single somewhat larger value. Then, with increasing values of H, B' is a trifle

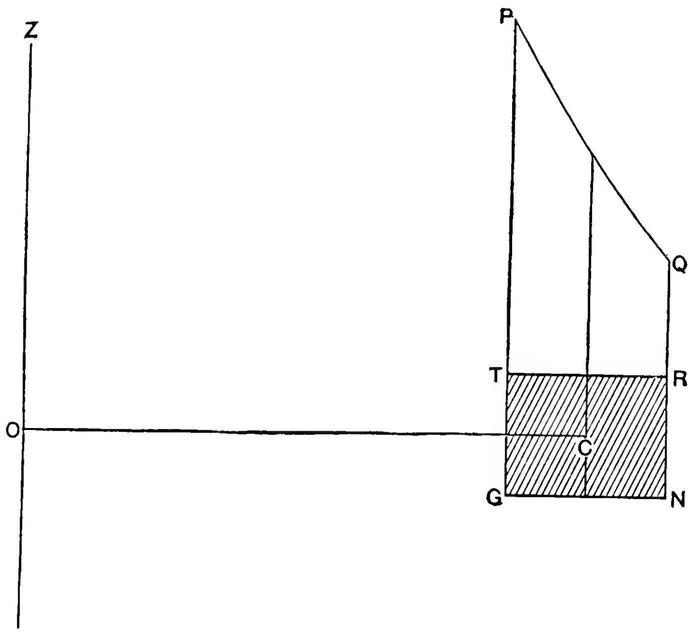

FIGURE 2.

smaller than B''; but the ratio B'/B'' soon approaches unity from the under side, and, for high excitations, is sensibly equal to one. It is evident, however, that the form of B'/B'' as a function of the average value of H in the ring must depend upon the dimensions of the latter as well as upon the magnetic properties of the material of which the ring is made.

If a ring of rectangular cross section, of the same inner and outer diameters as the toroid just described, be made of the Norway iron, and if the excitation be made such that the average value of the magnetic field in the metal at the centre (C) of the cross section is unity, the values of H and B already found may be used to draw the curve PQ, Figure 2. The area under this curve as computed by Simpson's

TABLE III.

Ring of the Pure Annealed Norway Iron. (Rectangular Cross Section. Inner Radius, 2 a; Outer Radius, 3a.)

H_0.	H_m.	H_1.	B_0.	B_m.	B_1.	H'.	B'.	B''.
0.5	0.4	0.33	400	280	190	0.405	267	255
1.0	0.8	0.67	2150	1140	710	0.811	1235	1195
1.5	1.2	1.00	4600	3155	2150	1.216	3435	3230
2.0	1.6	1.33	6600	5020	3835	1.622	5090	5130
3.0	2.4	2.00	9480	7950	6600	2.433	7970	8040
4.0	3.2	2.67	11280	9870	8660	3.244	9910	9950
6.0	4.8	4.00	13400	12350	11280	4.866	12340	12410
8.0	6.4	5.33	14300	13630	12880	6.488	13610	13680
10.0	8.0	6.67	14940	14300	13760	8.110	14310	14330

H_0 and B_0 are the values of the magnetic force and of the induction at the inner surface of the ring; H_1, B_1 and H_m, B_m the values of the same quantities at the outer surface of the ring and at the mean radius, respectively. H' is the mean value of the magnetic field in the steel. B' is the mean value of the induction in the ring as obtained by mechanical integration from a diagram of ascending reversals for the steel, and B'' is the value of the induction corresponding to H' as shown by the same diagram. The table shows the error made by using, B'/H' instead of the exact value B''/H' for the permeability corresponding to H'.

TABLE IV.

Thinner Ring of the Annealed Norway Iron. (Rectangular Cross Section. Inner Radius, 4 a; Outer Radius, 5 a.)

H_0.	H_m.	H_1.	B_0/B_2.	B.	H'.	B'.	B''.
0.62	0.555	0.50	1.59	405	0.558	502	490
0.94	0.833	0.75	1.90	1275	0.837	1325	1310
1.25	1.111	1.00	1.58	2702	1.116	2728	2740
2.50	2.222	2.00	1.25	7365	2.231	7386	7404
3.75	3.333	3.00	1.15	10130	3.347	10140	10150
6.25	5.555	5.00	1.08	13060	5.578	13070	13080
10.00	8.888	8.00	1.04	14600	8.924	14610	14620

TABLE V.

Ring of the Bessemer Steel. (Rectangular Cross Section. Inner Radius, 2 a ; Outer Radius, 3 a.)

H_0.	H_m.	H_1.	B_0.	B_m.	B_1.	B_0/B_1.	H'.	B'.	B''.
1.0	0.8	0.67	280	201	170	1.65	0.811	209	205
2.0	1.6	1.33	725	502	385	1.88	1.622	521	510
3.0	2.4	2.00	1920	1060	725	2.65	2.433	1150	1100
4.0	3.2	2.67	3760	2260	1360	2.77	3.244	2365	2330
6.0	4.8	4.00	7060	5350	3760	1.88	4.866	5370	5470
8.0	6.4	5.33	9030	7530	6190	1.46	6.488	7550	7630
10.	8.0	6.67	10490	9030	7800	1.35	8.110	9060	9130
20.	16.	13.33	13360	12660	11830	1.06	16.219	12630	12710

Rule appears to be 2185, and the value of B corresponding to the average value 1.0069 of H is also 2185, so for these dimensions and for this particular excitation, the error represented by $B' - B''$ seems non existent.

For an excitation great enough to make the value of H at the mean radius 2, a process similar to that just described shows that B' would be 6630 and B'', 6650; but if H_c were made 5, the value of B' would be 12560 and B'', 12590. The difference in this instance is less than one quarter of one per cent of either quantity and lies within the limits of error of most magnetic measurements made upon ring specimens. For work that must be very accurate, rings much thinner than this one — in which the ratio of the outer radius to the inner radius is 4/3 — are usually employed, and the error is then practically negligible for almost all excitations.

If magnetic measurements are to be made upon rings of the dimensions sometimes used in practical permeameters, the errors arising from the difference between B' and B'' become relatively important as appears from Tables III and V, where the results for two practical cases are given. A comparison of Tables III and IV will show how fast the error decreases when the ring is made thinner.

My thanks are due to the Trustees of the Bache Fund of the National Academy of Sciences for the loan of apparatus.

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NOTE ON KIRCHHOFF'S LAW.

By Griffith C. Evans.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

NOTE ON KIRCHHOFF'S LAW.

By Griffith C. Evans.

Presented by George W. Pierce. Received May 27, 1910.

Kirchhoff's Law in its simple statement says that the energy radiated at a given temperature divided by the absorption coefficient for that temperature is the same for all bodies of the same shape and size, that is, equal to the energy radiated by a perfectly absorbing body of the same shape and size at that temperature. The relation is easily shown to hold for total radiation, and depends merely on the First and Second Laws of Thermodynamics. But for separate wave lengths, a reduction of the law to a firm thermodynamic basis, though often attempted,[1] seems not to have been completely obtained. Unwarrantable assumptions, sometimes tacit, seem unavoidably to have been introduced.

The present paper, in order to show the nature of the assumptions that must be introduced, attempts to set up a system of assumptions from which the law will rigidly follow.

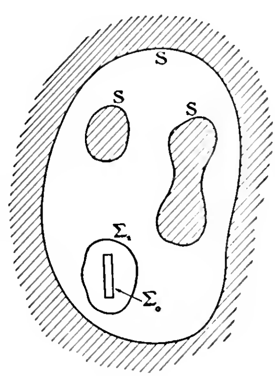

Figure 1.

Pursuing in the main a method of proof due to E. Pringsheim,[2] we take now a body S, in which there is a cavity (see Figure 1). This body need not be all in one piece nor need the cavity be filled homogeneously with one medium. Suppose, however, that there is in the cavity a simple closed surface Σ_1, enclosing another closed surface Σ_0 and not touching it at any point. The space between Σ_0 and Σ_1 is supposed filled with a medium

[1] For such considerations see H. Kayser: Handb. der Spectroscopie, II, 13 ff.

[2] E. Pringsheim: Einfache Herleitung des Kirchhoffschen Gesetzes, Verh. deutsche Phys. Ges. **3**, 81. Also Kayser: Spectroscopie, p. 37.

For a criticism of this proof see M. P. Rudski: Zeitschr. f. Wiss. Photog. **3**, 217, 1905. A reply by Pringsheim, ibid. p. 281.

that is fixed rigidly with Σ_1 wherever in the cavity Σ_1 may be. Σ_0 is fixed rigidly in this medium. We shall regard Σ_0 as a slab whose dimensions are small in regard to the other dimensions of the cavity, and whose thickness is small in comparison to its width and height. This surface Σ_0 we consider filled with various substances; we may have it so that the two faces consist of different materials. Let us denote the two surfaces by σ_1 and σ_2, each of area σ.

We have two quantities of energy to consider in regard to Σ_0, that which passes out of Σ_0, and that that passes in. The amounts of energy that in unit time pass out of Σ_0 at wave lengths between $\lambda = a$ and $\lambda = b$ through the two faces σ_1 and σ_2 we can denote respectively by

$$\sigma \int_a^b E_{\sigma_1} (\lambda)\, d\lambda + H_{21} \Big]_a^b \tag{1}$$

and

$$\sigma \int_a^b E_{\sigma_2} (\lambda)\, d\lambda + H_{12} \Big]_a^b \tag{2}$$

where $E_{\sigma_1}(\lambda)$ and $E_{\sigma_2}(\lambda)$ are functions of the substance of the element of surface, of its temperature, and of the nature of the medium in which it is immersed, but not of the temperature or position of other bodies in its neighborhood. $H_{21}\Big]_a^b$ and $H_{12}\Big]_a^b$ are two functions that depend on the surrounding conditions and approach zero as Σ_0 is removed from the influence of other radiating bodies; they represent the amounts of energy at wave lengths between $\lambda = a$ and $\lambda = b$, transmitted through Σ_0 from σ_2 to σ_1 and from σ_1 to σ_2 respectively. We shall denote the total amounts of energy transmitted by H_{21} and H_{12}. These assumptions that we have made in regard to $E_{\sigma_1}(\lambda)$ and $E_{\sigma_2}(\lambda)$ constitute essentially Prevost's Law of Exchange.

The amounts of energy at wave lengths between $\lambda = a$ and $\lambda = b$ that in unit time reach Σ_0 at the two faces σ_1 and σ_2 from outside, we can denote respectively by

$$\sigma \int_a^b e_{\sigma_1}(\lambda)\, d\lambda \tag{3}$$

and

$$\sigma \int_a^b e_{\sigma_2}(\lambda)\, d\lambda, \tag{4}$$

and the total amounts of energy that in unit time enter Σ_0 through the two faces σ_1 and σ_2 respectively by

$$\sigma \int_b^\infty A_1(\lambda)\, e_{\sigma_1}(\lambda)\, d\lambda + H_{12} \tag{5}$$

and

$$\sigma \int_b^\infty A_2(\lambda)\, e_{\sigma_2}(\lambda)\, d\lambda + H_{21}. \tag{6}$$

$A_1(\lambda)$ and $A_2(\lambda)$ are called absorption coefficients and denote functions that depend on the nature and temperature of σ_1 and σ_2, and possibly on the temperature of the body S. The existence of such functions is assumed. In the physical case the functions E, e, and A seem to be continuous in λ. And accordingly although it is not strictly necessary for the development of the proof, we shall make this assumption. We assume also that the functions E, e, and A are positive or zero at every λ, and that the functions A are $\leqq 1$.

Before seeking to obtain relations among the several kinds of energies, it is well to consider more in detail the nature of the functions $e_{\sigma_1}(\lambda)$ and $e_{\sigma_2}(\lambda)$. Let us denote by $\sigma \int_0^\infty e_1(\lambda)\, d\lambda$ the amount of energy that would reach σ' — the section of Σ_0 half way between σ_1 and σ_2 — from the σ_1 side, and $\sigma \int_0^\infty e_2(\lambda)\, d\lambda$ the amount of energy that would reach σ' from the σ_2 side, if Σ_0 were filled merely with the same medium that occupies the space between Σ_0 and Σ_1.

Let

$$e_{\sigma_1}(\lambda) = e_1(\lambda) + \delta_{\sigma_1}(\lambda) \tag{7}$$

$$e_{\sigma_2}(\lambda) = e_2(\lambda) + \delta_{\sigma_2}(\lambda). \tag{8}$$

Now as we change the slab Σ_0 in size, the functions $\delta_{\sigma_1}(\lambda)$ and $\delta_{\sigma_2}(\lambda)$ will change. If the body Σ_0 is small, the energy that reaches it will depend little on its own influence,[3] and $\delta_{\sigma_1}(\lambda)$ and $\delta_{\sigma_2}(\lambda)$ will be small. We make the assumption that as Σ_0 approaches zero, keeping the same relative proportions, both σ_1 and σ_2 approach zero, and approach zero in such a way that

$$\Delta_{\sigma_1} = \int_0^\infty |\, \delta_{\sigma_1}(\lambda)\, |\, d\lambda$$

and (9)

$$\Delta_{\sigma_2} = \int_0^\infty |\, \delta_{\sigma_2}(\lambda)\, |\, d\lambda$$

both approach zero.

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[3] See, however, p. 103.

Let us now put our apparatus in operation. At the start we put everything that has a temperature (ether, of course, has not) at one temperature T, and throughout succeeding time maintain S at temperature T. The body Σ_0 will thereafter, according to the Second Law of Thermodynamics, remain at temperature T. According to the First Law of Thermodynamics the amounts of energy absorbed and emitted by Σ_0 are equal. We have then the equation

$$\sigma\int_0^\infty E_{\sigma_1}(\lambda)\,d\lambda + H_{21} + \sigma\int_0^\infty E_{\sigma_2}(\lambda)\,d\lambda + H_{12} = \qquad (10)$$
$$\sigma\int_0^\infty A_1(\lambda)\,e_{\sigma_1}(\lambda)\,d\lambda + H_{12} + \sigma\int_0^\infty A_2(\lambda)\,e_{\sigma_2}(\lambda)\,d\lambda + H_{21}$$

or

$$\int_0^\infty E_{\sigma_1}(\lambda)\,d\lambda + \int_0^\infty E_{\sigma_2}(\lambda)\,d\lambda =$$
$$\int_0^\infty A_1(\lambda)\,e_{\sigma_1}(\lambda)\,d\lambda + \int_0^\infty A_2(\lambda)\,e_{\sigma_2}(\lambda)\,d\lambda. \qquad (11)$$

We can at this point deduce that

$$\int_0^\infty E_{\sigma_1}(\lambda)\,d\lambda + \int_0^\infty E_{\sigma_2}(\lambda)\,d\lambda =$$
$$\int_0^\infty A_1(\lambda)\,e_1(\lambda)\,d\lambda + \int_0^\infty A_2(\lambda)\,e_2(\lambda)\,d\lambda. \qquad (12)$$

For if this equation did not hold, but if, on the other hand, the left hand member were equal to the right hand member plus a quantity ϵ, we could by taking Σ_0 small enough secure a contradiction from equation (11), and the assumption that Δ_{σ_1} and Δ_{σ_2} approach 0 as Σ_0 decreases indefinitely.

If we replace S by any other solid body fulfilling the conditions we get an equation

$$\int^\infty E_{\sigma_1}(\lambda)\,d\lambda + \int_0^\infty E_{\sigma_2}(\lambda)\,d\lambda =$$
$$\int_0^\infty A_1(\lambda)\,e_1'(\lambda)\,d\lambda + \int_0^\infty A_2(\lambda)\,e_2'(\lambda)\,d\lambda. \qquad (13)$$

The question now arises as to what is the relation among the e's, — $e_1(\lambda)$, $e_2(\lambda)$, $e_1'(\lambda)$, $e_2'(\lambda)$. If the functions $A_1(\lambda)$ and $A_2(\lambda)$ were perfectly arbitrary it could easily be shown that all the functions $e(\lambda)$ are identical; but we have no right to make such an assumption.[4]

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[4] See page 104. This is in fact the assumption in the proof of E. Pringsheim.

The principal difficulty of the proof lies precisely in this step. The purpose is to make as modest a requirement as possible for the function $A(\lambda)$. We shall assume that there exist, or can be constructed, physical bodies such that the resulting assemblage of absorption coefficients is what we may call densely distributed between some two absorption coefficients that are distinct for every value of λ. We define densely distributed as follows :

Let $A'(\lambda)$ and $A''(\lambda)$ be two functions positive or zero and continuous for all positive values of λ. Let $f(\lambda)$ be any function finite and continuous for all positive values of λ, such that

$$A'(\lambda) \geqq f(\lambda) \geqq A''(\lambda), \qquad \lambda > 0,$$

or

$$A'(\lambda) \leqq f(\lambda) \leqq A''(\lambda), \qquad \lambda > 0,$$

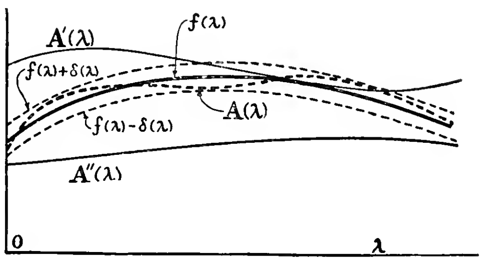

FIGURE 2.

and let δ be any quantity > 0. We say then that the functions $A(\lambda)$ are densely distributed between $A'(\lambda)$ and $A''(\lambda)$ provided that no matter what $f(\lambda)$ and δ are taken, there is an $A(\lambda)$ such that

$$|f(\lambda) - A(\lambda)| \leqq \delta, \quad 0 < \lambda.$$

We shall assume that $A_1(\lambda)$ and $A_2(\lambda)$ are independently densely distributed between two functions $A_1'(\lambda)$ and $A_1''(\lambda)$, and two functions $A_2'(\lambda)$ and $A_2''(\lambda)$ respectively, where

$$A_1''(\lambda) > A_1'(\lambda), \quad 0 < \lambda,$$

and

$$A_2''(\lambda) > A_2'(\lambda), \quad 0 < \lambda.$$

Suppose now that $e_1'(\lambda) \mp e_1(\lambda)$ at some value

$$\lambda = \lambda_0, \quad 0 < \lambda_0.$$

It will then be unequal to $e_1(\lambda)$ throughout a small neighborhood $\lambda_1 \leqq \lambda \leqq \lambda_2$, — let us say that $e_1'(\lambda) - e_1(\lambda) > \epsilon > 0$ throughout the interval.

Now if we subtract (12) from (13) we have

$$\int_0^\infty A_1(\lambda)\,[e_1'(\lambda) - e_1(\lambda)]\,d\lambda + \int_0^\infty A_2(\lambda)\,[e_2'(\lambda) - e_2(\lambda)]\,d\lambda = 0. \quad (14)$$

In particular we have

$$\int_0^\infty A_1'(\lambda)\,[e_1'(\lambda) - e_1(\lambda)]\,d\lambda + \int_0^\infty A_2'(\lambda)\,[e_2'(\lambda) - e_2(\lambda)]\,d\lambda = 0,$$

whence

$$\int_0^\infty [A_1(\lambda) - A_1'(\lambda)]\,[e_1'(\lambda) - e_1(\lambda)]\,d\lambda + \int_0^\infty [A_2(\lambda) - A_2'(\lambda)]\,[e_2'(\lambda) - e_2(\lambda)]\,d\lambda = 0. \quad (15)$$

On account of the freedom of choice for $A_1(\lambda)$ and $A_2(\lambda)$ we can now get a contradiction out of (15). For we can choose $A^1(\lambda)$ in reference to $A_1'(\lambda)$ and $A_2(\lambda)$ in reference to $A_2'(\lambda)$ in such a way that the only significant part of the integrals in (15) will be

$$\int_{\lambda_1}^{\lambda_2} [A_1(\lambda) - A_1'(\lambda)]\,[e_1'(\lambda) - e_1(\lambda)]\,d\lambda,$$

which can be made unequal to zero. But this contradicts (15). Hence $e_1'(\lambda_0) = e_1(\lambda_0)$ and $e_1'(\lambda) \equiv e_1(\lambda)$.

And not only for every body S but also for every position of Σ_0 in the cavity is $e_1'(\lambda) \equiv e_1(\lambda)$. For different positions of Σ_0 can be regarded as the same position of Σ_0 with different S. Hence in particular $e_1(\lambda) = e_2(\lambda)$. We have then the theorem that the character of the radiation impinging on either face of any element of surface σ anywhere in the cavity S is a function merely of the temperature of the cavity and the nature of the medium in which the element of surface is immersed.

If we denote the function $e_1(\lambda) = e_2(\lambda)$ by $e(\lambda)$, we may call $e(\lambda)$ perfectly black radiation. It is approximately the radiation emitted through a hole of unit area in the bounding surface of a large cavity whose interior walls are kept at the uniform temperature T.

We can now easily prove Kirchhoff's Law. Suppose that in the cavity S we have any body K, enclosed by a surface S, just outside the surface of K. A surface S_1' is assumed outside S_1; between K and S_1' there is an arbitrary medium. K itself may be solid, liquid or gaseous, though if K is liquid or gaseous the medium in which it is enclosed must be solid. We have then, at the temperature T,

$$\iint_{(S_1)} E(\lambda)\, dS_1 + \iint_{(S_1)} [1 - A(\lambda)]\, e(\lambda)\, dS_1 = \iint_{(S_1)} e(\lambda)\, dS_1 \qquad (17)$$

since the amount of radiation of wave length λ reaching any element of surface dS, *from either side* is $e(\lambda)$.

Hence

$$\iint_{(S_1)} E(\lambda)\, dS_1 = \iint_{(S_1)} A(\lambda)\, e(\lambda)\, dS_1, \qquad (18)$$

which is an expression of Kirchhoff's Law. The expression looks more natural if we denote by $\overline{A}(\lambda)$

$$\overline{A}(\lambda) = \frac{\iint_{(S_1)} A(\lambda)\, e(\lambda)\, dS_1}{\iint_{(S_1)} e(\lambda)\, dS_1};$$

we then have

$$\iint_{(S_1)} E(\lambda)\, dS_1 = \overline{A}(\lambda) \iint_{(S_1)} e(\lambda)\, dS_1. \qquad (19)$$

If the $E(\lambda)$ is a function of the element of surface only, we can by taking a spherical K of the material under investigation deduce from symmetry that

$$E(\lambda) = A(\lambda)\, e(\lambda).$$

Discussion of the Assumptions.

On page 99 certain assumptions are made in regard to the functions $\delta_{\sigma_1}(\lambda)$ and $\delta_{\sigma_2}(\lambda)$,— namely, that as the body Σ_0 decreases in size, $\delta_{\sigma_1}(\lambda)$ and $\delta_{\sigma_2}(\lambda)$ approach zero as a limit in such a way that

$$\Delta_{\sigma_1} = \int_0^\infty |\, \delta_{\sigma_1}(\lambda)\, |\, d\lambda$$

and

$$\Delta_{\sigma_2} = \int_0^\infty |\, \delta_{\sigma_2}(\lambda)\, |\, d\lambda$$

both approach zero. This amounts in effect to assuming that

$\sigma(\Delta_{\sigma_1} + \Delta_{\sigma_2})$, the difference in the radiation reaching the surface Σ_0 when there is and when there is not a different material in Σ_0 from the medium between Σ_0 and Σ_1, is an infinitesimal of the second order. This supposition seems more reasonable than it really is.

For let us suppose, if it is possible, that the body S is composed of a material which emits and absorbs only a partial spectrum, i.e. there are wave lengths for which S emits and absorbs no energy. With no body Σ_0 and ether for the medium throughout, the radiation $e(\lambda)$ reaching any element of surface within the cavity will not contain energy of these wave lengths which the body S does not emit. But if we insert a body Σ_0, however small, which has a complete spectrum, the energy of the forbidden wave lengths will build up until it reaches such intensity that the amount absorbed by the small body Σ_0 will be equal to the amount emitted by it. Since both of these quantities will therefore be infinitesimals of the first order, $\delta_1(\lambda)$ and $\delta_2(\lambda)$ will not approach zero at all as Σ_0 decreases in size.

We cannot therefore regard the invariance of the function $e(\lambda)$ as itself a statement of the non-existence of bodies of partial spectra.[5] For that invariance depends upon the assumption that Δ_{σ_1} and Δ_{σ_2} approach zero as Σ_0 becomes smaller and smaller, and hence gives no contradiction when S is a body of partial spectrum.

However, bodies of partial spectra, if they exist, must satisfy Kirchhoff's Law. For the analysis of pages 101 and 102 applies. And a body that has a partial emission spectrum must have an absorption spectrum extending over precisely the same region of wave lengths; and vice versa.

The second assumption of importance is the supposition that there can be constructed a dense distribution of absorption coefficients between some two distinct functions of λ. This assumption does not by any means demand the existence of an arbitrary absorption coefficient. Indeed the requirements of the proof can be met by a denumerable system[6]. And yet we cannot be certain of the possibility of physically constructing even this denumerable set.

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[5] See, however, in this connection W. Wien, Annalen d. Physik, **52**, 163, 1894.

[6] That is, a system of the same order of infinity as the natural numbers, or the rational fractions.

If $A'(\lambda)$ and $A''(\lambda)$ (see p. 8) have the same limit at ∞, a denumerable set sufficient for the demands of the proof can be set up by requiring that $f(\lambda)$ be continuous with continuously turning tangent (except at a finite number of points), and by replacing, in the definition of density, the two curves $f(\lambda) \pm \delta(\lambda)$ by two curves distant δ from the curve $f(\lambda)$.

Although the present proof depends, indeed, on this assumption of the physical existence of ideal elements, the proof is not completely invalidated by the lack of such a system. There is in this instance the considerable advantage that the existence of even a finite physical system approximately fulfilling the conditions of the postulated ideal system carries with it the approximate fulfilment of the thesis. Kirchhoff's Law is approximately satisfied in any case.

It is worth while noting that, although establishing a particular property of any particular substance, namely that its emission and absorption coefficient must stand in a certain fixed relation, the proof, on account of its nature, holds only in so far as the particular substance is a member of a physical system that contains as a subset the dense system postulated.[7] The properties of the individual are determined by the environment.

Finally there is a third assumption that needs some attention. For on page 103, in deducing equation (17), which holds for any particular value of λ, it is assumed that the wave length of energy is unchanged by reflection. And Kirchhoff's Law will therefore hold for any bodies whose surfaces are such that they do not change wave length by reflection.

In certain cases this assumption about the nature of the body itself is equivalent to a condition on the environment. For if the body under consideration be a member of a dense system of absorption coefficients, the above assumption may be replaced by postulating that a single member of that system be known to obey Kirchhoff's Law. And from this postulate the other, that wave length be unchanged by reflection, may be deduced. The two, though they do not look alike, are in a sense mathematically equivalent.

Conclusion.

The fruitfulness of Kirchhoff's Law depends upon the degree to which the absorption coefficient is invariant in regard to different conditions of the body or its surroundings, i.e., for instance in regard to (1) different temperatures of the absorbing body, and (2) different in-

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[7] Some proofs err in assuming the existence of bodies of partial spectra or in attempting to construct them; e. g.: Kirchhoff (1859) and Drude (1900).

In regard to other proofs it may be noted that those of B. Stewart and Prevostaye are not logically complete, and that Kirchhoff's second proof, besides being involved, is not strictly thermodynamic and assumes the existence of bodies with questionable properties.

For detailed references see Kayser's Spectroscopie II, pages 7–31.

tensities of radiation reaching the surface from outside [corresponding to different temperatures of the body S]. For in general the absorption coefficient has to be measured under one set of conditions when it is desired to apply the results to another set.

Hence it may seem that Kirchhoff's Law holds very well for one set of bodies and very poorly for another set, if the absorption coefficient is regarded as a constant. The preceding analysis shows, however, that carefully stated, the law holds as well for one substance as another, as well for gases and liquids as for solids.

In closing I should like to acknowledge my indebtedness to Professor G. W. Pierce of Harvard University for helpful criticism.

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CONTRIBUTIONS FROM THE RESEARCH LABORATORY OF PHYSICAL CHEMISTRY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY. — No 59.

THE ELECTROMOTIVE FORCE PRODUCED IN SOLUTIONS BY CENTRIFUGAL ACTION.

By Richard C. Tolman.

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Presented by Professor A. A. Noyes. Received June 17, 1910.

Contents.

1. Work of Previous Investigators and Purpose of this Research.

If the passage of an electric current is associated with the actual transfer of matter along the conductor, a number of phenomena, depending upon the inertia of this matter, are to be expected. The nature of these phenomena was completely discussed by Maxwell; but they have not yet been detected in metallic conductors.[1] In electrolytic conductors, however, the experiments of Hittorf have shown that there is an actual transfer of matter through the solution, accompanying the passage of a current. As an example of the electrical effects which may accompany such a motion of the carriers of electricity or ions, let us consider a solution of silver nitrate placed in a vertical tube with silver electrodes at the top and bottom. The passage of a current of electricity through this solution from the lower to the upper electrode will be accompanied by a raising of the silver ions in the solution and an approximately equal lowering of the considerably lighter nitrate ions. The net effect produced by the current will be the lifting of a certain weight of material from the lower electrode to the upper; and the work thus done against gravity must correspond to a definite electromotive force which will oppose the passage of the current in this direction.

Effects of this kind were first predicted by Colley,[2] and have also been considered by various other investigators.[3] Colley himself was

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[1] Maxwell, Treatise on Elec. & Mag., 3rd edition, Vol. II, 211 et seq.; Lodge, Modern Views of Elec. & Mag., 3rd edition, 89; Nichols, Phys. Zeitschr. **7**, 640 (1906).

[2] Colley, Journal der St. Petersburg chem. und phys. Gesellschaften **7**, 333, 1875; Pogg. Ann. **157**, 370 (1876); Ibid. **157**, 624 (1876); Wied. Beibl. **5**, 457 (1881); Wied. Ann., **17**, 55 (1882).

[3] Lodge, Phil. Mag. **2**, 367 (1876); Hertz, Wied. Ann. **14**, 590 (1881); Des Coudres, Wied. Ann. **49**, 284 (1893); Ibid. **57**, 232 (1896). The electromotive force which is produced in salt solutions by the action of gravity must be carefully distinguished from the actual changes in the concentration of the solute which gravity will produce. The difference in concentration between the solution in the upper and lower ends of a vertical tube, or the central and peripheral portions of a rotating tube can be calculated from simple thermodynamic considerations. It is to be expected, however, that this difference in *concentration* will be very *slowly* established (see Des Coudres, l. c. below). The difference in *potential* between the two ends of the solution is an *immediate* phenomenon which occurs as soon as the tube is set up. It is evident that when the final change in the concentration of a salt solution has completed itself there will no longer be any potential difference between the upper and lower portion of the solution. A complete bibliography of the theoretical and experimental work which has been done in this field follows: Gay Lussac, Ann. chim. phys. **11**, 306 (1819); Gouy et Chaperon, Ann. chim. phys. **12**, 384 (1887); Nernst, Zeitschr. f. phys. Chem. **2**, 637 (1888); Duhem, Journ. de phys. **7**, 391 (1888); T. v. Turin, Journ. der russ. Geschichte, **24**, 90 (1892), Wied. Beibl. **17**, 16 (1893); Des Coudres, Wied. Ann. **49**, 284 (1893); Ibid. Wied. Ann. **55**, 213 (1895); Van der Waals, Zeitschr. f. phys.

able to detect the existence of potential differences between the upper and lower electrodes placed in solutions of silver nitrate and cadmium iodide. The small electromotive forces produced in these solutions by gravity are masked by the presence of much larger variable potential differences which are due to otherwise undetectable differences between the two electrodes used. The effect of gravity is only perceived when a series of measurements is made of the deflections of a very sensitive galvanometer, the positions of the upper and lower electrodes being interchanged between each two measurements by a reversal of the tube containing the electrolyte. The potential differences, which Colley thus found, were of the order of magnitude theoretically predicted. Later he also showed the presence of a momentary electromotive force produced by the sudden stopping of a falling tube containing cadmium iodide solution.

The experimental problem was next attacked by Des Coudres who substituted centrifugal force for the weaker action of gravity. He made use of a rotating apparatus in which were placed two tubes with electrodes connected in series, containing cadmium iodide solution. The electrodes were placed 31 cm. and 9 cm. from the center and at 5.8 revolutions per second he obtained an electromotive force of 155 micro-volts. Individual measurements, however, varied at times more than 10% from the mean. Although the substitution of centrifugal force for the force of gravity greatly increases the size of the effect which can be obtained, it is accompanied by the introduction of new errors, produced, for example, by the unequal heating of the central and peripheral portions of the apparatus, as well as by other difficulties. The method was abandoned by Des Coudres in favor of measurements on specially constructed gravity cells. These measurements are of a considerable degree of accuracy and the results will be given in detail.

As we shall see in the next section, it is possible to derive a relation connecting the electromotive force produced by the action of gravity and the transference number of the electrolyte. The measurements of Des Coudres were made for the purpose of comparing the transference

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Chem. **5**, 157 (1890); Bredig Zeitschr. f. phys. Chem. **17**, 459 (1895); Lobry de Bruyn et van Calcar, Rec. d. travaux chim. d. Pays-Bas, **23**, 218 (1904); Franklin & Freudenberger, Trans. Am. Electrochem. Soc. **8**, 29 (1905); Earl of Berkeley & C. V. Burton, Phil. Mag. **17**, 606 (1909); Gibbs, The Scientific Papers of, Longmans, Green & Co. (1906), Vol. I, 144.

As to experimental results it may be said that the work of van Calcar and de Bruyn is in complete disagreement with the theory. The result reported by the Earl of Berkeley and Dr. Burton is, however, of the predicted order of magnitude.

numbers calculated from electromotive force with those determined by the Hittorf method. In his first gravitational experiment he again made use of cadmium iodide solution which owing to complex ion formation apparently has a transference number greater than unity, and hence gives a large electromotive force. In the experiment the solution was placed in a glass tube with the electrodes at the two ends about 91 cm. apart. The gravitational electromotive force was deduced from a series of readings of galvanometer deflection, the positions of the upper and lower electrodes being interchanged between each two measurements. The transference number of the solution was found by calculation to be 1.236, differing by at least five per cent from that determined in the analytical way.

In order to test the theory on more simple solutions such as the chlorides of the alkali metals, which do not have abnormal values of the transference number, it was necessary to compensate for the decreased size of the effect by the construction of an apparatus with greater difference in height between the electrodes. The electrodes were placed in glass vessels and connected together by a rubber tube full of the solution. The electrode containers could be raised and lowered at will, permitting a difference in level of 377 cm.

The following table gives a summary of his results, including a comparison between the value T_a for the transference number of the anion as calculated from his results and as determined by the Hittorf method. The measurements were made with calomel electrodes (in the case of $CdCl_2$ with Cd amalgam electrodes as well). The e.m.f. is expressed in microvolts per centimeter difference in height.

Salt.	% Conc.	e. m. f.	T_a	T_a Hittorf
KCl	16.8	+0.0510	0.50	0.52
NaCl	19.9	−0.0315	0.66	0.65
LiCl	17.3	−0.109	0.77	0.75
HCl	3.56	−0.0218	0.150	0.175
$BaCl_2$	17.0	+0.170	0.64	0.65
$CdCl_2$ (Cal. Elect.)	30.1	−0.183	1.10	0.82
$CdCl_2$ (Amalg. Elect.)	30.1	−0.221	1.02	

Des Coudres estimates that the uncertainty in the values of the electromotive forces given is not greater than ±0.009 microvolts.

In the present investigation, a new attempt was made to increase the accuracy of this method of determining transference numbers, by substituting for the effect of gravity the force produced by a centrifugal machine of considerable power. As will be seen in the sequel, the average deviation between successive measurements has in this way been reduced to a very small amount. An estimation of the actual error in the results is, however, somewhat difficult to make.

2. Derivation of an Expression for the Electromotive Force.

In order to derive an expression for the electromotive force produced in an electrolytic solution by the action of gravity, let us consider a vertical tube of height h, filled with the solution and provided with electrodes at the top and bottom. If E is the potential difference in volts between the upper and lower electrodes produced by the action of gravity, then by allowing one faraday of electricity to flow under this electromotive force we could obtain the external work $10^7 EF$ ergs. The passage of this electricity through the solution is accompanied, however, by the transfer of a certain net weight of material from the upper electrode to the lower one. From the principles of energetics, it is evident that this external energy $10^7 EF$ which we could obtain will just be sufficient to restore the solution to its original condition, that is, will do the work of raising the transferred material back from the lower electrode to the upper one. For the sake of definiteness, let us suppose that the electrolyte is a solution of an iodide ($C^+ I^-$) and that we are using iodine electrodes, consisting in practice of platinum electrodes with a small amount of iodine dissolved in the solution. If, now, we let one faraday of electricity flow through the solution, we know from the experiments of Faraday and of Hittorf that one atomic weight of iodine or M_I grams will be liberated at the anode or upper electrode and will disappear from the lower electrode, and at the same time that there will be a change in the ratio of salt to water at the two electrodes such that $T_c M_s$ gms. of salt will have apparently been transferred from the anode to the cathode, where T_c is the ordinary or Hittorf transference number of the cation and M_s is the molecular weight of the salt. In order, therefore, to restore the solution to its original condition of uniform concentration, it is necessary to raise $T_c M_s$ gms. of salt from the lower electrode to the upper one at the same time lowering one atomic weight of iodine. If we raise and lower these substances *through* the solution it is evident that they will be buoyed up by force equal to the weight of the volume of solution which they displace. Hence if g is the acceleration due to gravity, and d the density of the solution, the downward forces acting respectively on the salt and the iodine will be $g\ T_c\ M_s\ (1 - v_s\ d)$ and $g\ M_I\ (1 - v_{I_2}\ d)$ where v_s and v_{I_2} are the "partial" specific volumes [4] of the substances. Equating the external electrical work to the work done against these forces in transferring the substance from one electrode to the other, we have the desired relation [5]

$$10^7\ E\ F = h\ g\ [T_c M_s (1 - v_s d) - M_I (1 - v_{I_2} d)]. \qquad (1)$$

If instead of using a gravity cell we should rotate our solution n times per second, with electrodes at r_1 and r_2, since the centrifugal force acting on one gram at any radius r is $4\ \pi^2\ n^2\ r$, the work done in carrying one gram from r_2 to r_1 would evidently be

$$\int_{r_1}^{r_2} 4\ \pi^2\ n^2\ r\ dr = 2\ \pi^2\ n^2\ (r_2^2 - r_1^2)\ ^{6}$$

and our equation for electromotive force becomes

$$10^7 EF = 2\ \pi^2\ n^2\ (r_2^2 - r_1^2)\ [T_c M_s (1 - v_s d) - M_I\ (1 - v_{I_2} d)]. \qquad (2)$$

We see that the value of the electromotive force which is to be measured is proportional to the factor hg or $2\pi^2\ n^2\ (r^2_2 - r_1^2)$. In the experiments of Des Coudres on gravitational cells, the factor hg was 360,000, while with his rotating apparatus the value of the corresponding factor was 583,000. In the centrifugal experiments which are to be described in this paper the value of the factor was raised to 114,000,000.

The Effect of Hydration.

It has been shown by the careful researches of Buchböck [7] and especially of Washburn [8] that it is possible to distinguish between the ordinary Hittorf transference number and the so-called "true" transference number in which the motion of the ions is referred to a non-

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[4] For a definition of "partial" volumes, see page 131.

[5] In order to derive this equation, any reversible method might be used for restoring the solution to its original condition of uniform concentration. For example, the apparent difficulty of moving the substances *through* the solution can be obviated by removing them from the solution with the help of semi-permeable membranes. It is easy to show, however, that the relation derived is the same.

[6] It may also be pointed out that the expression $2\pi^2 n^2 (r_2^2 - r_1^2)$ is the increase in kinetic energy when one gram of material is transferred from r_1 to r_2.

[7] Buchböck, Z. physik. Chem. **55**, 563 (1906).

[8] Washburn, Tech. Quart. **21**, 164 (1908), Journ. Amer. Chem. Soc. **31**, 322 (1909).

electrolytic indicator dissolved in the solution instead of to the water. If water is carried by the current, owing to hydration of the ions, then the two transference numbers will be different.

The Hittorf transference number gives, however, the actual number of equivalents of salt which apparently disappear in the neighborhood of one electrode and appear at the other when one faraday of electricity is sent through the solution, and hence the Hittorf transference number gives us the amount of salt which must be moved in order to restore the solution to its original condition of uniform concentration. From a consideration of the method by which the equation for the electromotive force was derived, it is obvious that the *Hittorf* transference number is the one which has been determined in this research.

3. Kinetic Derivation of the Electromotive Force Expression.

In order to derive the equations used in this article, we have considered the production of the electromotive force in a rotating solution from a thermodynamic standpoint. The fact that more work was needed to send a current through a salt solution from the outer to the inner electrodes than in the reverse direction, could be predicted from thermodynamic principles, and the electromotive force, corresponding to this work, could be calculated merely from a knowledge of certain properties of the solution such as transference number and density, which can be experimentally determined. It is also instructive, however, to look at the question from a "kinetic" or molecular point of view. The fact that an electromotive force *is spontaneously* produced by centrifugal force gives us a real knowledge of the internal structure of an electrolytic conductor. It is, indeed, the most striking proof of the existence of free ions in an electrolyte.

Considered from a "kinetic" point of view, a solution of potassium iodide contains free potassium ions and free iodide ions. The iodide ions, however, corresponding to their greater atomic weight, are much denser than the potassium ions, and hence when the solution is rotated they move more readily towards the outer portion of the solution and charge it negatively. In fact from a consideration of the forces acting on the ions in the solution, it is possible to derive the same equation for the electromotive force of a gravity cell as that already obtained from thermodynamic reasoning. The method of proof is similar to that used by Nernst in his consideration of the diffusion cell.

Consider an iodide (C^+I^-) of molecular weight M_s dissociating into the ions C^+ and I^- of atomic weights M_C and M_I. Let v_s be the partial specific volume of the salt in solution and v_C and v_I be the same quantities for the positive and negative ions respectively. Now let us subject a solution of this salt to the action of gravity. If the positive ion C^+ is denser than the iodide ion, it will tend to move downward through the solution more rapidly and will produce a potential gradient in the solution $-dE_1/dh$. As this potential gradient is produced, however, it tends to decrease the downward velocity of the positive ion and increase the velocity of the negative ion so that under the final potential gradient produced they will move downward through the solution with equal velocities. We may now proceed to derive expressions for these equal velocities.

The total downward force acting on one mol of positive ions is the weight (gM_c), minus the buoyant force exerted by the solution $(gM_c v_c d)$, minus the electrical repulsion $(10^7\, F dE_1/dh)$ corresponding to the potential gradient. If u and v are the velocities with which the positive and negative ions move under unit force, the velocity with which they will move under the actual forces can now be calculated by simple multiplication, since the validity of Ohms law in solutions shows us that the velocity with which the ions move is proportional to the force acting on them.

Equating the velocities of the negative and positive ions, we have[9]

$$u\left(gM_c\,(1 - v_c d) - 10^7\,F\frac{dE_1}{dh}\right) = v\left(g\,M_I\,(1 - v_I d) + 10^7\,F\frac{dE_1}{dh}\right).$$

Solving for dE_1/dh and integrating between the limits o and h, where h is the difference in height between the electrodes, we have

$$10^7\,E_1 F = hg\left[\frac{u}{u+v}M_c\,(1 - v_c d) - \left(1 - \frac{u}{u+v}\right)M_I\,(1 - v_I d)\right]. \quad (3)$$

In order to obtain the actual electromotive force between the electrodes, we must consider not only the potential gradient in the solution, but also the potential drops which occur directly at the electrodes due to the electrode reaction $\frac{1}{2}\,I_2 + \ominus = I$. Since this reaction is accompanied by the change in volume $M_I\,(v_I - v_{I_2})$ and takes place under the difference in pressure between the upper and lower electrodes which is

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[9] It should be noted that when the concentration of the salt in the lower portion of the tube has appreciably increased, the concentration gradient adds new forces tending to slacken the downward motion of the ions. When the final equilibrium is reached the concentration gradient will be such that no potential gradient exists. We are interested, however, in the original condition before appreciable concentration changes have taken place.

equal to hgd, we must add to the electromotive force E_1 as given in equation (3), the electromotive force E_2 given by the equation below, which is derived by equating the external electrical work to the work produced by the change in volume

$$10^7 E_2 F = M_I (v_I - v_{I_2}) hgd.$$

Noticing that

$$\frac{u}{u+v} = T_c,$$

we obtain

$$10^7 (E_1 + E_2) F = hg [T_c M_c (1 - v_c d) + T_c M_I (1 - v_I d) - M_I (1 - v_{I_2} d)]$$

or
$$10^7 EF = hg [T_c M_s (1 - v_s d) - M_I (1 - v_{I_2} d)],$$

which is the same equation we originally obtained by thermodynamic reasoning.

The real interest attached to this kinetic consideration is the almost absolute proof it offers that some degree of dissociation or at least polarization of the salt molecules exists in aqueous solutions. Since unless the positive and negative components of the salt can move relative to one another we cannot see how a potential gradient is set up by centrifugal separation. The method gives, of course, no idea of the magnitude of the degree of ionization.

4. Description of Apparatus.

The general arrangement of the rotating apparatus is shown in Figure 1. It consists of a steam turbine A, with vertical shaft, driving the rotator B, which contains the tubes of solution. Electrical connection with the electrodes in the solution was made through the mercury contacts C.

The Steam Turbine.

The turbine used was a thirty horse-power de Laval loaned for this investigation by the General Electric Co. The turbine wheel was ten inches in diameter. A 2-inch steam pipe D leads into the annular space E from which the nozzles lead up to the turbine wheel. The holes F are opposite the openings to the nozzles and are threaded to receive the bonnets of the nozzle valves (not shown in the drawing). A 3-inch exhaust pipe leads from the exhaust chamber G, on the side away from that shown in the drawing. The turbine is supported on the legs H, bolted to the cross beams I. It is also bolted to the block J. Largely for the sake of safety, the apparatus was re-designed

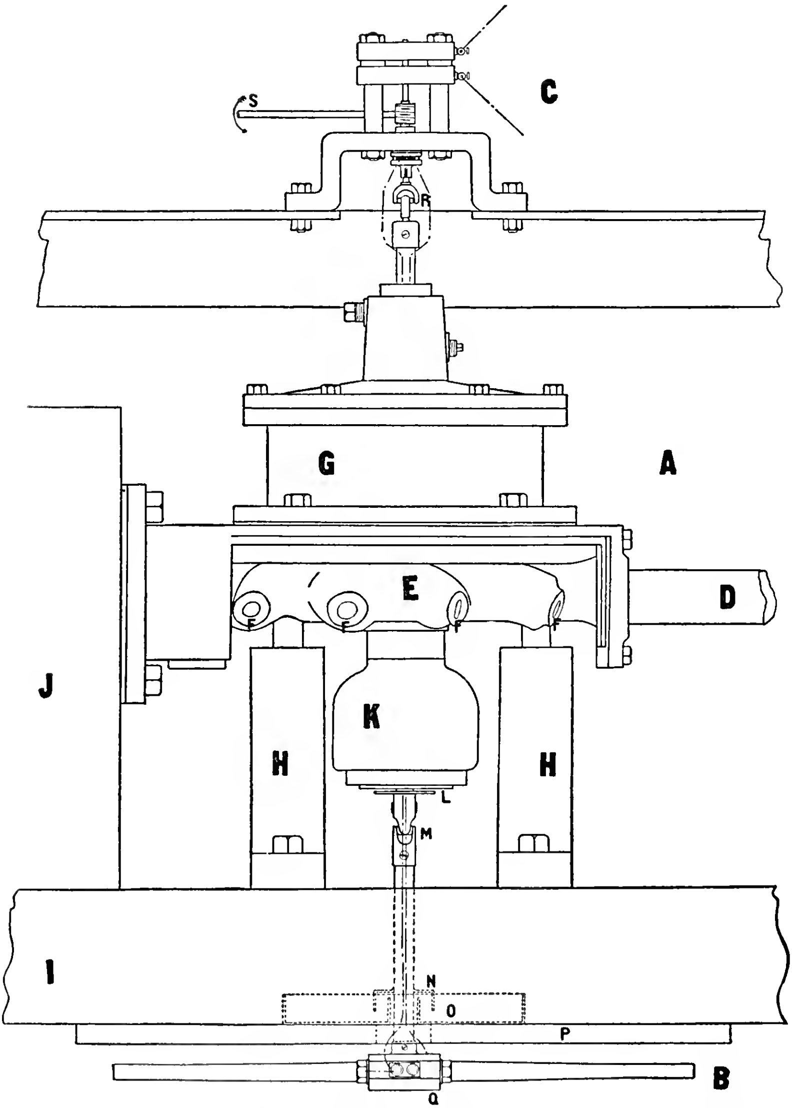

FIGURE 1. General Arrangement of the Rotating Apparatus. ⅕ size.

with a vertical shaft and was placed in a specially constructed pit. The vertical shaft necessitated the design of some form of thrust bearing to support the rotating system.

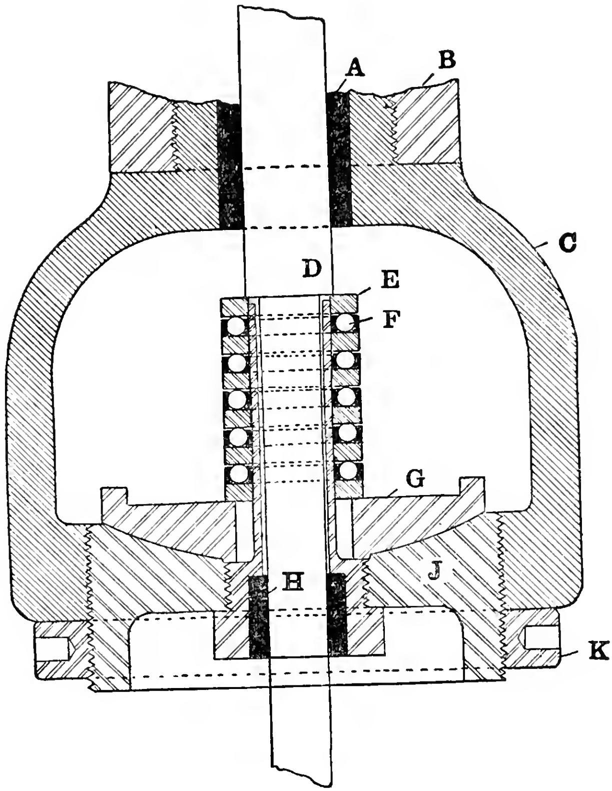

FIGURE 2. The Thrust Bearing. ½ size.

The Thrust Bearing.

The thrust bearing used was contained in the case K, Figure 1. The details are shown in Figure 2. It consists of a series of five ball bearings, placed one above the other, so as to distribute the total relative motion which must be cared for. Each ball bearing consists of a series of balls F, in a brass cage, between two hardened steel washers. A shoulder on the shaft D rests on the topmost washer E, and the lowest bearing rests on the levelling washer G, which has a spherical seat. The nut K locks the adjustment of the bearing at the proper height. The lateral bearings A and H as well as the top bearing of the turbine itself, consist of eminently satisfactory graphite-lined bushings supplied by the Graphite Lubricating Co., Bound Brook, N. J. The lubrication of the ball bearings was also with graphite which was applied with oil in the form of a paste.

This form of ball bearing was tried only after a number of bearings, including one specially designed for the purpose by The Standard Roller Bearing Co., had failed. The individual ball bearings of which this bearing was constructed were stock 13/16 inch "ball thrust collar" bearings made by the above firm. The ingenious idea of replacing the series of disks sometimes used in step bearings by a series of ball bearings came in a conversation with Dr. C. A. Kraus of this laboratory, to whom, in general, I am under the deepest obligation for an intelligent and sympathetic understanding of the many difficulties involved in the construction of an apparatus of this kind.

The "Spinning Top."

The rotator B, Figure 1, is two feet in diameter and consists of two hollow steel arms, screwed into a central hub P. The rotator is hung on a shaft 13/16 inch diameter and 8–1/2 inches long. This shaft is flexibly connected to the turbine shaft with a Hooke joint M, which permits the rotator to revolve about its own center of gravity. The arrangement is in the nature of a "spinning top." In order to prevent a precessional motion of the top which would quickly raise the rotator shaft to a horizontal position, the shaft was steadied at the point N by a system of cords not shown in Figure 1, but indicated in plan in Figure 3. The two cords MN and OP prevent motion of the shaft in the direction AA, and two others not shown in the figure are arranged to prevent motion in the direction BB. The cords were drawn tight against the shaft through the stationary supports at W, X, Y, and Z. The cords were braided cotton 5/32 inch in diameter and the wear on them, strange as it may seem, was very inconsiderable. This arrangement of cords was adopted after experimenting with many forms of steadying bearing, in which it was attempted to prevent the precessional motion with rubber washers, springs, or pneumatic dash pots.

This plan of driving the rotator as a spinning top must be considered as one of the distinctive features of the apparatus. By this means the rotator was driven up to a speed of 7850 revolutions per minute, giving a rim speed of nearly 50,000 feet per minute. At this speed there was a certain amount of vibration of the steadying cords. In general, however, it was found possible to raise the speed at which this vibration took place by moving the supports W, X, Y, and Z nearer to the shaft, and there is no doubt that the apparatus could be arranged to go to still higher speeds. As will be seen later, for other reasons, the actual measurements had to be made at speeds of 5000 revolutions or less.

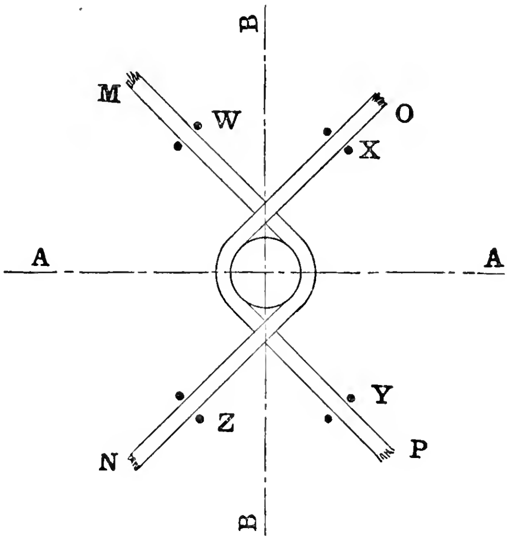

FIGURE 3. The Steadying Cords.

The fact to be specially noticed with regard to this "spinning-top method" is that it permits the driving at a very high speed of a rotator which has not been specially adjusted for either "stationary" or "running" balance. The method was adopted after considerable experimenting with a rotator driven on a shaft with fixed bearings. Although this rotator had been put into "stationary" balance, it caused so great a vibration that one of the bearings gave way at a speed under 2000 revolutions per minute, wrecking a machine upon which a considerable amount of labor had been expended.

The mathematical theory of a symmetrical top rotating with one fixed point and acted upon by gravity alone has been completely developed. It has further been shown by H. Lamb[10] that, in the case of a rotating top hanging below its fixed point, as in this apparatus, the effect of *viscous forces* in the Hooke joint is to produce a gradually increasing precessional motion which would finally raise the axis of the top horizontal. This is the precessional motion which was prevented by the steadying device described above. It must be noticed in general, in designing rotating apparatus of this kind, that the actual motion may differ considerably from that pre-

[10] H. Lamb, Proc. Roy. Soc., **80**, A 168 (1908).

dicted by the dynamical equations since, among other things for example, the latter take no account of restraining forces such as were introduced in this case by the steadying cords.

Some experiments, on a small scale, were made with rotating tops, driven through a Hooke joint from below. There were indications that this would be a most satisfactory method of driving an unbalanced rotor. The whole field is a very profitable one for research.

As a practical detail, it should be pointed out that the rotator shaft must be made short enough not to reach its own period of vibration at the speed employed. It was found impracticable to run this rotator with a shaft 22 inches long.

The Rotator.

Fig. 4 gives a detail of one of the two arms which screw into the central hub of the rotator. These arms were made from seamless steel tubing bored out to one-half inch inside diameter. The peripheral end of the tube was closed by screwing in a plug of steel, the threads of which were figured to withstand the shear at 8000 revolutions per minute, produced by the combined outward action of the centrifugal force of the plug and that of the contents of the tube, assuming them to be a liquid of specific gravity 1.3. In order to prevent leakage of liquid through the screw threads, the plug was silver soldered in position.

The thickness of wall at the peripheral end of the tube was enough to prevent bursting as calculated by Clavarino's formula for thick hollow cylinders. This thickness of wall was also great enough to sustain the outward tension as far as the point B, 3 inches from the end of the tube. From this point on towards the center the tube had to gradually increase in diameter in order to sustain the constantly increasing load. If r is the radius at any point where the section is s, D the density, f the tensile strength of the material, and n the number of revolutions, evidently $fds = 4\pi^2 n^2 r D s dr$. Integrating this equation and solving, the proper dimensions for the inner end of the arm were found. The screw thread on the inner end of the arm was sufficient to prevent the arm from shearing out from the hub.

The Solution Tube.

The solutions experimented on were iodides with iodine electrodes. The actual tubes which contained the solutions were made of glass. This was the only material found which was an insulator and at the same time was not attacked by the iodine. The latter condition was of great importance since the slightest loss of iodine in the neighborhood of either of the electrodes created a large potential difference between them. The tubes were carefully annealed and were floated in oil inside the rotator arms; nevertheless, the end thrust produced by the centrifugal force was great enough to break many tubes even at speeds as low as five thousand revolutions per minute.

With the hope of obtaining some material which would stand higher speeds, a great many other materials for tubes were investigated, including porcelain, paraffined paper and wood, celluloid, vulcanite, and steel tubes lined with paraffine, sulphur, and enamel but none was found

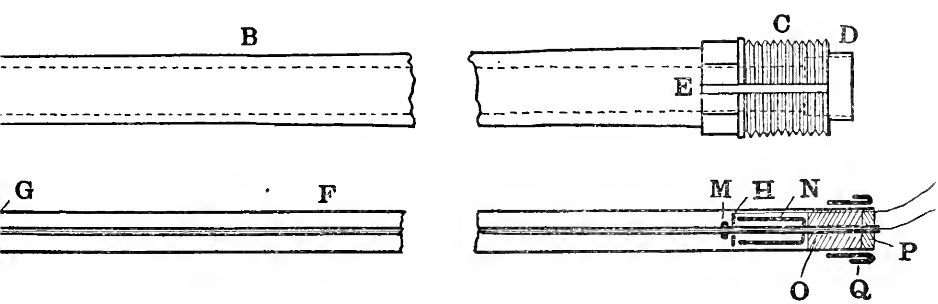

FIGURE 4. Rotator Arm and Solution Tube. ½ size.

satisfactory. Tubes of vulcanite, which has a specific gravity only slightly greater than water, practically floated in the steel arm without danger of breakage, and many experiments were made with them. There seemed, however, to be a distribution of iodine between the hard rubber and the solution which made it very difficult to keep the iodine concentration the same at the two electrodes, and they were finally discarded in favor of glass.

Figure 4 gives a detail of one of the tubes with its electrodes. The tubes were annealed by carefully wrapping them in asbestos and heating in a large electric furnace to about 500° C. and then allowing the furnace to cool, which took a day and a half. The electrodes were platinum. The outside electrode G rested on the bottom of the tube. The connecting wire had to be platinum-iridium in order to stand the centrifugal force. This wire was insulated from the solution by a glass tube which is slightly enlarged at M. On the enlargement M rests the inner electrode H. N shows the cross section of a glass thimble which is slipped down on to the electrode. A bubble of air was entrained in this thimble to allow for temperature expansion, and to provide a means of stirring the solution. Melted ceresin which had been specially purified was poured on top of the thimble at O and this was covered by a thin layer of a molten cement P which consisted of a mixture of wood tar and shellac.[11]

The tube is made to slip into the rotator arm and rests at the outside end on a disk of rubber packing. It is surrounded by oil to equalize the hydrostatic pressure. A short piece of rubber tubing is slipped onto the glass tube and turned back on itself as shown in the figure at Q. The tube projects from the end of the arm far enough so that the turned-back rubber just meets the end of the arm at D, and a piece of "bill-tie" tubing is slipped over the rubber tubing and the arm to prevent leakage of oil.

The leads from the electrodes come out from the rotator in the grooves E and are covered with small rubber tubing. Ordinary optical tubing was not a good enough insulator, but insulation was used which was stripped from 22-gauge rubber-covered wire supplied by the Simplex Electrical Co. of Cambridgeport. The wires were attached to binding posts on the rotator hub, from which electrical connection was carried up to the mercury contacts.

The Electrical Connections.

The path taken by the electrical connections is indicated by dot and dash lines in Figure 1. Leads from the binding posts on the rotator hub pass up through the center of the hollow rotator and turbine shafts and make connection with the mercury contacts. The wire used was 22-gauge rubber covered, as ordinary insulation would not stand the severe conditions of temperature and the effect of moisture.

At L in Figure 1 is a brass disk about 2-1/2 inches in diameter fastened to the rotating shaft. This disk prevents any leakage of water from the turbine trickling down the Hooke joint and getting into the hollow rotator shaft and thus interfering with the insulation. O is a stationary pan about 10 inches in diameter with a central hole for the shaft to pass through. This pan catches accidental dripping from the turbine and protects the hub of the rotator and its binding posts. Further protection is provided by a copper disk N with turned-down edges. This disk rotates with the shaft and covers the central hole in the pan O. The disk also radiated the heat from the turbine and from the friction of the steadying cords, which otherwise traveled down the shaft and produced bad temperature differences between the central and peripheral electrodes.

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[11] This useful cement was discovered by Dr. C. A. Kraus and Mr. R. D. Mailey of this laboratory.

The Mercury Contacts.

Connection between the leads, which came up through the hollow shafts, and the external measuring circuit was made through the mercury contacts C indicated in Figure 1 and shown in detail in Figure 5. The leads from the electrodes were made fast to the binding posts M and N, and electric connection was made through the small rotating

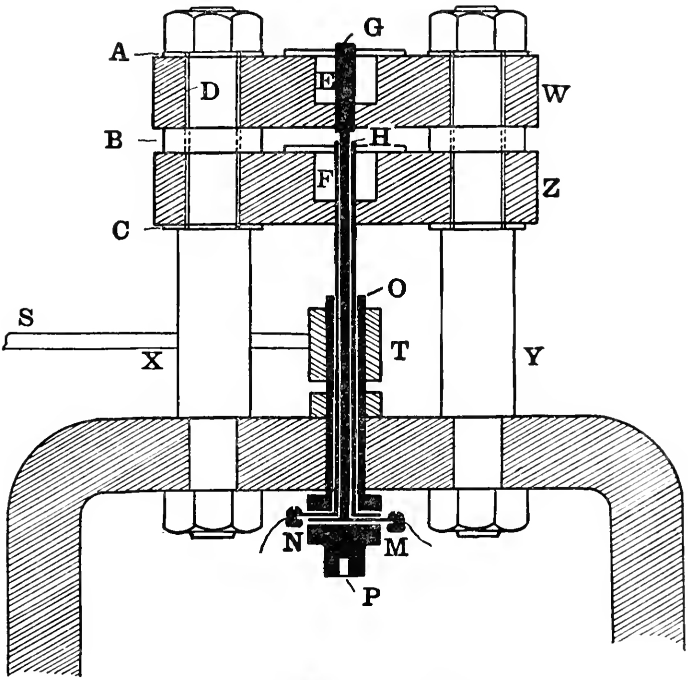

FIGURE 5. The Mercury Contacts. ½ size.

shafts G and H with mercury, which was placed in the circular troughs E and F in the steel blocks W and Z. The two small shafts are insulated from one another and from the outside driving shaft O by fine rubber tubing, and at the bottom by vulcanite disks. The steel blocks are insulated from each other and their supports X and Y by vulcanite washers and sleeves. Connection is made from the steel blocks to the measuring instruments.

The apparatus was specially designed with the purpose of eliminating the production of electromotive forces at the point of contact between the moving and stationary parts. For this reason, the shafts were made as small as possible, 3/16 inches in diameter, thus reducing the relative velocity at the point of contact, mercury was chosen as the contact substance, and the whole apparatus was made of the same material, steel, throughout. Measurements of the electromotive forces occurring in these mercury contacts, were made by driving the apparatus with the contacts short circuited. The potential differences found were in the neighborhood of 1/20 millivolt.

The Magneto.

Returning again to Figure 5, T is a worm driving the shaft S through a worm wheel not seen in the drawing. Measurements of the speed of rotation of the apparatus were made by reading the voltage produced by a magneto driven from this shaft S. This was a small three-bar magneto manufactured a number of years ago by the Holtzer Cabot Co. of Brookline. This was the least satisfactory part of the apparatus, since the voltage readings at a constant speed were liable to small, sudden fluctuations largely due to poor contact at the commutator. Both graphite and woven wire commutator brushes were tried. The accuracy of the speed determinations, however, seemed to be of the same order of magnitude as that of the other measurements, and it did not seem advisable to procure one of the newer magnetos or any of the more costly forms of apparatus for speed measurement. A small motor was arranged for driving the magneto independently, and the magneto was standardized after each series of measurements with the help of a stop watch and suitable counter.

The Electrical Measuring Instruments.

The potential differences obtained were of the order of a few millivolts. For their measurements, a Leeds and Northrup pontentiometer and a suitable galvanometer were used. A cadmium element supplied by the Weston Electric Co. was used as a standard cell, and this was further compared with another Weston cell both at the beginning and end of the measurements. In the actual experiments a reading of the voltmeter which gave the speed of the apparatus, and the potentiometer reading were taken as nearly simultaneously as possible.

5. The Method of Procedure.

With the apparatus described, measurements were made on solutions of potassium, sodium, lithium, and hydrogen iodides. The solutions contained exactly 1 mol of the salt and 1/100 mol of I_2 in a kilogram of water. In the case of hydrogen iodide, owing to the oxidation of the acid, there was a gradual increase in the amount of I_2 present in the solution. A rough colorimetric analysis of the solution as finally used showed the presence of about 2/100 mol I_2 per kilogram of water.

The glass tubes which contained the rotating solutions have already been described. In the experiments as finally performed only one tube was used, an approximate counterbalance being placed in the other arm of the rotator. By connecting two tubes in series, it would have been possible to double the electromotive force to be measured. This would not have greatly increased the certainty of the measurements, however, since, on stopping the rotator, the residual potential differences between the two electrodes were always found to be in the same direction, and by connecting two tubes in series the size of the error as well as that of the potential difference to be measured would have been increased.

Before filling the tubes, they were carefully rinsed with some of the solution to be used. The electrodes were heated to incandescence in a blast lamp and placed in the solution without being touched by the fingers.[12] By using care, it is possible in this way to reduce the original electromotive force between the two electrodes to the neighborhood of 0.2 millivolt or less. The small variable electromotive forces which do persist are probably partly due to differences in temperature between the two ends of the solution. As already described the tubes were sealed with purified ceresin. This was almost the only material found whose presence near one of the electrodes did not produce a large electromotive force.

After the apparatus had been set up ready for rotation it was tested for insulation. This is very important, since any leakage between the leads coming from the solution would apparently have decreased the size of the electromotive force produced. The test was carried out by disconnecting one of the leads from its binding post on the hub of the rotator and applying a drop of $1\frac{1}{2}$ volts at the other end of the leads where they joined the measuring system. A galvanometer was in series with the potential drop to measure the current leaking from one lead to the other.[13]

In general, for the final experiments the galvanometer was absolutely stationary, and if there was more than a trace of a deflection, the

[12] Laurie, Zeits. f. phys. Chem. **64**, 617 (1908).

[13] In order to make the test more thorough the lead which had been disconnected from the binding post was connected to the steel rotator. Until considerable experience had been gained, it was very difficult to eliminate leaks between the wires coming from the electrodes and the rotator.

trouble was eliminated before making a run. A deflection of one millimeter on the galvanometer scale would have corresponded to a resistance of about 1.5×10^9 ohms between the leads.

In the experiments a number of readings of the residual electromotive force were taken with the rotator stationary. It was then brought up to speed and a new series of readings commenced as soon as possible. After running for several minutes, the steam was shut off and more readings of the residual electromotive force commenced as soon as the rotator had stopped. The voltmeter attached to the magneto was placed near the potentiometer so that a determination of the speed of the apparatus was made immediately after each measurement. The approximate time when each measurement was taken was also recorded.

At the end of a day's measurements, the magneto was standardized with a stop watch and counter. Usually, about a half a dozen standardizing runs were made at speeds in the neighborhood of those used in the actual measurements. Each run lasted about a minute. The stop watch was started when some even figure appeared on the counter, and stopped similarly about a minute later. In general, five readings of the voltmeter were made, one before the stop watch was started, the next three at intervals of fifteen seconds, and the last one after the watch had been stopped. It was not easy to hold the speeds constant enough to make standardizing runs of over a minute desirable.

6. The Experimental Results.

In the following tables are given the data on which the calculations for each of the salts investigated are based. The first column gives the time when the observation was taken, the second column, the reading of the voltmeter, V (in decivolts), which indicated the speed of the apparatus, and the third column, the potential difference in millivolts, $E \times 10^3$, this being called negative when in the opposite direction from that produced by the centrifugal force. The radii of the two electrodes r_2 and r_1 are also given, and the data for the standardization of the magneto. In the case of all salts investigated, the inside electrode was positive with respect to the outside one during rotation.

It is a striking fact that, upon stopping the rotator the residual electromotive force is always found to be in the opposite direction from that produced by the centrifugal force. It will also be seen from an examination of the data that there is a general tendency for this residual electromotive force to increase somewhat in magnitude and then in the course of a few minutes gradually to disappear. The average magnitude reached by this residual electromotive force is 0.2 to 0.3 millivolts.

At first sight, it might seem possible to explain this residual potential difference by assuming that the outer electrode was heated more by the friction of the air, during rotation, than the inner one; since as a matter of fact, a difference in temperature of 1° *C.* would have produced a residual electromotive force of about 0.25 millivolts in the direction actually found.[14] This explanation, however, would not account for the fact that the residual electromotive force tends to increase after the machine has been stopped, and the very conditions removed which were supposed to create the temperature difference.[15] This increase in the magnitude of the residual electromotive force after stopping the machine was much easier to follow and the final value reached was much larger, in some earlier experiments where *vulcanite* tubes were used instead of glass for containing the solution.[16] This might indicate that the gradual increase of electromotive force after stopping the rotation is due to the gradual emergence into the solution of some constituent which had been forced into the pores of the tube by the centrifugal force or the pressure. Whatever the true explanation, the phenomenon is so complicated that it seemed best not to hazard a guess as to the probable size of the residual electro-

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[14] In order to determine the value of the electromotive forces produced by temperature differences between the electrodes, the thermoelectric power was carefully measured for a number of different circuits of the type, — platinum → salt solution: salt solution → platinum. The writer hopes to present the results of measurements of this kind in a later paper.

[15] Attempts were made to actually measure the temperature difference between the two electrodes, by placing in the tube an ordinary thermoelectric circuit with iron-nickel junctions at the outer and inner ends. Connection between this thermoelectric circuit and the measuring instruments was made through the mercury contacts already described. It was found, however, that the small electromotive forces arising in these mercury contacts were large enough to obscure those produced by the iron-nickel junctions, and the method was abandoned. There seem to be no practical metallic junctions of higher thermoelectric power than the iron-nickel combination.

In some earlier experiments very definite temperature effects were produced by heat which traveled down the rotator shaft. These effects were eliminated, however, by the copper radiating disk N, Fig. 5, already described, see p. 124.

[16] In these experiments made with vulcanite tubes, there was also the difference that the rotator was driven in a closed case instead of in the open air. Since the air in the case was considerably heated by the rotation, larger temperature differences might have been expected between the two electrodes. Nevertheless, this would not account at all for the increase in residual electromotive force after the rotator had been brought to rest.

motive force *during* rotation and no correction was made for it in the calculations

Further development of the centrifugal method should be in the direction of eliminating these residual potential differences between the electrodes. This could best be done by having the liquid circulate through the apparatus so as to pass from one electrode to the other and thus assure the same conditions at both.

TABLE I.

(Dec. 2, 1909.)

Solution: Molal KI, $\frac{1}{100}$ Molal I_2.

r_2 = 29.40 cm., r_1 = 4.3 cm.

Standardization of magneto: — $\frac{\text{Rev. of rotator per second}}{\text{Voltage of magneto}}$ = 1.050 (7 expts. av. dev. ± 0.0145).

Time.	V = Voltage of magneto.	$E \times 10^3$ = E. M. F.	$n = V \times 1.050$ = rev. per sec.	$\frac{E}{n^2} \times 10^6$.	Dev. from mean.
10.54	0	−0.30	...	...	...
11.12	0	−0.10	...	...	...
.23	0	−0.50	...	...	...
.25	0	−0.55	...	...	...
.28	60.0	+2.58	62.9	652	− 5
.29	58.5	2.47	61.4	655	− 2
.30	59.0	2.52	61.9	658	+ 1
.30½	59.8	2.55	62.8	647	−10
.31½	0	−0.15	...	...	...
.32	0	−0.30	...	...	...
.34	0	−0.18	...	...	...
.36	0	−0.14	...	...	...
.40	0	−0.17	...	...	...
.41½	71.5	+3.65	75.0	649	− 8
.42	71.2	3.73	74.7	668	−11
.43½	72.0	3.73	75.5	655	− 2
.44	72.7	3.90	76.2	672	+15
.45	73.1	3.93	76.7	668	+11
.46	0	−0.25	...	...	...
.47	0	−0.18	...	...	...
.49	0	+0.05	...	...	...
.50½	0	0.17	...	...	...
.52½	0	0.25	...	...	...
.55	78.0	4.38	81.9	653	− 4
.55½	77.5	4.35	81.3	658	+ 1
.56	77.0	4.25	80.8	651	− 6
.57	...	4.25	...	...	...
Average				657.1	± 6.3

7. Ratio of the Electromotive Force to the Square of the Number of Revolutions per Second.

From equation (2) it is evident that the electromotive force produced by the rotation should increase as the square of the number of revolutions per second, that is, E/n^2 for a given solution should be a constant. The degree of the constancy of this quantity is illustrated by the fifth column in Tables I–IV which gives the values of E/n^2 as calculated from the data. Considering the separate runs in a series of measurements, we see a tendency for the individual measurements of the *first* run to show the largest deviations from the mean. There is probably some connection between this and the fact that the value reached by the residual electromotive force is also largest after the first run in a series. In many of the individual runs there is a tendency for the electromotive force to decrease somewhat during the run. This would correspond to the gradual production of a negative residual electromotive force. As already pointed out, the nature of these residual potential differences is too uncertain to permit of a trustworthy correction.

8. The Partial Volumes of Iodine and the Iodides.

Before making a calculation of the transference numbers from the electromotive force data which we have just considered, a knowledge of the partial volumes of iodine in iodide solutions and of the iodides in aqueous solution is necessary.

The partial specific volume of any constituent of a solution may be defined as the increase in volume of the solution when one gram of the constituent in question is added to a quantity of the solution so large that the addition causes no appreciable change in concentration. In the language of mathematics, if the addition of Δm grams of the constituent at the concentration under consideration produces an increase of Δv cc. in the volume of the solution, the partial volume of the substance may be defined as the limit approached by $\Delta v/\Delta m$ as Δm approaches zero.

The quantity $\Delta v/\Delta m$ and its limit the partial volume dv/dm may be obtained by the same experimental methods used for the determination of the specific gravity of solutions or, indeed, may be calculated from specific gravity data if such are available.

The purpose of this section is to show the method of calculating partial volumes from picnometer weighings or from specific gravity data and also to present the results of some experimental determinations of the partial volume of iodine in potassium iodide solution as well as the

TABLE II.

(Dec. 23, 1909.)

Solution: Molal NaI, $\frac{1}{100}$ molal I_2.

$r_2 = 29.45$ cm., $r_1 = 4.2$ cm.

Standardization of Magneto: $\frac{\text{Rev. of rotator per second}}{\text{Voltage of magneto}} = 0.8695$ (8 exp. av. dev. ± .0035).

Time.	V = Voltage of magneto.	$E \times 10^3$ = E. M. F.	$n = V \times 0.8695$ = rev. per sec.	$\frac{E}{n^2} \times 10^6$.	Dev. from mean.
10.12	0	−0.16	...	...	...
.15	0	−0.04	...	...	...
.22	0	−0.35	...	...	...
.25½	66.2	+3.00	57.6	904	+15
.26½	65.9	2.95	57.3	898	+ 9
.27	65.9	2.83	57.3	862	−27
.28	65.9	2.83	57.3	862	−27
.28½	66.2	2.92	57.6	880	− 9
.30	0	−0.24	...	...	...
.30½	0	−0.28	...	...	...
.31¼	0	−0.17	...	...	...
.32½	0	0.00	...	...	...
.35	73.2	+3.61	63.7	889	0
.35½	74.2	3.70	64.5	889	0
.36½	74.8	3.75	65.1	885	− 4
.37	75.1	3.79	65.3	888	− 1
.38	76.0	3.88	66.1	887	− 2
.39½	0	−0.26	...	...	...
.40	0	−0.26	...	...	...
.41	0	−0.17	...	...	...
.42	0	−0.07	...	...	...
.43½	0	−0.00	...	...	...
.45	0	+0.03	...	...	...
.46	0	0.05	...	...	...
.48½	74.5	3.70	64.8	881	− 8
.49	74.2	3.72	64.5	893	+ 4
.50	74.8	3.72	65.1	877	−12
.50½	74.5	3.71	64.8	883	− 6
.51	74.5	3.70	64.8	881	− 8
.52	74.0	3.66	64.4	882	− 7
.53½	0	−0.15	...	...	...
.54½	0	−0.10	...	...	...
.55½	0	−0.06	...	...	...
.57	0	−0.05	...	...	...
.58½	0	−0.06	...	...	...
.60½	0	−0.10	...	...	...
11.03	80.0	+4.37	69.5	905	+16
.03½	81.2	4.53	70.6	910	+21
.04½	82.8	4.60	72.0	888	− 1
.05	83.2	4.65	72.3	889	0
.06	84.0	4.77	73.0	895	+ 6

TABLE II. — *Continued.*

Time.	V = Voltage of magneto.	$E \times 10^3$ = E. M. F.	$n = V \times 0.8695$ = rev. per sec.	$\frac{E}{n^2} \times 10.^6$	Dev. from mean.
.06½	84.1	4.80	73.1	899	+ 10
.07½	0	−0.23	...	...	...
.08	0	−0.23	...	...	...
.10	0	−0.04	...	...	...
.11½	0	+0.12	...	...	...
.14	0	0.17	...	...	...
.16	0	0.17	...	...	...
.18½	74.1	3.73	64.5	897	+ 8
.19¼	75.0	3.85	65.2	905	+16
.20	74.8	3.76	65.1	887	− 2
.20¾	74.2	3.73	64.5	896	+ 7
.21½	73.9	3.68	64.2	892	+ 3
.22½	0	−0.24	...	...	...
.23½	0	−0.18	...	...	...
.28	0	+0.18	...	...	...
Average				889.0	±8.5

partial volumes of potassium, sodium, lithium, and hydrogen iodides in molal solution, as calculated from existing data.

The partial volume for any solute may be determined from successive weighings of a picnometer filled with solutions of different concentrations. Let V be the volume in ccm. held by the picnometer, and let it contain M gm. of solution when the percentage concentration of the solute in question is $P \times 100$, and M' gm. of solution of concentration $P' \times 100$. The amount of solvent in the picnometer when it is weighed containing solution of concentration P is $(1-P)\,M$, hence the volume of a solution of concentration P' containing the same amount of solvent would be $\frac{(1-P)\,M}{(1-P')\,M'}\,V$, and the amount of the solute in that solution would be $\frac{(1-P)\,M}{(1-P')\,M'}\,P'M'$ leading to the relation

$$\frac{\Delta v}{\Delta m} = \frac{\frac{(1-P)M}{(1-P')M'}V - V}{\frac{(1-P)M}{(1-P')M'}P'M' - PM} = \frac{(1-P)\,MV - (1-P')\,M'V}{(1-P)MP'M' - (1-P')\,M'PM}. \tag{3}$$

It is evident that the percentage accuracy in the determination of this quantity $\Delta v/\Delta m$ will be greater the greater the quantities Δv and

TABLE III.

(Dec. 10, 1909.)

Solution: Molal LiI, $\frac{1}{100}$ molal I_2.

r_2 = 29.45 cm., r_1 = 4.2 cm.

Standardization of Magneto: — $\frac{\text{Rev. of rotator per second}}{\text{Voltage of magneto}}$ = 0.8605 (6 exp. av. dev. ± .0040).

Time.	V = Voltage of magneto.	$E \times 10^3$ = E. M. F.	$n = V \times 0.8605$ =rev. per sec.	$\frac{E}{n^2} \times 10^6$.	Dev. from mean.
10.10	0	−0.2	...	...	...
.24	0	−0.20	...	...	...
.29	0	−0.15	...	...	...
.33	0	−0.30	...	...	...
.36	59.2	+3.25	50.9	1253	+70
.36½	60.2	3.26	51.8	1217	+34
.37	61.2	3.30	52.6	1192	+ 9
.37½	61.5	3.30	52.9	1179	− 4
.38¼	61.9	3.31	53.3	1163	−20
.39	0	−0.20	...	....	...
.40	0	−0.5	...	....	...
.41½	0	−0.4	...	....	...
.44	0	−0.13	...	....	...
.45½	64.1	+3.65	55.1	1202	+19
.46	64.8	3.72	55.8	1193	+10
.46½	65.1	3.68	56.0	1172	−11
.47½	66.0	3.76	56.8	1165	−18
.48	66.3	3.77	57.1	1157	−26
.49	0	−0.15	...	....	...
.49½	0	−0.2	...	....	...
.50	0	−0.25	...	....	...
.51	0	−0.25	...	....	...
.51½	0	−0.25	...	....	...
.53	0	−0.22	...	....	...
.56	0	−0.25	...	....	...
.58½	69.5	+4.23	59.8	1182	− 1
.59	69.3	4.22	59.6	1188	+ 5
.60	69.2	4.18	59.5	1180	− 3
11.00½	68.7	4.13	59.1	1183	0
.01	68.4	4.05	58.8	1171	−12
.01¾	68.3	4.04	58.75	1170	−13
.02¾	0	−0.14	...	....	...
.03¼	0	−0.18	...	....	...
.04	0	−0.14	...	....	...
.05½	0	+0.10	...	....	...
.07½	0	0.15	...	....	...
.09½	0	0.16	...	....	...
.11	0	0.09	...	....	...
.13	73.8	4.80	63.5	1190	+ 7
.13¼	74.0	4.80	63.7	1182	− 1
.14	74.0	4.75	63.7	1169	− 4

TABLE III. — *Continued.*

Time.	V = Voltage of magneto.	$E \times 10^3$ = E. M. F.	n = V × 0.8605 = rev. per sec.	$\frac{E}{n^2} \times 10^6$.	Dev. from mean.
.14½	74.1	4.77	63.75	1173	−10
.15½	74.5	4.84	64.1	1178	− 5
.16	74.8	4.85	64.4	1170	−13
.17	0	−0.19	...	....	...
.17½	0	−0.27	...	....	...
.18	0	−0.30	...	....	...
.19	0	−0.30	...	....	...
.20½	0	−0.28	...	....	...
.24	0	−0.32	...	....	...
.39	0	−0.36	...	....	...
...	...	...	...	....	...
...	...	...	...	....	...
...	...	...	...	....	...
.44	0	−0.20	...	....	...
.46	0	−0.03	...	....	...
.48	80.1	+5.62	68.9	1184	+ 1
.48½	79.8	5.61	68.7	1189	+ 6
.49	79.3	5.48	68.25	1177	− 6
.49½	79.1	5.47	68.1	1179	− 4
.50½	78.5	5.38	67.5	1180	− 3
.52	0	−0.16	...	....	...
.52½	0	−0.25	...	....	...
.53½	0	−0.30	...	....	...
.55	0	−0.25	...	....	...
.56½	0	−0.25	...	....	...
.58	0	−0.25	...	....	...
12.00	84.1	+6.30	72.4	1202	+19
.00½	84.5	6.33	72.7	1198	+15
.01	84.1	6.18	72.4	1180	− 3
.02	84.0	6.15	72.25	1178	− 5
.02½	84.1	6.16	72.4	1175	− 8
.03	0	−0.18	...	....	...
.04	0	−0.35	...	....	...
.04½	0	−0.35	...	....	...
.05½	0	−0.25	...	....	...
.06¾	0	−0.07	...	....	...
.09½	0	+0.15	...	....	...
Average				1183	±11.7

TABLE IV.

(Nov. 20, 1909.)

Solution: Molal HI, approx. $\frac{M}{50}$ I_2.

r_2 = 29.43 cm., r_1 = 4.51 cm.

Standardization of Magneto: — $\frac{\text{Rev. of rotator per second}}{\text{Voltage of magneto}}$ = 1.0125 (6 exp. av. dev. ± .0095).

Time.	V = Voltage of magneto.	$E \times 10^3$ = E. M. F.	$n = V \times 0.0125$ = rev. per sec.	$\frac{E}{n^2} \times 10^6$.	Dev. from mean.
10.53	0	−0.38	...	...	...
11.10	0	−0.45	...	...	...
.17	0	+0.50	...	...	...
.18½	63.0	1.55	63.8	380	+ 4
.19½	70.0	1.83	70.8	365	−11
.20	70.2	1.90	71.0	377	+ 1
.20½	70.0	1.90	70.8	379	+ 3
.21½	0	−0.17	...	...	...
.22½	0	−0.25	...	...	...
.24	0	−0.22	...	...	...
.25	0	−0.15	...	...	...
.26	0	−0.1	...	...	...
.28	0	−0.08	...	...	...
.29½	73.5	+2.13	74.4	385	+ 9
.30½	76.0	2.17	76.9	367	− 9
.30	76.1	2.25	77.0	379	+ 3
.31½	75.5	2.25	76.4	385	+ 9
.32	75.8	2.25	76.7	382	+ 6
.33	0	−0.12	...	...	...
.33½	0	−0.20	...	...	...
.35	0	−0.16	...	...	...
.36½	0	−0.12	...	...	...
.38 (?)	0	−0.01	...	...	...
.45	78.5	+2.35	79.4	373	− 3
.46	79.0	2.33	79.9	365	−11
.46½	78.5	2.40	79.4	381	+ 5
.47	78.2	2.35	79.1	376	0
.48	0	−0.10	...	...	...
.48½	0	−0.15	...	...	...
.49½	0	−0.15	...	...	...
.50½	0	−0.10	...	...	...
.52	0	−0.07	...	...	...
.53	0	−0.02	...	...	...
.54½	79.0	+2.45	79.9	384	+ 8
.55	80.5	2.45	81.4	370	− 6
.56	81.0	2.45	81.9	366	−10
.57	0	−0.10	...	...	...
.57½	0	−0.15	...	...	...
.58	0	−0.15	...	...	...
.60	0	−0.07	..	...	...
12.03	0	+0.06	...	...	...
Average				375.9	±6.1

Δm; in other words, the farther apart the concentrations P and P' are taken. On the other hand, if we are interested in the partial volume of the solute at the concentration P, we must determine the limit of $\Delta v/\Delta m$ as P' is brought nearer and nearer to P. A satisfactory solution of the problem may often be obtained by determining $\Delta v/\Delta m$ for several different values of P', if possible making it in some cases larger and in others smaller than P. These values will then permit a close estimate of the limit of $\Delta v/\Delta m$ at the concentration P.

If a calculation of the quantity $\Delta v/\Delta m$ from specific gravity data is desired, equation (3) has to be changed merely by the substitution of the densities d and d' for the weights M and M' and by placing the volume V equal to unity, giving the formula

$$\frac{\Delta v}{\Delta m} = \frac{(1-P)d - (1-P')d'}{(1-P)dP'd' - (1-P')d'Pd} \qquad (4)$$

For the purpose of determining the partial volume of iodine in iodide solutions picnometer weighings were made of a solution of approximately quarter normal potassium iodide containing varying quantities of iodine. The partial volume was desired in solutions very dilute in iodine ($\frac{M}{100}$ I_2), and hence the solutions were made as dilute in iodine as was consistent with a reasonable degree of accuracy in the calculation of $\Delta v/\Delta m$ between pure potassium iodide solution and the iodine solution in question. It is especially desirable to make the determinations with solutions as dilute as the accuracy of the method will permit, since the exact measurements of Kohlrausch and Hallwachs [17] have shown that for several substances there is a rapid change in partial volumes when considerable dilutions are reached.

For the measurements, two separate solutions were made up from the same KI solution each containing about 1.5% I_2. The concentration of the solutions was determined by weighing the iodine used and transferring it directly from a glass-stoppered weighing tube into the solution. The amount of solution used was also determined by weight. From each of these original solutions two more were prepared by dilution with weighed quantities of the pure potassium iodide solution, giving in all six iodine solutions whose densities were determined. The density of the original KI solution was also determined.

The volume of the picnometer used was about 21 cc., and was exactly determined by weighing the picnometer filled with pure water. The

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[17] Wied. Ann. **53**, 14 (1894).

picnometer used was of the form recommended by Ostwald and Luther and the setting of the meniscus in the capillary was made in a thermostat kept constant at 25° C. For each solution, at least two weighings were made, the picnometer being replaced in the thermostat and the meniscus reset between weighings. The weighings were made immediately after removing the picnometer from the thermostat and wiping off the surface. The successive weighings of the same solution usually agreed within two tenths of a milligram. Proper corrections were made for the buoyancy of the air. The iodine was prepared by resubliming with potassium iodide.

The results are presented in the following table :

No. of Sol.	$\% I_2$	Density	$\Delta v/\Delta m$
1	1.630	1.03955	0.2370
2	1.506	1.03842	0.2390
1′	1.099	1.03525	0.2362
2′	1.014	1.03452	0.2410
1″	0.8292	1.03310	0.2385
2″	0.7538	1.03250	0.2374
0	0	0.02660	0

The last column gives the value $\Delta v/\Delta m$ between a solution containing no iodine and one of the concentration indicated. No evidence is present of a systematic variation of $\Delta v/\Delta m$ with the concentration. Omitting the value of $\Delta v/\Delta m$ for solution no. 2′, the average value is $\Delta v/\Delta m = 0.2376 \pm 0.0004 \left(= \frac{\text{average dev.}}{\sqrt{5}}\right)$, and this was taken as the partial specific volume of iodine at great dilutions in potassium iodide solution. The specific volume of pure iodine from the data in Landolt and Börnstein is 0.202.

In the following tables the results are given for some calculations of $\Delta v/\Delta m$ for potassium, sodium, lithium and hydrogen iodides in aqueous solution at 20° C. from the data of Wegner and of Perkin given in Landolt and Börnstein.

POTASSIUM IODIDE (WEGNER).

$P \times 100$.	$P' \times 100$.	$d_{20/4}$.	$d'_{20/4}$.	$\Delta v / \Delta m$.
10.27	15.12	1.0784	1.1205	0.283
10.27	20.04	1.0784	1.1665	0.285
15.12	20.04	1.1205	1.1665	0.286

For a concentration of one mol per kg. water $P \times 100 = 14.2$, dv/dm taken as 0.284 cc.

Specific volume of the solid (L & B) 0.326 cc.

SODIUM IODIDE (WEGNER).

$P \times 100$.	$P' \times 100$.	$d_{20/4}$.	$d'_{20/4}$.	$\Delta v / \Delta m$.
6.25	9.28	1.0483	1.0743	0.240
6.25	14.70	1.0483	1.1239	0.242
9.28	14.70	1.0743	1.1239	0.244

For a concentration of one mol per kg. water $P \times 100 = 13.0$, dv/dm taken as 0.244 cc.

Specific volume of the solid (L & B) 0.282.

LITHIUM IODIDE (WEGNER).

$P \times 100$.	$P' \times 100$.	$d_{20/4}$.	$d'_{20/4}$.	$\Delta v / \Delta m$.
0	5.90	0.9997	1.0437	0.262
0	10.54	0.9997	1.0825	0.2615
5.90	10.54	1.0437	1.0825	0.261

For a concentration of one mol per kg. water $P \times 100 = 11.8$, dv/dm taken as 0.2605 cc.

Specific volume of the solid (L & B) 0.246.

Hydrogen Iodide (Perkin).

$P \times 100$.	$P' \times 100$.	$d_{20/20}$.	$d'_{20/20}$.	$\Delta v/\Delta m$.
0	20.77	1.0000	1.1754	0.2815
0	31.77	1.0000	1.2962	0.2805
20.77	31.77	1.1754	1.2962	0.2795

For a concentration of one mol per kg. water $P \times 100 = 11.3$, dv/dm taken as 0.2815 cc. (20°) or 0.282 true cc.

It is interesting to note in the cases of potassium and sodium iodides that the specific volume of the pure substance is greater than the partial specific volume of the substance in solution, and in these cases there is a slight but definite increase in the partial volume as the solution becomes more concentrated, that is, as it approaches the pure salt. Moreover, in the case of lithium iodide, although here the partial volume in solution is greater than the volume of the pure substance it also approaches the latter as the solution becomes more concentrated.

9. Calculation of the Transference Number.

Substituting for π its value, and for F the value 96580, equation (2) may be written:

$$T_c = \left(\frac{4.895 \times 10^{10}}{(r_2^2 - r_1^2)} \frac{E}{n^2} + M_I (1 - v_{I_2} d) \right) \div M_s (1 - v_s d).$$

Using the average values of E/n^2 the transference number of the cation T_c was calculated for each of the salts. The data and results are given below:[18]

Solution.	$\frac{E}{n^2} \times 10^6$.	$(r_2^2 - r_1^2)$	M_s	v_s	M_I.	v_{I_2}.	d_s.	T_c.	P. E.
$\frac{M}{1}$ KI, $\frac{M}{100}$ I_2	657.1	846.0	166.0	0.284	126.9	0.2376	1.115	0.486	±0.001
$\frac{M}{1}$ NaI, $\frac{M}{100}$ I_2	889.0	849.3	149.9	0.244	126.9	0.2376	1.110	0.385	±0.001
$\frac{M}{1}$ LiI, $\frac{M}{100}$ I_2	1183.	849.3	133.9	0.2605	126.9	0.2376	1.096	0.268	±0.0013
$\frac{M}{1}$ HI, $\frac{M}{50}$ I_2	375.9	846.2	127.9	0.282	126.9	0.2376	1.090	0.816	±0.001

18 The values of E/n^2 are, of course, all negative since the current tends to flow from the outer to the inner electrode.

The Accuracy of the Results.

The last column in the table gives the extremely small "probable error" introduced into the value of the transference number, by the deviations between the different measurements of E/n^2. It was obtained by dividing the mean deviations by the square root of the number of observations. It is not a satisfactory measure of the reliability of the transference numbers, since there is considerable probability that the "residual" electromotive force existing between the electrodes during rotation were more likely to be in one direction than the other. The value ± 0.010 may be taken as a fairer measure of the probable accuracy of the determinations. In the case of KI, this would correspond to an average error of about 0.1 millivolt in the electromotive force.

The Effect of Pressure on the Results.

In deriving the equation on page 114 for the potential difference of a cell under the influence of centrifugal force, the tacit assumption was made that the quantities v_s, v_{I_2} and d are, throughout the solution, the same as those calculated from density measurements made at atmospheric pressure. This is not strictly true, since there is considerable pressure produced in the solution by the centrifugal force. The following equation in which the quantities are considered as variables is exact.

$$10^7 EF = \int_{r_1}^{r_2} 4\,\pi^2 n^2\, r\,[T_c M_s (1 - v_s d) - M_I (1 - v_{I_2} d)]\, dr \qquad (5)$$

The derivation of the equation is obvious. $10^7 EF$ is the external work obtained when one faraday of electricity flows through the solution under the potential difference E produced by the centrifugal action. This quantity of energy is equal to the work required to restore the solution to its original condition. To accomplish this $T_c M_s$ gm. of salt must be brought back from r_2 to r_1, and this quantity, multiplied by $v_s d$, is the number of gm. of displaced solution which will automatically move in the opposite direction as we move the salt through the solution, M_I and $v_{I_2} d$ are the same quantities for the iodine, which has to be transferred from r_1 to r_2. The centrifugal force acting on the material which is to be moved is the product of the mass into $4\pi^2 n^2 r$ and the total work done is obtained by multiplying by dr, and integrating between r_2 and r_1. If the integration is made on the assumption that v_s, v_{I_2} and d do not vary with r, that is, with the pressure, the equation actually used is obtained. As a matter of fact both the density of the solution and the partial volume of the salt will vary with the pressure, and hence as we move the salt through the solution from one electrode to the other at each level it will displace a different weight of solution, and will hence be buoyed up by a different amount. We have no data for the variation of v_s and v_{I_2} with the pressure, but an upper limit for the error which has been introduced by taking them as constant, may be calculated.

Assuming that $v_s d$ and $v_{I_2} d$ vary linearly with the pressure, since the pressure equals $2\pi^2 n^2 r^2 d$ [19] where d is approximately unity, we may write the equations

$$v_s d = \underline{v_s d}_0 (1 + 2\pi^2 n^2 r^2 a_1),$$

$$v_{I_2} d = \underline{v_{I_2} d}_0 (1 + 2\pi^2 n^2 r^2 a_2),$$

where $\underline{v_s d}_0$ and $\underline{v_{I_2} d}_0$ are the values for zero pressure, and a_1 and a_2 are the linear coefficients for the fractional change of $v_s d$ and $v_{I_2} d$ with the pressure. Substituting into equation (5) and integrating we obtain

$$E = \frac{2\pi^2 n^2}{10^7 F} (r_2^2 - r_1^2) \left[T_c M_s (1 - \underline{v_s d}_0) - M_I (1 - \underline{v_{I_2} d}_0) \right]$$

$$- \frac{2\pi^4 n^4}{10^7 F} (r_2^4 - r_1^4) (T_c M_s a_1 \underline{v_s d}_0 - M_I a_2 \underline{v_{I_2} d}_0).$$

The second term in the expression is seen to be the error introduced into the calculation of the electromotive force, by neglecting the change of the density and partial volumes with the pressure. Since the partial volumes will probably decrease with pressure and partly neutralize the increase in density, to obtain an upper limit for the error let us put a_1 and $a_2 = 4 \times 10^{-12}$, the value for the fractional change in the density of water per $\frac{\text{dyne}}{\text{cm}^2}$. Making this substitution, the value of the above term becomes only 0.00018 millivolts, for the experiment on potassium iodide, at 80 revolutions per second. We see that no appreciable error has been introduced by neglecting the change in the density and the partial volumes produced by pressure.

Before leaving the consideration of the pressure gradient in the tube, it must be pointed out that the transference number determined in these experiments is the transference number which exists when the solution is actually under the influence of that particular pressure gra-

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[19] This is strictly true only when the liquid reaches way to the center of the rotating apparatus.

dient. Owing to their enormous "internal pressure" the properties of liquids are, however, in general but little affected by changes in the external pressure.

The Effect of the Dissolved Iodine on the Results.

In the case of the alkali iodides one per cent of the $\bar{I}$ ion was changed into I_3 ion by the iodine present. In the HI solution, owing to oxidation about twice as much iodine was present. The transference number would, however, be only slightly affected by the small admixture of I_3 ion.[20]

10. Comparison of the Results with other Transference Measurements.

The available data on the transference numbers of iodides are very few, and, of these, many are vitiated by the use of membranes in the apparatus.[21] The most satisfactory data for comparison are the values determined by Washburn[22] for the Hittorf transference numbers of the alkali *chlorides* (at a concentration 1.2 — 1.3 molal), and the value for the Hittorf transference number of hydrochloric acid (at a concentration 1.0 molal) which can be calculated from Buchböck's determination of true transference number and hydration. This comparison is made in Table V. The transference numbers for the chlorides and iodides at infinite dilution, calculated from conductivity data are also given in the table, and finally, values obtained for $\frac{N}{10}$ KI and $\frac{N}{10}$ NaI by Dennison[23] using the method of Dennison and Steele and a value by Bein[24] for $\frac{N}{20}$ KI. This last is the only available datum for these iodides obtained by the Hittorf method without the use of membranes.

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[20] For data on the mobility of the I_3 ion see Burgess & Chapman, J. Chem. Soc. Trans. **85**, 1305 (1904), Bray & MacKay, Journ. Amer. Chem. Soc., **32**, 914 (1910).

[21] See McBain (Proc. Wash. Acad. Sci. **9**, 1–78 (1907); University of Toronto Studies, Papers from the Chem. Laboratories, No. 67, for a complete collection of the experimental data on transference numbers.

[22] Washburn, Tech. Quart. **21**, 164 (1908); Journ. Amer. Chem. Soc., **31**, 322 (1909); Buchböck, Z. physik. Chem. **55**, 563 (1906).

[23] Dennison, Trans. Faraday Soc., **5**, 165 (1909). It has been shown by Lewis, J. Am. Chem. Soc. **32**, 862 (1910), that the method of Dennison and Steele gives, after applying a calculable correction, the Hittorf transference number and not the true transference number as stated by Washburn.

[24] Bein, Z. physik. Chem. **27**, 1 (1898).

TABLE V.

TRANSFERENCE NUMBER OF THE ANION.

Halide of	Iodide, Centrifugal.	Chloride, Washburn. Buchböck.	Iodide, Infinite Dilution.[25]	Chloride, Infinite Dilution,[25]	Iodide, Dennison.	Iodide, Bein.
K	0.514	0.518	0.507	0.503	0.514	0.505
Na	0.615	0 634	0.604	0.601	0.624	....
Li	0.732	0.722	0.665	0.662	....	....
H	0.184	0.160 [26]	0.174	0.172	....	....

The agreement between the results presented in the table for the iodides with those for the chlorides is satisfactory. The greatest deviation occurs in the values for the two halogen acids. Hydrochloric acid,[27] however, is known to be abnormal in its behavior since the transference number passes through a minimum at a concentration below normal, and the same effect might occur in hydriodic acid solution. Furthermore, since the electromotive force measured was so small in the case of hydriodic acid any constant error in the nature of a residual electromotive force might have had a large effect.

11. SUMMARY.

In this article an apparatus and procedure have been described for determining transference numbers by the centrifugal method, first

[25] These values for the transference number at infinite dilution were calculated from the latest conductivity data of Kohlrausch, Zt. f. Electrochem. **13**, 333 (1907).

[26] This value for the Hittorf transference number of molal HCl is calculated from Buchböck's determination of true transference number and hydration by the relation connecting those quantities as developed by Washburn. Washburn himself gives 0.18 for the Hittorf transference of HCl, a value which was taken from Kohlrausch's tables, but it is probably considerably too high. For HCl, 0.97 molal, Riesenfeld u. Reinhold, Zts. f. Phys. Chem. **68**, 440 (1909), obtain the value 0.155, and Hopfgartner, Zts. Phys. Chem. **25**, 115 (1898), obtains 0.159 for 0.9 molal HCl.

[27] Riensenfeld u. Reinhold, Zts. f. phys. Chem. **68**, 440 (1909).

tried by Des Coudres. The method consists in the measurement of the electromotive force produced between electrodes placed at the central and peripheral ends of a rotating tube containing the electrolyte. An equation can be derived, connecting this electromotive force and the transference number of the salt with the speed of rotation, the density of the solution, and the molecular weight and the "partial" specific volume of the substances involved, quantities which can be independently determined.

Some of the details of the construction of the rotating apparatus may have general interest. A distinctive feature of the apparatus was the method of driving the rotator as a "spinning top," which had not been specially balanced and which was hung below its fixed point of support. A simple arrangement of cords was devised for steadying the shaft of the rotating top and preventing precessional motion. A thrust bearing suitable for high speeds is also described. It consists of a series of ball bearings which distribute the total relative motion. An equation is given for calculating the dimensions of a rotating arm of uniform strength with the cross section increasing in size towards the center of rotation.

With this apparatus measurements were made of the electromotive force produced by the rotation of molal solutions of potassium, sodium, lithium, and hydrogen iodides. As predicted from the equations, the electromotive force was found to increase proportionately to the square of the speed of rotation. From the data the transference number was calculated for the four solutions, and found to agree as well as could be expected, with the available results of other methods of determination.

It was pointed out in connection with a kinetic derivation of the electromotive force relation, that the production of an electromotive force by centrifugal force is a proof of the presence of free ions in an electrolytic solution or at least of a certain degree of electrical polarization in the molecules.

The writer desires to express his gratitude to Professor A. A. Noyes, the Director of the Research Laboratory, whose interest and support made possible the completion of this research. Thanks are also due to Professor Elihu Thomson and Dr. Sanford A. Moss of the General Electric Company, through whose kind offices was loaned the steam turbine used in this investigation. The writer also had the benefit of Dr. Moss's extensive experience with high-speed rotation, and is indebted to Professor George B. Haven of the Institute for his assistance in checking a large number of calculations of machine design which were made before the final apparatus was constructed.

The construction of the apparatus was made with the help of two grants of money from the Cyrus M. Warren Fund of the American Academy of Arts and Sciences. The Chemical Department of the Institute was also very generous in its support of this costly research.

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FEEDING REACTIONS OF THE ROSE CORAL (ISOPHYLLIA).

By F. W. Carpenter.

FEEDING REACTIONS OF THE ROSE CORAL (ISOPHYLLIA).[1]

By F. W. Carpenter.

Presented by E. L. Mark. Received July 5, 1910.

Introduction.

A number of investigators have studied in detail the structure of the polyps of the Madreporaria, the group of corals which includes the great majority of reef-building forms. Comparatively little attention has been paid, however, to the activities of the living zoöids, although the ultimate result of these activities, the formation of immense quantities of limestone, is a remarkable biological phenomenon, and one of very considerable geological importance.

The present paper is intended as a contribution to the field of coral physiology, and is concerned mainly with the reactions which follow the stimulation of the polyp by means of nutrient substances. Since the corals belong to the phylum of the Coelenterata, in which we have learned to look for the first appearance of a differentiated mechanism for neuro-muscular reflexes, the facts here recorded have at least an incidental bearing on the question of the phylogenetic origin of the nervous system.

The work on which the paper is based was done at the Bermuda Biological Station for Research in the summer of 1909. During a period of several weeks, observations were made on the feeding behavior of rose-coral polyps belonging to the genus Isophyllia. Of this genus three species are recorded by Verrill (: 02) for the Bermuda Islands, viz., *dipsacea*, *fragilis*, and *multiflora*, the first two being much more common than the last. Since the specific characters lie in the skeleton, it is not possible to determine with certainty the species of a living colony, but it is known from the subsequent examination of several skeletons that *dipsacea* occurred in the material used, and in all probability *fragilis* was also represented. The question of species

[1] Contributions from the Bermuda Biological Station for Research, No. 20; also Contributions from the Zoölogical Laboratory, University of Illinois, under the direction of Henry B. Ward, No. 4.

is here an unimportant one, as the feeding reactions of the various members of the genus are doubtless essentially the same.

The rose corals at Bermuda form small convex colonies, seldom measuring more than five or six inches in diameter. They occur abundantly in shallow water near shore, often in quiet bays partially shut off from the open sea, where the plankton, which apparently furnishes them with food, is considerably diminished in quantity. Their suc-

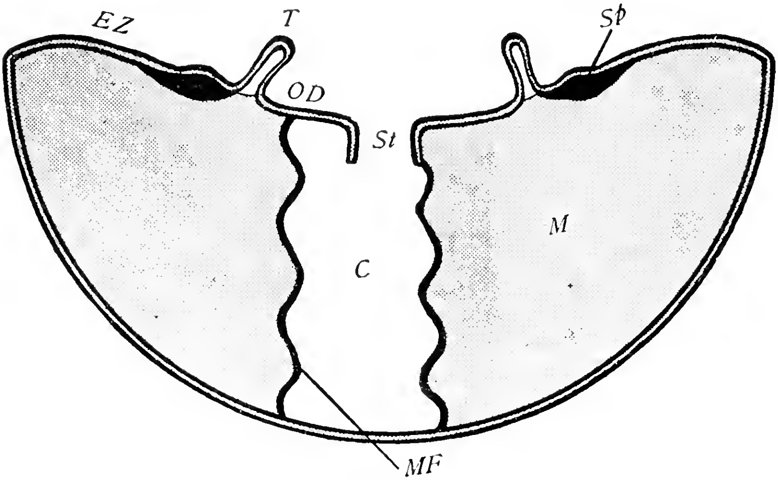

FIGURE 1. Diagrammatic vertical section through an expanded polyp. *C*, coelenteron or gastrocoelomic cavity; *EZ*, edge zone; *M*, mesentery; *MF*, mesenterial filament; *OD*, oral disk; *Sp*, sphincter muscle; *St*, stomodaeum; *T*, tentacle.

cessful adaptation to these apparently unfavorable situations, in which most corals do not thrive, suggests a very efficient mechanism for availing themselves of such food as may be present.

The individual polyps of the rose-coral colony are comparatively large, the oral surface often measuring an inch in diameter. The structural features of this surface are shown in the Plate, which is from a photograph of a small portion of a partially expanded colony. In the center of the polyp is the mouth (*M*), a small oval or circular aperture, which opens into the stomodaeum. Bordering the mouth is the flat ciliated oral disk (*OD*). This is bounded by a circle of short tentacles (*T*), external to which is a thickened ciliated edge zone (*EZ*), raised somewhat above the level of the oral disk, and forming the wide rim which completes the oral surface peripherally. A diagrammatic vertical section through an expanded polyp (Figure 1) reveals its cup-like shape, and shows the short stomodaeum (*St*) leading into the coelenteron (*C*), which is partially subdivided by the radially placed mesenteries (*M*). Each mesentery, whether complete or incomplete, bears along its free edge a coiled mesenterial filament (*MF*). The inner margin of the edge zone in such a section shows the cut ends of a large circular endodermal muscle (*Sp*), which acts as a sphincter. In his monograph on West Indian madreporarian polyps, Duerden (:02) states that this sphincter muscle is more strongly developed in Isophyllia than in any other genus of corals studied by him.

Owing to its large size the rose-coral polyp lends itself readily to a study of its reactions to food materials. The colonies thrive well in the laboratory, where they may be kept alive in running sea-water for weeks. The best time for experimenting is at night, when the polyps become fully expanded, and remain so in diffuse artificial light. However, during the daytime all the feeding reactions may be obtained from the partially contracted polyps, except those of the tentacles, the latter then being withdrawn under the cover of the inner margin of the edge zone.

The Feeding Reactions.

Movements of Oral Surface and Mesenteries. When a stream of sea-water gently ejected from a pipette falls on the oral disk of a polyp, there is usually no perceptible response. If a drop of a concentrated solution of Liebig's extract of meat in sea-water is similarly applied, there is an immediate contraction of the retractor muscles of the mesenteries, which draws the oral disk downward. Simultaneously a vigorous contraction of the sphincter muscle takes place, and the edge zone is drawn inward over the oral region, above which it forms a temporary roof (Figure 2). Under the influence of a strong stimulus the constantly diminishing central opening in this roof may finally be obliterated by the meeting of the edges of the aperture, and the oral disk below may thus become completely hidden from sight.

Eversion of Stomodaeum and Extrusion of Mesenterial Filaments. While this muscular reaction is taking place, and before the oral disk is concealed, the stomodaeum may be observed to become everted, and to project into the supra-discal space (Figure 2). The eversion of the stomodaeum is accompanied by the extrusion of mesenterial filaments through the mouth, together with the portions of the mesenteries to which they are attached. Not only are mesenterial filaments thrust through the oral aperture, but they are also extruded through the discal wall itself (Figure 2). Duerden (:02), who has observed this behavior of the mesenterial filaments in many madreporarian polyps, is convinced by his histological studies that there are, however, no permanent port-holes, or cinclides, in the body wall. He believes the mesenterial filaments may make temporary apertures for themselves in any part of the oral disk or wall of the column. When the filaments are withdrawn the orifices close after a few minutes, and leave no traces of their former existence.

The eversion of the stomodaeum, accompanied by partial extrusion of mesenterial filaments through the mouth, may follow a much less

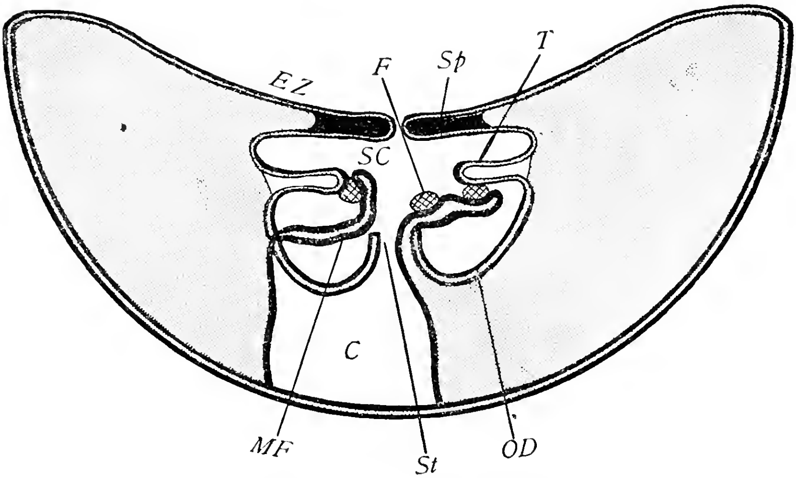

FIGURE 2. Diagrammatic vertical section through a contracted feeding polyp.

C, coelenteron; *EZ*, edge zone, folded over; *F*, food particle; *MF*, mesenterial filament, extruded; *OD*, oral disk; *Sp*, sphincter muscle, contracted; *St*, stomodaeum, everted; *SC*, supra-discal cavity; *T*, tentacle.

vigorous stimulus than the one just described. The reaction occurs when a piece of filter paper soaked in diluted meat juice is dropped on the oral disk. The latter becomes depressed, although the stimulus is not sufficient to cause an overfolding of the edge zone. Under these conditions the gullet may be seen to stretch toward the stimulating object. If it reaches the filter paper, this may become partially inclosed by the everted stomodaeum, and may lie for some time in contact with the mesenterial filaments emerging from it, without being actually swallowed. On comparatively few occasions have the bits of filter paper been observed to pass into the coelenteron of the polyp.

Reactions of the Tentacles. After sundown the tentacles of rose-coral polyps kept in the laboratory become expanded, and may be experimented upon in diffuse artificial light. If exposed to the direct rays of an acetylene bicycle lamp the tentacles contract after a few minutes, but they remain extended in diffuse light of sufficient intensity to render them clearly visible to the observer.

When the knob-like extremity of a tentacle is touched with a splinter of wood the latter immediately becomes affixed to the end of the tentacle, which is known to be loaded with nematocysts. Possibly the wood is empaled by the numerous nematocyst threads, or it may be held by some adhesive substance. However this may be, the splinter becomes so securely fastened that considerable force is needed to pull it away. A tentacle will often be drawn out two or three times its own length before the splinter is freed, and I have sometimes feared it would be torn from the body in the operation.

If an expanded tentacle is touched on one side with the splinter, there is a quick muscular response to the contact stimulus. The tentacle immediately bends in the direction of the stimulating object, which it affixes to the terminal knob.

Similar results follow the use of a bit of filter paper in place of the splinter, but if the paper is first dipped in meat extract, in addition to the tentacular reaction the oral disk begins to sink and the edge zone to fold over. In this reflex the distance of the receptor (the end of the tentacle) from the effector (the endodermal body muscles) is worthy of note.

Action of the Cilia. Both the oral disk and the edge zone are abundantly provided with cilia. Carmine particles dropped anywhere on their surfaces are rapidly transferred to the outer margin of the polyp, showing that the usual beat of the cilia is toward the periphery. Carmine particles dropped at the very margin of the mouth pass immediately into the stomodæum, the cilia of which appear to beat constantly inward.

When carmine grains which have previously been soaked in meat extract are dropped on the oral disk they also are usually carried to the peripheral margin of the polyp, even though the stimulation of the nutrient material may be sufficient to cause a slow sinking of the oral surface. I repeated this experiment many times with different polyps, and occasionally succeeded in obtaining a reversal of the cilia, which caused the carmine grains to move toward, and not away from, the mouth. I was, however, unable to obtain consistent reactions, and never could predict what the result would be. Analyses of the brand of meat extract used (Liebig's) indicate that it contains several of the substances shown by Parker (:05[a] and :05[b]) to be the effective stimuli in producing a reversal of the cilia on the lip zone of Metridium. Of these substances potassium chloride, creatine, and albumoses are present in the extract (Street : 08). The failure to obtain uniform reversals may have been due to the varying differences in the degree of dilution of the stimulating materials, or to unknown factors associated with the physiological condition of the polyps. As will be seen later, ciliary action plays an insignificant part in the feeding process of the rose coral, so that a prompt and definite reversal of the cilia of the oral disk is not a necessary factor in the operation of securing food.

Normal Feeding Behavior. This brief analysis of the responses of the polyp to chemical and tactile stimuli leads to an understanding of the way in which these various reactions are combined in normal feeding behavior.

It is generally believed that coral polyps take as food the small organisms found in the marine plankton, although there is little direct evidence in support of this view. Examination of the cœlentera of preserved specimens rarely reveals food material, the scarcity of which in the digestive cavities of the Madreporaria has been commented upon by Duerden, Hickson, Bourne, and Fowler (Pratt : 05). It has even been suggested that the zoöchlorellæ living in symbiotic relation with many corals may elaborate a part of the latter's food, and thus compensate for deficiencies in the supply from outside.

On the other hand it is to be noted that the diurnal expansion and contraction of coral polyps — their alternating periods of activity and quiescence — coincide respectively with the appearance and disappearance of the plankton in the surrounding waters. This can readily be observed in the case of the rose corals. During the daytime, when the sea about them is comparatively free from small organisms, the polyps appear partially contracted. No tentacles are to be seen. But after dark, when the myriad components of the plankton begin to swarm in the water, one may wade along the shore at low tide, and with the aid of a water glass and light from a bicycle lamp, he may readily see that the polyps are fully expanded, with tentacles well out. They have every appearance of being on the alert for food.

In order that I might observe under favorable conditions a polyp in the act of feeding upon what we may suppose to be its natural food, I placed a small colony in a dish of sea-water under a low-power binocular microscope. This was done by lamplight at about half-past nine in the evening when the polyps were well expanded. I then poured into the dish a small quantity of living plankton which a half hour before had been obtained with a tow-net in the locality from which the corals had been collected. The plankton consisted of small Crustacea, including many copepods, larvæ of various kinds, Protozoa, and other minute marine organisms. Almost immediately the tentacles of the polyp under observation began to turn in various directions. Most of the forms in the plankton were too small to be followed by the eye under the low-power lenses used, but I actually saw one copepod struck by and affixed to the knobbed end of a tentacle. Many organisms doubtless struck against the oral surface of the polyp. In a very few seconds the stomodæum became everted, mesenterial filaments were protruded, the disk sank, and the edge zone commenced to fold in toward the center. The latter movement continued until the tentacles and oral disk were completely roofed over and concealed from sight. In this condition the polyp remained for some time.

What takes place within the contracted polyp during this period of quiescence cannot be directly observed. But it seems to me highly probable that most of the organisms caught and held by the tentacles, or trapped by the overfolded edge zone, do not reach the gastrocœlomic cavity. I believe they are retained in the superficial chamber bounded by the oral disk below and the overfolded edge zone above, and here digested through contact with the extruded mesenterial filaments. The latter are known to be the digestive organs of the Actinozoa. They act upon proteid food by means of a tryptic ferment contained in the secretions of their gland cells, and then ingest or absorb the partially or wholly digested material (Jordan, :07; Pratt, :05). The mesenterial filaments can easily reach to all parts of the superficial chamber, and so be brought into contact with the food held by the tentacles. The attachment of the organisms to the tentacles appears from experiment to be so secure that it is difficult to see how the captured bodies can be discharged except by being digested away.

The mesenterial filaments are themselves well supplied with nematocysts, and these may be useful in securing any free organisms met with in the upper chamber. A few such organisms which are still at liberty to move may make their way into the diminished cœlenteron, although the cilia of the everted stomodaeum are no longer in a position to produce inhalent currents; but it seems probable that the greater part of the plankton captured by the polyp undergoes extra-coelenteric digestion in the way described.

When, under natural conditions, the polyp is feeding on small organisms, it is probable that a number of these must be captured by the tentacles, or come into contact with the oral disk, before a cumulative stimulus is obtained sufficient to cause the contraction of the sphincter and mesenterial muscles, and the extrusion of the mesenterial filaments. A few small carmine grains soaked in meat extract and dropped on the oral disk fail to bring about muscular reactions, even though the granules come into contact with the tentacles when being carried peripherally by the surface cilia. Furthermore, unless the stimulus is a strong one, the edge zone will not fold inward far enough to form a complete roof. As has been seen, a piece of filter paper soaked in dilute meat extract is not a sufficient stimulus to bring about the contraction of the sphincter muscle, although it does cause a sinking of the oral disk, with eversion of the gullet, and partial extrusion of mesenterial filaments through the mouth. But the roofing over of the oral disk is not a necessary part of the process of extra-coelenteric digestion, provided the food organisms are prevented from escaping by the tentacles or by the mesenterial filaments themselves.

As may now be seen, the cilia of the oral surface apparently play an unimportant rôle in the feeding process. Their chief function seems to be that of keeping the oral surface clean. Their abundance, and their persistency in driving foreign particles toward the periphery, may be correlated with the comparatively small amount of surface mucus, which in such polyps as Favia and Fungia forms a well-developed protective layer over the oral disk (Duerden, :06).

After the rose-coral polyp has completed the digestion of its food in the supra-discal cavity, the stomodaeum and mesenterial filaments are retracted, and the sphincter and mesenterial muscles relaxed. Water is then drawn in through the mouth by the action of the stomodaeal cilia, and the polyp expands into its resting condition. The cilia of the oral surface now come into play, and transport peripherally any undigested particles that may have been left on the oral disk. Such particles, as may be seen by watching carmine grains, are carried across the edge zone, and deposited in the grooves (*IG*, Plate) which separate adjacent polyps.

Feeding Reactions in Other Corals. In its method of taking food, as above described, the rose-coral zoöid differs from the polyps of the three other madreporarian genera whose feeding reactions have been observed. Two of these, Favia and Fungia, were studied by Duerden (:06) in the Hawaiian Islands. The oral disks of Favia and Fungia are not ciliated, but are well provided with gland cells, which secrete on the external surface an abundant coating of mucus. In this mucus foreign particles become embedded. Ordinarily the beat of the stomodaeal cilia is outward, but under the stimulus of meat juice the cilia reverse, and the strong inhalent currents thus produced waft into the mouth shreds and patches of the mucus, together with the foreign materials entangled or embedded in it. The writer evidently considers that these foreign materials furnish the polyp with its food.

The feeding reactions of the third genus, Caryophyllia, have been described briefly by Carlgren (:05). The cilia of the oral surface of this coral beat inward near the mouth, and outward in the peripheral region external to the tentacles. Food is brought by tentacular action into the circum-oral region, whence it is conveyed by cilia to the mouth and stomodaeum. Contractions of the oral disk and peristaltic movements of the gullet assist in the swallowing of large objects.

The feeding process in the related group of alcyonarian corals has been studied by Pratt (:05), who shows that in *Alcyonium digitatum* food is taken into the gastro-coelomic cavity by means of the muscular activities of the tentacles and gullet. Cilia are not mentioned as being concerned in the process. Within the body of the zoöid both intercellular and intracellular digestion take place through the agency of the mesenterial filaments.

It therefore appears that in all coral polyps heretofore investigated the food reaches the coelenteron and is here digested by the mesenterial filaments. But in Isophyllia the evidence points to an improvement in the method of appropriating food, which makes the polyp of this genus a very efficient plankton trap. It apparently does not risk losing its prey by attempting to transfer it by tentacular or ciliary movements to the mouth, and thence through the gullet to the usual digestive cavity. On the contrary, it affixes its prey firmly to its tentacles, and then, inclosing its captives in a more or less complete, though temporary, chamber formed on the external surface, it proceeds, by means of its mesenterial filaments, to digest its food *in situ*.

Allusion has been made to the statements of the morphologists in regard to the scarcity of food fragments in the coelentera of preserved madreporarian polyps. Whatever the reason for this may be in the case of other corals, in Isophyllia the failure to find such materials is readily explained by the extra-coelenteric digestion of at least the greater part of its food.

Evidences of Nervoid Transmission. The term "nervoid" is here used in preference to "nervous" because of the failure of the histologists to demonstrate differentiated tissues of undoubted nervous character in the polypal wall of the great majority of the Madreporaria, including Isophyllia. Duerden (:02) mentions the presence of a distinct ectodermal nerve layer in the tentacles of some species, — "Cladocera, Madrepora, and probably others," — but in his special description of a representative of the genus Isophyllia no reference is made to such a layer. A weak ectodermal muscle layer seems, however, to be of constant occurrence in the tentacles of the Madreporaria. As far as the polypal wall is concerned, Duerden and Ayres (:05) state that previous to their discovery of differentiated ectodermal nervous tissue in Coenopsammia no ectodermal columnar nerve or muscle layer had been found in any madreporarian polyp.

In spite of the apparent absence of adjustor tissues in the body wall of Isophyllia, a transmission of at least a nervoid character takes place from receptor to effector through intervening portions of the body. The evidence for this rests on the results of experiments with both chemical and tactile stimuli.

As has been stated in the preceding pages, the application of meat extract to the surface ectoderm is followed by an immediate contraction of the sphincter and mesenterial muscles, both of which are, according to Duerden (:02), situated in the endoderm. An impulse of some kind must, therefore, pass through the intervening mesogloea, which in Isophyllia is a layer of considerable thickness.

A longer distance must be traversed by the nervoid impulse when the retractor muscles of the mesenteries respond to a chemical stimulus affecting the distal end of a tentacle. This reaction, following the application of a bit of filter paper dipped in meat juice, has already been described.

There is evidence, furthermore, that the transmission of impulses is not confined to the polyp stimulated, but may extend throughout the colony. When meat extract is applied to a single polyp a widespread effect is produced. For example, in one experiment a small amount of concentrated meat extract was placed during the daytime on the central polyp of a colony with twenty-five oral apertures. The stimulating material soon spread over two adjacent polyps, which, with the central one, became tightly contracted. Meanwhile stomodaea began to be everted and mesenterial filaments extruded all over the colony, which in consequence presented, after two or three minutes, a very ragged appearance. The sphincter and retractor muscles of these outlying polyps were not appreciably affected.

It must be admitted that the explanation of this reaction as due to nervoid impulses from the central polyps is open to the objection that the stimulus may have been local, owing to the diffusion of the meat juice externally over the surface of the colony, or internally through the communicating coelentera. This objection cannot be raised, however, in the case of a colonial reaction induced by a tactile stimulus applied to a single polyp. Especially at night, when the corals are most sensitive, the touching of a centrally situated polyp with a piece of wood or a glass rod will often be followed not only by an immediate local muscular response, but also by a sudden contraction of the mesenterial muscles throughout the colony. The result is seen in the general sinking of the oral surfaces.

While the reaction in the zoöid actually touched might be accounted for by the direct stimulation of the endodermal muscles by mechanical pressure exerted through the overlying tissues, this explanation will not apply to the reaction in distant polyps. Nor can the movements in the outer portions of the colony be the result of the strain exerted by the contracting central polyp on contiguous polyps, and by these, when they in turn contract, on zoöids still more distant. For, as we have seen, a central polyp in a colony, contracting because of the direct application of a chemical stimulus, does so without affecting the retractor muscles of neighboring polyps. And, finally, the wide-spread reaction cannot be explained by assuming that the protoplasm of the colony is remarkably sensitive to slight molecular vibrations set up in it by touching one of the polyps. If this were so, the colonies would constantly be responding by obvious muscular contractions to vibrations in the surrounding medium or in the solid substratum on which they rest. That such reactions do not occur is evident from repeated observation.

When we turn from these experimental evidences of nervoid transmission to the histology of the Madreporaria, our attention is naturally directed to the mesogloea, through which we infer the impulses must pass, and in which we should, therefore, expect to discover at least some trace of transmitting tissues. According to Duerden (:02) "the mesogloea of coral polyps has generally been described as a perfectly structureless layer"; but the following quotation from page 416 of his monograph on West Indian Madreporaria shows that this statement does not hold for the form with which we are concerned:

"In large polyps, such as *Isophyllia dipsacea*, and also in Maeandrina, the mesogloea is rather thick, and minute connective-tissue cells occur sparsely throughout. In sections the cells are circular or oval in shape, with a central nucleus, and minute prolongations extend in all directions; many of these reach one or other of the surfaces of the layer, and there come into contact with the ectodermal or endodermal cells. In some instances the processes extend right across from one layer to the other, but are mostly disposed in an irregular stellate manner."

These "connective-tissue cells" are probably homologous with those forming the mesogloeal cell plexus in the related Alcyonaria. This plexus was formerly considered to be nervous in function, but the studies of Pratt (:05) on the digestive organs of the Alcyonaria indicate that the amoeboid cells whose processes give rise to the network are concerned with the ingestion and transportation of food. By means of these wandering cells "nutriment may be conveyed from the digestive endoderm cells of the zoöids to every portion of the colony." The system, therefore, in the opinion of the writer, "must be regarded as a nutritive as well as a sensitive plexus." More recently Kassianow (:08) has demonstrated in *Alcyonium digitatum* a well-developed ectodermal nervous system with ganglion and sense cells. He denies a nervous function to the branched cells of the mesogloea.

Returning to the histological structure of Isophyllia, it can be said that the so-called "connective-tissue cells" which bridge the mesogloea from ectodermal receptors to endodermal effectors fulfill, topographically at least, the requirements of a primitive correlating segment in a reflex mechanism. The ontogenetic history of these cells, and their exact structural interrelations, have not been worked out, so far as I am aware. But the alluring theory presents itself that these branching cells have migrated from the ectoderm, and, spreading throughout the mesogloea, have assumed contact relations one with another by means of their processes; and though they may not have become highly specialized nervous elements, we may suppose that their protoplasm has retained and augmented its primitive endowment of irritability. Possibly other primitive amoeboid characters have also been retained, such as the capacity for ingesting food, or for moving about by means of pseudopods, — both of which are suggested by Pratt's observations on the cells of the mesogloeal plexus of Alcyonaria.

The theory of the phylogenetic origin of the nervous system as outlined by Parker (:10), calls for the appearance of an adjustor tissue made up of primitive synaptic neurones as the next step in the process after the development of a sub-ectodermal nervous network, such as occurs in sea-anemones. Future studies, especially those of a histogenetic character, may reveal that this condition is realized in the loose aggregation of branching cells found in the mesogloea of coral polyps.

Summary.

1. When the rose-coral polyp is stimulated by the application of concentrated meat extract to the oral disk, the latter is drawn downward by the contraction of the retractor muscles of the mesenteries, and the margin of the oral surface is folded inward over the disk by the action of a well-developed sphincter muscle.

2. Meanwhile the stomodaeum is everted, and the mesenterial filaments are extruded both through the mouth and through temporary apertures in the oral disk.

3. The tentacles react quickly to contact stimulation, and affix the object which touches them to their knob-like distal ends, which are

heavily loaded with nematocysts. When the end of a tentacle is chemically stimulated with meat extract, the retractor muscles of the polyp contract.

4. Carmine grains dropped on the oral surface of an expanded polyp are transferred by ciliary action to the periphery. When the particles of carmine have previously been soaked in meat juice, the cilia usually continue to beat in an outward direction; occasionally, however, they reverse their effective strokes. The chief function of the cilia seems to be that of keeping the oral surface clean.

5. When plankton is fed to a polyp the small organisms are affixed by the tentacles, the oral disk sinks, and the marginal zone folds inward until it completely roofs over the tentacles and the depressed oral disk. Into the superficial chamber thus formed, the stomodaeum and mesenterial filaments project, and here the mesenterial filaments, which are the digestive organs of the polyp, probably digest and ingest or absorb the captured plankton, little of which appears to find its way into the reduced gastro-coelomic cavity. Extra-coelenteric digestion apparently takes place, therefore, in rose-coral polyps.

6. There is experimental evidence of the transmission of impulses of at least a nervoid character from ectodermal receptor cells through the mesogloea to endodermal effectors (muscles). This transmission is not confined to a single polyp, but may pass from one polyp to another.

7. It is known that branching cells (so-called "connective-tissue cells") occur in the mesogloea of Isophyllia. These extend from the ectoderm to the endoderm, and so have the topographical relations of adjustor cells, placing the receptor in communication with the effector. In the absence of exact information as to the origin, mutual relationships, and functions of these cells, it is nevertheless suggested that future studies may show them to be primitive synaptic neurones.

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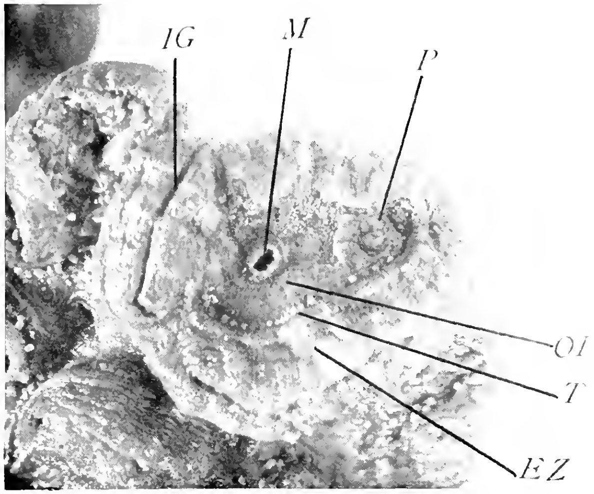

Small portion of a colony of Isophyllia showing oral surface of a partially expanded polyp. $\times 1\frac{2}{3}$.

EZ, edge zone; *IG*, interpolypal groove; *M*, mouth; *OD*, oral disk; *P*, small polyp which has arisen by fission from the larger polyp on the left; *T*, tentacles.

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CONTRIBUTIONS FROM THE RESEARCH LABORATORY OF PHYSICAL CHEMISTRY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY. — NO. 58.

ON FOUR-DIMENSIONAL VECTOR ANALYSIS, AND ITS APPLICATION IN ELECTRICAL THEORY.

BY GILBERT N. LEWIS.

CONTRIBUTION FROM RESEARCH LABORATORY OF PHYSICAL CHEMISTRY OF THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.— No. 58.

ON FOUR-DIMENSIONAL VECTOR ANALYSIS, AND ITS APPLICATION IN ELECTRICAL THEORY.

By Gilbert N. Lewis.

Received June 28, 1910.

The great generalization of Einstein, known as the principle of relativity, and its interpretation by Minkowski, have opened a new domain of natural science. The apparent artificiality and paradox of some of the consequences of the relativity theory disappear completely when with Minkowski we regard the science of kinematics as identical with the geometry of four-dimensional space.

Minkowski [1] and, following him, Abraham [2] have made an important beginning in the use of four-dimensional vector analysis. In general, however, Minkowski used for his more important deductions, not the vectorial method, but the matrix calculus of Cayley. This was undoubtedly due to the restricted and specialized character of our present vector analysis, for the vector method, permitting as it does a ready survey, and often a visualization of the results to which it leads, has shown its superiority over all other methods in several branches of physics, and there can be no doubt that it is also peculiarly well adapted to the solution of the new problems introduced by Minkowski.

I shall attempt to show in this paper what simple changes must be made in our present system of vector analysis to make it immediately adaptable to a space of higher dimensions. Only such changes will be made as are imperatively demanded by the nature of the problem, and these few changes will, I believe, recommend themselves, not only because of the increased generality of the resulting analysis, but because they restore many features of the original, and much neglected, system of Grassmann.[3]

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[1] Göttingen, Nachricht., 1908, p. 53.

[2] Rendiconti di Palermo, **30**, 1 (1910).

[3] References to Grassmann will be to the edition of 1894, Teubner, Leipzig.

In the second section several of the most useful formulae of four-dimensional vector analysis will be presented, and the last section will be devoted to some applications of these formulae in electromagnetic theory and the theory of relativity.

The Vector Analysis of Three Dimensions.

The simplest type of quantity which is distinguished from others of its class by magnitude and direction is the familiar line-vector, a one-dimensional quantity which we shall call a vector of the first order, or in brief, a 1-vector.

Just as two parallel line-vectors of the same length are regarded as equal, so two parallel plane surfaces of the same area are also considered equal. A plane area[4] constitutes a vector of the second order, or a 2-vector.

In general in a space of n dimensions we may distinguish 0-vectors or scalars; 1-vectors, 2-vectors, 3-vectors, etc., up to the n-vectors, which, like the 0-vectors, have no direction and may therefore be called pseudo-scalars.

In three-dimensional space the only true vectors which exist are 1-vectors and 2-vectors. Moreover, every 2-vector determines uniquely the 1-vector normal to it. In common vector analysis the 2-vector is regarded as equivalent to and replaceable by its normal 1-vector of the same magnitude, and therefore this analysis deals solely with 1-vectors. This simplification has certain obvious advantages which, however, are for the most part superficial. In some cases moreover it leads to difficulties.[5] In any case it must be abandoned when we pass to space of higher dimensions, where a 2-vector no longer uniquely determines a 1-vector.

Our first departure, then, from common vector analysis will consist in distinguishing between vectors of different orders. A 1-vector will

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[4] For simplicity we may deal only with the straight vectors (straight line, plane surface, etc.) since any curve terminating in two points may be regarded as equivalent to the straight line terminating in the same points, and a curved surface terminating in a *plane* closed curve, as equivalent to the plane area having the same boundary. Such a vector as a curved surface bounded by a closed curve which does not lie in a plane we shall not consider here.

[5] See, for example, the discussion of scalars and pseudo-scalars in Abraham-Föppl, Theorie der Elektrizität, p. 22–23. We shall have frequent occasion to cite this standard work, which contains an admirable presentation of the current system of vector analysis, as well as of electrical theory. References are to the edition of 1901 (Teubner, Leipzig).

be represented by a small letter in heavy type (e. g., **a**, **s**). A 2-vector will be represented by a capital letter in heavy type (e. g., **A**, **S**).

Let us consider a coördinate system of three perpendicular axes, x_1, x_2, x_3, and represent by $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$, the three unit vectors in these three directions. If the lengths of the components of a 1-vector **a** on the three axes are a_1, a_2, a_3, then

$$\mathbf{a} = a_1\mathbf{k}_1 + a_2\mathbf{k}_2 + a_3\mathbf{k}_3. \tag{1}$$

In general the addition and subtraction of 1-vectors follow the law.[6]

$$\mathbf{a} \pm \mathbf{b} = (a_1 \pm b_1)\,\mathbf{k}_1 + (a_2 \pm b_2)\,\mathbf{k}_2 + (a_3 \pm b_3)\,\mathbf{k}_3. \tag{2}$$

Similarly we may project a surface vector, or 2-vector, upon the three coördinate planes determined by x_1, x_2 ; x_1, x_3 ; x_2, x_3. The unit 2-vectors in these planes we will denote by $\mathbf{k}_{12}$, $\mathbf{k}_{13}$, $\mathbf{k}_{23}$,[7] and the areas of the projections of a 2-vector **A** by A_{12}, A_{13}, A_{23}. Then,

$$A = A_{12}\mathbf{k}_{12} + A_{13}\mathbf{k}_{13} + A_{23}\mathbf{k}_{23} \tag{3}$$

$$A \pm B = (A_{12} \pm B_{12})\,k_{12} + (A_{13} \pm B_{13})\,k_{13} + (A_{23} \pm B_{23})\,k_{23}. \tag{4}$$

Further we may adopt the convention,

$$\mathbf{k}_{12} = -\,\mathbf{k}_{21}\,;\ \mathbf{k}_{13} = -\,\mathbf{k}_{31}\,;\ \mathbf{k}_{23} = -\,\mathbf{k}_{32} \tag{5}$$

which requires the further convention,

$$A_{12} = -\,A_{21}\,;\ A_{13} = -\,A_{31}\,;\ A_{23} = -\,A_{32}. \tag{6}$$

Just as $\mathbf{k}_1$ represents a vector of unit length, $\mathbf{k}_{12}$ one of unit area, $\mathbf{k}_{123}$ will represent unit volume. It is the unit 3-vector or, in three-dimensional space, the unit pseudo-scalar. We shall adopt the convention

$$\mathbf{k}_{123} = \mathbf{k}_{312} = \mathbf{k}_{231} = -\,\mathbf{k}_{132} = -\,\mathbf{k}_{213} = -\,\mathbf{k}_{321}. \tag{7}$$

Equations (5) and (7) may be expressed in the following general rule which we shall also adopt in space of higher dimensions : Interchanging any two adjacent subscripts of a unit vector changes the sign of the vector.

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[6] The addition both of 1-vectors and 2-vectors is best defined geometrically. (See Grassmann, Ausdehnungslehre von 1844, p. 78.) The introduction of coördinate axes brings a foreign element into pure vector analysis, but on the other hand it will permit us to translate the vector equations more readily into Cartesian equations.

[7] In this case we depart from our rule that 2-vectors shall be represented by capital letters. On account of the subscripts there will be no confusion.

Multiplying any vector by a scalar, multiplies its magnitude by that scalar. Multiplication of a vector by a scalar follows the laws of association, commutation and distribution.

$$m\mathbf{a} = ma_1\mathbf{k}_1 + ma_2\mathbf{k}_2 + ma_3\mathbf{k}_3 \tag{8}$$

In multiplying one vector by another two kinds of product are to be distinguished, which, following Grassmann, we shall call the inner and outer products,[8] and define as follows.

The inner product follows the distributive and commutative laws. It is in general a vector and its order is the difference between the orders of the factors. (Thus the inner product of two 1-vectors is a 0-vector, or scalar, the inner product of a 1-vector and a 2-vector is a 1-vector.)

The outer product follows the distributive and associative laws. It is a vector of which the order is the sum of the orders of the factors. (Thus the outer product of two 1-vectors is a 2-vector, that of a 1-vector and a 2-vector is a 3-vector, which in three-dimensional space is a pseudo-scalar.)

The inner product of two vectors will be indicated merely by their juxtaposition, for example, $\mathbf{ab}$; $\mathbf{AB}$; $\mathbf{aA}$.

The outer product will be indicated by a cross[9] placed between two vectors, for example, $\mathbf{a}\times\mathbf{b}$; $\mathbf{A}\times\mathbf{B}$; $\mathbf{a}\times\mathbf{A}$.

Since both kinds of products follow the distributive law they may be completely defined by the rules governing the multiplication of the simple unit vectors. The rules for inner multiplication are as follows,

$$\left.\begin{array}{ll}
\mathbf{k}_1\mathbf{k}_1 = 1\,; & \mathbf{k}_1\mathbf{k}_2 = \mathbf{k}_2\mathbf{k}_1 = 0 \\
\mathbf{k}_{12}\mathbf{k}_{12} = 1\,; & \mathbf{k}_{12}\mathbf{k}_{13} = \mathbf{k}_{13}\mathbf{k}_{12} = 0 \\
\mathbf{k}_{123}\mathbf{k}_{123} = 1\,; & \\
\mathbf{k}_2\mathbf{k}_{12} = \mathbf{k}_{12}\mathbf{k}_2 = \mathbf{k}_1\,; & \mathbf{k}_1\mathbf{k}_{23} = \mathbf{k}_{23}\mathbf{k}_1 = 0 \\
\mathbf{k}_3\mathbf{k}_{123} = \mathbf{k}_{123}\mathbf{k}_3 = \mathbf{k}_{12}\,; & \\
\mathbf{k}_{23}\mathbf{k}_{123} = \mathbf{k}_{123}\mathbf{k}_{23} = \mathbf{k}_1\,; &
\end{array}\right\}. \tag{9}$$

These statements may be generalized, and the following rules will hold also for unit vectors mutually perpendicular, in space of any dimensions :

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[8] The terms scalar product and vector product would obviously be misnomers in the present system.

[9] This symbol for the outer product is used by Gibbs and his followers (see Gibbs, Collected Papers; Wilson-Gibbs, Vector Analysis; Coffin, Vector Analysis), and has several advantages over the more awkward square brackets [a, b] frequently used to express the outer product. The brackets were used by Grassmann, but had a far more general significance than the product defined above. (Ausdehnungslehre von 1862, p. 28.)

If each factor has a subscript which the other has not, the inner product is zero.

The inner product of two identical unit vectors is equal to unity.

In the remaining case, one factor having a higher order than the other, the subscripts of the former should be transposed until those subscripts occurring at the right are the same and in the same sequence as in the factor of lower order. These common subscripts are then cancelled and a unit vector with the remaining subscripts, in the sequence in which they stand, forms the inner product. Thus for example,

$$\mathbf{k}_{13}\mathbf{k}_{123} = -\,\mathbf{k}_{13}\mathbf{k}_{213} = -\,\mathbf{k}_{2}.$$

From these rules we obtain immediately the equations,

$$\mathbf{ab} = a_1b_1 + a_2b_2 + a_3b_3. \qquad (10)$$

$$\mathbf{aa} = \mathbf{a}^2 = a_1^{\,2} + a_2^{\,2} + a_3^{\,2}. \qquad (11)$$

$$\mathbf{AB} = A_1B_1 + A_2B_2 + A_3B_3. \qquad (12)$$

These products are scalars. On the other hand the product $\mathbf{aA}$ is a 1-vector lying in the plane $\mathbf{A}$ and perpendicular to the projection of $\mathbf{a}$ upon $\mathbf{A}$, namely,

$$\mathbf{aA} = \mathbf{Aa} = (A_{12}a_2 + A_{13}a_3)\,\mathbf{k}_1 + (A_{21}a_1 + A_{23}a_3)\,\mathbf{k}_2 + (A_{31}a_1 + A_{32}a_2)\,\mathbf{k}_3. \qquad (13)$$

Finally, the product of a pseudo-scalar and a 1-vector is the perpendicular 2-vector, that of a pseudo-scalar and a 2-vector is the perpendicular 1-vector, the product in each case having a magnitude equal to the product of the magnitudes of the factors.

The rules for outer multiplication may likewise be stated by stating the rules for the unit vectors.

$$\begin{gathered}\mathbf{k}_1\times\mathbf{k}_2 = \mathbf{k}_{12}\,;\ \mathbf{k}_1\times\mathbf{k}_{23} = \mathbf{k}_{12}\times\mathbf{k}_3 = \mathbf{k}_{123},\\ \mathbf{k}_1\times\mathbf{k}_1 = 0\,;\ \mathbf{k}_1\times\mathbf{k}_{12} = 0\,;\ \mathbf{k}_{12}\times\mathbf{k}_{12} = 0\,;\ \mathbf{k}_{12}\times\mathbf{k}_{13} = 0\,; \qquad (14)\\ \mathbf{k}_1\times\mathbf{k}_{123} = 0\,;\ \mathbf{k}_{12}\times\mathbf{k}_{123} = 0\,;\ \mathbf{k}_{123}\times\mathbf{k}_{123} = 0.\end{gathered}$$

These statements may be generalized and the following rules will hold also for unit, mutually perpendicular, vectors in space of any dimensions :

If two unit vectors possess any subscript in common, their outer product is zero.

In all other cases the outer product is a unit vector having all the subscripts of both factors, in the sequence in which they occur in the factors.

From these rules[10] we obtain the equations,

$$\mathbf{a}\times\mathbf{a} = 0. \tag{15}$$

$$\mathbf{a}\times\mathbf{b} = -\,\mathbf{b}\times\mathbf{a} = (a_1 b_2 - a_2 b_1)\,\mathbf{k}_{12} + (a_1 b_3 - a_3 b_1)\,\mathbf{k}_{13} + (a_2 b_3 - a_3 b_2)\,\mathbf{k}_{23}. \tag{16}$$

$$\mathbf{a}\times\mathbf{A} = \mathbf{A}\times\mathbf{a} = (a_1 A_{23} + a_2 A_{31} + a_3 A_{12})\,\mathbf{k}_{123}. \tag{17}$$

When the 2-vector in (17) is expressed as a product of two 1-vectors, $\mathbf{b}\times\mathbf{c}$, that equation becomes

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\times\mathbf{c} = \mathbf{a}\times\mathbf{b}\times\mathbf{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\,\mathbf{k}_{123}. \tag{18}$$

$$\text{and } \mathbf{a}\times(\mathbf{a}\times\mathbf{b}) = 0. \tag{18a}$$

We thus see that $\mathbf{a}\times\mathbf{b}$ represents the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$, and $\mathbf{a}\times\mathbf{b}\times\mathbf{c}$ the parallelopiped determined by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

It is important at this point to rewrite equation (13) using $\mathbf{b}\times\mathbf{c}$ in place of $\mathbf{A}$; expanding and rearranging the terms gives

$$\mathbf{a}\,(\mathbf{b}\times\mathbf{c}) = \mathbf{a}\mathbf{b}\times\mathbf{c}\,^{11} = (\mathbf{a}\mathbf{c})\,\mathbf{b} - (\mathbf{a}\mathbf{b})\,\mathbf{c}. \tag{19}$$

These equations (18) and (19) deserve especial attention, for they show the only essential difference between our system and the common system of vector analysis. The two systems give the same result for the outer multiplication of two 1-vectors and for the inner product of two 1-vectors or two 2-vectors. But the meanings of the outer and inner products of $\mathbf{a}$ and $\mathbf{b}\times\mathbf{c}$ are just reversed in the two systems.

Finally we find from our rules for unit vectors the outer product of two 2-vectors,

$$\mathbf{A}\times\mathbf{B} = 0. \tag{20}$$

By our general principle the outer product must in this case have the order 2 + 2, and a 4-vector cannot exist in three-dimensional space.[12]

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[10] The rules here given are somewhat redundant. For example, the distributive law, and $\mathbf{a}\times\mathbf{a} = 0$, alone suffice to prove $\mathbf{a}\times\mathbf{b} = -\,\mathbf{b}\times\mathbf{a}$, for

$$(a + b) \times (a + b) = 0 = \mathbf{a}\times\mathbf{a} + \mathbf{b}\times\mathbf{b} + \mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{a} = \mathbf{a}\times\mathbf{b} + \mathbf{b}\times\mathbf{a}.$$

See Grassmann, Ausdehnungslehre von 1844, p. 87.

[11] The parentheses may be removed simply because $(\mathbf{a}\mathbf{b})\times\mathbf{c}$ has no meaning.

[12] In ordinary vector analysis a meaning is given to the outer product of $(\mathbf{a}\times\mathbf{b})$ and $(\mathbf{c}\times\mathbf{d})$. It represents a vector determined by the line of intersection of the surface $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}\times\mathbf{d}$. We have seen that in n-dimensional space an n-vector has some properties of a scalar or vector of the order $n-n$. So we may modify our rules of multiplication so that the product of a p-vector and a

The inner product of $(\mathbf{a}\times\mathbf{b})$ and $(\mathbf{c}\times\mathbf{d})$ may be obtained from the preceding equations, and is the same as in ordinary vector analysis,

$$(\mathbf{a}\times\mathbf{b})\ (\mathbf{c}\times\mathbf{d}) = (\mathbf{ac})\ (\mathbf{bd}) - (\mathbf{bc})\ (\mathbf{ad}). \tag{21}$$

The differential operator ∇ ("del") we may define in the usual way,[13] namely,

$$\nabla = \mathbf{k}_1 \frac{\partial}{\partial x_1} + \mathbf{k}_2 \frac{\partial}{\partial x_2} + \mathbf{k}_3 \frac{\partial}{\partial x_3}. \tag{22}$$

Since the scalar operator $\frac{\partial}{\partial x}$, when applied to a single variable, can be treated as an algebraic quantity, the operator ∇ may be treated formally as a 1-vector, and we may derive a number of important equations by substituting ∇ for $\mathbf{a}$ or $\mathbf{b}$ in the preceding equations. Thus from (8) we obtain from the scalar ϕ the function known as gradient of ϕ

$$\nabla\phi = \mathbf{k}_1 \frac{\partial\phi}{\partial x_1} + \mathbf{k}_2 \frac{\partial\phi}{\partial x_2} + \mathbf{k}_3 \frac{\partial\phi}{\partial x_3}. \tag{23}$$

Combined with a 1-vector by inner and outer multiplication we obtain by equations (10) and (16), the functions known as the divergence and curl, respectively.

$$\nabla\mathbf{a} = \frac{\partial a_1}{\partial x_1} + \frac{\partial a_2}{\partial x_2} + \frac{\partial a_3}{\partial x_3}. \tag{24}$$

$$\nabla\times\mathbf{a} = \left(\frac{\partial a_2}{\partial x_1} - \frac{\partial a_1}{\partial x_2}\right)\mathbf{k}_{12} + \left(\frac{\partial a_3}{\partial x_1} - \frac{\partial a_1}{\partial x_3}\right)\mathbf{k}_{13} + \left(\frac{\partial a_3}{\partial x_2} - \frac{\partial a_2}{\partial x_3}\right)\mathbf{k}_{23}. \tag{25}$$

Evidently $\nabla\phi$ is a 1-vector, $\nabla\mathbf{a}$ a scalar, and $\nabla\times\mathbf{a}$ a 2-vector.

By equations (13) and (17) we may write expressions for $\nabla\mathbf{A}$ (a 1-vector), and $\nabla\times\mathbf{A}$ (a 3-vector, or pseudo scalar).

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q-vector, when $p + q > n$, is a vector of the order $p + q - n$. Such a product, which Grassmann calls "regressive," is formed according to a new set of rules and may best be regarded as a new type of product entirely distinct from the regular or "progressive" outer product. It is possible in the system here described to avoid the introduction of this new kind of product. Thus

$$((\mathbf{a}\times\mathbf{b})\ \mathbf{k}_{123}) \times (\ (\mathbf{c}\times\mathbf{d}\ \mathbf{k}_{123}) = \mathbf{e}\mathbf{k}_{123}),$$

where $\mathbf{e}$ is the 1-vector obtained in the ordinary vector analysis as the outer product of $(\mathbf{a}\times\mathbf{b})$ and $(\mathbf{c}\times\mathbf{d})$.

[13] Like other vector quantities and operators ∇ may be simply defined without reference to coördinates. See, for example, Wilson, Bull. Amer. Math. Soc. (2) **16**, 415 (1910).

$$\nabla \mathbf{A} = \left(\frac{\partial A_{12}}{\partial x_2} + \frac{\partial A_{13}}{\partial x_3}\right) \mathbf{k}_1 + \left(\frac{\partial A_{21}}{\partial x_1} + \frac{\partial A_{23}}{\partial x_3}\right) \mathbf{k}_2 + \left(\frac{\partial A_{31}}{\partial x_1} + \frac{\partial A_{32}}{\partial x_2}\right) \mathbf{k}_3. \quad (26)$$

$$\nabla \times \mathbf{A} = \left(\frac{\partial A_{23}}{\partial x_1} + \frac{\partial A_{31}}{\partial x_2} + \frac{\partial A_{12}}{\partial x_3}\right) \mathbf{k}_{123}. \quad (27)$$

When the quantity operated upon by ∇ contains two or more variables it may be expanded in terms of its components and these scalar quantities may then be differentiated in the ordinary way. We thus obtain such equations as the following:

$$\nabla (\phi \mathbf{a}) = \phi (\nabla \mathbf{a}) + \mathbf{a} (\nabla \phi), \quad (28)$$

$$\nabla \times (\phi \mathbf{a}) = \phi (\nabla \times \mathbf{a}) + (\nabla \phi) \times \mathbf{a}, \quad (29)$$

$$\nabla \times (\mathbf{a} \times \mathbf{b}) = \mathbf{b} \times (\nabla \times \mathbf{a}) - \mathbf{a} \times (\nabla \times \mathbf{b}). \quad (30)$$

By the above rules new operators may be formed from ∇ such as $\mathbf{a} \nabla$, $\mathbf{A} \nabla$, and $\nabla \nabla$ or ∇^2 which may be applied to any scalar or vector. The last is the well-known Laplacian operator and may obviously be expanded by equation (11),

$$\nabla^2 = \frac{\partial^2}{\partial x_1{}^2} + \frac{\partial^2}{\partial x_2{}^2} + \frac{\partial^2}{\partial x_3{}^2}. \quad (31)$$

Other operations involving ∇ twice are $\nabla (\nabla \mathbf{a})$ and $\nabla (\nabla \times \mathbf{a})$ or $\nabla \nabla \times \mathbf{a}$.

These quantities are connected by an important equation which we obtain by expanding according to (13), (23) and (10), namely

$$\nabla \nabla \times \mathbf{a} = \nabla (\nabla \mathbf{a}) - \nabla^2 \mathbf{a}. \quad (32)$$

Finally we have from (18a) and (15) the important identities,

$$\nabla \times (\nabla \times \mathbf{a}) = 0. \quad (33)$$

$$\nabla \times (\nabla \phi) = 0. \quad (34)$$

Equations (32), (33) and (34) are evidently equivalent to the familiar equations:[14]

$$\text{curl}^2\, \mathbf{a} = \text{grad}\,(\text{div}\, \mathbf{a}) - \nabla^2 \mathbf{a},$$
$$\text{div curl}\, \mathbf{a} = 0.$$
$$\text{curl grad}\, \phi = 0.$$

[14] Abraham-Föppl, I, equations 95, 94, 91 a.

Here as elsewhere our equations differ from those in common use whenever the product of a 1-vector and a 2-vector is concerned.

The Vector Analysis of Four Dimensions.

The revised system of three-dimensional vector analysis has been elaborated somewhat fully in the preceding section, since the methods there adopted may be used without any modification in developing the vector analysis of space of higher dimensions.

Let us consider a four-dimensional space in which any two points uniquely determine a straight line, any three points not in a line uniquely determine a plane, and any four points not in a plane uniquely determine a straight or Euclidean 3-space. This may be called a Euclidean four-dimensional space.

In such a space let us construct four mutually perpendicular coordinate axes, x_1, x_2, x_3, x_4. The 1-vectors of unit length in these four directions we may call $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$, $\mathbf{k}_4$. Each pair of axes determines a plane, thus forming six coördinate planes. The 2-vectors of unit area parallel to these planes we may call $\mathbf{k}_{12}$, $\mathbf{k}_{13}$, $\mathbf{k}_{14}$, $\mathbf{k}_{23}$, $\mathbf{k}_{24}$, $\mathbf{k}_{34}$. These six planes are mutually perpendicular. Moreover the plane $\mathbf{k}_{12}$ is *completely* perpendicular to the plane $\mathbf{k}_{34}$, in the sense that every line in $\mathbf{k}_{12}$ is perpendicular to every line in $\mathbf{k}_{34}$. The same is true of the pairs $\mathbf{k}_{23}$, $\mathbf{k}_{14}$, and $\mathbf{k}_{13}$, $\mathbf{k}_{24}$.

Each set of three axes determines a straight 3-space and the four coördinate 3-spaces thus determined may be represented by the unit 3-vectors $\mathbf{k}_{123}$, $\mathbf{k}_{124}$, $\mathbf{k}_{134}$, $\mathbf{k}_{234}$. Finally all four axes together determine the unit 4-vector or pseudo-scalar, $\mathbf{k}_{1234}$.

A 1-vector may be represented as the sum of its projections on the four axes,

$$\mathbf{a} = a_1\mathbf{k}_1 + a_2\mathbf{k}_2 + a_3\mathbf{k}_3 + a_4\mathbf{k}_4. \tag{35}$$

A 2-vector may be represented as the sum of its projections on the six coördinate planes.

$$\mathbf{A} = A_{12}\mathbf{k}_{12} + A_{13}\mathbf{k}_{13} + A_{14}\mathbf{k}_{14} + A_{23}\mathbf{k}_{23} + A_{24}\mathbf{k}_{24} + A_{34}\mathbf{k}_{34}. \tag{36}$$

A 3-vector may likewise be represented as the sum of its four projections on the coördinate 3-spaces.[15]

The addition and subtraction of vectors follow the same rules as in the case of three dimensions (equations 2 and 4). Moreover both

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[15] We shall not use the 3-vectors often enough in this paper to justify the introduction of a special symbol for them.

forms of multiplication are completely defined by the distributive law, and by the rules already given for the transposition of subscripts, and for inner and outer multiplication among the unit vectors. We may therefore write at once a large number of equations, of which some of the more important are the following,

$$\mathbf{A}\times\mathbf{B} = \mathbf{B}\times\mathbf{A} = (A_{12}B_{34} + A_{13}B_{24} + A_{14}B_{23} + A_{23}B_{14} + A_{24}B_{13} + A_{34}B_{12})\mathbf{k}_{1234}. \quad (37)$$

$$\mathbf{a}\times\mathbf{A} = \mathbf{A}\times\mathbf{a} = (a_1A_{23} + a_2A_{31} + a_3A_{12})\mathbf{k}_{123} + (a_1A_{24} + a_2A_{41} + a_4A_{12})\mathbf{k}_{124} + (a_1A_{34} + a_3A_{41} + a_4A_{13})\mathbf{k}_{134} + (a_2A_{34} + a_3A_{42} + a_4A_{23})\mathbf{k}_{234}. \quad (38)$$

$$\mathbf{a}\times\mathbf{b} = (a_1b_2 - a_2b_1)\mathbf{k}_{12} + (a_1b_3 - a_3b_1)\mathbf{k}_{13} + \ldots \quad (39)$$

$$\mathbf{a}\times\mathbf{a} = 0. \quad (40)$$

Here also $\mathbf{a}\times\mathbf{b}$ evidently represents the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$, so $\mathbf{a}\times\mathbf{b}\times\mathbf{c}$ will represent a parallelopiped, and $\mathbf{a}\times\mathbf{b}\times\mathbf{c}\times\mathbf{d}$ a parallel four-dimensional figure. It is very important to observe that all of our four-dimensional vector equations may be given simple geometrical definitions, and retain complete validity whatever set of coördinate axes be arbitrarily chosen.

Some of the more important inner products are the following :

$$\mathbf{ab} = \mathbf{ba} = a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4. \quad (41)$$

$$\mathbf{AB} = \mathbf{BA} = A_{12}B_{12} + A_{13}B_{13} + A_{14}B_{14} + A_{23}B_{23} + A_{24}B_{24} + A_{34}B_{34}. \quad (42)$$

$$\mathbf{aA} = \mathbf{Aa} = (A_{12}a_2 + A_{13}a_3 + A_{14}a_4)\mathbf{k}_1 + (A_{21}a_1 + A_{23}a_3 + A_{24}a_4)\mathbf{k}_2 + \ldots \quad (43)$$

$$(\mathbf{a}\times\mathbf{b})\ (\mathbf{c}\times\mathbf{d}) = (\mathbf{ac})\ (\mathbf{bd}) - (\mathbf{bc})\ (\mathbf{ad}). \quad (44)$$

$$\mathbf{a}\ (\mathbf{b}\times\mathbf{c}) = (\mathbf{ac})\ \mathbf{b} - (\mathbf{ab})\ \mathbf{c}. \quad (45)$$

This is a 1-vector lying in the plane $\mathbf{b}\times\mathbf{c}$ and perpendicular to the projection of $\mathbf{a}$ thereon. So $\mathbf{a}$ $(\mathbf{b}\times\mathbf{c}\times\mathbf{d})$ is a 2-vector lying in the 3-space $\mathbf{b}\times\mathbf{c}\times\mathbf{d}$ and perpendicular to the projection of $\mathbf{a}$ on that 3-space ; $(\mathbf{a}\times\mathbf{b})$ $(\mathbf{c}\times\mathbf{d}\times\mathbf{e})$ is a 1-vector in the 3-space $\mathbf{c}\times\mathbf{d}\times\mathbf{e}$ and perpendicular to the projection of $\mathbf{a}\times\mathbf{b}$ thereon.

The inner product of any vector with unit pseudo-scalar, $\mathbf{k}_{1234}$, is another vector of the same magnitude which may be called its *complement*. The complement of a scalar is a pseudo-scalar, and vice versa. The complement of a 1-vector is a 3-vector normal to it, and vice versa.

The complement of a 2-vector is the completely perpendicular 2-vector. In the vector analysis at present in use it is customary to identify a vector with its complement, and this is also done by Abraham[16] in the paper in which he makes use of four-dimensional vector analysis. In our present analysis there is no advantage to be gained by this step, which may cause much confusion.

As in the case of three dimensions we may define a differential operator, having the form of a 1-vector, as follows:

$$\Diamond = \mathbf{k}_1 \frac{\partial}{\partial x_1} + \mathbf{k}_2 \frac{\partial}{\partial x_2} + \mathbf{k}_3 \frac{\partial}{\partial x_3} + \mathbf{k}_4 \frac{\partial}{\partial x_4}. \tag{46}$$

This operator[17] $\Diamond$ (read "quad") may be treated like a simple vector under the same conditions as in the case of ∇. We thus obtain a number of important equations such as the following.

$$\Diamond \phi = \mathbf{k}_1 \frac{\partial \phi}{\partial x_1} + \mathbf{k}_2 \frac{\partial \phi}{\partial x_2} + \mathbf{k}_3 \frac{\partial \phi}{\partial x_3} + \mathbf{k}_4 \frac{\partial \phi}{\partial x_4}. \tag{47}$$

$$\Diamond \mathbf{a} = \frac{\partial a_1}{\partial x_1} + \frac{\partial a_2}{\partial x_2} + \frac{\partial a_3}{\partial x_3} + \frac{\partial a_4}{\partial x_4}. \tag{48}$$

$$\Diamond \times \mathbf{a} = \left(\frac{\partial a_2}{\partial x_1} - \frac{\partial a_1}{\partial x_2}\right) \mathbf{k}_{12} + \left(\frac{\partial a_3}{\partial x_1} - \frac{\partial a_1}{\partial x_3}\right) \mathbf{k}_{13} + \ldots \tag{49}$$

These three expressions correspond to gradient, divergence and curl in three-dimensional analysis. We may also apply $\Diamond$ to vectors of higher orders, for example, by (43) and (38),

$$\Diamond \mathbf{A} = \left(\frac{\partial A_{12}}{\partial x_2} + \frac{\partial A_{13}}{\partial x_3} + \frac{\partial A_{14}}{\partial x_4}\right) \mathbf{k}_1 + \left(\frac{\partial A_{21}}{\partial x_1} + \frac{\partial A_{23}}{\partial x_3} + \frac{\partial A_{24}}{\partial x_4}\right) \mathbf{k}_2 + \ldots \tag{50}$$

$$\Diamond \times \mathbf{A} = \left(\frac{\partial A_{23}}{\partial x_1} + \frac{\partial A_{31}}{\partial x_2} + \frac{\partial A_{12}}{\partial x_3}\right) \mathbf{k}_{123} + \ldots \tag{51}$$

We may form other operators like, $\mathbf{A}\Diamond$, a 1-vector operator, and the scalar operators $\mathbf{a}\,\Diamond$, and $\Diamond^2$. The last is the very important operator which Lorentz in a special case calls the d'Alembertian operator,

$$\Diamond^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2} + \frac{\partial^2}{\partial x_4^2}. \tag{52}$$

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16 Abraham (loc. cit.).

17 The operator $\Diamond$ has the same scalar components as the operator *lor* used by Minkowski.

Any of these operators may be applied to a scalar or to a vector of any order.

Two other important operations are connected with $\Diamond^2$ by the formula analogous to (32):

$$\Diamond(\Diamond\times\mathbf{a}) = \Diamond\Diamond\times\mathbf{a} = \Diamond(\Diamond\,\mathbf{a}) - \Diamond^2\mathbf{a}. \qquad (53)$$

And we have here also two important identities

$$\Diamond\times(\Diamond\times\mathbf{a}) = 0, \qquad (54)$$

$$\Diamond\times(\Diamond\,\phi) = 0. \qquad (55)$$

In the same way that we obtained equations (28), (29), (30), we find

$$\Diamond(\phi\mathbf{a}) = \phi(\Diamond\,\mathbf{a}) + \mathbf{a}\,(\Diamond\,\phi), \qquad (56)$$

$$\Diamond\times(\phi\mathbf{a}) = \phi\,(\Diamond\times\mathbf{a}) + (\Diamond\,\phi)\times\mathbf{a}, \qquad (57)$$

$$\Diamond\times(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\times(\Diamond\times\mathbf{a}) - \mathbf{a}\times(\Diamond\times\mathbf{b}). \qquad (58)$$

These equations will suffice to illustrate how readily the generalized vector analysis of the preceding section may be applied in a space of any dimensions.

Some Applications of Four-Dimensional Vector Analysis in the Theory of Electricity.

The principle of relativity as interpreted by Minkowski can be summed up in the statement [18] that a Euclidean four-dimensional space is determined by the coördinates, x, y, z, and ict, where i is the unit of imaginaries, $\sqrt{-1}$; and c is the velocity of light. The whole science of kinematics is merely the geometry of this four-dimensional space. As Minkowski himself has shown, there is no domain in which this new conception is more fruitful than in the science of electricity and magnetism.

Let us consider a system composed of electric charges moving in free space. The density of charge at any point we may call $\frac{\varrho}{c}$ in electromagnetic units, and if we call the velocity of the charge $\mathbf{v}$, then $\frac{\varrho}{c}\mathbf{v}$ represents the current density at a point. This 1-vector $\frac{\varrho}{c}\mathbf{v}$ lies wholly in the 3-space x, y, z.

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[18] This statement is subject to certain restrictions that we will not discuss.

Following Minkowski we may define a 1-vector, $\mathbf{q}$, in the space x, y, z, ict (or x_1, x_2, x_3, x_4) of which the projection on the 3-space is $\frac{\varrho}{c}\mathbf{v}$ and the scalar component along the x_4 (or ict) axis is $i\varrho$, by the equation,

$$\mathbf{q} = \frac{\varrho}{c}\mathbf{v} + i\,\varrho\,\mathbf{k}_4, \tag{59}$$

or

$$\mathbf{q} = \frac{\varrho}{c}v_1\mathbf{k}_1 + \frac{\varrho}{c}v_2\mathbf{k}_2 + \frac{\varrho}{c}v_3\mathbf{k}_3 + i\,\varrho\,\mathbf{k}_4. \tag{60}$$

Furthermore, from the electrical force $\mathbf{e}$, and the magnetic force $\mathbf{h}$, we shall find it convenient to define two new 2-vectors, $\mathbf{E}$ and $\mathbf{H}$, by the equations,[19]

$$\mathbf{E} = -\,i\mathbf{e}\times\mathbf{k}_4, \tag{61}$$

and

$$\mathbf{H} = \mathbf{h}\mathbf{k}_{123}. \tag{62}$$

$\mathbf{H}$ is the 2-vector complementary to $\mathbf{h}$ in the 3-space x, y, z. From these definitions we have

$$H_{12} = h_3\,;\; H_{23} = h_1\,;\; H_{31} = h_2, \tag{63}$$

$$E_{14} = -\,ie_1\,;\; E_{24} = -\,ie_2\,;\; E_{34} = -\,ie_3. \tag{64}$$

From $\mathbf{H}$ and $\mathbf{e}$ we may define[20] the vector potential $\mathbf{a}$ and the scalar potential ϕ by the familiar equations,

$$\mathbf{H} = \nabla\times\mathbf{a}, \tag{65}$$

$$-\,\mathbf{e} = \nabla\phi + \frac{1}{c}\frac{\partial\mathbf{a}}{\partial t}. \tag{66}$$

Finally we shall define a new 1-vector m by the equation,

$$\mathbf{m} = \mathbf{a} + i\phi\mathbf{k}_4, \tag{67}$$

or

$$\mathbf{m} = a_1\mathbf{k}_1 + a_2\mathbf{k}_2 + a_3\mathbf{k}_3 + i\phi\mathbf{k}_4. \tag{68}$$

Thus m is a vector of which the projection on the 3-space x, y, z is the vector potential, and of which the scalar component in the ict direction is the scalar potential multiplied by $\sqrt{-1}$.

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[19] Compare in this connection the discussion of $\mathbf{e}$ as a "polar" vector, $\mathbf{h}$ as an "axial" vector, in Abraham-Föppl (1, p. 243).

[20] This definition is evidently not complete, since $\mathbf{a}$ and ϕ are derived from $\mathbf{H}$ and $\mathbf{e}$ by a process of integration. We shall return to this point.

We shall call **m** the *fundamental electromagnetic 1-vector*, and show that the four important field equations of Maxwell and Lorentz, as well as other well-known equations, are all contained in the strikingly simple formula,

$$\Diamond\Diamond \times \mathbf{m} = \mathbf{q}. \tag{69}$$

In addition to this equation which states the experimental facts, we have from (54) the important identity

$$\Diamond \times \Diamond \times \mathbf{m} = 0. \tag{70}$$

The quantity $\Diamond \times \mathbf{m}$, which might be called the four-dimensional curl of **m**, is the fundamental electromagnetic 2-vector.[21] We will give it the symbol **M**,

$$\Diamond \times \mathbf{m} = \mathbf{M}. \tag{71}$$

Expanding $\Diamond \times \mathbf{m}$ by equation (49) gives

$$\mathbf{M} = \mathbf{k}_{12}\left(\frac{\partial a_2}{\partial x_1} - \frac{\partial a_1}{\partial x_2}\right) + \mathbf{k}_{13}\left(\frac{\partial a_3}{\partial x_1} - \frac{\partial a_1}{\partial x_3}\right) + \mathbf{k}_{23}\left(\frac{\partial a_3}{\partial x_2} - \frac{\partial a_2}{\partial x_3}\right)$$
$$+ \mathbf{k}_{14}\left(\frac{\partial i\phi}{\partial x_1} - \frac{\partial a_1}{\partial x_4}\right) + \mathbf{k}_{24}\left(\frac{\partial i\phi}{\partial x_2} - \frac{\partial a_2}{\partial x_4}\right) + \mathbf{k}_{34}\left(\frac{\partial i\phi}{\partial x_3} - \frac{\partial a_3}{\partial x_4}\right). \tag{72}$$

The first three terms are evidently equal to the curl of **a**; the last three may be put in the form

$$\left(i\mathbf{k}_1 \frac{\partial \phi}{\partial x_1} + i\mathbf{k}_2 \frac{\partial \phi}{\partial x_2} + i\mathbf{k}_3 \frac{\partial \phi}{\partial x_3} - \frac{\partial a}{\partial x_4}\right) \times \mathbf{k}_4;$$

and further collecting terms, and writing ict for x_4, gives

$$i\left(\nabla\phi + \frac{1}{c}\frac{\partial \mathrm{a}}{\partial t}\right) \times \mathbf{k}_4.$$

Hence,

$$\mathbf{M} = \nabla \times \mathrm{a} + i\left(\nabla\phi + \frac{1}{c}\frac{\partial \mathrm{a}}{\partial t}\right) \times \mathbf{k}_4, \tag{73}$$

and thus by equations (61), (65) and (66)

$$\mathbf{M} = \mathbf{H} + \mathbf{E}. \tag{74}$$

This equation gives a better idea of the physical significance of the 2-vector $\Diamond \times$ **m**, or **M**. **H** is a 2-vector lying wholly in the x, y, z 3-space. **E** is a 2-vector perpendicular to the x, y, z 3-space in a plane determined by the 1-vectors **e** and $\mathbf{k}_4$. We may therefore write,

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[21] This is the equivalent of the f or F of Minkowski.

$$\mathbf{M} = M_{12}\mathbf{k}_{12} + M_{13}\mathbf{k}_{13} + M_{23}\mathbf{k}_{23} + M_{14}\mathbf{k}_{14} + M_{24}\mathbf{k}_{24} + M_{34}\mathbf{k}_{34},$$
$$= H_{12}\mathbf{k}_{12} + H_{13}\mathbf{k}_{13} + H_{23}\mathbf{k}_{23} + E_{14}\mathbf{k}_{14} + E_{24}\mathbf{k}_{24} + E_{34}\mathbf{k}_{34}. \qquad (75)$$

By our fundamental equation (69) we have

$$\Diamond \mathbf{M} = \mathbf{q}.$$

Expanding this equation by (50) with the aid of (75) gives four equations

$$\left(\frac{\partial H_{12}}{\partial x_2} + \frac{\partial H_{13}}{\partial x_3} + \frac{\partial E_{14}}{\partial x_4}\right)\mathbf{k}_1 = \frac{\rho}{c}\, v_1\mathbf{k}_1,$$
$$\left(\frac{\partial H_{21}}{\partial x_1} + \frac{\partial H_{23}}{\partial x_3} + \frac{\partial E_{24}}{\partial x_4}\right)\mathbf{k}_2 = \frac{\rho}{c}\, v_2\mathbf{k}_2, \qquad (76)$$
$$\left(\frac{\partial H_{31}}{\partial x_1} + \frac{\partial H_{32}}{\partial x_2} + \frac{\partial E_{34}}{\partial x_4}\right)\mathbf{k}_3 = \frac{\rho}{c}\, v_3\mathbf{k}_3,$$
$$\left(\frac{\partial E_{41}}{\partial x_1} + \frac{\partial E_{42}}{\partial x_2} + \frac{\partial E_{43}}{\partial x_3}\right)\mathbf{k}_4 = \rho\, i\mathbf{k}_4.$$

Collecting the first three equations into one, with the aid of (25), (63) and (64), and writing ict for x_4 gives

$$\nabla \times \mathbf{h} - \frac{1}{c}\frac{\partial \mathbf{e}}{\partial t} = \frac{\rho}{c}\,\mathbf{v}, \qquad (77)$$

and the last equation by (24) and (64) changing E_{41} to $-E_{14}$, etc., gives

$$\nabla \mathbf{e} = \rho. \qquad (78)$$

By a similar expansion of equation (70)

$$\Diamond \times \mathbf{M} = 0,$$

we find with the aid of (49)

$$\nabla \times \mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} = 0, \qquad (79)$$

and

$$\nabla \mathbf{h} = 0. \qquad (80)$$

Equations 77–80, in the more familiar notation, are the well-known equations,

$$\text{curl } \mathbf{h} - \frac{1}{c}\frac{\partial \mathbf{e}}{\partial t} = \frac{\rho}{c}\,\mathbf{v}, \qquad (\alpha)$$

$$\text{div } \mathbf{e} = \rho, \qquad (\beta)$$

$$\text{curl } \mathbf{e} + \frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} = 0, \qquad (\gamma)$$

$$\text{div } \mathbf{h} = 0. \qquad (\delta)$$

Let us return to the discussion of the complete definition of the vector **m**. All that we have hitherto said of this vector is comprised in the statement,

$$\Diamond \times \mathbf{m} = \mathbf{E} + \mathbf{H}.$$

It is evident that this equation does not completely define **m**, for in general if **m**′ is a vector satisfying the equation

$$\Diamond \times \mathbf{m}' = \mathbf{E} + \mathbf{H},$$

we may superpose upon the field of the vector **m**′ the field of another vector **m**″ for which

$$\Diamond \times \mathbf{m}'' = 0.$$

Then if $\mathbf{m} = \mathbf{m}' + \mathbf{m}''$, we also have

$$\Diamond \times \mathbf{m} = \mathbf{E} + \mathbf{H}.$$

Suppose now [22] that **m**″ be so chosen that at every point in the field

$$\Diamond \mathbf{m}'' = - \Diamond \mathbf{m}'.$$

Then **m** satisfies the two equations,

$$\Diamond \times \mathbf{m} = \mathbf{E} + \mathbf{H},$$
$$\Diamond \mathbf{m} = 0. \qquad (81)$$

We may, therefore, without in any way modifying what has preceded, complete the definition of **m** by the equation (81). This equation combined with (67) gives the well-known expression,

$$\nabla \mathbf{a} + \frac{\partial i\phi}{\partial x_4} = 0, \qquad (82)$$

or

$$\operatorname{div} \mathbf{a} + \frac{1}{c}\frac{\partial \phi}{\partial t} = 0.$$ [23]

Now by equations (53) and (69),

$$\Diamond \times \mathbf{m} = \Diamond(\Diamond\,\mathbf{m}) - \Diamond^2 \mathbf{m} = \mathbf{q},$$

or by (81)

$$\Diamond^2 \mathbf{m} = - \mathbf{q}. \qquad (83)$$

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[22] This is not offered as a rigorous proof, for we have assumed that a field can be chosen with pre-determined values of $\Diamond \mathbf{m}''$ and $\Diamond \times \mathbf{m}''$.

[23] Abraham-Föppl, II, equation (30).

This is another simple form of our fundamental equation. Substituting for **m** by (67) gives the important equations[24]

$$\nabla^2\mathbf{a} - \frac{1}{c^2}\frac{\partial^2\mathbf{a}}{\partial t^2} = -\frac{\rho}{c}\,\mathbf{v}, \tag{84}$$

$$\nabla^2\phi - \frac{1}{c}\frac{\partial^2\phi}{\partial t^2} = -\rho. \tag{85}$$

Let us emphasize once more that all the equations of this section are mere definitions, or purely mathematical deductions, with the sole exception of the one equation which embodies the experimental facts, namely,

$$\Diamond\Diamond\times\mathbf{m} = \mathbf{q}.$$

In conclusion let us consider what is meant by the rotation of the axes in this four-dimensional space. The theory of relativity, as here employed, is equivalent to the statement that our four-dimensional vector equations are invariant in any orthogonal transformation of the axes x, y, z, ict.

The axis ict is characterized by the equation $\frac{\partial x}{\partial t} = \frac{\partial y}{\partial t} = \frac{\partial z}{\partial t} = 0$ and may be regarded as the four-dimensional locus ("Weltlinie") of a point at rest. A straight line, making a small angle with this axis in the plane passing through x and ict, is the locus of a point in uniform motion along the x axis. Taking this line as a new axis (ict') and in place of x, a new axis x', perpendicular to y, z, and ict', we have a new coördinate system in which our fundamental equation (69) retains complete validity. In other words, as Einstein pointed out, the equations of the electromagnetic field remain true, whatever point is arbitrarily chosen as a point of rest.

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[24] Abraham-Föppl, II, Equations (30 a) and (30 b).

CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE RESISTIVITY OF HARDENED CAST IRON AS A MEASURE OF ITS TEMPER AND OF ITS FITNESS FOR USE IN PERMANENT MAGNETS.

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It has long been known that the specific electrical resistance of a piece of soft tool steel is materially less than that of the same piece after it has been hardened, and that the relaxing of the temper of any piece of steel or iron makes the specific resistance less; but the first systematic study of this phenomenon was made by Messrs. Barus and Strouhal whose work is summarized in Bulletin 14 of the U. S. Geological Survey.[1]

In one experiment which these gentlemen made upon rods of "English Silver" steel, 0.15 cm. in diameter and all originally glass-hard, different pieces were tempered by heating them to different fairly high temperatures, as indicated by the oxide tints on their surfaces, and were then cooled. When the specimens thus treated were tested it appeared that the harder the temper, the higher was the specific resistance (s) referred to the centimeter cube, and the lower the temperature coefficient (a) of the specific resistance. In the case of a certain glass-hard rod, s, in microhms, was 45 and a was 0.0016; while in a thoroughly annealed rod of the same lot, s was 16 and a about 0.0040. From these

[1] Wiedemann, Lehre von der Elektricität, Vol. I, p. 502; Benoit, Comptes Rendus. **76**, 342 (1873); Matthiessen and Vogt, Pogg. Ann., **122**, 10 (1864); Auerbach, Wied. Ann., **5** (1878); Wied. Ann., **8**, 479 (1879); Callendar, Phil. Trans. 1887; Strouhal and Barus, Wied. Ann., **11**, 976 (1880); Barus, Phil. Mag., **8**, 341 (1879); Chernoff, Vortrag gehalten in der Russischen Technischen Gesellschaft, 1868; Jarolimek, Dingler's Journal, **221**, 436 (1876); Jarolimek and Ackermann, Zeitschrift für das Chemische Grossgewerbe, 1880; Percy-Wedding, Eisenhüttenkunde, II, p. 130, 1864; Karsten, Karsten und von Dechen's Archiv, **25**, 223 (1853); Barus and Strouhal, Bulletin of the United States Geological Survey, No. 14; Caron, Comptes Rendus, **56**, 43 (1863); Barus, Physical Review, **30**, 348 (1910).

TABLE I.

s.	*a.*	*s.*	*a.*
10	0.0050	30	0.0024
12	46	32	22
14	42	34	21
16	39	36	20
18	36	38	19
20	34	40	18
22	32	42	17
24	29	46	16
26	27	50	15
28	26	60	14

and similar experiments, Barus and Strouhal made out a table connecting s and a which they subsequently found to fit other kinds of steel pretty well. Some of their results are given in Table I.

If corresponding values of s and a be used as coördinates, a fairly smooth curve results, and the mean values of 79 for s and 0.0013 for a which Barus and Strouhal got for three pieces of cast iron which they tested, yield a point which seems to lie closely enough upon the prolongation of this curve. It appears also that the values of s and a which Matthiessen, Vogt, and Benoit obtained for different kinds of wrought iron agree numerically with the values for steel; and some persons have thought that it is possible to determine the position of any piece of iron or steel in the scale of mechanical hardness, without any knowledge of the percentage of combined carbon, by finding s alone.

For bar magnets or for simple bent magnets, fine tool steel, or better, some of the kinds of special magnet steel, serve very well, but if a permanent magnet is required of such a shape that the steel has to be heated red hot a number of times during the process of forging and before it is made glass-hard, irregular temper thus introduced into the material often shows itself in the presence of irregular magnetization when the magnet is finally charged, and this sometimes makes the magnet worthless. For this and other reasons, some makers of electrical instruments are now using chilled cast iron for such magnets, and these have usually proved to be satisfactory. They are cheap, they can be made quite as strong as tool steel magnets of the same dimensions, they are very permanent after they have once been aged, and the temperature coefficients of their magnetism are almost always much smaller than those of forged steel magnets. Cast iron for permanent magnets must, however, be really hard, and, unfortunately, mechanical tests of the hardness of this metal are often deceptive; it seems desirable, therefore, to inquire whether the electric resistivity of a piece of chilled cast iron is a criterion of its temper.

This paper gives briefly a few of the results of a large number of observations made originally with the object of testing the relative efficiencies of different methods of hardening cast iron for magnets, in use in the Jefferson Laboratory. The details of this work have mainly a local interest and are not enumerated here, but some general facts may be useful to persons who have to make such magnets for themselves.

Each of the test pieces was a rod about 30 cms. long and a little less than 0.6 cm. in diameter. These were all milled down from stouter pieces about 1.5 cms. in diameter which were usually cast in sets of a dozen from a grid pattern to insure that they should be of the same kind of iron. Different specimens from the same grid, however, often showed different resistivities before they were annealed and occasionally one or two pieces from a grid would differ sensibly from the other pieces after all had been softened with great care. These differences are to be expected, as Karsten showed long ago, for the outer layers of a mass of chilled cast iron sometimes contain a greater proportion of combined carbon than the inner layers in which most of the carbon may be free, and an unequal chilling of a grid in the mould would naturally make the material slightly different in different parts. It is easy in practice to avoid abnormal specimens. All the test pieces were prepared, annealed, and hardened by Mr. George W. Thompson, the mechanician of the Jefferson Laboratory, whose experience in treating cast iron extends over many years.

The measurements of the specific resistances of the rods (usually three for each specimen) were mostly made with the help of a standard Kelvin Double Bridge, but in a few cases the test piece was connected in series with a standard manganin resistance bar and a constant storage battery, and the small potential drop across a measured length of the rod was compared with the corresponding drop across the standard. Three commutators were used with this apparatus so that the effects of disturbing electromotive forces at the contacts might be avoided. The ultimate standard was Wolff No. 2718 furnished with the certificate of the Reichsanstalt.

In the determinations of the temperature coefficients of resistivity two large tanks of water were used. One of these was approximately at room temperature. The water in the other, which was kept in constant motion by a set of four propellers run by a small motor, was heated to a constant definitely determined temperature by means of a Simplex Electric Heater attached to a 110 volt circuit and dominated through a relay by a delicate thermostat. The annealing effects of very hot water upon hard cast iron had to be avoided, but the water in the second tank was usually made uncomfortably warm for the hand.

In making cast iron magnets, it is very necessary that the iron just before it is chilled shall be much hotter than it is safe to heat ordinary tool steel in making it hard. Dr. Campbell, of the National Physical Laboratory, Teddington, Middlesex, England, finds that a temperature of 1000° C. has been sufficient for the iron he has used, but some specimens of American iron seem to work best at a slightly higher temperature, just below the melting point. If a massive piece of cast iron weighing, say, fifty pounds be heated thus hot and then chilled in a proper bath, the material, as magnetic tests can be made to show, becomes hard throughout, whereas it is practically impossible to make a similar piece of tool steel glass-hard inside. The experiments of Chernoff upon a certain kind of steel, made more than forty years ago, showed that if the temperature from which the steel was chilled was made higher and higher, from, say, 400° C., the hardening effect was almost inappreciable until a cherry red was reached, when suddenly the chilled specimen was found to be glass-hard. It is not very surprising, therefore, that cast iron shows very little temper when chilled from a temperature of 800° C. or 900° C., but may easily be made glass-hard if its temperature just before the chilling is high enough, say 1050° C. for some kinds.

The rods were heated for the hardening, under a compressor blast, in a special gas furnace made for the purpose by Messrs. J. Connors and J. Coulson, and most of them were placed inside an iron tube to protect them from direct exposure to the flames. In annealing the rods they were packed in iron filings inside an iron tube closed at the ends by screw caps and heated thoroughly to a white heat for possibly 30 minutes before the tube was packed in ashes for many hours. Although the work was done with the greatest care, it soon appeared that it is usually impossible, at least by this particular annealing process, to bring a piece of cast iron once made glass-hard back to as low a resistivity as it originally had, and if the piece be repeatedly hardened and annealed, its resistivity in the relaxed state increases every time the cycle is passed through. The diameter of the piece also increases perceptibly much as the cast iron bars of a fire box grate grow longer with hard use. Two or three examples will show the complicated nature of the phenomena involved.

Two test pieces from the Broadway Iron Works, Cambridgeport, were annealed as they came from the foundry and then had resistivities 102.5 and 102.7 and a diameter of 0.574 cm. After both had been hardened, the resistivities at about 20° C. were 122.5 and 122.0, and after they had been again through the annealing furnace their resistivities were 108.7 and 107.1. The fourth time they were relaxed the specific resistances were 112.6 and 112.6, and their average diameters about 0.578 and 0.576. When they were finally hardened again, the resistivities were 136.7 and 137.8 and both diameters were 0.581. It did not seem worth while to carry the process farther.

TABLE II.

CAST IRON ROD FOUR TIMES HARDENED AND ANNEALED.

H.	*B.*	*H.*	*B.*
1.13	57	7.70	964
1.40	79	9.15	1521
2.03	120	13.2	2910
3.31	222	20.6	4585
4.40	326	32.7	6030
5.75	518	42.4	6430
6.54	681		

Another rod, presumably of a very different kind of iron, began with a diameter of 0.574 and after four annealings had a mean diameter of 0.578. Its resistivity in the relaxed state rose in four steps from 93.9 to 102.5; the first time it was hardened its resistivity was 112.0, the last time 116.5.

In the three cases here mentioned the specimens would cut common window glass easily the first time they were hardened; they were mechanically too soft to scratch the same glass when, having been repeatedly hardened and relaxed, they were finally hardened so that they had a higher resistivity than at first.

Another rod from the same foundry had a resistivity of 102.0 when it was first annealed, and a resistivity of 119.8 when it was hardened for the first time. After an hour in steam at 100° this fell to 118.0, and after five hours farther steaming to 116.6. The second time it was annealed the rod had a resistivity of 106.5, and the third time of 107.2.

The temperature coefficient of the resistivity of the first rod spoken of above was 0.00102 when the rod was soft; the third rod had a temperature coefficient of 0.00094.

Cast iron which has been several times hardened and annealed is finally in its annealed state not so permeable as once-annealed soft cast iron is. Table II gives the results of tests upon a rod of resistivity 98.3 which has been four times heated white hot and chilled and then annealed.

If the process of heating and chilling a number of cast-iron rods be carried out many times in succession without proper annealing after each chilling, there does not seem to be a progressive increase in the resistivity; the results are anomalous.

Several kinds of chilling baths were used for hardening the cast iron, among them ice cold water, cold brine, sulphuric acid and water, an acid bath (X) the constitution of which is a trade secret, but which, I understand, has been much used in commercial work; mineral oil, and paraffine.

It has long been known that in the hardening of tool steel from a dull red heat, it is much more important that the fall of the temperature of the piece down to say 300° C. shall be quickly brought about than that the rest of the journey to room temperatures shall be rapid. It is not difficult to cool quickly a slender rod, but a large piece of hot metal suddenly immersed in a water bath is immediately surrounded by a layer of steam and, unless the water be very vigorously stirred as in die hardening, the metal may remain red hot for a comparatively long time. Many attempts have been made by varying the chemical nature of the bath to lessen the effect of the steam cloak, and some persons have used a bath of easily fusible metal for the first part of the chilling process (as is now the practice for some of the new high power steels), and have completed the cooling in a water bath, the temperature of which within wide limits seems to be unimportant.

In the light of the behavior of steel, it seemed unlikely that in the hardening of cast iron from a temperature much higher than can be used with ordinary tool steel, there would be much advantage in making the hardening baths especially cold, and experience justified this assumption. Sometimes the hardening bath was chilled with ice, but usually it was used at room temperatures or even lukewarm.

For rods of the dimensions of the test pieces I used, water, brine, sulphuric acid and water, and the X mixture seemed almost equally effective in making the cast iron glass-hard, whether resistivity or magnetic permeability of the hardened piece was used as the criterion.

For massive pieces of iron the N mixture, which certainly is very good, is said to work more uniformly than a water bath. Several specimens which were chilled in iced water and iced brine developed minute cracks which showed in irregularities when the rods were magnetized, but these, which were tested before the construction of the special gas furnace, may not have been uniformly heated. The oil bath was nearly as good, so far as increasing the resistivity of the specimen, as the water bath, but the hardened pieces did not seem so hard mechanically. The melted paraffine wax, at as low a temperature as would keep the wax liquid, also increased the resistivity of a specimen chilled in it, provided it had not been hardened before, quite as much as the water bath, but a piece thus hardened would not scratch glass.

Most of the pieces of American cast iron which I have tested had, when soft, resistivities referred to the centimeter cube, which at 0° C.

TABLE III.

Grid.	*s.*	*a.*
I	76.5	0.00104
II	86.1	0.00106
III	89.9	0.00099
IV	94.2	0.00084

would lie between 73 microhms and 104 microhms. These pieces when hardened for the first time had resistivities which at the same temperature lay between 80 and 126. Nine pieces of American cast iron tested when soft by Barus and Strouhal had on the average a resistivity at 20° C. of about 79.1 microhms with a temperature coefficient of 0.00120. Four grids, typical of the softer kinds of iron which I have used, gave on the average when soft at the same temperature the results which appear in Table III.

To show the effect of hardening upon the temperature coefficient of the resistivity, I may instance six specimens with three different coefficients when hard. (See Table IV.)

When a number of steel bars of the same length and cut from the same long rod are hardened and are then magnetized in the same solenoid and aged, it frequently happens, as is well known, that the ultimate magnetic moments of the bars differ somewhat widely from one another; and the same thing is true of magnets made from cast-iron rods cut from the same grid. In Table V are given the magnetic

TABLE IV.

Rod.	s'.	a'.	s''.	a''.
1	94	0.00086	114	0.00070
2	97	0.00085	117	0.00070
3	79	0.00105	103	0.00086
4	75	0.00108	102	0.00086
5	88	0.00103	113	0.00091
6	86	0.00106	117	0.00091

s' and s'' are the resistivities at 20° C. in the soft and in the glass-hard states, respectively; a' and a'' are the temperature coefficients.

moments (M) and the temperature coefficients of the moments (k) of eight typical bar magnets which have been tested with great care by Mr. John Coulson.

TABLE V.

Grid.	Rod.	M.	k.	Chilling bath.	s.
A	1	1550	0.00022	Water	110
A	2	1580	22	Water	104
B	3	1720	24	Water	94
B	4	1740	26	Water	94
B	5	1560	26	H_2SO_4 and water	96
C	6	1410	27	H_2SO_4 and water	109
C	7	1500	27	H_2SO_4 and water	106
C	8	1600	27	"X" mixture	106

Mr. Coulson tested at the same time three magnets of the same dimensions as these but made of glass-hard Stubs tool steel. They had on the average a moment of about 1690 and a temperature coefficient of about 0.00095, which is more than three times as large as the corresponding value for cast iron.

After the moments of the eight cast-iron magnets had been determined, the rods were thoroughly demagnetized inside a solenoid through which a long series of currents, gradually decreasing in intensity and alternating in direction, could be sent. Then each was placed inside another solenoid and an HB diagram was found for it by the method of ascending reversals with the aid of a small test coil about its centre and a ballistic galvanometer of period sufficiently long for the purpose. Each rod was about 50.9 of its own diameters long and, according to

TABLE VI.

H.	*B.*	*H.*	*B.*
10	310	70	4250
20	720	80	4950
30	1290	90	5550
40	2100	100	5950
50	2870	120	6480
60	4100		

the formula of Dr. Shuddemagen for the end corrections of rods of these dimensions, the actual magnetic intensity (H) inside the metal at the centre is equal to $H' - 0.00107\ B$, where H' is the force inside the solenoid when the rod is removed. It is possible, therefore, to

TABLE VII.

Rod.	H_{1000}.	B_{120}.
A 1	26.3	6020
A 2	25.9	6200
B 3	26.5	6480
B 4	25.9	6480
B 5	25.2	6180
C 6	25.9	5960
C 7	26.1	6045
C 8	25.8	6150

determine very approximately the relative values of H and B from the observed values of H' and B, and the computation has been made by Mr. Coulson for these rods. The results of this work show that though the moments of the magnets differed so much among themselves, the permeabilities of the pieces of metal for excitations up to $H = 50$, at least, are much the same. Magnet B4, for instance, had a moment much larger than the moment of C6, but the value of H corresponding to an induction of 1000 was in each case about 25.9.

For the rod B4 the relation between H and B is indicated approximately at all events by the numbers given in Table VI.

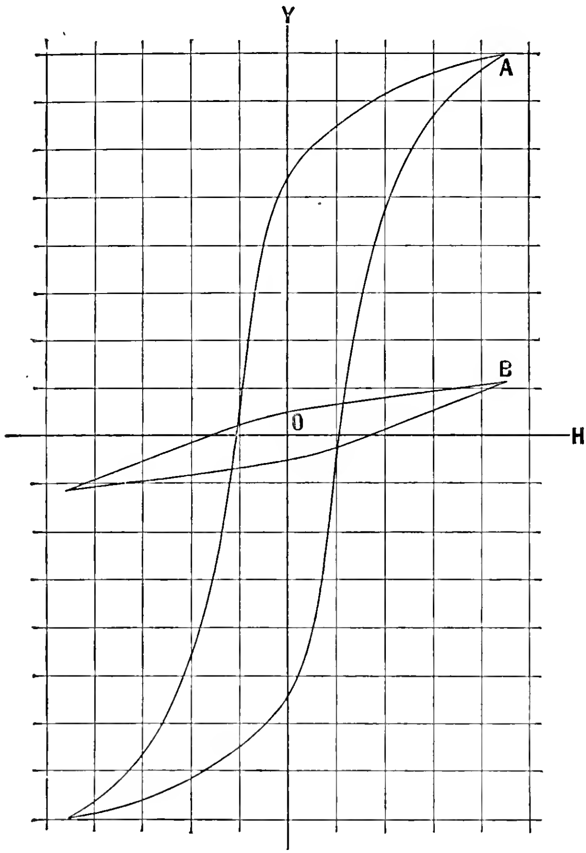

FIGURE 1.

Table VII gives under H_{1000} the value of the excitation corresponding to $B = 1000$, and under B_{120} the value of the induction corresponding to $H = 120$ for all the rods.

The specimens used were cast at different times in order that they might fairly represent the best mixtures used by the foundries from which they came, and in view of this fact the near agreement of the measurements recorded in this table is very striking. The differences are not greater than one might expect to find in a number of rods of fine polished drill rod from the same lot. For the present discussion it is of interest to notice that the permeabilities of the hard rods seem not to be connected in any obvious way with the resistivities. For any single specimen of cast iron, however, it is well known that hardening usually decreases the permeability especially at comparatively low excitations, and Figure 1 shows a rough kind of hysteresis diagram which I obtained some years ago for a cast-iron frame of several kilograms weight. Curve A corresponds to the soft state and B to the

TABLE VIII.

H.	*B*. (Rod hard.)	*B*. (Rod soft.)
100	6800	9650
200	8850	11160
300	10310	12460
400	11420	13550
500	12130	14400
600	12660	14980
10000	25650	28250
11000	26600	29300
12000	27500	30400
13000	28450	31300
14000	29400	32050
15000	30350	33600

hardened state of the same piece of iron. At high excitations the difference is not so striking but is very real.

Table VIII gives approximately the results of some measurements made two or three years ago upon cylinders and isthmuses of a certain kind of cast iron from the Broadway Iron Foundry. It must be clearly understood, however, that this applies only to iron which has once been through the annealing and subsequent hardening. A repetition of the process makes the hardened iron mechanically softer. As we have seen, a piece of cast iron properly hardened for the first time makes as strong a permanent magnet as a piece of Stubs Drill Rod does, but if the cast iron be several times hardened it becomes incapable of retaining the charge given it in the solenoid and the resulting magnet is perhaps only half as strong as the steel magnet.

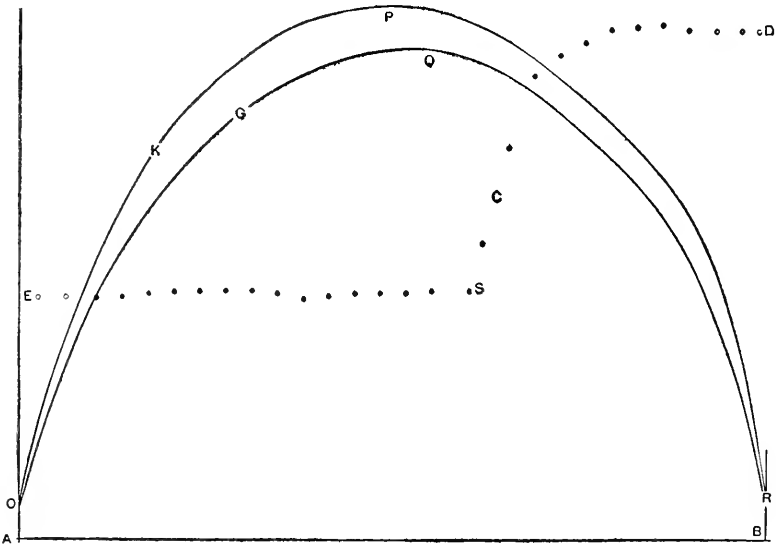

FIGURE 2.

The same phenomenon appears in the case of tool steel, though it is not very easy to harden a piece of tool steel glass-hard a number of times in succession without working it under the hammer to avoid the appearance of minute cracks in the metal.

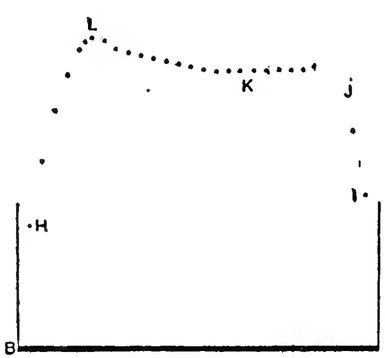

FIGURE 3.

For many years small magnets made of cast iron as it comes from the founder have been used in toys and in small "magnetos," but such magnets are not nearly permanent and are not so strong at the outset as similar magnets made of properly chilled iron. A certain annealed rod which I tested had when magnetized to saturation a moment of 605 on a certain scale, but a few minutes in boiling water reduced this to 455; when the rod had been hardened and again magnetized, its moment on the same scale as before was 831 and boiling reduced this to 740. The same magnetized castings are tested year after year in the Jefferson Laboratory, and so far as my experience goes, a properly hardened and aged magnet made of cast iron is quite permanent if it is exposed to such fields as that

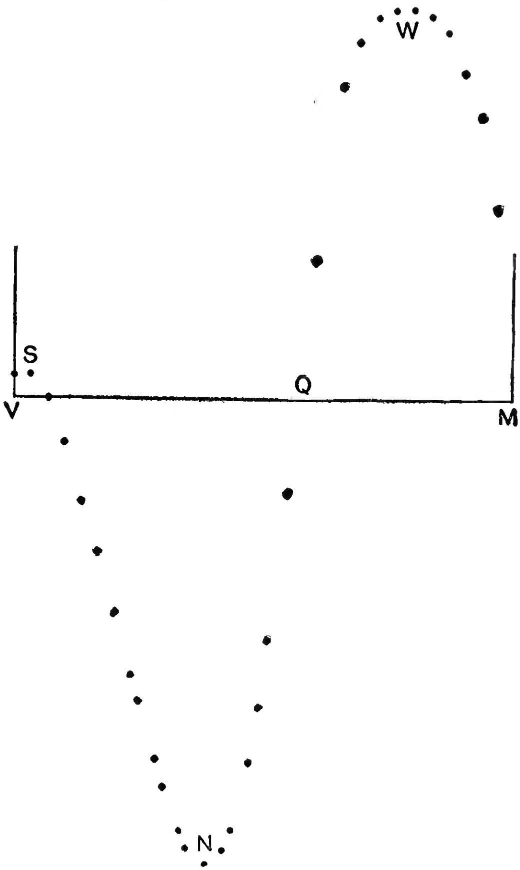

FIGURE 4.

of the earth, and mechanical shocks do not injure them in any way, if the metal is not broken or abraded.

Although a knowledge of the resistivity of a piece of cast iron tells very little about its temper unless one knows also its resistivity in the annealed state, yet the resistivity of different portions of the same piece is a trustworthy measure of the uniformity of temper. Tried by this test, many a piece of steel which has been hardened with care proves to be far from homogeneous.

Occasionally great differences of resistivity may be found in a magnetized steel rod which yields a fairly uniform iron-filing diagram.

The curve OKPR of Figure 2 shows the induction flux (B) at different points of the axis of a rod of Crescent Polished Drill Rod 29 cm. long and 0.5 cm. diameter just after it had been magnetized to saturation in a solenoid. Curve OGQR shows the same quantity after the rod had been exposed to steam for some time. AB is the common base of these curves. The distribution is in each case nearly uniform, and the iron-filing curve seems entirely so, but the resistivity of the metal is far from uniform, as the dotted diagram ESCD shows. This was obtained by measuring the resistances of a large number of very short lengths of the rod and determining from the results values for the resistance per centimeter at about thirty points on the axis. Of course a small portion at each end could not be treated in this way, and the fact is indicated by the open dots. One end of this bar was in the soft state in which this excellent steel comes in the market; the other end had been heated red hot and chilled, so that its resistivity was quite double that of the soft end. This magnet was not so strong as a hardened magnet of this steel should be, but was otherwise normal enough.

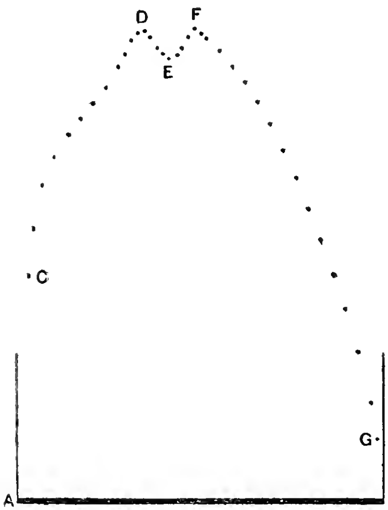

FIGURE 5.

Sometimes the iron-filing diagram belonging to a bar magnet seems very irregular when the distribution of magnetism in the metal is not very abnormal. Figure A shows a filing diagram belonging to a piece of Crescent steel of the same dimensions as that just described, while Figure 3 shows the values of B at different points in the axis. The "centre of gravity" of the magnetism is in this case not far distant from the middle of the bar. This same bar was remagnetized by rubbing a point near its centre upon one pole of a large motor, and then gave a filing diagram represented by Figure B. Here there are real consequent poles, and the distribution of the induction flux in the bar is shown by Figure 4. After this rod had been demagnetized as well as possible in a solenoid by the use of a series of currents alternating in direction and gradually decreasing in intensity, and then had been magnetized again to saturation in a solenoid as before, Diagram A came back again.

Another unequally hardened steel rod of the same kind gave the filing diagram shown in Figure C, and in this case the distribution of magnetism was that indicated in Figure 5.

Figure 6 shows in the curve HYU, of which the horizontal line through E is the base, the resistivity of a rod of cast iron of the dimensions of the specimens used in this investigation. For this particular piece the resistivity at one end corresponded to the annealed state and at the other end to glass-hardness. After this rod had been magnetized in a solenoid, the distribution of magnetism in it was that represented by the dotted curve GZX. This rod when magnetized irregularly on the motor gave the diagram LCK, but when the rod was demagnetized and again magnetized in the solenoid, the distribution GZX returned. It is interesting to notice that in the cases shown in Figures 4 and 6, the motor gave a smooth distribution of B while the solenoid gave an irregular one. When real consequent poles are pres-

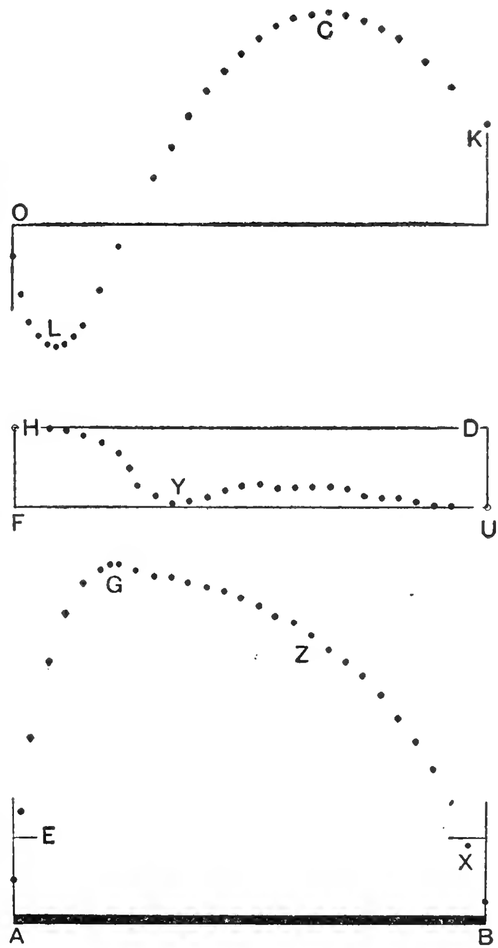

FIGURE 6.

ent, the value of B is at its greatest smaller than in the case of the solenoid magnetization.

Figure 7 shows in the curve PADQ the distribution of magnetism in an unequally hardened cast-iron rod when the magnetization took place in a long solenoid. Curve PDBQ shows on an exaggerated scale the distribution when the rod was magnetized between the poles of a large electromagnet. The greatest value of B was in this latter case about two thirds the corresponding value when the solenoid was used. In all the instances I have met, the solenoid gave the greatest value of B

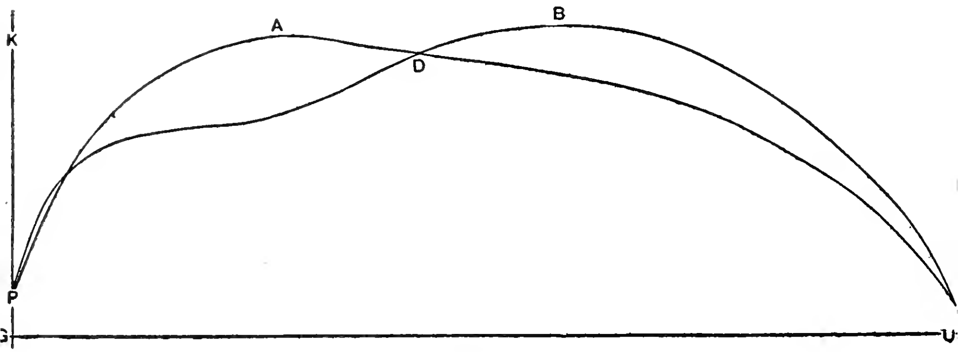

FIGURE 7.

and any other distribution gave an appreciably smaller value. Table IX gives the resistivity at points distant n cm. from the end of the rod which corresponds to G in Figure 7. It is evident that one end of the rod is glass-hard and the other very soft.

The most common form of irregularity in a cast-iron bar magnet seems to consist, if one may judge from a filing diagram, in a simple displacement of the magnetic centre from the geometric centre towards one end of the axis. This usually corresponds to a comparatively slight difference of resistivity along the bar. This case may be illustrated by a rod (K) which had once been hardened irregularly and then had been rehardened as uniformly as possible. In all such cases it is extremely difficult to get rid of the effects of careless hardening, though the irregularity may come up in a slightly different form. The next table (X) gives the resistivity of the metal, and, on an arbitrary scale, the value of B at a point distant n cm. from one end of this bar, which was 29 cm. long.

Table XI gives the resistivities and the relative values of B on the axis of a cast-iron magnet (Q) made of a rod hard in the middle and soft at the ends.

TABLE IX.

n.	*s.*	*n.*	*s.*
3	0.000121	16	0.000097
4	122	17	94
5	121	18	94
6	121	19	92
7	120	20	92
8	117	21	92
9	115	22	93
10	111	23	94
11	104	24	94
12	097	25	94
13	097	26	93
14	097	27	93
15	097		

TABLE X.

n.	*s.*	*B.*	*n.*	*s.*	*B.*
0		60	15	127	551
1		227	16	126	560
2		317	17	126	568
3	121	368	18	127	572
4	118	405	19	129	578
5	119	430	20	129	580
6	118	448	21	129	578
7	120	460	22	129	573
8	121	470	23	129	565
9	121	480	24	126	535
10	122	493	25	125	505
11	123	509	26	124	453
12	123	520	27		374
13	126	532	28		260
14	126	541	29		72

TABLE XI.

n.	s.	B.	n.	s.	B.
0		40	15	155	412
1		140	16	150	429
2		207	17	147	445
3	117	261	18	141	458
4	119	305	19	128	448
5	117	339	20	119	433
6	118	371	21	118	417
7	120	394	22	117	399
8	121	416	23	116	359
9	132	434	24	115	330
10	149	440	25	113	292
11	153	428	26		257
12	153	417	27		198
13	156	409	28		135
14	158	407	29		37

If the material used in these experiments may be considered typical of the so-called "pure cast iron" from good foundries, it appears, then, that an annealed casting may have at room temperatures a resistivity, referred to the centimeter cube, as low as 0.000073 or as high as 0.000104; that it is always possible to make the specimen glass-hard throughout by heating it to a temperature a little below the melting point and chilling it in a suitable bath; and that the process, as Barus and Strouhal showed, is always accompanied by an increase in resistivity. This increase is sometimes only about ten per cent of the original value, though it is oftener nearly twenty-five per cent and may rise somewhat higher. Only one kind of iron that I used resisted successfully a noticeable relaxation of temper in the hardened pieces by prolonged boiling in water. Of two pieces of iron from the same pouring, which have equal resistivities when first annealed, that one has the higher resistivity, after both have been hardened, which has the lower magnetic permeability. Tests of mechanical hardness are difficult to make upon cast iron and often disagree with the resistivity test. A repetition of the annealing and hardening process increases somewhat the size of a specimen and increases the resistivity for both the annealed and the chilled states, but in the hardened state the iron is never so hard mechanically as at the first hardening, and the bar loses in great measure its magnetic retentiveness, as do most kinds of tool steel which have been through the same experience. Many kinds of chilling liquids serve to make cast iron glass-hard, but for massive pieces cold water seems not to give such uniform results as the acid bath used by some professional hardeners. The temperature coefficient (a) of the resistivity of every one of my specimens was decreased by the hardening, though this does not seem to have been the case for the special cast iron used by Barus and Strouhal, which had a larger coefficient (120) than any I used. The coefficient a is not always smallest in that one of a number of specimens of cast iron which has the largest resistivity.

Castings from different sources often show when glass-hard a very close agreement in magnetic permeability, though their resistivities and the temperature coefficients of the resistivities may differ widely. The temperature coefficient of the magnetic moment of a cast-iron bar magnet is usually not more than one third as large as that of a similar magnet made of tool steel.

A uniformly hardened cast-iron or steel rod may have been irregularly magnetized, but if it be thoroughly demagnetized and then carefully remagnetized in a solenoid, its magnetism will become regular. Only irregular hardening seems to lead to persistently irregular magnetization in the case of a bar magnet, though the use nowadays of electromagnetic crane lifters sometimes magnetizes iron and steel rods in a manner which is difficult to deal with in the laboratory. Even an irregularly hardened slender rod may usually be demagnetized well enough for all practical purposes in a solenoid which carries currents alternating in direction and gradually decreasing in intensity, but large thick pieces are very tenacious of charges once given to them. The shield of a certain Rubens Panzer galvanometer in use in the Jefferson Laboratory was twice heated white hot and was kept hot for some time in a vain attempt to get rid of a slight magnetization. The resistivity of different portions of a casting gives trustworthy information about the uniformity of the hardening. Occasionally, as in a case cited above, an irregularly hardened piece of tool steel may be magnetized nearly normally, but usually irregular hardening leads to an irregular distribution of the magnetism which shows itself in an abnormal iron-filing diagram. An unusual filing diagram does not however, as some instances given show, always indicate that the distribution of the magnetic induction in the bar is very irregular.

My thanks are due to the Trustees of the Bache Fund of the National Academy of Sciences who have kindly lent me some of the apparatus used in making the observations described in this paper.

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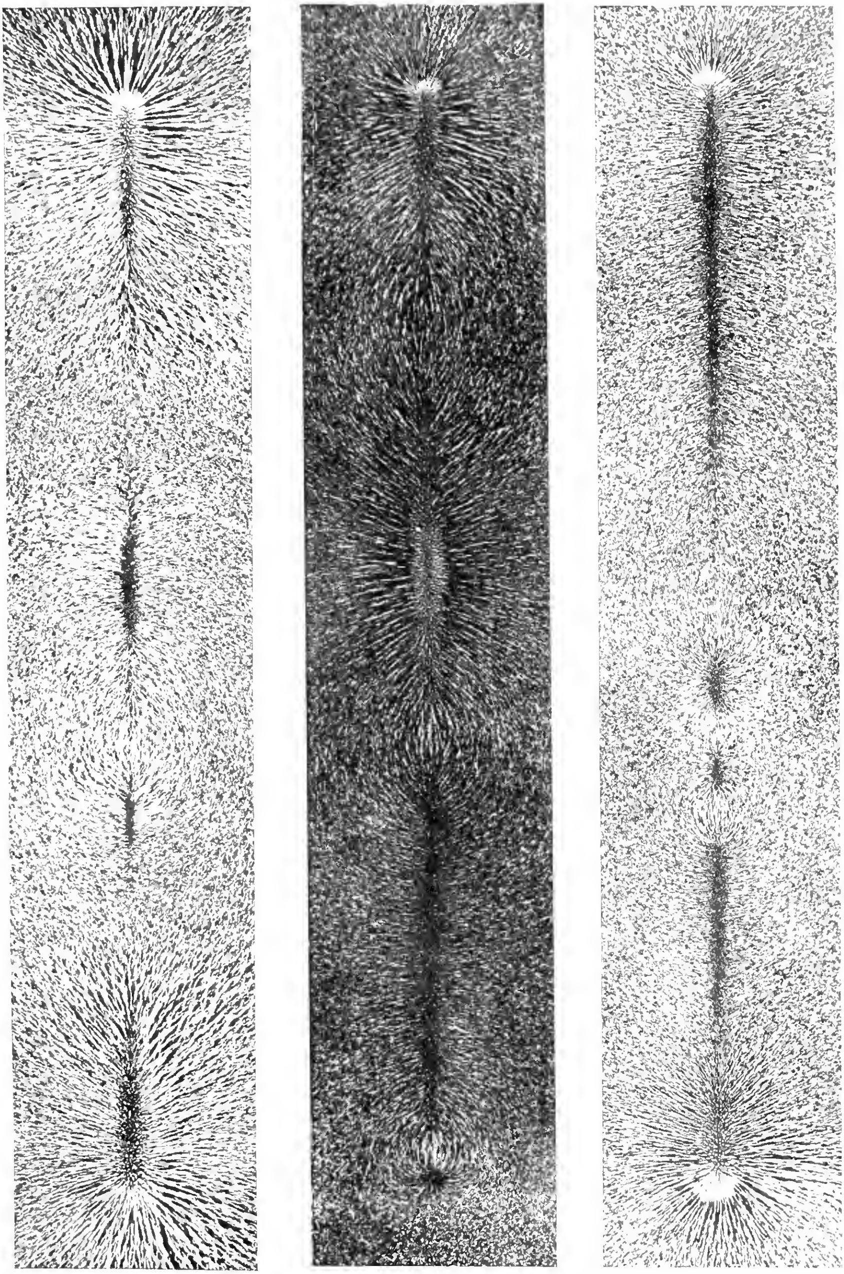

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE MAGNETIC PERMEABILITIES AT LOW EXCITATIONS OF TWO KINDS OF VERY PURE SOFT IRON.

By B. Osgood Peirce.

THE MAGNETIC PERMEABILITIES AT LOW EXCITATIONS OF TWO KINDS OF VERY PURE SOFT IRON.

By B. Osgood Peirce.

Presented May 11, 1910. Received August 26, 1910.

More than a year ago, I had occasion to study the magnetic properties under very high excitations of a piece of Norway iron[1] (P), which proved when analyzed to be extraordinarily pure. The tests made in the Chemical Laboratory of Harvard University by Mr. E. R. Riegel, for nickel, cobalt, tungsten, and even manganese, as well as for the metals of Groups IV and V, were all negative. There was less than 0.03 per cent of carbon, less than 0.047 per cent of phosphorus, less than 0.03 per cent of silicon, and less than 0.003 per cent of sulphur. A slender rod of this remarkable iron, of which we had originally a round bar five centimeters in diameter and thirty-four centimeters long, had, when annealed, an extremely high permeability under excitations above 200, but, because of the local reluctance at the joints, it did not prove easy to determine the permeability of this rod in a yoke at low excitations. The metal showed to the eye a fibrous structure with striae parallel to the length of the bar, as if minute quantities of scale had been included in the bar in the rolling; and it seemed likely that the specific reluctance to magnetization across the grain of the iron, would be greater than to magnetization parallel to the grain. Under these circumstances it was probable that the permeability of a ring, so cut from the metal that its axis should be parallel to the grain, would appear low. It happened, however, that I had two such rings, but that there was not enough of the iron left to make rings with axes perpendicular to the grain, and I was forced to get what information I could from them, though it soon became evident that for excitations above five gausses the permeability fell below what commercial Norway iron should show.

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[1] American Journal of Science, **28**, July, 1909.

This paper gives the results of some tests made at low excitations, which are interesting because of the great susceptibility which the rings showed in fields less than two gausses, and compares the magnetic behavior of this metal with that of a ring of the so-called "American Ingot Iron," which I obtained through the kindness of Dr. P. W. Bridgman. This well-known iron, which was made by the American Rolling Mill Company of Middletown, Ohio, seems to be perfectly homogeneous, and, according to the makers, contains less than 0.03 per cent of impurities all told.

All the rings were very accurately made by Mr. G. W. Thompson, the mechanician of the Jefferson Laboratory. The external diameters of the Norway iron rings were 5.000 cm. and 4.996 cm. respectively; their thicknesses were 0.250 cm. and 0.254 cm., and their breadths were 1.2204 cm. and 1.210 cm. The measurements were made with the help of Zeiss Comparator No. 3196 and a set of auxiliary gauges. After each ring had been measured, a coil of very fine double-silk-covered copper wire was wound on the metal in a single layer and then baked in shellac. Over this was wound, usually in two layers, the exciting coil of well-insulated wire nearly one millimeter in diameter. The ballistic galvanometers were of the moving coil type, and had periods amply long enough [2] for the work. The fine coil on the ring was always in simple circuit with the galvanometer and the secondary coil of a standard of self inductance tested by the Bureau of Standards.

The maximum value of the permeability (5480) which I obtained for the first ring tested seemed so high that at first I suspected that there was some error in the determination, so I changed the galvanometer, and then took off the coils and wound on new ones with different numbers of turns; but when the result was unchanged and the second ring gave values for the ordinates of the HB diagram which were practically indistinguishable from those obtained from the first ring, there seemed to be no doubt that the work had been accurately done. The two rings lay side by side in the original bar, and both must have had nearly the same discontinuities. Table I, founded upon several hundred separate determinations, gives values of the permeability of the metal obtained from 35 different excitations of the first ring and 25 of the second. A ring of very pure annealed iron from the Armstrong Works at Elswick gave in the hands of Wilson the same maximum value of the permeability as the rings just mentioned; but apart from the reports of some tests upon thin pieces of electrolytically deposited iron,

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[2] Peirce, These Proceedings, **44**, 1909 (283).

I have found no other records of permeabilities so high as this.[3] For excitations above six gausses, however, the rings were distinctly less permeable than good iron should be, and this anomalous behavior is perhaps due to discontinuities across the lines of magnetization.

TABLE I.

Annealed Ring of Norway Iron (P), Axis parallel to the Axis of the Original Bar. Measurements made by the Method of Ascending Reversals.

H.	*B.*	*B/H.*	*I.*	*H.*	*B.*	*B/H.*	*I.*
0.05	22	440	1.7	1.70	9040	5320	719
0.10	54	540	4.3	1.80	9340	5190	743
0.15	92	613	7.3	1.90	9640	5070	767
0.20	145	725	11.3	2.00	9870	4935	785
0.25	205	820	16	2.20	10300	4680	820
0.30	280	933	22	2.40	10690	4450	851
0.35	390	1114	31	2.60	11010	4230	876
0.40	495	1238	39	2.80	11310	4040	900
0.45	620	1377	49	3.00	11560	3850	920
0.50	790	1580	63	3.50	12010	3430	956
0.55	990	1800	79	4.00	12310	3080	980
0.60	1260	2100	100	4.50	12550	2790	991
0.65	1585	2440	126	5.00	12760	2550	1016
0.70	1980	2830	158	6.00	13060	2180	1039
0.75	2405	3205	191	7.00	13300	1900	1058
0.80	2850	3560	227	8.00	13510	1690	1075
0.85	3310	3890	263	9.00	13700	1520	1090
0.90	3780	4200	301	10.00	13870	1387	1103
0.95	4255	4480	339	20.00	14950	748	1188
1.00	4730	4730	376	30.00	15520	517	1233
1.10	5620	5110	499	40.00	15850	396	1258
1.20	6380	5320	508	50.00	16080	322	1276
1.30	7070	5440	563	60.00	16300	271	1292
1.40	7670	5480	610	70.00	16510	236	1308
1.50	8200	5470	653	80.00	16700	209	1322
1.60	8640	5400	687				

[3] Stoletow, Ann. d. Physik, **146**, 1872 (439); Riecke, Ann. d. Physik, **149**, 1873; Rowland, Phil. Mag., **46**, 1873 (140); Roessler, Inaugural Diss., Zürich, 1892; Holz, Ann. d. Physik, **8**, 1876 (353); Ewing, Magnetic Induction in Iron and Other Metals; Bauer, Ann. d. Physik, **11**, 1880 (349); G. vom Hofe, Ann. d. Physik, **37**, 1889 (482); Lehmann, Ann. d. Physik, **48**, 1893 (406); Benedicks, Ann. d. Physik, **6**, 1901; Lydall and Pocklington, Proc. Roy. Soc., **52**, 1892;

A well annealed isthmus of this iron cut lengthwise of the bar gave for *I*, under excitations as high as 18000 gausses, a final value of 1795, and an unannealed rod tested in a yoke gave 1730. These remarkable values point to a much higher permeability at medium excitations than the rings just mentioned show.

TABLE II.

Ring of Annealed "American Ingot Iron." Measurements made by the Method of Ascending Reversals.

H.	*B*.	*B*/*H*.	*I*.	*H*.	*B*.	*B*/*H*.	*I*.
0.1	35	175	2.8	4.0	11900	2975	947
0.3	106	530	8.4	4.5	12450	2777	991
0.2	198	660	15.7	5.0	12850	2570	1022
0 4	324	810	26	5.5	13200	2400	1050
0.5	482	964	38	6.0	13500	2250	1074
0.6	688	1147	55	6.5	13780	2120	1096
0.7	990	1414	79	7.0	13960	1994	1111
0.8	1325	1656	105	7.5	14180	1891	1128
0.9	1720	1911	137	8.0	14350	1794	1142
1.0	2320	2320	185	8.5	14480	1763	1152
1.5	5150	3433	410	9.0	14600	1622	1162
2.0	7340	3670	584	9.5	14720	1550	1171
2.5	9050	3620	720	10.0	14830	1483	1180
3.0	10220	3406	813	20.0	15850	792	1261
3.5	11150	3186	887				

Table IV shows corresponding values of *H* and *B* for a ring of annealed "American Ingot Iron" cut out by Mr. Thompson from a large plate of the metal. The outside diameter of the ring was 5.012 cm., its thickness was 0.283 cm., and its breadth about 2.116 cm. There were 112 turns in the testing coil and 74 turns in the exciting coil.

Table III gives for comparison the results of the determinations of the permeabilities of a number of different specimens of soft iron by different observers. Some of the numbers which I have obtained graphically from the published figures are only approximately correct.

Columns 7 and 9 give the records of observations made upon two small rings of very pure iron given by Colonel Dyer of the Elswick works to

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Wilson, Proc. Roy. Soc., **62**, 1898; du Bois, Ann. d. Physik, **51**, 1894 (537); Taylor Jones, Ann. d. Physik, **54**, 1895 (641); Phil. Mag., **39**, 1895 (254); Stefan, Wiener Berichte, **81**, 1880 (89); Ewing, Roy. Soc. Trans.

TABLE III.

H.	B.											
	Norway Iron (P) Rings, 1 and 2.	Long Rod of Norway Iron. (R.)	American Ingot Iron.	Hopkinson's Annealed Wrought Iron. (Average.)	Lehmann's Swedish Iron Ring.	Rössler's Iron Ellipsoid.	Wilson's Ring of Pure Iron (No. 2).	Ewing's Soft Iron Wire.	Lydall and Pocklington's Ring of Pure Iron (No. 1).	Hopkinson's Ring.	Rowland's Norway Iron Ring.	Norway Iron (Q).
0.2	145	90	106	110	140	68	300	26	46	80	100	115
0.4	495	260	324	340	450	165	800	55	128	170	380	340
0.6	1260	570	688	740	1180	282	1400	100	467	330	1100	780
0.8	2850	1125	1325	1280	2050	410	2050	160	1020	550	2500	1750
1.0	4730	2150	2320	2400	3020	560	3400	230	2150	950	4500	3050
1.5	8200	4600	5150	4700	4700	1550	8000	500	6020	3650	6950	5450
2.0	9870	6600	7340	5900	6050	4350	10700	900	7720	5400	8650	7200
2.5	10850	8240	9050	6800	7200	6950	12000	2500	8920	6780	9900	8200
3.0	11560	9480	10220	7600	8300	8700	12800	5800	9850	7800	10800	9150
3.5	12010	10460	11150	8350	9150	9800	13400	8400	10600	8800	11450	10050
4.0	12310	11280	11900	9000	9700	10600	13800	10350	11300	9600	12020	10800
4.5	12550	11980	12450	9500	10150	11240	14150	11650	11850	10250	12500	11400
5.0	12760	12560	12850	9950	10650	11800	14360	12480	12350	10750	12070	11900
6.0	13060	13400	13500	10700	11400	13700	14700	13500	13200	11500	13700	12800
7.0	13300	13900	13960	11300	12050	13400	14950	14100	13900	12100	14100	13400
8.0	13510	14300	14350	11800	12550	13780	15100	14350	14400	12550	14400	13850
9.0	13700	14660	14600	12180	12900	13960	15250	14750	14800	12950	14680	14200
10.0	13870	14940	14830	12500	13280	14050	15320	14950	15050	13350	14950	14400

Sir Frederick Abel, who presented them to Dr. John Hopkinson. The tests upon the first ring by Messrs. Pocklington and Lydall seem to show that they did not anneal the iron; the remarkable measurements of Wilson upon the second ring were made after the iron had been softened. Norway Iron (R) was a long annealed rod about half an inch in diameter. This was tested in a solenoid. An analysis of Hopkinson's ring made by the Whitworths, showed manganese 0.143 per cent, phosphorus 0.271 per cent, sulphur 0.012 per cent, carbon 0.01 per cent and "slag" 0.436 per cent. The Elswick iron contained 0.1 per cent of manganese and 0.013 per cent of sulphur but no phosphorus and hardly a trace of carbon or other impurity. Norway Iron (Q) was an annealed ring cut from a bar of "pure iron" obtained in the Boston market.

My thanks are due to Professor John Trowbridge, who furnished me with the pure iron described above, and to the Trustees of the Bache Fund of the National Academy of Sciences who loaned me some of the apparatus used in the work.

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Cambridge, Mass.

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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

A REVISION OF THE ATOMIC WEIGHT OF NEODYMIUM.

FIRST PAPER. — THE ANALYSIS OF NEODYMIUM CHLORIDE.

By Gregory Paul Baxter and Harold Canning Chapin.

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Contents.

Introduction.

Investigations bearing upon the rare earths have always been particularly attractive on account of some unusual chemical and physical properties of these substances, their peculiar relations to the Periodic System, and in particular the important position which some of them have taken with relation to artificial illumination. To be sure any research involving the preparation of even one of the earths in a state of approximate purity is sure to be protracted, if not tedious. The very proof of the purity of a preparation is by no means easy to secure. The chief guides which have served in the determination of the purity of material have been the spectra, both absorption and emission, and the atomic weights. For a quantitative determination of purity the atomic weight is perhaps more frequently used than any other test. Hence for this reason as well as from theoretical considerations an exact knowledge of the atomic weights of the rare earths is very desirable.

The atomic weights of certain members of the group have received a singularly large amount of attention. Those of cerium and lanthanum, for instance, have been investigated more times than that of iron. Yet owing to the unusual difficulties in the preparation of pure material, as well as to troublesome analytical problems, the determinations of the atomic weights of the rare earths are probably of a lower order of accuracy than those of the common elements.

Among the standard methods which have heretofore been employed in the determination of atomic weights of the rare earths, the following are the most important:

The Analysis of the Oxalate. (Stolba.)[1] The dried oxalate is analyzed for the oxide by ignition, and for oxalic acid by titration with permanganate. The complete analysis of the salt is necessary on account of the invariable presence of an uncertain amount of crystal water even in the dried salt. Gibbs[2] has pointed out, on account of irregularity in the water content, the necessity of grinding thoroughly the whole specimen of oxalate which is to be used for analysis, in order to insure homogeneity of material. Furthermore it has been shown by Baxter and Griffin[3] and Baxter and Daudt[4] that the oxalate must be precipitated in acid solution, since in neutral or alkaline solution the oxalates of the alkalies and ammonium are carried down by the rare earth oxalate in considerable quantities. Brauner[5] has found that in some cases at any rate the oxalate hydrolyzes in contact with water, although the magnitude of the latter error is doubtful. In spite of these difficulties the oxalate method is as easy to carry out as any and probably the most satisfactory for approximate purposes.

The Analysis of the Sulphate. The sulphates have been used for atomic weight purposes in several ways. Bahr and Bunsen[6] first employed the method of dissolving weighed amounts of oxide in sulphuric acid, and weighing the sulphate after expelling the excess of acid. By the ignition of the oxalate it is doubtless possible to prepare the oxides in a very pure state if great care is taken to protect them from access to moisture or carbon dioxide, at any rate with elements which form trivalent compounds only. To prepare the sulphate in a neutral dry condition is a much more difficult proposition, owing to the retention of water and sulphuric acid if the temperature of heating is low, and

1 Sitzungsber. böhm. Ges., Dec. (1878)
2 Proc. Amer. Acad., 28, 261 (1893).
3 Jour. Amer. Chem. Soc., 28, 1684 (1906).
4 Ibid., 30, 563 (1908).
5 Trans. Chem. Soc., 81, 1266 (1902).
6 Lieb. Ann., 137, 5 (1866).

to decomposition with loss of sulphuric acid at higher temperatures. The investigations of Bailey,[1] Kruss,[2] Jones,[3] Brauner,[4] Wild,[5] Brill,[6] and Holmberg[7] leave this matter still in a somewhat unsettled state.

The difficulty in obtaining the anhydrous sulphate in a neutral condition applies also to the method of ignition of sulphate to oxide. Furthermore it is probable that the oxide prepared by ignition from the sulphate retains traces of sulphuric acid.[8]

A third method depending upon the use of the sulphates involves the determination of the water in the octahydrated sulphate. Here the difficulty in obtaining crystals free from included cells of mother liquor is superimposed upon that of insuring complete dehydration.

The Analysis of the Chloride. The analysis of the chlorides has been little used on account of the tendency for the formation of insoluble basic compounds during drying. As Matignon[9] has pointed out, if the dehydration of the chloride is conducted slowly and thoroughly enough in an atmosphere of dry hydrochloric acid gas, this difficulty disappears and a perfectly soluble product can be obtained, even after fusion. Hence on account of the accuracy with which halogen salts may be analyzed by precipitation with silver, this method is one of the best for atomic weight purposes. In testing the applicability of this method, we chose neodymium chloride for investigation, since neodymium compounds are comparatively easy to secure in an approximately pure state in large quantities. Furthermore, unlike its close relation praseodymium, neodymium does not readily change from the trivalent state.

Historical. The atomic weight of neodymium has already been determined many times, with discrepant results. This element and praseodymium, the other component of the old didymium, were first separated through the fractional crystallization of the double ammonium nitrates by Auer von Welsbach[10] in 1885. He determined their atomic weights by Bunsen's method of converting oxide to sulphate, but apparently interchanged his results in publication, as first suggested by Brauner, as he gave the value 140.8 for neodymium and 143.6 for praseodymium.

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1 Trans. Chem. Soc., **51**, 683 (1887).
2 Zeit. anorg. Chem., **3**, 46 (1893).
3 Amer. Chem. Jour., **28**, 31 (1902).
4 Trans. Chem. Soc., **81**, 1248 (1902).
5 Ibid., **38**, 195 (1904).
6 Ibid., **47**, 464 (1905).
7 Zeit. anorg. Chem., **53**, 124 (1907).
8 Brauner, loc. cit., 1263.
9 Compt. Rend., **133**, 289 (1901); **140**, 1182 (1905).
10 Sitzungsber. Akad. Wiss. Wien., **92**, 317 (1885).

In 1898 Brauner,[1] by converting oxide to sulphate, obtained the result 143.6 for neodymium. In this he included a correction for an impurity of 2.9 per cent of the corresponding praseodymium compound, estimated by comparison of the intensity of absorption spectra.

In the same year Boudouard[2] obtained a neodymium preparation by precipitation with potassium sulphate from some impure yttrium material. Apparently to identify the base rather than to establish its atomic weight, he analyzed the sulphate by converting it to oxide, with the result 143.0. Undoubtedly his material contained praseodymium, as no steps were taken to remove this inevitable impurity.

In 1898 Jones[3] also investigated both neodymium and praseodymium with material purified by crystallization as the double ammonium nitrate. Cerium and lanthanum were eliminated by precipitation as basic nitrates, and a correction for residual praseodymium was obtained by comparing the intensities of its absorption bands in the neodymium preparations with those of the same bands in dilute praseodymium solutions of known strength. The atomic weight was found by converting the oxide to sulphate. The average of twelve results, ranging in value from 143.46 to 143.62, was 143.55. Corrected for praseodymium content this value becomes 143.60.

In 1901, after further purification of his original material, Brauner[4] published another determination, which yielded the value 143.89. As this is the result of but one analysis and includes a correction of questionable accuracy for praseodymium impurity, it can hardly be given much weight.

Auer von Welsbach[5] in 1903 published the results of further study of both neodymium and praseodymium. His source of material was a large quantity of double ammonium nitrates obtained as by-products in the technical extraction of lanthanum. A long series of fractional crystallizations as double ammonium nitrate served to free the neodymium from all but a trace of praseodymium. With this material, undoubtedly of very high purity, three analyses were made by the Bunsen method of converting oxide to sulphate, from which the atomic weight values 144.55, 144.52, and 144.57 respectively were obtained. Unfortunately no details of procedure are given.

A point of especial interest in connection with Jones's and Brauner's results is Auer's observation that the praseodymium absorption spectrum

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[1] Proc. Chem. Soc., **14**, 70 (1898); Chem. News, **77**, 161 (1898).
[2] Compt. Rend., **126**, 900 (1898).
[3] Amer. Chem. Jour., **20**, 345 (1898).
[4] Proc. Chem. Soc., **17**, 66 (1901).
[5] Sitzungsber. Akad. Wiss. Wien., **112**, 1037 (1903).

is weakened considerably by lanthanum and cerium ammonium nitrates and other salts, as well as by concentrated nitric acid, the effect of which was mentioned in Auer's earlier paper. Clearly any quantitative comparison of the intensity of the praseodymium spectrum from solutions of praseodymium salts alone, with that from solutions consisting mainly of other rare earth salts, is unreliable.

Feit and Przibylla[1] dissolved weighed amounts of various rare earth oxides in sulphuric acid and titrated the excess of acid, using methyl orange as the indicator. The endpoint was interfered with by the pink color of the neodymium salt, but this difficulty was obviated by comparing the solutions with standards containing the same amounts of indicator and neutral neodymium salt. Their neodymium material was prepared by fractional crystallization of the double magnesium nitrate. This preparation showed no trace of the strongest praseodymium and samarium absorption bands, although the "hellblau" color of the oxide was considered the best proof of its purity. Their result for the atomic weight of neodymium was 144.60.

Holmberg[2] after a study of the rare earth salts of twelve organic acids, selected the metanitrobenzol sulphonate as particularly suitable for the purification of neodymium, and by one hundred and sixty-two series of crystallizations obtained thirteen fractions ranging in atomic weight from 144.0 to 145.3. The purest fractions were selected and the oxide obtained by ignition of the oxalate was weighed and converted into sulphate. The value 144.10 was obtained as the mean of six results varying from 144.03 to 144.15.

The International Committee on Atomic Weights have chosen as the most probable value 144.3, which represents an average of the more recent determinations.

The Separation of Neodymia from Other Rare Earths.

It is a well-known fact that when a salt of a mixture of rare earths is subjected to fractional crystallization, the less soluble salts tend to concentrate at the less soluble end of each series of fractions, and the more soluble salts at the opposite end of the series. In most cases it is less difficult to free an earth very thoroughly from more soluble impurities than it is to eliminate impurities which tend to concentrate in the crystals. An obvious although not generally recognized way out of the difficulty is to crystallize the substance in such different forms that impurities which tend to concentrate in the crystals in one case,

[1] Zeit. anorg. Chem., **43**, 202 (1905).
[2] Ibid., **53**, 83 (1907).

concentrate in the mother liquors in another, continuing the crystallization in any one form until impurities which accumulate in the mother liquors have been eliminated as completely as possible.

In the present instance the material was crystallized first as double ammonium nitrate from dilute nitric acid, then as nitrate from concentrated nitric acid. According to Auer von Welsbach,[1] in crystallization as double ammonium nitrates the bases separate in the order, lanthanum, cerium, praseodymium, neodymium, samarium, terbium- and yttrium-earths, while according to Demarçay,[2] in the crystallization of the nitrates from nitric acid the solubility first decreases with increasing atomic weight through gadolinium, 157.2, and then increases. In the crystallization as double ammonium nitrates samarium and the more soluble earths were eliminated, while in the crystallization of the nitrates from nitric acid the solubility of the first four of the above nitrates is reversed and the lanthanum, cerium and praseodymium were removed in the mother liquors.

Three and one half kilograms of neodymium ammonium nitrate, very kindly furnished by Dr. H. S. Miner of the Welsbach Light Company, served as a starting point in the preparation of pure material. The history of this material is somewhat uncertain, but, as nearly as can be determined, is as follows: Swedish cerite was decomposed with acid and the rare earths were precipitated as oxalates, from which the oxides were obtained by ignition. These oxides were then treated with nitric acid and the insoluble basic ceric salt was produced by heating. The lanthanum, praseodymium, neodymium, samarium, yttrium, etc., were obtained in solution. After the addition of ammonium nitrate the material had received, probably, about 150 to 250 crystallizations. Our treatment showed it to contain considerable lanthanum, cerium, and praseodymium, and a small amount of samarium. Probably traces of other rare earths were present, but no evidence of their existence was found even in the extreme fractions of the purified material.

In the further purification to which we subjected the double neodymium ammonium nitrate the usual method of fractional crystallization was followed: The whole material (1) was crystallized and the crystals (2) were separated from the mother liquor (3). The crystals were dissolved and again crystallized, the crystals becoming (4) and the mother liquor (5). The mother liquor (3) was evaporated to crystallization, and the crystals were combined with (5), while the mother liquor formed (6), and so on indefinitely. After the first few series of

[1] Sitzungsber. Akad. Wiss. Wien., **112**, 1043 (1903).
[2] Compt. Rend., **122**, 728 (1896); **130**, 1021 (1900).

crystallizations the number of fractions in each series was maintained nearly constant between twenty and twenty-five, and the different fractions were not allowed to vary much in size. As a rule the solutions were allowed to become supersaturated by cooling without crystallization, then they were seeded with tiny crystals of the purest neodymium salt available, and allowed to stand undisturbed over night in order that by the formation of large crystals the removal of mother liquor might be facilitated. On account of the extra labor and time involved, centrifugal drainage of the crystals is of little advantage where the composition of crystals and mother liquor differs as slightly as in the present case, and therefore was not employed. When the end fractions became small they were either temporarily removed and added to similar fractions subsequently obtained, or, if it could be plainly seen by the difference in color that they contained considerable impurity, they were rejected.

A diagram of the crystallizations as double nitrate is given on page 222. In any given series of crystallizations a lower number always indicates a less soluble fraction. A line not connecting an end fraction with any fraction in a subsequent series indicates rejection.

When eighty-six series of crystallizations had been completed, spectrograms were made of the absorption spectra of selected fractions in the last series, which contained fractions 1287 to 1309. These photographs were taken on Cramer's Trichromatic plates with a spectrograph employing glass lenses and prism and covered the range λ 650 — λ 380. The spectra indicated the presence of samarium in only a few of the more soluble fractions, 1306 to 1309. Praseodymium could be detected in all the fractions, although only traces were visible in the more soluble fractions.

In order to throw more light upon the progress of the purification approximate atomic weight determinations by the "permanganate" method[1] were carried out with portions of fractions 1290, 1299, 1303, and 1305. The oxalate was precipitated by adding a dilute solution of the double nitrate to a dilute solution of an excess of pure oxalic acid, and the precipitate was thoroughly washed, collected, and dried at 120°–130°. After very complete mixing of the highly crystalline material by grinding in an agate mortar, weighed portions of the oxalate were converted to oxide by ignition in platinum crucibles. Other portions, weighed out at the same time, were dissolved in dilute sulphuric acid and titrated with a standard solution of potassium permanganate. From the ratio $M_2O_3 : 3\ C_2O_4$, the atomic weight was calculated.

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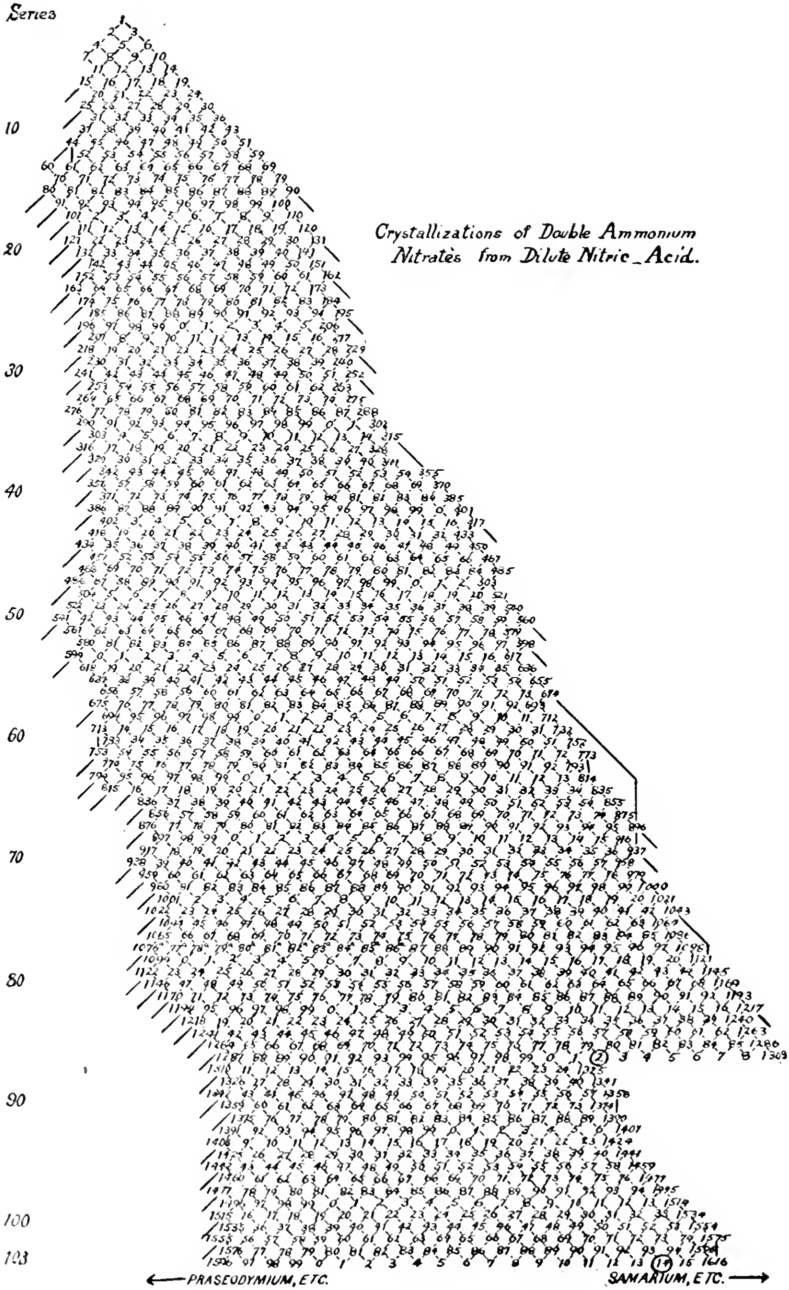
[1] See page 216.


Series
10
20
30
40
50
60
70
80
90
100
103
Crystallizations of Double Ammonium Nitrates from Dilute Nitric Acid.
← PRASEODYMIUM, ETC.
SAMARIUM, ETC. →

Fraction.		Per cent of Nd_2O_3.	Per cent of C_2O_4.	Atomic Weight.
1290		0.5334	0.4193	
		0.5334	0.4194	
			0.4195	
	Average	0.5334	0.4194	143.9
1299		0.5454	0.4274	
		0.5452	0.4281	
			0.4278	
			0.4279	
	Average	0.5453	0.4278	144.3
1303		0.5412	0.4258	
		0.5412	0.4258	
	Average	0.5412	0.4258	143.8
		0.5391	0.4217	
		0.5390	0.4228	
			0.4218	
			0.4221	
	Average	0.5390	0.4221	144.6
		0.5379	0.4212	
			0.4212	
	Average	0.5379	0.4212	144.6
	Average of second and third values . .			144.6
1305		0.5447	0.4269	
		0.5448	0.4272	
			0.4272	
			0.4269	
	Average	0.5448	0.4271	144.4
		0.5414	0.4236	
			0.4242	
	Average	0.5414	0.4239	144.6
	Average			144.5

Fractions 1299 to 1305 were evidently very nearly identical and probably nearly pure, while fraction 1290 apparently contained about ten per cent of praseodymium, cerium and lanthanum.

At this point fraction 1302, which seemed on the whole to be the purest in this series, was set aside for more careful examination and analysis. Fractions 1303 to 1309 also, which contained the bulk of the remaining samarium and yttrium earths, were separated from the others, and the fractionation was continued as before.

No further accumulation of samarium could be detected, after seventeen more series of crystallizations had been completed, in a spectrographic examination of the more soluble fractions of the one hundred and third series. Nevertheless the two most soluble fractions, 1615 and 1616, were rejected, and the next, 1614, was removed for subsequent analysis.

The sharpness with which the samarium is removed from the neodymium by the crystallization of the double ammonium nitrates is striking in comparison with the difficulty in eliminating the last traces of praseodymium in the same way. After the removal of only eight fractions, 1302 to 1309, the last of which was comparatively rich in samarium, seventeen series of crystallizations failed to reveal any residual samarium. On the other hand, when the fractions 1303 to 1309 were subjected to further fractional crystallization by themselves, the least soluble fraction of the fourteenth series was found to contain about one half per cent of praseodymium, estimated spectrographically as described later. By the same method fraction 1302, which contained more praseodymium than any one of the seven crystallized, was found to contain only 0.2 per cent of this impurity. Demarçay[1] and Feit and Przibylla[2] have emphasized the fact that neodymium and samarium may be separated with equal facility by crystallization as double magnesium nitrates.

At this point it seemed reasonably certain that practically all the more soluble rare earth impurities had been eliminated, while evidence of praseodymium could still be seen in the absorption spectra of all the fractions. For the removal of the praseodymium and at the same time cerium and lanthanum, if present, crystallization of the nitrates from concentrated nitric acid was next undertaken, since in this process these elements accumulate in the mother liquors.[3]

The ammonium nitrate was removed from the different fractions by evaporation and ignition, concentrated nitric acid was added, and the crystallization was carried through sixty-seven series. The same numbering was continued, except that the order of the one hundred and third series was reversed to give the less soluble fractions the lower numbers as before, 1613 becoming 1613a and 1596 becoming 1630. During these sixty-seven series no fractions were rejected at the less soluble end, although the extreme mother liquor was frequently re-

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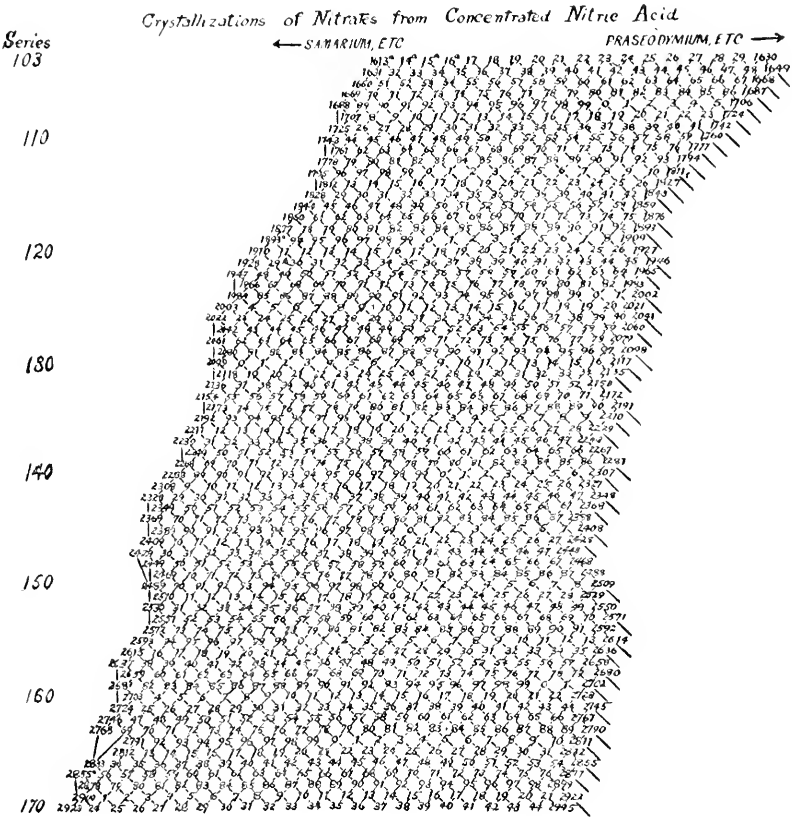
[1] Compt. Rend., **130**, 1021 (1900).

[2] Loc. cit., 207.

[3] Demarçay, loc. cit.

jected. Thus any nitrate less soluble than neodymium nitrate would concentrate in the extreme crystal fraction of the last series. A diagram of this treatment follows:



The fractions of the last series, 2923 to 2945, were then converted to oxide by precipitation as oxalate from acid solution and ignition of the oxalate. The different fractions of oxide were all of the same lilac blue color. We could detect no difference in the color of the oxides of fractions found to be essentially free from praseodymium and those containing several tenths of a per cent of this impurity.

The final fractions were next subjected to careful comparative spectrographic examination, great pains being taken to have the conditions of experimentation the same for all the fractions. One gram of every third fraction was dissolved in nitric acid, and, after the excess of acid had been evaporated, the solutions were diluted to the same volume, above five cubic centimeters. Photographs of the absorption spectra were made, the solutions being contained in test tubes of the same diameter, and various lengths of exposure being employed. The samarium band λ 401 was faintly visible in fraction 2923 only. This is not surprising, however, since, as has been stated before, no fractions had been removed from the less soluble end of the series in the course of all the sixty-seven series of crystallizations from concentrated nitric acid. Praseodymium could not be detected in fractions 2923 to 2932, but its strongest absorption band, λ 444, appeared with regularly increasing intensity in fractions 2935 to 2944.

In order to find out how much praseodymium can be detected in this way, and at the same time to estimate the proportion of praseodymium in the latter fractions, spectrograms were made, on one plate and with the same exposure, of the spectra of most of the solutions previously examined, and also of others similarly prepared from fractions 1302 and 1614, which had been removed at earlier stages of the fractionation. To a solution of fraction 2926, in which no praseodymium could be detected in the first photograph, 0.1 per cent of praseodymium oxide was added in solution, and the original concentration was restored by evap oration. The absorption spectrum of this specimen was photographed on the same plate as above, and again after the praseodymium content had been increased to 0.2, 0.5 and 1.0 per cent respectively. In the negative the first addition of 0.1 per cent made a distinct impression. From this it was concluded that fractions 2923 to 2932 contained less than that proportion of praseodymium. This assumption can scarcely be objected to on the ground that a considerable amount of praseodymium in fraction 2926 might have been brought within the limits of detection by means of a relatively small increment, because the intensity of the praseodymium band appeared to be proportional to the amounts of praseodymium added to the original solution.

Since these standards and the other solutions differed little except in praseodymium content, those giving praseodymium spectra of equal intensity could be safely assumed to contain the same percentage of this impurity. The results of the comparison of the different spectrograms are as follows:

Fraction	2935	2938	2941	2944	1302	1614
Per cent of Pr. . .	0.05	0.1	0.2	0.5	0.2	0.15

Since a proportion of one tenth of a per cent of praseodymium lowers the atomic weight of neodymium by only four one thousandths of a unit, the purer fractions were obviously as pure as it is worth while to make them. None of them in all probability were entirely free from praseodymium. In fact, visual spectroscopic examination of both fraction 2929 and 2931 with very strong absorption showed them to contain perceptible amounts of this impurity.

It is reasonable to suppose that cerium and lanthanum nitrates, which are more soluble in concentrated nitric acid than praseodymium nitrate, were removed from the neodymium even more completely than praseodymium. Evidence upon this point is more difficult to secure than in the case of praseodymium, on account of the lack of absorption bands of cerium and lanthanum in the visible spectrum. The following test for cerium was carried out with fraction 2944, which contained 0.5 per cent of praseodymium. The hydroxide was precipitated by the addition of an excess of sodium hydroxide, and, after washing by decantation, it was treated with saturated chlorine water for some time. A small quantity of undissolved hydroxide was filtered off and dissolved in nitric acid. To the nearly neutral solution a sufficient amount of a solution of calcium hypochlorite was added to produce a slight permanent precipitate. This precipitate was pink with no tinge of yellow. The same solution after the addition of a trace of cerium and testing with hypochlorite as before yielded a precipitate distinctly colored with the characteristic yellow of ceric hydroxide. Fraction 2944 was evidently nearly if not entirely free from cerium and the less soluble fractions of the series must have been purer still.

The Absorption Spectrum of Aqueous Solutions of Neodymium Salts.

The absorption spectra of two of the purest fractions, 2929 and 2931, were examined visually with a Hilger wave-length spectroscope. While no very great accuracy is claimed for the measurements, it is probable that the values are correct within one-millionth of a millimeter, for the spectroscope is capable of nearly ten times this accuracy for wave-lengths in the neighborhood of λ 500.

In the first two columns of the following table are given the values for the maxima of absorption observed during the gradual dilution of very concentrated, slightly acid solutions of the nitrates, each value having been obtained in the most dilute solutions in which the band could be observed. The spectra of the two fractions were identical.

The band λ 580 can be seen only at certain medium concentrations while the band λ 547 is visible only with very concentrated solutions. The praseodymium band λ 444 also was faintly visible only in the most

Baxter and Chapin Fractions 2929 and 2931. (1909).	Forsling and Holmberg. (1907).	Rech and Muthmann. (1906).	Auer von Welsbach. (1903).
			745 to 725
689	689.5		687
679	677.5	679.6	678
673	672.0	670.8	673
636	636.0	636.6	635
628	627.8	628.6	629
625	625.4	624.8	626
622	621.7	622.2	622
587		586.5	
583		583.2	583
580		580.8	
578	578.8 to 578.0	578.5	579
575	575.4	575.4	576
574	573.5	573.6	575
571	571.6	571.7	572.5
547			567.5
531	532.3	532.0	532
525		525.4	525
521	521.6	522.2	522.7
520	520.4	520.5	521
		514.5	
512	512.4	512.2	511.7
509	508.7	508.9	508.3
487	489.0 to 486.0	488.0	
480	479.9	479.8	
475	474.8 to 474.2	475.4	475.4
469	468.7	469.0	469.6
461	461.0	461.1	461
444 Pr		444.2	
		434.2	
433	433.0	432.9	432
		429.5	
		428.1	
427.5	427.1	427.2	426.5
418	418.3	418.2	417.8

concentrated solutions. The two bands λ 580 and λ 547 disappear if the nitrate is changed to chloride; otherwise the nitrate and chloride give maxima in essentially the same positions, although the relative intensities of the bands are somewhat different in the two cases.

The second, third, and fourth columns of the foregoing table represent the corresponding results obtained by Forsling from Holmberg's neodymium preparation,[1] by Rech from Muthmann's preparation,[2] and by Auer

[1] Zeit. anorg. Chem., **53**, 83 (1907).

[2] Zeit. Wiss. Photographie, Photophysik u. Photochemie, **3**, 411 (1906).

von Welsbach from his own material.[1] The last were taken from the drawings which were Auer's only published record, and since they are subject to two personal errors are less exact than the others. Being obtained, however, from very carefully prepared material, they should represent at any rate the number and the general location of the bands in the neodymium spectrum. Rech's measurements, since his material was less carefully prepared, include lines belonging to praseodymium and possibly to other elements.

While some of these bands are sharp, and appear in the same positions in solutions of various concentrations, others fade away gradually on the edges, and sometimes have their maxima shifted by changes in the concentration. Consequently the disagreement as to the exact positions of bands is due in many cases merely to differences in the conditions of observation. Furthermore, Forsling and Rech used chloride solutions, whereas Auer von Welsbach and the writers used nitrate solutions. On the whole the agreement between the different sets of values is very satisfactory. The bands between λ 587 and λ 571 are too close together to be distinguished from one another under any but the most favorable conditions. The double band λ 575 and λ 574 in particular separates only under rather limited conditions. The band at λ 579 in one of Auer's drawings may be intended to represent two lines at λ 580 and λ 577, and is thus interpreted by Kayser. Auer's band λ 567.5 appears very faintly in but one of his four drawings. There is a complete absence of shadow at this point in another drawing which represents a more concentrated solution, and which one would expect consequently to show the band. It is possible therefore that its appearance in the first print was merely accidental. The band λ 525 in concentrated solutions merges with λ 522 and in dilute it becomes invisible, so that it is not surprising that it was not observed by Forsling and Holmberg. The band λ 514.5, given by Rech alone, is coincident with no prominent band of any other rare earth, nor could it be detected in any of our preparations. The band λ 488 which both Forsling and Rech observed, as has already been stated, can be seen only with very strong absorption. This is doubtless the reason that it was not observed by Auer.

The presence or absence of the predominant impurities, praseodymium, cerium, lanthanum and samarium, in our best material has already been discussed. It is of some interest to consider other earths having prominent absorption bands in the visible spectrum.[2] Since none of

[1] Sitzungsber. Akad. Wiss. Wien., **112**, 1037 (1903).

[2] The following data are obtained from drawings by Urbain, Jour. de Chim. Phys., **4**, 31, 105, 232, 321 (1906).

these could have been present in quantity, their absorption bands obviously could be found only in concentrated solutions. The marked broadening and running together of the neodymium bands in concentrated solutions makes the detection of bands due to impurities difficult, except when these occur in regions in the spectrum comparatively free from neodymium bands. Of the three strong erbium bands λ 650, λ 540, and λ 522, the last two are completely masked by the neodymium in concentrated solution. The band λ 650, however, occurs in a region favorable for observation, and could not be detected even in the most concentrated solution. In the case of holmium the three strongest bands, λ 639, λ 535, and λ 485, all fall within the limits of neodymium bands at high concentrations. There was no evidence of the strongest, λ 535, in solutions of average concentrations. The strongest dysprosium band λ 451, which falls just within the broad neodymium band λ 484–λ 453, as it appeared in our most concentrated solutions, could not be detected. The terbium band λ 488 practically coincides with the neodymium band 487 which could be seen in our preparations only at the highest concentrations. Since this band could be seen of the same intensity in all our preparations, it is improbable that it represents an impurity of terbium. The sharp thulium band λ 700 does not fall within limits of any neodymium band, and we could find no evidence of its presence. Since the absorption of europium solutions in the visible spectrum is faint, and since all the stronger bands are masked completely by the bands of a concentrated neodymium solution, we have no spectroscopic evidence concerning the absence of this element. It may be added that, even in the extreme fractions of our crystal series, we could not detect the presence of any of these rarer elements. Further study of the absorption spectrum of neodymium is planned with the aid of a large quartz spectrograph, which has recently become available for us.

The Preparation of Neodymium Chloride.

The neodymium oxide fractions prepared as above were converted to chloride as follows: Fractions 1302 and 1614 were dissolved in the purest nitric acid in platinum vessels, the solutions were filtered, and the oxalate was precipitated from acid solution by means of the purest oxalic acid. After the oxalate had been washed by decantation and drained by suction upon a porcelain Gooch crucible employing a small disk of filter paper in place of an asbestos mat, the salt was dried and ignited in a platinum dish over a blast lamp. The oxide was dissolved in a quartz dish in hydrochloric acid which had been distilled with the use of a quartz condenser, and the chloride was three times crystallized, with centrifugal drainage in platinum Gooch crucibles[1] in each case. The crystal yield was brought to a maximum by treating the solutions at 0° with hydrochloric acid gas obtained by warming concentrated solutions of the acid and led to the solutions through a quartz tube. Drying over sulphuric acid in a vacuum desiccator and powdering in an agate mortar completed the preparation for the final dehydration and analysis.

On account of the small quantity of material in each fraction of the final series, adjacent fractions were combined. The combinations actually analysed were 2923–2924, 2926–2927–2928, 2932–2933, and 2936–2937. A suspicion of contamination by platinum during the ignition of the oxides in platinum led to a slight change in the purification of these fractions. The original oxide was dissolved in hydrochloric acid in a quartz dish, and hydrogen sulphide, generated by the action of sulphuric acid on ferrous sulphide and thoroughly scrubbed with water, was passed into the hot, slightly acid solution. A slight black precipitate of platinum sulphide was formed in every case. The oxalate was precipitated from the filtered and boiled solution, and was collected and dried as before. In the ignition of this oxalate an electrically heated quartz muffle, kindly loaned to us by Dr. H. H. Willard, was used. Since the temperature of the muffle was very uniform, a lower temperature could be employed for the decomposition of the oxalate. The product of decomposition, which contained a large proportion of carbonate when obtained in this way, was dissolved in hydrochloric acid in a quartz dish and was crystallized as before. The first mother liquor, which in the case of fractions 1302 and 1614 had been slightly tinged with yellow, was in the case of all the other fractions free from such evidence of dissolved platinum.

The Preparation of Pure Silver.

Pure silver was prepared by methods which have already been found to be very effective. Since these methods have been described in detail several times in papers from this laboratory,[2] only a brief outline of the processes is given here. Crude silver was dissolved in nitric acid, precipitated as chloride, reduced with alkaline invert sugar, and fused on charcoal. After cleansing by scrubbing with sand and etching with nitric acid, the buttons were again dissolved in nitric acid and the metal was precipitated from solution with ammonium formate. The

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1 Baxter, Jour. Amer. Chem. Soc., **30**, 286 (1908).

2 See especially Richards and Wells, Pub. Car. Inst., No. **28**, 16 (1905); Jour. Amer. Chem. Soc., **27**, 472.

thoroughly washed product was fused with a blowpipe upon a crucible of the purest lime. Electrolytic deposition with silver nitrate as the electrolyte and with a dissolving anode of the pure silver buttons followed, and the electrolytic crystals were fused in a current of pure hydrogen on a boat of the purest lime. The resulting bars of metal were first cleansed with nitric acid and then were sawed into pieces of a convenient size with a fine jeweller's saw. After the fragments had been etched with nitric acid until free from surface contamination with iron from the saw, they were thoroughly washed with ammonia and pure water, and finally heated to about 300° C. in a vacuum. The silver was preserved in a desiccator over solid potassium hydroxide.

Pure silver nitrate was prepared by crystallization of the commercial product until free from chloride. That used in analyses 6, 7, 8, and 15 was a portion of material purified by Dr. Grinnell Jones for work upon the atomic weight of phosphorus.[1]

In the preparation of reagents the precautions usual in exact work were taken. The ordinary distilled water of the laboratory was twice redistilled, once from alkaline permanganate and once alone, through block tin condensers. Hydrochloric and nitric acids were distilled through quartz condensers, in the case of the hydrochloric acid the first and last runnings being rejected, in the case of the nitric acid two distillations being carried out, the first third being rejected in each distillation. By careful tests it was shown that nitric acid distilled in this way does not contain more than the merest trace of chlorine if the original acid is nearly free from the latter element.

Quartz or platinum utensils were employed wherever glass would have introduced objectionable impurities, and electrical heaters were used whenever the products of combustion of illuminating gas were to be avoided. In the crystallization of solids centrifugal drainage was always used to assist in the mechanical removal of mother liquor from crystals, except in the fractional crystallization of the neodymium material, where it would have been of little assistance.

The Drying of the Neodymium Chloride.

The drying of the neodymium chloride for analysis presented several difficulties. Matignon[2] states that if the chloride is first completely dehydrated and then fused in a current of dry hydrochloric acid gas, the product is completely soluble in water. Our earlier experiments

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[1] Baxter and Jones, Proc. Amer. Acad., **45**, 137 (1909); Jour. Amer. Chem. Soc., **31**, 298.

[2] Compt. Rend., **133**, 289 (1901).

did not confirm this statement, for in several instances salt which apparently had been very completely dried, after fusion in a current of hydrochloric acid gas was found to contain a considerable amount of basic salt. Since, however, material which had been dried at temperatures slightly below the melting point of the anhydrous salt gave absolutely clear solutions, the expedient was adopted of drying the salt for analysis as completely as possible without fusion, with the expectation of determining the water in subsequent experiments. During later experiments for determining the residual water it was found that the preliminary drying had not been effective in the earlier experiments, and that salt which has been dried as for analysis and then fused in a current of hydrochloric acid gas does give a perfectly clear solution. This information was unfortunately acquired too late to be of direct use in preparing the salt for analysis. Since, however, the dried but unfused salt was found to contain a constant very small proportion of moisture, and since a correction was applied for this moisture, no appreciable error could have been introduced by the incomplete drying. It is hoped, nevertheless, before long to institute further analyses with material which has been fused.

The apparatus employed in the drying of the neodymium chloride was essentially that used by numerous investigators in this laboratory for the dehydration of chlorides. Hydrochloric acid gas was generated by the action of concentrated sulphuric acid upon concentrated hydrochloric acid, and was dried by passage through five towers about thirty centimeters long and four centimeters in diameter filled with glass beads moistened with concentrated sulphuric acid. Nitrogen was prepared by Wanklyn's method of passing air through concentrated ammonia solution and then over hot copper. The excess of ammonia was removed and the nitrogen was purified and dried by passing over dilute sulphuric acid, silver nitrate solution, solid caustic potash, concentrated sulphuric acid, and resublimed phosphorus pentoxide. Nitrogen prepared in this way always contains hydrogen,[1] produced by the catalytic decomposition of the excess of ammonia in passing over the hot copper, in quantities which vary with the excess of ammonia, the temperature of the copper, as well as the length of the copper layer and the speed of the gases. The hydrogen could not have had any effect upon the neodymium chloride, however, and is an advantage in preventing the attacking of the platinum boat. Air was purified and dried by reagents similar to those used in the purification of the nitrogen. The hydrochloric acid apparatus was constructed wholly of glass,

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[1] Our attention was first called to this fact by Dr. R. C. Wells.

and the nitrogen and air purifying trains had short rubber connections only at the beginning. Any one of these gases or any desired mixture could be delivered through the tube in which the boat containing the salt was placed. This tube could be heated to constant temperatures in an oven consisting of two superimposed solid aluminum blocks.[1]

According to Matignon,[2] at about 105° C. neodymium chloride loses five of its six molecules of water of crystallization rapidly, while the sixth molecule appears at 160°. We have determined the melting point of the hexahydrated chloride (made from fraction 2931) to be 124°. Thus the drying of the chloride may be caused to take place by a process of double efflorescence, a condition very favorable for the complete elimination of the water. Richards [3] has pointed out that a hydrated salt dried wholly by efflorescence can be freed from moisture much more effectively than one which has been allowed to melt during the dehydration, hence care was taken to keep the temperature below 124° until the greater part of the water had been expelled.

The powdered crystals of neodymium chloride were placed in a platinum boat which had been previously weighed in a weighing bottle, and the boat was heated in a current of mixed nitrogen and hydrochloric acid gases at gradually increasing temperatures. The boat was contained in a hard glass tube which formed part of a "bottling apparatus," [4] and this in turn was attached to the systems for purifying the various gases. The temperature was kept below 124° until the larger portion of the water had been expelled. Then it was gradually raised to 200°, and finally the salt was heated at about 330° for one hour in a current of pure hydrochloric acid gas. During the heating there was never the slightest appearance of melting. In the first five analyses the final temperature was higher than in the subsequent ones, and was not far below the fusing point of the anhydrous salt. After the tube had been allowed to cool, the hydrochloric acid was displaced by nitrogen and finally by air, and the boat was transferred to the weighing bottle without exposure to moist air by means of the bottling apparatus. After long standing in a desiccator near the balance the boat and contents were weighed.

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[1] Baxter and Coffin, Proc. Amer. Acad., **44**, 184 (1909); Jour. Amer. Chem. Soc., **31**, 296.

[2] Compt. Rend., **133**, 289 (1901).

[3] Zeit. physik. Chem., **46**, 194 (1903).

[4] Richards and Parker, Proc. Amer. Acad., **32**, 59 (1896).

The Method of Analysis.

In order to dissolve the chloride the boat was first transferred to a large dry Erlenmeyer flask, a column of bulbs was inserted in the neck of the flask and enough water to cover the boat was introduced through the bulbs. These precautions were taken because of the violence of the action of the water upon the finely divided anhydrous salt, in order to avoid both mechanical loss of material and possible evaporation of hydrochloric acid owing to hydrolysis at the high temperature which results at the moment when the salt comes in contact with water. That this error is not a serious one is evident from the fact that although in the first five analyses the use of the column of bulbs was omitted, yet the results of these analyses are not markedly different from the others. The solution was allowed to stand in the flask for some time in order to ensure absorption of any volatilized hydrochloric acid. Every specimen of salt dried and dissolved in this way gave a perfectly clear solution.

The solution of the neodymium chloride was diluted to a volume of about one liter with water containing a small amount of nitric acid. Then an equally dilute solution of a very nearly equivalent amount of pure silver nitrate containing nitric acid was added to the solution of the chloride. The glass-stoppered precipitating flask containing the solution was thoroughly shaken and allowed to stand for about twenty-four hours, and then an excess of silver nitrate of about one tenth of a gram was added to complete the precipitation.

After standing until the supernatant liquid was perfectly clear, the precipitate was thoroughly washed, first with dilute silver nitrate solution and then with very dilute nitric acid in the earlier analyses, with pure water in the later ones, and was collected upon a weighed Gooch-Munroe-Neubauer crucible. Then, after the crucible and contents had been dried in an electric air bath at about 150° for eighteen hours, they were weighed.

The moisture retained by the dried silver chloride was estimated in each analysis by determining the loss in weight during the fusion of the dried silver chloride in a weighed porcelain crucible. The weight of silver chloride dissolved in the filtrate and silver nitrate wash-waters was assumed to be 0.00003 gram per liter.[1] That dissolved in the acid or aqueous washings together with the ammonia rinsings of the precipitating flask was determined by comparison with standard solutions in

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[1] Richards and Wells, Pub. Car. Inst., No. **28**, 30 (1905); Jour. Amer. Chem. Soc., **27**, 487.

a nephelometer, the usual precautions being taken to secure uniformity of precipitation.[1]

When the ratio of the neodymium chloride to metallic silver was to be determined a quantity of silver equivalent to the chloride within a very few tenths of a milligram was weighed out and dissolved in nitric acid in a flask fitted with a column of bulbs to prevent loss of material by spattering. The quantity of silver was adjusted exactly by means of a hundredth normal solution of silver. After the silver solution had been freed from oxides of nitrogen it was diluted to a volume of one liter and was quantitatively added to the chloride solution. The precipitate was caused to settle completely by occasional shaking and standing for several days. Then portions of the mother liquor were tested for excess of silver or chloride by the addition of hundredth normal chloride or silver nitrate solutions, and comparison of the opalescence produced in a nephelometer. Deficiencies in either chloride or silver were made up by means of the dilute standard solutions of these substances until finally exactly equivalent amounts of chloride and silver were present in the solution.

In all except the first two titrations the solubility of the silver chloride in the mother liquor was very much decreased by cooling the solution with ice water, a method of increasing the accuracy of nephelometric observation which has been devised recently by Richards and Willard and used successfully in their revision of the atomic weight of lithium.[2]

As soon as equilibrium had been reached in a titration, an excess of one tenth of a gram of silver nitrate was added and the silver chloride was determined gravimetrically as described above. Corrections were of course applied for any chloride added in the titration.

The Determination of the Moisture Retained by the Neodymium Chloride.

Although in several cases where hydrated salts have been dried by efflorescence it has been shown that the moisture retained by the salt is negligible in quantity,[3] in the present instance careful experiments were instituted to test this point thoroughly. The direct determination of the water by absorption with phosphorus pentoxide, after fusion of the salt, was complicated in the case of neodymium chloride, since,

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[1] Richards and Wells, Amer. Chem. Jour., **31**, 235 (1904); **35**, 510 (1906). Richards and Stähler, Pub. Car. Inst., No. **69**, 20 (1907); Jour. Amer. Chem. Soc., **29**, 635.

[2] Pub. Car. Inst., No. **125**, 30 (1910); Jour. Amer. Chem. Soc., **32**, 32.

[3] Baxter and Coffin, Jour. Amer. Chem. Soc., **28**, 1587 (1906); Baxter and Tilley, Ibid., **31**, 212 (1901).

if the salt is fused in nitrogen, after being dried, hydrochloric acid is liberated owing to hydrolysis with the residual moisture. Fusion in a current of hydrochloric acid gas is not permissible because of the action of this gas upon the phosphorus pentoxide [1] necessary for the absorption of the water.

Fortunately neodymium chloride is only very slightly volatile at temperatures not far above its fusing point,[2] so that it is possible to estimate the moisture content of the dried salt by fusion in a current of hydrochloric acid gas and determination of the loss in weight. The trace of chloride volatilized during the fusion can then be collected and weighed.

For these experiments a glass tube is obviously unsuited owing to the action of the hydrochloric acid upon the glass at the fusing point of the chloride and consequent vaporization of alkali chloride into the neodymium chloride as well as into the cooler parts of the tube. Hence in these experiments a quartz tube was substituted for glass. This tube was apparently not attacked by the hydrochloric acid gas during a long series of experiments.

The exact procedure was as follows: The boat with a maximum charge of neodymium chloride crystals was heated in a current of nitrogen and hydrochloric acid gases, and was bottled and weighed exactly as in preparing the salt for the chloride analyses. After being weighed the boat was replaced in the quartz tube, and when the air had been completely displaced by hydrochloric acid gas, the tube was electrically heated by means of a removable mica sleeve wound with the resistance wire "Nichrome," until the salt was fused. The boat was then bottled in dry air a second time and reweighed. In nearly every experiment a barely visible condensation of concentrated hydrochloric acid solution took place in the cool portion beyond the boat.

Weight of $NdCl_3$.	Loss in Weight during Fusion.	Percentage Loss in Weight.
gm.	gm.	
5.36	0.00045	0.0084
6.73	0.00052	0.0077
5.93	0.00050	0.0084
Average		0.0082

[1] Baxter and Hines, Jour. Amer. Chem. Soc., **28**, 779 (1906).

[2] Neodymium chloride fuses at 785° and is not volatile at 1000° according to Matignon. Compt. Rend., **133**, 289 (1901); **140**, 1340 (1905).

The loss in weight of the salt found in this way is caused in part by a slight sublimation during the fusion, the magnitude of which was not determined in the first series of experiments. Since the neodymium chloride employed in the moisture determinations was not of so high grade of purity as that used in the chloride analyses, the volatilized material probably consisted in part of other chlorides. Hence the proportion of moisture calculated from the loss in weight is certainly a maximum. In a second series of experiments, after the fusion of the salt, the sublimate in the quartz tube was dissolved in water and the solution was evaporated to dryness in the weighed platinum boat, and after drying at 135° in hydrochloric acid gas the boat was reweighed.[1] The weights thus obtained were subtracted from the loss in weight of the salt found in the same experiment. When the quartz tube was heated alone in hydrochloric acid gas, no sublimate was formed, showing that the volatilized substance did not come from the quartz.

Weight of $NdCl_3$.	Loss in Weight during Fusion.	Weight of Volatile Matter.	Per cent of Moisture.
gm.	gm.	gm.	
6.24	0.00050	0.00030	0.0032
7.19	0.00052	0.00030	0.0031
Average			0.0032

It is to be noted that the percentage loss in weight in the second series of experiments is identical with that of the first series, although the per cent of moisture is less than half the percentage loss in weight during fusion. The effect upon the atomic weight of neodymium of correcting for the moisture is to lower the atomic weight by eight thousandths of a unit.

The percentage of moisture found is nearly identical in magnitude with that obtained from iodine pentoxide which had been made by efflorescence of iodic acid, 0.0023.[2]

The fused neodymium chloride seemed to be perfectly soluble in water, the probable reason for the better success in these experiments than in the earlier ones being the more efficient drying of the salt before fusion.

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[1] The moisture retained by neodymium chloride at this temperature is only 7.5 per cent of its weight. A higher temperature was avoided in order that no portion of the residue might volatilize.

[2] Baxter and Tilley, Jour. Amer. Chem. Soc., **31**, 212 (1909).

The Specific Gravity of Neodymium Chloride.

Although Matignon[1] had already found the density of anhydrous fused neodymium chloride at 18° referred to water at 4° to be 4.195, for the purpose of confirmation this constant was redetermined by displacement of toluol. On account of the difficulty of removing air from porous material the salt was fused for these determinations instead of being dried. The pycnometer employed was one of special form, modified by Baxter and Hines.[2]

The salt was first carefully dried and then finally fused in a current of hydrochloric acid gas in a small platinum boat, as previously described. The boat and contents were then weighed in a small weighing bottle to which they were transferred by means of the bottling apparatus. The salt was next quickly covered with dry toluol and a special pycnometer stopper was substituted for the ordinary stopper of the weighing bottle. Entangled air was removed by exhaustion in a desiccator and finally the toluol was adjusted to a mark in a capillary outlet tube in the usual way, after the temperature of the system had been carefully adjusted to 25°. In order to avoid evaporation of the toluol through the ground joint of the pycnometer this was made tight by means of a weighed quantity of syrupy phosphoric acid. The toluol content of the system being known, the displaced toluol could be found. The specific gravity of the toluol at 25° referred to water at 4° was determined in the same pycnometer to be 0.8636, while Jones[3] in earlier experiments with a different pycnometer found the specific gravity of the same sample to be 0.8633.

Weight of $NdCl_3$ in Vacuum.	Weight of Toluol displaced in Vacuum.	Specific Gravity of $NdCl_3$. 25°/4°.
gm.	gm.	
1.9967	0.4177	4.128
3.6878	0.7700	4.136
3.2569	0.6798	4.137
Average		4.134

A part of the difference between this value and Matignon's can be explained by the different temperatures at which the determinations were made.

[1] Compt. Rend., **140**, 1340 (1905).

[2] Amer. Chem. Jour., **31**, 222 (1904).

[3] Baxter and Jones, Proc. Amer. Acad., **45**, 155 (1910); Jour. Amer. Chem. Soc., **31**, 314.

If the weights are assumed to have the specific gravity at 8.3,[1] Matignon's value for the density leads to a vacuum correction per gram of substance of +0.000141, while the value 4.13 necessitates a correction of +0.000145. The difference amounts to only four ten thousandths of a per cent, which corresponds to a difference of only one thousandth of a unit in the atomic weight of neodymium. Hence it is of no importance which correction is employed.

The following table indicates the corrections for the buoyant effect of the air applied to the weights of the various substances.

	Specific Gravity.	Vacuum Correction.
Weights	8.3	
Toluol	0.863	+0.00126
$NdCl_3$	4.134	+0.000145
AgCl	5.56	+0.000071
Ag	10.49	−0.000031

All weighings were made on a nearly new balance, Troemner No. 10, which is employed exclusively in atomic weight researches. Weighings were made by substitution for counterpoises as nearly as possible like the objects weighed both in weight and in volume. The weights were carefully standardized to hundredths of a milligram by the method described by Richards.[2]

Results and Discussion.

In order to show that no appreciable error occurred owing either to occlusion by silver chloride or from loss of silver chloride in solution, the ratio between the silver used and the silver chloride obtained has been calculated in the five pairs of analyses for which the data are available.

Analyses.	Ag : AgCl.
8 and 16	0.752663
10 and 17	0.752726
12 and 19	0.752626
13 and 20	0.752686
14 and 21	0.752621
Average	0.752664

[1] Baxter, Proc. Amer. Acad., **42**, 209 (1906).
[2] Jour. Amer. Chem. Soc., **22**, 144 (1900).

ATOMIC WEIGHT OF NEODYMIUM.

Series I. $NdCl_3 : 3\ AgCl$.

Ag = 107.880 Cl = 35.457.

No. of Analysis.	Fraction.	Corrected Weight of $NdCl_3$ in Vacuum	Weight of AgCl in Vacuum.	Loss on Fusion.	AgCl Dissolved in Wash Water.	AgCl added in Titration.	Corrected Weight of AgCl in Vacuum.	Ratio $NdCl_3 : 3AgCl$.	Atomic Weight of Neodymium.
		gm.	gm.	gm.	gm.	gm.	gm.		
1	1302	3.16218	5.42334	0.00030	0.00242		5.42546	0.582841	144.257
2	1302	2.93305	5.02969	0.00047	0.00304		5.03226	0.582850	144.262
3	1302	2.99149	5.12978	0.00019	0.00236		5.13195	0.582915	144.289
4	1302	2.62468	4.50114	0.00030	0.00254		4.50338	0.582825	144.250
							Average	0.582858	144.265
5	1614	2.38439	4.08848	0.00025	0.00241		4.09064	0.582889	144.278
6	1614	2.55827	4.38789	0.00029	0.00131		4.38891	0.582894	144.280
7	1614	3.59114	6.15981	0.00026	0.00140		6.16095	0.582887	144.277
							Average	0.582890	144.278
8	2923–4	4.27402	7.33208	0.00023	0.00218	0.00200	7.33203	0.582925	144.293
9	2926–7–8	4.69459	8.06313	0.00029	0.00160	0.01000	8.05444	0.582857	144.264
10	2926–7–8	2.79574	4.80313	0.00025	0.00142	0.00800	4.79630	0.582895	144.280
							Average	0.582876	144.272
11	2932–3	2.59810	4.45553	0.00039	0.00227		4.45741	0.582872	144.270
12	2932–3	3.42064	5.87107	0.00021	0.00196	0.00410	5.86872	0.582860	144.265
13	2932–3	4.11855	7.06994	0.00018	0.00172	0.00630	7.06518	0.582936	144.298
							Average	0.582889	144.278
14	2936–7	3.95958	6.79583	0.00017	0.00179	0.00400	6.79345	0.582853	144.262
15	2936–7	2.71834	4.66295	0.00011	0.00111		4.66395	0.582841	144.257
							Average	0.582847	144.260
	General Average							0.582876	144.272

ATOMIC WEIGHT OF NEODYMIUM.

SERIES II. $NdCl_3 : 3\ Ag$.

Ag = 107.880. Cl = 35.457.

Number of Analysis.	Fraction.	Corrected Weight of $NdCl_3$ in Vacuum.	Weight of Ag in Vacuum.	Weight of Ag added.	Corrected Weight of Ag in Vacuum.	Ratio $NdCl_3 : 3Ag$.	Atomic Weight of Neodymium.
		gm.	gm.	gm.	gm.		
16	2923–4	4.27402	5.51825	0.00030	5.51855	0.774482	144.283
17	2926–7–8	2.79574	3.60989	0.00041	3.61030	0.774379	144.249
18	2932–3	2.77391	3.58159	0.00037	3.58196	0.774411	144.259
19	2932–3	3.42064	4.41676	0.00019	4.41695	0.774435	144.267
20	2932–3	4.11855	5.31786	0.00000	5.31786	0.774475	144.280
					Average	0.774440	144.269
21	2936–7	3.95958	5.11284	0.00050	5.11289	0.774431	144.266
22	2936–7	4.64435	5.99649	0.00041	5.99699	0.774447	144.271
					Average	0.774439	144.269
	General Average					0.774437	144.268
	Average of all the Results in Series I and II						144.270

RESULTS AVERAGED BY FRACTIONS.

Fraction.	1302.	1614.	2923–4.	2926–7–8.	2932–3.	2936–7.	Average.
$NdCl_3 : 3\ AgCl$	144.265	144.278	144.293	144.272	144.278	144.260	144.274
$NdCl_3 : 3\ Ag$			144.283	144.249	144.269	144.269	144.268
Average of all Individual Analyses . . .	144.265	144.278	144.288	144.264	144.273	144.264	144.272
Corrected for Praseodymium	144.273	144.284	144.288	144.264	144.273	144.268	144.275

The average ratio of silver to silver chloride is within four thousandths of a per cent of that obtained by Richards and Wells,[1] 0.752634 : 1.000000.

The results in the preceding tables add further evidence to that already obtained by the spectroscope, that all the samples analyzed were of a high degree of purity. Fraction 2923–4 contained all the rare earth impurities of higher atomic weight than neodymium which accumulated in sixty-seven series of crystallizations, and actually was found to contain a trace of samarium, while fraction 2936–7 contained more praseodymium, probably not more than 0.1 per cent, than any other fraction of the last series which was analyzed. The first impurity raises the atomic weight and the second lowers it; yet the difference in the average results of these two extreme fractions is only 0.024. The fractions 1302 and 1614 gave results differing from those of fractions known to be slightly purer by amounts no larger than the experimental error. When corrected for the proportion of the chief impurity, praseodymium, the results agree satisfactorily for all the samples except fraction 2923–4, which certainly contained an unestimated proportion of samarium.

The results obtained from fractions 1302, 1614, and 2923–4 naturally are less reliable than those obtained from the other three fractions. The corrected average for fractions 2926–7–8, 2932–3, 2936–7, is, however, 144.268, which differs from the corrected average for all six fractions, 144.275, by less than 0.01. It is evident that the value 144.27 can safely be taken to represent with accuracy the atomic weight of the purest material which we have succeeded in preparing. This value is essentially identical with the one chosen by the International Committee upon Atomic Weights, 144.3, and lies midway between the results of Auer von Welsbach and Feit and Przibylla on the one hand and that of Holmberg on the other.

We are indebted particularly to the Carnegie Institution of Washington for pecuniary assistance in carrying out this investigation, and also to the Cyrus M. Warren Fund for Research in Harvard University for indispensable platinum vessels, as well as to the Welsbach Light Company for the neodymium material.

Summary.

1. It is emphasized that crystallization in more than one form is advisable for the preparation in a pure state of salts of rare earths.
2. The preparation of a very pure neodymium salt is described.

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[1] Pub. Car. Inst., No. 28, (1905); Jour. Amer. Chem. Soc., **28**, 456.

3. The absorption spectrum of neodymium is described.

4. A method of obtaining anhydrous neodymium chloride is given.

5. The specific gravity of anhydrous neodymium chloride is found to be 4.134 at 25°/4°.

6. The most probable value of the atomic weight of neodymium is found to be 144.27, if silver is taken at 107.88; with silver at 107.87 neodymium becomes 144.26.

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CONTRIBUTIONS FROM THE ROGERS LABORATORY OF PHYSICS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

LIV. — *ON THE EQUILIBRIUM OF THE SYSTEM CONSISTING OF CALCIUM CARBIDE, CALCIUM CYANAMIDE, CARBON, AND NITROGEN.*

By M. de Kay Thompson and Robert H. Lombard.

Investigations on Light and Heat made or published, wholly or in part, with Appropriations from the Rumford Fund.

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LIV. — ON THE EQUILIBRIUM OF THE SYSTEM CONSISTING OF CALCIUM CARBIDE, CALCIUM CYANAMIDE, CARBON, AND NITROGEN.

By M. de Kay Thompson and Robert H. Lombard.[1]

Presented by H. M. Goodwin, October 12, 1910. Received September 24, 1910.

1. Introduction.

Since the discovery by Frank and Caro [2] that pure nitrogen is absorbed by impure calcium carbide at high temperature, according to the reversible reaction:

$$CaC_2 + N_2 \rightleftarrows CaCN_2 + C,$$

calcium carbide has been employed on a commercial scale for the fixation of atmospheric nitrogen. This method yields four times as much nitrogen for a given amount of power as the direct oxidation in an electric arc. Calcium cyanamide can be used directly as a fertilizer, or it may be the starting point for the manufacture of other nitrogen compounds. In consequence of the importance of this reaction, we were led to undertake the determination of the equilibrium pressure of the nitrogen in the above reaction for a number of different temperatures. In this system there are three components, and four phases; consequently, according to the phase rule there will be one degree of freedom; or for every temperature, a corresponding pressure at which equilibrium will exist.

The method of carrying out this investigation was to heat a graphite crucible containing graphite, calcium carbide, and calcium cyanamide, in an Arsem vacuum furnace [3] in an atmosphere of nitrogen. The

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[1] The following research was carried out under a grant from the Rumford Fund of the American Academy of Arts and Sciences. Grateful acknowledgment is hereby made to the trustees of this fund.

[2] Z. f. angew. Chem., **19**, 853 (1906).

[3] Trans. Am. Electroch. Soc., **9**, 163 (1906).

heating current was taken from a transformer in steps of 12 volts, and fine adjustment of the current was obtained by a carbon plate rheostat. The pressure was indicated by a mercury manometer, one arm of which was connected with the furnace; the other was evacuated and sealed off. The arrangement of the apparatus is shown in Figure 1. It was intended to measure the temperature of the crucible by means of a

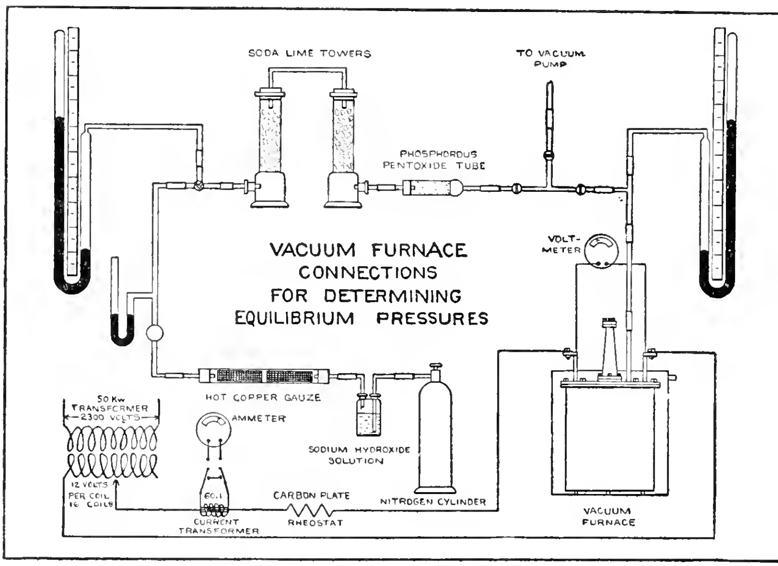

Figure 1.

thermo-electric junction entering the furnace through the tower projecting above the water in which the furnace was immersed, and two or three such runs were made. The wires of the couple were insulated from each other by different kinds of tubes. One wire was covered with a tube one millimeter in diameter, and this tube and the other wire were placed in a larger tube sealed at the end covering the junction. These tubes were long enough to reach up into the tower when their lower end was resting in the crucible. The hot end of the tube surrounding the wires of the thermo-electric couple was covered with a short graphite tube to protect it from the material in the crucible. No tubes that were available, however, would stand a temperature above 1200° C. Even Berlin porcelain was spoilt after a few hours' heating. Carbon seemed to have penetrated the whole mass of the porcelain where it was surrounded by the graphite spiral heater, and as much of the couple as was heated was also spoilt. It was therefore necessary to give up the direct measurement by the thermo-couple and use a Wanner optical pyrometer. The instrument used in this investigation had just been calibrated by the Bureau of Standards, with results that were not materially different from the calibration furnished by the makers. The amyl acetate lamp was not sufficiently constant, however, and

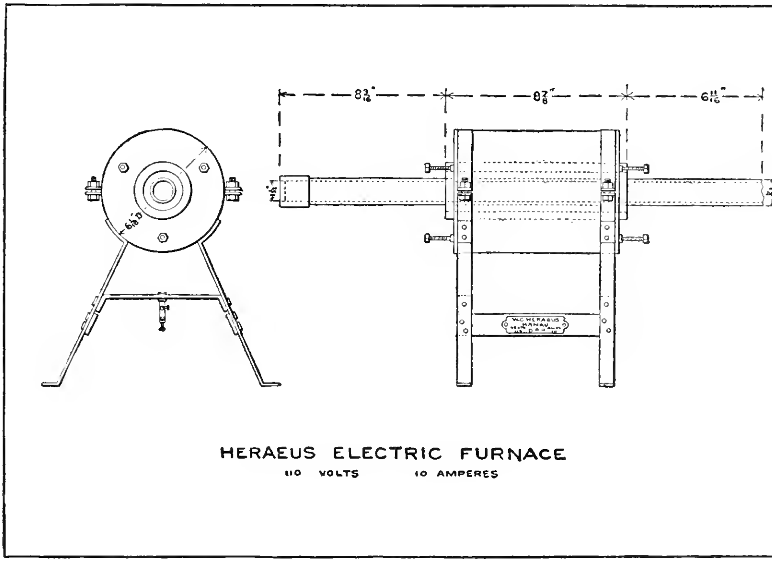

Figure 2.

therefore adjustment of the current in the small electric lamp of the Wanner pyrometer was made as follows: A furnace, shown in Figure 2, was made by wrapping platinum foil one fourth inch wide and 0.001 inch thick about a Royal Meissen porcelain tube of 2.54 centimeters inside diameter. A cylindrical piece of carbon was placed in the furnace, and a small porcelain tube was passed through the larger one and through a hole in the carbon. The smaller tube was to hold a thermo-electric couple, the junction of which was in the same plane as one of the faces of the carbon. The temperature of this face of the carbon was therefore given directly by the thermo-electric couple. In order to adjust the current in the Wanner pyrometer, the reading of the Wanner was set to agree with that of the couple, a piece of glass cut from the same plate as that closing the end of the tower of the vacuum furnace was placed in the groove in the pyrometer provided for holding the smoked glass sometimes used for increasing the range of the instrument, and the pyrometer was sighted on the carbon. The current was then adjusted till the two fields in the pyrometer had the same appearance. The small furnace was kept heated during each run, and before measuring the temperature of the vacuum furnace a comparison was always made with the thermo-electric couple. It was found by this means that the current in the lamp as adjusted from day to day was much more constant than when the amyl acetate lamp was used as a standard, but it will be seen from the tables below that the current was not absolutely constant. This must have been due to a variation in the electric lamp itself. The following table shows the effect of a change of the current in the lamp on the apparent temperature as measured by the pyrometer. The temperature of the body measured was held constant in the vacuum furnace. The current is given in arbitrary units equal approximately to tenths of an ampere.

TABLE I.

Current in Wanner Pyrometer.	Temperature.
6.0	1304
6.1	1295
6.2	1282
6.3	1263
6.4	1263
6.5	1241
6.6	1226

The current could not be set better than to 0.1 of a unit, corresponding on the average to 10 degrees.

In order to see how the temperature varied in the auxiliary furnace, the temperature was taken with the junction by placing it at different distances from the carbon block, which was placed near the center of the tube. The results are given in Figure 3, which shows that no appreciable error would be introduced if the junction had not been exactly in the same plane as the surface of the carbon plate.

The calcium carbide used in the following experiments was all made at one time from Merck's lime and turnings from Acheson graphite electrodes. It was analyzed by the loss in weight method and found $78\frac{1}{4}$ per cent pure. It was ground up and passed through a sieve with 36 meshes to the linear inch before the analyses were made.

The method of carrying out the experiments was as follows: the crucible was filled nearly to the top with the carbide, was covered with a graphite lid with a hole in it, and then placed in the furnace. The furnace was pumped out and the charge heated to a temperature at which the experiment was to be made (or higher) until all gases were driven off. Nitrogen, made by the Linde process and 90 per cent pure, was let into the furnace after passing it over hot copper, soda, lime, and phosphorous pentoxide. After allowing the nitrogen to enter the furnace until the pressure was 2 or 3 centimeters, it was pumped

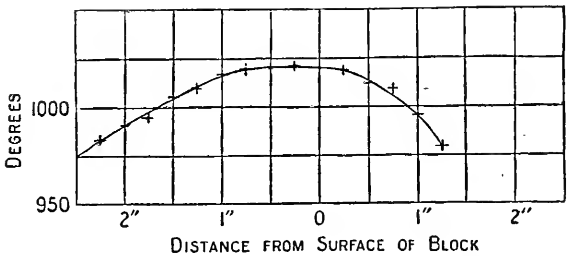

FIGURE 3. Temperature in Heraeus furnace at different distances from the surface of carbon block used for calibrating the Wanner pyrometer. The surface faced the pyrometer towards the left, in the figure.

out to about 1 or 2 millimeters. This was repeated once or twice to remove all other gases. The furnace was then filled with nitrogen to a pressure that was certainly greater than the equilibrium pressure. The furnace was usually hot when filled. The furnace was frequently filled in the afternoon and the run started the following morning. On heating with an excess of nitrogen some of it would be absorbed, producing calcium cyanamide and carbon from the carbide, so that all three solid phases were then present. After equilibrium had been reached from this side, nitrogen was pumped out, and the equilibrium was approached from the other side. The mean of the two results was taken as the best value of the equilibrium obtainable.

2. Experimental Data.

A Siemens and Halske millivoltmeter of 471 ohms resistance was used for measuring the temperature with a platinum platinum-rhodium junction. The millivoltmeter was calibrated and the following table contains the corrected value of the scale reading with the corresponding temperature.

TABLE II.

Calibration of Thermo-electric Junction.

	Temperature Centigrade of Melting Point.	Millivolts.	Log. of Temperature.	Log. of Millivolts.
Sulphur	447.7	3.56	2.6481	0.5515
Antimony	630.7	5.38	2.7998	0.7307
Aluminum	656.0	5.59	2.8169	0.7474
Copper	1083.0	10.19	3.0346	1.0081

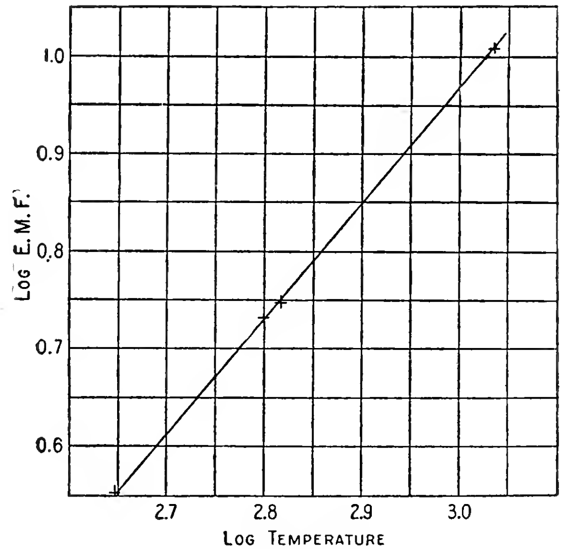

Figure 4. Calibration of thermo-electric couple.

The logarithms of the temperatures and pressures are seen from Figure 4 to lie on a straight line.

The pressures given in the following tables are in millimeters of mercury, and the temperatures in centigrade degrees. The numbers of the experiments are those recorded in the laboratory note-book. These tables contain only an abbreviated record of the experiments. Under the heading "Kilowatts" is given the product of current and amperes divided by 1000, not taking into account the power factor of the furnace.

Experiment 4.

The following experiment was carried out before the method of standardizing the Wanner pyrometer described above had been adopted.

Temperature measurements of the charge were therefore made with different values of the current in the lamp of the Wanner pyrometer, and the correct value, with its corresponding temperature, was determined subsequently from Table I.

	Time.	Kilowatts.	Pressure.
Apr. 28.	1.23 P. M.	8.36	317.8
	1.50 "	8.26	276.3

Some nitrogen was pumped out, in order not to use up all the carbide.

	1.52 P. M.	8.52	264.0
	4.45 "	8.52	267.4
Apr. 29.	12.35 "	8.52	267.8
	2.22 "	8.53	270.0

Here the pressure had become nearly constant, so nitrogen was admitted.

	2.25 P. M.	8.48	277.0
	4.45 "	8.52	273.3

The equilibrium pressure is therefore between 270 and 273, or 271.5 millimeters of mercury. The temperature, determined as explained above, was 1263° C.

Experiment 6.

A fresh charge of 33.7 grams of carbide was used in the following experiment.

	Time.	Kilowatts.	Pressure.
May 11.	2.55 P. M.	7.65	281.7
	4.05 "	7.66	224.0
	4.37 "	7.66	223.6

The pressure had become nearly constant, so nitrogen was admitted.

	4.45 P. M.	7.66	226.3
	5.10 "	7.66	225.1
May 12.	12.40 "	7.70	218.5
	1.00 "	7.00	222.7
	2.22 "	7.58	223.0

Nitrogen was again pumped out.

	2.23 P. M.	7.58	214.1
	4.30 "	7.52	224.7
May 13.	1.45 "	7.56	224.6
	2.45 "	7.61	223.5

The pressure had become nearly constant at 224 millimeters, so nitrogen was admitted.

Time.	Kilowatts.	Pressure.
2.55 P. M.	7.61	236.0
5.05 "	7.40	224.7

The mean temperature was 1223°, with the current in the lamp of the Wanner pyrometer 6.4 units. The equilibrium pressure was determined twice from each direction, and was found to be 224 millimeters.

Experiment 7.

The following experiment was made with the same charge as in 6.

	Time.	Kilowatts.	Pressure.
May 17.	2.35 P. M.	8.98	275.3
	3.45 "	8.98	282.9

The pressure was increasing very slowly, possibly all calcium cyanamide having been decomposed. Nitrogen was admitted.

	3.50 P. M.	8.98	316.5
	4.55 "	8.98	304.2
May 18.	1.30 "	8.92	305.6
	3.00 "	8.78	305.1

The pressure had become nearly constant. Nitrogen was therefore admitted.

	3.02 P. M.	8.78	329.2
	4.45 "	8.71	311.8
	5.00 "	8.71	313.0
May 19.	12.30 "	8.78	312.5
	1.13 "	8.71	312.9

The pressure was nearly constant. Some nitrogen was then pumped out.

1.18 P. M.	8.71	293.8
3.35 "	8.71	307.0

The pressure had become nearly constant. Some nitrogen was therefore admitted.

3.38 P. M.	8.71	317.4
4.20 "	8.71	313.0

Absorption was taking place very slowly. In order to see whether 313 or 305 was nearer the true value, nitrogen was pumped out.

4.34 P. M.	8.71	309.0
4.55 "	8.71	310.3

The results of this experiment are somewhat inconsistent, as the pressure was decreasing at 305 and increasing at 310 millimeters. The mean of these is 308 millimeters, and the mean temperature was 1297° C. The current in the lamp of the Wanner pyrometer was from 6.3 to 6.5 units.

Experiment 8.

	Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
June 14.	1.55 P. M.	9.86	444.0	5.80	1371°
	2.15 "	9.86	384.8	. .	. .
	2.45 "	9.69	373.8	5.83	1379
	3.15 "	9.73	375.3	5.88	1373
	3.45 "	9.73	375.4	5.82	1371
	4.45 "	9.65	376.1	5.80	1377
June 15.	8.45 A. M.	9.67	. .	. .	. .
	9.15 "	9.71	374.2	. .	. .
	10.30 "	9.68	376.5	5.80	1380

The pressure had become constant. Nitrogen was therefore pumped out.

	Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
	11.13 A. M.	9.64	366.6	. .	. .
	11.30 "	9.65	370.2	5.88	1374
	12.30 P. M.	9.62	373.4	. .	. .
	1.30 "	9.65	377.3	5.84	1377
	4.30 "	9.55	379.3	. .	. .

It is seen that the pressure decreased until it became constant at about 376 millimeters, and after pumping out to 367 it increased to 379. The average mean is 378 millimeters, and the average temperature 1378° C.

The interior of the furnace after these runs was found covered with a thick white deposit.

The charge in the crucible gave tests in each case for carbide and calcium cyanamide. The latter was tested for by adding ammonia and silver nitrate, which, with cyanamide, give a yellow precipitate of silver cyanamide. The white powder taken from the walls of the furnace also gave the yellow precipitate on adding ammonia and silver nitrate to a solution of this substance.

Experiment 9.

In this run an attempt was made to measure an equilibrium at about 1440°. It was not successful, however, as will be seen from the following results.

Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
10.15 A. M.	10.60	. .	. .	. .
10.20 "	10.88	650	5.75	1433
10.30 "	10.94	587	. .	. .
11.30 "	10.94	513	. .	. .
12.55 P. M.	10.94	514	5.80	1420
Some nitrogen pumped out.				
1.00 P. M.	10.94	458	. .	. .
1.15 "	10.94	468	. .	. .
2.00 "	10.94	469	. .	. .
Some nitrogen pumped out.				
2.05 P. M.	10.94	460	. .	. .
2.45 "	10.94	461	. .	. .
More nitrogen pumped out.				
2.50 P. M.	10.94	427	. .	. .
3.45 "	10.94	432.7	5.92	1448

It is evident that, though the reaction goes in both directions at this temperature, it is so slow that it is difficult to tell whether an equilibrium has been reached or not. The reason for this may have been that the charge had become very hard and could not be dug out of the crucible, and appeared as though it had been somewhat fused together. This would make it difficult for the nitrogen to penetrate its entire mass. The average temperature of this run was 1437°. A later attempt at 1448° was more successful, and is given in Experiment 15.

Experiment 10.

Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature
June 21. 3.10 A. M.	5.58	222.2	. .	. .
3.40 "	. .	174.8	5.90	1171
4.30 "	5.58	157.0	5.90	1165
4.53 "	5.58	150.5	. .	. .
June 22. 8.30 "	5.62	149.2	5.80	1164
10.30 "	5.72	144.0	. .	. .
11.30 "	5.69	141.0	5.80	1172
1.45 P. M.	5.69	133.8	5.82	1162
4.00 "	5.56	127.6	5.80	1159
4.45 "	5.64	125.8	5.82	. .
June 23. 8.30 "	5.72	125.5	5.88	1159
10.00 "	5.70	124.5	7.85	1155

The pressure had become constant. Some nitrogen was therefore pumped out to get the equilibrium from the other side.

Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
10.03 P. M.	5.69	102.2	. .	. .
12.30 "	5.71	104.2	. .	. .

The reaction went so slowly that it did not seem possible to get an equilibrium from this side at this temperature. The temperature was therefore raised.

1.00 P. M.	6.32	116.9	5.88	1199
2.00 "	6.31	142.0	. .	. .

The pressure was increasing rapidly, and in order not to use up all the calcium cyanamide, some nitrogen was admitted.

2.05 P. M.	6.27	160.0	. .	. .
2.30 "	6.30	160.4	. .	. .

Since the pressure remained constant, more nitrogen was admitted.

2.45 P. M.	6.30	189.7	. .	. .
3.00 "	6.31	181.0	5.87	1194
4.00 "	6.27	171.1	5.86	1193
4.45 "	. .	168.1	5.90	1193
June 24. 8.30 A. M.	6.33	166.5	. .	. .
10.00 "	6.34	166.5	. .	. .

This value, 166.5 millimeters, seems to be the equilibrium. In order to get it from the other side some nitrogen was pumped out.

10.05 A. M.	6.29	152.6	. .	. .
10.30 "	6.30	155.0	5.87	1197
11.30 "	6.27	159.0	5.80	1198
1.00 P. M.	6.29	160.0	5.88	1193
3.00 "	6.32	159.6	. .	. .

The pressure increased to only 160 millimeters in place of 166. In order to see which was nearer the correct value, nitrogen was admitted.

3.03 P. M.	6.32	166.3	. .	. .
3.45 "	6.31	164.0	5.88	1199
4.55 "	6.28	162.4	. .	. .
June 25. 8.30 A. M.	0.00	123.0	. .	. .
9.00 "	6.34	162.8	. .	. .
10.30 "	6.33	162.9	. .	. .

In these last data the absorption took place to 162 millimeters. Combining this with the expansion to 160, the mean is 161 millimeters.

With the same charge the following data were then taken at a higher temperature, after admitting some nitrogen.

Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
June 27. 9.00 A. M.	8.77	298.0	. .	. .
9.45 "	7.50	280.9	5.90	1271
11.50 "	7.53	274.6	5.80	1290
12.45 P. M.	7.51	274.3	. .	. .

Here the pressure had nearly reached the equilibrium value, as it was decreasing very slowly. In order to get equilibrium from the other side, some nitrogen was pumped out.

12.40 P. M.	7.51	257.6	. .	. .
1.30 "	7.53	263.5	5.92	1280
3.15 "	7.53	269.1	5.90	1280
4.10 "	7.54	270.8	5.90	1280

The equilibrium therefore lies between 274 and 271, the mean of which is 273 millimeters. The mean temperature was 1278° C.

There was a great deal of white powder on the walls of the furnace after this long heating.

The following run was made using a heating spiral of twice the cross-section and half the length, heated by a direct current.

Experiment 11.

Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
9.00 A. M.	7.86	330.1	5.80	1360
11.00 "	7.80	279.5	6.08	1310
12.55 P. M.	7.77	282.4	6.01	1320
2.00 "	7.71	282.8		

The equilibrium was approached only from one side, when the experiment was stopped on account of poor regulation of the temperature by this arrangement. The equilibrium found was 283 millimeters at a mean temperature of 1308° C.

Experiment 12.

The object of the following experiment was to measure an equilibrium between 1000° and 1100°. Since equilibrium could not be obtained in both directions at 1160 without a catalyzer, calcium chloride was added to the charge for this experiment, to act as a catalyzer.[4] The charge consisted of 22.5 grams of calcium carbide and 2.5 grams of dry calcium chloride, well mixed together.

	Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
July 1.	8.27 A. M.	4.07	151.8	. .	. .
	8.37 "	4.10	128.7	6.03	1050
	9.30 "	4.10	71.1	. .	. .
	10.30 "	4.08	44.9	6.00	1050
	12.45 P. M.	4.10	25.9	. .	. .
	2.00 "	4.08	24.2	6.12	1050
	2.30 "	4.08	23.7	. .	. .

Since the pressure had become fairly constant, some nitrogen was pumped out.

	2.35 P. M.	4.10	19.6	. .	. .
	4.45 "	4.07	21.8	6.10	1050

The constant pressure from the other direction is 22 millimeters. The mean value at 1053° is 22.8 millimeters. With the same charge the following experiment was made at a higher temperature.

July 5.	11.30 A. M.	4.97	29.7	. .	. .
	12.15 "	4.99	50.8	6.09	1105
	1.45 "	4.99	56.3	5.98	1117
	3.00 "	4.99	57.7	6.10	1115

Here the pressure had become nearly constant, so nitrogen was admitted.

	3.15 P. M.	4.99	108.7	. .	. .
	4.50 "	4.98	78.5	. .	. .
July 6.	8.30 A. M.	4.97	74.4	. .	. .
	2.00 "	5.01	67.2	5.90	1115

Absorption was still taking place, but so slowly that the experiment was stopped. The mean of 67.2 and 57.7 is 62.5 millimeters at 1117° C.

Experiment 13.

The charge for this experiment was 22.5 grams of calcium carbide and 2.5 grams of calcium chloride.

[4] That the chlorides of the alkali earth metals act in this way was discovered by Polzenius; see Bredig, Z. f. Elektroch., **13**, 69 (1907).

	Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
July 8.	8.05 A. M.	0.	167.5	. .	25
	8.10 "	4.96	199.7	. .	. .
	10.00 "	4.98	120.7	. .	. .

Some nitrogen pumped out, so as to reach equilibrium sooner.

10.03 A. M.	5.02	108.0	. .	. .
11.00 "	4.98	98.5	. .	. .

More nitrogen pumped out.

11.05 A. M.	5.01	93.8	. .	. .
2.00 P. M.	5.00	86.3	6.02	1110

More nitrogen pumped out.

2.05 P. M.	5.01	83.2	. .	. .
2.30 "	4.98	83.0	. .	. .

The pressure remained constant, and was then reduced to 73.4 and then to 70 and remained constant at both values. It was then reduced still more.

	4.02 P. M.	4.98	45.2	. .	. .
	4.50 "	4.98	48.0	. .	. .
July 9.	8.10 A. M.	5.00	35.3	. .	. .
	8.30 "	4.99	47.8	. .	. .
	9.45 "	5.00	50.9	6.05	1107
	10.00 "	. .	51.1	. .	. .
	10.45 "	. .	51.3	. .	. .

At a pressure of 86.3 millimeters, nitrogen was being slowly absorbed, and at 51.3 the pressure had become constant starting with a lower value. The mean of these is 68.3 millimeters, and the mean temperature was 1114°. No good reason why the velocity of the reaction was so slow in this case can be given.

Experiment 15.

	Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
July 13.	8.45 A. M.	0.	318.6	. .	. .
	8.50 "	10.9	415.	. .	. .
	9.30 "	10.9	428.5	6.1	1441
	10.00 "	10.9	429.5	. .	. .

It seemed that not enough nitrogen had been let into the furnace to start with, so more was admitted.

Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
10.22 A. M.	10.9	465.	. .	. .
11.00 "	10.9	449.0	. .	. .
11.30 "	10.9	448.0	6.1	1448

Absorption had taken place and the pressure had become constant. Nitrogen was then pumped out.

12.10 P. M.	10.9	430.5	. .	. .
12.30 "	10.9	433.5	6.1	1451
2.40 "	10.9	438.0	. .	. .

The pressure had become constant. Nitrogen was again admitted to see if the pressure would again diminish to 449.

2.43 P. M.	10.9	476.0	. .	. .
3.00 "	10.9	469.8	. .	. .
3.15 "	10.9	471.2	6.1	1445

It did not reach 449 but stopped at 470. On admitting more nitrogen, very little absorption took place, as seen below.

3.50 P. M.	10.9	496.3	. .	. .
4.15 "	10.9	495.7	. .	. .
4.30 "	10.9	494.5	. .	. .

This run may be summed up as follows : absorption took place from 465 to 449 millimeters, but when pumped out to 430.5 millimeters, the pressure increased to only 438 millimeters. After this the pressure remained constant at nearly any value. This may have been because the charge was changed by heating into a hard, solid mass, thus reducing the surface in contact with the nitrogen. This would explain why the reaction took place with a noticeable velocity only during the first part of the experiment.

The average of 449 and 438 is 444 millimeters, and the average temperature is 1448°C.

Experiment 16.

The charge in this experiment consisted of 21.5 grams of calcium carbide.

	Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
Aug. 3.	11.31 A. M.	8.83	434	. .	. .
	11.45 "	8.74	408	6.10	1353
	12.30 P. M.	8.83	389	. .	. .
	2.00 "	8.83	389	5.9–6.0	1379

The pressure had become constant, so nitrogen was pumped out.

Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
2.01 P. M.	8.74	373.3	. .	. .
2.20 "	8.70	379.9	6.00	1352
2.45 "	8.83	382.8	5.98	1368
3.30 "	8.85	384.2	5.9–6.0	1380
4.00 "	8.74	386.1	6.00	1361
4.30 "	8.70	386.0	. .	. .

The average temperature was 1372° C and pressures obtained from opposite directions were 386 and 389, the mean of which is 387.5 millimeters.

Experiment 17.

The charge consisted of 23.7 grams of carbide.

	Time.	Kilowatts.	Pressure.	Current in Wanner Pyrometer.	Temperature.
Aug. 5.	8.35 A. M.	00.00	328.1	. .	25
	9.00 "	10.30	. .	. .	. .
	9.05 "	10.30	423.0	. .	. .
	9.15 "	09.95	415.0	. .	. .
	11.05 "	09.95	381.0	. .	. .
	12.00 "	10.00	378.1	5.9	1396
	1.00 P. M.	10.20	379.0	5.9	1382
	1.45 "	10.20	379.7	. .	. .

The pressure had become constant, so nitrogen was pumped out.

1.50 P. M.	10.2	354.9	. .	. .
2.45 "	10.2	362.1	5.9	1387
4.05 "	10.2	366.1	6.0	1385
4.45 "	10.2	369.9	6.0	1390

The pressure was rising slowly when the experiment was stopped. The equilibrium is therefore nearer 380 than 370. However, the mean of these two, 375, is taken as the result of this run. The average temperature was 1385° C. The charge had become hard on the top but less so below the surface.

Experiment 18.

The disadvantage of the Arsem vacuum furnace for experiments of this kind is that the volume is so great that it takes quite a long time for the solid substance to absorb or evolve enough gas to reach equilibrium pressure. It was therefore decided to try an experiment in the Heraeus furnace that had been used as an auxiliary furnace for calibrating the Wanner pyrometer. Of course this could be used only at comparatively low temperature. It was glazed only on the inside, and it was anticipated that it would leak.

The charge was contained in a graphite boat 1.5 inches long. It consisted of 6.14 grams of a mixture of calcium carbide and calcium chloride containing 10 per cent of the latter. The ends of the porcelain tube of the furnace were closed with rubber stoppers; the stoppers and ends of the tube were shellacked. A small glazed porcelain tube for holding the thermo-electric couple extended through the larger tube and the two stoppers. The junction was pushed in until it was opposite the graphite boat, in the middle of the furnace.

August 10th, at 5 P. M., the furnace was pumped out to a pressure of 1 millimeter and left till August 11th, 16 hours later, when it had leaked to 13.1 millimeters. The furnace was then heated to about 1070° and pumped out a number of times, after which the gases remaining were swept out with nitrogen. While still hot, nitrogen was admitted to 110 millimeters. Absorption was taking place rapidly, and the following observations were taken.

Time.	Pressure.	Temperature.
2.08 P. M.	110.0	. .
2.10 "	32.0	1074
2.13 "	23.2	1074
2.20 "	21.5	1074
2.25 "	21.6	1074
2.32 "	22.1	1074
2.40 "	22.8	1077
2.45 "	23.2	1077

It seems that here the absorption of nitrogen at first overbalanced the leakage. The following day, minima of 26.8, 30, and 31.5 millimeters were obtained, starting with pressures of 145, 165, and between 100 and 200 millimeters respectively; and the best value of course would be obtained by starting very little above the equilibrium pressure, so that there would not be so much time for leakage to take place.

Although these determinations were somewhat in error from the leakage, this error was evidently not great, as the pressures agreed well with those obtained in the vacuum furnace. The temperature measurements were more reliable than the others, as they were made directly with the thermo-electric junction. The lowest pressure, 21.5

millimeters, is presumably the most reliable. The result of this run is therefore 21.5 millimeters at 1074° C.

The results of the above experiments are collected in the table below, and are plotted in Figure 5. The ordinates are pressures in centimeters of mercury and the abscissae are the temperatures in centigrade degrees. The pressures are not reduced to zero degrees as this correction is much less than the experimental error.

TABLE III.

Number of Experiment.	Pressure in Centimeters of Hg.	Temperature Centigrade.
4	27.0	1263
6	22.4	1223
7	30.8	1297
8	37.8	1378
10	12.5	1160
10	16.1	1196
10	27.3	1278
11	28.3	1308
12	2.3	1053
12	6.3	1117
13	6.8	1114
15	44.4	1448
16	38.8	1372
17	37.5	1385
18	2.2	1074

It is remarkable that the relation between the temperature and pressure is linear through such a great range in temperature. It would seem that at lower temperatures the line representing this relation must become concave upward, and it is hoped that in the near future it will be possible to carry out some experiments on this equilibrium below 1000 degrees, to test this point.

In order to compute the heat Q, evolved by the reaction $CaC_2 + N_2 \rightleftarrows CaCN_2 + C$, taken from left to right, it is not allowable to use the integrated van't Hoff equation

$$4.57 \log \frac{p_2}{p_1} = Q \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

because this equation is derived on the assumption that Q is constant, which is not the case in this reaction. If Q is computed from the differential equation

$$\frac{dp}{dt} = \frac{Q}{T(V - V_1)},$$

in which T is the absolute temperature, V is the volume of the carbide and nitrogen and V_1 is that of calcium cyanamide and carbon on the other side of the equation, and $\frac{dp}{dt}$ is the pressure gradient at the tem-

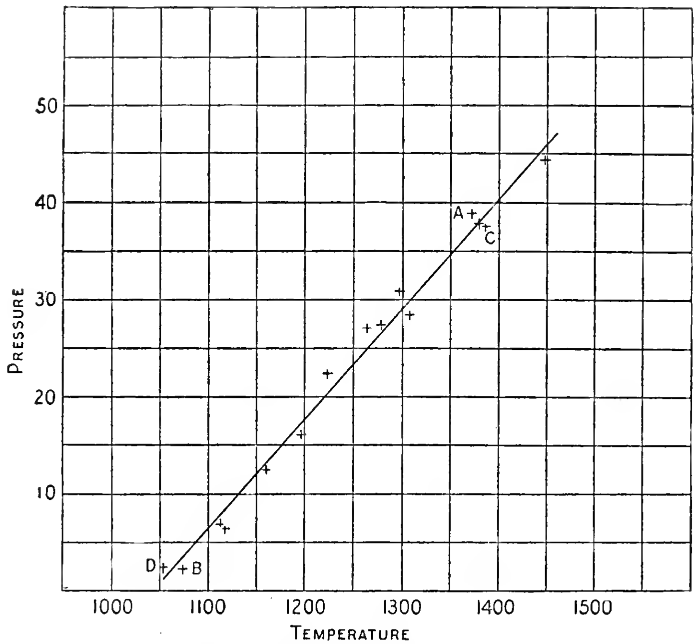

FIGURE 5. Plot showing pressures and temperatures at which equilibrium exists. Pressures are in centimeters of mercury; temperatures in centigrade degrees.

perature T, Q is found to have the values given in Table IV. The volume V_1 is negligible in comparison with V. Omitting V_1 the equation may be put in the following form :

$$Q = \frac{RT^2}{p} \frac{dp}{dt}.$$

The value of R is 1.99 calories, and the unit in which p is expressed cancels out.

TABLE IV.

SHOWING THE CHANGE IN THE HEAT OF THE REACTION WITH THE TEMPERATURE.

Temperature Centigrade.	Q in Gram Calories.
1100	63,900
1200	27,000
1300	18,800
1400	15,200

The value of $\frac{dp}{dt}$ is of course constant for every temperature.

The line as drawn in Figure 5 seems to represent the observations better than any other. The maximum variation in the slope would be represented by the lines AB and CD. From the value of $\frac{dp}{dt}$ obtained from these lines, the heat was calculated for 1200 degrees and found to be 29,700 and 25,800 calories respectively. This shows that the value calculated from the best representative line, 27,500, cannot be in error by more than 2000 calories, or 7 per cent.

It seems evident from the fact that the heat effect is greater the lower the temperature, that it would be advisable to work at as low a temperature as possible in the fixation of nitrogen by this method, because less heat would be required from external sources. This would be true if the velocity of the reaction did not decrease with decreasing temperature, for this might more than balance the benefit due to the increase in the heat of the reaction.

In view of the enormous increase in the heat of the reaction with decreasing temperature, as computed from these measurements, it is desirable that the heat should be measured in some other way. This might be done, as suggested by Mr. S. Peacock, by combustion experiments in a bomb. This would give indirectly the heat of the reaction at room temperature, which would be expected to be much larger than the values calculated above.

The free energy increase of the reaction

$$CaC_2 + N_2 \rightleftarrows CaCN_2 + C$$

at 1450° C., taken from left to right is

$$\begin{aligned} \Delta F &= -RT \log \tfrac{76}{46} \\ &= -4.57 \times 1720 \times 0.218 \\ &= -1714 \text{ gram calories.} \end{aligned}$$

At 1100° C., $\Delta F = -6700$ calories.

The errors in the temperature measurements are probably the main cause of the deviations of the single determinations from the best representative line. If all the error is assumed due to the temperature measurements, the average deviation of a single point from the line is only 12 degrees, and the largest deviation is 19 degrees. These are not greater than the experimental errors in the temperature measurements.

The thermo-electric couple had been cut off several times since the original calibration. It was subsequently calibrated as follows :

TABLE V.

Date.	Temperature.	Millivolts at Date given.	Millivolts at Original Calibration.
July 2	Sulphur point	3.60	3.60
August 13	Copper point	10.08	10.19

The original calibration had therefore remained practically constant throughout the experiments.

After each experiment for determining the equilibrium pressure, the furnace walls were covered inside with a white powder. A similar result was obtained in determining the equilibrium of the reaction $CaO + 3C \rightleftarrows CaC_2 + CO$.[5] It was noticeable, however, that there was more powder formed with calcium cyanamide for a given temperature and given duration of heating than with calcium carbide. It seemed probable that calcium cyanamide is volatile, especially after it was found that the white powder gave a yellow precipitate of silver cyanamide on adding silver nitrate to an ammoniacal solution of the powder. It was therefore thought desirable to make an analysis of this material.

Some of the pure white powder was analyzed for nitrogen by the Kjeldahl method, taking about one gram for analysis. The digestion with chemically pure sulphuric acid was continued for five hours near the boiling point ; immediately after, potassium permanganate was added. The two determinations gave 12.38 and 11.67 per cent nitrogen, corresponding to 34.3 per cent calcium cyanamide.

To get a check on this value, the calcium cyanamide was determined by the method of Perotti.[6] This method consists in allowing the substance to stand over night in water, filtering and washing, adding ammonia, and then an excess of silver nitrate. A yellow precipitate of silver cyanamide is produced which is filtered off, and the

[5] Trans. Am. Electroch. Soc., **15**, 197 (1909).

[6] Chemisches Centralblatt, **2**, 1059 (1905).

excess of silver nitrate is titrated with ammonium sulphocyanate. The sample taken was not perfectly white, but from its gray color evidently contained some carbon which had also distilled to the wall of the furnace. The residue obtained on treating the white powder with water was of a dark grey color. It was evidently carbon and calcium oxide, for (1) it dissolved slightly in a fresh portion of water and gave a distinct precipitate when ammonia and ammonuim oxalate were added; (2) on treating some that had been exposed to the air with hydrochloric acid it dissolved partly with effervescence, leaving a black residue behind; and (3) when the hydrochloric acid solution was neutralized with ammonia and ammonium oxalate was added, a large quantity of precipitate was formed. The analysis of the water solution for calcium cyanamide gave 38.59, 39.52, and 38.84 per cent corresponding to 13.51, 13.84, and 13.60 per cent nitrogen. An analysis of a sample of the same material used in the Kjeldahl determinations gave only 25.96 per cent calcium cyanamide, corresponding to 9.09 per cent nitrogen. Since this did not agree with the determinations by the Kjeldahl method or with the values obtained by the same method for other samples, analyses were made by the Perotti method of two more samples. The perfectly white material used in the Kjeldahl method could not be checked again as it had all been used up. The results of the analyses of the powder obtained in Experiment 16, were 15.96 and 15.35 per cent calcium cyanamide, corresponding to 5.59 and 5.38 per cent nitrogen. Analyses of the material obtained in Experiment 17 gave 29.10 and 28.95 per cent calcium cyanamide, corresponding to 10.19 and 10.13 per cent nitrogen. Experiments 16 and 17 were made at about the same temperature and still the amount of calcium cyanamide in the powder distilled to the cold part of the furnace was quite different.

Two determinations of calcium in the white powder used in the Kjeldahl determinations were made by first destroying the organic matter with fuming nitric acid and then decomposing the calcium nitrate by ignition in a platinum crucible and weighing as the oxide. The fuming nitric acid had to be added very slowly as it attacked the white powder with great violence, sometimes causing it to flame. The results were 39.93 and 39.27 per cent calcium, the average of which is 39.60 per cent. From this and the fact that the content of nitrogen was 12.1 per cent, the amounts of calcium oxide and of calcium cyanamide in the first sample of white powder analyzed were as follows:

Calcium cyanamide	34.5 per cent
Calcium oxide	30.5 " "
Unaccounted for	35.0 " "

Time did not permit of making a combustion analysis to see how the carbon checked up. This sample apparently contained very little free carbon, for it was of a cream white color.

Since calcium cyanamide was present in the powder distilled to the walls of the furnace, when heated with carbon it should produce calcium carbide with the evolution of nitrogen. In order to see whether this actually takes place, 0.4 gram of the powder was mixed with 0.9 gram Acheson graphite and heated in the vacuum furnace. The results are the following:

Experiment 14.

	Time.	Kilowatts.	Pressure.
July 11.	11.00 A. M.	0.	2.8
	11.12 "	0.33	10.
	11.20 "	0.78	. .
	11.35 "	. .	19.0
	11.35 "	1.27	. .
	11.45 "	. .	23.8
	11.45 "	1.8	. .
	12.00 "	. .	28.8
	12.00 "	2.7	. .
	12.20 P. M.	. .	67.
	12.20 "	5.3	. .
	12.35 "	8.0	. .
	12.45 "	. .	183.
	2.15 "	. .	195.

The temperature measured at 12.35 P. M., with the current in the Wanner 6.1 units, was 1287° C.

On opening the furnace after this run no white deposit was found on the walls, and the material in the crucible seemed to be hardly changed in appearance. It gave no test for carbide with an ammoniacal cuprous chloride solution, nor for cyanamide with silver nitrate. It is to be noticed, however, that a great deal more gas was evolved from this mixture on heating than on heating a sample of carbide, from which occluded gases always escape. For example, in Experiment 15, on heating carbide to 1450° C, these gases produced a pressure of 127 millimeters, which is considerably less than the 195 millimeters produced from the white powder and carbon at 1287°. It therefore seems certain that the large amount of gas evolved in Experiment 14 was due to a chemical reaction and not to occluded gases alone. A possible explanation of the absence of carbide in the crucible is that some oxygen got into the furnace. This would be changed to carbon monoxide which would decompose any carbide formed. The result of the experiment is, however, not conclusive.

Since calcium oxide was found in the white powder, an experiment was carried out to see to what extent lime would distill when heated alone in a vacuum. A graphite crucible was filled with pieces of Merck's lime about the size of a pea, and was heated in a vacuum for three hours at 1531 degrees. On opening the furnace it was found covered on the inside with a thin greyish-white powder. On collecting this it filled about one half inch of an ordinary test tube, and was placed in a desiccator several days before testing. It was then dissolved in hydrochloric acid and tested for calcium with ammonia and ammonium oxalate; a large amount of precipitate of calcium oxalate was formed. This experiment therefore shows that lime distills at about 1500 degrees in a vacuum. This doubtless is the cause of the thin layers always found in the experiments on the equilibrium of the reaction $CaO + 3C \rightleftarrows CaC_2 + CO$. It was not known at the time of the determination of this equilibrium that lime is distilled in the neighborhood of 1500 degrees.[7]

3. Summary.

The equilibrium pressure of nitrogen in the reaction

$$CaC_2 + N_2 \rightleftarrows CaCN_2 + C$$

was determined from both sides for a number of temperatures between 1050° C. and 1450° C. The results are plotted in Figure 5. It was found that the heat of the reaction varies enormously with the temperature. The values calculated from the plot in Figure 5 are given in Table IV.

The free energy increase of the reaction taken from left to right is −1714 gram calories at 1450° C. and −6700 calories at 1100° C.

Calcium cyanamide distills the cold parts of the vacuum furnace at temperatures as low as 1050° C. Pure lime distills appreciably in the neighborhood of 1500° C.

Electrochemical Laboratory, Rogers Laboratory of Physics,
Massachusetts Institute of Technology,
Boston, Mass., September 23, 1910.

[7] These Proceedings, **45**, 449 (1910).

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THE NATURE OF SOME SUPPOSED ALGAL COALS.

By Edward C. Jeffrey.

With Five Plates.

THE NATURE OF SOME SUPPOSED ALGAL COALS.[1]

By Edward C. Jeffrey.

Presented April 14, 1909. Received Aug. 12, 1910.

Early in the last decade of the nineteenth century MM. Renault and Bertrand published simultaneously their views as to the constitution of so-called boghead and similar coals.[2] In the bituminous schists of Autun are apparent even to the naked eye glistening spherical or oval bodies often arranged in layers, which readily reveal themselves under the microscope, in thin sections made in the manner of the rock sections of the petrographer and mineralogist. These objects are of considerable size, their dimensions ranging between 189 and 225 micra in length and between 95 and 115 micra in breadth. They have been named on account of their shape and locality *Pila bibractensis*. The individuals of this species when examined microscopically sometimes show more or less clearly a central cavity, which is surrounded by a wall sculptured with numerous alveoli, opening to the outside but having no communication with the internal space. The two authors cited agreed in regarding these structures as representing the remains of colonial gelatinous green Algae. The central cavity, which always appears in the better preserved specimens, was regarded as the equivalent of the hollow space in the midst of a colony of the living Volvox, or an allied genus of the colonial Algae. The substance surrounding the cavity, according to the hypothesis of the French authors, represents the gelatinous wall of a former algal colony, while the external openings or alveoli are imagined to correspond in their position and relations to the once living individuals of the colony. The hypothesis of the algal nature of the organism found in such abundance in the bituminous schists of Autun, was later extended by its authors to a wide range of similar organisms occurring in bogheads and cannels, as well as bituminous schists and oil-shales from various parts of the world. Their most notable subsequent communications on this sub-

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[1] Contributions from the Phanerogamic Laboratories of Harvard University, No. 23.

[2] Rénault, B., Communication faite sur le Boghead, Soc. Hist. Nat. Autun, 1892; Bertrand, C. E., *Pila bibractensis* et le Boghead d'Autun, Soc. Hist. Nat. Autun, 1892.

ject are as follows: C. E. Bertrand, *Reinschia australis* et Premières Remarques sur le Kérosene Shale de la Nouvelle Galles du Sud;[3] C. E. Bertrand, Les Charbons Humiques et les Charbons de Purins;[4] B. Renault, Sur Quelques Organismes des Combustibles Fossiles.[5] The contribution of Renault last cited is monumental in its character, representing as it does the labor of nearly a quarter of a century and dealing with most of the kinds of coal, which show structure when examined microscopically. Renault published from time to time, in the Bulletins of the Museum at Paris, his views as to the microscopic structure and related qualities of coal. Bertrand has recently written a very clear popular account of the views of Renault and himself.[6]

The ideas as to the constitution of certain coals cited above have not remained confined to their country of origin but have been adopted to a considerable degree in Europe and even in America. Professor Potonie, of the Royal School of Mines in Berlin, has taken up the Renault-Bertrand hypothesis and elaborated it in a number of publications. His views as to the origin of coal are stated with sufficient fulness in a pamphlet which has been of such general interest that it has gone through a number of editions.[7] Professor Potonie not only adopts the views as to the origin of boghead coals, oil-shales and bituminous schists advocated by the French authors cited above, but extends them to cannel coal, which Bertrand and Renault in common with other investigators have regarded as composed of the compressed remains of the spores of vascular cryptogams. The German author has gone further too than the French in describing recent peats which he supposes to be of algal origin and thus comparable with the bogheads and oil-shales of older geological periods. Dr. Davis White of the United States' Geological Survey has given not long since an admirably lucid account of recent views as to the origin of the various types of coal, including bogheads and oil-shales.

The present writer was led to investigate the subject of bogheads, oil-shales and other supposedly algal coals by reading the article of Dr. White cited above. In the investigations here described methods

[3] Bulletin de la Société, Hist. Nat. Autun, 1894.

[4] Travaux et Mém., Université de Lille, **6**, Memoir 21, 1898.

[5] Bulletin de la Société de l'Industrie Minerale, Serie 3, Tome 13, 4me. Livraison, 1899; Tome 14, 1re. Livraison, 1900, with atlas of 30 folio plate containing a large number of photomicrographs.

[6] Notions Nouvelles sur la Formation des Charbons de Terre, Revue du Mois, **3**, No. 15, pp. 323–41, Paris, 1907.

[7] H. Potonie, Die Entstehung der Steinkohle u. verwandter Bildungen einschliesslich des Petroleums. Vierte verbesserte u. erweiterte Auflage, Berlin, 1907.

have been used which admit of the securing of very numerous and even serial thin sections of the various coals studied. Previous observers have labored under the disadvantage of having to form their views from isolated and not very thin microscopic sections, obtained by the grinding methods of the lapidary. In the present research it has been found possible to adapt the more precise and delicate methods of the Biologist to the remains of organisms long extinct, which compose the mass of some of our most important if not most abundant coals. It will be well to give some account of the methods adopted before beginning the account of the observed facts.

Methods used in the Investigation.

It was found in all cases more advantageous to work with material softened by various treatments so that it could be cut successfully on the microtome. Control sections were used in many cases, prepared by the usual method, to make sure that the process of softening had not essentially modified the microscopic structures found in the various coals examined. The same process of softening was not found to be applicable to all kinds of coal. In the case of cannel or canneloid coals, treatment for a week or more with alcohol of seventy *per cent*, saturated with caustic soda or potash and kept at a temperature of from sixty to seventy centigrade, was found sufficient for the preliminary softening. After careful removal of the caustic alkali by repeated treatments with hot alchohol, it was generally found expedient to treat for two or three weeks with the strongest hydrofluoric acid. After washing out the acid, the small fragments of cannel are embedded in the usual way in Schering's celloidin and cut into thin sections on the sliding microtome. (The Jung-Thoma was found very useful for this purpose, on account of its rigidity.) The sections must in many cases be at least five micra thin. Thicker sections do not show details of structure with sufficient clearness nor can they be as advantageously photographed by the microscope as the thinner ones. In well-prepared material it is quite possible to cut serial sections of five micra. Individual sections thinner than five micra may be readily obtained. The preparation of serial sections has the advantage of making it possible to follow the structures observed through a number of sections, by which means their real nature can be more clearly and accurately elucidated.

In the case of the more resistent cannels and particularly in the case of those coals known as bogheads and oil-shales, more vigorous methods had to be adopted. It was found necessary to treat some of the American cannels and particularly American bogheads from Kentucky, with aqua regia and in some cases with a similar combination of acids in which the hydrochloric acid was replaced by hydrofluoric. The last reagent is recommended as the most effectual one which has been found for softening and disintegrating even the hardest coals, such as anthracite, etc. In this fluid, both the nitric and hydrofluoric acid are used of full strength. The last method was found particularly advantageous in preparing the bogheads of Autun and the so-called oil-shale of New South Wales for microscopic examination. Sometimes it is an advantage to return the coals to alkaline alcohol after treatment with the various acids described. In such cases care must be taken to wash out all the acid and to thoroughly dehydrate the pieces of coal before transferring them to alkali in alcohol, as otherwise they suffer disastrous swelling. In all instances, no matter what devices were used for softening the coals, they were embedded in celloidin before cutting. The presence of any free acid or alkali in the material is disastrous to the knife in the first instance or to the consistency of the celloidin in the second. After the sections are cut they are dehydrated in a mixture of absolute alcohol and chloroform, to avoid softening the celloidin matrix. After clearing in benzole or xylol, they are mounted in balsam. In a few instances, such as the oil-shale of New South Wales, where the sections are very light colored, it was found advantageous to mount in glycerine jelly. In the case of serial sections, the best procedure is to lay the sections on a slide as they come off the microtome knife and then dehydrate and clear them carefully in their order. The usual methods of cutting celloidin series do not answer in the case of coal, as from the nature of the material frequently sections become folded or torn in transferring to glass. As not above four or five sections are ordinarily needed in series in a given case, to elucidate structural features, the method described above is not so laborious as it might appear.

The Structure of Cannel Coal.

It will be well as a preliminary to the observations on so-called algal coals (bogheads, oil-shales, bituminous schists, etc.) to describe the structure shown by ordinary cannel coal. It has long been known and has been made particularly clear by the monumental investigations of Renault, that what is usually known as cannel coal is composed very largely of the flattened spores of vascular cryptogams. Figure 1, Plate 1 shows the structure of cannel coal as seen in sections vertical to the layering in a moderately high microscopic magnification. The linear light bodies in the dark matrix are the flattened spores. The remains of the originally rotund central cavity can be seen as a dark line. Figure 2, Plate 1, shows the same coal in section parallel to the planes of layering under the same microscopic enlargement. The spores appear in this plane in rounded outline and are much fewer in number. The explanation of the circular outline and the smaller number of apparent spores is afforded by the vertical section. In many cases it is possible in sections parallel to the layering of cannel to make out the presence of spores still in tetrads or in the case of isolated spores to distinguish triradiate ridges resembling those which mark contact surface of the reproductive bodies in many living vascular cryptogams. Some instances of these appearances are illustrated in the subsequent parts of this article, and need not be discussed here. The cannel just described is from Kentucky and is quite typical of the coals of this nature derived from this and other States. European cannels have also been studied but in general they show structure less clearly than the American cannels which have been examined. This is perhaps one of the causes of the apparent misinterpretations of some of the European writers on the microscopic structure of coal.

The Structure of Kentucky Boghead.

In most of the samples of cannel coal from Kentucky which have come under my observation, there occur, to a greater or less extent, bodies generally pale yellow in color but sometimes brown. These viewed in certain planes of section have the alveolar appearance, which is characteristic of the supposed Algae of Betrand, Renault, and Potonie. Those coals in which the imagined Algae become most abundant are known as bogheads, in accordance with the terminology of Renault.

It has been considered desirable in this connection to illustrate a boghead of unquestionable authenticity. I am indebted to Dr. David White of the United States' Geological Survey for some excellent material of Kentucky boghead of Pottsville age, which has been utilized in making the figures which are described in the paragraphs immediately following. A comparison with the figures and descriptions of Renault's monumental work show clearly that it is his *Pila kentuckyana* or a very closely allied species.

Figure 3, Plate 1, shows a number of the supposed Algae, denominated under the generic appellation Pila, somewhat highly magnified. They are variously grouped and scattered throughout the figure, a particularly striking clump appearing on the upper left hand of the figure. Most of the individuals show a mottled appearance, the dark spots corresponding in position to the supposed agal individuals constituting the colony. Figure 4, Plate 1, shows a rather extensive field under a low magnification. On the dark background formed by the so-called fundamental substance of the coal may be distinguished various lighter areas, some of which figure as Algae, according to the Renaultian view, and others, even with the low magnification used, are of such a rounded or angular contour that they obviously represent the crushed remains of spores of the vascular cryptogams. The spongy or mottled light bodies represent the supposititious Algae. As will be shown later, the two kinds of appearances belong together, and only the absence of thin serial sections could have prevented the distinguished French author, cited above, from recognizing this state of affairs. Figure 5, Plate 1, shows part of the field reproduced in the foregoing figure, on a considerably higher scale of magnification. Figure 6, Plate 1, shows a little above the horizontal middle line two bodies, which are obviously cryptogamous spores. The one on the right partakes of the spongy or mottled appearance, presented by the supposed Algae. Elsewhere, especially in the lower part of the figure, are seen the remains of additional bodies, which take their place among the reputed Algae of Renault. Figure 7, Plate 2, shows part of the last figure more highly magnified to bring out more clearly the structural features of the bodies represented in the foregoing figure. In our next illustration, Figure 8, Plate 2, is represented another field under moderate magnification. On the right is a light body representing a fragment of some broken-up plant, a kind of remains with which both cannel and boghead coals abound. On the extreme left is seen a cluster of structures representing a tetrad of spores. In the median region of the figure are several of Renault's putative Algae. Figure 9, Plate 2, shows the tetrad from the left of the foregoing figure, somewhat mroe highly magnified. It is now apparent that on the extreme right of the tetrad one of the spores is cut through the back and the two anglewise plane surfaces. Another spore of the tetrad appears below and is obviously of the same nature as Renault's Alga *Pila kentuckyana*. The third visible member of the tetrad is less clearly seen on the upper side and is not so obviously of the organization represented by the supposed Algae. It will be well to defer the interpretation of the last figure until a number of other illustrations of the imagined Algae of Renault have been examined. Figure 10, Plate 2, represents a particularly striking view of a spongy or mottled mass, which at the same time from its contour and angles, is obviously the spore of a vascular cryptogam viewed from the plane of section through its angular internal face. The plane of section is so fortunate, in the compressed condition of the spore, as to show at the same time part of the alveolar outer wall of the spore. In juxtaposition to the body, which is clearly a spore, are two of the putative Algae. In Figure 11, Plate 2, is shown another view of a spore-like body, there more obviously a spore. On the left is the projecting internal face of the spore, seen in section as an angle. The back of the spore is sculptured. In the remainder of the field are a number of the Alga-like bodies. The magnification in the present figure is about one half greater than in the last. Figure 12, Plate 2, represents, together with a number of the putative Algae, a body with the same alveolar parietal structure, which is at the same time obviously a spore. Figure 13, Plate 3, shows one of the spores so numerous in the boghead coal under examination, in face view. The three ridges which mark the surfaces of contact with the three originally present sister spores of the tetrad can be very clearly made out. Such clear views of the spore face are comparatively rare, whether from the condition of compression of the spores or from the loss by decay of the thinner angular inner face of the spore, it is not easy to determine. The outlines of the spore in the last figure are rough, showing the alveolar nature of the free surface resembling that found in the spores of many of the existing Lycopodiales. Figure 14, Plate 3, shows in the center another spore in the same plan of section as that in the last figure. The magnification is very much higher for the purpose of bringing out the rough surface of the spore shown on the margins of the figure, particularly on the lower side. Above and below the central object are small portions of two other spores appearing as mottled alveolar structures. Figure 15, Plate 3, shows a complete tetrad of the spores under consideration in an oblique plane of section, which partially passes through the tetrad and partially reveals its free surface. Of three spores appearing in the section through the tetrad, two show at least part of their rough outer surface, while in case of the third none of this is apparent.

It will probably be apparent to the unprejudiced reader, from the figures of free spores and tetrads of spores in various planes of section already shown, that the supposed Alga, *Pila kentuckyana* of Renault, as figured in Figure 4, Plate 23, of his monograph, in reality represents sections through the rough alveolar surface of the reproductive bodies of vascular cryptogams. Although the conclusion thus indicated seems clear from the examination of reasonably numerous and well-prepared sections of coal containing the species under discussion, I have not allowed such evidence alone to suffice. As explained in the introduction, numerous serial sections were cut, in which it was possible to follow the appearance of the imagined Algae as well as the obvious spores as seen in successive planes. On account of the compression of these objects, resulting from age-long pressure, coöperating with the softening effect of gradual decay, they are so thin that they rarely appear in more than two thin horizontal sections, more rarely in three. By the study of serial sections it was perfectly clear that the supposed Alga, *Pila kentuckyana* of Renault, and the obvious spores of vascular cryptogams, found in the same sections of coal, were different appearances presented by the same object. When only the free surfaces of the spores appear in the plane of section, as is more often the case, then their roughened exterior presents a mottled or alveolar appearance, which has been interpreted by a number of European observers in this and parallel cases as representing the bodies of gelatinous colonial Algae. When the angular or rotund aspects of the spores appear in section, their real nature is perfectly obvious. The error made by those who have been able to study only a few comparatively thick sections of the coals in question, results from not correlating the two sorts of appearances; an error scarcely to be wondered at from the nature of the preparations at their disposal.

The Structure of Scotch Boghead Coals.

The microscopic structure of the bogheads of Scotland is of particular interest because coals of this general type were earliest recognized here and the classic example Torbanite from Torbane Hill may be regarded as the original boghead, from the standpoint of scientific recognition. Through the kindness of Captain Baird G. Halberstadt, F. G. S., of Pottsville, Pennsylvania, I have had the opportunity of examining authentic material of a number of the Scotch coals of this group.

Figure 16, Plate 3, shows part of a section parallel to the planes of layering in a boghead coal, so called, from the Armadale deposits, Bathgate, Linlithgow, Scotland. The magnification, which is moderate, reveals the presence of a number of bodies of a nature similar to *Pila kentuckyana*, described in previous paragraphs. Like *Pila kentuckyana*, they have an alveolar mottled appearance in certain planes of section, while in others they reveal a central cavity circular or angular in contour, as the case may be, and finally in some cases the bodies in question are grouped together in obvious tetrads or reveal the triradiate face characteristic of most Lycopodineous spores. All these conditions are revealed in the various parts of Figure 16, as may be ascertained by the use of a hand lens. Near the middle line of the figure above and below may be seen tetrads in various planes of section. Two of these in the upper and lower region of the photograph are particularly striking. Figure 17, Plate 2, shows two upper tetrads somewhat highly magnified. In the lower of these the alveolar structure of the walls of the spores is specially prominent, the plane of section being particularly favorable. There seems to be no reason for doubt that here we have to do with the structures from Scotch and other cannels, designated by Renault as *Pila scotica.* It is apparently beyond question from the few illustrations of this species which it has been thought necessary to introduce that in the case of *Pila scotica* as in that of the American species *P. kentuckyana* we have to do with spores of vascular cryptogams and not with anything approaching in the remotest way colonial gelatinous Algae. By the methods used in connection with the present investigation it has been found possible to secure large numbers of extremely thin section, which in case of doubt may be made serial. The large number of sections has made it feasible to choose those in which the supposed Algae are in the condition of tetrads, and thus reveal their nature as spores of vascular plants. Had the technique adopted here been available to Renault, there seems to be little doubt that he would have escaped the error of attributing the structures, which he designated *Pila scotica*, to algal affinities.

Among the various appearances present by the sections of Armadale coal are certain much larger spores, likewise not infrequently found in tetrads, and manifesting the same alveolar structure in certain planes of section through the spore wall as the species described above. These appear to be what Renualt described as *Thylax britannicus.*[8] It has not been thought necessary to illustrate these structures, as their identity as spores of vascular cryptogams is entirely beyond question.

Figure 18, Plate 3, shows a section vertical to the plane of layering of another coal from the Bathgate region, but not from the Armadale mine, for which the writer is indebted also to Captain Halberstadt. The organisms in this coal are much more numerous and are less well preserved than in the case of the so-called Pilas described above.

Near the center of the section but a little below on the right, is seen with unusual clearness one of the structures, which compose the coal at present under discussion almost to the exclusion of the so-called fundamental substance, which makes up the mass of the bogheads hitherto described. It can be made out that the structure in question is very much flattened but that the age-long pressure and chemical change have not obliterated the original central cavity. Less clearly marked cavities surrounded by less well-preserved walls are

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[8] Op. cit., Pl. 21, figs. 1 and 2.

seen in several other instances in the section, while in the case of most of the organisms, the structure has suffered a good deal from the ravages of pressure and chemical metamorphosis. Figure 19, Plate 4, shows another section in the vertical plane of the same coal presenting somewhat similar appearances. This preparation, however, shows some of the bodies composing the coal in a darkened and chemically much modified condition, especially in the upper part of the field. Figure 20, Plate 4, represents a section through this boghead parallel to the planes of layering. Obviously some of the parts are better preserved than are others. In the well-preserved regions, which are distinguishable in figure by their lighter color, there is the same mottled appearance, which is characteristic of tangential sections of the spores known in the investigations of Renault as *Pila kentuckyana* and *P. scotica.* In the darker parts of the figure, the characteristic organisms of the coal have largely lost their structure and the accompanying swelling has almost completely obliterated the surface sculpture. Figure 21 illustrates another horizontal section of the coal under discussion, somewhat more highly magnified. On the upper left hand can be seen one of the constituting organisms of the coal in plane of section showing at once the profile and the face view of its wall. The profile view is the one sharply focussed and it presents all the appearance of a section through the wall of a macrospore of one of the existing Lycopodiales. In view of what has been learned regarding the structure of the American and Scotch species of Pila in the foregoing paragraphs, it can scarcely be doubted that the micro-organisms of the coal at present under discussion are also of the nature of spores of vascular cryptogams. The general arguments against the algal nature of these and similar bodies may, however, profitably be deferred to the end of the article.

Figure 22, Plate 4, shows a section parallel to the planes of stratification of the classic Scotch boghead, known as Torbanite. I owe this material as well as that of the other Scotch coals examined to the kindness of Captain Halberstadt. In this case the micro-organisms are in the condition of disorganization, which is generally found in the French bituminous schists to be discussed below. Certain faint lines are the only indication of structure presented by the light-colored bodies appearing in the microscopic field represented by the figure.

The Boghead of Autun.

Figure 23, Plate 4, shows the horizontal view of one of the micro-organisms of the boghead of Autun, first studied by Renault and Bertrand. The structure is almost obliterated by swelling and chemical metamorphosis, and only faint lines of alveolation indicate the nature of the original organization. Through the kindness of M. Bayle, Director of the Compagnie Lyonnaise des Schistes Bitumineux, I have received an abundant supply of the boghead from Autun, containing the organism described by Renault and Bertrand under the name of *Pila bibractensis*. These samples came both from the beds of Margenne and Thélots. It is an unfortunate circumstance, which beyond question has had a bearing on the views as to the nature of boghead coals, that the first of these to be minutely studied, *viz.*, that of Autun, is characterized by structural elements, which are obviously in an extremely bad condition of preservation. In none of the material which has passed under my observation have I found the component structures well organized. This seems to have been the condition of the material studied by Renault, to judge from the figures published in the atlas accompanying his work cited above. Some of the vertical sections published by Bertrand, however, present a better condition of preservation. This is notably the case in the reproduction of one of Bertrand's figures in Potonie's work on coal cited at the beginning of this article.[9] The accompanying horizontal aspect of the coal, however, presents the usual bad condition of preservation. Figure 24, Plate 4, shows a somewhat highly magnified vertical section of the boghead from Autun. In the lowermost of the organisms there is some indication of the presence of a central cavity. The walls of the structures in question are all in a swollen condition. Figure 25, Plate 5, shows a number of the individuals of Renault's *Pila bibractensis* under a low magnification. It may be stated in general of the boghead of Autun, that it is largely composed of organisms, which are in a disintegrated and swollen condition and which are consequently hard to interpret. It seems particularly unfortunate that this boghead was the first to be carefully studied microscopically.

Oil-Shale of New South Wales.

Figure 26, Plate 5, illustrates the structure of the oil-shale of New South Wales in section vertical to the plane of layering. This boghead, like those of Torbane Hill and Balbardie, Scotland, is almost completely made up of the organisms interpreted by Renault as Algae and named *Reinschia australis*. The organisms are generally not completely flattened in this plane of section and are often very much distorted and folded. The usual absence of complete flattening of the micro-organisms in bogheads composed very largely or almost entirely

[9] Potonie, op. cit., p. 23, fig. 11b.

of the disputed structures, together with the often convoluted and folded condition of these under the same condition, constitutes a strong argument against the algal nature of the latter, as it is hardly conceivable that gelatinous Algae in mass, even if preserved by a bituminous antiseptic, should have escaped complete collapse under the enormous and age-long pressure to which they have been subjected. Even so resistant a substance as wood ultimately collapses completely under pressure where it occurs as lignite. Figure 27, Plate 5, illustrates the appearance of the boghead under consideration, as viewed under a considerable magnification. The horizontal middle line of the figure is occupied by part of a single convoluted individual of *Reinschia australis*. It is a noteworthy fact that the organisms of the later bogheads, such as those of Autun and New South Wales, are very much larger in size than those found in the true Carboniferous coals of the same general structure. This contrast is illustrated clearly by a comparison of *Pila kentuckyana* or *Pila scotica* with *Pila bibractensis* or *Reinschia australis*, the two latter species being many times the dimensions of those mentioned first. Figure 28, Plate 5, shows the structure of a horizontal section of the boghead of New South Wales as viewed under a low magnification. A little above the horizontal middle line of the figure, two of the micro-organisms stand out with particular clearness. Figure 29, Plate 5, shows one of these considerably more highly magnified. The plane of section reveals both the profile and face view of the organism, showing the alveolar structure of the wall, found in all moderately well-preserved individuals. In those which have lost the usual yellowish hue and have turned some shade of brown, the structure has usually more or less completely disappeared.

The Structure of Coking Coals.

It is a well-known fact that certain coals are well adapted for coking purposes. In coals of this type when high temperatures are reached in the coking oven, the coal melts and is transformed under appropriate conditions into the substance known as coke, which is of course virtually a mineral charcoal. The property of melting, when subjected to heat, is one which is likewise characteristic of cannel and boghead coals. It has been suggested in the case of the last that the gelosic substance presented by the colonial framework of their supposed algal components is responsible for the readiness of fusion at high temperatures. Such an explanation is scarcely apposite in the case of the similarly fusible cannel coals. Even if we accept the point of view of Potonie and include the cannel coals likewise under the heading of algal or sapropelic coals, there appears to be considerable difficulty in accounting for the fact that the supposed algal constituents in true cannel coals are either insignificant in proportion to the unquestionable spores or are entirely absent. With a view to putting the validity of the algal hypothesis of the origin of fusible coals to the test, I have examined a number of coking coals microscopically, by the same general methods as those used in the case of cannels and bogheads. It is not my intention at the present time to attempt to describe the composition of coking coals as seen under the microscope, but certain facts, however, may be appropriately referred to. In general coking coals consist of dull and bright layers, which vary as to their relative thickness and general distribution. It has been found in those cases examined that the dull portions of coking coals represent wood in a more or less modified, but still clearly recognizable condition, while the bright parts of such coals are composed of wood in a high degree of modification and disintegration. Figure 30, Plate 5, shows a horizontal section through the layering of the dull region of a coking coal, known commercially as "No. I. Pennsylvania Coking." In the middle vertical line of the figure may be seen the end of a wood tracheid, showing unquestionable bordered pits. I have found similar appearances in the dull layers of other coking coals, notably Pocahontas coal. The bright parts of coking coal present the same composition with a much greater modification, both structural and chemical, of the wood elements. It is thus apparent that there is no necessary relation between the fusibility of coal and the presence of organisms of an algal nature, since cannel coal, which has few or none of the organisms, considered to be Algae, and coking coal, which is made up entirely of the remains of wood, both are fusible coals. It should be added that the structures appearing to the right and left of the vertical middle line are likewise wood elements, although they are not clearly recognizable as such in figure 30, Plate 5.

Conclusions.

It is appropriate, after the description of the organization of boghead coals from various parts of the world and from different levels of the Paleozoic given in the foregoing paragraphs, to discuss the algal hypothesis of the origin of these coals. It may be pointed out that there is unanimity among the various observers as to the common and similar organization of the structures present in these coals. It follows that if the best preserved ones and those which by reason of their size are most easily studied in thin sections of coal, turn out not to be of algal affinities, that a similar conclusion must be applied to the remaining organisms, which either by reason of their large size or imperfect condition of preservation cannot be so satisfactorily subjected to microscopic investigation. It is apparently beyond question that both *Pila kentuckyana* and *Pila scotica* represent the spores of vascular cryptogams. This conclusion has been reached from the study of the numerous thin sections which are readily prepared by the methods employed in the present investigation. Abundance of material makes it clear that the bodies which have been interpreted as colonial gelatinous Algae, in reality represent certain planes of section through the rough-coated spores of vascular cryptogams. When the structures in question are cut in favorable planes and of sufficient thinness, it becomes clear that the algal semblances represent tangential sections of the rough external surface of spores of vascular plants. The real nature of the supposed Algae is further made clear by their possession of the triradiate ridge characteristic of tetrahedral spores. Moreover, in certain instances the plane of section has been observed to pass at once through the plane anterior faces and the rough, rounded external one. Further, in serial sections, which may be prepared by the methods described above, the putative Algae may be seen to present at once the alveolar appearance which has been interpreted as indicating algal affinities and the form and triradiate ridges, which clearly indicate their identity as spores of vascular cryptogams. If any further evidence were needed as to their true nature, it would be furnished by their occasional occurrence in actual tetrads, a condition which, in connection with the other data derived from the study of thin sections, makes it impossible to regard them as anything else but spores.

In the case of the larger supposed Algae the case is not so clear, on account of the distorted and often swollen condition in which they occur, as well as by reason of the difficulty of interpreting objects of greater dimensions by means of thin sections. The writers who have studied the various supposed algal structures of Paleozoic coals are, however, agreed in the conclusion that they all belong to the same category. I am myself entirely in accord with this opinion. The view of the uniformity of the organisms under discussion is further strengthened by the fact that gradations in the condition of preservation occur in the case of the larger Pilas and of Reinschia, which show clearly in the condition of best preservation the greatest resemblance to *Pila kentuckyana* and *Pila scotica*. Unfortunately it was such badly preserved species as *Pila bibractensis* and *Reinschia australis* which were first studied microscopically.

Another very important argument against the algal character of the characteristic structural constituents of so-called boghead coals is afforded by the extreme hypotheses which this interpretation demands. For example, M. Bertrand in his work cited above [10] states that the oil-shale of New South Wales, forming a layer fifteen feet in thickness, composed practically entirely of the organisms known as *Reinschia australis*, must have been laid down a single season during a period of low water. He makes similar statements in regard to the thick bituminous deposits of Autun. It is inconceivable that such a huge mass of algal matter, which in its fresh condition must have been enormously greater in volume, should have been accumulated and synchronously preserved in so short a time. The problem of the preservation of this great amount of gelosic substance is not rendered easier by the supposition that the antiseptic was bituminous in its character and consequently must have been poorly soluble or quite insoluble in water. It is easy to imagine the preservation of logs and even of the harder parts of animals in asphaltic lakes, such as have been found in California, South America and certain of the West Indian Islands; but the best developed scientific imagination would find it difficult to picture enormous masses of gelatinous matter, impregnated rapidly and completely by preservatives of a bituminous nature. It moreover seems clear that the intervention of bituminous matter in the process of the formation of such coals from gelatinous Algae is absolutely essential, for as Bertrand has pointed out, the contraction in the organisms constituting boghead coals although great (from 1/7 to 1/24 in volume), would only correspond to a proportion of gelatinous substance from 105 to 360 *per mille;* whereas the amount of gelatin in ordinary dry commercial gelatin is from 700 to 800 *per mille.* He concluded that the deficiency must be made up by the infiltrated bituminous matter.

Further, even if it be granted that bituminous matter actually enters into the transformation of gelatinous Algae into boghead coals as the hypothesis originated and elaborated by Renault, Bertrand and Potonie demands, the question of the origin of this substance in connection with large accumulations of Algae in widely separated parts of the world and at remote geological epochs constitutes a very grave difficulty. It has been variously suggested that the bituminous substance originates from the remains of animals or from a transformation of a part of the algal substance itself into bituminous matter. In view of the relatively insignificant amount of animal matter compared with vegetable matter on the surface of the world at the present time we are scarcely justified in drawing the inference that the remains of animals

[10] Charbons Humiques et Charbons de Purins, p. 2.

gave rise to bitumen, especially as there is every reason to believe that the disproportion between animal and vegetable matter must have been very much greater in the Paleozoic than in the present epoch. If the Algae themselves gave rise to the bituminous matter, we would expect to find them locally more or less completely transformed into this substance. No cases of this kind exist so far as I am aware.

Another strong argument against the algal hypothesis of the origin of boghead coals is the fact that cannel coals, which are practically identical with them in chemical composition, are recognized to be composed predominantly of the spores of vascular cryptogams. Potonie[11] recognizes this identity of origin of cannel and boghead coals, since he states that they are both "sapropelic" in their nature, that is, they were both laid down in open quiet water and are both bituminous in their chemical composition. It is not open to doubt that the bituminous character of cannel coal is mainly, if not entirely, due to the enormous quantities of the remains of resinous spores of vascular cryptogams which it contains. Boghead coals are like cannel coals in their chemical composition, differing only in the more richly bituminous characteristics, which they present. As has been pointed out in the foregoing paragraphs, they are likewise notable for the greater proportion of substance showing structure under the microscope. There appears in fact to be a definite relation between the amount of structural elements and the proportion of bituminous matter found in such coals. It appears clear from the description of the micro-organisms of boghead coals, especially such of these as show them in a comparatively good condition of preservation, that the bodies in question represent the spores (in most cases apparently the macrospores) of vascular cryptogams. The greater concentration of the bituminous substance in so-called boghead coals, moreover, is related to a much scantier occurrence of such coals, which, as compared with cannels, occur in beds of very restricted area.

Apparently as a result of all the considerations brought forward in the statement of the conclusions drawn from the present investigation, we must regard the so-called boghead coals as essentially composed of the remains of spores of vascular cryptogams and thus as closely resembling cannels, which they in general differ from, only in the greater concentration and larger size of the constituent spores. The less abundant occurrence and more purely sporal composition of boghead coals is doubtless to be attributed to the nature of the component

[11] Entstehung d. Steinkohle u. Verwandter Bildungen, Einschliesslich des Petroleums, p. 20.

spores, which in contrast to those occurring in cannels are predominantly of large size, and are as a consequence in all probability to be regarded as macrospores. The tendency of water action, under which it is universally agreed both cannels and bogheads were laid down, would be to bring about a greater degree of concentration of the larger and heavier bodies, the macrospores.

At the present time the algal hypothesis is applied not only to the origin of boghead coals, but is extended also to the question of the origin of petroleum. If we assume that those bodies, which afford on distillation the greatest amount of petroleum, are as it were the mother substance of petroleum, the conclusion cannot be avoided that boghead coals and similar substances are the source of petroleum-like compounds. As has been pointed out above, we cannot with a due consideration of the microscopic structure of boghead coals regard them as composed of remains of Algae. The algal hypothesis of the origin of petroleum consequently, so far as it rests on the structural components of bogheads, falls to the ground. It is further invalidated by the relatively small quantities of boghead coals found throughout the world. The result of the present investigations is to show that bogheads are essentially similar in their composition to the much more abundant cannels. Consequently we are able to draw not only on the structure of the relatively small amounts of boghead coals for a hypothesis as to the origin of petroleum, but also upon the relatively abundant cannels, which are widely distributed in the Northern Hemisphere, where petroleum deposits are likewise abundant. The conclusion appears obvious that the innumerable spores of Paleozoic Pteridophyta laid down in enormous quantities on the bottoms of the shallow lakes or lagoons, in which the Coal Period proper abounded, have furnished the raw material from which in the course of countless years, as a result of great pressure and perhaps of high temperature as well, the enormously valuable petroleum products have been elaborated.

The discussion of the possibility of the formation of algal peats under modern conditions has not been entered upon in the present article because that subject will be considered in a subsequent communication.

Summary.

1. The organisms found in abundance in boghead coals are not of the nature of colonial gelatinous Algae, as has been asserted by Renault, Bertrand and Potonie on the basis of the examination of a small number of insufficiently thin sections of such coals.

2. The bodies in question, as revealed in thin serial sections, made

by improved technique on the microtome, are spores of vascular cryptogams.

3. The proof that the constituent micro-organisms of boghead coals are not Algae but spores, overthrows the algal hypothesis of the origin of petroleum and similar substances.

4. It appears clear that petroleum products have been derived, mainly at any rate, from the waxy and resinous spores of vascular cryptogams laid down on the bottoms of the shallow lakes of the Coal Period. These lacustrine layers, either as cannels, bogheads or bituminous shales, according to the sporal composition and the admixture of earthy matter, are the mother substance of petroleum. Pressure and temperature either separately or combined, in the presence of permeable strata, have brought about the distillation of petroleum from such deposits.

EXPLANATION OF THE PLATES.

PLATE 1.

FIGURE 1. Vertical section of Kentucky cannel coal. (× 180.)

FIGURE 2. Horizontal section of Kentucky cannel coal. (× 180.)

FIGURE 3. Horizontal section of Kentucky boghead coal, showing the supposed colonial Alga, *Pila kentuckyana*. (× 180.)

FIGURE 4. Horizontal section of another preparation of the same. (× 40.)

FIGURE 5. Part of Figure 4 more highly magnified. (× 120.)

FIGURE 6. The same showing another part. (× 120.)

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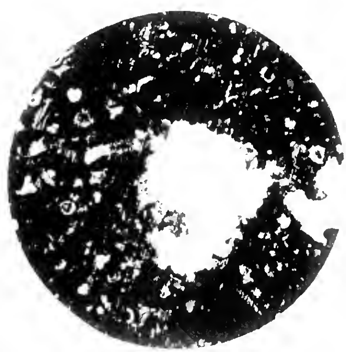

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PLATE 2.

Figure 7. Part of the same. (× 180.)

Figure 8. Part of another horizontal section showing *Pila kentuckyana.* (× 120.)

Figure 9. More highly magnified view of the same showing a tetrad of spores. (× 180.)

Figure 10. Another horizontal section of the same showing spores and the supposed Algæ. (× 180.)

Figure 11. Another of the same showing a very characteristic spore and a considerable number of the supposed Algæ. (× 180.)

Figure 12. Another preparation of the same, showing a spore-like body and fragments of numerous supposititious Algæ. (× 180.)

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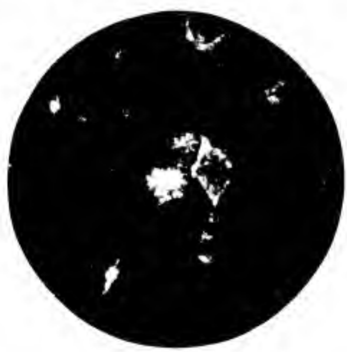

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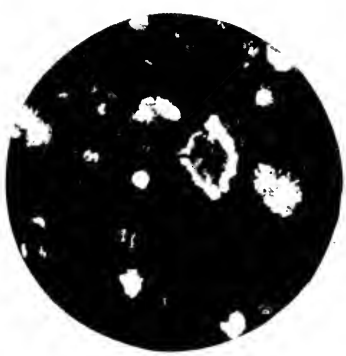

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PLATE 3.

FIGURE 13. Another of the same showing one spore with triradiate ridge and other spores and supposed Algæ. (× 180.)

FIGURE 14. A spore of the same highly magnified. (× 500.)

FIGURE 15. A tetrad of the same. (× 180.)

FIGURE 16. Horizontal section showing numerous examples of *Pila scotica*, some in tetrads. (× 130.)

FIGURE 17. Upper portion of Figure 16 more highly magnified. (× 180.)

FIGURE 18. Vertical section of boghead from Balbardie, Scotland. (× 120.)

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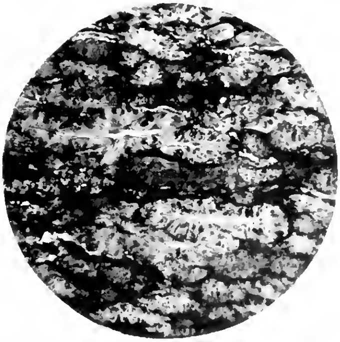

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PLATE 4.

FIGURE 19. Another of the same. (× 120.)

FIGURE 20. Horizontal section of the same. (× 120.)

FIGURE 21. Horizontal section of the same. (× 180.)

FIGURE 22. Horizontal section of Torbanite. (× 120.)

FIGURE 23. Horizontal section of boghead from Autun. (× 120.)

FIGURE 24. Vertical section of boghead from Autun. (× 120.)

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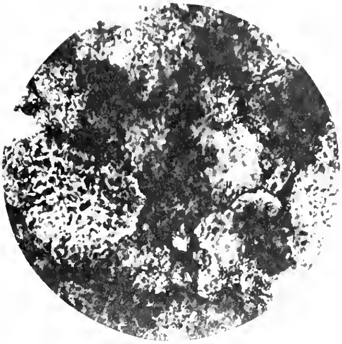

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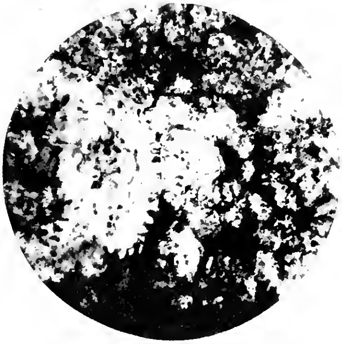

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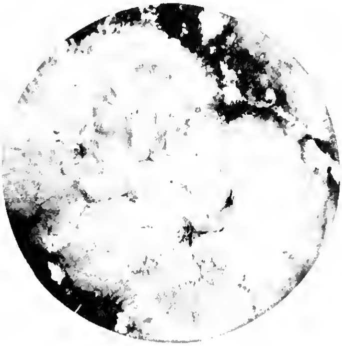

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PLATE 5.

FIGURE 25. Horizontal section of boghead from Autun. (× 80.)

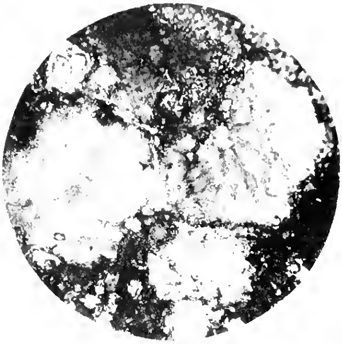
FIGURE 26. — Vertical section of oil-shale from New South Wales, showing masses of *Reinschia australis*. (× 40.)

FIGURE 26. Part of the same more highly magnified. (× 180.)

FIGURE 28. Horizontal section of oil-shale from New South Wales. (× 40.)

FIGURE 29. Part of the same. (× 120.)

FIGURE 30. Horizontal section of Pennsylvania coking coal showing presence of wood elements. (× 180.)



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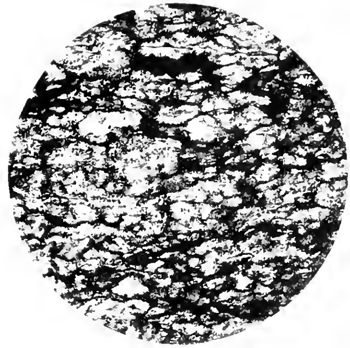

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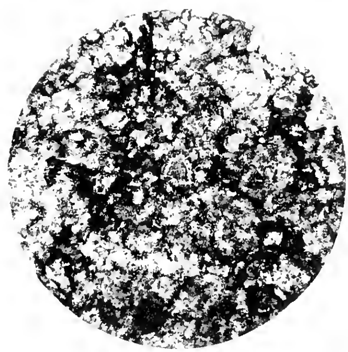

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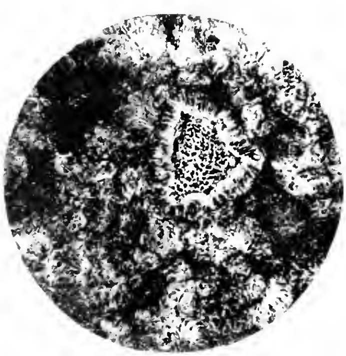

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THEORY OF COUPLED CIRCUITS, UNDER THE ACTION OF AN IMPRESSED ELECTROMOTIVE FORCE, WITH APPLICATIONS TO RADIOTELEGRAPHY.

By George W. Pierce.

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THEORY OF COUPLED CIRCUITS, UNDER THE ACTION OF AN IMPRESSED ELECTROMOTIVE FORCE, WITH APPLICATIONS TO RADIOTELEGRAPHY.

By George W. Pierce.

Presented October 12, 1910. Received October 20, 1910.

I. Introduction.

Persistent, or sustained, electric oscillations have recently come into extensive use in wireless telegraphy. With these oscillations, which are produced continuously while the transmitting key is depressed, tens of thousands of waves arrive at the receiving station during even the production of a single dot of the Morse code. This permits the establishment of a practically steady state at the receiving station, so that by the use of these persistent oscillations the mathematical treatment of the problem of the resonance conditions at the receiving station reduces to a problem of forced vibration.

The exact solution for the radiotelegraphic circuits, however, still presents considerable difficulty on account of the effect of the distributed capacity of the antennae. An approximation to a solution of the practical problem can be obtained by supposing that the antenna of the receiving station of the practical case can be replaced by a localized capacity so that the circuits become those represented in Figure 1. While this simplified system is a considerable departure from the actual practical circuits, calculations made from the simplified circuits seem nevertheless to be of importance, because the resonance in the simplified system is sharper than in the actual circuits, and the simplified computations thus afford a means of assigning certain theoretical limits to practical attainments.

It is the purpose of the present communication to give a solution of the equations representing the flow of electricity in a system of circuits of the form of Figure 1, under the action of a sinusoidal impressed electromotive force at e, and to make from this solution deductions in regard to the wireless telegraphic receiving station. It will not be necessary to neglect the resistances of the system, and, in fact, the influence of the resistances upon the resonance relations and upon the resultant current is the most interesting part of the investigation. The results of the mathematical treatment are illustrated by numerical examples.

II. An Experiment on Received Current.

In pursuing the mathematical development I have received aid from an examination of some experimental data previously published, and I

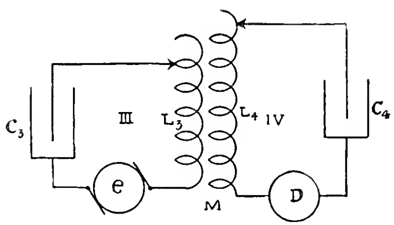

Figure 1. Diagram of coupled circuits with impressed e. m. f.

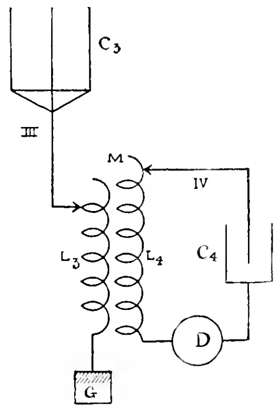

Figure 2. Electro-magnetically coupled wireless telegraph receiving station.

take the liberty of presenting one set of these experimental results [1] as an introduction to the theoretical investigation. These experimental data were obtained five years ago with a spark-discharge method of excitation instead of with a persistent source of waves, so that the experimental case is to be regarded in this connection merely as an aid to a concrete statement of the problem under consideration.

The experiments were made with a receiving station of the electromagnetically connected type, like that shown in Figure 2. With electric waves of fixed period arriving from a distant sending station the receiving station of Figure 2 was attuned by adjustment of the condenser C_4 which is in the side-circuit. Readings of the received current in the side-circuit were made by the use of a low-resistance high-frequency

[1] G. W. Pierce, Physical Review, **20**, 220, 1905.

dynamometer D. The inductances L_3 and L_4 and their mutual inductance were kept constant. Since, however, a single variation at the receiving circuit — namely, the variation of the condenser C_4 — is not sufficient to disclose the resonance conditions at the receiving station, the length of the receiving antenna was also given various values. The results are plotted in the curves of Figure 3, and were obtained as follows:

With a given length (23.8 meters) of the four-wire receiving antenna the capacity C_4 was set at a particular value and the deflection of the

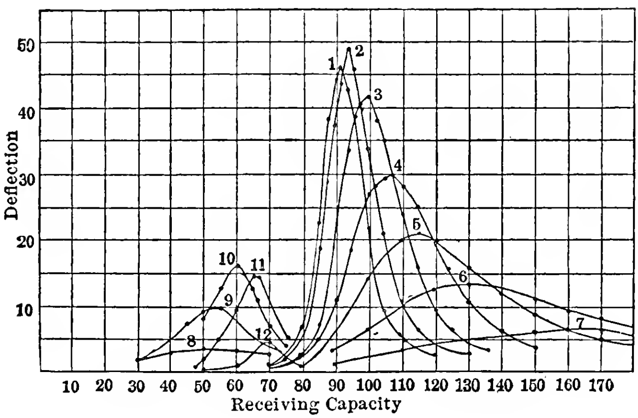

FIGURE 3. Family of experimental curves obtained with various heights of antenna (each height giving one of the curves), and various values of receiving capacity C_4.

dynamometer D was read. The capacity C_4 was then set at another value and the deflection of the dynamometer was read. Thus keeping the length of antenna at 23.8 meters, I took the readings of the dynamometer for a whole series of values of C_4. The results are shown in Curve 1 of Figure 3, in which the deflections of the dynamometer (current square) are plotted against values of "receiving capacity" C_4 (in arbitrary units). We shall call this kind of a curve a "resonance curve." When this resonance curve (Curve 1) was completed, the antenna was shortened to 20.8 meters and a second resonance curve, Curve 2, Figure 3, was taken. Again shortening the antenna successively to 17.8, 15.8, 14.8, 13.8, and 12.8 meters, I obtained the curves 3, 4, 5, 6, and 7 respectively of Figure 3.

From this it is seen that for each particular capacity of the antenna, varied by varying the length, there is a characteristic capacity C_4 that gives a maximum of current. We shall call the condenser capacity that gives a maximum of current the "resonant C_4" for a given C_3. In seeking an expression for the condition for resonance, it is to be noticed that, as C_3 has been diminished, the value of the "receiving capacity" C_4 required for resonance has been increased (compare

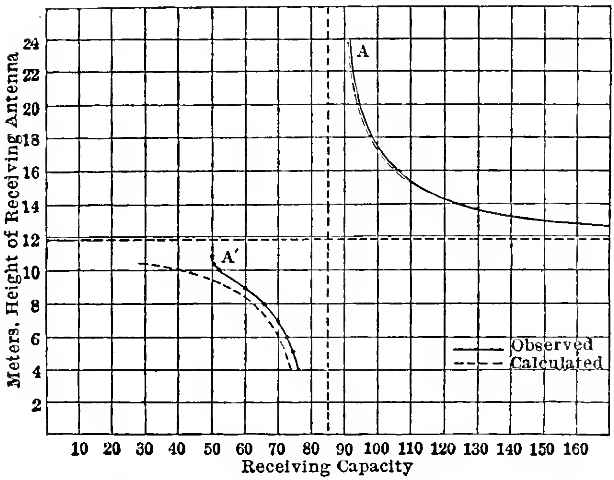

FIGURE 4. Relation of height of Antenna to C_4 for resonance.

Curves 1 to 7). On passing to a smaller length of antenna, in the neighborhood of 11.8 meters, the value of C_4 for resonance became very great and the amount of current received became inappreciably small. With this length of antenna, 11.8 meters, the condition of the receiving circuit was at its worst, for upon further decreasing the length of antenna to 10.5 meters resonance reappeared in the form of Curve 8. The resonance relation had undergone a discontinuity and the capacity C_4 for resonance had jumped back toward the origin. Now, decreasing the length of the antenna further to 10, 9, 8, and 7 meters successively, I obtained the resonance curves 9, 10, 11, and 12 respectively[2] of Figure 3.

[2] Curves 9 to 12 are plotted magnified about five times in comparison with Curves 1 to 8. In examining these curves it should be borne in mind that the decrease of length or height of the receiving antenna carries with it a double effect; namely, (1) a decrease of *reach* of the antenna into the field of force, and (2) a shift of the resonance relations of the receiving station with respect to the incident waves. We are at present concerned only with the second of these effects.

The relation between the height of antenna, H, and the capacity C_4 required for resonance was found to be approximately represented by the empirical equation:

$$(1) \qquad (H - 11.8)(C_4 - 84.6) = 88,$$

which is the equation of an equilateral hyperbola with axes at $H = 11.8$ and $C_4 = 84.6$. The nature of the agreement between the observed and the calculated values is shown in Figure 4. Evidently the relation expressed in equation (1), though an interesting approximation, is not exact.

III. Theoretical Treatment.

Let us now turn from the experiment to the theory of the oscillation. The problem undertaken is the investigation of the relative current in the detector circuit (Circuit IV of Figure 1) for various adjustments of the constants of Circuit III and Circuit IV. In the theoretical treatment this carries with it (1) a determination of the adjustments that must be made to obtain resonance, (2) a determination of the adjustment for best resonance, (3) a determination of the effect of the resistances on the resonance relations and on the amount of current receivable, (4) a discussion of the resistance that a detector must have for greatest sensitiveness, (5) a computation of the amount of disturbing current that will be obtained from an undesired source of waves, and (6) a quantitative judgment as to the sharpest selectivity that can be attained by circuits of the form of Figure 2.

As stated in the Introduction, in treating these general questions it has been found necessary to depart from strict observance of the actual practical wireless-telegraphic conditions and to assume the capacity of the antenna, which is a distributed capacity in practical wireless telegraphy, to be replaceable by a localized capacity C_3. This modification of the problem will not completely destroy the validity of the discussion, because the simplified problem enables us to derive certain important conclusions in regard to the problem with the less simple conditions.

Referring to the localized-capacity circuits of Figure 1, and supposing an electromotive force $E \cos \omega t$ at e, the differential equations of the current in the two circuits are

$$(2) \qquad L_3 \frac{\partial x}{\partial t} + M \frac{\partial y}{\partial t} + R_3 x + \frac{1}{C_3} \int x dt = E \cos \omega t,$$

$$(3) \qquad L_4 \frac{\partial y}{\partial t} + M \frac{\partial x}{\partial t} + R_4 y + \frac{1}{C_4} \int y dt = 0,$$

in which x and y are the values of the current in the Circuits III and IV respectively. Eliminating x between these equations we have

$$(4)\quad (L_3L_4 - M^2)\frac{\partial^4 y}{\partial t^4} + (R_4L_3 + R_3L_4)\frac{\partial^3 y}{\partial t^3} + \left(\frac{L_3}{C_4} + \frac{L_4}{C_3} + R_3R_4\right)\frac{\partial^2 y}{\partial t^2} + \left(\frac{R_3}{C_4} + \frac{R_4}{C_3}\right)\frac{\partial y}{\partial t} + \frac{y}{C_3C_4} = - EM\omega^3 \sin \omega t.$$

If for brevity we call the left-hand member of equation (4) $f(y)$, the complete solution of (4) is any particular solution of (4) plus the general solution of

$$(5)\qquad f(y) = 0.$$

Now the general solution of (5) involves exponentials with negative exponents as multipliers, and becomes zero after a few oscillations, so that all we need for the current y in case a large number of oscillations are performed, as with a persistent source of waves, is the "steady-state" solution for equation (4).

In order to get the steady-state solution of (4) let us write, in the place of (4), the equation

$$(6)\quad (L_3L_4 - M^2)\frac{\partial^4 y}{\partial t^4} + (R_3L_4 + R_4L_3)\frac{\partial^3 y}{\partial t^3} + \left(\frac{L_3}{C_4} + \frac{L_4}{C_3} + R_3R_4\right)\frac{\partial^2 y}{\partial t^2} + \left(\frac{R_3}{C_4} + \frac{R_4}{C_3}\right)\frac{\partial y}{\partial t} + \frac{y}{C_3C_4} = - EM\omega^3 e^{i\omega t},$$

which is (4) with $\sin \omega t$ replaced by an appropriate exponential with imaginary exponent.

Our required solution for (4) can then be obtained from a solution of (6) by getting y from (6), rationalizing the result, taking the imaginary part and dividing by i.

Now a particular solution of (6) is seen to be of the form of

$$(7)\qquad y = Ye^{i\omega t},$$

in which Y is to be determined by substitution of (7) in equation (6).

Making this substitution we obtain

$$(8)\qquad AY\omega^4 - BiY\omega^3 - CY\omega^2 + DiY\omega + FY = - EM\omega^3,$$

where A, B, C, D, and F are the coefficients of equation (6). Whence

$$(9)\qquad Y = \frac{- EM\omega^3}{(A\omega^4 - C\omega^2 + F) - (B\omega^3 - D\omega)\,i},$$

or writing the real part of the denominator as P, the imaginary as Qi,

$$(10)\qquad Y = \frac{-EM\omega^3}{P - Qi};$$

whence from equation (7),

$$(11)\qquad y = \frac{-EM\omega^3 e^{i\omega t}}{P - Qi},$$

$$= \frac{-EM\omega^3(\cos\omega t + i\sin\omega t)}{P - Qi}.$$

Rationalizing equation (11), taking only the imaginary part and dividing by i, we have

$$y = \frac{-EM\omega^3(P\sin\omega t + Q\cos\omega t)}{P^2 + Q^2},$$

$$(12)\qquad = \frac{-EM\omega^3\sin\left(\omega t + \tan^{-1}\frac{Q}{P}\right)}{\sqrt{P^2 + Q^2}}.$$

Replacing P and Q in the denominator by their values in terms of the constants of the circuits, we have

$$(13)\quad y = \frac{-EM\omega^3\sin\left(\omega t + \tan^{-1}\frac{Q}{P}\right)}{\sqrt{\left\{(L_3L_4-M^2)\omega^4-\left(\frac{L_3}{C_4}+\frac{L_4}{C_3}+R_3R_4\right)\omega^2+\frac{1}{C_3C_4}\right\}^2+\left\{(R_4L_3+R_3L_4)\omega^3-\left(\frac{R_4}{C_3}+\frac{R_3}{C_4}\right)\omega\right\}^2}}$$

Equation (13) is a well-known solution of equation (4), and gives the value of the current y in Circuit IV after the effect of the free-period initial disturbance has subsided. It is seen that the current y in Circuit IV is sinusoidal, with the frequency of the e.m.f. impressed on Circuit III. We shall concern ourselves only with the absolute value of the amplitude Y of this current.

Dividing numerator and denominator of equation (13) by ω^2 and factoring, we obtain

$$(14)\quad Y = \frac{EM\omega}{\sqrt{\left\{\left(L_3\omega-\frac{1}{C_3\omega}\right)\left(L_4\omega-\frac{1}{C_4\omega}\right)-M^2\omega^2-R_3R_4\right\}^2+\left\{R_4\left(L_3\omega-\frac{1}{C_3\omega}\right)+R_3\left(L_4\omega-\frac{1}{C_4\omega}\right)\right\}^2}}$$

Now let

$$(15)\qquad U = L_3\omega - \frac{1}{C_3\omega},\qquad V = L_4\omega - \frac{1}{C_4\omega}.$$

Then equation (14) may be written

$$(16) \qquad Y = \frac{EM\omega}{\sqrt{(UV - M^2\omega^2 - R_3R_4)^2 + (R_4U + R_3V)^2}}.$$

This is another form of the expression for the current in Circuit IV, after the effect of the free-period initial disturbance has subsided. From equation (16) it can be seen that L_3 and C_3 enter in the equation only in the form U; L_4 and C_4 enter only in the form V.

In seeking the dependence of Y on the constants of the circuits let us now follow mathematically the steps taken in the experiment above described.

First, we shall take a fixed value of U and determine what value of V makes Y a maximum. This value of V that makes Y a maximum we shall call the "resonant value of V." The resonant value of V is obtained by making

$$(17) \qquad \frac{\partial Y}{\partial V} = 0.$$

This gives

$$(18) \qquad (UV - M^2\omega^2 - R_3R_4)\,U + (R_4U + R_3V)\,R_3 = 0$$

or

$$(19) \qquad V = \frac{M^2\omega^2 U}{U^2 + R_3^2}.$$

Equation (19) gives the value of V for resonance with any particular given value of U. This relation will be further examined in a later section.

By the use of equation (19), when we have a given fixed value of U we can calculate the value of V for resonance. The current in Circuit IV for this resonant value of V may be obtained by substituting the value of V given by equation (19) into the equation for Y (equation (16)). If we designate this resonant value of Y by Y_{max}, we have

$$Y_{max} = \frac{EM\omega}{\sqrt{\left(\frac{M^2\omega^2U^2}{U^2 + R_3^2} - M^2\omega^2 - R_3R_4\right)^2 + \left(R_4U + \frac{R_3M^2\omega^2U}{U^2 + R_3^2}\right)^2}}$$

$$= \frac{EM\omega}{\sqrt{\left(\frac{R_3^2M^2\omega^2}{U^2 + R_3^2} + R_3R_4\right)^2 + \left(\frac{R_3M^2\omega^2U}{U^2 + R_3^2} + R_4U\right)^2}}$$

Factoring the denominator of this expression, we have

$$(20) \qquad Y_{max} = \frac{EM\omega}{\left(\frac{R_3M^2\omega^2}{U^2 + R_3^2} + R_4\right)\sqrt{R_3^2 + U^2}}.$$

For any given value of U and the necessary other constants of the circuits, equation (19) enables us to compute the resonant adjustment of Circuit IV, and equation (20) gives the amplitude of the received current at the resonant adjustment. We have thus found the best V and the best Y *for a given U*. It is proposed next to find what would be the best value to give to U, while also keeping V at its best value, and thus to determine the best possible Y, which we shall call $Y_{max\,max}$.

To obtain the best U we must apply to equation (20) the condition

$$\frac{\partial Y_{max}}{\partial U} = 0.$$

This gives

$$-\frac{R_3 M^2\omega^2 \cdot 2\,U}{(U^2 + R_3^2)^2}\sqrt{R_3^2 + U^2} + \left(\frac{R_3 M^2\omega^2}{U^2 + R_3^2} + R_4\right)\frac{U}{\sqrt{R_3^2 + U^2}} = 0,$$

or

$$- R_3 M^2\omega^2 U + R_4 U^3 + R_3^2 R_4 U = 0. \tag{20 a}$$

Whence, omitting for the present the case of $U = 0$, which is treated on page 302, we have, after dividing by U and transposing,

$$U_{opt} = \pm \sqrt{\frac{R_3}{R_4}(M^2\omega^2 - R_3 R_4)}, \tag{21}$$

in which the subscript "opt" is introduced to designate the optimum value. At the same time V must satisfy equation (19) which combined with equation (21) gives

$$V_{opt} = \pm \sqrt{\frac{R_4}{R_3}(M^2\omega^2 - R_3 R_4)}. \tag{22}$$

According to the conditions imposed by equation (19) U_{opt} and V_{opt} must either both be positive or both be negative. They cannot have opposite signs.

Equations (21) and (22) show what values to give U and V in order to obtain the largest possible current (which we shall call $Y_{max\,max}$) in Circuit IV. The value of the current, under these conditions, is found by substituting the optimum value of U, namely U_{opt} of equation (21), into the equation for Y_{max} (equation 20). When this is done, we have, after simplification,

$$Y_{max\,max} = \frac{E}{2\sqrt{R_3 R_4}}. \tag{23}$$

Equation (23) gives the max max current in Circuit IV provided equations (21) and (22) express the optimum resonance relation. Instead of (21) an alternative possible solution of equation (20_a) is

$$U_{opt} = 0.$$

Under this condition, according to equation (19)

$$V_{opt} = 0, \text{ also.}$$

This would give by equation (20)

$$(Y_{max\ max})_0 = \frac{EM\omega}{M^2\omega^2 + R_3R_4}. \tag{23 a}$$

The question arises under what conditions $(Y_{max\ max})_0$ of equation (23 a) is greater than $Y_{max\ max}$ of equation (23). The answer is seen to be that

$$(Y_{max\ max})_0 > Y_{max\ max}$$

when

$$\frac{EM\omega}{M^2\omega^2 + R_3R_4} > \frac{E}{2\sqrt{R_3R_4}};$$

that is, when

$$M\omega + \frac{R_3R_4}{M\omega} < 2\sqrt{R_3R_4}.$$

Squaring,

$$M^2\omega^2 + 2R_3R_4 + \frac{R_3^2R_4^2}{M^2\omega^2} < 4R_3R_4,$$

i. e.,

$$M^2\omega^2 - 2R_3R_4 + \frac{R_3R_4}{M^2\omega^2} < 0.$$

Extracting square root,

$$M\omega - \frac{R_3R_4}{M\omega} < 0,$$

$$M^2\omega^2 - R_3R_4 < 0. \tag{23 b}$$

In this case, the conditions (21) and (22) would give imaginary value of U_{opt} and V_{opt}.

Whence we conclude, that we are to use equations (21), (22) and (23) as solutions of the resonance problems, whenever $M^2\omega^2 > R_3R_3$. In other cases the alternative values

$$U_{opt} = 0, \quad V_{opt} = 0, \quad \text{and} \quad (Y_{max\ max})_0 = \frac{EM\omega}{M^2\omega^2 + R_3R_4}$$

are to be used.

IV. Numerical Examination of the General Resonance Relation.

The equations above derived we shall now submit to numerical examination. Let us first examine the general resonance relation as expressed in equation (19). Replacing U and V in equation (19) by

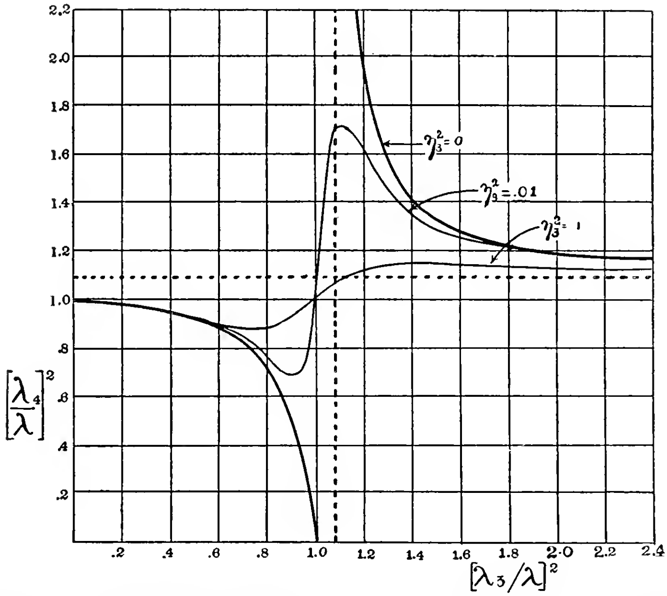

Figure 5. Theoretical relation of λ_4 to λ_3 for resonance. The different curves are for different values of damping in Circuit III. Given $\tau = .29$.

their values (15), we have the general resonance relation in the following form:

$$L_4\omega - \frac{1}{C_4\omega} = \frac{M^2\omega^2\left(L_3\omega - \dfrac{1}{C_3\omega}\right)}{\left(L_3\omega - \dfrac{1}{C_3\omega}\right)^2 + R_3^2}, \tag{24}$$

or

$$1 - \left(\frac{\omega_4}{\omega}\right)^2 = \frac{\tau^2\left\{1 - \left(\dfrac{\omega_3}{\omega}\right)^2\right\}}{\left\{1 - \left(\dfrac{\omega_3}{\omega}\right)^2\right\}^2 + \eta_3^2}, \tag{25}$$

in which

$$(26)\quad \begin{cases} \omega_4^2 = \dfrac{1}{L_4 C_4}, \\ \omega_3^2 = \dfrac{1}{L_3 C_3}, \\ \tau^2 = \dfrac{M^2}{L_3 L_4}, \\ \eta_3 = \dfrac{R_3}{L_3 \omega}, \\ \eta_4 = \dfrac{R_4}{L_4 \omega}. \end{cases}$$

TABLE I.

SHOWING RESONANT VALUES OF $(\omega/\omega_4)^2$ CORRESPONDING TO VARIOUS VALUES OF $(\omega/\omega_3)^2$. THE COEFFICIENT OF COUPLING, τ, IS ASSUMED TO BE .29.

$(\omega/\omega_3)^2$.	$(\omega/\omega_4)^2$ for.			$(\omega/\omega_3)^2$.	$(\omega/\omega_4)^2$ for.		
	$\eta_3^2 = 0.$	$\eta_3^2 = .01.$	$\eta_3^2 = .1.$		$\eta_3^2 = 0.$	$\eta_3^2 = .01.$	$\eta_3^2 = .1.$
2.50	1.16	1.17	1.12	.95	.373	.75	.96
2.00	1.20	1.19	1.13	.90	.54	.70	.93
1.66	1.27	1.25	1.15	.82	.70	.75	.89
1.43	1.39	1.33	1.15	.77	.78	.80	.88
1.25	1.72	1.51	1.13	.71	.82	.83	.88
1.17	2.27	....	....	.67	.85	.86	.89
1.11	6.25	1.72	1.08	.50	.93	.93	.93
1.05	−1.47	1.50	1.04	.30	.96	.96	.96
1.00	0.00	0.00	0.00	.25	.97	.97	.97

Some numerical calculations are given in Table I and plotted in Figure 5. Assuming $\tau = .29$, three different values of η_3^2 have been employed in calculating Table I, by the aid of equation (25) namely, $\eta_3^2 = 0$, .01, and .1, respectively.

Table I and Figure 5 give the resonant values of $\left(\dfrac{\omega}{\omega_4}\right)^2$ corresponding to different values of $\left(\frac{\omega}{\omega_3}\right)^2$. In examining the table and the figure it should be remembered that

$$(27)\quad \begin{cases} \left(\frac{\omega}{\omega_4}\right)^2 = \left(\frac{\lambda_4}{\lambda}\right)^2 = L_4C_4\omega^2, \\ \left(\frac{\omega}{\omega_3}\right)^2 = \left(\frac{\lambda_3}{\lambda}\right)^2 = L_3C_3\omega^2, \end{cases}$$

in which ω and λ are respectively the angular velocity and the wave length of the incident waves; λ_3 and λ_4 are the wave lengths corresponding to the natural period of Circuits III and IV respectively when standing alone. The abscissas of the curves of Figure 5 are values of $\left(\frac{\lambda_3}{\lambda}\right)^2$, or $\left(\frac{\omega}{\omega_3}\right)^2$, which equals $L_3C_3\omega^2$; the ordinates are values of $\left(\frac{\lambda_4}{\lambda}\right)^2$, or $\left(\frac{\omega}{\omega_4}\right)^2$, which equals $L_4C_4\omega^2$. For fixed values of L_3, L_4, and ω the abscissas and ordinates are, therefore, proportional to C_3 and C_4 respectively.

V. The General Resonance Relation (*continued*). Special Case Where $\eta_3^2 = 0$.

An examination of equation (25) shows that when $\eta_3^2 = 0$ the curve of $\left(\frac{\lambda_3}{\lambda}\right)^2$ vs. $\left(\frac{\lambda_3}{\lambda}\right)^2$ for resonance is an equilateral hyperbola, with horizontal asymptote at $\left(\frac{\lambda_4}{\lambda}\right)^2 = \frac{1}{1-\tau^2}$, and vertical asymptote at $\left(\frac{\lambda_3}{\lambda}\right)^2 = \frac{1}{1-\tau^2}$. This curve with its asymptotes is also plotted in Figure 5. In the part of the curves plotted in Figure 5, even when η_3^2 is not equal to zero the curve does not appreciably depart from the equilateral hyperbola provided $\eta_3^2 \overline{\gtrless} .001$. The corresponding curve in C_4 vs. C_3 for $\eta_3^2 = 0$, or $\eta_3^2 \overline{\gtrless} .001$, has its asymptotes at $C_3 = \frac{1}{(1-\tau^2)L_3\omega^2}$ and $C_4 = \frac{1}{(1=\tau^2)L_4\omega^2}$, and the equation giving the resonance relation of C_4 to C_3 in this case, obtained by a transformation of equation (25), is

$$(28)\quad \left(C_3 - \frac{1}{(1-\tau^2)L_3\omega^2}\right)\left(C_4 - \frac{1}{(1-\tau^2)L_4\omega^2}\right) = \frac{\tau^2}{(1-\tau^2)^2 L_3L_4\omega^4}.$$

This is the equation to which the empirical equation (1) corresponds, provided the height of antenna H is proportional to the capacity of the antenna C_3, assumed localized. The condition that C_3 should be localized, and particularly the conditions that the incident waves should be undamped, persistent and single-valued, were not fulfilled in the illustrative experiment cited above, and therefore it does not seem important to enter into a detailed comparison of equation (28) with equation (1). This has been partially done in the previous account of the experiment.[3]

The manner in which the resonance relation is affected by the resistance of Circuit III, corresponding to the antenna circuit, is shown in the curves marked $\eta_3^2 = .01$ and $\eta_3^2 = .1$ of Figure 5. The resistance R_4 of Circuit IV, corresponding to the detector circuit, is without effect in determining the form of these curves, which represent the general resonance relation. On the other hand, R_4 does have an effect in determining at which point of the curve of resonance relation the resonance is best, and R_4 is also significant in determining the sharpness of resonance. Some computations on this subject are given below.

VI. On the Optimum Resonance Relation.

Let us next examine the conditions for best resonance. We are still concerned merely with the steady-state vibration of the coupled circuits of Figure 1, under the action of the impressed sinusoidal electromotive force. The conditions for best resonance provided $M^2\omega^2 > R_3R_4$ are given in the equations (21) and (22), which after substitution from (15) become respectively

(29) $$\left(L_3\omega - \frac{1}{C_3\omega}\right)_{opt} = \pm \sqrt{\frac{R_3}{R_4}(M^2\omega^2 - R_3R_4)},$$

and

(30) $$\left(L_4\omega - \frac{1}{C_4\omega}\right)_{opt} = \pm \sqrt{\frac{R_4}{R_3}(M^2\omega^2 - R_3R_4)}.$$

Dividing both sides of equation (29) by $L_3\omega$ and employing the notation of equation (26) we have

(31) $$1 - \left(\frac{\omega_3}{\omega}\right)^2_{opt} = \pm\, \eta_3 \sqrt{\frac{\tau^2}{\eta_3\eta_4} - 1}.$$

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[3] G. W. Pierce, Physical Review, **20**, 220, 1905.

In like manner equation (30) becomes

$$(32)\qquad 1-\left(\frac{\omega_4}{\omega}\right)^2_{opt}=\pm\,\eta_4\sqrt{\frac{\tau^2}{\eta_3\eta_4}-1}.$$

Replacing the ratio of angular velocities by the reciprocal ratio of wave lengths, in accordance with equations (27), we may transform equations (31) and (32) into the following equations:

$$(33)\qquad \left(\frac{\lambda_3}{\lambda}\right)^2_{opt}=\frac{1}{1\pm\eta_3\sqrt{\dfrac{\tau^2}{\eta_3\eta_4}-1}},$$

and

$$(34)\qquad \left(\frac{\lambda_4}{\lambda}\right)^2_{opt}=\frac{1}{1\pm\eta_4\sqrt{\dfrac{\tau^2}{\eta_3\eta_4}-1}}.$$

It should be borne in mind that in order to get properly corresponding resonant values of λ_3 and λ_4, one must use either the plus sign in both of the equations or else the minus sign in both the equations. If one employs the plus sign in one of the equations and the minus sign in the other equation, the values of λ_3 and λ_4 so obtained are not appropriate simultaneous adjustments for best resonance. This is seen by an examination of equation (19).

By an examination of the discussion on p. 302 it will be seen that the optimum condition in the form of equations (33) and (34) can be attained only provided

$$(35)\qquad \frac{\tau^2}{\eta_3\eta_4}>1.$$

In order to facilitate the computation of λ_3 and λ_4 for various values of τ, η_3, η_4 and λ, equations (33) and (34) may be put in the form

$$(36)\qquad \left(\frac{\lambda_3}{\lambda}\right)_{opt}=\frac{1}{\sqrt{1\pm\phi_3}},\quad\text{where } \phi_3=\eta_3\sqrt{\frac{\tau^2}{\eta_3\eta_4}-1},$$

and

$$(37)\qquad \left(\frac{\lambda_4}{\lambda}\right)_{opt}=\frac{1}{\sqrt{1\pm\phi_4}},\quad\text{where } \phi_4=\eta_4\sqrt{\frac{\tau^2}{\eta_3\eta_4}-1}.$$

The following table (Table II) gives the values of $\left(\frac{\lambda_3}{\lambda}\right)_{opt}$ for various values of ϕ_3.

TABLE II.

VALUES OF $\left(\frac{\lambda_3}{\lambda}\right)_{opt.}$ CORRESPONDING TO DIFFERENT VALUES OF ϕ_3. COMPUTED FROM EQUATIONS (36) AND (37).

ϕ_3.	$\left(\frac{\lambda_3}{\lambda}\right)_{opt.}$ using + sign.	$\left(\frac{\lambda_3}{\lambda}\right)_{opt.}$ using — sign.	ϕ_3.	$\left(\frac{\lambda_3}{\lambda}\right)_{opt.}$ using + sign.	$\left(\frac{\lambda_3}{\lambda}\right)_{opt.}$ using — sign.
.0	1.000	1.000	1.4	.646	Imaginary
.1	.953	1.054	1.5	.632	"
.2	.913	1.118	1.6	.620	"
.3	.877	1.196	1.7	.608	"
.4	.845	1.292	1.8	.597	"
.5	.817	1.414	1.9	.587	"
.6	.791	1.581	2.0	.577	"
.7	.767	1.825	2.1	.568	"
.8	.746	2.236	2.2	.559	"
.9	.725	3.162	2.3	.550	"
1.0	.707	∞	2.4	.542	"
1.1	.690	Imaginary	2.5	.535	"
1.2	.675	"	2.6	.527	"
1.3	.660	"			

These results are plotted in the curves of Figure 6, with $\left(\frac{\lambda_3}{\lambda}\right)_{opt}$ as ordinates and ϕ_3 as abscissas. The same curves give the values of $\left(\frac{\lambda_4}{\lambda}\right)_{opt}$ as ordinates provided the abscissas are read as values of ϕ_4. If the lower branch of the curve is used in obtaining the values of $\left(\frac{\lambda_3}{\lambda}\right)_{opt}$, the same branch must be employed in finding the corresponding values of $\left(\frac{\lambda_4}{\lambda}\right)_{opt}$. In like manner the top branch of the curve may be employed in both cases, provided the value of ϕ_3 or ϕ_4 is less than unity. For values equal to unity the wave length for the top branch of the curve is infinite, and for greater values the wave lengths of the top branch become imaginary. In this case the optimum wave-length adjustment becomes single-values and must be read from the lower branch of the curve.

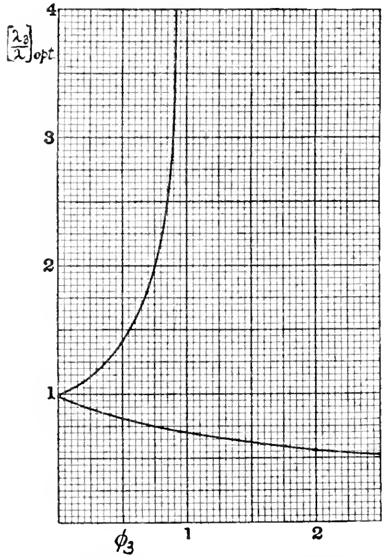

FIGURE 6. Auxiliary curve to assist in calculation of the optimum resonance adjustment. $\phi_3 = \eta_3 \sqrt{\frac{\tau^2}{\eta_3\eta_4} - 1}$. This curve also gives optimum value of λ_4 if ϕ_3 is replaced by $\phi_4 = \eta_4 \sqrt{\frac{\tau^2}{\eta_3\eta_4} - 1}$.

As an example of the manner of using the auxiliary curve (Figure 6) in the actual calculation of the optimum values of λ_3 and λ_4, let us take a special case. Suppose $\tau = .30$, $\eta_3 = .1$, let us give various values to η_4, and compute the corresponding optimum wave-length adjustments of the Circuits III and IV. A tabulation of the computation follows as Table III.

In compiling this table the values of ϕ_3 and ϕ_4 corresponding to the different values of η_4 were calculated by equations (36) and (37). The corresponding wave-length ratios were then taken from the curve of Figure 6.

The results contained in Table III are plotted in Figure 7. In a similar way the resonance relations for various values of τ and of η_3 may also be obtained, but the single example here computed and plotted serves to show the manner in which the coefficient of coupling and the damping factors contribute to determine the optimum resonance adjustment of the two circuits. The important facts to be noted are the following:

1. With given values of the coefficient of coupling and the damping factors of the two circuits the adjustment for best resonance is in general double valued. One may in general get best resonance either by setting both circuits to a wave length appropriately longer than that of the incident waves, or by setting both circuits to a wave length appropriately shorter than the incident waves.

TABLE III.

COMPUTATION OF OPTIMUM RESONANCE VALUES IN A SPECIAL CASE, IN WHICH
$\tau = .30$
$\eta_3 = .1$
η_4 IS GIVEN VARIOUS VALUES

η_4	ϕ_3	ϕ_4	$\left(\frac{\lambda_3}{\lambda}\right)_{opt.}$	$\left(\frac{\lambda_4}{\lambda}\right)_{opt.}$	$\left(\frac{\lambda_3}{\lambda}\right)_{opt.}$	$\left(\frac{\lambda_4}{\lambda}\right)_{opt.}$
.01	.943	.094	.720	.955	4.07	1.053
.02	.662	.133	.777	.940	1.73	1.070
.03	.539	.162	.806	.930	1.47	1.090
.04	.464	.186	.827	.917	1.37	1.110
.05	.412	.206	.840	.910	1.30	1.123
.06	.374	.224	.850	.905	1.26	1.135
.07	.345	.242	.862	.900	1.23	1.148
.08	.319	.255	.870	.895	1.205	1.152
.09	.300	.270	.875	.890	1.196	1.168
.1	.282	.282	.882	.882	1.180	1.180
.2	.187	.374	.917	.853	1.110	1.262
.3	.141	.423	.935	.837	1.078	1.310
.4	.112	.448	.950	.832	1.060	1.345
.5	.089	.445	.960	.830	1.048	1.340
.6	.071	.426	.970	.837	1.036	1.320
.7	.054	.378	.975	.852	1.032	1.262
.8	.035	.280	.980	.885	1.010	1.175
.9	.000	.000	1.000	1.000	1.000	1.000

Either pair of values under the brace is to be employed simultaneously for best resonance.

2. The adjustment for best resonance is materially influenced by the resistances of the two circuits. If, for example, with fixed incident

waves one tunes a wireless telegraph system of the coupled type to resonance with the use of a given detector, and then changes to a detector of different resistance, it is necessary to shift the wave length of both of the circuits in order to bring the system back to its best adjustment. This is a familiar experience, and serves to explain the

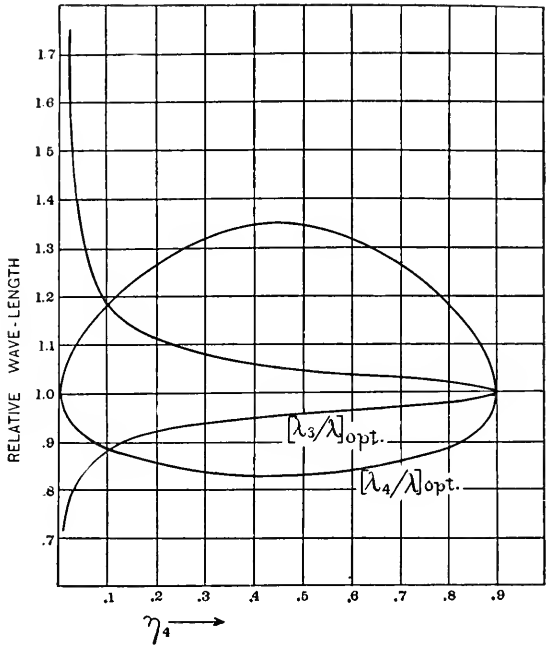

FIGURE 7. Relation of optimum wave-length adjustment to damping in Circuit IV, for a given value of η_3 and τ. ($\eta_3 = .1$, $\tau = .30$).

fact that with a vacuum detector of the type of Fleming's valve or Deforest's Audion or Hewitt's Mercury-arc detector one may attune the receiving station without change of its capacity or inductance merely by bringing a magnet up near the gaseous path of the detector. This merely changes the resistance of the circuit, and when the change is made in the proper direction and to the proper extent the system is thereby brought into resonance. The same thing may be effected in some cases by changing the strength of the local heating or ionizing current employed with detectors of this type.

3. The shifting of the resonance relation with change of resistance of the circuit has an influence on the sharpness of resonance when one

employs any of the ordinary detectors, because the resistance of the detectors is a function of the received current. This is a complex phenomenon and will enter into the consideration of sharpness of resonance, which is discussed below.

VII. On the Sharpness of Resonance and on the Possibility of Preventing Interference.

By reference to the previous pages it is seen that in order to obtain the best resonance, which gives a maximum of current in Circuit IV, it is necessary to adjust the period of both Circuit III and Circuit IV. We have given an expression for the value of the maximum current (that at best resonance) in equation (23), and we have also obtained a general expression for the current in Circuit IV (equation (16)), so that it is now possible to plot the current as best resonance is approached, and to form an estimate of the sharpness of resonance, whenever the constants of the circuits are known. This may conveniently be done in either of two ways, — which I shall classify as Case I and Case II, as follows:

Case I. Let us assume that the Circuit III is put at its best value (equation (21)), and let us compute the current in Circuit IV as the wave length of Circuit IV is varied. This corresponds to fixing the antenna circuit and tuning with the detector circuit.

Case II. Assume Circuit IV to be set at its best value (equation (22)) and compute the current in Circuit IV as the wave length of Circuit III is varied. This corresponds to fixing the constants of the detector circuit and tuning with the antenna circuit.

In either case we must know certain constants of the circuits, and I shall carry through the computation for both cases with several sets of constants. First it is necessary to transform the equations into suitable forms for making the computations.

Development of Equations for computing Case I. — The general expression for the amplitude of current in Circuit IV is given in equation (16); namely,

$$(16) \qquad Y = \frac{EM\omega}{\sqrt{(UV - M^2\omega^2 - R_3R_4)^2 + (R_4U + R_3V)^2}}.$$

Let us combine with this the condition that U shall have its optimum value, equation (22),

$$(22) \qquad U_{opt} = \pm\sqrt{\frac{R_3}{R_4}(M^2\omega^2 - R_3R_4)},$$

where $M^2\omega^2 > R_3 R_4$, and let us suppose that V has, in general, not its optimum value, but a value k times its optimum value; that is,

$$(38) \qquad V = k \cdot V_{opt},$$

in which k is a variable parameter, which may be positive, negative, whole, or fractional.

Equation (16) then becomes

$$(39) \quad Y_{(U_{opt})} = \frac{EM\omega}{\sqrt{(k \cdot U_{opt} V_{opt} - M^2\omega^2 - R_3 R_4)^2 + (R_4 U_{opt} + R_3 \cdot k \cdot V_{opt})^2}}.$$

Replacing U_{opt} and V_{opt} by their values (equations (21) and (22)) we have after simplification

$$(40) \qquad Y_{(U_{opt})} = \frac{E}{\sqrt{(k-1)^2 (M^2\omega^2 - R_3 R_4) + 4 R_3 R_4}}.$$

Now dividing the square of equation (40) by the square of equation (23) we have

$$(41) \qquad \left(\frac{Y_{(U_{opt})}}{Y_{max\ max}}\right)^2 = \frac{1}{\frac{(k-1)^2}{4}\left(\frac{M^2\omega^2}{R_3 R_4} - 1\right) + 1} = \frac{1}{\frac{(k-1)^2}{4}\left(\frac{\tau^2}{\eta_3\eta_4} - 1\right) + 1}.$$

This equation gives the current in terms of the parameter k provided $\tau^2 > \eta_3\eta_4$. Let us now obtain the wave lengths in terms of the same parameter. The values of V and V_{opt} from equations (15) and (22) substituted in (38) give

$$(42) \qquad L_4\omega - \frac{1}{C_4\omega} = \pm k \sqrt{\frac{R_4}{R_3}(M^2\omega^2 - R_3 R_4)}.$$

Dividing (42) by $L_4\omega$ gives

$$(43) \qquad 1 - \left(\frac{\omega_4}{\omega}\right)^2 = \pm k \sqrt{\frac{R_4}{R_3}(M^2\omega^2 - R_3 R_4)} = \pm k\eta_4 \sqrt{\frac{\tau^2}{\eta_3\eta_4} - 1}.$$

Whence

$$(44) \qquad \left(\frac{\lambda_4}{\lambda}\right)^2 = \frac{1}{1 \pm k\eta_4\sqrt{\frac{\tau^2}{\eta_3\eta_4} - 1}}$$

$$= \frac{1}{1 \pm k\phi_4}, \quad \text{where} \quad \phi_4 = \eta_4\sqrt{\frac{\tau^2}{\eta_3\eta_4} - 1}.$$

This equation (44) gives the relation of the wave lengths to the parameter k, and is applicable only provided U has its optimum value and $\tau^2 > \eta_3\eta_4$. The condition that V have its optimum value is conveniently expressed in the form of equation (36) above. Equations (44) and (36) must both be used with the same sign in order to be simultaneously correct.

In case $\tau^2 \overline{\overline{<}} \eta_3\eta_4$, equations (44) and (41) cannot be employed. In this case U_{opt} and V_{opt} are both zero (see page 302), and a special investigation is necessary. This proves to be simple. Let us take equation (16), make $U = U_{opt} = 0$, and we have

$$(45) \qquad Y_{(U_{opt})} = \frac{EM\omega}{\sqrt{(M^2\omega^2 + R_3R_4)^2 + R_3^2 V^2}}.$$

Expressing this result in terms of $Y_{max\,max}$ by dividing equation (45) by equation (23), we have

$$(46) \qquad \left(\frac{Y_{(U_{opt})}}{Y_{max\,max}}\right) = \frac{M^2\omega^2 \cdot 4\,R_3R_4}{(M^2\,\omega^2 + R_3R_4)^2 + R_3^2\left(L_4\omega - \frac{1}{C_4\omega}\right)^2}$$

$$= \frac{M^2\omega^2 \,/\, R_3R_4}{\frac{1}{4}\left(\frac{M^2\omega^2}{R_3R_4} + 1\right)^2 + \frac{1}{4}\frac{L_4^2\omega^2}{R_4^2}\left\{1 - \left(\frac{\omega_4}{\omega}\right)^2\right\}^2}$$

$$= \frac{\tau^2 \,/\, \eta_3\eta_4}{\frac{1}{4}\left(\frac{\tau^2}{\eta_3\eta_4} + 1\right)^2 + \frac{1}{4\,\eta_4^2}\left\{1 - \left(\frac{\lambda}{\lambda_4}\right)^2\right\}^2}.$$

Equation (46) is to be employed in place of (41) whenever

$$\tau^2 \,/\, \eta_3\eta_4 \overline{\overline{<}} 1.$$

An interesting case arises when $\tau^2 \,/\, \eta_3\eta_4 = 1$. Equation (46) then simplifies to

$$(47) \qquad \left(\frac{Y_{(U_{opt})}}{Y_{max\,max}}\right)^2 = 1 + \frac{1}{\frac{1}{4\,\eta_4^2}\left\{1 - \left(\frac{\lambda}{\lambda_4}\right)^2\right\}^2}.$$

Equation (47) holds only when $\tau^2 = \eta_4\eta_4$; that is, $M^2\omega^2 = R_4 R_4$. In this case also $U_{opt} = V_{opt} = 0$.

Computation and Discussion of Case I. In the above paragraphs the equations have been derived for the current developed in Circuit

TABLE IV.

$$\text{Given } \tau = .30 \qquad \tau^2 = .09$$
$$\eta_3 = .01 \qquad \frac{\tau^2}{\eta_3\,\eta_4} = 900$$
$$\eta_4 = .01$$

$$\left(\frac{Y(U_{opt})}{Y_{max\ max}}\right)^2 = \frac{1}{\dfrac{(k-1)^2}{4}\left(\dfrac{\tau^2}{\eta_3\eta_4} - 1\right) + 1} = \frac{1}{225(k-1)^2 + 1}$$

$$\left(\frac{\lambda_4}{\lambda}\right) = \frac{1}{1 \pm k\eta_4\sqrt{\dfrac{\tau^2}{\eta_3\eta_4} - 1}} = \frac{1}{1 \pm .3\,\mathrm{k}}.$$

k.	λ_4/λ With plus sign.	λ_4/λ With minus sign.	$\left[\frac{Y}{Y_{mm}}\right]^2$	k.	λ_4/λ With plus sign.	λ_4/λ With minus sign.	$\left[\frac{Y}{Y_{mm}}\right]^2$
1.8	.806	1.490	.007	.95	.882	1.183	.640
1.7	.814	1.428	.009	.9	.887	1.170	.307
1.6	.822	1.387	.012	.8	.898	1.147	.100
1.5	.831	1.348	.018	.7	.909	1.125	.047
1.4	.839	1.313	.027	.6	.921	1.104	.027
1.3	.848	1.280	.047	.5	.933	1.084	.018
1.2	.857	1.250	.100	.4	.945	1.066	.012
1.1	.868	1.222	.307	.3	.958	1.048	.009
1.05	.872	1.208	.640	.2	.971	1.030	.007
1.0	.877	1.196	1.000				

IV for any adjustment of this circuit, while Circuit III is kept at its optimum value. This corresponds to keeping the antenna circuit of a wireless telegraph station set at its best value for given conditions and tuning the system by variation of Circuit IV. Assuming any given constants of the two circuits and any given wave length of incident waves, we can now make computations which will give the shape and form of the resonance curve. In making the calculations if $M^2\omega^2 > R_3R_4$ we can take convenient values of the parameter k and calculate the wave-length adjustment corresponding to the given values of k (equation 44) and also the relative current for the same values of k (equation 41). The results will give relative current corresponding to various wave-length adjustments.

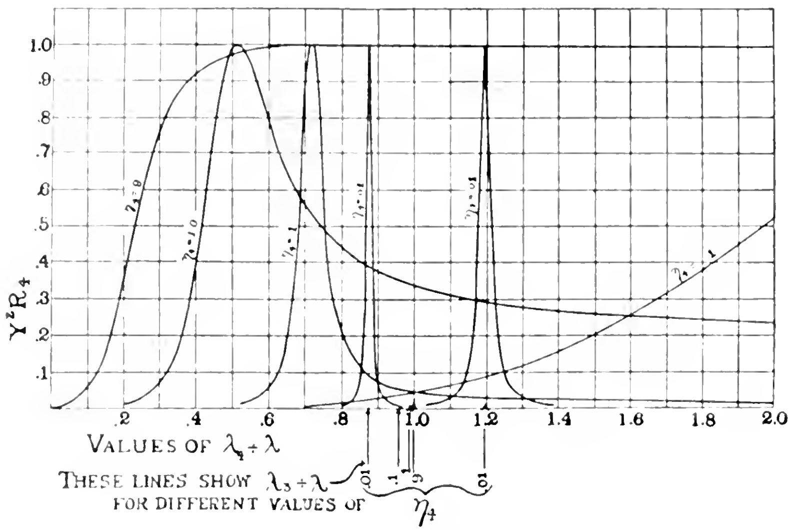

FIGURE 8. Resonance curves giving theoretical relation of current-square times resistance of Circuit IV to wave-length adjustment in the neighborhood of the optimum adjustment of Circuit IV when Circuit III is at optimum — for $\eta_3 = .01$, $\tau = .30$.

Case I with $\eta_4 = .01$, $\eta_3 = .01$, $\tau = .30$. — A sample set of computed results assuming $\tau = .30$, $\eta_3 = .01$ and $\eta_4 = .01$ is given in Table IV.

These results are plotted as the two curves marked "$\eta_4 = .01$" of Figure 8. The abscissas are values of the wave-length adjustment relative to the wave length of the incident waves (λ_4/λ). The ordinates are values of $\left(\frac{Y}{Y_{max\,max}}\right)^2$ but since $Y_{max\,max} = \frac{E}{2\sqrt{R_3R_4}}$, and since all the curves of the figure are plotted with constant R_3 (i.e., constant η_3), constant E, and constant M, it is perhaps more instructive to regard the ordinates of Figure 8 as relative values of Y^2R_4, and they are so designated in the figure. Referring again to the figure, these two curves marked "$\eta_4 = .01$" are obtained as resonance curves of square of current in Circuit IV plotted against wave-length adjustment of Circuit IV, and are obtained by setting Circuit III at its best value and leaving it at that value during the tuning. The appropriate best-value settings of the wave length of Circuit III are indicated by the positions of the two lines marked "$\eta_4 = .01$" at the lower margin of the figure. The short-wave adjustment of λ_3 (at $\lambda_3/\lambda = .877$) is required for the short-wave resonance curve (with its maximum at $\lambda_4/\lambda = .877$), and the long-wave adjustment of λ_3 (at $\lambda_3/\lambda = 1.196$) is required for the long-wave resonance curve (with maximum at $\lambda_4/\lambda = 1.196$). It is seen that in this particular case, with $\eta_3 = \eta_4 = .01$, the resonant adjustment of Circuit III and that of Circuit IV have the same wave-length $\lambda_3 = \lambda_4$; and because of the smallness of the damping factors η_3 and η_4, the two curves are sharp.

Case I (continued). Assuming again $\tau = .30$, $\eta_3 = .01$, *but with* $\eta_4 = .1$. — Suppose now that we give to Circuit IV a higher resistance so that $\eta_4 = .1$. This may be done by using a higher resistance detector in Circuit IV. This will cut down the maximum value of the current in Circuit IV, but will leave the square of the current times the resistance (namely, Y^2R_4) the same as before, so that the curve in terms of Y^2R_4 will have the same maximum amplitude for $\eta_4 = .1$ as for $\eta_4 = .01$. Complete computations from equations (44) and (41) show that the curve will have the form and position given in Figure 8 and marked "$\eta_4 = .1$." The corresponding appropriate adjustment of Circuit III is given by the line marked "$\eta_4 = .1$" of the lower margin. The curve going out to the right, also marked "$\eta_4 = .1$" in Figure 8, is a part of another possible resonance curve in this case. This second resonance curve culminates beyond the limit of the figure with its maximum at $\lambda_4/\lambda = 4.18$, and requires the adjustment of λ_3/λ at 1.05.

The results of the computations in this case show in an interesting manner the necessity of tuning both circuits to get resonance, and show how markedly the adjustment of Circuit IV may be affected by the adjustment of Circuit III; since with the constants here assumed, the change of Circuit III from $\lambda_3/\lambda = .96$ to $\lambda_3/\lambda = 1.05$ necessitates the shifting of λ_4/λ from .716 to 4.18. The resonance in the former case is sharp, and that in the latter case is very dull.

Case I (continued). Assuming again $\tau = .30$, $\eta_3 = .01$, *while* η_4 *is made* $= 1$. We obtain the resonance curve marked $\eta_4 = 1.0$ (Figure 8) with appropriate adjustment of λ_3/λ_4 at .986. In this case the second resonance value in the region of long waves is imaginary.

Case I (continued). $\tau = .30$, $\eta_3 = .01$, *while* $\eta_4 = 9$. — This is the special case requiring the use of equation (47) and gives the curve marked " $\eta_4 = 9$," requiring λ_3/λ to be 1.0. This curve is almost flat on top and for this condition tuning with Circuit IV is impossible. There is left here, as with all the curves of this figure, the alternative of leaving Circuit IV fixed at its best value and tuning with Circuit III. This is Case II, which we presently come to consider. It is proposed first, however, to present a new set of curves under Case I with a

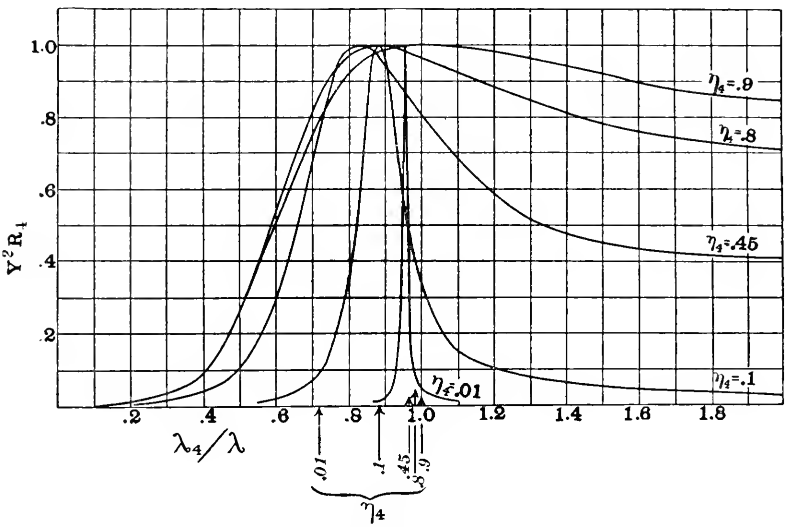

FIGURE 9. Curves similar to Figure 8, but with $\eta_3 = .1$, $\tau = .30$.

larger damping factor (η_3) in Circuit III, corresponding to the antenna circuit.

Case I (continued). Assuming $\tau = .30$, $\eta_3 = .1$, *and* η_4 *various.*— The curves for these conditions are plotted in Figure 9 for only the short-wave adjustment. The corresponding long-wave adjustments are presented in Figure 10.

The Equations and Computation of Case II. — It is now proposed to set Circuit IV at its best value, with any given constants of circuits, and to tune with Circuit III. The equations giving the current in Circuit IV for various values of wave length of Circuit III, are obtained from (41) and (44) by a simple interchange of subscripts, — giving

$$(48) \qquad \left(\frac{Y(\nu_{opt})}{Y_{max\ max}}\right)^2 = \frac{1}{\frac{(k-1)^2}{4}\left(\frac{\tau^2}{\eta_3\eta_4} - 1\right) + 1},$$

where k is determined by the equation

$$(49)\qquad \left(\frac{\lambda_3}{\lambda}\right)^2 = \frac{1}{1 \pm k\eta_3\sqrt{\frac{\tau^2}{\eta_3\eta_4} - 1}}.$$

By a comparison of (48) and (49) with equations (41) and (44) it will be seen that the method of Case II will give sharper resonance

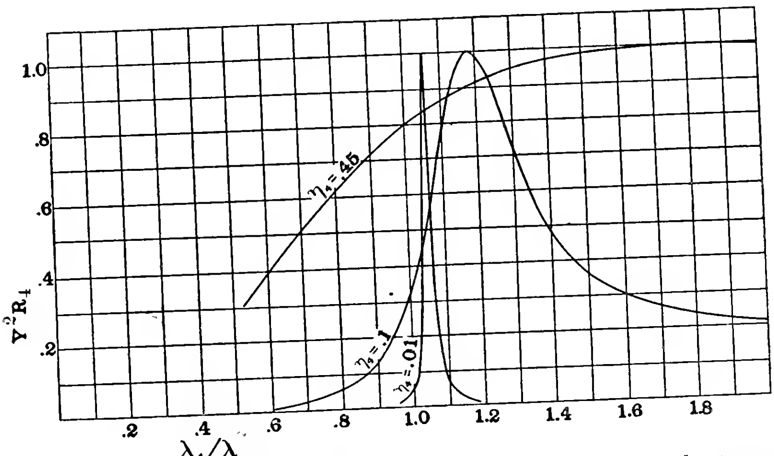

FIGURE 10. Long-wave adjustments corresponding to the short-wave adjustments of Figure 9.

than that of Case I whenever $\eta_3 < \eta_4$. With the high resistance detectors in ordinary use in wireless telegraphy, η_3 is generally much less than η_4, and the sharp tuning is best attained by the method of Case II; that is, by fixing the condenser circuit (Circuit IV) by successive approximations to a setting somewhere near its best value for the given incident waves, and then making the final adjustment by changing Circuit III.

Three numerical examples are given in the curves of Figure 11. For all these curves τ is assumed to be .30, and η_4 is taken as 1.0. The values of η_3 are marked on the respective curves.

Application to Actual Wireless Telegraphic Cases. — Now the corresponding actual wireless-telegraph resonance curves taken with coupled circuits, one of which has distributed capacity, will be less sharp than the corresponding curves here computed and drawn, so that as soon as we know the damping constants and the coefficient of coupling of the wireless-telegraph circuits we can select from the curves here computed a set of curves that will be more selective than the wireless-telegraph curves, and we can in this way fix a limit to the sharpness of resonance that can be attained in practice. Some computations of this character on the extent to which interference can be prevented under certain conditions, assumed as practical, have been published elsewhere.[4]

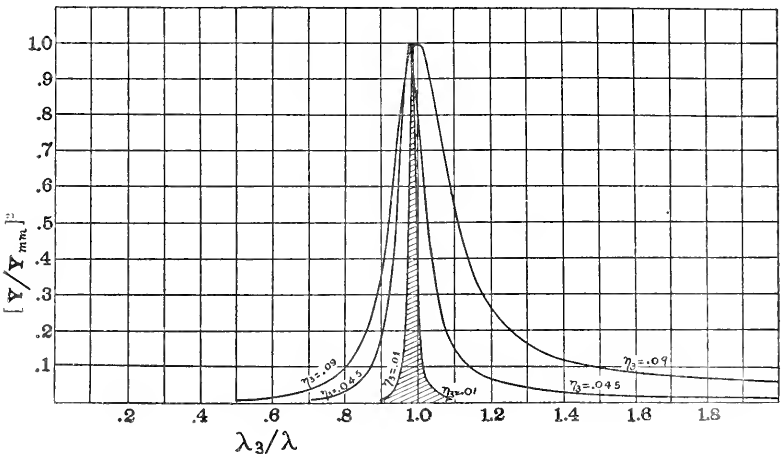

FIGURE 11. Resonance curves obtained by tuning with Circuit III while Circuit IV is at best value. $\tau = .30$, $\eta_4 = 1$.

VIII. THE MAXIMUM CURRENT, AND DETECTOR RESISTANCE.

Equation (23) is an expression for the maximum current that can be obtained in Circuit IV, when the inductances and capacities of Circuits III and IV are given their best values.

An interesting fact that can be obtained from an examination of equation (23) is that the square of the current multiplied by R_4 gives a quantity independent of R_4. This means that the heat developed in Circuit IV is independent of R_4; that is to say, the same amount of heat is developed in Circuit IV, at best resonance, whether a high-resistance or a low-resistance detector is employed. This means that, if the detector is an instrument for measuring heat, a low-resistance detector would be as sensitive as a high-resistance detector if it were not for the fact that the lower the resistance of the detector the less the proportion of the total heat that is developed in the detector itself,

[4] Pierce, Principles of Wireless Telegraphy, McGraw-Hill Book Company, New York, 1910.

because of the larger development of heat in the rest of the Circuit IV. Since the resonance is sharper with the low-resistance circuit, the resistance of a thermal detector, provided its indications are proportional merely to the heat developed in it, ought to be as low as is consistent with the localization of a large part of the energy in the detector; that is, for example, in order to get 9/10 of the maximum effect, the resistance of the detector, if its indications are proportional merely to the heat developed in it, ought to be nine times the high-frequency resistance of the rest of Circuit IV.

Similar considerations apply to a detector of the electrodynamometer type. If the deflections of the electrodynamometer are proportional to $n^2 Y^2$, where n is the number of turns of wire in the coil, and if the size of the channel of windings is fixed so that the resistance R of the detector is $\frac{\rho l}{S}$, l and S being the length and cross section of the wire in the coil, then we have

$$l = 2\,\pi r \cdot n,$$

in which r is the mean radius of the windings; and approximately

$$S = \frac{A}{n},$$

A being the area of the channel.

Therefore,
$$R = \frac{\rho\, 2\,\pi r n^2}{A},$$

or
$$R \sim n^2.$$

Whence if the deflection,

$$D \sim n^2 Y^2,$$
$$D \sim R\ Y^2.$$

In this, R is the resistance of the detector alone. Now according to equation (23) the quantity of $R_4 Y^2$ is not changed by changing R_4. Hence if the resistance R is made nine times the resistance of the rest of Circuit IV, the deflection of the high-frequency dynamometer will be 9/10 as large as would be obtained with a detector of very high resistance, and the resonance with the low resistance detector will be much sharper than with the detector of very high resistance.

However, it must be borne in mind that this conclusion holds only between different detectors of the same type, and presupposes that the factor by which $R Y^2$ is to be multiplied to get the deflection or other indication is independent of R. In the case of the dynamometer or hot-wire ammeter the factor of convertibility of the energy of rapid alternations into deflection is probably fairly constant but is small; whereas, with certain other types of detectors, notably the electrolytic and the crystal rectifiers the convertability of the energy of rapid alternations into direct current energy is not constant, and appears to be relatively large only provided the resistance of the detector is large. This has constrained wireless telegraphic practice to high-resistance detectors, with the consequent deficiency in sharpness of resonance. The analysis given in the present paper shows that there is no inherent necessity in using these high-resistance detectors provided only detectors of lower resistance can be found with a large efficiency in converting electric energy of rapid alternations into energy of direct current or slowly-varying periodic current.

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THE ACTION OF MERCURY ON STEEL AT HIGH PRESSURES.

By P. W Bridgman.

With a Plate.

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THE ACTION OF MERCURY ON STEEL AT HIGH PRESSURES.

By P. W. Bridgman.

Presented by John Trowbridge, November 9, 1910. Received October 28, 1910.

Under ordinary circumstances steel and mercury are inert with respect to each other, as is shown by the possibility of carrying mercury for indefinite periods of time in steel flasks. But there seems to be a widely spread notion that under higher pressures there may be some action not operative at lower pressures. This possibility is usually ascribed to the extraordinary mobility of the mercury molecule. For instance, every one who has had experience in making joints for pressures of a few atmospheres knows that mercury will easily find its way through holes impervious to water or less viscous fluids. It has therefore been thought probable that under higher pressures the easily moving mercury molecule might be forced through the very pores of the solid metal itself, and that in consequence it might be impossible to hold mercury at all in metal receptacles at high pressures. This view has received its highest confirmation from some often cited experiments of Amagat. Amagat[1] has described how in one case mercury was forced by a pressure of 3000 atmospheres in a fine spray through 8 cm. of cast steel, in which no flaw could be afterward detected with the microscope. Amagat explained this effect in the way suggested above by assuming that the mercury was forced by the high pressure through the very intermolecular pores of the solid steel. It is worthy of notice that it was found possible to avoid this difficulty merely by making another apparatus with thicker steel parts. There is also work by Cailletet and Collardeau[2] on the vapor pressure of mercury at high temperatures that seems to demand in explanation

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[1] Amagat, Ann. de Chim. et Phys. (6), **29**, 87–88 (1893); also Compt. Rend. March 2, 1885.

[2] Cailletet, Colardeau and Riviere, Compt. Rend. **130**, 1585–1591 (1900).

that the mercury was forced through solid steel by comparatively low pressures at sufficiently high temperatures.

In measurements undertaken by the author of various physical constants at high pressures, this question of the action of mercury and steel became of vital importance. For instance, the methods adopted to measure compressibility assumed that there was no penetration of the mercury into the steel containing vessel, as do also the methods used more recently in determining the variation with pressure of the freezing temperature of mercury and its change of volume on freezing. The preliminary work at low pressures made it seem probable that at least over the pressure range used by Amagat the effect described by him does not really exist in the grades of steel used by him, and that the observed effect was due more likely to flaws in the steel. At the same time it was found that at higher pressures there is undoubtedly an effect important enough to demand the redesigning of the apparatus for the measurement of the change of volume on freezing. It is the purpose of this paper to describe the various experiments made to prove the undoubted existence of the effect, and to offer a qualitative explanation. The effect was run across only incidentally, and it was examined only so much as was necessary for the work in hand. No endeavor has been made to make the experimental investigation or the explanation complete, as this would lead too far afield.

The effect was first found during an attempt to measure the change of volume of mercury on freezing by a method similar in many respects to that of Tammann.[3] It was found that cylinders of hardened chrome nickel steel would support very much less internal pressure when this pressure was transmitted by mercury than when the transmitting fluid was some other liquid such as water. The pressure might be less in the ratio of three or four to one; thus cylinders which stood without breaking 24000 atmospheres when the pressure was transmitted to the interior by a mixture of water and glycerine broke on the next application of pressure at 5–8000 atmos. if the transmitting fluid were mercury. These few preliminary experiments under varying conditions made the existence of an effect seem probable, but pointed to nothing conclusively. It might well be that there was a flaw running the entire length of the steel bar from which all these pieces were cut, into which the mercury forced its way in consequence of its greater mobility, in preference to the water, or it might be that there was here a fatigue effect, the steel breaking more readily on the second application of pressure with the mercury because of the exceedingly high pressure to which it had been previously exposed by

[3] Tammann, Kristallisieren und Schmelzen (1903), p. 204.

the water. This explanation, however, was opposed by all previous experience with this steel. In any event, the effect of the mercury was entirely different from that found by Amagat, as there was never any tendency for the mercury to squirt through the steel, but there was always sudden rupture, the cylinder cracking down one side in a plane containing the axis. To show conclusively that a cylinder of hardened nickel steel will really not stand so much pressure when the transmitting liquid is mercury as when it is some other liquid such as water, the following experiment was undertaken. A bar of this special steel (Krupp Special Chrome Nickel Steel E. F. 60.0) was cut into twelve pieces each $8\frac{1}{2}''$ long and $2''$ diameter. (See Figure 1.) The pieces were numbered and their orientation in the original bar carefully noted. They were then each pierced with a $\frac{1}{2}''$ hole reamed to size, turned on the outside true with the hole, and hardened by heating to a bright cherry red and quenching in a heavy tempering oil. Every other cylinder (Nos. 1, 3, 5, 7, 9, 11) was filled with mercury and tested by applying pressure to the mercury by means of a piston actuated by a hydraulic press. The test conditions of these different cylinders were varied somewhat by changing the rapidity with which pressure was applied; in other respects the conditions were the same. The other cylinders were tested in a similar way, except that the fluid transmitting the pressure was not mercury; being in four of the six cases water and glycerine, in the others ether and carbon disulphide respectively. The pressure in the test cylinders was determined from the pressure of the fluid actuating the hydraulic ram, multiplying in the ratio of the areas of the two pistons. An unknown error is introduced here by the friction of the packing, but in other experiments with similar cylinders in which the pressure inside the small cylinders was measured directly it was found that the error so introduced was nearly constant and easy to correct for. The correction so found was used in the results to be given. In any event, the correction is less than the irregularities introduced by other causes.

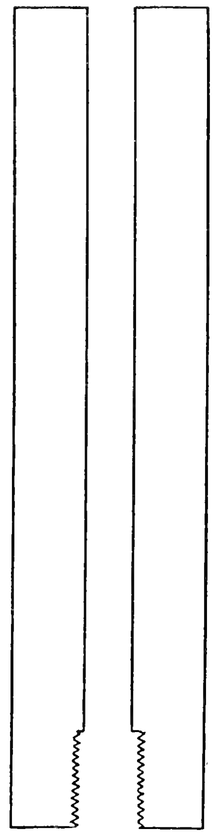

FIGURE 1. Form of the test cylinders broken with mercury.

The accompanying table (see Table I) shows the results found with the six cylinders containing mercury. The pressure was increased more slowly for the higher numbered cylinders; with the four first the

TABLE I.

No. of Cylinder.	Breaking Pressure $\frac{\text{Kgm}}{\text{cm}^2}$.	Rate of Increase of Pressure.	Total Euration.	Location of Crack.	Remarks.
1	10250	1250 kgm. in 30 sec.	$3\frac{1}{2}$ min.	$-123°$	Pressure on 1, 3,5, 7, was increased uniformly
3	4750	" " "	$1\frac{1}{3}$ "	$+107°$	
5	3250	1250 kgm. in 60 sec.	$1\frac{2}{3}$ "	$+64°$	Broke on second trial
7	3375	625 kgm. in 60 sec.	$4\frac{1}{4}$ "	$+11°$	Leaked on first at 2750
9	4000	1500 for $3\frac{1}{2}$ min. 2750 for 10 min.; breaks at 4000 after $\frac{1}{2}$ min.	14 "	$-25°$	Pressure on 9 and 11 increased discontinuously
11	3000	At 2750 for 46 min. Breaks at 3000 after 5 min.	51 "	$-106°$	

pressure was increased uniformly at the rate indicated, while with the two others it was increased in discontinuous steps as shown. In making the tests, pressure was first pushed to 1500 atmos. and kept there for several minutes to make sure that there was no leak and everything was in working order. The time given in the duration column is exclusive of the time occupied by the preliminary application of 1500. It is evident that the data admit of no quantitative comparison as the great discrepancy between tests 1 and 3 made under as nearly as possible the same conditions shows. However, these two tests were made with a high rate of increase of pressure. Those made with a slower rate show much more consistent results. In general it appears that the slower the rate of increase of pressure the lower the bursting pressure, which apparently has as its lower limit about 3000 atmos. The data are not inconsistent with the view that there is a critical pressure which will produce rupture if applied for an infinite time; pressures above this produce rupture in constantly less times.

The location of the crack is an important consideration. The crack in each cylinder is in an axial plane, extending the entire length of the cylinder. It was located by measuring the angular distance from a fiducial line marked the length of the bar from which the pieces were cut. The diagram (see Figure 2) showing the location of the crack in the several cylinders makes clear that the rupture does not take place along a flaw in an axial plane extending originally throughout the entire bar. Furthermore, since the manner of fracture of each cylinder demands that the flaw be in an axial plane if the fracture is due to a flaw, and since it seems improbable that a flaw throughout the length of one cylinder should not extend into the neighboring cylinders, the conclusion seems justified that the rupture is not due to a flaw.

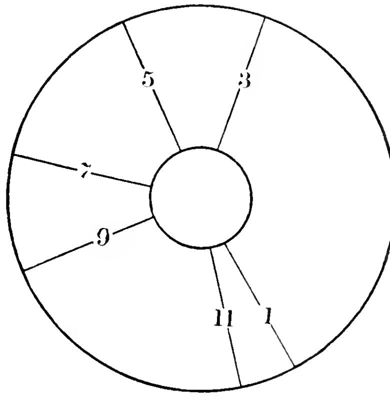

FIGURE 2. Orientation of the crack with respect to the original bar in the six cylinders broken with mercury.

Now compare with this the tests for the other set of six cylinders from the same bar. (See Table II.)

The difference between these two sets of tests is sufficiently striking. Not one of the cylinders filled with a fluid other than mercury was broken during the tests, although in several cases the test was terminated by an irrelevant accident. The

TABLE II.

No. of cylinder.	Maximum Pressure $\frac{Kgm}{cm^2}$.	Manner of applying Pressure.	Remarks.	Fluid.
2	23500	Rate of increase not noted	Packing plug blows out	Glycerine and water
4	19000	From 1500 to 15000 at rate of 1250 in 30 sec. Held at 15000 for 2½ min., 17500 for 3 min.	Packing plug blows out	Glycerine and water
6	15200	From 1500 to 11000 uniformly in 4½ min. Held here for 3 min., then at 13500 for 1 min.	Terminated by leak	Glycerine and water
8	24000	15 min. in reaching max. Held here for 6 min.	Leak after 6 min.	Glycerine and water
10	20250	Held at 16500 for 2 min.; at 20250 for 45 sec.	Plug blows out	Ether
12	24000	1 hour in reaching max. at uniform rate. Held her for 8 min.	No leak	CS_2

maximum pressure reached by three of the cylinders without breaking was 24000 atmos., over 8 times the lowest breaking pressure of the cylinders filled with mercury, and nearly $2\frac{1}{2}$ times the maximum breaking pressure. Greater pressure than 24000 atmos. was not applied because the limit of the press was reached. Of course the stress produced in a cylinder by internal fluid pressure depends only on the pressure in the fluid and not on the nature of the fluid. It must be, then, that the mercury takes some special part in producing rupture quite apart from the stress produced by it.

The magnitude of the fluid pressures mentioned here requires brief comment, because without a word of explanation it may seem so large as to cast discredit on the accuracy of all the data. In the first place, a steel of the kind described above with the indicated heat treatment is the only steel known to the author that will stand the pressures used. The best grades of carbon tool steels will not stand much more than 18000 atmos. As to the measurement of the pressure, similar apparatus has been used up to 13000 atmos. and the pressure measured directly with an absolute gauge with an accuracy of $\frac{1}{10}$ per cent. Absolute measurements have not been made above this, and it is possible that the friction of the packing may become unexpectedly large, although no evidence of this has been found. However, it makes no difference what the friction above 13000 is; the fact stands incontestable, for the breakage produced by mercury is in a region open to easy direct measurement, while the other cylinders have stood a pressure certainly several-fold greater.

All these results so far were obtained with hardened chrome nickel steel. Search was now made for the same effect in other steels. A cylinder of nickel steel of similar dimensions to the above, but left soft, was tried. This was filled with mercury. Pressure was kept at 8000 for 3 hours, then pushed gradually to 15000, where the increasing non-elastic stretch became so great as to let the mercury past the packing. The behavior here was exactly like that of a similar cylinder filled with water and stretched beyond the elastic limit: the elastic limit in the two cases was the same, as also the manner of yield and the shape into which the cylinder was deformed. Apparently, then, mercury exerts no selective action on the soft nickel steel. A similar cylinder of bessemer steel filled with mercury was left exposed to 3500 atmos. for 14 hours, and subsequently the pressure was increased until the mercury blew past the packing, exactly as for similar cylinders filled with water. It should be noted that non-elastic yield occurs for bessemer steel at pressures much lower than 3500, perhaps as low as 2000 atmos. The explanation finally adopted attaches some significance to the value of the elastic limit. The two cylinders above were tested at pressures beyond this limit. To test whether a soft cylinder would be broken by a pressure under the elastic limit if the pressure were sufficiently prolonged, a cylinder of soft nickel steel filled with mercury was exposed for three weeks to 4000 atmos. without rupture. At the end of three weeks, pressure was increased and rupture took place at 12000 atmos. The break was remarkable, as the cylinder showed no preliminary stretch as all soft steels do, but snapped like a piece of hardened tool steel. This may have been due to fatigue from the prolonged application of pressure, or it may be that weakening by the action of mercury had started at the lower pressure and that the cylinder would have finally broken at 4000, had the application been sufficiently prolonged.

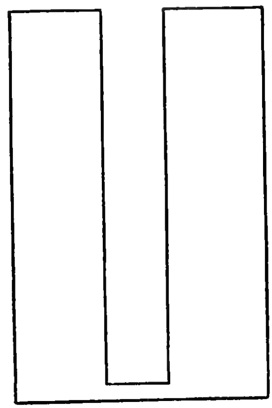

FIGURE 3. Another form of test cylinder for water or mercury, made of hardened nickel steel.

An attempt was made to make a series similar to that on the nickel steel on cylinders cut from a bar of tool steel containing 1.25 per cent carbon. Several of these cylinders cracked in hardening so that a complete set of observations could not be made. The few successful tests made with the hardened tool steel cylinders, however, confirm the conclusions reached with the hardened nickel steel. Thus one cylinder with no hardening flaw burst at 4000 atmos. when filled with mercury, and a similar cylinder, which had a hardening flaw in it, broke at 6500 on the second application of pressure when filled with water.

Another series of tests was now tried which gave the clue to the final explanation. Pressure was applied to cylinders of the form shown (see Figure 3), in which the end opposite the piston is left solid, and may be of varying thickness. In the first tests this bottom was left only $\frac{1}{8}''$ thick. The cylinders were made of the same nickel steel as above, hardened in oil. One, filled with mercury, ruptured by blowing out of the bottom at a pressure of 2400 atmos. The bottom blew out of a second in which the transmitting fluid was water, at a pressure of 12700 atmos. The manner of rupture was very different in the two cases. When water was used, the bottom was blown out in the form of a clean punching, slightly less in diameter than the hole and slightly bulged as one would expect. (See Figure 4.) On the other hand, the piece blown out by the mercury was in the form of a conical cap twice the diameter of the hole, as indicated in Figure 5. The structure of the ruptured metal from A to B was very coarse and granular, remarkably like the structure of brass made rotten by mercury. Between the grains of the steel were minute drops of mercury, and the fractured surface was partially wetted by mercury when plunged beneath a mercury surface. The only clean break was around the edge of the cap (BC), where the rupture took the form of a clean shear as for the piece broken with water.

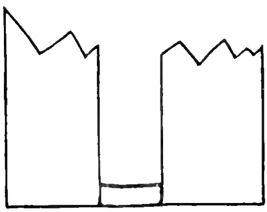

FIGURE 4. Form of the break of the cylinder of Figure 3 when the fluid exerting the pressure is water.

The corresponding tests with tool steel cylinders were hard to carry out because of loss of the cylinders on hardening. However, two successful tests were made. The bottom of one cylinder, when filled with mercury, blew off at 2700 atmos., while a similar cylinder filled with water did not fail until 5200 atmos. The manner of failure was different from that of the nickel steel, the detached piece being in the form of a frustum of a cone, as shown in Figure 6. The form of this cap was the same for both water and mercury, the only difference being that the one fractured with mercury showed unmistakable amalgamation over a limited region (AB). This, together with the curious fracture of the nickel steel piece, suggested the amalgamation of steel by pressure as a possible explanation.

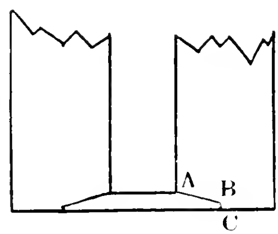

FIGURE 5. Form of the break of the cylinder of Figure 3 when the fluid exerting the pressure is mercury.

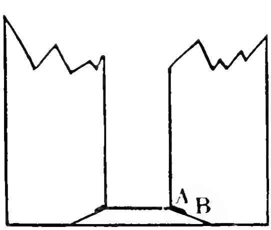

FIGURE 6. Form of the break in a cylinder like that of Figure 3 of hardened tool steel when the fluid exerting the pressure is mercury.

Experiment was also made on two cylinders of tool steel similar to the above, except that the bottom was $\frac{1}{2}''$ thick instead of $\frac{1}{8}''$. The cylinder filled with water stretched non-elastically and leaked at a pressure of 8000 atmos., while that filled with mercury failed at 4000 by the blowing out of mercury along a crack. This break was more like that described by Amagat than any other in the course of these experiments, but even here the break was distinctly visible, while Amagat states that he could find no trace of the crack with a microscope. (See Figure 7.)

In all these tests of the soft steel cylinders the punched out pieces were found more or less amalgamated around the sheared edges. In particular, when the cylinder of soft tool steel last mentioned was cut so that the fissure was exposed, it showed beautiful amalgamation over the entire surface of the rupture. This led to a short investigation of the possibility of amalgamating iron or steel. It has been known for some time that mercury is capable of dissolving a small quantity of iron, and that conversely an iron surface may be amalgamated by mercury.[4] This amalgamation is a matter of some difficulty, which may be made to take place by certain chemical or electrical reactions, but under ordinary conditions does not occur at all. It does not seem to have been noticed that under proper conditions the amalgamation of iron is a matter of the greatest ease, the difficulty under ordinary conditions being due apparently to a thin protecting layer of oxide. The following experiment showed strikingly how great the affinity between clean iron and mercury is. A piece of iron was broken underneath a mercury surface so that the freshly ruptured surface came directly in contact with the mercury. The diagram illustrates the form of experiment. (See Figure 8.) The test piece, in the shape of a thin hollow cylinder, is covered with mercury in an iron receptacle, and broken by forcing a wedge into the hole. In every case the broken surface is brightly and completely amalgamated, the mercury wetting it exactly as water wets a surface to which it adheres. The same result was obtained with hard and soft nickel steel, hard and soft tool steel, bessemer steel

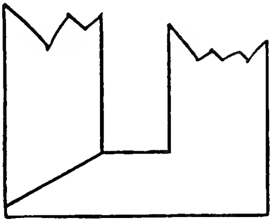

FIGURE 7. Form of the fracture in a cylinder of soft tool steel when the fluid exerting the pressure is mercury.

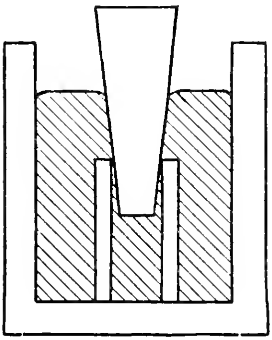

FIGURE 8. Apparatus for fracturing steel under the surface of mercury.

[4] See, for example, Richards, Wilson, and Garrod-Thomas, Pub. Carnegie Inst. Wash., No. 118, 54, 1909; also Richards and Garrod-Thomas, ZS. Phys. Chem., **72**, 181 (1910).

both in its natural state and quenched from a red heat, and with cast iron.[5] The same effect is shown by the same grades of steel when broken off by twisting under a mercury surface, showing that the amalgamation does not depend on the nature of the rupturing stress. This effect was shown in a striking way by one of the hardened tool steel cylinders. (See Plate.) This cylinder broke on the first application of pressure at 1500 atmos. Examination showed a small hardening crack at A from which the crack spread into the sound metal. The mercury, escaping under pressure through the freshly opened crack, produced nearly complete amalgamation of the entire section of the cylinder, as the photograph shows.

But on the other hand, all attempts failed to amalgamate the freshly broken surface if it had once come in contact with the air. A test piece like the above was supported directly over a mercury surface so that the fragments fell immediately into the mercury when rupture occurred. No trace of amalgamation was obtained here, as was also no trace with several modified forms of the experiment in which immersion was not so immediate.

Experiments were now made to find whether the amalgamation so produced might be made to work its way through the mass of the metal. Fragments of the above amalgamated test pieces were sealed into a bulb containing mercury so designed that the pieces should be kept below the surface of the mercury. The air was boiled out, the bulb sealed, and the whole kept at a temperature of 180° C. for three hours in an oil bath. The pieces were then examined for amalgamation by breaking them across. The hardened nickel steel piece showed amalgamation throughout its entire mass, the bessemer showed isolated splotches of amalgamation, while none of the others showed any effect. Similar amalgamated bits from the broken test pieces were now submerged in mercury, which was subjected to a pressure of 6500 atmos. at room temperature for six hours. All of the specimens, except the soft tool steel, now showed amalgamation throughout the interior. The hardened nickel steel piece was amalgamated completely, while the amalgamation of the others was not so perfect, being confined to patches near the surface.

Further, pressure by itself is not capable of producing amalgamation

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[5] Aluminum may also be amalgamated by rupturing under the surface of mercury. Aluminum so treated, when exposed to the air, shows the characteristic tree-like growth of the oxide. The same effect is also shown in nickel and cobalt. It should be remarked that the amalgamated iron surface shows no tendency to oxidize in the air, but keeps its silver luster untarnished at least for months.

on a steel surface or in the mass of the metal unless the amalgamation has been started on the surface by some other means. This was shown by subjecting rods of the three kinds of steel (hard and soft nickel steel, hard and soft tool steel, soft and quenched bessemer) to a pressure of about 6500 atmos. for twelve hours or more under mercury. These rods were scoured bright with fine emery paper immediately before being plunged into the mercury. The fracture after pressure treatment showed not the slightest trace of amalgamation. Reference may be made to some earlier experiments in which no gain of weight could be detected in steel pieces subjected to hydrostatic pressure in mercury. The possibility of amalgamation by pressure was also tried in another form. A hollow cylinder of hardened nickel steel was submerged in mercury, and pressure applied to the outside. The only difference between this and the case of the solid rod is that in the former the stress throughout the metal is not uniform, as it is in the latter. It was thought conceivable that mercury might be forced through metal in which the stress was not the same in every direction, while it might show no tendency to work its way through a mass in which the stress was already hydrostatic. The experiment, however, showed no amalgamation in this case either.

This test for amalgamation by examining the nature of the fracture showed itself so easy to apply and so unmistakable in its indications that it was now applied to the examination of the cylinders which had formed the subject of the first tests. The possibility of the amalgamation of the cylinders as an explanation had at first been discarded because the inner wall of the cylinder, where it was to be most expected, showed no indications of any amalgamation and because attempts to detect the presence of mercury throughout the mass of the metal by microscopic analysis of the polished and etched cross section had given no result. Professor Sanveur had been kind enough to examine four test pieces cut from different cylinders, three of which were ruptured with mercury and one with water. He found martensitic structure in the three former pieces, and only a very fine granular structure in the other. There was no visible trace of mercury in the pores, nor anything to suggest amalgamation. The differences of structure might be due merely to the slight differences in heat treatment occasioned by the separate hardening of the cylinders. More careful regulation of the conditions would be necessary to settle this point.

The fracture test was applied by cutting a scarf about $\frac{1}{4}''$ deep all around the cylinder with a thin emery wheel, and then breaking the cylinder at this scarf with a hammer. All the cylinders broken with the mercury showed the same characteristics. (See Figure 9.) The crack was in a radial plane; surrounding this crack on either side was a band within which the coarser structure and the silver luster showed that the steel had been amalgamated. Besides this band of amalgamation flanking the crack, which was present in every cylinder, there were other irregular splotches of amalgamation growing either from the central hole or from the flanking band. No cases were found of isolated islands of amalgamation in the midst of untouched metal. The diagram gives an idea of typical forms of this amalgamation, although so many varieties were seldom found in one specimen. In one specimen the amalgamation had grown in the form of a nearly complete ring with no contact with the interior except through the infected region about the crack, in the manner shown in the drawing. The tool steel cylinders showed the same effect, except that the amalgamated band about the crack was not so broad as for the nickel steel, and the other patches of amalgamation were less numerous. The peculiar fracture indicating amalgamation was found in none of the cylinders which were broken in the absence of mercury. Several such cylinders were examined, some of hard or soft tool steel, and some of nickel steel, broken with water after the repeated action of the maximum pressure of 24000 atmos.

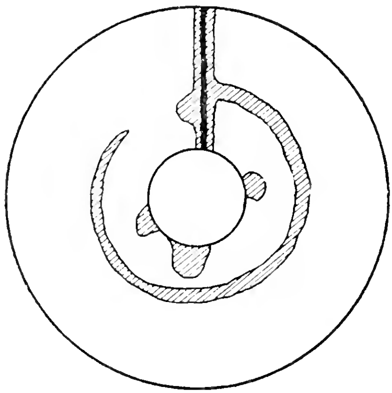


FIGURE 9. The form of the amalgamated region in cylinders like those of Figure 1 when broken with mercury.

Such are the experimental facts which must be explained. It seems evident that the premature breaking of the cylinders filled with mercury was due to the weakening of the steel produced by amalgamation. The fact of amalgamation seems sufficiently proved by the study of the fracture. That amalgamated steel would be weaker than the untouched steel seems obvious enough without the necessity of special experiment to prove it The fact was proved incidentally several times, however, when parts of the steel packing appliances which had been used with the mercury cylinders were used over again with the cylinders filled with water. The packing plugs in these circumstances always broke at a pressure very much less than the normal breaking pressure and with a fracture showing bright amalgamation. In explanation of the amalgamation of these cylinders we have in the first place the strong natural affinity between iron and mercury. This is prevented from coming into play under ordinary circumstances by a thin layer of dirt on the surface. But it seems reasonable to suppose that amalgamation will start in the mass of the metal if the mercury can be once forced into the pores, since under these circumstances the iron and mercury coming into contact with each other would be clean and the natural chemical affinity come into play. The argument consists in showing that in those cases where amalgamation took place the conditions were such as to favor the introduction of mercury into the pores of the steel, even if the surface were not amalgamated. Then after amalgamation is once started in the mass of the metal it is assisted in the rapidity of its growth from the amalgamated region by the action of hydrostatic pressure.

This discussion demands a slight consideration of the nature of the strain in a hollow cylinder exposed to internal fluid pressure. The stresses in the metal of the cylinder consist of a pressure (negative) across planes perpendicular to the radii, and a tension (positive) across radial planes. These stresses are greatest arithmetically at the interior surface, but the algebraic sum is constant throughout the mass of the cylinder. This has as a consequence that the volume strain in the cylinder is a dilation and is everywhere constant, so that the pores are opened up by the action of the stress and the entrance of mercury facilitated. This holds while the strain remains elastic. But when the internal pressure exceeds a certain value so that at the inner surface the algebraic difference between the radial pressure and the circumferential tension exceeds a critical value depending on the elastic limit, the strain becomes inelastic, the tension changes over to a pressure so that both principle stresses become compressions, the volume strain changes from a dilatation to a compression, and the pores close up. So that with a steel of low elastic limit the type of stress may change, giving a volume compression, at a lower value of the fluid pressure and therefore at a smaller preliminary volume dilatation than in a steel of higher elastic limit. This view as to the nature of the stress in a thick cylinder stressed beyond the elastic limit is supported by many other experiments on the bursting of thick cylinders. An account of these experiments will be published in another paper.

The difference found in the rupture points between soft and hardened steel is to be ascribed to two causes. One is the greater intrinsic ease of driving amalgamation through a mass of hardened steel by hydrostatic pressure. This was proved by the experiments on the broken amalgamated test pieces. It may be due in part to chemical difference between the hard and soft steel, but is almost certainly also due in part to the greater porosity of the hardened steel. It is well known that steel decreases in density on hardening. The other cause is the higher elastic limit of the hardened steel and the consequent wider opening of the pores before non-elastic closing sets in. For facility of comparison the elastic limits of the cylinders of the various grades of steel are given (see Table III), on the usual assumption that the elastic limit is determined by the maximum stretch at the interior. This assumption is probably not very near the truth, so the results can be expected to show only qualitative agreement. This table does not apply to the

TABLE III.

Kind of Steel.	Assumed Elastic Limit in Tension $\frac{\text{lbs}}{\text{in}^2}$.	Internal Pressure required to reach Elastic Limit. Atmos.	Minimum Bursting Pressure with Mercury. Atmos.
Bessemer . . .	40000	2000	Same as with water
Soft tool steel .	75000	4000	Same as with water
Hard tool steel .	150000	8000	4000
Soft nickel steel	90000	4800	Same as with water
Hard nickel steel	225000	12000	3000

cylinders in which the bottoms were blown out, as here the stress is of a different nature and the above values of the elastic limit do not hold. It appears from the table that only those cylinders were broken in which the stress was below the elastic limit. That the cylinders of bessemer and soft nickel steel were not broken may be explained by supposing that the pressure required to open the pores wide enough to force the mercury in is higher than the elastic limit.

It remains to explain the form of the amalgamated region found in the hardened nickel steel cylinders; i. e., bands on either side of the crack. It seems certain that slight inequalities in the structure of the steel will greatly affect the ease of amalgamation. This seems proved by the splotches of amalgamation found throughout the broken test pieces amalgamated by hydrostatic pressure and throughout the broken cylinders. Conceive, then, of a hollow cylinder filled with mercury subjected to pressure. The pressure expands the pores of the metal, and in consequence of the high mobility of the mercury molecule the mercury is forced into these pores through the layer of dirt lining the hole. At certain places where the metal is more susceptible the amalgamation spreads more rapidly than it does at others. But now the metal is weakened at each of these infected places, and the type of strain is modified as it would be by the presence of a flaw. The strain will be redistributed, the brunt of the strain coming at the point farthest removed from the center, where consequently the pores of metal will be still further distended. That is, at this point the amalgamation will proceed most rapidly. It is evident that the continuation of this process will produce a band of amalgamation travelling out along the radius. When the amalgamation reaches the outside, or approaches sufficiently close, the metal gives way, the crack appearing through the midst of the weakest region, that is, through the center of the amalgamated band. It is evident that when the process of amalgamation has once started in this way, it will proceed more and more rapidly as the resisting thickness of sound metal becomes less, thus accounting for the smallness of the other amalgamated regions. One cylinder was found, however, in which an amalgamated patch had worked its way nearly half way to the outside diametrically opposite the crack.

Failure to produce amalgamation in the rods subjected to hydrostatic pressure is to be explained by the fact that neither can the amalgamation begin at the surface, because of the thin layer of dirt, nor can the mercury force its way into the steel to begin amalgamation there because the interstices in the metal are closed up by the hydrostatic pressure. The same argument of course applies still more to the hollow cylinder submerged in mercury and subjected to pressure on the outside.

The experiments with the cylinders in which the bottoms were blown off are to be explained in the same way. The amalgamation grows most rapidly in the direction in which the distension is greatest, which in this case is diagonally from the corner of the hole. As the amalgamation proceeds it carries the hydrostatic pressure with it. When the region over which this pressure acts has extended so far that the sound metal left can no longer support the stress, it gives way as usual by a clean shear. The fact that mercury was forced through the bottom of a cylinder of soft tool steel (Figure 7), while soft tool steel cylinders of the form of Figure 1 were unbroken by the action of pressure, is probably to be explained by the different strain types in the two cases. In the case of Figure 7 distension in a direction diagonally from the corner of the hole is great enough to allow amalgamation before the pores are closed up by viscous yield. The point has not been worked out in greater detail, however. The disconcerting experiments in which a clean punching was blown out of the bottom of a soft cylinder with the sides of the punching amalgamated, are to be explained by the fact previously noted that a freshly fractured surface is amalgamated by mere contact with mercury. Breaking across of the punchings showed no evidence of amalgamation in the mass of the punching, so that all the amalgamation must have taken place after rupture. A similar experiment on the hardened nickel steel punching mentioned above showed more or less complete honeycombing of the metal with mercury.

This is as far as the explanation has been carried. Enough has been done to show that there is here a genuine effect, so that pressure can not be transmitted directly by mercury in hardened steel cylinders, and that the effect is due to amalgamation. One-sided pressure is necessary to start this amalgamation, so that when steel is entirely surrounded by mercury there is no danger of amalgamation or of penetration of the mercury into the pores. This fact was made use of in modifying the design of the apparatus spoken of in the first part of the paper.

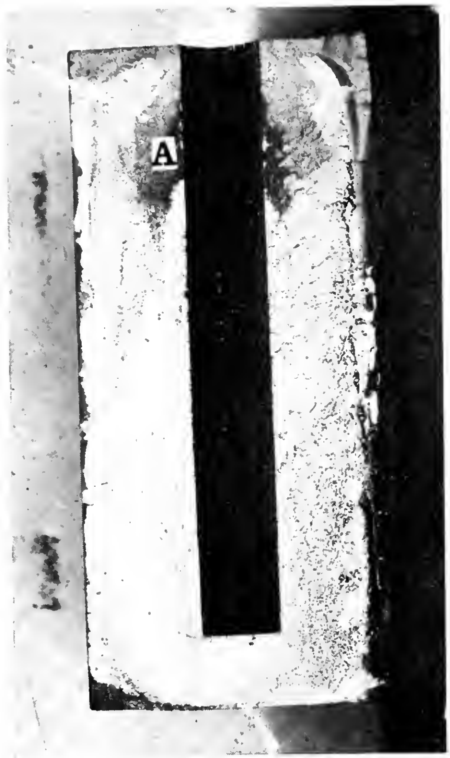
Summary.

The fact has been established that cylinders of hardened steel will burst at very much lower than the natural bursting pressure when the fluid exerting the pressure is mercury. Soft steel cylinders show the effect hardly at all, the yield point being reached before the pressure can be raised high enough to produce the effect. The fact that this rupture is due to the amalgamation of the steel is established by the examination of the fracture of such cylinders. The unexpectedly great affinity between steel and mercury was established by the complete amalgamation of surfaces broken under mercury, and the enormous effect of the slightest contact with the air was shown. When this amalgamation is once started, the rapidity with which it spreads through the metal is greatly increased by the action of hydrostatic pressure. The spread of mercury through the mass of the steel and the subsequent destruction of the hollow cylinders is produced by two causes, both of which must act together. One is the natural chemical affinity between mercury and steel, shown by the ready amalgamation of freshly broken surfaces. But the amalgamation is never started by the action of pressure alone. In all those cases in which we have had amalgamation, we have had in addition to the chemical affinity a strain of such a nature as to distend the pores of the metal. This allows the entrance of mercury into the pores so that amalgamation may begin, and also facilitates its further growth, which is most rapid in the

direction in which the pores are most distended. It was shown in detail that in all cases in which rupture occurs the strain is of such a type as to distend the metal, and that on the other hand in all those cases in which amalgamation is not produced by pressure, the strain is such as to compress the metal, closing up the pores.

This work was done in the course of an experiment on the thermal properties of mercury and water under high pressure, the expenses of which were partially defrayed by a liberal appropriation from the Rumford Fund of the American Academy of Arts and Sciences.

Jefferson Physical Laboratory, Harvard
University, Cambridge, Mass.
October, 1910.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INFINITESIMAL PROPERTIES OF LINES IN S_4 *WITH APPLICATIONS TO CIRCLES IN* S_3.

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Introduction.

In a recent paper (to be published in the American Journal, 1911) the author discussed properties of systems of lines in S_4 involving differentials of the first order, that is, simple tangency, and interpreted the results in the geometry of the circle in space of three dimensions. It is the purpose of the present paper to discuss properties of lines and circles involving second and higher differentials. The methods of the first paper are not applicable to this case. Here lines are represented by points of a six-dimensional spread which stands in a space of nine dimensions. The problem is, then, simply one of differential geometry in higher space.

1. *The coordinates.*[1] In S_4 let the line be defined by two points (homogeneous coordinates). Then the coordinates of a line are the two row determinants of the matrix,

$$\left\| \begin{matrix} x_1 & x_2 & x_3 & x_4 & x_5 \\ y_1 & y_2 & y_3 & y_4 & y_5 \end{matrix} \right\|.$$

Adopting the usual notation

$$p_{ik} = x_i y_k - x_k y_i,$$

we see that there are ten coordinates, since

$$p_{ik} = -p_{ki}, \qquad p_{ii} = 0.$$

These ten coordinates are connected by the five quadratic relations

$$(1) \qquad \Omega_i \equiv p_{kl}p_{mn} + p_{km}p_{nl} + p_{kn}p_{lm} = 0$$

[1] Castelnuovo, "Ricerche di geometria della retta nello spazio a quattro dimensione," Atti del Reale Istituto Veneto, series 7, **2**, 1890–91. In this paper only linear systems of lines have been considered.

($i = 1, 2, 3, 4, 5$, and $k\ l\ m\ n$ is a permutation of the remaining numbers after i is chosen). It is well known, however, that of these five relations Ω_i only three are distinct.

Now, letting p_{ik} be the ten homogeneous coordinates of a point in a space of nine dimensions S_9, the points which represent lines in S_4 are the points common to the five quadrics

$$\Omega_i = 0.$$

The intersection of these quadrics is known to be of order five and dimensions six. In our discussions, then, we shall be interested only in the points of this variety which we shall indicate by Φ. The following properties can be easily verified.

(*a*) The hyperplanes

$$(2) \qquad \sum_1^5{}_i\, \xi_i p_{ik} = 0, \qquad k = 1, 2, 3, 4, 5,$$

cut Φ in quadrics $V_4{}^2$ of order two and dimensions four ; hence Φ contains ∞^4 such quadrics.

(*b*) The S_3 defined by

$$p_{ik} = l_1 p_{ik}' + l_2 p_{ik}'' + l_3 p_{ik}''' + l_4 p_{ik}'''',$$

where p', p'', p''', p'''' correspond to lines passing through a common point, lies in Φ.

(*c*) The S_2 defined by

$$p_{ik} = l_1 p_{ik}' + l_2 p_{ik}'' + l_3 p_{ik}''',$$

where p', p'', p''' represent lines which lie in the same plane, also lies in Φ. It is easily seen that the planes (*c*) and the spaces (*b*) do not in general intersect. It can also be easily shown that two quadrics (*a*) intersect in a plane (*c*).

A pencil of lines in S_4 is represented by a line which is contained in Φ. The points of a quadric (*a*) represent the lines of S_4 which lie in a space of three dimensions S_3. A space (*b*) represents the lines in S_4 which pass through a fixed point, and a plane (*c*) represents the lines which lie in a fixed plane.

The lines which cut a fixed line p_{ik}' are represented by the intersection of Φ with its tangent S_6 at the point p_{ik}'. (Here p_{ik}' is used to denote a line in S_4 or a point on Φ.)

$$(3) \qquad \sum \frac{\partial \Omega_i}{p_{kl}} p_{kl}' = 0.$$

The intersection is of order three and dimensions four. The lines which cut two fixed lines will be represented by the intersection of two such tangent spaces S_6 with Φ; this is an ordinary quadric contained in Φ.

2. *Directions in ruled space of four dimensions.* In ruled space of four dimensions as in three dimensions a direction through a given line of a system is defined as the Chasles correlation which a ruled surface belonging to the system and passing through the given line determines; that is to say, a direction through a given line is determined by the given line and a line infinitely near to it. In the S_9 defined by the coordinates p_{ik} this corresponds to the tangent lines to the curves traced on the variety Φ, but since the points of S_9 which lie outside of Φ have no significance, the tangent lines in general will have none. We can, however, consider the polar of such a line. This polar will intersect Φ in an ordinary quadric cone, and since it can be looked upon as the intersection of two tangent spaces S_6 to Φ at infinitely near points of the given line, the points of this cone will represent lines of S_4 which cut two infinitely near lines, that is, will represent the special congruence determined by the two infinitely near lines. This congruence represents the Chasles correlation determined by the two lines. If the tangent line to Φ lies entirely in Φ, the quadratic cone and consequently the congruence will degrade. A curve all of whose tangents lie in Φ will represent a developable in S_4 because all the Chasles correlations are degenerate.

We saw that a tangent S_6 cuts Φ in a V_4^3. Then, if this S_6 is cut by an arbitrary hyperplane, the V_4^3 will be cut in a variety ϕ_3^3 of order three and dimensions three. Each point of Σ, the intersection of S_6 and the hyperplanes, will represent a direction through the line r corresponding to the point of tangency. The points of ϕ_3^3 will correspond to the special directions through r, that is, to the degenerate projectivities. The variety ϕ_3^3 and the space Σ in which it stands can be used instead of the ones used by the author in the paper previously referred to, and all the results there obtained can be obtained here. Properties of lines involving differentials of the first order are disposed of in this manner.

I. Properties of Five-Parameter Families of Lines.

3. In the following, for the convenience of writing, the coordinates $x_1, x_2 \ldots x_{10}$ will be used instead of $p_{12}, p_{13} \ldots p_{45}$. Also a variety on Φ and the system of lines which it represents will be denoted by the same symbols where there can be no ambiguity. The symbols $C_1, C_2 \ldots C_5$ will be used to denote linear systems of lines.

Consider a variety V_5 traced on Φ; that is, let the coordinates x be functions of the five parameters $u_1, u_2 \ldots u_5$. Here and following all the functions used are supposed to be continuous and to possess all partial derivatives of the n^{th} order. The tangent lines to V_5 will generate an S_5 determined by the six points [2]

$$(\pi_5) \qquad (x), \quad \left(\frac{\partial x}{\partial u_1}\right), \quad \left(\frac{\partial x}{\partial u_2}\right), \quad \left(\frac{\partial x}{\partial u_3}\right), \quad \left(\frac{\partial x}{\partial u_4}\right), \quad \left(\frac{\partial x}{\partial u_5}\right).$$

Through this S_5 will pass ∞^4 hyperplanes which are tangent likewise to V_5 (an hyperplane is said to be tangent to a variety if it contains the tangent space to the variety). Hence, *In each line of a five-parameter family V_5 there are ∞^4 C_5's tangent to it.*

The osculating planes of all the curves traced on V_5 which have the same tangent line in x generate an S_6 which contains the space π_5 tangent to V_5, as may be seen as follows:

Using Segre's[3] notation, the osculating planes will be determined by the three points

$$(4) \qquad f = 0,$$

$$(5) \qquad \Sigma f_i du_i = 0,$$

$$(6) \qquad \Sigma f_{ik} du_i du_k + \Sigma f_i d^2 u_i = 0.$$

Tangent lines are lines which join (4) to any point of the space determined by f_1, f_2, f_3, f_4, f_5 and hence generate π_5. Now, if the du_i are held fixed (that is, the direction of the tangent is fixed) and the d^2u_i are allowed to vary, we see that the osculating plane will always stand in the S_6 defined by

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[2] A tangent line joins the points x and $x + dx$, or expanding x and $x + \Sigma \frac{\partial x}{\partial u_i} du_i$. As the du_i determine the direction along V_5, the space generated by these lines will be the S_5 named.

[3] "Su una classa di superficie ecc." Atti di Torino, 1907. Points are represented by their equations in hyperplanar coordinates or also with the first member of these equations. Thus

$$f = \Sigma \xi^{(i)} x^{(i)},$$

where $\xi^{(i)}$ are the hyperplanar coordinates. The point which describes a variety shall be represented by f. We may also speak of the point x, represented by f, or $\Sigma \xi^{(i)} x^{(i)} = 0$. Subscripts are reserved to denote derivatives; thus,

$$f_{ijk} = \frac{\partial^3 f}{\partial u_i \partial u_j \partial u_k}.$$

$$(7)\qquad f=0,\ f_1=0,\ f_2=0,\ f_3=0,\ f_4=0,\ f_5=0,$$
$$\Sigma f_{ik}du_i du_k=0.$$

Since there are ∞^4 planes obtained allowing d^2u_i to vary, this S_6 must be the locus. From (7) it is evident that the S_6 contains the tangent space π_5, which is determined by

$$f=0,\ f_1=0\ \ldots\ f_5=0.$$

We will call the space defined by (7) an *hyperosculating* S_6. Now, if an S_6 containing π_5 is given, then a direction (values of du_i) can be determined so that the S_6 will hyperosculate V_5 along this direction. It is only necessary for the du_i to satisfy the relations

$$(8)\qquad \begin{aligned}(\xi,\ \Sigma f_{ik}du_i du_k)&=0,\\ (\xi',\ \Sigma f_{ik}du_i du_k)&=0,\\ (\xi'',\ \Sigma f_{ik}du_i du_k)&=0,\end{aligned}$$

where the symbol $(\xi, x)=0$ is used for $\Sigma\,\xi^{(i)}x^{(i)}=0$ and the S_6 is defined as the intersection of the three hyperplanes

$$(\xi,x)=0,\quad (\xi',x)=0,\quad (\xi'',x)=0.$$

Then the tangents to V_5 along which the S_6 is hyperosculating are determined by the values of du_i which satisfy (8). There are ∞^1 values of du_i which satisfy (8), and the tangents so defined generate a cone of order eight. Therefore *an* S_6 *passing through* π_5 *will hyperosculate* V_5 *along* ∞^1 *directions forming a cone of order eight.* This shows that the hyperosculating S_6 cuts V_5 in a surface which has a conical point of order eight. It is also evident that a hyperplane passing through a hyperosculating S_6 will cut V_5 in a V_4 which has a conical point of order two, since only the first of equations (8) would have to be satisfied. Similarly, an S_7 which contains the hyperosculating S_6 will cut V_5 in a V_3 which has a conical point of order four, since only two of equations (8) would have to be satisfied.

4. We have just seen that a hyperplane passing through π_5 cuts V_5 in a V_4 with a conical point in x, the point of tangency. Let us now examine how this cone varies as the hyperplane varies. The coordinates of the hyperplane will be

$$\xi_i=\lambda\alpha_i+\mu\beta_i+\nu\gamma_i+\eta\delta_i$$

(α, β, γ, δ are the coordinates of four hyperplanes passing through π_5). The cones therefore will form a linear series of three parameters and hence will have sixteen generators in common. *These are the sixteen directions upon which rest many of the infinitesimal properties of higher order of five-parameter families of lines.*

The tangent space π_5 cuts V_5 in a curve which has a multiple point of order sixteen in x. The tangents at the multiple point are the sixteen directions noted above.

These sixteen directions correspond to the asymptotic directions on a surface in S_3 both in the sense that two successive tangent spaces π_5 intersect in a line joining the points of contact and in the sense that tangents to V_5 in one of these directions have three-point contact. The latter property can be seen by deriving the condition that a tangent line have three-point contact. From equations (4), (5), (6) we see that this would simply lead to the condition that the point

$$\Sigma f_{ik} du_i du_k = 0$$

should lie in the tangent π_5, because in that case the quantities d^2u_i could be so chosen that the three-point (4,) (5,) (6), that is, (x), $(dx,)$ (d^2x), would lie on a line. The condition is, then, the vanishing of the determinants of the matrix

$$\left\| x \quad \frac{\partial x}{\partial u_1} \quad \frac{\partial x}{\partial u_2} \quad \frac{\partial x}{\partial u_3} \quad \frac{\partial x}{\partial u_4} \quad \frac{\partial x}{\partial u_5} \quad \sum \frac{\partial x}{\partial u_i \partial u_k} du_i du_k \right\|$$

and this is exactly the condition that defines the sixteen directions above.

5. In order to see that the tangent spaces at two consecutive points of V_5 along one of these sixteen directions intersect along the line joining the points of contact, we will first examine what corresponds to conjugate directions on V_5.[4] If we consider the points of V_5 as determined by the hyperplanes passing through them, then any point will be given by

$$(\xi, x) = 0,$$

where x is a function of the five u's. Then, if the tangent π_5 be looked upon as the intersection of four hyperplanes, the condition that the hyperplanes pass through x is

$$(9) \qquad (\xi', x) = 0, \quad (\xi'', x) = 0, \quad (\xi''', x) = 0, \quad (\xi^{\text{IV}}, x) = 0,$$

[4] The argument here is the same as that used by Segre for discussing a V_3 in S_5 in his 1907–8 lectures on "Geometria della retta."

and if this S_5 is tangent to V_5,

$$(10)\qquad \left(\xi', \frac{\partial x}{\partial u_1}\right) = 0,\quad \left(\xi', \frac{\partial x}{\partial u_2}\right) = 0,\quad \left(\xi', \frac{\partial x}{\partial u_3}\right) = 0,$$
$$\left(\xi', \frac{\partial x}{\partial u_4}\right) = 0,\quad \left(\xi', \frac{\partial x}{\partial u_5}\right) = 0,$$

and a similar set of equations for the hyperplanes ξ'', ξ''', ξ^{IV}. The S_5 infinitely near is determined by

$$(11)\qquad (\delta\xi', x) = 0,\ (\delta\xi'', x) = 0,\ (\delta\xi''', x) = 0,\ (\delta\xi^{\text{IV}}, x) = 0.$$

These two spaces will intersect in a line. If the point dx infinitely near to x is to lie in (11),

$$(12)\qquad (\delta\xi', dx) = 0\ \ .\ \ .\ \ .\ \ (\delta\xi^{\text{IV}}, dx) = 0.$$

Now, using δu_i to denote the direction of the line of intersection of (10) and (11), and du_i to denote the direction of the line joining x and $x + dx$, equations (12) take the form

$$(13)\qquad \left(\sum \frac{\partial \xi'}{\partial u_i}\delta u_i,\ \ \sum \frac{\partial x}{\partial u_i} du_i\right) = \sum\left(\frac{\partial \xi'}{\partial u_i}, \frac{\partial x}{\partial u_j}\right)\delta u_i du_j = 0,$$

and similar equations for the other ξ's.

Differentiating (9) and (10) partially with respect to the u's, we have

$$\left(\frac{\partial \xi'}{\partial u_i}, \frac{\partial x}{\partial u_j}\right) + \left(\xi', \frac{\partial^2 x}{\partial u_i \partial u_j}\right) = 0.$$

Substituting these values in (13),

$$(14)\qquad \sum\left(\xi', \frac{\partial^2 x}{\partial u_i\partial u_j}\right)\delta u_i du_j = \left(\xi', \sum\frac{\partial^2 x}{\partial u_i\partial u_j}\right) = 0.$$

Now, if we put $du_i = \delta u_i$ we have the system of equations (8); hence that is the condition that should be verified if the two consecutive tangent spaces are to intersect in the line joining the points of contact.

Since V_5 is contained in Φ (the intersection of the five quadrics Ω_i), the tangents to V_5 which have three-point contact must lie entirely in Φ. The sixteen lines discussed above then represent pencils of lines in S_4 and may be called *osculating* pencils, since they contain three lines of V_5 infinitely near. Further the planes of these pencils are inflexional for the cones of the system V_5 passing through the vertices of the pencils. Then

Through a line p of V_5 pass sixteen cones of V_5 which have inflexional planes, and sixteen curves of V_5 lying in planes passing through p have the line p for inflexional tangent.

The tangent π_5 to V_5 at x cuts V_5 in a curve which has in x a multiple point of order sixteen; hence *The ∞^4 linear series C_5 tangent V_5 in a line p cut V_5 in a ruled surface having p for multiple generator of order sixteen.*

6. The locus of all osculating S_4's of the curves traced on V_5 and passing through x and having the same osculating S_3 is a hyperplane S_8. To show this let us consider the S_4 as defined by the points

$$(x),\ (dx),\ (d^2x),\ (d^3x),\ (d^4x),$$

which in hyperplanar coordinates are

$$f = 0, \tag{4}$$

$$\Sigma f_i du_i = 0, \tag{5}$$

$$\Sigma f_{ij} du_i du_j + \Sigma f_i d^2 u_i = 0, \tag{6}$$

$$\Sigma f_{ijk} du_i du_j du_k + 3\,\Sigma f_{ij} d^2 u_i du_j + \Sigma f_i d^3 u_i = 0, \tag{15}$$

$$\Sigma f_{ijkl} du_i du_j du_k du_l + 6\,\Sigma f_{ijk} du_i du_j d^2 u_k + 3\,\Sigma f_{ij} d^2 u_i d^2 u_k + 4\,\Sigma f_{ij} d^3 u_i du_j + \Sigma f_i d^4 u_i = 0. \tag{16}$$

If $d^4 u_i$ are allowed to vary and all the other differentials are held fixed by reasoning similar to that previously used, it will be seen that the osculating S_4 will generate an S_8 determined by the nine points

$$f = 0,\ f_1 = 0\ \ldots\ f_5 = 0,\quad \Sigma f_{ij} du_i du_j = 0,$$

$$\Sigma f_{ijk} du_i du_j du_k + 3\,\Sigma f_{ij} d^2 u_i du_j = 0,$$

$$\Sigma f_{ijkl} du_i du_j du_k du_l + 6\,\Sigma f_{ijk} du_i du_j d^2 u_k + 3\,\Sigma f_{ij} d^2 u_i d^2 u_k + \Sigma f_{ij} du_i d^3 u_j = 0.$$

We see that this S_8 contains the hyperosculating S_6 and the tangent S_5. In a similar manner it is seen that the osculating S_3's which have an osculating plane fixed generate an S_7 which is contained in the S_8 and which contains the hyperosculating S_6. If du_i defines one of the sixteen principal directions, the above S_8 becomes an S_7, etc.

We saw that a hyperosculating S_6 defined by any direction through a point x hyperosculates along a whole cone of directions passing through x. If, however, the given direction is one of the sixteen principal directions, the hyperosculating S_6 becomes indeterminate; that is, there is an unlimited number corresponding to this direction. But an S_6 may be determined in this case which hyperosculates four points. There are sixteen such S_6's.

In ruled space of four dimensions we have:

All the linear series C_1 defined by taking a line p of V_5 and four lines consecutive to it, if only the last of the four consecutive lines is allowed to vary, are contained in a linear series C_5.

The tangent congruence to V_5 (the base of the ∞^4 linear systems C_5 tangent to V_5 in p) has three-line contact with all the ruled surfaces passing through p in one of the sixteen principal directions. There is a C_3 which has four-line contact with all the surfaces traced on V_5 tangent to a principal direction. There are sixteen such series C_3. Each one contains the tangent congruence.

A curve all of whose tangent lines lie in Φ we saw represents a developable surface in S_4; hence, *The lines of V_5 can be grouped into ∞^4 developable surfaces in sixteen ways; that is, sixteen of these developables pass through each line.*[5] *Each tangent plane to one of these developables osculates V_5.* (The pencils of lines lying in the tangent plane and having their vertices on the edge of regression contain three infinitely near lines of V_5.)

There are sixteen curves of V_5 (curves enveloped by lines of V_5) *having a given line p for tangent line such that the osculating planes containing p determine the sixteen osculating pencils of which p is a part.*

7. *Other important directions.* Equations (14) set up a correspondence between two directions t and t' defined by du_i and δu_i. This correspondence is reciprocal, and we saw that the coincidences define the sixteen principal directions. This correspondence between t and t', however, is not projective, as can best be seen as follows. Consider V_5 as the intersection of four hypersurfaces in S_9. If the tangent space π_5 is cut by an arbitrary hyperplane t and t' are represented by points in this S_4, the tangent hyperplanes to the hypersurfaces at points of V_5 cut V_5 in V_4's having a conical point. The cones formed by the tangent lines at these conical points are represented in the S_4 (intersection of π_5 by a hyperplane) by quadrics. Then t' is the polar of t with respect to these four quadrics; hence,

When t describes a line, t′ describes a curve generated by the intersection of corresponding hyperplanes in S_4 of four projective pencils and therefore is a curve of order four.

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[5] This system of sixteen developables does not contain all the developables which may pass through a given line, but there are no others having the properties mentioned above.

The sixteen principal directions are represented in the S_4 by the sixteen points of intersection of the four quadrics.

The correspondence between t and t' will not be unique if t coincides with one of the vertices of the self-conjugate five-point with respect to any two of the quadrics. By choosing the defining hypersurfaces differently the self-conjugate points can be chosen in ∞^2 different ways These points are in fact the vertices of the cones belonging to the three-parameter family of quadrics defined by the four quadrics above. Now, if du_i be taken as the homogeneous coordinates of the points of the S_4 above, it is seen at once that the coordinates of the vertices of these cones are given by

$$\sum_h \left(\xi, \frac{\partial^2 x}{\partial u_h \partial u_k}\right) \partial u_h = 0, \qquad k = 1, 2, 3, 4, 5. \tag{17}$$

(Two parameters are independent.) We shall next show that these directions are such that along them a hyperplane has two-point contact with V_5.

The condition that the hyperplane $(\xi, x) = 0$ be tangent to V_5 at x is given by

$$(\xi, x) = 0, \qquad \left(\xi, \frac{\partial x}{\partial u_1}\right) = 0 \;\ldots\; \left(\xi, \frac{\partial x}{\partial u_5}\right) = 0. \tag{18}$$

If it also passes through $x + dx$, then

$$(\xi, x + dx) = (\xi, x) + (\xi, dx) = (\xi, x) + \sum \left(\xi, \frac{\partial x}{\partial u_i}\right) du_i = 0.$$

From (18) it is at once seen that this condition is verified. Now, in order that the hyperplane be tangent at $x + dx$, we must have

$$\left(\xi, \frac{\partial x}{\partial u_i} + \frac{\partial}{\partial u_i} dx\right) = \left(\xi, \frac{\partial x}{\partial u_i}\right) + \left(\xi, \sum \frac{\partial^2 x}{\partial u_i \partial u_k} du_k\right) = 0;$$

hence from (18)

$$\left(\xi, \sum \frac{\partial^2 x}{\partial u_i \partial u_k} du_k\right) = \sum \left(\xi, \frac{\partial^2 x}{\partial u_i \partial u_k}\right) du_k = 0,$$

which is equation (17) again. Hence the directions defined by (17) are those along which a hyperplane may have two points of contact with V_5. These hyperplanes are called *stationary hyperplanes.*

Through each point of V_5 *pass* ∞^2 *curves such that along these curves hyperplanes have two-point contact with* V_5.

Tangent π_5's at two consecutive points of one of these curves do not intersect in a line but in general in a plane. Then in S_4 we have the following theorem:

Through any line p *of* V_5 *may be passed* ∞^2 *ruled surfaces in such a manner that there are linear series* C_5 *tangent to* V_5 *in two lines of the ruled surface infinitely near.*

II. Four-Parameter Families V_4.

8. Let V_4 be the variety on Φ whose points represent the family of lines. The tangent lines to V_4 at a point x generate a linear space π_4 of four dimensions. Through π_4 pass ∞^6 S_7's and hence, *There are* ∞^6 *linear series* C_4 *tangent to a four-parameter family of lines in a line of the family.*

The osculating spaces of the curves on V_4 which pass through a given point x are determined by the points

(1) $$f = 0,$$

(2) $$\Sigma f_i du_i = 0,$$

(3) $$\Sigma f_{ik} du_i du_k + \Sigma f_i d^2 u_i = 0,$$

(4) $$\Sigma f_{ijk} du_i du_j du_k + 3 \Sigma f_{ik} du_i d^2 u_k + \Sigma f_i d^3 u_i = 0,$$

(5) $$\Sigma f_{ijkl} du_i du_j du_k du_l + 6 \Sigma f_{ijk} du_i du_j d^2 u_k + 3 \Sigma f_{ij} d^2 u_i d^2 u_j + 4 \Sigma f_{ij} du_i d^3 u_j + \Sigma f_i d^4 u_i = 0.$$

By the same reasoning previously used it is at once seen that the osculating S_4's which have the same osculating S_3 in common generate an S_7 and the osculating planes having a tangent line in common generate an S_5.

The S_5 generated by the osculating planes having the tangent line (having direction du_i) in common is determined by the points

$$f = 0, \quad f_1 = 0, \quad f_2 = 0, \quad f_3 = 0, \quad f_4 = 0, \quad \Sigma f_{ik} du_i du_k = 0.$$

It is then the space which joins π_4 to the point

(6) $$\Sigma f_{ik} du_i du_k = 0.$$

Now, if du_i are allowed to vary, (6) will generate a variety of three dimensions and order eight[6] V_3^8, and hence the osculating planes of

[6] See Segre. "Prelimnai di una teoria della varità luoghi di spazi," Rendiconto di Palermo, **30**, 104 (1910).

all the curves which pass through a given point x will generate a variety of order eight and dimensions eight V_8^8, which has the tangent space π_4 for vertex.

Then in S_4

All the C_1's which contain a fixed line p of V_4 and two fixed lines infinitely near to it are contained in a C_4.

There is a C_2 which contains all the lines infinitely near, to infinitesimals of the second order, to a given line of V_4 in a given direction (direction being defined by two lines infinitely near of the first order.)

If the directions are varied, the C_2's above will generate a five-parameter family of lines of order eight which has the lines of a ruled surface C_1 for double lines.

9. *Noteworthy directions on V_4.* Through each point of V_4 pass ∞^2 important directions. In these directions there are curves whose osculating plane has four consecutive points in common with the V_4. If this is the case, the points (1), (2), (3), (4) must lie in the same plane. If du_i and d_2u_i are held fixed for the moment and the d^3u_i are allowed to vary, it is at once seen that if these four points lie in a plane then it is necessary that the points

$$(1), (2), (3), \Sigma f_{ijk}\, du_i du_j du_k + 3\,\Sigma f_{ik} du_i d^2u_k = 0,$$

$$\Sigma f_i d^3u_i = 0,$$

lie in the same S_3, and consequently the points

(7) $$f = 0, \quad f_1 = 0, \quad f_2 = 0, f_3 = 0, \quad f_4 = 0,$$

(6) $$\Sigma f_{ik} du_i du_k = 0,$$

(8) $$\Sigma f_{ijk}\, du_i du_j du_k + 3\,\Sigma f_{ik} du_i d^2u_k = 0,$$

will lie in an S_5. Conversely it is easily shown that if these points lie in an S_5 then values of d^3u_i can be determined so that the original four points (1), (2), (3), (4) will lie in a plane. Now, writing the last term of (8) in the form

$$d^2u_1 \Sigma f_{1k} du_k + d^2u_2 \Sigma f_{2k} du_k + d^2u_3 \Sigma f_{3k} du_k + d^2u_4 \Sigma f_{4k} du_k,$$

and noting that (6) can be written in a similar form where the d^2u_i are replaced by du_i we conclude that if the points (6), (7), (8) lie in an S_5 it is necessary that the ten points

(7) $$f = 0, \quad f_1 = 0, \quad f_2 = 0, \quad f_3 = 0, \quad f_4 = 0,$$

(9) $$\Sigma f_{ik} du_i = 0, \qquad k = 1, 2, 3, 4,$$

(10) $$\Sigma f_{ijk} du_i du_j du_k = 0,$$

will lie in an S_8, and conversely, if these ten points lie in an S_8, d^2u_i and d^3u_i can be so determined that the original four points will lie in a plane. The condition that these ten points lie in a plane is

$$(11) \qquad \left| x \ \frac{\partial x}{\partial u_1} \ \ldots \ \frac{\partial x}{\partial u_4} \ \Sigma f_{ik} du_k \ \ldots \ \Sigma f_{ijk} du_i du_j du_k \right| = 0.$$

Hence, if the du_i be considered as the homogeneous coordinates of the directions of the tangent lines at a point of V_4, we see that *Through each point of V_4 pass ∞^2 directions whose tangent lines form a cone of order seven such that along these directions there are curves whose osculating planes cut V_4 in four consecutive points.*

Then in S_4 we have

Through each line p of a four-parameter family V_4 pass ∞^2 directions such that there is a ruled surface tangent to each of these directions (containing the two infinitely near lines which determine the direction) *and possessing the property that four infinitely near generators are cut by one line.*

It is known that four lines determine a C_0 (the intersection of six C_5's), but through four infinitely near lines of one of these ruled surfaces pass a whole pencil of C_0's. There are C_1's which have five lines infinitely near in common with V_4.

A four-parameter family of lines can be generated by ∞^3 ruled surfaces such that four consecutive generators are cut by a line. This can be done in ∞^2 different ways.

10. Another set of *important* directions are those along which a tangent C_4 to V_4 has double contact. In S_9 a tangent S_7 to a V_4 is defined by the equations

$$(12) \qquad \begin{aligned} &(\xi, x) = 0, \quad \left(\xi, \frac{\partial x}{\partial u_1}\right) = 0 \ \ldots \ \left(\xi, \frac{\partial x}{\partial u_5}\right) = 0, \\ &(\xi, x) = 0, \quad \left(\xi, \frac{\partial x}{\partial u_1}\right) = 0 \ \ldots \ \left(\xi', \frac{\partial x}{\partial u_5}\right) = 0. \end{aligned}$$

If this S_7 passes through $x + dx$,

$$(\xi, x + dx) = 0, \qquad (\xi', x + dx) = 0.$$

Expressing dx in terms of du_i and applying (12), it is at once evident that these equations are satisfied. Now, if S_7 is also tangent at this point,

$$\left(\xi, \frac{\partial}{\partial u_i}(x + dx)\right) = 0, \qquad i = 1, \ldots 4.$$

$$\left(\xi', \frac{\partial}{\partial u_i}(x + dx)\right) = 0, \qquad i = 1, \ldots 4.$$

Expanding these equations and making use of (12), this condition reduces to

$$(13) \qquad \begin{aligned} &\sum \left(\xi, \frac{\partial^2 x}{\partial u_i \partial u_k}\right) du_k = 0, \qquad i = 1, \ldots 4. \\ &\sum \left(\xi', \frac{\partial^2 x}{\partial u_i \partial u_k}\right) du_k = 0, \qquad i = 1, \ldots 4. \end{aligned}$$

For these equations to be compatible requires five conditions, but as we saw a tangent S_7 depends upon six constants (six ξ's are arbitrary); hence there are ∞^1 solutions of (13), that is, there are ∞^1 S_7's which have two-point contact with V_4.

Then in S_4 *There are ∞^1 C_4's tangent to V_4 in a line p which are also tangent to V_4 in a line infinitely near to p.*

The consecutive tangent π_4's to V_4 (in S_9) along one of the directions above have a line in common. This line of intersection and the line joining the point of contact are in a sense conjugate directions.

III. Three-Parameter Families V_3.

11. The tangent space π_3 to V_3 at a point x is of three dimensions; hence there are ∞^9 S_6's passing through π_3 (and consequently tangent to V_3). Then in S_4 *There are ∞^9 C_3's tangent to a three-parameter family of lines V_3 in a given line.*

The osculating S_4's which have a fixed osculating S_3 in common generate an S_6. Then in S_4 *The ruled surfaces C_1, which have in common four infinitely near fixed lines of V_3 generate a C_3.*

12. The osculating planes which pass through a given direction generate an S_4, and if the direction is varied these S_4's will generate a cone V_6^4 having π_3 for vertex. The S_4 is determined by the points

$$f = 0, \quad f_1 = 0, \quad f_2 = 0, \quad f_3 = 0, \qquad \Sigma f_{ik} du_i du_k = 0.$$

Then if du_i is varied (that is, if the tangent line is varied) the cone is generated by the S_4's which project π_3 from the points of the surface generated by

$$\Sigma f_{ik} du_i du_k = 0,$$

and hence is a cone of order four and dimensions six. Then

In a three-parameter family of lines all the lines infinitely near a given line to infinitesimals of the second order lie in a three-parameter family of order four which has the lines represented by the intersection of π_3 with Φ for double lines.

In general there are no osculating planes of V_3 which hyperosculate. Hence in S_4 four consecutive lines of a three-parameter family are not in general cut by a fifth line.

13. *Special directions.* There are ∞^1 directions such that hyperplanes can be found which are tangent to V_3 in three consecutive points. Let the three points be

$$x, \qquad x' = x + dx, \qquad x'' = x + 2\,dx + d^2x.$$

If the hyperplane $(\xi, x) = 0$ is tangent at x,

$$(1) \quad (\xi, x) = 0, \quad \left(\xi, \frac{\partial x}{\partial u_1}\right) = 0, \quad \left(\xi, \frac{\partial x}{\partial u_2}\right) = 0, \quad \left(\xi, \frac{\partial x}{\partial u_3}\right) = 0,$$

and if it passes through x',

$$(\xi, x + dx) = 0,$$

which from (1) is seen to be true. Now, if (ξ, x) is tangent at x',

$$\left(\xi, \frac{\partial(x + dx)}{\partial u_i}\right) = 0,$$

which on expanding and applying (1) reduces to

$$\sum\left(\xi, \frac{\partial^2 x}{\partial u_i \partial u_k}\right)du_k, \qquad i = 1, 2, 3.$$

If the hyperplane passes through x'', it is at once seen that

$$\sum\left(\sum\left(\xi, \frac{\partial^2 x}{\partial u_i \partial u_k}\right)du_i\right)du_k = 0,$$

which is seen to be satisfied if (2) is satisfied; that is, if the hyperplane is tangent at x', it passes through x''. If it is also tangent at x'',

$$\left(\xi, \frac{\partial}{\partial u_i}(x + 2\,dx + d^2x)\right) = 0.$$

Expanding and using the previous relations, this reduces to

$$\left(\xi, \sum_{ij}\frac{\partial^3 x}{\partial u_i \partial u_j \partial u_k}\,du_i du_j\right) + 3\left(\xi, \sum\frac{\partial^2 x}{\partial u_i \partial u_j}\,du_i d^2u_j\right) = 0.$$

The second term of this expression vanishes if (2) is satisfied; hence

$$\left(\xi,\ \sum_{ij} \frac{\partial^3 x}{\partial u_i \partial u_j \partial u_k} du_i du_j\right) = 0, \qquad k = 1, 2, 3.$$

The hyperplane (ξ, x) will then have three points of contact infinitely near if the ten equations (1), (2), (3) are satisfied. This requires one relation, and hence there are ∞^1 directions du_i of V_3 generating a cone of order nine along which a hyperplane has three infinitely near points of contact with V_3. Then

In a three-parameter family of lines through each point pass ∞^1 *directions such that a* C_5 *will be tangent in three lines of the system infinitely near.*

IV. Application to the Geometry of the Circle.

14. If the equation of the sphere is written in the form

$$t(x^2 + y^2 + z^2) + 2(ax + by + cz) + 2d = 0,$$

the quantities a, b, c, d, t may be taken as the coordinates of the sphere. The spheres of the pencil determined by two spheres (a, b, c, d, t), (a', b', c', d', t') will have coordinates

$$(a + \lambda a', b + \lambda b', c + \lambda c', d + \lambda d', t + \lambda t').$$

Then, if we consider the circle defined by this pencil as representing the pencil, we see that the circle in ordinary space corresponds to the line in S_4 joining the points

$$(a, b, c, d, t), (a', b', c', d', t').$$

The coordinates of the circle can then be taken as the two-row determinants of the matrix

$$\begin{Vmatrix} a & b & c & d & t \\ a' & b' & c' & d' & t' \end{Vmatrix}.$$

These coordinates are identical with those used of the line in S_4.

The lines in S_4 which lie in an S_3 correspond to the circles which cut a given sphere orthogonally.[7]

[7] For the explanation of this and the following statements the reader is referred to the paper of the author previously referred to.

Two lines in S_4 which intersect correspond to circles which lie on the same sphere.

A pencil of lines in S_4 corresponds to a pencil of circles on a sphere.

All the lines in S_4 which pass through a given point correspond to all the circles on a given sphere.

The lines in S_4 which lie in a plane correspond to the circles which pass through two fixed points.

Developable surfaces in S_4 correspond to annular surfaces and ruled surfaces to circled surfaces.

We are now able to interpret the preceding theorems in S_4 in circles.

15. FIVE-FOLD INFINITE SYSTEMS OF CIRCLES U_5. *In each circle C of U_5 there are ∞^4 linear systems A_5 tangent to it. Each tangent A_5 cuts U_5 in a circled surface which has C for multiple generator of order* 16.

The circles of U_5 can be grouped into ∞^4 annular surfaces such that the osculating pencil of circles (the pencil determined by C and the two points in which it cuts the curve corresponding to the cuspidal edge of a developable)[8] *has three circles infinitely near in common with U_5.* This can be done in sixteen different ways.

Through each circle C of U_5 pass sixteen annular surfaces having the pencil of spheres determined by C for tangent pencil such that the osculating congruence determined by three infinitely near circles of the annular surface determine the sixteen osculating pencils of which C is a part.

Through each circle C of U_5 pass ∞^2 circled surfaces such that there are linear series A_5 tangent to U_5 in two consecutive circles of such surfaces.

16. FOUR-PARAMETER FAMILIES U_4. *There are ∞^6 linear series A_4 tangent to U_4 in a given circle C.*

All the linear circled surfaces[9] *A_1 which contain C and two circles of U_4 infinitely near to it generate a linear series C_4.*

All the circles of U_4 infinitely near C to infinitesimals of second order, in a given direction (defined by two consecutive circles) *lie in a linear series A_2. If the direction is varied, the linear series will generate a U_5 having all the circles of a circled surface for double elements.*

Through each circle C of U_4 pass ∞^2 directions such that there are circled surfaces tangent to each direction (the circled surfaces contain

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[8] See "Circles orthogonal to a given sphere," Annals of Mathematics, series 2, **8**, 57.

[9] The term linear circled surface is used to indicate the circled surface common to five-linear circle complexes. Linear does not apply to the order or class of the surface.

the two infinitely near circles which define the direction) *such that four infinitely near generators are cut in two points by one circle.*

The system U_4 *can be generated in* ∞^2 *ways by* ∞^3 *circled surfaces having the above property.*

There are ∞^2 *linear series* A_4 *tangent to* U_4 *in a circle* C *and a circle infinitely near to* C.

17. THREE-PARAMETER FAMILIES U_3. *There are* ∞^9 *linear series* A_3 *tangent to* U_3 *in a given circle* C.

The linear circled surfaces which have C *and three consecutive circles to* C *in common generate a linear series* A_3.

The circles infinitely near to the first and second order to a given article C *of* U_3 *lie in a three-parameter family of order four having five circles of a penticycle for double elements.*

There are ∞^1 *directions such that a linear system* A_5 *will have three-circle contact with* U_3.

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THE INDETERMINATE PRODUCT.

By H. B. Phillips.

THE INDETERMINATE PRODUCT.[1]

By H. B. Phillips.

Presented by H. W. Tyler, November 9, 1910. Received November 12, 1910.

The analysis considered in this paper is applicable to many linear systems, such as linear forms, matrices, vectors, etc. For convenience of language it is here stated in terms of linear spaces.

A theorem involving in its statement only linear spaces can be expressed in terms of two processes, the determination of the space of certain spaces and the determination of the space common to certain spaces. It may involve two relations, of spaces contained in a space of a certain order and of spaces intersecting in a space of a certain order. We develop, after Grassmann, an analysis which expresses symbolically these relations and the results of these processes.

If a, b, c are points and λ, μ, ν numbers,

$$\lambda a + \mu b + \nu c,$$

interpreted according to matrix theory, represents a point in the space of a, b, c and, if λ, μ, ν take all values, represents all points in that space. This expression vanishes only when the points lie in a lower space. Thus with undetermined multipliers we can express the relations desired. This method is usually very clumsy, however, and the peculiar excellence of Grassmann's system consists in replacing these sums with unknown coefficients by products without them.

For this purpose we consider what Gibbs called an indeterminate product. It has the following properties:

$$(1) \quad A + B = B + A,$$
$$(2) \quad (A + B) + C = A + (B + C),$$
$$(3) \quad A(B + C) = AB + AC,$$
$$(4) \quad (AB)C = A(BC),$$
$$(5) \quad \lambda A = A\lambda,$$
$$(6) \quad 0A = A0 = 0,$$

[1] The method discussed in this paper was suggested by Gibbs in his vice-presidential address before the American Association for the Advancement of Science (Scientific Papers, **2**, 109).

where A, B, C are multiple quantities or numbers and λ a number. An equality that is a consequence of these relations alone will be called an identity.

We assume a space containing all the points of a given discussion. Points in this space are expressible as linear functions (in the matrix sense) of others. In the statement of a linear problem these relations between points are the only ones that occur. Hence an expression will be provably (and therefore defined as) zero only when it is reduced by linear relations identically to zero.

Let $f(a_1, a_2 \ . \ . \ . \ a_n)$ be a rational integral function of the n points a_i and

$$f(a_1 a_2 \ . \ . \ . \ a_n) = 0$$

There must then (by definition) be a set of points $b_1 \ . \ . \ . \ b_m$, in terms of which the a's are linearly expressible, such that

$$f(a_1 \ . \ . \ . \ a_n) = \phi(b_1 \ . \ . \ . \ b_m) \equiv 0.$$

If any of the b's are linear functions of the others we may suppose them replaced so that $b_1 \ . \ . \ . \ b_m$ are linearly independent. If $n > m$, the a's (expressible in terms of a smaller number of points) must satisfy a linear relation. If $n = m$ and the a's are linearly independent, the equations expressing them in terms of b's can be solved and the points b_i expressed in terms of the a's. Then, since an identity transforms into an identity,

$$\phi(b_1 \ . \ . \ . \ b_m) = f(a_1 \ . \ . \ . \ a_n) \equiv 0.$$

If $n < m$ and the a's linearly independent, the equations expressing a's in terms of b's can be solved for n b's, and since that part of $\phi(b_1 \ . \ . \ . \ b_m)$ containing these must vanish identically, $f(a_1 \ . \ . \ . \ a_n) \equiv 0$ as before. Thus if $f(a_1 \ . \ . \ . \ a_n)$ really contains the a's, these points can not be linearly independent. If they are replaced by any linearly independent set of points b_i, the new expression, since it vanishes, must vanish identically. Therefore, if a rational integral function of n points vanishes, these points satisfy a linear relation, and the given function reduces identically to zero when the n points are expressed as linear functions of *any* set of linearly independent points.

To form expressions that do vanish when the points are linearly related we use ordered determinants, i.e., determinants expanded like ordinary determinants with all products ordered, first term being taken from first column, second term from second column, etc. Thus,

$$\begin{vmatrix} a_1 & a_1 \\ a_2 & a_2 \end{vmatrix} = a_1 a_2 - a_2 a_1.$$

Such a determinant has certain properties of ordinary determinants. Thus, if two rows are equal, the determinant is zero; its value is not changed when a multiple of one row is added to another row; it can be expanded in terms of minors taken from first m columns. As the above example shows rows and columns are not interchangeable.

The necessary and sufficient condition that $a_1 \ldots a_n$ satisfy a linear relation is

$$\begin{vmatrix} a_1 & a_1 & \ldots & a_1 \\ a_2 & a_2 & \ldots & a_2 \\ . & . & . & . \\ . & . & . & . \\ a_n & a_n & \ldots & a_n \end{vmatrix} = 0.$$

For if any expression vanishes, we have shown that the a's are linearly related. Conversely, if they satisfy a linear relation, since one row is a linear function of the others, the determinant is zero.

We represent the above determinant by the symbols $[a_1 \ldots a_n]$ or $(a_1 \ldots a_n)$. This may be regarded as a product of the points $a_1 \ldots a_n$. Gibbs called it the combinatorial product because it has the property (characteristic of Grassmann's combinatorial) of changing sign when two of the a's are interchanged.

Similarly we define a combinatorial product

$$[f(a_1 \ldots a_n)\, \phi(b_1 \ldots b_m)],$$

when f and ϕ are any rational integral functions. The two expressions are multiplied distributively and each product replaced by the sum of all permutations which leave the order of the a's and the order of the b's unchanged, the sign being negative when the permutation is odd. Thus the combinatorial product of a_1a_2 and b_1b_2 is

$$a_1a_2b_1b_2 + a_1b_1b_2a_2 + b_1b_2a_1a_2$$
$$+ b_1a_1a_2b_2 - a_1b_1a_2b_2 - b_1a_1b_2a_2.$$

From the preceding definition it follows that

$$[(a_1 \ldots a_n)\,(a_{m+1} \ldots a_n)] = (a_1 \ldots a_m).$$

For the left hand member expresses that every permutation of $a_1 \ldots a_m$ is to be placed in every position among the letters of every permutation of $a_{m+1} \ldots a_n$. That is equivalent to the right side which represents all permutations of $a_1 \ldots a_n$ and the rule of sign is the same for both. It should be noted that $(a_1 a_2 b_1 b_2)$ is the combinatorial product of $(a_1 a_2)$ with $(b_1 b_2)$ and not of $a_1 a_2$ with $b_1 b_2$.

Let $A = (a_1 \ldots a_n) \neq 0$. If x is a linear function of $a_1 \ldots a_{n1}$, i.e., if x lies in the space of $a_1 \ldots a_n$,

$$(Ax) = (a_1 \ldots a_n x) = 0.$$

Hence $(Ax) = 0$ is the equation of that space and we may represent the space by the symbol A.

If

$$A = (a_1 \ldots a_n), \quad B = (b_1 \ldots b_m),$$
$$(AB) = (a_1 \ldots a_n b_1 \ldots b_m).$$

If the points $a_1 \ldots a_n b_1 \ldots b_m$ are independent, i.e., if the spaces do not intersect, (AB) represents the space determined by A and B. If the spaces A and B intersect, $(AB) = 0$. Because of this property of vanishing under incidence (property characteristic of Grassmann's progressive product) Gibbs called this product progressive. The progressive product represents our first linear process and by its vanishing gives the first linear relation.

We can now express the condition that two spaces intersect. To distinguish different types or degrees of incidence we need other products. We define as the regressive product in a space of order p the result of multiplying two functions distributively and replacing each term of the result by the sum of terms gotten by permuting, as in the progressive product, the first p letters in each term. If $A = (a_1 \ldots a_n)$, $B = (b_1 \ldots b_m)$

$$(AB)_p = \begin{vmatrix} a_1 a_1 & \ldots & a_1 0 & \ldots & 0 \\ a_2 a_2 & \ldots & a_2 0 & \ldots & 0 \\ \cdots & & \cdots & & \\ a_n a_n & \ldots & a_n 0 & \ldots & 0 \\ b_1 b_1 & \ldots & b_1 & \ldots & b_1 \\ \cdots & & \cdots & & \\ b_m b_m & \ldots & b_m & \ldots & b_m \end{vmatrix}$$

there being p elements not zero in each row of the A's.

If $b_i \ldots b_j$ is any combination of b's, $b_k \ldots b_l$ the remaining ones,

$$B = \Sigma \pm (b_i \ldots b_j)(b_k \ldots b_l),$$

the sign being minus when

$$b_i \ . \ . \ . \ b_j b_k \ . \ . \ . \ b_l$$

is an odd permutation of $b_1 \ . \ . \ . \ b_m$.

Multiplication by A gives

$$(AB)_p = \Sigma \pm (Ab_i \ . \ . \ . \ b_j)\,(b_k \ . \ . \ . \ b_l).$$

This is the fundamental identity of Grassmann,[2] many special cases of which are used in geometry.

Let A and B be two spaces intersecting in the space

$$C = (c_1 \ . \ . \ . \ c_k).$$

We can determine n–k points a_i and m–k points b_i such that

$$A = (a_1 \ . \ . \ . \ a_{n-k} c_1 \ . \ . \ . \ c_k), \quad B = (b_1 \ . \ . \ . \ b_{m-k}\, c_1 \ . \ . \ . \ c_k).$$

We expand the product (AB) by the formula given above. If $p > m + n - k$, there are two c's equal in each of the prefactors and the result is zero. If $p = m + n - k$, there is just one term that does not vanish, giving

$$\begin{aligned}(AB)_p &= (a_1 \ . \ . \ . \ a_{n-k} c_1 \ . \ . \ . \ c_k b_1 \ . \ . \ . \ b_{m-k})\,(c_1 \ . \ . \ . \ c_k)\\ &= DC,\end{aligned}$$

where D is the space containing both A and B and C the space common to them. If $p < m + n - k$, there are a number of terms in the expansion. The determinants which are prefactors are all different. The same is true of postfactors. Since the points in it are linearly independent, the expression can not factor or vanish. Hence if $A \neq 0$, $B \neq 0$, $[AB]_p = 0$ is the necessary and sufficient condition that A and B lie in a space of order less than p. If the containing space is of order p, $[AB]_p$ is the product of that space and the common space.

Progressive products are used when the number of factors is equal to or less than p, regressive when that number is greater than p. Hence it causes no confusion to use the same notation $[AB]$ for both.

Expressions occurring in the discussions of linear geometry are always homogeneous. Upon multiplying two such expressions regressively, each term of the result is either zero or of the form DC where D is the space in which we are working. Since it is a factor of all

[2] Ausdehnungslehre (1862), p. 83.

such expressions, the equations connecting them are not changed by assuming D scalar. This amounts to making all progressive products of p letters scalar.

The regressive product represents the space common to two spaces or vanishes when they are contained in a space of order less than p. This product represents symbolically the second fundamental linear process, and by its vanishing the second type of relation.

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VECTOR-DIAGRAMS OF OSCILLATING-CURRENT CIRCUITS.

By A. E. Kennelly.

VECTOR-DIAGRAMS OF OSCILLATING-CURRENT CIRCUITS.

By A. E. Kennelly.

Presented November 9, 1910. Received November 25, 1910.

Although, as the appended bibliography indicates, the analysis of oscillating-current circuit phenomena has received much attention, and has attained a considerable degree of development, yet the methods of vector-diagrams do not seem to have been applied to it. It is believed that these vector-diagram methods offer marked advantages to the student, and particularly to students of electrical engineering who have already become familiar with the use of vector-diagrams in connection with alternating-current circuits.

Definition.

An oscillating-current circuit may be defined as a circuit which, in undergoing a change of energy, carries a current that oscillates periodically about an ultimate value. In such a circuit not only the electric current, but also the emf. quantity, power, and energy oscillate freely. For brevity, the term oscillating current may be designated by the letters *o. c.* In practice, an o. c. circuit comprises a condenser as an essential element, the energy oscillating, or tending to oscillate, between the electric and magnetic types, at a frequency determined solely by the constants of the circuit. The oscillations decay in amplitude, energy being expelled irrecoverably from the circuit, during the process, by a regular sequence of dwindling impulses. These impulses may be either of the joulean or *hertzian* type, or of both combined.

Simple Resistanceless Oscillating-Current Circuits.

The simplest type of o. c. circuit comprises a condenser AB, Figure 1, of capacity or permittance c farads, inserted in a circuit of negligible resistance, containing a total inductance of l henrys. Let n be the free-oscillation frequency of the circuit, in cycles per second, and let $\omega = 2\pi n$ be the free-oscillation angular velocity of the circuit, in radians per second. Then the reactance jX_l of the inductance DE at this frequency will be

$$jX_l = jl\omega \qquad \text{ohms} \quad (1)$$

where $j = \sqrt{-1}$, the quadrantal operator.

The reactance $-jX_c$ of the condenser at the same frequency will be

$$-jX_c = -j\frac{1}{c\omega} = -j\frac{s}{\omega} \qquad \text{ohms} \quad (2)$$

where $s = 1/c$ is the elastance of the condenser in darafs.[1]

Discharging and Charging Oscillations. — An oscillatory circuit may be excited into activity either when energy is added to it, or when energy is withdrawn from it. In the former case *charging oscillations*, and in the latter case *discharging oscillations* are produced. Thus the circuit may be initially devoid of electric or magnetic energy, and a certain constant potential difference of U_0 volts, as from a storage battery, may be inserted in the circuit between the terminals TT, Figure 1. The condenser will then be charged by an oscillatory process, or series of charging oscillations.

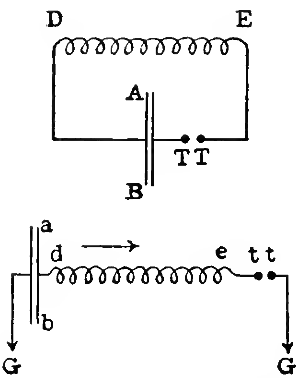


FIGURE 1. Diagram of connections of simple oscillating-current circuit, and schematic representation of the same.

Again, the condenser may be initially charged to a potential difference of U_0 volts, and allowed to discharge by short-circuiting the terminals TT. Discharging oscillations will then be produced. Or, the condenser may be initially without charge, but the inductance DE (Figure 1) may be charged by allowing a steady current of I_0 amperes to pass through it from some external source. If then the gap TT be closed, and the source of charging current is suddenly withdrawn sparklessly, discharging oscillations will be produced in the circuit.

Discharging oscillations may even be considered as taking place in an active initially energised circuit, in the limiting case where the circuit is assumed to be resistanceless; so that energy ceases to be dissipated during the discharge.

In any of the above cases, the circuit selects the frequency of free

[1] Prof. V. Karapetoff seems to have been the first writer to have had the courage to print the useful term *daraf* as a unit of elastance the reciprocal of a *farad*. "The Electric Circuit," Ithaca, N. Y. 1910, p. 72. See also "Electrical Papers," O. Heaviside, 1892, Vol. 2, p. 125.

oscillation such that the reactance of the circuit remains zero. This is the law of all oscillating-current circuits, whether they contain small or large resistance, up to the aperiodic limit. Consequently:

$$jX_l - jX_c = 0 \qquad \text{ohms} \quad (3)$$

$$jl\omega = j\frac{1}{c\omega} = j\frac{s}{\omega} \qquad \text{ohms} \quad (4)$$

whence

$$\omega = \frac{1}{\sqrt{lc}} = \sqrt{\frac{s}{l}}. \qquad \text{radians/sec.} \quad (5)$$

When condensers are connected in parallel, their permittances are more readily dealt with; and when connected in series, their elastances.

Discharging Oscillations of a Simple Resistanceless Oscillating-Current Circuit. — The oscillatory system of Figure 1 may be given its initial stock of energy either electrically or magnetically; that is, either by giving an initial electric charge of Q_0 coulombs to the condenser, or by exciting a total initial linked magnetic flux $\Phi_0 = I_0 l$ ampere-henrys in the coil, where I_0 is the initial exciting current-strength, supposed to be suddenly withdrawn from the coil without loss of energy in sparking. With respect to amplitude, the discharging oscillations of the coil will be the same as those of the condenser, provided that:

$$\Phi_0 = Q_0 z_0 \qquad \text{volt-seconds} \quad (6)$$

where $z_0 = \sqrt{\frac{l}{c}} = \sqrt{ls}$ is the surge impedance of the system. With respect to phase, however, the discharging oscillations of the excited coil will be in quadrature with those of the excited condenser. In cases where both the condenser and the coil are initially excited, and are allowed to discharge simultaneously, each may be considered independently, and the two sets of oscillations may then be summed.

General Rotating Vector-Diagram of Simple Resistanceless Oscillating-Current Discharging Circuit. — In Figure 2, let $O\overline{U}_0'$ represent to volt-scale the initial p. d. applied to the condenser of Figure 1, and producing therein an initial electric charge Q_0. Then $O\overline{U}_0$ will be the equal and opposite p. d. of U_0 volts, tending to discharge the condenser. The direction of the discharging p. d. OU_0 may be taken as the direction of reference or voltage phase-standard, and XOX as the axis of reference. The vector $O\overline{I}_0$ then represents the discharging current established by the discharging p. d. $O\overline{U}_0$. The vector system $O\overline{U}_0$, $O\overline{I}_0$, $O\overline{E}_0$ is to be pivoted about the point O, and, starting at time

$t = 0$ from the position shown, to rotate positively in the plane $\overline{U}_0 \overline{I}_0 \overline{E}_0$ with the angular velocity ω, as determined by formula (5). At any time t seconds after the release of the discharge, the orthogonal projections on XOX of the three vectors $O\overline{U}_0$, $O\overline{I}_0$, and $O\overline{E}_0$ will represent the corresponding instantaneous values of the discharging p. d. at condenser terminals, the current strength, and the emf. of self-induction in the coil. The vector discharging current therefore rotates in quadra-

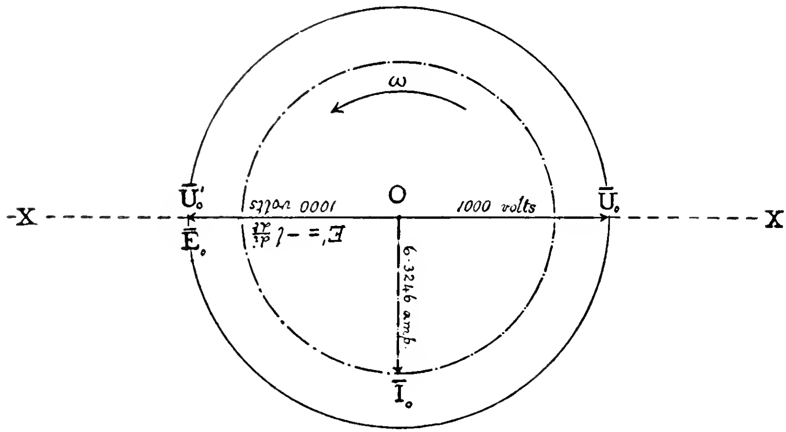


FIGURE 2. Rotative vector-diagram of simple resistanceless oscillating-current circuit.

ture between the two opposed and equal electromotive forces of discharge and of self-induction, developing with them reactive power and cyclic energy; but with no dissipated energy, under the assumption of negligible resistance.

Figure 3 presents a series of stationary vectors; ω, V, I, P, W. The vector ω is drawn in the $-j$ direction, and with a length $\omega = 1/\sqrt{cl} = \sqrt{s/l}$ radians-per-second, according to formula (5). If we take, as an example, a condenser of $c = 4$ microfarads ($s = 0.25$ megadarat), charged to an initial p. d. of $U_0 = 1000$ volts, and discharged through a resistanceless inductance of $l = 0.1$ henry, we find $\omega = 1581.14$ radians per second, corresponding to a frequency of $n = 251.646$ cycles per second, and a complete period of $T = 0.003,974$ second.

Starting from the initial position of Figure 2, the subsequent position of any vector in the system, after the lapse of t seconds, is determined by multiplying the vector by

$$\epsilon^{-(-j\omega t)} = \epsilon^{j\omega t} = \epsilon^{j\frac{t}{\sqrt{cl}}} = \epsilon^{j1581.14t}, \qquad \text{numeric} \angle \quad (7)$$

The admittance of the oscillating-current circuit is given by the Y vector, Figure 3, which is drawn in the $-j$ direction to a scale of mhos, and a length of $c\omega$ mhos. In the case considered $Y = -j0.006,324,6$ mho.

The initial vector oscillating current is given by the I vector, Figure 3, which is drawn in the $-j$ direction to a scale of amperes, and a length of $\overline{U}_0 Y$, as the amplitude, or maximum cyclic value, of the p. d. at condenser terminals. Or, expressed in terms of the quantity of electricity in the condenser, $\overline{I}_0 = \overline{Q}_0 \omega$, where $\overline{Q}_0$ is the initial condenser charge in coulombs, and the vector amplitude or maximum cyclic quantity. $\overline{Q}_0$ is also $Q\sqrt{2}$, where Q is the root-mean-square of the oscillating condenser charge, in coulombs. In the case considered, $\overline{I}_0$ is $-j$ 6.3246 amperes, and $I = 4.472,14$ r. m. s. amperes. This shows that in Figure 2 the current vector $O\overline{I}_0 = 6.3246$ max. cyclic amperes, lies 90° in phase behind the discharging p. d. $O\overline{U}_0$.

The oscillatory power in the circuit is given by the P vector, Figure 3. Since the current is in quadrature with the emfs. in a resistanceless circuit, the power will be wholly reactive or non-dissipative. The P_m vector is drawn in the $-j$ direction, to a scale of watts, and to a length of $P_m = \frac{\overline{U}_0 \overline{I}_0}{2} = UI$ units on this scale. In the case considered $P_m = -j3162.3$ watts. This is the maximum cyclic value, or amplitude, of the power of the condenser. The power is positive when the condenser is doing work, or discharging, and is negative when the condenser is receiving energy from the magnetic field of the coil, or is charging.

The oscillatory energy in the circuit is given by the W vector. This vector is drawn in the $= -j$ direction, to a scale of joules, and to a length of $W_m = P/2\omega$ units on that scale. In the case considered, $W_m = -j1$ joule. This is the maximum cyclic value, or amplitude, of the energy delivered by the condenser into the magnetic field of the coil at each oscillation.

The time variation of the various oscillating quantities is represented in Figure 5, for one complete cycle of the current and p. d. The sinusoid u, u, u, of 1000 volts amplitude, is the graph of the condenser p. d., commencing at $U_0 = 1000$ volts. The sinusoid e is the emf. of self-induction in the coil, and is always equal and opposite to the p. d. of the condenser. The sinusoid ii, of 6.3246 amperes amplitude, is the graph of the discharging current, and is 90° in phase behind the discharging p.d. The sinusoid p, of double frequency, and of amplitude

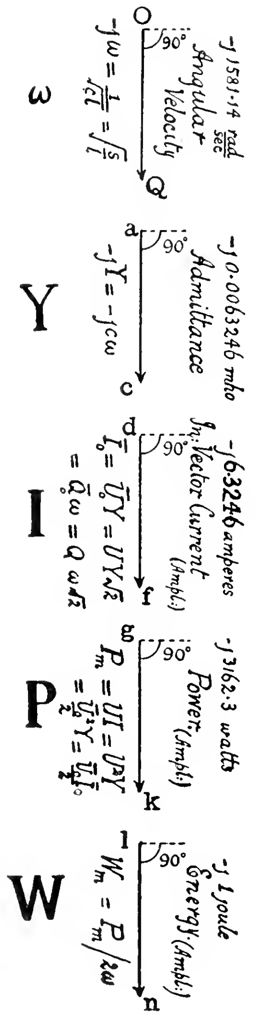


FIGURE 3. Stationary vector-diagrams of angular velocity, admittance, r. m. s., current strength, maximum cyclic power, and maximum cyclic energy in a simple resistanceless oscillating-current circuit to phase of p. d. as standard.

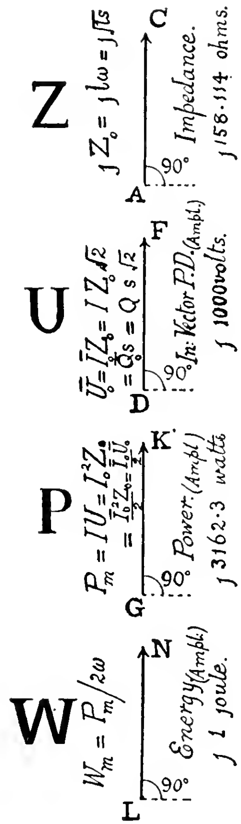


FIGURE 4. Stationary vector-diagrams of impedance, r. m. s., potential difference, maximum cyclic power, and maximum cyclic energy in a simple resistanceless oscillatory-current circuit to phase of current as standard.

3.1623 kilowatts, is the power developed in the condenser. It starts from zero, in the positive direction, at the moment of release. The opposite sinusoid p_l, of double frequency, and also of amplitude 3.1623 kw., is the power developed in the inductance. The condenser and coil are reciprocally and successively generator and recipient in respect to power.

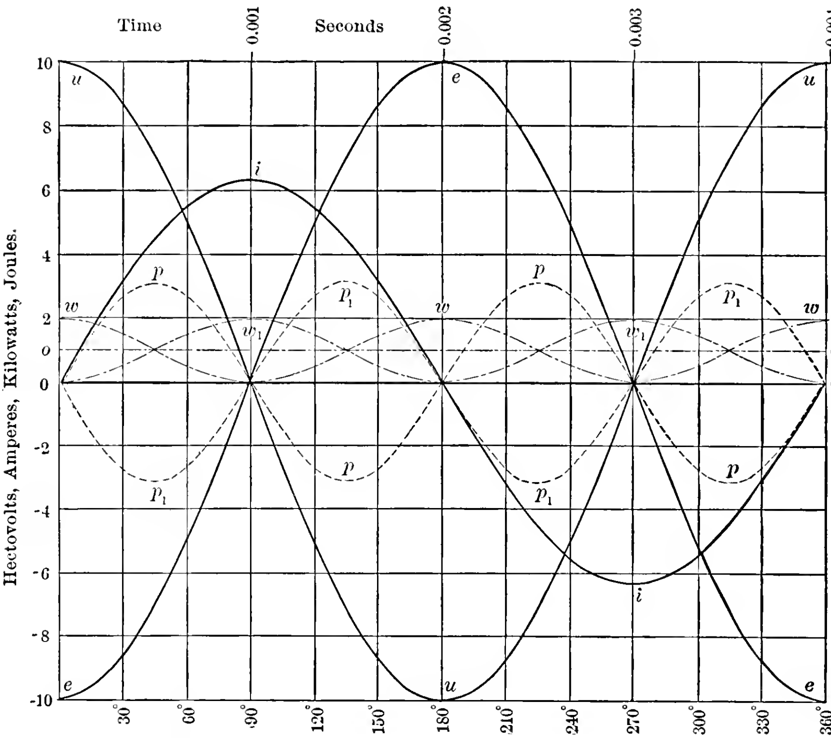


FIGURE 5. Analysis of p. d., current, power, and energy in a simple resistanceless discharging circuit.

The sinusoid w_l, of double frequency, is the graph of the energy expended by the condenser and stored in the coil. It has an amplitude of 1 joule above and below the line *oo* as axis. The total energy in the coil, therefore, reckoned from zero, is

$$2\ W_m = \frac{UI}{\omega} = \frac{U_0 I_0}{2\ \omega} = \frac{1}{2} l I_0^2. \qquad \text{joules} \quad (8)$$

The opposite sinusoid w_c, also of double frequency, and 1 joule amplitude, is the graph of energy in the condenser. It is evident that at every instant

$$w_c + w_l = W_0 \qquad \text{joules (9)}$$

where W_0 is the initial energy $U_0^2 c/2$ of the system, or 2 joules in the case considered.

All of the vectors in Figure 3 are stationary, and in the series I, P, and W, are all drawn to the phase of the discharging p. d. $\overline{U}_0$ as standard. This means that the vector current i is 90° behind that of u (Figure 5), the condenser power p is quadrature power, or purely reactive power, and the delivered energy W_m is also quadrature or reactive energy.

With respect to current-phase as standard, we have the series of vectors in Figure 4, commencing with Z_0 the impedance of the circuit, which is drawn in the $+j$ direction to a scale of ohms and a length of $l\omega$ on this scale. In the case considered, $Z_0 = j158.114$ ohms. Z_0 is thus a purely reactive impedance, or reactance.

The vector $\overline{U}_0$, Figure 4, represents the amplitude or initial value of the p. d. at condenser terminals. It is drawn in the $+j$ direction, or is 90° ahead of the current, and to a scale of volts, to a length of $\overline{I}_0 Z_0 = IZ_0\sqrt{2}$ on that scale, where I is the r.m. s. value of the vector $\overline{I}_0$. In the case considered, $\overline{I}_0 = 6.3246$ amplitude amperes and $I = 4.472$ r. m. s. amperes. In terms of the electric quantity, however, it may also be expressed as $\overline{Q}_0 s = Qs\sqrt{2}$ volts. In the case considered, this vector amplitude p. d. $\overline{U}_0$ is 1000 volts, with an effective or r. m. s. value of $U = 707.1$ volts.

The vector P_m in Figure 4 represents the amplitude, or maximum cyclic value, of the oscillatory power of the condenser. It is drawn in the $+j$ direction, being leading quadrature power with respect to current phase, to watt scale, and to a length of $\frac{\overline{I}_0 \overline{U}_0}{2} = IU$ units on that scale. In the case considered, $P_m = j\,3162.3$ watts.

The vector W_m in Figure 4 represents the amplitude or maximum cyclic value of the oscillatory energy expended by the condenser. It differs only from the vector W_m of Figure 3, by being drawn in the $+j$ instead of in the $-j$ direction. This is because the energy and power are leading quadrature quantities with reference to the current, but lagging quadrature quantities with respect to the discharging p. d.

The stationary vector-diagrams of Figures 3 and 4 may be considered as graphically corresponding to the following vector equations:

With respect to the potential difference as standard of phase,

$$\bar{I}_0 = \bar{U}_0 Y \,\overline{|90^\circ} = \bar{Q}_0 \omega \,|90^\circ, \qquad \text{maximum cyclic amperes}$$

$$P_m = \frac{\bar{U}_0 \bar{I}_0}{2} \overline{|90^\circ} = UI \overline{|90^\circ} = \frac{\bar{U}_0^{\,2} Y}{2} |90^\circ = U^2 Y |90^\circ, \qquad \text{max. cyclic watts}$$

$$W_m = \frac{P_m}{2\,\omega} \overline{|90^\circ} = \frac{\bar{U}_0^{\,2} Y}{4\,\omega} \overline{|90^\circ} = \frac{UI}{2\,\omega} \overline{|90^\circ}. \qquad \text{maximum cyclic joules}$$

With respect to the current as standard of phase:

$$\bar{U}_0 = \bar{I}_0 Z_0 \underline{|90^\circ} = \bar{Q}_0 s \underline{|90^\circ}, \qquad \text{maximum cyclic volts} \quad (10)$$

$$P_m = \frac{\bar{I}_0 \bar{U}_0}{2} \underline{|90^\circ} = \frac{\bar{I}_0^{\,2} Z_0}{2} \underline{|90^\circ} = IU \underline{|90^\circ} = I^2 Z_0 \underline{|90^\circ}, \qquad \text{max. cyc. watts} \quad (11)$$

$$W_m = \frac{P_m}{2\,\omega} \underline{|90^\circ} = \frac{\bar{I}_0^{\,2} Z_0}{4\omega} \underline{|90^\circ} = \frac{IU}{2\omega} \underline{|90^\circ}. \qquad \text{maximum cyclic joules} \quad (12)$$

The vector-diagrams I, P, and W, of Figure 3, as well as U, P, and W, of Figure 4, may also be treated as rotative vector-diagrams, for deducing instantaneous values of their respective quantities projectively. Thus in the case of I, Figure 3., we may rotate the vector I_0 about the point d, in the positive direction, with angular velocity ω, commencing at time $t = 0$, from the position shown. The instantaneous orthogonal projections of the vector on the dotted line, or "real" axis, will then give the corresponding instantaneous o. c. strengths.

In the case of the P vector, Figure 3, the vector gk may be rotated about the point g with positively directed angular velocity $2\,\omega$, commencing from the position shown in Figure 3, at time $t = 0$. Instantaneous orthogonal projections on the dotted line, or "real" axis, will then give the corresponding instantaneous oscillating-current power, as indicated by the double-frequency sinusoid p in Figure 5. This is the power of the condenser. The same rotating power-vector may also be used to project the power of the inductance, if displaced in phase by 180°, that is, if it starts from rest in the diametrically opposite position to that shown in Figure 3.

In the case of the W vector, Figure 3, the vector ln may be rotated in the positive direction about the point l with angular velocity 2ω. If the vector starts, at time $t = 0$, from the position indicated in Figure 3, projections must be taken on the line ln, or axis of imaginaries. Instantaneous projections of the vector on this axis will then mark off instantaneous values of the energy in the condenser ($u^2c/2$ joules).

If the rotating vector be displaced 180° in phase, its projections will mark off instantaneous values of energy, ($i^2l/2$ joules) in the inductance. For reasons that will be evident on considering oscillating-current circuits containing resistance, it is preferable to describe a circle upon a diameter LSC, Figure 6, with the radius $ln = W_m$, Figure 3, and rotate this circle in the positive direction, with the angular velocity 2ω, at the same time rolling it along the axis OY, drawn through the point L. The orthogonal projection of the center S upon OY will mark time, as indicated in Figure 6 both to a scale of seconds, and to degrees of energy phase. The projection of the point C in the circle on the OX axis, commencing at c, will mark off a distance Oc corresponding to the instantaneous energy $u^2c/2$ joules in the condenser. Similarly, the projection of the opposite point L in the circle on the OX axis, commencing at l, will mark off a distance Ol corresponding, on the same scale, to the instantaneous energy $i^2l/2$ joules in the inductance.

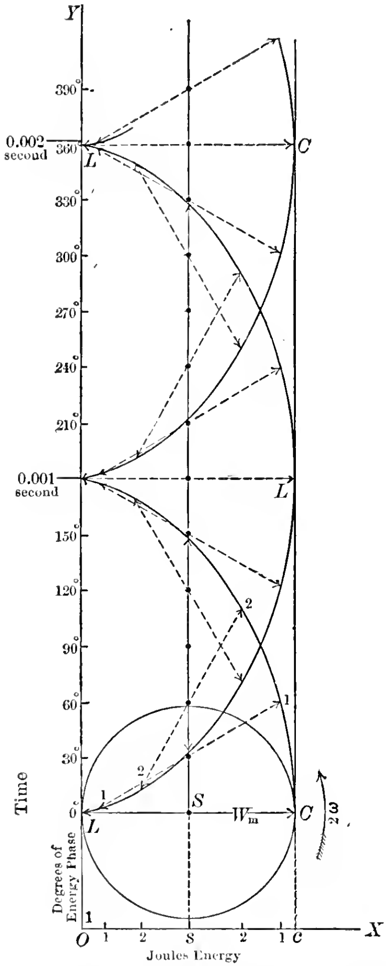


FIGURE 6. Rotating and rolling vector-diagram of the condenser energy, the inductance energy, and the semi-system energy in a resistanceless oscillating-current circuit.

If we connect the points L and C by a straight line, and take the middle point, it will coincide, in the resistanceless case here considered, with the center S of the rolling circle. Consequently, the projection of the point S on the OX axis at s, will mark off a length Os, equal to half the sum of the instantaneous energy in the condenser and in the inductance, i. e., equal to half the instantaneous energy remaining in the system. We may call this quantity Os, for brevity, the semi-system-energy. Since there can be no dissipation of energy from such a circuit devoid of joulean and hertzian resistance, the semi-system-energy Os does not vary with time as the circle rolls along the OY axis. CCC and LLL are cycloids differing 180° in phase.

Inductance-Discharging Oscillations in a Simple Resistanceless Oscillating-Current Circuit. — We have thus far considered condenser-discharging oscillations. If, however, the inductance be charged with current and magnetic energy from a separate source, and this source is suddenly and sparklessly removed, while the condenser circuit Figure 1 is closed at $T\,T$, the inductance will set up a series of discharging oscillations. If we assume that the initial steady current strength I_0, at the moment of release, is equal to the maximum cyclic value of the current in the case already considered, then the oscillations of the inductance-discharging system will differ only in phase from the oscillations of the condenser-discharging system already discussed. Thus, if the current in the inductance were 6.3246 amperes at the instant of release, and the condenser were initially without charge, the oscillations of the system would be those of Figure 5, except that the time would start from the instant denoted by 90° in that diagram. If the initial direction of the current in the coil were reversed, the starting point in time would be at 270° in Figure 5. Consequently, with condenser-discharging oscillations, we start in Figure 5 with $t = 0$, either from the phase of 0° or of 180°, according to the direction of the p. d. impressed upon the condenser; while with inductance-discharging oscillations, we start with $t = 0$, either from the phase of 90° or of 270°, according to the direction of the current impressed upon the coil. The sequence of all the phenomena will then remain in each case as presented in Figure 5.

Not only Figures 2 and 5, but also Figures 3 and 4 apply equally to inductance and condenser-discharging oscillations. The diagrams of Figure 4 apply more directly, however, to inductance discharges, and those of Figure 3 to condenser discharges, because in the former the initial current is the independent common variable; while in the latter initial p. d. is the independent common variable. Thus, if in Figure 1 the inductance was initially charged with a current of 10 amperes, and an energy of 5 joules, the maximum cyclic current $\overline{I}_0$ would be 10 amperes, the r. m. s. or virtual current, I, 7.071 amperes. The maximum cyclic oscillating p. d. $\overline{U}_0$, in the absence of resistance, would be 1581.14 volts, the r. m. s., or virtual, p. d. U, would be 1118.0 volts, the maximum cyclic power P_m would be 7905.9 watts, and the maximum cyclic condenser or inductance energy 7905.7/3162.3 = 2.5 joules. The semi-system-energy would also be 2.5 joules.

Charging Oscillations in a Simple Resistanceless Oscillating-Current Circuit. — If with the system of Figure 1 initially unchanged, we suddenly impress upon the circuit, assumed resistanceless, between the terminals TT, a constant potential difference U_0, also assumed resistanceless, as from a storage battery of large cells, then both the condenser and the inductance will be subjected to charging oscillations. In this case, if the impressed p. d. U_0 is the same as that already assumed for the initial p. d. of the discharging condenser, the conditions represented in Figure 5 will apply to the charging oscillations, except in regard to phase, and to the meaning of the zero line OO. The sign of the oscillations will be reversed, or the phase displaced 180°, from those corresponding to a p. d. of the direction Ou, Figure 5. That is, an impressed p. d. having the + direction will set up from the start the same system of oscillations as those from the discharge of a condenser impressing a + direction of p. d. on the circuit. The zero line OO of Figure 5, in regard to p. d. and to condenser energy, instead of representing zero p. d. and zero condenser energy, will also have to be interpreted respectively in the charging case, as the constant value of impressed p. d., and the mean energy of the condenser under the impressed p. d. That is, the horizontal line through − 10 will be the zero line of p. d. if the impressed p. d. is 1000 volts tending to make the condenser p. d. positive.

Simple Oscillating-Current Circuits Containing Resistance.

When such a circuit as that shown in Figure 1 is allowed to discharge through a known total resistance r ohms, including both joulean and hertzian resistances (all types that involve dissipation of power in proportion to the square of the current), the first step is to find the resistanceless angular velocity ω of Figure 3, that is to determine the angular velocity of discharge on the basis of no resistance ($r = o$). Let this value of resistanceless angular velocity be denoted by ω_0. We then proceed to determine the angular velocity ω in the presence of the actual resistance r.

Let $$\rho = r/2 \qquad \text{ohms} \quad (13)$$

be the semi-resistance of the circuit, and

$$\tau = l/\rho \qquad \text{seconds} \quad (14)$$

will then be a time-constant, which may, for convenience, be called the *oscillation time-constant* of the circuit, as distinguished from the ordinary time-constant l/r when the circuit is non-oscillatory and the condenser is short-circuited. The oscillation time-constant is thus double the ordinary time-constant. If, as in Figure 7, we take $r = 200$ ohms in the same circuit as has been considered in Figures 3, 4 and 5, $\tau = 0.001$ second.

The time-constant reciprocal of the oscillating-current circuit is

$$\lambda = \frac{1}{\tau} = \frac{\rho}{l} \cdot \text{ seconds}^{-1} \,. \quad (15)$$

In the ω-diagram of Figure 7, draw $OP = \lambda$, to a suitable scale of reciprocal-seconds, in the direction of reference, or along the real axis in the positive direction. From P draw a line PQ in the $-j$ direction, or perpendicular to OP. With center O, and radius OQ equal, on the adopted scale, to the value of the resistanceless angular velocity

$$\omega_0 = \sqrt{s/l},$$

obtained as in Figure 3, intersect the line PQ in Q. Then the intercept PQ will measure, to scale, the angular velocity ω of the oscillation in the circuit with the resistance r present.

Or analytically,

$$\omega = \sqrt{\omega_0^2 - \lambda^2} = \omega_0 \sin \phi \quad \text{radians per sec.} \quad (16)$$

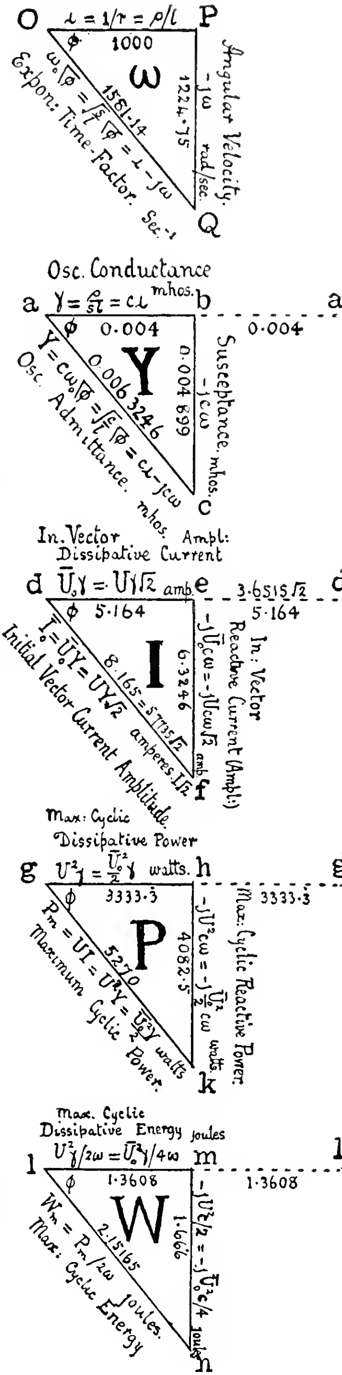


FIGURE 7. Stationary vector-diagrams of angular velocity, admittance, vector current strength, maximum cyclic power, and energy in a simple o. c. circuit containing resistance. Phase of p. d., standard.

and T, the period of current and p. d. is $2\pi/\omega$. seconds (17)
In the case considered $\omega = 1224.75$ radians per second, and $T = 0.00513$ second.

The matter may be viewed in another direction by considering that with the resistanceless circuit of Figure 3, the vector condition of the p. d. in the circuit, at any instant t seconds after release is

$$u = \bar{U}_0 \epsilon^{-t(-j\omega_0)} = \bar{U}_0 \epsilon^{j\omega_0 t} \qquad \text{volts} \quad (18)$$

where $-j\omega_0$ of Figure 3 is the factor of the time in the exponential variable $\epsilon^{j\omega_0 t}$. The effect of introducing a resistance r into the circuit is to alter (18) to

$$u = \bar{U}_0 \epsilon^{-t(\omega_0 \overline{|\phi})} = \bar{U}_0 \epsilon^{-t(\lambda - j\omega)} = \bar{U}_0 \epsilon^{-\lambda t + j\omega t}. \qquad \text{volts} \quad (19)$$

That is, the exponential time-factor $-j\omega_0$ of the resistanceless case (Figure 3) is deflected from the $-j$ direction to a direction making an angle $\overline{\backslash\phi}$ with the direction of reference; such that

$$\overline{\backslash\phi} = \cos^{-1}(\lambda/\omega_0). \qquad \text{radians or degrees} \quad (20)$$

In the case considered, $\phi = 50°.46'.06'' = 0.8861$ radian.

The rotative vector-diagram of the resistant circuit is shown in Figure 8. OU_0, measured to scale along the axis of projection $-XOX$, is the initial p. d. between condenser terminals at release. $O\bar{U}_0$ is the initial position of the vector p. d. whose projection is OU_0. In a certain sense, $O\bar{U}_0$ is a fictitious vector; because it has a value OU_0 cosec $\phi = 1291$ volts, which is greater than the initial p. d. at the moment of release; but, owing to the effect of damping, this seeming inconsistency gives rise to no error in the result. $O\bar{E}_0$ is the vector emf. of self-induction in its initial position. Midway between $O\bar{U}_0$ and $O\bar{E}_0$ lies the vector current $\bar{I}_0$, whose projection on XOX is initially zero. The cophase component Od (5.164 amperes), of the current along $O\bar{U}_0$, is the dissipative component taking power from the discharging p. d. while the component in quadrature thereto, $d\bar{I}_0$ is the reactive component, taking reactive power from $O\bar{U}_0$. The "drop" of vector $\bar{I}_0 r$ volts in the circuit would thus be a vector in line with $\bar{I}_0$ and terminating at the point r. This would also be the resultant of the two vectors $O\bar{E}_0$ and $O\bar{U}_0$. If we take a vector $-Ir$ drop $= OR$, or Or reversed, we have three vectors $O\bar{E}_0$, $O\bar{U}_0$, and OR whose vector sum is zero. This triple set of vectors is to be rotated about the center O, with the uniform angular velocity ω, obtained from Figure 7. But instead of rotating in simple circles, as in the resistanceless case of Figure 2, the three vectors of Figure 8 rotate in equiangular spirals, the angle of each spiral being ϕ as defined in equation 20. That is,

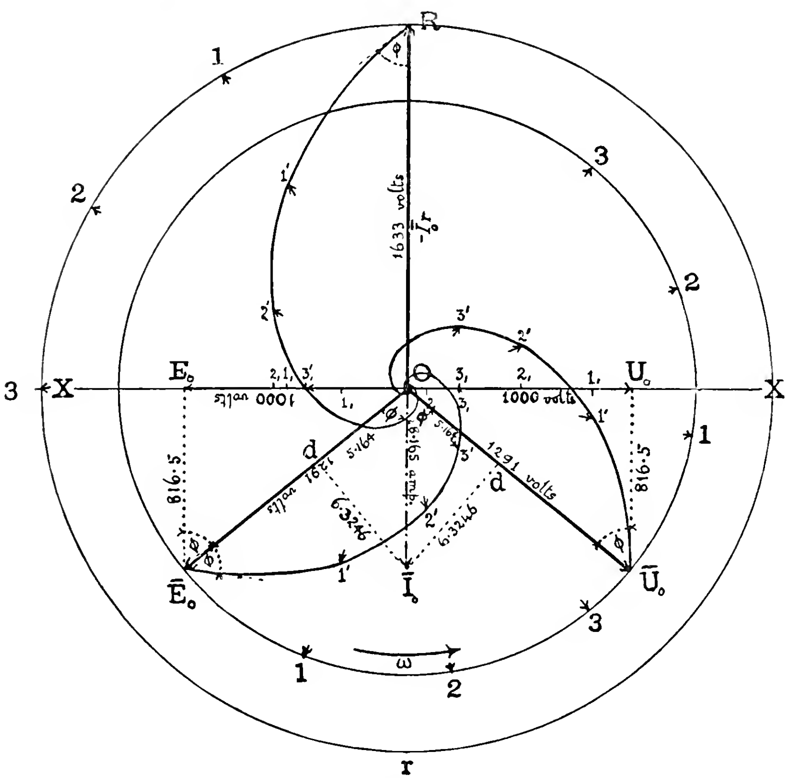


FIGURE 8. Rotative vector-diagram of an oscillating-current circuit containing resistance. Instant of release of condenser charge.

the tangent to the spiral at any point makes with the radius vector the constant angle ϕ. The vectors rotating with the uniform angular velocity ω of formula (16), describe equiangular spirals because energy is dissipated from the system in the resistance r, and each vector shrinks with time at the uniform exponential rate $\epsilon^{-\mu t}$; or falls to $1/\epsilon$th of its value in a time τ equal to the oscillatory time-constant. Since, however, all three vectors shrink at the same exponential rate, and since their vector sum in Figure 8 is initially zero, their vector sum will always be zero, and the sums of their projections on the $-XX$ axis will also always be zero. That is, at all times

$$u - l\frac{di}{dt} - ir = 0. \qquad \text{volts} \quad (21)$$

It is noteworthy that in Figure 8, both the discharging p. d. $O\bar{U}_0$ and the emf. of self-induction make an angle ϕ with the current $O\bar{I}_0$, the former leading and the latter lagging. Each of these emfs. therefore develops power on the current. This is a general condition of the o. c. circuit, different from that of the a. c. circuit, in which the emf. of self-induction exerts no dissipative power on the current, being in quadrature therewith.

We may, however, dispense with the equiangular spirals of Figure 8 by assuming that all the vectors rotate in circles with uniform angular velocity ω, provided we apply to their instantaneously projected values on XOX, the proper damping factor $\epsilon^{-\mu t}$ for the instant considered.

The positions of the three vector emfs. and also their projections on $-XX$, are indicated in Figure 8 for three successive instants angularly separated by 30°, or 0.000,427,5 second.

Returning to the stationary vectors of Figure 7, if we multiply the ω-diagram by the condenser-capacity c, we obtain the oscillatory admittance diagram Y to a scale of mhos. The oscillatory admittance is numerically the same as in the resistanceless case of Figure 3, (0.006,324,6 mho), but makes a negative angle ϕ with the reference axis, instead of 90°. The oscillatory conductance is the real component, and the oscillatory susceptance the $-j$ component, of the oscillatory admittance.

Multiplying the Y diagram by $\bar{U}_0$, the vector amplitude of the initial discharging p. d., we obtain the current or I diagram, $d\,e\,f$, of Figure 7. The initial vector oscillatory current $\bar{I}_0$ is 8.165 amperes, corresponding to a r. m. s. initial oscillatory current I of 5.7735 amperes. The reactive component $e\,f$ is the same as in the resistanceless case. The dissipative component $d\,e$ is the component in phase with the discharging p. d. $O\bar{U}_0$ Figure 8. A like component is also in phase with the self-inductive emf. OE; so that the total equivalent component of dissipation current would be $d\,d'$ Figure 7, or 10.328 amperes maximum cyclic initial value.

Multiplying the I diagram, Figure 7, by $\frac{\bar{U}_0}{2}$, we obtain the stationary-vector power-diagram P, or the watts triangle $g\,h\,k$, which may be drawn to any suitable scale of watts. This gives the maximum cyclic power P_m, of which the component $h\ k$ is the maximum cyclic reactive power, or the maximum cyclic power in the inductance; while the real component $g\ h$ is the maximum cyclic power expended in resistance by the discharging p. d. $O\bar{U}_0$, Figure 8. But a like dissipation of power occurs under the influence of the emf. of self-induction $O\bar{E}_0$, so that the total undamped maximum cyclic dissipative power in the circuit is $g g'$, Figure 7, of 6666.6 watts.

Finally, if we divide the P diagram by $2\ \omega$, or twice the resistant angular velocity, we obtain the W diagram of Figure 7, or the triangle $l\ m\ n$, which may be drawn to a suitable scale of joules. The stationary vector $l\ n$ is the undamped maximum cyclic energy W in the oscillatory circuit as measured, at condenser terminals, or 2.151,65 joules in the case considered. The $-j$ component, or 1.66 joules, is the undamped maximum cyclic energy in the reactance, and the real component $l\ m = 1.3608$ is the undamped maximum cyclic energy dissipated by the discharging p. d. $O\bar{U}_0$, Figure 8, on the oscillatory current. But a like amount of energy will be dissipated by the self-inductive emf. OE. Consequently, the total undamped maximum cyclic dissipative energy in the circuit will be $l\ l'$, Figure 7, or 2.7216 joules.

The condition of either the vector p. d. $O\bar{U}_0$ (Figure 7), the vector self-inductive emf. $O\bar{E}_0$, or the vector current $O\bar{I}_0$ after t seconds, is obtained by applying the exponential $(\epsilon^{-(\iota - j\omega)t})$, as in (19). This exponential may be expressed as $\epsilon^{-\iota t}.\ \epsilon^{j\omega t}$, the first of which is a damping-factor, and the second a rotating factor. The diagrams of Figure 7 apply only to the effects of the rotating factor, as though no damping existed. That is, they represent undamped oscillating quantities, or quantities which would be projected on $-XOX$ by the rotation of the vectors $O\bar{U}_0$, $O\bar{E}_0$, OR, in pure circles, instead of in spirals. The damping-factor $\epsilon^{-\iota t}$ to be applied, is represented in Figure 9, which is drawn on semi-logarithm paper, i.e., on paper ruled to a logarithmic scale of ordinates, but to a uniform scale of abscissas. The ordinates give the damping-factor, and the abscissas the time from release, to a scale of degrees, and also of seconds. The straight line OA represents the damping-factor for voltage and currents. The straight line OB represents the damping-factor for powers and energies. The time in which the voltage and current fall to $1/\epsilon$th of their initial value is τ, the oscillatory time-constant, or 0.001 second in the case considered. The number of radians through which the vectors of Figure 8 must rotate in order to shrink to $1/\epsilon$th of their initial values, i. e., the time-angle for a damping-factor of $\epsilon^{-1} = 0.367,88$, is $l\omega/\rho = \tan\phi$, or, in the case considered, 1.22475 radians $= 70.2°$.

Figure 10 shows the undamped time-values of the various quantities considered, or the values before applying the damping-factor. The sinusoid $\bar{u}$, of 1291 volts amplitude, corresponds, to the projection of

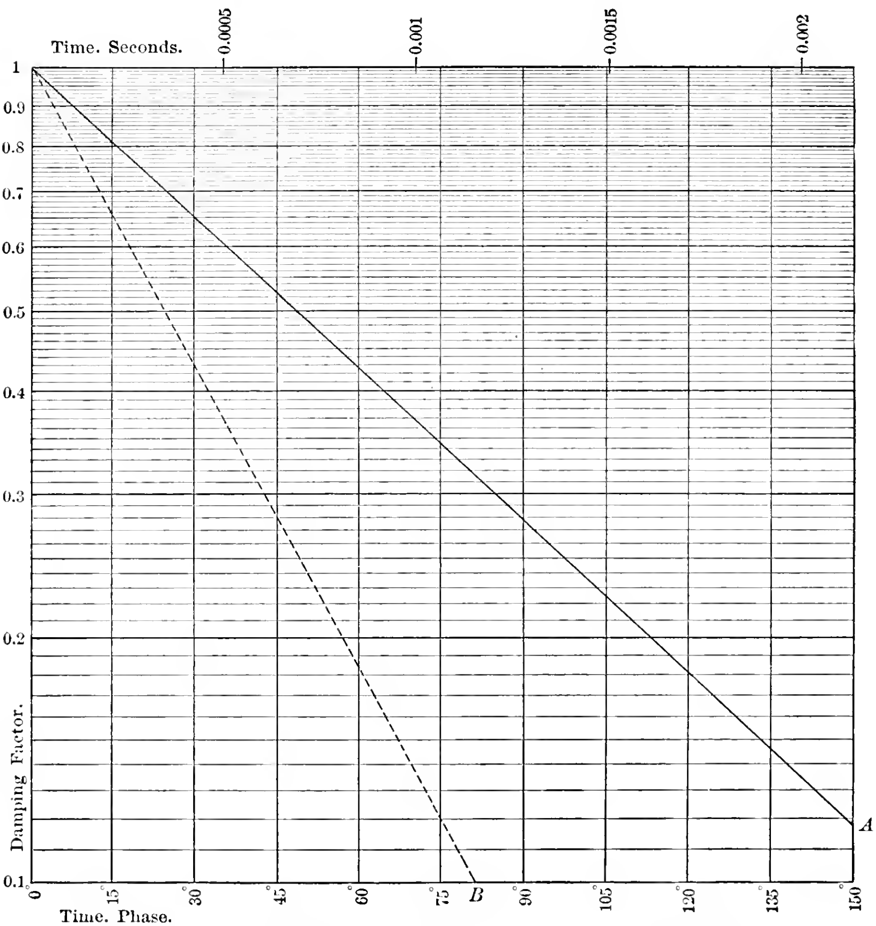


FIGURE 9. Damping-factors at different time-intervals after release.

$O\bar{U}_0$, Figure 8, on the XOX axis. Similarly, the sinusoid $\bar{e}$, of 1291 volts amplitude, corresponds to the projection of OE_0, Figure 8, on the same axis. The heavy sinusoid $\bar{i}$, of 8.165 amperes amplitude, gives the undamped current, starting positively from zero at O. The emfs.

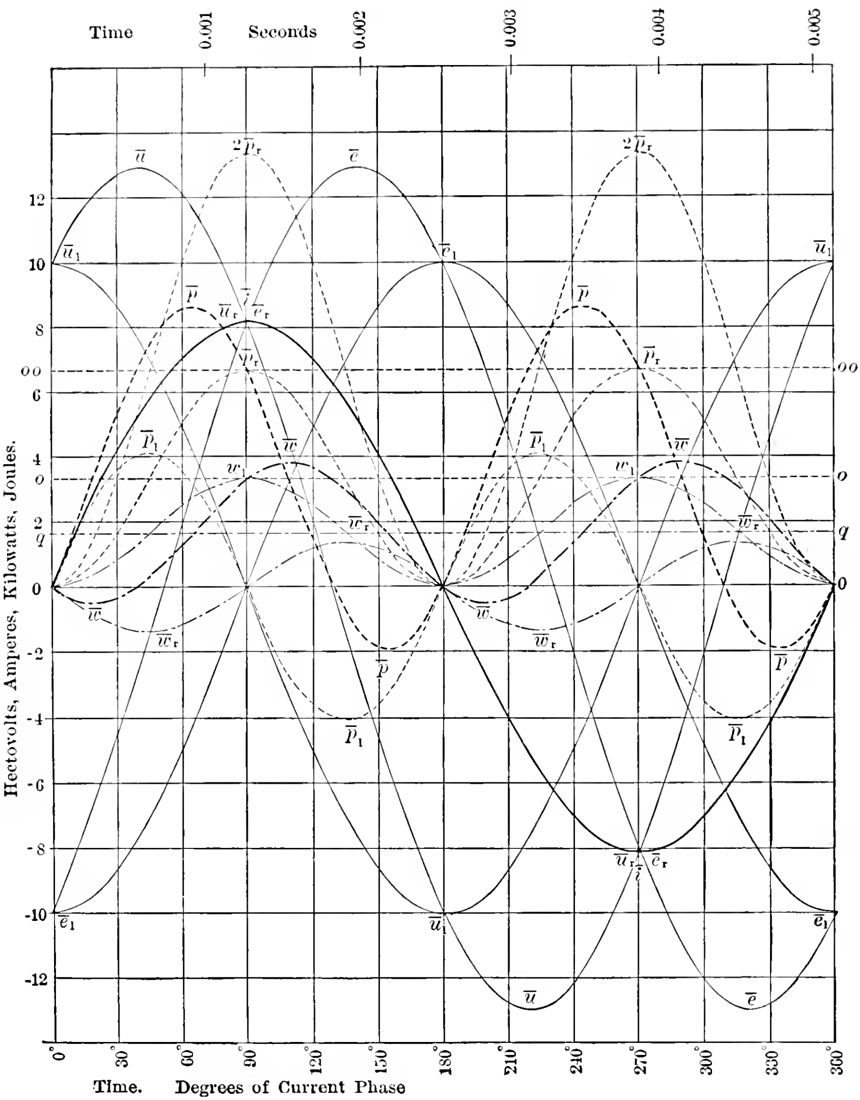


FIGURE 10. Curves of potential difference, current, power and energy in simple oscillating-current circuit containing resistance, leaving damping out of account.

$\bar{u}$ and $\bar{e}$, being out of phase with the current, are divided each into two components $\bar{u}_r$ $\bar{u}_l$ and $\bar{e}_r$ $\bar{e}_l$, the former being in phase with the current, the latter in quadrature therewith. The maximum cyclic values of $\bar{u}_r$ and $\bar{e}_r$ will be 816.5 volts each as at U, Figure 12, and the maximum cyclic values of $\bar{u}_l$ and $\bar{e}_l = 1000$ volts each. The undamped power of the cophase components $\bar{i}$ and $\bar{u}_r$ is the double-frequency $\bar{p}_r$ of 3333.3 watts amplitude about the zero line oo, itself elevated 3333.3 watts. The power of $\bar{i}$ and $\bar{e}_r$ will be an identical sinusoid. The total power of cophase components is thus the sinusoid 2 $\bar{p}_r$ of 6666.6 watts amplitude, about the zero line oo, oo, itself elevated 6666.6 watts. The reactive power expended by the quadrature voltage component of $\bar{u}_l$ upon the current $\bar{i}$, is the double-frequency sinusoid $\bar{p}_l$ of 4082.5 watts as at P, Figure 12. This power is in the magnetic field of the reactance. The total power exerted by $\bar{u}$ upon $\bar{i}$ is the heavy double-frequency sinusoid $\bar{p}$, of 5270 watts amplitude, about the zero line oo elevated 3333.3 watts.

The undamped energy of reactance magnetic flux is the double frequency sinusoid w_l, of 1.6̇6 joules amplitude, about the zero line qq, elevated 1.66 joules. The undamped energy of dissipation in resistance due to $\bar{u}_r$ and $\bar{i}$, is the double-frequency sinusoid w_r of 1.3608 joules. An identical sinusoid would represent the energy of dissipation due to $\bar{e}_r$ and $\bar{i}$; so that the total unattenuated energy of dissipation would be a double-frequency sinusoid of 2.7216 joules amplitude, as at W, Figure 12. The total undamped energy of $\bar{u}$ acting on $\bar{i}$ is the heavy double-frequency sinusoid $\bar{w}$, of 2.151,65 joules amplitude, about the zero line qq.

If we apply the attenuation-factors of Figure 9 to the ordinates of Figure 10, that is, multiplying the currents and voltages by $^{-\alpha t}$, while multiplying the powers and energies by $\epsilon^{-2\alpha t}$, we obtain the curves of Figure 11 (with the exceptions of w and w_r). This diagram represents the actual succession of events in the oscillating-current circuit. The p. d. at terminals falls along u. The emf. of self-induction pursues the opposite curve e. The current follows the heavy line i, reaching a maximum of 3.04 amperes near to 45° of its phase, or 0.00064 second after release. The components of voltage in phase with the current — u_r and e_r — both coincide with its curve. The quadrature components of voltage are u_l and e_l. The total component of voltage in phase with the current follows the curve marked 2 u_r. The power curve p reaches a maximum of 2200 watts at about 60° of its phase. This power rapidly subsides, and only crosses the zero line in feeble measure. The total dissipative power is shown by the curve 2 p_r.

The energy in the inductance follows the curve w_l. This corresponds to $\overline{w}_l$, in Figure 10, after applying the factor $\epsilon^{-2\alpha t}$. The total energy dissipated in the resistance, and the total energy expended by the con-

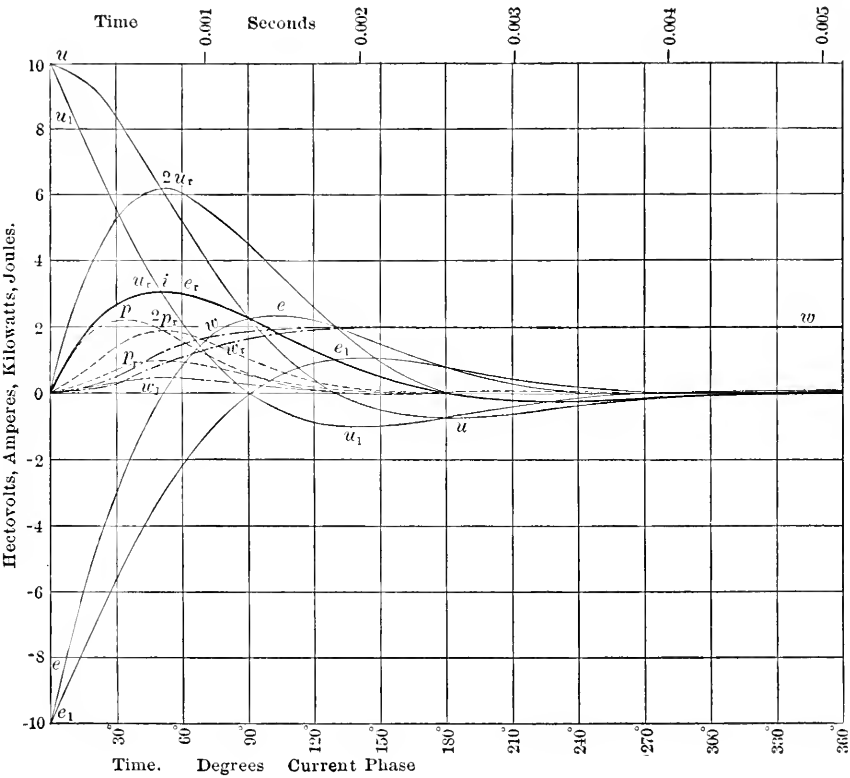
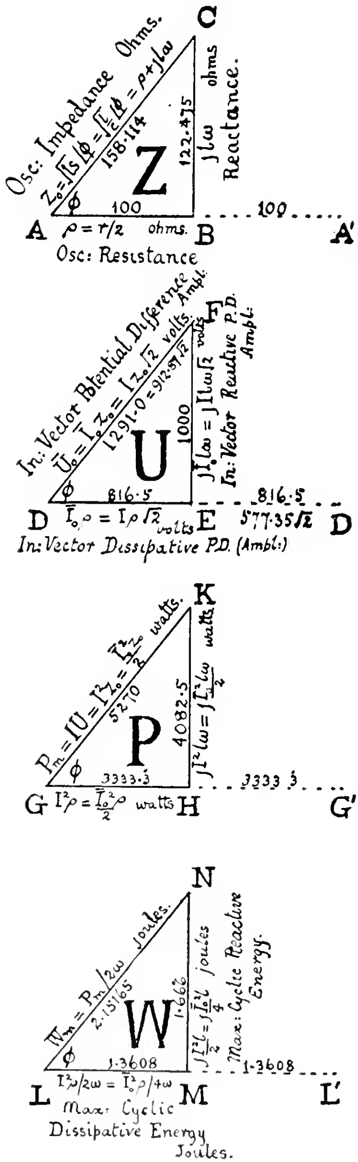


FIGURE 11. Curves of potential difference, current, power and energy in simple oscillating-current circuit containing resistance after applying damping-factors to the ordinates of Figure 10.

denser, follow respectively the curves w_r and w, which do not correspond with the cyclic curves $\overline{w}_r$ and $\overline{w}$ of Figure 10, being energy summations instead of instantaneous values. Consequently, the curves of $\overline{w}_r$, and $\overline{w}$ in Figure 10 must be interpreted otherwise than by the application of an attenuation-factor.

With respect to the phase of the oscillating-current as standard, we have the series of stationary vector-diagrams of Figure 12. The oscil-



latory impedance of the circuit is $Z = \rho + jl\omega$ ohms, or $Z_0 \underline{/\phi}$; where Z_0 is the impedance in the resistanceless condition of Figure 4. The total resistance r of the circuit is represented by AA'. The initial vector potential difference has an amplitude $\bar{U}_0 = \bar{I}_0 Z_0$ volts. The total initial drop of potential resistance is indicated by DD'. The undamped maximum cyclic power is indicated at P, and the undamped cyclic energy at W. The P and W diagrams to standard current phase are inverted with respect to those of standard p. d. phase, because the undamped power lags behind the p. d. but leads the undamped current.

The diagrams in Figures 7 and 12 of the oscillating-current circuit correspond to similar diagrams in connection with the alternating-current circuit.[2] The U, I, P, and W diagrams are stationary vector-diagrams, but they may be converted into rotating vector-diagrams. Thus I may be made a rotating vector-diagram by mounting it on the point d as center, and giving a positive angular velocity ω to the triangle, starting with df in the $-j$ direction. The projection of df on the real axis, after applying the damping-factor $\epsilon^{-\Delta t}$, will

FIGURE 12. Stationary vector-diagrams of impedance, vector potential difference, maximum cyclic power and energy in a simple o. c. circuit containing resistance. Phase of current, standard.

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[2] "Vector Power in Alternating-Current Circuits" by A. E. Kennelly. Trans. American Inst. Elect. Engrs. June, 1910. Vol. 29.

give the actual current at any instant. This is equivalent to rotating dJ in an equiangular spiral of angle ϕ in the manner of Figure 8.

Similar treatment will convert U into a rotating vector-triangle, except that DF should start from the position OU_0 of Figure 8.

The P diagram, in either Figure 7 or Figure 12, can be made into a rotating vector-diagram, by rotating the triangle with angular velocity 2ω about the vertex k or K respectively. Thus, taking the power triangle ghk of Figure 7, we mount it on an axis at k, Figure 13, and draw kr equal and parallel to hg. The three-vector system, kh, kg, and kr is then allowed to rotate with the angular velocity 2ω, in the case considered 2449.5 rad/sec, starting from the position shown, when the p. d. vector OU_0, Figure 8, starts with angular velocity ω from the position KH, or parallel to OY. The orthogonal projections on $-XOX$ of the three vectors kh, kg, and kr, then define at any instant the undamped reactive, total, and dissipative power, under the action of $O\bar{U}_0$ on the oscillatory current. The total undamped power $\bar{p} = 5270$ watts thus lags $\frac{\pi}{2} + \phi$ behind the p. d. vector in terms of power phase, or $\frac{\pi}{4} + \frac{\phi}{2}$ in terms of p. d. phase (70° .23′). The total undamped power is measured on the OX axis from the point O and oscillates between the limits $O\bar{p} = 8603.3$ and $O-\bar{p} = -1936.6$ watts, as shown in Figure 10.

The undamped dissipative power $\bar{p}_r$ oscillates between the limits $O\bar{p}_r = 6666.6$ watts and zero. The total undamped dissipative power is $2\,\bar{p}_r = 13{,}333.3$ watts, owing to the separate actions of $O\bar{U}_0$ and OE_0, Figure 8.

The undamped reactive power $\bar{p}_l$ is reckoned from o as zero, and oscillates between ± 4082.5 watts.

The damping-factor $\epsilon^{-2\mu t}$ must be applied to $\bar{p}$ and $\bar{p}_r$ as measured from O, and to p_l as measured from o, along the OX axis.

If we employ the phase of the oscillating current as standard, and as taken in Figure 10, we mount the power triangle GHK of Figure 12 on an axis at K, and rotate it, as before, at angular velocity 2ω, starting from the position GHK shown in Figure 13 when the current vector $O\bar{I}_0$ of Figure 8 is passing through the position KHo with angular velocity ω. The total power $\bar{p}$ will then lag $\frac{\pi}{2} - \phi$ behind the current vector, in terms of power-phase, or $\frac{\pi}{4} - \frac{\phi}{2}$, in terms of current phase

(19° 37′) as indicated in Figure 10. The $\overline{p}_r$ vector Kr then follows, 90° of power phase or 45° of current phase, behind the current at release. The vector KH, or undamped reactive power vector, Figure 13, starts in phase with the current.

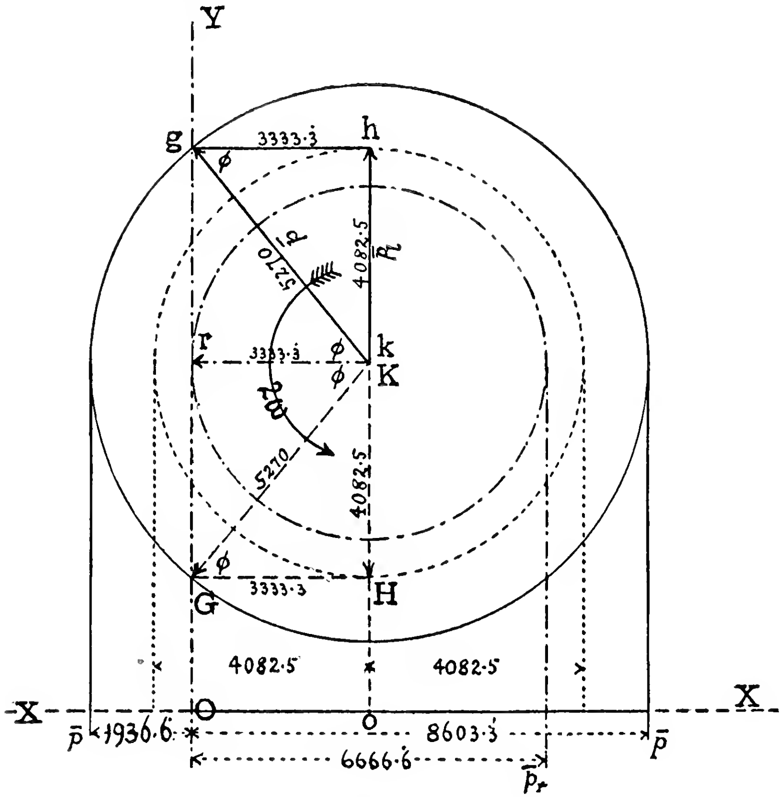

FIGURE 13. Rotative vector power diagram for p. d. standard phase and for current standard phase.

The stationary vector-diagrams of Figure 7 may be understood as involving the following phase relations, with respect to the phase of the undamped vector p. d. $O\overline{U}_0$, Figure 8.

$$\overline{I}_0 = \overline{U}_0 Y \,\overline{\backslash\phi} = UY\sqrt{2}\,\overline{\backslash\phi} = \overline{U}_0 c\omega_0 \,\overline{\backslash\phi} \qquad \text{max. cyclic amperes} \qquad (22)$$

$$P_m = UI\,\overline{\backslash\phi} = \frac{\overline{U}_0\overline{I}_0 Y}{2}\,\overline{\backslash\phi} = \frac{\overline{U}_0^2}{2}\,Y\,\overline{\backslash\phi} \qquad \text{max. cyclic watts} \qquad (23)$$

That is, the undamped current lags ϕ° of p. d. phase behind the discharging undamped p. d. The undamped power reaches its positive

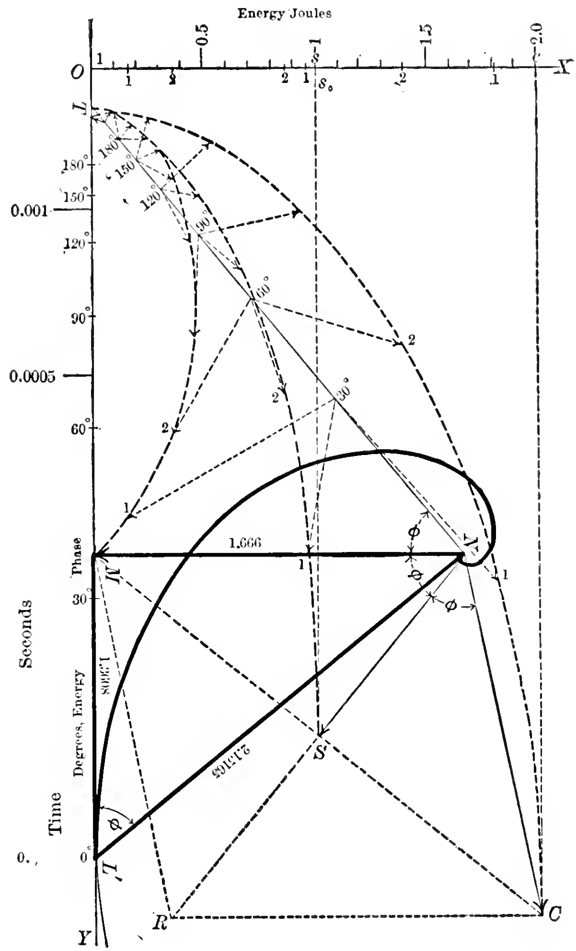

FIGURE 14. Rotating and rolling vector-diagram of the condenser energy, the inductance energy, the semi-system energy and the semi-dissipated energy in an oscillating-current circuit.

maximum ϕ° of its phase behind the discharging undamped p. d. Similarly in Figure 12, we may write the relations

$$\bar{U}_0 = \bar{I}_0 Z \,\underline{/\phi} = IZ\sqrt{2}\,\underline{/\phi} = \bar{I}_0 l\omega_0\,\underline{/\phi} \qquad \text{max. cyclic volts} \quad (24)$$

$$P_m = IU\,\underline{/\phi} = \frac{\bar{I}_0^{\,2} Z}{2}\,\underline{/\phi} = \frac{\bar{I}_0 \bar{U}_0 Z}{2}\,\underline{/\phi}. \qquad \text{max. cyclic watts} \quad (25)$$

Or the discharging p. d. is $\phi°$ of current phase ahead of the current and the undamped power reaches its maximum $\phi°$ of its phase ahead of the current, all being considered as undamped.

In order to derive a rotative energy vector-diagram, we take the triangle lmn of Figure 7, or $L'MN$ of Figure 12 and lay $L'M$ along the OY axis as shown in Figure 14. With the point N as pole, the equiangular spiral NL' is drawn, of angle ϕ. This will be tangent to the OY axis at L'. We then draw a vector $NC = NM$ and making with NM an angle of 2ϕ. The mid-point S of the straight line NC is then connected to N by the vector NS. We may call NC the condenser-energy vector, NM the reactive-energy vector, and NS the semi-system-energy vector; i. e., the vector of the half sum of the energy in the condenser and reactance. We now rotate the three vectors and the spiral, with angular velocity 2ω, while permitting the spiral to roll along the axis OY. The successive turns of the spiral are to be capable of rolling on this axis, as by displacing them infinitesimally out of the plane of the paper, like the wards of a conical band spring. The vectors NC, NS, and NM are also to shrink as they rotate by application of the damping-factor $\epsilon^{-2\iota t}$. Then the projections of C, M, and S, on the OX axis, will define the instantaneous energy in the condenser, reactance and semi-system respectively. The path of the pole N will be the straight line NT, pursued with damped velocity. The paths C and M will be exponential cycloids, that of S an exponential trochoid. The positions of C, S, and M are traced in Figure 14 for several energy phase intervals of 30°, or 0.000,213,8 second, the first two of which are marked 1, 2, on each curve. It will be seen that taking the energy scale along OX conformably with that of LMN, the condenser energy starts at 2 joules, and after 60° or 0.000,427,6 second, it falls to 1.3811 joules. The reactive energy ol commences at zero and after 60° power and energy phase it rises to 0.3544 joule. The semi-system energy Os starts at 1 joule, and after 60° falls to 0.8678 joule. The displacement $s_0 s$ is therefore half the dissipated energy = 0.1322 joule. The total dissipated energy at this instant is thus $2\,s_0 s$ = 0.2645 joule. All three vectors finally terminate and shrink into the point T. The distance $TL' = NL'/\cos\phi$, and NT is perpendicular to NL'.

The fundamental differential equation for quantity q is satisfied by

$$q = A\epsilon^{-(\iota + j\omega)t} + B\epsilon^{-(\iota - j\omega)t}, \qquad \text{coulombs} \quad (26)$$

where A and B are integration constants, while ι and ω follow from the construction of the triangle OPQ, Figure 7.

Choosing the constants consistently with the discharge of the condenser initially charged to potential U_0 volts, the potential at time t is

$$u = U_0 \operatorname{cosec} \phi \, \epsilon^{-\iota t} \sin(\omega t + \phi) \qquad \text{volts} \quad (27)$$

$$= \bar{U}_0 \epsilon^{-\iota t} \sin(\omega t + \phi), \qquad \text{volts} \quad (28)$$

from which q follows by the relation $q = u/s = uc$ coulombs. $\bar{U}_0$ is the initial value of the vector discharging p. d. as obtained from Figures 8 and 12.

The instantaneous current is

$$i = Q_0 \omega \operatorname{cosec}^2 \phi \, \epsilon^{-\iota t} \sin \omega t \qquad \text{amperes} \quad (29)$$

$$= U_0 c \omega_0 \operatorname{cosec} \phi \, \epsilon^{-\iota t} \sin \omega t \qquad \text{amperes} \quad (30)$$

$$= \frac{\bar{U}_0}{z_0} \epsilon^{-\iota t} \sin \omega t = \bar{I}_0 \epsilon^{-\iota t} \sin \omega t, \qquad \text{amperes} \quad (31)$$

where $$\bar{I}_0 = U_0 / l\omega = \bar{U}_0 / z_0 = \bar{U}_0 c \omega_0. \qquad \text{amperes} \quad (32)$$

The emf. of self-induction in the circuit at any instant is

$$e = U_0 \operatorname{cosec} \phi \, \epsilon^{-\iota t} \sin(\omega t - \phi) \qquad \text{volts} \quad (33)$$

$$= \bar{U}_0 \epsilon^{-\iota t} \sin(\omega t - \phi). \qquad \text{volts} \quad (34)$$

The instantaneous power of the condenser in the circuit is

$$p = ui = \bar{U}_0 \bar{I}_0 \epsilon^{-2\iota t} \sin \omega t \cdot \sin(\omega t + \phi) \qquad \text{watts} \quad (35)$$

$$= \frac{\bar{U}_0 \bar{I}_0}{2} \epsilon^{-2\iota t} [\cos \phi - \cos(2\,\omega t + \phi)] \qquad \text{watts} \quad (36)$$

$$= UI \epsilon^{-2\iota t} [\cos \phi - \cos(2\,\omega t + \phi)]. \qquad \text{watts} \quad (37)$$

The instantaneous power of the reactance in the circuit is

$$p' = ei = \bar{U}_0 \bar{I}_0 \epsilon^{-2\iota t} \sin \omega t \cdot \sin(\omega t - \phi) \qquad \text{watts} \quad (38)$$

$$= \frac{\bar{U}_0 \bar{I}_0}{2} \epsilon^{-2\iota t} [\cos \phi - \cos(2\,\omega t - \phi)] \qquad \text{watts} \quad (39)$$

$$= UI \epsilon^{-2\iota t} [\cos \phi - \cos(2\,\omega t - \phi)]. \qquad \text{watts} \quad (40)$$

The total dissipation power of the condenser and reactance is after t seconds,

$$2p_r = p + p' = \bar{U}_0^2 \gamma \epsilon^{-2\alpha t}(1 - \cos 2\omega t), \qquad \text{watts} \quad (41)$$

$$= \bar{I}_0^2 \rho \epsilon^{-2\alpha t}(1 - \cos 2\,\omega t), \qquad \text{watts} \quad (41a)$$

where $\gamma = c\alpha$, the oscillation conductance of the circuit. The energy in the condenser t seconds after release, if $W_l = mn$, Figure 7, $= MN$, Figure 12, is

$$w_c = \frac{\bar{U}_0^2}{4} c \epsilon^{-2\alpha t}[1 - \cos(2\,\omega t + 2\,\phi)] \qquad \text{joules} \quad (42)$$

$$= W_l \epsilon^{-2\alpha t}[1 - \cos(2\,\omega t + 2\phi)]. \qquad \text{joules} \quad (43)$$

The energy in the reactance t seconds after release is

$$w_l = \frac{\bar{U}_0^2}{4} c \epsilon^{-2\alpha t}(1 - \cos 2\,\omega t) \qquad \text{joules} \quad (44)$$

$$= W_l \epsilon^{-2\alpha t}(1 - \cos 2\,\omega t). \qquad \text{joules} \quad (45)$$

The total energy of the system t seconds after release is

$$w = w_c + w_l = \frac{\bar{U}_0^2}{2} c \epsilon^{-2\alpha t}[1 - \cos(2\,\omega t + \phi)\cos\phi] \quad \text{joules} \quad (46)$$

$$= 2\,W_l \epsilon^{-2\alpha t}[1 - \cos(2\,\omega t + \phi)\cos\phi]. \quad \text{joules} \quad (47)$$

The semi-system energy w_s at time t is therefore

$$w_s = W_l \epsilon^{-2\alpha t}[1 - \cos(2\,\omega t + \phi)\cos\phi]. \quad \text{joules} \quad (48)$$

The initial energy of the system is

$$W_0 = \frac{\bar{U}_0^2 c}{2}\sin^2\phi = 2\,W_l \sin^2\phi = 2\,W_m \sin^8\phi = \frac{\bar{U}_0^2 c}{2}. \quad \text{joules} \quad (49)$$

The total loss by dissipation is at any instant

$$w_d = W_0 - w$$

$$= \frac{\bar{U}_0^2 c}{2}\{\sin^2\phi - [1 - \cos(2\,\omega t + \phi)\cos\phi]\epsilon^{-2\alpha t}\}. \quad \text{joules} \quad (50)$$

This is the total expenditure of energy in I^2r up to time t. At any complete energy cycle, when $\cos(2\omega t + \phi) = \cos\phi$, and $t = m\frac{T}{2}$,

$$w_d = \frac{\bar{U}_0^2 c}{2}\left[\sin^2\phi\left(1 - \epsilon^{-2\iota m\frac{T}{2}}\right)\right] = W_0\left(1 - \epsilon^{-\iota mT}\right). \quad \text{joules} \quad (51)$$

The energy dissipated in the first energy cycle, when $t = T/2$,

$$w_1 = W_0\left(1 - \epsilon^{-\iota T}\right). \quad \text{joules} \quad (52)$$

The dissipation in successive energy cycles 1st, 2d, 3rd, 4th, etc., is

$$w_1, \quad w_1\epsilon^{-\iota T}, \quad w_1\epsilon^{-2\iota T}, \quad w_1\epsilon^{-3\iota T}, \quad \text{etc.} \quad \text{joules} \quad (53)$$

The total ultimate energy dissipated is

$$W_0 = w_1\left(1 + \epsilon^{-\iota T} + \epsilon^{-2\iota T} + \ldots\right) \quad \text{joules} \quad (54)$$

$$= \frac{w_1}{1 - \epsilon^{-\iota T}} \quad \text{joules} \quad (55)$$

With the values in the case considered of $\bar{U}_0 = 1291$, $U_0 = 1000$, $c = 4 \times 10^{-6}$, $\omega = 1224.75$, $\phi = 50° \ 46'$, $\iota = 1000$, $T = 0.00513$, $W_0 = 2$, we have, for the attenuation factor of one power period or semi-period of p. d., $\epsilon^{-\iota T} = 0.005,987$. The energy dissipated by I^2r in the first energy cycle is thus $2 \times 0.994,01 = 1.988,02$ joules. The second cycle dissipates $0.005,987 \times 1.988,02 = 0.0119$ joule. Each successive cycle dissipates 0.5987 per cent of the amount dissipated in the last preceding cycle. It is thus evident that in a damped oscillatory discharge, a relatively large fraction of the energy is rejected from the system in the first half-cycle of voltage or current, i. e., the first complete energy cycle.

Logarithmic Decrement.

If v be any vector oscillating-quantity of the type

$$v = V_0\epsilon^{-\iota t}\sin(\omega t + \phi), \quad \text{Ph. Q.} \quad (56)$$

such as a voltage, current, or force.

Then the rotating vector of this quantity $V_0\epsilon^{-\iota t}\epsilon^{j\omega t}$, in passing from one assigned position to another, in a time t_1 seconds, decreases from the first to the second value by the exponential, or damping factor, $\epsilon^{-\iota t_1}$. The exponent, ιt_1, of the damping may be defined as the Naperian logarithm of the decrement during the interval, or simply as the log-decrement in the interval. If the log-decrement be denoted by δ, then

$$\delta = \perp t_1. \qquad \text{numeric} \quad (57)$$

If the rotating vector moves through one radian at the actual angular velocity ω, the time t_1 occupied in the passage will be $1/\omega$ seconds; so that

$$\delta_1 = \frac{\perp}{\omega} = \cot \phi. \qquad \text{numeric} \quad (58)$$

If the rotating vector moves through a half-cycle, semi-revolution, or π radians, the time occupied in the passage will be $t_1 = \pi/\omega$ seconds; so that

$$\delta_\pi = \pi \frac{\perp}{\omega} = \frac{\perp}{2\,n} = \pi \cot \phi, \qquad {}^3\text{numeric} \quad (59)$$

which is the log-decrement of any two successive elongations of the vector's projection in opposite directions on the axis of reference $X'X$. If the vector moves through a whole cycle, revolution, or $2\,\pi$ radians, the time occupied will be $t_1 = 2\,\pi/\omega$ seconds, and

$$\delta_{2\pi} = 2\,\pi \frac{\perp}{\omega} = \frac{\perp}{n} = \frac{T}{\tau} = 2\,\pi \cot \phi, \qquad \text{numeric} \quad (60)$$

which is the log-decrement of any two successive elongations of the vector's projection in one and the same direction on the reference axis. Consequently, from any pair of successive maxima of the oscillating quantity v, the log-decrement is obtainable, and from this the angle ϕ of the equiangular spiral and of the stationary vectors for the quantity is obtainable. In the case considered, $\delta_1 = 0.8165$, $\delta_\pi = 2.5651$, and $\delta_{2\pi} = 5.1302$.

Root of Mean Square of Oscillating-Current Quantities.

If v be any vector oscillating quantity of the type $V_0 \epsilon^{-\perp t} \sin \omega t$, such as an oscillating-voltage, current, or force; then if the damping coefficient $\perp$ is taken as zero, the square root of the mean square of the quantity during any integral number of cycles is $V = V_0/\sqrt{2}$, as in ordinary alternating-current theory. If, for instance, the initial voltage is 1000, and there were no damping, the r. m. s. voltage of the o. c.

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[3] The condition expressed in (59) was first pointed out by Prof. Clerk-Maxwell, in a somewhat different manner. See Bibliography.

circuit as shown by an ideally perfect voltmeter would be 707.1 volts. The question is what would such an instrument show in the presence of a known damping? The reading of the instrument would evidently fall as the damping continued, and, in most practical cases, would fall very rapidly; so that the inquiry must be limited to a certain definite instant during the charge or discharge; or at least to a certain definite interval of time within the process. At any instant we have

$$v^2 = V_0^2 \epsilon^{-2 \alpha t} \sin^2 \omega t. \qquad \text{(Ph. Q.)}^2 \quad (61)$$

The time integral of this square from time $t = 0$ to $t = \infty$ is

$$\int_0^\infty v^2 dt = \frac{V_0^2}{4 \alpha} \left(\frac{\omega^2}{\alpha^2 + \omega^2} \right) = \frac{V_0^2}{4 \alpha} (1 - \cos^2 \phi). \quad \text{(Ph. Q.)}^2 \text{ secs.} \quad (62)$$

But after the lapse of a time t_1 seconds, including m complete periods of the oscillating quantity, its value will have become

$$v_{mT} = V_0 \epsilon^{-\alpha m T} \sin \omega t = V_0 \epsilon^{-\alpha t_1} \sin \omega t, \qquad \text{Ph. Q.} \quad (63)$$

where T is the period of the oscillation, and m is any positive integer; so that

$$\int_0^{t_1} v^2 dt = \frac{V_0^2}{4 \alpha} (1 - \cos^2 \phi)(1 - \epsilon^{-2 \alpha t_1}). \qquad \text{(Ph. Q.)}^2 \text{ secs.} \quad (64)$$

The mean of this square during the interval is

$$\frac{1}{t_1} \int_0^{t_1} v^2 dt = \frac{V_0^2}{4 \alpha t_1} (1 - \cos^2 \phi)(1 - \epsilon^{-2 \alpha t_1}), \qquad \text{(Ph. Q)}^2 \quad (65)$$

and the root of this mean square during the interval is

$$V = \sqrt{\frac{1}{t_1} \int_0^{t_1} v^2 dt} = \frac{V_0}{\sqrt{2}} \left(\sin \phi \sqrt{\frac{1 - \epsilon^{-2 \alpha t_1}}{2 \alpha t_1}} \right)$$

$$= \frac{V_0 \epsilon^{-\alpha \frac{t_1}{2}}}{\sqrt{2}} \left(\sin \phi \sqrt{\frac{\sinh \alpha t_1}{\alpha t_1}} \right). \qquad \text{Ph. Q.} \quad (66)$$

But $V_0 / \sqrt{2}$ is the r. m. s. value of the undamped oscillating quantity, and $V_0 \epsilon^{-\alpha \frac{t_1}{2}} / \sqrt{2}$ would be the r. m. s. value of the oscillating quantity at the mid-interval if it were to continue thereat throughout undamped. Consequently the r. m. s. value of an oscillating quantity $V_0\epsilon^{-\alpha t}\sin\omega t$ during any number of complete periods, is either the r. m. s. value of its initial undamped vector multiplied by

$$\left(\sin\phi\sqrt{\frac{1-\epsilon^{-2\alpha t_1}}{2\alpha t_1}}\right)$$

or the r. m. s. value of its mid-interval damped vector, multiplied by $\sin\phi\sqrt{\frac{\sinh \alpha t_1}{\alpha t_1}}$. After αt_1 passes the numeric 2, the first of these rapidly converges to $\frac{\sin\phi}{\sqrt{2\alpha t_1}}$, so that when the oscillatory damping is rapid, the r. m. s. value of the oscillatory quantity varies inversely as the square root of the interval during which the summation is effected. If the summation be confined to a single period, starting with radius vector V_0, $m = 1$ and

$$V = \sqrt{\frac{1}{T}\int_0^T v^2 dt} = \frac{V_0}{\sqrt{2}}\left(\sin\phi\sqrt{\frac{1-\epsilon^{-2\alpha T}}{2\alpha T}}\right)$$

$$= \frac{V_0\epsilon^{-\alpha\frac{T}{2}}}{\sqrt{2}}\left(\sin\phi\sqrt{\frac{\sinh \alpha T}{\alpha T}}\right). \qquad \text{Ph. Q.} \quad (67)$$

In the case considered, the current is $i = 8.165\ 2^{-1000t}\sin 1224.75t$. If we take the first complete period of $T = 0.00513$ second, with

$$\sin\phi = 0.7746;\ \text{and}\left(\sin\phi\sqrt{\frac{1-\epsilon^{-2\alpha T}}{2\alpha T}}\right) = 0.7746\sqrt{\frac{1}{10.26}} = 0.242;$$

so that the r. m. s. value of the current during the first complete damped oscillation would be 1.396 amperes.

If the oscillating quantity is a cosinusoid of the type

$$v = V_0\epsilon^{-\alpha t}\cos\omega t. \qquad \text{Ph. Q.} \quad (68)$$

The integral to infinite time of its square:

$$\int_0^\infty v^2 dt = \frac{V_0^2}{4\alpha}(1+\cos^2\phi). \qquad (\text{Ph. Q.})^2 \quad (69)$$

So that by the same reasoning as before

$$V = \sqrt{\frac{1}{t_1}\int_0^{t_1} v^2 dt} = \frac{V_0}{\sqrt{2}}\sqrt{\frac{(1 + \cos^2\phi)\,(1 - \epsilon^{-2\alpha t_1})}{2\alpha t_1}}$$

$$= \frac{V_0}{\sqrt{2}}\,\epsilon^{-\alpha\frac{t_1}{2}}\sqrt{(1 + \cos^2\phi)\left(\frac{\sinh \alpha t_1}{\alpha t_1}\right)}, \quad \text{Ph. Q.} \quad (70)$$

which differs from the preceding corresponding expression (66) in the substitution of $\sqrt{1 + \cos^2\phi}$ for $\sqrt{1 - \cos^2\phi}$. By the combination of

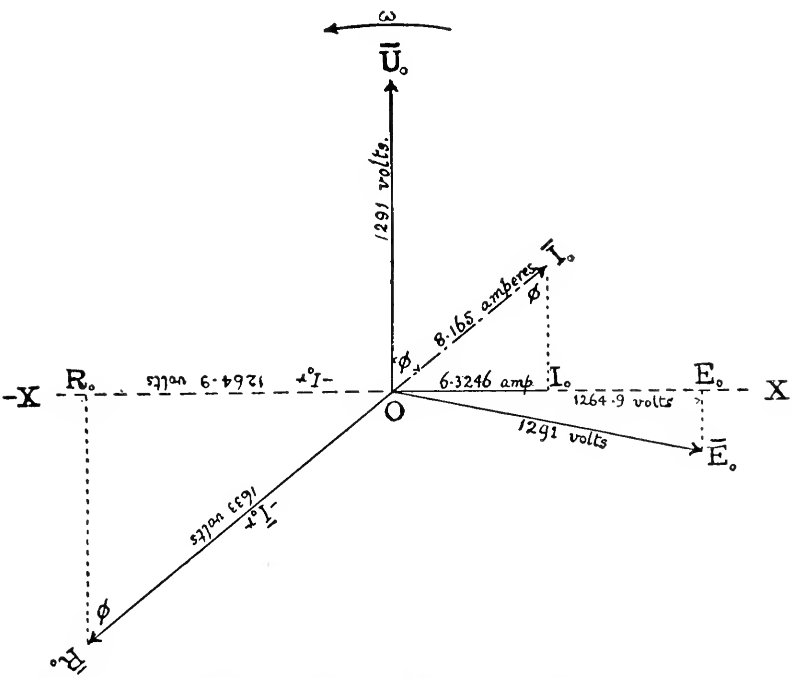

FIGURE 15. Rotative vector-diagram of an oscillating-current circuit containing resistance. Instant of release of reactance charge.

formulas (66) and (70) the r. m. s. values of oscillating quantities containing a sum of sine and cosine terms can be evaluated.

Discharging Oscillations from Reactance. — If with the condenser (Figure 1) uncharged, and resistance r in the circuit, we charge the reactance with magnetic energy, by passing through it a steady current of I_0 amperes, we may then allow the reactance to discharge through the condenser and resistance. We first find ω_0 the angular velocity of discharge for $r = 0$ and then determine ω by the stationary vector-diagram of Figure 7, taking the full resistance r into account.

Upon the axis $-XOX$ of reference, Figure 15, lay off the initial value OI_0 of the current in the reactance, at the instant of release, assumed in this case to be 6.3246 amperes. The initial energy in the reactance will be $lI_0^2/2 = 2$ joules. Lay off an angle $XO\bar{I}_0 = 90_0 - \phi$, and a vector initial current $\bar{I}_0 = I_0$ cosec $\phi = 8.165$ amperes. All of the diagrams in Figures 7 and 12 now apply. The U diagram gives us the initial position of the condenser p. d. vector $O\bar{U}_0 = 1291$ volts. The projection of this on $-XOX$ is $OU_0 = 0$, corresponding to the uncharged condition of the condenser. $O\bar{E}_0$ is the initial position of the vector inductive emf. in the reactance, and the initial self-inductance emf. in the circuit is $OE_0 = 1264.9$ volts propelling the discharging current. The initial Ir drop in the circuit coincides in phase with $\bar{I}_0$, and taken negatively, extends to $O\bar{R}_0 = 1633$ volts. The projection of this on $-XOX$ gives an initial Ir emf. in the circuit of 1264.9 volts just equilibrating the emf. of self-induction. The entire vector system is to be considered as starting to rotate at angular velocity ω. If the diagram is to include the effects of damping, then each vector must rotate in an equiangular spiral of angle ϕ as indicated in Figure 8. But if we apply independent damping factors, the vectors in Figure 15 may rotate in circles as undamped quantities.

It will be seen that Figure 15 corresponds to Figure 8 except that it is advanced $\pi - \phi$ in phase. Thus a given energy charge discharged from the condenser, in this case 2 joules, with an initially uncharged reactance, will give rise to precisely the same rotative vector-diagram as the same energy discharged from the reactance, except in regard to the phase of the diagram. The rotative diagrams of U, I, P, and W will also be the same in either case; except that in the rotating and rolling vector-diagram Figure 14, NM and NC interchange in significance, NM being the reactance-energy vector in one case and the condenser-energy vector in the other.

Simultaneous Discharging Oscillations from Condenser and Reactance. — It is possible for a circuit like that of Figure 1, containing resistance, inductance, and capacity in simple series, to be released with an electric charge in the condenser, and an independent magnetic charge in the coil. The discharge which follows is then in part a condenser discharge, and in part a reactance discharge. If we know the initial charges, we can obtain the vector-diagram of the discharge by determining the ω-diagram of the system (Figure 7) and then making two separate rotative vector-diagrams, one, like Figure 8, for the discharging condenser and the other, like Figure 15, for the discharging reactance. These two vector-diagrams are now to be combined vectorially, into a new resultant vector-diagram, which will represent the behavior of the mixed discharge. Since each of the component diagrams obeys the geometry of Figures 7 and 12, the resultant diagram will also obey it. The rotative vector-diagrams U, I, P, and W will also follow, but the W diagram of Figure 13 will be ambiguous, except in regard to the dissipated energy.

It is evident, moreover, that since the energy of a simple resistance-reactance-condensance oscillator exchanges harmonically from the electric to the magnetic form, after correcting for dissipation, any initial state of assigned separate electric and magnetic energies must correspond to some phase of a discharge from a certain single stock of energy, electric or magnetic.

Charging Oscillations of Circuit Containing Resistance. — If in the circuit of Figure 1, assumed to possess resistance, and with no initial charge, we insert a constant charging p. d. between the terminals t, t, in such a direction as will cause a subsequent discharge to flow in the positive sense of the arrow $d\ e$; then both the charging emf. and the initial direction of the charging current must be reckoned negative, or in the sense e, d. The charging vector-diagram will then be the same as that of Figure 8, with a phase difference of 180^0, or read upside down. The stationary vector-diagrams of Figures 7 and 12 apply as before, as well as the rotative vector-diagrams of Figures 13 and 14. In the last-named, however, the condenser energy Oc must be counted from c to O, or in the reversed direction, since the energy in the condenser is initially nil and increases to its final full value.

We may consider that Figure 8 is the vector-diagram of a charging condenser system in which the constant impressed emf. between the terminals $t\ t$ (Figure 1) is positive, or in the direction of the arrow $d\ e$. If as before $c = 4 \times 10^{-6}$ farad; $r = 200$ ohms; $l = 0.1$ henry and $E = 1000$ volts, we first find $\omega_0 = 1581.14$ radians per second as in Figure 3, then the angle ϕ, and $\omega = 1581.14 \overline{\backslash 50^\circ\ 46'}$, as in Figure 7.

We then lay off $OU_0 = E = 1000$ volts in Figure 8 and $O\bar{U}_0 = OU$.

$\operatorname{cosec} \phi \overline{\backslash \frac{\pi}{2} - \phi} = 1291 \overline{\backslash 39^\circ\ 14'}$ volts. This is the vector charging p. d.

The vector current follows from Figure 7 as 8.165 amperes, and the vector emf. of self-induction symmetrical with OU_0 in regard to the current. The graphs of Figures 10 and 11 then apply as before, except that in Figure 11 the p. d. must be read from the top line as zero, since the potential of the condenser at d, Figure 1, commences at zero and falls to a final value of — 1000 volts. The other graphs in Figure 11 remain unaltered.

The charging oscillations of a simple circuit containing resistance differ only from the discharging oscillations with the same resistance, in regard to phase direction, and to the absolute numerical values of the condenser potential u, condenser charge q, and condenser energy W_0.

Condensers and Reactances in Series.

If we have a circuit (Figure 16), containing a plurality of condensers, or of reactances, or of both, in simple series, containing also resistances, and subject to charging or discharging oscillations, we may compute the behavior of the system as follows:

Let r' = sum of the individual joulean resistances. (ohms).
r'' = sum of the individual hertzian resistances. (ohms).
$r = r' + r''$ = the sum of all the individual resistances. (ohms).
$s = s_1 + s_2 + s_3$ etc., the sum of the individual elastances. (darafs).
$l = l_1 + l_2 + l_3$ etc., the sum of the individual inductances. (henrys).

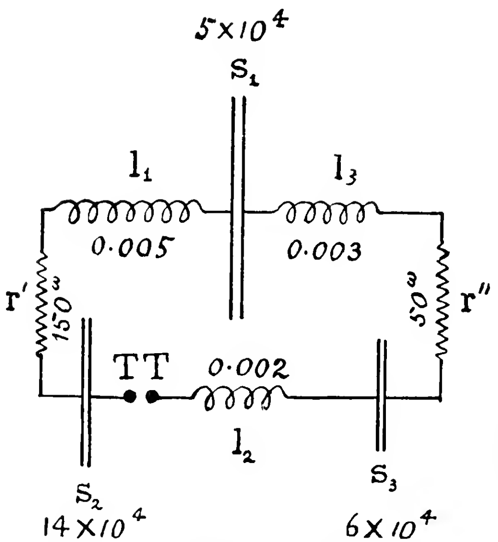

Figure 16. Simple series oscillation circuit of composite elements. Inductances in dekahenrys.

Then if we have only charging oscillations to consider, under the action of a constant impressed emf. E, inserted between terminal TT, we may replace the multiple element system by the equivalent single element system of Figure 1 with resistance r, elastance s, and inductance l.

Discharging Oscillations of Condenser in Simple Series Circuit of Multiple Elements. — Let one of the condensers, say s_1, Figure 16, be initially charged with a quantity Q_1 coulombs to an initial potential $U_0 = Q_1 s_1$ volts, the rest of the system being without charge. Then, after release, the discharge of condenser s_1 will charge the other condenser oppositely, in such a manner as to check the oscillation. The oscillations will take place about a condition of voltage equilibrium, such that the emf. of the discharging condenser is equal and opposite to the sum of the emfs. in the other condensers. If the quantity necessary to flow through the circuit in order to attain the condition of emf. equilibrium is q_e coulombs, then

$$(Q_0 - q_e)\, s_1 = q_e\, (s_2 + s_3), \qquad \text{volts} \quad (71)$$

or

$$q_e = Q_0 \frac{s_1}{s_1 + s_2 + s_3} = Q_0 \frac{s_1}{s}. \qquad \text{coulombs} \quad (72)$$

This is the oscillatory part of the charge in s_1. The remainder would reside permanently in the condensers, if there were no dielectric leakage.

Since the passage of q_e coulombs attains the point of equilibrium about which oscillation takes place, the first swing, neglecting damping, carries $2\, q_e$ coulombs through the circuit.

Of the initial stock of energy $W_0 = Q_0^2 s_1/2$ joules, the portion subject to oscillation is, neglecting damping,

$$W_0 \frac{s_1}{s_1 + s_2 + s_3} = W_0 \frac{s_1}{s}, \qquad \text{joules} \quad (73)$$

This portion is alternately electric and magnetic energy. The remainder persists in the electric form, disregarding leakage.

If the impressed voltage E', instead of being applied initially to the component condenser s_1, were applied initially to the equivalent resultant condenser s of Figure 1, whose elastance is the sum of the component elastances, the charge taken by s would be

$$q_0 = Q_0 \frac{s_1}{s}, \qquad \text{coulombs} \quad (74)$$

and the energy of the charge would be

$$w_0 = W_0 \frac{s_1}{s}. \qquad \text{joules} \quad (75)$$

But we have seen that these are precisely the amounts of oscillation-charge and oscillation-energy available in the case of the charged component condenser. Hence we obtain the following rule for the treatment of composite simple series circuits with component condenser discharges: — Form the equivalent single-element series circuit (Figure 1). Impress the same initial emf. on the single condenser as would be impressed on the component condenser. The discharging oscillations of the single-element system will then be identical with those that would occur in the composite system. After the oscillations have subsided, there will be in the composite system a residual electric energy to take into account, which does not appear in the equivalent single-element system.

Thus in the case of Figure 16, let $s_1 = 5 \times 10^4$, $s_2 = 14 \times 10^4$, $s_3 = 6 \times 10^4$ darafs, $l_1 = 0.05$, $l_2 = 0.02$, $l_3 = 0.03$ henry; $r' = 150$ ohms, $r'' = 50$ ohms, and let an initial charge of 0.02 coulomb be given to s_1 by an impressed emf. of 1000 volts, the other elements being without charge. The initial electric energy of the system $W_0 = 10$ joules. To find the oscillation of the system, we form the equivalent single-element system (Figure 1) with $s = 25 \times 10^4$ darafs, $l = 0.1$ henry, $r = 200$ ohms, and impress 1000 volts initially on the condenser s. This will take a charge of 0.004 coulomb, and an electric energy of 2 joules. These are the oscillation-charge and oscillation-energy of the composite system considered. The oscillations of the system are the same as those indicated in Figures 7 to 15. After the oscillations have subsided, there will be a residual energy of 8 joules in the system, neglecting dielectric leakage, 0.016 coulomb at 800 volts in s_1, —0.004 coulomb at —560 volts, in s_2, and —0.004 coulomb at —240 volts in s_3.

Non-Oscillatory Condenser Discharges.

Although the non-oscillatory discharge of a condenser lies outside of the title of this paper, an outline of the case may be admitted, not only in order to complete the discussion, but also to present therein certain important analogies to the oscillating-current discharge.

If in Figure 12, the semi-circuit resistance ρ is increased until it is equal to z_0, the resistanceless impedance, both the reactance $jl\omega$, the angle ϕ, and the angular velocity ω disappear; so that the discharge becomes non-oscillatory and aperiodic. If ρ is increased beyond this aperiodic value, the discharge continues to be non-oscillatory, but becomes what may be called *ultraperiodic*. We may first consider the ultraperiodic case.

In Figure 17 let op be the exponential time-factor of an ultraperiodic circuit. About op as diameter, construct the semicircle oqp. With center o, and radius $oq = \omega_0$, the resistanceless angular velocity, intersect the semicircle in q. Then the chord pq is the non-oscillatory or hyperbolic angular velocity Ω of the currents in the circuit.[4] In the case of a circuit resistance $r = 500$ ohms, the hyperbolic angular velocity will be $\Omega = 1936.492$ hyperbolic radians per second, as shown at OPQ, Figure 17.

Analytically, $$\Omega = \sqrt{\iota^2 - \omega_0^2} \qquad \text{hyp. radians/sec.} \qquad (76)$$

and the hyperbolic period T of the current and p. d. is $2\pi/\Omega$ seconds.

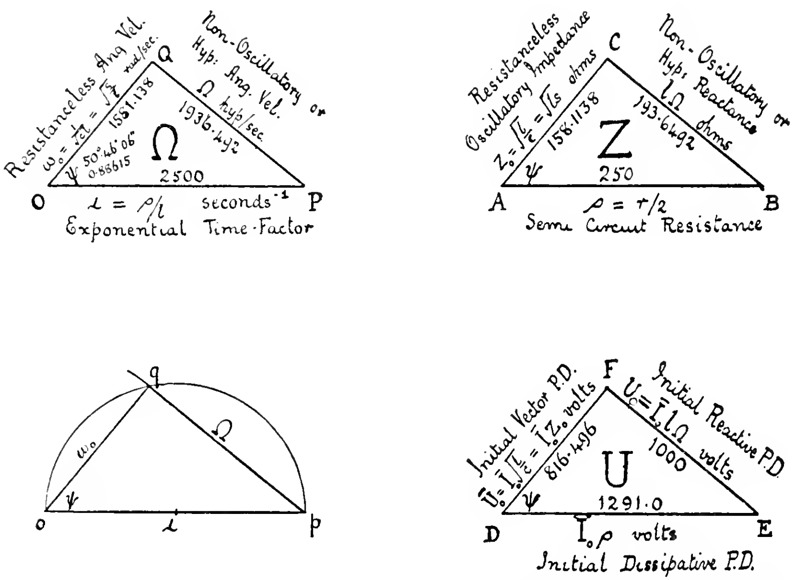

FIGURE 17. Stationary vector-diagrams of hyperbolic angular velocity, impedance, and discharging potential difference in a simple non-oscillating condenser circuit with ultraperiodic resistance.

Similarly, the Z-triangle of Figure 17 has a base $AB = \rho = r/2 = 250$ ohms, and a side AC equal to the resistanceless oscillatory impedance $\sqrt{ls} = 158.1138$ ohms, as in Figures 4 and 12. The remaining side CB opposite to the angle ψ and perpendicular to AC, represents the non-oscillatory or hyperbolic reactance 193.6492 ohms. Operating upon the Z triangle by multiplication with 5.164 amperes, the initial vector-current, we obtain the U triangle DEF, Figure 17.

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[4] For the first publication of the conception of hyperbolic angular velocity of discharges in ultraperiodic circuits, we are indebted to Dr. Alexander Macfarlane. See appended Bibliography.

Figure 18 shows the rotative vector-diagram of voltage and current for the ultraperiodic case considered, and corresponding to Figure 8, the rotative vector-diagram for the oscillating-current case. Lay down the p. d. triangle DEF of Figure 17 at $\bar{I}_0 U_0 O$, Figure 18, to voltage scale as shown. On $O\bar{I}_0$ as semi-axis, construct the rectangular hyperbola $H\bar{E}_0\bar{I}_0\bar{U}_0H'$, whose asymptotes OA and OA' make angles of 45° with $O\bar{I}_0$. From U_0, draw a parallel to $O\bar{I}_0$, intersecting the hyperbola at $\bar{U}_0$. Join $O\bar{U}_0$. Then the angle $\bar{I}_0 O\bar{U}_0 = \psi$ will be the gudermannian of the hyperbolic sector $\bar{I}_0 O\bar{U}_0$; or the hyperbolic sector angle $\bar{I}_0 O\bar{U}_0$ the anti-gudermannian of ψ. From the opposite corresponding point $\bar{E}_0$ of the hyperbola, draw the straight line $O\bar{E}_0$. Then $O\bar{U}_0$ represents the initial vector discharging p. d. in the circuit considered, $O\bar{E}_0$ the initial vector emf. of self-induction, and $O\bar{I}_0$ the initial vector discharging current, corresponding respectively to the vectors of the same denomination in Figure 8. These three vectors, starting at the positions shown, at the moment when the condenser of 4 microfarads, after being charged at 1000 volts p. d., is released through $l = 0.1$ henry and $r = 500$ ohms, run along the hyperbola $H\bar{I}_0H'$, in the positive direction, with uniform hyperbolic angular velocity $\Omega = 1936.492$ hyps. per second. That is, the sectorial areas described by each vector in each second of time are constant and equal to 1936.492 hyperbolic radians, taking the length of the semi-axis $O\bar{I}_0$ as unity. Consequently, at any instant, the sectorial areas $\bar{E}_0 O\bar{I}_0$ and $\bar{I}_0 O\bar{U}_0$ remain equal to that shown, this area being the hyperbolic angle of either, and equal to 1.03172 hyps. $= gd.^{-1}\ 50°.\ 46'.\ 06''$. As the three vectors $O\bar{E}_0$, $O\bar{I}_0$ and $O\bar{U}_0$ rotate with this uniform angular velocity in contact with the hyperbola, they continually approach the asymptote OA', without ever actually reaching it.

The resultant, or vector sum of $O\bar{U}_0$ and $O\bar{E}_0$ is $Or = 2582$ volts, and is equal to the initial vector product of the discharging current $O\bar{I}_0$ and the resistance r of the circuit. This vector also rotates with uniform hyp. angular velocity Ω, over the rectangular hyperbola $h\ r\ h'$, in the direction rh, corresponding to Or, Figure 8. As in Figure 8, the negative of vector Or, or $-\bar{I}_0\ r$, should be drawn in the direction $\bar{I}_0\ O$; but is omitted from the diagram for economy of space. It may be demonstrated that this negative extension of Or, rotating positively over a rectangular hyperbola, the image of $h\ r\ h'$, will always be in vector equilibrium with $O\bar{E}_0$ and $O\bar{U}_0$, so that the geometrical sum of these three vectors at any instant will be zero, just as in the case of Figure 8.

The orthogonal projections of OE_0, $-Or$, and OU_0 give the instantaneous values of the discharging p. d., the Ir drop, and the emf.

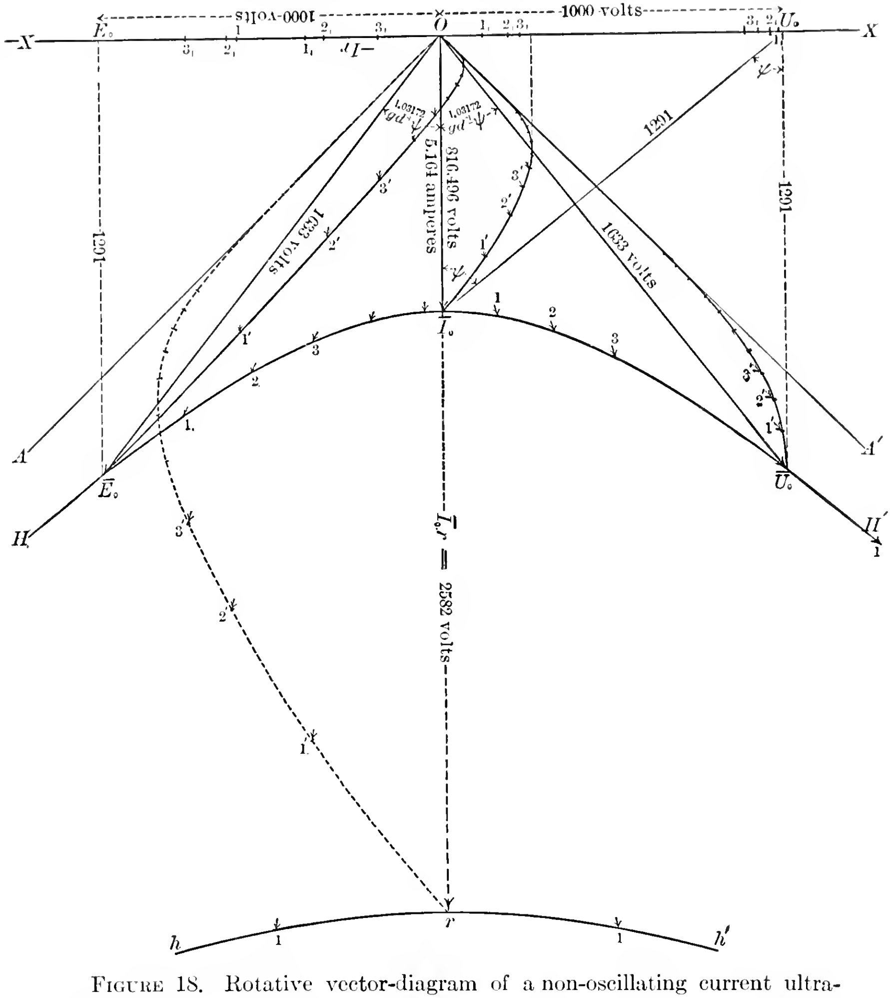

FIGURE 18. Rotative vector-diagram of a non-oscillating current ultra-periodic circuit containing resistance. Instant of release of condenser charge.

of self-induction respectively, after applying the damping factor $\epsilon^{-\mu t}$. That is, we may take the instantaneous XX projections of these undamped vectors, and then apply the damping factor for the instant considered; or, we may shrink the vectors by the application of the damping factor before taking projections, as in Figure 8. In the case considered, the damping coefficient $\alpha = 2500$, as in Figure 17; so that applying the damping factor ϵ^{-2500t}, we obtain the curved lines of Figure 18. The dotted line $o3'2'1'r$ is drawn as though with negative rotation of Or, to simulate the projective effect of a negative vector $- I_0\, r$.

The points 1, 2, and 3 on the hyperbolas, indicate the positions of the various vectors after the lapse of 1, 2 and 3 ten-thousandths of a second respectively, after release. The corresponding points 1′, 2′, 3′ on the curved lines, give the termini of the same vector as reduced by damping, and the projections of the latter, 1, 2, 3, on the XX axis, give the corresponding instantaneous values in the circuit of the discharging p. d. u, the ir drop, the current, and the emf. of self-induction, in the same manner as in Figure 8. It will be observed that while the undamped vectors all increase in length without limit, the actually projected values under the dominating influence of the damping factor, diminish in time without limit. In particular, the current i reaches a maximum when the vector $O\bar{I}_0$ has swept over 1.0317 hyp. radians, in a time 1.0317/1936.492 = 0.000,532,8 second. At the same instant, the self-inductive emf. vector $O\bar{E}_0$ will have reached the transverse axis $O\bar{I}_0$, and its projection on XX will momentarily vanish. Consequently, there will be no emf. of self-induction in the circuit at this instant; because the current is stationary for that instant, being about to diminish. After this instant, the self-inductive emf. changes sign, and propels the current along with the discharging p. d.

It will be noticed that as in the oscillatory-current case, both the discharging voltage and the voltage of self-induction exert dissipative activity upon the discharging current.

It may also be noticed that although the orthogonally-projecting rotating-crank vector-diagrams used in this paper are convenient and useful devices for representing the actions in o. c. circuits, the polar-coordinate vector-diagram, sometimes called the "time" diagram, is not so well adapted for this purpose. The undamped vector quantities in the periodic case may indeed be represented by circles on the polar-coordinate diagram; but the corresponding damped quantities are represented by spirals that are not so easily interpreted as equiangular spirals. Moreover, in the ultraperiodic case, the hyperbolic analogue is missing in the polar-coordinate diagram; so that apparently there is no analogy presented in polar-coordinate representation between the periodic and ultraperiodic cases. It would seem, therefore, that the orthogonally projected "crank-diagram" or "clock-diagram" has wider applications, in these respects, than the polar-coordinate vector-diagram.

Analytically, we have the following relations : —

The fundamental differential equation for quantity q is satisfied by

$$q = A\epsilon^{-(\alpha-\Omega)t} + B\epsilon^{-(\alpha+\Omega)t} \qquad \text{coulombs} \quad (77)$$

where A and B are integration constants, while α and Ω follow from the construction of the triangle OPQ of Figure 17. Choosing the constants consistently with the discharge of a condenser initially charged to potential U_0 volts, the discharging p. d. after t seconds is

$$u = U_0 \cot\psi\, \epsilon^{-\alpha t} \sinh(\Omega t + gd^{-1}\psi) \qquad \text{volts} \quad (78)$$

$$= \bar{U}_0 \epsilon^{-\alpha t} \sinh(\Omega t + gd^{-1}\psi) \qquad \text{volts} \quad (79)$$

form which q follows by the relation $q = u/s = uc$ coulombs. $\bar{U}_0$ is the initial vector value of the discharging p. d. by Figures 17 and 18.

The instantaneous current i is

$$i = \bar{I}_0 \epsilon^{-\alpha t} \sinh \Omega t \qquad \text{amperes} \quad (80)$$

where

$$\bar{I}_0 = U_0/l\Omega = \bar{U}_0/z_0. \qquad \text{amperes} \quad (81)$$

The current i will therefore be a maximum when

$$\tanh \Omega t = \Omega\tau = \sin\psi, \qquad \text{numeric} \quad (82)$$

or

$$\Omega t = gd^{-1}\psi. \qquad \text{hyp. radians} \quad (83)$$

The emf. of self-induction in the circuit at any instant is

$$e = U_0 \cot\psi\, \epsilon^{-\alpha t} \sinh(\Omega t - gd^{-1}\psi) \qquad \text{volts} \quad (84)$$

$$= \bar{U}_0 \epsilon^{-\alpha t} \sinh(\Omega t - gd^{-1}\psi). \qquad \text{volts} \quad (85)$$

The apparent resistance of the circuit u/i is

$$Z = \rho + l\Omega \coth \Omega t. \qquad \text{ohms} \quad (86)$$

That is, the apparent resistance of the circuit, judging from the discharging p. d. and the discharging current, commences at ∞ and tends rapidly to the limit $(\rho + l\Omega)$ ohms.

The instantaneous power of the condenser in the circuit is

$$p = U_0 \bar{I}_0 \cot\psi\, \epsilon^{-2\alpha t} \sinh \Omega t \cdot \sinh(\Omega t + gd^{-1}\psi) \qquad \text{watts} \quad (87)$$

$$= \bar{U}_0 I_0 \epsilon^{-2\alpha t} \sinh \Omega t \cdot \sinh(\Omega t + gd^{-1}\psi). \qquad \text{watts} \quad (88)$$

A complete series of stationary vector-diagrams might be presented following those of Figure 17, and corresponding to those of Figures 7 and 12; but in view of the relative simplicity of formulas (78) to (88), such vector-diagrams have more theoretical than practical interest.

Aperiodic Case.

When ρ, the semi-resistance of the circuit, is just equal to the surge impedance $z_0 = \sqrt{ls}$ of the same; then the circuit is aperiodic, and there is neither a circular angular velocity ω, nor a hyperbolic angular velocity Ω. The aperiodic case may also be represented by a special

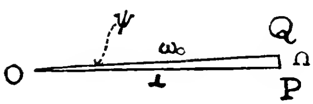

Figure 19. Aperiodic case as limiting condition of ultra-periodic circuit, when $\omega_0 = \perp$. Ω stationary vector-diagram.

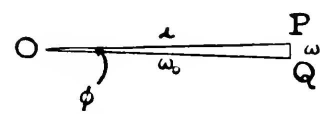

Figure 20. Aperiodic case as limiting condition of o. c. circuit, when $\perp = \omega_0$. ω stationary vector-diagram.

rotating vector-diagram as has been suggested by Macfarlane; but it is easier, for practical purposes, to treat it as a limiting case of the ultra-periodic circuit. We have by (78)

$$u = U_0 \cot\psi\, \epsilon^{-\perp t} \sinh(\Omega t + gd^{-1}\psi). \qquad \text{volts} \quad (89)$$

Now let Ω become very small, as in Figure 19. Consequently, ψ becomes very small; so that $\cot\psi$ may be replaced by $1/\psi$, and $\sinh(\Omega t + gd^{-1}\psi)$ by $(\Omega t + gd^{-1}\psi)$.

Hence $$u_{\psi=0} = \frac{U_0}{\psi} \epsilon^{-\perp t}(\Omega t + gd^{-1}\psi). \qquad \text{volts} \quad (90)$$

But $\Omega = \perp\psi$ and $gd^{-1}\psi = \psi$ when ψ approaches zero; thus

$$u_{\psi=0} = \frac{U_0}{\psi}\epsilon^{-\perp t}(\perp\psi t + \psi) = U_0\epsilon^{-\perp t}(\perp t + 1), \quad \text{volts} \quad (91)$$

and $$q_{\psi=0} = Q_0\epsilon^{-\perp t}(\perp t + 1), \qquad \text{coulombs} \quad (92)$$

$$i_{\psi=0} = -\frac{dq}{dt} = Q_0\perp^2 t\epsilon^{-\perp t}. \qquad \text{amperes} \quad (93)$$

This is a maximum when $t = \tau$; when

$$i_m = Q_0 \iota \epsilon^{-1} = \frac{Q_0}{\tau}\epsilon^{-1}. \qquad \text{amperes} \quad (94)$$

The power in the circuit is

$$p = U_0 Q_0 \iota^2 t \epsilon^{-2\iota t}(\iota t + 1). \qquad \text{watts} \quad (95)$$

We may also derive (91) from the limiting case of the oscillatory discharge. Taking (27), we have

$$u = U_0 \operatorname{cosec}\phi\, \epsilon^{-\iota t} \sin(\omega t + \phi), \qquad \text{volts} \quad (96)$$

and if ω becomes very small, as in Figure 20, the angle ϕ becomes very small; so that $\operatorname{cosec}\phi$ may be replaced by $1/\phi$ and $\sin(\omega t + \phi)$ by $(\omega t + \phi)$. Hence

$$u_{\phi=0} = \frac{U_0}{\phi}\epsilon^{-\iota t}(\omega t + \phi). \qquad \text{volts} \quad (97)$$

But $\omega = \iota\phi$ when ϕ approaches zero; thus,

$$u_{\phi=0} = \frac{U_0}{\phi}\epsilon^{-\iota t}(\iota\phi t + \phi) = U_0 \epsilon^{-\iota t}(\iota t + 1), \qquad \text{volts} \quad (98)$$

as in (91). So that the aperiodic case may be computed either as the limiting oscillatory case with $\omega = 0$, or as the limiting ultraperiodic case with $\Omega = 0$.

Summary of Conclusions.

The orthogonal-projection or rotating-crank vector-diagram of the ordinary a. c. circuit applies also, by extension, to the o. c. circuit.

With the aid of the stationary vector-diagrams, the principal features of any given o. c. case may be simply and speedily deduced.

By making the above stationary vector-diagrams rotative, and subsequently applying the proper damping-factor, the process of oscillation in any given o. c. circuit may be readily visualised.

By interpreting the above diagrams and formulas hyperbolically, the corresponding properties of the ultraperiodic case may be readily computed and visualised. That is, the rotating-crank vector-diagram of the ordinary a. c. circuit applies also, by extension, to the non-oscillatory ultraperiodic condenser circuit.

The properties of the condenser circuit, whether oscillatory or ultraperiodic, are intimately connected with a certain angle, $cos^{-1}(\rho/z)$ or $cosh^{-1}(\rho/z)$, connected with the circuit constants.

The polar-coordinate type of vector-diagram is not so conveniently adapted to the o. c. circuit as the crank-projection diagram.

The aperiodic case may be treated as the limiting case either of the oscillatory, or of the ultraperiodic, circuit.

List of Symbols Employed.

A. B.	Constants of integration (coulombs).
a. c.	abbreviation for alternating current.
c, c_1, c_2, c_3	Permittance of a condenser, or of each of several condensers (farads).
γ	Oscillation conductance of an o. c. circuit (mhos).
$\delta_1, \delta_\pi, \delta_{2\pi}$	Logarithmic decrement of an oscillating quantity $\left(\log_\epsilon \frac{V_0}{v}\right)$ during the angular interval of 1 radian, π radians (semi-period), and 2π radians (complete period), of the radius vector (numeric).
ϵ	$= 2.71828. \ . \ . \ .$
$\bar{E}_0$	Initial vector amplitude of emf. of self-induction (volts ∠).
E_0	Initial emf. of self-induction in circuit (volts).
E'	Charging emf. impressed upon a condenser (volts).
$e, \bar{e}$	Instantaneous value of emf. of self-induction in an o. c. circuit, and the undamped value of same (volts).
$\bar{e}_r, e_r$	Undamped and damped instantaneous values of self-induction emf. in phase with the current (volts).
I_0	Initial current strength in a circuit (amperes).
$\bar{I}_0$	Initial vector amplitude oscillating current (amperes ∠).
I	$=\bar{I}_0/\sqrt{2}$, the r. m. s. value of initial vector current amplitude (amperes).
$\bar{i}, i$	Undamped and damped instantaneous currents (amperes).
j	$= \sqrt{-1}$, quadrantal operator.
$l, l_1, l_2, l_3,$	Inductance of an o. c. circuit, and of particular parts thereof (henrys).
m	any positive integer (numeric).
n	Frequency of oscillation of a circuit (cycles per second).
o. c.	abbreviation for oscillating-current.
π	$= 3.14159 \ . \ . \ .$ (numeric)
$Ph. Q.$	Any oscillatory physical quantity pertaining to an o. c. circuit.

P_m. $\bar{p}$, p	Undamped maximum cyclic power developed by condenser, undamped instantaneous value of same, and damped instantaneous value of same (watts).
$\bar{p}_l$, p_l	Undamped and damped instantaneous values of power in inductance (watts).
$\bar{p}_r$, p_r	Undamped and damped instantaneous values of power developed by condenser in the resistance of an o. c. circuit (watts).
$p' = ei$	Instantaneous power of the emf. of self-induction (watts).
$2\bar{p}_r$, $2p_r$	Total undamped, and total damped, intantaneous values of power developed in the resistance of an o. c. circuit by condenser and inductance combined (watts).
Q_0. q_0	Initial charge in a condenser (coulombs).
$\bar{Q}_0$	Initial vector amplitude of electric charge in condenser (coulombs $\angle$).
Q	$= \bar{Q}_0/\sqrt{2}$, the r. m. s. value of the initial vector amplitude (coulombs).
Q_1	Initial charge of one among several condensers in series (coulombs).
q	Instantaneous charge in condenser (coulombs).
q_e	Quantity required to flow through an o. c. circuit in order to establish p. d. equilibrium (coulombs).
r'	Joulean resistance in an o. c. circuit (ohms).
r''	Hertzian resistance in an o. c. circuit (ohms).
$r = r' + r''$	Total resistance in an o. c. circuit (ohms).
r. m. s.	Square root of mean square of an oscillatory quantity.
ρ	$= r/2$, Semi total of resistance in an o. c. circuit (ohms).
Φ_0	Total initial magnetic flux linked with a discharging circuit counting all of the turns in the same (volt-seconds).
ϕ	Phase angle in an o. c. circuit, and angle of a spiral (radians or degrees).
ψ	Phase angle in an ultraperiodic circuit (radians or degrees).
$gd^{-1}\psi$	Antigudermannian of a circular angle, or the hyperbolic angle of which ψ is the gudermannian (hyp. radians).
s, s_1, s_2, s_3,	$= 1/c$, elastance of a condenser, or of each of several condensers (darafs).
t	Elapsed time from the release of an o. c. system (seconds).
t_1	Time interval (seconds).
T	Period of an o. c. circuit (seconds).
τ	$= l/\rho$, Oscillation time-constant of an o. c. circuit (seconds).

$\perp$	$= 1/\tau = \rho/l$, time-constant-reciprocal of an o. c. circuit (seconds $^{-1}$).
$\bar{U}_0$	Initial vector amplitude of the discharging p. d. in an o. c. circuit (volts $\angle$).
U	$= \bar{U}_0/\sqrt{2}$, the r. m. s. value of the initial vector discharging p. d. (volts).
U_0	Initial discharging p. d. in a circuit (volts).
$\bar{u}$, u,	Undamped and damped time-values of condenser p. d. in an o. c. circuit (volts).
$\bar{u}_r$, u_r	Undamped and damped time-values of the component of the condenser p. d. in phase with the current (volts).
V_0, V, v	Initial, r. m. s., and instantaneous values of any oscillating quantity, pertaining to an o. c. circuit.
W_0	Initial energy in a condenser or in a reactance (joules).
W_l	Initial vector amplitude of cyclic energy in reactance, *mn*, Figure 7, *MN*, Figures 12 and 14 (joules).
W_m	Undamped maximum cyclic value of energy in a condenser or reactance (joules).
$\bar{w}$, w	Undamped and damped time-values of energy in condenser or reactance (joules).
w_1	Energy dissipated in a circuit in the first energy cycle (joules).
w_c, w_l	Energy in a condenser, and in a reactance, at a specified time (joules).
w_d	Total expenditure of energy by dissipation from a circuit up to t seconds (joules).
w_0	Energy of charge communicated to a series of condensers (joules).
w_s	Semi-system energy at time t (joules).
XX, YY	Cartesian rectangular coordinates.
X_l, X_c	Reactance of an inductance and of a condenser in an o. c. circuit (ohms).
Y	Admittance of an o. c. circuit (mhos).
Z, z	Impedance of an o. c. circuit (ohms).
Z_0, $z_0 = \sqrt{l s}$	Impedance of an o. c. circuit devoid of resistance (ohms).
ω_0	Angular velocity of an o. c. circuit with its resistance ignored (circular radians per second).
ω	Angular velocity of an o. c. circuit in the presence of its actual resistance (circular radians per second).
Ω	Angular velocity of an ultraperiodic circuit (hyperbolic radians per second).

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CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE, E. L. MARK, DIRECTOR. — No. 218.

DIVISION OF LABOR AMONG ANTS.

By Edith N. Buckingham.

With a Plate.

CONTRIBUTIONS FROM THE ZOÖLOGICAL LABORATORY OF THE MUSEUM OF COMPARATIVE ZOÖLOGY AT HARVARD COLLEGE, UNDER THE DIRECTION OF E. L. MARK. — No. 218.

DIVISION OF LABOR AMONG ANTS.[1]

By Edith N. Buckingham.

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Table of Contents.

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I. Introduction.

It is my purpose to state in the present paper the results of some observations and experiments on division of labor among ants, especially in relation to size and structural differences of individual workers and of classes. These studies have been carried on, with interruptions, since September, 1904, chiefly at the Museum of Comparative Zoölogy, at Cambridge, Mass., under the guidance of Dr. E. L. Mark, to whom I am deeply grateful for kind and valuable assistance, but also in part at several other places, especially in summer, viz., at the Marine Biological Laboratory, Woods Hole, Mass., at the Biological Laboratory of the Brooklyn Institute of Arts and Sciences at Cold Spring Harbor, Long Island, at Ogunquit, Maine, and at Randolph, N. H. I wish to express my obligations to Dr. W. M. Wheeler for help in identifying species and for many suggestions.

II. Historical.

Many writers have for a long time maintained that among ants there is a division of labor correlated with differences of size and structure. While some of this work has been done either by experiment or observation in the laboratory, or in the field, with this question as the main point, much of it has been more or less incidental, the student giving attention to this subject only as a side issue of other work.

There are some differences of opinion as to the functions of the different classes. Probably this is largely due to the fact that the observations were in many cases made upon different species. There are, however, certain facts regarding the existence of classes which seem to be established by previous observations.

Different classes certainly exist in various species of ants; but the division into classes is not sharp in all species in which they are present.

Where structural differences are not well defined, there is not a sharp distinction in function: For example, classes 1 and 2 of Anomma (Savage, '**47**, p. 5); some species of Eciton (Bates, '**92**, p. 355; Belt, '**88**, p. 357); different sizes of workers other than soldiers in Holcomyrmex (Wroughton, '**92**, p. 15); some species of Camponotus (Pricer, :**08**, p. 192).

In those species where the classes are well defined, the small workers are generally admitted to be the real workers. In Anomma they engage in carrying pupae and prey (Savage, '**47**, pp. 5–11; '**50**, p. 197); in *Eciton crassicornus* and *E. vastator*, in repairing the nest, and in *E. hamata* and *E. mexicana*, in marching beside the column (Bates, '**92**, p. 362; Belt, '**88**, p. 22; Sumichrast, '**68**, pp. 43–44); in Atta, in the charge of the "razzias" (Sumichrast, '**68**, p. 44); in *A. sexdens*, in tending queen larvae and leaf-cutting (Forel, '**97**, p. 331); in *A. cephalotes*, in leaf-cutting (Forel, l. c.); in *A. structor*, in carrying in seeds (Moggridge, '**73**, p. 49); in *A. fervens*, in leaf-cutting (Wheeler, :**01**[a], p. 200); in Solenopsis, in constructing covered ways (Rothney, '**89**, p. 366); in *Pheidologeton laboriosus*, in being in the open, fighting, making covered ways (Rothney, '**89**, p. 369); in *Pheidole megacephala*, in attacking prey, carrying home prey (Heer, '**52**, p. 3); in *P. instabilis*, in foraging, excavating, caring for the brood, assisting callows to emerge (Wheeler, :**07**[b], p. 4); in Pheidole in general, in fighting, going into the field, feeding the majors (Wheeler, :**02**, p. 770); in Holcomyrmex, in harvesting grain (Wroughton, '**92**, p. 15); in *Œcophylla smaragdina*, in carrying larvae for shuttles (Doflein, :**05**, p. 502); in Camponotus, in fighting outside the nest (Forel, '**74**, p. 354); in *Camponotus ligniperdus*, *C. herculeanus*, and *C. pubescens*, in carrying larvae and pupae (Forel, l. c.); in Myrmecocystus, in fighting (Escherich, :**06**, p. 46); in *Formica sanguinea*, in all the domestic duties (White, '**95**, p. 67).

In regard to the functions of the large workers, there is evidence that they engage in a variety of occupations, but in some cases there is a great difference of opinion as to what they do. For example, in most species they appear to be for defence, since they are thus reported for Anomma (Savage, '**47**, p. 5; '**50**, p. 197); *Eciton erratica*, *E. vasta-*

tor, *E. crassicornus* (Bates, '**92**, pp. 355 and 362), *Pheidole megacephala* (Heer, '**52**, p. 3), *P. pallidula* (Forel, '**74**, p. 384), *P. instabilis* (Wheeler, :**07**[b], p. 4), Pheidole in general (Lubbock, '**77**; '**79**, p. 69; '**82**, p. 20; but Wheeler, :**02**, p. 770, says that in some species of this genus they are cowardly), Pogonomyrmex (McCook, '**79**, p. 196; Wheeler, :**05**[d], p. 384), Camponotus (Forel, '**74**, p. 354; McCook, '**76**, p. 286; Alcott, '**97**; Lüderwaldt, :**09**, p. 311), *Formica sanguinea* (White, '**95**, p. 67), Atta (Forel, '**97**, p. 331; Wheeler, :**07**[a], p. 675; Belt, '**88**, p. 83), Œcophylla (Doflein, :**05**, p. 502), and *Formicoxenus nitidulus* (Forel, '**86**, p. 132). They are said to form arches with their bodies in Anomma (Savage, '**47**, p. 5); to march beside the column in Anomma (Savage, l. c.; '**50**, p. 197), *Eciton drepanophora* (Bates, '**92**, p. 360), *E. hamata* (Belt, '**88**, p. 22), and *Atta fervens;* to perform some sort of guard duty in *Eciton mexicana* (Sumichrast, '**68**, pp. 43–44), Pheidole (Reichenbach, '**96**, p. xcv), Pogonomyrmex (McCook, '**79**, p. 196), and especially Colobopsis, the wood borers (Forel, '**96**, p. 486), in which the soldiers close the entrance of the nest with their heads and allow no strangers to enter (Escherich, :**06**, p. 46; Forel, '**86**, p. 132; :**03**, p. 83; :**05**, pp. 453–454; Wheeler, :**04**[a], p. 44; :**10**, pp. 182, 211–212; :**05**[f]; Lubbock, '**82**); to attack prey in Anomma (Savage, '**47**); to cut up prey or seeds in Pheidole (Heer, '**52**; Reichenbach, '**96**; Wheeler, :**02**, p. 770; :**10**, p. 279; :**07**[b], p. 4), Aphaenogaster and Pogonomyrmex (Forel, '**86**, p. 132); to attend to various aspects of building in *Eciton mexicana* (Sumichrast, '**68**) and Pheidologeton (Rothney, '**89**, p. 369); to care for the young in Eciton and Myrmecocystus (Escherich, :**06**, p. 46); to grind leaves in *Atta cephalotes* Forel, '**97**, p. 331); to cut grass in Pogonomyrmex (McCook, '**79**, p. 22); to carry seeds in *Atta structor* (Moggridge, '**73**, p. 49); to carry out refuse in Camponotus (Pricer, :**08**, p. 192); to pull the edges of a leaf together in mending the nest in Œcophylla (Doflein, :**05**, p. 506); to perform in general the severer duties of the colony in Formicoxenus (Forel, '**86**, p. 132) and Pogonomyrmex (McCook, '**79**, p. 22); to become repletes in Myrmecocystus (Wheeler, :**08**[b], p. 378); to lay eggs in *Aphaenogasta fulva* (Fielde, :**01**) and in Camponotus (Pricer, :**08**).

As examples of difference of opinion held by competent observers as to the function of the large workers in the same species, the following may be cited. In Atta they are thought by Bates to be passive defenders of the rest of the colony, but Wheeler contends that they are aggressive soldiers. In Pheidole generally they are thought by Lubbock to be fighters, but Wheeler states that, while in some species of this genus they are for defence, in others they are not.

In certain species there seems to be a third class, which is in size and structure intermediate between these two, while in function it sometimes resembles the large and sometimes the small.

Often the same occupation is seen to belong to one class in one species and to another in another species.

That in general the greater amount of work is done by a few workers is the contention of Miss Fielde (:**03**[d], p. 621) and of Forel ('**74**, p. 151). Lubbock ('**82**), working on foraging in *Formica fusca*, came to the same conclusion, but made no study of the relation of activities to size. Pricer thinks that in Camponotus it is the medium-sized ants which accomplish most, whereas Forel believes that the small ones do the most work. But I know of no case where in a given species a large number of functions have been studied in relation to class distinctions.

Therefore without further evidence we are not in a position to state definitely the relation of size and form to division of labor.

III. Polymorphism of Ants.

A. Polymorphism in General.

In order to study satisfactorily the correlation between polymorphism and division of labor, it is necessary to define what is meant by polymorphism. Wheeler (:**10**, p. 86) says that only those animals properly represent the phenomena of polymorphism "in which characteristic intraspecific and intrasexual groups of individuals may be recognized, or, in simpler language, those species in which one or both of the sexes appear under two or more distinct forms."

According to Wheeler (see also Wheeler, :**07**[b], p. 85) and to Escherich (:**06**, p. 45) polymorphism is commonly supposed to be due to a physiological division of labor.

Though both sexes of ants show some tendency to polymorphism, such as is described for males by Wheeler (:**04**[d]), by Forel (:**04**[a] ; :**04**[b]), and by Emery ('**86** ; :**06**), polymorphism is not common in the male sex, and we are not here concerned with that manifestation of it. If, however, we confine our attention to the females of a colony of ants, it will be found that they are, as a rule, markedly polymorphic, consisting of two chief divisions, queens and workers ; although in some cases (certain Ponerinae) there are no apparent distinctions between the queen and the worker classes, the two forms being connected by individuals of an intermediate character (Wheeler, :**00**[a], p. 1 ; :**10**, pp. 242–243). Wheeler (:**00**[a]) feels certain "that forms externally indistinguishable from the workers commonly function as females." Again, he (:**03**[c], p. 6) says of *Leptothorax emersoni* that many workers approach the queens in size, possess ocelli, and probably function as queens. In *Leptothorax tuberum*, Fletcher ('89) found that workers lay eggs. Workers, though very rarely, may even bear vestiges of wings (Wheeler, :05[e]). Emery ('94, p. 54), too, states that in most Ponerinae the workers are only slightly differentiated from the queens, and that in several species there are intermediate forms between workers and queens. Furthermore, that some species have a wingless queen which is transitional between the queen and workers. Such a queen is also mentioned by Wheeler (:04[e], p. 251). Moreover, the head of the normal, winged queens of such ants as have two different sorts of workers is almost always less developed than that of the largest worker (Camponotus, Pheidologeton, Pheidole, etc.); only seldom (certain Colobopsis and Cryptocerus) does the head of the queen resemble that of the largest worker in size and form. It furthermore appears (Emery, '94, p. 55) that the degree of development of the sexual glands is correlated with the development of the head.

As a rule, however, egg-laying belongs to the queen, while the workers attend to the general needs of the nest, though some workers do occasionally lay eggs. In a good many species they seem, according to Miss Holliday (:03), to be anatomically if not physiologically adapted for this. Indeed, Wheeler (:06[a], p. 298) says that when workers are well fed, they readily become fertile, and that they can, and often do, produce normal young from unfecundated eggs. But if the workers may lay eggs, so, on the other hand, according to Wheeler (:03[a]), "the females [Leptothorax Mayr] live almost like the workers, being merely somewhat less inclined to work." But in some cases (e. g. in *Formica consocians, difficilis, sanguinea*, etc.) the queen has so far lost her powers of performing the general work of the nest that she is dependent on the workers of another species to bring up her first brood (Wheeler, :04[b], p. 359; :04[c]; :05[a]; :05[b]; :05[d], p. 399), and it is doubtful whether in some species she alone could found a colony (Wheeler, :05[a]). Again, through becoming parasites, the workers of some species in several genera (Anergates, Epoecus, Tomognathus, Sympheidole, Epipheidole) have secondarily disappeared (Forel, '95, p. 145; Wheeler, :04[d]).

Nor are the workers of a given species themselves always alike. Emery ('94), Wheeler (:07[b], pp. 53–57; :08[a], pp. 44–47; :10, pp. 92–99), and Forel ('95, pp. 142 et seq., and :04[a]) describe several castes of females in many species, and these authors would account for the great gap between the extreme types of workers in a species by the disappearance of intermediate forms (Emery, '94; Wheeler, :08[a], pp. 58–59; :10, p. 112; Forel, '86, p. 132; '95; :04[a], p. 574). On the other hand, there are species in which there is still a completely graded series from the smallest to the largest workers (see also Emery, '**96**, p. 397).

Occasionally the fertile winged queens (Emery, '**96**, p. 399) are of different sizes, as, for example, in *Camponotus abdominalis* F. and *C. dorylus* subsp. *confuscus* Emery; and the differences here are parallel to those of the workers. Again (Wheeler and McClendon, :**03**; Wheeler, :**03**[b]), some species may have two distinct forms of queen without intermediates, and Wheeler (:**03**[b], p. 650) is inclined to believe this due to mutation. Moreover, intermediates, though usually abnormal, are found between the queen and the largest workers (see also Forel, :**04**[a], p. 574; Wasmann, '**90**, pp. 301 et seq.; '**95**; :**02**; :**09**[a], pp. 46, 52, 60, et seq.; Viehmeyer, :**04**; Muckermann, :**04**; and Wheeler, :**01**[b]; :**10**, pp. 406–408). Hence, taking ants as a whole, there is evidence of a long series of gradations from the smallest workers to the largest queens.

When there are modifications of workers of the same species, they are (Emery, '**96**, p. 398), first, modifications of size with but slight differences of form of body and mandibles; but on approaching the maximum size the form of the mandibles changes rapidly, though by a series of gradations. Individuals showing such gradations are difficult to get together because of the scarcity of soldiers, and especially of forms intermediate between them and the worker-major. There are also differences of form and sculpture of the head and other parts of the body. According to Forel ('**95**, p. 143), the large worker may be distinguished by its enormous increase in size or by the peculiar form of its head or its mandibles, which are suited to breaking seeds, obstructing the nest opening, or fighting, etc. If the head has no such adaptation, the ant shows no peculiarities.

Wasmann ('**90**, p. 300) says that with differences of size are correlated differences of sculpture of separate parts of the body, but especially of the back and the head. Emery (:**01**[a]; :**01**[b], p. 54, foot-note, and :**04**, pp. 588–589) states that the smallest forms of driver ants, as well as of other species of Dorylus, differ in the structure of the head and its appendages, even in the number of antennal joints, from the medium-sized and small ants, and that these differ from the largest specimens in having smaller heads and toothed mandibles.

It is an interesting fact, as Emery ('**94**, p. 55) points out, that in the Ponerinae, which are considered morphologically as the stem form of ants, there is no striking polymorphism of the worker classes (Emery knows it only in the case of Melissotarsus). But polymorphism of the worker does appear in many genera of all the other groups, and it therefore seems that polymorphism among ants has originated polyphyletically. It is furthermore probable that, since the large workers more closely resemble the queen, they are more primitive than the small ones. Emery, then, classifies ants according to the condition of the workers as follows :

I. Ants with only large workers.

II. Ants with large and small workers.

a. The extremes connected by intermediate forms.

b. The large and small workers without intermediate forms.

III. Ants with only small workers, which are very different from the queen ; due to dropping out of large workers.

IV. Ants with one kind of workers, which are smaller than the queen through increase of size of queen.

V. Ants from which have disappeared the worker class because of parasitism.

According to this classification, only the species which fall within the second group offer more than one form of workers in the same species. Hence, I have chosen as examples of workers with extremes connected by intermediate forms, two species of Camponotus (*C. americanus* and *C. herculeanus pictus*). As the only available species in this region of the country which falls under the division "large and small workers without intermediate forms" is *Pheidole pilifera*, I have also selected that species. In addition, I succeeded in collecting *P. vinelandica* from New Jersey, and in obtaining *P. dentata* through the kindness of Mr. Carl Hartman of Austin, Texas.

B. Polymorphism of the Species Studied for Division of Labor.

1. *Camponotus.* — In Camponotus the workers form a continuous series from the smallest, which have heads much longer than broad, to the largest, whose heads are nearly as broad as long. This was established for *Camponotus americanus* by measuring nearly 500 individuals, most of which were alive. To secure significant and reliable results requires the selection of suitable and easily identified points on the head and careful attention to the *position* of the parts during measurements. In using the microscope to view the head and to get the distance between points in its outline it is especially necessary to have the heads of the individuals which are to be compared with one another held in the same relative position. It was found practicable to do this by pressing the head firmly into a small mass of rather soft bees' wax mounted on a glass slide. Usually the wax thus employed held the head in the desired position, but sometimes it was found necessary to push the fore feet of the animal also into the wax to prevent the ant from pulling its head away. The aim in orienting the

head was to place it as nearly as possible in a horizontal position for the purpose of viewing its dorsal aspect. This was done in the case of the transverse axis by placing the head so that the outer margin of the two eyes was equally distant from the outline of the adjacent side of the head; in the case of the anterio-posterior axis, by bringing the anterior margin of the clypeus and the posterior margin of the head to lie at the same focus. These two points can both be seen distinctly

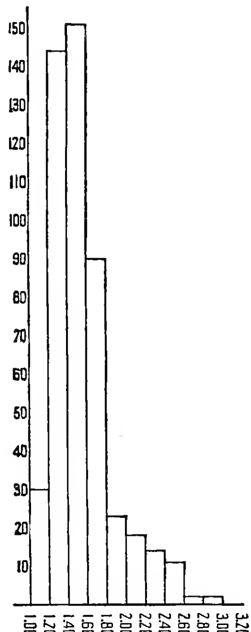


FIGURE 1.

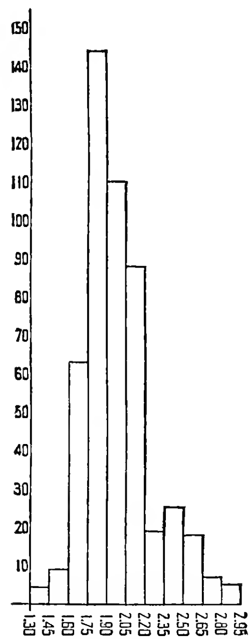


FIGURE 2.

only when the head is in one position. In making the measurements I used a Zeiss A* objective so set as to give a magnification of about 45 diameters at a projection distance of 430 mm. With this arrangement the whole length of the head of the largest individual could be covered by the ocular micrometer, by means of which the measurements were made. To avoid accidental errors measurements were taken more than once, and recorded in terms of divisions of ocular micrometer. In all cases they were ultimately calculated in millimeters. By "length" is meant the distance measured in a straight line in the median plane between the posterior margin of the head and the anterior margin of the clypeus; by "breadth" or "width" is meant the distance between the lateral outlines of the head measured along a line tangent to the posterior margin of the eyes.

The first diagram (Figure 1) is a frequency polygon, giving the results

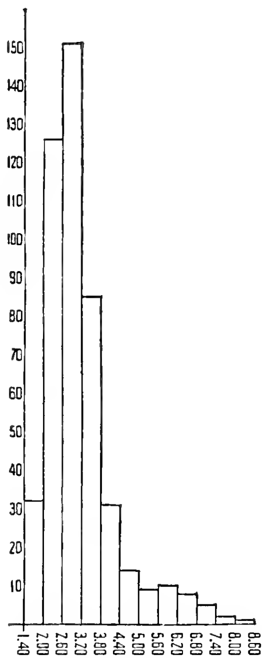


FIGURE 3.

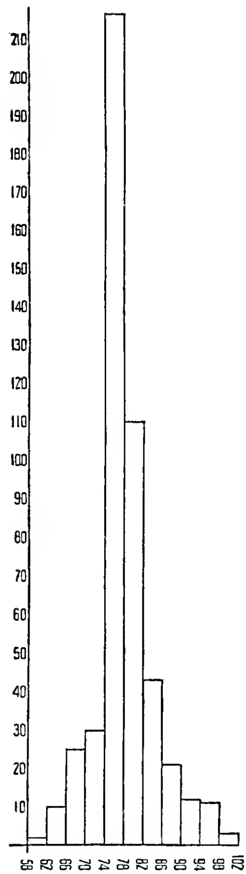


FIGURE 4.

of the measurements of width, Figure 2 that of length, Figure 3 that of length multiplied by width, and Figure 4 the ratios found by dividing width by length. In constructing the polygons the ants were divided into classes. For the length a difference of 0.2 mm. was chosen as the basis, the first class containing all individuals in which the length of the head fell between 1.00 mm. and 1.20 mm.; the second class between 1.20 and 1.40 mm., and so on. The interval adopted for width (Figure 2) was 0.15 mm.; that for area 0.6 sq. mm.; and for ratios 4 per cent of length. The ordinates show the number of individuals falling within each class. None of the resulting "curves" shows any signs of two or more maxima, except that of Figure 2, where the indication is too slight to be of significance, so that there is no evidence from these measurements that there are well marked classes. When a given colony was plotted by itself, the results were substantially like those here recorded for the combination of several colonies, and each curve was therefore similar. Consequently it seems to me that no objection can be raised to combining the results of several colonies, as I have here done.

I have also tried to find evidence of distinct types or classes based on other grounds, such as the number of teeth on the mandibles and the ratio of the width between the eyes to the width between the insertions of the antennae, but with equally small success.

I next turned my attention to *Camponotus herculeanus pictus.* Here, too, although I did not make such measurements as are described above for *C. americanus*, there seems to be a complete, graded series. This fact is easily seen from the Plate, which was made from a series of photographs taken with the aid of a microscope. All exhibit the animal in the same position and under the same magnification, — about 9 diameters, — so that they represent accurately variations of size and shape. But not relying on this single view of the head, since all the parts were not visible in that position, studies were made with the microscope from different sides. When necessary in comparing the size of the various organs measurements were made by means of the ocular micrometer.

1. The most striking difference (Plate, Figures 2–20) between the large and the small worker is that of size, the heads of the large workers being not only actually but also relatively larger than those of the smaller ones.

2. Not only is the head of the larger workers larger, but it is also of a somewhat different shape. Thus, while the head of the small worker is somewhat longer than broad, the head of the large worker is more nearly equal in the two dimensions, or even a little broader than long in the largest specimens, thus resembling *Camponotus americanus.* Moreover, the posterior margin of the small heads is convex backward, whereas that of the large workers is more nearly straight, or even slightly concave backward. It might be more nearly correct to say of the latter that it presents a backward convexity of the two ends of the pos-

terior margin, probably due to the greater development of the great jaw muscles, as in this respect the large worker seems to resemble the condition of the soldier in those species where there is a real soldier. Moreover, there is another difference in the general shape of the head, in that its greatest diameter dorso-ventrally is less in proportion to the length of the head in the larger workers; in other words, the heads of the larger workers are somewhat shallower.

3. In regard to the differences of some of the organs of the head, it may be seen at a glance that the length of the antennae relative to the size of the head is much less in the larger ants, they being hardly any longer by actual measurement than in the smaller ants. The number of joints in the antennae seems to be here, as in *Camponotus americanus*, always the same, unlike Dorylus (Emery, :01[a]). There are, however, some differences in the antennal parts. For example, in the smaller ants the scape is slightly longer than the width of the head, while in the larger individuals it is somewhat shorter. In heads of intermediate size the condition is variable; sometimes the head is wider than the length of the scape, sometimes the reverse is true, regardless of the size of the head. In the larger ants the funiculus is only slightly longer than in the smaller ones, and in proportion to the size of the head it is much shorter. In proportion to the length of the scape it is shorter in the larger ants. The scape itself is also of a different shape in the two extremes; in the large ants it is larger at the distal end than at the proximal end, whereas in the small workers it is nearly uniform in thickness throughout its length.

4. Again, it was found that in the larger ants the compound eyes are set slightly further back, as may be seen especially in side view, and that they are also somewhat further from the margin of the head.

5. The clypeus presents a good deal of difference in the two extremes; in the larger ants it is much thinner dorso-ventrally and only slightly arched on the dorsal surface, while in the smaller ants the arch becomes higher and more angular. Moreover, there is a considerable difference, as seen from the dorsal side, in the shape of its outline, which is nearly rectangular in the large individuals, and somewhat hexagonal in the small.

6. In the larger workers the frontal carinae (Wheeler, :10, p. 18) are slightly farther apart in proportion to the total length of the head.

7. The mandibles are not very unlike in the two extreme sizes, but are, on the whole, more strongly built in the large workers. All the teeth are fairly large in the large workers, but in the small workers the outside ones are the stronger, though the difference in strength is not great.

8. But in each of these characteristics there is a graded series, so that a distinction into classes cannot be made; indeed, it sometimes happens that in a single individual of the middle size some characters more clearly resemble those of the large workers, while others are more like those of the small. These observations tend strongly to confirm those made by measurements on *Camponotus americanus*. They are, so far as I am aware, the only observations of the sort made on this genus.

In order to see how much the *queens* (Plate, Figure 1) of *Camponotus pictus* differed from the *workers* in regard to these same structures, I made similar observations on them, comparing them with the largest workers, which it is evident they more closely resemble than they do the small ones. The results follow:

1. The head of the queen is here somewhat, though not much, larger than it is in the largest worker. In regard to its shape, as compared with that of the worker, it is only slightly broader in proportion to its length, about as much as we should expect from its increased size. The posterior margin of the head of the queen resembles very closely that of the largest worker. The dorso-ventral axis of the head is slightly shorter than in that of the worker.

2. The length of the antennae, in proportion to the size of the head, is less in the queen. The antennal joints are of the same number in worker and queen. If the length of the scape is compared to the width of the head, it is found that in the queen it is hardly, if any, shorter than in the large worker. The funiculus of the queen is slightly shorter in proportion to the length of the scape. In proportion to the size of the head it is also somewhat shorter, being actually of about the same length in both forms. The scape in the queen is still thicker at the distal end than in the large workers.

3. The compound eyes are slightly further back than in the large worker, and resemble the condition in the small worker more closely in being a little nearer the margin than in the large worker.

4. The clypeus is even flatter in the queen than in the large worker and slightly more indented by the cheeks; the arch is curved in both.

5. The frontal carinae are slightly further apart in the queen than in the large worker.

6. The mandibles of the queen very closely resemble those of the large worker.

So far, these characters make it appear as though the queen were merely at one end of a long series of females. But in this species other characters, such as the more developed ovaries, the presence of wings, etc., show that there is a noticeable break between the queen and the largest worker.

However, in nearly all the differences among the workers of this species, it is necessary to observe that there is no break in the series, no tendency to form separate classes. There is, rather, a continuous variation from one size to the next, forming, as Wheeler (:07[b], p. 77) says, a series of intermediates between the very large and the very small.

Although one would not expect the duties of classes in this species to be as distinct from one another as they are in those polymorphic species where there are sharp morphological differences, still it seemed possible that polymorphism might be associated with some recognizable division of labor. Furthermore, as already stated, it was thought well to compare the activities of two forms, in one of which (as in *Camponotus americanus* and *C. pictus*) there is a graded series; and in the other (as in certain species of Pheidole) there are dimorphic classes without intermediate forms between the large-headed soldiers and the workers (Emery, :02, p. 719; Wheeler, :10, p. 559).

2. *Pheidole.* — In Pheidole there is, as a rule, no intermediate form between the small workers of ordinary proportions and the soldiers (Emery, :02, p. 719; Wheeler, :10, p. 559), though such intermediate individuals are occasionally found. By "soldier" is meant, according to Forel ('95, p. 143), merely a large worker which, through complete dropping out of the intermediate forms and through adaptation to precise functions, has become differentiated from the small worker. It is often so different from the worker proper as to be taken for another species. In some species, as *Ponera edouardi*, there are, indeed, two forms of soldier (Forel, '95, p. 145). In Pheidole, however, there is only one. The differences between the heads of the soldier (Plate, Figure 22) and the worker (Figure 23) of *Pheidole pilifera* may be described as follows:

1. A great difference in the size of the head, that of the soldier being much larger, not only actually, in correspondence with its larger body, but even relatively; it is so heavy and clumsy, that it is held bent downward at a much greater angle than that of the small worker.

The thickness of the heads measured dorso-ventrally does not differ materially. When the dorsal surface of the head is examined, however, it is found that the proportions of its outline are quite different; it is nearly square in the small workers, and quite oblong in the soldiers. The posterior margin is only slightly cordate in the small ants; but in the soldiers it is deeply indented in the centre, where ends a median dorsal groove, which begins about the middle of the head. This groove lies between the muscles of the mandibles, which cause a pair of longitudinal mounds; these are wanting in the small workers. The head of the soldier is much redder, except for the black eyes and the dark regions around the mouth. It is also covered with stronger ridges of chitin and is more pubescent.

2. The antennae are, relatively to the whole head, much longer in the small ants. The number of joints is the same for both classes, but in the small ants the three distal joints are larger than in the soldiers. The scapes are in form very similar in the two castes, but are relatively shorter in the soldier. The funiculus, although actually slightly shorter, is, in proportion to the size of the head, much longer in the small workers.

3. The frontal carinae differ slightly in the two forms, being nearly parallel in the small workers, but diverging posteriorly somewhat in the soldiers.

4. The clypeus in the small ants resembles a triangle, somewhat curved outward in front, but in the soldier it has four sides, of which the two lateral diverge anteriorly, and its anterior margin is slightly notched in the middle.

5. The compound eyes are further forward in the soldier than in the worker.

6. The mandibles in the small ants are slender, and bear teeth, of which the outside one is especially large, but in the soldier they are very strongly built, blunt, straight-edged, without teeth, and somewhat sharp on the edges which come in contact with each other, very much resembling hatchets.

When the queen (Plate, Figure 21) of this species was examined, it was found that, while she resembles the worker more closely in certain characteristics, she is, on the whole, more like the soldier.

1. Her whole body is larger than that of the soldier, suggesting that, as in *Camponotus americanus* and *C. pictus*, she represents one end of a series from which some members have, probably, dropped out. But there is more difference between majors and queens in this species than in Camponotus, and even within each of the two classes, soldiers and minors, — supposedly connected by missing forms, — the gradation is not as uniform as it is throughout the Camponotus series.

2. Her head is both relatively and actually smaller than that of the soldier. On the other hand, the shape, both in regard to its general proportions and to the posterior margin, more nearly resembles that of the worker; but the chitinous ridges and the general color closely resemble those of the soldier, though they are less marked.

3. Certain organs of the head also resemble those of the soldier more closely, viz., the antennae and their parts, the angle which the

frontal carinae make with each other, the shape of the clypeus, and the pubescence, this being even more marked than in the soldiers.

4. On the other hand, the mandibles are toothed and in shape are much like those of the small workers, and the mandibular muscles do not cause mounds on the dorsal surface of the head.

5. There are three well marked ocelli, and the compound eyes are much larger than those of either soldiers or workers.

6. From these facts it seems probable that the large workers originally resembled the queen more than did the small workers, and later developed to an exaggerated extent certain of her characteristics, which the small worker, on the other hand, lost.

IV. Methods in General.

In order that I might not be biased when considering the subject of division of labor among ants as a whole by conclusions deduced from a single method of work, it was thought well to make observations under several different conditions, partly by placing the ants in more, partly in less natural surroundings. My observations may thus be arranged in three groups as follows: A. work with Fielde (:00; :04[a]) nests, with aluminum nests (Buckingham, :09) and other apparatus; B. Work with Barth (:09) nests; C. Out-door work.

A. *Work with Fielde Nests, Aluminum Nests and Other Apparatus.*

The Fielde and aluminum nests used are shown in plan in Figure 6 (p. 447); they were ten inches long and six inches wide.

1. *Marking.* — In order to study the activities of each member of a colony, when experimenting with ants in the Fielde nests and in my own, the following method was employed: Each individual was marked and its head measurements were recorded together with the data of the various experiments, so that it might be possible to tell which sizes were concerned in the different activities and to keep account of the special activities of each individual. Length of head multiplied by width of head was used as a criterion of size. The classes of the species of Pheidole which I used are so distinct that I have thought no marking necessary, and have simply recorded the numbers of ants of each class taking part in each activity. With Camponotus, however, the case was quite different, since it was impossible to establish natural groups or classes, owing to the continuous gradation of the forms into one another.

A common way of marking ants, described by Miss Fielde (:03[a], p. 610, foot-note), is to affix to them colored paint by means of varnish. I have never succeeded in making this method work well, since the paint often falls off or is removed by the ant or its companions. Especially is this likely to happen in the course of experiments extending over considerable time, as mine have done. I have had no better success with trying to affix other substances to the ants. The following

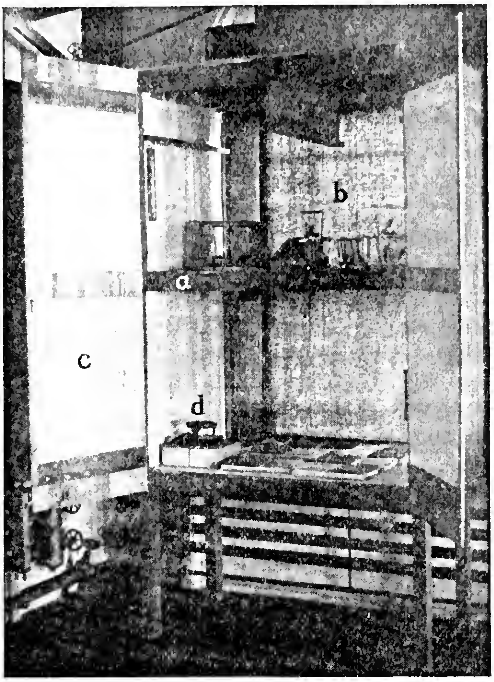


FIGURE 5.

method, however, works well with species of large ants: Pieces of colored sewing-silk were split into their component strands. One or, if different colors were to be combined, two or more of these were tied with a single knot into a loop. The legs of the insect were then grasped by the thumb and forefinger of the left hand, while with the right hand the loop was slipped over the abdomen, adjusted immediately back of the last pair of legs, and fastened by completing a square knot, care being taken not to make the loop around the ant too tight. A small pair of forceps is indispensable in adjusting and tying the loop. Provided that the thread is sufficiently slender, I cannot see that its presence makes any difference in the behavior of the ant after the first few minutes, and the number of individuals which can be thus marked is practically unlimited, for by varying the single or combined colors, many different combinations can be secured.

2. *Heating.* — In carrying out this set of observations the ants were kept during the summer at ordinary room temperature. In winter, however, in order to maintain a state of greater activity than they would have had at room temperature, especially at night, it was deemed expedient to keep them at temperatures which approximated those of summer, the extremes being about 60° and 90° Farenheit. They were therefore placed, when not under observation, in an artificially heated chamber (Figure 5). This chamber, made of matched boards $\frac{3}{8}$ in. thick, was about five feet high, three and a half feet wide, and two and a half feet from front to back, and raised on wooden legs about two feet above the floor. The front of the chamber was composed of two doors (*c*), and the back was open, but was placed against the wall of the room in such a way as to surround a west window that was provided with two sashes enclosing an air space between them. The floor, the roof, the two ends and the two doors were lined with sheets of asbestos. To give more surface on which to place the nests and to allow at the same time free circulation of the heated air, a shelf (*a*) made of slats was placed midway between the floor and roof. Since the afternoon sun sometimes made the temperature in the chamber dangerously high, and since daylight excites ants, the window was supplied with a red curtain (*b*), so arranged that it could be easily raised or lowered. The curtain was used continuously except occasionally for a few moments when a little extra light was needed while caring for the ants. Although the doors could be tightly closed, they were usually left slightly ajar, thus affording better ventilation and avoiding danger of too great a rise of temperature. Heat was supplied by an electric stove (*d*), the current being taken from the lighting circuit. The ants which I was studying were kept on the floor of the chamber, where the temperature was somewhat lower than on the shelf. The latter was used by Mr. I. A. Field for the ants he was rearing in connection with his studies on spermatogenesis. The chamber was built for receiving ants and in accordance with plans worked out by Mr. Field and the director of the laboratory. While the ants were kept in this chamber at such times as they were not under immediate inspection, all observations were made at ordinary room temperatures, under the influence of daylight.

B. Work with Barth Nests.

1. *Environment in General.* — It appeared that there are some activities of ants which cannot be satisfactorily studied in natural nests, and in order to watch the insects under more natural conditions than were used for observations under division *A* (pp. 440–442), and to subject them, though in the laboratory, to an environment as nearly as possible resembling that of their wild homes, Barth (:09) nests were selected. Here the ants soon burrowed their chambers and galleries in earth between two glass jars, a smaller set within a larger, the outside of which was always covered with thick black paper, except at such times as the nests were examined. To prevent the escape of the ants, screens of wire gauze were placed over the tops of the jars, which were otherwise open, allowing free access of air. During the period that these observations were made the nests were kept continuously in a photographic dark room, with black walls, and a small window through which daylight entered for some hours each day. In order to produce the darkness normal to ants underground, this daylight was closed out when the black paper was removed from the nests during observation. The light needed in studying the movements of the ants was secured by a 16-C. P. incandescent filament enclosed in a bulb of ruby glass. This lamp served also another purpose, viz. to stimulate the ants slightly, and thus make them more lively.

2. *Influences of Heat and Light.* — That this stimulus was due to heat and not to light was proved by the following experiment, tried on two different colonies of *Camponotus herculeanus pictus.* The results of both were so nearly alike that I give only those of one. When the temperature of the earth in the nest was 20° C., by a thermometer previously placed there, and the ants were quiet, a water screen $2\frac{1}{2}$ inches thick was placed between the nest and the ruby bulb, which was about four inches distant from the nest. After five minutes the ants were still perfectly quiet. The screen was then removed. Immediately the ants began to move vigorously, and at the end of five minutes more the glass felt warmer to my hand where the light shone on it than in other places. The temperature in the earth near the ants had now gone up to 22° C., and the ants were carrying the larvae toward the outer glass of the nest. The water screen was now replaced, and the ants examined again at the end of another five minutes, when, though the thermometer still registered 22° C., the ants were perfectly quiet. After five minutes more with the screen still in place, the room having become in general warmer, the thermometer in the earth registered 22+° C., and the ants were perfectly quiet. Black paper

was now placed around the nest to keep out the light, the lamp being left in the same position, but the screen having been removed. After five minutes the thermometer reading was 25° C., and the ants were in commotion. I then poured cold water over the outside of the nest till the ants became fairly quiet, the thermomoter standing at 22.5° C. When the thermometer had reached 20° C., the lamp was again placed near the nest, and at the first intimation of motion among the ants a reading of the thermometer was taken; it showed 21° C. But previously, when there was light and no *rise* of temperature the ants were quiet at 22+° C. The nest was now cooled to 19° C. and the ants began to move at 20° C. It therefore seems fair to assume that the stimulus is not simply heat, but a *rise* of temperature. To see the effect of light other than red, at the same power, a 16–C. P. incandescent lamp with an ordinary glass bulb was now used, at the same distance that the ruby light had previously been, with the water screen in place. At the end of two minutes the ants were disturbed, but they seemed to be less disturbed by the light of a 16–C. P. incandescent lamp than by its heat. From these two experiments it seems clear that we are dealing here with the stimulus of heat from the ruby bulb rather than of light. Heat is really a natural stimulus to ants, as they are in the habit of coming up under stones or to other warm parts of the nest, and of taking there the eggs, larvae and pupae to warm them. Moreover, in the Barth nest I found that so long as the heat was not too intense, the ants took their young toward it; if, however, it became very intense, from placing the lamp nearer the nest, they carried them away. So long as the behavior of the ants indicated that the warmth was favorable to them, so long, I think, we may safely say that it was a natural stimulus.

C. Out-door Work.

That I might study ants under entirely normal conditions, I worked during the spring, summer, and autumn of 1909 on out-door colonies of *Camponotus herculeanus pictus* at Randolph, N. H., and of *Pheidole pilifera* at Cambridge, Mass. In order to obtain as natural results as possible, the observations were made in many cases without touching the nest at all, and in no case was there any disturbance of the ant when it could be avoided. To this end, before making observations, I often waited for a time after seating myself on the ground so that the ants might recover from any stimulus to activity caused by my approach. In making notes or in preserving ants, all individuals sharing in any activity under the influence of any excitement, other than that which might naturally be expected in such an activity, were so recorded. To gain as much evidence as possible from the observations, two methods of study were used, viz. (1) field notes, and (2) captured ants.

1. *Field Notes.* — First, at the time the observations were made notes were taken, giving general impressions of the sorts of individuals engaged in various pursuits, and these notes were later carefully summed up and compared for the different colonies. Although such notes are necessarily in many respects less accurate for Camponotus, because of the nature of polymorphism in the species here used, than is the evidence derived from the second method, to be described below, yet they reveal more precisely the numbers of ants engaged in the various activities, and are, I think, in the main fairly correct.

2. *Captured Ants.* — The second method employed in the field work, used particularly in connection with *Camponotus herculeanus pictus*, was as follows: Because the workers of this species form a graded series (p. 438), it was in many cases impossible to judge with accuracy as to the size of ants engaged in particular activities, and it was, therefore, thought well to distinguish, for the purpose of later study, those ants which shared in any given activity. Accordingly, small vials of commercial alcohol were used, into each of which were dropped all ants of a particular colony engaged in a given activity on any day, and each bottle was provided with a label giving the colony, the activity, and the date of capture of all ants contained therein. For convenience each ant was later mounted on a separate pin with a label containing the above data.

D. Activities Tested.

Although the activities of the ants studied have not all been tested with each method of observation, some of them have. I give here the whole category of activities which were examined by any method: 1. foraging, either (*a*) presence in the field, or (*b*) actually carrying food; 2. partaking of different foods; 3. feeding themselves; 4. regurgitating food to others, likewise receiving regurgitated food; 5. licking others, likewise being licked; 6. tending the young (eggs, larvae, or pupae); 7. building, (*a*) digging, (*b*) carrying earth; 8. carrying other ants, likewise being carried by other ants; 9. surrounding the queen; 10. fighting; 11. responding to disturbances of the nest; 12. guarding; 13. scavangering.

Though the nature of most of these activities seems clear from the names given them, a few, perhaps, need explanation. For example, "preparing food" means tearing it into smaller pieces or crushing it so that it can be carried into the nest, or so that it is suitable for eating. "Regurgitating food" is a fairly common habit among ants; when they have been feeding, they sometimes disgorge some of the food to their companions. The process of "licking" is minutely described by McCook ('79, p. 125) for *Pogonomyrmex barbatus* as cleaning each other. The ant which is doing the licking or cleaning passes all over the body of the other ant with her mouth parts, which are constantly in active motion. The ant which is being licked stands still, apparently content to have the process carried on. Wheeler (:03[c], pp. 43–44) says in regard to licking in Leptothorax that there can be little doubt that the ants obtain some substance from the body of the Myrmicas, but it is difficult to ascertain its nature. He thinks it may be a secretion of the cutaneous glands, or merely the salivary secretion that has been spread over the Myrmicas by the mutual licking in which they indulge. Later he (Wheeler :07[c], p. 70) speaks of it as an "oleaginous secretion." By "responding to disturbances of the nest" I mean such disturbances, for example, as knocking on logs in which *Camponotus pictus* for the most part lives, jarring the artificial nests, or tearing open log nests or earth nests. "Guarding" signifies such habits for the protection of the nest as are found in Colobopsis (see p. 428). Although these guards allow all the inhabitants of the nest free passage, they nevertheless keep out all intruders.

V. Studies of Camponotus.

A. *Camponotus americanus.*

In the observations on the various activities of *Camponotus americanus* I have tried to discover (1) the proportion of the whole number of worker ants which engaged in a given activity, (2) the extent to which each individual was engaged in that activity, and (3) the relation, if any, between the size of the individuals and the nature of their activities.

1. *Foraging.*

1. *Methods.* — In order to determine to what extent this species is attracted by food near at hand, and which individuals respond to this stimulus, any arrangement which would allow one to know definitely which individuals, if any, had had recourse to the food (or at least to the chamber containing it) during a fixed interval of time would afford a basis for judging of the general effect of food as a stimulus and for determining the particular ants or classes of ants stimulated by it. The method adopted to attain these ends was as follows: The ants

were confined in the Fielde nests already mentioned (Figure 6). In all cases the chambers were connected with each other by two passages, one at each end of the partition. In each of the chambers was a moist sponge, and both chambers were kept moderately dark by pasteboard covers laid over the glass. The ants were placed in chamber *A*, the food in chamber *B*. One of the passages between *A* and *B* was blocked by a plug of cotton, while in the other was placed a trap-door which allowed the ants to pass from *A* to *B*, but not in the reverse direction. The trap-door (Figures 7–11) was made as follows: A piece of mica, *a*, slightly narrower than the passage, rested on a fulcrum, *b*, in such a way that one end touched the floor of the nest in chamber *A*, while the other came against the roof (*c*) of the passage in chamber *B*. The roof, *c*, covered the whole passage between *A* and *B*. In Figure 8, a front

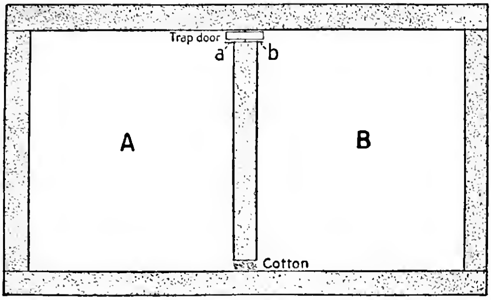


FIGURE 6.

elevation, are shown, besides the parts already mentioned, *d*, the outside wall of the nest in cross section, and *e*, the cross partition between *A* and *B*, also in section. The fulcrum (*b*) consisted of a short piece of straightened watch spring just long enough to fit easily across the passage, and supported, edge up, between two strips of thin card-board, cut as shown in Figure 9, where *b* is the steel watch spring, and *h* one of the supporting cards. The dotted lines (*g*) show where the card was bent at right angles to form the parallel wings, *f*, which were glued to the glass wall of the passage (Figure 8, *d* and *e*). The portions *i* of the card-board remained in a plane parallel to *b* and hence perpendicular to *f*. Both edges of the mica plate (*a*) were notched at the middle (Figure 11) to receive the projecting edges, *i*, of the vertical card-board. This arrangement prevented the slipping of the mica plate on the fulcrum, while allowing freedom of motion in a vertical direction. The two arms of the mica plate were of unequal length, the longer and heavier arm projecting into chamber *A*. Consequently, when the plate was undisturbed, this arm rested on the floor of chamber *A*; but when an ant, ascending the incline of the plate, passed beyond the fulcrum, its weight was added to that of the shorter arm, and by the time it had come near the end of the shorter arm, the combined weight of the two

FIGURE 7.

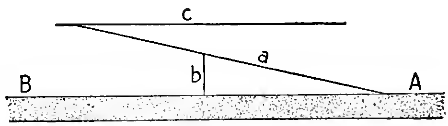


FIGURE 8.

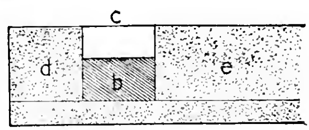


FIGURE 9.

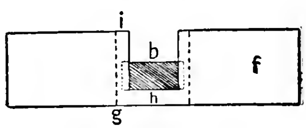


FIGURE 10.

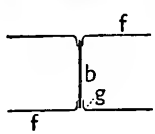
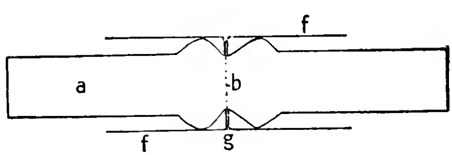


FIGURE 11.



was sufficient to tip the plate and bring the *B* end into contact with the floor. As soon as the ant had passed from the plate to the floor of chamber *B*, the weight of the long arm caused the plate to swing back into its original position. Figure 10 shows, in horizontal section, at the level of the uncovered portion of the watch spring (*b*), the fulcrum without the mica plate. As the mica plate was somewhat longer than the end of the partition, the latter was prolonged by a vertical piece of paper (*a*, *b*, Figure 6) parallel with the edge of the plate. This prevented ants from getting under the plate from the *A* side, and interfered with any attempt of a second ant to get on the *B* end of the plate while it was being held down by the weight of the ant entering *B*.

To eliminate as far as possible other factors than that of food, the latter was placed alternately in chamber *B* and in chamber *A*, the same number of observations being made for each position.

2. *Observations.* — In these experiments ten different colonies were made use of. Records in regard to the feeding activity were made every 24 hours with the exception of certain Sundays and a few holidays. It is clearly necessary, however, to exclude observations made on any day succeeding one when no observations had been made, because the time interval during which the ants had had an opportunity to respond was in such a case 48 hours instead of 24 hours, the usual interval. At the beginning of each observation the ants were all placed in chamber *A*, and the food was placed half the time in chamber *A*, half the time in chamber *B*. A "*series*" of observations consists of a number (50 or 25) of separate observations all made under the same conditions as to the relation of insects to the food-chamber. The maximum number of observations in a week was only five, except during those periods when records were made on Sundays as well as week days, in which cases *seven* observations a week were possible. During the earlier experiments on this species, 50 observations were included in each series, but later the number was reduced to 25. A "*set*" of observations includes two or more "series," one of the series being made while the food was in chamber *A*; the other while it was in *B*. The ants at the beginning of every experiment were, as stated, all in one chamber (*A*). After the lapse of 24 hours the number of individuals which had migrated into chamber *B* was noted. The results of a series of 50 (or 25) such observations were combined as follows: First was computed what per cent of the whole number of ants under observation made their way into the unoccupied chamber during each observation. These per cents for the 50 (or 25) observations were then averaged. The result is shown in Table I. When the food was in chamber *B*, these values give a partial measure of the stimulus to migration caused by food; but, assumably, the migration under this condition was not due exclusively to the stimulus of food. To eliminate as far as possible all other factors except food, experiments were also made with the food in chamber *A*. The individuals making their way from chamber *A* to chamber *B* under this condition would clearly not be attracted to *B* by food, and the number of them may fairly be taken as an approximate measure of the number of individuals which made their way into the food-chamber (when that chamber was *B*) independently of the influence of food. In the earlier sets of observations (viz. with 50 observations in a series) *three* series were combined into a set (colonies 4, 17, 20, and 24); in the later sets only *two* series (colonies

TABLE I.[1]

(24-Hour Intervals.)

Colony	4	4	4	17	17	17	20	20	20	24	24	24	42	42	51	51	52	52	53	53	54	54	55	55
Chamber containing food	B	A	B	B	A	B	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	B
No. of observations	50	50	50	50	50	50	50	50	50	50	50	50	25	25	25	25	25	25	25	25	25	25	25	25
Av. no. of workers in colony	7.6	5.3	3.3	28.3	49.5	20.2	78.9	57.7	53.1	56.6	55.9	20.6	13	10	17	12	19	9	30	21	38	27	7	5
Per cent of workers entering B (whether containing food or not)	27.1	10.6	33.8	25.4	6.8	25.7	6.7	1.2	6.5	3.2	29.2	8.9	37	17	17	22	16	23	15	38	19	3	21	38
Av. of per cent of 2 "sets" in one chamber	..	30.5	..	..	25.7	..	..	6.6	..	..	6.1	..	..	..	..	..	..	..	..	..	..	..	..	..
Av. no. of winged queens	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	4	4	5	5	1	1	..	..
Per cent of winged queens entering B (whether containing food or not)	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0	0	27	58	0	0	..	..
Av. no. of deälated queens	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	0	0	1	1	1	1	..	..
Per cent of deälated queens entering B (whether containing food or not)	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	28	48	0	0	..	..

Av. per cent when food was in B, 21.2. Av. per cent when food was in A, 15.6.

[1] Owing to the death of some members of a colony, the number varied. The average number in a colony was determined by taking the sum of the individuals found in the nest on successive days and dividing that by the number of observations.

42, 51, 53, 54, 55). Usually the three-series set embraced two series with the food in *B* and one with the food in *A*, but in one instance (colony 24) the reverse was true, two series of observations having been made with food in *A*, and only one with food in *B*. These three series were so arranged as to eliminate as far as possible any tendency of the colony to a change in its activities during several weeks of observation, by putting one of the two series *before*, the other *after* the odd series. Although the results may differ considerably in different experiments, and the proportions found may be so variable as to show that we are dealing with rough approximations only, still the numerical relations are the only basis we have to go by, and are not less reliable than vague statements that there are "many" or "very many" more engaged in this than in that activity. Accordingly, I shall give the numerical results of my observations, which are of necessity the basis of my general statements of proportions.

In those cases where two series of observations were made with the food in the same chamber, it was found that there is not much difference in the per cents resulting in the two cases (27.1 and 33.8; 25.4 and 25.7; 6.7 and 6.5; 3.2 and 8.9). In experiments with colonies 4, 17, 24, 42, and 54 there was a much larger number of ants entering *B* when the food was in *B* than when it was in *A*, as we should expect; but, for some reason which I have been unable to explain, the reverse was true in colonies 51, 52, 53 and 55. However, the average of these per cents was higher when the food was in *B* (21.2 per cent) than when it was in *A* (15.6 per cent). This would leave a difference of 5.6 per cent, which may be assumed as the per cent which went to *B* for food alone. It will be noticed that the per cent of ants entering *B* is fairly low, showing that not every ant went each day to the food chamber, and, indeed, some ants did not enter *B* at all. This is shown by an examination of the details of a series of observations in colony 54. The ants whose numbers are not recorded here died before this experiment was performed. Nos. 25, 28, 35, 37, 39, 45, 47 and 48 did not enter chamber *B* at all, while out of a possible 33 times, No. 30 entered *B* twenty-two times; No. 2, twenty times; Nos. 21 and 22, seventeen times each; Nos. 10 and 12, sixteen times each; Nos. 3 and 52, fifteen times each; No. 9, eleven times; No. 20, eight times; No. 44, six times; No. 14, five times; Nos. 8 and 13, three times each; Nos. 11, 17, 19, 31, 34, and 42, twice each; and Nos. 15, 19, 33, 36, and 40, once each. There were, in fact, instances in all the colonies of failure on the part of some individuals to enter the food-chamber. It might be urged that any one individual does not need food every day, and can readily live for some time without it, as has

TABLE II.

(48-Hour Intervals.)

Colony	4	4	4	17	17	17	20	20	20	24	24	24	42	42	51	51	52	52	53	53	54	54	55	55
Chamber containing Food	B	A	B	B	A	B	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A
Av. no. of workers	8.1	..	..	52	..	16.8	84.4	..	52.4	..	45	20.7	12.8	10.2	20.4	12	23.6	9.3	31.2	20.9	39.2	27.3	7	5
Per cent of workers entering B (whether containing food or not)	36.4	..	..	34.3	..	28	6	..	4.6	..	32.7	6.2	40.4	26.2	29.2	26.4	14.4	32.7	16.7	43.9	23.8	7.7	32	52.8

Av. per cent when food was in B, 24.3.

Av. per cent when food was in A, 28.

been shown by Janet ('98) and by Fielde (: 04[b]); but in my experiments many individuals did not enter the food-chamber for a long period, fifty days, and while it is possible that they may exist for such a period when deprived of food, I do not believe that with food within their reach they would, as a rule, go so long without eating. Moreover, it is certain that not every individual is dependent for its food on a personal visit to the food-chamber, because I have often seen ants, immediately after being taken from *B* (containing food) and placed back in *A*, regurgitate food to their fellows, so that the failure of a particular ant to visit the food-chamber by no means warrants the conclusion that it has remained without nutrition. Many of the nests contained queens, males, and larvae, all of which throve, the ants being able to rear their young in chamber *A* provided proper food was kept in *B*, and provided the ants which entered *B* were placed back in *A*. Colony 4 was the only one without a queen, but as the ants even in this colony flourished, it cannot be said that in all cases the food is supplied by queens, either through their fat-bodies or otherwise as is done when a queen is founding a colony.

Table II was made up in exactly the same manner as Table I, except that the intervals between observations were 48 hours instead of 24. In the cases of colonies 24, 42, 51, and 54, the per cents of ants entering *B* were larger when the food was in *B* than when it was in *A*; the reverse was true in colonies 52, 53, and 55. The deficiencies in the records of colonies 4, 17, 20, and 24 are due to the fact that these colonies were examined every 24 hours, during at least one series when the food was in *A* or in *B*. A comparison of the results of these two tables shows that there is one colony (51) from which during the 24-hour periods more ants entered *B* when the food was in *A* than when it was in *B*; but which in the 48-hour periods gave the opposite result. All those colonies which in the 24-hour periods showed more ants entering *B* when the food was in *B* than when it was in *A*, showed the same results in the 48-hour periods. In the 48-hour periods, as in the 24-hour periods, when there was more than one series with food in *B*, the per cents of ants in *B* were fairly similar in the two cases (34.3 and 28 ; 6 and 4.6), nor are the results very different in a given colony (17 and 20) for the 24- and 48-hour periods. On the whole, when we compare the results in the 24-hour periods and the 48-hour periods, it will be seen that there were somewhat more ants entering *B* during the 48-hour periods than during the 24-hour periods. In the 48-hour periods there were rather more ants entering *B* when the food was in *A* than when it was in *B*, as opposed to the reverse condition shown for the 24-hour periods. I believe this to be due to the fact that there were several colonies (4, 17, 20, and 24) which, throughout one or more series, were examined daily (consequently no 48-hour periods) and these were colonies which, during the 24-hour periods showed more ants entering *B* when the food was in *B*.

To ascertain the constancy of individuals in regard to foraging, I noted the number of times which each ant had the opportunity (one opportunity in 24 hours) to go into *B*, and also the number of times which it availed itself of this opportunity. From the ratio of the two numbers, expressed in per cents, was subtracted a similar ratio ascertained when the food was in *A*, since we must assume that the ants entering *B* when the food was in *A* did so for some other purpose than that of obtaining food, and that an equal number would presumably have passed from *A* to *B* when the food was in *B* for similar reasons, i. e. *not* for food. These results are given in Table III, where the second column shows the number of ants engaged in foraging for each of the several per cents from −78 to 85. For the sake of convenience the results of the corresponding operation for the activities of tending the young (third column) and building (fourth column) are incorporated in the same table. The results on foraging would have been more convincing had more ants in all colonies entered *B* when the food was in *B* than when it was in *A*. But as has been stated, it was frequently found that an individual ant entered *B* more often when the food was in *A* than when it was in *B*.

In compiling Table III, I have disregarded in respect to every activity all individuals which died before the end of the experiment. There were many ants which did not engage in foraging at all (76 out of 195, or about 39 per cent), while only a few entered *B* a large part of the possible number of times — only 8 out of 195, for example, taking advantage of more than half their opportunities. This suggests some sort of division of labor, and thus agrees with Lubbock's ('82) observations, although the number of individuals which constantly engaged in this activity was much larger in my studies than in his. This difference may have been due in part to our having used different species of ants, but also in part to differences in our methods of experiment-

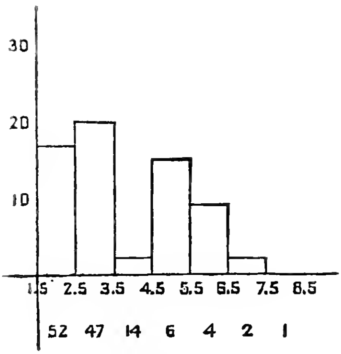


FIGURE 12.
(Foraging.)

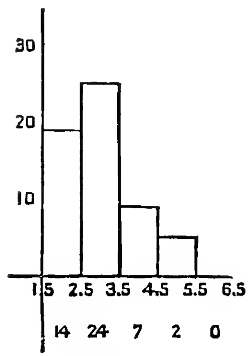


FIGURE 13.
(Tending the Young.)

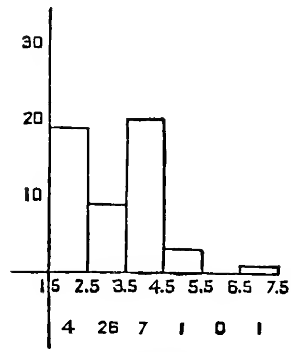


FIGURE 14.
(Building.)

ing and observing. His ants had perfect freedom to go to and from the food, being therefore able immediately to return with food to their fellows; whereas mine had to wait for a longer or shorter time before returning, in some cases probably nearly twenty-four hours, with the result that those of my ants which entered the food-chamber were for much of the time of no use as a source of food supply to the rest of the colony, in consequence of which other ants probably went in search of food, making the total number of food-seekers somewhat larger than it would have been but for this restraint. His observations of unrestrained ants were made once an hour; mine, made on ants that were free to enter the food chamber, but could not leave it, were made only once a day. However, I do not think this wholly explains matters, for on days immediately following those on which no record had been taken, there were not many more foraging ants than after an interval of only twenty-four hours, which seems to me to show that nearly all individuals likely to frequent the food-chamber had done so during the first twenty-four hours. Moreover, the artificial heating of my ants may have had something to do with the difference in our results, for while my ants were thus stimulated to unusual activity, his experiments were carried on at room temperature in winter.

TABLE III.

Given %.	Foraging.	Tending Young.	Building.	Given %.	Foraging.	Tending Young.	Building.	Given %.	Foraging.	Tending Young.	Building.	Given %.	Foraging.	Tending Young.	Building.	Given %.	Foraging.	Tending Young.	Building.	
−78	1	..	..	−44	..	..	..	−10	1	..	..	24	1	1	..	58	1	..	..	
−77	..	..	..	−43	..	..	..	− 9	1	..	..	25	1	..	1	59	..	..	1	
−76	..	..	..	−42	..	..	..	− 8	..	..	..	26	1	2	..	60	..	1	..	
−75	..	..	..	−41	..	..	..	− 7	2	..	..	27	..	..	..	61	1	..	..	
−74	..	..	..	−40	..	..	..	− 6	..	..	..	28	2	1	..	62	..	..	..	
−73	..	..	..	−39	..	..	..	− 5	..	..	..	29	1	..	1	63	..	..	..	
−72	..	..	..	−38	..	..	..	− 4	3	..	..	30	1	3	..	64	..	..	..	
−71	1	..	..	−37	..	..	..	− 3	1	..	..	31	1	..	..	65	..	..	..	
−70	..	..	..	−36	1	..	..	− 2	1	..	..	32	..	2	..	66	..	..	..	
−69	..	..	..	−35	1	..	..	− 1	..	..	..	33	2	..	..	67	..	..	..	
−68	..	..	..	−34	..	..	..	0	76	7	5	34	..	1	..	68	..	..	..	
−67	..	..	..	−33	..	..	..	1	..	..	4	35	1	..	1	69	..	..	..	
−66	..	..	..	−32	..	..	..	2	20	2	5	36	2	1	..	70	..	..	..	
−65	1	..	..	−31	..	..	..	3	9	..	3	37	..	..	..	71	..	..	..	
−64	1	..	..	−30	1	..	..	4	2	2	2	38	1	1	..	72	..	..	..	
−63	..	..	..	−29	1	..	..	5	3	..	1	39	..	..	..	73	..	..	..	
−62	..	..	..	−28	..	..	..	6	6	4	1	40	1	1	..	74	1	1	..	
−61	1	..	..	−27	..	..	..	7	5	..	3	41	..	..	..	75	..	..	..	
−60	..	..	..	−26	1	..	..	8	..	2	1	42	..	..	..	76	..	..	..	
−59	..	..	..	−25	..	..	..	9	5	..	1	43	..	..	..	77	..	..	..	
−58	..	..	..	−24	..	..	..	10	..	3	1	44	..	..	1	78	..	..	..	
−57	1	..	..	−23	..	..	..	11	2	..	2	45	..	..	..	79	1	..	..	
−56	..	..	..	−22	..	..	..	12	..	2	1	46	3	1	..	80	..	..	..	
−55	1	..	..	−21	..	..	..	13	..	..	..	47	..	..	1	81	..	..	..	
−54	..	..	..	−20	1	..	..	14	1	1	1	48	..	..	..	82	1	..	..	
−53	..	..	..	−19	2	..	..	15	4	..	1	49	..	..	..	83	..	..	..	
−52	..	..	..	−18	..	..	..	16	..	1	1	50	..	2	..	84	..	..	..	
−51	..	..	..	−17	..	..	..	17	..	..	..	51	1	..	..	85	1	..	..	
−50	..	..	..	−16	..	..	..	18	3	2	1	52	..	..	..	..				
																	195	46	43	Total no. ants engaged in each activity
−49	..	..	..	−15	..	..	..	19	1	..	..	53	..	..	..	..	92	39	38	No. reacting positively
−48	..	..	..	−14	..	..	..	20	..	..	..	54	..	1	..	..	27	0	0	No. reacting negatively
−47	..	..	..	−13	1	..	..	21	2	..	..	55	..	..	..	..	76	7	5	No. not reacting
−46	..	..	..	−12	..	..	..	22	..	..	2	56	..	..	..	..				
																	8	5	1	No. reacting positively more than 50 per cent
−45	1	..	..	−11	..	..	..	23	3	1	1	57	1	..	..	..	..	..	..	

I do not find from these experiments any very striking evidence of a correlation between size and the habit of foraging. I have plotted frequency polygons (Figures 12–14) for three of the various activities studied. For this purpose the ants experimented with were divided into six classes, based on the size of the head expressed in sq. mm. (length × breadth). The class values are indicated along the axis of abcissas. The values along the axis of ordinates are the average per cent of times (the opportunities being 100 per cent) which ants of these sizes entered into the activities of foraging (Figure 12), tending the young (Figure 13), and building (Figure 14), respectively. The figures below the middle of each class show the number of individuals in that class. There seems to be no marked difference as to the amount of foraging among the various classes, except that the largest ants did not engage much in that activity. It might be objected that the colonies should not have been classed together in this way, since there might be differences in behavior of the classes of different colonies, depending on the number of ants of each size within the colony, numbers which change with the age and amount of food of the colony. But here the colonies, being all small, were in approximately the same condition, and were kept in the same environment. Moreover, such a combining of colonies would tend not to exaggerate but to obliterate differences in behavior of the different classes, and yet such differences are more or less evident.

2. *Tending the Young.*

1. *Methods.* — In endeavoring to ascertain which ants were most active in tending the young, both passages at the ends of the partition in the Fielde nest were left open. The ants were placed in chamber *A*, and this was left partially darkened until the ants had collected in the darkest part of the chamber. Then the screen was removed, allowing the daylight to fall upon the insects. In order to cause ants more quickly to pick up the young, a gentle current of air was pumped into the corner of the nest where the ants were congregated. The pumping was done by the following method: A five-gallon glass carboy was used as a compression chamber for the air. Into this the air was pumped by means of a foot-bellows, connected to the carboy by a heavy rubber tube. Another tube leading from the carboy terminated in a piece of glass tubing drawn out to a fine point, which was inserted through a small hole in one of the two roof-panes of the nest.

Four colonies were used in this experiment. Each colony was observed for 50 one-minute periods. During each period a record was made of all ants seen carrying eggs or larvae (there were no pupae). The average number [2] of workers in each colony was then found (Table IV, line 3) and finally the per cent of workers which were active in tending the young (line 4). In a similar manner, line 5 gives the average number of winged queens in the colony, line 7 the average

[2] See page 450, foot-note 1.

number of deälated queens, and lines 6 and 8 the per cent of times that these were respectively active in tending the young.

2. *Observations.* — I may note here the following incidental observations :

(1) Ants often appeared to touch the young, even with the antennae, without seeming to notice them.

(2) Unless violently disturbed, the ants did not remove the young to a definitely safe place, but wandered about with them in their mouths in an apparently aimless way.

(3) After an experiment had continued for some minutes, the ants were not as much disturbed by the current of air as they were at first.

TABLE IV.

No. of the Colony	42	51	54	55
No. of observations	50	50	50	50
Av. no. of workers in colony	5	10	23	24
Per cent of workers tending young	17	8	31	9
Av. no. of winged queens in colony	. .	2	1	. .
Per cent winged queens tending young . .	. .	0	2	. .
Av. no. of deälated queens in colony . . .	. .	1	1	1
Per cent deälated queens tending young . .	. .	0	10	2

This is in agreement with Turner's (: **07**, p. 408) statement in regard to sound, and Fielde's (: **03**[b], p. 493) in regard to other stimuli.

Unfortunately the number of individuals in a colony in this experiment was small. However, it was found (Table IV) that only a small per cent (17, 8, 31, 9) of the workers were occupied in tending the young. The per cent of workers engaged in this activity seems to be quite independent of the number of individuals in a colony.

A similar conclusion to that reached in regard to foraging (pp. 453–455), — viz. that a few individuals are more active than the rest, — is also to be drawn from my experiments on tending the young, as shown in the third column of Table III (p. 455). The methods used in experimenting allow no minus results in this activity, or in building (column 4), such as were found in foraging. Out of 46 indviduals experimented with, 7 did not share at all in tending the young, and only 5 reacted more than half the time. On the whole, this resembles the results obtained in regard to foraging.

Figure 13 (p. 454), constructed as described on pages 455 and 456, seems to show that there is a somewhat greater activity on the part of the smaller individuals. So far as these observations go, this might be considered, as in certain aspects of foraging (pp. 453–455), to be due to a great alertness of some individuals in more than one activity, which certainly exists, and will be discussed later (pp. 462–463); but in corroboration of field experiments on *C. pictus* (pp. 468–469), it also seems to me to point clearly to a greater activity on the part of the smallest ants of *C. americanus* in tending the young.

FIGURE 15.

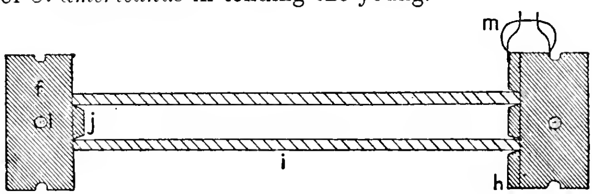
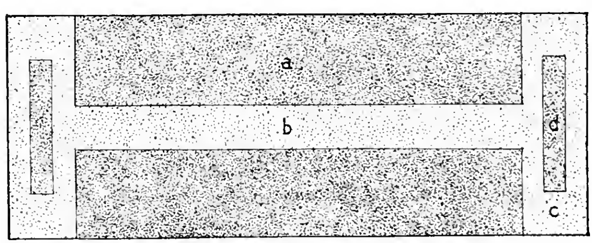


FIGURE 16.



It is somewhat peculiar that both deälated and even winged queens, as well as workers, took part in this activity.

3. *Building.*

1. *Methods.* — (*a*) *Horizontal nest.* Two kinds of nest were used in investigating this part of the problem. The first was a Fielde nest, such as was used in previously described experiments. In one chamber, kept dark except during observations, was placed damp earth, collected with the ants, so as to be of an appropriate sort. The nest was nearly filled with the earth, only enough room being left to enable the ants to walk comfortably between the earth and the roof. The other chamber was exposed to daylight and contained the food. I call this the horizontal nest.

(*b*) *Vertical nest.* The second kind of nest (Figures 15–17) was vertical, and consisted of a plaster-of-Paris base, two end pieces of wood let into two sockets (Figure 16, *d*) in the base, two sides of glass, and a wooden cover. The base (Figure 16) was $6\frac{1}{2}$ inches long, $2\frac{1}{2}$ inches wide, and $\frac{5}{8}$ inch thick. The thickness of the base was increased above the level at *a* $1\frac{1}{8}$ inches at the ends (*c*) and along the middle of its

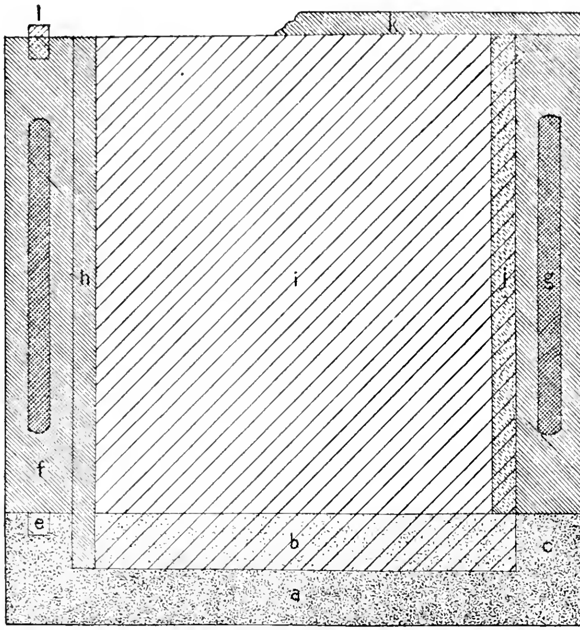


FIGURE 17.

upper surface (*b*). In each end thickening (*c*) there was a groove, or elongated mortise (*d*), which received the tenon (*e*, Figure 17) of the lower end of the wooden end piece. In the upper end of each of these pieces was inserted a cylindrical glass peg, *l* (Figure 17), to hold in place the wooden cover, *k*, which had bored in it holes corresponding in size to the peg and in such positions as to hold the end pieces parallel to each other. The end pieces were $5\frac{1}{2}$ inches long, $1\frac{1}{2}$ inches wide and $\frac{11}{16}$ inch thick. To the inner surface of each was securely fastened a strip of wood (*j*) $\frac{3}{16}$ inch thick and as wide ($\frac{1}{2}$ inch) as the median thickening (*b*) of the base. This strip of wood and the thickening (*b*) of the base served as stops to keep apart the two glass plates (*i*, Figure 15), 5 inches × 6 inches, which formed the sides of the chamber. When one of the glass sides had been firmly pressed against the median "stops," the two wooden strips (*h*, Figures 17, 15) were pressed against the pane and clamped in position by steel clamps (*m*, Figure 15), one jaw of which rested in a vertical groove (*g*, Figure 17) cut in the edge of the end piece, as shown in section at *m* (Figure 15). When both panes were thus clamped in position, a fairly tight chamber ½ inch thick and 5 inches × 6 inches in area was completed. This was nearly filled with earth. All wooden pieces were infiltrated with paraffin to prevent warping, as the base of the nest was occasionally placed in water to keep the

TABLE V.

No. of the Colony	54	54	53	53	52	51	51	42	42
Kind of Nest	H	V	H	V	V	H	V	H	V
No. of observations	150	150	150	150	150	150	150	150	150
Av. no. of workers in colony	22	21	17	17	2	9	5	4	2
Per cent of workers building	5	13	5	14	59	17	15	13	38
Av. no. of winged queens in colony	1	1	4	3	1	1	1	5	4
Per cent of winged queens building	0	0	4	0	3	65	0	2	0
Av. no. deälated queens in colony	1	1	2	3	1	1	2	5	6
Per cent deälated queens building	0	3	0	0	7	0	8	3	4

earth damp. In case either pane of glass needed cleaning, the nest was laid on its side with that pane uppermost, and the plate was then exchanged for a clean one by unclamping the wooden strips of that side.

2. *Observations.* — The colonies observed in this activity were especially small, so that it is not safe, perhaps, to lay much stress on the results, except in relation to other work on this subject. I give them, therefore, merely as additional evidence.

From these experiments (Table V) [3] I found, again, that the per cent of individuals engaged in this activity is, as a rule, small, and is independent of the number of individuals present in the nest. In most cases (colonies 54, 53, and 42) the per cent was smaller in the

[3] Table V was constructed in substantially the same manner as Table IV (p. 457).

horizontal nest (*H*), but in colony 51 it was smaller in the vertical (*V*). For my problem, however, this is unimportant.

In Table III, column 4, it may be noted that out of 43 ants tested in building, five did not engage at all, while only one reacted more than half the time, the largest per cent being 59. As in tending the young, the methods employed allow only positive results.

Figure 14 (p. 454) shows a distinct correlation between size of the individual and activity. It will be noticed that this activity is almost confined to the smaller sizes (1.5 mm.–4.5 mm.).

TABLE VI.

Classes,		1		2		3		4	
		1.40–3.19 sq. mm.		3.20–4.99 sq. mm.		5.00–6.79 sq. mm.		6.80–8.59 sq. mm.	
No. of Colony.	No. of observations.	No. of Individuals.	Per cent active.	No. of Individuals.	Per cent active.	No. of Individuals.	Per cent active.	No. of Individuals.	Per cent active.
90	50	2	29	2	28	2	26	2	30
89	50	10	20	10	18	10	17	10	32

But here again, as in tending the young, both deälated and winged queens took part.

4. *Fighting.*

1. *Methods.* — It was impracticable to deal here with the colony as a whole, since it was impossible in the confusion which occurred to distinguish readily the sizes which took part. In order to discover which of four different known sizes (classes) was more active in fighting, a few individuals of each size were placed in four separate Stender dishes, one dish for each size. Each dish measured 5.5 cm. in diameter and contained the same number (given in Table VI) of ants. The limits of the head sizes of the individuals in each of the four classes is indicated in sq. mm. Into each dish in succession was introduced

for a period of thirty seconds a worker from another colony of *Camponotus americanus*, and the number of ants attacking it was noted for each dish.

2. *Observations.* — Table VI gives the results of these experiments. The first column gives the number of the colony, the second the number of observations made on each colony. The results for each of the four classes of ants are shown separately; for each class the first column shows the whole number of individuals of that class under observation, the second column the per cent of activity.[4]

Two colonies were observed in these experiments, and in both cases it was found that the largest size took part slightly more often than the other sizes. They were, too, more savage in their attacks. For the first time, then, we seem to have an indication of greater activity on the part of the large workers. This might, of course, be due to differences in surrounding conditions, for in the other activities Fielde nests were used (pp. 446, 456, 458), and in fighting, Stender dishes; but in view of work done in the field on *Camponotus pictus* (pp. 468–471), I hardly think that this is the explanation. It seems to me, rather. that it is due to division of labor, though here, again, the distinction is not hard and fast, as all sizes are very active, but the largest workers most so.

5. *Relations of Classes to Various Activities.*

Having discussed foraging, tending the young, building, and fighting in their various aspects, attention should now be directed to the specific question of the relation between size and the various activities. If we examine Figures 12–14 we see that, while the correlation between the size of the ants and the three first mentioned activities is not in all cases striking, nevertheless, on the whole, the smaller ants are the more active in each of these functions. This is even more evident when the curves given in Figures 12–14 are combined (Figure 18) so as to show the total activity for these three occupations for each size. In the case of fighting, on the contrary, it is the larger individuals which show the greater activity.

It is another question whether a *given individual* shares in all activities, for it is conceivable that certain individuals of a given size-class confine their attention to one mode, and others of the same size-class exhibit theirs in another mode of performance. As a matter of fact, I

[4] The per cent active was found in each case by first ascertaining the number of ants active during each observation; the per cent which this number was of the whole number of ants in a given dish was then found, and finally these per cents were averaged for each dish from each colony.

have frequently seen the same ant occupied first in one and then in another activity. Indeed, some individuals were about equally concerned in two or more activities. When, however, we consider all the evidence on the various activities, we are warranted in the conclusion that in fighting, the largest ants predominate ; in building, the small and middle-sized ones ; in tending the young, the smaller individuals ; and in foraging, all sizes except the largest. From field evidence on *Camponotus pictus* I can corroborate these conclusions.

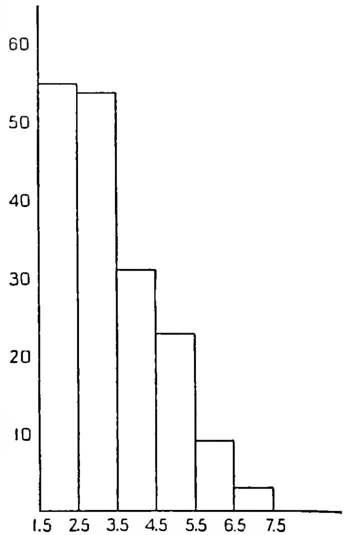


FIGURE 18.

And yet it is evident that certain individuals, regardless of size, are much more alert in each activity than others ; this at first sight suggests a division of labor not correlated with size differences. In Figure 19 individual ants, arranged on the axis of abcissas in the order of the size of the head, are designated by the letters of the alphabet ; at the left the ordinates indicate the per cents of the possible number of times which each of these twenty-one individuals (*A*–*U*) was engaged in the several activities, these being plotted as solid black lines. The upper curve shows "building," the second, "tending the young," the third, "foraging," and the fourth (bottom) is the sum of the other three curves, which thus gives the total activity of each individual for these three occupations. The dotted curve shows the various head sizes in sq. mm. (length × width), the values being indicated by the figures in the right margin. It may be seen at a glance : (1) that there is no very obvious correlation between any of the activities when compared with one another (curves 1, 2, and 3) ; (2) that there is no close correlation between any of the various activities and the size of the head (compare curves 1, 2, and 3 with the dotted curve) ; and (3) that certain individuals are in general much more active than others (bottom curve). The fourth curve shows that, on the whole, the smaller individuals are the more active. While there is a slight preponderance of certain sizes (D = 2.30 sq. mm.) in certain activities (building), there is also an individual diligence in several activities, and this seems to be more or less independent of size.

In regard to the behavior of queens, it may be said that it was only rarely that they shared in any of the activities described above, and then it was usually under more or less exciting influences. This activity was, in all probability, a reversion in the case of deälated individuals to the condition of the queen when founding a colony, and in the cases of winged individuals to the condition in the parental nest (see Wheeler, :06[a]).

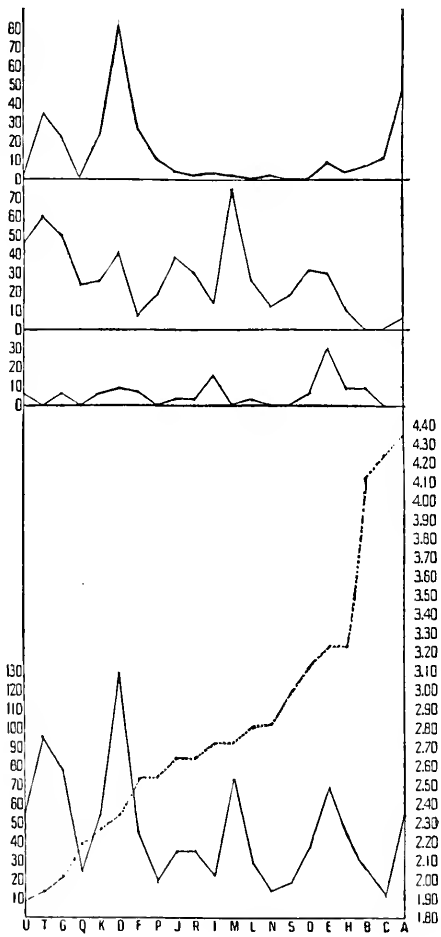


FIGURE 19.

6. *Conclusions in Regard to Camponotus americanus.*

From the foregoing account it appears that:

1. Males were never active in the duties of the nest.

2. The proportion of workers occupied in any given activity seems to be independent of the number of individuals in the colony, when the colony is as small as those here studied.

3. Queens, even when retaining their wings, may take part in certain activities in small colonies, although the colony has reached such an age as to contain all classes.

4. A few individuals are very constant in a certain activity, or in even more than one, the majority being much less active.

5. Some individuals are about equally concerned in two or more activities.

6. Too few individuals were available in making up Figures 12–14 to show division of labor as clearly for this species as is seen in *C. pictus*

when examined in the field (pp. 467–471). There seems here to be a greater activity on the part of some individuals than of others, irrespective of size; this is probably due to psychological differences of individuals, as suggested by Escherich (:06, p. 45). But there seems also to be some evidence of a division of labor correlated with polymorphism; this is not marked by hard and fast lines, since all classes may share in any activity, but by a *preponderance* (*a*) of large ants in fighting, (*b*) of those of medium and small size in building, (*c*) of those of small size in tending the young, and (*d*) of those of small and medium size in foraging.

7. The small and medium-sized workers are, in general, much more active than the largest ones.

B. Camponotus herculeanus pictus.

i. In Barth Nests.

1. *Methods.* — In Barth nests, 8 inches high × 6 inches in diameter, with ¾ inch between the jars, five colonies of *C. herculeanus pictus* were studied, all of which had been previously studied in the field, the object being to compare the two sets of studies with a view to learning if the ants behaved differently in the laboratory from what they did in the field. In other words, the field studies afforded a control for those in the laboratory. Therefore, not only those activities which had been unsatisfactorily examined in the field were studied, but also all others. The colonies were necessarily smaller than those in the natural nests, as it was impossible to collect the whole of a colony. I attribute to this diminution of the size of the colony the fact that, as a whole, the ants in captivity were less active than those in the field, for it will later be seen (p. 471) that in the normal state the smaller colonies of *Camponotus herculeanus pictus* were less active than the larger.

2. *Observations.* — In the Barth nests I have never seen any of these ants carrying in food, *feeding* themselves, or preparing food. More of the intermediate sizes than of any other size were seen outside of the earth at the top of the nest, or running about in the bottom of the well, i. e. the inside jar, a condition which may be compared to foraging. The other two sizes about equalled each other in this activity.

In *regurgitating* and receiving regurgitated food it is difficult to see in which direction the food passes. But as I have seen this process occur between all possible combinations of majors, intermediates, and minors, no one class can be the sole regurgitators; I rather suspect that it is more common for the two smaller classes to regurgitate, and for the majors to receive. The reasons are as follows: First, the majors were several times seen drumming on the heads of these two classes, both immediately before this activity took place and during its performance, whereas this drumming was never seen in the other two classes. Apparently the same stimulus is used by ants to make aphids give out secretions (Forel, '**74**, p. 251) and to make ants regurgitate (l. c., p. 243. See also Wheeler, :**01**[c], p. 438). This leads me to think that it is the ant desiring food, rather than the one giving it, which does this tapping. Secondly, throughout all my observations on this species and *C. americanus*, it is the class of large ants which has less to do with collecting food than the other classes. Thirdly, the large ants are, on the whole, so much more lazy than the intermediates and minors that it is characteristic for them to take food which is easily obtained from other ants near at hand rather than to seek it or prepare it for themselves. Of course they probably do often obtain other food, and the other classes certainly sometimes receive regurgitated food; but for the reasons stated, I am inclined to think that in general the majors are more apt to receive regurgitated food, the intermediates and minors to give it. On one occasion I saw an intermediate and a queen, on another a minor and a queen engaged in regurgitation, but I am unable to say which way the food went during the process.

In *licking*, all three classes took part, but the majors slightly less often. The majors were never licked, and the minors seldom, the intermediates being most often the objects of this activity. There were not many individuals, however, which shared in this activity, so that this evidence may not have great weight by itself; it may, however, help to confirm other evidence.

Practically the same conditions in regard to *tending the young* hold true here as in the same colonies out of doors, viz. that there are many minors, few intermediates and no majors so engaged. This was particularly noticeable in colony R 18, where the nest consisted of two chambers, an upper and a lower, with a short gallery connecting the two, and another gallery leading to the surface. It was a marked fact that day after day the large and some of the intermediate ants stayed in the lower chamber, while the other intermediates and all the small ants were in the upper chamber with the larvae. Moreover, when the ants were stimulated with the heat from the electric bulb (pp. 440–443), it was nearly always the small ants which carried the larvae toward the heat, or away from it when it was too great. In regard to the chambers and galleries, I have noticed both here and in out-of-door colonies that ants do not, as a rule, collect in the galleries, but use these rather as passages; they are inclined to huddle together in the chambers.

In *building*, both in digging and in carrying earth, it was the majors and intermediates which performed the greater part, the minors hardly sharing at all. Digging is here similar to that described for *Pheidole pilifera* (p. 485).

Carrying (and being carried) was not a frequent occurrence, and even when it did occur, it was, in these nests, rather a dragging than a carrying. In the few cases where it was observed, minors were dragging intermediates or majors, and no minors were dragged.

Any unemployed ants may collect in the chamber with the queen, regardless of their size, and I have no reason to think that one size is more apt to do this than is another.

When the nest is jarred, or the earth on top broken, ants of all sizes become much excited. Here there did not seem to be a difference in the reaction of ants of different sizes as there was in the out-door nests.

Nothing resembling guarding was observed.

ii. In Out-door Nests.

1. *Field Notes.*

1. *Methods.* — In considering these notes it should be remembered that the records were made at the time the ants were captured, and therefore before any conclusions could have been deduced from them. Later, the ants were arranged in series, and they will be described further on (pp. 469–471). The results of these field observations are of course additional to those obtained by actually collecting and arranging other ants in series. So far as they agree with the results deduced from collected material (see pp. 469–471), they strengthen the conclusions drawn from that material. The classes had to be determined arbitrarily, and were designated as "majors," "intermediates," and "minors." The numbers were also, as a rule, only approximate, and were classed as "many," "some," and "few," except where actual numbers are given, when they always fell under "few." The approximate numbers of individuals of each class occupied each day in each activity, then, were observed, and a summary was made first for each colony, and later a summary for all the colonies taken together. The lack of records in several of these activities (regurgitating and receiving regurgitated food, licking and being licked, and as a rule, surrounding the queen) is due to the fact that it is difficult, and often impossible, to observe them in an undisturbed out-door nest, and one must therefore depend upon as natural conditions as possible in the laboratory to show facts concerning them. Though digging is not represented here, nevertheless I succeeded in capturing ants engaged in this activity

(p. 469). Fighting is probably partially represented by "disturbed by knocking."

2. *Observations.* — In the out-door work on *Camponotus herculeanus pictus* it was evident that many more ants, probably even a greater proportion of the colony, took part in the various activities than was the case with *Camponotus americanus* in the laboratory; and yet, when compared with the total population, it was noticeable that few were engaged in any kind of work at one time. This was best seen in foraging; for no matter how many ants were in the field, when the nest was opened a vastly greater number of individuals was revealed within.

In regard to those activities which were successfully observed in the field, it was found that the intermediates and minors take a much more active part than do the majors, and on the whole, the intermediates are here slightly more active than the minors.

Except for those ants which were captured, only two, an intermediate and a major, were seen actually carrying in food.

The majors were least active in "tending the young," the intermediates more active, and the minors most so.

As for building, all sizes were seen carrying earth, though the two smaller sizes excelled in this, especially the minors. No ants were observed digging, except those which were captured.

The act of carrying I think is most frequently manifested when the ants are somewhat excited, either by some disturbance or by the presence of food. This activity is on the whole not very common, but in these out-of-door nests, the process seems to be one of actually carrying rather than of dragging, which was observed in the Barth nests (p. 467).

My impression is very strong that, while all classes are disturbed when the nest is knocked upon or broken into, and while all will attack an intruder vigorously, it is on the whole the majors in this species which exhibit most zest in this response. They gnash their jaws, curl their abdomens forward under the rest of the body in order to throw poison on the enemy, and appear very savage. If they once get hold of an enemy, they cling with bull-dog tenacity, keeping a firm grip with their mandibles even after death. While all this is also true to some extent of the intermediates and the minors, it is the majors which are the fiercest.

The note-book evidence, then, seems to show that the minors are the most active in tending the young, somewhat larger ants in foraging and in building, and that the majors are more or less reserved for fighting. There is, however, no hard and fast division between classes either morphologically or physiologically, and therefore, all that one can say is that ants of certain sizes predominate in a given activity.

Another very striking conclusion which may be drawn from the field notes is that on the whole the majors are relatively inactive in all occupations except fighting, while the other two sizes taken together are very active. Only a small proportion of the colony is at one time occupied in any given activity.

2. *Captured Ants.*

1. *Methods.* — Ants of twenty-two colonies of *Camponotus herculeanus pictus* were studied in their natural out-door nests, as described on pages 467–469. When insects from the same nests had been killed, they were, as mounted specimens, arranged in series according to size, and a permanent record of each series was secured by means of photographs showing the insects' natural size. This was done to show particular facts given below. One series consisted of ants engaged in foraging (*a*, all the ants seen in the field; *b*, ants seen carrying food in the field); a second, tending the young; a third, building (*a*, carrying earth; *b*, digging; *c*, digging under excitement); a fourth, surrounding the queen; a fifth, carrying other ants; a sixth, being carried by other ants; a seventh, fighting, i. e. rushing out when the nest was disturbed, or running about excitedly five minutes after the nest had been disturbed. In order to compare the various colonies studied for each activity, the ants of each colony engaged in a given activity were arranged by themselves, according to size, from the smallest to the largest. Other series were also made, where no attention was paid to colonies, all the ants taking part in each occupation being arranged in a single series.

2. *Observations.* — In the case of each occupation it was found that there was a perfectly graded series of sizes, i. e. with no division into classes; this means that all sizes take part in each activity which was tested. This was especially marked when all the colonies were arranged in a single series.

But when the various occupations were compared with one another, it was found that, although ants of various sizes shared in each activity, in some activities there is much more of a tendency for the large ants to take part, while the smaller ants are more inclined to share in other activities. The smaller ants, for example, are much more apt to tend the young, while the larger ones are more prone to rush out when the nest is disturbed. Thus, I was able to group the series representing the several activities in such a way that the series in which the small ants predominated were at one end, and those in which the large predominated at the other. The several series fell into the following order, beginning with those in which the smallest sizes predominated: (1) tending the young, (2) foraging, including ants actually carrying food, — which was done neither by the very large nor the very small, — (3) surrounding a very young queen, carrying other ants, and being carried by other ants (the ants in these series being of about equal size), (4) building, (5) responding to external disturbances. It should be stated, however, that the three activities listed under 3, as well as that of actually carrying food, had only a few individuals in each. Of the other activities, tending the young was exhibited by 114 individuals, foraging by 96, building by 42, and responding to disturbances by 56. A series was also made of 74 individuals which were tending the young under excitement, i. e. when their nests were disturbed; these ants were, on the whole, much larger than those which were collected under normal conditions of tending the young. To watch this activity under normal conditions is more difficult than to watch several of the other activities; but in some colonies the ants brought the young near openings in the nests, or in carrying them from one part of the nest to another, had to pass exposed places, so that I was thus able to observe and capture them. I was also able on some occasions to open the nest suddenly enough to see ants tending the young before they became disturbed. Although from the shape of the heads of all sizes of individuals one would not expect to find any specialized guards, such as are found in Colobopsis (Lubbock, '82; Forel, '74; :03, p. 83; :05, pp. 453–454; Escherich, :06, p. 46; Wheeler, :04ᵃ, :10, pp. 184, 211–212), nevertheless I made an examination to see if there were any indication that special guards existed. This species does not appear to guard the entrances in any way, but, with the exception of those individuals which are going in and out, all individuals are more apt to stay quietly huddled together in chambers within the nest. By trimming off little by little the decayed wood of the nest, along the passages, working from the outside in, and taking great care to create no disturbance, I have succeeded in making this observation repeatedly. No queens were engaged in any of the activities, probably because here there were a sufficient number of workers to attend to the needs of the colony.

In order to see how much influence the size of the colony had on the sort of individuals participating in certain occupations, I noted, at the time the ants were collected, the size of the colony. This had to be estimated roughly, and when statements of the size of a given colony were recorded on different days, these separate estimates were considered in making up the final estimate; colonies were classed as very large, large, small, and very small, but none of them were so small as those of *C. americanus*, which had been previously used. The mounted ants, arranged in series for each occupation according to colonies, were then compared colony with colony, keeping in mind the size of the original out-door colony from which they were captured. It was found that in regard to foraging, the sorts of individuals were practically the same, irrespective of the size of the colony. There was, however, this difference in behavior, that the large colonies sent proportionately more workers into the field, probably because large colonies are apt to be less timid than small ones. Forel ('74, p. 249) has also noticed this fact.

While also proportionally more ants were active in tending the young in the large colonies, I failed to find any correlation between the *size* of the workers engaged in this activity and the numbers in the colony. In tending the young under excitement and in responding to disturbances, there were hardly any ants from small colonies, so that a comparison between large and small colonies could not be made for these activities. So few ants were seen bringing in food that it was impossible to ascertain whether such correlation exists in that activity or not. As it does not, however, exist in ants seen in the field, when those without food are considered together, it is highly probable that it is also wanting here. In carrying other ants, in being carried by other ants, and in carrying earth, no correlation was found.

Of course it must be remembered that, — as Wheeler (:02) found in Pheidole and Pricer (:08) in certain species of Camponotus, — in colonies which are extremely small because of poor nourishment, due to the youth of the colony or to other causes, there are no large workers, or at most only a few, and hence all the duties must be performed by the small ants. My remarks in regard to lack of correlation between the size of the colony and the sorts of ants engaged in various activities would therefore apply only to such colonies as have all the sizes which are normal to them. Also, there are, in proportion to the whole number of active ants, rather fewer large ants engaged in most of the activities in these wild colonies than in the small colonies in artificial nests described above.

iii. Correlation between Age and Function.

1. *Methods.* — When an ant emerges from the pupal state it is very light in color, but begins almost immediately to grow darker, taking, according to the species, a longer or shorter time to reach its ultimate color tone. McCook ('79, p. 20), Forel ('74, p. 262), Pérez (:00, p. 769), and Moreno (:00) claim that ants, when they first emerge, do not go out or fight, but attend to duties within the nest. That there is some

difference in regard to capability and strength of individuals of a given age in different species is stated by Wheeler (:00[b], p. 63). He says that newly hatched queens, as well as worker callows, of Stigmatomma are soon able to run about and to join in the labors of the colony; and they are not so feeble as the callows of more specialized ants. He also says (Wheeler, :07[b], p. 87) that the callows of many ants confine themselves to caring for the larvae and pupae, and do not forage till later.

The difference of brightness occurs in *Camponotus herculeanus pictus*, especially in the thorax, and it has occurred to me to make use of the specimens collected at Randolph, N. H. (see p. 445), in order to see if there is correlation between brightness and various occupations. Of course, as Wasmann (:09[b], p. 39) points out, there is some individual variation in the length of time required for the color to reach its maximum darkness, but it is probable that on the whole the lighter ants are younger, and their strength less developed.

In order to resolve this problem I proceeded as follows: The mounted specimens which had been captured at Randolph, N. H., sharing in particular activities, were first classified according to those activities, and then the ants of each activity were arranged according to color, and compared.

2. *Observations.* — On the whole, the number of light or brightly colored ants was small, but they were of all classes. When the separate activities were observed, the following facts were noticed:

1. In foraging, there were none of the very youngest, though there were a few somewhat lighter than those of the deepest color.

2. There were a few ants engaged in "tending the young" which showed signs of being *very callow*, since they were of such light color that even the legs were very pale. I remember noticing when I collected these that they had not obtained their full strength. There were a good many ants of different sizes in which the thorax had a very bright color, and thus seemed to be young. Most of those which were tending the young under excitement had reached their maximum darkness, though there were a few which were fairly young.

3. Of the few ants which were taken in the act of surrounding a queen, none had obtained their full color, and yet not any were very light colored. This was probably a young brood, the first from that queen, for the queen herself showed signs of being young, and there were only a very few ants in the colony.

4. While most of the ants engaged in building were of full color, there were a very few brightly colored ones.

5. While some of the ants which were being carried were of the maximum depth of color, it is somewhat curious that the majority

were rather light-colored, and some gave evidence of being very young, their color being in general pale, even in the legs. On the other hand, those which were doing the carrying were almost universally dark, and none of them were very bright. Those which were being carried were probably more or less weak.

6. When those which responded to disturbances of the nest were examined, it was found that all had at least nearly reached the maximum depth of color, and most of them had quite reached that stage.

Thus it may be seen that, as has been claimed for other species and for ants in general, the young workers of this species sometimes build, and sometimes are carried about; but they are especially engaged in tending the young (eggs, larvae, or pupae). Occasionally young individuals, though not the youngest, carry other ants, or are found in the field, but only the fully colored ants share in defence.

iv. Conclusions for Camponotus herculeanus pictus.

To summarize the facts concerning *Camponotus herculeanus pictus* when studied in Barth nests or by any of the methods used in the field :

1. No males were seen to share in the duties of workers.

2. The very youngest ants take part especially in tending the eggs, larvae, and pupae, and sometimes in building, and they are apt to be carried by their companions, but they do not go into the field until they are somewhat darker than the very young stages, though they may do so while still retaining some brightness of color. The ants of this species do not, however, enter into the defence of the nest until they are dark.

3. Only a small proportion of the whole number of ants in a colony is engaged at one time in any activity.

4. Though more ants in proportion to the size of the colony are active in large colonies than in small ones, nevertheless this does not seem to make a difference in the kind of individuals engaged in different sorts of work, provided that the colony is of sufficient age to contain ants of all sizes. It must be remembered, however, that in the small colonies in artificial nests rather more large ants, in proportion to the whole number participating, were engaged in the various activities than was the case in out-door nests.

5. All classes may take part in carrying food, but the intermediates are the ones which do this most. Intermediates are also more likely to be found in the field, apparently foraging.

6. While individuals of all classes regurgitate food and receive regurgitated food, it is *probable* that the two smaller classes, especially

the intermediates, are more addicted to regurgitating, and the majors to receiving regurgitated food.

7. In tending the young, the majors are least active, the intermediates more so, and the minors most so. When excited, larger ants enter into this activity somewhat more than they do under normal conditions.

8. In building, all sizes were seen carrying earth, though the two smaller sizes especially excelled in this. In digging, the intermediates were also the more active. The evidence in regard to building is somewhat conflicting, the field notes indicating minors and the Barth nests majors and intermediates as the more active. As the captured ants can, after all, be studied by more exact methods than those in the field, I am inclined to accept as correct the results obtained with the use of nests.

9. Carrying and being carried seem to take place most frequently when the ants are somewhat excited, either by some unnatural disturbance or by the presence of food. These activities are, however, not very common. The process is sometimes one of dragging, but often it is one of actually carrying, and the minors and majors seem often to be transported by other ants, and the intermediates to be carrying them.

10. All three classes took part in licking, but the majors slightly less than the other two classes. The majors seemed never to be licked, and the minors seldom, the intermediates being most often the objects of this activity. The evidence on this activity was, however, slight; nevertheless, it is probable that majors, too, are licked.

11. Ants of all sizes may surround the queen.

12. While all classes will attack an intruder when the nest is disturbed, the majors are the most savage.

13. There is nothing in *C. herculeanus pictus* resembling the process of guarding seen in Colobopsis.

14. There is no hard and fast distinction between classes, either morphologically or physiologically — for in each activity there is a perfectly graded series with no break into classes — but only a *preponderance* of ants of certain sizes in certain activities.

15. The duties in which the small ants, i. e. intermediates and minors, excel may be called the "household duties," and foraging.

16. The majors are relatively inactive in all occupations except fighting, while the other two sizes taken together are very active.

VI. Studies of Pheidole.

A. *Pheidole pilifera.*

i. Experiments with Ants in Aluminum Nests and Other Apparatus.

1. *Methods in General.*

I made use of four colonies of *Pheidole pilifera* from Cambridge, Mass. In order to ascertain whether in these ants there is any correlation between habits on the one hand and size and structure on the other, an attempt was made, as with *Camponotus americanus* (pp. 446–465), to discover what proportion of each class (see pp. 438–440) took part in each of several activities.

In most cases I have endeavored to ascertain (a) the proportion of the whole number of working ants which engage in a given activity, and (b) the relation, if any, between the size of the individuals and the sort of activities which they showed; but in fighting, in carrying, and in partaking of different kinds of food, I have considered only the second of these two matters.

All experiments, unless otherwise stated, were performed in aluminum nests ten inches long, six inches wide, and half an inch deep, kept under the conditions described on pages 440–442, and in all cases both chambers were exposed to daylight and ordinary room temperature during observations. At other times the chamber not containing food was darkened by a pasteboard cover and the nest was placed in the heated chamber.

2. *Feeding Themselves.*

1. *Methods.* — To make sure that all the ants which partook of food were observed, the food, on paraffined paper, was placed in the chamber with the ants, the paper and the food being removed whenever the ants were not under observation. To give the ants an environment as natural as possible, the chamber in which they lived was provided with damp earth. During observations a record was made of the number of ants of each class which partook of food during one hundred and fifty periods of one minute each.

2. *Observations.* — For convenience of comparison, the results of the observations on the various activities of the several species are all placed together in Table VII. Except for fighting, carrying, being carried, and kinds of food, of which mention will be made in appropriate places, this table was compiled as follows. The first column in the table gives the number of the colony, and the first column under each activity shows the number of observations made upon the colony concerning that activity.

TABLE VII.

Species.	No. of the Colony.	Feeding Themselves.							Regurgitating.							Receiving Regurgitated Food.							Licking.						
		No. of Observations.	Workers.		Sol-diers.		Minors.		No. of Observations.	Workers.		Sol-diers.		Minors.		No. of Observations.	Workers.		Sol-diers.		Minors.		No. of Observations.	Workers.		Sol-diers.		Minors.	
			Av. no.	Av. % active.	Av. no.	Av. % active.	Av. no.	Av. % active.		Av. no.	Av. % act.ve.	Av. no.	Av. % active.	Av. no.	Av. % active.		Av. no.	Av. % active.	Av. no.	Av. % active.	Av. no.	Av. % active.		Av. no.	Av. % active.	Av. no.	Av. % active.	Av. no.	Av. % active.
P. pilifera.	81	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
	86	150	282	0.003	8	0.	274	0.003	100	89	0.01	5	0.	84	0.01	100	89	0.01	5	0.	84	0.01	100	89	0.2	5	0.	84	0.2
	87	..	..	..	..	..	..	..	100	39	0.	7	0.	32	0.	100	39	0.	7	0.	32	0.	100	39	0.7	7	0.	32	0.7
	91	..	..	..	..	..	..	..	100	173	0.01	6	0.	167	0.01	100	173	0.01	6	0.	167	0.01	100	173	0.5	6	0.	167	0.5
P. dentata.	82	150	42	0.2	21	0.03	21	0.3	150	48	0.03	23	0.	25	0.1	150	48	0.03	23	0.03	25	0.03	150	48	0.2	23	0.	25	0.3
	83	150	6	2.3	3	0.	3	0.4	150	13	0.1	4	0.	9	0.1	150	13	0.1	4	0.	9	0.1	150	13	1.7	4	1.	9	2.9
	84	150	40	0.	4	0.	36	0.	150	59	0.01	5	0.	54	0.01	150	59	0.01	5	0.	54	0.01	150	59	0.4	5	0.	54	0.4
P. vinelandica.	58	150	17	1.1	9	1.8	8	0.3	150	21	0.03	11	0.1	10	0.	150	21	0.03	11	0.	10	0.1	150	21	2.3	11	1.1	10	3.8
		Being Licked.							Tending Young.							Building.							Surrounding Queen.						
P. pilifera.	81	..	..	..	..	..	..	..	..	..	..	..	..	..	..	150	12	0.8	4	1.4	8	0.6	..	..	..	..	..	..	..
	86	100	89	0.2	5	0.	84	0.2	..	..	..	..	..	..	..	100	112	2.	5	1.	107	2.	..	..	..	..	..	..	..
	87	100	39	0.6	7	0.4	32	0.6	..	..	..	..	..	..	..	100	38	0.4	6	0.	32	0.4	..	..	..	..	..	..	..
	91	100	173	0.5	6	1.	167	0.5	100	193	2.6	6	0.	187	2.6	100	157	0.2	6	0.	151	0.2	..	..	..	..	..	..	..

P. dentata.	82	150	48	0.2	23	0.1	25	0.2	150	42	0.3	21	0.2	21	0.6	150	42	6.9	21	3.2	21	11.1	..	..	..	..	..	..	..
	83	150	13	2.2	4	3.3	9	1.7	150	16	2.	5	0.	11	2.5	150	6	3.2	3	0.7	3	5.7	..	..	..	..	..	..	..
	84	150	59	0.3	5	0.1	54	0.3	150	70	3.1	5	0.9	65	3.9	150	43	0.8	4	1.1	38	0.9	..	..	..	..	..	..	..
P. vinelandica.	58	150	21	2.2	11	2.6	10	1.6	150	141	2.4	28	0.	113	3.1	150	113	6.3	26	1.2	87	7.2	41	74	12.4	20	11.9	54	13.4

		Guarding.							Fighting.					Carrying and being carried.					Kinds of Food.								
		No. of Observations.	Workers.		Soldiers.		Minors.		No. of Observations.	Soldiers.		Minors.		No. of Observations.	Soldiers.		Minors.		No. of Ants Feeding.	Meal-worm.		Honey.		Grass-seed.		Apple.	
			Av. no.	Av. % active.	Av. no.	Av. % active.	Av. no.	Av. % active.		Av. no.	Av. % active.	Av. no.	Av. % active.		Av. no.	Av. % active.	Av. no.	Av. % active.		% Sold.	% Min.	% Sold.	% Min.	% Sold.	% Min.	% Sold.	% Min.
P. pilifera.	81	150	4	0.	2	0.	2	0.	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
	86	100	108	0.2	5	0.	103	0.2	50	8	27.9	8	25.6	3	6	0.	163	7.3	1	100.	0.	0.	0.	0.	0.	0.	0.
	87	100	34	0.1	6	0.	28	0.1	50	11	18.7	11	6.3	..	..	..	..	..	..	..	..	..	..	..	..	..	..
	91	100	154	0.03	6	0.2	148	0.03	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
P. dentata.	82	150	42	1.2	21	0.9	21	1.7	..	..	..	..	..	..	..	..	..	..	12	0.	16.7	8.3	50.	0.	0.	0.	25.
	83	150	6	2.2	3	1.3	3	2.9	..	..	..	..	..	1	4	0.	9	11.	3	0.	33.3	0.	33.3	0.	0.	0.	33.3
	84	150	40	0.	4	0.	36	0.	50	4	24.5	4	6.	10	5	0.	49	3.	0	0.	0.	0.	0.	0.	0.	0.	0.
P. vinelandica.	58	150	47	0.7	18	1.4	29	0.3	..	..	..	..	..	..	..	..	..	..	15	66.7	0.	20.	13.3	0.	0.	..	..

The second column under each activity gives the average number of workers in the colony, the fourth and sixth the average numbers of the two classes of workers (soldiers and minors, respectively) in the nest during the observations on that activity. It was necessary to take the *average* number rather than the *total* number, because of the death of some individuals between series of observations when the whole of an experiment on a colony could not be carried out at one time. In many cases this was equal to the total number of individuals at the beginning of the experiment, no deaths having occurred. The third, fifth and seventh columns under each activity give the average per cent of the workers as a whole and of the respective classes (soldiers and minors) which took part in each activity. These were found by dividing the sum of the per cents of those which were active at each observation by the total number of observations, usually 150. Dots signify that a particular experiment was not made upon the colony opposite which the dots appear, as, for instance, opposite colony 81, "feeding," "regurgitating," etc. This was usually due to the death of the whole colony before the experiment on that form of activity could be carried out, but in the case of "surrounding the queen" the absence of records is due to the fact that most of the colonies had no queen. Colonies 81, 86, 87, and 91 were *Pheidole pilifera;* 82, 83, and 84 were *P. dentata;* while colony 58 was *P. vinelandica.*

Of *Pheidole pilifera*, only one colony (86) was tested in the matter of feeding. Of all the workers, including both soldiers and minors, the per cent engaged in feeding was small (0.003 per cent). In this case, the soldiers did not take part at all.

3. *Regurgitating and Receiving Regurgitated Food.*

1. *Methods.* — As these two sets of activities are necessarily reciprocal, they were noted at the same time. At two-minute intervals the number of ants of each class engaged in regurgitating food was recorded, and likewise the number of each class which were receiving regurgitated food. As the ants were inclined when opportunity offered to hide in the earth, a wet sponge, to provide both drinking water for them and dampness for the atmosphere, was substituted for earth. To prevent the ants from hiding in this sponge it was wrapped in cheese cloth, the edges of which were securely sewed together so as to leave no openings. When this was done great care had to be taken to prevent the sponge from drying. This sponge was placed in one chamber and food in the other.

2. *Observations.* — Three colonies of *Pheidole pilifera* were observed in these experiments. Here, again, there were very few ants (0.01 per

cent) actively or passively engaged, and in one of the three colonies (87) there was no such activity during my observations. In each of the other two colonies the percentage engaged in this process was about equal, but as there was considerable difference in the whole number (89 and 173) of ants in these colonies, it follows that the number engaged is not closely correlated with the number of ants in the colony.

In this species I saw no soldiers either regurgitating or receiving regurgitated food. In one nest, as before stated, this activity was not seen at all; in the others, minors fed minors, 0.01 per cent giving and 0.01 per cent receiving food in each colony. In this species, then, so far as my observations go, regurgitating is limited to the minors.

4. *Licking and Being Licked.*

1. *Methods.* — These two sets of activities, being also reciprocal, were observed at the same time. In one chamber was a wet sponge covered with cheese cloth; in the other, food. Observations were made once every two minutes, and the number of ants of each class, both those licking other ants, and those being licked, was recorded separately.

2. *Observations.* — One hundred observations were made on each of three colonies. Here, too, only a few ants licked or were licked (0.2 per cent, 0.7 per cent, 0.5 per cent; and 0.2 per cent, 0.6 per cent, 0.5 per cent), and the per cents of ants so engaged in the different colonies of the species do not agree closely. In this species of Pheidole the proportion of ants engaged seems to be independent of the total number of ants in the colony.

In all cases in this species the minors were more active than the soldiers, as the latter did not lick other ants at all. In the matter of being licked, two colonies gave more minors (0.2 per cent $>$ 0 per cent; 0.6 per cent $>$ 0.4 per cent), and one, more soldiers (0.5 per cent $<$ 1 per cent).

5. *Tending the Young.*

1. *Methods.* — In this experiment it was not practicable to have earth on the floor of the nest, as the ants were inclined to make use of the earth in which to hide their young; therefore a wet sponge covered with cheese cloth was put into one of the chambers, while the food was placed in the other. At intervals of one minute a count was made of the number of ants of each class holding young in their mouths, or otherwise seeming to care for them. As an individual in possession of young frequently held them for a long time, the young, after each ob-

servation, were taken from any ants carrying them, and placed on the floor of the nest, so as to give all ants as nearly as possible an equal chance to pick them up.

2. *Observations.* — One hundred observations were made on a single colony of *P. pilifera.* The other colonies contained no young. In this colony only a few workers (2.6 per cent) were engaged in this activity, no soldiers sharing in it.

6. *Building.*

1. *Methods.* — In order to watch the ants while building, one chamber was provided with damp earth. This was placed on the floor of the chamber, and was of such depth that the ants could not wholly secrete themselves in it. Food was placed in the other chamber. As an aid in counting the ants, the pane of glass which covered the chamber containing earth was ruled into half-inch squares. Observations were made at five-minute intervals in the case of one colony (81); in all other cases the interval was two minutes. At each observation a count was made of all ants of each class which were either digging or carrying earth.

2. *Observations.* — One hundred and fifty observations were made on colony 81, and on the others (86, 87, 91) one hundred each.

Although the per cent of active ants here was somewhat greater than in the activities previously considered, still it was small (0.8, 2, 0.4, 0.2). It was not correlated with the whole number of ants in the colony.

In two colonies (87 and 91) the soldiers did not build at all, and in one of the others (86) the minors were the more active (2 per cent > 1 per cent), in the remaining colony (81) the soldiers were the more active (0.6 per cent < 1.4 per cent). Considering all the colonies together, the greater activity was decidedly on the part of the minors.

7. *Surrounding the Queen.*

As there were no queens in any of the colonies of *Pheidole pilifera*, I was obliged to omit observations in regard to this subject on this species.

8. *Guarding.*

1. *Methods.* — For the purpose of observing which individuals served as guards to protect or warn the colony in time of danger, the ants were allowed to build a nest of earth in one chamber; food was placed in the other. In order to stimulate the ants and to discover whether any individuals were stationed as guards at the entrances of the earth

nest, and if so, to which class or classes they belonged, the entrances to the nest were probed with a bristle either once every five minutes (colony 81) or once every minute (colonies 86, 87, 91). The number of ants of each class emerging from these entrances at each observation was recorded. Often the first individual to come from the entrance attacked the bristle with widely opened jaws, in some instances even clinging to the bristle. Sometimes, however, it rushed out excitedly, with open jaws, and then hurried back into the hole, whence it often emerged again, sometimes followed almost immediately by one or more other ants. In such cases the class to which these accompanying ants belonged was also recorded, though they were not considered as belonging among the original guards.

2. *Observations.* — Four colonies of *Pheidole pilifera* were experimented on by making one hundred and fifty observations on colony 81, and one hundred observations on each of colonies 86, 87, and 91.

Only a very small per cent (0 ; 0.2 ; 0.1 ; 0.3 respectively) were engaged in this activity, one colony giving no response ; moreover, the number was not correlated with the whole number of ants in the colony.

In two of the responsive colonies (86 and 87) of this species, the soldiers were not represented ; in the other they were slightly more active than the minors ($0.2 > 0.03$) ; but we may say of this species that on the whole the minors were the more active.

9. *Fighting.*

1. *Methods.* — In this activity it was impossible to deal with per cents of the colony as a whole, because when an enemy was introduced, all the ants of the colony rushed about in such a lively and excited manner that it was out of the question to count even those which attacked it. Accordingly the soldiers of the colony were placed in a Stender dish, 5.5 cm. in diameter, and an equal number of minors were placed in another Stender dish of the same size. If a minor was killed, another was put in its place, but if a soldier was killed, the number of minors was reduced by removing one of the minors, so that the number of ants in the two dishes was constantly equal. The observations consisted in counting for one-minute periods the number of ants clinging to an enemy which was placed for that length of time alternately with the soldiers and with the minors. The enemy introduced was an ant from a colony of *Camponotus americanus.*

2. *Observations.* — Fifty observations were made on each of two colonies of *Pheidole pilifera* (colonies 86 and 87). Under "Fighting," and "Carrying and Being Carried," in Table VII the first column gives

the number of observations, the second and fourth the average number of soldiers and of minors respectively used in the experiment, and the third and fifth columns the average per cents of soldiers and of minors respectively which took part in this activity.

It will be noticed that in fighting, the per cents of ants attacking an enemy are fairly large (27.9, 18.7 ; 25.6, 6.3) and that the soldiers were much more active than the minors.

10. *Carrying and Being Carried.*

1. *Methods.* — These activities occurred so rarely that I have not experimented in regard to them. However, when I have seen one ant carrying another, I have recorded the fact, as well as the class to which each belonged, and the number of each class present in the colony at the time of the observation, so that I might judge of the proportions, which would be in some degree comparable with those in the other activities. Though these observations were few, perhaps too few to make it safe to draw conclusions from them when they stand by themselves, nevertheless, in connection with other observations on the same activities, they may have some value.

2. *Observations.* — Carrying was observed to take place in only one colony (86) of *P. pilifera.* In this colony no soldiers were active in either carrying or being carried. The per cent of minors (7.3 per cent) engaged in carrying and being carried was therefore the same, and one table answers for both.

11. *Partaking of Different Kinds of Food.*

1. *Methods.* — According to the accounts of Reichenbach ('96, p. xcv) and Wheeler (:02, p. 770), it seems probable that different classes of ants have different tastes in regard to various *kinds* of food and different ways of treating it. Accordingly, while making my other observations on feeding, I have tried to discover whether one class interests itself in certain kinds of food and the other class in other kinds. To this end I recorded the class to which each ant seen at the food belonged, and also the kind of food of which it was partaking.

2. *Observations.* — One colony (86) of *P. pilifera* was observed, and in this colony only one ant (a soldier) ate, its food being a meal-worm. During these observations no ant ate grass-seed. That this species does eat grain, and even grass-seed, I feel sure from Wheeler's (:05^{d}, p. 380) observations and from my own observations in the field; moreover Wheeler states that in the grain-storing species of Pheidole, the soldiers act as seed crushers for the community.

12. *Relation of Classes to Various Activities.*

From the foregoing account it is evident that, at least under the conditions employed, both classes of *Pheidole pilifera* share in most of the activities which were examined. In treatment of seed, no activity was shown by either class. The soldiers of this species took no part in either feeding, tending the young, regurgitating, receiving regurgitated food, licking, carrying or being carried; but they were more active than the minors in fighting. In being licked, the two classes shared nearly equally, but in all other activities the minors excelled (feeding, regurgitating, licking, tending the young, building, and carrying), occupations which may be called "household duties." Carrying, in which the minors alone engaged, may be correlated with the size of the head, for the body of a minor would be better balanced when performing this duty than would that of a soldier. Guarding, though shared apparently to a greater extent by the minors, seems to be more or less accidental, and does not here consist — as in Colobopsis and a species of Pheidole recorded by Reichenbach — of blocking the entrances of the nests with the head. If it did, we should expect to find always the same sort of worker at the same opening, which is not the case in any of the species of Pheidole which I have studied. The act of guarding seems here to be rather the result of the fact that, as ants pass in and out, or are otherwise near an opening, they rush out when disturbed. Now, the soldiers are not apt to leave the nest, and hence are probably not as often near the openings, which would account for the greater number of minors found guarding.

But if the minors are pre-eminently occupied with household duties, the soldiers seem to be concerned with occupations requiring strength of jaw, such as fighting, and though the minors help them in this, there is much difference between the two classes in regard to this activity. Moreover, when in the presence of the enemy, the soldiers ran about snapping their jaws in a much more ferocious way than did the minors, and they never ran away, whereas the minors frequently did. We may, then, say that in this species the minors are, in general, more concerned with "household duties," and the majors with fighting, and, as Wheeler has shown, with crushing seed.

ii. Studies of Pheidole pilifera in Barth Nests.

1. *Methods.* — The methods here employed were exactly like those described for *Camponotus pictus* (pp. 443 et seq., 465), except for the difference in size of the nests. These nests were 5 inches in diameter,

$4\frac{3}{4}$ inches high, and had $\frac{1}{4}$ inch space between the two jars. Two colonies were under observation.

2. *Observations.* — There were neither queens nor young of *Pheidole pilifera* in the nests, and hence evidence on the activities of surrounding the queen and tending the young is necessarily wanting. Only a small part of the colony took part at any one time in any of the activities observed.

Though I several times tried the same experiment with the meal-worm which was tried in the field (p. 486), I never got a response. This experiment consisted in placing a meal-worm near the nest, and noticing whether either class broke it to pieces.

Unfortunately the ants all died before I was able to try any experiments on fighting. I had purposely left this experiment till the last, because of its destructive outcome. However, if responding to a disturbance of the nest caused by knocking on the glass or tampering with the earth at the top of the nest be any indication, it may be said that both classes respond pretty generally every time; but the soldiers seem to be much more excited than the minors by such stimuli.

I did not see any scavengers in these nests.

In regard to foraging, the minors were seen in the well and on top of the earth much oftener than the soldiers; the latter kept almost wholly underground.

No soldier was ever seen carrying in food or preparing seed, but a few minors were noticed doing both of these things, especially the carrying of food. Once, however, when a minor brought in a seed and placed it near a soldier, the latter became excited, waving its antennae in the air. Presently it left the chamber, but returned almost immediately, and for a short time remained near the seed perfectly quiet. Soon it left again, and did not return while I watched the nest; where it went I was unable to see. It did not at any time touch the seed, and the cause of its excitement may have been something wholly different. Once I saw a soldier carry a seed from one part of the nest to another; and on another occasion four small ants each tried unsuccessfully to drag a soldier toward a pile of seed. When the small ants broke up seed, they sometimes pulled at different parts of the same husk, and sometimes each attacked a seed alone, pulling off pieces of the husk with its jaws. Chaff was commonly found in the deeper parts of the nest and under the well. Though "breakfast foods" of various kinds ("Quaker" oats, hominy, rice, corn meal) were placed on the surface of the nests, I never saw the ants split these. They were carried in, however, especially the Quaker oats, but the ants seemed to much prefer grass-seed.

On only a few occasions did I see minors feeding themselves, and soldiers never; nor did I see the latter receive any regurgitated food.

The small ants were much more likely to lick other ants than were the soldiers, of which I saw only one engaged in this performance. The minors were just as much inclined to lick one class as the other.

Though I did not see many ants engaged in building, there was a division of this kind of labor between the two classes. The carrying of earth was all done by the minors, whereas all the digging observed was the work of soldiers (four individuals, not all from the same colony). The digging consisted of using the front feet, much as a dog does in digging a hole in the ground, and also in breaking off pieces of earth with the jaws. I have also seen soldiers, by squeezing their way through narrow passages in the galleries, loosen earth, which the minors then carried away.

Only minors have been seen dragging other ants, and only soldiers were being dragged. Had those which were being dragged been small enough, they probably would have been carried, for this activity seems to correspond to that which is properly called "carrying" in cases where the ants being transported are small; such, for example, as were observed in this species, both in aluminum nests and in the field, and likewise in Camponotus (p. 468).

To sum up the facts concerning the relative participation of the two classes in the various activities as far as observed in Barth nests, it may be stated that all the digging was done by soldiers, and that they were more excited in responding to disturbances of the nest, but that in all other functions they were less active, not sharing at all in some of them. There was no evidence of guarding.

iii. Out-door Work on Pheidole pilifera.

1. *Methods.*

Essentially the same methods were used in studying *Pheidole pilifera* as in that of *Camponotus pictus*, except that, owing to the distinctness of classes, it was possible to make and record final observations as to class without the necessity of killing and preserving for subsequent study so large a number of ants observed in the different activities. There are occasionally individuals intermediate in form between the extreme classes, but they are so rare as to cause no danger of confusion. I have had so little chance to observe them that I am not prepared to say just what their functions are. Frequently records were made of the number of ants of each class in a colony seen engaged in a given occupation during a certain period, usually of five minutes' duration. Sometimes ants of this species were captured and preserved in alcohol and then mounted in series, as described above for Camponotus (p. 469). Eighteen colonies were studied.

2. *Observations.*

1. *Field notes.* — In these studies, as in those on *Camponotus pictus*, certain activities were not recorded. This was due to the fact that, for the most part, these activities are carried on below ground, and hence are not to be seen without disturbing the nest.

In regard to the other activities, it appeared that here as in *Camponotus pictus* when studied in the field, the proportion of ants engaged in any activity at one time was very small in relation to the total number of ants belonging to the colony. On the whole, the minors are much more alert in all activities, it being only rarely that the soldiers take any part at all. Thus, carrying food, tending the young, carrying other ants, fighting and scavengering are functions in which only the minors take part; while in other duties (foraging, cutting up meal-worms, carrying earth, and being carried), though both classes participate, the minors exhibit the greater activity. I have never seen the soldiers eating in the field. There is, moreover, no function which belongs exclusively to the soldiers. The question is whether there is any function to which this class is exclusively adapted. Wheeler (:02, p. 770) says that in Pheidole, the soldiers of the grain-storing species crush seed, breaking up the hard shells, while those of the carnivorous species cut up the hard chitinous joints of insects. He states that, in some species, the soldiers deserve their name, while in others they are cowardly. I have never seen *Pheidole pilifera* in the field tearing seeds apart or crushing them. As to cutting to pieces large insects, I have frequently seen the soldiers pass by large insects which they found in the field, or the meal-worms which I placed near the entrances of the nests, whereas the same food was frequently carried in by the minors, either in separate bits or as a whole, by the combined action of a number of ants. Only once have I seen a soldier engaged in cutting up an insect. On this occasion I placed a meal-worm, which was dry and had been partly eaten by its comrades, near the entrance of the nest where many minors were going and coming. Almost immediately two minors began to feed on it, and others soon joined them. After five minutes, when one had eaten, it entered the nest, and almost immediately a number of other minors emerged and began to eat the meal-worm. Then they began to break off bits of the insect and to carry it into the nest. All the ants which emerged from the entrance now went to the meal-worm. After fifteen minutes the first soldier made its appearance, and with its huge jaws began to cut the meal-worm, always returning to the nest after every three or four cuts had been made. After a time the ants tried to carry the whole carcass into the nest, but found it too large, so they only succeeded in moving it to the entrance. I do not feel sure whether or not the soldier carried any pieces into the nest, but the minors certainly did, and they also broke off little bits. If the soldier carried nothing back into the nest, I am at a loss to explain why it kept returning there. This is the only time I have seen such a performance. It would seem that while the soldiers of this species sometimes cut up insects in the field, it is not very common for them to do so; but perhaps this sort of work is carried on to a greater extent within the nest.

In regard to fighting, I tried a number of times throwing down a colony of Lasius (sp ?) upon a Pheidole nest. Sometimes no fight followed, the ants of the two species not happening to meet each other; but when a struggle did ensue, the soldiers were much more sluggish than the minors, which attacked Lasius vigorously. Even when the soldiers came in contact with the enemy, they did not succeed in grasping them, though they snapped at them. Possibly the Lasius workers were too quick for them, or the jaws of the Pheidole soldiers are not adapted for grasping the legs of Lasius. However, I have already (pp. 481–482) given evidence to show that the soldiers of *Pheidole pilifera* do fight. Forel ('**74**, p. 246) describes the different manner in which ants fight with enemies larger or smaller than themselves. When the enemy is larger, the small ants try to grasp its antennae or its legs, which is just what happened when Pheidole was fighting with Camponotus. But Lasius, though slightly larger than Pheidole, was more nearly of the size of Pheidole, and the small workers were therefore better able to cope with Lasius than with Camponotus.

From the field-notes on *Pheidole pilifera*, then, it appears that the minors are, on the whole, much more active than the soldiers. The soldiers in the field appeared very sluggish, moved more slowly than the minors, and were rarely occupied.

2. *Captured Ants.* — The studies on captured specimens of *Pheidole pilifera* yielded the following results: Out of a total number of 288 ants collected in the field, around the natural nests, only four were soldiers. Of these 288 ants, 92 minors and only one soldier carried food. There were thirty-seven minors and no soldiers captured building, i. e. carrying earth out of the nests; five minors and no soldiers tending the young under conditions causing excitement; one soldier being carried, and two minors carrying. From this it is evident that,

at most, the soldiers take little part in these occupations. Of the minors which were taken empty-mouthed in the field, one was hardly half the normal size, and was very light in color.

iv. Conclusions from all Studies on Pheidole pilifera.

1. Only a small portion of the whole colony is engaged in any of the activities of the colony at any one time, whether observed in the laboratory or in the field, though in the field the proportion is larger than in the laboratory.

2. The proportion of ants engaging in any particular activity is probably not closely correlated with the number of ants in the colony, though in very large colonies more ants take part than in very small ones.

3. The size of the colony does not seem to make much difference as to kind of ants engaging in the various activities, so long as both classes are present, and there is not great difference in the sizes of the colonies. Moreover, the classes behave in essentially the same manner in this respect whether the colonies are studied in the laboratory or in the field. There are, however, more ants of each kind active in the out-door nests than in the artificial ones, and on the whole, a greater proportion of soldiers are active in the laboratory than in the field.

4. The minors were much more active than the soldiers in obtaining food for themselves.

5. No soldiers were seen regurgitating or receiving regurgitated food, but minors regurgitated to minors.

6. No individuals were seen to eat grass-seed, but a soldier ate meal-worm when caged in an aluminum nest.

7. Minors alone took part in carrying in food, with the exception of one soldier which was captured, though soldiers were occasionally seen in the field, and once a soldier was observed carrying a seed from one part of a Barth nest to another.

8. In scavengering, only minors took part.

9. No soldiers broke or crushed grass-seed, though such seed is found in the nests, and minors carry it home. Minors, however, have been seen breaking seed.

10. Both soldiers and minors were seen cutting up a meal-worm.

11. The minors were active in licking, but I saw only one soldier taking part in this occupation, this one being in a Barth nest. Both classes were licked, but the minors somewhat more frequently than the soldiers.

12. No soldiers showed any evidence of taking part in tending the

young, even in the out-door colonies when excited, but the minors were fairly active.

13. The greater activity in building was decidedly on the part of the minors, though both classes carried earth and both dug. Probably the soldiers, with their large jaws, are better fitted to do this work than they are to share in some other functions, such as tending the young, for example, and hence their relatively increased activity in this duty.

14. There was no guarding of the kind observed in Colobopsis by Wheeler and by Forel. However, when the entrances of the nests were disturbed, both classes were inclined to rush out, though the minors were somewhat the more active.

15. When confined in Stender dishes it was found: First, that the proportion of ants engaged in fighting is unusually large for both soldiers and minors. This is probably due in part to the difference of methods from those employed for the other activities, the ants being here confined in so small a space as to have a very large chance of stimulation; and in part to the nature of the activity, which is of such a character that we should expect a large proportion of individuals to share in it. Secondly, the soldiers were much more active in fighting than were the minors, and they were more successful in grasping the legs and antennae of Camponotus, while the minors seized the body hairs of their enemies. On the other hand, in the field, the minors seemed more successful in fighting with Lasius. The soldiers were much more active in responding to disturbances caused by knocking on the nest.

16. In aluminum nests no soldiers were active in carrying or being carried. In Barth nests, soldiers were dragged more commonly than minors, but did not drag or carry other ants.

17. The minors show greater activity in foraging and in all "household duties."

18. The minors are much more lively in those activities in which they share than are the soldiers.

19. There is no hard and fast line between the duties undertaken by separate classes, and in most duties all individuals are capable of sharing. There is, nevertheless, in each activity a *preponderance* of ants of a certain class taking part.

B. Studies on Other Species of Pheidole.

1. *Methods.* — Three colonies of *Pheidole dentata*, from Austin, Texas, and one of *Pheidole vinelandica*, from Highlands, New Jersey, were studied in exactly the same way as *Pheidole pilifera* in aluminum

nests and other apparatus, except for slight differences in the number of observations or in the length of time of the observations. I had no opportunity for studying these ants by the other methods used on *P. pilifera.*

2. *Observations.* — Since the conclusions for these two species in the main closely resembled those for *P. pilifera*, I have considered them merely as a control, and I shall give only the points in which they differ from *P. pilifera.* The numerical values are given in Table VII.

I. *Pheidole dentata.*

1. In *Pheidole dentata*, though the minors were more active in regurgitating, as also in receiving regurgitated food, soldiers appeared to take some part in receiving regurgitated food.

2. Occasionally soldiers were seen carrying larvae.

3. No soldiers were observed carrying other ants, and only minors were carried.

4. Only minors ate meal-worms. Both classes ate honey.

5. No soldiers and no minors were seen to crush, break, or eat grass-seed. This is what we should expect, since *Pheidole dentata* is a purely carnivorous species.

II. *Pheidole vinelandica.*

1. Surrounding the Queen. (a.) *Methods.* — This was the only species in which I had a colony containing a queen. The ants of this genus have, like other ants, a tendency to gather in groups around the queen. The number of ants of each class sharing in this grouping was noted once every five minutes. To keep the ants from secreting themselves under the sponge, thus making observation difficult, the sponge was removed. There was food in one chamber.

(b.) *Observations.* — Only 41 observations were made on the colony of *P. vinelandica*, because the queen died before further observations were possible. The proportion of workers surrounding the queen was somewhat higher (12.4 per cent) than in any of the other activities.

In this case both classes shared in the activity, but the minors were more active than the soldiers (13.4 per cent > 11.9 per cent).

2. Though the soldiers, when confined in artificial nests, *seemed* more active in feeding, the minors of this species appeared to me to be much more numerous around the natural nest in the field, even in proportion to their numbers in the colony. If more than one colony had been accessible at the time of the experiments in the artificial nest, the results with this species might have resembled those with the other species more than these records show. There are two possible expla-

nations for the apparent greater activity of the soldiers of this species in feeding: First, the amount of difference between the two classes may not be real, both being about equally concerned in this activity. Secondly, the greater activity of the minors in the field may be due to the fact that they collect more food than they need for themselves, and later regurgitate it from their crops to other individuals. In the case of colony 58 they may have taken only enough for themselves, so that they were here less active than usual, and in consequence, the soldiers appeared relatively more active. In view of the facts that the minors usually regurgitate and soldiers receive, that soldiers at best give out food only very seldom, and that soldiers of *Pheidole pilifera*, and probably also of this species, appear only rarely in the field, I am inclined toward the second explanation.

3. The soldiers were more licked than were the minors.

4. Though, as in *P. pilifera*, the minors were most concerned in tending the young, on one occasion a soldier was seen with young in its mouth.

5. In surrounding the queen, both classes took part, the minors being somewhat more active than the soldiers. This was the only colony of Pheidole in which this activity was tested.

6. Though the nature of guarding resembled that found in *P. pilifera*, the soldiers in *P. vinelandica* seemed much more active than the minors.

7. Though carrying and being carried were looked for, no cases were observed.

8. No individuals of either class crushed or broke up grass-seed.

VII. Discussion.

Having given in the historical part of this paper some account of the work of other writers on division of labor among ants, and having recorded my own observations on several species, we may now state such conclusions concerning the division of labor as seem warranted by the facts already established.

1. It will be remembered that Forel ('95, p. 146) and Escherich (:06, p. 45) state for ants in general that polymorphism goes hand in hand with division of labor. Many examples of this are cited in the historical part (II) of this paper.

Of the species which I have studied, *Camponotus americanus* and *C. herculeanus pictus* are examples of a graded series of workers extending from the smallest to the largest with no break in the line, and in many characteristics the queen closely resembles the largest worker.

Pheidole pilifera, *P. vinelandica* and *P. dentata*, on the contrary, belong to the group described by Emery ('94, p. 55) as having two classes of workers, the extremes not being connected by intermediates.

The large size of the larger workers of Camponotus, especially the great size of the head and its disproportionate width, which are correlated with greater strength of jaw, adapts them particularly for combatting enemies and for work requiring great strength. The small workers, owing to their size, are better adapted to pursuits which require agility. However, their jaws and the general proportions of their bodies probably fit them better for carrying loads, such as larvae, pupae, earth, wood, food, etc. In the workers, the position of the eyes, the depth of the head, and the shape of its posterior margin are probably caused by the greater development of the clypeus and the mandibles. But in all these structures there is a gradual transition in size from one end of the series to the other.

In *Pheidole pilifera*, however, the case is different. Though the same parts of the head that are variable in *C. herculeanus pictus* are also subject to variation in this species, there is not a graded series; there are only three well-marked conditions, a queen, a large worker and a small worker. The small worker, especially because of its weak mandibles, which are toothed, seems adapted to only the lighter work of the colony, while the soldier, with its peculiar mandibles, is better adapted for heavy work or fighting. The queen resembles in some respects the small worker, in others the large one. The characters of the queen which least resemble those of the small worker, become especially prominent in the soldier, which has, in addition, certain modifications of structure depending on the enormously developed mandibles. It seems that we have a case in harmony with the view of Emery ('94) and of Wheeler (:07[b] ; :10), that the large workers resembling the queen were the primitive form, that the small workers were developed later as the result of gradual diminution in size, and that the intermediate forms then disappeared, leaving the extremes as two separate classes, and that in addition the large workers became enormously developed along certain lines.

2. These forms, Camponotus and Pheidole, have been compared with each other for the purpose of discovering, if possible, whether there is any difference in the distribution of functions in the classes of the two genera. With this end in view a larger number of activities has been studied in each group than has hitherto been examined in any one species. Though it was found that artificial conditions made some difference in the proportion of active ants, there does not seem to be

much difference in respect to the *sort* of individuals which participate *most* in each occupation, for the results in this respect seem to agree fairly well for each species, whether studied in the laboratory or in the field. This statement must, of course, be understood as applying to such colonies only as have attained sufficient size to contain individuals of all classes. Furthermore, it was found in both Camponotus and Pheidole that in the small colonies which were used in the laboratory the larger ants did more work than they do in the field. This is probably due to a reversion on their part to queen instincts (p. 464). It was found that in both genera the investigated activities of the classes generally overlap; that is, the activities are not limited exclusively to one class, but probably all individuals share more or less in all activities. In general, however, there appears to be in certain activities a preponderance of large ants, and in certain others of small ones. For example, in *Camponotus americanus* the small individuals seem to be engaged especially in tending the young, the minors and intermediates together in building, the intermediates in foraging, and the majors in fighting. In *C. pictus* the minors are especially engaged in tending the young; the intermediates in carrying food, in foraging, and in carrying other ants; and the minors and intermediates together in regurgitating (probably), in building, and in licking other ants. In *Pheidole pilifera* there is no intermediate class, but all these duties are performed by the small workers. Moreover, when, as occasionally happens, intermediates are found, they more often approximate the size of the small than the large workers, and even the shape of the head is not wholly like that of the soldiers. In Camponotus there is a slightly greater break between the functions of the majors and those of other ants than there is anywhere else in the worker series, though there is less difference in structure between the extremes than there is in Pheidole. Nor is there as much difference in behavior between the majors and minors of Camponotus as between those of Pheidole. It therefore seems as if the two classes of smaller individuals, the minors and intermediates, taken together, share in the work which in Pheidole is accomplished by the small ants. Again, the soldiers of Pheidole and the majors of Camponotus both seem to take a more active part in fighting than do the other workers of either genus. In some species of Pheidole, however, according to Wheeler (:02, p. 770), the soldiers are cowardly and not inclined to fight. Furthermore it has been established by Wheeler (l. c.) that in Pheidole the soldiers of some species prepare insect food, or grain, so that it may be carried home or eaten. I, also, have seen some evidence of this. The small workers, as I have shown, also undoubtedly share in this. It is seldom that the soldiers of Pheidole are seen in the field, and when they are, they are usually idle.

3. It was found in *Camponotus americanus* and I have no reason to doubt it is likewise true of the other species of this genus and of the species of Pheidole which I studied, and, indeed, for ants in general) that a few individuals were much more active than the others, not only in foraging, as was proved by Lubbock ('82) for certain species, but also in building and in tending the young; and these more active ants were not all of the same size. Moreover, some of them were about equally concerned in more than one activity. This is probably due to such individual psychical differences as those to which Escherich (:06, p. 45) refers, and is in agreement with the statement of Fielde (:03[d], p. 621) that "a few individuals among ants do most of the work undertaken."

4. In both Camponotus and Pheidole, there were many more ants in the colony than were active at any one time. This was true not only in artificial nests, but also, though to a less extent, in the field. Lubbock ('82) found this limitation of activity to be very striking in his artificial nests of *Formica fusca* and Polyergus containing "*the usual slaves*," though the numbers of ants in his colonies were much smaller than those in mine. The artificial conditions, which were unavoidable in such nests as either of us have used, must evidently be responsible for the very small number of ants which were active under these circumstances, as compared with the conditions out of doors. But the proportion of active ants in my colonies was much larger than in his. Although it was found that the proportion of individuals which were active in natural nests was much greater than in the artificial nests of the laboratory, nevertheless, when compared with the size of the whole colony, the number is surprisingly small even in the natural nests. In Camponotus it is, to judge from my own observations, somewhat smaller than in Pheidole.

It was also found that the number of individuals which are active in a colony is not closely correlated with the whole number in the colony, for often more ants are found to be active in a very small colony than in one slightly larger. It is certain, however, that if a very large and a very small colony are compared, the large colony will be found to have many more individuals at work than the small one.

5. In regard to the kind of work done by individuals at different ages, it may be said that my studies on *C. herculeanus pictus* confirm the statements of Forel ('74, p. 262), Peréz (:00, p. 769), Wheeler (:07[b], p. 87), and others, that ants when they emerge from the pupal state attend especially to nursing and somewhat to other duties within the

nest, but that they do not go into the field until they have nearly reached their darkest color, and that they do not fight until they have completely attained that condition. It has been stated by Fielde (:03[c], p. 321) that if ants from different colonies, even of different species, are placed together within twelve hours after hatching, and if they touch all the other ants with their antennae during the three days following, they will live together peaceably. She further states (Fielde, :02, p. 541) that before an ant is five days old it has all its reflexes determined. The fact that ants when very young do not fight at all would account for the possibility of forming mixed colonies, as just described. Probably by the time their fighting instincts are developed, they have become so used to one another that they have no inclination to fight. The same would probably also be true of ants emerging from the pupal state under similar conditions in out-door colonies.

6. Some attention was paid to the kinds of activity in which the queens of colonies of various sizes engaged, in order that queens and workers might be compared. While the queens of those species of Camponotus which were studied are capable of bringing up colonies and of doing all the work necessary for this, it was found that in large colonies they commonly serve merely as egg-layers, except possibly in time of danger; but in small colonies, even in those which are of sufficient age to contain workers of all sizes, their instincts occasionally cause them to revert to other occupations, and to share in such activities as building and tending the young. Indeed, Wheeler (:04[a], p. 156) found a queen of Colobopsis guarding the entrances of the nest, and says (:03[c], p. 45), "So far as I was able to observe, both the virgin and the deälated females of Leptothorax behaved in all respects like the workers," and, "it should also be noted that in artificial nests consisting exclusively of female Myrmicas and Leptothorax colonies, the former behaved in nearly all respects like the workers." In *Trachymyrmex septentrionalis* he (Wheeler, :06[b], p. 99) saw old, deälated females doing work, and he also reports (:07[a], p. 741) the same fact for *Atta (Trachymyrmex) turrifex.* McCook ('79, p. 148) states that the queen of Pogonomyrmex probably assists somewhat in the nursing of the young, and may contribute something of her strength to the extension of the formicary bounds. This reversion of the queen to her earlier occupations seems to hold true not only for deälated individuals, which must work when founding a colony, but also for winged queens. That the work in the case of the latter is due to virgin instincts, seems probable from Wheeler's statements (:06[a]). The deälated and winged queens do not seem to differ in respect to those activities which were noted. Possibly if I had used some such species as *Formica consocians* (Wheeler,

:04[b]; :05[a], p. 5), the group of which *Formica rufa* is the centre (Wheeler, :05[b]), or Polyergus (Wheeler, :05[a], p. 3), — where the queen cannot bring up her first brood, but must depend for that on ants of another species, — the queens would not have been found engaged in any work; but Wheeler (:08[a], p. 65) makes the statement that even queens which cannot bring up their first brood show threptic instincts. The essential rôle of the queen, however, even in *Camponotus herculeanus pictus* and *C. americanus*, seems to be egg-laying rather than attending to any other duties, and she is, in consequence, rather passive, and is never found outside the nest except at the time of swarming. I had no opportunity to study the queen of *Pheidole pilifera*, for I have never seen a queen of this species outside the nest, and only once found one in the ground. They seem to be very timid and shy, and I have no reason to think that they take an active part in the work of the nest, even though, according to Wheeler (:02, p. 768), they can found their own colonies.

7. In regard to the class or classes which do most of the work, it is noticeable that in both species of Camponotus the two smaller classes, and in all the species of Pheidole studied the small workers, predominated in all the "household duties" and in foraging. Furthermore, they move with greater rapidity and seem in every way much more lively, except when a fight occurs. On the other hand, the large workers in both genera were relatively sluggish. When the nature of polymorphism is considered, this seems to be most reasonable, for the large workers, especially in Camponotus, are much more like the queen in structure than are the small workers. Pricer (:08) has recently shown that in Camponotus, at any rate, the large workers make their appearance in a colony only slightly before the sexual forms, while the small workers appear as the first brood, and Wheeler (:02) several years ago stated the same fact for Pheidole. As soon as this first brood has reached maturity, the queen gives over all the work to them, and thereafter is even less active than the largest workers. But when a colony has reached sufficient size to produce large workers, there are more individuals than are needed to carry on the work, and the large individuals, being less well adapted for the various occupations, come to take on not only the queen's duties, some of them laying eggs, but also approach her in instinct, becoming less employed than the smaller workers. We have already seen that it is especially in times of danger that the queen's instinct to work returns to her (Escherich, :06, p. 45; Wheeler, :05[c], p. 271), and that she then becomes more active; or, if a queen is deprived of her first brood (Wheeler, :06[a]; :10, p. 527), she begins to bring up another brood, precisely as in the first instance, pro-

vided her body still contains sufficient food-tissue. I find that the large workers and the soldiers resemble the queens in being active in time of danger, for then they come to the front and enter vigorously — in most species even more vigorously than the small workers — into the defence of the colony, and in small colonies they share more in the work.

This passivity of the large workers, except in case of need, seems to be a valuable adaptation for the safety of the colony as a whole, in that there are thus always a certain number of well-adapted individuals which are ready to put all their efforts into the defence in a sudden emergency. The other workers may be more or less spent by their occupations, but the large workers, or soldiers, as the case may be, are continually fresh and ready for the fray. Another way in which this passivity on the part of the soldiers seems a profitable adaptation is in guarding, especially in such species as Colobopsis. Were the soldiers of a lively disposition, it would be impossible for them to remain for a long time quietly keeping their heads in the entrances of the nests, and hence a much larger number of individuals would be necessary to carry on this work.

8. Males were never active in Camponotus, and none were found in Pheidole.

9. All observers seem to agree, more or less completely, that feeding in the larval stage is in some way responsible for differences of classes in ants, the chief reasons for such a view being as follows: (*a*) Such differences have been shown to be the result of feeding in both bees and termites. (*b*) When a young queen ant of a species in which the workers are polymorphic establishes a colony, she necessarily rears her workers with a small amount of food; this first brood always consists of small individuals. (*c*) Poorly nourished larvae do not starve, but pupate and become small ants. As some organs begin to develop at one stage of larval growth and others at other stages, it follows that some organs are likely to become influenced by lack of food more than others. In ants provided with meagre nourishment, nutrition is often not sufficient to stimulate certain organs to develop at all. The consequent difference in structure results in the establishment of classes. This may be seen in cases where there is a long series of intermediate forms, as well as in cases where the classes are more obviously separated from one another, for Wheeler (:**02**) has shown that there is less variation in cases where there is not a plentiful food supply. He (Wheeler, :**10**, p. 106) also states that "the grossly mechanical withdrawal by parasites of food substances already assimilated by the larva produces changes of the same kind as those which distinguish the worker from

the queen." (*d*) As Escherich (:06, p. 49) points out, if such great differences as exist between the queen and worker of Carebara had arisen suddenly (through mutation), unquestionably the destruction of the species would have taken place, for the small worker is naturally governed by a wholly different sort of conduct from the queens, a wholly different manner of feeding, of building, etc. If in a colony of Colobopsis, according to Escherich, there had never been any transitional forms between the workers proper and the soldiers, the latter might not know how to work.

There does not seem to be unanimity of opinion as to whether these differences are due to qualitative or quantitative changes in feeding. Emery is inclined to believe that qualitative feeding may play a considerable part, since it is known that this is the case in bees; but Forel ('95, :04[a]) thinks it is not safe to depend on this analogy, since it has not been proved — but only conjectured — that different forms of ants can be created by different qualities of food; ants have, moreover, neither comb nor cells, such as bees have, but must make within their own crops the differentiation of such food as they give to the larvae. It seems to me that Forel's argument need not necessarily hold, inasmuch as there may be some specialization on the part of the ants whereby the larvae are furnished by regurgitation with food of different qualities. It is not impossible that a given ant may, at least for a certain time, busy herself in feeding larvae which are to be raised as queens, while another is engaged in rearing a particular kind of worker, etc., or certain larvae may (Wheeler, :08[a], p. 50; :10, p. 103) assimilate only certain kinds of food. These are, it is true, only speculative suggestions, for which there is at present no direct evidence, so far as I know. However, I am inclined to think that quantitative feeding may alone be sufficient to account for class distinctions, as the organs which tend to develop late may fail to receive sufficient nutriment before pupating and therefore fail to develop. Though Wheeler (:00[a], pp. 27–28; :02; :10, pp. 104–107) urges that there is much ground for believing it not only possible, but probable, that definite forms may arise from differences in feeding, he found (:00[a]) that in Ponerine ants the feeding habits were such as to make it improbable that a qualitative difference of food exists. Later he (Wheeler, :07[b]; :08[a]; :10) gave reasons for believing the differences of food to be quantitative rather than qualitative, but adds, "a direct causal connection between under feeding on the one hand and ontogenetic loss or development of characters on the other hand, has not been satisfactorily established."

Granting that some kind of food difference, probably quantitative, exists, there are three principal views in regard to the amount of influence it has. The view of Spencer ('94) and his followers is that any fertilized egg may develop into a queen or worker, or any modification of either, or any transitional stage between the two, merely by a difference of feeding in the larval stage. The larvae which pupate before reaching their maximum growth become workers, the others, queens; and the several castes of workers are produced from larvae which pupate at different stages.

Emery ('94, '96, :04) and his followers go farther than this, claiming that the germ-plasm, being plastic, is capable of different degrees and kinds of development according to the stimuli (especially food) acting on the larvae. This peculiarity of the germ-plasm, which is laid down in every female egg, is somatogenic and not blastogenic, and is transmitted as an ability of germ-plasm to develop variously in the individual according to the conditions.

Finally, there are those who hold with Weismann ('92, pp. 455–497) that, since the queen and the workers are very different, the eggs producing them must contain in the germ-plasm a different "determinant" for each form. The stimulus which calls forth one or another of these forms seems to be difference of feeding in the larval state, but such feeding alone would not be a sufficient cause to produce such differences.

It seems to me that while feeding, probably only quantitative, is effective in bringing about the various castes in a species, it is a stimulus rather than a cause, for certain structures of the workers and queens of some species show that morphologically the queen and worker of a given species vary independently of each other, and the same thing is true of distinct soldier and worker castes. These facts make it appear that, as Wheeler says, "while adaptive characters in stature, sculpture, pilosity, and color must depend for their ontogenetic development on the nourishment of the larva, it is equally certain that they have been acquired and fixed during the phylogeny of the species. In other words, nourishment, temperature, and other environmental factors merely furnish the conditions for their attainment of characters predetermined by heredity." Wheeler feels that we are therefore "compelled to agree with Weismann that the characters that enable us to differentiate the castes must be somehow represented in the egg. We may grant this, however, without accepting his conception of representative units."

The worker characters are inherited, because the workers are capable of laying eggs (according to Wheeler, much more often than has been supposed), and these eggs develop into males, through which the worker characters are transmitted.

But the morphological is not the only point of view from which to regard this question. As has already been said (p. 429), polymorphism has been commonly attributed to a division of labor, and if this is so, it presupposes a difference of instinct among otherwise similar workers before structural differences arose. Such a physiological difference seems to be illustrated by the experiments of Lubbock on monomorphic forms, which have already been cited; Wheeler (:10, p. 370), indeed, states that such a division of labor probably exists in various forms and degrees among all ants with monomorphic workers. Also, Figure 19 (p. 464), based on the conditions in a graded series, seems to indicate something of the same kind, for here there seems to be a difference in activities not yet entirely correlated with size.

Reichenbach (:08) maintains that the fundaments of all the instincts must be present in each ant egg; if the offspring is a male, the female instincts do not mature. Wheeler (:06[a]) states that most, if not all, of the characters of the worker are quantitatively, but not qualitatively, different from those of the queen; i. e. the worker does not differ from the queen as a mutant, but as a fluctuating variant, which has been produced by imperfect feeding in the larval stages. The same idea is again suggested by him (Wheeler, :06[b]) when he says that the queen is the epitome of the instincts of the ant colony, etc. According to the view of Weismann, that there must be "determinants" to cause the different castes of workers, it would be difficult to account for the fact that form and function are not more closely correlated, even in species like *Pheidole pilifera*, where morphologically the classes are very distinct. If, on the other hand, we substitute for "determinants" a different potentiality of the same germ-plasm to develop into any one of several forms, according to more or less favorable conditions, we should expect, what we have seen to be the case, that while there is a greater tendency for each class to perform certain duties, there is an overlapping of the functions. Each individual seems to have potentially an instinct to perform the duties even of those classes in which, owing to structural differences, we should not expect it to take part. As it is with the queen (when the workers which normally perform certain duties are present, she does not perform those duties), so it is, to a less extent, with the workers; for though they do not take an equal part in the work to which another class is especially adapted, they do share in it to a certain extent, unless physically incapacitated. We may say, then, that while there are some differences in the duties of the different classes of workers and of the queen, all female ants have an instinct — which may become more or less latent — to perform all the duties of the colony; while the male, coming from an egg which is potentially different, is distinguished from all the female forms in structure and in those instincts which undergo development.

VIII. Summary.

The conclusions for certain parts of this paper have already been given, and need only to be referred to here. Those on Polymorphism have not been summarized; they are stated for Camponotus on pp. 432–438, for Pheidole on pp. 438–440; the summaries are to be found as follows: *Camponotus americanus* (pp. 464, 465); *Camponotus herculeanus pictus* (pp. 473, 474); *Pheidole pilifera* (pp. 488, 489); *Pheidole dentata* (p. 490); *Pheidole vinelandica* (pp. 490, 491).

In addition to these conclusions, certain other facts have been established.

1. Not only certain temperatures, but also increments of temperature are an effective stimulus for ants.

2. The fact that callow ants do not fight, may account for the possibility of placing together young ants of different species to form mixed colonies.

3. Large ants are more apt to share in the work of a very small than of a very large colony, showing in this respect a resemblance to the queen.

4. It is natural that the large workers and the soldiers should be more sluggish in their activities and motions, since they more closely resemble the queen than do the small workers.

5. This sluggishness is a valuable adaptation, in that there are thus always some ants which are not in an exhausted condition by reason of constant labors, and therefore are ready to defend the colony.

6. Inactivity is also advantageous in such forms as Colobopsis, where guarding in one position for a long time would otherwise become irksome, and more individuals would be required to perform this work.

7. In Pheidole, where intermediate classes have probably dropped out, the small ants perform all the duties which in Camponotus are done by the small and intermediate workers together.

8. The overlapping of the duties of the different classes of ants is an additional reason for believing that the classes arise from eggs with a similar potentiality.

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EXPLANATION OF PLATE

All the figures were photographed by means of a Zeiss A* objective, giving a magnification of about 9 diameters.

FIGURES 1–20. Heads of *Camponotus herculeanus pictus*.

FIGURE 1 — Queen.

FIGURES 2–20 — Workers.

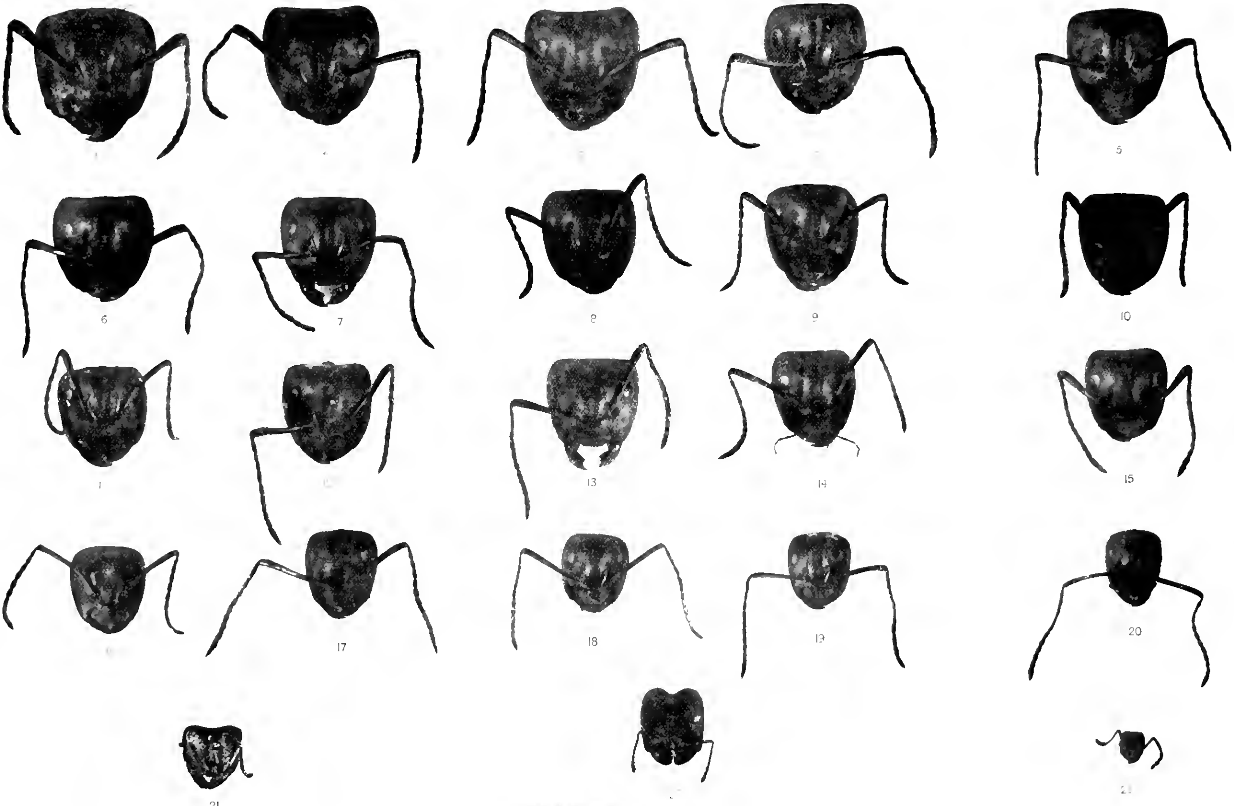
FIGURES 21–23 — Heads of *Pheidole pilifera*.

FIGURE 21 — Queen.

FIGURE 22 — Soldier.

FIGURE 23 — Minor worker.

For further description see text pp. 435 ff., 438 ff.



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CONTRIBUTIONS FROM THE CHEMICAL LABORATORY OF HARVARD COLLEGE.

A METHOD FOR DETERMINING HEAT OF EVAPORATION AS APPLIED TO WATER.

By Theodore W. Richards and J. Howard Mathews.

Investigations on Light and Heat published with Aid from the Rumford Fund.

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By Theodore W. Richards and J. Howard Mathews.

Presented by Theodore W. Richards. Received December 24, 1910.

Among usual calorimetric measurements none has been in the past less satisfactory than the measurement of heat of vaporization. The methods employed have been almost as untrustworthy as they have been numerous. The results for water are indeed not very divergent, but in most other cases there is little or no concordance. In the case of ethyl formate, for example, the variation in the values given by different experimenters is over ten per cent; in the case of ethyl acetate, the extreme difference is over thirteen per cent. These cases have been selected because they concern substances carefully investigated by many observers; less carefully studied cases might have been selected which show even greater discrepancies. Evidently most of the results are in error; but which are the correct ones? The newest values are by no means necessarily the best, for wide discrepancies have appeared in the most recent work. Part of the errors were undoubtedly due to impurity in the materials, and some to faults in the methods.

Desiring really to know the heats of evaporation of a few liquids, we felt hopeless concerning our ability to select among these discordant figures. New research was evidently needed, involving great care in purification of material, and careful choice of the best details of experimentation. The present paper contains an account of the evolution of a satisfactory method.

A brief account of previous methods may well precede the description of our own experimental work, as this was based partly on the successes and partly on the failures of others.

The methods used for measurements of heats of evaporation are of two classes. In one the energy used in the process of vaporization is measured; in the other the energy given up by the condensation of vapor is evaluated; each procedure should of course yield the same result. Most of the methods used, particularly the earlier ones, belong to the latter class; because, although those belonging to the first class have the advantage of being independent of the values for the specific heats of the substances used, they are more complicated in manipulation and calculation, and none has proved to be entirely satisfactory.

Joseph Black[1] was the first to make the interesting observation, that in the formation of the vapor phase from the liquid phase heat is absorbed. His first experiments, though extremely crude, were sufficient to show that in the case of water the amount of heat absorbed by this transformation is considerable. Somewhat later Black and Irvine made further experiments in this direction, and found the value 520 calories per gram, which is surprisingly close to the now generally accepted value, considering the crude method with which they worked. In 1781 Watt[2] for a short time attacked the subject at the suggestion of Black, and about fifteen years later returned to it and made a number of measurements, the details and results of which he published He pointed out that heat is lost through radiation, but found no method of measuring this loss. He showed also that the condenser gains heat by conduction when connected directly to the boiler by means of a metallic tube, and sought to overcome this source of error through making the connection by means of a cork, so that metallic contact was avoided. The average of eleven separate determinations gave him the value 525.2 calories, or 625.2 from zero,[3] the values varying from 612.9 to 637.1, a difference of about four per cent; but he expressed his opinion that the true value is not far from 633 calories.

Somewhat later Count Rumford[4] made three experiments concerning the same constant, finding the mean value 667 as the total heat of evaporation from zero, — a result much higher than Watt's. Rumford also determined the heat of evaporation of two or three organic liquids, but these results have no value, since, as he admits, the substances were not pure. In this work he sought to overcome the error due to radiation by starting a determination with the calorimeter water at a temperature as much below the surrounding temperature as it would be above it at the completion of the determination, on the assumption that the gain in heat during the first half of the total time

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[1] See Regnault's Experiences, Part I, p. 635 (1847).

[2] Regnault, Mém. de l'Inst. de France, **21**, 635 (1847); Robinson's Mechanical Philosophy, **2**, 5 (1822).

[3] This latter number includes the heat necessary to raise the gram of water from 0°C to 100°C.

[4] See Biot's Traité de Physique, Vol. 4, p. 710.

would be exactly counterbalanced by the loss in the second half. This practice has recently been found to be of very doubtful advantage.[5] In 1818 Ure[6] found the value 637.5 calories; and Despretz,[7] in 1823, published the value 631 as the result of one series of measurements, and 640 as the result of a second series, his apparatus having been similar to that used by Rumford. The next work was done by Brix[8] in 1842, who pointed out many of the causes of error and sought to correct them mathematically. His value for water was 640, including the heat given up by the water between 100° and 0°, — a result very close to the most probable figure.

In 1847 Regnault[9] published his exhaustive memoir upon this subject, which surpassed in detailed precaution any preceding work. As an average of thirty-eight separate determinations he obtained the value 636.67 calories, the individual values varying between 635.6 and 638.4.

The work of Andrews[10] may be considered somewhat more in detail, because his method was similar to those which have since been usually employed. He distilled the liquid from a screened retort into a spiral condenser placed in a calorimeter, and noted the rise in temperature. The water equivalent of his calorimeter, water, and utensils was only about 280 grams, the amount of water distilled was a little less than two grams, and there was no device to prevent particles of water from being carried over in the vapor, so that the method was still evidently in its infancy. His thermometric precautions and his efforts to apply corrections for heat lost and gained by radiation were very crude, as was also his method for determining specific heats; but nevertheless, for alcohol, he obtained 202.4 at the boiling point, a value which is but three per cent lower than the present accepted value. His two figures for water, 531 and 543 calories (not including the heat required to warm the water), obtained at different rates, were less satisfactory; premature condensation evidently took place in his apparatus.

Favre and Silbermann[11] shortly afterwards measured the heats of vaporization of a number of organic liquids as well as of water, but the amount of material used was very small and the method in general unsatisfactory, so that their results are of but doubtful value.

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[5] T. W. Richards and L. L. Burgess, J. Am. Chem. Soc., **32**, 449 (1910).
[6] Ure, Phil. Trans., 1818, p. 385.
[7] Despretz, Ann. Chim. Phys., (2) **24**, 323 (1823).
[8] Brix, Pogg. Ann., **55**, 341 (1842).
[9] Regnault, Mém. de l'Inst. de France, **21**, 638.
[10] Andrews, Pogg. Ann., **75**, 501 (1848).
[11] Favre and Silbermann, Ann. Chim. Phys. (3) **37**, 461 (1853).

The next work of importance is that of Berthelot,[12] whose method (a modification of Andrews's) is so well known as to need no description. The vaporizer was placed immediately over the calorimeter and heated by a ring burner, the tube for delivering the vapor passing through the ring. His apparatus, although ingenious, has grave defects, some of which have been pointed out by Schiff[13] and by Kahlenberg.[14] Superheating of the vapor, the most glaring defect, was suspected by the former critic, but the experimental demonstration was first made by the latter. Another source of error is direct radiation from the burner into the calorimeter, for which, however, an approximate although unsatisfactory correction may be applied. There was no device to prevent the vapor which came off from the liquid before it reached the boiling point from gaining access to the condenser.

Passing over the work of Schall,[15] who used the method of Favre and Silbermann, we come to the work of Robert Schiff,[16] who sought to avoid the causes for error in the Berthelot apparatus by removing the source of heat from the neighborhood of the calorimeter, and interposing a small silver trap just before the entrance of the vapor into the condenser. By this ingenious device, particles of unvaporized material that might be carried over by the current of vapor, and most of the liquid resulting from a premature condensation of the vapor, would be caught and prevented from going into the condenser. Some of the earlier experimenters had sought to prevent prematurely condensed liquid from reaching the condenser by making a sharp bend in the vapor delivery tube, so that condensed liquid would run back into the vaporizer, but Schiff's device has the decided advantage that the trap can be placed much closer to the condenser, so that the amount of condensation between this point and the condenser will be smaller. That this was a distinct improvement is apparent in his results; they were among the best that have been made. While the method of Schiff is undoubtedly better than any similar methods previously used, it is objectionable in the case of liquids of higher boiling point, as Kahlenberg has shown. The danger here arises from the fact that a substance of high boiling point condenses in the trap to such an extent that it finally overflows and runs into the condenser.

Another objection, heretofore not mentioned, lies in the fact that the vapor, just before it reaches the calorimeter, passes through a zone,

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12 Berthelot, C. R., **85**, 646.
13 Schiff, Liebig's Ann., **234**, 338 (1886).
14 Kahlenberg, Jour. Phys. Chem., **5**, 215 (1895).
15 Schall, Ber. Chem. Ges., **17**, 2199 (1884).
16 Schiff, l. c.

which, though heated, is not quite as high in temperature as the boiling point, and consequently premature condensation is not wholly avoided. By no means all of the heat thus lost finds its way into the calorimeter. The result of this defect is to give a value lower than the true heat of vaporization.

In the hands of Louguinine[17] the apparatus of Schiff has been so perfected as to make it one of the most satisfactory heretofore used. The tube leading to the trap was made large in order to prevent the clogging with condensed liquid. The proximity of the hot trap to the calorimeter necessitated a correction for heat gained therefrom through radiation and conduction. In Schiff's work this correction appears to have been omitted, but Louguinine evaluated this cause of error by keeping the trap hot while he prevented both vapor and liquid from entering the condenser. In this way the heat gained per minute by radiation and conduction can be determined fairly well, and, knowing the time during which vapor enters the condenser and gives up heat, a suitable correction can be made for the heat gained by radiation and conduction during the same period. Louguinine made the distance from the trap to the condenser very short (15 mm.) in order to reduce the premature condensation of the vapor, but our experience indicates that even then he could not have been wholly successful. The fact that his results for the heat of vaporization of water are fairly concordant does not prove the absence of error from this cause. To reduce condensation to zero, the distance between trap and condenser should also be reduced to zero, — an obvious impossibility. Because, as has been said, the effect of the error is to make the heat of vaporization appear less than it really is; Louguinine's three values, 535.61, 537.61, and 538.51 calories per gram are probably too low.

About fourteen years ago J. A. Harker[18] published an elaborate and interesting account of his work. His early rejected experiments demonstrated more conclusively than ever the danger of premature condensation, and he concluded that this cause of error is inevitable, when the vapor is introduced from above. In his later work, fearing that minute drops of water exist in vapor as ordinarily formed in distillation, he passed the vapor through coils in a heated oven, and then through a zone kept at a constant temperature about three degrees above the boiling point. Finally the vapor was admitted into the condensing coil of the calorimeter, through the side of the latter, the temperature being taken at the point of introduction by means of

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17 Ann. Chim. Phys., (7) **7**, 251 (1896).

18 Mem. Manchester Lit. and Phil. Soc., (4) **10**, 38 (1896).

a calibrated thermo-couple. The apparatus was ingenious, but the conduction of heat into the calorimeter from the entering tube must have been considerable, and heat was undoubtedly gained also by direct radiation from the black interior surface of the ebonite chamber at the opening in the side of the calorimeter.

He concluded from his experiments, which he wished to be considered as merely preliminary, that the true value for the heat of vaporization of water (not including the heat given out in cooling the water) is about 540 calories. Unfortunately, his data, corrections, and calculations, are not given.

Kahlenberg,[19] in an almost simultaneous investigation, sought to retain all the good qualities of the method of Berthelot, and at the same time endeavored to eliminate its sources of error by an ingenious modification, using the heat generated from an electric current conveyed by a resistance wire immersed under the liquid to be vaporized, in order to produce the vaporization. Thus superheating is avoided, as well as the presence of any flame near the calorimeter. This method, undoubtedly one of the best heretofore used, has yielded fairly good results, which are nevertheless undoubtedly somewhat vitiated by premature condensation.

Other methods which should be considered are the electrical methods of Marshall and Griffiths,[20] of Marshall and Ramsay,[21] of Brown,[22] and of Henning.[23]

The method of Marshall and Griffiths is exceedingly complicated, and the heat of vaporization of but one substance, benzene, has been obtained. The values were determined for the temperature of 50°C., 40°C., 30°C., and 20°C., and the value at the boiling point obtained by extrapolation. Upon this value for benzene, thus obtained, was based all the work done by Marshall and Ramsay. This latter method was merely a comparative method. The electric current was sent through two similar vaporizers in series, and the amount of liquid distilled from each was weighed. The results obtained all depended, therefore, upon the value of the heat of vaporization of one substance, and for this one substance different experimenters had found values differing by over sixteen per cent. The investigators state that they were unable to obtain any satisfactory results for water, perhaps because of electrolysis and its consequent consumption of electrical energy.

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19 Kahlenberg, Journal Phys. Chem., **5**, 215 (1895).
20 Marshall and Griffiths, Phil. Mag. (5) **41**, 1 (1896).
21 Marshall and Ramsay, Phil. Mag. (5) **41**, 38 (1896).
22 Brown, Jr. Chem. Soc., **83**, 987 (1903).
23 Henning, Ann. d. Phys. (4) **21**, 849 (1906).

Brown, working with a single apparatus, passed a current of measured value for a measured time through a wire of known resistance, immersed under the liquid. The portion evaporated was weighed; thus all the data for calculating the heat of evaporation were obtained. As in Ramsay's and Marshall's method the liquid must be at the boiling point before the measured current is passed through, but no knowledge of the specific heat of the liquid is necessary. The results obtained by Brown agree well among themselves, but are even higher in value than those obtained by Marshall and Ramsay.

F. Henning[24] used a method similar in principle to that used by Brown, but superior in detail, because greater precautions were taken to provide against possible errors. The original paper must be consulted for particulars, most of which do not immediately concern the present investigation. The greatest difficulty in this interesting and painstaking work seems to have been that considerable heat was necessarily carried away by the conducting wires, and there seems to have been no entirely satisfactory method for measuring or calculating this uncertain quantity. As the mean of his determinations made at an average temperature of 100.59°, Henning found the heat of vaporization of a gram (weighed in vacuum) of water to be 538.25 calories at 15°. A number of determinations were made also at reduced pressures, but they do not concern us here.

A modification of this electrical method has been proposed by A. Cameron Smith,[25] who suggested that the electrically heated vaporizer be suspended from one arm of an analytical balance. This apparatus is suitable only for lecture demonstrations; many possible sources of error tend to diminish its accuracy.

Among these many methods, Kahlenberg's modification of Berthelot's seemed to be preferable, partly because of its simplicity, and partly because it retains the advantage of the electrical method of heating while avoiding the disadvantages. Hence we used this method as the basis of our own, seeking to discover and correct any sources of error which might still remain in it.

The Evolution of the Present Apparatus.

One of the most serious causes of error in all calorimetric work is the more or less uncertain correction for cooling. Hence one of the first steps of the present research was the application of the new method of adiabatic calorimetry to the problem.[26] This method, first put into

[24] Loc. cit. [25] Proc. Roy. Soc., Edinburgh, **24**, 450 (1903).

[26] Richards, Forbes, and Henderson, Proc. Am. Acad., **41**, 1 (1905). See also Richards and Jesse, as well as Richards and Burgess, Jour. Am. Chem. Soc., **32**, 268 (1910) and **32**, 431 (1910).

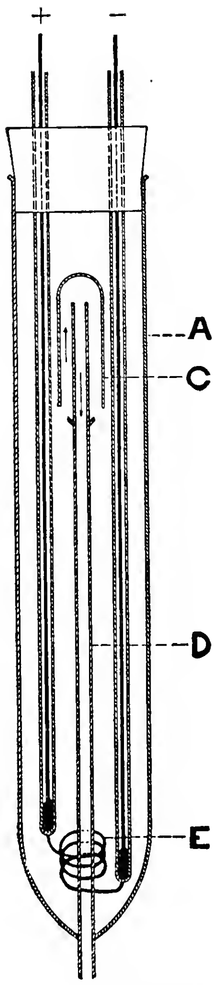

Figure 1. First Modification of Kahlenberg's Vaporizer.

The hood or trap (C) over the delivery tube helps to eliminate mechanically carried drops.

practice about six years ago at Harvard, has since been used successfully in several calorimetric processes, such as the determination of specific heats, heats of neutralization, combustion, and solution. The method consists in warming the surroundings of the calorimeter at the same rate and to the same extent as the calorimeter itself during its operation. This is accomplished by surrounding the calorimeter proper by a watertight jacket, which is immersed in a larger vessel containing dilute alkali. Into the latter can be delivered the requisite amount of acid, the bath being stirred rapidly so as to insure uniformity of temperature. By such a system it is possible to prevent heat-exchange to or from the calorimeter proper, and the thermometer is stationary during both the initial and the final readings.

For the measurements of the heat of vaporization a modified form of Kahlenberg's apparatus was combined with the adiabatic method of calorimetry. The exact form of apparatus finally used was reached only by degrees, several successive improvements having been introduced. The first modification of Kahlenberg's vaporizer consisted in the interposition of a trap or hood (C in Figures 1, 3, 4 and 5) between the boiling liquid and the condenser, to prevent drops of unvaporized material from being carried into the latter by the lively current of vapor. The objection to such a trap as that used by Schiff (namely, premature condensation of vapor) was avoided by keeping the trap entirely enclosed by the vapor of the boiling liquid, and therefore at the same temperature. Furthermore it was made in such a form as to prevent any danger of its filling and running over.

With this apparatus a number of preliminary determinations were made, with a condenser and calorimeter to be described presently, and a striking systematic irregularity was observed in the results. The heat of vaporization of benzene, for example, appeared to be about 89 calories when 25 grams took 14 minutes to evaporate, but as much as 93 calories when the time was shortened to 2.5 minutes, with corresponding results for intervening rates. The results are plotted in the accompanying diagram.

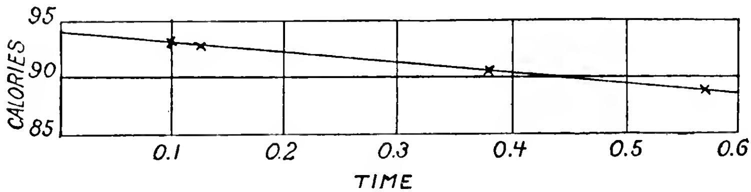

FIGURE 2. The Effect of Speed of Vaporization on the Results. (Benzene in the Vaporizer shown in Figure 1.)
In the direction of ordinates are plotted the observed values for the heat of vaporization of a gram of benzol in small calories; in the direction of abscissæ are plotted the times (in fractions of a minute) needed for the vaporization of that quantity.

The ordinates represent heats of vaporization in calories, and the abscissæ, the time in minutes required to vaporize one gram. By a comparatively short linear extrapolation we obtain the value 94.1 cal. as the heat of vaporization of benzene. It will be shown in a subsequent paper that this value is very near the most probable value obtained afterwards with better apparatus. Water was found to exhibit precisely the same phenomenon. Previous investigators have not taken into consideration this variation of result produced by varying the rate of distillation, and the oversight undoubtedly accounts for much of the wide variation in the published results.

Consideration of the various possible complications which might bring about this time-effect led to the conclusion that it was probably due to premature condensation between the vaporizer and the condensing coil, and the consequent loss of heat from the vapor thus condensed. Evidently such a loss must be directly proportional to the time required for the vapor to pass through the zone of premature condensation. Further, in accordance with Newton's Law of Cooling, the loss of heat should be proportional to the difference in temperature between the boiling point of the substance and the environment of this zone, — a conclusion later verified by the facts.

Attempts were next made to eliminate as far as possible the zone of premature condensation. The vaporizer was set into an asbestos shield made in the form of a frustum of a cone, the space between the asbestos and the glass being closely packed with cotton, and the outside of the cone covered with bright tin foil to cut down the radiation. In order to place the vaporizer as low as possible and at the same time to prevent its becoming wetted, a shallow glass cup was placed immediately below the asbestos shield. The cup also served to prevent loss of heat by evaporation of the calorimeter water at its surface, or by evaporation of water rising on the vaporizer stem by capillary action or other cause, such evaporation being caused by the presence of the hot tube carrying the vapor. In this manner the distance between the boiling liquid within the vaporizer and the calorimeter water was reduced to a little less than one centimeter.

In spite of the precautions to prevent radiation, the correction for heat gained by the calorimeter due to this cause was increased from about 0.002° to 0.008° per minute, a quantity which was very carefully determined and applied to the results.

This vaporizer yielded results which gave when plotted a line less steep than the previous one. The modification had evidently improved the results, but had not wholly eliminated the cause or causes of error.

In the space above a boiling liquid in a flask, a mist can often be observed. This mist is caused by radiation of heat from the walls of the vessel; and the question arose as to whether the difficulty in obtaining uniform results at various speeds might not be due in part to the formation of such a mist within the vaporizer. The mist would not be entirely caught by the trap, and would thus introduce minute drops of water into the condenser, possibly proportional in amount to the time required for distillation.

In order to discover whether any difficulty arose from this cause, the vaporizer, above the asbestos cone, was surrounded by a jacket through which live steam was passed throughout the measurement. This must have prevented the formation of a mist within the vaporizer; but the results were no better.

The ejection of fine drops from the surface of the boiling liquid — drops which might be swept along by the current of vapor — could not have been the cause of the particular trouble in question. Such an action would seem more likely when the boiling is rapid than when it is slow, an outcome exactly the opposite to that actually observed. Doubtless such an ejection occurs to a slight extent, but in our trap its effect had undoubtedly been much reduced. As will be seen, our final result shows that we have been at least as fortunate as others in eliminating this danger.

Harker, in his desperation on account of obvious premature condensation, resorted to superheating the vapor, in order to insure its being perfectly dry; accordingly we, too, tested this doubtful device. A coil of fine platinum wire was introduced into our vapor delivery tube, extending nearly its whole length, and a small current was sent through the wire. Several determinations made at different speeds and with different strengths of current through the coil, gave higher values than before. With a constant strength of current, the slower the speed, the higher was the result; and with a constant rate of evaporation, the greater the current the higher was the result. Evidently superheating occurred. With some modifications of the parts and the construction of special thermometers it would have been possible to take the temperature of the vapor as it entered the calorimeter; but the unsatisfactory nature of these experiments offered no temptation to the further prosecution of this line of attack.

Being still convinced that the principal premature condensation took place in the narrow zone between the vaporizer and the condenser, we next sought to reduce further this distance, and to protect the vapor passing through it from loss of heat. A new vaporizer having a vacuum jacket was constructed as shown in Figure 3, the walls (B) of the jacket being about one centimeter from the walls (A) of the vaporizer, save at the bottom where the distance was about eight millimeters. Both the outer wall of the vaporizer and the inner wall of

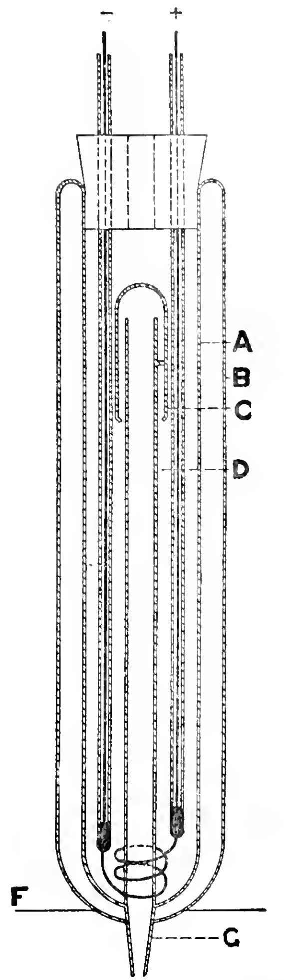

FIGURE 3. A Further Modification of Kahlenberg's Vaporizer.

The compartment containing the liquid and coiled platinum wire for heating is surrounded by a silvered vacuum jacket (AB). The trap (C) is retained. The condenser is attached at G. The vaporizer is immersed in the water of the calorimeter as far as the water-line at F.

the jacket were brightly silvered to a distance of about five centimeters above the vapor exit. The vaporizer was set so low that the level (F) of the calorimeter water came up on its jacket to a height of a few millimeters. This brought the boiling liquid in the vaporizer to within one centimeter of the calorimeter water, largely protected the vapor from condensation in this zone, and prevented most of the evaporation from the surface of the calorimeter water at the line of immersion. The vaporizer projected a short distance through the adiabatic cover, and, since the projecting part was silvered, the amount of radiation was not excessively large. The space around the vaporizer was closely packed with cotton. With this apparatus, values were obtained which were slightly higher than those previously obtained with the earlier forms of apparatus, when made at the same rate; but they were not as consistent as desired, partly because of the fact that the diameter of the exit of the delivery tube was much too large.

Another attempt to reach a better result led to the cutting off of still more of the delivery tube below, the thermal protection being provided only by a rubber cup, cut from a child's ball, placed between the vaporizer and condenser. The rubber cup was packed with cotton, as was also the space around the vaporizer. After a few determinations, however, the apparatus was discarded, because the correction for radiation and conduction was very large (0.040° per minute), and also because of the difficulty in making a tight joint.

The form of apparatus finally adopted combined all the advantages of the preceding forms, and is shown in Figure 4. The boiling liquid was surrounded by a vacuum jacket whose walls were about one centimeter from the walls of the boiling compartment, save at the bottom where the space was but five millimeters across. In addition to the hood covering the upper end of the vapor delivery tube, the tube was also provided with another trap to catch and retain any liquid that might in any way gain access thereto. This trap was placed as low as possible so that the distance between it and the condenser might be reduced to a minimum, yet it was surrounded by the boiling liquid in order to prevent condensation within it. The heating coil was placed so low as to make sure that the liquid surrounding the trap was at the boiling point. If prematurely condensed liquid now gained access to the condenser, it must come entirely from condensation in the short distance between the trap and the calorimeter water, — because all previously condensed liquid was caught in the trap. After an experiment the trap usually contained a few drops of liquid, sufficient to have introduced appreciable error, had it been allowed to reach the condenser.

The interior of the vacuum jacket was brightly silvered to a height of four or five centimeters, the silvering being on both walls, so that any heat passing from the boiling liquid to the calorimeter water by radiation had to pass through two brightly silvered surfaces and a vacuum space. The conduction of heat to the calorimeter through the glass itself cannot be prevented, but was made small by having the glass as light as was consistent with the strength demanded. The proper correction was always applied for heat gained by the calorimeter in these two ways, the necessary observations being always determined before each measurement of latent heat. For several minutes previous to admitting the vapor into the condenser, readings of the temperature were made at intervals of one minute, until the increase became constant and of certain value. The nature of the problem is such as to make this correction absolutely necessary, for the vaporizer must be brought close to the calorimeter water, and radiation and conduction across this small distance cannot be prevented.

After many experiments had been made, not only with water but also with higher boiling substances, the accidental cracking of the vaporizer jacket near the top destroyed its high vacuum; nevertheless the correction for the combined radiation and conduction was then found to be but little greater than before. Apparently the two brightly silvered surfaces effectually prevented radiation.

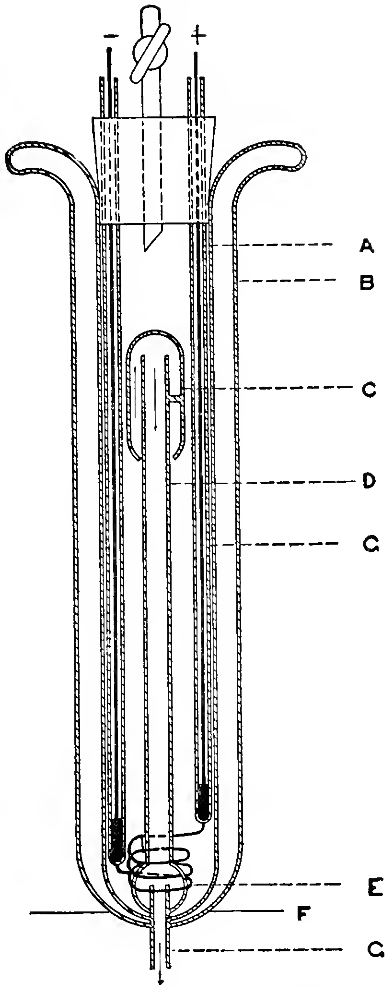

Figure 4. Final Form of Vaporizer (½ actual size).

A vacuum jacket (AB), silvered inside, surrounds the hot vessel, which is provided with a stopcock above. The delivery tube has two traps, one over its top (C), and another (E) as near as possible to the water of the calorimeter below. The condenser is attached at the very bottom at G, and F is the waterline, as in Figure 3.

The heating coil had a resistance of about 0.7 ohm, and was supplied with a suitably controlled current of from twelve to eighteen amperes from eight large storage cells. The ends of the coil were sealed into the ends of small glass tubes within which were stout copper wires, contact being made by mercury. It is necessary that the copper wires be heavy so that they may not become heated, and thus superheat the vapor coming into contact with the glass tubes encasing them.

The temperatures at which distillations took place were read from small standardized Anschütz thermometers, whose mercury threads were entirely within the vapor, so that no correction for projecting mercury thread was necessary. The bulb of the thermometer was placed opposite the entrance to the hood, in order to measure the temperature of the vapor actually admitted — a point especially emphasized by Louguinine, who nevertheless merely inferred the temperature from the barometric pressure and the coefficients expressing the dependence of boiling point on pressure.

Another point, usually neglected, is worthy of brief notice. Before the liquid comes to the boiling point, an appreciable quantity of vapor may pass over and be condensed. Obviously this may introduce error, since the assumption is made, in calculating the result, that all the vapor was at the boiling point of the liquid. Moreover, the heat of vaporization is different at different temperatures. In the present experiments this cause of error was eliminated by providing the vaporizer with an outlet and stopcock above, and by passing a very slow current of dried air backwards through the condenser coil and vaporizer until the liquid was boiling at a lively rate, and the temperature of the whole interior was quite at the boiling point of the liquid. This air prevented the vapor from passing into the condenser, so that no premature condensation was possible; its initial temperature was kept close to that of the calorimeter.

Preliminary experiments were made to find whether or not the proximity of the hot coil of platinum wire might superheat the vapor passing in its downward course to the condenser. The wire was first made into a coil about 3 cm. in diameter, and a thermometer suspended therein, the bulb of the thermometer being at the center. The coil and thermometer bulb were then immersed in distilled water in a large open test tube, the current was connected, and the temperature noted at which boiling occurred. The coil was then made about 2 cm. in diameter and the process repeated; finally the wire was coiled as tightly as possible around the thermometer bulb, without actual contact, and the temperature at which boiling took place was noted as before. It was found that there was no danger of superheating from this source, as the temperature recorded by the thermometer was exactly the same in all three cases. As long as the wire is covered with

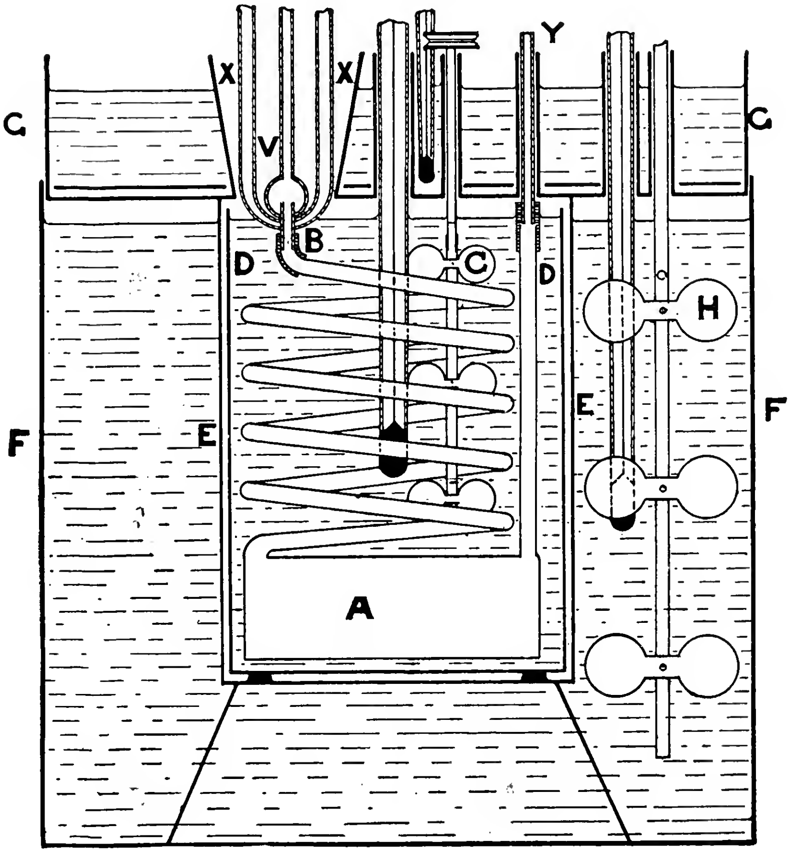

FIGURE 5. The Calorimeter.
The vaporizer (V) is set within a large hole (XX) in the cover. At B is attached the condenser A immersed in water contained in the calorimeter. Between the jacketing vessels E and F is dilute alkali, into which sulphuric acid is dropped in order that the temperature of the surroundings should keep pace with that of the calorimeter proper. C is a stirrer within the calorimeter, H one in the outside vessel.

liquid, it matters not whether the resistance wire is coiled loosely or closely around the tube through which the vapor is passing. Hence vapor passing through a tube similarly encircled by hot wire must be free from superheating.

The calorimeter, of about 1.5 liters capacity, was made of thin nickel-plated sheet copper, highly burnished on the outside. It was almost filled with water, and is shown in Figure 5 (D, D). The condenser (A) within the calorimeter was constructed of block tin, the joints being soldered by tin only; it consisted of a spiral tube, 1 meter long and 3 millimeters in internal diameter, coiled in four turns, with a tin cylinder 10 centimeters long and 3 centimeters in diameter at the bottom to serve as a receptacle for the condensed liquid. The tin cylinder had an outlet tube, leading directly up to the air of the room. The tin coil and cylinder together weighed 436.7 grams. The outlet tube or beak of the vaporizer was attached to the worm by a short piece of pure rubber tubing (B). Various preliminary experiments were made as to the position of this joint in relation to the water of the calorimeter, but the details need not be given. Finally, the arrangements shown in the figure was adopted; the silvered jacket of the vaporizer was immersed to the depth of about a centimeter under the water. The water-line was thus protected from heat by the vacuum jacket, and abnormal evaporation and cooling at this point were therefore avoided.

The stirrer (C) within the calorimeter was of the propeller pattern, having six blades, each 1 centimeter long. It was made of copper, and, in order to prevent loss of heat by conduction, extended only to the surface of the water, where it joined a shaft of hard rubber to which was attached the driving mechanism. The scarcely perceptible evolution of heat from the stirrer, being directly proportional to the time of the experiment, was included with the warming due to the proximity of the vaporizer in a single time-correction, and thus eliminated from the result.

The heat capacity of the solid parts of the calorimeter was equivalent to 53.4 grams of water, the separate parts amounting to the following quantities. The finished copper calorimetric vessel weighed 299.95 grams. It was "plated" inside and out with a thin electrolytic film of burnished nickel, which has a specific heat so near that of copper as to cause no appreciable difference in the heat capacity of the whole. The copper vessel was soldered with about 4 grams of solder, the solder consisting half of lead and half of tin, and having therefore a specific heat of about half that of copper. The calorimeter vessel with its nickel and solder was therefore approximately equivalent to the pure copper vessel, weighing 298 grams, and had a heat capacity equivalent to 27.7 grams of water, the specific heat of copper at 21° being about

0.093. It may be noted that the weight of the copper vessel need not be known within two or three grams, for this corresponds to the limit of possible accuracy of the thermometric part of the experimentation. The pure tin condenser weighed 436.7 grams, and therefore had a heat capacity equivalent to 23.6 grams of water, the specific heat of tin at 21° being about 0.054.[27] The thermometer was found by the method of Ostwald-Luther[28] to have a heat capacity equivalent to 1.4 grams of water; and the copper stirrer, weighing 7.7 grams, had a heat capacity of 0.7 on the same basis. These weights have as their sum 53.4 grams.

An important matter of detail lay in the discovery of the time needed for equalization of temperature between the hot liquid accumulating in the condenser and the water of the calorimeter. In order to test this, a glass funnel was substituted for the vaporizer at B, and 17 grams of water were poured little by little through this funnel during 5 minutes. There being no heated object near the calorimeter, the radiation-effect was negligible, and the rise in temperature of the calorimeter water was due only to the hot water introduced.

Immediately after the addition, the reading of the thermometer was a trifle over 1°; in another minute the thermometer read 1.101°; in yet another minute 1.104°. In the 3 minutes following, the thermometer rose 0.001° each minute, and finally remained perfectly constant at 1.107°. Thus in 5 minutes after the last portion of water had been added a constant temperature had been attained, showing that with the rate of stirring usually adopted, this time was sufficient for complete equalization of the temperature within the calorimeter.

The calorimeter was surrounded by a narrow air-space, bounded by a copper can (E) with a burnished nickel-plated lining. This was immersed in a much larger vessel (F) of about ten liters capacity, which contained dilute crude sodium hydroxide. The outer vessel was provided with a basin-shaped cover (G) of about four liters capacity, through which were several openings for thermometers, vaporizer (V), stirrer, etc. The bottom of the cover was coated with bright tin-foil. In this way the calorimeter was entirely surrounded by a uniform temperature, except where the vaporizer protruded through the cover. A powerful stirrer (H), revolving at the rate of 250 revolutions per minute and driven by an electric motor, kept the lower alkaline solution in violent agitation, while the solution in the cover was more gently agi-

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[27] The specific heats of copper and tin usually given (0.094 and 0.055 respectively) correspond to the range between 20° and 100°. The values given above take account of the decrease with decreasing temperature.

[28] Ostwald-Luther, Handbuch, p. 300 (1910).

tated by means of a perforated ring of heavy sheet copper, lifted by an electric motor and allowed to fall by its own weight. The stirring in the cover need not be so energetic as that below, since the temperature in this part of the bath need not be so accurately adjusted as in the lower compartment.

As the temperature in the calorimeter rose, because of the condensation of the vaporized liquid, concentrated sulphuric acid was run from burettes into both jacketing compartments, so that the environment around the calorimeter was changed in temperature as fast as the calorimeter itself. The details are so similar to those of the other applications of this adiabatic method that they need not be reviewed. The stirring was so effective that the thermometers in different parts of the bath all rose at practically the same rate.

It was found possible to follow the rising temperature of the calorimeter within 0.05° in the lower compartment and within 0.1° in the upper compartment throughout the entire course of the experiment. Suitable tests proved that with so small a difference in temperature there was no danger of appreciable heat-exchange with the surroundings during the brief progress of a determination.

The rise in temperature of the calorimeter was measured by a carefully standardized Beckmann thermometer, having a slender bulb and rather a long scale. Readings to 0.001° were made with certainty by means of a lens. The standardization was effected with very great care by comparison with two Baudin thermometers standardized by the Bureau des Poids et Mesures of Paris. The comparison was conducted by immersing all the thermometers in the water of the adiabatic calorimeter surrounded by its jacket. The calorimeter was covered tightly with a non-conducting cover through which the thermometers and stirrer passed, and the thermometer to be studied was immersed to the depth at which it was subsequently to be used. All the thermometers were placed as closely together as possible in order to insure their having the same temperature. The temperature of the calorimeter water could be held perfectly constant for any desired length of time, so that there was no error due to the lag in any of the thermometers. Readings were made (after gently tapping the thermometers) at every $\frac{1}{10}$° mark, and the corrections were calculated with due account of the corrections for the various standardized thermometers used. The Baudin thermometers had previously been found to agree very closely with an accurate Fuess thermometer, which in its turn had been carefully studied by the Reichsanstalt of Berlin.

The various parts of the apparatus having been described in detail, the actual execution of the experimental work may now be considered.

As may have been inferred, many preliminary determinations were made in order to determine the best conditions, and the various dangers to be avoided. Water, having been the liquid most studied by others, was chosen as the most desirable substance with which to test the apparatus and method.

Over thirty trials were made with the object of finding the best position of the vaporizer and testing various devices to prevent radiation.

Finally the following method of experimentation was adopted: The pan-like copper cover (G, Figure 5) of the calorimeter jacket was placed temporarily upon a raised stand which permitted easy access to its lower surface: the vaporizer (V) containing the liquid to be investigated was adjusted into its place (packing the space X between its silvered envelope and the copper cover with cotton wool), the empty weighed condenser (A) was then attached to the beak of the vaporizer (B) as it protruded beneath the elevated copper cover, and the thermometer and stirrer were arranged in their orifices. Meanwhile the calorimeter had been almost filled with a weighed amount of water at about 20°, and the jacketing crude alkaline solutions had been adjusted at exactly the same temperature. When all was ready, the copper cover with the suspended condenser was placed into position over the calorimeter, the condenser being immersed very carefully in the calorimeter water. The silvered beak of the vaporizer itself dipped about a centimeter under the surface, as shown in the diagram. The slow backward current of air, which served to prevent the access of vapor to the condenser until all was ready, was then driven through the apparatus, and at the same time the heating electrical current was turned on within the vaporizer. When the liquid was actively boiling, a careful study of the radiation-conduction-effect on the calorimeter was made. This having been accomplished, the actual experiment could begin: the stopcock (K, Figure 4) above was closed and the backward current of air discontinued. The rapid current of vapor was thus suddenly switched into the condenser, and the temperature of the calorimeter began to rise at a rapid rate. This rate was continuously matched in the environing alkali by admitting acid in suitable quantities, and the experiment proceeded very smoothly until a rise of about 4 degrees had been accomplished. Finally the stopcock (K, Figure 4) above the vaporizer was suddenly opened, the posterior outlet (Y, Figure 5) of the condenser closed, and the electric current cut off. The vapor forming was then free to escape into the air, and when boiling stopped, the condensed liquid was not drawn back into the vaporizer by the sudden vacuum formed upon cessation of boiling. For at least five minutes afterwards, the temperature of the calorimeter was read every minute with as great care as at first. After dismounting the apparatus the condensing coil was dried outside and weighed; and the experiment was thus brought to a close.

The increasing temperature and the increasing heat capacity of the calorimetric system during an experiment cause complications in the calculation which have usually received insufficient attention. At the beginning, the heat capacity of the calorimeter is that of the solid apparatus and the water in which the coil is immersed. As the experiment proceeds, this heat capacity is augmented by the liquid which collects within the condenser. At the end of the distillation the heat capacity of the calorimetric system reaches its maximum. Evidently the value used in the calculation must be taken as the initial heat capacity plus the heat capacity of half the condensed liquid.

Heat is radiated and conducted from the hot vaporizer into the calorimetric system, and correction for this unavoidable complication must be applied. The correction is primarily based upon the measurement of the heat gained during the preliminary minutes before the experiment has been begun, in the usual fashion; but it must be remembered that this value does not apply exactly to the end of the experiment, because the calorimeter has then risen in temperature, and therefore cannot take so much heat from the vaporizer as before. For example, if the temperature of the calorimeter is 20° at first and 24° at the conclusion of the experiment, and if the vaporizer has a temperature of 100°, it is evident that the difference of temperature between the vaporizer and the calorimeter is 80° at first and only 76° at the end. Hence, if the calorimeter gains 0.009° during each preliminary minute, it will be expected to gain only $76/80 \times 0.009° = 0.0085$ during each final minute, and intermediate values during the intervening period. The method of correction for this changing effect is obvious and was easily applied. This practice was justified by the actual results, for the warming effects of the hot vaporizer was always found to be less after the experiment than before. The diminution was manifest even before the vaporizer itself had cooled considerably.

The changing condition of the calorimeter involved a similar detail in the calculation of the heat given out by cooling the condensed liquid from its boiling point to the temperature of the condenser. The heat actually measured in the calorimeter was due of course not only to the heat given out by condensation, but also to that given out by the condensed liquid in falling from the temperature of the vaporizer to that of the calorimeter. The first portion of condensed liquid is cooled to the initial temperature of the calorimeter, the last portion only to the somewhat higher final temperature. Obviously here again the mean temperature must be taken in the calculation. Thus, if the experiment began at 20° and ended at 24° and the barometer indicated 760 mm., the first drop of water was cooled through 80°, the last through only 76°. Obviously here again the average value, 78°, should be taken in the calculation. It is easy to show by means of the calculus that all these compromises in the calculation are legitimate.

The amount of water evaporating into the small air space around the calorimeter during an experiment was negligible both in weight and in thermal effect.

There follows, as a typical example, the complete data and computation of a single case, the second given in the table, selected at random from results.

For five minutes preceding the experiment, the calorimeter gained at a perfectly constant rate of 0.009° per minute. When this was certain the vapor was turned into the condensing coil, and after five minutes more the electric current of 14 amperes was stopped, and the vapor prevented from gaining further access to the coil. In yet five minutes the calorimeter had settled down once more to a rate of increase in temperature corresponding to that observed at first, that is to say, 0.0085° per minute, clearly due only to radiation and condensation from the vaporizer. The heat from the actual condensation had all been imparted to the calorimeter, hence the experiment was considered as concluded. The several data and the simple calculation depending upon them are given below. The barometric pressure was exactly 760.0, hence the steam entered the condenser at 100.0°.

In calculating the heat given out in cooling a gram of water from 100° to the mean temperature of the calorimeter (21.42°) the figures of Barnes [29] were employed, because they probably represent most nearly the present standard of temperature. By graphic integration, the average specific heat of water over this range was found to be 1.0012 times the value at 21.4°; hence is calculated the value 78.67 calories above, corresponding to a fall of 78.58°. The 21° calorie is apparently about 0.9985 times the 15° calorie, hence in terms of the latter the result would be 535.2.

Six determinations were conducted in this way with this vaporizer, two of them being run very slowly and the others with increasing speed. For the slowest about 11 or 12 amperes were usually necessary, and 17 or 18 amperes were required for the fastest. The results are rearranged in the table and renumbered in order of speed, beginning with the fastest, so as to make the dependence of the results upon the

[29] Landolt and Börnstein, Tabellen, pp. 393 and 810 (Berlin, 1905).

Heat of Vaporization of Water.

Determination No. 2.

Time when Reading was taken	Observed Thermom. Reading in Water in Calorimeter (uncorrected).	Increase in Temperature per Minute.
1.21	0.090°	
1.22	0.098°	0.008°
1.23	0.107°	0.009°
1.24	0.116°	0.009°
1.25	0.125°	0.009°
1.26	0.134°	0.009°
1.27	0.143°	0.009°
1.28	0.152°	0.009°
1.29	0.161°	0.009°
	Average,	0.009°
1.29	Vapor turned into coil.	
1.34	Electric current stopped.	
1.35	5.100°	. . .
1.36	5.117°	0.017°
1.37	5.128°	0.009°
1.38	5.138°	0.010°
1.39	5.147° [30]	0.009°
1.40	5.155°	0.008°
1.41	5.164°	0.009°

	Reading of Thermometer.	Correction to reduce Reading to true Temperature.	Corrected Temperature Hydrogen Scale.
Final temperature	5.147°	+18.787	23.934°
Initial temperature	0.161°	+18.760	18.921°
Total rise in temperature			= 5.013°
Rise due to radiation and conduction (2.5 × 0.009° + 7.5 × 0.0085)			= 0.086°
Rise in temperature due solely to condensation of vapor and cooling of resulting liquid to temperature of calorimeter			= 4.927°

Water equivalent of calorimeter and fittings	53.4 grams
" " water in calorimeter	1252.9
" " ½ the liquid condensed	5.26
Total water equivalent	1311.56 grams

[30] Point at which rise of temperature due to condensation of vapor ceased. Further increase of temperature, since the rise did not exceed 0.0085° per minute, was due to radiation from vaporizer.

Weight of condensing coil and condensed liquid	447.219
Weight of condensing coil alone	436.705
Weight of vapor condensed (liquid vaporized)	10.514 grams

Total heat effect per gram $= \frac{1311.6 \times 4.927}{10.514}$ = 614.63 cals. at 21°

Heat given out by cooling 1 gram from 100.0° to 21.42° = 78.67 cals. at 21°

Uncorrected heat of vaporization of *one gram* of water = 536.0 cals. at 21°

speed more clearly manifest. The first five experiments made early in December, 1907, were all consecutive; but that numbered 6 was executed long after the others (in January, 1908), after the vacuum jacket had been used for higher boiling liquids, and had unfortunately cracked. Except for its somewhat larger warming correction, no essential difference could be detected between its results and that of the preceding experiment, carried out at the same rate. The last experiment was conducted under a pressure of 757 millimeters of mercury, the others were practically at normal pressure of 760 millimeters.

The table explains itself.

Heat of Evaporation of Water.

Series I, with Vaporizer I.

No. of experiment.	Observed weight condensed water (in air).	Time in minutes.	Time for one gram.	Rise of temperature corrected for radiation from vaporizer.	Water equivalent total.	Difference between temperature of vaporization and mean temperature.	Heat of vaporization + heat of cooling to temperature of calorimeter.	Heat of vaporization of 1 gram (uncorrected) in 21° calories.
1	8.319	3.0	0.36	3.921°	1307.6	78.9	616.3	537.4
2	10.514	5.0	0.48	4.927°	1311.6	78.6	614.6	536.0
3	8.283	4.25	0.51	3.895°	1307.8	78.8	614.9	536.1
4	9.558	8.5	0.89	4.473°	1309.3	78.6	612.6	534.0
5	8.368	9.0	1.07	3.896°	1310.2	78.3	609.9	531.6
6	8.398	9.0	1.07	3.916°	1309.4	78.4	610.5	532.1

It will be noticed that the figure in the last column falls from 537.4 calories per gram, the result of the fastest determination, to 531.8 calories, in the two slowest, — a change of about 1 per cent.

This definite march of the results seems to be referable only to the loss of heat by premature condensation in the part of the tube between the trap and the condenser, — a defect in the method which is inevitable, for the vaporizer could hardly be brought closer to the calorimeter than it was in these experiments.

When the results are plotted, the values for the heat of vaporization being laid out in the direction of ordinates, and the time required for vaporization of 1 gram laid out in the direction of abscissæ, the linear

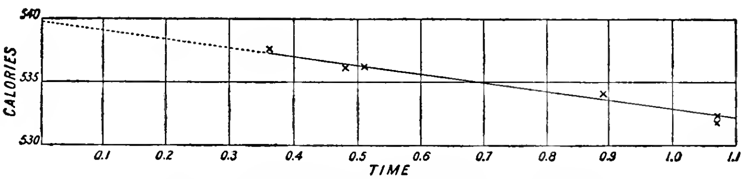

FIGURE 6. The Heat of Evaporation of Water; First Series.
Time in fractions of a minute is plotted in the direction of abscissæ, and heat of evaporation (in 21° calories) in the direction of ordinates. The dotted line is an extrapolation, giving the value for a hypothetical instantaneous experiment.

tendency of the results is manifest. The greatest departure of any single result from the straight line representing their average tendency is only 0.6 calorie, or 0.1 per cent of the total thermal quantity being measured. This corresponds to an error of thermometric reading of 0.004°. The agreement therefore is as close as could be expected.

By extrapolation to zero time the value 539.6 (cal. 21°) is obtained for the heat of vaporization of a gram of water weighed in air — a value from which the effect of premature condensation must have been eliminated, because there is every reason to believe that this disturbing phenomenon is directly dependent upon the time consumed in the experiment, and that if the experiment could be performed instantaneously the error would wholly disappear. This value becomes 538.8 in terms of the calorie at 15°.

Inspection of the curve shows that the loss of heat in 1 minute must have been 7.0 small calories from this apparatus under these circumstances, and each of the results is evidently to be reduced to a common basis by adding to it this value multiplied by the fraction of a minute required for the vaporization of 1 gram. The results, then, become respectively 539.9, 539.4, 539.7, 540.2, 539.1 and 539.6 in the mean 539.6 (cal. 21°), essentially the same as the value found by the graphic extrapolation, upon which indeed the values are directly dependent.

Feeling that it was desirable to test these conclusions in another apparatus, a new vaporizer was made, — unfortunately, however, with a somewhat wider and heavier exit tube. In this apparatus both the warming effect of the vaporizer upon the calorimeter and the loss of heat from premature condensation were greater than before — the

Heat of Evaporation of Water.

Series II, with Vaporizer II.

No. of experiment.	Observed weight condensed water (in air).	Time in minutes.	Time for one gram.	Rise of temperature corrected for radiation from vaporizer.	Water equivalent total.	Difference between temperature of vaporization and mean temperature.	Heat of vaporization + heat of cooling to temperature of calorimeter.	Heat of vaporization of 1 gram (uncorrected) in 21° calories.
7	7.435	3.0	0.40	3.484°	1309.0	78.62	613.5	534.9
8	7.170	3.0	0.42	3.376°	1306.9	78.95	615.4	536.4
9	8.740	4.5	0.52	4.091°	1308.6	78.54	612.5	534.0
10	8.625	5.0	0.58	4.034°	1308.6	78.59	612.1	533.5
11	9.040	10.0	1.11	4.190°	1310.1	78.69	607.3	528.6
12	9.150	10.0	1.09	4.250°	1307.8	78.54	607.5	529.0

former instead of being about 0.009° per minute became about 0.013, and the correction for premature condensation also increasing from 7 calories per minute to nearly 9.7 calories per minute. The determinations were also somewhat less concordant than before, as was to have been expected on account of the larger correction for radiation and conduction. Nevertheless they add valuable confirmatory evidence to the results given in the previous series, and accordingly are recorded in the following table arranged in the same way as those. Experiments 7, 8, 9, 10 and 11 were consecutive, and were made on the 7th and 8th of April, 1908. Experiment 12 was made at another time, but accords satisfactorily with 11, having about the same rate. During Nos. 7, 9 and 10 the barometer stood at 763 millimeters; during Nos. 8 and 11 at 766 millimeters, and during 12 at 762 millimeters.

On extrapolating the steeper but fairly consistent straight line drawn through these determinations, the value 539.3 (cal. 21°) is obtained for the heat of vaporization of a gram of water, a value only about 0.06 per cent lower than that found in the previous series. A part of this difference is due to the slightly higher temperature of boiling in the second series, caused by the greater pressure.

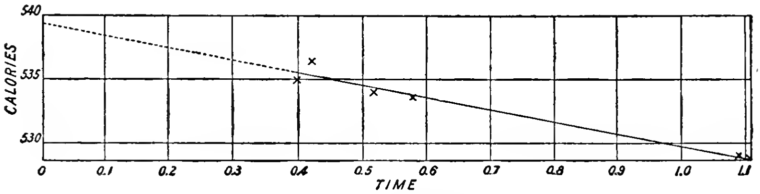

Figure 7. The Heat of Evaporation of Water; Second Series.
Time in fractions of a minute is plotted in the direction of abscissae, and heat of evaporation (in 21° calories) in the direction of ordinates. The dotted line is an extrapolation, giving the value for a hypothetical instantaneous experiment.

The individual determinations may be corrected by adding to them 9.65 multiplied by the time needed for vaporization of 1 gram, because the loss of heat is found through extrapolation to have been 9.65 small calories per minute. Corrected in this way the five determinations become respectively 538.8, 540.5, 539.0, 539.1, 539.3, on the average 539.3 as given before. The average of the results given in this series and those given in the previous series is 539.45. As the second decimal place has no significance, this may be rounded off to 539.5, giving the first series preference, partly because the pressure was more nearly normal, and partly because the apparatus was undoubtedly better and the series more concordant. This value becomes 538.7 in terms of the calorie at 15°. If the weight of the water is reduced to the vacuum standard, both of these figures are diminished by 0.11 per cent. The resulting values may be given in a brief table, together with the same quantity expressed in terms of the c. g. s. units : —

Final Result.

Latent heat of evaporation at 100° of 1.0000 gram (in vacuum) of water equals

538.9 $cal._{21°}$ or 0.5389 $Cal._{21°}$
538.1 $cal._{15°}$ or 0.5381 $Cal._{15°}$
2251 joules or 2.251 kilojoules

In all the subsequent work the first vaporizer was used, even although its vacuum had been destroyed; and it continued to give excellent results with other substances. The communication of these results must be left for a future publication, partly because the necessary specific heats of the liquids are not yet well enough known. One point, however, in connection with the results may advantageously be mentioned here, because it strengthens highly one's faith in the method of extrapolation employed to eliminate the effect of premature condensation. The loss of heat per gram was 7 calories per minute with steam in the best apparatus (Vaporizer I). With liquids of lower boiling point the loss should be less; with liquids of higher boiling point the loss should be more, if the inference is really justified. As a matter of fact precisely this phenomenon was noticed with different liquids. For example in the case of benzene, where the difference between the temperature of the vaporizer and that of the calorimeter amounted to 60° instead of to about 80°, the loss per minute amounted to about 5.2 calories per minute,[31] a figure strictly proportional to the number 7.0 found in the case of water. Again, with ethyl butyrate, boiling at 122°, the difference between the temperature of the vaporizer and that of the calorimeter was 102° instead of 80°, a fall of temperature which should correspond to a loss of heat of about 9 calories per minute, and the actual loss was found to be about 9.2 calories per minute, an amount as close as could be expected to the computed result. Moreover, in the case of methyl formate, boiling at 32°, the time of the experiment made practically no difference at all in the observed value for the heat of vaporization. Thus it seems perfectly clear that the march in the results is really due to an illicit loss of heat, and that the method employed for correcting the results is the best that can be devised. In conclusion the remark may be made that unfortunate as this inevitable difficulty with the method is, it is no more unfortunate than similar difficulties which come into any other method for determining the latent heat of vaporization. As has been more than once pointed out, the very nature of the problem renders impossible a method wholly free from some sort of correction. Even Henning's far more complicated method had its own difficulties of a somewhat similar kind, as a perusal of his paper will show.

The comparison of our value for the heat of vaporization of a gram of water weighed in vacuum, 538.9 $cal._{21°}$ (or 538.1 $cal._{15°}$), with the

[31] This figure applies only to Vaporizer I, not to the preliminary form used in the benzene series given on p. 13. In the early form the loss per minute was over 9 calories with benzene.

work of others, speaks strongly in its favor. The value given by Regnault, 536.7, is unquestionably too low. Henning's value, 538.25 $cal._{15}$ was found at 100.6°. Corrected to 100°, this would be 538.7 $cal._{15°}$, a value only a trifle above ours. Joly,[32] Harker,[33] and Smith,[34] likewise obtained values near 540.

In conclusion we are glad to express our gratitude to the Carnegie Institution of Washington, for generous pecuniary assistance.

Summary.

1. The method of Berthelot and Kahlenberg for determining heats of vaporization has been modified in such a way as to diminish greatly the errors inherent in the experimentation. A Dewar vessel was used as a vaporizer and the calorimetric work was strictly adiabatic.

2. A serious persistent disturbing effect, amounting to about 0.1 per cent of the total per minute, was eliminated by conducting experiments at different speeds and extrapolating the results to a hypothetical instantaneous experiment from which the disturbing effect may be supposed to be eliminated, because this was found to depend essentially upon time. The disturbing effect was probably premature condensation in the very narrow zone between the vaporizer and the condenser.

3. The heat of vaporization of a true gram of water was found by this method to be 538.9 $cal._{21°}$ or 2.251 kilojoules per gram. A gram molecule therefore requires 9.707 $Cal._{21°}$ or 40.54 kilojoules, when the vaporization is conducted at 100° (O = 16.000, 1 $Cal._{21°}$ = 4.177 kilojoules).

4. Comparison of this figure with the results of others shows that the method is trustworthy and suitable for general use.

5. Numerous other liquids also have been used in the apparatus, and consistent results with them have been obtained. These will be communicated in a future paper, when the specific heats of the liquids have been determined.

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[32] Phil. Trans., **186**, 322 (1895). Dependent on Joly's value for the mean calorie between 12° and 100° as determined by the steam calorimeter.

[33] Loc. cit.

[34] Loc. cit.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

THE EFFECTS OF SUDDEN CHANGES IN THE INDUCTANCES OF ELECTRIC CIRCUITS AS ILLUSTRATIVE OF THE ABSENCE OF MAGNETIC LAG AND OF THE VON WALTENHOFEN PHENOMENON IN FINELY DIVIDED CORES. CERTAIN MECHANICAL ANALOGIES OF THE ELECTRICAL PROBLEMS.

By B. Osgood Peirce.

With a Plate.

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By B. Osgood Peirce.

Presented May 11, 1910. Received December 31, 1910.

In making some kinds of electrical measurements, one occasionally needs to alter abruptly the inductances of a circuit and to inquire what the effect of the change is upon the march of the currents which the circuit is carrying. If the circuit happens to be a simple one with no magnetic metals and no other circuits near, and if the whole change takes place in a sufficiently short time, it is easy to compute the magnitude and the direction of the corresponding change in the current. If, however, the circuit is complex, or affected by the presence of other inductive circuits in the neighborhood, and if the duration of the change in inductance is long compared with the various time constants which enter, the problem may be much more difficult; though if there be no magnetic metals in the field, the principles laid down more than forty years ago by Maxwell[1] in his dynamical theory of the electromagnetic field, and soon afterwards elaborated and illustrated by Rayleigh and others, point the way to the solution.

In most cases which present themselves in practice, there are masses of magnetizable metal in the form of cores, near the circuits to be studied, and it is often difficult, even if one knows something about the magnetic properties and the history of the cores, to predict exactly what the effects of a given sudden change in the inductances will be.

[1] Maxwell, Philosophical Transactions, Dec. 1864; Rayleigh, Philosophical Magazine, **38**, 1869, **39**, 1870, **30**, 1890.

This paper discusses first, with the help of mechanical analogies, a few simple and familiar cases of circuits without cores, with the purpose of emphasizing some facts to be met with also when cores are present, and then gives a number of diagrams obtained from the photographic records of oscillographs in circuits which contained large electromagnets some with solid and some with divided cores. Changes in the inductances of a circuit which contains one or more electromagnets often involve the moving of comparatively large masses of metal, and it is obviously impossible to make such changes instantaneously even though they may be carried out in intervals which are not long relatively to the time constants of the circuit. In solid cores, also, eddy currents tend to mask the effects of sudden changes in the conformation of the circuit, and this, with the fact that the susceptibility of the iron depends not only upon the intensity of the present excitation, but also upon the past experiences of the metal, leads to considerable differences in the magnetic behavior of a circuit according as it does or does not "contain iron." The diagrams show these differences and illustrate some typical conditions which arise in practical work.

It is well known that the final flux of magnetic induction through a solid iron core is not determined by the intensity of the excitation alone even when its magnetic condition at the outset is given, but depends in many cases upon the manner of application of the given excitation, — whether it be made suddenly, by small steps at intervals, or by slow, continuous rise. The diagrams are interesting in this connection because they show that when the core is fairly well divided, the forms of two distinct portions of a current curve, interrupted by a sudden change of inductances, are often almost identical with corresponding portions of two current curves obtained without any such interruption, the one with the original inductances, the other with the final ones.

It may be well to consider briefly at the outset the very simplest case, the familiar one of a single circuit, without iron, of fixed resistance, r ohms, and of inductance originally equal to L_0 henries, which contains a constant electromotive force of E volts and is carrying at the time $t = 0$, a current of C_0 amperes. At this instant ($t = 0$) let the inductance begin to change according to some law, and progressing always in the same direction, let it attain at the time T the given value L_1, after which it shall remain constant.

In order to illustrate graphically the effect of making the given change in inductance in longer or shorter time intervals, it will be convenient to use three rectangular axes for t, C, and T respectively, and to represent the value (L) of the inductance at any time $0 < t < T$, by the expression

$$L_0 + (L_1 - L_0) f\left(\frac{t}{T}\right), \qquad (1)$$

where $f(0) = 0$, $f(1) = 1$, and for $0 < x < 1$, $f'(x) > 0$.

The line OK in the tT plane (Figure 1) has the equation $t = T$, and after the value ($T = OF$) has been fixed for T, the course of the current during the change of inductance may be shown by a curve (HQS) in a plane (FRS), the equation of which is $T = T'$. FH, which measures the ordinates of the line AG in the CT plane represents the initial current (C_0) at the time $t = 0$, and the ordinates of the curve MB in the plane COK represent for different values of T the intensities of the current at the end of the change in inductance, when the form of the function f is the same. The vertical distance AB shows the magnitude of the sudden change in the current strength when the change in L is supposed to be instantaneous and is the same whatever form f has.

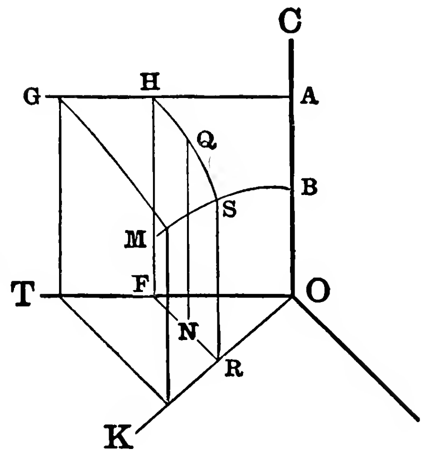

FIGURE 1.

During the interval $0 < t < T$, the current is to be found, of course, by solving the equation

$$E - \frac{d\,(LC)}{dt} = rC, \qquad (2)$$

— where L has the variable value given above — and making the constant of integration such that $C = C_0$ when $t = 0$. After the epoch $t = T$, the intensity of the current satisfies the equation

$$E - L_1 \cdot \frac{dC}{dt} = rC, \qquad (3)$$

in which the coefficients are constants. Equation (2) may be written in the form

$$\frac{dC}{dt} + \frac{r + L'}{L} \cdot C = \frac{E}{L}, \tag{4}$$

where $L' = dL/dt$, and if we make use of the usual notation,[2] and put $P = (r + L')/L$, $Q = E/L$, we shall have in general

$$C = e^{-p} (M + \int_0^t Qe^{r} dt), \tag{5}$$

where $$p = \int_0^t P \cdot dt = \log\left(\frac{L}{L_0}\right) + \int_0^t \frac{rdt}{L}. \tag{6}$$

That is, if $0 \leqq t \leqq T$,

$$C_t = \frac{L_0}{L_t} e^{-\int_0^t \frac{rdt}{L}} (C_0 + \frac{E}{L_0} \int_0^t e^{\int_0^t \frac{rdt}{L}} \cdot dt), \tag{7}$$

and, in particular,

$$C_T = \frac{L_0}{L_1} e^{-\int_0^T \frac{rdt}{L}} (C_0 + \frac{E}{L_0} \int_0^T e^{\int_0^t \frac{rdt}{L}} \cdot dt). \tag{8}$$

In this expression L is to progress always in the same direction from L_0 to L_1, and cannot pass through the value zero, so that the limit of C_T as T approaches zero has the familiar value

$$\underset{T \doteq 0}{\text{Limit}}\ C_T = \frac{L_0 C_0}{L_1}, \tag{9}$$

which might have been found directly by integrating (2) with respect to t from 0 to T; the electromagnetic momentum has no sudden change. Equation (9) follows immediately, of course, when one makes use of the usual analogies between the phenomena of ordinary mechanics and those of electromagnetism. Equation (2) is in form like the equation of motion of a system the mass of which changes with the time in a certain given manner and which is under the action of a constant accelerating force and a retarding force proportional to the velocity. Let a moving mass L grow steadily during its motion by the gradual accretion of small particles which, originally at rest, are suddenly made part of the moving system, much as the links of a fine chain which has been lying on a table are successively set in motion when one end of the chain is lifted more and more; or let the mass L decrease steadily by the loss of small particles each of which leaves

[2] Forsyth, Treatise on Differential Equations, § 14.

the system with a parting push which reduces its own velocity to zero and speeds its late companions on their way; then, if C is the velocity of the moving mass, E the accelerating force, and rC the retarding force, the equation of motion will be $d(LC)/dt = E - rC$, that is, (2).

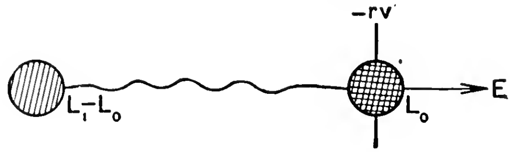

FIGURE 2.

If, when its velocity is C_0, the mass of such a system be instantaneously changed from L_0 to L_1, the principle of the conservation of momentum in impact shows that if C_1 is the velocity immediately after the impulsive change, $L_0C_0 = L_1C_1$.

The conventional diagram shown in Figure 2 indicates the nature of this simple mechanical problem. L_0 is a mass furnished with a stiff vane of such a size as to make the air resistance (which is proportional to the velocity) equal to r units when the mass is moving with unit velocity. L_0 is urged to the right by the constant force E and is retarded by a force rv. A slack inextensible string connects L_0 with another mass $L_1 - L_0$, and when the string becomes taut, the impulsive change in the velocity of L_0 corresponds to the change in the current in the inductive circuit when the inductance is impulsively changed from L_0 to L_1.

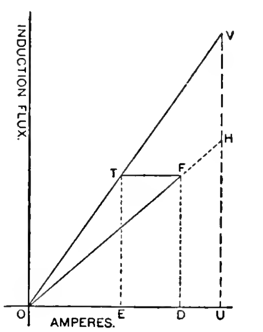

FIGURE 3. The line OFTV represents the change of the induction flux linked with a circuit without iron, when the inductance is suddenly increased.

If the induction flux, N, in a circuit which contains no iron be plotted against the current, the resulting locus is a straight line through the origin, the slope of which is the self-inductance of the circuit. If, then, the lines (OH, OV, Figure 3) corresponding to L_0 and L_1 be drawn, and if when the rising current has attained the value C_0, the inductance be supposed to change suddenly to L_1, the induction flux through the circuit preserves its value unchanged while the current falls from C_0 to C_1, and the point in the diagram which gives the state of the circuit moves from F to T.

If, as is approximately the case with some circuits which have open cores made of very finely divided soft iron, the hysteresis diagram is extremely narrow, so that the inductance may be considered to be a definite function of the current strength, we may represent two different states of the circuit by lines like OFR and OKS of Figure 4. It is then easy to see that in this case also a sudden change from one state to the other when the current had the value OD $= C_0$ would leave the induction flux through the circuit momentarily unchanged while the current fell to OE $= C_1$, and the point which represents the state of the circuit would suddenly move from F to K.

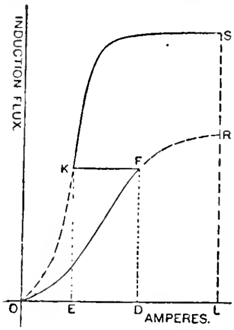

FIGURE 4. The line OFKS represents the growth of the induction flux in a magnet with finely divided core when the inductance of the circuit is suddenly increased.

If, at any instant, the total flux of magnetic induction through any simple circuit, which may or may not contain iron, is N (maxwells), if r is the resistance of the circuit in ohms, C, the current in amperes, and E, the applied electromotive force in volts,

$$E - \frac{1}{10^8} \cdot \frac{dN}{dt} = rC, \tag{10}$$

or

$$\frac{dN}{dt} = 10^8 \cdot r \left(\frac{E}{r} - C \right), \tag{11}$$

and if the final value (E/r) of the current be denoted by C', and the change in N during the time interval t_1 to t_2 by $N_{1,2}$,

$$N_{1,2} = r \cdot 10^8 \int_{t_1}^{t_2} (C' - C)\, dt. \tag{12}$$

If, now, C be plotted against the time (as in an oscillograph diagram) in a curve s (Figure 5) in which l centimeters parallel to the axis of abscissas represent one second, and an ordinate m centimeters long one ampere, the curve will have a horizontal asymptote (CY) at a distance (KC) corresponding to E/r amperes from the time axis, and, if OK represents the time t_1 and OL the time t_2, the area FGDC, or $A_{1,2}$, expressed in square centimeters, is equal to

$$lm \int_{t_1}^{t_2} (C' - C)\, dt, \tag{13}$$

and
$$N_{1,2} = \frac{10^8}{lm} (r \cdot A_{1,2}). \tag{14}$$

The curve ONJ of Figure 6, which has been carefully drawn to scale, represents the growth of the current with the time in a circuit without

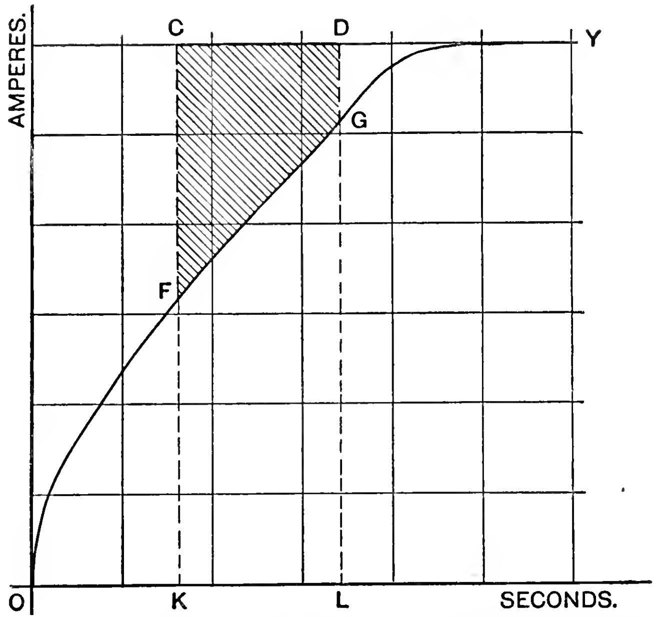

FIGURE 5. If l centimeters parallel to the horizontal axis represent one second, and an ordinate m centimeters long one ampere, $A \cdot 10^8 \cdot r/lm$ (where A is the area, in square centimeters, of CDGF) represents the change in the magnetic flux through the circuit during the interval KL.

iron, of resistance r and inductance L. The curve OPT represents the current in the same circuit when the inductance has been increased to $4L$, while the resistance is the same as before. If, when the inductance of the circuit is L, the current rises in the time OU to the value UN, and if then the inductance is instantly increased to $4L$, the current falls to UF and then rises again in the manner indicated by the curve FG, which is the curve OPT moved to the right through a distance OL just great enough to make its ordinate at the time OU equal to one fourth of UN. Since the area between the curve and its asymptote is proportional to the inductance flux through the circuit, it is clear without any of the reasoning of the preceding paragraphs, that there cannot be any impulsive change of the induction flux when the inductance is suddenly increased. A glance at the figure shows, however, that the rate of increase of the induction suddenly becomes much greater than it was just before the change.

Curve ODE of Figure 7 shows the manner of growth of the current in another simple circuit of fixed inductance, $4L$. If, at the time OW, when the current has attained half its final strength, and the induction

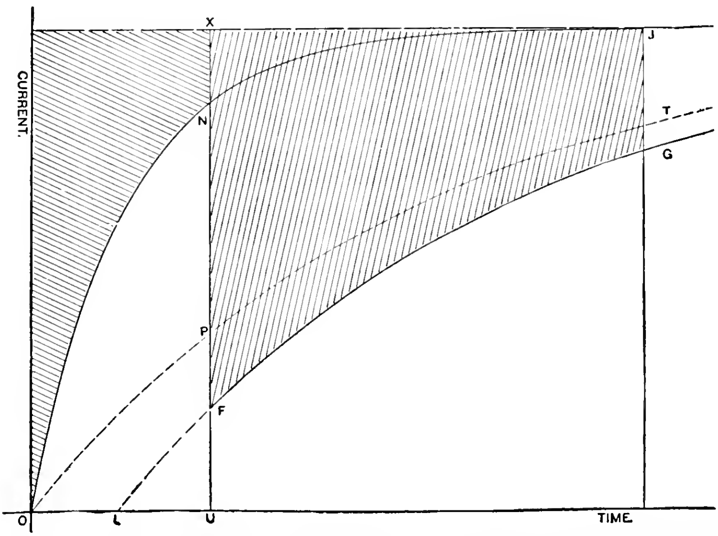

FIGURE 6. The line ONJ represents the current in a circuit of inductance L without iron. OPT shows the form of the current in the same circuit when the inductance has been increased to $4L$. ONFG is the current when the inductance is suddenly changed from L to $4L$ at the time OU.

flux through the circuit is represented on the scale indicated by equation by OABD, the inductance be suddenly changed to L, the current suddenly becomes four times as strong as it was and then falls in a manner shown by the curve ST. The flux through the circuit just after the change is already twice as large as it will be eventually when the current reaches its final value, OA, and it decreases by an amount represented by the area BST, which is half the area AODB. Just before the change the flux was increasing with the time at a rate represented by the length of the line BD; just after the change it decreases at a rate represented by the line BS, which is twice as long as DB.

In the case of a circuit which does not contain iron, an increase of inductance without an increase of the resistance usually involves a change of the conformation of the circuit, and this generally requires a considerable fraction of a second, at least, to bring about, so that the formula (9) cannot be used to determine the current strength at the end of the inductance change. To illustrate this fact we may assume that the change from L_0 to L_1 in the time T is brought about at a

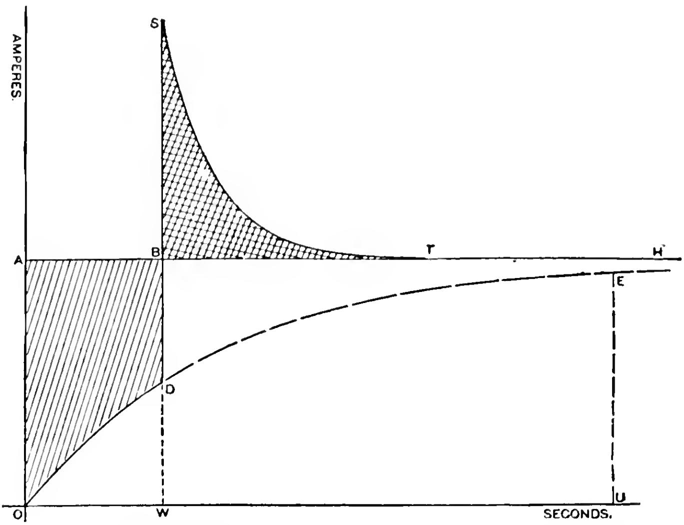

FIGURE 7. ODE shows the current in a simple circuit of fixed inductance, $4L$. If at the time OW, when the current has attained half its final intensity, the inductance is suddenly reduced to L, the course of the current will be ODBST.

constant rate so that $L = L_0 + t\,(L_1 - L_0)/T$, and the strength of the current at the time t is given by the equation

$$C = \left(\frac{L_0}{L}\right)^m \cdot \left(C_0 + \frac{ET}{m\,(L_1 - L_0)} \cdot \frac{L^m - L_0^{\,m}}{L_0^{\,m}}\right), \tag{15}$$

where

$$m = \frac{rT + L_1 - L_0}{L_1 - L_0}. \tag{16}$$

If, now, $C = E/r$ amperes, $L_0 = 2$ henries, $L_1 = 4$ henries, then, according as T is one second, half a second, one tenth of a second, or one hundredth of a second, the value (C_1) of the current at the end of the interval T is $0.980 \cdot C_0$, $0.962 \cdot C_0$, $0.836 \cdot C_0$, or $0.569 \cdot C_0$, whereas C_1

would be $0.5 \cdot C_0$, if the change in the inductance had been instantaneous.

Figure 8 shows in TW the relative changes in the current in this circuit from $t = 0$ to $t = T$, when T is one tenth of a second, and in TZ the changes when T is one one-hundredth of a second. If the change were instantaneous the course of the current in one tenth of a second would correspond to the line TRU.

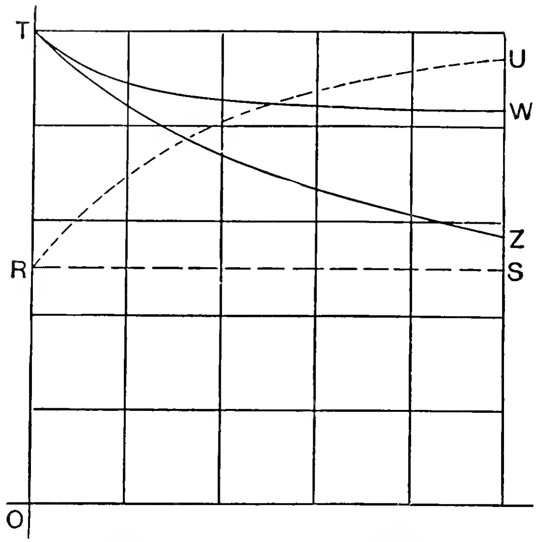

FIGURE 8. If in a certain inductive circuit, without iron, the inductance be instantaneously doubled, the course of the current in the next tenth of a second will be TRU. TW shows the current if the doubling be brought about by a continuous change going on at a constant rate during the whole interval. TZ shows on a different time scale, the course of the current for a hundredth of a second, if during this interval the inductance be changed at a constant rate which results at the end in its being doubled.

We may next consider the somewhat less simple circuit indicated in Figure 9_a, consisting of three parallel branches each of which has self-inductance, but no two of which have mutual inductance. Let r, r_1, r_2 be the resistances of the branches, L, L_1, L_2 their inductances, E, E_1, E_2 the constant electromotive forces of the generators in them, and C, C_1, C_2 the currents. At the time $t = 0$, when the currents and the inductances have given values, let the inductances begin to change according to given laws each of which can be expressed by an equation similar to (1), and let them attain, at the time T, other given values, which they thereafter keep. It is evident that any instant during the interval $0 < t < T$,

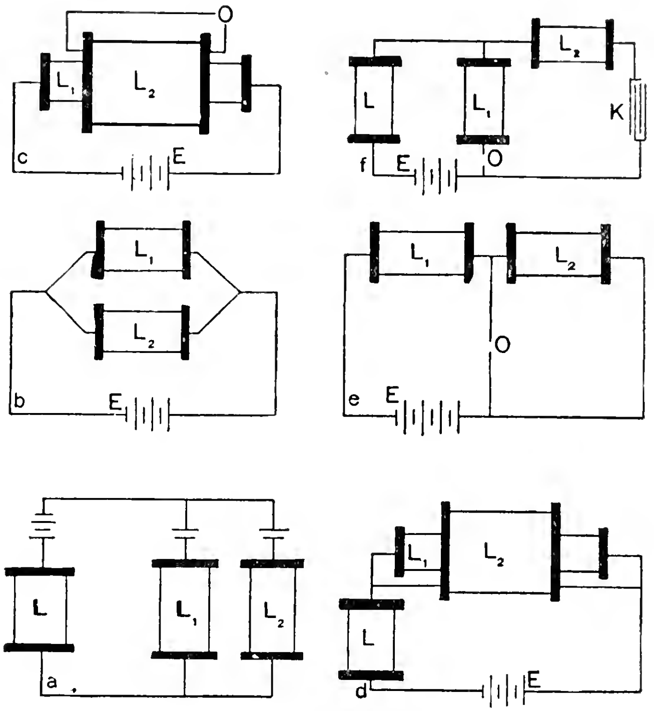

FIGURE 9.

$$\left.\begin{aligned} E + E_1 - \frac{d\,(LC)}{dt} - \frac{d\,(L_1C_1)}{dt} &= rC + r_1C_1, \\ E + E_2 - \frac{d\,(LC)}{dt} - \frac{d\,(L_2C_2)}{dt} &= rC + r_2C_2, \\ C = C_1 + C_2, & \end{aligned}\right\} \qquad (17)$$

or, if we represent differentiation with respect to the time by accents,

$$\left.\begin{aligned} (L + L_1)\,C'_1 + LC'_2 + (L' + L'_1 + r + r_1)\,C_1 + (L' + r)\,C_2 &= E + E_1, \\ LC'_1 + (L + L_2)\,C'_2 + (L' + r)\,C_1 + (L' + L'_2 + r + r_2)\,C_2 &= E + E_2. \end{aligned}\right\} \qquad (18)$$

If, from these equations and others obtained by differentiating them with respect to the time, C_2 and its derivatives be eliminated, we shall get a differential equation of the second order for C_1 in which the inductances and their derivatives are known functions of t, and the initial values of C_1 and C'_1 are also known. This new equation may be found by equating to zero the determinant

$$\begin{vmatrix} L & 2L'+r & L'' & (L+L_1)C'''_1+(2L'+2L'_1+r+r_1)C''_1+(L''+L''_1)C_1 \\ L+L_2 & 2L'+2L'_2+r+r_2 & L''+L''_2 & LC'''_1+(2L'+r)C''_1+L''C_1 \\ 0 & L & L'+r & (L+L_1)C'_1+(L'+L'_1+r+r_1)C_1-E-E_1 \\ 0 & L+L_2 & L'+L'_2+r+r_2 & LC'_1+(L'+r)C_1-E-E_2 \end{vmatrix} \quad (19)$$

and, although it may be somewhat simplified, it generally proves rather intractable. If, however, the interval T is so short that the changes in the inductances may be regarded as impulsive, the corresponding changes in the currents may be found immediately, for if the equations be integrated with respect to the time from $t=0$ to $t=T$, and if T be made to approach zero, while the currents remain finite, it appears that $LC+L_1C_1$ and $LC+L_2C_2$ have the same values just after the impulsive change in the inductances as they had just before the change. The induction flux through each circuit chosen for the equations remains unchanged by the sudden change of inductances.

It is easy to find a number of different problems in mechanics each of which yields equations of motion of the form (17), and is, therefore, analogous in a sense to the electromagnetic problem under consideration. Such an analogy, even though it be difficult to embody it in a working model, sometimes makes clearer to a person already familiar with mechanical principles the nature of the phenomena which he is to look for in interpreting his electrical equations. It will do no harm if, in imagining a mechanical system which is to serve this purpose, we postulate the existence o. flexible, inextensible, massless strings, or even, at a pinch, the existence of stiff, nearly massless rods, or of pulley wheels so light that their moments of inertia shall be negligible. It is often desirable to imagine the motions of the masses which in the mechanical system represent the inductances in the electrical problem, to be hindered by retarding forces proportional to the velocities, to represent the electrical resistances. The resistance which the air offers to a body moving through it with a constant velocity not greater than 50 cms. per second is very nearly proportional to that velocity; and since the velocities which in the mechanical case correspond to the currents are usually much smaller than that, the resistance may be sufficiently well indicated by thin wings or vanes of proper size attached to the masses.

In the arrangement shown in Figure 10 the masses L, L_1, L_2, are urged towards the bottom of the diagram by forces of intensity E, E_1, E_2. The lines drawn across the masses indicate wings of such shapes as to make the resistances due to the air r, r_1, r_2 dynes respectively, when the corresponding velocities are one centimeter per second. It is evident from the geometry of the figure that the velocity of L downward is equal to the sum of the velocities of L_1 and L_2 upward. The tension of the string attached to L and passing over the massless pulley A is at every instant half that of the cord which is attached to the massless pulley B, and equal to the tension of the cord which connects L_1 and L_2. The equations of motion of the masses are of the form (17). If, as a consequence of applied forces or impulses, the string should become slack, the analogy between the mechanical and the electromagnetic problems would disappear, and it is sometimes convenient to imagine the masses attached to taut endless strings in some such manner as is shown in Figure 11. It is very easy to construct a model of this kind which will work fairly well if one uses for masses properly loaded roller skates which move about on the level top of a table. The masses may be connected by fine catgut passing around small, cheap pulleys with vertical axes mounted on the table.

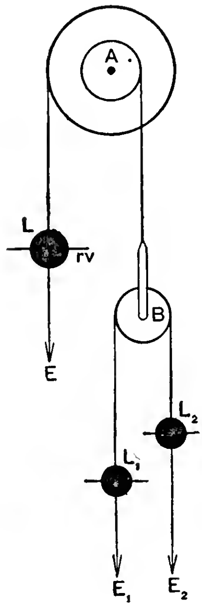

FIGURE 10.

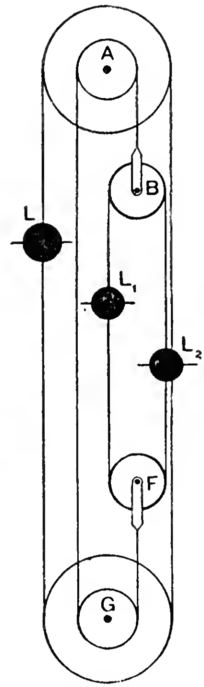

FIGURE 11.

A special case of some practical interest is that indicated in Figure 9_b, where the terminals of a battery without sensible self-inductance are connected by two inductive branches in parallel. The currents are given by the equations

$$L_1 L_2 \frac{d^2 C_1}{dt^2} + [(r + r_1) L_2 + (r + r_2) L_1] \frac{dC_1}{dt} + (r_1 r_2 + r r_1 + r r_2) C_1 = r_2 E \quad (20)$$

$$L_1 L_2 \frac{d^2 C_2}{dt^2} + [(r + r_1) L_2 + (r + r_2) L_1] \frac{dC_2}{dt} + (r_1 r_2 + r r_1 + r r_2) C_2 = r_1 E,$$

and it is clear that if the inductances are suddenly changed, the products $L_1 C_1$ and $L_2 C_2$ are continuous, and if, in particular, only one of the inductances is altered, the current in the parallel branch is itself

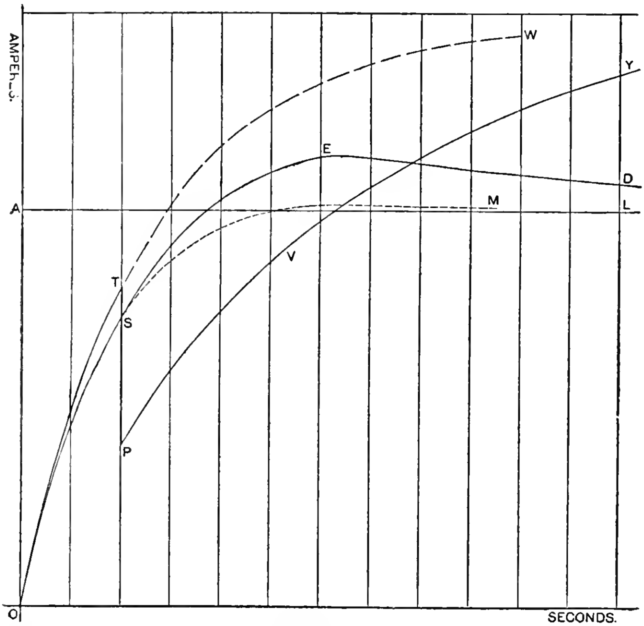

FIGURE 12. The lines OTW, OSM, show the forms of the currents in two parallel inductive resistances which connect the terminals of a storage battery. When at a given instant, the inductance of one of the parallel branches is suddenly doubled, the current in it changes its value abruptly and takes the course OTPVY, while the current in the other (OSED) suffers no sudden change in strength.

continuous. Figures 12 and 13 are drawn to scale for two typical cases which indicate well enough what is usually to be expected. In both diagrams $L_1 = 1$, $L_2 = 1$, $r = 12$, $r_1 = 20$, $r_2 = 30$, $E = 120$ (or these quantities are to be in the proportions here given). The final values of C_1 and C_2 are 3 amperes and 2 amperes.

In the case which corresponds to Figure 12 the battery circuit is closed at a given instant, and 0.02 seconds afterwards, when C_1 has attained the value 1.607 and C_2 the value 1.457, L_1 is suddenly changed from 1 to 2. As a consequence, C_1 falls suddenly to 0.8035, while C_2 remains momentarily unchanged. Before the change, the currents were given by the equations

$$C_1 = 3 - \tfrac{120}{104}\, e^{-24t} - \tfrac{120}{65}\, e^{-50t}, \quad C_2 = 2 + \tfrac{120}{156}\, e^{-24t} - \tfrac{360}{130}\, e^{-50t}, \qquad (21)$$

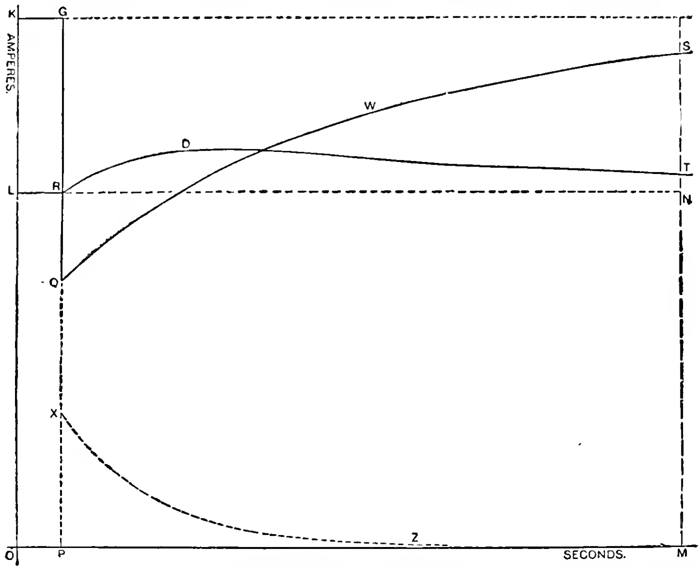

FIGURE 13. After the currents in two parallel inductive resistances which connect the terminals of a storage battery have become steady, at the values OK, OL, the inductance of one of the branches is suddenly doubled so that the current in it takes the course KGQWS. The current in the other branch takes the continuous form LRDT and approaches its final value from above.

and afterwards by the approximate equations

$$\begin{aligned} C_1 &= 3 - 1.912\, e^{-13.48t} - 0.284\, e^{-44.52t}, \\ C_2 &= 2 + 0.803\, e^{-13.48t} - 1.346\, e^{-44.52t}. \end{aligned} \qquad (22)$$

The line OTPVY shows the course of C_1, and OSED the course of C_2. It will be observed that C_2 approaches its final value from above.

If the change in inductance is made after the currents have attained their final values, the courses of C_1 and C_2 will be those indicated in Figure 13 by the lines KGQS and LRDT. If after the currents have reached their steady values, the main circuit be suddenly broken, C_1

and C_2 instantly acquire equal and opposite values, and the subsequent course of C_1 is given by the equation

$$C_1 = \frac{E(L_1 r_2 - L_2 r_1)}{r_1 r_2 + r r_1 + r r_2} e^{-\frac{r_1+r_2}{L_1+L_2}t}. \tag{23}$$

See in Figure 13 the line GRXZ.

If the terminals of an open battery circuit of inductance L and of resistance r be connected by a number of inductive conductors in parallel, of resistances r_1, r_2, r_3, r_4, etc., and of inductances L_1, L_2, L_3, L_4, etc., and if sudden changes be made in the inductances, the quantities

$$LC + L_1C_1,\ LC + L_2C_2,\ LC + L_3C_3,$$

etc., will be continuous. If L is negligible, and if only some of the other inductances be impulsively changed, the currents in the other branches will be continuous.

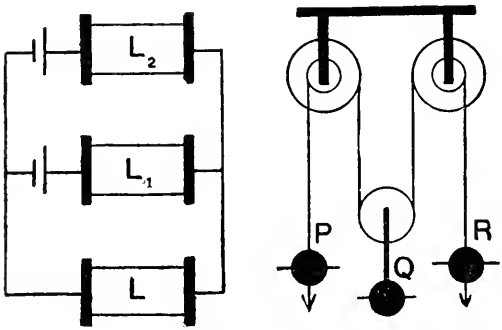

FIGURE 14.

If, in the arrangement shown in Figure 14, the masses P, Q, R are numerically equal to L_1, L, L_2, respectively, if the velocities of P and R in the direction of the bottom of the page are C_1, C_2, and if the dimensions of the vanes attached to the masses are such that the air offers resistance of r_1, r, r_2 times the velocities to the motion of P, Q, and R, the equations of motion of the masses are identical with the current equations for the electrical circuit shown in the figure.

The currents in two neighboring circuits (Figure 9_c) of self-inductances L_1, L_2, and mutual inductance M, which contain the electromotive forces E_1, E_2, are given by the familiar equations

$$\begin{aligned} E_1 - L_1\frac{dC_1}{dt} - M\frac{dC_2}{dt} - r_1C_1 = 0, \\ E_2 - L_2\frac{dC_2}{dt} - M\frac{dC_1}{dt} - r_2C_2 = 0, \end{aligned} \tag{24}$$

and any impulsive changes in the inductances cause such sudden changes in the current as will keep $L_1C_1 + MC_2$ and $L_2C_2 + MC_1$ momentarily unchanged.

Many different working models have been made to illustrate the simple electrical problems which concern two such circuits. Of these some of the best known are due to Maxwell, Rayleigh, J. J. Thomson, Webster, and Boltzmann.

The original model of Maxwell, now in the Cavendish Laboratory, is represented by Figure 15_a, taken from Gray's Absolute Measurements in Electricity and Magnetism, where an excellent account of the apparatus and its theory may be found.

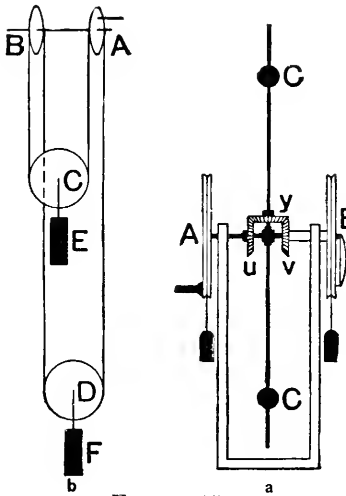

FIGURE 15.

In Lord Rayleigh's model, shown in Figure 15_b, "two similar pulleys A, B, turn upon a piece of round steel fixed horizontally. Over these is hung an endless cord, and the two bights carry similar pendent pulleys, C, D, from which again hang weights, E, F. . . . In the electrical analogy, the rotary velocity of A corresponds to a current in a primary circuit, that of B to a current in the secondary. . . . In the absence of friction there is nothing to correspond to electrical resistance, so that the conductors must be looked on as perfect. If x and y denote the circumferential velocities, in the same direction, of the pulleys A, B, where the cord is in contact with them, $\frac{1}{2}(x + y)$ is the vertical velocity of the pendent pulleys. Also $\frac{1}{2}(x - y)$ is the circumferential velocity of C, D, due to rotation, at the place where the cord engages. If the diameter be here $2a$, the angular velocity is $(x - y)/2a$. Thus, if M be the total mass of each pendent pulley and attachment, Mk^2, the moment of inertia of the revolving parts, the whole kinetic energy corresponding to each is

$$\frac{1}{2} M \left\{ \frac{(x+y)^2}{4} + \frac{k}{a^2}\left(\frac{(x-y)^2}{4}\right) \right\}. \qquad (25)$$

For the energy of the whole system, we should have the double of this, and, if it were necessary to include them, terms proportional to x^2 and y^2, to represent the energy of the fixed pulleys."

Here $L_1 = L_2 = a^2 + k^2$, $M = a^2 - k^2$, and, if there were no magnetic leakage, k would need to be zero.

Figure 16 represents the model of Professor Sir J. J. Thomson.[3] "It consists of three smooth, parallel, horizontal steel bars on which masses m_1, m_2, m slide, the masses being separated from the bars by friction wheels; the three masses are connected together by a light rigid bar which passes through holes in swivels fixed on the upper part of the masses; the bar can slide backwards and forwards through these holes, so that the only constraint imposed by the bar is to keep the masses in a straight line."

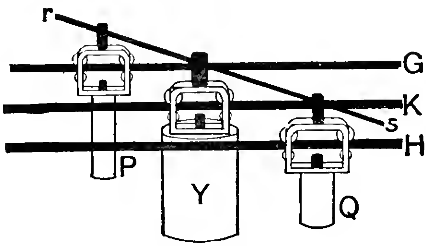

FIGURE 16.

If x'_1, x'_2 are velocities of m_1, m_2, in the same direction, the velocity of M, if it be midway between m_1 and m_2, is $\frac{1}{2}(x'_1 + x'_2)$, and the kinetic energy is of the form

$$\tfrac{1}{2} L_1 x_1'^2 + M x'_1 \cdot x'_2 + \tfrac{1}{2} L_2 x_2'^2, \tag{26}$$

where $L_1 = m_1 + \frac{1}{4} m$, $L_2 = m_2 + \frac{1}{4} m$, $M = \frac{1}{4} m$.

Professor Webster's model is a modification of that of Thomson. "If the middle weight, instead of rolling on a fixed rail, roll on the bar connecting the two other carriages, the coefficients of induction will vary with the position of the middle mass, and moving it along its bar while one of the outer masses is moving will cause the other to move. The centrifugal force tending to make the middle mass roll along its bar will represent the magnetic forces between the currents."

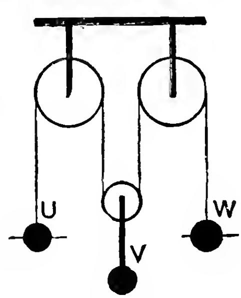

FIGURE 17.

The very elaborate and ingenious model of Boltzmann is described at length in the first fifty pages of his Vorlesungen über Maxwell's Theorie der Elektricität und des Lichtes.

The general features of another simple model illustrative of this electrical problem are shown in Figure 17. The mass of U is $L_1 - M$, that of V is $4M$, and that of W, $L_2 - M$. In Figure 18 the strings

[3] J. J. Thomson, Elements of the Mathematical Theory of Electricity and Magnetism, Chapter XI.; Webster, Science, Dec. 1895; The Theory of Electricity and Magnetism, § 71.

are represented as stretched over four small pulleys to keep them taut. I have found that this model made of three weighted roller skates, moving over a level table top, and connected in the manner indicated by cords passing around such small cheap pulleys as are obtainable at any ironmonger's shop, may be made to work extremely well. The effects of sudden changes of inductance can be directly observed by dropping suitable masses into the skates as they move. In Figure 14, which illustrates the same problem, the mass of Q is M, and those of P and Q are $L_1 - M$, $L_2 - M$, respectively. Q should have no vane.

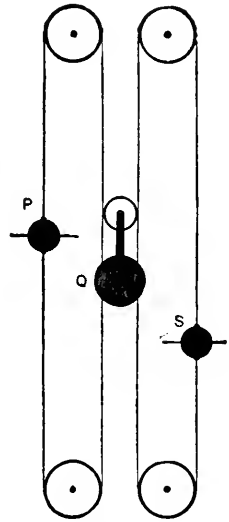

FIGURE 18.

Scores of other models, more or less simple of construction, can easily be devised. It is to be noticed, however, that in some of the models which have been used to illustrate this problem, the masses representative of some of the combinations of the inductances would need to be negative if they were to correspond to cases which occasionally arise in electrical engineering.

If either of the two neighboring circuits contains no battery, the corresponding value of E in equations (24) is to be put equal to zero. Figure 19 is drawn for the case of an induction coil without iron and with no cell in the secondary circuit. The self-inductances of the two circuits are equal. The dotted curve, P, shows the form of the current induced in the secondary circuit when the primary circuit, which has been carrying a steady current, is suddenly broken. If, after a few seconds, the primary circuit containing its battery be closed again, the current in the secondary circuit will have the general form of either Q or S. Q, R, and S are drawn for mutual inductances respectively half as great, nine tenths as great, and equal to, the inductance of either circuit. P is drawn for $M = L/2$, and corresponds, therefore, to Q; the areas V and W are equal. Curves like P corresponding to R and S could be found by exaggerating all of P's ordinates in the ratio 9/5, or the ratio 2.

Figures 20, 21, 22, 23, and 24 illustrate some phenomena which are frequently encountered in the practical use of neighboring inductive circuits. The curves have been drawn to scale for certain numerical values of the resistances, and the inductances so chosen as to make the results typical. There is no iron in either circuit, and only one circuit, the primary, contains a battery.

In Figure 20 the current in the primary circuit is drawn above OX, in the curve OJAZ, and the current in the secondary circuit beneath MN. Each of the circuits has a self-inductance of 2 henries, and the

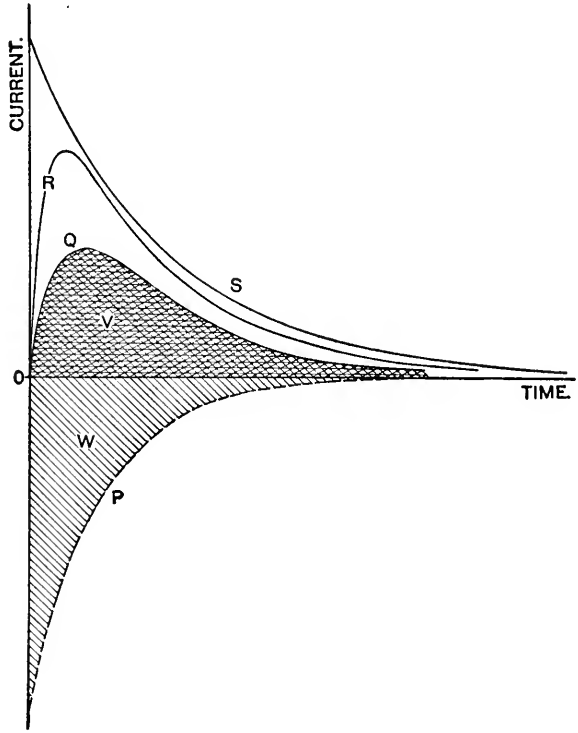

FIGURE 19. The curves Q, R, S represent for different relative values of the mutual inductance the current induced in the secondary circuit of a certain induction coil without iron, when the primary circuit is suddenly closed.

resistances are 30 ohms and 40 ohms. The mutual inductance is at first $\sqrt{2}$, and the currents are given by the equations

$$C_1 = 4 - 2.4\, e^{-10t} - 1.6\, e^{-60t}, \qquad C_2 = \frac{-2.4}{\sqrt{2}}\,(e^{-10t} - e^{-60t}), \quad (27)$$

until the time $OG = 1/20$, when all the inductances are suddenly doubled. The currents are then given by the equations

$$C'_1 = 4 - 1.928\, e^{-5t} - 0.840\, e^{-30t}, \qquad C'_2 = 0.891\, e^{-30t} - 1.361\, e^{-5t}. \quad (28)$$

In the case represented by Figure 21, $r_1 = 30$, $r_2 = 40$, $M = \sqrt{2}$, $L_2 = 2$. At the beginning $L_1 = 2$, but at the time OA this is suddenly changed to 4. Before the change in L_1 the currents are given by the equations

$$C_1 = 4 - 2.4\,e^{-10t} - 1.6\,e^{-60t}, \qquad C_2 = \frac{-2.4}{\sqrt{2}}\,(e^{-10t} - e^{-60t}), \qquad (29)$$

Just before the impulse, $C_1 = 2.465$, and $C_2 = -0.945$; just after, the current in the primary is about 0.822 and the secondary current has the small positive value 0.217. The new currents satisfy the equations

$$C'_1 = 4 - 2.634\,e^{-20t/3} - 0.543\,e^{-30t}, \quad C'_2 = -0.932\,e^{-20t/3} + 1.149\,e^{-30t}, \quad (30)$$

very nearly. C_2 is plotted below TQ.

Figure 22 shows the manner of growth of two neighboring currents, when $r = 30$, $r = 40$, $L = 2$, $L = 2$, and when M, which is at first

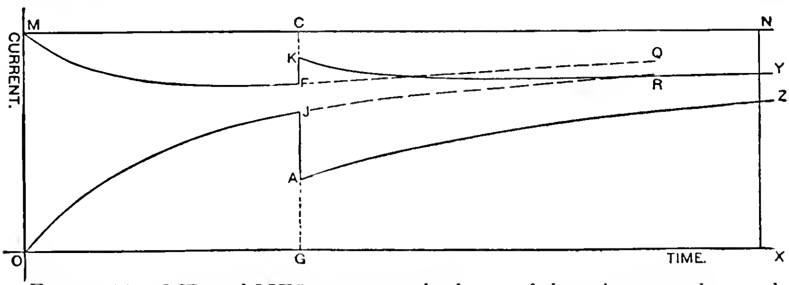

FIGURE 20. OJR and MFQ represent the forms of the primary and secondary currents in a certain induction coil without iron when the primary circuit is closed at the origin of time. If at the time OG, the self-inductances of both circuits and the mutual inductance of the two are suddenly doubled, the currents take the forms OJAZ and MFKY.

$\sqrt{2}$, is suddenly changed to zero at the time OA. When M is changed, the current in the primary circuit suddenly falls from 2.465 to 1.797, and the current in the secondary circuit, which has been negative, rises from -0.945 to $+0.798$. After the change, the currents are given by the simple equations

$$C'_1 = 4 - 2.203\,e^{-15t}, \qquad C'_2 = 0.798\,e^{-20t}. \qquad (31)$$

Figure 23 exhibits the effects of a sudden change in the value of the mutual inductance between the two circuits already described under Figures 21 and 22, from $\sqrt{2}$ to 1.9, while the other inductances remain unaltered. The primary current is shown by the curve OKRGS, and

OX shows its final value. The second current is represented by the curve ADWU plotted under AN and displaced to the right so that the sudden increase in the absolute strength at the time of the change in M may appear. The flux of magnetic induction through the primary circuit is represented on the usual scale by the shaded area. The

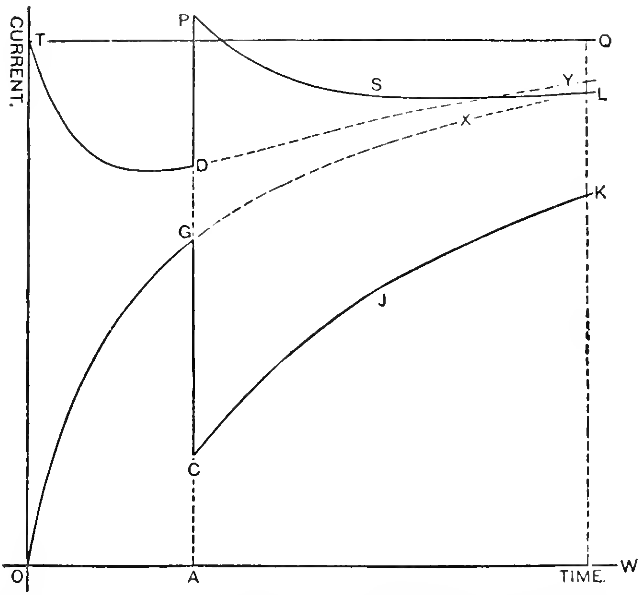

FIGURE 21. At the time OA, the self-inductance of the primary circuit of a certain induction coil without iron is suddenly doubled, while the self-inductance of the secondary circuit and the mutual inductance of the two remain unchanged. OGCJK shows the course of the primary current and TDPSL that of the secondary current.

black area points to a decrease in this flux which goes on from the time XF to the time XG, when the current falls below its final value. The induction flux linked with each of the two circuits is plotted against the time in Figure 24. These quantities are shown to be continuous at the time of change in the inductance, as, of course, they should be.

Figure 25 shows another arrangement of two neighboring circuits and an analogous mechanical system. The gap O is closed at first, but is suddenly opened when the current has become steady. The mass W moves alone under the action of a force E which urges it in the direction of the bottom of the page, and the air resistance. The motion soon becomes steady, but when the string which connects W to X becomes taut the motion is suddenly changed.

Figure 9_d represents a circuit consisting of three parallel branches, each of which has self-inductance and may contain a battery, and two of which have mutual inductance. If L, L_1, L_2 are the self-inductances, r, r_1, r_2 the resistances, C, C_1, C_2 the currents, E, E_1, E_2 the electro-

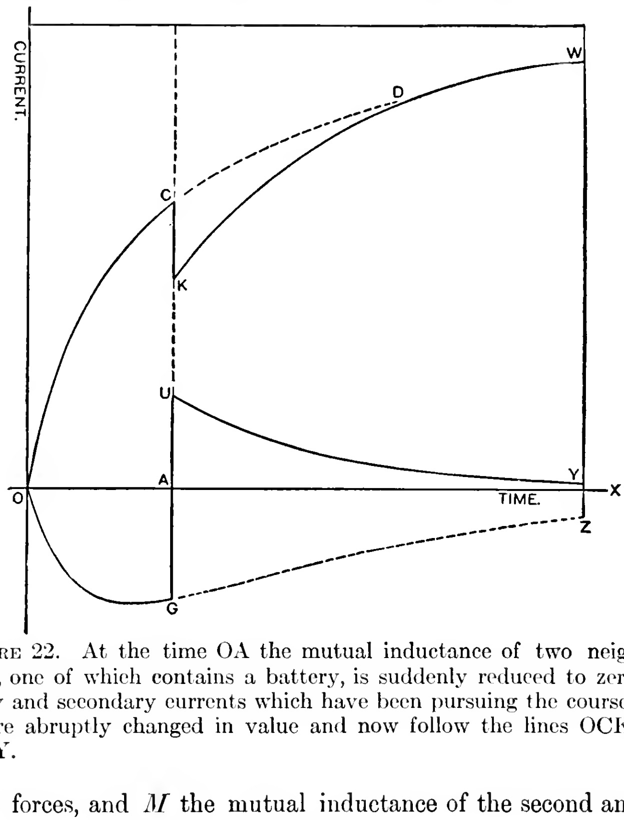

FIGURE 22. At the time OA the mutual inductance of two neighboring circuits, one of which contains a battery, is suddenly reduced to zero. The primary and secondary currents which have been pursuing the courses OCD, OGZ are abruptly changed in value and now follow the lines OCKW and OGAUY.

motive forces, and M the mutual inductance of the second and third branches, the currents satisfy the equations

$$(L + L_1)\frac{dC_1}{dt} + (L + M)\frac{dC_2}{dt} + (r + r_1)\,C_1 + rC_2 = E + E_1, \tag{32}$$

$$(M + L)\frac{dC_1}{dt} + (L + L_2)\frac{dC_2}{dt} + rC_1 + (r + r_2)\,C_2 = E + E_2,$$

$$C = C_1 + C_2.$$

Any sudden changes in the inductances cause such sudden changes in the currents as shall keep $[(L + L_1) C_1 + (L + M) C_2]$ and

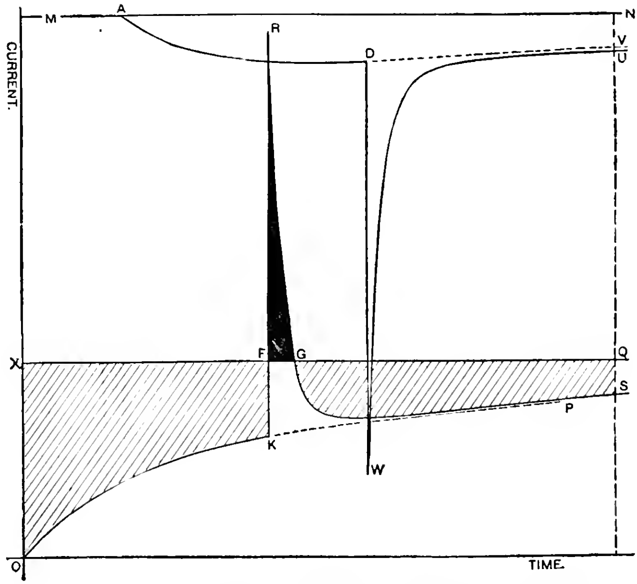

FIGURE 23. The two circuits of a certain induction coil without iron have equal self-inductances (L, L) and the mutual inductance $L\sqrt{2}$. At the time XF the mutual inductance is suddenly increased to $(1.9)L$, and the currents which have been following the curves OKP, ADV, take the forms KRGS, DWU.

$[(L + M) C_1 + (L + L_2) C_2]$ momentarily unchanged. In a case frequently met with in practice, there is no appreciable inductance in the

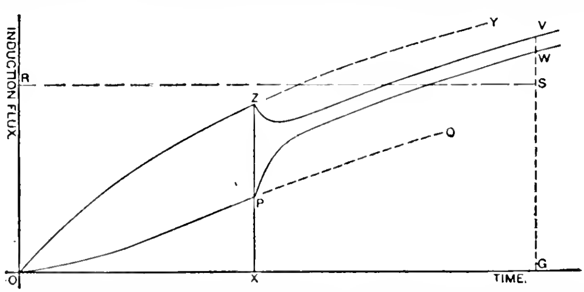

FIGURE 24. OPW and OZV show, as functions of the time, the fluxes of magnetic induction linked with the two circuits which carry the currents represented in the last figure.

first branch and no electromotive forces in the other branches, so that $L = 0$, $E_1 = 0$, $E_2 = 0$, and any instantaneous change in the inductances will leave

$$(L_1C_1 + MC_2)$$

and $$(MC_1 + L_2C_2)$$

momentarily unchanged.

If in Figure 14

$$P = L_1 - M,$$

$$Q = L + M,$$

$$R = L_2 - M,$$

and if the vanes are of such dimensions as to make the air resistance r_1, r, r_2 when the bodies to which they are attached have unit velocities, the equations of motion of the mechanical system are of the form (32), if E is applied upward to Q.

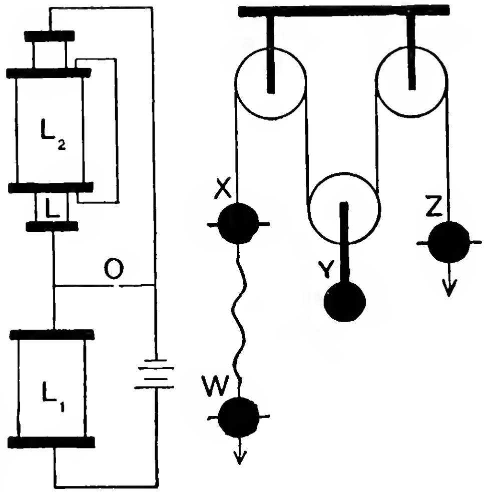

FIGURE 25.

Figure 26 illustrates a special case under this problem where $r = 1$, $r_1 = 20$, $r_2 = 30$, $L_1 = 2$, $L_2 = 3$, $M = 0$, up to the time OC, when by a sudden change in the conformation of the circuit, M is made equal to 2. Before the change $C_1 = 1.986$, $C_2 = 1.324$; the change in M leaves C_1 momentarily unchanged but suddenly reduces C_2 to zero. After the impulse the currents are given by the equations

$$C'_1 = 3 - 1.717\, e^{-5.959t} + 0.703\, e^{-54.541t},$$
$$C'_2 = 2 - 1.428\, e^{-5.959t} - 0.572\, e^{-54.541t}, \qquad (33)$$

very nearly.

If in the arrangement shown in Figure 9_e, the gap O, which has been closed by a stout wire, is suddenly opened, the current falls impulsively to a value which keeps the induction flux through the battery circuit momentarily unchanged.

The mechanical system shown in Figure 27 is analogous to the electrical circuit indicated in Figure 9_f. The gap O, which has been closed, is supposed to be opened at a given signal. The spring S is the analogue of the condenser K.

A circuit which contains an electromagnet has, of course, no definite inductance in the sense of the ratio of the flux of magnetic induction linked with the circuit to the intensity of the current, for this ratio is different for different current strengths, and for a given electromagnet, and a given current depends upon the previous magnetic history of the core.

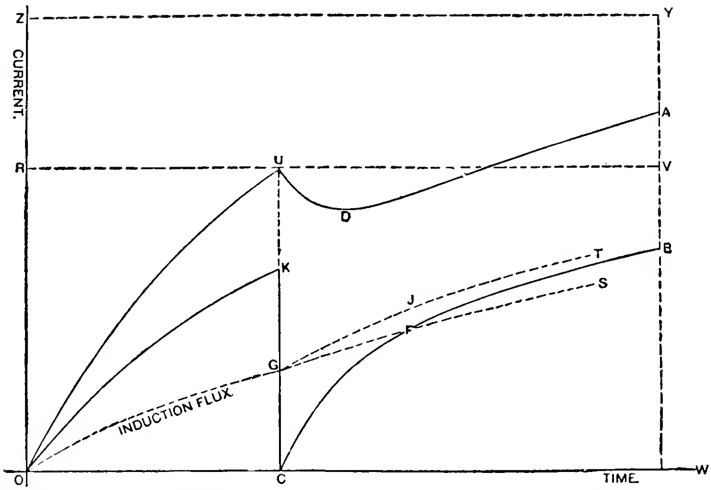

FIGURE 26. Two parallel coils of resistances 20 ohms and 30 ohms and of inductances 2 and 3 respectively, connect, in parallel, the terminals of a storage battery of 1 ohm internal resistance. At the beginning there is no mutual inductance between the branch circuits and the currents follow the curves OU, OK. At the time OC, the conformation of the circuit is suddenly changed so as to introduce a mutual inductance of 2, and as a result, the courses of the two currents are altered: the first follows henceforth the line UDA, the second the line KCFB. The inductions linked with the parallel branches are shown by the dotted curves.

In the case of a single circuit without iron the magnetic flux which accompanies a changing current is at every instant the same as it would be under a steady current of the intensity which the changing current then has. If, however, a second circuit closed on itself is brought into such a position that the two circuits have a mutual inductance, a changing current in the first circuit induces a current in the other which contributes to the flux through the first. If, therefore, an electromagnet has a solid core, the eddy currents induced in it while a current is growing or decreasing in the exciting coil affect the amount of the flux through the core, and it is not possible to obtain a hysteresis

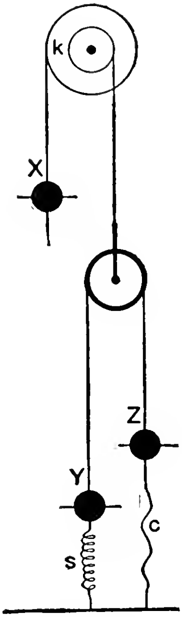

FIGURE 27.

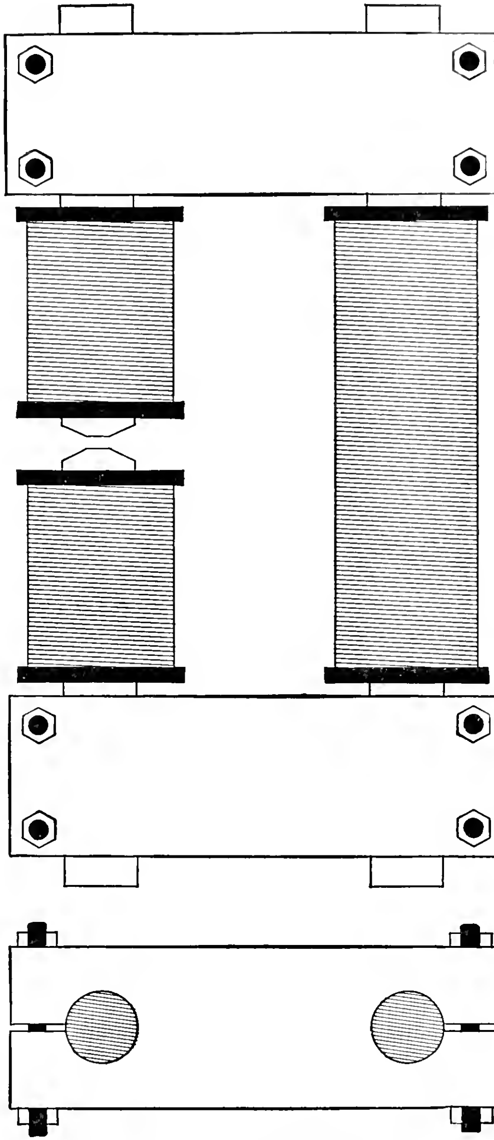

FIGURE 28. The electromagnet TP which has a solid core weighing about 300 kilograms.

diagram for the iron directly from the records of an oscillograph in the coil circuit. It is difficult, indeed, to obtain by any method a satisfactory magnetic curve for such a core, for if the iron starts from a given magnetic state, it is possible to get very different magnetic fluxes from a given exciting current by building up the current more or less slowly. In any useful examination of the magnetic properties of a solid piece of iron which is to be used for any practical purpose, it is essential that the metal be made to go over the same magnetic journeys which it will later be required to make, and at the same speed.

Before we discuss this anomalous magnetization more carefully, we may stop for some moments to study the records of an oscillograph in circuits which contained either the electromagnet TP, Figure 28, which has a solid core, or a certain toroid (DN), about 41 centimeters in mean diameter, the core of which was made of about 25 kilograms of fine, soft, varnished iron wire. It will be seen from Figures 29, 30, 31, 32, 33, 34 that the phenomena are in general what we should expect

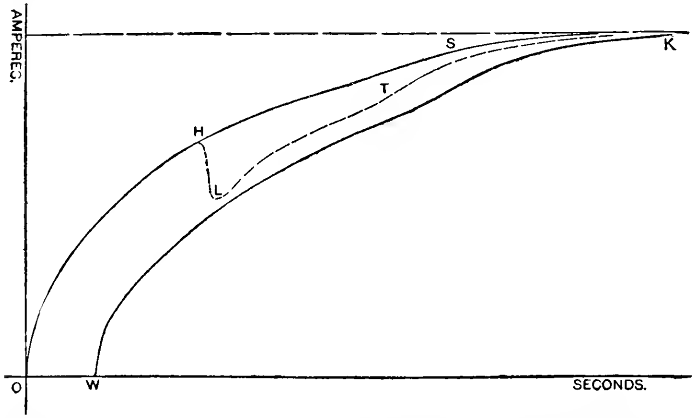

FIGURE 29. The curve OHS shows the manner of growth of a current in the coil of the magnet TP when the poles are separated by about three inches. WK shows the rise of the same current when the poles are nearly closed by the insertion of a planed block of iron between them. HLT shows the effect of suddenly dropping the block in while the current is growing.

to find in similar circuits without iron, though eddy currents and the time taken to make the mechanical changes modify somewhat the courses of the currents in the exciting circuits.

THE ANOMALOUS MAGNETIZATION OF IRON.

In 1863 von Waltenhofen first called attention to the fact that if an increasing current (C) ending in the maximum value (C') be sent through a long solenoid, the final value of the magnetic moment of a bar of soft iron in the solenoid, which was at the outset demagnetized, will depend not only upon the final strength of the current, but also upon the manner of growth of C in attaining this intensity. This moment will be greater if the current be suddenly applied in full strength than if it be made to grow slowly, either continuously or by short steps. If, after the current has remained steady for a short time at the strength C' it be made to decrease to zero, the residual moment of the bar will be less if the circuit be suddenly opened than if the decrease be made slowly by introducing more and more resistance.[4] If the soft iron bar to be magnetized was stout and relatively short, von Waltenhofen was sometimes able to reverse the direction of the remanent magnetism by a sudden break of the circuit. In one instance where the length of the bar was about ten centimeters and the diam-

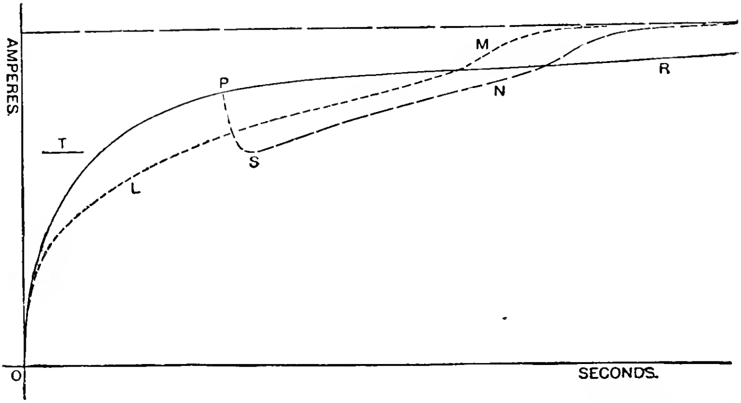

FIGURE 30. The short coils of the magnet TP are in series with one another and with a battery and an oscillograph. The current follows the line OPR or the line OLM according as the long coil of the magnet is open or closed. When at the point P, the long coil circuit is suddenly closed, the current in the battery follows the line PSN, which despite eddy currents is not very unlike the upper part of OLM.

eter about two centimeters, the magnetic moment while the current was passing was about 45 units, and about —0.20 when the current had been stopped. It seemed to von Waltenhofen that these phenomena could not be due to the induced currents caused by the sudden changes in the exciting current, and he explained them as consequences of the inertia of the molecular magnets turning in a viscous medium. This view seems to have been taken by Fromme, Auerbach, Ewing,

[4] Von Waltenhofen, Poggendorff's Ann. **120**, 1863; Fromme, Poggendorff's Ann., Ergbnd. **7**, 1876; Wied. Ann. **4**, 1878, **5**, 1878, **13**, 1881, **18**, 1883, **44**, 1891; Bartoli and Alessandri, Nuovo Cimento, **8**, 1880; Righi, Mem. di Bologna, **1**, 1880; Peuckert, Wied. Ann. **32**, 1887; Auerbach, Wied. Ann. **14**, 1881, **16**, 1882; Winkelmann's Handbuch der Physik, Band 5; Wiedemann, Lehre von der Elektricität, Band IV; Ewing, Magnetic Induction, § 84; Gumlich und Schmidt, Electrotechnische Zeitschrift, **21**, 1905.

Peuckert, Zielinski and others who have written upon the subject, while G. Wiedemann thought that his researches and those of Righi showed that eddy currents in the iron, and alternating currents induced in the exciting coil accounted best for the observed facts. In describing experiments upon this so-called anomalous magnetization, Wiedemann distinguishes between the permanent moment (P), that is, the remanent moment after the current has ceased, and the total moment (T), which

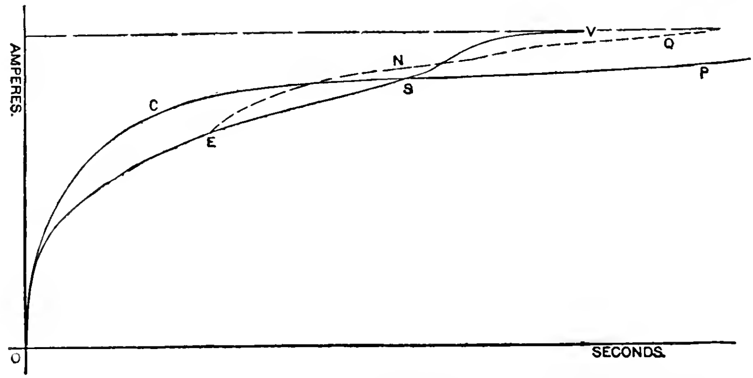

FIGURE 31. The short coils of the magnet TP are in series with one another and with a battery and an oscillograph. The current follows the line OCP or OESV according as the long coil of the magnet is open or closed on itself. When at the point E the long coil which has been closed is suddenly opened, the oscillogram is of the form ENQ which on account of the disturbing effects of eddy currents is not like the upper part of the curve OCSP.

is the moment when the current is steady at its highest value. This last quantity is regarded as the sum of the permanent moment and a moment (V) which vanishes with the current. The suffix a attached to P or T denotes that this moment has been reached after a gradual change in the current, while the suffix f denotes that the current has been suddenly opened or closed. According to Wiedemann, T_a is always smaller than T_f, and P_f than P_a, but these differences are much larger for short stout rods than for relatively long ones, where they become insignificant. $(P_a - P_f)/P_a$ is smaller in the case of a rod made of a bundle of insulated soft iron wires than in the case of a solid rod of the same dimensions. As C' is made larger, $T_f - T_a$ attains a maximum and then sometimes decreases slightly. If the rod to be magnetized is surrounded by a thick metal tube in which eddy currents can be induced, P_f is slightly increased, especially if the current be first slowly raised to C and then suddenly stopped. If shorter and shorter iron rods of a given diameter are tested, P_f gradually decreases to zero under given value of C', and then changes sign; the inversion comes with longer rods when C' is weak than when it is strong. If, with a given rod, C' be gradually increased, the negative moment finally decreases and changes sign. After this there is no inversion and P_f is positive.

It appears from Fromme's experiments that the von Waltenhofen

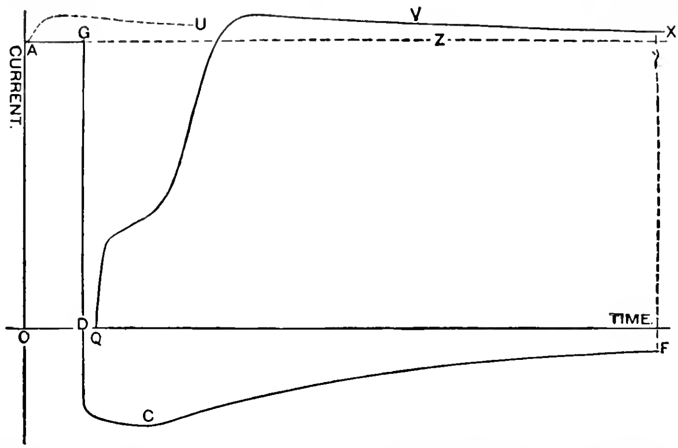

FIGURE 32. The toroid (DN) and the electromagnet (TP) with its poles separated by a gap of about eight centimeters, were placed in parallel across the terminals of a storage battery, with an oscillograph in the toroid branch. QVX shows the course of the current in the toroid which approaches its final value from above (See Figure 12). GCF, with its irregularities of curvature, shows the current in the toroid when the battery circuit was suddenly broken. AU shows the form of the temporary current induced in the toroid when an iron block was suddenly dropped into the gap between the jaws of the electromagnet, after the currents in the circuit had become steady.

effect is often less marked in straight, finely divided cores than in solid ones, and we may inquire how greatly the division of a straight core may be expected to facilitate the changes in the field (H) within the iron, due to given changes in the exciting circuit. It is clear that if the circuit of an electromagnet be suddenly broken, the decay of the electromagnetic field in the core is much less rapid when the core is solid and eddy currents induced in it shield the inner filaments, than when it is made of wire. Indeed, if eddy currents were non-existent, the field would fall instantaneously to zero, in the absence of magnetic lag, when the current in the coil ceased to flow. If the exciting coil remains closed and some change is suddenly made in its resistance or in the electromotive force applied to it, the change in the current and therefore the change in the field in the iron caused by the current cannot be made instantaneous, even if eddy currents be wholly shut out, and, though dividing up the core has its effects, we cannot expect them to be so striking as in the case where the exciting circuit is open.

Let us consider a very long, uniformly wound solenoid consisting of N turns of insulated copper wire per centimeter of its length, wound closely upon a long, soft iron prism of square cross-section ($2a \times 2a$)

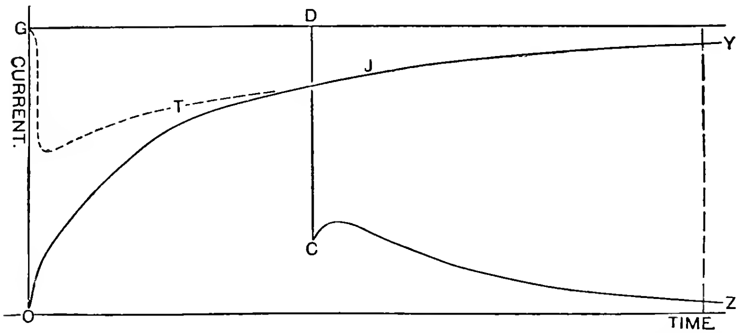

FIGURE 33. The toroid (DN) and the electromagnet (TP), with its jaws separated by a gap of about eight centimeters, were placed in parallel across the poles of a storage battery, with an oscillograph in the electromagnet branch. OJY shows the manner of growth of the current in TP and DCZ the manner of decay of the current when the battery circuit was suddenly broken. If, after the currents in the circuit have become steady, a block of iron was suddenly dropped into the gap in the core of the electromagnet, the induced current took the form GT.

built up compactly of a large number of straight, varnished filaments or "wires" of square cross-section ($c \times c$), with their axes parallel to that of the prism, which shall be used as the z axis. The electric resistance of the solenoid per centimeter of its length parallel to the z axis shall be w, the constant applied electromotive force per centimeter of the axis shall be E, and the intensity of the current in the coil shall be C.[5] Within the core, the magnetic field (H) will have everywhere and always the direction of the axis of the prism, and if q is the current flux at any instant at any point in the iron, ρ the specific resistance of the metal, and μ its magnetic permeability, which for the present purpose shall be regarded as having a fixed uniform value, $q_x = 0$, $q_y = 0$, $H_x = 0$, $H_y = 0$, $H_z = H$, $4\pi q = \text{Curl } H$,

[5] Peirce, These Proceedings, **43**, 5, 1907.

$$\frac{\partial H}{\partial t} = \frac{\rho}{4\pi\mu}\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y}\right). \tag{34}$$

When there are no eddy currents in the core, the intensity (H) of the magnetic field has at every point of the iron the boundary value $H_S = 4\,\pi NC$, but in general H varies from point to point. The flux of magnetic induction through the turns of the coil per centimeter of

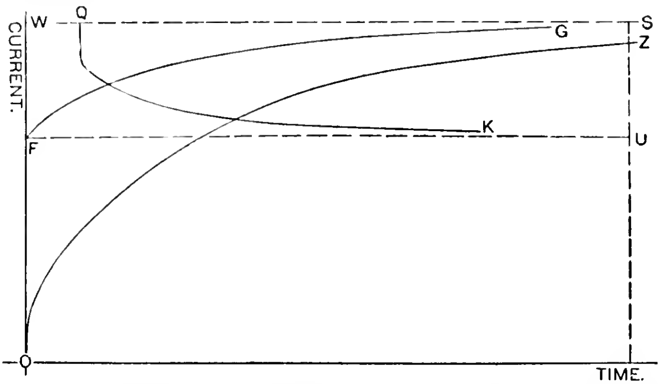

FIGURE 34. Two electromagnets were placed in series with each other and with an oscillograph and a storage battery, and a shunt (S) of small resistance was provided for one of the magnets. OZ shows the form of the battery current when S was closed, QK the fall of this current when S was suddenly opened after the original current had become steady, and FG the rise of the current to its old value when the shunt is again closed.

its length parallel to the z axis and N times the induction flux through the core are practically equal, and we may write

$$E - \frac{dp}{dt} = E - \mu N \int\int \frac{\partial H}{\partial t} \cdot dx dy = w \cdot C = \frac{w \cdot H_S}{4\,\pi N}, \tag{35}$$

or,

$$E = \frac{w \cdot H_S}{4\,\pi N} + \frac{\mu\rho N}{4\pi\mu}\int\int\left(\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2}\right)dx dy, \tag{36}$$

where the integration extends over a cross-section of the core.

The vector H is always perpendicular to its curl, and the intensity of the component of the current at any point in the iron, in any direction s, parallel to the xy plane at any instant, is equal to $1/4\,\pi$ times the value at that point, at that instant, of the derivative of H in a direction parallel to the xy plane, and 90° in counter clockwise rotation ahead of s.

Along any curve in the iron parallel to the xy plane, H must be constant if there is no flow of electricity across the curve. At every instant, therefore, the value of H at the boundary common to any two filaments must be everywhere equal to H_0. If the coil circuit is broken, H must be constantly zero at the surface of every filament.

$$E = \frac{c H_0}{4\pi N} - \frac{\sigma c}{4\pi N}\int\int\left(\frac{\partial H}{\partial x} - \frac{\partial H}{\partial y}\right)dx\,dy, \qquad (37)$$

where the integral is to be taken over the section of one filament.

Eddy currents in any filament which are the same in direction and intensity at all points in any line parallel to the axis do not affect in any way the magnetic field outside the filament.

If after a steady current E_0/r has been running in the solenoid, the circuit be instantaneously broken, the value of H falls from $4\pi N E_0/r$ to 0 at the surface of the prism and at the surface of every filament, and H is given by the equation

$$H = \frac{16 H_0}{\pi^2}\sum_{j=1}^{j=\infty}\sum_{k=1}^{k=\infty}\frac{e^{-\lambda t}}{(2j-1)(2k-1)}\cdot\sin\frac{(2k-1)\pi x}{c}\cdot\sin\frac{(2j-1)\pi y}{c}, \qquad (38)$$

$$\lambda = \frac{\pi \sigma}{4\mu c^2}\left[(2k-1)^2 + (2j-1)^2\right], \qquad (39)$$

in every filament. The origin is at one corner of the cross-section of the filament and the x and y axes are two sides of the section; k and j are integers, and $H_0 = 4\pi N E_0/r$.

If we differentiate both members of equation (38) with respect to the time, and integrate the result over the cross-section of a filament, we get for the average value for the whole wire, at the time t, of the rate of change of the magnetic force in the iron,

$$\frac{1}{c^2}\int_0^c\int_0^c\frac{dH}{dt}\,dx\,dy = \frac{4\sigma H_0}{\mu c^2}\sum_{j=1}^{j=\infty}\sum_{k=1}^{k=\infty}\left[(2j-1)^2 + (2k-1)^2\right]\frac{e^{-\lambda t}}{(2j-1)^2(2k-1)^2} = \frac{4\sigma H_0}{\mu c^2}\cdot M. \qquad (40)$$

At the origin of time M has the same maximum value for all values of μ and for different values of c [illegible]. M has the same numerical value at times t_1, t_2, such that $t_1/\mu_1 c_1^2 = t_2/\mu_2 c_2^2$, provided μ has the same value in both cases. The smaller the values of μ and c the sooner does M for a given wire attain a given value. At $t = 0$, when the change of H in the iron is most rapid at all points, the average value of $\delta H/\delta t$ throughout the core is inversely proportional to ε, so that the average rate at which H is changing is 100 times as great when the core consists of filaments only one millimeter square as when the filaments are a centimeter square. This example illustrates the fact that when the main circuit of an electromagnet is suddenly broken, the changes in excitation to which the iron in a divided core is subject are far more violent than those which the particles of a solid core encounter. It is to be noticed that the average value of $\delta H/\delta t$ given above is proportional to the electrical conductivity of the iron and to the intensity of the steady current in the exciting coil before the break.

In a similar manner it is possible to show that if a given current in the exciting coil of an electromagnet be changed by a sudden increase or decrease in the resistance of the circuit while the applied electromotive force is unaltered, the whole given change of the magnetizing field in the core takes place somewhat more quickly if the core is finely divided than if it is solid. The general fact is, of course, evident without computation.

If the square core of a solenoid the area of the cross-section of which is A square centimeters be made of a bundle of infinitely long straight iron wires placed close together, and if after a steady current of intensity E/r has been running for some time through the circuit so that there is a magnetic field of uniform intensity $H_1 = 4\pi NE/r$ in the core, the resistance of the solenoid circuit be suddenly changed to C ohms per centimeter of length of the coil, the current in the coil will gradually change to E/C, and the field in the core finally reaches the uniform value $H_2 = 4\pi NE/C$. At any instant the field in so much of the space A as is occupied by air is $4\pi NI$; the eddy currents in the small wires act like solenoidal current sheets and do not affect the field outside the wires. Within each wire there are, of course, eddy currents, and at every point in the iron at every instant the field intensity, H, must satisfy the equation (1).

The induction flux, φ, through the solenoid per centimeter of its length is

$$4\pi NI(A - n^2F) + \mu \Sigma \int\int H \, dx \, dy, \qquad (41)$$

where n^2 is the number of wires in the core and F is the area of the cross-section of each of them. The double integral is to be extended over the cross-sections of all the wires.

Since $$E - \frac{d\varphi}{dt} = CI,$$

$$w'C + (A - n^2B)\, 4\pi N^2 \cdot \frac{dC}{dt} + \mu N \int\!\!\int \frac{\partial H}{\partial t} \cdot dS = E, \tag{42}$$

and if H_S represents the strength of the magnetic field in the air space within the solenoid, and $A - n^2B$ is written $h \cdot A$,

$$H_S - \frac{4\,\pi NE}{w'} + \frac{4\,\pi N^2 hA}{w'} \cdot \frac{dH_S}{dt} + \frac{4\,\pi\mu n^2 N^2}{w'} \int\!\!\int \frac{\partial H}{\partial t} \cdot dS = 0, \tag{43}$$

where the double integral is to be taken over the cross-section of a single filament. If we put $H = H' + H_\infty$ and $H_S = H'_S + H_\infty$, the last equation becomes

$$H'_S + \frac{4\,\pi N^2 hA}{w'} \cdot \frac{dH'_S}{dt} + \frac{4\,\pi\mu n^2 N^2}{w'} \int\!\!\int \frac{\partial H'}{\partial t} \cdot dS, \tag{44}$$

in which H' satisfies at every point the equation

$$\frac{\partial H'}{\partial t} = \frac{\rho}{4\,\pi\mu}\left(\frac{\partial^2 H'}{\partial x^2} + \frac{\partial^2 H'}{\partial y^2}\right) = \frac{\rho}{4\,\mu\pi}\left\{\frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \cdot \frac{\partial H'}{\partial r}\right)\right\} \tag{45}$$

where r is the distance from the axis of the wire in which the point lies. We are to find a function H' which satisfies equations (44), (45), which, when $t = 0$, is everywhere equal to $H_0 - H_\infty$, and which vanishes everywhere when t is infinite.

If
$$\varpi = \sum Le^{-\beta^2 t} \cdot J_0(mr), \tag{46}$$

in which either m or β may be chosen at pleasure and the other computed from the equation

$$m^2\rho = 4\,\pi\mu\beta^2, \tag{47}$$

and if for m in (46) we use the successive roots of the transcendental equation

$$J_0(mb)\left(1 - \frac{N^2 hA\rho \cdot m^2 b^2}{\mu w' b^2}\right) = \frac{2\,\pi n^2 N^2 \rho}{w'} \cdot mb \cdot J_1(mb), \tag{48}$$

where b is the radius of the wire, ϖ satisfies equations (44), (45) and vanishes when t is infinite.

Without any consideration of the question of a possible development of unity in terms of an infinite series of Bessel's Functions of the form $J_0(mr)$, where the m's have the values just mentioned, it is clear[6] that,

[6] Byerly, Annals of Mathematics for April, 1911.

within the comparatively short range from 0 to b, unity may be represented with sufficient accuracy by a few terms (sometimes two) of the form

$$L_1 \cdot J_0(m_1 r) + L_2 \cdot J_0(m_2 r) + L_3 \cdot J_0(m_3 r) + \ldots = \sum L_k \cdot J_0(m_k r), \quad (49)$$

so that
$$H = H_\infty + (H_0 - H_\infty) \sum L_k \cdot e^{-\beta^2 t} \cdot J_0(m_k r) \quad (50)$$

gives the value of the magnetic field at the time t at any desired point in the wire in question, and, therefore, at any desired point in any other wire of the core.

$$\frac{\partial H}{\partial t} = \frac{-(H_0 - H_\infty)\rho}{4\pi\mu} \sum L_k \cdot m^2 e^{-\beta^2 t} \cdot J_0(mr), \quad (51)$$

and if this be integrated over the cross-section of a wire and divided by πb^2, the result,

$$\Omega = \frac{-(H_0 - H_\infty)\rho}{2\pi\mu b^2} \sum L_k \cdot e^{-\beta^2 t} \cdot mb \cdot J_1(mb), \quad (52)$$

will represent the average value in the whole core, at the time t, of the time rate of change of the magnetic field. An example will best show the meaning of these rather intractable expressions.

Suppose the core of a long solenoid of square cross-section, ten centimeters on a side, to be built up of straight, round iron rods one millimeter in diameter placed close together; then $h = 0.2146$, $b = 0.05$, $n = 100$. If the resistance of the solenoid coil per centimeter of its length is $\frac{1}{16}$ of an ohm, the specific resistance of the iron 9950 absohms, the number of turns of wire per centimeter of the solenoid 10, and the value of the permeability of the iron 100, then $mb = x$ satisfies the equation

$$J_0(x) \cdot (1 - 1.3666\, x^2) = 1000\, x \cdot J_1(x), \quad (53)$$

and the first root $x = 0.04465$ will suffice, for $m = 0.8930$; and $J_0(0.8930\, r)$ differs from unity by less than one tenth of one per cent over the whole range from $r = 0$ to $r = b$, and from (50)

$$\frac{H - H_\infty}{H_0 - H_\infty} = e^{-6.315t} \cdot J_0(mr), \quad (54)$$

very approximately. In the case of a core of the same cross-sectional area (0.7854 A), and the same permeability, but wholly without eddy currents, it is easy to show that

$$H = H_\infty + (H_0 - H_\infty) \cdot e^{-kt}, \quad (55)$$

where $k = w't/4\pi N^2 \cdot D$, $\quad D = A\,[4 + \pi(\mu - 1)]/4$.

For this problem, (55) yields

$$\frac{H - H_\infty}{H_0 - H_\infty} = e^{-6.316t}, \tag{56}$$

and a comparison of (54) and (56) shows that the eddy currents in a core of this wire, one millimeter in diameter, have practically no effect in slowing the changes in magnetism of the iron.

If the core of the given solenoid were made up of rods one centimeter in diameter, mb or x would be given as the roots of the equation

$$J_0(x)\cdot(1 - 0.01366\,x^2) = 10\,x\cdot J_1(x), \tag{57}$$

and it is not very difficult to show by a process of trial and error from Meissel's *Tafel der Bessel'schen Functionen*, that the first three of these roots are approximately equal to 0.4411, 3.8525, 7.0204, and that the corresponding values of $J_0(x)$ and $J_1(x)$ are 0.951946, $-$ 0.402672, 0.300112, and 0.215229, $-$0.008352, 0.001444.

If, with these roots, we wish to determine such a set of coefficients, (L_1, L_2, L_3) as shall make the mean square of the difference between unity and $\Sigma L \cdot J_0(mr)$ as small as possible, for the range from $r = 0$ to $r = b$, we have to solve the equations

$$A_1 \cdot L_1 + B_{12} \cdot L_2 + B_{13} \cdot L_3 = C_1, \quad B_{12} \cdot L_1 + A_2 \cdot L_2 + B_{23} \cdot L_3 = C_2$$

$$B_{13} \cdot L_1 + B_{23} L_2 + A_3 L_3 = C_3, \qquad \text{where}$$

$$C_1 = 2\pi b^2 \cdot J_1(x_1)/x_1, \qquad A_1 = \pi b^2 \{[J_0(x_1)]^2 + [J_1(x_1)]^2\},$$

$$B_{12} = b^2\,[x_1 \cdot J_0(x_2) \cdot J_1(x_1) - x_2 \cdot J_0(x_1) \cdot J_1(x_2)]/(x_1^2 - x_2^2),$$

as Professor Byerly's theorems show. The computation here indicated shows that $\Omega = 6.096$ or 0.320, approximately, according as $t = 0$ or $t = 0.1$, whereas, if eddy currents were wholly cut out, the corresponding values would be 6.316 and 0.336. These figures illustrate the comparatively slight effect of subdividing the core in the particular case here considered. The results would, of course, be somewhat different numerically, with different assumed values for the constants of the circuit.

It is clear that the inversion of sign in the magnetic moment of a straight iron bar, when the magnetic excitation is suddenly removed, accompanies, at least, a large demagnetizing factor due to the ends of the bar, and no one seems to have observed the phenomenon in the case of closed cores. In rings, however, as in straight bars, the ultimate value of the intensity (I) of magnetization depends very much upon the manner in which the given exciting current is made to attain its final strength.

The experiments of Rücker[7] upon small solid iron toroids seem to show that at moderate excitations there may be a difference of from 6 per cent to as much as 30 per cent in the final flux density due to a given current, according as the current is applied suddenly or by

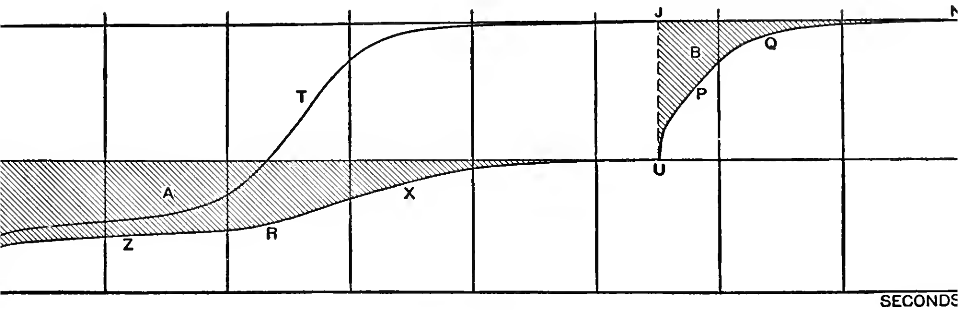

FIGURE 35.

many short steps, and, unlike some other observers, he found a very real difference ($T_f - T_a$), though a smaller one than in the case of the solid metal, for a toroid with core of fine iron wire (Blumendraht). In the case of a large electromagnet with solid, closed core, weighing altogether more than 1500 kilograms, Babbitt found, by a very ingenious method of procedure, a difference of 17.4 per cent between the final flux density in the iron caused by the sudden application of a given current, and the growth from nothing of the same current in 56 steps. The cross-section of this massive core is more than 450 square centimeters in its narrowest part, and eddy currents are so much in evidence that quite two minutes are required for a "suddenly applied" current to attain its steady value.

Babbitt also carried out a long series of very accurate measurements extended over several months, upon two small toroids of fine, carefully annealed iron wire, and upon a toroid weighing more than 40 kilograms made of very well softened iron wire about half a millimeter in diameter. His results show conclusively that if one of these softened and demagnetized cores has been first put through the cycle due to a given excitation a considerable number of times to obliterate the effects

[7] Babbitt, These Proceedings, **46**, 1911; Rücker, Inaugural Dissertation, Halle-Wittenberg, 1905.

of the past experiences of the iron, the form of the hysteresis diagram is precisely the same, whether the half cycle be carried out by one reversal of the main switch or in a very large number of steps. In general agreement with these results are some less accurate ones which I obtained three years ago in experimenting upon a transformer which has an exciting coil of 1394 turns and a core of about 120 square centimeters in cross-section, built up of thin strips of varnished sheet

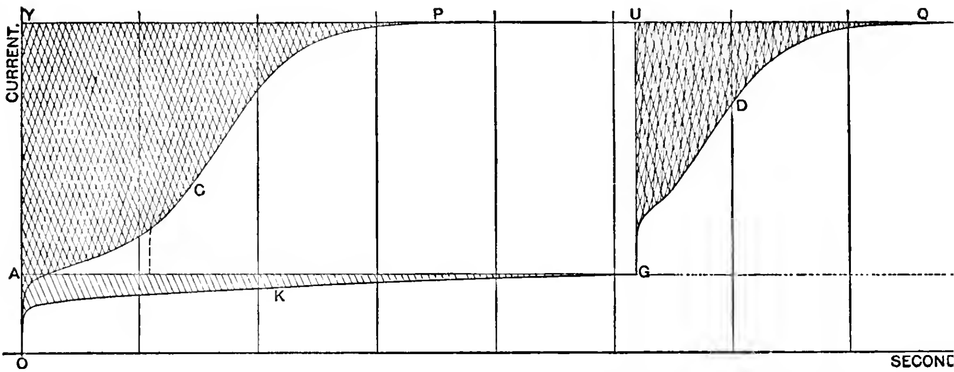

FIGURE 36. Growth from an originally neutral core of a current in a transformer with a laminated core. The effects of eddy currents are here noticeable.

iron about ten centimeters wide. This transformer was connected in simple circuit with a storage battery and a rheostat besides a suitable oscillograph. When the circuit was suddenly closed, with such a resistance (x) in the rheostat that the final strength of the current was about 1.10 amperes, the current curve was of the form R as shown in Figure 35, and when after a few seconds x was suddenly removed, so as to bring the final strength of the current up to about 2.30 amperes, the current curve was Q. When the whole journey was made without x the current curve was T. The sum of the flux changes represented by the shaded areas as measured by a Coradi "Grand Planimètre Roulant et à Sphère" was 1126, while the flux change corresponding to the area above the curve T was 1130. The core was not sufficiently well divided to avoid all evidence of eddy currents, for the curve Q does not exactly conform throughout with the upper part of T. This is shown more clearly in Figure 36, taken with the same transformer. Here the area of the shaded portion above K multiplied by the resistance then in the circuit should be equal to so much of the area above C, multiplied by the resistance belonging to it, as lies to the left of the dotted line which rises at about 1.1 seconds after the circuit was closed, and is an exact copy of the curve D moved to the left. This curve coincides with C for a large part of its course, but has a trifle less area above it than that portion of C has which lies to the right of the ordinate at which the lowest part of the dotted curve begins. The shape of D just at the beginning points to the existence of eddy currents.

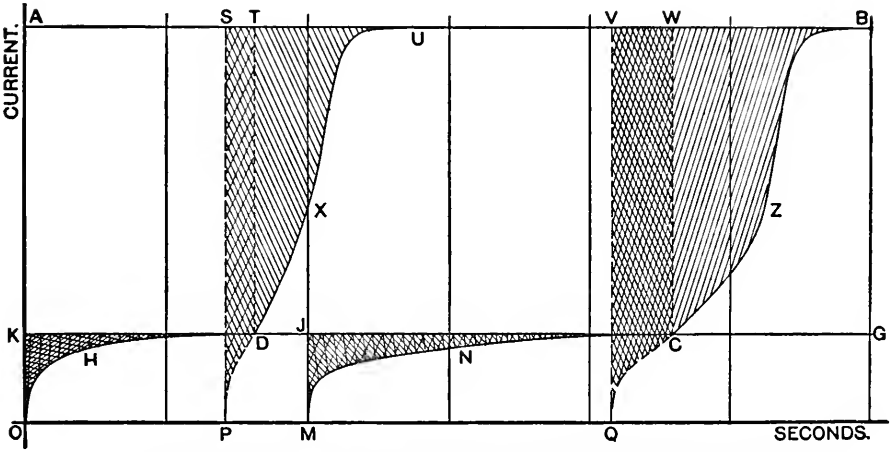

FIGURE 37. Current curves for a toroid with fine wire core. The second part of a two-stage current is exactly the same as if the current were allowed to grow at once to its final value.

To test more thoroughly the effect upon the flux of magnetic induction through the core of the transformer, of building up the current in different ways, I first measured with great care, by aid of a modified Rubens-du Bois "Panzer Galvanometer," the changes of this flux for a quick reversal of an excitation of 1812 ampere turns. I then measured by means of the planimeter a long series of oscillograph records obtained by reversing the same excitation by a considerable number of steps. All the testing instruments were different in the two cases, and no comparison was possible until the final results were reached and were found to differ from one another by only one part in fourteen hundred. The labor of reducing the oscillograms was so great that this close agreement must be considered accidental, but there can be little doubt, I think, that the flux change due to the single step and the sum of the changes due to the long series of steps which together cover the same change of excitation were practically indistinguishable.

Figure 37 shows copies of oscillograms taken with a number of toroids in series. The core of each toroid was made of perhaps fifteen kilograms of very soft, varnished iron wire, about one tenth of a millimeter in diameter. The curves OHD, PDXU were taken when the

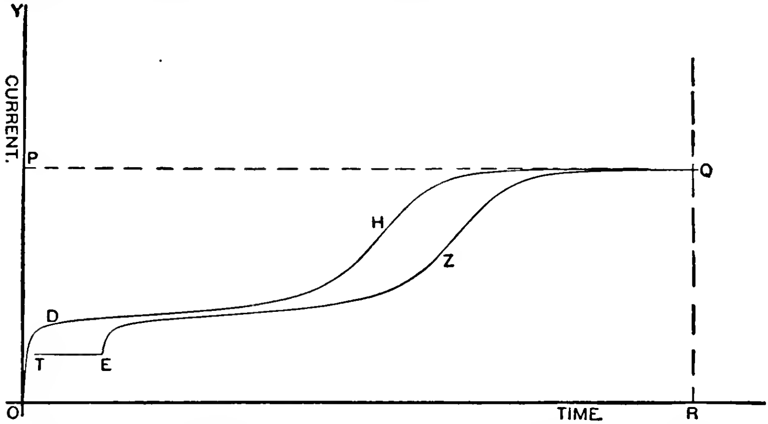

FIGURE 38. ODHQ represents the curve of growth of a current in the exciting coil of the toroid DN, if the circuit was suddenly closed when its resistance was r. If the circuit was first closed with a higher resistance $(r+s)$, which corresponded to a steady current of intensity OT, and if the resistance s was suddenly shunted out, the current rose to the intensity OP in the manner indicated by the curve EZQ, which, as Figure A shows, is of exactly the same form as the upper portion of ODHQ.

cores had been thoroughly demagnetized just before the experiment; the curves MNC, QCZB after the core had been put a number of times

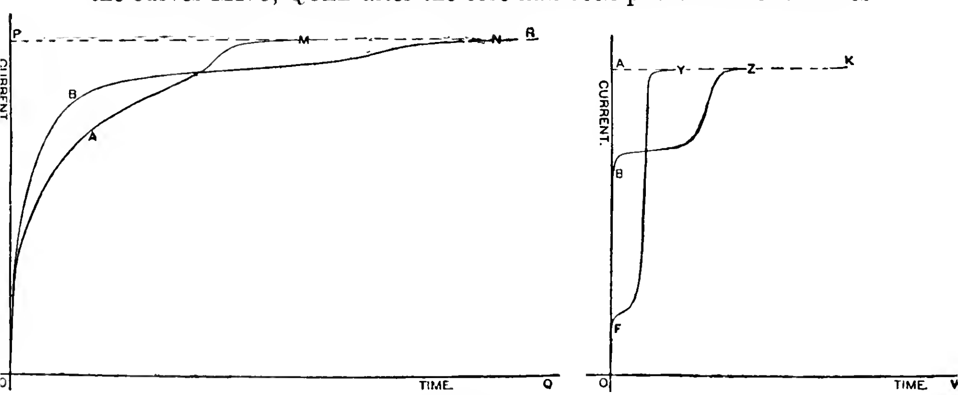

FIGURE 39.

through the cycle corresponding to the excitation used. The toroids were in simple circuit with a storage battery, an oscillograph, and a rheostat of resistance x; when the circuit was suddenly closed the current grew to the final value corresponding to this resistance by the curve OHD or MNC, as the case might be. When at the proper time the rheostat resistance was suddenly shunted out of the circuit, the current rose to the value OA by the curve DXU or the curve CZB. If x had been shunted out at the start the current curve had the shape accurately represented, when the starting point had been shifted just

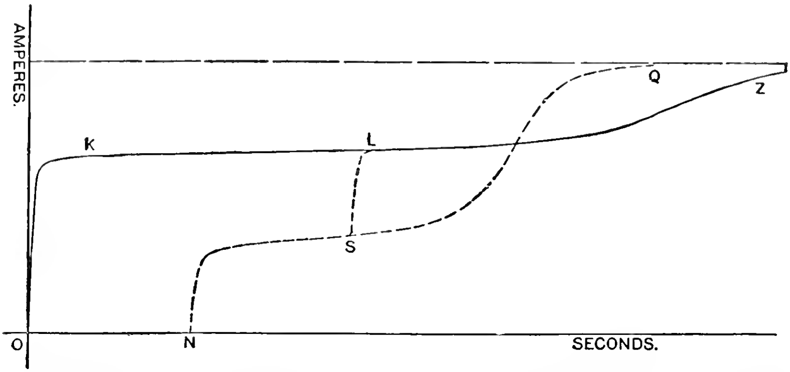

FIGURE 40. The current in one coil of the toroid DN which is in series with a battery follows the line OKLZ or the line NSQ, according as another coil on the same core is open or closed. When this last coil which has been closed is suddenly opened while the battery current is rising, this changes abruptly and follows exactly the upper part of the line OKLZ.

far enough to the right, by PDXU or QCZB. It was not possible to detect any difference between the curves DXU and CZB and the upper parts of the curves obtained with x all the time out of circuit. This figure was drawn by superposing several oscillograms, for it is very difficult after one curve has been taken upon the sensitized paper carried by the revolving drum to start another curve some time afterwards at such a point that it shall coincide with the upper part of the first one. This feat has, however, just been accomplished in another case by Mr. John Coulson, who made the records shown in Figures A and B, and has helped me in most of the experimental work of this paper. Figure 38, drawn from another photograph, shows the two curves which coincide in A. The oscillograph was in circuit with the coil of a large toroid of about 41 centimeters in diameter, the core of which is made of soft, varnished iron wire about half a millimeter in diameter. Each record shows a current curve obtained by applying the electromotive force directly to the circuit, and the second part of a current diagram when an extra resistance, at first in the circuit, was suddenly shunted out.

There seems to be in these cases neither a magnetic time lag nor any sensible von Waltenhofen effect.

If an electromagnet has two exciting coils, and if one of them be attached to the terminals of a battery, the form of the battery current

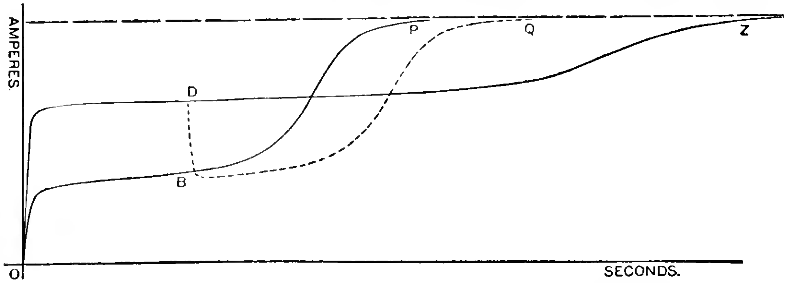

FIGURE 41. The current in one coil of the toroid DN which is in series with an oscillograph and a battery, follows the curve ODZ or the curve OBP, according as another coil wound on the same core is open or closed in itself. When at the point *D*, the second coil which has been open is suddenly closed, the oscillograph record gives the curve DBQ which except at the very beginning can be exactly superposed upon the upper part of the line OBP.

will depend upon whether the second coil is open or closed on itself, and the difference is usually noticeable even when the magnet has a

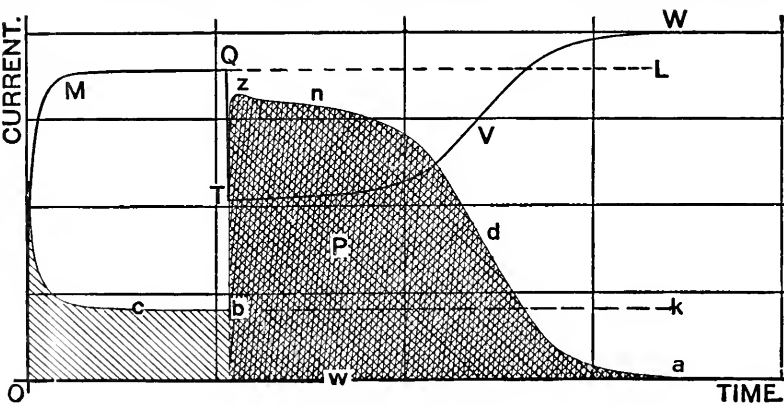

FIGURE 42.

large solid core in which eddy currents are being induced. Figure 39 shows curves taken under the two conditions just mentioned for both the electromagnet TP and the toroid DN. To determine whether the closing of the second coil in the case of the electromagnet where strong eddy currents already existed changed the amount of the final flux through the circuit, Mr. Coulson has measured with great care a number of oscillograms taken with this apparatus, and finds the area between the asymptote and the curve OAM to be 6216 on the scale of his planimeter, while the area above the curve OBN is 6214 on the same scale. The areas above the curves agree within a small fraction of one per cent, as they were expected to do.

Figures 40 and 41 exhibit oscillograms taken with the toroid, DN, under sudden opening and closing of the second coil, and these show no signs of von Waltenhofen effects. Figure 42 gives the records of two oscillographs, one in the primary circuit of a toroid which has a core made of soft iron wire only one tenth of a millimeter in diameter, the other in a secondary coil, when a third coil, wound on the same core, was suddenly closed.

In early experiments upon the phenomenon of the reversal of moment in short rods magnetized in a solenoid, when the current was suddenly stopped, it was observed that if the rod had been previously magnetized permanently in the direction in which the current magnetized it, reversal never occurred, but that it always appeared, under favorable circumstances, if the direction of the previous magnetization was the opposite of that which the current gave it. This and like results has led many physicists to think that the molecules of the iron, when the exciting force due to the current is suddenly removed, return to the positions which they had just before the current acted upon them, but that the motion is so much resisted by frictional forces that the kinetic energy is lost when the particles have swung slightly beyond the positions of equilibrium where they are held by the friction. Wiedemann believed, on the other hand, that when the exciting circuit of an electromagnet is suddenly opened, the rise and decay of the Oeffnungsextrastrom induces in the mass of the iron, currents, alternating in direction and decreasing in intensity, and that the magnetization of a rod due to the original current is reversed in sign, under favorable circumstances, by a weaker current in the opposite direction. In the case of closed rings, where demagnetizing factors are absent, anomalous magnetization seems to appear only when eddy currents in the iron so shield the particles inside the mass that they are never exposed to sudden changes in the intensity of the exciting magnetic field.

My thanks are due to the Trustees of the Bache Fund of the National Academy of Sciences, who have lent me some of the apparatus used in measuring the oscillograms mentioned in this paper.

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THE INTERNAL RESISTANCE OF THE LEAD ACCUMULATOR.

By Harry W. Morse and Ledyard W. Sargent.

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Presented by John Trowbridge, December 14, 1910. Received January 3, 1911.

General Discussion.

1. The internal resistance of a lead accumulator is not a factor of importance in the practical operation of storage batteries. In a small pocket battery it is only a few hundredths of an ohm: in medium sized cells it sinks to a few thousandths, and in large cells, such as are used in regulation and central station work, it is of the order of a few hundred-thousandths of an ohm. In almost every case the cell resistance is so low in comparison with the other resistances in the working circuit that it can be neglected as far as practical calculation is concerned.

But the resistance of a cell and the changes which take place in this factor are of much interest when the behavior of the cell is being studied from a more general point of view. A lead accumulator seems a very simple system indeed at first glance, but it is in fact a very complex one, concealing many problems difficult of solution. While the cell is at rest, fully charged, and therefore containing plates which are good conductors from their center out to the boundary of the electrolyte, its resistance is very nearly that of the acid between these plates, and it can be calculated with fair approximation. But when the cell is working, either on charge or discharge, its condition is most variable. The acid must diffuse through a series of fine channels, and great differences of acid concentration in the different layers of the plates will be set up. Then, too, the particles of lead and lead peroxide, themselves good conductors, will be more or less completely coated with lead sulphate — a bad conductor. The acid concentration will, moreover, be a function of the *rate* at which the cell is working, and the thickness of the lead sulphate coating over a particle of lead or lead peroxide at any given point in the plate will also be a function of the rate.

2. An ideal section of a storage battery of two plates is shown in

Figure 1. *A* is the plate, and its resistance can be assumed to remain fairly constant, except for its temperature coefficient. This factor has a small value within the range of practical operating conditions. *C* is that part of the electrolyte which does not lie very close to the plates. Its total concentration is determined accurately by Faraday's Law and its resistance is a function only of this concentration and of the temperature. *B* is the most active, variable, and interesting part of the cell. It includes the active material, that part of the electrolyte which is in the pores of this material, and that part of the electrolyte which is

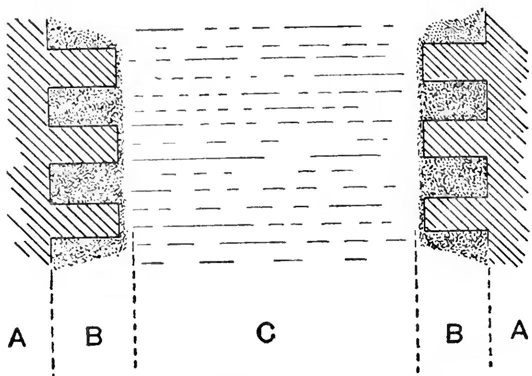

FIGURE 1. Section through storage cell.

near the plates. For ordinary rates large concentration changes can be assumed to extend not more than a millimeter or two from the outside surface of the active material before they are equalized by mixture with the main body of the electrolyte.

When the cell is at rest, its resistance may be considered as being made up of three parts. *A*, metallic in nature, and therefore with a negative temperature coefficient, but constant at a fixed temperature. *B*, also metallic in its nature, and sufficiently low so that the resistance of the electrolyte in its pores can be neglected in comparison with it. *C*, a purely electrolytic resistance, with positive temperature coefficient, but constant at a fixed temperature.

When the cell is working at a constant rate and fixed temperature, *A* remains constant. *C* is a function only of the total concentration of acid in the cell, and can be made constant by using small plates and a large body of electrolyte. The resistance of *B* now becomes more complex, and may be considered to break up into two parts, one a function of the condition of the active material and the other a function of the concentration of the electrolyte in the pores of the active material and in the layer in immediate contact with its surface.

The change in B during charge will, moreover, be wholly different from the change during discharge, for in both cases the changes in the active material *follow the electrolyte.* During charge the active material first to enter into reaction is that at the surface of the plate, provided the plate has not been allowed to "sulphate" so completely that the conductivity through the active mass has been very greatly reduced. Since the action is taking place at the surface the electrolyte does not have to diffuse far through narrow channels. But as the diffusion path increases and the cell becomes more fully charged, concentrated acid is produced in the pores of the mass. In spite of the decrease in resistance due to this better conducting acid it is still the plate itself which does most of the conducting. It is therefore to be expected that the change of resistance during the charge of a healthy storage cell will not be large.

During discharge a very different state of affairs exists. In this case also action begins at the surface of the plate, where electrolyte is available for the reaction. But as discharge proceeds, and the area of activity recedes into the interior of the active material, acid is used up within the plate and the concentration of the active part of the electrolyte decreases. To this loss is added the loss of conductivity of the plate itself, for the particles of lead and lead peroxide in the outer layers have now been covered with sulphate and more or less completely insulated from each other. The result is as if the distance between the plates had been increased, for the plate surface which is actually carrying current is now well back in the interior of the mass of active material, instead of at the actual outside surface.

It should be kept in mind also that the equalization of concentration differences in a working plate is not cared for wholly by diffusion. While this is a potent factor always, it can be shown to be insufficient to account for the facts. A considerable part of the equalization is probably cared for by local concentration cells at various parts of the plate, and these local actions depend for their efficiency on good conductivity of the plate between the points where the concentration differences exist. Any change in the conducting power will affect the rate of equalization of acid and will displace the area of action within the plate.

Viewed in this way we should hardly expect any very large or very rapid changes in cell resistance, nor any of the peculiar maxima of resistance at various points in charge and discharge which appear in some of the older reports on the subject. It is probable that polarization was not eliminated in these measurements.

On long standing, a storage cell may acquire a very high resistance indeed as the result of complete "sulphation." This term means that the active lead sulphate formed during discharge has gradually changed into the crystalline inactive form and that crystals of this form have completely covered the particles of lead and lead peroxide with an insulating coating. Authentic cases of cells of considerable size, with internal resistance as high as 10 ohms, are known. But under the usual conditions of charge and discharge the sulphate retains its "active" state, and even after standing discharged for a month or more no great change in internal resistance is usually to be observed [see § 15].

3. In its ordinary work, a storage cell is discharged only until the plate potential sinks to about 1.7–1.8 volts. This means usually that only about one quarter of the active material in the plates has entered into reaction and that the increased resistance in the active material is due rather to separation of the particles by sulphate coatings than to complete transformation of the active material at any point into insulating material (sulphate). On charge these sulphate coatings and bridges are rapidly broken down, and the decrease in resistance is therefore much more rapid than that corresponding to change in electrolyte within the plate.

After a period of discharge, with corresponding change in resistance, the cell recovers its original e. m. f. along a curve which is somewhat like a diffusion curve [§ 13]. This curve is made steeper by the equalizing effect of local action as explained in 2. It also recovers its original resistance along a somewhat similar curve. These facts indicate clearly the *dynamic* nature of the whole cell activity, for evidently the change in resistance as well as the change in e. m. f. is fundamentally a function of acid concentration and diffusion. The particles of active material cannot have been completely covered by insulating layers, for on standing the plate returned to its original condition as measured by e. m. f. and resistance.

We must evidently think of the particles of lead and lead peroxide as covered with a spongy or powdery layer of lead sulphate, with interstices so small that diffusion cannot overcome the effect of even a small current and its accompanying exhaustion of acid within the pores. As long as no current is flowing, and when recovery has been allowed to take place completely, the total active surface has not been greatly reduced by the changes in the plate, nor has the "active surface" been far removed from its original plane. But the passage of even a small discharge current causes exhaustion of acid in the pores to such an extent that nearly pure water intervenes between the electrolyte and the conducting material of the plate, and the result is a considerable increase in resistance.

While it is true that the concentration and therefore the resistance of the main body of the electrolyte is completely determined by Faraday's Law, the passage of the same quantity of electricity may result very differently in various types of cells. Change in concentration and resistance will be great in those types which have large weight of plates in proportion to their content of electrolyte. They will be small in the types where weight is not a factor to be considered and where a large excess of acid is maintained.

Other factors which may effect resistance will be discussed in connection with the data of this paper.

4. On page 611 will be found a list of references on the subject of the resistance of galvanic cells and accumulators. Many of these papers were written at a time when the difficulty of such measurements was not understood, and it may be said that the research of Nernst and Haagn (1896) was pioneer work, and that they showed for the first time how to eliminate the disturbing factors of the problem. The measurements of Dolazalek and Gahl (1901) are still more accurate and include data on several types of cell and on various rates of charge and discharge.

Our attention has been turned largely toward the temperature coefficients of resistance under various conditions, for we wished to exhibit as clearly as possible the dynamic nature of the phenomena in the lead accumulator. After investigation of the other methods of measurement, we adopted the form of bridge described by Ayres [**20**]. Ayres himself did not use the bridge for measurements on storage cells, nor indeed for any very low resistances of electrolytic nature. But we have found this type of bridge, with slight alteration, to be most satisfactory for the measurement of electrolytic resistances of the order of 0.01 to 0.10 ohm.

The Bridge and Auxiliary Apparatus.

5. The bridge connections are shown in Figure 2, together with the circuit used in charging and discharging the cell and the secondary bridge on which resistances were measured after balance had been obtained. The bridge itself is symmetric, and contains a meter slide wire of manganin of about 0.40 ohm resistance. At opposite ends of this wire connection is made to the source of alternating current and to the two other arms of the bridge. The cell arm contains the cell under investigation in series with a capacity C_1: the other arm con-

tains a similar and nearly equal capacity C_2 and the variable resistance R. C_1 and C_2 are rolled paraffin paper condensers of about 22 microfarads each, composed of 10 units of 2.2. mf. connected in parallel by No. 11 copper wires soldered at each point and also to the bridge connections. The entire wiring of the bridge was non-inductive.

The cell was maintained at constant temperature in a thermostat and was connected by soldered No. 11 wires terminating in a heavy mercury-metal switch-block so arranged that the cell could be placed

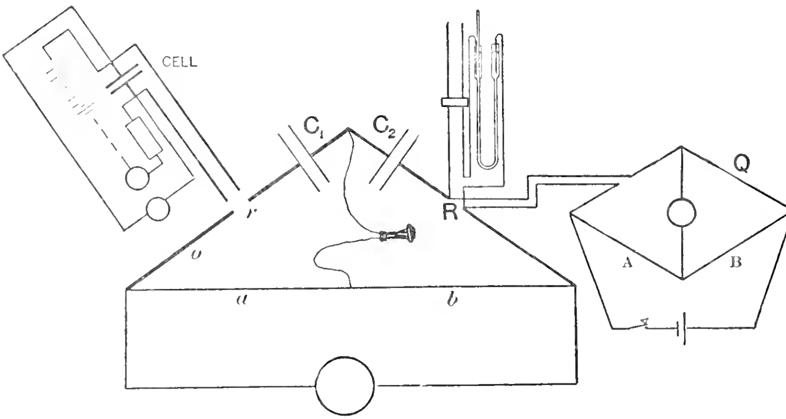

Figure 2. Bridge connections for measuring low electrolytic resistances.

in circuit, or cut out, without change in the block resistance and without short-circuiting the cell. The ¼-inch wires of the block switch were kept continually under mercury whichever way the switch was turned, thus keeping contacts bright when the cell was out.

The variable resistance R consisted of two parts: One of two parallel wires shunted by a heavy sliding block, and the other, in series with this, was a vertical U-tube of glass containing mercury, as shown in Figure 3. The connecting wires dip into the mercury in the two arms and the cross-section of one arm was decreased or increased by lowering or raising the glass rod. This smoothly variable resistance proved to be of the greatest aid in obtaining the close settings desired.

In parallel with R was a box Wheatstones bridge and galvanometer. R could be thrown over into the X-arm of this bridge by means of a mercury switch, but as the box bridge had a resistance of about 1000 ohms it was found unnecessary to cut it out while making settings on the cell, and during the later measurements the switch was removed, leaving the connections as shown in Figure 2. The ratio of the fixed arm of this auxiliary bridge was 1 : 1000 and resistances could be measured on it to 0.0001 ohm.

During the majority of our measurements a resistance of a few tenths of an ohm ((O) in the figure) was kept in the cell arm in series with C_1 and for some measurements of high resistances it was found necessary to introduce a similar resistance in series with the slide-wire and mercury resistances of R.

Measurements were made with "cell in" and "cell out" within as short a time as possible, in order to eliminate any possible changes, and measurements were repeated several times in each case. We were thus measuring the difference between [cell + R_1] and R_1, where R_1 is the resistance of O plus the connections in that arm.

Various sources of alternating current were tried, but none was wholly satisfactory. We had no source of pure sine-waves at our disposal and some trouble from harmonics was experienced. This was removed by using a transformer and various combinations of capacities in the circuit.

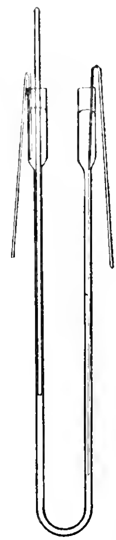

FIGURE 3. Variable mercury resistance.

THEORY OF THIS TYPE OF BRIDGE.

6. The complete theory of this type of bridge may be found in Ayres's paper [20], but our method of operation was necessarily somewhat different from his because of the small magnitudes of the resistances we had to measure.

For the ideal condition of balance we have

$$r/R = a/b = C_2/C \tag{1}$$

where the bridge is non-inductive, the resistances of connections are negligible, a and b are the bridge-wire readings, C_2 is the capacity of the "known" arm, C is the resultant of the capacity of the cell with C_1, r is the resistance of the cell and its leads, and R is that of the opposite arm of the bridge.

Taking into consideration the resistances of the various connections, and that of the coil o inserted in the cell arm, we have, for both bridges, the following equations :

$$\frac{a}{b} = \frac{o + r + u}{R + v}, \tag{2}$$

where u and v are the resistances of the connections on the cell side and the R side respectively;

$$(3) \qquad R + w = Q\frac{A}{B} = Q\,(0.001) = q,$$

where w is the resistance of connection to the auxiliary bridge. Combining (2) and (3) gives

$$(4) \qquad o + r = (q - w + v)\frac{a}{b} - u.$$

Upon cutting out the cell and short circuiting that arm of the bridge, we have

$$(5) \qquad o = \left(Q'\frac{A}{B} - w + v\right)\frac{a'}{b'} - u = (q' - w + v)\frac{a'}{b'} - u.$$

Subtracting (5) from (4) gives

$$(6) \qquad r = \frac{a}{b}\left(q - q'\frac{a'b}{ab'}\right) + \frac{a}{b}\left(\frac{a'b}{ab'} - 1\right)w - \frac{a}{b}\left(\frac{a'b}{ab'} - 1\right)v.$$

During the first part of the investigation a lay between the limits 506 mm. and 507 mm., and during the second part between 481 mm. and 486 mm. In most cases the difference between a and a^1 was less than 0.5 mm., and it was usually about 0.3 mm.

In view of this, the terms containing w and v can at once be neglected. The difference between $q - q'$, the value actually recorded as resistance, and $(q - q')\frac{a'b}{ab'}$, is $(\frac{a'b}{ab'} - 1)\,q'$, which does not exceed 0.0005 ohm, since q' did not exceed 0.25 and the value of the parenthesis was in no case greater than 0.002.

The following observations are deemed interesting in connection with the fact that the bridge reading was not the same with the cell in as when the cell was out.

1. The bridge reading with "cell in" a is smaller than a'.
2. Both with "cell in" and "cell out," the insertion of a condenser (5.2 MF) in parallel with the leads to the cell-switch causes an increase in the value of a or a' as the case may be.
3. Decreasing the frequency by 50% causes no change in a, but a' is thereby decreased.
4. A condenser in parallel with C_1 causes a and a' to decrease.

The last observation is to be expected from the theory of the bridge. It is hard to see how any capacity possessed by the cell could act so as to make a smaller than a'; rather the opposite effect would be expected. The supposition of an inductance in the cell would explain this change in the bridge reading, but a change in the frequency did not affect a, while it did affect a'.

The explanation would seem to be as follows: The configuration of R suggests inductance, and an approximate calculation shows $2\pi n L_2$ to be about 0.01. Now the value of $1/2\pi n C_2$ is about 5. A change of 0.5 mm. from a to a' corresponds to a change of 0.01 in $1/2\pi n C_2$. Calculation gives about 0.01 for $2\pi n L$ for the leads from the cell-switch to the cell, corresponding to $2\pi n L_2$ for the C_2 arm of the bridge. From these considerations it would appear that a rather than a' is more nearly the point where $C_2/C = a/b$, for, with "cell in," the inductive reactance of the C_2 arm is balanced by that of the cell leads.

Since C_1 and C_2 are about the same, and since C_2 has an inductance with it, while C_1 has not, the reactance of the C_2 arm will increase with decrease of frequency faster than the reactance of the C_1 arm; in short, the reactance of the C_2 arm will become greater than that of the C_1 arm, and a' will diminish with decrease in frequency. On the other hand, with "cell in," the two reactances will change equally with change in frequency, that is, a will not change.

7. From the average of all our measurements we have compiled the values for the open circuit resistance of our cell at various temperatures. They are as follows:

TABLE I.

Open Circuit Resistance of Planté Plates at Various Temperatures.

Temperature.	Resistance.	Temperature.	Resistance.
0.0 C°	0.0686	25.0	0.0448
8.5	0.0579	30.0	0.0420
10.0	0.0562	45.0	0.0362
20.0	0.0483	55.0	0.0335

These values are plotted in the full-line curve of Figure 4. In the dotted-line curve on the same figure are given the values for sulphuric acid of the same concentration as that used in our cell. We have chosen the two points indicated by larger circles as the points from which to calculate the remainder of the curve, which is plotted from the empirical formula

$$K_t = K_o\ (1 + ct + c^1t^2)$$

It will be seen that the cell resistance and the acid resistance fall very closely together from zero to 30°. Above this point the cell re-

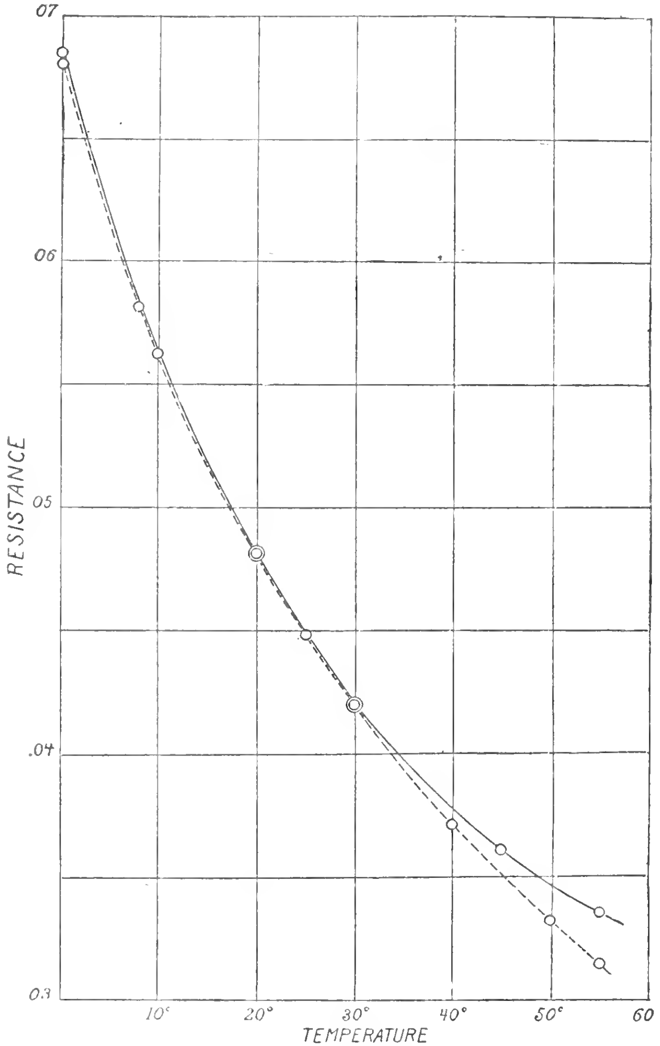

FIGURE 4. Temperature-resistance curves for electrolyte and cell.

sistance does not decrease as rapidly with rise of temperature as does the acid. As a matter of fact the two points at zero do not fall together. So it is evident that both at high and low temperatures the cell resistance is higher than that calculated for the pure acid. This figure indicates merely the shape of the two curves with reference to each other, and by no means indicates that the resistance of the cell is the same as that of sulphuric acid of the same concentration between a pair of platinum electrodes placed in the same geometrical position as the "average plate surfaces." The plate resistance is not negligible and its negative temperature coefficient may account for the higher resistance at the higher temperature in part at least. That the resistance of the plate itself enters is clearly shown by the fact the pasted

TABLE II.

Internal Resistance of a Planté Cell during Discharge at 2 Amperes. Temperature 8.5° C.

Time.	Cell in.	Cell out.	Cell.	Voltage.
0	.2882	.2304	.0578	{ 2.09 open-circuit { 1.85 after closing
10	.2895	.2317	.0578	1.83
22	.2926	.2306	.0620	1.77
37	.2961	:2316	.0645	1.70
52	.3080	.2313	.0767	1.55
62	.3142	.2316	.0836	1.36
71	.3158	.2315	.0843	1.18
77	.3160	.2316	.0844	0.97
81	.3164	.2315	.0849	0.80
86	.3191	.2314	.0877	0.63
96	.3656	.2315	.1341	0.30 [reversed]

plates of slightly greater area, placed as nearly as possible the same distance apart, show a decidedly greater resistance on open circuit than that of these Planté plates, which have a solid lead web and lead ribs running their whole length. The cells with pasted plates have nearly .01 ohm more resistance.

8. The values of Table 2 are plotted in the upper curve of Figure 5, and they show the characteristic course of changes in this particular type of plate. The cell changes its internal resistance a little more than 100 percent during complete discharge, and by the time the last value on the list is reached the cell voltage has dropped to zero.

The most interesting thing about the curve is the flat place which occurs after 60–80 minutes of discharge. This is quite characteristic of Planté plates with ribs, but it has never been noticed before by any of the observers who have worked on storage cells. Indication of such a flat place may be seen in some of Haagn's curves, but it would seem that he thought them due to errors of measurement.

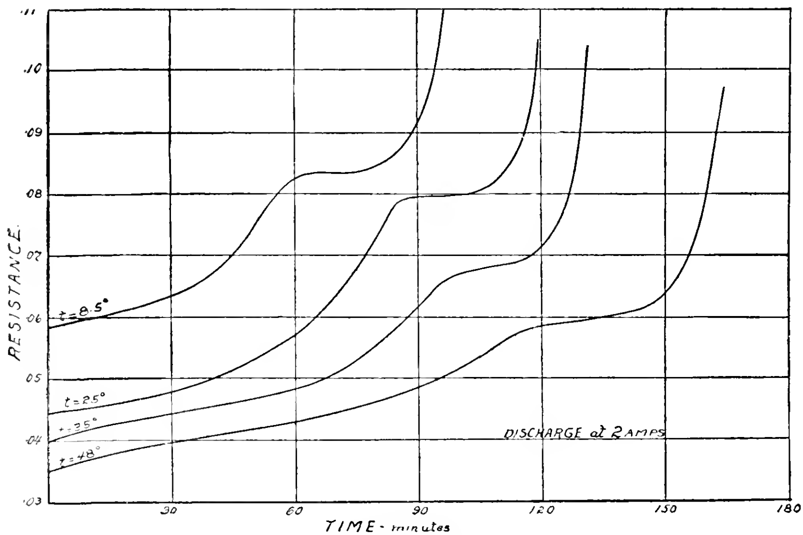

FIGURE 5. Curves of resistance of Planté cell during discharge at various temperatures.

The reason for the appearance of such a change in curvature seems quite obvious. These plates are of the Gould type, and they have ribs spun from the lead of the originally flat plate. These ribs are changed by formation to form active material, which lies close to the ribs at their tops but which forms a solid mass down toward the center web of the plate. During the first part of the discharge the electrolyte finds active material on the ribs and diffuses largely into the almost open space between them. As this material is used up the action moves further into the plate and presently reaches the mass of material which fills the bottom of the grooves. Here for a time there is material enough at a practically constant distance from the surface of the plate to supply the action, and when this is used up the resistance rises very rapidly and the plate potential shows that the cell is completely discharged.

TABLE III.

Planté Cell. Discharged at 2 Amperes.

Time.	Cell.	Voltage.	Time.	Cell.	Voltage.
Temperature 25° C.					
Open circuit	.0448	..	76	.0690	1.62
0	.0448	..	92	.0796	1.43
8	.0453	1.89	103	.0800	1.28
15	.0472	1.88	110	.0823	1.16
32	.0481	..	120	.1061	0.26
48	.0527	1.78	126	.1365	0.00
62	.0570	1.70			
Temperature 35° C.					
Open circuit	.0400	..	94	.0650	1.57
8	.0399	1.92	105	.0678	1.42
19	.0430	1.91	110	.0682	1.38
33	.0452	1.87	121	.0720	1.25
49	.0465	1.84	127	.0800	0.65
67	.0492	1.77	134	.1300	0.30
82	.0579	1.70	147	.1901	0.00
Temperature 48° C.					
Open circuit	.0351	..	112	.0570	1.65
6	.0358	1.96	120	.0589	1.57
23	.0395	1.95	131	.0598	1.52
37	.0406	1.90	136	.0600	1.47
46	.0420	1.87	144	.0620	1.37
63	.0431	1.84	153	.0653	1.00
79	.0460	1.80	159	.0945	0.52
92	.0488	1.76	173	.1757	0.12
103	.0520	1.70	178	..	0.00

As will be seen later, paste plates show no such flat place in their curves. [§ 11.]

The values for this Planté cell, discharged at 2 amperes at various temperatures [Table III], are all plotted on the curves of Figure 5. The corresponding voltage curves will be found in Figure 6.

The characteristic points appear in each curve, and the flat place moves toward a later point in the discharge curve, as might be ex-

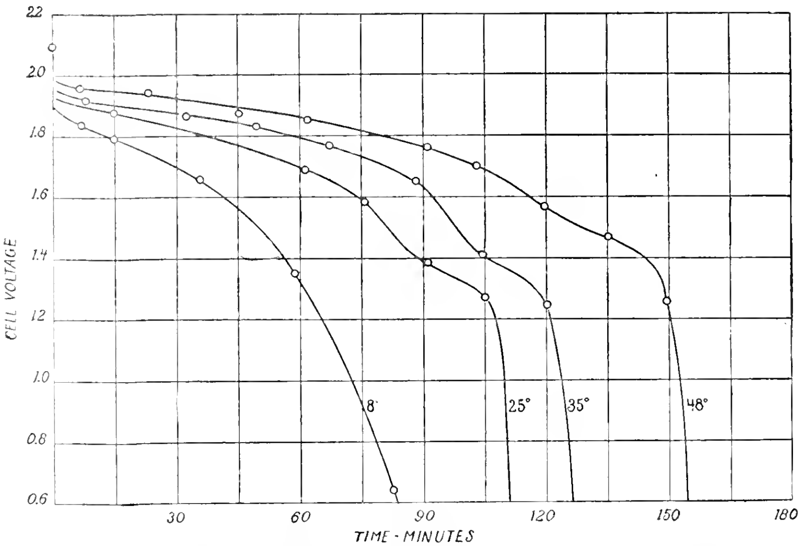

FIGURE 6. Voltage curves of Planté cell during discharge at various temperatures.

pected, when the temperature is raised. This means merely that the material of the plate can be better reached and utilized at the higher temperature, and that therefore more of the active material on the ribs enters the reaction, leaving the mass of material at the bottom of the grooves for a later period of the discharge.

It is by no means easy to be sure that the cell has reached the steady state corresponding to any given temperature, except by maintaining it at the new temperature for several hours. The lag of resistance behind its final value when temperature is quickly changed is very noticeable and caused us much trouble. We finally found it best to keep the cell for at least six hours at the new temperature before trying to take a discharge curve.

It will be noticed that the curve for 25° C. does not fit in very well with the rest of the set. Most of our measurements at this temperature were made at the beginning of the work, and it is quite possible that some lag error has caused this slight variation. The values for resistances at 8.5° were still harder to fix, but the curve given is the average of so many measurements that its correctness is fairly certain. At the higher temperatures the lag becomes much less troublesome. The explanation which suggests itself is that this slow change corresponds

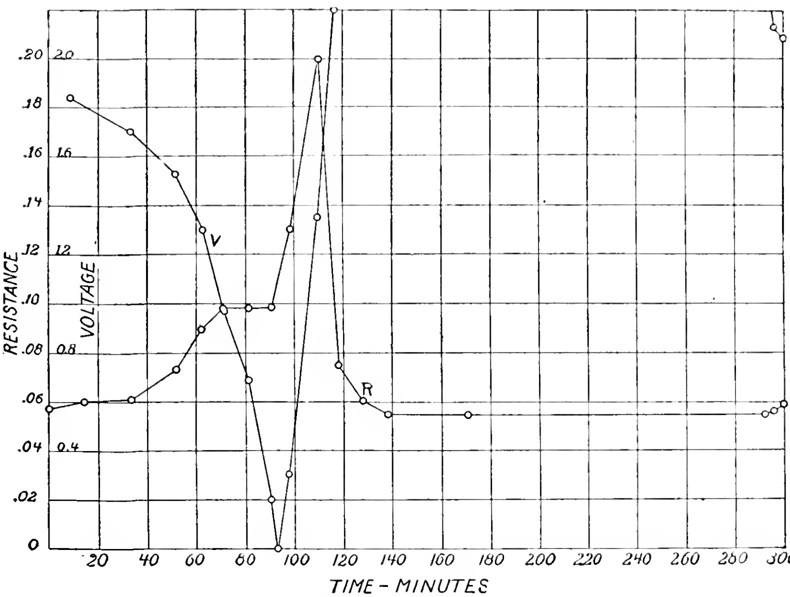

FIGURE 7. Resistance and voltage curves for a Planté cell during complete discharge and reversal.

to a chemical reaction. Probably lead sulphate is dissolved as the temperature is raised and precipitated as it is lowered. This would be just such a process as would result in the slow adjustment of the cell resistance to a new temperature.

This explanation is made more probable by some experiments we have made in an ordinary conductivity vessel with platinum electrodes, filled with sulphuric acid from our cell. Some pieces of lead and lead peroxide from a partially discharged cell were placed in the vessel and the conductivity followed after a change in temperature. The lag of resistance behind temperature is just as noticeable in this arrangement as in the more complicated cell under investigation, and there seems to be no doubt that the explanation given is the correct one. A similar lag in the e. m. f. following a change in temperature is to be expected, and this has been noticed by Dolazalek during careful determination of the temperature coefficient of the e. m. f. of a lead storage cell.

The curves of cell voltage during discharge are given in Figure 6, and the flat place in the curve is evident in all except the curve for 8.5°. It is possible that we missed it in this case by not taking points

TABLE VI.

COMPLETE REVERSAL AT 2 AMPERES. TEMPERATURE 8.5° C.

Time.	Cell.	Voltage.	Time.	Cell.	Voltage.
Open circuit	.0577	. . .	110	.2000	−1.36
8	.0585	1.83	118	.0751	−2.30
14	.0600	1.78	128	.0604	−2.33
33	.0610	1.70	138	.0554	−2.35
51	.0739	1.53	147	.0571	−2.37
62	.0904	1.30	171	.0545	−2.44
71	.0980	0.97	293	.0541	−2.56
81	.0980	0.69	Opened	circuit	
91	.0986	0.20	296	.0564	−2.13
93	. . .	0.00	300	.0586	−2.08
98	.1302	−0.31 Reversed			

close enough together, as it is quite evident in the curve of Figure 7, which was drawn from another run at the same temperature.

9. Two criteria have long been considered most pertinent as describing the condition of a storage cell. One of these is the density of the electrolyte, and if care has been taken to keep this density right, it is possible to judge accurately of the condition of the cell by an examination of the electrolyte at a given point in the cell cycle. Usually the density is measured at full charge. It could as well be measured at the end of discharge or at any other known point in the cycle, provided the curves describing the relation between density and voltage

for a given current and at constant temperature were accurately known.

The other criterion of the condition of the cell now in use is the plate potential. In order that this may give useful information it is necessary to have a discharge curve, on which are plotted the cell voltages at various times of discharge at constant current and constant temperature. Once in possession of such a curve, measurement of

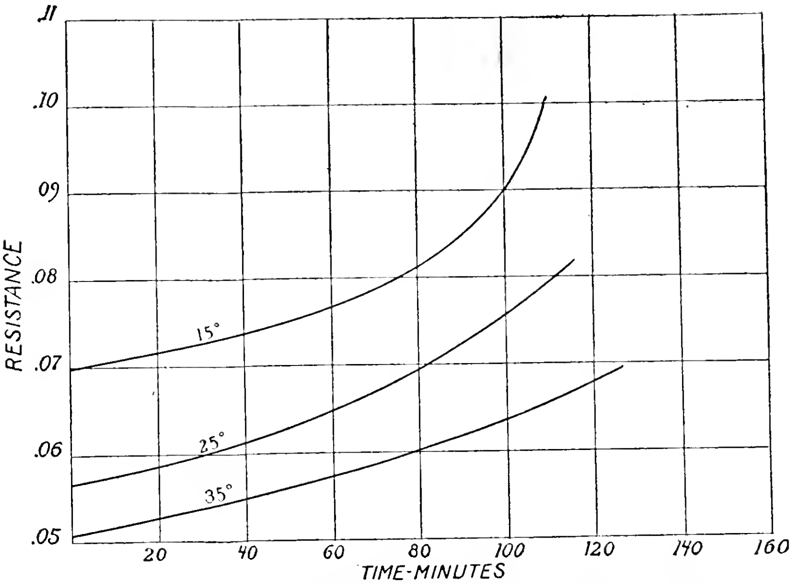

FIGURE 8. Curves showing resistance of paste plate cell during discharge at various temperatures.

cell voltage, at a known time after discharge at constant rate has begun, gives accurate information as to the condition of the cell.

It is evident that a measurement of the cell resistance serves equally well to determine the condition of the cell, for the resistance-time curves of Figure 5 are equally as characteristic as the voltage-time curves of Figure 6. The greater difficulty of measurement of cell resistance will probably prevent any practical application of this fact.

Reversal of a Planté Cell.

10. It was considered a matter of interest to follow a cell at one temperature through complete discharge and then on to more or less complete reversal. If the explanation given on page 591 is correct, it

was to be expected that the resistance would decrease very rapidly after passing through a maximum near the end of complete discharge of the cell, and that the e. m. f. would also reverse rapidly and soon attain nearly its maximum with sign opposite the normal direction in the cell. Table VI. and Figure 7 show how well this is realized:

The data of the above table is plotted in the curves of Figure 7, and

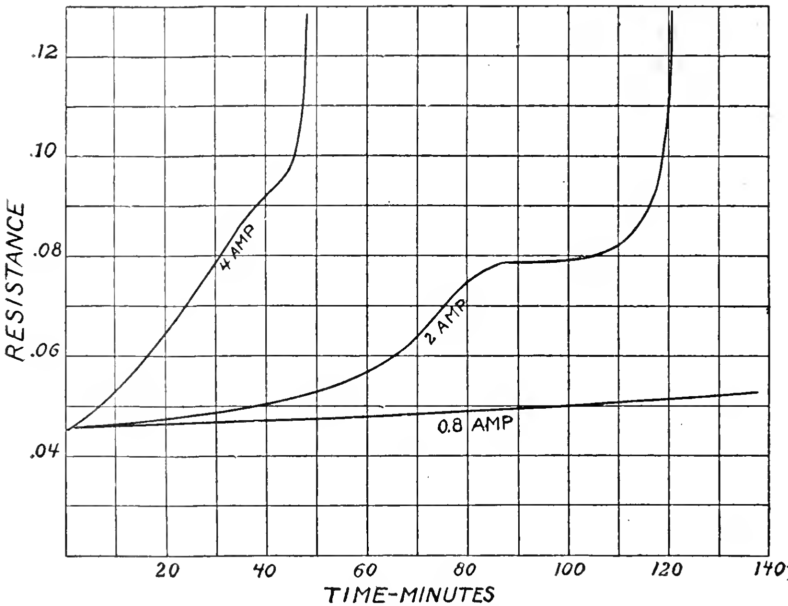

Figure 9. Resistance of Planté plate cell at various discharge rates.

they indicate qualitatively but clearly the course of the changes in the cell. It should be mentioned that the sharp point at the maximum of resistance (about .20 ohm) is only a rough approximation, as it was quite impossible to follow the change in this portion of the curve rapidly enough to insure accuracy. It is, however, quite certain that the point given is lower than the true maximum value.

Pasted Plates.

11. New pasted plates were thoroughly charged and put through several cycles until their capacity was approximately constant. They were then set up in a cell with separation of the plates as nearly as possible that of the Planté cells. The curves given in Figure 8 show the course of the resistance during discharge at 2 amperes at various

temperatures. The points of special interest to be noted are the higher resistance of the cell and the smoothness of the curves. These pasted plates were made to have as nearly as possible the same capacity as the Planté cell at the 2-ampere discharge rate.

12. The course of the resistance of our Planté cell at constant temperature, but at various rates of discharge, is clearly shown by the

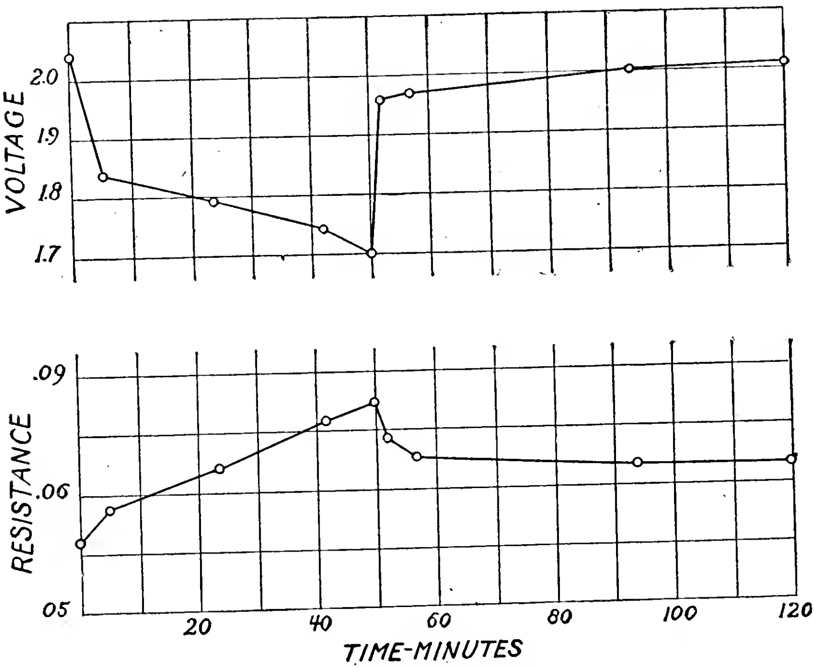

FIGURE 10. Voltage and resistance during partial discharge followed by recovery.

curves of Figure 9. It should be noted that these curves could be used as criteria of cell condition quite as well as voltage curves — and that the cell capacities corresponding to the different rates are clearly given by a statement of the cell resistance, corresponding to the rate in question, at which discharge must be stopped.

RECOVERY.

13. Curves showing resistance and voltage during partial discharge and subsequent recovery on open circuit are given in Figure 10. These curves are characteristic of the general shape of all recovery curves.

Overcharge.

14. The resistance during overcharge seems to be a most variable factor. It rises and falls a thousandth of an ohm or so in one cell but has no regular course.

The disturbing factor must be the escaping gases as there is no other action going on in the cell.

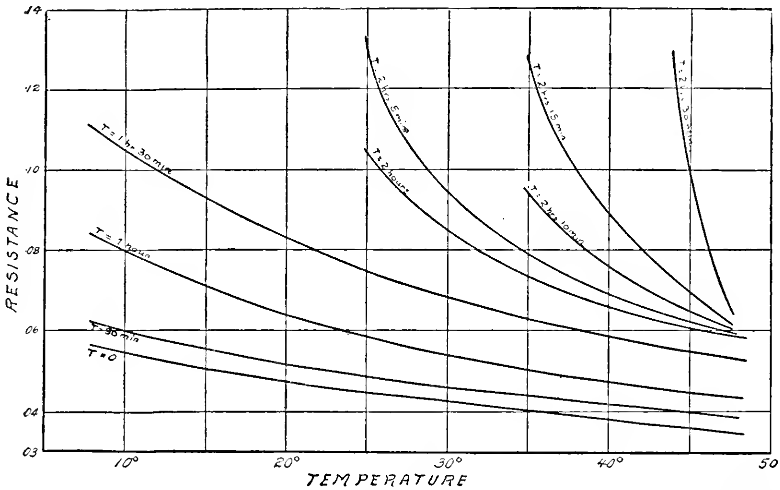

Figure 11. Temperature effect on resistance after various periods of discharge.

Sulphated Plate.

15. Measurements were made over a period of a month or more on a nearly discharged plate. It showed no characteristic course whatever but varied from day to day over a range of several thousandths of an ohm, rising slightly during the first three weeks and then falling again. We evidently did not arrive at a sulphated condition in the usual understanding of the word.

The Temperature-effect on Resistance.

16. The curves of Figure 5 show the course of change of resistance during discharge at constant rate and constant temperature. They are isothermals. From these it is possible to construct *isochronal* curves of resistance *vs.* temperature, the isochronal curve corresponding to the fact that the same total amount of active material has been effected in each case, but at a different temperature. These curves are of special

interest because they show how *dynamic* a thing the momentary equilibrium in such a cell must be. A set of these derived curves is given in Figure 11. The points on the curve T = 0 are for open circuit and they give the temperature-resistance curve of Figure 4. The points on the curve T = 30 are resistances taken from the isothermal curves as they are cut by a vertical line at 30 minutes of discharge, and so on for the other curves.

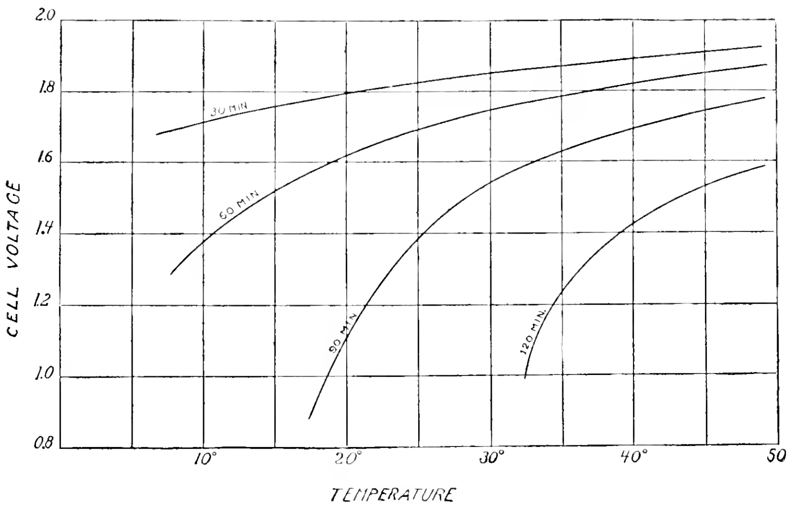

FIGURE 12. Temperature effect on voltage after various periods of discharge.

The slope of the line T = 0 gives the temperature-coefficient of resistance at the temperature corresponding to the point where the slope is determined. The slope at any point on one of the other curves is the temperature coefficient corresponding to the temperature where the slope is taken, but for all the curves except T = 0 the condition of the cell is one of dynamic momentary equilibrium. The slopes are nevertheless temperature coefficients corresponding to this particular state in the cell and they are of great interest in disclosing the condition of the cell. In fact it would be hard to find a more striking expression of the complex system under observation than that given by these curves.

The change in temperature-effect is enormous. At T = 0 the coefficient is about 1.5% per degree C. At T = 150 minutes the coefficient has reached 23% per degree C

The Temperature Effect on Cell Voltage.

17. It must also be true that the isochronal lines of temperature and cell voltage will show a somewhat similar course. For while the resistance is a more complicated function of cell condition than the e. m. f. both factors must be largely determined by diffusion if final analysis were possible. Sections have been made of the isothermal voltage curves of Figure 6 at various times during discharge, and these

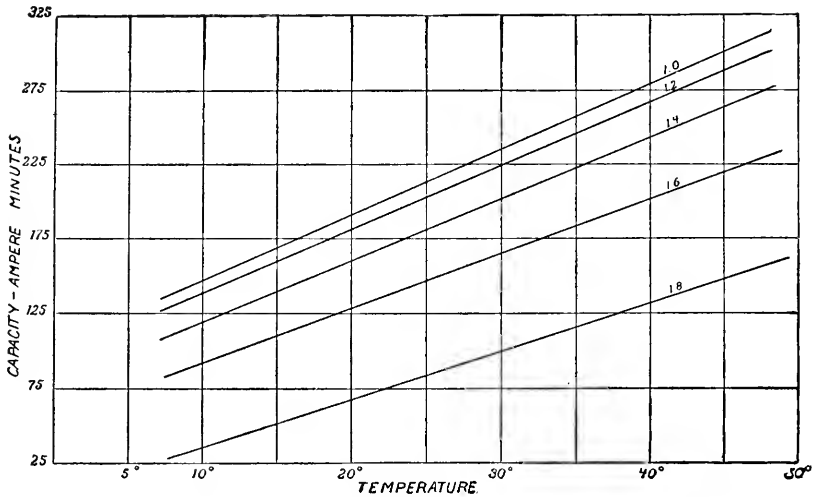

Figure 13. Temperature effect on capacity. Planté plates. One hour rate of discharge.

are plotted in the curves of Figure 12. No attempt has been made to determine the e.m.f. of the fully charged cell at various temperatures.

At 35° C. the coefficient of a cell which has been under discharge for 30 minutes is about 0.006 volts per degree, while that of a cell which has been running at the same rate for two hours is 0.62 volts per degree. [The open circuit temperature coefficient for this acid concentration is about .0003 volts per degree (Dolazalek).]

It should be kept in mind in considering this and the results indicated by the curves of Figure 11, that the cell contains throughout the course of each of the curves, exactly the same amount of each of its constituents. The same quantity of electricity has passed through it, and equal amounts of lead, lead peroxide and lead sulphate are present. The total amount of acid in the whole bulk of the electrolyte is also the same in each case. The only difference is in the distribution of the materials in the cell.

Temperature Coefficient of Capacity.

18. From the data of these measurements it is also possible to calculate the temperature coefficient of the capacity of this type of plate. The values so obtained are plotted in the curves of Figure 13. It will be seen that they are nearly straight lines, and that they remain straight for various end voltages. In practice such plates would hardly ever be discharged below 1.6 volts at this rate, and the temperature coefficient of capacity, for plates discharged to this point, is about 3.5 percent per degree. This coefficient is only applicable to plates of this particular type, and to this discharge rate. For other plate types and various rates the temperature coefficient of capacity has been found to vary from 1.5 to 3.5 percent per degree.

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NOTES ON THE ELECTRICAL CONDUCTIVITY OF ARGENTIC SULPHIDE.

By Hammond Vinton Hayes.

NOTES ON THE ELECTRICAL CONDUCTIVITY OF ARGENTIC SULPHIDE.

By Hammond Vinton Hayes.

Presented February 8, 1911. Received January 18, 1911.

The peculiar action of the sulphide of silver as a conductor of electricity was described by Faraday [1] in 1833. Later Hittorf,[2] in 1851, repeated Faraday's experiments and confirmed the results of the earlier work. In 1902 Streintz,[3] in a study of the conductivity of compressed powders, investigated the behavior of compressed silver sulphide in an electrical circuit and found all of the phenomena described by Faraday and Hittorf. All of these observers attribute to silver sulphide a negative coefficient of resistance for heat, Faraday speaking of it as "an extraordinary case . . . which is in direct contrast with the influence of heat upon metallic bodies."

I have recently conducted a long series of experiments upon the electrical resistance of compressed silver sulphide and, as a result of my observations, am led to the belief that many, if not all, of the phenomena described by earlier experimenters may be explained as due primarily to the condition of the contacts between the specimen of silver sulphide and the electrodes used to connect it into the electrical circuit.

In order to study the behavior of this material I have found it convenient to consider the specimen and its electrodes during three distinct stages or phases of their condition. These will be briefly described.

1st Condition. When the specimen is first placed in circuit it has an extremely high resistance. Upon heating the specimen its conductivity increases with extreme rapidity and, when the source of heat is removed, the conductivity frequently falls to nearly that which originally existed. As an illustration of this action the figures given by Streintz of test upon a sample may be quoted. "At a temperature of 100° the specimen showed a resistance from 3,000 to 1,000 ohms; at 150°, from 400 to 150 ohms; at 220° the resistance measured only from .1 to .2

[1] Faraday's Experimental Researches in Electricity, Vol. 1, 432 et seq.
[2] Hittorf, Pogg. Ann., Vol. 84, 1851.
[3] Streintz, Ann. der Physik., Vol. 3, 1900, and Vol. 8, 1902.

of an ohm; and from 230° upward, it appeared to have practically the same resistance." As also noted by Faraday the application of external heat is not necessary, as by the use of a source of current of sufficient voltage the requisite current can be obtained to heat the specimen sufficiently and "break down" its initial resistance.

2nd Condition. After the initial high resistance has been broken down and a current of some strength has passed, I have found no case where the specimen attained its original high resistance. Usually, after cooling, the resistance is but a fraction of that which originally existed but is practically always several times greater than that which exists when the specimen has become heated by a flow of current through it. This stage, where the specimen shows a gradual lowering of apparent resistance with increasing current or an increase in apparent resistance with a diminishing current, but never returns to the abnormally high initial resistance, is termed the second condition of the test.

3rd Condition. After the current has passed through the specimen for some time, — or usually more rapidly if the direction of the current has been reversed,— the characteristics of the sulphide of silver specimen in its second condition are no longer found, and the resistance of the specimen changes little with changes in current strength or with heat. This peculiarity of what is here termed the "third condition" was described by Faraday, who also noted that the properties of the specimen described under the second condition "could not be renewed until a fresh surface of the sulphuret had been applied to the positive pole. This was in consequence of peculiar results of decomposition."

Streintz's tests were made upon small cylinders of compressed silver sulphide approximately two centimeters in length and fifteen hundredths of a square centimeter in cross-section. Electrodes of platinum foil were placed at the two ends of the cylinder and two supplementary electrodes were embedded within the specimen. In my experiments plates of compressed sulphide were used, which were rectangular in shape and roughly three-quarters of an inch in length by five-sixteenths of an inch in width; the average thickness of the plates was about one thirty-second of an inch. Electrodes of platinum, silver and copper were used both in the form of fine wires or of foil. The terminals were pressed into the plates of silver sulphide and the tests conducted both while the specimen and its terminals were in the press and after they had been removed. Many of the specimens studied by me had been prepared by subjecting them to a pressure of 500,000 pounds to the square inch. The pressure used in forcing the terminals into the specimen was approximately 25,000 pounds.

The electrical measurements were made by the use of a Weston milliammeter and voltmeter, included in circuit with the specimen, and a battery having an electro-motive force of 20 volts. The voltmeter was connected about the terminals of the specimen and the current was controlled by the use of resistance coils placed in the direct circuit with specimen, ammeter and battery. A telephone was also so arranged that it could be connected with the terminals of the specimen under test.

An extremely large number of measurements was made on many specimens under varying conditions. The characteristics of this material, as described above, were found throughout the tests and agree with those found and recorded by former experimenters. The use of the telephone in my experiments, however, has, I believe, thrown some new light upon the action which is taking place within the specimen of sulphide of silver during the passage of the electrical current through it.

Consideration of Condition I. I have frequently found that specimens had an initial resistance so high that with 20 volts no measurements of current could be made with the instruments at hand. When connected with a 110-volt lighting circuit there was an immediate fall in resistance and thereafter the 20-volt source sufficed for current measurements. Placing a heated soldering iron in the vicinity of the specimen would frequently be equally effective in lowering the initial resistance of a specimen.

The following table gives characteristic current and voltage readings of specimens when in the first condition.

TABLE I.

Voltage at Terminals.	Current Amperes.	Apparent Resistance.
18.	.06	300. ohms.
17.	.28	61. ohms.
18.	.34	53. ohms.
16.	.38	42. ohms.
4.5	.85	5.3 ohms.

This action appeared to be equally conspicuous with platinum, steel, silver or copper electrodes. During the time that the resistance of this specimen was high, violent "sputtering" or "burning" noises were audible in the telephone, and these noises entirely ceased at the time of making the last measurement recorded above. The noises heard in the telephone were characteristic of those frequently present when a current is passing through an imperfect contact. The subsidence of these noises was indicative of the formation of more perfect contacts.

The existence of these burning noises shows the presence of an imperfect connection between the specimen and one or both of its electrodes and that arcing is taking place at one or both of these places. An examination of all specimens tested has shown that violent action of some kind has taken place at the point of connection between the sulphide pellet and the electrode connected with the positive pole of the battery; the surface of the specimen has been blackened and some of the fine wires of the electrode have been burned away. On the other hand, the negative ends of the specimens and the electrodes by means of which they have been connected with the negative pole of the battery have been invariably bright and clean.

The probable explanation of the physical action which takes place during the first condition is that despite the close mechanical association of the electrodes and the specimen there is no electrical connection, and an arc must take place between one or both electrodes and the specimen before a current can pass. To produce such an arc requires ionization either by heat or by a sufficient difference of potential at the point or points of discontinuity. It appears from all tests that this action takes place most readily at the negative electrode of the specimen, and that gradually a more or less perfect connection is established at this point. When this action takes place the excessive apparent resistance of the specimen is broken down, and the specimen may be considered as passing from the first into the second condition. In support of this explanation of this phenomenon, the following tests may be described.

The direction of the current through a specimen, the resistance of which had been broken down, was reversed. The noises in the telephone which previously had been present were no longer apparent, and after a short time the changes in resistance characteristic of the second condition ceased to exist and the specimen passed into the third condition. Again, a specimen which was in the third condition had one of its electrodes removed and, after the surface of the specimen had been carefully cleaned where the electrode had previously been, it was clamped under great pressure in the jaws of a steel vise with soft copper surfaces. The figures obtained in this test are given below.

From these figures it will be seen that a perfect connection had been established between the original terminal and the sulphide, but that a good connection had not been made at the terminal which had been clamped in the vise. During the test, while the current was passing from the vise to the sulphide, the noise in the telephone was very great, but when the current was passing through the imperfect connection in the opposite direction no noises could be detected. After the

TABLE II.

Current from Vise to Ag_2S.			Current from Ag_2S to Vise.		
Voltage at Terminals.	Current Amperes.	Apparent Resistance.	Voltage at Terminals.	Current Amperes.	Apparent Resistance.
1.10	.210	5.2	.5	.210	2.4
2. to 3.	.350	..	.48	.390	1.2
1. to 3.	.520	..	.5	.540	.9
2.	.650	3.			
2.5 to	.850 to	..			
3.5	.900	..			

current had passed for a short time a good connection was established at the terminal in the vise and the third condition of the action was established.

What actually takes place at the negative electrode I have not determined. Both Faraday and Hittorf were inclined to the belief that conduction through the sulphide of silver is electrolytic, and Hittorf states that silver was found by him at the junction with the negative electrode. Hittorf also found that, in similar tests made with the oxide of copper, antennae of copper were thrown out from the specimen at the negative pole.

Consideration of Condition II. The second condition exists when a more or less perfect connection has been established at the negative electrode and the contact at the positive terminal is still faulty. All tests of specimens in this condition show the characteristics of measurements of currents and potentials at imperfect contacts in an electrical circuit. The really remarkable peculiarity of the silver sulphide is that contact cannot be established even when the electrodes and the specimen are pressed together under a weight of many tons.

Attention might be called to the fact that there is present a condition of "unilateral" conduction such as has been extensively studied by Professor Pierce,[4] the magnitude of the action being dependent upon the degree to which perfect contact has been obtained at the negative terminal of the specimen. This differential action is shown in Table II., and Table III. given below as a fairly characteristic set of measurements made upon specimens in the second condition. It should be noted that the repeated reversals of current gradually establish perfect connections at the two terminals of the specimen, and thus rapidly obliterate the unilateral conduction and tend to establish the third condition.

[4] Pierce, Physical Review, **25** (1907); **28** (1909).

TABLE III.

			Current Reversed.		
Voltage at Terminals.	Current Amperes.	Apparent Resistance.	Voltage at Terminals.	Current Amperes.	Apparent Resistance.
.42	072	5.8	.27	.072	3.7
.54	.110	4.9	.36	.110	3.2
.82	.200	4.1	.54	.202	2.6
1.08	.310	3.5	.72	.310	2.3
1.04	.350	3.0	.70	.350	2.0
1.04	.405	2.5	.80	.410	1.9
1.16	.480	2.4	.90	.500	1.8
1.38	.640	2.1	1.04	.640	1.6
1.60	.850	1.8	1.30	.830	1.5
1.90	1.350	1.4	1.60	1.300	1.2

It is this second condition which has been specially studied by previous experimenters, and it has been due to the measurements made while the contacts between the electrodes and the specimen were in the condition above described that the negative coefficient of resistance for heat has been attributed to silver sulphide. Increasing temperature of the specimen or increasing difference of potential at its terminals simply promotes the ionization at the imperfect contact or contacts and permits a greater flow of current.

Consideration of Condition III. The condition which I have called the third is that in which fairly good connections have been established between both electrodes and the specimen, and is characterized by an absence of noise in the telephone, with the current passing through the specimen in either direction. The figures given in Table IV. are those obtained from continued tests of the sample, the apparent resistances of which are given in Table III. after it had passed into the third condition.

TABLE IV.

Voltage at Terminals.	Current Amperes.	Apparent Resistance.	Voltage at Terminals.	Current Amperes.	Apparent Resistance.
.08	.072	1.11	.08	.072	1.11
.122	.110	1.11	.122	.110	1.11
.36	.31	1.16	.35	.305	1.15
.58	.500	1.16	.58	.500	1.16
.74	.63	1.17	.74	.63	1.17
1.00	.85	1.17	1.00	.85	1.17

The characteristics of the second condition, it will be seen, have entirely disappeared, and the resistance with increasing current or temperature instead of falling appears to have increased. As stated by Faraday, the removal of the positive electrode from the specimen, and the scraping of the surface so as to present a new layer of the sulphide restores in every case the specimen to the properties previously possessed by it under the second condition.

The conclusions which may be drawn from the above are :

(a) That electrical connection cannot be made between compressed silver sulphide and electrodes of platinum, steel, silver or copper simply by pressure.

(b) That conduction through compressed silver sulphide cannot be obtained until sufficient ionization has occurred at the contacts to allow the passage of an arc.

(c) That connection, when a current has passed, is most readily and completely obtained at the negative terminal of the specimen.

(d) That it is probable that, when perfect connections have been established between the sulphide of silver and its electrodes, the sulphide has a positive rather than a negative coefficient of resistance for increasing temperatures.

Boston, January 25, 1910.

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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

ON THE ELECTROMAGNETIC AND THE THERMOMAGNETIC TRANSVERSE AND LONGITUDINAL EFFECTS IN SOFT IRON.

By Edwin H. Hall and L. L. Campbell.

Investigations on Light and Heat published with Aid from the Rumford Fund.

ON THE ELECTROMAGNETIC AND THE THERMOMAGNETIC TRANSVERSE AND LONGITUDINAL EFFECTS IN SOFT IRON.

By Edwin H. Hall and L. L. Campbell.

Presented February 8, 1911. Received February 8, 1911.

For many years past the electrical and thermal properties of a certain kind of wrought iron have been, from time to time, under investigation in this laboratory, and a number of papers [1] giving the methods and results of this study have already been published. The present paper has to do with the so-called Hall effect and the allied effects in the same kind of iron. These are so numerous and so unfamiliar to ordinary observation that it seems well to give an enumeration and brief description of them, somewhat like that given by H. Zahn [2] in his comprehensive paper on such phenomena.

In each of Figures 1–4 the rectangle, with very short side projections, or arms, represents pretty accurately, halfscale, the broadside of one of our "plates" of iron, which were each about 5 cm. long between heavy terminal blocks of copper (see Figure 7), and about 2 cm. wide. The circle indicates the size of the flat face of each pole of the electromagnet, the arrow showing the direction of the magnetizing current.

Description of the Transverse Effects.

Hall Effect. — Figure 1, in which EE is the longitudinal electrical current, and $E'E'$ is the transverse electrical current due to the action of the magnet, shows the Hall effect, positive, according to the ordinary convention as to the sign, in this case, the equipotential line which would naturally extend straight across between the arms being rotated

[1] These Proceedings, **41**, May, 1905; **42**, March, 1907.

[2] Über galvanometrische und thermomagnetische Effekte, Jahr. d. Rad. u. Elek., **5**, 166–218 (1908).

by the magnetic action through a small angle in the direction of flow of the Amperian magnetizing current.

The letter commonly used for the Hall effect coefficient is R; we shall use also ${}_eT_e$ in the same sense, T indicating *transverse*, the initial e indicating that the *longitudinal* flow is *electrical*, the final e indicating that the *transverse* flow is *electrical*. If we let

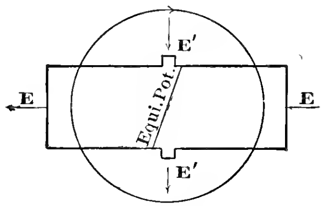

FIGURE 1.

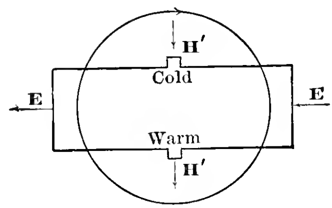

FIGURE 2.

$C =$ the longitudinal current, in absolute units,
$H =$ " intensity of the field " " "
$\Delta P' =$ " diff. of pot. set up between the arms,
$w =$ " width of the plate in cm.,
$t =$ " thickness of the plate in cm.,

we have by definition

$$ {}_eT_e \text{ (or } R) = \frac{\Delta P'}{w} \div \frac{C}{wt} H = \frac{t\Delta P'}{CH}. \tag{1}$$

Ettingshausen Effect. — In Figure 2, with the longitudinal electrical current EE and the magnetic field just as in Figure 1, we take note of a transverse difference of temperature, $\Delta\theta'$, established by magnetic action. This is the Ettingshausen effect. $\Delta\theta'$, as here represented, is in such a direction that a transverse current of heat would flow in the same direction as the transverse electrical current of Figure 1. In this case the Ettingshausen effect, like the Hall effect of Figure 1, is said to be positive.

Zahn uses P for the Ettingshausen coefficient. We shall use also, in the same sense, ${}_eT_h$, the T, as before, standing for *transverse*, the e for *electrical*, longitudinal, the h for *thermal*, transverse. We have by definition

$$ {}_eT_h \text{ (or } P) = \frac{\Delta\theta'}{w} \div \frac{C}{wt} H = \frac{t\Delta\theta'}{CH}. \tag{2}$$

Nernst Effect. — In Figure 3, with the magnetic field as in Figures 1–2, we have a longitudinal heat-current, HH, and we take note of a transverse difference of potential, $\Delta P'$, established by magnetic action. This is the Nernst effect. This $\Delta P'$, as here represented, is in the same direction as the $\Delta P'$ of Figure 1, in which the longitudinal current was electrical. In this case, we shall call the Nernst effect positive; Zahn would call it negative. Zahn uses Q for the Nernst

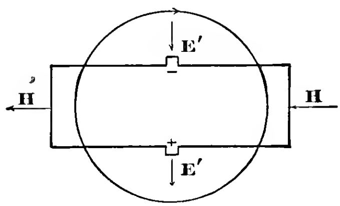

FIGURE 3.

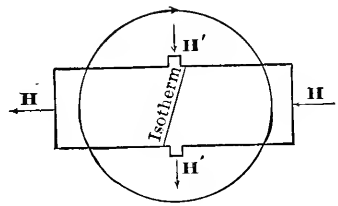

FIGURE 4.

coefficient with his convention as to the sign. We shall use ${}_hT_e$ as equivalent to his $-Q$, the prefix h indicating a *thermal* longitudinal flow, the T_e indicating a transverse electrical action. We have by definition

$$ {}_hT_e \text{ (or } -Q) = \frac{\Delta P'}{w} \div \frac{d\theta}{dl} H, \qquad (3) $$

where $\frac{d\theta}{dl}$ = the temperature-gradient along the plate.

Leduc Effect. — In Figure 4, with the magnetic field and the longitudinal heat-current as in Figure 3, we take note of the transverse difference of temperature, $\Delta\theta'$, established by magnetic action. This is the Leduc effect. This $\Delta\theta'$, as here represented, is in the same direction as the $\Delta\theta'$ of Figure 2, in which the longitudinal current was electrical. In this case, the Leduc effect is called positive. Zahn uses S for the Leduc coefficient. We shall use also, in the same sense, ${}_hT_h$. We have by definition

$$ {}_hT_h \text{ (or } S) = \frac{\Delta\theta'}{w} \div \frac{d\theta}{dl} H, \qquad (4) $$

The longitudinal effects are defined later, in connection with an account of our work on them.

Description of Apparatus.

Each of the iron plates mentioned above was brazed at each end into a thick copper block, as in Figures 7 and 8. After being so fixed each plate was measured in thickness at nine points, the distribution of which is well shown by Figure 5, the distance between (1) and (2), (2) and (3), (4) and (5), etc., being appoximately 1.5 cm.

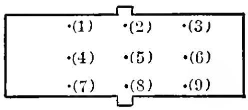

Figure 5.

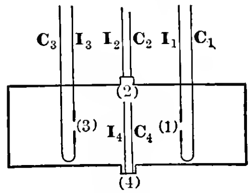

Figure 6.

Plate 1.— The measurements of Plate 1 were

Length between coppers . . . . 5.1 cm.,
Width " " . . . . 1.98 cm.,
Arms about 0.12 cm. long and about 0.10 cm. wide,

	cm.	Pt.	cm.	Pt.	cm.	Pt.
Thickness	0.1490 at	1	0.1460 at	2	0.1505 at	3
	0.1490 "	4	0.1480 "	5	0.1495 "	6
	0.1475 "	7	0.1450 "	8	0.1500 "	9
Means	0.1485		0.1463		0.1500	

As the part of the plate which is particularly important in the transverse effects, the effects that interest us most, is the narrow transverse strip including the arms, the thickness of this part is to be especially considered. This, according to our measurements, is about 0.1463 cm.; but as the method of measurement, by means of vernier calipers with faces of considerable width, must tend to give a maximum value, we take off about 1 per cent from the quantity just given and call the mean thickness of the median transverse strip 0.145 cm.

To Plate 1, eight wires were attached (Figure 6) by means of copper electrolytically deposited. Four of these, C_1, C_2, C_3, C_4, were of copper about 0.017 cm. in diameter, the others, I_1, I_2, I_3, I_4, were of annealed iron wire about 0.020 cm. in diameter. This iron wire was drawn from a portion of the same bar that furnished the plates 1 and 2. These fine iron wires were joined to somewhat heavier iron wires of the same source 15 cm. or more above the iron plate, and the heavier wires, perhaps a meter long, were soldered to copper wires, the junctions being kept in a water bath. The distance between junctions (1) and (3) could not be very accurately determined. It was measured and taken as 2.78 cm. The wires C_1 and C_3 ran down from the points of attachment, passed under the lower edge of the plate, and came up across it on the other face. The distance between C_2 and I_2 and between C_4 and I_4 was perhaps 0.05 cm., each wire being attached to the outer end of the arm. On the face shown in Figure 6, a flat sheet of asbestos about 0.15 cm. thick, arranged so as not to disturb the wires, was cemented by means of asphaltum or melted shellac (the latter was used with Plate 2), the whole being subjected to a temperature high enough to soften the cement and allowed to cool with sufficient pressure on the asbestos sheet to fix it flat in place.

The width of Plate 1 is the direction of the fibre, or grain, of the original iron bar.

Plate 2. — The measurements of Plate 2 were

Length between coppers	. . . .	5.15 cm.,
Width	. . . .	2.01 " ,

Arms about 0.10 cm. wide and 0.10 cm. long,

	cm.	Pt.	cm.	Pt.	cm.	Pt.
Thickness (Figure 5)	0.134	at 1	0.135	at 2	0.138	at 3
	0.138	" 4	0.142	" 5	0.145	" 6
	0.140	" 7	0.143	" 8	0.146	" 9
Means	0.137		0.140		0.143	

The face of Plate 2 was smoother at the last than the face of Plate 1, and we have taken as the mean thickness of the arm strip of (2) the mean of the measurements as here recorded, 0.140 cm. The thickness is several per cent greater on one edge of the plate than on the other, but it does not appear that any serious harm can come from this inequality.

The iron wires of Plate 1 (see Figure 6) were with Plate 2 replaced by constantan ("advance") wires, K_1, K_2, etc., about 0.020 cm. in diameter, which had been annealed by heating to bright incandescence by means of an electric current. The distance from junction (1) to junction (3) was now made about 2.62 cm., in order to have them a little nearer the centre of the magnetic field than the corresponding junctions of Plate 1 had been. Wires C_1 and C_2 did not here pass under the lower edge of the plate, but followed the course shown in Figure 6, strips of mica fastened by shellac, melted on, being used for insulation. The wires themselves were bedded in shellac in their course across the plate. The plate was finally backed, on the side shown, with a flat sheet of asbestos about 0.15 cm. thick, according to the description already given.

The length of Plate 2 is the direction of the fibre of the original bar.

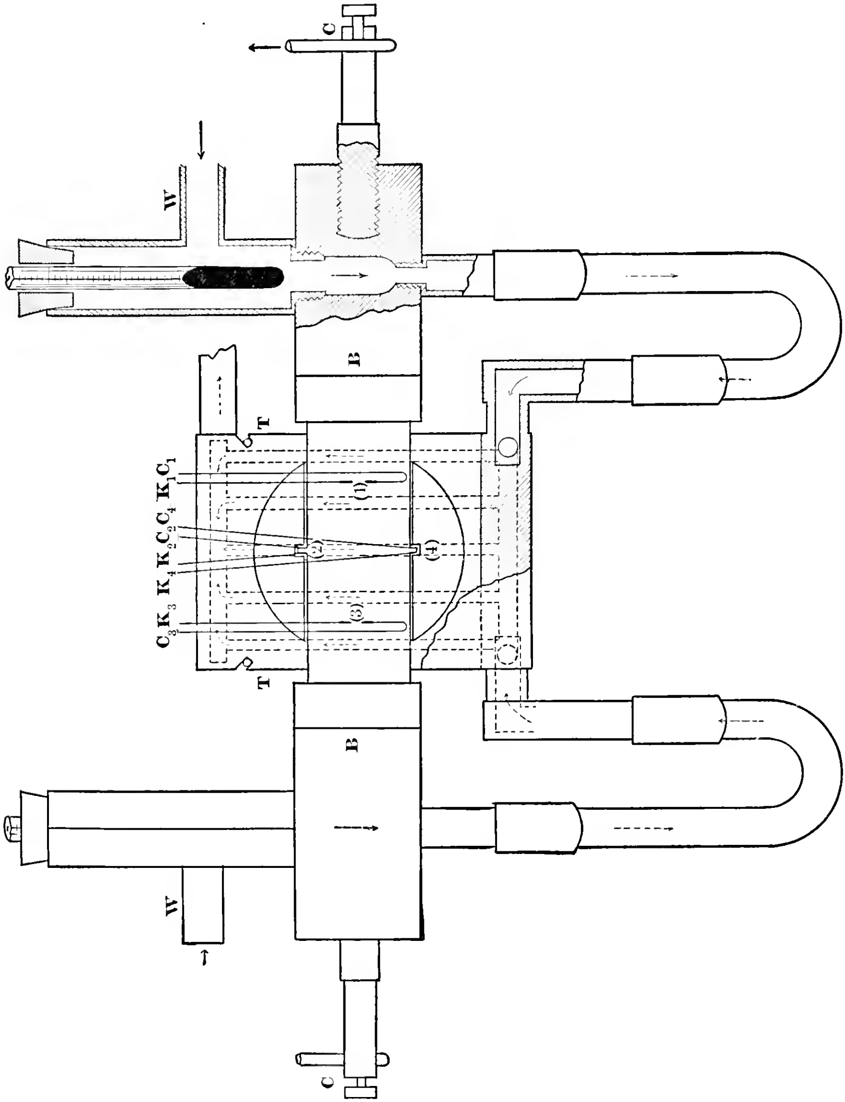

FIGURE 7.

Mounting the Plates in the Field.

Figures 7 and 8 show the method of mounting the plates in the magnetic field for the electromagnetic experiments. *T T* is a trough of soft iron consisting of a rectangular one-piece[3] block about 4.5 cm. long, 6.5 cm. tall and 2.0 cm. wide, with a channel 0.5 cm. wide and 5.5 cm. deep cut through it from end to end, leaving an uncut part 1 cm. thick below. Through holes drilled in this uncut portion two streams of water, entering at the ends and meeting at the middle, reach ten vertical passages, five in each wall of the trough. The passages in each wall lead into a horizontal passage at the top, and the two streams of water issuing from the apparatus are reunited into one beyond the range of our figures.

The object of this water circulation is to control the temperature of the trough, and so that of our iron plate, which lies in the channel of the

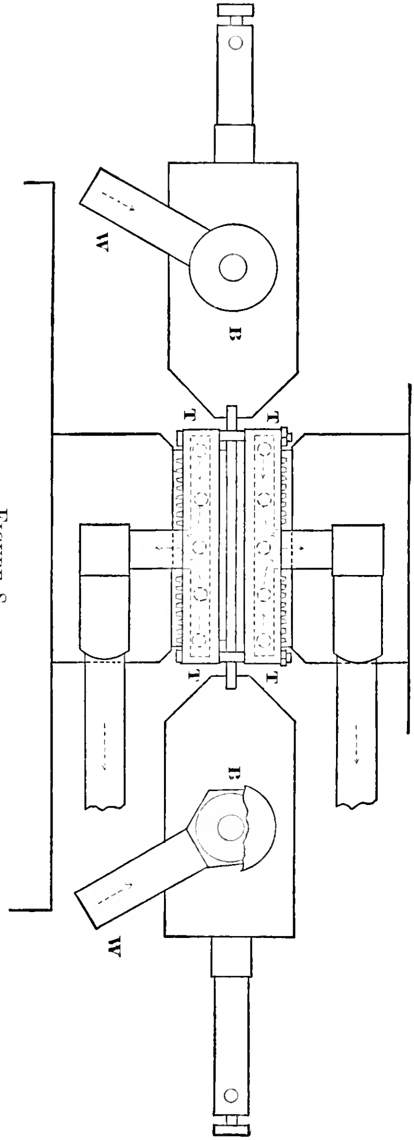

Figure 8.

[3] The trough first used was not from one piece, but was a composite affair soldered together. It leaked more or less, and after much trial the new trough was substituted for it March 26, 1910. Nearly all our observations on the transverse and longitudinal effects made before that date we regard as tentative only and reject from our final results.

trough, during the electromagnetic observations; for in these it is desirable to have all parts at the same temperature, except as differences may be set up by the magnetic action that we are studying.

The asbestos pad, indicated by a dotted strip in Figure 8, at the back of the plate bears against one face of the channel in the trough. Between the plate and the other face of this channel is a space about 0.2 cm. wide, into which passes the single-turn "test-coil," of 1 cm. radius, by means of which the strength of the magnetic field is measured. The test-coil is in circuit with a ballistic galvanometer and the throw produced when it is suddenly withdrawn from the field is easily and accurately measured. The semicircular pieces above and below the plate are of soft iron of nearly the same thickness as the plate, their object being to insure uniformity of magnetic field over the whole width of the iron under observation. Care is taken to have the plate a little nearer to one face of the channel than to the other, so that it may not be pulled forward on the test-coil side by magnetic action. At each end of the trough near the top is a slot which holds a small brass bar, having a head at one end and a nut at the other and carrying a stud 0.5 cm. long on its middle. This device prevents the walls of the trough from bending toward or from each other during the magnetic action.

During thermomagnetic observations also, with a longitudinal gradient of temperature, the plate was at first kept in the iron trough, without, of course, circulation of the water therein. It appeared, however, after a time, that the nearness of the walls of the trough affected the longitudinal temperature-gradient, and after May 21, 1910, the trough was discarded during thermomagnetic measurements. At this time disks of cork (see Figure 8) about 0.2 cm. thick were cemented to the flat faces of the pole pieces of the magnet, which faces are 4.0 cm. in diameter, and the exposed surface of each disk was cross-cut with a file or saw, in order to reduce the naturally low heat-carrying power of the cork. The asbestos pad at the back of the iron plate was then placed against the cork disk on one pole of the magnet, and the other pole was brought somewhat toward the plate, but not near enough to pull the plate away from the other pole. Most of the thermomagnetic data which we use were obtained after this arrangement had been adopted.

The cork disks remained in use during all the subsequent measurements, even when the iron trough was put back for repetition of electromagnetic observations. Previous to their introduction a facing of asbestos had been used between the pole pieces and the sides of the trough.

The tubes W and W show where the two streams of water enter the apparatus. The course of these streams is first downward past the thermometer-bulbs, which give the temperature readings called T_n and T_s, through the massive copper blocks B and B. Then, if electromagnetic experiments are in order, the course followed is that shown by the arrows in Figure 7; but if a temperature-gradient for thermomagnetic work is to be established and maintained, the two streams, one warm and the other cold, pass off at once from B and B to the sink.

The longitudinal electric current, when one is used, enters by one of the heavy wires C, C, and passes out by the other.

Plate 1 was studied at intervals from June 5, 1909, to June 29, 1910. All the observations we have to publish on Plate 2 were made during December, 1910.

Calibration of the Thermo-Electric Couples.

The "thermo-electric height" between copper and our iron was determined some years ago for various temperatures ranging from 27° to 71°, and from this set of data a curve was plotted which gives the thermo-electric force of a copper-iron couple within these limits. As our iron wires were drawn from a piece of the same iron bar that furnished the iron used in determining these thermo-electric heights, and as they were annealed after drawing, it was assumed that the curve just described would serve for couples made by joining these wires with copper; that is, we assumed the iron wires to be thermo-electrically so like the plate to which we attached them, that we should be justified in ignoring any thermo-electric forces which might arise from differences between the plate and this wire.

But to our surprise and disappointment we found that the thermo-electric force of the couple I_1–*plate*–I_3 was about one tenth-part as that of the couple C_1–*plate*–C_3. This discovery, which was not made at the beginning of our work, taken with the fact that the thermo-electric quality of iron is considerably affected by magnetization, led to the rejection of iron wires and the use of constantan wires in their place, when we came to the preparation of Plate 2.

Experiments to determine whether constantan wire would be appreciably effected in its thermo-electric quality by magnetism were conducted as follows: A piece of annealed constantan wire was fastened by means of shellac, applied melted, within a narrow glass tube across, or through, which ran side-tubes carrying streams of water, one at a temperature near 16° degrees, the other near 76°.

The distance between the streams was about 6 cm., so that the gradient of temperature in the wire between must have been about 10° per cm. The ends of the wire, which was about 130 cm. long, were soldered to copper wires, and the junctions were placed in a suitable water bath, the copper wires leading to the galvanometer. The part of the iron wire lying between the streams of hot and cold water was placed across the space between the magnetic poles in a field the strength of which, for a distance of perhaps 4 cm., was not far from 8000.

Galvanometer readings taken with "field off" and "field on" showed that the change of thermo-electric quality produced in the unequally heated constantan wire by magnetic action, if any such change occurred, was probably too small to yield a change of 3×10^{-8} volt in the thermo-electric force of the circuit.

Four couples of copper and annealed constantan wires were tested for calibration purposes. The temperatures used were those of the tap-water (about 12.5°), boiling ether (about 35.5°), boiling alcohol (about 80°), and boiling water. The method used was substantially that which has been described in previous papers.[4] None of the junctions came into direct contact with any one of the liquids or its vapor, each being protected by an arrangement like that shown in Figure 10. The thermometers used had been studied with considerable care in previous work.

The four couples agreed so closely that for no one of the temperature intervals used in the calibration did the result from any one couple differ so much as one per cent from the mean result given by all. This mean follows:

Temperature interval.	Mean e. m. f. per degree.
11.6°–34.5°	3911 absolute
34.6°–79.1°	4145 "
79.3°–99.9°	4527 "

The thermo-electric current runs from constantan to copper at the hot junction; that is, at ordinary temperatures the copper line on the thermo-electric diagram lies between the iron line and the constantan line.

Plotting a curve through the three points given by the data shown above, we get from it the following table for copper-constantan: —

[4] These Proceedings, **41**, p. 42.

Temperature.	Thermo-electric height (microvolts per degree).	Temperature.	Thermo-electric height (microvolts per degree).
0° C.	38.04	60° C.	41.72
10°	38.16	70°	42.71
20°	38.94	80°	43.91
30°	39.50	90°	45.33
40°	40.14	100°	47.10
50°	40.88		

MEASUREMENT OF THE TRANSVERSE EFFECTS.

In giving the results of our observations we shall report on the various transverse effects in the order in which we have defined them. Relations, theoretical or empirical, between some of them will be discussed later, with especial reference to the formulas of Moreau and of Voigt.

Hall Effect. — With Plate 1 we found

1910.	$_eT_e$.	θ.	C_p.	H.
Mar. 29	+ 714 × 10^{-5}	12.7°	2.9	6500
" 30	1024 "	53.0°	"	"

θ here is the temperature of the water flowing through the copper blocks at the end of the plate and the iron trough in which the plate lay. C_p is approximately the strength of the current through the plate, expressed in absolute units; H is approximately the strength of the magnetic field at the middle part of the plate, expressed in absolute units.

The temperature-coefficient of $_eT_e$, as derived from the data here given, is

$$(1024 - 714) \div 714\,(53 - 12.7) = 0.0108.$$

Thermo-electric action, due to the Ettingshausen transverse temperature-difference acting in the all-iron thermo-electric couple I_2–*plate*–I_4 of Figure 6, is negligible in its influence on the Hall effect as here observed.

With Plate 2, using the circuit C_2–*plate*–C_4, we found

1910.	$_eT_e$.	θ.	C_p.	H.
Dec. 3	+ 807 × 10^{-5}	13.1°	2.94	5150
" 22	781 "	12.7°	2.97	5460
	794 " .	12.9°	2.96	5300
" 10	1486 × 10^{-5}	84.5°	2.99	5040
	(1464) "			

After the observations of December 10, on the Hall effect, it was found that the plate had been several millimeters too high in the magnetic field during the observations of December 3 and 10, so that the strength of the field to which it had been exposed was somewhat uncertain. Proper adjustment was made December 22, and later in the same day the Hall effect was measured again at low temperature. The coefficient found December 22 was about 3.3 per cent lower than that of December 3, though the temperature on the later date was only 0.4° lower, which would account for scarcely 0.5 per cent difference. For the value of ${}_eT_e$ at the mean low temperature, 12.9°, we have taken the mean, 794×10^{-5} of the values found on December 3 and December 22. For the value of ${}_eT_e$ at the high temperature, 84.5°, we have taken the value found December 10 reduced by 1.5 per cent, thus getting 1464×10^{-5}.

For the temperature-coefficient of ${}_eT_e$ in Plate 2, we get

$$(1464 - 794) \div 794\ (84.5 - 12.9) = 0.0118.$$[5]

Ettingshausen Effect. — With Plate 1 we undertook to measure this effect by using the east and the west coils of our galvanometer differentially, connecting the I_2I_4 circuit with the west coil and the C_2C_4 circuit with the east coil and taking care to have the total resistance of one circuit equal to that of the other. This arrangement we called (I_2-I_4) *versus* (C_2-C_4). Lest there might be error from a possible accidental advantage of one coil of the galvanometer over the other in action upon the needle, a series of observations made with this arrangement was always coupled with a (C_2-C_4) *vs.* (I_2-I_4) series, in which the $C_2\,C_4$ circuit was connected with the west coil and the I_2I_4 circuit with the east coil.

The theory of this test was that the Hall effects in the two circuits would just neutralize each other, leaving the Ettingshausen effect in the $C_2\,C_4$ circuit to give an account of itself in the net result. This experimental method was not so good as that used in the study of the

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[5] Dr. Alpheus W. Smith gave in the Physical Review for Jan., 1910, the results of his study of the Hall effect in various metals through a large range of temperature. Taking data from the diagrams of his paper, we find as the temperature-coefficient of the Hall effect

0.0088	between 22° and 100° in "Kahlbaum iron,"
0.0140	" 23° " " " electrolytic iron.

The mean of these values, 0.0114, agrees very well with the values found by us in Plates 1 and 2.

Ettingshausen effect in Plate 2, and the results here attained are not entitled to any great confidence. They are

1910.	$_eT_h$.		θ.	C_p.	H.
Mar. 29	-375	$\times 10^{-10}$	12.7	2.86	6500
" 30	-1120	"	53.1	2.85	"

It seems inadvisable to calculate a temperature-coefficient of $_eT_h$ from these data.

With Plate 2 we put the circuit C_4K_4 into connection with the west coil of the galvanometer and the circuit C_2K_2 into connection with the east coil, as in Figure 9.[6] We shall call this arrangement C_4K_4 *vs.* C_2K_2, or (4) *vs.* (2). Series with this arrangement were coupled with series having a (2) *vs.* (4) arrangement, the galvanometer connections being exchanged in the shift from one arrangement to the other. The results are

1910.	$_eT_h$.		θ.	C_p.	H.
Dec. 9	-500 [492]	$\times 10^{-10}$	12.2°	2.94	5000
" 10	621 [612]	"	83.9°	30.0	"

The numbers given in brackets, which are the ones to be finally taken, are reached by deducting 1.5 per cent from the numbers immediately above, the deduction being made for reasons given in connection with $_eT_e$.

From these data we get the following temperature-coefficient of $_eT_h$ in Plate 2:

$$(612 - 492) \div 492\ (83.9 - 12.2) = 0.0034.$$

The transverse differences of temperature (between the arms) actually produced by electromagnetic action in the cases here described in Plate 2 were about 0.0053° at the low temperature and 0.0067° at the high temperature. Whether these temperature-differences are set up instantaneously by the magnetic action or whether they grow for a time after the full magnetic field-strength is on, is a question we cannot at present answer with full confidence. The galvanometer was usually read about 45 to 60 seconds after the magnet-current, C_m, was put on. We have not observed any tendency toward galvanometer

[6] Proportions are not observed in this figure, nor is the device for keeping the two wires separate, except at their soldered junction, within the glass tubes in V shown. For this detail see Figure 10.

drift of such a character as to indicate that we should have got larger Ettingshausen effects by waiting longer for these effects to be established.

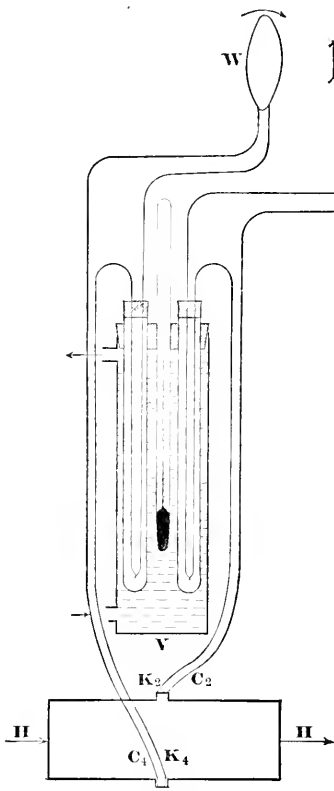

FIGURE 9.

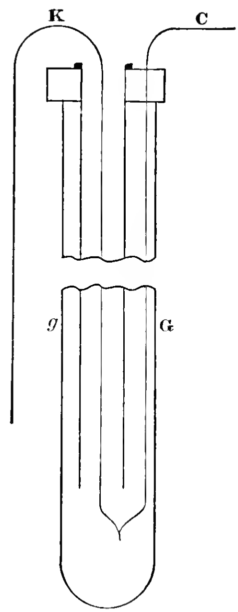

FIGURE 10.

The Nernst Effect. — With Plate 1 we at first used the iron trough already described, during the thermomagnetic tests. We now reject all the Nernst-effect observations made with this trough between the poles. The remaining results are

1910.	${}_hT_e$.	Arm θ.	$\frac{d\theta}{dl}$.	H.
June 4	-944×10^{-6}	45.7°	11.3°	8400
" 10	928 "	44.8°	11.3°	8700
Means	-936 "	45.3°	11.3°	8550

The "arm θ," that is, the temperature of the median cross-line of the plate, was found by connecting either the C_2I_2 circuit or the C_2I_4 circuit with the galvanometer and then bringing the outer junction of the thermo-electric couple to such a temperature as to make the current in the circuit zero.

The temperature-gradient $\frac{d\theta}{dl}$ along the plate was found by connecting the thermo-electric circuit $C_1-plate-C_3$ with the galvanometer.

The transverse potential difference, the Nernst effect, was found by means of the circuit $I_2-plate-I_4$, the assumption being made that the thermo-electric difference between the iron plate and the iron wires could safely be neglected, so that no account need be taken here of the fact that a transverse temperature-difference, the Leduc effect, is set up simultaneously with the Nernst effect. It now seems very doubtful whether this assumption was justified; but as we have good reason for supposing that the error thus introduced into the value of ${}_hT_e$ was not more than 10 per cent, and as we are not sure of the sign of this error, we leave ${}_hT_e$ for Plate 1 as we have found it.

With Plate 2 the conditions were better. We found arm θ and $d\theta/dl$ in essentially the same way as before. We found the transverse e. m. f., $\Delta E'$ let us say, produced in the circuit $C_2-plate-C_4$ by magnetic action and deducted from this the amount, a large part of the whole, which from a study of the Leduc effect we found to be attributable to a thermo-electric force set up in the circuit $C_2-plate-C_4$ by magnetic action. If we call this e. m. f. $\Delta_l E'$ and call the true Nernst transverse difference of potential $\Delta P'$, we have

$$\Delta P' = \Delta E' - \Delta_l E.$$

Thus at 31° we found $\Delta P' = 132.3 - 54.5 = 77.8$ absolute,
and at 60° " " $\Delta P' = 137.0 - 50.3 = 86.7$ "

The results are

1910.	${}_hT_e$.	Arm θ.	$\frac{d\theta}{dl}$.	H.
Dec. 28	-880×10^{-6}	30.1°	8.4 (N–S)	5550
" 29	840 "	31.7°	8.3 (S–N)	"
Means	860 "	31.0°	8.4	5550
Dec. 30	940×10^{-6}	59.3°	7.7 (S–N)	5560
" 30	1020 "	60.3°	7.7 (N–S)	5470
Means	$-$ 980 "	60.0°	7.7	5500

Reference to formula (3) defining ${}_hT_e$ will show that it contains the factor $1 \div \left(w \times \frac{d\theta}{dl}\right)$, where w is the width. Now as the Nernst $\Delta P'$ is found between two points separated by the width of the main plate plus the length of both arms, it might seem as if w must be taken as this whole distance. On the other hand, as the flow of heat in the arms follows a somewhat dubious course, one may well doubt whether the whole length of the arms should be included. Fortunately, the fact that the width w occurs in combination with the gradient $\frac{d\theta}{dl}$ relieves us from this difficulty. The longitudinal gradient which counts is the gradient on the narrow strip running across from arm to arm and out into the arms, and this gradient will bear the same ratio to the measured gradient $\frac{d\theta}{dl}$ that the width of the main part of the plate bears to the effective width at the arms. Accordingly, we can take w as the width of the plate without the arms and $\frac{d\theta}{dl}$ as the gradient which would exist if the arms were absent, that is, practically, the gradient as measured by means of the circuit C_1 – *plate* – C_3. This we have done, here and in dealing with the Leduc effect.

The temperature-coefficient of ${}_hT_e$ we find to be

$$(980 - 860) \div 860\ (60 - 31) = 0.0048.$$

The Leduc Effect. — All of the Leduc observations on Plate 1 were made with the iron trough in use. For this reason, and the further reason that the dubious thermo-electric quality of our iron wire was involved in them, these observations are entitled to far less weight than those made with Plate 2.

The method used with both plates was essentially that indicated in Figure 9, the (2) circuit and the (4) circuit being used differentially, one connected with the west coil of the galvanometer, the other connected with the east coil.

With Plate 1 we found

1910.	${}_hT_h$.	Arm θ.	$\frac{d\theta}{dl}$.	H.
Mar. 31	$+453 \times 10^{-9}$	43 (?)	11.0 (N–S)	6500
Apr. 2	410 "	44 (?)	11.3 "	"
Means	432 "	43.5(?)	11.2	6500
Apr. 1	421×10^{-9}	43 (?)	11.3 (S–N)	6500
" 2	438 "	44 (?)	11.3 "	"
" 9	420 "	45 (?)	11.3 "	"
Means	426 "	44 (?)	11.3	6500
Final means	+429 "	44 (?)	11.3	6500

With Plate 2 we found

1910.	${}_hT_h$.	Arm θ.	$\frac{d\theta}{dl}$.	
Dec. 28	+ 549 × 10^{-9}	29.8°	8.3 (N–S)	5600
" 29	586 "	31.3°	8.6 (S–N)	5550
Means	+ 568 "	30.6°	8.5	5600
Dec. 30	+ 614 × 10^{-9}	59.2°	7.9 (S–N)	5470
" "	649 "	61.1°	7.7 (N–S)	5550
Means	+ 632 "	60.2°	7.8	5500

The temperature-coefficient of ${}_hT_h$ in Plate 2 is, according to these figures,

$$(632-568) \div 568\ (60.2-30.6) = 0.0038.$$

The + sign of ${}_hT_h$ means, according to our definition, that the isothermal lines are by the magnetic field rotated in the same direction in which the magnetizing current flows.

The difference of temperature actually produced between the arms by this action was about 0.0054°.

The magnet current was usually put on about 45 seconds before the final galvanometer reading showing the Leduc effect was taken. Professor Campbell, who did most of the experimental work in this research, is under the impression that the Leduc effect did not attain its maximum value as soon as the full field-strength was reached, but that it continued, according to the testimony of the galvanometer, to grow for a time after that moment. We cannot speak positively, as yet, on this interesting particular.

Relations of ${}_eT_e$, ${}_eT_h$, etc.: The Moreau Formula and its Suggestions; the Voigt Formula.

Attempts have been made by various writers to correlate some or all of the various effects which are dealt with in this paper and to connect them with some of the more generally familiar electrical and thermal properties of metals. Naturally the electron theory has been used of late in such efforts; but we shall consider first a suggestion or proposition made by Moreau,[7] about ten years since, in which no mention of the electron theory was made or intended. Indeed, Moreau says, "As, moreover, the variations of K' [${}_eT_h$] are explained by those of c [${}_eT_e$], since the Hall effect is the primordial phenomenon, one is justified in supposing that the molecular state of the plate has a consider-

[7] C. R., **130** (1900), pp. 122–124, 412–414, 562–565.

able influence on the value of c. This seems to me a new proof that the Hall phenomenon is due to a deformation of the plate under the influence of the magnetic field, and it is enough to explain the differences, often very great, observed between the values of c, for the same body, by different experimenters."

It is difficult to see how anyone who has tried the Hall effect with a variety of conditions as to the manner of supporting the metal under examination can attribute this effect to any ordinary distortion of the material. If distortion, or deformation, is the cause of the phenomenon in question, this distortion must be something as remote from ordinary bending and twisting as are the changes of molecular orientation which we imagine to accompany or constitute magnetization.

We might, therefore, pay little attention to the suggestion of Moreau, if he had not given and supported with some experimental evidence an exceedingly simple formula,

$$ {}_eT_e \div \rho = {}_hT_e \div s, \qquad (5) $$

in which ${}_eT_e =$ the Hall coefficient,
${}_hT_e =$ " Nernst "
$\rho =$ " specific electrical resistance,
$s =$ " mechanical equivalent of the Thomson-effect coefficient, that is, the amount of heat energy (ergs) absorbed by the unit current (absolute) of electricity per second in going, according to the ordinary convention of current direction, through the metal from a point where the temperature is T degrees C. to a point where it is $(T + 1)$ degrees C.

This formula of Moreau, if it could be substantiated by further experimental evidence, would prove disturbing to the point of view from which the present definition of the Hall coefficient was framed, even if it did not lead to the conclusion that the Hall effect and its various allied effects are due to a deformation of some kind in the material showing these phenomena. For it is to be noted that in the definition of ${}_eT_e$ the relation of the transverse potential-gradient to longitudinal *current-strength*, not to the longitudinal potential-gradient, is expressed. The reasons for this are two: 1st, a notion held by the discoverer at the time of the discovery that the new effect was due to the action of magnetic force on a *current* of electricity, as such, not to the action of such force on a medium containing lines of electrical force,[8]

[8] At the suggestion of Professor Rowland an attempt was made to rotate by magnetic force the equipotential lines of a piece of glass under electrical stress. The result was negative. — E. H. H.

as such; 2d, the fact that it was much easier to measure current-strength than potential-gradient, in the longitudinal direction, in metal sheets of uneven thickness and uncertain quality such as were often used in the early days of the Hall effect.

It is true that before long it began to be seen that metals of high resistance had, as a rule, high values of ${}_eT_e$, and this fact raised the question whether high transverse potential-gradient might not go naturally with high longitudinal potential-gradient as such. But it was found that, in non-magnetic metals at least, rise of temperature, increasing the resistance and the longitudinal potential-gradient, did not as a rule, if ever, increase[9] the Hall effect proportionally. Accordingly, there has seemed to be no sufficient reason for recasting the definition of ${}_eT_e$ by introducing the potential-gradient instead of the longitudinal current-strength.

But the formula of Moreau suggests, or perhaps is suggested, by such a change of view as this recasting of ${}_eT_e$ would imply. To prove this statement an argument leading to this formula will now be given.

We will treat the matter first in its qualitative aspect; and we will take the case of our soft iron, in order to be the more definite. Figure 1, which is correct for this iron, shows a +Hall effect, the equipotential lines being rotated in the direction of the Amperian current of the field. Such a rotation establishes a transverse potential-gradient, the lower edge of the plate attaining thus the higher potential.

Turning now to Figure 3 and remembering that in iron the Thomson-effect coefficient, s, is negative, so that heat is absorbed by an electrical current when it flows (in the ordinary conventional sense) from high temperature to low temperature in iron, we see that in the present case we must, in order that an electrical current may *not* flow along the plate, have the cold end of the plate at a higher potential than the hot end; that is, we have, under the conditions of Figure 3, a potential-gradient opposite in direction to the temperature-gradient and opposite to the potential-gradient along the plate in Figure 1. Accordingly, our magnetic field, rotating the equipotential lines of this *static* Thomson-effect potential-gradient, would make in our iron the upper edge of the plate electrically positive as compared with the lower edge, thus giving what we call a negative Nernst effect. The Nernst effect in our iron is negative, according to our convention as to signs, and the

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[9] "The temperature-coefficient of the Hall effect in gold, zinc, platinum, silver and aluminum has . . . been determined and it has been found that with the exception of aluminum there is in each of these cases a decrease in the effect as the temperature is raised from −190° to about 22°." Dr. Alpheus W. Smith, Physical Review, Jan., 1910.

signs at least of the Moreau formula are correct in this case, ${}_eT_e$ and ρ being both + and ${}_hT_e$ and s both − .

The quantitative argument is very simple. Writing $\frac{dP}{dl}$ for the potential-gradient which maintains the current C through a plate of width w, thickness t, and specific resistance ρ, we have, using absolute units, $C = \frac{dP}{dl} \times \frac{wt}{\rho}$, and accordingly

$$ {}_eT_e = \frac{\Delta P'}{w} \div \frac{C}{wt} H = \left(\frac{\Delta P'}{w} \div \frac{dP}{dl} \right) \frac{\rho}{H}. \qquad (6)$$

Taking, as before (equation (3)),

$$ {}_hT_e = \frac{\Delta P'}{w} \div \frac{d\theta}{dl} H, $$

and observing that the *static* Thomson-effect potential-gradient is

$$ s \frac{d\theta}{dl} = \frac{dP}{dl}, $$

we have

$$ {}_hT_e = \left(\frac{\Delta P'}{w} \div \frac{dP}{dl} \right) \frac{s}{H}. \qquad (7)$$

If, now, we can assume that the equipotential lines of the electric current in the Hall effect are by magnetic action rotated just as far as the static thermo-electric equipotential lines of the heat current in the Nernst effect, we shall have the parenthesis in equation (6) equal to the parenthesis in (7), and then, dividing (6) by (7), we shall get

$$ {}_eT_e \div {}_hT_e = \rho \div s, \quad \text{or} \quad {}_eT_e \div \rho = {}_hT_e \div s, \qquad (8)$$

which is the equation of Moreau.

Before proceeding further it will be well to assemble in one table the various coefficients under discussion, each evaluated, as accurately as our knowledge of the rates of change with temperature will enable us to evaluate it, for various temperatures. We have, the last line relating to Plate 1 and the other lines to Plate 2,

Temp.	${}_eT_e$	${}_eT_h$	${}_hT_h$	${}_hT_e$	ρ	s	S
20°	861×10^{-5}	-505×10^{-10}	545×10^{-9}	-815×10^{-6}	12550	−851	1429
40°	1048 "	538 "	588 "	897 "	13730	961	1370
60°	1235 "	572 "	630 "	980 "	14910	1079	1307
80°	1422 "	604 "	672 "	1062 "	16090	1203	1240
100°	1609 "	638 "	714 "	1144 "	17270	1333	1171
45°	963 "	−971(?) "	430(?) "	−935 "			

(I.)

Previous work [10] has given us 11365 as the value of ρ at 0 C., with a temperature-coefficient 0.00519, and a Thomson-effect formula, running from 32° C. to 182° C., from which we get

$$s = -(107\ T + 2\ T^2)\ 419 \times 10^{-5}, \qquad (9)$$

T being the absolute temperature.

S of the last column is the estimated "thermo-electric height" of our iron relative to lead. It will be discussed at considerable length in connection with the formula of Voigt.

Testing the Moreau formula with the data given, we find

	Temp.	$({}_eT_e \div \rho) = A$.	${}_hT_e \div s = B$.	$(A - B) \div A$.	
Plate 2	20° C.	$+ 686 \times 10^{-9}$	$+ 958 \times 10^{-9}$	$- 0.40 -$	(II.)
	40°	763 "	934 "	$- 0.22 +$	
	60°	828 "	908 "	$- 0.10 -$	
	80°	884 "	883 "	$- 0.00$	
	100°	932 "	858 "	$+ 0.08 -$	
Plate 1	45°	+ 686 "	+ 944 "	$- 0.38 -$	

Not all of the temperature-coefficients are sufficiently well known to justify us in pushing the comparison through a greater range of temperature, and in fact the values here set down for A and B are subject to an error of several per cent, probably. But when all reasonable allowance for error has been made, it seems unlikely that the Moreau formula can hold in this iron through any considerable range of temperature. The difference between the result from Plate 1 at 45° and that for Plate 2 at or near the same temperature is less significant as throwing a doubt upon the accuracy of our work than it would be if the two plates had been cut in the same direction from the original iron bar. Plate 2 is far more important, not merely because it has been studied under better conditions, but because the main current in it was in the direction of the fibres of the iron, the direction along which the measurements for ρ and for s have been made.

It appears that the Moreau formula holds in our iron for one temperature, which in Plate 2 is near 80° C. Below this temperature the static Thomson equipotential lines appear to be rotated through a somewhat greater angle, by a given strength of field, than the equipotential lines of an ordinary electric current; but above this temperature the opposite appears to hold.

Comparison of ${}_eT_e \div \rho$ in Table II with ${}_hT_h$ in Table I shows an

[10] These Proceedings, **42**, March, 1907.

approach to equality, which may be interpreted as meaning that the equipotential lines of the longitudinal electric current in the Hall effect and the isothermal lines of the longitudinal heat-current in the Leduc effect are rotated in the same direction and to nearly the same extent, at certain temperatures. Detailed comparison gives

Temp.	$({}_eT_e \div \rho) = A$	${}_hT_h = B$	$(A - B) \div A$	
20°	$+ 686 \times 10^{-9}$	$+ 545 \times 10^{-9}$	+ 0.21 −	(III.)
40°	763 "	588 "	0 23 −	
60°	828 "	630 "	0.24 −	
80°	884 "	672 "	0.24 −	
100°	932 "	714 "	0.23 +	

It would seem from this showing that, within the range of temperature here considered, the isothermal lines are rotated in the Leduc effect about three-fourths as far as the equipotential lines are rotated in the Hall effect, the strength of magnetic field being the same on both cases.

Comparison of Tables II and III indicates that, within the range of temperature shown, the static Thomson equipotential lines are rotated farther in the Nernst effect than the isothermal lines are simultaneously turned in the Leduc effect, but that the excess noted diminishes rapidly with rise of temperature and seems likely to become zero or negative with further rise.

As we have in the Moreau formula a suggestion at least of some relation between ${}_eT_e$ and ${}_hT_e$, we might expect to find evidence of a reciprocal relation between ${}_hT_h$ and ${}_eT_h$; that is, between the Leduc effect and the Ettingshausen effect. Reflection shows, however, that one link in the chain of reasoning, if it deserves such a name, is here missing. We go from the Hall effect to the Nernst effect by way of the Thomson effect, the known difference of electric potential which accompanies flow of heat or difference of temperature in iron under normal conditions. But no corresponding difference of temperature necessarily accompanying flow of electricity or difference of electric potential is known to us. On the other hand, if we could find by inspection of the various coefficients before us evidence of a relation between ${}_hT_h$ and ${}_eT_h$, we might perhaps take this as a clue to what could be called the reciprocal of the Thomson effect, and could be described as a temperature-gradient due to potential-gradient. The possibility of such a discovery makes it desirable to examine closely all the relations which a study of our various tables of data suggest.

A look at the uppermost line of Table I shows that, for the temperature 20° C. at least, we have, very nearly,

$$_eT_e \times {}_eT_h = {}_hT_h \times {}_hT_e, \text{ or } {}_eT_e \div {}_hT_e = {}_hT_h \div {}_eT_h. \qquad (10)$$

If we test this formula through the various temperatures we find

Temp.	$({}_eT_e \div {}_hT_e) = A$	$({}_hT_h \div {}_eT_h) = B$	$(A - B) \div A$	
20°	− 10.56	− 10.79	− 0.02 +	(IV.)
40°	11.68	10.93	+ 0.06 +	
60°	12.60	11.01	+ 0.13 −	
80°	13.39	11.11	+ 0.17 +	
100°	14.07	11.19	+ 0.20 +	

The approximate equality here indicated appears to be closer than that of the Moreau formula. But is it anything more than accident? Going back to the definitions of ${}_eT_e$, etc., we find, putting $\Delta P_1'$ for the transverse potential-difference of the Hall effect and $\Delta P_2'$ for the transverse potential-difference of the Nernst effect,

$${}_eT_e \div {}_hT_e = \left(\frac{\Delta P_1'}{w} \div \frac{C}{wt}\right) \div \left(\frac{\Delta P_2'}{w} \div \frac{d\theta}{dl}\right) = \frac{\Delta P_1'}{\Delta P_2'} \times \left(\frac{d\theta}{dl} \div \frac{C}{wt}\right). \quad (11)$$

The dimensions of this quotient A are those of a temperature-gradient divided by a current density.

We find also

$${}_hT_h \div {}_eT_h = \left(\frac{\Delta\theta'_2}{w} \div \frac{d\theta}{dl}\right) \div \left(\frac{\Delta\theta'_1}{w} \div \frac{C}{wt}\right) = \frac{\Delta\theta'_2}{\Delta\theta'_1} \times \left(\frac{C}{wt} \div \frac{d\theta}{dl}\right). \quad (12)$$

The dimensions of this quotient B are the reciprocal of those of the quotient A, and accordingly it appears probable that we have only an accidental coincidence in the approach to equality shown by A and B.

If we undertake to explain the Ettingshausen effect as due to the magnetic rotation of the isothermal lines of a longitudinal temperature-gradient set up by the electric current along the plate, we see that, as the coefficient ${}_eT_h$ is about 0.09 as large as ${}_hT_h$, we must assume our temperature-gradient in question to be about 0.09 of a degree per centimeter in our plate when the electric current density therein is 1. Such a condition, as a normal attendant of a current in iron, could not have escaped the attention of ordinary experience. If it were set up by the act of magnetization, it would be detected in the longitudinal tests presently to be described.

Is it, then, possible that the Ettingshausen transverse effect is due to an actual displacement of the electric current in our plate, the flow, under the action of the magnet, being stronger along one edge than along the other edge of the plate? This seems very unlikely, especially in view of the fact that, in Plate 2 at least, the resistance appears to be slightly, very slightly, decreased by magnetization. Indeed, the whole amount of heat generated per minute per unit length of the plate by our current C_p was about 0.6 calorie, and the change in this amount due to the magnetization was probably not so much as 0.00002 calorie.

There is at least one other point of view which is worth trying. The transverse potential-gradient set up in the Hall effect is, apparently, like the Thomson potential-gradient, *static*, maintained without flow of electricity down the gradient, though there may be, if the circuit is properly closed, flow of electricity up this gradient. In the Thomson effect this static condition is attended by, and is apparently due to, a temperature-gradient of the opposite slope. Is it perhaps true that wherever such a static potential-gradient exists in a metal there is an accompanying temperature-gradient, and are the two gradients in iron always of opposite slope? The Ettingshausen temperature-slope is opposite in our iron to the potential slope of the Hall effect, and to this extent encourages our question; but when we come to the ratio of the two slopes and compare this with the ratio of the two slopes in the Thomson effect, we find a great difference. In the latter case the ratio is our *s*. The exhibit follows:

Temp.	$({}_eT_e \div {}_eT_h.)$	$s.$	$({}_eT_e \div {}_eT_h) \div s.$	
20°	− 170500	− 851	200.4	(V.)
40°	194800	961	202.7	
60°	216000	1079	200.1	
80°	235000	1203	193.3	
100°	252200	1333	189.2	

We have at least the satisfaction of finding that the ratio of the two ratios, although each changes a good deal, nearly 50 per cent, remains pretty nearly constant,[11] especially at the lower temperatures, where the values of ${}_eT_h$ are probably more accurate than for the higher ones. Indeed, the whole change from 20° to 100° is less than 6 per cent. But the transverse temperature-gradient is only one two-hundredth part of what it should be to make the transverse ratio equal to *s*.

It would be difficult to go farther in this line of inquiry without

[11] It should be remembered, however, that the temperature-coefficient of ${}_eT_h$ cannot be regarded as accurately determined.

introducing the electron theory, which we do not propose to discuss in this paper.

The Voigt Formula. — Before the appearance of Moreau's equation Voigt[12] had been led by theoretical considerations, not involving consideration of electrons, to propose a formula very like that of Moreau, which, using our own symbols in part, we shall write thus :[13]

$$ {}_eT_c \div \rho = {}_hT_e \div \Theta'. \tag{13}$$

Θ', replacing the s of the Moreau formula, is a quantity the definition of which we shall discuss at considerable length, as a number of refer-

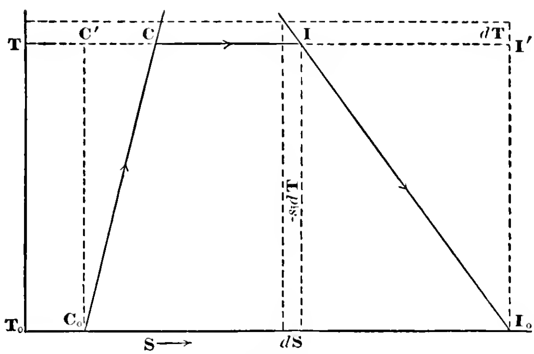

FIGURE 11.

ences to it which we have seen do not make its meaning clear.[14] We cannot well show the significance of Θ' without the use of a diagram.

The ordinary thermo-electric diagram has two rectangular axes, the horizontal one representing temperature, the vertical one a variable not usually named here, but which, as one of us pointed out several years ago,[15] is really entropy. Accordingly we make our diagram (Figure 11) as an ordinary temperature-entropy diagram is made, with temperature, absolute C., vertical and entropy (ergs $\div$ T) horizontal. In accordance with common practice the T axis is assumed to be identical

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[12] Wied. Annalen, **67** (1899).

[13] Voigt has a — sign in his equation, which we avoid by making our ${}_hT_e$ equal his $-Q$.

[14] For example, Zahn appears to confuse Θ', which $= \partial\Theta / \partial T$, with Θ itself.

[15] Proceedings, A. A. A. S., **54** (1904). The paper contains much that I regret publishing, but in its early pages there is this one suggestion which still seems to me valuable. — E. H. H.

with the line for lead. The representative lines for copper and iron are here made straight, though in a later figure they are shown as curves. In accordance with the ordinary interpretation of a temperature-entropy diagram, unit quantity of electricity, ten coulombs, flowing (in the ordinary sense) from C_0 to C in copper or from I to I_0 in iron absorbs heat, and *vice versa*, the Thomson effect; flowing from C in copper to I in iron, it absorbs heat, and *vice versa*, the Peltier or Seebeck effect.

If now we denote by $-\Theta_c$[16] the area $T_0TCC_0T_0$ and by $-\Theta_i$ the area $T_0TII_0T_0$, the total, or net, thermo-electric force, in a copper-iron couple having one junction at T_0 (absolute zero) and the other junction at T, is represented by the area

$$C_0CII_0C_0 = -\Theta_i + \Theta_c. \qquad (14)$$

If we let $\alpha_c =$ the line T_0C_0 and $\alpha_i = T_0I_0$, we can write

$$-\Theta_c = \alpha_c T + \tfrac{1}{2}\beta_c T^2, \qquad (15)$$

and

$$-\Theta_i = \alpha_i T + \tfrac{1}{2}\beta_i T^2, \qquad (16)$$

where $\beta_c = (C'C \div T)$ and $\beta_i = -(II' \div T)$.

These equations (15) and (16) correspond to equation (55) of Voigt's paper.

It is evident from our diagram that, if we allow a very small increase of temperature, from T to $T + dT$, and if we let S_i represent the entropy of ten coulombs at I, *the conventional zero of entropy being so taken that* $S_i = TI$, we have

$$-\frac{d\Theta_i}{dT}dT = S_i dT, \quad \text{or} \quad -\frac{d\Theta_i}{dT} = S_i. \qquad (17)$$

Now the Θ' of the Voigt formula (13) is $\frac{d\Theta_i}{dT}$, and is therefore equal to our $-S_i$. It is numerically equal to the "thermo-electric height" of iron relative to lead.[17]

The relation of Θ'_i to s_i, the mechanical equivalent of the Thomson-effect coefficient, is readily found. The mechanical equivalent of the Thomson-effect heat absorbed by ten coulombs in rising in iron from

[16] We use the — sign here in order to conform to the use of Voigt.

[17] At least is has been so taken by Moreau in his empirical test of the Voigt formula, without protest, so far as we know, from Voigt, though the latter does not, perhaps, say explicitly that his Θ'_m for any metal m has reference to lead.

T to $T + dT$ is $s_t dT$, or rather, since s_t is essentially —, the mechanical equivalent of the heat *given out* in this change is $-s_t dT$, and this is represented by the area between the two adjacent vertical lines of our diagram. Evidently this area is equal to the area between the two adjacent horizontal lines from I' to I, that is, to $(a_t + \Theta'_t)\,dT$, and so we get

$$-s_t = \Theta'_t + a_t, \quad \text{or} \quad \Theta'_t + s_t = -a_t. \qquad (18)$$

That is, the sum of Voigt's Θ'_t and the s_t, which Moreau uses in place of Θ'_t, is a constant, represented in Figure 11 by the line $T_0 I_0$ taken as a negative quantity. As we have already identified Θ'_t numerically with the line TI, taken as a negative quantity, we see that s_t is represented by the line II' taken as a negative quantity.

It must be remembered that this simple relation between s_t and Θ'_t results from the assumption made, for the time, in drawing Figure 11, and made also by Voigt, that the iron line is straight. The actual relation between s_t and Θ'_t is, according to our experiments, more complex.

The argument for Voigt's proposition, as expressed in equation (13), we shall not undertake to give; and even when we try to put it to an empirical test by means of our data, we do so with a serious *a priori* doubt as to its value. What real significance can we attach to the formula of Voigt? His $-\Theta'$, the "thermo-electric height" of iron with respect to lead, cannot be taken as a *fundamental* datum for iron. We have, to be sure, identified $-\Theta'$ with our S; but this S does not profess to be the absolute value of the thermo-electric entropy for iron. It is merely what we get for iron when we arbitrarily take the thermo-electric entropy of lead to be zero.

We have proceeded as follows: Assuming the "thermo-electric height" of "galvanoplastic" copper relative to lead to be 380×10^{-8} microvolts, or 380 absolute, at 20° C., from the observations of Matthiessen, and taking the thermo-electric height of our iron with respect to modern commercial copper wire (assumed to be thermo-electrically like Matthiessen's copper) to be 1049 absolute at 20° C., according to our own observations, we get 1429 absolute as the thermo-electric height of our iron with respect to lead at 20° C. But "thermo-electric heights" are merely isothermal entropy-differences, if we reckon entropy, as we do, in ergs $\div$ T. Accordingly in Figure 12 we take the T-axis as the lead line and at 20° C. lay off from this axis a distance 380 to give us the 20°-point on our copper line, and a distance of 1429 to give us the 20°-point on the iron line. The other points of the iron line must be found by means of our formula (9) for the Thomson effect in iron.

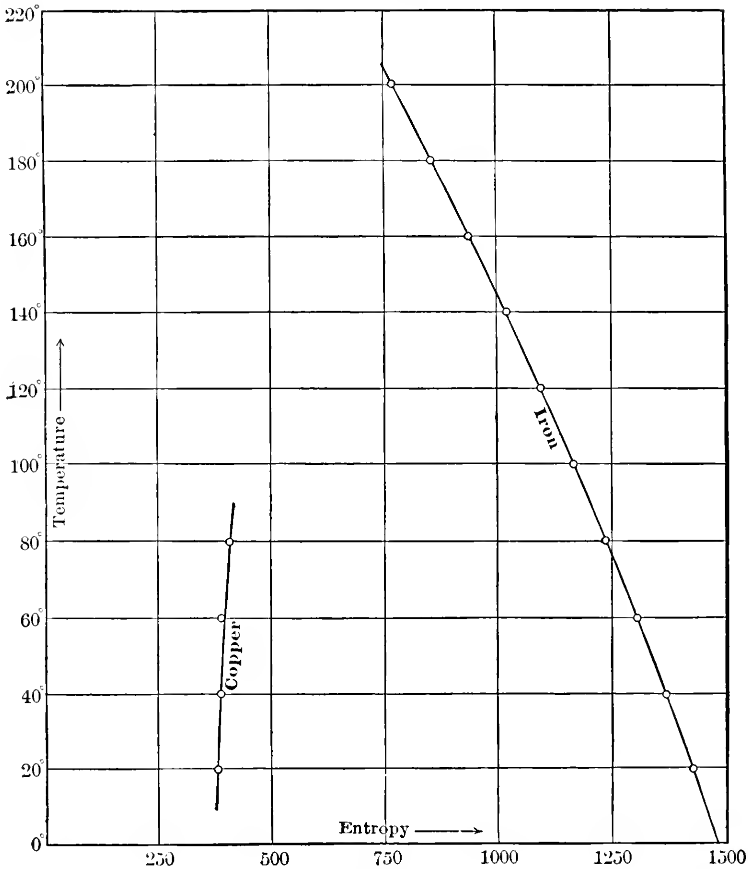

FIGURE 12.

Taking from Figure 11 the relation $TdS = sdT$, we get from (9)

$$dS = -419 \times 10^{-5} (107 + 2T) dT \ldots \quad (19)$$

Integrated this gives

$$S = K - (107T + T^2) \times 419 \times 10^{-5} \ldots \quad (20)$$

The value of S being taken, on grounds already shown, as 1429 at 20° C., we find the value of K to be 1921. Accordingly the values of S have been calculated which are given in the following table :

Temp.	S (entropy).	Temp.	S.	
0° C.	1486	120° C.	1098	
20°	1429	140°	1021	
40°	1370	160°	941	(VI.)
60°	1307	180°	858	
100°	1171	200°	771	

With these values of S the iron line of Figure 12 has been plotted. Various points of the copper line have been found by measuring to the left from the corresponding points of the iron line distances representing the thermo-electric heights of iron with respect to copper at the various temperatures. The iron line is decidedly concave toward the left, the copper line, according to our data, has a much smaller inclination from the vertical than the iron line, indicating a much smaller Thomson effect in copper than in iron, and is slightly concave toward the right.

If we now, taking our values of S for certain temperatures, e. g. 20°, 40°, 60°, 80°, 100°, test the Voigt formula for these temperatures, we find,

Temp.	$(- {}_eT_e S \div \rho) = A$.	${}_hT_e = B$.	$(A - B) \div A$.	
20°	$- 980 \times 10^{-6}$	$- 815 \times 10^{-6}$	0.17 —	
40°	1046 "	897 "	0.14 —	
60°	1083 "	980 "	0.10 —	(VII.)
80°	1096 "	1062 "	0.03 +	
100°	1091 "	1144 "	— 0.05 —	

Comparison of this table with Table II. will show that the Voigt formula, with the value of $- S$ (or Θ') found as we have found it, agrees with our data considerably better than the Moreau formula does. Each formula is right in our case as to signs[18] and each is satisfied by our data at some one temperature, which is near 80° C. for the Moreau formula and not very many degrees higher for the Voigt formula. Above the temperature of agreement ${}_hT_e$ is too small for the Moreau formula and too large for the Voigt formula. It is to be remembered that neither ${}_eT_e$ nor ${}_hT_e$ has been determined very accurately, nor is the temperature-coefficient of either precisely known. It seems to us, however, very unlikely that our errors of measurement are great enough to account for the disagreement between either formula and our data. It is probable that neither formula is correct.

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[18] There may be some doubt as to the signs. See the Postscript to this paper.

Of the two, the Moreau formula seems to us the more likely to be useful, as suggestive of true relations which it does not perfectly express. The value of s_i, which we need to know in order to use this formula, we do know or can find. It is a definite property of iron and of iron alone. But the value of Voigt's Θ_i', as we have shown, is merely a property of iron with relation to lead. Now we know that the lead line cannot be regarded as the real boundary of the thermo-electric diagram. The lines for certain metals lie beyond it, below it in the ordinary diagram, to the left of it in such a diagram as that of Figure 11. These metals have a negative thermo-electric height, a negative thermo-electric entropy, as regards lead. Reckoning thermo-electric heights from lead, taking lead as having zero thermo-electric entropy, is quite as arbitrary as taking the freezing point of water for the zero of temperature. We may do injustice here; it is possible that we have not seen the exact meaning of Voigt's Θ'.

However useful the formula of Moreau may be, his thesis that the Hall effect and all the transverse effects associated with it are due to some deformation of the metal plate in the magnetic field appears to be incompetent to explain the observed facts, especially the Ettingshausen effect, and, if taken alone, misleading rather than helpful. It is altogether probable that the electron theory must be used, if all the phenomena observed are to be accounted for.

The Longitudinal Effects.

All of the transverse effects just considered change direction with change of direction of the magnetic field, the sign of the coefficient in each case remaining unchanged. The associated longitudinal effects, which are now to be described, remain, as the argument from symmetry would predict, unchanged in direction with change of direction of the field. One might, from this absence of dependence on the sign of the field magnetism, expect the longitudinal effects to be proportional to the square of the field-strength, and this may be true of some of these effects in some metals, but in general it appears not to be true. The fact seems to be, however, that the longitudinal effects are much less simple functions of field-strength than the transverse effects are and sometimes change direction with change of intensity of the field. Whether they are strictly or very nearly proportional to the longitudinal gradients of potential and temperature respectively, our own experiments do not enable us to say. We shall, however, assume this proportionality and shall accordingly define a certain coefficient for each of the two longitudinal effects which we have found in iron. For the sake of completeness we shall describe two other longitudinal effects which have been observed, though we have not yet made sure of them in iron.[19]

No one of the four longitudinal effects has any assigned name. Each must be designated by a rather long descriptive title.

Longitudinal Electromagnetic Potential-Difference. — With the magnetic field and the EE current as in Figure 1, we may take note of a change of longitudinal potential-gradient, due to transverse magnetic action, without change of strength of the longitudinal current. Of this effect Zahn says, "As its magnitude is found to be proportional to the strength of the primary [longitudinal] current-strength, it may be taken as a change of resistance."

We shall call this effect positive when it produces an increase of resistance. We take as the coefficient

$$ {}_eL_e = \Delta R \div RH, \tag{21}$$

where R = the normal resistance of a certain length of the plate and ΔR = the increase of resistance of the same length of the plate.

In studying this effect in Plate 1 we connected points (1) and (3) of the plate (see Figure 6), by means of the iron wires, with our galvanometer, introducing into the galvanometer circuit e. m. f. from another circuit, so as to get an approximate balance of forces at the start, thus keeping the scale of the galvanometer within the field of view. Then we applied the magnetic field and observed the deflection produced, which was due to an increase or decrease in one of our counter-balancing e. m. f.'s, the other being not affected by the magnet. This method is troublesome, because it is difficult to keep the requisite degree of equilibrium through an extended series of observations.

This operation gave us for Plate 1 values of ${}_eL_e$ which are exhibited in the following table. The numbers given in the last three columns are approximate only.

1909.	${}_eL_e$.	Arm θ.	C_p.	H.
July 8	$+ 164 \times 10^{-10}$	22°5	2.96	10500
" 9	148 "	22.0	"	10900
Means	$+ 156 \times 10^{-10}$	22°	2.96	10700

With Plate 2 a different method was used. Copper wires connected with points (1) and (3) of the plate led to one coil of the galvanometer, while wires from two other points, A and B (see Figure 13), of the plate-circuit led to the other coil. By means of the shunts S_1 and S_2

[19] But see p. 664, footnote.

an approximate balance of the galvanometer forces was obtained which remained fairly constant when the span of copper wire AB was kept in a bath of oil to prevent sudden changes of temperature. With this arrangement the following result was obtained with Plate 2 :

	eLe.	Arm θ.	C_p.	H.
Dec. 24, 1910	-59×10^{-10}	13°0	3.0	5400

The value of R was about 113,000, absolute, or 0.000113 ohm, and that of ΔR about -3, absolute. This is a small quantity, and it may therefore be well to give a few details as to the observations. Five series, or "runs" were made, three with the current C_p flowing south through the plate, and two with this current flowing north. It is not safe to depend on observations made without such reversal of the main current; for there is always a possibility of some illegitimate action of the electromagnet circuit on the galvanometer which can be eliminated by combining two series of data, one obtained with the current in the plate running north, the other with this current running south. The five series mentioned gave the following mean galvanometer deflections :

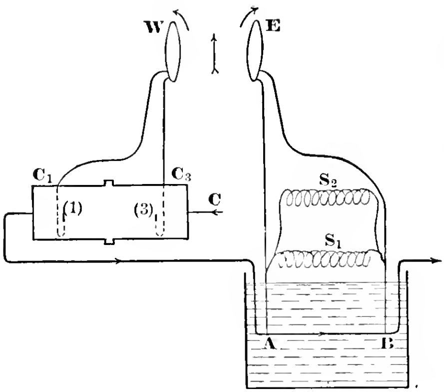

FIGURE 13.

	With C_p north.	With C_p south.
	+ 0.33 cm.	+ 0.11 cm.
	+ 0.24 "	− 0.21 "
		− 0.22 "
Means	+ 0.29 "	− 0.11 "

Final mean $= \frac{1}{2}(0.29 + 0.11) = 0.20$ cm.

The first series with C_p south, which gives a mean deflection discordant in sign with the others, was the shortest series and internally the worst of the five.

One other possibility of error must be considered here. The copper wires C_1 and C_3, being connected with the iron plate, form a thermo-electric couple, and if the action of the magnet should make a difference of temperature amounting to 0.01 degree between the junctions (1) and (3), the thermo-electric action of this copper-iron couple would produce as great a deflection of the galvanometer as that given above, which was taken to indicate a change of resistance. Accordingly, it was necessary to consider the possible magnitude of this second longitudinal effect, which will presently be considered.

It is to be noted that Plate 1 shows in a field of 10,700 an *increase* of resistance which we may represent, on an arbitrary scale, by 16, while Plate 2 shows in a field of 5400 a *decrease* of resistance which we may represent on the same scale by 3. Zahn says, "In ferromagnetic metals there is in weak fields an increase of resistance, in stronger fields a decrease. According to the results of Grunmach for iron and cobalt and a preliminary publication of Blake on measurements in nickel, the initial increase is perhaps due to longitudinal components [of magnetism]." When it is remembered that in one of our plates, (1), the resistance is measured at right angles with the fibres or grain of the iron, while in (2) it is measured in the direction of the fibres, it need not, perhaps, be considered strange that the change of resistance caused by magnetic action is opposite in direction in the two cases. In both cases the change is very small.

We now proceed to the other longitudinal effect which is so intimately connected with the one just discussed.

Longitudinal Electromagnetic Temperature-Difference. — With the magnetic field and the EE current as in Figure 1, we may look for a difference of temperature, $\Delta\theta'$, established by magnetic action between two points d cm. apart along the plate. We have spent much time in looking for this effect, especially in Plate 1, being led on by accidental phenomena which simulated it rather persistently, though with discrepancies that were suspicious, until a slight change in the disposition of the plate between the poles of the magnet wiped out the apparent temperature change which we had been studying and possibly gave a slight apparent change in the opposite direction.

With Plate 2 we gave only one day, a whole day, to this particular question. The method used was the following: Copper-constantan couple (1), having one junction at point (1) on the plate and the other in a glass tube placed in a water-bath, was put in circuit with one coil of our galvanometer. Couple (3), with one junction at point (3) on the plate and the other in the water-bath, was put in circuit with the other coil of the galvanometer, opposing the action of couple (1) and nearly balancing it. Then the magnetic field was brought into action and the consequent deflections of the galvanometer were observed.

The day's work gave a net deflection of 0.03 cm. as the result of magnetic action, — a quantity smaller than its own probable error. This deflection, if considered reliable, would indicate a temperature-difference of 0.0008° C., — about a tenth of the temperature-difference that would be needed to account for the apparent change of resistance which has been described above.

The coefficient of the longitudinal effect here looked for but not found may be defined as follows:

$$_{e}L_{h} = \frac{\Delta\theta'}{d} \div \frac{C_{p}}{wt} H. \tag{22}$$

Longitudinal Thermomagnetic Potential-Difference. — With the magnetic field and the *HH* current as in Figure 3, we may take note of the difference of potential, $\Delta P'$, set up by magnetic action between two points d cm. apart along the plate. We shall call this effect positive when the potential-gradient thus established is in the same direction as the temperature-gradient along the plate. As the coefficient of this effect we have

$$_{h}L_{e} = \frac{\Delta P'}{d} \div \frac{d\theta}{dl} H, \tag{23}$$

where $\frac{d\theta}{dl}$ means the temperature-gradient along the plate.

We have not looked for this effect in Plate 2. In Plate 1 we studied it at considerable length, because of a disagreement between our observations and those reported by Zahn on the authority of Houllevigue and of Moreau. He says, "The phenomenon is complicated here [in ferromagnetic metals], for the sign is dependent on the field-strength. With small H the potential-gradient established is in the same direction as the temperature-gradient; it reaches a maximum at ordinary temperature [*bei mittlerer Temperatur*] for a field-strength of about 3000 gausses, then vanishes, in nickel for H about 4500, in iron and steel at about double this field-strength, and takes, for higher fields, the opposite direction. The reversal of sign occurs at a lower field-strength the higher the mean temperature of the plate is."

We used at first the iron wires I_1 and I_3, connected with points (1) and (3) on the plate. The outer ends of these wires were permanently joined to copper wires, in junctions like the one shown in Figure 10, and from these junctions the copper wires led to the galvanometer. After such a balance of thermo-electric forces in this circuit as to keep the scale of the galvanometer in the field of vision had been effected, the magnetic force was brought into action. An effect was observed;

but we soon found that the thermo-electric behavior of the iron wires, as well as that of the plate, was affected in such a way by the magnetization that it was impossible to get satisfactory results with this arrangement.

Accordingly we attached two other iron wires, one to each of the copper blocks B and B of Figure 7, near the ends of the iron plate but well outside the powerful intrapolar region of the magnetic field, within which the iron wires previously used lay. Working with this arrangement we got very considerable effects which showed that transverse magnetization produced in the plate a change of quality or condition such as to make the copper block at the hotter end of the plate electrically positive compared with the copper block at the colder end. This difference of potential, $\Delta P'$, we could and did measure, but there was, and still is, some uncertainty as to the value which should be assigned to d in this case. It should not be the whole length of the plate, for the whole length was not subjected to the full strength, H, of the magnetic field. We have taken d as 80 per cent of the whole length of the plate between the blocks, and have reckoned ${}_hL_e$ accordingly. Each value of this coefficient, as given below, was found from two sets of observations, one made with the heat-current flowing south, the other with it flowing north. As usual, the numbers given in the last three columns are approximate only.

1910.	${}_hL_e$.	Arm θ.	$\frac{d\theta}{dl}$.	H.
June 14–17	80×10^{-5}	47°	13.7	12000
	83 "	47°	13.7	8200
	211 "	47°	13.7	2000
28–29	69 "	48°	13	12000
	73 "	48°	13	7900
	187 "	48°	13	1900

The relation of ${}_hL_e$ to H in these two groups of data is shown by means of the two curves of Figure 14. The × points are for the earlier observations, the o points for the later ones. The two curves are very like, and neither suggests a reversal of sign of the effect in question at any stage of magnetization. Why one curve runs notably lower than the other through its whole course we cannot say with full confidence. It is, however, a fact that in the period of the June 14–17 observations the arrangement for getting the sensitiveness of the galvanometer was under suspicion. In the warm, moist atmosphere of a Cambridge summer the problem of insulation in delicate electrical measurements becomes considerable. Leakages occur to a disturbing extent where during the months of artificial heating of the laboratory they need not be regarded. There was perceptible leakage from the magnet-circuit into the galvanometer-circuit during June. Efforts were made to eliminate errors from such disturbances by combining series of observations taken under a variety of conditions, but we cannot be sure that these efforts were entirely successful. It is unlikely, however, that they affected vitally the results at which we arrived.

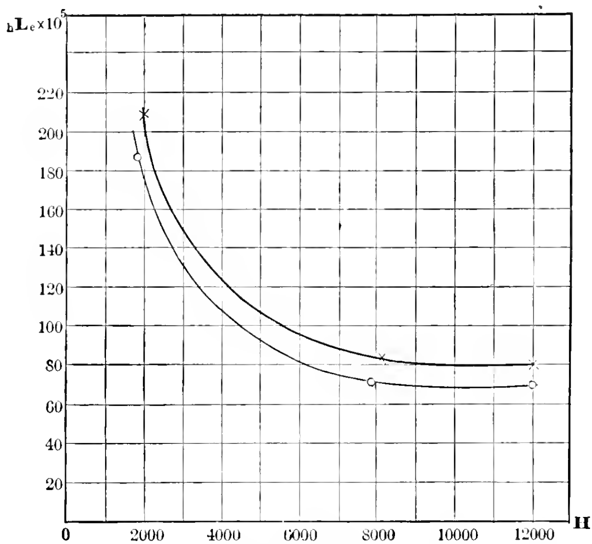

FIGURE 14.

Longitudinal Thermomagnetic Temperature-Difference. — With the magnetic field and the *HH* current as in Figure 3, we may look for an increase or decrease of the temperature-gradient along the plate as the result of magnetic action. This effect was looked for in Plate 1, but its existence was not proved. The method used was substantially the same as that illustrated in Figure 9 for determining a transverse temperature-difference. That is, circuits (1) and (3), acting thermo-electrically only, were set against each other, each having one coil of the galvanometer. The great initial difference of temperature between the junction (1) and the junction (3) made it necessary to place the outer junctions of the (1) circuit and of the (3) circuit in baths at different temperatures.

We thought at one time, in April, 1910, that we had found a rather large positive value for the coefficient in question, which may be defined as follows :

$$ {}_hL_h = \frac{\Delta\theta'}{d} \div \frac{d\theta}{dl} H, \qquad (24) $$

$\Delta\theta'$ being the increase of temperature-difference between two points d cm. apart along the plate.

Critical examination of the case, however, showed that in using, as we must with Plate 1, iron wires connected with points (1) and (3), we had brought in a source of error so great as to make the result entirely uncertain.

We hope to try the experiment with Plate 2, using now the constantan-copper junctions (1) and (3).[20]

Summary.

1. A new and descriptive nomenclature is proposed and used for the various transverse and longitudinal coefficients. (See equations (1), (2), (3), (4), (23), (24), (25), (26).) For example, the Hall-effect coefficient is called ${}_eT_e$, the initial subscript e indicating that the longitudinal current is electrical, the T indicating that the effect observed is transverse, the final subscript e indicating that this transverse effect is electrical. The Ettingshausen coefficient is called ${}_eT_h$, the Nernst coefficient ${}_hT_e$, the Leduc coefficient ${}_hT_h$. Each of the four transverse effects is called + when the current to which it gives rise in the exterior part of a transverse circuit leaves the metal plate in the direction of the "ponderomotive" force acting on this plate or in what would be the direction of this force if longitudinal heat-flow were replaced by longitudinal electric flow. This accords with the usual convention regarding the sign of the Hall effect, but is opposite to the usual convention regarding the sign of the Nernst effect.

2. Measurements of ${}_eT_e$, ${}_eT_h$, ${}_hT_e$, ${}_hT_h$, and of their temperature-coefficients through a small range, have been made in a single plate of soft iron, for which the electrical conductivity, the Thomson-effect coefficient, and the thermo-electric height relative to copper, were already well known through previous study of pieces from the same bar that furnished the plate. (See Table I.)

3. The Moreau formula ${}_eT_e \div \rho = {}_hT_e \div s$, in which ρ is the specific resistance, absolute, of the iron, and s is the number of ergs of heat-energy absorbed by ten coulombs in rising through one centigrade degree in iron (the Thomson effect), is discussed theoretically and is

[20] See footnote, p. 664.

tested by means of the data for iron. (See Table II.) The formula is true for one temperature, near 80° C., according to our data, but departs progressively from the truth with descent from this temperature and probably with ascent above it. Below the temperature mentioned the first member of the Moreau equation, as here given, is too small; above this temperature it is, probably, too large. Hence Moreau's theory, which would account for the Hall effect by a deformation of the plate under magnetic action (this deformation producing a real rotation of the equipotential lines of the longitudinal electric current), and would account for the Nernst effect as a like and equal rotation of the Thomson-effect equipotential lines of a longitudinal temperature-flow, appears to be ill-founded and, if taken literally, unsafe, though it is likely to be useful in a suggestive way.

4. Following the suggestion of the Moreau formula, we tested the equation ${}_eT_e \div \rho = {}_hT_h$ for various temperatures, to see whether the isothermal lines of the longitudinal temperature-flow are, in a field of given strength, rotated in the same direction and to the same extent as the equipotential lines of a longitudinal electric flow. (See Table III.) The direction is the same in the two cases, but the equipotential lines are, according to this test, rotated about one and a quarter times as far as the isothermal lines, this ratio remaining nearly constant through the whole range of temperature considered, 20° to 100°. (See Table IV.)

5. "Rotation" cannot account for the Ettingshausen effect, since there is no corresponding longitudinal condition to be rotated, none, that is, which can be regarded as adequate to produce the observed transverse effect. On the other hand, the ratio of the Hall transverse potential-gradient to the Ettingshausen transverse temperature-gradient appears to be almost strictly proportional, through a considerable range of temperature, to the Thomson effect s. Thus we have, approximately, ${}_eT_e \div {}_eT_h = 200\, s$, from 20° to 80° or higher. (See Table V.) It should be said, however, that the temperature-coefficient of ${}_eT_h$ cannot be regarded as accurately known. The suggestion is made that the transverse potential-gradient, which is like the Thomson-effect potential-gradient in being *static* (not attended by flow down the gradient), may be the cause of the transverse temperature-gradient, whereas in the Thomson effect the temperature-gradient causes the potential-gradient. It seems likely that the electron theory must here be used, but the attempt is not made in this paper.

6. The Voigt formula ${}_eT_e \div \rho = {}_hT_e \div \Theta'$, in which Θ', the "thermo-electric height" of lead relative to the metal, takes the place of the Thomson effect s of the Moreau formula, is considered. The iron here

studied has never been tested against lead directly, but from Matthiessen's value of the thermo-electric height of "galvanoplastic" copper relative to lead, and from the known thermo-electric height of this iron relative to copper wire of the present day, values of Θ' have been estimated for various temperatures, from 20° to 100°. With these values of Θ', and with values of the other factors taken from or reckoned from direct observations on the iron here studied, the Voigt formula has been tested. (See Table VII.) Like the Moreau formula, it seems to be correct at one temperature, between 80° and 90°, and to be untrue at other temperatures. Unlike the Moreau formula, it fails because the left-hand member is too large below this particular temperature. No constant added to the value of Θ' would make the formula hold true with varying temperature. In this connection it is pointed out that, the ordinary thermo-electric diagram being merely a temperature-entropy diagram (with the temperature-axis horizontal and the entropy-axis vertical, unfortunately contrary to the familiar custom of ordinary thermodynamics), thermo-electric heights are merely entropy-differences. Accordingly, Voigt's Θ', the thermo-electric height of lead relative to any metal, cannot be regarded as a fundamental datum for the metal; for certainly we cannot suppose the entropy of electricity to be zero in lead, as we know there are metals which have a negative thermo-electric height, a negative electric entropy, relative to lead.

7. A thermo-electric diagram is given for copper and iron in which the temperature-axis is made vertical and the entropy-axis horizontal. This diagram shows graphically, by curvature of the iron-line, the law of change of the Thomson-effect coefficient with change of temperature, which law was in an earlier paper expressed algebraically. (See Figure 12.)

8. The longitudinal effect which is shown as a change of resistance of the iron plate by magnetization in the direction of its thickness, the coefficient of which effect is called $_eL_e$, was observed in two iron plates, one cut with its width parallel to the fibres, or grain, of the iron, the other with its length parallel to these fibres. In the former plate there was a very slight *increase* of resistance in a field of 10,700 absolute units, in the latter a still slighter *decrease* of resistance in a field of 5400.

9. The longitudinal effect which is shown as a change of longitudinal potential-gradient, due to magnetization in the direction of the thickness of the plate, in a plate along which a heat-current is flowing, was observed in one of the plates mentioned in (8), the one first described there. It was not looked for in the other plate. The coefficient of this effect is called $_hL_e$. (See equation (25).) According to Zahn, who quotes Houllevigue and Moreau as authorities, the potential-

gradient established in iron by magnetic action is in the same direction as the temperature-gradient in comparatively weak fields, up to perhaps 9000, but is in the opposite direction in stronger fields. In the iron here studied the potential-gradient produced by the magnetization is in the same direction as the temperature-gradient all the way from a field-strength of 1900, the lowest tried, to 12000, the highest tried, though the coefficient ${}_hL_e$ is much larger for weak fields than for strong fields (see Figure 14).

10. The other two longitudinal effects, the coefficients of which may be called ${}_eL_h$ and ${}_hL_h$, were looked for diligently in one plate, but their existence there was not proven.[21]

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Postscript added March 11, 1911. — The objection which we have made to the use of Θ'_m, the "thermo-electric height" of lead with respect to any metal m, as a fundamental property of the metal m, seems to me so plain that I have been at great pains to make sure that we have not fallen into some gross blunder of interpretation of Voigt's theory. There can be no question, however, that Moreau takes Voigt's Θ'_m in this way, and I have seen no protest against his doing so. There is possibly room for doubt as to whether Voigt had *lead* in mind or some other, unnamed, material of zero Thomson effect, with reference to which his Θ'_m is to be reckoned, but this question is of little importance.

Apparently Moreau does not criticise Voigt's theoretical discussion. He seems merely to point out that it is different from his own, and to test each empirically by means of data which he has taken from various sources. He says, "I consider a circuit [*chaine*] formed by a metal M and lead, the junctions being at different temperatures. At a point (x, y, z) of M, if X, Y, Z are the components of the thermo-electric electromotive force per unit of length, one has, according to M. Voigt,

$$X = \Theta' \frac{\partial T}{\partial x}, \quad Y = \Theta' \frac{\partial T}{\partial y}, \quad Z = \Theta' \frac{\partial T}{\partial z}, \quad \text{where } \Theta' = \frac{\partial \Theta}{\partial T}. \tag{1}$$

Θ is a function of the absolute temperature T, and is characteristic of the metal M."

"Let us suppose that a thin slice of the metal M is placed in a mag-

[21] It now appears that magnetization parallel to the thickness decreases notably the longitudinal temperature-gradient in Plate 2. March 11, 1911.

netic field H, perpendicular to the lines of force, the plane XY coinciding with the plane of the slice, which will be traversed by a current of heat going in the direction OY. Equations (1) give

$$X = 0, \qquad Y = \Theta'_m \frac{\partial T}{\partial Y}, \qquad Z = 0.$$

"M. Voigt supposes that the force Y turns under the action of the field as the electromotive force of the primary current in the Hall phenomenon turns. One obtains, then, along the axis OX, a transverse electromotive force X, which, referred to the unit of length, is

$$X_1 = -\frac{C}{\rho} \Theta'_m \frac{\partial T}{\partial y} H$$

(where C is the Hall coefficient, ρ the resistivity of the metal), or, according to equation (3) $[\Phi = \Theta'_m]$,

$$X_1 = -\frac{C}{\rho} \Phi \frac{\partial T}{\partial y} H. \tag{4}$$

"This formula (4) gives the thermo-electric effect according to M. Voigt."

We thus get as the expression, according to Voigt, for K, the Nernst coefficient,

$$K^{22} = X_1 \div \frac{\partial T}{\partial y} H = -\frac{C}{\rho} \Phi,$$

and this is the expression which Moreau tests by means of data which he gives. For example, he gives the value of Φ as -1619 for iron and -152 for copper, evidently taking Φ as the thermo-electric height of *lead with respect to* the other metals.

As to his own formula for K, Moreau says, contrasting his point of view with that of Voigt, "By assuming that, *only*, the thermo-electric electromotive forces relative to the Thomson effect turn under the action of the field, I have obtained the formula

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22 Moreau does not expressly say that $K = -\frac{C}{\rho}\Phi$ according to Voigt. He does, however, put the values of $-\frac{C}{\rho}\Phi$ alongside his own values of K for comparison. Moreau may have been in doubt as to the sign of the Voigt formula.

$$X_2 = KH \frac{\partial T}{\partial y}, \qquad \text{or} \qquad K = \frac{\sigma C}{\rho},$$

σ being the specific heat of electricity."

Moreau, then, seems to admit that Voigt's Y is a real thermo-electric force,

$$\frac{\partial \Theta_m}{\partial T} \frac{\partial T}{\partial y}, \qquad \text{or} \qquad \frac{\partial \Theta}{\partial y},$$

directed along the Y-axis at any point in the metal M, though not the force which he considers significant in the Nernst effect. But it seems to me that this supposed total electromotive force at any point in the metal M is a fiction. The only electromotive forces which we have reason to suppose existing in an unequally heated piece of metal *in open circuit*, as a metal is when tested for the Nernst effect, are those found in the Thomson effect. When we have two unequally heated metals united in a thermo-electric circuit, the total electromotive force at any point in either metal is likely to be something different from that which the Thomson effect alone at that point would account for, but we have no sufficient reason for supposing it to be actually the $\frac{\partial \Theta_m}{\partial T} \frac{\partial T}{\partial y}$ mentioned above. If, for example, we are considering iron and if, for simplicity, we assume that $\partial T / \partial y = 1$, the thermo-electric e. m. f. in the iron at temperature T is, according to Voigt, represented by the length of the line TI, taken as a negative quantity, in Figure 11 of our paper. Now it is true that, if we make this assumption and then integrate around the whole circuit, which we will suppose to be of copper and iron, we shall get the area $C_0CII_0C_0$ as the total e. m. f. of the circuit, which result will be correct. But we should arrive at precisely the same correct integral result if we placed the T_0T line of Figure 11 indefinitely far to the left or to the right of its present position, which would have the effect of increasing or decreasing indefinitely the value of the expression $\partial \Theta_m / \partial T$, the supposed force at any individual point in either metal. I can see, then, no objective reality in Voigt's supposed force Y.

Moreau was, I think, the first to put Voigt's formula for the value of the Nernst coefficient to the numerical test. He gives,[23] the three bottom lines being from his own observations,

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[23] C. R., **130**, 564 (1900).

	C_{20}. (Nernst.)	Φ_{20}.	$-\frac{C}{\rho}\Phi_{20}$.	$K = \frac{\sigma C}{\rho}$.	K (obs.). (Nernst.)	[σ_{20}.[24]]
Bismuth	−10.1	+8800 or +6400	+0.337 or +0.246	+0.149	+0.196	−1875
Antimony	+0.192	−2240	+0.0097	+0.0090	+0.0094	+2074
Nickel	−0.024	+2280	+0.0039	+0.0026	+0.0073	−1482
Cobalt	+0.0046	+2200	−0.00094	−0.00175	+0.00154	−4073
Iron	+0.0113	−1619	+0.0018	−0.00156	−0.00156	−1412
Steel	+0.0175	−1731	+0.00114	−0.00062	−0.00060	− 952
Copper	−0.00052	−152	−0.000047	−0.000084	−0.000073	+ 275
Zinc	−0.00041	−279	−0.000018	−0.000046	−0.000054	+ 697
	C_0.	Φ_0.	$-\frac{C_0}{\rho_0}\Phi_0$.		K_0 (obs.).	
Soft iron	+0.00837	−1062	+0.000636		−0.000646	
Soft steel	+0.00662	−1351	+0.000814		−0.000596	
Cobalt	+0.00399	+1318	−0.000512		−0.00146	

It will be seen that K, which = our ${}_hT_e$ and Zahn's $-Q$, as calculated from the formula of Moreau and given in the fourth column agrees in sign with K as observed by Nernst in every case except that of cobalt, and that in this case direct observation by Moreau himself gives the same sign which his formula gives. In other words, according to Moreau his own formula gives the right sign for K in every case. Zahn, however, agrees with Nernst in his observation of the sign of the Nernst effect in cobalt.

Moreau finds the Voigt formula, or at least the expression $-\frac{C\Phi}{\rho}$, to give the same signs as his own formula in all the cases examined except those of iron and steel, but I am inclined to the opinion that he has made a mistake of sign in the use of the Voigt formula. It seems to me that the Voigt formula gives the right sign for K in iron and

[24] C. R., **130**, 124 (1900).

steel and the wrong sign in the other cases. I have not found any perfectly explicit statement by Voigt himself of the relation between the Nernst coefficient and the Hall coefficient, but I reason as follows: If we consider iron and take a case in which $\partial T/\partial y$ is positive, we have, as we saw a little distance back, Y as a negative quantity; that is, according to Voigt, there is a thermo-electric e. m. f. acting in the direction of decreasing y. But with our iron in open circuit, as it would be for the Nernst-effect observations, we have no flow of electricity along the iron, the natural thermo-electric e. m. f. being balanced, or, as Voigt says, *compensated*, by a static charge having a negative potential-gradient in the positive direction of y. If, now, we think of the magnetic field as rotating the equipotential lines of this potential-gradient in the direction of the Amperian current, as the equipotential lines of the primary electric current are rotated in the Hall effect in iron, we get what in our paper has been called, and what Moreau would call, a *negative* Nernst effect.

According to this argument, which deals with a puzzling question and may be incorrect in its conclusion, all the signs in the third column of Moreau's table, as given above, should be changed, and in every case for which the specific heat of electricity within a metal and the thermo-electric height of the metal with respect to lead have the same sign the Moreau formula and the Voigt formula will predict opposite signs for the Nernst effect.

The Moreau formula seems to me profoundly suggestive, though not strictly correct. But the fact that the Voigt formula, which I believe to be fundamentally wrong in one of its factors, gives results numerically so like those of Moreau, and so like the values given by direct observation, is enough to warn us to be cautious in examining every theory and testing every formula relating to the matters here dealt with.

E. H. H.

Proceedings of the American Academy of Arts and Sciences.

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ON THE OPACITY OF CERTAIN GLASSES FOR THE ULTRA VIOLET.

By Louis Bell.

With Two Plates.

Investigations on Light and Heat published with Aid from the Rumford Fund.

ON THE OPACITY OF CERTAIN GLASSES FOR THE ULTRA-VIOLET.

By Louis Bell.

Presented February 8, 1911. Received February 8, 1911.

The absorption of ultra-violet radiations by various media has been the subject of numerous investigations which need not here be recounted. So far as modern optical glasses are concerned our knowledge rests chiefly on the researches of Krüss,[1] who investigated a series of ten typical Jena glasses by means of a fluorescent-screen polarizing photometer. His results extended to w. l. 309 $\mu\mu$ and included thicknesses of 1, 10, and 100 mm. of glasses ranging from a very light boro-silicate crown of index $n_D = 1.51$ to a dense silicate flint having $n_D = 1.67$. At the lower limit of wave length there was practically complete absorption by all these glasses in a thickness of 1 cm. and partial transmissibility for the lighter glasses in thicknesses of 1 mm. The most striking thing about his results was the generally similar form of the absorption curves and the very rapid increase of absorption below w. l. 350 $\mu\mu$ to 340 $\mu\mu$. For greater wave lengths than the first mentioned, optical glasses, save for the heaviest flints, are moderately transparent, as is found in ordinary practice. The writer, indeed, many years ago, photographed the solar spectrum down to Cornu's line O, w. l. 344 $\mu\mu$, through a large 45° prism of moderately dense flint, $n_D = 1.62$. Zschimmer[2] has added to Krüss' work an interesting investigation of the so-called Uviol glasses and some other optical glasses, with special reference to the value of the Uviol glass in transmitting the shorter wave lengths in the ultra-violet. His spectrograms show in some of the Uviol glasses transmission to as low as w. l. 265 $\mu\mu$ in a thickness of 2 mm., and down to about w. l. 295 $\mu\mu$ in a thickness of 10 mm. This shows, from the standpoint of transmissibility, a great improvement over the best of the ordinary glasses, spectra through which usually terminate at about w. l. 300 $\mu\mu$. Some extremely light crowns are a trifle more transparent.

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[1] Ztschr. f. Instrumentenkunde, **23**, 197, 229.

[2] Ibid., **23**, 360.

In the course of the investigation here undertaken, for example, the writer reached w. l. 289 $\mu\mu$ through 5 mm. of Parra-Mantois "crown extra léger," $n_D = 1.476$. The existence of glasses of this order of transparency is, as Zschimmer has pointed out, of importance in astrographic and spectrographic work in which it may be desirable to take advantage of all the rays which are transmitted by the atmosphere. As is well known, the atmospheric absorption becomes exceedingly strong below about w. l. 310 $\mu\mu$, and extinguishes the solar spectrum at about w. l. 295 $\mu\mu$. Below this point, absorption is complete for the whole thickness of the atmosphere, and as one nears w. l. 200 $\mu\mu$ it is practically complete, even for layers of a few cm. in thickness. This is the region in which the atmospheric absorption is displayed by very powerful ionization, which must therefore be a striking phenomenon in the upper regions of the atmosphere, since the solar light, composed as it is of well-known metallic radiations extending far into the ultra-violet, fails to show these only in virtue of atmospheric absorption.

The work of Krüss and of Zschimmer was carried on by means of spectrographs constructed of quartz, or quartz and fluorspar, and they both used, as a source, the quartz mercury lamp, which is by far the best available source for such investigations, inasmuch as when the lamp is held at a uniform current it gives a wonderfully steady radiation, which is not true of any other source rich in ultra-violet rays.

From another standpoint, that of the investigation of glasses particularly adapted to cut off the ultra-violet, which are of interest to the student of physiological optics, several investigators have recently attacked the problem. In particular, important researches have been published by Voege,[3] Hallauer,[4] and Schanz and Stockhausen.[5] The studies from the ophthalmological side have unfortunately mostly been made with open electric arcs of various kinds, which are subject to so considerable fluctuations, even under the most favorable circumstances, as to make the comparison of different spectrograms, even when taken in immediate succession, somewhat untrustworthy, although the distribution of intensity in a single spectrogram may be judged with considerable confidence.

In fact the quantitative value of spectrographic studies is somewhat limited, even with the steadiest sources of radiation, on account of the different performance of different makes of photographic plates and the varying action of developing and fixing baths. One can, however,

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[3] The Illuminating Engineer, London, **2**, 205, 543.

[4] Archiv. of Ophthalmology, **21**, 1.

[5] Ztschr. f. Augenheilkunde, **23**, 397.

by uniform methods of treatment, get results which have a comparative value great enough to decide most of the important questions which come up for consideration in studying ultra-violet absorptions.

Having a quartz lamp and spectrograph set up for another research, it seemed desirable to the writer to take advantage of this apparatus for the preliminary investigation of some of the glasses commonly used in the practice of American opticians, since the data already referred to apply chiefly to glasses exploited mainly upon the Continent. The apparatus used was the concave-grating spectrograph of the Rogers Laboratory of Physics, kindly placed at the writer's disposal by the Director. It is fitted with a Rowland concave grating of approximately 1.75 meters radius of curvature, mounted in the ordinary Rowland manner. The grating itself has a ruled surface 24 × 50 mm. ruled with about 14,438 lines per inch. The plateholder takes plates up to 3 × 13 cm., and the whole instrument is set up in a light-tight room with the slit and the source outside. The lamp employed was one of the ordinary quartz mercury lamps of the French Cooper-Hewitt Co., operating at about 200 volts. In these investigations it was used at a normal current of 4 amperes. The lamp was set up about 90 cm. from the slit, and the tube was focussed upon it by a quartz lens of about 20 cm. focal length. An ammeter was kept in circuit with the lamp so that the current could be adjusted to a uniform value by a rheostat. The path of the rays from lamp to plateholder was thus slightly less than 4.5 meters. Inasmuch as only the violet and ultra-violet portion of the spectrum was thus investigated, ordinary photographic plates (Seed's No. 27) were used. These were developed, each set of exposures together, in a large tray, with 5 per cent rodinal for 5 minutes, and fixed together in stock hypo-solution. Except for special purposes of comparison in which other exposures are stated, the plates were exposed for a uniform period of 5 minutes and developed as soon as each particular set had been exposed, the spectrograph room being fitted with facilities for doing this. The printing was likewise done in groups to secure uniformity.

The glasses investigated included a few optical glasses which seemed to be of interest, and a group of colored glasses of the kinds frequently used for protective spectacles in this country. In working close to the quartz lamp, which was so situated that one had to work at times within half a meter, the operator's eyes were protected by spectacles composed of two of the glasses referred to later, but such protection seems to be quite unnecessary in ordinary working around the laboratory with this lamp. In fact the writer was convinced, during a long period of experimenting, that the dangers to the eye from the quartz lamp, although not negligible under extreme conditions, have been very much exaggerated. These observations in fact quite confirm the recent statements of Birch-Hirschfeld[6] on this subject. The ordinary care which one exercises in keeping intensely brilliant lights out of the eye at short range seems also sufficient to avert any trouble from the specific effects of extreme ultra-violet radiations.

The accompanying plates, all taken with a rather wide slit, about .2 mm., summarize the results obtained.

Figure 1, Plate 1, shows the ultra-violet spectrum of the quartz lamp from about w. l. 410 $\mu\mu$ to about w. l. 310 $\mu\mu$, as shown by the approximate wave-length scale above. It will be noted that, save for the brilliant triplet near w. l. 365 $\mu\mu$, the region of the ultra-violet, down to the double at about w. l. 313, has very few strong lines, and the region between the two groups mentioned is especially barren. There is in fact much less total strength apparent in the part of the quartz-lamp spectrum shown on these plates, than appears in the spectra of the sun and of ordinary electric arcs, the strong mercury lines being few. It is only in the extreme ultra-violet beyond the range shown in Plate 1, that the mercury spectrum is particularly brilliant.

Figure 2, Plate 1, shows the same spectrum when cut down by a polished plate of A.O. crown glass 6.2 mm. thick. The weakening of the further ultra-violet is here very marked, especially in case of the doublet at w. l. 313 $\mu\mu$. The absorption near the other end of the strip is comparatively small and, in fact, the glass may be said to be moderately transparent to near w. l. 313 $\mu\mu$. No lines of the mercury spectrum of less w. l. than this appear on this plate or on any other of the plates reproduced herewith. In other words, all the glasses tested and shown in these spectrograms cut off completely all radiation of w. l. less than that stated. The A.O. crown is one of the glasses commonly used for spectacle lenses in American practice.

Figure 3, Plate 1, is the quartz arc spectrum as reduced by a medium tint of ordinary commercial "smoke" glass 5.6 mm. thick. In this case everything of less w. l. than about 360 $\mu\mu$ is completely obliterated. The strong group near w. l. 365 $\mu\mu$ gets through with a fair degree of brilliancy, and a similar moderate absorption extends into the violet and elsewhere into the visible spectrum.

Figure 4, Plate 1, is the spectrum as reduced by a plate of B. & L. crown 4.2 mm. thick, a glass also much used for spectacles and comparable with Figure 2. It has, however, a slightly higher refractive index and shows more absorption in the extreme lines and not quite as

[6] XI Congresso Internazionale di Oftalmologia, p. 623.

much, owing perhaps to its less thickness, in the left-hand side of the spectrum. It is, however, moderately transparent to the ultra-violet.

Figure 5, Plate 1, is the result obtained from a very pale amethyst glass in a thickness of 2.8 mm. This is a glass which has been considerably used for protective spectacles in which great density may not be desirable. It absorbs a trifle more of the extreme rays than do the clear crown glasses, and conspicuously more from the violet to and below w. l. 360 $\mu\mu$. The appearance of the spectrogram suggests somewhat selective absorption, but it is little more effective as regards absorption in the ultra-violet generally than the common smoke glass.

Figure 6, Plate 1, is a protective glass of a curious light yellowish-pink hue, which has been considerably used under the name of Arundel tint. It shows considerably stronger absorption than the amethyst glass, especially for the shorter rays; the doublet at w. l. 313 $\mu\mu$ is completely wiped out. The next conspicuous line at w. l. 334 $\mu\mu$ is very greatly reduced in intensity, as is indeed the remainder of the ultra-violet shown in this spectrogram. It also reduces the blue end of the visible spectrum slightly, although its general tint is very light.

Figure 7, Plate 1, is the quartz arc spectrum of Figure 1 as reduced by a slip of Fieuzal glass 4 mm. thick. It completely obliterates the whole ultra-violet region and the blank spectrogram is introduced here merely for sake of contrast. It is a medium tint of yellowish-green glass, cutting out much of the violet and blue and reducing considerably the blue-green, although, since it retains all the rays of highest luminosity in the spectrum, it does not cut down the total light sufficiently to serve as a protection against arc lamps or other extremely brilliant lights. It is, however, a pleasant and effective glass where considerable reduction in intensity is not required.

Figure 8, Plate 1, also a complete blank, was produced by a sheet of ordinary commercial amber glass of medium tint and 2 mm. thick. Its absorption of the ultra-violet is complete, and is strong in the violet, blue, and blue-green. Its tint is due to letting through the red a little more freely than does the Fieuzal glass just considered, which in general properties it resembles rather closely. It is not sufficiently dense to protect adequately against the dazzling effect of very brilliant sources of light in the tint here examined, but is dense enough to serve all ordinary purposes well. Its extinguishment of the ultra-violet, however, is most complete, a property which it probably shares with many other varieties of yellowish and orange-tinted glasses. A slip of a reddish amber selenium glass, such as is sometimes used for railway signals, 3.5 mm. thick, likewise cut off the whole ultra-violet region, giving a blank spectrogram. This last-mentioned glass absorbed power-fully, clear down into the green, and, while fairly transparent to the rays of high luminosity, was preferable to either of the glasses just described for reducing the painful glare of powerful illuminants.

Figure 9, Plate **1**, taken at the end of this series, shows again the spectrum of the bare quartz tube with an exposure of 15 seconds. The plate therefore, in this case, received just five per cent of the total energy recorded on Figure 1, Plate **1**. The line near w. l. 391 $\mu\mu$ appears faintly in the negative, but is lost in the reproduction. The line near w. l. 405 $\mu\mu$ does not appear on this plate since, in this case, as in some of the others, the plateholder had been shifted by a small amount in order to reach down a little further into the ultra-violet. A comparison of this plate with those showing the effects of the absorbing glasses gives a vivid idea of the extent of the absorption produced by the glasses, especially those having even slight coloration.

In connection with these figures of Plate **1** should be considered Figure 8 of Plate **2**, exposed under parallel conditions for 15 minutes. This shows the extreme ultra-violet portion of the quartz lamp spectrum, the last line visible in the reproduction being at w. l. 230 $\mu\mu$. With the exception of the doublet at w. l. 313 $\mu\mu$, this portion of the ultra-violet was obliterated by the absorption of the glass in all the spectrograms shown. The relatively great richness of the spectrum in the extreme ultra-violet portion down to about w. l. 230 $\mu\mu$, where atmospheric absorption became very powerful, is conspicuous in this figure. In the lower portion of this spectrum are indicated the so-called "abiotic" radiations, which are particularly active in cell destruction, and as the quartz lamp here tested was put to use for bactericidal experiments, this portion of the ultra-violet was of special interest. As Cernovodeanu and Henri [7] have shown, the bactericidal effects are practically confined to wave lengths below 270, and increase beyond this point with enormous rapidity, an effect very possibly corresponding to the powerful ionization produced in this part of the extreme ultra-violet.

Figures 1 to 7, inclusive, of Plate **2**, are not strictly comparable with the figures of Plate **1**, since they were taken at a later period, after the spectrograph had been in use for other work. The apparatus was set up anew. The grating was turned over, giving a somewhat more brilliant first order spectrum, and the brightest portion of the quartz tube was focussed upon the slit. The exposure in this series was, except when noted, 5 minutes, as before, but the intensities were notably higher.

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[7] Elektrotechnik u. Maschinenbau, **28**, 69.

Figure 1 shows the spectrum of the bare tube under these new conditions at the same current of 4 amperes used in all the spectrograms, and needs no further comment.

Figure 2, Plate 2, is an interesting illustration of the powerful absorption produced by a modern anastigmat lens in the ultra-violet portion of the spectrum. It was obtained merely by substituting the photographic lens for the quartz lens previously used in focussing the tube upon the slit. The particular lens used was a Zeiss Unar, series 1 B No. 5, of 155 mm. equivalent focus. This lens is composed of 4 thin separate elements, having an aggregate mean thickness of between 10 and 11 mm. It will be noted that the absorption for wave lengths less than 365 $\mu\mu$ is very active, and that all the ultra-violet lines are somewhat weakened. The practical significance of the spectrogram is that even the first-class modern photographic lens is practically almost opaque to wave lengths below 365 $\mu\mu$, and it points out the necessity of using special glasses and special constructions for spectrographic or astrographic work. For there is ample intensity in the ultra-violet spectrum of daylight to give trouble were the lens fully transparent down to the limit of the rays ordinarily transmitted by the atmosphere. As Zschimmer has shown,[8] one can work clear down to this limit with the Uviol glasses, which of course should be achromatized with this point in view. The Unar lens, like some others of recent type with separate lenses, suffers more from loss of light by reflections at the multiple surfaces than would the older anastigmats with thick cemented lenses, but probably much more than makes up for this loss by the lessened thickness of glass. The loss from reflections is quite a serious matter, inasmuch as the refractive index of an ordinary crown at say w. l. 350 $\mu\mu$ to 340 $\mu\mu$ is in the vicinity of 1.55 and that of ordinary flints is about 1.65. Reckoning the losses by Fresnel's formula, $\left(\frac{n-1}{n+1}\right)^2$, without applying additive corrections for the obliquity of incidence, it appears that in a 4-lens separated system the loss of light by reflections will amount to nearly 35 per cent in this region of the spectrum ; and assuming a thickness of 10 mm. for the crowns and an equal amount for the flints, the absorptive loss for the former can scarcely be less than 20 per cent and that in the latter scarcely less than 70 per cent, so that on the whole hardly 20 per cent of the incident light would reach the plate. Evidently the increased absorption due to the greater thickness of glass would more than overbalance the gain from reduced reflections in the case of anastigmat lenses of the common symmetrical cemented type in

[8] Loc. cit.

which the total thickness would be at least doubled for the same aperture.

Figure 3, Plate 2, shows the absorption of a slip of clear green signal glass, 2.4 mm. thick. This particular glass gives absorption in the violet and in the red and orange, and transmits not over 15 per cent of the total light. The absorption in the further ultra-violet is complete, and even the group at w. l. 365 $\mu\mu$ gets through enormously weakened. The particular protective spectacles previously mentioned were made of a layer of this glass and of the reddish amber selenium glass before mentioned, of equal thickness. They completely suppressed the ultra-violet, violet, blue, much of the green and also most of the red and orange, leaving a nearly monochromatic strip in the most luminous part of the spectrum. The resulting combination transmitted ample light to enable one to read instruments and notes, or even to read a newspaper in fairly strong light, but cut down the intensity to a point that enabled one to look with comfort at short range into the quartz lamp or the most powerful commercial flaming arcs.

Figure 4, Plate 2, is the same amber glass as Figure 8, Plate 1, taken with the stronger illumination. Two of the lines in the violet came faintly through, but the ultra-violet was completely suppressed.

Figure 5, Plate 2, is the same glass as in Figure 7, Plate 1. This as before completely suppressed the ultra-violet and let through the two violet lines very faintly.

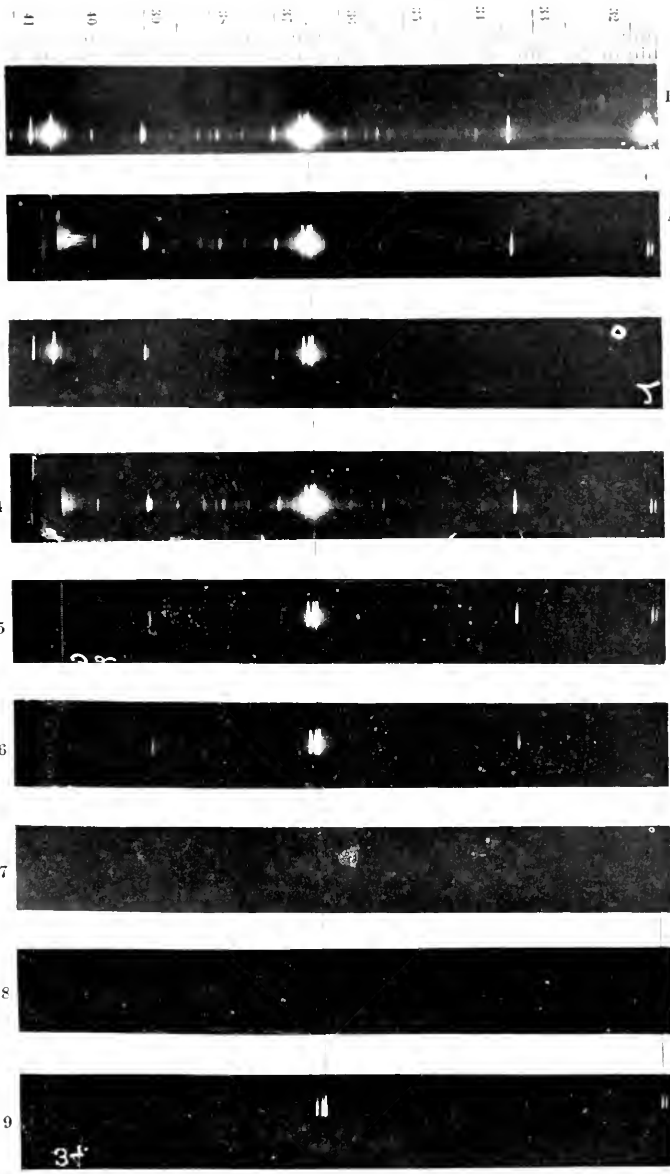
Figure 6, Plate 2, is the spectrum taken through a slip of No. 1 Euphos glass, 4 mm. thick. This is a light yellowish green in color, suppressing the blue and violet somewhat less completely than the two previous glasses, but like them blocking out the ultra-violet effectively. None of the four glasses just described cuts down the intense light of the quartz lamp or arc lamps, when viewed at short range, sufficiently to avert ocular distress after a short exposure. For adequate protection against such sources considerably deeper shades of all of them are necessary and are readily available. It must be borne in mind in this connection that among arc lamp workers and operators of electric furnaces cases of the so-called *ophthalmia electrica*, which is chargeable to the extreme ultra-violet rays, chiefly the abiotic radiations already referred to, are rare compared with distressing symptoms due to the luminous rays. Protective glasses, therefore, as light in tint as these of Plate 2 are, as a matter of practical experience, of comparatively little service although they suppress the ultra-violet quite completely. Deeper shades of any of them may be effective.

Figure 7, Plate 2, is an exposure of the bare quartz tube for 5 seconds. This spectrogram received therefore 1.66 per cent of the total energy received by Figure 1, Plate 2, and well exhibits the effectiveness of the absorption produced by the glasses considered in the whole ultra-violet region. Without going here into the matter of any specific effects due to the action of the ultra-violet rays upon the eye it is interesting to compare the results here obtained with those derived from some of the other protective glasses recently brought to notice. From the very interesting studies of Hallauer[9] it appears that the so-called enixanthos glass shows absorption very similar to that of the Fieuzal, while Hallauer's own glass possesses similar characteristics in the ultra-violet, but like the amber glass here described carries a stronger absorption into the blue. The Gonin rose glass apparently resembles the pinkish glass of Figure 6, Plate 1, while the blue glass is still more transparent to the shorter wave lengths. Hallauer's gray glass apparently corresponds quite closely to the smoke of Figure 3, Plate 1. The commercial red glass tested by Voege[10] cut off the ultra-violet completely in a thickness of 2.9 mm. Broadly the red, amber, yellowish and yellowish-green glasses which absorb noticeably in the blue and violet also carry increased absorption through the ultra-violet, while the blue and green glasses which transmit somewhat freely in the blue and violet also transmit a perceptible amount of ultra-violet. This latter class of glasses is undesirable for protective use for other reasons than those connected with the ultra-violet, however, since a predominant blue tone in the transmitted light is both of uncomfortably low luminosity and greatly reduces the contrasts in most objects viewed. Of the amber and greenish glasses all will transmit, when of light tint or very thin, a limited amount of the more refrangible rays, including some ultra-violet, as would readily be found by a very prolonged exposure. The ultra-violet radiations transmitted by any of them when of sufficient thickness or density to be of service in softening the intensity of the visible rays is extremely trivial in amount, so small as to be utterly negligible in practice, as the spectrograms given show, and all of them, even the lightest in tint, suppress very thoroughly the only radiations in the ultra-violet that are certainly known to have a specific harmful effect. To sum up this matter of protection against the ultra-violet: All ordinary glasses, even the clear optical glasses, suppress all the ultra-violet radiations certainly known to have a specific harmful effect upon the human eye. As to the rest of the ultra-violet, say from w. l. 300 $\mu\mu$ to the visible rays, there is little clinical evidence of injurious effect, and even this little does not gain in convincing quality from its

[9] Loc. cit.

[10] Loc. cit.

too frequent association with perfervid commendation of somebody's special variety of protective glass. If as a matter of precaution it be thought desirable in certain cases to cut off the whole ultra-violet region, there is a liberal choice of entirely effective glasses available. For this purpose the common commercial amber glasses such as are used for spectacles and signal lenses seem to be quite as effective as those especially made for such use, the choice between them being chiefly a matter of taste. The very lightest tints of all of them undoubtedly may transmit faint traces of the ultra-violet rays nearest the visible spectrum, but when there is density enough materially to soften the visible light these traces disappear. For protection against powerful sources of light at short range only the deepest tints reduce the visible light sufficiently, and these are particularly opaque to the ultra-violet.



1 Bare tube. 5 minutes.

2 A.O. Crown. 6.2 mm.

3 Medium smoke. 5.6 mm.

4 B. and L. Crown. 4.2 mm.

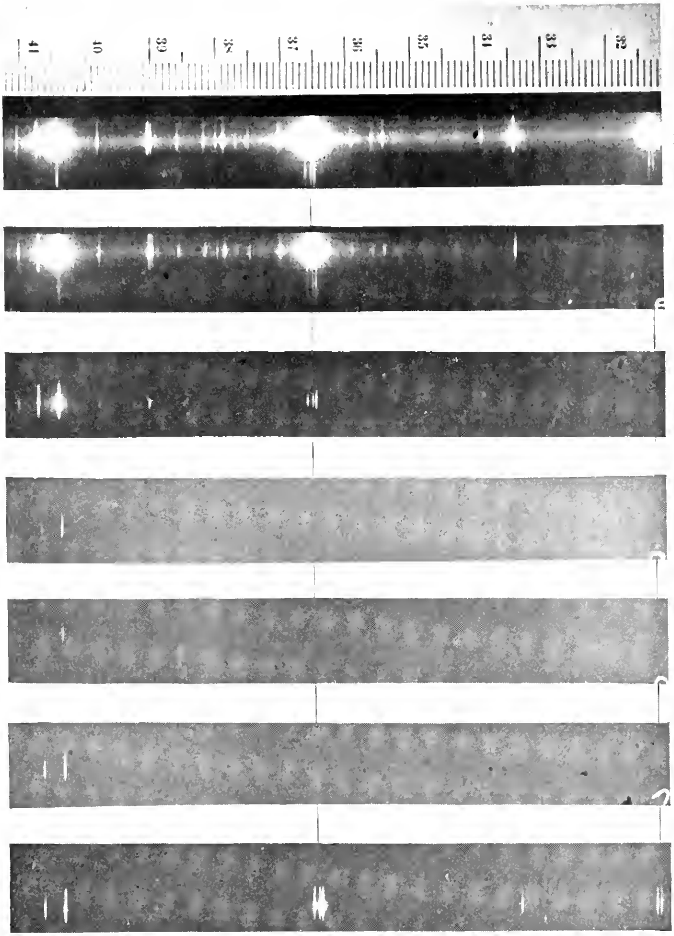
5 Amethyst glass. 2.8 mm.

6 Pinkish glass. 4.6 mm.

7 Fieuzal glass. 4.0 mm.

8 Amber glass. 2.0 mm.

9 Bare tube. 15 seconds.



1 Bare tube. 5 minutes

2 Unar lens. 5 minutes.

3 Green glass. 2.4 mm.

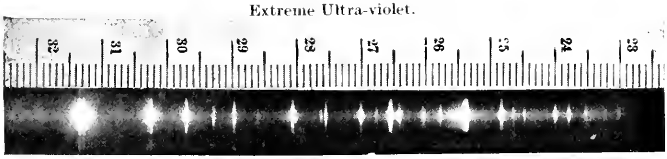
4 Amber glass 2.0 mm.

5 Fieuzal glass 4 mm.

6 Euphos glass 4 mm.

7 Bare tube. 5 seconds.

Extreme Ultra-violet.



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"Abiotic" radiations.

Proceedings of the American Academy of Arts and Sciences.

Vol. XLVI. No. 25. — May, 1912.

RECORDS OF MEETINGS, 1910–1911.

REPORT OF THE COUNCIL: BIOGRAPHICAL NOTICE.

Henry Pickering Bowditch. By Walter B. Cannon.

RUMFORD PREMIUM.

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(Title Page and Table of Contents.)

RECORDS OF MEETINGS.

Nine hundred ninety-ninth Meeting.

OCTOBER 12, 1910. — STATED MEETING.

The PRESIDENT in the chair.

There were forty-five Fellows and one guest present.

The following letters and circulars were read by the Corresponding Secretary: — letters from Messrs. Archibald C. Coolidge, Roland B. Dixon, Worthington C. Ford and Edward C. Moore, accepting Resident Fellowship; from Sir David Gill, accepting Foreign Honorary Membership; a notice from his family of the death of F. von Recklinghausen; from the Instituts Solvay, asking recognition of the new society; from the tenth International Congress of Geography to be held in Rome in October, 1911; a notice of the formation of a society for the development of experimental sciences, in connection with the Imperial Moscow University; a circular notice to the President of the Academy that he had been elected a member of the organizing Committee for the Eighth International Congress of Applied Chemistry, to be held in the United States in 1912; a request from the Director of the Civic Pageant of the Boston 1915 organization for assistance for the Pageant, to be held in November; a program of the first Universal Race Congress, to be held in London, July 26–29, 1911; a notice of various International Horticultural Expositions to be held in Turin in 1911, in connection with the International Exposition of Industry and Labor; a letter from Professor Hugo Münsterberg to Professor Watson, announcing the formal opening of the Amerika-Institut of the German Government.

The Chair announced the following deaths: Robert Amory, Resident Fellow in Class II., Section 3; George P. Fisher, Class III., Section 3, and Melville W. Fuller, Class III., Section 1, Associate Fellows; Frederick J. Furnivall, Class III., Section 4, and F. von Recklinghausen, Class II., Section 4, Foreign Honorary Members.

The following gentlemen were elected Resident Fellows:

Alfred Church Lane, of Cambridge, Class II., Section 1 (Geology, Mineralogy, and Physics of the Globe).

Winthrop John Vanleuven Osterhout, of Cambridge, Class II., Section 2 (Botany).

Dr. Louis Bell called the attention of the Academy to the fact that the next meeting would be the one thousandth meeting of the Academy.

It was

Voted, that a committee be appointed to consider the celebration of the event.

On motion of the Recording Secretary it was

Voted, to meet on adjournment, November 9, 1910.

The following communication was given by Dr. Percival Lowell: "Investigations of Halley's Comet at Flagstaff Observatory."

The following papers were read by title:

"Theory of Coupled Circuits, under the Action of an Impressed Electromotive Force, with Applications to Radio Telegraphy." By G. W. Pierce.

"A Revision of the Atomic Weight of Neodymium. First Paper. — The Analysis of Neodymium Chloride." By G. P. Baxter and H. C. Chapin.

"On the Equilibrium of the System consisting of Calcium Carbide, Calcium Cyanide, Carbon, and Nitrogen." By M. de Kay Thompson and R. H. Lombard. Presented by H. M. Goodwin.

One thousandth Meeting.

NOVEMBER 9, 1910. — ADJOURNED STATED MEETING.

The PRESIDENT in the Chair.

There were thirty-eight Fellows and one guest present.

The Corresponding Secretary read letters from Messrs. Alfred C. Lane and W. J. V. Osterhout, accepting Resident Fellowship, and a circular from the Boston Vocation Bureau, announcing the first national conference to consider the question of the choice of vocations by the young men and women of this country, to be held in Boston November 15 and 16, 1910.

The Council reported that it had considered Professor Percival Lowell's charge that Professor Edward C. Pickering had suppressed information in regard to Professor Lowell's candidate, — Mr. Lampland, — and having considered Professor Pickering's testimony in regard to the same, found that the charge was entirely unfounded, and passed the following vote: — "That the Council has considered the charges presented by Professor Lowell and the new counter-evidence from Professor Pickering, and considers the charges unfounded. The error arose by reason of forgetfulness on the part of Professor Davidson."

The President read the following letter from Mr. G. R. Agassiz:

My dear Prof. TROWBRIDGE,

Percy Lowell has just telephoned me that you were waiting to call a meeting of your American Academy Committee, until you heard from us. You remember that I said that we (my brothers and I) thought it advisable not to make a formal proposal till we were in a position to pay the bequest. Matters move so slowly in these matters that it is still impossible for us to say how soon this will be. It certainly will not be before Mr. Bowditch leaves.

You may remember that the question will be for the Academy to decide whether it prefers to have a fund of $50,000 — or the house Mr. Agassiz intended to build for them. Should the Academy decide on the latter alternative (after they have received our proposal) they will doubtless find that the conditions will be something like this:

If they care to devote the $50,000, bequeathed to them unconditionally by Mr. Agassiz, toward building from the plans that Mr. Agassiz had prepared — then his children would pay the rest of the cost — and furnish a suitable fund to furnish the house.

This is in no way an official communication, but is written in the hope of recalling to you what our actions will in all probability be. Furthermore I should like to have this matter known as widely as possible among the members of the Academy, as, should you decide to build, we should doubtless like the assurance that such a course meets with the approval of a very considerable majority of its members.

Yours very truly,

G. R. AGASSIZ.

The following recommendations from the Committee on Policy were read: —

That the Academy hopes in a short time to be in a position heartily to accept the conditions of the Agassiz heirs, informally outlined concerning the building.

1. That the number of Resident Fellows be gradually increased by one hundred.

2. That not more than twenty-five Fellows be added annually, and not over four in any section.

3. That a committee of five, including the President and Recording Secretary, be appointed to take charge of meetings.

4. That the functions of the Council be enlarged in order to give it supervision over all affairs of the Academy.

5. That if, at 8.30, business is in progress, it be postponed, and the communication announced, be called.

No action on the above recommendations was taken by the Academy.

The President appointed Dr. Louis Bell and Professor W. M. Davis a Committee on the Celebration of the one thousandth meeting.

The following communication was given: —

"The Supposed Recent Subsidence of the Massachusetts Coast," by Professor D. W. Johnson.

The following papers were presented by title: —

"The Pegmatites of the Riebeckite-Aegirite Granite of Quincy, Mass.; their Minerals, Structure and Origin." By C. H. Warren and Charles Palache.

"The Vector Diagram of the Oscillating-Current Circuit." By A. E. Kennelly.

"Infinitesimal Properties of Lines in S_4 with Applications to Circles in S_3." By C. L. E. Moore. Presented by H. N. Tyler.

"The Indeterminate Product." By H. B. Phillips. Presented by H. N. Tyler.

"A Fundamental Theorem Regarding Curves on Reguli." By W. E. Story.

"The Action of Mercury on Steel at High Pressures." By P. W. Bridgman. Presented by John Trowbridge.

One thousand and first Meeting.

DECEMBER 14, 1910.

The Academy met at the University Club, to celebrate its one thousandth Meeting.

The PRESIDENT in the chair.

There were present seventy-five Resident Fellows, one Associate Fellow and twenty-two guests.

The dinner was typical of colonial times — being copied largely from one given on Forefather's Day in Plymouth, 1769. The menu, which is of historic interest, follows on page 688:

After the dinner the President, Professor John Trowbridge, congratulated the members upon the large attendance which testified to the vitality of a body which, after 130 years, could show such strength, and called upon the Recording Secretary to read the record of the last meeting. Several members objected, and desired the record of the first meeting. This was accordingly read, as follows:

May 30, 1780. By an Act of the Great and General Court of the State of Massachusetts, passed y^{e} 3^{d} of May, 1780, a Society was incorporated and established by the name of American Academy of Arts and Sciences. In the Act, the Philosophical chamber in the University of Cambridge was determined to be the place where the Fellows of the Academy should hold their first meeting and the Honorable James Bowdoin Esq^{r} was authorised and impowered to fix the time for holding the said meeting and to notify the same. He having done this by advertisement in the public newspapers the fellows of the said Academy held their first meeting at the place aforesaid on the 30th of May, 1780.

At the meeting the Act of Incorporation was first published. The Rev^{d} President Langdon then prayed, after which the Fellows passed the following votes: —

1. That the Honorable James Bowdoin, Esq. be President until the next meeting of the Academy.

2. That M^{r} Caleb Gannett be Secretary until the next meeting of the Academy.

3. That Ebenezer Storer, Esq^{r} be Treasurer until the next meeting of the Academy.

4. That the Honorable James Bowdoin, Esq^{r}, Mr Caleb Gannett, the Rev^{d} Samuel Langdon, D. D., the Honorable John Pickering Esq^{r}.,

THE REPAST

A Baked Indian Whortleberry Pudding

SALMON AN OYSTER PYE

A Venifon Pafty

A HAMM RABBIT & ONIONS

CYDER PVNCH

Turkeys, Roft

PEASE BAKED BEANS

A DISH OF SVKOVTTAHHASH

Divers Pyes

JELLYS & SVLLABVBS CRANBERRY TARTS

CHEESE

Stephen Sewall, Esq^{r}., the Rev^{d} Edward Wigglesworth & the Rev^{d} Samuel Williams, be a Committee to agree upon the names, number and duties of the several officers they shall judge necessary or convenient to the Academy and the tenure or estate they shall respectively have in their offices: Also to prepare such rules, orders and bye-laws, as they shall judge necessary or convenient for the well-ordering and good government of the Academy. Also to consider of the times, places and manner of convening the Fellows of the Academy and the number of Fellows which shall be present to constitute a meeting of the Academy: to devise a common seal for the Academy, to consider for what causes fines shall be levied & what the fines shall be, and to report their proceedings upon the premises at the adjournment of this meeting.

5. That this meeting be adjourned to the second Wednesday of July next at ten o'clock in this place.

The President called attention to the words in the minutes, "The Rev. President Langdon then prayed," and asked Rev. Edward H. Hall if he could remove an odium scientiæ from the academy in abandoning this custom — in the hope that he would assure the members that although they were birds of passage — lone wandering — they were not lost.

Reverend Mr. Hall then spoke as follows:

The Revd. President Langdon then prayed.

It is well that the ravages of time have preserved for us this brief statement, for so far as appears, that was the first and perhaps the last time that the divine blessing was invoked within the Academy walls. There was no special intent in this, for the absence of prayer was by no means unknown in the early Puritan gatherings, and we find Governor Bowdoin, the first president of the Academy closing his inaugural address with a full evangelical invocation of almighty favor and support; at the same time, in view of the perfunctory character so often assumed in this service, it is a satisfaction to remember this relation of religion to science through the whole history of the Academy; each paying due honor to the other with as little intrusion as possible into the other's domain. Science I suppose pays best homage to religion by showing its reverence for the laws of the universe without attempting to force its spiritual interpretation upon them.

Meantime it is pleasant to note that the American Academy turned at once to Harvard College in initiating its career. The Rev. Samuel Langdon was not one of the most noted of our early presidents.

Falling as it did in the stormy hours of the incipient Revolution, his administration was broken from the start by the turmoil of war and the constant removals of family and possessions from town to town, from Cambridge to Watertown, to Concord, and again to Cambridge. So disturbed was the college life that all public ceremonial was restricted, and President Langdon entered upon his office without any formal inauguration. Even lottery tickets, the collegiate resource in days of need, remained on the hands of the authorities unsold, and the Corporation had to purchase two thousand tickets on their own account. The college needed a firm hand at the helm, and there is reason to think that this quiet country parson, called from a Portsmouth parish, notwithstanding his fine intellectual quality and ample learning, found himself hardly in his element among exuberant students aflame with patriotic ardor. No serious outbreaks are recorded in Langdon's time, it is true; but the college officials were pained early in his administration, when the Tory students amused themselves by "bringing India tea into commons and drinking it to show their loyalty."

On the whole, however, the brief period of Langdon's presidency passed quietly, reflecting honor upon the president's scholarship and learning, as well on the whole as on his administrative zeal. He was by no means lacking in public activity, taking his full share, if the accounts are correct, in the political affairs where college and state had to act together. He is credited with an active part in 1779 in framing the three articles of the constitution confirming the privileges of the college and defining the change in the position and functions of the Overseers. The provision whereby, in connection with the Governor, Council and Senate, the ministers of the Congregational churches of Cambridge, Watertown, Charlestown, Boston, Roxbury and Dorchester succeeded to the functions of the Board of Overseers, has hardly vindicated its wisdom, and has long given way to something better, but otherwise the interests of Harvard College have found themselves admirably guarded by the constitution of 1779.

Such being the condition of college affairs, it was with great astonishment that the Corporation Aug. 29, 1780, received the resignation of President Langdon after six years' service. Nothing had pointed to this event, the college was in no crisis, nor was any special cause given by the president for this sudden withdrawal from his post. In a letter of singular dignity and detachment, free from all accusations, and dwelling with simple pathos upon the hopes of greater usefulness which he had cherished in "serving the noble cause of country and liberty, and the important interests of Religion and literature," dwelling also upon the "severe labors he had gone through since entering upon his office," he declares that his "taste for youthful studies is decreasing" and pleads for a "more retired situation." The next day a meeting of the Overseers was called, no discussion seems to have followed, no regrets were expressed, the Overseers simply voted their thanks and acknowledged the value of his services, and the resignation was accepted. It reads as if the Corporation and Overseers were so accustomed to presidential resignations that no formalities were thought necessary, and the only thing to do was to choose a new head and start afresh.

What did it all mean? Of course there was more than appeared upon the surface, and by degrees a situation disclosed itself altogether unique in the annals of the college. So far as appears the relation of the president to his students was friendly and their respect for his scholarship and learning was great; but religious and political dissensions were rife, and the students caught many of the catch-words of their elders. He was thought skeptical and called by some a deist. Apparently a small body of students, otherwise discontented, took up these charges and formed a combination against him. A meeting of the three upper classes was called, and resolutions were unanimously passed, charging President Langdon with "impiety, heterodoxy, unfitness for the office of preacher of the Christian religion, and still more for that of President." Twelve students were appointed to wait upon the President and invite him to resign his office. Two days afterwards he detained the students after morning prayers and, with unexampled humility, told them that he should resign (Quincy ii. 179). At once, the solemn crisis having passed, these same students, with equal unanimity, passed resolutions of entire confidence in his ability and character, and great sympathy with his needs.

In this extraordinary episode it is easy for those who are acquainted with student life to trace fairly well the probable course of events. With all sympathy with the weary and tormented president, we cannot help detecting in his immediate surrender to the situation, in the very absence of recrimination or reproach, and his quick retirement from the contest, the evidence of fine scholarly instincts perhaps, but of a man consciously out of place amid the rudenesses and frequent brutalities of college life. It was evidently President Langdon's misfortune to be called out of the quiet of a country parsonage into the turmoil of the American Revolution. With these singular incidents in our thoughts, it is easy to understand why one of his biographers, cited by Quincy, declared that President Langdon "wanted dignity and authority," another "that he wanted judgment and a spirit of government."

With this single event of Samuel Langdon's career, almost the only important one that history has preserved, we take our leave of the 16th President of Harvard College. Slight as it is, it presents the college in a curious light, and in no respect more curious than in its relations with the patient president who bore himself with such scholarly calm through the turmoils of riotous days. It is pleasant to remember that the last seventeen years of his life, from 1781 to 1797, were spent in the congenial seclusion of a small country parsonage.

After the completion of Mr. Hall's address, the President read the following letter from Professor William W. Goodwin, a former President of the Academy.

Some of the pleasantest recollections of my first years as Tutor and Professor are connected with the Academy, of which I was made a member by the kindness of my beloved teacher and friend, Cornelius Conway Felton, soon after I returned to Cambridge from Germany in 1856. I shall never forget the enjoyment of the earliest meetings which I attended. The first was at the house of Dr. Jacob Bigelow, then President of the Academy. Another memorable Meeting was when the venerable Ex-Vice-President, Josiah Quincy, invited the Academy to his house on Park street (I think it was on his ninetieth birthday), and entertained us by reminiscences of his early life in Boston and of his Presidency of Harvard College. After the meetings the Cambridge party, among whom were apt to be Professors Felton, Lane, Lovering, Agassiz, and Jeffries Wyman, with Morrill Wyman, B. A. Gould and others, hastened to Brattle street and filled the Cambridge omnibus to overflowing; or occasionally, when the moon was full, made up a party to walk home to Cambridge.

At about this time a controversy arose between some of its members who felt that the increasing interest taken in the social meetings in private houses interfered with the more important scientific duties of the Academy as a learned society, and those who (like Professor Felton) "could see no harm in a glass of wine and an oyster." The social meetings became less and less frequent, however, when the Academy became more pleasantly settled in quarters of its own, and now, when we have a comfortable home in our own house in Boston, the old controversy has become a matter of early history.

I beg you to present to the Academy my deep regrets that I cannot be with them on Wednesday evening. As one of its older members, one who remembers always with pleasure that he had for a few years the honor of being its President, I feel a just pride in its honorable and useful past; and I hope that, as the years bring round the anniversary of its two thousandth meeting, the members then assembled may be able to feel that its traditions of fruitful work in the cause of science and the arts have been faithfully preserved and its field of useful service to humanity has been much enlarged.

The Corresponding Secretary, Professor Edwin H. Hall, was next called upon to report recent correspondence of the Academy; but on account of the commemorative character of the meeting, he excused himself from the current correspondence, and instead, presented to the meeting three letters received by the Academy one hundred and thirty years ago. These letters follow:

JAMAICA PLAIN JULY 8th 1780.

SIR,

Permit me, as a Friend to all Establishments which have a Tendency to promote useful Knowledge, to congratulate you upon the Institution of the American Academy of Arts and Sciences; and to add my warmest Wishes for the Success of so laudable a Design.

As it will doubtless be a Part of the Society's Plan to form a Museum of the natural Productions of the Country, as well as to investigate the Properties of each, and the Uses to which it may be applied, I beg Leave to send you a Piece of the Asbestos, and some Pyrites, both Natives of Pennsylvania. The former I received from the ingenious David Rittenhouse, Esquire, who informed me he had used some of the Filaments of it (soon after it was taken out of the Earth) as Wick for a Lamp; but having been long exposed to the Air, it now wears the Appearance of the Petrefaction. Even in this State it is fissile, and may be easily split length-wise.

The Pyrites I collected at Lancaster, where they are found in great Plenty: they contain Sulphur; and, notwithstanding the Regularity and Polish of some of them, are in their natural State.

My employment under Congress, as Surveyor General of the Post Officies and Roads in the Eastern District, frequently affords me Opportunities of collecting Specimens of various Kinds; and should these now sent prove acceptable, I shall take the Liberty, from Time to Time, of adding others as they occur.

I have the Honor to be, Sir

Your most obedient and very huml Servt

EBEN HAZARD

BOSTON, Nov^r 6th 1780.

HONOURED SIR,

As I find my Name inserted among the names of the Members of the *New Academy* erected among us; and I perceive your Honour to be the chosen President of it; I think it my Duty to inform you, and so the Academy, that I esteem it honorary to be reckoned worthy to be placed as a Member among you.

I am too far advanced in Life to attend the Meetings of the Academy. And although I dare not form the Wish, *O mihi praeteritas referat ut Jupiter Annos!* yet I must own that, if I was thirty or forty years younger, it would have been a much greater Satisfaction to me to have been numbered among you.

However I am heartily willing and strongly desirous, as far as it may be in my Power, to promote the Design, and support the Credit, Reputation and usefulness, of the Society.

And, in order to show my Good-will and Respect in the Body I have hastily collected the Methods and Rules, that have been proposed to be observed and followed by two foreign Academies; and have added a Proposal or two of my own.

And, if the Academy, over which I trust you will worthily preside, should think it worth the while to ask me to employ my Thoughts or use my Pen, on any Subject that may be subservient towards a Furtherance of the Arts and Sciences; I shall endeavour, as old as I am, by the Divine Will and Assistance to comply with the Requisition from them.

One of the *Academy of Sciences at Paris*, it was I think M^r *Frontenelle*, called the Same *Une Corpse des Esprits*, A Body of Spirits, or Geniuses: I wish from my Heart, that our Academy may prove and continue to be, such a Body, to their own Honour and the Enlightening and Rejoicing of Others.

I wish you Honoured Sir, and all the Brethren of our Academy, the Presence and Blessing of Heaven in all your Projections and Endeavours to promote useful Sciences, as well as at the same Time Religion and Virtue; and am, with great Respect,

Your most humble and obedient Servant

SAMUEL MATHER.

P. S. I found your Honours Name among the first Members of the French Academy: But he spelt it Baudoin: Which seems to be the right way of Spelling it. He was a learned Man.

LONDON, July 21st 1781.

DEAR SIR,

I think myself much honoured by the favour of your letter dated the 28th of Feby last wch I received about a month ago. I am made very happy by the information it contains, that in the midst of war and the most important struggle that a people were ever engaged in, a new Academy for promoting arts and Sciences has been established at Boston. In compliance with your desire, I have communicated the incorporating Act and list of members to the President and Secretaries of the Royal Society, attended with a letter of my own stating the contents of your letter to me, and the hopes wch the American Academy entertain that the Royal Society, governed by the neutrality of Philosophy, will favour it with its encouragement. I do not yet know certainly what notice will be taken of these communications. The reply that has been reported to me from the President is, that it has not been customary to lay before the Royal Society notices of the institution of any Societies whatever.

I am obliged to be cautious in communicating the inaugural oration of your honourable and worthy President on account of some political passages in it. For my own part I approve and admire these passages; and I request the favour of you to deliver my best respects to the Author. I have delivered your letters to Dr Morell and Mr Maskelyne. I have likewise got a Friend to communicate to the Society of Arts and Commerce the copy of the incorporating Act which you intended for them.

I am at present very busy in preparing for the Press a fourth edition of my Treatise on Life-annuities and Reversionary paymts I shall enlarge it to two volumes, and when out to Press (wch I am afraid will not be till the beginning of next Summer) I shall endeavour to get it convey'd to you in hopes of the honour of its being accepted as a testimony of my respect for the American Academy. This work having been of some use I am anxious about making it as complete as possible. With this view I am collecting all the Observations I can get on population, the increase of mankind, and the duration of human life in different Situations.

All that can be worth communicating to you in the Philosophical and Astronomical way is published in the numbers of the Philosophical Transactions of the Royal Society which come out every half year. What has lately most engaged attention is the new Star discovered near Auriga by Mr Herschel, a gentleman at Bath, who has for some time been very curious and diligent in watching the Heavens. This

Star was at first taken for a comet; and the Astronomer Royal once expected that it would have passed over the disk of the Sun at the beginning of last month. But he has since told me, that it is doubted whether it may not be a planet never before discovered moving at a much greater distance from the Sun than *Saturn*. It has been for some time hid by the Sun's rays. Should it appear again, Something more certain will probably be determined concerning it.

D^r Priestly never went farther in his History of Philosophy than Electricity and Optics. He has been for some time wholly employed in making experim^ts on the different Sorts of air. In this branch of Philosophy he has made several very important discoveries, an account of which he has given in five Octavo Volumes, the last published this Summer. One of the most important facts which he has discovered is the effect of vegetation, aided by the action (not of *heat* but) of *light* in purifying, preserving and restoring common air constantly injured and diminished by the breathing of animals, the burning of fires, putrefaction and other causes. In the day time and particularly in Sunshine, the purest kind of air is emitted by the leaves of trees and all vegetables; and this emission is more or less copious in proportion to the vigour of the vegetation and the force of the Sun's light. In the night and in the dark it ceases entirely. D^r Priestly is going on with these experim^ts, and very probably another volume will be published in a little time.

If you think that my best respects and wishes will be acceptable to the members of your Academy, I beg you would deliver them. No one can observe with a more earnest attention than I do all that now passes in America. With much gratitude and the greatest regard I am, S^r, your most obedient and humble Ser^t

RICH^D PRICE,

Deliver my very respectful complim^ts to the venerable D^r Chauncy. D^r Winthrop was my correspondent. With pain I reflect that he is no more in this world to promote virtue, liberty and Science. But we are all following him. God grant that we may leave the world wiser and better for us.

A copy of this letter was sent by another conveyance.

The President called attention to the rich store of historical documents possessed by the Academy, of which the letters read were but a small portion. There are letters from General Washington acknowledging his election to the Academy; a letter from Priestly, the discoverer of oxygen; several from Count Rumford, and a host of others.

The paper announced for the evening was on "Earthquakes"; but Professor Jaggar, who was to present it, moved that in consequence of the festivities it be omitted.

The President introduced the toastmaster of the evening, Professor E. C. Pickering. Professor Pickering spoke as follows:

This is the third celebration of its kind by the American Academy of Arts and Sciences. The first, eighty years ago, the Semicentennial, also included a dinner. My knowledge of it is derived only from hearing it discussed fifty years later. The second celebration, the Centennial, is well remembered by many of us. When the American Academy was founded in 1780, there was only one scientific society in America, the American Philosophical Society, founded thirty-seven years earlier, at the initiative of Franklin. Curiously enough, just thirty-seven years later, the third Society of its kind was established, the New York Academy of Sciences.

Of the sixty-two charter members of the American Academy, three were under thirty years of age, and two were over seventy. The Academy once elected a man who was twenty-one years old, and thus came within four months of having a member who was legally an infant. Youthful membership led to long terms, the longest, that of Dr. Jacob Bigelow, extending over sixty-seven years. The shortest, of but a few hours, was that of Mr. Horace Mann. It gave him much pleasure to learn that he had been elected, although he died the same night. The term of one of the charter members, Theodore Parsons of Newbury, appears to have been negative, as he is stated to have been lost at sea in 1779, the year before the Academy was founded. The terms of several of our members have exceeded half a century, including those of three now living. The term of the senior member, Professor Francis H. Storer, fifty-three years, exceeds by a few minutes only that of his twin member, President Eliot. Our former President, Professor Goodwin, has been a member for fifty-one years.

The usefulness, in fact the justification, of an Academy like this is not in holding meetings, or in reading scientific or literary papers. Such work is only local and temporary. The real objects of the Academy should be the increase and diffusion of knowledge, the first by research, the second by its publications. Research is the most important of all. I take great pride, as a member of the Rumford Committee for nearly forty years, in the list of investigations we have been able to aid. But it is pitiable to consider the many cases that have come before the Committee where admirable work must be abandoned for lack of a few hundred dollars. No more valuable contribution to Science, or memorial to this meeting, could be made than the gift to the Academy of a Fund, whether large or small, to be administered like the Rumford Fund, but without the conditions which sometimes restrict the usefulness of that most valuable gift.

About thirty years ago, the Proceedings of the Academy filled twenty volumes, and its Memoirs ten. At that time I urged the preparation of an index to the whole, as the principal objection to publication in our Proceedings is that papers are buried in them, and are likely to be forgotton or overlooked. The need of such an index is much greater now than at that time. We shall soon have filled fifty volumes of Proceedings, and fifteen of Memoirs.

The first toast I shall propose to you is "The Foundation of the Academy." This is a question of History, and the Academy has a vigorous younger sister, the Massachusetts Historical Society, which is still in the prime of life, since it is but a little over a hundred years old. Many persons are active members of both societies, and I will ask one of them, Mr. Andrew McFarland Davis, to respond to this toast.

The paper of Mr. Davis contained the following interesting account of the incorporation of the Academy and of the time when it came into existence: —

The place at which the first meeting of the Academy should be held, was designated as the philosophical chamber of the University of Cambridge. This was in all probability the Jefferson Laboratory, rather than the Emerson Hall, of that day and was undoubtedly under the roof of the present Harvard Hall. Rev. Samuel Williams, who had succeeded John Winthrop as Hollis Professor of Mathematics and Natural Philosophy, was one of the incorporators of the Academy. He had accompanied Winthrop to Newfoundland twenty years before to observe the transit of Venus, and later he had given lessons in Natural Philosophy to Benjamin Thompson, better known by the title which he afterward acquired of Count Rumford.

To a certain extent the definition given of the purposes of the Academy betrays the limitations imposed upon investigators of those days by their surroundings, their education and the primitive state of knowledge on most of the subjects, to the study of which they promised their attention. Eighty-seven per cent of the persons named in the act of incorporation were graduates of Harvard College. A glance at the curriculum of that institution ought to show how far this body of men, who it may be inferred from the language of the preamble to the act were regarded as men of genius and learning, had been furnished opportunity for special preparation to cope with the numerous questions suggested in this list.

The Hollis professorship of mathematics and natural philosophy, founded in 1727, provided the only instructor in that institution whose function it was to deal with any of these topics. Professorships instituted after that date could have had no influence in the mental preparation of these investigators but it may be of interest to note the slow awakening of the college to the necessities of students along these lines. A professorship of Chemistry and Materia Medica was instituted in 1783; of Natural Religion, Moral Philosophy and Civil Polity, in 1789; of Natural History, in 1805; of Sciences applied to the useful arts, in 1816; of History, ancient and modern, in 1839; and of Astronomy and Mathematics, in 1842.

The annual income which the Academy could receive was limited to £500 from real property and £2,000 from personal. This measure of value was in silver at 6*s.* 8*d.* per ounce, and amounted to a little less than $8,500. The annual dues were to be paid in Spanish Milled dollars in specie or an equivalent in bills of the current exchange.

The seal of the Academy was not adopted for some years thereafter. The erect figure upon it represents Minerva. Instruments of husbandry, as well as a quadrant and a telescope, are to be seen in the foreground. A corn-field to the left, a town in the distance; a ship under sail approaching it; overhead the sun rising above a cloud. Of this seal those who were describing it said: "The device represents the situation of a new country depending principally on agriculture, but attending at the same time to arms, commerce and the Sciences." One correction in the language used in this description suggests itself. The word "arms" in the phrase "arms, commerce and the sciences," should obviously be arts. The peculiar growth in the lower left-hand portion of the seal represents the corn-field, and was intended to be symbolical of agriculture. Commerce found recognition in the ship and science was obliged to content herself with the quadrant and telescope.

Elections in the Academy were in those days formidable affairs. The polls were to be opened at three P. M. and after the choice of scrutineers, the ballotting was to begin. The ballot box was to remain open until five P. M. and was then to be closed. The voter was required to fold his ballot and hand it in this form to the President, whose duty it was to put the ballot in the box and at the same time to check the name of the voter upon his list of members. A majority was required for election. Ties were settled by drawing lots.

The relations of the Academy to the public in those early days were absolutely different from the aloofness and reticence which prevail to-day. The aid of outsiders was deliberately sought for, and the results were communicated to the newspapers of the day. The records for January 31, 1781, show that "The Reverend Samuel Williams, having been directed by the Council to prepare an invitation to the public to communicate to the Academy any experiments, observations and productions of nature or art, adapted to the ends of its institution; and to lay the same before the Academy at the next meeting for their approval, in order to its publication; reported as follows:

"The Academy have the pleasure to inform the public that they have received the following communications, viz." Then follows a list of papers submitted at the meeting. Notices of meetings were required to be published.

Our associate, Abner C. Goodell, has collated in a note in the edition of the Province Laws which he edited, a few facts relative to the proceedings which took place prior to the passage of the act incorporating the Academy. It appears that on the 21st of March, 1776, a resolve was passed in the Continental Congress recommending to the "assemblies, conventions, and councils or committees of safety" of the several colonies to "take the earliest measures for erecting and establishing in each and every colony, a society for the improvement of agriculture, arts, manufactures and commerce, and to maintain a correspondence between such societies, that the rich and numerous natural advantages of this country for supporting its inhabitants may not be neglected."

So far as our own society is concerned, John Adams claimed to have first suggested its organization, in the course of a conversation with Rev. Dr. Samuel Cooper, at a dinner given in 1779, by the corporation of Harvard College in honor of Chevalier de la Luzerne, the French ambassador, and his suite. In his recollections, written thirty years after the event, Mr. Adams describes how he enlisted the services of Dr. Cooper in the propagation of his idea and plan, which was done so effectually that as he says "the first Legislature under the new Constitution adopted and established it by law." In this statement his memory served him false. We have already seen that the act was passed before the new Constitution came into operation.

In a "country dependent principally upon agriculture," to adopt the language of the person who described the seal of the Academy, before the days of the application of electricity, for power, light or heat; before the development of the steam engine, whether of the stationary or locomotive type; the modes of life, the methods of business, the opportunities for scientific investigation, the subjects to be investigated, and the methods to be employed were so different from those of to-day that it requires positive effort to reproduce the limitations imposed upon the men of 1780 by their surroundings. Washington is to all intents and purposes as near Boston for us of to-day as was Worcester to our forefathers in 1780, the true measure of the distance being not the number of miles which intervene, but the ease with which they can be traversed. At the time of the revolution, says one writer, the stage-coach was unknown on this continent — a statement open to question, but still so near the truth that it may be quoted for its practical definition of the condition of passenger transportation at that time. Travel was effected either on horseback or in the private chaise, caleche or coach. Communities under these conditions were necessarily provincial, interchange of thought was restricted and there was nothing to stimulate investigation. Real estate and bonds and notes were the only avenues open for investment of funds. When Ebenezer Storer opened his accounts as Treasurer of Harvard College, he charged himself with certain real estate and with two hundred and nineteen personal notes and bonds, the latter being the bulk of the income-yielding property of the College. Turn to the statement of our Treasurer and see the field covered by his investments to-day, and you will have as good a picture of contrast between then and now as can be given. Think for a moment the way in which the opportunity for scientific investigation must have been increased by the utilization of capital as shown on our Treasurer's books.

This period of bucolic simplicity continued for about half a century during which there was no material change in the lives and daily habits of our people. Things were not, however, stationary. This half century saw the beginning of the organization of combined capital. The water power of the country was utilized. Canals were dug. Stage-coach lines were established and the country was prepared for the great revolution which was to follow from the construction of steam railways and the stimulation of industry through the combination of capital in the form of corporations.

Some of us older members of the Academy can remember when the only public method of travel in New England was the stage-coach. As a boy, I well remember the daily passing my father's house of the stage which furnished means for the transportation of passengers between Worcester and Boston. The influence of the development of the facilities for travel since that time are far reaching. The various meetings of scientific men which take place in the holiday season were absolutely impossible then and the world lost the stimulus to research arising from the contact of different persons from different parts of the country working along parallel lines which must necessarily follow these personal encounters of experts.

If we turn to the records of our meetings of to-day we find that there are presented many papers which are read only by title. Not a few of us have envied our Recording Secretary for the faculty which he possesses of reading with rapid facility the titles of these papers, the obscure terms of some of which convey no information to the hearer of the subject of the investigation, and leave no other impression behind than doubt as to whether there can be many persons beside the writer who can tell what the paper is about. This brings to our attention the fact that with the growth of scientific knowledge there has come partition of subjects and specialization of investigations. Every man no longer knows everything. The farmer of to-day is not necessarily carpenter, blacksmith, harness-maker and cobbler as well.

The change that has been produced within the period of my own life, in our social, industrial and scientific conditions by the mere development of photography is in itself an adequate explanation of the rapid progress in knowledge which has brought about the contrast between the simple earnest groping for knowledge on the part of our forefathers and the marvellous studies in the region of the incomprehensible submitted for our consideration to-day. Before the days of Daguerre, the portrait, the miniature, and the black silhouette were the only means at command to register the likeness of a relative or friend, the inadequate silhouette being the substitute for the snap-shot. Strip from the walls of our homes the records at different periods of life of the appearance of the several members of the family, what a void would be left in the household! The industrial arts, astronomy, the diagnosis of diseases, the study of the motion of animals and of birds, indeed pretty much every form of scientific study, has found the use of the photograph contributory to its advance. An aviator — if the word is permissible — crosses the English Channel. The camera records his flight and his picture — in mid-air, high above the water of the channel — is published in an illustrated magazine. A would-be assassin assaults a distinguished public officer, on the crowded deck of an outgoing steamer. The picture of the scene is secured while the smoke still hangs round the barrel of the pistol and is laid before the readers of a daily newspaper. The ineffectual efforts of a favorite foot-ball eleven are captured and offered by the Sunday papers as a feeble solace to their disappointed admirers.

All this is familiar to every one and all know that photography is but one of a number of scientific accomplishments, the development of which has helped to bring us where we are. The story of their progress serves to emphasize the difference between scientific knowledge and scientific possibilities in 1780 and in 1910.

The advance of our investigators has carried them farther and farther from the central starting point of ignorance on the radius which measures the circle of knowledge, but with every outward step that has been taken the contact of the circumference of this area of conquered territory with the unknown beyond has become larger and larger. The proceedings of the thousand meetings of the Academy contain a record of the conquest of a large part of the field which we now occupy and there is no sign as yet that our students flatter themselves that the unconquered space of the beyond has given up all its secrets.

Following Mr. Davis's address the Toastmaster called for the second toast:

The second toast I propose to you is "The First Extant Communication Presented to the Academy." There is one name, more prominent than any other in the science of Physics, which has repeated itself in different individuals, in no way connected with each other. There is no danger that this Academy will forget the name of Benjamin Thompson, Count Rumford, the greatest name in science that America produced during the eighteenth century, with the possible exception of the other Benjamin — Franklin. In the nineteenth century, Sir William Thomson stands preëminent, now better known as Lord Kelvin. At the present time, no name stands higher than that of Sir Joseph Thomson of Cambridge, England, the worthy successor of Maxwell and Rayleigh. All three Thomsons were Foreign Honorary Members of this Academy, but to them we may add a fourth, who like Lord Kelvin gave a practical turn to his work. I will ask Professor Elihu Thomson to respond to this toast.

Professor Thomson responded as follows:

The first Academy paper, brief as it was, dealt with an extremely important subject, the making of steel. It was an art comparatively new to the Western world. Reliable information was needed, and this the paper furnished. There is no occasion to dwell upon the present importance of this art for it now dominates all our industries. The United States stands preëminent to-day as a steel producing and consuming nation. Indeed, our great manufacturing industries, our railroads, our steamships, our electrical plants, and our buildings largely rest, so to speak, on foundations of iron and steel. The telegraph shapes its signals by an iron magnet. Even the telephone speaks to us through a steel magnet and an iron diaphragm.

The earth itself is probably mostly composed of iron, for we have reason to believe that we are living on a layer of cooled and modified slag surrounding a still hot spherical iron ingot five thousand miles or more in diameter, the product of the celestial furnaces. We get our usable iron from the surface flecks of rust here and there.

The art of steel making, as first practiced, was merely an art. There was no science to assist, as it long antedated metallurgical science. The processes were the result of accident or empirical trials. Only within the half century just passed has steel production been brought under control of scientific methods, and only within the past twenty-five years has there been reached a just understanding of the nature of the changes involved. That singularly valuable property of true steel which enables its hardness, elasticity and toughness to be adjusted by a simple heat treatment, has received its explanation. There is no need to emphasize the enormous importance of the property of hardening and tempering which is the characteristic most prized in tool steel. Chemical analysis and microscopic examination of etched faces or sections have revealed in large measure the actual structure of the steel in its various states. Such microscopic examination constitutes the comparatively new science of metallography, which has in recent years contributed so much to our knowledge of metals and alloys, but more particularly the structure of steel.

We now know that not only is steel a complex product, but that its properties, as varied by hardening and tempering, or heat treatment, depend upon the greater or less predominance and upon the distribution of certain chemical constituents, the relative amounts of which are not the same even in the same bar when heated to different temperatures. Names are given to these constituents; ferrite, cementite, pearlite, martensite or hardenite, graphite; and they are, except the last, composed of iron associated with certain proportions of carbon in combination. By sudden cooling of red hot steel to harden it, we catch and fix, as it were, the components in their relation as existing in the hot metal, and before they can adjust their proportions to correspond with that normal to cold metal. In the hot metal so fixed by sudden cooling, there is a preponderance of a hard constituent, martensite or hardenite. If the hot steel had been slowly cooled, then there is time for a reproportioning in which the hard martensite disappears and gives place to a mass composed largely of softer constituents.

Still more valuable is that property of tempering, without which steel tools would be either too hard and brittle, or too soft and flexible.

By moderate reheating of chilled and hardened steel a partial change from the unstable hard condition may be effected; a partial recovery of the condition and proportion of constituents normal to soft or slowly cooled steel. This is tempering, and allows just such a degree of hardness to be retained as is needed, while the brittleness is diminished.

A curious physical fact recently found by Prof. Carl Barus is that during the lapse of a sufficient time all hardened steel will anneal or soften itself automatically, even at ordinary temperatures, but that the change will be accelerated as we raise the temperature. Fortunately the period for this self annealing in the cold is long; but it follows that if we were to dig up steel tools which had been hardened say two or three thousand years ago, we ought to find them much softer than they were originally.

Our scientific knowledge of steel is, of course, not yet complete. Steels with new constituents and more valuable properties are continually being made. For example may be mentioned the so-called air hardening or high speed steels which have added so enormously to the productiveness of machinery and labor in the working of iron and steel itself, and metals generally. Iron and steel production goes on at a rapid rate of increase. Our consumption per capita per annum is already several hundred pounds, and the amount is larger, if I mistake not, in the United States than in any other country. Perhaps we waste more.

The first Academy paper which I have reread describes that method of steel making known as the cementation process. Even until very recently this was the method almost universally relied upon for the production of high grades of tool steel, but within a decade or so the art of refining steel has reached such an advanced stage that great quantities of fine steel are made in what are known as basic open hearth furnaces. Lastly, the advent of the electric arc furnace, some of them dealing with charges of 30,000 pounds or more at a time, has rendered possible refining at very high temperatures, far beyond those of ordinary combustion furnaces. The more perfect control and exclusion of deleterious gases allows separation of impurities and control of composition so as to ensure the product having the qualities desired. In these furnaces electric arcs are maintained on a scale many thousands of times greater than the arcs used in lighting our streets. The enormous output of heat and light is shut up within the furnace walls, or it would be insupportable, insufferable. Its energy goes to raise the temperature of the molten steel bath, and the energy itself may be drawn from a water power.

As in the case of steel making, many an art has been discovered more or less accidentally, and then practiced empirically for a long time. When, however, scientific investigation discloses the laws and principles upon which it is founded, improvement takes place, as it has in steel making, by leaps and bounds. In the future, science should lead, not follow. Our Academy formed in 1780 brought together the leaders in scientific thought of the time, and it was organized for the spread of scientific knowledge. Can it not go on from this, its thousandth meeting, doing its part under improved auspices, adding to its influence and importance and assisting in those inevitable great advances which are the promise of the future of science?

The Toastmaster: "The next toast I propose to you is 'The First Research Undertaken by the Academy.' I offer this toast with pleasure, partly because of its professional interest to me, and partly because, as I have already said, I believe that the principal function of this Academy should be research. I will ask my colleague, Professor Robert W. Willson, to respond to this toast."

The First Research Undertaken by the Academy.

From the records of the Academy it appears that it was "Voted August 30, 1780, that the Hon. Thomas Cushing, Esqr., The Hon. Henry Gardner, Esqr., and Cotton Tufts, Esqr., be a Committee to confer with the Reverend and Honorable Congregation of the University of Cambridge upon pursuing measures to procure an accurate observation of the Solar eclipse in October next, in the eastern part of this State, and, in case it should be judged expedient, to join with the congregation aforesaid in an application to the Great and General Court for such assistance as may in the best manner affect the design."

Some years ago Dr. B. A. Gould, the founder of the Astronomical Journal, told me that its publication was delayed for six months, to await the completion of a research by Professor Benjamin Peirce on the Development of the Perturbative Function, in order that the issue of the first number, which was wholly occupied by this paper, might show that it was to be a worthy rival of the great European Journal whose field it entered.

In like manner we may look on this first research undertaken by the Academy, as foreshadowing the important place which this Society has since occupied in the development of the Sciences in America.

It may be of interest to know that the first number of the Journal, treating, as it did, of a subject far beyond the comprehension of all but a very few of the very learned, was probably consigned to the waste basket by most of the recipients, and has at all events become extremely rare. It is in fact lacking in Professor Pierce's own set of volumes of which I am now the fortunate possessor.

Such a fate was not to be feared for the account of our "first research." There must always be a strong popular interest in a total eclipse of the Sun, especially if it occurs so near at hand that it may be seen without a long and expensive journey.

Nowadays we all anxiously scan the newspapers to learn whether our observers, sent half round the world, perhaps, to accomplish their utmost in a short five minutes, have had a clear sky at the critical moment.

The first eclipse of the sun in the present century occurred on the 18th of May, 1911. Its track passed over the Indian Ocean and it was successfully observed at the Island of Mauritius and in Sumatra at about noon of that date. Had direct telegraphic communication been established, this success might easily have been known in Boston in time to be announced in the morning papers and read at our breakfast tables on that same 18th day of May. What would the Rev. Samuel Williams, Hollisian Professor of Mathematics and Natural Philosophy, have thought of this modern miracle! — familiar though he doubtless was with the times of Hezekiah, when the Lord did the thing that he had spoken: "Behold, I will bring again the shadow of the degrees, which is gone down in the sundial of Ahaz, ten degrees backward. So the sun returned ten degrees by which degrees it was gone down."

I select the name of Samuel Williams from the list of Reverend Professors who one hundred and thirty years ago shed a lustre upon the University and formed a considerable part of the membership of the Academy, because he was naturally chosen to be the leader of their expedition.

Successor of the gentle and learned Rev. John Winthrop who had died in the previous year, he taught the Astronomy of his day to such undergraduates as were competent to pursue that vigorous study, and from his pupils six were chosen, together with a graduate who had just taken his degree, as assistants in his observations. His own account is as follows:

"Observations of a solar eclipse, October 27, 1780, made on the east side of *Long-Island* in *Penobscot-Bay*."

"A total eclipse of the sun is a curious and uncommon phenomenon. From the principles of Astronomy it is certain that a central eclipse will happen, in some part of the earth, in the course of every year: But it is but seldom that a total eclipse of the sun is seen in any particular place. A favorable opportunity presenting for viewing one of these eclipses on October 27, 1780, the American Academy of Arts and Sciences, and the University at Cambridge, were desirous to have it properly observed in the eastern parts of the State, where, by calculation, it was expected it would be total. With this view they solicited the government of the Commonwealth, that a vessel might be prepared to convey observers to *Penobscot-Bay;* and that application might be made to the officer who commanded the *British* garrison there, for leave to take a situation convenient for this purpose.

"Though involved in all the calamities and distresses of a severe war, the government discovered all the attention and readiness to promote the cause of science, which could have been expected in the most peaceable and prosperous times; and passed a resolve, directing the Board of War to fit out the Lincoln galley to convey me to *Penobscot*, or any other port at the eastward, with such assistants as I should judge necessary.

"Accordingly, I embarked October 9 with Mr. Stephen Sewall, Professor of the Oriental Languages, James Winthrop, Esq., Librarian, Fortesque Vernon, A. B., and Messrs. Atkins, Davis, Hall, Dawson, Rensalear, and King, Students in the University. We took with us an excellent clock, an astronomical quadrant of 2–1/2 feet radius, made by Sissons, several telescopes, and such other apparatus as were necessary.

"On the 17th we arrived in *Penobscot-Bay*. The vessel was directed to come to anchor in a cove in the east side of *Long-Island*: after several attempts to find a better situation for observations, we fixed on this place as the most convenient we had reason to expect: And on the 19th we put our instruments on shore, near the house of Mr. *Shubael Williams*, where the following observations were made."

[A note to the foregoing paragraph gives us a clear picture of the difficulties due to the existing state of war :

"As an officer who commanded at Penobscot in his answer to the application of the government, had limited us to a time wholly inadequate to our purpose, from the 25th to the 30th of October, we were obliged to make a second application for leave to enter *Penobscot-Bay*. Leave was granted, but with a positive order to have no communication with any of the inhabitants, and to depart on the 28th, the day after the eclipse. Being thus retarded and embarrassed by military orders, and allowed no time after the eclipse to make any observations, it became necessary to set up our apparatus and begin our observations without any further loss of time. In the course of which we received every kind of assistance from Capt. Henry Mowatt, of the Albany, which it was in his power to give."]

I pass over these further details of the account which are mainly of interest to professional astronomers, but I should read this early account of the phenomenon which has been referred to since 1836 as "Baily's Beads," after the English Astronomer who described their appearance at the total eclipse of that year.

Professor Williams after recording his micrometric observations of the diminishing width of the solar crescent, continues thus :

"Immediately after the last observation, the sun's limb became so small as to appear like a circular thread, or rather like a very fine horn. Both the ends lost their acuteness, and seemed to break off in the form of small drops or stars; some of which were round, and others of an oblong figure. They would separate to a small distance : Some would appear to run together again, and others diminish until they wholly disappeared. Finding it very difficult to measure the lucid part any longer, I observed again in the larger telescope, looking out for the total immersion.

"After viewing the sun's limb about a minute, I found almost the whole of it thus broken or separated into drops, a small part only in the middle remaining connected. Plate I, Figure VII. This appearance remained about a minute, when one of my assistants, who was looking at the sun with his naked eye, observed that the light was increasing."

He then resumed his proper task of making measures of the width of the increasing crescent until the end of the eclipse.

Other phenomena observed are as follows :

"From the beginning of the eclipse unto the time of the greatest obscuration, the *colour* and *appearance* of the sky was gradually changing from an azure blue to a more dark or dusky colour, until it bore the appearance and gloom of night."

"The degree of darkness was greater than was to be expected, considering the sun was not wholly obscured. *Venus* appeared in the west; *Jupiter* was seen near the sun; *Lucida* near the zenith, and *Aridef* in the north-east near the horizon, appeared very bright."

[There seems to be some mistake here, both the stars mentioned were in the north-east, while the star near the zenith was *Arcturus*, "Bonus dormitat Homerus," as the learned professor himself might have said.]

"Several others of the fixt stars were also seen whose situations were not particularly noted. Objects at a small distance appeared confused, and we were obliged to make use of candles to count our clock. But as soon as the greatest obscuration was past, it was universally remarked that the increase of the light was much more rapid than that of the darkness had been.

"As the darkness increased a *chill* and *dampness* were very sensibly felt. To ascertain the quantity of *dew* that fell on a square foot during the eclipse, we cut two pieces of very fine soft paper exactly twelve inches square. Having weighed them in a nice balance, we placed them on an horizontal board in the open air. Just after the greatest obscuration we weighed one of them again, and found its weight was increased by the *dew* that had fallen upon it, four and one-half grains *Troy*. At the end of the eclipse we took up the other, and found its weight increased by the *dew* that lay upon it, but 3 grains; 1-1/2 grains being evaporated as the light and heat of the sun increased. By a similar experiment, the quantity of *dew* that fell upon a square foot the night before was found to be 6-1/2 grains; the night after the eclipse, 7 grains. Thus, in 1 hour and 19 minutes, when the light and heat of the sun were rapidly decreasing, there fell two-thirds as much *dew* as fell the night before, or the night after the eclipse."

Observations of the Fahrenheit thermometer showed a fall from 58° at the beginning of the eclipse to 48° at the greatest obscuration.

"To this we may add, so unusual a darkness, dampness and chill, in the midst of the day, seemed to spread a general amazement among all sorts of animals. Nor could we ourselves observe such unusual phenomena without some disagreeable feelings."

The account closes thus: "The longitude of the place of our observation agrees very well with what we had supposed in our calculations. But the latitude is near half a degree less than what the map of that part of the country had let us to expect. On this account our situation, instead of falling within the limits of total darkness proved to be very near the southern extremity."

I hope it will not tire you if I read from the account of James Winthrop, preserved in the archives of the Academy, but, so far as I know, never printed:

An Account of the Proceedings of the Company sent by the General Court of Massachusetts to observe the Solar Eclipse at Penobscot on 27 Oct. 1780; By James Winthrop.

On Monday 9th Oct. 1780, We sailed from Boston in the sloop Lincoln-Galley, a vessel belonging to the Government. The Company consisted of ten persons besides passengers & marines. Stormy weather prevented our arrival at Camden till the fourteenth. Having obtained leave from Genl Wadsworth, commander of the Massachusetts

Forces in that department, a flag was sent from thence on the fifteenth to the British Commanders at Penobscot, with letters from the Reverend Professor Williams, desiring permission to enter their harbor immediately, as our business was solely to promote the interest of Science, which is the common interest of all mankind. Mr Vernon took charge of the letters. On the 16th he returned with the answers of Lieut. Col. Campbell & Captain Mowatt. They both permitted us to come up the Bay immediately, & to anchor our Vessel in Williams' cove on the east side of Long Island, about three leagues from the British Fort. From thence we were directed to go in our boat to the Albany, which lay near the fort, before we should land. Capt. Mowatt very politely offered us every assistance in his power towards promoting the business we had in view. Colonel Campbell received us with evident reluctance. His strict prohibitions of all communication with the Inhabitants put it out of our power to procure the smallest articles of refreshment, or any building to secure our apparatus. In consequence of these orders we went up the Bay to Williams' cove on the 17th & on the next day in our boat to the Albany. We found no convenient place for our observations about the harbor. Being uncertain of our Longitude and Latitude, & surrounded by almost perpetual fogs, We determined to tarry at Long Island. With permission we took up part of an house & barn which were made to answer our purpose. On the 19th & 20th we set up our clock & other instruments. Till the 24th the weather was so thick, that we had no opportunity to make any observations, either for regulating the clock or ascertaining the Latitude. The Variation we observed daily by means of a meridian drawn the first day of our being ashore. All our observations made it 11°, 55′ West. In the year 1761 Doctor Winthrop observed the Variation to be 8° West at Fort Pownal about twelve miles N. E. from our place of Observation. Whence it appears, that contrary to the common opinion the Variation has been increasing in the eastern parts of this State for several years past.

"The 24th 25th & 26th were spent in regulating the clock, & determining the latitude. The latitude appears from the mean of several observations of the Sun & Fixed Stars to be 44°, 17′, 8″, which is not so far northerly as the maps represent.

"On the Twenty seventh the weather was perfectly fine. In the morning we took several altitudes. A little before eleven o'clock the observers took their places: But as a compleat account of their Observations will be presented by Mr Professor Williams, to mention them particularly here would be needless. I shall confine myself to physical appearances; & content myself with remarking that the eclipse was greater than we had reason to expect at that place. As the Eclipse increased Fahrenheit's Thermometer, which at the beginning stood at 58, gradually fell to $48\frac{1}{2}$ where it stood at the time of the greatest obscuration. The weather was sensibly colder though the change was gradual. Our Prospect became confined, & distant objects were lost while those about us assumed a gloomy appearance. The Sky, particularly at the northeast, appeared as if charged with a thick fog. The Darkness was so great, that a lanthorn was necessary in counting the Clock; yet we had light enough abroad to distinguish countenances without any difficulty, & to write down our observations. Shades were better defined than they ever are by moon-light; & the Sun, even at the greatest obscuration, shone with such dazzling brightness, that those of the company, who looked without a colored glass, could hardly perceive the Eclipse.

"Venus was seen clearly by the whole company shining with a vivid light. Several other Stars were seen, among which were Lucida Lyrâ & Aridef. The part of the Sun which remained uneclipsed was not more than one eightieth part of his diameter, & one eighth of his Circumference. The upper cusp was ragged & the lower one rounded. At the upper point appeared two drops as bright as Stars of the first magnitude. At the end of the eclipse the Thermometer had risen to 58, where it was when the eclipse begun. The dew which fell on a sheet of paper twelve inches square was found immediately after the middle of the Eclipse to be $4\frac{1}{2}$ grains. On another sheet at the end of the Eclipse it was 3 grains. The preceding night it was $6\frac{1}{2}$, & the night following 7 grains. On the 28th we packed our Instruments, & sent them aboard the Vessel. The next day we set sail, & on the 10th of November arrived in Boston."

It is evident from what I have read to you that the Academy was fortunate in the personnel of its party, — it was well equipped and prepared to observe any new and unexpected phenomena which might occur. We can only guess what they might have seen and reported if they had found a more suitable place of observation than was permitted by the military conditions imposed on them. That they would have found by their first day's observations for latitude that it was advisable to move further to the northeast, perhaps to the fort itself, seems probable, and in that case there would have been an unusually efficient group of men upon the spot.

To say nothing of their elders, the general character of the six undergraduates who accompanied the party is shown by their later lives. Two were members of the Academy, two were members of Congress, and one a Justice of the Supreme Court of New Hampshire. The graduate, Mr. Fortesque Vernon, died in 1790.

May I add that it is a peculiar gratification to myself, and will certainly be to some others present, that we have a personal acquaintance with at least two of the instruments which were used on this adventure.

In the collections of the Department of Physics at Cambridge is the two-foot reflector used by Professor Williams; on it is a silver plate bearing the arms of Pepperell and Sparhawk, — both good examples of canting heraldry. It was the gift of Sir William Pepperell after the destruction of the collection of Philosophical Instruments in Harvard Hall in 1764.

It is furnished with a divided object glass micrometer by Dollond, the precursor of the modern heliometer. A similar micrometer, the one with which Professor Williams made his measurements, was attached to the smaller reflector of one foot focus. This little instrument now has a place in the Faculty Room at Cambridge, just under the portrait of John Winthrop, in which it is faithfully depicted as an artistic accessory.

Gentlemen, it is almost no exaggeration to say that to-day, at our thousandth meeting, we have a thousandfold advantage over the founders. In the interval — back of us, before them — lies the century wonderful of science. But theirs were no small beginnings. I congratulate the Academy on its first research.

The last toast proposed was "The Millenium," responded to by Professor George L. Kittredge.

The celebration closed with thanks to Dr. Louis Bell and to Professor W. M. Davis for their inception of the quaint celebration of the one thousandth meeting, and for their success in making it a memorable one.

The following papers were presented by title: —

"On the Internal Resistance of the Lead Accumulation." By H. W. Morse and L. W. Sargent. Presented by John Trowbridge.

"The Wave Potential of a Circular Line of Sources." By A. G. Webster.

"Division of Labor among Ants." By Edith N. Buckingham. Presented by E. L. Mark.

"A New Method for the Study of Elastic Hysteresis." By A. G. Webster and T. L. Porter.

"The Action of Metals on Ketoric Chlorides of the Aromatic Series." By J. F. Norris.

One thousand and second Meeting.

January 11, 1911. — Stated Meeting.

The Academy met in its Hall.

The President in the chair.

There were thirty-eight Fellows present.

The Corresponding Secretary presented the following: — a letter from Professor W. G. Farlow, representative of the Academy at the Third International Botanical Congress, enclosing a printed report of the Congress; a notice of the death of Angelo Mosso, from the Reale Accademia delle Scienze, Torino; a letter from Mr. Loammi F. Baldwin, of Woburn, notifying the Academy of the bequest in the will of Mrs. Catherine Rumford (Baldwin) Griffith, of mementos of Count Rumford; the felicitations of the new year, from the Museo nacional de Arqueologia, Historia y Etnologia, Mexico; a circular letter from the Nobel committee of the Svenska Akademien, requesting the distribution of some enclosed circulars regarding the competition for the Nobel Prize in Literature.

The President read the following letter from the sons of the late Professor Agassiz: —

14 Ashburton Place, Boston, Mass.,
January 7, 1911.

President and Members of the American Academy.

Gentlemen, — From a letter of our father's to Professor Trowbridge, dated October 16th, 1909, and published in Vol. 45, No. 21, of your Proceedings, it is evident that Mr. Agassiz did not intend to endow the building that he proposed to give to the Academy.

We therefore take pleasure in making the following proposal: Should the Academy care to devote the $50,000 (bequeathed to the Academy unconditionally by Mr. Agassiz) toward the construction of the building he proposed to give them, we on our part will give No. 26 Newbury Street clear of all mortgages, and pay the balance of the cost of construction, provided this sum does not exceed $30,000. (You will notice by the enclosed letter from the Architects that the estimated cost of the building is approximately $74,000.) We will pay the Architect's fee, and give the sum of $9,000 for furnishing the building.

As our interest in this matter is to carry out Mr. Agassiz's wishes as nearly as we can interpret them, we should like to have it understood that the Academy, in case it favors our proposal, will make no changes from the plan which we hand you herewith without consulting us.

As we should regret to feel that we had induced the Academy to adopt a measure that was opposed by any considerable minority of its members, we would like, in case the Academy accepts our offer, the assurance that such a course meets with the approval of at least three-quarters of its active members.

We trust that the members of the Academy will give themselves ample time to consider carefully how they expect to meet the additional expenses of the maintenance of a larger building; and that they will also consider whether they would not prefer to have the use of a fund of $50,000 rather than the proposed building.

We enclose plan of the building and short description of the specifications.

Respectfully yours,
(signed) G. R. Agassiz,
Max. Agassiz,
R. L. Agassiz.

On the motion of Dr. Ernst it was

Voted, I. That the thanks of the Academy be returned to the sons of Professor Agassiz for the letter transmitted by the President.

II. That the letter be referred to the Committee on Policy for action in connection with the circular of information and inquiry to be sent out by it.

The President made a brief statement of the probable annual expense of running the proposed new building; and also a statement of the number of Resident Fellows at stated periods during the last one hundred years, showing that the Resident Fellowship had not increased for forty years, whereas there were a great many who were eligible to Fellowship in the Academy.

The following gentlemen were elected members of the Academy: —

Richard Cockburn Maclaurin, of Boston, to be a Resident Fellow in Class I., Section 2.

William Curtis Farabee, of Cambridge, to be a Resident Fellow in Class III., Section 2.

Alfred Marston Tozzer, of Cambridge, to be a Resident Fellow in Class III., Section 2.

Albert Matthews, of Boston, to be a Resident Fellow in Class III., Section 4.

On motion of the Recording Secretary, it was

Voted, To appropriate the following sums from the income of the General Fund: —

For Expenses of meetings	$100
For Treasurer's office	25
For General Expenses, at the discretion of the President	50

The following letter to the President was read: —

PROF. JOHN TROWBRIDGE.

Dear Sir, — At a meeting of the Committee of the University Museum of Harvard University, held 4 January, 1911, it was Voted, To invite the American Academy of Arts and Sciences to hold a Meeting in the spring of 1911 in the University Museum.

Yours truly,
SAMUEL HENSHAW, for the Committee.

On the motion of W. M. Davis, it was

Voted, That a committee be appointed to plan and carry out the meeting in Cambridge.

The President appointed Messrs. Wolff, Jeffrey, Mark, Henshaw, Putnam, and W. M. Davis a committee on the Cambridge meeting.

The following communication was given: —

"An Investigation of the Cartago Earthquake in Costa Rica," by Professor T. A. Jaggar, Jr.

The following papers were presented by title: —

"A Revision of the Atomic Weight of Iron. Third paper. — The Analysis of Ferrous Bromide." By G. P. Baxter, Thorbergur Thorvaldson, and Victor Cobb.

"A Revision of the Atomic Weight of Iron. Fourth paper. — The Atomic Weight of Meteoric Iron." By G. P. Baxter and Thorbergur Thorvaldson.

"Buddhaghosa's Way of Purity analysed: first third, Morality." By C. R. Lanman.

On motion of the Recording Secretary it was

Voted, To meet on adjournment on the second Wednesday in February.

One thousand and third Meeting.

FEBRUARY 8, 1911.—ADJOURNED STATED MEETING.

The PRESIDENT in the chair.

There were thirty-six Fellows present.

In the absence of the Corresponding Secretary, the President read the following letters and circulars: — letters accepting Fellowship from W. C. Farabee, A. M. Tozzer, Albert Matthews, R. C. Maclaurin; a circular from the Royal Frederick University of Christiania, inviting the Academy to send a delegate to its one hundredth anniversary celebration to be held September 5–6, 1911; a circular of the Fourth International Congress of Philosophy to be held at Bologna, April 6–11, 1911; a circular from the Boston 1915 Committee, asking a delegate to the first annual meeting of the Education Conference to be held March 16, 1911; a card of invitation to the President to attend a meeting in memory of Henry Charles Lea, at Philadelphia, January 20th.

The President announced the following deaths: — Samuel H. Butcher, Foreign Honorary Member in Class III., Section 4; Leonard P. Kinnicutt, Resident Fellow in Class I., Section 3,— a member of the Warren Committee since its formation in 1893, and chairman of that Committee since 1902.

The President reported that the Committee on Policy had voted against allowing any member to be enrolled in two different Sections of the Academy at the same time, and that the Council accepted that report.

The President read the following: —

The Committee on Policy submits the following recommendations as its report, and will move their adoption.

1. To accept the offer of the Agassiz heirs, whereby we should secure a dignified and comfortable home, with the probability of increased activity and wider influence, the methods of raising the funds needed for the increased cost of maintenance to be left for future consideration.

Note 1. In reply to a recent circular 133 Fellows voted in favor of the above recommendation, 10 voted against it, 56 did not vote.

Note 2. A committee on endowment has been appointed, from which a preliminary report is expected.

2. To amend the statutes in accordance with the bill now before the Legislature, then to increase the Massachusetts membership by 20 or 25 a year until we have perhaps 300 such members; and thereafter to increase membership as the Academy shall then determine.

3. To amend the statutes so that we may gradually increase the American membership outside of Massachusetts to such a number — for example, 300 or 400 — as shall justify our name: "The American Academy of Arts and Sciences"; and thus in some measure return to the original intention of the Academy.

4. To place all American members in one list, all to have the right of attending meetings, presenting papers, and taking part in discussions. All Massachusetts members to pay an annual fee, and to have the further rights of voting on the affairs of the Academy, publishing in the publications of the Academy and receiving the publications if desired. American members residing outside of Massachusetts also to have these additional rights on payment of corresponding fees. All American members to be called Fellows, and their names to be published in one list, subdivided into Classes and Sections as at present, but without indicating which are paying and which non-paying members.

5. That the functions of the Council be enlarged, in order to give it supervision over all the affairs of the Academy, not otherwise specified by the statutes.

6. That a committee of five, including the President and the Recording Secretary, be appointed to take charge of meetings.

Note. Recommendations 5 and 6 were reported to the Academy November 9th, 1910.

If the above recommendations are adopted by the Academy, proposed alterations of the statutes will be submitted, for reference to a committee.

The President reported that the Committee on endowment — Messrs. C. P. Bowditch, E. C. Clarke, and the President — had been promised four thousand dollars without effort on the part of the Committee, and he saw no reason why an endowment of fifty thousand dollars might not be raised.

On motion of W. M. Davis, seconded by A. G. Webster, it was

Voted, To adopt the first recommendation of the Committee on Policy, as follows: —

"Accept the offer of the Agassiz heirs, whereby we secure a dignified and comfortable home, with the probability of increased activity and wider influence, the methods of raising the funds needed for the increased cost of maintenance to be left for future consideration."

On motion of W. R. Livermore it was

Voted, To adopt the second recommendation of the Committee, as follows: —

"Amend the statutes in accordance with the bill now before the Legislature, then to increase the Massachusetts membership by 20 or 25 a year until we have perhaps 300 such members; and thereafter to increase membership as the Academy shall then determine."

On motion of Louis Bell, it was

Voted, That the fourth recommendation be laid on the table.

On motion of A. G. Webster it was

Voted, to adopt the fifth recommendation, as follows: —

"That the functions of the Council be enlarged, in order to give it supervision over all affairs of the Academy, not otherwise specified by the Statutes."

On motion of A. G. Webster it was

Voted, To adopt the sixth recommendation, as follows: —

"That a committee of five, including the President and the Recording Secretary, be appointed to take charge of meetings."

On motion of A. G. Webster it was

Voted, That the third recommendation be laid on the table.

Professor Davis presented proposed alterations of the Statutes to accord with the recommendations of the Committee on Policy just adopted, and moved that they be referred to a Committee.

The President referred the proposed alteration of the Statutes to a Committee consisting of W. M. Davis and H. H. Edes.

The meeting then adjourned to the Reception room where the following paper was given by Professor Kuno Francke: — "The Historical Aspect of Mediaeval German Mysticism."

The following papers were presented by title: —

"The Opacity of Certain Glasses to the Ultra Violet." By Louis Bell.

"On the Electromagnetic and Thermomagnetic Transverse and Longitudinal Effects in Soft Iron." By Edwin H. Hall and L. L. Campbell.

"Notes on the Electrical Conductivity of Argentic Sulphide." By Hammond V. Hayes.

"Investigation of Temperature Errors caused by the Protection Tube in Cooling-Curve Measurements." By Harvey C. Hayes. Presented by J. Trowbridge.

"Determination of the Altitudes of Aeroplanes." By Robert W. Willson.

One thousand and fourth Meeting.

March 8, 1911. — Stated Meeting.

The President in the chair.

There were thirty-seven Fellows present.

The Corresponding Secretary read a letter from Mr. F. J. Stimson, resigning Fellowship; also the preliminary announcement of the Eighth International Congress of Applied Chemistry, to take place in Washington and New York September 4–13, 1912.

The Chair announced the death of Judge Francis C. Lowell, Resident Fellow in Class III., Section 1.

The report of the Committee on the amendments of the Statutes was read and accepted, and the amendments were adopted, as follows:

The first three printed lines of Chapter I., Article 2 are amended so as to read:

"2. The number of Resident Fellows residing in the Commonwealth of Massachusetts shall not exceed four hundred, of whom there shall not be more than one hundred and fifty in any one of the three classes."

The last five printed lines of Chapter II., Article 2 are amended so as to read:

"It shall be the duty of this Council to exercise a discreet supervision over all nominations and elections, and in general to supervise all the affairs of the Academy, not explicitly reserved to the Academy as a whole, or intrusted to special Committees by these statutes. With the consent of the Fellow interested, the Council shall have the power to make transfers between the several sections of the same Class, reporting their action to the Academy."

The last paragraph of Chapter II., Article 3, and the whole of Article 4, are amended so as to read:

"At the Annual Meeting, the Council shall submit to the Academy for its action, a report recommending the appropriations which in the opinion of the Council should be made for the various purposes of the Academy. Special appropriations may be made in any stated meeting of the Academy on the recommendation of the Council.

"4. If any office shall become vacant during the year, the vacancy shall be filled by the Council for the unexpired term."

Article 5 of Chapter III. is repealed.

Chapter V., Article 7 is amended so as to read as follows:

"7. The House Committee to consist of three Fellows. This Committee shall have charge of all expenses connected with the House, including the general expenses of the Academy not specifically assigned to other Committees, but not including the expenses of meetings. This Committee shall report to the Council in March in each year on the appropriations needed for the coming year. All bills incurred by this Committee within the limits of the appropriations made by the Academy shall be approved by the Chairman of the House Committee."

A new Article, No. 8, is added, as follows:

"8. The Committee on Meetings, to consist of the President, the Recording Secretary and three Fellows. This Committee shall have charge of plans for meetings of the Academy. It shall report to the Council in March in each year on the appropriations needed for the coming year. All bills incurred by this Committee within the limits of the appropriations made by the Academy shall be approved by the Chairman of the Committee on Meetings."

The present Article 8 becomes Article 9.

The present Article 9 becomes Article 10.

Chapter VI., Article 1 is amended so as to read:

"1. The Corresponding Secretary shall conduct the correspondence of the Academy, recording or making an entry of all letters written in its name, and preserving on file all letters which are received; and at each meeting of the Council he shall present the letters which have been addressed to the Academy since the last meeting, and at the next meeting of the Academy he shall present such of these letters as the Council may determine. Under the direction of the Council, he shall keep a list of the Resident Fellows, Associate Fellows, and Foreign Honorary Members, arranged in their Classes and in Sections in respect to the special sciences in which they are severally proficient; and he shall act as secretary to the Council."

Standing Vote 10 is amended so as to read:

"10. No report of any paper presented at a meeting of the Academy shall be published by any member without the consent of the author, and no report shall in any case be published by any member in a newspaper as an account of the proceedings of the Academy, without the consent and approval of the Council previously given."

It was

Voted, to adopt the third and fourth recommendations of the Committee on Policy, read at the meeting of February 8th, with the addition of the words "(United States)" after the word American in line one of Recommendations 4, as follows:

"To amend the statutes so that we may gradually increase the American membership outside of Massachusetts to such a number — for example, 300 or 400 — as shall justify our name: 'The American Academy of Arts and Sciences;' and thus in some measure return to the original intention of the Academy."

"To place all American (United States) members in one list, all to have the right of attending meetings, presenting papers, and taking part in discussions. All Massachusetts members to pay an annual fee, and to have the further rights of voting on the affairs of the Academy, publishing in the publications of the Academy, and receiving the publications if desired. American members residing outside of Massachusetts also to have these additional rights on payment of corresponding fees. All American members to be called Fellows, and their names to be published in one list, subdivided into Classes and Sections as at present, but without indicating which are paying and which non-paying members."

The Corresponding Secretary presented proposed alterations of the Statutes to accord with the recommendations of the Committee on Policy just adopted, together with other alterations recommended by the Council, and moved that they be referred to a Committee.

The President referred the proposed alterations of the Statutes to a Committee consisting of H. H. Edes and W. M. Davis.[1]

Dr. Ernst said that as the adoption by the Academy of the several recommendations of the Committee on Policy necessitated so many alterations in the Statutes, other changes might, in consequence, suggest themselves to the Committee; and he

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[1] Professor Davis, having been called to Europe soon after his appointment, resigned from the Committee, whereupon the President appointed Professor Elihu Thomson and Dr. Henry Lefavour to serve with Mr. Edes.

expressed the hope that it would feel warranted in incorporating in its report such further changes as it may deem expedient. Whereupon it was

Voted, That the Committee be authorized and requested to act in accordance with Dr. Ernst's suggestion.

It was

Voted, To appropriate from the income of the Publication Fund, five hundred (500) dollars for use of the Publication Committee during the present year.

It was also

Voted, To appropriate from the income of the General Fund, twenty (20) dollars for General expenses.

William Wallace Campbell, of Mt. Hamilton, Cal., was elected an Associate Fellow in Class I., Section 1 (Mathematics and Astronomy).

The President reported that the Charter of the Academy had been further amended by the Massachusetts Legislature as follows:

[Chapter 47.]

COMMONWEALTH OF MASSACHUSETTS.

In the year One Thousand Nine Hundred and Eleven.

An Act Relative to the American Academy of Arts and Sciences.

Be it enacted by the Senate and House of Representatives in General Court assembled and by the authority of the same, as follows: —

SECTION 1. Section four of chapter forty-six of the acts of the year seventeen hundred and seventy-nine, as amended by section one of chapter one hundred and twenty-nine of the acts of the year nineteen hundred and ten, is hereby further amended by striking out the word "three," in the last line, and inserting in place thereof the word: — four, — so as to read as follows: — SECTION 4. That the fellows of the said academy may from time to time elect such persons to be fellows thereof as they shall judge proper; and that they shall have full power and authority from time to time to suspend, expel or disfranchise any fellow of the said academy who shall by his conduct render himself unworthy of a place in that body, in the judgment of the academy; and also to settle and establish the rules, forms and conditions of election, suspension, expulsion and disfranchisement: provided, that the number of the said academy, who are inhabitants of this state, shall not at any one time be more than four hundred nor less than forty.

SECTION 2. Section six of said chapter forty-six, as amended by section two of said chapter one hundred and twenty-nine, is hereby further amended by striking out the word "one," in the seventh line, and inserting in place thereof the word: — two, — and by striking out the word "three," in the eighth line, and inserting in place thereof the word: — five, — so as to read as follows: — SECTION 6. That the fellows of the said academy may, and shall forever hereafter be deemed capable in the law of having, holding and taking, in fee-simple or any less estate, by gift, grant, devise or otherwise, any lands, tenements or other estate, real and personal: provided, that the said real estate shall not exceed in value the sum of two hundred thousand dollars, and the said personal estate shall not exceed in value the sum of five hundred thousand dollars; all the sums mentioned in the preceding section of this act to be valued in silver at the rate of six shillings and eight pence by the ounce: and the annual interest and income of the said real and personal estate, together with the fines and penalties aforesaid, shall be appropriated for premiums, to encourage improvements and discoveries in agriculture, arts and manufactures, or for other purposes consistent with the end and design of the institution of the said academy, as the fellows thereof shall determine.

SECTION 3. This act shall take effect upon its passage.

HOUSE OF REPRESENTATIVES, February 14, 1911.

Passed to be enacted. JOSEPH WALKER, *Speaker*.

IN SENATE, February 15, 1911.

Passed to be enacted. ALLEN T. TREADWAY, *President*.

February 17, 1911.

Approved. EUGENE N. FOSS.

Office of the Secretary,

BOSTON, March 2, 1911.

A true copy.

Witness the Great Seal of the Commonwealth.

WM. M. OLIN,

[Seal.] *Secretary of the Commonwealth.*

The President appointed the following gentlemen a Nominating Committee: —

Harry M. Goodwin, of Class I.

Charles S. Minot, of Class II. (Chairman)

George F. Moore, of Class III.

It was

Voted, To meet on adjournment April 12th.

The following communication was given by Dr. D. G. Lyon: "Harvard Excavations at Samaria in 1910," illustrated by lantern slides.

The following paper was presented by title: —

Contributions from the Gray Herbarium of Harvard University, new series, No. XXXIX. I. On the Classification of certain Eupatorieae; II. Revision of the Genus Barroetia; III. On some hitherto undescribed or misplaced Compositae. By B. L. Robinson.

One thousand and fifth Meeting.

Adjourned Stated Meeting. — April 12, 1911.

The Academy met in the Geological Lecture Room of the University Museum, Cambridge, at the invitation of the Committee of the Museum.

The President in the chair.

There were fifty-five Fellows, and forty-three guests present.

The following letters and circulars were presented: — from the Verein für Naturkunde, Cassel, an invitation to the celebration of its seventy-fifth anniversary; from the Committee of the American Year Book Corporation, asking that a representative of the Academy attend its meeting, March 25th, and asking for authority to add the name of the representative of the Academy to the Corporation; from the tenth International Congress of Geography to be held at Rome, Oct. 15–22, 1911 (Circular No. 2).

The Chair announced the death of Henry Pickering Bowditch, Resident Fellow in Class II., Section 3; and of Samuel Franklin Emmons, Associate Fellow in Class II., Section 1.

It was

Voted, To transfer the money now appropriated from the income of the Rumford Fund for Books and binding, to that of Periodicals and binding.

The chairman of the Rumford Committee stated that the Rumford Premium was granted to Mr. Charles Gordon Curtis for his "improvements in the utilization of heat as work in the steam-turbine."

The President presented the medals to Mr. Curtis.

Mr. Curtis in receiving the medals addressed the Academy on the "Development of the Steam Turbine in the United States."

Professor George A. Reisner gave an illustrated account of the Archaeological Survey of Lower Nubia.

The following paper was presented by title:

"The Nature of Volcanic Action." By Reginald A. Daly.

One thousand and sixth Meeting.

May 10, 1911. — Annual Meeting.

The President in the chair.

Thirty-six Fellows present.

The Chair announced the death of Thomas Wentworth Higginson, Resident Fellow in Class III., Section 4; of Charles Otis Whitman, Associate Fellow in Class II., Section 3; and of J. H. van't Hoff, Foreign Honorary Member in Class I., Section 3.

The Corresponding Secretary read the following letter and circulars: — from Lucien Carr, resigning Fellowship; from the Committee of the International Congress of Orientalists, inviting delegates to the 16th Session to be held at Athens, April 7th–14th, 1912; from the Committee of the first Universal Race Congress, to be held in London, July 26th–29th, 1911.

The following report of the Council was read: —

Since the last report of the Council eighteen deaths have been reported: — five Resident Fellows, — Robert Amory, Leonard Parker Kinnicutt, Francis Cabot Lowell, Henry Pickering Bowditch, Thomas Wentworth Higginson; seven Associate Fellows, — Cyrus Ballou Comstock, William Price Craighill, William Wirt Howe, Melville Weston Fuller, George Park Fisher, Samuel Franklin Emmons, Charles Otis Whitman; six Foreign Honorary Members, — Friedrich von Recklinghausen, Samuel Henry Butcher, William Huggins, Robert Koch, Maurice Levy, Jacobus Henricus van't Hoff.

Two Resident Fellows have resigned.

New members elected are: — Resident Fellows, six; Associate Fellows, one.

The roll of the Academy now includes 194 Resident Fellows, 76 Associate Fellows, and 55 Foreign Honorary Members.

The annual report of the Treasurer was read, of which the following is an abstract: —

General Fund.

Receipts.

Balance, May 1, 1910	$975.63	
Investments	1,799.40	
Assessments	1,860.00	
Admission fees	90.00	
Sale of chairs	35.00	$4,760.03

Expenditures.

Expense of House	$944.98	
Expense of Library	2,438.71	
Expense of Meetings	259.59	
Treasurer	164.31	
Insurance	48.00	
Moving	100.29	
Rent, 711 Boylston St.	18.75	
General expenses of Society	174.21	
Income transferred to principal	173.05	$4,321.89
Balance, May 1, 1911		438.14
		$4,760.03

Rumford Fund.

Receipts.

Balance, May 1, 1910	$1,359.31	
Investments	3,008.28	$4,367.59

Expenditures.

Research	$1,650.00	
Books, periodicals and binding	272.66	
Publication	398.92	
Medals	400.00	
Sundries	3.50	
Income transferred to principal	144.41	$2,869.49
Balance May 1, 1911		1,498.10
		$4,367.59

C. M. Warren Fund.

Receipts.

Balance, May 1, 1910	$783.48	
Investments	385.63	$1,169.11

Expenditures.

Research	$200.00	
Sundries	4.90	
Income transferred to principal	16.49	
Charged to reduce premium on bonds	50.00	$271.39
Balance, May 1, 1911		897.72
		$1,169.11

Publication Fund.

Receipts.

Balance, May 1, 1910	$1,321.95	
Appleton Fund investments	664.06	
Centennial Fund investments	2,331.67	
Sale of publications	332.78	$4,650.46

Expenditures.

Publication	$3,370.43	
Sundries	67.50	
Income transferred to principal	146.35	$3,584.28
Balance, May 1, 1911		1,066.18
		$4,650.46

The following reports were also presented: —

Report of the Librarian.

During the construction of the new building, the Academy has obtained Room 5, of the office-building 711 Boylston Street, as temporary quarters for the Assistant Librarian.

In March the unbound periodicals, the pamphlets, and all the stock of Proceedings and Memoirs which was in the basement of 28 Newbury Street were safely and expeditiously removed under the efficient supervision of the Assistant Librarian and Charles E. Wilder, who has been an assistant in the Library out of school and college hours since 1904.

The number of volumes added to the shelves during the past year is three hundred and twenty-seven, making thirty-one thousand, three hundred and forty-three. The number of volumes added includes two hundred and seventy-seven received by gift and exchange, thirty-six purchased by the General Fund, and fourteen by the Rumford Fund.

Eighty-five books have been borrowed from the library up to the time the stack building was closed, March 25th.

The expenses charged to the library are as follows: — Miscellaneous, $371.95 (which includes $104.62 for cataloguing); Binding, $539.55 General, and $69.55 Rumford, Funds; Periodical subscriptions $483.71 General, and $164.34 Rumford, Funds, as the cost of subscriptions and binding.

The appropriation of $50 from the income of the Rumford Fund plus $68.17, the unexpended balance from last year for Books and binding, was transferred to the account of the appropriation for Periodicals and binding.

A. Lawrence Rotch, *Librarian.*

May 10, 1911.

Report of the Rumford Committee.

During the past year the following persons have received grants of the sums specified in aid of researches on light or heat: —

October 12, 1910. Dr. P. W. Bridgman, of the Jefferson Physical Laboratory, for the continuance of his research on the thermal and optical properties of bodies under extreme pressures, additional,	$400
Professor Charles L. Norton, of the Massachusetts Institute of Technology, for his researches on thermal insulation	400
January 11, 1911. Professor Joel Stebbins, of the University of Illinois, for the continuance of his researches with the selenium photometer, additional,	200
Professor M. A. Rosanoff, of Clark University, for his investigations on the fractional distillation of binary mixtures, additional,	300
February 8, 1911. Dr. Daniel F. Comstock, of the Massachusetts Institute of Technology, for his research on the possible effect of the motion of the source on the velocity of light	100
Professor Gilbert N. Lewis, of the Massachusetts Institute of Technology, for his research on the free energy changes in chemical reactions	150
Professor Robert W. Wood, of the Johns Hopkins University, for his researches on the optical properties of vapors	150

Since the last annual meeting of the Academy, the following papers have been published in the Proceedings at the expense of the Rumford Fund :

Vol. 46, No. 4. "Note on Kirchoff's Law." By G. C. Evans.

Vol. 46, No. 11. "On the Equilibrium of the System consisting of Calcium Carbide, Calcium Cyanamide, Carbon, and Nitrogen." By M. DeK. Thompson and R. H. Lombard.

Vol. 46, No. 19. "A Method for Determining Heat of Evaporation as applied to Water." By T. W. Richards and J. H. Mathews.

Vol. 46, No. 23. "On the Electromagnetic and the Thermomagnetic Transverse and Longitudinal Effects in Soft Iron." By E. H. Hall and L. L. Campbell.

Vol. 46, No. 24. "On the Opacity of Certain Glasses for the Ultra-Violet." By L. Bell.

On October 12, it was voted that the Chairman of the Committee be requested to bring up to date the pamphlet concerning the Rumford Fund, published by the Academy in 1905.

The endeavors of the Committee to procure fac-simile copies of the inscriptions upon the reverse of the Rumford Medals awarded prior to 1899, in order that replicas in bronze may be made, have been successful. All but one of the medals have been located and photographs or plaster casts of these are gradually being secured. Since the date mentioned, a bronze replica of each medal awarded has been made at the same time as those to be presented.

Reports of progress in their respective researches which have been aided from the Rumford Fund have been received from Messrs. P. W. Bridgman, W. W. Campbell, A. L. Clark, D. F. Comstock, W. J. Fisher, E. B. Frost, L. J. Henderson, L. R. Ingersoll, N. A. Kent, F. E. Kester, G. N. Lewis, C. E. Mendenhall, C. L. Norton, J. A. Parkhurst, T. W. Richards, M. A. Rosanoff, F. A. Saunders, J. Stebbins, M. DeK. Thompson, and F. W. Very.

At a meeting of the Committee held on January 11, it was unanimously voted for the first time and at a meeting held on February 8, for the second time, to recommend to the Academy the award of the Rumford Premium to Professor James Mason Crafts, for his researches in high temperature themometry and the exact determination of new fixed points on the thermometric scale.

Charles R. Cross, *Chairman.*

May 10, 1911.

Report of the C. M. Warren Committee.

The C. M. Warren Committee beg to report that during the past year an additional grant of $50 has been made to Dr. J. Elliot Gilpin, of Johns Hopkins University, for a continuation of his work upon the "Study of the Nature and Source of Petroleum."

Dr. Gilpin has presented a somewhat extended report of work already accomplished, and has outlined his plans for future investigations.

During the year Professor James F. Norris has published his work on the "Structure of Triphenylmethyl," in aid of which a grant of $250 was made by the Warren Committee in 1908.

Reports have been received from Dr. Frederic Bonnet, Jr., Professor W. L. Jennings, and Dr. E. W. Washburn, regarding the progress of researches for which grants have been made from the Warren Fund. None of these researches is as yet sufficiently advanced to permit publication of results.

The Committee has suffered a severe loss during the past year in the removal by death of Dr. Leonard Parker Kinnicutt, for many years one of its most active members, and for the past few years its chairman. The members of the Committee desire to record their sense of appreciation of the efficient service which Dr. Kinnicutt rendered to the Academy, and their sense of personal loss in the termination of his helpful and genial companionship.

H. P. TALBOT, *Chairman.*

May 10, 1911.

REPORT OF THE PUBLICATION COMMITTEE.

Between May 1, 1910, and May 1, 1911, there were published six numbers of Volume XLV. (Nos. 16–21) and twenty-four numbers of Volume XLVI. of the Proceedings. The total publication amounted to 912 + iv pages, with twelve plates. Of these publications, six numbers (No. 18, Vol. XLV., and Nos. 4, 11, 19, 23, and 24 of Vol. XLVI.) have been authorized by the Rumford Committee to be published at the expense of the Rumford Fund.

There was available for use of the Committee on Publication an unexpended balance from last year of $444.49, together with an appropriation of $3,000, and an amount of $23.88 from sales of publications between March 4 and May 1, 1910, and $312.14 from sales between May 1, 1910, and March 3, 1911, — in all, $3,780.51 from the Publication Fund. Bills against this fund to the amount of $3,425.43 have been approved by the Chairman of the Committee, and have been submitted to the Treasurer. This leaves an unexpended balance of $354.68.

Bills aggregating $387.50 incurred in publishing Rumford papers, have been forwarded to the Rumford Committee.

G. W. PIERCE, *Chairman.*

May 10, 1911.

Report of the House Committee.

During the year the house has been occupied as usual, with the exception of the first floor. The building was abandoned on March 25th, and has since been demolished. A temporary office has been secured.

At the annual meeting, May 11, 1910, $1,200 was appropriated for House expenses. In March, 1911, the Meeting-room chairs were sold for $35, making $1,235 for use during the year.

Of this amount $1,182.07 has been expended, leaving a balance of $52.93 toward the expenses of the coming year.

The expenses of the coming year will not be great as heretofore, the largest amount will be that of rent of the present office on Boylston Street — $450, with a few incidentals.

A. G. Webster, *Chairman.*

May 10, 1911.

Financial Report of the Council.

The income for the year 1911–12, as estimated by the Treasurer, is as follows: —

General Fund	Investments	$2,742.13	
	Assessments	1,800.00	$4,542.13
Publication Fund	Appleton Fund	$631.06	
	Centennial Fund	2,334.17	$2,965.23
Rumford Fund	Investments		$2,940.53
Warren Fund	Investments		$336.88

The above estimates, less 5 per cent to be added to the capital, leave an income available for appropriation as follows: —

General Fund	$4,315.03
Publication Fund	2,816.97
Rumford Fund	2,793.50
Warren Fund	320.04

The following appropriations are recommended: —

General Fund.

House expenses	$600	
Library expenses	1,600	
Books, periodicals, and binding	1,200	
Expenses of meetings	200	
Treasurer's office	150	
General expenses	400	$4,150

PUBLICATION FUND.

Publication		$2,500

RUMFORD FUND.

Research	$1,000	
Periodicals, books and binding	250	
Publication	700	
To be used at discretion of Committee	800	$2,750

C. M. WARREN FUND.

Research		$300

On the suggestion of the Assistant Treasurer, the amount recommended for appropriation from the income of the General Fund for Books, periodicals and binding was reduced to $1,000, and the amount recommended for House expenses was raised to $800.

It was then

Voted, To appropriate for the purposes named the following sums: —

From the income of the General Fund	$4,150
From the income of the Publication Fund	2,500
From the income of the Rumford Fund	2,750
From the income of the Warren Fund	300

On motion of the Corresponding Secretary it was

Voted, That the assessment for the ensuing year be ten dollars ($10).

On the recommendation of the Rumford Committee it was

Voted, To award the Rumford Premium to Professor James Mason Crafts, for his researches in high temperature thermometry and the exact determination of fixed points on the thermometric scale.

On the motion of Dr. Bell, it was

Voted, That the Chairman of the House Committee and the Chairman of the Library Committee be constituted, with the President, a Building Committee, to have full power to act for the Academy in all that pertains to the new building, to have authority to add to their number as may seem desirable, and to report progress at each meeting of the Academy. A majority of the members shall constitute a quorum and meetings may be called at any time by any member observing occasion for it.

It was proposed by Professor W. M. Davis, that all meetings of the Academy may be Stated or Business meetings, and that nominations may be either read at a meeting or sent out in print with the notice for the meeting.

It was

Voted, To refer Mr. Davis's proposition to the Committee on the Revision of the Statutes.

The annual election resulted in the choice of the following officers and committees:

JOHN TROWBRIDGE, *President.*
ELIHU THOMSON, *Vice-President for Class I.*
HENRY P. WALCOTT, *Vice-President for Class II.*
JOHN C. GRAY, *Vice-President for Class III.*
EDWIN H. HALL, *Corresponding Secretary.*
WILLIAM WATSON, *Recording Secretary.*
CHARLES P. BOWDITCH, *Treasurer.*
A. LAWRENCE ROTCH, *Librarian.*

Councillors for Three Years.

HARRY W. TYLER, of Class I.
THOMAS A. JAGGAR, JR., of Class II.
ARTHUR FAIRBANKS, of Class III.

Finance Committee.

JOHN TROWBRIDGE,
ELIOT C. CLARKE,
FRANCIS BARTLETT.

Rumford Committee.

CHARLES R. CROSS, ARTHUR G. WEBSTER,
EDWARD C. PICKERING, ELIHU THOMSON,
ERASMUS D. LEAVITT, ARTHUR A. NOYES.
LOUIS BELL.

C. M. Warren Committee.

HENRY P. TALBOT, WILLIAM H. WALKER,
GREGORY P. BAXTER, ARTHUR A. NOYES,
CHARLES R. SANGER, GEORGE D. MOORE,
JAMES F. NORRIS.

The following Standing Committees were chosen: —

Publication Committee.

GEORGE W. PIERCE, of Class I.
WALTER B. CANNON, of Class II.
ALBERT A. HOWARD, of Class III.

Library Committee.

HARRY M. GOODWIN, of Class I.
SAMUEL HENSHAW, of Class II.
HENRY W. HAYNES, of Class III.

Auditing Committee.

HENRY H. EDES, WORTHINGTON C. FORD.

House Committee.

ARTHUR G. WEBSTER, A. LAWRENCE ROTCH,
LOUIS DERR.

Mr. Andrew McFarland Davis exhibited a specimen of the Chinese paper currency of about the date 1375, and explained what is known about the currency of that period.

The following papers were presented by title: —

"Anomalous Magnetization in Iron." By B. O. Peirce.

"The Electrical Inductivities of Certain Poor Conductors." By B. O. Peirce.

"On the von Waltenhofen Effect in Soft Iron Rings." By L. A. Babbitt. Presented by B. O. Peirce.

"Calanoid Copepoda from the Bermuda Islands." C. O. Esterly. Presented by E. L. Mark.

AMERICAN ACADEMY OF ARTS AND SCIENCES.

REPORT OF THE COUNCIL. — PRESENTED MAY 10, 1911.

BIOGRAPHICAL NOTICE.

REPORT OF THE COUNCIL.

Since the last report of the Council eighteen deaths have been reported: — five Resident Fellows, — Robert Amory, Leonard Parker Kinnicutt, Francis Cabot Lowell, Henry Pickering Bowditch, Thomas Wentworth Higginson; seven Associate Fellows, — Cyrus Ballou Comstock, William Price Craighill, William Wirt Howe, Melville Weston Fuller, George Park Fisher, Samuel Franklin Emmons, Charles Otis Whitman; six Foreign Honorary Members, — Friedrich von Recklinghausen, Samuel Henry Butcher, William Huggins, Robert Koch, Maurice Levy, Jacobus Henricus van't Hoff.

HENRY PICKERING BOWDITCH.

At the death of Henry Pickering Bowditch, on March 13, 1911, there passed away a man who, as soldier, scientist, and public-spirited citizen, had led a life of enviable usefulness and permanent achievement. He was born in Boston April 4, 1840, grandson of Nathaniel Bowditch, the distinguished writer on mathematics and navigation; and son of Ingersoll Bowditch, a merchant honored for integrity and generosity. Through his mother, he was descended from the bold and patriotic Col. Timothy Pickering, Secretary of State under Washington.

At the age of seventeen he entered Harvard College, and after four years of study received the A. B. degree in 1861. In the fall of that year he had begun, in the Lawrence Scientific School, the study of chemistry, but then the needs of his country appealed to him so strongly that he volunteered his services for the War. In November he was appointed second lieutenant of the First Massachusetts Cavalry. From January, 1862, when his regiment was sent to the front, until the close of the conflict, he was in active service. He took part in the battle of Secessionville, and his regiment was in the reserve at Fredericksburg. In June, 1862, he was commissioned first lieutenant, and in May, 1863, captain. Thereafter he was in the battles of Aldie, Culpepper, and Rapidan Station; and at New Hope Church, November 27, 1863, he was shot in the right forearm. During the winter of 1863–64 convalescence from his wound kept him from the field. In February, 1864, he was honorably discharged; but almost immediately he joined the service again, as major in the Fifth Massachusetts Cavalry (colored), and with that regiment entered Richmond April 3, 1865.

Major Henry L. Higginson has recorded his memory of Captain Bowditch as he then appeared, — a handsome, refined, and home-bred looking youth, often reserved and even unbending in his manner, but unflagging in his faithfulness and unflinching in his courage.

Except for the end to be sought, however, service in the army was unpleasant to him, and on June 3, 1865, soon after the close of the War, he resigned his command.

In the autumn of 1865, he resumed his studies at the Lawrence Scientific School and came under the inspiring influence of Professor Jeffries Wyman. Although later that fall he entered the Harvard Medical School, he continued to receive the stimulus of Professor Wyman's instruction by pursuing between terms under that famous teacher's direction the subject of comparative anatomy. In 1866 he received the A. M. degree, and in 1868 was graduated from the Medical School.

On receiving his medical degree Dr. Bowditch went abroad to study physiology. He was fortunate in coming into relations with two of the foremost physiologists in the last century, — Claude Bernard, in Paris, and Carl Ludwig, in Leipzig. Bernard did not influence him nearly so much as did Ludwig, who, besides being an enthusiastic investigator, was a most warm-hearted and lovable character. In the Leipzig laboratory at that time and shortly afterwards was a remarkable group of young men, including Mosso, Kronecker, Cyon, Brunton, and Lankester. These men who, as the years passed, took high positions in Switzerland, Italy, France and England, maintained throughout their lives the close friendships established by the delightful days in Leipzig.

Under Ludwig's leadership Dr. Bowditch experienced for the first time the joy and the thrill of scientific search and discovery. As a result he published two papers, one of them, on peculiarities of the activity of cardiac muscle, destined to be famous. In that paper he reported that cardiac muscle differs from other muscle in contracting each time with its full force or not at all — following the "all-or-nothing" law, as it has been called, — and further, that repeated uniform stimulation causes a "treppe" effect, an increasing vigor of contraction, so that the record of the response rises like a stair. The generalization that activity of an organ is favorable to further activity has grown out of this observation on the heart, made forty years ago.

In September, 1871, Dr. Bowditch returned to Boston from Leipzig, bringing with him, as his wife, Selma Knauth, the daughter of Franz Theodor and Fanny Elizabeth Knauth, at whose home he, with other American students, had been hospitably welcomed. On his return he became Assistant Professor of Physiology in the Harvard Medical School, and in October, 1871, began his service. Previous to that time Oliver Wendell Holmes had given, at the end of his lectures on anatomy, a half-dozen lectures on the functions of the body. In medical circles in Boston at that period there was little appreciation of the ideals of the investigator in scientific medicine; there was only one laboratory and that was the dissecting room. Dr. Bowditch had to create not only his own laboratory, but also his own atmosphere. He secured for his uses two rooms in the attic of the Medical School building on North Grove Street, and fitted them with physiological apparatus. That was the first physiological laboratory, for the use of students, in the United States.

These rooms might perhaps be better designated the first laboratory for experimental medicine established in this country, for every phase of experimental medical work was there represented. Charles S. Minot carried on investigations in general biology, J. Ott in experimental pharmacology, J. C. Warren in experimental pathology, G. Stanley Hall and W. F. Southard in experimental psychology, O. K. Newell in experimental surgery, and W. P. Lombard, J. J. Putnam assisted by William James, C. S. Minot, G. M. Garland, C. H. Williams, J. W. Warren, F. H. Hooper, and F. W. Ellis in physiological researches. The hospitality of the laboratory was unbounded; indeed the first careful work in bacteriology in this country was begun there. With Dr. Bowditch's inspiration every scientific interest of a complete modern medical school was stimulated. From the start the emphasis was on productive scholarship. In the preface to the first collection of papers published from the laboratory the announcement was made that the contributions were presented in a volume, "not from any exaggerated idea of their value and importance, but with the hope that, by calling attention to the facilities offered in the laboratory for original research, a greater number of workers may be encouraged to attempt the investigation of the many physiological problems now pressing for a solution."

Dr. Bowditch's own investigations were almost as varied as those of the men who worked with him. His training in the Leipzig school, which was characterized by the application of physical principles to bodily processes, gave full play to his unusual inventive faculties. Simultaneous records on a single kymograph were suggested by him while in Leipzig, and this suggestion is said to have first directed the attention of Ludwig to the young American's abilities. The contriving of the Bowditch clock was another product of the Leipzig experience. In the Harvard laboratory the invention of a new form of induction apparatus, with the secondary coil turning at various angles to the primary, and the devising of a new form of plethysmograph to register changes in the volume of organs, testify to his ingenuity.

That apparatus is a means to an end, however, he never forgot, and for many years he was engaged in researches of striking range and originality. Besides his investigations of the peculiar functions of cardiac muscle, he demonstrated the indefatigability of nerves, made interesting observations on the knee-jerk and conditions affecting it, performed experiments which showed the force of ciliary motion, studied the effects of different rates and intensities of stimulation on the action of vaso-motor nerves, conducted an exhaustive examination of the rate of growth of school-children, and collaborated with workers in still other fields of physiological science.

In the teaching of physiology Dr. Bowditch's instruction was marked by wide learning, clear discussion of controverted questions, cautious inference when convincing demonstration was not forthcoming, and orderly exposition. His lectures were unusually well illustrated, by methods which made lasting impressions. The sending of students to original sources for material for physiological theses was a notable contribution to educational procedure. The conferences at which the theses were read and the weekly quizzes were delightfully informal and conversational. For thirty-five years Dr. Bowditch continued his helpful relations to students of the Harvard Medical School. In 1876 he was made Professor of Physiology; and in 1903, when the George Higginson Professorship of Physiology was established, he was appointed first occupant of the chair.

Further service to physiology he performed by assisting to found the American Physiological Society, of which he was the second president. The character of the Society as an association of active investigators is largely the result of his initiative. His example and his frank appreciation of original work as it was newly reported at meetings of the Society were an important source of encouragement to young men beginning physiological investigation. The American Journal of Physiology also received his support from the time it was first suggested. Both the American Physiological Society and its journal have been of incalculable value in stimulating physiological and biochemical research in this country.

Dr. Bowditch's relations to medicine were not, however, confined to the advancement of physiology alone, taken even in the wide sense in which he regarded it. His first larger allegiance was to the Harvard Medical School. He was prominent in the movement during the late 70's to secure a new home for the School, and helped to obtain the necessary funds for the building occupied from 1882 to 1906, at the corner of Boylston and Exeter Streets. In 1883 he was persuaded that as Dean he would have opportunities of increased usefulness to medicine. During the next ten years while he held the deanship, important changes were brought to pass in medical education. Bacteriology was introduced as a regular study, a pioneer venture under his leadership. The four years' required course was adopted, another forward step which the Harvard Medical School was among the first to take. A further important innovation was the calling of men from other universities to assume positions in the School, — Dr. W. H. Howell came from Michigan to be Associate Professor of Physiology, and Dr. W. T. Councilman came from Johns Hopkins to be Professor of Pathology. Although Dr. Bowditch resigned from the deanship in 1893, he never ceased to be interested in the larger problems of medical instruction. During his later years he became a strong advocate of greater freedom of election in medical study. Two of his addresses, "Reform in Medical Education" and "The Medical School of the Future," are admirable statements of the principles of sound teaching.

He was one of the first to foresee the necessity of enlarging the Harvard Medical School and bringing it into closer relations to hospitals. He became an enthusiastic promotor of the plan to make an important medical center of the Longwood Group of medical institutions, and with Dr. John Collins Warren, was astonishingly successful in securing funds for the realization of that great vision.

Among the most valuable of the larger services to medicine which Dr. Bowditch performed was his defense of animal experimentation against repeated attempts to pass unreasonable legislative restrictions. The success of the medical profession in Massachusetts in overcoming the misguided zeal of ignorant agitators has given heart to the profession elsewhere. The methods which were here used in meeting the petitioners for hostile legislation are now being employed in other states. In an address on "The Advancement of Medicine by Research" Dr. Bowditch clearly exposed the methods of the antivivisectionists, and presented an illuminating statement of the great benefits which had been secured for mankind by animal experimentation.

In spite of his large interest in medical research and education, he maintained useful relations with public affairs non-professional in character. From 1877 to 1881 he was a member of the Boston School Committee. He was President of the Boston Children's Aid Society and helped to broaden its scope and importance. Between 1895 and 1902 he was happily engaged as trustee of the Boston Public Library. As a member of the Committee of Fifty on the Alcohol Problem, he submitted a report on the character of public school instruction regarding the effect of alcohol, and urged that that instruction should be made to accord with scientific fact.

For his important services he was widely honored. He was elected a fellow of the American Academy of Arts and Sciences in 1872. During the year 1877 he was its Recording Secretary, and from 1881 to 1883 a member of its Council. For twenty-two years he served on the Library Committee. He was also a member of the American Philosophical Society of Philadelphia, the National Academy of Sciences, and many other scientific bodies in this country. The Royal Society of Medicine and Natural Science of Brussels, the Academy of Science of Rome, and other foreign societies enrolled him among their members. The University of Cambridge made him Doctor of Science in 1898; and Edinburgh (1898), Toronto (1903), Pennsylvania (1904), and Harvard (1906) gave him the degree of Doctor of Laws.

Everyone who came in close contact with Dr. Bowditch was impressed by his rare combination of sure and sober judgment with vigorous will and readiness of action, — qualities which made him a natural leader. Furthermore his mind was fertile with ingenious and effective ways to secure the accomplishment of worthy ends. He was eminently single-minded; the matter in hand was always the important matter to be attended to. Persons who knew him well recall that he seldom spoke of the past, almost never of his experiences in the War, and rarely of his earlier researches. The forward look to the fulfilment of plans already started was characteristic of him to the last. These qualities of energetic leadership were tempered by unfailing courtesy, fairness, and good-will, and warmed by a delightful sense of humor. These lovable characteristics brought to him the friendship and lifelong devotion of the foremost men of medical science, as well as of his students and his associates in various activities. Friendship was to him a blessing to be cultivated. He rejoiced in having his friends with him at his beautiful home in Jamaica Plain, and in his summer camp in the Adirondacks, or in going to be with them. Comrades of his Leipzig days visited him thus, as well as Sir Michael Foster, Professor Mosso, and Professor Waller. And he frequently renewed associations with them in Europe.

Although during the years from 1906 to his death Dr. Bowditch's vigor and activity were more and more limited by the advances of a paralysing disease, his mind remained clear and he continued much interested, almost until the last, in plans he had outlined for the Medical School. In spite of the growing physical weakness which must have clouded for a time his energetic spirit during those years, Dr. Bowditch's life as a whole must be regarded as essentially a happy life, happy in durable achievements, happy in the affections of close friends, and in the tender devotion of his family. When it became necessary for him to face the inevitable, he accepted his fate with cheerful patience and with gentlest consideration for those who ministered to him. Perhaps as the end approached he recalled the words which he wrote in a memorial to his friend, Theodore Lyman, years before: —

"I remember, Mr. President, when a young man, looking around among the men of my generation and considering whose lot in life seemed to me, on the whole, the most enviable. I came to the conclusion that Theodore Lyman was, of all my acquaintances, the man for whom the future seemed to hold out the brightest promise.

"In vigorous health, with a personality — physical, mental, and moral — which endeared him to all who came in contact with him, happily married, with instincts and powers which led him to the highest callings, to the service of his country in the field and in legislative halls, with tastes for the study of the natural sciences and abundant means to gratify them, there seemed to be nothing lacking to make his life an ideally happy one.

"Then, when the shadow of a slow, insidious disease fell upon him it seemed for a time as if his life were but to afford another illustration of the old Greek saying that no man is to be judged happy before his death; but when I saw how bravely he met the advances of his enemy, and with what courageous cheerfulness he interested himself in the pursuits of his friends and in the active life around him in which he could no longer share, I could not help feeling that a happiness was reserved for him higher than any of which the Greek philosopher had dreamed or I, as a young man, had formed a conception — the happiness of knowing that by the force of his example he had helped to raise those who came under its influence to a higher and nobler life."

W. B. Cannon.

List of Publications by Dr. H. P. Bowditch.

1871 Über die Eigenthümlichkeiten der Reizbarkeit, welche die Muskelfasern des Herzens zeigen. Arb. a. d. physiol. Anst. zu. Leipz., 1871, 139–176. Also: Ber. d. k. sachs. Gesellsch. d. Wissensch. Math. phys. Kl., 1871

1872 Über die Interferenz des retardirenden und beschleunigenden Herznerven. Arb. a. d. physiol. Anst. zu. Leipz., 1872. 259–280.

1873 The lymph spaces in fasciæ. Proc. Am. Acad., Feb. 11, 1873.

1874 Dr. Bowditch and Minot, C. S. The influence of anæsthetics on the vasomotor centres. Boston M. & S. J., 1874, xc, 493–498.

1875 A new form of inductive apparatus. Proc. Am. Acad., Oct. 12, 1875.

1876 Force of ciliary motion. Boston M. & S. J., 1876, xcv, 159–164.

1877 The growth of children. 8th Annual Report of the State Board of Health of Mass., Boston, 1877, 275.

1878 Does the apex of the heart contract automatically? J. of Physiol., 1878, i, 104–107.

1879 The growth of children, a supplementary investigation, with suggestions in regard to methods of research. 10th Annual Report of the State Board of Health of Mass., Boston, 1879, 33–62.

1879–80 Dr. Bowditch and Garland, G. M. The effect of the respiratory movements on the pulmonary circulation. J. of Physiol., 1879–80, ii, 91–109.

1879 A new form of plethysmograph. Proc. Am. Acad., May 14, 1879.

1879–80 Physiological apparatus in use at the Harvard Medical School. J. of Physiol., 1879–80, ii, 202–205.

1881 The relation between growth and disease. Trans. Am. Med. Ass'n., 1881, xxxii, 371–377.

1880–82 Dr. Bowditch and Southard, W. F. A comparison of sight and touch. J. of Physiol., 1880–82, iii, 232–245. Also abstract of same.

1880–82 Dr. Bowditch and Hall, G. S. Optical illusions of motion. J. of Physiol., 1880–82, iii, 297–307. Abstract of above.

1882 Dr. Bowditch and Harris, F. A. On the collection of data at autopsies. Boston M. & S. J., 1882, cvii, 365.

1883 Dr. Bowditch and Warren, J. W. Plethysmographische Untersuchungen über die Gefässnerven der Extremitäten. Centralbl. f. d. med. Wissensch., 1883, xxi, 513.

1885 Note on the nature of nerve-force. J. of Physiol., 1885, vi, 133–135.

1886 What is nerve-force? Proc. Am. Ass'n. Adv. Sc., 1886, xxxv, 237–246.

1886 Dr. Bowditch and Warren, J. W. Plethysmographic experiments on the vasomotor nerves of the limbs. J. of Physiol., 1886, vii, 416–450.

1886 Vaso-motor nerves of the limbs. Abstract of foregoing paper. Proc. Am. Ass'n. Adv. Sc., 1886, xxxv, 270.
1887 The action of sulphuric ether on the peripheral nervous system. Am. J. Med. Sc., Apr., 1887.
1888 The reinforcement and inhibition of the knee-jerk. Boston M. & S. J., 1888, cxviii, 542–543.
1890 Dr. Bowditch and Warren, J. W. The knee-jerk and its physiological modifications. J. of Physiol., 1890, xi, 25–64.
1890 The physique of women in Massachusetts. 21st Annual Report of the State Board of Health of Mass., 1889, Boston, 1890, 287–304.
1890 Über den Nachweis der Unermüdlichkeit des Säugethiernerven. Arch. f. Physiol., 1890, 505–508.
1890 The growth of children studied by the method of percentile grades. Internat. Med. Congress, Berlin, 1890.
1891 The growth of children studied by Galton's method of percentile grades. 22d Annual Report of the State Board of Health of Mass., Boston, 1891, 479–522.
1894 Are composite photographs typical pictures? McClure's Mag. Sept., 1894.
1895 A card catalogue of scientific literature. Science, Feb. 15, 1895.
1896 The advancement of medicine by research. Boston M. & S. J., 1896, cxxxiv, 577–581. Proc. of Mass. Med. Soc., 1896.
1897 Memoir of Charles Edouard Brown-Sequard. (1817–1894.) Read before the National Academy, April, 1897.
1898 Reform in medical education. Science, Dec. 30, 1898; and Boston M. & S. J., 1898, cxxxix, 643–646.
1898 Apparatus for illustrating movements of the eye. Boston Soc. Med. Sci., June, 1898.
1900 The medical school of the future. Trans. of the 5th Congress of the American Physicians and Surgeons, May, 1900; and Philadelphia Medical Journal, May 5, 1900.
1904 The study of physiology. Univ. of Pa. Medical Bulletin, June 1904.

Two Resident Fellows have resigned.

New members elected are: — Resident Fellows, six; Associate Fellows, one.

The roll of the Academy now includes 194 Resident Fellows, 76 Associate Fellows, and 55 Foreign Honorary Members.

RUMFORD PREMIUM.

In conformity with the terms of the gift of Benjamin, Count Rumford, granting a certain fund to the American Academy of Arts and Sciences, and with a decree of the Supreme Judicial Court for carrying into effect the general charitable intent and purpose of Count Rumford, as expressed in his letter of gift, the Academy is empowered to make from the income of said fund, as it now exists, at any Annual Meeting, an award of a gold and a silver medal, being together of the intrinsic value of three hundred dollars, as a premium to the author of any important discovery or useful improvement in light or in heat, which shall have been made and published by printing, or in any way made known to the public, in any part of the continent of America, or any of the American islands; preference being always given to such discoveries as shall, in the opinion of the Academy, tend most to promote the good of mankind; and to add to such medals, as a further premium for such discovery and improvement, if the Academy see fit so to do, a sum of money not exceeding three hundred dollars.

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