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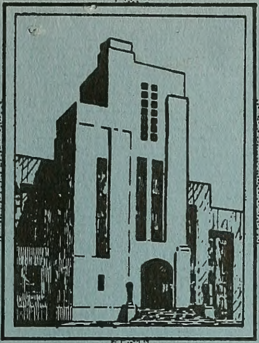

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SEA TESTS OF THE USCGC UNIMAK PART 2 - STATISTICAL PRESENTATION OF THE MOTIONS, HULL BENDING MOMENTS, AND SLAMMING PRESSURES FOR SHIPS OF THE AVP TYPE

by

N.H. Jasper, Dr. Eng., and R.L. Brooks, CDR, USN

STRUCTURAL MECHANICS LABORATORY
RESEARCH AND DEVELOPMENT REPORT

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NS 731-037

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ABSTRACT

The motions and hull-girder bending moments which a ship of the general form and size of the AVP10 Class may be expected to experience over a wide range of operating conditions are presented in statistical form. The data are based on extensive measurements made on the USCGC UNIMAK during sea trials in the North Atlantic Ocean. The methods of statistics have been employed in the planning of the at-sea phases of the trials and in the collection, analysis, and presentation of the large amount of data. From the test results, data are derived for this type of ship for use in design and operating problems involving bending moments, hull motions, and slamming pressures. Formulas are given for use in estimating probable maximum values of moments and motions.

INTRODUCTION

The Bureau of Ships initiated a long-range investigation of strains in ships at sea for the purpose of evaluating and improving methods for the design of the ship girder and its structural components.[1] Instrumentation has been developed and installed on various types of ships to collect information on the wave loads, stresses, and motions which ships experience in service. During the winter of 1954 and 1955, measurements were carried out on the USCGC UNIMAK (formerly AVP31) during operation in the North Atlantic Ocean. One of the main objectives of this work is the collection of sufficient data on ship motions and longitudinal hull-girder stresses to determine, by statistical methods, the frequency distributions of these quantities for different combinations of sea conditions, ship speed, and ship heading relative to the waves. For a complete background and general discussion of these trials, see Reference 2.

This report presents the distributions of the motions and bending moments* to be utilized for design purposes. To devise these distributions, it is necessary to specify the ship operations for which the vessel is to be designed. The term "mission" will be used here to define the ship's assigned operational pattern. One component of this mission is the aggregate of sea conditions under which the vessel must operate. It will be assumed that the ship will operate in the North Atlantic Ocean inasmuch as this probably represents more severe sea conditions than the vessel will actually experience and thus is on the safe side.

Accordingly, the probable speeds and headings at which these ships would be expected to operate under wartime conditions and the fraction of time the ships would spend at each of the various conditions were estimated by the skippers of a number of vessels of this class.

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[1]References are listed on page 42.

*The hull bending moments due to flexure in the longitudinal plane of the ship were deduced from the strain measurements and the section modulus applicable to the strain-gage location.

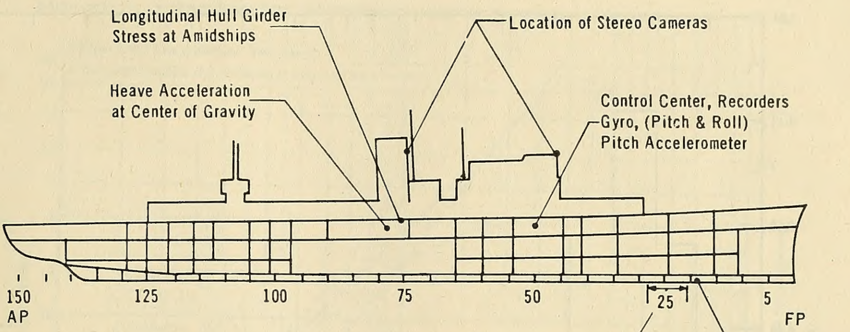
TABLE 1

Estimated Wartime Operating Conditions

The data for the WAVP vessels have been developed on the basis of a detailed analysis of ships' logs. For the AVP vessels data are based on estimates made by officers having experience in this type over a wide range of operating conditions. Values for individual ships were evaluated for mutual consistency and then averaged for each sea state and speed range. Sea states are defined in Reference 4.

Ocean	Ship Speed knots	Ship Reporting	Percentage of Time Operating at the Given Speed*							
			Sea State 2 Significant Wave Height 6 ft		Sea State 3 Significant Wave Height 7-9 ft		Sea State 4 Significant Wave Height 16 ft		Sea State 5 Significant Wave Height 21 ft	
Atlantic	7	WAVP370		19.7		0		6.2		47.9
		WAVP374		16.7		9.5		4.3		6.6
		WAVP378		16.5		30.0		22.0		14.8
		WAVP381		22.4		9.8		25.0		70.0
		WAVP382	15.74	13.5	15.01	14.6	17.75	14.7	35.53	32.6
		WAVP383	average	22.1	average	41.2	average	39.8	average	27.4
		AVP38		10.0		10.0		10.0		10.0
		AVP41		5.0		5.0		20.0		75.0
Pacific		COMAIRPAC	1 average		1 average		45 average		95 average	
Atlantic	10	WAVP370		13.9		0		19.2		17.8
		WAVP374		9.8		2.9		15.2		19.9
		WAVP378		15.9		8.9		31.4		13.7
		WAVP381	17.29	17.3	15.16	17.0	28.53	13.8	28.89	20.0
		WAVP382	average	10.4	average	4.9	average	9.5	average	34.8
		WAVP383		11.0		17.6		9.1		9.9
		AVP38		50.0		50.0		70.0		90.0
		AVP41		10.0		20.0		60.0		25.0
Pacific		COMAIRPAC	3 average		3 average		45. average		5. average	
Atlantic	14	WAVP370		26.6		29.2		23.8		24.7
		WAVP374		30.4		37.2		46.4		41.6
		WAVP378		48.7		45.3		38.1		58.9
		WAVP381		27.7		12.2		17.5		10.0
		WAVP382	35.17	23.0	37.95	67.9	28.54	36.6	22.58	21.2
		WAVP383	average	20.0	average	11.8	average	25.9	average	24.2
		AVP38		40.0		40.0		20.0		0
		AVP41		65.0		60.0		20.0		0
Pacific		COMAIRPAC	95. average		95. average		10 average		0 average	
Atlantic	17	WAVP370		39.8		70.8		50.8		9.6
		WAVP374		43.1		50.4		34.1		31.9
		WAVP378	31.80	18.9	31.88	15.8	25.18	8.5	13.00	12.6
		WAVP381	average	32.6	average	61.0	average	43.7	average	0
		WAVP382		53.1		12.6		39.2		11.4
		WAVP383		46.9		29.4		25.2		38.5
		AVP38		0		0		0		0
		AVP41		20.0		15.0		0		0
Pacific		COMAIRPAC	1 average		1 average		0		0	

* **For each ship, the percentages add up to 100 percent for each sea state.**



Midship Section Modulus (for location of strain gage)	11,000 ft-in.^{2}
Midship Section Moment of Inertia	761 ft^4
Block Coefficient	0.571
Midship Section Area Coefficient	0.972
Prismatic Coefficient	0.588
Waterplane Area Coefficient	0.703

Figure 1 - Profile and Characteristics of USCGC UNIMAK (AVP10-Class Vessel)

The information received from these officers is summarized in Table 1. These estimates were primarily based on an examination of ships' logs.

The sea conditions will be specified in terms of a significant wave height.* Estimates of the significant wave heights are made by weather observers stationed on a number of weather ships at various locations in the Atlantic Ocean. These observations have been made at 3-hr intervals since 1947. It has been found that the frequency distribution of these significant wave heights is approximately logarithmically normal.[3] The Weather Bureau's observations of significant wave heights have been utilized here to evaluate the sea conditions to be expected in the North Atlantic Ocean.

During the at-sea phases, oscillographic recordings were made of actual variations of roll and pitch angle, heave accelerations, and hull strains as the ship responded to wave-induced loads. In general, 1/2-hr continuous records were taken for each combination of ship speed, heading, and sea condition. Typical oscillograms are shown in Appendix A. Instruments were located as shown in Figure 1.

The pressures incident to slamming acting on the ship's bottom were measured by seven pressure gages installed on the UNIMAK.[5] Similar but more limited data were obtained during trials[6] of a sister ship, the USCGC CASCO.

*The significant wave height was obtained by averaging the observed highest wave in each of a number of groups of waves. Note that the term "significant height" as used here is not synonymous with the statistical meaning of "significant" value which is defined as the average of the upper third highest values.

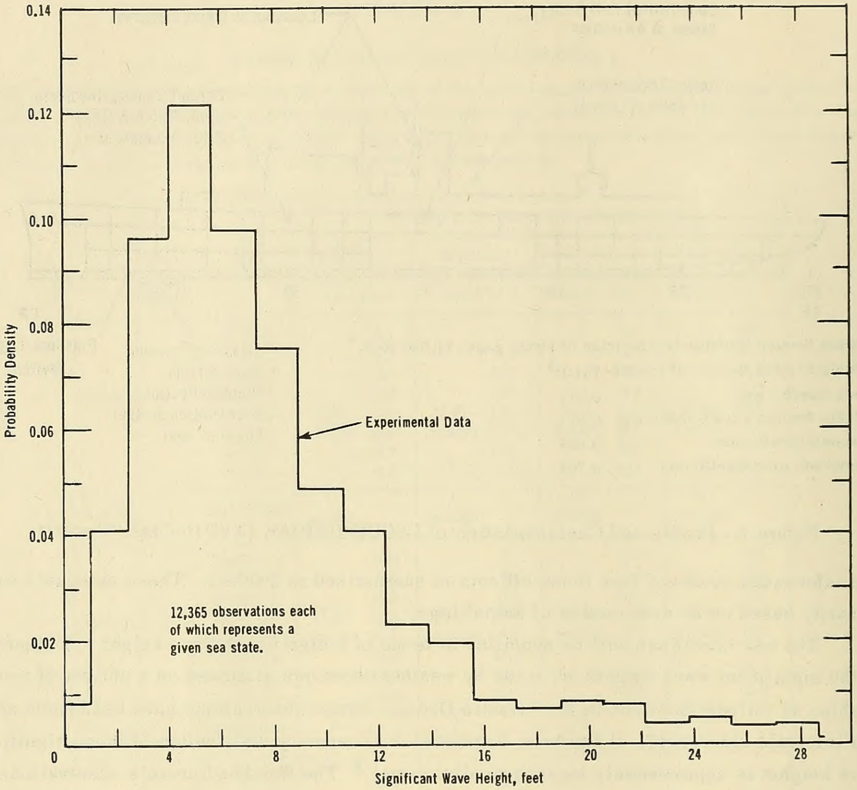


Figure 2a - Distribution Function

STATISTICAL BACKGROUND

The wave heights, ship motions, and hull bending moments experienced under a given set of conditions will be described or specified in terms of their distribution patterns or, mathematically speaking, their distribution functions.

For illustrative purposes, consider one of the variables, for example, wave height. All wave heights are considered to be members of a statistical "population." The distribution function (d.f.) of wave heights indicates the relative probability $p(x)$ of encountering a wave of a given height as a function of that height. Figure 2a illustrates this distribution function. (Similar illustrations are given for the ship motions in Figures 3 through 6.) The area under the curve up to a value x_i is the integral of the d.f. up to the value $x = x_i$; it is equal to the fraction of all members of the population of wave heights which have a height less than x_i.

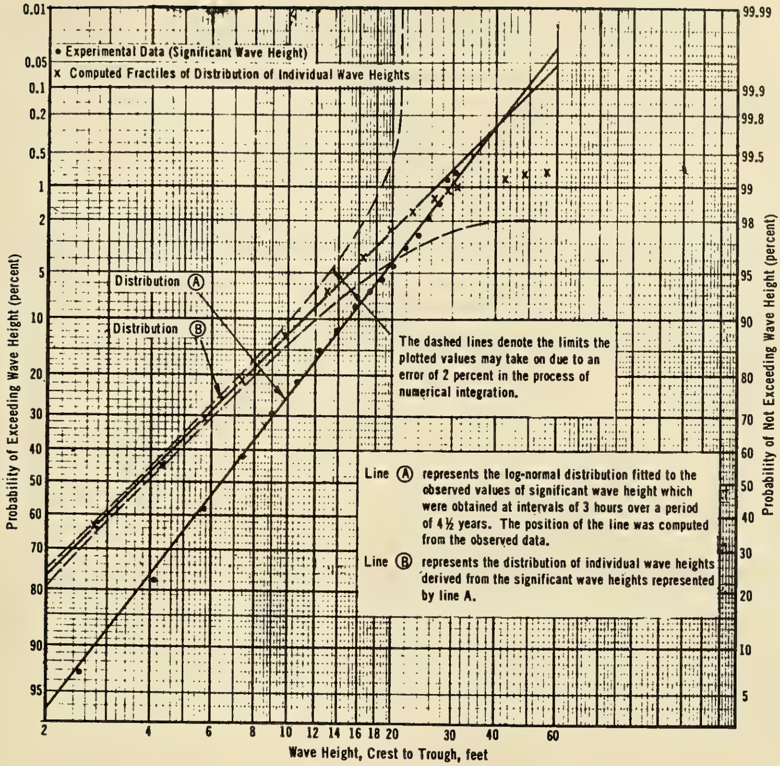


Figure 2b - Cumulative Distribution Function

Figure 2 - Distribution of Heights of Ocean Waves at Weather Station C, 52° N 37° W, North Atlantic Ocean

This distribution is based on 12,365 observations made over a period of 4½ years by U.S. Weather Bureau personnel.

Mathematically

$$P(x) = \int_0^x p\,dx \text{ and } P(x \to \infty) = \int_0^\infty p\,dx = 1 \qquad [1]$$

P is a function of x, and this function is designated as the cumulative distribution function (c.d.f.) of x. $P(x)$ is numerically equal to the probability that a value chosen at random from the population is less than x.

A discussion of the statistical methods utilized here is given in References 3 and 7.

There is considerable evidence[3] to indicate that the distribution of wave heights corresponding to any *one* given sea condition is of the one-parameter type known as the Rayleigh distribution which is defined as

$$P(x) = 1 - e^{-x^2/E}$$

where E is independent of x. Thus the probability is defined by a single number* E. On the other hand, when the heights of all waves experienced over a long period of time, say over several years, are considered, then the evidence indicates that the logarithm of the wave height is approximately normally distributed, that is, the two-parameter log-normal distribution describes the situation. The log-normal distribution is defined as follows:

$$p(\log x)\, d(\log x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\log x - \mu)^2}{2\sigma^2}} d(\log x)$$

where u is the mean value of log x and σ is the standard deviation of log x.

Reference 3 shows that these two types of distributions also describe the response of the ship to the waves. For the sake of brevity, the distributions applicable to homogeneous conditions of the sea, ship speed, and course will be called "short-term" distributions, whereas the function which represents the distribution when the seas, ship speeds, and courses are allowed to vary over a range of conditions, will be designated as "long-term" distributions.

The distribution pattern will, at a glance, give the probability of exceeding any given magnitude of motion or stress. It also can be applied to the prediction of the largest magnitude to be expected in a given number of variations. For application to design for endurance strength, the distribution pattern can be utilized as a load spectrum. Illustration of these applications will be given in a later section.

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*E is the mean value of x^2.

DERIVATION OF DISTRIBUTIONS OF SHIP MOTIONS AND LONGITUDINAL BENDING MOMENTS OF THE HULL

It will be assumed without further discussion that the short-term distribution of wave-induced ship motions and stresses may be represented by the one-parameter Rayleigh distribution and that the corresponding long-term distributions are approximated by the two-parameter log-normal distribution. Evidence to support these hypotheses is presented in Reference 3.

Typical distribution patterns of variation* in pitch angle are shown in Figures 3 through 6. In all, 129 similar sets were analyzed. Pertinent results are given in Tables 2 through 6 for variations of pitch angle, pitch acceleration, roll angle, heave acceleration, and the hull girder stress in the main deck amidships due to bending of the ship in a longitudinal plane normal to the deck.

It is interesting to note that all cumulative Rayleigh distributions (for example, those shown in Figures 4 and 6) become identical if $v^2 = x^2/E$ is plotted against the probability instead of plotting x directly. Utilizing this artifice it is necessary to know only the value of E corresponding to a particular sea condition, ship speed, and heading in order to obtain the probability of exceeding any value of x from a single graph (Figure 4) which is equally applicable to wave heights, ship motions, and hull stresses. The values of E for various ship operations are given in Tables 2 through 6. Table 7 gives factors which, together with the value E, permit making statistical predictions as discussed later.

We now proceed to utilize the short-term distributions, each of which is characterized by a value of E, as building blocks in order to construct the long-term frequency distribution patterns of the ship responses to the sea applicable to *wartime service in the North Atlantic Ocean*. (It should be noted that the distribution patterns for other "missions" can be readily computed from the data given in this report.) Each of these short-term distributions will be weighted in accordance with the relative fraction of time spent at given sea state (f_2), at the given heading to the sea (f_3), and at the given ship speed (f_1). For example, if tests have indicated that the ship will experience $N = 480$ pitch variations per hour in a State 2 sea when heading directly into the waves at a speed of 10 knots, then one may expect that $n = f_1 f_2 f_3 N$ = (0.33) (0.34) (0.125) 480 = 6.73 variations of pitch angle per hour, out of the average number of variations per hour, can be attributed to this set of environmental conditions over an average year's operation in the assigned mission.

These calculations are carried out in Tables 8 through 11. Each horizontal line in these tables gives the data corresponding to a given set of environmental conditions. The probabilities $(1-P)$ of exceeding given values of pitch angle, etc., are computed and tabulated in columns 10 through 18. The total number of variations per hour which, over the average

*Throughout this report, a variation is taken to mean the peak-to-peak variation of the variable.

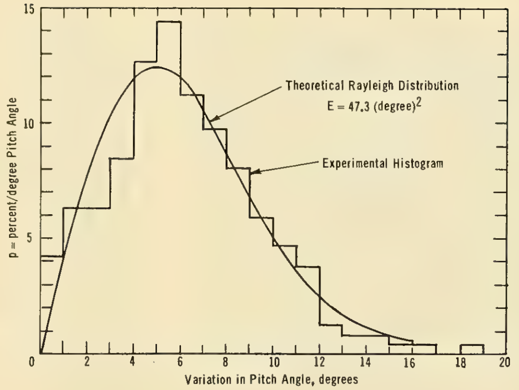


Figure 3 - Distribution of Variation in Pitch Angle (Sample 1) for USCGC UNIMAK

Conditions at time of test were: State 5 sea, head seas, ship speed 7½ knots, significant wave height 21 ft. The ship pitched at a rate of about 330 variations per hour. The test sample consisted of 236 variations in pitch angle. This sample is representative of 129 samples that were analyzed. Each such sample has been developed from analysis of oscillograms covering continuous periods of one-half hour or thereabout.

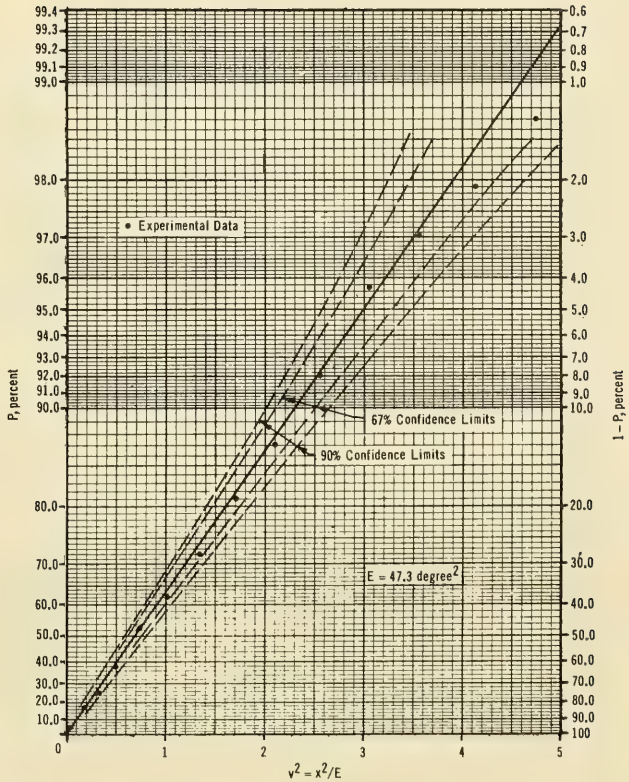


Figure 4 - Cumulative Distribution of Variation in Pitch Angle (Sample 1) for USCGC UNIMAK

Conditions at time of test were: State 5 sea, head seas, ship speed 7½ knots.

See Figure 3 for corresponding histogram. The plotted confidence limits do not include an allowance for errors of measurement.

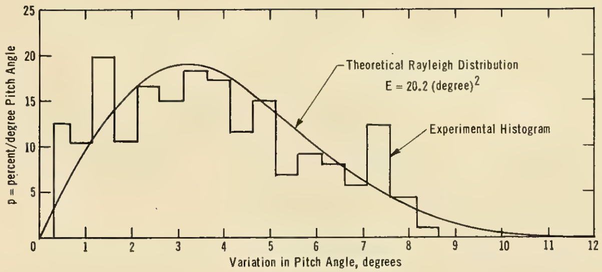


Figure 5 - Distribution of Variation in Pitch Angle (Sample 2) for USCGC UNIMAK

Conditions at time of test were: State 5 sea, quarter following seas, ship speed 7½ knots, significant wave height 21 ft. The ship pitched at a rate of about 370 variations per hour. The test sample consisted of 175 variations in pitch angle. This sample the poorest fit to the Rayleigh distribution of the 129 samples analyzed.

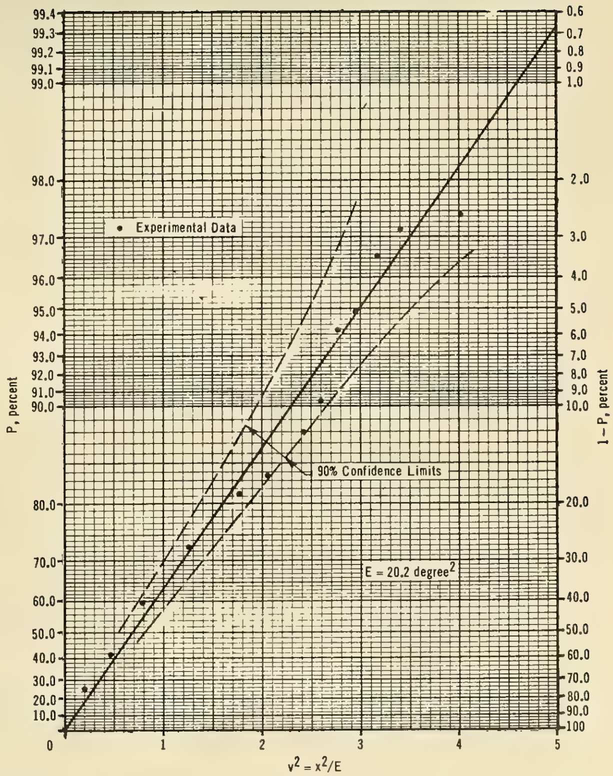


Figure 6 - Cumulative Distribution of Variation in Pitch Angle (Sample 2) for USCGC UNIMAK

Conditions at time of test were: State 5 sea, quarter following seas, ship speed 7½ knots. See Figure 5 for corresponding histogram.

year, will exceed each given level are obtained by summing the product of column 9 with columns 10 through 18 over all environmental conditions. The last line in the table gives the probability of exceeding any one of the given magnitudes, for the long-term distribution. The latter values are plotted on the cumulative probability distribution charts in Figures 7 through 10.

The straight lines shown on these charts have *not* been drawn by eye through the plotted points but have been computed directly from the percentages represented by the plotted points under the assumption that the long-term distribution is of the log-normal type. A sample calculation is given in Appendix B. The rather good fit of the computed line to the plotted points indicates that this assumption is reasonable. One would expect that the points corresponding to the more extreme values would lie above the theoretical line because by far the greatest contribution to the computed probability for these extreme values derives from the more severe sea conditions. It is apparent that if data had been available for more severe seas than State 5, the probabilities of exceeding the higher values would have been increased whereas the plotted points representing probabilities of exceeding low or medium large values would not have been affected to any noticeable extent.

The value of E corresponding to any short-term distribution may readily be used to predict the most probable maximum value of the motion or stress expected in any given number of oscillations. Longuet-Higgins[8] has shown that the largest probable value out of N measurements is $\sqrt{E}$ times a constant if the population is of the Rayleigh type, where the constant is a function of N only. For large values of N, the constant is nearly equal to $\sqrt{\log_e N}$. Table 7 gives the value of the constant by which $\sqrt{E}$ must be multiplied. A comparison of predicted and measured maximum values, utilizing this method, is given in Tables 2 through 6. There appears to be a satisfactory agreement.

The wave-induced hull-girder stresses can be converted to the corresponding vertical bending moments amidships by making use of the midship section modulus which is applicable to the strain-gage location (23.8 ft above baseline, 10 ft above the location of the neutral axis). Tests have indicated [2,6] that the deckhouse of the AVP vessel is fully effective in resisting bending, thus resulting in a section moment of inertia of 761 ft^4 which corresponds to a section modulus applicable to the strain-gage location of 11,000 ft-in^2. This value of the section modulus has been used to convert wave induced stresses to wave-induced bending moments.

TABLE 2

Basic Statistical Data on Pitch Angles

Sea State (Est'd)	Significant Wave Height ft	Heading of Waves Relative to Ship	Ship Speed knots*	N Number of Variations per Hour	Minutes Sampled	E deg^2	Predicted Maximum Value for 1-hr Operation	Maximum Measured Peak-to-Peak Variation deg	Number of Variations in Sample from Which Maximum Was Obtained	Predicted Maximum Peak-to-Peak Variation	Ratio Predicted Maximum to Measured Maximum
			7-7 1/2								
2	6	Head	10	480	30	4.00	5.0	4.8	240	4.68	0.98
		Seas	14	555	32	4.48	5.3	5.3	296	5.04	0.95
			17								
			7-7 1/2								
2	6	Quarter	10	404	37	1.97	3.4	3.1	249	3.3	1.06
		Head	14	514	32	1.86	3.5	3.3	274	3.24	0.98
		Seas	17								
			7-7 1/2								
2	6	Beam	10	561	29 1/2	5.40	5.9	6.2	276	5.5	0.89
		Seas	14	643	32	3.75	4.9	4.8	343	4.69	0.98
			17								
			7-7 1/2								
2	6	Quarter	10								
		Following	14	583	29	1.68	3.3	3.5	282	3.08	0.88
		Seas	17								
			7-7 1/2								
2	6	Following	10								
		Seas	14	578	32	1.86	3.5	3.8	308	3.27	0.86
			17								
			10	583	26	10.35	8.1	7.6	253	7.56	0.99
3	7-9	Head	14	603	32	9.62	7.8	7.3	321	7.49	1.03
		Seas	17	616	27	9.59	7.9	6.7	277	7.34	1.09
3	7-9	Quarter	10	587	27 1/2	1.79	3.4	3.1	269	3.16	1.02
		Head	14								
		Seas	17	592	37	2.02	3.6	3.9	365	3.45	0.89
			10	542	29	3.52	4.7	4.3	262	4.44	1.03
3	7-9	Beam	14								
		Seas	17	298	26	1.08	2.5	3.3	129	2.29	0.70
		Quarter	10								
3	7-9	Following	14								
		Seas	17	523	31	1.85	3.4	3.9	270	3.16	0.85
3	7-9	Following	14								
		Seas	17	504	23	1.28	2.8	2.4	193	2.59	1.08
		Head									
4	16	Seas	14	578	30	26.12	12.9	11.6	289	12.1	1.00
		Head	7 1/2	517	27 1/2	47.32	17.2	18.0	236	16.1	0.90
5	21	Seas	10								
			14	583	24	56.00	18.9	15.5	235	17.4	1.16
		Quarter	7 1/2	541	38 1/2	35.13	14.9	13.9	347	14.3	1.03
5	21	Head	10								
		Seas	14	577	28 1/2	47.14	17.3	16.5	274	16.3	0.99
5	21	Beam	7 1/2	364	30	16.78	9.9	8.8	182	9.34	1.06
		Seas									
		Quarter	7 1/2	368	28 1/2	20.24	10.9	9.8	175	10.2	1.04
5	21	Following	10	296	29	28.00	12.6	11.5	143	11.8	1.02
		Seas	14	302	27	17.2	9.9	8.8	136	9.16	1.04
		Following	7 1/2	291	33	31.12	13.3	11.8	160	12.58	1.06
5	21	Seas	10								
			14	273	31	20.54	10.8	10.8	140	10.1	0.94

*The speed is the nominal speed read from a calibration curve of propeller rpm versus knots; 7 1/2, 10, 14, and 17 knots correspond to 94, 127, 185, and 230 rpm, respectively.

TABLE 3

Basic Statistical Data on Pitch Accelerations

Sea State (Est'd)	Significant Wave Height ft	Heading of Waves Relative to Ship	Ship Speed knots*	N Number of Variations per Hour	Minutes Sampled	E $\left(\frac{rad}{sec^2}\right)^2$	Predicted Maximum Value for 1-hr Operation	Maximum Measured Peak-to-Peak Variation rad/sec^2	Number of Variations in Sample from Which Maximum Was Obtained	Predicted Maximum Peak-to-Peak Variation	Ratio Predicted Maximum to Measured Maximum
2	6	Head Seas	10	704	30	0.0015	0.099	0.093	352	0.093	1.00
			14	786	32	0.0024	0.126	0.123	419	0.121	0.99
2	6	Quarter Head Seas	10	761	32	0.0015	0.100	0.093	406	0.095	1.02
			14	872	32	0.0014	0.097	0.097	465	0.093	0.960
2	6	Beam Seas	10	708	29 1/2	0.0033	0.147	0.144	348	0.139	0.97
			14	740	32	0.0033	0.148	0.140	395	0.140	1.00
2	6	Quarter Following Seas	14	890	29	0.0016	0.104	0.11	430	0.099	0.90
2	6	Following Seas	14	635	32	0.00028	0.043	0.047	339	0.0404	0.86
3	7-9	Head Seas	10	694	26	0.0109	0.267	0.28	301	0.250	0.90
			14	765	32	0.0161	0.327	0.322	396	0.310	0.97
			17	862	27	0.0171	0.341	0.357	368	0.319	0.90
3	7/9	Quarter Head Seas	10	779	27 1/2	0.0022	0.121	0.13	357	0.114	0.88
			17	900	37	0.0028	0.138	0.15	555	0.133	0.89
3	7-9	Beam Seas	10	763	29	0.0030	0.142	0.161	369	0.133	0.83
			17	760	26	0.0029	0.139	0.174	329	0.130	0.75
3	7-9	Quarter Following Seas	17	696	31	0.001	0.081	0.075	338	0.076	1.00
3	7-9	Following Seas	17	629	23	0.00072	0.068	0.064	241	0.063	0.98
4	16	Head Seas	14	780	30	0.0222	0.384	0.39	390	0.35	0.90
5	21	Head Seas	7 1/2	624	27 1/2	0.0173	0.334	0.425	284	0.341	0.80
			14	692	24	0.0308	0.450	0.47	278	0.42	0.89
5	21	Quarter Head Seas	7 1/2	615	38 1/2	0.0140	0.300	0.314	395	0.29	0.92
			14	711	28 1/2	0.0327	0.465	0.47	338	0.437	0.93
5	21	Beam Seas	7 1-2	458	30	0.0039	0.155	0.161	229	0.145	0.90
5	21	Quarter Following Seas	7 1/2	702	28 1/2	0.0032	0.145	0.146	333	0.136	0.93
			10	660	29	0.0023	0.123	0.129	319	0.115	0.89
			14	714	27	0.0023	0.123	0.131	322	0.115	0.88
5	21	Following Seas	7 1/2	524	33	0.0013	0.090	0.068	288	0.085	1.25
			14	470	31	0.0004	0.050	0.058	243	0.047	0.81

*The speed is the nominal speed read from a calibration curve of propeller rpm versus knots; 7 1/2, 10, 14, and 17 knots correspond to 94, 127, 185, and 230 rpm, respectively.

TABLE 4

Basic Statistical Data on Roll Angles

Sea State (Est'd)	Significant Wave Height ft	Heading of Waves Relative to Ship	Ship Speed knots*	N Number of Variations per Hour	Minutes Sampled	E deg^2	Predicted Maximum Value for 1-hr Operation	Maximum Measured Peak-to-Peak Variation deg	Number of Variations in Sample from Which Maximum Was Obtained	Predicted Maximum Peak-to-Peak Variation	Ratio Predicted Maximum to Measured Maximum
2	6	Head Seas	10	380	30	19.0	10.7	9.4	190	9.96	1.06
			14	362	32	18.3	10.4	10.5	193	9.80	0.93
2	6	Quarter Head Seas	10	355	37	31.0	13.5	13.1	219	12.9	0.99
			14	340	32	26.0	12.4	12.2	191	11.7	0.96
2	6	Beam Seas	10	388	29 1/2	17.8	10.3	10.6	191	9.66	0.91
			14	368	32	19.0	10.6	10.3	196	10.0	0.97
2	6	Quarter Following Seas	14	335	29	24.0	11.9	10.9	172	11.1	1.02
2	6	Following Seas	14	326	32	21.0	11.0	10.3	174	10.4	1.01
3	7-9	Head Seas	10	436	26	20.6	11.2	10.6	189	10.4	0.98
			14	400	32	17.5	10.3	9.7	213	9.66	1.00
			17	448	27	9.8	7.75	7.2	202	7.23	1.00
3	7-9	Quarter Head Seas	10	209	27 1/2	6.5	6.0	6.0	96	5.45	0.91
			17	339	37	7.5	6.6	6.7	209	6.34	0.95
3	7-9	Beam Seas	10	260	30	6.9	6.2	8.7	130	5.8	0.67
			17	482	26	13.0	9.0	9.2	209	8.35	0.91
3	7-9	Quarter Following Seas	17	285	31	19.3	10.5	10.9	147	9.8	0.90
3	7-9	Following Seas	17	355	23	15.8	9.6	9.1	136	8.81	0.97
4	16	Head Seas	14	348	30	7.8	6.8	6.9	174	6.35	0.92
5	21	Head Seas	7 1/2	369	27 1/2	119.	26.6	24.4	168	24.7	1.01
			14	398	24	84.8	22.5	20.6	160	20.7	1.01
5	21	Quarter Head Seas	7 1/2	388	38 1/2	75.3	21.2	18.1	249	21.0	1.16
			14	408	28 1/2	64.3	19.8	23.0	194	18.4	0.80
5	21	Beam Seas	7 1/2	364	30	171.	31.8	25.0	182	29.9	0.84
5	21	Quarter Following Seas	7 1/2	384	28 1/2	176.	32.4	38.8	182	30.2	0.78
			10	372	29	148.	29.6	24.0	180	27.8	1.15
			14	360	27	114.	25.9	21.9	162	24.1	1.10
5	21	Following Seas	7 1/2	356	33	150.	29.6	24.4	196	28.1	1.15
			14	340	31	60.	18.8	18.5	176	17.6	0.95

*The speed is the nominal speed read from a calibration curve of propeller rpm versus knots; 7 1/2, 10, 14, and 17 knots correspond to 94, 127, 185, and 230 rpm, respectively.

TABLE 5

Basic Statistical Data on Stresses

Sea State (Est'd)	Significant Wave Height ft	Heading of Waves Relative to Ship	Ship Speed knots*	N Number of Variations per Hour	Minutes Sampled	E $\left(\frac{\text{kips}}{\text{in.}^2}\right)$	Predicted Maximum Value for 1-hr Operation kips	Maximum Measured Peak-to-Peak Variation kips/in.^{2}	Number of Variations in Sanple from Which Maximum Was Obtained	Predicted Maximum Peak-to-Peak Variation	Ratio Predicted Maximum to Measured Maximum
2	6	Head Seas	10	670	30	0.22	1.20	1.2	335	1.13	0.94
			14	784	32	0.31	1.44	1.5	418	1.37	0.91
2	6	Quarter Head Seas	10	256	37	0.21	1.08	1.2	158	1.03	0.86
			14	1004	32	0.24	1.30	1.3	536	1.22	0.94
2	6	Beam Seas	10	522	29 1/2	0.37	1.52	1.6	257	1.43	0.89
			14	666	32	0.29	1.38	1.7	355	1.31	0.77
2	6	Quarter Following Seas	14	640	27	0.22	1.19	1.5	288	1.11	0.74
2	6	Following Seas	14	392	32	0.36	1.46	1.4	209	1.39	0.99
3	7-9	Head Seas	10	621	26	1.12	2.70	2.6	269	2.5	0.96
			14	778	32	1.28	2.92	2.8	414	2.84	1.01
			17	991	27	1.21	2.88	2.9	446	2.72	0.94
3	7-9	Quarter Head Seas	10	864	27 1/2	0.11	0.86	0.9	396	0.81	0.90
			17	711	37	0.16	1.02	1.0	438	0.985	0.99
3	7-9	Beam Seas	10	503	29	0.19	1.09	0.94	243	1.02	1.08
			17	685	26	0.18	1.09	0.97	297	1.01	1.04
3	7-9	Quarter Following Seas	17	459	31	0.15	0.96	1.0	237	0.905	0.91
3	7-9	Following Seas	17	360	23	0.09	0.72	0.59	138	0.666	1.13
4	16	Head Seas	14	768	30	1.17	2.8	3.1	384	2.64	0.85
5	21	Head Seas	7 1/2	581	27 1/2	2.81	4.2	3.9	265	3.96	1.02
			14	790	24	3.50	4.8	4.5	318	4.48	1.00
5	21	Quarter Head Seas	7 1/2	598	38 1/2	2.34	3.9	3.9	384	3.73	0.96
			14	694	28 1/2	3.00	4.4	4.0	330	4.16	0.96
5	21	Beam Seas	7 1/2	532	30	1.3	2.8	2.7	266	2.69	1.00
5	21	Quarter Following Seas	7 1/2	449	28	0.86	2.3	2.0	212	2.14	0.94
			10	328	29	1.12	2.5	2.3	159	2.38	1.03
			14	344	27	1.01	2.4	2.3	155	2.25	0.98
5	21	Following Seas	7 1/2	325	33	1.90	3.3	3.6	179	3.14	0.87
			14	300	31	1.57	3.0	3.1	155	2.82	0.91

*The speed is the nominal speed read from a calibration curve of propeller rpm versus knots; 7 1/2, 10, 14, and 17 knots correspond to 94, 127, 185, and 230 rpm, respectively.

TABLE 6

Basic Statistical Data on Heave Accelerations

Sea State (Est'd)	Significant Wave Height ft	Heading of Waves Relative to Ship	Ship Speed knots*	N Number of Variations per Hour	Minutes Sampled	E $g's^2$	Predicted Maximum Value for 1-hr Operation g's	Maximum Measured Peak-to-Peak Variation g's	Number of Variations in Sample from Which Maximum Was Obtained	Predicted Maximum Peak-to-Peak Variation	Ratio Predicted Maximum to Measured Maximum
3	7-9	Head Seas	10	448	26	0.0107	0.252	0.26	239	0.251	0.97
			14	590	32	0.0186	0.35	0.32	315	0.321	1.01
			17	582	27	0.0221	0.37	0.37	262	0.35	0.95
5	21	Head Seas	7 1/2	433	27	0.0221	0.37	0.51	197	0.341	0.67
			10	518	27	0.0272	0.41	0.41	233	0.385	0.94
			14	514	24 1/2	0.0382	0.49	0.49	210	0.451	0.92
5	21	Quarter Head Seas	7 1/2	466	38 1/2	0.0182	0.33	0.29	299	0.322	1.11
			10	583	28	0.0245	0.40	0.44	272	0.371	0.84
			14	565	27 1/2	0.0498	0.57	0.62	259	0.525	0.85
5	21	Beam Seas	7 1/2	433	28	0.0143	0.29	0.26	202	0.276	1.06
5	21	Quarter Following Seas	7 1/2	418	32	0.0147	0.30	0.35	223	0.282	0.81
			10	552	30	0.0131	0.29	0.34	226	0.267	0.78
			14	504	27	0.0135	0.29	0.32	227	0.270	0.84

*The speed is the nominal speed read from a calibration curve of propeller rpm versus knots; 7 1/2, 10, 14, and 17 knots correspond to 94, 127, 185, and 230 rpm, respectively.

TABLE 7

Constants Required for Prediction of Probable Maximum Value in a Sample from a Rayleigh Distribution

Let x_{max} be the most probable maximum value of x taken from a sample containing N values of x. Then, according to Longuet-Higgins[6] $x_{max} = \sqrt{\theta}\sqrt{E}$ where $\theta = \log_e N - \log_e \left[1 - \frac{1}{2\theta}\left(1 - e^{-\theta}\right)\right]$. For large N, $\theta \approx \log_e N$.

Sample Size N	$\sqrt{\theta}$	Sample Size N	$\sqrt{\theta}$
1	0.707	1000	2.642
2	1.030	2000	2.769
5	1.366	5000	2.929
10	1.583	10,000	3.044
20	1.778	20,000	3.155
50	2.010	50,000	3.296
100	2.172	100,000	3.400
200	2.323		
500	2.509		

TABLE 8

Derivation of Predicted Distribution Pattern for Variations in Pitch Angle for Wartime Duty in North Atlantic Ocean

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Sea State	Significant Wave Height*	Relative Heading of Waves	Ship Speed**	Weighting Factors†				Average Number of Variations per Hour Contributed by Each Operating Condition	Probability of Exceeding Given Magnitude of Variation†† (1-P)									E (deg)^2 Mean Square Variation
	ft		knots	f_1	f_2	f_3	N	$f_1 f_2 f_3 N = n$	½°	1°	2°	4°	6°	8°	10°	14°	18°	
	<4	All	All		0.20			111.56	0.795	0.397	0.0247							1.08 (estimated)
2	6	H	10^{7}	0.33	0.34	0.125	430	16.73	0.94	0.78	0.368	0.0184	0.124^{-3}	0.111^{-6}	0.014^{-9}	0.053^{-20}		4.00
		H	14^{17}	0.67		0.125	555	15.80	0.94	0.79	0.410	0.0284	0.333^{-3}	0.611^{-6}	0.204^{-9}	0.715^{-20}		4.48
		QH	10^{7}	0.33		0.25	404	11.33	0.88	0.60	0.134	0.00030						1.97
		QH	14^{17}	0.67		0.25	514	29.28	0.87	0.59	0.117	0.00018						1.86
		B	10^{7}	0.33		0.25	561	15.74	0.96	0.83	0.476	0.0518	1.29^{-3}	7.22^{-6}	9.0^{-9}	1.69^{-20}		5.40
		B	14^{17}	0.67		0.25	643	36.62	0.94	0.77	0.344	0.0139	0.068^{-3}	0.039^{-6}				3.75
		QF	$14^{7,10,17}$	1.00		0.25	583	49.56	0.86	0.55	0.093	0.0001						1.68
		F	$14^{7,10,17}$	1.00		0.125	578	24.57	0.87	0.59	0.117	0.0002						1.86
3	9	H	10^{7}	0.30	0.29	0.125	583	6.34	0.976	0.90	0.680	0.213	0.0310	0.206^{-2}	0.638^{-4}	0.219^{-8}		10.35
		H	14	0.38		0.125	603	8.32	0.975	0.90	0.659	0.189	0.0234	0.130^{-2}	0.304^{-4}	0.138^{-8}		9.62
		H	17	0.32		0.125	616	6.71	0.975	0.90	0.658	0.189	0.0235	0.128^{-2}	0.299^{-4}	0.136^{-8}		9.59
	7	QH	10^{7}	0.30		0.25	587	12.8	0.870	0.57	0.107	0.0001						1.79
		QH	17^{14}	0.70		0.25	592	30.07	0.882	0.61	0.138	0.0004						2.02
		B	10^{7}	0.30		0.25	542	11.82	0.931	0.75	0.322	0.011						3.52
		B	17^{14}	0.70		0.25	298	15.14	0.795	0.40	0.025							1.08
		QF	$17^{7,10,14}$	1.00		0.25	523	37.92	0.872	0.58	0.115	0.0002						1.85
		F	$17^{7,10,14}$	1.00		0.125	504	18.30	0.822	0.46	0.044							1.28
4	16	H	$14^{7,10,17}$	1.00	0.11	1.00	578	63.58	0.99	0.96	0.860	0.532	0.252	0.0862	0.0217	0.550^{-3}	0.420^{-5}	26.12
5	21	H	7	0.36	0.06	0.125	517	1.36	0.994	0.978	0.920	0.705	0.467	0.259	0.122	16.0^{-3}	1.08^{-3}	47.32
		H	$14^{10,17}$	0.64		0.125	583	2.80	0.995	0.983	0.930	0.750	0.520	0.318	0.168	30.3^{-3}	3.08^{-3}	56.00
		QH	7	0.36		0.25	541	2.92	0.990	0.972	0.893	0.635	0.358	0.162	0.058	3.76^{-3}	0.10^{-3}	35.13
		QH	$14^{10,17}$	0.64		0.50	577	11.1	0.994	0.978	0.918	0.704	0.466	0.258	0.119	15.6^{-3}	1.03^{-3}	47.14
		B	7	0.36		0.25	364	1.97	0.985	0.944	0.787	0.385	0.117	0.022	0.003			16.78
		QF	7	0.36		0.25	368	1.99	0.986	0.950	0.820	0.452	0.169	0.043	0.007			20.24
		QF	10	0.28		0.25	296	1.24	0.991	0.965	0.867	0.564	0.276	0.102	0.028	0.91^{-3}	0.01^{-3}	28.00
		QF	14^{17}	0.36		0.25	302	1.63	0.985	0.943	0.430	0.394	0.124	0.023	0.003	0.01^{-3}		17.20
		F	7	0.36		0.125	291	0.786	0.990	0.969	0.878	0.598	0.315	0.127	0.040	1.82^{-3}	0.03^{-3}	31.12
		F	$14^{10,17}$	0.64		0.125	273	1.31	0.986	0.950	0.823	0.459	0.174	0.044	0.008	0.07^{-3}		20.54
Number of Variations per Hour Exceeding Given Level (Summing over all conditions)								539.3	483	346	148	56.9	26.5	10.5	3.61	0.330	0.0221	
Percentage of Variations Exceeding Given Level								100	89.6	64.2	28.0	10.5	4.91	1.95	0.669	0.0612	0.0041	

*Column 2 gives the estimated significant wave height. This value was estimated by taking the mean value of the highest waves in the center of each group of well-formed waves.

**Column 4 gives the nominal speed taken from a calibration curve of shaft rpm versus knots. The exponents in this column represent ship speeds for which no data is available and for which the experimental data corresponding to the indicated speed is assumed to apply for purposes of the table.

†f_1 is the fraction of time the ship will operate at a given speed for a given sea state and includes all headings relative to the predominant wave direction.
f_2 is the fraction of time that a given sea state will be expected, on the average.
f_3 is the fraction of time the ship will make a given heading to the sea, for a given sea state and speed.
N is the number of variations expected per hour of operation under the condition considered. This value of N is taken from the tables of basic data.

††The exponents in columns 10-18 denote the power of 10 by which the given number is to be multiplied.

TABLE 9

Derivation of Predicted Distribution Pattern for Variations in Pitch Acceleration for Wartime Duty in North Atlantic Ocean

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Sea State	Significant Wave Height*	Relative Heading of Waves	Ship Speed**	Weighting Factors†				Average Number of Variations per Hour Contributed by Each Operating Condition	Probability of Exceeding Given Magnitude of Variation††(1-P) (Rad/Sec²)									E (Rad/sec²)² Mean Square Variation
	ft		knots	f_1	f_2	f_3	N	$f_1 f_2 f_3 N = n$	0.002	0.005	0.01	0.02	0.05	0.10	0.20	0.50	1.00	
	<4	All	All		0.20			157.58		0.7790	0.370	0.0183						0.0001
2	6	H	10^{7}	0.33	0.34	0.125	704	9.874		0.98350	0.9350	0.7660	0.189	0.00128	2.56×10^{-12}			0.0015
	↓	H	14^{17}	0.67	↓	0.125	786	22.385		0.98964	0.9592	0.8465	0.354	0.0155	57×10^{-9}			0.0024
	↓	QH	10^{7}	0.33	↓	0.25	761	21.348		0.98350	0.9350	0.7660	0.189	0.00128	2.56×10^{-12}			0.0015
	↓	QH	14^{7}	0.67	↓	0.25	872	49.660		0.98230	0.9311	0.7516	0.168	0.00079	0.38×10^{-12}			0.0014
	↓	B	10^{7}	0.33	↓	0.25	708	19.859		0.99245	0.9702	0.8859	0.469	0.0482	5.43×10^{-6}	1.2×10^{-33}		0.0033
	↓	B	14^{17}	0.67	↓	0.25	740	42.143		0.99245	0.9702	0.8859	0.469	0.0482	5.43×10^{-6}	1.2×10^{-33}		0.0033
	↓	QF	$14^{7,10,17}$	1.00	↓	0.25	890	75.650		0.98450	0.9395	0.7788	0.210	0.00192	13.8×10^{-12}			0.0016
	↓	F	$14^{7,10,17}$	1.00	↓	0.125	635	26.989		0.91457	0.7000	0.2400	0.00013	0.32×10^{-15}				0.00028
3	7-9	H	10^{7}	0.30	0.29	0.125	694	7.548		0.99771	0.99087	0.9640	0.795	0.400	0.0256	0.11×10^{-9}		0.0109
	↓	H	14	0.38	↓	0.125	765	10.537		0.99845	0.99380	0.9755	0.856	0.538	0.0832	0.178×10^{-6}		0.0161
	↓	H	17	0.32	↓	0.125	862	10.000		0.99856	0.99416	0.9768	0.864	0.558	0.0962	0.437×10^{-6}		0.0171
	↓	QH	10^{7}	0.30	↓	0.25	779	16.944		0.98870	0.9556	0.8338	0.321	0.0106	12.6×10^{-9}			0.0022
	↓	QH	17^{14}	0.70	↓	0.25	900	45.675		0.99111	0.9650	0.8669	0.410	0.0281	0.59×10^{-6}			0.0028
	↓	B	10^{7}	0.30	↓	0.25	763	16.596		0.99170	0.9673	0.8752	0.434	0.0358	1.64×10^{-6}			0.0030
	↓	B	17^{14}	0.70	↓	0.25	760	38.570		0.99142	0.9662	0.8712	0.422	0.0319	1.03×10^{-6}			0.0029
	↓	QF	$17^{7,10,14}$	1.00	↓	0.25	696	50.460		0.7790	0.370	0.0183	13.8×10^{-12}					0.00010
	↓	F	$17^{7,10,14}$	1.00	↓	0.125	629	22.803		0.97530	0.8704	0.5740	0.0310	0.90×10^{-6}				0.00072
4	16	H	$14^{7,10,17}$	1.00	0.11	1.00	780	85.80		0.99887	0.99551	0.98213	0.8936	0.6378	0.1650	12.7×10^{-6}		0.0222
5	21	H	7½	0.36	0.06	0.125	624	1.685		0.99856	0.99429	0.9772	0.8653	0.5615	0.0990	0.53×10^{-6}		0.0173
	↓	H	$14^{10,17}$	0.64	↓		692	3.322		0.99919	0.99678	0.98710	0.9220	0.723	0.2728	0.30×10^{-3}	0.008×10^{-12}	0.0308
	↓	QH	7½	0.36	↓	0.25	615	3.323		0.99821	0.99287	0.9718	0.8364	0.490	0.0574	0.0175×10^{-6}		0.0140
	↓	QH	$14^{10,17}$	0.64	↓	0.50	711	13.651		0.99924	0.99696	0.98782	0.92645	0.736	0.2942	0.48×10^{-3}	0.51×10^{-12}	0.0327
	↓	B	7½	0.36	↓	0.25	458	2.473		0.99360	0.9747	0.9026	0.527	0.077	35×10^{-6}			0.0039
	↓	QF	7½	0.36	↓	0.25	702	3.791		0.99221	0.9693	0.8826	0.458	0.044				0.0032
	↓	QF	10	0.28	↓	0.25	660	2.772		0.98918	0.9575	0.8404	0.338	0.013				0.0023
	↓	QF	14^{17}	0.36	↓	0.25	714	3.856		0.98918	0.9575	0.8404	0.338	0.013				0.0023
	↓	F	7½	0.36	↓	0.125	524	1.415		0.98097	0.9260	0.7350	0.146	0.46×10^{-3}				0.0013
	↓	F	$14^{10,14}$	0.64	↓	0.125	470	2.259		0.93950	0.7790	0.368	0.0019	13.8×10^{-12}				0.0004
Number of Variations per Hour Exceeding Given Magnitude								768.97		715.34	608.13	465.50	241.26	91.306	21.47	0.00865		
Percentage of Variations Exceeding Given Level										93.03	79.1	60.5	31.4	11.87	2.79	0.0001		

*Column 2 gives the estimated significant wave height. This value was estimated by taking the mean value of the highest waves in the center of each group of well-formed waves.

**Column 4 gi\ · the nominal speed taken from a calibration curve of shaft rpm versus knots. The exponents in this column represent ship speeds for which no data is available and for which the exp.. imental data corresponding to the indicated speed is assumed to apply for purposes of the table.

†f_1 is the fraction of time the ship will operate at a given speed for a given sea state and includes all headings relative to the predominant wave direction.
f_2 is the fraction of time that a given sea state will be expected, on the average.
f_3 is the fraction of time the ship will make a given heading to the sea, for a given sea state and speed.
N is the number of variations expected per hour of operation under the condition considered. This value of N is taken from the tables of basic data.

††The exponents in columns 10-18 denote the power of 10 by which the given number is to be multiplied.

TABLE 10

Derivation of Predicted Distribution Pattern for Variations in Roll Angle for Wartime Duty in North Atlantic Ocean

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	15	17	18
Sea State	Significant Wave Height* ft	Relative Heading of Waves	Ship Speed** knots	Weighting Factors†				Average Number of Variations per Hour Contributed by Each Operating Condition	Probability of Exceeding Given Magnitude of Variation ††(1-P)								E $(deg)^2$ Mean Square of Variations
				f_1	f_2	f_3	N	$f_1 f_2 f_3 N = n$	1°	2°	5°	10°	15°	20°	30°	40°	
	<4			1.00	0.20	1.00		70.11	0.859	0.528	0.0213	0	0	0	0	0	6.5 (estimated)
2	6	H	10^{7}	0.33	0.34	0.125	380	5.38	0.949	0.81	0.268	0.00518	0.0000072	0			19.0
		H	14^{17}	0.67		0.125	362	10.31	0.946	0.805	0.255	0.00425	0.0000028	0			18.3
		QH	10^{7}	0.33		0.25	355	9.96	0.968	0.878	0.447	0.0397	0.000703	0.00000252	0	0	31.0
		QH	14^{17}	0.67		0.25	340	19.36	0.961	0.858	0.383	0.0215	0.000175	0.00000021	0	0	26.0
		B	10^{7}	0.33		0.25	388	10.88	0.946	0.799	0.245	0.0029	0.0000031	0	0	0	17.8
		B	14^{17}	0.67		0.25	368	20.96	0.949	0.81	0.268	0.00518	0.0000072	0	0	0	19.0
		QF	$14^{7,10,17}$	1.00		0.25	335	28.48	0.960	0.845	0.353	0.0155	0.0000833	0.0595×10^{-6}	0	0	24.0
	↓	F	$14^{7,10,17}$	1.00	↓	0.125	326	13.86	0.953	0.825	0.304	0.00855	0.0000215	0	0	0	21.0
3	9	H	10^{7}	0.30	0.29	0.125	436	4.71	0.952	0.877	0.438	0.037	0.000598	0.00000188	0	0	20.6
		H	14	0.38		0.125	400	5.51	0.946	0.795	0.240	0.0025	0.00000252	0	0	0	17.5
		H	17	0.32		0.125	448	5.20	0.902	0.653	0.078	0.0000304	0	0	0	0	9.8
	7	QH	10^{7}	0.30		0.25	209	4.55	0.859	0.528	0.0213	0.00000021	0	0	0	0	6.5
		QH	17^{14}	0.70		0.25	339	17.22	0.873	0.579	0.0343	0.00000163	0	0	0	0	7.5
		B	10^{7}	0.30		0.25	260	5.66	0.865	0.559	0.0268	0.00000052	0	0	0	0	6.9
		B	17^{14}	0.70		0.25	482	24.49	0.925	0.735	0.146	0.000455	0.00000012	0	0	0	13.0
		QF	$17^{7,10,14}$	1.00		0.25	285	20.66	0.950	0.812	0.366	0.00565	0.0000089	0	0	0	19.3
	↓	F	$17^{7,10,14}$	1.00	↓	0.125	355	12.87	0.940	0.775	0.206	0.00179	0	0	0	0	15.8
4	16	H	$14^{7,10,17}$	1.00	0.11	1.00	578	63.58	0.880 55.95	0.598 38.02	0.046 2.92	0.0000025 0.00016	0 0	0 0	0 0	0 0	7.8
5	21	H	7	0.36	0.06	0.125	369	0.996	0.925	0.966	0.809	0.428	0.1515	0.0348	0.00052	0.00000152	119.0
		H	$14^{10,17}$	0.64		0.125	398	1.910	0.987	0.952	0.734	0.308	0.0704	0.00901	0.000025	0.0000000025	84.8
		QH	7	0.36		0.25	388	2.095	0.988	0.948	0.717	0.265	0.0505	0.00476	0.00000658	0	75.3
		QH	$14^{10,17}$	0.64		0.50	408	7.834	0.985	0.940	0.678	0.211	0.0303	0.0020	0.00000067	0	64.3
		B	7	0.36		0.25	364	1.966	0.943	0.975	0.863	0.557	0.268	0.096	0.00520	0.0000855	171.0
		QF	7	0.36		0.25	384	2.074	0.943	0.976	0.868	0.566	0.278	0.103	0.00600	0.000114	176.0
		QF	10	0.28		0.25	372	1.562	0.938	0.971	0.845	0.509	0.219	0.0672	0.00227	0.0000202	148.0
		QF	14^{7}	0.36		0.25	360	1.944	0.925	0.967	0.802	0.415	0.139	0.0301	0.000373	0.00000083	114.0
		F	7	0.36		0.125	356	0.961	0.938	0.971	0.847	0.513	0.223	0.0695	0.00250	0.0000237	150.0
	↓	F	$14^{10,17}$	0.64	↓	0.125	340	1.632	0.985	0.935	0.660	0.1885	0.0233	0.00128	0.00000031	0	60.0
Number of Variations per Hour Exceeding Given Level (Summing over all conditions)								376.7	344.8	268.3	78.6	9.50	2.61	0.712	0.0299	0.000462	
Percentage of Variations Exceeding Given Level									91.52	71.22	20.87	2.52	0.694	0.189	0.0079	0.00012	

*Column 2 gives the estimated significant wave height. This value was estimated by taking the mean value of the highest waves in the center of each group of well-formed waves.

**Column 4 gives the nominal speed taken from a calibration curve of shaft rpm versus knots. The exponents in this column represent ship speeds for which no data is available and for which the experimental data corresponding to the indicated speed is assumed to apply for purposes of the table.

†f_1 is the fraction of time the ship will operate at a given speed for a given sea state and includes all headings relative to the predominant wave direction.
f_2 is the fraction of time that a given sea state will be expected, on the average.
f_3 is the fraction of time the ship will make a given heading to the sea, for a given sea state and speed.
N is the number of variations expected per hour of operation under the condition considered. This value of N is taken from the tables of basic data.

††The exponents in columns 10-18 denote the power of 10 by which the given number is to be multiplied.

TABLE 11

Derivation of Predicted Distribution Pattern for Variation in Hull Stress Due to Longitudinal Bending, at the Main Deck, Amidships for Wartime Duty in North Atlantic Ocean

1	2	3	4	5		7	8	9	10	11	12	13	14	15	16	17	18	19
Sea State	Significant Wave Height*	Relative Heading of Waves	Ship Speed**	Weighting Factors†				Average Number of Variations per Hour Contributed by Each Operating Condition	Probability of Exceeding Given Magnitude of Variation††(1 - P)									E (kips/in²)² Mean Square Variation
									Kips per Square Inch									
	ft		knots	f_1	f_2	f_3	N	$f_1 f_2 f_3 N = n$	0.2	0.5	1.0	1.5	2.0	4.0	8.0	0.7		
	<4				0.20		646	129.24	0.69541	0.1031	0.00011							0.11
2	6	H	10^{7}	0.33	0.34	0.125	670	9.40	0.835	0.321	0.0106	0.0000358	0.0123 × 10⁻⁶	0.025 × 10⁻³⁰		0.108		0.22
		H	14^{17}	0.67			784	22.32	0.880	0.447	0.0398	0.000705	2.52 × 10⁻⁶	0.038 × 10⁻²¹		0.206		0.31
		QH	10^{7}	0.33		0.25	256	7.18	0.826	0.304	0.0094	0.0000250	0.00525 × 10⁻⁶			0.097		0.21
		QH	14^{17}	0.67			1004	57.18	0.846	0.353	0.0156	0.0000855	0.057 × 10⁻⁶			0.130		0.24
		B	10^{7}	0.33			522	14.64	0.898	0.509	0.0672	0.00228	20.7 × 10⁻⁶	0.17 × 10⁻¹⁸		0.266		0.37
		B	14^{17}	0.67			666	37.93	0.871	0.422	0.0320	0.000430	1.12 × 10⁻⁶	1.0 × 10⁻²⁴		0.184		0.29
		QF	$14^{7,10,17}$	1.00			640	54.40	0.835	0.321	0.0106	0.0000358	0.0123 × 10⁻⁶	0.025 × 10⁻³⁰		0.108		0.22
		F	$14^{7,10,17}$	1.00		0.125	392	16.66	0.896	0.495	0.0622	0.00192	15.0 × 10⁻⁶	0.049 × 10⁻¹⁸		0.256		0.36
3	7-9	H	10^{7}	0.30	0.29	0.125	621	6.75	0.96487	0.800	0.4095	0.1343	0.0280	0.624 × 10⁻⁶		0.646		1.12
		H	14	0.38			778	10.72	0.96918	0.8227	0.4580	0.1725	0.0439	3.72 × 10⁻⁶		0.672		1.28
		H	17	0.32			991	11.50	0.96749	0.8135	0.4378	0.1555	0.0368	1.82 × 10⁻⁶		0.667		1.21
		QH	10^{7}	0.30		0.25	864	18.79	0.69541	0.1031	0.000113	1.30 × 10⁻⁹	0.158 × 10⁻¹⁵			0.0116		0.11
		QH	17^{14}	0.70			711	36.09	0.77881	0.2095	0.00192	0.78 × 10⁻⁶	0.014 × 10⁻⁹			0.0467		0.16
		B	10^{7}	0.30			503	10.94	0.81037	0.2681	0.00520	7.2 × 10⁻⁶	0.72 × 10⁻⁹			0.076		0.19
		B	17^{14}	0.70			685	34.76	0.80064	0.2495	0.00388	3.75 × 10⁻⁶	0.238 × 10⁻⁹			0.066		0.18
		QF	$17^{7,10,14}$	1.00			459	33.28	0.76610	0.1890	0.00128	0.308 × 10⁻⁶	2.63 × 10⁻¹²			0.038		0.15
		F	$17^{7,10,14}$	1.00		0.125	360	13.05	0.64123	0.0621	0.000015	0.0138 × 10⁻⁹	0.048 × 10⁻¹⁸			0.0043		0.09
4	16	H	$14^{7,10,17}$	1.00	0.11	1.00	768	84.48	0.96637	0.80775	0.4255	0.1460	0.0328	1.16 × 10⁻⁶		0.658		1.17
5	21	H	7	0.36	0.06	0.125	581	1.57	0.98588	0.91474	0.7010	0.4490	0.2410	3.35 × 10⁻³	0.125 × 10⁻⁹	0.8400		2.81
		H	$14^{10,17}$	0.64			790	3.80	0.98863	0.93096	0.7518	0.5260	0.3190	10.4 × 10⁻³	11.6 × 10⁻⁹	0.8692		3.50
		QH	7	0.36		0.25	598	3.23	0.98301	0.90067	0.6525	0.3824	0.1810	1.08 × 10⁻³	0.0013 × 10⁻⁹	0.8112		2.34
		QH	$14^{10,17}$	0.64		0.50	694	13.32	0.98675	0.91996	0.7167	0.4722	0.2640	4.83 × 10⁻³	0.55 × 10⁻⁹	0.8493		3.00
		B	7	0.36		0.25	532	2.87	0.96968	0.82508	0.4633	0.1773	0.0460	4.50 × 10⁻⁶		0.6860		1.30
		QF	7	0.36			449	2.43	0.95453	0.74777	0.3125	0.0730	0.0096	8.2 × 10⁻⁹		0.5660		0.86
		QF	10	0.28			328	1.38	0.96487	0.8000	0.4095	0.1340	0.0280	0.624 × 10⁻⁶		0.646		1.12
		QF	14^{17}	0.36			344	1.86	0.96116	0.78064	0.3718	0.1078	0.0190	0.125 × 10⁻⁶		0.6155		1.01
		F	7	0.36		0.125	325	0.88	0.97914	0.87657	0.5910	0.3060	0.1220	0.220 × 10⁻³	1.52 × 10⁻¹⁵	0.7725		1.90
		F	$14^{10,17}$	0.64			300	1.44	0.97484	0.85280	0.5293	0.2385	0.0782	0.038 × 10⁻³		0.7320		1.57
Number of Variations per Hour Exceeding Given Magnitude (Summing over all Conditions)								642.09	531.71	246.75	74.96	28.89	9.994	0.113		141.60		
Percentage of Variations Exceeding Given Level									82.8	38.43	11.67	5.00	1.56	0.0018		22.05		

*Column 2 gives the estimated significant wave height. This value was estimated by taking the mean value of the highest waves in the center of each group of well-formed waves.

**Column 4 gives the nominal speed taken from a calibration curve of shaft rpm versus knots. The exponents in this column represent ship speeds for which no data is available and for which the experimental data corresponding to the indicated speed is assumed to apply for purposes of the table.

†f_1 is the fraction of time the ship will operate at a given speed for a given sea state and includes all headings relative to the predominant wave direction.
f_2 is the fraction of time that a given sea state will be expected, on the average.
f_3 is the fraction of time the ship will make a given heading to the sea, for a given sea state and speed.
N is the number of variations expected per hour of operation under the condition considered. This value of N is taken from the tables of basic data.

††The exponents in columns 10-18 denote the power of 10 by which the given number is to be multiplied.

DESIGN AND OPERATIONAL CONDITIONS FOR WARTIME SERVICE

In the discussion of the statistical background, it was pointed out that the distribution patterns readily give the probability of exceeding any given magnitude of the motion or stress and that the distribution pattern can be utilized as a load spectrum for endurance strength calculations. In this section the methods will be applied to determine design and operational conditions for wartime service. These determinations are based on the following assumptions.

1. The vessel will be operating in the North Atlantic Ocean. The observations of sea conditions at weather station C (52°N37°W), see Figure 2, are considered typical of conditions in the North Atlantic and are assumed to represent the conditions the ships will encounter in service.

2. The ship operating speeds for the different sea conditions are taken to be the average of the estimates obtained from ship's officers of a number of ships; see Table 1.

3. All headings of the ship relative to the predominant wave direction are assumed equally likely, excepting only that seas coming approximately off the beam are considered unlikely for combinations of high speeds and rough seas.

"LONG-TERM" DISTRIBUTIONS OF SHIP MOTION, HULL BENDING MOMENT, AND WAVE HEIGHT

The charts of Figures 7 through 10 give the probability of exceeding and of not exceeding any given value of stress or motion if all the motions or stresses are considered to which the vessel is subjected over a period of a number of years. For example, only 3 percent of all variations in roll angle would, on the average, exceed a value of 10 deg peak-to-peak; see Figure 9. Figure 2 gives similar data for significant* wave heights to be expected in the North Atlantic Ocean. These distributions may be considered valid up to maximum variations of 25 deg in pitch, 56 deg in roll, 0.8 rad/sec^2 in pitch acceleration, and 40,000 ft-tons in bending moment.

PREDICTIONS OF SHIP RESPONSE TO WAVES FOR GIVEN CONDITIONS

It is difficult to make reliable estimates on the basis of the available data because the specification of the sea state and of the relative heading of the ship to the sea are somewhat indefinite. Nevertheless the following formulas may be used to make estimates which should be fairly good if the environmental conditions are similar to those for which measurements were made.

The most frequent magnitude of the variation will be 0.707 $\sqrt{E}$. The average magnitude of the variation will be 0.886 $\sqrt{E}$. The significant magnitude (average of the upper third of the

*The significant wave height is used to denote a sea state. It is estimated as indicated in the footnote on page 3.

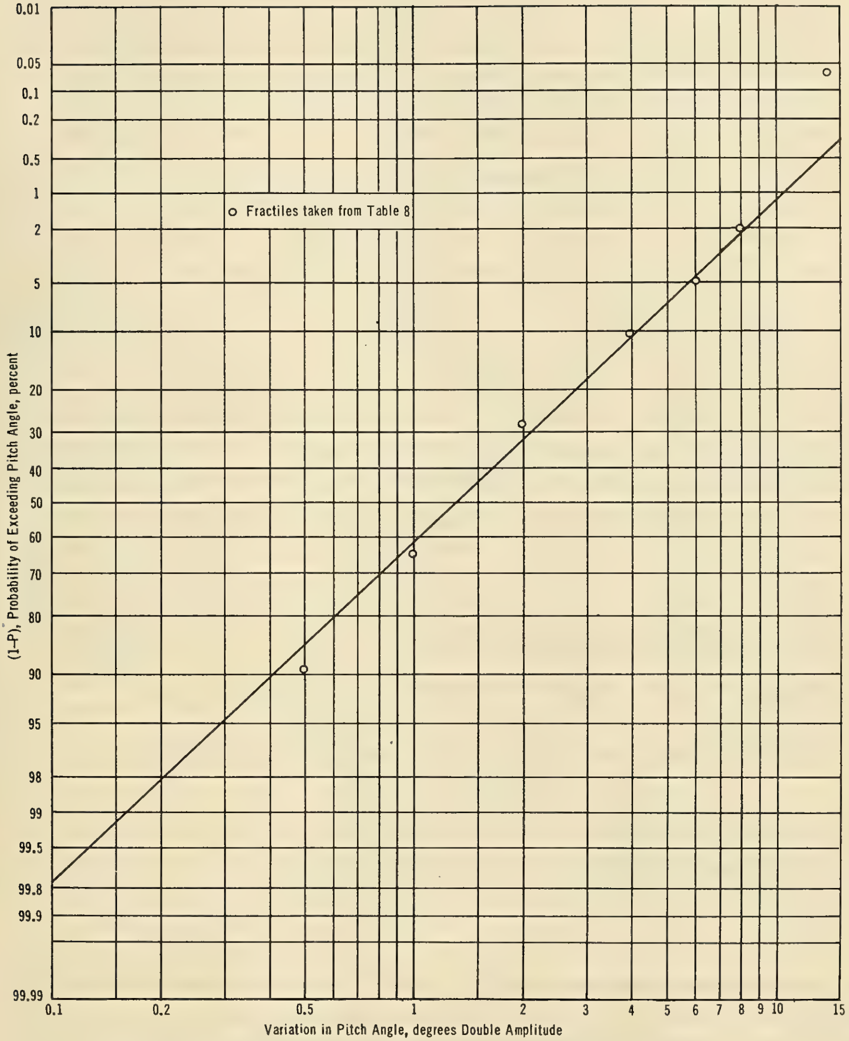


Figure 7 - Long-Term Cumulative Distribution of Pitch Angle for Wartime Service, North Atlantic Ocean

This log-normal distribution is computed on the basis of the data represented by the plotted points and it corresponds to the following parameters (see Appendix B for calculations).

Mean value of $\log_{10}$ (variation in pitch angle) = 0.1227

Standard deviation of $\log_{10}$ (variation in pitch angle) = 0.3898

It is probably valid up to 25 deg.

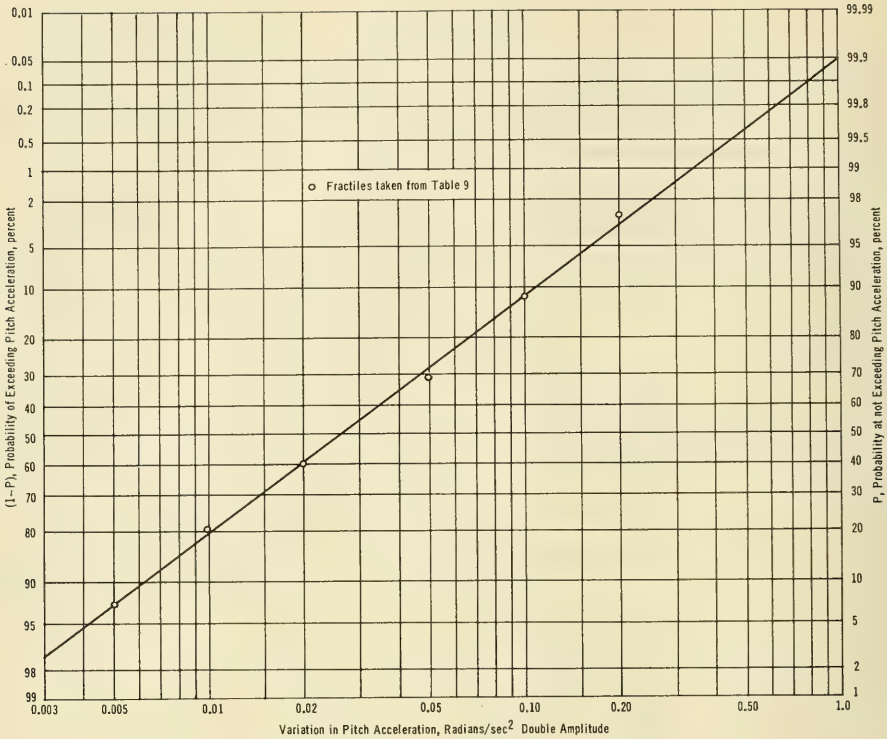


Figure 8 - Long-Term Cumulative Distribution of Pitch Acceleration for Wartime Service, North Atlantic Ocean

This log-normal distribution is computed on the basis of the data represented by the plotted points and it corresponds to the following parameters:

Mean value of $\log_{10}$ (pitch acceleration in rad/sec^2) = 2.4314

Standard deviation of $\log_{10}$ (pitch acceleration in rad/sec^2) = 0.4895

It is probably valid up to 0.80 rad/sec^2.

waves having the largest magnitudes) will be 1.416 $\sqrt{E}$. The most probable magnitude of the largest of 50 variations will be 2.01 $\sqrt{E}$. The most probable magnitude of the largest of 500 variations will be 2.509 $\sqrt{E}$. The most probable magnitude of the largest of N variations will be (constant) $\sqrt{E}$ where the value of the constant is given in Table 7. For large values of N, the constant is approximately equal to $\sqrt{\log_e N}$.

The values of E corresponding to various combinations of sea condition, ship speed, and heading are given in Tables 2 through 6.

PREDICTION OF EXTREME VALUES

It may be desired to estimate the largest value of ship motions or hull girder stress

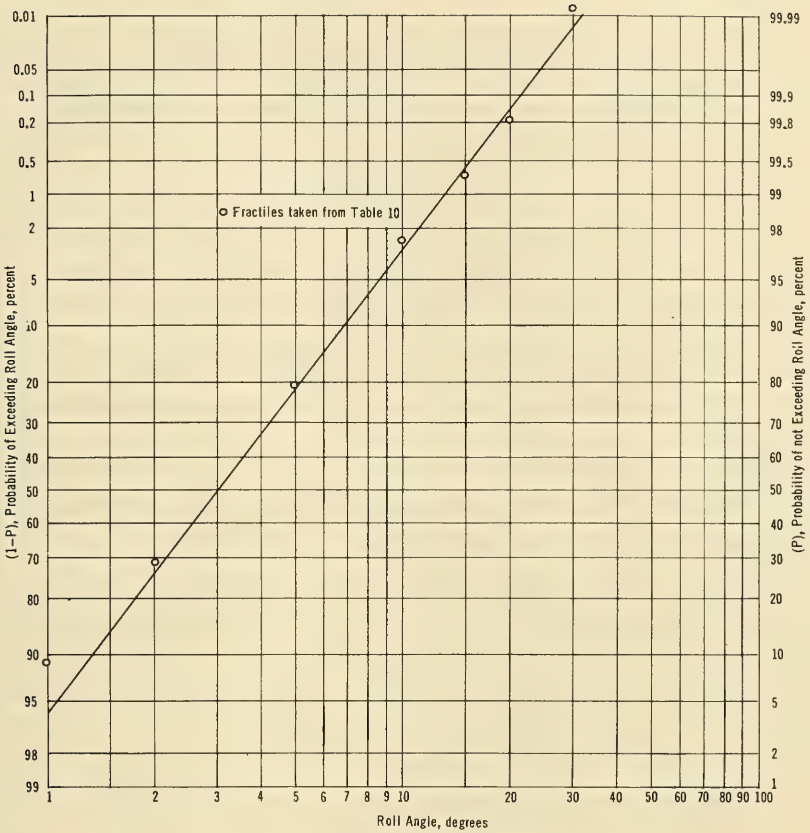


Figure 9 - Long-Term Cumulative Distribution of Roll Angle
for Wartime Service, North Atlantic Ocean

This log-normal distribution is computed on the basis of the data represented by the plotted points and it corresponds to the following parameters:

Mean value of $\log_{10}$ (variation in roll angle in degrees) = 0.4868

Standard deviation of $\log_{10}$ (variation in roll angle in degrees) = 0.2741

It is probably valid up to 56 deg.

that a ship structure is expected to experience in a given number of variations or over a given period of time.* The cumulative probability functions plotted in Figure 4 (Rayleigh distribution) and Figures 7 through 10 (log-normal distributions) may be used to find the probability of exceeding (1-*P*) or of not exceeding (*P*) a given value of the variable.

In the preceding section a convenient formula is given for estimating the most probable magnitude of the largest variation out of *N* variations; this formula is applicable only when the

*The average number of variations expected per hour of sea operation are given in Tables 8 through 11, column 8.

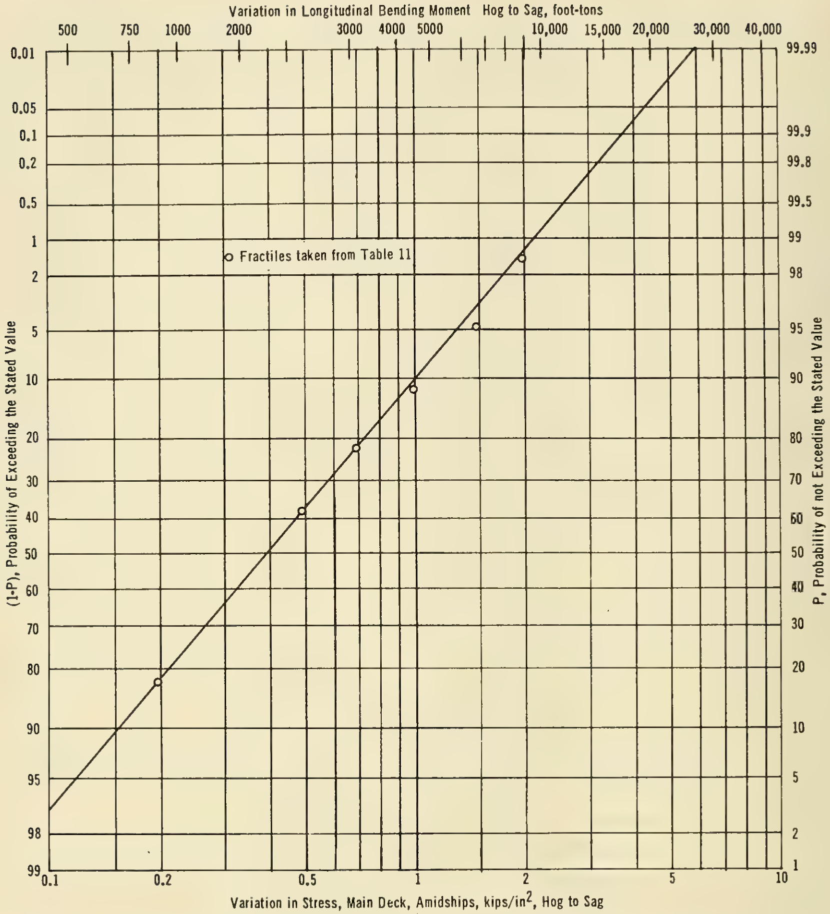


Figure 10 - Long-Term Cumulative Distribution of Longitudinal Bending Moment, Amidships, for Wartime Service, North Atlantic Ocean

This log-normal distribution is computed on the basis of the data represented by the plotted points and it corresponds to the following parameters:

Mean value of $\log_{10}$ (variation in stress in kips/in.^{2}) = $\overline{1}.6016$

Standard deviation of $\log_{10}$ (variation in stress in kips/in.^{2}) = 0.3229

It is probably valid up to 40,000 ft-tons.

conditions of ship speed, heading, speed, and sea are steady. The following is a more general approach which may be applied to any distribution provided that its cumulative distribution function $P(x)$ is known.

The following formula, developed in Reference 3, gives the fraction f of all samples of size N [belonging to a given distribution specified by $P(x)$] which will have at least one value of $x > x_{m_1}$. The formula is valid only if $P(x_{m_1})$ has a value close to unity, that is, the formula is designed to estimate the values which occur rarely. The formula is

$$f = 1 - e^{-N[1 - P(x_{m_1})]}$$

or alternatively

$$1 - P(x_{m_1}) = \frac{-\log_e (1 - f)}{N}$$

where $[1 - P(x_{m_1})]$ is the probability of exceeding x_{m_1} in the fraction f of all samples of size N each. Knowing $[1 - P(x_{m_1})]$, it is easy to compute the corresponding value of the variable x_{m_1}.

In order to estimate the largest values of motion and bending moments for design purposes we will use this formula to estimate the value x_{m_1} which, on the average, is exceeded by the fraction f of all similar ships during their service life. Thus f represents the risk of exceeding x_{m_1}.

It will be assumed that the worst combination of operating conditions is the most severe of those listed in Tables 1 through 6, viz., a State 5 sea characterized by a significant wave height estimated to be 21 ft. The values of E_m specifying the corresponding Rayleigh distributions are listed in Table 12. Assume that the ship will be subjected to these operating conditions for a duration of 12 hours, experiencing V variations in this period of time, and that this situation will be repeated n times during the service life of the ship; therefore $N = nV$. For the Rayleigh distribution we have

$$[1 - P(x_{m_1})] = e^{-x_{m_1}^2 / E_m}$$

Substitution in the expression for f gives:

$$1 - f = \exp\,[-e^{-y}] \quad \text{where } y = \frac{x_{m_1}^2}{E_m} - \log_e N$$

Table 1 of Reference 9 tabulates the values of $\exp\,[-e^{-y}]$ as a function of y. Thus, for a specified risk f of exceeding x_{m_1} one may look up the corresponding value of y and then solve for the desired value of x_{m_1} from the relation

$$x_{m_1}^2 = E_m\,[y + \log_e N]$$

As an example consider the maximum value for the variation in roll angle.

From Table 12: $E_m = 176\ (\text{deg})^2$, $V = 4600$

If we take $f = 0.001$ and $n = 10$, then Reference 9 gives $y = 7.0$. Therefore:

$$x_{m_1} = [176\,(7.0 + 10.74)]^{1/2} = 56 \text{ deg (port to starboard)}$$

TABLE 12 - Maximum Values of Ship Motion and Longitudinal Bending Moment for Use in Design Calculations

All values given refer to the peak-to-peak variation.

Quantity	Condition for Which the Extreme Value is Predicted					V Number of Variations in 12 Hours for Steady Conditions	Estimated Most Probable Largest Variation in 12 Hours $\sqrt{E \log_e V}$	Maximum Variation for Operating Life of Ship $f = 0.001$	Largest Measured Variation from Table 5, Ref. 2†	Maximum Variation for Design Purposes
	Sea State	Significant Wave Height	Direction of Seas	Shaft RPM	E_m					
Roll Angle	5	21	QF	94	176 (deg)^2	4600	39°	56°	50°	56°
Pitch Angle	5	21	H	185	56 (deg)^2	7000	22°	32°**	20°	about 25°
*Roll Angle	5	21	QF	94						40°
*Pitch Angle	5	21	QF	94						10°
Pitch Acceleration	5	21	QH	185	0.0327 (rad/sec^2)^2	8500	0.54 rad/sec^2	0.78 rad/sec^2	0.77 rad/sec^2	0.8 rad/sec^2
Heave Acceleration	5	21	QH	185	0.0498 (gravity unit)^2	6800	0.66 g	1.0 g	1.0 g	1.0 g
Longitudinal Bending Moment	5	21	H	185	85×10^6 (ft-tons)^2	9500	28,000 ft-tons	40,000 ft-tons	33,000 ft-tons	40,000 ft-tons
Longitudinal Bending Stress	5	21	H	185	3.50 (kips/in^2)^2	9500	5.7 kips/in^2	8.0 kips/in^2	6.7 kips/in^2	8.0 kips/in^2

*This is believed to be a most severe condition of simultaneous roll and pitch. The data are taken from Figure 5b of Reference 2.

**This estimate (32°) is believed to be outside the range within which the statistical estimation is valid and therefore the value is discarded.

†These values are nearly the largest magnitudes obtained under any conditions experienced during the tests reported in Reference 2.

QH - Quarter Following; QF - Quarter Head; H - Head

Maximum estimated values for the other variables have been computed similarly, taking $f = 0.001$ and $n = 10$. They are listed in Table 12 together with the largest values measured at any time during the rough water sea trials reported in References 2 and 6. We will take the larger of the statistically estimated and the measured values as the suggested maximum value to use for design purposes.

For some design problems, it is necessary to make an estimate of the extreme conditions of simultaneous pitch and roll. It is unlikely that the maximum conditions of pitch and roll listed in the last column of Table 12 will occur simultaneously. The most severe combinations of simultaneous roll and pitch angle, measured during the sea trials,[2] was 40 deg double amplitude in roll together with 10 deg double amplitude in pitch, see Table 12. It is suggested that this combination be used as an extreme condition for design purposes. Predicted extreme values should be used with caution because the method eventually breaks down by predicting too extreme a value. This occurs because, in practical application the theoretical distribution cannot be relied upon at the extreme ranges of the function. For example, the prediction of 32 deg for the extreme value of pitch angle variation is probable unrealistic as nonlinear behavior of the pitch motion will probably set a lower limit than this.

The extreme values listed in the last column of Table 12 may be used to set an interim upper limit to the validity of predicted extreme values.

DESIGN LOADS FOR BOTTOM STRUCTURE TO WITHSTAND SLAMMING LOADS

A detailed analysis of the loads, stresses, and deflection for the bottom plating of an AVP vessel incident to slamming has been made by Greenspon.[5] On the basis of the latter study and photographs of an AVP during slamming,[2] it is estimated that the part of the bottom structure extending from the keel to the turn of the bilge in the forward quarter length of the ship may be subjected to occasional localized slamming or pounding pressures. In some locations within this area, the pressure attains values of the order of 300 psi. For the purpose of design calculations, the time variation of the pressure is such that it may be assumed to act statically. The following design loads are given as an interim recommendation:

a. For the design of bottom panels: Assume that the pressure attains a maximum value which varies linearly from 300 psi at the bow to 30 psi at a distance of 0.25 L from the bow:

$$p_0 = 300 - 1080 \; \frac{x}{L}, \quad \frac{x}{L} < 0.25$$

where p_0 is the design pressure for plates,

x is the location of the plate measured from the bow in feet, and

L is the length of the ship in feet.

b. For the design of transverse framing: Assume that a static uniform pressure of intensity $p_0/2$ acts on the bottom over a strip one frame space in width.

c. For the design of longitudinal stiffeners or framing: Assume that a static pressure acts on the bottom plating supported by the longitudinal equal to $p_0/2$.

If the bottom plating is to be designed to keep the stresses within the elastic limit, then the simple formulas and tables of Reference 10 may be conveniently used for the calculation of the maximum stresses and deflections in plates loaded transversely. The ultimate load for transversely loaded panels may be estimated by utilizing the simple method given by Greenspon.[11]

DISCUSSION

The data of Tables 2 through 6 furnish a basis for working up distributions of motion and hull bending moment for assigned service missions of ships of this type. In the present instance, these basic data have been utilized to predict wartime service conditions for operation in the North Atlantic; the procedure for making this prediction is carried out in Tables 8 through 11, as a guide for similar analyses of other missions. It should be pointed out that the prediction for "Wartime North Atlantic" service is greatly influenced by the estimated operational speeds provided by the U.S. Coast Guard. Inspection of Table 1 indicates that the estimates of service operating speeds made by COMAIRPAC vary considerably from those provided by ships in service in Atlantic waters.

It should be emphasized that the sea state is a variable which is most difficult to define. In the present case, variability of the sea conditions, for a given sea state, has been minimized by conducting the tests for a given sea state during one continuous time interval with the exception of the data for the State 3 Sea, for which two different time periods were required. The estimates of the sea state, as defined by the scale given in Reference 4, were made by the Weather Bureau observers who were assigned to the USCGC UNIMAK during the trials. In order to give a more quantitative idea of the sea state, sample stereophotographs together with photogrammetric analyses thereof are reproduced in Appendix C for sea states experienced during the trials of the UNIMAK (States 2, 4, and 5).

Inasmuch as the evaluation of sea states made by ships' officers and reported in Table 1, as well as that made by weather observers during the sea tests, depended upon visual observations, one may expect reasonable agreement in the severity of the sea as defined by the sea state. In any case, no better basis for making the synthesis given in Tables 8 through 11 was available to the authors.

The general method of synthesis utilized in this report could also be applied to data obtained from model tests of ships in waves rather than from full-scale test data. This would be a more flexible arrangement in that a wider variety of conditions could economically be covered by model tests than would be feasible with full-scale tests. Furthermore, the problem of measurement would be less difficult.

The order of magnitude of the pressures measured on the USCGC UNIMAK were also experienced by a sister ship, the USCGC CASCO (formerly AVP 12), during earlier sea tests in 1951. The indications are that the measured pressures are typical of the loading to be expected in heavy seas for this type of ship.

ACKNOWLEDGMENTS

The cooperation of the Commandant, U.S. Coast Guard and the Commanders, Air Force, U.S. Pacific and U.S. Atlantic Fleets made it possible to obtain realistic operational estimates of the speeds and headings under which ships of the AVP class are expected to operate in service. The Naval Photographic Interpretation Center and the Naval Hydrographic Office made the analyses of stereophotographs given in Appendix C. Mr. B.M. Wigle of the Vibrations Division, Taylor Model Basin, assisted in the calculations.

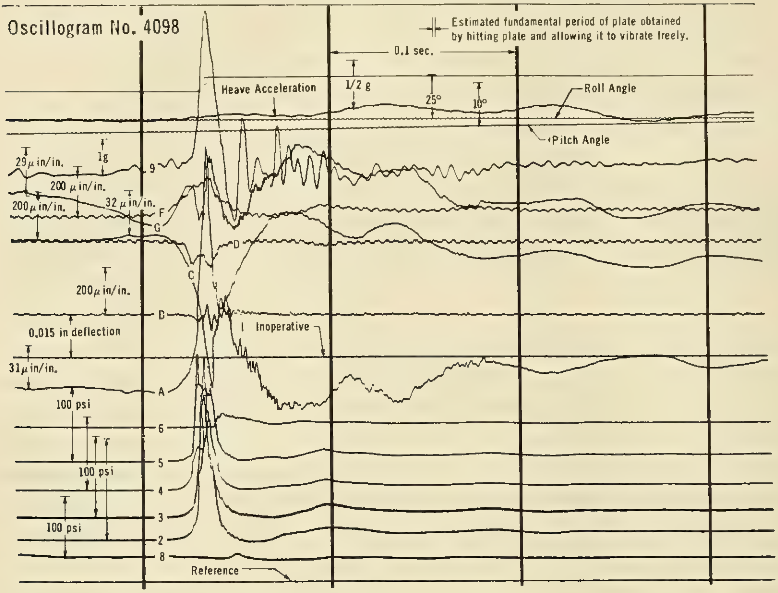
APPENDIX A

SAMPLE OSCILLOGRAMS

The two oscillograms of Figure 11a indicate the variation of pressures on the bottom plating with time and the consequent strains and deflections in the plate. The response to slamming is also indicated in the oscillograms of pitch acceleration and hull girder stress amidships in Figure 11e.

The maximum peak-to-peak variations in stress measured in the keel 15 in. aft of Frame 23 were 3500 psi, associated with presently undefined higher modes of hull structure which were excited subsequent to slamming at the bow.

Figure 11 - Samples of Records Taken During the Tests



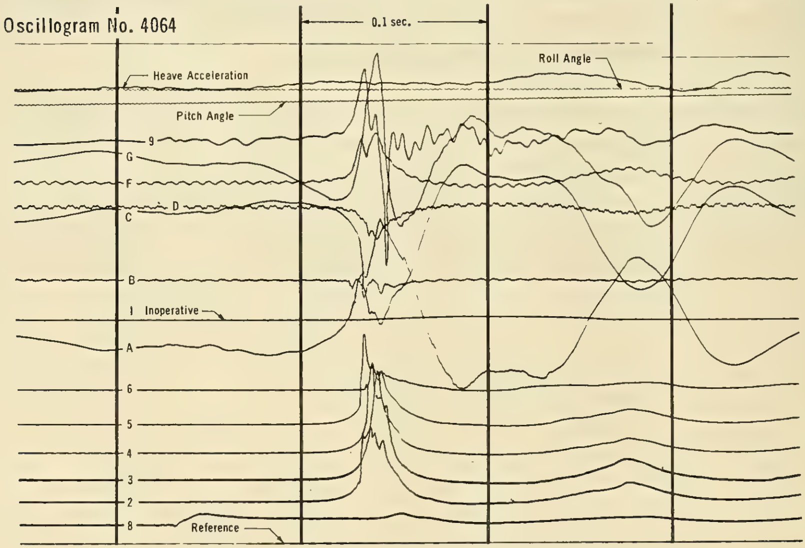


Figure 11a - Variation of Pressures in the Bottom Plating with Time

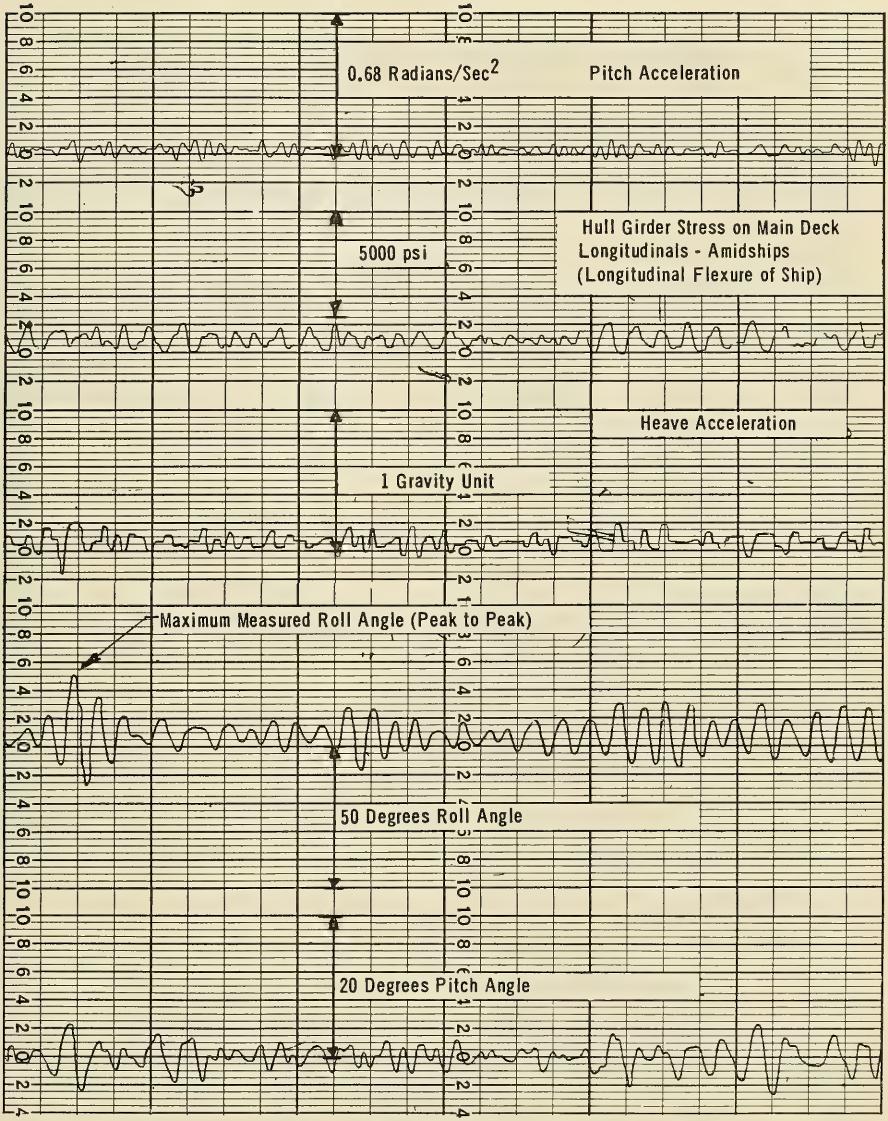


Figure 11b - Oscillogram at Time of Maximum Measured Roll Angle 16 Jan 55

Maximum Roll Angle	40 deg	Ships Speed	7.5 knots
Significant Wave Ht	21 ft	Relative Heading of Seas	135 deg
Wind Velocity	32 knots	Heading of Waves	090 deg
Wind Direction	090 deg	Heading of Ship	315 deg

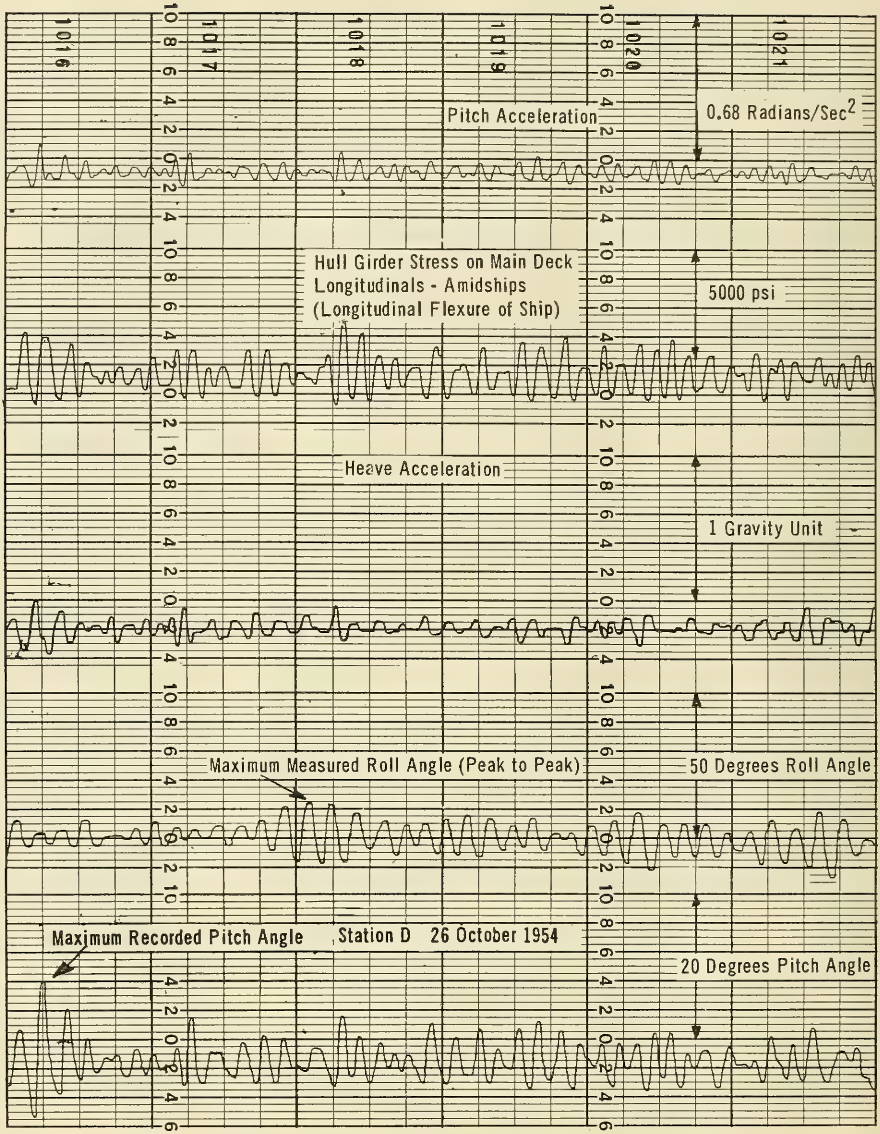


Figure 11c - Oscillogram at Time of Maximum Measured Pitch Angle 26 Oct 1954

Maximum Pitch Angle	18½ deg	Ships Speed	0.0 knots (Patent Log)
Significant Wave Height	25-29 ft	Relative Heading of Seas	000 deg
Wind Velocity	50 knots	Direction from Which Waves Come	260 deg
Wind Direction	270 deg	Heading of Ship	260 deg

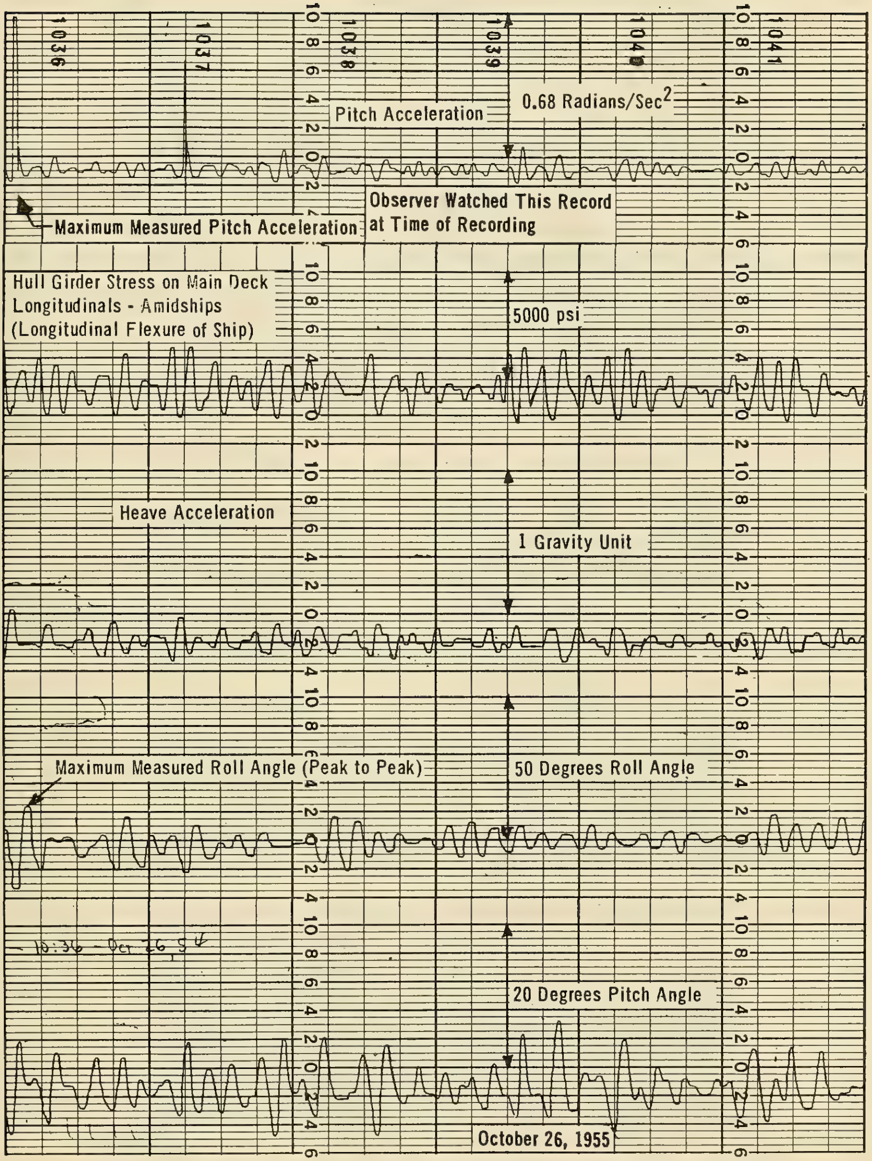


Figure 11d - Oscillogram at Time of Maximum Measured Pitch Acceleration 26 Oct 1954

Maximum Pitch Acceleration	0.77 rad/sec^2	Ships Speed	0.0 knots (Patent Log)
Significant Wave Height	29 ft	Relative Heading of Seas	010 deg
Wind Velocity	50 knots	Direction from Which Waves Come	270 deg
Wind Direction	270 deg	Heading of Ship	260 deg

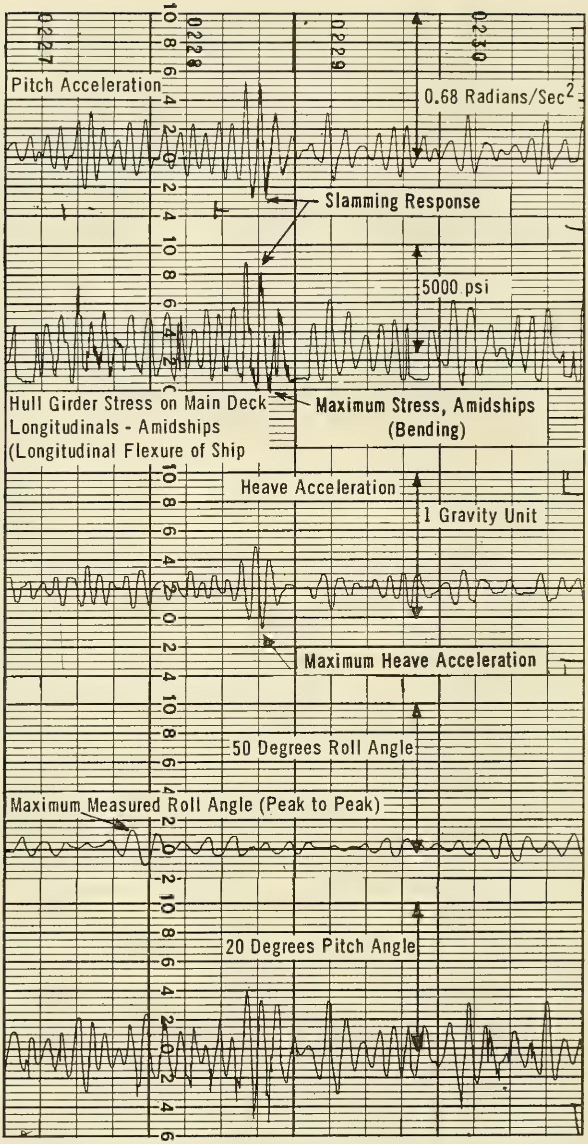


Figure 11e - Oscillogram at Time of Maximum Measured Stress and Heave Acceleration 1 Feb 1955

Maximum Stress	5900 psi	Ships Speed	14.2 knots
Maximum Heave Acceleration	0.55 g	Relative Heading of Seas	015 deg
Significant Wave Height	20 ft	Direction from Which Waves Come	090 deg
Wind Velocity	32 knots	Heading of Ship	075 deg
Wind Direction	079 deg		

APPENDIX B

SAMPLE CALCULATIONS

In the derivation of the long-term distributions of ship motions and bending moments, it is necessary to fit a log-normal distribution to the truncated histograms computed in Tables 8 through 11. This requires the calculation of the mean value and the standard deviation from the truncated data.

The method and tables of Reference 7 are applied in making these calculations as indicated below. In the calculations, the symbols used are: σ for standard deviation, y and z for the parameters needed to enter Table IX of Reference 7, z being an estimate of the point of truncation.

LONG-TERM DISTRIBUTION OF VARIATION IN PITCH ANGLE

The mean value and standard deviation of the long-term distribution of the variation in pitch angle are computed from the data given in Table 8. These data are truncated at a pitch angle of 1/2 deg (θ = 5).

Variation deg × 10 θ	$\log_{10}\theta$ at End of Class Interval	$\log_{10}\theta$ at Center of Class Interval h	$\log_{10}\theta$ Measured from Point of Truncation $x = h - h_T$	$x^2 = (h - h_T)^2$	Percent of Variations Falling within Class N	Nx^2	Nx
0	$-\infty$						
5	$0.6990 = h_T$	0.8495	0.1505	0.0227	25.4	0.577	
10	1.0000	1.1505	0.4515	0.2039	36.2	7.381	
20	1.3010	1.4516	0.7526	0.5664	17.5	9.912	
40	1.6021	1.6902	0.9912	0.9825	5.6	5.502	
60	1.7782	1.8407	1.1417	1.3035	2.96	3.858	
80	1.9031	1.9516	1.2526	1.5690	1.28	2.008	
100	2.0000	2.0731	1.3741	1.8882	0.608	1.148	
140	2.1461	2.2007	1.5017	2.2551	0.0571	0.129	
180	2.2553				89.605	30.515	44.792

In accordance with the procedure outlined on page 29 of Reference 7, we have:

$$y = \frac{\Sigma N x^2 \Sigma N}{2(\Sigma N x)^2} = \frac{30.515\,(89.61)}{2\,(44.79)^2} = 0.6815$$

$z = -1.087$ (from Table IX of Reference 7, corresponding to $y = 0.6815$)

$g(z) = 0.7798$ (from Table IX of Reference 7)

$$\sigma \approx s = \frac{\Sigma N x}{\Sigma N}\, g(z) = \frac{44.79\,(0.7798)}{89.61} = 0.3898$$

Theoretical percentage of truncation = 13.9 percent (from Table II of Reference 7, corresponding to $z = -1.087$).

Mean value of $x = -zs =$ 1.087 (0.3898) = 0.4237 = $\bar{x}$.

Mean value of $h = 0.6990 + \bar{x} = 0.6990 + 0.4237 = 1.1227$, $\theta = 13.26$

Mean value of θ = antilog of 1.1227 = 13.26

Therefore the mean value of the pitch angle = 13.26 ÷ 10 = 1.326°

The value of the variate h corresponding to a probability of 97.5 percent is 1.96 standard deviations greater than the mean value of h. Let this value be $h_{97.5}$. Then $h_{97.5} = \log(\theta_{97.5}) = 1.1227 + 1.96\ (0.3898) = 1.8867$, $\theta_{97.5} = 77.0$. Therefore the pitch angle corresponding to a probability of 97.5 percent is 7.7 deg. The two sets of values of pitch angle and probability (1.326 deg, 0.50), (7.7 deg, 0.975) define the straight line in Figure 7 which represents the log-normal distribution of pitch angle.

APPENDIX C

PHOTOGRAPHIC DEFINITION OF SEA CONDITIONS

Photographs were taken with stereo-cameras during the sea voyages at estimated State 2, 4, and 5 seas. Photogrammetric analysis results in wave profiles at various distances from the centerline of the ship; see Figure 13. By use of these profiles, it is possible to obtain quantitative values of wave height and wave length which can then be treated statistically to determine, for example, the mean value of the one-third highest waves, commonly referred to as the significant wave height.

Figure 12a - Photograph 525F, Sea State 2

Figure 12b - Photograph 551F, Sea State 4

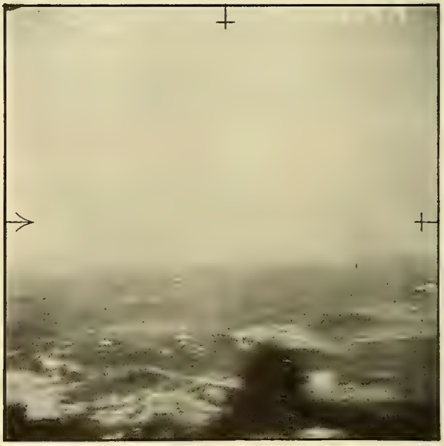


Figure 12c - Photograph 515A, Sea State 5

Figure 12 - Wave Photographs

The stereo cameras were located 52 ft apart approximately 56 ft above the ship's baseline.

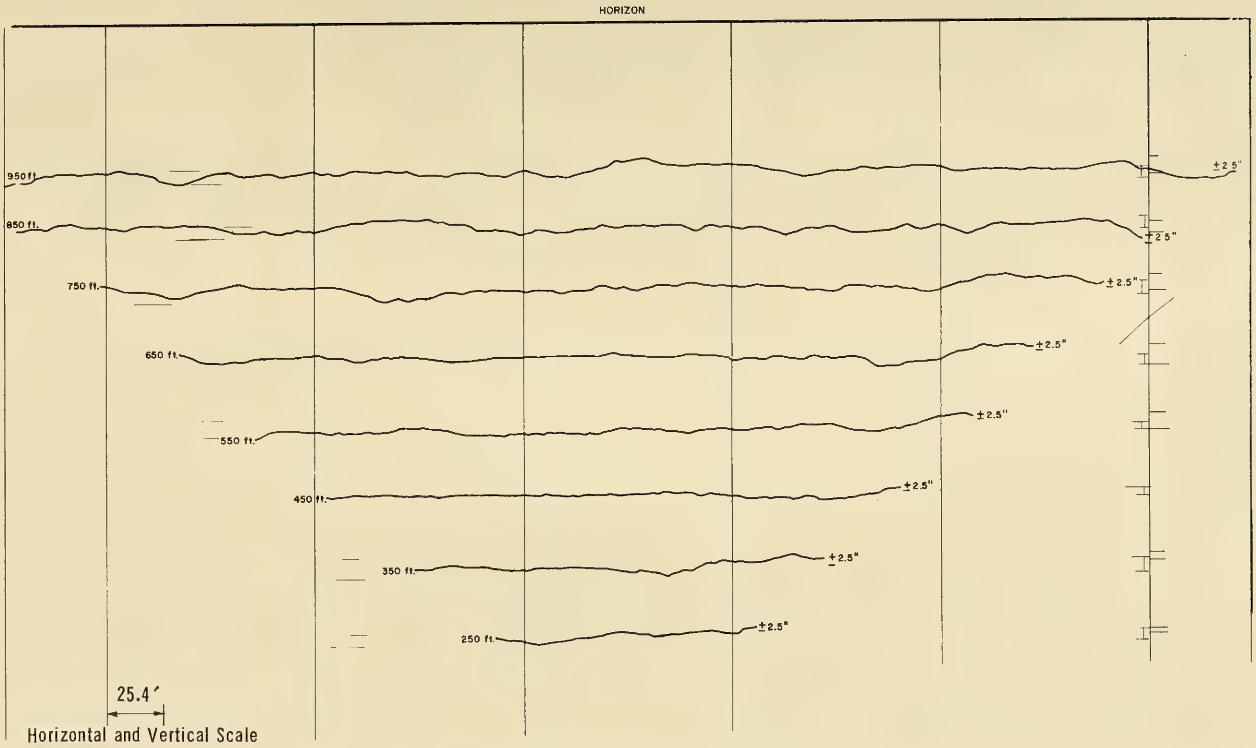


Figure 13a - Derived from Photograph 525, Sea State 2

(Photograph taken on January 28, 1955, at 1516 Greenwich time)

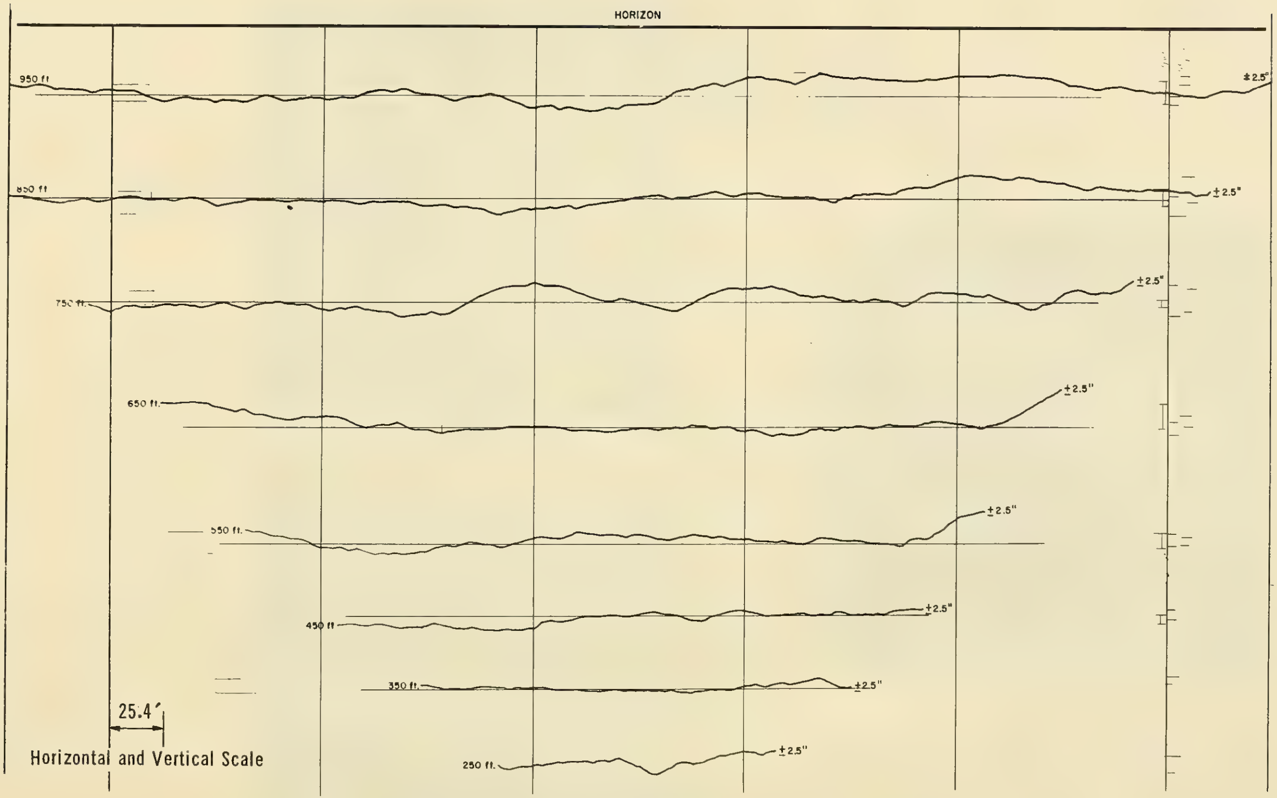


Figure 13b - Derived from Photograph 551, Sea State 4

(Photograph taken on February 2, 1955, at 1639 Greenwich time)

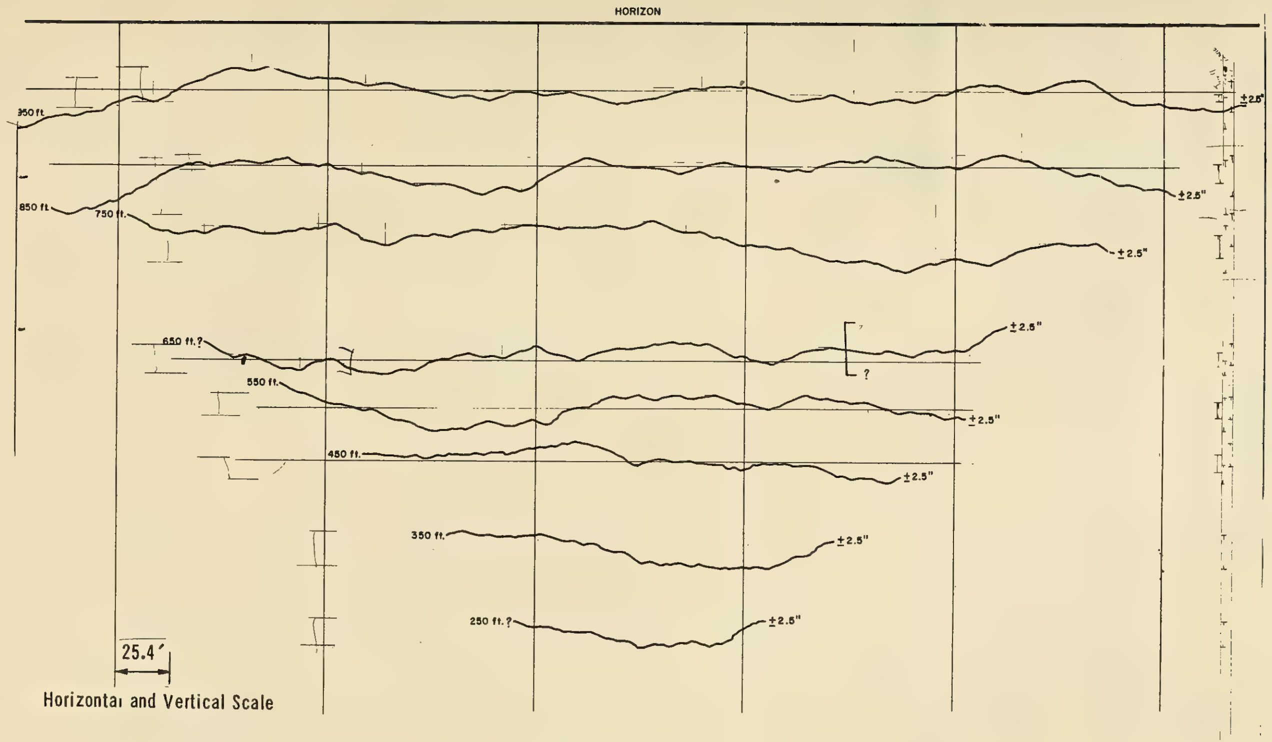


Figure 13c - Derived from Photograph 515, Sea State 5

(Photograph taken on January 16, 1955, at 1605 Greenwich time)

Figure 13 - Wave Profiles Derived from Stereo-Photographs

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The motions and hull-girder bending moments which a ship of the general form and size of the AVP10 class may be expected to experience over a wide range of operating conditions are presented in statistical form. The data are based on extensive measurements made on the USCGC UNIMAK during sea trials in the North Atlantic Ocean. The methods of statistics have been employed in the planning of the at-sea phases of the trials and in the collection, analysis, and presentation of the large amount of data. From

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4. Ships – Motion – Statistical analysis
5. Ship hulls – Bending moments
6. Ship hulls – Slamming – Pressure distribution
7. UNIMAK (U.S. Coast Guard Cutter)

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