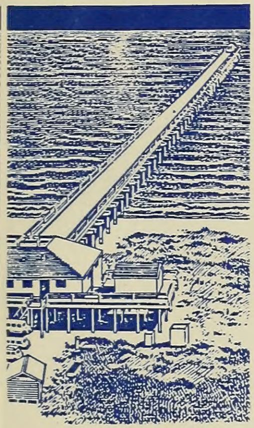
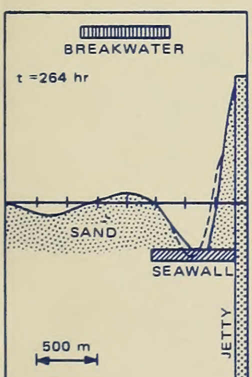
US Army Corps
of Engineers





TECHNICAL REPORT CERC-86-3

SEAWALL BOUNDARY CONDITION IN NUMERICAL MODELS OF SHORELINE EVOLUTION

by

Hans Hanson

Department of Water Resources Engineering
Lund Institute of Technology
University of Lund
Box 118
Lund, Sweden, S-221-00

and

Nicholas C. Kraus

Coastal Engineering Research Center
Waterways Experiment Station, Corps of Engineers
PO Box 631, Vicksburg, Mississippi 39180-0631

April 1986

Final Report

Approved For Public Release; Distribution Unlimited

Prepared for

DEPARTMENT OF THE ARMY
US Army Corps of Engineers
Washington, DC 20314-1000

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: Technical Report CERC-86-3
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): SEAWALL BOUNDARY CONDITION IN NUMERICAL MODELS OF SHORELINE EVOLUTION
5. TYPE OF REPORT & PERIOD COVERED: Final report
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s):
Hans Hanson
Nicholas C. Kraus
8. CONTRACT OR GRANT NUMBER(s):
9. PERFORMING ORGANIZATION NAME AND ADDRESS:
Department of Water Resources Engineering
Lund Institute of Technology
University of Lund
Box 118, Lund, Sweden S-221-00
and US Army Engineer Waterways Experiment Station
Coastal Engineering Research Center
PO Box 631, Vicksburg, Mississippi 39180-0631
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS:
11. CONTROLLING OFFICE NAME AND ADDRESS:
DEPARTMENT OF THE ARMY
US Army Corps of Engineers
Washington, DC 20314-1000
12. REPORT DATE: April 1986
13. NUMBER OF PAGES: 59
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report):
Approved for public release; distribution unlimited
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):
18. SUPPLEMENTARY NOTES:
Available from National Technical Information Service, 5285 Port Royal Road, Springfield, Virginia 22161.
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Sea-walls (LC)
Coast changes--Mathematical models (LC)
Beach erosion--Mathematical models (LC)
Shore protection (LC)
Coastal engineering (LC)
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
This report gives a complete description of the method of Hanson and Kraus (1985) for implementing the seawall boundary condition in the shoreline change numerical model. The physical basis and governing principles are discussed in a descriptive way in Parts I and II. Technical details of the shoreline model and implementation of the seawall boundary condition are given in Part III. Example calculation and computer programs for both the explicit and implicit numerical solution schemes are also given.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(*When Data Entered*)

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE(*When Data Entered*)

PREFACE

The investigation described in this report was authorized as a part of the Civil Works Research and Development Program by the Office, Chief of Engineers (OCE), US Army. The work was performed under the work unit Numerical Modeling of Shoreline Response to Coastal Structures, which is part of the Shore Protection and Restoration Program. Mr. J. H. Lockhart, Jr., and Mr. John G. Housley were the OCE Technical Monitors.

The study was conducted from 1 October 1984 through 30 April 1985 by Dr. Nicholas C. Kraus, Research Physical Scientist, Coastal Engineering Research Center (CERC), US Army Engineer Waterways Experiment Station (WES), in conjunction with related engineering studies by Mr. Hans Hanson of the University of Lund, Sweden. This report presents the overall results of these efforts. The CERC portion of the study was under the general supervision of Dr. Robert W. Whalin, former Chief, and Dr. James R. Houston, present Chief, CERC, and former Chief, Research Division, and Manager, Shore Protection and Restoration Program; Mr. Charles C. Calhoun, Jr., Assistant Chief, CERC; Mr. H. Lee Butler, Chief, Coastal Processes Branch; and Dr. S. Rao Vemulakonda, Principal Investigator, Numerical Modeling of Shoreline Response work unit. Ms. Joan Pope, Research Physical Scientist, Coastal Structures Evaluation Branch of CERC, made a critical review of an early version of the manuscript. Comments on this publication are invited.

COL Allen F. Grum, USA, was Director of WES at the time of publication of this report. Dr. Whalin was Technical Director.

CONTENTS

LIST OF FIGURES

SEAWALL BOUNDARY CONDITION IN NUMERICAL MODELS OF SHORELINE EVOLUTION

PART I: INTRODUCTION

Overview

1. This report provides potential users with a complete description of the method developed by Hanson and Kraus (1985) for implementing the seawall boundary condition in the shoreline change numerical model. Example runs are included so that users may test their programs. Computer programs written in FORTRAN 77 are given and explained for both explicit and implicit finite-difference numerical solution schemes.

2. The governing principles for the seawall boundary condition are summarized in Part I. The physical basis of the seawall boundary condition is discussed in a general and descriptive way in Part II. Parts I and II provide background material and can be understood without knowledge of numerical modeling. Technical details of the shoreline numerical model and implementation of the seawall boundary condition are given in Part III. Two example calculations and a discussion of numerical accuracy and efficiency are given in Part IV. The computer programs are described in Part V and listed in Appendix A.

Purpose of Seawalls

3. Chronic erosion is found along many portions of the coast of the United States and other coasts of the world. Coastal erosion is caused by diverse factors. These include rise in mean sea level, increase in severity of incident waves, change in local magnitude and direction of incident waves (as produced, e.g., by a newly installed coastal structure), loss of sediment supply from rivers and cliffs, and interruption of the local littoral drift by structures. If the cause of undesirable erosion in an area cannot be eliminated or corrected, then buildings, roads, and other resources will eventually become endangered, and some degree of shore protection must be undertaken. Chapter 1 of the Shore Protection Manual (SPM 1984) contains a detailed discussion of the causes of coastal erosion and their remedial measures.

4. The shore can be protected against erosion through the use of coastal structures, nonstructural procedures, such as beachfill, or a combination of structures and nonstructural methods (SPM 1984; US Army Corps of Engineers 1981). In situations where extensive damage may occur because of storm waves and water intrusion, or where nonstructural procedures are not feasible, then seawalls, bulkheads, and coastal dikes are commonly constructed for beach erosion control and for preventing inundation. If the word "seawall" is used to describe any man-made or natural object which functions as a nonerodible barrier along the shoreline, the concept of "seawall" encompasses true seawalls, coastal dikes, storm surge barriers, shore-connected breakwaters, bulkheads, revetments, and rocky coastal cliffs. A coast may contain several such seawalls, and their presence must be taken into account when assessing the long-term (order of years) evolution of the shoreline.

5. It is also necessary to estimate the impact of a proposed seawall in the design process for shore protection. Even a wide sandy beach cannot erode indefinitely; at some point in time the beach material will be exhausted, and permanent structures and resources will become exposed to wave action and inundation. In such a situation, emergency protective measures will be taken, most likely by the construction of a revetment, bulkhead, or seawall. A numerical model of shoreline change must allow for the real world situation of the ultimate presence of a seawall.

Seawalls and the Shoreline Change Model

6. Numerical models provide a powerful means for making quantitative estimations of shoreline evolution. In particular, the so-called "one-line" numerical model, originating from the work of Pelnard-Considere (1954), has been widely applied in recent years. Kraus (in preparation) gives an annotated bibliography of the literature on one-line models. The term "one-line" typically refers to the shoreline; therefore, this model is often called the "shoreline" model. Despite the large number of applications of the shoreline model, representation of the action of a seawall in the model has received little attention. A seawall imposes a constraint, or boundary condition, on the solution (shoreline position) obtained with the model.

7. The most obvious boundary condition imposed by a seawall is that the beach fronting the wall cannot move landward of it. Also, a seawall prevents the sediment contained behind it from entering the littoral system,

thereby modifying the sand transport rate along the beach and possibly starving the adjacent beach through the elimination of potential littoral material. In an extreme case, if the level of the beach in front of a seawall drops, waves will reflect from the wall instead of dissipating on the beach. Standing waves can cause local scour that may temporarily increase transport alongshore or offshore, until a new, steeper equilibrium profile is achieved. The integrity of the seawall may be threatened when the beach elevation drops.

8. In the literature, there has been very little discussion on representation of a seawall in the shoreline model or in other models. Essentially all of the work reported to date has been conducted by engineers associated with coastal engineering in Japan. More than 25 percent of Japan's 34,000-km-long (21,000-mile) coastline is protected by seawalls, coastal dikes, armor blocks, and similar structures (Ogawara 1983).

9. In the early 1970's, Hashimoto et al. (1971) discussed the behavior of the longshore sand transport rate in front of a seawall armored by blocks. They recommended the longshore transport rate be set to zero if the shoreline reaches the seawall. Ozasa and Brampton (1980) treated the loss of berm in front of a seawall and devised prescriptions for introducing the action of a seawall in the shoreline numerical model. In essence, their procedure also consists of setting the longshore sand transport rate equal to zero at calculation points where the berm has been removed and the shoreline has retreated to the seawall. Hanson and Kraus (1980) gave a procedure in the form of a simple shoreline adjustment, but this alone is unsatisfactory because it does not conserve sand volume. Tanaka and Nadaoka (1982) noted that the procedure of setting the transport rate to zero is not correct. They proposed two alternative methods, but unfortunately their methods appear to be arbitrary and incomplete.

10. Recently, Hanson and Kraus (1985) have given an outline of a well-tested procedure for representing the action of a seawall in balance with the capability of the shoreline numerical model and in accordance with three general principles. The present report gives a complete description of their method. The physical reasoning behind the method is discussed in Part II. The principles upon which the method is based are:

a. The shoreline in front of a seawall cannot recede landward of the seawall.

b. Sand volume must be conserved.

c. The direction of sand transport alongshore must be preserved in accordance with the natural direction of the potential local transport.

11. Although the above-listed principles are easy to understand, their implementation in a computer program is considerably involved, in particular, for b and c. The present report describes well-tested algorithms for implementing the seawall boundary condition in a general manner.

Limitations of the Method

12. The seawall constraint should be formulated on the same level of idealization as the shoreline model. Thus, it is not appropriate in the model to consider wave reflection and sea bottom scouring, and settling, flanking, and collapse of the seawall (for further discussion on one or more of these topics, see Sato, Tanaka, and Irie 1969; Silvester 1977; Toyoshima 1979; Walton and Sensabaugh 1979). It should be stressed that the procedure described here possesses the same limitations as well as the same advantages as the shoreline model. The seawall boundary condition is only valid to the extent the shoreline model is valid.

13. One of the most restrictive assumptions made in deriving the shoreline model is that the beach profile remains unchanged and moves seaward or shoreward in parallel to itself (an assumption of equilibrium of the profile). In nature, however, if a beach erodes to reach a vertical or nearly vertical seawall, due to wave reflection and scouring, the beach slope immediately in front of a seawall is expected to become steeper than the slope on the adjoining beach without structures or steeper than the original beach before the seawall was built.

14. The above discussion notwithstanding, examples can be found in the field of the growth and recovery of formerly eroded beaches fronting rough-faced sloping seawalls (Toyoshima 1979); nearly vertical seawalls (O'Brien 1985); and even a vertical seawall (Berrigan 1985a,b). Because of an apparent lack of data in these cases, however, cause and effect have not been clearly distinguished. That is, it is not known with certainty whether the seawalls promoted growth of the beaches in front of them or if, e.g., sediment transport conditions changed to bring back the beaches with no relation to the seawall, the seawall only initially functioning to protect the land behind it. A combination of the two scenarios is also possible. Toyoshima (1979) states

that a rough-faced sloping and, ideally, permeable seawall will promote recovery by dissipating wave energy, similar to the functioning of a natural beach.

15. Based on the results of their laboratory experiments, Hattori and Kawamata (1977) found that a necessary condition for the naturally occurring restoration of an eroded beach backed by a seawall is that a surf zone exist seaward of the wall. Essentially the same conclusion had been reached in an earlier laboratory study by Chestnutt and Schiller (1971). Clearly, results of simulations incorporating the seawall boundary condition in a shoreline model must be interpreted with caution.

16. In order to account for an alongshore variation in beach slope, a mechanism to allow for cross-shore sand transport and a more complicated numerical scheme than that used in the shoreline model are required. Numerical models now exist which account for cross-shore transport in a schematic way. The "2-line" model of Bakker (1969) and Bakker et al. (1971), and the "N-line" model of Perlin and Dean (1978, 1983) are examples. Such models can, in principle, more realistically represent the beach slope in front of a seawall than can the shoreline model.

17. At present, however, these models, although more sophisticated than the shoreline model, have limitations for engineering use stemming from lack of knowledge of the physical mechanism of cross-shore sand transport. Numerical instability and long computer run times are the main technical problems encountered. Relatively short calculation time is an appealing feature of the shoreline model. This feature, plus its demonstrated versatility for handling a wide range of boundary conditions, ensures the use of the shoreline model as an engineering tool in the foreseeable future.

18. In summary for this section, to the extent that changes in beach cross section can be neglected in comparison to changes in beach planform, the shoreline model is a useful engineering tool for systematically investigating and estimating shoreline evolution over time periods of several months to several years. If seawalls are located along the coast, because of possible significant changes in beach cross section, particular caution should be exercised in interpreting model results.

19. As progress is made, it will become desirable to incorporate the seawall boundary condition in models more sophisticated than the shoreline model. This task may prove to be difficult. Experience and familiarity with the implementation of the seawall boundary condition in the shoreline model should provide useful guidance.

PART II: BACKGROUND FOR THE SEAWALL BOUNDARY CONDITION

Action of a Seawall on a Beach

20. There is remarkably little quantitative information available on the behavior of real beaches backed by seawalls. It has been long known that under certain wave conditions, a vertical seawall will accelerate erosion of the beach in front of it (see Russell and Inglis 1953; Sato, Tanaka, and Irie 1969). Scour is the primary cause of this erosion. Sand is scoured from the sea bottom in front of a vertical seawall by the standing wave system produced by wave reflection at the wall. Any current, such as the longshore current, can then transport the mobilized sand out of the area. If there is a continued net loss of sand over a long period of time, the end result is that the beach in front of the seawall can no longer maintain the natural equilibrium profile and the beach slope will become steeper. Walton and Sensabaugh (1979) discuss this and other processes believed to enhance erosion of beaches backed by vertical or nearly vertical seawalls.

21. On the laboratory scale, it has been amply demonstrated that a seawall does not always produce erosion when introduced in the active wave zone of a beach in equilibrium with the existing waves. A brief discussion will now be given of three experiments (Dorland 1940, Chestnutt and Schiller 1971, and Hattori and Kawamata 1977) performed using sand beaches in two-dimensional wave flumes.

22. Dorland (1940) used moderately steep waves in an attempt to reproduce storm conditions. He placed a vertical seawall at the shoreline of a beach which had been allowed to attain equilibrium under constant wave action, scooped out part of the bed in front of the seawall, and then continued applying the waves. In the two such experiments performed, the outer bar moved landward and the scooped out area partially filled with sand from the offshore. In a third series of runs using three sets of wave conditions varying cyclically, Dorland similarly found that the scooped out beach partially recovered.

23. Chestnutt and Schiller (1971) found that maximum erosion occurred if a seawall was placed on an equilibrium beach in a "critical" region lying from about $0.5\ x_b$ to $0.67\ x_b$, where x_b is the width of the surf zone, as measured from the shoreline. When the seawall was moved to a position shoreward of the critical region, the profile immediately seaward of the wall began

to accrete, i.e., the previously wave-scoured region tended to be filled. Chestnutt and Schiller point out that the surf zone width depends, in part, on the wave period. Other factors being the same, the surf zone will be wider for longer period waves. Therefore, whether or not a seawall will tend to promote erosion or accretion depends on the wave conditions, which usually have a marked seasonal variation.

24. Hattori and Kawamata (1977) recorded beach profile changes on a laboratory beach with and without a vertical seawall. For given wave conditions, the beach was allowed to attain equilibrium before introduction of the seawall. Incident wave steepness was varied for a fixed location of the seawall relative to the initial shoreline. They found the existence of a surf zone to be a necessary condition for recovery of an eroded seawall-backed beach. This result is in agreement with the findings of Chestnutt and Schiller (1971). The existence of a surf zone implies minimum wave reflection at the seawall. Hattori and Kawamata also found that the restoring wave conditions for a seawall-backed beach are similar to those for a natural laboratory beach without a seawall.

25. Movable bottom laboratory experiments are difficult to interpret because of scale effects, and longshore processes were absent in the experiments under discussion. Nevertheless, a reasonable conclusion to be drawn from the aforementioned work is that an eroded beach in front of a seawall tends to recover when the mean water level is low, the waves have mild steepness, and a sediment supply exists in the offshore. Toyoshima (1979), O'Brien (1985), and Berrigan (1985a,b) give examples of prototype beaches backed by seawalls which have become stable or have recovered to some degree.

26. The interaction between beaches and seawalls is far from understood. A focused and intensive field monitoring effort is definitely needed as a first step toward achieving quantitative understanding of the influence of a seawall on the shoreline and beach profile. Without data, quantitative understanding and numerical modeling of the processes involved will be limited and suspect.

Seawall at Oarai Beach, Japan

27. The physical picture for the seawall boundary condition formulated by Hanson and Kraus (1985) is based on general observations of the shoreline

change at the seawall located south of Oarai Harbor, Ibaraki Prefecture, Japan. A location map is given in Figure 1. Shoreline change at this site has been extensively documented and numerically modeled (Kraus, Hanson, and Harikai 1985). There are two seawalls on this north-south oriented sandy beach facing the Pacific Ocean. The north seawall is a continuous massive concrete wall 2 km (1.24 miles) long and 5 m (16.4 ft) high from base to crown. Portions of the north seawall at Oarai are shown in Figures 2 and 3. The face of the north seawall is mildly curved outward and armor blocks have sometimes been placed at the foot of the wall when the beach eroded. The south seawall is similarly constructed and 800 m (0.5 miles) long. Beach change at the north seawall has mainly been studied.

28. When the shoreline reaches the seawall, the local beach slope becomes slightly steeper than the typical nearshore slope on this coast (which itself varies between approximately 1/50 and 1/70 from the beach face to the wave breaker line). The change in beach slope is mild and appears to be negligible for purposes of applying the shoreline model. No drastic alteration in beach characteristics occurs and the beach is exposed at low tide (Figure 3). At high tide, when the shoreline has receded to the seawall, broken waves slap against the face of the wall.

29. Although the shoreline may reach the seawall at some location, it has been inferred on the basis of the observed and modeled long-term shoreline change that sand moves alongshore through the area to be deposited adjacent to a large groin at Oarai Harbor (Kraus, Harikai, and Kubota 1981; Mizumura 1982; Kraus and Harikai 1983; Kraus, Hanson, and Harikai 1985). Since alongshore variations in the slope of the beach in front of the seawall are small, the seawall does not appreciably alter the pattern of wave breaking. A surf zone usually exists in front of the seawall and the capacity for waves to move sand alongshore is retained. Sand is transported in the direction of the wave-induced longshore current, and the beach in front of the seawall has been observed to periodically erode and recover.

Idealized Seawall Boundary Condition

30. From the observations described above, Hanson and Kraus (1985) developed the concept of the idealized functioning of a seawall for use with the shoreline model. They concluded that once the shoreline reaches a seawall at

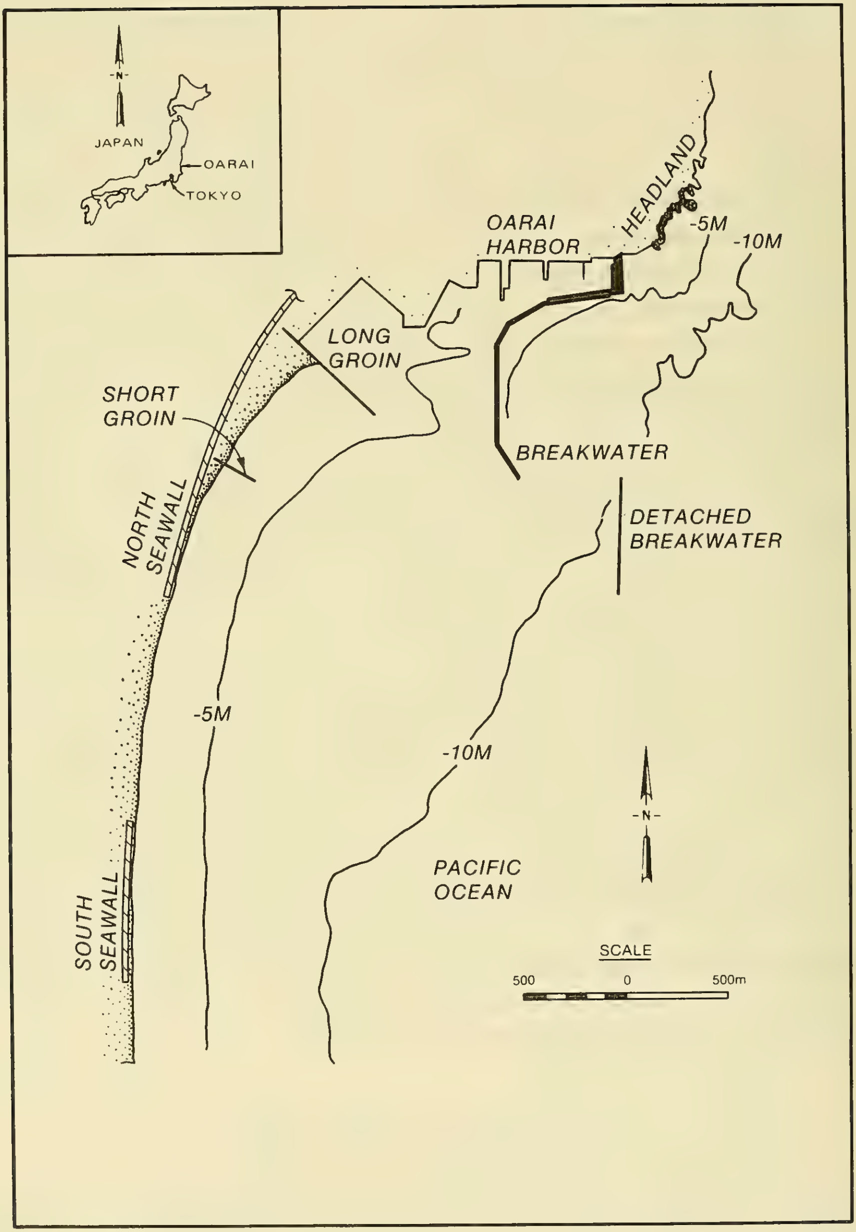


Figure 1. Location map for the beach and seawalls at Oarai Harbor, Japan

Figure 2. North seawall at Oarai, Japan, May 1980

Figure 3. South end of north seawall at Oarai, Japan, May 1980
(Seawall face in lower right portion of photograph)

a particular location, sand cannot originate from that area. There can be a net gain, but no net loss, for a beach area in contact with a seawall (since it is assumed in the shoreline model that the beach level does not drop below the water line and that the beach slope does not change). However, sand can move alongshore through such an area, passing into and out of its boundaries, according to the natural direction of transport. Sand can also be deposited in front of a seawall, thus allowing the beach to recover.

31. In the shoreline model, it would be incorrect to set the transport rate equal to zero at a location where the shoreline makes contact with a seawall, as done in most previous treatments. Rather, the transport rate should be adjusted to allow calculation cells in contact with a seawall to transfer sand in order to conserve total sand volume and preserve the direction of its transport.

32. On real beaches, sand is not always transported in the same direction over the full length of the beach. Changes in the direction of transport may be produced, for example, by longshore variations in wave direction and wave height as caused by refraction over an irregular bottom, or by diffraction at structures and headlands. Therefore, at one or more areas along a beach, it is possible that a net amount of sand is moving out of the area. The ways in which this can occur, and implications for shoreline change in the presence of a seawall, are described in the section <u>Model Input Requirements and Boundary Conditions,</u> in Part III.

PART III: SHORELINE MODEL AND THE SEAWALL BOUNDARY CONDITION

Shoreline Model Review

33. The theory of the shoreline model originated with Pelnard-Considere (1954). He assumed that the beach bottom, not necessarily of planar slope, always remains in equilibrium and, as a consequence, moves in parallel to itself down to a certain depth, herein called the depth of closure. Therefore, one contour, or "line," is sufficient to describe changes in beach planform. This line is conveniently taken as the shoreline. Pelnard-Considere did not develop a numerical model but did give closed-form mathematical solutions for certain idealized cases and verified the results through laboratory experiments. Details of the numerical formulation of the model may be found in, e.g., Komar (1976, 1983), Le Méhauté and Soldate (1978) and Hanson and Kraus (1980).

34. The purpose of the shoreline model is to simulate long-term evolution of the shoreline or the beach planform. The governing equation for the shoreline position is obtained from the continuity equation for beach sediment (assumed to be cohesionless sand). A predictive formula for the sand transport rate is necessary to solve the governing equation. Sand transport and the resultant shoreline change depend on the local wind, waves, and currents, beach planform, boundary conditions, and constraints such as the one produced by a seawall. It will be assumed here that the longshore sand transport is produced solely by obliquely incident waves; other transport mechanisms are possible, such as coastal, tidal, and wind-generated currents.

35. In the present work, it will be sufficient to use the equation for the shoreline position in its most basic form:

$$\frac{\partial y}{\partial t} + \frac{1}{D} \frac{\partial Q}{\partial x} = 0 \qquad (1)$$

where

- y = shoreline position, m
- t = time, s
- D = depth of closure, m
- Q = volume rate of longshore sediment transport, m^3/s
- x = distance alongshore, m

For simplicity, only longshore transport of sand is considered. It is straightforward to generalize Equation 1 to formally include contributions for cross-shore transport, as well as sediment sources and sinks. An equation given by Hallermeier (1979, 1983) for a limiting depth of sand motion in terms of the incident wave conditions has been recommended by Kraus and Harikai (1983) for use as the depth of closure (see also Kraus 1984).

Model Input Requirements and Boundary Conditions

36. In order to solve Equation 1, three kinds of information are required: (a) the initial location of the shoreline with respect to some coordinate system (Figure 4) in which the x-axis is oriented along the trend of the coast and the y-axis points offshore, (b) an expression for the longshore sand transport rate, Q, and (c) boundary conditions for either y or Q at the two lateral ends of the beach. Of these, the initial position of the shoreline is readily obtained or assumed.

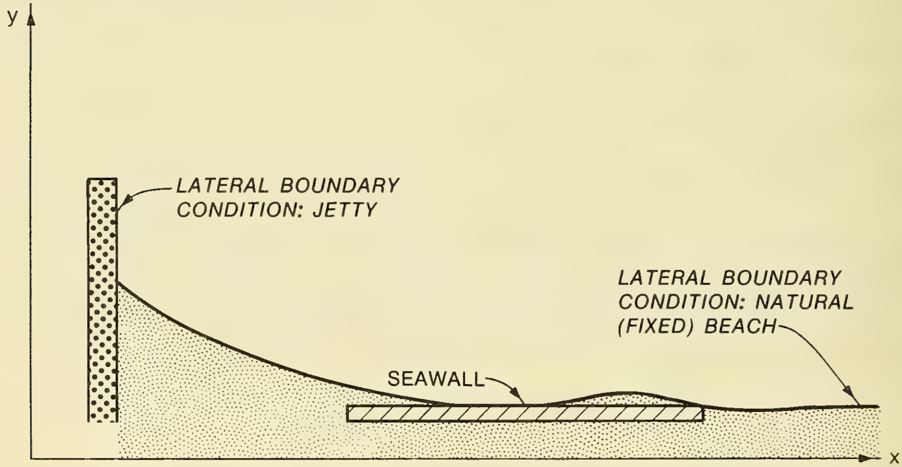


Figure 4. Definition sketch for coordinate system, shoreline, seawall, and lateral boundary conditions

37. The longshore transport rate, Q, is usually calculated from the "CERC" formula (SPM 1984, Chapter 4):

$$Q = K' \left(H^2 C_g\right)_b \sin 2\,\theta_{bs} \tag{2a}$$

$$K' = \frac{K}{16(S - 1)a'} \left(\frac{1}{r}\right)^{5/2} \quad (2b)$$

where

- K = dimensionless empirical coefficient (of order 0.4)
- H = significant wave height, m
- C_g = wave group velocity, m/s
- θ_{bs} = angle of breaking waves to the shoreline, deg
- S = ratio of sand density to water density
- a' = volume of solids/total volume
- r = conversion factor from Root Mean Square (RMS) to significant wave height, if necessary (equals 1.416)

The subscript b indicates quantities at wave breaking. The group velocity at breaking is calculated from:

$$\left(C_g\right)_b = \left(\frac{g\ H_b}{\gamma}\right)^{1/2} \quad (3)$$

where

- g = acceleration of gravity, m/s^2
- γ = ratio of wave height to water depth at breaking, approximately equal to 0.78

38. The angle θ_{bs} is the angle of the breaking waves to the shoreline. It is equal to the difference between the angle the breaking waves makes with the x-axis and the angle the shoreline makes with the x-axis:

$$\theta_{bs} = \theta_b - \tan^{-1}\left(\frac{\partial y}{\partial x}\right) \quad (4)$$

where

- θ_b = angle of breaking waves to x-axis, deg

39. Common lateral boundary conditions are $Q = 0$ at an impermeable barrier such as a long jetty or groin, and $\partial Q/\partial x = 0$ on a beach that has a stable (fixed) shoreline position. The latter boundary condition on Q can also be expressed as $\partial y/\partial t = 0$ (see Equation 1).

40. In addition to lateral boundary conditions, which are necessary to solve any problem, it is sometimes required to constrain the solution, i.e., restrict movement of the shoreline position. For example, the shoreline along

the beach backed by a seawall cannot recede behind the wall. In this report, the seawall constraint is referred to as a boundary condition although it is not a boundary condition in a true sense.

41. Three terms will be defined to distinguish important transport situations which can occur at a seawall.

Minus area (Figure 5a)

42. The expression "minus" area (minus calculation cell in the numerical model) is applied if, at a given time, sand is transported out of both sides of the area. If a minus area occurs where the shoreline has eroded to a seawall, then the sand transport rate must be corrected in such a manner as to conserve sand volume and preserve direction of transport, in order to pass information about the lateral boundary conditions. In the method described in this report, transport rate corrections along the beach are made in the direction of sediment transport, i.e., in the downdrift direction. Therefore, minus cells are starting points for corrections.

Plus area (Figure 5b)

43. If sand is moving into an area from both sides at a given time, this condition defines a "plus" area (plus calculation cell in the numerical model). The terminology "plus cell" describes the reverse situation of a minus cell; consequently, transport rate corrections end at plus cells (or at lateral boundaries).

Regular area (Figure 5c)

44. The most common situation is for a certain quantity of sand to enter one side of an area and for a slightly different quantity of sand to leave the area on the opposite side. This is called a "regular" area (regular cell in the numerical model). Sand volume and direction of transport must be preserved whether or not there is a local net gain or net loss of material. If the shoreline in a regular area is in contact with a seawall, no more sand can leave the cell than enters it. If the converse occurs, causing the nonphysical movement of the shoreline to a position landward of the seawall, the transport rates must be corrected in an appropriate manner to move the shoreline position to the seawall.

45. If a wide beach exists in front of a seawall, it is not necessary to distinguish between minus areas, plus areas, and regular areas. These three concepts become important only when the shoreline makes contact with a seawall.

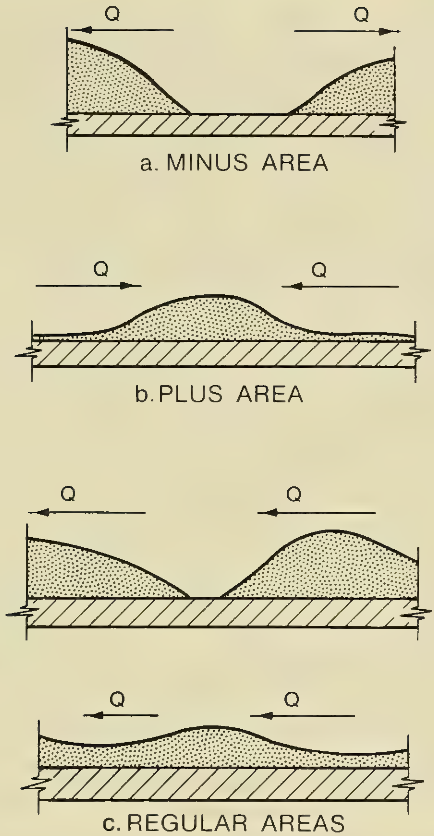


Figure 5. Conceptual diagram showing minus, plus, and regular areas

Explicit Numerical Model

46. Equation 1 will be discretized using a staggered grid representation, as shown in Figure 6. For convenience, Equation 1 is reproduced here.

$$\frac{\partial y}{\partial t} + \frac{1}{D}\frac{\partial Q}{\partial x} = 0 \qquad (1)$$

The x-axis, which runs parallel to the trend of the shoreline, is divided into N calculation cells by $N + 1$ cell faces (solid vertical lines in Figure 6), with a general cell denoted by i . On this grid, Q-points and y-points are defined alternately. Q-points define calculation cell faces and y-points lie at the centers of cells. Subscripts denote locations of points along the beach. Both Q-grid points and y-grid points are separated by a constant distance Δx alongshore; the distance between a Q-point and an adjacent y-grid point is $\Delta x/2$. Lateral boundary conditions must be specified at the ends of the grid, e.g., at Q_1 and Q_{N+1}. Alternatively, it is possible to specify boundary conditions at y_1 and y_N, or impose a condition on y at one end of the grid and a condition on Q at the other end.

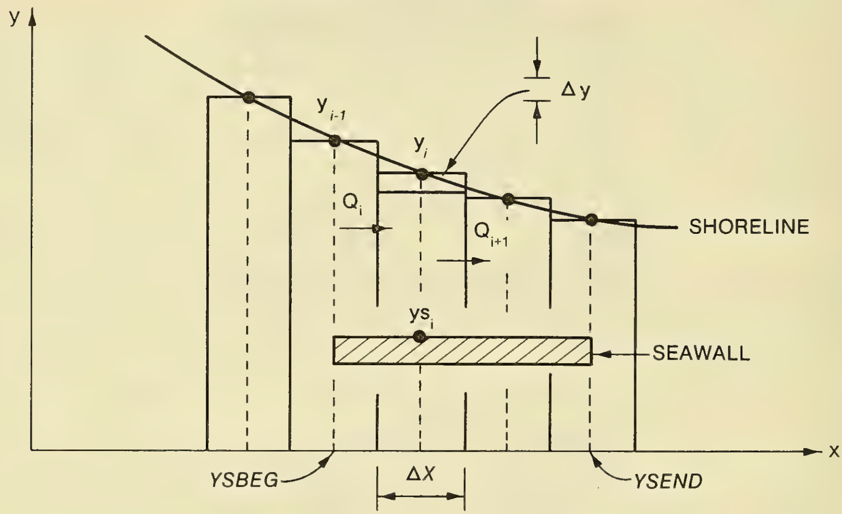


Figure 6. Definition sketch for finite difference discretization

47. For simplicity, only one seawall will be considered. Its beginning and ending coordinates on the x-axis are denoted by YSBEG and YSEND, respectively, as shown in Figure 4. A general y-position at the seawall is denoted by ys_i.

48. In a standard explicit scheme, Equation 1 is discretized as

$$y_i' = 2B\,(Q_i - Q_{i+1}) + y_i \tag{5}$$

where

$B = \Delta t/(2D\Delta x)$, s/m^2

Δt = time step, s

Δx = space interval, m

49. For notational convenience, a prime on a quantity will denote its value at the next (future) time step; an unprimed quantity is evaluated at the present time step. Quantities at the present time step are known. In customary notation, the next time step is denoted by a superscript $n + 1$ and the present time step is denoted by a superscript n. The customary notation will be used in certain applications to follow.

50. For the purpose of implementing a boundary condition, or constraint, the explicit model is convenient since (a) only immediately neighboring values of Q_i and y_i are involved, and (b) the implementation only involves the present shoreline position and present transport rates; no quantities at the next time step are used.

51. If the shoreline moves landward of the position of the seawall at a certain grid point, thus violating the seawall constraint, the longshore sand transport rate must be corrected to conserve sand volume. The (nonphysical) erosion, or retreat, of the shoreline to a position behind a seawall, as shown in Figure 7, results in a nonphysical additional transport of sand out of the associated calculation cell. The transport rates at the cell faces must therefore be corrected to prevent the shoreline from moving behind the seawall. The correction must be made with consideration of the direction of transport at the two faces of the particular cell violating the seawall constraint. Only minus cells and regular cells may require correction. The seawall constraint is never violated at a plus cell, because the shoreline always advances in a plus cell.

52. The calculation procedure is described in detail next. An overview is as follows. First, the transport rates along the beach are calculated in order to determine the transport directions and to identify minus, plus, and regular cells. Then, as required, corrections start at either a seawall boundary or the first minus cell encountered in the search. After the starting cell is corrected, corrections to regular cells are made as necessary following the direction(s) of the longshore transport, until either a plus cell or a lateral boundary is reached. This procedure is repeated at each time step.

Correction at a
minus cell (Figure 7a)

53. Since correction is necessary, the shoreline position y_i' lies behind the seawall. The general principle governing transport corrections is that the transport rate at a downdrift cell face should be reduced to a value

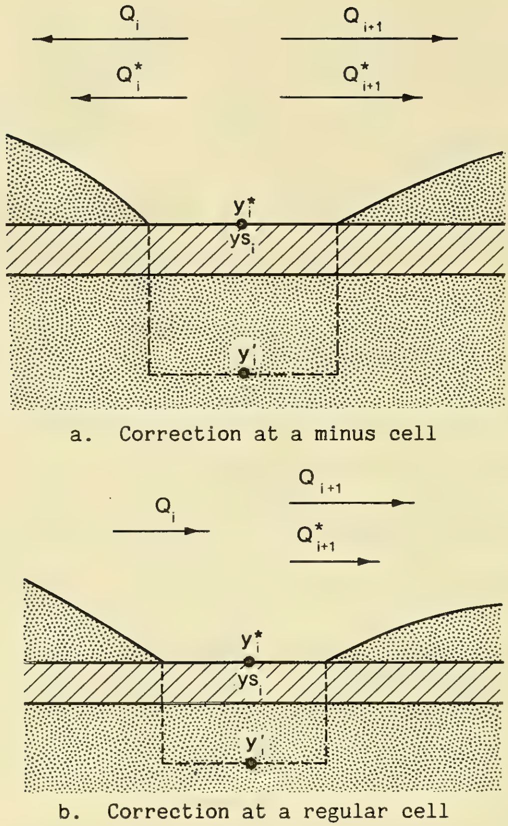


Figure 7. Conceptual diagram showing shoreline and transport corrections at minus and regular cells

that will place the shoreline at the seawall. In a minus cell, the transport rates at both cell faces are directed outward; therefore, both need to be adjusted. It does not appear that the adjustments can be specified in a unique way. Hanson and Kraus (1985) calculate corrected transport rates as equal proportions of the original rates, as follows (with corrected quantities denoted by a superscript asterisk):

$$Q_i^* = Q_i \frac{y_i - ys_i}{y_i - y_i'} \qquad (6a)$$

$$Q_{i+1}^* = Q_{i+1} \frac{y_i - ys_i}{y_i - y_i'} \qquad (6b)$$

The logic behind Equation 6 is perhaps more clearly understood by rearranging terms, to give, for example,

$$\frac{Q_i}{y_i - y_i'} = \frac{Q_i^*}{y_i - ys_i} \qquad (7)$$

from which it is seen that, whereas Q_i causes the shoreline to move from y_i to y_i', the corrected transport rate Q_i^* moves the shoreline from y_i to ys_i (as required).

54. By substitution of Equation 6 into Equation 5, with Q_i and Q_{i+1} replaced by Q_i^* and Q_{i+1}^*, respectively, it is verified that the corrected shoreline position is

$$y_i^* = ys_i \qquad (8)$$

Since the adjustment was made through use of the continuity equation, the procedure conserved sand volume.

Correction at a
<u>regular cell (Figure 7b)</u>

55. With corrections at the minus cell completed, adjustments continue for cells on both sides, following the direction of transport. The transport rate at an updrift face will have previously been corrected and should not be corrected again. Assuming for the purpose of explanation that the transport rate through a particular cell is in the positive x-direction, the adjusted downdrift transport rate Q_{i+1}^* is obtained by setting the new position y_i' equal to ys_i in Equation 5, to give

$$ys_i = y_i + 2B\left(Q_i - Q_{i+1}^*\right) \qquad (9)$$

The corrected transport rate is then

$$Q_{i+1}^* = Q_i - \frac{ys_i - y_i}{2B} \qquad (10)$$

As a simple check, insertion of Q^*_{i+1} from Equation 10 into Equation 5 gives the following desired result for the corrected shoreline position:

$$y^*_i = ys_i \tag{11}$$

Again, this is the mathematical statement of the shoreline constraint.

56. After Q^*_{i+1} and y^*_i are obtained, calculation moves to the next grid point to determine y'_{i+1} in similar manner. Calculation proceeds from cell to cell in the downdrift direction along the seawall until either a plus cell or the end of the seawall is encountered. If a plus cell is encountered calculation of the shoreline position continues without necessity for correction until another minus point or the end of the seawall is encountered.

57. On the other side of the original minus cell, where the transport rate is in the negative x-direction, analogous corrections are made to transport rates as described above, i.e., to Q_i . This allows determination of y'_{i-1} . The calculation then proceeds from cell to cell in the downdrift direction.

58. The programs YSEXP and CORRE, discussed in Part V and listed in Appendix A, calculate shoreline change with the explicit numerical scheme for a beach backed by a seawall.

Implicit Numerical Model

59. Compared to the straightforward development for the explicit scheme, as presented in the previous subsection, representation of the seawall constraint in an implicit numerical scheme is extraordinarily complex. In an implicit scheme, values of the new Q_i are solved for simultaneously, over the whole grid, in terms of the old Q_i and other quantities. Thus, in checking to determine whether the seawall constraint has been violated, the time level halfway between the old and new time levels is involved. In the explicit method, transport rates of only those cells in contact with a seawall need to be corrected; in the implicit scheme, correction of one cell will affect all cells downdrift (whether in front of the seawall or not) and thus all cells downdrift require correction. Correction of all downdrift cells increases the complexity and execution time of the computation.

60. As already discussed, the direction of sand transport must be

preserved when correction of the transport rate is made to satisfy the seawall constraint. Since, in general, the transport direction can reverse along a beach, in an implicit scheme the transport rate must be solved for twice, starting independently from each of the two lateral boundaries. This doubles the number of calculations performed, even if no corrections are required, and greatly reduces the speed advantage the implicit method normally holds over the explicit solution method. Kraus and Harikai (1983) discuss and compare the relative efficiencies of the explicit and implicit numerical schemes for the shoreline model without inclusion of the seawall constraint. A similar comparison of relative efficiency, including operation of the seawall constraint, is given in the examples discussed in Part IV.

61. The finite difference equations in an implicit scheme will be derived for calculating shoreline change in the presence of a seawall. The grid and notation are the same as those used in the explicit scheme, described in the previous subsection. As the starting point, Equation 1 is rewritten to give equal weight to present and future values:

$$\frac{\partial y}{\partial t} = - \frac{1}{2} \left(\frac{1}{D} \frac{\partial Q}{\partial x} + \frac{1}{D'} \frac{\partial Q'}{\partial x} \right) \tag{12}$$

In finite difference form, Equation 12 becomes

$$y_i' = B' \left(Q_i' - Q_{i+1}' \right) + yc_i \tag{13}$$

where

$$yc_i = y_i + B \left(Q_i - Q_{i+1} \right) \tag{14}$$

The quantity yc_i can be interpreted as the shoreline position midway between y_i and y_i' ; it is known since it only contains values at the present time step and input data. The quantity $B' = \Delta t/(2D' \Delta x)$ differs from the unprimed version in that it contains the depth of closure at the new time step, which can be calculated from the new wave conditions.

62. It is possible to solve Equation 13 by an iterative procedure between the y_i' and the Q_i' , as done for example, by Le Méhauté and Soldate (1978). A computationally faster approach is to express the Q_i' in terms of the y_i' through linearization of Equation 2. Such a linearization is

expected to provide an accurate approximation under typical wave conditions, for which the breaking wave angle is small (less than 30). The linearization method was introduced by Perlin and Dean (1978) for use with the CERC formula, Equation 2. The method was extended by Kraus and Harikai (1983) to account for an additional contribution arising from a systematic change in breaking wave height alongshore (Ozasa and Brampton 1980), as caused, e.g., by wave diffraction. These references should be consulted for details. The final result is that the transport rate at the new time step can be expressed in the form

$$Q'_i = E'_i \,(y'_{i-1} + y'_i) + F'_i \tag{15}$$

where E'_i and F'_i are functions of the incident wave parameters. Substitution of Equation 13 into Equation 15 gives a tridiagonal system of equations for the Q'_i. A tridiagonal system can be solved by an efficient standard algorithm, called the double-sweep algorithm. The solution is based on the following recurrence relation:

$$Q'_i = EE'_i \, Q'_{i+1} + FF'_i \tag{16}$$

where

$$EE'_i = \frac{B'_i}{1 + B'_i \,(2 - EE'_{i-1})} \tag{17}$$

$$FF'_i = \frac{F'_i + E'_i \,(yc_{i-1} - yc_i) + B'_i \, FF'_{i-1}}{1 + B'_i \,(2 - EE'_{i-1})} \tag{18}$$

$$B'_i = B \, E'_i \tag{19}$$

63. The solution procedure, prior to making any corrections to account for the seawall, is as follows:

- a. Specify a boundary condition at $i = 1$ in terms of EE'_i and FF'_i.
- b. Solve Equations 17 and 18 for i = 2 to N, in ascending order. This constitutes the first sweep.
- c. Specify a boundary condition for Q'_{N+1}.

d. Solve Equation 16 for $i = N$ to 1 , in descending order. This step is the second sweep through the grid.

e. Substitute the Q_i' into Equation 13 to obtain the new shoreline positions, y_i' .

64. The shoreline positions thus obtained at each time step must be compared with the position of the seawall to determine if the seawall constraint was violated. If so, then the shoreline position and associated transport rates must be corrected. In general, when making corrections to satisfy the seawall constraint, it is necessary to calculate the Q_i' in ascending order, as well as descending order, so that transport corrections can be made in either direction. The above procedure must be repeated by using a recurrence relation similar to Equation 16, but which allows calculation of Q_i' from the boundary condition at $i = 1$. This relation has the form:

$$Q_i' = PP_i' \, Q_{i-1}' + RR_i' \tag{20}$$

The quantities PP_i' and RR_i' depend on PP_{i+1}' and RR_{i+1}' , respectively. These quantities are defined similarly to EE_i' and FF_i' in Equations 17 and 18, and will not be written here. Expressions for these quantities and their solution scheme can be found in program YSIMP, discussed in Part V and listed in Appendix A.

65. The time evolution of y_i' in the implicit scheme is shown pictorially in Figure 8a. For comparison, the analogous picture for the explicit scheme is given in Figure 8b. The shoreline positions y_i are assumed to be the same in both cases. In the implicit scheme, it is seen that both the present values (time level n) and the future values of Q (time level $n + 1$), entering through $\partial y/\partial t$ in Equation 12, are used to calculate the shoreline change from y_i to y_i' . Since the shoreline change rates $\partial y/\partial t$ are constant during the time increment Δt , the shoreline change over a time step is a straight line.

66. The shoreline position midway (in time) between y_i and y_i' was previously denoted as yc_i . It is seen that y-points lie on a straight line between two adjacent yc-points. Hence, yc-points represent possible extremes in shoreline position. The important implication of this is that in the implicit scheme the seawall constraint must be formulated in terms of the yc_i and not the y_i .

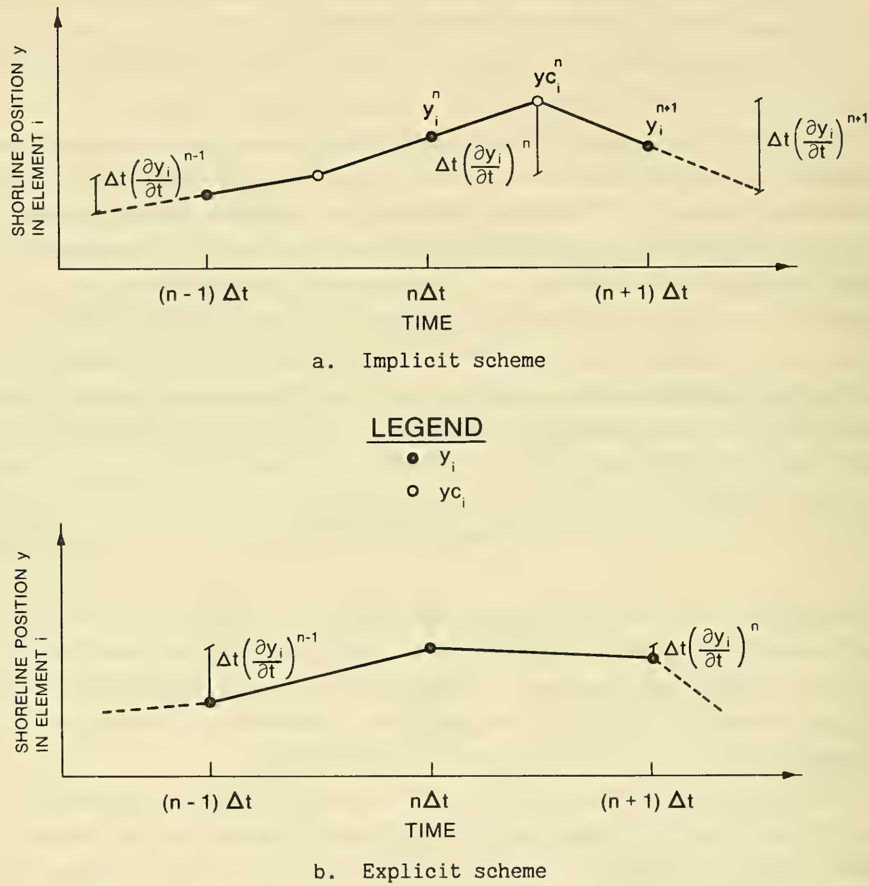


Figure 8. Schematic diagram of the time evolution of a representative shoreline position coordinate

67. Given shoreline position y_i, position yc_i' can be calculated as (see Figure 6a and Equation 12),

$$yc_i' = y_i + \frac{\Delta t}{2}\left(\frac{\partial y_i}{\partial t}\right)^n + \Delta t\left(\frac{\partial y_i}{\partial t}\right)^{n+1}$$

$$= \Delta t\left(\frac{1}{2D}\frac{\partial Q}{\partial x} + \frac{1}{D'}\frac{\partial Q'}{\partial x}\right) + y_i \tag{21}$$

since there is a half time step between yc_i' and y_i and a full time step between yc_i and yc_i' . In finite difference form, Equation 21 becomes:

$$yc_i' = 2B' \, (Q_i' - Q_{i+1}') + yc_i \tag{22}$$

A major goal has been achieved by arriving at Equation 22, because the seawall constraint must be formulated in terms of yc-points. The implementation of the constraint is similar to that for the explicit scheme, and only an outline will be given.

Correction at a minus area

68. As in the explicit scheme, transport adjustments start at a minus cell and from there are performed in the direction of transport. For the minus cell itself, the adjustment resembles that expressed by Equation 6 and reads as follows:

$$Q_i^* = \frac{yc_i - ys_i}{2(yc_i - y_i')} Q_i' \tag{23a}$$

$$Q_{i+1}^* = \frac{yc_i - ys_i}{2(yc_i - y_i')} Q_{i+1}' \tag{23b}$$

Substitution of these corrected values into Equation 22, and using Equation 13, verifies that the desired result has been obtained, i.e.,

$$yc_i' = ys_i \tag{24}$$

Finally, the corresponding corrected shoreline position is computed from Equation 13 as

$$y_i^* = \frac{ys_i + yc_i}{2} \tag{25}$$

The corrected position is thus found to lie halfway between the previous extremal position, yc_i , and the seawall.

Correction at a regular cell, positive transport

69. Corrections are made by moving in the positive x-direction. Since the transport rate into the cell has already been adjusted in connection with

the previous (updrift) cell, only the transport rate out of the cell must be adjusted in order to satisfy Equation 20. This equation contains information about the upstream boundary condition. Before any adjustments are made at cell face $i + 1$, Equation 20 reads

$$Q'_{i+1} = PP'_{i+1} Q^*_i + RR'_{i+1} \tag{26}$$

where Q^*_i is the corrected rate made for the previous cell. This relation holds unless the seawall constraint was violated. If so, then Q'_{i+1} must be adjusted by setting yc'_i equal to ys_i in Equation 22, thus giving

$$ys_i = 2B' (Q^*_i - Q^*_{i+1}) + yc_i \tag{27}$$

This is easily solved for the corrected transport rate for the downdrift cell:

$$Q^*_{i+1} = Q^*_i - \frac{ys_i - yc_i}{2B'} \tag{28}$$

The procedure used to arrive at Equations 26-28 is continued in the downdrift direction until either a plus cell or a boundary is encountered.

Correction at a regular
cell, negative transport

70. The procedure used here for making corrections downdrift, in the negative-x direction (on the other side of the minus cell), is very similar to the procedure described immediately above. The new transport rate at cell face i is given by Equation 16, i.e.,

$$Q'_i = EE'_i Q^*_{i+1} + FF'_i \tag{29}$$

Then the corrected transport is found to be

$$Q^*_i = Q^*_{i+1} + \frac{ys_i - yc_i}{2B'} \tag{30}$$

This procedure is repeated downstream until a plus cell or a boundary is encountered.

PART IV: EXAMPLE CALCULATIONS

General Comments

71. Two examples are presented. These hypothetical situations demonstrate applications of the shoreline model with an operative seawall boundary condition and allow checking of user implementations of the programs given in Appendix A. An attempt was made to give semi-plausible examples while also preserving clarity. This resulted in two idealized cases for which most of the common structures and boundary conditions could be included. The first example is that of an initially straight shoreline bounded on one side by a jetty. The beach is protected by the combination of a detached breakwater and a straight seawall segment. The second example is a curved pocket beach lying between two headlands and protected by a curved seawall. Hanson and Kraus (1985) show results of several other sample calculations.

72. In the examples, the wave field is introduced artificially; the breaking wave height and breaking wave angle were fabricated "by hand" to achieve the desired trends in shoreline movement in order to exercise the seawall constraint algorithms. The breaking wave data are set in the subprogram INDATA which is given in Appendix A. Values of the time and space steps and other parameters are entered via FORTRAN DATA statements. The names of parameters and variables closely follow the notation of the main text of this report. The important exceptions are: the angle "theta," denoted as "Z," and the empirical coefficient "K," denoted as "K1" in the program.

73. Both examples can be run using either the explicit or the implicit numerical scheme, programs YSEXP and YSIMP, respectively, in Appendix A. In the latter part of these programs a calculation is made to check sand volume conservation. It can be verified that volume is conserved to within truncation error.

Stability

74. Before proceeding to the examples, the stability properties of the shoreline model are briefly reviewed. It can be shown (e.g., Hanson and Kraus 1980; Kraus and Harikai 1983) that for small breaking wave angles and constant wave height, Equation 1, together with Equation 2, reduces to the functional form of the heat equation, the governing equation derived by Pelnard-Considere (1954). The accuracy and stability properties of numerical

schemes for solving this equation are well known. Generally speaking, numerical accuracy can be improved somewhat by taking a smaller time step for a given space step, assuming negligible numerical truncation error. Increased computer execution time is the price paid for using smaller time steps. Therefore, one wants to balance speed of the calculation with numerical accuracy.

75. Numerical accuracy should be distinguished from "physical" accuracy. Numerical accuracy is a measure of how well a finite difference scheme reproduces the solution of a differential equation; physical accuracy is a measure of how well the differential equation (and the numerical solution if one is employed) describes the process of interest.

76. For an explicit scheme, there is a stringent limitation (the Courant condition) on the size of the largest possible time step, other variables being held constant. For small breaking wave angles, in the present case this condition is

$$R_s \le \frac{1}{2} \tag{31a}$$

where

$$R_s = \frac{2\, K'\, \Delta t \left(H^2 C_g\right)_b}{D\, (\Delta x)^2} \tag{31b}$$

The quantity R_s was called the "stability parameter" by Kraus and Harikai (1983). Equation 31a is an adequate indicator of stability in most applications, since breaking wave angles are usually small. The stability parameter gives an estimate of the numerical accuracy of the solution, with accuracy typically increasing for decreasing values of R_s .

Example 1: Jetty and Detached Breakwater

77. The initial condition is shown in Figure 9a. The initially straight 2,000-m stretch of beach is protected by a shore-parallel, detached breakwater and a seawall connected to a long jetty. The seawall is set back 7 m from the initial shoreline. The jetty is assumed to be sufficiently long so as to act as a complete littoral barrier. The breakwater is drawn in Figure 9 to aid visual understanding; in actuality, it lies much farther offshore. The seawall and breakwater have been constructed to prevent erosion

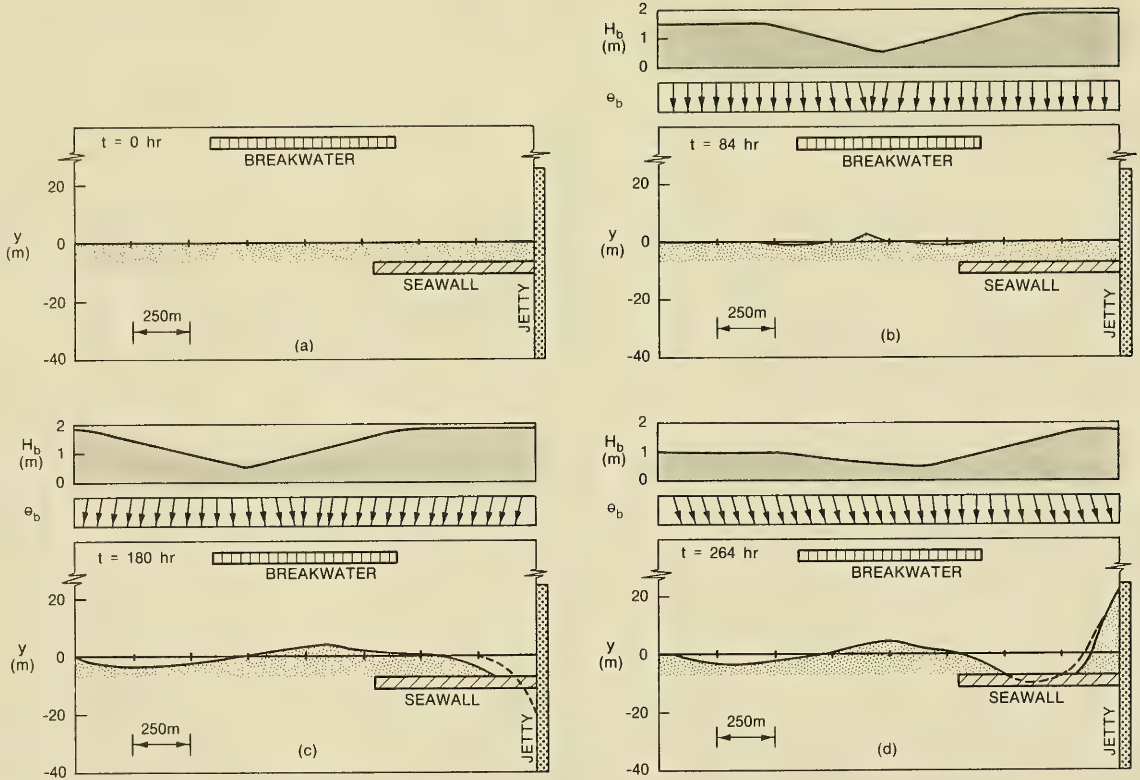


Figure 9. Hypothetical example of shoreline change behind a detached breakwater and in the presence of a seawall

of the beach adjacent to the jetty. The beach on the far left side of the figure is assumed to be outside the area of influence of the structures, and therefore its position remains fixed.

78. Waves arrive at the site as shown in Figures 9b, c, and d. In these figures, the longshore distributions of the breaking wave height and breaking wave angle are displayed in graphic form above the related beach planform. The local breaking wave height and angle are mainly controlled by the detached breakwater. The shoreline that would result if there were no seawall is indicated by a dashed line.

79. Figure 9b shows the result of waves arriving almost normal to the shoreline for a period of 84 hr. Convergence of waves behind the detached breakwater causes a bulge, or salient, to form. The wave direction then changes, Figure 9c, and waves arrive obliquely from the right for an elapsed time of 180 hr. This results in a loss of sand on the beach next to the jetty. The seawall prevents the shoreline from eroding farther landward immediately next to the jetty; the price paid is that more sand is removed from along the front of the seawall. Finally, as shown in Figure 9d, the wave direction changes again and waves arrive obliquely from the left. The wave shadow zone behind the detached breakwater also shifts and the potential region for erosion moves to the middle of the seawall. Sand returns next to the jetty, and an eroded sector forms at the middle of the seawall.

80. Although differences in shoreline positions with and without the seawall are moderate in this example, by altering the input wave conditions (e.g., by increasing the difference in breaking wave angle between applied wave conditions) a much greater disparity in resultant shorelines can be generated.

Example 2: Pocket Beach

81. The initial shoreline configuration for this example is shown in Figure 10a. A curved pocket beach approximately 2 km long is bounded by two long headlands which contain the littoral transport. A curved seawall is located 4 m landward of the initial shoreline.

82. Waves first arrive obliquely from the right side of the figure for 126 hr to produce the planform shown in Figure 10b. As a result, beach material moves toward the left headland. The seawall has protected the area on the right side of the beach, as seen by the shoreline change that would have

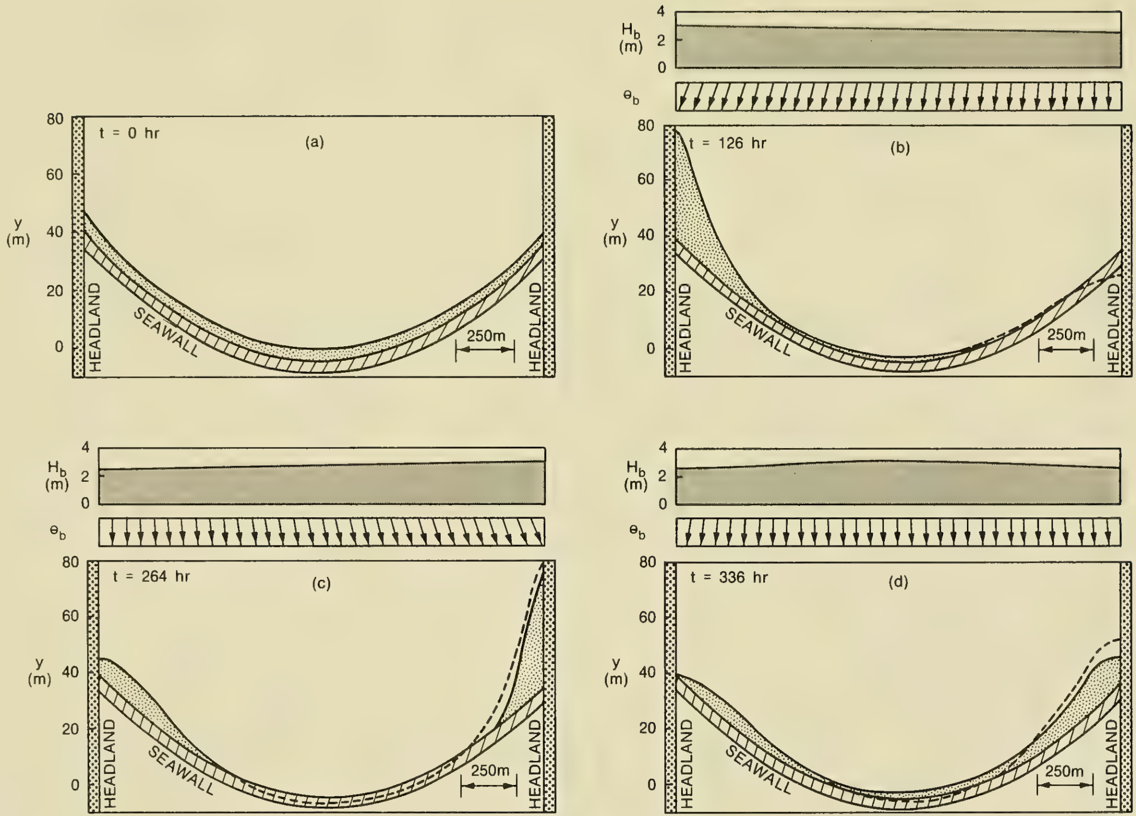


Figure 10. Hypothetical example of shoreline change along a curved pocket beach backed by a seawall

occurred without the seawall (dashed line). The incident waves then swing in direction and arrive obliquely from the left for 138 hr, as seen in Figure 10c. Sand is transported past the center of the seawall to form a wide beach adjacent to the right headland. The beach planform in (c) is not a mirror image of (b) because, although the waves were mirror images, the initial shoreline conditions were different.

83. In Figure 10c, the seawall is protecting approximately half of the shore, and much of the eroded sector is still located on the right side. Intuition might have suggested more erosion on the leftmost side since the more recent waves were from the left. However, the interaction between waves and shoreline is nonlinear (Equation 2, the sine dependence), and the calculated change is different than might be expected. Finally, almost normally incident waves arrive to the coast for 72 hr, to give the result shown in Figure 10d. The beach has essentially returned to its initial planform, Figure 10a. A beach again exists all along the front of the seawall.

84. In this example, the seawall protected the beach under episodes of oblique wave incidence, preventing excessive landward retreat of the shoreline. The seawall therefore worked to promote recovery of the beach (compare solid and dashed lines in Figure 10d). It should be cautioned that this result is partially an artifact of the assumption of an equilibrium (constant) profile. In nature, the beach profile in an eroded area would probably become steeper than the average beach profile; it then might take a longer duration of the normally incident waves to cause the beach to recover.

Comparison of Accuracy and Efficiency of the Explicit Scheme and the Implicit Scheme

85. The configuration of Example 2 was used to compare the numerical accuracy and efficiency of the explicit and implicit numerical solution schemes when operating under the seawall constraint. Although the results are necessarily site-dependent, experience has shown the trends to be representative and the conclusions qualitatively correct. Kraus and Harikai (1983) gave a similar comparison of explicit and implicit numerical schemes for shoreline models without the seawall constraint.

86. The results of the comparison are shown in Table 1. The wave input used was that in Figure 10b and run for 120 hr. The values of key parameters were the same as in the previous examples: maximum wave height $H_{max} = 3$ m ,

T = 8 s, DX = 50 m, and D = 6 m. In the comparison, the time step, DT, was varied and the reference or standard case was taken to be the explicit scheme with DT = 6 hr. The relative accuracy with respect to the reference result, Δy_r, where Δy is the change in shoreline position between final and initial positions, is given at three locations on the left side of the beach.

Table 1

Stability and Accuracy of Explicit and Implicit Numerical Schemes with an Operative Seawall

Δt hr	Stability parameter R_s	Relative Execution Time	$\frac{\Delta y - \Delta y_r}{\Delta y_r}$ (percent) i = 1	i = 10	i = 20
		Explicit Scheme			
1	0.08	5.30	-0.6	-3.1	0.0
2	0.17	2.72	-0.5	-2.8	0.0
4	0.34	1.43	-0.2	-1.2	0.0
6	0.51	1.00	0.0	0.0	0.0
8	0.67	unstable			
		Implicit Scheme			
6	0.51	2.24	-0.5	-3.1	0.0
12	1.01	1.19	-0.2	-2.0	0.0
24	2.02	0.67	-0.7	0.0	0.0
60	5.05	0.35	13.4	2.3	14.7
120	10.11	0.24	22.1	-7.9	23.3

87. The results in Table 1 are qualitatively similar to those given by Kraus and Harikai (1983). The explicit model is computationally faster than the implicit model per time step; however, larger time steps can be taken with the implicit model while preserving reasonable numerical accuracy, allowing a potential overall speed advantage. For example, the implicit model with a time step of 24 hr and stability parameter of 2.02 is about 30 percent faster than the reference explicit result, yet still has acceptable numerical accuracy. Engineering judgment must be exercised on a case-by-case basis to decide if a 24-hr time step will give acceptable physical accuracy. In a similar comparison without a seawall, Kraus and Harikai (1983) found the implicit model with a 6-hr time step to be comparable in accuracy and execution time to the reference explicit model with the same time step. As was discussed in Part III, the implicit model suffers a loss in efficiency when the seawall boundary condition is operative.

PART V: EXPLANATION OF COMPUTER PROGRAMS

General Comments

88. Here, an explanation is given of main operations performed in four of the five FORTRAN programs given in Appendix A. The programs are set up to compute the examples presented in Part IV. The final shoreline positions calculated in the examples are given in Part IV so that user implementations of the programs can be checked.

89. The programs constitute the foundation of a "1-line model" and calculate shoreline change on a beach backed by a seawall by means of either the explicit or the implicit numerical scheme. In order to run the programs for a general case, wave information is needed to calculate the longshore sediment transport along the beach in question. Specifically, the breaking wave height and angle along the beach are required. The breaking wave field must be obtained from a wave calculation program such as a refraction program or from a combined refraction and diffraction program if large coastal structures are involved. It was beyond the scope of this report to include a numerical wave model. The breaking wave field will also be influenced by the plan shape of the beach (the so-called sediment-wave interaction), which changes with time. Numerical wave models and their relation to the shoreline change model are discussed by Kraus (1983).

90. The five subprograms are called by a main program. Input wave data for the examples are fabricated in subroutine INDATA. The subroutine INDATA is elementary and will not be discussed. The longshore sand transport rate, computed by means of Equation 2, is calculated in subroutines YSEXP (explicit solution scheme) and YSIMP (implicit solution scheme). Shoreline change in the presence of a seawall is computed in subroutines CORRE (explicit) and CORRI (implicit). These latter two routines correct both the transport rate and shoreline position as described in Part III.

91. Many of the algorithms are repeated in the subroutines. Comments are given once for each generic type of algorithm. For clarity, the programs are arranged to calculate for only one continuous seawall of arbitrary length and configuration. They can easily be generalized to handle any number of seawalls.

92. In the explanations, the names of variables and line numbers refer to those in the indicated programs. Line numbers in parentheses refer to the

explicit program version. The names of most key variables in the programs are the same as those used in the main text of this report. They are again defined here to make the explanation more self-contained. The programs themselves contain a large number of comment statements describing the operations performed in distinct program segments.

Programs YSEXP and YSIMP

93. Lines 170-190 (150-170): These statements initialize basic parameters. YSBEG and YSEND define the beginning and end grid points of the seawall (Figure 4), with YSBEG < YSEND. The grid spacing is DX (in meters) and the time step is DT (in hours). NTIMES specifies the number of timesteps and IT1 and IT2 denote timesteps when the wave data are changed in the examples. DENOM is the value of physical quantities in the denominator of Equation 2b, evaluated for quartz sand. K1 is the empirical coefficient (K) in Equation 2b. The wave period is denoted by T (seconds).

94. Lines 250-310 (240-290): Specify initial shoreline and seawall positions for a straight beach and seawall.

95. Lines 370-450 (350-430): Specify initial shoreline and seawall positions for a curved beach and seawall.

96. Line 570 (530): Call in wave data and renew as specified.

97. Line 730 (680): Calculate closure depth, DCLOS, from wave conditions.

98. Lines 790-830 (780-810): These lines specify boundary conditions for the simple cases of a fixed beach position and an impermeable long groin (jetty, headland).

99. Lines 850-960 (720-750): Calculation of the longshore transport rate.

100. Lines 1080-1150: In the implicit model, in order to make corrections in both directions, a reversed double sweep is necessary. The longshore transport rates in the arrays Q and QQ should be equal; a checking procedure is provided to verify this.

101. Lines 1270 (890): After the shoreline position is calculated, each y_i must be checked to see if it violates the seawall constraint. The subroutines CORRI and CORRE are called to do the check and to correct the shoreline positions and transport rates as necessary.

102. Line 1300 (970): This program segment is an error checking calculation to verify that sand volume was conserved. It also accounts for sand that may have entered the system at the boundaries.

Programs CORRE and CORRI

103. Subroutines CORRE and CORRI are called by YSEXP and YSIMP, respectively. They recalculate the transport rate due to the possible limited volume of sand in front of a seawall and adjust the position of the shoreline accordingly.

104. Line 200 (190): A branch is made according to whether the transport rate Q_i is less than, greater than, or equal to zero. A branch is necessary because the corrections must be performed in the direction of sand transport.

105. Line (200): This and similar lines correspond to Equation 5.

106. Line 260: Corresponds to Equation 20.

107. Lines 270-310 (260-310): If the intermediate shoreline position YC (for the explicit scheme, position Y) is seaward of the seawall, no correction is necessary. If not, the downstream transport rate Q_{i+1} must be corrected in order to conserve sand volume. The position YC (Y) is then set to the corresponding position of the seawall.

108. Lines 540-680 (540-680): Calculate as described above, but for the reversed transport direction.

109. Lines 700-800 (700-800): Corrections at a minus point are computed. Sand cannot be generated in a minus cell located at a seawall. Therefore, the transport rates at both cell faces are corrected so that the shoreline will not move landward of the seawall.

110. Lines 820-950 (820-970): This program segment operates in the same manner as similar segments previously described, except that here the calculation is done in order of decreasing index since the transport is in the negative x-direction. Calculation starts at the point to the left (lower i-values) of the minus cell and continues downstream until a plus cell is encountered.

111. Lines 970-1100: After corrections are completed for grid points within the domain of the seawall, the same procedure must be carried out for the unprotected (unstructured) parts of the beach, if any. This step is necessary for the implicit scheme, since all values of Q are solved at once. It is not required in the explicit scheme, for which corrections are completely determined point by point, at the present time step.

REFERENCES

Bakker, W. T. 1969. "The Dynamics of a Coast with a Groin System," Proceedings of 11th Coastal Engineering Conference, American Society of Civil Engineers, pp 492-517.

Bakker, W. T., Klein Breteler, E. H. J., and Roos, A. 1971. "The Dynamics of a Coast with a Groyne System," Proceedings of 12th Coastal Engineering Conference, American Society of Civil Engineers, pp 199-218.

Berrigan, P. D. 1985a. "The Taraval Vertical Seawall," Shore and Beach, Journal of the American Shore and Beach Preservation Association, Vol 53, No. 1, pp 2-7.

______________. 1985b. "Seasonal Beach Changes at the Taraval Seawall," Shore and Beach, Journal of the American Shore and Beach Preservation Association, Vol 53, No. 2, pp 9-15.

Chestnutt, C. B., and Schiller, R. E., Jr. 1971. "Scour of Simulated Gulf Coast Sand Beaches Due to Wave Action in Front of Sea Walls and Dune Barriers," Texas A&M University, COE Report No. 139, TAMU-SG-71-207, 54 pp.

Dorland, G. M. 1940. "Equilibrium Sand Slopes in Front of Sea Walls," MS Thesis, Department of Civil Engineering, University of California, Berkeley, 43 pp.

Hallermeier, R. F. 1979. "Uses for a Calculated Limit Depth to Beach Erosion," Proceedings of 16th Coastal Engineering Conference, American Society of Civil Engineers, pp 1493-1512.

__________. 1983. "Sand Transport Limits in Coastal Structure Design," Proceedings of Coastal Structures '83, American Society of Civil Engineers, pp 703-716.

Hanson, H., and Kraus, N. C. 1980. "Numerical Model for Studying Shoreline Changes in the Vicinity of Coastal Structures," Report No. 3040, Department of Water Resources Engineering, University of Lund, Sweden, 44 pp.

_________. 1985. "Seawall Constraint in the Shoreline Numerical Model," Journal of Waterway, Port, Coastal and Ocean Engineering," American Society of Civil Engineers, Vol 111, No. 6, pp 1079-1083.

Hashimoto, H., et al. 1971. "Study on the Prediction of the Longshore Transport Rate," Report of the 25th Engineering Meeting of the Ministry of Construction," Japan, pp 517-541. (In Japanese.)

Hattori, M., and Kawamata, K. 1977. "Experiments on Restoration of Beaches Backed by Seawalls," Coastal Engineering in Japan, Vol 20, Japan Society of Civil Engineers, pp 55-68.

Komar, P. D. 1976. Beach Processes and Sedimentation, Prentice-Hall, Inc., Englewood Cliffs, N. J., pp 252-261.

__________. 1983. "Computer Models of Shoreline Changes," in (P. D. Komar, ed.), CRC Handbook of Coastal Processes and Erosion," CRC Press, Inc., Boca Raton, Fla., pp 191-204.

Kraus, N. C. 1983. "Applications of a Shoreline Prediction Model," Proceedings of Coastal Structures '83, American Society of Civil Engineers, pp 632-645.

Kraus, N. C. 1984. Discussion of "Wave Data Discretization for Shoreline Processes," by B. Le Méhauté, J. D. Wang, and C. C. Lu, Journal of Waterway, Port, Coastal and Ocean Engineering, Vol 110, No. 1, American Society of Civil Engineers, pp 128-130.

__________. in preparation. "Bibliography of the Shoreline Numerical Model," US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center.

Kraus, N. C., Hanson, H., and Harikai, S. 1985. "Shoreline Change at Oarai Beach: Past, Present and Future," Proceedings of 19th Coastal Engineering Conference, American Society of Civil Engineers, pp 2107-2123.

Kraus, N. C., and Harikai, S. 1983. "Numerical Model of the Shoreline Change at Oarai Beach," Coastal Engineering, Vol 7, No. 1, pp 1-28.

Kraus, N. C., Harikai, S. and Kubota, K. 1981. "Numerical Modeling of the Breaking Waves and Shoreline Change at Oarai Beach," Proceedings of 28th Japanese Coastal Engineering Conference, Japan Society of Civil Engineers, pp 295-299. (In Japanese.)

Le Méhauté, B., and Soldate, M. 1978. "A Numerical Model for Predicting Shoreline Changes," Miscellaneous Report No. 80-6, US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, 72 pp.

Mizumura, K. 1982. "Shoreline Change Estimates Near Oarai, Japan," Journal Waterway, Port, Coastal and Ocean Engineering, American Society of Civil Engineers, Vol 108, No. 1, pp 47-64.

O'Brien, M. P. 1985. Keynote Address, Proceedings of 19th Coastal Engineering Conference, American Society of Civil Engineers, pp 1-12.

Ogawara, M. 1983. "Status of Coastal Engineering Works (in Japan), Present and Future," Kaigan (Coast), Vol 23, All-Japan Coast Association, pp 1-7. (In Japanese.)

Ozasa, H., and Brampton, A. H. 1980. "Mathematical Modeling of Beaches Backed by Seawalls," Coastal Engineering, Vol 4, No. 1, pp 47-64.

Pelnard-Considere, R. 1954. "Essai de Theorie de L'evolution des Formes de Rivages en Plages de Sable et de Galets," 4th Journees de l'Hydraulique, Les Energies de la Mer, Question III, Rapport No. 1, pp 289-298.

Perlin, M., and Dean, R. G. 1978. "Prediction of Beach Planforms with Littoral Controls," Proceedings of 16th Coastal Engineering Conference, American Society of Civil Engineers, pp 1818-1838.

__________. 1983. "A Numerical Model to Simulate Sediment Transport in the Vicinity of Structures," Miscellaneous Report No. 83-10, US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, 119 p.

Russell, R. C. H., and Inglis, C. 1953. "The Influence of a Vertical Wall on a Beach in Front of It," Proceedings of the Minnesota International Hydraulics Convention, pp 221-226.

Sato, S., Tanaka, N., and Irie, I. 1969. "Study on Scouring at the Foot of Coastal Structures," Proceedings of 11th Coastal Engineering Conference, American Society of Civil Engineers, pp 579-598.

Shore Protection Manual. 1984. 4th ed., 2 vols, US Army Engineer Waterways

Experiment Station, Coastal Engineering Research Center, US Government Printing Office, Washington, DC.

Silvester, R. 1977. "The Role of Wave Reflection in Coastal Processes," Proceedings of Coastal Sediments '77, American Society of Civil Engineers, pp 639-654.

Tanaka, N., and Nadaoka, K. 1982. "Development and Application of a Numerical Model for the Prediction of Shoreline Changes," Technical Note 436, Port and Harbor Research Institute, Ministry of Transport, Japan, 40 pp. (In Japanese.)

Toyoshima, O. 1979. "Effectiveness of Sea Dikes with Rough Slope," Proceedings of 16th Coastal Engineering Conference, American Society of Civil Engineers, pp 2528-2539.

US Army Corps of Engineers. 1981. "Low Cost Shore Protection: Final Report on Shoreline Erosion Control Demonstration Program," Coastal Engineering Research Center, 830 pp.

Walton, T. L., and Sensabaugh, W. 1979. "Seawall Design on the Open Coast," Florida Sea Grant College, Report No. 29, 24 pp.

APPENDIX A: COMPUTER PROGRAM LISTINGS

```
C* Program YSEXP calculates shoreline change according to one line
C* theory, taking into account the effects of a seawall.
      INTEGER YSBEG,YSEND
      REAL K1,KAP1
      DIMENSION Y(40),YS(40),Q(41),Z(40),H(40),YO(40)
      DATA YSBEG/26/,YSEND/40/,DX/50./,DT/6./
      DATA DENOM/2.362   /,NTIMES/44/,N/40/,IT1/15/,IT2/31/
      DATA K1/0.12/,T/8.0/,G/9.806/,GAMMA/0.78/,RADIUS/12000./
      WRITE(*,*) '*******EXPLICIT CALCULATION*******'
      WRITE(*,*) 'YSBEG=',YSBEG,'           YSEND=',YSEND
C*
C*
C* Initialize arrays
C* Straight shoreline
      DO 100 I=1,N
         Q(I)=0.
         Y(I)=0.
100   CONTINUE
      DO 105 I=1,N
        YS(I)=-7.
105   CONTINUE
      Q(N+1)=0.
      DCLOS=0.
      GOTO 120
C* Curved shoreline
      DO 110 I=1,N
        BET=ASIN(FLOAT(21-I)*DX/RADIUS)
        Y(I)=RADIUS*(1.-COS(BET))
        YO(I)=Y(I)
110   CONTINUE
      DO 115 I=YSBEG,YSEND
        YS(I)=Y(I)-4.
115   CONTINUE
120   CONTINUE
C*
      WRITE(*,10) (YS(I),I=1,N)
      KAP1=K1/(16.*DENOM)
C* QL=longshore transport rate over open boundary
C*     QL=0
      DO 200 IT=1,NTIMES+1
         IF(IT.EQ.1.OR.IT.EQ.IT1.OR.IT.EQ.IT2) IC=1
C* Subroutine INDATA computes relevant input wave data
C* at any desired time step.
         IF(IC.EQ.1) CALL INDATA(IT,IT1,IT2,H,Z,N,DT)
         IF(IT.EQ.NTIMES+1) THEN
           IC=2
           IHOURS=(IT-1)*INT(DT)
           WRITE(*,40)
           WRITE(*,*) 'FINAL CONDITIONS (after ',IHOURS,' hours)'
         ENDIF
         IF(IC.GE.1) THEN
           WRITE(*,40)
           WRITE(*,30) (Y(I),I=1,N)
           WRITE(*,40)
           WRITE(*,20) (Q(I),I=1,N)
         ENDIF
         IF(IC.EQ.2) GOTO 999
         IC=0
         DCLOS=2.28*H(1)-68.5*(H(1)/T)**2/G
         B=DT*3600./(2.*DCLOS*DX)
         B2=2.*B
C*
         DO 300 I=2,N
           ZBS=Z(I)-ATAN((Y(I)-Y(I-1))/DX)
           Q(I)=H(I)**2*SQRT(G/GAMMA*H(I))*KAP1*SIN(2*ZBS)
```

```
300      CONTINUE
C* Boundary conditions:
C* Pinned beach
          Q(1)=Q(2)
C* Groin(s)
C*        Q(1)=0.
          Q(N+1)=0.
C*
          IF (YSBEG.GE.3) THEN
             DO 400 I=1,YSBEG-2
                Y(I)=Y(I)-B2*(Q(I+1)-Q(I))
 400         CONTINUE
          ENDIF
C* Correction of shoreline in front of seawall if necessary
          CALL CORRE(YSBEG,YSEND,Q,B2,Y,YS)
          IF(YSEND.NE.N) THEN
             DO 500 I=YSEND+1,N
                Y(I)=Y(I)-B2*(Q(I+1)-Q(I))
 500         CONTINUE
          ENDIF
C*
C*
C* Error calculation (DIFF: closed boundaries, AROUT: open boundary)
C*     QL=QL+Q(1)
 200   CONTINUE
 999   CONTINUE
       DIFF=0.
       AAREA=0.
       DO 600 I=1,N
          DIFF=DIFF+YO(I)-Y(I)
          AAREA=AAREA+ABS(YO(I)-Y(I))
 600   CONTINUE
       ERROR=DIFF/AAREA
C*     AROUT=QL*DT*3600./DCLOS-DIFF*DX
C*     ERROR=AROUT/AAREA
C*
C*** Output ***
       WRITE(*,*)
       WRITE(*,*) 'LOST SAND VOLUME=',ERROR*100,'  %'
C*     WRITE(*,*) '(QL*DT/D-AREA)/ABSAREA*100=',ERROR*100,'%'
 10    FORMAT(1X,'SEAWALL POSITION'/(1X,10F8.2))
 20    FORMAT(1X,'LONGSHORE TRANSPORT'/(1X,10F8.4))
 30    FORMAT(1X,'SHORELINE POSITION'/(1X,10F8.2))
 40    FORMAT(//)
       STOP
       END
```

```
      SUBROUTINE CORRE(YSBEG,YSEND,Q,B2,Y,YS)
C* CORRE recalculates transport rates (Q) due to limited sand
C* volume in front of a seawall and adjusts the shoreline
C* position as necessary. Explicit calculation scheme.
C*
      INTEGER YSBEG,YSEND
      REAL Q(41),Y(40),YS(40)
C*
      I=YSBEG
      IF(Q(I).GT.0) THEN
         Y(I-1)=Y(I-1)-B2*(Q(I)-Q(I-1))
C*
C* Q positive: Calc of shoreline Y with correction of Q and Y
C* as necessary.
C*
 10      IF(Q(I+1).GE.0) THEN
            Y(I)=Y(I)-B2*(Q(I+1)-Q(I))
            IF(Y(I).LT.YS(I)) THEN
               DIFF=YS(I)-Y(I)
               Q(I+1)=Q(I+1)-DIFF/B2
               Y(I)=YS(I)
            ENDIF
            I=I+1
            IF(I.EQ.YSEND+1) GOTO 100
         GOTO 10
         ENDIF
         K=I
         I=I+1
         IF(I.EQ.YSEND+1) THEN
            Y(I-1)=Y(I-1)-B2*(Q(I)-Q(I-1))
            GOTO 100
         ENDIF
         IF(I.EQ.YSEND) THEN
            I=I+1
            GOTO 30
         ENDIF
      ELSE
         K=YSBEG-1
         IF(YSBEG.EQ.1) K=1
      ENDIF
C*
C* Q negative: Search for a minus point. If absent, calc Y
C* for the right end element. Correct Q as necessary.
C*
 20   IF(Q(I+1).LT.0) THEN
         I=I+1
         IF(I.EQ.YSEND) THEN
            IF(Q(I+1).LE.0) THEN
               Y(I)=Y(I)-B2*(Q(I+1)-Q(I))
               IF(Y(I).LT.YS(I)) THEN
                  DIFF=YS(I)-Y(I)
                  Q(I)=Q(I)+DIFF/B2
                  Y(I)=YS(I)
               ENDIF
               GOTO 30
            ENDIF
         ENDIF
      GOTO 20
      ENDIF
C*
C* Minus point: Corr of Q out of the element if shoreline moves
C* behind seawall.
C*
      Y(I)=Y(I)-B2*(Q(I+1)-Q(I))
      IF(Y(I).LT.YS(I)) THEN
```

```
         DIFF=YS(I)-Y(I)
         QDIFF=Q(I+1)-Q(I)
         Q(I)=Q(I)-DIFF/B2*(Q(I)/QDIFF)
         Q(I+1)=Q(I+1)-DIFF/B2*(Q(I+1)/QDIFF)
         Y(I)=YS(I)
      ENDIF
C*
C* Calc of Y starting from element to the left of minus
C* point or boundary. Q is negative.
C*
 30   DO 40 J=I-1,K,-1
         Y(J)=Y(J)-B2*(Q(J+1)-Q(J))
         IF(Y(J).LT.YS(J).AND.J.GE.YSBEG) THEN
            DIFF=YS(J)-Y(J)
            Q(J)=Q(J)+DIFF/B2
            Y(J)=YS(J)
         ENDIF
 40   CONTINUE
      I=I+1
      IF(I.GE.YSEND+1) GOTO 100
C*
C* Calc of Y starting from element to the right of minus
C* point or boundary. Q is positive.
C*
      GOTO 10
 100  CONTINUE
      RETURN
      END
```

```
C* Program YSIMP is an implicit version of program YSEXP and
C* calculates shoreline change according to one line theory,
C* taking into account the effects of a seawall.
      INTEGER YSBEG,YSEND
      REAL K1,KAP1
      DIMENSION Z(40),Y(40),YS(40),YO(40),Q(41),YCOLD(40),E(40),F(40)
      DIMENSION EP(40),FP(40),BP(40),P(41),R(41),QQ(41),H(40)
      DATA YSBEG/1/,YSEND/40/,DX/50./,DT/6./
      DATA DENOM/2.362   /,NTIMES/56/,N/40/,IT1/22/,IT2/45/
      DATA K1/0.12/,T/8.0/,G/9.806/,GAMMA/0.78/,RADIUS/12000./
      WRITE(*,*) '*******IMPLICIT CALCULATION*******'
      WRITE(*,*) 'YSBEG=',YSBEG,'             YSEND=',YSEND
C*
C* Initialize arrays
C* Straight shoreline
      DO 100 I=1,N
         Q(I)=0.
         Y(I)=0.
 100  CONTINUE
      DO 105 I=YSBEG,YSEND
         YS(I)=-7.
 105  CONTINUE
      Q(N+1)=0.
      DOLD=0.
      DCLOS=0.
C*    GOTO 120
C* Curved shoreline
      DO 110 I=1,N
         BET=ASIN(FLOAT(21-I)*DX/RADIUS)
         Y(I)=RADIUS*(1.-COS(BET))
         YO(I)=Y(I)
110   CONTINUE
      DO 115 I=YSBEG,YSEND
         YS(I)=Y(I)-4.
115   CONTINUE
120   CONTINUE
C*
      WRITE(*,10) (YS(I),I=1,N)
      KAP1=K1/(16.*DENOM)
C* QL=longshore transport rate over open boundary
C*     QL=0.
C* C=correction term in continuity calculation
C*     C=1.0
      DO 200 IT=1,NTIMES+1
         IF(IT.EQ.1.OR.IT.EQ.IT1.OR.IT.EQ.IT2) IC=1
C* Subroutine INDATA computes relevant input wave data
C* at any desired time step.
         IF(IC.EQ.1) CALL INDATA(IT,IT1,IT2,H,Z,N,DT)
         IF(IT.EQ.NTIMES+1) THEN
           IC=2
           IHOURS=(IT-1)*INT(DT)
           WRITE(*,40)
           WRITE(*,*) 'FINAL CONDITIONS (after ',IHOURS,' hours)'
         ENDIF
         IF(IC.GE.1) THEN
           WRITE(*,40)
           WRITE(*,30) (Y(I),I=1,N)
           WRITE(*,40)
           WRITE(*,20) (Q(I),I=1,N)
         ENDIF
         IF(IC.EQ.2) GOTO 999
         IC=0
         DOLD=DCLOS
         DCLOS=2.28*H(1)-68.5*(H(1)/T)**2/G
         B=DT*3600./(2.*DCLOS*DX)
```

```
            BOLD=DT*3600./(2.*DOLD*DX)
            YCOLD(1)=Y(1)+BOLD*(Q(1)-Q(2))
C* Boundary conditions:1
C* Groin causing Q(1)=0.
            E(1)=0.
            F(1)=0.
C* Pinned beach as Q(1)=Q(2)
C*           E(1)=1.
C*           F(1)=0.
C*
            DO 300 I=2,N
               YCOLD(I)=Y(I)+BOLD*(Q(I)-Q(I+1))
               ZS=ATAN((Y(I)-Y(I-1))/DX)
               Z2=2.*Z(I)
               PWR=H(I)**2*SQRT(G/GAMMA*H(I))
               EP(I)=PWR*KAP1*2*COS(Z2)*(COS(ZS))**2/DX
               FP(I)=PWR*KAP1*SIN(Z2)*(2*(COS(ZS))**2-1.)
               BP(I)=B*EP(I)
               DEN=1.+BP(I)*(2.-E(I-1))
               E(I)=BP(I)/DEN
               F(I)=(FP(I)+EP(I)*(YCOLD(I-1)-YCOLD(I))+BP(I)*F(I-1))/DEN
  300       CONTINUE
C* Boundary condition 2: groin
            Q(N+1)=0.
C*
             DO 400 I=N,1,-1
                Q(I)=E(I)*Q(I+1)+F(I)
                IF(IT.EQ.1) THEN
                  YCOLD(I)=Y(I)+B*(Q(I)-Q(I+1))
                ENDIF
   400       CONTINUE
C*** Reversed double sweep ***
C* Boundary conditions 3: groin
             P(N+1)=0.
             R(N+1)=0.
C*
             DO 500 I=N,2,-1
                P(I)=BP(I)/(1.+BP(I)*(2.-P(I+1)))
                R(I)=(FP(I)+EP(I)*(YCOLD(I-1)-YCOLD(I))+BP(I)*R(I+1))/
     &                    (1.+BP(I)*(2.-P(I+1)))
   500       CONTINUE
C* Boundary condition 4 (alt 1: closed boundary, alt 2: open)
             QQ(1)=0.
C*           QQ(1)=R(2)/(1.-P(2))
C*
             DO 550 I=2,N+1
                QQ(I)=P(I)*QQ(I-1)+R(I)
                CHECK=ABS(QQ(I)-Q(I))
C*              IF(CHECK.GT.0.0005) WRITE(*,*) 'TRANSPORT CALC. DIFFER'
   550       CONTINUE
C*
C*Correction of shoreline in front of seawall if necessary
             CALL CORRI(YSBEG,YSEND,Q,B,YCOLD,E,F,P,R,Y,YS,N)
C*
C*
C* Error calculation (DIFF: closed boundaries, AROUT: open boundary)
C*        IF(IT.EQ.NTIMES) C=0.5
C*        QL=QL+C*Q(1)
  200     CONTINUE
  999     CONTINUE
          DIFF=0.
          AAREA=0.
          DO 600 I=1,N
             DIFF=DIFF+YO(I)-Y(I)
             AAREA=AAREA+ABS(YO(I)-Y(I))
   600    CONTINUE
```

```
      ERROR=DIFF/AAREA
C*     AROUT=QL*DT*3600./DCLOS-DIFF*DX
C*     ERROR=AROUT/AAREA
C*
C*** Output ***
      WRITE(*,*)
      WRITE(*,*) 'LOST SAND VOLUME=',ERROR*100,'  %'
C*    WRITE(*,*) '(QL*DT/DCLOS)/ABS(AREA) *100=',ERROR*100,'  %'
 10   FORMAT(1X,'SEAWALL POSITION'/(1X,10F8.2))
 20   FORMAT(1X,'LONGSHORE TRANSPORT'/(1X,10F8.4))
 30   FORMAT(1X,'SHORELINE POSITION'/(1X,10F8.2))
 40   FORMAT(//)
      STOP
      END
```

```
      SUBROUTINE CORRI(YSBEG,YSEND,Q,B,YCOLD,E,F,P,R,Y,YS,N)
C* CORRI recalculates transport rates (Q) due to limited sand
C* volume in front of a seawall and adjusts the shoreline
C* position as necessary. Implicit calculation scheme.
C*
      INTEGER YSBEG,YSEND
      REAL Q(41),Y(40),YS(40),YCOLD(40)
      REAL E(40),F(40),P(41),R(41)
C*
      I=YSBEG
      IF(Q(I).GT.0) THEN
C*
C* Q positive: Calc of shoreline with correction of Q and Y
C* as necessary.
C*
 10      IF(Q(I+1).GE.0) THEN
            Q(I+1)=P(I+1)*Q(I)+R(I+1)
            YC=2*B*(Q(I)-Q(I+1))+YCOLD(I)
            IF(YC.LT.YS(I)) THEN
               DQ=(YS(I)-YCOLD(I))/(2*B)
               Q(I+1)=Q(I)-DQ
            ENDIF
            Y(I)=B*(Q(I)-Q(I+1))+YCOLD(I)
            I=I+1
            IF(I.EQ.YSEND+1) GOTO 100
         GOTO 10
         ENDIF
         K=I
         I=I+1
         IF(I.EQ.YSEND+1) THEN
            Y(I-1)=B*(Q(I-1)-Q(I))+YCOLD(I-1)
            GOTO 100
         ENDIF
         IF(I.EQ.YSEND) THEN
            I=I+1
            GOTO 30
         ENDIF
      ELSE
         K=YSBEG
C*
C* Q negative: Search for minus piont. If absent, calc Y
C* for right end element. Correct Q as necessary.
C*
      ENDIF
 20   IF(Q(I+1).LT.0) THEN
         I=I+1
         IF(I.EQ.YSEND) THEN
            IF(Q(I+1).LE.0) THEN
               YC=2*B*(Q(I)-Q(I+1))+YCOLD(I)
               IF(YC.LT.YS(I)) THEN
                  DQ=(YS(I)-YCOLD(I))/(2*B)
                  Q(I)=Q(I+1)+DQ
               ENDIF
               Y(I)=B*(Q(I)-Q(I+1))+YCOLD(I)
               GOTO 30
            ENDIF
         ENDIF
      GOTO 20
      ENDIF
C*
C* Minus point: Corr of Q out of the element if shoreline moves
C* behind seawall.
C*
      YC=2*B*(Q(I)-Q(I+1))+YCOLD(I)
      IF(YC.LT.YS(I)) THEN
```

```
            DQ=(YS(I)-YCOLD(I))/(2*B)
            QDIFF=Q(I)-Q(I+1)
            Q(I)=Q(I)*DQ/QDIFF
            Q(I+1)=Q(I+1)*DQ/QDIFF
         ENDIF
         Y(I)=B*(Q(I)-Q(I+1))+YCOLD(I)
C*
C* Calc of Y starting from element to the left of minus
C* point or boundary. Q is negative.
C*
 30      DO 40 J=I-1,K,-1
            Q(J)=E(J)*Q(J+1)+F(J)
            YC=2*B*(Q(J)-Q(J+1))+YCOLD(J)
            IF(YC.LT.YS(J).AND.J.GE.YSBEG) THEN
               DQ=(YS(J)-YCOLD(J))/(2*B)
               Q(J)=Q(J+1)+DQ
            ENDIF
            Y(J)=B*(Q(J)-Q(J+1))+YCOLD(J)
 40      CONTINUE
         I=I+1
         IF(I.GE.YSEND+1) GOTO 100
C*
C* Calc of Y starting from element to the right of minus
C* point or boundary. Q is positive.
C*
         GOTO 10
 100     CONTINUE
         DO 110 I=YSBEG-1,1,-1
            Q(I)=E(I)*Q(I+1)+F(I)
            Y(I)=B*(Q(I)-Q(I+1))+YCOLD(I)
 110     CONTINUE
         IF(YSEND.NE.N) THEN
         DO 120 I=YSEND+1,N
            Q(I+1)=P(I+1)*Q(I)+R(I+1)
            Y(I)=B*(Q(I)-Q(I+1))+YCOLD(I)
 120     CONTINUE
         ENDIF
         RETURN
         END
```

```
      SUBROUTINE INDATA(IT,IT1,IT2,H,Z,N,DT)
C* SPECIFIES WAVE HEIGHTS AND ANGLES AT SPECIFIED TIME STEPS
      DIMENSION H(40),Z(40),A(40)
C*
C*IDUM=0:
C*Detached breakwater version. The program gives representative
C*wave data (H,Z) simulating effect of shore parallell detached
C*breakwater 16*DX offshore and running from I=12 to I=28.
C*Inital beach is straight line.
C*
C*IDUM=1:
C*Represents an initially circular beach with no offshore structures.
C*
      IDUM=1
      DTR=3.141593/180.
      IF(IT.EQ.IT1) GOTO 20
      IF(IT.EQ.IT2) GOTO 30
C*
C*WAVE ANGLES
C************
C*Case 1: Unaffected breaking angle = 0 deg.
      IF(IDUM.EQ.1) GOTO 14
      DO 10 I=1,12
      Z(I)=0.
10    CONTINUE
      DO 11 I=13,19
      Z(I)=Z(I-1)+10./7.
11    CONTINUE
      Z(20)=-10.
      DO 12 I=21,28
      Z(I)=Z(I-1)+10./8.
12    CONTINUE
      DO 13 I=29,N
      Z(I)=0.
13    CONTINUE
      GOTO 50
C*Alternative case 1: Unaffected angle = -20 deg.
14    CONTINUE
      DO 15 I=1,N
      Z(I)=-20.*FLOAT(40-I)/40.
15    CONTINUE
      GOTO 50
C*
C*Case 2: Unaffected breaking angle = -10 deg.
20    CONTINUE
      IF(IDUM.EQ.1) GOTO 27
      DO 21 I=1,8
      Z(I)=-10.
21    CONTINUE
      DO 22 I=9,14
      Z(I)=Z(I-1)+10./6.
22    CONTINUE
      DO 23 I=15,18
      Z(I)=Z(I-1)+2.5
23    CONTINUE
      DO 24 I=19,20
      Z(I)=Z(I-1)-5.
24    CONTINUE
      DO 25 I=21,23
      Z(I)=Z(I-1)-2.5
25    CONTINUE
      DO 26 I=24,40
      Z(I)=-10.
26    CONTINUE
```

```
      GOTO 60
C*Alternative case 2: Unaffected angle = 20 deg.
27    CONTINUE
      DO 28 I=1,N
      A(I)=-Z(N+1-I)/DTR
28    CONTINUE
      DO 29 I=1,N
      Z(I)=A(I)
29    CONTINUE
      GOTO 60
C*
C*Case 3: Unaffected breaking angle = 15 deg.
30    CONTINUE
      IF(IDUM.EQ.1) GOTO 35
      DO 31 I=1,18
      Z(I)=15.
31    CONTINUE
      DO 32 I=19,28
      Z(I)=Z(I-1)-1.5
32    CONTINUE
      DO 33 I=29,33
      Z(I)=Z(I-1)+2.5
33    CONTINUE
      DO 34 I=34,40
      Z(I)=15.
34    CONTINUE
      GOTO 70
C*Alternative case 3: Unaffected angle = 0 deg.
35    CONTINUE
      DO 36 I=1,N
      Z(I)=0.
36    CONTINUE
      GOTO 70
C*
C*WAVE HEIGHTS
C*************
C*Case 1
50    CONTINUE
      IF(IDUM.EQ.1) GOTO 55
      DO 51 I=1,10
      H(I)=1.50
51    CONTINUE
      DO 52 I=11,20
      H(I)=H(I-1)-0.1
52    CONTINUE
      DO 53 I=21,33
      H(I)=H(I-1)+1.35/14.
53    CONTINUE
      DO 54 I=34,40
      H(I)=1.85
54    CONTINUE
      GOTO 100
C*Alternative case 1
55    CONTINUE
      DO 56 I=1,N
      H(I)=3.0-FLOAT(I-1)*.5/40.
C*     H(I)=2.5
56    CONTINUE
      GOTO 100
C*
C*Case 2
60    CONTINUE
      IF(IDUM.EQ.1) GOTO 65
      DO 61 I=1,2
      H(I)=1.85
61    CONTINUE
```

```
      DO 62 I=3,16
      H(I)=H(I-1)-1.35/14.
62    CONTINUE
      DO 63 I=17,29
      H(I)=H(I-1)+1.35/14.
63    CONTINUE
      DO 64 I=30,40
      H(I)=1.85
64    CONTINUE
      GOTO 100
C*Alternative case 2
65    CONTINUE
      DO 66 I=1,N
      A(I)=H(N+1-I)
66    CONTINUE
      DO 67 I=1,N
      H(I)=A(I)
67    CONTINUE
      GOTO 100
C*
C*Case 3
70    CONTINUE
      IF(IDUM.EQ.1) GOTO 75
      DO 71 I=1,12
      H(I)=1.0
71    CONTINUE
      DO 72 I=13,24
      H(I)=H(I-1)-0.5/12.
72    CONTINUE
      DO 73 I=25,37
      H(I)=H(I-1)+1.35/14.
73    CONTINUE
      DO 74 I=38,N
      H(I)=1.85
74    CONTINUE
      GOTO 100
C*Alternative case 3
75    CONTINUE
      DO 76 I=1,20
      A(I)=H(2*I)
76    CONTINUE
      DO 77 I=21,N
      A(I)=A(N-I+1)
77    CONTINUE
      DO 78 I=1,N
      H(I)=A(I)
78    CONTINUE
C*
C*OUTPUT
C*
100   CONTINUE
      IHOURS=(IT-1)*INT(DT)
      WRITE(*,600)
      IF(IT.EQ.1) WRITE(*,*) 'INITIAL CONDITIONS'
      IF(IT.EQ.IT1.OR.IT.EQ.IT2) THEN
         WRITE(*,*) 'CONDITIONS AFTER',IHOURS,'HOURS'
      ENDIF
      WRITE(*,*) 'WAVE HEIGHTS'
      WRITE(*,602) (H(I),I=1,N)
      WRITE(*,600)
      WRITE(*,*) 'WAVE ANGLES'
      WRITE(*,602) (Z(I),I=1,N)
      WRITE(*,600)
      DO 200 I=1,N
      Z(I)=Z(I)*DTR
200   CONTINUE
600   FORMAT(//)
602   FORMAT((1X,10F8.2))
      RETURN
      END
END OF FILE
```

APPENDIX B: NOTATION

a'	Volume of solids/total volume
b	Subscript denoting breaking condition
C_g	Wave group velocity, m/s
D	Depth of profile closure, m
g	Acceleration resulting from gravity, m/s^2
H	Wave height, m
i	Subscript denoting position alongshore
K	Dimensionless empirical coefficient in the longshore sediment transport rate formula
n	Superscript denoting time level
N	Total number of calculation cells in the model
Q	Total volumetric longshore sediment transport rate, m^3/s
r	Conversion factor from RMS to significant wave height
R_s	Stability parameter, m^2/s
S	Ratio of density of solids to density of water
t	Time, s
x	Position alongshore, m
y	Position on-offshore; shoreline position, m
yc	Extremal, internally calculated shoreline position, used in the implicit numerical solution scheme, m
ys	Position of seawall on-offshore, m
γ	Ratio of wave height to water depth at breaking
Δt	Time step, s
Δx	Space step alongshore, m
Δy	Space increment on-offshore, m
θ_{bs}	Angle of breaking waves to the shoreline, deg
θ_b	Angle of breaking waves to the x-axis, deg
*	Superscript denoting a corrected value
'	Superscript denoting a quantity at next time step

dV	Volume of solid-material action
b	Subscript denoting breaking condition
c_g	Wave group velocity, m/s
D	Depth of profile closure, m
g	Acceleration resulting from gravity, m/s^2
H	Wave height, m
i	Subscript denoting position alongshore
K	Dimensionless empirical coefficient in the longshore sediment transport rate formula
n	Superscript denoting time level
N	Total number of calculation cells in the model
Q	Total volumetric longshore sediment transport rate, m^3/s
r	Conversion factor from rms to significant wave height
R_s	Stability parameter, m^3/s
s	Ratio of density of solids to density of water
t	Time, s
x	Position alongshore, m
y	Position offshore; shoreline position, m
y'	External, internally calculated shoreline position, used in the implicit numerical solution scheme, m
y_s	Position of seawall, offshore, m
γ	Ratio of wave height to water depth at breaking
Δt	Time step, s
Δx	Space step alongshore, m
Δy	Space increment offshore, m
θ_{bs}	Angle of breaking waves to the shoreline, deg
θ_b	Angle of breaking waves to the x-axis, deg
$'$	Superscript denoting a calculated value
$*$	Superscript denoting a quantity as with time step