SMITHSONIAN

CONTRIBUTIONS TO KNOWLEDGE.

VOL. XVIII.

EVERY MAN IS A VALUABLE MEMBER OF SOCIETY, WHO, BY HIS OBSERVATIONS, RESEARCHES, AND EXPERIMENTS, PROCURES KNOWLEDGE FOR MEN.—SMITHSON.

CITY OF WASHINGTON:
PUBLISHED BY THE SMITHSONIAN INSTITUTION.
MDCCCLXXIII.

COLLINS, PRINTER,
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ADVERTISEMENT.

This volume forms the eighteenth of a series, composed of original memoirs on different branches of knowledge, published at the expense, and under the direction, of the Smithsonian Institution. The publication of this series forms part of a general plan adopted for carrying into effect the benevolent intentions of James Smithson, Esq., of England. This gentleman left his property in trust to the United States of America, to found, at Washington, an institution which should bear his own name, and have for its objects the "*increase* and *diffusion* of knowledge among men." This trust was accepted by the Government of the United States, and an Act of Congress was passed August 10, 1846, constituting the President and the other principal executive officers of the general government, the Chief Justice of the Supreme Court, the Mayor of Washington, and such other persons as they might elect honorary members, an establishment under the name of the "Smithsonian Institution for the increase and diffusion of knowledge among men." The members and honorary members of this establishment are to hold stated and special meetings for the supervision of the affairs of the Institution, and for the advice and instruction of a Board of Regents, to whom the financial and other affairs are intrusted.

The Board of Regents consists of three members *ex officio* of the establishment, namely, the Vice-President of the United States, the Chief Justice of the Supreme Court, and the Mayor of Washington, together with twelve other members, three of whom are appointed by the Senate from its own body, three by the House of Representatives from its members, and six persons appointed by a joint resolution of both houses. To this Board is given the power of electing a Secretary and other officers, for conducting the active operations of the Institution.

To carry into effect the purposes of the testator, the plan of organization should evidently embrace two objects: one, the increase of knowledge by the addition of new truths to the existing stock; the other, the diffusion of knowledge, thus increased, among men. No restriction is made in favor of any kind of knowledge; and, hence, each branch is entitled to, and should receive, a share of attention.

The Act of Congress, establishing the Institution, directs, as a part of the plan of organization, the formation of a Library, a Museum, and a Gallery of Art, together with provisions for physical research and popular lectures, while it leaves to the Regents the power of adopting such other parts of an organization as they may deem best suited to promote the objects of the bequest.

After much deliberation, the Regents resolved to divide the annual income into two parts—one part to be devoted to the increase and diffusion of knowledge by means of original research and publications—the other part of the income to be applied in accordance with the requirements of the Act of Congress, to the gradual formation of a Library, a Museum, and a Gallery of Art.

The following are the details of the parts of the general plan of organization provisionally adopted at the meeting of the Regents, Dec. 8, 1847.

DETAILS OF THE FIRST PART OF THE PLAN.

I. To increase Knowledge.—*It is proposed to stimulate research, by offering rewards for original memoirs on all subjects of investigation.*

1. The memoirs thus obtained, to be published in a series of volumes, in a quarto form, and entitled "Smithsonian Contributions to Knowledge."

2. No memoir, on subjects of physical science, to be accepted for publication, which does not furnish a positive addition to human knowledge, resting on original research; and all unverified speculations to be rejected.

3. Each memoir presented to the Institution, to be submitted for examination to a commission of persons of reputation for learning in the branch to which the memoir pertains; and to be accepted for publication only in case the report of this commission is favorable.

4. The commission to be chosen by the officers of the Institution, and the name of the author, as far as practicable, concealed, unless a favorable decision be made.

5. The volumes of the memoirs to be exchanged for the Transactions of literary and scientific societies, and copies to be given to all the colleges, and principal libraries, in this country. One part of the remaining copies may be offered for sale; and the other carefully preserved, to form complete sets of the work, to supply the demand from new institutions.

6. An abstract, or popular account, of the contents of these memoirs to be given to the public, through the annual report of the Regents to Congress.

II. TO INCREASE KNOWLEDGE.—*It is also proposed to appropriate a portion of the income, annually, to special objects of research, under the direction of suitable persons.*

1. The objects, and the amount appropriated, to be recommended by counsellors of the Institution.

2. Appropriations in different years to different objects; so that, in course of time, each branch of knowledge may receive a share.

3. The results obtained from these appropriations to be published, with the memoirs before mentioned, in the volumes of the Smithsonian Contributions to Knowledge.

4. Examples of objects for which appropriations may be made:—

(1.) System of extended meteorological observations for solving the problem of American storms.

(2.) Explorations in descriptive natural history, and geological, mathematical, and topographical surveys, to collect material for the formation of a Physical Atlas of the United States.

(3.) Solution of experimental problems, such as a new determination of the weight of the earth, of the velocity of electricity, and of light; chemical analyses of soils and plants; collection and publication of articles of science, accumulated in the offices of Government.

(4.) Institution of statistical inquiries with reference to physical, moral, and political subjects.

(5.) Historical researches, and accurate surveys of places celebrated in American history.

(6.) Ethnological researches, particularly with reference to the different races of men in North America; also explorations, and accurate surveys, of the mounds and other remains of the ancient people of our country.

I. TO DIFFUSE KNOWLEDGE.—*It is proposed to publish a series of reports, giving an account of the new discoveries in science, and of the changes made from year to year in all branches of knowledge not strictly professional.*

1. Some of these reports may be published annually, others at longer intervals, as the income of the Institution or the changes in the branches of knowledge may indicate.

2. The reports are to be prepared by collaborators, eminent in the different branches of knowledge.

3. Each collaborator to be furnished with the journals and publications, domestic and foreign, necessary to the compilation of his report; to be paid a certain sum for his labors, and to be named on the title-page of the report.

4. The reports to be published in separate parts, so that persons interested in a particular branch, can procure the parts relating to it, without purchasing the whole.

5. These reports may be presented to Congress, for partial distribution, the remaining copies to be given to literary and scientific institutions, and sold to individuals for a moderate price.

The following are some of the subjects which may be embraced in the reports:—

I. PHYSICAL CLASS.

1. Physics, including astronomy, natural philosophy, chemistry, and meteorology.
2. Natural history, including botany, zoology, geology, &c
3. Agriculture.
4. Application of science to arts.

II. MORAL AND POLITICAL CLASS.

5. Ethnology, including particular history, comparative philology, antiquities, &c.
6. Statistics and political economy.
7. Mental and moral philosophy.
8. A survey of the political events of the world; penal reform, &c.

III. LITERATURE AND THE FINE ARTS.

9. Modern literature.
10. The fine arts, and their application to the useful arts.
11. Bibliography.
12. Obituary notices of distinguished individuals.

II. To diffuse Knowledge.—*It is proposed to publish occasionally separate treatises on subjects of general interest.*

1. These treatises may occasionally consist of valuable memoirs translated from foreign languages, or of articles prepared under the direction of the Institution, or procured by offering premiums for the best exposition of a given subject.

2. The treatises to be submitted to a commission of competent judges, previous to their publication.

DETAILS OF THE SECOND PART OF THE PLAN OF ORGANIZATION.

This part contemplates the formation of a Library, a Museum, and a Gallery of Art.

1. To carry out the plan before described, a library will be required, consisting, 1st, of a complete collection of the transactions and proceedings of all the learned societies of the world; 2d, of the more important current periodical publications, and other works necessary in preparing the periodical reports.

2. The Institution should make special collections, particularly of objects to verify its own publications. Also a collection of instruments of research in all branches of experimental science.

3. With reference to the collection of books, other than those mentioned above, catalogues of all the different libraries in the United States should be procured, in order that the valuable books first purchased may be such as are not to be found elsewhere in the United States.

4. Also catalogues of memoirs, and of books in foreign libraries, and other materials, should be collected, for rendering the Institution a centre of bibliographical knowledge, whence the student may be directed to any work which he may require.

5. It is believed that the collections in natural history will increase by donation, as rapidly as the income of the Institution can make provision for their reception; and, therefore, it will seldom be necessary to purchase any article of this kind.

6. Attempts should be made to procure for the gallery of art, casts of the most celebrated articles of ancient and modern sculpture.

7. The arts may be encouraged by providing a room, free of expense, for the exhibition of the objects of the Art-Union, and other similar societies.

8. A small appropriation should annually be made for models of antiquity, such as those of the remains of ancient temples, &c.

9. The Secretary and his assistants, during the session of Congress, will be required to illustrate new discoveries in science, and to exhibit new objects of art; distinguished individuals should also be invited to give lectures on subjects of general interest.

In accordance with the rules adopted in the programme of organization, each memoir in this volume has been favorably reported on by a Commission appointed for its examination. It is however impossible, in most cases, to verify the statements of an author; and, therefore, neither the Commission nor the Institution can be responsible for more than the general character of a memoir.

The following rules have been adopted for the distribution of the quarto volumes of the Smithsonian Contributions:—

1. They are to be presented to all learned societies which publish Transactions, and give copies of these, in exchange, to the Institution.

2. Also, to all foreign libraries of the first class, provided they give in exchange their catalogues or other publications, or an equivalent from their duplicate volumes.

3. To all the colleges in actual operation in this country, provided they furnish, in return, meteorological observations, catalogues of their libraries and of their students, and all other publications issued by them relative to their organization and history.

4. To all States and Territories, provided there be given, in return, copies of all documents published under their authority.

5. To all incorporated public libraries in this country, not included in any of the foregoing classes, now containing more than 10,000 volumes; and to smaller libraries, where a whole State or large district would be otherwise unsupplied.

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Columbus Delano.	*The Secretary of the Interior.*

TABLE OF CONTENTS.[1]

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[1] Each memoir is separately paged and indexed.

PAGE

ARTICLE IV. OBSERVATIONS ON TERRESTRIAL MAGNETISM AND ON THE DEVIATIONS OF THE COMPASSES OF THE UNITED STATES IRON-CLAD MONADNOCK DURING HER CRUISE FROM PHILADELPHIA TO SAN FRANCISCO, IN 1865 AND 1866. By WM. HARKNESS, M.D. Accepted for publication September, 1871. Published April, 1873. 4to. pp. 225. With 2 Diagrams.

SECTION I. Introduction.

SECTION II. Descriptions of Stations.

SECTION III. Astronomical Observations.

SECTION IV. Observations on Terrestrial Magnetism.

Determination of constants peculiar to the portable declinometer, D. 22.

SECTION V. Observations on the Magnetism of the Ship.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.
222

TABLES AND RESULTS

OF THE

PRECIPITATION, IN RAIN AND SNOW,

IN THE

UNITED STATES:

AND AT SOME STATIONS IN ADJACENT PARTS OF NORTH AMERICA,
AND IN CENTRAL AND SOUTH AMERICA.

COLLECTED BY THE SMITHSONIAN INSTITUTION, AND DISCUSSED UNDER DIRECTION OF

JOSEPH HENRY, SECRETARY.

BY
CHARLES A. SCHOTT,
ASSISTANT U. S. COAST SURVEY; MEM. AM. PHIL. SOCIETIES OF PHILADELPHIA AND WASHINGTON,
AND OF ACADEMY OF SCIENCES OF CATANIA, SICILY.

WASHINGTON CITY:
PUBLISHED BY THE SMITHSONIAN INSTITUTION.
[MARCH 1872.]

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CONTENTS.

INTRODUCTION.

THE following memoir contains in tabulated form the abstracts of all the records of observations of the rain-fall which have been made from the early settlement of this country down to the close of the year 1866, so far as they could be obtained. These records are from about one thousand two hundred stations, and consist of the observations made under the direction of the Smithsonian Institution, assisted since 1854 by the Patent Office and Department of Agriculture; of those by the Medical Department of the United States Army, of those by the United States Survey of the North and Northwest Lakes, of those made by the New York University System, by the Franklin Institute in Philadelphia, and also of those by other scientific institutions and individuals. For a more definite account of the various sources of information we would refer to subsequent pages. It is proper, however, that we should here express our obligations for the valuable co-operation of the Medical Department of the Army under Surgeon-General Barnes, who has given us free access to all the unpublished records, and also for that of the Department of Agriculture under the Commissioner, General Capron.

These materials were placed in the hands of Mr. Charles A. Schott, of the United States Coast Survey, whom we had previously employed in the reduction and discussion of the observations of Kane, Hayes, McClintock, and others, which have been published from time to time in the Smithsonian Contributions to Knowledge. The tables which were prepared as the basis for future study are in themselves of much utility in regard to the practical knowledge which they afford of the special climatology of the United States, while the deductions which have been drawn from the discussion of the facts presented in these tables, and which are principally expressed in the maps of the distribution of the rain-fall, are intimately connected with the agriculture, commerce, and mechanical industry of the country, while they constitute a valuable contribution to the physical geography of the globe.

The aim was to collect information on the rain-fall from all sources of observation, whether published or in manuscript, but the effort could only be approximately successful, since, however diligent may have been the search, much must remain undiscovered. On this account the tables have been so constructed as to admit of continued additions and corrections.

The degree of precision of such observations, made, as they have been, by so many persons of different knowledge and experience, must, of course, vary within a considerably wide range; still it would appear from the results of the discussions of Mr. Schott that they are sufficiently exact to afford satisfactory evidence of definite laws of the rain-fall, such as he has exhibited by curves for different sections of the country. The rain-fall, which apparently is one of the most irregular of atmospheric phenomena, is nevertheless referable, from numerous observations through long periods, to definite phases of occurrence, both in time and quantity, and even the imperfections of the instruments, provided they give results in excess as well as in deficiency, in a measure compensate each other.

This memoir is one of a series embodying the results of all the labors of the Smithsonian Institution in regard to the meteorology of the United States. These will include not only all the observations which have been made under its own direction, but also the discussion of all that have been made by other parties. The whole series will embrace the tabulation and discussion of observations on the temperature, atmospheric pressure, direction and force of the wind, moisture of the air, and miscellaneous phenomena. The discussion of the temperature and the winds, which has been in progress for several years, is now well advanced, and will be published in part during the year 1872.

Since the completion of this memoir an effort has been made to increase the number of observations, and for this purpose a simple form of rain-gauge has been adopted, and of this about four hundred have been distributed to different parts of the country.

The publication of this memoir will, it is hoped, tend still further to awaken attention to this subject, and to secure a still larger number of voluntary observers, which will enhance the value of the records, since this depends both on the number and on the duration of the series.

JOSEPH HENRY,

Secretary S. I.

WASHINGTON, D. C., 1871.

THE RAIN-FALL IN THE UNITED STATES.

COLLECTION AND TABULATION OF RECORDS AND RESULTS.

It is intended in the following pages to present, as a contribution towards the advancement of American climatology, the leading facts connected with the aqueous precipitation over the area of the United States; also to include such scanty records from other parts of North America and from Central and South America as could be collected. It has been the aim to bring together from all sources records of observations of the fall of water, in rain or snow, from the earliest observers to those of the present time.

The first work, after collecting the material, was to form tables giving a summary of monthly amounts of rain-fall, arranged for each State and Territory separately, for each year. These tables served as the basis from which most of the deductions given in this memoir are derived; but, on account of their voluminous character and consequent great expense of printing, they are not published at present. They can be referred to, however, at the Institution, for more minute information[1] respecting the nature of the record than could conveniently be given in the printed tables, which contain all that is known or essential to the true valuation of each series. Tables A and B of this memoir are directly derived from these manuscript tables, and present their general contents in a more condensed form.

As early as 1853 and 1854 the Smithsonian Institution had collected a large number of rain records, and tabulated the monthly and annual amounts, and given other information respecting the rain-fall for stations occupied up to that period. This collection was placed in my hands in August, 1867, was revised and enlarged, and the records and results, generally, were brought up to the close of the year 1866. As no definite limit for closing the records was assigned, the tables at many stations include those for the year 1867, and, for some few localities, those for a part of 1868 also. Additions to any of the tables can readily be made hereafter, as date and reference are given for each series.

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[1] In these manuscript tables any break in the regularly noted series for any month is indicated by dashes, and when there was no precipitation during the interval a zero is inserted. In the monthly summaries the insertion of a star (*) indicates incomplete record, the sign (†) indicates a doubtful result, and the sign (‡) an approximate result. The series of Smithsonian records of 1860 were in part lost or damaged by fire, in January, 1865; many of these, however, have since been replaced by the observers when called upon for duplicates.

SPECIAL REFERENCES TO RECORDS.

The earliest observations of the rain-fall, known to have been made within the present limits of the United States, do not go back further than the year 1738; in the eighteenth century we have but nine stations, which, with the exception of Charleston, S. C., afforded no results till after 1772. Records of the rain-fall do not become continuous at any one station till after the commencement of the present century, but the continuity may be maintained for one station or another within the United States from about 1791 to the present time. After 1825, when the New York University system of meteorological observations was inaugurated, the records become more numerous. In 1836 was commenced the system of observations of the rain-fall under the direction of the Medical Department of the United States Army, which system, like the former, is continued to the present time, and constitutes a most important contribution to our knowledge in this branch of meteorology. In 1839 observations of the rain-fall were made under the auspices of the Franklin Institute, Pennsylvania, and maintained for a few years. In 1847 it was proposed by the present Secretary of the Smithsonian Institution to establish a system of extended meteorological observations, which went into operation in 1849. In connection with the United States Patent Office, the Smithsonian Institution published, in extenso, the meteorological observations for the years 1854 to 1859 inclusive, and the records of the large and still increasing corps of observers co-operating with the Institution constitute now the most bulky part of our meteorological collections. A valuable series of observations was commenced in 1859 under the direction of the superintendents of the survey of the North and Northwest Lakes, which under the direction of the United States Engineer Corps is still continued. The following references to sources from which material has been drawn, and which relate to the various principal systems enumerated above, deserve to be specially mentioned:—

The annual reports of the Regents of the University of the State of New York (Albany, see specially vol. for 1855; the twentieth report appeared in 1868); the Army Meteorological Register for twelve years, from 1831 to 1842 inclusive, compiled from observations made by the officers of the Medical Department, at the military posts of the United States, prepared by Surgeon-General T. Lawson (Washington, 1851); the Army Meteorological Register for twelve years, from 1843 to 1854 inclusive, compiled and prepared as the preceding volume, and containing an appendix of a report on the distribution of rain, illustrated by five rain-charts, based upon the reports furnished by the military posts (Washington, 1855); the Statistical Report on the Sickness and Mortality in the Army of the United States, from 1855 to 1860, compiled and prepared under the direction of Surgeon-General T. Lawson, by R. H. Coolidge, M.D., U. S. A. (Washington, 1860); Results of Meteorological Observations made under the direction of the U. S. Patent Office and the Smithsonian Institution, for the years 1854 to 1859 inclusive, Vol. I. (Washington, 1861); and the Annual Report on the Survey of the North and Northwest Lakes, for the year ending June 30, 1867, by Bvt. Brig.-Gen. W. F. Raynolds. Among manuscripts of large extent, those kindly furnished by Capt. (now Gen.) G. G. Meade and Col. (late Lieut.-Col.) J. D. Graham, Top. Eng. U. S. A., Superintendents of the Lake Survey (1858–1863), and by Bvt. Maj.-Gen. J. K. Barnes, Surgeon-General U. S. A., for eight years (1860–1867), were of great value, especially the latter, as comprising records at the military stations situated *west* of the Mississippi, upon which stations our meteorological information for the western part of the United States almost wholly depends. The records of the Smithsonian observers from 1860 to 1868 were available in manuscript; these, and the materials mentioned above, together with many series published or communicated by public institutions or by private individuals, are incorporated in the following tables and reductions.

The unit of measure of rain-fall is the English inch; in the few exceptional cases in which the record is given in units of the metric system, the latter has been changed to the former for the sake of uniformity.

DESCRIPTION OF RAIN-GAUGES AND DIRECTIONS FOR OBSERVING.

Particulars respecting the kind of rain and snow gauge, the method of observing, and other remarks and directions, will be found in the following extracts.

Rain-gauges at the Military Posts of the United States, and Regulations for their Use.—The following extract is from the preface of the Army Meteorological Register, from 1831 to 1842 inclusive (Washington, 1851): "In 1836 rain-gauges were furnished to many of the posts, by which the daily falls of rain and snow would be measured, and entered upon the tables in inches and fractions of an inch. The instrument employed is the conical rain-gauge of De Witt; and observations are ordered to be made immediately after every shower or fall of rain or snow. The following are the instructions issued by the Department for its observers: The instrument used to measure the quantity of rain which falls is the conical rain-gauge. It is to be kept remote from all elevated structures, at a distance at least equal to its height, and still further off where it can be conveniently done. It is to be suspended in a circular opening, made in a board, which is to be fixed to a post, eight feet from the ground; the opening to be five inches in diameter, and bevelled, so as to fit the cone of the gauge, into which the conical cap is to be placed, base downwards, to prevent evaporation. The measurement is made by putting down perpendicularly to the bottom of the gauge the measuring stick and applying it from its point to the water-mark on the scale, which will express the quantity in inches or their decimals. The graduation of the scale is by hundredths of an inch for the first three-tenths of an inch, and above that by tenths and half-tenths. Parts of degrees will be measured by the eye, and set down in decimals. If a rain continue for any length of time, the quantity in the gauge will be measured at suitable intervals, before the water rises high in it, and the measurements summed up at the close.

"In freezing weather, when the rain-gauge cannot be used out of doors, it must be taken into the room, and a tin vessel will be substituted for receiving the snow, rain, or sleet that may then fall. This vessel must have its opening exactly equal to that of the rain-gauge, and widen downwards to a sufficient depth, with a considerable slope. It should be placed where nothing can obstruct the descending snow from entering, and where no drift-snow can be blown into it. During a continued snowstorm the snow may be occasionally pressed down. The contents of the vessel must be melted by placing it near the fire, with a cover to prevent evaporation, and the water produced poured into the gauge to ascertain its quantity, which must then be entered on the register."

A description of the conical rain-gauge is given in Silliman's Journal of Science and Arts for April, May, and June, 1832.

Directions issued July 1, 1844, from the Surgeon-General's Office, hold the senior medical officer on duty at each military station officially responsible for the accuracy of the meteorological observations made at the station.

The following additional direction is given in the introduction to the Army Meteorological Register from 1843 to 1854 inclusive (Washington, 1855): "At every fall of rain, snow, hail, or sleet the time of its commencement and end will be recorded, and the quantity which fell is indicated by the rain-gauge."

Rain-gauges adopted by the Smithsonian Institution.—The following is an extract from the "Directions for Meteorological Observations adopted by the Smithsonian Institution, for the first-class Observers," in the annual Smithsonian Report for 1855 (Washington, 1856): "The ombrometer, or rain-gauge, is a funnel accompanied by a graduated cylindrical glass vessel, and by a reservoir. It should be placed in an open space. Trees, high buildings, and other obstacles, if too near, may have a considerable influence in increasing or diminishing the quantity of rain which falls into the funnel. The surface of the receiver should be placed horizontally about six inches above the ground." Next follows a simple mode of establishing this gauge, accompanied by a wood-cut of the same. Directions for observing are given as follows:—

"To make the observation, remove the funnel and pour the water from the jug into the large graduated glass cylinder. The opening of the funnel being one hundred square inches, one inch of rain in depth gives one hundred cubic inches of water; and each division of the glass containing a cubic inch of water, each of them represents a hundredth of an inch of rain fallen into the ombrometer. These degrees are large enough to permit us to estimate the thousandths of an inch, etc. etc.

"The snow-gauge should be supported vertically, in an open place, between three short wooden posts, its opening being about two feet from the ground. It should be employed in the following manner: When only a small quantity of snow falls, or of snow alternating with rain, or of dry and fine snow, driven by the wind, it should be collected in the snow-gauge, as would be done in the ombrometer. But when the snow falls in a sufficient quantity to cover the ground more than an inch deep, the vessel must be emptied, and plunged, mouth downwards, into the snow, until the rim reaches the bottom. A plate of tinned iron or a small board may then be passed between the ground and the mouth of the gauge, and the whole reversed. In this way a cylinder of snow, of which the base is superficially one hundred inches, will be cut out and received into the vessel. The operation may be facilitated by placing on the ground a platform of strong board or plank, two or three feet square, on which the snow is received. The place selected for this purpose must be one where the snow has not been heaped up or swept away by the wind, and where it presents, as near as possible, the mean depth of the layer that has fallen. In order to take only the snow which may fall in the interval between two observations, the board should be swept after each measurement, and the place designated by stakes. The collected snow must be melted by placing the gauge, covered with a board to prevent evaporation, in a warm room, and the quantity of water produced measured by pouring it into the glass cylinder. The rain water and melted snow water must be separately entered in the journal. During abundant rain-falls it is well to measure the water more than once a day, or at least immediately after the rain, and the quantity of rain fallen, together with the time it has lasted, is to be noted. When it freezes, it will be necessary to protect the receiver by filling in the interior of the barrel with straw."

"A series of observations has been made at the Smithsonian Institution with rain-gauges of different sizes and different forms, the result of which, as far as the observations have been carried, is to induce a preference for the smallest gauges. The one which was first distributed by the Institution and the Patent Office to the observers (represented on p. 229 of the Secretary's Report for 1855), consists of a funnel terminated above by a cylindrical brass ring, bevelled into a sharp edge at the top, turned perfectly round in a lathe, and of precisely five inches diameter. The rain which falls within this ring is conducted into a two-quart bottle placed below to receive it. To prevent any water which may run down on the outside of the funnel from entering the bottle, a short tube is soldered on the lower part of the former and incloses the neck of the latter. The funnel and bottle are placed in a box or small cask, sunk to the level of the ground, which is covered with a board having a circular hole in its centre to receive and support the funnel. To prevent the rain-drops which may fall on this board from spattering into the mouth of the funnel, some pieces of old cloth or carpet may be tacked upon it."

"The object of placing the receiving ring so near the surface of the earth, is to avoid eddies caused by the wind, which might disturb the uniformity of the fall of rain."

"In the morning, or after a shower of rain, the bottle is taken up, and its contents measured in the graduated tube, and the quantity in inches and parts recorded in the register. The gauge or tube which was first provided for this purpose will contain when full only one-tenth of an inch of rain, the divisions indicating hundredths and thousandths of an inch. As this, however, was found to be too small for convenience, another gauge which contains an inch of rain, and indicating tenths and hundredths, was sent to observers."

Another and simpler form afterwards adopted by the Institution and Patent Office is one of those which have been experimented on at the Institution. It is a modification of a gauge which was received from Scotland, and which has been recommended by Mr. Robert Russell. It consists (see figure on p. 230 of the Secretary's Annual Report for 1855) of a large brass cylinder two inches in diameter, to catch the rain; a smaller brass cylinder for receiving the water and reducing the diameter of the column to allow of greater accuracy in measuring the height;

a whalebone scale divided by experiment, so as to indicate tenths and hundredths of an inch of rain. A wooden cylinder is to be inserted permanently in the ground for the protection and ready adjustment of the instrument.

To ascertain the amount of water produced from snow, a column of the depth of the fall of snow, of the same diameter as the mouth of the funnel, should be melted and measured as so much rain. The simplest method of obtaining a column of snow for this purpose, is to procure a tin tube about two feet long, having one end closed, and precisely of the diameter of the mouth of the gauge.

From measurements of this kind, repeated in several places when the depth of the snow is unequal, an average quantity may be obtained.

To facilitate transportation, the larger cylinder is attached to the smaller by a screw-joint.

A still simpler gauge has lately been devised by the Secretary of the Institution; it consists of a cylindrical tube two and a half or three inches in diameter and nine inches in length, closed with a *conical* bottom or else provided with a small cylinder projecting from a flat bottom, the object of which would be to give precise indications of the *smallest* quantities, and, with the same instrument, measure the largest amount which falls in any one shower. But, for ordinary observers, it is thought that nothing is better than a simple cylinder, three or four inches in diameter, and a wooden measuring-rod divided into inches and tenths. Instruments of the kind last mentioned are now being distributed among observers.

To measure thousandths of inches is considered useless labor, since no two gauges can be made to give results agreeing in the hundredths of inches, owing to the irregularity in the rain-fall itself.

EXPLANATORY REMARKS ON RECORDS.

The value or precision of the observations collected from such various sources must necessarily have a wide range, and while the greater portion is supposed to come up to a standard precision, a smaller portion partakes more or less of uncertainty, either from want of reliable instruments or from defective observing. In a phenomenon so very irregular as that of the rain-fall, it is difficult to separate defective observations from correct ones; rejections of observations have, therefore, been resorted to very sparingly, and only in such few instances when other records have most undoubtedly proved their unreliability. It may be well briefly to enumerate the principal sources of error to which observations of the rain-fall were found to be liable: A defective construction of the gauge; error in the graduation or scale; improper location of gauge in reference to surrounding objects; and allowance, by rule, for water fallen as snow, instead of measuring the melted snow (or hail). Rain-gauges as commonly used, may be supposed liable to no greater error than about two per cent.; this limit includes error introduced by a defect in the graduation or in the manner of applying a measuring-rod. The most frequent cause of difference in the results by two observers at adjacent stations will generally be found in the manner of exposure of their gauges; the proper location with reference to level ground and surrounding objects, as trees, houses, or hills being left to the judgment of observers, the rule has not always been observed to place the gauge *horizontally* at a distance from any such object of at least twice the amount of its elevation above the location of the gauge. The effect on the wind of elevated objects is to produce eddies, often felt at great distances, as may frequently be seen by the whirling motion of dust. The rule generally adopted of allowing one-tenth of the depth of snow as its equivalent amount in water, is, of course, a very rough one, as the allowance must depend upon the temperature of the snow, the form and size of the snowflakes, and the depth fallen, since the snow will be compressed by its own weight. For precise measures, the water resulting from the melted snow should always be recorded. The loss by evaporation of the water collected in the gauge, or of the snow when melting, can easily be guarded against. Respecting the elevation of the gauges above the general level of the place, so very little information could be obtained that no special mention of it is made in the tables; they were generally on the ground, or, at most, but a foot above. There can hardly be too many observers even within quite a limited area, for instance, such as is covered by a city of moderate extent; the results here collected give abundant proof, not only of large differences in the amount collected in single showers, in consequence of their not unfrequently sharp definition of extent, but also in the aggregate monthly and annual amounts. There are a few records of western stations which, in some parts, leave it doubtful whether a blank indicates that no rain fell or whether no observations were made; not to introduce any uncertainty in the deductions, and in the absence of better information in such cases, it was assumed that no observations were taken. A more embarrassing case to the computer was the uncertainty, in many instances, whether melted snow had been included by the observer in his result or not. This question was not unfrequently difficult to decide, but, upon the whole, the discussion of the annual periodicity of the rain gave sufficient proof that no serious error has crept into the results from that source of defective information. No results at stations, when such omissions were known to exist, were admitted in the discussion.

The following tables, A and B, show the average amount of precipitation for each month, season, and year, and the amounts for a series of years, in the United States, some adjacent parts of North America, and in Central and South America. They have been, as previously stated, deduced from the manuscript tables retained in the Smithsonian Institution for future use and reference.

TABLES

OF

AQUEOUS PRECIPITATION

FOR

MONTHS, SEASONS, AND YEARS.

(A.)

EXPLANATION OF TABLE (A).

The headings of the different columns of the table, in general, sufficiently explain their contents, but a few additional remarks may be made, principally relating to the position of the stations and to the continuity and combination of records.

The latitude and longitude of many stations near the sea-coast, and especially of the military posts there situated, are given on the authority of the U. S. Coast Survey; positions near the Great Lakes are from the survey of the lakes by officers of the U. S. A.; other positions and all those in the interior are taken from the best sources available, and frequently depend on the authority of the observers. In the absence of any trigonometrical survey of the States (excepting Massachusetts) and Territories, these assigned geographical positions are subject to much uncertainty; that in latitude may reach ±5′ and that in longitude ±12′ for the more extreme cases, and may reach half these amounts in ordinary cases. Probably few will be found exceeding these limits.

The height of the place of observation above the ocean is given on the authority of the observer, unless corrected from other and apparently more reliable sources. These data are known in many instances to be quite defective, but there are now no means at hand to improve them.

The tabular means given for any one month are derived from all monthly amounts in the different years, and, unless the whole series of observations is unbroken, may differ in weight, since they may be derived from a greater or less number of years, and, for the same reason, of discontinuity in the observations; the numbers in the column headed "Extent" do not always correspond to the *time elapsed* from the date of commencement to that of the close of the series, as given in the column following. The column headed "Extent" shows the number of years and months of *actual* observation. The contents of the table is generally brought up to the year 1867.

When more than one series for any limited locality was on record, either covering the same period or for different years, consolidated results are given at the bottom of the table, at least for the more prominent localities. For these consolidated results, the several monthly values observed for any one month and for any one year were first combined by taking the mean, and these monthly means for the several years were afterwards united. This mode of combination must necessarily be followed on account of the annual inequality in the rain-fall. The observer's name is stated, except for the series of observations taken by the U. S. Army, and for those taken by the New York University system, and necessarily for a few other stations where the observer's name was not given. The observations of the army were generally taken by the assistant surgeon of the post, and those of the

State of New York by the professors of the academies. The ABBREVIATIONS used in the last column headed "Reference" are principally the following:—

Arm. Met. Reg., 1855,	denotes the U. S. Army Meteorological Register, Washington, 1855.
M. D. of U. S. A., 1860,	denotes the U. S. Medical Statistics forming the third collection of the Army Meteorological Publications, Washington, 1860.
P. O. and S. I., Vol. I.,	denotes the Results of the Meteorological Observations made under the direction of the Patent Office and the Smithsonian Institution, Washington, 1861.
Surv. of N. and N. W. Lakes,	denotes Manuscript and Annual Report of the Survey of the N. and N. W. Lakes under the direction of the U. S. Topographical Engineers.
Sm. Obs.	denotes the Manuscripts by the observers operating in connection with the Smithsonian Institution.
Sm. Coll'n.	denotes Manuscripts collected at different times by the Institution.
N. Y. Acad. Sys. Reg. Rep.	denotes the Reports by the Regents of the Universities of New York.
M. S. from S. G. O.	denotes the latest Observations at Military Stations west of the Mississippi River, obtained from the Surgeon-General's Office, U. S. A., 1867–8.
Amer. Alm.	denotes American Almanac, Boston.
Smith. Con. Know.	denotes Smithsonian Contributions to Knowledge, Washington.
Sill's Journ.	denotes American Journal of Science, New Haven.
Mem. Am. Acad.	denotes Memoirs of the American Academy of Science, Boston.
Trans. Phil. Soc.	denotes Transactions of the American Philosophical Society, Phila.
London Gard. Ch.	denotes London Gardener's Chronicle, London, England.

TABLES.

MAINE.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Latitude.	Longitude.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Kent	47°15′	68°35′	575	3.75	2.60	1.77	1.06	2.63	1.36	7.72	2.57	1.36	4.41	3.86	3.36
2. Hancock Bar'cks	46 07	67 49	620	2.86	1.91	1.84	2.83	2.95	3.82	4.83	3.27	2.74	3.92	3.29	2.71
3. Fort Sullivan	44 54	66 59	78	3.17	3.19	3.16	2.79	2.92	2.15	4.29	3.62	3.17	3.30	3.39	4.26
4. Eastport	44 54	66 59	40	2.65	0.95	2.25	3.15	4.50	4.00	2.70	5.82	2.20	6.20	3.15	2.52
5. Gardiner	44 11	69 46	76	3.53	2.86	3.58	3.41	3.88	3.39	3.19	3.88	2.90	3.86	3.73	3.88
6. Fort Preble	43 39	70 13	53	3.37	3.39	2.92	4.14	5.05	3.39	2.78	4.11	3.31	4.25	4.37	4.06
7. Brunswick	43 54	69 57	74	3.24	2.75	3.72	3.44	4.55	3.69	3.65	4.37	3.03	3.74	4.65	3.85
8. Bath	43 55	69 49	50	4.31	3.37	2.88	2.89	3.59	2.55	2.31	3.07	1.77	3.35	3.27	3.72
9. Saco	43 31	70 26	69	3.69	3.26	3.72	2.12	5.52	2.69	3.57	4.76	2.76	4.34	3.81	4.87
10. Hampden	44 44	68 50	180	4.00	2.55	4.53	2.39	4.43	3.09	4.72	. .	. .	. .	. .	. .
11. Biddeford	43 30	70 27	46	3.40	3.88	3.23	4.39	5.74	3.48	3.09	4.57	3.36	5.24	4.18	5.19
12. Carmel	44 47	69 00	175	2.20	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
13. Fryeburg[1]	44 03	70 58	. .	2.45	3.18	2.12	3.52	3.86	3.29	2.17	5.59	3.69	4.84	5.99	4.41
14. Perry	45 00	67 06	100	3.86	3.41	4.34	3.81	5.03	3.89	3.68	3.44	3.20	5.73	6.31	3.59
15. Steuben[2]	44 31	67 58	50	4.35	4.62	4.76	4.56	5.50	3.56	3.76	3.51	3.79	4.56	4.96	5.33
16. Bucksport	44 34	68 47	90	2.38	3.67	2.20	3.31	5.40	2.80	2.67	4.29	3.82	5.53	5.71	2.58
17. Cornish[3]	43 40	70 44	780	3.70	3.86	4.41	4.07	3.36	3.60	4.21	4.67	2.99	4.46	4.91	3.61
18. Belfast	44 24	69 00	. .	2.70	1.88	2.67	3.53	3.20	3.45	. .	0.90	3.76	3.73	4.55	4.27
19. Monson	45 11	69 35	. .	3.16	2.44	. .	. .	. .	2.75	4.38	5.12	3.44	5.87	3.04	3.25
20. Portland	43 38	70 15	187	5.04	2.05	4.82	4.34	3.51	3.16	5.14	4.67	3.44	4.58	2.80	5.08
21. Dexter	44 55	69 32	700	3.24	2.94	4.69	2.91	2.88	3.31	3.75	1.60	2.78	3.32	4.31	2.99
22. Exeter (and East Exeter)	44 57	68 59	190	3.22	3.02	4.78	2.35	3.54	2.15	3.03	2.31	2.40	. .	. .	. .
23. Lisbon	44 00	70 04	130	2.75	3.43	4.69	3.64	3.70	3.54	3.28	3.56	3.83	4.77	5.45	3.36
24. Kennebec Arsn'l	44 19	69 46	. .	3.55	2.25	1.23	5.33	2.82	2.79	2.91	5.10	1.09	3.14	0.30	4.30
25. Norway	44 10	70 35	. .	3.41	1.77	1.31	. .	. .	. .	. .	. .	. .	. .	. .	. .
26. Vassalboro'*	44 28	69 47	. .	4.71	2.07	2.77	. .	. .	. .	. .	. .	. .	. .	. .	2.67
27. New Sharon	44 37	70 04	. .	. .	. .	1.90	. .	. .	. .	. .	. .	. .	. .	. .	. .
28. Topsham	43 54	69 57	100	3.10	1.85	5.35	6.80	. .	. .	. .	. .	. .	. .	. .	1.85
29. Bethel	44 20	70 51	650	3.51	3.47	3.77	4.93	3.91	2.62	6.12	1.56	3.19	5.08	. .	2.01
30. North Bridgeton	44 03	70 45	300	3.35	1.45	2.30	. .	. .	. .	. .	. .	. .	. .	. .	. .
31. Foxcroft Acad'y	45 12	69 13	. .	. .	. .	. .	. .	. .	4.05	6.24	3.23	4.52	6.43	. .	. .
32. West Waterville	44 30	69 45	250	3.65	3.03	4.50	2.98	3.07	2.07	3.27	3.29	2.97	3.68	3.71	3.15
33. Williamsburg	45 21	69 07	. .	4.03	. .	3.00	3.27	5.07	. . .	5.45	7.11	7.35	4.67	3.78	3.66
34. Lee	45 25	68 17	. .	3.22	5.09	6.76	3.72	6.60	1.42	2.39	2.00	3.58	3.92	4.17	3.43
35. Standish	43 45	70 30	280	1.48	5.04	5.70	2.66	4.11	3.42	2.79	2.01	1.26	4.63	3.74	2.58
36. Rumford	44 30	70 40	600	. .	. .	. .	. .	. .	. .	. .	. .	. .	2.65	4.38	3.75

NEW HAMPSHIRE.

Name of Station.	Latitude.	Longitude.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Constitution	43 04	70 42	40	2.42	2.64	2.16	3.44	3.44	3.01	2.40	3.80	2.43	3.29	3.23	3.32
2. Hanover, Dartmouth College	43 42	72 17	530	2.86	2.70	2.98	3.13	3.46	3.89	3.41	3.79	3.17	4.01	3.40	3.52
3. Laconia	43 39	71 30	. .	3.25	3.31	3.65	3.56	3.86	3.72	3.64	4.14	4.27	4.19	3.07	3.92
4. Lake Village	43 35	71 30	. .	3.44	2.68	3.57	3.36	3.71	3.52	3.94	3.90	4.18	4.44	3.06	3.81
5. Concord[4]	43 12	71 29	374	2.92	3.34	2.38	3.49	3.97	2.72	3.68	4.17	3.43	4.26	3.62	3.01
6. Great Falls	43 17	70 52	250	4.34	6.32	. .	5.48	1.13	3.53	1.56	0.22	5.13	3.62	7.79	. .
7. Londonderry[5]	42 53	71 20	300	3.69	3.93	2.47	4.50	5.53	2.33	2.07	4.89	2.07	5.26	5.01	3.86
8. Manchester[6]	42 59	71 28	300	2.74	3.94	2.07	4.78	4.83	1.94	2.86	6.33	3.84	3.68	4.06	4.10
9. Salmon Falls	43 30	71 00	. .	2.71	3.80	3.42	4.07	2.09	2.15	1.65	0.19	5.52	3.11	6.13	1.91
10. North Barnstead	43 38	71 27	. .	2.43	1.88	2.70	1.91	4.60	1.70	2.37	3.59	2.56	3.78	3.45	2.02

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[1] January and December, 1849, no record; January, March, July, August, September, October, November, December, 1850, no record; last seven months of 1851 no record.

[2] In 1850, May and June only observed.

[3] The mean value is given for all corresponding observations by the two observers.

* Oak Grove Seminary.

MAINE.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	5.46	11.65	9.63	9.71	36.45	1	0	Sept. 1844;	Aug. 1845	Surgeon	Army Met. Reg., 1855.
2	7.62	11.92	9.95	7.48	36.97	9	3	June, 1836;	Aug. 1845	"	" " " "
3	8.87	10.06	9.86	10.62	39.41	8	8	Jan. 1841;	Aug. 1853	Ass't Surgeon	" " " "
4	9.90	12.52	11.55	6.12	40.09	2	0	Jan. 1833;	Dec. 1834	?	Amer. Alma., 1836.
5	10.87	10.46	10.49	10.27	42.09	26	11	Jan. 1837;	Dec. 1866	R. H. Gardiner and F. Gardiner	Sm. Coll'n (MS.).
6	12.11	10.28	11.93	10.82	45.14	8	10	Nov. 1840;	Aug. 1853	Ass't Surgeon	Army Met. Reg., 1855.
7	11.71	11.71	11.42	9.84	44.68	32	1	April, 1808;	Dec. 1859	P. Cleaveland	Smith. Con. Know., 1867.
8	9.36	7.93	8.39	11.40	37.08	9	0	Jan. 1832;	Dec. 1840	J. Hayden	Am. Alma. 1842.
9	11.36	11.02	10.91	11.82	45.11	8	0	1843;	1851	Batchelder & Garland	Am. Alma. & Blodget's Clim.
10	11.35	..	..	..	..	0	7	Jan. 1844;	July, 1844	J. Herrick	Sm. Coll'n.
11	13.36	11.14	12.78	12.47	49.75	4	1	July, 1848;	Feb. 1854	J. G. Garland	Am. Alma., 1850 and fol.
12	..	..	..	..	..	0	1	Jan. 1854;		J. J. Bell	P. O. & S. I. Vol. 1, 1861.
13	9.50	11.05	14.52	10.04	45.11	5	0	Feb. 1849;	May, 1856	G. B. Barrows	S. C. & P. O. & S. I. Vol. 1, '61.
14	13.18	11.01	15.24	10.86	50.29	9	6	Feb. 1854;	Dec. 1866	W. D. Dana	P. O. & S. I. Vol. 1, '61, & S. O.
15	14.82	10.83	13.31	14.30	53.26	12	2	May, 1850;	Nov. 1866	J. D. Parker	S. C. & P. O. & S. I. & S. O.
16	10.91	9.76	15.06	8.63	44.36	4	0	1849;	1852	R. Buck	Sm. Coll'n (MS.).
17	11.84	12.48	12.36	11.17	47.85	11	2	April, 1856;	Dec. 1866	G. W. Guptill and S. West.	P. O. & S. I. Vol. 1, & S. O.
18	9.40	..	12.04	8.85	..	1	10	Sept. 1859;	May, 1863	G. E. Brackett	" " " " "
19	..	12.25	12.35	8.85	..	0	10	Sept. 1856;	Nov. 1857	B. F. Wilbur	P. O. & S. I. Vol. 1.
20	12.67	12.97	10.82	12.17	48.63	3	7	Feb. 1856;	Dec. 1859	H. Willis	" " " "
21	10.48	8.66	10.41	9.17	38.72	4	6	June, 1858;	June, 1863	B. F. Wilbur	P. O. & S. I. Vol. 1, & S. O.
22	10.67	7.49	..	..	..	1	0	Jan. 1858;	Sept. 1861	S. Gilman and Dr. J. B. Wilson	" " " " "
23	12.03	10.38	14.05	9.54	46.00	7	8	April, 1859;	Dec. 1866	A. P. Moore	" " " " "
24	9.38	10.80	4.53	10.10	34.81	1	4	May, 1857;	Aug. 1858	Med. Off.	M. D. of U. S. A. 1860.
25	..	..	..	..	..	0	3	Feb. 1860;	Jan. 1861	G. W. Verrill, Jr.	Sm. Obs.
26	..	..	..	9.45	..	0	9	Feb. 1860;	Mar. 1862	J. Van Blarcom	" "
27	..	..	..	..	..	0	1	Mar. 1860;		Dr. J. F. Pratt	" "
28	..	..	..	6.80	..	0	5	Dec. 1860;	April, 1861	W. Johnson	" "
29	12.61	10.30	..	8.99	..	1	1	Jan. 1861;	Feb. 1862	Rev. A. G. Gaines	" "
30	..	..	..	..	..	0	3	Jan.	Mar. 1861	M. Gould	" "
31	..	13.52	..	..	..	0	5	June,	Oct. 1863	M. Pitman	" "
32	10.55	8.63	10.36	9.83	39.37	3	5	Aug. 1863;	Dec. 1866	B. F. Wilbur	" "
33	11.34	..	15.80	..	..	1	1	Oct. 1863;	Dec. 1866	E. Pitman and H. W. Pitman	" "
34	17.08	5.81	11.67	11.74	46.30	2	3	June, 1864;	Dec. 1866	E. Pitman	" "
35	12.47	8.22	9.63	9.10	39.42	1	4	May, 1865;	Dec. 1866	J. P. Moulton	" "
36	..	..	..	..	..	0	3	Oct.	Dec. 1866	W. Pettingill	" "

NEW HAMPSHIRE.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	9.04	9.21	8.95	8.38	35.58	12	5	April, 1836;	Sept. 1853	Ass't Surgeon	Army Met. Reg. 1855.
2	9.57	11.09	10.58	9.08	40.32	19	0	Jan. 1835;	Dec. 1854	Prof. Ira and A. A. Young	S. C., P. O., & S. I. Vol. 1.
3	10.57	11.50	11.53	10.48	44.58	4	0	Jan. 1857;	Dec. 1866	Cot. & Wool Man. Co.	S. Coll'n.
4	10.64	11.36	11.68	9.93	43.61	4	0	Jan. 1857;	Dec. 1866	" " " " "	" "
5	9.84	10.57	11.31	9.27	40.99	8	9	Jan. 1849;	Dec. 1866	Dr. W. Prescott, E. P. Colby, H. E. Sawyer, Jno. T.	P. O. & S. I. Vol. 1, & S. O.
6	..	5.31	16.54	..	..	0	10	June, 1854;	June, 1855	H. E. Sawyer [Wheeler	P. O. & S. I. Vol. 1.
7	12.50	9.29	12.34	11.48	45.61	6	11	Mar. 1849;	Feb. 1867	R. C. Mack	S. C. & P. O. & S. I. Vol. 1.
8	11.68	11.13	11.58	10.78	45.17	4	9	Feb. 1852;	Mar. 1861	S. N. Bell	S. C. & P. O. & S. I. Vol. 1, & S. O.
9	9.58	3.99	14.76	8.42	36.75	1	0	Jan.	Dec. 1854	G. B. Sawyer	P. O. & S. I. Vol. 1.
10	9.21	7.66	9.79	6.33	32.99	3	1	Mar. 1856;	Nov. 1866	R. F. Hanscom, C. H. Pitman	P. O. & S. I. Vol. 1, & S. O.

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[4] Earlier series of Smithsonian Collection consists of five years, 1849–1853.

[5] In earlier series of Smithsonian Collection, January, February, December, 1849, August, September, October, 1850, omitted; series extends to 1853.

[6] Older series of Smithsonian Collection from February to March, 1852, and from August, 1852, to July (inclusive) 1854, extending two years and two months.

NEW HAMPSHIRE.—CONTINUED.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
11. Stratford	44°40′	71°34′	1000	2.77	2.52	3.47	2.48	3.31	3.24	4.42	3.84	3.98	3.64	3.21	3.08
12. West Enfield	43 30	72 00	..	3.72	1.46	2.50	5.37	3.25	3.69	9.25	4.28	3.69	5.85	2.96	1.75
13. Claremont	43 22	72 21	539	2.96	3.26	4.99	3.32	3.96	2.93	3.94	4.92	3.51	4.15	2.38	3.37
14. Francestown	42 59	71 45	..	..	..	..	..	..	2.37	5.26	..	1.48	7.91	..	..
15. Shelburn	44 23	71 06	700	2.84	1.59	..	..	..	..	..	..	..	5.10	..	..
16. Mount Washington	44 16	71 18	6293	..	..	..	..	..	3.87	5.47	7.67	..	..	..	..
17. Farmington	43 20	71 07	300	4.91	..	..	..	..	..	..	..	..	..	..	..
18. London Ridge[1]	43 20	71 25	475	5.50	3.12	..	4.52	4.55	6.20	5.50	3.92	2.50	..	7.80	3.80
19. Littleton and North Littleton	44 20	71 40	..	2.59	1.49	2.16	1.90	4.68	0.75	5.17	4.60	1.80	2.43	1.05	2.30
20. Portsmouth	43 05	70 46	38	..	..	..	..	..	..	..	..	4.09	1.81	2.35	0.59
21. Portsmouth	43 05	70 46	50	3.39	2.47	2.04	1.01	3.02	2.72	2.20	3.41	1.99	3.76	2.29	1.58

VERMONT.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fayetteville[2]	42 56	72 40	280?	4.94	3.67	4.00	4.54	4.70	4.66	5.06	4.81	5.37	4.15	5.67	3.60
2. Burlington	44 28	73 11	346	1.76	1.84	2.04	2.10	3.29	3.41	4.38	3.29	3.60	3.72	2.62	2.10
3. St. Johnsbury	44 25	72 00	540	2.33	1.97	1.92	2.48	1.67	4.92	3.39	2.70	3.66	3.63	3.15	3.68
4. Brandon	43 45	73 00	460	2.81	2.47	2.64	2.36	2.83	2.71	4.55	4.66	3.18	3.18	3.00	2.18
5. Craftsbury	44 40	72 30	1100	2.44	2.39	2.95	2.66	3.02	3.10	4.35	4.75	3.95	3.31	3.37	2.95
6. Norwich	43 42	72 21	..	..	..	..	..	..	..	..	..	..	..	1.37	..
7. Shelburne	44 23	73 11	150	1.90	4.40	1.31	4.45	3.87	4.58	5.53	6.42	2.67	3.90	2.95	2.62
8. Woodstock	43 36	72 35	650	2.50	1.45	1.67	2.78	4.95	3.06	4.94	4.33	3.44	5.25	2.96	3.25
9. Lunenburg	44 28	71 41	1124	3.22	2.09	3.17	2.51	4.27	2.02	3.45	3.37	4.12	3.84	3.37	4.15
10. Springfield	43 18	72 32	300	4.54	2.37	3.40	3.59	3.53	4.03	5.15	4.13	2.70	3.94	3.02	2.75
11. Calais	44 22	72 09	..	..	4.10	1.70	1.06	..	..	..	..	..	..	..	..
12. Brookfield	44 02	72 35	1000	..	2.17	3.97	2.86	5.78	..	..	..	..	..	..	..
13. Middlebury	43 59	73 10	398	1.68	2.90	2.28	2.37	2.99	2.39	3.92	3.68	5.30	3.17	2.97	2.97
14. Randolph	43 34	72 35	700	1.72	4.02	0.58	1.73	1.87	3.64	2.81	2.80	5.12	2.33	3.54	2.76
15. Barnet	44 18	72 05	952	..	..	..	2.06	3.25	4.75	4.50	3.50	..	..	2.75	1.00
16. Rutland	43 37	72 58	500	3.50	2.78	3.10	3.01	4.72	3.91	2.31	2.11	2.48	5.66	4.10	3.49
17. East Montpelier[3]	44 20	72 38	540	..	..	..	..	..	..	..	..	..	..	..	..

MASSACHUSETTS.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Watertown Arsenal	42 22	71 09	100	2.87	2.86	3.30	3.70	3.75	3.61	2.64	4.41	3.00	3.85	3.98	4.11
2. Fort Independence	42 20	71 00	50	3.47	2.36	2.13	3.95	3.73	3.41	3.16	4.18	2.75	2.90	3.48	3.62
3. Springfield	42 06	72 36	200	2.26	3.18	2.30	3.35	5.06	3.28	4.79	4.17	2.92	4.72	4.36	3.40
4. Worcester (Asylum)	42 16	71 49	528	3.87	3.23	3.63	3.92	4.08	3.27	4.00	5.19	3.56	4.18	4.10	3.89
5. Cambridge, Harvard Observatory[4]	42 23	71 07	71	4.04	3.04	3.75	3.91	3.48	3.40	3.65	5.35	3.97	3.59	4.14	4.07
6. Amherst	42 22	72 34	267	3.07	3.01	3.18	3.33	4.05	3.77	4.35	4.59	2.98	3.88	3.91	3.78
7. New Bedford	41 39	70 56	90	3.34	3.30	3.56	3.53	3.64	2.83	2.98	3.94	3.29	3.36	3.87	3.78
8. Nantucket	41 17	70 06	30	3.92	2.81	3.25	4.02	3.75	3.27	2.48	2.72	3.16	3.57	3.38	4.77
9. Cambridge[4]	42 23	71 07	71	3.50	2.62	2.52	2.73	5.86	2.08	2.22	2.28	3.79	2.47	1.85	3.48
10. Boston[5]	42 22	71 04	..	3.18	3.37	3.66	3.18	3.56	2.70	3.72	3.39	3.25	2.90	3.33	3.30
11. Boston	42 22	71 04	82	..	..	..	..	..	..	..	..	..	..	..	..

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[1] Jan. and July rough measures. [2] These results apparently overmeasured.

[3] Snow reduced to water.

[4] Cambridge series combined: Jan. 3.95; Feb. 2.97; March, 3.55; April, 3.72; May, 3.86; June, 3.19; July, 3.42; August, 4.86; Sept. 3.94; Oct. 3.40; Nov. 3.77; Dec. 3.97; Spring, 11.13; Summer, 11.47; Autumn, 11.11; Winter, 10.89; Year, 44.60; Extent, 30 yrs. 11 mos.; Date, 1784—1867.

NEW HAMPSHIRE.—Continued.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
11	9.26	11.50	10.83	8.37	39.96	9	3	Mar. 1856;	Dec. 1866	B. Brown.	P. O. and S. I. Vol. 1, & S. O.
12	11.12	17.22	12.50	6.93	47.77	2	1	July, 1856;	Dec. 1858	N. Purmort.	P. O. and S. I. Vol. 1.
13	12.27	11.79	10.04	9.59	43.69	6	5	Sept. 1857;	Dec. 1866	F. N. Freeman & A. Chase	P. O. and S. I. Vol. 1, & S. O.
14	..	..	..	..	..	0	4	June—October,	1857	Dr. M. N. Root	P. O. and S. I. Vol. 1.
15	..	..	..	..	..	0	5	Oct. 1858;	Feb. 1862	F. Odell	P. O. and S. I. Vol. 1, & S. O.
16	..	17.01	..	..	..	0	3	June—August,	1859	J. H. Hall	P. O. and S. I. Vol. 1.
17	..	..	..	..	..	0	1	Jan. 1861;		L. Bell	Sm. Obs.
18	..	15.62	..	12.42	..	0	11	Jan. 1862;	Feb. 1863	Dr. Isaac S. French	“ “
19	8.74	10.52	5.28	6.38	30.92	1	9	Mar. 1863;	July, 1864	R. C. Whiting, R. Smith	“ “
20	..	..	8.25	..	..	0	4	September—	Dec. 1866	J. Hatch	“ “
21	6.07	8.33	8.04	7.44	29.88	4	0	June, 1843; to	1848	Surgeon Delany	Sm. Coll'n.

VERMONT.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	13.24	14.53	15.19	12.21	55.17	7	5	May, 1826;	Sept. 1833	Gen. Field	M. S. and Sill's Journ.
2	7.43	11.08	9.94	5.70	34.15	27	1	Jan. 1828;	Nov. 1864	Prof. Z. Thompson and McK. Petty [banks	M. S. and Hist. of Vermont and S. O.
3	6.07	11.01	10.44	7.98	35.50	2	9	Jan. 1854;	Jan. 1861	J.K.Colby, J.P.& F.Fair-	P. O. and S. I. Vol. 1, & S. O.
4	7.83	11.92	9.36	7.46	36.57	9	5	Mar. 1856;	Dec. 1866	D. Buckland and H. Buckland	“ “ “ “ “ “
5	8.63	12.20	10.63	7.78	39.24	10	7	Jan. 1856;	Dec. 1866	J. A. Paddock	“ “ “ “ “ “
6	..	..	..	..	..	0	1	Nov. 1856;		A. Jackman	P. O. and S. I. Vol. 1.
7	9.63	16.53	9.52	8.92	44.60	1	9	Mar. 1856;	Dec. 1857	G. Bliss	“ “ “ “
8	9.40	12.33	11.65	7.20	40.58	2	0	Jan. 1857;	Dec. 1858	C. Marsh	“ “ “ “
9	9.95	8.84	11.33	9.46	39.58	4	10	Sept. 1859;	Dec. 1866	H. A. Cutting	P. O. and S. I. Vol. 1, & S. O.
10	10.52	13.31	9.66	9.66	43.15	2	4	Dec. 1860;	Nov. 1863	J. W. Chickering	Sm. Obs.
11	..	..	..	..	..	0	3	Apr. 1862;	Mar. 1864	J. K. Toby	“ “
12	..	..	..	..	..	0	4	Feb.	May, 1863	T. F. Pollard	“ “
13	7.64	9.99	11.44	7.55	36.62	2	10	Mar. 1864;	Dec. 1866	H. A. Sheldon	“ “
14	4.18	9.25	10.99	8.50	32.92	1	2	Nov. 1865;	Dec. 1866	C. L. Paine	“ “
15	..	12.75	..	..	..	0	7	April,	Dec. 1866	Dr. B. F. Eaton	“ “
16	10.83	8.33	12.24	9.77	41.17	1	0		1789	Williams	Hist. Vermont.
17	..	..	..	10.73	..	6	0	1846;	1851	B. J. Wheeler	Pat. Off. Rep.

MASSACHUSETTS.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	10.75	10.66	10.83	9.84	42.08	7	3	June, 1836;	Nov. 1844	Assistant Surgeon	Army Met. Reg. 1855. ['60.
2	9.81	10.75	9.13	9.45	39.14	8	3	July, 1851;	Dec. 1859	Assistant Surgeon	A. M. R. '55, & M. D. U. S. A.
3	10.71	12.24	12.00	8.84	43.79	11	0	Jan. 1848;	Dec. 1866	L. C. Allin, F. A. Brewer, and J. Weatherhead	S. C.; P. O. and S. I. Vol. 1, and S. O.
4	11.63	12.46	11.84	10.99	46.92	26	0	Jan. 1841;	Dec. 1867	Dr. J. Draper, J. Cotton, and others	MS. in S. C. and P. O. and S. I. Vol. 1.
5	11.14	12.40	11.70	11.15	46.39	25	11	Jan. 1841;	Dec. 1867	W. C. Bond and J. Winlock	Am. Alm. and S. C.
6	10.56	12.71	10.77	9.86	43.90	32	1	Aug. 1835;	Dec. 1867	Prof. E. S. Snell	MS. in S. C.
7	10.73	9.75	10.52	10.42	41.42	54	3	Oct. 1813;	Dec. 1867	S. Rodman, E. T. Tucker	“ “ “ [and S. O.
8	11.02	8.47	10.11	11.50	41.10	13	2	Jan. 1847;	Mar. 1861	W. Mitchell	S. C., P. O. and S. I. Vol. 1,
9	11.11	6.58	8.11	9.60	35.40	5	0	1784;	1788	Williams	Williams' Hist. of Vermont.
10	10.40	9.81	9.48	9.85	39.54	20	0	Jan. 1818;	Dec. 1837	Dr. E. Hale	MS. com. by Prof. Lovering.
11	..	..	..	..	43.35	12	0	1841;	1852	J. P. Hall	Bond MS.

[6] Boston, observations in Boston and at Fort Independence, Boston Habor, combined: Jan. 3.26; Feb. 3.08; March, 3.22; April, 3.40; May, 3.61; June, 2.90; July, 3.55; August, 3.63; Sept. 3.09; Oct. 2.90; Nov. 3.37; Dec. 3.39; Spring, 10.23; Summer, 10.08; Autumn, 9.36; Winter, 9.73; Year, 39.40; Extent, 28 yrs. 3 mos.; Date, 1818—1859.

MASSACHUSETTS.—CONTINUED.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
12. Williamstown[1]	42°43′	73°13′	725	1.78	2.03	2.74	1.93	2.84	3.29	3.62	2.68	3.63	1.98	2.30	1.94
13. Williamstown, Hopkins' Observatory[1].	42 43	73 13	686	2.47	1.89	2.48	3.09	3.94	2.94	4.79	5.02	3.58	3.60	3.38	3.40
14. Chelsea Hospital	42 25	71 00	40	4.75	2.10	3.10	4.60	3.00	1.50	1.50	2.80	2.00	2.10	2.60	1.25
15. Mendon	42 06	71 34	..	2.91	2.60	2.33	3.41	4.20	3.32	3.82	4.75	3.35	2.93	3.30	2.75
16. Waltham[2]	42 23	71 15	..	..	..	..	3.85	3.61	3.24	3.60	4.47	..	..	..	..
17. North Attleboro'	41 52	71 23	175	3.72	3.00	1.61	5.01	4.72	2.47	3.56	4.82	3.60	3.22	3.59	4.70
18. Newburyport	42 48	70 52	42	3.39	3.18	2.01	3.63	3.71	2.52	2.67	6.53	1.83	3.74	3.50	5.03
19. Princeton	42 28	71 53	1113	3.31	3.96	1.87	4.98	3.00	2.57	4.47	4.96	2.83	4.82	4.40	3.59
20. Newbury	42 45	70 55	25	1.05	..	..	..	0.58	..	..	..	..	..	..	0.70
21. Richmond[3]	42 23	73 20	1100	6.20	7.07	6.30	4.44	8.54	10.03	8.11	5.49	5.12	6.11	8.27	7.06
22. Southwick[4]	42 02	72 48	265	2.97	5.13	2.30	4.18	4.60	2.60	4.18	4.66	2.80	4.80	4.53	3.45
23. Southwick	42 02	72 48	265	2.90	3.39	2.28	6.67	5.19	3.46	5.50	1.67	3.40	7.57	5.77	4.70
24. Westfield	42 06	72 48	180	3.99	3.00	3.60	3.47	4.66	4.00	4.91	4.90	2.95	4.50	3.88	4.29
25. Wood's Hole	41 32	70 40	25	3.06	7.14	3.90	5.83	..	..	..	..	..	..	..	..
26. Weymouth	42 12	70 56	150	5.06	..	1.30	7.67	4.32	2.22	2.91	8.00	4.57	3.32	1.83	5.23
27. Bridgewater Normal School	42 00	71 00	150	3.77	2.26	3.94	3.89	2.70	3.98	2.77	2.79	2.92	2.54	2.09	3.79
28. Lawrence	42 42	71 10	143	3.75	2.69	3.56	4.28	4.05	3.09	3.82	5.28	3.25	3.27	3.72	4.20
29. Canton	42 12	71 08	90	4.22	2.78	..	..	..	..	..	..	..	..	2.60	4.98
30. Florida	42 42	73 10	2500	3.43	..	..	..	..	..	..	..	..	..	..	..
31. Plainfield	42 30	72 56	..	..	..	2.75	..	..	..	..	..	..	..	..	..
32. Topsfield	42 38	71 57	..	3.31	2.66	4.22	2.27	2.90	2.39	5.26	2.82	3.10	3.18	3.55	3.56
33. Clinton	42 25	71 42	..	2.25	..	3.40	..	2.47	6.19	5.12	5.69	7.62	2.03	..	..
34. Fitchburg	42 35	71 50	484	0.80	3.12	6.31	3.95	3.71	2.57	5.96	4.18	2.38	4.91	4.83	0.35
35. Sandwich	41 45	70 30	20	5.28	2.35	5.84	3.06	3.44	1.70	3.98	7.57	2.59	2.56	5.54	4.08
36. Falmouth	41 33	70 37	20	..	..	3.89	..	..	..	..	..	..	..	..	..
37. Baldwinsville	42 37	72 05	847	3.44	1.13	3.69	2.43	6.53	1.75	1.58	4.67	2.31	4.84	4.31	3.68
38. Fall River	41 43	71 09	200	..	..	..	..	..	..	..	..	..	..	6.45	2.06
39. West Dennis	41 40	70 11	25	..	..	..	..	..	..	..	..	2.32	1.94	..	..
40. Georgetown	42 42	71 00	225	..	0.90	..	..	..	..	..	..	..	..	..	0.70
41. Lunenburg	42 35	71 43	450	..	..	..	..	..	..	..	..	..	..	..	1.10
42. Kingston	42 00	70 45	60	..	..	..	..	..	..	6.75	3.65	5.80	3.21	3.13	3.02
43. Charlestown[5]	42 21	71 03	60	2.66	2.22	4.08	3.20	3.33	2.36	2.88	3.42	3.03	3.52	2.86	2.27
44. Stow	42 30	71 35	250	2.50	2.78	3.98	3.52	4.05	2.68	3.84	3.20	3.13	4.06	2.33	3.03
45. Barnstable	41 42	70 23	20	3.12	6.33	..	7.01	7.09	2.08	3.51	4.66	3.75	4.50	4.12	6.50
46. West Springfield	42 10	72 48	300	4.18	3.67	2.94	4.29	8.21	3.55	5.28	5.94	5.78	4.03	2.56	2.04
47. Framingham	42 18	71 27	150	5.73	2.80	..	..	4.11	1.34	2.52	5.36	2.28	6.76	3.97	3.49
48. Lynn	42 28	70 57	..	2.05	3.14	2.56	4.31	3.53	2.76	1.98	4.64	2.93	4.20	4.12	3.36
49. Bradford[6]	42 50	71 06	..	1.98	1.67	3.25	2.40	1.55	1.47	1.98	3.65	2.00	5.40	2.90	3.08

RHODE ISLAND.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Adams	41 29	71 20	40	4.07	3.85	4.35	4.84	4.40	3.51	3.54	4.60	3.09	4.73	5.29	5.18
2. Providence	41 50	71 23	150	3.42	2.88	3.46	3.59	3.49	3.13	3.08	3.93	3.09	3.35	4.04	3.92
3. Providence	41 49	71 25	87	5.02	3.14	4.46	4.93	2.73	4.25	4.73	2.81	5.13	3.46	4.72	1.62
4. Newport	41 28	71 21	25	3.37	5.41	5.11	1.74	4.50	4.13	1.81	3.78	2.60	3.49	4.52	4.91
5. North Scituate	41 50	71 34	300	..	..	..	7.61	4.45	2.01	..	..	..	..	..	..

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[1] Williamstown series combined: Jan. 2.16; Feb. 1.95; March, 2.60; April, 2.58; May, 3.39; June, 3.10; July, 4.32; Aug. 4.09; Sept. 3.60; Oct. 2.98; Nov. 2.97; Dec. 2.85; Spring, 8.57; Summer, 11.51; Autumn, 9.55; Winter, 6.96; Year, 36.59; Extent, 19 yrs. 2 mos.; Date, 1816—1867.

[2] Observations extend only over five months (April—August) each year; the monthly means are those of 42 years.

[3] The amount recorded appears excessive.

MASSACHUSETTS.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs.	mos.	DATE. Beginning.	End.	OBSERVER.	REFERENCE.
12	7.51	9.59	7.91	5.75	30.76	8	0	1816;	1823	Dewey and Williams	Mem. Am. Acad.; S. C.
13	9.51	12.75	10.56	7.76	40.58	11	2	Sept. 1854;	Dec. 1867	W. P. Alcott, T. E. Braston, Prof. A. Hopkins	MS. in S. C. & P. O. & S. I. Vol. I.
14	10.70	5.80	6.70	8.10	31.30	1	10	Jan. 1861;	Dec. 1862	Patten and Fox, Surgs. U. S. N.	Sm. Col.
15	9.94	11.89	9.58	8.26	39.67	8	2	Feb. 1854;	Nov. 1866	Dr. J. G. Metcalf	P. O. and S. I. Vol. I, & S. O.
16	..	11.31	..	..	..	17	6	Apr. 1824;	Aug. 1865	Boston Manuf. Co.	Waltham Free Press, 1865.
17	11.34	10.85	10.41	11.42	44.02	4	3	Sept. 1852;	Mar. 1857	H. Rice	S. C. & P. O. & S. I. Vol. I.
18	9.35	11.72	9.07	11.60	41.74	4	8	July, 1852;	Aug. 1858	Dr. H. C. Perkins	“ “ “ “ “
19	9.85	12.00	12.05	10.86	44.76	3	5	Jan. 1854;	Dec. 1857	J. Brooks	P. O. and S. I. Vol. I.
20	..	..	...	..	..	0	3	May, 1864;	Jan. 1865; Dec. 1866	J. H. Caldwell	Sm. Obs.
21	19.28	23.63	19.50	20.33	82.74	6	1	Jan. 1854;	Dec. 1866	W. Bacon	P. O. and S. I. Vol. I, & S. O.
22	11.08	11.44	12.13	11.55	46.20	3	8	May, 1849–51;	Aug. 1852; July, 1853	A. Holcomb	Sm. Col.
23	14.14	10.63	16.74	10.99	52.50	2	4	Jan. 1854;	May, 1857	A. Holcomb	P. O. and S. I. Vol. I.
24	11.73	13.81	11.33	11.28	48.15	10	5	Dec. 1854;	May, 1866	Rev. E. Davis.	P. O. and S. I. Vol. I, & S. O.
25	..	..	..	..	..	0	5	Jan. 1854;	Apr. 1855	B. R. Gifford.	P. O. and S. I. Vol. I.
26	13.29	13.13	9.72	..	..	1	4	Aug. 1856;	Jan. 1859	Dr. N. Q. Tirrell	“ “ “ “
27	10.53	9.54	7.55	9.82	37.44	2	10	June, 1856;	Jan. 1861	C. M. Barrows	P. O. and S. I. Vol. I, & S. O.
28	11.89	12.19	10.24	10.64	44.96	8	8	Jan. 1856;	Dec. 1866	J. Fallon	“ “ “ “ “
29	..	..	..	11.98	..	0	5	Jan. 1857;	Jan. 1858	D. H. Ellis	P. O. and S. I. Vol. I.
30	..	..	..	..	..	0	1	Jan. 1857;		L. F. Whitcomb	“ “ “ “
31	..	..	..	..	..	0	1	Mar. 1857;		F. Shaw	“ “ “ “
32	9.39	10.47	9.83	9.53	39.22	5	9	Apr. 1860;	Dec. 1866	N. W. Brown	Sm. Obs.
33	..	17.00	..	..	..	0	8	May, 1860;	Mar. 1861	Dr. G. M. Morse	“ “
34	13.97	12.71	12.12	4.27	43.07	1	0	1861		G. Raymond	“ “
35	12.34	13.25	10.69	11.71	47.99	2	1	Apr. 1863;	Dec. 1866	Dr. N. Barrows	“ “
36	..	..	..	..	..	0	1	Mar. 1863;		Dr. N. Barrows	“ “
37	12.65	8.00	11.46	8.25	40.36	2	3	Aug. 1863;	Dec. 1865	Rev. E. Dewhurst	“ “
38	..	..	..	..	..	0	2	Nov.	Dec. 1861	C. C. Terry	“ “
39	..	..	..	..	..	0	2	Sept.	Oct. 1864	E. Tappan	“ “
40	..	..	..	..	..	0	2	Feb. 1865;	Dec. 1866	H. M. Nelson	“ “
41	..	..	..	..	..	0	1	Dec. 1866;		G. A. Cunningham	“ “
42	..	..	12.14	..	..	0	6	July,	Dec. 1866	G. S. Newcomb	“ “
43	10.61	8.66	9.41	7.15	35.83	11	0	1792;	1802	Barrett	Mem. Am. Acad., Blodget's [Climatology.
44	11.55	9.72	9.52	8.31	39.10	10	0	1795;	1804	Newell	“ “ “ “
45	..	10.25	12.37	15.95	..	0	11	Aug. 1852;	Aug. 1853	B. R. Gifford	Sm. Coll.
46	15.44	14.77	12.37	9.89	52.47	1	0	Nov. 1849;	Oct. 1850	. . . .	“ “
47	..	9.22	13.01	12.02	..	0	10	May, 1849;	Feb. 1850	G. A. Hyde	“ “
48	10.40	9.38	11.25	8.55	39.58	4	0	1849;	1853	Jacob Batchelder	“ “
49	7.20	7.10	10.30	6.73	31.33	2	0	1772;	1773	Williams	Trans. Phil. Soc. Vol. 2, 1773.

RHODE ISLAND.

	Spring.	Summer.	Autumn.	Winter.	Year.	yrs.	mos.	Beginning.	End.	Observer.	Reference.
1	13.59	11.65	13.11	13.10	51.45	12	0	Oct. 1841;	Oct. 1859	Assistant Surgeon	Army Met. Reg. 1855, and M. D. of U. S. A. 1860.
2	10.54	10.14	10.48	10.22	41.38	35	1	Dec. 1831;	Dec. 1866	Prof. A. Caswell	Sm. Con. to Kn. 1860, and MS. in Sm. Coll'n.
3	12.12	11.79	13.31	9.78	47.00	2	7	June, 1860;	July, 1863	H. C. Sheldon	Sm. Obs.
4	11.35	9.72	10.61	13.69	45.37	1	4	Sept. 1865;	Dec. 1866	W. H. Crandall	“ “
5	..	..	..	..	..	0	3	1854		H. C. Sheldon	P. O. and S. I. Vol. I.

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[4] Monthly record not accessible.

[5] The gauge is said to have been badly placed in this case, receiving only the water falling vertically.

[6] Observations made with great care; gauge three inches in diameter; snow and ice carefully melted.

NEW JERSEY.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Lambertville[1]	40°23′	74°56′	96	3.22	3.12	3.22	3.19	4.29	3.38	4.07	4.94	3.90	3.41	3.21	3.87
2. Newark	40 45	74 10	35	3.52	3.34	3.46	3.67	4.36	3.20	3.66	4.72	3.47	3.58	3.79	4.08
3. Bloomfield	40 49	74 08	120	3.39	2.99	2.68	4.05	4.61	3.64	4.19	4.54	2.41	3.30	3.47	3.11
4. Burlington	40 05	75 10	60	3.72	3.83	3.39	4.33	5.21	4.09	3.05	4.27	3.88	3.13	2.91	4.83
5. Sergeantville	40 29	75 03	. .	5.20	2.18	4.67	7.75	10.99	5.84	3.55	6.15	1.05	. .	1.17	. .
6. Navesink, Highlands	40 24	73 59	111	5.68	2.32	3.61	4.52	5.74	2.00	1.65	2.61	. .	. .	. .	. .
7. Mount Holly	40 00	74 47	30	0.72	. .	1.20	. .	. .	. .	. .	. .	. .	. .	. .	. .
8. Long Branch	40 20	74 05	10	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	10.20	2.50
9. Junction Rancocus & Delaware Rivers .	40 03	75 11	15	2.87	3.41	3.37	3.07	4.24	2.66	3.56	3.51	5.24	3.14	2.97	4.79
10. Jamesburg	40 22	74 27	. .	. .	. .	. .	. .	. .	. .	. .	2.52	. .	. .	. .	. .
11. Passaic Valley . . .	40 53	74 12	140	4.89	2.62	5.54	4.58	8.45	4.91	10.03	. .	. .	. .	. .	4.02
12. Cole's Landing . . .	39 54	75 02	50	2.84	4.19	4.03	2.93	6.73	3.18	2.44	3.73	6.73	2.66	2.98	4.63
13. Greenwich	39 20	75 25	30	3.05	4.78	3.74	2.67	4.64	1.73	2.47	2.51	4.98	2.68	2.76	3.72
14. Seaville	39 20	74 42	18	. .	8.90	3.90	4.30	. .	2.80	3.33	3.20	4.80	6.55	. .	. .
15. Paterson	40 10	74 52	60	1.20	4.12	1.98	2.28	3.73	4.69	4.85	4.30	5.62	5.20	3.62	4.36
16. Trenton	40 13	74 45	. .	2.80	6.25	2.21	4.02	4.68	3.66	4.26	4.80	4.60	4.80	3.98	5.12
17. Middletown	40 26	74 10	. .	5.72	2.55	1.16	2.50	4.50	4.10	3.40	1.70	2.50	4.00	3.13	1.37
18. Branchburg	40 33	74 40	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	4.45
19. Dover	40 54	74 32	652	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	3.48
20. New Brunswick . .	40 30	74 27	80	1.65	4.76	1.68	3.08	4.30	2.91	2.91	7.07	5.84	3.83	2.61	2.83
CONNECTICUT.															
1. Fort Trumbull . . .	41 21	72 05	23	3.64	2.76	4.67	2.63	3.60	2.49	3.34	4.82	1.94	5.50	5.72	3.58
2. Hartford	41 46	72 41	60	3.38	3.56	3.71	3.30	4.07	2.83	3.80	3.35	3.06	4.00	4.86	3.87
3. New Haven[2]	41 18	72 55	50	3.51	3.81	3.43	3.34	4.07	3.30	4.45	3.96	4.15	3.64	3.74	3.38
4. Middletown	41 33	72 39	192	3.41	3.24	3.18	3.35	3.96	2.22	3.50	6.60	2.13	5.09	3.70	3.82
5. Middletown	41 33	72 39	175	4.33	3.60	4.25	3.17	4.45	3.89	4.78	3.93	3.85	2.91	4.07	3.60
6. Pomfret	41 52	72 00	587	3.97	3.32	3.53	3.62	3.77	3.86	4.59	4.64	3.47	3.76	4.24	4.34
7. Salisbury[3]	41 59	73 18	737	2.29	2.79	2.62	2.89	4.62	4.01	4.71	3.60	3.26	4.68	4.90	4.67
8. Saybrook, Lynde Pt. L. H.	41 16	72 20	10	5.47	2.48	3.27	4.84	3.58	3.89	3.55	5.79	3.51	3.43	4.23	4.80
9. West Cornwall . . .	41 50	73 21	1000	2.00	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
10. Georgetown	41 15	73 25	300	. .	. .	1.98	3.62	5.63	3.25	2.95	13.75	4.25	1.88	3.32	6.21
11. New London	41 21	72 04	90	3.13	1.68	1.80	5.14	4.95	2.72	4.00	5.19	3.85	2.89	4.15	5.21
12. Wallingford.	41 27	72 50	133	4.85	3.02	4.35	4.43	5.09	4.82	4.20	6.62	4.36	3.05	2.98	4.26
13. Canton	41 49	72 54	750	4.24	3.87	6.14	1.81	2.76	6.93	9.29	4.36	3.39	4.04	5.46	2.56
14. Plymouth	41 40	73 03	. .	3.25	3.33	4.36	2.77	3.72	1.67	8.91	4.31	2.81	4.23	5.45	3.78
15. Groton	41 31	72 12	20	1.65	4.86	4.45	2.60	5.81	4.30	2.30	3.93	4.06	3.91	3.68	5.28
16. New Haven[2]	41 18	72 55	50	2.99	4.31	3.44	2.90	5.03	3.31	2.73	5.03	2.41	3.39	3.27	3.51
NEW YORK.															
1. Fort Hamilton[4] . . .	40 36	74 02	25	3.17	3.41	3.24	3.69	4.38	3.83	3.58	4.31	3.16	2.58	3.47	3.73
2. Fort Columbus[5] . . .	40 41	74 01	23	3.03	2.77	3.30	3.34	4.71	3.76	3.41	4.78	3.36	3.30	3.41	4.07
3. West Point, M. Ac'd.[6]	41 24	73 57	167	3.49	3.22	3.25	4.11	4.98	3.50	4.28	5.18	3.31	4.21	4.07	4.05

[1] Mean annual amount 1838 to 1860, from 22 years, 43.99.

[2] New Haven consolidated series: Jan. 3.42; Feb. 3.89; March, 3.43; April, 3.27; May, 4.23; June, 3.30; July, 4.19; Aug. 4.15; Sept. 3.88; Oct. 3.60; Nov. 3.67; Dec. 3.40; Spring, 10.93; Summer, 11.64; Autumn, 11.15; Winter, 10.71; Year, 44.43; Extent, 25 yrs. 2 mos.; Date, 1804—1868.

[3] 1844 to 1851 complete, last half of 1852, January, April, May, June, July, August, September, 1854.

NEW JERSEY.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.		Observer.	Reference.
						yrs.	mos.	Beginning.	End.		
1	10.70	12.39	10.52	10.21	43.82	17	2	July, 1843;	Aug. 1860	L. H. Parsons	Am. Alm., Sm. Col. and P.O. and S. I. Vol. 1.
2	11.49	11.58	10.84	10.94	44.85	23	5	May, 1843;	Dec. 1866	W. A. Whitehead	S.C., P.O. & S. I. Vol. 1, & S.O.
3	11.34	12.37	9.18	9.49	42.38	10	4	Mar. 1849;	Dec. 1862	R. L. Cooke	" " " " "
4	12.93	11.41	9.92	12.38	46.64	4	7	July, 1856;	Dec. 1866	Dr. E. R. Schmidt, Rev. A. Frost, and J. C. Deacon	P.O. and S. I. Vol. 1, and S.O.
5	23.41	15.54	. .	. .	. .	1	0	Jan. 1857;	Mar. 1858	J. T. Sergeant	P. O. and S. I., Vol. 1.
6	13.87	6.26	. .	. .	. .	0	8	Jan.	Aug. 1861	Prof. L. Harper	Sm. Obs.
7	. .	. .	. .	. .	. .	0	2	Jan.	Mar. 1861	Dr. M. J. Rhees	" "
8	. .	. .	. .	. .	. .	0	2	Nov.	Dec. 1861	H. A. Stokes	" "
9	10.68	9.73	11.35	11.07	42.83	3	7	May, 1863;	Dec. 1866	T. J. Beans	" "
10	. .	. .	. .	. .	. .	0	1	Aug. 1863		Rev. W. M. Wells	" "
11	18.57	. .	. .	11.53	. .	1	2	Dec. 1863;	July, 1865	W. Brooks	" "
12	13.69	9.35	12.37	11.66	47.07	2	10	Mar. 1864;	Dec. 1866	J. S. Lippincott and S. Wood	" "
13	11.05	6.71	10.42	11.55	39.73	2	10	Mar. 1864;	Dec. 1866	R. C. Sheppard	" "
14	. .	9.33	. .	. .	. .	1	0	Mar. 1865;	Oct. 1866	B. Cole	" "
15	7.99	13.84	14.44	9.68	45.95	1	5	Aug. 1865;	Dec. 1866	W. Brooks	" "
16	10.91	12.72	13.38	14.17	51.18	1	4	Sept. 1865;	Dec. 1866	E. R. Cook	" "
17	8.16	9.20	9.63	9.64	36.63	1	0	June, 1831;	May, 1832	. . . . .	Sm. Coll'n.
18	. .	. .	. .	. .	. .	0	1	Dec. 1866		J. Fleming	Sm. Obs.
19	. .	. .	. .	. .	. .	0	1	Dec. 1866		H. Shriver	" "
20	9.06	12.89	12.28	9.24	43.47	1	0	1866		G. H. Cook	" "

CONNECTICUT.

1	10.90	10.65	13.16	9.98	44.69	3	5	Jan. 1843;	May, 1846	Assistant Surgeon	Army Met. Reg. 1855.
2	11.08	9.98	11.92	10.81	43.79	6	9	July, 1846;	Mar. 1853	Chas. H. Hoadley	Sm. Col.
3	10.84	11.71	11.53	10.70	44.78	21	3	Jan. 1804;	July, 1829	Prof. Olmstead and others, Yale College	MS. furn. by Prof. E. Loomis.
4	10.49	12.32	10.92	10.47	44.20	3	6	Mar. 1849;	Sept. 1852	Prof. A. W. Smith	Sm. Col.
5	11.87	12.60	10.83	11.53	46.83	7	3	Sept. 1858;	Dec. 1866	Prof. J. Johnston	P. O. and S. I. Vol. 1, and S.O.
6	10.92	13.09	11.47	11.63	47.11	11	11	Jan. 1854;	Dec. 1866	Rev. D. Hunt	" " " " "
7	10.13	12.32	12.84	9.75	45.04	9	1	Jan. 1844;	Sept. 1854	Dr. O. Plumb	S.C. and P. O. and S. I. Vol. 1.
8	11.69	13.23	11.17	12.75	48.84	6	5	Feb. 1854;	May, 1861	J. Rankin	P.O. and S. I. Vol. 1, and S.O.
9	. .	. .	. .	. .	. .	0	1	Jan. 1854		T. S. Gold	P. O. and S. I. Vol. 1.
10	11.23	19.95	9.45	. .	. .	0	10	March,	Dec. 1856	A. B. Hull	" " " "
11	11.89	11.91	10.89	10.02	44.71	1	11	Jan. 1856;	Feb. 1858	Rev. T. Edwards	" " " "
12	13.87	15.64	10.39	12.13	52.03	5	3	April, 1856;	July, 1862	B. F. Harrison	P. O. and S. I. Vol. 1. and S.O.
13	10.71	20.58	12.89	10.67	54.85	1	7	Dec. 1861;	July, 1863	J. Case	Sm. Obs.
14	10.85	14.89	12.49	10.36	48.59	1	11	July, 1862;	May, 1864	D. W. Learned	" "
15	12.86	10.53	11.65	11.79	46.83	1	0	1866		Rev. E. Dewhurst	" "
16	11.37	11.07	9.07	10.81	42.32	3	11	June, 1864;	Apr. 1868	. . . .	MS. furn. by Prof. E. Loomis.

NEW YORK.

1	11.31	11.72	9.21	10.31	42.55	18	8	Jan. 1839;	Dec. 1859	Assistant Surgeon	Arm. Met. Reg. 1855, M. D. of U. S. A. 1860.
2	11.35	11.95	10.07	9.87	43.24	24	0	Jan. 1836;	Dec. 1859	" "	" " " " "
3	12.34	12.96	11.59	10.76	47.65	20	0	Jan. 1840;	Dec. 1859	" "	" " " " "

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[4] At the Narrows, New York Bay and Harbor. For New York consolidated series see end of table.

[5] At Fort Wood, Bedloe's Island, New York Harbor, from January, 1837, to September, 1839. Fort Columbus is on Governor's Island, New York Harbor.

[6] The observations of 1836–7–8–9 are omitted as doubtful. Location on the west bank of Hudson River, about 50 statute miles from the sea, surrounded by hills from 600 to 1400 feet in height.

NEW YORK.—CONTINUED.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
4. Watervliet Arsenal[1] .	42°43′	73°43′	50	2.07	2.08	2.19	2.92	3.55	3.73	3.51	3.10	3.24	3.00	2.93	2.33
5. Plattsburg Barracks .	44 41	73 25	186	1.38	1.20	2.18	2.56	3.63	3.51	3.22	3.30	3.72	3.67	2.66	2.37
6. Madison Barracks. .	43 57	76 15	262	2.35	1.98	2.59	2.31	2.66	3.05	4.00	2.67	3.24	3.76	2.95	2.16
7. Fort Ontario	43 20	76 40	310	2.30	2.13	1.82	1.75	2.18	3.06	2.59	2.04	2.41	4.23	3.00	3.00
8. Fort Niagara	43 15	79 08	262	2.21	1.99	2.30	2.09	2.73	2.56	3.25	2.78	3.34	2.57	2.58	2.35
9. Buffalo Barracks . .	42 53	78 54	660	3.33	1.54	3.08	2.59	2.83	2.77	3.05	3.41	4.94	4.72	3.88	2.66
10. Flatbush, Erasmus Hall	40 37	74 02	54	3.42	2.95	3.55	3.69	3.84	3.89	3.68	4.26	3.18	3.58	3.81	3.67
11. East Hampton, Clinton Academy . . .	40 58	72 28	16	2.93	2.30	2.54	3.82	3.66	2.98	2.59	3.22	3.27	3.65	3.16	3.17
12. North Salem	41 20	73 38	361	2.93	2.40	3.04	3.15	4.06	3.40	4.03	3.75	3.15	4.35	3.24	3.32
13. North Salem	41 20	73 38	361	. .	. .	. .	3.09	4.07	2.68	. .	12.10	3.03	1.53	1.60	3.49
14. New York Hospital .	40 45	74 02	. .	2.93	2.43	3.92	1.36	4.68	3.81	3.61	4.26	3.77	4.37	2.15	4.80
15. Lowville	43 46	75 32	847	2.38	2.54	1.98	2.14	2.79	3.42	3.52	3.01	2.91	3.33	3.01	2.47
16. Jamaica, Union Hall, or Clinton Hall . .	40 42	73 50	30	2.50	2.23	2.83	3.05	3.54	3.69	3.94	4.09	3.38	3.48	3.49	2.85
17. Newburgh Academy.	41 31	74 05	150	2.74	2.16	2.56	2.08	4.33	3.47	3.64	3.14	2.87	3.54	3.15	2.46
18. Newburgh	41 31	74 00	65	1.06	5.02	2.85	2.05	5.70	4.75	4.10	3.94	2.50	3.02	3.42	2.92
19. Albany Academy[2]. .	42 39	73 44	130	2.77	2.59	2.82	3.12	3.85	4.48	4.39	3.44	3.34	3.69	3.24	2.94
20. Goshen, Farm. Hall	41 20	74 11	425	2.50	2.42	2.52	2.14	3.30	3.53	2.98	2.55	2.74	2.95	2.27	3.40
21. Albany[2].	42 39	73 45	75	2.69	3.60	1.89	2.26	5.77	3.24	3.98	1.37	2.80	4.59	2.61	1.67
22. Utica	43 06	75 13	473	2.89	2.66	2.70	3.29	3.31	4.62	4.74	3.65	3.48	2.84	3.61	3.35
23. Pompey.	42 56	76 05	1300	1.69	1.60	1.26	1.77	3.08	4.21	4.12	3.19	2.93	3.23	2.10	1.57
24. Pompey Hill	42 52	76 09	1737	2.24	2.08	1.09	. .	2.80	2.45	1.70	. .	. .	8.02	3.40	3.46
25. Penn Yan	42 42	77 08	740	1.38	1.44	1.65	2.39	3.06	3.37	3.14	2.83	2.75	2.61	2.11	1.69
26. Lansingburg	42 47	73 40	30	2.29	2.07	2.16	2.40	2.78	3.92	3.55	2.52	3.02	3.19	2.82	2.59
27. Rochester.	43 08	77 51	500	2.06	3.02	1.76	2.66	2.91	2.72	3.04	2.10	2.04	3.36	2.26	2.14
28. Rochester Univers.[3].	43 08	77 51	516	2.10	1.98	2.13	2.49	3.07	3.02	3.24	2.83	3.30	3.22	2.74	2.51
29. Rochester[4]	43 08	77 51	516	2.49	2.31	2.97	2.73	3.12	2.92	3.37	3.22	3.36	3.64	2.58	2.96
30. Geneva[5].	42 53	77 02	567	1.64	1.08	1.82	3.31	2.91	3.17	3.40	3.54	2.51	3.06	2.25	2.03
31. Geneva	42 53	77 02	567	1.36	1.36	1.61	3.03	4.18	3.22	2.09	4.07	3.84	3.15	2.35	1.43
32. Buffalo[6].	42 53	78 55	600	2.46	2.56	3.45	2.91	3.53	2.46	2.79	3.02	3.65	3.16	2.99	3.40
33. Buffalo	42 53	78 56	629	1.74	3.43	1.26	1.21	1.64	1.60	1.10	2.19	3.41	4.03	3.41	2.25
34. Buffalo	42 53	78 55	585	2.21	1.83	2.80	2.40	3.15	1.70	2.55	3.48	3.79	3.19	2.43	2.81
35. New York, Inst. for the Deaf & Dumb[7]	40 43	73 58	159	4.19	4.16	3.87	4.95	4.83	5.17	4.86	3.92	3.89	4.05	4.28	4.48
36. New York, Inst. for the Deaf & Dumb	40 43	73 58	165	2.97	4.07	3.85	2.48	5.18	2.59	4.38	4.55	2.81	3.87	3.91	4.07
37. Champion.	44 05	75 37	. .	3.41	1.67	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
38. Martinsburg.	43 43	75 34	. .	. .	. .	2.96	2.62	6.24	. .	. .	. .	. .	. .	. .	. .
39. Seneca Falls	42 54	76 50	463	1.44	2.24	2.38	2.82	3.30	2.26	4.25	3.07	2.70	3.97	3.89	3.09
40. Ovid, or Seneca Collegiate Institute . .	42 41	76 52	800	1.82	1.58	1.48	3.63	3.48	7.13	3.88	2.57	3.15	4.73	3.03	1.49
41. Seneca Falls[8]	42 54	76 50	. .	2.31	2.02	2.95	2.32	3.88	4.95	3.06	1.68	1.49	4.16	3.21	4.20

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[1] This series is supposed incomplete in the measure of water falling as snow. Location west bank of Hudson River above Albany.

[2] Albany consolidated series: Jan. 2.76; Feb. 2.67; March, 2.76; April, 3.06; May, 3.92; June, 4.44; July, 4.37; August, 3.37; Sept. 3.32; Oct. 3.73; Nov. 3.22; Dec. 2.90; Spring, 9.74; Summer, 12.18; Autumn, 10.27; Winter, 8.33; Year, 40.52; Extent, 28 yrs. 4 mos.; Date, 1826—1866.

[3] Rochester, consolidated series: Jan. 2.14; Feb. 2.18; March, 2.16; April, 2.51; May, 3.01; June, 3.04; July, 3.22; Aug. 2.67; Sept. 3.09; Oct. 3.25; Nov. 2.69; Dec. 2.60; Spring, 7.68; Summer, 8.93; Autumn, 9.03; Winter, 6.92; Year, 32.56; Extent, 35 yrs. 1 mo.; Date, 1836—1866. (Series 1831–38, 1837—1866, 1860—1866.)

[4] Observations by Prof. Dewey at the University; locality for the other observer not known.

[5] Geneva consolidated series: Jan. 1.60; Feb. 1.12; March, 1.79; April, 3.27; May, 3.11; June, 3.18; July, 3.19; Aug.

NEW YORK.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs.	mos.	DATE. Beginning.	End.	OBSERVER.	REFERENCE.
4	8.66	10.34	9.17	6.48	34.65	17	4	Apr. 1836;	Dec. 1854	Assistant Surgeon.	Arm. Met. Reg. 1855.
5	8.37	10.03	10.05	4.95	33.40	9	1	Jan. 1840;	Apr. 1852	" "	" " " "
6	7.56	9.72	9.95	6.49	33.72	6	2	Jan. 1840;	Apr. 1852	" "	" " " "
7	5.75	7.69	9.64	7.43	30.51	8	11	Jan. 1844;	Sept. 1856	" "	Arm. Met. Reg. 1855, M. D. of U. S. A. 1860.
8	7.12	8.59	8.49	6.55	30.75	17	5	Oct. 1841;	Dec. 1866	Assistant Surgeon; L. Leffman	Arm. Met. Reg. 1855, P. O. and S. I. Vol. 1, Survey N. and N.W. Lakes, 1867.
9	8.50	9.23	13.54	7.53	38.80	3	1	July, 1842;	July, 1845	Assistant Surgeon	Arm. Met. Reg. 1855.
10	11.08	11.83	10.57	10.04	43.52	36	2	Jan. 1826;	Dec. 1866	Profs. of Acad. Rev. W. W. Howard, E. T. Mack	N. Y. Acad. Sys., P. O. and S. I. Vol. 1, and Sm. Obs.
11	10.02	8.79	10.08	8.40	37.29	16	8	Jan. 1828;	Aug. 1852	Profs. of Acad.	N. Y. Acad. Sys.
12	10.25	11.18	10.74	8.65	40.82	20	0	Jan. 1830;	Dec. 1852	J. F. Jenkins	" " "
13	. .	. .	6.16	. .	. .	0	8	1856		M. J. Lobdell	P. O. and S. I. Vol. 1.
14	9.96	11.68	10.29	10.16	42.09	4	0	Jan. 1847;	Dec. 1850	Dascey	Am. Alm. 1850, and foll.
15	6.91	9.95	9.25	7.39	33.50	21	8	Jan. 1827;	Dec. 1858	Profs. of Acad.; J. C. House	N. Y. Acad. Sys.; P. O. and S. I. Vol. 1.
16	9.42	11.72	10.35	7.58	39.07	25	0	Jan. 1827;	Dec. 1852	Profs. of Acad.	N. Y. Acad. Sys.
17	8.97	10.25	9.56	7.36	36.14	20	0	Jan. 1830;	Apr. 1852	" " "	" " "
18	10.60	12.79	8.94	9.00	41.33	1	10	Mar. 1865;	Dec. 1866	J. H. Gardiner	Sm. Obs.
19	9.79	12.31	10.27	8.30	40.67	27	0	Jan. 1826;	Dec. 1852	Beck and Cook	N. Y. Acad. Sys.
20	7.96	9.06	7.96	8.32	33.30	10	0	1834—1849		Profs. of Acad.	N. Y. Acad. Sys., Reg. Rep.
21	9.92	8.59	10.00	7.96	36.47	1	4	Jan. 1865;	Apr. 1866	H. M. Paine	Sm. Obs.
22	9.30	13.01	9.93	8.90	41.14	21	11	1826;	Dec. 1852	Profs. of Acad.	N. Y. Acad. Sys., R. R.
23	6.11	11.52	8.26	4.86	30.75	16	3	Jan. 1830;	Dec. 1852	" " "	" " " "
24	. .	. .	. .	7.78	. .	0	9	May, 1856;	Mar. 1858	J. F. Kendall, S. M. Ingalls	P. O. and S. I. Vol. 1.
25	7.10	9.34	7.47	4.51	28.42	38	8	Jan. 1829;	Oct. 1867	Dr. H. P. Sartwell	MS. in Sm. Coll. and R. R. N. Y. Acad. Sys.
26	7.34	9.99	9.03	6.95	33.31	20	0	1826—1846		Profs. of Acad.	N.Y. Acad. Sys., Blodget's Cl.
27	7.33	7.86	7.66	7.22	30.07	5	3	Jan. 1831;	July, 1838	Dr. E. S. Marsh	MS. in Sm. Coll. presented by I. A. Lapham.
28	7.69	9.09	9.26	6.59	32.63	30	0	Jan. 1837;	Dec. 1866	L. Wetherell & others	Am. Alm. 1867, etc., N. Y. Aca. Sys. Reg. Rep. 1868.
29	8.82	9.51	9.58	7.76	35.67	7	0	Jan. 1860;	Dec. 1866	Prof. C. Dewey, Dr. M. M. Mathews	Reg. Rep. 1868, Sm. Obs.
30	8.04	10.11	7.82	4.75	30.72	15	10	Jan. 1841;	Oct. 1856	Rev. W. D. Wilson	Sm. Coll'n (N. Y. Aca. Sys.).
31	8.82	9.38	9.34	4.15	31.69	3	0	Jan. 1864;	Dec. 1866	" " "	Sm. Obs.
32	9.89	8.27	9.80	8.42	36.38	10	8	Sept. 1855;	Dec. 1867	E. O. Salisbury, W. Ives	P. O. and S. I. Vol. 1, Sm. Col.
33	4.11	4.89	10.85	7.42	27.27	1	0	1832		Prof. of Acad.	N. Y. Aca. Sys. Reg. Rep.
34	8.35	7.73	9.41	6.85	32.34	7	6	July, 1859;	Dec. 1866	W. S. King, E. Dorr	Surv. N. & N.W. Lakes, Gen. Reynolds, U. S. A.
35	13.65	13.95	12.22	12.83	52.65	11	10	Jan. 1854;	Dec. 1866	Prof. O. W. Morris	P.O. and S. I. Vol. 1, Sm. Obs.
36	11.51	11.52	10.59	11.11	44.73	5	3	Part 1844, '46, '48 to '51		" "	MS. Sm. Coll'n.
37	. .	. .	. .	. .	. .	0	2	1844		Dr. F. B. Hough	" " "
38	. .	. .	. .	. .	. .	0	3	1844		" " "	" " "
39	8.50	9.58	10.56	6.77	35.41	2	6	1850 to May 1852, Sept. Oct. 1852		John P. Fairchild	" " "
40	8.59	13.58	10.91	4.89	37.97	2	7	Jan. 1856;	Sept. 1858	J. W. Chickering	P. O. and S. I. Vol. 1.
41	9.15	9.69	8.86	8.53	36.23	1	0	1852		. . . .	MS. in Sm. Col., N.Y. Ac. Sys.

3.62; Sept. 2.72; Oct. 3.07; Nov. 2.27; Dec. 1.93; Spring, 8.17; Summer, 9.99; Autumn, 8.06; Winter, 4.65; Year, 30.87; Extent, 18 yrs. 10 mos.; Date, 1841—1866.

[6] Buffalo, consolidated series: Jan. 2.26; Feb. 2.39; March, 3.07; April, 2.66; May, 3.25; June, 2.21; July, 2.59; August, 2.86; Sept. 3.57; Oct. 3.11; Nov. 2.83; Dec. 3.04; Spring, 8.98; Summer, 7.66; Autumn, 9.51; Winter, 7.69; Year, 33.84; Extent, 11 yrs. 8 mos.; Date, 1832—1867. See Climatology of Buffalo, by W. Ives, 1867.

[7] New York Institution for the Deaf and Dumb, consolidated series: Jan. 3.83; Feb. 4.13; March, 3.86; April, 4.22; May, 4.93; June, 4.41; July, 4.71; August, 4.10; Sept. 3.57; Oct. 4.00; Nov. 4.17; Dec. 4.36; Spring, 13.01; Summer, 13.22; Autumn, 11.74; Winter, 12.32; Year, 50.29; Extent, 17 yrs.; Date, 1844—1866. After 1860 the elevation is given as 105 feet; the amount of rain in series 2 appears abnormal and excessive.

[8] Perhaps identical with Ovid.

NEW YORK.—CONTINUED.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
42. Boonville	43°29′	75°25′	. .	. .	. .	. .	. .	. .	3.00	1.02	3.01	3.96	. .	. .	. .
43. Alfred	42 15	78 06	. .	1.96	1.69	2.26	. .	. .	. .	. .	. .	. .	. .	. .	. .
44. Elmira	42 05	76 53	860	1.60	0.65	2.32	3.60	2.45	3.18	2.28	1.85	2.07	1.38	2.58	1.92
45. Jamestown	42 06	79 18	1364	2.71	2.63	2.53	3.79	3.89	4.19	5.90	3.43	3.08	3.39	4.14	4.21
46. Jamestown	42 05	79 29	1454	2.73	3.05	3.10	3.65	4.40	3.90	5.40	5.90	6.35	4.55	5.35	5.55
47. Plattsburg Academy	44 42	73 26	156	1.25	0.95	1.33	1.95	0.84	4.51	2.77	2.34	1.60	2.68	2.29	2.98
48. Syracuse	43 03	76 10	407	2.26	2.10	2.68	2.81	3.16	3.66	3.53	2.53	3.62	3.91	3.39	3.10
49. Ithaca	42 27	76 30	417	1.81	1.76	2.51	3.00	3.54	3.83	3.31	2.99	3.41	3.34	2.87	2.34
50. Oxford	42 28	75 32	961	2.47	2.21	2.27	2.52	3.60	4.27	3.77	3.34	3.35	3.36	2.73	2.47
51. Homer, Cortland Ac.	42 38	76 11	1095	1.72	2.89	2.66	3.41	5.06	4.36	6.51	3.04	4.39	4.36	4.79	3.79
52. Hudson	42 15	73 47	150	2.36	1.97	3.22	2.28	3.09	3.59	3.66	2.61	2.13	3.85	2.73	3.03
53. Mexico	43 27	76 14	331	2.60	2.12	2.23	1.53	2.76	2.56	3.16	2.08	3.38	4.16	3.40	3.71
54. Liberty	41 50	74 55	1530	1.64	5.72	4.93	7.48	3.39	4.77	2.73	3.53	2.79	2.86	3.88	4.46
55. Mexico	43 27	76 14	423	3.09	. .	0.86	2.16	3.65	1.52	2.17	3.34	5.95	4.19	3.37	5.16
56. Liberty	41 45	74 46	1474	1.45	1.22	1.87	2.40	. .	. .	. .	. .	. .	. .	. .	. .
57. Ogdensburg	44 43	75 34	232	2.10	2.65	3.66	2.97	2.45	5.02	3.38	3.43	2.61	2.32	3.34	3.70
58. Delhi	42 16	74 58	1384	1.67	0.92	3.36	3.59	2.59	2.73	4.36	2.97	2.70	3.45	4.59	4.23
59. Clinton, Hamilt. Col.	43 03	75 25	1127	2.27	2.50	2.29	2.07	2.94	3.54	3.68	2.60	3.30	3.42	2.72	2.56
60. Blackwell's Island[1]	40 45	73 57	29	6.46	1.41	2.67	4.59	5.53	3.80	5.53	5.60	3.32	2.65	2.27	3.61
61. Gouverneur, H. Schl.	44 25	75 35	400	2.30	2.45	1.67	1.79	2.20	2.82	2.56	1.99	2.84	3.21	2.18	1.88
62. Gouverneur[2]	44 19	75 29	. .	2.32	2.56	2.81	2.68	2.88	2.59	3.16	2.86	4.37	3.28	3.93	3.50
63. N.Y. U.S. Nav. Hos.	40 41	74 01	56	2.66	3.00	3.23	2.72	3.68	3.54	2.54	2.56	3.58	3.29	3.87	3.63
64. Sackett's Harbor	43 55	75 57	266	1.98	2.46	2.89	2.95	3.36	2.51	2.98	3.10	3.44	4.01	4.13	2.92
65. Charlotte	43 13	77 51	273	1.73	1.72	2.28	2.41	2.92	2.43	2.90	2.95	2.89	3.03	2.56	2.09
66. Beaver Creek	41 20	74 50	700	. .	. .	5.75	. .	4.50	3.92	2.45	1.83	7.02	2.98	5.40	0.50
67. Chatham	42 26	73 30	. .	1.10	2.80	. .	14.00	. .	. .	. .	. .	. .	. .	. .	. .
68. Falconer	42 05	79 10	. .	2.90	3.00	2.60	. .	. .	. .	. .	. .	. .	. .	. .	. .
69. Houseville	43 40	75 32	900	1.96	2.92	3.03	3.03	3.03	2.13	4.45	3.49	4.03	4.55	4.00	2.43
70. Lodi[3]	43 37	76 54	. .	1.25	1.34	0.91	3.22	2.35	3.59	3.50	1.92	2.39	2.76	1.81	2.31
71. Madrid	44 43	75 33	280	1.62	2.40	2.60	4.21	2.54	3.85	1.98	3.39	4.41	1.66	. .	. .
72. Sag Harbor, L. Isl'd	41 00	72 20	40	3.68	2.60	3.37	4.45	3.47	2.41	3.38	3.67	3.62	3.88	4.12	4.42
73. Madrid	44 43	75 33	280	1.82	2.13	2.20	1.34	2.40	2.44	4.27	1.84	2.07	3.49	3.28	3.10
74. Smithville	44 00	76 01	. .	4.71	5.39	2.99	. .	. .	. .	. .	. .	. .	. .	3.35	. .
75. Spencertown	42 19	73 41	750	2.59	1.68	1.22	4.62	4.26	4.07	3.97	4.09	3.00	2.96	2.66	3.43
76. Troy	42 44	73 40	55	3.78	2.00	2.61	3.02	3.61	3.79	4.60	3.25	2.97	3.04	2.82	2.06
77. Canton	44 38	75 15	304	3.40	1.15	0.98	1.80	2.83	2.83	3.55	3.48	2.93	4.80	. .	5.37
78. Troy, Rensselaer In.	42 44	73 37	58	2.72	2.58	3.08	5.87	2.62	2.29	2.62	1.10	. .	. .	2.35	2.47
79. Angelica	42 15	78 01	1500	2.85	1.16	. .	4.40	. .	8.01	. .	1.83	4.82	1.77	. .	1.90
80. Beverly[4]	41 22	73 57	180	3.01	2.89	3.77	4.47	5.05	4.09	4.14	5.65	3.84	3.47	3.99	3.83
81. Cazenovia	42 55	75 46	1260	3.25	3.08	3.92	4.35	3.26	4.12	4.59	4.02	3.90	4.29	3.17	3.15
82. Clinton	43 03	75 22	600	2.41	2.23	2.59	4.65	3.66	3.56	3.54	4.75	3.02	3.83	3.78	3.47
83. West Concord	42 32	78 52	2000	. .	. .	1.50	. .	. .	. .	. .	. .	. .	. .	1.00	. .
84. Fishkill Landing	41 31	73 59	42	2.98	2.05	3.15	2.97	3.68	3.99	4.00	4.40	3.16	2.66	3.97	2.92
85. Fordham, St. John's College	40 54	73 57	147	. .	1.70	. .	. .	. .	. .	3.65	. .	0.58	4.45	2.63	3.65

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[1] Opposite New York; Penitentiary Hospital.

[2] Gouverneur, consolidated series: Jan. 2.30; Feb. 2.48; March, 1.94; April, 1.98; May, 2.37; June, 2.77; July, 2.70;

NEW YORK.—Continued.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.	Observer.	Reference.
						yrs.	mos.	Beginning. End.		
42	..	7.03	..	..	..	0	4	1852	....	MS. in Sm. Col., N. Y. Ac. Sys.
43	..	..	..	..	..	0	3	1852	....	“ “ “ “ “
44	8.37	7.31	6.03	4.17	25.88	1	6	Apr. 1851; Oct. 1852	Profs. of Acad.	N. Y. Aca. Sys.
45	10.21	13.52	10.61	9.55	43.89	2	2	Nov. 1850; Dec. 1852	“ “ “	N. Y. Ac. Sys., MS. in Sm. Col.
46	11.15	15.20	16.25	11.33	53.93	2	1	Jan. 1864; Feb. 1866	S. W. Roe	Sm. Obs.
47	4.12	9.62	6.57	5.18	25.49	2	0	Dec. 1850; Mar. 1856	J. W. Taylor	N. Y. Ac. Sys. MS. in Sm. Col. and P. O. and S. I. Vol. 1.
48	8.65	9.72	10.92	7.46	36.75	9	5	1840–46; Sept. 1850–52	Profs. of Acad.	N. Y. Ac. Sys. Reg. Rep. and MS. in Sm. Coll'n.
49	9.05	10.13	9.62	5.91	34.71	18	11	1830 to Nov. 1852	“ “ “	“ “ “ “ “ “
50	8.39	11.38	9.44	7.15	36.36	20	0	1830 to 1846, 1850–52	“ “ “	“ “ “ “ “ “
51	11.13	13.91	13.54	8.40	46.98	2	6	Sept. 1850; Dec. 1852; and Jan. and Feb. 1856	E. C. Reed	N. Y. Aca. Sys. MS.
52	8.59	9.86	8.71	7.36	34.52	15	0	1830; Dec. 1852	Profs. of Acad.	N. Y. Aca. Sys.
53	6.52	7.80	10.94	8.43	33.69	14	3	1837; Dec. 1852	“ “ “	“ “ “
54	15.80	11.03	9.53	11.82	48.18	2	3	Oct. 1850; Dec. 1852	Watkins	N. Y. Aca. Sys. MS.
55	6.67	7.03	13.51	..	..	0	11	Mar. 1856; Jan. 1857	J. R. French	P. O. and S. I. Vol. 1.
56	..	..	..	..	..	0	4	1856	J. Felt, Jr.	“ “ “ “
57	9.08	11.83	8.27	8.45	37.63	5	8	Sept. 1850; June, 1860	W. E. Guest	N. Y. Ac. Sys. MS., and P. O. and S. I. Vol. 1, and Sm. Obs.
58	9.54	10.06	10.74	6.82	37.16	1	3	Sept. 1851; Dec. 1852	McKoon	N. Y. Aca. Sys. MS.
59	7.30	9.82	9.44	7.33	33.89	19	0	1830–'6, '39, '42–1852	Prof. O. Root	N. Y. Ac. Sys. MS. & Reg. Rep.
60	12.79	14.93	8.24	11.48	47.44	2	0	Nov. 1855; Nov. 1857	Dr. W. W. Sanger	P. O. and S. I. Vol. 1.
61	5.66	7.37	8.23	6.63	27.89	18	1	1830; Dec. 1855	Profs. of Aca. P. O. Williams	N. Y. Aca. Sys., Reg. Rep., P. O. and S. I. Vol. 1.
62	8.37	8.61	11.58	8.38	36.94	5	10	Jan. 1861; Dec. 1866	C. H. Russell	Sm. Obs.
63	9.63	8.64	10.74	9.29	38.30	5	1	Dec. 1860; Dec. 1866	J. A. Lockwood, T. L. Smith, J. C. Palmer	MS. in Sm. Coll'n, Sm. Obs.
64	9.20	8.59	11.58	7.36	36.73	7	6	July, 1859; Dec. 1866	H. Metcalf	P. O. and S. I. Vol. 1, Surv. N. and N. W. Lakes.
65	7.61	8.28	8.48	5.54	29.91	7	6	July, 1859; Dec. 1866	A. Mulligan	“ “ “ “ “
66	..	8.20	15.40	..	..	0	9	1854	C. S. Woodward	P. O. and S. I. Vol 1.
67	..	..	..	..	..	0	3	1854	C. T. Chase	“ “ “ “
68	..	..	..	..	..	0	3	1854	L. A. Langdon	“ “ “ “
69	9.09	10.07	12.58	7.31	39.05	5	2	Mar. 1849; Dec. 1866	W. D. Yale	MS. Sm. Coll'n, P. O. and S. I. Vol. 1, and Sm. Obs.
70	6.48	9.01	6.96	4.90	27.35	4	5	Jan. 1854; Dec. 1858	J. Lefferts	P. O. and S. I. Vol. 1.
71	9.35	9.22	..	..	..	1	4	Jan. 1854; Oct. 1859	E. A. Dayton	“ “ “ “
72	11.29	9.46	11.62	10.70	43.07	8	5	Mar. 1849; Dec. 1858	E. N. Byram	MS. in Sm. Col. and P. O. and S. I. Vol. 1.
73	5.94	8.55	8.84	7.05	30.38	2	10	1849—1852	E. A. Dayton	MS. in Sm. Coll'n.
74	..	..	..	..	..	0	7	Jan. 1854; Nov. 1856	J. E. Breed	P. O. & S. I. Vol. 1.
75	10.10	12.13	8.62	7.70	38.55	5	3	July, 1854; June, 1861	Rev. T. C. P. Hyde, L. S. Packard and others	P. O. & S. I. Vol. 1, & Sm. Coll'n.
76	9.24	11.64	8.83	7.84	37.55	3	5	Jan. 1861; July, 1866	J. W. Heimstreet	Sm. Obs.
77	5.61	9.81	..	9.92	..	1	4	Apr. 1855; Sept. 1858	E. W. Johnson	P. O. & S. I. Vol. 1.
78	11.57	6.01	..	7.77	..	1	6	Jan. 1854; July, 1866	Prof. E. H. Allen, Prof. D. Green, W. Fenton, W. L. Haskin	P. O. & S. I. Vol. 1, & Sm. Obs.
79	..	..	..	5.91	..	0	11	Jan. 1856; June, 1857	Dr. E. M. Alba	P. O. & S. I. Vol. 1.
80	13.29	13.88	11.30	9.73	48.20	8	10	Feb. 1856; Dec. 1866	T. B. Arden	P. O. & S. I. Vol. 1, & Sm. Obs.
81	11.53	12.73	11.36	9.48	45.10	5	7	Oct. 1856; June, 1865	A. White.	“ “ “ “ “ “ “
82	10.90	11.85	10.63	8.11	41.49	6	1	Jan. 1856; Mar. 1865	Prof. O. Root, Dr. H. M. Paine	“ “ “ “ “ “ “
83	..	..	..	..	..	0	2	1856	L. Woodward	P. O. & S. I. Vol. 1.
84	9.80	12.39	9.79	7.95	39.93	8	4	Jan. 1856; Oct. 1866	W. H. Denning	P. O. & S. I. Vol. 1, & Sm. Obs.
85	..	..	7.66	..	..	0	8	July, 1856; Feb. 1862	Prof. C. Pernot, Dr. S. Spooner, J. Aubier, Prof. A. T. Monroe	“ “ “ “ “ “ “

August, 2.22; Sept. 3.22; Oct. 3.23; Nov. 2.64; Dec. 2.30; Spring, 6.29; Summer, 7.69; Autumn, 9.09; Winter, 7.08; Year, 30.15; Extent, 23 yrs. 11 mos.; Date, 1830—1866.

[3] Or Townsendville, also Covert.

[4] Also known as Philipstown and Ardenia.

NEW YORK.—Continued.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
86. Lake	43°15′	73°33′	..	2.90	1.78	1.67	4.09	4.75	4.84	5.51	3.97	3.70	4.58	3.27	3.68
87. Oswego	43 28	76 34	250	3.29	3.37	3.96	4.28	4.22	3.23	3.82	3.50	4.19	4.50	4.58	4.00
88. Bellport, L. Island	40 44	72 54	15	5.42	4.75	4.07	3.14	2.95	3.75	3.04	3.73	6.03	2.74	3.84	1.96
89. Wampsville[1]	43 04	75 50	500	3.88	3.19	3.78	5.40	6.15	5.65	7.34	8.15	7.20	6.27	5.93	3.99
90. Watertown	43 56	75 52	268	1.65	0.21	1.65	1.44	3.72	1.41	1.83	3.71	3.84	3.83	1.85	2.05
91. Fort Edward	43 13	73 42	..	3.72	1.72	2.84	4.00	5.26	7.85	4.92	4.71	4.27	8.57	6.11	5.76
92. McGrawville	42 34	76 11	1450	3.91	0.68	3.50	2.29	4.22	10.89	2.51	2.88	3.20	..	..	..
93. Plainville	43 00	77 16	..	4.57	4.74	2.08	5.32	4.77	7.26	..	..	..	..	..	..
94. Waterford	42 47	73 41	70	3.42	1.97	2.95	3.60	3.15	4.24	4.58	3.55	2.95	2.89	2.61	2.58
95. Wellsville	42 07	78 06	1480	..	..	..	2.25	2.50	..	3.53	3.69	5.50	..	..	2.62
96. Saratoga	43 06	74 00	960	1.50	..	..	3.30	7.10	2.80	2.90	..	3.60	..	..	..
97. Dansville	42 38	77 46	672	2.11	3.62	1.99	1.80	3.27	1.20	..	1.61	5.75	2.52	1.32	2.66
98. Schenectady, Union College	42 49	73 55	..	4.91	2.49	2.45	2.98	3.50	4.10	3.16	2.67	1.98	3.39	1.94	4.67
99. Schenectady	42 49	73 55	300	..	..	..	3.37	..	1.77	..	..	..	..	..	..
100. Wilson	43 20	78 56	250	0.84	1.38	..	..	..	..	..	..	..	..	0.90	1.37
101. Palermo	43 25	77 26	327	2.66	3.45	3.35	2.07	2.55	6.80	2.90	2.60	4.73	4.70	5.28	4.20
102. Sing Sing	41 09	73 56	125	1.79	3.56	0.82	1.50	6.18	2.27	2.43	5.29	3.85	4.48	2.66	2.00
103. Montgomery[2]	41 32	74 00	300	2.64	2.22	2.24	2.32	3.05	4.06	3.36	2.62	2.53	3.28	2.74	3.12
104. Potsdam, St. Lawrence Academy	44 40	75 01	394	1.40	1.06	1.48	1.70	3.02	3.31	4.03	2.81	3.11	3.34	1.93	1.44
105. Bridgewater	42 55	75 17	1286	4.26	2.85	3.01	4.26	3.32	5.37	4.82	2.74	2.56	4.37	2.12	4.35
106. Cazenovia Acad.[3]	42 55	75 46	1260	2.46	2.07	2.60	2.83	3.76	4.50	4.11	3.59	3.40	3.64	3.00	2.88
107. Malone	44 50	74 23	645	1.78	2.18	2.16	2.44	2.86	3.61	3.35	2.12	3.31	3.18	2.07	2.39
108. Somerville	44 10	75 25	412	3.09	1.66	2.17	2.17	2.70	1.47	4.34	2.42	2.33	3.77	2.63	1.93
109. Bellville Union Ac.[4]	43 45	76 10	300	1.76	1.93	1.35	1.97	2.39	2.55	2.69	2.40	3.37	4.00	2.97	2.14
110. Oneida Institute	43 08	75 14	824	3.07	1.59	1.48	2.32	2.54	3.31	3.27	3.08	2.69	3.72	2.15	1.97
111. Mt. Pleasant	41 09	73 47	125	2.16	1.50	2.55	3.57	3.63	3.33	4.31	3.83	3.03	3.27	2.44	2.67
112. Kingston Academy[5]	41 55	74 02	188	2.92	1.93	2.74	2.15	3.43	3.49	3.72	2.81	2.26	3.11	3.37	3.17
113. Redhook	42 02	73 56	..	2.86	1.54	2.39	3.18	3.03	3.99	4.31	2.83	2.45	2.75	2.46	2.36
114. Kinderhook Acad.	42 22	73 43	125	2.21	1.53	2.48	2.97	3.41	4.55	4.35	3.35	2.94	3.25	2.69	2.75
115. Poughkeepsie, Dutchess Co. Aca.	41 41	73 55	..	3.34	2.08	3.22	2.90	3.60	3.66	4.09	4.41	2.54	3.94	3.35	3.23
116. Cambridge, Washington Co. Acad.	43 03	73 23	500	2.90	2.59	2.03	2.98	3.49	4.30	3.58	4.18	2.76	3.48	2.80	2.14
117. Granville	43 20	73 17	250	2.06	1.44	1.79	2.07	3.52	3.18	3.52	2.89	2.65	2.88	3.06	2.46
118. Hartwick	42 38	75 01	1100	2.35	1.74	2.25	4.45	3.52	3.65	3.80	2.87	2.18	3.72	3.17	2.62
119. Canajoharie	42 53	74 35	284	2.70	1.30	0.85	1.30	3.91	4.52	3.60	1.42	2.00	3.64	2.76	..
120. Fairfield Academy	43 05	74 55	1185	2.69	1.79	2.36	2.53	3.09	4.29	4.21	3.65	3.08	3.56	2.46	2.74
121. Cherry Valley Aca.	42 48	74 47	1335	2.66	2.57	2.80	3.03	4.04	4.35	4.24	3.45	3.55	3.90	3.18	3.07
122. Johnstown	43 00	74 23	250	3.14	2.72	3.78	2.63	3.34	4.35	4.12	3.35	2.72	3.36	3.57	3.29
123. Lewiston[6]	43 09	79 10	280	1.38	1.30	1.54	1.64	2.19	2.72	2.27	2.10	2.68	2.44	1.69	1.15
124. Springville	42 30	78 50	1060	1.18	2.37	1.84	3.95	2.17	3.95	3.60	3.64	4.60	5.82	3.03	2.30
125. Fredonia Academy	42 26	79 24	710	2.04	1.82	1.99	1.93	3.32	3.83	3.34	3.78	4.46	4.31	3.27	2.96
126. Fredonia	42 26	79 24	715	2.05	2.19	..	..	..	..	..	..	..	..	..	2.45
127. Cuba	42 15	78 30	1502	2.78	1.96	3.72	3.13	2.54	3.43	2.21	3.27	3.47	2.52	2.73	2.97
128. Palmyra	43 05	77 16	450	1.16	0.82	1.03	2.24	1.66	3.50	2.88	3.05	3.22	3.66	1.80	0.54
129. Canandaigua	42 50	77 15	813	3.00	3.43	2.31	2.42	4.91	3.74	3.30	3.47	2.83	3.13	2.82	2.56
130. Milville	43 08	78 20	600	2.47	1.85	1.68	1.95	2.13	2.84	2.88	2.24	4.61	3.33	3.09	2.45
131. Middlebury Acad.	42 49	78 10	800	1.46	1.77	2.26	2.46	2.92	3.40	3.30	2.81	2.83	2.88	2.56	1.79
132. Henrietta, Monroe County	43 06	77 51	600	1.66	1.82	1.06	3.22	1.84	3.48	1.53	3.01	3.42	3.99	1.38	0.54
133. Gaines	43 17	78 15	414	2.60	1.72	3.49	2.64	2.47	3.74	3.79	2.40	3.12	1.65	3.01	2.83

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[1] Also called Oneida, amount probably overmeasured.

[2] Near Newburgh, and possibly that station.

[3] Or Oneida Conference Seminary, consolidated series: Jan. 2.65; Feb. 2.31; March, 2.96; April, 3.15; May, 3.64; June, 4.41; July, 4.19; August, 3.68; Sept. 3.48; Oct. 3.79; Nov. 3.03; Dec. 2.95; Spring, 9.75; Summer, 12.28; Autumn, 10.30; Winter, 7.91; Year, 40.24; Extent, 24 yrs. 7 mos.; Date, 1830—1865.

[4] (Jefferson County) Dove, in Pogg. Ann., gives: Jan. 2.07; Feb. 1.81; March, 1.77; April, 1.84; May, 2.74; June, 2.66; July, 2.09; August, 2.63; Sept. 2.53; Oct. 4.40; Nov. 3.03; Dec. 2.45; Spring, 6.35; Summer, 7.38; Autumn, 9.96; Winter, 6.33; Year, 30.02; Extent, 9 years; Date, ?

NEW YORK.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs.	mos.	DATE. Beginning.	End.	OBSERVER.	REFERENCE.
86	10.51	14.32	11.55	8.36	44.74	1	11	Dec. 1856;	Nov. 1858	P. Reid	P. O. & S. I. Vol. 1.
87	12.46	10.55	13.27	10.66	46.94	8	3	Apr. 1856;	Dec. 1866	J. H. Hart, W. S. Malcolm	P. O. & S. I. Vol. 1, & Sm. Obs.
88	10.16	10.52	12.61	12.13	45.42	2	6	Aug. 1857;	June, 1862	H. W. Titus	“ “ “ “ “ “ “
89	15.33	21.14	19.40	11.06	66.93	8	7	Mar. 1856;	Dec. 1866	Dr. S. Spooner.	“ “ “ “ “ “ “
90	6.81	6.95	9.52	3.91	27.19	1	0	1856		Dr. P. O. Williams	P. O. & S. I. Vol. 1.
91	12.10	17.48	18.95	11.20	59.73	1	7	Nov. 1857;	May, 1859	Prof. S. Sias	“ “ “ “
92	10.01	16.28	. .	. .	. .	0	9	1857		J. M. Smith	“ “ “ “
93	12.17	. .	. .	. .	. .	0	6	1857		J. H. Norton	“ “ “ “
94	9.70	12.37	8.45	7.97	38.49	4	11	June, 1857;	May, 1863	J. C. House	P. O. & S. I. Vol. 1, & Sm. Obs.
95	. .	. .	. .	. .	. .	0	6	May, 1857;	Apr. 1858	H. M. Sheerar	P. O. & S. I. Vol. 1.
96	. .	. .	. .	. .	. .	0	6	Apr. 1858;	Sept. 1859	W. H. Riker	“ “ “ “
97	7.06	. .	9.59	8.39	. .	1	5	Aug. 1859;	Jan. 1863	J. J. Brown	P. O. & S. I. Vol. 1, & Sm. Obs.
98	8.93	9.93	7.31	12.07	38.24	4	0	Jan. 1835;	Dec. 1838	Profs. of Aca.	N. Y. Aca. Sys., Reg. Rep.
99	. .	. .	. .	. .	. .	0	3	June, 1859;	April and June, 1864	H. A. Schauber, R. M. Fuller	P. O. & S. I. Vol. 1, & Sm. Obs.
100	. .	. .	. .	3.59	. .	0	7	Nov. 1859;	Feb. 1861	E. S. Holmes	“ “ “ “ “ “ “
101	7.97	12.30	14.71	10.31	45.29	2	11	Jan. 1860;	Dec. 1866	E. B. Bartlett	Sm. Obs.
102	8.50	9.99	10.99	7.35	36.83	2	3	Mar. 1849;	Aug. 1852	C. F. Maurice	MS. in Sm. Coll'n.
103	7.61	10.04	8.55	7.98	34.18	11	0	1830;	1842	Profs. Acad.	N. Y. Ac. Sys., Reg. Rep.
104	6.20	10.15	8.38	3.90	28.63	20	0	1828;	1848	“ “	N. Y. Ac. Sys., Blodget's Clim.
105	10.59	12.93	9.05	11.46	44.03	4	0	1834;	1837	“ “	N. Y. Ac. Sys., Reg. Rep.
106	9.19	12.20	10.04	7.41	38.84	19	0	1830;	1849	“ “	“ “ “ “ “
107	7.46	9.08	8.56	6.35	31.45	9	0	1830–31;	1839–45	“ “	“ “ “ “ “ “
108	7.04	8.23	8.73	6.68	30.68	3	0	Apr. 1849;	Apr. 1852	F. B. Hough	MS. in Sm. Coll'n.
109	5.71	7.64	10.34	5.83	29.52	7	0	1830;	1842	Profs. Acad.	N. Y. Ac. Sys., Reg. Rep.
110	6.34	9.66	8.56	6.63	31.19	6	0	1844–6;	1850–52	“ “	“ “ “ “ “
111	9.75	11.47	8.74	6.33	36.29	13	0	1830;	1844	“ “	“ “ “ “ “
112	8.32	10.02	8.74	8.02	35.10	19	0	1830;	1849	“ “	“ “ “ “ “
113	8.60	11.13	7.66	6.76	34.15	11	0	1830;	1842	“ “	“ “ “ “ “
114	8.86	12.25	8.88	6.49	36.48	17	0	1830;	1846	“ “	“ “ “ “ “
115	9.72	12.16	9.83	8.65	40.36	15	0	1830;	1849	“ “	“ “ “ “ “
116	8.50	12.06	9.04	7.63	37.23	11	0	1830;	1843	“ “	“ “ “ “ “
117	7.38	9.59	8.59	5.96	31.52	15	0	1834;	1849	“ “	“ “ “ “ “
118	10.22	10.32	9.07	6.71	36.32	10	0	1830;	1849	“ “	“ “ “ “ “
119	6.06	9.54	8.40	. .	. .	3	0	1833;	1835	“ “	“ “ “ “ “
120	7.78	12.15	9.10	7.22	36.45	17	0	1828;	1849	“ “	“ “ “ “ “
121	9.87	12.04	10.63	8.30	40.84	13	0	1830;	1851	“ “	“ “ “ “ “
122	9.75	11.82	9.65	9.15	40.37	12	0	1830;	1845	“ “	“ “ “ “ “
123	5.37	7.09	6.81	3.83	23.10	14	0	1830;	1850	“ “	“ “ “ “ “
124	7.96	11.19	13.45	5.85	38.45	2	0	1834;	1835	“ “	“ “ “ “ “
125	7.24	10.45	12.04	6.82	36.55	16	0	1830;	1848	“ “	N. Y. Aca. Sys., Reg. Rep., Blodget's Clim.
126	. .	. .	. .	. .	. .	0	3	Dec. 1863;	Feb. 1864	D. J. Pratt	Sm. Obs.
127	9.39	8.91	8.72	7.71	34.73	2	0	1840;	1841	Talcott	N. Y. Ac. Sys., Reg. Rep.
128	4.93	9.43	8.68	2.52	25.56	2	0	1834;	1835	Profs. Acad.	“ “ “ “ “
129	9.64	10.51	8.78	8.99	37.92	7	0	1830;	1837	“ “	“ “ “ “ “
130	5.76	7.96	11.03	6.77	31.52	7	0	1841;	1847	“ “	“ “ “ “ “
131	7.64	9.51	8.27	5.02	30.44	17	0	1826;	1848	Dewey.	Blodget's Clim.
132	6.12	8.02	8.79	4.02	26.95	2	0	1835;	1836	Profs. Acad.	N. Y. Ac. Sys., Reg. Rep.
133	8.60	9.93	7.78	7.15	33.46	4	0	1839;	1842	“ “	“ “ “ “ “

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[5] Blodget, in his Climatology, gives: Jan. 3.26; Feb. 2.21; March, 2.97; April, 2.53; May, 3.70; June, 3.84; July, 4.09; August, 2.68; Sept. 2.24; Oct. 3.11; Nov. 3.46; Dec. 3.34; Spring, 9.20; Summer, 10.61; Autumn, 8.81; Winter, 8.81; Year, 37.43; Extent, 19 years; Date, 1829—1849.

[6] This amount is supposed too small, and due to the placing of the gauge on a roof fifty feet from the ground; Mr. Bell, the observer, suggests that the locality may be subject to a somewhat small amount of rain.

NEW YORK.—Continued.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
134. Onondaga[1]	42°59′	76°06′	1260	2.01	1.49	1.82	2.12	3.20	3.74	3.12	3.62	2.76	3.10	2.66	1.99
135. Ledyard or Cayuga	42 43	76 37	447	1.44	0.99	1.66	2.30	2.76	3.33	3.21	3.03	3.46	2.74	2.40	1.50
136. Auburn	42 55	76 28	650	2.50	2.04	2.13	2.22	3.45	3.57	3.13	3.23	3.20	3.38	2.85	2.72
137. Auburn	42 55	76 28	650	3.30	1.60	2.70	1.25	. .	. .	. .	. .	. .	. .	. .	. .
138. Oyster Bay	40 50	73 49	50	2.68	1.11	3.58	5.28	7.36	3.49	4.40	2.09	2.37	1.28	1.98	2.12
139. Plattsburg Acad.	44 40	73 26	180	3.05	3.28	4.63	2.81	2.35	3.74	2.53	2.81	3.31	4.53	2.43	2.63
140. Mexico	43 27	76 14	331	2.09	1.97	2.14	1.47	2.80	2.43	3.09	2.10	3.02	4.00	3.18	3.19
141. Prattsburg, Franklin Academy	42 34	77 20	1494	1.95	1.82	2.37	2.71	3.13	3.94	3.27	2.63	3.40	2.91	2.33	2.48
142. Baldwinsville	43 04	76 40	. .	0.80	0.90	2.30	. .	. .	. .	. .	. .	. .	. .	. .	1.60
143. Hermitage	42 45	78 16	1500	5.19	3.64	4.28	3.76	4.64	3.40	4.96	5.10	3.70	4.80	4.76	3.50
144. Waverly Village	42 22	78 59	1300	2.34	1.42	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
145. New York City, 5th Avenue	40 45	73 59	79	3.91	2.35	3.78	5.22	5.11	. .	. .	. .	. .	. .	. .	. .
146. New York City, 2d Avenue	40 44	73 59	41	5.08	2.98	4.00	3.70	3.42	6.37	2.95	1.59	2.15	4.54		3.06
147. Lima	42 53	77 51	. .	. .	1.00	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
148. Constantia	43 17	76 04	424	. .	. .	. .	. .	. .	. .	. .	2.67	. .	. .	. .	. .
149. Theresa	44 12	75 48	365	2.17	2.23	3.36	3.27	3.91	2.15	2.05	2.76	4.28	3.87	3.87	3.75
150. Suffern	41 30	74 30	. .	. .	. .	1.59	. .	. .	. .	. .	. .	. .	. .	. .	. .
151. South Hartford	43 18	73 21	400	1.78	2.76	3.66	3.24	5.04	2.82	2.99	5.77	3.68	2.57	4.74	3.70
152. South Trenton	43 10	74 56	835	3.57	5.16	4.61	4.00	3.81	4.75	4.51	4.64	5.26	4.65	3.62	4.20
153. Fort Ann	43 03	73 46	1430	5.51	1.55	2.97	4.60	3.71	. .	4.45	2.75	4.09	4.15	2.83	2.51
154. Throg's Neck	40 49	73 50	19	1.92	2.96	2.75	2.24	3.80	2.66	2.11	3.72	4.90	2.29	3.88	4.19
155. Brookhaven, Moriches	40 49	72 36	13	4.29	6.37	5.36	3.45	5.32	2.38	3.64	3.72	5.15	3.58	4.71	6.70
156. Argyle	43 18	73 29	290	. .	. .	. .	4.23	2.53	1.04	2.95	6.69	. .	. .	. .	. .
157. St. Francis Xavier's College, N. Y.	40 44	73 59	113	. .	. .	1.34	2.87	4.32	3.64	1.20	1.66	5.14	3.41	3.73	1.96
158. Columbia College, New York	40 43	74 05	100	1.75	4.81	2.75	2.68	3.78	2.65	1.67	3.84	2.97	2.43	2.46	4.00
159. Little Genesee	42 00	78 20	1500	. .	. .	. .	. .	. .	. .	1.87	0.48	3.50	. .	4.30	3.23
160. Germantown	42 . .	73 40	. .	. .	. .	. .	2.50	4.30	6.80	1.10	6.20	6.50	3.50	5.80	6.30
161. N. Hammond	44 30	75 40	. .	. .	. .	. .	. .	. .	8.74	3.21	9.63	10.44	4.14	6.78	6.11
162. Warsaw	42 40	78 10	. .	. .	. .	2.47	2.54	1.79	. .	. .	. .	. .	. .	. .	. .
163. Depauville, near	44 10	76 03	350	3.32	3.98	3.20	2.29	3.74	4.08	2.81	5.30	5.01	5.00	5.23	4.21
164. Pierrepont Manor	43 48	76 13	617	2.15	1.74	2.94	2.80	3.15	2.70	3.56	3.28	3.70	4.60	3.87	3.16

[1] Same station from longer series: Jan. 1.93; Feb. 1.41; March, 1.76; April, 2.01; May, 3.13; June, 3.52; July, 3.58; Aug. 3.79; Sept. 2.76; Oct. 3.28; Nov. 2.54; Dec. 1.88; Spring, 6.90; Summer, 10.89; Autumn, 8.58; Winter, 5.22; Year, 31.59; Reference, Dove, in Pog. Ann. Vol. 4, 1855.

PENNSYLVANIA.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Alleghany Arsenal at Pittsburg	40°29′	79°59′	704	2.06	2.16	2.58	3.14	3.67	3.84	3.05	3.34	2.65	2.79	2.73	3.22
2. Carlisle Barracks[2]	40 12	77 14	500	2.99	2.48	2.68	2.70	4.03	3.64	4.11	3.22	2.37	2.26	2.38	3.64
3. Fort Mifflin[3]	39 52	75 12	20	2.43	2.74	4.63	3.44	4.90	2.87	4.77	4.98	3.19	3.72	3.51	4.23
4. Gettysburg	39 49	77 18	624	3.14	2.60	3.00	3.55	3.85	3.46	3.40	3.40	2.98	3.03	3.05	3.43
5. Philadelphia[4]	39 57	75 11	60	3.16	3.28	3.42	4.48	4.96	4.74	3.58	4.52	4.29	2.99	3.56	3.76

[2] Observations between March, 1844, and April, 1846, are inaccurate, and were omitted in the summary of the Army Meteorological Register.

[3] Observations of 1845 were omitted in the summary, these measures being erroneous.

NEW YORK.—Continued.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
134	7.14	10.48	8.52	5.49	31.63	11	0	1832;	1843	Profs. Acad.	N. Y. Ac. Sys., Reg. Rep.
135	6.72	9.57	8.60	3.93	28.82	9	0	1830–34;	1842–46	“ “	“ “ “ “ “
136	7.80	9.93	9.43	7.26	34.42	22	0	1827;	1849	“ “	N. Y. Ac. Sys., Blodget's Clim.
137	. .	. .	. .	. .	. .	0	6	Jan. 1860;	Apr. 1861	J. B. Dill	Sm. Obs.
138	16.22	9.98	5.63	5.91	37.74	3	0	1835;	1837	Profs. Acad.	N. Y. Ac. Sys., Reg. Rep.
139	9.79	9.08	10.27	8.96	38.10	3	0	1847;	1849	“ “	N. Y. Ac. Sys., Blodget's Clim.
140	6.41	7.62	10.20	7.25	31.48	12	0	. . . . .		“ “	N. Y. Ac. Sys., Dove in Pogg. Ann.
141	8.21	9.84	8.64	6.25	32.94	9	0	. . . . .		“ “	“ “ “ “ “ “
142	. .	. .	. .	3.30	. .	0	4	Feb. 1860;	Jan. 1861	J. Bowman	Sm. Obs.
143	12.68	13.46	13.26	12.33	51.73	3	10	Oct. 1860;	Aug. 1864	A. A. Hibbard	“ “
144	. .	. .	. .	. .	. .	0	2	1861		W. Flint and J. Curtis	“ “
145	14.11	. .	. .	. .	. .	0	5	1861		F. I. Slade	“ “
146	11.12	10.91	. .	11.12	. .	1	4	Jan. 1861;	Dec. 1863	C. C. Wakely	“ “
147	. .	. .	. .	. .	. .	0	1	1861		Prof. S. A. Lattimore	“ “
148	. .	. .	. .	. .	. .	0	1	1861		S. Clark	“ “
149	10.54	6.96	12.02	8.15	37.67	5	2	Aug. 1861;	Dec. 1866	S. O. Gregory	“ “
150	. .	. .	. .	. .	. .	0	1	1863		J. H. Warren	“ “
151	11.94	11.58	10.99	8.24	42.75	3	5	Aug. 1863;	Dec. 1866	G. M. Ingalsbe	“ “
152	12.42	13.90	13.53	12.93	52.78	3	2	Aug. 1863;	Dec. 1866	S. Barrows	“ “
153	11.28	. .	11.07	9.57	. .	1	9	Nov. 1863;	May, 1866	P. A. McMore	“ “
154	8.79	8.49	11.07	9.07	37.42	1	2	Dec. 1863;	Feb. 1866	E. Morris, F. M. Rogers	“ “
155	14.13	9.74	13.44	17.36	54.67	2	10	Mar. 1864;	Dec. 1866	S. E. Smith	“ “
156	. .	10.68	. .	. .	. .	0	5	1864		G. M. Hunt	“ “
157	8.53	6.50	12.28	. .	. .	1	4	Apr. 1864;	Dec. 1866	Rev. J. M. Aubier, J. Hogan	“ “
158	9.21	8.16	7.86	10.56	35.79	1	9	Jan. 1865;	Dec. 1866	Prof. C. A. Joy	“ “
159	. .	. .	. .	. .	. .	0	5	1866		D. Edwards	“ “
160	. .	14.10	15.80	. .	. .	0	9	1866		S. W. Roe	“ “
161	. .	. .	21.58	21.36	. .	0	7	1866		C. A. Wooster	“ “
162	6.80	. .	. .	. .	. .	0	3	1865		J. P. Morse	“ “
163	9.23	12.19	15.24	11.51	48.17	1	4	Sept. 1865;	Dec. 1866	H. Haas	“ “
164	8.89	9.54	12.17	7.05	37.65	6	10	Jan. 1859;	Oct. 1865	W. C. Pierrepont	20th An. Rep. Reg. Univer. N. Y.

New York City and Harbor consolidated series: Jan. 3.35; Feb. 3.28; March, 3.40; April, 3.67; May, 4.62; June, 3.93; July, 3.77; August, 4.34; Sept, 3.37; Oct. 3.25; Nov. 3.60; Dec. 4.01; Spring, 11.69; Summer, 12.04; Autumn, 10.22; Winter, 10.64; Year, 44.59; Extent, 31 years; Date, 1836—1866. (Series at Fort Columbus, Fort Hamilton, the Deaf and Dumb Hospital, Blackwell's Island, the U. S. Naval Hospital.)

PENNSYLVANIA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	9.39	10.23	8.17	7.44	35.23	22	7	Aug. 1836;	Dec. 1859	Assist. Surgeon	Army Met. Reg. 1855, and M. D. of U. S. A. 1860.
2	9.41	10.97	7.01	9.11	36.50	10	6	Apr. 1848;	Nov. 1859	“ “	“ “ “ “ “
3	12.97	12.62	10.42	9.40	45.41	6	9	Jan. 1843;	Oct. 1853	“ “	Army Met. Reg. 1855
4	10.40	10.26	9.06	9.17	38.89	25	2	Jan. 1839;	Feb. 1865	Prof. M. Jacobs, Rev. D. Eyster, H. E. Jacobs	Sm. Col., Jour. Frankl. Inst., P. O. and S. I. Vol. 1, S. O.
5	12.86	12.84	10.84	10.20	46.74	16	6	July, 1851;	Dec. 1867	Prof. J. A. Kirkpatrick	Journ. Franklin Institute.

‘ Philadelphia, mean by series of 43 years observed at Pennsylvania Hospital, and by Prof. Kirkpatrick: Latitude, 39° 57′; Longitude, 75° 11′; Height, 55 feet; Jan. 3.25; Feb. 2.97; March, 3.45; April, 3.66; May, 3.99; June, 4.16; July, 3.96; August, 4.46; Sept. 3.64; Oct. 3.34; Nov. 3.48; Dec. 3.69; Spring, 11.10; Summer, 12.58; Autumn, 10.46; Winter, 9.91; Year, 44.05; Extent, 43 years; Date, 1825—1867.

PENNSYLVANIA.—CONTINUED.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
6. Spring Mill, near Philadelphia[1]	40°07′	75°17′	..	..	..	..	..	..	..	..	..	..	..	..	..
7. Philadelphia, Penn'a Hospital[2]	39 57	75 11	50	3.23	2.82	3.59	3.30	3.72	3.82	4.10	4.21	3.44	3.43	3.56	3.75
8. Morrisville, near Philadelphia[3]	40 13	74 52	30	3.29	2.77	3.33	3.87	3.94	4.09	3.62	4.05	3.48	3.65	4.28	3.29
9. Philadelphia, Girard College	39 58	75 10	133	2.83	1.45	2.51	3.16	2.40	2.55	4.85	5.09	2.12	2.78	2.73	3.08
10. Philadelphia, U. S. Navy Hospital[4]	39 56	75 10	36	2.36	2.26	2.99	4.44	4.99	4.26	2.93	4.29	3.99	3.66	3.44	3.62
11. Philadelphia, U. S. Navy Yard[4]	39 57	75 09	..	2.62	2.90	1.25	2.50	1.29	0.98	2.60	4.95	3.63	3.19	2.75	1.52
12. Newtown	40 15	74 57	..	3.14	2.89	3.67	5.20	4.05	4.18	5.10	6.87	2.96	3.33	3.72	5.32
13. Chromedale, near Media[5]	39 55	75 27	196	2.79	3.15	2.49	3.96	4.80	3.94	3.96	4.98	3.08	3.27	3.39	4.37
14. Germantown	40 03	75 10	100	2.18	3.58	3.07	2.62	2.87	3.22	4.25	3.48	3.27	3.50	3.01	3.05
15. Germantown	40 03	75 10	..	..	..	..	..	..	..	3.68	9.17	..	3.83	..	4.33
16. Harrisburg[6]	40 16	76 50	320	2.79	2.45	3.12	3.91	4.43	4.80	3.87	3.38	3.50	3.73	2.52	3.03
17. Lancaster Colliery[7] .	40 45	76 30	920	1.95	0.84	1.12	3.69	4.69	3.63	4.39	3.47	1.58	1.74	2.91	3.55
18. Shamokin	40 45	76 30	700	2.97	1.81	2.85	4.17	6.05	5.57	4.08	4.90	3.16	3.46	3.39	2.81
19. Canonsburg, Jefferson College[8]	40 17	80 10	850	2.46	2.10	2.70	3.15	3.47	4.34	3.46	3.09	4.08	2.51	2.58	2.77
20. Mifflintown	40 34	77 28	..	3.62	..	2.60	3.82	3.11	3.05	2.73	2.74	1.62	4.61	2.17	5.38
21. Norristown	40 08	75 19	153	3.82	3.17	2.61	5.10	4.50	4.41	3.67	4.22	4.34	2.96	3.71	3.97
22. West Chester	39 59	75 35	150	2.80	3.49	4.09	3.38	4.56	4.40	4.43	4.48	4.11	3.73	3.62	3.85
23. West Chester	39 57	75 35	478	..	..	..	5.00	2.31	3.28	4.81	..	..	3.29	3.61	3.94
24. Somerset	40 02	79 03	2195	2.45	2.34	2.00	4.26	4.76	4.10	3.00	3.42	2.98	2.93	3.50	3.00
25. Fleming (Centre) . .	40 55	77 53	780	2.74	2.48	3.67	4.27	3.65	4.16	3.45	3.87	4.16	3.92	3.03	3.01
26. Huntingdon	40 35	78 03	734	1.67	1.37	1.57	4.13	5.44	3.86	2.91	2.89	2.78	2.02	3.28	4.20
27. Easton	40 43	75 16	320	2.97	2.58	3.27	3.48	5.57	3.91	3.96	3.98	3.74	3.09	4.66	4.35
28. Chambersburg . . .	39 58	77 45	618	3.29	2.80	2.15	4.72	5.27	1.82	3.28	4.24	4.39	4.02	3.86	1.64
29. Indiana	40 40	79 10	1321	2.22	1.85	2.35	3.98	3.18	3.46	2.97	5.30	4.00	0.90	2.02	3.08
30. Erie	42 08	80 12	640	2.17	2.01	3.78	2.02	2.19	5.79	3.79	..	3.15	4.37	5.67	2.27
31. Bedford	40 01	78 30	..	2.46	1.61	1.65	3.43	3.47	2.90	2.33	2.17	2.78	1.88	3.76	1.73
32. Brookville	41 12	79 08	..	..	..	..	..	..	..	3.15	0.70	2.90	2.30	..	..
33. Bustleton	40 05	75 01	..	1.08	..	..	..	..	..	..	..	..	..	..	..
34. Byberry[9]	40 05	74 58	70	4.43	3.84	4.64	6.15	6.01	5.21	5.88	2.61	4.04	4.46	5.63	3.61
35. Ceres	42 00	78 25	1440	2.95	3.45	2.35	3.50	5.40	5.55	4.87	3.13	5.33	8.20	5.33	4.72
36. Moss Grove	41 40	79 51	1400	2.14	3.02	2.57	4.44	2.91	4.23	1.94	3.05	2.75	2.69	3.16	4.10
37. Pittsburg	40 27	79 59	..	3.94	1.12	3.01	3.49	3.01	5.46	3.45	2.38	1.89	2.51	2.01	2.85
38. Pittsburg	40 27	79 59	850	4.70	3.50	3.30	5.28	4.10	3.98	3.47	3.38	3.26	3.91	3.70	5.10
39. Pittsburg[10]	40 27	79 59	1026	2.12	2.18	2.10	3.58	3.82	3.74	3.38	3.99	2.46	2.14	2.85	2.12

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[1] Monthly means not accessible; melted snow is included in the annual amount. Position, 12 statute miles north of Philadelphia.

[2] The gauge in use from 1825 to 1838 (Dr. Conrad's observations) was made by Lukens; it was a hollow inverted cone of sheet zinc, with a graduated brass measuring rod; after 1838 a gauge by Francis was used.

[3] From 1790 to 1825 there are no observations in the months from January to August inclusive. Observations from 1850 (or 1852) to end of series by E. Hance, at Fallsington, near Morrisville, latitude 40° 12′, longitude 74° 53′.

PENNSYLVANIA.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.		Observer.	Reference.
						yrs.	mos.	Beginning.	End.		
6	..	..	..	..	35.63	21	0	1810;	1830	P. Legaroux	Am. Alm. 1839; Darby's Gaz.
7	10.61	12.13	10.43	9.80	42.97	30	3	Jan. 1824;	Dec. 1865	Dr. Conrad, Dr. Swift and H. Eachers	Sm. Coll'n and S. O.
8	11.14	11.76	11.41	9.35	43.66	45	1	Oct. 1790;	Dec. 1866	Ch. Pierce & E. Hance	MS. in S. C. and P. O. and S. I. Vol. 1, and S. O.
9	8.07	12.49	7.63	7.36	35.55	5	1	June, 1840;	June, 1845	Dr. A. D. Bache	Exec. Doc. Senate, 1847.
10	12.42	11.48	11.09	8.24	43.23	7	1	Feb. 1857;	June, 1865	J. M. Greene, J. M. Foltz, W. Johnson, D. Harlan, S. Maulsby	MS. in Sm. Coll'n and S. O.
11	5.04	8.53	9.57	7.04	30.18	1	8	Apr. 1843;	Dec. 1845	. . . . .	MS. in Sm. Coll'n.
12	12.92	16.15	10.01	11.35	50.43	4	2	Mar. 1839;	June, 1843	L. H. Parsons	Am. Alm. 1843; Jour. Frank. Institute.
13	11.25	12.88	9.74	10.31	44.18	9	7	Jan. 1849;	Jan. 1859	J. Edwards, J. H. Smedley	MS. in Sm. Coll'n, P. O. and S. I. Vol. 1.
14	8.56	10.95	9.78	8.81	38.10	9	0	June, 1819;	July, 1828	Haines	Darby's U. S. Gazetteer.
15	..	..	..	..	..	0	4	1843		C. J. Wister	MS. in Sm. Coll'n.
16	11.46	12.05	9.75	8.27	41.53	14	8	Aug. 1840;	Dec. 1866	Dr. J. Heisely, W. O. Hickok	Journ. Frank. Inst., P. O. and S. I. Vol. 1, Sm. Obs.
17	9.50	11.49	6.23	6.34	33.56	2	0	1856 and 1858		P. Friel	P. O. and S. I. Vol. 1, and MS. in Sm. Coll'n.
18	13.07	14.55	10.01	7.59	45.22	6	1	Jan. 1857;	Jan. 1863	" "	Journ. Frank. Inst., P. O. and S. I. Vol. 1, and Sm. Obs.
19	9.32	10.89	9.17	7.33	36.71	10	3	Jan. 1839;	Dec. 1866	Campbell, J. R. Williams, Prof. W. Smith	Journ. Frank. Inst., P. O. and S. I. Vol. 1, MS. and S. O.
20	9.53	8.52	8.40	..	..	1	5	July, 1839;	May, 1843	J. A. A. Kinkead	Journ. Frank. Inst., MS.
21	12.21	12.30	11.01	10.96	46.48	8	11	Feb. 1839;	July, 1863	L. E. Corson, Rev. J. Grier Ralston	Journ. Frank. Inst., MS., P.O. and S. I. Vol. 1. and S. O.
22	12.03	13.31	11.46	10.14	46.94	10	0	1817;	1827	Darlington	Jour. Fr. In. (Blodget's Clim.)
23	..	..	..	..	..	1	2	Apr. 1841;	Dec. 1845	Jeffries, E. W. Beans	Journ. Frank. Inst., MS.
24	11.02	10.52	9.41	7.79	38.74	8	0	Feb. 1840;	Dec. 1861	G. Mowry, (Dr. F. Chorpenning in 1856)	Journ. Frank. Inst. MS., P. O. and S. I. Vol. 1, and S. O.
25	11.59	11.48	11.11	8.23	42.41	6	0	Jan. 1859;	Dec. 1866	J. F. M., S. Brugger	Journ. Frank. Inst., P. O. and S. I. Vol. 1, Sm. Obs.
26	11.14	9.66	8.08	7.24	36.12	3	5	April, 1840;	Feb. 1860	J. Miller, Dr. W. Brewster	Journ. Frank. Inst.
27	12.32	11.85	11.49	9.90	45.56	5	1	April, 1846;	Dec. 1859.	Green, S. J. Coffin, G. S. Houghton	MS. P. O. and S. I. Vol. 1, Journ. Frank. Inst.
28	12.14	9.34	12.27	7.73	41.48	1	11	Feb. 1860;	Apr. 1862	W. Heyser.	Journ. Fr. Inst. and Sm. Obs.
29	9.51	11.73	6.92	7.15	35.31	1	11	Aug. 1839;	Jan. 1860	Whyte, W. D. Hildebrand	Journ. Frank. Inst.
30	7.99	..	13.19	..	..	1	1	Dec. 1839;	Dec. 1860	Park & Reid, B. Grant	" " "
31	8.55	7.40	8.42	5.80	30.17	7	5	Jan. 1853;	Dec. 1861	S. Brown, Rev. H. Heckerman	MS. in Sm. Coll'n and P. O. and S. I. Vol. 1, Sm. Obs.
32	..	..	..	..	..	0	4	1854		D. S. Deering	P. O. and S. I. Vol. 1.
33	..	..	..	..	..	0	1	1854		I. C. Martindale	" " " "
34	16.80	13.70	14.13	11.88	56.51	4	5	Jan. 1854;	Dec. 1863	J. Comly, J. W. Saurman, I. C. Martindale	P. O. and S. I. Vol. 1, Sm. Obs.
35	11.25	13.55	18.86	11.12	54.78	3	9	Mar. 1849;	Mar. 1854	Dr. R. P. Stevens	MS. in S.C., P.O. & S.I. Vol. 1.
36	9.92	9.22	8.60	9.26	37.00	5	4	Jan. 1852;	Feb. 1857	F. Schreiner	" " " " "
37	9.51	11.29	6.41	7.91	35.12	1	4	Sept. 1840;	Dec. 1841	Bakewell	Journ. Frank. Inst.
38	12.68	10.83	10.87	13.30	47.68	2	6	Part of 1849 and 1850		Dr. H. Smyser	MS.
39	9.50	11.11	7.45	6.42	34.48	6	5	Apr. 1853;	Dec. 1859	W. W. Wilson, Dr. H. Smyser, W. Martin, and others	MS. and P. O. S. I. Vol. 1.

[4] At these stations it is doubtful whether melted snow is included or not.

[5] For the Delaware County Institute of Science; also known as Lima.

[6] The means of the two series were taken for the monthly values in 1857–8–9–61–62–63–64.

[7] Near Shamokin.

[8] The mean values are taken for the two series in 1861.

[9] Results doubtful.

[10] Oakland Station and Marine Hospital; the latter station in 1858 and 1859. The height given is that of Oakland Station; the elevation of the hospital is not known.

PENNSYLVANIA.—CONTINUED.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
40. Pittsburg[1]	40°28′	80°07′	937	2.46	2.60	2.30	1.90	0.40	1.00	1.50	3.13	5.35	4.47	4.36	3.38
41. Pocopson	39 54	75 37	218	3.40	3.56	3.12	4.24	4.92	4.44	3.18	3.92	4.20	3.16	3.57	4.04
42. Randolph	41 28	80 10	1720	3.14	2.45	2.85	3.05	3.35	6.57	3.50	..	4.02	4.50	4.55	..
43. Sugar Grove	42 00	79 20	1450	2.90	2.86	2.23	4.15	4.33	..	..	4.40	3.20	3.19	4.64	1.95
44. Warrior's Mark	40 39	78 14	..	..	..	1.44	5.84	3.62	5.24	3.94	..	..	..	..	..
45. Haverford	40 00	75 20	..	2.89	2.49	2.54	..	2.90	5.57	2.98	5.63	..	..	4.51	4.41
46. West Haverford and Haverford College	40 00	75 21	400	4.05	3.36	3.57	5.58	3.85	4.37	3.45	3.35	4.97	3.82	4.09	3.98
47. Youngsville	41 50	79 20	1185	..	2.50	..	..	..	..	..	..	..	..	..	..
48. Pottsville	40 41	76 09	..	5.43	2.64	..	3.10	5.48	2.78	7.00	4.82	6.06	0.86	..	..
49. Pottsville	40 41	76 24	..	3.33	3.13	..	2.57	2.38	9.46	8.43	2.16	4.76	3.99	2.20	5.79
50. Carlisle	40 12	77 12	498	2.99	2.35	1.45	4.70	3.34	4.36	..	..	..	1.07	2.42	5.44
51. Carlisle, Dickinson College	40 12	77 11	500	2.51	1.41	3.35	4.20	4.56	6.00	2.75	3.12	4.77	2.70	3.40	3.57
52. Lewisburg, Univers.	40 58	76 58	..	2.58	2.05	2.79	3.35	4.57	4.49	3.45	3.47	3.62	1.95	2.93	3.62
53. Meadville	41 39	80 11	1088	1.93	2.34	3.00	3.17	3.82	6.05	3.80	3.82	2.23	4.38	2.20	3.74
54. Nazareth	40 43	75 21	530	5.85	1.58	3.30	3.52	5.07	2.08	5.28	6.35	5.32	2.33	3.98	0.66
55. Mount Joy	40 08	76 32	..	3.56	1.84	3.19	3.65	4.44	3.78	2.93	3.32	3.71	2.41	2.85	3.22
56. Murraysville	40 28	79 35	960	1.22	2.95	2.53	3.72	6.65	4.65	3.02	3.60	2.14	2.51	3.63	4.36
57. Westtown	39 57	75 34	550	3.45	2.12	6.50	4.13	9.05	5.20	2.22	5.30	1.41	3.38	3.50	4.75
58. Linden	41 10	77 11	..	..	2.91	4.42	4.43	..	..	..	..	..	..	2.89	..
59. Reading	40 19	75 56	262	..	0.96	0.88	3.08	4.57	3.14	3.81	3.16	2.36	2.47	2.14	3.84
60. Altoona	40 37	78 22	1620	..	..	..	..	..	..	..	..	..	3.59	3.63	9.72
61. Altoona	40 35	78 22	1208	..	..	1.60	13.90	..	..	..	..	..	..	..	..
62. Berwick	41 05	76 15	583	2.31	1.64	3.04	4.08	3.51	1.50	3.49	4.55	4.66	2.84	2.34	2.36
63. Worthington	40 52	79 39	..	..	..	..	..	..	..	3.75	4.00	5.88	3.87	1.75	..
64. Worthington, near	40 50	79 31	1050	5.50	3.06	2.96	4.19	3.64	4.04	4.04	2.96	5.19	3.34	6.13	5.16
65. Lancaster	40 02	76 20	350	3.43	2.40	2.61	4.22	3.11	3.75	2.83	3.85	3.08	2.75	2.68	3.45
66. Northumberland	40 55	76 49	500	2.64	1.97	3.80	4.10	3.47	4.08	3.94	4.13	2.00	2.09	2.58	4.34
67. Bellefonte	40 55	77 49	..	..	1.91	2.87	2.26	5.92	4.93	4.68	4.15	3.36	2.06	2.68	4.23
68. Rose Cottage	41 07	79 09	..	3.00	2.80	3.08	1.07	..	..	8.93	5.90	5.25	0.90	..	3.24
69. Danville	40 58	76 39	..	..	1.75	..	..	..	..	..	..	..	1.34	..	..
70. Smithport	41 54	78 33	..	3.06	1.99	4.20	2.10	2.73	4.71	4.01	2.99	2.89	2.00	2.34	4.80
71. Silver Lake	41 55	76 01	..	1.20	1.44	1.87	2.81	2.24	3.03	2.27	2.71	2.81	1.80	1.16	2.61
72. Stroudsburg	40 58	75 16	..	3.59	..	..	5.81	3.41	4.08	4.23	4.25	4.04	1.00	4.00	4.51
73. York	39 58	76 40	..	..	..	..	..	..	3.16	2.36	3.24	..	1.85	2.38	..
74. Butler	40 54	79 50	850	3.43	1.95	3.12	2.93	3.29	2.83	2.12	4.53	2.61	2.69	3.14	3.42
75. Wilkesbarre	41 14	75 56	..	..	2.19	..	..	3.13	..	..	..	..	..	..	1.99
76. Uniontown	39 54	79 42	..	5.83	1.31	3.63	3.56	..	4.26	..	..	..	3.10	4.70	5.36
77. Port Carbon	40 43	76 06	..	..	1.04	3.38	4.30	3.07	4.69	3.66	5.54	2.12	3.56	2.79	3.61
78. Ebensburg	40 31	78 45	..	..	1.21	..	3.46	5.12	3.57	3.04	2.60	2.99	3.26	3.37	2.15
79. Franklin	41 24	79 55	..	2.02	1.46	4.13	1.99	3.42	3.82	2.16	3.52	1.42	2.40	2.44	2.81
80. Beaver	40 43	80 20	..	..	2.47	..	2.88	2.52	5.24	1.55	2.86	..	3.47	2.41	..
81. Warren	41 57	79 14	..	..	..	..	2.03	1.37	4.81	3.85	..	..	..	5.08	..
82. West Greenfield	41 05	79 54	..	..	..	..	..	2.98	..	..	..	..	..	..	..
83. Milford	41 17	74 50	..	..	..	..	..	..	..	3.66	..	..	..	..	..
84. Punxsutawny	40 59	79 00	..	..	..	..	..	..	..	..	..	..	1.44	..	..
85. Whitehall, near[2]	40 40	75 26	250	2.20	1.65	1.35	..	..	..	..	..	..	..	..	1.80
86. Philadelphia, Frankford Arsenal	39 59	75 11	30	3.07	2.51	2.61	3.60	3.30	4.49	3.78	5.15	3.25	2.59	3.23	3.29
87. Darby	39 55	75 17	120	2.49	2.84	3.92	3.89	3.97	2.60	4.00	5.84	2.62	3.15	2.80	4.58
88. McKean	41 55	78 26	1500	3.07	1.57	5.07	2.74	1.90	3.96	2.33	3.75	2.92	3.58	2.33	2.30
89. Manchester	40 32	80 03	750	2.90	3.00	1.00	1.50	4.19	3.26	3.13	1.27	4.88	3.95	2.44	5.18
90. Hollidaysburg	40 28	78 23	1200	2.97	4.46	0.99	4.05	3.59	1.05	1.19	5.04	1.78	3.02	3.28	1.93
91. Union Canal[3]	40 20	76 30	480	..	..	..	..	..	..	..	..	..	..	..	..
92. Sewickleyville	40 38	80 14	656	4.90	4.63	1.51	4.61	2.85	4.03	5.94	3.46	5.27	2.92	4.31	0.95
93. Latrobe[3]	40 14	79 29	922	5.96	3.78	3.56	5.41	3.43	4.23	3.48	5.74	8.96	2.88	6.26	1.52

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[1] Also known as Troy Hill.

[2] West of Whitehall Station.

[3] St. Vincent's College.

PENNSYLVANIA.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.		Observer.	Reference.
						yrs.	mos.	Beginning.	End.		
40	4.60	5.63	14.18	8.44	32.85	1	8	Jan. 1856;	Dec. 1866	V. Scriba, Dr. R. Muller, G. Albree	P. O. and S. I. Vol. 1, & S. O.
41	12.28	11.54	10.93	11.00	45.75	12	9	Jan. 1854;	Dec. 1866	F. Darlington	“ “ “ “ “ “
42	9.25	..	13.07	..	..	1	2	June, 1854;	Feb. 1856	O. T. Hobbs	P. O. and S. I. Vol. 1.
43	10.71	..	11.03	7.71	..	0	10	Aug. 1853;	May, 1854	W. O. Blodget	MS. and P. O. and S. I. Vol. 1.
44	10.90	..	..	..	..	0	5	1854		J. R. Lowrie	P. O. and S. I. Vol. 1.
45	..	14.18	..	9.79	..	1	10	Jan. 1839;	Dec. 1841	. . . . .	Journ. Frank. Inst.
46	13.00	11.17	12.88	11.39	48.44	6	4	Jan. 1854;	June, 1863	Dr. P. Swift	P. O. and S. I. Vol. 1, and S.O.
47	..	..	..	..	..	0	1	1854		Dr. A. C. Blodget	P. O. and S. I. Vol. 1.
48	..	14.60	..	..	..	0	9	1839		Porter	Journ. Frank. Inst.
49	..	20.05	10.95	12.25	..	0	11	1855		Dr. A. Heger	P. O. and S. I. Vol. 1.
50	9.49	..	..	10.78	..	1	8	Jan. 1839;	Dec. 1841	Allen	Journ. Frank. Inst.
51	12.11	11.87	10.87	7.49	42.34	2	5	Jan. 1856;	Sept. 1859	Prof. W. C. Wilson	P. O. and S. I. Vol. 1.
52	10.71	11.41	8.50	8.25	38.87	5	10	Feb. 1856;	Dec. 1866	Prof. C. S. James	P. O. and S. I. Vol. 1, & S. O.
53	9.99	13.67	8.81	8.01	40.48	3	11	Jan. 1839;	Sept. 1858	Dick & Limber, T. H. Thickstun	Jr. Fr. In., P. O. & S. I. Vol. 1.
54	11.89	13.71	11.63	8.09	45.32	2	4	Feb. 1856;	May, 1862	H. A. Brickenstein, J. C. Harvey, O. T. Huebner	P. O. and S. Vol. 1, and S. O.
55	11.28	10.03	8.97	8.62	38.90	7	10	Aug. 1857;	Dec. 1866	Dr. J. R. Hoffer	“ “ “ “ “ “
56	12.90	11.27	8.28	8.53	40.98	2	0	Apr. 1857;	Mar. 1859	F. L. Stewart	P. O. and S. I. Vol. 1.
57	19.68	12.72	8.29	10.32	51.01	1	8	July, 1857;	Mar. 1859	S. Alsop	“ “ “ “
58	..	..	..	..	..	0	4	Nov. 1858;	Apr. 1859	J. Barrett	“ “ “ “
59	8.53	10.11	6.97	..	..	1	7	Apr. 1839;	Apr. 1858	Egelman, Dr. J. B. Peale, C. Hahn	Jour. Fr. In. P.O. & S.I. Vol. 1.
60	..	..	..	..	..	0	3	1859		W. R. Boyers	P. O. and S. I. Vol. 1.
61	..	..	..	..	..	0	2	1863		T. H. Savery	Sm. Obs.
62	10.63	9.54	9.84	6.31	56.32	2	6	June, 1859;	Jan. 1865	J. Eggert	P. O. and S. I. Vol. 1.
63	..	..	11.50	..	..	0	5	1859		S. Scott	“ “ “ “
64	10.79	11.04	14.66	13.72	50.21	2	6	Feb. 1860;	July, 1862	“ “	Sm. Obs.
65	9.94	10.43	8.51	9.28	38.16	5	0	Jan. 1839;	Dec. 1843	“ “	Journ. Frank. Inst.
66	11.37	12.15	6.67	8.95	39.14	2	11	Jan. 1839;	Dec. 1841	Hewston	“ “ “
67	11.05	13.76	8.10	..	..	2	1	Feb. 1839;	Dec. 1841	Harris	“ “ “
68	..	..	..	9.04	..	0	9	Jan. 1839;	Mar. 1840	Gaskell	“ “ “
69	..	..	..	..	..	0	2	1839		Frick	“ “ “
70	9.03	11.71	7.23	9.85	37.82	2	4	Feb. 1839;	Aug. 1841	Atkins & Chatwick	“ “ “
71	6.92	8.01	5.77	5.25	25.95	2	6	Mar. 1839;	Dec. 1841	Rase	“ “ “
72	..	12.56	9.04	..	..	1	4	Apr. 1839;	Dec. 1841	Stokes	“ “ “
73	..	8.76	..	..	..	0	5	1839		Mason	“ “ “
74	9.34	9.48	8.44	8.80	36.06	5	1	Nov. 1839,	Dec. 1841, and 1849–50–51	Mechling	Jour. Fr. In. & MS. in S. Coll'n.
75	..	..	..	..	..	0	3	Dec. 1839;	May, 1841	Dennis & Maxwell	Journ. Frank. Inst.
76	..	..	..	12.50	..	0	10	Dec. 1839;	June, 1841	Weethee	“ “ “
77	10.75	13.89	8.47	..	..	1	3	Aug. 1840;	Dec. 1841	P. C. of Lyceum	“ “ “
78	..	9.21	9.62	..	..	1	5	Feb. 1840;	Dec. 1841	Lewis	“ “ “
79	9.54	9.50	6.26	6.29	31.59	1	7	Feb. 1840;	Dec. 1841	Connelly	“ “ “
80	..	9.65	..	..	..	0	11	Apr. 1840;	Aug. 1841	. . . . .	“ “ “
81	..	..	..	..	..	0	5	May, 1840;	Apr. 1841	Brown & King	“ “ “
82	..	..	..	..	..	0	1	1840		Campbell	“ “ “
83	..	..	..	..	..	0	1	1840		. . . . .	“ “ “
84	..	..	..	..	..	0	1	1841		. . . . .	“ “ “
85	..	..	..	5.65	..	0	4	Feb. 1860;	Mar. 1861	E. Kohler	“ “ “
86	9.51	13.42	9.07	8.87	40.87	8	0	1836;	1843	Mordecai, Capt. U.S.A.	“ “ “
87	11.78	12.44	8.57	9.91	42.70	4	0	1849;	1852	John Jackson	MS. in Sm. Coll'n.
88	9.71	10.04	8.83	6.94	35.52	1	6	1840 and part of 1841		. . . . .	Journ. Frank. Inst.
89	6.69	7.66	11.27	11.08	36.70	1	0	in 1849 and 1850		Corydon Marks	MS. in Sm. Coll'n.
90	8.63	7.28	8.08	9.36	33.35	1	0	1853		J. R. Lowrie	“ “ “
91	..	..	..	..	38.40	14	0	1835;	1848	Lehman	Roberts, Engineer.
92	8.97	13.43	12.50	10.48	45.38	1	1	Jan. 1861;	Jan. 1862	J. I. Travelli, G. H. Tracy	Sm. Obs.
93	12.40	13.45	18.10	11.26	55.21	1	6	Jan. 1861;	Jan. 1862	Prof. R. Mueller	“ “

[3] Monthly means not accessible.

PENNSYLVANIA.—Continued.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
94. Towanda	41°47′	76°34′	840	1.15	1.00	. .	. .	. .	. .	. .	10.00	. .	. .	. .	. .
95. Latrobe	40 20	79 15	569	. .	. .	. .	. .	3.43	. .	. .	. .	. .	. .	. .	. .
96. Carpenters	41 37	76 53	. .	. .	. .	. .	. .	. .	. .	. .	3.36	1.52	. .	. .	. .
97. Susquehanna Depot	42 00	75 38	800	. .	. .	. .	. .	. .	1.38	6.41	. .	. .	. .	. .	. .
98. Tioga	41 53	77 15	1000	1.54	1.59	4.37	3.13	5.66	3.57	4.87	4.20	5.37	2.40	3.05	1.67
99. Mooreland T'nship	40 00	75 11	250	2.17	5.10	3.07	2.42	5.15	4.18	4.12	3.72	5.98	3.70	3.64	2.84
100. Grampian Hills[1]	41 . .	78 40	1400	2.90	3.41	5.47	3.33	3.06	3.70	4.84	4.91	5.09	3.24	3.42	4.25
101. Blooming Grove	41 20	75 00	. .	1.26	7.85	1.65	2.10	6.12	8.60	3.25	3.95	2.45	3.45	5.30	3.15
102. Oxford	39 50	75 50	575	. .	. .	. .	. .	7.40	5.66	5.57	1.90	2.25	. .	. .	. .
103. Ephrata	40 08	76 14	. .	3.87	4.57	1.42	2.46	2.92	7.31	1.70	3.73	5.87	1.95	3.45	2.19
104. Stevensville	41 45	76 35	300	. .	. .	. .	. .	1.58	4.34	3.00	3.02	4.33	2.20	3.58	2.61

DELAWARE.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Georgetown	38 43	75 22	. .	3.00	3.03	3.71	3.93	4.28	5.02	2.66	5.40	3.34	3.22	3.65	4.90
2. Milford	38 55	75 27	25	1.20	. .	. .	. .	4.56	1.85	. .	. .	. .	. .	. .	. .
3. Fort Delaware	39 35	75 34	10	2.60	0.67	1.53	3.08	2.66	4.53	2.15	3.46	2.01	1.39	1.29	3.84
4. Wilmington	39 44	75 33	115	5.05	4.20	7.07	4.95	7.80	2.55	4.85	3.40	7.88	2.90	4.80	8.15

MARYLAND and DISTRICT OF COLUMBIA.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort McHenry	39 16	76 35	36	2.80	2.65	3.68	3.45	3.66	3.32	3.33	4.18	3.21	3.33	3.43	4.06
2. Fort Severn	38 59	76 29	20	3.96	3.02	3.13	1.97	4.55	4.03	4.14	3.83	8.42	3.46	4.29	3.81
3. Washington City[2]	38 54	77 01	70	2.04	2.50	2.46	2.88	2.12	2.26	3.35	2.55	1.83	1.31	1.66	2.13
4. Fort Washington	38 43	77 02	60	2.37	3.81	3.17	5.30	4.10	2.16	3.90	6.78	2.86	2.08	5.28	3.21
5. Washington City, Capitol Hill[3]	38 53	77 00	90	4.46	2.74	2.56	4.04	3.86	2.93	3.92	3.69	3.52	3.55	3.09	2.86
6. Washington City, U. S. N. Observatory[4]	38 54	77 03	110	2.69	2.43	2.93	3.47	3.53	3.48	3.82	3.86	2.88	3.54	2.65	2.68
7. Washington City, Smithson. Inst.	38 53	77 01	40	3.06	2.57	2.07	3.82	4.27	4.54	3.61	4.62	3.38	2.49	2.23	4.39
8. District of Columbia[5]	38 57	76 55	275	1.58	2.01	0.31	3.36	7.09	1.08	4.35	7.05	1.62	1.70	0.89	5.54
9. Baltimore	39 18	76 36	. .	2.75	3.35	4.25	2.54	3.71	3.38	3.99	3.43	4.20	3.15	3.18	2.91
10. Baltimore	39 17	76 37	. .	2.76	2.94	3.66	3.35	4.58	3.20	5.19	4.85	3.03	3.43	3.62	3.87
11. Georgetown, D. C.	38 55	77 04	. .	2.39	2.18	3.68	4.52	2.98	3.62	5.36	4.48	2.00	3.08	3.01	2.08
12. Elkton	39 38	75 58	35	. .	. .	. .	. .	. .	. .	3.23	. .	. .	. .	. .	3.49
13. Sykesville, Schellman Hall	39 23	76 57	700	3.53	3.03	3.94	4.42	5.93	3.98	4.54	3.78	4.22	3.88	3.63	3.65
14. Frederick	39 24	77 18	400	3.52	2.74	2.53	3.94	3.88	4.74	2.96	2.48	4.07	2.14	2.38	2.96
15. Frederick	39 24	77 18	550	2.19	2.44	3.67	6.15	2.43	3.55	6.53	6.24	2.86	1.85	5.51	4.65
16. Chestertown, Washington College	39 13	76 03	80	2.99	1.90	3.20	4.69	4.42	3.82	2.98	7.01	3.37	2.86	3.56	2.07
17. Bladensburg	38 57	76 58	105	2.98	2.32	3.86	3.96	3.89	3.50	4.30	3.21	3.07	2.54	2.64	3.00
18. Annapolis	38 58	76 29	20	3.66	3.27	3.92	4.90	4.77	4.72	4.94	4.25	4.49	3.82	3.47	4.13
19. Leitersburg	39 30	77 30	. .	3.81	2.36	2.68	4.55	3.57	3.94	2.43	3.05	4.47	2.54	2.82	3.04
20. St. Mary's City	38 10	76 30	45	4.02	3.34	2.67	4.90	3.77	2.89	3.88	5.35	2.71	2.77	3.46	2.82
21. Agricultural College	38 59	76 56	. .	4.50	4.63	2.42	4.62	4.00	5.47	5.20	5.97	3.53	2.70	6.60	1.83
22. Woodlawn	39 39	74 04	. .	3.22	5.70	3.86	3.36	4.86	7.16	4.93	2.55	5.02	4.69	2.79	4.46

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[1] Near Pennsville.

[2] This series is reported in the first publication from the Medical Department. Observations to 1827, by Rev. R. Little; for 1828–9 results from Elliot's Historical Sketches (Washington, 1830), observer not known. Some measures appear defective.

[3] Observations by Lieut. Gilliss on Capitol Hill, north of Capitol.

[4] Series at the U. S. Naval Observatory; observations from 1846 to 1861, by Lieut. M. F. Maury, U. S. N. Measures of 1849, and perhaps those of 1848, appear defective; those of 1849 were excluded. Observations from 1861 to 1867, by Com. J. M. Gilliss, U. S. N., and Com. B. F. Sands, U. S. N., directors.

[5] N. E. corner of District of Columbia, across eastern branch of Potomac, about seven statute miles from the Capitol.

PENNSYLVANIA.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs. mos.	DATE. Beginning. End.	OBSERVER.	REFERENCE.
94	..	..	..	..	..	0 3	1861	I. H. Kingsbury, S. J. Coffin.	Sm. Obs.
95	..	..	..	..	..	0 1	1861	W. R. Boyers	" "
96	..	..	..	..	..	0 2	1862	E. L. McNett	" "
97	..	..	..	..	..	0 2	1863	H. H. Atwater	" "
98	13.16	12.64	10.82	4.80	41.42	2 11	Jan. 1864; Dec. 1866	E. T. Bentley	" "
99	10.64	12.02	13.32	10.11	46.09	2 6	June, 1864; Dec. 1866	A. Spencer	" "
100	11.86	13.45	11.75	10.56	47.62	2 6	July, 1864; Dec. 1866	E. Fenton	" "
101	9.87	15.80	11.20	12.26	49.13	1 7	May, 1865; Dec. 1866	J. Grathwohl	" "
102	..	13.13	..	..	..	0 5	1865	Dr. H. Duffield	" "
103	6.80	12.74	11.27	10.63	41.44	1 0	1866	W. H. Spera	" "
104	..	10.36	10.11	..	..	0 8	1866	J. R. Dutton	" "

DELAWARE.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs. mos.	DATE. Beginning. End.	OBSERVER.	REFERENCE.
1	11.92	13.08	10.21	10.93	46.14	2 6	July, 1857; Dec. 1859	Dr. D. W. Mauld	P. O. and S. I., Vol. I.
2	..	..	..	..	..	0 3	Jan., May, June, 1858	R. A. Martin	" " " "
3	7.27	10.14	4.69	7.11	29.21	4 5	Jan. 1855; May, 1859	Assist. Surgeon	M. D. of U. S. A., 1860.
4	19.82	10.80	15.58	17.40	63.60	1 10	Jan. 1864; Oct. 1865	Dr. U. D. Hedges	Sm. Obs.

MARYLAND and DISTRICT OF COLUMBIA.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs. mos.	DATE. Beginning. End.	OBSERVER.	REFERENCE.
1	10.79	10.83	9.97	9.51	41.10	23 2	May, 1836; June, 1859	Assist. Surgeon	Army Met. Reg. 1855, and M. D. of U. S. A. 1860.
2	9.65	12.00	16.17	10.79	48.61	2 5	Mar. 1843; July, 1845	" "	Army Met. Reg. 1855.
3	7.46	8.16	4.80	6.67	27.09	5 9	Jan. 1824; Dec. 1829	Rev. R. Little	Sm. Col. & J. Elliot's Hist. Sket.
4	12.57	12.84	10.22	9.39	45.02	2 5	May, 1851; Sept. 1853	Assist. Surgeon	Army Met. Reg. 1855.
5	10.46	10.54	10.16	10.06	41.22	3 11	July, 1838; June, 1842	Lieut. Gilliss, U. S. N.	Sen. Doc. 28th Cong. 1844–5.
6	9.93	11.16	9.07	7.80	37.96	19 0	Jan. 1846; Dec. 1867	Lieut. M. F. Maury, Com. J. M. Gilliss, & Com. B. F. Sands, U. S. N.	Am. Alm. 1848 and foll., also MS. in Sm. Col. and records of observers.
7	10.16	12.77	8.10	10.02	41.05	5 1	Jan. 1854; Dec. 1859	Observer of Institution	P. O. and S. I. Vol. I, 1861.
8	10.76	12.48	4.21	9.13	36.58	1 0	Sept. 1857; Aug. 1858	J. Wiessner	MS. in Sm. Coll'n.
9	10.50	10.80	10.53	9.01	40.84	8 3	1817; June, 1837	Dr. L. Brantz	Pr. Jour. in Sm. Coll'n.
10	11.59	13.24	10.08	9.57	44.48	9 7	Jan. 1846; Aug. 1859	Dr. Edmundson and Prof. A. M. Mayer	Sm. Coll'n, P. O. & S. I. Vol. I.
11	11.18	13.46	8.09	6.65	39.38	2 1	Feb. 1860; Feb. 1863	Rev. C. B. Mackee	Sm. Obs.
12	..	..	..	..	..	0 2	Dec. 1843; July, 1844	F. Finch	Sm. Coll'n.
13	14.29	12.30	11.73	10.21	48.53	14 1	1849; Oct. 1855	H. M. Baer	Sm. Coll'n, P. O. and S. I. Vol. I, and Sm. Obs.
14	10.35	10.18	8.59	9.22	38.34	9 3	Jan. 1854; Sept. 1866	H. E. Hanshew and H. M. Baer	P.O. and S. I. Vol. I, and S.O.
15	12.25	16.32	10.22	9.28	48.07	1 0	1852	Lewis F. Steiner	MS. Sm. Coll'n.
16	12.31	13.81	9.79	6.96	42.87	5 0	June, 1855; July, 1864	Prof. J. R. Dutton	P. O. and S. I. Vol. I, and S.O.
17	11.71	11.01	8.25	8.30	39.27	7 5	June, 1856; Aug. 1865	B. O. Lowndes	" " " " "
18	13.59	13.91	11.78	11.06	50.34	11 11	Jan. 1857; Dec. 1866	W. R. Goodman	" " " " "
19	10.80	9.42	9.83	9.21	39.26	3 9	Sept. 1858; June, 1862	J. E. Bell	" " " " "
20	11.34	12.12	8.94	10.18	42.58	5 0	Jan. 1861; May, 1866	Rev. J. Stephenson	Sm. Obs.
21	11.04	16.64	12.83	10.96	51.47	1 2	Feb. 1861; July, 1862	Dr. M. Johns	" "
22	12.08	14.64	12.50	13.38	52.60	2 0	Jan. 1865; Dec. 1866	J. O. McCormick	" "

VIRGINIA.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Latitude.	Longitude.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fortress Monroe[1] . .	37°00′	76°18′	8	3.37	2.72	3.30	2.98	3.89	4.32	5.34	5.66	4.65	2.92	3.31	4.58
2. Berryville.	39 13	77 56	. .	1.81	1.77	1.54	4.53	5.45	4.75	5.20	2.61	1.48	2.20	2.21	3.93
3. Powhattan Hill . . .	37 33	77 40	200	2.11	2.12	2.10	2.89	3.30	3.21	3.69	4.93	2.32	2.28	2.38	2.48
4. Richmond[2]	37 32	77 27	172	0.77	1.77	4.84	5.80	1.86	2.82	3.58	7.44	2.37	0.87	2.89	3.28
5. Portsmouth	36 50	76 19	34	4.24	4.28	3.11	4.72	4.97	4.72	4.89	5.78	4.96	2.76	3.01	4.76
6. Portsmouth U. S. Naval Hospital[3] . . .	36 51	76 18	18	3.47	5.01	1.86	3.68	2.44	2.54	1.74	1.93	3.94	3.77	6.00	4.35
7. Alexandria	38 48	77 01	56	2.86	2.50	1.38	3.23	3.90	3.36	3.40	4.14	2.69	2.67	1.97	2.80
8. Alleghany County[4] . .	37 45	80 02	2000	1.82	2.82	3.05	2.86	3.74	2.30	4.46	3.22	1.86	2.23	3.19	3.82
9. Crichton's store[5]. . .	36 40	77 46	500	3.63	2.72	2.58	2.73	3.01	4.11	3.24	4.39	2.17	1.91	2.13	3.76
10. Lynchburg	36 30	79 07	575	. .	5.60	5.20	3.70	2.70	5.80	. .	1.60	2.60	. .	. .	. .
11. Anthony's Creek[4]. .	37 50	79 55	. .	2.31	3.03	3.37	2.53	4.44	2.26	4.05	3.36	1.62	2.50	3.06	4.43
12. Montcalm	38 05	78 21	450	. .	. .	. .	. .	. .	. .	4.37	. .	1.50	1.62	2.88	1.50
13. Smithfield	37 02	76 37	100	4.30	2.70	3.67	3.16	4.07	4.20	5.32	4.68	4.23	3.84	3.02	3.96
14. Ruthven	37 21	77 33	. .	2.45	2.34	3.06	4.07	5.13	2.13	4.33	4.76	4.93	2.40	2.71	3.04
15. Winchester	39 15	78 10	. .	2.47	2.05	2.08	4.74	5.22	3.78	4.29	4.37	4.21	2.76	2.94	3.81
16. Rose Hill.	38 00	76 57	250	2.13	4.32	1.68	3.62	4.05	4.55	5.03	3.70	2.90	1.88	1.68	2.92
17. Williamsburg	37 15	76 40	100	3.19	2.05	3.95	3.68	2.87	3.75	4.50	9.15	4.76	3.63	2.62	2.88
18. Rougemont	38 05	78 21	450	4.35	2.69	2.96	4.79	5.28	5.37	4.14	4.28	5.49	3.03	2.84	5.26
19. Hartwood[6]	38 15	77 34	350	3.97	3.19	3.04	2.15	5.72	6.91	2.01	4.04	5.23	2.37	4.54	4.19
20. Lewinsville	38 56	77 11	180	3.46	2 45	4.07	4.75	2.53	4.20	1.85	1.44	4.50	2.33	2.07	3.52
21. Meadow Dale . . .	38 23	79 35	. .	1.89	2.40	3.01	5.90	2.80	5.17	3.78	2.20	1.40	4.72	4.50	4.20
22. Mossy Creek	38 20	79 05	. .	. .	. .	. .	6.00	. .	. .	. .	. .	. .	. .	. .	. .
23. Montross	38 07	76 54	200	. .	. .	. .	5.23	3.78	6.00	3.34	5.76	4.06	0.50	. .	. .
24. Westwood	37 33	77 27	. .	4.05	4.10	2.46	3.40	2.89	1.44	6.25	5.37	2.06	2.12	3.50	5.46
25. Mount View	38 00	78 30	521	4.75	3.55	4.50	6.15	4.07	. .	1.80	6.00	8.00	4.00	2.10	4.30
26. Charlottesville . . .	38 01	78 26	. .	1.84	4.00	3.63	2.38	3.06	2.38	3.10	7.00	1.14	3.54	3.13	3.31
27. King George C. H. .	38 16	77 10	50	0.43	3.80	3.73	3.73	1.36	3.15	2.87	2.05	1.25	1.27	2.25	1.03
28. Genito Mills[7]	36 30	77 30	200	4.08	4.79	6.03	4.65	4.37	3.45	3.14	5.03	1.57	3.03	2.55	5.16
29. Fruit Farm	39 00	77 50	. .	. .	0.80	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
30. Wytheville	36 55	81 00	2287	6.33	5.29	2.33	3.60	2.37	3.14	. .	2.56	. .	9.37	4.26	4.00
31. Lexington, Virginia Mil. Inst.	37 41	79 25	1000	3.89	3.70	2.02	. .	. .	. .	. .	. .	. .	. .	. .	. .

WEST VIRGINIA.

Name of Station.	Latitude.	Longitude.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. White Sulph. Springs	37 45	80 20	2000	2.40	3.22	3.73	2.93	3.41	3.16	4.68	2.79	1.56	2.16	3.58	3.92
2. Ashland near Buffalo	38 34	82 10	600	1.88	2.60	3.01	4.73	3.53	3.89	4.90	3.08	3.23	1.77	3.24	3.54
3. Lewisburg	37 49	80 28	2000	2.88	2.76	2.56	2.66	2.17	2.52	3.37	3.32	3.58	2.02	4.00	3.91
4. Sheetz Mills	39 25	78 40	. .	2.20	1.69	2.11	3.56	4.27	2.41	3.46	3.57	2.51	2.55	1.87	2.46
5. New Creek Depot .	39 25	79 00	. .	. .	3.30	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
6. Kanawha	38 53	81 25	. .	1.98	3.46	1.87	4.05	3.30	4.95	1.47	2.60	1.35	2.83	2.46	4.52
7. Poplar Grove[8] . . .	38 20	81 26	720	4.75	2.93	3.64	5.69	3.59	5.67	4.49	6.02	2.87	4.51	4.65	7.03
8. Peach Grove Lodge	39 15	81 00	1100	3.48	2.25	1.70	2.74	5.52	2.36	2.22	3.02	1.71	. .	. .	. .
9. Trout Run Valley[9]. .	39 30	78 30	1720	2.47	2.26	2.16	4.83	3.50	4.91	3.63	2.83	1.41	2.18	1.18	3.61
10. Salem	39 20	80 01	1100	1.50	1.62	1.52	. .	. .	. .	3.72	4.30	1.60	3.60	3.56	7.86
11. Wirt Court	39 05	81 26	. .	2.84	1.14	1.42	3.92	6.43	3.79	4.87	3.14	0.76	4.00	3.11	7.64
12. Cross Creek[10]	40 19	80 31	. .	2.11	2.36	2.51	5.55	2.74	3.64	3.50	5.61	2.47	2.49	2.64	3.34
13. Point Pleasant . . .	38 50	82 09	480	2.87	. .	1.91	8.17	3.70	5.89	. .	. .	. .	. .	. .	. .
14. Wheeling.	40 07	80 42	. .	2.36	1.33	1.01	7.19	. .	. .	. .	. .	. .	. .	1.71	2.13

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[1] Observations of 1851–2–3–4 omitted.

[2] Observations of February, 1860, at an elevation of 83 feet.

[3] Observations in March, April, May, June, 1865, by A. A. Henderson, in Norfolk.

[4] Registers ordered by the Board of Public Works of Virginia. The result was sent in MS. by Dr. W. N. Patton, of Lewisburg, for the first two stations; that at Anthony's Creek is from the report of the James River and Kanawha Canal Co. for 1852. Five gauges were placed by this authority, but those here given embrace all the results yet published.

VIRGINIA.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.		Observer.	Reference.
						yrs.	mos.	Beginning.	End.		
1	10.17	15.32	10.88	10.67	47.04	19	4	July, 1836;	Dec. 1859	Assist. Surgeon.	Army Met. Reg. 1855, and M. D. of U. S. A. 1860.
2	11.52	12.56	5.89	7.51	37.48	2	6	July, 1856;	Dec. 1858	E. & Dr. R. Kownslar	MS. in Sm. Coll'n.
3	8.29	11.83	6.98	6.71	33.81	14	4	Jan. 1850;	Dec. 1867	E. T. Tayloe	Am. Alm. and MS. in S.Coll'n
4	12.50	13.84	6.13	5.82	38.29	2	9	Aug. 1850;	Feb. 1860	D. Turner & J. Appleyard	MS. in Sm. Coll'n and S. O.
5	12.80	15.39	10.73	13.28	52.20	5	0	May, 1852;	Dec. 1859	Prof. N. B. Webster	MS. in Sm. Coll'n and P. O. and S. I. Vol. 1.
6	7.98	6.21	13.71	12.83	40.73	2	1	Jan. 1861;	Dec. 1866	Surg. S. Barrington, A. A. Henderson, P. S. Wales	Sm. Coll'n.
7	8.51	10.90	7.33	8.16	34.90	5	6	Jan. 1853;	June, 1858	B. Hallowell and F. Miller	Blodget's Clim. and P. O. and S. I. Vol. 1.
8	9.65	9.98	7.28	8.46	35.37	4	6	Sept. 1847;	Feb. 1852	Dickson .	Sm. Coll'n from pub. reports.
9	8.32	11.74	6.21	10.11	36.38	7	0	Jan. 1854;	Mar. 1861	R. F. Astrop	P. O. and S. I. Vol. 1, and S.O.
10	11.60	..	..	..	..	0	7	1854, Feb. Sept.		A. Nettleton	P. O. and S. I. Vol. 1.
11	10.34	9.67	7.18	9.77	36.96	4	0	Sept. 1847;	Aug. 1851	. . . . . .	Pub. Rep. of Virginia.
12	..	..	6.00	..	..	0	5	1854, July, Dec.		C. J. Meriwether	P. O. and S. I. Vol. 1.
13	10.90	14.20	11.09	10.96	47.15	6	8	July, 1854;	Mar. 1861	Dr. J. R. Purdie	" " " "
14	12.26	11.22	10.04	7.83	41.35	2	4	Aug. 1856;	May, 1859	J. C. Ruffin	" " " "
15	12.04	12.44	9.91	8.33	42.72	4	11	Mar. 1856;	Apr. 1861	J. W. Marvin	P. O. and S. I. Vol. 1, and S.O.
16	9.35	13.28	6.46	9.37	38.46	2	5	Apr. 1857;	Dec. 1859	G. W. Upshaw	P. O. and S. I. Vol. 1.
17	10.50	17.40	11.01	8.12	47.03	5	0	1772—1777		Madison	Jefferson's Notes on Va.
18	13.03	13.79	11.36	12.30	50.48	3	9	May, 1857;	Apr. 1861	G. C. Dickinson	P. O. and S. I. Vol. 1, and S. O.
19	10.91	12.96	12.14	11.35	47.36	2	8	Mar. 1858;	Mar. 1861	A. Van Doren	" " " " " "
20	11.35	7.49	8.90	9.43	37.17	1	5	June, 1858;	Oct. 1859	Rev. C. B. McKee	P. O. and S. I. Vol. 1.
21	11.71	11.15	10.62	8.49	41.97	1	2	Jan. 1858;	Feb. 1859	J. Slaven	" " " "
22	..	..	..	..	..	0	1	April, 1858		J. Hotchkiss	" " " "
23	..	15.10	..	..	..	0	7	April—Oct. 1859		E. E. Spence	" " " "
24	8.75	13.06	7.68	13.61	43.10	2	1	May, 1859;	Feb. 1862	C. J. Meriwether	P. O. and S. I. Vol. 1, and S. O.
25	14.72	..	14.10	12.60	..	1	2	Jan. 1860;	Apr. 1861	J. R. Abell	Sm. Obs.
26	9.07	12.48	7.81	9.15	38.51	3	0	1849—1852		C. J. Meriwether	MS. in Sm. Coll'n.
27	8.82	8.07	4.77	5.26	26.92	1	0	1851		Not known	Am. Alm.
28	15.05	11.62	7.15	14.03	47.85	4	4	May, 1849;	Aug. 1853	R. F. Astrop	MS. in Sm. Coll'n.
29	..	..	..	..	..	0	1	Feb. 1860		J. Pickett	Sm. Obs.
30	8.30	..	..	15.62	..	0	10	May, 1860;	Apr. 1861	W. D. Roedel	" "
31	..	..	..	..	..	0	3	Jan. Feb. Mar. 1861		W. K. Park	" "

WEST VIRGINIA.

	Spring.	Summer.	Autumn.	Winter.	Year.	yrs.	mos.	Beginning.	End.	Observer.	Reference.
1	10.07	10.63	7.30	9.54	37.54	5	6	Sept. 1847;	Feb. 1853	Watts and Estill	MS. by Dr. Patten in Sm. C.
2	11.27	11.87	8.24	8.02	39.40	5	11	Jan. 1852;	Dec. 1866	Prof. G. R. Rossiter and S. Couch, C. L. Roffe	MS. in Sm. Coll'n, P. O. and S. I. Vol. 1, and S. O.
3	7.39	9.21	9.60	9.55	35.75	6	0	Sept. 1852;	Mar. 1861	Dr. T. Patton, J. W. Stalnaker	Report of Com. of Agr., H. Clark, MS.
4	9.94	9.44	6.93	6.35	32.66	9	11	Jan. 1856;	Dec. 1865	. . . . .	Report of Com. of Agr., H. Clark.
5	..	..	..	..	..	0	1	Feb. 1854		M. McDonald	P. O. and S. I. Vol. 1.
6	9.22	9.02	6.64	9.96	34.84	3	0	Jan. 1856;	July, 1859	D. L. Ruffner, W. C. Reynolds	" " " "
7	12.92	16.18	12.03	14.71	55.84	3	3	Apr. 1857;	Feb. 1861	J. E. Kendall	P. O. and S. I. Vol. 1, and S.O.
8	9.96	7.60	..	..	..	0	9	Jan. Sept. 1856		W. C. Quincy	P. O. and S. I. Vol. 1.
9	10.49	11.37	4.77	8.34	34.97	1	11	Jan. 1856;	May, 1861	D. H. Ellis	P. O. and S. I. Vol. 1, and S.O.
10	..	..	8.76	10.98	..	0	9	July, 1857;	Mar. 1858	J. C. Wells	P. O. and S. I. Vol. 1.
11	11.77	11.80	7.87	11.62	43.06	1	10	Jan. 1857;	Dec. 1858	Dr. J. W. Hoff	" " " "
12	10.80	12.75	7.60	7.81	38.96	1	7	Sept. 1858;	June, 1860	B. D. Sanders	P. O. and S. I. Vol. 1, and S. O.
13	13.78	..	..	..	..	0	5	Jan. June, 1859		W. R. Boyers	P. O. and S. I. Vol. 1.
14	..	..	..	5.82	..	0	6	Nov. 1859;	Apr. 1860	G. P. Lockwood	P. O. and S. I. Vol. 1, and S. O.

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[5] See also Genito or Genito Mills.

[6] Also known as "Falmouth," and "Peach Lawn."

[7] See Crichton's store; the two localities are close together; the results are kept separate owing to the great difference in the annual mean amounts.

[8] Also known as Kanawha Salines.

[9] Also known as Crack Whip.

[10] Also Holliday's Cove; the cove about three miles north of the creek, observed September and October, 1858.

NORTH CAROLINA.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Johnston . . .	33°55′	78°01′	20	2.64	1.75	3.49	0.59	2.75	2.44	5.18	7.90	9.86	2.42	4.04	2.95
2. Thornbury[1]	36 20	77 21	. .	2.53	2.23	3.43	2.84	2.04	1.71	2.76	. .	. .	. .	2.41	2.15
3. Gaston[2]	36 32	77 45	. .	3.00	2.69	3.18	4.19	4.75	2.65	4.45	4.78	3.42	2.54	3.10	4.65
4. Chapel Hill, University of N. C. . . .	35 54	79 17	. .	3.66	3.24	3.54	3.39	3.57	2.34	3.99	3.96	5.21	2.32	3.15	4.34
5. Murfreesborough . .	36 30	77 06	. .	3.43	3.17	2.57	4.27	2.98	3.17	3.33	2.17	3.48	1.43	2.56	3.20
6. Fort Macon.	34 41	76 40	20	0.71	1.96	0.89	2.53	2.07	0.71	7.13	0.49	. .	. .	. .	. .
7. Asheville	35 37	82 29	2250	1.80	2.40	1.10	2.44	2.30	4.60	6.00	3.80	1.25	. .	. .	3.83
8. Warrenton	36 30	78 15	. .	. .	. .	. .	2.10	. .	2.65	5.26	2.03	3.72	2.56	1.51	. .
9. Davidson College. .	35 30	80 54	850	4.40	4.28	2.57	3.41	3.04	3.04	2.66	2.68	9.80	3.38	2.18	4.14
10. Marlborough	35 28	75 36	. . .	3.83	5.79	2.86	5.21	2.78	3.95	5.31	3.96	. .	. .	. .	. .
11. Goldsboro'	35 20	77 51	102	4.80	4.70	2.86	6.00	7.42	1.59	1.54	11.46	2.03	6.61	3.93	3.38
12. Wilson	35 41	77 47	105	. .	2.37	1.11	6.35	1.85	4.90	6.64	3.26	4.94	2.95	2.45	2.25
13. Trinity College . . .	34 45	78 30	400	3.69	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
14. Attaway Hill	35 25	80 00	850	. .	. .	. .	5.56	4.12	3.67	7.33	2.16	5.63	2.61	. .	. .
15. Raleigh.	35 47	78 38	. .	. .	. .	. .	. .	. .	. .	. .	1.81	5.50	5.06	2.40	3.56
16. Near Statesville. . .	35 30	80 30	. .	. .	. .	. .	. .	. .	4.81	2.90	2.00	7.25	4.00	5.50	4.15

SOUTH CAROLINA.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Moultrie. . . .	32 46	79 51	25	2.59	2.51	4.18	1.75	3.82	4.34	6.40	7.63	4.90	2.14	2.11	3.14
2. Charleston[3]	32 47	79 56	25	2.23	2.93	2.84	1.81	3.35	4.74	6.07	7.16	5.59	3.12	2.16	3.11
3. Charleston[4]	32 47	79 56	25	2.51	2.32	3.77	1.66	3.55	3.64	5.78	6.87	4.69	2.62	1.78	2.73
4. Camden[5]	34 17	80 33	275	3.73	2.77	5.00	3.03	4.34	4.38	6.74	7.05	3.68	2.37	3.05	4.42
5. St. John's.	33 18	79 56	50	3.10	2.56	3.02	2.12	3.70	3.96	6.95	6.47	4.38	2.23	2.31	2.67
6. All Saints[6]	33 29	79 17	20	3.83	3.15	3.58	2.55	2.00	4.05	7.31	4.15	4.93	2.51	2.76	3.00
7. Abbeville (near) . .	34 13	82 28	. .	4.34	2.53	2.28	2.21	2.42	1.81	1.19	3.12	1.52	2.62	1.43	4.11
8. Robertsville	32 40	80 26	50	0.60	1.18	7.38	0.50	2.99	5.43	3.63	5.80	4.63	1.57	0.40	3.03
9. Fulton	33 40	80 31	150	3.96	4.17	3.50	4.34	6.17	4.61	7.37	6.51	5.33	2.61	1.91	3.83
10. Aiken	33 32	81 34	563	4.39	3.70	3.41	2.29	4.00	3.49	5.70	6.82	3.26	0.86	2.49	4.13
11. Columbia	33 59	81 02	315	4.19	4.47	7.21	3.20	3.51	2.61	3.58	7.19	2.89	1.40	2.29	4.63
12. Edisto Island	32 34	80 18	23	1.24	1.52	5.22	0.32	1.86	2.82	. .	6.13	0.42	2.34	1.15	1.66
13. Mount Pleasant. . .	32 47	79 55	20	. .	0.92	2.28	. .	. .	. .	. .	. .	. .	. .	. .	. .
14. Beaufort	32 26	80 41	15	3.00	1.68	4.85	1.49	3.56	6.10	4.62	. .	. .	. .	3.24	1.58
15. Hilton Head	32 14	80 43	15	3.63	2.13	6.83	1.00	1.66	5.57	4.90	3.66	3.66	2.70	2.76	1.53
16. Kirkwood (near Camden)	34 17	80 33	250	3.54	2.52	6.04	4.91	2.60	3.18	4.67	11.45	3.28	1.31	5.09	3.76
17. Barratsville	34 10	82 20	500	6.05	4.80	7.67	6.20	4.50	3.06	7.30	4.75	1.10	3.75	2.35	5.15

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[1] Also known as Jackson.

[2] Also known as Green Plains.

[3] Observations for 1738 to 1748 from "Description of South Carolina," London, 1761; observations for 1749 to 1752 from Philosophical Transactions, 1753, and observations 1753 to 1759 from Chalmer's South Carolina. All reductions were made by the Smithsonian Institution. The observations between 1738 and 1748 are by Dr. Lining.

[4] For the years 1856–57–58 the mean values from Dr. J. Johnson's and Dr. J. Dawson's records were used in the above series. [Add to the reference: Printed slips by City Register and Sm. Obs.]

NORTH CAROLINA.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning. End.	Observer.	Reference.
1	6.83	15.52	16.32	7.34	46.01	1	7	Jan. 1844; July, 1845	Assist. Surgeon	Arm. Met. Reg. 1855.
2	8.31	..	..	6.88	..	1	1	Jan. 1854; Apr. 1855	Rev. F. Fitzgerald and Prof. D. Morelle	P. O. and S. I. Vol. 1.
3	12.12	11.88	9.06	10.34	43.40	4	8	Aug. 1856; Mar. 1861	Dr. G. F. Moore	P. O. & S. I. Vol. 1, & Sm. Obs.
4	10.50	10.29	10.68	11.24	42.71	3	11	Apr. 1856; Apr. 1861	Dr. J. Phillips	" " " " " " "
5	9.82	8.67	7.47	9.80	35.76	3	7	Oct. 1856; Apr. 1861	Rev. A. McDowell	" " " " " " "
6	5.49	8.33	..	..	..	0	8	Jan.—Aug. 1849	Assist. Surgeon U.S.A.	MS. in Sm. Coll'n.
7	5.84	14.40	..	8.03	..	0	10	July, 1857; June, 1858	W. W. McDowell	P. O. and S. I. Vol 1.
8	..	9.94	7.79	..	..	0	7	April—Nov. 1857	Dr. W. M. Johnston	" " " "
9	9.02	8.38	15.36	12.82	45.58	1	9	Jan. 1858; Dec. 1859	Prof. W. C. Kerr	" " " "
10	10.85	13.22	..	..	..	0	8	Jan.—Aug. 1858	R. H. Drysdale	" " " "
11	16.28	14.59	12.57	12.88	56.32	1	1	Mar. 1860; Mar. 1861	Prof. E. W. Adams	Sm. Obs.
12	9.31	14.80	10.34	..	..	0	11	Feb.—Dec. 1866	E. W. Adams	" "
13	..	..	..	..	..	0	1	Jan. 1861	B. Craven	" "
14	..	13.16	..	..	..	0	7	April—Oct. 1861	F. J. Kron	" "
15	..	..	12.96	..	..	0	5	Aug.—Dec. 1866	F. P. Brewer	" "
16	..	9.71	16.75	..	..	0	7	June—Dec. 1866	T. A. Allison	" "

SOUTH CAROLINA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning. End.	Observer.	Reference.
1	9.75	18.37	9.15	8.24	45.51	17	1	Nov. 1842; Dec. 1859	Assist. Surgeon	Arm. Met. Reg. 1855, M. D. of U. S. A. 1860.
2	8.00	17.97	10.87	8.27	45.11	22	0	Jan. 1738; Dec. 1759	Dr. Lining and others	Phil. Trans. 1753, Chalmer's S.C. & Descrip. of S.C. 1761
3	8.98	16.29	9.09	7.56	41.92	19	7	Jan. 1841; Oct. 1861	J. Ryan, Dr. J. L. Dawson, Dr. J. Johnson, Dr. G. S. Pelzer	Am. Alm. 1841–53, Blodget's Clim., P. O. and S. I. Vol. 1.
4	12.37	18.17	9.10	10.92	50.56	8	2	May, 1849; Dec. 1857	T. Carpenter, and Dr. J. A. Young	Sm. Col. & P. O. & S. I. Vol. 1.
5	8.84	17.38	8.92	8.33	43.47	13	2	Jan. 1846; Mar. 1861	H. W. & T. P. Ravenel	Sm. Col. (Agr. Soc.) P. O. and S. I. Vol. 1. and S. O.
6	8.13	15.51	10.20	9.98	43.82	8	4	Jan. 1851; Apr. 1861	Rev. A. Glennie	S.C., P.O. & S. I. Vol. 1, & S.O.
7	6.91	6.12	5.57	10.98	29.58	1	0	July, 1838; June, 1839	T. Parker	Am. Alm. 1840.
8	10.87	14.86	6.60	4.81	37.14	1	0	1843	Smith	Agric'l Register.
9	14.01	18.49	9.85	11.96	54.31	7	0	Jan. 1818; Dec. 1824	J. Dyson	MS. in Sm. Coll'n.
10	9.70	16.01	6.61	12.22	44.54	6	1	May, 1854; Feb. 1861	H. W. Ravenel, and J. H. Cornish	P. O. & S. I. Vol. 1, & Sm. Obs.
11	13.92	13.38	6.58	13.29	47.17	2	8	Jan. 1850; Sept. 1859	F. H. Harleston, C. C. Tew, J. B. White, and E. H. Barton	MS. S. C., P. O. & S. I. Vol. 1.
12	7.40	..	3.91	4.42	..	0	11	Feb. 1856; Jan. 1857	E. A. Fuller, and Dr. E. N. Fuller	P. O. and S. I. Vol. 1.
13	..	..	..	..	..	0	2	Feb. Mar. 1857	Dr. E. N. Fuller	" " " "
14	9.90	..	..	6.26	..	1	0	Jan. 1864; Mar. 1865	Dr. M. M. Marsh	Sm. Obs.
15	9.49	14.13	9.12	7.29	40.03	1	3	Apr. 1864; June, 1865	J. W. Abert, Lt. C. R. Suter	" " "
16	13.55	19.30	9.68	9.82	52.35	3	0	1850–1–2	McRae	Journal (Sm. Coll'n).
17	18.37	15.11	7.20	16.00	56.68	1	10	June, 1849; Mar. 1851	Dr. J. P. Barratt	MS. in Sm. Coll'n.

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[4] Charleston combined series: Jan. 2.36; Feb. 2.64; March, 3.28; April, 1.74; May, 3.45; June, 4.22; July, 5.93; Aug. 7.02; Sept. 5.18; Oct. 2.89; Nov. 1.99; Dec. 2.93; Spring, 8.47; Summer, 17.17; Autumn, 10.06; Winter, 7.93; Year, 43.63; Extent, 41 yrs. 7 mos.; Date, 1738—1759 and 1841—1861.

[5] See also Kirkwood.

[6] Also known as Waccamaw.

GEORGIA.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Augusta Arsenal[1]	33°28′	81°53′	600?	2.74	2.43	4.69	1.47	0.82	1.30	0.79	1.57	1.43	1.89	1.19	3.88
2. Oglethorpe Barracks	32 04	81 06	40	3.57	2.18	7.11	2.91	3.43	4.65	8.79	8.06	4.07	1.95	1.19	3.42
3. Savannah[2]	32 05	81 05	42	2.90	2.52	3.71	2.01	4.93	4.57	7.71	8.34	4.62	2.24	1.77	3.00
4. Whitemarsh Island	32 03	81 02	18	2.86	2.22	2.85	2.38	3.93	3.72	5.06	6.08	4.82	1.51	2.21	2.67
5. Sparta	33 17	83 09	550	6.28	5.08	4.01	3.00	4.41	3.08	4.93	6.60	4.02	2.07	4.36	6.27
6. Culloden	32 51	84 13	825	5.32	7.30	7.62	1.64	1.90	1.65	9.81	1.13	9.39	3.15	7.79	7.54
7. Perry	32 31	83 46	325	1.40	2.86	2.52	2.36	2.88	3.69	4.78	8.20	1.25	1.51	9.21	3.46
8. Milledgeville	33 07	83 20	577	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	1.14	7.09
9. Augusta	33 28	81 54	470	2.70	. .	5.49	2.71	2.32	3.52	4.31	8.33	5.60	1.29	2.13	5.15
10. La Grange	33 02	84 55	. .	0.21	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
11. Thomaston[3]	32 56	84 30	750	3.62	3.48	6.50	2.14	1.27	2.41	5.33	5.72	1.71	0.84	6.02	4.17
12. Athens[4]	33 57	83 25	870	3.13	2.60	3.20	1.81	3.25	4.02	5.58	3.12	1.51	3.47	1.57	3.28
13. Athens[4]	33 58	83 30	850	5.49	4.69	4.43	5.41	3.80	3.68	4.60	5.83	4.49	. .	. .	6.96
14. Factory Mills	33 40	84 46	. .	4.45	0.49	0.92	2.01	5.60	2.86	. .	. .	. .	. .	. .	. .
15. Hillsborough	33 13	83 45	566	8.26	4.17	4.07	3.18	2.22	4.77	. .	. .	0.12	0.55	5.35	4.22
16. The Rock	32 52	84 23	833	2.45	0.41	2.10	3.02	5.37	0.90	5.10	4.64	0.51	0.67	5.87	5.79
17. Thomson	33 26	82 28	. .	2.06	2.64	4.71	2.51	2.88	. .	. .	. .	. .	. .	. .	4.56
18. Atlanta	33 45	84 18	1050	3.91	4.84	4.52	4.84	5.87	4.19	3.44	5.41	7.24	1.62	4.29	8.26
19. Boston	31 00	84 00	. .	. .	5.30	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
20. Cuthbert	. .	. .	. .	. .	. .	. .	. .	. .	1.66	2.16	3.37	. .	. .	. .	. .
21. Dalton	34 50	85 00	775	11.12	13.50	5.07	. .	. .	. .	. .	. .	. .	. .	. .	. .

FLORIDA.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Marion, St. Augustine	29 54	81 18	25	2.09	1.63	2.34	1.56	2.00	4.27	3.24	3.03	5.85	2.42	1.29	2.08
2. Fort Shannon[5]	29 34	81 39	25	0.93	2.64	6.16	2.47	2.86	6.54	7.35	7.60	4.33	3.78	1.60	1.42
3. Fort Pierce (Capron)	27 30	80 20	30	4.33	4.01	3.16	2.77	3.93	10.05	5.92	6.65	6.93	4.74	3.07	2.46
4. Key West, Salt W'rks[6]	24 34	81 47	4	2.41	1.67	2.24	1.02	1.94	5.40	4.15	4.23	5.82	3.93	1.54	2.32
5. New Smyrna	28 54	81 02	20	. .	0.42	1.73	0.63	1.14	2.75	7.23	4.47	3.15	2.46	. .	. .
6. Fort Myers	26 38	82 00	50	3.32	3.00	3.73	2.55	2.91	11.99	9.05	7.85	7.80	1.43	0.72	2.20
7. Fort Brooke	28 00	82 28	20	2.29	2.97	3.24	1.69	3.03	6.82	11.64	9.55	5.56	2.18	1.93	2.73
8. Fort Meade	28 01	82 00	80	1.07	1.01	1.64	1.78	4.01	7.79	7.55	6.35	4.85	1.50	0.56	1.79
9. Cedar Keys[7]	29 07	83 03	35	2.80	5.30	1.80	1.40	0.90	6.40	4.07	11.88	4.97	3.80	3.17	2.01
10. Fort Barrancas	30 21	87 18	20	3.87	4.95	5.87	2.94	4.05	4.66	6.80	7.23	5.25	2.41	6.05	3.12
11. Key West Barracks[6]	24 34	81 48	10	2.29	1.46	2.84	1.23	2.72	6.20	3.85	4.55	7.04	5.67	2.58	2.11
12. Key West[6]	24 34	81 48	20	2.37	0.68	1.90	1.05	1.19	4.49	1.23	2.81	6.57	10.45	0.67	1.29
13. Key West[6]	24 34	81 48	10	1.53	1.05	1.45	1.13	5.26	2.68	2.36	3.96	4.45	3.30	2.32	1.16
14. Key West[6]	24 34	81 48	10	. .	. .	. .	. .	1.03	1.85	1.20	8.80	3.95	8.51	0.90	1.31
15. Knox Hill[8]	30 30	85 30	148	2.72	4.43	2.42	2.47	2.73	4.72	6.25	5.67	3.35	2.77	3.33	3.21
16. Cedar Keys[7]	29 08	83 03	35	3.60	2.17	3.19	1.85	0.79	4.18	8.79	6.87	6.12	3.07	2.54	1.76
17. Indian Key	24 53	80 41	10	2.36	0.87	1.30	2.00	3.29	3.30	4.52	4.22	6.57	6.31	3.66	0.06
18. Jacksonville	30 20	81 39	14	3.04	2.52	4.53	2.72	3.66	5.18	7.17	8.72	6.42	2.07	3.55	3.14
19. Warrington N. Yard	30 21	87 16	12	3.66	3.32	3.63	5.04	6.18	5.84	8.93	9.40	2.66	2.54	4.82	5.12

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[1] Includes the year 1845 of extraordinary drought.

[2] See also Oglethorpe Barracks and Whitemarsh Island.

[3] See also observations at the Rock.

[4] Consolidated series: Jan. 3.80; Feb. 3.48; March, 3.66; April, 3.16; May, 3.46; June, 3.92; July, 5.16; August, 4.02; Sept. 2.50; Oct. 3.47; Nov. 1.57; Dec. 4.02; Spring, 10.28; Summer, 13.10; Autumn, 7.54; Winter, 11.30; Year, 42.22.

[5] January and February, 1841, at Fort Ried, a few miles distant.

[6] Key West, consolidated series: Jan. 2.12; Feb. 1.28; March, 1.99; April, 1.25; May, 3.13; June, 4.79; July, 3.02; August, 4.10; Sept. 5.69; Oct. 4.97; Nov. 2.01; Dec. 1.88; Spring, 6.37; Summer, 11.91; Autumn, 12.67; Winter, 5.28;

GEORGIA.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning. End.	Observer.	Reference.
1	6.98	3.66	4.51	9.05	24.20	2	0	May, 1844; Apr. 1846	Asst. Surgeon	Arm. Met. Reg. 1855.
2	13.45	21.50	7.21	9.17	51.33	4	8	Jan. 1843; Dec. 1850	" "	" " " "
3	10.65	20.62	8.63	8.42	48.32	23	2	Aug. 1836; Oct. 1859	Dr. J. F. Posey, A. G. Oemsler	Am. Alm. 1838-57, P. O. and S. I. Vol. 1.
4	9.16	14.86	8.54	7.75	40.31	11	5	Sept. 1849; Apr. 1861	R. T. Gibson	S.C., P.O. & S. I. Vol. 1. & S.O.
5	11.42	14.61	10.45	17.63	54.11	9	0	Mar. 1852; Apr. 1861	Dr. E. M. Pendleton	" " " " " " "
6	11.16	12.59	20.33	20.16	64.24	1	9	Oct. 1852; June, 1854	Prof. J. Darby	Sm. Coll'n, P. O. & S. I. Vol. 1.
7	7.76	16.67	11.97	7.72	44.12	2	3	Apr. 1851; July, 1853	Dr. Geo. F. Cooper	Agr. Rep.
8	. .	. .	. .	. .	. .	0	2	Nov. Dec. 1843	J. M. Cotting	Sm. Coll'n.
9	10.52	16.16	9.02	. .	. .	1	2	July, 1854; Dec. 1859	W. Haines, Dr. W. H. Doughty	P. O. & S. I. Vol. 1.
10	. .	. .	. .	. .	. .	0	1	Jan. 1855	. . . .	" " "
11	9.91	13.46	8.57	11.27	43.21	1	7	June, 1856; Dec. 1859	Dr. J. Anderson	" " "
12	8.26	12.72	6.55	9.01	36.54	4	6	Jan. 1845; June, 1849	McCay	Blodget's Clim.
13	13.64	14.11	. .	17.14	. .	1	11	Mar. 1857; Sept. 1859	Prof. J. D. Easter	P. O. & S. I. Vol. 1.
14	8.53	. .	. .	. .	. .	0	6	Jan.—June, 1857	F. T. Simpson	" " "
15	9.47	. .	6.02	16.65	. .	0	10	Sept. 1857; June, 1858	E. S. Glover	" " "
16	10.49	10.64	7.05	8.65	36.83	1	0	1857	Dr. J. Anderson	" " "
17	10.10	. .	9.26	. .	. .	0	6	Dec. 1858; May, 1859	Dr. W. T. Grant	" " "
18	15.23	13.04	13.15	17.01	58.43	2	0	1859 and Nov. 1865, to Dec. 1866	Dr. J. G. Westmoreland and F. Deckner	P. O. and S. I. Vol. 1, and S.O.
19	. .	. .	. .	. .	. .	0	1	Feb. 1861	Rev. W. Blewett	Sm. Obs.
20	. .	. .	. .	. .	. .	0	3	June, July, Aug. 1860	C. C. Seavey	" "
21	. .	. .	. .	. .	. .	0	3	Jan. Feb. March, 1861	Dr. J. R. M'Afee	" "

FLORIDA.

1	5.90	10.54	9.56	5.80	31.80	3	6	Jan. 1844; Feb. 1852	Asst. Surgeon	Army Met. Reg. 1855.
2	11.49	21.49	9.71	4.99	47.68	3	0	Jan. 1841; Dec. 1843	" "	" " " "
3	9.86	22.62	14.74	10.80	58.02	6	6	Mar. 1852; May, 1858	" "	Army Met. Reg. 1855, and M. D. of U. S. A. 1860.
4	5.20	13.78	11.29	6.40	36.67	11	7	May, 1851; Feb. 1864	W. C. Dennis	MS. in Sm. Coll'n, P. O. and S. I. Vol. 1, and S. O.
5	3.50	14.45	. .	. .	. .	0	9	Feb. to Oct. 1853	Asst. Surgeon	Army Met. Reg. 1855.
6	9.19	28.89	9.95	8.52	56.55	7	5	Jan. 1851; June, 1858	" "	Army Met. Reg. 1855, and M. D. of U. S. A. 1860.
7	7.96	28.01	9.67	7.99	53.63	17	0	Jan. 1840; July, 1858	" "	" " " " "
8	7.43	21.69	6.91	3.87	39.90	3	7	May, 1851; Nov. 1854	" "	" " " " "
9	4.10	22.35	11.94	10.11	48.50	2	6	July, 1840; Dec. 1842	" "	" " " " "
10	12.86	18.69	13.71	11.94	57.20	8	5	July, 1842; June, 1855	" "	" " " " "
11	6.79	14.60	15.29	5.86	42.54	12	9	Jan. 1837; Dec. 1859	" "	" " " " "
12	4.14	8.53	17.69	4.34	34.70	2	0	May, 1843; Apr. 1845	G. Wurdemann, U. S. [C. S.	MS. in Sm. Coll'n.
13	7.84	9.00	10.07	3.74	30.65	6	3	1832—1838	Coll'r W. A. Whitehead	MS. in Sm. Coll'n & Am. Alm.
14	. .	11.85	13.36	. .	. .	0	11	May, to Dec. 1843, Aug. to Oct. 1849	" " "	Am. Alm.
15	7.62	16.64	9.45	10.36	44.07	1	7	May, 1853; Nov. 1855	J. Newton	Am. Alm. & P. O. S. I. Vol. 1.
16	5.83	19.84	11.73	7.53	44.93	7	10	May, 1851; Mar. 1861	Judge A. Steele	MS. in Sm. Coll'n, P. O. and S. I. Vol. 1, and S. O.
17	6.59	12.04	16.54	3.29	38.46	2	0	Jan. 1836; Dec. 1837	Chas. Howe	Am. Alm. 1839.
18	10.91	21.07	12.04	8.70	52.72	6	0	June, 1851; June, 1866	Dr. A. S. Baldwin	MS. in Sm. Coll'n P. O. and S. I. Vol. 1, and S. O.
19	14.85	24.17	10.02	12.10	61.14	4	2	Jan. 1854; Dec. 1859	J. Pearson and others	P. O. & S. I. Vol. 1.

Year, 36.23; Extent, 24 years; Date, 1832—1864. Series by Whitehead, by Wurdemann, by Dennis, and by Asst. Surgeons. For the years 1852–9 the mean by two series is included.

[7] Also known as Atsena Otie; consolidated series: Jan. 3.45; Feb. 2.80; March, 2.93; April, 1.74; May, 0.82; June, 4.59; July, 7.61; Aug. 8.23; Sept. 5.78; Oct. 3.27; Nov. 2.71; Dec. 1.84; Spring, 5.49; Summer, 20.43; Autumn, 11.76; Winter, 8.09; Year, 45.77; Extent, 10 years; Date, 1840—1861. By Army Surgeons and A. Steele.

[8] Also known as Orange Hill.

FLORIDA.—CONTINUED.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
20. Pensacola	30°24′	87°13′	18	5.67	4.67	2.37	6.54	5.62	1.49	8.25	8.69	1.66	3.56	6.47	4.28
21. St. Augustine. . . .	29 54	81 19	8	1.19	2.69	5.97	1.66	2.90	4.29	6.35	6.74	3.21	3.89	1.99	2.28
22. Gainesville[1].	29 35	82 26	184	3.95	2.50	3.79	1.70	2.84	5.76	7.55	6.87	3.94	1.77	2.23	2.01
23. Lake City[2]	30 12	82 37	185	4.73	4.25	7.63	4.90	3.81	4.31	11.76	10.08	10.68	2.70	7.50	6.03
24. Micanopy.	29 35	82 31	78	1.30	4.34	7.52	0.93	0.84	3.75	10.76	10.92	7.70	1.77	3.00	1.53
25. Fort Dallas	25 55	80 20	20	6.02	1.62	4.85	2.44	5.97	5.37	9.37	4.15	6.78	7.93	2.40	2.14
26. Fort Deynaud . . .	26 30	81 30	. .	5.56	2.34	4.56	0.70	4.08	9.42	6.14	12.50	3.36	2.59	1.04	3.01
27. Barrancas Barracks .	30 20	87 18	20	4.18	2.04	7.85	4.04	2.95	3.52	8.47	8.71	4.62	5.08	4.91	7.41

ALABAMA.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Fort Mitchell. . . .	32 25	86 30	. .	2.29	5.52	4.61	6.70	6.30	7.08	1.75	4.28	1.22	0.11	3.86	1.48
2. Mt. Vernon Arsenal	31 12	88 02	200	6.67	5.64	5.17	4.30	3.98	6.19	6.60	7.23	3.73	3.96	6.82	5.85
3. Mobile	30 41	88 02	30	8.89	5.07	5.86	4.95	3.43	5.06	4.37	8.59	4.67	2.64	6.58	4.31
4. Florence	34 48	87 45	. .	6.19	4.75	4.75	2.87	3.81	4.87	6.25	3.12	1.44	9.75	1.75	12.81
5. Huntsville	34 43	86 46	600	5.80	4.72	5.89	5.02	3.97	5.12	4.59	4.87	3.49	2.83	3.67	4.91
6. Greene Springs . . .	32 50	87 46	500	3.54	4.02	4.86	3.11	3.90	4.20	3.71	5.55	2.99	2.37	5.59	4.94
7. Monroeville	31 33	87 25	150	3.68	6.69	4.65	5.52	7.04	4.95	6.89	7.30	2.74	1.56	5.72	4.15
8. Tuscaloosa (Univer.)	33 12	87 28	245	3.27	1.73	2.24	. .	. .	3.67	2.18	2.51	4.16	3.02	2.62	1.36
9. Wetokaville	33 20	86 00	. .	12.75	8.81	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
10. Auburn.	32 37	85 34	. .	3.72	1.33	4.12	1.58	3.11	4.18	3.36	4.80	1.10	1.46	8.15	6.37
11. Carlowville.	32 10	87 15	400	5.50	3.64	5.57	5.28	3.13	5.59	2.90	6.08	3.94	2.65	5.83	5.55
12. Greensborough . . .	32 40	87 34	350	6.38	4.84	4.38	4.31	4.32	2.85	3.79	5.70	4.54	1.76	6.07	6.63
13. Montgomery	32 25	86 22	162	7.76	6.68	7.00	4.00	2.08	6.46	. .	. .	0.60	3.78	5.17	7.23
14. Selma	32 25	86 51	200	5.23	5.43	10.97	5.09	1.61	4.33	2.31	4.47	4.12	5.16	3.86	9.70
15. Moulton (Baptist Female Institute.)	34 36	87 25	643	. .	7.32	5.89	6.23	5.18	5.01	0.90	. .	. .	. .	. .	1.73
16. Orville	32 24	87 06	200	. .	. .	. .	. .	. .	. .	. .	. .	. .	1.82	2.86	5.46
17. Bon Secour	30 25	87 50	12	. .	. .	. .	. .	. .	. .	. .	. .	. .	1.00	. .	5.35
18. Spring Hill College	30 41	88 09	158	. .	. .	. .	. .	. .	6.57	. .	4.67	. .	. .	. .	. .
19. Greensboro', S. Univ.	32 40	87 40	. .	4.89	5.36	4.79	4.54	3.18	3.69	2.84	5.49	3.05	2.19	4.68	5.70

MISSISSIPPI.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Pass Christian . . .	30 20	89 25	20	. .	. .	. .	. .	. .	5.18	5.36	3.50	5.70	1.97	. .	. .
2. East Pascagoula . .	30 20	88 42	20	. .	. .	. .	. .	5.55	1.64	7.63	5.04	3.29	4.82	1.83	. .
3. Vicksburg[3]	32 23	90 56	350	5.34	4.74	4.33	3.68	4.06	3.31	4.02	3.77	3.27	2.61	4.80	4.94
4. Vicksburg[4]	32 23	90 56	350	5.40	3.30	5.00	4.80	8.60	2.40	7.00	1.20	4.15	2.46	4.25	6.52
5. Natchez[5]	31 34	91 21	264	2.84	4.80	3.22	4.68	2.63	1.92	4.84	3.16	4.12	2.21	3.27	4.99
6. Natchez	31 34	91 25	264	6.30	4.29	4.73	4.64	5.55	4.88	5.40	3.28	5.19	3.60	4.54	5.92
7. Natchez[6]	31 34	91 25	264	4.71	8.62	5.22	6.29	3.60	3.06	3.84	4.70	3.41	3.05	3.73	5.94
8. Churchhill	31 50	91 25	250	2.69	5.28	3.47	3.01	3.47	3.30	3.72	4.20	3.91	2.40	3.86	6.66
9. Columbus.	33 30	88 28	227	5.05	5.70	4.78	5.76	4.93	4.10	4.08	2.91	3.11	1.53	6.03	9.15

[1] Also recorded "Alashua County."

[2] Also known as Alligator.

[3] Observations of 1840–41 by N. W. Hatch; the gauge is elevated three and a half feet above level of ground.

[4] Vicksburg consolidated series: Jan. 5.34; Feb. 4.65; March, 4.37; April, 3.75; May, 4.34; June, 3.26; July, 4.20; Aug. 3.60; Sept. 3.33; Oct. 2.59; Nov. 4.73; Dec. 5.14; Spring, 12.46; Summer, 11.06; Autumn, 10.65; Winter, 15.13; Year, 49.30; Extent, 16 years; Date, 1840—1867.

FLORIDA.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.		Observer.	Reference.
						yrs.	mos.	Beginning.	End.		
20	14.53	18.43	11.69	14.62	59.27	3	5	Aug. 1849;	Dec. 1852	J. Pearson	Sm. Coll'n.
21	10.53	17.38	9.09	6.16	43.16	3	8	Nov. 1855;	Dec. 1859	Dr. P. B. Mauran	P. O. and S. I. Vol. 1.
22	8.33	20.18	7.94	8.46	44.91	4	7	June, 1856;	Feb. 1861	J. B. Bailey	P. O. and S. I. Vol. 1, and S. O.
23	16.34	26.15	20.88	15.01	78.38	2	8	Dec. 1857;	Oct. 1866	E. R. Ives	" " " " " "
24	9.29	25.43	12.47	7.17	54.36	1	7	June, 1858;	Dec. 1859	Dr. J. B. Bean	P. O. and S. I. Vol. 1.
25	13.26	18.89	17.11	9.78	59.04	3	5	Jan. 1855;	May, 1858	Asst. Surgeon	M. D. of U. S. A. 1860.
26	9.34	28.06	6.99	10.91	55.30	2	3	Feb. 1855;	Apr. 1858	" "	" " " " "
27	14.84	20.70	14.61	13.63	63.78	3	8	Jan. 1855;	Dec. 1859	" "	" " " " "

ALABAMA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.		Observer.	Reference.
						yrs.	mos.	Beginning.	End.		
1	17.61	13.11	5.19	9.29	45.20	1	3	July, 1836;	Sept. 1837	Asst. Surgeon	Army Met. Reg. 1855
2	13.45	20.02	14.51	18.16	66.14	15	5	Oct. 1840;	Dec. 1859	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
3	14.24	18.02	13.89	18.27	64.42	2	0	Jan. 1841;	Dec. 1842	Dr. S. B. North	Am. Alm. 1843–4.
4	11.43	14.24	12.94	23.75	62.36	1	0	1849		B. R. Gifford	MS. in Sm. Coll'n.
5	14.88	14.58	9.99	15.43	54.88	12	0	Jan. 1831;	Dec. 1842	Rev. Dr. Allen	MS. in Sm. Coll'n, also Drake's Val. Miss.
6	11.87	13.46	10.95	12.50	48.78	6	9	Dec. 1854;	Dec. 1867	Prof. H. Tutwiler, J. W. A. Wright	P.O. and S. I. Vol. 1, and S.O. and MS.
7	17.21	19.14	10.02	14.52	60.89	5	5	Mar. 1849;	Nov. 1855	S. J. Cumming	Sm. Coll'n and P. O. and S. I. Vol. 1.
8	. .	8.36	9.80	6.36	. .	0	11	Jan. 1854;	Mar. 1855	Prof. M. Tuomey and G. Benagh	P. O. and S. I. Vol. 1.
9	. .	. .	. .	. .	. .	0	2	Jan. Feb. 1854		B. F. Holley	" " " "
10	8.81	12.34	10.71	11.42	43.28	2	6	Jan. 1855;	Dec. 1857	Prof. J. Darby	" " " "
11	13.98	14.57	12.42	14.69	55.66	3	3	June, 1856;	July, 1860	Dr. H. L. Alison	P. O. and S. I. Vol. 1, and S.O.
12	13.01	12.34	12.37	17.85	55.57	4	4	June, 1856;	Dec. 1861	R. B. Waller	" " " " " "
13	13.08	. .	9.55	21.67	. .	1	5	Oct. 1858;	Apr. 1861	Rev. J. A. Shepherd	P. O. and S. I. Vol. 1, and S. C.
14	17.67	11.11	13.14	20.36	62.28	1	6	Apr. 1858;	Sept. 1859	Dr. S. K. Jennings	P. O. and S. I. Vol. 1.
15	17.30	. .	. .	. .	. .	0	11	Mar. 1859;	Dec. 1866	A. D. Hunt, Prof. J. Shackelford, T. M. Peters, A. J. Harris	P. O. and S. I. Vol. 1, and S. O.
16	. .	. .	. .	. .	. .	0	3	Oct. Nov. Dec. 1859		Dr. S. K. Jennings	P. O. and S. I. Vol. 1.
17	. .	. .	. .	. .	. .	0	2	Oct. Dec. 1866		W. J. Vankirk	Sm. Obs.
18	. .	. .	. .	. .	. .	0	2	June, Aug. 1866		A. Cornette	" "
19	12.51	12.02	9.92	15.95	50.40	12	7	Jan. 1855;	Apr. 1868	N. T. Lupton	MS. in Sm. Coll'n.

MISSISSIPPI.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.		Observer.	Reference.
						yrs.	mos.	Beginning.	End.		
1	. .	14.04	. .	. .	. .	0	11	July, 1848;	July, 1860	Asst. Surgeon and Rev. J. A. Shepherd	Army Met. Reg. 1855, and Sm. Coll'n.
2	. .	14.31	9.94	. .	. .	1	7	Aug. 1849;	Sept. 1853	Asst. Surgeon	Army Met. Reg. 1855.
3	12.07	11.10	10.68	15.02	48.87	14	8	Jan. 1840;	Sept. 1854	A. L. Hatch	Am. Alm. 1843, and MS. in Sm. Coll'n.
4	18.40	10.60	10.86	15.22	55.08	1	3	Oct. 1866;	Dec. 1867	Army Surgeon	MS. from Surg. Gen. Office.
5	10.53	9.92	9.60	12.63	42.68	4	11	Feb. 1799;	Dec. 1803	W. Dunbar	Trans. Phil. Soc.
6	14.92	13.56	13.33	16.51	58.32	8	0	Jan. 1840;	Dec. 1847	Dr. H. Tooley	Am. Alm. 1842, and fol.
7	15.11	11.60	10.19	19.27	56.17	5	5	Dec. 1856;	Dec. 1866	J. E. Smith, R. M'Cary and W. M'Cary	P. O. and S. I. Vol. 1, and S. O.
8	9.95	11.22	10.17	14.63	45.97	5	7	Jan. 1850;	July, 1855	Dr. F. B. Coleman	Am. Alm. 1856.
9	15.47	11.09	10.67	19.90	57.13	4	7	May, 1855;	Dec. 1859	J. S. Lull	P. O. and S. I. Vol. 1.

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[5] At Forest, four miles east of Natchez: these with other meteorological observations were communicated by the author to President Jefferson, and by him to the Am. Phil. Soc.

[6] Natchez, consolidated series: Jan. 5.02; Feb. 5.63; March, 4.44; April, 5.11; May, 4.20; June, 3.55; July, 4.81; Aug. 3.70; Sept. 4.40; Oct. 3.06; Nov. 3.94; Dec. 5.69; Spring, 13.75; Summer, 12.06; Autumn, 11.40; Winter, 16.34; Year, 53.55; Extent, 18 years; Date, 1799—1866.

MISSISSIPPI.—CONTINUED.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
10. Lake Washington . .	33°00′	91°06′	. .	. .	. .	. .	. .	. .	. .	1.70	0.09	. .	. .	. .	. .
11. Jackson.	32 23	90 08	350	5.94	5.87	3.39	4.65	3.50	4.57	6.22	3.39	0.89	2.24	7.15	6.79
12. Oxford (University of Miss.)	34 20	89 25	300	2.61	3.59	1.93	5.15	1.70	5.63	4.01	5.44	1.11	1.45	8.67	2.34
13. Port Gibson[1]	31 51	91 02	. .	4.79	4.20	3.31	5.31	. .	. .	. .	3.95	4.35	1.88	5.78	. .
14. Paulding[2].	32 03	89 10	. .	4.64	2.40	4.42	4.03	2.35	3.92	2.71	3.72	1.68	2.39	3.39	7.20
15. Hernando	34 48	89 55	275	. .	. .	. .	. .	. .	. .	. .	. .	. .	0.89	3.17	5.17
16. Prairie Line[2]	32 10	89 10	215	3.70	7.88	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
17. Holly Springs . . .	34 45	89 30	. .	. .	. .	. .	. .	. .	. .	. .	2.70	1.67	1.95	. .	2.61

LOUISIANA.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Fort Wood	30 08	89 51	20	5.57	1.88	6.48	5.37	4.28	7.03	5.63	4.64	4.13	5.65	8.72	4.15
2. Fort Pike.	30 10	89 38	10	4.79	2.62	5.76	4.40	4.84	8.52	7.92	3.44	8.43	3.72	6.81	4.62
3. New Orleans Bar'ks[3]	29 57	90 01	10	5.01	2.97	3.92	3.50	4.16	4.66	6.34	6.09	2.91	3.00	4.25	4.22
4. Baton Rouge	30 26	91 18	41	5.08	4.20	4.38	4.26	5.02	5.08	6.74	6.72	3.50	2.91	6.02	6.25
5. Fort Jesup	31 33	93 32	80?	4.70	2.76	4.72	4.86	3.80	4.61	3.36	2.97	3.02	3.80	2.92	4.03
6. New Orleans[3]. . . .	29 57	90 04	20	3.99	3.81	3.78	3.83	3.33	4.27	7.10	5.47	4.43	3.17	3.88	3.90
7. West Feliciana . . .	30 38	91 20	96	5.60	4.85	5.60	8.00	5.45	3.70	6.10	5.04	3.80	2.80	4.00	6.40
8. Rapides.	31 08	92 20	76	6.50	3.15	4.20	4.65	6.40	6.85	8.45	5.30	1.25	3.60	6.80	10.25
9. Plaquemine.	30 20	91 18	30	7.00	4.00	4.40	5.80	5.50	8.70	10.60	7.00	3.30	2.08	4.00	4.70
10. St. Francisville . . .	30 43	91 27	80	4.08	3.52	3.96	7.29	5.30	3.76	6.37	2.95	3.15	3.67	5.15	6.07
11. St. Francisville . . .	30 49	91 27	80	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	4.00
12. Monroe.	32 20	92 10	100	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
13. Black River (Plant'n)	31 30	91 46	108	7.54	4.88	3.84	4.60	2.98	4.38	2.98	3.31	1.49	4.18	4.25	9.55
14. Moss Grove "	31 37	91 47	68	. .	. .	. .	. .	. .	3.33	1.43	4.76	. .	7.43	. .	. .

TEXAS.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Fort Belknap	33 08	98 48	1600	0.47	2.29	1.32	0.88	4.21	3.98	2.49	2.97	2.77	2.92	2.65	1.10
2. Fort Worth	32 40	97 25	1100	1.56	4.54	3.61	4.30	6.59	3.73	2.38	2.69	2.06	3.29	4.14	1.97
3. Phantom Hill[1] . . .	32 30	99 45	2300	0.26	0.80	0.54	0.45	2.85	2.90	1.15	0.03	2.55	3.41	1.34	0.94
4. Fort Chadbourne . .	32 02	100 05	2120	0.94	1.37	0.85	1.53	3.39	2.56	1.71	2.27	3.33	2.03	1.70	1.21
5. Fort Graham	31 56	97 26	900	1.42	5.24	4.55	4.53	2.90	2.71	2.15	2.06	0.80	4.24	4.73	5.25
6. Fort Croghan	30 40	98 31	1000	1.44	4.61	4.72	3.88	3.01	3.33	3.39	1.08	2.24	2.11	3.89	2.86
7. Fort Martin Scott . .	30 10	99 05	1300	0.80	2.98	5.82	6.48	2.31	5.18	1.25	0.78	1.31	1.07	2.68	1.87
8. Fort Mason	30 50	99 10	1200	0.74	1.46	1.07	2.32	2.97	2.37	3.89	4.18	4.67	2.39	1.16	1.76
9. Fort Terrett	30 23	100 16	1320	0.80	1.54	1.15	0.95	3.97	5.13	3.36	1.72	2.91	4.21	0.64	0.77
10. Fort McKavett . . .	30 55	100 05	2060	0.98	1.87	1.45	1.12	2.23	2.03	2.33	1.53	3.66	2.81	1.00	1.78
11. San Antonio	29 25	98 25	600	1.52	4.18	2.30	2.23	2.71	3.37	1.72	3.14	4.37	1.40	3.26	2.73
12. Corpus Christi . . .	27 47	97 27	20	1.98	2.76	0.62	4.01	4.68	5.63	4.89	3.35	6.34	1.70	0.90	0.91
13. Fort Ewell	28 05	98 57	200	0.76	4.73	0.71	1.12	5.11	7.85	2.90	2.43	4.96	2.36	0.49	1.16

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[1] Also known as Elliott's Academy.

[2] These stations are either close together or identical.

[3] New Orleans, consolidated series: Jan. 4.53; Feb. 3.35; March, 3.86; April, 3.64; May, 3.81; June, 4.49; July, 6.70; August, 5.74; Sept. 3.70; Oct. 3.09; Nov. 4.07; Dec. 4.07; Spring, 11.31; Summer, 16.93; Autumn, 10.86; Winter, 11.95;

MISSISSIPPI.—Continued.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs. mos.	Date. Beginning. End.	Observer.	Reference.
10	..	..	..	..	..	0 2	July, Aug. 1854	Rev. J. A. Shepherd	P. O.& S. I. Vol. 1.
11	11.54	14.18	10.28	18.60	54.60	4 2	June, 1849; Dec. 1855	A.R. Green, Hatch and others.	MS. at S. Inst. and P. O. and S. I. Vol. 1.
12	8.78	15.08	11.23	8.54	43.63	1 6	Jan. 1855; June, 1856	Prof. L. Harper	P. O. and S. I. Vol. 1.
13	..	..	13.01	..	..	0 10	Aug. 1855; Apr. 1857	Prof. J. B. Elliott	“ “ “ “
14	10.80	10.35	7.46	14.24	42.85	1 6	Feb. 1858; Sept. 1859	Rev. E. S. Robinson	“ “ “ “
15	..	..	..	..	..	0 3	Oct. Nov. Dec. 1859	Dr. W. M. Johnston	“ “ “ “
16	..	..	..	..	..	0 2	Jan. Feb. 1861	Rev. E. S. Robinson	Sm. Obs.
17	..	..	..	..	..	0 3	1867	Asst. Surgeon	MS. from Surg. Gen. Office.

LOUISIANA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs. mos.	Date. Beginning. End.	Observer.	Reference.
1	16.13	17.30	18.50	11.60	63.53	2 11	Apr. 1843; Apr. 1846	Asst. Surgeon	Army Met. Reg. 1855.
2	15.00	19.88	18.96	12.03	65.87	3 5	Apr. 1843; Aug. 1849	“ “	“ “ “ “
3	11.58	17.09	10.16	12.20	51.03	17 9	Jan. 1839; Dec. 1859	“ “	Army Met. Reg. 1855, and M. D. of U. S. A. 1860
4	13.66	18.54	12.43	15.53	60.16	15 1	Jan. 1843; Oct. 1859	“ “	“ “ “ “ “ “
5	13.38	10.94	9.74	11.49	45.55	9 11	Jan. 1836; Dec. 1845	“ “	Army Met. Reg. 1855
6	10.94	16.84	11.48	11.70	50.96	15 8	Aug. 1833; Mar. 1861	Dr. E. H. Barton, also D. T. Lillie, T. Harrison	Sill. Journ. 1837, Am. Alm. 1838-45, Am. Med. Soc. Philada. 1856, Sm. Coll'n, P. O. & S. I. Vol. 1, & S. O.
7	19.05	14.84	10.60	16.85	61.34	13 0	1820—1833	.....	Report by Dr. E. H. Barton, State Med. Soc. 1851.
8	15.25	20.60	11.65	19.90	67.40	3 0	1848—1850	.....	“ “ “ “ “ “
9	15.70	26.30	9.38	15.70	67.08	6 0	1845—1850	.....	“ “ “ “ “ “
10	16.55	13.08	11.97	13.67	55.27	5 0	.....	.....	San. Rep. by Dr. Barton.
11	..	..	..	..	..	0 1	1856	B. R. Gifford	P. O. & S. I. Vol. 1.
12	..	..	..	..	54.00	10 0	1808—1818	.....	Report by Dr. Barton.
13	11.42	10.67	9.92	21.97	53.98	2 3	Dec. 1856; May, 1859	Dr. A. R. Kilpatrick	P. O. & S. I. Vol. 1.
14	..	9 52	..	..	..	0 4	1860	Dr. E. Merrill	Sm. Obs.

TEXAS.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs. mos.	Date. Beginning. End.	Observer.	Reference.
1	6.41	9.44	8.34	3.86	28.05	5 10	Oct. 1852; Dec. 1858	Asst. Surgeon.	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
2	14.50	8.80	9.49	8.07	40.86	3 9	Dec. 1849; Aug. 1853	“ “	Army Met. Reg. 1855.
3	3.84	4.08	7.30	2.00	17.22	1 6	Sept. 1852; Feb. 1854	“ “	“ “ “ “
4	5.77	6.54	7.06	3.52	22.89	8 7	May, 1852; Dec. 1860	“ “	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
5	11.98	6.92	9.77	11.91	40.58	3 6	Mar. 1850; Aug. 1853	“ “	Army Met. Reg. 1855.
6	11.61	7.80	8.24	8.91	36.56	4 3	June, 1849; Aug. 1853	“ “	“ “ “ “
7	14.61	7.21	5.06	5.65	32.53	2 3	Jan. 1850; Mar. 1852	“ “	“ “ “ “
8	6.36	10.44	8.22	3.96	28.98	5 1	Apr. 1852; Dec. 1860	“ “	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
9	6.07	10.21	7.76	3.11	27.15	1 9	Apr. 1852; Dec. 1853	“ “	Army Met. Reg. 1855.
10	4.80	5.89	7.47	4.63	22.79	6 6	Apr. 1852; Mar. 1859	“ “	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
11	7.24	8.23	9.03	8.43	32.93	6 7	Sept. 1849; Dec. 1860	“ “	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
12	9.31	13.87	8.94	5.65	37.77	3 0	Jan. 1846; Mar. 1856	“ “	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
13	6.94	13.18	7.81	6.65	34.58	2 0	Oct. 1852; Sept. 1854	“ “	Army Met. Reg. 1855.

Year, 51.05; Extent, 23 years; Date, 1833—1861. Series by Dr. Barton, by Lillie, U. S. Army observations and at Medical Purveying Office.

Elevation of gauge, 1836–44, 40 feet above ground; 1848–55, 16 feet; 1861, 35 feet; at the barracks, 10 feet above Gulf.

[4] Clear Fork of Brazos.

TEXAS.—Continued.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
14. Fort Merrill	28°17′	98°00′	150	0.23	2.09	0.09	1.62	3.43	4.10	5.71	4.12	4.25	1.18	1.42	1.29
15. Fort Brown[1]	25 53	97 26	50	1.61	1.84	1.23	0.61	1.64	3.62	2.34	3.50	6.58	4.79	1.98	1.73
16. Ringgold Barracks .	26 23	98 47	521	1.19	1.02	0.75	1.20	2.06	3.48	1.92	2.54	3.01	2.31	0.78	1.06
17. Fort McIntosh[2] . . .	27 30	99 29	806	0.58	1.28	0.56	0.78	2.22	2.76	2.05	1.36	2.58	1.88	0.97	0.82
18. Fort Duncan	28 42	100 30	1460	0.29	1.30	1.19	0.60	1.58	4.58	2.47	1.85	3.72	1.34	1.24	1.07
19. Fort Inge	29 10	99 47	845	0.70	1.96	1.50	1.52	2.36	4.51	2.66	2.50	2.44	2.77	1.67	0.87
20. Fort Lincoln	29 22	99 33	900	0.13	4.00	3.50	1.86	2.89	2.07	0.99	0.39	1.54	1.36	2.01	0.98
21. Fort Clark	29 17	100 25	1000	0.50	1.65	1.01	1.38	1.51	2.98	0.89	2.61	4.33	1.56	0.91	3.05
22. Fort Bliss & El Paso	31 47	106 30	3830	0.10	0.95	0.33	0.00	0.13	0.23	1.02	2.00	2.79	0.88	0.87	0.26
23. Huntsville[3]	30 41	95 29	. .	2.20	0.54	2.35	0.82	1.22	4.18	2.40	0.72	4.43	1.67	0.53	2.87
24. Gonzales[4]	29 28	97 39	150	5.60	5.30	3.00	8.60	2.00	2.70	3.90	1.00	0.30	3.40	1.50	4.70
25. Pin Oak	30 00	97 15	. .	5.00	5.50	10.00	7.50	6.25	1.00	0.50	0.00	1.25	4.50	3.00	2.25
26. Chappell Hill	30 15	96 20	542	. .	. .	. .	0.90	5.15	4.05	2.90	5.35	. .	. .	. .	. .
27. New Braunfels[5] . . .	29 42	98 15	. .	1.52	2.03	1.00	1.93	4.67	2.62	2.23	2.05	3.94	3.12	1.77	0.94
28. Austin[6]	30 19	97 46	650	2.25	2.78	2.64	2.33	3.74	2.39	1.44	2.56	4.80	2.70	1.81	1.66
29. Austin[6]	30 15	97 47	. .	0.05	0.82	2.82	3.45	2.27	1.56	. .	. .	4.81	0.45	3.90	2.27
30. Helena	29 00	97 56	600	0.10	. .	0.05	. .	. .	. .	. .	. .	. .	. .	. .	. .
31. Washington	30 26	96 15	. .	4.14	0.76	1.40	0.40	1.26	5.20	1.51	2.36	4.67	3.08	0.82	4.63
32. Goliad	28 35	97 15	50	3.03	0.53	1.41	0.00	6.55	1.97	1.41	0.43	1.33	4.23	1.42	2.82
33. Larissa	31 45	95 50	755	7.75	. .	. .	. .	. .	1.75	. .	0.10	1.31	. .	6.13	6.43
34. Union Hill	30 11	96 31	542	5.86	2.40	2.63	0.97	0.93	3.10	1.72	2.73	2.65	4.78	1.67	2.56
35. Bonham.	33 40	96 13	435	. .	. .	. .	. .	. .	. .	. .	. .	. .	1.92	0.68	2.09
36. Cross Roads	30 21	97 31	672	. .	5.11	0.89	1.59	0.20	. .	. .	. .	1.52	1.20	2.15	3.40
37. Round Top[7].	30 06	96 37	. .	4.35	2.36	1.82	2.35	0.74	1.51	0.53	2.91	4.11	3.04	2.73	1.20
38. Sisterdale	29 54	98 35	1000	1.02	0.45	1.14	0.16	1.72	4.37	1.19	1.68	7.10	1.46	0.10	3.11
39. Webberville	30 14	97 34	394	. .	. .	. .	0.50	. .	. .	. .	. .	10.75	. .	0.00	. .
40. Parsons Seminary at Webberville	30 14	97 34	394	3.00	0.50	2.62	. .	. .	. .	. .	. .	. .	. .	. .	. .
41. Fort Lancaster . . .	30 42	101 25	2350	1.30	0.41	0.35	2.51	1.98	3.59	1.96	3.46	4.88	2.91	0.93	1.97
42. Camp Cooper	31 01	99 00	. .	. .	3.01	0.06	0.42	2.46	2.75	6.25	9.35	1.64	3.38	. .	3.85
43. Camp Hudson . . .	30 05	101 07	. .	0.76	0.13	0.11	1.42	1.60	2.13	0.08	3.33	0.99	0.97	0.19	0.42
44. Fort Davis	30 26	103 37	4700	0.68	0.81	0.43	1.14	0.87	1.36	3.39	3.75	4.45	0.91	0.42	0.50
45. Camp Verde	30 00	99 10	1400	1.02	1.26	2.60	1.47	2.34	2.67	2.19	5.80	4.77	1.99	2.16	2.38
46. Camp Colorado[8]. . .	31 55	99 17	. .	0.92	0.75	0.63	2.42	1.14	1.22	1.03	5.44	4.27	1.99	2.46	2.08
47. Fort Quitman	30 40	105 00	3710	0.15	0.78	0.01	0.21	0.10	0.06	1.20	1.11	1.75	0.22	0.24	0.43
48. Fort Stockton[9] . . .	30 20	102 30	. .	. .	. .	0.03	1.45	1.10	0.30	0.00	2.53	1.25	0.98	0.13	0.20
49. Gilmer, near[10]. . . .	32 46	94 50	950	. .	2.27	1.93	3.30	4.59	0.90	0.48	6.63	0.72	9.59	10.67	2.21
50. Aransas Canal[11]. . .	27 47	97 08	15	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	1.79	. .
51. Burkeville	31 00	93 32	. .	. .	1.45	4.35	5.95	. .	. .	. .	. .	. .	. .	. .	. .

[1] Near Matamoras.

[2] Laredo.

[3] The annual mean is from the years 1858 and 1859.

[4] The annual mean is from the years 1850, 1851, 1852.

[5] Formerly known as New Wied.

TEXAS.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs.	mos.	DATE. Beginning.	End.	OBSERVER.	REFERENCE.
14	5.14	13.93	6.85	3.61	29.53	3	3	Apr. 1851;	Nov. 1855	Asst. Surgeon	Ar. Met. Reg. 1855, M. D. of U. S. A. 1860.
15	3.48	9.46	13.35	5.18	31.47	9	8	Jan. 1850;	Dec. 1860	“ “	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
16	4.01	7.94	6.10	3.27	21.32	9	9	Sept. 1849;	Dec. 1860	“ “	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
17	3.56	6.17	5.43	2.68	17.84	9	6	July, 1849;	Dec. 1858	“ “	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
18	3.37	8.90	6.30	2.66	21.23	10	0	Oct. 1849;	Dec. 1860	“ “	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
19	5.38	9.67	6.88	3.53	25.46	7	4	Nov. 1849;	Dec. 1860	“ “	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
20	8.25	3.45	4.91	5.11	21.72	1	10	Sept. 1850;	July, 1852	“ “	Army Meteorological Register, 1855.
21	3.90	6.48	6.80	5.20	22.38	8	2	Aug. 1852;	Dec. 1860	“ “	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
22	0.46	3.25	4.54	1.31	9.56	8	0	Aug. 1850;	Dec. 1867	“ “	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
23	4.39	7.30	6.63	5.61	34.98	1	0	Jan. 1859;	Dec. 1859	T. Gibbs	MS. in Sm. Coll'n.
24	13.60	7.60	5.20	15.60	36.00	1	0	1850		Bennett	“ “ “
25	23.75	1.50	8.75	12.75	46.75	1	0	1856		Dr. W. H. Gantt	P. O. and S. I. Vol. 1.
26	. .	12.30	. .	. .	. .	0	5	1866		“ “ “	Sm. Obs.
27	7.60	6.90	8.83	4.49	27.82	5	1	Jan. 1854;	Dec. 1859	Prof. J. C. Ervendberg	P. O. & S. I. Vol. 1.
28	8.71	6.39	9.31	6.69	31.10	9	8	Mar. 1856;	Dec. 1866	J. Van Nostrand, Dr. S. K. Jennings, S. Palm	P.O. and S. I. Vol. 1, and S.O.
29	8.54	. .	9.16	3.14	. .	1	0	Feb. 1866;	June, 1867	Assist. Surgeon	MS. from Surg. Gen. Office.
30	. .	. .	. .	. .	. .	0	2	1857		J. C. Brightman	P. O. and S. I. Vol. 1.
31	3.06	9.07	8.57	9.53	30.23	1	11	Feb. 1857;	Dec. 1859	B. H. Rucker	“ “ “ “
32	7.96	3.81	6.98	6.38	25.13	1	0	1858		J. C. Brightman	“ “ “ “
33	. .	. .	. .	. .	. .	0	6	Aug. 1858;	Dec. 1859	F. L. Yoakum	“ “ “ “
34	3.63	7.55	9.10	10.82	31.10	2	4	Mar. 1858;	Mar. 1861	Dr. W. H. Gantt	P. O. and S. I. Vol. 1, and S.O.
35	. .	. .	. .	. .	. .	0	3	1859		Prof. S. Sias	P. O. and S. I. Vol. 1.
36	2.68	. .	4.87	. .	. .	0	10	Nov. 1859;	Nov. 1860	F. S. Wade	P. O. and S. I. Vol. 1, and S. O.
37	4.91	4.95	9.88	7.91	27.65	2	4	Jan. 1859;	Apr. 1861	B. Schumann	“ “ “ “ “ “
38	3.02	7.24	8.66	4.58	23.50	1	0	1859		E. Kapp	P. O. and S. I. Vol. 1.
39	. .	. .	. .	. .	. .	0	3	1859		C. W. Yellowby	“ “ “ “
40	. .	. .	. .	. .	. .	0	3	1861		Prof. C. W. Yellowby	Sm. Obs.
41	4.84	9.01	8.72	3.68	26.25	4	6	July, 1856;	Dec. 1806	Assist. Surgeon	M.D. of U.S.A. 1860, and MS. from S.G.O.
42	2.94	18.35	. .	. .	. .	1	3	Feb. 1857;	Oct. 1859	“ “	M.D. of U.S.A. 1860.
43	3.13	5.54	2.15	1.31	12.13	2	7	May, 1858;	Dec. 1860	“ “	M. D. of U. S. A. 1860, MS. from S. G. O.
44	2.44	8.50	5.78	1.99	18.71	5	11	Jan. 1855;	Dec. 1860	“ “	M. D. of U. S. A. 1860, MS. from S. G. O.
45	6.41	10.66	8.92	4.66	30.65	4	7	Jan. 1857;	Dec. 1867	“ “	M. D. of U. S. A. 1860, MS. from S. G. O.
46	4.19	7.69	8.72	3.75	24.35	3	11	Jan. 1857;	Dec. 1860	“ “	M. D. of U. S. A. 1860, MS. from S. G. O.
47	0.32	2.37	2.21	1.36	6.26	1	11	Jan. 1859;	Dec. 1860	“ “	M. D. of U. S. A. 1860, MS. from S. G. O.
48	2.58	2.83	2.36	. .	. .	1	1	Oct. 1859;	Dec. 1860	“ “	M. D. of U. S. A. 1860, MS. from S. G. O.
49	9.82	8.01	20.98	. .	. .	1	3	Mar. 1860;	June, 1861	J. M. Glasco	Sm. Obs.
50	. .	. .	. .	. .	. .	0	1	1860		F. Kaler	“ “
51	. .	. .	. .	. .	. .	0	3	1861		Dr. N. P. West	“ “

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[6] Consolidated series: Jan. 2.00; Feb. 2.76; March, 2.56; April, 2.26; May, 3.61; June, 2.33; July, 1.44; Aug. 2.56; Sept. 4.80; Oct. 2.70; Nov. 1.77; Dec. 1.71; Spring, 8.43; Summer, 6.33; Autumn, 9.27; Winter, 6.47; Year, 30.50; Extent, 10 years, 1 month; Date, 1856—1867.

[7] Farm two miles southeast of Roundtop.

[8] Or Fort Colorado.

[9] Or Camp Stockton.

[10] Three miles west of Gilmer.

[11] On dredge-boat A. Hawley, Nueces County.

TENNESSEE.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Nashville	36°10′	86°49′	533	5.18	4.26	4.98	4.61	4.91	4.56	4.06	4.30	4.49	3.78	3.92	2.97
2. Memphis[1]	35 08	90 00	262	3.26	6.64	4.18	3.42	3.37	3.15	1.83	2.87	1.50	2.94	3.55	5.10
3. Memphis Navy Y'd[2]	35 08	90 00	221	3.30	6.60	4.20	3.40	3.40	3.10	1.80	2.90	1.50	2.90	3.50	5.10
4. Memphis[1]	35 08	90 00	262	2.91	4.94	4.70	6.00	5.88	4.85	4.86	3.11	2.17	1.49	3.35	4.98
5. Glenwood[3]	36 28	87 13	481	3.63	4.13	4.10	4.63	4.17	3.74	3.57	3.61	2.90	2.53	4.18	5.44
6. Knoxville, East Tennessee University	35 59	83 54	1000	5.45	2.95	3.05	2.86	3.03	6.10	3.61	3.80	1.84	2.75	2.30	2.02
7. Lebanon, Cumberland University	36 15	86 15	480	3.43	6.03	2.98	4.52	3.86	2.85	2.17	4.44	1.81	1.03	3.73	6.61
8. Greenville.	36 08	82 47	. .	. .	. .	. .	. .	. .	. .	8.75	16.87	. .	. .	. .	. .
9. Friendship	35 50	89 25	. .	. .	. .	1.94	4.85	4.70	6.50	8.11	. .	. .	. .	. .	. .
10. Walnut Grove . . .	36 00	82 53	1350	2.33	. .	. .	. .	. .	. .	. .	. .	. .	. .	5.84	3.46
11. Pomona.	36 00	85 00	2200	0.37	0.20	. .	. .	. .	. .	. .	. .	. .	. .	. .	0.82
12. University Place, Cumberland Mt.	35 12	86 00	2000	6.83	7.05	3.73	1.53	3.86	3.61	2.30	4.14	4.14	2.26	7.80	3.82
13. Winchester	35 12	86 15	. .	1.19	0.73	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
14. Austin	36 20	86 20	2000	5.62	3.02	1.93	6.60	4.89	. .	. .	. .	. .	. .	. .	. .

ARKANSAS.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Smith	35 23	94 29	460	2.09	2.70	2.61	4.84	4.51	4.15	3.62	3.35	3.00	3.21	3.63	2.65
2. Washington	33 44	93 41	660	4.72	4.99	5.11	5.47	4.69	3.91	4.71	4.09	3.40	4.09	4.86	4.46
3. Camden	33 32	92 48	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	3.30	4.98	. .
4. Buckhorn	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	3.00	7.12	3.00	1.23	. .
5. Little Rock Arsenal	34 42	92 15	. .	2.00	9.38	2.95	5.92	10.90	0.50	2.20	. .	. .	0.60	0.20	. .
6. Union County[4] . . .	33 18	93 00	. .	9.14	6.23	5.25	11.88	7.24	3.26	2.63	13.33	1.25	3.50	6.25	11.12
7. Helena (near) . . .	34 33	90 45	. .	2.07	6.47	8.75	5.40	7.38	8.87	3.20	5.74	9.16	4.69	4.75	8.15

KENTUCKY.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Springdale	38 07	85 24	570	3.46	3.63	4.36	4.22	4.32	5.11	4.26	4.09	3.19	3.06	3.82	5.06
2. Danville	37 40	84 30	950	3.82	3.40	3.89	3.79	5.19	4.05	4.47	4.24	2.73	2.05	3.30	4.70
3. Millersburg	38 40	84 27	804	3.32	2.44	4.50	3.67	4.01	3.07	3.15	5.09	2.77	2.49	3.46	3.15
4. Ballardsville	38 26	85 31	461	4.10	1.43	2.91	3.59	6.40	5.72	2.90	4.86	3.20	3.12	4.28	2.74
5. Paris	38 16	84 07	810	2.35	3.48	1.96	3.92	3.67	4.26	3.39	3.86	2.16	2.36	4.00	5.69
6. Bardstown . . . . . St. Joseph College.	37 52	85 18	. .	2.90	4.44	2.59	4.91	4.60	3.45	4.58	5.15	2.44	2.29	3.26	9.36
7. Louisville.	38 10	85 40	450	4.29	5.22	4.60	5.59	4.28	4.96	2.37	2.10	3.13	3.43	3.62	4.53
8. Chilesburg[6]	38 04	84 06	850	4.51	4.02	3.54	4.53	3.79	3.89	3.94	4.02	3.56	2.42	3.86	4.67
9. Lexington	38 06	84 18	940	2.75	9.22	9.31	4.05	5.95	3.71	5.80	5.52	1.97	0.87	3.57	6.69
10. Newport Barracks .	39 05	84 29	500	2.10	2.40	1.79	4.50	6.19	5.79	3.16	4.02	2.87	2.52	5.04	5.60
11. Prospect Hill	38 40	83 33	700	5.51	3.50	4.62	2.90	3.33	3.44	5.63	5.70	1.33	3.25	1.88	6.12
12. Beech Fork	37 48	85 00	. .	. .	. .	. .	. .	. .	3.60	2.10	3.60	2.50	1.00	5.00	3.12
13. Nicholasville	37 58	84 18	940	5.74	4.42	4.68	5.46	3.04	2.75	4.11	4.14	2.03	2.31	4.61	2.06
14. Campbell County[7] . .	39 04	84 24	812	. .	. .	. .	. .	5.10	3.30	6.34	5.64	2.74	. .	. .	. .
15. Taylorsville.	38 00	85 10	600	. .	. .	. .	1.33	1.21	4.07	6.23	. .	. .	. .	. .	. .

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[1] Memphis, consolidated series: Jan. 3.06; Feb. 5.58; March, 4.51; April, 4.90; May, 4.63; June, 4.00; July, 3.34; August, 2.99; Sept. 1.88; Oct. 2.11; Nov. 3.43; Dec. 5.03; Spring, 14.04; Summer, 10.33; Autumn, 7.42; Winter, 13.67; Year, 45.46; Extent, 7 years; Date, 1849—1861.

[2] This series and the first one do not appear to be composed of independent observations.

TENNESSEE.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs.	mos.	DATE. Beginning. End.	OBSERVER.	REFERENCE.
1	14.50	12.92	12.19	12.41	52.02	7	3	June, 1839; Sept. 1850	Prof. J. Hamilton	Am. Alm. 1841–46, and S. C.
2	10.97	7.85	7.99	15.00	41.81	3	0	Aug. 1849; Aug. 1852	R. Harris	MS. in Sm. Coll'n.
3	11.00	7.80	7.90	15.00	41.70	3	0	1851—1853	Pearson	Blodget's Climatology.
4	16.58	12.82	7.01	12.83	49.24	3	10	Mar. 1850; Mar. 1861	R. A. Marr, Dr. D. F. Wright, Dr. W. J. Tuck, and Dr. R. W. Mitchell	U. S. Naval Observatory Vol. III, Met. Rep. Memp., P. O. and S. I. Vol. I, and S. O.
5	12.90	10.92	9.61	13.20	46.63	13	10	Mar. 1851; Dec. 1866	Prof. W. M. Stewart	MS., P. O. and S. I. Vol. I, and Sm. Obs.
6	8.94	13.51	6.89	10.42	39.76	1	9	Jan. 1854; Oct. 1856	Prof. G. Cooke and Prof. T. L. Griswold	P. O. and S. I. Vol. I.
7	11.36	9.46	6.57	16.07	43.46	2	6	Nov. 1850; Mar. 1855	Prof. A. P. Stewart, Prof. B. C. Jillson	MS., P. O. & S. I. Vol. I.
8	..	..	..	..	..	0	2	1843	.....	MS.
9	11.49	..	..	..	..	0	5	1855	Dr. R. T. Carver	P. O. and S. I. Vol I.
10	..	..	..	..	..	0	3	Nov. Dec. 1856, Jan.'57	J. B. Bean	" " " "
11	..	..	..	1.39	..	0	3	1860	J. W. Dodge and son	Sm. Obs.
12	9.12	10.05	14.20	17.70	51.07	1	3	Jan. 1860; Mar. 1861	C. R. Barney	" "
13	..	..	..	..	..	0	2	1860	S. W. Houghton	" "
14	13.42	..	..	..	..	0	5	1861	Dr. S. K. Jennings	" "

ARKANSAS.

	Spring.	Summer.	Autumn.	Winter.	Year.	yrs.	mos.	Beginning. End.	OBSERVER.	REFERENCE.
1	11.96	11.12	9.84	7.44	40.36	21	6	May, 1837; Feb. 1861	Assist. Surgeon	Arm. Met. Reg. 1855, M. D. of U. S. A. 1860, and MS. from S. G. O.
2	15.63	12.93	12.14	14.00	54.50	21	9	Jan. 1840; Sept. 1867	Dr. N. D. Smith	Sm. Con. to Know. 1860, and Sm. Obs.; MS. from S. G. O.
3	..	..	..	..	..	0	2	1855	J. J. McElrath	P. O. & S. I. Vol. I.
4	..	..	11.35	..	..	0	4	1859	A. Younger	" " " "
5	19.77	..	..	..	..	0	9	1867	Assist. Surgeon	MS. from S. G. O.
6	24.37	19.22	11.00	26.49	81.08	1	0	1850	Ross	Patent Office Report.
7	21.53	17.81	18.60	16.69	74.63	1	1	Dec. 1865; Dec. 1866	O. F. Russell	Sm. Obs.

KENTUCKY.

	Spring.	Summer.	Autumn.	Winter.	Year.	yrs.	mos.	Beginning. End.	OBSERVER.	REFERENCE.
1	12.90	13.46	10.07	12.15	48.58	24	3	July, 1841; Dec. 1866	L. Young and Mrs. L. Young	MS. in Sm. Coll'n, Am. Alm. 1848, P. O. and S. I. Vol. I, and S. O.
2	12.87	12.76	8.08	11.92	45.63	8	0	Jan. 1854; Dec. 1866	Prof. O. Beatty	P. O. and S. I. Vol. I, and S. O.
3	12.18	11.31	8.72	8.91	41.12	4	0	Jan. 1854; Apr. 1862	Rev. G. S. Savage	" " " " " " "
4	12.90	13.48	10.60	8.27	45.25	1	7	July, 1856; Jan. 1862	Dr. J. Swain	" " " " " " "
5	9.55	11.51	8.52	11.52	41.10	3	11	Jan. 1856; Dec. 1859	Dr. L. G. Ray	P. O. and S. I. Vol. I.
6	12.10	13.18	7.99	16.70	49.97	2	8	Jan. 1858; Oct. 1861	J. H. Lunemann, B. Schlicker, T. H. Miles	P.O. and S. I. Vol. I, and S. O.
7	14.47	9.43	10.18	14.04	48.12	3	0	Apr. 1858; Dec. 1862	Rev. S. R. Williams	P. O. and S. I. Vol. I.
8	11.86	11.85	9.84	13.20	46.75	9	0	Jan. 1858; Dec. 1866	Dr. S. D. Martin	Sm. Obs.
9	19.31	15.03	6.41	18.66	59.41	1	4	1859 and 1867	Rev. S. R. Williams	P. O. & S. I. Vol. I, & Sm. Col.
10	12.48	12.88	10.43	10.10	45.89	5	0	Jan. 1855; Dec. 1859	Surgeon U. S. A.	M. D. of U. S. A. 1860.
11	10.85	14.77	6.46	15.13	47.21	1	10	Mar. 1849; Dec. 1850	O. Beatty	MS. in Sm. Coll'n.
12	..	9.30	8.50	..	..	0	6	1860	Dr. C. D. Case	Sm. Obs.
13	13.18	11.00	8.95	12.22	45.35	2	4	Jan. 1861; June, 1863	J. McD. Mathews	" "
14	..	15.28	..	..	..	0	5	1861	M. G. Williams	" "
15	..	..	..	..	..	0	4	1866	H. C. Mathis	" "

[3] Also known as Glenwood Cottage, near Clarksville.

[4] These observations perhaps not quite reliable.

[6] About five and a half miles west of Winchester.

[5] Near Louisville.

[7] On Ohio River eight miles above Cincinnati.

OHIO.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Steubenville	40°25′	80°41′	670	2.94	2.75	3.38	3.53	3.85	4.01	3.89	3.97	3.48	3.18	3.16	3.34
2. Marietta[1]	39 25	81 29	580	2.90	3.06	3.18	3.54	4.13	4.35	4.45	3.94	3.13	3.26	3.18	3.58
3. Marietta	39 25	81 29	. .	2.17	1.66	2.80	2.25	3.07	3.43	1.96	3.07	1.51	3.86	1.76	3.46
4. Marietta	39 25	81 31	630	5.05	1.81	1.93	6.04	3.37	2.59	4.00	3.45	3.96	3.89	4.70	1.76
5. Cincinnati[2]	39 06	84 28	582	3.38	3.41	3.84	3.45	4.64	5.22	4.47	4.51	3.10	3.34	3.53	4.54
6. Cincinnati[3]	39 06	84 28	582	3.35	3.51	3.93	3.66	4.55	5.01	4.37	4.32	3.10	3.32	3.48	4.29
7. Cincinnati	39 06	84 28	480	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	5.80
8. Cincinnati	39 06	84 28	480	1.15	5.67	4.11	3.80	3.43	3.67	2.65	3.45	2.29	2.76	3.42	5.50
9. Cincinnati	39 06	84 28	. .	. .	. .	. .	6.39	3.72	. .	3.82	5.25	. .	3.93	4.54	2.68
10. Cincinnati	39 06	84 30	588	2.89	2.61	3.89	4.13	3.76	2.70	4.60	4.10	4.64	2.44	2.62	3.20
11. Cincinnati, Woodward High School.	39 06	84 29	588	2.56	[illegible].29	3.10	3.76	4.05	3.66	3.59	3.49	3.92	2.71	3.31	3.29
12. College Hill, Farmer's College.	39 07	84 35	800	3.12	2.69	4.73	4.99	4.90	3.78	4.59	3.74	4.05	2.55	4.53	4.41
13. Gallipolis	39 00	82 01	520	2.42	1.97	2.39	3.94	3.92	3.37	4.20	3.32	4.01	2.19	2.77	3.85
14. Portsmouth[4]	38 44	82 51	468	2.90	2.87	3.33	2.77	3.83	4.38	3.91	3.18	2.40	2.79	2.89	3.50
15. Portsmouth	38 45	82 50	523	2.90	3.37	1.61	4.35	3.84	3.10	2.19	4.88	1.67	2.25	3.37	3.89
16. Portsmouth	38 45	82 50	537	2.76	2.76	4.50	2.64	4.63	1.63	2.59	2.67	2.95	2.79	3.24	4.24
17. Cleveland (near)	41 42	81 36	625	2.69	2.98	2.15	1.53	2.78	3.00	3.77	1.95	3.28	1.79	2.64	3.77
18. Cleveland[5]	41 30	81 42	643	1.88	1.77	2.51	2.78	3.55	4.55	2.82	3.23	3.65	2.23	3.70	3.06
19. Cleveland[5]	41 30	81 47	645	2.95	2.15	2.98	3.31	3.17	3.43	3.06	3.25	4.13	2.82	3.64	3.23
20. East Cleveland	41 31	81 38	653	2.87	1.71	2.02	2.27	1.27	2.97	4.25	1.93	2.33	1.85	2.74	3.49
21. Toledo	41 39	83 32	604	2.12	2.11	3.54	3.47	3.66	3.55	4.12	3.32	5.32	2.40	3.11	2.74
22. Kelley's Island	41 36	82 43	587	1.88	1.49	2.57	3.25	2.85	3.18	3.51	2.85	3.91	2.48	2.80	2.47
23. Bowling Green	41 22	83 40	700	2.90	3.45	6.25	0.90	6.10	6.25	7.20	3.90	8.85	3.50	6.05	5.06
24. Bowling Green	41 27	83 45	700	2.78	2.40	4.40	3.98	3.76	3.62	2.74	3.19	2.02	3.02	3.10	3.55
25. Dayton, Cooper Sem.	39 44	84 10	860	1.49	1.15	1.15	1.09	5.10	3.57	3.17	2.83	1.62	1.98	3.75	. .
26. Dayton	39 44	84 11	860	2.65	1.90	. .	2.70	0.91	. .	3.27	. .	. .	. .	. .	. .
27. Urbana, University	40 06	83 43	1015	2.57	2.31	3.16	3.54	4.30	4.37	3.34	3.32	4.45	2.39	3.30	3.26
28. New Lisbon	40 45	80 45	961	2.17	2.22	3.06	2.75	2.70	4.42	2.77	3.73	4.06	2.22	2.58	3.39
29. Hillsborough	39 13	83 30	1150	2.65	2.78	2.71	3.61	4.41	4.08	4.19	3.62	3.72	2.43	3.41	4.29
30. Lebanon	39 28	84 10	717	4.15	2.62	5.48	1.90	4.23	4.42	4.25	3.94	3.29	4.45	2.76	4.61
31. St. Clairsville	40 10	80 55	600	2.63	5.75	3.17	4.13	4.87	5.58	2.15	3.81	4.41	3.47	3.06	7.13
32. New Athens, Franklin College.	40 15	81 05	. .	. .	. .	. .	. .	. .	5.51	0.93	. .	. .	5.73	. .	2.25
33. Columbus	39 57	83 03	870	2.60	. .	. .	2.70	1.50	. .	3.33	2.97	3.05	2.58	3.68	2.07
34. Northwood, Geneva Hall[6]	40 28	83 45	1170	1.84	1.83	3.72	4.74	4.42	4.73	2.67	2.58	1.78	2.96	4.12	3.72
35. Germantown	39 30	84 20	720	2.21	3.45	2.85	3.01	2.01	2.90	3.94	2.63	2.58	2.13	3.85	3.19
36. Germantown (near)	39 37	84 11	. .	. .	. .	. .	. .	1.50	1.77	4.72	0.94	. .	1.64	3.21	2.82
37. Granville	40 03	82 34	995	2.47	3.65	3.35	3.64	3.53	5.59	4.82	6.52	2.70	3.03	4.11	5.21
38. Keene	40 23	81 53	. .	2.66	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
39. Keene	40 25	81 53	1000	3.37	2.91	3.31	3.66	3.20	2.84	3.04	4.05	2.01	2.51	2.38	3.19

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[1] Observations between 1817 and 1823 by J. Wood; between 1826 and 1863 by Dr. S. P. Hildreth; after 1863 by Dr. G. O. Hildreth.

[2] Cincinnati, consolidated series: Jan. 3.06; Feb. 3.17; March, 3.68; April, 3.67; May, 4.38; June, 4.48; July, 4.20; August, 4.21; Sept. 3.35; Oct. 3.29; Nov. 3.41; Dec. 3.97; Spring, 11.73; Summer, 12.89; Autumn, 10.05; Winter, 10.20; Year, 44.87; Extent, 31 years; Date, 1835—1866.

[3] The same series as that marked [2], but extending over 22 years and 2 months, details for the last two years and two months not on file.

OHIO.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	10.76	11.87	9.82	9.03	41.48	37	1	Dec. 1830;	Dec. 1867	Roswell Marsh	MS. in Sm. Coll'n.
2	10.85	12.74	9.57	9.54	42.70	47	11	Nov. 1817;	Dec. 1867	J. Wood, Dr. S. P. Hildreth, and Dr. G. O. Hildreth	MS. in Sm. Coll'n, also Sm. Con. to Kn. No. 120.
3	8.12	8.46	7.13	7.29	31.00	1	5	Jan. 1854;	May, 1855	J. W. Andrews	P. O. & S. I. Vol. I.
4	11.34	10.04	12.55	8.62	42.55	2	7	Feb. 1860;	Aug. 1862	D. P. Adams	Sm. Coll'n.
5	11.93	14.20	9.97	11.33	47.43	18	0	Jan. 1835;	Dec. 1852	Prof. Ray	MS. in Sm. Coll'n.
6	12.14	13.70	9.90	11.15	46.89	20	2	Jan. 1835;	Feb. 1855	" "	Blodget's Clim.
7	..	..	..	..	48.63	16	0	1840—1855		John Lea	Horticulturist, etc. in Blodget's Clim.
8	11.34	9.77	8.47	12.32	41.90	2	1	Jan. 1851;	Dec. 1852 and Dec. 1855	J. Lea	P. O. & S. I. Vol. I, Hort. Rep.
9	..	..	..	..	..	0	9	Apr. 1843;	Aug. 1844	M. G. Williams	MS. in Sm. Coll'n.
10	11.78	11.40	9.70	8.70	41.58	6	9	Jan. 1859;	Dec. 1866	R. C. Phillips, J. H. Phillips	P. O. and S. I. Vol. I, and S. O.
11	10.91	10.74	9.94	8.14	39.73	10	0	Jan. 1855;	Dec. 1866	F. W. Hurtt, G. W. Harper	" " " " "
12	14.62	12.11	11.13	10.22	48.08	8	2	Jan. 1854;	Dec. 1866	Prof. R. S. Bosworth, G. S. Ormsby, Prof. J. H. Wilson, J. W. Hammit, L. B. Tuckerman	" " " " "
13	10.25	10.89	8.97	8.24	38.35	5	5	Apr. 1854;	Dec. 1866	Dr. G. W. Livesay, A. P. Rodgers	" " " " "
14	9.93	11.47	8.08	9.27	38.75	19	6	Jan. 1830;	Sept. 1852	Dr. Hempstead	MS. in Sm. Coll'n.
15	9.80	10.17	7.29	10.16	37.42	4	4	Jan. 1856;	July, 1863	J. H. Poe and Dr. D. B. Cotton	P. O. and S. I. Vol. I. and S. O.
16	11.77	6.89	8.98	9.76	37.40	2	4	May, 1863;	Aug. 1865	L. Engelbrecht	Sm. Obs.
17	6.46	8.72	7.71	9.44	32.38	1	6	Oct. 1850;	June, 1853	Edw. Wade	MS.
18	8.84	10.60	9.58	6.71	35.73	10	6	May, 1855;	Dec. 1866	G. A. Hyde & Mrs. Hyde	P. O. & S. I. Vol. I, & Sm. Col.
19	9.46	9.74	10.59	8.33	38.12	7	6	July, 1859;	Dec. 1866	B. A. Stanard	Survey N. & N. W. Lakes.
20	5.56	9.15	7.92	8.07	30.70	1	0	1862		Mrs. M. A. Pillsbury	Sm. Obs.
21	10.67	10.99	10.83	6.97	39.46	6	0	Jan. 1861;	Dec. 1866	Dr. J. B. Trembley	Sm. Obs. & An. Met. Synopsis.
22	8.67	9.54	9.19	5.84	33.24	7	6	July, 1859;	Dec. 1866	G. C. Huntington	MS. in Sm. Coll'n.
23	13.25	17.35	18.40	11.41	60.41	1	0	Feb. 1866;	Jan. 1867	J. Clarke	Sm. Coll'n.
24	12.14	9.55	8.14	8.73	38.56	4	6	May, 1858;	Dec. 1863	Dr. W. R. Peck	P. O. & S. I. Vol. I, & Sm. Obs.
25	7.34	9.57	7.35	..	..	1	5	Jan. 1856;	Nov. 1858	Dr. J. C. Fisher, L. Groneweg	P. O. and S. I. Vol. I.
26	..	..	..	..	..	0	5	1845		M. G. Williams	Sm. Coll'n.
27	11.00	11.03	10.14	8.14	40.31	11	0	Jan. 1854;	Dec. 1866	Prof. M. G. Williams	P. O. and S. I. Vol. I. and S. O.
28	8.51	10.92	8.86	7.78	36.07	8	4	Jan. 1855;	Dec. 1866	J. F. Benner	" " " " "
29	10.73	11.89	9.56	9.72	41.90	7	5	Apr. 1855;	Dec. 1866	C. C. James, J. Mc D. Mathews	" " " " "
30	11.61	12.61	10.50	11.38	46.10	3	5	July, 1843;	Mar. 1850	J. C. Hatfield	MS. in Sm. Coll'n.
31	12.17	11.54	10.94	15.51	50.16	1	8	Mar. 1850;	Oct. 1851	Harris	Patent Office Report.
32	..	..	..	..	..	0	4	July, 1843;	June, 1844	J. P. Mason	MS. Franklin College.
33	..	..	9.31	..	..	1	2	Apr. 1843;	Sept. 1845	T. Kennedy, J. Greiner	MS. in Sm. Coll'n.
34	12.88	9.98	8.86	7.39	39.11	2	4	Feb. 1854;	Mar. 1861	Rev. R. Shields, J. C. Smith	P. O. and S. I. Vol. I, and S. O.
35	7.87	9.47	8.56	8.85	34.75	6	0	Dec. 1850;	Dec. 1856	L. Groneweg	Blodget's Clim. & P. O. & S. I. Vol. I.
36	..	7.43	..	..	..	0	7	1866		J. S. Binkerd	P. O. and S. I. Vol. I.
37	10.52	16.93	9.84	11.33	48.62	7	2	Jan. 1850;	Feb. 1857	Prof. S. N. Sanford	Blodget's Clim. & P. O. & S. I. Vol. I.
38	..	..	..	..	..	0	1	1854		E. Spooner	P. O. and S. I. Vol. I.
39	10.17	9.93	6.90	9.47	36.47	3	0	June, 1849;	July, 1852	E. C. Bidwell	MS. in Sm. Coll'n.

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[4] Portsmouth, consolidated series: Jan. 2.89; Feb. 2.97; March, 3.10; April, 3.10; May, 3.92; June, 3.89; July, 3.51; Aug. 3.32; Sept. 2.33; Oct. 2.70; Nov. 2.99; Dec. 3.61; Spring, 10.12; Summer, 10.72; Autumn, 8.02; Winter, 9.47; Year, 38.33; Extent, 26 years; Date, 1830—1865.

[5] Cleveland, consolidated series: Jan. 2.33; Feb. 2.03; March, 2.49; April, 3.20; May, 3.52; June, 4.27; July, 3.04; Aug. 3.36; Sept. 3.64; Oct. 2.82; Nov. 3.80; Dec. 3.11; Spring, 9.21; Summer, 10.67; Autumn, 10.26; Winter, 7.47; Year, 37.61; Extent, 11 yrs. 8 mos.; Date, 1855—1866.

[6] Belle Centre, Geneva Hall.

OHIO.—CONTINUED.

NAME OF STATION.	Lat.	Long.	Height in feet.	MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
40. Mt. Vernon	40°25′	82°31′	. .	1.43	2.45	3.22	2.72	3.15	. .	. .	. .	. .	. .	. .	1.35
41. Norwalk	41 13	82 43	. .	2.95	1.89	2.46	2.59	2.98	3.43	4.13	3.30	3.24	2.73	2.40	2.04
42. Oberlin[1]	41 20	82 15	800	1.51	2.33	3.06	3.54	2.72	3.38	3.97	4.04	3.63	3.57	2.78	1.34
43. Oberlin[1]	41 20	82 15	800	1.44	1.82	1.39	3.02	3.35	5.72	3.05	3.33	2.96	2.81	3.36	2.26
44. Perrysburg	41 39	83 40	. .	2.00	1.18	1.75	4.91	2.98	5.56	7.78	2.44	4.73	2.25	4.84	3.38
45. Savannah	41 12	82 31	1098	2.54	3.32	2.84	4.07	4.70	5.86	3.97	2.95	3.05	2.81	4.44	4.04
46. Yankeetown	40 00	84 32	700	. .	. .	. .	. .	. .	5.25	. .	1.75	. .	. .	. .	. .
47. Zanesville	39 58	82 29	700	2.71	2.64	2.08	1.91	5.06	5.71	4.43	3.41	3.43	3.80	3.61	2.16
48. Arcola[2]	41 50	81 00	. .	4.24	. .	1.02	3.18	5.38	5.62	3.75	2.25	3.59	7.78	6.75	4.77
49. Hiram	41 20	81 08	1290	2.51	2.21	1.77	3.21	3.82	4.29	3.15	2.66	2.56	3.20	3.03	4.28
50. Jackson, Jackson Co.	39 07	82 32	666	. .	. .	. .	2.20	5.04	4.10	4.31	6.51	5.89	1.03	2.95	. .
51. Jackson, Monroe Co.	39 34	81 00	540	1.73	4.45	2.12	5.15	4.67	4.55	2.20	6.17	2.70	2.70	2.61	5.86
52. Madison[2]	41 50	81 00	620	2.41	2.71	3.26	5.11	4.53	4.43	4.26	4.64	4.18	4.44	6.12	2.91
53. Madison[2]	41 49	81 10	. .	. .	2.40	. .	. .	6.39	. .	. .	. .	. .	. .	. .	. .
54. Newark	40 06	82 28	825	2.00	3.40	2.76	2.53	2.19	2.30	2.83	2.77	1.85	1.81	3.08	3.14
55. Bellefontaine	40 21	83 40	1040	1.76	2.78	1.44	7.97	3.89	5.99	2.05	2.39	2.47	3.69	4.48	4.11
56. Cheviot	39 07	84 34	. .	. .	2.79	0.76	1.24	3.06	2.07	3.94	1.69	3.22	. .	. .	. .
57. Collingwood	41 45	83 34	. .	. .	. .	. .	. .	5.20	5.27	2.75	2.45	2.21	1.97	3.63	1.30
58. Jefferson	41 44	80 52	. .	. .	. .	2.54	5.60	3.06	4.23	2.80	3.47	1.14	4.21	5.04	5.18
59. Scioto	38 40	82 49	468	. .	. .	. .	. .	1.98	. .	. .	. .	. .	. .	. .	. .
60. Sidney	40 21	84 11	. .	0.83	. .	0.31	. .	8.90	1.44	. .	1.99	2.72	4.65	3.31	3.41
61. West Bedford	40 18	82 01	876	1.36	2.01	1.10	. .	. .	. .	. .	. .	1.76	1.99	2.63	2.57
62. Edinburg	41 20	81 00	520	2.33	2.00	1.98	4.44	6.01	5.24	5.25	3.95	1.02	3.78	8.37	4.44
63. Franklin	39 30	84 00	. .	1.66	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
64. Medina	41 07	81 42	1206	1.05	2.11	1.60	4.40	5.13	5.21	6.97	2.46	1.27	3.39	4.33	5.36
65. Ripley	38 47	83 31	574	3.35	3.33	4.10	4.03	6.91	4.06	4.77	3.48	3.90	2.84	3.26	4.39
66. Welchfield	41 23	81 12	1205	3.73	3.49	4.03	4.47	4.61	4.54	4.86	4.39	3.91	4.05	5.23	4.63
67. Avon	41 27	82 04	840	2.27	3.99	3.98	4.28	3.23	4.23	2.10	1.74	4.09	2.05	5.35	4.23
68. Hudson, Western Reserve College	41 15	81 24	1137	2.19	2.59	3.53	2.82	3.05	3.91	2.41	3.77	2.65	2.74	2.94	2.66
69. Lancaster[4]	39 40	82 40	. .	2.43	. .	. .	3.87	11.18	7.27	5.77	4.82	0.71	2.37	2.71	6.85
70. New Concord	40 03	81 44	. .	5.06	3.81	2.87	4.38	4.53	10.45	4.48	4.34	1.73	2.98	2.49	4.66
71. Montville	41 07	81 37	1255	2.99	2.15	2.42	3.15	2.77	2.76	2.67	2.63	2.02	1.71	3.24	2.63
72. Westerville	40 04	83 10	. .	2.70	1.93	2.82	3.68	3.77	3.97	2.99	4.19	4.30	2.00	3.74	3.10
73. Brecksville	41 23	81 40	800	. .	. .	. .	. .	. .	. .	. .	. .	. .	1.81	5.10	2.60
74. Elk Run[5]	40 41	80 44	1152	3.23	1.71	3.30	2.93	2.31	3.67	2.89	3.97	4.43	2.39	2.70	3.11
75. Bucksville	41 15	81 30	800	. .	2.66	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
76. Freedom	41 13	81 08	1100	2.56	4.37	2.27	4.52	6.53	5.98	4.84	3.36	4.02	2.93	6.51	1.46
77. Iberia	40 46	82 51	1160	. .	. .	. .	. .	. .	. .	. .	. .	. .	3.37	2.48	. .
78. North Bend	39 08	84 35	800	4.09	2.77	3.39	4.24	3.76	2.01	4.94	2.08	3.64	2.51	4.22	3.33
79. Troy	40 03	84 06	1003	2.99	3.46	6.66	9.14	5.74	5.09	2.82	4.30	2.22	2.84	4.12	3.18
80. Sandusky	41 32	82 42	. .	1.51	1.53	2.47	3.20	3.08	3.20	3.51	3.14	4.23	2.88	2.66	2.55
81. Bethel	39 00	84 00	555	2.92	2.34	4.38	4.29	3.87	3.65	5.07	3.50	6.32	2.63	2.74	5.37
82. Rockport	41 30	81 40	665	2.90	2.00	0.00	. .	. .	. .	. .	. .	. .	. .	. .	. .
83. Austinburg	41 54	80 51	816	2.32	2.75	3.88	3.53	2.71	1.87	4.76	3.30	4.42	2.64	3.85	3.82
84. Hartford, Halcyon Academy	40 13	82 38	. .	3.38	2.98	1.98	4.16	3.11	3.29	2.84	0.98	1.14	2.68	3.31	2.36
85. Cincinnati[3]	39 07	84 31	900	2.75	1.99	3.07	3.94	6.65	3.85	4.73	6.46	2.74	3.98	. .	. .
86. Seville	39 59	81 47	1075	3.20	2.13	2.67	2.06	3.15	0.78	1.72	3.86	3.40	3.46	2.05	2.04
87. Norton	41 04	81 39	1200	. .	. .	1.02	3.54	2.52	2.29	1.79	. .	. .	. .	. .	. .
88. New Westfield	41 13	83 49	692	. .	4.68	. .	4.55	6.78	6.40	1.49	2.98	3.91	4.10	3.03	5.93
89. Coshocton	40 18	81 53	765	5.62	2.88	. .	. .	. .	. .	. .	. .	. .	3.45	4.01	0.84
90. Kenton	40 06	83 50	1000	. .	. .	. .	4.96	3.40	7.75	8.62	14.35	. .	3.31	6.01	7.71

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[1] Oberlin, consolidated series: Jan. 1.47; Feb. 2.03; March, 2.10; April, 3.24; May, 3.08; June, 4.56; July, 3.51; Aug. 3.69; Sept. 3.29; Oct. 3.19; Nov. 3.11; Dec. 1.71; Spring, 8.42; Summer, 11.76; Autumn, 9.59; Winter, 5.21; Year, 34.98; Extent, 6 years, 6 mos.; Date, 1849—1857.

[2] Also known as Unionville and as Madison, the relative positions of these localities not known, approx. altitude 620 feet.

[3] Mt. Auburn Inst.

OHIO.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning. End.	Observer.	Reference.
40	9.09	..	..	5.23	..	0	9	Jan. 1854; Apr. 1855	F. A. Benton.	P. O. and S. I. Vol. I.
41	8.03	10.86	8.37	6.88	34.14	5	7	Oct. 1854; Dec. 1866	G. A. Hyde and Rev. A. Newton	P. O. and S. I. Vol. I, and S. O.
42	9.32	11.39	9.98	5.18	35.87	3	0	Mar. 1849; Aug. 1850 June, 1851—1852	Prof. J. H. Fairchild and others	Sm. Coll'n.
43	7.76	12.10	9.13	5.52	34.51	3	5	Jan. 1854; Nov. 1857	Prof. J. H. Fairchild	P. O. and S. I. Vol. I.
44	9.64	15.78	11.82	6.56	43.80	1	11	Apr. 1854; Sept. 1866	F. & D. K. Hollenbeck	" " " "
45	11.61	12.78	10.30	9.90	44.59	7	8	Mar. 1854; July, 1863	Dr. J. Ingram	P. O. and S. I. Vol. I, and S. O.
46	..	..	..	..	..	0	2	1854	A. Jacque	P. O. and S. I. Vol. I.
47	9.05	13.55	10.84	7.51	40.95	2	0	May, 1854; Feb. 1857	Dr. J. G. F. Holston	" " " "
48	9.58	11.62	18.12	..	..	1	4	Jan. 1855; Aug. 1856	A. Cunningham	" " " "
49	8.80	10.10	8.79	9.00	36.69	3	8	Sept. 1855; Oct. 1860	Rev. S. L. Hillier, S. M. Luther	P. O. and S. I. Vol. I, and S.O.
50	..	..	14.92	9.87	..	0	11	Apr. 1855; May, 1858	M. Gilmer, S. B. Wood	P. O. and S. I. Vol. I.
51	11.94	12.92	8.01	12.04	44.91	2	0	Jan. 1858; Dec. 1859	E. D. Johnson	" " " "
52	12.90	13.33	14.74	8.03	49.00	6	2	Feb. 1855; Feb. 1863	A. C. King (A. Cunningham)	P. O. and S. I. Vol. I, and S. O.
53	..	..	..	..	..	0	2	1858	Rev. L. S. Atkins	P. O. and S. I. Vol. I.
54	7.48	7.90	6.74	8.54	30.66	2	0	Jan. 1855; Aug. 1853	L. M. Dayton, I. Dille	P. O. and S. I. Vol. I, and S. O.
55	13.30	10.43	10.64	8.65	43.02	2	8	Jan. 1856; Sept. 1860	J. Shaw	" " " " " "
56	5.06	7.70	..	..	..	0	8	1856	E. Hannaford	P. O. and S. I. Vol. I.
57	..	10.47	7.81	..	..	1	3	June, 1856; June, 1858	H. and S. E. Bennett	" " "
58	11.20	10.50	10.39	..	..	0	11	Mar. 1856; Mar. 1857	J. D. Herrick	" " "
59	..	..	..	..	..	0	1	1856	J. H. Poe	" " "
60	..	..	10.68	..	..	0	10	Sept. 1856; Dec. 1867	J. Shaw	" " "
61	..	..	6.38	5.94	..	0	7	Sept. 1856; Mar. 1857	H. D. McCarty	" " "
62	12.43	14.44	13.17	8.77	48.81	1	8	Apr. 1857; Dec. 1858	S. Sanford	" " "
63	..	..	..	..	..	0	1	1857	Dr. W. L. Schenck	" " "
64	11.13	14.64	8.99	8.52	43.28	1	7	Mar. 1857; Sept. 1858	Rev. L. F. Ward	" " "
65	15.04	12.31	10.00	11.07	48.42	4	6	Oct. 1857; Dec. 1866	J. Ammen	P. O. and S. I. Vol. I, and S. O.
66	13.11	13.79	13.19	11.85	51.94	8	8	Mar. 1857; Mar. 1866	B. F. Abell	" " " "
67	11.49	8.07	11.49	10.49	41.54	1	3	Oct. 1858; Dec. 1859	Rev. L. F. Ward	P. O. & S. I. Vol. I.
68	9.40	10.09	8.33	7.44	35.26	11	4	Sept. 1838; June, 1863	E. Loomis and others[3]	Sill. Jour. Vol. 41 and 49, and P. O. and S. I. Vol. I.
69	..	17.86	5.79	..	..	0	10	April, 1858; Jan. 1859	H. W. Jaeger, W. E. Davis	P O. and S. I. Vol. I.
70	11.78	19.27	7.20	13.53	51.78	1	0	May, 1849; Apr. 1850	S. G. Irvine	MS. in Sm. Coll'n.
71	8.34	8.06	6.97	7.77	31.14	3	5	Oct. 1858; Feb. 1863	W. P. Clark	P.O. and S. I. Vol. I, and S. O.
72	10.27	11.15	10.04	7.73	39.19	7	3	Jan. 1858; Dec. 1866	Prof. J. Haywood	" " "
73	..	..	..	..	..	0	3	1859	S. L. Hillier	P. O. and S. I. Vol. I.
74	8.54	10.53	9.52	8.05	36.64	6	4	Aug. 1859; Dec. 1866	S. B. McMillan	" " "
75	..	..	..	..	..	0	1	1861	S. L. Hillier	Sm. Obs.
76	13.32	14.18	13.46	8.39	49.35	1	11	May, 1859; May, 1862	H. M. Davidson	P.O. and S. I. Vol. I, and S.O.
77	..	..	..	..	..	0	2	1859	S. T. Boyd	P. O. and S. I. Vol. I.
78	11.39	9.03	10.37	10.19	40.98	3	6	Nov. 1859; July, 1863	A. A. Warder	P.O. and S. I. Vol. I, and S.O.
79	21.54	12.21	9.18	9.63	52.56	3	4	Jan. 1859; Oct. 1863	C. L. McClung	" " " "
80	8.75	9.85	9.77	5.59	33.96	9	0	Jan. 1859; Dec. 1867	T. Neill	MS. in Sm. Coll'n.
81	12.54	12.22	11.69	10.63	47.08	5	5	Jan. 1860; Dec. 1866	G. W. Crane	Sm. Obs.
82	..	..	..	..	..	0	3	1860	E. Colbrunn	" "
83	10.12	9.93	10.91	8.89	39.85	3	0	Nov. 1862; Jan. 1866	J. G. Dole and others[6]	" "
84	9.25	7.11	7.13	8.72	32.21	3	0	Mar. 1860; Mar. 1863	M. Sperry	" "
85	13.66	15.04	..	..	..	0	10	1861	E. T. Tappan	" "
86	7.88	6.36	8.91	7.37	30.52	1	1	Jan. 1861; Dec. 1862	L. F. Ward	" "
87	7.08	..	..	..	..	0	5	1861	A. S. Steever	" "
88	..	10.87	11.04	..	..	0	10	Apr. 1862; Feb. 1863	A. E. Jerome	" "
89	..	..	..	9.34	..	0	5	Oct. 1861; Feb. 1862	T. H. Johnson	" "
90	..	30.72	..	..	..	0	11	Apr. 1862; Dec. 1866	Dr. C. H. Smith	" "

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[3] Observers for and after 1858: Prof. C. A. Young, E. W. Childs, H. J. Clark, E. W. Stuart, W. Pettingill, J. C. Elliott

[4] W. E. Davis' observation in January, 1859, in lat. 39° 46′, long. 82° 36′.

[5] Also known as East Fairfield, N. E. qr. of Range 2, Sect. 11.

[6] C. S. S.Griffing, D. S. Alvord, E. D. Winchester.

OHIO.—CONTINUED.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
91. Saybrook	41°50′	80°54′	650	1.60	2.22	0.90	..	5.00	7.04	2.50	5.85	4.32	2.37	3.64	3.77
92. Milnersville[1]	40 10	81 45	..	2.25	2.05	2.62	2.45	1.68	2.75	2.37	2.92	4.13	1.42	2.17	2.69
93. Wellington	41 08	82 13	875	..	2.43	..	..	1.00	..	1.00	..	..	..	..	..
94. Kingston	39 29	83 00	692	3.10	1.70	3.76	2.03	4.59	2.76	2.77	2.52	5.16	1.82	2.37	3.10
95. Eaton	39 50	84 26	1200	3.55	..	2.18	3.82	10.30	3.49	6.91	..	10.09	1.45	2.15	3.19
96. Cuyahoga Falls	42 00	81 00	..	..	2.00	..	..	..	..	..	..	..	..	3.07	0.95
97. Marion	40 35	83 07	1077	2.72	2.22	4.10	2.23	3.04	4.06	5.44	2.71	8.86	2.01	2.37	3.24
98. Pine Valley Mills	40 20	80 38	1000	..	..	7.93	0.66	..	..	..	..	..	..	..	..

MICHIGAN.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Detroit, Barracks[2]	42 20	82 58	580	2.14	1.38	2.86	2.92	2.73	3.91	3.20	2.18	3.31	2.04	2.06	1.30
2. Detroit,[2] Met. Obs'y	42 20	83 00	562	1.27	1.46	2.67	2.45	2.36	2.97	3.18	2.95	3.21	2.71	2.00	1.97
3. Detroit,[2] Mar. Hosp.	42 24	82 58	597	2.20	1.66	2.98	3.04	2.71	2.74	2.56	3.73	3.44	3.61	2.89	1.96
4. Detroit	42 24	82 58	597	2.89	3.30	3.51	4.00	3.47	4.99	6.03	3.43	5.10	3.56	4.14	3.76
5. Fort Gratiot[3]	42 59	82 25	598	2.19	1.76	2.82	2.51	2.69	3.52	3.33	2.87	4.09	2.66	2.10	1.80
6. Fort Mackinac	45 51	84 33	728	1.36	0.84	1.21	1.14	2.15	2.67	3.48	2.85	3.00	2.06	1.99	1.21
7. Fort Brady	46 30	84 43	600	1.76	1.05	1.30	1.73	2.03	2.71	3.58	3.30	4.50	3.21	2.97	2.18
8. Copper Falls Mines[4]	47 25	88 36	1230	5.00	1.88	1.35	1.80	1.89	2.84	2.00	3.89	6.80	3.60	4.30	6.56
9. Brooklyn	42 06	83 35	1020	2.01	2.59	1.86	5.03	7.00	1.50	2.00	3.95	3.26	3.57	2.53	3.87
10. St. James, Beaver Island	45 44	85 27	..	1.25	..	1.75	0.75	2.31	7.32	2.88	3.92	1.48	1.95	0.93	1.20
11. Fort Wilkins[5]	47 30	88 00	630	0.83	1.72	3.84	2.95	1.63	1.12	..	..	..	0.50	2.91	2.45
12. Ann Arbor University of Michigan[6]	42 17	83 43	891	1.68	1.36	1.82	4.64	3.13	4.83	4.97	1.02	4.26	2.94	3.18	2.65
13. Flint Scientific Inst.	42 58	83 39	..	2.84	1.46	1.74	2.41	2.33	5.62	2.60	1.18	4.00	2.79	3.61	2.14
14. Eagle River Mine	47 20	88 02	1020	3.75	1.55	0.75	0.83	..	1.74	2.00	3.40	1.60	2.20	..	5.55
15. Thunder Bay Island	45 02	83 11	610	3.20	2.65	2.83	3.07	2.03	2.33	2.86	2.54	3.58	2.86	2.75	2.87
16. Battle Creek	42 20	85 10	750	1.84	2.19	2.04	2.99	3.21	3.75	3.21	2.62	2.91	2.34	2.95	1.83
17. Monroe City[7]	41 54	83 19	551	1.65	1.37	2.81	2.61	2.61	2.46	3.24	3.42	3.88	2.37	2.20	2.41
18. Monroe	41 58	83 22	584	2.27	3.22	2.80	1.49	2.14	2.95	2.38	3.52	3.99	2.49	2.33	2.21
19. Tawas City[8]	44 16	83 31	583	1.73	0.91	1.98	2.26	1.82	1.82	1.72	2.24	2.39	2.15	2.13	1.46
20. Grand Haven	43 05	86 13	588	1.64	1.28	1.16	1.06	3.07	1.69	1.66	3.87	3.29	3.06	2.64	1.58
21. Ontonagon	46 52	89 31	630	2.45	1.42	1.41	1.84	2.18	2.21	2.24	2.99	2.52	2.11	1.85	2.57
22. Cooper[9]	42 40	85 31	690	2.56	1.15	4.76	5.06	6.30	4.82	3.02	4.83	6.46	3.15	2.75	2.03
23. Grand Rapids	43 00	85 40	750	2.97	2.07	3.45	3.08	4.10	4.47	2.51	2.48	3.46	2.95	3.76	4.35

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[1] Also called New Birmingham.

[2] Detroit, consolidated series: Jan. 1.85; Feb. 1.40; March, 2.78; April, 2.81; May, 2.66; June, 3.47; July, 3.17; Aug. 2.61; Sept. 3.12; Oct. 2.41; Nov. 2.18; Dec. 1.59; Spring, 8.25; Summer, 9.25; Autumn, 7.71; Winter, 4.84; Year, 30.05; Extent, 21 years, 1 month; Date, 1836—1866. The series of 6 years, 10 months, between 1848 and 1856, is omitted, as the amount appears too large.

[3] Position from Survey of N. and N. W. Lake.

OHIO.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs. mos.	DATE. Beginning.	End.	OBSERVER.	REFERENCE.
91	. .	15.39	10.33	7.59	. .	1 7	May, 1862;	Jan. 1866	Rev. L. S. Atkins, J. B. Frazer	Sm. Obs.
92	6.75	8.04	7.72	6.99	29.50	4 5	June, 1862;	Dec. 1866	Rev. D. Thompson	“ “
93	. .	. .	. .	. .	. .	0 3	1863		L. F. Ward	“ “
94	10.38	8.05	9.35	7.90	35.68	3 1	Nov. 1863;	Dec. 1866	Prof. J. Haywood	“ “
95	16.30	. .	13.69	. .	. .	0 10	Jan. 1864;	July, 1865	T. J. Larsh	“ “
96	. .	. .	. .	. .	. .	0 3	Nov. 1864;	Feb. 1865	D. M. Rankin	“ “
97	9.37	12.21	13.24	8.18	43.00	1 8	May, 1865;	Dec. 1866	Dr. H. A. True	“ “
98	. .	. .	. .	. .	. .	0 2	1866		D. M. Tweedy	“ “

MICHIGAN.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs. mos.	DATE. Beginning.	End.	OBSERVER.	REFERENCE.
1	8.51	9.29	7.41	4.82	30.03	12 4	Apr. 1836;	May, 1861	Asst. Surgeon	Army Met. Reg. 1855.
2	7.48	9.10	7.92	4.70	29.20	7 5	Jan. 1859;	Dec. 1866	E. P. Austin, J. Brennan, and others	Survey N. and N. W. Lakes.
3	8.73	9.03	9.94	5.82	33.52	2 8	Apr. 1858;	July, 1862	Dr. Z. Pitcher, L. S. Horton, E. B. Chapman, and others	P. O. and S. I. Vol. 1, and S. O.
4	10.98	14.45	12.80	9.95	48.18	6 10	Dec. 1848;	Nov. 1856	Dr. W. A. Raymond, Rev. G. Duffield	MS. in Sm. Coll'n, P. O. and S. I. Vol. 1.
5	8.02	9.72	8.85	3.75	32.34	11 1	July, 1836;	Aug. 1859	Asst. Surg. and Lieut. C. N. Turnbull	Army Met. Reg. and Survey N. and N. W. Lakes.
6	4.50	9.00	7.05	3.41	23.96	13 1	July, 1836;	Dec. 1859	Asst. Surgeon	Army Met. Reg. 1855, and M. D. of U. S. A. 1860.
7	5.06	9.59	10.68	4.99	30.32	17 7	July, 1836;	Sept. 1856	“ “	Army Met. Reg. 1855, and M. D. of U. S. A. 1860.
8	5.04	8.73	14.70	13.44	41.91	1 7	Dec. 1855;	Aug. 1857	C. S. Whittlesey	MS. and P. O. and S. I. Vol. 1.
9	13.89	7.45	9.36	8.47	39.17	1 2	Sept. 1852;	Mar. 1854	Dr. M. K. Taylor	MS. in Sm. Coll'n.
10	4.81	14.12	4.36	. .	. .	1 9	Sept. 1852;	May, 1856	J. J. Strang	MS. and P. O. and S. I. Vol. 1.
11	8.42	. .	. .	5.00	. .	0 9	Oct. 1845;	June, 1846	Asst. Surgeon	From Surg. Gen. Office. U. S. A.
12	9.59	10.82	10.38	5.69	36.48	2 11	Jan. 1853;	Aug. 1856	L. Woodruff, Prof. A. Winchell	MS. and P. O. and S. I. Vol. 1.
13	6.48	9.40	10.40	6.44	32.72	1 9	Apr. 1854;	Dec. 1855	Dr. D. Clark, Dr. M. Miles	P. O. and S. I., Vol. 1.
14	. .	7.14	. .	10.85	. .	0 10	Dec. 1855;	Oct. 1856	J. S. Morgan, M. A. Goff	MS. and P. O. and S. I. Vol. 1.
15	7.93	7.73	9.19	8.72	33.57	7 5	Aug. 1858;	Dec. 1865	C. N. Turnbull, J. Carr, W. P. Smith, I. I. Malden	Survey N. and N. W. Lakes.
16	8.24	9.58	8.20	5.86	31.88	9 2	Mar. 1849;	Dec. 1859	Dr. W. M. Campbell	Sm. Coll'n, P. O. and S. I. Vol. 1, and Sm. Obs.
17	8.03	9.12	8.45	5.43	31.03	7 6	July, 1859;	Dec. 1866	J. Lane	Survey N. and N. W. Lakes.
18	6.43	8.85	8.81	7.70	31.79	6 5	Jan. 1854;	Dec. 1866	H. J. Whelpley and F. E. Whelpley, G. W. Bowlsby, Rev. W. Bowlsby	P. O. and S. Vol. 1, and S. O.
19	6.06	5.78	6.67	4.10	22.61	8 4	Sept. 1858;	Dec. 1866	J. Oliver, C. H. Whittemore	Survey N. and N. W. Lakes, and P. O. and S. I. Vol. 1.
20	5.29	7.22	8.99	4.50	26.00	3 11	Sept. 1859;	July, 1863	H. Squier	Survey N. and N. W. Lakes.
21	5.43	7.44	6.48	6.44	25.79	5 11	Aug. 1859;	June, 1865	H. Selby and H. B. Smith	“ “ “ “ “
22	16.12	12.67	12.36	5.74	46.94	3 4	June, 1854;	Oct. 1862	O. C. Walker	P. O. and S. I. Vol. 1.
23	10.63	9.46	10.17	9.39	39.65	6 2	Apr. 1854;	Dec. 1866	A. O. Currier, L. H. Streng, E. A. Strong, E. S. Holmes	P. O. and S. I. Vol. 1, and S. O.

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[4] Near Eagle Harbor.

[5] Copper Harbor, Keewenaw Point.

[6] The earlier series, 1849—1852, December, omitted, the rain only being measured. The observations by L. Woodruff were taken three and a half miles E. S. E. from Ann Arbor.

[7] Or Monroe Piers.

[8] Ottawa Point.

[9] At this station the quantity of rain is apparently overmeasured.

MICHIGAN.—Continued.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Latitude.	Longitude.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
24. Saugatuck	42°30′	85°50′	..	2.20	2.50	0.94	..	..	..	..	..	4.02	4.58	3.51	3.51
25. Holland	42 42	86 11	..	2.90	2.02	3.31	2.62	3.56	2.13	3.16	3.44	5.75	3.38	3.25	3.19
26. Newark	42 30	86 00	..	..	..	..	2.46	..	..	..	..	..	..	..	..
27. Romeo	42 44	83 00	714	1.01	1.85	0.89	1.97	2.25	4.53	0.00	1.42	1.23	0.98	3.24	3.32
28. Marquette	46 33	87 23	710	2.10	2.17	1.99	2.60	1.90	3.21	2.79	3.02	2.80	2.68	2.63	2.12
29. Marquette	46 32	87 23	630	2.91	2.28	2.24	3.81	3.66	3.51	3.26	2.84	4.56	2.39	3.24	2.25
30. New Buffalo	41 45	86 46	661	2.53	1.74	3.64	4.55	4.81	3.67	2.68	4.86	5.48	3.77	3.98	2.49
31. Port Huron	42 53	82 24	606	..	1.45	..	..	..	..	..	..	..	..	..	..
32. Lake George	46 15	85 00	..	..	..	..	..	..	..	3.68	..	2.94	..	..	..
33. St. Mary's River	46 20	84 10	585	..	..	..	..	..	5.43	..	..	..	..	..	..
34. Ypsilanti	42 15	83 47	735	1.41	1.72	3.29	3.29	3.00	3.05	1.78	3.65	4.03	2.72	2.70	2.23
35. Mill Point	43 06	86 10	..	2.75	2.55	1.15	..	..	..	..	..	..	..	..	..
36. Redford Centre	42 28	83 10	650	..	..	..	..	..	6.00	2.92	5.60	..	..	..	..
37. Eureka Valley	47 00	88 00	800	5.82	1.80	..	..	..	..	..	..	3.81	..	..	4.55
38. State Agricul. Coll. near Lansing	42 42	84 33	895	1.22	1.45	2.75	2.51	2.71	4.30	3.11	2.55	3.72	2.31	1.95	2.06
39. Sugar Island	45 02	83 09	574	1.20	0.96	3.66	..	..	3.47	..	..	..	..	..	0.98
40. Litchfield Village	42 01	84 46	1040	..	..	..	..	..	4.63	1.85	2.66	10.30	7.87	4.75	2.80
41. Houghton	46 40	88 30	600	..	..	..	..	..	..	..	2.23	..	..	..	..

INDIANA.

Name of Station.	Latitude.	Longitude.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Richmond	39 49	84 46	800	1.71	3.06	2 95	3.61	4.39	4.15	2.50	3.89	1.37	3.03	4.08	3.14
2. Richmond[1]	39 52	84 44	850	2.57	2.77	2.41	3.94	4.34	3.75	3.76	3.79	4.98	2.65	3.86	3.92
3. Laporte	41 40	86 41	650	1.35	..	3.50	1.25	6.12	2.50	2.88	9.62	2.25	4.00	8.37	1.30
4. South Bend	41 39	86 21	600	2.07	2.01	3.64	3.84	2.78	2.82	4.30	3.30	3.86	2.36	4.25	3.55
5. Indianapolis	39 45	86 08	698	3.05	2.13	4.74	4.72	5.32	2.28	3.43	2.84	3.54	3.56	5.67	3.55
6. New Harmony	38 08	87 50	350	4.31	4.04	3.38	4.52	2.61	4.41	3.54	4.84	2.80	2.84	1.62	3.94
7. New Harmony	38 08	87 50	350	2.77	2.51	3.40	3.50	3.76	4.09	3.11	3.19	3.87	2.25	3.21	3.09
8. Logansport	40 45	86 13	625	1.55	3.41	3.66	3.24	4.43	2.81	1.60	3.89	3.23	2.57	3.84	3.11
9. Cannelton	37 57	86 41	380	1.53	2.70	2.35	3.96	3.61	4.54	2.20	3.07	2.47	2.80	3.18	4.05
10. Michigan City	41 44	86 55	622	1.14	1.11	1.91	2.86	4.72	4.41	3.54	3.72	2.46	2.55	1.30	1.98
11. Green Castle	39 39	86 46	..	1.15	1.00	..	..	..	..	..	..	..	..	..	..
12. Kendallville	40 28	85 13	975	..	3.40	3.06	6.37	6.50	6.92	7.07	2.98	4.38	..	..	..
13. Lafayette	40 25	86 49	620	..	..	..	..	5.90	..	..	..	..	..	..	..
14. New Albany	38 17	85 45	..	3.88	1.43	1.74	2.88	3.93	2.71	2.66	3.49	3.02	2.99	4.63	4.28
15. Evansville	38 08	87 29	390	3.05	..	1.04	6.65	6.14	2.80	5.44	1.78	3.01	..	..	..
16. Madison	38 45	85 20	450	3.48	3.44	6.59	6.19	7.17	4.40	2.92	6.11	5.69	1.09	2.16	8.62
17. Notre Dame	41 45	86 10	..	..	..	..	..	..	..	..	..	4.49	3.92	3.43	..
18. Mishawaka	41 39	86 02	..	..	..	..	3.56	2.10	13.15	1.15	6.20	9.60	5.70	..	..
19. Cadiz[2]	39 55	85 20	1060	2.69	2.51	5.57	5.09	2.81	2.31	5.55	1.65	4.16	3.50	4.14	2.27
20. Rockville	39 47	87 13	1100	2.70	0.76	3.75	3.00	3.68	1.84	3.06	2.12	2.25	5.25	3.57	2.58
21. Spiceland	39 48	85 18	1025	2.03	1.97	4.63	3.47	3.97	3.30	5.22	3.17	6.90	2.03	3.83	3.05
22. Newcastle	39 53	85 16	1000	2.32	2.12	4.53	4.39	3.48	2.34	0.82	3.64	5.88	2.20	4.02	3.47
23. Muncie	40 12	85 16	1000	2.10	2.34	2.40	4.10	1.25	..	..	5.92	11.72	3.42	4.65	3.92
24. Independence	40 30	85 45	..	..	..	..	..	2.62	2.80	..	..	..	..	..	..
25. Rensselaer	40 56	87 12	..	1.05	3.28	3.78	2.85	2.95	6.80	5.07	6.25	6.25	1.70	9.05	3.15
26. Pennville	40 35	85 00	1000	0.80	0.45	..	..	..	..	..	..	..	..	..	1.25
27. Vevay	38 46	84 59	..	4.06	1.67	5.27	3.71	6.65	3.86	5.82	1.72	10.88	1.51	2.73	4.89
28. Farmer's Institute	40 20	86 57	..	..	..	..	..	4.12	4.45	6.75	6.44	6.75	1.75	..	..
29. Columbia City	41 10	85 25	..	1.29	2.05	6.55	1.75	6.12	3.06	4.94	5.69	6.75	2.15	1.98	2.54
30. Aurora	39 04	84 54	480	2.17	1.31	4.90	1.91	1.56	2.88	4.94	2.65	10.74	1.52	3.26	3.00
31. Merom	39 10	87 40	..	..	..	..	..	..	2.30	..	..	..	2.74	3.20	3.55

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[1] Observations by E. B. Rambo, from August, 1862, to January, 1863, are included in the above series.

MICHIGAN.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs.	mos.	DATE. Beginning. End.	OBSERVER.	REFERENCE.
24	..	..	12.11	8.21	..	0	7	Sept. 1855; Mar. 1856	L. H. Streng	P. O. and S. I. Vol. 1.
25	9.49	8.73	12.38	8.11	38.71	5	2	June, 1856; Dec. 1866	“ “	P. O. and S. I. Vol. 1, & S. O.
26	..	..	..	..	..	0	1	1856	“ “	P. O. and S. I. Vol. 1.
27	5.11	5.95	5.45	6.18	22.69	1	3	Jan. 1856; Mar. 1857	Dr. S. L. Andrews	“ “ “ “
28	6.49	9.02	8.11	6.39	30.01	7	5	July, 1859; Dec. 1866	H. S. Bacon	Survey N. and N. W. Lake.
29	9.71	9.61	10.19	7.44	36.95	4	8	Sept. 1857; July, 1863	P. White, Dr. G. H. Blaker	P.O. and S. I. Vol. 1, and S. O.
30	13.00	11.21	13.23	6.76	44.20	3	6	July, 1857; May, 1862	J. B. Crosby	“ “ “ “ “ “
31	..	..	..	..	..	0	1	1858	J. Allen	P. O. and S. I. Vol. 1.
32	..	..	..	..	..	0	2	1859	A. W. Whipple, and E. Perrault	“ “ “ “
33	..	..	..	..	..	0	1	1859	. . . . . .	“ “ “ “
34	9.58	8.48	9.45	5.36	32.87	4	7	Jan. 1859; Sept. 1864	C. S. Woodard	P. O. and S. I. Vol. 1, & S. O.
35	..	..	..	..	..	0	3	1861	L. M. S. Smith	Sm. Obs.
36	..	14.52	..	..	..	0	3	1861	Dr. C. C. Smith	“ “
37	..	..	..	..	..	0	4	Sept. 1863; Feb. 1864	W. Van Orden	“ “
38	7.97	9.96	7.98	4.73	30.64	3	5	Aug. 1863; Dec. 1866	Prof. R. C. Kedzie	“ “
39	..	..	..	3.14	..	0	6	Mar. 1865; Feb. 1866	J. W. Paxton	“ “
40	..	9.14	22.92	..	..	0	7	1866	R. Bullard	“ “
41	..	..	..	..	..	0	1	1866	J. B. Minick	“ “

INDIANA.

	Spring.	Summer.	Autumn.	Winter.	Year.	yrs.	mos.	Beginning. End.	Observer.	Reference.
1	10.95	10.54	8.48	7.91	37.88	6	1	Apr. 1851; July, 1859	W. W. Austin, J. Moore	MS. in S. C. & P. O. & S. I. Vol. 1.
2	11.69	11.30	11.49	9.26	43.74	15	0	Jan. 1852; Dec. 1866	J. Valentine	MS. in Sm. Coll'n and S. O.
3	10.87	15.00	14.62	..	..	0	11	Mar. 1849; Jan. 1850	R. M. Newkirk	MS. in Sm. Coll'n.
4	10.26	10.42	10.47	7.63	38.78	3	0	May, 1862; June, 1865	J. H. Dayton, R. Burroughs	Sm. Obs.
5	14.78	8.55	12.77	8.73	44.83	4	2	Jan. 1861; Feb. 1865	R. Mayhew	MS. in Sm. Coll'n and S. O.
6	10.51	12.79	7.26	12.29	42.85	2	0	July, 1826; July, 1828	Dr. Troost	Darby U. S. G.
7	10.66	10.39	9.33	8.37	38.75	12	5	Jan. 1853; Dec. 1866	J. Chappelsmith	MS., P.O. & S. I. Vol. 1, & S.O.
8	11.33	8.30	9.64	8.07	37.34	4	10	July, 1854; June, 1863	E. L. Berthoud, C. B. Laselle, I. Bartlett, T. B. Helm	P. O. and S. I. Vol. 1, and S.O.
9	9.92	9.81	8.45	8.28	36.46	3	6	Jan. 1857; Dec. 1861	H. Smith	“ “ “ “ “ “
10	9.49	11.67	6.31	4.23	31.70	2	5	Mar. 1857; Sept. 1860	C. S. Woodard, W. Woodbridge, B. D. Angell, H. Blake	P. O. and S. I. Vol. 1, and Survey N. and N. W. Lakes.
11	..	..	..	..	..	0	2	1854	Prof. J. Tingley	P. O. and S. I. Vol. 1.
12	15.93	16.97	..	..	..	0	8	1854	J. Knauer, W. B. Coventry	“ “ “
13	..	..	..	..	..	0	1	1854	A. H. Bixby	“ “ “
14	8.55	8.86	10.64	9.59	37.64	2	11	Apr. 1856; Oct. 1865	C. Barnes, Dr. A. Martin, Dr. E. S. Crosier	P. O. and S. I. Vol. 1, and S. O.
15	13.83	10.02	..	..	..	0	8	1858	J. F. Crisp	P. O. and S. I. Vol. 1.
16	19.95	13.43	8.94	15.54	57.86	2	1	Jan. 1858; July, 1866	C. Barnes, Rev. S. Collins	P. O. and S. I. Vol. 1, and S.O.
17	..	..	11.84	..	..	0	3	1858	T. Vagnier	P. O. and S. I. Vol. 1.
18	..	20.50	..	..	..	0	7	1859	G. C. Munfield	“ “ “
19	13.47	9.51	11.80	7.47	42.25	2	6	Jan. 1860; Feb. 1863	W. Dawson	Sm. Obs.
20	10.43	7.02	11.07	6.04	34.56	1	7	Jan. 1860; Feb. 1864	H. H. Anderson	“ “
21	12.07	11.69	12.76	7.05	43.57	3	8	May, 1863; Dec. 1866	W. Dawson	“ “
22	12.40	6.80	12.10	7.91	39.21	1	10	June, 1863; Apr. 1865	T. B. Redding	“ “
23	7.75	..	19.79	8.36	..	1	2	Oct. 1863; Dec. 1866	E. J. Rice, Dr. G. W. H. Kemper	“ “
24	..	..	..	..	..	0	2	1864	O. Free	“ “
25	9.58	18.12	17.00	7.48	52.18	1	2	July, 1864; Sept. 1865	Dr. J. H. Loughridge	“ “
26	..	..	..	2.50	..	0	3	Dec. 1864; Feb. 1865	M. Griest	“ “
27	15.63	11.40	15.12	10.62	52.77	1	11	Feb. 1865; Dec. 1866	C. G. Boerner	“ “
28	..	17.64	..	..	..	0	6	1865	J. E. Windle	“ “
29	14.42	13.69	10.88	5.88	44.87	1	4	Sept. 1865; Dec. 1866	Dr. F. and L. McCoy	“ “
30	8.37	10.47	15.52	6.48	40.84	1	0	1866	G. Sutton	“ “
31	..	..	..	..	..	0	4	1866	T. Holmes	“ “

[2] One mile south of Cadiz.

ILLINOIS.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Athens	39°52′	89°56′	800	2.51	2.20	2.59	4.12	4.84	5.72	3.36	2.96	3.27	2.51	2.77	2.77
2. Pekin[1]	40 36	89 45	..	2.69	2.48	2.98	3.92	3.54	3.81	4.88	3.81	3.86	2.48	3.11	3.69
3. York Neck	40 05	91 33	..	1.42	1.99	3.04	7.51	2.68	6.06	5.75	2.18	5.25	3.51	1.56	3.45
4. Warsaw[2]	40 20	91 31	550	0.58	3.50	2.24	4.27	5.75	4.39	4.99	2.53	2.07	3.68	3.28	2.90
5. Chicago	41 54	87 38	591	1.09	5.43	2.15	2.47	4.89	4.14	..	..	..	2.92	3.98	3.86
6. Carthage[3]	40 23	91 17	..	0.58	4.63	1.26	1.14	1.74	2.03	0.92	..	..	..	..	..
7. Joliet	41 30	88 09	..	..	1.75	..	3.25	0.12	..	5.62	5.00	..	1.12	..	..
8. Batavia	41 52	88 20	636	1.77	2.12	3.30	2.89	4.22	3.56	4.93	2.14	4.11	2.75	3.30	1.59
9. Manchester	39 33	90 34	683	3.31	3.13	3.24	3.44	4.81	3.09	3.17	2.45	3.35	3.04	2.37	2.39
10. Upper Alton	38 55	90 10	650	1.84	1.43	2.96	2.30	3.25	2.43	3.10	2.71	2.23	1.81	2.41	2.06
11. Alton	38 57	90 14	650	2.38	3.93	4.27	4.90	3.43	2.60	3.59	4.92	4.44	1.44	2.35	2.25
12. Augusta	40 12	90 58	500	2.30	2.29	2.61	4.73	4.11	3.90	3.86	2.63	4.57	2.86	1.99	3.29
13. Brighton[4]	39 00	90 13	..	1.62	2.58	1.80	1.75	2.30	4.07	3.00	2.55	3.67	2.57	2.53	1.49
14. Marengo	42 14	88 38	842	1.37	1.98	3.13	2.36	4.08	4.35	4.07	4.77	5.02	2.57	2.59	2.29
15. Ottawa	41 20	88 47	500	2.35	2.47	2.72	3.34	3.83	3.54	4.30	3.41	3.36	2.78	2.54	2.55
16. Peoria	40 43	89 30	440	1.78	2.23	3.03	3.61	3.36	2.65	3.92	3.77	3.95	2.43	2.08	3.02
17. Peoria, near	40 38	89 45	512	3.12	2.14	3.95	4.40	2.65	2.17	2.42	3.10	4.36	2.25	1.40	1.43
18. Riley	42 11	88 33	760	1.98	2.30	2.84	3.80	3.88	3.12	4.98	4.10	4.86	2.52	2.41	2.66
19. West Salem	38 30	88 00	..	2.86	3.47	2.04	3.80	4.50	4.50	4.85	3.42	2.83	1.74	4.09	4.13
20. Aurora	41 48	88 22	696	1.76	1.47	2.76	2.83	3.94	3.35	4.25	3.51	3.97	3.09	2.16	1.80
21. West Urbana	40 09	88 17	550	1.02	1.74	2.01	5.25	4.70	4.55	2.39	3.59	3.74	2.00	3.21	2.41
22. Wheaton[5]	41 49	88 06	682	2.09	2.08	4.34	4.67	4.73	2.74	2.90	4.90	4.31	2.90	3.56	2.48
23. Winnebago	42 17	89 12	900	2.10	1.88	2.55	3.51	3.66	3.90	4.27	4.14	4.75	2.90	2.01	2.16
24. Edgington	41 25	90 46	686	..	1.77	0.12	5.31	4.16	1.93	2.81	0.90	3.66	1.20	6.80	..
25. Elgin	42 03	88 16	777	1.65	2.12	3.96	3.99	5.38	3.09	3.45	2.79	4.85	1.94	3.16	1.33
26. Evanston	42 03	87 38	614	1.40	1.73	2.00	2.37	1.60	2.00	3.38	3.33	3.32	2.45	0.66	0.54
27. Sandwich	41 31	88 30	665	2.78	2.65	4.28	3.81	5.89	4.51	6.66	5.14	5.52	3.51	2.49	2.93
28. Waynesville	40 16	89 07	..	1.53	1.52	3.08	5.75	7.12	6.00	3.88	1.62	4.00	2.63	3.12	2.22
29. Lebanon[6]	38 37	89 56	500	2.62	2.46	3.73	4.07	4.86	4.53	1.27	3.54	3.25	3.06	1.49	3.05
30. Osceola	41 16	90 16	..	2.61	3.58	2.35	6.52	3.94	5.30	6.60	6.00	3.30	..	4.65	2.43
31. Olney	38 45	88 05	..	..	..	..	..	..	..	..	..	4.00	1.94	..	..
32. Lee Centre	41 49	89 42	..	..	..	..	..	..	..	..	..	..	..	..	4.55
33. Galesburg	40 45	90 10	795	2.12	1.68	2.66	3.91	2.34	3.66	4.11	3.46	4.84	2.84	1.12	2.30
34. Jacksonville	39 30	90 06	676	5.85	0.90	4.15	2.10	3.25	4.15	2.70	3.20	4.95	2.25	0.75	1.10
35. Channahon	41 15	88 16	630	..	..	3.28	4.16	1.58	1.81	..	..	..	..	..	..
36. Highland	38 43	89 48	620	3.83	1.94	3.46	4.74	3.95	3.40	3.61	3.59	2.92	4.31	2.98	3.72
37. Elmira	41 12	90 15	..	3.03	2.00	2.12	3.25	3.36	2.31	4.00	5.39	6.20	2.06	0.97	2.18
38. Waverly[7]	39 40	90 05	680	2.08	2.77	3.11	2.97	2.36	2.14	4.98	1.99	4.02	2.85	3.05	3.35
39. Hoyton	38 30	89 00	..	0.50	..	..	4.22	1.35	4.37	6.00	2.62	2.00	1.50	3.50	3.30
40. Wyanet[8]	41 30	89 45	..	1.09	2.71	2.75	4.02	1.73	3.63	5.57	5.33	7.17	2.57	1.17	2.57
41. Elmore[9]	40 56	90 04	612	1.13	1.60	1.98	2.25	4.90	3.35	3.41	2.52	6.31	2.04	5.18	2.40
42. Dubois[10]	38 14	89 16	..	1.65	3.27	5.15	1.80	3.11	6.00	4.88	1.90	7.57	3.40	2.02	4.40
43. Murrayville	39 34	90 28	683	..	..	..	4.92	1.50	3.29	6.97	0.55	5.82	2.25	0.00	..
44. Pleasant Ridge (Nursery)	41 15	89 15	550	..	..	..	..	..	..	..	7.75	..	..	..	1.30
45. Golconda	37 41	88 46	..	0.50	1.20	5.88	2.05	3.33	..	3.32	1.67	7.83	0.87	3.20	3.48
46. Magnolia, near	41 15	89 15	500	..	..	..	..	..	..	..	..	..	..	2.31	3.91

[1] At Orchard Farm near Pekin; the position of Pekin, as stated, is in latitude 40° 35′, longitude 89° 43′.

[2] Observations two and a half miles east of Warsaw.

[3] At Prairie Garden near Carthage.

[4] Piasa Farms.

[5] Or Illinois Institute.

ILLINOIS.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	11.55	12.04	8.55	7.48	39.62	15	11	Jan. 1843;	Dec. 1858	J. Hall	P. O. Rep., MS., P. O. & S. I. Vol. 1.
2	10.44	12.50	9.45	8.86	41.25	10	7	Jan. 1855;	Oct. 1865	J. H. Riblet	MS. in S. C. and P. O. and S. I. Vol. 1, and S. O.
3	13.23	13.99	10.32	6.86	44.40	2	0	Jan. 1864;	Dec. 1865	V. P. Gay	MS. in Sm. Coll'n.
4	12.26	11.91	9.03	6.98	40.18	2	2	May, 1856;	July, 1858	B. Whitaker	MS. in Sm. Coll'n and P. O. and S. I. Vol. 1.
5	9.51	..	..	10.38	..	1	0	Mar. 1844;	Oct. 1862	S. Meacham, G. D. Hiscox, A. M. Byrne	Sm. Coll'n, P. O. & S. I. Vol. 1, and Sm. Obs.
6	4.14	..	..	..	..	0	7	1857		S. J. Wallace	MS. in Sm. Coll'n.
7	..	..	..	..	..	0	7	Oct. 1843;	July, 1845	Dr. M. K. Brownson	" " "
8	10.41	10.63	10.16	5.48	36.68	2	8	Mar. 1854;	Feb. 1861	Prof. W. Coffin, T. Mead	P. O. and S. I. Vol. 1, and S. O.
9	11.49	8.71	8.76	8.83	37.79	10	8	July, 1854;	Dec. 1866	J., R. E., C. W. Grant	" " " " "
10	8.51	8.24	6.45	5.33	28.53	7	0	May, 1849;	Oct. 1863	P. P. Brown, Dr. J. James, A. C. Trible	MS., P. O. and S. I. Vol. 1, and Sm. Obs.
11	12.60	11.11	8.23	8.56	40.50	1	4	May, 1849;	Aug. 1850	Norton Johnson	MS.
12	11.45	10.39	9.42	7.88	39.14	9	6	Mar. 1856;	Dec. 1866	Dr. S. B. Mead	P. O. and S. I. Vol. 1, and S. O.
13	5.85	9.62	8.77	5.69	29.93	2	10	Jan. 1856;	Feb. 1859	Rev. W. V. Eldridge	P. O. and S. I. Vol. 1.
14	9.57	13.19	10.18	5.64	38.58	6	4	Apr. 1856;	July, 1866	O. P. and J. S. Rogers	P. O. and S. I. Vol. 1, and S. O.
15	9.89	11.25	8.68	7.37	37.19	10	3	Mar. 1856;	Dec. 1866	Dr. J. O. Harris	" " " " "
16	10.00	10.34	8.46	7.03	35.83	9	11	Jan. 1856;	Dec. 1866	Dr. F. Brendel	" " " " "
17	11.00	7.69	8.01	6.69	33.39	1	2	Jan. 1861;	Feb. 1862	M. A. Breed	Sm. Obs.
18	10.52	12.20	9.79	6.94	39.45	9	8	May, 1856;	Dec. 1866	E. Babcock	P. O. and S. I. Vol. 1, and S. O.
19	10.34	12.77	8.66	10.46	42.23	4	4	Feb. 1856;	Oct. 1860	H. A. Titze	" " " " "
20	9.53	11.11	9.22	5.03	34.89	4	4	Oct. 1857;	Dec. 1866	A. J. Babcock, A. and E. D. Spaulding	" " " " "
21	11.96	10.53	8.95	5.17	36.61	2	8	Jan. 1857;	Dec. 1859	Dr. J. Swain	P. O. and S. I. Vol. 1.
22	13.74	10.54	10.77	6.65	41.70	2	9	Dec. 1857;	Sept. 1861	Prof. G. H. Collier	P. O. and S. I. Vol. 1, and S. O.
23	9.72	12.31	9.66	6.14	37.83	9	6	Mar. 1857;	Dec. 1866	J. W. Tolman	" " " " "
24	9.59	5.64	11.66	..	..	1	9	Feb. 1858;	Oct. 1861	Dr. E. H. Bowman, E. H. Bowman, Jr.	" " " " "
25	13.33	9.33	9.95	5.10	37.71	2	11	Feb. 1858;	July, 1862	J. B. Newcomb	" " " " "
26	5.97	8.71	6.43	3.67	24.78	1	9	Feb. 1858;	Nov. 1866	H. G. Meacham, C. E. Smith, W. H. Morrison, J. H. Gill	" " " " "
27	13.98	16.31	11.52	8.36	50.17	7	9	Dec. 1858;	Dec. 1866	Dr. N. E. Ballou	" " " " "
28	15.95	11.50	9.75	5.27	42.47	1	1	Feb. 1858;	Mar. 1859	J. E. Cantril	P. O. and S. I. Vol. 1.
29	12.66	9.34	7.80	8.13	37.93	1	8	Nov. 1859;	June, 1862	Rev. N. E. Cobleigh	P. O. and S. I. Vol. 1, and S. O.
30	12.81	17.90	..	10.84	..	1	4	Jan. 1860;	May, 1861	Dr. J. S. Pashley	Sm. Obs.
31	..	..	..	..	..	0	2	1860		H. H. Brickenstein	" "
32	..	..	..	..	..	0	1	1860		E. D. Straus	" "
33	8.91	11.23	8.80	6.10	35.04	5	9	Feb. 1861;	Dec. 1866	W. Livingston	" "
34	9.50	10.05	7.95	7.85	35.35	1	1	Mar. 1861;	Mar. 1862	T. Dudley	" "
35	9.02	..	..	..	..	0	4	1861		J. Fitch	" "
36	12.15	10.60	10.21	9.49	42.45	3	0	Mar. 1861;	Mar. 1864	A. F. Bandelier, Jr.	" "
37	8.73	11.70	9.23	7.21	36.87	2	11	May, 1862;	Dec. 1866	O. A. Blanchard	" "
38	8.44	9.11	9.92	8.20	35.67	4	3	May, 1862;	Dec. 1866	T. Dudley	" "
39	..	12.99	7.00	..	..	1	0	Apr. 1864;	June, 1866	J. Ellsworth, O. J. Marsh	" "
40	8.50	14.53	10.91	6.37	40.31	2	5	Aug. 1864;	Dec. 1866	E. S. Phelps, Jr., and L. E. Phelps	" "
41	9.13	9.28	13.53	5.13	37.07	2	0	Sept. 1864;	Dec. 1866	W. H. Adams	" "
42	10.06	12.78	12.99	9.32	45.15	1	11	Jan. 1865;	Dec. 1866	W. C. Spencer	" "
43	..	10.81	8.07	..	..	0	8	1865		J. and E. Grant	" "
44	..	..	..	..	..	0	2	Aug. 1865;	Dec. 1866	V. Aldrich	" "
45	11.26	..	11.90	5.18	..	0	11	1866		W. V. Eldridge	" "
46	..	..	..	..	..	0	2	1866		H. K. Smith	" "

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[6] McKendree College.

[7] Observations of 1866, at Loamie, Sangamon County, height 675, position near Waverly.

[8] Four miles N. W. of Wyanet.

[9] Defective Record.

[10] Near Coloma.

WISCONSIN.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Howard	44°30′	88°05′	620	1.19	0.87	1.70	3.33	3.97	4.93	5.48	4.01	3.11	2.36	2.37	1.30
2. Fort Winnebago	43 31	89 28	770	0.91	0.82	1.07	2.26	2.25	4.24	4.21	3.01	3.62	2.00	2.01	1.09
3. Fort Crawford	43 05	91 00	642	1.19	1.24	1.92	2.99	2.72	3.74	3.48	4.65	4.06	2.04	1.80	1.57
4. Milwaukee[1]	43 03	87 55	591	1.33	1.10	1.94	2.79	3.33	3.90	3.28	2.71	3.09	2.25	2.57	1.84
5. Milwaukee[1]	43 03	87 55	591	2.05	1.81	2.05	2.62	3.13	2.74	3.56	2.46	3.50	2.48	2.04	1.95
6. Milwaukee	43 03	87 57	630	1.88	1.27	2.41	3.15	4.29	2.98	4.75	3.26	4.65	3.18	2.16	1.75
7. Lowell	43 21	88 48	..	0.30	3.60	0.75	2.85	2.70	2.90	3.10	5.65	1.80	3.65	2.05	0.90
8. Superior	46 46	92 03	660	0.95	1.44	1.45	2.07	2.62	2.34	3.15	3.19	3.66	2.67	1.31	0.79
9. Superior	46 46	92 03	680	..	..	..	1.40	3.14	2.04	2.97	4.50	3.50	3.11	..	0.35
10. Beloit, College	42 30	89 03	750	1.98	1.21	1.77	3.04	3.30	3.24	3.94	3.48	3.32	2.24	2.34	1.79
11. Ceresco	43 50	88 57	917	..	..	..	2.70	4.95	..	8.62	3.47	2.52	2.79	6.50	..
12. Janesville	42 42	89 00	768	1.74	2.34	1.09	1.78	4.42	4.47	3.31	2.60	2.98	2.54	1.90	1.88
13. Beloit[2]	42 30	89 03	750	2.35	1.61	3.22	4.09	6.42	5.26	7.15	5.46	3.84	2.65	4.19	2.11
14. Platteville	42 45	90 45	800	1.47	2.39	2.16	3.26	4.46	5.59	5.81	2.23	3.50	2.38	3.44	2.62
15. Appleton, Lawrence University	44 10	88 35	800	0.75	1.26	1.54	2.81	3.11	4.04	3.75	2.45	2.84	2.49	2.10	3.92
16. Baraboo	43 29	89 44	800	1.12	1.18	2.10	3.27	4.61	4.38	3.98	2.45	1.70	4.86	2.05	1.51
17. Ashland or Bay City[3]	46 33	91 00	610	2.24	1.47	2.48	4.93	6.09	4.61	3.72	5.62	5.88	4.87	3.18	1.36
18. Norway	42 50	88 10	753	0.83	3.55	0.63	4.75	5.25	7.75	3.10	2.35	4.60	1.46	6.70	6.57
19. Waukesha	42 55	88 11	812	..	..	0.40	2.53	2.39	..	0.93	1.86	2.19	1.83	2.90	..
20. Falls of St. Croix	45 30	92 40	660	..	..	..	..	..	..	..	..	..	..	..	0.75
21. Menasha	44 13	88 18	..	3.35	0.84	2.87	..	..	..	..	..	..	5.12	2.56	..
22. New London	44 21	88 45	..	1.94	1.68	1.53	4.20	4.42	4.19	3.50	3.38	4.38	1.69	1.99	1.85
23. Bayfield	46 18	90 50	..	1.80	2.10	4.22	..	..	..	..	..	2.17	3.12	1.23	..
24. Green Bay, near	44 30	88 06	584	..	..	..	..	1.44	4.06	2.70	2.11	5.17	3.97	3.38	..
25. Green Bay	44 34	88 07	732	0.93	1.36	1.80	3.25	0.98	2.85	4.69	2.27	6.66	1.39	3.47	2.17
26. Kenosha	42 35	87 50	600	2.29	1.44	2.54	3.49	4.48	3.20	3.26	2.06	5.47	1.80	2.29	1.15
27. Manitowoc	44 07	87 45	658	2.02	1.80	1.84	2.49	1.89	3.12	3.97	4.00	2.52	2.83	2.07	1.74
28. Rocky Run	43 26	89 19	..	2.43	1.60	1.85	2.80	4.59	3.19	4.07	4.21	3.10	3.21	1.41	1.66
29. Wausau	45 00	89 40	..	..	0.81	..	6.90	..	..	3.60	..	..	..	..	..
30. Springdale	43 44	89 16	..	1.40	3.37	..	..	..	..	..	..	..	..	..	..
31. Madison, University Building	43 05	89 23	1068	1.88	1.30	2.05	2.76	1.65	1.20	..	..	1.81	3.25	1.50	1.70
32. Summit	43 06	88 33	780	2.03	1.83	2.37	3.32	5.42	2.91	4.65	1.41	4.79	2.84	1.86	1.45
33. Dartford	43 30	89 25	..	..	..	3.30	..	..	..	..	..	..	..	..	..
34. Weyaumega	44 15	88 50	870	3.72	0.65	2.85	2.94	0.94	6.30	6.20	5.35	8.63	5.50	0.10	1.07
35. Embarrass	44 51	88 37	..	2.28	1.22	2.36	1.92	2.04	4.11	4.80	4.38	3.28	2.85	2.69	2.15
36. Delavan	42 39	88 37	935	1.01	1.60	2.45	2.75	1.68	3.38	2.94	6.97	4.60	2.10	0.75	1.78
37. New Holstein	43 45	88 08	..	0.50	..	..	..	..	..	..	..	..	..	2.30	..
38. Plymouth	43 44	88 07	870	1.30	1.90	3.40	3.10	2.20	4.60	3.95	5.45	6.60	3.90	0.85	1.35
39. Rural	44 15	89 05	910	..	..	..	7.73	..	..	..	..	..	..	..	..

MINNESOTA.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Snelling	44 53	93 10	820	0.87	0.54	1.24	2.38	3.06	3.72	3.73	3.05	3.50	1.36	1.52	0.85
2. Fort Ripley (Gaines)	46 10	94 24	1130	0.71	0.63	1.34	1.61	2.91	4.18	3.98	2.74	3.46	1.43	1.42	0.70
3. Fort Ridgely	44 30	94 45	1230	1.61	1.29	1.72	1.88	3.01	2.57	2.64	3.90	3.17	1.49	1.20	1.21
4. Hazlewood[4]	45 00	95 30	..	1.25	0.69	1.74	2.22	3.82	4.31	3.65	3.88	3.11	1.63	1.73	1.04
5. St. Anthony's Falls	45 00	93 15	820	0.80	0.05	2.48	4.13	5.37	2.69	4.66	2.05	3.19	..	0.70	..

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[1] Milwaukee consolidated series: Jan. 1.56; Feb. 1.33; March, 1.97; April, 2.74; May, 3.27; June, 3.55; July, 3.36; Aug. 2.73; Sept. 3.23; Oct. 2.36; Nov. 2.37; Dec. 1.93; Spring, 7.98; Summer, 9.64; Autumn, 7.96; Winter, 4.82; Year, 30.40; Extent, 23 yrs. 4 mos.; Date, 1841—1866. From two series by Dr. Lapham.

WISCONSIN.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning. End.	Observer.	Reference.
1	9.00	14.42	7.84	3.36	34.62	7	6	Jan. 1836; May, 1852	Asst. Surgeon	Army Met. Reg. 1855.
2	5.58	11.46	7.63	2.82	27.49	9	0	Aug. 1836; Aug. 1845	“ “	“ “ “ “
3	7.63	11.87	7.90	4.00	31.40	9	3	May, 1836; Aug. 1845	“ “	“ “ “ “
4	8.06	9.89	7.91	4.27	30.13	16	4	Jan. 1841; Dec. 1859	Dr. E. S. Marsh, I. A. Lapham, and Dr. Carl Winkler	Bulletin of Wis. Agr. & Mech. Association, 1860.
5	7.80	8.76	8.02	5.81	30.39	7	5	Aug. 1859; Dec. 1866	Dr. I. A. Lapham	Surv. of N. and N.W. Lakes.
6	9.85	10.99	9.99	4.90	35.73	5	6	May, 1861; Dec. 1866	Dr. C. Winkler	Sm. Obs.
7	6.30	11.65	7.50	4.80	30.25	1	0	1857	N. C. Daniels	Am. Alm. 1859.
8	6.14	8.68	7.64	3.18	25.64	7	9	July, 1856; Dec. 1866	L. Washington, G. R. Stuntz, E. H. Bly	P. O. and S. I. Vol. 1, Survey N. and N. W. Lakes.
9	..	9.51	..	..	..	1	9	Apr. 1860; June, 1863	W. Mann	Sm. Obs.
10	8.11	10.66	7.90	4.98	31.65	10	7	Jan. 1854; Dec. 1866	Prof. W. Porter, H. S. Kelsey, H. D. Porter, and others	P. O. and S. I. Vol. 1, and S.O.
11	..	..	11.81	..	..	0	7	July, 1854; May, 1855	M. E. Baker	P. O. and S. I. Vol. 1.
12	7.29	10.38	7.42	5.96	31.05	4	11	Jan. 1854; Dec. 1858	J. F. Willard	“ “ “ “
13	13.73	17.87	10.68	6.07	48.35	4	0	1850—1853	S. P. Lathrop	MS. in Sm. Coll'n.
14	9.88	13.63	9.32	6.48	39.31	6	10	Aug. 1851; Nov. 1859	Dr. J. L. Pickard	MS. and P. O. and S. I. Vol. 1.
15	7.46	10.24	7.43	5.93	31.06	4	7	Jan. 1856; Sept. 1861	Prof. R. Z. Mason	P. O. and S. I. Vol. 1, and S. O.
16	9.98	10.81	8.61	3.81	33.21	1	6	Jan. 1852; June, 1853	B. F. Mills	MS. in Sm. Coll'n.
17	13.50	13.95	13.93	5.07	46.45	4	2	July, 1856; Mar. 1862	Dr. E. Ellis	P.O. and S. I. Vol. 1, and S. O.
18	10.63	13.20	12.76	10.95	47.54	1	1	Mar. 1856; Mar. 1857	J. E. Himoe	P. O. and S. I. Vol. 1.
19	5.32	..	6.92	..	..	0	8	1856	Prof. S. A. Bean, Dr. L. C. Slye	“ “ “ “
20	..	..	..	..	..	0	1	1857	M. T. W. Chandler	“ “ “ “
21	..	..	..	..	..	0	5	Oct. 1857; Mar. 1858	Col. D. Underwood	“ “ “ “
22	10.15	11.07	8.06	5.47	34.75	1	7	Jan. 1857; Aug. 1858	J. E. Breed	“ “ “ “
23	..	..	6.52	..	..	0	6	Sept. 1858; Mar. 1859	J. H. Nourse	“ “ “ “
24	..	8.87	12.52	..	..	1	1	June, 1858; Nov. 1859	D. Underwood	“ “ “ “
25	6.03	9.81	11.52	4.46	31.82	1	9	Jan. 1864; Sept. 1865	F. Deckner	Sm. Obs.
26	10.51	8.52	9.56	4.88	33.47	4	0	Mar. 1858; June, 1863	Rev. J. Gridley	P.O. and S. I. Vol. 1, and S. O.
27	6.22	11.09	7.42	5.56	30.29	4	1	Feb. 1858; Dec. 1866	J. Lüps	“ “ “ “ “
28	9.24	11.47	7.72	5.69	34.12	6	10	Nov. 1859; Dec. 1866	W. W. Curtis	“ “ “ “ “
29	..	..	..	..	..	0	3	1859	Dr. W. A. Gordon	P. O. and S. I. Vol. 1.
30	..	..	..	..	..	0	2	1860	S. Armstrong	Sm. Obs.
31	6.46	..	6.56	4.88	..	1	6	Jan. 1861; May, 1864	Prof. J. W. Sterling, W. Fellows	“ “
32	11.11	8.97	9.49	5.31	34.88	2	0	Jan. 1861; June, 1863	A. W. Clarke	“ “
33	..	..	..	..	..	0	1	1861	J. H. Powers	“ “
34	6.73	17.85	14.23	5.44	44.25	1	5	Mar. 1861; Aug. 1866	M. Parker	“ “
35	6.32	13.29	8.82	5.65	34.08	2	11	Jan. 1864; Dec. 1866	J. E. Breed	“ “
36	6.88	13.29	7.45	4.39	32.01	2	4	Sept. 1864; Dec. 1866	L. Eddy	“ “
37	..	..	..	..	..	0	2	Nov. 1864; Jan. 1865	F. Hachez	“ “
38	8.70	14.00	11.35	4.55	38.60	1	8	Jan. 1865; Dec. 1866	G. Moeller	“ “
39	..	..	..	..	..	0	1	1865	J. C. Hicks	“ “

MINNESOTA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning. End.	Observer.	Reference.
1	6.68	10.50	6.38	2.26	25.82	22	4	July, 1836; Dec. 1867	Asst. Surgeon	Army Met. Reg. 1855, MD. of U.S.A. 1860 & MS. fr'm S.G.O.
2	5.86	10.90	6.31	2.04	25.11	16	10	Jan. 1850; Dec. 1867	“ “	“ “ “ “ “
3	6.61	9.11	5.86	4.11	25.69	13	0	July, 1853; Dec. 1866	“ “	“ “ “ “ “
4	7.78	11.84	6.47	2.98	29.07	5	2	May, 1854; July, 1862	Rev. S. R. Riggs, A.W. Huggins, & J. B. Riggs	P.O. and S. I. Vol. 1, and S.O.
5	11.98	9.40	..	..	..	0	10	1854	Dr. C. F. Anderson	P. O. and S. I. Vol. 1.

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[2] Amount of rain unusually high in this series.

[3] Also called Odanah.

[4] Or Lac-qui-parle, also called (after 1860) Oomahoo and Dakota Mission.

MINNESOTA.—Continued.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
6. Red Wing, Hamlin University	44°34′	92°30′	..	..	..	..	4.77	..	..	..	..	..	..	1.37	1.25
7. Princeton	45 50	93 45	..	2.55	0.83	1.83	2.76	4.06	3.80	2.50	3.88	4.98	1.54	2.15	..
8. Lake Winnebigoshish	47 30	94 40	..	0.97	7.07	0.85	..	1.42	..	..	..	..	..	..	..
9. Wabashaw	44 30	92 15	850	2.13	0.63	1.20	..	3.40	4.11	7.10	1.30	..	..	..	1.44
10. Beaver Bay	47 12	91 18	1270	2.61	0.86	1.82	2.78	3.29	2.76	2.65	3.15	3.10	2.15	1.02	1.01
11. Burlington	47 01	92 30	645	1.51	1.71	2.26	2.47	6.47	4.33	5.42	4.75	3.48	4.24	2.34	1.37
12. Chatfield	43 50	92 25	900	0.86	3.72	3.62	4.95	5.40	6.11	4.24	3.26	4.69	2.69	2.58	1.24
13. Forest City	45 45	96 00	..	0.70	..	1.52	1.98	5.95	2.69	5.24	5.30	3.58	2.38	1.94	1.15
14. Stillwater	45 04	92 45	756	..	..	..	..	..	..	..	..	..	..	1.54	..
15. St. Cloud	45 45	94 23	..	..	..	..	..	6.83	6.29	2.08	1.54	5.28	..	..	..
16. Itasca	45 16	93 32	856	1.20	2.12	1.30	2.55	5.85	2.95	3.45	..	..	..	..	4.40
17. White Bear Lake, near	45 36	95 26	..	..	1.47	1.04	..	3.20	..	..	..	..	..	..	0.27
18. Hastings	44 45	93 00	..	1.49	0.57	1.46	3.41	1.28	5.42	2.43	2.51	2.35	1.63	2.55	0.41
19. Pope County[1]	45 43	95 28	..	0.80	0.54	3.37	5.41	3.13	..	..	..	..	..	..	..
20. Douglas County[2]	45 48	95 23	..	..	..	..	..	..	2.96	..	..	..	..	..	..
21. Saint Paul	44 55	93 05	800	0.98	0.72	1.67	1.98	1.98	3.40	3.93	3.78	1.88	2.31	1.27	1.19
22. New Ulm	44 16	94 26	1500	0.83	1.08	1.91	3.04	2.49	3.33	5.62	4.29	1.67	2.04	0.77	0.77
23. Sibley	44 31	94 25	..	3.50	..	0.61	1.93	1.88	2.55	3.69	..	8.01	3.76	1.41	0.70
24. Minneapolis	44 58	93 10	856	2.05	0.19	1.07	2.59	0.07	7.27	1.54	5.74	2.19	1.94	2.35	0.33

IOWA.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Fort Atkinson	43 00	92 00	700	0.71	0.83	2.54	4.68	5.04	6.68	8.67	5.08	2.81	1.51	0.50	0.73
2. Fort Des Moines	41 32	93 38	780	0.95	0.67	1.42	3.83	1.61	6.58	2.01	2.34	3.58	1.00	0.32	2.25
3. Fort Dodge (Clarke)	42 28	94 03	880	0.65	0.42	1.43	3.04	3.45	5.16	1.57	1.42	2.55	3.26	2.38	1.49
4. Iowa City	41 37	91 30	621	2.08	1.69	2.92	4.46	3.89	3.91	4.12	5.01	5.17	4.95	2.99	1.93
5. Fort Madison, near[3].	40 37	91 28	600	2.01	2.68	3.01	3.79	4.32	4.38	4.33	4.31	4.71	3.19	2.74	2.49
6. Muscatine[4]	41 26	91 05	586	1.83	2.47	3.14	4.33	4.45	4.77	3.75	5.27	4.05	3.39	2.91	2.52
7. Poultney	42 40	91 21	..	1.06	0.96	2.01	3.17	5.09	4.52	5.59	3.04	2.98	2.48	2.43	1.04
8. Dubuque[5]	42 30	90 40	666	2.04	1.64	2.01	2.09	3.00	3.69	3.97	2.87	3.97	3.30	2.11	1.55
9. Quasqueton	42 23	91 43	888	0.74	0.52	1.33	0.32	3.96	4.20	3.15	2.29	2.16	1.79	..	0.20
10. St. Mary's	41 00	95 45	1200	..	1.25	..	..	..	..	..	..	..	..	..	..
11. Bellevue	42 15	90 25	..	1.54	2.04	2.21	2.87	4.69	3.93	2.52	2.69	2.70	3.07	3.61	2.58
12. Border Plains	42 36	94 05	..	0.38	1.75	2.75	4.50	5.15	5.62	6.34	3.96	2.48	5.12	1.15	1.10
13. Pleasant Plain	41 07	91 55	950	1.19	1.54	2.38	3.43	4.01	4.06	5.31	3.92	3.23	4.08	2.09	1.57
14. Fairfield	41 01	91 57	940	1.90	2.47	4.17	4.85	5.07	5.37	6.40	3.73	3.98	3.14	4.96	1.90
15. Maquoketa	42 04	90 41	..	..	..	..	..	..	..	..	..	..	..	4.75	..
16. Rossville	43 10	91 21	1400	1.53	0.91	2.47	4.03	4.15	7.40	5.20	2.45	2.67	1.56	3.56	1.31
17. Sioux City	42 35	96 27	1258	0.67	0.43	0.98	2.97	3.16	3.09	3.94	3.51	3.54	1.45	1.55	0.37
18. Burlington	40 49	91 07	486	2.30	0.60	..	..	..	..	..	1.60	3.29	0.55	0.78	..
19. Lyons City[6]	41 50	90 10	630	1.82	3.09	3.24	4.87	3.29	3.41	4.97	5.56	4.94	3.55	1.91	3.07
20. Fayette Village	42 51	91 51	1000	..	0.55	..	..	..	..	..	..	..	..	..	..
21. Hesper	43 30	91 46	720	1.87	1.65	1.59	..	..	..	..	..	3.75	4.00	3.09	0.60
22. Franklin	42 45	87 16	..	1.50	2.35	..	..	..	..	..	..	..	..	..	..
23. Mt. Vernon	41 58	91 28	..	2.00	1.90	..	..	..	..	..	..	..	..	..	..
24. Davenport, Griswold College	41 31	90 40	737	1.86	1.78	2.39	5.09	3.22	4.41	4.24	5.06	4.24	3.11	1.21	2.85

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[1] Township 126 N. Range 38 W. Sec. 17.

[2] Township 127 N. Range 37 W.

[3] Four miles N. W. of Fort Madison.

[4] Formerly Bloomington.

[5] Dubuque, consolidated series: Jan. 1.81; Feb. 1.48; March, 2.13; April, 2.31; May, 3.69; June, 4.59; July, 4.04; Aug.

MINNESOTA.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series. yrs.	mos.	DATE. Beginning. End.	OBSERVER.	REFERENCE.
6	. .	. .	. .	. .	. .	0	3	Nov. 1855; Apr. 1856	Rev. J. Brooks	P. O. & S. I. Vol. 1.
7	8.65	10.18	8.67	. .	. .	2	9	Jan. 1856; Aug. 1860	O. E. Garrison, S. M. Byers	P.O. and S. I. Vol. 1, and S.O.
8	. .	. .	. .	. .	. .	0	4	1857	Rev. B. F. Odell	P. O. and S. I. Vol. 1.
9	. .	12.51	. .	4.20	. .	0	8	Dec. 1857; Aug. 1858	Rev. S. L. Hillier	" " " "
10	7.89	8.56	6.27	4.48	27.20	6	7	Nov. 1858; Dec. 1866	T. Clarke, H. Wieland	P. O. and S. I. Vol. 1, and S.O.
11	11.20	14.50	10.06	4.59	40.35	2	7	Jan. 1858; Oct. 1860	A. A. Hibberd	" " " " "
12	13.97	13.61	9.96	5.82	43.36	1	8	May, 1859; May, 1861	T. F. Thickstun	" " " " "
13	9.45	13.23	7.90	. .	. .	3	8	June, 1858; Apr. 1866	A.C. Smith, H.L.Smith	" " " " "
14	. .	. .	. .	. .	. .	0	1	1858	A. Van Vorhes	P. O. and S. I. Vol. 1.
15	. .	9.91	. .	. .	. .	0	8	May, 1860; Aug. 1861	O. E. Garrison	Sm. Obs.
16	9.70	. .	. .	7.72	. .	0	8	Dec. 1860; July, 1861	O. H. Kelley	" "
17	. .	. .	. .	. .	. .	0	4	1861	O. E. Garrison	" "
18	6.15	10.36	6.53	2.47	25.51	1	0	June, 1861; May, 1862	T. F. Thickstun	" "
19	11.91	. .	. .	. .	. .	0	5	1862	O. E. Garrison	" "
20	. .	. .	. .	. .	. .	0	1	1862	" "	" "
21	5.63	11.11	5.46	2.89	25.09	5	0	Aug. 1859; Dec. 1866	A. B. Paterson, J. W. Heimstreet	" "
22	7.44	13.24	4.48	2.68	27.84	2	9	Mar. 1864; Dec. 1866	C. Roos	" "
23	4.42	. .	13.18	. .	. .	1	3	May, 1865; Nov. 1866	C.W. & C.E. Woodbury	" "
24	3.73	14.60	6.48	2.57	27.38	1	0	1866	W. Cheney	" "

IOWA.

	Spring.	Summer.	Autumn.	Winter.	Year.	yrs.	mos.	Date.	Observer.	Reference.
1	12.26	20.43	4.82	2.27	39.78	2	1	May, 1844; May, 1846	Asst. Surgeon	Army Met. Reg. 1855.
2	6.86	10.93	4.90	3.87	26.56	1	5	June, 1844; Feb. 1846	" "	" " " "
3	7.92	8.15	8.19	2.56	26.82	1	10	Aug. 1851; May, 1853	" "	" " " "
4	11.27	13.04	13.11	5.70	43.12	7	4	June, 1857; Dec. 1866	Dr. W. Reynolds, Prof. T. S. Parvin	P. O. and S. I. Vol. 1, and S.O.
5	11.12	13.02	10.64	7.18	41.96	17	7	Mar. 1848; Dec. 1866	Daniel McCready	MS., P.O. & S.I. Vol. 1, & S.O.
6	11.92	13.79	10.35	6.82	42.88	19	0	Jan. 1846; July, 1866	T. S. Parvin, Rev. J. Ufford, J. P. Walton, S. Foster	Am. Alm. 1849, P. O. and S. I. Vol. 1, and S.O.
7	10.27	13.15	7.89	3.06	34.37	3	7	Apr. 1853; Aug. 1858	Rev. B. F. Odell	MS., P. O. and S. I. Vol. 1.
8	7.10	10.53	9.38	5.23	32.24	12	2	Oct. 1852; Dec. 1866	Dr. A. Horr	P. O. and S. I. Vol. 1, and S. O.
9	5.61	9.64	. .	1.46	. .	1	4	Jan. 1854; Oct. 1855	Dr. E. C. Bidwell	P. O. and S. I. Vol. 1.
10	. .	. .	. .	. .	. .	0	1	1854	D. E. Read	" " " "
11	9.77	9.14	9.38	6,16	34.45	4	6	Jan. 1856; Aug. 1860	J. C. Fory	P. O. and S. I. Vol. 1, and S.O.
12	12.40	15.92	8.75	3.23	40.30	1	11	July, 1856; Sept. 1859	W. K. Goss	P. O. and S. I. Vol. 1.
13	9.82	13.29	9.40	4.30	36.81	9	0	Jan. 1856; Sept. 1865	T. McConnel	P. O. and S. I. Vol. 1, and S. O.
14	14.09	15.50	12.08	6.27	47.94	2	8	May, 1857; Dec. 1859	S. McBeth, Dr. J. M. Shaffer	P. O. and S. I. Vol. 1.
15	. .	. .	. .	. .	. .	0	1	1857	E. F. Hobart	" " " "
16	10.65	15.05	7.79	3.75	37.24	2	0	Nov. 1857; Dec. 1859	C. D. Beaman	" " " "
17	7.11	10.54	6.54	1.47	25.66	3	2	Sept. 1857; Mar. 1863	Dr. J. J. Saville, A. J. Millard	P. O. and S. I. Vol. 1, and S.O.
18	. .	. .	4.62	. .	. .	0	6	Aug. 1859; Feb. 1860	J. M. Corse	" " " " " "
19	11.40	13.94	10.40	7.98	43.72	6	10	Aug. 1859; Dec. 1866	Dr. A. T. Hudson, Dr. P. J. Farnsworth	" " " " " "
20	. .	. .	. .	. .	. .	0	1	1860	J. M. McKenzie	Sm. Obs.
21	. .	. .	10.84	4.12	. .	0	7	Sept. 1860; Mar. 1861	H. B. Williams	" "
22	. .	. .	. .	. .	. .	0	2	1861	D. Beal	" "
23	. .	. .	. .	. .	. .	0	2	1861	Prof. A. Collin	" "
24	10.70	13.71	8.56	6.49	39.46	5	6	Jan. 1861; Dec. 1866	Dr. J. Langer, W. P. Dunwoody, J. Chamberlain, G. B. Pratt, and others	" "

2.96; Sept. 3.85; Oct. 3.08; Nov. 2.13; Dec. 1.40; Spring, 8.13; Summer, 11.59; Autumn, 9.06; Winter, 4.69; Year, 33.47; Extent, 15 years; Date, 1851—1866.

[6] Also known as Clinton.

IOWA.—Continued.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
25. Algona	43°01′	94°04′	1500	0.53	3.58	1.00	2.80	3.23	3.82	2.40	5.76	2.58	1.60	2.10	1.20
26. Bangor	42 00	93 00	. .	. .	. .	. .	. .	. .	. .	. .	3.62	13.00	. .	. .	. .
27. Brookside or Byron .	42 29	91 50	. .	1.77	3.64	3.05	3.62	4.60	4.77	5.92	5.29	4.78	4.65	2.27	2.06
28. Vernon Springs . . .	43 20	92 12	. .	. .	1.22	1.18	. .	3.89	1.14	. .	. .	2.20	2.94	0.71	1.51
29. Spring Grove, near .	42 32	93 20	. .	2.40	2.69	1.37	4.19	3.07	10.54	5.60	4.11	3.83	3.36	1.94	1.69
30. Mount Pleasant . . .	40 58	91 38	. .	0.75	0.44	2.14	4.24	3.20	0.77	6.25	1.85	. .	. .	. .	3.12
31. Monticello, near . .	42 15	91 15	800	1.58	1.65	2.55	4.04	2.00	5.41	4.35	4.48	2.41	3.03	2.51	1.99
32. Waterloo	42 30	92 22	. .	. .	. .	. .	. .	. .	. .	2.75	. .	1.87	. .	. .	. .
33. Guttenburg	42 48	91 10	. .	0.09	2.53	. .	1.40	. .	. .	. .	3.00	6.80	2.17	3.80	2.72
34. Des Moines	41 35	93 37	830	1.30	0.63	0.68	4.25	0.75	4.00	0.50	3.12	3.43	4.00	0.37	0.91
35. Manchester	42 30	91 30	925	2.64	0.66	0.97	2.08	1.35	4.67	6.51	4.79	3.01	3.72	0.82	1.25
36. Clinton	41 40	90 15	630	2.80	4.10	3.75	2.90	2.65	4.80	6.20	7.75	. .	3.60	0.75	3.25
37. Vawtersgrove	41 30	95 35	1500	. .	. .	. .	. .	3.31	9.12	3.62	2.75	6.50	1.87	0.75	1.44
38. Harrisgrove	41 40	95 50	900	. .	. .	. .	. .	1.20	3.10	3.00	1.30	4.40	. .	. .	. .
39. Dubuque[1]	42 30	90 40	666	0.96	0.84	2.63	3.19	6.46	8.48	4.30	3.34	3.36	1.63	2.28	0.80

MISSOURI.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Jefferson Barracks .	38 28	90 15	472	2.27	2.16	3.01	3.13	4.67	4.97	4.79	3.86	3.27	3.10	3.23	2.42
2. St. Louis Arsenal . .	38 34	90 15	450	1.99	2.24	3.85	4.04	4.93	6.47	4.07	3.47	2.44	3.19	3.23	2.71
3. St. Louis	38 37	90 16	481	2.06	2.57	3.75	3.85	4.82	5.21	3.92	3.91	3.08	3.06	2.93	3.02
4. St. Louis, College Hill[2]	38 40	90 15	475	2.20	1.85	2.86	2.70	2.73	3.58	3.22	2.84	3.86	2.53	1.35	2.67
5. Edina	40 10	92 18	. .	1.74	1.35	2.30	3.81	1.94	3.88	3.31	2.12	4.63	1.82	1.18	2.40
6. Rolla	37 58	91 50	. .	1.92	3.70	2.40	6.12	2.85	2.37	6.36	1.50	13.75	2.18	0.58	1.10
7. Hannibal	39 44	91 23	. .	0.31	1.23	2.90	2.15	5.75	4.50	1.25	1.84	1.90	2.91	0.50	0.89
8. Cape Girardeau[3] . .	37 20	89 34	. .	4.00	4.83	. .	. .	4.39	2.71	5.85	4.97	0.94	2.63	5.94	3.11
9. Palmyra[4]	39 47	91 37	. . .	. .	. .	. .	. .	1.87	1.72	2.76	1.89	2.37	0.51	4.49	5.66
10. St. Joseph	39 40	94 40	. .	1.13	. .	. .	. .	1.20	1.94	2.89	2.95	. .	2.95	. .	. .
11. Springfield	37 12	93 12	. .	. .	1.50	3.10	3.26	. .	. .	4.48	. .	. .	. .	. .	. .
12. Rhineland[5]	38 42	91 46	800	. .	2.44	0.84	2.04	1.36	. .	. .	. .	. .	. .	. .	1.08
13. Cassville	36 41	93 56	3000	. .	1.82	1.80	3.40	4.51	1.68	1.69	0.80	. .	0.85	2.18	1.97
14. Paris, near	39 30	92 00	700	3.80	1.50	0.90	1.80	1.20	4.55	1.40	1.80	2.40	1.30	0.50	1.30
15. Stockton	37 36	93 48	800	. .	4.38	. .	1.25	. .	. .	. .	. .	. .	. .	. .	1.95
16. Warrenton	38 37	91 15	825	. .	0.41	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
17. Dundee[6]	38 30	91 10	536	2.55	5.61	14.50	11.50	5.75	7.00	. .	3.75	. .	4.50	5.50	4.87
18. Tower Grove, near St. Louis	38 36	90 20	500	2.15	1.74	3.75	3.95	2.86	3.22	3.23	2.62	5.43	2.27	2.50	3.48
19. Luray, near	40 28	91 57	. .	. .	. .	3.69	3.65	. .	. .	. .	. .	. .	. .	. .	. .
20. Wyaconda Prairie .	40 12	91 37	. .	1.64	1.84	4.12	7.61	1.78	3.33	5.41	2.81	6.46	4.64	1.83	3.89
21. Harrisonville	38 38	94 25	. .	0.85	2.53	2.82	6.98	4.12	6.30	7.34	4.12	7.03	2.91	2.25	2.16
22. Athens	40 30	91 45	. .	1.31	3.10	3.07	4.73	1.19	2.19	3.95	1.70	2.96	4.95	2.75	1.43
23. Laborville	38 33	90 43	. .	. .	1.30	3.73	16.10	7.00	2.10	. .	. .	. .	. .	. .	. .
24. Allenton, near . . .	38 29	90 45	482	0.65	4.27	5.92	3.78	3.83	3.95	4.78	2.60	6.13	2.52	2.69	2.20
25. Easton	39 46	94 22	. .	1.73	0.97	. .	. .	1.10	8.40	6.67	4.43	6.64	2.74	0.63	1.46
26. Edinburg	40 07	93 50	. .	. .	. .	. .	. .	. .	. .	. .	. .	2.79	2.10	3.00	2.25
27. Union	38 25	91 09	616	3.20	. .	2.17	4.39	2.50	4.36	5.64	1.46	7.20	2.40	1.11	2.05

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[1] Dubuque, consolidated series: Jan. 1.81; Feb. 1.48; March, 2.13; April, 2.31; May, 3.69; June, 4.59; July, 4.04; Aug. 2.96; Sept. 3.85; Oct. 3.08; Nov. 2.13; Dec. 1.40; Spring, 8.13; Summer, 11.59; Autumn, 9.06; Winter, 4.69; Year, 33.47; Extent, 15 years; Date, 1851—1866.

[2] This series appears to give a smaller amount than other stations in the vicinity, it has therefore not been combined with the preceding one.

IOWA.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
25	7.03	11.98	6.28	5.31	30.60	3	4	June, 1861;	July, 1865	Dr. F. McCoy and E. McCoy	Sm. Obs.
26	..	..	..	..	..	0	2	1861		Isaac M. Gidley	“ “
27	11.27	15.98	11.70	7.47	46.42	4	7	Apr. 1862;	Dec. 1866	A. C. Wheaton, D. B. Wheaton	“ “
28	..	..	5.85	..	..	0	8	Sept. 1862;	June, 1863	G. Marshall	“ “
29	8.63	20.25	9.13	6.78	44.79	3	2	Nov. 1863;	Dec. 1866	N. Townsend.	“ “
30	9.58	8.87	..	4.31	..	0	10	Dec. 1863;	Dec. 1864	E. L. Briggs, H. Briggs	“ “
31	8.59	14.24	7.95	5.22	36.00	2	2	July, 1864;	Dec. 1866	C. Mead	“ “
32	..	..	..	..	..	0	2	1864		T. Steed.	“ “
33	..	..	12.77	5.34	..	0	9	Aug. 1864;	Oct. 1865	P. Dorweiler	“ “
34	5.68	7.62	7.80	2.84	23.94	1	4	Sept. 1865;	Dec. 1866	Rev. J. A. Nash	“ “
35	4.40	15.97	7.55	4.55	32.47	1	4	Sept. 1865;	Dec. 1866	A. Mead	“ “
36	9.30	18.75	..	10.15	..	1	2	Oct. 1865;	Dec. 1866	P. J. Farnsworth	“ “
37	..	15.49	9.12	..	..	0	8	1866		A. F. Bryant	“ “
38	..	7.40	..	..	..	0	5	1866		J. F. Stern	“ “
39	12.28	16.12	7.27	2.60	38.27	3	0	1851—1853		Dr. A. Horr	MS. in Sm. Coll'n.

MISSOURI.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	10.81	13.62	9.60	6.85	40.88	20	11	July, 1840;	July, 1862	Asst. Surgeon	Ar. Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
2	12.82	14.01	8.86	6.94	42.63	18	8	July, 1836;	Nov. 1856	“ “	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
3	12.42	13.04	9.07	7.65	42.18	28	3	Jan. 1837;	Dec. 1867	Dr. G. Engelmann, Dr. Wislizenus	MS. in Sm. Coll'n, P. O. and S. I. Vol. 1, & Sm. Obs.
4	8.29	9.64	7.74	6.72	32.39	6	5	Feb. 1860;	Dec. 1866	J. H. Lunemann and others	Sm. Obs.
5	8.05	9.31	7.63	5.49	30.48	7	8	May, 1859;	Dec. 1866	J. C. Agnew	Sm. Coll'n.
6	11.37	10.23	16.51	6.72	44.83	1	0	Apr. 1866;	Mar. 1867	H. Ruggles	“ “
7	10.80	7.59	5.31	2.43	26.13	1	4	Jan. 1854;	May, 1855	O. H. P. Lear	P. O. and S. I. Vol. 1.
8	..	13.53	9.51	11.94	..	1	1	Oct. 1856;	Jan. 1858	Rev. J. Knoud	“ “ “ “
9	..	6.37	7.37	..	..	1	1	June, 1856;	Nov. 1857	G. P. Comings	“ “ “ “
10	..	7.78	..	..	..	0	6	May, 1857;	Jan. 1858	E. B. Neely	“ “ “ “
11	..	..	..	..	..	0	4	July, 1857;	Apr. 1858	J. A. Stephens	“ “ “ “
12	4.24	..	..	..	..	0	5	Dec. 1859;	May, 1860	C. Vogel	P. O. and S. I. Vol. 1, and S. O.
13	9.71	4.17	..	..	..	1	2	Feb. 1860;	June, 1861	M. L. Wyrick	Sm. Obs.
14	3.90	7.75	4.20	6.60	22.45	1	4	Feb. 1860;	Feb. 1862	W. F. Maxey	“ “
15	..	..	..	..	..	0	4	Feb. 1860;	Feb. 1861	W. Wells	“ “
16	..	..	..	..	..	0	1	1860		M. A. Tidswell	“ “
17	31.75	..	..	13.03	..	0	11	May, 1860;	June, 1861	S. S. Bailey	“ “
18	10.56	9.07	10.20	7.37	37.20	2	5	Jan. 1861;	Jan. 1864	A. Fendler	“ “
19	..	..	..	..	..	0	2	1861		B. P. Hanan	“ “
20	13.51	11.55	12.93	7.37	45.36	4	1	Apr. 1862;	Aug. 1866	G. P. Ray	“ “
21	13.92	17.76	12.19	5.54	49.41	2	10	Oct. 1863;	Dec. 1866	J. Christian	“ “
22	8.99	7.84	10.66	5.84	33.33	2	1	Jan. 1864;	July, 1866	J. T. Caldwell	“ “
23	26.83	..	..	..	..	0	5	1864		W. Muir	“ “
24	13.53	11.33	11.34	7.12	43.32	1	11	July, 1864;	Dec. 1866	A. Fendler	“ “
25	..	19.50	10.01	4.16	..	1	7	Sept. 1864;	Nov. 1866	P. B. Sibley	“ “
26	..	..	7.89	..	..	0	4	1866		J. E. Vertrees	“ “
27	9.06	11.46	10.71	..	..	0	11	1866		B. Moore	“ “

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[3] At St. Vincent's College.

[4] St. Paul's College.

[5] Vogel's farm near Rhineland.

[6] These observations probably overmeasured.

INDIAN TERRITORY.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Towson	34°00′	95°33′	300	3.13	2.97	4.38	5.33	5.84	5.78	4.62	3.96	3.41	4.59	4.23	2.84
2. Fort Washita	34 14	96 38	645	1.77	2.72	2.53	3.40	5.31	4.33	3.24	2.86	3.56	2.94	3.52	1.86
3. Fort Gibson	35 48	95 07	560	1.96	2.32	2.39	4.24	4.61	4.00	2.55	2.81	2.37	3.91	3.08	2.13
4. Fort Arbuckle	34 27	97 09	1000	0.89	2.98	1.12	2.39	4.46	3.60	3.13	4.12	3.38	2.08	2.97	1.57
5. Baptist Mission	35 . .	97 . .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	2.20	4.10
NEBRASKA.															
1. Fort Kearny	40 38	98 57	2360	0.59	0.43	1.25	2.26	4.30	3.69	4.74	2.70	2.29	1.57	0.97	0.46
2. Omaha	41 15	96 10	1300	1.47	1.33	1.77	3.72	4.69	4.24	6.48	4.74	2.06	2.42	1.31	0.79
3. Bellevue	41 08	95 50	. .	1.31	1.43	2.95	3.39	2.86	3.40	3.89	1.67	3.06	2.12	1.12	1.33
4. Nebraska City	40 40	95 43	1005	. .	. .	. .	. .	5.69	4.22	2.08	1.49	3.50	2.13	2.27	0.20
5. Pioneer Grove	41 20	95 57	1400	. .	. .	. .	0.74	2.13	3.19	2.75	. .	. .	. .	. .	. .
6. Washington County[1]	41 22	96 12	1025	2.22	0.60	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
7. Rock Bluffs	40 53	95 54	1100	3.30	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
8. Fontenelle	41 31	96 45	1000	. .	. .	. .	. .	. .	. .	. .	. .	1.05	0.90	2.35	2.90
9. Nursery Hill	40 40	95 50	1266	. .	7.75	3.86	6.72	2.94	. .	. .	. .	. .	. .	. .	. .
10. Ionia	42 41	96 54	3000	. .	. .	. .	. .	. .	. .	6.50	1.50	. .	. .	. .	. .
11. Glendale	40 55	96 05	1300	. .	. .	2.53	3.19	2.85	. .	2.80	3.22	5.65	. .	1.65	1.60
KANSAS.															
1. Fort Scott	37 45	94 45	1000	1.92	1.18	1.79	3.70	7.08	8.13	4.55	3.69	2.30	2.66	3.46	1.69
2. Fort Leavenworth	39 21	94 48	896	0.92	1.33	1.32	2.47	3.47	5.48	3.79	3.91	3.62	1.84	2.23	1.36
3. Fort Atkinson	37 47	100 14	2330	0.04	0.49	0.96	3.38	9.34	4.35	. .	2.80	3.85	6.81	1.39	1.60
4. Fort Riley	39 03	96 35	1300	0.77	1.01	0.75	1.74	3.01	3.93	3.00	3.22	3.09	1.21	1.15	0.74
5. Council City	38 42	95 50	. .	0.65	3.60	2.60	0.33	2.69	1.55	3.24	3.86	. .	. .	. .	. .
6. Leavenworth City	39 15	94 52	787	1.85	1.19	2.24	2.90	5.51	11.72	10.55	3.24	5.47	4.69	2.99	1.59
7. Burlingame	38 42	95 45	. .	1.43	0.77	2.24	1.97	3.00	5.40	3.43	3.24	1.60	3.71	1.76	1.91
8. Manhattan[2]	39 13	95 45	1000	1.24	1.05	2.42	2.35	3.66	4.47	4.51	3.53	3.01	1.62	1.38	1.29
9. Neosho Falls	38 03	95 31	. .	1.25	1.42	1.61	2.59	5.46	5.75	4.48	2.68	3.06	0.55	0.71	1.62
10. Mapleton	38 04	94 51	. .	3.02	. .	. .	. .	3.09	4.57	4.25	1.10	1.70	. .	. .	. .
11. Topeka	39 03	95 39	. .	. .	. .	2.62	3.50	1.12	5.25	2.62	. .	. .	. .	. .	. .
12. Fort Larned	38 10	98 57	1932	0.27	1.12	0.48	0.98	3.82	4.54	5.21	2.92	2.50	1.28	1.34	0.42
13. Fort Harker	38 43	98 15	. .	0.24	0.15	0.13	0.30	4.21	1.65	2.74	. .	. .	. .	. .	. .
14. Wyandotte	39 08	94 40	707	2.32	1.99	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
15. Gardner	38 47	95 00	800	. .	0.85	2.85	4.25	0.25	5.89	5.08	1.38	3.52	3.78	. .	1.26
16. Lecompton	39 03	95 09	825	1.50	. .	. .	3.77	8.22	8.10	5.00	. .	. .	. .	. .	. .
17. Lawrence	38 58	95 12	800	1.22	0.68	. .	. .	4.32	6.71	5.14	1.56	2.33	1.52	1.31	2.15
18. Olathe	38 50	94 40	. .	1.33	2.80	2.72	5.36	6.39	9.68	9.64	6.17	9.18	3.65	4.42	2.92
19. Atchison	39 42	95 00	1000	. .	1.40	. .	. .	. .	. .	. .	. .	. .	. .	4.40	4.40
20. Council Grove	38 42	96 32	1000	1.00	0.10	1.45	4.85	8.60	11.70	. .	2.05	5.05	2.53	1.35	1.20
21. Avon	38 08	95 35	775	. .	. .	. .	4.43	3.14	5.46	3.25	. .	. .	. .	. .	. .

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[1] Section 14.

[2] Observations from October, 1864, to December, 1866, at the Manhattan Agricultural College.

INDIAN TERRITORY.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.		Observer.	Reference.
						yrs.	mos.	Beginning.	End.		
1	15.55	14.36	12.23	8.94	51.08	13	10	June, 1836;	Apr. 1854	Assist. Surgeon	Army Met. Reg. 1855.
2	11.24	10.43	10.02	6.35	38.04	15	6	Aug. 1843;	Dec. 1860	" "	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
3	11.24	9.36	9.36	6.41	36.37	20	5	July, 1836;	June, 1857	" "	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860.
4	7.97	10.85	8.43	5.44	32.69	9	9	Oct. 1850;	Sept. 1867	" "	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
5	..	..	..	..	..	0	2	1860		H. F. Buckner	Sm. Obs.
					NEBRASKA.						
1	7.81	11.13	4.83	1.48	25.25	14	3	Mar. 1849;	May, 1866	Assist. Surgeon	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
2	10.18	15.46	5.79	3.59	35.02	2	9	June, 1857;	Feb. 1861	W. N. Byers, J. G. Rain, J. T. Allen	P. O. and S. I. Vol. 1, and S. O.
3	9.20	8.96	6.30	4.07	28.53	8	2	Mar. 1858;	Dec. 1866	W. Hamilton	" " " " " "
4	..	7.79	7.90	..	..	0	8	1859		E. E. Mason	P. O. and S. I. Vol. 1.
5	..	..	..	..	..	0	4	Apr. 1860;	July, 1861	J. G. Rain	Sm. Obs.
6	..	..	..	..	..	0	2	1861		A. M. J. Bowen	" "
7	..	..	..	..	..	0	1	1861		H. C. Pardee	" "
8	..	..	4.30	..	..	0	4	1863		J. Evans	" "
9	13.52	..	..	..	..	0	4	1865		R. O. Thompson	" "
10	..	..	..	..	..	0	2	1865		L. T. Hill	" "
11	8.57	..	..	..	..	0	8	1866		Dr. A. L. & J. Child	" "
					KANSAS.						
1	12.57	16.37	8.42	4.79	42.15	10	3	Jan. 1843;	Mar. 1853	Assist. Surgeon	Army Met. Reg. 1855.
2	7.26	13.18	7.69	3.61	31.74	30	6	May, 1836;	Dec. 1867	" "	Ar. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
3	13.68	..	12.05	2.13	..	1	0	Aug. 1852;	Aug. 1853	" "	Army Met. Reg. 1855.
4	5.50	10.15	5.45	2.52	23.62	13	11	Nov. 1853;	Dec. 1867	" "	Arm. Met. Reg. 1855, M. D. of U.S.A. 1860, MS. from S.G.O.
5	5.62	8.65	..	..	..	0	8	Feb. 1857;	Jan. 1858	E. Fish	P. O. and S. I. Vol. 1.
6	10.65	25.51	13.15	4.63	53.94	2	5	Nov. 1857;	Dec. 1866	E. L. Berthoud, H. D. McCarty, Dr. J. Stayman	P. O. and S. I. Vol. 1, and S. O.
7	7.21	12.07	7.07	4.11	30.46	2	10	Feb. 1858;	Mar. 1861	E. and L. Fish	" " " " " " "
8	8.43	12.51	6.01	3.58	30.53	7	7	Jan. 1858;	Dec. 1866	I. T. Goodnow, H. L. Denison, B. F. Mudge	" " " " " " "
9	9.66	12.91	4.32	4.29	31.18	2	7	Mar. 1859;	Sept. 1861	B. F. Goss	" " " " " " "
10	..	9.92	..	..	..	0	6	1858		Dr. S. O. Himoe	P. O. and S. I. Vol. 1.
11	7.24	..	..	..	..	0	5	1858		F. W. Giles	" " "
12	5.28	12.67	5.12	1.81	24.88	4	11	Jan. 1861;	Dec. 1867	Assist. Surgeon	MS. from S. G. O.
13	4.64	..	..	..	..	0	7	1867		" "	" " " "
14	..	..	..	..	..	0	2	1860		J. H. Millar	Sm. Obs.
15	7.35	10.35	..	..	..	1	2	Apr. 1860;	Feb. 1862	G. F. Merriam, J. Scott	" "
16	..	..	..	..	..	0	5	Jan. 1861;	July, 1866	W. A. McCormick, D. G. Bacon	" "
17	..	13.41	5.16	4.05	..	2	2	Jan. 1861;	Dec. 1864	W. J. R. Blackman, A. N. Fuller, W. L. G. Soule	" "
18	14.47	25.49	17.25	7.05	64.26	2	10	Jan. 1864;	Dec. 1866	W. Beckwith	" "
19	..	..	..	..	..	0	3	1866		Dr. H. B. & C. Horn	" "
20	14.90	..	8.93	2.30	..	0	11	1866		Dr. A. Woodworth	" "
21	..	..	..	..	..	0	4	1866		A. Crocker	" "

DAKOTA.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Randall	43°01′	98°12′	1245	0.49	0.42	0.99	1.10	2.67	2.30	1.78	2.56	2.43	1.09	0.47	0.30
2. Fort Pierre	44 23	100 12	1456	0.50	1.18	0.46	1.63	2.19	0.48	1.18	1.66	1.29	1.08	1.39	0.47
3. Fort or Camp Abercrombie	46 27	96 28	. .	0.52	0.66	1.03	1.82	1.82	2.18	3.01	2.30	1.07	1.30	0.99	0.64
4. Fort Wadsworth. . .	45 43	97 30	. .	. .	. .	. .	. .	7.96	13.80	8.80	2.37	1.65	. .	. .	. .
5. Fort Buford	48 01	103 58	1900	. .	0.70	0.08	0.06	0.42	1.27	. .	0.54	0.10	0.46	0.58	1.12
6. Fort Stevenson . . .	47 30	101 30	. .	. .	. .	. .	. .	. .	. .	. .	0.13	0.03	0.57	0.67	0.97
7. Yankton Ind'n Agency	42 52	98 24	1900	. .	0.22	3.10	5.75	5.90	12.00	. .	. .	. .	. .	. .	1.00
8. Yankton	42 51	97 30	. .	. .	. .	. .	. .	. .	3.62	. .	. .	. .	. .	. .	. .
9. Mouth Cherry[1]. . . .	44 20	100 10	1500	. .	. .	. .	9.30	6.32	. .	. .	. .	. .	. .	. .	. .

WYOMING TERRITORY.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Laramie	42 12	104 31	4519	0.61	0.46	0.84	1.06	3.74	1.90	1.63	1.37	1.17	0.97	0.84	0.57
2. Fort Phil. Kearney . .	44 30	106 50	6000	. .	. .	6.30	2.05	5.00	5.75	2.00	0.30	1.30	1.80	0.42	0.33
3. Deer Creek Agency .	42 49	106 00	5000	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	1.33	0.66
4. Gilbert's Trading Post	42 28	108 40	7400	2.00	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
5. Sweet Water Bridge .	42 30	107 25	7000	. .	. .	. .	2.00	7.40	. .	. .	. .	. .	. .	. .	. .
6. Camp Scott	41 18	110 32	. .	0.32	0.26	0.25	0.52	1.31	1.30	. .	. .	. .	. .	. .	. .

COLORADO.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Massachusetts .	37 32	105 23	8365	0.34	0.86	0.61	1.15	1.19	0.71	2.01	2.84	1.80	0.87	3.61	1.07
2. Fort Garland	37 32	105 40	8365	0.11	0.21	0.33	0.34	0.33	0.77	1.29	1.27	0.76	0.33	0.24	0.13
3. Fort Lyon	38 08	102 50	4000	0.32	0.12	0.16	2.09	4.84	1.40	2.53	0.37	0.04	0.00	0.07	0.15
4. Fort Morgan.	40 15	103 46	4500	0.31	. .	. .	0.00	2.55	0.73	0.52	0.60	0.00	0.19	. .	. .
5. Montgomery.	39 . .	106 . .	. .	4.55	1.00	3.70	5.56	11.73	. .	. .	. .	. .	. .	. .	2.10
6. Mountain City. . . .	39 35	105 40	. .	. .	0.25	0.97	0.60	. .	. .	. .	. .	. .	4.00	3.20	. .
7. Golden City	39 45	105 20	5240	. .	. .	. .	. .	5.40	3.50	3.94	0.50	2.20	. .	. .	. .

MONTANA.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Helena City	46 45	111 50	4150	3.10	0.47	1.76	2.20	4.30	3.50	0.70	0.20	1.80	2.61	0.50	1.00
2. Camp Cook	47 43	109 38	. .	. .	. .	0.09	0.51	. .	. .	1.09	0.13	0.97	1.57	0.40	. .
3. Fort C. F. Smith . .	45 20	107 56	. .	0.51	0.43	1.13	0.22	2.21	1.57	0.15	0.02	0.22	0.25	4.77	1.65

NEW MEXICO.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Craig	33 26	107 08	4576	0.26	0.34	0.42	0.09	1.10	0.61	2.58	2.92	2.36	1.08	0.50	0.41
2. Fort Fillmore. . . .	32 14	106 15	3937	0.05	0.42	0.17	0.13	0.18	0.47	1.89	1.80	1.53	0.55	0.94	0.29
3. Fort Webster	32 48	108 05	6350	0.40	1.00	0.07	2.23	1.14	2.98	3.67	2.75	2.36	0.80	1.87	0.19
4. Fort Thorn	32 47	107 21	4500	0.48	0.86	0.50	0.10	0.24	0.21	2.65	3.79	4.23	0.33	0.71	0.53
5. Fort Conrad	33 34	107 09	4576	0.06	0.11	0.14	0.04	0.33	0.78	1.28	1.18	1.25	0.50	0.78	0.31
6. Socorro	34 10	106 50	4560	0.04	0.48	0.60	0.43	0.07	0.09	0.84	1.30	0.24	1.81	1.34	0.62

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[1] Near Fort Pierre.

DAKOTA.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs. mos.	Date. Beginning. End.	Observer.	Reference.
1	4.76	6.64	3.90	1.21	16.51	8 10	Jan. 1857; Dec. 1867	Assist. Surgeon	M. D. of U. S. A. 1860, MS. from S. G. O.
2	4.28	3.32	3.76	2.15	13.51	1 10	July, 1855; Apr. 1857	" "	M.D. of U.S.A. 1860.
3	4.67	7.49	3.36	1.82	17.34	6 11	Aug. 1860; Dec. 1867	" "	MS. from Surg. Gen. Office.
4	..	24.97	..	..	..	0 5	1867	" "	" " " " "
5	0.51	..	1.14	..	..	0 10	1867	" "	" " " " "
6	..	..	1.27	..	..	0 5	1867	" "	" " " " "
7	14.75	..	..	..	..	0 9	Feb. 1860; May, 1861	F. N. Greenwood	Sm. Obs.
8	..	..	..	..	..	0 1	1862	G. W. Lawson	" "
9	..	..	..	..	..	0 2	1861	M. C. Rousseau	" "

WYOMING TERRITORY.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs. mos.	Date. Beginning. End.	Observer.	Reference.
1	5.64	4.90	2.98	1.64	15.16	12 6	Sept. 1849; Jan. 1864	Assist. Surgeon	Ar. Met. Reg. 1855, M. D. of U. S. A. MS. from S. G. O.
2	13.35	8.05	3.52	..	..	0 10	1867	" "	MS. from Surg. Gen. Office.
3	..	..	..	..	..	0 2	1859	Maj. T. S. Twiss	P. O. and S. I. Vol. I.
4	..	..	..	..	..	0 1	1859	C. H. Miller	" " " "
5	..	..	..	..	..	0 2	1864	A. F. Ziegler	Sm. Obs.
6	2.08	..	..	..	..	0 6	1858	Assist. Surgeon	M. D. of U. S. A. 1860.

COLORADO.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs. mos.	Date. Beginning. End.	Observer.	Reference.
1	2.95	5.56	6.28	2.27	17.06	5 1	Oct. 1852; July, 1858	Assist. Surgeon	Ar. Met. Reg. 1855, M. D. of U. S. A. 1860.
2	1.00	3.33	1.33	0.45	6.11	6 4	Oct. 1858; Oct. 1867	" "	M. D. of U. S. A. 1860, MS. from S. G. O.
3	7.09	4.30	0.11	0.59	12.09	1 0	1867	" "	MS. from Surg. Gen. Office.
4	..	1.85	..	..	..	0 8	1867	" "	" " " " "
5	20.99	..	..	..	..	0 6	Dec. 1863; May, 1864	J. Suttrell	Sm. Obs.
6	..	..	..	..	..	0 5	Oct. 1860; Apr. 1861	Dr. W. T. Ellis	" "
7	..	7.94	..	..	..	0 5	1860	J. McDonald, M. S. Blunt	" "

MONTANA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs. mos.	Date. Beginning. End.	Observer.	Reference.
1	8.26	4.40	4.91	4.57	22.14	1 0	1866	A. C. Wheaton	Sm. Obs.
2	..	..	2.94	..	..	0 9	Oct. 1866; Nov. 1867	Assist. Surgeon	MS. from Surg. Gen. Office.
3	3.56	1.74	5.24	2.59	13.13	1 0	1867	" "	" " " " "

NEW MEXICO.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs. mos.	Date. Beginning. End.	Observer.	Reference.
1	0.61	6.11	3.94	1.01	11.67	9 7	Jan. 1855; Dec. 1867	Assist. Surgeon	M. D. of U. S. A. 1860, MS. from S. G. O.
2	0.48	4.16	3.02	0.76	8.42	8 3	Sept. 1851; Dec. 1859	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
3	3.44	9.40	5.03	1.59	19.46	1 11	Feb. 1852; Dec. 1853	" "	Army Met. Reg. 1855.
4	0.84	6.65	5.27	1.87	14.63	4 11	Jan. 1854; Dec. 1858	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
5	0.51	3.24	2.53	0.48	6.76	3 9	Oct. 1851; May, 1855	" "	Army Met. Reg. 1855.
6	1.10	2.23	3.39	1.14	7.86	1 9	Nov. 1849; Aug. 1851	" "	" " " "

NEW MEXICO.—Continued.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
7. Albuquerque	35°06′	106°38′	5032	0.31	0.23	0.28	0.32	0.23	1.37	1.05	1.93	0.83	0.81	0.40	0.36
8. Cebolleta[1]	35 15	107 20	6200	0.30	1.61	0.36	0.75	0.12	0.14	0.55	1.22	3.60	1.59	0.68	1.13
9. Fort Marcy[2]	35 41	106 02	6846	0.47	1.34	0.74	0.81	0.72	1.55	2.77	3.28	1.97	1.17	0.98	0.85
10. Las Vegas	35 35	105 16	6418	. .	4.23	0.01	1.06	2.82	0.00	5.10	. .	0.00	0.00	0.12	3.12
11. Fort Union	35 54	104 57	6670	0.58	0.37	0.55	0.49	0.75	2.83	5.97	5.16	2.56	1.03	1.14	0.45
12. Camp Burgwin[3]. . .	36 26	105 26	. .	0.72	0.47	0.51	0.81	0.25	0.36	0.85	1.71	1.01	0.44	0.97	0.55
13. Fort Stanton	33 29	105 38	. .	0.46	1.15	0.86	0.58	0.37	1.63	3.03	6.15	2.85	1.19	0.85	1.23
14. Fort McRae	33 18	107 03	4500	. .	. .	. .	1.03	5.00	3.00	. .	. .	. .	. .	. .	. .
15. Fort Sumner	34 20	104 00	. .	0.20	0.47	0.48	0.23	0.64	2.03	4.62	2.16	1.42	1.38	0.58	1.42
16. Fort Wingate. . . .	35 10	107 45	. .	0.35	0.62	0.29	0.32	0.10	0.00	4.74	3.51	0.70	2.22	0.07	1.03
17. Fort Selden.	32 23	106 55	. .	0.10	0.60	0.60	0.00	0.01	. .	4.31	0.80	1.81	0.26	0.43	0.20
18. Fort Bayard	32 40	108 25	4450	. .	. .	0.67	0.20	. .	0.00	4.45	2.19	1.60	0.75	0.55	1.50

UTAH.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Great Salt Lake City[4]	40 46	112 06	4320	1.68	2.25	2.47	1.39	1.34	1.49	2.54	1.01	1.37	2.20	2.35	3.76
2. Fort Bridger	41 20	110 23	6656	0.22	0.12	0.27	1.05	0.80	0.50	0.52	0.57	0.51	0.37	0.64	0.55
3. Camp Floyd	40 13	112 08	4860	0.28	0.63	0.50	0.60	0.65	0.20	1.40	0.34	0.69	0.66	1.23	0.16
4. Camp Douglas . . .	40 39	111 42	4800	2.77	1.53	2.87	1.11	1.58	0.60	0.84	0.60	0.99	1.79	1.71	4.18
5. Tonaquint[5]	37 11	113 50	. .	1.43	0.79	0.33	0.28	0.55	0.03	0.63	0.83	0.56	1.08	0.75	1.12

ARIZONA.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Defiance . . .	35 43	109 10	6500	0.98	0.70	0.84	0.67	0.52	0.74	2.44	2.73	1.86	0.70	1.16	0.87
2. Fort Buchanan . . .	31 40	110 55	5330	1.25	1.43	0.15	0.98	0.00	0.34	6.22	6.92	2.27	1.33	1.00	1.22
3. Fort Mojave	35 06	114 35	604	0.51	0.25	0.25	0.05	0.00	0.03	0.04	0.01	0.07	0.00	0.88	0.42
4. Fort Whipple. . . .	34 27	112 11	5700	1.72	1.16	8.00	0.06	1.17	0.00	2.70	2.38	3.65	0.33	0.14	1.25
5. Camp Goodwin . . .	32 52	109 51	. .	4.80	2.08	1.87	1.33	0.01	0.00	3.47	3.73	5.85	2.27	2.40	4.97
6. Camp Grant.	32 54	110 40	. .	1.90	1.00	0.50	. .	0.00	. .	. .	. .	3.10	0.00	3.33	0.33
7. Camp McDowell . .	33 46	111 36	. .	0.88	0.16	2.11	0.03	0.00	0.00	2.97	1.18	1.62	0.14	0.17	2.90
8. Camp L. Tucson . .	32 13	110 53	. .	. .	.	. .	. .	0.00	0.00	2.90	1.40	0.60	. .	. .	1.70
9. Camp Skull Valley .	34 45	112 30	5000	3.11	1.11	3.09	0.30	. .	. .	. .	. .	. .	. .	. .	. .
10. Camp Wallen. . . .	31 31	110 11	. .	4.60	2.30	0.60	0.00	0.00	0.00	6.66	2.50	0.20	0.00	0.00	0.90
11. Camp McPherson. .	34 45	112 18	3726	. .	. .	. .	. .	0.13	0.00	2.81	1.23	0.18	0.03	0.98	4.09

IDAHO.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Boisé	43 56	116 04	. .	2.20	0.53	1.71	1.26	1.17	0.68	0.07	0.42	0.60	0.41	1.26	2.97
2. Fort Lapway	46 18	116 54	. .	1.37	1.42	1.43	0.87	1.24	2.98	0.27	0.26	1.32	0.57	2.25	3.02

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[1] In October, 1851, position removed to Laguna, latitude 35° 03′, longitude 107° 14′, altitude 6000 feet.

[2] Near Santa Fé.

[3] Or Cantonment Burgwin, nine miles northward from Taos.

[4] The observations of 1857 in longitude 112° 10′, same latitude and elevation.

NEW MEXICO.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	EXTENT of series.		DATE.		OBSERVER.	REFERENCE.
						yrs.	mos.	Beginning.	End.		
7	0.83	4.35	2.04	0.90	8.12	12	2	Feb. 1850;	July, 1867	Asst. Surgeon	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
8	1.23	1.91	5.87	3.04	12.05	2	1	Dec. 1849;	Dec. 1851	" "	Army Met. Reg. 1855.
9	2.27	7.60	4.12	2.66	16.65	12	6	Feb. 1850;	Dec. 1865	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860, and MS. from S. G. O.
10	3.89	. .	0.12	. .	. .	0	11	Apr. 1850;	July, 1851	" "	Army Met. Reg. 1855.
11	1.79	13.96	4.73	1.40	21.88	11	5	Sept. 1851;	Mar. 1863	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
12	1.57	2.92	2.42	1.74	8.65	5	9	Sept. 1854;	May, 1860	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
13	1.81	10.81	4.89	2.84	20.35	5	0	Jan. 1856;	Dec. 1860	" "	M. D. of U. S. A. 1860, MS. from S. G. O.
14	. .	. .	. .	. .	. .	0	3	1864		" "	MS. from Surg. Gen. Office.
15	1.35	8.81	3.38	2.09	15.63	3	1	May, 1864;	Nov. 1867	" "	" " " " "
16	0.71	8.25	2.99	2.00	13.95	2	5	Jan. 1865;	Dec. 1867	" "	" " " " "
17	0.61	. .	2.50	0.90	. .	1	4	Jan. 1866;	Dec. 1867	" "	" " " " "
18	. .	6.64	2.90	. .	. .	0	9	1867		" "	" " " " "

UTAH.

	Spring.	Summer.	Autumn.	Winter.	Year.	yrs.	mos.	Beginning.	End.	OBSERVER.	REFERENCE.
1	5.20	5.04	5.92	7.69	23.85	5	8	Feb. 1857;	Dec. 1866	H. E. & W. W. Phelps	P.O. and S.I. Vol. 1, and S.O.
2	2.12	1.59	1.52	0.89	6.12	6	3	July, 1858;	Nov. 1867	Assist. Surgeon	M. D. of U. S. A. 1860, MS. from S. G. O.
3	1.75	1.94	2.58	1.07	7.34	2	6	July, 1858;	Dec. 1860	" "	M. D. of U. S. A. 1860, MS. from S. G. O.
4	5.56	2.04	4.49	8.48	20.57	3	3	Mar. 1864;	Dec. 1867	" "	MS. from S. G. O.
5	1.16	1.49	2.39	3.34	8.38	2	6	Jan. 1861;	Apr. 1866	H. Pearce, G. A. Burgon	Sm. Obs.

ARIZONA.

	Spring.	Summer.	Autumn.	Winter.	Year.	yrs.	mos.	Beginning.	End.	OBSERVER.	REFERENCE.
1	2.03	5.91	3.72	2.55	14.21	8	5	May, 1852;	Dec. 1860	Assist. Surgeon	Army Met. Reg. 1855, M. D. of U. S. A. 1860, and MS. from S. G. O.
2	1.13	13.48	4.60	3.90	23.11	2	5	Aug. 1857;	Dec. 1859	" "	M. D. of U. S. A. 1860.
3	0.30	0.08	0.95	1.18	2.51	2	2	Aug. 1859;	Oct. 1866	" "	M. D. of U. S. A. 1860, MS. from S. G. O.
4	9.23	5.08	4.12	4.13	22.56	1	4	June, 1865;	Dec. 1867	" "	MS. from S. G. O.
5	3.21	7.20	10.52	11.85	32.78	1	6	Jan. 1866;	Dec. 1867	" "	" " " "
6	. .	. .	6.43	3.23	. .	0	9	Sept. 1866;	Oct. 1867	" "	" " " "
7	2.14	4.15	1.93	3.94	12.16	1	4	Sept. 1866;	Dec. 1867	" "	" " " "
8	. .	4.30	. .	. .	. .	0	6	1867		" "	" " " "
9	. .	. .	. .	. .	. .	0	4	1867		" "	" " " "
10	0.60	9.16	0.20	7.80	17.76	1	0	1867		" "	" " " "
11	. .	4.04	1.19	. .	. .	0	8	1867		" "	" " " "

IDAHO.

	Spring.	Summer.	Autumn.	Winter.	Year.	yrs.	mos.	Beginning.	End.	OBSERVER.	REFERENCE.
1	4.14	1.17	2.27	5.70	13.28	2	5	Mar. 1864;	Dec. 1867	Assist. Surgeon	MS. from S. G. O.
2	3.54	3.51	4.14	5.81	17.00	2	11	Jan. 1864;	Nov. 1866	" "	" " " "

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[6] Also called St. George, is probably identical with Heberville (1861), which latter place has the same position assigned, same observer; series do not overlap.

NEVADA.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Latitude.	Longitude.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Churchill . . .	39°17′	119°19′	4284	1.46	0.32	0.01	0.11	0.06	0.22	1.55	0.22	0.11	0.35	1.04	0.22
2. Fort Ruby	40 01	115 35	5922	2.24	0.23	1.63	1.57	2.04	0.59	0.59	1.82	0.53	3.33	0.94	1.76
3. Camp McGarry . . .	41 40	119 00	6000	2.46	0.41	2.78	0.43	3.72	0.28	0.00	0.22	1.14	1.24	1.81	3.45
4. Camp McDermit . .	41 58	117 40	4700	3.54	0.64	0.48	0.22	0.29	0.16	..	..	0.00	0.59	0.77	1.76
5. Camp W. Scott . . .	41 34	117 30	..	2.26	0.54	0.50	0.29	..	0.26	0.00	0.38	1.52	..	1.97	6.17

CALIFORNIA.

NAME OF STATION.	Latitude.	Longitude.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Sacramento	38 34	121 28	82	3.51	2.65	3.21	1.66	0.88	0.06	0.03	0.01	0.08	0.47	2.13	4.87
2. Sacramento	38 34	121 28	82	6.31	0.50	3.60	1.51	0.01	..	0.00	0.00	0.00	0.61	3.54	3.70
3. Fort Yuma	32 44	114 36	200	0.18	0.54	0.17	0.08	0.00	0.00	0.34	0.34	1.09	0.10	0.30	0.32
4. Fort San Diego . . .	32 42	117 14	150	1.19	2.02	0.96	0.67	0.39	0.08	0.17	0.20	0.06	0.23	1.19	2.00
5. San Luis Rey	33 13	117 25	20	0.09	0.95	0.21	..	..	..	0.00	0.00	0.00	0.20	3.28	2.22
6. Ranchos del Chino and de Jurupa[1]	34 00	117 30	1000	0.95	1.50	3.12	0.33	1.14	0.00	0.00	0.09	0.00	0.00	1.67	4.77
7. Fort Tejon[2].	34 53	118 53	3240	1.62	3.19	2.49	3.19	1.17	0.21	0.02	0.16	2.02	1.00	1.49	2.97
8. Monterey	36 36	121 52	140	1.68	1.50	3.27	0.63	0.53	0.13	0.08	0.00	0.01	0.33	1.31	2.73
9. Monterey	36 36	121 52	42	3.32	0.85	3.01	1.29	0.93	0.09	0.12	0.04	0.06	0.29	2.07	3.22
10. Fort Miller	37 00	119 40	402	1.65	1.76	4.29	1.91	1.05	0.00	0.00	0.00	0.03	0.28	2.62	5.40
11. San Francisco, Presidio[3]	37 48	122 26	150	3.51	2.71	2.88	1.76	0.70	0.03	0.02	0.00	0.07	0.60	2.19	4.35
12. San Francisco[3] . . .	37 48	122 27	130	4.54	2.87	3.09	1.83	0.76	0.05	0.00	0.03	0.11	0.53	2.99	4.72
13. Benicia Barracks . .	38 03	122 08	64	2.65	2.00	2.53	1.58	0.65	0.13	0.03	0.02	0.09	0.46	1.80	3.11
14. Camp Far West . . .	39 07	121 18	175	3.46	0.63	6.35	2.23	0.43	0.00	0.00	0.00	1.15	0.06	1.97	4.32
15. Fort Reading	40 30	122 05	674	4.87	3.27	3.91	3.92	2.85	0.31	0.00	0.06	0.15	0.69	3.30	5.78
16. Fort Humboldt . . .	40 45	124 10	50	5.78	5.96	5.30	2.82	1.24	0.47	0.19	0.07	0.59	1.78	4.13	7.59
17. Fort Jones	41 36	122 52	2570	2.88	4.10	2.77	1.25	1.21	0.70	0.11	0.10	0.20	1.40	2.59	4.39
18. Stockton	37 57	121 17	..	2.87	4.00	1.43	1.62	0.78	0.29	0.03	0.00	0.00	0.36	0.85	1.46
19. Crescent City	41 45	124 11	12	..	..	..	..	..	..	..	..	..	..	..	4.99
20. Downieville	39 27	..	2200	..	..	..	..	..	..	..	0.01	0.07	4.19	11.32	6.77
21. Santa Clara[4]	37 20	122 00	98	..	..	..	..	..	..	..	..	..	..	3.25	1.29
22. Union Ranche . . .	39 25	121 30	..	..	..	..	3.08	3.00	0.04	0.40	0.00	0.06	..	7.21	2.63
23. Fort Crook	41 06	121 25	3390	3.20	3.76	3.39	1.60	1.25	0.64	0.32	0.01	0.61	1.13	3.26	4.51
24. Fort Ter-Waw . . .	41 49	124 12	..	9.97	8.78	4.50	6.07	3.63	0.18	2.41	0.71	3.81	5.64	12.71	11.52

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[1] Longitude of Rancho de Jurupa 117° 25′.

[2] At Fort Tejon the record for November and December, 1855, received from two sources differs; the Army Register, of 1860, gives 0.00 and 7.50 respectively. The elevation given in 1855 has not been repeated in the later volumes; number probably doubtful.

NEVADA.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.		Observer.	Reference.
						yrs.	mos.	Beginning.	End.		
1	0.18	1.99	1.50	2.00	5.67	2	5	Jan. 1862;	Aug. 1865	Assist. Surgeon	MS. from S. G. O.
2	5.24	3.00	4.80	4.23	17.27	1	9	May, 1863;	Mar. 1866	" "	" " " "
3	6.93	0.50	4.19	6.32	17.94	1	6	Jan. 1866;	Dec. 1867	" "	" " " "
4	0.99	..	1.36	5.94	..	0	11	Feb. 1866;	June, 1867	" "	" " " "
5	..	0.64	..	8.97	..	0	10	1867		" "	" " " "

CALIFORNIA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series.		Date.		Observer.	Reference.
						yrs.	mos.	Beginning.	End.		
1	5.75	0.10	2.68	11.03	19.56	18	0	Sept. 1849;	Aug. 1867	Dr. T. M. Logan	Met. Reports in Sm. Coll'n.
2	5.12	..	4.15	10.51	..	1	6	July, 1849;	Apr. 1863	Assist. Surgeon, Dr. F. W. Hatch	Army Met. Reg. 1855, An. Rep. Sm. Inst. for 1854, MS. from S. G. O.
3	0.25	0.68	1.49	1.04	3.46	9	9	Jan. 1851;	Dec. 1867	Assist. Surgeon	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
4	2.02	0.45	1.48	5.21	9.16	13	0	Jan. 1850;	Apr. 1866	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
5	..	..	3.48	3.26	..	0	10	July, 1850;	Aug. 1851	" "	Army Met. Reg. 1855.
6	4.59	0.09	1.67	7.22	13.57	1	11	July, 1851;	Mar. 1854	" "	" " "
7	6.85	0.39	4.51	7.78	19.53	4	8	Mar. 1855;	Aug. 1864	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
8	4.43	0.21	1.65	5.91	12.20	4	2	May, 1847;	Aug. 1852	" "	Army Met. Reg. 1855.
9	5.23	0.25	2.42	7.39	15.29	4	0	Jan. 1860;	Dec. 1866	Dr. C. A. Canfield	Sm. Obs.
10	7.25	0.00	2.93	8.81	18.99	6	9	July, 1851;	June, 1858	Assist. Surgeon	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
11	5.34	0.05	2.86	10.57	18.82	13	1	Mar. 1850;	Dec. 1867	Assist. Surgeon and D. F. Parkinson	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O. and Sm. Obs. in 1863 and 1864.
12	5.68	0.08	3.63	12.13	21.52	13	11	Dec. 1850;	Dec. 1866	M. Walthall, Jr., Dr. H. Gibbons, Dr. W. O. Ayres	9th An. Rep. Sm. Inst. 1855, Am. Alm. 1859, P. O. and S. I. Vol. 1, & Sm. Obs.
13	4.76	0.18	2.35	7.76	15.05	13	7	Nov. 1849;	Dec. 1864	Assist. Surgeon	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
14	9.01	0.00	3.18	8.41	20.60	2	3	Jan. 1850;	Mar. 1852	Dr. R. V. Abbott, U. S. A.	9th An. Rep. Sm. Inst., Army Met. Reg. 1855.
15	10.68	0.37	4.14	13.92	29.11	3	9	Apr. 1852;	Mar. 1856	Assist. Surgeon	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
16	9.36	0.73	6.50	19.33	35.92	11	2	Jan. 1854;	Nov. 1866	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
17	5.23	0.91	4.19	11.37	21.70	5	0	Jan. 1853;	June, 1858	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
18	3.83	0.32	1.21	8.33	13.69	3	6	Jan. 1854;	Dec. 1857	Dr. R. K. Ried, and M. Walthall	P. O. and S. I. Vol. 1, Am. Alm. 1859.
19	..	..	..	..	..	0	1	1859		R. B. Randall	P. O. and S. I. Vol. 1.
20	..	..	15.58	..	..	0	7	Nov. 1859;	Dec. 1860	Dr. T. R. Kibbe	P. O. and S. I. Vol. 1, and S. O.
21	..	..	..	..	..	0	2	1859		Prof. O. S. Frombes	P. O. and S. I. Vol. 1.
22	..	0.44	..	..	..	1	0	June, 1859;	Sept. 1860	J. Slaven	P. O. and S. I. Vol. 1, and S. O.
23	6.24	0.97	5.00	11.47	23.68	8	3	Jan. 1858;	Oct. 1867	Assist. Surgeon	M. D. of U. S. A. 1860, MS. from S. G. O.
24	14.20	3.30	22.16	30.27	69.93	2	3	Apr. 1859;	Sept. 1861	" "	M. D. of U. S. A. 1860, MS. from S. G. O.

[3] Combined series: Jan. 4.71; Feb. 3.09; March, 3.22; April, 1.80; May, 0.67; June, 0.05; July, 0.01; Aug. 0.01; Sept. 0.10; Oct. 0.50; Nov. 2.61; Dec. 4.92; Spring, 5.69; Summer, 0.07; Autumn, 3.21; Winter, 12.72; Year, 21.69; Extent, 17 years, 10 months; Date, 1850—1868. The observations by Tennent, between September, 1853, and June, 1868, are included.

[4] University of the Pacific.

CALIFORNIA.—Continued.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
25. Alcatraz Island . . .	37°49′	122°24′	. .	8.10	2.85	1.20	0.45	0.25	0.03	0.00	0.00	0.00	0.19	1.12	4.98
26. Camp Gaston	41 10	123 15	. .	11.00	7.68	6.59	5.77	0.81	0.92	0.24	0.05	0.92	1.42	9.84	16.96
27. Fort Bidwell	41 51	120 06	4680	6.58	4.15	3.88	1.30	0.00	0.60	0.10	1.40	2.80	0.80	3.10	8.20
28. Drum Barracks . . .	33 49	118 15	32	2.78	1.03	2.87	0.06	0.00	0.00	0.00	0.60	0.00	0.00	0.30	4.19
29. Fort Babbitt	36 22	119 17	. .	1.29	0.90	. .	0.00	0.00	. .	. .	. .	. .	0.00	0.85	. .
30. Fort Wright	39 45	123 08	. .	9.74	5.68	5.92	1.50	0.22	0.00	0.00	0.00	0.79	1.54	9.66	18.59
31. Fort Point	37 48	122 28	. .	6.03	3.10	2.12	1.50	0.66	0.00	0.00	0.00	0.50	0.27	2.50	9.00
32. Camp Lincoln . . .	41 55	124 15	. .	26.23	11.29	4.59	10.06	0.60	1.12	0.05	0.00	0.75	1.34	10.17	15.39
33. Camp Independence	36 50	118 11	4800	0.00	1.63	4.76	0.53	0.76	0.00	0.01	1.15	. .	. .	. .	12.19
34. Camp Union	38 32	121 28	54	4.00	0.92	0.32	0.64	0.54	0.08	0.45	0.00	. .	. .	. .	. .
35. Meadow Valley[1] . .	40 20	120 15	4000	7.41	6.05	6.21	3.20	1.98	1.09	2.40	1.13	1.53	2.41	12.10	11.52
36. Folsom	. .	. .	. .	. .	. .	4.60	0.60	0.50	0.10	0.00	0.00	. .	. .	. .	. .
37. Marysville	39 21	121 30	80	7.63	. .	. .	. .	0.53	. .	0.04	. .	. .	. .	. .	2.34
38. San Benito	36 . .	121 30	140	4.91	1.88	0.36	0.37	0.48	0.17	. .	. .	. .	0.05	2.15	4.79
39. Santa Barbara . . .	34 24	119 42	20	. .	. .	0.13	0.28	. .	. .	. .	0.11	. .	. .	. .	. .
40. San Francisco[2] . . .	37 48	122 24	290	8.05	4.44	2.62	1.59	0.49	0.05	0.00	0.03	0.08	0.39	3.01	5.80

OREGON.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Orford	42 44	124 29	50	9.08	5.79	6.17	7.53	4.56	1.33	0.56	1.22	2.34	7.31	10.27	14.43
2. Fort Lane	42 24	122 30	2000	2.82	0.92	1.85	1.75	1.62	0.72	0.00	. .	. .	0.32	3.37	6.55
3. Fort Dalles[3]	45 36	120 55	350	4.36	2.39	1.97	0.74	0.94	0.53	0.36	0.27	1.46	0.98	3.38	4.36
4. Astoria	46 11	123 49	50	27.00	10.95	6.10	4.38	5.95	2.85	0.00	1.15	1.87	6.70	13.20	6.20
5. Oregon City	45 20	122 24	200	7.93	5.17	7.15	1.92	3.37	1.41	0.15	0.94	2.24	3.46	5.12	8.75
6. Portland	45 24	122 30	150	. .	8.95	7.85	1.39	1.51	0.13	0.22	0.87	. .	. .	. .	. .
7. Fort Hoskins	45 02	123 32	. .	12.73	10.03	9.07	2.86	2.76	1.92	0.37	0.36	3.18	3.36	8.34	11.73
8. Fort Yamhill	45 21	123 30	. .	9.51	7.31	7.66	3.03	2.41	1.61	0.36	0.42	2.69	2.96	7.55	10.08
9. Fort Umpqua[4]	43 42	124 10	8	10.28	9.91	9.73	4.01	3.09	2.18	0.30	0.38	2.27	3.66	9.71	11.89
10. Blockhouse	44 25	123 30	. .	12.70	12.30	16.06	4.82	5.40	2.23	0.30	0.00	10.12	6.96	14.07	11.33
11. Fort Stevens	46 12	123 57	. .	15.56	13.77	2.56	5.70	0.88	1.67	4.58	0.29	2.54	9.28	7.30	21.69
12. Camp Watson . . .	44 13	119 45	. .	. .	. .	. .	. .	. .	0.19	0.22	. .	0.94	0.35	0.92	3.69
13. Wallamet[5]	44 56	123 01	120	8.39	. .	. .	. .	2.54	. .	. .	. .	. .	1.76	. .	. .
14. Auburn	44 45	118 15	3300	. .	. .	. .	. .	. .	2.37	1.00	1.72	. .	. .	. .	. .
15. Albany, near	44 22	123 00	600	14.15	4.60	11.71	1.69	5.33	. .	0.01	0.75	. .	2.80	4.88	8.22
16. Fort Klamath	42 40	121 54	4200	4.58	3.47	4.77	0.20	1.50	0.00	0.30	1.40	. .	1.40	7.30	5.00

WASHINGTON TERRITORY.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Vancouver[6] . .	45 40	122 30	50	6.18	3.73	3.93	2.47	2.45	1.83	1.38	0.52	1.65	2.38	5.14	7.18
2. Fort Steilacoom . . .	47 10	122 25	300	6.90	5.03	4.90	3.32	1.75	1.75	0.47	1.20	2.37	3.26	6.13	6.90
3. Fort Walla-Walla .	46 03	118 20	. .	2.51	2.38	1.49	0.96	2.24	1.15	0.62	0.31	1.60	0.98	2.40	2.84
4. Fort Bellingham . .	48 45	122 30	88	3.04	2.59	3.51	2.10	1.58	1.19	0.43	1.20	2.24	2.22	3.36	6.21
5. Fort Simcoe	46 14	120 40	. .	2.31	2.86	1.22	0.33	0.29	0.26	0.07	0.52	0.44	0.58	0.78	0.95

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[1] Or Silver Creek.

[2] At Tennent's residence on Leavenworth Street.

[3] Or Dalles of Columbia.

[4] Elevation, as given above, doubtful, but small.

CALIFORNIA.—Continued.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning. End.	Observer.	Reference.
25	1.90	0.03	1.31	15.93	19.17	2	7	Jan. 1862; Dec. 1867	Assist. Surgeon	MS. from S. G. O.
26	13.17	1.21	12.18	35.64	62.20	4	5	Jan. 1862; Dec. 1867	" "	" " " "
27	5.18	2.10	6.70	18.93	32.91	1	3	Jan. 1864; Dec. 1867	" "	" " " "
28	2.93	0.60	0.30	8.00	11.83	2	3	Jan. 1865; Dec. 1867	" "	" " " "
29	..	..	..	..	..	0	6	1865	" "	" " " "
30	7.64	0.00	11.99	34.01	53.64	2	7	Jan. 1865; Dec. 1867	" "	" " " "
31	4.28	0.00	3.27	18.13	25.68	1	8	Jan. 1866; Dec. 1867	" "	" " " "
32	15.25	1.17	12.26	52.91	81.59	1	3	Sept. 1866; Dec. 1867	" "	" " " "
33	6.05	1.16	..	13.82	..	0	9	1867	" "	" " " "
34	1.50	0.53	..	..	..	1	1	Jan. 1865; July, 1867	" "	" " " "
35	11.39	4.62	16.04	24.98	57.03	4	4	Jan. 1861; Nov. 1867	J. H. Whitlock, M. D. Smith	Sm. Obs.
36	0.60	..	..	..	..	0	6	1861	S. V. Blakeslee	" "
37	..	..	..	..	..	0	4	Jan. 1862; July, 1863	W. C. Belcher	" "
38	1.21	..	..	11.58	..	1	3	Nov. 1861; May, 1863	Dr. C. A. Canfield	" "
39	..	..	..	..	..	0	3	1864	Dr. W. W. Hays	" "
40	4.70	0.08	3.48	18.29	26.55	8	2	Sept. 1853; Dec. 1854; Sept. 1861; June, 1868	Thomas Tennent	San Francisco Bul. and MS. copy.

OREGON.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning. End.	Observer.	Reference.
1	18.26	3.11	19.92	29.30	70.59	2	10	June, 1852; July, 1856	Assist. Surgeon	Army Met. Reg. 1855, M. D. of U. S. A. 1860.
2	5.22	..	..	10.29	..	1	4	Jan. 1855; July, 1856	" "	M. D. of U. S. A. 1860.
3	3.65	1.16	5.82	11.11	21.74	12	7	July, 1850; Nov. 1865	" "	Army Met. Reg. 1855, M.D. of U.S.A. 1860, MS. from S.G.O.
4	16.43	4.00	21.77	44.15	86.35	1	2	Aug. 1850; Sept. 1851	" "	Army Met. Reg. 1855.
5	12.44	2.50	10.82	21.85	47.61	3	0	1851 and Apr. 1857; Mar. 1859	Rev. G. H. Atkinson	MS. in Sm. Coll'n.
6	10.75	1.22	..	..	..	0	10	Apr. 1858; Aug. 1859	G. H. Stebbins	P. O. and S. I. Vol. 1.
7	14.69	2.65	14.88	34.49	66.71	6	9	Dec. 1856; Sept. 1864	Assist. Surgeon	M. D. of U. S. A. 1860, MS. from S. G. O.
8	13.10	2.39	13.20	26.90	55.59	9	3	Nov. 1856; Apr. 1866	" "	M. D. of U. S. A. 1860, MS. from S. G. O.
9	16.83	2.86	15.64	32.08	67.41	5	10	Aug. 1856; May, 1862	" "	M. D. of U. S. A. 1860, MS. from S. G. O.
10	26.28	2.53	31.15	36.33	96.29	1	8	Mar. 1858; Dec. 1859	" "	M. D. of U. S. A. 1860.
11	9.14	6.54	19.12	51.02	85.82	1	0	1867	" "	MS. from S. G. O.
12	..	..	2.21	..	..	0	6	1867	" "	" " " "
13	..	..	..	..	..	0	3	May, 1861; Oct. 1863 Jan. 1864	T. H. Crawford, P. L. Willis	Sm. Obs.
14	..	5.09	..	..	..	0	4	June, 1864; Aug. 1865	S. H. W. Hindman	" "
15	18.73	..	..	26.97	..	0	10	1866	" " "	" "
16	6.47	1.70	..	13.05	..	1	2	Jan. 1865; Mar. 1866	Assist. Surgeon	MS. from S. G. O.

WASHINGTON TERRITORY.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning. End.	Observer.	Reference.
1	8.85	3.73	9.17	17.09	38.84	16	4	Dec. 1849; Dec. 1867	Assist. Surgeon	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
2	9.97	3.42	11.76	18.83	43.98	16	0	Nov. 1849; Dec. 1867	" "	Army Met. Reg. 1855, M. D. of U. S. A. 1860, MS. from S. G. O.
3	4.69	2.08	4.98	7.73	19.48	8	7	Jan. 1857; May, 1867	" "	M. D. of U. S. A. 1860, MS. from S. G. O.
4	7.19	2.82	7.82	11.84	29.67	2	2	June, 1857; July, 1859	" "	M. D. of U. S. A. 1860.
5	1.84	0.85	1.80	6.12	10.61	2	0	Apr. 1857; Apr. 1859	" "	" " " "

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[5] Wallamet University (Willamette?).

[6] Also called Columbia Barracks.

WASHINGTON TERRITORY.—Continued.

Mean Amount of Precipitation in Rain and Melted

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
6. Fort Cascades. . . .	45°39′	121°50′	..	8.56	8.74	7.47	5.83	4.51	1.79	0.40	0.94	5.65	5.15	9.48	6.05
7. Fort Townsend . . .	48 05	122 45	135	1.94	1.70	0.98	1.20	1.33	0.15	..	..	..	..	..	2.34
8. Camp Simiahmoo . .	49 01	122 46	11	5.61	2.68	4.10	2.12	1.84	1.41	0.68	0.80	3.91	7.88	2.57	0.00
9. Fort Colville	48 42	118 02	..	1.27	1.07	0.28	0.52	1.25	1.13	1.54	0.35	0.40	0.76	0.82	0.44
10. San Juan Island . .	48 28	123 01	150	3.99	3.57	1.98	1.81	0.67	0.57	1.08	0.15	1.23	4.07	3.64	4.77
11. Cape Disappointment	46 17	124 02	30	11.17	8.28	7.75	4.46	2.16	2.13	4.55	0.52	4.44	6.74	12.70	10.00
12. Neeah Bay[1].	48 22	124 37	40	18.17	10.10	15.97	7.47	4.69	6.90	2.22	2.50	8.93	8.37	19.43	18.60

ALASKA.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Yukon	66 30	145 00	..	0.65	1.45	0.56	..	2.09	1.20	2.68	..	..	..	..	..
2. Sitka[2].	57 03	135 18	20	7.43	6.99	4.95	5.39	4.17	3.65	4.43	7.58	10.32	11.30	9.37	7.81

BRITISH NORTH AMERICA.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Fort Simpson	61 51	120 25	300	0.77	1.44	0.56	0.76	..	1.11	..	..	..	..	..	..
2. Red River Settlement	50 06	97 00	653	..	..	..	0.44	4.12	8.90	8.71	7.25	6.00	..	..	..
3. Moose Factory . . .	51 15	80 45	30	0.92	0.80	1.06	1.29	0.95	2.28	1.16	0.90	4.26	1.81	1.57	2.07
4. Fort Peel's River . .	67 10	135 00	..	..	..	6.19	4.20	3.19	8.00	4.00	3.50	..	1.15	0.25	..
5. Fort Norman	64 00	124 00	200	..	..	..	..	7.37	..	..	..	..	..	..	..

NEW FOUNDLAND.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. St. John's	47 35	52 40	170	4.50	3.19	4.72	3.45	3.65	5.03	3.52	5.42	6.00	6.17	7.93	4.72

NOVA SCOTIA.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Albion Mines	45 34	62 42	120	3.19	3.99	4.26	3.02	2.54	2.02	2.50	3.52	3.14	6.31	5.08	4.06
2. Wolfville	45 06	64 25	80	3.09	2.10	3.70	3.37	3.38	3.96	2.49	4.43	3.72	4.01	3.94	3.69
3. Windsor, King's Col.	44 59	64 07	200	..	1.40	2.50	1.58	2.11	1.95	3.33	2.08	1.33	4.59	4.21	1.86
4. Halifax	44 39	63 37	8	3.77	4.72	..	2.04	..	..	..	..	..	..	..	..

NEW BRUNSWICK.

Name of Station.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
1. Thurston, near Dunbar[3]	46 00	67 00	..	2.23	2.07	1.92	1.37	2.03	2.39	2.10	2.36	3.02	3.00	2.65	2.26
2. St. John	45 17	66 04	135	4.92	4.40	3.89	3.84	4.81	1.82	3.05	4.86	5.41	3.48	6.04	4.60

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[1] In August, 1863, and afterwards, observations were made at a point seventeen feet above half tide.

WASHINGTON TERRITORY.—CONTINUED.

SNOW FOR EACH MONTH, SEASON, AND THE YEAR.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
6	17.81	3.13	20.28	23.35	64.57	3	1	May, 1858;	May, 1861	Asst. Surgeon	M. D. of U. S. A. 1860, MS. from S. G. O.
7	3.51	..	..	7.15	..	1	2	Jan. 1859;	May, 1861	" "	M. D. of U. S. A. 1860, MS. from S. G. O.
8	8.06	2.89	14.36	8.29	33.60	1	4	Mar. 1859;	June, 1860	" "	M. D. of U. S. A. 1860, MS. from S. G. O.
9	2.05	3.02	1.98	2.78	9.83	4	4	Jan. 1861;	Nov. 1867	" "	MS. from S. G. O.
10	4.46	1.80	8.94	12.33	27.53	2	11	Jan. 1864;	Dec. 1867	" "	" " " "
11	14.37	7.20	23.88	29.45	74.90	2	0	Aug. 1864;	Nov. 1867	" "	" " " "
12	28.13	11.62	36.73	46.87	123.35	3	4	May, 1863;	Sept. 1866	J. G. Swan	Sm. Obs.

ALASKA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	..	..	..	..	..	0	6	1861		R. Kennicott	Sm. Obs.
2	14.51	15.66	30.99	22.23	83.39	16	3	May, 1847;	Nov. 1867	Asst. in charge Obser'y	Annals of the Central Physical Observatory of Russia, A. T. Kuppfer, Pogg. An. Vol. 4, 1855, and Ex. Doc. No. 177, H. of R. 40th Cong. 2d Ses.

BRITISH NORTH AMERICA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	..	..	..	..	..	0	9	Jan. 1861;	Apr. 1862	A. Flett, W. W. Kirby	Sm. Obs.
2	..	24.86	..	..	..	1	2	June, 1855;	Aug. 1861	D. Gunn	P.O. and S. I. Vol. 1, and S. O.
3	3.30	4.34	7.64	3.79	19.07	1	5	Jan. 1861;	May, 1862	J. Mackenzie	Sm. Obs.
4	13.58	15.50	..	..	..	0	8	Mar. to Nov. 1863		A. Flett	" "
5	..	..	..	..	..	1	5	May, 1862		" "	" "

NEW FOUNDLAND.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	11.82	13.97	20.10	12.41	58.30	4	8	Mar. 1856;	Feb. 1864	J. Delaney and son	P. O. and S. I. Vol. 1, and S.O.

NOVA SCOTIA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	9.82	8.04	14.53	11.24	43.63	5	10	June, 1843;	Mar. 1854	H. Poole	MS. & P.O. & S. I. Vol. 1, 1861.
2	10.45	10.88	11.67	8.88	41.88	8	0	Sept. 1855;	Dec. 1865	Stuart, Hartt, Higgins	P. O. and S. I. Vol. 1, and S. O.
3	6.19	7.36	10.13	..	..	1	8	May, 1857;	June, 1863	Hensley, How, and Everett	" " " " "
4	..	..	..	..	..	0	3	Apr. 1859;	Jan. Feb. 1861	R. J. Nelson	" " " " "

NEW BRUNSWICK.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	5.32	6.85	8.67	6.56	27.40	12	0	Jan. 1841;	Dec. 1852	R. Mossman	London Gard. Ch.
2	12.54	9.73	14.93	13.92	51.12	4	6	Aug. 1860;	Dec. 1866	G. Murdoch	Sm. Obs.

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[2] Mag. and Met. Observatory at Japonski Island, opposite Sitka.

[3] Probably no account of snow in any form.

CANADA EAST.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. St. Martin's	45°32′	73°36′	118	2.47	2.97	2.37	4.37	4.93	3.85	5.72	4.52	6.25	5.79	4.92	3.03
2. Montreal	45 30	73 36	57	3.52	3.84	3.32	2.72	3.49	2.88	3.78	3.73	3.35	4.23	3.33	3.57
3. Sherbrook and Hatley[1]	45 25	72 00	. .	0.77	2.49	0.36	2.45	2.24	1.61	2.08	2.33	. .	. .	. .	. .
4. Windsor	45 28	72 30	288	2.90	2.44	0.48	2.78	2.06	2.73	4.34	0.95	4.57	1.40	2.17	2.36

CANADA WEST.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Toronto, Observatory	43 39	79 23	342	2.58	2.55	2.40	2.71	3.25	2.84	3.44	2.87	3.54	2.57	3.35	3.07
2. Kingston, Queen's College	44 13	76 30	294	0.97	2.55	1.77	2.65	1.92	2.39	3.31	3.04	2.45	1.21	2.06	2.87
3. Michipicoton	47 56	85 06	660	1.71	1.71	3.02	2.61	1.53	1.75	1.62	2.50	3.87	1.56	4.06	1.87
4. Niagara	43 09	79 20	270	3.73	3.19	2.64	1.63	3.49	1.13	. .	. .	3.94	2.84	4.08	2.14

MEXICO.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Matamoras	25 52	97 27	55	1.57	2.32	2.46	2.26	2.22	3.63	2.37	1.65	7.05	4.47	4.50	2.24
2. Cordova[2]	19 26	97 51	860	2.26	2.26	4.09	3.48	5.53	22.19	16.43	15.94	19.72	12.00	4.54	3.64
3. Mexico, city of . . .	19 30	99 01	7665	0.00	. .	0.20	0.55	1.35	7.66	1.24	0.41	3.92	1.36	1.27	. .
4. Chihuahua	28 38	106 30	4640	0.09	1.50	0.26	0.00	0.23	1.39	8.52	6.03	5.23	1.05	1.14	0.00
5. Vera Cruz[3]	19 12	96 09	50	5.10	0.00	0.00	0.50	31.40	21.20	59.70	35.90	38.90	8.00	4.50	0.40
6. Vera Cruz	19 12	96 09	26	3.83	1.81	0.20	. .	0.32	. .	. .	. .	17.04	14.77	2.81	0.64
7. Mirador[2]	19 15	96 12	3500	1.39	1.53	2.80	1.76	6.12	15.95	12.53	12.57	14.30	9.31	4.70	1.96
8. Minatitlan[4]	17 59	94 07	60	. .	. .	0.10	0.15	1.82	. .	. .	. .	. .	. .	. .	. .
9. San Juan Bautista[5] .	17 47	92·36	40	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	3.16	0.93
10. Frontera, Baar of Tabasco	18 32	92 40	12	5.30	0.84	0.87	2.46	2.19	8.22	6.84	3.55	6.11	22.40	8.85	4.88

WEST INDIES AND BERMUDAS.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Up Park Camp, Jamaica	17 59	76 56	225	1.74	0.01	0.12	. .	. .	. .	. .	. .	. .	2.71	2.81	10.00
2. Nassau, New Providence[6]	25 05	77 21	80	1.97	1.27	0.54	1.70	7.00	6.33	4.17	6.65	7.74	5.90	1.38	2.10
3. Ponce, Porto Rico .	17 56	66 35	23	. .	11.00	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
4. Philipsburg, St. Martin	18 05	63 03	. .	4.79	3.29	3.50	3.69	6.01	7.70	4.64	8.97	9.63	16.31	9.12	8.67
5. Sombrero	18 37	63 27	45	0.58	0.64	0.77	0.43	1.74	2.18	1.96	1.63	2.40	2.61	1.44	0.46
6. Port of Spain, Trinidad	10 39	61 34	16	2.14	4.29	. .	. .	. .	. .	. .	. .	. .	5.28	12.92	6.18
7. Matanzas, Cuba . .	23 02	81 43	50	3.18	0.77	0.63	1.92	2.32	5.35	9.57	11.50	7.80	7.47	3.38	1.40
8. St. George's, Bermuda	32 23	64 40	123	3.26	3.38	2.65	2.84	3.62	2.63	5.00	4.58	6.77	7.64	7.10	5.43
9. Shelby Bay, “	32 28	64 32	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	6.04
10. St. George's, “	32 23	64 40	123	5.63	5.06	3.76	3.10	3.30	5.34	4.65	3.37	3.93	10.90	4.19	5.28
11. Ireland Is'd,[7] “	32 23	64 38	. .	4.91	4.82	3.80	4.19	4.57	2.41	3.75	4.28	5.03	7.74	5.40	4.64
12. Bermuda, Roy. Nav. Hospital	32 23	64 40	. .	7.23	3.78	3.74	3.46	3.11	4.07	2.67	6.25	5.54	4.24	1.45	3.51
13. Havana.	23 09	82 22	50	3.17	1.94	1.70	2.41	3.40	5.94	5.63	2.66	4.75	4.93	1.80	1.43
14. Grand Turk, and Turk's Island	21 31	71 08	15	. .	. .	. .	. .	2.93	1.75	0.44	. .	. .	. .	. .	. .
15. St. Thomas	18 21	64 56	. .	3.10	1.55	2.17	2.40	0.87	0.83	2.25	4.71	8.20	3.06	4.44	1.60
16. San Fernando . . .	21 30	77 00	554	2.74	0.59	2.49	2.09	20.28	14.56	. .	. .	. .	. .	. .	. .
17. Port au Prince, Hayti	18 32	72 21	171	1.27	2.35	3.55	9.52	12.42	4.50	4.10	5.51	6.10	6.22	3.93	1.49

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[1] First five months at Hatley in latitude 45° 15′.

[2] Record in millimetres, changed to inches.

[3] Mean annual amount from nine years, 1822 to 1830, 183.20 inches. (Blodget's Meteorology) Mayer's Hist. of Mexico.

CANADA EAST.

Snow for each Month, Season, and the Year.

.	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning. End.	Observer.	Reference.
1	11.67	16.09	16.96	8.47	53.19	9	8	Apr. 1851; Jan. 1862	Dr. C. Smallwood	S.C., P.O. & S. I. Vol. 1, & S.O.
2	9.53	10.39	10.91	10.93	41.76	9	6	May, 1831; June, 1863	Robertson, Cord, Hall	S.C., Am. Alm., P. O. and S. I. Vol. 1, S. O.
3	5.05	6.02	. .	. .	. .	0	8	Jan. to Aug. 1836	Z. Thompson	Sm. Coll'n.
4	5.32	8.02	8.14	7.70	29.18	1	0	1806	Fowler	Hist. Vermont.

CANADA WEST.

1	8.36	9.15	9.46	8.20	35.17	25	10	Jan. 1840; Dec. 1865	Capt. Lefroy, Cherriman, G. T. Kingston	Results Toronto Mag. Obs.
2	6.34	8.74	5.72	6.39	27.19	2	2	Jan. 1858; Mar. 1860	J. Williamson	P. O. and S. I. Vol. 1, and Rep. Director of Kingston Obs'y.
3	7.16	5.87	9.49	5.29	27.81	2	5	Nov. 1863; May, 1866	C. Rankin	Sm. Obs.
4	7.76	. .	10.86	9.06	. .	1	0	Feb. 1861; June, 1863	H. Phillipps	" "

MEXICO.

1	6.94	7.65	16.02	6.13	36.74	5	8	May, 1843; Apr. 1851	Dr. J. L. Berlandier	MS. in Sm. Coll'n.
2	13.10	54.56	36.26	8.16	112.08	5	0	Jan. 1859; Dec. 1864	J. A. Hieto	P. O. and S. I. Vol. 1, S. O.
3	2.10	9.31	6.55	. .	. .	0	10	1856	Prof. L. C. Ervendberg	P. O. and S. I. Vol. 1.
4	0.49	15.94	7.42	1.59	25.44	3	0	1843—1845	Potts	Dr. Wislizenus
5	31.90	116.80	51.40	5.50	205.60	1	0	1830	. . . . .	Mayer, History Mexico.
6	. .	. .	34.62	6.28	. .	1	0	Sept. 1856; Mar. 1858	Dr. H. Berendt	P. O. and S. I. Vol. 1.
7	10.68	41.05	28.31	4.88	84.92	7	4	Sept. 1857; Dec. 1866	C. Sartorius	P. O. and S. I. Vol. 1, and S.O.
8	2.07	. .	. .	. .	. .	0	3	1859	C. Laszlo	P. O. and S. I. Vol. 1.
9	. .	. .	. .	. .	. .	0	2	1862	" "	Sm. Obs.
10	5.52	18.61	37.36	11.02	72.51	1	9	Mar. 1863; July, 1865	" "	" "

WEST INDIES AND BERMUDAS.

1	. .	. .	. .	10.15	. .	0	6	Oct. 1855; Mar. 1856	Director Met. Obs'y	MS. in Sm. Coll'n.
2	9.24	17.15	15.02	5.34	46.75	3	3	Jan. 1840; Aug. 1845	J. C. Lees	" " "
3	. .	. .	. .	. .	. .	0	1	1844	W. A. Mitchell	" " "
4	13.20	21.31	35.06	16.75	86.32	2	10	Jan. 1862; Dec. 1864	A. A. Julien, L. C. L. Huntington	" " "
5	2.94	5.77	6.45	1.68	16.84	1	8	Feb. 1863; Oct. 1865	A. A. Julien, M. Brayton	" " "
6	. .	. .	. .	12.61	. .	0	5	Oct. 1856; Feb. 1857	Geolog. Surveyor	P. O. and S. I. Vol. 1.
7	4.87	26.42	18.65	5.35	55.29	1	0	1835	A. Mallory	Sill. Jour. Vol. 31, 1837.
8	9.11	12.21	21.51	12.07	54.90	2	5	Dec. 1856; Dec. 1859	By order of Com. R. E.	MS. in Sm. Coll'n, Records Met. Obs'y.
9	. .	. .	. .	. .	. .	0	1	1857	J. B. Arnold	P. O. and S. I. Vol. 1.
10	10.16	13.36	19.02	15.97	58.51	4	0	1836—1839	Capt. Page, R. E.	Records Met. Observatory.
11	12.56	10.44	18.17	14.37	55.54	7	8	Jan. 1852; Dec. 1859	Capt. Supt.	Bermuda R. Gaz., Hamilton.
12	10.31	12.99	11.23	14.52	49.05	1	0	1852	S. D. Wells, Asst. Surg. R. N.	MS. in Sm. Coll'n.
13	7.51	14.23	11.48	6.54	39.76	5	0	1811—1815	. . . . .	Sagra's Hist. of Cuba (Blodget's Climatology).
14	. .	. .	. .	. .	. .	0	3	1861	A. G. Carothers	Sm. Obs.
15	5.44	7.79	15.70	6.25	35.19	3	0	1842—1844	Knox	Reg. Rep.
16	24.86	. .	. .	. .	. .	0	6	1840	. . . . .	Am. Alm.
17	25.49	14.11	16.25	5.11	60.96	4	5	Aug. 1863; Dec. 1867	Dr. A. Ackermann	Petermann's Geog. Mith. x, 1868.

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[4] Chinameca. [5] Central District, Tabasco. [6] Bahama Islands.
[7] H. M. Naval Yard, by authority of the Capt. Sup't, J. D. A., J. G. Calder, and T. G. Wells.

CENTRAL AMERICA.

MEAN AMOUNT OF PRECIPITATION IN RAIN AND MELTED

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Port of Limon, Costa Rica	10°00′	83°03′	..	23.33	6.83	3.67	3.33	4.00	4.42	5.00	9.38	..	1.73	12.71	4.83
2. Guatemala[1]	14 40	90 30	..	0.37	0.04	1.38	2.08	6.38	12.09	9.64	8.14	8.40	5.42	0.55	0.18
3. San José, Costa Rica	9 54	84 06	3772	1.11	0.20	..	2.40	4.61	12.51	9.65	8.32	10.24	14.00	3.91	2.93
4. Aspinwall	9 23	79 53	7	2.27	1.46	0.47	2.77	13.25	13.28	9.65	13.71	15.19	12.28	24.42	12.69
5. Belize, Br. Honduras	17 29	88 13	..	6.12	3.15	1.68	1.81	0.78	6.87	8.04	7.32	7.09	13.70	7.93	7.86

SOUTH AMERICA.

NAME OF STATION.	Lat.	Long.	Height in feet.	Jan.	Feb.	March.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1. Rustenburg Plantat'n[2]	..	..	..	12.63	11.70	5.19	5.15	14.02	13.02	10.13	5.08	3.36	2.48	4.44	10.18
2. Demerara, Br. Guiana	6 50	58 11	36	..	..	..	..	..	..	3.79	..	..	..	..	..
3. Georgetown, " "	6 49	58 12	..	15.88	5.54	4.68	6.58	9.24	10.05	..	..	..	..	..	..
4. Asuncion, Paraguay .	–25 16	57 45	..	7.37	5.91	5.62	5.79	1.17	1.55	0.76	1.59	..	..	..	3.76
5. Catharina Sophia, Surinam	5 48	56 47	..	10.48	2.79	6.45	7.11	10.00	9.17	8.26	3.85	2.75	2.93	5.66	8.49
6. Caracas, Venezuela[3] .	10 31	66 55	2923	0.17	0.00	0.72	3.57	0.51	3.90	4.74	4.66	8.65	5.96	4.04	0.09

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[1] Colegio-Seminario a cargo de los P. P. de la Comp. de Jesus. Measures converted from millimetres into inches.

[2] Gov't Plant'n, also called Vossenburg, Colony of Surinam, Dutch Guiana.

CENTRAL AMERICA.

Snow for each Month, Season, and the Year.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	11.00	18.80	..	34.99	..	0	11	Oct. 1865;	Aug. 1866	F. Valentini	MS. in Sm. Coll'n.
2	9.84	29.87	14.37	0.59	54.67	3	0	Jan. 1857;	Dec. 1859	Fathers of College	Rep. of College Ob's.
3	..	30.48	28.15	4.24	..	2	5	Jan. 1862;	Dec. 1866	C. N. Riotte, Dr. A. Von Frantzius	Sm. Obs.
4	16.49	36.64	51.89	16.42	121.44	4	0	Oct. 1862;	Dec. 1866	Dr. W. T. White, Dr. J. P. Kluge	" "
5	4.27	22.23	28.72	17.13	72.35	5	9	Aug. 1862;	Apr. 1868	S. Cockburn	MS. and Report on the River Belize, May, 1867.

SOUTH AMERICA.

	Spring.	Summer.	Autumn.	Winter.	Year.	Extent of series. yrs.	mos.	Date. Beginning.	End.	Observer.	Reference.
1	24.36	28.23	10.28	34.51	97.38	3	8	May, 1861;	Dec. 1865	C. J. Hering	Sm. Obs.
2	..	..	..	..	..	0	1	1843		D. Blair	Sm. Coll'n.
3	20.50	..	..	...	..	0	6	1854		J. Dawes	" "
4	12.58	3.90	..	17.04	..	0	9	Dec. 1853;	Aug. 1854	K. E. Hopkins	Sm. Coll'n., P.O.& S.I. Vol. 1.
5	23.56	21.28	11.34	21.76	77.94	3	10	Feb. 1856;	Dec. 1859	C. T. Hering	P. O. and S. I. Vol. 1.
6	4.80	13.30	18.65	0.26	37.01	1	0	1860		Dr. Ibarra	An. Rep. Sec. Sm. Inst'n for 1867.

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[3] French inches were converted into English inches.

TABLES

OF

AQUEOUS PRECIPITATION

FOR

SERIES OF YEARS.

(B.)

EXPLANATION OF TABLE (B).

An asterisk affixed to any number indicates that it is derived from *less* than twelve months of observations, in no case, however, are annual amounts given for which more than *three* months had to be found by interpolation, which was effected either by using the observed amount at the nearest station during the time required, or by using the means from adjacent years for the same month or months. The first mode of interpolation is quite reliable, and was always preferred to the second. Blanks for any one year indicate either no observations or an insufficiency (that is, less than nine months) of record during the year. The annual means at the bottom of each column are taken from the preceding table (A), whenever the series was not continuous.

I. MAINE.

Year.	Brunswick.	Gardiner.	Steuben.	Perry.	Hancock Barracks.	Cornish.	Bath.	Fort Preble.	Fort Sullivan.	Saco.	Lisbon.	Eastport.	Dexter.	Biddeford.	Portland.
1808	52.95*	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1809	42.31	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1810	28.09	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1811	38.21*	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1812	40.71	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1813	37.44	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1814	48.20	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1815	27.42*	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1816	32.40*	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1817	39.29*	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1818	26.38	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1832	...	...	...	...	...	...	49.04	...	...	...	...	...	...	...	...
1833	...	...	...	...	...	...	41.53	...	...	...	...	43.00	...	...	...
1834	...	...	...	...	...	...	31.99	...	...	...	...	37.20	...	...	...
1835	...	...	...	...	...	...	30.96	...	...	...	...	...	...	...	...
1836	...	...	...	...	...	...	37.55	...	...	...	...	...	...	...	...
1837	...	32.45	...	...	35.68	...	33.40	...	...	...	...	...	...	...	...
1838	...	30.19	...	...	38.37	...	34.91*	...	...	...	...	...	...	...	...
1839	54.19	41.05	...	...	38.77	...	...	...	...	...	...	...	...	...	...
1840	36.12	41.15	...	...	40.59	...	...	...	...	...	...	...	...	...	...
1841	48.18	38.52	...	...	36.97	...	...	36.97	38.11	...	...	...	...	...	...
1842	54.98	38.13	...	...	37.60	...	...	36.64	40.65	...	...	...	...	...	...
1843	66.93	46.30	...	...	41.33	...	...	41.61*	35.46	...	...	...	...	...	...
1844	50.32	38.32	...	...	30.80	...	...	32.66	37.54	41.07	...	...	...	...	...
1845	75.64	39.18	...	...	...	...	...	38.37*	...	57.19	...	...	...	...	...
1846	43.46	35.31	...	...	...	...	...	...	...	36.45	...	...	...	...	...
1847	61.17	44.90	...	...	...	...	...	...	...	43.81	...	...	...	...	...
1848	59.37	48.86	...	...	...	...	...	...	...	...	...	...	...	...	...
1849	39.31	39.56	...	...	...	...	...	...	...	...	...	...	...	45.59	...
1850	57.44	51.47	...	...	...	...	...	58.60	47.17	...	...	...	...	59.57	...
1851	47.37	50.81	...	...	...	...	...	55.10	36.56	...	...	...	...	44.82	...
1852	55.27	46.65	...	...	...	...	...	60.02	39.80	...	...	...	...	...	...
1853	...	...	...	...	...	...	...	47.59*	...	...	...	...	...	...	...
1854	46.14	...	47.14*	51.06*	...	...	...	...	...	...	...	...	...	...	...
1855	44.58	39.23	...	51.26*	...	...	...	...	...	...	...	...	...	...	...
1856	32.78	37.97	62.43*	49.29	...	46.29*	...	...	...	...	...	...	...	...	44.04*
1857	42.88	46.20	...	58.95*	...	58.07	...	...	...	...	...	...	...	...	49.67
1858	45.54	47.04	43.18*	36.13	...	44.56	...	...	...	...	...	...	...	...	44.45
1859	48.44	48.11	63.49*	61.78	...	50.64	...	...	...	...	53.32*	...	44.46*	...	52.92*
1860	...	...	...	...	...	50.94	...	...	...	...	42.57	...	30.99*	...	...
1861	...	39.92*	56.88*	47.18	...	46.74	...	...	...	...	41.77	...	42.51	...	...
1862	...	39.24	54.93*	44.84	...	38.71	...	...	...	...	43.40	...	37.86	...	...
1863	...	49.46*	52.33*	...	...	51.39*	...	...	...	...	52.49*	...	...	...	...
1864	...	40.95	49.15	45.89	...	45.69	...	...	...	...	43.96	...	...	...	...
1865	...	41.87	47.65	...	...	45.23	...	...	...	...	41.46	...	...	...	...
1866	...	45.66	55.01*	...	...	45.55	...	...	...	...	49.10	...	...	...	...
	44.68	42.09	53.26	50.29	36.97	47.85	37.08	45.14	39.41	45.11	46.00	40.09	38.72	49.75	48.63

MAINE.—CONTINUED.					II. NEW HAMPSHIRE.											
Year.	Kennebec Arsenal.	West Waterville.	Lee.	Standish.	Hanover.	Fort Constitution.	Stratford.	Concord.	Laconia.	Lake Village.	Londonderry.	Manchester.	Salmon Falls.	West Enfield.	North Barnstead.	Claremont.
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1835	...	...	...	...	31.65	...	...	...	...	...	...	...	...	...	...	...
1836	...	...	...	...	37.87	30.93*	...	...	...	...	...	...	...	...	...	...
1837	...	...	...	...	35.45	28.16	...	...	...	...	...	...	...	...	...	...
1838	...	...	...	...	40.62	31.84	...	...	...	...	...	...	...	...	...	...
1839	...	...	...	...	32.05	26.93	...	...	...	...	...	...	...	...	...	...
1840	...	...	...	...	45.08	29.54*	...	...	...	...	...	...	...	...	...	...
1841	...	...	...	...	38.22	...	...	...	...	...	...	...	...	...	...	...
1842	...	...	...	...	47.82	33.79	...	...	...	...	...	...	...	...	...	...
1843	...	...	...	...	55.98	34.15	...	...	...	...	...	...	...	...	...	...
1844	...	...	...	...	47.71	31.65	...	...	...	...	...	...	...	...	...	...
1845	...	...	...	...	38.33	...	...	...	...	...	...	...	...	...	...	...
1846	...	...	...	...	37.23	...	...	...	...	...	...	...	...	...	...	...
1847	...	...	...	...	45.86	...	...	...	...	...	...	...	...	...	...	...
1848	...	...	...	...	48.49	...	...	...	...	...	...	...	...	...	...	...
1849	...	...	...	...	38.20	...	...	...	...	...	...	...	...	...	...	...
1850	...	...	...	...	40.31	50.73	...	...	...	...	...	...	...	...	...	...
1851	...	...	...	...	41.49	45.35	...	...	...	...	...	...	...	...	...	...
1852	...	...	...	...	35.66	42.49	...	...	...	...	...	...	...	...	...	...
1853	...	...	...	...	...	37.32*	...	...	...	...	...	...	...	...	...	...
1854	...	...	...	...	34.57	...	...	...	...	...	41.45*	44.99	36.75	...	...	...
1855	...	...	...	...	...	...	...	...	...	...	43.85*	...	...	...	...	...
1856	...	...	...	...	...	...	...	34.13*	...	...	...	...	...	...	...	...
1857	34.47*	...	...	...	...	...	...	...	49.08	48.17	...	46.40*	...	46.62*	...	...
1858	35.14*	...	...	...	...	...	43.24*	38.33*	42.55	41.73	...	...	...	38.00*	...	...
1859	...	...	...	...	...	...	35.32	...	46.68	44.82	...	...	...	...	...	...
1860	...	...	...	...	...	...	30.90	...	...		...	...	...	...	...	...
1861	...	...	...	...	...	...	45.10	...	...	...	...	...	...	...	...	46.73
1862	...	...	...	...	...	...	39.98	...	...	...	...	...	...	...	...	38.78
1863	...	...	...	...	...	...	42.82*	...	...	...	...	...	...	...	...	44.25
1864	...	34.60	...	...	...	...	41.73	...	...	...	...	...	...	...	...	43.01
1865	...	37.32	46.71	40.22*	...	...	38.51	...	...	...	...	...	...	...	36.49	45.18
1866	...	44.76	44.63*	38.64*	...	...	50.45	...	40.07	39.70	...	...	...	...	32.96*	47.52
	34.81	39.37	46.30	39.42	40.32	35.58	39.96	40.99	44.58	43.61	45.61	45.17	36.75	47.77	32.99	43.69

III. VERMONT.												IV. MASSACHUSETTS.			
Year.	Fayetteville.	Burlington.	Craftsbury.	Brandon.	Lunenburg.	St. Johnsbury.	Shelburn.	Woodstock.	Springfield.	Middlebury.	Randolph.	Year.	New Bedford.	Boston.	Cambridge, Harvard Observat'y.
...	...	...	...	...	...	...	...	...	...	...	...	1814	38.30	...	...
...	...	...	...	...	...	...	...	...	...	...	...	1815	36.30	...	...
...	...	...	...	...	...	...	...	...	...	...	...	1816	39.23	...	...
...	...	...	...	...	...	...	...	...	...	...	...	1817	38.52	...	...
...	...	...	...	...	...	...	...	...	...	...	...	1818	36.24	42.99	...
...	...	...	...	...	...	...	...	...	...	...	...	1819	35.26	35.48	...
...	...	...	...	...	...	...	...	...	...	...	...	1820	36.73	44.18	...
...	...	...	...	...	...	...	...	...	...	...	...	1821	40.57	36.89	...
...	...	...	...	...	...	...	...	...	...	...	...	1822	37.14	27.20	...
...	...	...	...	...	...	...	...	...	...	...	...	1823	53.24	46.43	...
...	...	...	...	...	...	...	...	...	...	...	...	1824	42.08	35.98	...
...	...	...	...	...	...	...	...	...	...	...	...	1825	33.86	32.41	...
...	...	...	...	...	...	...	...	...	...	...	...	1826	48.69	41.68	...
1827	66.40	...	...	...	...	...	...	...	...	...	...	1827	55.91	44.39	...
1828	66.30	43.30	...	...	...	...	...	...	...	...	...	1828	34.70	33.98	...
1829	43.60	...	...	...	...	...	...	...	...	...	...	1829	58.14	48.09	...
1830	51.90	...	...	...	...	...	...	...	...	...	...	1830	57.48	44.62	...
1831	62.20	...	...	...	...	...	...	...	...	...	...	1831	54.39	50.87	...
1832	46.80	39.59	...	...	...	...	...	...	...	...	...	1832	43.83	46.68	...
1833	50.42*	49.44	...	...	...	...	...	...	...	...	...	1833	37.89	39.71	...
...	...	...	...	...	...	...	...	...	...	...	...	1834	40.11	38.03	...
...	...	...	...	...	...	...	...	...	...	...	...	1835	41.97	35.48	...
...	...	...	...	...	...	...	...	...	...	...	...	1836	38.07	35.71	...
...	...	...	...	...	...	...	...	...	...	...	...	1837	34.73	29.98	...
1838	...	30.83	...	...	...	...	...	...	...	...	...	1838	34.03	31.57	...
1839	...	27.89	...	...	...	...	...	...	...	...	...	1839	39.44	34.82	...
1840	...	37.19	...	...	...	...	...	...	...	...	...	1840	44.08	...	...
1841	...	32.68	...	...	...	...	...	...	...	...	...	1841	44.98	...	40.86
1842	...	33.85	...	...	...	...	...	...	...	...	...	1842	34.73	...	40.65
1843	...	26.58	...	...	...	...	...	...	...	...	...	1843	45.04	...	52.52
1844	...	31.21	...	...	...	...	...	...	...	...	...	1844	36.21	...	33.16
1845	...	36.04	...	...	...	...	...	...	...	...	...	1845	42.72	...	50.51
1846	...	29.66	...	...	...	...	...	...	...	...	...	1846	30.68	...	30.04
1847	...	38.55	...	...	...	...	...	...	...	...	...	1847	40.81	...	48.21
1848	...	31.38	...	...	...	...	...	...	...	...	...	1848	36.21	...	43.03
1849	...	26.35	...	...	...	...	...	...	...	...	...	1849	31.87	40.30	40.97
1850	...	37.51	...	...	...	...	...	...	...	...	...	1850	51.78	53.98	54.14
1851	...	31.83	...	...	...	...	...	...	...	...	...	1851	49.79	44.31	42.11
1852	...	28.82	...	...	...	...	...	...	...	...	...	1852	41.01	47.94	40.51
1853	...	33.05	...	...	...	...	...	...	...	...	...	1853	35.07	48.86	53.83
1854	...	25.45	...	...	...	26.05*	...	...	...	...	...	1854	47.84	45.71	45.17
1855	...	38.30	...	...	...	...	...	...	...	...	...	1855	36.44	44.19	47.65
1856	...	26.82	39.48*	34.68*	...	...	40.65*	...	...	...	...	1856	32.98	52.16	53.81
1857	...	28.69*	47.10	43.68	...	...	48.17	45.45	...	...	...	1857	38.59	56.87	57.92
1858	...	...	38.40	32.92	...	...	...	35.70	...	...	...	1858	42.42	52.67	45.46
1859	...	...	34.37	31.49	...	...	...	...	...	...	...	1859	48.90	56.70	59.34
1860	...	...	37.70*	36.17*	...	...	...	...	...	...	...	1860	38.35	51.46	45.09
1861	...	42.56	40.45	42.93	...	...	...	...	43.46	...	...	1861	44.76	50.07	50.28
1862	...	40.82*	39.13	32.84*	39.19*	...	...	...	40.80	...	...	1862	41.36	61.06	57.21
1863	...	39.47*	36.02*	46.11	...	...	...	...	...	...	...	1863	44.40	67.78	56.42
1864	...	35.74*	39.50	...	41.16.	...	...	...	...	37.38*	...	1864	39.52	49.30	39.46
1865	...	...	36.53	...	43.36	...	...	...	...	35.20	...	1865	44.62	47.83	43.59
1866	...	...	42.99	35.37*	35.57*	...	...	...	...	37.03	33.30	1866	38.54	50.70	35.49*
...	...	...	...	...	...	...	...	...	...	...	...	1867	45.61	55.64	41.71
	55.17	34.15	39.24	36.57	39.58	35.50	44.60	40.58	43.15	36.62	32.92		41.42	44.99	46.39

MASSACHUSETTS.—Continued.

Year.	Amherst.	Worcester Asylum.	Springfield.	Fort Independence.	Watertown Arsenal.	Nantucket.	Chelsea, U.S. Hosp.	Mendon.	North Attleboro'.	Newburyport.	Princeton.	Southwick.	Westfield.	Williamstown.	Bridgewater.
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1836	40.76	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1837	38.70	...	...	...	32.06	...	...	...	...	...	...	...	...	...	...
1838	39.81	...	...	...	43.99	...	...	...	...	...	...	...	...	...	...
1839	42.83	...	...	...	42.89	...	...	...	...	...	...	...	..	...	...
1840	47.01	...	...	...	48.08	...	...	...	...	...	...	...	...	...	...
1841	41.63	51.11	...	...	44.45*	...	...	...	...	...	...	...	...	...	...
1842	38.05	40.78	...	...	...	...	...	...	...	...	...	...	...	...	...
1843	51.58	48.67	...	...	48.41	...	...	...	...	...	...	...	...	...	...
1844	40.58	37.34	...	...	39.50*	...	...	...	...	...	...	...	...	...	...
1845	39.74	42.49	...	...	...	...	...	...	...	...	...	...	...	...	...
1846	34.92	34.60	...	...	...	...	...	...	...	...	...	...	...	...	...
1847	47.81	49.00	...	...	...	32.44*	...	...	...	...	...	...	...	...	...
1848	41.38	38.53	41.38	...	...	34.60	...	...	...	...	...	...	...	...	...
1849	38.45	37.39	34.36	...	...	27.08	...	...	...	...	...	...	...	...	...
1850	55.57	55.49	53.80	...	...	42.85	...	...	...	...	...	...	...	...	...
1851	43.50	43.79	45.22	...	...	41.82	...	...	...	...	...	...	...	...	...
1852	43.60	61.83	40.44	30.22	...	39.77	...	...	...	...	...	...	...	...	...
1853	50.82*	59 64	51.36	41.04	...	42.00	...	...	...	...	...	...	...	...	...
1854	47.35	59.25	50.37	38.64	...	47.43	...	...	45.68	35.92*	46.26	50.84	...	...	...
1855	48.18	58.62	47.21	35.78	...	48.93	...	...	44.93	42.22	46.24	50.82*	48.39	54.51	...
1856	40.13	46.94	...	46.90*	...	35.70	...	...	45.42	46.67	42.02*	...	47.57	43.90*	...
1857	47.66	53.24	...	42.37	...	49.88	...	...	...	...	...	...	53.33	...	41.17*
1858	42.59	40.93	...	37.67	...	41.93	...	42.49*	...	...	...	...	43.12	34.76	30.27*
1859	51.29	48.77	...	46.65	...	48.35	...	...	...	...	...	...	54.29	43.65*	...
1860	39.64	48.63	...	...	...	...	...	...	...	...	...	...	...	41.35	...
1861	46.82	42.31	...	...	...	...	35.60	39.86*	...	...	...	...	52.52	44.50	...
1862	49.43	44 09	...	...	...	...	27.00*	37.58*	...	...	...	...	49.66	...	...
1863	54.20*	...	...	...	...	...	...	43.66*	...	...	...	...	59.09*	...	...
1864	33.87	...	32.52*	...	...	...	...	28.45*	...	...	...	...	34.16	31.36	...
1865	42.86	38.52	...	...	...	...	...	36.04*	...	...	...	...	43.26	36.70*	...
1866	45.01*	42.31	...	...	...	...	...	43.15*	...	...	...	...	...	35.58	...
1867	46.11	48.94	...	...	...	...	...	...	...	...	...	...	...	33.94	...
	43.90	46.92	43.79	39.14	42.08	41.10	31.30	39.67	44.02	41.74	44.76	18.65	48.15	36.59	37.44

MASSACHUSETTS.—Continued.										V. RHODE ISLAND.			
Year.	Lawrence.	Sandwich.	Baldwinsville.	Topsfield.	Fitchburg.	West Springfield.	Waltham.	Lake Cochituate.	Lowell.	Year.	Providence.	Fort Adams.	Newport.
...	...	...	...	...	...	...	...	...	...	1832	39.46	...	...
...	...	...	...	...	...	...	...	...	...	1833	34.14	...	...
...	...	...	...	...	...	...	...	...	...	1834	41.92	...	...
...	...	...	...	...	...	...	...	...	...	1835	30.96	...	...
...	...	...	...	...	...	...	...	...	...	1836	37.87	...	...
...	...	...	...	...	...	...	...	...	...	1837	31.62	...	...
...	...	...	...	...	...	...	...	...	...	1838	37.88	...	...
...	...	...	...	...	...	...	...	...	...	1839	36.75	...	...
...	...	...	...	...	...	...	...	...	...	1840	41.19	...	...
...	...	...	...	...	...	...	...	...	...	1841	47.86	...	...
...	...	...	...	...	...	...	...	...	...	1842	37.71	54.79	...
...	...	...	...	...	...	...	...	...	...	1843	42.50	67.61	...
...	...	...	...	...	...	...	...	...	...	1844	35.00	49.07	...
...	...	...	...	...	...	...	...	...	...	1845	43.16	42.69	...
...	...	...	...	...	...	...	...	...	...	1846	30.51	35.76*	...
...	...	...	...	...	...	...	...	...	...	1847	48.50	...	...
...	...	...	...	...	...	...	...	...	...	1848	40.48	...	...
1849	...	...	...	...	...	...	40.74	...	51.09	1849	34.69	42.63	...
1850	...	...	...	...	...	52.47	62.13	...	45.68	1850	51.49	54.15	...
1851	...	...	...	...	...	...	41.00	...	41.00	1851	43.38	52.76	...
1852	...	...	...	...	...	...	42.24	45.93	42.78	1852	38.58	57.33	...
1853	...	...	...	...	...	...	45.04	55.86	43.92	1853	53.27	65.59*	...
1854	...	...	...	...	...	...	41.29	43.15	42.08	1854	46.25	...	...
1855	...	...	...	...	...	...	40.63	34.96	46.65	1855	39.05	...	...
1856	58.70	...	...	...	...	...	42.33	40.80	44.23	1856	40.97	...	...
1857	56.59	...	...	...	...	...	44.04	63.10	50.70	1857	44.75	...	...
1858	41.36	...	...	...	...	...	37.40	48.66	36.76	1858	44.51	43.35*	...
1859	47.50	...	...	...	...	...	48.49	49.02	47.96	1859	45.16	52.82*	...
1860	...	...	...	41.77*	...	...	...	55.44	46.79	1860	38.24	...	...
1861	...	...	...	33.95	43.07	...	...	46.44	43.13	1861	44.25	...	...
1862	42.70*	...	...	...	...	...	...	49.69	44.44	1862	50.09	...	...
1863	53.92*	54.15*	...	...	...	...	53.66	69.30	55.09	1863	54.17	...	...
1864	39.07	41.67	39.67*	33.93*	...	...	36.56	42.60	39.38	1864	36.83	...	...
1865	...	...	38.20	38.34	...	...	35.84	49.46	38.10	1865	42.71	...	...
1866	38.01	...	...	39.99	...	...	43.46	62.32	39.77	1866	44.05	...	45.92
1867	...	...	...	...	...	...	41.40	56.25	45.71	1867	47.04	...	...
	44.96	47.99	40.36	39.22	43.07	52.47	43.52	50.81	44.48		41.54	51.45	45.37

VI. CONNECTICUT.

Year.	New Haven.	Pomfret.	Fort Trumbull.	Hartford.	Middletown.	Saybrook.	New London.	Wallingford.	Canton.	Plymouth.	Groton.
1804	43.30	...	...	...	...	...	...	...	...	...	...
1805	41.07*	...	...	...	...	...	...	...	...	...	...
1806	38.61	...	...	...	...	...	...	...	...	...	...
1807	45.32	...	...	...	...	...	...	...	...	...	...
1808	49.40	...	...	...	...	...	...	...	...	...	...
1809	44.55	...	...	...	...	...	...	...	...	...	...
1810	39.40	...	...	...	...	...	...	...	...	...	...
1811	47.66	...	...	...	...	...	...	...	...	...	...
1812	44.17	...	...	...	...	...	...	...	...	...	...
1813	53.40	...	...	...	...	...	...	...	...	...	...
1814	56.06	...	...	...	...	...	...	...	...	...	...
1815	50.64	...	...	...	...	...	...	...	...	...	...
1816	38.00	...	...	...	...	...	...	...	...	...	...
1817	43.38	...	...	...	...	...	...	...	...	...	...
1818	38.02	...	...	...	...	...	...	...	...	...	...
1819	33.89	...	...	...	...	...	...	...	...	...	...
1820	46.19	...	...	...	...	...	...	...	...	...	...
1821	44.64	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...
1827	51.38	...	...	...	...	...	...	...	...	...	...
1828	51.22*	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...
1843	...	...	52.78	...	...	...	...	...	...	...	...
1844	...	...	40.29	...	...	...	...	...	...	...	...
1845	...	...	42.81	...	...	...	...	...	...	...	...
1846	...	...	...	...	...	...	...	...	...	...	...
1847	...	...	...	45.60	...	...	...	...	...	...	...
1848	...	...	...	39.94	...	...	...	...	...	...	...
1849	...	...	...	39.84	...	...	...	...	...	...	...
1850	...	...	...	57.14	...	...	...	...	...	...	...
1851	...	...	...	41.99	...	...	...	...	...	...	...
1852	...	...	...	39.95	...	...	...	...	...	...	...
1853	...	...	...	...	...	...	...	...	...	...	...
1854	...	57.14	...	...	...	...	...	...	...	...	...
1855	...	42.68	...	...	...	45.19*	...	...	...	...	...
1856	...	47.69	...	...	...	48.95*	41.67	55.33*	...	...	...
1857	...	51.68*	...	...	...	49.97	47.23*	57.38*	...	...	...
1858	...	41.83	...	...	...	43.06*	...	41.64	...	...	...
1859	...	53.73	...	...	53.33	58.16	...	57.11	...	...	...
1860	...	...	...	...	...	40.83*	...	48.68	...	...	...
1861	...	44.85	...	...	47.19	...	...	...	...	...	...
1862	...	40.96	...	...	48.33	...	...	...	51.04	...	...
1863	...	56.74	...	...	52.79*	...	...	...	...	57.19	...
1864	...	37.40	...	...	37.08	...	...	...	...	...	...
1865	41.89	45.10	...	...	42.51	...	...	...	...	...	...
1866	46.98	45.82	...	...	46.27	...	...	...	...	...	46.83
1867	45.44	...	...	...	...	...	...	...	...	...	...
	44.43	47.11	44.69	43.79	45.97	48.84	44.71	52.03	54.85	48.59	46.83

VII. NEW YORK.

Year.	Flatbush.	East Hampton.	North Salem.	Jamaica.	Newburg.	Albany.	Utica.	Pompey.	Penn Yan.	Rochester.	Fort Hamilton.	Fort Columbus.	Westpoint.	Watervliet Arsenal.	Plattsburg Barracks.
1826	...	...	...	...	...	33.12	...	...	...	...	...	...	...	...	...
1827	...	...	...	...	...	49.80	...	...	...	...	...	...	...	...	...
1828	...	30.91	...	48.91	...	37.66	36.57	...	...	...	...	...	...	...	...
1829	...	42.56	...	45.83	...	38.07	36.16	...	32.39	...	...	...	...	...	...
1830	53.47	46.65	43.37	43.32	34.83	41.85	46.19	31.66*	30.07	...	...	...	...	...	...
1831	42.52	38.49	43.04	37.75	...	39.54	37.85	29.47*	29.90	34.60	...	...	...	...	...
1832	42.82	41.82	40.93	39.15	43.16	44.45	49.90	26.44	30 70	31.45	...	...	...	...	...
1833	46.76	36.29	43.26	36.67	41.17	41.74	37.79	30.14	25.89	30.30	...	...	...	...	...
1834	39.47	40.12	33.50	34.06	29.23	32.45	34.52	...	22.39	22.90	...	...	...	...	...
1835	38.10	30.55	34.99	28.78	25.04	40.44	38.58	33.27	26.51	32.70	...	...	...	...	...
1836	43.89	35.50	...	36.57	30.27	44.60	33.16	23.48	27.85	...	...	27.57	...	34.69*	...
1837	34.66	35.23	...	32.13	25.32*	41.17	27.54*	30.30	24.56	31.05	...	65.51	...	32.04	...
1838	41.11	28.06	30.87	33.70	48.24	42.03	...	23.21	31.43	25.69	...	41.90	...	30.80	...
1839	42.90	46.39	...	33.44	44.54	38.11	35.44	25.54	31.30	30.09	51.11	42.97	...	29.73	...
1840	35.93	43.32	37.38	35.47	38.98	44.39	45.81	32.79	29.54	29.39	...	29.80	47.82	32.28	35.70
1841	52.12	43.62	39.78	44.57	...	37.85	41.95	25.28	24.04	32.33	...	42.08	53.94	35.79	24.88*
1842	47.04	33.07	45.72	41.59	30.63	45.29	56.69	29.84	31.50	33.24	...	33.98	50.62	44.93*	29.02
1843	50.19	29.46	50.17	33.66	33.51	48.35	40.47	31.38	29.18	30.21	44.18	41.37	50.60	42.04*	40.92
1844	39.07	...	34.90	44.72	31.87	32.45	35.67	...	20.10	26.17	32.64	36.38	48.37	...	39.28
1845	32.14	...	39.50	33.71	30.35*	40.44	40.28	...	24.40	34.43	35.11	34.08	43.38	36.54	27.61
1846	44.00	...	45.67	35.94	34.75	39.85	36.37	...	28.93	37.13	52.69	48.91	46.58	31.49	...
1847	45.93	...	44.80	44.47	29.82	41.38	...	...	27.72	36.14	62.69	64.85	35.05	31.80	...
1848	33.41	...	35.65	32.76	34.72	48.22	...	...	22.57	32.03	34.13	36.80	50.79	39.93	...
1849	32.47	...	42.65	30.01	33.72	36.72	...	...	22.97	33.37	29.75	31.74	38.25	33.19*	...
1850	48.58	...	55.63	...	55.63	50.97	40.39	45.08	31.72	38.47	45.89	54.53	59.94	33.22	43.90
1851	40.82	...	35.38	...	45.41	34.66	49.97	40.26	26.81	24.97	37.87	40.88	40.60	34.08	36.20
1852	47.46	...	39.02	35.30	...	31.92	44.31*	41.33*	25.52*	35.04	39.66	43.84	43.31	31.14	...
1853	...	...	...	...	...	...	...	...	29.47	32.52	55.24	52.20	63.56	42.82	...
1854	...	...	...	...	...	...	...	...	19.66	29.42	44.30	45.18	47.06	27.50	...
1855	...	...	...	...	...	...	...	...	29.96	33.93	40.37	43.95	49.28	...	...
1856	39.90	...	...	...	...	...	...	...	21.66	26.42	36.27	38.65	47.41	...	...
1857	49.75	...	...	...	...	...	...	...	44.90	43.03	42.41	40.70	47.19	...	...
1858	43.23	...	...	...	...	...	...	...	26.78	35.97	36.05	37.45	43.94	...	...
1859	58.92	...	...	...	...	...	...	...	29.15	28.50	43.09	62.41	43.69	...	...
1860	...	...	...	...	...	...	...	...	25.62	30.11	...	...	...	...	...
1861	47.40	...	...	...	...	...	...	...	32.74	35.99	...	...	...	...	...
1862	...	...	...	...	...	...	...	...	31.46	40.21	...	...	...	...	...
1863	33.13*?	...	...	...	...	...	...	...	40.32	31.21	...	...	...	...	...
1864	33.33	...	...	...	...	...	...	...	31.97	40.84	...	...	...	...	...
1865	47.41	...	...	...	40.59*	38.14	...	...	31.77	35.27	...	...	...	...	...
1866	34.72*	...	...	...	40.89	...	...	...	26.85	36.07	...	...	...	...	...
1867	...	...	...	...	...	...	...	...	28.45*	...	...	...	...	...	...
	43.52	37.29	40.82	39.07	36.61	40.52	41.14	30.75	28.42	32.56	42.55	43.24	47.65	34.65	33.40

NEW YORK.—Continued.

Year.	Madison Barracks.	Fort Ontario.	Fort Niagara.	Buffalo Barracks.	New York Hospital.	Lowville.	Geneva.	Buffalo.	N. Y. Inst. for Deaf and Dumb.	Seneca Falls	Jamestown.	Syracuse.	Oxford.	Homer.	Hudson.
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1836	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1837	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1838	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1839	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1840	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1841	32.37	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1842	37.31	...	50.49	...	...	...	...	...	...	...	...	...	...	...	...
1843	33.77	...	37.07	44.70	...	...	...	...	...	...	...	...	...	...	...
1844	29.23	34.58	18.86?	34.79	...	...	...	...	...	...	...	...	...	...	...
1845	32.37	35.26*	20.61?	...	...	...	...	...	...	...	...	...	...	...	...
1846	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1847	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1848	...	...	...	...	44.76	...	...	...	...	...	...	...	...	...	...
1849	...	26.96	...	...	37.61	...	...	...	...	...	...	...	...	...	...
1850	57.52?	34.63	39.74	...	51.50	...	...	...	...	...	...	...	...	...	...
1851	53.79?	31.95	25.91	...	...	...	...	...	...	...	...	...	...	...	...
1852	...	32.54	37.04	...	...	...	30.48*	...	...	36.23	38.33*	41.44	35.44	45.28	36.15
1853	...	...	26.47	...	...	...	...	...	...	...	...	...	...	...	...
1854	...	...	29.54*	...	...	...	...	...	42.78	...	...	...	...	...	...
1855	...	25.93	...	...	...	...	...	...	52.93	...	...	...	...	...	...
1856	...	32.14*	...	...	...	...	27.46*	...	39.42	32.66	...	...	...	...	...
1857	...	...	...	...	...	47.32	...	...	55.79*	44.33	...	...	...	...	...
1858	...	...	...	...	...	35.38	...	41.95	51.87	...	...	...	...	...	...
1859	...	...	...	...	...	...	...	32.71	55.60	...	...	...	...	...	...
1860	...	...	29.13	...	...	...	...	30.04	...	...	...	...	...	...	...
1861	...	...	33.93	...	...	...	...	38.61	57.07	...	...	...	...	...	...
1862	...	...	28.73	...	...	...	...	36.67	53.45	...	...	...	...	...	...
1863	...	...	26.72	...	...	...	...	29.75	57.02	...	...	...	...	...	...
1864	...	...	30.38	...	...	...	32.97	40.96	48.00	...	52.70	...	...	...	...
1865	...	...	24.41	...	...	...	33.15	33.55	62.87	...	54.84*	...	...	...	...
1866	...	...	31.49	...	...	...	28.95	39.45	54.48*	...	...	...	...	...	...
1867	...	...	...	...	...	...	...	31.44	...	...	...	...	...	...	...
	33.72	30.51	30.75	38.80	42.09	33.50	30.87	33.84	50.29	36.61	48.91	36.75	36.36	46.98	34.52

NEW YORK.—CONTINUED.

Year.	Mexico.	Liberty.	Ogdensburg.	Delhi.	Clinton.	Blackwell's Isl'd, Pen'y Hospital.	Gouverneur High Sch'l.	New York U.S. Naval Hospital.	Sackett's Harbor.	Charlotte.	Lodi.	Sag Harbor.	Spencertown.	Troy.	Beverly.	Cazenovia.	Clinton.
1838	...	...	...	...	...	...	19.67	...	...	...	...	...	...	...	...	...	...
1839	...	...	...	...	...	...	23.48	...	...	...	...	...	...	...	...	...	...
1840	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1841	...	...	...	...	...	...	22.67	...	...	...	...	...	...	...	...	...	...
1842	...	...	...	...	...	...	19.42	...	...	...	...	...	...	...	...	...	...
1843	...	...	...	...	...	...	15.73	...	...	...	...	...	...	...	...	...	...
1844	...	...	...	...	...	...	25.46	...	...	...	...	...	...	...	...	...	...
1845	...	...	...	...	...	...	33.48	...	...	...	...	...	...	...	...	...	...
1846	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1847	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1848	...	...	...	...	...	...	25.70	...	...	...	...	...	...	...	...	...	...
1849	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1850	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1851	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1852	39.72	47.82	37.14	36.36	45.11	...	...	...	...	...	...	...	...	...	...	...	...
1853	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1854	...	...	...	...	...	...	28.98	...	...	...	20.18	62.62	...	...	...	...	...
1855	...	...	...	...	...	...	25.66	...	...	...	37.00	40.52*	42.49	...	...	...	...
1856	...	...	...	...	...	40.25	...	...	...	...	...	42.04*	38.40	...	...	...	31.73*
1857	...	...	40.45*	...	...	54.34*	...	...	...	...	33.38*	37.21	44.77*	...	...	58.05*	63.04*
1858	...	...	37.49*	...	...	...	...	...	...	...	...	43.90*	...	...	42.46	...	41.82
1859	...	...	36.58*	...	...	...	...	...	...	...	...	...	35.69*	...	49.72	41.62	...
1860	...	...	...	...	...	...	...	...	34.80	25.54	...	...	...	...	49.95*	...	...
1861	...	...	...	...	...	...	50.75	35.94	45.90	33.42	...	...	...	39.06	51.76	43.44	...
1862	...	...	...	...	...	...	33.28	37.19	28.76	29.44	...	...	...	32.39	...	45.60*	30.67
1863	...	...	...	...	...	...	30.83*	...	38.53	24.83	...	...	...	...	...	...	39.77*
1864	...	...	...	...	...	...	36.62	31.58	41.54	35.10	...	...	...	...	41.16	...	38.35
1865	...	...	...	...	...	...	27.78	41.73*	30.62	31.63	...	...	...	...	47.68	...	...
1866	...	...	...	...	...	...	...	46.64	38.92	32.23	...	...	...	...	48.49	...	...
	33.69	48.18	37.63	37.16	33.89	47.44	30.15	38.30	36.73	29.91	27.35	43.07	38.55	37.55	48.20	45.10	41.49

NEW YORK.—CONTINUED.

Year.	Fishkill Landing.	Lake.	Oswego.	Fort Edward.	Wampsville.	Watertown.	Bellport.	Waterford.	Pierrepont Manor.	Palermo.	Hermitage.	Theresa.	South Hartford.	South Trenton.	Brookhaven.	Columbia College, N. York.	Depauville.
1856	...	...	...	...	...	27.19	...	...	...	...	...	...	...	...	...	...	...
1857	...	47.78*	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1858	30.06	42.36*	...	61.75	...	...	...	35.23	...	...	...	...	...	...	...	...	...
1859	41.84	...	...	...	72.62*	...	51.94	34.99	39.11	...	...	...	...	...	...	...	...
1860	...	...	...	...	69.97*	...	...	...	34.62	...	...	...	...	...	...	...	...
1861	40.76	...	50.05	...	73.53	...	...	39.32	42.74	...	52.30	...	...	...	...	...	...
1862	47.33	...	44.59*	...	63.71	...	...	35.90	29.29	...	47.91	31.18	...	...	...	...	...
1863	48.78*	...	56.16	...	65.74	...	...	...	39.55	...	53.13	34.67*	...	...	...	...	...
1864	38.71	...	46.79	...	60.95	...	...	...	39.19	...	...	45.50	49.19	52.04*	51.08*	...	...
1865	40.81	...	43.32	...	64.99	...	...	...	38.99*	43.41	...	33.86	43.67	52.33	56.23	37.36*	...
1866	37.26*	...	43.23	...	64.22	...	...	...	...	51.85	...	42.27*	34.98	55.22	56.66	34.17	47.17
	39.93	44.74	46.94	59.73	66.93	27.19	45.42	38.49	37.65	45.29	51.73	37.67	42.75	52.78	54.67	35.79	48.17

VIII. NEW JERSEY.

Year.	Newark.	Lambert-ville.	Bloomfield.	Burlington.	Junc. Rancocus & Delaware Rivers.	Coles' Landing.	Greenwich.	Paterson.	Trenton.	New Brunswick.
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1839	...	44.01	...	...	...	...	...	...	...	...
1840	...	41.61	...	...	...	...	...	...	...	...
1841	...	57.37	...	...	...	...	...	...	...	...
1842	...	41.86	...	...	...	...	...	...	...	...
1843	...	51.32	...	...	...	...	...	...	...	...
1844	40.21	40.33	...	...	...	...	...	...	...	...
1845	36.57	42.88	...	...	...	...	...	...	...	...
1846	51.58	45.18	...	...	...	...	...	...	...	...
1847	53.84	51.04	...	...	...	...	...	...	...	...
1848	36.77	34.18	...	...	...	...	...	...	...	...
1849	40.05	43.74	43.63*	...	...	...	...	...	...	...
1850	51.15	53.26	49.67	...	...	...	...	...	...	...
1851	41.38	32.44	35.85	...	...	...	...	...	...	...
1852	43.54	45.22	40.52	...	...	...	...	...	...	...
1853	52.34	42.94	49.83	...	...	...	...	...	...	...
1854	43.48	43.16	37.57	...	...	...	...	...	...	...
1855	44.27	45.18	32.89	...	...	...	...	...	...	...
1856	34.54	32.33	27.32	...	...	...	...	...	...	...
1857	49.36	48.66	71.03*	51.80	...	...	...	...	...	...
1858	40.42	40.32	35.66*	...	...	...	...	...	...	...
1859	57.05	47.64	...	...	...	...	...	...	...	...
1860	44.97	...	...	...	...	...	...	...	...	...
1861	43.46	...	...	...	...	...	...	...	...	...
1862	44.64	...	43.00	...	...	...	...	...	...	...
1863	47.27	...	...	...	...	...	...	...	...	...
1864	38.50	...	...	45.26*	42.18	48.34*	37.92*	...	...	...
1865	52.33	...	...	47.75	47.01	55.13	43.66	...	...	...
1866	39.97	...	...	47.60	40.11	37.78	37.57	48.51	54.82	43.47
1867	...	...	...	...	...	...	...	...	...	...
	44.85	43.99	42.38	46.64	42.83	47.07	39.73	45.95	51.18	43.47

IX. PENNSYLVANIA.

Year.	Philadelphia.[1]	Morrisville.	Spring Mill.	Alleghany Arsenal.
	Phila. and vicinity.			
1810	...	...	32.66	...
1811	...	...	34.97	...
1812	...	...	39.30	...
1813	...	...	35.63	...
1814	...	...	43.14	...
1815	...	...	34.67	...
1816	...	...	27.95	...
1817	...	...	36.01	...
1818	...	...	30.18	...
1819	...	...	23.35	...
1820	...	...	39.61	...
1821	...	...	32.18	...
1822	...	...	29.86	...
1823	...	...	41.85	...
1824	...	...	38.74	...
1825	29.57	29.34*	...	...
1826	35.14	40.00	...	...
1827	38.50	39.50	...	...
1828	37.97	38.75	...	...
1829	41.85	42.00	...	...
1830	45.07	44.75	...	...
1831	43.94	41.50	...	...
1832	39.87	40.25	...	...
1833	48.55	48.87	...	...
1834	34.24	35.08*	...	...
1835	39.30	40.00	...	...
1836	42.66	43.25	...	...
1837	39.04	37.00*	...	35.66
1838	45.29	44.25	...	...
1839	43.74	44.75	...	25.62
1840	47.40	47.65*	...	28.64
1841	55.50	57.00	...	34.84
1842	48.58	45.25	...	41.54
1843	46.95	46.50	...	38.71
1844	40.17	39.00	...	32.55
1845	40.02	39.75	...	31.89
1846	44.39	44.50	...	47.79
1847	45.09	...	...	46.22
1848	35.00	...	...	34.14
1849	42.10	...	...	34.81
1850	54.54	51.98	...	37.41
1851	37.93	38.70	...	29.64
1852	45.96	52.50	...	41.36
1853	41.78	...	...	33.63
1854	45.23	45.00	...	26.67
1855	44.66	43.10	...	43.33
1856	33.52	30.34	...	26.59
1857	48.44	47.80	...	38.96
1858	41.06	40.10	...	36.21
1859	54.77	51.65	...	34.08*
1860	45.40	...	...	...
1861	46.41	43.00	...	...
1862	45.66	42.80	...	...
1863	49.64	46.90	...	...
1864	46.35	36.00	...	...
1865	55.29	43.50	...	...
1866	43.57	41.15	...	...
1867	62.94	...	...	...
	44.05	43.66	35.63	35.23

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[1] This series is a combination of results at the Pennsylvania Hospital and that of Prof. Kirkpatrick; the mean is taken for the years 1851, 52, 53, 64, 65, where they overlap. The observations at the Girard College, at the U. S. Naval Hospital, and at the Philadelphia Navy Yard, are inferior in accuracy.

PENNSYLVANIA.—Continued.

Year.	Carlisle Barracks.	Fort Mifflin.	Gettysburg.	Philadel'a U.S. Naval Hospital.	Philadel'a Navy Y'd.	Philadel'a Girard Col.	Newtown.	Chromedale.	Harrisburg.	Lancaster Colliery.	Shamokin.	Canonsburg.	Norristown.	Fleming.	Somerset.
1838	...	...	...	...	...	...	38.00	...	...	...	...	...	...	...	...
1839	...	...	38.07	...	...	...	44.01	...	...	...	...	...	...	...	...
1840	...	...	34.32	...	...	...	51.61	...	...	...	...	...	...	...	33.74*
1841	...	...	45.44	...	...	42.13	57.34	...	...	...	...	...	...	...	31.73*
1842	...	...	36.24	...	...	36.12	41.85	...	...	...	...	...	...	...	...
1843	...	41.04	47.64	...	40.01*	39.43	...	...	...	...	...	...	...	...	...
1844	...	46.82	31.18	...	...	31.35	...	...	...	...	...	...	...	...	...
1845	...	...	30.22	...	28.01*	...	...	...	27.97*	...	...	...	...	...	...
1846	...	...	52.27	...	...	...	...	...	...	...	...	...	...	...	44.07
1847	...	...	50.89	...	...	...	...	...	...	...	...	...	...	...	...
1848	35.83*	...	33.62	...	...	...	...	...	...	...	...	...	...	...	...
1849	23.94	42.34	31.55	...	...	...	...	43.02	...	...	...	...	...	...	...
1850	47.36	54.50*	45.64	...	...	...	...	53.79	...	...	...	...	...	...	...
1851	32.76*	38.04	31.00	...	...	...	...	40.77	...	...	...	...	...	...	...
1852	39.24	51.33	43.34	...	...	...	...	47.30	...	...	...	...	...	...	...
1853	25.81*	40.47*	32.86	...	...	...	...	40.81	...	...	...	...	...	...	...
1854	...	...	32.87	...	...	...	...	45.36	43.69*	...	...	...	43.79	...	...
1855	40.53*	...	47.63	...	...	...	...	46.58	...	...	...	...	48.14	...	...
1856	36.97	...	28.62	...	...	...	...	34.29	27.28*	28.76	...	23.97*	38.04	...	...
1857	42.30	...	38.22	57.59*	...	...	...	45.60	...	...	57.01	40.84	47.55	...	...
1858	33.56	...	45.64	37.38*	...	...	...	...	...	38.48	38.48	34.86	39.33	...	37.35
1859	41.77*	...	42.42	53.63*	...	...	...	...	47.75*	...	41.97	40.27	46.95	39.75	36.86
1860	...	...	38.18	52.40	...	...	...	...	40.71*	...	47.58	...	...	45.26	49.32
1861	...	...	37.01	40,00	...	...	...	...	39.30	...	46.18	36.88	47.05	41.75	40.07
1862	...	...	31.89	43.90*	...	...	...	...	39.41	...	39.56	...	45.03	42.33	...
1863	...	...	...	33.82	...	...	...	...	56.78*	...	...	32.00	...	42.43	...
1864	...	...	...	...	...	...	...	...	51.05	...	...	38.78	...	44.70	...
1865	...	...	...	...	...	...	...	...	45.79	...	...	43.96	...	41.26	...
1866	...	...	...	...	...	...	...	...	39.13	...	...	39.86*	...	41.93	...
	36.50	45.41	38.89	43.23	30.18	35.55	50.43	44.18	41.53	33.56	45.22	36.71	46.48	42.41	38.74

PENNSYLVANIA.—Continued.

Year.	Hunting-don.	Easton.	Chambers-burg.	Bedford.	Byberry.	Moss Grove.	Pittsburg.	Pocopson.	West Haverford.	Pottsville.	Lewisburg.	Meadville.	Mount Joy.	Murrays-ville.	Worthing-ton (near).
1839	...	...	...	...	...	...	...	...	...	44.89*	...	...	...	...	...
1840	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1841	32.28	...	...	...	...	...	35.90	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1853	...	...	...	19.32	...	...	36.52*	...	...	...	...	...	...	...	...
1854	...	...	...	17.43*	...	34.23*	23.26	45.90	...	...	...	...	...	...	...
1855	...	...	...	33.04*	...	43.67*	45.92	49.56	...	50.20*	...	...	...	...	...
1856	...	36.52*	...	19.72*	...	34.60*	28.82	34.35	...	...	26.34*	...	...	...	...
1857	...	43.26	...	...	...	...	36.07	46.72	47.38*	...	38.02	44.34	...	41.32*	...
1858	34.87	38.56	...	...	...	...	35.95*	44.36	44.19*	...	39.86	38.48*	44.41	36.01	...
1859	...	51.66	...	34.17*	...	...	36.69	50.66	54.89	...	41.51	...	45.81*	...	...
1860	...	...	44.73*	44.65	51.62*	...	...	52.97*	...	...	...	...	...	...	35.88*
1861	...	...	37.24*	40.37	73.64	...	...	41.99	45.66*	...	...	...	49.88	...	54.42
1862	...	...	...	...	50.56	...	...	42.73	49.28*	...	...	...	34.76*	...	...
1863	...	...	...	...	60.11*	...	...	51.84*	...	...	...	...	45.65*	...	...
1864	...	...	...	...	...	...	...	45.11	...	...	...	...	33.75	...	...
1865	...	...	...	...	...	...	...	48.60	...	...	45.93	...	31.19*	...	...
1866	...	...	...	...	...	...	26.71	40.47	...	...	41.16*	...	27.02*	...	...
	36.12	45.56	41.48	30.17	56.51	37.00	37.09	45.75	48.44	47.54	38.87	40.48	38.90	40.98	50.21

PENNSYLVANIA.—CONTINUED.												X. DELAWARE.		
Year.	Lancaster.	Northumberland.	Smithport.	Silver Lake.	Butler.	Sewickleyville.	Latrobe.	Tioga.	Mooreland.	Pennsville (near).	Ephrata.	Georgetown.	Fort Delaware.	Wilmington.
1839	39.95	37.29	42.24*	31.10*	...	...	...	...	...	...	...	...	...	...
1840	35.47	41.33*	36.57	30.34*	37.47*	...	...	...	...	...	...	...	...	...
1841	37.45	39.73	...	25.35*	36.57	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1855	...	...	...	...	...	...	...	...	...	...	...	...	39.56	...
1856	...	...	...	...	...	...	...	...	...	...	...	...	31.46	...
1857	...	...	...	...	...	...	...	...	...	...	...	...	25.96	...
1858	...	...	...	...	...	...	...	...	...	...	...	37.22	17.14	...
1859	...	...	...	...	...	...	...	...	...	...	...	54.42	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1861	...	...	...	...	...	43.13	50.91	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1864	...	...	...	...	...	...	...	46.16*	...	...	...	...	...	58.90
1865	...	...	...	...	...	...	...	43.39	46.95	48.13	...	...	...	64.63*
1866	...	...	...	...	...	...	...	36.02	44.51	46.86	41.44	...	...	...
	38.16	39.14	37.82	25.95	36.06	45.38	55.21	41.42	46.09	47.62	41.44	46.14	29.21	63.60

XI. MARYLAND AND DISTRICT OF COLUMBIA.

Year.	Washington and Dist. of Columbia.	Baltimore.	Fort McHenry.	Fort Washington.	Fort Severn.	Sykesville.	Frederick.	Chestertown.	Bladensburg.	Annapolis.	Leitersburg.	St. Mary's City.	Woodlawn.
1817	...	48.55	...	...	...	...	...	...	...	...	...	...	...
1818	...	32.60	...	...	...	...	...	...	...	...	...	...	...
1819	...	28.75	...	...	...	...	...	...	...	...	...	...	...
1820	...	42.50	...	...	...	...	...	...	...	...	...	...	...
1821	...	50.20	...	...	...	...	...	...	...	...	...	...	...
1822	...	29.20	...	...	...	...	...	...	...	...	...	...	...
1823	...	44.55	...	...	...	...	...	...	...	...	...	...	...
1824	29.00	42.28	...	...	...	...	...	...	...	...	...	...	...
1825	24.45	...	...	...	...	...	...	...	...	...	...	...	...
1826	...	...	...	...	...	...	...	...	...	...	...	...	...
1827	26.39*	...	...	...	...	...	...	...	...	...	...	...	...
1828	23.55	...	...	...	...	...	...	...	...	...	...	...	...
1829	43.09	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...
1836	47.27	...	...	...	...	...	...	...	...	...	...	...	...
1837	31.80	...	45.00	...	...	...	...	...	...	...	...	...	...
1838	35.10	...	47.10	...	...	...	...	...	...	...	...	...	...
1839	37.65	...	51.70	...	...	...	...	...	...	...	...	...	...
1840	35.73*	...	37.50	...	...	...	...	...	...	...	...	...	...
1841	45.11	...	43.90	...	...	...	...	...	...	...	...	...	...
1842	...	...	35.10	...	...	...	...	...	...	...	...	...	...
1843	...	...	48.79	...	50.84*	...	...	...	...	...	...	...	...
1844	...	...	32.46	...	40.76	...	...	...	...	...	...	...	...
1845	...	...	28.39	...	...	...	...	...	...	...	...	...	...
1846	41.61	62.04	46.66	...	...	...	...	...	...	...	...	...	...
1847	35.82	45.44	33.01	...	...	...	...	...	...	...	...	...	...
1848	23.24	37.82	34.42	...	...	...	...	...	...	...	...	...	...
1849	...	35.22	30.63	...	...	...	...	...	...	...	...	...	...
1850	...	50.38	44.80	...	...	...	...	...	...	...	...	...	...
1851	...	36.41	38.10	...	...	...	...	...	...	...	...	...	...
1852	36.44	55.38	51.50	50.42	...	...	48.07	...	...	...	...	...	...
1853	31.75	38.67*	36.00	42.89*	...	...	...	...	...	...	...	...	...
1854	28.29	...	59.20	...	...	43.67*	32.89	...	...	...	...	...	...
1855	29.76	...	29.31	...	...	53.80	34.06	...	...	...	...	...	...
1856	33.63	...	22.87	...	...	34.64*	30.65	...	...	...	...	...	...
1857	41.64	...	38.37	...	...	...	42.27	...	37.20*	51.64	...	...	...
1858	39.01	35.76*	46.06	...	...	...	47.26	31.54*	...	38.59	...	...	...
1859	42.32	...	...	...	...	56.26	47.73	...	42.52*	51.45	37.31	...	...
1860	40.31	...	...	...	...	59.60	...	...	36.41*	...	36.86*	...	...
1861	44.34	...	...	...	...	46.15	33.77	44.32	34.91*	54.06	38.77	57.42	...
1862	41.72	...	...	...	...	47.35	35.87	46.08*	36.78*	52.73	...	44.85	...
1863	43.56	...	...	...	...	63.50*	...	...	53.50*	44.88*	...	38.18*	...
1864	34.02	...	...	...	...	41.80	...	...	...	42.26	...	34.67*	...
1865	44.69	...	...	...	...	47.54*	...	...	...	57.59	...	38.94*	52.11
1866	46.85	...	...	...	...	...	37.48*	...	...	60.53	...	...	53.09
1867	53.45	...	...	...	...	...	...	...	...	...	...	...	...
	37.52	42.33	41.10	45.02	48.61	48.53	38.34	42.87	39.27	50.34	39.26	42.58	52.60

XII. VIRGINIA.

Year.	Fortress Monroe.	Berryville.	Powhatan Hill.	Portsmouth, also Naval Hospital.	Alexandria.	Alleghany County.	Crichton's Store.	Smithfield.	Ruthven.	Winchester.	Rose Hill.	Rougemont.	Hartwood.	Meadow Dale.
1837	40.70	...	...	...	...	...	...	...	...	...	...	...	...	...
1838	44.94	...	...	...	...	...	...	...	...	...	...	...	...	...
1839	72.00	...	...	...	...	...	...	...	...	...	...	...	...	...
1840	74.10	...	...	...	...	...	...	...	...	...	...	...	...	...
1841	65.31	...	...	...	...	...	...	...	...	...	...	...	...	...
1842	67.06	...	...	...	...	...	...	...	...	...	...	...	...	...
1843	47.31	...	...	...	...	...	...	...	...	...	...	...	...	...
1844	41.68	...	...	...	...	...	...	...	...	...	...	...	...	...
1845	47.77	...	...	...	...	...	...	...	...	...	...	...	...	...
1846	50.39	...	...	...	...	...	...	...	...	...	...	...	...	...
1847	48.93	...	...	...	...	...	...	...	...	...	...	...	...	...
1848	29.44	...	...	...	...	35.74	...	...	...	...	...	...	...	...
1849	32.64	...	...	...	...	32.54	...	...	...	...	...	...	...	...
1850	49.76	...	37.95	...	...	38.46	...	...	...	...	...	...	...	...
1851	26.86?	...	26.94	...	...	32.85	...	...	...	...	...	...	...	...
1852	27.24?	...	37.74	...	...	...	...	...	...	...	...	...	...	...
1853	26.74?	...	28.81	...	40.55	...	...	...	...	...	...	...	...	...
1854	19.32?	...	30.05	...	35.03	...	44.35	...	...	...	...	...	...	...
1855	24.47?*	...	30.08	...	33.32	...	40.16*	41.72	...	...	...	...	...	...
1856	40.42	...	29.17	54.48*	29.89	...	36.47	48.50	...	...	...	...	...	...
1857	42.88	37.75	32.23	54.41	37.65	...	34.09	47.20	37.12	42.91	36.50*	...	...	...
1858	37.24	38.16	31.39	44.25	...	...	32.16	43.77	...	38.82	...	39.74	...	40.87
1859	39.74*	...	34.59	52.58	...	...	30.71	52.56	...	43.00	41.15	55.79	...	...
1860	...	...	33.79	...	...	...	33.07*	48.03*	...	48.46*	...	...	43.12	...
1861	...	...	44.79	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1866	...	...	...	46.80	...	...	...	...	...	...	...	...	...	...
1867	...	...	43.03	...	...	...	...	...	...	...	...	...	...	...
	47.04	37.48	33.81	48.92	34.90	35.37	36.38	47.15	41.35	42.72	38.46	50.48	47.36	41.97

XIII. WEST VIRGINIA.

Year.	White Sulphur Springs.	Sheetz Mills.	Ashland.	Lewisburg.	Kanawha.	Poplar Grove.	Wirt Court-house.
1844	...	...	...	...	...	...	...
	...	...	...	...	...	...	...
1848	39.30	...	...	...	...	...	...
1849	35.39	...	...	...	...	...	...
1850	39.08	...	...	...	...	...	...
1851	34.46	...	...	...	...	...	...
1852	34.21	...	...	...	...	...	...
1853	...	...	...	...	...	...	...
1854	...	...	35.34	...	...	...	...
1855	...	...	34.64*	...	...	...	...
1856	...	20.26	...	30.74*	29.25	...	...
1857	...	32.20	43.70*	45.02*	...	38.76*	34.98
1858	...	25.55	...	...	34.43*	...	52.95*
1859	...	34.80	...	33.26	...	65.89*	...
1860	...	40.80	...	36.78	...	74.06*	...
1861	...	38.05	...	...	...	...	...
1862	...	32.72	...	...	...	...	...
1863	...	41.35	...	...	...	...	...
1864	...	25.15	...	...	...	...	...
1865	...	35.56*	...	...	...	...	...
1866	...	...	35.74*	...	...	...	...
	37.54	32.66	39.40	35.75	34.84	55.84	43.06

XIV. NORTH CAROLINA.

Year.	Fort Johnston.	Gaston.	Chapel Hill.	Murfrees-borough.	Davidson College.
1844	50.05	...	...	...	...
	...	...	...	...	...
1848	...	...	...	...	...
1849	...	...	...	...	...
1850	...	...	...	...	...
1851	...	...	...	...	...
1852	...	...	...	...	...
1853	...	...	...	...	...
1854	...	...	...	...	...
1855	...	...	...	...	...
1856	...	...	40.98*	...	...
1857	...	36.32	38.66*	29.22	...
1858	...	40.83	43.94	32.54	45.44
1859	...	49.69	42.18	43.58	46.76*
1860	...	46.13	...	...	...
1861	...	...	...	...	...
1862	...	...	...	...	...
1863	...	...	...	...	...
1864	...	...	...	...	...
1865	...	...	...	...	...
1866	...	...	...	...	...
	46.01	43.40	42.71	35.76	45.58

XV. SOUTH CAROLINA.

Year.	Charleston.	Fulton.	Fort Moultrie.	Camden.	St. John's.	All Saints.	Robertsville.	Aiken.	Columbia.
1738	49.27	...	...	...	...	...	...	...	...
1739	65.96	...	...	...	...	...	...	...	...
1740	39.81	...	...	...	...	...	...	...	...
1741	52.07	...	...	...	...	...	...	...	...
1742	36.01	...	...	...	...	...	...	...	...
1743	39.75	...	...	...	...	...	...	...	...
1744	48.32	...	...	...	...	...	...	...	...
1745	50.15	...	...	...	...	...	...	...	...
1746	39.65	...	...	...	...	...	...	...	...
1747	44.56	...	...	...	...	...	...	...	...
1748	51.60	...	...	...	...	...	...	...	...
1749	54.42	...	...	...	...	...	...	...	...
1750	56.16	...	...	...	...	...	...	...	...
1751	50.82	...	...	...	...	...	...	...	...
1752	42.88	...	...	...	...	...	...	...	...
1753	47.36	...	...	...	...	...	...	...	...
1754	37.22	...	...	...	...	...	...	...	...
1755	43.29	...	...	...	...	...	...	...	...
1756	31.69	...	...	...	...	...	...	...	...
1757	44.44	...	...	...	...	...	...	...	...
1758	30.62	...	...	...	...	...	...	...	...
1759	36.37	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...
1818	...	34.30	...	...	...	...	...	...	...
1819	...	40.80	...	...	...	...	...	...	...
1820	...	61.80	...	...	...	...	...	...	...
1821	...	77.90	...	...	...	...	...	...	...
1822	...	64.60	...	...	...	...	...	...	...
1823	...	47.30	...	...	...	...	...	...	...
1824	...	53.60	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...
1841	53.94	...	...	...	...	...	...	...	...
1842	42.09	...	...	...	...	...	...	...	...
1843	54.72	...	42.82	...	...	...	37.14	...	...
1844	36.39	...	33.98	...	...	...	...	...	...
1845	46.44	...	43.51	...	...	...	...	...	...
1846	44.34	...	43.45	...	45.83	...	...	...	...
1847	47.83	...	65.31	...	35.55	...	...	...	...
1848	43.40	...	56.87	...	29.28	...	...	...	...
1849	30.69	...	39.78	...	24.93	...	...	...	...
1850	23.69	...	35.80	54.60	39.29	...	...	...	56.69
1851	33.14	...	39.11	...	42.59	46.55	...	...	...
1852	49.72	...	51.26	53.12	56.98	53.11*	...	...	...
1853	43.48	...	45.65	46.10	48.80	42.42	...	...	...
1854	37.62	...	39.76	51.58	44.45	...	...	...	...
1855	34.85	...	43.66	42.60	40.76	30.97	...	40.82	...
1856	41.48	...	40.75	53.94	57.48	32.73	...	54.49*	...
1857	34.62	...	40.28	41.68	42.16	40.12	...	35.87	...
1858	45.03	...	47.48	...	47.96*	51.14	...	45.92	36.57*
1859	52.33*	...	63.72	...	43.73	44.41	...	44.62	...
1860	...	...	...	...	...	...	...	...	...
1861	42.59	...	...	...	...	...	...	...	...
	43.63	54.31	45.51	50.56	42.85	43.82	37.14	44.54	47.17

XVI. GEORGIA.

Year.	Savannah.	Augusta Arsenal.	Oglethorpe Barracks.	Whitemarsh Island.	Sparta.
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1837	37.68	...	...	...	...
1838	38.50	...	...	...	...
1839	25.98*	...	...	...	...
1840	47.08	...	...	...	...
1841	69.83	...	...	...	...
1842	63.67	...	...	...	...
1843	43.66	...	43.51	...	...
1844	48.62	...	57.56	...	...
1845	37.60	19.13	38.05	...	...
1846	56.64	...	57.46*	...	...
1847	69.93	...	...	...	...
1848	56.92	...	...	...	...
1849	50.41	...	...	...	...
1850	55.27	...	63.99*	37.84	...
1851	43.72	...	...	29.79	...
1852	54.85	...	...	49.93	49.03*
1853	46.89	...	...	45.56	48.75
1854	43.14	...	...	41.18	44.35
1855	38.59	...	...	37.36	51.52
1856	45.38	...	...	38.74	51.87
1857	33.50	...	...	30.83*	39.56
1858	53.20	...	...	52.35	63.08
1859	54.19*	...	...	45.65	61.31*
1860	...	...	...	32.95*	60.16*
	...	...	...	...	...
1866	...	...	...	...	...
	48.32	24.20	51.33	40.31	54.11

GEORGIA.—Continued.					XVII. **FLORIDA.**										
Year.	Culloden.	Perry.	Thomaston.	Atlanta.	Key West.	Fort Marion.	Fort Shannon.	Fort Pierce.	Knox Hill.	Fort Myers.	Fort Brooke.	Fort Meade.	Cedar Keys.	Indian Key.	Jacksonville.
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1833	...	...	...	...	27.56	...	...	...	...	...	...	...	...	...	...
1834	...	...	...	...	36.11	...	...	...	...	...	...	...	...	...	...
1835	...	...	...	...	30.09	...	...	...	...	...	...	...	...	...	...
1836	...	...	...	...	24.40	...	...	...	...	...	...	...	...	35.15	...
1837	...	...	...	...	38.95	...	...	...	...	...	...	...	...	41.77	...
1838	...	...	...	...	20.49	...	...	...	...	...	...	...	...	...	...
1839	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1840	...	...	...	...	...	...	...	...	...	...	89.86	...	...	...	...
1841	...	...	...	...	...	...	54.13	...	...	...	48.89	...	54.20	...	...
1842	...	...	...	...	...	...	45.67	...	...	...	45.96	...	44.20	...	...
1843	...	...	...	...	...	...	43.27	...	...	...	56.28	...	...	...	...
1844	...	...	...	...	38.40	29.01	...	...	...	...	51.67	...	...	...	...
1845	...	...	...	...	...	29.87*	...	...	...	...	53.06	...	...	...	...
1846	...	...	...	...	...	...	...	...	...	...	54.20	...	...	...	...
1847	...	...	...	...	...	...	...	...	...	...	46.20	...	...	...	...
1848	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1849	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1850	...	...	...	...	...	...	...	...	...	...	46.36*	...	...	...	...
1851	...	32.99*	...	...	53.36	33.30*	...	...	...	61.55	...	...	...	...	...
1852	...	57.70	...	...	46.40	...	...	47.19*	...	74.27	69.26	46.34	55.57	...	...
1853	57.60	...	...	...	45.82	...	...	96.39	...	82.64	44.77	18.27	52.89*	...	...
1854	...	...	...	...	44.64	...	...	46.79	39.92	44.75	77.44	55.31*	39.62	...	56.30
1855	...	...	...	...	44.64	...	...	48.96*	53.19*	54.66	37.90*	...	34.52	...	...
1856	...	...	...	...	40.29	...	...	58.03	...	49.03	53.73*	...	34.41	...	52.31
1857	...	...	...	...	38.87	...	...	54.43	...	40.20	35.93*	...	...	...	49.04*
1858	...	...	...	...	35.76	...	...	...	...	...	...	...	50.80*	...	46.94*
1859	...	...	43.81	64.54	31.56	...	...	...	...	...	...	...	60.34*	...	46.88
1860	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1861	...	...	...	...	25.91	...	...	...	...	...	...	...	...	...	...
1862	...	...	...	...	33.62	...	...	...	...	...	...	...	...	...	...
1863	...	...	...	...	32.25*	...	...	...	...	...	...	...	...	...	...
1864	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1865	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1866	...	...	...	51·68*	...	...	...	...	...	...	...	...	...	...	...
	64.24	44.12	43.21	58.43	36.23	31.80	47.68	58.02	44.07	56.55	53.63	39.90	45.77	38.46	52.72

FLORIDA.—CONTINUED.								XVIII. **ALABAMA.**								
Year.	Warrington Navy Y'd.	St. Augustine.	Gainesville.	Lake City.	Micanopy.	Fort Dallas.	Ft. & Bar'ks Barrancas.	Huntsville.	Mt. Vernon Arsenal.	Mobile.	Florence.	Green Springs.	Auburn.	Carlowville.	Greensboro'.	Greensboro' University.
1830	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1831	...	...	...	...	...	...	...	43.46	...	...	...	...	...	...	...	...
1832	...	...	...	...	...	...	...	46.33	...	...	...	...	...	...	...	...
1833	...	...	...	...	...	...	...	67.67	...	...	...	...	...	...	...	...
1834	...	...	...	...	...	...	...	63.14	...	...	...	...	...	...	...	...
1835	...	...	...	...	...	...	...	60.29	...	...	...	...	...	...	...	...
1836	...	...	...	...	...	...	...	54.75	...	...	...	...	...	...	...	...
1837	...	...	...	...	...	...	...	47.08	...	...	...	...	...	...	...	...
1838	...	...	...	...	...	...	...	48.32	...	...	...	...	...	...	...	...
1839	...	...	...	...	...	...	...	29.08	...	...	...	...	...	...	...	...
1840	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1841	...	...	...	...	...	...	...	...	...	73.90	...	...	...	...	...	...
1842	...	...	...	...	...	...	...	...	...	54.94	...	...	...	...	...	...
1843	...	...	...	...	...	...	64.16	...	76.38	...	...	...	...	...	...	...
1844	...	...	...	...	...	...	59.58	...	...	...	...	...	...	...	...	...
1845	...	...	...	...	...	...	41.23*	...	68.52	...	...	...	...	...	...	...
1846	...	...	...	...	...	...	74.91*	...	65.36	...	...	...	...	...	...	...
1847	...	...	...	...	...	...	...	...	71.43	...	...	...	...	...	...	...
1848	...	...	...	...	...	...	...	...	49.55	...	...	...	...	...	...	...
1849	...	...	...	...	...	...	...	...	79.23	...	62.36	...	...	...	...	...
1850	...	...	...	...	...	...	...	...	49.55	...	...	...	...	...	...	...
1851	...	...	...	...	...	...	44.07*	...	48.77	...	...	...	...	...	...	...
1852	...	...	...	...	...	...	...	...	51.49	...	...	...	...	...	...	...
1853	...	...	...	...	...	...	53.50*	...	106.57	...	...	...	...	...	...	...
1854	43.00	...	...	...	...	...	50.92	...	62.31	...	...	...	...	...	...	...
1855	49.96	...	...	...	...	68.32	77.68	...	59.76	...	...	39.27*	37.85*	...	...	37.60
1856	79.83*	...	...	...	...	52.80	53.56	...	58.15	...	...	53.98*	44.14*	...	...	52.23
1857	65.41*	40.85	37.24	...	...	59.47	...	...	49.82	...	...	50.19	...	47.44	52.40*	47.96
1858	...	49.83*	58.43	79.69	...	...	...	...	64.35	...	...	52.49*	...	46.84*	47.55*	43.60
1859	...	47.53	45.70	75.55	45.46	...	70.09	...	64.94	...	...	57.56*	...	59.17*	59.38	53.45
1860	...	...	38.60*	...	...	...	...	...	...	...	...	...	...	...	...	54.81
1861	...	...	...	...	...	...	...	...	...	...	...	...	...	...	51.25	49.93
1862	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	43.16
1863	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	54.62
1864	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	53.62
1865	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	56.59*
1866	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	55.58*
1867	...	...	...	...	...	...	...	...	...	...	...	40.48	...	...	...	48.30*
	61.14	43.16	44.91	78.38	54.36	59.04	59.20	54.88	66.14	64.42	62.36	48.78	43.28	55.66	55.57	50.40

Mount Vernon Arsenal, amount in 1853 doubtful.

	XIX. **MISSISSIPPI.**					XX. **LOUISIANA.**						XXI. **TEXAS.**				
Year.	Natchez.	Vicksburg.	Church Hill.	Columbus.	Oxford.	Fort Wood.	Fort Pike.	New Orleans.	Baton Rouge.	Fort Jesup.	Black River.	Ft. Belknap.	Ft. Worth.	Phantom Hill.	Fort Chadbourne.	Ft. Graham.
1799	43.48*	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1800	31.09	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1801	45.50	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1802	57.92	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1803	37.56	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1836	...	...	...	...	...	...	...	52.25	...	49.85	...	...	...	...	...	...
1837	...	...	...	...	...	...	...	...	...	49.54	...	...	...	...	...	...
1838	...	...	...	...	...	...	...	...	...	47.22	...	...	...	...	...	...
1839	...	...	...	...	...	...	...	46.42	...	45.36	...	...	...	...	...	...
1840	48.48	46.35	...	...	...	...	...	44.28	...	37.12	...	...	...	...	...	...
1841	59.78	48.47	...	...	...	...	...	49.92	...	55.95	...	...	...	...	...	...
1842	43.52	46.85	...	...	...	...	...	47.55	...	41.88	...	...	...	...	...	...
1843	78.73	60.28	...	...	...	64.96*	105.96*	60.99	61.07	52.99	...	...	...	...	...	...
1844	45.91	37.21	...	...	...	46.84	44.26	49.22*	44.09*	41.42	...	...	...	...	...	...
1845	53.00	49.86	...	...	...	57.27*	...	...	50.60	34.92*	...	...	...	...	...	...
1846	61.79	52.69	...	...	...	...	...	...	116.40	...	...	...	...	...	...	...
1847	75.32	59.77	...	...	...	...	...	53.51	...	...	...	...	...	...	...	...
1848	...	51.30	...	...	...	...	...	53.41	...	...	...	...	...	...	...	...
1849	...	43.88	...	...	...	...	...	54.75	55.97*	...	...	...	...	...	...	...
1850	...	50.11	57.40	...	...	...	...	51.13	77.02*	...	...	...	37.95	...	...	35.84*
1851	...	43.40	44.16	...	...	...	...	52.47	41.34	...	...	...	42.49	...	...	41.17
1852	...	43.32	37.00	...	...	...	...	43.88	53.46	...	...	...	43.36	...	...	44.86
1853	...	51.58	52.72	...	...	...	...	62.19	52.78*	...	...	25.14	...	14.13	27.46	...
1854	...	...	49.87	...	...	...	...	52.84	60.62	...	...	20.65	...	...	34.27	...
1855	...	...	...	...	41.46	...	...	41.92	49.02	...	...	19.21	...	...	27.43*	...
1856	...	...	...	51.71*	...	...	...	67.12	67.74	...	...	35.92	...	...	20.03	...
1857	...	...	...	53.97	...	...	...	47.12	49.99	...	49.70*	41.61	...	...	20.07	...
1858	...	...	...	57.16	...	...	...	60.70	63.78*	...	60.65*	...	...	...	10.34	...
1859	47.89	...	...	60.12	...	...	...	49.40	50.93*	...	...	...	...	...	21.39	...
1860	...	...	...	...	...	...	...	...	...	...	...	...	...	...	12.83	...
1861	51.55	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1862	41.78*	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1863	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1864	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1865	61.43	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1866	71.45	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1867	...	48.87	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	53.55	49.30	45.97	57.13	43.63	63.53	65.87	51.05	60.16	45.55	53.98	28.05	40.86	17.22	22.89	40.58

TEXAS.—CONTINUED.

Year.	Ft. Croghan.	Ft. Martin Scott.	Ft. Mason.	Ft. Terrett.	McKavett.	San Antonio.	Ft. Ewell.	Ft. Merrill.	Fort Brown and Matamoras.	Ringgold Barracks.	Fort McIntosh.	Ft. Duncan.	Fort Inge.	Ft. Lincoln.	Ft. Clark.
1845	...	...	...	...	...	...	...	...	36.09*	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1849	...	...	...	...	...	...	...	...	38.50*	...	...	...	...	...	...
1850	25.60	32.52	...	...	...	29.98	...	...	26.03	17.64	17.35	17.98	24.48	...	...
1851	44.87	26.07	...	...	...	30.84	...	18.48*	28.31	14.08	14.23	14.12	20.07*	20.58	...
1852	45.71	...	42.80*	43.00*	40.03*	35.12*	...	...	28.80	17.82	10.53	28.37	32.82	...	...
1853	...	...	30.28*	24.83	...	...	39.34	...	26.53	22.58	22.78	33.31	37.64	...	29.14
1854	...	...	...	...	16.77	...	33.83*	35.24	50.60	29.05	23.90	22.00	24.05	...	14.42
1855	...	...	...	...	16.18	...	...	27.60*	60.80	25.65*	22.98	17.32	...	...	22.16
1856	...	...	21.26*	...	24.29	...	...	...	26.14	24.58	18.71	17.92	...	...	15.79
1857	...	...	34.90	...	22.12	...	...	...	20.70	12.60	16.41	28.29	...	...	41.60*
1858	...	...	19.73	...	21.62	28.56	...	...	24.36	19.88	10.29	16.01	...	...	19.98
1859	...	...	...	...	...	31.98	...	...	...	...	...	...	18.06	...	17.75
1860	...	...	...	...	...	22.31	...	...	...	...	...	...	16.77*	...	20.23*
	36.56	32.53	28.98	27.15	22.79	32.93	34.58	29.53	33.44	21.32	17.84	21.23	25.46	21.72	22.38

TEXAS.—Continued.

Year.	Ft. Bliss.	Huntsville.	Pin Oak.	New Braunfels.	Austin.	Washington.	Goliad.	Round Top.	Sisterdale.	Fort Lancaster.	Camp Hudson.	Fort Davis.	Camp Verde.	Camp Colorado.	Ft. Quitman.
1854	...	...	...	25.70	...	...	...	...	...	...	...	...	...	...	...
1855	11.31*	...	...	25.42	...	...	...	...	...	...	...	21.21	...	...	...
1856	21.81	...	46.75	29.83*	21.10*	...	...	...	...	...	...	25.94	...	...	...
1857	13.73*	...	...	27.86*	33.13*	...	...	...	...	39.58	...	19.96*	30.28	16.64	...
1858	5.00	46.04	...	...	36.37	...	25.13	...	...	28.21	...	14.12	18.60*	18.75*	...
1859	4.83	23.93	...	25.60*	28.24	27.32	...	26.18	23.50	24.89	11.88*	22.55	19.76	20.80	8.31
1860	2.46*	...	...	...	...	...	...	25.74	...	13.37	13.85	8.52	37.95	40.27	4.18
1861	...	...	...	...	28.69	...	...	...	...	...	...	...	...	...	...
1862	...	...	...	...	22.18	...	...	...	...	...	...	...	...	...	...
1863	...	...	...	...	34.70	...	...	...	...	...	...	...	...	...	...
1864	...	...	...	...	25.16	...	...	...	...	...	...	...	...	...	...
1865	...	...	...	...	40.51*	...	...	...	...	...	...	...	...	...	...
1866	...	...	...	...	41.65	...	...	...	...	...	...	...	...	...	...
1867	2.84	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	9.56	34.98	46.75	27.82	31.10	30.23	25.13	27.65	23.50	26.25	12.13	18.71	30.65	24.35	6.26

XXII. ARKANSAS. XXIII. TENNESSEE. XXIV. KENTUCKY.

	XXII. ARKANSAS.			XXIII. TENNESSEE.					XXIV. KENTUCKY.						
Year.	Fort Smith.	Washington.	Helena. (near)	Nashville.	Memphis.	Glenwood.	Knoxville.	University Place.	Springdale.	Danville.	Millersburg.	Ballards-ville.	Paris.	Bardstown.	Louisville.
1838	27.30	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1839	42.59	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1840	46.23	67.50	...	59.14	...	...	...	...	...	...	...	...	...	...	...
1841	53.60*	54.50	...	61.70	...	...	...	...	...	...	...	...	...	...	...
1842	38.79	41.60	...	53.59	...	...	...	...	40.71	...	...	...	...	...	...
1843	37.97	63.40	...	58.31	...	...	...	...	48.67	...	...	...	...	...	...
1844	32.93	45.50	...	42.27	...	...	...	...	40.40	...	...	...	...	...	...
1845	33.36	46.70	...	55.00	...	...	...	...	43.28	...	...		...	...	...
1846	45.73	41.40	...	...	...	...	...	...	47.80	...	...		...	...	...
1847	49.04	58.30	...	...	...	...	...	...	50.12	...	...	...	...	...	...
1848	44.36	60.00	...	...	...	...	...	...	58.36	...	...	...	...	...	...
1849	57.54	70.40	...	...	...	...	...	...	45.27	...	...	...	...	...	...
1850	...	65.70	...	...	47.56*	...	...	...	67.10	...	...	...	...	...	...
1851	...	41.50	...	...	...	...	...	...	42.34	...	...	...	...	...	...
1852	61.03	69.50	...	...	...	...	...	...	51.91	...	...	...	...	...	...
1853	24.34	55.20	...	...	...	...	...	...	35.56	...	...	...	...	...	...
1854	37.98	59.20	...	...	...	43.04	46.23	...	46.20	48.16*	34.66*	...	...	...	...
1855	33.91	47.60	...	...	...	44.10	...	...	45.71*	40.18*	33.60*	...	...	...	...
1856	29.48	46.20	...	...	...	39.51	...	...	30.91	...	...	...	28.86	...	...
1857	38.83	57.10	...	...	43.01*	43.45	...	...	46.70	42.34*	...	...	48.76*	...	...
1858	...	56.70	...	...	53.55	53.68	...	...	49.90	...	...	...	39.22	53.12	44.21*
1859	...	46.10	...	...	52.30*	45.58	...	...	51.14	45.11*	...	...	46.21	51.88*	...
1860	28.66*	62.27	...	...	...	...	...	48.41	...	...	...	...	...	...	...
1861	...	...	...	...	...	48.05	...	...	49.56	45.73*	42.12	43.84	...	34.51*	41.38*
1862	...	...	...	...	...	49.55	...	...	60.14	...	...	...	...	...	50.36
1863	...	...	...	...	...	54.37	...	...	48.34*	...	...	...	...	...	...
1864	...	...	...	...	...	41.33	...	...	45.18	...	...	...	...	...	...
1865	...	...	...	...	...	59.98	...	...	62.15	52.34	...	...	...	...	...
1866	...	...	77.57	...	...	44.80	...	...	59.80	...	...	...	...	...	...
	40.36	54.50	74.63	52.02	45.46	46.63	39.76	51.07	48.58	45.63	41.12	45.25	41.10	49.97	48.12

KENTUCKY.—Continued.					XXV. **OHIO.**										
Year.	Lexington.	Newport Barracks.	Chilesburg.	Nicholas-ville.	Marietta.	Steubenville.	Cincinnati.	Portsmouth.	College Hill.	Gallipolis.	Cleveland.	Toledo.	Kelley's Island.	Bowling Green.	Urbana.
1818	...	...	...	...	50.92	...	...	...	...	...	...	...	...	...	...
1819	...	...	...	...	36.30	...	...	...	...	...	...	...	...	...	...
1820	...	...	...	...	39.11	...	...	...	...	...	...	...	...	...	...
1821	...	...	...	...	43.32	...	...	...	...	...	...	...	...	...	...
1822	...	...	...	...	43.38	...	...	...	...	...	...	...	...	...	...
1823	...	...	...	...	50.08*	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1826	...	...	...	...	41.60	...	...	...	...	...	...	...	...	...	...
1827	...	...	...	...	41.48	...	...	...	...	...	...	...	...	...	...
1828	...	...	...	...	49.50	...	...	...	...	...	...	...	...	...	...
1829	...	...	...	...	39.52	...	...	...	...	...	...	...	...	...	...
1830	...	...	...	...	37.26	...	...	26.50	...	...	...	...	...	...	...
1831	...	...	...	...	53.54	43.55	...	42.30	...	...	...	...	...	...	...
1832	...	...	...	...	48.33	39.75	...	45.40	...	...	...	...	...	...	...
1833	...	...	...	...	40.37	35.65	...	36.70	...	...	...	...	...	...	...
1834	...	...	...	...	34.66	38.78	...	29.50	...	...	...	...	...	...	...
1835	...	...	...	...	42.46	38.15	52.15	25.50	...	...	...	...	...	...	...
1836	...	...	...	...	36.75	39.15	57.39	30.60	...	...	...	...	...	...	...
1837	...	...	...	...	43.86	35.39	42.71	47.80	...	...	...	...	...	...	...
1838	...	...	...	...	35.48	28.16	39 45	36.50	...	...	...	...	...	...	...
1839	...	...	...	...	33.32	28.02	30.62	27.20	...	...	...	...	...	...	...
1840	...	...	...	...	39.09	37.56	47.34	41.64	...	...	...	...	...	...	...
1841	...	...	...	...	42.82	31.27	41.05	43.98	...	...	...	...	...	...	...
1842	...	...	...	...	42.07	41.19	41.29	41.78	...	...	...	...	...	...	...
1843	...	...	...	...	41.76	41.04	51.19	55.05	...	...	...	...	...	...	...
1844	...	...	...	...	36.64	38.67	41.94	36.30	...	...	...	...	...	...	...
1845	...	...	...	...	33.90	38.44	46.38	40.13	...	...	...	...	...	...	...
1846	...	...	...	...	46.27	52.21	53.52	45.50	...	...	...	...	...	...	...
1847	...	...	...	...	52.30	57.28	65.18	38.66*	...	...	...	...	...	...	...
1848	...	...	...	...	43.18	50.25	49.68	40.18	...	...	...	...	...	...	...
1849	...	...	...	...	42.89	47.32	52.97	41.64	...	...	...	...	...	...	...
1850	...	...	...	...	52.36	46.98	54.76	56.79	...	...	...	...	...	...	...
1851	...	...	...	...	34.94	28.59	31.70	28.43	...	...	...	...	...	...	...
1852	...	...	...	...	46.50	49.30	54.06	39.11*	...	...	...	...	...	...	...
1853	...	...	...	...	37.04	35.50	...	...	...	...	...	...	...	...	...
1854	...	...	...	...	38.80	30.11	...	...	47.72*	41.70*	...	...	...	...	41.35
1855	...	50.91	...	...	45.75	47.93	...	...	...	37.02	...	...	...	...	55.17*
1856	...	27.72	...	...	32.46	32.35	25.49	...	...	...	26.09	...	...	...	30.85
1857	...	46.33	...	...	40.64	54.98	34.89	...	...	33.09*	39.75	...	...	...	39.82
1858	...	58.08	42.47	...	61.84	50.49	49.19	...	...	...	44.16	...	...	...	41.02
1859	...	46.58	47.87	...	48.55	49.68	45.11	46.38	51.43*	...	38.91	...	...	35.05	36.51
1860	...	...	37.00	...	39.91	46.48	...	...	...	...	37.18	...	25.88	...	...
1861	...	...	47.75	46.68	46.41	38.78	42.36	42.44	42.76*	...	33.90	40.87	30.05	31.49	...
1862	...	...	47.33	45.22	42.70	42.36	38.15	38.32*	...	...	36.43	44.85	41.92	53.08*	37.61
1863	...	...	45.88	...	37.06	38.74	40.58*	...	48.69*	...	33.72	33.92	31.97	29.72*	36.56
1864	...	...	41.03	...	40.94	49.22	33.31	36.82	47.37	...	33.23	37.17	37.41	...	32.03
1865	...	...	60.36	...	48.84	48.03	43.98	...	53.20*	43.89*	32.12	39.55	34.04	...	46.04
1866	...	...	51.18	...	47.27	48.94	49.85	...	50.89	39.55*	48.65	40.67	38.18	59.26*	49.62
1867	58.18	...	...	...	46.71	34.14	...	...	...	...	...	31.06	...	...	...
	59.41	45.89	46.75	45.35	42.70	41.48	44.87	38.33	48.08	38.35	37.61	39.46	33.24	41.72	40.31

OHIO.—CONTINUED.

Year.	New Lisbon.	Hills-borough.	German-town.	Granville.	Norwalk.	Oberlin.	Perrysburg.	Savannah.	Hiram.	Jackson, Monroe Co.	Madison.	Bellefon-taine.	Edinburg.	Ripley.	Welchfield.
1854	...	...	44.43	41.69	...	28.02	...	...	...	...	...	...	...	...	...
1855	...	...	47.00	54.88	...	48.06	55.18	61.66*	...	...	...	...	...	...	...
1856	...	28.10*	24.10	25.63	...	24.59*	...	28.85	30.47	...	...	...	...	...	...
1857	...	42.12	...	...	...	...	...	44.01	39.27*	...	49.23*	...	49.02*	...	58.11*
1858	42.45	47.29	...	...	...	...	...	51.71	34.71	43.20	62.12*	54.19	49.26*	45.11	49.05
1859	40.18	51.93	...	...	...	...	...	50.57	...	46.67	51.16	35.33	...	...	56.15
1860	...	...	...	...	...	...	...	...	...	...	47.97	...	...	...	56.78*
1861	35.28	...	...	...	...	...	...	34.39	...	...	47.03	...	...	...	46.63
1862	28.76	...	...	...	40.25*	...	...	44.97	...	...	36.93	...	...	...	45.45
1863	27.36*	...	...	...	28.18*	...	...	...	...	...	...	...	...	...	44.68
1864	30.11	35.35	...	...	32.70	...	...	...	...	...	...	...	...	40.14*	53.51
1865	41.37	48.13	...	...	32.15	...	...	...	...	...	...	...	...	60.85	56.29
1866	46.99	46.61	...	...	45.43*	...	...	...	...	...	...	...	...	...	...
	36.07	41.90	34.75	48.62	34.14	34.98	43.80	44.59	36.69	44.91	49.00	43.02	48.81	48.42	51.94

OHIO.—CONTINUED.

Year.	Avon.	Hudson.	Montville.	Westerville.	Elk Run.	North Bend.	Troy.	Sandusky.	Bethel.	Austinburg.	Hartford.	Milnersville.	Kingston.	Marion.
1839	...	33.02	...	...	...	...	...	...	...	...	...	...	...	...
1840	...	42.25	...	...	...	...	...	...	...	...	...	...	...	...
1841	...	26.96	...	...	...	...	...	...	...	...	...	...	...	...
1842	...	39.36	...	...	...	...	...	...	...	...	...	...	...	...
1843	...	28.96	...	...	...	...	...	...	...	...	...	...	...	...
1844	...	33.25	...	...	...	...	...	...	...	...	...	...	...	...
1845	...	33.08	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1857	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1858	...	...	...	45.40	...	...	...	...	...	...	...	...	...	...
1859	42.00	41.38	30.77	45.20	...	...	46.60	33.62	...	...	...	...	...	...
1860	...	...	...	...	...	38.02	...	30.92	45.36	...	28.25*	...	...	...
1861	...	54.88	28.62	30.97	35.38	42.19	56.11	29.18	...	...	30.80	...	...	...
1862	...	32.16	32.89	43.34	30.14	46.16*	62.21	39.58	...	...	35.72	...	...	...
1863	...	...	...	32.37*	35.63*	...	...	26.36	41.19*	46.81*	...	27.28*	...	...
1864	...	...	...	...	33.58	...	...	34.23	47.53	41.86	...	34.61*	29.98	...
1865	...	...	...	41.22*	40.70	...	...	40.00	47.32	32.41	...	31.51	41.48	...
1866	...	...	...	45.31*	46.73	...	...	42.83	47.22	...	...	37.93	34.72	49.70
1867	...	...	...	...	...	...	...	28.89	...	...	...	...	...	...
	41.54	35.26	31.14	39.19	36.64	40.98	52.56	33.96	47.08	39.85	32.21	29.50	35.68	43.00

XXVI. MICHIGAN.

Year.	Detroit.	Fort Gratiot.	Fort Mackinac.	Fort Brady.	Ann Arbor.	Flint.	Thunder Bay Island.	Monroe Piers.	Monroe.	Battle Creek.	Tawas City.	Grand Haven.	Ontonagon.	Cooper.	Grand Rapids.
1836	40.38*	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1837	27.22	...	...	36.92	...	...	...	...	...	...	...	...	...	...	...
1838	...	...	...	34.37	...	...	...	...	...	...	...	...	...	...	...
1839	24.28	...	...	24.01	...	...	...	...	...	...	...	...	...	...	...
1840	29.63	42.74	...	33.09	...	...	...	...	...	...	...	...	...	...	...
1841	27.16	32.95	...	24.11*	...	...	...	...	...	...	...	...	...	...	...
1842	31.84	39.62	...	26.40	...	...	...	...	...	...	...	...	...	...	...
1843	27 60	29.46	...	27.25	...	...	...	...	...	...	...	...	...	...	...
1844	33.10	33.74	...	34.45	...	...	...	...	...	...	...	...	...	...	...
1845	21.51	26.72	19.82	29.76	...	...	...	...	...	...	...	...	...	...	...
1846	...	...	17.61	29.91*	...	...	...	...	...	...	...	...	...	...	...
1847	...	...	23.15	31.63	...	...	...	...	...	...	...	...	...	...	...
1848	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1849	33.49	...	20.34	...	27.32 ?	...	...	...	...	...	...	...	...	...	...
1850	25.40	25.75	26.93	35.66	22.90 ?	...	...	...	...	...	...	...	...	...	...
1851	...	31.80	11.70	45.30	35.64 ?	...	...	...	...	...	...	...	...	...	...
1852	...	....	...	...	25.76 ?	...	...	...	...	...	...	...	...	...	...
1853	...	...	37.22	21.74	...	...	...	...	...	...	...	...	...	...	...
1854	46.49	...	35.40	29.16	28.90	24.08*	...	...	...	25.73	...	...	...	...	...
1855	71.19 ?	...	33.73	22.80*	46.17	40.54	...	...	...	42.94	...	...	...	...	52.06
1856	28.46*	...	19.11*	24.98*	...	...	...	...	...	18.75*	...	...	...	33.66	35.33*
1857	...	...	...	...	...	...	...	...	...	31.23	...	...	...	...	36.58
1858	35.52*	...	...	...	...	...	...	...	...	35.75	...	...	...	...	35.69
1859	28.93	...	22.55	...	...	...	35.14	...	...	31.86	20.62	...	...	...	41.71*
1860	28.27	...	...	...	...	...	27.88	27.75	...	...	17.29	26.63	21.21	...	...
1861	38.40	...	...	...	...	...	36.10	35.17	28.39*	...	21.42	25.01	34.46	...	...
1862	31.53	...	...	...	...	...	36.94	33.23	33.69*	...	24.71	27.14	27.03	...	...
1863	30.57	...	...	...	...	...	31.03	30.51	...	...	30.81	...	23.94	...	...
1864	25.88	....	...	...	...	...	31.93	29.18	28.96	...	19.01	...	22.08	...	...
1865	21.10	...	...	...	...	...	31.33	33.36	26.68	...	27.35	...	..	...	...
1866	31.72	...	...	...	...	...	...	31.14	28.59	...	19.56	...	...	...	...
1867	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	30.05	32.34	23.96	30.32	36.48	32.72	33.57	31.03	31.79	31.88	22.61	26.00	25.79	33.66	39.65

MICHIGAN.—CONTINUED.

Year.	Holland.	Romeo.	Marquette.	New Buffalo.	Ypsilanti.	State Agr'l College.
1852	...	...	...	...	...	...
1853	...	...	...	...	...	...
1854	...	...	...	...	...	...
1855	...	...	...	...	...	...
1856	...	21.37	...	...	...	...
1857	...	...	...	...	...	...
1858	...	...	38.41	52.42	...	...
1859	...	...	41.25	43.12	35.42	...
1860	37.07*	...	25.79	...	...	...
1861	36.18	...	39.78	...	40.14	...
1862	35.55*	...	29.19	...	29.02	...
1863	39.35*	...	35.60	...	27.50*	...
1864	...	...	22.32	...	...	27.99
1865	...	...	26.08	...	...	29.92
1866	47.09*	...	29.49	...	...	39.62
	38.71	22.69	31.99	44.20	32.87	30.64

XXVII. INDIANA.

Year.	Richmond.	South Bend.	Indianapolis.	New Harmony.	Logansport.	Cannelton.	New Albany.	Madison.	Cadiz.
1852	46.36	...	...	...	...	...	...	...	...
1853	31.97	...	...	...	...	...	...	...	...
1854	41.83	...	...	35.58	...	...	...	...	...
1855	56.77	...	...	48.11	...	...	...	...	...
1856	24.66	...	...	23.18	24.58	...	...	...	...
1857	39.42	...	...	39.66	39.87	30.60*	...	...	...
1858	55.39	...	...	48.71	50.07	44.24*	...	46.54	...
1859	45.49	...	...	44.33	41.89	36.39	...	...	...
1860	38.29	...	...	...	...	...	...	...	40.01*
1861	39.45	...	39.56	37.49	...	34.61	...	...	41.40
1862	50.22	...	56.48	46.34	...	...	...	...	...
1863	36.97	40.02*	40.11	34.98*	...	...	...	...	...
1864	34.08	37.10	33.78	33.71	...	...	32.94	...	...
1865	52.37	...	...	45.23	...	...	49.56*	...	...
1866	56.52	...	...	37.32	...	...	...	...	...
	43.32	38.78	44.83	38.75	37.34	36.46	37.64	57.86	42.25

	INDIANA.—Continued.						**XXVIII. ILLINOIS.**								
Year.	Spiceland.	Newcastle.	Independence.	Vevay.	Columbia City.	Aurora.	Athens.	Pekin.	York Neck.	Warsaw.	Batavia.	Manchester.	Upper Alton.	Augusta.	Brighton.
1843	...	...	...	...	...	...	40.93	...	...	...	...	...	...	...	...
1844	...	...	...	...	...	...	48.17	...	...	...	...	...	...	...	...
1845	...	...	...	...	...	...	43.04	...	...	...	...	...	...	...	...
1846	...	...	...	...	...	...	44.90	...	...	...	...	...	...	...	...
1847	...	...	...	...	...	...	32.61	...	...	...	...	...	...	...	...
1848	...	...	...	...	...	...	44.22	...	...	...	...	...	...	...	...
1849	...	...	...	...	...	...	38.49	...	...	...	...	...	...	...	...
1850	...	...	...	...	...	...	40.69	...	...	...	...	...	...	...	...
1851	...	...	...	...	...	...	46.91	...	...	...	...	...	...	...	...
1852	...	...	...	...	...	...	38.26	...	...	...	...	...	...	...	...
1853	...	...	...	...	...	...	34.83	...	...	...	...	...	...	...	...
1854	...	...	...	...	...	...	31.13	...	...	...	...	...	16.60*	...	...
1855	...	...	...	...	...	...	43.52	50.21	...	...	...	48.03	32.39	...	...
1856	...	...	...	...	...	...	25.12	27.89	...	...	...	31.02	27.18*	42.01*	...
1857	...	...	...	...	...	...	32.49*	32.60	...	24.79	...	33.76	...	28.75	17.73
1858	...	...	...	...	...	...	47.27	60.70	...	...	52.72	49.41	...	51.05	31.27
1859	...	...	...	...	...	...	...	30.30	...	...	31.03	42.37	38.65*	44.88	...
1860	...	...	...	...	...	...	...	36.40	...	...	...	...	...	...	...
1861	...	...	...	...	...	...	...	37.25*	...	...	...	30.03	...	32.47	...
1862	...	...	54.61	...	...	...	...	51.73	...	...	...	39.83	...	43.93*	...
1863	...	...	31.57	...	...	...	...	36.33*	...	...	...	34.35*	...	30.62*	...
1864	34.52	36.16	...	...	...	...	...	34.24	44.42	...	...	27.40	...	35.80	...
1865	47.30	...	...	53.24*	...	...	...	48.50*	44.42	...	...	...	...	42.00	...
1866	58.00	...	...	52.55	46.19	40.84	...	...	...	...	...	43.70	...	39.34	...
	43.57	39.21	43.39	52.77	44.87	40.84	39.62	41.25	44.42	40.18	36.68	37.79	28.53	39.14	29.93

ILLINOIS.—Continued.

Year.	Marengo.	Ottawa.	Peoria.	Riley.	West Salem.	Aurora.	West Urbana.	Wheaton.	Winnebago.	Elgin.	Sandwich.	Lebanon.	Galesburg.	Highland.	Elmira.
1856	41.48*	32.43*	27.91	...	31.97*	...	...	...	...	...	...	...	...	...	...
1857	35.27*	35.34*	30.45	...	41.49*	...	31.71*	...	42.25*	...	...	...	...	...	...
1858	49.15*	46.87	53.36	47.49*	51.00	47.50	46.11*	51.38	45.16	45.35*	...	...	...	...	...
1859	30.65*	27.89	29.38	25.51	41.02	30.47	34.06	30.78	26.51	28.79	38.96*	...	...	...	...
1860	...	27.07	...	32.58*	...	42.86*	...	...	33.78*	...	56.27	...	...	...	...
1861	33.16	38.89	33.36	37.51	...	...	...	...	44.51	40.44*	68.70	34.83	35.38*	34.64*	...
1862	39.55	55.71	46.95	37.87	...	...	...	...	45.20*	...	70.39	...	42.75	59.50	...
1863	...	32.44*	34.36*	40.46*	...	...	...	...	33.24*	...	50.51*	...	27.61*	36.04*	...
1864	...	29.53	30.87	46.90	...	...	...	...	28.90	...	38.37	...	32.73	...	...
1865	...	37.17*	37.02	49.90	...	...	...	...	44.82	...	34.34	...	36.29	...	38.03
1866	...	32.76	36.43	44.72	...	...	...	...	38.91	...	36.12	...	31.97	...	34.42
	38.58	37.19	35.83	39.45	42.23	34.89	36.61	41.70	37.83	37.71	50.17	37.93	35.04	42.45	36.87

ILLINOIS.—CONTINUED.					XXIX. **WISCONSIN.**										
Year.	Waverly.	Wyanet (near).	Elmore.	Dubois.	Ft. Howard.	Fort Winnebago.	Fort Crawford.	Milwaukee.	Lowell.	Superior City.	Beloit.	Janesville.	Platteville.	Appleton.	Ashland.
1836	...	...	...	...	37.64	...	...	...	...	...	...	...	...	...	...
1837	...	...	...	...	40.55	31.34	34.05	...	...	...	...	...	...	...	...
1838	...	...	...	...	37.56	27.88	21.96	...	...	...	...	...	...	...	...
1839	...	...	...	...	31.28	28.95	32.70*	...	...	...	...	...	...	...	...
1840	...	...	...	...	30.48	27.12	31.25	...	...	...	...	...	...	...	...
1841	...	...	...	...	...	28.45	31.05	34.32	...	...	...	...	...	...	...
1842	...	...	...	...	...	24.51	38.34	...	...	...	...	...	...	...	...
1843	...	...	...	...	...	22.80	25.58	...	...	...	...	...	...	...	...
1844	...	...	...	...	...	30.59*	39.06	32.50	...	...	...	...	...	...	...
1845	...	...	...	...	...	...	...	20.54	...	...	...	...	...	...	...
1846	...	...	...	...	...	...	...	25.26	...	...	...	...	...	...	...
1847	...	...	...	...	...	...	...	22.45	...	...	...	...	...	...	...
1848	...	...	...	...	...	...	...	33.52	...	...	...	...	...	...	...
1849	...	...	...	...	...	...	...	31.09	...	...	...	...	...	...	...
1850	...	...	...	...	...	...	...	26.41	...	...	...	...	...	...	...
1851	...	...	...	...	31.87	...	...	30.40	...	...	...	...	...	...	...
1852	...	...	...	...	...	...	...	29.33	...	...	...	...	...	...	...
1853	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1854	...	...	...	...	...	...	...	31.38*	...	...	...	36.47	...	...	...
1855	...	...	...	...	...	...	...	34.16	...	...	...	31.14	...	...	...
1856	...	...	...	...	...	...	...	29.02	...	...	27.89	22.05	29.61*	23.37	...
1857	...	...	...	...	...	...	...	30.89	30.25	...	33.30*	27.41	34.82	35.69	...
1858	...	...	...	...	...	...	...	44.86	...	...	47.09*	45.69*	47.26	36.39	43.09*
1859	...	...	...	...	...	...	...	28.86	...	...	22.74*	...	26.63	27.82*	53.57*
1860	...	...	...	...	...	...	...	24.02	...	29.52	...	...	...	...	44.84*
1861	...	...	...	...	...	...	...	33.47	...	35.24	37.64	...	...	...	43.69*
1862	...	...	...	...	...	...	...	41.41	...	21.96	39.59	...	...	...	...
1863	38.17*	...	...	...	...	...	...	33.56	...	17.70	29.14*	...	...	...	...
1864	28.85	...	...	...	...	...	...	29.80	...	20.97	30.81	...	...	...	...
1865	38.71*	50.00	34.46*	46.10*	...	...	...	31.86	...	30.06	24.50	...	...	...	...
1866	34.33	34.32	32.44*	44.21	...	...	...	36.18	...	25.74	30.10	...	...	...	...
	35.67	40.31	37.07	45.15	34.62	27.49	31.40	30.40	30.25	25.64	31.65	31.05	39.31	31.06	46.45

	WISCONSIN—Continued.								XXX. MINNESOTA.								
Year.	Green Bay.	Kenosha.	Manitowoc.	Rocky Run.	Summit.	Embarrass.	Delavan.	Plymouth.	Ft. Snelling.	Ft. Ripley.	Ft. Ridgely.	Hazlewood.	Beaver Bay.	Burlington.	Saint Paul.	New Ulm.	Minneapolis.
1837	...	...	...	...	...	...	...	...	24.02	...	...	...	...	...	...	...	...
1838	...	...	...	...	...	...	...	...	27.72	...	...	...	...	...	...	...	...
1839	...	...	...	...	...	...	...	...	21.19	...	...	...	...	...	...	...	...
1840	...	...	...	...	...	...	...	...	23.17	...	...	...	...	...	...	...	...
1841	...	...	...	...	...	...	...	...	21.67	...	...	...	...	...	...	...	...
1842	...	...	...	...	...	...	...	...	26.52*	...	...	...	...	...	...	...	...
1843	...	...	...	...	...	...	...	...	23.79	...	...	...	...	...	...	...	...
1844	...	...	...	...	...	...	...	...	30.24	...	...	...	...	...	...	...	...
1845	...	...	...	...	...	...	...	...	25.34	...	...	...	...	...	...	...	...
1846	...	...	...	...	...	...	...	...	26.10	...	...	...	...	...	...	...	...
1847	...	...	...	...	...	...	...	...	21.80	...	...	...	...	...	...	...	...
1848	...	...	...	...	...	...	...	...	23.18	...	...	...	...	...	...	...	...
1849	...	...	...	...	...	...	...	...	49.69	...	...	...	...	...	...	...	...
1850	...	...	...	...	...	...	...	...	25.50	35.32	...	...	...	...	...	...	...
1851	...	...	...	...	...	...	...	...	23.42	36.14*	...	...	...	...	...	...	...
1852	...	...	...	...	...	...	...	...	15.07	34.52	...	...	...	...	...	...	...
1853	...	...	...	...	...	...	...	...	20.47	26.12	...	...	...	...	...	...	...
1854	...	...	...	...	...	...	...	...	26.59	18.49	25.30	...	...	...	...	...	...
1855	...	...	...	...	...	...	...	...	24.78	23.55	34.78	...	...	...	...	...	...
1856	...	...	...	...	...	...	...	...	22.62	25.33	23.20	...	...	...	...	...	...
1857	...	...	...	...	...	...	...	...	32.09	...	38.38	...	...	...	...	...	...
1858	...	...	...	...	...	...	...	...	...	19.81	22.52	31.93*	...	30.72*	...	...	...
1859	...	27.55	...	...	...	...	...	...	...	26.00	32.85	25.36*	33.55	40.50*	...	...	...
1860	...	...	...	39.04*	...	...	...	...	...	30.61	16.97	...	...	...	...	...	...
1861	...	35.25	...	36.63	32.67	...	...	...	...	32.42	23.05	28.56*	34.56	...	...	...	...
1862	...	36.92	...	38.82	...	...	...	...	...	14.39	30.05	...	24.94*	...	...	...	...
1863	...	...	...	31.48*	...	...	...	...	...	17.20	18.17	...	23.39*	...	15.36*	...	...
1864	29.69	...	26.78	20.63	...	28.87	...	...	...	12.06*	14.46	...	20.59*	...	14.86	23.78*	...
1865	32.68*	...	31.59	31.52	...	40.58*	31.29	...	...	25.11*	34.94*	...	25.92*	...	38.14	35.32	...
1866	...	...	27.24	39.67*	...	35.49	31.42	37.50	...	...	...	...	27.10*	...	27.14	22.43	27.38
1867	...	...	...	...	...	...	...	...	...	30.34	...	...	...	...	...	...	...
	31.82	33.47	30.29	34.12	34.88	34.08	32.01	38.60	25.82	25.11	25.69	29.07	27.20	40.35	25.09	27.84	27.38

XXXI. IOWA.

Year.	Fort Atkinson.	Fort Dodge.	Iowa City.	Fort Madison.	Muscatine.	Poultney.	Dubuque.	Bellevue.	Pleasant Plain.	Fairfield.	Rossville.	Sioux City.	Lyons City.	Daven-port.	Algona.
1845	34.83	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1846	...	...	...	...	34.55	...	...	...	...	...	...	...	...	...	...
1847	...	...	...	...	28.50	...	...	...	...	...	...	...	...	...	...
1848	...	...	...	50.56*	39.62	...	...	...	...	...	...	...	...	...	...
1849	...	...	...	54.14	59.11	...	...	...	...	...	...	...	...	...	...
1850	...	...	...	50.36	49.08	...	...	...	...	...	...	...	...	...	...
1851	...	...	...	45.32	74.20	...	...	...	...	...	...	...	...	...	...
1852	...	25.85	...	52.22	60.42	...	...	...	...	...	...	...	...	...	...
1853	...	...	...	...	44.92	35.62*	...	...	...	...	...	...	...	...	...
1854	...	...	...	46.26	23.66	33.21	27.77	...	...	...	...	...	...	...	...
1855	...	...	...	48.05*	31.13	31.01	29.13	...	...	...	...	...	...	...	...
1856	...	...	...	43.86	41.94	...	...	31.15*	43.72	...	...	...	...	...	...
1857	...	...	...	27.54	34.85	...	...	36.05	...	...	...	...	...	...	...
1858	...	...	43.24	51.10	58.45	...	47.19	51.51	59.88*	65.90	40.23*	35.61*	...	...	...
1859	...	...	...	37.46	35.96	...	29.72*	27.97	39.95	42.34	31.29	...	...	...	...
1860	...	...	...	32.23	...	...	26.84	...	27.02	...	...	...	33.29	...	...
1861	...	...	47.51	31.01	42.89	...	38.47	...	40.01	...	...	25.71*	39.98	36.78*	...
1862	...	...	45.42	37.56	53.16	...	35.00	...	33.49	...	...	18.58*	46.10	42.23	...
1863	...	...	33.15*	30.07	...	...	33.92	...	21.55*	...	...	...	41.53*	...	30.83*
1864	...	...	46.23	31.17	36.88*	...	25.07	...	30.07*	...	...	...	42.62*	29.93*	24.43*
1865	...	...	45.19	44.56	34.21	...	36.21	...	...	...	...	...	62.51	30.88	...
1866	...	...	44.49	41.85	...	...	36.25	...	...	...	...	...	42.77*	57.18	...
	39.78	26.82	43.12	41.96	42.88	34.37	33.47	34.45	36.81	47.94	37.24	25.66	43.72	39.46	30.60

	IOWA.—Continued.					**XXXII. MISSOURI.**								
Year.	Brookside.	Spring Grove (near).	Monticello (near).	Des Moines.	Manchester.	Jefferson Barracks.	St. Louis Arsenal.	St. Louis.	St. Louis, College Hill.	Edina.	Hannibal.	Tower Grove, near St. Louis.	Wyaconda.	Harrison-ville.
1837	...	...	...	...	...	...	26.88	27.00	...	...	...	...	...	...
1838	...	...	...	...	...	...	24.08	28.36	...	...	...	...	...	...
1839	...	...	...	...	...	...	41.99*	47.45	...	...	...	...	...	...
1840	...	...	...	...	...	...	...	41.49	...	...	...	...	...	...
1841	...	...	...	...	...	36.27	39.64	42.72	...	...	...	...	...	...
1842	...	...	...	...	...	29.18	...	31.69	...	...	...	...	...	...
1843	...	...	...	...	...	32.58	31.06	34.79	...	...	...	...	...	...
1844	...	...	...	...	...	37.27	51.11	45.73	...	...	...	...	...	...
1845	...	...	...	...	...	32.91	39.49	38.19	...	...	...	...	...	...
1846	...	...	...	...	...	36.01	56.18	44.23	...	...	...	...	...	...
1847	...	...	...	...	...	...	65.06	47.60*	...	...	...	...	...	...
1848	...	...	...	...	...	35.10	62.86	45.80*	...	...	...	...	...	...
1849	...	...	...	...	...	38.58	71.54	45.71	...	...	...	...	...	...
1850	...	...	...	...	...	39.61	35.13	50.50	...	...	...	...	...	...
1851	...	...	...	...	...	36.13	34.45	42.84	...	...	...	...	...	...
1852	...	...	...	...	...	55.13	40.21	46.63	...	...	...	...	...	...
1853	...	...	...	...	...	36.08	28.74	30.89	...	...	...	...	...	...
1854	...	...	...	...	...	43.27	31.08	40.63	...	...	27.68	...	...	...
1855	...	...	...	...	...	46.58*	53.58	50.37	...	...	...	...	...	...
1856	...	...	...	...	...	...	31.75*	42.08	...	...	...	...	...	...
1857	...	...	...	...	...	41.01	...	39.03	...	...	...	...	...	...
1858	...	...	...	...	...	49.96	...	68.83	...	...	...	...	...	...
1859	...	...	...	...	...	51.58	...	61.38	...	...	...	...	...	...
1860	...	...	...	...	...	40.08	...	29.79	25.61*	20.88	...	...	...	...
1861	...	...	...	...	...	51.86	...	38.03	30.91	28.08	...	35.06	...	...
1862	47.12*	...	...	...	...	...	...	44.00	36.67	34.45	...	40.14	52.12*	...
1863	49.24*	...	...	...	...	...	...	37.28*	30.24*	24.72	...	...	31.13*	...
1864	42.53	36.88	...	...	...	...	...	33.42*	31.09*	26.42	...	...	42.84	33.97*
1865	50.88	50.02	...	...	...	...	...	46.88	39.21	40.46	...	...	46.68*	58.50
1866	42.36	42.53	40.10	21.18	31.73	...	...	41.58*	39.66	40.94	...	...	...	58.37
1867	...	...	...	...	...	...	...	37.76	...	...	...	...	...	...
	46.42	44.79	36.00	23.94	32.47	40.88	42.63	42.18	32.39	30.48	26.13	37.20	45.36	49.41

XXXIII. **INDIAN TERRITORY.**					XXXIV. **KANSAS.**								
Year.	Fort Towson.	Fort Washita.	Fort Gibson.	Fort Arbuckle.	Fort Scott.	Fort Leavenworth.	Leavenworth City.	Fort Riley.	Burlingame.	Manhattan.	Neosho Falls.	Fort Larned.	Olathe.
1837	43.80	...	34.66	...	...	38.45	...	...	...	...	...	...	...
1838	34.40	...	18.84	...	...	26.28	...	...	...	...	...	...	...
1839	66.00	...	33.74*	...	...	33.32	...	...	...	...	...	...	...
1840	55.70	...	55.82	...	...	32.14	...	...	...	...	...	...	...
1841	66.62*	...	36.25	...	...	25.90*	...	...	...	...	...	...	...
1842	73.36	...	29.11	...	...	26.29	...	...	...	...	...	...	...
1843	55.90	...	35.61	...	44.53	15.94	...	...	...	...	...	...	...
1844	46.54	36.00	37.58	...	62.60	48.12	...	...	...	...	...	...	...
1845	38.08	34.70	26.27	...	61.59	34.56	...	...	...	...	...	...	...
1846	...	49.75	33.85	...	34.04	23.75	...	...	...	...	...	...	...
1847	...	37.45	24.10	...	34.64	21.03	...	...	...	...	...	...	...
1848	...	35.28	37.73	...	29.25	37.99	...	...	...	...	...	...	...
1849	...	64.29	52.65	...	45.43	42.85	...	...	...	...	...	...	...
1850	49.65	41.34	35.01	...	30.03	27.07	...	...	...	...	...	...	...
1851	48.97*	31.52	52.26	24.24	33.01	37.81	...	...	...	...	...	...	...
1852	...	47.07	51.65	46.02	46.52	36.53	...	...	...	...	...	...	...
1853	42.10	30.46	25.88	26.68	...	25.20	...	...	...	...	...	...	...
1854	...	43.28	28.82	28.22	...	24.40	...	16.93	...	...	...	...	...
1855	...	21.81	35.66	29.69	...	27.55	...	26.25	...	...	...	...	...
1856	...	29.04	39.85*	41.93	...	42.72	...	24.84	...	...	...	...	...
1857	...	33.17	...	47.49	...	31.71	...	17.98	...	...	...	...	...
1858	...	...	...	...	...	59.65	59.91	31.97	38.50*	37.66*	...	...	...
1859	...	...	...	27.96	...	38.84	...	23.47	31.18*	36.21	34.73*	...	...
1860	...	...	...	19.73	...	19.38	...	13.36	14.23*	...	19.64	...	...
1861	...	...	...	...	...	27.27	...	31.68	...	38.11*	39.57*	18.01	...
1862	...	...	...	...	...	29.50	...	20.21	...	26.91*	...	17.66*	...
1863	...	...	...	...	...	30.40*	...	27.48	...	32.90*	...	...	...
1864	...	...	...	...	...	15.93*	...	14.43*	...	20.20*	...	...	46.59 ?
1865	...	...	...	...	...	50.88*	...	22.21*	...	33.04*	...	38.14*	87.84 ?
1866	...	...	...	...	...	...	49.41	27.05*	...	...	...	...	58.31 ?
1867	...	...	...	...	...	25.72*	...	30.25	...	...	...	18.70*	...
	51.08	38.04	36.37	32.69	42.15	31.74	53.94	23.62	30.46	30.53	31.18	24.88	64.26 ?

XXXV. NEBRASKA.				XXXVI. DAKOTA.			XXXVI½ WYOMING.	XXXVII. MONTANA.		XXXVIII. COLORADO.		
Year.	Fort Kearny.	Omaha.	Bellevue.	Fort Randall.	Fort Pierre.	Fort Abercrombie.	Fort Laramie.	Fort C. F. Smith.	Helena City.	Fort Massachusetts.	Fort Garland.	Fort Lyon.
1849	45.76*	...	...	...	...	...	...	...	...	...	...	...
1850	25.07	...	...	...	...	...	9.43*	...	...	...	...	...
1851	26.44	...	...	...	...	...	9.89*	...	...	...	...	...
1852	20.65	...	...	...	...	...	31.42	...	...	...	...	...
1853	29.90	...	...	...	...	...	30.78	...	...	16.23*	...	...
1854	26.65	...	...	...	...	...	22.26	...	...	...	...	...
1855	25.05*	...	...	...	...	...	18.99	...	...	18.81*	...	...
1856	29.10	...	...	...	12.56	...	15.02	...	...	13.87	...	...
1857	28.62	...	...	16.16	...	...	6.15	...	...	14.66	...	...
1858	26.14	48.38*	48.89*	21.30	...	...	7.90	...	...	...	...	...
1859	16.10	21.06	18.44*	15.70	...	...	6.26	...	...	...	11.35	...
1860	16.85	...	19.23	18.83	...	...	...	...	...	...	6.62	...
1861	19.34	...	23.92	20.28	...	23.39	...	...	...	...	5.44	...
1862	22.10*	...	26.56	19.00	...	11.38	12.70	...	...	...	6.33	...
1863	...	...	...	8.83*	...	13.40	5.11*	...	...	...	3.00*	...
1864	...	...	19.89*	...	...	16.85	...	...	...	...	...	...
1865	...	...	26.56*	...	...	18.92*	...	...	...	...	...	...
1866	...	...	24.14	...	...	...	...	...	22.14	...	...	...
1867	...	...	...	15.02*	...	20.05*	...	13.13	...	...	2.74*	12.09
	25.25	35.02	28.53	16.51	13.51	17.34	15.16	13.13	22.14	17.06	6.11	12.09

XXXIX. NEW MEXICO.

Year.	Fort Craig.	Fort Fillmore.	Fort Webster.	Fort Thorn.	Fort Conrad.	Socorro.	Albuquerque.	Cebolleta.	Fort Marcy.	Fort Union.	Cantonment Burgwin.	Fort Stanton.	Fort Sumner.	Fort Wingate.
1850	...	...	...	...	...	7.12	4.91*	9.69	...	...	...	...	...	...
1851	...	...	...	...	...	...	...	15.12	...	...	...	...	...	...
1852	...	12.41*	29.73*	...	8.63	...	14.59*	...	...	26.64	...	...	...	...
1853	...	9.04	8.79	...	7.86	...	7.10	...	21.77	13.43	...	...	...	...
1854	...	6.07	...	14.60	5.75	...	12.51	...	24.80	14.37	...	...	...	...
1855	7.89	7.51	...	13.44	...	...	...	...	24.18	18.57	7.53	...	...	...
1856	12.56	9.22	...	13.51	...	...	4.15	...	23.07	20.21	3.86	16.81	...	...
1857	13.43	10.40	...	20.55*	...	...	5.20	...	8.52	20.94	7.98	28.70	...	...
1858	4.63	5.11	...	10.58	...	...	16.30	...	11.35	22.79	11.91	18.76	...	...
1859	24.58	5.52	...	...	...	...	5.95	...	9.49	24.54	12.70	23.81	...	...
1860	6.62*	...	...	...	...	...	3.78	...	8.83	14.44	...	13.65	...	...
1861	10.65	...	...	...	...	...	...	...	15.79*	41.14?*	...	...	...	...
1862	21.44*	...	...	...	...	...	...	...	...	21.37	...	...	...	...
1863	...	...	...	...	...	...	...	...	7.75	...	...	...	...	...
1864	...	...	...	...	...	...	6.18*	...	20.72*	...	...	...	...	...
1865	...	...	...	...	...	...	...	...	23.15	...	...	...	27.25	10.16*
1866	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1867	...	...	...	...	...	...	...	...	...	...	...	...	8.92*	22.00
	11.67	8.42	19.46	14.63	6.76	7.86	8.12	12.05	16.65	21.88	8.65	20.35	15.63	13.95

	XL. **ARIZONA.**						XLI. **UTAH.**				XLII. **IDAHO.**		XLIII. **NEVADA.**
Year.	Fort Defiance.	Fort Buchanan.	Fort Mojave.	Camp Goodwin.	Camp McDowell.	Camp Wallen.	Great Salt Lake.	Fort Bridger.	Camp Floyd.	Camp Douglas.	Fort Boise.	Fort Lapway.	Fort Churchill.
1853	13.87	...	...	...	...	...	...	...	...	...	...	...	...
1854	22.44	...	...	...	...	...	...	...	...	...	...	...	...
1855	17.07*	...	...	...	...	...	...	...	...	...	...	...	...
1856	11.63	...	...	...	...	...	...	...	...	...	...	...	...
1857	13.06	...	...	...	...	...	14.70*	...	...	...	...	...	...
1858	11.97	16.08	...	...	...	...	...	...	...	...	...	...	...
1859	11.44*	25.82	...	...	...	...	20.27*	6.81*	11.28	...	...	...	...
1860	11.84*	...	2.23	...	...	...	...	5.05	4.83	...	...	...	...
1861	...	...	...	...	...	...	22.22*	...	...	...	...	...	...
1862	...	...	...	...	...	...	...	...	...	...	...	...	6.77
1863	...	...	...	...	...	...	...	...	...	...	...	...	...
1864	...	...	...	...	...	...	23.87 ?	7.34	...	...	...	14.05	...
1865	...	...	...	...	...	...	22.67	4.62*	...	16.86	11.53	11.57*	...
1866	...	...	...	...	...	...	38.20	...	...	17.54*	...	...	...
1867	...	...	...	27.93	14.97	17.76	...	...	...	28.00	...	...	...
	14.21	23.11	2.51	32.78	12.16	17.76	23.85	6.12	7.34	20.57	13.28	17.00	5.67

XLIV. CALIFORNIA.

Year.	Sacramento.	Fort Yuma.	San Diego.	Ranchos del Chino and de Jurupa.	Monterey.	Fort Miller.	San Francisco (consolidated series).	Benicia Barracks.	Camp Far West.	Fort Reading.	Fort Humboldt.	Fort Jones.	Stockton.	Fort Tejon.
1850	19.50	...	7.84	...	10.19	...	...	13.20*	20.38	...	...	...	...	...
1851	15.10	...	7.49	...	...	...	15.12	15.30	19.47	...	...	...	...	...
1852	27.00	...	11.87	...	...	49.35	25.64	17.57	...	...	...	...	...	...
1853	20.00	1.78	7.88	8.20	...	18.40	19.16	11.80	...	25.23	...	17.72*	...	...
1854	21.83	4.50	12.06	...	...	9.71*	20.66	12.37	...	15.91	29.01	12.94	20.33*	...
1855	18.56	1.80*	13.37	...	...	13.16	24.60	17.54*	...	37.36*	38.97	22.85*	...	14.15* ?
1856	14.23	1.53	10.14	...	...	12.67	18.10	12.10	...	...	27.24	19.44	12.59	34.22
1857	16.91	0.64*	6.89*	...	...	12.27	17.73	12.15	...	...	32.21	26.33*	11.56	9.83
1858	16.79	2.09	8.94	...	...	...	20.15	12.39	...	...	41.68	...	...	24.13
1859	16.89	4.84	...	...	...	...	21.13	13.67	...	...	42.75	...	...	18.19*
1860	19.71	...	...	...	17.31*	...	20.68	14.49	...	...	40.91	...	...	...
1861	20.92	...	...	...	...	...	23.08	...	...	...	34.54*	...	...	...
1862	27.44	...	7.11*	...	...	...	36.03	20.04*	...	...	...	...	...	...
1863	12.19	...	...	...	...	...	15.07	...	...	...	37.49	...	...	...
1864	19.27	...	7.42*	...	17.08	...	19.84	...	...	...	31.03*	...	...	...
1865	11.15	...	8.53	...	8.16	...	11.73	...	...	...	31.50	...	...	...
1866	26.52	...	...	...	21.56	...	34.04	...	...	...	...	...	...	...
1867	...	2.94	...	...	...	...	28.62	...	...	...	...	...	...	...
	19.56	3.46	9.16	13.57	13.74	18.99	21.69	15.05	20.60	29.11	35.92	21.70	13.69	19.53

	CALIFORNIA.—CONTINUED.								XLV. **OREGON.**						
Year.	Fort Crook.	Fort Ter-Waw.	Camp Gaston.	Fort Bidwell.	Drum Barracks.	Camp Wright.	Fort Point.	Meadow Valley.	Dalles of Columbia.	Oregon City.	Fort Hoskins.	Fort Yamhill.	Fort Umpqua.	Blockhouse.	Fort Stevens.
1851	...	...	...	...	...	...	...	...	...	51.26	...	...	...	...	...
1852	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1853	...	...	...	...	...	...	...	...	14.48	...	...	...	...	...	...
1854	...	...	...	...	...	...	...	...	12.39	...	...	...	...	...	...
1855	...	...	...	...	...	...	...	...	11.90*	...	...	...	...	...	...
1856	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1857	...	...	...	...	...	...	...	...	29.34	43.65*	65.06	57.78	62.95	...	...
1858	22.38	...	...	...	...	...	...	...	43.65	43.09	67.97	62.00	73.17	98.80*	...
1859	20.39	63.95*	...	...	...	...	...	...	35.96*	...	71.04	55.63	69.15	93.78*	...
1860	25.89	75.31	...	...	...	...	...	...	21.32	...	...	50.91	55.73	...	...
1861	21.33*	...	...	...	...	...	...	...	28.85	...	77.39*	63.39*	89.46	...	...
1862	26.71	...	39.34	...	...	...	...	...	15.97*	...	45.80	56.76	...	...	...
1863	...	...	63.44	...	...	...	...	...	14.00	...	74.64	53.54*	...	...	...
1864	...	...	65.17*	...	...	...	...	54.28*	...	...	60.81*	43.35*	...	...	...
1865	26.52*	...	66.32*	...	4.42	33.28	...	52.41*	22.18*	...	...	54.90	...	...	...
1866	...	...	...	...	...	...	27.58*	96.81*	...	...	...	...	...	...	...
1867	22.42*	...	...	41.80	...	63.80*	22.37*	...	...	...	...	...	...	...	85.82
	23.68	69.93	62.20	32.91	11.83	53.64	25.68	57.03	21.74	47.61	66.71	55.59	67.41	96.29	85.82

	XLVI. **WASHINGTON TERRITORY.**										XLVII. **ALASKA.**
Year.	Fort Vancouver.	Fort Steilacoom.	Fort Walla-Walla.	Fort Bellingham.	Fort Simcoe.	Fort Cascades.	Fort Colville.	San Juan Island.	Cape Disappointment.	Neah Bay.	Sitka.
1848	...	...	...	...	...	...	...	...	...	...	90.95
1849	...	...	...	...	...	...	...	...	...	...	74.87*
1850	38.40	33.31	...	...	...	...	...	...	...	...	95.81
1851	...	39.32	...	...	...	...	...	...	...	...	72.44
1852	56.09*	48.79 ?	...	...	...	...	...	...	...	...	78.17*
1853	42.07	57.01 ?	...	...	...	...	...	...	...	...	90.94
1854	31.73*	70.21 ?	...	...	...	...	...	...	...	...	87.18
1855	45.43*	...	...	...	...	...	...	...	...	...	...
1856	52.57	...	...	...	...	...	...	...	...	...	87.51
1857	47.57*	39.53	17.89	...	...	...	...	...	...	...	89.66
1858	40.46	50.14	19.12	31.29	13.76	...	...	...	...	...	81.09
1859	38.86	38.72	18.13	...	...	60.97	...	...	...	...	81.77
1860	34.97	30.62	20.50	...	...	57.17	...	...	...	...	...
1861	44.09	36.67*	40.57	...	...	...	15.56	...	...	...	58.68
1862	29.49	25.75	39.27*	...	...	...	10.60	...	...	...	85.93
1863	42.41	45.09	5.29*	...	...	...	3.66*	...	...	...	74.33
1864	31.48	30.65	8.38*	...	...	...	...	20.41	...	109.53*	...
1865	25.91*	37.23*	...	...	...	...	...	...	70.74*	121.30	...
1866	...	...	...	...	...	...	...	...	...	...	...
1867	35.13	58.47*	...	...	...	...	14.65*	30.90	83.54*	126.50*	...
	38.84	43.98	19.48	29.67	10.61	64.57	9.83	27.53	74.90	123.35	83.39

BRITISH NORTH AMERICA.

Year.	St. John's, New Foundland.	Albion Mines, Nova Scotia.	Wolfville, Nova Scotia.	Thurston, New Brunswick.	St. John, New Brunswick.	St. Martins, Canada East.	Montreal, Canada East.	Toronto, Canada West.	Kingston, Canada West.	Michipicoton, Canada West.	Moose Factory, Hudson Bay Territory.
1831	...	...	...	...	...	...	39.55 ?	...	...	...	...
1832	...	...	...	...	...	...	30.88 ?	...	...	...	...
1833	...	...	...	...	...	...	40.26 ?	...	...	...	...
1834	...	...	...	...	...	...	24.10 ?	...	...	...	...
1835	...	...	...	...	...	...	38.05 ?	...	...	...	...
1836	...	...	...	...	...	...	27.84*	...	...	...	...
1837	...	...	...	...	...	...	23.37	...	...	...	...
1838	...	...	...	...	...	...	23.59	...	...	...	...
1839	...	...	...	...	...	...	20.67	...	...	...	...
1840	...	...	...	...	...	...	32.49	31.86 ?	...	...	...
1841	...	...	...	26.40	...	...	...	37.30 ?	...	...	...
1842	...	...	...	23.40	...	...	...	43.44 ?	...	...	...
1843	...	...	...	29.20	...	...	...	50.21	...	...	...
1844	...	...	...	26.30	...	...	...	33.38*	...	...	...
1845	...	...	...	24.40	...	...	...	27.93	...	...	...
1846	...	...	...	32.50	...	...	...	38.60	...	...	...
1847	...	...	...	23.20	...	...	...	36.97	...	...	...
1848	...	...	...	32.20	...	...	...	26.83	...	...	...
1849	...	33.56*	...	26.30	...	...	...	36.55	...	...	...
1850	...	46.14	...	20.80	...	...	...	35.48	...	...	...
1851	...	43.95	...	25.30	...	50.47*	...	30.75	...	...	...
1852	...	49.32	...	38.80	...	53.02	...	40.93	...	...	...
1853	...	47.32	...	...	...	55.71	...	29.88	...	...	...
1854	...	...	...	...	...	49.99	...	32.74	...	...	...
1855	...	...	...	...	...	51.66	...	41.53	...	...	...
1856	...	...	31.44*	...	...	45.48	35.59	28.05	21.65	...	...
1857	62.09*	...	41.89*	...	...	54.98*	44.63*	40.58	31.34	...	...
1858	41.90*	...	...	...	...	55.56*	38.47	32.59	28.77	...	...
1859	66.99	...	77.31*	...	...	60.23	49.11*	39.76	24.43	...	...
1860	...	...	...	...	...	...	...	28.00	...	...	...
1861	55.05	...	...	...	...	53.56	53.52	34.46	33.09	...	19.03
1862	...	...	36.77*	...	...	...	38.83	34.05	...	...	...
1863	...	...	...	...	50.89	...	...	32.77	...	...	...
1864	...	...	42.25*	...	49.69	...	...	36.94	...	24.06	...
1865	...	...	39.02*	...	50.22	...	...	32.93	...	31.01	...
1866	...	...	37.87*	...	52.59	...	...	...	...	...	...
	58.30	43.63	41.88	27.40	51.12	53.19	35.28	35.17	27.86	27.81	19.07

	MEXICO.				WEST INDIES AND BERMUDA.					CENTRAL AMERICA.			SOUTH AMERICA.	
Year.	Matamoras.	Cordova.	Vera Cruz.	Mirador.	Nassau, Bahama Islands.	H. M. Nav. Y'd, Ireland Island, & St. George's Met. Obs. and N. Hospital, Bermuda.	Philipsburg, St. Martin.	Matanzas, Cuba.	Port-au-Prince, Hayti.	Guatemala.	Belize, Brit. Honduras.	Aspinwall.	Rustenburg, Governm't Plant'n, Dutch Guiana.	Catharina Sophia, Surinam.
1830	...	...	205.60	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1835	...	...	...	...	...	...	...	55.29	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1841	...	...	...	...	49.99	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1845	36.09*	...	...	...	...	...	...	...	...	...	...	...	...	...
	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1849	38.50*	...	...	...	...	...	...	...	...	...	...	...	...	...
1850	31.31*	...	...	...	...	...	...	...	...	...	...	...	...	...
1851	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1852	...	...	...	...	...	49.44	...	...	...	...	...	...	...	...
1853	...	...	...	...	...	67.78	...	...	...	...	...	...	...	...
1854	...	...	...	...	...	47.09	...	...	...	...	...	...	...	...
1855	...	...	...	...	...	50.10	...	...	...	...	...	...	...	...
1856	...	...	...	...	...	45.47	...	...	...	...	...	...	...	64.88*
1857	...	...	...	...	...	59.66	...	...	...	54.52	...	...	...	74.81*
1858	...	...	...	...	...	64.32	...	...	...	50.18	...	...	...	86.24
1859	...	104.01	...	75.33	...	51.45	...	...	...	59.36	...	...	...	85.89
1860	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1861	...	129.44	...	109.75*	...	...	...	...	...	...	...	...	...	...
1862	...	103.46	...	71.41	...	...	104.72	...	...	...	...	...	103.92*	...
1863	...	104.52	...	72.77	...	...	64.99	...	...	...	53.20	109.36*	117.23*	...
1864	...	118.94	...	101.77	...	...	...	...	60.68	...	78.02	115.74*	...	...
1865	...	...	...	86.69	...	...	...	...	66.88	...	75.08	107.43	...	...
1866	...	...	...	55.52	...	...	...	...	67.56	...	67.92	129.62	...	...
1867	...	...	...	...	...	...	...	...	50.29	...	88 84	...	...	...
	36.74	112.08	183.20[1]	84.92	46.75	55.34	86.32	55.29	60.96	54.67	72.35	121.44	97.38	77.94

[1] Mean of nine years.

GENERALIZATION OF TABULAR RESULTS AND CONSTRUCTION OF RAIN-CHARTS OF THE UNITED STATES.[1]

The mutual relations of the preceding tabular results can best be shown by a graphical representation, and I therefore proceed at once to the explanation of the three rain-charts accompanying this paper, which exhibit the most obvious results of the co-ordination.

Construction and Analysis of the Rain-Charts of the United States.—From the material collected in the general table of results, the geographical distribution of rain over the area of the United States is shown on the accompanying three charts, for the average amount fallen during the year as well as for the summer and winter seasons. For other parts of the Western Continent the observations are too scanty even to attempt a graphical representation.

To explain the construction of these charts it suffices to show it for the one exhibiting the annual amount. All stations at which the observations extended over four years and more, were plotted by their co-ordinate latitude and longitude, and against the dot was written the nearest full inch of the amount of rain (and melted snow); curves were then drawn with a free hand among these dots, for a certain amount of rain, uniting all places at which this amount is recorded. These curves, designated as isohyetal lines, and constructed in the manner of contour lines generally, are given for *equal intervals* of four inches, an amount which resulted from a consideration of the probable error of the results. If drawn too close, the curves exhibit temporary or accidental inflections which only tend to confuse, and if drawn too far apart there is danger of losing part of the permanent features of these rain-curves. Next, all stations, having between one and four years of record, were plotted and marked with a cross, and the amount noted as before; these latter numbers were only used to complete, modify, and generally improve the curves previously constructed, and have of course less weight than the numbers first plotted. The increasing amount of rain, as shown by the increasing figures written against each curve, is also indicated by somewhat heavier lines. This chart exhibits the results from nearly 790 stations. Those for the extreme seasons depend nearly upon the same number.

General Character of the Distribution of Rain on the Average during the Year.—The most striking features of the phenomenon of rain, as delineated on the chart, are the apparent precision and continuity in the law of its distribution, and the great variation or range in its amount. Thus the curve, passing over places where the annual fall amounts to 40 inches, can be followed from New Brunswick, on the Bay of Fundy, to Texas; the isohyetal line of 36 inches, similarly, runs without interruption from the St. Lawrence River to the mouth of the Rio Grande. The regularity of the curves is sufficiently distinct to mark out everywhere the progression in the deposition of the aqueous vapor. The annual amount varies from four inches in the Yuma and Gila Deserts, at the head of the Gulf of California, to 80 inches

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[1] For the original skeleton maps the Institution is indebted to the U. S. Coast Survey. The projection used is that known as the "polyconic," explained in the Coast Survey Report for 1859, Appendix No. 33, or Report for 1853, Appendix No. 39.

and more on the Pacific Coast in Washington Territory; on the Gulf Coast 64 inches appears to be the maximum amount, and 48 on the Atlantic Coast.

The principal supply of rain over the United States comes from the Gulf of Mexico; its diffused vapor can be traced from the eastern slope of the Rocky Mountains to the Great Lakes: while the supply of vapor from the Atlantic Ocean is distinctly traceable over that area lying north and east of Virginia. All States and Territories west of the Rocky Mountains receive their supply of rain from the condensed vapors of the Pacific Ocean.

There are distinct localities of entry of *maximum* rain from each of these basins of supply; the vapors from the Pacific are deposited within a remarkably well-defined coast region between latitude 41° and our boundary at the Straits of Juan de Fuca; the rain pours down with great intensity on the coast between the mouth of Columbia River and Cape Flattery. It is surprising how little rain falls on the Pacific coast between San Diego and Cape Mendocino, and how quickly the atmosphere becomes drained of its vapor as we leave the coast and proceed inland in latitudes north of 41°. The coast range of mountains here act powerfully as condensers by forcing the air up their western slopes.

The densest part of the Gulf vapor is thrown over the delta of the Mississippi River, and as far east as longitude 86° its *axis of diffusion* can be traced distinctly to the west end of Lake Erie; it is inclined towards the northeast for two reasons—the effect of the earth's rotation on a flow from the south, and the influence generally of the prevailing westerly winds. A second sweep over the country occurs in southern Florida, most likely due to the immediate proximity of the Gulf Stream; and there is a third, as yet undefined, influx, passing through Georgia and South Carolina.

The condensation of vapor from the Atlantic is most apparent a *short distance inland* at the following localities: Along the coast of Maine near Eastport and near Portland, in central Connecticut, western Massachusetts, and extending to southern Vermont, and near the entrance of Chesapeake Bay. Upon the whole, hills and mountain ranges appear to have a comparatively small directive influence upon the distribution of the rain. Florida, which may be considered as almost perfectly flat, exhibits well-defined bounding lines of rain distribution. River courses also seem to influence the amount of rain, as along the Rio Grande. At the mouth of the Hudson River, the curves become suddenly contracted; and some similar feature can perhaps be traced out on the Mississippi delta near New Orleans. Beyond furnishing by their evaporation a supply to the general fund of moisture, the Great Lakes do not appear to exercise any direct influence; on the yearly average the rains along their borders are not increased. There is even a remarkably small amount of rain-fall in northern New York, close to Lake Ontario. The effect of equalizing the temperature produced by all large bodies of water has no doubt a direct influence upon the distribution of rain; the greater and more sudden the variations in temperature, the greater, comparatively, the rain-fall.

The laws of the distribution of the rain-fall, as far as they depend upon the *changes* of temperature and direction of wind, can be studied to better advantage by means of the two charts showing the distribution in summer and in winter, than by that for the year, since the latter necessarily brings out the resultant phenomena, and should consequently be of greater complexity than either of those for the extreme seasons. With a few exceptions, presently to be noticed, the distribution of rain in the extreme seasons is not very dissimilar from that of the year as a whole.

Distribution of Rain in Summer.—The leading feature is the transfer of the maximum rains to the peninsula of Florida. On its western or Gulf coast, we find twenty-eight inches recorded during June, July, and August; proceeding northerly and easterly, this amount gradually diminishes to less than one-half in Virginia. The peninsula of Florida is subject to the regular rainy season of the tropics, attaining its maximum about one month after the sun's greatest northern declination. There is a fresh development of rain over northern Indiana, northern Illinois, northeastern Iowa, and extending as far as central Minnesota, with one of its foci of sixteen inches, directly south of Lake Michigan. This is most probably due to a direct influence of the lake. The region is well traced out by the twelve and fourteen inch curves, the former running up along the western shore of the lake. On the chart, the ten and twelve-inch curves appear quite complex in their windings.

In California, we have a large area, extending from the Colorado to the Sacramento River, where no rain falls during the three summer months; at most, there may be an occasional sprinkling, often hardly enough to wet the gauge. Even in the most southerly part of the State, we find nothing analogous to the rainy season noticed in similar latitudes in Georgia. This may be explained by the different directions of the wind. On the Atlantic coast, as far up as latitude 32½° north (on the average during the year and still higher in the summer), the (easterly) trade-winds blow constantly; on the Pacific coast, on the contrary, all along the peninsula of California, and on the coast of California proper, as high north as latitude 41°, the prevailing wind during the year is from the northward and westward. And it is also to be noted that the ocean currents partake of the same difference in direction—the Gulf Stream on the Atlantic side running *northerly*, and the coast current on the Pacific side running *southeasterly;* the former being a *warm*, the latter a *cold* current. The California current is a branch of the north Pacific easterly current, which divides, off the coast of Oregon, sending one branch towards the coast of Washington and Alaska Territories, and the other skirting the coast of California.

Distribution of Rain in Winter.—In this season, we find the influx of moisture from the Gulf of Mexico over the Mississippi delta in its full development; there is a heavy precipitation of rain west of the river, and covering Arkansas and adjacent parts of Missouri. The current of moisture sweeping over Alabama and Mississippi, after running due north about 250 kilometres, bifurcates. One branch, of sixteen inches rain, skirts the southern and eastern spurs of the Alleghany range, and extends into North Carolina, with twelve inches; its southern boundary is along the extreme region of the trade-winds: the other branch passes along the opposite side of the Alleghany mountains, depositing twenty inches in Alabama, eighteen in Tennessee, sixteen in Kentucky, and twelve near the banks of the Ohio River.

On the chart, the curve of eight inches may be followed from Texas to New Brunswick, Canada.

A reference to Prof. Coffin's Wind Chart,[1] showing the average resulting yearly direction of the wind, very plainly exhibits the intimate connection subsisting between rain-fall and wind. On that chart, the direction and area of the trade-wind are marked out as passing over the Mississippi delta from the S.E.; after reaching latitude 33°, the direction becomes more southerly, and further north changes gradually into S.W. in Illinois; it then turns west near the southern terminus of Lake Michigan, having completely changed its character into the anti-trade. The reader's attention may also be called to the S.E. winds blowing directly across the gulf coast of Texas. The system of winds on the Pacific coast of the United States is very imperfectly known; it appears, however, that the coast of Oregon and of Washington Territory is directly exposed to the westerly winds coming from the Pacific, and producing, on the immediate sea-coast, the profuse winter rains. The sudden increase in rain north of latitude 41°, from twenty to forty-four inches, near the Straits of Juan de Fuca, is due to the direct impinging on the coast-range of mountains, of the anti-trade, heavily charged with moisture; this wind is deflected, as well as the general line of the coast, south of latitude $40\frac{1}{2}$°, and hence blows nearly *parallel* with the coast. In consequence of this change in direction, the rains have diminished to ten inches and less when we reach the latitude of the Bay of Monterey, south of San Francisco.

These (extra-tropical) winter rains extend along our Pacific coast as far south as San Diego in striking contrast with the (intra-tropical) summer rains as experienced in our lower latitudes on the Atlantic coast.

The rain-charts for the extreme meteorological seasons give the hyetal curves for every even inch, but in a few exceptional cases on the Pacific slope an intermediate curve of an odd inch has been introduced.

It was not deemed of sufficient importance to construct rain-charts for the intermediate or transition seasons of spring and autumn, since the curves for these seasons must necessarily partake in a great measure of the characters of both extremes; if their construction should become desirable, the material is given, ready to hand, in the general table of the Rain-Fall.

The connection of the rain-fall with the direction of the wind and the configuration and topography of the country could be studied to better advantage if we possessed reliable charts of the resultant direction of the wind for *each* of the extreme seasons, and if a plain hypsometrical map, showing the form of the surface by *contour* lines drawn for given and equal intervals, could be made the basis of our rain-charts. The character of the surface, whether wooded, under culture, or arid, should also be indicated on such a map.

Respecting the general accuracy and reliability of the results exhibited by the rain-charts, the necessary information will be found further on in the discussion of the variations in the rain-fall. Here, however, we meet with a distinct hyetal

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[1] Smithsonian Contributions to Knowledge. Winds of the Northern Hemisphere, by J. H. Coffin, A.M. Washington, November, 1853.

region of rather an exceptional character. According to the statements of Dr. Gibbons and other meteorologists,[1] the most rainy point during the winter is in a direct line with the trend of the southern coast, or about S. S. E. This is nearly the direction of the great basin formed by the Sacramento and San Joaquin River, and (nearer to the coast) of the Bay of San Francisco and of the Valley of the Salinas River. These great hypsometrical features tend to keep up the direction of the rainy south-easterly winds, as high up as latitude $40\frac{1}{2}°$, beyond which limit a different state of relation between rain and wind prevails. The warm S. E. wind from the region of the Gulf of California and the western coast of Lower California deposits a very small amount of rain in the latitude of San Diego (33°), but parts with its moisture gradually and more freely on its way to the higher and colder latitudes. The chart is here very imperfect for want of material, and considering the configuration of the country, and the variation in the rain-fall, as influenced by height, much labor remains to be expended before a complete analysis and a perfect graphical representation of the distribution of rain can be made out. The west and northwest winds are prevalent during the dry season, and it would seem to be in accordance with facts that this season commences later and terminates earlier in the higher than in the lower latitudes.

THE ANNUAL FLUCTUATION IN THE RAIN-FALL.

In the preceding charts of the distribution of the rains in summer and winter we already have had striking examples of the changes in the amount of rain during these extreme seasons. We now propose to follow these changes from month to month, or to develop the annual fluctuation. The data for this investigation are furnished by the tables, series B, from which we select those stations where the rain-record extends over the longest series of years. It is only by the combination of results for a longer period that we can hope to exhibit this inequality with desirable clearness.

The following three diagrams for Marietta, San Francisco, and Boston are designed to show the *regularity* in the progression, from month to month, of the annual fluctuation; the *diversity* of distribution during the year in regard to number of maxima and minima as well as to their epochs of recurrence; and the *range* of the yearly variation.

To facilitate comparison all diagrams in this article are drawn to the same scale, excepting those plotted from ratios. The apparent irregularities in the curves can only be lessened by a series of observations extending over many years, and in the present discussion no station was admitted with less than fifteen years of record. To render the comparisons, as far as practicable, independent of these accidental irregularities, the mean monthly rain-fall was expressed analytically by means of Bessel's circular function.

$$R = A + B_1 \, sin \, (\theta + C_1) + B \, sin_2 \, (2\theta + C_2) + B_3 \, sin \, (3\theta + C_3) + \ldots .$$

where R = rain-fall in inches for any one month.

A = its mean monthly amount.

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[1] See an article "Winds and Rains of California, by H. S. Warner," in the Proceedings of the American Association for the Advancement of Science, Baltimore meeting, 1858.

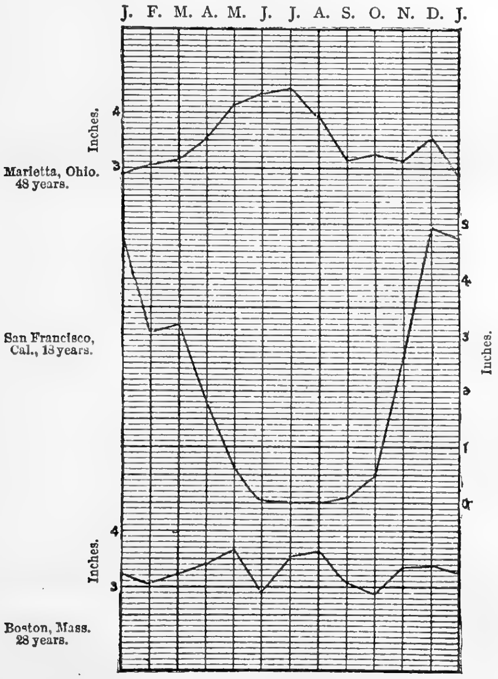
B_1 B_2 B_3 . . . the parameter of its fluctuations for the periods of one, one-half, and one-third of a year.

$\theta =$ arc proportional to the time and inversely proportional to the length of the period, which is represented by 360°. This angle consequently counts at the rate of 30° a month.

C_1 C_2 C_3 . . . angular constants having reference to the epochs of their respective periods.

Three terms (involving θ and its multiples) of this function are quite sufficient when applied to the fluctuations in the rain-fall.

Diagram 1.



The following table contains the computed values of A B_1 B_2 B_3 C_1 C_2 C_3 for all stations having fifteen or more years of record. The angle θ counts from January 1.

The probable error of any computed monthly value, as given in the last column, is derived from the approximate formula $0.845 \frac{\Sigma\Delta}{\sqrt{n(n-7)}}$ in which $\Sigma\Delta =$ sum of differences of observed and computed monthly values, all taken as positive. The condition $\Sigma\Delta = 0$ is satisfied; $n = 12$. Of the 12 values given by observation at any station, 7 are necessary for determining the constants.

Station.	Lat.	Long.	Height above sea.	Epochs.	Years of record.	A	B_1	B_2	B_3	C_1	C_2	C_3	Prob. error.
			feet.		years.	in.	in.	in.	in.	°	°	°	in.
Gardiner, Me.	44°10′	69°46′	76	1837–66	27	3.51	0.09	0.26	0.18	148	172	145	±0.30
Brunswick, Me.	43 54	69 57	74	1808–59	32	3.72	0.26	0.44	0.32	233	171	174	.40
Hanover, N. H.	43 42	72 17	530	1835–54	19	3.36	0.39	0.25	0.06	220	167	253	.24
Burlington, Vt.	44 29	73 11	346	1828–64	27	2.84	1.10	0.10	0.25	233	163	300	.24
Worcester, Mass.	42 16	71 49	528	1841–67	26	3.91	0.33	0.04	0.39	205	300	140	.34
Cambridge, Mass.	42 23	71 07	71	1784–1867	31	3.72	0.24	0.08	0.52	193	353	116	.24
Boston, Mass.	42 22	71 04	..	1818–59	28	3.28	0.08	0.07	0.26	292	91	142	.19
New Bedford, Mass.	41 39	70 56	90	1813–67	54	3.45	0.18	0.14	0.29	109	237	139	.21
Amherst, Mass.	42 22	72 34	267	1835–67	32	3.66	0.47	0.28	0.28	233	125	187	.29
Williamstown, Mass.	42 43	73 13	705	1816–67	19	3.05	0.88	0.11	0.28	234	66	161	.22
Providence, R. I.	41 50	71 23	150	1831–66	35	3.45	0.17	0.20	0.36	131	178	151	.18
New Haven, Conn.	41 18	72 55	50	1804–68	25	3.70	0.25	0.14	0.04	239	8	52	.28
East Hampton, N.Y.	40 58	72 28	16	1828–52	17	3.11	0.11	0.47	0.34	245	210	84	.17
Jamaica, N. Y.	40 42	73 50	30	1827–52	25	3.26	0.72	0.16	0.19	236	167	194	.13
Flatbush, N. Y.	40 37	74 02	54	1826–66	36	3.63	0.24	0.18	0.22	251	149	161	.31
Fort Hamilton, N.Y.	40 36	74 02	25	1839–59	19	3.55	0.38	0.25	0.33	298	108	123	.30
Fort Columbus, N.Y.	40 41	74 01	23	1836–59	24	3.60	0.42	0.25	0.43	254	146	112	.42
New York City, N. Y.	40 43	74 00	50	1836–66	31	3.72	0.32	0.27	0.28	281	132	115	.25
North Salem, N. Y.	41 20	73 38	361	1830–52	20	3.40	0.51	0.28	0.03	231	180	202	.42
Westpoint, N. Y.	41 24	73 57	167	1840–59	20	3.97	0.43	0.24	0.40	241	165	132	.44
Newburgh, N. Y.	41 31	74 05	150	1830–52	20	3.01	0.57	0.39	0.27	243	152	337	.40
Poughkeepsie, N. Y.	41 41	73 55	..	1830–49	15	3.36	0.51	0.28	0.24	234	116	139	.47
Kingston, N. Y.	41 55	74 02	188	1829–49	19	3.02	0.26	0.62	0.23	246	128	247	.30
Hudson, N. Y.	42 13	73 46	150	1830–52	15	2.88	0.34	0.36	0.42	245	152	265	.42
Kinderhook, N. Y.	42 22	73 43	125	1830–46	17	3.04	0.98	0.46	0.17	254	136	240	.26
Albany, N. Y.	42 39	73 44	130	1826–66	28	3.38	0.70	0.35	0.26	253	138	302	.11
Watervliet Ars. N. Y.	42 43	73 43	50	1836–54	17	2.89	0.73	0.30	0.09	252	172	341	.08
Lansingburg, N.Y.	42 47	73 40	30	1826–46	20	2.78	0.57	0.35	0.29	236	139	300	.20
Granville, N. Y.	43 20	73 17	250	1834–49	15	2.63	0.71	0.50	0.03	234	149	254	.20
Fairfield, N. Y.	43 05	74 55	1185	1828–49	17	3.04	0.88	0.36	0.12	241	104	302	.29
Clinton, N. Y.	43 00	75 20	1127	1830–52	19	2.82	0.56	0.17	0.37	227	125	310	.18
Utica, N. Y.	43 07	75 13	473	1826–52	22	3.43	0.68	0.49	0.23	250	105	218	.21
Lowville, N. Y.	43 46	75 32	847	1827–58	22	2.79	0.56	0.23	0.29	219	113	300	.09
Gouverneur, N. Y.	44 25	75 35	400	1830–66	24	2.51	0.41	0.08	0.35	191	215	332	.13
Potsdam, N. Y.	44 40	75 01	394	1828–48	20	2.39	1.24	0.18	0.30	239	154	318	.29
Cazenovia, N. Y.	42 55	75 46	1260	1830–65	25	3.35	0.76	0.23	0.14	249	149	286	.18
Oxford, N. Y.	42 28	75 32	961	1830–52	20	3.03	0.81	0.29	0.28	247	132	338	.07
Pompey, N. Y.	42 56	76 05	1300	1830–52	16	2.56	1.26	0.37	0.42	243	111	320	.12
Auburn, N. Y.	42 55	76 28	650	1827–49	22	2.87	0.61	0.29	0.22	227	150	7	.14
Ithaca, N. Y.	42 27	76 37	417	1830–52	19	2.89	0.78	0.42	0.10	250	199	322	.11
Geneva, N. Y.	42 53	77 02	567	1841–66	19	2.57	0.96	0.32	0.24	250	203	128	.32
Penn Yan, N. Y.	42 42	77 11	740	1829–67	39	2.37	0.91	0.27	0.10	253	178	344	.03
Rochester, N. Y.	43 08	77 51	516	1831–66	35	2.71	0.47	0.21	0.12	228	180	329	.14
Middlebury, N. Y.	42 49	78 10	800	1826–48	17	2.54	0.77	0.22	0.22	251	199	272	.03
Fort Niagara, N. Y.	43 15	79 08	262	1841–66	17	2.56	0.42	0.04	0.03	228	15	79	.24
Fredonia, N. Y.	42 26	79 24	710	1830–48	16	3.05	1.10	0.40	0.36	209	178	349	.23
Toronto, Up. Can.	43 39	79 23	342	1840–65	23	2.93	0.32	0.17	0.12	215	142	120	.30
Newark, N. J.	40 45	74 10	35	1843–66	23	3.74	0.17	0.07	0.40	222	141	124	.31
Lambertville. N. J.	40 23	74 56	96	1843–60	17	3.65	0.51	0.18	0.38	235	40	111	.31
Morrisville, Pa.	40 12	74 53	30	1825–66	31	3.64	0.35	0.33	0.15	243	181	172	.23
Philadelphia, Pa..	39 57	75 11	60	1824–67	43	3.67	0.45	0.15	0.22	260	110	136	.14
Gettysburg, Pa.	39 49	77 18	624	1839–65	25	3.24	0.27	0.26	0.23	287	151	113	.09
Baltimore, Md.	39 18	76 37	..	1818–59	28	3.53	0.32	0.05	0.29	257	267	168	.25
Fort McHenry, Md.	39 16	76 34	36	1836–59	23	3.43	0.19	0.14	0.42	235	189	153	.29
Washington, D. C.	38 54	77 03	110	1824–67	28	3.13	0.53	0.06	0.15	272	48	150	.22
Fortress Monroe, Va.	37 00	76 18	8	1836–59	19	3.92	0.95	0.66	0.54	232	60	136	.30
Charleston, S. C.	32 47	79 56	25	1738–1861	42	3.64	1.77	1.34	0.33	238	19	141	.43
Fort Moultrie, S. C.	32 46	79 51	25	1842–59	17	3.79	1.77	1.36	0.63	247	22	156	.58
Savannah, Ga.	32 05	81 05	42	1836–59	23	4.03	2.21	1.52	0.68	253	32	145	.60
Fort Brooke, Flo.	28 00	82 28	20	1840–58	17	4.47	3.42	2.57	1.04	247	37	206	.34
Key West, Flo.	24 34	81 48	13	1832–64	24	3.02	1.69	0.30	0.91	219	276	1	.52
Mt. Vernon Arsn. Ala.	31 12	88 02	200	1840–59	15	5.51	0.26	1.28	0.75	145	67	204	.58
New Orleans, La.	29 57	90 02	20	1833–61	23	4.25	0.74	0.93	0.60	248	58	181	.29
Baton Rouge, La.	30 26	91 18	41	1843–59	15	5.01	0.33	1.17	1.03	237	86	170	.41
Natchez, Miss.	31 34	91 25	264	1799–1866	18	4.46	0.70	0.43	0.39	44	44	124	.53
Vicksburg, Miss.	32 23	90 56	350	1840–67	16	4.11	0.76	0.56	0.34	58	83	144	.38
Springdale, Ken.	38 07	85 24	570	1841–66	24	4.05	0.43	0.51	0.36	324	120	178	.36
Richmond, Ind.	39 49	84 44	850	1852–66	15	3.64	0.49	0.28	0.44	250	211	116	.47
Cincinnati, Ohio	39 06	84 25	582	1835–66	31	3.74	0.51	0.30	0.17	279	123	164	.22
Portsmouth, Ohio	38 42	82 53	468	1830–65	26	3.19	0.37	0.44	0.11	310	120	178	.22

Stations.	Lat.	Long.	Height above sea.	Epochs.	Years of record.	A	B_1	B_2	B_3	C_1	C_2	C_3	Prob. error.
			feet.		years.	in.	in.	in.	in.	°	°	°	in.
Marietta, Ohio	39°25′	81°29′	580	1817–67	48	3.56	0.60	0.34	0.13	273	114	194	±0.14
Steubenville, Ohio	40 25	80 41	670	1830–67	37	3.46	0.51	0.12	0.15	266	132	144	.10
Alleghany Arsn. Pa.	40 32	80 02	704	1836–59	23	2.94	0.53	0.40	0.15	270	158	134	.24
Detroit, Mich.	42 20	82 58	580	1836–66	21	2.50	0.75	0.17	0.09	266	241	253	.31
Fort Brady, Mich.	46 30	84 43	600	1836–56	18	2.53	1.33	0.07	0.24	207	271	54	.32
Fort Ripley, Min.	46 19	94 19	1130	1850–67	17	2.09	1.65	0.25	0.21	257	97	331	.36
Fort Snelling, Min.	44 53	93 10	820	1836–67	22	2.15	1.57	0.13	0.19	258	129	80	.31
Milwaukee, Wis.	43 03	87 55	591	1841–66	23	2.53	0.92	0.30	0.07	257	177	69	.17
Dubuque, Io.	42 30	90 40	666	1851–66	15	2.79	1.32	0.20	0.45	255	161	338	.25
Muscatine, Io.	41 26	91 05	586	1846–66	19	3.57	1.27	0.32	0.26	264	247	115	.32
Fort Madison, Io.	40 37	91 28	600	1848–66	18	3.50	1.17	0.21	0.17	260	271	65	.20
Athens, Ill.	39 52	89 56	800	1843–58	16	3.30	1.16	0.74	0.33	291	159	28	.31
St. Louis Ars. Mo.	38 40	90 10	450	1836–56	19	3.55	1.35	0.79	0.41	291	162	268	.38
St. Louis, Mo.	38 37	90 16	481	1837–67	28	3.51	1.08	0.41	0.11	287	171	230	.30
Jefferson Barr'ks, Mo.	38 28	90 15	472	1840–62	21	3.41	1.20	0.44	0.19	267	140	270	.18
Ft. Leavenworth, Ka.	39 21	94 48	896	1836–67	31	2.64	1.74	0.39	0.19	257	121	351	.45
Ft. Gibson, Ind. T.	35 48	95 03	560	1836–57	20	3.03	0.69	0.91	0.23	290	201	356	.33
Ft. Smith, Ark.	35 23	94 29	460	1837–61	22	3.36	0.79	0.66	0.25	282	194	193	.30
Ft. Washita, Ind. T.	34 14	96 38	645	1843–60	16	3.17	0.94	0.68	0.38	279	195	351	.43
Washington, Ark.	33 44	93 41	660	1840–67	22	4.54	0.65	0.10	0.36	18	207	196	.28
San Francisco, Ca.	37 48	122 26	170	1850–68	18	1.81	2.37	0.59	0.52	70	87	142	.24
Sacramento, Ca.	38 35	121 28	82	1849–67	18	1.63	2.04	0.45	0.58	67	100	150	.39
Ft. Vancouver, W. T.	45 40	122 30	50	1849–67	16	3.24	2.46	1.08	0.39	81	125	131	.40
Ft. Steilacoom, W.T.	47 10	122 25	300	1849–67	16	3.66	2.96	0.45	0.24	82	136	154	.42
Sitka, Ala. T.	57 03	135 18	20	1847–67	16	6.95	3.07	1.25	0.56	146	274	25	.35

The observed and computed (by the above formulæ) values of monthly rain-fall at each of the 95 stations are shown graphically on the accompanying plates I. to V., the former by dots, the latter by continuous curves which are obtained by computing the rain-fall for periods of thirty days, or for each month.

The preceding results, by means of plates I to V, can be further generalized by combining into groups all stations exhibiting the same characteristics in their annual distribution of rain. For this purpose each observed monthly amount, at any station, was divided by the average monthly amount[1] at the station; we thus obtain the annual fluctuation in a manner which admits of more ready comparison.

The distinctive groups, as far as our material at present permits of their recognition, are the following:—

TYPE I.—*Atlantic sea-coast from Portland to Washington.*

Characteristics: Three nearly equal maxima, about the middle of May, August, and December, and one principal minimum about the beginning of February. The range between the extreme monthly values is small. The August maximum is generally the highest.

Month.	Gardiner, 27 years.	Brunswick, 32 years.	Worcester, 26 years.	Cambridge, 31 years.	Boston, 28 years.	New Bedford, 54 years.	Providence, 35 years.	Flatbush, 36 years.	Fort Hamilton, 19 years.	Fort Columbus, 24 years.	New York, 31 years.	Westpoint, 20 years.	Newark, 23 years.	Lambertville, 17 years.	Philadelphia, 43 years.	Baltimore, 28 years.	Fort McHenry, 23 years.	Washington, 28 years.	Mean of 18 Stations.
January	1.01	.87	.99	1.06	1.00	.97	.99	.94	.89	.84	.90	.88	.94	.88	.89	.79	.82	.91	.92
February	.82	.74	.83	.80	.94	.96	.84	.81	.96	.77	.88	.81	.89	.86	.81	.84	.77	.84	.84
March	1.02	1.00	.93	.96	.98	1.03	1.01	.98	.91	.91	.92	.82	.93	.88	.94	1.12	1.07	.92	.96
April	.97	.93	1.00	1.00	1.04	1.02	1.04	1.02	1.04	.93	.99	1.04	.98	.87	1.00	.95	1.01	1.13	1.00
May	1.11	1.22	1.04	1.04	1.10	1.06	1.01	1.06	1.24	1.31	1.24	1.25	1.17	1.18	1.09	1.11	1.07	1.10	1.13
June	.97	.99	.84	.86	.89	.82	.91	1.07	1.08	1.05	1.06	.88	.86	.93	1.13	.97	.97	1.06	.96
July	.91	.98	1.02	.92	1.08	.87	.89	1.01	1.01	.95	1.01	1.08	.98	1.12	1.08	1.10	.97	1.25	1.01
August	1.11	1.18	1.33	1.31	1.10	1.14	1.14	1.18	1.21	1.33	1.17	1.30	1.26	1.35	1.22	1.17	1.21	1.20	1.22
September	.82	.81	.91	1.06	.94	.95	.89	.88	.89	.93	.91	.83	.93	1.07	.99	1.01	.94	.91	.92
October	1.10	1.01	1.07	.92	.89	.98	.97	.99	.73	.92	.88	1.06	.96	.94	.91	.95	.97	1.01	.96
November	1.06	1.25	1.05	1.02	1.03	1.12	1.17	1.05	.98	.95	.97	1.03	1.01	.88	.95	.96	1.00	.83	1.02
December	1.10	1.04	.99	1.07	1.03	1.10	1.14	1.01	1.05	1.13	1.08	1.02	1.09	1.06	1.01	1.05	1.18	.85	1.06

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[1] The quantity called A in the preceding equations.

The ratios in the last column, which are derived from an aggregate of 525 years, are represented by dots on diagram 2. The heavy continuous curve is drawn among these dots with a free hand. Comparing the curves at different places we notice that any one of the three maxima may become the highest, while the principal minimum, with hardly an exception, falls into midwinter.

Type II.—*Hudson River Valley, Vermont, northern and western New York; from Newburg to Burlington, and from Hanover to Fredonia.*

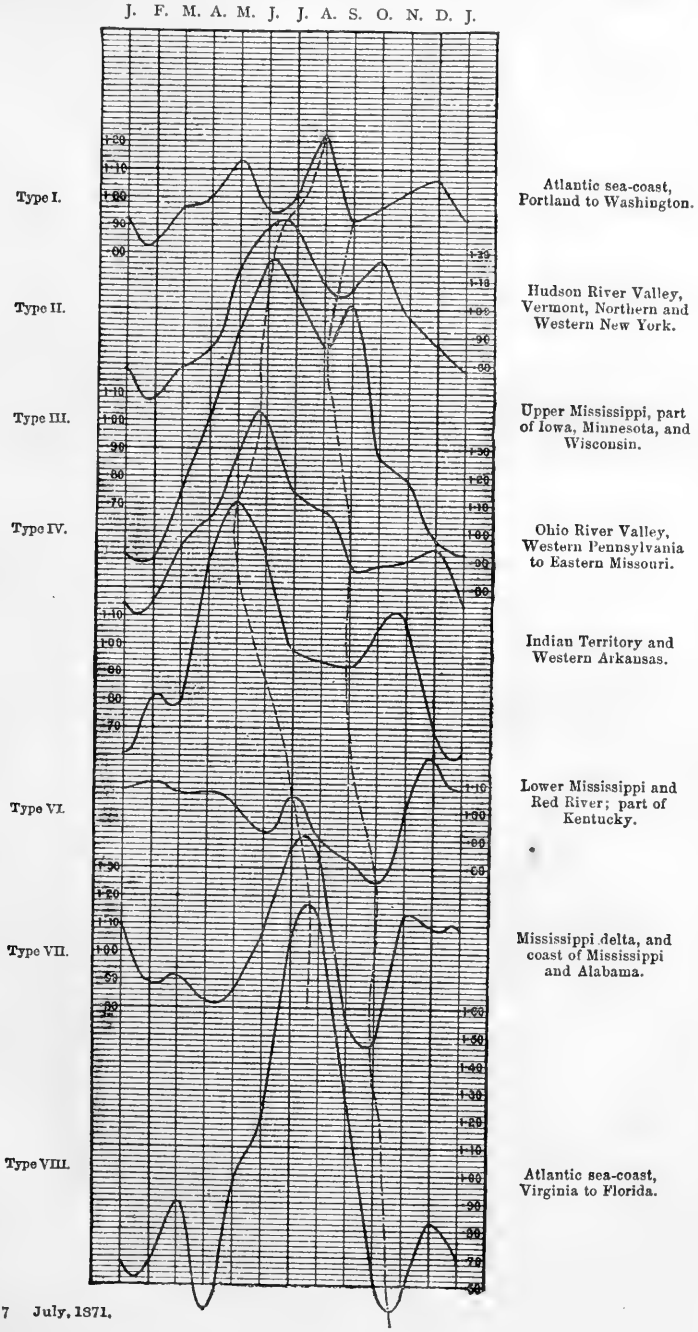
Characteristics: Two maxima, early in July and about middle of October, and a principal minimum early in February. Ordinary range between the extremes.

Month.	Newburg, 20 years.	Poughkeepsie, 15 years.	Kingston, 19 years.	Hudson, 15 years.	Kinderhook, 17 years.	Albany, 28 years.	Watervliet Arsenal, 17 years.	Lansingburg, 20 years.	Granville, 15 years.	Hanover, 19 years.	Burlington, 27 years.	Fairfield, 17 years.	Clinton, 19 years.	Utica, 22 years.
January	.91	1.00	1.02	.82	.73	.82	.72	.83	.78	.85	.62	.89	.80	.84
February	.72	.62	.69	.69	.50	.79	.72	.75	.55	.80	.65	.59	.89	.78
March	.85	.96	.95	1.12	.82	.82	.76	.78	.68	.89	.72	.78	.81	.79
April	.69	.86	.78	.79	.98	.91	1.01	.86	.79	.93	.74	.83	.73	.96
May	1.44	1.07	1.18	1.07	1.12	1.16	1.23	1.00	1.34	1.03	1.16	1.02	1.04	.97
June	1.15	1.09	1.21	1.25	1.50	1.31	1.29	1.41	1.21	1.16	1.19	1.41	1.25	1.35
July	1.21	1.22	1.29	1.27	1.43	1.29	1.22	1.28	1.34	1.01	1.54	1.39	1.30	1.38
August	1.04	1.31	.91	.91	1.10	1.00	1.07	.91	1.10	1.13	1.16	1.20	.92	1.06
Sept.	.96	.76	.75	.74	.97	.98	1.12	1.09	1.01	.94	1.27	1.01	1.17	1.02
October	1.18	1.17	1.03	1.34	1.07	1.10	1.04	1.15	1.10	1.19	1.31	1.17	1.21	.83
Nov.	1.05	1.00	1.13	.95	.89	.95	1.01	1.02	1.16	1.01	.92	.81	.96	1.05
Dec.	.82	.96	1.08	1.05	.91	.86	.81	.93	.94	1.05	.74	.90	.91	.98

Month.	Lowville, 22 years.	Governeur, 24 years.	Potsdam, 20 years.	Cazenovia, 25 years.	Oxford, 20 years.	Pompey, 16 years.	Auburn, 22 years.	Ithaca, 19 years.	Geneva, 19 years.	Penn Yan, 39 years.	Rochester, 35 years.	Middlebury, 17 years.	Fredonia, 16 years.	Mean of 27 Stations.
January	.85	.92	.59	.79	.82	.66	.87	.63	.62	.58	.79	.58	.67	.78
February	.91	.99	.44	.69	.73	.63	.71	.61	.44	.61	.81	.70	.60	.69
March	.71	.77	.62	.88	.75	.49	.74	.87	.70	.70	.80	.89	.65	.79
April	.77	.79	.71	.94	.83	.69	.78	1.04	1.27	1.01	.92	.97	.63	.86
May	1.00	.95	1.26	1.09	1.19	1.20	1.20	1.22	1.21	1.29	1.11	1.15	1.09	1.14
June	1.23	1.10	1.39	1.32	1.41	1.64	1.24	1.33	1.24	1.42	1.12	1.34	1.26	1.29
July	1.26	1.08	1.69	1.25	1.24	1.61	1.09	1.15	1.24	1.33	1.19	1.30	1.10	1.28
August	1.08	.88	1.18	1.10	1.10	1.25	1.13	1.03	1.41	1.19	.98	1.11	1.07	1.09
Sept.	1.04	1.28	1.30	1.04	1.11	1.15	1.12	1.18	1.06	1.16	1.14	1.11	1.46	1.07
October	1.18	1.29	1.40	1.13	1.11	1.26	1.18	1.16	1.19	1.10	1.20	1.13	1.41	1.17
Nov.	1.08	1.05	.81	.91	.90	.82	.99	.99	.88	.89	.99	1.01	1.07	.97
Dec.	.89	.92	.61	.88	.82	.61	.95	.81	.75	.71	.96	.70	.97	.87

This curve, derived on the aggregate from 564 years of observations, is presented on diagram 2. In a few exceptional cases, the second or autumn maximum surpasses the first or summer maximum in height, and the intermediate minimum approaches that of midwinter. Compared with Type I, curves like those at Amherst and Williamstown would seem to indicate that, in proceeding from the coast westwards, the May maximum gradually disappears, while the two remaining maxima occur nearly two months earlier.

DIAGRAM 2.—TYPE-CURVES OF THE ANNUAL FLUCTUATION IN RAIN FALL.



TYPE III.—*Upper Mississippi River, from Fort Madison, southern Iowa, to Fort Ripley, central Minnesota, and including part of Wisconsin.*

Characteristics: A principal maximum about the end of June, a secondary maximum about the middle of September, and a principal minimum about the beginning of February. Range, tolerably large.

Months.	Fort Ripley, 17 years.	Fort Snelling, 22 years.	Dubuque, 15 years.	Milwaukee, 23 years.	Muscatine, 19 years.	Fort Madison, 18 years.	Mean of 6 Stations.
January	.34	.41	.65	.62	.51	.57	.52
February	.30	.25	.53	.53	.69	.77	.51
March	.64	.58	.76	.78	.88	.86	.75
April	.77	1.11	.83	1.08	1.21	1.08	1.01
May	1.39	1.42	1.32	1.29	1.25	1.23	1.32
June	2.00	1.73	1.64	1.40	1.34	1.25	1.56
July	1.90	1.74	1.45	1.33	1.05	1.24	1.45
August	1.31	1.42	1.06	1.08	1.48	1.23	1.26
September	1.66	1.63	1.38	1.28	1.13	1.35	1.40
October	.68	.63	1.11	.93	.95	.91	.87
November	.68	.71	.76	.94	.81	.78	.78
December	.34	.39	.50	.76	.71	.71	.57

The curve is derived, on the aggregate, from 114 years of observation; it does not materially differ from that of Type II; the maximum in October appears now shifted into September, and the whole range of the variation is increased. Comparing the curves individually, there is a tendency to a single maximum, the earlier of the two; at one station they appeared to merge into one, and at another, the later maximum is higher than the earlier one. More observations are required to render this type more perfect.

TYPE IV.—*Ohio River Valley; from Alleghany Arsenal, western Pennsylvania, to St. Louis, eastern Missouri.*

Characteristics: One principal maximum, and one principal minimum; the former early in June, the latter early in February. Range, moderate.

Month.	Alleghany Arsenal, 23 years.	Steubenville, 37 years.	Marietta, 48 years.	Cincinnati, 31 years.	Portsmouth, 26 years.	Athens, 16 years.	St. Louis Arsenal, 19 years.	St. Louis, 28 years.	Jefferson Barracks, 21 years.	Mean of 9 Stations.
January	.70	.85	.82	.82	.90	.76	.56	.59	.67	.74
February	.73	.79	.86	.85	.93	.67	.63	.73	.63	.76
March	.88	.98	.90	.98	.97	.78	1.09	1.07	.88	.95
April	1.07	1.02	.99	.98	.97	1.25	1.14	1.10	.92	1.05
May	1.25	1.11	1.16	1.17	1.23	1.47	1.39	1.37	1.37	1.28
June	1.31	1.16	1.22	1.20	1.22	1.73	1.82	1.48	1.46	1.40
July	1.04	1.12	1.25	1.12	1.10	1.02	1.14	1.11	1.40	1.14
August	1.14	1.15	1.11	1.13	1.04	.90	.98	1.11	1.13	1.08
September	.90	1.01	.88	.90	.73	.99	.69	.88	.96	.88
October	.95	.92	.92	.88	.84	.76	.90	.87	.91	.88
November	.93	.91	.89	.91	.94	.84	.91	.83	.95	.90
December	1.09	.97	1.01	1.06	1.13	.84	.76	.86	.71	.94

The Type-curve IV is derived, on the aggregate, from 249 years of observation. The individual curves show considerable deviations in the time of the principal maximum and in the amount of the secondary maximum.

TYPE V.—*Indian Territory and Western Arkansas; Forts Gibson, Smith, and Washita.*

Characteristics: Two maxima, the principal one about the middle of May, the secondary one early in November, and a principal minimum early in January. Range tolerably moderate.

Month.	Fort Gibson, 20 years.	Fort Smith, 22 years.	Fort Washita, 16 years.	Means of 3 Stations.
January	.65	.62	.56	.61
February	.77	.80	.86	.81
March	.79	.78	.80	.79
April	1.40	1.44	1.07	1.30
May	1.52	1.34	1.68	1.51
June	1.32	1.23	1.37	1.31
July	.84	1.08	1.92	.98
August	.93	1.00	.90	.94
September	.78	.89	1.12	.93
October	1.29	.96	.93	1.06
November	1.01	1.08	1.11	1.07
December	.70	.79	.59	.69

TYPE VI.—*Lower Mississippi and Red Rivers; also Kentucky, Natchez, Vicksburg, Washington, Ark., and Springdale, Ken.*

Characteristics: One principal maximum and one principal minimum, the former early in December, the latter about the middle of October. Range small. Secondary maximum in July, secondary minimum in June.

Month.	Springdale, 24 years.	Washington, 22 years.	Vicksburg, 16 years.	Natchez, 18 years.	Means of 4 Stations.
January	.85	1.04	1.30	1.13	1.08
February	.90	1.10	1.13	1.26	1.10
March	1.08	1.13	1.06	1.00	1.07
April	1.04	1.20	.91	1.14	1.07
May	1.07	1.03	1.06	.94	1.02
June	1.26	.86	.79	.80	.93
July	1.05	1.04	1.02	1.08	1.05
August	1.01	.90	.88	.83	.90
September	.79	.75	.81	.99	.83
October	.76	.90	.63	.69	.75
November	.94	1.07	1.15	.88	1.01
December	1.25	.98	1.25	1.27	1.19

Type-curve VI depends on the aggregate of 80 years of observations. The individual curves show considerable variations in the earlier months of the year; after May the character of the type-curve is exhibited by each. This type probably extends over a large area, and is closely related to the following one.

Type VII.—*Mississippi, Delta, and Gulf Coast of Alabama and Mississippi; New Orleans, Baton Rouge, Mount Vernon Arsenal.*

Characteristics: Two maxima, the principal one about the end of July, the secondary one early in December, and a principal minimum early in October. Towards the end of April there occurs a secondary minimum. Range moderate.

Month.	New Orleans, 23 years.	Mt. Vernon Arsenal, 15 years.	Baton Rouge, 15 years.	Mean of 3 Stations.
January	1.06	1.21	1.01	1.09
February	.79	1.02	.84	.89
March	.91	.94	.87	.91
April	.86	.78	.85	.83
May	.90	.72	1.00	.87
June	1.06	1.12	1.01	1.06
July	1.57	1.20	1.35	1.37
August	1.35	1.31	1.34	1.33
September	.87	.68	.70	.75
October	.73	.72	.58	.68
November	.96	1.24	1.20	1.13
December	.96	1.06	1.25	1.09

The above type-curve depends upon a short series of observations, but the three curves agree remarkably well, excepting the non-appearance of a subordinate minimum at the Alabama station in February. This type is intermediate between Types VI and VIII.

Type VIII.—*Southeastern Coast of the United States. Virginia to Florida.*

Characteristics: One principal maximum, late in July or early in August, with two small adjacent minima, about the middle of April and late in October. The subordinate maxima occur in March and December. Range, very large.

Month.	Fortress Monroe, 19 years.	Charleston, 42 years.	Fort Moultrie, 17 years.	Savannah, 23 years.	Fort Brooke, 17 years.	Mean of 5 Stations.
January	.86	.65	.68	.72	.51	.68
February	.70	.73	.66	.63	.66	.68
March	.85	.90	1.10	.92	.72	.90
April	.76	.48	.46	.50	.38	.52
May	.99	.95	1.01	1.22	.68	.97
June	1.10	1.16	1.14	1.13	1.53	1.21
July	1.36	1.63	1.69	1.91	2.60	1.84
August	1.44	1.93	2.02	2.07	2.14	1.92
September	1.19	1.42	1.29	1.15	1.24	1.26
October	.74	.79	.56	.56	.49	.63
November	.84	.55	.56	.44	.43	.56
December	1.17	.81	.83	.74	.61	.83

The aggregate number of years used in forming the type-curve is 118. The individual curves exhibit well the characteristic summer maximum; the other features partake of more or less variations. That the central maximum of the three maxima of Type I corresponds to the principal maximum of Type VIII is very well shown by the curves for Fort McHenry, Washington, and Fortress Monroe on plate 3. The dividing line between Types I and VIII is intermediate between the two last-named stations and well defined.

TYPE IX.—*Western Coast of the United States, from the Bay of San Francisco to Puget's Sound.*

Characteristics: A most decided minimum during the summer months, amounting at some places almost to an absence of rain, and a well-marked maximum late in December. Range, excessive.

Month.	San Francisco, 18 years.	Sacramento, 18 years.	Fort Vancouver, 16 years.	Fort Steilacoom, 16 years.	Mean of 4 Stations.
January	2.60	2.15	1.91	1.88	2.14
February	1.71	1.63	1.15	1.37	1.46
March	1.78	1.97	1.21	1.34	1.58
April	.99	1.02	.76	.91	.92
May	.37	.54	.76	.48	.54
June	.03	.04	.57	.48	.28
July	.01	.02	.43	.13	.15
August	.01	.01	.16	.33	.13
September	.06	.05	.51	.65	.32
October	.28	.29	.74	.89	.55
November	1.44	1.31	1.59	1.68	1.50
December	2.72	2.99	2.22	1.88	2.45

The four curves on plate 5 are sufficiently characteristic to require no further illustration. This type appears to have no relation whatever to any of the preceding ones, excepting perhaps the existence of a maximum in December.

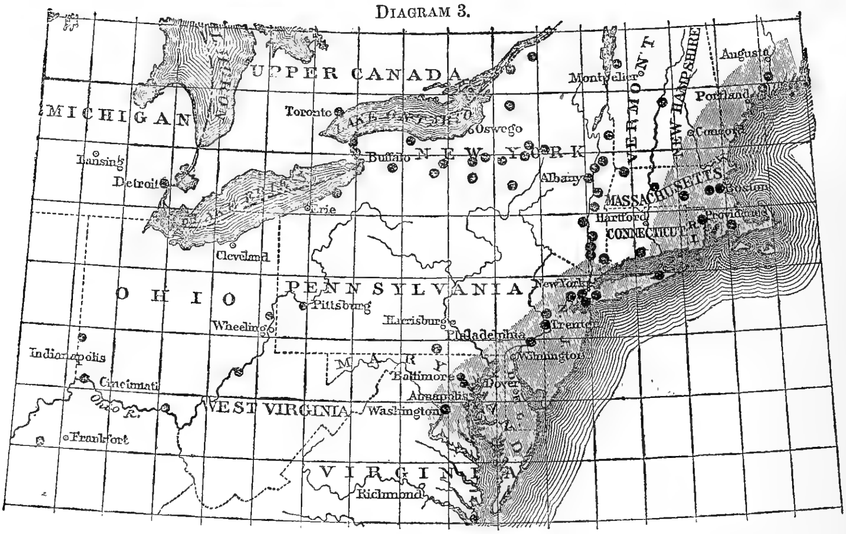
TYPE X.—*Alaska, as observed at Sitka.*

Characteristics: A minimum in June and a maximum in October. Range, excessive.

This curve is shown on plate 5; Types IX and X can not be connected for want of intermediate stations, but it seems probable that the maximum of IX in December is shifted to October in X. The minima of IX and X fall in the same season of the year.

The geographical distribution of the rain stations, from which the type-curves were derived, is very unequal. These stations are most numerous in the State of New York and in adjacent States; their location is shown by dots on the accompanying sketch, diagram 3; the strip of shaded area along the sea-coast exhibits the extent of the annual distribution having three maxima and one principal minimum, as given by Type-curve I. These annual fluctuations are so blended that an exact limit for any one type can not, in general, be assigned, though the above boundary for Type I appears quite distinct.

DIAGRAM 3.



The lines of dashes connecting the several type-curves on the preceding diagram (2) are an attempt at further generalization by showing the existence and connection of a principal maximum in summer, followed by a minimum in autumn. We can further trace out a secondary maximum late in the year, and a minimum, sometimes of a primary, sometimes of a secondary character, about the second month.

Large tracts of country are still destitute of stations of long series, but covered with a sufficient number of stations of short series to enable us to make out the character of the annual fluctuation. The following stations have been thus combined into groups:—

Group 1.—Central Pennsylvania. 10 stations: Bedford, Somerset, Fleming, Moss Grove, Butler, Huntingdon, Shamokin, Harrisburg, Carlisle Barracks, Canonsburg.

Group 2.—West Virginia and adjacent part of Virginia. 7 stations: Winchester, White Sulphur Springs, Ashland, Lewisburg, Sheetsmills, Alleghany County, Anthony's Creek.

Group 3.—Southeastern Virginia. 5 stations: Williamsburg, Crichton's store, Genito Mills, Smithfield, Powhatan Hill.

Group 4.—Western Florida. 4 stations: Fort Barrancas, Warrington Navy Yard, Pensacola, Barrancas Barracks.

Group 5.—Alabama and adjacent part of Mississippi. 4 stations: Huntsville, Greene Springs, Greensboro', Columbus.

Group 6.—Central Texas. 5 stations: Fort Belknap, Fort Chadbourn, Fort Mason, Fort McKavett, Fort Lancaster.

Group 7.—Southeastern Texas. 10 stations: Fort Brown, Ringgold Barracks, Fort McIntosh, Fort Duncan, Fort Inge, San Antonio, Fort Clark, New Braunfels, Matamoras.

Group 8.—Tennessee. 3 stations: Nashville, Glenwood, Memphis.

Group 9.—Kentucky. 4 stations: Danville, Millersburg, Chilesburg, Paris.

Group 10.—Ohio. 14 stations: Cleveland, Urbana, Bowling Green, Kelley's Island, Toledo, Granville, Norwalk, Savannah, Oberlin, Madison, Hudson, Welchfield, Westerville, Sandusky.

Group 11.—Southern Michigan. 5 stations: Holland, Grand Rapids, Battle Creek, Monroe City, Monroe.

Group 12.—Northern Michigan. 4 stations: Fort Mackinac, Thunder Bay Island, Tawas City, Marquette.

Group 13.—Indiana. 5 stations: New Harmony, Indianapolis, Logansport, Cannelton, Spiceland.

Group 14.—Illinois. 11 stations: Pekin, Manchester, Upper Alton, Augusta, Marengo, Ottawa, Peoria, Riley, Winnebago, Sandwich, Galesburg.

Group 15.—Wisconsin. 6 stations: Fort Howard, Fort Winnebago, Fort Crawford, Beloit College, Platteville, Rocky Run.

Group 16.—Iowa. 5 stations: Iowa City, Pleasant Plain, Lyons City, Davenport, Brookside.

Group 17.—Eastern Nebraska and Kansas. 4 stations: Fort Scott, Fort Riley, Bellevue, Manhattan.

Group 18.—Eastern New Mexico. 3 stations: Fort Marcy, Fort Union, Fort Stanton.

Group 19.—Western New Mexico. 4 stations: Fort Craig, Fort Fillmore, Albuquerque, Camp Burgwin.

The stations on the Pacific Slope are yet too scattered, and at too different elevations to admit of combination.

The results for each group are contained in the following tables. The monthly values are given in terms of the monthly average, as was done in the preceding tables of the type-curves.

Group No.		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Mean lat.		40.°6	38.°3	37.°0	30.°4	33.°4	31.°5	28.°2	35.°9	38.°2	41.°2	42.°4	45.°4	39.°3	40.°9	43.°3	41.°7	39.°3	35.°0	34.°3
Mean long.		78.4	80.2	77.3	87.3	87.7	99.9	98.8	88.0	84.3	82.5	84.7	84.7	86.4	89.5	89.6	91.2	95.8	105.5	106.4
No. of years.		76	41	37	19	37	32	82	28	25	109	34	34	29	99	50	33	40	28	36
Ratio of Annual Fluctuation.	*J.*	.82	.73	.98	.87	1.10	.42	.50	.99	.96	.72	.81	.93	.65	.70	.56	.50	.50	.31	.45
	F.	.68	.84	.81	.74	1.13	.70	.90	1.16	.92	.72	.75	.70	.76	.70	.49	.67	.45	.58	.47
	M.	.82	.92	1.04	.98	1.15	.47	.63	1.13	.95	.92	1.00	.86	1.12	.94	.63	.80	.76	.44	.45
	A.	1.18	1.10	.97	.92	1.05	.78	.65	1.18	1.09	1.08	.89	1.02	1.13	1.11	1.07	1.23	1.07	.39	.44
	M.	1.31	1.23	.99	.94	.91	1.38	1.12	1.14	1.14	1.10	1.08	.92	1.26	1.22	1.29	1.09	1.60	.37	.25
	J.	1.28	.94	1.06	.77	.97	1.34	1.60	1.02	1.05	1.25	1.09	1.08	1.02	1.11	1.51	1.18	1.91	1.23	.91
	J.	.99	1.34	1.12	1.61	.86	1.14	.94	.91	1.03	1.09	1.01	1.17	.93	1.35	1.63	1.40	1.53	2.40	2.07
	A.	1.09	1.10	1.59	1.69	1.07	1.32	1.09	.91	1.18	1.13	1.07	1.11	.97	1.15	1.30	1.42	1.17	2.98	2.72
	S.	.99	.86	.85	.71	.72	1.80	1.89	.77	.78	1.09	1.39	1.34	1.19	1.32	1.25	1.28	1.10	1.51	1.86
	O.	.89	.73	.83	.68	.51	1.22	1.16	.70	.64	.88	.94	1.01	.79	.85	.86	1.16	.73	.69	.94
	N.	.97	1.01	.72	1.10	1.13	.69	.79	.96	1.00	1.09	1.01	1.04	1.18	.72	.81	.60	.69	.60	.91
	D.	1.00	1.20	1.03	.99	1.40	.74	.72	1.12	1.25	.94	.97	.82	1.01	.83	.60	.66	.49	.52	.52
Average annual am't.		in 38.0	in 37.2	in 42.5	in 60.4	in 52.8	in 25.8	in 26.8	in 48.0	in 43.6	in 40.0	in 34.6	in 28.4	in 40.2	in 38.3	in 33.1	in 41.9	in 31.2	in 19.6	in 9.2

As the above numbers give the character of the annual fluctuation almost by inspection, but few remarks are required respecting these secondary results.

The rains in central Pennsylvania (group 1) partake of the combined characters of Types I and IV, as do also those in West Virginia (group 2); in southeastern Virginia (group 3) the rains are distributed according to Types VII and VIII; in western Florida (group 4) according to Type VII; in Alabama (group 5) according to Type VI, which modified gulf type is a very remarkable one, owing to its very low summer maximum. The curves for central and southeastern Texas (groups 6 and 7) are peculiar; they rise to two maxima, in June and September (the principal one), and fall to a principal minimum in January. The range is very large. The distribution in Tennessee (group 8) and in Kentucky (group 9) is a reproduction of Type VI; the Ohio (group 10) and Indiana (group 13) curves approximate more towards Type II, and Illinois (group 14) towards Type III. The curves for southern and northern Michigan (groups 11 and 12) are alike, and present a peculiar type, which may be called the lake-type, exhibiting one maximum in September, and a minimum in February, with a moderate range. The Wisconsin (group 15) and Iowa (group 16) curves approach Type III; also the Nebraska and Kansas (group 17) curve, the secondary September maximum, however, disappeared. The distribution of rain in New Mexico (groups 18 and 19) is similar to that given in Type VIII, with a high maximum in August and low minima in January and May.

By means of the preceding ratios of the annual distribution and the annual amount given on the chart by hyetal curves, the *normal amount* of rain for *every month* may readily be obtained for any desired locality.

Permanency of the Annual Fluctuation.—The combinations made in the preceding investigation imply that no sensible change has taken place in the law of the annual distribution within the period of observation; that such is really the case remains to be proved. The material collected which mainly refers to our own times, and hardly reaches back into the past century, is evidently insufficient for a full investigation, and obliges us to be satisfied, for the present, with a less complete proof; but it can be shown that the secular change, if any, in the annual distribution, must be very small. The best proof of the invariability of the annual fluctuation, during the past hundred years, is furnished by the observations at Charleston, S. C. between 1738–1759, and 1841–1861; these observations are represented by the equations:—

For the 22 years in the 18th century,

$$R = + 3.76 + 1.93 \sin(\theta + 239°) + 1.16 \sin(2\theta + 20°) + 0.22 \sin(3\theta + 130°,)$$

and for the 20 years in the present century,

$$R = + 3.49 + 1.59 \sin(\theta + 243°) + 1.19 \sin(2\theta + 14°) + 0.46 \sin(3\theta + 147°,)$$

which equations give nearly identical results. That the differences are within the probable error of observation becomes apparent by a comparison of the modern results among themselves by means of station Fort Moultrie in Charleston Harbor, where we have a 17 year series between 1842 and 1859. The expression for Fort Moultrie is

$$R = + 3.19 + 1.77 \sin(\theta + 247°) + 1.36 \sin(2\theta + 22°) + 0.63 \sin(3\theta + 156°.)$$

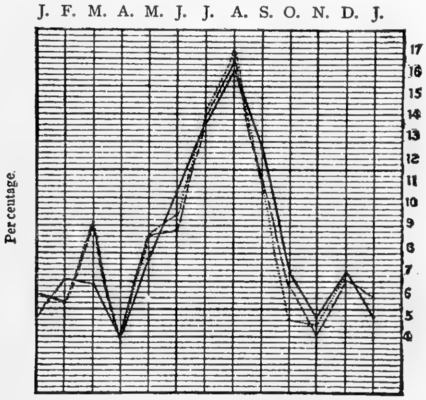
The following table gives the *monthly per centage* of the mean *annual* rain-fall obtained by dividing, in each series, the amount in each month by that of the year, and shifting the decimal point.

MONTHLY PER CENTAGE OF RAIN-FALL.

Month.	(*a*) 1738–59	(*b*) 1841–61	(*c*) 1842–59	Mean of *b* and *c*.
January	4.9	5.9	5.7	5.8
February	6.5	5.5	5.5	5.5
March	6.3	9.0	9.2	9.1
April	4.0	4.0	3.8	3.9
May	7.4	8.5	8.4	8.5
June	10.5	8.7	9.5	9.1
July	13.5	13.8	14.1	13.9
August	15.9	16.4	16.8	16.6
September	12.4	11.2	10.8	11.0
October	6.9	6.3	4.7	5.5
November	4.8	4.2	4.6	4.4
December	6.9	6.5	6.9	6.7

The numbers of columns (*a*), (*b*), and (*c*) are plotted on the accompanying diagram 4, the first by a full line, the second by dashes, and the third by dots. The accordance of these lines is sufficiently close to indicate no material change.

DIAGRAM 4.



In the following tables the monthly percentage of the annual rain-fall is given for successive periods for two of the longest series available, and for a third station selected on account of its geographical position.

Epoch.	Jan.	Feb.	March.	April.	May.	June.	July.	August.	Sept.	Oct.	Nov.	Dec.
New Bedford, Mass. 54 years, 1813–68.												
1813–19	6.6	10.8	8.7	10.1	9.7	8.2	3.9	9.2	10.4	6.0	10.5	6.0
1820–29	7.2	8.9	9.8	7.2	8.9	6.1	6.8	11.8	7.1	9.2	8.9	8.0
1830–39	9.0	6.6	7.8	8.0	9.6	7.7	7.5	9.8	7.2	8.7	8.3	9.8
1840–49	7.9	7.3	8.8	7.7	7.8	6.7	6.0	9.0	7.2	9.6	11.1	11.0
1850–59	9.0	7.1	7.0	11.2	8.4	5.6	9.8	7.6	9.4	6.3	8.6	9.9
1860–68	7.9	8.6	9.5	7.3	8.9	7.5	7.6	9.2	7.3	7.9	9.4	9.0
Marietta, Ohio. 48 years, 1818–67.												
1818–23	5.5	7.6	9.3	7.6	9.3	9.3	12.6	9.5	8.0	7.5	8.1	5.7
1826–29	6.3	11.1	7.9	10.7	7.3	10.5	9.8	7.7	5.7	6.9	6.2	9.8
1830–39	7.0	7.2	6.2	5.8	10.5	13.0	12.1	9.3	7.0	7.1	8.1	6.7
1840–49	6.8	5.1	7.7	6.9	8.9	11.5	10.1	9.0	8.4	8.5	6.7	10.4
1850–59	5.7	7.9	6.1	10.0	10.8	9.2	8.3	9.9	6.9	7.4	8.0	9.8
1860–67	9.2	6.6	8.7	9.9	9.6	7.2	10.0	9.1	7.3	8.0	6.9	7.5
Sacramento, Cal. 18 years, 1849–67.												
1849–55	12.5	12.1	24.6	10.9	3.1	0.2	0.0	0.0	0.9	1.4	10.7	23.8
1856–61	13.9	14.9	14.2	7.3	5.9	0.6	0.6	0.0	0.0	4.5	11.4	26.7
1862–67	29.2	13.9	8.3	6.3	4.9	0.0	0.0	0.0	0.0	1.1	13.6	22.7

It will be seen that in the above specimens of successive annual fluctuations, the incidental irregularities even for decennial means are yet far too great to allow a recognition of any secular change in the annual inequality, which, at any rate, must be quite small.

Causes of the Annual Fluctuation.—The subject of the annual fluctuation may be closed with a few remarks respecting the cause of the same. Of the various forms which we have seen the annual inequality to assume, there is for the eastern part of the United States an evident tendency of a maximum about summer and of a minimum about winter, while on the Pacific Slope the reverse of this obtains. In the first named locality two intermediate maxima, and particularly one minimum in autumn, combine in various degrees with the principal extremes to shape the annual fluctuation. The capacity of the atmosphere for rain is a maximum in summer, when it holds the greatest absolute amount of vapor, and a minimum in winter, when the vapor is least in amount; but the *relative* humidity, which is more directly related to rain, at these seasons is the reverse of the above distribution. The temperature *changes*, as measured by daily or monthly extremes, are greater in winter than in summer, and supposing the air saturated with moisture, any fall in temperature will precipitate rain or snow; these temperature changes are generally brought about by changes in the direction of the wind, and this is one of the causes of the annual fluctuation; its principal origin, however, must be sought in the *relative frequency* of those winds which are charged with the vapors derived directly from the surrounding oceans. We thus require, for the study of the rain-fall, a classification of the winds, as recorded at the time of rain or snow, together with the annual fluctuation of the wind for each given direction. To be of value such tables must extend over a long series of years.

At Marietta, Ohio, for which place I have discussed[1] the relative frequency and the relation of wind to rain, the results are in entire accordance with the rain charts; the summer rains are accompanied by S. W. and W. winds, and the winter rains by E. and S. E. winds. The relative frequency of the S. W. reaches a maximum in July, and that of the S. E. a minimum in December or January, in accordance with the rain curve at this place. A similar correspondence between the charts and the rainy directions can be traced at Brunswick, Me.;[2] here I found the rainy wind in winter to be the N. E., and in summer the S. E., also the S. W., and less frequently the N. E.

At Toronto, Canada West, the diagram of the observed annual fluctuation is of a remarkable zigzag character and of small range (see Plate 2, bottom); its most expressive feature is a minimum about the close of winter. The very elaborate tabulation of meteorological results at the Toronto Observatory,[3] by G. T. Kingston, Director, shows that the winds most accompanied by rain or snow are from the E. N. E., E., E. S. E. and S. E. in winter, and from the E. N. E., S. W., and W. S. W. in summer, as observed between 1853 and 1862. There are consequently two opposing rainy directions in summer. The annual variation in the relative frequency exhibits one or two maxima, and as many minima.

Distribution of Rainy Days in the Year.—The number of rainy days in a year was collected from the records, and tabulated to be available in the shape of averages for each month, and for various geographical positions. Less time was expended in this collection than in that of the amount for two reasons: First, it became evident that all observers are not agreed in the manner of recording days of precipitation. While some only note days of settled or continuous rains, others include all days of precipitation, even if hardly a measurable amount fell, and days, otherwise clear, on which thunder and rain storms occurred, lasting but a small fraction of an hour. Storms of the latter kind, for instance, frequently pass in summer, over this city from the westward, in the afternoon, with remarkable regularity, for a number of days in succession, and if the average temperature be increasing they will occur earlier each succeeding day. It is proper to make a distinction, by separate entries, between these two extreme cases, and thus the differences in records may be greatly reduced. In some records, where rainy and snowy days are given separately, the number of days of precipitation becomes doubtful when it rains and snows on the same day. In the first two Army Registers it would seem

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[1] Smithsonian Contributions to Knowledge (120). Results of Meteorological Observations made at Marietta, Ohio, between 1826 and 1859, by S. P. Hildreth, M. D., etc. etc. Washington, June, 1868.

[2] Smithsonian Contributions to Knowledge (204). Results of Meteorological Observations, made at Brunswick, Me., between 1807 and 1859, by P. Cleaveland, LL.D., etc. etc. Washington, June, 1867.

[3] Results of Meteorological Observations made at the Magnetical Observatory, Toronto, Canada, W., during the years 1854 to 1859 inclusive, also during the years 1860–1–2. Two volumes. Toronto, 1864.

to have been the practice, at some stations, to divide the sum of rainy hours by 24, to obtain the number of rainy days; it is often doubtful whether a bar — is to be interpreted "no record," or "no precipitation." Secondly, it was thought unnecessary to develop separately the laws to which the number of rainy days appear subject, since they are nearly identical with those of the amount, and generally but slightly depending upon the seasons. In my discussion of the rain-fall at Marietta,[1] Ohio, it is shown that the copiousness of precipitation is nearly the same throughout the year. The following table gives, for each month, the average quantity in any one day, as derived from thirty-two years of record.

Month.	No. of rainy days.	Average fall of rain (or snow).
January,	6.1	0.44 inches.
February,	5.9	.51 "
March,	6.7	.44 "
April,	7.5	.44 "
May,	8.4	.50 "
June,	9.2	.51 "
July,	8.3	.53 "
August,	7.6	.53 "
September,	6.2	.52 "
October,	6.8	.48 "
November,	6.4	.52 "
December,	6.5	.55 "

Dividing the average monthly amount by the average monthly number of rainy days, the quotient is nearly the same for the extreme sections; the *summer rains* are slightly *heavier* than the winter rains. The same is the case at Brunswick,[2] Me., as shown by the following table of daily averages (of rain and snow) copied from my discussion and derived from thirty-two years of record.

January	0.33	July	.51
February . . .	.34	August	.65
March	.42	September . . .	.51
April	.46	October	.54
May	.51	November . . .	.55
June	.51	December . . .	.42

The heaviest rains therefore fall in August, the lightest in January and February, epochs in which the air's capacity for vapor is greatest and least, respectively.

The following figures are copied from Tables IX and X of Prof. Caswell's Meteorological Summaries,[3] for Providence, R. I., to which I have added the ratios in the last column, derived from twenty-eight years of observation.

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[1] Results of Meteorological Observations made at Marietta, Ohio, between 1817 and 1859, by J. Wood and Dr. S. P. Hildreth. Smithsonian Contributions to Knowledge, No. 120. Washington, 1868.

[2] Results of Meteorological Observations made at Brunswick, Me., between 1807 and 1859, by Prof. P. Cleaveland. Smithsonian Contributions to Knowledge, No. 204. Washington, 1867.

[3] Meteorological Observations made at Providence, R. I., between 1831 and 1860, by Prof. A. Caswell. Smithsonian Contributions to Knowledge. Washington, 1860.

Month.	Average amount.	Average No. of rainy or snowy days.	Ratio.
	inches.		
January,	3.19	8.34	0.38
February,	2.76	8.00	.34
March,	3.20	7.07	.45
April,	3.56	8.86	.40
May,	3.42	8.45	.41
June,	3.04	8.59	.35
July,	3.00	7.90	.38
August,	4.05	9.41	.43
September,	2.93	6.63	.44
October,	3.39	6.63	.51
November,	3.88	7.27	.53
December,	3.93	9.43	.42

Here the autumn rains appear heavier than any others; those in mid-winter are the lightest.

The following figures are from Tables LXXXVII and LXXXVIII of Director Kingston's Results of Meteorological Observations made at the Magnetical Observatory at Toronto, Canada West (Toronto, 1864), derived from twenty-two years of record, ending with 1862.

Month.	Amount of rain.	No. of days of rain.	Ratio for rain.	Ratio for rain and snow.
	inches.			
January,	1.27	5	0.25	0.16
February,	1.03	4	.26	.18
March,	1.55	6	.26	.16
April,	2.39	9	.27	.22
May,	3.08	11	.28	.28
June,	2.96	12	.25	.25
July,	3.67	10	.37	.37
August,	3.03	10	.30	.30
September,	3.79	11	.34	.34
October,	2.53	12	.21	.19
November,	3.19	10	.31	.22
December,	1.64	5	.33	.17

Here, also, we note a slight increase in the average daily rain-fall in the warmer season, and a slight decrease in the colder season. The above examples, drawn from long records and from stations in different geographical relations, suffice to prove the above assertion as to the nearly uniform copiousness of average daily precipitation, with a slight excess in the warmer months, and a slight deficiency in the colder months, over the average amount. The last table also exhibits the comparatively light precipitation when in the form of snow.

The average number of days of precipitation, during a year, are given in the following table, arranged according to States and Territories. In compiling this Table only those records were admitted which were continuous during a year.

Locality.	Aggregate years.	Days.	Locality.	Aggregate years.	Days.
Maine	89	93	Kentucky	10	89
New Hampshire	15	76	Ohio	118	116
Vermont	26	89	Michigan	87	117
Massachusetts & Connecticut	26	98	Indiana and Illinois	10	107
Rhode Island	50	96	Wisconsin	48	89
New York	214	109	Minnesota	47	89
New Jersey	12	118	Iowa	19	98
Pennsylvania	93	119	Missouri	49	70
Delaware, Maryland, and District of Columbia	58	83	Indian Territory	60	73
Virginia	37	85	Kansas	43	77
North and South Carolina	52	89	Nebraska	10	75
Georgia	18	83	Wyoming	8	72
Florida	77	91	New Mexico[1]	47	56
Alabama	17	98	Arizona	8	75
Mississippi and Louisiana	50	92	California[2]	47	50
Texas	111	58	Oregon[3]	16	131
Arkansas	15	75	Washington Territory[3]	19	132
			Alaska	14	235

At Toronto, Canada, the number of rainy and snowy days averages 162 in a year, as derived from 23 years of record; at Hamilton, Canada, ten years of record gave 122 days.

For Colorado, Dakota, and Utah, from three, four, and one year of record, we have 51, 72, and 102 days of precipitation, respectively.

From the above table, and by means of the ratios of amount for each month, compared with the mean amount, we can readily deduce the probable number of rainy days in any given month; thus, for the month of May, in the State of New York, for instance, we have the average number of rainy days in a month $= \frac{109}{12}$ by preceding table, and the monthly ratio 1.14 from Table, Type II, hence, number of rainy days to be expected in May $= 1.14$ times $\frac{109}{12} = 10$.

The aggregate number of years upon which the average numbers for rainy and snowy days of the preceding table depend is 1620.

SECULAR CHANGE OF THE RAIN-FALL.

The question whether the annual rain-fall is gradually increasing or diminishing, stationary, of a periodic character, or apparently irregular, is one of great interest, scientifically as well as practically. The preceding table B has been specially prepared for the investigation of these changes of the annual amount of rain, and if these records should not be of sufficient extent, they will at least form a basis for

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[1] Average very uncertain, the precipitation depending too much upon the *elevation* and other circumstances. Number of rainy and snowy days vary between 31 and 93.

[2] The extremes, Fort Yuma 11 days, and Fort Humboldt 82 days, also Fort Crook 83 days, are not included in the tabular mean. Average hardly admissible without further specification, as in the case of New Mexico.

[3] Averages uncertain without specification.

future investigators. This table is derived from table A, and contains all stations where records extend over one or more years; the resulting mean values given at the bottom of each column are taken from the monthly mean sums of table A, and will therefore coincide with the means of the annual values of table B, in case the observations suffered no interruption. In order not to lose observations extending over nine or more months in any year, the values for the remaining months were interpolated, either by substituting the average values at those months, as resulting from the whole series, or by substituting, when practicable, amounts observed at an adjacent station. Such interpolated annual amounts have less weight, and are distinguished by an asterisk; they may be omitted, if desirable, in rigorous investigations. The fact that comparatively few of the records, extending over a number of years, at any station, are free from occasional interruptions, greatly increases the labor of reduction, and seriously interferes with the value of the whole record. The difficulty in attempting to interpolate any monthly amount, even within the limits of a city, will appear in the following sample of comparisons of annual amounts, as given by three observers at different stations in San Francisco:—

1853.	By Assist. Surg. U. S. A.	18.70 inches.
	“ Mr. Walthall and Dr. Gibbons	19.70 “
1854.	“ Assist. Surg. U. S. A.	17.00 “
	“ Mr. Walthall and Dr. Gibbons	22.12 “
1855.	“ Assist. Surg. U. S. A.	19.26 “
	“ Mr. Tennent	26.39 “
1856.	“ Assist. Surg. U. S. A.	14.40 “
	“ Mr. Tennent	22.31 “
1857.	By Assist. Surg. U. S. A.	16.04 inches.
	“ Dr. Ayres	12.97 “
	“ Mr. Tennent	20.96 “
1858.	“ Assist. Surg. U. S. A.	15.95 “
	“ Dr. Ayres	17.76 “
	“ Mr. Tennent	23.46 “
1859.	“ Assist. Surg. U. S. A.	16.34 “
	“ Dr. Ayres	25.41 “
	“ Mr. Tennent	21.39 “
1860.	“ Assist. Surg. U. S. A.	20.19 “
	“ Mr. Tennent	21.18 “

From these comparisons the great diversity in the corresponding monthly amounts may be inferred. Quite a remarkable case of an extraordinary fall of rain, which occurred at Charleston, S. C., in August, 1859, with a sharply defined boundary, is recorded in the statistical report on the sickness and mortality in the army of the United States, between 1855 and 1860, by Brevet Brigadier-General T. Lawson, page 504. "If *two inches* could fall during *one and half an hour* in one part of the city (at the U. S. Arsenal), while but 0.35 fell at the same time about *two miles* distant (City Registrar, Tradd Street), it may be readily believed that even if only *six and three-quarter inches* fell in Charleston during thirty hours, 16.45 inches, or even twice that quantity, may have fallen in forty-seven and one-half hours at Sullivan's Island, which is *seven miles* distant."

If the rain-fall is recorded by more than one observer in any place, for the same month or year, the mean has been taken and inserted in Table B.

Before making use of this table with a view of a study of the permanency in the amount of the rain-fall during a long series of years, it will be desirable to inquire into the degree of uncertainty, or the probable error, of the annual sums and the mean amount there given. A cursory examination of the table reveals the fact of a limiting variation in the annual amount at any place to the extent of nearly one-half and of one and a half of the mean quantity, which itself, however, is subject to a considerable uncertainty even when derived from a long series. Supposing the changes in the yearly amounts to be due to accidental circumstances, and if subject to periodic changes, that these be small and extend over a comparatively long time, we may apply without sensible error the formulæ

$$e = 0.845 \frac{\Sigma\Delta}{\sqrt{n^2 - n}} \text{ and } r = \frac{e}{\sqrt{n}}$$

where e the probable error of any annual amount and r that of the mean of the whole series of n years; $\Sigma\Delta$ equals the sum of the annual differences from the mean irrespective of sign.

Applying these formulæ to the Marietta, Ohio, results we find the probable error of any observed annual amount $= \pm 4.1$ inches, of the mean amount from forty-eight years of observation $= \pm 0.59$ inches, and it would take a hundred years of record to reduce the probable error of the mean to 0.4 of an inch, as shown in the following table:—

Probable Error in Mean Annual Amount of Rain-fall, at Marietta, Ohio.

Deduced from	1 year of	observation	. . . . .	± 4.1
" "	5 years	"	. . . . .	1.8
" "	10 "	"	. . . . .	1.3
" "	20 "	"	. . . . .	0.9
" "	30 "	"	. . . . .	0.7
" "	40 "	"	. . . . .	0.6
" "	50 "	"	. . . . .	0.5
" "	100 "	"	. . . . .	0.4

Supposing these numbers to express the average variability in the rain-fall of a country, the successive curves on a rain chart showing the annual distribution should not be for a less difference than about three inches, even if we employ results from stations occupied for at least five years, otherwise the graphical representation becomes confused and expresses temporary instead of permanent features. In accordance with the above principle, the proper selection of curves, as constructed on each of the three accompanying rain-charts, was readily determined.

The following table contains, for a number of selected stations in different parts of the country and of long record, the probable error e of the amount of rain fallen in any one year, also the ratio of this quantity to the average yearly amount, or $\frac{e}{R}$. These fractions admit of comparison for different localities, and are an index of the greater or less variability in the rain-fall from year to year. The last column gives the probable error r, or the amount of uncertainty which attaches to the resulting average rain-fall as made out from the present record.

Station.	Location.	No. of years of record.	e	$\frac{e}{R}$	r
			inches.		inches.
Gardiner,	Maine,	27	±3.9	±.09	±0.8
Brunswick,	Maine,	31	7.6	.17	1.4
Hanover,	New Hampshire,	19	4.3	.11	1.0
Burlington,	Vermont,	27	4.3	.13	1.3
Worcester,	Massachusetts,	25	5.5	.12	1.1
Boston,	Massachusetts,	41	6.1	.14	0.9
New Bedford,	Massachusetts,	54	4.5	.11	0.6
Amherst,	Massachusetts,	32	3.8	.09	0.7
Providence,	Rhode Island,	36	4.1	.10	0.7
Jamaica,	New York,	23	4.4	.12	0.9
Flatbush,	New York,	32	4.8	.11	0.8
New York,[1]	New York,	30	7.6	.17	1.4
Newburg,	New York,	22	5.5	.15	1.2
Albany,	New York,	28	3.5	.09	0.7
Penn Yan,	New York,	39	3.1	.11	0.5
Rochester,	New York,	35	3.0	.09	0.5
Toronto,	Canada West,	26	3.7	.10	0.7
Morrisville,	Pennsylvania,	37	3.7	.09	0.6
Philadelphia,	Pennsylvania,	43	4.1	.09	0.6
Gettysburg,	Pennsylvania,	24	5.3	.14	1.1
Baltimore,[2]	Maryland,	30	6.6	.16	1.2
Washington,	Dist. Columbia,	30	5.3	.14	1.0
Fortress Monroe,	Virginia,	23	10.7	.23	2.2
Charleston,	South Carolina,	42	4.4	.12	0.8
Savannah,	Georgia,	23	7.5	.15	1.6
Key West,	Florida,	19	5.9	.16	1.4
New Orleans,	Louisiana,	20	4.5	.09	1.0
Fort Brown,[3]	Texas,	11	8.5	.26	2.6
Springdale,	Kentucky,	23	5.5	.11	1.2
Cincinnati,	Ohio,	28	6.2	.14	1.2
Portsmouth,	Ohio,	27	5.2	.14	1.0
Marietta,	Ohio,	48	4.1	.10	0.6
Steubenville,	Ohio,	37	5.6	.14	0.9
Detroit,	Michigan,	22	3.8	.12	0.8
Fort Snelling,	Minnesota,	21	3.0	.12	0.7
Milwaukee,	Wisconsin,	23	3.5	.12	0.7
Muscatine,	Iowa,	18	9.0	.21	2.1
St. Louis,	Missouri,	31	5.5	.13	2.0
Fort Leavenworth,	Kansas,	30	6.8	.21	1.2
Fort Gibson,	Indian Territory,	20	6.4	.18	1.4
Washington,	Arkansas,	21	7.0	.13	1.5
Fort Laramie,	Wyoming,	17	6.7	.45	1.9
Fort Kearny,	Nebraska,	14	4.2	.17	1.1
Fort Marcy,	New Mexico,	12	5.6	.33	1.6
San Francisco,	California,	17	4.1	.19	1.0
Fort Vancouver,	Washington Ter.	16	6.7	.18	1.7
Fort Steilacoom,	Washington Ter.	15	8.7	.19	2.3
Sitka,	Alaska,	14	6.7	.08	1.8

The numbers in the column headed $\frac{e}{R}$ do not indicate, upon the whole, any very great variation from the *average irregularity* in the rain-fall as observed from year

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[1] Fort Columbus, Institution of Deaf and Dumb.

[2] Baltimore City, Fort McHenry.

[3] Fort Brown and Matamoras.

to year; in the New England States and along the sea-board as far south as Washington, the probable uncertainty in *any one annual amount* is 12 per cent., and the same holds good for the Ohio and eastern part of the Mississippi Valleys, and in the region of the Lakes; along our southern Atlantic coast, and along the Gulf coast, the probable error is 17 per cent; west of the Mississippi it is 23 per cent.; in California, Oregon, and Washington Territory, it is 19 per cent.; and at Sitka, Alaska, but 8 per cent. of the annual amount.

The figures in the last column of the above table vary from about $\pm\frac{1}{2}$ to about $\pm 2\frac{1}{2}$ inches, and on the average indicate a probable error of $\pm 1\frac{1}{4}$ inch in the resulting annual amount from the longer series. In the construction of the annual hyetal curves on the accompanying chart, results from series as short as four years were admitted, and upon the average these curves must be considered as being liable to a probable error of ±2 or 3 inches, less or more, according to location, as indicated by the ratio $\frac{e}{R}$.

The probable uncertainty of the hyetal curves of the extreme seasons may be estimated at $\frac{1}{3}$ of that given above.

Returning to the consideration of a secular change in the annual amounts, we might, for limited localities where the rain-fall is nearly the same, examine for every year the difference from the mean amount; but for our purpose it becomes necessary to employ the *ratio* of each annual to the mean amount in order to render the results in different localities strictly comparable.

The following table contains these ratios for a number of stations selected on account of their extent of record.

	Maine.		N.H	Vt.	Massachusetts.								R.I.	Con.	New York.		
	Gardiner.	Brunswick.	Hanover.	Burlington.	New Bedford.	Cambridge.	Boston.	Amherst.	Worcester.	Waltham.	Lake Cochituate.	Lowell.	Providence.	New Haven.	Flatbush.	East Hampton.	North Salem.
1800	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	...	...	...	....	...	...	...	...	...	...	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	...	...	...	...	...	.98	...	...	...
5	...	...	...	...	...	...	...	...	...	...	...	...	...	.92	...	...	...
6	...	...	...	...	...	...	...	...	...	...	...	...	...	.87	...	...	...
7	...	...	...	...	...	...	...	...	...	...	...	...	...	1.02	...	...	...
8	...	1.19	...	...	...	...	...	...	...	...	...	...	...	1.11	...	...	...
9	...	.95	...	...	...	...	...	...	...	...	...	...	...	1.00	...	...	...
1810	...	.63	...	...	...	...	...	...	...	...	...	...	...	.89	...	...	...
1	...	.86	...	...	...	...	...	...	...	...	...	...	...	1.07	...	...	...
2	...	.91	...	...	...	...	...	...	...	...	...	...	...	1.00	...	...	...
3	...	.84	...	...	...	...	...	...	...	...	...	...	...	1.20	...	...	...
4	...	1.08	...	...	.93	...	...	...	...	...	...	...	...	1.26	...	...	...
5	...	.61	...	...	.88	...	...	...	...	...	...	...	...	1.14	...	...	...
6	...	.73	...	...	.95	...	...	...	...	...	...	...	...	.86	...	...	...
7	...	.88	...	...	.93	...	...	...	...	...	...	...	...	.98	...	...	...
8	...	.59	...	...	.87	...	.95	...	...	...	...	...	...	.86	...	...	...
9	...	...	...	...	.85	...	.79	...	...	...	...	...	...	.76	...	...	...
1820	...	...	...	...	.89	...	.98	...	...	...	...	...	...	1.04	...	...	...
1	...	...	...	...	.98	...	.82	...	...	...	...	...	...	1.00	...	...	...
2	...	...	...	...	.90	...	.61	...	...	...	...	...	...	...	...	...	...
3	...	...	...	...	1.29	...	1.03	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	1.02	...	.80	...	...	...	...	...	...	...	...	...	...
5	...	...	...	...	.82	...	.72	...	...	...	...	...	...	...	...	...	...
6	...	...	...	...	1.18	...	.93	...	...	...	...	...	...	...	...	...	...
7	...	...	...	...	1.35	...	.99	...	...	...	...	...	...	1.16	...	...	...
8	...	...	...	1.27	.84	...	.75	...	...	...	...	...	...	1.18	...	.83	...
9	...	...	...	...	1.40	...	1.07	...	...	...	...	...	...	...	...	1.14	...
1830	...	...	...	...	1.39	...	.99	...	...	...	...	...	...	...	1.23	1.25	1.06
1	...	...	...	...	1.31	...	1.13	...	...	...	...	...	...	...	.98	1.03	1.05
2	...	...	...	1.16	1.06	...	1.04	...	...	...	...	...	.95	...	.99	1.12	1.00
3	...	...	...	1.45	.92	...	.88	...	...	...	...	...	.82	...	1.07	.97	1.06
4	...	...	...	...	.97	...	.85	...	...	...	...	...	1.01	...	.91	1.08	.82
5	...	...	.79	...	1.01	...	.79	...	...	...	...	...	.75	...	.88	.82	.86
6	...	...	.94	...	.92	...	.80	.93	...	...	...	...	.91	...	1.01	.95	...
7	.77	...	.88	...	.84	...	.67	.88	...	...	...	...	.76	...	.80	.94	...
8	.72	...	1.01	.90	.82	...	.70	.91	...	...	...	...	.91	...	.95	.75	.76
9	.97	1.21	.80	.82	.95	...	.77	.98	...	...	...	...	.89	...	.99	1.24	...
1840	.98	.81	1.12	1.09	1.07	...	...	1.07	...	...	...	...	.99	...	.83	1.16	.92
1	.91	1.08	.95	.96	1.09	.88	...	.95	1.09	...	...	...	1.15	...	1.20	1.17	.98
2	.91	1.23	1.19	.99	.84	.88	...	.87	.87	...	...	...	.91	...	1.08	.89	1.12
3	1.10	1.50	1.39	.78	1.09	1.13	...	1.17	1.04	...	...	...	1.03	...	1.15	.79	1.23
4	.91	1.13	1.19	.92	.88	.71	...	.92	.80	...	...	...	.84	...	.90	...	.86
5	.93	1.69?	.93	1.06	1.03	1.09	...	.90	.91	...	...	...	1.04	...	.74	...	.97
6	.84	.97	.92	.87	.74	.66	...	.80	.74	...	...	...	.74	...	1.01	...	1.12
7	1.07	1.37	1.14	1.13	.99	1.04	...	1.09	1.05	...	...	...	1.17	...	1.06	...	1.10
8	1.16	1.33	1.20	.92	.88	.93	...	.94	.82	...	...	...	.98	...	.77	...	.87
9	.94	.88	.95	.77	.77	.88	.90	.88	.80	.94	...	1.15	.84	...	.75	...	1.05
1850	1.22	1.29	1.00	1.10	1.25	1.17	1.20	1.26	1.18	1.43	...	1.03	1.24	...	1.12	...	1.36
1	1.21	1.06	1.03	.93	1.20	.91	.99	.99	.93	.94	...	.92	1.05	...	.94	...	.87
2	1.11	1.24	.89	.85	1.00	.87	1.06	.99	1.32	.97	.90	.96	.93	...	1.09	...	.96
3	...	...	...	.97	.85	1.16	1.08	1.16	1.27	1.04	1.10	.99	1.28	...	...	...	...
4	...	1.03	.86	.74	1.16	.97	1.02	1.08	1.26	.95	.85	.95	1.12	...	...	...	...
5	.93	1.00	...	1.1[illegible]	.88	1.03	.98	1.10	1.[illegible]5	.94	.69	1.05	.94	...	...	...	...
6	.90	.73	...	.79	.80	1.16	1.16	.92	1.00	.97	80	1.00	.99	...	.92	...	...
7	1.10	.96	...	.84	.93	1.25	1.26	1.09	1.14	1.01	1.24	1.14	1.08	...	1.14	...	...
8	1.12	1.02	...	...	1.02	.98	1.17	.97	.87	.86	.96	.83	1.07	...	.99	...	...
9	1.14	1.08	...	...	1.18	1.28	1.26	1.17	1.04	1.12	.97	1.08	1.09	...	1.35	...	...
1860	...	...	...	...	.93	.97	1.14	.90	1 04	...	1.09	1.05	.92	...	...	...	...
1	.95	...	...	1.25	1.08	1.08	1.11	1.07	.90	...	.91	.97	1.06	...	1.09	...	...
2	.93	...	...	1.20	1.00	1.23	1.36	1.13	.94	...	.98	1.00	1.21	...	...	...	...
3	1.18	...	...	1.16	1.07	1.22	1.50	1.24	...	1.23	1.37	1.24	1.30	...	.76	...	...
4	.97	...	...	1.05	.95	.85	1.10	.77	...	.84	.84	.89	.89	...	.77	...	...
5	.99	...	...	...	1.08	.94	1.06	.98	.82	.82	.97	.86	1.03	.94	1.09	...	...
6	1.09	...	...	...	.93	.76	1.13	1.03	.90	1.00	1.23	.89	1.06	1.06	.80	...	...
1867	...	...	...	...	1.10	.90	1.24	1.05	1.04	.95	1.11	1.03	1.13	1.02	...	...	...

	New York.—Continued.													**Canada.**		**N. J.**	
	Jamaica.	Newburgh.	Albany.	Utica.	Pompey.	Penn Yan.	Rochester.	Fort Hamilton.	Ft. Columbus & Inst. for D. &D.; N. York.	Westpoint.	Watervliet Arsenal.	Fort Niagara.	Gouverneur.	Montreal.	Toronto.	Newark.	Lambertville.
1800	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1810	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1820	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	...	...	.82	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	...	...	1.23	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	1.25	...	.93	.89	...	...	...	...	...	...	...	...	...	...	...	...	...
9	1.17	...	.94	.88	...	1.14	...	...	...	...	...	...	...	...	...	...	...
1830	1.11	.95	1.03	1.12	1.03	1.06	...	...	...	...	...	...	...	...	...	...	...
1	.97	...	.98	.92	.96	1.05	1.06	...	...	...	...	...	...	1.12	...	...	...
2	1.00	1.18	1.10	1.21	.86	1.08	.97	...	...	...	...	...	...	.87	...	...	...
3	.94	1.13	1.03	.92	.98	.91	.93	...	...	...	...	...	...	1.14	...	...	...
4	.87	.80	.80	.84	...	.79	.70	...	...	...	...	...	...	.68	...	...	...
5	.74	.69	1.00	.94	1.08	.93	1.00	...	...	...	...	...	...	1.08	...	...	...
6	.94	.83	1.10	.81	.76	.98	...	...	.64	...	1.09	...	...	.79	...	...	...
7	.82	.69	1.03	.67	.99	.86	.95	...	1.52?	...	.92	...	...	.66	...	...	...
8	.86	1.32	1.04	...	.75	1.11	.79	...	.97	...	.89	...	.65	.67	...	...	.87
9	.85	1.22	.94	.86	.83	1.10	.92	1.20	.99	...	.86	...	.78	.59	...	...	1.00
1840	.91	1.06	1.08	1.11	1.06	1.04	.91	...	.69	1.00	.93	...	...	.92	.91	...	.95
1	1.14	...	.94	1.02	.83	.85	.99	...	.97	1.13	1.03	...	.75	...	1.06	...	1.30
2	1.06	.84	1.12	1.38	.97	1.11	1.02	...	.79	1.06	1.30	1.61	.65?	...	1.24	...	.95
3	.86	.92	1.19	.99	1.02	1.03	.93	1.04	.96	1.06	1.21	1.20	.52?	...	1.43	...	1.17
4	1.14	.87	.80	.87	...	.71	.80	.77	.84	1.02	...	.61	.85	...	.95	.90	.92
5	.86	.83	1.00	.98	...	.86	1.06	.82	.79	.91	1.06	.67	1.11	...	.79	.81	.97
6	.92	.95	.98	.89	...	1.02	1.14	1.24	1.13	.98	.91	...	...	...	1.09	1.15	1.03
7	1.14	.81	1.02	...	...	.98	1.11	1.47	1.50	.74	.92	...	...	...	1.05	1.20	1.16
8	.84	.95	1.19	...	...	.79	.98	.80	.85	1.07	1.15	...	.86	...	.76	.82	.78
9	.77	.92	.91	...	...	.81	1.03	.70	.74	.80	.96	...	...	...	1.04	.89	.99
1850	...	1.52	1.26	.98	1.46	1.12	1.18	1.08	1.26	1.26	.96	1.29	...	...	1.01	1.14	1.21
1	...	1.24	.86	1.22	1.31	.95	.77	.89	.94	.85	.98	.85	...	...	.87	.92	.74
2	.90	...	.79	1.08	1.34	.90	1.08	.93	1.01	.91	.90	1.20	...	...	1.16	.97	1.03
3	...	...	...	...	...	1.04	1.00	1.30	1.21	1.33	1.24	.86	...	...	.85	1.17	.97
4	...	...	...	...	...	.69	.90	1.04	.95	.99	.79	.96	.96	...	.93	.97	.98
5	...	...	...	...	...	1.05	1.04	.95	1.03	1.03	...	...	.85	...	1.18	.99	1.03
6	...	...	...	...	...	.76	.81	.85	.83	1.00	...	...	...	1.01	.80	.77	.74
7	...	...	...	...	...	1.58	1.32	1.00	1.02	.99	...	...	...	1.27	1.15	1.10	1.11
8	...	...	...	...	...	.95	1.10	.85	.95	.92	...	...	...	1.09	.93	.90	.92
9	...	...	...	...	...	1.03	.88	1.01	1.27	.92	...	...	...	1.39	1.13	1.27	1.08
1860	...	...	...	...	...	.90	.92	...	...	...	...	.95	...	...	.80	1.00	...
1	...	...	...	...	...	1.15	1.10	...	1.13	...	...	1.10	1.68	1.52	.98	.97	...
2	...	...	...	...	...	1.11	1.24	...	1.06	...	...	.94	1.11	1.10	.97	1.00	...
3	...	...	...	...	...	1.42	.96	...	1.13	...	...	.87	1.02	...	.93	1.05	...
4	...	...	...	...	...	1.12	1.25	...	.96	...	...	.99	1.22	...	1.05	.86	...
5	...	1.11	.94	...	...	1.12	1.08	...	1.25	...	...	.79	.92	...	.94	1.17	...
6	...	1.12	...	...	...	.95	1.11	...	1.08	...	...	1.02	...	...	...	.89	...
1867	...	...	...	...	...	1.01	...	...	...	...	...	...	...	...	...	...	...

	Pennsylvania.					Md.	D.C.	Va.		S. C.			Ga.		Florida.		
	Philadelphia.	Morrisville.	Spring Mill.	Gettysburg.	Allegheny Arsenal.	Baltimore & Ft.McHenry.	Washington.	Fortress Monroe.	Powhatan Hill.	Charleston.	Ft. Moultrie.	St. Johns.	Savannah.	Whitemarsh Island.	Key West.	Ft. Brooke.	Ft.& Bar'ks, Barrancas.
1800	...	...	...	...	...	...	...	...	1738	1.13	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...	9	1.51	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...	1740	.91	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...	...	1	1.20	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	2	.83	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	3	.91	...	...	...	...	...	...	...
6	...	...	...	...	...	...	...	...	4	1.11	...	...	...	...	...	...	...
7	...	...	...	...	...	...	...	...	5	1.15	...	...	...	...	...	...	...
8	...	...	...	...	...	...	...	...	6	.91	...	...	...	...	...	...	...
9	...	...	...	...	...	...	...	...	7	1.02	...	...	...	...	...	...	...
1810	...	...	.92	...	...	...	...	...	8	1.18	...	...	...	...	...	...	...
1	...	...	.98	...	...	...	...	...	9	1.25	...	...	...	...	...	...	...
2	...	...	1.11	...	...	...	...	...	1750	1.29	...	...	...	...	...	...	...
3	...	...	1.00	...	...	...	...	...	1	1.17	...	...	...	...	...	...	...
4	...	...	1.21	...	...	...	...	...	2	.98	...	...	...	...	...	...	...
5	...	...	.97	...	...	...	...	...	3	1.09	...	...	...	...	...	...	...
6	...	...	.79	...	...	...	...	...	4	.85	...	...	...	...	...	...	...
7	...	...	1.01	...	...	1.15	...	...	5	.99	...	...	...	...	...	...	...
8	...	...	.85	...	...	.77	...	...	6	.73	...	...	...	...	...	...	...
9	...	...	.66	...	...	.68	...	...	7	1.02	...	...	...	...	...	...	...
1820	...	...	1.11	...	...	1.01	...	...	8	.70	...	...	...	...	...	...	...
1	...	...	.90	...	...	1.19	...	...	1759	.84	...	...	...	...	...	...	...
2	...	...	.84	...	...	.69	...	...		...	...	...	...	...	...	...	...
3	...	...	1.18	...	...	1.05	...	...	...	...	...	...	...	...	...	...	...
4	...	...	1.09	...	...	1.00	.78	...	...	...	...	...	...	...	...	...	...
5	.67	.67	...	...	...	...	.65	...	...	...	...	...	...	...	...	...	...
6	.80	.92	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	.87	.90	...	...	...	...	.70	...	...	...	...	...	...	...	...	...	...
8	.86	.89	...	...	...	...	.63	...	...	...	...	...	...	...	...	...	...
9	.95	.96	...	...	...	...	1.15	...	...	...	...	...	...	...	...	...	...
1830	1.02	1.02	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	1.00	.95	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	.90	.92	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	1.11	1.12	...	...	...	...	...	...	...	...	...	...	...	...	.76	...	...
4	.78	.80	...	...	...	...	...	...	...	...	...	...	...	...	1.00	...	...
5	.89	.92	...	...	...	...	...	...	...	...	...	...	...	...	.83	...	...
6	.97	.99	...	...	...	...	1.26	...	...	...	...	...	...	...	.68	...	...
7	.89	.85	...	...	1.01	1.10	.85	.87	...	...	...	...	.78	...	1.08	...	...
8	1.03	1.01	...	...	...	1.15	.94	.95	...	...	...	...	.80	...	.57	...	...
9	.99	1.02	...	.98	.73	1.26	1.01	1.53	...	...	...	...	.54	...	...	...	...
1840	1.08	1.09	...	.88	.81	.91	.95	1.58	...	...	...	...	.97	...	...	1.68	...
1	1.26	1.30	...	1.17	.99	1.07	1.21	1.39	...	1.24	...	...	1.45	...	...	.94	...
2	1.10	1.04	...	.93	1.18	.85	...	1.43	...	.96	...	...	1.32	...	...	.86	...
3	1.07	1.07	...	1.23	1.10	1.19	...	1.01	...	1.26	.94	...	.90	...	...	1.05	1.08
4	.91	.89	...	.80	.92	.79	...	.89	...	.83	.75	...	1.01	...	1.06	.96	1.01
5	.91	.91	...	.78	.91	.69	...	1.02	...	1.06	.96	...	.78	...	...	.99	.70
6	1.01	1.02	...	1.34	1.35	1.30	1.11	1.07	...	1.02	.95	1.07	1.17	...	...	1.01	1.27
7	1.02	...	...	1.31	1.31	.94	.96	1.04	...	1.10	1.44	.83	1.45	...	...	.86	...
8	.79	...	...	.86	.97	.87	.62	.63	...	1.00	1.25	.68	1.18	...	...	...	...
9	.95	...	...	.81	.99	.79	...	.70	...	.70	.87	.58	1.04	...	...	...	...
1850	1.24	1.19	...	1.17	1.06	1.14	...	1.06	1.13	.54	.79	.92	1.14	.94	...	.87	...
1	.86	.89	...	.80	.84	.89	...	.57	.79	.76	.86	.99	.91	.74	1.47	...	.74
2	1.04	1.20	...	1.12	1.18	1.28	.97	...	1.12	1 14	1.13	1.33	1.14	1.24	1.28	1.29	...
3	.95	...	...	.84	.96	.89	.85	...	.85	1.00	1.00	1.14	.97	1.13	1.27	.84	.91
4	1.03	1.03	...	.84	.76	1.44	.75	...	.89	.86	.87	1.04	.90	1.02	1.23	1.44	.86
5	1.01	.99	...	1.23	1.23	.72	.79	...	.90	.80	.96	.95	.80	.93	1.23	.71	1.31
6	.76	.70	...	.73	.75	.56	.90	.86	.88	.95	.90	1.34	.94	.96	1.11	1.00	.91
7	1.10	1.09	...	.98	1.11	.93	1.11	.91	.95	.80	.89	.98	.70	.76	1.07	.67	...
8	.93	.92	...	1.17	1.03	.98	1.04	.79	.93	1.03	1.04	1.12	1.11	1.30	.99	...	...
9	1.24	1.18	...	1.09	.97	...	1.13	.85	1.02	1.20	1.40	1.02	1.12	1.13	.87	...	1.18
1860	1.03	...	...	.98	...	...	1.08	...	1.00	...	...	...	...	.82	...	...	...
1	1.05	.98	...	.95	...	...	1.18	...	1.32	.98	...	...	...	...	.72	...	...
2	1.04	.98	...	.82	...	...	1.11	...	...	...	...	...	...	...	.93	...	...
3	1.13	1.07	...	...	...	...	1.16	...	...	...	...	...	...	...	.89	...	...
4	1.05	.82	...	...	...	...	.91	...	...	...	...	...	...	...	...	...	...
5	1.25	1.00	...	...	...	...	1.19	...	...	...	...	...	...	...	...	...	...
6	.99	.94	...	...	...	...	1.25	...	...	...	...	...	...	...	...	...	...
7	1.43	...	...	...	...	...	1.43	...	1.27	...	...	...	...	...	...	...	...

	Alabama.			Miss.		La.		Texas.					Ark.		Ten.	Ken.
	Huntsville.	Mt. Vernon Arsenal.	Greensboro' University.	Natchez.	Vicksburg.	New Orleans.	Baton Rouge.	Ft. Brown & Matamoras.	Ringgold Barracks.	Fort McIntosh.	Fort Duncan.	Austin.	Fort Smith.	Washington.	Glenwood.	Springdale.
1799	...	...	...	.81	...	...	...	...	...	...	...	...	...	...	...	...
1800	...	...	...	.58	...	...	...	...	...	...	...	...	...	...	...	...
1	...	...	...	.85	...	...	...	...	...	...	...	...	...	...	...	...
2	...	...	...	1.08	...	...	...	...	...	...	...	...	...	...	...	...
3	...	...	...	.70	...	...	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1810	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1820	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1830	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	.79	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	.85	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	1.23	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	1.15	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	1.10	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	1.00	...	...	...	...	1.02	...	...	...	...	...	...	...	...	...	...
7	.86	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	.88	...	...	...	...	...	...	...	...	...	...	...	.68	...	...	...
9	.53	...	...	...	...	.91	...	...	...	...	...	...	1.05	...	...	...
1840	...	...	...	.91	.94	.87	...	...	...	...	...	...	1.14	1.24	...	...
1	...	...	...	1.12	.99	.98	...	...	...	...	...	...	1.33	1.00	...	...
2	...	...	...	.81	.95	.85	...	...	...	...	...	...	.96	.76	...	.84
3	...	1.16	...	1.47	1.22	1.20	1.01	...	...	...	...	...	.94	1.16	...	1.00
4	...	...	...	.86	.76	.96	.73	...	...	...	...	...	.82	.84	...	.83
5	...	1.04	...	.99	1.01	...	.84	1.08	...	...	...	...	.83	.86	...	.89
6	...	.99	...	1.15	1.07	...	1.93	...	...	...	...	...	1.13	.76	...	.98
7	...	1.08	...	1.41	1.21	1.05	...	...	...	...	...	...	1.21	1.07	...	1.03
8	...	.75	...	...	1.04	1.05	...	...	...	...	...	...	1.10	1.10	...	1.20
9	...	1.20	...	...	.89	1.07	.93	1.15	...	...	...	...	1.42	1.29	...	.93
1850	...	.75	...	...	1.02	1.00	1.28	.78	.83	.98	.85	...	...	1.21	...	1.38
1	...	.74	...	...	.88	1.03	.69	.85	.66	.79	.67	...	...	.76	...	.87
2	...	.78	...	...	.88	.86	.89	.86	.84	.59	1.34	...	1.51	1.28	...	1.07
3	...	1.61	...	...	1.05	1.22	.88	.80	1.06	1.28	1.57	...	.60	1.01	...	.73
4	...	.95	...	...	...	1.03	1.01	1.52	1.36	1.34	1.04	...	.94	1.09	.92	.95
5	...	.90	.75	...	...	.82	.82	1.82	1.21	1.29	.82	...	.84	.87	.95	.94
6	...	.88	1.04	...	...	1.31	1.13	.78	1.15	1.05	.84	.68	.73	.85	.85	.64
7	...	.75	.95	...	...	.92	.83	.62	.59	.93	1.33	1.07	.96	1.05	.93	.96
8	...	.97	.87	...	...	1.19	1.06	.73	.93	.58	.76	1.17	...	1.04	1.15	1.03
9	...	.98	1.06	.89	...	.97	.85	...	...	...	...	.91	...	.85	.98	1.05
1860	...	...	1.09	...	...	...	...	...	...	...	...	...	.71	1.14	...	...
1	...	...	.99	.96	...	...	...	...	...	...	...	.92	...	...	1.03	1.02
2	...	...	.86	.78	...	...	...	...	...	...	...	.72	...	...	1.06	1.24
3	...	...	1.08	...	...	...	...	...	...	...	...	1.12	...	...	1.17	.99
4	...	...	1.06	...	...	...	...	...	...	...	...	.81	...	...	.89	.93
5	...	...	1.12	1.15	...	...	...	...	...	...	...	1.30	...	...	1.29	1.28
6	...	...	1.10	1.33	...	...	...	...	...	...	...	1.34	...	...	.96	1.23
7	...	...	.96	...	.99	...	...	...	...	...	...	...	...	...	...	...

	Ohio.				Mich.		Indiana.		Illinois.			Wis.	Minnesota.			Iowa.	
	Marietta.	Steubenville.	Cincinnati.	Portsmouth.	Detroit.	Fort Brady.	Richmond.	New Harmony.	Athens.	Ottawa.	Pekin.	Milwaukee.	Ft. Snelling.	Fort Ripley.	Ft. Ridgely.	Ft. Madison.	Muscatine.
1799	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1800	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1810	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	1.19	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9	.85	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1820	.92	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	1.01	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	1.02	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	1.17	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	.98	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	.97	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	1.16	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9	.93	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1830	.87	...	...	.69	...	...	...	...	...	...	...	...	...	...	...	...	...
1	1.25	1.05	...	1.11	...	...	...	...	...	...	...	...	...	...	...	...	...
2	1.13	.96	...	1.19	...	...	...	...	...	...	...	...	...	...	...	...	...
3	.95	.86	...	.96	...	...	...	...	...	...	...	...	...	...	...	...	...
4	.81	.93	...	.77	...	...	...	...	...	...	...	...	...	...	...	...	...
5	1.00	.92	1.16	.67	...	...	...	...	...	...	...	...	...	...	...	...	...
6	.86	.94	1.28	.80	1.34	...	...	...	...	...	...	...	...	...	...	...	...
7	1.03	.85	.95	1.25	.91	1.22	...	...	...	...	...	...	.93	...	...	...	...
8	.83	.68	.88	.95	...	1.13	...	...	...	...	...	...	1.07	...	...	...	...
9	.78	.67	.68	.71	.81	.79	...	...	...	...	...	...	.82	...	...	...	...
1840	.92	.91	1.06	1.09	.99	1.09	...	...	...	...	...	...	.90	...	...	...	...
1	1.01	.75	.91	1.15	.91	.80	...	...	...	...	...	1.13	.84	...	...	...	...
2	.99	.99	.92	1.09	1.06	.87	...	...	...	...	...	...	1.03	...	...	...	...
3	.98	.99	1.14	1.44	.92	.90	...	...	1.04	...	...	...	.92	...	...	...	...
4	.86	.93	.93	.95	1.10	1.14	...	...	1.23	...	...	1.07	1.17	...	...	...	...
5	.79	.93	1.03	1.05	.72	.98	...	...	1.09	...	...	.68	.98	...	...	...	...
6	1.08	1.26	1.19	1.19	...	.99	...	...	1.13	...	...	.83	1.01	...	...	...	.81
7	1.23	1.38	1.45	1.01	...	1.04	...	...	.82	...	...	.74	.85	...	...	...	.66
8	1.01	1.21	1.11	1.05	...	...	...	...	1.12	...	...	1.10	.90	...	...	1.21	.93
9	1.00	1.14	1.18	1.09	1.11	...	...	...	.97	...	...	1.02	1.92	...	...	1.29	1.38
1850	1.23	1.13	1.22	1.48	.85	1.18	...	...	1.03	...	...	.87	.99	1.41	...	1.20	1.14
1	.82	.69	.71	.74	...	1.50	...	...	1.19	...	...	1.00	.91	1.44	...	1.08	1.73
2	1.09	1.19	1.20	1.02	...	...	1.07	...	.97	...	...	.97	.59	1.38	...	1.25	1.41
3	.87	.86	...	...	...	.72	.74	...	.88	...	...	...	.79	1.04	...	...	1.05
4	.91	.73	...	...	1.54	.96	.97	.92	.79	...	...	1.03	1.03	.73	.99	1.10	.55
5	1.07	1.16	...	...	...	.75	1.31	1.24	1.10	...	1.22	1.12	.96	.94	1.35	1.15	.73
6	.76	.78	.57	...	.95	.82	.57	.60	.63	.87	.67	.96	.88	1.01	.91	1.05	.98
7	.95	1.33	.77	...	...	...	.91	1.02	.82	.95	.79	1.01	1.25	...	1.49	.66	.81
8	1.45	1.22	1.10	...	1.18	...	1.28	1.26	1.19	1.26	1.47	1.48	...	.79	.88	1.22	1.36
9	1.14	1.20	1.01	1.21	.96	...	1.05	1.14	...	.75	.74	.95	...	1.04	1.28	.89	.84
1860	.94	1.12	...	...	.94	...	.88	...	...	.73	.88	.79	...	1.22	.66	.77	...
1	1.09	.94	.94	1.11	1.28	...	.91	.97	...	1.05	.91	1.10	...	1.29	.90	.74	1.00
2	1.00	1.02	.85	1.00	1.05	...	1.16	1.19	...	1 50	1.25	1.36	...	.57	1.17	.89	1.24
3	.87	.93	.90	...	1.01	...	.85	.90	...	.87	.88	1.11	...	.69	.73	.72	...
4	.96	1.19	.74	.96	.86	...	.79	.87	...	.79	.83	.98	...	.48	.56	.74	.86
5	1.15	1.16	.98	...	.70	...	1.21	1.17	...	1.00	1.18	1.05	...	1.00	1.36	1.06	.80
6	1.11	1.18	1.11	...	1.05	...	1.31	.96	...	.88	...	1.19	...	...	...	1.00	...
7	1.09	.82	...	...	...	...	...	...	...	...	...	...	...	1.21	...	...	...

	Iowa.	Missouri.			Indian Territory.			Kansas.		Nebr.	Wyo.
	Dubuque.	Jefferson Barracks.	St. Louis Arsenal.	St. Louis.	Fort Towson.	Washita.	Ft. Gibson.	Ft. Leavenworth.	Fort Riley.	Ft. Kearny.	Ft. Laramie.
1836	...	...	...	...	...	...	...	...	...	...	...
7	...	...	.63	.64	.86	...	.95	1.21	...	...	...
8	...	...	.56	.67	.67	...	.52	.83	...	...	...
9	...	...	.99	1.12	1.29	...	.93	1.05	...	...	...
1840	...	...	...	.98	1.09	...	1.53	1.01	...	...	...
1	...	.89	.93	1.01	1.30	...	1.00	.82	...	...	...
2	...	.73	...	.75	1.44	...	.80	.83	...	...	...
3	...	.79	.73	.82	1.09	...	.98	.51	...	...	...
4	...	.91	1.20	1.08	.91	.95	1.03	1.52	...	...	...
5	...	.80	.93	.90	.74	.91	.72	1.09	...	...	...
6	...	.88	1.32	1.05	...	1.31	.93	.75	...	...	...
7	...	...	1.53	1.13	...	.98	.66	.66	...	...	...
8	...	.86	1.48	1.09	...	.93	1.04	1.20	...	...	...
9	...	.94	1.68	1.08	...	1.69	1.45	1.35	...	1.81	...
1850	...	.97	.82	1.20	.97	1.09	.96	.85	...	.99	.62
1	...	.88	.81	1.01	.96	.83	1.43	1.19	...	1.05	.66
2	...	1.35	.95	1.11	...	1.24	1.42	1.15	...	.82	2.07
3	...	.88	.68	.73	.82	.80	.71	.79	...	1.19	2.03
4	.83	1.06	.73	.96	...	1.14	.79	.77	.72	1.05	1.47
5	.87	1.14	1.26	1.19	...	.57	.98	.87	1.11	1.00	1.25
6	...	...	.75	1.00	...	.76	1.09	1.35	1.05	1.15	.99
7	...	1.01	...	.92	...	.87	...	1.00	.76	1.14	.40
8	1.41	1.22	...	1.63	...	...	...	1.88	1.36	1.03	.52
9	.89	1.26	...	1.45	...	...	...	1.23	1.00	.64	.41
1860	.80	.98	...	.71	...	...	...	.61	.56	.67	...
1	1.15	1.27	...	.90	...	...	...	.86	1.34	.77	...
2	1.05	...	...	1.04	...	...	...	.93	.86	.87	.84
3	1.01	...	...	.88	...	...	...	.96	1.16	...	.34
4	.75	...	...	.79	...	...	...	.51	.61	...	...
5	1.08	...	...	1.11	...	...	...	1.61	.94	...	...
6	1.08	...	...	.99	...	...	...	...	1.15	...	...
7	...	...	...	.89	...	...	...	.81	1.28	...	...

	New Mexico.			California.					Oreg.	Wash. Ter.		Alas.
	Albuquerque.	Fort Marcy.	Fort Union.	Sacramento.	San Diego.	San Francisco.	Benicia Barracks.	Fort Humboldt.	Dalles of Columbia.	Fort Vancouver.	Fort Steilacoom.	Sitka.
1848	...	...	...	...	...	...	...	...	...	...	...	1.09
9	...	...	...	...	...	...	...	...	...	...	...	.90
1850	.60	...	...	1.00	.86	...	.89	...	...	.99	.76	1.15
1	...	...	...	.78	.82	.70	1.01	...	...	...	.89	.87
2	1.80	...	1.22	1.38	1.30	1.18	1.18	...	...	1.44	1.11	.94
3	.88	1.31	.61	1.03	.86	.88	.78	...	.67	1.08	1.30	1.09
4	1.54	1.49	.66	1.12	1.32	.95	.83	.81	.57	.82	1.60	1.04
5	...	1.46	.85	.95	1.46	1.13	1.16	1.09	.55	1.17	...	...
6	.51	1.38	.93	.73	1.11	.83	.81	.76	...	1.35	...	1.05
7	.64	.52	.96	.87	.75	.82	.81	.90	1.35	1.22	.89	1.07
8	2.01	.68	1.04	.86	.98	.93	.83	1.16	2.01	1.04	1.14	.97
9	.73	.57	1.12	.86	...	.98	.90	1.19	1.66	1.00	.88	.98
1860	.47	.53	.66	1.01	...	.95	.97	1.14	.99	.90	.69	...
1	...	.95	1.88	1.07	...	1.06	...	.96	1.33	1.13	.83	.70
2	...	...	.98	1.40	.78	1.66	1.33	...	.73	.76	.59	1.03
3	...	.47	...	.62	...	.70	...	1.04	.65	1.09	1.03	.89
4	.76	1.25	...	.99	.81	.91	...	.86	...	.81	.69	...
5	...	1.39	...	.57	.93	.54	...	.88	1.02	.67	.85	...
6	...	...	...	1.36	...	1.57	...	...	...	...	...	...
7	...	...	...	...	...	1.32	...	...	...	.90	1.33	...

To free the ratios of the preceding table from the accidental irregularities, and to exhibit the nature of the fluctuations from year to year more distinctly, we unite them into groups formed of stations where the annual rain-fall appears subject to the same laws. The number and extent of these groups may be recognized by means of the investigation of the annual distribution as shown by the type curves, and the proper selection of stations so combined may be verified, to some extent, by the agreement within the accidental irregularities of the ratios themselves. These means, the number of stations from which they were derived, and the fourth order of successive means are given in the following table for a number of groups or types. The successive means[1] are introduced to smooth down the incidental irregularities of the phenomenon as a means for the further study of the annual fluctuation.

Group I is composed of stations on the Atlantic sea-board from Maine to Virginia (see Diagram 3); Group II covers the State of New York, it includes also Hanover, Burlington, Amherst, Montreal, and Toronto; Group III is formed by Forts Ripley and Snelling, Dubuque, Milwaukee, Muscatine, Fort Madison, and Ottawa; Group IV covers the Ohio Valley from Pittsburg westward, and includes Eastern Missouri; Group V is formed by Forts Gibson, Washita, Towson, Smith, and Washington, Ark.; Group VII includes New Orleans, Baton Rouge, Mount Vernon Arsenal, Huntsville, Greensboro', and Fort Barrancas; Group VIII comprises the Southern coast from Virginia to Florida; and Group IX the Pacific coast stations San Diego, San Francisco, Sacramento, Benicia Barracks, and Fort Humboldt.

Results from Groups I, II, IV are tolerably trustworthy, but the remaining ones can only be taken for rough approximations on account of an insufficiency of stations.

The decimal points are omitted in the tabular quantities; they are expressed, therefore, in percentage of the mean amount.

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[1] The following formulæ may also be employed, occasionally, to effect the same; they involve no special supposition as to curvature (being based upon the consideration of straight lines), and are directly derived from the average ordinate $y_m = \frac{1}{2}(y_1 + y_2)$;
for three ordinates $y_m = \frac{1}{4}(y_1 + 2y_2 + y_3)$;
for four " $y_m = \frac{1}{8}(y_1 + 3y_2 + 3y_3 + y_4)$;
for five " $y_m = \frac{1}{16}(y_1 + 4y_2 + 6y_3 + 4y_4 + y_5)$;
for six " $y_m = \frac{1}{32}(y_1 + 5y_2 + 10y_3 + 10y_4 + 5y_5 + y_6)$;
for seven " $y_m = \frac{1}{64}(y_1 + 6y_2 + 15y_3 + 20y_4 + 15y_5 + 6y_6 + y_7)$; and so on;
for $n+1$ ordinates the coefficients are those of the n^{th} power of a binomial and corresponding to successive means of the n^{th} order.

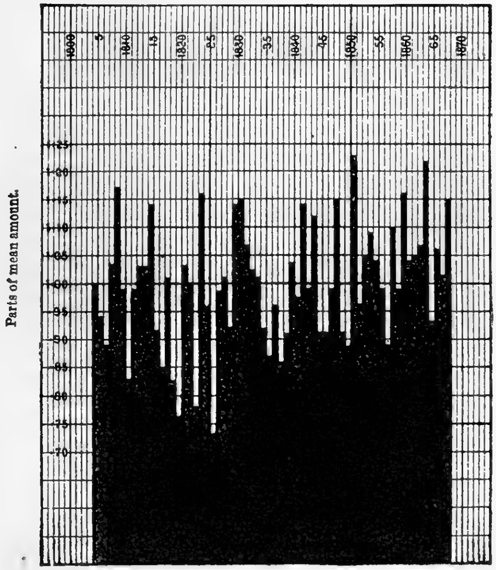
At what order of successive means we should stop depends entirely on the nature of the irregularities; care must be taken not to eliminate by the process any of the characteristic features.

	GROUP I.			GROUP II.			GROUP III.			GROUP IV.			GROUP V.			GROUP VII.			GROUP VIII.			GROUP IX.		
	Sea-coast Maine to Virginia.			State of New York and adjacent parts of Canada, N.H., Mass. and Vt.			Parts of Iowa Minnesota, Ill., Wis.			Ohio Valley, Ohio, Ind., Ill., Kent. and part of Mo.			Indian Ter. and Ark.			Louisiana, Alab., West. Flor.			Sea-coast Virginia to Florida.			Sea-coast California.		
	Per cent. of mean amount.	No. of stat'ns.	4th order of suc. means.	Per cent. of mean amount.	No. of stat'ns.	4th order of suc. means.	Per cent. of mean amount.	No. of stat'ns.	4th order of suc. means.	Per cent. of mean amount.	No. of stat'ns.	4th order of suc. means.	Per cent. of mean amount.	No. of stat'ns.	4th order of suc. means.	Per cent. of mean amount.	No. of stat'ns.	4th order of suc. means.	Per cent. of mean amount.	No. of stat'ns.	4th order of suc. means.	Per cent. of mean amount.	No. of stat'ns.	4th order of suc. means.
1804	100	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	94	1	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	89	1	96	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	104	1	102	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	117	2	106	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9	99	2	101	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1810	83	3	94	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
1	99	3	96	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
2	103	3	101	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
3	103	3	104	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
4	114	4	103	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
5	92	4	97	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
6	85	4	92	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
7	101	5	90	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
8	83	6	87	...	...	...	...	...	...	120	1	...	...	...	...	...	...	...	...	...	...	...	...	...
9	76	5	87	...	...	...	...	...	...	86	1	...	...	...	...	...	...	...	...	...	...	...	...	...
1820	103	5	93	...	...	...	...	...	...	93	1	95	...	...	...	...	...	...	...	...	...	...	...	...
1	100	5	94	...	...	...	...	...	...	102	1	100	...	...	...	...	...	...	...	...	...	...	...	...
2	78	4	96	...	...	...	...	...	...	103	1	105	...	...	...	...	...	...	...	...	...	...	...	...
3	116	4	97	...	...	...	...	...	...	118	1	107	...	...	...	...	...	...	...	...	...	...	...	...
4	96	5	94	...	...	...	...	...	...	...	...	106	...	...	...	...	...	...	...	...	...	...	...	...
5	73	5	89	...	...	...	...	...	...	...	...	104	...	...	...	...	...	...	...	...	...	...	...	...
6	98	4	91	81	1	...	...	...	...	99	1	103	...	...	...	...	...	...	...	...	...	...	...	...
7	101	6	96	122	1	...	...	...	...	98	1	103	...	...	...	...	...	...	...	...	...	...	...	...
8	92	8	102	102	3	105	...	...	...	117	1	102	...	...	...	...	...	...	...	...	...	...	...	...
9	114	7	108	98	3	102	...	...	...	94	1	97	...	...	...	...	...	...	...	...	...	...	...	...
1830	115	8	111	103	5	101	...	...	...	79	2	95	...	...	...	...	...	...	...	...	...	...	...	...
1	107	8	108	100	6	102	...	...	...	115	3	101	...	...	...	79	1	...	...	...	...	...	...	...
2	102	9	104	104	8	102	...	...	...	110	3	102	...	...	...	85	1	...	...	...	...	...	...	...
3	101	9	99	106	8	97	...	...	...	93	3	97	...	...	...	123	1	106	...	...	...	...	...	...
4	92	9	94	76	6	90	...	...	...	85	3	93	...	...	...	115	1	112	...	...	...	...	...	...
5	87	9	91	93	8	87	...	...	...	95	4	93	...	...	...	110	1	108	...	...	...	...	...	...
6	96	10	90	89	9	87	...	...	...	98	4	93	...	...	...	101	2	99	...	...	...	...	...	...
7	86	11	90	84	10	87	93	1	...	92	7	89	90	2	...	86	1	90	83	2	...	...	...	...
8	91	14	93	90	11	88	107	1	...	77	6	84	62	3	...	88	1	83	87	2	...	...	...	...
9	104	16	98	88	12	91	82	1	93	82	7	86	109	3	100	72	2	82	103	2	105	...	...	...
1840	97	16	103	101	12	95	90	1	93	97	6	92	125	4	112	87	1	86	127	2	120	...	...	...
1	114	18	105	93	11	100	99	2	97	96	8	95	116	4	112	98	1	91	136	3	125	...	...	...
2	99	17	106	113	12	104	103	1	100	95	8	98	99	4	105	85	1	95	124	3	118	...	...	...
3	112	18	103	109	12	101	92	1	100	102	11	99	104	4	98	111	4	97	103	4	105	...	...	...
4	91	18	98	85	11	95	112	2	98	102	11	100	91	5	92	90	3	97	87	4	96	...	...	...
5	91	17	96	93	12	92	83	2	92	96	11	104	81	5	91	86	3	103	95	4	97	...	...	...
6	99	19	100	95	10	95	88	3	87	115	11	110	103	4	96	140	3	111	106	5	103	...	...	...
7	115	18	102	102	9	98	75	3	90	118	11	114	98	4	103	106	2	109	117	5	104	...	...	...
8	91	18	99	96	10	98	104	4	102	112	11	113	104	4	113	90	2	102	95	5	96	...	...	...
9	89	20	100	91	9	100	123	3	113	110	11	110	146	4	119	107	3	99	78	5	88	...	...	...
1850	123	20	105	117	12	104	112	5	118	115	11	106	106	4	115	101	3	95	93	7	86	92	3	...
1	96	20	106	99	12	104	123	5	117	90	11	102	99	4	110	80	4	90	80	7	91	83	4	...
2	105	23	105	101	11	101	112	5	110	111	12	97	136	4	107	84	3	93	118	6	100	126	4	101
3	109	17	105	101	7	97	96	3	100	83	10	93	79	5	100	115	4	98	102	6	100	89	4	102
4	104	19	102	86	8	96	88	6	94	88	11	93	99	4	91	96	4	99	93	6	95	101	5	102
5	99	20	98	109	5	97	96	6	94	117	12	94	82	4	87	92	5	98	89	6	93	116	5	101
6	89	21	98	84	6	100	96	6	98	71	12	93	86	4	89	105	5	97	98	7	93	85	5	94
7	110	21	102	120	6	105	94	5	103	96	12	99	96	3	93	86	4	96	85	7	95	83	5	90
8	99	21	106	100	5	106	125	6	105	126	12	109	104	1	95	102	4	98	105	7	100	95	5	93
9	116	20	108	111	5	103	89	6	100	112	11	109	85	1	...	101	5	102	111	7	103	98	4	98
1860	104	11	108	88	5	103	86	5	96	93	6	103	93	2	...	109	1	102	91	2	...	102	4	102
1	105	15	108	116	7	106	105	6	100	102	10	101	...	...	...	99	1	98	115	2	...	103	3	107
2	107	14	111	109	7	109	110	6	103	109	9	99	...	...	...	86	1	97	...	...	...	129	4	106
3	122	13	110	109	6	108	88	5	102	91	8	95	...	...	...	108	1	101	...	...	...	79	3	95
4	93	14	106	103	6	108	110	6	102	90	9	93	...	...	...	106	1	107	...	...	...	89	4	88
5	106	16	104	115	6	108	100	6	105	91	8	97	...	...	...	112	1	108	...	...	...	73	4	100
6	102	16	...	104	5	...	104	4	...	114	7	...	...	...	...	110	1	...	...	...	...	146	2	...
7	115	11	...	102	2	...	121	1	...	94	3	...	...	...	...	96	1	...	127	1	...	132	1	...

The tabular mean results are graphically exhibited in Diagrams 5, 6, and 7 for the more reliable groups.

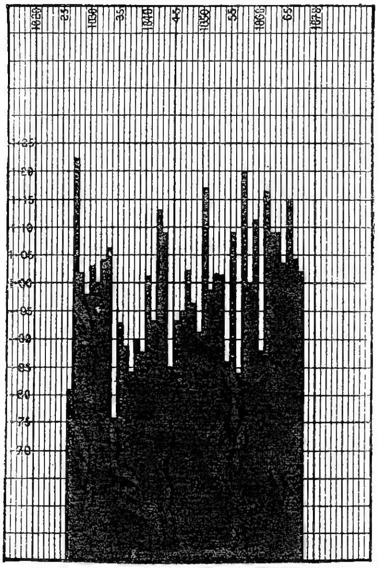
The irregularities in the successive annual amounts of rain are very great, yet they do not wholly obliterate the indications of a conformity to general laws; thus Diagrams 5 and 6 distinctly indicate an increase of rain, on the average, since 1835, and show a certain tendency to an arrangement of groups of years of drought followed by unusually wet years. The fluctuations of rain, as shown on Diagrams 5 and 6, are evidently similar, and these groups might have been combined into one; the fluctuations in the Ohio Valley are of quite a different character.

Diagram 5.



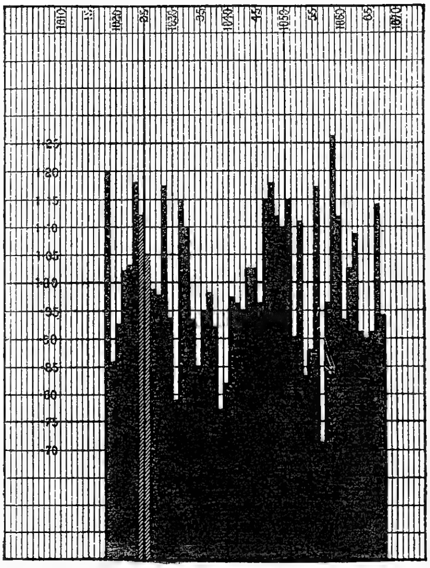
FLUCTUATIONS IN THE ANNUAL RAIN FALL
Group I.—*Atlantic Sea-coast, Maine to Maryland.*

DIAGRAM 6.



FLUCTUATIONS IN THE ANNUAL RAIN FALL
Group II.—*State of New York with some adjacent localities.*

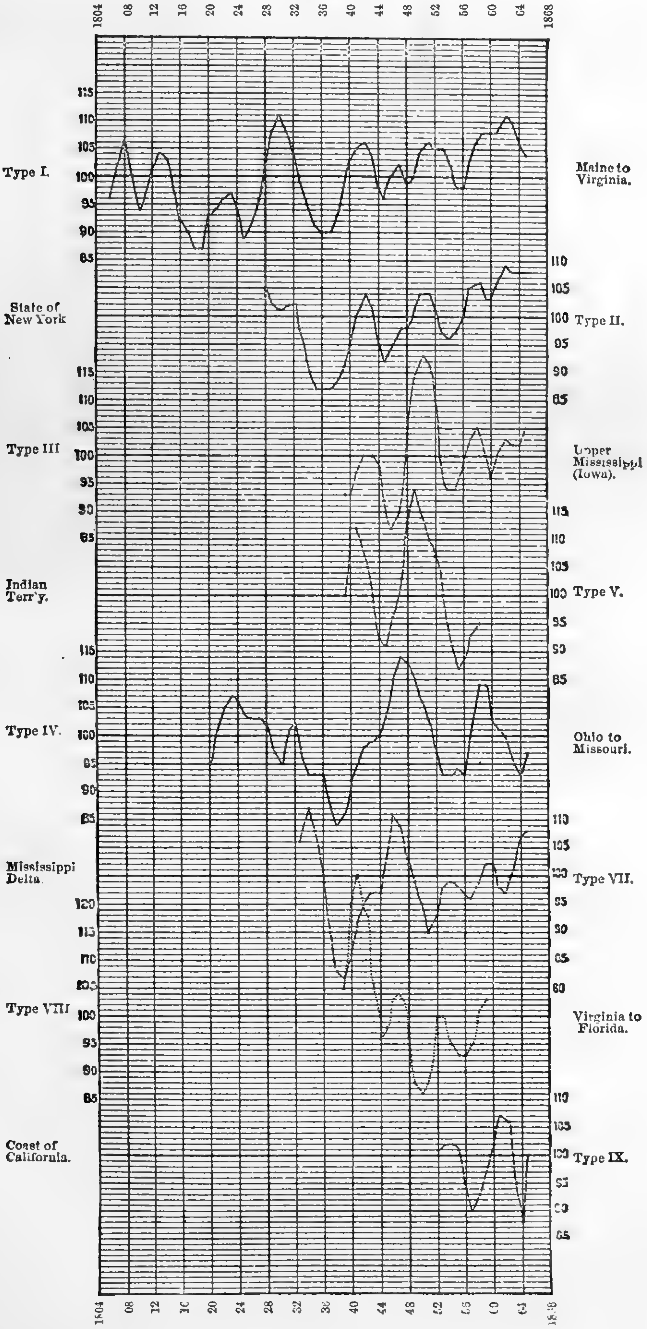
DIAGRAM 7.



FLUCTUATIONS IN THE ANNUAL RAIN FALL
Group IV.—*Valley of the Ohio River; Ohio to Eastern Missouri.*

These fluctuations can be compared and studied to better advantage by means of a graphical representation of the *successive means* (of the 4th order as given in the table); these are given in Diagram 8.

DIAGRAM 8.


1804
08
12
16
20
24
28
32
36
40
44
48
52
56
60
64
1868
Type I.
Maine to Virginia.
State of New York
Type II.
Type III
Upper Mississippi (Iowa).
Indian Terr'y.
Type V.
Type IV.
Ohio to Missouri.
Mississippi Delta.
Type VII.
Type VIII
Virginia to Florida.
Coast of California.
Type IX.
115
110
105
100
95
90
85
120

The observed succession of annual amounts of rain-fall on the Atlantic coast from Maine to Virginia, and in the interior of the State of New York, seems to be governed by the same circumstances or laws, as is evident by a comparison of the curves Type I and Type II; and indeed Type III, region of the upper Mississippi (Iowa), bears some resemblance to these curves, but is not yet sufficiently developed to be pronounced identical in character. The curve of Type I points to a gradual increase of the average annual precipitation from about 1818 to the present time; whether or not we have reached the maximum of this long fluctuation or secular change cannot yet be determined, but it is probable that we have passed it. Besides this greater change, which extends over several decades, and which undoubtedly is of a periodic character, there are smaller undulations covering a period of but a few years; these latter waves are superposed upon the larger one, and appear of variable parameter, those near the minimum of the greater period being greatest, and decreasing gradually to the present time. As our records increase in extent this general character of the rain-fall will no doubt receive some modification; the above description answers, however, to its aspect within the period now available for discussion.

The variations in the rain-fall, from year to year, in the Ohio Valley, and as far west as Missouri, are different from those just noticed, though they are not of an opposite character. Type IV indicates *no* secular increase, and the secondary waves appear larger than in Type I; the remarkable period of droughts about 1836, as well as the less conspicuous or relative one about 1855, are common to the two regions. A comparison with Type VII of the Mississippi delta and Alabama leads to the inference that the law of succession of dry and wet years along the region of the Ohio partakes largely of the character of that on the Gulf coast. On our southern Atlantic coast the distribution is different from any of the above types; the rain-fall here seems to have been on the decrease (see Type VIII). Type IX, for the coast of California, is too limited in extent to be analyzed.

The wavelike irregularities presented in Diagram 8 were compared with a curve representing the state of the sun's annual average activity in the production of spots, a phenomenon which may possibly have some indirect connection with the variations in the annual rain-fall. The decided minimum in precipitation about the years 1837–8 corresponds to the decided maximum of solar disturbance at this epoch; but the comparison of the two phenomena about the period 1854–5 leads to an opposite conclusion. This last epoch is one of minimum rain-fall, as well as of a minimum in the number of spots occurring in 1855–6. The two curves appear, of course, to coincide for some years about these epochs, yet it is plain that either there is no such connection between the two phenomena, as has been supposed, or else the accidental and local irregularities in the rain-fall are not sufficiently eliminated to allow of the recognition of the law regulating the secular changes.

Taking a general view of the results of this investigation, it must be admitted that, while they exhibit, in a concise form, the broader features of the laws of distribution of rain in the United States, they must still be regarded as but first approximations, considering the irregularity in the phenomenon itself, and the comparative scantiness and frequent discontinuity of our records. While these deductions follow from the present collection and combination of available material, they may stimulate to more extended observations and serve as a basis of more detailed investigations hereafter.

LIST OF METEOROLOGICAL STATIONS

AT WHICH THE RAIN-FALL WAS OBSERVED.

Missouri—*Continued.*
Easton, 66
Edina, 66, 113
Edinburg, 66
Hannibal, 66, 113
Harrisonville, 66, 113
Jefferson Barracks, 66, 113, 127, 130, 152
Laborville, 66
Luray, 66
Palmyra, 66
Paris, 66
Rhineland, 66
Rolla, 66
St. Joseph, 66
St. Louis, 66, 66, 66, 113, 113, 113, 113, 127, 127, 130, 130, 145, 152, 152
St. Paul's College, 66, 67
St. Vincent's College, 66, 67
Springfield, 66
Stockton, 66
Tower Grove, 66, 113
Union, 66
Vogel's Farm, 66, 67
Warrenton, 66
Wyaconda, 113
Wyaconda Prairie, 66

Montana, 70, 71, 115
Camp Cook, 70
Fort C. F. Smith, 70, 115
Helena, 70, 115

Nebraska, 68, 69, 115, 135, 142, 145, 152
Bellevue, 68, 115, 135
Fontanelle, 68
Fort Kearny, 68, 115, 145, 152
Glendale, 68
Ionia, 68
Nebraska City, 68
Nursery Hill, 68
Omaha, 68, 115
Pioneer Grove, 68
Rock Bluffs, 68
Washington County, 68

Nevada, 74, 75, 116
Camp McDermit, 74
Camp McGarry, 74
Camp W. Scott, 74
Fort Churchill, 74, 116
Fort Ruby, 74

New Hampshire, 16, 17, 18, 89, 142, 145, 147, 154
Claremont, 18, 89
Concord, 16, 89
Farmington, 18
Fort Constitution, 16, 89
Francestown, 18
Great Falls, 16
Hanover, 16, 89, 126, 128, 145, 147
Laconia, 16, 89
Lake Village, 16, 89
Londonderry, 16, 89
London Ridge, 18
Littleton, 18
Manchester, 16, 89
Mount Washington, 18
North Barnstead, 89
North Littleton, 18
Portsmouth, 18, 18
Salmon Falls, 16, 89
Shelburn, 18
Stratford, 18, 89
West Enfield, 18, 18, 89

New Jersey, 22, 23, 97, 126, 142, 148
Bloomfield, 22, 97
Branchburg, 22
Burlington, 22, 97
Coles' Landing, 22, 97
Dover, 22
Greenwich, 22, 97
Jamesburg, 22
Junction of Rancocus and Delaware Rivers, 22, 97
Lambertville, 22, 97, 126, 127, 148
Long Branch, 22
Middletown, 22
Mount Holly, 22
Navesink Highlands, 22
Newark, 22, 97, 126, 127, 148
New Brunswick, 22, 97
Passaic Valley, 22
Paterson, 22, 97
Seaville, 22
Sergeantville, 22
Trenton, 22, 97

New Mexico, 70, 71, 72, 73, 115, 135, 135, 142, 142, 152
Albuquerque, 72, 115, 135, 152
Camp Burgwin, 72, 135
Cantonment Burgwin, 72, 115, 135
Cebolleta, 72, 115
Fort Bayard, 72
Fort Conrad, 70, 115
Fort Craig, 70, 115, 135
Fort Fillmore, 70, 115, 135
Fort Marcy, 72, 115, 135, 145, 152
Fort McRae, 72
Fort Selden, 72
Fort Stanton, 72, 115, 135
Fort Sumner, 72, 115
Fort Thorn, 70, 115
Fort Union, 72, 115, 135, 152
Fort Webster, 70, 115
Fort Wingate, 72, 115
Las Vegas, 72
Santa Fé, 72
Socorro, 70, 115

New York, 22, 23, 24, 25, 26, 26, 27, 28, 29, 30, 30, 31, 94, 95, 96, 96, 121, 128, 129, 133, 142, 142, 142, 145, 147, 148, 153, 154, 155, 156, 157, 158
Albany, 24, 24, 24, 94, 126, 128, 145, 148
Alfred, 26
Angelica, 26
Ardenia, 26, 27
Argyle, 30
Auburn, 30, 30, 126, 128
Baldwinsville, 30
Beaver Creek, 26
Bellport, 28, 96
Bellville, 28
Beverly, 26, 96
Blackwell's Island, 26, 96
Boonville, 26
Bridgewater, 28
Brookhaven, 30, 96
Buffalo, 24, 24, 24, 24, 95, 95
Cambridge, 28
Canajoharie, 28
Canandaigua, 28
Canton, 26
Cayuga, 30
Cazenovia, 26, 28, 96, 126, 128
Champion, 24
Charlotte, 26, 96
Chatham, 26
Cherry Valley, 28

New York—*Continued.*
Clinton, 26, 26, 96, 96, 126, 128
Columbia College, 96
Constantia, 30
Covert, 26, 27
Cuba, 28
Dansville, 28
Delhi, 26, 26, 96
Depauville, 30, 96, 96
East Hampton, 24, 94, 126, 147
Elmira, 26
Fairfield, 28, 126, 128
Falconer, 26
Fishkill Landing, 26, 96
Flatbush, 24, 94, 126, 127, 145, 147
Fordham, 26
Fort Ann, 30
Fort Columbus, 22, 94, 126, 127, 145, 148
Fort Edward, 28, 96
Fort Hamilton, 22, 24, 94, 95, 126, 126, 127, 148, 148
Fort Niagara, 24, 95, 126, 148
Fort Ontario, 24, 95
Fredonia, 28, 28, 126, 128
Gaines, 28
Geneva, 24, 24, 24, 95, 126, 128
Germantown, 30
Goshen, 24
Gouverneur, 26, 26, 26, 96, 126, 128, 148
Granville, 28, 126, 128
Hamilton College, 96
Hartwick, 28
Henrietta, 28
Hermitage, 30, 96
Homer, 26, 95
Houseville, 26
Hudson, 26, 95, 126, 128
Hudson River Valley, 129
Ithaca, 26, 126, 128
Jamaica, 24, 94, 126, 145, 148
Jamestown, 26, 26, 95
Johnstown, 28
Kinderhook, 28, 126, 128
Kingston, 28, 126, 128
Lake, 28, 96
Lansingburg, 24, 126, 128
Ledyard, 30
Lewiston, 28
Lima, 30
Little Genesee, 30
Lodi, 26, 96
Lowville, 24, 95, 126, 128
Madison Barracks, 24, 95
Madrid, 26, 26
Malone, 28
Martinsburg, 24
McGrawville, 28
Mexico, 26, 26, 30, 96
Middlebury, 28, 126, 128
Millville, 28
Montgomery, 28
Moriches, 30
Mt. Pleasant, 28
Newburg, 24, 24, 28, 94, 126, 128, 145, 148
New York City, 24, 24, 24, 26, 30, 30, 30, 30, 95, 95, 96, 96, 126, 127, 145, 148
North Hammond, 30
North Salem, 24, 24, 94, 126, 147
Ogdensburg, 26, 96
Oneida, 28, 28
Onondaga, 30
Oswego, 28, 96
Ovid or Seneca Inst., 24
Oxford, 26, 95, 126, 128
Oyster Bay, 30

ALPHABETICAL LIST OF METEOROLOGICAL OBSERVERS.

INDEX.

ANNUAL FLUCTUATION IN THE RAIN FALL.

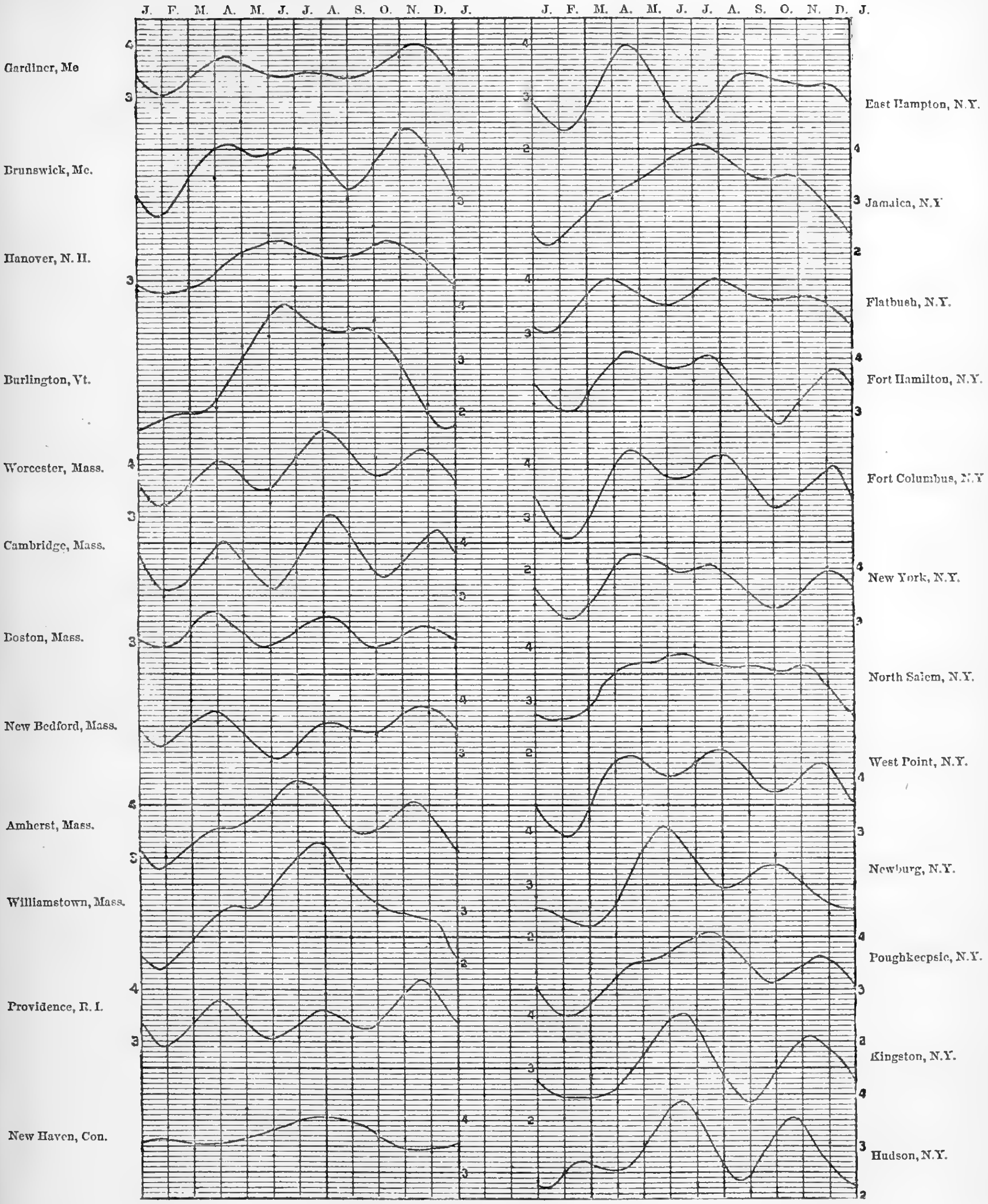


PLATE II.

ANNUAL FLUCTUATION IN THE RAIN FALL.

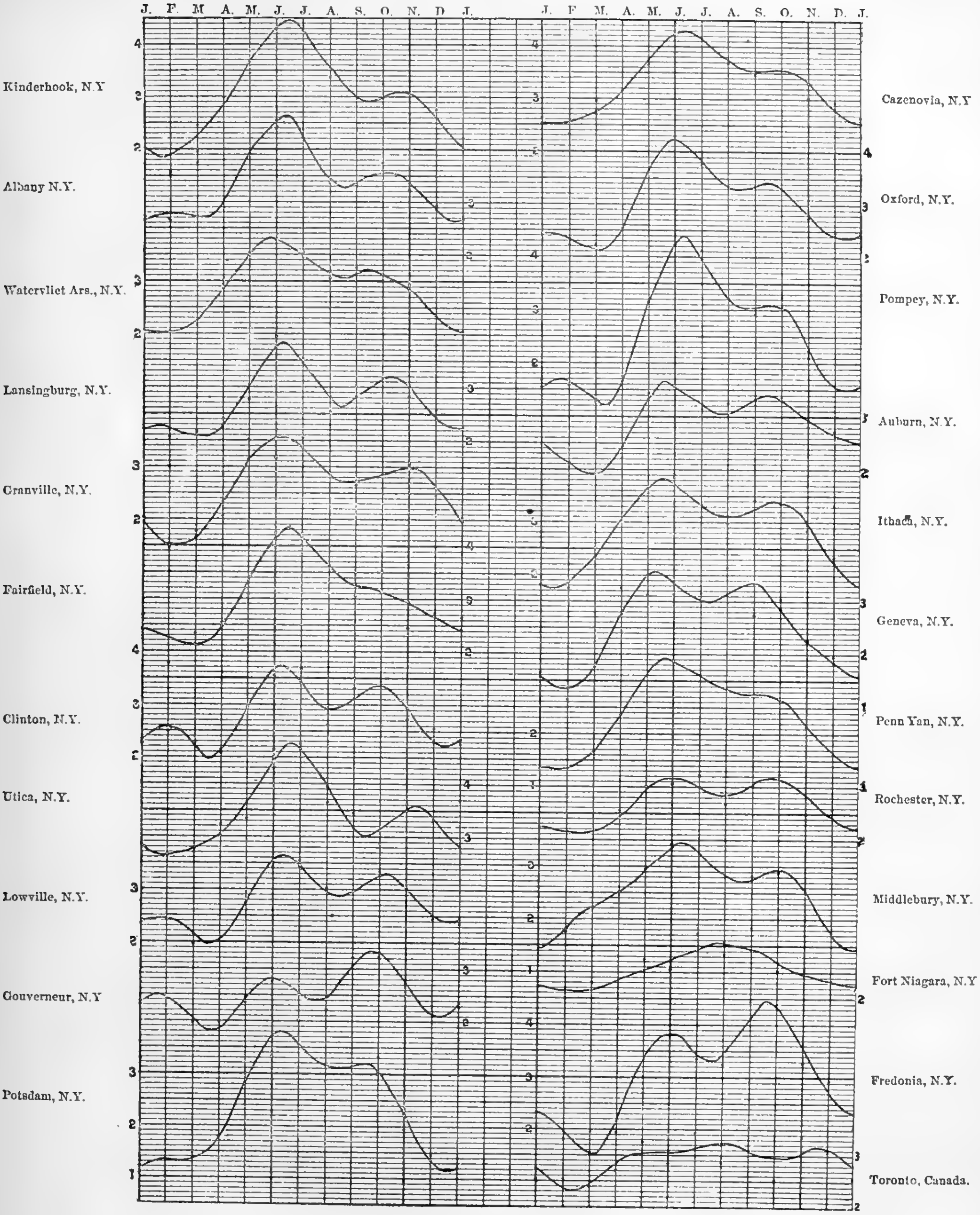


PLATE III.

ANNUAL FLUCTUATION IN THE RAIN FALL.

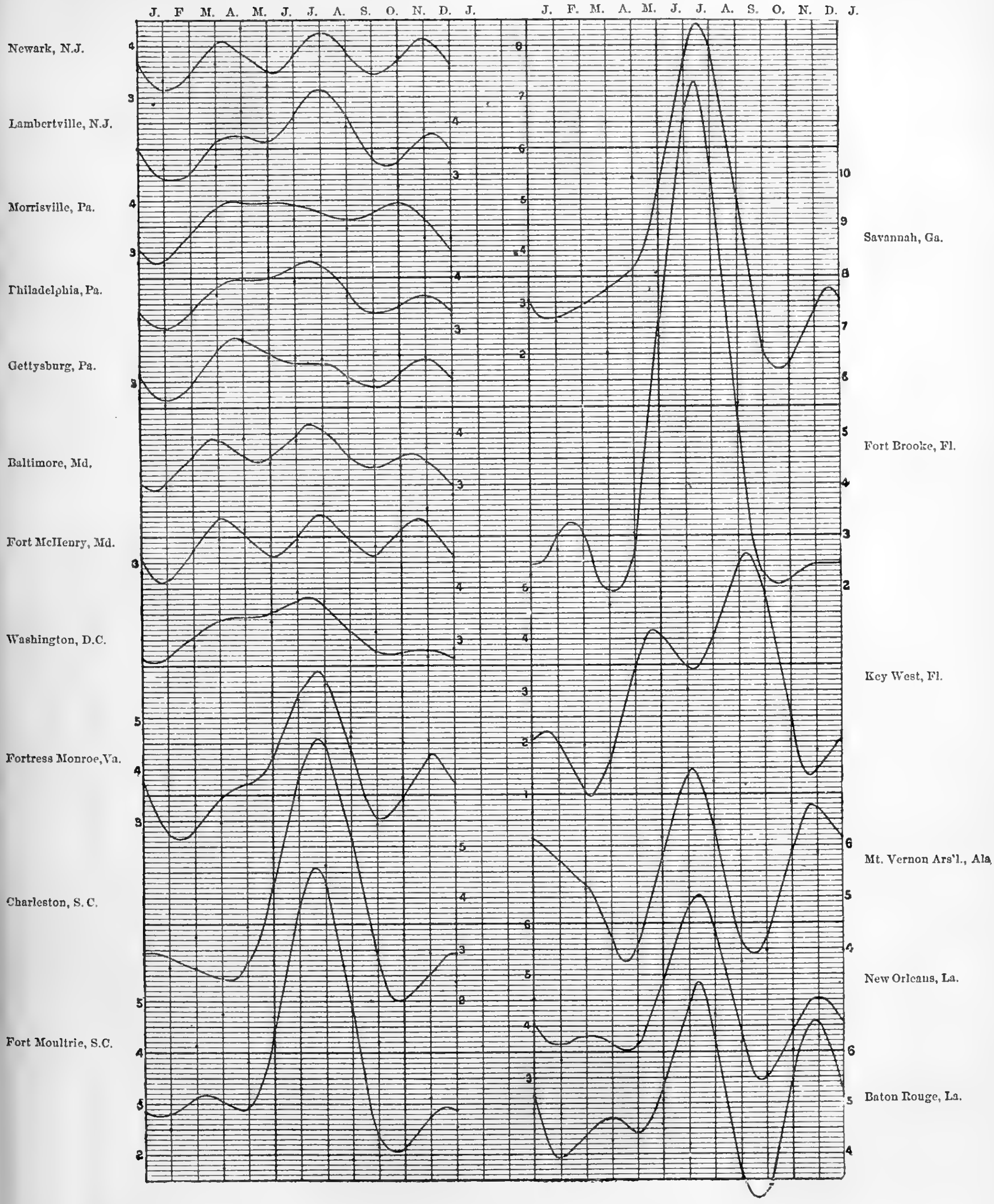


PLATE IV.

ANNUAL FLUCTUATION IN THE RAIN FALL.

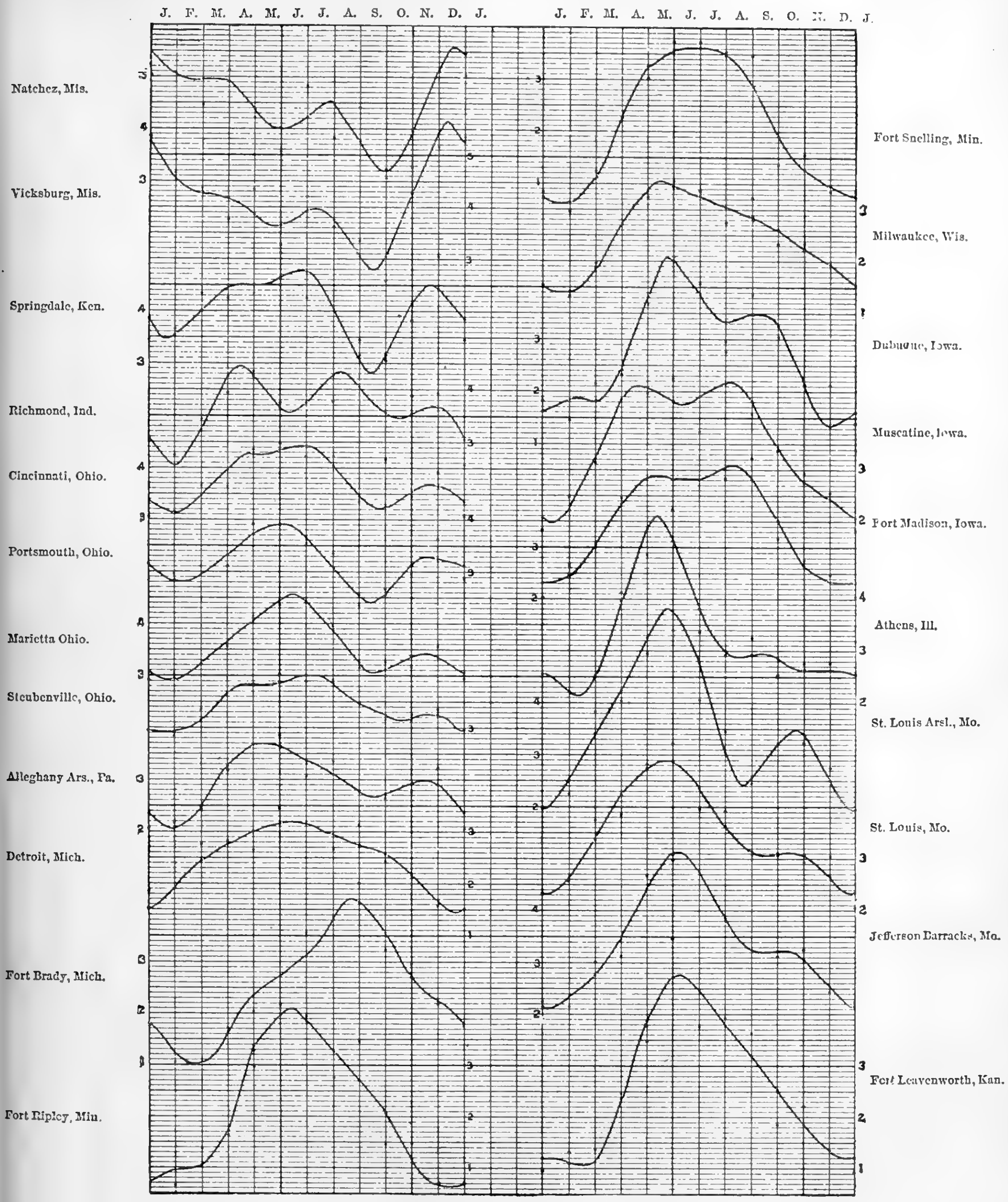
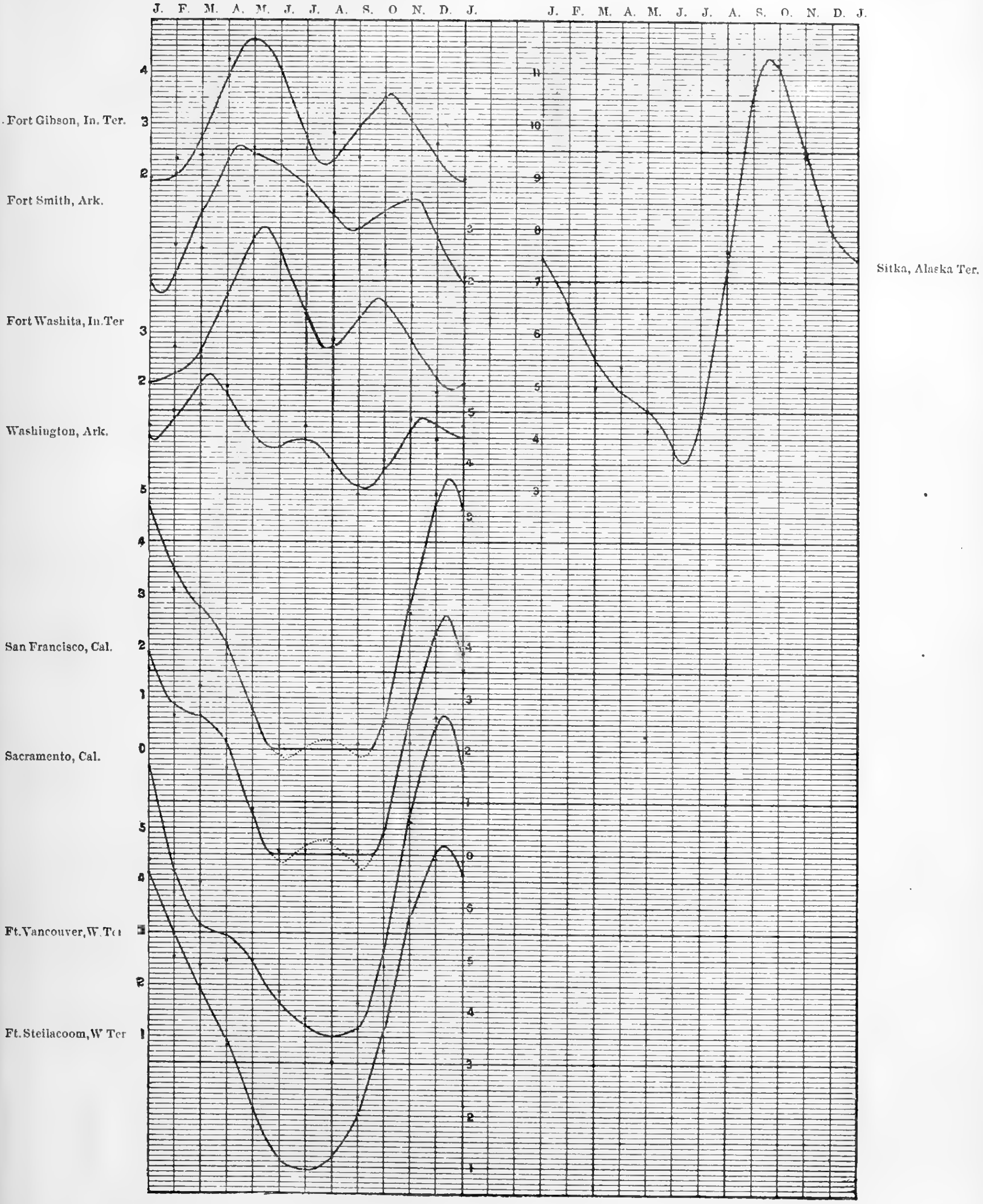


PLATE V.

ANNUAL FLUCTUATION IN THE RAIN FALL.

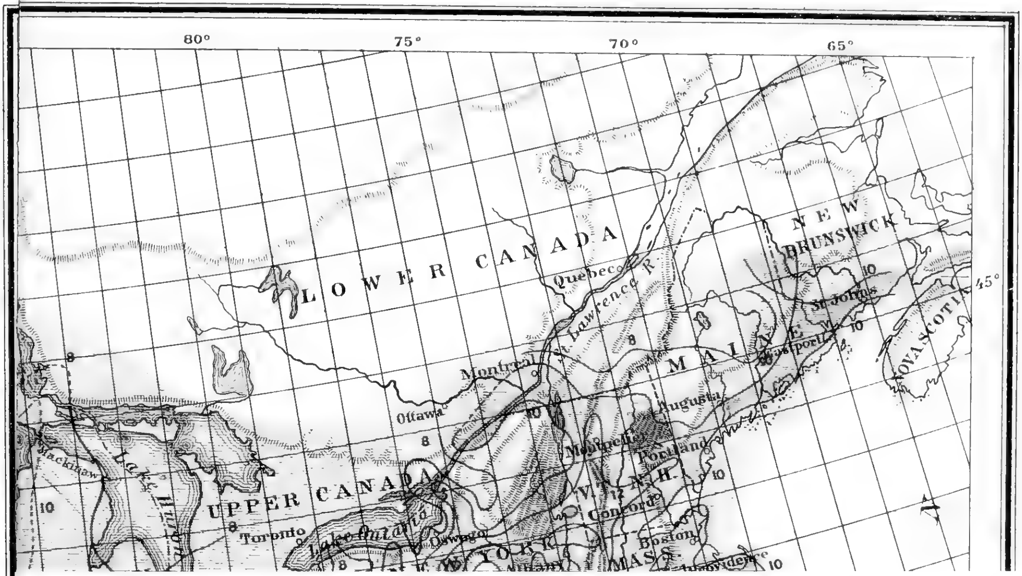


S

E SUMMER (JUN.JUL.AUG.)

le for the Smithsonian Institution.

SUMMER

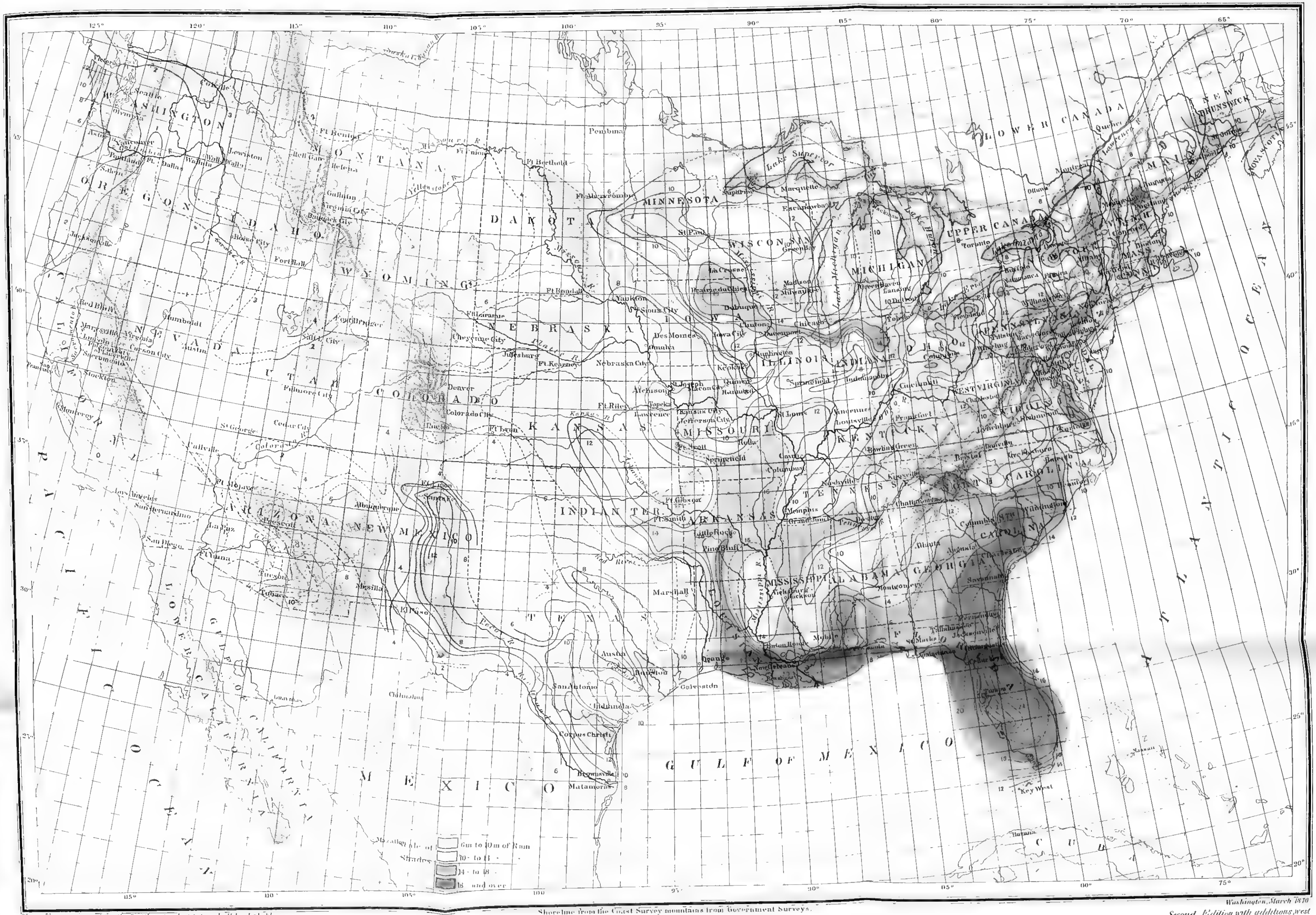


RAIN-CHART OF THE UNITED STATES

SHOWING THE DISTRIBUTION BY ISOHYETAL LINES OF THE MEAN PRECIPITATION IN RAIN FOR THE SUMMER (JUN. JUL. AUG.)

Constructed under the direction of PROF. JOSEPH HENRY, SECY. SM. IN. from materials collected, and observations made for the Smithsonian Institution,
by Chas. A. Schott Asst U.S. Coast Survey in Aug. 1868

SUMMER



Base Chart engraved for the Smithsonian Institution by H. Lindenkohl

Shoreline from the Coast Survey mountains from Government Surveys.
Scale 1/10,000,000
The numbers in blue indicate the rain fall in English inches
the broken lines are approximations
The precipitation on the mountain ranges west of Longitude 101
though often considerable is too imperfectly known to be delineated

[illegible]

Washington, March 1870

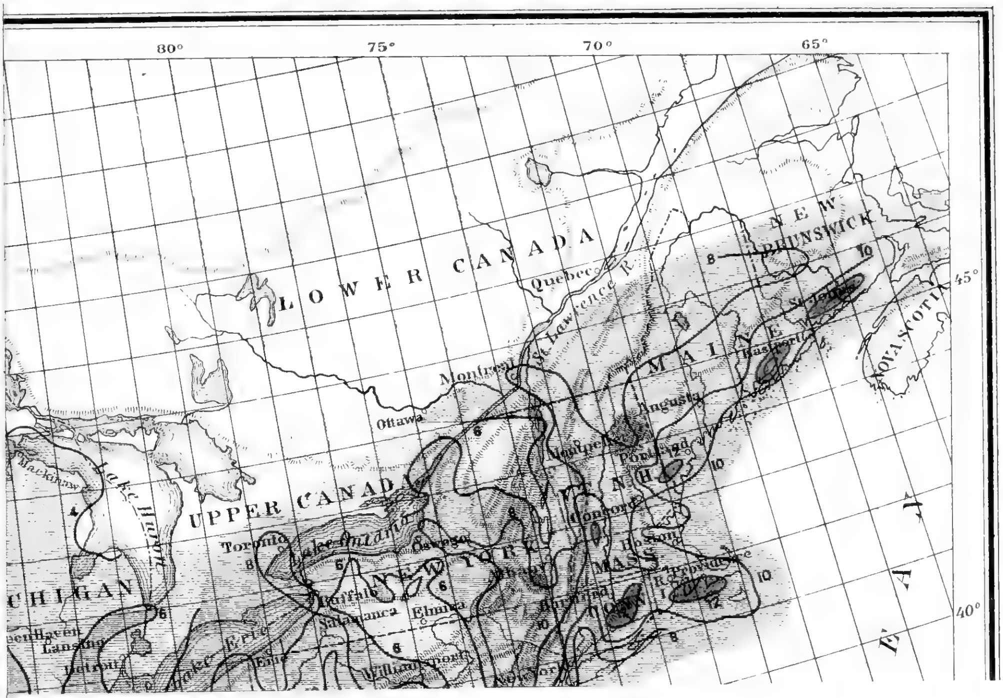
Second Edition with additions, west of longitude 95° to 1872.

ES

NOW FOR THE WINTER (DEC. JAN. FEB.)

de for the Smithsonian Institution.

WINTER



RAIN-CHART OF THE UNITED STATES

SHOWING THE DISTRIBUTION BY ISOHYETAL LINES OF THE MEAN PRECIPITATION IN RAIN AND MELTED SNOW FOR THE WINTER (DEC. JAN. FEB.)

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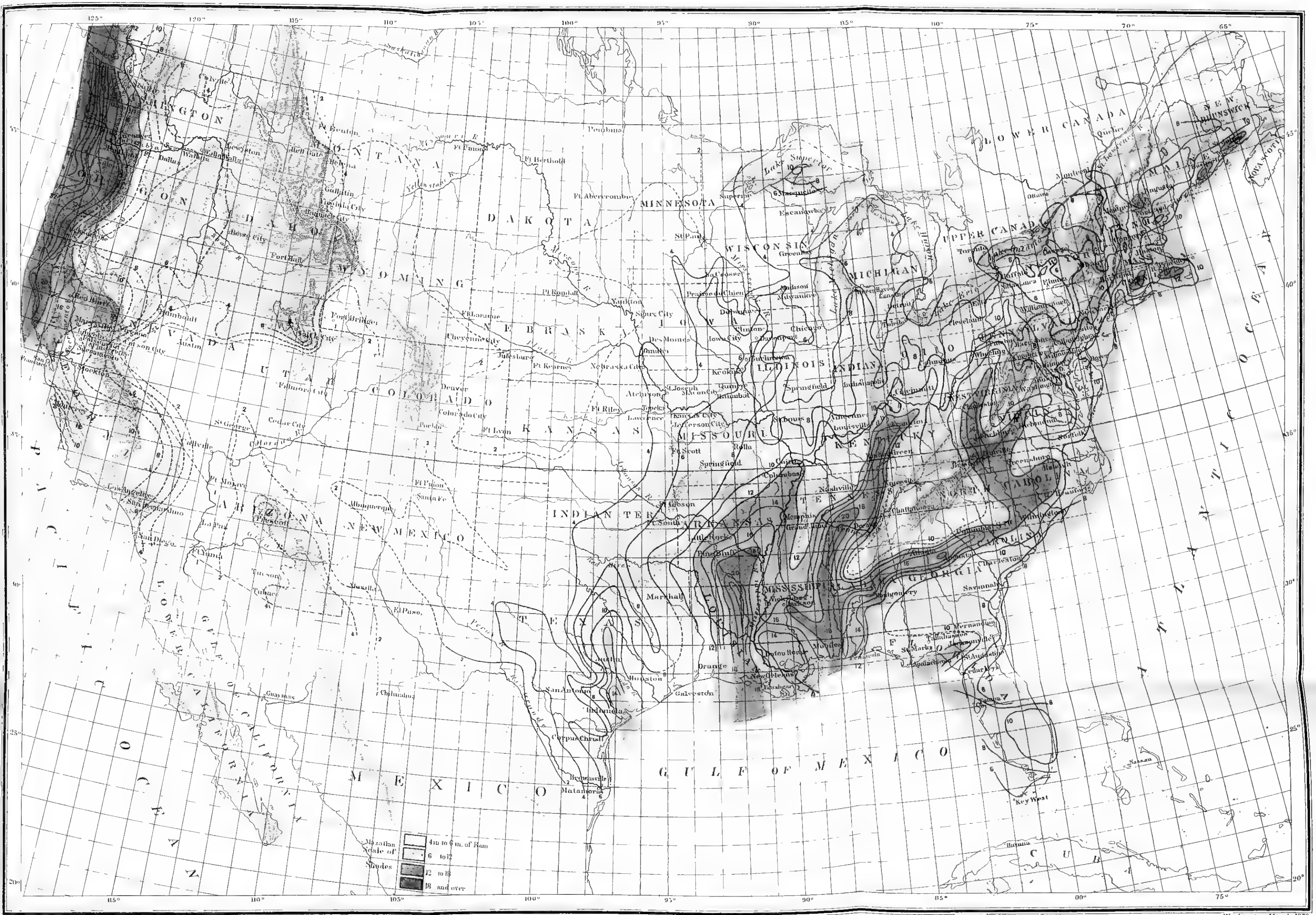
WINTER



Base Chart, engraved for the Smithsonian Institution by H. Lindenkohl

Shoreline from the Coast Survey, mountains from Government Surveys
Scale 10,000,000.

The numbers in blue indicate the rain fall in English inches
the broken lines are approximations
The precipitation on the mountain ranges west of Longitude 101°
though often considerable is too imperfectly known to be delineated.

Washington, March 1870.

Second Edition with additions west of longitude 95°, to 1872.

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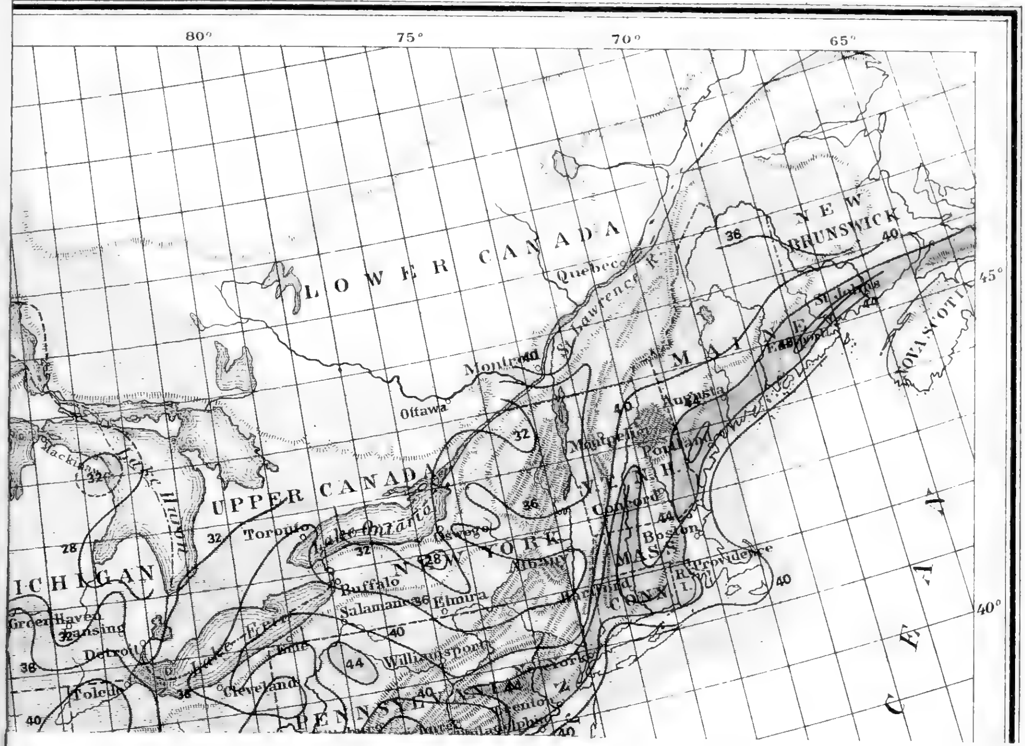
ES

MELTED SNOW FOR THE YEAR.

ade for the Smithsonian Institution.

FOR THE YEAR.



RAIN-CHART OF THE UNITED STATES

SHOWING THE DISTRIBUTION BY ISOHYETAL LINES OF THE MEAN PRECIPITATION IN RAIN AND MELTED SNOW FOR THE YEAR.

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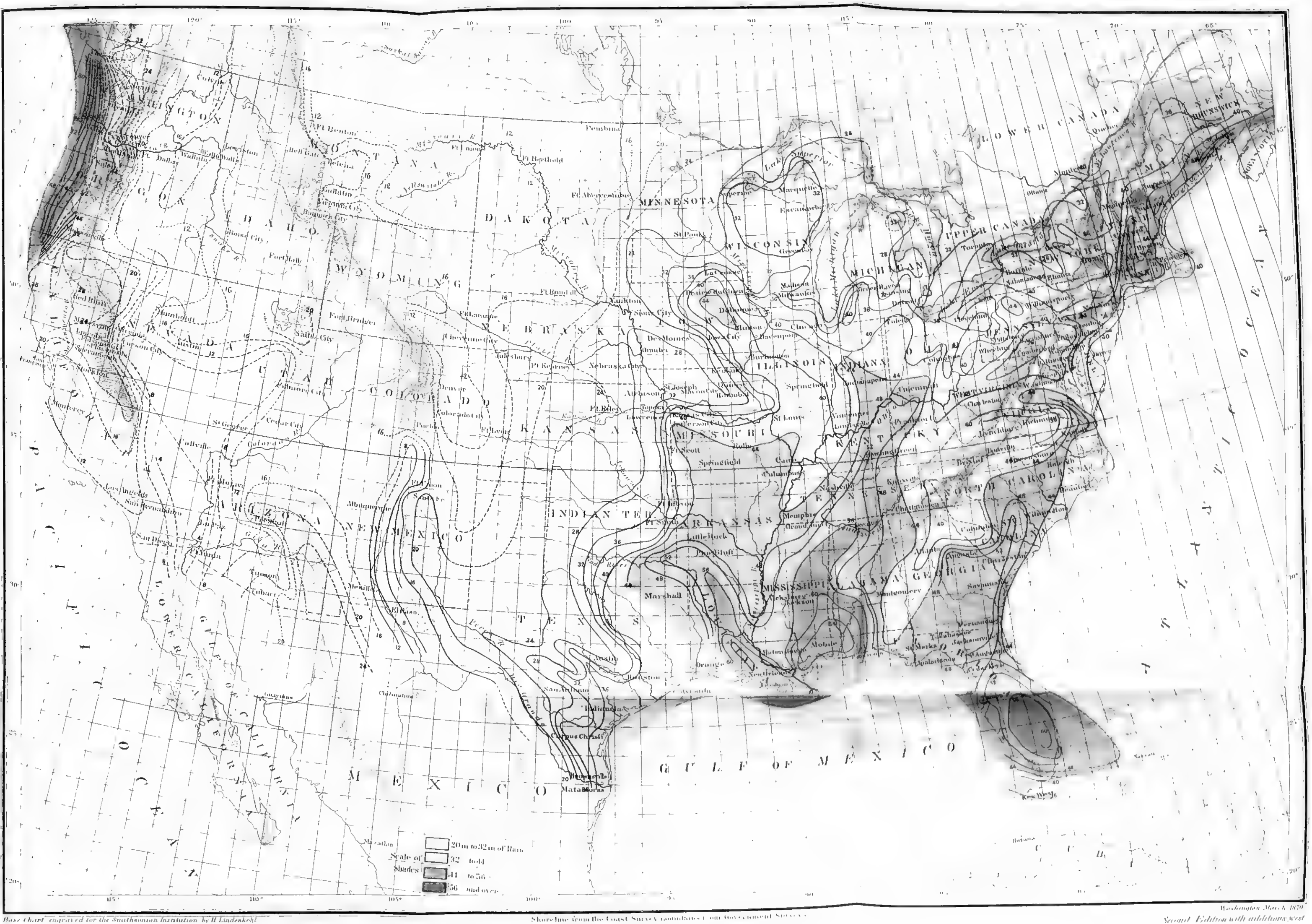
by Chas. A. Schott Asst. U.S. Coast Survey in Aug. 1868.

FOR THE YEAR.



Base Chart engraved for the Smithsonian Institution by H. Lindenkohl

Shoreline from the Coast Survey, boundaries from Government Surveys.
Scale 10 000 000
The numbers in blue indicate the rain fall in English inches
the broken lines are approximations
[illegible]
[illegible]

Washington March 1870.
Second Edition with additions west of longitude 95 to 1872.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.
232

MEMOIR

ON THE

SECULAR VARIATIONS OF THE ELEMENTS

OF THE

ORBITS OF THE EIGHT PRINCIPAL PLANETS,

MERCURY, VENUS, THE EARTH, MARS, JUPITER, SATURN, URANUS, AND NEPTUNE;

WITH TABLES OF THE SAME;

TOGETHER WITH THE

OBLIQUITY OF THE ECLIPTIC, AND THE PRECESSION OF THE EQUINOXES
IN BOTH LONGITUDE AND RIGHT ASCENSION.

BY
JOHN N. STOCKWELL, M.A.

[ACCEPTED FOR PUBLICATION, DECEMBER, 1870.]

ADVERTISEMENT.

This memoir was presented to the Institution by Professor J. H. C. Coffin, Superintendent of the American Nautical Almanac, and was afterwards submitted to Prof. S. Newcomb of the Naval Observatory, both of whom recommended its adoption as one of the Smithsonian Contributions to Knowledge. The appropriation for printing at the time it was presented having been temporarily exhausted, a friend of science, who does not allow his name to be mentioned, furnished the means for its immediate publication.

JOSEPH HENRY,
Secretary.

Smithsonian Institution,
May, 1872.

PREFACE.

THE computations of the secular inequalities, the results of which are given in the following Memoir, were commenced about ten years ago, and have been continued, during many interruptions, till the present time. In the spring of 1870 the calculations were so far advanced that the greater part of the Memoir was put in form for the press; but funds for printing it were not then available, and the computations were suffered to languish till late in autumn, when provision was made through the Smithsonian Institution for its publication. The work was then resumed, completed, and forwarded to the printer without delay; but an unexpected interval of leisure occurring during the process of publication, I availed myself of the opportunity thus presented, and prepared an additional chapter containing tabular values of the elements of the planetary orbits, together with the formulæ necessary for their convenient application. It is believed that this additional chapter will contribute largely to the usefulness of the work, and be found of great practical value in the researches of astronomers.

J. N. STOCKWELL.

CLEVELAND, May, 1872.

INTRODUCTION.

THE reciprocal gravitation of matter produces disturbances in the motions of the heavenly bodies, causing them to deviate from the elliptic paths which they would follow, if they were attracted only by the sun. The determination of the amount by which the actual place of a planet deviates from its true elliptic place at any time is called the problem of planetary perturbation. The analytical solution of this problem has disclosed to mathematicians the fact that the inequalities in the motions of the heavenly bodies are produced in two distinct ways. The *first* is a direct disturbance in the elliptic motion of the body; and the *second* is produced by reason of a variation of the *elements* of its elliptic motion. The elements of the elliptic motion of a planet are six in number, viz: the mean motion of the planet and its mean distance from the sun, the eccentricity and inclination of its orbit, and the longitude of the node and perihelion. The first two are invariable· the other four are subject to both periodic and secular variations.

The inequalities in the planetary motions which are produced by the direct action of the planets on each other, and depend for their amount only on their distances and mutual configurations, are called *periodic inequalities*, because they pass through a complete cycle of values in a comparatively short period of time; while those depending on the variation of the elements of the elliptic motion are produced with extreme slowness, require an immense number of ages for their full development, and are called *secular inequalities.* The general theory of all the planetary inequalities was completely developed by La Grange and La Place nearly a century ago; and the particular theory of each planet for the periodic inequalities was given by La Place in the *Mécanique Céleste.*

The determination of the periodic inequalities of the planets has hitherto received more attention from astronomers than has been bestowed upon the secular inequalities. This is owing in part to the immediate requirements of astronomy, and also in part to the less intricate nature of the problem. It is true that an approximate knowledge of the secular inequalities is necessary in the treatment of the periodic inequalities; but since the secular inequalities are produced with such extreme slowness, most astronomers have been content with the supposition that they are developed uniformly with the time. This supposition is sufficiently near the truth to be admissible in most astronomical investigations during the comparatively short period of time over which astronomical observations or human history extends; but since the values of these variations are derived from the equations of the differential variations of the elements at a particular epoch, it follows that they afford us no knowledge respecting the ultimate condition of the planetary system, or even a near approximation to its actual condition at a time only comparatively remote from the epoch of the elements on which they are founded. But aside from any considerations connected with the immediate needs of practical astronomy, the study of the secular inequalities is one of the most interesting and important departments of physical science, because their indefinite continuance in the same direction would ultimately seriously affect the stability of the planetary system. The demonstration that the secular inequalities of the planets are not indefinitely progressive, but may be expressed analytically by a series of terms depending on the *sines* and *cosines* of angles which increase uniformly with the time, is due to La Grange and La Place. It therefore follows that the secular inequalities are periodic, and differ from the ordinary periodic inequalities only in the length of time required to complete the cycle of their values. The amount by which the elements of any planet may ultimately deviate from their mean values can only be determined by the simultaneous integration of the differential equations of these elements, which is equivalent to the summation of all the infinitesimal variations arising from the disturbing forces of all the planets of the system during the lapse of an infinite period of time.

The simultaneous integration of the equations which determine the instantaneous variations of the elements of the orbits gives rise to a complete equation in which the unknown quantity is raised to a power denoted by the number of planets, whose mutual action is considered. La Grange first showed that if any of the roots of this equation were equal or imaginary, the finite expressions for the values of the elements would contain terms involving arcs of circles or exponential quantities, without the functions of *sine* and *cosine*, and as these terms would increase indefinitely with the time, they would finally render the orbits so very eccentrical that the stability of the planetary system would be destroyed. In order to determine whether the roots of the equation were all real and unequal, he substituted the approximate values of the elements and masses which were employed by astronomers at that time in the algebraic equations, and then by determining the roots he found them to be all real and unequal. It, therefore, followed, that for the particular values of the masses employed by La Grange, the equations which determine the secular variations contain neither arcs of a circle nor exponential quantities, without the signs of *sine* and *cosine;* whence it follows that the elements of the orbits will perpetually oscillate about their mean values This investigation was valuable as a first attempt to fix the limits of the variations of the planetary elements; but, being based upon values of the masses which were, to a certain extent, gratuitously assumed, it was desirable that the important truths which it indicated should be established independently of any considerations of a hypothetic character. This magnificent generalization was effected by La Place. He proved that, whatever be the relative masses of the planets, the roots of the equations which determine the periods of the secular inequalities will all be real and unequal, provided the bodies of the system are subjected to this one condition, *that they all revolve round the sun in the same direction.* This condition being satisfied by all the members of the solar system, it follows that the orbits of the planets will never be very eccentrical or much inclined to each other by reason of their mutual attraction. The important truths in relation to the forms and positions of the planetary orbits are embodied in the two following theorems by the author of the *Mécanique Céleste:* I. *If the mass of each planet be multiplied by the product of the square of the eccentricity and square root of the mean distance, the sum of all these products will always retain the same magnitude.* II. *If the mass of each planet be multiplied by the product of the square of the inclination of the orbit and the square root of the mean distance, the sum of these products will always remain invariable.* Now, these quantities being computed for a given epoch, if their sum is found to be small, it follows from the preceding theorems that they will always remain so; consequently the eccentricities and inclinations cannot increase indefinitely, but will always be confined within narrow limits.

In order to calculate the limits of the variations of the elements with precision, it is necessary to know the correct values of the masses of all the planets. Unfortunately, this knowledge has not yet been attained. The masses of several of the planets are found to be considerably different from the values employed by La Grange in his investigations. Besides, he only took into account the action of the six principal planets which are within the orbit of Uranus. Consequently, his solution afforded only a first approximation to the limits of the secular variations of the elements.

The person who next undertook the computation of the secular inequalities was Pontécoulant, who, about the year 1834, published the third volume of his *Théorie Analytique du Système du Monde.* In this work he has given the results of his solution of this intricate problem. But the numerical values of the constants which he obtained are totally erroneous on account of his failure to employ a sufficient number of decimals in his computation. Our knowledge of the secular variations of the planetary orbits was, therefore, not increased by his researches.

In 1839 Le Verrier had completed his computation of the secular inequalities of the seven principal planets. This mathematician has given a new and accurate determination of the constants on which the amount of the secular inequalities depends; and has also given the coefficients for correcting the values of the constants for differential variations of the masses of the different planets. This investigation of Le Verrier's has been used as the groundwork of most of the subsequent corrections of the planetary elements and masses, and deservedly holds the first rank as authority concerning the secular variations of the planetary orbits. But Le Verrier's researches were far from being exhaustive, and he failed to notice some curious and interesting relations of a permanent character in the secular variations of the orbits of Jupiter, Saturn, and Uranus. Besides, the planet Neptune had not then been discovered; and the action of this planet considerably modifies the secular inequalities which would otherwise take place. We will now briefly glance at the results of our own labors on the subject.

On the first examination of the works of those authors who had investigated this problem, we perceived that the methods of reducing to numbers those analytical integrals which determine the secular variations of the elements were far from possessing that elegance and symmetry of form which usually characterize the formulas of astronomy. The first step, therefore, was to devise a system of algebraic equations, by means of which we should be enabled to obtain the values of the unknown quantities with the smallest amount of labor. It was soon found to be impracticable to deduce algebraic formulas for the constants, by the elimination of eight unknown quantities from as many linear symmetrical equations, of sufficient simplicity to be used in the deduction of exact results. It therefore became necessary to abandon the idea of a direct solution of the equations, and to seek for the best approximate method of obtaining rigorous values of the unknown quantities. This we have accomplished as completely as could be desired, and by means of the formulas which we have obtained, it is now possible to determine the secular variations of the planetary elements with less labor, perhaps, than would be necessary for the accurate determination of a comet's orbit. The methods and formulas are given in detail in the following Memoir.

After computing anew the numerical coefficients of the differential equations of the elements, we have substituted them in these equations, and have obtained, by means of successive approximations, the rigorous values of the constants corresponding to the assumed masses and elements. The details of the computation are given in the Memoir referred to, and it is unnecessary to speak of them here. We shall, therefore, only briefly allude to some of the conclusions to which our computations legitimately lead.

The object of our investigation has been the determination of the numerical values of the secular changes of the elements of the planetary orbits. These elements are four in number, viz: the eccentricities and inclinations of the orbits, and the longitudes of the nodes and perihelia. The questions that may legitimately arise in regard to the eccentricities and inclinations relate chiefly to their magnitude at any time; but we may also desire to know their rates of change at any time, and the limits within which they will perpetually oscillate. In regard to the nodes and perihelia, it is sometimes necessary to know their relative positions when referred to any plane and origin of coördinates; and also their mean motions, together with the amount by which their actual places can differ from their mean places. With respect to the magnitudes and positions of the elements, together with their rates of change, we may observe that our equations will give them for any required epoch, by merely substituting in the formulas the interval of time between the epoch required and that of the formulas, which is the beginning of the year 1850. A tabulation of the various planetary elements, of sufficient extent to supply the needs of the astronomer, is given at the close of the work. A similar tabulation of the elements of the earth's orbit of sufficient extent to be useful in geological investigations, does not come within the scope of our work; and we leave the computation of the elements for special epochs to those investigators who may need them in their researches. We shall here give the limits between which the eccentricities and inclinations will always oscillate, together with the mean motions of the perihelia and nodes on the fixed ecliptic of 1850; and shall also give the inclinations and nodes referred to the invariable plane of the planetary system.

For the planet Mercury, we find that the eccentricity is always included within the limits 0.1214943 and 0.2317185. The mean motion of its perihelion is 5″.463803; and it performs a complete revolution in the heavens in 237,197 years. The maximum inclination of his orbit to the fixed ecliptic of 1850 is 10° 36′ 20″, and its minimum inclination is 3° 47′ 8″; while with respect to the invariable plane of the planetary system, the limits of the inclination are 9° 10′ 41″ and 4° 44′ 27″. The mean motion of the node of Mercury's orbit on the ecliptic of 1850, and on the invariable plane, is in both cases the same, and equal to 5″.126172, making a complete revolution in the interval of 252,823 years. The amount by which the true place of the node can differ from its mean place on the ecliptic of 1850 is equal to 30° 8′, while on the invariable plane this limit is only 18° 31′.

For the planet Venus, we find that the eccentricity always oscillates between 0 and 0.0706329. Since the theoretical eccentricity of the orbit of Venus is a vanishing element, it follows that the perihelion of her orbit can have no mean motion, but may have any rate of motion, at different times, between nothing and infinity, both direct and retrograde. The position of her perihelion cannot therefore be determined within given limits at any very remote epoch by the assumption of any particular value for the mean motion, but it must be determined by direct computation from the finite formulas. The maximum inclination of her orbit to the ecliptic of 1850 is 4° 51′, and to the invariable plane it is 3° 16′.3; while the mean motion of her node on both planes is indeterminate, because the inferior limit of the inclination is in each case equal to nothing.

A knowledge of the elements of the earth's orbit is especially interesting and important on account of the recent attempts to establish a connection between geological phenomena and terrestrial temperatures, in so far as the latter is modified by the variable eccentricity of her orbit. The amount of light and heat received from the sun in the course of a year depends to an important extent on the eccentricity of the earth's orbit; but the distribution of the same over the surface of the earth depends on the relative position of the perihelion of the orbit with respect to the equinoxes, and on the obliquity of the ecliptic to the equator. These elements are subject to great and irregular variations; but their laws can now be determined with as much precision as the exigencies of science may require. We will now more carefully examine these elements, and the consequences to which their variations give rise.

As we have already computed the eccentricity of the earth's orbit at intervals of 10,000 years during a period of 2,000,000 years, by employing the constants which correspond to the assumed mass of the earth increased by its twentieth part, we here give the elements corresponding to this increased mass. We therefore find that the eccentricity of the earth's orbit will always be included within the limits of 0 and 0.0693888; and it consequently follows that the *mean* motion of the perihelion is indeterminate, although the actual motion and position at any time during a period of 2,000,000 years can be readily found by means of the tabular value of that element. The eccentricity of the orbit at any time can also be found by means of the same table.

The inclination of the apparent ecliptic to the fixed ecliptic of 1850 is always less than 4° 41′; while its inclination to the invariable plane of the planetary system always oscillates within the limits 0° 0′ and 3° 6′. It is also evident that the mean motion of the node of the apparent ecliptic on the fixed ecliptic of 1850, and also on the invariable plane, is wholly indeterminate.

The mean value of the precession of the equinoxes on the fixed ecliptic, and also on the apparent ecliptic, in a Julian year, is equal to 50″.438239; whence it follows, that the equinoxes perform a complete revolution in the heavens in the average interval of 25,694.8 years; but on account of the secular inequalities in their motion, the time of revolution is not always the same, but may differ from the mean time of revolution by 281.2 years. We also find that if the place of the equinox be computed for any time whatever, by using the mean value of precession, its place when thus determined can never differ from its true place to a greater extent than 3° 56′ 26″. The maximum and minimum values of precession in a Julian year are 52″.664080 and 48″.212398, respectively, and since the length of the tropical year depends on the annual precession, it follows that the maximum variation of the tropical year is equal to the mean time required for the earth to describe an arc which is equal to the maximum variation of precession. Now this latter quantity being 4″.451682, and the sidereal motion of the earth in a second of time being 0″.041067, it follows that the maximum variation of the tropical year is equal to 108.40 seconds of time. In like manner, if we take the difference between the present value of precession and the maximum and minimum values of the same quantity, we shall find that the tropical year may be shorter than at present by 59.13 seconds, and longer than at present by 49.27 seconds. We also find that the tropical year is now shorter than in the time of Hipparchus, by 11.30 seconds.

The obliquity of the equator to the apparent ecliptic, and also to the fixed ecliptic of 1850, has also been determined. The variations of this element follow a law similar to that which governs the variation of precession, although the maximum values of the inequalities are considerably smaller than those which affect this latter quantity. The mean value of the obliquity of both the apparent and fixed ecliptics to the equator is 23° 17′ 17″. The limits of the obliquity of the apparent ecliptic to the equator are 24° 35′ 58″ and 21° 58′ 36″; whence it follows that the greatest and least declinations of the sun at the solstices can never differ from each other to any greater extent than 2° 37′ 22″. And here we may mention a few, among the many happy, consequences which result from the spheroidal form of the earth. Were the earth a perfect sphere there would be no precession or change of obliquity arising from the attraction of the sun and moon; the equinoctial circle would form an invariable plane in the heavens, about which the solar orbit would revolve with an inclination varying to the extent of twelve degrees, and a motion equal to the planetary precession of the equinoctial points. The sun, when at the solstices, would, at some periods of time, attain the declination of 29° 17′ for many thousands of years; and again, at other periods, only to 17° 17′. The seasons would be subject to vicissitudes depending on the distance of the tropics from the equator, and the distribution of solar light and heat on the surface of the earth would be so modified as essentially to change the character of its vegetation, and the distribution of its animal life. But the spheroidal form of the earth so modifies the secular changes in the relative positions of the equator and ecliptic that the inequalities of precession and obliquity are reduced to less than one-quarter part of what they would otherwise be. The periods of the secular changes, which, in the case of a spherical earth, would require nearly two millions of years to pass through a complete cycle of values, are now reduced to periods which vary between 26,000 and 53,000 years. The secular motions which would take place in the case of a spherical earth are so modified by the actual condition of the terrestrial globe that changes in the position of the equinox and equator are now produced in a few centuries, which would otherwise require a period of many thousands of years. This consideration is of much importance in the investigation of the reputed antiquity and chronology of those ancient nations which attained proficiency in the science of astronomy, and the records of whose astronomical labors are the only remaining monument of a highly intellectual people, of whose existence every other trace has long since passed away. For it is evident that, if these changes were much slower than they are, a much longer time would be required in order to produce changes of sufficient magnitude to be detected by observation, and we should be unable to estimate the interval between the epochs of elements which differed by only a few thousand years, since they would manifestly be so nearly identical with our own that the value of legitimate conclusions would be greatly impaired by the unavoidable errors of the observations on which they were based.

The duration of the different seasons is also greatly modified by the eccentricity of the earth's orbit. At present the sun is north of the equator scarcely 186½ days, and south of the same circle about 178¾ days; thus making a difference of 7¾ days between the length of the summer and winter at present. But when the eccentricity of the orbit is nearly at its maximum, and its transverse axis also passes through the solstices, both of which conditions have, in past ages, been fulfilled, the summer, in one hemisphere, will have a period of 198¾ days, and a winter of only 166½ days, while, in the other hemispheres, these conditions will be reversed; the winter having a period of 198¾ days, and a summer of only 166½ days. The variations of the sun's distance from the earth in the course of a year, at such times, are also enormous, amounting to almost one-seventh part of its mean distance —a quantity scarcely less than 13,000,000 of miles!

Passing now to the consideration of the elements of the planet Mars, we find that the eccentricity of his orbit always oscillates within the limits 0.018475 and 0.139655; and the mean motion of his perihelion is 17″.784456. The maximum inclination of his orbit to the fixed ecliptic of 1850, and to the invariable plane of the planetary system, is 7° 28′ and 5° 56′ respectively. The minimum inclination to both planes being nothing, the mean motion of the node is indeterminate.

The secular variations of the orbits of Jupiter, Saturn, Uranus, and Neptune present some curious and interesting relations. These four planets compose a system by themselves, which is practically independent of the other planets of the system.

The maximum and minimum limits of the eccentricity of the orbits of these four planets are as follows:—

	Maximum eccentricity.	Minimum eccentricity.
Jupiter	0.0608274	0.0254928
Saturn	0.0843289	0.0123719
Uranus	0.0779652	0.0117576
Neptune	0.0145066	0.0055729

The maximum and minimum inclinations of their orbits to the invariable plane of the planetary system have the following values:—

	Maximum inclination.	Minimum inclination.
Jupiter	0° 28′ 56″	0° 14′ 23″
Saturn	1 0 39	0 47 16
Uranus	1 7 10	0 54 25
Neptune	0 47 21	0 33 43

The perihelia and nodes of their orbits have the following mean motions in a Julian year of 365¼ days:—

	Mean motion of perihelion.	Mean motion of node on the invariable plane.
Jupiter	+ 3″.716607	—25″.934567
Saturn	+22 .460848	— 25 .934567
Uranus	+ 3 .716607	— 2 .916082
Neptune	+ 0 .616685	— 0 .661666

But the most curious relation developed by this investigation pertains to the relative motions and positions of the perihelia and nodes of the three planets, Jupiter, Saturn, and Uranus. These relations are expressed by the two following theorems:—

I. *The mean motion of Jupiter's perihelion is exactly equal to the mean motion of the perihelion of Uranus, and the mean longitudes of these perihelia differ by exactly* 180°. II. *The mean motion of Jupiter's node on the invariable plane is exactly equal to that of Saturn, and the mean longitudes of these nodes differ by exactly* 180°.

We also perceive that the mean motion of Saturn's perihelion is very nearly six times that of Jupiter and Uranus, and this latter quantity is very nearly six times that of Neptune; or, more exactly, 985 times the mean motion of Jupiter's perihelion are equal to 163 times that of Saturn, and 440 times the mean motion of Neptune's perihelion are equal to 73 times that of Jupiter and Uranus. The perihelion of Saturn's orbit performs a complete revolution in the heavens in 57,700 years; the perihelia of Jupiter and Uranus in 348,700 years; while that of Neptune requires no less than 2,101,560 years to complete the circuit of the heavens. In like manner the nodes of Jupiter and Saturn, on the invariable plane, perform a complete revolution in 49,972 years; that of Uranus in 444,432 years; while the node of Neptune requires 1,958,692 years to traverse the circumference of the heavens. The motions of the nodes are retrograde, and those of the perihelia are direct; thus conforming to the same law of variation as that which obtains in the corresponding elements of the moon's motion.

We may here observe that the law which controls the motions and positions of the perihelia of the orbits of Jupiter and Uranus is of the utmost importance in relation to their mutual perturbations of Saturn's orbit. For, in the existing arrangement, the orbit of Saturn is affected only by the *difference* of the perturbations by Jupiter and Uranus; whereas, if the mean places of the perihelia of these two planets were the same, instead of differing by 180°, the orbit of Saturn would be affected by the sum of their disturbing forces. But notwithstanding this favoring condition, the elements of Saturn's orbit would be subject to very great perturbations from the superior action of Jupiter, were it not for the comparatively rapid motion of its perihelion; its equilibrium being maintained by the very act of perturbation. Indeed, the stability of Saturn's orbit depends to a very great extent upon the rapidly varying positions of its transverse axis. For, if the motions of the perihelia of Jupiter and Saturn were very nearly the same, the action of Jupiter on the eccentricity of Saturn's orbit would be at its maximum value during very long periods of time, and thereby produce great and permanent changes in the value of that element. But, in the existing conditions, the rapid motion of Saturn's orbit prevents such an accumulation of perturbation, and any increase of eccentricity is soon changed into a corresponding diminution. The same remark is also applicable to the perturbations of the forms of the orbits of Jupiter and Uranus by the disturbing action of Saturn; for the secular variations of Jupiter's orbit depend almost entirely upon the influence of Saturn, because the planet Neptune is too remote to produce much disturbance, and the mean disturbing influence of Uranus on the eccentricity of Jupiter's orbit is identically equal to nothing, by reason of the relation which always exists between the perihelia of their orbits. We may here observe that the eccentricity of the orbit of Saturn always increases, while that of Jupiter diminishes, and *vice versâ.*

The consequences which result from the mutual relations which always exist between the nodes of Jupiter and Saturn, on the invariable plane of the planetary system, are no less interesting or remarkable with respect to the *position* of the orbit of Uranus than those which result from the permanent relation between the perihelia of Jupiter and Uranus are with respect to the form of the orbit of Saturn. The mean disturbing force of Saturn on the inclination of the orbit of Uranus is about four times that of Jupiter; but as these two planets always act on the inclinations in opposite directions, it follows that the joint action of the two planets is equivalent to the action of a single planet at the distance of Saturn and having about three-fourths of his mass; so that the orbit of Uranus might attain a considerable inclination from the superior action of Saturn if allowed to accumulate during the lapse of an unlimited time, at its maximum rate of variation depending on the action of this planet. But such an accumulation of perturbation is rendered forever impossible by reason of the comparatively rapid motion of the nodes of Jupiter and Saturn, with respect to that of Uranus, on the invariable plane. By reason of this rapid motion, the secular changes of the inclination of the orbit of Uranus pass through a complete cycle of values in the period of 56,300 years. The corresponding cycle of perturbation in the eccentricity of Saturn's orbit is 69,140 years. It is the rapid motion of the orbit with respect to the forces in the one case, and the rapid motion of the forces with respect to the orbit in the other, that gives permanence of form and position to the orbits of Saturn and Uranus.

The mean angular distance between the perihelia of Jupiter and Uranus is exactly 180°; but the conditions of the variations of these elements are sufficiently elastic to allow of a considerable deviation on each side of their mean positions. The perihelion of Jupiter may differ from its mean place to the extent of 24° 10′, and that of Uranus to the extent of 47° 33′; and therefore the longitudes of the perihelia of these two planets can differ from 180° to the extent of 71° 43′. The nearest approach of the perihelia of these two planets is, therefore, 108° 17′.

In like manner the longitudes of the nodes of Jupiter and Saturn, on the invariable plane, can suffer considerable deviations from their mean positions. The actual position of Jupiter's node may differ from its mean place to the extent of 19° 38′; while that of Saturn may deviate from its mean place to the extent of 7° 7′. It therefore follows that their longitudes on the invariable plane can differ from 180° by only 26° 45′. Their nearest possible approach is 153° 15′, while their present distance apart is 166° 27′.

The inequalities in the eccentricity of Neptune's orbit are very small, and the two principal ones have periods of 613,900 years, and 418,060 years respectively. Strictly speaking, the periods of the secular inequalities of the eccentricities and perihelia are the same for all the planets; and the same remark is equally applicable to the nodes and inclinations. But the principal inequalities of the several planetary orbits are different, unless they are connected by some permanent relation, similar to that which exists between the perihelia of Jupiter and Uranus, or the nodes of Jupiter and Saturn. Thus the principal inequalities affecting the inclination of the orbits of Jupiter and Saturn have the same periods for each planet, and these periods are, for the two principal inequalities, 51,280 years, and 56,303 years. In like manner the principal inequalities in the eccentricities of Jupiter and Saturn depend on their mutual attraction, and have a period of 69,141 years. The secular inequalities of those orbits which have no vanishing elements are composed of terms, of very different orders of magnitude; and it frequently happens that two or three of these terms are greater than the sum of all the remaining ones. In such cases the variation of the corresponding element very approximately conforms to a much simpler law, and the maxima and minima repeat themselves according to definite and well-defined cycles. But with regard to the orbits of Venus, the Earth, and Mars, the rigorous expressions of the eccentricities and inclinations are composed of twenty-eight periodic terms, having coefficients of considerable magnitude; and this circumstance renders the law of their variations extremely intricate.

The method we have adopted for finding the coefficients of the corrections of the constants, depending on finite variations of the different planetary masses, consists in supposing that each planetary mass receives in succession a finite increment, and then finding the values of all the constants corresponding to this increased mass in the same manner as for the assumed masses. By this means we have a set of values corresponding to the assumed masses, and another set corresponding to a finite increment to each of the planetary masses. Then, taking the difference between the two sets of constants, and dividing by the increment, which produced it, we get the coefficient of the variation of the constants for any other finite increment of mass to the same planet; but, on account of the importance of the earth's mass, and the probable inaccuracy of its assumed value, we have prepared separate solutions corresponding to the several increments of $\frac{1}{20}$, $\frac{2}{20}$, and $\frac{3}{20}$ of its assumed mass; and a comparison of the values which the different solutions give for the superior limit of the eccentricity of the earth's orbit has suggested the inquiry whether there may not be some unknown physical relation between the masses and mean distances of the different planets. The same peculiarity in the elements of the orbit of *Venus* is also found to depend upon particular values of the mass of that planet. But without entering into details in regard to the peculiarity referred to, we here give the several values of the masses of these two planets which we have employed in our computations, and the corresponding values of the superior limit of the eccentricity of their orbits.

For Venus.		For the Earth.	
Mass m'	*Maximum* e'	Mass m''	*Maximum* e''.
m'_0	0.070633	m''_0	0.067735
$m'_0(1+\frac{1}{20})$	0.074872	$m''_0(1+\frac{1}{20})$	0.069389
$m'_0(1+\frac{2}{20})$	0.076075	$m''_0(1+\frac{2}{20})$	0.069649
$m'_0(1+\frac{3}{20})$	0.075029	$m''_0(1+\frac{3}{20})$	0.068089
$m'_0(1+\frac{4}{20})$	0.072098		

These numbers show that if the mass of Venus were to be increased, the superior limit of the eccentricity of her orbit would also increase until it had attained a maximum value, after which a further increase of her mass would diminish that limit; and the same remark is also applicable to the eccentricity of the earth's orbit.

The above numbers indicate that the superior limit of the eccentricity of the orbit of Venus is a maximum if the mass of that plant is equal to $m'_0\left(1+\frac{2.04}{20}\right)$; or, if $m'=\frac{1}{354490}$ of the sun's mass; and the superior limit of the eccentricity of the earth's orbit is a maximum if the earth's mass is equal to $m''_0\left(1+\frac{1.643}{20};\right)$ or, if $m''=\frac{1}{340700}$ of the sun's mass. But this value of the earth's mass corresponds to a solar parallax of 8″.730, a value closely approximating to the recent determinations of that element.

If, then, the mass of *Venus is* equal to $\frac{1}{354490}$, and the mass of the *earth is* equal to $\frac{1}{340700}$, it follows that the orbits of these two planets will ultimately become more eccentric from the mutual attraction of the other planets, than they would for any other values of their respective masses; and we may now inquire whether such coincidence between the superior limits of the eccentricities and the masses of these two planets has any physical significance, or is merely accidental.

Since the mean motions and mean distances of the planets are invariable, and independent of the eccentricities of the orbits, it would seem that there could be no connection between these elements, by means of which the stability of the system might be secured or impaired; but a more careful examination shows that, although the mean motions or times of revolution of the planets are invariable, their actual velocities, or the variation of their mean velocities, depends wholly on the eccentricities; and, were any of the planetary orbits to become extremely elliptical, the velocity of the planet would be subject to great variations of velocity,—moving with very great rapidity when in perihelion, and with extreme slowness when in the neighborhood of its aphelion; and it is evident that when the planet was in this latter position a small foreign force acting upon it might so change its velocity as to completely change the elements of its orbit, by causing it to fall upon the sun or fly off into remoter space. A system of bodies moving in very eccentrical orbits is, therefore, one of manifest instability; and if it can also be shown that a system of bodies moving in circular orbits is one of unstable equilibrium, it would seem that, between the two supposed conditions, a system might exist which should possess a greater degree of stability than either. The idea is thus suggested of the existence of a system of bodies in which the masses of the different bodies are so adjusted to their mean distances as to insure to the system a greater degree of permanence than would be possible by any other distribution of masses. The mathematical expression of a criterion for such distribution of masses has not yet been fully developed; and the preceding illustrations have been introduced here, more for the purpose of calling the attention of mathematicians and astronomers to this interesting problem than for any certain light we have yet been able to obtain in regard to its solution.

MEMOIR

ON

THE SECULAR VARIATIONS OF THE ELEMENTS OF THE ORBITS OF THE EIGHT PRINCIPAL PLANETS,

MERCURY VENUS THE EARTH MARS, JUPITER, SATURN, URANUS, AND NEPTUNE.

CHAPTER I.

ON THE SECULAR VARIATIONS OF THE ECCENTRICITIES AND PERIHELIA.

1. We shall assume as the basis of our computation the following differential equations, which determine the instantaneous variations of the eccentricities and places of the perihelia of the planetary orbits at any time. These equations are demonstrated by **La Place**, in Book II, Chapter VII, of the *Mécanique Céleste;* and by **Pontécoulant**, in Book II, Chapter VIII, of his *Théorie Analytique du Système du Monde*, and are as follows:—

$$\left.\begin{aligned}
\frac{dh}{dt} &= \left\{(0,1)+(0,2)+(0,3)+\&c.\right\} l - \boxed{0,1}\,l' - \boxed{0,2}\,l'' - \boxed{0,3}\,l''' - \&c.;\\
\frac{dl}{dt} &= -\left\{(0,1)+(0,2)+(0,3)+\&c.\right\} h + \boxed{0,1}\,h' + \boxed{0,2}\,h'' + \boxed{0,3}\,h''' + \&c.;\\
\frac{dh'}{dt} &= \left\{(1,0)+(1,2)+(1,3)+\&c.\right\} l' - \boxed{1,0}\,l - \boxed{1,2}\,l'' - \boxed{1,3}\,l''' - \&c.;\\
\frac{dl'}{dt} &= -\left\{(1,0)+(1,2)+(1,3)+\&c.\right\} h' + \boxed{1,0}\,h + \boxed{1,2}\,h'' + \boxed{1,3}\,h''' + \&c.;\\
&\&c.
\end{aligned}\right\} \quad (A)$$

If we denote by e, e', e'', &c., ϖ, ϖ', ϖ'', &c., the eccentricities and longitudes of the perihelia of the orbits of *Mercury*, *Venus*, the *Earth*, &c., we shall have the following equations for the determination of these quantities:—

$$\left.\begin{aligned}
h&=e\sin\varpi, & h'&=e'\sin\varpi', & h''&=e''\sin\varpi'', \ \&c.,\\
l&=e\cos\varpi. & l'&=e'\cos\varpi', & l''&=e''\cos\varpi'', \ \&c.
\end{aligned}\right\} \quad (1)$$

Whence we deduce

$$e^2=h^2+l^2,\quad e'^2=h'^2+l'^2,\quad e''^2=h''^2+l''^2,\ \&c.;\ \tan\varpi=\frac{h}{l},\ \tan\varpi'=\frac{h'}{l'},\ \&c. \quad (2)$$

If h, l, h', l', &c., are determined by the integrals of equations (A), for any time whatever, and substituted in equations (2), we shall obtain the corresponding values of e, e', &c., ϖ, ϖ', &c.

2. Now to find the integrals of equations (A), we shall suppose

$$\left.\begin{array}{lll} h=N\sin(gt+\beta), & h'=N'\sin(gt+\beta) & h''=N''\sin(gt+\beta), \text{ \&c.,} \\ l=N\cos(gt+\beta), & l'=N'\cos(gt+\beta) & l''=N''\cos(gt+\beta), \text{ \&c.} \end{array}\right\} (3)$$

If these values be substituted in equations (A), they will become,

$$\left.\begin{array}{l} Ng=\{(0,1)+(0,2)+(0,3)+\text{ \&c.}\}\,N-\boxed{0,1}N'-\boxed{0,2}N''-\boxed{0,3}N'''-\text{ \&c.} \\ N'g=\{(1,0)+(1,2)+(1,3)+\text{ \&c.}\}\,N'-\boxed{1,0}N-\boxed{1,2}N''-\boxed{1,3}N'''-\text{ \&c.} \\ N''g=\{(2,0)+(2,1)+(2,3)+\text{ \&c.}\}\,N''-\boxed{2,0}N-\boxed{2,1}N'-\boxed{2,3}N'''-\text{ \&c.} \\ \text{\&c.} \end{array}\right\} \text{(B)}$$

If we suppose the number of planets whose mutual action is considered, to be i, the number of these equations will be i; and by eliminating the constant quantities N, N', N'', &c., we shall obtain a final equation in g of the degree i.

3. The quantities $(0,1)$ and $(1,0)$, $\boxed{0,1}$, $\boxed{1,0}$; $(0,2)$ and $(2,0)$, $\boxed{0,2}$, $\boxed{2,0}$; $(1,2)$ and $(2,1)$; $\boxed{1,2}$ and $\boxed{2,1}$, &c., have some remarkable relations with each other, which not only facilitate their computation, but render the equations resulting from the elimination of N, N', N'', &c., much shorter and more commodious. The general expression for $(0,1)$ is

$$(0,1)=-\frac{3m'na^2a'(a,a')'}{4(a'^2-a^2)^2}. \qquad (4)$$

In this equation n and a denote the mean motion and mean distance of *Mercury*, m' denotes the mass of *Venus* and a' its mean distance from the sun. If we change n, a into n', a', and m', a' into m, a, respectively, $(0,1)$ will change it into $(1,0)$, and we shall have

$$(1,0)=-\frac{3mn'a'^2a(a',a)'}{4(a'^2-a^2)^2}. \qquad (5)$$

Now since $(a, a')'=(a', a)'$, equations (4) and (5) will give

$$(1,0)=(0,1)\frac{m}{na}\cdot\frac{n'a'}{m'}; \qquad (6)$$

we shall also have

$$\left.\begin{array}{l} (2,0)=(0,2)\dfrac{m}{na}\cdot\dfrac{n''a''}{m}; \\ (3,0)=(0,3)\dfrac{m}{na}\cdot\dfrac{n'''a'''}{m'''}. \\ \text{\&c.} \end{array}\right\} \qquad (7)$$

The same relations also hold with respect to the quantities $\boxed{0,1}$, $\boxed{1,0}$, $\boxed{0,2}$, $\boxed{2,0}$, &c., so that we shall also have

$$\left.\begin{array}{l} \boxed{1,0}=\boxed{0,1}\dfrac{m}{na}\cdot\dfrac{n'a'}{m'}; \\ \boxed{2,0}=\boxed{0,2}\dfrac{m}{na}\cdot\dfrac{n''a''}{m''}; \\ \boxed{3,0}=\boxed{0,3}\dfrac{m}{na}\cdot\dfrac{n'a'''}{m'''}. \\ \text{\&c.} \end{array}\right\} \qquad (8)$$

Therefore when we have computed the quantities $(0,1)$, $(0,2)$, $(0,3)$, &c., or the coefficient for an interior planet, depending on the action of an exterior planet, we shall obtain the corresponding coefficient for an exterior planet, depending on the action of an interior planet, by means of equations (6), (7), and (8).

Equations (6) and (7) may be written as follows:—

$$\left.\begin{aligned} &(1,0)\,\frac{m'}{n'a'} = (0,1)\,\frac{m}{na} \\ &(2,0)\,\frac{m''}{n''a''} = (0,2)\,\frac{m}{na} \\ &\&c. \end{aligned}\right\} \qquad (9)$$

We shall also similarly have

$$\left.\begin{aligned} &(2,1)\,\frac{m''}{n''a''} = (1,2)\,\frac{m'}{n'a'} \\ &(3,1)\,\frac{m'''}{n'''a'''} = (1,3)\,\frac{m'}{n'a'} \\ &(3,2)\,\frac{m'''}{n'''a'''} = (2,3)\,\frac{m''}{n''a''} \\ &\&c. \end{aligned}\right\} \qquad (10)$$

We shall therefore have

$$\left.\begin{aligned} &(1,2)(2,0)(0,1)=(2,1)(1,0)(0,2), \\ &(1,3)(3,2)(2,1)=(3,1)(1,2)(2,3), \\ &(3,0)(0,2)(2,3)=(0,3)(3,2)(2,0), \\ &(3,1)(1,0)(0,3)=(1,3)(3,0)(0,1), \\ &\&c. \end{aligned}\right\}. \qquad (11)$$

We shall also have the following products of four factors

$$\left.\begin{aligned} &(0,1)(1,2)(2,3)(3,0)=(1,0)(0,3)(3,2)(2,1), \\ &(0,1)(1,3)(3,2)(2,0)=(1,0)(0,2)(2,3)(3,1), \\ &(0,2)(2,1)(1,3)(3,0)=(2,0)(0,3)(3,1)(1,2), \\ &(1,2)(2,3)(3,4)(4,1)=(2,1)(1,4)(4,3)(3,2), \\ &\&c \end{aligned}\right\}. \qquad (12)$$

And of five factors we have

$$\left.\begin{aligned} &(0,1)(1,2)(2,3)(3,4)(4,0)=(1,0)(0,4)(4,3)(3,2)(2,1), \\ &(0,1)(1,2)(2,4)(4,3)(3,0)=(1,0)(0,3)(3,4)(4,2)(2,1), \\ &(1,2)(2,3)(3,4)(4,5)(5,1)=(2,1)(1,5)(5,4)(4,3)(3,2), \\ &(1,2)(2,4)(4,3)(3,5)(5,1)=(2,1)(1,5)(5,3)(3,4)(4,2), \\ &\&c. \end{aligned}\right\}. \qquad (13)$$

And in like manner we may form the products of six, or any number of factors. Equations (11), (12), and (13) are very useful in reducing two terms to a single one. We may in like manner form the following equations:—

$$\left.\begin{aligned} &\boxed{0,1}\,\boxed{1,2}\,\boxed{2,0}=\boxed{1,0}\,\boxed{0,2}\,\boxed{2,1}, \\ &\boxed{1,3}\,\boxed{3,2}\,\boxed{2,1}=\boxed{3,1}\,\boxed{1,2}\,\boxed{2,3}, \\ &\boxed{2,3}\,\boxed{3,0}\,\boxed{0,2}=\boxed{3,2}\,\boxed{2,0}\,\boxed{0,3}, \\ &\boxed{3,1}\,\boxed{1,0}\,\boxed{0,3}=\boxed{1,3}\,\boxed{3,0}\,\boxed{0,1}, \\ &\&c. \end{aligned}\right\}. \qquad (14)$$

$$\left.\begin{array}{l}
\boxed{0,1}\,\boxed{1,2}\,\boxed{2,3}\,\boxed{3,0}=\boxed{1,0}\,\boxed{0,3}\,\boxed{3,2}\,\boxed{2,1},\\
\boxed{0,1}\,\boxed{1,3}\,\boxed{3,2}\,\boxed{2,0}=\boxed{1,0}\,\boxed{0,2}\,\boxed{2,3}\,\boxed{3,1},\\
\boxed{0,2}\,\boxed{2,1}\,\boxed{1,3}\,\boxed{3,0}=\boxed{2,0}\,\boxed{0,3}\,\boxed{3,1}\,\boxed{1,2},\\
\boxed{1,2}\,\boxed{2,3}\,\boxed{3,4}\,\boxed{4,1}=\boxed{2,1}\,\boxed{1,4}\,\boxed{4,3}\,\boxed{3,2},\\
\quad\&c.
\end{array}\right\}. \qquad (15)$$

$$\left.\begin{array}{l}
\boxed{0,1}\,\boxed{1,2}\,\boxed{2,3}\,\boxed{3,4}\,\boxed{4,0}=\boxed{1,0}\,\boxed{0,4}\,\boxed{4,3}\,\boxed{3,2}\,\boxed{2,1},\\
\boxed{0,1}\,\boxed{1,2}\,\boxed{2,4}\,\boxed{4,3}\,\boxed{3,0}=\boxed{1,0}\,\boxed{0,3}\,\boxed{3,4}\,\boxed{4,2}\,\boxed{2,1},\\
\boxed{1,2}\,\boxed{2,3}\,\boxed{3,4}\,\boxed{4,5}\,\boxed{5,1}=\boxed{2,1}\,\boxed{1,5}\,\boxed{5,4}\,\boxed{4,3}\,\boxed{3,2},\\
\boxed{1,2}\,\boxed{2,4}\,\boxed{4,3}\,\boxed{3,5}\,\boxed{5,1}=\boxed{2,1}\,\boxed{1,5}\,\boxed{5,3}\,\boxed{3,4}\,\boxed{4,2},\\
\quad\&c.
\end{array}\right\} \qquad (16)$$

4. The quantities $(0,1)$, $(0,2)$, $(0,3)$, &c., $(1,0)$, $(1,2)$, $(1,3)$, &c.; $\boxed{0,1}$, $\boxed{0,2}$, $\boxed{0,3}$, &c., $\boxed{1,0}$, $\boxed{1,2}$, $\boxed{1,3}$, &c.; depend on the masses and mean distances of the different planets. The analytical expressions of $(0,1)$ and $\boxed{0,1}$ are as follows: *Méc. Cél.* [1076], [1082], *Bowditch's Translation.*

$$(0,1)=-\frac{3m'n\alpha^2 b^{(1)}_{-\frac{1}{2}}}{4(1-\alpha^2)^2}; \qquad (17)$$

$$\boxed{0,1}=-\frac{3m'n\alpha\{(1+\alpha^2)b^{(1)}_{-\frac{1}{2}}+\frac{1}{2}\alpha b^{(0)}_{-\frac{1}{2}}\}}{2(1-\alpha^2)^2}. \qquad (18)$$

In these equations n denotes the mean motion of the disturbed planet; m' the mass of the disturbing planet; α the ratio of the mean distances of the inner and outer planets, $b^{(0)}_{-\frac{1}{2}}$ and $b^{(1)}_{-\frac{1}{2}}$ depend entirely on α, and are given by equations [989] *Méc. Cél.* If we reduce the coefficients of the different powers of α to decimal numbers we shall have the following equations to determine $b^{(0)}_{-\frac{1}{2}}$ and $b^{(1)}_{-\frac{1}{2}}$.

$$\left.\begin{array}{l}
\frac{1}{2}b^{(0)}_{-\frac{1}{2}}=1+0.25\alpha^2+0.015625\alpha^4+0.003906249\alpha^6+0.001525878\alpha^8\\
\qquad +0.0007476805\alpha^{10}+0.0004205703\alpha^{12}+0.0002596378\alpha^{14}\\
\qquad +0.0001714015\alpha^{16}+0.0001190288\alpha^{18}+0.00008599836\alpha^{20}\\
\qquad +0.00006414336\alpha^{22}+0.00004910978\alpha^{24}+0.00003843058\alpha^{26}\\
\qquad +0.00003063663\alpha^{28}+0.00002481567\alpha^{30}+\&c.
\end{array}\right\}. \qquad (18)$$

$$\left.\begin{array}{l}
b^{(1)}_{-\frac{1}{2}}=-\alpha+0.125\alpha^3+0.015625\alpha^5+0.004882812\alpha^7+0.002136230\alpha^9\\
\qquad +0.001121521\alpha^{11}+0.0006608960\alpha^{13}+0.0004219114\alpha^{15}\\
\qquad +0.0002856691\alpha^{17}+0.0002023490\alpha^{19}+0.0001485425\alpha^{21}\\
\qquad +0.0001122509\alpha^{23}+0.00008688650\alpha^{25}+0.00006862602\alpha^{27}\\
\qquad +0.00005514591\alpha^{29}+0.00004497838\alpha^{31}+\&c.
\end{array}\right\}. \qquad (19)$$

If we now multiply equation (19) by $1+\alpha^2$, and equation (18) by α, and put

$$\{(1+\alpha^2)b^{(1)}_{-\frac{1}{2}}+\tfrac{1}{2}\alpha b^{(0)}_{-\frac{1}{2}}\}\alpha=b^{(0)}, \qquad (20)$$

we shall have

$$\left.\begin{array}{l}
b^{(0)}=-0.625\alpha^4+0.15625\alpha^6+0.024414061\alpha^8+0.008544920\alpha^{10}\\
\qquad +0.004005431\alpha^{12}+0.002202987\alpha^{14}+0.0013424452\alpha^{16}\\
\qquad +0.0008789820\alpha^{18}+0.0006070469\alpha^{20}+0.0004368899\alpha^{22}\\
\qquad +0.0003249368\alpha^{24}+0.0002482472\alpha^{26}+0.0001939431\alpha^{28}\\
\qquad +0.00015440856\alpha^{30}+0.00012493996\alpha^{32}+\&c.
\end{array}\right\}. \qquad (21)$$

It will manifestly be unnecessary to compute $b_{-\frac{1}{2}}^{(0)}$ separately, since it is already included in the value of $b^{(0)}$. Therefore taking logarithmic coefficients of equations (19) and (21), we shall have the two following working formulæ for the computation of $b^{(0)}$ and $b_{-\frac{1}{2}}^{(1)}$.

$$\left.\begin{aligned} b^{(0)} = -\alpha^4\{0.625 &- [9.1938200]\alpha^2 - [8.3876400]\alpha^4 - [7.9317080]\alpha^6 \\ &- [7.6026493]\alpha^8 - [7.3430119]\alpha^{10} - [7.1278964]\alpha^{12} \\ &- [6.9439800]\alpha^{14} - [6.7832222]\alpha^{16} - [6.6403720]\alpha^{18} \\ &- [6.5117989]\alpha^{20} - [6.3948844]\alpha^{22} - [6.2876743]\alpha^{24} \\ &- [6.1886714]\alpha^{26} - [6.0967014]\alpha^{28} - \&c.\} \end{aligned}\right\} . \quad (22)$$

$$\left.\begin{aligned} b_{-\frac{1}{2}}^{(1)} = -\alpha\{1 &- [9.0969100]\alpha^2 - [8.1938200]\alpha^4 - [7.6886700]\alpha^6 \\ &- [7.3296480]\alpha^8 - [7.0498073]\alpha^{10} - [6.8201332]\alpha^{12} \\ &- [6.6252212]\alpha^{14} - [6.4558633]\alpha^{16} - [6.3061010]\alpha^{18} \\ &- [6.1718509]\alpha^{20} - [6.0501898]\alpha^{22} - [5.9389523]\alpha^{24} \\ &- [5.8364888]\alpha^{26} - [5.7415133]\alpha^{28} - [5.6530038]\alpha^{30} - \&c.\} \end{aligned}\right\} . \quad (23)$$

Then we shall have

$$(0,1) = -\frac{3m'n\alpha^2 b_{-\frac{1}{2}}^{(1)}}{4(1-\alpha^2)^2}, \qquad (24)$$

$$\boxed{0,1} = -\frac{3m'nb^{(0)}}{2(1-\alpha^2)^2}. \qquad (25)$$

5. To reduce the preceding formulæ to numbers we shall assume the following values of the invariable elements of the eight principal planets.

Invariable Elements of the Eight Principal Planets.

	Masses.	Mean motions in a Julian year.	Mean distances from the sun.
Mercury	$m = \frac{1+\mu}{4865751}$	$n = 5381016''.2$	$a = 0.3870987$
Venus	$m' = \frac{1+\mu'}{390000}$	$n' = 2106641.438$	$a' = 0.7233323$
The Earth . . .	$m'' = \frac{1+\mu''}{368689}$	$n'' = 1295977.440$	$a'' = 1.0000000$
Mars.	$m''' = \frac{1+\mu'''}{2680637}$	$n''' = 689050.9023$	$a''' = 1.5236878$
Jupiter	$m^{IV} = \frac{1+\mu^{IV}}{1047.879}$	$n^{IV} = 109256.719$	$a^{IV} = 5.202798$
Saturn	$m^{V} = \frac{1+\mu^{V}}{3501.6}$	$n^{V} = 43996.127$	$a^{V} = 9.538852$
Uranus	$m^{VI} = \frac{1+\mu^{VI}}{24905}$	$n^{VI} = 15424.5094$	$a^{VI} = 19.183581$
Neptune	$m^{VII} = \frac{1+\mu^{VII}}{18780}$	$n^{VII} = 7873.993$	$a^{VII} = 30.03386$

From these quantities we shall obtain the following logarithms

m	log. 93.3128501;	a	log. 9.58782172;
m'	log. 94.4089354;	a'	log. 9.85933786;
m''	log. 94.4333398;	a''	log. 0.00000000;
m'''	log. 93.5717620;	a'''	log. 0.18289600;
m^{IV}	log. 96.9796889;	a^{IV}	log. 0.71623697;
m^{V}	log. 96.4557335;	a^{V}	log. 0.97949611;
m^{VI}	log. 95.6037135;	a^{VI}	log. 1.28292968;
m^{VII}	log. 95.7263044;	a^{VII}	log. 1.47761116;

n	log. 6.7308643;	$m \div na$	log. 86.9941641;
n'	log. 6.3235906;	$m' \div n'a'$	log. 88.2260070;
n''	log. 6.1125974;	$m'' \div n''a''$	log. 88.3207424;
n'''	log. 5.8382513;	$m''' \div n'''a'''$	log. 87.5506147;
n^{IV}	log. 5.0384481;	$m^{IV} \div n^{IV}a^{IV}$	log. 91.2250039;
n^{V}	log. 4.6434145;	$m^{V} \div n^{V}a^{V}$	log. 90.8328229;
n^{VI}	log. 4.1882114;	$m^{VI} \div n^{VI}a^{VI}$	log. 90.1325724;
n^{VII}	log. 3.8961950;	$m^{VII} \div n^{VII}a^{VII}$	log. 90.3524982.

The values of a, a', a'', &c., give the following values of α and $1-\alpha^2$.

For *Mercury* and	*Venus*	$\alpha=0.5351603$	log. 9.72848386;	$1-\alpha^2$ log. 9.8534569;
"	*Earth*	$\alpha=0.3870987$	log. 9.58782172;	$1-\alpha^2$ log. 9.9294980;
"	*Mars*	$\alpha=0.2540538$	log. 9.40492572;	$1-\alpha^2$ log. 9.9710237;
"	*Jupiter*	$\alpha=0.07440202$	log. 8.87158475;	$1-\alpha^2$ log. 9.9975892;
"	*Saturn*	$\alpha=0.04058127$	log. 8.60832561;	$1-\alpha^2$ log. 9.9992842;
"	*Uranus*	$\alpha=0.02017864$	log. 8.30489204;	$1-\alpha^2$ log. 9.9998231;
"	*Neptune*	$\alpha=0.01288874$	log. 8.11021056;	$1-\alpha^2$ log. 9.9999279.

For *Venus* and	*Earth*	$\alpha=0.7233323$	log. 9.85933786;	$1-\alpha^2$ log. 9.6783274;
"	*Mars*	$\alpha=0.4747247$	log. 9.67644186;	$1-\alpha^2$ log. 9.8890979;
"	*Jupiter*	$\alpha=0.1390276$	log. 9.14310089;	$1-\alpha^2$ log. 9.9915235;
"	*Saturn*	$\alpha=0.07583011$	log. 8.87984175;	$1-\alpha^2$ log. 9.9974955;
"	*Uranus*	$\alpha=0.03770580$	log. 8.57640818;	$1-\alpha^2$ log. 9.9993822;
"	*Neptune*	$\alpha=0.02408390$	log. 8.38172670;	$1-\alpha^2$ log. 9.9997480.

For *Earth* and	*Mars*	$\alpha=0.6563025$	log. 9.81710400;	$1-\alpha^2$ log. 9.7553161;
"	*Jupiter*	$\alpha=0.1922043$	log. 9.28376303;	$1-\alpha^2$ log. 9.9836522;
"	*Saturn*	$\alpha=0.1048344$	log. 9.02050389;	$1-\alpha^2$ log. 9.9952006;
"	*Uranus*	$\alpha=0.0521279$	log. 8.71707032;	$1-\alpha^2$ log. 9.9988183;
"	*Neptune*	$\alpha=0.03329575$	log. 8.52238884;	$1-\alpha^2$ log. 9.9995183.

For *Mars* and	*Jupiter*	$\alpha=0.2928593$	log. 9.46665903;	$1-\alpha^2$ log. 9.9610571;
"	*Saturn*	$\alpha=0.1597349$	log. 9.20339989;	$1-\alpha^2$ log. 9.9887751;
"	*Uranus*	$\alpha=0.07942665$	log. 8.89996632;	$1-\alpha^2$ log. 9.9972515;
"	*Neptune*	$\alpha=0.05073232$	log. 8.70528484;	$1-\alpha^2$ log. 9.9988808.

For *Jupiter* and *Saturn*	$\alpha=0.5454324$	log. 9.73674086;	$1-\alpha^2$ log. 9.8466486;
“ *Uranus*	$\alpha=0.2712110$	log. 9.43330729;	$1-\alpha^2$ log. 9.9668195;
“ *Neptune*	$\alpha=0.1732311$	log. 9.23862581;	$1-\alpha^2$ log. 9.9867677.
For *Saturn* and *Uranus*	$\alpha=0.4972404$	log. 9.69656643;	$1-\alpha^2$ log. 9.8766519;
“ *Neptune*	$\alpha=0.3176033$	log. 9.50188495;	$1-\alpha^2$ log. 9.9538216.
For *Uranus* and *Neptune*	$\alpha=0.6387318$	log. 9.80531852;	$1-\alpha^2$ log. 9.7723377.

6. If we now substitute these several values of α inequations (22) and (23), we shall obtain the following values of the quantities $b^{(0)}$ and $b^{(1)}_{-\frac{1}{2}}$.

Mercury and *Venus*	log. $b^{(0)}=98.6758747n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.7120142n$;
“ *Earth*	log. $b^{(0)}=98.1301668n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.5794468n$;
“ *Mars*	log. $b^{(0)}=97.4084445n$;	log. $b^{(1)}_{-\frac{1}{2}}=99,4013786n$;
“ *Jupiter*	log. $b^{(0)}=95.2816171n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.8712839n$;
“ *Saturn*	log. $b^{(0)}=94.2290035n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.6082362n$;
“ *Uranus*	log. $b^{(0)}=93.0154041n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.3048699n$;
“ *Neptune*	log. $b^{(0)}=92.2367042n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.1102016n$.
Venus and *Earth*	log. $b^{(0)}=99.1656339n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.8275394n$;
“ *Mars*	log. $b^{(0)}=98.4754676n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.6636495n$;
“ *Jupiter*	log. $b^{(0)}=96.3661735n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.1420477n$;
“ *Saturn*	log. $b^{(0)}=65.3146217n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.8795292n$;
“ *Uranus*	log. $b^{(0)}=94.1013583n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.5763310n$;
“ *Neptune*	log. $b^{(0)}=93.3227239n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.3816952n$.
The *Earth* and *Mars*	log. $b^{(0)}=99.0105934n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.7915113n$;
“ *Jupiter*	log $b^{(0)}=96.9268788n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.2817435n$;
“ *Saturn*	log. $b^{(0)}=95.8766987n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.0199061n$;
“ *Uranus*	log. $b^{(0)}=94.6638660n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.7169227n$;
“ *Neptune*	log. $b^{(0)}=93.8853150n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.5223286n$.
Mars and *Jupiter*	log. $b^{(0)}=97.6529713n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.4619255n$;
“ *Saturn*	log. $b^{(0)}=96.6066892n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.2020080n$;
“ *Uranus*	log. $b^{(0)}=95.3950591n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.8996234n$;
“ *Neptune*	log. $b^{(0)}=94.6167398n$;	log. $b^{(1)}_{-\frac{1}{2}}=98.7051450n$.
Jupiter and *Saturn*	log. $b^{(0)}=98.7074558n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.7195915n$;
“ *Uranus*	log. $b^{(0)}=97.5209527n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.4292578n$;
“ *Neptune*	log. $b^{(0)}=96.7470972n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.2369875n$.
Saturn and *Uranus*	log. $b^{(0)}=98.5532200n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.6824668n$;
“ *Neptune*	log. $b^{(0)}=97.7921436n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.4963019n$.
Uranus and *Neptune*	log. $b^{(0)}=98.9667121n$;	log. $b^{(1)}_{-\frac{1}{2}}=99.7812070n$.

7. We must now substitute the values of $b^{(1)}_{-\frac{1}{2}}$, and the corresponding values of α, in equation (24); and change m', successively into m'', m''', m^{IV}, &c., and we shall obtain the values of $(0,1)$, $(0,2)$, $(0,3)$, &c., or the coefficient of the action of each of the planets on *Mercury*. The characters 0, 1, 2, 3, 4, 5, 6, 7, refer respectively to *Mercury*, *Venus*, the *Earth*, *Mars*, *Jupiter*, *Saturn*, *Uranus*, and *Neptune*. Then changing n into n', and m', into m'', m''', &c., we shall obtain the values of $(1,2)$, $(1,3)$, &c., or the action of each outer planet on *Venus*. And in like manner we shall obtain $(2,3)$, $(2,4)$, $(2,5)$, &c.; $(3,4)$, $(3,5)$, &c., or the action of each outer planet on each inner one. We shall then obtain $(1,0)$, $(2,0)$, $(2,1)$, &c., or the action of an inner planet on an outer one by means of the equations (6), (7), (10), &c.

In this manner we have obtained the following results:—

$$\begin{array}{lll}
(0,1)=(1+\mu'). & 2''.9986729 & \text{log. } 0.4769291; \\
(0,2)=(1+\mu''). & 0\ .8617070 & \text{log. } 9.9353596; \\
(0,3)=(1+\mu'''). & 0\ .0279815 & \text{log. } 8.4468702; \\
(0,4)=(1+\mu^{IV}). & 1\ .6028375 & \text{log. } 0.2048895; \\
(0,5)=(1+\mu^{V}). & 0\ .0772642 & \text{log. } 8.8879781; \\
(0,6)=(1+\mu^{VI}). & 0\ .0013324 & \text{log. } 7.1246469; \\
(0,7)=(1+\mu^{VII}). & 0\ .0004603 & \text{log. } 6.6629969.
\end{array}$$

$$\begin{array}{lll}
(1,0)=(1+\mu). & 0''.1758273 & \text{log. } 9.2450862; \\
(1,2)=(1+\mu''). & 6\ .6305873 & \text{log. } 0.8215520; \\
(1,3)=(1+\mu'''). & 0\ .1020355 & \text{log. } 9.0087513; \\
(1,4)=(1+\mu^{IV}). & 4\ .2028443 & \text{log. } 0.6235433; \\
(1,5)=(1+\mu^{V}). & 0\ .1988873 & \text{log. } 9.2986071; \\
(1,6)=(1+\mu^{VI}). & 0\ .0034100 & \text{log. } 7.5327484; \\
(1,7)=(1+\mu^{VII}). & 0\ .0011765 & \text{log. } 7.0706089.
\end{array}$$

$$\begin{array}{lll}
(2,0)=(1+\mu). & 0''.0406239 & \text{log. } 8.6087813; \\
(2,1)=(1+\mu'). & 5\ .3310972 & \text{log. } 0.7268166; \\
(2,3)=(1+\mu'''). & 0\ .2982001 & \text{log. } 9.4745078; \\
(2,4)=(1+\mu^{IV}). & 7\ .0682646 & \text{log. } 0.8493128; \\
(2,5)=(1+\mu^{V}). & 0\ .3265163 & \text{log. } 9.5139049; \\
(2,6)=(1+\mu^{VI}). & 0\ .0055565 & \text{log. } 7.7447989; \\
(2,7)=(1+\mu^{VII}). & 0\ .0019144 & \text{log. } 7.2820328.
\end{array}$$

$$\begin{array}{lll}
(3,0)=(1+\mu). & 0''.0077700 & \text{log. } 7.8904196; \\
(3,1)=(1+\mu'). & 0\ .4832186 & \text{log. } 9.6841436; \\
(3,2)=(1+\mu''). & 1\ .7564488 & \text{log. } 0.2446355; \\
(3,4)=(1+\mu^{IV}). & 14\ .6598964 & \text{log. } 1.1661309; \\
(3,5)=(1+\mu^{V}). & 0\ .6313987 & \text{log. } 9.8003037; \\
(3,6)=(1+\mu^{VI}). & 0\ .0105215 & \text{log. } 8.0220791; \\
(3,7)=(1+\mu^{VII}). & 0\ .0036105 & \text{log. } 7.5575701.
\end{array}$$

$(4,0)=(1+\mu)$. 0″.0000942.0 log. 5.9740497;
$(4,1)=(1+\mu')$. 0 .0042125.6 log. 7.6245464;
$(4,2)=(1+\mu'')$. 0 .0088115.3 log. 7.9450513;
$(4,3)=(1+\mu''')$. 0 .0031027.1 log. 7.4917417;
$(4,5)=(1+\mu^{V})$. 7 .3963746.3 log. 0.8690189;
$(4,6)=(1+\mu^{VI})$. 0 .0757628.5 log. 8.8794563;
$(4,7)=(1+\mu^{VII})$. 0 .0240169.3 log. 8.3805175.

$(5,0)=(1+\mu)$. 0″.0000112.0 log. 5.0493193;
$(5,1)=(1+\mu')$. 0 .0004918.0 log. 6.6917912;
$(5,2)=(1+\mu'')$. 0 .0010042.1 log. 7.0018244;
$(5,3)=(1+\mu''')$. 0 .0003296.8 log. 6.5180955;
$(5,4)=(1+\mu^{IV})$.18 .2473541 log. 1.2611999;
$(5,6)=(1+\mu^{VI})$. 0 .2782820.5 log. 9.4444852;
$(5,7)=(1+\mu^{VII})$. 0 .0687398.9 log. 8.8372088.

$(6,0)=(1+\mu)$. 0″.0000009.688 log. 93.9862386;
$(6,1)=(1+\mu')$. 0 .0000422.85 log. 95.6261830;
$(6,2)=(1+\mu'')$. 0 .0000856.98 log. 95.9329689;
$(6,3)=(1+\mu''')$. 0 .0000275.50 log. 95.4401214;
$(6,4)=(1+\mu^{IV})$. 0 .9373198.2 log. 99.9718878;
$(6,5)=(1+\mu^{V})$. 1 .3955188.2 log. 0.1447357;
$(6,7)=(1+\mu^{VII})$. 0 .4332570.2 log. 99.6367457.

$(7,0)=(1+\mu)$. 0″.0000002.0168 log. 93.3046628;
$(7,1)=(1+\mu')$. 0 .0000087.926 log. 94.9441177;
$(7,2)=(1+\mu'')$. 0 .0000177.94 log. 95.2502770;
$(7,3)=(1+\mu''')$. 0 .0000056.975 log. 94.7556866;
$(7,4)=(1+\mu^{IV})$. 0 .1790701.5 log. 99.2530232;
$(7,5)=(1+\mu^{V})$. 0 .2077464.0 log. 99.3175335;
$(7,6)=(1+\mu^{VI})$. 0 .2611078.3 log. 99.4168199.

In like manner formulas (25) and (8) will give the following values of $\boxed{0,1}$, $\boxed{1,0}$; $\boxed{0,2}$, $\boxed{2,0}$; $\boxed{1,2}$, $\boxed{2,1}$; &c.

$\boxed{0,1}=(1+\mu')$.1″.926868 log. 0.2848519;
$\boxed{0,2}=(1+\mu'')$.0. 4087579 log. 99.6114662;
$\boxed{0,3}=(1+\mu''')$.0. 008812816 log. 97.9451147;
$\boxed{0,4}=(1+\mu^{IV})$.0. 1489646 log. 99.1730832;
$\boxed{0,5}=(1+\mu^{V})$.0. 00391854 log. 97.5931242;
$\boxed{0,6}=(1+\mu^{VI})$.0. 0000336068 log. 95.5264270;
$\boxed{0,7}=(1+\mu^{VII})$.0. 00000741495 log. 94.8701084.

$\boxed{1,0}=(1+\mu)$.0.″1129820 log. 99.0530090;
$\boxed{1,2}=(1+\mu'')$.5. 520785 log. 0.7420008;
$\boxed{1,3}=(1+\mu''')$.0. 0587105 log. 98.7687157;
$\boxed{1,4}=(1+\mu^{IV})$.0. 7286137 log. 99.8624973;
$\boxed{1,5}=(1+\mu^{V})$.0. 0188385 log. 98.2750461;
$\boxed{1,6}=(1+\mu^{VI})$.0. 00016069 log. 96.2059893;
$\boxed{1,7}=(1+\mu^{VII})$.0. 0000354172 log. 95.5492142.

$\boxed{2,0}=(1+\mu\ )$. $0''.0192703$ log. 98.2848879;
$\boxed{2,1}=(1+\mu'\ )$. $4\ .438798$ log. 0.6472654;
$\boxed{2,3}=(1+\mu'''\ )$. $0\ .2293041$ log. 99.3604119;
$\boxed{2,4}=(1+\mu^{IV})$. $1\ .690254$ log. 0.2279520;
$\boxed{2,5}=(1+\mu^{V}\ )$. $0\ .0427287$ log. 98.6307197;
$\boxed{2,6}=(1+\mu^{VI})$. $0\ .000361930$ log. 96.5586316;
$\boxed{2,7}=(1+\mu^{VII})$. $0\ .0000796657$ log. 95.9012715.

$\boxed{3,0}=(1+\mu\ )$. $0''.00244717$ log. 97.3886641;
$\boxed{3,1}=(1+\mu'\ )$. $0\ .2780404$ log. 99.4441080;
$\boxed{3,2}=(1+\mu''\ )$. $1\ .350640$ log. 0.1305396;
$\boxed{3,4}=(1+\mu^{IV})$. $5\ .307483$ log. 0.7248886;
$\boxed{3,5}=(1+\mu^{V}\ )$. $0\ .1256652$ log. 99.0992151;
$\boxed{3,6}=(1+\mu^{VI})$. $0\ .001043788$ log. 97.0186122;
$\boxed{3,7}=(1+\mu^{VII})$. $0\ .000228889$ log. 96.3596252.

$\boxed{4,0}=(1+\mu\ )$. $0''.0000087547$ log. 94.9422434;
$\boxed{4,1}=(1+\mu'\ )$. $0\ .00073030$ log. 96.8635004;
$\boxed{4,2}=(1+\mu''\ )$. $0\ .00210713$ log. 97.3236905;
$\boxed{4,3}=(1+\mu'''\ )$. $0\ .00112331$ log. 97.0504994;
$\boxed{4,5}=(1+\mu^{V}\ )$. $4\ .835390$ log. 0.6844315;
$\boxed{4,6}=(1+\mu^{VI})$. $0\ .0254429$ log. 98.4055666;
$\boxed{4,7}=(1+\mu^{VII})$. $0\ .00518090$ log. 97.7144056.

$\boxed{5,0}=(1+\mu\ )$. $0''.00000056815$ log. 93.7544654;
$\boxed{5,1}=(1+\mu'\ )$. $0\ .0000465833$ log. 95.6682302;
$\boxed{5,2}=(1+\mu''\ )$. $0\ .000131413$ log. 96.1186392;
$\boxed{5,3}=(1+\mu'''\ )$. $0\ .0000656156$ log. 95.8170069;
$\boxed{5,4}=(1+\mu^{IV})$.$11\ .92923$ log. 1.0766125;
$\boxed{5,6}=(1+\mu^{VI})$. 0.1671612 log. 99.2231355;
$\boxed{5,7}=(1+\mu^{VII})$. 0.0269346 log. 98.4303106.

$\boxed{6,0}=(1+\mu\ )$. $0''.000000024435$ log. 92.3880187;
$\boxed{6,1}=(1+\mu'\ )$. $0\ .0000019926$ log. 94.2994239;
$\boxed{6,2}=(1+\mu''\ )$. $0\ .00000558215$ log. 94.7468016;
$\boxed{6,3}=(1+\mu'''\ )$. $0\ .0000027331$ log. 94.4366545;
$\boxed{6,4}=(1+\mu^{IV})$. $0\ .3147736$ log. 99.4979981;
$\boxed{6,5}=(1+\mu^{V}\ )$. $0\ .8382741$ log. 99.9233860;
$\boxed{6,7}=(1+\mu^{VII})$. $0\ .3255696$ log. 99.5126438.

$\boxed{7,0}=(1+\mu\ )$. $0''.000000003249$ log. 91.5117743;
$\boxed{7,1}=(1+\mu'\ )$. $0\ .00000026468$ log. 93.4227230;
$\boxed{7,2}=(1+\mu''\ )$. $0\ .000000740484$ log. 93.8695157;
$\boxed{7,3}=(1+\mu'''\ )$. $0\ .000000361195$ log. 93.5577417;
$\boxed{7,4}=(1+\mu^{IV})$. $0\ .0386288$ log. 98.5869113;
$\boxed{7,5}=(1+\mu^{V}\ )$. $0\ .0814020$ log. 98.9106353;
$\boxed{7,6}=(1+\mu^{VI})$. $0\ .1962086$ log. 99.2927180.

8. The values of $(0,1)$, $(0,2)$, &c., $\boxed{0,1}$, $\boxed{0,2}$, &c., $(1,0)$, $(1,2)$, &c., $\boxed{1,0}$, $\boxed{1,2}$, &c., being substituted in equations (B), supposing μ, μ', μ'', &c., to be equal to nothing, we shall have a series of numerical equations which are perfectly symmetrical in form, between g and the unknown quantities N, N', N'', &c. If we then eliminate N, N', N'' from these equations, we shall obtain a final equation in g, of a degree equal to the number of original equations. The construction of this final equation in g is the most delicate and intricate problem connected with the actual determination of the secular inequalities. Theoretically speaking, this equation may be formed by eliminating the quantities N, N', N'', &c., by the ordinary methods of elimination in algebra. But this method, though direct and simple in theory, leads to impracticable operations when we attempt to apply it to the formation of the equation of the eighth degree which is necessary in the simultaneous determination of the secular inequalities of the eight principal planets. The only actual merit of this method seems to be that of leaving the algebraic values of N', N'', N''', &c., in the successive eliminations, in very good form for computation, when the value of g has been determined; while its defects are twofold, as follows: *First*, it introduces foreign facts depending on g, which raise the final equation in g, to a very high degree; and *secondly*, it necessitates the employment of a very great number of decimals in the successive eliminations, in order to obtain a near approximation to the correct value of the final equation. The method of *determinants* enables us to form the final equation in g without actually performing the eliminations of the unknown quantities N, N', N'', &c. It also enables us to estimate, in advance, the exact amount of labor necessary for forming the final equation arising from any number of linear symmetrical equations. In the year 1860, we published a short paper on this interesting branch of analysis in GOULD'S *Astronomical Journal*, Vol. VI. From the explanation and formulæ there given, it follows that each of the given equations contains one binomial term of the form $g—a$, and that each term of the final equation contains one factor from each of the given equations; and also that the whole number of terms in the final equation is equal to the continued product of the numbers 1, 2, 3, 4, &c., to n inclusive; n denoting the number of given equations. In the present case n is equal to *eight*; there being one equation corresponding to each of the eight principal planets. The whole number of terms in the equation of the eighth degree is therefore equal to $1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 = 40320$. There are therefore 40320 distinct terms in the equation of the eighth degree, each of which contains eight factors which are either monomial or binomial. They are distributed in the following order:—

1	term	having	8	binomial	factors	producing	9	monomial	terms.
28	terms	“	6	“	“	“	196	“	“
112	“	“	5	“	“	“	672	“	“
630	“	“	4	“	“	“	3150	“	“
2464	“	“	3	“	“	“	9856	“	“
7420	“	“	2	“	“	“	22260	“	“
14832	“	“	1	“	“	“	29664	“	“
14833	“	without binomial factors				“	14833	“	“
Total 40320							80640		

The equation of the eighth degree when completely developed is therefore composed of 80640 distinct monomial terms, each of which contains eight factors. The actual formation of this equation could therefore with difficulty be brought within the compass of an ordinary lifetime; and we must, therefore, seek for a shorter and more expeditious method of attaining results which seem to necessarily involve such an immense expenditure of labor.

9. For this purpose we shall resume equations (B) of § 2, and shall suppose

$$\left.\begin{aligned}
\boxed{0,0}&=g-\{(0,1)+(0,2)+(0,3)+\text{\&c.}\};\\
\boxed{1,1}&=g-\{(1,0)+(1,2)+(1,3)+\text{\&c.}\};\\
\boxed{2,2}&=g-\{(2,0)+(2,1)+(2,3)+\text{\&c.}\};\\
\boxed{3,3}&=g-\{(3,0)+(3,1)+(3,2)+\text{\&c.}\};\\
\boxed{4,4}&=g-\{(4,0)+(4,1)+(4,2)+\text{\&c.}\};\\
\boxed{5,5}&=g-\{(5,0)+(5,1)+(5,2)+\text{\&c.}\};\\
\boxed{6,6}&=g-\{(6,0)+(6,1)+(6,2)+\text{\&c.}\};\\
\boxed{7,7}&=g-\{(7,0)+(7,1)+(7,2)+\text{\&c.}\};
\end{aligned}\right\}\quad . \qquad (26)$$

By this means equations (B) will be reduced to the following:—

$$\left.\begin{aligned}
&\boxed{0,0}N+\boxed{0,1}N'+\boxed{0,2}N''+\boxed{0,3}N'''+\boxed{0,4}N^{IV}+\boxed{0,5}N^{V}+\boxed{0,6}N^{VI}+\boxed{0,7}N^{VII}=0,\\
&\boxed{1,1}N'+\boxed{1,0}N+\boxed{1,2}N''+\boxed{1,3}N'''+\boxed{1,4}N^{IV}+\boxed{1,5}N^{V}+\boxed{1,6}N^{VI}+\boxed{1,7}N^{VII}=0,\\
&\boxed{2,2}N''+\boxed{2,0}N+\boxed{2,1}N'+\boxed{2,3}N'''+\boxed{2,4}N^{IV}+\boxed{2,5}N^{V}+\boxed{2,6}N^{VI}+\boxed{2,7}N^{VII}=0,\\
&\boxed{3,3}N'''+\boxed{3,0}N+\boxed{3,1}N'+\boxed{3,2}N''+\boxed{3,4}N^{IV}+\boxed{3,5}N^{V}+\boxed{3,6}N^{VI}+\boxed{3,7}N^{VII}=0,\\
&\boxed{4,4}N^{IV}+\boxed{4,0}N+\boxed{4,1}N'+\boxed{4,2}N''+\boxed{4,3}N'''+\boxed{4,5}N^{V}+\boxed{4,6}N^{VI}+\boxed{4,7}N^{VII}=0,\\
&\boxed{5,5}N^{V}+\boxed{5,0}N+\boxed{5,1}N'+\boxed{5,2}N''+\boxed{5,3}N'''+\boxed{5,4}N^{IV}+\boxed{5,6}N^{VI}+\boxed{5,7}N^{VII}=0,\\
&\boxed{6,6}N^{VI}+\boxed{6,0}N+\boxed{6,1}N'+\boxed{6,2}N''+\boxed{6,3}N'''+\boxed{6,4}N^{IV}+\boxed{6,5}N^{V}+\boxed{6,7}N^{VII}=0,\\
&\boxed{7,7}N^{VII}+\boxed{7,0}N+\boxed{7,1}N'+\boxed{7,2}N''+\boxed{7,3}N'''+\boxed{7,4}N^{IV}+\boxed{7,5}N^{V}+\boxed{7,6}N^{VI}=0,
\end{aligned}\right\}.\,(B')$$

Now since the coefficients of N^{IV}, N^{V}, N^{VI}, and N^{VII} of the first four of the equations (B′) are independent of g, and also the coefficients of N, N', N'' and N''', in the last four of equations (B′), we may divide them into two distinct groups, and determine the values of g, N', N'', N''', &c., by successive approximations. We shall therefore suppose

$$\left.\begin{aligned}
b&=\boxed{0,4}N^{IV}+\boxed{0,5}N^{V}+\boxed{0,6}N^{VI}+\boxed{0,7}N^{VII},\\
b'&=\boxed{1,4}N^{IV}+\boxed{1,5}N^{V}+\boxed{1,6}N^{VI}+\boxed{1,7}N^{VII},\\
b''&=\boxed{2,4}N^{IV}+\boxed{2,5}N^{V}+\boxed{2,6}N^{VI}+\boxed{2,7}N^{VII},\\
b'''&=\boxed{3,4}N^{IV}+\boxed{3,5}N^{V}+\boxed{3,6}N^{VI}+\boxed{3,7}N^{VII},
\end{aligned}\right\}\quad . \qquad (27)$$

$$\left.\begin{aligned}
b_1&=\boxed{4,0}N+\boxed{4,1}N'+\boxed{4,2}N''+\boxed{4,3}N''',\\
b_2&=\boxed{5,0}N+\boxed{5,1}N'+\boxed{5,2}N''+\boxed{5,3}N''',\\
b_3&=\boxed{6,0}N+\boxed{6,1}N'+\boxed{6,2}N''+\boxed{6,3}N''',\\
b_4&=\boxed{7,0}N+\boxed{7,1}N'+\boxed{7,2}N''+\boxed{7,3}N'''.
\end{aligned}\right\}\quad . \qquad (28)$$

Substituting these quantities in equations (B′), they will become

$$\left.\begin{aligned}
&\boxed{0,0}N+\boxed{0,1}N'\ \boxed{0,2}N''+\boxed{0,3}N'''+b=0,\\
&\boxed{1,1}N'+\boxed{1,0}N\ \boxed{1,2}N''+\boxed{1,3}N'''+b'=0,\\
&\boxed{2,2}N''+\boxed{2,0}N\ \boxed{2,1}N'+\boxed{2,3}N'''+b''=0,\\
&\boxed{3,3}N'''+\boxed{3,0}N\ \boxed{3,1}N'+\boxed{3,2}N''+b'''=0;
\end{aligned}\right\}\quad . \qquad (B'')$$

$$\left.\begin{aligned}
&\boxed{4,4}N^{IV}+\boxed{4,5}N^{V}\ \boxed{4,6}N^{VI}+\boxed{4,7}N^{VII}+b_1=0,\\
&\boxed{5,5}N^{V}+\boxed{5,4}N^{IV}\boxed{5,6}N^{VI}+\boxed{5,7}N^{VII}+b_2=0,\\
&\boxed{6,6}N^{VI}+\boxed{6,4}N^{IV}\boxed{6,5}N^{V}+\boxed{6,7}N^{VII}+b_3=0,\\
&\boxed{7,7}N^{VII}+\boxed{7,4}N^{IV}\boxed{7,5}N^{V}+\boxed{7,6}N^{VI}+b_4=0.
\end{aligned}\right\}\ . \qquad (B''')$$

If we now suppose b, b', b'', &c. to be equal to nothing, and eliminate N', N'', and N''' from equations (B″), and N^{V}, N^{VI}, and N^{VII} from equations (B‴), the resulting equations will be divisible by N and N^{IV} respectively; and we shall have

$$\left.\begin{aligned}
&\boxed{0,0}\boxed{1,1}\boxed{2,2}\boxed{3,3}-\boxed{3,2}\boxed{2,3}\boxed{0,0}\boxed{1,1}-\boxed{3,1}\boxed{1,3}\boxed{0,0}\boxed{2,2}-\boxed{3,0}\boxed{0,3}\boxed{1,1}\boxed{2,2}\\
&-\boxed{2,1}\boxed{1,2}\boxed{0,0}\boxed{3,3}-\boxed{2,0}\boxed{0,2}\boxed{1,1}\boxed{3,3}-\boxed{1,0}\boxed{0,1}\boxed{2,2}\boxed{3,3}+\{\boxed{3,2}\boxed{2,1}\boxed{1,3}\\
&+\boxed{3,1}\boxed{1,2}\boxed{2,3}\}\boxed{0,0}+\{\boxed{3,0}\boxed{0,2}\boxed{2,3}+\boxed{3,2}\boxed{2,0}\boxed{0,3}\}\boxed{1,1}+\{\boxed{3,1}\boxed{1,0}\boxed{0,3}\\
&+\boxed{3,0}\boxed{0,1}\boxed{1,3}\}\boxed{2,2}+\{\boxed{2,0}\boxed{0,1}\boxed{1,2}+\boxed{2,1}\boxed{1,0}\boxed{0,2}\}\boxed{3,3}\\
&+\boxed{3,2}\boxed{2,3}\boxed{1,0}\boxed{0,1}+\boxed{3,1}\boxed{1,3}\boxed{2,0}\boxed{0,2}+\boxed{3,0}\boxed{0,3}\boxed{2,1}\boxed{1,2}\\
&-\{\boxed{3,0}\boxed{0,2}\boxed{2,1}\boxed{1,3}+\boxed{3,1}\boxed{1,2}\boxed{2,0}\boxed{0,3}\}-\{\boxed{3,1}\boxed{1,0}\boxed{0,2}\boxed{2,3}\\
&+\boxed{3,2}\boxed{2,0}\boxed{0,1}\boxed{1,3}\}-\{\boxed{3,0}\boxed{0,1}\boxed{1,2}\boxed{2,3}+\boxed{3,2}\boxed{2,1}\boxed{1,0}\boxed{0,3}\}
\end{aligned}\right\}=0; \tag{29}$$

$$\left.\begin{aligned}
&\boxed{4,4}\boxed{5,5}\boxed{6,6}\boxed{7,7}-\boxed{7,6}\boxed{6,7}\boxed{4,4}\boxed{5,5}-\boxed{7,5}\boxed{5,7}\boxed{4,4}\boxed{6,6}-\boxed{7,4}\boxed{4,7}\boxed{5,5}\boxed{6,6}\\
&-\boxed{6,5}\boxed{5,6}\boxed{4,4}\boxed{7,7}-\boxed{6,4}\boxed{4,6}\boxed{5,5}\boxed{7,7}-\boxed{5,4}\boxed{4,5}\boxed{6,6}\boxed{7,7}+\{\boxed{7,6}\boxed{6,5}\boxed{5,7}\\
&+\boxed{7,5}\boxed{5,6}\boxed{6,7}\}\boxed{4,4}+\{\boxed{7,4}\boxed{4,6}\boxed{6,7}+\boxed{7,6}\boxed{6,4}\boxed{4,7}\}\boxed{5,5}\\
&+\{\boxed{7,5}\boxed{5,4}\boxed{4,7}+\boxed{7,4}\boxed{4,5}\boxed{5,7}\}\boxed{6,6}+\{\boxed{6,4}\boxed{4,5}\boxed{5,6}+\boxed{6,5}\boxed{5,4}\boxed{4,6}\}\boxed{7,7}\\
&+\boxed{7,5}\boxed{5,7}\boxed{6,4}\boxed{4,6}+\boxed{7,4}\boxed{4,7}\boxed{6,5}\boxed{5,6}+\boxed{7,6}\boxed{6,7}\boxed{5,4}\boxed{4,5}\\
&-\{\boxed{7,4}\boxed{4,6}\boxed{6,5}\boxed{5,7}+\boxed{7,5}\boxed{5,6}\boxed{6,4}\boxed{4,7}\}-\{\boxed{7,5}\boxed{5,4}\boxed{4,6}\boxed{6,7}\\
&+\boxed{7,6}\boxed{6,4}\boxed{4,5}\boxed{5,7}\}-\{\boxed{7,4}\boxed{4,5}\boxed{5,6}\boxed{6,7}+\boxed{7,6}\boxed{6,5}\boxed{5,4}\boxed{4,7}\}
\end{aligned}\right\}=0. \tag{30}$$

Each of these equations is evidently of the fourth degree in g, and consequently has four roots, which may be found by any of the ordinary methods of finding the roots of numerical equations. These roots will be only approximate, because we have neglected b, b', b'', &c., in the determination of equations (29) and (30). If we substitute the approximate roots derived from equation (29) in any three of equations (B''), we can find by elimination the values of N', N'', and N''' in terms of N, which remains indeterminate. When N', N'', and N''' have been thus determined we must substitute their values in equations (28), and we shall obtain the values b_1, b_2, b_3, and b_4 in terms of N; and these quantities are then to be substituted in equations (B'''), together with the corresponding value of g; and we shall then obtain the values of N^{IV}, N^{V}, N^{VI}, and N^{VII}, in terms N. But instead of performing this operation separately for each of the roots, in the manner described above, it is better to deduce a system of algebraic equations, not only for the purpose of facilitating the numerical calculations, but also for the purpose of devising checks to the accuracy of the different parts of the computations.

10. If we now assume the following quantities

$$\left.\begin{aligned}
A=\boxed{0,0}\boxed{2,2}-\frac{\boxed{1,0}\boxed{0,3}}{\boxed{1,3}}\boxed{2,2}-\frac{\boxed{1,2}\boxed{2,3}}{\boxed{1,3}}\boxed{0,0}+\frac{\boxed{1,0}\boxed{0,2}\boxed{2,3}}{\boxed{1,3}}+\frac{\boxed{1,2}\boxed{2,0}\boxed{0,3}}{\boxed{1,3}}&\\
-\boxed{2,0}\boxed{0,2};&
\end{aligned}\right\} \tag{31}$$

$$\left.\begin{aligned}
A'=\boxed{0,0}\boxed{3,3}-\frac{\boxed{1,3}\boxed{3,2}}{\boxed{1,2}}\boxed{0,0}-\frac{\boxed{1,0}\boxed{0,2}}{\boxed{1,2}}\boxed{3,3}+\frac{\boxed{1,0}\boxed{0,3}\boxed{3,2}}{\boxed{1,2}}+\frac{\boxed{1,3}\boxed{3,0}\boxed{0,2}}{\boxed{1,2}}&\\
-\boxed{3,0}\boxed{0,3};&
\end{aligned}\right\} \tag{32}$$

$$\left.\begin{aligned}A''=&\boxed{0,0}\boxed{1,1}-\frac{\boxed{2,1}\boxed{1,3}}{\boxed{2,3}}\boxed{0,0}-\frac{\boxed{2,0}\boxed{0,3}}{\boxed{2,3}}\boxed{1,1}+\frac{\boxed{2,0}\boxed{0,1}\boxed{1,3}}{\boxed{2,3}}+\frac{\boxed{2,1}\boxed{1,0}\boxed{0,3}}{\boxed{2,3}}\\&-\boxed{1,0}\boxed{0,1};\end{aligned}\right\}(33)$$

$$\left.\begin{aligned}D=&\boxed{1,1}\boxed{2,2}-\frac{\boxed{0,2}\boxed{2,3}}{\boxed{0,3}}\boxed{1,1}-\frac{\boxed{0,1}\boxed{1,3}}{\boxed{0,3}}\boxed{2,2}+\frac{\boxed{0,1}\boxed{1,2}\boxed{2,3}}{\boxed{0,3}}+\frac{\boxed{0,2}\boxed{2,1}\boxed{1,3}}{\boxed{0,3}}\\&-\boxed{2,1}\boxed{1,2};\end{aligned}\right\}(34)$$

$$\left.\begin{aligned}D'=&\boxed{1,1}\boxed{3,3}-\frac{\boxed{0,3}\boxed{3,2}}{\boxed{0,2}}\boxed{1,1}-\frac{\boxed{0,1}\boxed{1,2}}{\boxed{0,2}}\boxed{3,3}+\frac{\boxed{0,1}\boxed{1,3}\boxed{3,2}}{\boxed{0,2}}+\frac{\boxed{0,3}\boxed{3,1}\boxed{1,2}}{\boxed{0,2}}\\&-\boxed{3,1}\boxed{1,3};\end{aligned}\right\}(35)$$

$$\left.\begin{aligned}D''=&\boxed{2,2}\boxed{3,3}-\frac{\boxed{0,2}\boxed{2,1}}{\boxed{0,1}}\boxed{3,3}-\frac{\boxed{0,3}\boxed{3,1}}{\boxed{0,1}}\boxed{2,2}+\frac{\boxed{0,3}\boxed{3,2}\boxed{2,1}}{\boxed{0,1}}+\frac{\boxed{0,2}\boxed{2,3}\boxed{3,1}}{\boxed{0,1}}\\&-\boxed{3,2}\boxed{2,3};\end{aligned}\right\}(36)$$

$$\left.\begin{aligned}A_1=&\boxed{4,4}\boxed{6,6}-\frac{\boxed{5,6}\boxed{6,7}}{\boxed{5,7}}\boxed{4,4}-\frac{\boxed{5,4}\boxed{4,7}}{\boxed{5,7}}\boxed{6,6}+\frac{\boxed{5,4}\boxed{4,6}\boxed{6,7}}{\boxed{5,7}}+\frac{\boxed{5,6}\boxed{6,4}\boxed{4,7}}{\boxed{5,7}}\\&-\boxed{6,4}\boxed{4,6};\end{aligned}\right\}(37)$$

$$\left.\begin{aligned}A_2=&\boxed{4,4}\boxed{7,7}-\frac{\boxed{5,7}\boxed{7,6}}{\boxed{5,6}}\boxed{4,4}-\frac{\boxed{5,4}\boxed{4,6}}{\boxed{5,6}}\boxed{7,7}+\frac{\boxed{5,4}\boxed{4,7}\boxed{7,6}}{\boxed{5,6}}+\frac{\boxed{5,7}\boxed{7,4}\boxed{4,6}}{\boxed{5,6}}\\&-\boxed{7,4}\boxed{4,7};\end{aligned}\right\}(38)$$

$$\left.\begin{aligned}A_3=&\boxed{4,4}\boxed{5,5}-\frac{\boxed{6,5}\boxed{5,7}}{\boxed{6,7}}\boxed{4,4}-\frac{\boxed{6,4}\boxed{4,7}}{\boxed{6,7}}\boxed{5,5}+\frac{\boxed{6,4}\boxed{4,5}\boxed{5,7}}{\boxed{6,7}}+\frac{\boxed{6,5}\boxed{5,4}\boxed{4,7}}{\boxed{6,7}}\\&-\boxed{5,4}\boxed{4,5};\end{aligned}\right\}(39)$$

$$\left.\begin{aligned}D_1=&\boxed{5,5}\boxed{6,6}-\frac{\boxed{4,6}\boxed{6,7}}{\boxed{4,7}}\boxed{5,5}-\frac{\boxed{4,5}\boxed{5,7}}{\boxed{4,7}}\boxed{6,6}+\frac{\boxed{4,5}\boxed{5,6}\boxed{6,7}}{\boxed{4,7}}+\frac{\boxed{4,6}\boxed{6,5}\boxed{5,7}}{\boxed{4,7}}\\&-\boxed{6,5}\boxed{5,6};\end{aligned}\right\}(40)$$

$$\left.\begin{aligned}D_2=&\boxed{5,5}\boxed{7,7}-\frac{\boxed{4,7}\boxed{7,6}}{\boxed{4,6}}\boxed{5,5}-\frac{\boxed{4,5}\boxed{5,6}}{\boxed{4,6}}\boxed{7,7}+\frac{\boxed{4,5}\boxed{5,7}\boxed{7,6}}{\boxed{4,6}}+\frac{\boxed{4,7}\boxed{7,5}\boxed{5,6}}{\boxed{4,6}}\\&-\boxed{7,5}\boxed{5,7};\end{aligned}\right\}(41)$$

$$\left.\begin{aligned}D_3=&\boxed{6,6}\boxed{7,7}-\frac{\boxed{4,7}\boxed{7,5}}{\boxed{4,5}}\boxed{6,6}-\frac{\boxed{4,6}\boxed{6,5}}{\boxed{4,5}}\boxed{7,7}+\frac{\boxed{4,7}\boxed{7,6}\boxed{6,5}}{\boxed{4,5}}+\frac{\boxed{4,6}\boxed{6,7}\boxed{7,5}}{\boxed{4,5}}\\&-\boxed{7,6}\boxed{6,7};\end{aligned}\right\}(42)$$

$$B=\left\{\boxed{2,2}-\frac{\boxed{1,2}\boxed{2,3}}{\boxed{1,3}}\right\}b;\quad B'=\left\{\boxed{3,3}-\frac{\boxed{1,3}\boxed{3,2}}{\boxed{1,2}}\right\}b;\quad B''=\left\{\boxed{1,1}-\frac{\boxed{2,1}\boxed{1,3}}{\boxed{2,3}}\right\}b;\quad(43)$$

$$\left.\begin{aligned}C&=\left\{\frac{\boxed{0,2}\boxed{2,3}}{\boxed{0,3}}-\boxed{2,2}\right\}\frac{\boxed{0,3}}{\boxed{1,3}}b'; & C'&=\left\{\frac{\boxed{0,3}\boxed{3,2}}{\boxed{0,2}}-\boxed{3,3}\right\}\frac{\boxed{0,2}}{\boxed{1,2}}b';\\C''&=\left\{\frac{\boxed{0,3}\boxed{2,1}}{\boxed{2,3}}-\boxed{0,1}\right\}b'; & C'''&=\left\{\frac{\boxed{0,2}\boxed{3,1}}{\boxed{3,2}}-\boxed{0,1}\right\}b';\end{aligned}\right\}(44)$$

$$\left.\begin{aligned}E&=\left\{\frac{\boxed{0,3}\boxed{1,2}}{\boxed{1,3}}-\boxed{0,2}\right\}b''; & E'&=\left\{\frac{\boxed{0,1}\boxed{1,3}}{\boxed{0,3}}-\boxed{1,1}\right\}\frac{\boxed{0,3}}{\boxed{2,3}}b'';\\E''&=\left\{\frac{\boxed{0,3}\boxed{3,1}}{\boxed{0,1}}-\boxed{3,3}\right\}\frac{\boxed{0,1}}{\boxed{2,1}}b''; & E'''&=\left\{\frac{\boxed{0,1}\boxed{3,2}}{\boxed{3,1}}-\boxed{0,2}\right\}b'';\end{aligned}\right\}(45)$$

$$\left.\begin{aligned} F &= \left\{\frac{\boxed{0,2}\boxed{1,3}}{\boxed{1,2}}-\boxed{0,3}\right\}b'''; & F' &= \left\{\frac{\boxed{0,1}\boxed{2,3}}{\boxed{2,1}}-\boxed{0,3}\right\}b'''; \\ F'' &= \left\{\frac{\boxed{0,2}\boxed{2,1}}{\boxed{0,1}}-\boxed{2,2}\right\}\frac{\boxed{0,1}}{\boxed{3,1}}b'''; & F''' &= \left\{\frac{\boxed{0,1}\boxed{1,2}}{\boxed{0,2}}-\boxed{1,1}\right\}\frac{\boxed{0,2}}{\boxed{3,2}}b'''; \end{aligned}\right\} (46)$$

$$B_1=\left\{\boxed{6,6}-\frac{\boxed{5,6}\boxed{6,7}}{\boxed{5,7}}\right\}b_1;\ B_2=\left\{\boxed{7,7}-\frac{\boxed{5,7}\boxed{7,6}}{\boxed{5,6}}\right\}b_1;\ B_3-\left\{\boxed{5,5}-\frac{\boxed{6,5}\boxed{5,7}}{\boxed{6,7}}\right\}b_1; \quad (47)$$

$$\left.\begin{aligned} C_1 &= \left\{\frac{\boxed{4,6}\boxed{6,7}}{\boxed{4,7}}-\boxed{6,6}\right\}\frac{\boxed{4,7}}{\boxed{5,7}}b_2; & C_2 &= \left\{\frac{\boxed{4,7}\boxed{7,6}}{\boxed{4,6}}-\boxed{7,7}\right\}\frac{\boxed{4,6}}{\boxed{5,6}}b_2; \\ C_3 &= \left\{\frac{\boxed{4,7}\boxed{6,5}}{\boxed{6,7}}-\boxed{4,5}\right\}b_2; & C_4 &= \left\{\frac{\boxed{4,6}\boxed{7,5}}{\boxed{7,6}}-\boxed{4,5}\right\}b_2; \end{aligned}\right\} (48)$$

$$\left.\begin{aligned} E_1 &= \left\{\frac{\boxed{4,7}\boxed{5,6}}{\boxed{5,7}}-\boxed{4,6}\right\}b_3; & E_2 &= \left\{\frac{\boxed{4,5}\boxed{5,7}}{\boxed{4,7}}-\boxed{5,5}\right\}\frac{\boxed{4,7}}{\boxed{6,7}}b_3; \\ E_3 &= \left\{\frac{\boxed{4,7}\boxed{7,5}}{\boxed{4,5}}-\boxed{7,7}\right\}\frac{\boxed{4,5}}{\boxed{6,5}}b_3; & E_4 &= \left\{\frac{\boxed{4,5}\boxed{7,6}}{\boxed{7,5}}-\boxed{4,6}\right\}b_3; \end{aligned}\right\} (49)$$

$$\left.\begin{aligned} F_1 &= \left\{\frac{\boxed{4,6}\boxed{5,7}}{\boxed{5,6}}-\boxed{4,7}\right\}b_4; & F_2 &= \left\{\frac{\boxed{4,5}\boxed{6,7}}{\boxed{6,5}}-\boxed{4,7}\right\}b_4; \\ F_3 &= \left\{\frac{\boxed{4,6}\boxed{6,5}}{\boxed{4,5}}-\boxed{6,6}\right\}\frac{\boxed{4,5}}{\boxed{7,5}}b_4; & F_4 &= \left\{\frac{\boxed{4,5}\boxed{5,6}}{\boxed{4,6}}-\boxed{5,5}\right\}\frac{\boxed{4,6}}{\boxed{7,6}}b_4; \end{aligned}\right\} (50)$$

$$\left.\begin{aligned} f &= B+C+E; & f' &= B'+C'+F; & f'' &= B''+C''+E'; \\ f''' &= B'+E''+F'; & f^{IV} &= B+E'''+F''; & f^{V} &= B''+C'''+F'''; \end{aligned}\right\} (51)$$

$$\left.\begin{aligned} f_1 &= B_1+C_1+E_1; & f_2 &= B_2+C_2+F_1; & f_3 &= B_3+C_3+E_2; \\ f_4 &= B_2+E_3+F_2; & f_5 &= B_1+E_4+F_2; & f_4 &= B_3+C_4+F_4; \end{aligned}\right\} (52)$$

we shall have the following system of equations for the rigorous determination of g, N', N'', N''', &c.—observing that the terms of equations (29) and (30), which are inclosed by braces, are reduced to single terms by means of equations (14) and (15).

$$\left.\begin{aligned} &\boxed{0,0}\boxed{1,1}\boxed{2,2}\boxed{3,3}-\boxed{3,2}\boxed{2,3}\boxed{0,0}\boxed{1,1}-\boxed{3,1}\boxed{1,3}\boxed{0,0}\boxed{2,2} \\ -&\boxed{3,0}\boxed{0,3}\boxed{1,1}\boxed{2,2}-\boxed{2,1}\boxed{1,2}\boxed{0,0}\boxed{3,3}-\boxed{2,0}\boxed{0,2}\boxed{1,1}\boxed{3,3} \\ -&\boxed{1,0}\boxed{0,1}\boxed{2,2}\boxed{3,3}+2\boxed{3,2}\boxed{2,1}\boxed{1,3}\boxed{0,0}+2\boxed{3,0}\boxed{0,2}\boxed{2,3}\boxed{1,1} \\ +2&\boxed{3,1}\boxed{1,0}\boxed{0,3}\boxed{2,2}+2\boxed{2,0}\boxed{0,1}\boxed{1,2}\boxed{3,3}+\boxed{3,2}\boxed{2,3}\boxed{1,0}\boxed{0,1} \\ +&\boxed{3,1}\boxed{1,3}\boxed{2,0}\boxed{0,2}+\boxed{3,0}\boxed{0,3}\boxed{2,1}\boxed{1,2}-2\boxed{3,0}\boxed{0,2}\boxed{2,1}\boxed{1,3} \\ -2&\boxed{3,1}\boxed{1,0}\boxed{0,2}\boxed{2,3}-2\boxed{3,0}\boxed{0,1}\boxed{1,2}\boxed{2,3} \end{aligned}\right\}=(\chi,\chi_1,\chi_2,\chi_3); \quad (53)$$

$$\left.\begin{aligned} &\boxed{4,4}\boxed{5,5}\boxed{6,6}\boxed{7,7}-\boxed{7,6}\boxed{6,7}\boxed{4,4}\boxed{5,5}-\boxed{7,5}\boxed{5,7}\boxed{4,4}\boxed{6,6} \\ -&\boxed{7,4}\boxed{4,7}\boxed{5,5}\boxed{6,6}-\boxed{6,5}\boxed{5,6}\boxed{4,4}\boxed{7,7}-\boxed{6,4}\boxed{4,6}\boxed{5,5}\boxed{7,7} \\ -&\boxed{5,4}\boxed{4,5}\boxed{6,6}\boxed{7,7}+2\boxed{7,6}\boxed{6,5}\boxed{5,7}\boxed{4,4}+2\boxed{7,4}\boxed{4,6}\boxed{6,7}\boxed{5,5} \\ +2&\boxed{7,5}\boxed{5,4}\boxed{4,7}\boxed{6,6}+2\boxed{6,4}\boxed{4,5}\boxed{5,6}\boxed{7,7}+\boxed{7,5}\boxed{5,7}\boxed{6,4}\boxed{4,6} \\ +&\boxed{7,4}\boxed{4,7}\boxed{6,5}\boxed{5,6}+\boxed{7,6}\boxed{6,7}\boxed{5,4}\boxed{4,5}-2\boxed{7,4}\boxed{4,6}\boxed{6,5}\boxed{5,7} \\ -2&\boxed{7,5}\boxed{5,4}\boxed{4,6}\boxed{6,7}-2\boxed{7,4}\boxed{4,5}\boxed{5,6}\boxed{6,7} \end{aligned}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7); \quad (54)$$

$$\left.\begin{array}{l}
\left\{\frac{\boxed{0,2}}{\boxed{1,2}}D'f-\frac{\boxed{0,3}}{\boxed{1,3}}Df'\right\}\div\left\{\frac{\boxed{0,3}}{\boxed{1,3}}-\frac{\boxed{0,2}}{\boxed{1,2}}\right\}N\\
=\left\{\frac{\boxed{0,1}}{\boxed{2,1}}D''f''-\frac{\boxed{0,3}}{\boxed{2,3}}Df'''\right\}\div\left\{\frac{\boxed{0,3}}{\boxed{2,3}}-\frac{\boxed{0,1}}{\boxed{2,1}}\right\}N\\
=\left\{\frac{\boxed{0,2}}{\boxed{3,2}}D'f^{IV}-\frac{\boxed{0,1}}{\boxed{3,1}}D''f^{V}\right\}\div\left\{\frac{\boxed{0,1}}{\boxed{3,1}}-\frac{\boxed{0,2}}{\boxed{3,2}}\right\}N
\end{array}\right\}=(\chi,\chi_1,\chi_2,\chi_3);\quad(55)$$

$$\left.\begin{array}{l}
\left\{\frac{\boxed{4,6}}{\boxed{5,6}}D_2f_1-\frac{\boxed{4,7}}{\boxed{5,7}}D_1f_2\right\}\div\left\{\frac{\boxed{4,7}}{\boxed{5,7}}-\frac{\boxed{4,6}}{\boxed{5,6}}\right\}N^{IV}\\
=\left\{\frac{\boxed{4,5}}{\boxed{6,5}}D_3f_3-\frac{\boxed{4,7}}{\boxed{6,7}}D_1f_4\right\}\div\left\{\frac{\boxed{4,7}}{\boxed{6,7}}-\frac{\boxed{4,5}}{\boxed{6,5}}\right\}N^{IV}\\
=\left\{\frac{\boxed{4,6}}{\boxed{7,6}}D_2f_5-\frac{\boxed{4,5}}{\boxed{7,5}}D_3f_6\right\}\div\left\{\frac{\boxed{4,5}}{\boxed{7,5}}-\frac{\boxed{4,6}}{\boxed{7,6}}\right\}N^{IV}
\end{array}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7);\quad(56)$$

$$N'=\frac{AN+f}{\left\{\boxed{0,3}\div\boxed{1,3}\right\}D};\qquad N'=\frac{A'N+f'}{\left\{\boxed{0,2}\div\boxed{1,2}\right\}D'};\quad(57)$$

$$N''=\frac{A''N+f''}{\left\{\boxed{0,3}\div\boxed{2,3}\right\}D};\qquad N''=\frac{A'N+f'''}{\left\{\boxed{0,1}\div\boxed{2,1}\right\}D''};\quad(58)$$

$$N'''=\frac{AN+f^{IV}}{\left\{\boxed{0,1}\div\boxed{3,1}\right\}D''};\qquad N'''=\frac{A''N+f^{V}}{\left\{\boxed{0,2}\div\boxed{3,2}\right\}D'};\quad(59)$$

$$\left.\begin{array}{l}
N=\frac{\boxed{1,2}\,\boxed{0,3}\,Df'-\boxed{1,3}\,\boxed{0,2}\,D'f''}{\boxed{1,3}\,\boxed{0,2}\,AD'-\boxed{1,2}\,\boxed{0,3}\,A'D}=\frac{\boxed{2,1}\,\boxed{0,3}\,Df'''-\boxed{2,3}\,\boxed{0,1}\,D''f''}{\boxed{2,3}\,\boxed{0,1}\,A''D''-\boxed{2,1}\,\boxed{0,3}\,A'D}\\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad=\frac{\boxed{3,2}\,\boxed{0,1}\,D''f^{V}-\boxed{3,1}\,\boxed{0,2}\,D'f^{IV}}{\boxed{3,1}\,\boxed{0,2}\,AD'-\boxed{3,2}\,\boxed{0,1}\,A''D''}
\end{array}\right\};\quad(60)$$

$$N^{V}=\frac{A_1N^{IV}+f_1}{\left\{\boxed{4,7}\div\boxed{5,7}\right\}D_1};\qquad N^{V}=\frac{A_2N^{IV}+f_2}{\left\{\boxed{4,6}\div\boxed{5,6}\right\}D_2};\quad(61)$$

$$N^{VI}=\frac{A_3N^{IV}+f_3}{\left\{\boxed{4,7}\div\boxed{6,7}\right\}D_1};\qquad N^{VI}=\frac{A_2N^{IV}+f_4}{\left\{\boxed{4,5}\div\boxed{6,5}\right\}D_3};\quad(62)$$

$$N^{VII}=\frac{A_1N^{IV}+f_5}{\left\{\boxed{4,5}\div\boxed{7,5}\right\}D_3};\qquad N^{VII}=\frac{A_3N^{IV}+f_6}{\left\{\boxed{4,6}\div\boxed{7,6}\right\}D_2};\quad(63)$$

$$\left.\begin{array}{l}
N^{IV}=\frac{\boxed{4,7}\,\boxed{5,6}\,D_1f_2-\boxed{4,6}\,\boxed{5,7}\,D_2f_1}{\boxed{4,6}\,\boxed{5,7}\,A_1D_2-\boxed{4,7}\,\boxed{5,6}\,A_2D_1}=\frac{\boxed{4,7}\,\boxed{6,5}\,D_1f_4-\boxed{4,5}\,\boxed{6,7}\,D_3f_3}{\boxed{4,5}\,\boxed{6,7}\,A_3D_3-\boxed{4,7}\,\boxed{6,5}\,A_2D_1}\\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad=\frac{\boxed{4,5}\,\boxed{7,6}\,D_3f_6-\boxed{4,6}\,\boxed{7,5}\,D_2f_5}{\boxed{4,6}\,\boxed{7,5}\,A_1D_2-\boxed{4,5}\,\boxed{7,6}\,A_3D_3}
\end{array}\right\}.\quad(64)$$

11. Equations (53) to (64) are entirely rigorous. They are, moreover, under a very simple and convenient form for the computation of N', N'', N''', &c.; and, as there are duplicate and independent formulæ for all these quantities, any error that may accidentally creep into the computation of one formula is at once detected by computation of the other. They have also this additional advantage: sometimes one of the formulæ for N, N', N'', &c., gives value for these quantities of the form

$N=\frac{a-a'}{a''}$, in which a is very nearly equal to a', and the computation of these quantities cannot be readily effected by logarithms with sufficient precision to give their *difference* correct to more than three or four significant figures; and, in all these cases, the other formula for the same quantity gives a value which is free from this source of error.

The computation of the successive approximations to the values of the required quantities is then arranged as follows:—

We first find the roots of equation (53), on the supposition that χ is equal to nothing. We shall designate these roots by g, g_1, g_2, and g_3, and the corresponding values of the second member by χ, χ_1, χ_2, and χ_3. The roots of equation (54) are also designated by g_4, g_5, g_6, and g_7, and the corresponding values of the second member by χ_4, χ_5, χ_6, and χ_7. When g, g_1, g_2, &c. have been determined, we must transform equation (53) into others whose roots shall be smaller by the values g, g_1, g_2, and g_3. Then if we denote the corrections to be applied to g, g_1, &c., in order to obtain their correct values by δg, δg_1, δg_2, and δg_3, we shall have a system of equations for the determination of δg, δg_1, δg_2, and δg_3, of the following form:—

$$\left.\begin{array}{l}\delta g^4+a\,\delta g^3+b\,\delta g^2+c\,\delta g=\chi\,; \\ \delta g_1^4+a'\,\delta g_1^3+b'\,\delta g_1^2+c'\,\delta g_1=\chi_1; \\ \delta g_2^4+a''\,\delta g_2^3+b''\,\delta g_2^2+c''\,\delta g_2=\chi_2; \\ \delta g_3^4+a'''\delta g_3^3+b'''\delta g_3^2+c'''\delta g_3=\chi_3;\end{array}\right\}\quad. \qquad (65)$$

$$\delta g+\delta g_1+\delta g_2+\delta g_3+\delta g_4+\delta g_5+\delta g_6+\delta g_7=0. \qquad (66) \quad \text{[Equation of condition.]}$$

The equations for the determination of δg_4, δg_5, δg_6, and δg_7 are entirely similar. Then, having determined the approximate values of g, g_1, g_2, &c., we must substitute them in succession in equations (31—42) inclusive, and we shall obtain the values of A, A', A'', &c., D, D', D'', &c., which are to be substituted in equations (57—59), and we shall obtain the approximate values of N', N'', and N'''. These quantities are then to be substituted in equations (28), and we shall get the values of b_1, b_2, b_3, and b_4; which quantities, together with the value of g, are to be substituted in equations (47—50) inclusive, and we obtain B_1, B_2, B_3, C_1, C_2, &c., &c. Then equation (52) will give f_1, f_2, &c., which are to be substituted in equations (61—64), and we shall obtain N^{IV}, N^{V}, N^{VI}, and N^{VII}. Equations (27) will then give b, b', b'', and b''', which are to be substituted along with g in equations (43—46). Then equations (51) will give f, f', f'', &c., which being substituted in equations (54) will give the value of χ, on which the value of δg depends in the first of equations (65). When δg has been determined we must add it to the approximate value of g, and repeat the whole computation with the corrected value of g, and by this means we shall obtain the correct values of g, N', N'', N''', &c. In like manner we shall obtain g_1, N_1', N_1'', N_1''', &c.; g_2, N_2', N_2'', &c.

12. We shall now reduce the preceding formulæ to numbers, and illustrate by a numerical example the extreme simplicity of the formulæ (which appear so unwieldy in their algebraic form), and the comparative facility with which the required quantities can be obtained.

Substituting the values of $(0,1)$, $(0,2)$, $(0,3)$, &c., given in § 7, in equations (26), we shall get the following values of $\boxed{0,0}$, $\boxed{1,1}$, $\boxed{2,2}$, &c.:—

$$\left.\begin{array}{ll} \boxed{0,0}=g-\ \ 5''.5702558; & \boxed{4,4}=g-\ \ 7''.5123754; \\ \boxed{1,1}=g-11\ .3147682; & \boxed{5,5}=g-18\ .5962129; \\ \boxed{2,2}=g-13\ .0721730; & \boxed{6,6}=g-\ \ 2\ .7662522; \\ \boxed{3,3}=g-17\ .5528645; & \boxed{7,7}=g-\ \ 0\ .6479569. \end{array}\right\}. \tag{67}$$

These give

$$\boxed{0,0}\boxed{1,1}=g^2-16.8850240.g+\ 63.02615319170556; \tag{68}$$
$$\boxed{0,0}\boxed{2,2}=g^2-18.6424288.g+\ 72.81534747185340; \tag{69}$$
$$\boxed{0,0}\boxed{3,3}=g^2-23.1231203.g+\ 97.77394528773910; \tag{70}$$
$$\boxed{1,1}\boxed{2,2}=g^2-24.3869412.g+147.90860736529860; \tag{71}$$
$$\boxed{1,1}\boxed{3,3}=g^2-28.8676327.g+198.60659306350890; \tag{72}$$
$$\boxed{2,2}\boxed{3,3}=g^2-30.6250375.g+229.45408138955850; \tag{73}$$

$$\boxed{4,4}\boxed{5,5}=g^2-26.1085883.g+139.70173232312266; \tag{74}$$
$$\boxed{4,4}\boxed{6,6}=g^2-10.2786276.g+\ 20.78112497747588; \tag{75}$$
$$\boxed{4,4}\boxed{7,7}=g^2-\ \ 8.1603323.g+\ \ 4.86769547582026; \tag{76}$$
$$\boxed{5,5}\boxed{6,6}=g^2-21.3624651.g+\ 51.44181484629338; \tag{77}$$
$$\boxed{5,5}\boxed{7,7}=g^2-19.2441698.g+\ 12.04954446242401; \tag{78}$$
$$\boxed{6,6}\boxed{7,7}=g^2-\ \ 3.4142091.g+\ \ 1.79241220013018; \tag{79}$$

$$\boxed{0,0}\boxed{1,1}\boxed{2,2}\boxed{3,3}=\left.\begin{array}{r} g^4-47.5100615.g^3+809.584727769664.g^2 \\ -5804.515976137376.g+14461.60808412039 \end{array}\right\}; \tag{80}$$

$$\boxed{4,4}\boxed{5,5}\boxed{6,6}\boxed{7,7}=\left.\begin{array}{l} g^4-29.5227974.g^3+230.634324285266.g^2 \\ -523.768277980466.g+250.403089395286 \end{array}\right\}. \tag{81}$$

13. We shall now give the computation of equations (31—50), and also of (53) and (54) in full, because we can then readily correct them for any assumed changes in the adopted masses; and observe at the same time that the coefficients in equations (55—64) are independent of the masses of the planets, and consequently are not affected by their supposed variation.

Computation of A.

$$\begin{array}{lrl} & \boxed{0,0}\boxed{2,2}= & g^2-18.642428800.g+\ \ 72.815347472 \\ -\boxed{0,0}\boxed{1,2}\boxed{2,3}\div\boxed{1,3}= & & -21.562395123.g+120.10805649 \\ -\boxed{2,2}\boxed{1,0}\boxed{0,3}\div\boxed{1,3}= & & -\ \ 0.016959303.g+\ \ \ 0.22169494 \\ +\boxed{1,0}\boxed{0,2}\boxed{2,3}\div\boxed{1,3}= & & +\ \ \ 0.1803729465 \\ +\boxed{1,2}\boxed{2,0}\boxed{0,3}\div\boxed{1,3}= & & +\ \ \ 0.01596936 \\ -\boxed{2,0}\boxed{0,2} & = & -\ \ \ 0.00787688 \\ \text{Sum of terms} & A= & g^2-40.22178322.g+193.3335643 \end{array}$$

Computation of A'.

$$\begin{aligned}
&\boxed{0,0}\boxed{3,3} &&= g^2 - 23.123120300.g + 97.77394529\\
-&\boxed{0,0}\boxed{1,3}\boxed{3,2}\div\boxed{1,2} &&= -\ 0.014363307.g +\ 0.0800073\\
-&\boxed{3,3}\boxed{1,0}\boxed{0,2}\div\boxed{1,2} &&= -\ 0.008365164.g +\ 0.1468326\\
+&\boxed{1,0}\boxed{0,3}\boxed{3,2}\div\boxed{1,2} &&= +\ 0.0002436\\
+&\boxed{1,3}\boxed{3,0}\boxed{0,2}\div\boxed{1,2} &&= +\ 0.0000106\\
-&\boxed{3,0}\boxed{0,3} &&= -\ 0.0000216\\
&\text{Sum of terms} \quad A' &&= g^2 - 23.14584877.g + 98.0010178
\end{aligned}$$

Computation of A''.

$$\begin{aligned}
&\boxed{0,0}\boxed{1,1} &&= g^2 - 16.885024000.g + 63.02615319\\
-&\boxed{0,0}\boxed{2,1}\boxed{1,3}\div\boxed{2,3} &&= -\ 1.136499372.g +\ 6.33059222\\
-&\boxed{1,1}\boxed{2,0}\boxed{0,3}\div\boxed{2,3} &&= -\ 0.000740612.g +\ 0.00837985\\
+&\boxed{2,0}\boxed{0,1}\boxed{1,3}\div\boxed{2,3} &&= +\ 0.00950700\\
+&\boxed{2,1}\boxed{1,0}\boxed{0,3}\div\boxed{2,3} &&= +\ 0.01927424\\
-&\boxed{1,0}\boxed{0,1} &&= -\ 0.21770124\\
&\text{Sum of terms} \quad A'' &&= g^2 - 18.022263984.g + 69.17620526
\end{aligned}$$

Computation of D.

$$\begin{aligned}
&\boxed{1,1}\boxed{2,2} &&= g^2 - 24.386941200.g + 147.908607365\\
-&\boxed{1,1}\boxed{0,2}\boxed{2,3}\div\boxed{0,3} &&= -10.635634399g. + 120.339738\\
-&\boxed{2,2}\boxed{0,1}\boxed{1,3}\div\boxed{0,3} &&= -12.836685468.g + 167.803373\\
+&\boxed{0,1}\boxed{1,2}\boxed{2,3}\div\boxed{0,3} &&= +276.789684\\
+&\boxed{0,2}\boxed{2,1}\boxed{1,3}\div\boxed{0,3} &&= +\ 12.087392\\
-&\boxed{2,1}\boxed{1,2} &&= -\ 24.5056485\\
&\text{Sum of terms} \quad D &&= g^2 - 47.859261067.g + 700.423146
\end{aligned}$$

Computation of D'.

$$\begin{aligned}
&\boxed{1,1}\boxed{3,3} &&= g^2 - 28.867632700.g + 198.606593\\
-&\boxed{1,1}\boxed{0,3}\boxed{3,2}\div\boxed{0,2} &&= -\ 0.029119780.g +\ 0.329484\\
-&\boxed{3,3}\boxed{0,1}\boxed{1,2}\div\boxed{0,2} &&= -26.024746029.g + 456.808841\\
+&\boxed{0,1}\boxed{1,3}\boxed{3,2}\div\boxed{0,2} &&= +\ 0.373801\\
+&\boxed{0,3}\boxed{3,1}\boxed{1,2}\div\boxed{0,2} &&= +\ 0.033095\\
-&\boxed{3,1}\boxed{1,3} &&= -\ 0.016324\\
&\text{Sum of terms} \quad D' &&= g^2 -\ 54.92149851.g + 656.135490
\end{aligned}$$

Computation of D''.

$$\begin{aligned}
&\boxed{2,2}\boxed{3,3} &&= g^2 - 30.625037500.g + 229.4540814\\
-&\boxed{2,2}\boxed{0,3}\boxed{3,1}\div\boxed{0,1} &&= -\ 0.001271660.g +\ 0.0166233\\
-&\boxed{3,3}\boxed{0,2}\boxed{2,1}\div\boxed{0,1} &&= -\ 0.941628729.g +\ 16.5282815\\
+&\boxed{0,3}\boxed{3,2}\boxed{2,1}\div\boxed{0,1} &&= +\ 0.0274200\\
+&\boxed{0,2}\boxed{2,3}\boxed{3,1}\div\boxed{0,1} &&= +\ 0.0135249\\
-&\boxed{3,2}\boxed{2,3} &&= -\ 0.3097073\\
&\text{Sum of terms} \quad D'' &&= g^2 - 31.56793789.g + 245.7302238
\end{aligned}$$

Computation of A_1.

$$\begin{array}{rl}
\boxed{4,4}\,\boxed{6,6} & = g^2 - 10.278627600.g + 20.78112498 \\
-\boxed{4,4}\,\boxed{5,6}\,\boxed{6,7} \div \boxed{5,7} & = -2.020545804.g + 15.17909859 \\
-\boxed{6,6}\,\boxed{5,4}\,\boxed{4,7} \div \boxed{5,7} & = -2.294602698.g + 6.34744976 \\
+\boxed{5,4}\,\boxed{4,6}\,\boxed{6,7} \div \boxed{5,7} & = +3.66870084 \\
+\boxed{5,6}\,\boxed{6,4}\,\boxed{4,7} \div \boxed{5,7} & = +0.01012112 \\
-\boxed{6,4}\,\boxed{4,6} & = -0.00800875 \\
\text{Sum of terms} \quad A_1 & = g^2 - 14.593776102.g + 45.97848654
\end{array}$$

Computation of A_2.

$$\begin{array}{rl}
\boxed{4,4}\,\boxed{7,7} & = g^2 - 8.160332300.g + 4.86769548 \\
-\boxed{4,4}\,\boxed{5,7}\,\boxed{7,6} \div \boxed{5,6} & = -0.031614994.g + 0.23750370 \\
-\boxed{7,7}\,\boxed{5,4}\,\boxed{4,6} \div \boxed{5,6} & = -1.815697936.g + 1.17649401 \\
+\boxed{5,4}\,\boxed{4,7}\,\boxed{7,6} \div \boxed{5,6} & = +0.07254385 \\
+\boxed{5,7}\,\boxed{7,4}\,\boxed{4,6} \div \boxed{5,6} & = +0.00015836 \\
-\boxed{4,7}\,\boxed{7,4} & = -0.00020013 \\
\text{Sum of terms} \quad A_2 & = g^2 - 10.00764523.g + 6.35419527
\end{array}$$

Computation of A_3.

$$\begin{array}{rl}
\boxed{4,4}\,\boxed{5,5} & = g^2 - 26.108588300.g + 139.70173232 \\
-\boxed{4,4}\,\boxed{6,5}\,\boxed{5,7} \div \boxed{6,7} & = -0.069351012.g + 0.52099084 \\
-\boxed{5,5}\,\boxed{6,4}\,\boxed{4,7} \div \boxed{6,7} & = -0.005009102.g + 0.09315032 \\
+\boxed{6,4}\,\boxed{4,5}\,\boxed{5,7} \div \boxed{6,7} & = +0.12592049 \\
+\boxed{6,5}\,\boxed{5,4}\,\boxed{4,7} \div \boxed{6,7} & = +0.15913300 \\
-\boxed{5,4}\,\boxed{4,5} & = -57.68249007 \\
\text{Sum of terms} \quad A_3 & = g^2 - 26.182948414.g + 82.91843690
\end{array}$$

Computation of D_1.

$$\begin{array}{rl}
\boxed{5,5}\,\boxed{6,6} & = g^2 - 21.362465100.g + 51.44181485 \\
-\boxed{5,5}\,\boxed{4,6}\,\boxed{6,7} \div \boxed{4,7} & = -1.598839244.g + 29.732355 \\
-\boxed{6,6}\,\boxed{4,5}\,\boxed{5,7} \div \boxed{4,7} & = -25.138334450.g + 69.538973 \\
+\boxed{4,5}\,\boxed{5,6}\,\boxed{6,7} \div \boxed{4,7} & = +50.793156 \\
+\boxed{4,6}\,\boxed{6,5}\,\boxed{5,7} \div \boxed{4,7} & = +0.110881 \\
-\boxed{6,5}\,\boxed{5,6} & = -0.140127 \\
\text{Sum of terms} \quad D_1 & = g^2 - 48.09963879.g + 201.477053
\end{array}$$

Computation of D_2.

$$\begin{array}{rl}
\boxed{5,5}\,\boxed{7,7} & = g^2 - 19.244169800.g + 12.04954446 \\
-\boxed{5,5}\,\boxed{4,7}\,\boxed{7,6} \div \boxed{4,6} & = -0.039953699.g + 0.7429875 \\
-\boxed{7,7}\,\boxed{4,5}\,\boxed{5,6} \div \boxed{4,6} & = -31.768769968.g + 20.5847937 \\
+\boxed{4,5}\,\boxed{5,7}\,\boxed{7,6} \div \boxed{4,6} & = +1.0043694 \\
+\boxed{4,7}\,\boxed{7,5}\,\boxed{5,6} \div \boxed{4,6} & = +0.0027708 \\
-\boxed{7,5}\,\boxed{5,7} & = -0.0021925 \\
\text{Sum of terms} \quad D_2 & = g^2 - 51{,}052893467.g + 34.3822734
\end{array}$$

Computation of D_3.

$$\begin{aligned}
\boxed{6,6}\,\boxed{7,7} &= g^2-3.4142091000.g+1.7924122001\\
-\boxed{6,6}\,\boxed{4,7}\,\boxed{7,5}\div\boxed{4,5} &= -0.0000872187.g+0.0002412688\\
-\boxed{7,7}\,\boxed{4,6}\,\boxed{6,5}\div\boxed{4,5} &= -0.0044108379.g+0.0028580329\\
+\boxed{4,7}\,\boxed{7,6}\,\boxed{6,5}\div\boxed{4,5} &= +0.0001762293\\
+\boxed{4,6}\,\boxed{6,7}\,\boxed{7,5}\div\boxed{4,5} &= +0.0001394486\\
-\boxed{7,6}\,\boxed{6,7} &= -0.0638795429\\
\text{Sum of terms}\quad D_3 &= g^2-3.4187071566.g+1.7319476368
\end{aligned}$$

Computation of B, B', and B''.

$$\begin{aligned}
\boxed{2,2} &= g--13^{\prime\prime}.0721730\\
-\boxed{1,2}\,\boxed{2,3}\div\boxed{1,3} &= -21\,.5623951\\
\text{Sum}\quad &= B\div b=g-34\,.6345681;
\end{aligned}$$

$$\begin{aligned}
\boxed{3,3} &= g-17^{\prime\prime}.552864500\\
-\boxed{1,3}\,\boxed{3,2}\div\boxed{1,2} &= -\ 0\,.014363307\\
\text{Sum}\quad &= B'\div b=g-17\,.567227807;
\end{aligned}$$

$$\begin{aligned}
\boxed{1,1} &= g-11^{\prime\prime}.31476820\\
-\boxed{2,1}\,\boxed{1,3}\div\boxed{2,3} &= -\ 1\,.13649937\\
\text{Sum}\quad &= B''\div b=g-12\,.45126757.
\end{aligned}$$

Computation of C, C', C'', and C'''.

$$\begin{aligned}
-\boxed{2,2} &= 13^{\prime\prime}.0721730-g\\
+\boxed{0,2}\,\boxed{2,3}\div\boxed{0,3} &= 10\,.6356344\\
\text{Sum}\quad &= 23\,.7078074-g
\end{aligned}$$

$$\text{log.}\ \{\boxed{0,3}\div\boxed{1,3}\}=[9\,.176390].$$

$$\text{Therefore}\quad C'=\{23^{\prime\prime}.7078074-g\}\times[9\,.1763990]b'.$$

$$\begin{aligned}
-\boxed{3,3} &= 17\,.55286450-g\\
+\boxed{0,3}\,\boxed{3,2}\div\boxed{0,2} &= 0\,.02911978\\
\text{Sum}\quad &= 17\,.58198428-g
\end{aligned}$$

$$\text{log.}\ \{\boxed{0,2}\div\boxed{1,2}\}=[\ 8\,.8694654\,]$$

$$\text{Therefore}\quad C'=\{17^{\prime\prime}.58198428-g\}\times[8\,.8694654]b'.$$

$$\begin{aligned}
+\boxed{0,3}\,\boxed{2,1}\div\boxed{2,3} &= +\ 0.170596\\
-\boxed{0,1} &== -\ 1.926868\\
\text{Sum}\quad &= C''\div b'=-\ 1.756272
\end{aligned}$$

$$\text{Therefore}\quad C''=-[0.2445917]b'.$$

$$\begin{aligned}
+\boxed{0,2}\,\boxed{3,1}\div\boxed{3,2} &= +\ 0.084146\\
-\boxed{0,1} &= -\ 1.926868\\
\text{Sum}\quad &= C'''\div b'=-\ 1.842722
\end{aligned}$$

$$\text{Therefore}\quad C'''=-[0.2654598]b'.$$

Computation of B_1, B_2, and B_3.

$$\begin{aligned}
\boxed{6,6} &= g-\ 2^{\prime\prime}.7662522\\
-\boxed{5,6}\,\boxed{6,7}\div\boxed{5,7} &= -\ 2\,.0205458\\
\text{Sum}\quad &= B_1\div b_1=g-\ 4\,.7867980;
\end{aligned}$$

$$\begin{aligned}
\boxed{7,7} &= g-\ 0^{\prime\prime}.64795690\\
-\boxed{5,7}\,\boxed{7,6}\div\boxed{5,6} &= -\ 0\,.0316149937\\
\text{Sum}\quad &= B_2\div b_1=g-\ 0\,.6795718937;
\end{aligned}$$

$$\begin{aligned}
\boxed{5,5} &= g-18\,.5962129\\
-\boxed{6,5}\,\boxed{5,7}\div\boxed{6,7} &= -\ 0\,.069351012\\
\text{Sum}\quad &= B_3\div b_1=g-18\,.665563912.
\end{aligned}$$

Computation of C_1, C_2, C_3, and C_4.

$$\begin{aligned}
-\boxed{6,6} &= 2^{\prime\prime}.7662522\quad -g\\
+\boxed{4,6}\,\boxed{6,7}\div\boxed{4,7} &= 1\,.598839244\\
\text{Sum}\quad &= 4\,.365091444-g
\end{aligned}$$

$$\text{log.}\ \{\boxed{4,7}\div\boxed{5,7}\}=[9\,.2840950]$$

$$\text{Therefore}\quad C_1=\{4^{\prime\prime}.365091444-g\}\times[9\,.2840950]b_2.$$

$$\begin{aligned}
-\boxed{7,7} &= 0\,.6479569\quad -g\\
+\boxed{4,7}\,\boxed{7,6}\div\boxed{4,6} &= 0\,.0399536996\\
\text{Sum}\quad &= 0\,.6879105996-g
\end{aligned}$$

$$\text{log.}\ \{\boxed{4,6}\div\boxed{5,6}\}=[9\,.1824311]$$

$$\text{Therefore}\quad C_2=\{0^{\prime\prime}.6879105996-g\}\times[9\,.1824311]b_2.$$

$$\begin{aligned}
\boxed{4,7}\,\boxed{6,5}\div\boxed{6,7} &= +\ 0.01333975\\
-\boxed{4,5} &= -\ 4.835390\\
\text{Sum}\quad &= C_3\div b_2=-\ 4.822050
\end{aligned}$$

$$\text{Therefore}\quad C_3=-[0.6832317]b_2.$$

$$\begin{aligned}
\boxed{4,6}\,\boxed{7,5}-\boxed{7,6} &= +\ 0.01055562\\
-\boxed{4,5} &= -\ 4.835390\\
\text{Sum}\quad &= C_4\div b_2=-\ 4.8248344
\end{aligned}$$

$$\text{Therefore}\quad C_4=-[0.6834824]b_2.$$

Computation of E, E', E'', and E'''.

$+\boxed{0,3}\boxed{1,2}\div\boxed{1,3}=+\ 0.8287048$
$-\boxed{0,2} \qquad =-\ 0.4087579$
Sum $=E\div b''=+\ 0.4199469$
Therefore $E=+[9.6231944]b''$.

$-\boxed{1,1}=+\ 11''.3147682-g$
$+\boxed{0,1}\boxed{1,3}\div\boxed{0,3}=+\ 12\ .8366855$
Sum $=+\ 24\ .1514537-g$
log. $\{\boxed{0,3}\div\boxed{2,3}\}=\ [\ 8\ .5847028]$
Therefore $E'=\ \{24''.1514537-g\}\times$
$[\ 8\ .5847028]b''$.

$-\boxed{3,3}=+\ 17''.5528645-g$
$+\boxed{0,3}\boxed{3,1}\div\boxed{0,1}=+\ \ 0\ .00127166$
Sum $=+\ 17\ .55413616-g$
log. $\{\boxed{0,1}\div\boxed{2,1}\}=\ [9\ .6375865]$
Therefore $E''=\ \{17''.55413616-g\}\times$
$[9\ .6375865]b''$.

$\boxed{0,1}\boxed{3,2}\div\boxed{3,1}=+\ 9.360164$
$-\boxed{0,2}=-\ 0.408758$
Sum $=E'''\div b''=+\ 8.951406$
Therefore $E'''=+[0.9518913]b''$.

Computation of F, F', F'', and F'''.

$\boxed{0,2}\boxed{1,3}\div\boxed{1,2}=+\ 0.004346915$
$-\boxed{0,3} \qquad =-\ 0.008812816$
Sum $=F\div b'''=-\ 0.004465901$
Therefore $F=-[7.6499091]b'''$.

$\boxed{0,1}\boxed{2,3}\div\boxed{2,1}=+\ 0.09954018$
$-\boxed{0,3} \qquad =-\ 0.00881282$
Sum $=F'\div b'''=+\ 0.09072736$
Therefore $F'=+[8.9577383]b'''$.

$-\boxed{2,2}=+13''.07217300-g$
$+\boxed{0,2}\boxed{2,1}\div\boxed{0,1}=+\ \ 0\ .94162873$
Sum $=+14\ .01380173-g$
log. $\{\boxed{0,1}\div\boxed{3,1}\}=\ [0\ .8407439]$
Therefore $F''=\ \{14''.01380173-g\}\times$
$[0\ .8407439]b''$.

$-\boxed{1,1}=+11''.3147682-g$
$+\boxed{0,1}\boxed{1,2}\div\boxed{0,2}=+26\ .0247460$
Sum $=+37\ .3395142-g$
log. $\{\boxed{0,2}\div\boxed{3,2}\}=\ [9\ .4809266]$
Therefore $F'''=\ \{37''.3395142-g\}\times$
$[9\ .4809266]b'''$.

Computation of E_1, E_2, E_3, and E_4.

$+\boxed{4,7}\boxed{5,6}\div\boxed{5,7}=+\ 0.03215367$
$-\boxed{4,6} \qquad =-\ 0.02544290$
Sum $=E_1\div b_3=+\ 0.00671077$
Therefore $E_1=\ [7.8267723]b_3$.

$-\boxed{5,5}=+18''.5962129-g$
$+\boxed{4,5}\boxed{5,7}\div\boxed{4,7}=+25\ .1383344$
Sum $=+43\ .7345473-g$
log. $\{\boxed{4,7}\div\boxed{6,7}\}=\ [8\ .2017618]$
Therefore $E_2=\ \{43\ .7345473-g\}\times$
$[8\ .2017618]b_3$.

$-\boxed{7,7}=+0\ .6479569-g$
$+\boxed{4,7}\boxed{7,5}\div\boxed{4,5}=+0\ .000047219$
Sum $=+0\ .648004119-g$
log. $\{\boxed{4,5}\div\boxed{6,5}\}=\ [0\ .7610455]$
Therefore $E_3=\ \{0''.648004119-g\}\times$
$[0\ .7610455]b_3$.

$\boxed{4,5}\boxed{7,6}\div\boxed{7,5}=+11.655051$
$-\boxed{4,6} \qquad =-\ 0.025443$
Sum $=E_4\div b_3=+11.629608$
Therefore $E_4=\ [1.0655652]b_3$.

Computation of F_1, F_2, F_3, and F_4.

$\boxed{4,6}\boxed{5,7}\div\boxed{5,6}=+\ 0.004099602$
$-\boxed{4,7} \qquad =-\ 0.005180905$
Sum $=F_1\div b_4=-\ 0.001081303$
Therefore $F_1=-[7.0339474]b_4$.

$\boxed{4,5}\boxed{6,7}\div\boxed{6,5}=+\ 1.8779726$
$-\boxed{4,7} \qquad =-\ 0.0051809$
Sum $=F_2\div b_4=+\ 1.8727917$
Therefore $F_2=+[0.2724895]b_4$.

$-\boxed{6,6}=+2''.7662522-g$
$+\boxed{4,6}\boxed{6,5}\div\boxed{4,5}=+0\ .004410838$
Sum $=+2\ .770663038-g$
log. $\{\boxed{4,5}\div\boxed{7,5}\}=\ [1\ .7737962]$
Therefore $F_3=\ \{2''.770663038-g\}\times$
$[1\ .7737962]b_4$.

$-\boxed{5,5}=+18\ .5962129-g$
$+\boxed{4,5}\boxed{5,6}\div\boxed{4,6}=+31\ .76876997$
Sum $=+50\ .364982867-g$
log. $\{\boxed{4,6}\div\boxed{7,6}\}=\ [\ 9\ .11284867]$
Therefore $F_4=\ \{50''.3649829-g\}\times$
$[\ 9\ .1128486]b_4$.

Computation of the Equations of the Fourth Degree.

$-\boxed{2,1}\boxed{1,2}\boxed{0,0}\boxed{3,3}=-24.50564851.g^2+566.64705848.g-2396.01393568$

$-\boxed{3,2}\boxed{2,3}\boxed{0,0}\boxed{1,1}=-0.30970734.g^2+5.22941589.g-19.51966234$

$-\boxed{1,0}\boxed{0,1}\boxed{2,2}\boxed{3,3}=-0.21770124.g^2+6.66710860.g-49.95243773$

$-\boxed{3,1}\boxed{1,3}\boxed{0,0}\boxed{2,2}=-0.01632389.g^2+0.30431699.g-1.18862983$

$-\boxed{2,0}\boxed{0,2}\boxed{1,1}\boxed{3,3}=-0.00787688.g^2+0.22738681.g-1.56439984$

$-\boxed{3,0}\boxed{0,3}\boxed{1,1}\boxed{2,2}=-0.00002157.g^2+0.00052594.g-0.00318986$

$+2\boxed{3,2}\boxed{2,1}\boxed{1,3}\boxed{0,0}=+0.70396439.g-3.92126177$

$+2\boxed{2,0}\boxed{0,1}\boxed{1,2}\boxed{3,3}=+0.40998748.g-7.19645470$

$+2\boxed{3,1}\boxed{1,0}\boxed{0,3}\boxed{2,2}=+0.00055368.g-0.00723785$

$+2\boxed{3,0}\boxed{0,2}\boxed{2,3}\boxed{1,1}=+0.00045875.g-0.00519060$

$+\boxed{1,0}\boxed{0,1}\boxed{3,2}\boxed{2,3}=+0.06742367$

$+\boxed{2,1}\boxed{1,2}\boxed{3,0}\boxed{0,3}=+0.00052850$

$+\boxed{2,0}\boxed{0,2}\boxed{3,1}\boxed{1,3}=+0.00012858$

$-2\boxed{3,0}\boxed{0,1}\boxed{1,2}\boxed{2,3}=-0.01193875$

$-2\boxed{3,1}\boxed{1,0}\boxed{0,2}\boxed{2,3}=-0.00588878$

$-2\boxed{3,0}\boxed{0,2}\boxed{2,1}\boxed{1,3}=-0.00052136$

Sum of preceding terms $=-25.05727943.g^2+580.19077701.g+2479.32266834$

Add $\boxed{0,0}\boxed{1,1}\boxed{2,2}\boxed{3,3}=$

$$g^4-47.5100615.g^3+809.58472777.g^2-5804.51597614.g-14461.60808412$$

Sum is the value of equation (53).

Whence we get $\left.\begin{array}{l} g^4-47.5100615.g^3+784.52744834.g^2 \\ -5224.32519913.g+11982.28541576 \end{array}\right\}=(\chi,\chi_1,\chi_2,\chi_3).$ (82)

In like manner,

$-\boxed{5,4}\boxed{4,5}\boxed{6,6}\boxed{7,7}=-57.682490072.g^2+196.940082515.g-103.390798939$

$-\boxed{6,5}\boxed{5,6}\boxed{4,4}\boxed{7,7}=-0.140126895.g^2+1.143482030.g-0.682095055$

$-\boxed{7,6}\boxed{6,7}\boxed{4,4}\boxed{5,5}=-0.063879543.g^2+1.667804687.g-0.924082807$

$-\boxed{6,4}\boxed{4,6}\boxed{5,5}\boxed{7,7}=-0.008008749.g^2+0.154121732.g-0.096501781$

$-\boxed{7,5}\boxed{5,7}\boxed{4,4}\boxed{6,6}=-0.002192532.g^2+0.022536218.g-0.045563277$

$-\boxed{7,4}\boxed{4,7}\boxed{5,5}\boxed{6,6}=-0.000200132.g^2+0.004275316.g-0.010295162$

$+2\boxed{6,4}\boxed{4,5}\boxed{5,6}\boxed{7,7}=+0.508856230.g-0.329716905$

$+2\boxed{7,5}\boxed{5,4}\boxed{4,7}\boxed{6,6}=+0.010061979.g-0.027833971$

$+2\boxed{7,6}\boxed{6,5}\boxed{5,7}\boxed{4,4}=+0.008860222.g-0.066561313$

$+2\boxed{7,4}\boxed{4,6}\boxed{6,7}\boxed{5,5}=+0.000639958.g-0.011900801$

$+\boxed{5,4}\boxed{4,5}\boxed{7,6}\boxed{6,7}=+3.684731101$

$+\boxed{6,5}\boxed{5,6}\boxed{7,4}\boxed{4,7}=+0.000028044$

$+\boxed{6,4}\boxed{4,6}\boxed{7,5}\boxed{5,7}=+0.000017559$

$-2\boxed{7,4}\boxed{4,5}\boxed{5,6}\boxed{6,7}=-0.020330689$

$-2\boxed{7,5}\boxed{5,4}\boxed{4,6}\boxed{6,7}=-0.016087486$

$-2\boxed{7,4}\boxed{4,6}\boxed{6,5}\boxed{5,7}=-0.000044382$

Sum of preceding terms $=-57.896897923.g^2+200.460720887.g-109.937035864$

Add $\boxed{4,4}\boxed{5,5}\boxed{6,6}\boxed{7,7}=$

$$g^4-29.5227974.g^3+230.634324285.g^2-523.768277980.g+250.403089395$$

Sum is value of equation (54).

Whence we get $\left.\begin{array}{l} g^4-29.5227974.g^3+172.737426362.g^2 \\ -323.307557093.g+140.466053531 \end{array}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7).$ (83)

Equations (55–64) reduced to numbers are as follows, in which the numbers inclosed in brackets are logarithms.

$$N' = [0.8236010]\frac{AN+f}{D} = [1.1305346]\frac{A'N+f'}{D'}; \quad (84)$$

$$N'' = [1.4152972]\frac{A''N+f''}{D} = [0.3624135]\frac{A'N+f'''}{D''}; \quad (85)$$

$$N''' = [9.1592561]\frac{AN+f^{IV}}{D''} = [0.5190734]\frac{A''N+f^{V}}{D'}; \quad (86)$$

$$N = \frac{[9.1763990]Df' - [8.8694654]D'f}{[8.8694654]AD' - [9.1763990]A'D}; \quad (87)$$

$$N = \frac{[8.5847028]Df''' - [9.6375865]D''f''}{[9.6375865]A''D'' - [8.5847028]A'D}; \quad (88)$$

$$N = \frac{[0.8407439]D''f^{V} - [9.4809266]D'f^{IV}}{[9.4809266]AD' - [0.8407439]A''D''}; \quad (89)$$

$$N^{V} = [0.7159050]\frac{A_1N^{IV}+f_1}{D_1} = [0.8175689]\frac{A_2N^{IV}+f_2}{D_2}; \quad (90)$$

$$N^{VI} = [1.7982382]\frac{A_3N^{IV}+f_3}{D_1} = [9.2389545]\frac{A_2N^{IV}+f_4}{D_3}; \quad (91)$$

$$N^{VII} = [8.2262038]\frac{A_1N^{IV}+f_5}{D_3} = [0.8871514]\frac{A_3N^{IV}+f_6}{D_2}; \quad (92)$$

$$N^{IV} = \frac{[9.2840950]D_1f_2 - [9.1824311]D_2f_1}{[9.1824311]A_1D_2 - [9.2840950]A_2D_1}; \quad (93)$$

$$N^{IV} = \frac{[8.2017618]D_1f_4 - [0.7610455]D_3f_3}{[0.7610455]A_3D_3 - [8.2017618]A_2D_1}; \quad (94)$$

$$N^{IV} = \frac{[1.7737962]D_3f_6 - [9.1128486]D_2f_5}{[9.1128486]A_1D_2 - [1.7737962]A_3D_3}; \quad (95)$$

$$\left.\begin{array}{l} \left\{[9.9882719]D'f - [0.2952055]Df'\right\}\dfrac{1}{N} \\ = \left\{[8.9873765]Df''' - [0.0402602]D''f''\right\}\dfrac{1}{N} \\ = \left\{[8.6595749]D'f^{IV} - [0.0193922]D''f^{V}\right\}\dfrac{1}{N} \end{array}\right\} = (\chi, \chi_1, \chi_2, \chi_3); \quad (96)$$

$$\left.\begin{array}{l} \left\{[0.5787943]D_2f_1 - [0.6804582]D_1f_2\right\}\dfrac{1}{N^{IV}} \\ = \left\{[7.4419161]D_1f_4 - [0.0011998]D_3f_3\right\}\dfrac{1}{N^{IV}} \\ = \left\{[7.3400015]D_2f_5 - [0.0009491]D_3f_6\right\}\dfrac{1}{N^{IV}} \end{array}\right\} = (\chi_4, \chi_5, \chi_6, \chi_7); \quad (97)$$

We shall here repeat and number the equations which we have computed, for convenience of future reference. By this means we shall obtain the following

Fundamental Equations for the Adopted Masses.

$$A = g^2 - 40.22178322\ .g + 193.3335643; \qquad (98)$$
$$A' = g^2 - 23.14584877\ .g + 98.0010178; \qquad (99)$$
$$A'' = g^2 - 18.02226398\ .g + 69.17620526; \qquad (100)$$
$$A_1 = g^2 - 14.593776102\ .g + 45.97848654; \qquad (101)$$
$$A_2 = g^2 - 10.00764523\ .g + 6.35419527; \qquad (102)$$
$$A_3 = g^2 - 26.182948414\ .g + 82.91843690; \qquad (103)$$

$$D = g^2 - 47.859261067\ .g + 700.423146; \qquad (104)$$
$$D' = g^2 - 54.92149851\ .g + 656.135490; \qquad (105)$$
$$D'' = g^2 - 31.56793789\ .g + 245.7302238; \qquad (106)$$
$$D_1 = g^2 - 48.09963879\ .g + 201.477053; \qquad (107)$$
$$D_2 = g^2 - 51.052893467\ .g + 34.3822734; \qquad (108)$$
$$D_3 = g^2 - 3.4187071566.g + 1.7319476368; \qquad (109)$$

$$\left.\begin{array}{l} B = \{g - 34.6345681\}b;\ B' = \{g - 17.567227807\}b; \\ \qquad\qquad\qquad\qquad\quad B'' = \{g - 12.4512675772\}b; \end{array}\right\} \quad (110)$$

$$\left.\begin{array}{l} C = \{23.7078074 - g\}[9.1763990]b'; \\ C' = \{17.58198428 - g\}[8.8694654]b'; \\ C'' = -[0.2445917]b'; \\ C''' = -[0.2654598]b'; \end{array}\right\} \quad (111)$$

$$\left.\begin{array}{l} E = +[9.6231944]b'' \\ E' = \{24.1514537 - g\}[8.5847028]b''; \\ E'' = \{17.55413616 - g\}[9.6375865]b''; \\ E''' = +[0.9518913]b''; \end{array}\right\} \quad (112)$$

$$\left.\begin{array}{l} F = -[7.6499091]b'''; \\ F' = +[8.9577383]b'''; \\ F'' = \{14.01380173 - g\}[0.8407439]b'''; \\ F''' = \{37.3395142 - g\}[9.4809266]b'''; \end{array}\right\} \quad (113)$$

$$\left.\begin{array}{l} B_1 = \{g - 4.7867980\}b_1;\ B_2 = \{g - 0.6795718937\}b_1; \\ \qquad\qquad\qquad\qquad\quad B_3 = \{g - 18.665563912\}b_1; \end{array}\right\} \quad (114)$$

$$\left.\begin{array}{l} C_1 = \{4.365091444 - g\}[9.2840950]b_2; \\ C_2 = \{0.6879105996 - g\}[9.1824311]b_2; \\ C_3 = -[0.6882317]b_2; \\ C_4 = -[0.6834824]b_2; \end{array}\right\} \quad (115)$$

$$\left.\begin{array}{l} E_1 = +[7.8267723]b_3; \\ E_2 = \{43.7345473 - g\}[8.2017618]b_3; \\ E_3 = \{0.648004119 - g\}[0.7610455]b_3; \\ E_4 = +[1.0655652]b_3; \end{array}\right\} \quad (116)$$

$$\left.\begin{array}{l} F_1 = -[7.0339474]b_4; \\ F_2 = +[0.2724895]b_4; \\ F_3 = \{2.770663038 - g\}[1.7737962]b_4; \\ F_4 = \{50.3649829 - g\}[9.1128486]b_4; \end{array}\right\} \quad (117)$$

$$\left.\begin{aligned}
b &=\{0.1489647\ldots[9.1730832]\}N^{IV}+[7.5931242]N^{V}+[95.5264270]N^{VI}\\
&\qquad+[94.8701084]N^{VII}\\
b' &=\{0.7286137\ldots[9.8624973]\}N^{IV}+[8.2750461]N^{V}+[96.2059893]N^{VI}\\
&\qquad+[95.5492142]N^{VII}\\
b'' &=\{1.690254\ \ldots[0.2279520]\}N^{IV}+[8.6307197]N^{V}+[96.5586316]N^{VI}\\
&\qquad+[95.9012715]N^{VII}\\
b''' &=\{5.307482\ \ldots[0.7248886]\}N^{IV}+[9.0992151]N^{V}+[97.0186122]N^{VI}\\
&\qquad+[96.3596252]N^{VII}
\end{aligned}\right\}\quad(118)$$

$$\left.\begin{aligned}
b_1 &=\{0.000008754742\ \ldots[94.9422434]\}N+[96.8635004]N'\\
&\qquad+[97.3236905]N''+[97.0504994]N''';\\
b_2 &=\{0.0000005681531\ \ldots[93.7544654]\}N+[95.6682302]N'\\
&\qquad+[96.1186392]N''+[95.8170069]N''';\\
b_3 &=\{0.00000002443536\ \ldots[92.3880187]\}N+[94.2994239]N'\\
&\qquad+[94.7468016]N''+[94.4366545]N''';\\
b_4 &=\{0.000000003249184\ldots[91.5117743]\}N+[93.4227230]N'\\
&\qquad+[93.8695157]N''+[93.5577417]N''';
\end{aligned}\right\}\quad(119)$$

We have given the natural and logarithmic coefficients of N and N^{IV}, in the values of b, b', b'', &c., because the values of the other quantities are determined in functions of these, and they will therefore be wanted.

14. If we now suppose the second members of equations (82) and (83) to be equal to nothing, we shall obtain the following values of g, g_1, g_2, &c.:—

$$\begin{aligned}
g &= 5''.46370645; & g_4 &= 0''.61668516;\\
g_1 &= 7.24769852; & g_5 &= 2.72772365;\\
g_2 &= 17.01424590; & g_6 &= 3.71780374;\\
g_3 &= 17.78441063; & g_7 &= 22.46058485.
\end{aligned}$$

We must now transform equation (82) in four others whose roots shall be respectively less by g, g_1, g_2, and g_3. Putting δg for the root of the first transformed equation, we shall have the following equation to determine δg.

$$\delta g^4-25.6552\delta g^3+184.8969\delta g^2-253.8812\delta g+\chi=0.$$

But since δg, δg_1, &c. are very small quantities, we may neglect δg^3, δg^4, in these transformed equations, and we shall then get by dividing by the coefficients of δg

$$\delta g=\frac{\chi}{253.8812}+\frac{184.8969}{253.8812}.\delta g^2.$$

We may first neglect the last term of this equation, and we shall obtain a first approximation to the value of δg, with which we can compute the last term of the equation. If we perform the same process with the other roots, we shall obtain the following equations for determining δg, δg_1, δg_2, &c.

$$\delta g=+[7.59537\ ]\chi+[9.8623\ ]\delta g^2\ ; \qquad (120)$$

$$\delta g_1=-[7.73616\ ]\chi_1-[9.5602\ ]\delta g_1^2; \qquad (121)$$

$$\delta g_2=+[8.061074]\chi_2+[0.04511]\delta g_2^2; \qquad (122)$$

$$\delta g_3=-[8.00000\ ]\chi_3-[0.16856]\delta g_3^2; \qquad (123)$$

$$\delta g_4=+[7.84466\ ]\chi_4+[9.92529]\delta g_4^2; \qquad (124)$$

$$\delta g_5=-[8.38464\ ]\chi_5+[9.76863]\delta g_5^2; \qquad (125)$$

$$\delta g_6=+[8.23997\ ]\chi_6-[0.10692]\delta g_6^2; \qquad (126)$$

$$\delta g_7=-[6.09265\ ]\chi_7-[9.17554]\delta g_7^2. \qquad (127)$$

15. We shall now give the computation of N', N'', &c., for the root g. Using $g=5''.46370645$, we get $g^2=29.8520882$. Substituting these in equations (98–109), we get the following values of A, A', &c., D, D', &c.

$A=+\ 3.425636$	log. 0.5347412;	$D=+468.78628$	log. 2.6709749;
$A'=+\ 1.390983$	" 0.1433218;	$D'=+385.91263$	" 2.5864890;
$A''=+\ 0.5599336$	" 9.7481365;	$D''=+103.10436$	" 2.0132764;
$A_1=-\ 3.9055338$	" 0.5916804n;	$D_1=-\ 31.473166$	" 1.4979404n;
$A_2=-18.472552$	" 1.2665269n;	$D_2=-214.70366$	" 2.3318394n;
$A_3=-30.285419$	" 1.4812336n;	$D_3=+\ 12.899760$	" 1.1105816.

Substituting A, A', A'', D, D', D'', in equations (84–86), and neglecting f, f', &c., we shall get the following computation of N',N'', and N'''.

Computation of N' & N''.			*Computation of N' & N'''.*			*Computation of N'' & N'''.*		
constant	log.	0.8236010	constant	log.	1.1305346	constant	log.	0.3624135
A	"	0.5347412	A'	"	0.1433218	A'	"	0.1433218
$1\div D$	"	7.3290251	$1\div D'$	"	7.4135110	$1\div D''$	"	7.9867236
A''	"	9.7481365	A''	"	9.7481365	A	"	0.5347412
constant	"	1.4152972	constant	"	0.5190734	constant	"	9.1592561
N'	"	8.6873673	N'	"	8.6873674	N''	"	8.4924589
N''	"	8.4924588	N'''	"	7.6807209	N'''	"	7.6807209

The computation of these quantities is thus seen to be correct, since independent formulæ give the same values. We may now use these values of N', N'', and N''', in equations (119), and we shall obtain the following values of b_1, b_2, b_3, and b_4.

$b_1=+0.0001151786N$	log. 96.0613718;
$b_2=+0.000007234609N$	" 94.8594150;
$b_3=+0.0000003080273N$	" 93.4885892;
$b_4=+0.00000004087918N$	" 92.6115022;

Equations (114–117) will now give,

$B_1=+0.00007796538N$	log. 95.8919018;
$B_2=+0.0005510300N$	" 96.7411752;
$B_3=-0.001520571N$	" 97.1820068n;
$C_1=-0.00000152882N$	" 94.1843555n;
$C_2=-0.00000525886N$	" 94.7208918n;
$C_3=-0.0000348856N$	" 95.5426467n;
$C_4=-0.0000349058N$	" 95.5428974n;
$E_1=+0.000000002067N$	" 91.3153615;
$E_2=+0.000000187594N$	" 93.2732190;
$E_3=-0.00000855646N$	" 94.9322943n;
$E_4=+0.00000358228N$	" 94.5541593;
$F_1=-0.0000000000442N$	" 89.6454496n;
$F_2=+0.000000076558N$	" 92.8839917;
$F_3=-0.00000653946N$	" 94,8155417n;
$F_4=+0.000000238018N$	" 93.3766095.

Substituting these quantities in equations (52) we get,

$f_1=+0.00007643863N$ log. 95.8833129;
$f_2=+0.0005457711N$ " 96.7370106;
$f_3=-0.001555269N$ " 97.1918055n;
$f_4=+0.0005425501N$ " 96.7344399;
$f_5=+0.00007500820N$ " 95.8751088;
$f_6=-0.001555239N$ " 97.1917972n.

With these values of f_1, f_2, &c., and A_1, A_2, A_3, D_1, D_2, and D_3, either of the equations (93–95) will give

$N^{IV}=-0.00005102365N$ log. 95.7077716n.

Therefore we shall easily find

$A_1N^{IV}=+0.0001992746N$, $A_2N^{IV}=+0.0009425372N$, $A_3N^{IV}=+0.001545273N$.

Then

$A_1N^{IV}+f_1=+0.0002757132N$; $A_2N^{IV}+f_2=+0.0014883083N$;
$A_3N^{IV}+f_3=-0.000009996N$;
$A_1N^{IV}+f_5=+0.0002742828N$; $A_2N^{IV}+f_4=+0.0014850873N$;
$A_3N^{IV}+f_6=-0.000009966N$.

Equations (90–92) will now give the following:—

Computation of N^V and N^{VI}.

constant	log.	0.7159050
$A_1N^{IV}+f_1$	"	96.4404576
$1\div D_1$	"	98.5020596n
$A_3N^{IV}+f_3$	"	94.9998262n
constant	"	1.7982382
N^V	"	95.6584222n
N^{VI}	"	95.3001240

Computation of N^V and N^{VII}.

constant	log.	0.8175689
$A_2N^{IV}+f_2$	"	97.1726929
$1\div D_2$	"	97.6681606n
$A_3N^{IV}+f_6$	"	94.9985209n
constant	"	0.8871514
N^V	"	95.6584224n
N^{VII}	"	93.5538329

Computation of N^{VI} and N^{VII}.

constant	log.	99.2389545
$A_2N^{IV}+f_4$	"	97.1717519
$1\div D_3$	"	98.8894184
$A'N^{IV}+f_5$	"	96.4381985
constant	"	98.2262038
N^{VI}	"	95.3001248
N^{VII}	"	93.5538207

The small differences in the different values of N^{VI} and N^{VII}, in this example, are owing to the circumstance that A_3N^{IV} is nearly equal to f_3 and f_6, and has a contrary sign, which renders $A_3N^{IV}+f_3$ and $A_3N^{IV}+f_6$ very small quantities. We must therefore reject these values and use those depending on f_4 and f_5.

Having found the values of N^{IV}, N^{V}, N^{VI}, and N^{VII}, we must now substitute them in equations (118), and we shall get the following values of b, b', b'', and b'''.

$$\begin{array}{lll}
b = -0.000007745108N & \text{log.} & 94.8890274n; \\
b' = -0.00003787069N & \text{“} & 95.5783033n; \\
b'' = -0.00008781746N & \text{“} & 95.9435809n; \\
b''' = -0.0002754383N & \text{“} & 96.4400243n.
\end{array}$$

Substituting these quantities, together with g, in equations (110–113), we get

$$\begin{array}{lll}
B = +0.0002259314N & \text{log.} & 96.3539766; \\
B' = +0.00009374307N & \text{“} & 95.9719391; \\
B'' = +0.00005411940N & \text{“} & 95.7333530; \\
C = -0.0001037110N & \text{“} & 96.0158248n; \\
C' = -0.00003397893N & \text{“} & 95.5312097n; \\
C'' = +0.00006651123N & \text{“} & 95.8228950; \\
C''' = +0.00006978516N & \text{“} & 95.8437631; \\
E = -0.0000368787N & \text{“} & 95.5667753n; \\
E' = -0.00006307260N & \text{“} & 95.7998407n; \\
E'' = -0.0004609025N & \text{“} & 96.6636091n; \\
E''' = -0.0007860900N & \text{“} & 96.8954722n; \\
F = +0.00000123008N & \text{“} & 94.0899334; \\
F' = -0.00002498980N & \text{“} & 95.3977626n; \\
F'' = -0.01632072N & \text{“} & 98.2127394n; \\
F''' = -0.002657125N & \text{“} & 97.4244119n.
\end{array}$$

These quantities being substituted in equations (51) we shall obtain

$$\begin{array}{lll}
f = +0.0000853417N & \text{log.} & 95.9311615; \\
f' = +0.00006099422N & \text{“} & 95.7852886; \\
f'' = +0.00005755803N & \text{“} & 95.7601059; \\
f''' = -0.0003921492N & \text{“} & 96.5934513n; \\
f^{IV} = -0.01688088N & \text{“} & 98.2273951n; \\
f^{V} = -0.002533220N & \text{“} & 97.4036729n.
\end{array}$$

Now substituting f, f', &c., D, D', D'', in equations (96), each one of them will give

$$\chi = +0.0243677.$$

This value of χ is to be substituted in equation (120), and we shall find

$$\delta g = +0''.00009598,$$

and consequently

$$g = 5''.4638024.$$

We have thus obtained a first approximation to the values of the required quantities. In order to get a closer approximation we must repeat the whole computation by using the corrected value of g, and the values of f, f', f'', &c., already found. But instead of computing new values of A, A', A'', &c., D, D', D'', &c., by the formulæ (98–109), it is much more convenient, as well as less laborious, to compute corrections to these quantities depending on δg by formulæ similar to the following:—

$$\left.\begin{array}{l}
\delta A = 2g\delta g + \delta g^2 - \delta g 40.22178 \ldots.; \\
\delta A' = 2g\delta g + \delta g^2 - \delta g 23.14584 \ldots.;
\end{array}\right\} \qquad (128)$$

Then we shall have

$$\left.\begin{array}{ll}\text{Corrected } A=A+\delta A; & \\ \text{“} \qquad A'=A'+\delta A'; & \\ \text{\&c.} & \end{array}\right\} \qquad (129)$$

In this manner we shall obtain for the root g,

$g = 5''.4638027;$
$N' = +0.04864325N$ log. $98.6870226;$
$N'' = +0.03104381N$ “ $98.4919751;$
$N''' = +0.004766696N$ “ $97.6782174;$
$N^{IV} = -0.00005096246N$ “ $95.7072504n;$
$N^{V} = -0.00004548871N$ “ $95.6579036n;$
$N^{VI} = +0.00001992532N$ “ $95.2994053;$
$N^{VII} = +0.000000357473N$ “ $93.5532432.$

By performing similar calculations with reference to g_1, g_2, g_3, &c., we shall obtain the following values:—

$g_1 = 7''.2484269;$
$N_1' = -0.7493120N_1$ log. $9.8746627n;$
$N_1'' = -0.5714185N_1$ “ $9.7569543n;$
$N_1''' = -0.0946700N_1$ “ $8.9762124n;$
$N_1^{IV} = +0.0003959309N_1$ “ $96.5976194;$
$N_1^{V} = +0.0004045142N_1$ “ $96.6069338;$
$N_1^{VI} = -0.0001020591N_1$ “ $96.0088516n;$
$N_1^{VII} = -0.000004173243N_1$ “ $94.6204737n.$

$g_2 = 17''.0143734;$
$N_2' = -7.644764N_2$ log. $0.8833640n;$
$N_2'' = +7.708429N_2$ “ $0.8869658;$
$N_2''' = +15.38340N_2$ “ $1.1870524;$
$N_2^{IV} = -0.0007220048N_2$ “ $96.8585401n;$
$N_2^{V} = -0.004362647N_2$ “ $97.6397511n;$
$N_2^{VI} = +0.000267212N_2$ “ $96.4269521;$
$N_2^{VII} = +0.00001963376N_2$ “ $95.2930035.$

$g_3 = 17''.7844562;$
$N_3' = -8.277302N_3$ log. $0.9178888n;$
$N_3'' = +10.207100N_3$ “ $1.0089024;$
$N_3''' = -49.61991N_3$ “ $1.6956560n;$
$N_3^{IV} = +0.0006685073N_3$ “ $96.8251062;$
$N_3^{V} = +0.006909513N_3$ “ $97.8394474;$
$N_3^{VI} = -0.0003927175N_3$ “ $96.5940804n;$
$N_3^{VII} = -0.00002909962N_3$ “ $95.4638874n.$

$g_4 = 0''.6166849$;
$N_4 = +0.1213144 N_4^{IV}$ log. 9.0839135;
$N_4' = +0.1843578 N_4^{IV}$ " 9.2656615;
$N_4'' = +0.2135417 N_4^{IV}$ " 9.3294826;
$N_4''' = +0.3454671 N_4^{IV}$ " 9.5384067;
$N_4^{V} = +1.127803 N_4^{IV}$ " 0.0522322;
$N_4^{VI} = +24.49974 N_4^{IV}$ " 1.3891617;
$N_4^{VII} = +157.8892 N_4^{IV}$ " 2.1983503.

$g_5 = 2''.7276592$;
$N_5 = +0.2925180 N_5^{IV}$ log. 9.4661526;
$N_5' = +0.2866292 N_5^{IV}$ " 9.4573205;
$N_5'' = +0.3000735 N_5^{IV}$ " 9.4772276;
$N_5''' = +0.3995364 N_5^{IV}$ " 9.6015563;
$N_5^{V} = +0.9103640 N_5^{IV}$ " 9.9592151;
$N_5^{VI} = +15.29769 N_5^{IV}$ " 1.1846258;
$N_5^{VII} = -1.497460 N_5^{IV}$ " 0.1753554*n*.

$g_6 = 3''.7166075$;
$N_6 = +0.5675117 N_6^{IV}$ log. 9.7539748;
$N_6' = +0.3847378 N_6^{IV}$ " 9.5851649;
$N_6'' = +0.3786204 N_6^{IV}$ " 9.5782040;
$N_6''' = +0.4354816 N_6^{IV}$ " 9.6389698;
$N_6^{V} = +0.7901060 N_6^{IV}$ " 9.8976854;
$N_6^{VI} = -1.0394174 N_6^{IV}$ " 0.0167908*n*;
$N_6^{VII} = +0.03291265 N_6^{IV}$ " 8.5173628.

$g_7 = 22''.4608479$;
$N_7 = -0.006236689 N_7^{IV}$ log. 97.7949541*n*;
$N_7' = +0.02030250 N_7^{IV}$ " 98.3075494;
$N_7'' = -0.1520658 N_7^{IV}$ " 99.1820317*n*;
$N_7''' = -0.9615594 N_7^{IV}$ " 99.9829761*n*;
$N_7^{V} = -3.091803 N_7^{IV}$ " 0.4902118*n*;
$N_7^{VI} = +0.1154716 N_7^{IV}$ " 99.0624752;
$N_7^{VII} = +0.00872852 N_7^{IV}$ " 97.9409405.

16. We have thus determined all the roots appertaining to the equation of the eighth degree, together with the ratios of the constant quantities N', N'', &c., to N; N_1', N_1'', &c., to N_1; and the ratios of the similar quantities relative to each of the other roots. The complete integrals of the equations (A) will therefore be the sums of all the corresponding terms depending on g, g_1, g_2, &c., and we shall therefore have

$$\left.\begin{array}{l}
h\,=N\sin(gt+\beta)+N_1\sin(g_1t+\beta_1)+N_2\sin(g_2t+\beta_2)+\&c.,\\
h'=N'\sin(gt+\beta)+N_1'\sin(g_1t+\beta_1)+N_2'\sin(g_2t+\beta_2)+\&c.,\\
h''=N''\sin(gt+\beta)+N_1''\sin(g_1t+\beta_1)+N_2''\sin(g_2t+\beta_2)+\&c.,\\
\&c.;\\[1ex]
l\,=N\cos(gt+\beta)+N_1\cos(g_1t+\beta_1)+N_2\cos(g_2t+\beta_2)+\&c.,\\
l'=N'\cos(gt+\beta)+N_1'\cos(g_1t+\beta_1)+N_2'\cos(g_2t+\beta_2)+\&c.,\\
l''=N''\cos(gt+\beta)+N_1''\cos(g_1t+\beta_1)+N_2''\cos(g_2t+\beta_2)+\&c.,\\
\&c.
\end{array}\right\}\text{(C)}$$

β, β_1, β_2, &c., being arbitrary constant quantities. These equations contain twice as many constant quantities as there are roots g, g_1, g_2, &c., of the differential equations (A). These constant quantities are indeterminate by analysis; but they may be deduced from the values of h, h', h'', &c., l, l', l'', &c., at any given epoch. If we suppose $t=0$, in equations (C), and also suppose the first members to be known quantities, the preceding equations will give the values of these constant quantities by direct elimination. But in order to determine them more conveniently, we shall resume the differential equations (A), and we shall multiply the first, third, fifth, &c., by $Nm\div na$, $N'm'\div n'a'$, $N''m''\div n''a''$, &c., and they will become

$$\left.\begin{array}{rl}
N\dfrac{m}{na}\dfrac{dh}{dt}= & N\dfrac{m}{na}\left\{(0,1)+(0,2)+(0,3)+\&c.,\right\}l\\
 & -N\dfrac{m}{na}\left\{\boxed{0,1}\,l'+\boxed{0,2}\,l''+\boxed{0,3}\,l'''+\&c.,\right\}\\
N'\dfrac{m'}{n'a'}\dfrac{dh'}{dt}= & N'\dfrac{m'}{n'a'}\left\{(1,0)+(1,2)+(1,3)+\&c.,\right\}l'\\
 & -N'\dfrac{m'}{n'a'}\left\{\boxed{1,0}\,l+\boxed{1,2}\,l''+\boxed{1,3}\,l'''+\&c.,\right\}\\
N''\dfrac{m''}{n''a''}\dfrac{dh''}{dt}= & N''\dfrac{m''}{n''a''}\left\{(2,0)+(2,1)+(2,3)+\&c.,\right\}l''\\
 & -N''\dfrac{m''}{n''a''}\left\{\boxed{2,0}\,l+\boxed{2,1}\,l'+\boxed{2,3}\,l'''+\&c.,\right\}\\
\&c. &
\end{array}\right\}(130)$$

If we add these equations, and in their sum substitute the values of $N\{(0,1)+(0,2)+(0,3)+\&c.\}$, $N'\{(1,0)+(1,2)+(1,3)+\&c.\}$, &c., deduced from equations (B), we shall obtain

$$\left.\begin{array}{l}
N\dfrac{m}{na}\dfrac{dh}{dt}+N'\dfrac{m'}{n'a'}\dfrac{dh'}{dt}+N''\dfrac{m''}{n''a''}\dfrac{dh}{dt}+\&c.\\
=g\left\{Nl\dfrac{m}{na}+N'l'\dfrac{m'}{n'a'}+N''l''\dfrac{m''}{n''a''}+\&c.\right\}\\
+N\left\{l'\left(\dfrac{m'}{n'a'}\boxed{1,0}-\dfrac{m}{na}\boxed{0,1}\right)+l''\left(\dfrac{m''}{n''a''}\boxed{2,0}-\dfrac{m}{na}\boxed{0,2}\right)+\&c.\right\}\\
+N'\left\{l\left(\dfrac{m}{na}\boxed{0,1}-\dfrac{m'}{n'a'}\boxed{1,0}\right)+l''\left(\dfrac{m''}{n''a''}\boxed{2,1}-\dfrac{m'}{n'a'}\boxed{1,2}\right)\right.\\
\qquad\left.+l'''\left(\dfrac{m'''}{n'''a'''}\boxed{3,1}-\dfrac{m'}{n'a'}\boxed{1,3}\right)+\&c.\right\}
\end{array}\right\}(131)$$

But each of the factors of l, l', l'', &c., in the terms of the second member of this equation, which are independent of g, becomes nothing by means of equations (9); and we shall therefore have

$$\left.\begin{aligned} &N\frac{m}{na}\frac{dh}{dt}+N'\frac{m'}{n'a'}\frac{dh'}{dt}+N''\frac{m''}{n''a''}\frac{dh''}{dt}+\text{\&c.} \\ &=g\left\{Nl\frac{m}{na}+N'l'\frac{m'}{n'a'}+N''l''\frac{m''}{n''a''}+N'''l'''\frac{m'''}{n'''a'''}+\text{\&c.}\right. \end{aligned}\right\} \quad (132)$$

If we substitute in this equation the preceding values of h, h', h'', &c., l, l', l'', &c., we shall have, by comparing the coefficients of the same cosines,

$$\left.\begin{aligned} &NN_1\frac{m}{na}+N'N_1'\frac{m'}{n'a'}+N''N_1''\frac{m''}{n''a''}+N'''N_1'''\frac{m'''}{n'''a'''}+\text{\&c.}=0; \\ &NN_2\frac{m}{na}+N'N_2'\frac{m'}{n'a'}+N''N_2''\frac{m''}{n''a''}+N'''N_2'''\frac{m'''}{n'''a'''}+\text{\&c.}=0; \\ &NN_3\frac{m}{na}+N'N_3'\frac{m'}{n'a'}+N''N_3''\frac{m''}{n''a''}+N'''N_3'''\frac{m'''}{n'''a'''}+\text{\&c.}=0; \\ &\text{\&c.} \end{aligned}\right\} \quad (133)$$

If we now multiply the preceding values of h, h', h'', &c., respectively $N\frac{m}{na}$, $N'\frac{m'}{n'a'}$, $N''\frac{m''}{n''a''}$, we shall have by means of equations (133),

$$\left.\begin{aligned} &N\frac{m}{na}h+N'\frac{m'}{n'a'}h'+N''\frac{m''}{n''a''}h''+N'''\frac{m'''}{n'''a'''}h'''+\text{\&c.} \\ &=\left\{N^2\frac{m}{na}+N'^2\frac{m'}{n'a'}+N''^2\frac{m''}{n''a''}+N'''^2\frac{m'''}{n'''a'''}+\text{\&c.}\right\}\sin(gt+\beta) \end{aligned}\right\} \quad (134)$$

In like manner the preceding values of l, l', l'', &c., will give,

$$\left.\begin{aligned} &N\frac{m}{na}l+N'\frac{m'}{n'a'}l'+N''\frac{m''}{n''a''}l''+N'''\frac{m'''}{n'''a'''}l'''+\text{\&c.} \\ &=\left\{N^2\frac{m}{na}+N'^2\frac{m'}{n'a'}+N''^2\frac{m''}{n''a''}+N'''^2\frac{m'''}{n'''a'''}+\text{\&c.}\right\}\cos(gt+\beta) \end{aligned}\right\} \quad (135)$$

If the origin of the time t, be fixed at the epoch for which the values of h, l, h', l', &c., are supposed to be known; the two preceding equations will give,

$$\tan\beta=\frac{N\frac{m}{na}h+N'\frac{m'}{n'a'}h'+N''\frac{m''}{n''a''}h''+N'''\frac{m'''}{n'''a'''}h'''+\text{\&c.}}{N\frac{m}{na}l+N'\frac{m'}{n'a'}l'+N''\frac{m''}{n''a''}l''+N'''\frac{m'''}{n'''a'''}l'''+\text{\&c.}} \quad (136)$$

Since N', N'', N''', &c. are given in terms of N, this indeterminate quantity will disappear from the expression of $\tan\beta$. Having found β, we may substitute it in either of equations (134) or (135), and we shall obtain the value of N; and consequently N', N'', N''', &c. will be known. We shall thus have the system of constant quantities corresponding to the root g. Changing in the preceding equations the root g successively into g_1, g_2, g_3, &c., we shall obtain the values of the constant quantities, corresponding to each of these roots.

If we now put the first members of equations (134) and (135) equal to x and y respectively and the coefficients of sin or cos $(gt+\beta)$ in the same equations equal to z, we shall have,

$$x=z\sin(gt+\beta); \qquad y=z\cos(gt+\beta). \tag{137}$$

Whence

$$\tan(gt+\beta)=x\div y. \tag{138}$$

17. In order to find the values of x and y, it is necessary to know the values of h and l. We shall therefore suppose that at the epoch of 1850, the eccentricities and places of the perihelia of the eight principal planets have the following values,

Mercury,	$e\ =0.2056179$	log. $e\ =9.3130610$;	$\varpi\ =\ 75°\ 7'\ 0''.0$;
Venus,	$e'\ =0.00684184$	log. $e'\ =7.8351730$;	$\varpi'\ =129\ 28\ 51\ .7$;
The Earth,	$e''\ =0.01677120$	log. $e''\ =8.2245642$;	$\varpi''\ =100\ 21\ 41\ .0$;
Mars,	$e'''\ =0.0931324$	log. $e'''\ =8.9691008$;	$\varpi'''\ =333\ 17\ 47\ .8$;
Jupiter,	$e^{IV}=0.0482388$	log. $e^{IV}=8.6833965$;	$\varpi^{IV}=\ 11\ 54\ 53\ .1$;
Saturn,	$e^{V}\ =0.0559956$	log. $e^{V}\ =8.7481539$;	$\varpi^{V}=\ 90\ \ 6\ 12\ .0$;
Uranus,	$e^{VI}=0.0462149$	log. $e^{VI}=8.6647821$;	$\varpi^{VI}=170\ 34\ 17\ .6$;
Neptune,	$e^{VII}=0.00917396$	log. $e^{VII}=7.9625568$;	$\varpi^{VII}=\ 50\ 16\ 39\ .1$.

Now since $h=e\sin\varpi$, $l=e\cos\varpi$, $h'=e'\sin\varpi'$, $l'=e'\cos\varpi'$, &c., we shall obtain the following values

$h\ =+0.198720$, log. $h\ =9.2989408$;
$h'\ =+0.0052808$, log. $h'\ =7.7226976$;
$h''\ =+0.0164977$, log. $h''\ =8.2174237$;
$h'''\ =-0.0418510$, log. $h'''\ =8.6217066n$;
$h^{IV}=+0.0099592$, log. $h^{VI}=7.9982241$;
$h^{V}\ =+0.0559955$, log. $h^{V}\ =8.7481532$;
$h^{VI}=+0.0075707$, log. $h^{VI}=7.8791377$;
$h^{VII}=+0.0070561$, log. $h^{VII}=7.8485672$;

$l\ =+0.052813$, log. $l\ =8.7227434$;
$l'\ =-0.0043502$, log. $l'\ =7.6385092n$;
$l''\ =-0.0030164$, log. $l''\ =7.4794898n$;
$l'''\ =+0.083199$, log. $l'''\ =8.9201199$;
$l^{IV}=+0.0471996$, log. $l^{IV}=8.6739377$;
$l^{V}\ =-0.00010988$, log. $l^{V}\ =6.0042715n$;
$l^{VI}=-0.0455906$, log. $l^{VI}=8.6588752n$;
$l^{VII}=+0.0058628$, log. $l^{VII}=7.7681049$.

We have already given the logarithms of $m\div na$, $m'\div n'a'$, $m''\div n''a''$, &c., in § 5; and if we add them successively to log. h and log. l, log. h' and log. l', &c., we shall obtain the following values of the logarithms of the constants, for the given epoch, which enter into the values of x and y.

$$\log.\ h\frac{m}{na}=86.2924049; \qquad \log.\ l\frac{m}{na}=85.7169075;$$

$$\log.\ h'\frac{m'}{n'a'}=85.9487046; \qquad \log.\ l'\frac{m'}{n'a'}=85.8645162n;$$

$$\log.\ h''\frac{m''}{n''a''}=86.5381661; \qquad \log.\ l''\frac{m''}{n''a''}=85.8002322n;$$

$$\log.\ h'''\frac{m'''}{n'''a'''}=86.1723213n; \qquad \log.\ l'''\frac{m'''}{n'''a'''}=86.4707346;$$

$$\log.\ h^{IV}\frac{m^{IV}}{n^{IV}a^{IV}}=89.2232280; \qquad \log.\ l^{IV}\frac{m^{IV}}{n^{IV}a^{IV}}=89.8989416;$$

$$\log.\ h^{V}\frac{m^{V}}{n^{V}a^{V}}=89.5809761; \qquad \log.\ l^{V}\frac{m^{V}}{n^{V}a^{V}}=86.8370944n;$$

$$\log.\ h^{VI}\frac{m^{VI}}{n^{VI}a^{VI}}=88.0117101; \qquad \log.\ l^{VI}\frac{m^{VI}}{n^{VI}a^{VI}}=88.7914476n;$$

$$\log.\ h^{VII}\frac{m^{VII}}{n^{VII}a^{VII}}=88.2010654; \qquad \log.\ l^{VII}\frac{m^{VII}}{n^{VII}a^{VII}}=88.1206031.$$

18. These quantities are now to be substituted in equations (136) and (137), in connection with the values of N', N'', N''', &c., corresponding to the different roots.

For the root $g=5''.4638027$, we find

$$x=+\frac{1847539}{10^{20}}N, \quad y=+\frac{64178.9}{10^{20}}N, \quad z=+\frac{10467760}{10^{23}}N^2.$$

Whence $\beta=88°\ 0'\ 37''.8$, and $\log.\ N=9.2470063$.

Therefore for root g, we have the following values,

$N\ =+0.1766064,$
$N'\ =+0.0085906,$
$N''\ =+0.0054825,$
$N'''=+0.00084182,$
$N^{IV}=-0.0000090002,$
$N^{V}\ =-0.0000080335,$
$N^{VI}=+0.0000035168,$
$N^{VII}=+0.000000063131.$

In like manner we shall find for the root $g_1=7''.2484269$,

$$x_1=+\frac{1654860.4}{10^{20}}N_1, \quad y_1=+\frac{4347600.6}{10^{20}}N_1, \quad z_1=+\frac{173037154}{10^{20}}N^2.$$

Whence $\beta_1=20°\ 50'\ 19''.4$, and $\log.\ N_1=8.4294913$.

Therefore for the root g_1 we have

$N_1\ =+0.02688384,$
$N_1'\ =-0.02014438,$
$N_1''\ =-0.01536192,$
$N_1'''\ =-0.00254509,$
$N_1^{IV}=+0.000010644,$
$N_1^{V}\ =+0.000010875,$
$N_1^{VI}=-0.000002744,$
$N_1^{VII}=-0.0000001122.$

For the root $g_2=17''.0143734$, we find,

$$x_2=-\frac{18893046}{10^{20}}N_2, \quad y_2=+\frac{40873423}{10^{20}}N_2, \quad z_2=+\frac{3.068840}{10^{10}}N_2^2.$$

$\beta_2=335°\ 11'\ 31''.4$, $\quad \log.\ N_2=7.1665150$.

$N_2\ =+0.001467287,$
$N_2'=-0.01121706,$
$N_2''=+0.01131047,$
$N_2'''=+0.02257186,$
$N_2^{IV}=-0.000001059,$
$N_2^{V}\ =-0.000006401,$
$N_2^{VI}=+0.000000392,$
$N_2^{VII}=+0.0000000288.$

For the root $g_3=17''.7844562$, we find,

$$x_3=+\frac{0.01310358}{10^{10}}N_3,\ y_3=-\frac{0.014106133}{10^{10}}N_3,\ z_3=+\frac{12.083015}{10^{10}}N_3^2$$

$$\beta_3=137^\circ\ 6'\ 36''.5, \qquad \log.\ N_3=7.2023283.$$

$$\begin{array}{ll}
N_3\ =+0.001593413, & N_3^{IV}=+0.0000010652, \\
N_3'=-0.01318916, & N_3^{V}\ =+0.0000110097, \\
N_3''=+0.01626412, & N_3^{VI}=-0.0000006258, \\
N_3'''=-0.0790650, & N_3^{VII}=-0.00000004636.
\end{array}$$

For the root $g_4=0''.6166849$, we find,

$$x_4=+\frac{3.3572052}{10^{10}}N_4^{IV},\ y_4=+\frac{1.360283}{10^{10}}N_4^{IV},\ z_4=+\frac{56970.44}{10^{10}}N_4^{IV2}.$$

$$\beta_4=67^\circ\ 56'\ 34''.9, \qquad \log.\ N_4^{IV}=95.8033371.$$

$$\begin{array}{ll}
N_4\ =+0.000007714, & N_4^{IV}=+0.000063582, \\
N_4'=+0.000011722, & N_4^{V}\ =+0.000071708, \\
N_4''=+0.000013577, & N_4^{VI}=+0.00155775, \\
N_4'''=+0.000021946, & N_4^{VII}=+0.01003893.
\end{array}$$

For the root $g_5=2''.7276592$, we find,

$$x_5=+\frac{0.6475797}{10^{10}}N_5^{IV},\ y_5=-\frac{0.1743025}{10^{10}}N_5^{IV},\ z_5=+\frac{345.0396}{10^{10}}N_5^{IV2}.$$

$$\beta_5=105^\circ\ 3'\ 52''.9, \qquad \log.\ N_5^{IV}=97.2886122.$$

$$\begin{array}{ll}
N_5\ =+0.000568545, & N_5^{IV}=+0.001943624, \\
N_5'=+0.000557100, & N_5^{V}\ =+0.00176940, \\
N_5''=+0.000583230, & N_5^{VI}=+0.02973295, \\
N_5'''=+0.000776548, & N_5^{VII}=-0.002910500.
\end{array}$$

For the root $g_6=3''.7166075$, we find,

$$x_6=+\frac{0.4583186}{10^{10}}N_6^{IV},\ y_6=+\frac{0.8566965}{10^{10}}N_6^{IV},\ z_6=+\frac{22.51127}{10^{10}}N_6^{IV2}.$$

$$\beta_6=28^\circ\ 8'\ 45''.8, \qquad \log.\ N_6^{IV}=98.6350825.$$

$$\begin{array}{ll}
N_6\ =+0.02449386, & N_6^{IV}=+0.04316011, \\
N_6'=+0.01660532, & N_6^{V}\ =+0.03410106, \\
N_6''=+0.01634130, & N_6^{VI}=-0.04486144, \\
N_6'''=+0.01879544, & N_6^{VII}=+0.001420513.
\end{array}$$

For the root $g_7=22''.4608479$, we find,

$$x_7=-\frac{1.0095026}{10^{10}}N_7^{IV},\ y_7=+\frac{0.7872144}{10^{10}}N_7^{IV},\ z_7=+\frac{81.86052}{10^{10}}N_7^{IV2}.$$

$$\beta_7=307^\circ\ 56'\ 50''.1, \qquad \log.\ N_7^{IV}=98.1941887.$$

$$\begin{array}{ll}
N_7\ =-0.00009753, & N_7^{IV}=+0.01563827, \\
N_7'=+0.00031750, & N_7^{V}\ =-0.04835044, \\
N_7''=-0.00237805, & N_7^{VI}=+0.001805776, \\
N_7'''=-0.01503712, & N_7^{VII}=+0.000136499.
\end{array}$$

If these values be substituted in equations (C), we shall have the complete values of h, h', h'', &c., l, l', l'', &c., from which we can obtain the eccentricity and place of the perihelia by means of the formulas,

$$\tan\varpi=h\div l;\quad e=\sqrt{h^2+l^2}=h\operatorname{cosec}\varpi=l\sec\varpi. \qquad (139)$$

19. If, in equations (C), we put, instead of h and l, their values, $e\sin\varpi$, and $e\cos\varpi$, we shall get

$$e\sin\varpi=N\sin(gt+\beta)+N_1\sin g_1t+\beta_1)+N_2\sin(g_2t+\beta_2)+\text{\&c.} \qquad (140)$$

$$e\cos\varpi=N\cos(gt+\beta)+N'\cos g_1t+\beta_1)+N_2\cos(g_2t+\beta_2)+\text{\&c.} \qquad (141)$$

Multiplying equation (140) by $\cos(gt+\beta)$, and (141) by $\sin(gt+\beta)$, the difference of their products will give

$$e\sin(\varpi-gt-\beta)=N_1\sin\{(g_1-g)t+\beta_1-\beta\}+N_2\sin\{(g_2-g)t+\beta_2-\beta\}+\text{\&c.} \quad (142)$$

If we multiply equation (140) by $\sin(gt+\beta)$, and (141) by $\cos(gt+\beta)$ the sum of their products will give

$$\left.\begin{aligned}&e\cos(\varpi-gt-\beta)=\\&\qquad N+N_1\cos\{(g_1-g)t+\beta_1-\beta\}+N_2\cos\{(g_2-g)t+\beta_2-\beta\}+\text{\&c.}\end{aligned}\right\}(143)$$

Dividing equation (142) by (143), we shall eliminate e, and get,

$$\tan(\varpi-gt-\beta)=\frac{N_1\sin\{(g_1-g)t+\beta_1-\beta\}+N_2\sin\{(g_2-g)t+\beta_2-\beta\}+\text{\&c.}}{N+N_1\cos\{(g_1-g)t+\beta_1-\beta\}+N_2\cos\{(g_2-g)t+\beta_2-\beta\}+\text{\&c.}} \quad (144)$$

When the sum $N_1+N_2+N_3+$&c., of the coefficients of the cosines of the denominator, all taken positively, is less than N, $\tan(\varpi-gt-\beta)$ cannot become infinite; the angle $\varpi-gt-\beta$ cannot therefore become equal to a right angle; consequently, the mean motion of the perihelion will be in that case equal to gt. It is also easy to see that when all the cosines of the denominator of $\tan(\varpi-gt-\beta)$ become equal to ±1, N_1, N_2, N_3, &c., being supposed positive, the denominator will be either a *maximum* or a *minimum*, and the numerator will be equal to nothing; $\tan(\varpi-gt-\beta)$ will then be equal to nothing; $\varpi-gt-\beta$ will therefore become equal to nothing, and equation (143) will become,

$$e=N\pm\{N_1+N_2+N_3+\text{\&c.}\}; \qquad (145)$$

Consequently, the maximum and minimum values of e will be

$$\left.\begin{aligned}\textit{maximum}\ e&=N+\ N_1+N_2+N_3+\text{\&c.};\\ \text{and}\ \textit{minimum}\ e&=N-\{N_1+N_2+N_3+\text{\&c.}\}\end{aligned}\right\}\qquad(146)$$

We shall now substitute the numbers which we have already computed, in these equations, for the purpose of determining the maximum and minimum values of the eccentricities, and the mean motions of the perihelia.

20. For the planet *Mercury*, we have,

Maximum $e=N+N_1+N_2+N_3+N_4+N_5+N_6+N_7=0.2317185$. One-half of this is 0.1158593, which being less than N, it follows that N exceeds the sum of all the remaining terms; consequently, the mean annual motion of Mercury's perihelion is equal to g or 5''.4638027, and performs a complete revolution in 237197 years; and the minimum value of the eccentricity is 0.1214943.

For the planet *Venus* we have,

Maximum $e'=N'+N_1'+N_2'+\&c.=0.0706329$. One-half of this is 0.0353165. As this number exceeds any *one* of the coefficients, N', N_1', N_2', &c., it follows that the perihelion of the orbit of Venus *has no mean motion*, and that the minimum value of its eccentricity is zero.

For the *Earth* we have,

Maximum $e''=N''+N_1''+N_2''+\&c.=0.0677352$. One-half of this is 0.0338676. As this number exceeds any *one* of the coefficients N'', N_1'', N_2'', &c., it follows that the perihelion of the Earth's orbit *has no mean motion*, and that the minimum value of the eccentricity is zero.

For the planet *Mars* we have,

Maximum $e'''=N'''+N_1'''+N_2'''+\&c.=0.1396547$. One-half of this is 0.0698274. As this number is less than N_3''', it follows that the perihelion of the orbit of Mars has a mean annual motion equal to g_3 or 17''.7844562; and that the minimum eccentricity of his orbit is equal to 0.0184753. We shall here observe that a small variation in the assumed mass of the Earth would produce a considerable variation in the limits of eccentricity and mean motion of the perihelion.

For the planet *Jupiter* we have,

Maximum $e^{IV}=N^{IV}+N_1^{IV}+N_2^{IV}+\&c.=0.0608274$. One-half of this is 0.0304137. As this number is less than N_6^{IV}, it follows that the perihelion of the orbit of Jupiter has a mean annual motion equal to g_6 or 3''.7166075; and that the minimum value of the eccentricity is equal to 0.0254928.

For the planet *Saturn* we have,

Maximum $e^{V}=N^{V}+N_1^{V}+N_2^{V}+\&c.=0.0843289$. One-half of this is equal to 0.0421644. As this number is less than N_7^{V}, it follows that the perihelion of Saturn's orbit has a mean annual motion equal to g_7 or 22''.4608479; and that the minimum value of the eccentricity is equal to 0.0123719.

For the planet *Uranus* we have,

Maximum $e^{VI}=N^{VI}+N_1^{VI}+N_2^{VI}+\&c.=0.0779652$. One-half of this is 0.0389826. As this number is less than N_6^{VI}, it follows that the perihelion of the orbit of Uranus has a mean annual motion equal to g_6 or 3''.7166075; and that the minimum value of the eccentricity is equal to 0.0117576.

For the planet *Neptune* we have,

Maximum $e^{VII}=N^{VII}+N_1^{VII}+N_2^{VII}+\&c.=0.0145066$. One-half of this is 0.0072533. As this number is less than N_4^{VII}, it follows that the perihelion of Neptune's orbit has a mean annual motion equal to g_4 or 0''.6166849; and that the minimum value of the eccentricity is equal to 0.0055712.

21. We see, by the preceding article, that the mean motions of the perihelia of *Jupiter* and *Uranus* are exactly equal. It follows from this circumstance that the

mean relative positions of their perihelia will always be the same; and we shall now inquire what their mean relative positions are. For this purpose we shall resume equation (144), and substitute in it the values corresponding to these two planets. By this means we shall get the following equations,

$$\left.\begin{array}{l}\tan(\varpi^{IV}-g_6t-\beta_6)= \\ \dfrac{N^{IV}\sin\{(g-g_6)t+\beta-\beta_6\}+N_1^{IV}\sin\{(g_1-g_6)t+\beta_1-\beta_6\}+\&c.}{N_6^{IV}+N^{IV}\cos\{(g-g_6)t+\beta-\beta_6\}+N_1^{IV}\cos\{(g_1-g_6)t+\beta_1-\beta_6\}+\&c.};\end{array}\right\}\quad(147)$$

$$\left.\begin{array}{l}\tan(\varpi^{VI}-g_6t-\beta_6)= \\ \dfrac{N^{VI}\sin\{(g-g_6)t+\beta-\beta_6\}+N_1^{VI}\sin\{(g_1-g_6)t+\beta_1-\beta_6\}+\&c.}{N_6^{VI}+N^{VI}\cos\{(g-g_6)t+\beta-\beta_6\}+N_1^{VI}\cos\{(g_1-g_6)t+\beta_1-\beta_6\}+\&c.}.\end{array}\right\}\quad(148)$$

Now, since the mean values of the numerators of these equations are each equal to nothing, and the signs of the denominators depend wholly on N_6^{IV} and N_6^{VI}; it follows that ϖ^{IV} will always be equal to ϖ^{VI}, if N_6^{IV} has the same sign as N_6^{VI}; and ϖ^{IV} will always differ from ϖ^{VI} by two right angles if N_6^{IV} and N_6^{VI} have different signs. According to the numbers which we have calculated, N_6^{IV} and N_6^{VI} have different signs; consequently, the mean longitudes of the perihelia of the orbits of Jupiter and Uranus differ by a semicircumference.

For the purpose of determining the maximum values of tan $(\varpi^{IV}-g_6t-\beta_6)$ and tan $(\varpi^{VI}-g_6t-\beta_6)$, we may suppose them to be of the following form,

$$\tan(\varpi^{IV}-g_6t-\beta_6)=\frac{\{N^{IV}+N_1^{IV}+N_2^{IV}+N_3^{IV}+N_4^{IV}+N_5^{IV}+N_7^{IV}\}\sin\alpha}{N_6^{IV}+\{N^{IV}+N_1^{IV}+N_2^{IV}+N_3^{IV}+N_4^{IV}+N_5^{IV}+N_7^{IV}\}\cos\alpha};\quad(149)$$

$$\tan(\varpi^{VI}-g_6t-\beta_6)=\frac{\{N^{VI}+N_1^{VI}+N_2^{VI}+N_3^{VI}+N_4^{VI}+N_5^{VI}+N_7^{VI}\}\sin\alpha}{N_6^{VI}+\{N^{VI}+N_1^{VI}+N_2^{VI}+N_3^{VI}+N_4^{VI}+N_5^{VI}+N_7^{VI}\}\cos\alpha};\quad(150)$$

the coefficients of cos α being supposed positive. These equations evidently attain their maximum values when cos α is equal to the quotient of its coefficient divided by the constant term of the denominator, taken negatively. If we reduce them to numbers, they will become

$$\tan(\varpi^{IV}-g_6t-\beta_6)=\frac{0.0176673\sin\alpha}{0.0431601+0.0176673\cos\alpha};\quad(151)$$

$$\tan(\varpi^{VI}-g_6t-\beta_6)=\frac{0.0331038\sin\alpha}{-0.0448614+0.0331038\cos\alpha}.\quad(152)$$

The first of these evidently attains a maximum value when $\alpha=\pm114°\ 10'$; and the second one when $\alpha=\pm42°\ 27'$. Consequently, we shall find

Maximum value of $(\varpi^{IV}-g_6t-\beta_6)=\pm24°\ 10'$;
and " " " $(\varpi^{VI}-g_6t-\beta\ )=180°\pm47°\ 33'$.

The nearest approach of the perihelia of these two planets will therefore be 108° 17'.

The mean annual motions of the perihelia of the four planets *Jupiter*, *Saturn*, *Uranus*, and *Neptune* being

Jupiter and *Uranus*	3″.7166075;
Saturn	22 .4608479;
Neptune	0 .6166849;

it follows that the mean motion of *Saturn's* perihelion is very nearly six times that of *Jupiter* and *Uranus;* and this latter quantity is very nearly six times that of *Neptune;* or, more exactly, 985 times the mean motion of *Jupiter's* perihelion is equal to 163 times that of *Saturn;* and 440 times the mean motion of *Neptune's* perihelion is equal to 73 times that of *Jupiter* and *Uranus.* It also follows that the perihelion of *Saturn* performs a complete revolution in the heavens in 57700 years; that of *Jupiter* and *Uranus* in 348700 years; while that of *Neptune* requires no less than 2101560 years to perform the circuit of the heavens.

22. Having determined the numerical values of all the constants which enter into the integrals of the differential equations (A), corresponding to the assumed masses, it now remains to determine the changes which would be introduced into these constants, on the supposition that the masses were changed from m to $m(1+\mu)$, m' to $m'(1+\mu')$, &c., or, on the supposition that the masses of all the planets were multiplied by a factor $(1+\alpha)$, α being any supposed correction of the mass of the planet, and greater than -1. If we suppose α, or μ, μ', μ'', &c. to be equal to -1, it is evident the whole system of differential equations would vanish. The effect of changing all the masses, in the ratio of 1 to $1+\alpha$, on the equation of the eighth degree, would be equivalent to multiplying the coefficients of the different powers of g by $(1+\alpha)^{8-n}$, n denoting the exponent of g in the given term. Consequently, the coefficient of g^8 would remain unaltered; that of g^7 would be multiplied by $1+\alpha$; that of g^6 would be multiplied by $(1+\alpha)^2$, &c.; while the term of the equation which is independent of g would be multiplied by $(1+\alpha)^8$. It is evident that these changes are such as would be produced by multiplying each of the roots of the equation by $1+\alpha$; consequently, we shall have the following theorem:—

If the masses of all the planets be simultaneously increased in the ratio of 1 *to* $1+\alpha$, *all the roots of the equation in* g *will be increased in the same ratio.*

It is also evident that, if the masses of all the planets be multiplied by $1+\alpha$, the values of A, A', A'', &c., D, D', D'', &c. will all be multiplied by $(1+\alpha)^2$; and, as they are all multiplied by the same quantity, it is manifest that the ratios of the quantities N, N', N'', &c., will remain unaltered. And since the ratios of N, N', N'', &c., remain unaltered, it is evident that $\tan\beta$ will be unchanged, and consequently the values of N, N', N'', &c. will not be changed. It therefore follows that, *if the masses of all the planets be simultaneously increased in any given ratio, the magnitudes of the secular inequalities will remain unchanged.*

To illustrate, we shall observe that if the masses of all the planets be supposed to be doubled, the intensity of the disturbing forces would be doubled; but, according to the preceding theorem, the roots g, g_1, g_2, &c. would be doubled; and consequently the disturbing forces would operate in the same direction during only one-half of the time; and since a double force acting during one-half the time produces the same effect as a single force acting during a double interval, it follows that the magnitude of the resulting inequalities will remain unchanged.

23. The rigorous determination of the separate effects of the corrections of the masses μ, μ', μ'', &c. on the values of the constants which determine the secular variations of the elements, when the masses simultaneously vary, is a much more difficult and intricate problem than that of the determination of the secular inequalities themselves. For, if we employ the masses in indeterminate forms, $m(1+\mu)$, $m'(1+\mu')$, $m''(1+\mu'')$, &c., instead of m, m', m'', &c., it is evident that the solutions of the differential equations would contain terms depending on μ, μ', μ'', &c., and on all the powers and products of these quantities up to μ^7, μ'^7, μ''^7, inclusive, in addition to the terms already calculated. And if we neglect the powers and products of μ, μ', μ'', &c., above the first, it is evident that our solution would be very imperfect, unless μ, μ', μ'', &c. were very small quantities. Unfortunately, this is not the case, for the masses of some of the planets are still very imperfectly known; and consequently the terms depending on the powers and products of μ, μ', μ'', &c. ought not to be neglected. There seems to be only one practicable method of determining the effects of the corrections of the masses on the values of the constants which we have already determined. And this method consists in supposing the mass of each planet, in succession, to be increased by a finite quantity, μ_0, and then determine anew all the constants in the same manner as with the assumed masses. If we then subtract the values of the constants which depend on the assumed masses, from the values of the constants which result from the corrected mass of the planet, we shall obtain the coefficient of the correction depending on μ, by dividing the difference of the constants by μ_0, or the finite variation of the mass of the planet. In this way we get the whole variation resulting from the assumed variation of mass, or, in other words, we retain the terms depending on all the *powers* μ_0, μ_0^2, μ_0^3, &c., and neglect only the terms depending on the products μ, μ', μ'', &c., when they simultaneously vary. As this is the method which we have adopted, we shall here give the resulting fundamental equations, together with the values of the constants determined by their solution.

24. We shall now suppose the mass of *Mercury* to be increased to two and one-half times its assumed value. In this case μ_0 will be equal to $1\frac{1}{2}$, and $m=\frac{1+\mu_0}{4865751}=\frac{2.5}{4865751}=\frac{1}{1946300.4}$. This value of the mass of *Mercury* is very nearly the same as that employed by astronomers, during the early part of the present century, and is doubtless considerably larger than the actual value. But as the perturbations produced by this planet are very small, a considerable variation of its mass will produce only a small variation in the values of the fundamental equations. We shall now compute the effect which this change in the mass of *Mercury* produces in the fundamental equations.

If we now put $1+\mu=2.5$ in the values of $(1,0)$, $(2,0)$, $(3,0)$, &c., $\boxed{1,0}$, $\boxed{2,0}$, $\boxed{3,0}$, &c., and substitute the resulting numbers in equations (26), we shall get the following values of $\boxed{0,0}$, $\boxed{1,1}$, $\boxed{2,2}$, &c.

$$\left.\begin{array}{ll}\boxed{0,0}=g-\ 5''.5702558; & \boxed{4,4}=g-\ 7''.5125167;\\ \boxed{1,1}=g-11\ .5785090; & \boxed{5,5}=g-18\ .5962297;\\ \boxed{2,2}=g-13\ .1331088; & \boxed{6,6}=g-\ 2\ .7662536;\\ \boxed{3,3}=g-17\ .5645194; & \boxed{7,7}=g-\ 0\ .6479572.\end{array}\right\}\quad(153)$$

These values give

$$\boxed{0,0}\boxed{1,1}=g^2-17.1487648.g+\ 64.4952569126;\qquad(154)$$

$$\boxed{0,0}\boxed{2,2}=g^2-18.7033646.g+\ 73.1547754652;\qquad(155)$$

$$\boxed{0,0}\boxed{3,3}=g^2-23.1347752.g+\ 97.83886606206;\qquad(156)$$

$$\boxed{1,1}\boxed{2,2}=g^2-24.7116178.g+152.06181843878;\qquad(157)$$

$$\boxed{1,1}\boxed{3,3}=g^2-29.1430284.g+203.37094595357;\qquad(158)$$

$$\boxed{2,2}\boxed{3,3}=g^2-30.6976282.g+230.67674429991;\qquad(159)$$

$$\boxed{4,4}\boxed{5,5}=g^2-26.1087464.g+139.70448617829;\qquad(160)$$

$$\boxed{4,4}\boxed{6,6}=g^2-10.2787703.g+\ 20.78152636644;\qquad(161)$$

$$\boxed{4,4}\boxed{7,7}=g^2-\ 8.1604739.g+\ \ 4.86778928589;\qquad(162)$$

$$\boxed{5,5}\boxed{6,6}=g^2-21.3624833.g+\ 51.44188735405;\qquad(163)$$

$$\boxed{5,5}\boxed{7,7}=g^2-19.2441869.g+\ 12.04956092697;\qquad(164)$$

$$\boxed{6,6}\boxed{7,7}=g^2-\ 3.4142108.g+\ \ 1.792413937146.\qquad(165)$$

$$\boxed{0,0}\boxed{1,1}\boxed{2,2}\boxed{3,3}=\left.\begin{array}{l}g^4-47.8463930.g^3+\ \ 821.59840712.g^2\\ -5935.67265019.g+14877.555887385\end{array}\right\}(166)$$

$$\boxed{4,4}\boxed{5,5}\boxed{6,6}\boxed{7,7}=\left.\begin{array}{l}g^4-29.5229572.g^3+230.63766404.g^2\\ -523.77824644.g+250.40826811\end{array}\right\}(167)$$

The difference between these values and the similar quantities depending on the assumed masses being denoted by Δ, we shall find the following values,

$$\left.\begin{array}{ll}\Delta\boxed{0,0}=\ \ 0. & \Delta\boxed{4,4}=-0.0001413,\\ \Delta\boxed{1,1}=-0.2637408, & \Delta\boxed{5,5}=-0.0000168,\\ \Delta\boxed{2,2}=-0.0609358, & \Delta\boxed{6,6}=-0.0000014,\\ \Delta\boxed{3,3}=-0.0116549, & \Delta\boxed{7,7}=-0.0000003.\end{array}\right\}\quad(168)$$

$$\left.\begin{array}{l}\Delta\boxed{0,0}\boxed{1,1}=-0.2637408.g+1.4691037;\\ \Delta\boxed{0,0}\boxed{3,3}=-0.0116549.g+0.06492077;\\ \Delta\boxed{1,1}\boxed{3,3}=-0.2753957.g+4.7643529;\\ \Delta\boxed{0,0}\boxed{2,2}=-0.0609358.g+0.33942800;\\ \Delta\boxed{1,1}\boxed{2,2}=-0.3246766.g+4.1532110\ \ ;\\ \Delta\boxed{2,2}\boxed{3,3}=-0.0725907.g+1.2226629\ \ ;\\ \Delta\boxed{4,4}\boxed{5,5}=-0.0001581.g+0.00275386;\\ \Delta\boxed{4,4}\boxed{7,7}=-0.0001416.g+0.00009381;\\ \Delta\boxed{5,5}\boxed{7,7}=-0.0000171.g+0.00001647;\\ \Delta\boxed{4,4}\boxed{6,6}=-0.0001427.g+0.00040139;\\ \Delta\boxed{5,5}\boxed{6,6}=-0.0000182.g+0.00007250;\\ \Delta\boxed{6,6}\boxed{7,7}=-0.0000017.g+0.0000017370.\end{array}\right\}\quad(169)$$

The following calculation of ΔA must serve as an example for the computation of all similar quantities.

$$
\begin{array}{lrl}
 & \Delta\boxed{0,0}\boxed{2,2} = & -\ 0.06093580.g + \ \ 0.33942800 \\
 & -\ \boxed{2,2}\boxed{1,0}\boxed{0,3}\,\mu \div \boxed{1,3} \quad = & \ \ \ \ 0.02543896.g + \ \ 0.33254241 \\
 & -\Delta\boxed{2,2}\boxed{1,0}\boxed{0,3}(1+\mu)\div\boxed{1,3} = & \qquad\qquad\qquad\quad +\ \ 0.00258360 \\
\text{Sum of last three terms of } A\times\mu = & & \qquad\qquad\qquad\quad +\ \ 0.28269800 \\
\text{Sum of terms} = & \Delta A = & -\ 0.08637476.g + \ \ 0.95725201 \\
\text{Computed} & & A=g^2-40.22178322.g+193.3335643 \\
\text{Corrected} & & A=g^2-40.30815798.g+194.2908163.
\end{array}
$$

If we proceed in like manner with A', A'', &c., we shall obtain the following system of fundamental equations for $\mu=+\frac{3}{2}$.

Fundamental Equations for $\mu=\frac{3}{2}$; *or for the case where* $m=\frac{1}{1946300.4}$.

$$A\ =g^2-40.30815798.g\ \ +194.2908163; \tag{170}$$
$$A'=g^2-23.17005142.g\ \ +\ \ 98.2867801; \tag{171}$$
$$A''=g^2-18.28711570.g\ \ +\ \ 70.37498708; \tag{172}$$
$$A_1=g^2-14.59391880.g\ \ +\ \ 45.9791766; \tag{173}$$
$$A_2=g^2-10.00778683.g\ \ +\ \ \ \ 6.35429409; \tag{174}$$
$$A_3=g^2-26.183106514.g\ +\ \ 82.92120064; \tag{175}$$

$$D\ =g^2-48.18393767.g\ \ +708.16361; \tag{176}$$
$$D'=g^2-55.19689421.g\ \ +661.210840; \tag{177}$$
$$D''=g^2-31.64052859.g\ \ +246.9639388; \tag{178}$$
$$D_1=g^2-48.09965699.g\ \ +201.477187; \tag{179}$$
$$D_2=g^2-51.05291057.g\ \ +\ \ 34.3823000; \tag{180}$$
$$D_3=g^2-\ 3.4187088566.g+\ \ 1.7319493752. \tag{181}$$

$$B=\{g-34.6955039\}b;\quad \left.\begin{array}{l} B'=\{g-17''.578882777\}b; \\ B''=\{g-12\ .71500846\ \}b; \end{array}\right\} \tag{182}$$

$$\left.\begin{array}{l}
C\ =\{23.7687432\ -g\}[9.1763990]b'; \\
C'=\{17.59363918-g\}[8.8694654]b'; \\
C''=-[0.2445917]b'; \\
C'''=-[0.2654598]b'.
\end{array}\right\} \tag{183}$$

$$\left.\begin{array}{l}
E\ =+[9.6231944]b''; \\
E'=\{24.4151946\ -g\}[8.5847028]b''; \\
E''=\{17.56579106-g\}[9.6375865]b''; \\
E'''=+[0.9518913]b''.
\end{array}\right\} \tag{184}$$

$$\left.\begin{array}{l}
F\ =-[7.6499091]b'''; \\
F'\ =+[8.9577383]b'''; \\
F''=\{14.07473753-g\}[0.8407439]b'''; \\
F'''=\{37.6032551\ -g\}[9.4809266]b'''.
\end{array}\right\} \tag{185}$$

$$B_1=\{g-4.7867994\}b_1;\quad \left.\begin{array}{l} B_2=\{g-\ 0.6795721937\}b_1; \\ B_3=\{g-18.665580712\ \}b_1; \end{array}\right\} \tag{186}$$

$$\left.\begin{aligned}C_1&=\{4.365092844\ -g\}[9.2840950]b_2;\\ C_2&=\{0.6879108996-g\}[9.1824311]b_2;\\ C_3&=-[0.6832317]b_2;\\ C_4&=-[0.6834824]b_2;\end{aligned}\right\}\qquad(186)$$

$$\left.\begin{aligned}E_1&=+[7.8267723]b_3;\\ E_2&=\{43.7345641\ \ -g\}[8.2017618]b_3;\\ E_3&=\{\ 0.648004419-g\}[0.7610455]b_3;\\ E_4&=+[1.0655652]b_3;\end{aligned}\right\}\qquad(187)$$

$$\left.\begin{aligned}F_1&=-[7.0339474]b_4;\\ F_2&=+[0.2724895]b_4;\\ F_3&=\{\ 2.770664438-g\}[1.7737962]b_4;\\ F_4&=\{50.3649997\ \ -g\}[9.1128486]b_4;\end{aligned}\right\}\qquad(188)$$

$$\left.\begin{aligned}g^4-47.8463930.g^3+796.20272817.g^2-5344.10960488.g\\ +12307.3911771\end{aligned}\right\}=(\chi,\chi_1,\chi_2,\chi_2);\qquad(189)$$

$$\left.\begin{aligned}g^4-29.5229572.g^3+172.74076612.g^2-\ 323.31739720.g\\ +140.47094066\end{aligned}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7).\qquad(190)$$

And lastly, the values of b_1, b_2, b_3, and b_4 become,

$$\left.\begin{aligned}b_1=\{0.00002188686\ldots\ldots[95.3401834]\}N&+[96.8635004]N'\\ +[97.3236905]\ N''&+[97.0504994]N''';\\ b_2=\{0.000001420383\ldots\ldots[94.1524054]\}N&+[95.6682302]N'\\ +[96.1186392]\ N''&+[95.8170069]N''';\\ b_3=\{0.0000000610884\ldots[92.7859587]\}N&+[94.2994239]N'\\ +[94.7468016]\ N''&+[94.4366545]N''';\\ b_4=\{0.00000000812296\ldots[91.9097143]\}N&+[93.4227230]N'\\ +[93.8695157]\ N''&+[93.5577417]N'''.\end{aligned}\right\}\qquad(191)$$

The values of b, b', b'', and b''' remain the same as in equations (118).

Equations (189) and (190) give, when the second members are put equal to 0,

$$\left.\begin{aligned}g&=\ 5''.34449540; & g_4&=\ 0''.616685510;\\ g_1&=\ 7\ .53805074; & g_5&=\ 2\ .72773208;\\ g_2&=17\ .12785195; & g_6&=\ 3\ .71791243;\\ g_3&=17\ .83599491; & g_7&=22\ .46062783.\end{aligned}\right\}\qquad(192)$$

The solutions of equations (170–191) will now give the following values, remembering that the coefficients of equations (84–97) remain unchanged. For the root g, we get,

$$g=5''.3446763;$$

$$\begin{aligned}N'\ &=+0.10318901N & &\text{log. } 99.0136340;\\ N''\ &=+0.0652640N & &\text{"}\quad 98.8146736;\\ N'''\ &=+0.01001010N & &\text{"}\quad 98.0004383;\\ N^{IV}&=-0.0001170589N & &\text{"}\quad 96.0684043n;\\ N^{V}\ &=-0.0001036091N & &\text{"}\quad 96.0153978n;\\ N^{VI}&=+0.0000476249N & &\text{"}\quad 95.6778341;\\ N^{VII}&=+0.000000750320N & &\text{"}\quad 93.8752464.\end{aligned}$$

For the root g_1, we get the following values,

$g_1 = 7''.5386995$;
$N_1' = -0.8746970 N_1$ log. 99.9418576n;
$N_1'' = -0.6900382 N_1$ “ 99.8388736n;
$N_1''' = -0.1163716 N_1$ “ 99.0658468n;
$N_1^{IV} = +0.0004334367 N_1$ “ 96.6369257;
$N_1^{V} = +0.0004535340 N_1$ “ 96.6566098;
$N_1^{VI} = -0.0001067081 N_1$ “ 96.0281973n;
$N_1^{VII} = -0.000004636406 N_1$ “ 94.6661815n.

For the root g_2, we get the following values,

$g_2 = 17''.1279859$;
$N_2' = -\ 7.663670 N_2$ log. 0.8844368n;
$N_2'' = +\ 7.458903 N_2$ “ 0.8726750;
$N_2''' = +18.20022 N_2$ “ 1.2600767;
$N_2^{IV} = -\ 0.0007514405 N_2$ “ 96.8758946n;
$N_2^{V} = -\ 0.004832818 N_2$ “ 97.6842004n;
$N_2^{VI} = +\ 0.0002927626 N_2$ “ 96.4665156;
$N_2^{VII} = +\ 0.00002153572 N_2$ “ 95.3331593.

For the root g_3, we get the following values,

$g_3 = 17''.8360300$;
$N_3' = -\ 8.244660 N_3$ log. 0.9161728n;
$N_3'' = +\ 9.718342 N_3$ “ 0.9875922;
$N_3''' = -39.93574 N_3$ “ 1.6013617n;
$N_3^{IV} = +\ 0.0004835988 N_3$ “ 96.6844852;
$N_3^{V} = +\ 0.005252281 N_3$ “ 97.7203480;
$N_3^{VI} = -\ 0.0002970608 N_3$ “ 96.4728453n;
$N_3^{VII} = -\ 0.00002202336 N_3$ “ 95.3428835n.

For the root g_4, we get the following values,

$g_4 = 0''.6166852$;
$N_4 = +\ 0.1197280 N_4^{IV}$ log. 9.0781956;
$N_4' = +\ 0.1807174 N_4^{IV}$ “ 9.2570000;
$N_4'' = +\ 0.2114780 N_4^{IV}$ “ 9.3252650;
$N_4''' = +\ 0.3450307 N_4^{IV}$ “ 9.5378578;
$N_4^{V} = +\ 1.127821 N_4^{IV}$ “ 0.0522403;
$N_4^{VI} = +\ 24.50074 N_4^{IV}$ “ 1.3891794;
$N_4^{VII} = +157.8951 N_4^{IV}$ “ 2.1983687.

For the root g_5, we get the following values,

$g_5 = 2''.7276680$;

$N_5 = +\ 0.2886746 N_5^{IV}$ log. 9.4604086;
$N_5' = +\ 0.2816212 N_5^{IV}$ “ 9.4496654;
$N_5'' = +\ 0.2969611 N_5^{IV}$ “ 9.4726996;
$N_5''' = +\ 0.3989168 N_5^{IV}$ “ 9.6008823;
$N_5^{V} = +\ 0.9103832 N_5^{IV}$ “ 9.9592242;
$N_5^{VI} = +15.29957 N_5^{IV}$ “ 1.1846793;
$N_5^{VII} = -\ 1.497633 N_5^{IV}$ “ 0.1754053*n*.

For the root g_6, we get the following values,

$g_6 = 3''.7167141$;

$N_6 = +0.5652238 N_6^{IV}$ log. 9.7522204;
$N_6' = +0.3828501 N_6^{IV}$ “ 9.5830288;
$N_6'' = +0.3770050 N_6^{IV}$ “ 9.5763472;
$N_6''' = +0.4350723 N_6^{IV}$ “ 9.6385614;
$N_6^{V} = +0.7901118 N_6^{IV}$ “ 9.8976886;
$N_6^{VI} = -1.039307 N_6^{IV}$ “ 0.0167438*n*;
$N_6^{VII} = +0.03290417 N_6^{IV}$ “ 8.5172509.

For the root g_7, we get the following values,

$g_7 = 22''.4608918$;

$N_7 = -0.006358233 N_7^{IV}$ log. 7.8033365*n*;
$N_7' = +0.02171584 N_7^{IV}$ “ 8.3367766;
$N_7'' = -0.1536641 N_7^{IV}$ “ 9.1865723*n*;
$N_7''' = -0.9634742 N_7^{IV}$ “ 9.9838401*n*;
$N_7^{V} = -3.091784 N_7^{IV}$ “ 0.4902090*n*;
$N_7^{VI} = +0.1154710 N_7^{IV}$ “ 9.0624732;
$N_7^{VII} = +0.00872848 N_7^{IV}$ “ 7.9409386.

25. We must now substitute the numbers we have computed, in the last article, in equations (134) and (135); making use of the logarithms of $\frac{m'}{n'a'}h$, $\frac{m}{n'a'}l'$, $\frac{m''}{n''a''}$, &c., which were used for the adopted masses, but using the following values of log. $\frac{m}{na}$, log. $\frac{m}{na}h$, log. $\frac{m}{na}l$.

$$\log.\ \frac{m}{na} = 87.3921041;\ \log.\ \frac{m}{na}h = 86.6903449;\ \log.\ \frac{m}{na}l = 86.1148475.$$

For the root $g = 5''.344676$, we get,

$$x = +\frac{4618325}{10^{20}}N;\ y = +\frac{259345}{10^{20}}N;\ z = +\frac{27356380}{10^{20}}N^2.$$

Whence $\beta = 86^\circ\ 47'\ 9''.2$; and log. $N = 9.2281141$.

$N = +0.1690885$; $N^{IV} = -0.0000198$;
$N' = +0.0174481$; $N^{V} = -0.0000175$;
$N'' = +0.0110355$; $N^{VI} = +0.00000805$;
$N''' = +0.0016926$; $N^{VII} = +0.000000127$.

For the root $g_1=7''.538699$, we get,

$$x_1=+\frac{4456102}{10^{20}}N_1;\quad y_1=+\frac{5531411}{10^{20}}N_1;\quad z_1=+\frac{253588600}{10^{20}}N_1^2;$$

Whence $\beta_1=38°\ 13'\ 16''.2$; log. $N_1=8.4434891$.

$$N_1=+0.0277645;\qquad N_1^{IV}=+0.00001203;$$
$$N_1'=-0.0242855;\qquad N_1^{V}=+0.00001259;$$
$$N_1''=-0.0191586;\qquad N_1^{VI}=-0.000002963;$$
$$N_1'''=-0.0032310;\qquad N_1^{VII}=-0.0000001287.$$

For the root $g_2=17''.127986$, we get,

$$x_2=-\frac{22856900}{10^{20}}N_2;\quad y_2=+\frac{49895050}{10^{20}}N_2;\quad z_2=+\frac{3.332267}{10^{10}}N_2^2;$$

Whence $\beta_2=335°\ 23'\ 35''.1$; log. $N_2=7.2167768$.

$$N_2=+0.0016473;\qquad N_2^{IV}=-0.00000124;$$
$$N_2'=-0.0126245;\qquad N_2^{V}=-0.00000796;$$
$$N_2''=+0.0122872;\qquad N_2^{VI}=+0.000000482;$$
$$N_2'''=+0.0299815;\qquad N_2^{VII}=+0.0000000355.$$

For the root $g_3=17''.836030$, we get,

$$x_3=+\frac{111304510}{10^{20}}N_3;\quad y_3=-\frac{112878600}{10^{20}}N_3;\quad z_3=+\frac{8.789905}{10^{10}}N_3^2.$$

Whence $\beta_3=135°\ 24'\ 8''.2$; log. $N_3=7.2561145$.

$$N_3=+0.0018035;\qquad N_3^{IV}=+0.000000872;$$
$$N_3'=-0.0148692;\qquad N_3^{V}=+0.00000947;$$
$$N_3''=+0.0175270;\qquad N_3^{VI}=-0.000000536;$$
$$N_3'''=-0.0720238;\qquad N_3^{VII}=-0.0000000397.$$

For the root $g_4=0''.6166852$, we get,

$$x_4=+\frac{3.358365}{10^{10}}N_4^{IV};\quad y_4=+\frac{1.360318}{10^{10}}N_4^{IV};\quad z_4=+\frac{56975.27}{10^{10}}N_4^{IV2}.$$

Whence $\beta_4=67°\ 56'\ 57''.9$; log. $N_4^{IV}=95.8034310$.

$$N_4=+0.00000761;\qquad N_4^{IV}=+0.00006359;$$
$$N_4'=+0.00001098;\qquad N_4^{V}=+0.00007172;$$
$$N_4''=+0.00001345;\qquad N_4^{VI}=+0.00155816;$$
$$N_4'''=+0.00002194;\qquad N_4^{VII}=+0.0100415.$$

For the root $g_5=2''.727688$, we get,

$$x_5=+\frac{0.6475863}{10^{10}}N_5^{IV};\quad y_5=-\frac{0.1743984}{10^{10}}N_5^{IV};\quad z_5=+\frac{345.0960}{10^{10}}N_5^{IV2}.$$

Whence $\beta_5=105°\ 4'\ 20''.9$; log. $N_5^{IV}=7.2885615$.

$$N_5=+0.00056101;\qquad N_5^{IV}=+0.0019434;$$
$$N_5'=+0.00054730;\qquad N_5^{V}=+0.0017692;$$
$$N_5''=+0.00057711;\qquad N_5^{VI}=+0.0297331;$$
$$N_5'''=+0.00077525;\qquad N_5^{VII}=-0.0029105.$$

For the root $g_6=3''.716714$, we get,

$$x_6=+\frac{0.4584871}{10^{10}}N_6^{IV};\ y_6=+\frac{0.8567341}{10^{10}}N_6^{IV};\ z_6=+\frac{22.51143}{10^{10}}N_6^{IV2}.$$

Whence $\beta_6=28^\circ\ 9'\ 13''.5$; log. $N_6^{IV}=8.6351298$.

$$\begin{array}{ll}
N_6\ =+0.0243977; & N_6^{IV}\ =+0.0431648;\\
N_6'\ =+0.0165257; & N_6^{V}\ =+0.0341050;\\
N_6''=+0.0162732; & N_6^{VI}=-0.0448615;\\
N_6'''=+0.0187798; & N_6^{VII}=+0.0014203.
\end{array}$$

For the root $g_7=22''.460892$, we get,

$$x_7=-\frac{1.009497}{10^{10}}N_7^{IV};\ y_7=+\frac{0.7872134}{10^{10}}N_7^{IV};\ z_7=+\frac{81.35929}{10^{10}}N_7^{IV2}.$$

Whence $\beta_7=307^\circ\ 56'\ 50''.6$; log. $N_7^{IV}=8.1941936$.

$$\begin{array}{ll}
N_7\ =-0.0000994; & N_7^{IV}\ =+0.0156384;\\
N_7'\ =+0.0003396; & N_7^{V}\ =-0.0483507;\\
N_7'\ =-0.0024030; & N_7^{VI}=+0.0018058;\\
N_7'''=-0.0150672; & N_7^{VII}=+0.00013650.
\end{array}$$

The difference between the values here given, and the values depending on the adopted masses, manifestly measures the increment arising from the supposition that $\mu=\frac{3}{2}$; and two-thirds of this difference is the coefficient of μ, in the expression for the values of the constants corresponding to any other value of μ.

26. We shall now suppose that $\mu'=+\frac{1}{20}$, and the mass of *Venus* will become $m'=(1+\frac{1}{20})\div390000=1\div371428.6$. Using this mass, we shall find,

$$\left.\begin{array}{ll}
\Delta\boxed{0,0}=-0'.1499336; & \Delta\boxed{4,4}=-0''.0002106;\\
\Delta\boxed{1,1}=\ \ 0.\qquad ; & \Delta\boxed{5,5}=-0.0000246;\\
\Delta\boxed{2,2}=-0.2665548; & \Delta\boxed{6,6}=-0.0000021;\\
\Delta\boxed{3,3}=-0.0241609; & \Delta\boxed{7,7}=-0.0000004.
\end{array}\right\}\ (193)$$

Whence,

$$\left.\begin{array}{ll}
\boxed{0,0}=g-\ \ 5''.7201894; & \boxed{4,4}=g-\ \ 7''.5125860;\\
\boxed{1,1}=g-11.3147682; & \boxed{5,5}=g-18.5962375;\\
\boxed{2,2}=g-13.3387278; & \boxed{6,6}=g-\ \ 2.7662543;\\
\boxed{3.3}=g-17.5770254; & \boxed{7,7}=g-\ \ 0.6479573.
\end{array}\right\}\ (194)$$

These quantities give,

$$\boxed{0,0}\boxed{1,1}=g^2-17.0349576.g+\ 64.7226171211; \qquad (195)$$
$$\boxed{0,0}\boxed{2,2}=g^2-19.0589172.g+\ 76.3000493710; \qquad (196)$$
$$\boxed{0,0}\boxed{3,3}=g^2-23.2972148.g+100.5439143766; \qquad (197)$$
$$\boxed{1,1}\boxed{2,2}=g^2-24.6534960.g+150.9246131399; \qquad (198)$$
$$\boxed{1,1}\boxed{3,3}=g^2-28.8917936.g+198.8799680465; \qquad (199)$$
$$\boxed{2,2}\boxed{3,3}=g^2-30.9157532.g+234.4551573443; \qquad (200)$$

$$\boxed{4,4}\boxed{5,5}=g^2-26.1088235.g+139.7058334952; \tag{201}$$

$$\boxed{4,4}\boxed{6,6}=g^2-10.2788403.g+20.7817233266; \tag{202}$$

$$\boxed{4,4}\boxed{7,7}=g^2-8.1605433.g+4.8678349406; \tag{203}$$

$$\boxed{5,5}\boxed{6,6}=g^2-21.3624918.g+51.4419219482; \tag{204}$$

$$\boxed{5,5}\boxed{7,7}=g^2-19.2441948.g+12.0495678407; \tag{205}$$

$$\boxed{6,6}\boxed{7,7}=g^2-3.4142116.g+1.79241466734. \tag{206}$$

$$\boxed{0,0}\boxed{1,1}\boxed{2,2}\boxed{3,3}=\left.\begin{array}{l} g^4-47.9507108.g^3+825.82631940.g^2 \\ -5994.8821218.g+15174.5513808 \end{array}\right\}; \tag{207}$$

$$\boxed{4,4}\boxed{5,5}\boxed{6,6}\boxed{7,7}=\left.\begin{array}{l} g^4-29.5230351.g^3+230.63929622.g^2 \\ -523.78311550.g+250.41078507 \end{array}\right\}. \tag{208}$$

We shall therefore obtain the following

Fundamental Equations for $\mu'=+\frac{1}{20}$; *or for* $m'=\frac{1}{371428.6}$.

$$A=g^2-40.63827162.g+200.0557143; \tag{209}$$

$$A'=g^2-23.31994327.g+100.7733425; \tag{210}$$

$$A''=g^2-18.22902235.g+71.3586722; \tag{211}$$

$$A_1=g^2-14.5939888.g+45.9795152; \tag{212}$$

$$A_2=g^2-10.00785623.g+6.35434212; \tag{213}$$

$$A_3=g^2-26.183183614.g+82.92255281; \tag{214}$$

$$D=g^2-48.76765014.g+728.640657; \tag{215}$$

$$D'=g^2-56.24689671.g+679.929057; \tag{216}$$

$$D''=g^2-31.8586536.g+250.7543893; \tag{217}$$

$$D_1=g^2-48.09966549.g+201.477252; \tag{218}$$

$$D_2=g^2-51.052918467.g+34.3823105; \tag{219}$$

$$D_3=g^2-3.4187096566.g+1.731950106. \tag{220}$$

$$B=\{g-34.9011230\}b;\quad \left.\begin{array}{l} B'=\{g-17.591388707\}b; \\ B''=\{g-12.508092541\}b; \end{array}\right\} \tag{221}$$

$$\left.\begin{array}{l} C=\{23.9743622-g\}[9.1763990]b'; \\ C'=\{17.60614518-g\}[8.8694654]b; \\ C''=-[0.2657810]b'; \\ C'''=-[0.2866491]b'; \end{array}\right\} \tag{222}$$

$$\left.\begin{array}{l} E=+[9.6231944]b''; \\ E'=\{24.7932880-g\}[8.5847028]b''; \\ E''=\{17.57829706-g\}[9.6375865]b''; \\ E'''=+[0.9518913]b''; \end{array}\right\} \tag{223}$$

$$\left.\begin{array}{l} F=-[7.6499091]b'''; \\ F'=+[8.9577383]b'''; \\ F''=\{14.28035653-g\}[0.8407439]b'''; \\ F'''=\{38.6407515-g\}[9.4809266]b'''; \end{array}\right\} \tag{224}$$

$$\left.\begin{array}{ll} B_1=\{g-4.7868001\}b_1; & B_2=\{g-\ 0.6795722937\}b_1; \\ & B_3=\{g-18.665588512\}b_1; \end{array}\right\} \qquad (225)$$

$$\left.\begin{array}{l} C_1=\ \{4.365093544\ \ -g\}[9.2840950]b_2; \\ C_2=\ \{0.68791009996-g\}[9.1824311]b_2; \\ C_3=-[0.6832317]b_2; \\ C_4=-[0.6834824]b_2; \end{array}\right\} \qquad (226)$$

$$\left.\begin{array}{l} E_1=+[7.8267723]b_3; \\ E_2=\ \{43.7345719\ \ -g\}[8.2017618]b_3; \\ E_3=\ \{\ 0.648004519-g\}[0.7610455]b_3; \\ E_4=+[1.0655652]b_3; \end{array}\right\} \qquad (227)$$

$$\left.\begin{array}{l} F_1=-[7.0339474]b_4; \\ F_2=+[0.2724895]b_4; \\ F_3=\ \{\ 2.770665138-g\}[1.7737962]b_4; \\ F_4=\ \{50.36500746\ \ -g\}[9.1128486]b_4; \end{array}\right\} \qquad (228)$$

$$\left.\begin{array}{r} g^4-47.9507108.g^3+799.53205638.g^2-5381.3548732.g \\ +12499.1914947 \end{array}\right\}=(\chi,\chi_1,\chi_2,\chi_3); \quad (229)$$

$$\left.\begin{array}{r} g^4-29.5230351.g^3+172.74239830.g^2-323.32220515.g \\ +140.47332174 \end{array}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7). \quad (230)$$

The values of b, b', b'', and b''' are given by equations (118); and the values of b_1, b_2, b_3, and b_4 are given by equations (119), by simply adding log. $(1+\frac{1}{20})=$ [0.0211893] to the coefficients of N'.

Putting equations (229) and (230), equal to nothing, they will give,

$$\left.\begin{array}{ll} g=\ 5''.59773937; & g_4=\ 0''.61668564; \\ g_1=\ 7\ .25215980; & g_5=\ 2\ .72773622; \\ g_2=17\ .20072233; & g_6=\ 3\ .71796565; \\ g_3=17\ .90008930; & g_7=22\ .46064759. \end{array}\right\} \qquad (231)$$

The equations just computed will give the following values:—

For the root g, we get,

$$g=5''.5978504;$$

$$\begin{array}{lll} N' & =+0.05341742N & \text{log. } 98.7276830; \\ N'' & =+0.03480002N & \text{“ } \ 98.5415794; \\ N''' & =+0.00540574N & \text{“ } \ 97.7328551; \\ N^{IV} & =-0.00005306593N & \text{“ } \ 95.7248158n; \\ N^{V} & =-0.00004782067N & \text{“ } \ 95.6796157n; \\ N^{VI} & =+0.00001988778N & \text{“ } \ 95.2985863; \\ N^{VII} & =+0.0000004029593N & \text{“ } \ 93.6052612. \end{array}$$

For the root g_1, we get,

$g_1 = 7''.2528745$;

$N_1' = -0.6557708 N_1$ log. $99.8167520n$;
$N_1'' = -0.5021007 N_1$ " $99.7007908n$;
$N_1''' = -0.08380783 N_1$ " $98.9232841n$;
$N_1^{IV} = +0.0003520860 N_1$ " 96.5466488;
$N_1^{V} = +0.0003598465 N_1$ " 96.5561173;
$N_1^{VI} = -0.00009068952 N_1$ " $95.9575571n$;
$N_1^{VII} = -0.000003712050 N_1$ " $94.5696138n$.

For the root g_2, we get,

$g_2 = 17''.2008675$;

$N_2' = -\ 7.2870812 N_2$ log. $0.8625536n$;
$N_2'' = +\ 7.5219298 N_2$ " 0.8763293;
$N_2''' = +21.345970 N_2$ " 1.3293159;
$N_2^{IV} = -\ 0.0008132064 N_2$ " $96.9102008n$;
$N_2^{V} = -\ 0.005455322 N_2$ " $97.7368204n$;
$N_2^{VI} = +\ 0.0003281043 N_2$ " 96.5160119;
$N_2^{VII} = +\ 0.00002415603 N_2$ " 95.3830256.

For the root g_3, we get,

$g_3 = 17''.9001137$;

$N_3' = -\ 7.82904 N_3$ log. $0.8937084n$;
$N_3'' = +\ 9.67313 N_3$ " 0.9855672;
$N_3''' = -33.37815 N_3$ " $1.5234623n$;
$N_3^{IV} = +\ 0.000347642 N_3$ " 96.5411317;
$N_3^{V} = +\ 0.004032944 N_3$ " 97.6056222;
$N_3^{VI} = -\ 0.000226715 N_3$ " $96.3554809n$;
$N_3^{VII} = -\ 0.00001681933 N_3$ " $95.2258087n$.

For the root g_4, we get,

$g_4 = 0''.61668534$;

$N_4 = +\ 0.1207828 N_4^{IV}$ log. 9.0820049;
$N_4' = +\ 0.1835444 N_4^{IV}$ " 9.2637411;
$N_4'' = +\ 0.2119770 N_4^{IV}$ " 9.3262888;
$N_4''' = +\ 0.3449880 N_4^{IV}$ " 9.5378038;
$N_4^{V} = +\ 1.127829 N_4^{IV}$ " 0.0522431;
$N_4^{VI} = +\ 24.50116 N_4^{IV}$ " 1.3891867;
$N_4^{VII} = +157.8978 N_4^{IV}$ " 2.1983761.

For the root g_5, we get,

$g_5 = 2''.7276716$;

$N_5 = +\ 0.2857445 N_5^{IV}$ log. 9.4559779;
$N_5' = +\ 0.2850941 N_5^{IV}$ " 9.4549882;
$N_5'' = +\ 0.2978305 N_5^{IV}$ " 9.4739691;
$N_5''' = +\ 0.3989201 N_5^{IV}$ " 9.6008859;
$N_5^{V} = +\ 0.9103906 N_5^{IV}$ " 9.9592278;
$N_5^{VI} = +15.30032 N_5^{IV}$ " 1.1847004;
$N_5^{VII} = -\ 1.497701 N_5^{IV}$ " $0.1754251n$.

For the root g_6, we get,

$$\begin{aligned}
&g_6=3''.7167656; && \\
&N_6 \;=+0.5404720N_6^{IV} && \log.\ 9.7327733; \\
&N_6' \;=+0.3822413N_6^{IV} && \text{“}\quad 9.5823376; \\
&N_6'' \;=+0.3758324N_6^{IV} && \text{“}\quad 9.5748787; \\
&N_6''' =+0.4347748N_6^{IV} && \text{“}\quad 9.6382644; \\
&N_6^{V} =+0.7901155N_6^{IV} && \text{“}\quad 9.8976906; \\
&N_6^{VI} =-1.039253N_6^{IV} && \text{“}\quad 0.0167214n; \\
&N_6^{VII}=+0.03290012N_6^{IV} && \text{“}\quad 8.5171974.
\end{aligned}$$

For the root g_7, we get,

$$\begin{aligned}
&g_7=22''.4609133; && \\
&N_7 \;=-0.006641048N_7^{IV} && \log.\ 7.8222366n; \\
&N_7' \;=+0.02354937N_7^{IV} && \text{“}\quad 8.3719794; \\
&N_7'' \;=-0.1585801N_7^{IV} && \text{“}\quad 9.2002488n; \\
&N_7''' =-0.9647544N_7^{IV} && \text{“}\quad 9.9844168n; \\
&N_7^{V} =-3.091771N_7^{IV} && \text{“}\quad 0.4902072n; \\
&N_7^{VI} =+0.1154698N_7^{IV} && \text{“}\quad 9.0624686; \\
&N_7^{VII}=+0.008728388N_7^{IV} && \text{“}\quad 7.9409340.
\end{aligned}$$

27. These numbers are now to be substituted in equations (134) and (135). We must also add the logarithm of $(1+\mu')$ to the logarithms of $\frac{m'}{n'a'}$, $\frac{m'}{n'a'}h'$, and $\frac{m'}{n'a'}l'$, which were used for the adopted masses; and by this means they will become,

$$\log.\frac{m'}{n'a'}=88.2471963;\ \log.\frac{m'}{n'a'}h'=85.9698939;\ \log.\frac{m'}{n'a'}l'=85.8857055n.$$

We shall therefore get for the root $g=5''.5978504$;

$$x=+\frac{1853795}{10^{20}}N,\quad y=+\frac{41624.4}{10^{20}}N,\quad z=\frac{10625807}{10^{20}}N^2.$$

Whence $\beta=88°\ 42'\ 49''.4$; $\log.\ N=9.2418092$.

$$\begin{aligned}
&N \;=+0.1745056; && N^{IV}=-0.0000092603; \\
&N' \;=+0.0093216; && N^{V} =-0.000008345; \\
&N'' =+0.0060728; && N^{VI}=+0.000003470; \\
&N''' =+0.00094333; && N^{VII}=+0.0000000703.
\end{aligned}$$

For the root $g_1=7''.252874$, we get,

$$x_1=+\frac{1689761}{10^{20}}N_1,\quad y_1=+\frac{3937391}{10^{20}}N_1,\quad z_1=\frac{138888320}{10^{20}}N_1^2.$$

Whence $\beta_1=23°\ 23'\ 37''.0$; $\log.\ N_1=98.4892507$.

$$\begin{aligned}
&N_1 \;=+0.0308497; && N_1^{IV}=+0.000010862; \\
&N_1' \;=-0.0202303; && N_1^{V} =+0.000011101; \\
&N_1'' =-0.0154897; && N_1^{VI}=-0.0000027974; \\
&N_1''' =-0.00258544; && N_1^{VII}=-0.00000011452.
\end{aligned}$$

For the root $g_2 = 17''.200867$, we get,

$$x_2 = -\frac{32718590}{10^{20}} N_2,\ y_2 = +\frac{57870410}{10^{20}} N_2,\ z_2 = \frac{3.742550}{10^{10}} N_2^2.$$

Whence $\beta_2 = 330^\circ\ 31'\ 1''.9$; log. $N_2 = 97.2495186$.

$N_2 = +0.00177621$; $N_2^{IV} = -0.0000014445$;
$N_2^{\prime} = -0.0129441$; $N_2^{V} = -0.0000096903$;
$N_2^{\prime\prime} = +0.0133612$; $N_2^{VI} = +0.00000058281$;
$N_2^{\prime\prime\prime} = +0.0379170$; $N_2^{VII} = +0.00000004291$.

For the root $g_3 = 17''.900114$, we get,

$$x_3 = +\frac{93612050}{10^{20}} N_3,\ y_3 = -\frac{95375820}{10^{20}} N_3,\ z_3 = \frac{7.000929}{10^{10}} N_3^2.$$

Whence $\beta_3 = 135^\circ\ 32'\ 5''.0$; log. $N_3 = 97.2807821$.

$N_3 = +0.0019089$; $N_3^{IV} = +0.00000066361$;
$N_3^{\prime} = -0.0149448$; $N_3^{V} = +0.00000769847$;
$N_3^{\prime\prime} = +0.0184650$; $N_3^{VI} = -0.00000043278$;
$N_3^{\prime\prime\prime} = -0.0637154$; $N_3^{VII} = -0.00000003210$.

For the root $g_4 = 0''.6166853$, we get,

$$x_4 = +\frac{3.357379}{10^{10}} N_4^{IV},\ y_4 = +\frac{1.360319}{10^{10}} N_4^{IV},\ z_4 = \frac{56977.20}{10^{10}} N_4^{IV2}.$$

Whence $\beta_4 = 67^\circ\ 56'\ 36''.7$; log. $N_4^{IV} = 95.8033066$.

$N_4 = +0.000007679$; $N_4^{IV} = +0.000063578$;
$N_4^{\prime} = +0.000011669$; $N_4^{V} = +0.000071705$;
$N_4^{\prime\prime} = +0.000013477$; $N_4^{VI} = +0.00155773$;
$N_4^{\prime\prime\prime} = +0.000021934$; $N_4^{VII} = +0.01003882$.

For the root $g_5 = 2''.727672$, we get,

$$x_5 = +\frac{0.6476120}{10^{10}} N_5^{IV},\ y_5 = -\frac{0.1744696}{10^{10}} N_5^{IV},\ z_5 = \frac{345.1504}{10^{10}} N_5^{IV2}.$$

Whence $\beta_5 = 105^\circ\ 4'\ 40''.0$; log. $N_5^{IV} = 97.2885211$.

$N_5 = +0.00055526$; $N_5^{IV} = +0.00194322$;
$N_5^{\prime} = +0.00055400$; $N_5^{V} = +0.00176909$;
$N_5^{\prime\prime} = +0.00057875$; $N_5^{VI} = +0.0297318$;
$N_5^{\prime\prime\prime} = +0.00077519$; $N_5^{VII} = -0.00291036$.

For the root $g_6 = 3''.716766$, we get,

$$x_6 = +\frac{0.4583190}{10^{10}} N_6^{IV},\ y_6 = +\frac{0.8566834}{10^{10}} N_6^{IV},\ z_6 = \frac{22.51091}{10^{10}} N_6^{IV2}.$$

Whence $\beta_6 = 28^\circ\ 8'\ 47''.2$; log. $N_6^{IV} = 98.6350843$.

$N_6 = +0.0233269$; $N_6^{IV} = +0.0431603$;
$N_6^{\prime} = +0.0164976$; $N_6^{V} = +0.0341016$;
$N_6^{\prime\prime} = +0.0162167$; $N_6^{VI} = -0.0448545$;
$N_6^{\prime\prime\prime} = +0.0187650$; $N_6^{VII} = +0.0014200$.

For the root $g_7=22''.460913$, we get,

$$x_7=-\frac{1.009492}{10^{10}}N_7^{IV},\ y_7=+\frac{0.7872137}{10^{10}}N_7^{IV},\ z_7=\frac{81.858790}{10^{10}}N_7^{IV2}.$$

Whence $\beta_7=307^\circ\ 56'\ 51''.1$; log. $N_7^{IV}=98.1941950$.

$$\begin{array}{ll}
N_7\ =-0.0001039; & N_7^{IV}=+0.0156385;\\
N_7'=+0.00036828; & N_7^{V}\ =-0.0483506;\\
N_7''=-0.00247996; & N_7^{VI}=+0.0018058;\\
N_7'''=-0.0150873; & N_7^{VII}=+0.0001365.
\end{array}$$

28. We shall now suppose $\mu''=+\frac{1}{20}$; or, the mass of Earth to be $m''=(1+\frac{1}{20})\div368689=1\div351132.4$. Using this mass, we shall find,

$$\left.\begin{array}{ll}
\Delta\boxed{0,0}=-0''.0430854; & \Delta\boxed{4,4}=-0''.0004406;\\
\Delta\boxed{1,1}=-0\ .3315294; & \Delta\boxed{5,5}=-0\ .0000502;\\
\Delta\boxed{2,2}=\ \ 0\ . \qquad\quad ; & \Delta\boxed{6,6}=-0\ .0000043;\\
\Delta\boxed{3,3}=-0\ .0878224; & \Delta\boxed{7,7}=-0\ .0000009.
\end{array}\right\}\quad(232)$$

Whence we get,

$$\left.\begin{array}{ll}
\boxed{0,0}=g-\ \ 5''.6133412; & \boxed{4,4}=g-\ \ 7''.5128160;\\
\boxed{1,1}=g-11\ .6462976; & \boxed{5,5}=g-18\ .5962631;\\
\boxed{2,2}=g-13\ .0721730; & \boxed{6,6}=g-\ \ 2\ .7662565;\\
\boxed{3,3}=g-17\ .6406869; & \boxed{7,7}=g-\ \ 0\ .6479578.
\end{array}\right\}\quad(233)$$

These quantities give the following equations,

$$\left.\begin{array}{l}
\boxed{0,0}\,\boxed{1,1}=g^2-17.2596388.g+\ \ 65.3746421455;\\
\boxed{0,0}\,\boxed{2,2}=g^2-18.6855142.g+\ \ 73.3785672744;\\
\boxed{0,0}\,\boxed{3,3}=g^2-23.2540281.g+\ \ 99.0231945721;\\
\boxed{1,1}\,\boxed{2,2}=g^2-24.7184706.g+152.2424170367;\\
\boxed{1,1}\,\boxed{3,3}=g^2-29.2869845.g+205.4486895058;\\
\boxed{2,2}\,\boxed{3,3}=g^2-30.7128599.g+230.6021109956;
\end{array}\right\}\quad(234)$$

$$\left.\begin{array}{l}
\boxed{4,4}\,\boxed{5,5}=g^2-26.1090791.g+139.7103029579;\\
\boxed{4,4}\,\boxed{6,6}=g^2-10.2790725.g+\ \ 20.7823760933;\\
\boxed{4,4}\,\boxed{7,7}=g^2-\ \ 8.1607738.g+\ \ \ \ 4.8679877272;\\
\boxed{5,5}\,\boxed{6,6}=g^2-21.3625196.g+\ \ 51.4420336761;\\
\boxed{5,5}\,\boxed{7,7}=g^2-19.2442209.g+\ \ 12.0495937275;\\
\boxed{6,6}\,\boxed{7,7}=g^2-\ \ 3.4142143.g+\ \ \ \ 1.79241747598.
\end{array}\right\}\quad(235)$$

$$\left.\begin{array}{l}
\boxed{0,0}\,\boxed{1,1}\,\boxed{2,2}\,\boxed{3,3}=g^4-\ \ 47.9724987.g^3\ +\ \ 826.06962154.g^2\\
\qquad\qquad-5987.951367523.g+15075.53048434
\end{array}\right\}\quad(236)$$

$$\left.\begin{array}{l}
\boxed{4,4}\,\boxed{5,5}\,\boxed{6,6}\,\boxed{7,7}=g^4-\ \ 29.5232934.g^3+230.64471166.g^2\\
\qquad\qquad-523.79928387.g+250.41918860
\end{array}\right\}\quad(237)$$

We shall therefore obtain the following

Fundamental Equations for $\mu''=+\frac{1}{20}$; *or, for* $m''=1\div 351132.4$.

$$\left.\begin{aligned}
A&=g^2-41.34298838.g+200.8870856;\\
A'&=g^2-23.27675657.g+99.2516206;\\
A''&=g^2-18.39687878.g+71.57390627;\\
A_1&=g^2-14.594221002.g+45.9806378;\\
A_2&=g^2-10.00808673.g+6.35450308;\\
A_3&=g^2-26.183439214.g+82.92703835;
\end{aligned}\right\}: \tag{238}$$

$$\left.\begin{aligned}
D&=g^2-48.72257219.g+727.694841;\\
D'&=g^2-55.34085031.g+665.272796;\\
D''&=g^2-31.70284173.g+247.7780603;\\
D_1&=g^2-48.09969329.g+201.477460;\\
D_2&=g^2-51.052944567.g+34.3823533;\\
D_3&=g^2-3.4187123566.g+1.7319529170.
\end{aligned}\right\} \tag{239}$$

$$\left.\begin{aligned}
B=\{g-35.7126879\}b;\quad B'&=\{g-17.655050207\}b;\\
B''&=\{g-12.782796972\}b;
\end{aligned}\right\} \tag{240}$$

$$\left.\begin{aligned}
C&=\{24.2395891-g\}[9.1763990]b';\\
C'&=\{17.66980668-g\}[8.8694654]b';\\
C''&=-[0.2445917]b';\\
C'''&=-[0.2654598]b'.
\end{aligned}\right\} \tag{241}$$

$$\left.\begin{aligned}
E&=+[9.6443837]b'';\\
E'&=\{24.4829831-g\}[8.5847028]b'';\\
E''&=\{17.64195856-g\}[9.6375865]b'';\\
E'''&=+[0.9730806]b''.
\end{aligned}\right\} \tag{242}$$

$$\left.\begin{aligned}
F&=-[7.6499091]b''';\\
F'&=+[8.9577383]b''';\\
F''&=\{14.06088317-g\}[0.8407439]b''';\\
F'''&=\{37.6710436-g\}[9.4809266]b'''.
\end{aligned}\right\} \tag{243}$$

$$\left.\begin{aligned}
B_1=\{g-4.7868023\}b_1;\quad B_2&=\{g-0.6795727937\}b_1;\\
B_3&=\{g-18.665614112\}b_1;
\end{aligned}\right\} \tag{244}$$

$$\left.\begin{aligned}
C_1&=\{4.365095744-g\}[9.2840950]b_2;\\
C_2&=\{0.6879114996-g\}[9.1824311]b_2;\\
C_3&=-[0.6832317]b_2;\\
C_4&=-[0.6834824]b_2;
\end{aligned}\right\} \tag{245}$$

$$\left.\begin{aligned}
E_1&=+[7.8267723]b_3;\\
E_2&=\{43.7345975-g\}[8.2017618]b_3;\\
E_3&=\{0.648005019-g\}[0.7610455]b_3;\\
E_4&=+[1.0655652]b_3;
\end{aligned}\right\} \tag{246}$$

$$\left.\begin{aligned}
F_1&=-[7.0339474]b_4;\\
F_2&=+[0.2724895]b_4;\\
F_3&=\{2.770667338-g\}[1.7737962]b_4;\\
F_4&=\{50.3650331-g\}[9.1128486]b_4;
\end{aligned}\right\} \tag{247}$$

$$\left.\begin{array}{r} g^4-47.9724987.g^3+799.77118048.g^2-5375.58619284.g \\ +12441.50566776 \end{array}\right\}=(\chi, \chi_1, \chi_2, \chi_2); \quad (248)$$

$$\left.\begin{array}{r} g^4-29.5232934.g^3+172.74781374.g^2-\ 323.33816841.g \\ +140.48125235 \end{array}\right\}=(\chi_4, \chi_5, \chi_6, \chi_7). \quad (249)$$

The values of b, b', b'', and b''' are given by equations (118); and the values of b_1, b_2, b_3, and b_4 are given by equations (119), by simply adding log. $(1+\frac{1}{20})=[0.0211893]$, to the coefficients of N''.

Putting equations (248) and (249), equal to nothing, they will give,

$$\left.\begin{array}{ll} g=\ 5''.5094502; & g_4=\ 0''.61668624; \\ g_1=\ 7.3146241; & g_5=\ 2.7277498; \\ g_2=17.2173969; & g_6=\ 3.7181422; \\ g_3=17.9310275; & g_7=22.4607151. \end{array}\right\} \quad (250)$$

The equations just computed, will now give the following values:

For the root g, we get,

$g=5''.5095453;$
$N' =+0.04709515N$ log. $98.6729762;$
$N'' =+0.03032428N$ " $98.4817906;$
$N''' =+0.004803897N$ " $97.6815864;$
$N^{IV} =-0.0000499083N$ " $95.6981728n;$
$N^{V} =-0.00004469284N$ " $95.6502379n;$
$N^{VI} =+0.00001922807N$ " $95.2839357;$
$N^{VII} =+0.0000003604268N$ " $93.5568171.$

For the root g_1, we get,

$g_1=7.3153801;$
$N_1' =-0.7533911N_1$ log. $9.8770206n;$
$N_1'' =-0.5814061N_1$ " $9.7644796n;$
$N_1''' =-0.0996916N_1$ " $8.9986586n;$
$N_1^{IV} =+0.0004072400N_1$ " $6.6098504;$
$N_1^{V} =+0.0004183444N_1$ " $6.6215340;$
$N_1^{VI} =-0.0001038256N_1$ " $6.0163044n;$
$N_1^{VII} =-0.000004308711N_1$ " $4.6343474n.$

For the root g_2, we get,

$g_2=17''.2175318;$
$N_2' =-\ 7.713321N_2$ log. $0.8872414n;$
$N_2'' =+\ 7.200735N_2$ " $0.8573768;$
$N_3''' =+19.05947N_2$ " $1.2801108;$
$N_3^{IV} =-\ 0.0007473015N_2$ " $96.8734959n;$
$N_2^{V} =-\ 0.005061101N_2$ " $97.7042450n;$
$N_2^{VI} =+\ 0.0003038889N_2$ " $96.4827148;$
$N_2^{VII} =+\ 0.00002237723N_2$ " $95.3498063.$

For the root g_3, we get,

$g_3=17''.9310572$;

$N_3' =- 8.306614N_3$	log. 0.9194240n;
$N_3'' =+ 9.36934N_3$	" 0.9717089;
$N_3''' =-37.82400N_3$	" 1.5777676n;
$N_3^{IV} =+ 0.000407767N_3$	" 96.6104120;
$N_3^{V} =+ 0.00487549N_3$	" 97.6880184;
$N_3^{VI} =- 0.0002732457N_3$	" 96.4365534n;
$N_3^{VII} =- 0.00002027655N_3$	" 95.3069941n.

For the root g_4, we get,

$g_4=0''.6166859$;

$N_4 =+ 0.1210308N_4^{IV}$	log. 9.0828960;
$N_4' =+ 0.1840970N_4^{IV}$	" 9.2650466;
$N_4'' =+ 0.2134317N_4^{IV}$	" 9.3292589;
$N_4''' =+ 0.3445194N_4^{IV}$	" 9.5372136;
$N_4^{V} =+ 1.127857N_4^{IV}$	" 0.0522542;
$N_4^{VI} =+ 24.50277N_4^{IV}$	" 1.3892152;
$N_4^{VII} =+157.9085N_4^{IV}$	" 2.1984055.

For the root g_5, we get,

$g_5=2''.7276841$;

$N_5 =+ 0.2888086N_5^{IV}$	log. 9.4606101;
$N_5' =+ 0.2846311N_5^{IV}$	" 9.4542824;
$N_5'' =+ 0.2991853N_5^{IV}$	" 9.4759404;
$N_5''' =+ 0.3984217N_5^{IV}$	" 9.6003430;
$N_5^{V} =+ 0.9104170N_5^{IV}$	" 9.9592404;
$N_5^{VI} =+15.30294N_5^{IV}$	" 1.1847750;
$N_5^{VII} =- 1.497941N_5^{IV}$	" 0.1754948n.

For the root g_6, we get,

$g_6=3''.7169230$;

$N_6 =+0.5533846N_6^{IV}$	log. 9.7430272;
$N_6' =+0.3799388N_6^{IV}$	" 9.5797136;
$N_6'' =+0.3762970N_6^{IV}$	" 9.5755308;
$N_6''' =+0.4342462N_6^{IV}$	" 9.6377360;
$N_6^{V} =+0.790124N_6^{IV}$	" 9.8976952;
$N_6^{VI} =-1.039090N_6^{VI}$	" 0.0166530n;
$N_6^{VII} =+0.0328877N_6^{IV}$	" 8.5170337.

For the root g_7, we get,

$g_7=22''.4609846$;

$N_7 =-0.006671589N_7^{IV}$	log. 7.8242293n;
$N_7' =+0.02618465N_7^{IV}$	" 8.4180469;
$N_7'' =-0.1544778N_7^{IV}$	" 9.1888659n;
$N_7''' =-0.9765513N_7^{IV}$	" 9.9896951n;
$N_7^{V} =-3.091724N_7^{IV}$	" 0.4902020n;
$N_7^{VI} =+0.1154676N_7^{IV}$	" 9.0624601;
$N_7^{VII} =+0.008728216N_7^{IV}$	" 7.9409255.

29. These numbers are now to be substituted in equations (134) and (135). We must also add the logarithm of $(1+\mu'')$ to those of $\frac{m''}{n''a''}$, $\frac{m''}{n''a''}h''$, and $\frac{m''}{n''a''}l''$, in order to obtain the numbers which are to be used in this computation.

For the root $g=5''.5095453$, we get,

$$x=+\frac{1853601}{10^{20}}N; \quad y=+\frac{73698.3}{10^{20}}N; \quad z=\frac{10443191}{10^{20}}.$$

Whence $\beta=87^\circ\ 43'\ 23''.3$; log. $N=9.2495260$.

$N\ =+0.1776340$;
$N'\ =+0.0083657$;
$N''=+0.0053866$;
$N'''=+0.00085332$;
$N^{IV}=-0.000008865$;
$N^{V}\ =-0.000007939$;
$N^{VI}=+0.000003416$;
$N^{VII}=+0.0000000640$.

For the root $g_1=7''.315380$, we get,

$$x_1=+\frac{1595251}{10^{20}}N_1; \quad y_1=+\frac{4450991}{10^{20}}N_1; \quad z_1=+\frac{180052670}{10^{20}}N_1^2.$$

Whence $\beta_1=19^\circ\ 43'\ 4''.3$; log. $N_1=98.4192990$.

$N_1\ =+0.0262603$;
$N_1'\ =-0.0197843$;
$N_1''=-0.0152679$;
$N_1'''=-0.00261793$;
$N_1^{IV}=+0.000010694$;
$N_1^{V}\ =+0.000010986$;
$N_1^{VI}=-0.0000027265$;
$N_1^{VII}=-0.00000011315$.

For the root $g_2=17''.217532$, we get,

$$x_2=-\frac{27629730}{10^{20}}N_2; \quad y_2=+\frac{51666060}{10^{20}}N_2; \quad z_2=\frac{3.432446}{10^{10}}N_2^2.$$

Whence $\beta_2=331^\circ\ 51'\ 47''.6$; log. $N_2=97.2322182$.

$N_2\ =+0.00170694$;
$N_2'\ =-0.0131662$;
$N_2''=+0.0122912$;
$N_2'''=+0.0325334$;
$N_2^{IV}=-0.0000012756$;
$N_2^{V}\ =-0.000008639$;
$N_2^{VI}=+0.0000005187$;
$N_2^{VII}=+0.0000000382$.

For the root $g_3=17''.931057$, we get,

$$x_3=+\frac{104020250}{10^{20}}N_3; \quad y_3=-\frac{108060520}{10^{20}}N_3; \quad z_3=\frac{8.174639}{10^{10}}N_3^2.$$

Whence $\beta_3=136^\circ\ 5'\ 29''.0$; log. $N_3=97.2635963$.

$N_3\ =+0.0018348$;
$N_3'\ =-0.0152412$;
$N_3''=+0.0171912$;
$N_3'''=-0.0694007$
$N_3^{IV}=+0.0000007482$;
$N_3^{V}\ =+0.000008946$;
$N_3^{VI}=-0.0000005014$;
$N_3^{VII}=-0.0000000372$.

For the root $g_4=0''.6166859$, we get,

$$x_4=+\frac{3.357579}{10^{10}}N_4^{IV};\quad y_4=+\frac{1.360360}{10^{10}}N_4^{IV};\quad z_4=\frac{56981.24}{10^{10}}N_4^{IV2}.$$

Whence $\beta_4=67^\circ\ 56'\ 38''.9$; log. $N_4^{IV}=95.8033000$.

$$\begin{aligned}
N_4\ &=+0.000007695; && N_4^{IV}=+0.000063577;\\
N_4'\ &=+0.00001170; && N_4^{V}\ =+0.000071706;\\
N_4''&=+0.000013569; && N_4^{VI}=+0.00155781;\\
N_4'''&=+0.000021904; && N_4^{VII}=+0.01003935.
\end{aligned}$$

For the root $g_5=2''.727684$, we get,

$$x_5=+\frac{0.6476505}{10^{10}}N_5^{IV};\quad y_5=-\frac{0.1746351}{10^{\ 0}}N_5^{IV};\quad z_5=\frac{345.2619}{10^{10}}N_5^{IV2}.$$

Whence $\beta_5=105^\circ\ 5'\ 26''.0$; log. $N_5^{IV}=97.2884328$.

$$\begin{aligned}
N_5\ &=+0.00056110; && N_5^{IV}=+0.00194282;\\
N_5'\ &=+0.00055299; && N_5^{V}\ =+0.00176878;\\
N_5''&=+0.00058126; && N_5^{VI}=+0.02973089;\\
N_5'''&=+0.00077406; && N_5^{VII}=-0.00291023.
\end{aligned}$$

For the root $g_6=3''.716923$, we get,

$$x_6=\frac{0.4583310}{10^{10}}N_6^{IV};\quad y_6=+\frac{0.8566739}{10^{10}}N_6^{IV};\quad z_6=+\frac{22.51056}{10^{10}}N_6^{IV2}.$$

Whence $\beta_6=28^\circ\ 8'\ 50''.4$; log. $N_6^{IV}=98.6350898$.

$$\begin{aligned}
N_6\ &=+0.0238845; && N_6^{IV}=+0.04316083;\\
N_6'\ &=+0.0163985; && N_6^{V}\ =+0.03410240;\\
N_6''&=+0.0162413; && N_6^{VI}=-0.04484797;\\
N_6'''&=+0.0187424; && N_6^{VII}=+0.00141946.
\end{aligned}$$

For the root g_7,$=22''.460985$, we get,

$$x_7=-\frac{1.009476}{10^{10}}N_7^{IV};\quad y_7=+\frac{0.7872105}{10^{10}}N_7^{IV};\quad z_7=+\frac{81.85731}{10^{10}}N_7^{IV2}.$$

Whence $\beta_7=307^\circ\ 56'\ 52''.2$; log. $N_7^{IV}=98.1941978$.

$$\begin{aligned}
N_7\ &=-0.0001043; && N_7^{IV}=+0.01563860;\\
N_7'\ &=+0.0004095; && N_7^{V}\ =-0.04835037;\\
N_7''&=-0.0024158; && N_7^{VI}=+0.00180575;\\
N_7'''&=-0.0152719; && N_7^{VII}=+0.000136497
\end{aligned}$$

30. The assumed mass of the earth was obtained on the hypothesis that the mean equatorial parallax of the sun is only 8″.50. Recent investigations indicate that the sun's parallax is greater by at least one-thirtieth part. Consequently the assumed mass of the earth is too small by its *one-ninth* or *one-tenth* part; and on account of the importance of this element in most astronomical theories, we shall here give another determination of the constants corresponding to an increment of $\frac{1}{10}$ to the earth's mass. This new determination will be useful as an indication to

what extent the variation of the constants is proportional to the variation of the masses; and will also enable us to interpolate the constants for intermediate values of the earth's mass.

If we now suppose $\mu''=+\frac{1}{10}$, we shall find $m''=\frac{1.1}{368689}=\frac{1}{335172}$. And by using this value of the mass we shall find,

$$\left.\begin{array}{ll}\Delta\boxed{0,0}=-0''.0861707, & \Delta\boxed{4,4}=-0''.0008812,\\ \Delta\boxed{1,1}=-0\,.6630587, & \Delta\boxed{5,5}=-0\,.0001004,\\ \Delta\boxed{2,2}=0\,. \qquad\quad , & \Delta\boxed{6,6}=-0\,.0000086,\\ \Delta\boxed{3,3}=-0\,.1756449, & \Delta\boxed{7,7}=-0\,.0000018.\end{array}\right\}\tag{251}$$

Whence we get,

$$\left.\begin{array}{ll}\boxed{0,0}=g-5''.6564265\,; & \boxed{4,4}=g-7''.5132566\,;\\ \boxed{1,1}=g-11\,.9778269\,; & \boxed{5,5}=g-18\,.5963133\,;\\ \boxed{2,2}=g-13\,.0721730\,; & \boxed{6,6}=g-2\,.7662608\,;\\ \boxed{3,3}=g-17\,.7285094\,; & \boxed{7,7}=g-0\,.6479587.\end{array}\right\}\tag{252}$$

These quantities will give the following equations,

$$\left.\begin{array}{l}\boxed{0,0}\,\boxed{1,1}=g^2-17.6342534.g+67.75169748957285\,;\\ \boxed{0,0}\,\boxed{2,2}=g^2-18.7285995.g+73.94178576978450\,;\\ \boxed{0,0}\,\boxed{3,3}=g^2-23.3849359.g+100.28001037565910\,;\\ \boxed{1,1}\,\boxed{2,2}=g^2-25.0499999.g+156.57622540085370\,;\\ \boxed{1,1}\,\boxed{3,3}=g^2-29.7063363.g+212.34901678822286\,;\\ \boxed{2,2}\,\boxed{3,3}=g^2-30.8006824.g+231.75014190892620.\end{array}\right\}\tag{253}$$

$$\left.\begin{array}{l}\boxed{4,4}\,\boxed{5,5}=g^2-26.1095699.g+139.71887363689278\,;\\ \boxed{4,4}\,\boxed{6,6}=g^2-10.2795174.g+20.78362721292128\,;\\ \boxed{4,4}\,\boxed{7,7}=g^2-8.1612153.g+4.86827997930242\,;\\ \boxed{5,5}\,\boxed{6,6}=g^2-21.3625741.g+51.44225250630864\,;\\ \boxed{5,5}\,\boxed{7,7}=g^2-19.2442720.g+12.04964299066071\,;\\ \boxed{6,6}\,\boxed{7,7}=g^2-3.4142195.g+1.79242275182896.\end{array}\right\}\tag{254}$$

$$\left.\begin{array}{r}\boxed{0,0}\,\boxed{1,1}\,\boxed{2,2}\,\boxed{3,3}=g^4-48.4349358.g^3+842.64887773.g^2\\ -6173.539244345.g+15701.465507779\end{array}\right\}\tag{255}$$

$$\left.\begin{array}{r}\boxed{4,4}\,\boxed{5,5}\,\boxed{6,6}\,\boxed{7,7}=g^4-29.5237894.g^3+230.655099078.g^2\\ -523.830290018.g+250.4352879666\end{array}\right\}\tag{256}$$

We shall therefore obtain the following

Fundamental Equations for $\mu''=+\frac{1}{10}$; *or for* $m''=1\div335172.$

$$\left.\begin{array}{l}A\;=g^2-42.46419343.g\;+208.5335061\,;\\ A'=g^2-23.40766437.g\;+100.5097899\,;\\ A''=g^2-18.77149338.g\;+74.00017359\,;\\ A_1=g^2-14.594665902.g+45.9827890\,;\\ A_2=g^2-10.00852823.g\;+6.35481085\,;\\ A_3=g^2-26{,}183930014.g+82.93563982\,;\end{array}\right\}\tag{257}$$

$$\left.\begin{aligned}
D &= g^2 - 49.585883207.g + 755.319136\,; \\
D' &= g^2 - 55.76020211.g + 674.468336\,; \\
D'' &= g^2 - 31.83774566.g + 249.8341677\,; \\
D_1 &= g^2 - 48.09974779.g + 201.477866\,; \\
D_2 &= g^2 - 51.052995667.g + 34.3824324\,; \\
D_3 &= g^2 - 3.4187175566.g + 1.7319581971.
\end{aligned}\right\} \qquad (258)$$

$$B = \{g - 36''.7908076\}b\,; \quad \left.\begin{aligned}
B' &= \{g - 17''.742872707\}b\,; \\
B'' &= \{g - 13\ .114326272\}b\,;
\end{aligned}\right\} \qquad (259)$$

$$\left.\begin{aligned}
C &= \{24.7713708 - g\}[9.1763990]b'\,; \\
C' &= \{17.75762918 - g\}[8.8694654]b'\,; \\
C'' &= -[0.2445917]b'\,; \\
C''' &= -[0.2654598]b'\,;
\end{aligned}\right\} \qquad (260)$$

$$\left.\begin{aligned}
E &= +[9\ 6645871]b''\,; \\
E' &= \{24.8145124 - g\}[8.5847028]b''\,; \\
E'' &= \{17.72978106 - g\}[9.6375865]b''\,; \\
E''' &= +[0.9932840]b''\,;
\end{aligned}\right\} \qquad (261)$$

$$\left.\begin{aligned}
F &= -[7.6499091]b'''\,; \\
F' &= +[8.9577383]b'''\,; \\
F'' &= \{14.10796460 - g\}[0.8407439]b'''\,; \\
F''' &= \{38.0025729 - g\}[9.4809266]b'''.
\end{aligned}\right\} \qquad (262)$$

$$B_1 = \{g - 4.78680657\}b_1\,; \quad \left.\begin{aligned}
B_2 &= \{g - 0.6795736737\}b_1\,; \\
B_3 &= \{g - 18.665664332\}b_1\,;
\end{aligned}\right\} \qquad (263)$$

$$\left.\begin{aligned}
C_1 &= \{4.365100014 - g\}[9.2840950]b_2\,; \\
C_2 &= \{0.6879123796 - g\}[9.1824311]b_2\,; \\
C_3 &= -[0.6832317]b_2\,; \\
C_4 &= -[0.6834824]b_2\,;
\end{aligned}\right\} \qquad (264)$$

$$\left.\begin{aligned}
E_1 &= +[7.8267723]b_3\,; \\
E_2 &= \{43.7346477 - g\}[8.2017618]b_3\,; \\
E_3 &= \{\ 0.648005897 - g\}[0.7610455]b_3\,; \\
E_4 &= +[1.0655652]b_3\,;
\end{aligned}\right\} \qquad (265)$$

$$\left.\begin{aligned}
F_1 &= -[7.0339474]b_4\,; \\
F_2 &= +[0.2724895]b_4\,; \\
F_3 &= \{\ 2.770671608 - g\}[1.7737962]b_4\,; \\
F_4 &= \{50.3650833 - g\}[9.1128486]b_4\,;
\end{aligned}\right\} \qquad (266)$$

$$\left.\begin{aligned}
&g^4 - 48.4349358.g^3 + 815.10927504.g^2 \\
&- 5528.66691090.g + 12909.37803021
\end{aligned}\right\} (= \chi, \chi_1, \chi_2, \chi_3)\,; \qquad (267)$$

$$\left.\begin{array}{l} g^4-29.5237894.g^3+172.758201154.g^2 \\ -323.368779974.g+140.4964513342 \end{array}\right\} (=\chi_4, \chi_5, \chi_6, \chi_7). \qquad (268)$$

The values of b, b', b'', and b''' are given by equations (118); and the values of b_1, b_2, b_3, and b_4 are given by equations (119), by simply adding log. $(1+\frac{1}{10})=0.0413927$ to the coefficient of N''.

Putting equations (267) and (268) equal to nothing, they will give,

$$\left.\begin{array}{ll} g=\ 5''.554906079, & g_4=\ 0''.616687321, \\ g_1=\ 7\ .378294649, & g_5=\ 2\ .727775991, \\ g_2=17\ .403271601, & g_6=\ 3\ .718480731, \\ g_3=18\ .098463471, & g_7=22\ .460845357. \end{array}\right\} \quad (268^1)$$

Equations (257–268), in connection with equations (84–97), will now give the following values:—

For the root g, we get,

$$\begin{array}{lll} g=5''.5550002, & & \\ N' =+0.04569346N & \text{log.} & 98.6598541; \\ N'' =+0.02968146N & \text{''} & 98.4724852; \\ N''' =+0.00482527N & \text{''} & 97.6853181; \\ N^{IV}=-0.00004897096N & \text{''} & 95.6899385n; \\ N^{V} =-0.00004399572N & \text{''} & 95.6434103n; \\ N^{VI}=+0.00001859827N & \text{''} & 95.2694725; \\ N^{VII}=+0.0000003632732N & \text{''} & 93.5602333. \end{array}$$

For the root g_1, we get,

$$\begin{array}{lll} g_1=7''.3790776, & & \\ N_1' =-0.755884N_1 & \text{log.} & 99.8784552n; \\ N_1'' =-0.590047N_1 & \text{''} & 99.7708867n; \\ N_1''' =-0.1045547N_1 & \text{''} & 99.0193438n; \\ N_1^{IV}=+0.0004179936N_1 & \text{''} & 96.6211696; \\ N_1^{V} =+0.0004316384N_1 & \text{''} & 96.6351200; \\ N_1^{VI}=-0.0001054817N_1 & \text{''} & 96.0231771n; \\ N_1^{VII}=-0.000004437763N_1 & \text{''} & 94.6471640n. \end{array}$$

For the root g_2, we get,

$$\begin{array}{lll} g_2=17''.4034121, & & \\ N_2' =-\ 7.766638N_2 & \text{log.} & 0.8902330n; \\ N_2'' =+\ 6.689090N_2 & \text{''} & 0.8253671; \\ N_2''' =+23.91929N_2 & \text{''} & 1.3787482; \\ N_2^{IV}=-\ 0.000785956N_2 & \text{''} & 96.8953983n; \\ N_2^{V} =-\ 0.005986225N_2 & \text{''} & 97.7771530n; \\ N_2^{VI}=+\ 0.0003529385N_2 & \text{''} & 96.5476990; \\ N_2^{VII}=+\ 0.00002604318N_2 & \text{''} & 95.4156940. \end{array}$$

For the root g_3, we get,

$g_3 = 18''.0984790$,
$N_3' = -\ 8.350533 N_3$ log. 0.9217142n;
$N_3'' = +\ 8.673780 N_3$ " 0.9382084;
$N_3''' = -28.56700 N_3$ " 1.4558646n;
$N_3^{IV} = +\ 0.0002228091 N_8$ " 96.3479330;
$N_3^{V} = +\ 0.003251222 N_3$ " 97.5120466;
$N_3^{VI} = -\ 0.000179346 N_3$ " 96.2536923n;
$N_3^{VII} = -\ 0.00001332988 N_3$ " 95.1248263n.

For the root g_4, we get,

$g_4 = 0''.6166870$,
$N_4 = +\ 0.1207555 N_4^{IV}$ log. 9.0819069;
$N_4' = +\ 0.1838474 N_4^{IV}$ " 9.2644574;
$N_4' = +\ 0.2133264 N_4^{IV}$ " 9.3290446;
$N_4''' = +\ 0.3435830 N_4^{IV}$ " 9.5360316;
$N_4^{V} = +\ 1.127906 N_4^{IV}$ " 0.0522730;
$N_4^{VI} = +\ 24.50543 N_4^{IV}$ " 1.3892623;
$N_4^{VII} = +157.9263 N_4^{IV}$ " 2.1984543.

For the root g_5, we get,

$g_5 = 2''.7277089$;
$N_5 = +\ 0.285290 N_5^{IV}$ log. 9.4552860;
$N_5' = +\ 0.2827626 N_5^{IV}$ " 9.4514220;
$N_5'' = +\ 0.2983537 N_5^{IV}$ " 9.4747313;
$N_5''' = +\ 0.397320 N_5^{IV}$ " 9.5991402;
$N_5^{V} = +\ 0.910471 N_5^{IV}$ " 9.9592660;
$N_5^{VI} = +15.30821 N_5^{IV}$ " 1.1849243;
$N_5^{VII} = -\ 1.498422 N_5^{IV}$ " 0.1756342n.

For the root g_6, we get,

$g_6 = 3''.7172386$;
$N_6 = +0.5402483 N_6^{IV}$ log. 9.7325934;
$N_6' = +0.3755153 N_6^{IV}$ " 9.5746276;
$N_6'' = +0.3741537 N_6^{IV}$ " 9.5730500;
$N_6''' = +0.4330305 N_6^{IV}$ " 9.6365185;
$N_6^{V} = +0.7901418 N_6^{IV}$ " 9.8977050;
$N_6^{VI} = -1.038754 N_6^{IV}$ " 0.0165127n;
$N_6^{VII} = +0.03286241 N_6^{IV}$ " 8.5166994.

For the root g_7, we get,

$g_7 = 22''.4611216$;
$N_7 = -0.007172393 N_7^{IV}$ log. 97.8556641n;
$N_7' = +0.0327495 N_7^{IV}$ " 98.5152046;
$N_7'' = -0.1572018 N_7^{IV}$ " 99.1964576n;
$N_7''' = -0.9919734 N_7^{IV}$ " 99.9965000n;
$N_7^{V} = -3.091662 N_7^{IV}$ " 0.4901920n;
$N_7^{VI} = +0.1154641 N_7^{IV}$ " 99.0624468;
$N_7^{VII} = +0.00872795 N_7^{IV}$ " 97.9409123.

31. These numbers are now to be substituted in equations (134) and (135). We must also add the logarithm of $(1+\mu'')$ or 0.0413927 to those of $\frac{m''}{n''a''}$, $\frac{m''}{n''a''}h''$, and $\frac{m''}{n''a''}l''$, in order to obtain the numbers to be used in this computation.

For the root $g=5''.5550002$, we get,

$$x=+\frac{1859252}{10^{20}};\ y=+\frac{982149.1}{10^{20}};\ z=\frac{10422045}{10^{20}}.$$

Whence $\beta=87^\circ\ 28''\ 12''.8$, log. $N=9.2518088$.

$N\ \ =+0.1785701$; $\qquad N^{IV}=-0.000008745$;
$N'\ =+0.0081595$; $\qquad N^{V}\ =-0.000007856$;
$N''=+0.0053002$; $\qquad N^{VI}=+0.000003321$;
$N'''=+0.00086522$; $\qquad N^{VII}=+0.0000000649$.

For the root $g_1=7''.3790776$, we get,

$$x_1=+\frac{1535521}{10^{20}};\ y_1=+\frac{4548921}{10^{20}};\ z_1=\frac{186590940}{10^{20}}.$$

Whence $\beta_1=18^\circ\ 39'\ 8''.9$; log. $N_1=98.4104497$.

$N_1\ =+0.0257306$; $\qquad N_1^{IV}=+0.000010755$;
$N_1'\ =-0.0194493$; $\qquad N_1^{V}\ =+0.000011106$;
$N_1''=-0.0151823$; $\qquad N_1^{VI}=-0.000002714$;
$N_1'''=-0.00269026$; $\qquad N_1^{VII}=-0.0000001142$.

For the root $g_2=17''.4034121$, we get,

$$x_2=-\frac{39188050}{10^{20}};\ y_2=+\frac{65870040}{10^{20}};\ z_2=\frac{4.079210}{10^{10}}.$$

Whence $\beta_2=329^\circ\ 15'\ 1''.2$; log. $N_2=97.2739118$.

$N_2\ =+0.0018789$; $\qquad N_2^{IV}=-0.000001477$;
$N_2'\ =-0.0145930$; $\qquad N_2^{V}\ =-0.000011248$;
$N_2''=+0.0125684$; $\qquad N_2^{VI}=+0.0000006631$;
$N_2'''=+0.0449428$; $\qquad N_2^{VII}=+0.00000004893$.

For the root $g_3=18''.0984790$, we get,

$$x_3=+\frac{82704460}{10^{20}};\ y_3=-\frac{81987110}{10^{20}};\ z_3=\frac{5.806087}{10^{10}}.$$

Whence $\beta_3=134^\circ\ 45'\ 1''.6$; log. $N_3=97.3022770$.

$N_3\ =+0.0020058$; $\qquad N_3^{IV}=+0.0000004469$;
$N_3'\ =-0.0167491$; $\qquad N_3^{V}\ =+0.0000065211$;
$N_3''=+0.0173974$; $\qquad N_3^{VI}=-0.0000003597$;
$N_3'''=-0.0572983$; $\qquad N_3^{VII}=-0.00000002674$.

For the root $g_4=0''.6166870$, we get,

$$x_4=+\frac{3.357912}{10^{10}};\quad y_4=+\frac{1.360959}{10^{10}};\quad z_4=\frac{56997.71}{10^{10}}.$$

Whence $\beta_4=67^\circ\ 56'\ 42''.3$; log. $N_4^{IV}=95.8032384$.

$$\begin{array}{ll}
N_4\ =+0.000007676; & N_4^{IV}\ =+0.000063565;\\
N_4'\ =+0.000011687; & N_4^{V}\ =+0.000071695;\\
N_4''=+0.000013561; & N_4^{VI}=+0.00155764;\\
N_4'''=+0.000021841; & N_4^{VII}=+0.01003850.
\end{array}$$

For the root $g_5=2''.7277089$, we get,

$$x_5=+\frac{0.6477216}{10^{10}};\quad y_5=-\frac{0.1749684}{10^{10}};\quad z_5=\frac{345.4842}{10^{10}}.$$

Whence $\beta_5=105^\circ\ 6'\ 59''.3$; log. $N_5^{IV}=97.2882538$.

$$\begin{array}{ll}
N_5\ =+0.00055404; & N_5^{IV}\ =+0.0019420;\\
N_5'\ =+0.00054913; & N_5^{V}\ =+0.0017682;\\
N_5''=+0.00057941; & N_5^{VI}=+0.0297289;\\
N_5'''=+0.00077160; & N_5^{VII}=-0.00290997.
\end{array}$$

For the root $g_6=3''.7172386$, we get,

$$x_6=+\frac{0.4583439}{10^{10}};\quad y_6=+\frac{0.8566511}{10^{10}};\quad z_6=\frac{22.50983}{10^{10}}.$$

Whence $\beta_6=28^\circ\ 8'\ 55''.1$; log. $N_6^{IV}=98.6350978$.

$$\begin{array}{ll}
N_6\ =+0.0233180; & N_6^{IV}\ =+0.0431616;\\
N_6'\ =+0.0162078; & N_6^{V}\ =+0.0341038;\\
N_6''=+0.0161491; & N_6^{VI}=-0.0448343;\\
N_6'''=+0.0186903; & N_6^{VII}=+0.00141840.
\end{array}$$

For the root $g_7=22''.4611216$, we get,

$$x_7=-\frac{1.009450}{10^{10}};\quad y_7=+\frac{0.7872062}{10^{10}};\quad z_7=\frac{81.85450}{10^{10}}.$$

Whence $\beta_7=307^\circ\ 56'\ 54''.3$; log. $N_7^{IV}=98.1942049$.

$$\begin{array}{ll}
N_7\ =-0.00011217; & N_7^{IV}\ =+0.0156388;\\
N_7'\ =+0.00051216; & N_7^{V}\ =-0.0483500;\\
N_7''=-0.0024585; & N_7^{VI}=+0.0018057;\\
N_7'''=-0.0155133; & N_7^{VII}=+0.00013650.
\end{array}$$

32. We shall now suppose $\mu'''=+1$; and the mass of *Mars* will become,

$$m'''=\frac{1+1}{2680637}=\frac{1}{1340318.5}.$$

Using this mass, we shall find,

$$\left.\begin{array}{ll}
\Delta\boxed{0,0}=-0''.0279815; & \Delta\boxed{4,4}=-0''.0031027;\\
\Delta\boxed{1,1}=-0\ .1020355; & \Delta\boxed{5,5}=-0\ .0003297;\\
\Delta\boxed{2,2}=-0\ .2982001; & \Delta\boxed{6,6}=-0\ .0000275;\\
\Delta\boxed{3,3}=\ \ 0\ .\qquad\qquad ; & \Delta\boxed{7,7}=-0\ .0000057.
\end{array}\right\}\quad(269)$$

Whence we get,

$$\left.\begin{array}{ll}
\boxed{0,0}=g-\ \ 5''.5982373\,; & \boxed{4,4}=g-\ \ 7''.5154781\,; \\
\boxed{1,1}=g-11\,.4168037\,; & \boxed{5,5}=g-18\,.5965426\,; \\
\boxed{2,2}=g-13\,.3703731\,; & \boxed{6,6}=g-\ \ 2\,.7662797\,; \\
\boxed{3,3}=g-17\,.5528645\,; & \boxed{7,7}=g-\ \ 0\,.6479626.
\end{array}\right\} \quad (270)$$

From these quantities we get the following equations,

$$\left.\begin{array}{l}
\boxed{0,0}\boxed{1,1}=g^2-17.0150410.g+\ \ 63.9139763201\,; \\
\boxed{0,0}\boxed{2,2}=g^2-18.9686104.g+\ \ 74.8505214033\,; \\
\boxed{0,0}\boxed{3,3}=g^2-28.1511018.g+\ \ 98.2651007657\,; \\
\boxed{1,1}\boxed{2,2}=g^2-24.7871768.g+152.6469250785\,; \\
\boxed{1,1}\boxed{3,3}=g^2-28.9696682.g+200.3976083692\,; \\
\boxed{2,2}\boxed{3,3}=g^2-30.9232376.g+234.6883473387\,;
\end{array}\right\} \quad (271)$$

$$\left.\begin{array}{l}
\boxed{4,4}\boxed{5,5}=g^2-26.1120207.g+139.7619086460\,; \\
\boxed{4,4}\boxed{6,6}=g^2-10.2817578.g+\ \ 20.7899145038\,; \\
\boxed{4,4}\boxed{7,7}=g^2-\ \ 8.1634407.g+\ \ \ \ 4.8697487299\,; \\
\boxed{5,5}\boxed{6,6}=g^2-21.3628223.g+\ \ 51.4432382846\,; \\
\boxed{5,5}\boxed{7,7}=g^2-19.2445052.g+\ \ 12.0498640941\,; \\
\boxed{6,6}\boxed{7,7}=g^2-\ \ 3.4142423.g+\ \ \ \ 1.79244578674.
\end{array}\right\} \quad (272)$$

$$\left.\begin{array}{r}
\boxed{0,0}\boxed{1,1}\boxed{2,2}\boxed{3,3}=g^4-47.9382786.g^3+824.7624793.g^2 \\
-5969.6589279.g+14999.865474416\,;
\end{array}\right\} \quad (273)$$

$$\left.\begin{array}{r}
\boxed{4,4}\boxed{5,5}\boxed{6,6}\boxed{7,7}=g^4-29.5262630.g^3+230.707120045.g^2 \\
-523.98540191.g+250.515644299.
\end{array}\right\} \quad (274)$$

We shall therefore obtain the following

Fundamental Equations for $\mu'''=+1$; *or, for* $m'''=1\div1340318.5$.

$$\left.\begin{array}{l}
A\ =g^2-40.54796482.g\ \ +195.9771436\,; \\
A'=g^2-23.18819358.g\ \ +\ \ 98.5732170\,; \\
A''=g^2-18.15228098.g\ \ +\ \ 70.09590592\,; \\
A_1=g^2-14.596906302.g+\ \ 45.9936083\,; \\
A_2=g^2-10.01075363.g\ \ +\ \ \ \ 6.35635695\,; \\
A_3=g^2-26.186380814.g+\ \ 82.97883004\,;
\end{array}\right\} \quad (275)$$

$$\left.\begin{array}{l}
D\ =g^2-48.259496667.g+710.074578\,; \\
D'=g^2-55.05265379.g\ \ +658.652503\,; \\
D''=g^2-31.86740965.g\ \ +250.7131091\,; \\
D_1=g^2-48.09999599.g\ \ +201.479694\,; \\
D_2=g^2-51.053228867.g+\ \ 34.3827873\,; \\
D_3=g^2-\ \ 3.4187403566.g+\ \ 1.7319812509.
\end{array}\right\} \quad (276)$$

$$\left.\begin{aligned} &B=\{g-34.9327682\}b; \quad B'=\{g-17.581591114\}b'; \\ &\phantom{B=\{g-34.9327682\}b; \quad} B''=\{g-12.553303072\}b; \end{aligned}\right\} \quad (277)$$

$$\left.\begin{aligned} C &= \{24.0060075 -g\}[9.1763990]b'; \\ C' &= \{17.61110406-g\}[8.8694654]b'; \\ C'' &= -[0.2445917]b'; \\ C''' &= -[0.2654598]b'; \end{aligned}\right\} \quad (278)$$

$$\left.\begin{aligned} E &= +[9.6231944]b''; \\ E' &= \{24.2534892 -g\}[8.5847028]b''; \\ E'' &= \{17.55540782-g\}[9.6375865]b''; \\ E''' &= +[0.9518913]b''; \end{aligned}\right\} \quad (279)$$

$$\left.\begin{aligned} F &= -[7.9509391]b'''; \\ F' &= +[9.2587683]b'''; \\ F'' &= \{14.31200183-g\}[0.8407439]b'''; \\ F''' &= \{37.4415497 -g\}[9.4809266]b'''; \end{aligned}\right\} \quad (280)$$

$$\left.\begin{aligned} &B_1=\{g-4.7868255\}b_1; \quad B_2=\{g- 0.6795775937\}b_1; \\ &\phantom{B_1=\{g-4.7868255\}b_1; \quad} B_3=\{g-18.665893612\}b_1; \end{aligned}\right\} \quad (281)$$

$$\left.\begin{aligned} C_1 &= \{4.365118944 -g\}[9.2840950]b_2; \\ C_2 &= \{0.6879162996-g\}[9.1824311]b_2; \\ C_3 &= -[0.6832317]b_2; \\ C_4 &= -[0.6834824]b_2; \end{aligned}\right\} \quad (282)$$

$$\left.\begin{aligned} E_1 &= +[7.8267723]b_3; \\ E_2 &= \{43.7348770 -g\}[8.2017618]b_3; \\ E_3 &= \{ 0.648009819-g\}[0.7610455]b_3; \\ E_4 &= +[1.0655652]b_3; \end{aligned}\right\} \quad (283)$$

$$\left.\begin{aligned} F_1 &= -[7.0339474]b_4; \\ F_2 &= +[0.2724895]b_4; \\ F_3 &= \{ 2.770690538-g\}[1.7737962]b_4; \\ F_4 &= \{50.365313 -g\}[9.1128486]b_4; \end{aligned}\right\} \quad (284)$$

$$\left.\begin{aligned} &g^4-47.9382786.g^3 +799.3791471.g^2 \\ &\quad -5382.3863166.g+12482.1014329 \end{aligned}\right\} =(\chi, \chi_1, \chi_2, \chi_3); \quad (285)$$

$$\left.\begin{aligned} &g^4-29.5262630.g^3 +172.81022212.g^2 \\ &\quad -323.52210151.g+140.57248634 \end{aligned}\right\} =(\chi_4, \chi_5, \chi_6, \chi_7). \quad (286)$$

The values of b, b', b'', and b''' are given by equations (118); and the values of b_1, b_2, b_3, and b_4 are given by equations (119), by simply adding [0.3010300] to the coefficients of N'''.

Putting equations (285) and (286), equal to nothing, they will give,

$$\left.\begin{array}{ll} g = 5''.4982776; & g_4 = 0''.616692122; \\ g_1 = 7\ .4032880; & g_5 = 2\ .7279047; \\ g_2 = 17\ .0209128; & g_6 = 3\ .7201830; \\ g_3 = 18\ .0158003; & g_7 = 22\ .4614832. \end{array}\right\} \qquad (287)$$

The equations just computed will now give the following values:—

For the root g, we get,

$$\begin{array}{lll} g = 5''.4983682; & & \\ N' & = +0.04574900N & \text{log. } 98.6603816; \\ N'' & = +0.02849177N & \text{`` } 98.4547195; \\ N''' & = +0.004428614N & \text{`` } 97.6462679; \\ N^{IV} & = -0.00004876604N & \text{`` } 95.6881175n; \\ N^{V} & = -0.00004363627N & \text{`` } 95.6398476n; \\ N^{VI} & = +0.00001885617N & \text{`` } 95.2754534; \\ N^{VII} & = +0.0000003497564N & \text{`` } 93.5437656. \end{array}$$

For the root g_1, we get,

$$\begin{array}{lll} g_1 = 7''.4040670; & & \\ N_1' & = -0.8078317N_1 & \text{log. } 9.9073209n; \\ N_1'' & = -0.6055137N_1 & \text{`` } 9.7821240n; \\ N_1''' & = -0.10224573N_1 & \text{`` } 9.0096452n; \\ N_1^{IV} & = +0.0004271000N_1 & \text{`` } 6.6305296; \\ N_1^{V} & = +0.0004419796N_1 & \text{`` } 6.6454022; \\ N_1^{VI} & = -0.0001073512N_1 & \text{`` } 6.0308069n; \\ N_1^{VII} & = -0.00000454067N_1 & \text{`` } 4.6571202n. \end{array}$$

For the root g_2, we get,

$$\begin{array}{lll} g_2 = 17''.0211649; & & \\ N_2' & = -\ 7.637284N_2 & \text{log. } 0.8829390n; \\ N_2'' & = +\ 7.417000N_2 & \text{`` } 0.8702282; \\ N_2''' & = +14.83900N_2 & \text{`` } 1.1714047; \\ N_2^{IV} & = -\ 0.001116683N_2 & \text{`` } 97.0479297n; \\ N_2^{V} & = -\ 0.006781928N_2 & \text{`` } 97.8313532n; \\ N_2^{VI} & = +\ 0.0004152514N_2 & \text{`` } 96.6183110; \\ N_2^{VII} & = +\ 0.00003050897N_2 & \text{`` } 95.4844276. \end{array}$$

For the root g_3, we get,

$$\begin{array}{lll} g_3 = 18''.0158631; & & \\ N_3' & = -\ 8.466438N_3 & \text{log. } 0.9277008n; \\ N_3'' & = +10.65257N_3 & \text{`` } 1.0274544; \\ N_3''' & = -26.00480N_3 & \text{`` } 1.4150536n; \\ N_3^{IV} & = +\ 0.0005716405N_3 & \text{`` } 6.7571230; \\ N_3^{V} & = +\ 0.007478025N_3 & \text{`` } 7.8737870; \\ N_3^{VI} & = -\ 0.0004156815N_3 & \text{`` } 6.6187606n; \\ N_3^{VII} & = -\ 0,00003086801N_3 & \text{`` } 5.4895086n. \end{array}$$

For the root g_4, we get,

$g_4 = 0''.61669170$;

$N_4 = +0.1217372 N_4^{IV}$ log. 9.0854233;
$N_4' = +0.1852979 N_4^{IV}$ “ 9.2678704;
$N_4'' = +0.2150961 N_4^{IV}$ “ 9.3326326;
$N_4''' = +0.345612 N_4^{IV}$ “ 9.5385887;
$N_4^{V} = +1.128136 N_4^{IV}$ “ 0.0523616;
$N_4^{VI} = +24.51803 N_4^{IV}$ “ 1.3894856;
$N_4^{VII} = +158.0099 N_4^{IV}$ “ 2.1986843.

For the root g_5, we get,

$g_5 = 2''.7278184$;

$N_5 = +0.2903833 N_5^{IV}$ log. 9.4629717;
$N_5' = +0.2858690 N_5^{IV}$ “ 9.4561671;
$N_5'' = +0.2999592 N_5^{IV}$ “ 9.4770622;
$N_5''' = +0.3995210 N_5^{IV}$ “ 9.6015396;
$N_5^{V} = +0.9107056 N_5^{IV}$ “ 9.9593780;
$N_5^{VI} = +15.33141 N_5^{IV}$ “ 1.1855820;
$N_5^{VII} = -1.500545 N_5^{IV}$ “ 0.1762490n.

For the root g_6, we get,

$g_6 = 3''.7186222$;

$N_6 = +0.5567669 N_6^{IV}$ log. 9.7456734;
$N_6' = +0.3805960 N_6^{IV}$ “ 9.5804642;
$N_6'' = +0.3754068 N_6^{IV}$ “ 9.5745022;
$N_6''' = +0.4351473 N_6^{IV}$ “ 9.6386363;
$N_6^{V} = +0.7902211 N_6^{IV}$ “ 9.8977486;
$N_6^{VI} = -1.0372986 N_6^{IV}$ “ 0.0159038n;
$N_6^{VII} = +0.03275252 N_6^{IV}$ “ 8.5152447.

For the root g_7, we get,

$g_7 = 22''.4619460$;

$N_7 = -0.00561240 N_7^{IV}$ log. 7.7491486n;
$N_7' = +0.01444352 N_7^{IV}$ “ 8.1596730;
$N_7'' = -0.1296412 N_7^{IV}$ “ 9.1127430n;
$N_7''' = -0.967188 N_7^{IV}$ “ 9.9855109n;
$N_7^{V} = -3.091169 N_7^{IV}$ “ 0.4901228n;
$N_7^{VI} = +0.1154386 N_7^{IV}$ “ 9.0623510;
$N_7^{VII} = +0.008726036 N_7^{IV}$ “ 7.9408170.

33. These numbers are now to be substituted in equations (134) and (135). We must also add the logarithm of $(2=1+\mu''')$ to those of $\frac{m'''}{n'''a'''}$, $\frac{m'''}{n'''a'''}h'''$, and $\frac{m'''}{n'''a'''}l'''$, in order to obtain the numbers which are to be used in this computation.

For the root $g=5''.4983682$, we get,

$$x=+\frac{1840691}{10^{20}}N;\quad y=+\frac{98053.1}{10^{20}}N;\quad z=+\frac{10390430}{10^{20}}N^2.$$

Whence $\beta=86°\ 57'\ 2''.7$; log. $N=9.2489632$.

$$\begin{array}{ll}
N\ =+0.1774036; & N^{IV}=-0.000008651; \\
N'\ =+0.00811605; & N^{V}\ =-0.000007741; \\
N''\ =+0.0050546; & N^{VI}=+0.0000033452; \\
N'''=+0.00078565; & N^{VII}=+0.00000006205.
\end{array}$$

For the root $g_1=7''.404067$, we get,

$$x_1=+\frac{1842718}{10^{20}}N_1;\quad y_1=+\frac{4337257}{10^{20}}N_1;\quad z_1=\frac{197199700}{10^{20}}N_1^2.$$

Whence $\beta_1=23°\ 1'\ 6''.9$; log. $N_1=8.3783427$.

$$\begin{array}{ll}
N_1\ =+0.0238970; & N_1^{IV}=+0.000010206; \\
N_1'\ =-0.0193047; & N_1^{V}\ =+0.000010562; \\
N_1''\ =-0.0144699; & N_1^{VI}=-0.000002565; \\
N_1'''=-0.0024433; & N_1^{VII}=-0.0000001085.
\end{array}$$

For the root $g_2=17''.021165$, we get,

$$x_2=-\frac{51010670}{10^{20}}N_2,\quad y_2=+\frac{80108830}{10^{20}}N_2,\quad z_2=\frac{3.698921}{10^{10}}N_2^2.$$

Whence $\beta_2=327°\ 30'\ 44''.5$; log. $N_2=97.4095166$.

$$\begin{array}{ll}
N_2\ =+0.00256754; & N_2^{IV}=-0.0000028671; \\
N_2'=-0.0196090; & N_2^{V}\ =-0.000017413; \\
N_2''=+0.0190434; & N_2^{VI}=+0.0000010662; \\
N_2'''=+0.0380997; & N_2^{VII}=+0.00000007833.
\end{array}$$

For the root $g_3=18''.015863$, we get,

$$x_3=+\frac{137961060}{10^{20}}N_3;\quad y_3=-\frac{149026100}{10^{20}}N_3;\quad z_3=\frac{8.388122}{10^{10}}N_3^2.$$

Whence $\beta_3=137°\ 12'\ 28''.8$; log. $N_3=97.3840054$.

$$\begin{array}{ll}
N_3\ =+0.00242106; & N_3^{IV}=+0.0000013840; \\
N_3'=-0.0204978; & N_3^{V}\ =+0.000018105; \\
N_3''=+0.0257905; & N_3^{VI}=-0.0000010064; \\
N_3'''=-0.0629592; & N_3^{VII}=-0.00000007473.
\end{array}$$

For the root $g_4=0''.6166917$, we get,

$$x_4=+\frac{3.359379}{10^{10}}N_4^{IV};\quad y_4=+\frac{1.360856}{10^{10}}N_4^{IV};\quad z_4=\frac{57058.06}{10^{10}}N_4^{IV2}.$$

Whence $\beta_4=67°\ 56'\ 51''.2$; log. $N_4^{IV}=95.8029370$.

$$\begin{array}{ll}
N_4\ =+0.000007733; & N_4^{IV}=+0.000063524; \\
N_4'=+0.000011770; & N_4^{V}\ =+0.000071663; \\
N_4''=+0.000013664; & N_4^{VI}=+0.00155748; \\
N_4'''=+0.000021955; & N_4^{VII}=+0.01003740.
\end{array}$$

For the root $g_5 = 2''.727818$, we get,

$$x_5 = +\frac{0.6479474}{10^{10}} N_5^{IV};\quad y_5 = -\frac{.1763111}{10^{10}} N_5^{IV};\quad z_5 = \frac{346.4665}{10^{10}} N_5^{IV2}.$$

Whence $\beta_5 = 105^\circ\ 13\ 19''.4$; log. $N_5^{IV} = 97.2873891$.

$$\begin{array}{ll} N_5 = +0.000562809; & N_5^{IV} = +0.00193816; \\ N_5' = +0.000554059; & N_5^{V} = +0.00176509; \\ N_5'' = +0.000581368; & N_5^{VI} = +0.0297147; \\ N_5''' = +0.000774334; & N_5^{VII} = -0.00290829. \end{array}$$

For the root $g_6 = 3''.718622$, we get,

$$x_6 = +\frac{0.4583136}{10^{10}} N_6^{IV},\quad y_6 = +\frac{0.8566921}{10^{10}} N_6^{IV},\quad z_6 = \frac{22.50708}{10^{10}} N_6^{IV2}.$$

Whence $\beta_6 = 28^\circ\ 8'\ 45''.3$; log. $N_6^{IV} = 98.6351607$.

$$\begin{array}{ll} N_6 = +0.0240344; & N_6^{IV} = +0.0431679; \\ N_6' = +0.0164295; & N_6^{V} = +0.0341122; \\ N_6'' = +0.0162055; & N_6^{VI} = -0.0447780; \\ N_6''' = +0.0187844; & N_6^{VII} = +0.0014139. \end{array}$$

For the root $g_7 = 22''.461946$, we get,

$$x_7 = -\frac{1.009108}{10^{10}} N_7^{IV};\quad y_7 = +\frac{0.7869276}{10^{10}} N_7^{IV};\quad z_7 = \frac{81.83667}{10^{10}} N_7^{IV2}.$$

Whence $\beta_7 = 307^\circ\ 56'\ 52''.8$; log. $N_7^{IV} = 98.1941499$.

$$\begin{array}{ll} N_7 = -0.0000878; & N_7^{IV} = +0.0156369; \\ N_7' = +0.0002258; & N_7^{V} = -0.0483362; \\ N_7'' = -0.0020272; & N_7^{VI} = +0.0018051; \\ N_7''' = -0.0151238; & N_7^{VII} = +0.0001364. \end{array}$$

34. We shall now suppose $\mu^{IV} = +\frac{1}{100}$; and the mass of *Jupiter* will become, $m^{IV} = \frac{1+\mu^{IV}}{1047.879} = \frac{1.01}{1047.879} = \frac{1}{1037.504}$. Using this value of *Jupiter's* mass, we shall find,

$$\left.\begin{array}{ll} \Delta\boxed{0,0} = -0''.0160284; & \Delta\boxed{4,4} = 0''. \qquad ; \\ \Delta\boxed{1,1} = -0\,.0420284; & \Delta\boxed{5,5} = -0\,.1824735; \\ \Delta\boxed{2,2} = -0\,.0706826; & \Delta\boxed{6,6} = -0\,.0093732; \\ \Delta\boxed{3,3} = -0\,.1465990; & \Delta\boxed{7,7} = -0\,.0017907. \end{array}\right\} \quad (288)$$

Whence we get,

$$\left.\begin{array}{ll} \boxed{0,0} = g - 5''.5862842; & \boxed{4,4} = g - 7''.5123754; \\ \boxed{1,1} = g - 11\,.3567966; & \boxed{5,5} = g - 18\,.7786864; \\ \boxed{2,2} = g - 13\,.1428556; & \boxed{6,6} = g - 2\,.7756254; \\ \boxed{3,3} = g - 17\,.6994635; & \boxed{7,7} = g - 0\,.6497476. \end{array}\right\} \quad (289)$$

From these quantities we get the following equations,

$$\left.\begin{aligned}
\boxed{0,0}\,\boxed{1,1}&=g^2-16.9430808.g+63.4422934092;\\
\boxed{0,0}\,\boxed{2,2}&=g^2-18.7291398.g+73.4197265812;\\
\boxed{0,0}\,\boxed{3,3}&=g^2-23.2857477.g+98.8742332985;\\
\boxed{1,1}\,\boxed{2,2}&=g^2-24.4996522.g+149.2607377924;\\
\boxed{1,1}\,\boxed{3,3}&=g^2-29.0562601.g+201.0092068986;\\
\boxed{2,2}\,\boxed{3,3}&=g^2-30.8423191.g+232.6214929780;
\end{aligned}\right\}\quad(290)$$

$$\left.\begin{aligned}
\boxed{4,4}\,\boxed{5,5}&=g^2-26.2910618.g+141.0725417557;\\
\boxed{4,4}\,\boxed{6,6}&=g^2-10.2880008.g+20.8515399746;\\
\boxed{4,4}\,\boxed{7,7}&=g^2-8.1621230.g+4.8811478864;\\
\boxed{5,5}\,\boxed{6,6}&=g^2-21.5543118.g+52.1225989505;\\
\boxed{5,5}\,\boxed{7,7}&=g^2-19.4284340.g+12.2014064196;\\
\boxed{6,6}\,\boxed{7,7}&=g^2-3.4253730.g+1.80345594215.
\end{aligned}\right\}\quad(291)$$

$$\left.\begin{aligned}
\boxed{0,0}\,\boxed{1,1}\,\boxed{2,2}\,\boxed{3,3}&=g^4-47.7853999.g^3+818.62769096.g^2\\
&-5898.0322091.g+14758.0410108;
\end{aligned}\right\}\quad(292)$$

$$\left.\begin{aligned}
\boxed{4,4}\,\boxed{5,5}\,\boxed{6,6}\,\boxed{7,7}&=g^4-29.7164348.g^3+232.93269093.g^2\\
&-530.6408472.g+254.418113702.
\end{aligned}\right\}\quad(293)$$

We shall therefore obtain the following

Fundamental Equations for $\mu^{IV}=+\frac{1}{100}$; *or for* $m^{IV}=\frac{1}{1037.504}$.

$$\left.\begin{aligned}
A&=g^2-40.3084942.g+194.2847527;\\
A'&=g^2-23.30847617.g+99.1027623;\\
A''&=g^2-18.08032078.g+69.61059287;\\
A_1&=g^2-14.626095329.g+46.1708070;\\
A_2&=g^2-10.02759291.g+6.38342153;\\
A_3&=g^2-26.365472005.g+83.71712664;
\end{aligned}\right\}\quad(294)$$

$$\left.\begin{aligned}
D&=g^2-47.971972067.g+703.129607;\\
D'&=g^2-55.11012591.g+662.354530;\\
D''&=g^2-31.78521949.g+249.0357671;\\
D_1&=g^2-48.29148549.g+202.685210;\\
D_2&=g^2-51.237157667.g+34.5983143;\\
D_3&=g^2-3.4298710566.g+1.7430000948.
\end{aligned}\right\}\quad(295)$$

$$\left.\begin{aligned}
B=\{g-34.7052507\}b;\quad B'&=\{g-17.713826807\}b;\\
B''&=\{g-12.49329597\}b;
\end{aligned}\right\}\quad(296)$$

$$\left.\begin{aligned}
C&=\{23.7784900-g\}[9.1763990]b';\\
C'&=\{17.72858328-g\}[8.8694654]b',\\
C''&=-[0.2445917]b';\\
C'''&=-[0.2654598]b'.
\end{aligned}\right\}\quad(297)$$

$$\left.\begin{array}{l}E\ =+[9.6231944]b'';\\ E'=\{24.1934821\ \ -g\}[8.5847028]b'';\\ E''=\{17.70073515-g\}[9.6375865]b'';\\ E'''=+[0.9518913]b''.\end{array}\right\}\qquad(298)$$

$$\left.\begin{array}{l}F\ =-[7.6499091]b''';\\ F'\ =+[8.9577383]b''';\\ F''=\{14.08448433-g\}[0.8407439]b''';\\ F'''=\{37.3815426\ \ -g\}[9.4809266]b'''.\end{array}\right\}\qquad(299)$$

$$B_1=\{g-4.7961712\}b_1;\quad \left.\begin{array}{l}B_2=\{g-\ 0.6813625937\}b_1;\\ B_3=\{g-18.848037412\ \}b_1;\end{array}\right\}\qquad(300)$$

$$\left.\begin{array}{l}C_1=\ \{4.374464644\ \ -g\}[9.2840950]b_2;\\ C_2=\ \{0.6897012996-g\}[9.1824311]b_2;\\ C_3=-[0.6832317]b_2;\\ C_4=-[0.6834824]b_2;\end{array}\right\}\qquad(301)$$

$$\left.\begin{array}{l}E_1=+[7.8267723]b_3;\\ E_2=\{43.9170208\ \ \ -g\}[8.2017618]b_3;\\ E_3=\{\ 0.649794819-g\}[0.7610455]b_3;\\ E_4=+[1.0655652]b_3;\end{array}\right\}\qquad(302)$$

$$\left.\begin{array}{l}F_1=-[7.0339474]b_4;\\ F_2=+[0.2724895]b_4;\\ F_3=\{\ 2.780036238-g\}[1.7737962]b_4;\\ F_4=\{50.5474564\ \ \ -g\}[9.1128486]b_4;\end{array}\right\}\qquad(303)$$

$$\left.\begin{array}{l}g^4-47.7853999.g^3+793.57041154.g^2\\ \quad-5313.7879554.g+12250.8363744\end{array}\right\}=(\chi,\chi_1,\chi_2,\chi_3);\qquad(304)$$

$$\left.\begin{array}{l}g^4-29.7164348.g^3+174.4588860.g^2\\ \quad-327.5400877.g+142.7433842\end{array}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7).\qquad(305)$$

The values of b_1, b_2, b_3, and b_4 are given by equations (119); and the values of b, b', b'', and b''' are given by equations (118), by simply adding log. $(1+\mu^{\prime v})=$ [0.0043214], to the coefficients of N''.

Putting equations (286) and (287), equal to nothing, they will give,

$$\left.\begin{array}{ll}g=\ 5''.48175018; & g_4=\ 0''.618590378;\\ g_1=\ 7\,.29977391; & g_5=\ 2\,.73702595;\\ g_2=17\,.09470611; & g_6=\ 3\,.72450143;\\ g_3=17\,.90916970; & g_7=22\,.63631704.\end{array}\right\}\qquad(306)$$

The equations just computed will now give the following values:

For the root g, we get,

$g = 5''.48184344;$

$N' = +0.04775507N$	log. $98.6790194;$
$N'' = +0.03031151N$	“ $98.4816076;$
$N''' = +0.004615663N$	“ $97.6642342;$
$N^{IV} = -0.00004974188N$	“ $95.6967222n;$
$N^{V} = -0.00004429498N$	“ $95.6463545n;$
$N^{VI} = +0.00001941068N$	“ $95.2880406;$
$N^{VII} = +0.000000351368N$	“ $93.5457621.$

For the root g_1, we get,

$g_1 = 7''.3005026;$

$N_1' = -0.7657460N_1$	log. $99.8840847n;$
$N_1'' = -0.5821200N_1$	“ $99.7650126n;$
$N_1''' = -0.09563476N_1$	“ $98.9806158n;$
$N_1^{IV} = +0.0003996932N_1$	“ $96.6017268;$
$N_1^{V} = +0.0004077858N_1$	“ $96.6104318;$
$N_1^{VI} = -0.00010221725N_1$	“ $96.0095242n;$
$N_1^{VII} = -0.000004220228N_1$	“ $94.6253359n.$

For the root g_2, we get,

$g_2 = 17''.0948281;$

$N_2' = -\ 7.697880N_2$	log. $0.8863712n;$
$N_2'' = +\ 7.832065N_2$	“ $0.8938764;$
$N_2''' = +13.95241N_2$	“ $1.1446493;$
$N_2^{IV} = -\ 0.0007072570N_2$	“ $6.8495773n;$
$N_2^{V} = -\ 0.004093204N_2$	“ $7.6120634n;$
$N_2^{VI} = +\ 0.0002502615N_2$	“ $6.3983940;$
$N_2^{VII} = +\ 0.00001841771N_2$	“ $5.2652356.$

For the root g_3, we get,

$g_3 = 17''.90922396;$

$N_3 = -\ 8.366200N_3$	log. $0.9225281n,$
$N_3'' = +10.511163N_3$	“ $1.0216508;$
$N_3''' = -56.62883N_3$	“ $1.7530376n;$
$N_3^{IV} = +\ 0.0008158954N_3$	“ $6.9116345;$
$N_3^{V} = +\ 0.008084869N_3$	“ $7.9076730;$
$N_3^{VI} = -\ 0.0004567941N_3$	“ $6.6597205n;$
$N_3^{VII} = -\ 0.00003392040N_3$	“ $5.5304609n.$

For the root g_4, we get,

$g_4 = 0''.61859008$;

$N_4 = +\ 0.1213314N_4^{IV}$ log. 9.0839733;
$N_4' = +\ 0.1844663N_4^{IV}$ " 9.2659172;
$N_4'' = +\ 0.2137640N_4^{IV}$ " 9.3299345;
$N_4''' = +\ 0.3456726N_4^{IV}$ " 9.5386650;
$N_4^{V} = +\ 1.125699N_4^{IV}$ " 0.0514222;
$N_4^{VI} = +\ 24.58977N_4^{IV}$ " 1.3907546;
$N_4^{VII} = +159.0425N_4^{IV}$ " 2.2015132.

For the root g_5, we get,

$g_5 = 2''.7369612$;

$N_5 = +\ 0.2919622N_5^{IV}$ log. 9.4653267;
$N_5' = +\ 0.2861383N_5^{IV}$ " 9.4565760;
$N_5'' = +\ 0.2997059N_5^{IV}$ " 9.4766953;
$N_5''' = +\ 0.3993596N_5^{IV}$ " 9.6013642;
$N_5^{V} = +\ 0.9082830N_5^{IV}$ " 9.9582212;
$N_5^{VI} = +15.32706N_5^{IV}$ " 1.1854589;
$N_5^{VII} = -\ 1.494938N_5^{IV}$ " 0.1746230n.

For the root g_6, we get,

$g_6 = 3''.7233071$;

$N_6 = +0.5627043N_6^{IV}$ log. 9.7502802;
$N_6' = +0.3824702N_6^{IV}$ " 9.5825976;
$N_6'' = +0.3767387N_6^{IV}$ " 9.5760403;
$N_6''' = +0.4346798N_6^{IV}$ " 9.6381695;
$N_6^{V} = +0.7887484N_6^{IV}$ " 9.8969386;
$N_6^{VI} = -1.044537N_6^{VI}$ " 0.0189238n;
$N_6^{VII} = +0.03309702N_6^{IV}$ " 8.5197888.

For the root g_7, we get,

$g_7 = 22''.6365781$;

$N_7 = -0.006141989N_7^{IV}$ log. 7.7883092n;
$N_7' = +0.01915236N_7^{IV}$ " 8.2822223;
$N_7'' = -0.1513593N_7^{IV}$ " 9.1800092n;
$N_7''' = -0.9658338N_7^{IV}$ " 9.9849024n;
$N_7^{V} = -3.128146N_7^{IV}$ " 0.4952871n;
$N_7^{VI} = +0.1158790N_7^{IV}$ " 9.0640050;
$N_7^{VII} = +0.008773014N_7^{IV}$ " 7.9431488.

35. These numbers are now to be substituted in equations (134) and (135). We must also add the logarithm of $(1+\mu^{IV})$ to those of $\frac{m^{IV}}{n^{IV}a^{IV}}$, $\frac{m^{IV}}{n^{IV}a^{IV}}h^{IV}$, and $\frac{m^{IV}}{n^{IV}a^{IV}}l^{IV}$, in order to obtain the numbers which are to be used in this computation.

For the root $g=5''.481843$, we get,

$$x=+\frac{1850169}{10^{20}}N;\quad y=+\frac{70884.5}{10^{20}}N;\quad z=\frac{10443875}{10^{20}}N^2.$$

Whence $\beta=87^\circ\ 48'\ 21''.4$; log. $N=9.2486682$.

$$\begin{aligned} N\ &=+0.1772834; & N^{IV}&=-0.000008818;\\ N'\ &=+0.0084662; & N^{V}\ &=-0.000007853;\\ N''&=+0.0053737; & N^{VI}&=+0.000003441;\\ N'''&=+0.00081828; & N^{VII}&=+0.00000006229. \end{aligned}$$

For the root $g_1=7''.300503$, we get,

$$x_1=+\frac{1630157}{10^{20}}N_1;\quad y_1=+\frac{4425088}{10^{20}}N_1;\quad z_1=\frac{179817860}{10^{20}}N_1^{\,2}.$$

Whence $\beta_1=20^\circ\ 13'\ 23''.8$; log. $N_1=8.4187229$.

$$\begin{aligned} N_1\ &=+0.0262254; & N_1^{IV}&=+0.000010482;\\ N_1'\ &=-0.0200820; & N_1^{V}\ &=+0.000010694;\\ N_1''&=-0.0152664; & N_1^{VI}&=-0.000002681;\\ N_1'''&=-0.00250806; & N_1^{VII}&=-0.00000011068. \end{aligned}$$

For the root $g_2=17''.094828$, we get,

$$x_2=-\frac{15347790}{10^{20}}N_2;\quad y_2=+\frac{36663280}{10^{20}}N_2;\quad z_2=\frac{2.973721}{10^{10}}N_2^{\,2}.$$

Whence $\beta_2=337^\circ\ 17'\ 6''.3$; log. $N_2=7.1259941$.

$$\begin{aligned} N_2\ &=+0.0013366; & N_2^{IV}&=-0.0000009453;\\ N_2'\ &=-0.0102888; & N_2^{V}\ &=-0.0000054709;\\ N_2''&=+0.0104682; & N_2^{VI}&=+0.00000033449;\\ N_2'''&=+0.0186485; & N_2^{VII}&=+0.00000002462. \end{aligned}$$

For the root $g_3=17''.909224$, we get,

$$x_3=+\frac{147160530}{10^{20}}N_3;\quad y_3=-\frac{160644600}{10^{20}}N_3;\quad z_3=\frac{14.88588}{10^{10}}N_3^{\,2}.$$

Whence $\beta_3=137^\circ\ 30'\ 30''.1$; log. $N_3=7.1654027$.

$$\begin{aligned} N_3\ &=+0.0014635; & N_3^{IV}&=+0.0000011941;\\ N_3'\ &=-0.0122442; & N_3^{V}\ &=+0.000011832;\\ N_3''&=+0.0153834; & N_3^{VI}&=-0.0000066853;\\ N_3'''&=-0.0828782; & N_3^{VII}&=-0.0000004964. \end{aligned}$$

For the root $g_4=0''.6185901$, we get,

$$x_4=+\frac{3.376737}{10^{10}}N_4^{IV};\quad y_4=+\frac{1.377873}{10^{10}}N_4^{IV};\quad z_4=\frac{57802.14}{10^{10}}N_4^{IV2}.$$

Whence $\beta_4=67^\circ\ 48'\ 7''.8$; log. $N_4^{IV}=95.7999963$.

$$\begin{aligned} N_4\ &=+0.000007655; & N_4^{IV}&=+0.0000630952;\\ N_4'\ &=+0.000011639; & N_4^{V}\ &=+0.000071026;\\ N_4''&=+0.0000134875; & N_4^{VI}&=+0.00155150;\\ N_4'''&=+0.000021810; & N_4^{VII}&=+0.01003482. \end{aligned}$$

For the root $g_5=2''.736961$, we get,

$$x_5=+\frac{0.6488003}{10^{10}}N_5^{IV};\ y_5=-\frac{0.1681608}{10^{10}}N_5^{IV};\ z_5=\frac{346.3853}{10^{10}}N_5^{IV2}.$$

Whence $\beta_5=104^\circ\ 31'\ 50''.1$; log. $N_5^{IV}=97.2866700$.

$$\begin{array}{ll}
N_5\ =+0.00056493; & N_5^{IV}=+0.00193495; \\
N_5'\ =+0.00055366; & N_5^{V}\ =+0.00175748; \\
N_5''=+0.00057992; & N_5^{VI}=+0.0296571; \\
N_5'''=+0.00077274; & N_5^{VII}=-0.00289263.
\end{array}$$

For the root $g_6=3''.723307$, we get,

$$x_6=-\frac{0.4594222}{10^{10}}N_6^{IV};\ y_6=+\frac{0.8649404}{10^{10}}N_6^{IV};\ z_6=+\frac{22.67899}{10^{10}}N_6^{IV2}.$$

Whence $\beta_6=27^\circ\ 58'\ 31''.7$; log. $N_6^{IV}=8.6353288$.

$$\begin{array}{ll}
N_6\ =+0.0243002; & N_6^{IV}=+0.0431846; \\
N_6'\ =+0.0165168; & N_6^{V}\ =+0.0340618; \\
N_6''=+0.0162693; & N_6^{VI}=-0.0451079; \\
N_6'''=+0.0187715; & N_6^{VII}=+0.0014293.
\end{array}$$

For the root $g_7=22''.636578$, we get,

$$x_7=-\frac{1.021673}{10^{10}}N_7^{IV};\ y_7=+\frac{0.7951379}{10^{10}}N_7^{IV};\ z_7=+\frac{83.56643}{10^{10}}N_7^{IV2}.$$

Whence $\beta_7=307^\circ\ 53'\ 32''.2$; log. $N_7^{IV}=8.1901156$.

$$\begin{array}{ll}
N_7\ =-0.0000952; & N_7^{IV}=+0.0154923; \\
N_7'\ =+0.0002967; & N_7^{V}\ =-0.0484622; \\
N_7''=-0.0023449; & N_7^{VI}=+0.0017952; \\
N_7'''=-0.0149630; & N_7^{VII}=+0.0001359.
\end{array}$$

36. We shall now suppose that $\mu^{V}=+\frac{1}{40}$; and the mass of *Saturn* will become, $m^{V}=\frac{1+\frac{1}{40}}{3501.6}=\frac{1}{3416.195}$. Using this value of *Saturn's* mass, we shall find,

$$\left.\begin{array}{ll}
\Delta\boxed{0,0}=-0''.0019316, & \Delta\boxed{4,4}=-0''.1849094, \\
\Delta\boxed{1,1}=-0\ .0049722, & \Delta\boxed{5,5}=-0\ .\qquad\quad , \\
\Delta\boxed{2,2}=-0\ .0081629, & \Delta\boxed{6,6}=-0\ .0348880, \\
\Delta\boxed{3,3}=-0\ .0157850, & \Delta\boxed{7,7}=-0\ .0051937.
\end{array}\right\}\ (307)$$

Whence we get,

$$\left.\begin{array}{ll}
\boxed{0,0}=g-\ \ 5''.5721874; & \boxed{4,4}=g-\ \ 7''.6972848; \\
\boxed{1,1}=g-11\ .3197404; & \boxed{5,5}=g-18\ .5962129; \\
\boxed{2,2}=g-13\ .0803359; & \boxed{6,6}=g-\ \ 2\ .8011402; \\
\boxed{3,3}=g-17\ .5686495; & \boxed{7,7}=g-\ \ 0\ .6531506.
\end{array}\right\}\ (308)$$

From these quantities we get the following equations,

$$\left.\begin{aligned}
\boxed{0,0}\boxed{1,1}&=g^2-16.8919278.g+63.0757148282;\\
\boxed{0,0}\boxed{2,2}&=g^2-18.6525233.g+72.8860828897;\\
\boxed{0,0}\boxed{3,3}&=g^2-23.1408369.g+97.8958073789;\\
\boxed{1,1}\boxed{2,2}&=g^2-24.4000763.g+148.0660067328;\\
\boxed{1,1}\boxed{3,3}&=g^2-28.8883899.g+198.8725515186;\\
\boxed{2,2}\boxed{3,3}&=g^2-30.6489854.g+229.8038367694.
\end{aligned}\right\}\qquad(309)$$

$$\left.\begin{aligned}
\boxed{4,4}\boxed{5,5}&=g^2-26.2934977.g+143.1403468927;\\
\boxed{4,4}\boxed{6,6}&=g^2-10.4984250.g+21.5611738841;\\
\boxed{4,4}\boxed{7,7}&=g^2-8.3504354.g+5.0274861855;\\
\boxed{5,5}\boxed{6,6}&=g^2-21.3973531.g+52.0905995219;\\
\boxed{5,5}\boxed{7,7}&=g^2-19.2493635.g+12.1461276134;\\
\boxed{6,6}\boxed{7,7}&=g^2-3.4542908.g+1.8295664023.
\end{aligned}\right\}\qquad(310)$$

$$\left.\begin{aligned}
\boxed{0,0}\boxed{1,1}\boxed{2,2}\boxed{3,3}=g^4-47.5409132.g^3+810.60000012.g^2\\
-5815.0364817.g+14495.04127448
\end{aligned}\right\}\qquad(311)$$

$$\left.\begin{aligned}
\boxed{4,4}\boxed{5,5}\boxed{6,6}\boxed{7,7}=g^4-29.7477885.g^3+235.7953005.g^2\\
-542.55408336.g+261.88476948
\end{aligned}\right\}\qquad(312)$$

We shall therefore obtain the following

Fundamental Equations for $\mu^{\text{v}}=+\frac{1}{40}$; *or, for* $m^{\text{v}}=\frac{1}{3416.195}$.

$$\left.\begin{aligned}
A&=g^2-40.2318777.g+193.4460881;\\
A'&=g^2-23.16356537.g+98.1230397;\\
A''&=g^2-18.02916778.g+69.22796584;\\
A_1&=g^2-14.813573502.g+47.2122074;\\
A_2&=g^2-10.19774833.g+6.52926208;\\
A_3&=g^2-26.369591592.g+84.94828460.
\end{aligned}\right\}\qquad(313)$$

$$\left.\begin{aligned}
D&=g^2-47.872396167.g+700.738213;\\
D'&=g^2-54.94225571.g+656.812394;\\
D''&=g^2-31.59188579.g+246.0948532;\\
D_1&=g^2-48.76298515.g+206.032362;\\
D_2&=g^2-51.852306416.g+35.1877225;\\
D_3&=g^2-3.4587888566.g+1.7691277905.
\end{aligned}\right\}\qquad(314)$$

$$\left.\begin{aligned}
B=\{g-34.6427310\}b;\quad B'&=\{g-17.58301281\}b;\\
B''&=\{g-12.4562398\}b;
\end{aligned}\right\}\qquad(315)$$

$$\left.\begin{aligned}
C&=\{23.7159703-g\}[9.1763990]b';\\
C'&=\{17.5977693-g\}[8.8694654]b';\\
C''&=-[0.2445917]b';\\
C'''&=-[0.2654598]b';
\end{aligned}\right\}\qquad(316)$$

$$\left.\begin{aligned}
E &=+[9.6231944]b'';\\
E' &= \{24.1564259-g\}[8.5847028]b'';\\
E'' &= \{17.56992116-g\}[9.6375865]b'';\\
E''' &=+[0.9518913]b'';
\end{aligned}\right\} \qquad (317)$$

$$\left.\begin{aligned}
F &=-[7.6499091]b''';\\
F' &=+[8.9577383]b''';\\
F'' &= \{14.02196463-g\}[0.8407439]b''';\\
F''' &= \{37.3444864-g\}[9.4809266]b'''.
\end{aligned}\right\} \qquad (318)$$

$$\left.\begin{aligned}
B_1=\{g-4.8216860\}b_1;\quad B_2&=\{g-0.6847655937\}b_1;\\
B_3&=\{g-18.667297687\}b_1;
\end{aligned}\right\} \qquad (319)$$

$$\left.\begin{aligned}
C_1 &= \{4.399979444-g\}[9.2840950]b_2;\\
C_2 &= \{0.6931042996-g\}[9.1824311]b_2;\\
C_3 &=-[0.6939556]b_2;\\
C_4 &=-[0.6942063]b_2;
\end{aligned}\right\} \qquad (320)$$

$$\left.\begin{aligned}
E_1 &=+[7.8267723]b_3;\\
E_2 &= \{44.3630057-g\}[8.2017618]b_3;\\
E_3 &= \{\ 0.653197819-g\}[0.7610455]b_3;\\
E_4 &=+[1.0655652]b_3;
\end{aligned}\right\} \qquad (321)$$

$$\left.\begin{aligned}
F_1 &=-[7.0339474]b_4;\\
F_2 &=+[0.2724895]b_4;\\
F_3 &= \{\ 2.805551038-g\}[1.7737962]b_4;\\
F_4 &= \{51.159202116-g\}[9.1128486]b_4;
\end{aligned}\right\} \qquad (322)$$

$$\left.\begin{aligned}
&g^4-47.5409132.g^3+785.54272070.g^2\\
&-5234.4038677.g+12012.62971359
\end{aligned}\right\}(=\chi,\ \chi_1,\ \chi_2,\ \chi_3); \qquad (323)$$

$$\left.\begin{aligned}
&g^4-29.7477885.g^3+176.4527823.g^2\\
&-334.71804394.g+146.97863935
\end{aligned}\right\}(=\chi_4,\ \chi_5,\ \chi_6,\ \chi_7). \qquad (324)$$

The values of b_1, b_2, b_3, and b_4 are given by equations (119); and the values of b, b', b'', and b''' are given by equations (118), by simply adding log. $(1+\mu^{\text{v}})=[0.0107239]$, to the coefficients of N^{v}.

Putting equations (323) and (324) equal to nothing, they will give,

$$\left.\begin{aligned}
g &= 5''.46587505; & g_4 &= 0''.622281636;\\
g_1 &= 7\,.25377728; & g_5 &= 2\,.76237256;\\
g_2 &= 17\,.02345660; & g_6 &= 3\,.78742019;\\
g_3 &= 17\,.79780427; & g_7 &= 22\,.57571411.
\end{aligned}\right\} \qquad (325)$$

The equations just computed will now give the following values:—

For the root g, we get,

$g = 5''.4659743$,

$N' = +0.04853754N$ log. 8.6860778;
$N'' = +0.03095683N$ “ 8.4907565;
$N''' = +0.004748442N$ “ 7.6765512;
$N^{IV} = -0.00005254717N$ “ $5.7205493n$;
$N^{V} = -0.00004692140N$ “ $5.6713710n$;
$N^{VI} = +0.00002117625N$ “ 5.3258490;
$N^{VII} = +0.000000363425N$ “ 3.5604147.

For the root g_1, we get,

$g_1 = 7''.2545148$,

$N_1' = -0.7512055N_1$ log. $9.8757588n$;
$N_1'' = -0.5726608N_1$ “ $9.7578974n$;
$N_1''' = -0.09479172N_1$ “ $8.9767704n$;
$N_1^{IV} = +0.0004012718N_1$ “ 6.6034387;
$N_1^{V} = +0.0004102662N_1$ “ 6.6130658;
$N_1^{VI} = -0.0001060996N_1$ “ $6.0257136n$;
$N_1^{VII} = -0.000004281016N_1$ “ $4.6315468n$.

For the root g_2, we get,

$g_2 = 17''.0235807$,

$N_2' = -\ 7.650735N_2$ log. $0.8837032n$;
$N_2'' = +\ 7.722107N_2$ “ 0.8877358;
$N_2''' = +15.22875N_2$ “ 1.1826643;
$N_2^{IV} = -\ 0.0007070363N_2$ “ $96.8494417n$;
$N_2^{V} = -\ 0.004279910N_2$ “ $97.6314346n$;
$N_2^{VI} = +\ 0.0002688755N_2$ “ 96.4295512;
$N_2^{VII} = +\ 0.00001969787N_2$ “ 95.2944192.

For the root g_3, we get,

$g_3 = 17''.7978505$,

$N_3' = -\ 8.286660N_3$ log. $0.9183798n$;
$N_3'' = +10.238030N_3$ “ 1.0102164;
$N_3''' = -50.30837N_3$ “ $1.7016403n$;
$N_3^{IV} = +\ 0.006648115N_3$ “ 7.8226985;
$N_3^{V} = +\ 0.006915978N_3$ “ 7.8398536;
$N_3^{VI} = -\ 0.0004030984N_3$ “ $6.6054110n$;
$N_3^{VII} = -\ 0.00002979690N_3$ “ $5.4741710n$.

For the root g_4, we get,

$g_4 = 0''.62228135;$

$N_4 = +\ 0.1215129 N_4^{IV}$ log. 9.0846223;
$N_4' = +\ 0.1845219 N_4^{IV}$ " 9.2660478;
$N_4'' = +\ 0.2136944 N_4^{IV}$ " 9.3297932;
$N_4''' = +\ 0.3455668 N_4^{IV}$ " 9.5385321;
$N_4^{V} = +\ 1.133130 N_4^{IV}$ " 0.0542796;
$N_4^{VI} = +\ 24.593006 N_4^{IV}$ " 1.3908116;
$N_4^{VII} = +160.63011 N_4^{IV}$ " 2.2058270.

For the root g_5, we get,

$g_5 = 2''.7623108;$

$N_5 = +\ 0.2979185 N_5^{IV}$ log. 9.4740975;
$N_5' = +\ 0.2890480 N_5^{IV}$ " 9.4609699;
$N_5'' = +\ 0.3020508 N_5^{IV}$ " 9.4800800;
$N_5''' = +\ 0.4005457 N_5^{IV}$ " 9.6026522;
$N_5^{V} = +\ 0.9164754 N_5^{IV}$ " 9.9621209;
$N_5^{VI} = +15.69060 N_5^{IV}$ " 1.1956394;
$N_5^{VII} = -\ 1.514217 N_5^{IV}$ " 0.1801882n.

For the root g_6, we get,

$g_6 = 3''.7862093;$

$N_6 = +0.6004756 N_6^{IV}$ log. 9.7784954;
$N_6' = +0.3937991 N_6^{IV}$ " 9.5952747;
$N_6'' = +0.3856959 N_6^{IV}$ " 9.5862450;
$N_6''' = +0.4382820 N_6^{IV}$ " 9.6417536;
$N_6^{V} = +0.794009 N_6^{IV}$ " 9.8998255;
$N_6^{VI} = -1.022221 N_6^{IV}$ " 0.0095448n;
$N_6^{VII} = +0.03054184 N_6^{IV}$ " 8.4848952.

For the root g_7, we get,

$g_7 = 22''.5759799;$

$N_7 = -0.006116321 N_7^{IV}$ log. 7.7864903n;
$N_7' = +0.01912057 N_7^{IV}$ " 8.2815008;
$N_7'' = -0.1503075 N_7^{IV}$ " 9.1769806n;
$N_7''' = -0.9432520 N_7^{IV}$ " 9.9746277n;
$N_7^{V} = -3.002313 N_7^{IV}$ " 0.4774559n;
$N_7^{VI} = +0.1143927 N_7^{IV}$ " 9.0583982;
$N_7^{VII} = +0.00864081 N_7^{IV}$ " 7.9365546.

37. These numbers are now to be substituted in equations (134) and (135). We must also add the logarithm of $1+\mu^{V}$, to those of $\frac{m^{V}}{n^{V}a^{V}}$, $\frac{m^{V}}{n^{V}a^{V}}h^{V}$, and $\frac{m^{V}}{n^{V}a^{V}}l^{V}$, in order to obtain the numbers to be used in this computation.

For the root $g=5''.465974$, we get,

$$x=+\frac{1834742}{10^{20}}N;\ y=+\frac{50945.0}{10^{20}}N;\ z=\frac{10464937}{10^{20}}N^2.$$

Whence $\beta=88^\circ\ 24''\ 34''.2$, log. $N=9.2440059$.

$N\ \ =+0.1753904;$ $\qquad N^{IV}=-0.000009216;$
$N'\ =+0.0085130;$ $\qquad N^{V}\ =-0.000008230;$
$N''\ =+0.0054295;$ $\qquad N^{VI}=+0.000003714;$
$N'''=+0.00083283;$ $\qquad N^{VII}=+0.00000006374.$

For the root $g_1=7''.254515$, we get,

$$x_1=+\frac{1718569}{10^{20}}N_1;\quad y_1=+\frac{4394108}{10^{20}}N_1;\quad z_1=\frac{173815680}{10^{20}}N_1^2.$$

Whence $\beta_1=21^\circ\ 21'\ 39''.0$; log. $N_1=8.4336898$.

$N_1\ =+0.0271450;$ $\qquad N_1^{IV}=+0.000010892;$
$N_1\ =-0.0203915;$ $\qquad N_1^{V}\ =+0.000011137;$
$N_1''=-0.0155449;$ $\qquad N_1^{VI}=-0.0000028801;$
$N_1'''=-0.00257312;$ $\qquad N_1^{VII}=-0.00000011621.$

For the root $g_2=17''.023581$, we get,

$$x_2=-\frac{18688360}{10^{20}}N_2;\quad y_2=+\frac{40529780}{10^{20}}N_2;\quad z_2=\frac{3.058100}{10^{10}}N_2^2.$$

Whence $\beta_2=335^\circ\ 14'\ 43''.7$; log. $N_2=7.1641841$.

$N_2\ =+0.0014594;$ $\qquad N_2^{IV}=-0.000001032;$
$N_2'\ =-0.0111657;$ $\qquad N_2^{V}\ =-0.000006246;$
$N_2''=+0.0112699;$ $\qquad N_2^{VI}=+0.0000003924;$
$N_2'''=+0.0222253;$ $\qquad N_2^{VII}=+0.00000002875.$

For the root $g_3=17''.797850$, we get,

$$x_3=+\frac{132834470}{10^{20}}N_3;\quad y_3=-\frac{143133400}{10^{20}}N_3;\quad z_3=\frac{12.343316}{10^{10}}N_3^2.$$

Whence $\beta_3=137^\circ\ 8'\ 14''.1$; log. $N_3=7.1992139$.

$N_3\ =+0.00158203;$ $\qquad N_3^{IV}=+0.0000010517;$
$N_3'\ =-0.0131097;$ $\qquad N_3^{V}\ =+0.000010491;$
$N_3''=+0.0161968;$ $\qquad N_3^{VI}=-0.0000006377;$
$N_3'''=-0.0795892;$ $\qquad N_3^{VII}=-0.00000004714.$

For the root $g_4=0''.6222813$, we get,

$$x_4=+\frac{3.414548}{10^{10}}N_4^{IV};\ y_4=+\frac{1.395571}{10^{10}}N_4^{IV};\ z_4=\frac{58943.27}{10^{10}}N_4^{IV2}.$$

Whence $\beta_4=67^\circ\ 46'\ 10''.2$; log. $N_4^{IV}=95.7964432$.

$N_4\ =+0.000007604;$ $\qquad N_4^{IV}=+0.000062581;$
$N_4'\ =+0.000011548;$ $\qquad N_4^{V}\ =+0.000070912;$
$N_4''=+0.000013373;$ $\qquad N_4^{VI}=+0.00153906;$
$N_4'''=+0.000021626;$ $\qquad N_4^{VII}=+0.0100524.$

For the root $g_5=2''.762311$, we get,

$$x_5=+\frac{0.6624109}{10^{10}}N_5^{IV};\ y_5=-\frac{0.1988506}{10^{0}}N_5^{IV};\ z_5=\frac{361.8939}{10^{10}}N_5^{IV2}.$$

Whence $\beta_5=106^\circ\ 42'\ 33''.6$; log. $N_5^{IV}=7.2812824$.

$$\begin{aligned}
N_5\ &=+0.00056935; & N_5^{IV}&=+0.00191110;\\
N_5'\ &=+0.00055240; & N_5^{V}\ &=+0.00175147;\\
N_5''&=+0.00057725; & N_5^{VI}&=+0.0299862;\\
N_5'''&=+0.00076548; & N_5^{VII}&=-0.00289381.
\end{aligned}$$

For the root $g_6=3''.786209$, we get,

$$x_6=+\frac{0.4675181}{10^{10}}N_6^{IV};\ y_6=+\frac{0.8555864}{10^{10}}N_6^{IV};\ z_6=\frac{22.61242}{10^{10}}N_6^{IV2}.$$

Whence $\beta_6=28^\circ\ 39'\ 12''.7$; log. $N_6^{IV}=8.6346522$.

$$\begin{aligned}
N_6\ &=+0.0258909; & N_6^{IV}&=+0.0431174;\\
N_6'\ &=+0.0169796; & N_6^{V}\ &=+0.0342356;\\
N_6''&=+0.0166303; & N_6^{VI}&=-0.0440755;\\
N_6'''&=+0.0188976; & N_6^{VII}&=+0.0013169.
\end{aligned}$$

For the root $g_7=22''.575980$, we get,

$$x_7=-\frac{1.004018}{10^{10}}N_7^{IV};\ y_7=+\frac{0.7872757}{10^{10}}N_7^{IV};\ z_7=\frac{79.68195}{10^{10}}N_7^{IV2}.$$

Whence $\beta_7=308^\circ\ 6'\ 3''.2$; log. $N_7^{IV}=8.2044480$.

$$\begin{aligned}
N_7\ &=-0.0000979; & N_7^{IV}&=+0.0160121;\\
N_7'\ &=+0.0003062; & N_7^{V}\ &=-0.0480733;\\
N_7''&=-0.00240674; & N_7^{VI}&=+0.0018317;\\
N_7'''&=-0.0151034; & N_7^{VII}&=+0.0001384.
\end{aligned}$$

38. We shall now suppose that $\mu^{VI}=+\frac{1}{20}$; and the mass of *Uranus* will become $m^{VI}=\frac{1+\mu^{VI}}{24905}=\frac{1}{23719}$. Using this value of the mass of *Uranus*, we shall find,

$$\left.\begin{aligned}
\Delta\boxed{0,0}&=-0''.0000666; & \Delta\boxed{4,4}&=-0''.0037881;\\
\Delta\boxed{1,1}&=-0\ .0001705; & \Delta\boxed{5,5}&=-0\ .0139141;\\
\Delta\boxed{2,2}&=-0\ .0002778; & \Delta\boxed{6,6}&=\ \ 0\ .\qquad\quad;\\
\Delta\boxed{3,3}&=-0\ .0005261; & \Delta\boxed{7,7}&=-0\ .0130554.
\end{aligned}\right\}\quad(326)$$

Whence we get,

$$\left.\begin{aligned}
\boxed{0,0}&=g-\ \ 5''.5703224; & \boxed{4,4}&=g-\ \ 7''.5161635;\\
\boxed{1,1}&=g-11\ .3149387; & \boxed{5,5}&=g-18\ .6101270;\\
\boxed{2,2}&=g-13\ .0724508; & \boxed{6,6}&=g-\ \ 2\ .7662522;\\
\boxed{3,3}&=g-17\ .5533906; & \boxed{7,7}&=g-\ \ 0\ .6610123.
\end{aligned}\right\}\quad(327)$$

From these quantities we get the following equations:—

$$\left.\begin{aligned}
\boxed{0,0}\boxed{1,1}&=g^2-16.8852611.g+63.0278564952;\\
\boxed{0,0}\boxed{2,2}&=g^2-18.6427732.g+73.8177655141;\\
\boxed{0,0}\boxed{3,3}&=g^2-23.1237130.g+97.7780448551;\\
\boxed{1,1}\boxed{2,2}&=g^2-24.3873895.g+147.9139794608;\\
\boxed{1,1}\boxed{3,3}&=g^2-28.8683793.g+198.6155386162;\\
\boxed{2,2}\boxed{3,3}&=g^2-30.6258414.g+229.4658349917;
\end{aligned}\right\}\quad(328)$$

$$\left.\begin{aligned}
\boxed{4,4}\boxed{5,5}&=g^2-26.1262905.g+139.8767572878;\\
\boxed{4,4}\boxed{6,6}&=g^2-10.2824157.g+20.7916038174;\\
\boxed{4,4}\boxed{7,7}&=g^2-8.1771758.g+4.9682765223;\\
\boxed{5,5}\boxed{6,6}&=g^2-21.3763792.g+51.4803047560;\\
\boxed{5,5}\boxed{7,7}&=g^2-19.2711393.g+12.3015228516;\\
\boxed{6,6}\boxed{7,7}&=g^2-3.4272645.g+1.82852672910.
\end{aligned}\right\}\quad(329)$$

$$\left.\begin{aligned}
\boxed{0,0}\boxed{1,1}\boxed{2,2}\boxed{3,3}=g^4-47.5111025.g^3+809.61901994.g^2&\\
-5804.87167416.g+14462.73971842;&
\end{aligned}\right\}\quad(330)$$

$$\left.\begin{aligned}
\boxed{4,4}\boxed{5,5}\boxed{6,6}\boxed{7,7}=g^4-29.5535550.g^3+231.24699197.g^2&\\
-527.16726514.g+255.76838948.&
\end{aligned}\right\}\quad(331)$$

We shall therefore obtain the following

Fundamental Equations for $\mu^{VI}=+\frac{1}{20}$; *or, for* $m^{VI}=\frac{1}{23719}$.

$$\left.\begin{aligned}
A&=g^2-40.22212762.g+193.3374230;\\
A'&=g^2-23.14644147.g+98.0051228;\\
A''&=g^2-18.02250108.g+69.17798438;\\
A_1&=g^2-14.698591492.g+46.9394976;\\
A_2&=g^2-10.02448873.g+6.47860075;\\
A_3&=g^2-26.200650614.g+83.09379428;
\end{aligned}\right\}\quad(332)$$

$$\left.\begin{aligned}
D&=g^2-47.859709367.g+700.433897;\\
D'&=g^2-54.92219511.g+656.158133;\\
D''&=g^2-31.56874179.g+245.7424731;\\
D_1&=g^2-48.19349485.g+205.563726;\\
D_2&=g^2-51.079862967.g+35.0495614;\\
D_3&=g^2-3.4319830985,g+1.7650873387.
\end{aligned}\right\}\quad(333)$$

$$\left.\begin{aligned}
B=\{g-34.6348459\}b;\quad B'&=\{g-17.567753907\}b;\\
B''&=\{g-12.45143805\}b;
\end{aligned}\right\}\quad(334)$$

$$\left.\begin{aligned}
C&=\{23.7080852-g\}[9.1763990]b';\\
C'&=\{17.58251038-g\}[8.8694654]b';\\
C''&=-[0.2445917]b';\\
C'''&=-[0.2654598]b';
\end{aligned}\right\}\quad(335)$$

$$\left.\begin{aligned}
E &=+[9.6231944]b'';\\
E' &= \{24.1516242 -g\}[8.5847028]b'';\\
E'' &= \{17.55466226-g\}[9.6375865]b'';\\
E''' &=+[0.9518913]b'';
\end{aligned}\right\} \tag{336}$$

$$\left.\begin{aligned}
F &=-[7.6499091]b''';\\
F' &=+[8.9577383]b''';\\
F'' &= \{14.01407953-g\}[0.8407439]b''';\\
F''' &= \{37.3396847 -g\}[9.4809266]b''';
\end{aligned}\right\} \tag{337}$$

$$\left.\begin{aligned}
B_1=\{g-4.88782529\}b_1;\ B_2&=\{g-\ 0.692627293\}b_1;\\
B_3&=\{g-18.679478012\}b_1;
\end{aligned}\right\} \tag{338}$$

$$\left.\begin{aligned}
C_1 &= \{4.445033406 -g\}[9.2840950]b_2;\\
C_2 &= \{0.7009659996-g\}[9.1824311]b_2;\\
C_3 &=-[0.6832317]b_2;\\
C_4 &=-[0.6834824]b_2;
\end{aligned}\right\} \tag{339}$$

$$\left.\begin{aligned}
E_1 &=+[7.8479616]b_3;\\
E_2 &= \{43.7484614 -g\}[8.2017618]b_3;\\
E_3 &= \{\ 0.661059519-g\}[0.7610455]b_3;\\
E_4 &=+[1.0867545]b_3;
\end{aligned}\right\} \tag{340}$$

$$\left.\begin{aligned}
F_1 &=-[7.0339474]b_4;\\
F_2 &=+[0.2724895]b_4;\\
F_3 &= \{\ 2.77088358-g\}[1.7737962]b_4;\\
F_4 &= \{50.3788970 -g\}[9.1128486]b_4;
\end{aligned}\right\} \tag{341}$$

$$\left.\begin{aligned}
&g^4-47.5111025.g^3 +784.56174052.g^2\\
&-5224.66611233.g+11983.3131284
\end{aligned}\right\}=(\chi, \chi_1, \chi_2, \chi_3); \tag{342}$$

$$\left.\begin{aligned}
&g^4-29.5535550.g^3 +173.33949329.g^2\\
&-325.77538456.g+143.38934238
\end{aligned}\right\}=(\chi_4, \chi_5, \chi_6, \chi_7). \tag{343}$$

The values of b_1, b_2, b_3, and b_4 are given by equations (119); and the values of b, b', b'', and b''' are given by equations (118), by simply adding log. $(1+\mu^{VI})=[0.0211893]$, to the coefficients of N^{VI}.

Putting equations (342) and (343), equal to nothing, they will give,

$$\left.\begin{aligned}
g &= 5''.46378145; & g_4 &= 0''.628004507;\\
g_1 &= 7\ .24790553; & g_5 &= 2\ .72658456;\\
g_2 &=17\ .01456031; & g_6 &= 3\ .72632806;\\
g_3 &=17\ .78485521; & g_7 &=22\ .47263787.
\end{aligned}\right\} \tag{344}$$

The equations just computed will now give the following values:—

For the root g, we get,

$g=5''.4638782;$

$N' =+0.04863950N$ log. 8.6869890;
$N'' =+0.03104071N$ “ 8.4919316;
$N''' =+0.004765953N$ “ 7.6781498;
$N^{IV} =-0.0000511782N$ “ 5.7090852n;
$N^{V} =-0.0000456230N$ “ 5.6591837n;
$N^{VI} =+0.00001999627N$ “ 5.3009490;
$N^{VII}=+0.000000318386N$ “ 3.5029538.

For the root g_1, we get,

$g_1=7''.2486360;$

$N_1' =-0.749378N_1$ log. 9.8747004n;
$N_1'' =-0.5714613N_1$ “ 9.7569868n;
$N_1''' =-0.0946740N_1$ “ 8.9762302n;
$N_1^{IV} =+0.0003968524N_1$ “ 6.5986290;
$N_1^{V} =+0.0004049153N_1$ “ 6.6073642;
$N_1^{VI} =-0.00010217875N_1$ “ 6.0093606n;
$N_1^{VII}=-0.000004032583N_1$ “ 4.6055833n.

For the root g_2, we get,

$g_2=17''.0146874;$

$N_2' =-\ 7.644965N_2$ log. 0.8833755n;
$N_2'' =+\ 7.708896N_2$ “ 0.8869922;
$N_2''' =+15.37836N_2$ “ 1.1869102;
$N_2^{IV} =-\ 0.0007260315N_2$ “ 6.8609555n;
$N_2^{V} =-\ 0.004354339N_2$ “ 7.6389222n;
$N_2^{VI} =+\ 0.0002668697N_2$ “ 6.4262993;
$N_2^{VII}=+\ 0.00001945905N_2$ “ 5.2891217.

For the root g_3, we get,

$g_3=17''.7849019;$

$N_3' =-\ 8.277612N_3$ log. 0.9179050n;
$N_3'' =+10.208104N_3$ “ 1.0089451;
$N_3''' =-49.64280N_3$ “ 1.6958563n;
$N_3^{IV} =+\ 0.0006759756N_3$ “ 6.8299311;
$N_3^{V} =+\ 0.006899148N_3$ “ 7.8387954;
$N_3^{VI} =-\ 0.0003922849N_3$ “ 6.5936016n;
$N_3^{VII}=-\ 0.00002886839N_3$ “ 5.4604225n.

For the root g_4, we get,

$g_4 = 0''.62800419;$

$N_4 = +\ 0.1217541 N_4^{IV}$ log. 9.0854837;
$N_4' = +\ 0.1847141 N_4^{IV}$ " 9.2665000;
$N_4'' = +\ 0.2138538 N_4^{IV}$ " 9.3301171;
$N_4''' = +\ 0.3456976 N_4^{IV}$ " 9.5386963;
$N_4^{V} = +\ 1.127368 N_4^{IV}$ " 0.0520657;
$N_4^{VI} = +\ 23.96983 N_4^{IV}$ " 1.3796650;
$N_4^{VII} = +153.5574 N_4^{IV}$ " 2.1862708.

For the root g_5, we get,

$g_5 = 2''.72651823;$

$N_5 = +\ 0.2923210 N_5^{IV}$ log. 9.4658600;
$N_5' = +\ 0.286536 N_5^{IV}$ " 9.4571790;
$N_5'' = +\ 0.2999964 N_5^{IV}$ " 9.4771160;
$N_5''' = +\ 0.3994943 N_5^{IV}$ " 9.6015106;
$N_5^{V} = +\ 0.9107594 N_5^{IV}$ " 9.9594036;
$N_5^{VI} = +14.68646 N_5^{IV}$ " 1.1669170;
$N_5^{VII} = -\ 1.519462 N_5^{IV}$ " 0.1816899n.

For the root g_6, we get,

$g_6 = 3''.7251312;$

$N_6 = +0.5714777 N_6^{IV}$ log. 9.7569992;
$N_6' = +0.3858590 N_6^{IV}$ " 9.5864288;
$N_6'' = +0.3794987 N_6^{IV}$ " 9.5792102;
$N_6''' = +0.4358324 N_6^{IV}$ " 9.6393195;
$N_6^{V} = +0.7893470 N_6^{IV}$ " 9.8972680;
$N_6^{VI} = -1.030466 N_6^{IV}$ " 0.0130335n;
$N_6^{VII} = +0.0357074 N_6^{IV}$ " 8.5527582.

For the root g_7, we get,

$g_7 = 22''.4729002;$

$N_7 = -0.00622292 N_7^{IV}$ log. 7.7939944n;
$N_7' = +0.02016987 N_7^{IV}$ " 8.3047032;
$N_7'' = -0.1518602 N_7^{IV}$ " 9.1814440n;
$N_7''' = -0.9593122 N_7^{IV}$ " 9.9819600n;
$N_7^{V} = -3.093542 N_7^{IV}$ " 0.4904560n;
$N_7^{VI} = +0.1154757 N_7^{IV}$ " 9.0624906;
$N_7^{VII} = +0.008683434 N_7^{IV}$ " 7.9386915.

39. These numbers are now to be substituted in equations (134) and (135). We must also add the logarithm of $1+\mu^{VI}$, to those of $\frac{m^{VI}}{n^{VI}a^{VI}}$, $\frac{m^{VI}}{n^{VI}a^{VI}}h^{VI}$, and $\frac{m^{VI}}{n^{VI}a^{VI}}l^{VI}$, in order to obtain the numbers which are to be used in this computation.

For the root $g=5''.463878$, we get,

$$x=+\frac{1846778}{10^{20}}N;\ y=+\frac{61805.2}{10^{20}}N;\ z=\frac{10467663}{10^{20}}N^2.$$

Whence $\beta=88^\circ\ 4'\ 59''.6$; log. $N=9.2468080$.

$N\ =+0.1765257;$ $\qquad N^{IV}=-0.0000090343;$
$N'\ =+0.0085861;$ $\qquad N^{V}\ =-0.0000080536;$
$N''\ =+0.0054795;$ $\qquad N^{VI}=+0.0000035299;$
$N'''=+0.00084131;$ $\qquad N^{VII}=+0.0000000562.$

For the root $g_1=7''.248636$, we get,

$$x_1=+\frac{1657215}{10^{20}}N_1;\ \ y_1=\frac{4358217}{10^{20}}N_1;\ z_1=\frac{173063980}{10^{20}}N_1^2.$$

Whence $\beta_1=20^\circ\ 49'\ 9''.6$; log. $N_1=8.4304273$.

$N_1\ =+0.0269418;$ $\qquad N_1^{IV}=+0.000010692;$
$N_1'\ =-0.0201896;$ $\qquad N_1^{V}\ =+0.000010909;$
$N_1''\ =-0.0153962;$ $\qquad N_1^{VI}=-0.000002753;$
$N_1'''=-0.00255069;$ $\qquad N_1^{VII}=-0.0000001086.$

For the root $g_2=17''.014687$, we get,

$$x_2=-\frac{18857840}{10^{20}}N_2,\ y_2=+\frac{40818170}{10^{20}}N_2,\ z_2=\frac{3.068621}{10^{10}}N_2^2.$$

Whence $\beta_2=335^\circ\ 12'\ 11''.5$; log. $N_2=7.1659197$.

$N_2\ =+0.0014652;$ $\qquad N_2^{IV}=-0.0000010637;$
$N_2'\ =-0.0112020;$ $\qquad N_2^{V}\ =-0.0000063803;$
$N_2''=+0.0112957;$ $\qquad N_2^{VI}=+0.00000039104;$
$N_2'''=+0.0225336;$ $\qquad N_2^{VII}=+0.00000002851.$

For the root $g_3=17''.784902$, we get,

$$x_3=+\frac{131044100}{10^{20}}N_3;\ y_3=-\frac{141058200}{10^{20}}N_3;\ z_3=\frac{12.091655}{10^{10}}N_3^2.$$

Whence $\beta_3=137^\circ\ 6'\ 27''.7$; log. $N_3=7.2020253$.

$N_3\ =+0.0015923;$ $\qquad N_3^{IV}=+0.0000010764;$
$N_3'\ =-0.0131805;$ $\qquad N_3^{V}\ =+0.0000109855;$
$N_3''=+0.0162544;$ $\qquad N_3^{VI}=-0.0000006246;$
$N_3'''=-0.0790463;$ $\qquad N_3^{VII}=-0.0000000460.$

For the root $g_4=0''.6280042$, we get,

$$x_4=+\frac{3.295099}{10^{10}}N_4^{IV};\ \ y_4=\frac{1.261747}{10^{10}}N_4^{IV};\ z_4=\frac{53937.18}{10^{10}}N_4^{IV2}.$$

Whence $\beta_4=69^\circ\ 2'\ 50''.4$; log. $N_4^{IV}=95.8156910$.

$N_4\ =+0.000007965;$ $\qquad N_4^{IV}=+0.000065417;$
$N_4'\ =+0.000011808;$ $\qquad N_4^{V}\ =+0.000073749;$
$N_4''=+0.000013989;$ $\qquad N_4^{VI}=+0.00156804;$
$N_4'''=+0.000022614;$ $\qquad N_4^{VII}=+0.01004537.$

For the root $g_5=2''.726518$, we get,

$$x_5=+\frac{0.6486453}{10^{10}}N_5^{IV};\ y_5=-\frac{0.1822083}{10^{10}}N_5^{IV};\ z_5=\frac{337.9588}{10^{10}}N_5^{IV2}.$$

Whence $\beta_5=105^\circ\ 41'\ 25''.3$; log. $N_5^{IV}=7.2996358$.

$N_5\ =+0.00058277$;
$N_5'\ =+0.00057135$;
$N_5''=+0.00059807$;
$N_5'''=+0.00079643$;
$N_5^{IV}=+0.00199359$;
$N_5^{V}\ =+0.00181568$;
$N_5^{VI}=+0.0292788$;
$N_5^{VII}=-0.0030292$.

For the root $g_6=3''.725131$, we get,

$$x_6=+\frac{0.4579378}{10^{10}}N_6^{IV},\ y_6=+\frac{0.8593662}{10^{10}}N_6^{IV},\ z_6=\frac{22.55046}{10^{10}}N_6^{IV2}.$$

Whence $\beta_6=28^\circ\ 3'\ 7''.9$; log. $N_6^{IV}=8.6352986$.

$N_6\ =+0.0246773$;
$N_6'\ =+0.0166620$;
$N_6''=+0.0163874$;
$N_6'''=+0.0188199$;
$N_6^{IV}=+0.0431816$;
$N_6^{V}\ =+0.0340853$;
$N_6^{VI}=-0.0444971$;
$N_6^{VII}=+0.00154190$.

For the root $g_7=22''.4729002$, we get,

$$x_7=-\frac{1.010089}{10^{10}}N_7^{IV};\ y_7=+\frac{0.7868583}{10^{10}}N_7^{IV};\ z_7=\frac{81.93419}{10^{10}}N_7^{IV2}.$$

Whence $\beta_7=307^\circ\ 55'\ 6''.8$; log. $N_7^{IV}=8.1938807$.

$N_7\ =-0.0000972$;
$N_7'\ =+0.0003152$;
$N_7''=-0.0023732$;
$N_7'''=-0.0149913$;
$N_7^{IV}=+0.0156272$;
$N_7^{V}\ =-0.0483433$;
$N_7^{VI}=+0.0018046$;
$N_7^{VII}=+0.0001357$.

40. We shall now suppose $\mu^{VII}=+\frac{1}{10}$; and the mass of *Neptune* will become, $m^{VII}=\frac{1+\frac{1}{10}}{18780}=\frac{1}{17072.73}$. Using this value of *Neptune's* mass, we shall find,

$$\left.\begin{array}{ll}\Delta\boxed{0,0}=-0''.0000460; & \Delta\boxed{4,4}=-0''.0024017;\\ \Delta\boxed{1,1}=-0\ .0001177; & \Delta\boxed{5,5}=-0\ .0068740;\\ \Delta\boxed{2,2}=-0\ .0001914; & \Delta\boxed{6,6}=-0\ .04333257;\\ \Delta\boxed{3,3}=-0\ .0003611; & \Delta\boxed{7,7}=\ \ 0\ .\end{array}\right\}\quad(345)$$

Whence we get,

$$\left.\begin{array}{ll}\boxed{0,0}=g-\ \ 5''.5703018; & \boxed{4,4}=g-\ \ 7''.5147771;\\ \boxed{1,1}=g-11\ .3148859; & \boxed{5,5}=g-18\ .6030869;\\ \boxed{2,2}=g-13\ .0723644; & \boxed{6,6}=g-\ \ 2\ .8095779;\\ \boxed{3,3}=g-17\ .5532256; & \boxed{7,7}=g-\ \ 0\ .6479569.\end{array}\right\}\quad(346)$$

From these quantities we get the following equations,

$$\left.\begin{aligned}
\boxed{0,0}\boxed{1,1}&=g^2-16.8851877.g+63.0273292956;\\
\boxed{0,0}\boxed{2,2}&=g^2-18.6426662.g+72.8170149476;\\
\boxed{0,0}\boxed{3,3}&=g^2-23.1235274.g+97.7767641555;\\
\boxed{1,1}\boxed{2,2}&=g^2-24.3872503.g+147.9123116292;\\
\boxed{1,1}\boxed{3,3}&=g^2-28.8681115.g+198.6127448410;\\
\boxed{2,2}\boxed{3,3}&=g^2-30.6255900.g+229.4621614386;
\end{aligned}\right\}\quad(347)$$

$$\left.\begin{aligned}
\boxed{4,4}\boxed{5,5}&=g^2-26.1178640.g+139.7980514254;\\
\boxed{4,4}\boxed{6,6}&=g^2-10.3243550.g+21.1133516636;\\
\boxed{4,4}\boxed{7,7}&=g^2-8.1627340.g+4.8692516739;\\
\boxed{5,5}\boxed{6,6}&=g^2-21.4126648.g+52.2668218260;\\
\boxed{5,5}\boxed{7,7}&=g^2-19.2510438.g+12.0539985182;\\
\boxed{6,6}\boxed{7,7}&=g^2-3.4575348.g+1.82048538639.
\end{aligned}\right\}\quad(348)$$

$$\left.\begin{aligned}
\boxed{0,0}\boxed{1,1}\boxed{2,2}\boxed{3,3}=g^4-\quad 47.5107777.g^3+\quad 809.60832632.g^2\\
-5804.7608117.g+14462.38720986;
\end{aligned}\right\}\quad(349)$$

$$\left.\begin{aligned}
\boxed{4,4}\boxed{5,5}\boxed{6,6}\boxed{7,7}=g^4-\ 29.5753988.g^3+231.92196050.g^2\\
-530.90381750.g+254.50030966.
\end{aligned}\right\}\quad(350)$$

We shall therefore obtain the following

Fundamental Equations for $\mu^{VII}=+\frac{1}{10}$; *or for* $m^{VII}=\frac{1}{17072.73}$.

$$\left.\begin{aligned}
A\ &=g^2-40.22202062.g\quad+193.3362269;\\
A'&=g^2-23.14625587.g\quad+98.0038404;\\
A''&=g^2-18.02242768.g\quad+69.17743373;\\
A_1&=g^2-14.639503502.g+46.4149812;\\
A_2&=g^2-10.01320843.g\quad+6.38683557;\\
A_3&=g^2-26.192224114.g+83.01495699;
\end{aligned}\right\}\quad(351)$$

$$\left.\begin{aligned}
D\ &=g^2-47.859570167.g+700.430559;\\
D'&=g^2-54.9219773.g\qquad+656.151042;\\
D''&=g^2-31.56849039.g\quad+245\ 7386441;\\
D_1&=g^2-48.14983849.g\quad+203.402186;\\
D_2&=g^2-51.063762837.g+34.5618232;\\
D_3&=g^2-3.4620415785.g+1.7536927202.
\end{aligned}\right\}\quad(352)$$

$$\left.\begin{aligned}
B=\{g-34.6347594\}b;\quad B'&=\{g-17''.567588907\}b;\\
B''&=\{g-12\ .451385272\}b;
\end{aligned}\right\}\quad(353)$$

$$\left.\begin{aligned}
C\ &=\{23.7079988\ -g\}[9.1763990]b';\\
C'&=\{17.58234538-g\}[8.8694654]b';\\
C''&=-[0.2445917]b';\\
C'''&=-[0.2654598]b';
\end{aligned}\right\}\quad(354)$$

$$\left.\begin{aligned}
E\ &=+[9.6231944]b'';\\
E'&=\{24.1515714\ -g\}[8.5847028]b'';\\
E''&=\{17.55449726-g\}[9.6375865]b'';\\
E'''&=+[0.9518913]b'';
\end{aligned}\right\}\quad(355)$$

$$\left.\begin{aligned}
F\ &=-[7.6499091]b''';\\
F'&=+[8.9577383]b''';\\
F''&=\{14.01399313-g\}[0.8407439]b''';\\
F'''&=\{37.3396319\ -g\}[9.4809266]b''';
\end{aligned}\right\}\quad(356)$$

$$B_1=\{g-4.8301237\}b_1;\quad \left.\begin{aligned}B_2&=\{g-\ 0.682733393\}b_1;\\ B_3&=\{g-18.672437912\}b_1;\end{aligned}\right\}\quad(357)$$

$$\left.\begin{aligned}
C_1&=\ \{4.408417144\ -g\}[9.2840950]b_2;\\
C_2&=\ \{0.6919059696-g\}[9.1824311]b_2;\\
C_3&=-[0.6832317]b_2;\\
C_4&=-[0.6834824]b_2;
\end{aligned}\right\}\quad(358)$$

$$\left.\begin{aligned}
E_1&=+[7.8267723]b_3;\\
E_2&=\{43.7414213\ \ -g\}[8.2017618]b_3;\\
E_3&=\{\ 0.648008841-g\}[0.7610455]b_3;\\
E_4&=+[1.0655652]b_3;
\end{aligned}\right\}\quad(359)$$

$$\left.\begin{aligned}
F_1&=-[7.0753401]b_4;\\
F_2&=+[0.3138822]b_4;\\
F_3&=\{\ 2.813958738-g\}[1.7737962]b_4;\\
F_4&=\{50.37185686\ -g\}[9.1128486]b_4.
\end{aligned}\right\}\quad(360)$$

$$\left.\begin{aligned}&g^4-47.5107777.g^3+784.55104690.g^2\\ &-5224.5598798.g+11982.99308371\end{aligned}\right\}=(\chi,\chi_1,\chi_2,\chi_3);\quad(361)$$

$$\left.\begin{aligned}&g^4-29.5753988.g^3+174.01843537.g^2\\ &-327.77137981.g+142.39163768\end{aligned}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7).\quad(362)$$

The values of b_1, b_2, b_3, and b_4 are given by equations (119); and the values of b, b', b'', and b''' are given by equations (118), by simply adding log. $(1+\mu^{VII})=$ [0.0413927], to the coefficients of N^{VII}.

Putting equations (361) and (362), equal to nothing, they will give,

$$\left.\begin{aligned}
g&=\ 5''.463758326; & g_4&=\ 0''.6142757037;\\
g_1&=\ 7\ .247841113; & g_5&=\ 2\ .770720524;\\
g_2&=17\ .014463042; & g_6&=\ 3\ .723848045;\\
g_3&=17\ .784715218; & g_7&=22\ .466554527.
\end{aligned}\right\}\quad(363)$$

The equations just computed will now give the following values:

For the root g, we get,

$g=5''.4638549$;
$N' =+0.04864067N$ log. 98.6869996;
$N'' =+0.03104168N$ “ 98.4919453;
$N''' =+0.004766193N$ “ 97.6781717;
$N^{\mathrm{IV}} =-0.00005107290N$ “ 95.7081905n;
$N^{\mathrm{V}} =-0.0000455608N$ “ 95.6585913n;
$N^{\mathrm{VI}} =+0.00002028325N$ “ 95.3071375;
$N^{\mathrm{VII}} =+0.0000003449032N$ “ 93.5376972.

For the root g_1, we get,

$g_1=7''.2485708$;
$N_1' =-0.7476334N_1$ log. 9.8746886n;
$N_1'' =-0.5714480N_1$ “ 9.7569768n;
$N_1''' =-0.09467274N_1$ “ 8.9762250n;
$N_1^{\mathrm{IV}} =+0.0003964014N_1$ “ 6.5981351;
$N_1^{\mathrm{V}} =+0.000404752N_1$ “ 6.6071885;
$N_1^{\mathrm{VI}} =-0.0001031014N_1$ “ 6.0132647n;
$N_1^{\mathrm{VII}} =-0.00000414784N_1$ “ 4.6178221n.

For the root g_2, we get,

$g_2=17''.0145899$;
$N_2' =-\ 7.644900N_2$ log. 0.8833718n;
$N_2'' =+\ 7.708750N_2$ “ 0.8869839;
$N_2''' =+15.37999N_2$ “ 1.1869559;
$N_2^{\mathrm{IV}} =-\ 0.0007239023N_2$ “ 6.8596800n;
$N_2^{\mathrm{V}} =-\ 0.004358588N_2$ “ 7.6393458n;
$N_2^{\mathrm{VI}} =+\ 0.0002678406N_2$ “ 6.4278763;
$N_2^{\mathrm{VII}} =+\ 0.0000195659N_2$ “ 5.2924994.

For the root g_3, we get,

$g_3=17''.7847617$;
$N_3' =-\ 8.277515N_3$ log. 0.9179000n;
$N_3'' =+10.20779N_3$ “ 1.0089319;
$N_3''' =-49.63566N_3$ “ 1.6957938n;
$N_3^{\mathrm{IV}} =+\ 0.0006721358N_3$ “ 96.8274570;
$N_3^{\mathrm{V}} =+\ 0.006905492N_3$ “ 97.8391946;
$N_3^{\mathrm{VI}} =-\ 0.0003936316N_3$ “ 96.5950900n;
$N_3^{\mathrm{VII}} =-\ 0.00002907741N_3$ “ 95.4635558n.

For the root g_4, we get,

$g_4=0''.6142754;$

$N_4 \;=+\;\;0.1212222N_4^{IV}$ log. 9.0835822;
$N_4' \;=+\;\;0.1842843N_4^{IV}$ “ 9.2654883;
$N_4'' \;=+\;\;0.2134764N_4^{IV}$ “ 9.3293498;
$N_4''' =+\;\;0.3454170N_4^{IV}$ “ 9.5383437;
$N_4^{V} =+\;\;1.127956N_4^{IV}$ “ 0.0522920;
$N_4^{VI}=+\;24.27190N_4^{IV}$ “ 1.3851038;
$N_4^{VII}=+145.26667N_4^{IV}$ “ 2.1621660.

For the root g_5, we get,

$g_5=2''.7706502;$

$N_5 \;=+\;0.2995950N_5^{IV}$ log. 9.4765344;
$N_5' \;=+\;0.2898491N_5^{IV}$ “ 9.4621720;
$N_5'' \;=+\;0.3027071N_5^{IV}$ “ 9.4810226;
$N_5''' =+\;0.4009023N_5^{IV}$ “ 9.6030386;
$N_5^{V} =+\;0.9054583N_5^{IV}$ “ 9.9568685;
$N_5^{VI}=+14.64440N_5^{IV}$ “ 1.1656714;
$N_5^{VII}=-\;1.406558N_5^{IV}$ “ 0.1481577n.

For the root g_6, we get,

$g_6=3''.7226566;$

$N_6 \;=+0.5703212N_6^{IV}$ log. 9.7561195;
$N_6' \;=+0.3855324N_6^{IV}$ “ 9.5860609;
$N_6'' \;=+0.3792427N_6^{IV}$ “ 9.5789172;
$N_6''' =+0.4357298N_6^{IV}$ “ 9.6392173;
$N_6^{V} =+0.7895770N_6^{IV}$ “ 9.8973944;
$N_6^{VI}=-1.083630N_6^{VI}$ “ 0.0348810n;
$N_6^{VII}=+0.0356831N_6^{IV}$ “ 8.5524627.

For the root g_7, we get,

$g_7=22''.4668172;$

$N_7 \;=-0.006229914N_7^{IV}$ log. 7.7944821n;
$N_7' \;=+0.02023760N_7^{IV}$ “ 8.3061590;
$N_7'' \;=-0.1519652N_7^{IV}$ “ 9.1817441n;
$N_7''' =-0.9604665N_7^{IV}$ “ 9.9824823n;
$N_7^{V} =-3.092542N_7^{IV}$ “ 0.4903156n;
$N_7^{VI}=+0.1157082N_7^{IV}$ “ 9.0633640;
$N_7^{VII}=+0.008726764N_7^{IV}$ “ 7.9408532.

41. These numbers are now to be substituted in equations (134) and (135). We must also add the logarithm of $1+\mu^{VII}$, to those of $\frac{m^{VII}}{n^{VII}a^{VII}}$, $\frac{m^{VII}}{n^{VII}a^{VII}}h^{VII}$, and $\frac{m^{VII}}{n^{VII}a^{VII}}l^{VII}$, in order to obtain the numbers which are to be used in this computation.

For the root $g=5''.463855$, we get,

$$x=+\frac{1847131}{10^{20}}N;\quad y=+\frac{63087.5}{10^{20}}N;\quad z=\frac{10467693}{10^{20}}N^2.$$

Whence $\beta=88^\circ\ 2'\ 37''.9$; log. $N=9.2468998$.

$N\ =+0.1765630;$
$N'\ =+0.0085881;$
$N''=+0.0054808;$
$N'''=+0.00084153;$
$N^{IV}=-0.0000090176;$
$N^{V}\ =-0.0000080444;$
$N^{VI}=+0.0000035813;$
$N^{VII}=+0.0000000609.$

For the root $g_1=7''.248571$, we get,

$$x_1=+\frac{1656245}{10^{20}}N_1;\quad y_1=+\frac{4351968}{10^{20}}N_1;\quad z_1=+\frac{173055600}{10^{20}}N_1^2.$$

Whence $\beta_1=20^\circ\ 50'\ 7''.8$; log. $N_1=8.4298727$.

$N_1\ =+0.0269075;$
$N_1'\ =-0.0201633;$
$N_1''=-0.0153761;$
$N_1'''=-0.0025474;$
$N_1^{IV}=+0.000010666;$
$N_1^{V}\ =+0.000010891;$
$N_1^{VI}=-0.00002774;$
$N_1^{VII}=-0.0000001116.$

For the root $g_2=17''.014590$, we get,

$$x_2=-\frac{18874270}{10^{20}}N_2;\quad y_2=+\frac{40848060}{10^{20}}N_2;\quad z_2=\frac{3.068734}{10^{10}}N_2^2.$$

Whence $\beta_2=335^\circ\ 12'\ 0''.8'$; log. $N_2=7.1662319$.

$N_2\ =+0.0014663;$
$N_2'\ =-0.0112099;$
$N_2''=+0.0113035;$
$N_2'''=+0.0225521;$
$N_2^{IV}=-0.0000010862;$
$N_2^{V}\ =-0.000006540;$
$N_2^{VI}=+0.0000004019;$
$N_2^{VII}=+0.0000000294.$

For the root $g_3=17''.784762$, we get,

$$x_3=+\frac{131051610}{10^{20}}N_3;\quad y_3=-\frac{141079200}{10^{20}}N_3;\quad z_3=\frac{12.088926}{10^{10}}N_3^2.$$

Whence $\beta_3=137^\circ\ 6'\ 45''.3$; log. $N_3=7.2021538$.

$N_3\ =+0.0015928;$
$N_3'\ =-0.0131842;$
$N_3''=+0.0162587;$
$N_3'''=-0.0790583;$
$N_3^{IV}=+0.0000010705;$
$N_3^{V}\ =+0.000010999;$
$N_3^{VI}=-0.000000627;$
$N_3^{VII}=-0.0000000463.$

For the root $g_4=0''.6142754$, we get,

$$x_4=+\frac{3.385189}{10^{10}}N_4^{IV};\quad y_4=+\frac{1.399525}{10^{10}}N_4^{IV};\quad z_4=\frac{53091.32}{10^{10}}N_4^{IV2}.$$

Whence $\beta_4=67^\circ\ 32'\ 18''.7$; log. $N_4^{IV}=95.8388230$

$N_4\ =+0.000008364;$
$N_4'\ =+0.000012715;$
$N_4''=+0.000014729;$
$N_4'''=+0,000023832;$
$N_4^{IV}=+0.000068996;$
$N_4^{V}\ =+0.000077824;$
$N_4^{VI}=+0.00167466;$
$N_4^{VII}=+0.01002282.$

For the root $g_5 = 2''.7706502$, we get,

$$x_5 = +\frac{0.6382110}{10^{10}} N_5^{IV};\quad y_5 = -\frac{0.1345391}{10^{10}} N_5^{IV};\quad z_5 = \frac{318.2862}{10^{10}} N_5^{IV2}.$$

Whence $\beta_5 = 101° \; 54' \; 14''.5$; log. $N_5^{IV} = 7.3115881$.

$N_5 = +0.00061393$; $\qquad N_5^{IV} = +0.00204922$;
$N_5' = +0.00059396$; $\qquad N_5^{V} = +0.00185548$;
$N_5'' = +0.00062031$; $\qquad N_5^{VI} = +0.0300096$;
$N_5''' = +0.00082154$; $\qquad N_5^{VII} = -0.00288234$.

For the root $g_6 = 3''.722656$, we get,

$$x_6 = \frac{0.4577640}{10^{10}} N_6^{IV};\quad y_6 = +\frac{0.8595159}{10^{10}} N_6^{IV};\quad z_6 = +\frac{22.63367}{10^{10}} N_6^{IV2}.$$

Whence $\beta_6 = 28° \; 2' \; 20''.5$; log. $N_6^{IV} = 8.6337215$.

$N_6 = +0.0245381$; $\qquad N_6^{IV} = +0.0430251$;
$N_6' = +0.0165876$; $\qquad N_6^{V} = +0.0339716$;
$N_6'' = +0.0163169$; $\qquad N_6^{VI} = -0.0466232$;
$N_6''' = +0.0187473$; $\qquad N_6^{VII} = -0.0015353$.

For the root $g_7 = 22''.466817$, we get,

$$x_7 = -\frac{1.009767}{10^{10}} N_7^{IV};\quad y_7 = +\frac{0.7872124}{10^{10}} N_7^{IV};\quad z_7 = +\frac{81.89131}{10^{10}} N_7^{IV2}.$$

Whence $\beta_7 = 307° \; 56' \; 23''.7$; log. $N_7^{IV} = 8.1940957$.

$N_7 = -0.0000974$; $\qquad N_7^{IV} = +0.0156349$;
$N_7' = +0.0003164$; $\qquad N_7^{V} = -0.0483516$;
$N_7'' = -0.0023759$; $\qquad N_7^{VI} = +0.0018091$;
$N_7''' = -0.0150168$; $\qquad N_7^{VII} = +0.0001364$.

42. We have thus obtained the values of all the constants, corresponding to the separate variations of the planetary masses. If we now subtract the values of the constants which correspond to the assumed masses from the values which result from the supposition that each planetary mass receives in succession a finite increment, and divide the difference of the constants by the supposed increment of mass, and connect together the different results, we shall have the following system of equations for the determination of the constants which correspond to any other assumed finite variation of the masses. The unit of the coefficients of μ, μ', μ'', &c., in the values of N, N', N'', &c., N_1, N_1', N_1'', &c., are the seventh decimal place of these coefficients.

$$\begin{aligned} g=&5^{\prime\prime}.463803-0^{\prime\prime}.079418\mu+2^{\prime\prime}.68094\mu^{\prime}+0^{\prime\prime}.91484\mu^{\prime\prime}+0^{\prime\prime}.034565\mu^{\prime\prime\prime}+1^{\prime\prime}.8040\mu^{\mathrm{IV}} \\ &+0^{\prime\prime}.08684\mu^{\mathrm{V}}+0^{\prime\prime}.00150\mu^{\mathrm{VI}}+0^{\prime\prime}.00052\mu^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} \beta=&88^{\circ}\ 0^{\prime}\ 38^{\prime\prime}-2939^{\prime\prime}\mu+50632^{\prime\prime}\mu^{\prime}-20688^{\prime\prime}\mu^{\prime\prime}-3815^{\prime\prime}\mu^{\prime\prime\prime}-73640^{\prime\prime}\mu^{\mathrm{IV}}+57456^{\prime\prime}\mu^{\mathrm{V}} \\ &+5236^{\prime\prime}\mu^{\mathrm{VI}}+1202^{\prime\prime}\mu^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N=&+0.1766064-50121\mu-420200\mu^{\prime}+205480\mu^{\prime\prime}+7970\mu^{\prime\prime\prime}+676800\mu^{\mathrm{IV}} \\ &-486480\mu^{\mathrm{V}}-16180\mu^{\mathrm{VI}}-4360\mu^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N^{\prime}=&+0.0085906+59050\mu+146200\mu^{\prime}-44980\mu^{\prime\prime}-4746\mu^{\prime\prime\prime}-124400\mu^{\mathrm{IV}}-31040\mu^{\mathrm{V}} \\ &-900\mu^{\mathrm{VI}}-250\mu^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N^{\prime\prime}=&+0.0054825+37020\mu+118060\mu^{\prime}-19180\mu^{\prime\prime}-4279\mu^{\prime\prime\prime}-108800\mu^{\mathrm{IV}}-21200\mu^{\mathrm{V}} \\ &-600\mu^{\mathrm{VI}}-170\mu^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N^{\prime\prime\prime}=&+0.0008418+5672\mu+20302\mu^{\prime}-2300\mu^{\prime\prime}-561.7\mu^{\prime\prime\prime}-23540\mu^{\mathrm{IV}}-3596\mu^{\mathrm{V}} \\ &-102\mu^{\mathrm{VI}}-29\mu^{\mathrm{VII}}; \end{aligned}$$

$$N^{\mathrm{IV}}=-0.0000090-72\mu-52\mu^{\prime}+27\mu^{\prime\prime}+3.5\mu^{\prime\prime\prime}+182\mu^{\mathrm{IV}}-86.4\mu^{\mathrm{V}}-6.8\mu^{\mathrm{VI}}-1.7\mu^{\mathrm{VII}};$$

$$N^{\mathrm{V}}=-0.0000080-63\mu-62\mu^{\prime}+19\mu^{\prime\prime}+2.9\mu^{\prime\prime\prime}+181\mu^{\mathrm{IV}}-78.4\mu^{\mathrm{V}}-4.0\mu^{\mathrm{VI}}-1.1\mu^{\mathrm{VII}};$$

$$N^{\mathrm{VI}}=+0.0000035+30\mu-9\mu^{\prime}-20\mu^{\prime\prime}-1.7\mu^{\prime\prime\prime}-76\mu^{\mathrm{IV}}-78.9\mu^{\mathrm{V}}+2.6\mu^{\mathrm{VI}}+6.4\mu^{\mathrm{VII}};$$

$$N^{\mathrm{VII}}=+0.0000000.6+0.4\mu+1.4\mu^{\prime}+0.2\mu^{\prime\prime}-0.0\mu^{\prime\prime\prime}-0.8\mu^{\mathrm{IV}}-0.2\mu^{\mathrm{V}}-1.4\mu^{\mathrm{VI}}-0.2\mu^{\mathrm{VII}}.$$

$$\begin{aligned} g_1=&7^{\prime\prime}.248427+0^{\prime\prime}.193515\mu+0^{\prime\prime}.08894\mu^{\prime}+1^{\prime\prime}33906\mu^{\prime\prime}+0^{\prime\prime}.155640\mu^{\prime\prime\prime}+5^{\prime\prime}.2076\mu^{\mathrm{IV}} \\ &+0^{\prime\prime}.2429\mu^{\mathrm{V}}+0^{\prime\prime}.00418\mu^{\mathrm{VI}}+0^{\prime\prime}.00144\mu^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} \beta_1=&20^{\circ}\ 50^{\prime}\ 19^{\prime\prime}+41718^{\prime\prime}\mu+171946^{\prime\prime}\mu^{\prime}-86900^{\prime\prime}\mu^{\prime\prime}+7848^{\prime\prime}\mu^{\prime\prime\prime}-221553^{\prime\prime}\mu^{\mathrm{IV}}+75188^{\prime\prime}\mu^{\mathrm{V}} \\ &-1396^{\prime\prime}\mu^{\mathrm{VI}}-115^{\prime\prime}\mu^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N_1=&+0.0268838+5871\mu+793180\mu^{\prime}-124700\mu^{\prime\prime}-29868\mu^{\prime\prime\prime}-658400\mu^{\mathrm{IV}} \\ &+104480\mu^{\mathrm{V}}+11600\mu^{\mathrm{VI}}+2370^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N_1^{\prime}=&-0.0201444-27607\mu-17180\mu^{\prime}+72020\mu^{\prime\prime}+8397\mu^{\prime\prime\prime}+62400\mu^{\mathrm{IV}}-98840\mu^{\mathrm{V}} \\ &-9120\mu^{\mathrm{VI}}-1890^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N_1^{\prime\prime}=&-0.0153619-25311\mu-25560\mu^{\prime}+18800\mu^{\prime\prime}+8920\mu^{\prime\prime\prime}+95500\mu^{\mathrm{IV}}-73200\mu^{\mathrm{V}} \\ &-6860\mu^{\mathrm{VI}}-1420^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N_1^{\prime\prime\prime}=&-0.0025451-4573\mu-8070\mu^{\prime}-14568\mu^{\prime\prime}+1018\mu^{\prime\prime\prime}+3730\mu^{\mathrm{IV}}-11212\mu^{\mathrm{V}} \\ &-1120\mu^{\mathrm{VI}}-231\mu^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N_1^{\mathrm{IV}}=&+0.0000106+9.3\mu+43.6\mu^{\prime}+10\mu^{\prime\prime}-4.4\mu^{\prime\prime\prime}-162\mu^{\mathrm{IV}}+99.4\mu^{\mathrm{V}}+9.6\mu^{\mathrm{VI}} \\ &+2.2\mu^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N_1^{\mathrm{V}}=&+0.0000109+11.4\mu+45.2\mu^{\prime}+22.2\mu^{\prime\prime}-3.1\mu^{\prime\prime\prime}-181\mu^{\mathrm{IV}}+104.8\mu^{\mathrm{V}}+6.8\mu^{\mathrm{VI}} \\ &+1.6\mu^{\mathrm{VII}}; \end{aligned}$$

$$\begin{aligned} N_1^{\mathrm{VI}}=&-0.0000027--14.6\mu-10.6\mu^{\prime}+3.6\mu^{\prime\prime}+1.8\mu^{\prime\prime\prime}+63\mu^{\mathrm{IV}}-54.4\mu^{\mathrm{V}}-1.8\mu^{\mathrm{VI}} \\ &-3.0\mu^{\mathrm{VII}}; \end{aligned}$$

$$N_1^{\mathrm{VII}}=-0.0000001-0.1\mu-0.5\mu^{\prime}-0.2\mu^{\prime\prime}+0.0\mu^{\prime\prime\prime}+1.5\mu^{\mathrm{IV}}-1.6\mu^{\mathrm{V}}+0.7\mu^{\mathrm{VI}}+0.1\mu^{\mathrm{VII}}.$$

$$\begin{aligned}g_2=&17''.014373+0''.075742\mu+3''.72988\mu'+4''.06318\mu''+0''.006792\mu'''+8''.0455\mu^{\mathrm{IV}}\\&+0''.36832\mu^{\mathrm{V}}+0''.00628\mu^{\mathrm{VI}}+0''.00217\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}\beta_2=&335^\circ\ 11'\ 31''+483''\mu-336590''\mu'-239676''\mu''-27647''\mu'''+753490''\mu^{\mathrm{IV}}+7692''\mu^{\mathrm{V}}\\&+802''\mu^{\mathrm{VI}}+294''\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_2=&+0.0014673+1200\mu+61784\mu'+47930\mu''+11002\mu'''-130700\mu^{\mathrm{IV}}-3160\mu^{\mathrm{V}}\\&-420\mu^{\mathrm{VI}}-90\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_2'=&-0.0112171-9383\mu-345400\mu'-339820\mu''-83919\mu'''+928300\mu^{\mathrm{IV}}+20560\mu^{\mathrm{V}}\\&+3020\mu^{\mathrm{VI}}+720\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_2''=&+0.0113105+6512\mu+410140\mu'+196140\mu''+77329\mu'''-842300\mu^{\mathrm{IV}}-16240\mu^{\mathrm{V}}\\&-2960\mu^{\mathrm{VI}}-700\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_2'''=&+0.0225719+49397\mu+3069020\mu'+1992300\mu''+155278\mu'''-3928400\mu^{\mathrm{IV}}\\&-138640\mu^{\mathrm{V}}-7660\mu^{\mathrm{VI}}+1980\mu^{\mathrm{VII}};\end{aligned}$$

$$N_2^{\mathrm{IV}}=-0.0000011-1.2\mu-77\mu'-43.4\mu''-18.1\mu'''+114^{\mathrm{IV}}+10.8\mu^{\mathrm{V}}-1.0\mu^{\mathrm{VI}}-2.7\mu^{\mathrm{VII}};$$

$$\begin{aligned}N_2^{\mathrm{V}}=&-0.0000064-10.4\mu-658\mu'-447.6\mu''-110.1\mu'''+930\mu^{\mathrm{IV}}+62\mu^{\mathrm{V}}+4.2\mu^{\mathrm{VI}}\\&-13.9\mu^{\mathrm{VII}};\end{aligned}$$

$$N_2^{\mathrm{VI}}=+0.0000004-0.6\mu+38\mu'+25.4\mu''+6.7\mu'''-58\mu^{\mathrm{IV}}+0.0\mu^{\mathrm{V}}-0.2\mu^{\mathrm{VI}}+1.0\mu^{\mathrm{VII}};$$

$$N_2^{\mathrm{VII}}=+0.0000000.3+0.0\mu+2.8\mu'+1.9\mu''+0.5\mu'''-4.2\mu^{\mathrm{IV}}+0.0\mu^{\mathrm{V}}-0.1\mu^{\mathrm{VI}}+0.1^{\mathrm{VII}}.$$

$$\begin{aligned}g_3=&17''.784456+0''.034383\mu+2''.31316\mu'+2''.93202\mu''+0.231407\mu'''+12''.4768\mu^{\mathrm{IV}}\\&+0''.53576\mu^{\mathrm{V}}+0''.00892\mu^{\mathrm{VI}}+0''.00306\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}\beta_3=&137^\circ\ 6'\ 36''.5-4099''\mu-113430''\mu'-73350''\mu''+352''\mu'''+143360''\mu^{\mathrm{IV}}+3904''\mu^{\mathrm{V}}\\&-176''\mu^{\mathrm{VI}}+88''\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_3=&+0.0015934+1401\mu+63100\mu'+48280\mu''+8277\mu'''-129900\mu^{\mathrm{IV}}-4560\mu^{\mathrm{V}}\\&-220\mu^{\mathrm{VI}}-60\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_3'=&-0.0131892-11200\mu-351120\mu'-410400\mu''-73086\mu'''+945000\mu^{\mathrm{IV}}\\&+31800\mu^{\mathrm{V}}+1740\mu^{\mathrm{VI}}+500\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_3''=&+0.0162641+8420\mu+440180\mu'+185420\mu''+95264\mu'''-880700\mu^{\mathrm{IV}}\\&-26920\mu^{\mathrm{V}}-1940^{\mathrm{VI}}-540\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_3'''=&-0.0790650+46942\mu+3069920\mu'+1932860\mu''+161058\mu'''-3813200\mu^{\mathrm{IV}}\\&-209680\mu^{\mathrm{V}}+3740\mu^{\mathrm{VI}}+670\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_3^{\mathrm{IV}}=&+0.0000011-1.3\mu-80.4\mu'-63.4\mu''+3.2\mu'''+128.9\mu^{\mathrm{IV}}-5.4\mu^{\mathrm{V}}+2.2\mu^{\mathrm{VI}}\\&+0.5\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_3^{\mathrm{V}}=&+0.0000110-10.3\mu-662.2\mu'-413\mu''+71\mu'''+823^{\mathrm{IV}}-207\mu^{\mathrm{V}}-4.8\mu^{\mathrm{VI}}\\&-1.1\mu^{\mathrm{VII}};\end{aligned}$$

$$\begin{aligned}N_3^{\mathrm{VI}}=&-0.0000006+0.6\mu+38.6\mu'+24.9\mu''-3.8\mu'''-42.7\mu^{\mathrm{IV}}-4.8\mu^{\mathrm{V}}+0.2\mu^{\mathrm{VI}}\\&-0.1\mu^{\mathrm{VII}};\end{aligned}$$

$$N_3^{\mathrm{VII}}=-0.0000000.5+0.0\mu+2.8\mu'+1.8\mu''-0.3\mu'''-3.3\mu^{\mathrm{IV}}-0.3\mu^{\mathrm{V}}+0.1\mu^{\mathrm{VI}}+0.0\mu^{\mathrm{VII}}.$$

$$g_4=0''.6166849+0\mu+0\mu'+0''.00002\mu''+0''.000007\mu'''+0''.1905\mu^{IV}+0''.22384\mu^{V}+0''.22638\mu^{VI}-0''.0241\mu^{VII};$$

$$\beta_4=67^\circ\ 56'\ 35''+16''\mu+34''\mu'+78''\mu''+16''\mu'''-50700''\mu^{IV}+25000''\mu^{V}+79500''\mu^{VI}-14560''\mu^{VII};$$

$$N_4\ =+0.0000077-\ 1\mu-\ 6\mu'-\ 4\mu''+\ 0\mu'''-\ 58\mu^{IV}-\ 44\mu^{V}+\ 50\mu^{VI}+\ 65\mu^{VII};$$
$$N_4'\ =+0.0000117-\ 5\mu-11\mu'-\ 4\mu''+\ 0\mu'''-\ 83\mu^{IV}-\ 70\mu^{V}+\ 17\mu^{VI}+100\mu^{VII};$$
$$N_4''\ =+0.0000136-\ 1\mu-20\mu'-\ 2\mu''+\ 1\mu'''-\ 90\mu^{IV}-\ 83\mu^{V}+\ 82\mu^{VI}+115\mu^{VII};$$
$$N_4'''\ =+0.0000219-\ 0\mu-\ 2\mu'-\ 8\mu''+\ 0\mu'''-136\mu^{IV}-128\mu^{V}+134\mu^{VI}+189\mu^{VII};$$
$$N_4^{IV}=+0.0000636+\ 0\mu+\ 0\mu'-\ 1\mu''-\ 1\mu'''-487\mu^{IV}-400\mu^{V}+368\mu^{VI}+541\mu^{VII};$$
$$N_4^{V}\ =+0.0000717+\ 0\mu+\ 0\mu'-\ 0\mu''-\ 0\mu'''-682\mu^{IV}-318\mu^{V}+408\mu^{VI}+612\mu^{VII};$$
$$N_4^{VI}=+0.0015578+\ 3\mu-\ 4\mu'+12\mu''-\ 3\mu'''-6250\mu^{IV}-7476\mu^{V}+2058\mu^{VI}+11691\mu^{VII};$$
$$N_4^{VII}=+0.0100389+17\mu-22\mu'+84\mu''-15\mu'''-4110\mu^{IV}+5388\mu^{V}+1288\mu^{VI}-\ 1611\mu^{VII}.$$

$$g_5=2''.727659+0''.000019\mu+0''.00026\mu'+0''.00050\mu''+0''.00016\mu'''+0''.9302\mu^{IV}+1''.38608\mu^{V}-0''.02282\mu^{VI}+0''.42991\mu^{VII};$$

$$\beta_5=105^\circ\ 3'\ 53''+19''\mu+940''\mu'+1860''\mu''+566''\mu'''-192300''\mu^{IV}+236840''\mu^{V}+45040''\mu^{VI}-113780''\mu^{VII};$$

$$N_5\ =+0.0005685-50\mu-2656\mu'-1488\mu''-57\mu'''-3612\mu^{IV}+322\mu^{V}+2844\mu^{VI}+4539\mu^{VII};$$

$$N_5'\ =+0.0005571-65\mu-620\mu'-822\mu''-30\mu'''-3436\mu^{IV}-1881\mu^{V}+2850\mu^{VI}+3686\mu^{VII};$$

$$N_5''\ =+0.0005832-41\mu-896\mu'-394\mu''-19\mu'''-3314\mu^{IV}-2392\mu^{V}+2968\mu^{VI}+3708\mu^{VII};$$

$$N_5'''\ =+0.0007765-9\mu-272\mu'-498\mu''-22\mu'''-3807\mu^{IV}-4428\mu^{V}+3976\mu^{VI}+4499\mu^{VII};$$

$$N_5^{IV}=+0.0019436-1\mu-80\mu'-160\mu''-55\mu'''-8674\mu^{IV}-13008\mu^{V}+9994\mu^{VI}+10560\mu^{VII};$$

$$N_5^{V}\ =+0.0017694-1\mu-62\mu'-124\mu''-43\mu'''-11920\mu^{IV}-7172\mu^{V}+9256\mu^{VI}+8608\mu^{VII};$$

$$N_5^{VI}=+0.0297330+1\mu-240\mu'-420\mu''-182'''-75800\mu^{IV}+101280\mu^{V}-90840\mu^{VI}+27660\mu^{VII};$$

$$N_5^{VII}=+0.0029105+0\mu+28\mu'+54\mu''+22\mu'''+17870\mu^{IV}+6676\mu^{V}-23736\mu^{VI}+2816\mu^{VII}.$$

$$g_6=3''.716607+0''.000071\mu+0''.00318\mu'+0''.00632\mu''+0''.002015\mu'''+0''.6700\mu^{IV}\quad 2''.78408\mu^{V}+0''.17048\mu^{VI}+0''.06049\mu^{VII};$$

$$\beta_6=28^\circ\ 8'\ 46''+19''\mu+24''\mu'+90''\mu''-0''.5\mu'''-61400''\mu^{IV}+73080''\mu^{V}-6760''\mu^{VI}-3850''\mu^{VII};$$

$$N_6\ =+0.0244939-642\mu-233400\mu'-121880\mu''-4595\mu'''-193700\mu^{IV}+558800\mu^{V}\quad 36680\mu^{VI}+4420\mu^{VII};$$

$$N_6'\ =+0.0166053-531\mu-21540\mu'-41360\mu''-1758\mu'''-88500\mu^{IV}+149720\mu^{V}+11340\mu^{VI}+1770\mu^{VII};$$

$$N_6'' = +0.0163413 - 454\mu - 24920\mu' - 20000\mu'' - 1358\mu''' - 72000\mu^{\mathrm{IV}} + 115600\mu^{\mathrm{V}} \\ +9220\mu^{\mathrm{VI}} - 2440\mu^{\mathrm{VII}};$$

$$N_6''' = +0.0187954 - 104\mu - 6080\mu' - 10600\mu'' - 110\mu''' - 23900\mu^{\mathrm{IV}} + 40840\mu^{\mathrm{V}} \\ +4900\mu^{\mathrm{VI}} - 4810\mu^{\mathrm{VII}};$$

$$N_6^{\mathrm{IV}} = +0.0431601 + 31\mu + 40\mu' + 140\mu'' + 78\mu''' + 24500\mu^{\mathrm{IV}} - 17080\mu^{\mathrm{V}} + 4300\mu^{\mathrm{VI}} \\ -13500\mu^{\mathrm{VII}};$$

$$N_6^{\mathrm{V}} = +0.0341011 + 26\mu + 100\mu' + 260\mu'' + 111\mu''' - 39300\mu^{\mathrm{IV}} + 53800\mu^{\mathrm{V}} - 3160\mu^{\mathrm{VI}} \\ -12950\mu^{\mathrm{VII}};$$

$$N_6^{\mathrm{VI}} = -0.0448614 - 0\mu + 1380\mu' + 2680\mu'' + 834\mu''' - 236100\mu^{\mathrm{IV}} + 314360\mu^{\mathrm{V}} \\ +72860\mu^{\mathrm{VI}} - 176180\mu^{\mathrm{VII}};$$

$$N_6^{\mathrm{VII}} = +0.0014205 - 1\mu - 100\mu' - 200\mu'' - 66\mu''' + 8800\mu^{\mathrm{IV}} - 41440\mu^{\mathrm{V}} + 24280\mu^{\mathrm{VI}} \\ +11480\mu^{\mathrm{VII}}.$$

$$g_7 = 22''.460848 + 0''.000029\mu + 0''.00130\mu' + 0''.00274\mu'' + 0''.001098\mu''' + 17''.5730\mu^{\mathrm{IV}} \\ 4''.60528\mu^{\mathrm{V}} + 0''.24104\mu^{\mathrm{VI}} + 0''.05969\mu^{\mathrm{VII}};$$

$$\beta_7 - 307^\circ\ 56'\ 50'' + 0''\mu + 20''\mu' + 42''\mu'' + 2''.7\mu''' - 19800''\mu^{\mathrm{IV}} + 22120''\mu^{\mathrm{V}} - 2060''\mu^{\mathrm{VI}} \\ -265''\mu^{\mathrm{VII}}$$

$$N_7 = -0.0000975 - 13\mu - 1280\mu' - 1360\mu'' + 97\mu''' + 2300\mu^{\mathrm{IV}} - 160\mu^{\mathrm{V}} + 60\mu^{\mathrm{VI}} \\ +10\mu^{\mathrm{VII}};$$

$$N_7' = +0.0003175 + 147\mu + 10160\mu' + 18400\mu'' - 911\mu''' - 20800\mu^{\mathrm{IV}} - 4520\mu^{\mathrm{V}} \\ -460\mu^{\mathrm{VI}} - 110\mu^{\mathrm{VII}};$$

$$N_7'' = -0.0023780 - 167\mu - 20400\mu' - 7560\mu'' + 3508\mu''' + 33100\mu^{\mathrm{IV}} - 11480\mu^{\mathrm{V}} \\ +960\mu^{\mathrm{VI}} + 210\mu^{\mathrm{VII}};$$

$$N_7''' = -0.0150371 - 201\mu - 10040\mu' - 46960\mu'' - 867\mu''' + 74100\mu^{\mathrm{IV}} - 26520\mu^{\mathrm{V}} \\ +9160\mu^{\mathrm{VI}} + 2030\mu^{\mathrm{VII}};$$

$$N_7^{\mathrm{IV}} = +0.0156383 + 1\mu + 40\mu' + 60\mu'' - 14\mu''' - 146000\mu^{\mathrm{IV}} + 149520\mu^{\mathrm{V}} - 2220\mu^{\mathrm{VI}} \\ -340\mu^{\mathrm{VII}};$$

$$N_7^{\mathrm{V}} = -0.0483504 - 2\mu + 40\mu' + 0\mu'' + 142\mu''' - 111800\mu^{\mathrm{IV}} + 110840\mu^{\mathrm{V}} + 1420\mu^{\mathrm{VI}} \\ -120\mu^{\mathrm{VII}};$$

$$N_7^{\mathrm{VI}} = +0.0018058 + 0\mu + 0\mu' + 0\mu'' - 7\mu''' - 10600\mu^{\mathrm{IV}} + 10360\mu^{\mathrm{V}} - 240\mu^{\mathrm{VI}} + 330\mu^{\mathrm{VII}};$$

$$N_7^{\mathrm{VII}} = +0.0001365 + 0\mu + 0\mu' + 0\mu'' - 1\mu''' - 600\mu^{\mathrm{IV}} + 760\mu^{\mathrm{V}} - 160\mu^{\mathrm{VI}} - 10\mu^{\mathrm{VII}}.$$

CHAPTER II.

ON THE SECULAR VARIATIONS OF THE NODES AND INCLINATIONS OF THE ORBITS.

1. THE secular variations of the nodes, and the inclinations of the orbits, are determined by the integration of a system of differential equations which are entirely similar in form to those from which the eccentricities and perihelia were obtained.

If we denote by ϕ, ϕ', ϕ'', &c., the inclinations, and by θ, θ', θ'', &c., the longitudes of the nodes of the planets, *Mercury*, *Venus*, the *Earth*, &c., and put

$$\left.\begin{array}{l} \tan\phi\sin\theta=p,\ \ \tan\phi'\sin\theta'=p'\ \ \tan\phi''\sin\theta''=p''\ \&c., \\ \tan\phi\cos\theta=q,\ \ \tan\phi'\cos\theta'=q'\ \ \tan\phi''\cos\theta''=q''\ \&c.; \end{array}\right\} (364)$$

we shall have the following system of differential equations for the determination of p, p', p'', &c., q, q', q'', &c.

$$\left.\begin{array}{l} \dfrac{dq}{dt}=\ \ \left\{(0,1)+(0,2)+(0,3)+\&c.\right\}p-(0,1)p'-(0,2)p''-(0,3)p'''-\&c., \\ \dfrac{dp}{dt}=-\left\{(0,1)+(0,2)+(0,3)+\&c.\right\}q+(0,1)q'+(0,2)q''+(0,3)q'''+\&c., \\ \dfrac{dq'}{dt}=\ \ \left\{(1,0)+(1,2)+(1,3)+\&c.\right\}p'-(1,0)p-(1,2)p''-(1,3)p'''-\&c., \\ \dfrac{dp'}{dt}=-\left\{(1,0)+(1,2)+(1,3)+\&c.\right\}q'+(1,0)q+(1,2)q''+(1,3)q'''+\&c., \\ \dfrac{dq''}{dt}=\ \ \left\{(2,0)+(2,1)+(2,3)+\&c.\right\}p''-(2,0)p-(2,1)p'-(2,3)p'''-\&c., \\ \dfrac{dp''}{dt}=-\left\{(2,0)+(2,1)+(2,3)+\&c.\right\}q''+(2,0)q+(2,1)q'+(2,3)q'''-\&c. \\ \&c. \end{array}\right\} (E)$$

To integrate these equations, we shall suppose

$$\left.\begin{array}{l} p=N\sin(gt+\beta),\quad p'=N'\sin(gt+\beta),\quad p''=N''\sin(gt+\beta),\ \&c., \\ q=N\cos(gt+\beta),\quad q'=N'\cos(gt+\beta),\quad q''=N''\cos(gt+\beta),\ \&c. \end{array}\right\} (365)$$

If we substitute these values of p, q, p', q', &c., in equations (E), they will become

$$\left.\begin{array}{l} -gN\ =\}(0,1)+(0,2)+0,3+\&c.\}N\ -(0,1)N'-(0,2)N''-(0,3)N'''-\&c.; \\ -gN'=\}(1,0)+(1,2)+1,3+\&c.\}N'-(1,0)N\ -(1,2)N''-(1,3)N'''-\&c.; \\ -gN''=\}(2,0)+(2,1)+2,3+\&c.\}N''-(2,0)N\ -(2,1)N'\ -(2,3)N'''-\&c.; \\ \&c. \end{array}\right\} (E')$$

These equations are similar in form to equations (B), except that g is negative. They will produce, by eliminating N', N'', &c., an equation in g, of the eighth degree.

One of the roots of this equation will evidently be equal to nothing, since equations (E′) will be satisfied by supposing $g=0$, and $N=N'=N''$, &c.

2. We shall now suppose,

$$\left.\begin{array}{l}(0,0)=g+(0,1)+(0,2)+(0,3)+\&c.,\\ (1,1)=g+(1,0)+(1,2)+(1,3)+\&c.,\\ (2,2)=g+(2,0)+(2,1)+(2,3)+\&c.,\\ \&c.;\end{array}\right\}\qquad(366)$$

$$\left.\begin{array}{l}-b\;=(0,4)N^{IV}+(0,5)N^{V}+(0,6)N^{VI}+(0,7)N^{VII},\\ -b'\;=(1,4)N^{IV}+(1,5)N^{V}+(1,6)N^{VI}+(1,7)N^{VII},\\ -b''=(2,4)N^{IV}+(2,5)N^{V}+(2,6)N^{VI}+(2,7)N^{VII},\\ -b'''=(3,4)N^{VI}+(3,5)N^{V}+(3,6)N^{VI}+(3,7)N^{VII},\end{array}\right\}\qquad(367)$$

$$\left.\begin{array}{l}-b_1=(4,0)N\;+(4,1)N'+(4,2)N''+(4,3)N''',\\ -b_2=(5,0)N\;+(5,1)N'+(5,2)N''+(5,3)N''',\\ -b_3=(6,0)N\;+(6,1)N'+(6,2)N''+(6,3)N''',\\ -b_4=(7,0)N\;+(7,1)N'+(7,2)N''+(7,3)N'''.\end{array}\right\}\qquad(368)$$

If we now substitute these quantities in equations (E′), they will become

$$\left.\begin{array}{l}(0,0)N\;-(0,1)N'-(0,2)N''-(0,3)N'''+b\;=0,\\ (1,1)N'-(1,0)N\;-(1,2)N''-(1,3)N'''+b'\;=0,\\ (2,2)N''-(2,0)N\;-(2,1)N'-(2,3)N'''+b''=0,\\ (3,3)N'''-(3,0)N\;-(3,1)N'-(3,2)N''+b'''=0,\end{array}\right\}\qquad(\mathrm{E}'')$$

$$\left.\begin{array}{l}(4,4)N^{IV}-(4,5)N^{V}-(4,6)N^{VI}-(4,7)N^{VII}+b_1=0,\\ (5,5)N^{V}-(5,4)N^{IV}-(5,6)N^{VI}-(5,7)N^{VII}+b_2=0,\\ (6,6)N^{VI}-(6,4)N^{IV}-(6,5)N^{V}-(6,7)N^{VII}+b_3=0,\\ (7,7)N^{VII}-(7,4)N^{IV}-(7,5)N^{V}-(7,6)N^{VII}+b_4=0.\end{array}\right\}\qquad(\mathrm{E}''')$$

These equations are similar to equations (B″) and (B‴) of § 9; and we may make use of equations (31–64) for their solution, if we suppose $\boxed{0,0}=(0,0)$, $\boxed{1,1}=(1,1)$, &c.; $\boxed{0,1}=-(0,1)$, $\boxed{0,2}=-(0,2)$, &c.; $\boxed{1,0}=-(1,0)$, $\boxed{1,2}=-(1,2)$, &c., in these equations. We have given the values of $(0,1)$, $(0,2)$, &c., $(1,0)$, $(1,2)$, &c., in § 7. The values of $(0,0)$, $(0,1)$, $(2,2)$, &c., are given by means of the corresponding values $\boxed{0,0}$, $\boxed{1,1}$, $\boxed{2,2}$, &c., in equations (67), by simply changing the sign of the numerical terms of the second member.

3. We shall now reduce equations (31–64) to numbers. The values of the products $(0,0)(1,1)$, $(0,0)(2,2)$, &c. are given by means of equations (68–79) by simply changing the sign of the coefficients of g.

Computation of A.

$$\begin{array}{rl}
(0,0)(2,2)= & g^2+18.6424288.g+\ 72.81534747 \\
-(0,0)(1,2)(2,3)\div(1,3)= & +19.3779798.g+107.9403044 \\
-(2,2)(1,0)(0,3)\div(1,3)= & +\ 0.0482175.g+\ \ 0.6303080 \\
+(1,0)(0,2)(2,3)\div(1,3)= & +\ \ 0.4427946 \\
+(1,2)(2,0)(0,3)\div(1,3)= & +\ \ 0.0738673 \\
-(2,0)(0,2)\qquad = & -\ \ 0.0350059 \\
\text{Sum of terms}\quad A= & g^2+38.0686261.g+181.867616
\end{array}$$

Computation of A′.

$$\begin{array}{rl}
(0,0)(3,3)= & g^2+23.1231203.g+97.7739453 \\
-(0,0)(1,3)(3,2)\div(1,2)= & +\ 0.0270293.g+\ 0.1505601 \\
-(3,3)(1,0)(0,2)\div(1,2)= & +\ 0.0228504.g+\ 0.4010900 \\
+(1,0)(0,3)(3,2)\div(1,2)= & +\ 0.0013033 \\
+(1,3)(3,0)(0,2)\div(1,2)= & +\ 0.0001030 \\
-(3,0)(0,3)\qquad = & -\ 0.0002174 \\
\text{Sum of terms}\quad A'= & g^2+23.1730000.g+98.3267843
\end{array}$$

Computation of A″.

$$\begin{array}{rl}
(0,0)(1,1)= & g^2+16.8850240.g+63.0261532 \\
-(0,0)(2,1)(1,3)\div(2,3)= & +\ 1.8241484.g+10.1609735 \\
-(1,1)(2,0)(0,3)\div(2,3)= & +\ 0.0038119.g+\ 0.0431310 \\
+(2,0)(0,1)(1,3)\div(2,3)= & +\ 0.0416825 \\
+(2,1)(1,0)(0,3)\div(2,3)= & +\ 0.0879559 \\
-(1,0)(0,1)\qquad = & -\ 0.5272484 \\
\text{Sum of terms}\quad A''= & g^2+18.7129843.g+72.8326477
\end{array}$$

Computation of D.

$$\begin{array}{rl}
(1,1)(2,2)= & g^2+24.3869412.g+147.9086074 \\
-(1,1)(0,2)(2,3)\div(0,3)= & +\ 9.1832668.g+103.9065348 \\
-(2,2)(0,1)(1,3)\div(0,3)= & +10.9347838.g+142.941385 \\
+(0,1)(1,2)(2,3)\div(0,3)= & +211.894020 \\
+(0,2)(2,1)(1,3)\div(0,3)= & +\ 16.751639 \\
-(2,1)(1,2)\qquad = & -\ 35.348315 \\
\text{Sum of terms}\quad D= & g^2+44.5049918.g+588.053871
\end{array}$$

Computation of D′.

$$\begin{array}{rl}
(1,1)(3,3)= & g_2+28.8676327.g+198.606593 \\
-(1,1)(0,3)(3,2)\div(0,2)= & +\ 0.0570369.g+\ \ 0.645360 \\
-(3,3)(0,1)(1,2)\div(0,2)= & +23.0739263.g+405.013500 \\
+(0,1)(1,3)(3,2)\div(0,2)= & +\ \ 0.623672 \\
+(0,3)(3,1)(1,2)\div(0,2)= & +\ \ 0.104041 \\
-(3,1)(1,3)\qquad = & -\ \ 0.049305 \\
\text{Sum of terms}\quad D'= & g^2+51.9985959.g+604.943861
\end{array}$$

Computation of D''.

$$\begin{aligned}
(2,2)(3,3) &= g^2+30.6250375.g+229.4540814\\
-(2,2)(0,3)(3,1)\div(0,1) &= +\ 0.0045090.g+\ \ 0.0589430\\
-(3,3)(0,2)(2,1)\div(0,1) &= +\ 1.5319589.g+\ 26.8902683\\
+(0,3)(3,2)(2,1)\div(0,1) &= \qquad\qquad\qquad +\ \ 0.0873762\\
+(0,2)(2,3)(3,1)\div(0,1) &= \qquad\qquad\qquad +\ \ 0.0414048\\
-(3,2)(2,3) &= \qquad\qquad\qquad -\ \ 0.5237732\\
\text{Sum of terms}\quad D'' &= g^2+32.1615054.g+256.0083004
\end{aligned}$$

Computation of A_1.

$$\begin{aligned}
(4,4)(6,6) &= g^2+10.2786276.g+20.7811250\\
-(4,4)(5,6)(6,7)\div(5,7) &= +\ 1.7539697.g+13.1764792\\
-(6,6)(5,4)(4,7)\div(5,7) &= +\ 6.3754170.g+17.6360114\\
+(5,4)(4,6)(6,7)\div(5,7) &= \qquad\qquad\qquad +\ 8.7135093\\
+(5,6)(6,4)(4,7)\div(5,7) &= \qquad\qquad\qquad +\ 0.0911343\\
-(6,4)(4,6) &= \qquad\qquad\qquad -\ 0.0710140\\
\text{Sum of terms}\quad A_1 &= g^2+18.4080144.g+60.3272452
\end{aligned}$$

Computation of A_2.

$$\begin{aligned}
(4,4)(7,7) &= g^2+8.16033230.g+4.8676955\\
-(4,4)(5,7)(7,6)\div(5,6) &= +0.06449760.g+0.4845300\\
-(7,7)(5,4)(4,6)\div(5,6) &= +4.96787889.g+3.218971\\
+(5,4)(4,7)(7,6)\div(5,6) &= \qquad\qquad\qquad +0.411199\\
+(5,7)(7,4)(4,6)\div(5,6) &= \qquad\qquad\qquad +0.003351\\
-(7,4)(4,7) &= \qquad\qquad\qquad -0.004301\\
\text{Sum of terms}\quad A_2 &= g^2+13.1927088.g+8.981445
\end{aligned}$$

Computation of A_3.

$$\begin{aligned}
(4,4)(5,5) &= g^2+26.1085883.g+139.7017323\\
-(4,4)(6,5)(5,7)\div(6,7) &= +\ 0.2214108.g+\ \ 1.663321\\
-(5,5)(6,4)(4,7)\div(6,7) &= +\ 0.0519589.g+\ \ 0.966238\\
+(6,4)(4,5)(5,7)\div(6,7) &= \qquad\qquad\qquad +\ \ 1.099942\\
+(6,5)(5,4)(4,7)\div(6,7) &= \qquad\qquad\qquad +\ \ 1.411586\\
-(5,4)(4,5) &= \qquad\qquad\qquad -134.964267\\
\text{Sum of terms}\quad A_3 &= g^2+26.3819580.g+\ \ 9.878552
\end{aligned}$$

Computation of D_1.

$$\begin{aligned}
(5,5)(6,6) &= g^2+21.3624651.g+\ 51.441815\\
-(5,5)(4,6)(6,7)\div(4,7) &= +\ 1.3667356.g+\ 25.416106\\
-(6,6)(4,5)(5,7)\div(4,7) &= +21.1694805.g+\ 58.560122\\
+(4,5)(5,6)(6,7)\div(4,7) &= \qquad\qquad\qquad +\ 37.130625\\
+(4,6)(6,5)(5,7)\div(4,7) &= \qquad\qquad\qquad +\ \ 0.302610\\
-(6,5)(5,6) &= \qquad\qquad\qquad -\ \ 0.388348\\
\text{Sum of terms}\quad D_1 &= g^2+43.8986812.g+172.462930
\end{aligned}$$

Computation of D_2.

$$\begin{aligned}
(5,5)(7,7) &= g^2+19.2441698.g+12.0495445\\
-(5,5)(4,7)(7,6)\div(4,6) &= +\ 0.0827715.g+\ 1.539238\\
-(7,7)(4,5)(5,6)\div(4,6) &= +27.1673827.g+17.603294\\
+(4,5)(5,7)(7,6)\div(4,6) &- \qquad\qquad\qquad\quad +\ 1.752231\\
+(4,7)(7,5)(5,6)\div(4,6) &= \qquad\qquad\qquad\quad +\ 0.018327\\
-(7,5)(5,7) &= \qquad\qquad\qquad\quad -\ 0.014281\\
\text{Sum of terms}\quad D_2 &= g^2+46.4943240.g+32.948353
\end{aligned}$$

Computation of D_3.

$$\begin{aligned}
(6,6)(7,7) &= g^2+3.4142091.g+1.79241220\\
-(6,6)(4,7)(7,5)\div(4,5) &= +0.0006746.g+0.00186605\\
-(7,7)(4,6)(6,5)\div(4,5) &= +0.0142946.g+0.00926231\\
+(4,7)(7,6)(6,5)\div(4,5) &= \qquad\qquad\qquad +0.00118319\\
+(4,6)(6,7)(7,5)\div(4,5) &= \qquad\qquad\qquad +0.00092197\\
-(7,6)(6,7) & \qquad\qquad\qquad\quad -0.11312682\\
\text{Sum of terms}\quad D_3 &= g^2+3.4291782.g+1.69251890
\end{aligned}$$

Computation of B, B', and B''.

$$\begin{aligned}
(2,2) &= g+13.072173\\
-(1,2)(2,3)\div(1,3) &= +19.37798\\
\text{Sum } &= B\div b = g+32.45015
\end{aligned}$$

$$\begin{aligned}
(3,3) &= g+17.5528645\\
-(1,3)(3,2)\div(1,2) &= +\ 0.0270293\\
\text{Sum } &= B'\div b = g+17.5798938
\end{aligned}$$

$$\begin{aligned}
(1,1) &= g+11.314768\\
-(2,1)(1,3)\div(2,3) &= +\ 1.824148\\
\text{Sum } &= B''\div b = g+13.138916.
\end{aligned}$$

Computation of C, C', C'', and C'''.

$$\begin{aligned}
-(2,2) &= -g-13.0721730\\
(0,2)(2,3)\div(0,3) &= -\ 9.183266\\
\text{Sum } &= -\{g+22.255439\}\\
\log.\ \{(0,3)\div(1,3)\} &= [9.4381189]\\
\text{Therefore } C &= -\{g+22.255439\}\times\\
&\qquad [9.4381189]b'.
\end{aligned}$$

$$\begin{aligned}
-(3,3) &= -g-17.5528645\\
(0,3)(3,2)\div(0,2) &= -\ 0.0570356\\
\text{Sum } &= -\{g+17.6099001\}\\
\log.\ \{(0,2)\div(2,1)\} &= [9.1138076]\\
\text{Therefore } C' &= -\{g+17.6099001\}\times\\
&\qquad [9.1138076]b'.
\end{aligned}$$

Computation of B_1, B_2, and B_3.

$$\begin{aligned}
(6,6) &= g+2.7662522\\
-(5,6)(6,7)\div(5,7) &= +1.7539698\\
\text{Sum } &= B_1\div b_1 = g+4.5202220;
\end{aligned}$$

$$\begin{aligned}
(7,7) &= g+0.6479569\\
-(5,7)(7,6)\div(5,6) & \quad +0.0644976\\
\text{Sum } &= B_2\div b_1 = g+0.7124545;
\end{aligned}$$

$$\begin{aligned}
(5,5) &= g+18.5962129\\
-(6,5)(5,7)\div(6,7) &= +\ 0.2214108\\
\text{Sum } &= B_3\div b_1 = g+18.8176237.
\end{aligned}$$

Computation of C_1, C_2, C_3, and C_4.

$$\begin{aligned}
-(6,6) &= -g-\ 2.7662522\\
(4,6)(6,7)\div(4,7) &= -\ 1.3667356\\
\text{Sum } &= -\{g+\ 4.1329878\}\\
\log.\ \{(4,7)\div(5,7)\} &= [9.5433087]\\
\text{Therefore } C_1 &= -\{g+\ 4.1329878\}\times\\
&\qquad [9.5433087]b_2.
\end{aligned}$$

$$\begin{aligned}
-(7,7) &= -g-\ 0.6479569\\
(4,7)(7,6)\div(4,6) &= -\ 0.0827715\\
\text{Sum } &= -\{g+\ 0.7307284\}\\
\log.\ \{(4,6)\div(5,6)\} &= [9.4349711]\\
\text{Therefore } C_2 &= -\{g+\ 0.7307284\}\times\\
&\qquad [9.4349711]b_2.
\end{aligned}$$

$(0,3)(2,1)\div(2,3)=-\ 0.5002407$
$-(0,1)=+\ 2.9986729$
Sum $=C''\div b'=+\ 2.4984322$
Therefore $C''=+[0.3976676]b'$.

$(0,2)(3,1)\div(3,2)=-\ 0.2370651$
$-(0,1)=+\ 2.9986729$
Sum $=C'''\div b'=+\ 2.7616078$
Therefore $C'''=+[0.4411620]b'$.

Computation of E, E', E'', *and* E'''.

$(0,3)(1,2)\div(1,3)=-\ 1.818323$
$-(0,2)=+\ 0.861707$
Sum $=E\div b''=-\ 0.956616$
Therefore $E=-[9.9807377]b''$.

$-(1,1)=-g-11.3147682$
$+(0,1)(1,3)\div(0,3)=\ -10.9347838$
Sum $=-\{g+22.2495520\}$
log. $\{(0,3)\div(2,3)\}=[8.9723624]$
Therefore $E'=-\{g+22.249552\}\times$
$[8.9723624]b''$.

$-(3,3)=-g-17.5528645$
$+(0,3)(3,1)\div(0,1)=\ -\ 0.0045090$
Sum $=-\{g+17.5573735\}$
log. $\{(0,1)\div(2,1)\}=[9.7501125]$
Therefore $E''=-\{g+17.5573735\}\times$
$[9.7501125]b''$.

$(0,1)(3,2)\div(3,1)=-10.89986$
$-(0,2)=+\ 0.86171$
Sum $=E'''\div b''=-10.03815$
Therefore $E'''=-[1.0016537]b''$.

Computation of F, F', F'', *and* F'''.

$(0,2)(1,3)\div(1,2)=-\ 0.01326047$
$-(0,3)=+\ 0.02798148$
Sum $=F\div b'''=+\ 0.01472101$
Therefore $F=+[8.1679376]b'''$.

$(0,1)(2,3)\div(2,1)=-\ 0.1677337$
$-(0,3)=+\ 0.0279815$
Sum $=F'\div b'''=-\ 0.1397522$
Therefore $F'=-[9.1453586]b'''$.

$(4,7)(6,5)\div(6,7)=-\ 0.0773584$
$-(4,5)=+\ 7.3963746$
Sum $=C_3\div b_2=+\ 7.3190162$
Therefore $C_3=+[0.8644527]b_2$.

$(4,6)(7,5)\div(7,6)=-\ 0.0602795$
$-(4,5)=+\ 7.3963746$
Sum $=C_4\div b_2=+\ 7.3360951$
Therefore $C_4=+[0.8654649]b_2$.

Computation of E_1, E_2, E_3, *and* E_4.

$(4,7)(5,6)\div(5,7)=-\ 0.09722854$
$-(4,6)=+\ 0.07576285$
Sum $E_1\div b_3=-\ 0.02146569$
Therefore $E_1=-[8.3317448]b_3$.

$-(5,5)=-g-18.5962129$
$(4,5)(5,7)\div(4,7)=\ -21.1694805$
Sum $=-\{g+39.7656934\}$
log. $\{(4,7)\div(6,7)\}=[8.7437718]$
Therefore $E_2=-\{g+39.7656934\}\times$
$[8.7437718]b_3$.

$-(7,7)=-g-\ 0.6479569$
$(4,7)(7,5)\div(4,5)=\ -\ 0.00067458$
Sum $=-\{g+\ 0.64863148\}$
log. $\{(4,5)\div(6,5)\}=[0.7242832]$
Therefore $E_3=-\{g+0.6486315\}\times$
$[0.7242832]b_3$.

$(4,5)(7,6)\div(7,5)=-\ 9.296196$
$-(4,6)=+\ 0.075763$
Sum $=E_4\div b_3=-\ 9.220433$
Therefore $E_4=-[0.9647514]b_3$.

Computation of F_1, F_2, F_3, *and* F_4.

$(4,6)(5,7)\div(5,6)=-\ 0.01871457$
$-(4,7)=+\ 0.02401693$
Sum $=F_1\div b_4=+\ 0.00530236$
Therefore $F_1=+[7.7244692]b_4$.

$(4,5)(6,7)\div(6,5)=-\ 2.296302$
$-(4,7)=+\ 0.024017$
Sum $=F_2\div b_4=-\ 2.272285$
Therefore $F_2=-[0.3564628]b_4$.

$-(2,2)=-g-13.072173$
$(0,2)(2,1)\div(0,1)=-\ 1.531958$
Sum $=-\{g+14.604131\}$
log. $\{(0,1)\div(3,1)\}=[0.7927855]$
Therefore $F''=-\{g+14.604131\}\times[0.7927855]b''$.

$-(1,1)=-g-11.3147682$
$(0,1)(1,2)\div(0,2)=-23.0739263$
Sum $=-\{g+34.3886945\}$
log. $\{(0,2)\div(3,2)\}=[9.6907241]$
Therefore $F'''=-\{g+34.3886945\}\times[9.6907241]b'''$.

$-(6,6)=-g+2.7662522$
$(4,6)(6,5)\div(4,5)=+0.0142946$
Sum $=-\{g+2.7805468\}$
log. $\{(4,5)\div(7,5)\}=[1.5514854]$
Therefore $F_3=-\{g+2.7805468\}\times[1.5514854]b_4$.

$-(5,5)=-g-18.5962129$
$(4,5)(5,6)\div(4,6)=-27.1673827$
Sum $=-\{g+45.7635956\}$
log. $\{(4,6)\div(7,6)\}\ [9.4626364]$
Therefore $F_4=-\{45.7635956+g\}\times[9.4626364]b_4$.

Computation of the Equations of the fourth degree.

$$\begin{aligned}
&-(2,1)(1,2)(0,0)(3,3)=-35.3483155.g^2-817.363351.g-3456.144265\\
&-(3,2)(2,3)(0,0)(1,1)=-0.5237732.g^2-8.843924.g-33.01142\\
&-(1,0)(0,1)(2,2)(3,3)=-0.5272484.g^2-16.147000.g-120.97930\\
&-(3,1)(1,3)(0,0)(2,2)=-0.0493054.g^2-0.919173.g-3.59019\\
&-(2,0)(0,2)(1,1)(3,3)=-0.0350059.g^2-1.010537.g-6.95240\\
&-(3,0)(0,3)(1,1)(2,2)=-0.0002174.g^2-0.005302.g-0.03216\\
&+2(3,2)(2,1)(1,3)(0,0)=-1.910880.g-10.64409\\
&+2(2,0)(0,1)(1,2)(3,3)=-1.615446.g-28.35570\\
&+2(3,1)(1,0)(0,3)(2,2)=-0.004755.g-0.06216\\
&+2(3,0)(0,2)(2,3)(1,1)=-0.003993.g-0.04518\\
&+(1,0)(0,1)(3,2)(2,3)=+0.27616\\
&+(2,1)(1,2)(3,0)(0,3)=+0.00768\\
&+(2,0)(0,2)(3,1)(1,3)=+0.00173\\
&-2(3,0)(0,1)(1,2)(2,3)=-0.09214\\
&-2(3,1)(1,0)(0,2)(2,3)=-0.04366\\
&-2(3,0)(0,2)(2,1)(1,3)=-0.00728
\end{aligned}$$

Sum of preceding terms $=-36.4838658.g^2-847.824361.g-3659.67438$
Add $(0,0)(1,1)(2,2)(3,3)=$
$g^4+47.5100615.g^3+809.5847278.g^2+5804.515976.g+14461.60808$
Sum is the value of equation (53).
Whence we get

$$\left.\begin{aligned}&g^4+47.5100615.g^3+773.1008620.g^2\\&+4956.691615.g+10801.93370\end{aligned}\right\}=(\chi,\chi_1,\chi_2,\chi_3). \quad (369)$$

In like manner we get

$$\begin{aligned}
&-(5,4)(4,5)(6,6)(7,7)=-134.9642669.g^2-460.796228.g-241.911599\\
&-(6,5)(5,6)(4,4)(7,7)=-0.3883479.g^2-3.169047.g-1.890359\\
&-(7,6)(6,7)(4,4)(5,5)=-0.1131268.g^2-2.953581.g-15.804014\\
&-(6,4)(4,6)(5,5)(7,7)=-0.0710140.g^2-1.366608.g-0.855687\\
&-(7,5)(5,7)(4,4)(6,6)=-0.0142805.g^2-0.146774.g-0.296764\\
&-(7,4)(4,7)(5,5)(6,6)=-0.0043007.g^2-0.091874.g-0.221237
\end{aligned}$$

$+2(6,4)(4,5)(5,6)(7,7)= \quad -\ 3.858530.g-\ 2.500161$

$+2(7,5)(5,4)(4,7)(6,6)= \quad -\ 0.182088.g-\ 0.503701$

$+2(7,6)(6,5)(5,7)(4,4)= \quad -\ 0.050095.g-\ 0.376332$

$+2(7,4)(4,6)(6,7)(5,5)= \quad -\ 0.011756.g-\ 0.218615$

$+\ (5,4)(4,5)(7,6)(6,7)= \quad +\ 15.268077$

$+\ (6,5)(5,6)(7,4)(4,7)= \quad +\ 0.001670$

$+\ (6,4)(4,6)(7,5)(5,7)= \quad +\ 0.001014$

$-2(7,4)(4,5)(5,6)(6,7)= \quad -\ 0.319377$

$-2(7,5)(5,4)(4,6)(6,7)= \quad -\ 0.248866$

$-2(7,4)(4,6)(6,5)(5,7)= \quad -\ 0.002603$

Sum of preceding terms $=-135.5553368.g^2-472.626581.g-249.878554$

Add $(4,4)(5,5)(6,6)(7,7)=$

$g^4+29.5227974.g^3+230.6343243.g^2+523.7682780.g+250.403089$

Sum is value of equation (54).

Whence we get

$$\left.\begin{array}{r} g^4-29.5227974.g^3+95.0789875.g^2 \\ +51.141697.g+0.524535 \end{array}\right\}=(\chi_4, \chi_5, \chi_6, \chi_7). \quad (370)$$

Equations (55–64) reduced to numbers are as follows, in which the numbers inclosed in brackets are logarithms.

$$N'=[0.5618811]\,\frac{AN+f}{D}=[0.8861924]\,\frac{A'N+f'}{D'}; \qquad (371)$$

$$N''=[1.0276376]\,\frac{A''N+f''}{D}=[0.2498875]\,\frac{A'N+f'''}{D''}; \qquad (372)$$

$$N'''=[9.2072145]\,\frac{AN+f^{IV}}{D''}=[0.3092759]\,\frac{A''N+f^{V}}{D'}; \qquad (373)$$

$$N=\frac{[9.4381189]\,D\,f'-[9.1138076]\,D'f}{[9.1138076]\,A\,D'-[9.4381189]\,A'\,D}; \qquad (374)$$

$$N=\frac{[8.9723624]Df'''-[9.7501125]D''f''}{[9.7501125]A''D''-[8.9723624]A'D}; \qquad (375)$$

$$N=\frac{[0.7927855]D''f^{V}-[9.6907241]D'f^{IV}}{[9.6907241]AD'-[0.7927855]A''D''}; \qquad (376)$$

$$N^{V}=[0.4566913]\,\frac{A_1N^{IV}+f_1}{D_1}=[0.5650289]\,\frac{A_2N^{IV}+f_2}{D}; \qquad (377)$$

$$N^{VI}=[1.2562282]\,\frac{A_3N^{IV}+f_3}{D_1}=[9.2757168]\,\frac{A_2N^{IV}+f_4}{D_3}; \qquad (378)$$

$$N^{VII}=[8.4485146]\,\frac{A_1N^{IV}+f_5}{D_3}=[0.5373636]\,\frac{A_3N^{IV}+f_6}{D_2}; \qquad (379)$$

$$N^{IV}=\frac{[9.5433087]D_1f_2-[9.4349711]D_2f_1}{[9.4349711]A_1D_2-[9.5433087]A_2D_1}; \qquad (380)$$

$$N^{IV}=\frac{[8.7437718]D_1f_4-[0.7242832]D_3f_3}{[0.7242832]A_3D_3-[8.7437718]A_2D_1}; \qquad (381)$$

$$N^{IV}=\frac{[1.5514854]D_3f_6-[9.4626364]D_2f_5}{[9.4626364]A_1D_2-[1.5514854]A_3D_3}. \tag{382}$$

$$\left.\begin{aligned}
&\Big\{[9.9546226]D'f\;-[0.2789339]Df\Big\}\frac{1}{N}\\
=&\Big\{[9.3015115]Df'''-[0.0792616]D''f''\Big\}\frac{1}{N}\\
=&\Big\{[8.9337058]D'f^{IV}-[0.0357672]D''f^{V}\Big\}\frac{1}{N}
\end{aligned}\right\}=(\chi,\chi_1,\chi_2,\chi_3); \tag{383}$$

$$\left.\begin{aligned}
&\Big\{[0.5477106]D_2f_1-[0.6560482]D_1f_2\Big\}\frac{1}{N^{IV}}\\
=&\Big\{[8.0240547]D_1f_4-[0.0045661]D_3f_3\Big\}\frac{1}{N^{IV}}\\
=&\Big\{[7.9147050]D_2f_5-[0.0035540]D_3f_6\Big\}\frac{1}{N^{IV}}
\end{aligned}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7). \tag{384}$$

If we repeat and number the formulæ which we have computed, we shall have the following

Fundamental Equations for the adopted masses.

$$A\;=g^2+38.0686261.g+181.867616; \tag{385}$$
$$A'=g^2+23.1730000.g+\;98.3267843; \tag{386}$$
$$A''=g^2+18.7129843.g+\;72.8326477; \tag{387}$$
$$A_1=g^2+18.4080144.g+\;60.3272452; \tag{388}$$
$$A_2=g^2+13.1927088.g+\;\;8.981445; \tag{389}$$
$$A_3=g^2+26.3819580.g+\;\;9.878552; \tag{390}$$

$$D\;=g^2+44.5049918.g+588.053871; \tag{391}$$
$$D'=g^2+51.9985959.g+604.943861; \tag{392}$$
$$D''=g^2+32.1615054.g+256.0083004; \tag{393}$$
$$D_1=g^2+43.8986812.g+172.462930; \tag{394}$$
$$D_2=g^2+46.4943240.g+\;32.948353; \tag{395}$$
$$D_3=g^2+\;3.4291782.g+\;\;1.69251890; \tag{396}$$

$$\left.\begin{aligned}
B=\{g+32,45015\}b;\quad B'&=\{g+17.5798938\}b;\\
B''&=\{g+13.138916\}b;
\end{aligned}\right\} \tag{397}$$

$$\left.\begin{aligned}
C\;&=-\{g+22.255439\}\;[9.4381189]b';\\
C'\;&=-\{g+17.6099001\}[9.1138076]b';\\
C''&=+[0.3976676]b';\\
C'''&=+[0.4411620]b';
\end{aligned}\right\} \tag{398}$$

$$\left.\begin{aligned}
E\;&=-[9.9807377]b'';\\
E'\;&=-\{g+22.249552\}\;[8.9723624]b'';\\
E''&=-\{g+17.5573735\}[9.7501125]b'';\\
E'''&=-[1.0016537]b'';
\end{aligned}\right\} \tag{399}$$

$$\left.\begin{aligned}
F &=+[8.1679376]b''';\\
F' &=-[9.1453586]b''';\\
F'' &=-\{g+14.604131\}\ [0.7927855]b''';\\
F''' &=-\{g+34.3886945\}[9.6907241]b''';
\end{aligned}\right\}\quad(400)$$

$$\left.\begin{aligned}
B_1=\{g+4.5202220\}b_1;\ B_2&=\{g+\ 0.7124545\}b_1;\\
B_3&=\{g+18.8176237\}b_1;
\end{aligned}\right\}\quad(401)$$

$$\left.\begin{aligned}
C_1&=-\{g+4.1329878\}[9.5433087]b_2;\\
C_2&=-\{g+0.7307284\}[9.4349711]b_2;\\
C_3&=+[0.8644527]b_2;\\
C_4&=+[0.8654649]b_2;
\end{aligned}\right\}\quad(402)$$

$$\left.\begin{aligned}
E_1&=-[8.3317448]b_3;\\
E_2&=-\{g+39.7656934\}[8.7437718]b_3;\\
E_3&=-\{g+\ 0.6486315\}[0.7242832]b_3;\\
E_4&=-[0.9647514]b_3;
\end{aligned}\right\}\quad(403)$$

$$\left.\begin{aligned}
F_1&=+[7.7244692]b_4;\\
F_2&=-[0.3564628]b_4;\\
F_3&=-\{g+\ 2.7805468\}[1.5514854]b_4;\\
F_4&=-\{g+45.7635956\}[9.4626364]b_4.
\end{aligned}\right\}\quad(404)$$

We shall also have

$$\left.\begin{aligned}
-b &=\{\ 1.6028375 \ldots\ldots [0.2048895]\}N^{IV}+[8.8879781]N^{V}\\
&\qquad\qquad +[7.1246469]\ N^{VI}+[6.6629969]N^{VII};\\
-b' &=\{\ 4.2028443 \ldots\ldots [0.6235433]\}N^{IV}+[9.2986071]N^{V}\\
&\qquad\qquad +[7.5327484]\ N^{VI}+[7.0706089]N^{VII};\\
-b'' &=\{\ 7.0682646 \ldots\ldots [0.8493128]\}N^{IV}+[9.5139049]N^{V}\\
&\qquad\qquad +[7.7447989]\ N^{VI}+[7.2820328]N^{VII};\\
-b''' &=\{14.659896\ \ldots\ldots [1.1661309]\}N^{IV}+[9.8003037]N^{V}\\
&\qquad\qquad +[8.0220791]\ N^{VI}+[7.5575701]N^{VII}.
\end{aligned}\right\}\quad(405)$$

$$\left.\begin{aligned}
-b_1 &=\{0.000094200\ \ \ldots\ldots [95.9740497]\}N+[7.6245464]N'\\
&\qquad\qquad +[\ 7.9450513]\ N''+[7.4917417]N''';\\
-b_2 &=\{0.0000112026\ \ldots\ldots [95.0493193]\}N+[6.6917912]N'\\
&\qquad\qquad +[\ 7.0018244]\ N''+[6.5180955]N''';\\
-b_3 &=\{0.00000096881 \ldots\ldots [93.9862386]\}N+[5.6261830]N'\\
&\qquad\qquad +[\ 5.9329689]\ N''+[5.4401214]N''';\\
-b_4 &=\{0.00000020167 \ldots\ldots [93.3046628]\}N+[4.9441177]N'\\
&\qquad\qquad +[\ 5.2502770]\ N''+[4.7556866]N'''.
\end{aligned}\right\}\quad(406)$$

4. If we now suppose the second members of equations (369) and (370) to be equal to nothing, we shall obtain the following values of g, g_1, g_2, &c.

$$\left.\begin{aligned}
g&=-\ \ 5''.1223003, & g_4&=-\ \ 0''.01045922,\\
g_1&=-\ \ 6\ .5863879, & g_5&=-\ \ 0\ .66298299,\\
g_2&=-17\ .3924594, & g_6&=-\ \ 2\ .91695653,\\
g_3&=-18\ .4089138, & g_7&=-25\ .93239866.
\end{aligned}\right\}\quad(407)$$

By means of equations (385–407) and (371–384), we now obtain the following values.

For the root g, we get,

$$g = -5''.126112,$$

$N' = +0.1227919N$	log. 9.0891696,
$N'' = +0.08795864N$	" 8.9442784,
$N''' = +0.01757422N$	" 8.2448760,
$N^{IV} = -0.0002076885N$	" 6.3174124n,
$N^{V} = -0.000264229N$	" 6.421980n,
$N^{VI} = +0.000231553N$	" 6.364650,
$N^{VII} = +0.00000640325N$	" 4.806400.

For the root g_1, we get,

$$g_1 = -6''.592128,$$

$N_1' = -0.2764636N_1$	log. 9.4416380n,
$N_1'' = -0.2229392N_1$	" 9.3481864n,
$N_1''' = -0.04673056N_1$	" 8.6696010n,
$N_4^{IV} = +0.0003357396N_1$	" 6.5260026,
$N_1^{V} = +0.0004742548N_1$	" 6.6760116,
$N_1^{VI} = -0.0002454315N_1$	" 6.3899302n,
$N_1^{VII} = -0.00001482107N_1$	" 5.1708794n.

For the root g_2, we get,

$$g_2 = -17''.393390,$$

$N_2' = -\ 5.563101N_2$	log. 0.7453170n,
$N_2'' = +\ 4.563350N_2$	" 0.6592838,
$N_2''' = +33.24578N_2$	" 1.5217364,
$N_2^{IV} = -\ 0.001649287N_2$	" 7.2172964n,
$N_2^{V} = -\ 0.01403826N_2$	" 8.1473134n,
$N_2^{VI} = +\ 0.001367028N_2$	" 7.1357772,
$N_2^{VII} = +\ 0.000157230N_2$	" 6.1965360.

For the root g_3, we get,

$$g_3 = -18''.408914,$$

$N_3' = -\ 6.098710N_3$	log. 0.7852380n,
$N_3'' = +\ 6.655888N_3$	" 0.8232060,
$N_3''' = -10.22309N_3$	" 1.0095825n,
$N_3^{IV} = -\ 0.0001958273N_3$	" 5.2918731n,
$N_3^{V} = -\ 0.0001514003N_3$	" 6.1801268,
$N_3^{VI} = +\ 0.00001260627N_3$	" 5.1005864,
$N_3^{VII} = +\ 0.00000140198N_3$	" 4.1467414.

For the root g_4, we get,

$$g_4 = 0, \text{ and}$$

$$N_4 = N_4' = N_4'' = N_4''' = N_4^{IV} = N_4^{V} = N_4^{VI} = N_4^{VII}.$$

For the root g_5, we get,

$$\begin{array}{lll}
g_5=-0''.661666, & & \\
N_5 =+1.232212N_5^{IV} & \text{log.} & 0.0906854, \\
N_5' =+1.131314N_5^{IV} & \text{``} & 0.0535832, \\
N_5'' =+1.108223N_5^{IV} & \text{``} & 0.0446270, \\
N_5''' =+1.049477N_5^{IV} & \text{``} & 0.0209706, \\
N_5^{V} =+0.9653287N_5^{IV} & \text{``} & 9.9846752, \\
N_5^{VI} =-0.9379252N_5^{IV} & \text{``} & 9.9721682n, \\
N_5^{VII} =-9.829270N_5^{IV} & \text{``} & 0.9925212n.
\end{array}$$

For the root g_6, we get,

$$\begin{array}{lll}
g_6=-2''.916082, & & \\
N_6 =+\ 3.557327N_6^{IV} & \text{log.} & 0.5511238, \\
N_6' =+\ 2.059163N_6^{IV} & \text{``} & 0.3136906, \\
N_6'' =+\ 1.845317N_6^{IV} & \text{``} & 0.2660709, \\
N_6''' =+\ 1.314187N_6^{IV} & \text{``} & 0.1186570, \\
N_6^{V} =+\ 0.8164588N_6^{IV} & \text{``} & 9.9119342, \\
N_6^{VI} =-20.11300N_6^{IV} & \text{``} & 1.3034768n, \\
N_6^{VII} =+\ 2.161663N_6^{IV} & \text{``} & 0.3347880.
\end{array}$$

For the root g_7, we get,

$$\begin{array}{lll}
g_7=-25''.934567, & & \\
N_7 =-0.04207851N_7^{IV} & \text{log.} & 8.6240604n, \\
N_7' =-0.04653315N_7^{IV} & \text{``} & 8.6677625n, \\
N_7'' =-0.4328950N_7^{IV} & \text{``} & 9.6363826n, \\
N_7''' =-1.468114N_7^{IV} & \text{``} & 0.1667598n, \\
N_7^{V} =-2.490709N_7^{IV} & \text{``} & 0.3963230n, \\
N_7^{VI} =+0.1093424N_7^{IV} & \text{``} & 9.0387886, \\
N_7^{VII} =+0.01225277N_7^{IV} & \text{``} & 8.0882343.
\end{array}$$

5. Having thus determined all the roots of the equation of the eighth degree, together with the ratios of the constant quantities N', N'', N''', &c., corresponding to each root, the complete integrals of equations (E) will be

$$\left.\begin{array}{l}
q=N\cos(gt+\beta)+N_1\cos(g_1t+\beta_1)+N_2\cos(g_2t+\beta_2)+\&c., \\
q'=N'\cos(gt+\beta)+N_1'\cos(g_1t+\beta_1)+N_2'\cos(g_2t+\beta_2)+\&c., \\
q''=N''\cos(gt+\beta)+N_1''\cos(g_1t+\beta_1)+N_2''\cos(g_2t+\beta_2)+\&c., \\
\&c.; \\
p=N\sin(gt+\beta)+N_1\sin(g_1t+\beta_1)+N_2\sin(g_2t+\beta_2)+\&c., \\
p'=N'\sin(gt+\beta)+N_1'\sin(g_1t+\beta_1)+N_2'\sin(g_2t+\beta_2)+\&c., \\
p''=N''\sin(gt+\beta)+N_1''\sin(g_1t+\beta_1)+N_2''\sin(g_2t+\beta_2)+\&c., \\
\&c.
\end{array}\right\}\quad (F)$$

The analysis of § 16 will conduct us to the following equations for the determination of the arbitrary constants corresponding to each root.

$$\left.\begin{aligned}&Np\frac{m}{na}+N'p'\frac{m'}{n'a'}+N''p''\frac{m''}{n''a''}+N'''p'''\frac{m'''}{n'''a'''}+\text{\&c.}\\&=\left\{N^2\frac{m}{na}+N'^2\frac{m'}{n'a'}+N''^2\frac{m''}{n''a''}+N'''^2\frac{m'''}{n'''a'''}+\text{\&c.}\right\}\sin(gt+\beta)\end{aligned}\right\}\quad(408)$$

$$\left.\begin{aligned}&Nq\frac{m}{na}+N'q'\frac{m'}{n'a'}+N''q''\frac{m''}{n''a''}+N'''q'''\frac{m'''}{n'''a'''}+\text{\&c.}\\&=\left\{N^2\frac{m}{na}+N'^2\frac{m'}{n'a'}+N''^2\frac{m''}{n''a''}+N'''^2\frac{m'''}{n'''a'''}+\text{\&c.}\right\}\cos(gt+\beta)\end{aligned}\right\}\quad(409)$$

And since, for the root g_4, we have $N_4=N_4'=N_4''=N_4'''$, &c., the system of equations similar to equations (133) will be divisible by $N_4\,N_4'$ &c., we shall have the following equations of condition:—

$$\left.\begin{aligned}&N\frac{m}{na}+N'\frac{m'}{n'a'}+N''\frac{m''}{n''a''}+N'''\frac{m'''}{n'''a'''}+\text{\&c.}=0;\\&N_1\frac{m}{na}+N_1'\frac{m'}{n'a'}+N_1''\frac{m''}{n''a''}+N_1'''\frac{m'''}{n'''a'''}+\text{\&c.}=0;\\&N_2\frac{m}{na}+N_2'\frac{m'}{n'a'}+N_2''\frac{m''}{n''a''}+N_2'''\frac{m'''}{n'''a'''}+\text{\&c.}=0;\\&\text{\&c.}\end{aligned}\right\}\quad(410)$$

Now putting the first members of equations (408) and (409), equal to x and y respectively, and the coefficient of sin or cos of $(gt+\beta)$, in the same equations equal to z, we shall have

$$\left.\begin{aligned}x&=z\sin(gt+\beta);\quad y=\cos(gt+\beta);\\\text{whence}\ \tan(gt+\beta)&=x\div y.\end{aligned}\right\}\quad(411)$$

6. In order to find the values of x and y, which are to be used in equations (411), we shall assume the following values of ϕ, ϕ', ϕ'', &c., θ, θ', θ'', &c., corresponding to the beginning of the year 1850, at which epoch $t=0$.

Mercury,	$\phi\ =7°$	0′	8″.2,	$\theta\ =\ 46°$	33′	3″.2;
Venus,	$\phi'\ =3$	23	34.4,	$\theta'\ =\ 75$	20	42.9;
The Earth,	$\phi''\ =0$	0	0.0,	$\theta''\ =\ 0$	0	0 ;
Mars,	$\phi'''\ =1$	51	2.3,	$\theta'''\ =\ 48$	23	36.8;
Jupiter,	$\phi^{IV}=1$	18	40.3,	$\theta^{IV}=\ 98$	54	20.5;
Saturn,	$\phi^{V}\ =2$	29	22.4,	$\theta^{V}\ =112$	19	20.6;
Uranus,	$\phi^{VI}=0$	46	29.9,	$\theta^{VI}=\ 73$	14	14.4;
Neptune,	$\phi^{VII}=1$	47	0.9,	$\theta^{VII}=130$	7	45.3;

Now, since $q=\tan\phi\cos\theta$, and $p=\tan\phi\sin\theta$, we find,

$p\ =+0.0891690$, log. $p\ =8.9502138$;
$p'\ =+0.0573577$, log. $p'\ =8.7585915$;
$p''\ =\ 0.$ log. $p''\ =-\infty$
$p'''\ =+0.0241597$, log. $p'''\ =8.3830911$;

$$
\begin{array}{ll}
p^{IV}=+0.0226127, & \log.\, p^{IV}=8.3543534; \\
p^{V}=+0.0402201, & \log.\, p^{V}=8.6044432; \\
p^{VI}=+0.0129519, & \log.\, p^{VI}=8.1123323; \\
p^{VII}=+0.0238090, & \log.\, p^{VII}=8.3767411; \\
\\
q=+0.0844678, & \log.\, q=8.9266911; \\
q'=+0.0149991, & \log.\, q'=8.1760651; \\
q''=\ 0. & \log.\, q''=-\infty\quad ; \\
q'''=+0.0214548, & \log.\, q'''=8.3315249; \\
q^{IV}=-0.0035434, & \log.\, q^{IV}=7.5494159n; \\
q^{V}=-0.0165138, & \log.\, q^{V}=8.2178478n; \\
q^{VI}=+0.0039012, & \log.\, q^{VI}=7.5911972; \\
q^{VII}=-0.0200698, & \log.\, q^{VII}=8.3025434n.
\end{array}
$$

Now, adding the logarithms of $m\div na$, $m'\div n'a'$, &c., which are given in § 5, we shall obtain the following values of the logarithms of the constants for the given epoch, which enter into the values of x and y.

$$
\begin{array}{ll}
\log.\, p\dfrac{m}{na}=85.9443779; & \log.\, q\dfrac{m}{na}=85.9208552; \\
\log.\, p'\dfrac{m'}{n'a'}=86.9845985; & \log.\, q'\dfrac{m'}{n'a'}=86.4020721; \\
\log.\, p''\dfrac{m''}{n''a''}=-\infty & \log.\, q''\dfrac{m''}{n''a''}=-\infty \\
\log.\, p'''\dfrac{m'''}{n'''a'''}=85.9337058; & \log.\, q'''\dfrac{m'''}{n'''a'''}=85.8821396; \\
\log.\, p^{IV}\dfrac{m^{IV}}{n^{IV}a^{IV}}=89.5793573; & \log.\, q^{IV}\dfrac{m^{IV}}{n^{IV}a^{IV}}=88.7744198n; \\
\log.\, p^{V}\dfrac{m^{V}}{n^{V}a^{V}}=89.4372661; & \log.\, q^{V}\dfrac{m^{V}}{n^{V}a^{V}}=89.0506707n; \\
\log.\, p^{VI}\dfrac{m^{VI}}{n^{VI}a^{VI}}=88.2449047; & \log.\, q^{VI}\dfrac{m^{VI}}{n^{VI}a^{VI}}=87.7237696; \\
\log.\, p^{VII}\dfrac{m^{VII}}{n^{VII}a^{VII}}=88.7292393; & \log.\, q^{VII}\dfrac{m^{VII}}{n^{VII}a^{VII}}=88.6550416n.
\end{array}
$$

7. These quantities are now to be substituted in equations (408) and (409), in connection with the values of N', N'', N''', &c., corresponding to the different roots.

For the root $g=-5''.126112$, we find,

$$x=+\frac{0.612517}{10^{14}}N;\quad y=+\frac{1.5865662}{10^{14}}N;\quad z=\frac{14.046564}{10^{14}}N^2.$$

Whence $\beta=211°\ 6'\ 26''.8$; and $\log.\, N=9.0830567$.

Therefore, for the root g, we have the following values,

$$
\begin{array}{ll}
N=+0.121076, & N^{IV}=-0.00002517, \\
N'=+0.0148671, & N^{V}=-0.00003200, \\
N''=+0.0106496, & N^{VI}=+0.00002804, \\
N'''=+0.0021278, & N^{VII}=+0.000000775.
\end{array}
$$

In like manner we shall find for the root $g_1=6''.592128$,

$$x_1=+\frac{0.692850}{10^{14}}N_1;\quad y_1=-\frac{0.6389472}{10^{14}}N_1;\quad z_1=\frac{33.24236}{10^{14}}N_1^2.$$

Whence $\beta_1=132^\circ\ 40'\ 57''.8$; and log. $N_1=8.4525867$.

Therefore, for the root g_1, we have,

$N_1\ =+0.028352,$
$N_1'\ =-0.007838,$
$N_1''\ =-0.006321,$
$N_1'''\ =-0.001325,$

$N_1^{IV}=+0.00000952,$
$N_1^{V}\ =+0.00001345,$
$N_1^{VI}=-0.00000696,$
$N_1^{VII}=-0.00000042.$

For the root $g_2=-17''.393390$, we find,

$$x_2=-\frac{6.863228}{10^{13}}N_2,\ y_2=+\frac{2.889470}{10^{13}}N_2,\ z_2=\frac{488.6204}{10^{12}}N_2^2.$$

Whence $\beta_2=292^\circ\ 49'\ 53''.2$; and log. $N_2=7.1829906$.

$N_2\ =+0.001524,$
$N_2'=-0.008478,$
$N_2''=+0.0069546,$
$N_2'''=+0.0506672,$

$N_2^{IV}=-0.00000251,$
$N_2^{V}\ =-0.00002140,$
$N_2^{VI}=+0.00000208,$
$N_2^{VII}=+0.00000024.$

For the root $g_3=-18''.408914$, we get,

$$x_3=-\frac{6.724398}{10^{13}}N_3;\ y_3=-\frac{2.217061}{10^{13}}N_3;\ z_3=\frac{1925.3617}{10^{13}}N_3^2.$$

Whence $\beta_3=251^\circ\ 45'\ 8''.6$; and log. $N_3=7.5655490$.

$N_3\ =+0.003677,$
$N_3'=-0.022428,$
$N_3''=+0.024477,$
$N_3'''=-0.0375951,$

$N_3^{IV}=-0.000000072,$
$N_3^{V}\ =-0.000000557,$
$N_3^{VI}=+0.0000000463,$
$N_3^{VII}=+0.000000005.$

For the root $g_4=0''$, we get,

$$x_4=+\frac{0.7256453}{10^{10}}N_4;\quad y_4=-\frac{0.2113461}{10^{10}}N_4;\quad z_4=\frac{27.24403}{10^{10}}N_4^2.$$

Whence $\beta_4=106^\circ\ 14'\ 18''.0$; log. $N_4=8.4431335$.

And $N_4=N_4'=N_4''=N_4'''=N_4^{IV}=N_4^{V}=N_4^{VI}=N_4^{VII}=+0.02774173$.

For the root $g_5=-0''.661666$, we get,

$$x_5=+\frac{0.1016992}{10^{10}}N_5^{IV};\ y_5=-\frac{0.2717211}{10^{10}}N_5^{IV};\ z_5=\frac{24.19165}{10^{9}}N_5^{IV}.$$

Whence $\beta_5=20^\circ\ 31'\ 24''.6$; and log. $N_5^{IV}=7.0789383$.

$N_5\ =+0.001478,$
$N_5'=+0.001357,$
$N_5''=+0.001329,$
$N_5'''=+0.001259,$

$N_5^{IV}=+0.0011994,$
$N_5^{V}\ =+0.0011577,$
$N_5^{VI}=-0.00112485,$
$N_5^{VII}=-0.0117882.$

For the root $g_6 = -2''.916082$, we get,

$$x_6 = +\frac{0.3678921}{10^{10}} N_6^{IV};\quad y_6 = -\frac{0.3544802}{10^{10}} N_6^{IV};\quad z_6 = \frac{580.9484}{10^{10}} N_6^{IV2}.$$

Whence $\beta_6 = 133^\circ\ 56'\ 10''.8$; and log. $N_6^{IV} = 6.9441833$.

$$\begin{array}{ll}
N_6 \ = +0.003128, & N_6^{IV} = +0.0008794, \\
N_6' \ = +0.001811, & N_6^{V} \ = +0.0007180, \\
N_6'' = +0.001623, & N_6^{VI} = -0.0176872, \\
N_6''' = +0.001156, & N_6^{VII} = +0.0019010.
\end{array}$$

For the root $g_7 = -25''.934567$, we get,

$$x_7 = -\frac{0.2996623}{10^{10}} N_7^{IV};\quad y_7 = +\frac{0.2203054}{10^{10}} N_7^{IV};\quad z_7 = \frac{59.03157}{10^{10}} N_7^{IV2}.$$

Whence $\beta_7 = 306^\circ\ 19'\ 21''.2$; log. $N_7^{IV} = 7.7993771$.

$$\begin{array}{ll}
N_7 \ = -0.0002652, & N_7^{IV} = +0.00630053, \\
N_7' \ = -0.0002932, & N_7^{V} \ = -0.0156928, \\
N_7'' = -0.0027275, & N_7^{VI} = +0.0006890, \\
N_7''' = -0.0092499, & N_7^{VII} = +0.00007720.
\end{array}$$

If these values be substituted in equations (F), we shall have the complete values of q, q', q'', &c., p, p', p'', &c., from which we can obtain the inclination of the orbits of all the planets to the fixed ecliptic of 1850, and the longitudes of the nodes, on the same plane and referred to the equinox of 1850, by the formulæ

$$\tan\phi = \sqrt{p^2+q^2};\quad \tan\theta = p \div q. \tag{412}$$

8. If we now substitute in equations (F), the values of q and p, we shall get

$$q = \tan\phi\cos\theta = N\cos(gt+\beta) + N_1\cos(g_1t+\beta_1) + N_2\cos(g_2t+\beta_2) + \text{\&c.}; \tag{413}$$

$$p = \tan\phi\sin\theta = N\sin(gt+\beta) + N_1\sin(g_1t+\beta_1) + N_2\sin(g_2t+\beta_2) + \text{\&c.} \tag{414}$$

Multiplying equations (413) by $\sin(gt+\beta)$, and (414) by $-\cos(gt+\beta)$, we shall get, by adding their products, and reducing

$$\left.\begin{array}{r}\tan\phi\sin(\theta-gt-\beta) = N_1\sin\{(g_1-g)t+\beta_1-\beta\} + N_2\sin\{(g_2-g)t \\ +\beta_2-\beta\} + \text{\&c.}\end{array}\right\} \quad 415$$

If we multiply (413) by $\cos(gt+\beta)$, and (414) by $\sin(gt+\beta)$, we shall get, by adding their products, and reducing

$$\left.\begin{array}{r}\tan\phi\cos(\theta-gt-\beta) = N + N_1\cos\{(g_1-g)t+\beta_1-\beta\} + N_2\cos\{(g_2-g)t \\ +\beta_2-\beta\} + \text{\&c.}\end{array}\right\} \quad (416)$$

Dividing equation (415) by (416) we eliminate $\tan\phi$, and find,

$$\tan(\theta-gt-\beta) = \left.\frac{N_1\sin\{(g_1-g)t+\beta_1-\beta\} + N_2\sin\{(g_2-g)t+\beta_2-\beta\} + \text{\&c.}}{N + N_1\cos\{(g_1-g)t+\beta_1-\beta\} + N_2\cos\{(g_2-g)t+\beta_2-\beta\} + \text{\&c.}}\right\} \quad (417)$$

When the sum $N_1+N_2+N_3+$ &c. of the coefficients of the cosines of the denominator, taken positively, is less than N, $\tan(\theta-gt-\beta)$ cannot become infinite; the angle $(\theta-gt-\beta)$ cannot become a right angle: consequently, the mean motion of the node will, in this case, be equal to gt. The analysis of § 19 being applied to equation (416), will show that

$$\left.\begin{aligned} \textit{maximum}\ \tan\phi &= N + N_1+N_2+N_3+\ \&c.;\\ \text{and}\ \textit{minimum}\ \tan\phi &= N - \{N_1+N_2+N_3+\ \&c.\}\end{aligned}\right\} \tag{418}$$

We shall now substitute the numbers which we have already computed, in these equations, for the purpose of determining the maximum and minimum values of the inclinations of the different orbits to the fixed ecliptic of 1850, and the mean motions of the nodes of the different planets on that plane.

9. For the planet *Mercury*, we have,

Maximum $\tan\phi = N + N_1 + N_2 + N_3 +$ &c.$=0.187242$. One-half of this is 0.093621, which being less than N, it follows that N exceeds the sum of all the remaining terms; consequently, the mean motion of Mercury's node is equal to g, or $-5''.126112$. The maximum inclination of his orbit to the ecliptic of 1850 is 10° 36′ 20″; and the minimum inclination is 3° 47′ 8″.

The substitution of the numbers for the other planets shows that the minimum inclinations of all the other planetary orbits to the ecliptic of 1850 are equal to nothing; consequently, the mean motions of the nodes on that plane are indeterminate. The maximum inclinations of the different orbits are as follows:—

	Max. inclination.			Max. inclination.	
Venus,	4°	51′	*Jupiter*,	2°	4′
Earth,	4	41	*Saturn*,	2	36
Mars,	7	28	*Uranus*,	2	42.5
			Neptune,	2	22.7

Having thus given the solution of the fundamental equations for the assumed masses, it now remains to determine the coefficients depending on the variation of the masses. This we shall do by using the same finite variations of the masses as were employed in finding the similar coefficients of the variations of the constants on which the eccentricities and perihelia depend.

10. If we now suppose that $\mu=+1.5$, we shall obtain the values of the fundamental quantities which are to be used in the computation by simply making all the terms of equations (153–169) positive. We shall then obtain the following

Fundamental Equations for $\mu=+\frac{3}{2}$: *or for* $m=\frac{1}{1946300.4}$.

$$\left.\begin{aligned} A\ &= g^2+38.201888.g\ +183.882325;\\ A' &= g^2+23.2189305.g+\ 98.9957893;\\ A'' &= g^2+18.9824430.g+\ 73.7725468;\\ A_1 &= g^2+18.4081571.g+\ 60.3279033;\\ A_2 &= g^2+13.1928504.g+\ \ 8.981549;\\ A_3 &= g^2+26.3821161.g+\ \ 9.881338;\end{aligned}\right\} \tag{419}$$

$$\left.\begin{aligned}
D &= g^2+44.8296684.g+595.295403\,;\\
D' &= g^2+52.2739916.g+609.992181\,;\\
D'' &= g^2+32.2340961.g+257.2490518\,;\\
D_1 &= g^2+43.8986994.g+172.463056\,;\\
D_2 &= g^2+46.4943411.g+\ \ 32.948378\,;\\
D_3 &= g^2+\ \ 3.4291799.g+\ \ \ 1.69252064.
\end{aligned}\right\}\qquad(420)$$

$$\left.\begin{aligned}
B=\{g+32.51109\}b\,;\quad B' &= \{g+17.5915487\}b\,;\\
B'' &= \{g+13.402657\}b\,;
\end{aligned}\right\}\qquad(421)$$

$$\left.\begin{aligned}
C &= -\{g+22.316375\}\ [9.4381189]b'\,;\\
C' &= -\{g+17.6215550\}[9.1138076]b'\,;\\
C'' &= +[0.3976676]b'\,;\\
C''' &= +[0.4411620]b'\,;
\end{aligned}\right\}\qquad(422)$$

$$\left.\begin{aligned}
E &= -[9.9807377]b''\,;\\
E' &= -\{g+22.5132928\}[8.9723624]b''\,;\\
E'' &= -\{g+17.5690284\}[9.7501125]b''\,;\\
E''' &= -[1.0016537]b''\,;
\end{aligned}\right\}\qquad(423)$$

$$\left.\begin{aligned}
F &= +[8.1679376]b'''\,;\\
F' &= -[9.1453586]b'''\,;\\
F'' &= -\{g+14.665067\ \}[0.7927855]b'''\,;\\
F''' &= -\{g+34.6524353\}[9.6907241]b'''\,;
\end{aligned}\right\}\qquad(424)$$

$$\left.\begin{aligned}
B_1=\{g+4.5202234\}b_1\,;\quad B_2 &= \{g+\ 0.7124548\}b_1\,;\\
B_3 &= \{g+18.8176405\}b_1\,;
\end{aligned}\right\}\qquad(425)$$

$$\left.\begin{aligned}
C_1 &= -\{g+4.1329892\ \}[9.5433087]b_2\,;\\
C_2 &= -\{g+0.73072874\}[9.4349711]b_2\,;\\
C_3 &= +[0.8644527]b_2\,;\\
C_4 &= +[0.8654649]b_2\,;
\end{aligned}\right\}\qquad(426)$$

$$\left.\begin{aligned}
E_1 &= -[8.3317448]b_3\,;\\
E_2 &= -\{g+39.7657102\ \}[8.7437718]b_3\,;\\
E_3 &= -\{g+\ 0.64863178\}[0.7242832]b_3\,;\\
E_4 &= -[0.9647509]b_3\,;
\end{aligned}\right\}\qquad(427)$$

$$\left.\begin{aligned}
F_1 &= +[7.7244692]b_4\,;\\
F_2 &= -[0.3564628]b_4\,;\\
F_3 &= -\{g+27.805482\}\ [1.5514854]b_4\,;\\
F_4 &= -\{g+45.7636124\}[9.4626364]b_4.
\end{aligned}\right\}\qquad(428)$$

$$\left.\begin{aligned}
&g^4+47.8463930.g^3+784.2708337.g^2\\
&\ +5058.994655.g+10978.29258
\end{aligned}\right\}=(\chi,\chi_1,\chi_2,\chi_3)\,;\quad(429)$$

$$\left.\begin{aligned}
&g^4+29.5229572.g^3+95.0823272.g^2\\
&\ \ +51.151359.g+0.529116
\end{aligned}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7).\quad(430)$$

The values of b, b', b'', and b''' are given by equations (405), and the values of b_1, b_2, b_3, and b_4 are given by equations (406), by merely multiplying the coefficients of N by $1+\mu=2.5$.

If we put equations (429) and (430) equal to nothing, they will give

$$
\begin{array}{ll}
g_1=-\ \ 4''.8105312, & g_4=-\ \ 0''.0105499, \\
g_2=-\ \ 7\ .0595858, & g_5=-\ \ 0\ .6629939, \\
g_3=-17\ .4356542, & g_6=-\ \ 2\ .9169622, \\
g_4=-18\ .5406218, & g_7=-25\ .9324509.
\end{array}
$$

The solutions of equations (419–430) will now give the following values—remembering that the coefficients of equations (371–384) remain unchanged.

For the root g, we get,

$$
\begin{array}{ll}
g\ =-4''.815328, & \\
N'\ =+0.2099057N & \text{log. } 9.3210242, \\
N''\ =+0.1471310N & \text{“ } 9.1677040, \\
N'''\ =+0.0292551N & \text{“ } 8.4662020, \\
N^{IV}=-0.000390605N & \text{“ } 6.5917376n, \\
N^{V}\ =-0.000486221N & \text{“ } 6.6868340n, \\
N^{VI}=+0.000495923N & \text{“ } 6.6954144, \\
N^{VII}=+0.000008719136N & \text{“ } 4.9404735.
\end{array}
$$

For the root g_1, we get,

$$
\begin{array}{ll}
g_1=-7''.064535, & \\
N_1'\ =-0.400690N_1 & \text{log. } 9.6028084n, \\
N_1''\ =-0.3382646N_1 & \text{“ } 9.5292564n, \\
N_1'''\ =-0.0725079N_1 & \text{“ } 8.8603854n, \\
N_1^{IV}=+0.000451081N_1 & \text{“ } 6.6542543, \\
N_1^{V}\ =+0.000660230N_1 & \text{“ } 6.8196951, \\
N_1^{VI}=-0.000300091N_1 & \text{“ } 6.4772529n, \\
N_1^{VII}=-0.0000202801N_1 & \text{“ } 5.3070710n.
\end{array}
$$

For the root g_2, we get,

$$
\begin{array}{ll}
g_2=-17''.436558, & \\
N_2\ =-\ \ 5.522000N_2 & \text{log. } 0.7420964n, \\
N_2''\ =+\ \ 4.240878N_2 & \text{“ } 0.6274557, \\
N_2'''\ =+37.23564N_2 & \text{“ } 1.5709589, \\
N_2^{IV}=-\ \ 0.001749727N_2 & \text{“ } 7.2429702n, \\
N_2^{V}\ =-\ \ 0.01522711N_2 & \text{“ } 8.1826175n, \\
N_2^{VI}=+\ \ 0.001476315N_2 & \text{“ } 7.1691790, \\
N_2^{VII}=+\ \ 0.0001698568N_2 & \text{“ } 6.2300830.
\end{array}
$$

For the root g_3, we get,

$g_3 = -18''.540625,$
$N_3' = -6.10629 N_3$ log. $0.7857773n$,
$N_3'' = +6.47848 N_3$ “ 0.8114730,
$N_3''' = -8.653764 N_3$ “ $0.9372052n$,
$N_3^{IV} = -0.0000396789 N_3$ “ $5.5985600n$,
$N_3^{V} = -0.000582834 N_3$ “ $6.7655450n$,
$N_3^{VI} = +0.0000498987 N_3$ “ 5.6980890,
$N_3^{VII} = +0.00000572183 N_3$ “ 4.7575350.

For the root $g_4 = 0''$, we get,

$N_4' = N_4'' = N_4''' = N_4^{IV} = \&c.$

For the root g_5, we get,

$g_5 = -0''.661636,$
$N_5 = +1.235046 N_5^{IV}$ log. 0.0916830,
$N_5' = +1.135283 N_5^{IV}$ “ 0.0551040,
$N_5'' = +1.110556 N_5^{IV}$ “ 0.0455402,
$N_5''' = +1.049956 N_5^{IV}$ “ 0.0211710,
$N_5^{V} = +0.965323 N_5^{IV}$ “ 9.9846725,
$N_5^{VI} = -0.938071 N_5^{IV}$ “ $9.9722356n$,
$N_5^{VII} = -9.829980 N_5^{IV}$ “ $0.9925526n$.

For the root g_6, we get,

$g_6 = -2''.916041,$
$N_6 = +\ 3.695208 N_6^{IV}$ log. 0.5676388,
$N_6' = +\ 2.162242 N_6^{IV}$ “ 0.3349042,
$N_6'' = +\ 1.91106 N_6^{IV}$ “ 0.2812740,
$N_6''' = +\ 1.327425 N_6^{IV}$ “ 0.1230098,
$N_6^{V} = +\ 0.816341 N_6^{VI}$ “ 9.9118716,
$N_6^{VI} = -20.12041 N_6^{IV}$ “ $1.3036368n$,
$N_6^{VII} = +\ 2.162624 N_6^{IV}$ “ 0.3349790.

For the root g_7, we get,

$g_7 = -25''.934626,$
$N_7 = -0.0421230 N_7^{IV}$ log. $8.6245191n$,
$N_7' = -0.0455785 N_7^{IV}$ “ $8.6587603n$,
$N_7'' = -0.4351129 N_7^{IV}$ “ $9.6386020n$,
$N_7''' = -1.469776 N_7^{IV}$ “ $0.1672216n$,
$N_7^{V} = -2.490698 N_7^{IV}$ “ $0.3963210n$,
$N_7^{VI} = +0.1080897 N_7^{IV}$ “ 9.0337842,
$N_7^{VII} = +0.01225267 N_7^{IV}$ “ 8.0882305.

11. Substituting these values in equations (408) and (409), we shall get the following values:—

For the root $g=-4''.815328$, we get,

$$x=+\frac{1.524075}{10^{14}}N;\quad y=+\frac{3.435486}{10^{14}}N;\quad z=\frac{36.65228}{10^{14}}N^2.$$

Whence $\beta=23^\circ\ 55'\ 15''.6$; and $\log. N=9.2108901$.

$$\begin{aligned}
N &=+0.162514, & N^{IV} &=-0.0000635,\\
N' &=+0.034037, & N^{V} &=-0.0000790,\\
N'' &=+0.023913, & N^{VI} &=+0.0000806,\\
N''' &=+0.004754, & N^{VII} &=+0.0000014.
\end{aligned}$$

For the root $g_1=-7''.064535$, we get,

$$x_1=+\frac{1.725880}{10^{14}}N_1;\quad y_1=-\frac{46}{10^{20}}N_1;\quad z_1=+\frac{75.88155}{10^{14}}N_1^2.$$

Whence $\beta_1=90^\circ\ 0'\ 7''.1$; and $\log. N_1=8.3568764$.

$$\begin{aligned}
N_1 &=+0.022744, & N_1^{IV} &=+0.0000103,\\
N_1' &=-0.009114, & N_1^{V} &=+0.0000150,\\
N_1'' &=-0.007694, & N_1^{VI} &=-0.0000068,\\
N_1''' &=-0.001649, & N_1^{VII} &=-0.000000046.
\end{aligned}$$

For the root $g_2=-17''.043656$, we get,

$$x_2=-\frac{6.709998}{10^{13}}N_2;\quad y_2=+\frac{3.568591}{10^{13}}N_2;\quad z_2=\frac{5820.032}{10^{13}}N_2^2.$$

Whence $\beta_2=298^\circ\ 0'\ 19''.7$; and $\log. N_2=7.1158842$.

$$\begin{aligned}
N_2 &=+0.001306, & N_2^{IV} &=-0.00000228,\\
N_2' &=-0.007211, & N_2^{V} &=-0.00001988,\\
N_2'' &=+0.005538, & N_2^{VI} &=+0.00000193,\\
N_2''' &=+0.048623, & N_2^{VII} &=+0.00000022.
\end{aligned}$$

For the root $g_3=-18''.540625$, we get,

$$x_3=-\frac{6.589852}{10^{13}}N_3;\quad y_3=-\frac{1.924650}{10^{13}}N_3;\quad z_3=\frac{1774.3737}{10^{13}}N_3^2.$$

Whence $\beta_3=253^\circ\ 43'\ 8''.0$; and $\log. N_3=7.5876055$.

$$\begin{aligned}
N_3 &=+0.003869, & N_3^{IV} &=-0.000000154,\\
N_3' &=-0.023626, & N_3^{V} &=-0.000002255,\\
N_3'' &=+0.025066, & N_3^{VI} &=+0.0000001931,\\
N_3''' &=-0.033482, & N_3^{VII} &=+0.0000002213.
\end{aligned}$$

For the root $g_4=0$, we get,

$$x_4=+\frac{0.7257772}{10^{10}}N_4;\quad y_4=-\frac{0.2112211}{10^{10}}N_4;\quad z_4=\frac{27.24551}{10^{10}}N_4^2.$$

Whence $\beta_4=106^\circ\ 13'\ 35''.2$; $N_4=+0.0277436$, $\log. N_4=8.4431625$.

For the root $g_5=-0''.661636$, we get,

$$x_5=+\frac{0.1018241}{10^{10}}N_5^{IV};\ y_5=+\frac{0.2719087}{10^{10}}N_5^{IV};\ z_5=\frac{24.19508}{10^9}N_5^{IV2}.$$

Whence $\beta_5=20^\circ\ 32'\ 21''.0$; and log. $N_5^{IV}=7.0792052$.

$$\begin{array}{ll} N_5\ =+0.001482, & N_5^{IV}=+0.00120007, \\ N_5'\ =+0.001362, & N_5^{V}\ =+0.00115846, \\ N_5''=+0.001333, & N_5^{VI}=-0.00112554, \\ N_5'''=+0.001260, & N_5^{VII}=-0.0117964. \end{array}$$

For the root $g_6=-2''.916041$, we get,

$$x_6=+\frac{0.3683810}{10^{10}}N_6^{IV},\ y_6=-\frac{0.3540483}{10^{10}}N_6^{IV},\ z_6=\frac{581.3949}{10^{10}}N_6^{IV}.$$

Whence $\beta_6=133^\circ\ 51'\ 48''.4$; and log. $N_6^{IV}=6.9438946$.

$$\begin{array}{ll} N_6\ =+0.003247, & N_6^{IV}=+0.0008788, \\ N_6'\ =+0.001900, & N_6^{V}\ =+0.0007174, \\ N_6''=+0.001679, & N_6^{VI}=-0.0176820, \\ N_6'''=+0.001166, & N_6^{VII}=+0.0019005. \end{array}$$

For the root $g_7=-25''.934657$, we get,

$$x_7=-\frac{0.2996641}{10^{10}}N_7^{IV};\ y_7=+\frac{0.2202989}{10^{10}}N_7^{IV};\ z_7=\frac{59.03124}{10^{10}}N_7^{IV2}.$$

Whence $\beta_7=306^\circ\ 19'\ 17''.6$; and log. $N_7^{IV}=7.7993764$.

$$\begin{array}{ll} N_7\ =-0.000265, & N_7^{IV}=+0.00630052, \\ N_7'\ =-0.000287, & N_7^{V}\ =-0.0156923, \\ N_7''=-0.002741, & N_7^{VI}=+0.0006891, \\ N_7'''=-0.007355, & N_7^{VII}=+0.00007720. \end{array}$$

12. For an increment of $\frac{1}{20}$, in the mass of *Venus*, we have the preliminary computations by merely making all the coefficients positive in equations (193-208). We shall then obtain the following

Fundamental Equations for $\mu'=+\frac{1}{20}$; *or for* $m'=\frac{1}{371428.6}$.

$$\left.\begin{array}{l} A\ =g^2+38.4851145.g+188.270582; \\ A'=g^2+23.3470945.g+101.1013581; \\ A''=g^2+18.9541253.g+\ \ 75.3044562; \\ A_1=g^2+18.4082271.g+\ \ 60.3282264; \\ A_2=g^2+13.1929198.g+\ \ \ \ 8.981600; \\ A_3=g^2+26.3821932.g+\ \ \ \ 9.882701. \end{array}\right\}\quad(431)$$

$$\left.\begin{array}{l} D\ =g^2+45.3182858.g+610.942268; \\ D'=g^2+53.1764531.g+626.087193; \\ D''=g^2+32.4522201.g+261.0475918; \\ D_1=g^2+43.8987079.g+172.463115; \\ D_2=g^2+46.4943490.g+\ \ 32.948389; \\ D_3=g^2+\ \ 3.4291807.g+\ \ \ \ 1.69252138. \end{array}\right\}\quad(432)$$

$$\left.\begin{array}{ll} B=\{g+32.71670\}b; & B'=\{g+17.6040547\}b; \\ & B''=\{g+13.230123\}b; \end{array}\right\} \qquad (433)$$

$$\left.\begin{array}{l} C\ =-\{g+22.521994\}\,[9.4381189]b'; \\ C'\ =-\{g+17.6340610\}[9.1138076]b'; \\ C''=+[0.4188569]b'; \\ C'''=+[0.4623513]b'; \end{array}\right\} \qquad (434)$$

$$\left.\begin{array}{l} E\ =-[9.9807377]b''; \\ E'\ =-\{g+22.7962912\}[8.9723624]b''; \\ E''=-\{g+17.5815344\}[9.7501125]b''; \\ E'''=-[1.0016537]b''; \end{array}\right\} \qquad (435)$$

$$\left.\begin{array}{l} F\ =+[8.1679376]b'''; \\ F'\ =-[9.1453586]b'''; \\ F''=-\{g+14.870686\}\,[0.7927855]b'''; \\ F'''=-\{g+35.542391\}\,[9.6907241]b'''; \end{array}\right\} \qquad (436)$$

$$\left.\begin{array}{ll} B_1=\{g+4.5202241\}b_1; & B_2=\{g+\ 0.7124549\}b_1; \\ & B_3=\{g+18.8176481\}b_1; \end{array}\right\} \qquad (437)$$

$$\left.\begin{array}{l} C_1=-\{g+4.1329899\}[9.5433087]b_2; \\ C_2=-\{g+0.7307288\}[9.4349711]b_2; \\ C_3=+[0.8644527]b_2; \\ C_4=+[0.8654649]b_2; \end{array}\right\} \qquad (438)$$

$$\left.\begin{array}{l} E_1=-[8.3317448]b_3; \\ E_2=-\{g+39.7657180\}[8.7437718]b_3; \\ E_3=-\{g+0.64863188\}[0.7242832]b_3; \\ E_4=-[0.9647509]b_3; \end{array}\right\} \qquad (439)$$

$$\left.\begin{array}{l} F_1=+[7.7244692]b_4; \\ F_2=-[0.3564628]b_4; \\ F_3=-\{g+2.7805489\}[1.5514854]b_4; \\ F_4=-\{g+45.763620\}[9.4626364]b_4; \end{array}\right\} \qquad (440)$$

$$\left.\begin{array}{l} g^4+47.9507108.g^3+787.5462101.g^2 \\ \quad +5098.436146.g+11226.89493 \end{array}\right\}=(\chi, \chi_1, \chi_2, \chi_3). \qquad (441)$$

$$\left.\begin{array}{l} g^4+29.5230351.g^3+95.0839594.g^2 \\ \quad +51.156083.g\ +\ 0.531357 \end{array}\right\}=(\chi_4, \chi_5, \chi_6, \chi_7). \qquad (442)$$

The values of b, b', b'', and b''' are given by equations (405); and the values of b_1, b_2, b_3, and b_4 are given by equations (406), by merely multiplying the coefficients of N' by $1+\mu'=1.05$.

If we put equations (441) and (442) equal to nothing, they will give

$$\begin{array}{ll} g\ =-\ 5''.1965445, & g_4=-\ 0''.0105949, \\ g_1=-\ 6\,.6295555, & g_5=-\ 0\,.6629993, \\ g_2=-17\,.4583971, & g_6=-\ 2\,.9169649, \\ g_3=-18\,.6662138, & g_7=-25\,.9324761. \end{array}$$

The solutions of equations (431–442) will now give the following values:—

For the root g, we get,

$g=-5''.201065$,
$N' = +0.1374398N$ log. 9.1381124,
$N'' = +0.1000556N$ " 9.0002416,
$N''' = +0.0201689N$ " 8.3046820,
$N^{IV} = -0.000233031N$ " $6.3674143n$,
$N^{V} = -0.000298026N$ " $6.4742546n$,
$N^{VI} = +0.000252528N$ " 6.4023092,
$N^{VII} = +0.00000754208N$ " 4.8774912.

For the root g_1, we get,

$g_1=-6''.6347695$,
$N_1' = -0.2372900N_1$ log. $9.3752793n$,
$N_1'' = -0.1935676N_1$ " $9.2868326n$,
$N_1''' = -0.0409512N_1$ " $8.6122666n$,
$N_1^{IV} = +0.000292034N_1$ " 6.4654333,
$N_1^{V} = +0.000413818N_1$ " 6.6168096,
$N_1^{VI} = -0.000211538N_1$ " $6.3253889n$,
$N_1^{VII} = -0.0000129222N_1$ " $5.1113360n$.

For the root g_2, we get,

$g_2=-17''.459300$,
$N_2' = -\ 5.234937N_2$ log. $0.7189114n$,
$N_2'' = +\ 4.210629N_2$ " 0.6243470,
$N_2''' = +40.01022N_2$ " 1.6021710,
$N_2^{IV} = -\ 0.00184238N_2$ " $7.2653793n$,
$N_2^{V} = -\ 0.01622134N_2$ " $8.2100866n$,
$N_2^{VI} = +\ 0.001569054N_2$ " 7.1956378,
$N_2^{VII} = +\ 0.000180556N_2$ " 6.2566120.

For the root g_3, we get,

$g_3=-18''.666221$,
$N_3' = -5.839411N_3$ log. $0.7663690n$,
$N_3'' = +6.569047N_3$ " 0.8175024,
$N_3''' = -7.879027N_3$ " $0.8964726n$,
$N_3^{IV} = -0.0000520881N_3$ " $5.7167383n$,
$N_3^{V} = -0.000963122N_3$ " $6.9836813n$,
$N_3^{VI} = +0.0000818396N_3$ " 5.9129634,
$N_3^{VII} = +0.00000942162N_3$ " 4.9741254.

For the root g_4, we get,

$g_4=0''$, and $N_4=N_4'=N_4''=$, &c.

For the root g_5, we get,

$$g_5 = -0''.661664,$$

$N_5 \;= +1.229553 N_5^{IV}$	log. 0.0897471,
$N_5' \;= +1.131670 N_5^{IV}$	" 0.0537194,
$N_5'' \;= +1.108860 N_5^{IV}$	" 0.0448769,
$N_5''' \;= +1.049663 N_5^{IV}$	" 0.0210502,
$N_5^{V} = +0.9653253 N_5^{IV}$	" 9.9846737,
$N_5^{VI} = -0.9380050 N_5^{IV}$	" 9.9722051n,
$N_5^{VII} = -9.829623 N_5^{IV}$	" 0.9925368n.

For the root g_6, we get,

$$g_6 = -2''.9160753,$$

$N_6 \;= +\; 3.484775 N_6^{IV}$	log. 0.5421747,
$N_6' \;= +\; 2.063762 N_6^{IV}$	" 0.3146597,
$N_6'' \;= +\; 1.853037 N_6^{IV}$	" 0.2678842,
$N_6''' \;= +\; 1.316456 N_6^{IV}$	" 0.1194064,
$N_6^{V} = +\; 0.8164332 N_6^{IV}$	" 9.9119207,
$N_6^{VI} = -20.11465 N_6^{IV}$	" 1.3035125n,
$N_6^{VII} = +\; 2.159171 N_6^{IV}$	" 0.3342872.

For the root g_7, we get,

$$g_7 = -25''.934665,$$

$N_7 \;= -0.0423304 N_7^{IV}$	log. 8.6266524n,
$N_7' \;= -0.0418973 N_7^{IV}$	" 8.6221861n,
$N_7'' \;= -0.4430757 N_7^{IV}$	" 9.6464778n,
$N_7''' \;= -1.470350 N_7^{IV}$	" 0.1674208n,
$N_7^{V} = -2.490680 N_7^{IV}$	" 0.3963178n,
$N_7^{VI} = +0.1093404 N_7^{IV}$	" 9.0387805,
$N_7^{VII} = +0.01225256 N_7^{IV}$	" 8.0882264.

13. If we now substitute these values in equations (408) and (409), we shall obtain the following quantities:—

For the root $g = -5''.201065$, we get,

$$x = +\frac{0.638032}{10^{14}} N; \quad y = \frac{1.6964973}{10^{14}} N; \quad z = \frac{15.329701}{10^{14}} N^2.$$

Whence $\beta = 20°\ 36'\ 29''.8$; and log. $N = 9.0727385$.

$N \;= +0.118233,$
$N' \;= +0.016251,$
$N'' \;= +0.011804,$
$N''' \;= +0.002385,$

$N^{IV} = -0.00002755,$
$N^{V} \;= -0.00003524,$
$N^{VI} = +0.00002986,$
$N^{VII} = +0.0000008918.$

For the root $g_1=-6''.634795$, we get,

$$x_1=+\frac{0.637032}{10^{14}}N_1;\quad y_1=-\frac{0.4606865}{10^{14}}N_1;\quad z_1=\frac{27.744169}{10^{14}}N_1^2.$$

Whence $\beta_1=125°\ 52'\ 26''.4$; and log. $N_1=8.4523391$.

$N_1\ =+0.028336,$
$N_1'\ =-0.006724,$
$N_1''\ =-0.005485,$
$N_1'''\ =-0.001160,$
$N_1^{IV}=+0.000008275,$
$N_1^{V}\ =+0.000011726,$
$N_1^{VI}=-0.000005994,$
$N_1^{VII}=-0.0000003662.$

For the root $g_2=-17''.459300$, we get,

$$x_2=-\frac{6.884438}{10^{13}}N_2,\ y_2=+\frac{3.578728}{10^{13}}N_2,\ z_2=\frac{6546.046}{10^{13}}N_2^2.$$

Whence $\beta_2=297°\ 28'\ 0''.1$; and log. $N_2=7.0738291$.

$N_2\ =+0.001185,$
$N_2'=-0.006205,$
$N_2''=+0.004991,$
$N_2'''=+0.047424,$
$N_2^{IV}=-0.000002184,$
$N_2^{V}\ =-0.000019227,$
$N_2^{VI}=+0.000001860,$
$N_2^{VII}=+0.000000214.$

For the root $g_3=-18''.666222$, we get,

$$x_3=-\frac{6.787567}{10^{13}}N_3;\ y_3=-\frac{1.953449}{10^{13}}N_3;\ z_3=\frac{1727.1619}{10^{13}}N_3^2.$$

Whence $\beta_3=253°\ 56'\ 39''.4$; and log. $N_3=7.6116607$.

$N_3\ =+0.004089,$
$N_3'=-0.023880,$
$N_3''=+0.026863,$
$N_3'''=-0.032221,$
$N_3^{IV}=-0.000000213,$
$N_3^{V}\ =-0.000003939,$
$N_3^{VI}=+0.000000335,$
$N_3^{VII}=+0.0000000385.$

For the root $g_4=0''$, we get,

$$x_4=+\frac{0.7256935}{10^{10}}N_4;\quad y_4=-\frac{0.2113335}{10^{10}}N_4;\quad z_4=\frac{27.24487}{10^{10}}N_4^2.$$

Whence $\beta_4=106°\ 14'\ 11''.1$; and log. $N_4=8.4431447$.
$N_4=N_4'=N_4''=N_4'''=N_4^{IV}=N_4^{V}=N_4^{VI}=N_4^{VII}=0.02774244.$

For the root $g_5=-0''.661664$, we get,

$$x_5=+\frac{0.1017327}{10^{10}}N_5^{IV};\ y_5=+\frac{0.2717512}{10^{10}}N_5^{IV};\ z_5=\frac{241.9334}{10^{10}}N_5^{IV2}.$$

Whence $\beta_5=20°\ 31'\ 39''.5$; and log. $N_5^{IV}=7.0789678$.

$N_5\ =+0.001475,$
$N_5'=+0.001357,$
$N_5''=+0.001330,$
$N_5'''=+0.001259,$
$N_5^{IV}=+0.00119941,$
$N_5^{V}\ =+0.00115783,$
$N_5^{VI}=-0.00112485,$
$N_5^{VII}=-0.0117895.$

For the root $g_6 = -2''.916075$, we get,

$$x_6 = +\frac{0.3678204}{10^{10}} N_6^{IV};\quad y_6 = -\frac{0.3543521}{10^{10}} N_6^{IV};\quad z_6 = \frac{581.0177}{10^{10}} N_6^{IV2}.$$

Whence $\beta_6 = 133^\circ\ 55'\ 53''.6$; and log. $N_6^{IV} = 6.9440119$.

$$\begin{aligned} N_6 &= +0.003063, & N_6^{IV} &= +0.000879047, \\ N_6' &= +0.001814, & N_6^{V} &= +0.000717683, \\ N_6'' &= +0.001629, & N_6^{VI} &= -0.0176817, \\ N_6''' &= +0.001157, & N_6^{VII} &= +0.00189801. \end{aligned}$$

For the root $g_7 = -25''.934665$, we get,

$$x_7 = -\frac{0.2996518}{10^{10}} N_7^{IV};\quad y_7 = +\frac{0.2202706}{10^{10}} N_7^{IV};\quad z_7 = \frac{59.03076}{10^{10}} N_7^{IV2}.$$

Whence $\beta_7 = 306^\circ\ 19'\ 9''.1$; and log. $N_7^{IV} = 7.7993490$.

$$\begin{aligned} N_7 &= -0.0002667, & N_7^{IV} &= +0.00630012, \\ N_7' &= -0.0002640, & N_7^{V} &= -0.0156916, \\ N_7'' &= -0.0027914, & N_7^{VI} &= +0.00068886, \\ N_7''' &= -0.0092634, & N_7^{VII} &= +0.000077193 \end{aligned}$$

14. For an increment of $\frac{1}{20}$, to the mass of the earth, we have the preliminary computations by merely making all the coefficients positive in equations (232–237). We shall then obtain the following

Fundamental Equations for $\mu'' = +\frac{1}{20}$; *or for* $m'' = 1 \div 351132.4$.

$$\left.\begin{aligned} A &= g^2 + 39.080610.g + 188.728586; \\ A' &= g^2 + 23.3039078.g + 99.5792048; \\ A'' &= g^2 + 19.0875991.g + 75.2609947; \\ A_1 &= g^2 + 18.4084593.g + 60.3292965; \\ A_2 &= g^2 + 13.1931503.g + 8.981769; \\ A_3 &= g^2 + 26.3824488.g + 9.887223; \end{aligned}\right\} \quad (443)$$

$$\left.\begin{aligned} D &= g^2 + 45.2956845.g + 610.444623; \\ D' &= g^2 + 52.4179477.g + 613.831274; \\ D'' &= g^2 + 32.3259257.g + 258.6223612; \\ D_1 &= g^2 + 43.8987357.g + 172.463309; \\ D_2 &= g^2 + 46.4943751.g + 32.948430; \\ D_3 &= g^2 + 3.4291834.g + 1.69252419. \end{aligned}\right\} \quad (444)$$

$$\left.\begin{aligned} B = \}g + 33.41905\}b;\quad B' &= \{g + 17.6677162\}b; \\ B'' &= \{g + 13.490445\}b; \end{aligned}\right\} \quad (445)$$

$$\left.\begin{aligned} C &= -\{g + 22.714602\}\ [9.4381189]b'; \\ C' &= -\{g + 17.6977225\}[9.1138076]b'; \\ C'' &= +[0.3976676]b'; \\ C''' &= +[0.4411620]b'; \end{aligned}\right\} \quad (446)$$

$$\left.\begin{aligned}
E\ &=-[0.0019270]b'';\\
E'&=-\{g+22.6010814\}[8.9723624]b'';\\
E''&=-\{g+17.6451959\}[9.7501125]b'';\\
E'''&=-[1.0228430]b'';
\end{aligned}\right\}\qquad(447)$$

$$\left.\begin{aligned}
F\ &=+[8.1679376]b''';\\
F'&=-[9.1453586]b''';\\
F''&=-\{g+14.680729\}\ [0.7927855]b''';\\
F'''&=-\{g+34.7402239\}[9.6907241]b''';
\end{aligned}\right\}\qquad(448)$$

$$\left.\begin{aligned}
B_1=\{g+4.5202263\}b_1;\ B_2&=\{g+\ 0.71245534\}b_1;\\
B_3&=\{g+18.8176739\}b_1;
\end{aligned}\right\}\qquad(449)$$

$$\left.\begin{aligned}
C_1&=-\{g+4.1329921\}[9.5433087]b_2;\\
C_2&=-\{g+0.7307293\}[9.4349711]b_2;\\
C_3&=+[0.8644527]b_2;\\
C_4&=+[0.8654649]b_2;
\end{aligned}\right\}\qquad(450)$$

$$\left.\begin{aligned}
E_1&=-[8.3317448]b_3;\\
E_2&=-\{g+39.7657436\}[8.7437718]b_3;\\
E_3&=-\{g+\ 0.6486324\}[0.7242832]b_3;\\
E_4&=-[0.9647509]b_3;
\end{aligned}\right\}\qquad(451)$$

$$\left.\begin{aligned}
F_1&=+[7.7244692]b_4;\\
F_2&=-[0.3564628]b_4;\\
F_3&=-\{g+\ 2.7805511\}[1.5514854]b_4;\\
F_4&=-\{g+45.7636454\}[9.4626364]b_4.
\end{aligned}\right\}\qquad(452)$$

$$\left.\begin{aligned}
&g^4+47.9724987.g^3+787.7904009.g^2\\
&\quad+5093.460923g+11190.32489
\end{aligned}\right\}=(\chi,\chi_1,\chi_2,\chi_3);\quad(453)$$

$$\left.\begin{aligned}
&g^4+29.5232934.g^3+95.0893749.g^2\\
&\quad+51.171764.g\ +\ 0.538790
\end{aligned}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7).\quad(454)$$

The values of b, b', b'', and b''' are given by equations (405); and the values of b_1, b_2, b_3, and b_4 are given by equations (406), by merely multiplying the coefficients of N'' by $1+\mu''=1.05$.

If we put the equations (453) and (454) equal to nothing, they will give

$$\begin{aligned}
g\ &=-\ 5''.1662610, & g_4&=-\ 0''.0107428,\\
g_1&=-\ 6\ .6253960, & g_5&=-\ 0\ .6630168,\\
g_2&=-17\ .5129033, & g_6&=-\ 2\ .9169739,\\
g_3&=-18\ .6679386, & g_7&=-25\ .9325598.
\end{aligned}$$

The solutions of equations (443–454) will now give the following values:—

For the root g, we get,

$g = -5''.170230$,

$N' = +0.1213375N$ log. 9.0839952,
$N'' = +0.0874828N$ " 8.9419226,
$N''' = +0.01800280N$ " 8.2553402,
$N^{IV} = -0.0002097093N$ " 6.3216176n,
$N^{V} = -0.0002676246N$ " 6.4275260n,
$N^{VI} = +0.0002299092N$ " 6.3615564,
$N^{VII} = +0.00000665906N$ " 4.8234126.

For the root g_1, we get,

$g_1 = -6''.631323$,

$N_1' = -0.2725185N_1$ log. 9.4353960n,
$N_1'' = -0.2210735N_1$ " 9.3445366n,
$N_1''' = -0.04781081N_1$ " 8.6795260n,
$N_1^{IV} = +0.000340372N_1$ " 6.5319537,
$N_1^{V} = +0.000482200N_1$ " 6.6832273,
$N_1^{VI} = -0.0002467402N_1$ " 6.3922400n,
$N_1^{VII} = -0.0000150588N_1$ " 5.1777901n.

For the root g_2, we get,

$g_2 = -17''.5137898$,

$N_2' = -\ 5.562224N_2$ log. 0.7452485n,
$N_2'' = +\ 4.103496N_2$ " 0.6131540,
$N_2''' = +38.24098N_2$ " 1.5825290,
$N_2^{IV} = -\ 0.001733292N_2$ " 7.2388717n,
$N_2^{V} = -\ 0.01569197N_2$ " 8.1956776n,
$N_2^{VI} = +\ 0.001509355N_2$ " 7.1787913,
$N_2^{VII} = +\ 0.0001737467N_2$ " 6.2399165.

For the root g_3, we get,

$g_3 = -18''.667944$,

$N_3' = -6.186960N_3$ log. 0.7914773n,
$N_3'' = +6.338887N_3$ " 0.8020130,
$N_3''' = -8.476533N_3$ " 0.9282183n,
$N_3^{IV} = -0.00004491846N_3$ " 5.6524248n,
$N_3^{V} = -0.0007949226N_3$ " 6.9003248n,
$N_3^{VI} = +0.00006740752N_3$ " 5.8287083,
$N_3^{VII} = +0.00000774975N_3$ " 4.8892878.

For the root g_4, we get,

$g_4 = 0''$, and
$N_4 = N_4' = N_4'' = N_4''' =$ &c.

For the root g_5, we get,

$g_5 = -0''.661665$,
$N_5 = +1.230450 N_5^{IV}$ log. 0.0900640,
$N_5' = +1.130316 N_5^{IV}$ " 0.0531999,
$N_5'' = +1.107793 N_5^{IV}$ " 0.0444583,
$N_5''' = +1.049700 N_5^{IV}$ " 0.0210647,
$N_5^{V} = +0.9653247 N_5^{IV}$ " 9.9846734,
$N_5^{VI} = -0.9380148 N_5^{IV}$ " $9.9722097n$,
$N_5^{VII} = -9.829793 N_5^{IV}$ " $0.9925444n$.

For the root g_6, we get,

$g_6 = -2''.916080$,
$N_6 = +\ 3.509603 N_6^{IV}$ log. 0.5452580,
$N_6' = +\ 2.043376 N_6^{IV}$ " 0.3103480,
$N_6'' = +\ 1.836884 N_6^{IV}$ " 0.2640819,
$N_6''' = +\ 1.315752 N_6^{IV}$ " 0.1191740,
$N_6^{V} = +\ 0.8164404 N_6^{IV}$ " 9.9119245,
$N_6^{VI} = -20.11413 N_6^{IV}$ " $1.3035012n$,
$N_6^{VII} = +\ 2.161811 N_6^{IV}$ " 0.3348177.

For the root g_7, we get,

$g_7 = -25''.934782$,
$N_7 = -0.04268995 N_7^{IV}$ log. $8.6303257n$,
$N_7' = -0.03535223 N_7^{IV}$ " $8.5484168n$,
$N_7'' = -0.4372758 N_7^{IV}$ " $9.6407554n$,
$N_7''' = -1.478718 N_7^{IV}$ " $0.1698852n$,
$N_7^{V} = -2.490648 N_7^{IV}$ " $0.3963122n$,
$N_7^{VI} = +0.1093379 N_7^{IV}$ " 9.0387706,
$N_7^{VII} = +0.01225226 N_7^{IV}$ " 8.0882161.

15. If we now substitute these values in equations (408) and (409), we shall obtain the following quantities:—

For the root $g = -5''.170189$, we get,

$$x = +\frac{0.581731}{10^{14}} N; \quad y = +\frac{1.5880256}{10^{14}} N; \quad z = \frac{14.050229}{10^{14}} N^2.$$

Whence $\beta = 20°\ 7'\ 8''.3$, and log. $N = 9.0805176$.

$N = +0.120370$, $N^{IV} = -0.00002524$,
$N' = +0.014605$, $N^{V} = -0.00003221$,
$N'' = +0.010530$, $N^{VI} = +0.00002767$,
$N''' = +0.002167$, $N^{VII} = +0.0000008015$.

For the root $y_1 = -6''.631323$, we get,

$$x_1 = +\frac{0.768973}{10^{14}} N_1; \quad y_1 = -\frac{0.6414600}{10^{14}} N_1; \quad z_1 = \frac{33.22066}{10^{14}} N_1^2.$$

Whence $\beta_1 = 129^\circ\ 50'\ 4''.4$; and log. $N_1 = 8.4791971$.

$$\begin{array}{ll} N_1\ = +0.030144, & N_1^{IV} = +0.00001026, \\ N_1'\ = -0.008215, & N_1^{V}\ = +0.00001454, \\ N_1'' = -0.006664, & N_1^{VI} = -0.000007438, \\ N_1''' = -0.001441, & N_1^{VII} = -0.0000004539. \end{array}$$

For the root $g_2 = -17''.5137898$, we get,

$$x_2 = -\frac{6.914692}{10^{13}} N_2; \quad y_2 = +\frac{3.461327}{10^{13}} N_2; \quad z_2 = \frac{6089.390}{10^{13}} N_2^2.$$

Whence $\beta_2 = 296^\circ\ 35'\ 29''.2$; and log. $N_2 = 7.1037540$.

$$\begin{array}{ll} N_2\ = +0.001270, & N_2^{IV} = -0.000002201, \\ N_2'\ = -0.007063, & N_2^{V}\ = -0.000019926, \\ N_2'' = +0.005211, & N_2^{VI} = +0.0000019167, \\ N_2''' = +0.048560, & N_2^{VII} = +0.0000002206. \end{array}$$

For the root $g_3 = -18''.667944$, we get,

$$x_3 = -\frac{6.844087}{10^{13}} N_3; \quad y_3 = -\frac{2.032363}{10^{13}} N_3; \quad z_3 = \frac{1783.3974}{10^{13}} N_3^2.$$

Whence $\beta_3 = 253^\circ\ 27'\ 40''.1$; and log. $N_3 = 7.6024178$.

$$\begin{array}{ll} N_3\ = +0.004003, & N_3^{IV} = -0.00000018, \\ N_3'\ = -0.024768, & N_3^{V}\ = -0.00000318, \\ N_3'' = +0.025376, & N_3^{VI} = +0.00000027, \\ N_3''' = -0.033934, & N_3^{VII} = +0.000000031. \end{array}$$

For the root $g_4 = 0''$, we get,

$$x_4 = +\frac{0.7256453}{10^{10}} N_4; \quad y_4 = -\frac{0.2113461}{10^{\,0}} N_4; \quad z_4 = \frac{27.24508}{10^{10}} N_4^2.$$

Whence $\beta_4 = 106^\circ\ 14'\ 18''.0$; and $N_4 = N_4' = N_4'' =$ &c.,
$= +0.02774066$, log. 8.4431168.

For the root $g_5 = -0''.6616647$, we get,

$$x_5 = +\frac{0.1016673}{10^{10}} N_5^{IV}; \quad y_5 = +\frac{0.2717444}{10^{10}} N_5^{IV}; \quad z_5 = \frac{241.9412}{10^{10}} N_5^{IV2}.$$

Whence $\beta_5 = 20^\circ\ 30'\ 57''.6$; and log. $N_5^{IV} = 7.0789098$.

$$\begin{array}{ll} N_5\ = +0.001476, & N_5^{IV} = +0.00119925, \\ N_5'\ = +0.001356, & N_5^{V}\ = +0.00115768, \\ N_5'' = +0.001329, & N_5^{VI} = -0.00112471, \\ N_5''' = +0.001259, & N_5^{VII} = -0.01178812 \end{array}$$

For the root $g_6 = -2''.916080$, we get,

$$x_6 = +\frac{0.3678561}{10^{10}} N_6^{IV}; \quad y_6 = -\frac{0.3544989}{10^{10}} N_6^{IV}; \quad z_6 = \frac{581.0130}{10^{10}} N_6^{IV}.$$

Whence $\beta_6 = 133^\circ\ 56'\ 26''.3$; and $\log.\ N_6^{IV} = 6.9441240$.

$$\begin{array}{ll} N_6 = +0.003086, & N_6^{IV} = +0.000879273, \\ N_6' = +0.001797, & N_6^{V} = +0.000717874, \\ N_6'' = +0.001615, & N_6^{VI} = -0.0176858, \\ N_6''' = +0.001157, & N_6^{VII} = +0.00190082. \end{array}$$

For the root $g_7 = -25''.934782$, we get,

$$x_7 = -\frac{0.2996357}{10^{10}} N_7^{IV}; \quad y_7 = +\frac{0.2203002}{10^{10}} N_7^{IV}; \quad z_7 = \frac{59.02984}{10^{10}} N_7^{IV}.$$

Whence $\beta_7 = 306^\circ\ 19'\ 27''.6$; and $\log.\ N_7^{IV} = 7.7993611$.

$$\begin{array}{ll} N_7 = -0.0002690, & N_7^{IV} = +0.00630030, \\ N_7' = -0.0002227, & N_7^{V} = -0.01569181, \\ N_7'' = -0.0027550, & N_7^{VI} = +0.000688861, \\ N_7''' = -0.0093164, & N_7^{VII} = +0.0000771929. \end{array}$$

16. For an increment of $\frac{1}{10}$, to the mass of the earth, we have the preliminary computations by merely making all the coefficients positive in equations (251–256). We shall then obtain the following

Fundamental Equations for $\mu'' = +\frac{1}{10}$; *or for* $m'' = 1 \div 335172$.

$$\left.\begin{array}{l} A = g^2 + 40.092595.g + 195.673045; \\ A' = g^2 + 23.4348156.g + 100.8391921; \\ A'' = g^2 + 19.4622137.g + 77.7179078; \\ A_1 = g^2 + 18.4089042.g + 60.3313475; \\ A_2 = g^2 + 13.1935918.g + 8.982096; \\ A_3 = g^2 + 26.3829396.g + 9.895893. \end{array}\right\} \quad (455)$$

$$\left.\begin{array}{l} D = g^2 + 46.0863772.g + 633.139825; \\ D' = g^2 + 52.8372995.g + 622.776921; \\ D'' = g^2 + 32.4903462.g + 261.2498776; \\ D_1 = g^2 + 43.8987902.g + 172.463686; \\ D_2 = g^2 + 46.4944263.g + 32.948508; \\ D_3 = g^2 + 3.4291886.g + 1.69252948. \end{array}\right\} \quad (456)$$

$$\left.\begin{array}{ll} B = \{g + 34.38795\} b; & B' = \{g + 17.7555387\} b \\ & B'' = \{g + 13.801975\} b \end{array}\right\} \quad (457)$$

$$\left.\begin{array}{l} C = -\{g + 27.173766\} [9.4381149] b'; \\ C' = -\{g + 17.7855450\} [9.1138076] b'; \\ C'' = +[0.3976676] b'; \\ C''' = +[0.4411620] b'. \end{array}\right\} \quad (458)$$

$$\left.\begin{aligned}
E\ &=-[0.0221304]b'';\\
E'\ &=-\{g+22.6010814\}[8.9723624]b'';\\
E''&=-\{g+17.7330184\}[9.7501125]b'';\\
E'''&=-[1.0430464]b''.
\end{aligned}\right\} \qquad (459)$$

$$\left.\begin{aligned}
F\ &=+[8.1679376]b''';\\
F'\ &=-[9.1453586]b''';\\
F''&=-\{g+14.757327\}\ [0.7927855]b''';\\
F'''&=-\{g+35.0517532\}[9.6907241]b'''.
\end{aligned}\right\} \qquad (460)$$

$$\left.\begin{aligned}
B_1=\{g+4.5202306\}b_1;\quad B_2&=\{g+\ 0.7124563\}b_1;\\
B_3&=\{g+18.8177241\}b_1;
\end{aligned}\right\} \qquad (461)$$

$$\left.\begin{aligned}
C_1&=-\{g+4.1329964\}[9.5433087]b_2;\\
C_2&=-\{g+0.7307302\}[9.4349711]b_2.\\
C_3&=+[0.8644527]b_2;\\
C_4&=+[0.8654649]b_2.
\end{aligned}\right\} \qquad (462)$$

$$\left.\begin{aligned}
E_1&=-[8.3317448]b_3;\\
E_2&=-\{g+39.7657938\}[8.7437718]b_3;\\
E_3&=-\{g+\ 0.6486333\}[0.7242832]b_3;\\
E_4&=-[0.9647514]b_3.
\end{aligned}\right\} \qquad (463)$$

$$\left.\begin{aligned}
F_1&=+[7.7244692]b_4;\\
F_2&=-[0.3564628]b_4;\\
F_3&=-\{g+\ 2.7805554\}[1.5514854]b_4;\\
F_4&=-\{g+45.7636960\}[9.4626364]b_4.
\end{aligned}\right\} \qquad (464)$$

$$\left.\begin{aligned}
&g^4+48.4349358.g^3+802.5743024.g^2\\
&\quad+5231.898890.g+11585.83042
\end{aligned}\right\}=(\chi,\ \chi_1,\ \chi_2,\ \chi_3). \qquad (465)$$

$$\left.\begin{aligned}
&g^4+29.5237894.g^3+95.0997623.g^2\\
&\quad+51.201831.g\ \ +\ 0.553045
\end{aligned}\right\}=(\chi_4,\ \chi_5,\ \chi_6,\ \chi_7). \qquad (466)$$

The values of b, b', b'', and b''' are given by equations (405); and the values of b_1, b_2, b_3, and b_5 are given by equations (406), by merely multiplying the coefficients of N'' by $1+\mu''=1+\frac{1}{10}$.

If we put equations (465) and (466) equal to nothing, they will give,

$$\begin{aligned}
g\ &=-\ 5''.2095599; & g_4&=-\ 0''.0110263;\\
g_1&=-\ 6\ .6631448; & g_5&=-\ 0\ .6630507;\\
g_2&=-17\ .6257463; & g_6&=-\ 2\ .9169913;\\
g_3&=-18\ .9364848; & g_7&=-25\ .9327210.
\end{aligned}$$

Equations (455–466) will now give the following values:—

For the root g, we get,

$$g = -5''.2136546;$$

$$\begin{aligned}
N' &= +0.1200684N & &\text{log.}\ 9.0794288; \\
N'' &= +0.0871180N & &\text{``}\ \ 8.9401080; \\
N''' &= +0.01844217N & &\text{``}\ \ 8.2658120; \\
N^{IV} &= -0.0002119225N & &\text{``}\ \ 96.3261770n; \\
N^{V} &= -0.000271274N & &\text{``}\ \ 96.4334082n; \\
N^{VI} &= +0.000228589N & &\text{``}\ \ 96.3590546; \\
N^{VII} &= +0.000006910472N & &\text{``}\ \ 94.8395077.
\end{aligned}$$

For the root g_1, we get,

$$g_1 = -6''.6692717;$$

$$\begin{aligned}
N_1' &= -0.2683056N_1 & &\text{log.}\ 9.4286296n; \\
N_1'' &= -0.2189168N_1 & &\text{``}\ \ 9.3402791n; \\
N_1''' &= -0.0487843N_1 & &\text{``}\ \ 8.6882800n; \\
N_1^{IV} &= +0.0003444146N_1 & &\text{``}\ \ 96.5370815; \\
N_1^{V} &= +0.000489310N_1 & &\text{``}\ \ 96.6895841; \\
N_1^{VI} &= -0.000247679N_1 & &\text{``}\ \ 96.3938898n; \\
N_1^{VII} &= -0.00001526832N_1 & &\text{``}\ \ 95.1837912n.
\end{aligned}$$

For the root g_2, we get,

$$g_2 = -17''.6265859;$$

$$\begin{aligned}
N_2' &= -\ 5.55453N_2 & &\text{log.}\ 0.7446477n; \\
N_2'' &= +\ 3.67668N_2 & &\text{``}\ \ 0.5654561; \\
N_2''' &= +43.0731N_2 & &\text{``}\ \ 1.6342060; \\
N_2^{IV} &= -\ 0.00179671N_2 & &\text{``}\ \ 97.2544790n; \\
N_2^{V} &= -\ 0.0172969N_2 & &\text{``}\ \ 98.2379690n; \\
N_2^{VI} &= +\ 0.00164470N_2 & &\text{``}\ \ 97.2160878; \\
N_2^{VII} &= +\ 0.000189469N_2 & &\text{``}\ \ 96.2775388.
\end{aligned}$$

For the root g_3, we get,

$$g_3 = -18''.9364959;$$

$$\begin{aligned}
N_3' &= -6.27941N_3 & &\text{log.}\ 0.7979189n; \\
N_3'' &= +6.06766N_3 & &\text{``}\ \ 0.7830214; \\
N_3''' &= -7.19812N_3 & &\text{``}\ \ 0.8572192n; \\
N_3^{IV} &= -0.0000461221N_3 & &\text{``}\ \ 95.6639090n; \\
N_3^{V} &= -0.00129815N_3 & &\text{``}\ \ 97.1133248n; \\
N_3^{VI} &= +0.000107625N_3 & &\text{``}\ \ 96.0319138; \\
N_3^{VII} &= +0.0000124176N_3 & &\text{``}\ \ 95.0940376.
\end{aligned}$$

For the root g_4, we get,

$$g_4 = 0'',\ N_4 = N_4' = N_4'' = \&c.$$

For the root g_5, we get,

$g_5 = -0''.6616623;$

$N_5 = +1.228747N_5^{IV}$ log. 0.0894623;
$N_5' = +1.12937N_5^{IV}$ " 0.0528360;
$N_5'' = +1.107383N_5^{IV}$ " 0.0442980;
$N_5''' = +1.049923N_5^{IV}$ " 0.0211576;
$N_5^{V} = +0.965320N_5^{IV}$ " 9.9846716;
$N_5^{VI} = -0.938100N_5^{IV}$ " 9.9722493n;
$N_5^{VII} = -9.83017N_5^{IV}$ " 0.9925608n.

For the root g_6, we get,

$g_6 = -2''.9160771;$

$N_6 = + 3.46445N_6^{IV}$ log. 0.5396342;
$N_6' = + 2.02862N_6^{IV}$ " 0.3072002;
$N_6'' = + 1.82900N_6^{IV}$ " 0.2622141;
$N_6''' = + 1.317308N_6^{IV}$ " 0.1196872;
$N_6^{V} = + 0.816421N_6^{IV}$ " 9.9119142;
$N_6^{VI} = -20.11546N_6^{IV}$ " 1.3035300n;
$N_6^{VII} = + 2.161956N_6^{IV}$ " 0.3348464.

For the root g_7, we get,

$g_7 = -25''.9350099;$

$N_7 = -0.0434072N_7^{IV}$ log. 8.6375619n;
$N_7' = -0.0232236N_7^{IV}$ " 8.3659300n;
$N_7'' = -0.442046N_7^{IV}$ " 9.6454670n;
$N_7''' = -1.489419N_7^{IV}$ " 0.1730168n;
$N_7^{V} = -2.490578N_7^{IV}$ " 0.3962995n;
$N_7^{VI} = +0.1093334N_7^{IV}$ " 9.0387530;
$N_7^{VII} = +0.0122517N_7^{IV}$ " 8.0881970.

17. If we now substitute these values in equations (408) and (409), we shall obtain the following quantities:—

For the root $g = -5''.2136546$, we get,

$$x = +\frac{5.51372}{10^{15}}; \qquad y = +\frac{15.903915}{10^{15}}; \qquad z = \frac{140.64952}{10^{15}}.$$

Whence $\beta = 19° \, 7' \, 7''.0$; log. $N = 9.0780045$.

$N = +0.1196753;$ $N^{IV} = -0.000025364;$
$N' = +0.0143703;$ $N^{V} = -0.000032468;$
$N'' = +0.0104268;$ $N^{VI} = +0.000027359;$
$N''' = +0.00220757;$ $N^{VII} = +0.00000082705.$

For the root $g_1=-6''.6692717$, we get,

$$x_1=+\frac{8.43328}{10^{15}};\quad y_1=-\frac{6.419195}{10^{15}};\quad z_1=\frac{331.3455}{10^{15}}.$$

Whence $\beta_1=127^\circ\ 16'\ 40''.5$; log. $N_1=98.5049615$.

$$\begin{aligned}
N_1\ &=+0.0319861; & N_1^{IV}&=+0.000011017;\\
N_1'&=-0.0085822; & N_1^{V}\ &=+0.000015651;\\
N_1''&=-0.0070024; & N_1^{VI}&=-0.000007922;\\
N_1'''&=-0.0015604; & N_1^{VII}&=-0.0000004884.
\end{aligned}$$

For the root $g_2=-17''.626586$, we get,

$$x_2=-\frac{6.947485}{10^{13}};\quad y_2=+\frac{4.015769}{10^{13}};\quad z_2=\frac{7425.592}{10^{13}}.$$

Whence $\beta_2=300^\circ\ 0'\ 37''.6$; log. $N_2=97.0339306$.

$$\begin{aligned}
N_2\ &=+0.0010813; & N_2^{IV}&=-0.000001943;\\
N_2'&=-0.0060059; & N_2^{V}\ &=-0.000018702;\\
N_2''&=+0.00397545; & N_2^{VI}&=+0.000001778;\\
N_2'''&=+0.0465733; & N_2^{VII}&=+0.0000002049.
\end{aligned}$$

For the root $g_3=-18''.9364959$, we get,

$$x_3=-\frac{6.960810}{10^{13}};\quad y_3=-\frac{1.901617}{10^{13}};\quad z_3=\frac{1696.1782}{10^{13}}.$$

Whence $\beta_3=254^\circ\ 43'\ 12''.9$; log. $N_3=97.6288181$.

$$\begin{aligned}
N_3\ &=+0.0042542; & N_3^{IV}&=-0.0000001962;\\
N_3'&=-0.0267139; & N_3^{V}\ &=-0.0000055201;\\
N_3''&=+0.0258130; & N_3^{VI}&=+0.0000004576;\\
N_3'''&=-0.0306223; & N_3^{VII}&=+0.0000000528.
\end{aligned}$$

For the root $g_4=0''$, we get,

$$x_4=+\frac{0.7256453}{10^{10}};\quad y_4=-\frac{0.2113461}{10^{10}};\quad z_4=\frac{27.24612}{10^{10}}.$$

Whence $\beta_4=106^\circ\ 14'\ 18''.0$; log. $N_4=98.4431002$.

$$N_4=N_4'=N_4''=N_4'''=N_4^{IV}=N_4^{V}=N_4^{VI}=N_4^{VII}=0.0277396.$$

For the root $g_5=-0''.6616623$, we get,

$$x_5=+\frac{0.1016437}{10^{10}};\quad y_5=+\frac{0.2717608}{10^{10}};\quad z_5=\frac{241.9590}{10^{10}}.$$

Whence $\beta_5=20^\circ\ 30'\ 37''.9$; log. $N_5^{IV}=97.0788886$.

$$\begin{aligned}
N_5\ &=+0.0014735; & N_5^{IV}&=+0.0011992;\\
N_5'&=+0.0013543; & N_5^{V}\ &=+0.0011576;\\
N_5''&=+0.0013280; & N_5^{VI}&=-0.0011248;\\
N_5'''&=+0.0012591; & N_5^{VII}&=-0.0117880.
\end{aligned}$$

For the root $g_6=-2''.9160771$, we get,

$$x_6=+\frac{0.3678168}{10^{10}},\quad y_6=-\frac{0.3545173}{10^{10}},\quad z_6=\frac{581.0884}{10^{10}}.$$

Whence $\beta_6=133^\circ\ 56'\ 42''.7$; log. $N_6^{IV}=96.9440543$.

$$\begin{array}{ll} N_6\ =+0.0030457; & N_6^{IV}=+0.0008812; \\ N_6'\ =+0.0017834; & N_6^{V}\ =+0.00071774; \\ N_6''=+0.0016079; & N_6^{VI}=-0.0176842; \\ N_6'''=+0.0011581; & N_6^{VII}=-0.0018963. \end{array}$$

For the root $g_7=-25''.9350099$, we get,

$$x_7=-\frac{0.2996051}{10^{10}};\quad y_7=+\frac{0.2202944}{10^{10}};\quad z_7=\frac{59.02778}{10^{10}}.$$

Whence $\beta_7=306^\circ\ 19'\ 35''.1$; log. $N_7^{IV}=97.7993434$.

$$\begin{array}{ll} N_7\ =-0.0002735; & N_7^{IV}=+0.0063000; \\ N_7'\ =-0.0001463; & N_7^{V}\ =-0.0156907; \\ N_7''=-0.0027849; & N_7^{VI}=+0.0006888; \\ N_7'''=-0.0093834; & N_7^{VII}=+0.00007719. \end{array}$$

18. If we now suppose the mass of *Mars* to be doubled, we shall have the preliminary computations by making all the coefficients of equations (269–274) positive. We shall then obtain the following

Fundamental Equations for $\mu'''=+1$; *or for* $m'''=1\div1340318.5$

$$\left.\begin{array}{l} A\ =g^2+38.394808.g\ +184.259392; \\ A'=g^2+23.2280108.g+\ 98.9712014; \\ A''=g^2+18.8430013.g+\ 73.7719022; \\ A_1=g^2+18.4111446.g+\ 60.3416523; \\ A_2=g^2+13.1958172.g+\ \ 8.983726; \\ A_3=g^2+26.3853904.g+\ \ 9.939432. \end{array}\right\}\quad(467)$$

$$\left.\begin{array}{l} D\ =g^2+44.9052274.g+596.989960; \\ D'=g^2+52.1576683.g+608.070284; \\ D''=g^2+32.4642145.g+260.9092264; \\ D_1=g^2+43.8990384.g+172.465387; \\ D_2=g^2+46.4946594.g+\ 32.948855; \\ D_3=g^2+\ 3.4292114.g+\ \ 1.69255259. \end{array}\right\}\quad(468)$$

$$\left.\begin{array}{ll} B=\{g+32.74835\}b; & B'=\{g+17.6069231\}b; \\ & B''=\{g+13.240952\}b; \end{array}\right\}\quad(469)$$

$$\left.\begin{array}{l} C\ =-\{g+22.553639\}\ [9.4381189]b'; \\ C'\ =-\{g+17.6669357\}[9.1138076]b'; \\ C''=+[0.3976676]b'; \\ C'''=+[0.4411620]b'. \end{array}\right\}\quad(470)$$

$$\left.\begin{aligned} E &= -[9.9807377]b''; \\ E' &= -\{g+22.3515875\}[8.9723624]b''; \\ E'' &= -\{g+17.5618825\}[9.7501125]b''; \\ E''' &= -[1.0016537]b''. \end{aligned}\right\} \qquad (471)$$

$$\left.\begin{aligned} F &= +[8.4689676]b'''; \\ F' &= -[9.4463886]b'''; \\ F'' &= -\{g+14.902331\}[0.7927855]b'''; \\ F''' &= -\{g+34.490730\}[9.6907241]b'''. \end{aligned}\right\} \qquad (472)$$

$$B_1=\{g+4.5202496\}b_1; \quad \left.\begin{aligned} B_2 &= \{g+\ 0.7124602\}b_1; \\ B_3 &= \{g+18.8179534\}b_1. \end{aligned}\right\} \qquad (473)$$

$$\left.\begin{aligned} C_1 &= -\{g+4.1330154\ \}[9.5433087]b_2; \\ C_2 &= -\{g+0.73073414\}[9.4349711]b_2; \\ C_3 &= +[0.8644527]b_2; \\ C_4 &= +[0.8654649]b_2. \end{aligned}\right\} \qquad (474)$$

$$\left.\begin{aligned} E_1 &= -[8.3317448]b_3; \\ E_2 &= -\{g+39.7660231\ \}[8.7437718]b_3; \\ E_3 &= -\{g+\ 0.64863718\}[0.7242832]b_3; \\ E_4 &= -[0.9647514]b_3. \end{aligned}\right\} \qquad (475)$$

$$\left.\begin{aligned} F_1 &= +[7.7244692]b_4; \\ F_2 &= -[0.3564628]b_4; \\ F_3 &= -\{g+\ 2.7805744\}[1.5514854]b_4; \\ F_4 &= -\{g+45.7639253\}[9.4626364]b_4. \end{aligned}\right\} \qquad (476)$$

$$\left.\begin{aligned} g^4+47.9382786.g^3+787.7033176.g^2 \\ +5108.828105.g+11271.52356 \end{aligned}\right\} =(\chi, \chi_1, \chi_2, \chi_3); \qquad (477)$$

$$\left.\begin{aligned} g^4+29.5262630.g^3+95.1517832.g^2 \\ +51.352675.g+0.624611 \end{aligned}\right\} =(\chi_4, \chi_5, \chi_6, \chi_7). \qquad (478)$$

The values of b, b', b'', and b''' are given by equations (405), and the values of b_1, b_2, b_3, and b_4 are given by equations (406), by merely multiplying the coefficients of N''' by $1+\mu'''=2$.

If we put equations (477) and (478) equal to nothing, they will give

$$\begin{aligned} g_1 &= -\ 5''.1830604, & g_4 &= -\ 0''.0124492, \\ g_2 &= -\ 6\ .7024834, & g_5 &= -\ 0\ .6632208, \\ g_3 &= -17\ .3261252, & g_6 &= -\ 2\ .9170763, \\ g_4 &= -18\ .7266096, & g_7 &= -25\ .9335168. \end{aligned}$$

The solutions of equations (467–478) will now give the following values:—

For the root g, we get,

$g=-5''.186718,$	
$N'=+0.1139908N$	log. 9.0568698,
$N''=+0.0802224N$	“ 8.9042956,
$N'''=+0.01622926N$	“ 8.2102986,
$N^{IV}=-0.000196341N$	“ 6.2930111n,
$N^{V}=-0.0002508766N$	“ 6.3994600n,
$N^{VI}=+0.000213949N$	“ 6.3303095,
$N^{VII}=+0.00000630073N$	“ 4.7993910.

For the root g_1, we get,

$g_1=-6''.708872,$	
$N_1'=-0.3009834N_1$	log. 9.4785424n,
$N_1''=-0.2389096N_1$	“ 9.3782336n,
$N_1'''=-0.05086216N_1$	“ 8.7063948n,
$N_1^{IV}=+0.0003713773N_1$	“ 6.5698154,
$N_1^{V}=+0.0005292475N_1$	“ 6.7236588,
$N_1^{VI}=-0.0002649250N_1$	“ 6.4231228n,
$N_1^{VII}=-0.00001649956N_1$	“ 5.2174723n.

For the root g_2, we get,

$g_2=-17''.327793,$	
$N_2'=-5.526427N_2$	log. 0.7424444n,
$N_2''=+4.250740N_2$	“ 0.6284646,
$N_2'''=+21.16062N_2$	“ 1.3255284,
$N_2^{IV}=-0.002056417N_2$	“ 7.3131111n,
$N_2^{V}=-0.01697217N_2$	“ 8.2297373n,
$N_2^{VI}=+0.001664117N_2$	“ 7.2211838,
$N_2^{VII}=+0.000191324N_2$	“ 6.2817698.

For the root g_3, we get,

$g_3=-18''.726623,$	
$N_3'=-6.280437N_3$	log. 0.7979898n,
$N_3''=+7.147022N_3$	“ 0.8541251,
$N_3'''=-8.117785N_3$	“ 0.9094376n,
$N_3^{IV}=+0.0000583566N_3$	“ 5.7660900,
$N_3^{V}=+0.00177227N_3$	“ 7.2485300,
$N_3^{VI}=-0.0001516876N_3$	“ 6.1809500n,
$N_3^{VII}=-0.0000176279N_3$	“ 5.2462000n.

For the root g_4, we get,

$g_4=0''$; and $N_4=N_4'=N_4''=$&c.

For the root g_5, we get,

$$g_5 = -0''.661664,$$

$N_5 = +1.229368 N_5^{IV}$ log. 0.0896819,
$N_5' = +1.129025 N_5^{IV}$ " 0.0527036,
$N_5'' = +1.105860 N_5^{IV}$ " 0.0436997,
$N_5''' = +1.049158 N_5^{IV}$ " 0.0208410,
$N_5^{V} = +0.9653170 N_5^{IV}$. " 9.9846700,
$N_5^{VI} = -0.9382057 N_5^{IV}$ " 9.9722980n,
$N_5^{VII} = -9.830882 N_5^{IV}$ " 0.9925925n.

For the root g_6, we get,

$$g_6 = -2''.916095,$$

$N_6 = +\ 3.479802 N_6^{IV}$ log. 0.5415546,
$N_6' = +\ 2.020979 N_6^{IV}$ " 0.3055617,
$N_6'' = +\ 1.810077 N_6^{IV}$ " 0.2576970,
$N_6''' = +\ 1.308654 N_6^{IV}$ " 0.1168248,
$N_6^{V} = +\ 0.8164215 N_6^{IV}$ " 9.9119144,
$N_6^{VI} = -20.11574 N_6^{IV}$ " 1.3035360n,
$N_6^{VII} = +\ 2.156995 N_6^{IV}$ " 0.3348492.

For the root g_7, we get,

$$g_7 = -25''.933517,$$

$N_7 = -0.04094460 N_7^{IV}$ log. 8.6121966n,
$N_7' = -0.04815456 N_7^{IV}$ " 8.6826374n,
$N_7'' = -0.4073554 N_7^{IV}$ " 9.6099734n,
$N_7''' = -1.473038 N_7^{IV}$ " 0.1682137n,
$N_7^{V} = -2.490000 N_7^{IV}$ " 0.3961992n,
$N_7^{VI} = +0.1092910 N_7^{IV}$ " 9.0385844,
$N_7^{VII} = +0.01224673 N_7^{IV}$ " 8.0880201.

19. If we now substitute these values in equations (408) and (409), we shall obtain the following quantities:—

For the root $g = -5''.186718$, we get,

$$x = +\frac{6.16813}{10^{15}} N;\ y = +\frac{15.530471}{10^{15}} N;\ z = \frac{134.20637}{10^{15}} N^2.$$

Whence $\beta = 21^\circ\ 39'\ 31''.5$; and log. $N = 9.0952088$.

$N = +0.124511,$ $N^{IV} = -0.000024449,$
$N' = +0.014194,$ $N^{V} = -0.000031240,$
$N'' = +0.0099895,$ $N^{VI} = +0.000026642,$
$N''' = +0.0020212,$ $N^{VII} = +0.00000078455.$

For the root $g_1 = -6''.708872$, we get,

$$x_1 = +\frac{6.90464}{10^{15}} N_1; \quad y_1 = -\frac{8.259874}{10^{15}} N_1; \quad z_1 = \frac{372.8292}{10^{15}} N_1^2.$$

Whence $\beta_1 = 140^\circ\ 6'\ 26''.4$; and log. $N_1 = 8.4605329$,

$N_1 = +0.028876,$

$N_1' = -0.0086913,$

$N_1'' = -0.0068988,$

$N_1''' = -0.0014686,$

$N_1^{IV} = +0.000010724,$

$N_1^{V} = +0.000015283,$

$N_1^{VI} = -0.0000076500,$

$N_1^{VII} = -0.00000047644.$

For the root $g_2 = -17''.327793$, we get,

$$x_2 = -\frac{6.999264}{10^{13}} N_2; \quad y_2 = +\frac{3.944520}{10^{13}} N_2; \quad z_2 = \frac{4077.099}{10^{13}} N_2^2.$$

Whence $\beta_2 = 299^\circ\ 24'\ 14''.3$; and log. $N_2 = 7.2945933$.

$N_2 = +0.0019706,$

$N_2' = -0.0108902,$

$N_2'' = +0.0083764,$

$N_2''' = +0.0416986,$

$N_2^{IV} = -0.0000040523,$

$N_2^{V} = -0.000033445,$

$N_2^{VI} = +0.0000032793,$

$N_2^{VII} = +0.00000037702.$

For the root $g_3 = -18''.726623$, we get,

$$x_3 = -\frac{6.863743}{10^{13}} N_3; \quad y_3 = -\frac{2.942093}{10^{13}} N_3; \quad z_3 = \frac{2202.062}{10^{13}} N_3^2.$$

Whence $\beta_3 = 246^\circ\ 47'\ 52''.7$; and log. $N_3 = 7.5303588$.

$N_3 = +0.0033912,$

$N_3' = -0.0212985,$

$N_3'' = +0.0242373,$

$N_3''' = -0.0275294,$

$N_3^{IV} = +0.0000001979,$

$N_3^{V} = +0.0000060102,$

$N_3^{VI} = -0.0000005144,$

$N_3^{VII} = -0.00000005978.$

For the root $g_4 = 0''$, we get,

$$x_4 = +\frac{0.7257311}{10^{10}} N_4; \quad y_4 = -\frac{0.2112699}{10^{10}} N_4; \quad z_4 = \frac{27.24758}{10^{10}} N_4^2.$$

Whence $\beta_4 = 106^\circ\ 13'\ 51''.5$; and log. $N_4 = 8.4431120$, $N_4 = 0.02774035$.

For the root $g_5 = -0.6616640$, we get,

$$x_5 = +\frac{0.1016924}{10^{10}} N_5^{IV}; \quad y_5 = +\frac{0.2718731}{10^{10}} N_5^{IV}; \quad z_5 = +\frac{241.9923}{10^{10}} N_5^{IV\,2}.$$

Whence $\beta_5 = 20^\circ\ 30'\ 42''.3$; and log. $N_5^{IV} = 7.0790118$.

$N_5 = +0.0014747,$

$N_5' = +0.0013543,$

$N_5'' = +0.0013265,$

$N_5''' = +0.0012585,$

$N_5^{IV} = +0.00119953,$

$N_5^{V} = +0.00115794,$

$N_5^{VI} = -0.00112520,$

$N_5^{VII} = -0.01179220.$

For the root $g_6=-2''.9160954$, we get,

$$x_6=+\frac{0.3679185}{10^{10}}N_6^{IV};\quad y_6=-\frac{0.3544211}{10^{10}}N_6^{IV};\quad z_6=\frac{581.1008}{10^{10}}N_6^{IV2}.$$

Whence $\beta_6=133^\circ\ 55'\ 46''.2$; and log. $N_6^{IV}=6.9440507$.

$$\begin{array}{ll} N_6\ =+0.0030592, & N_6^{IV}=+0.00087912, \\ N_6'\ =+0.0017767, & N_6^{V}\ =+0.00071774, \\ N_6''=+0.0015913, & N_6^{VI}=-0.0176843, \\ N_6'''=+0.0011505, & N_6^{VII}=+0.00190064. \end{array}$$

For the root $g_7=-25''.936797$, we get,

$$x_7=-\frac{0.2995996}{10^{10}}N_7^{IV};\quad y_7=+\frac{0.2201126}{10^{10}}N_7^{IV};\quad z_7=\frac{59.01481}{10^{10}}N_7^{IV2}.$$

Whence $\beta_7=306^\circ\ 18'\ 15''.6$; and log. $N_7^{IV}=7.7993080$.

$$\begin{array}{ll} N_7\ =-0.0002579, & N_7^{IV}=+0.00629953, \\ N_7'\ =-0.00030335, & N_7^{V}\ =-0.0156858, \\ N_7''=-0.00256615, & N_7^{VI}=+0.00068848, \\ N_7'''=-0.0092794. & N_7^{VII}=+0.000077149. \end{array}$$

20. For an increment of $+\frac{1}{100}$, to the mass of the *Jupiter*, we shall obtain the preliminary computations by merely making all the coefficients positive in equations (280–293). We shall then obtain the following

Fundamental Equations for $\mu^{IV}=+\frac{1}{100}$; *or for* $m^{IV}=1\div1037.504$.

$$\left.\begin{array}{l} A\ =g^2+38.155337.g\ +182.786001; \\ A'=g^2+23.3356274.g+\ 99.4308553; \\ A''=g^2+18.7710411.g+\ 73.2781865; \\ A_1=g^2+18.4811418.g+\ 60.7217122; \\ A_2=g^2+13.2441783.g+\ \ 9.040174; \\ A_3=g^2+26.5649511.g+\ \ 9.944072. \end{array}\right\}\qquad(479)$$

$$\left.\begin{array}{l} D\ =g^2+44.6177028.g+590.564857; \\ D'=g^2+52.1872233.g+610.731488; \\ D''=g^2+32.3787870.g+259.4006143; \\ D_1=g^2+44.0905279.g+173.591533; \\ D_2=g^2+46.6785882.g+\ 33.163967; \\ D_3=g^2+\ 3.4403421.g+\ \ 1.70359455. \end{array}\right\}\qquad(480)$$

$$\left.\begin{array}{ll} B=\{g+32.52083\}b; & B'=\{g+17.7264928\}b; \\ & B''=\{g+13.180944\}b. \end{array}\right\}\qquad(481)$$

$$\left.\begin{array}{l} C\ =-\{g+22.326122\}\ [9.4381189]b'; \\ C'\ =-\{g+17.7564991\}[9.1138076]b'; \\ C''=+[0.3976676]b'; \\ C'''=+[0.4411620]b'. \end{array}\right\}\qquad(482)$$

$$\left.\begin{aligned}
E\ &=-[9.9807377]b'';\\
E'\ &=-\{g+22.2915804\}[8.9723624]b'';\\
E''&=-\{g+17.7039725\}[9.7501125]b'';\\
E'''&=-[1.0016537]b''.
\end{aligned}\right\}\tag{483}$$

$$\left.\begin{aligned}
F\ &=+[8.1679376]b''';\\
F'\ &=-[9.1453586]b''';\\
F''&=-\{g+14.674814\}\,[0.7927855]b''';\\
F'''&=-\{g+34.4307229\}\,[9.6907241]b'''.
\end{aligned}\right\}\tag{484}$$

$$\left.\begin{aligned}
B_1=\{g+4.5295952\}b_1;\ \ B_2&=\{g+\ 0.7142452\}b_1;\\
B_3&=\{g+19.0000972\}b_1.
\end{aligned}\right\}\tag{485}$$

$$\left.\begin{aligned}
C_1&=-\{g+4.1423610\}\ \ [9.5433087]b_2;\\
C_2&=-\{g+0.73251914\}[9.4349711]b_2;\\
C_3&=+[0.8644527]b_2;\\
C_4&=+[0.8654649]b_2.
\end{aligned}\right\}\tag{486}$$

$$\left.\begin{aligned}
E_1&=-[8.3317448]b_3;\\
E_2&=-\{g+39.9481669\}[8.7437718]b_3;\\
E_3&=-\{g+0.6504222\}\ \ [0.7242832]b_3;\\
E_4&=-[0.9647514]b_3.
\end{aligned}\right\}\tag{487}$$

$$\left.\begin{aligned}
F_1&=+[7.7244692]b_4;\\
F_2&=-[0.3564628]b_4;\\
F_3&=-\{g+2.7899200\}[1.5514854]b_4;\\
F_4&=-\{g+45.946069\}[9.4626364]b_4.
\end{aligned}\right\}\tag{488}$$

$$\left.\begin{aligned}
g^4+47.7853999.g^3+782.14382516.g^2\\
+5044.303371.g+11057.20317
\end{aligned}\right\}=(\chi,\chi_1,\chi_2,\chi_3).\tag{489}$$

$$\left.\begin{aligned}
g^4+29.7164348.g^3+96.0269583.g^2\\
+51.793880.g\ +\ 0.533011
\end{aligned}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7).\tag{490}$$

The values of b_1, b_2, b_3, and b_4 are given by equations (406); and the values of b, b', b'', and b''' are given by equations (405), by merely multiplying the coefficients of N^{IV} by $1+\mu^{IV}=1.01$.

If we now put equations (489) and (490) equal to nothing, they will give

$$\begin{aligned}
g&=-\ \ 5''.1493604, & g_4&=-\ \ 0''.0104945,\\
g_1&=-\ \ 6\ .6300170, & g_5&=-\ \ 0\ .6646769,\\
g_2&=-17\ .5198308, & g_6&=-\ \ 2\ .9259522,\\
g_3&=-18\ .4861917, & g_7&=-26\ .1153112.
\end{aligned}$$

The solutions of equations (479–490) will now give the following values:—

For the root g, we get,

$g = -5''.153064$,

$N \ \ = +0.1198583N$ log. 9.0786680,
$N'' = +0.08550057N$ “ 8.9319690,
$N''' = +0.01695429N$ “ 8.2292796,
$N^{IV} = -0.0002019713N$ “ $6.3052894n$,
$N^{V} = -0.0002567140N$ “ $6.4094494n$,
$N^{VI} = +0.0002241425N$ “ 6.3505243,
$N^{VII} = +0.000006320077N$ “ 4.8007224.

For the root g_1, we get,

$g_1 = -6''.635874$,

$N_1' = -0.2841568N_1$ log. $9.4535581n$,
$N_1'' = -0.2283353N_1$ “ $9.3585790n$,
$N_1''' = -0.04747286N_1$ “ $8.6764453n$,
$N_1^{IV} = +0.000343187N_1$ “ 6.5355307,
$N_1^{V} = +0.0004843167N_1$ “ 6.6851294,
$N_1^{VI} = -0.0002492740N_1$ “ $6.3966770n$,
$N_1^{VII} = -0.0000151962N_1$ “ $5.1817351n$.

For the root g_2, we get,

$g_2 = -17''.520779$,

$N_2' = -\ 5.627642N_2$ log. $0.7503265n$,
$N_2'' = +\ 4.727661N_2$ “ 0.6746463,
$N_2''' = +31.11766N_2$ “ 1.4930069,
$N_2^{IV} = -\ 0.001595785N_2$ “ $7.2029742n$,
$N_2^{V} = -\ 0.01334776N_2$ “ $8.1254084n$,
$N_2^{VI} = +\ 0.001291801N_2$ “ 7.1111957,
$N_2^{VII} = +\ 0.0001489017N_2$ “ 6.1728995.

For the root g_3, we get,

$g_3 = -18''.486192$,

$N_3' = -\ 6.134120N_3$ log. $0.7877523n$,
$N_3'' = +\ 6.742900N_3$ “ 0.8288466,
$N_3''' = -11.29640N_3$ “ $1.0529402n$,
$N_3^{IV} = +\ 0.000000955724N_3$ “ 93.9803324,
$N_3^{V} = +\ 0.0001854001N_3$ “ 96.2681100,
$N_3^{VI} = -\ 0.0000169937N_3$ “ $95.2302878n$,
$N_3^{VII} = -\ 0.000002026216N_3$ “ $94.3066957n$.

For the root g_4, we get,

$g_4 = 0''$, and $N_4 = N_4' = N_4'' =$, &c.

For the root g_5, we get,

$$g_5=-0''.6633671,$$

$N_5\ =+1.231158N_5^{IV}$	log. 0.0903139,
$N_5'\ =+1.130398N_5^{IV}$	" 0.0532315,
$N_5''\ =+1.107341N_5^{IV}$	" 0.0442814,
$N_5'''=+1.049016N_5^{IV}$	" 0.0207820,
$N_5^{V}\ =+0.9653980N_5^{IV}$	" 9.9847064,
$N_5^{VI}=-0.9445346N_5^{IV}$	" 9.9752179n,
$N_5^{VII}=-9.899907N_5^{IV}$	" 0.9956311n.

For the root g_6, we get,

$$g_6=-2''.925086,$$

$N_6\ =+\ 3.536719N_6^{IV}$	log. 0.5486005,
$N_6'\ =+\ 2.047060N_6^{IV}$	" 0.3111304,
$N_6''\ =+\ 1.834513N_6^{IV}$	" 0.2635208,
$N_6'''=+\ 1.310090N_6^{IV}$	" 0.1173012,
$N_6^{V}\ =+\ 0.8166500N_6^{IV}$	" 9.9120360,
$N_6^{VI}=-20.25015N_6^{IV}$	" 1.3064432n,
$N_6^{VII}=+\ 2.169824N_6^{IV}$	" 0.3364244.

For the root g_7, we get,

$$g_7=-26''.117474,$$

$N_7\ =-0.04211995N_7^{IV}$	log. 8.6244879n,
$N_7'\ =-0.04851416N_7^{IV}$	" 8.6858685n,
$N_7''\ =-0.4329533N_7^{IV}$	" 9.6364410n,
$N_7'''=-1.477216N_7^{IV}$	" 0.1694440n,
$N_7^{V}\ =-2.515433N_7^{IV}$	" 0.4006128n,
$N_7^{VI}=+0.1096052N_7^{IV}$	" 9.0398312,
$N_7^{VII}=+0.01229432N_7^{IV}$	" 8.0897044.

21. If we now substitute these values in equations (408) and (409), we shall obtain the following quantities:—

For the root $g=-5''.153064$, we get,

$$x=+\frac{0.616929}{10^{14}}N;\ y=+\frac{1.5676783}{10^{14}}N;\ z=\frac{13.836148}{10^{14}}N^2.$$

Whence $\beta=21°\ 28'\ 52''.1$, and log. $N=9.0855076$.

$N\ =+0.121761,$	$N^{IV}=-0.0000245922,$
$N'\ =+0.014594,$	$N^{V}\ =-0.0000312577,$
$N''\ =+0.0104106,$	$N^{VI}=+0.0000272918,$
$N'''\ =+0.0020644;$	$N^{VII}=+0.000000769538.$

For the root $g_1 = -6''.6358742$, we get,

$$x_1 = +\frac{0.685590}{10^{14}} N_1; \quad y_1 = -\frac{0.6767428}{10^{14}} N_1; \quad z_1 = \frac{34.48222}{10^{14}} N_1^2.$$

Whence $\beta_1 = 134° \, 37' \, 40''.5$; and log. $N_1 = 8.4461821$.

$N_1 = +0.027937,$ $\qquad N_1^{IV} = +0.00000958766,$
$N_1' = -0.0079385,$ $\qquad N_1^{V} = +0.0000135304,$
$N_1'' = -0.0063791,$ $\qquad N_1^{VI} = -0.0000069640,$
$N_1''' = -0.00132626,$ $\qquad N_1^{VII} = -0.00000042454.$

For the root $g_2 = -17''.520779$, we get,

$$x_2 = -\frac{69.06713}{10^{14}} N_2; \quad y_2 = +\frac{26.31099}{10^{14}} N_2; \quad z_2 = \frac{4443.491}{10^{13}} N_2^2.$$

Whence $\beta_2 = 290° \, 51' \, 15''.3$; and log. $N_2 = 7.2209728$.

$N_2 = +0.0016633,$ $\qquad N_2^{IV} = -0.0000026543,$
$N_2' = -0.0093605,$ $\qquad N_2^{V} = -0.000022202,$
$N_2'' = +0.0078636,$ $\qquad N_2^{VI} = +0.0000021487,$
$N_2''' = +0.0517583,$ $\qquad N_2^{VII} = +0.00000024767.$

For the root $g_3 = -18''.486192$, we get,

$$x_3 = -\frac{67.51436}{10^{14}} N_3; \quad y_3 = -\frac{23.46891}{10^{14}} N_3; \quad z_3 = \frac{20391.171}{10^{14}} N_3^2.$$

Whence $\beta_3 = 250° \, 49' \, 54''.7$; and log. $N_3 = 7.5447249$

$N_3 = +0.0035053,$ $\qquad N_3^{IV} = +0.00000000335,$
$N_3' = -0.0215019,$ $\qquad N_3^{V} = +0.0000006499,$
$N_3'' = +0.0236359,$ $\qquad N_3^{VI} = -0.00000005957,$
$N_3''' = -0.0395973,$ $\qquad N_3^{VII} = -0.00000000710.$

For the root $g_4 = 0''$, we get,

$$x_4 = +\frac{7.294416}{10^{11}} N_4; \quad y_4 = -\frac{2.119410}{10^{11}} N_4; \quad z_4 = \frac{274.1191}{10^{11}} N_4^2.$$

Whence $\beta_4 = 106° \, 12' \, 4''.8$; and $N_4 = N_4' = N_4'' =$ &c.,
$= +0.02771087$, log. 8.4426502.

For the root $g_5 = -0''.6633671$, we get,

$$x_5 = +\frac{1.016103}{10^{11}} N_5^{IV}; \quad y_5 = +\frac{2.742753}{10^{11}} N_5^{IV}; \quad z_5 = \frac{245.2401}{10^{10}} N_5^{IV\,2}.$$

Whence $\beta_5 = 20° \, 19' \, 41''.0$; and log. $N_5^{IV} = 7.0765225$

$N_5 = +0.0014684,$ $\qquad N_5^{IV} = +0.00119268,$
$N_5' = +0.0013482,$ $\qquad N_5^{V} = +0.00115141,$
$N_5'' = +0.0013207,$ $\qquad N_5^{VI} = -0.00112652,$
$N_5''' = +0.0012511,$ $\qquad N_5^{VII} = -0.0118074.$

For the root $g_6 = -2''.925086$, we get,

$$x_6 = +\frac{0.3697416}{10^{10}} N_6^{IV};\quad y_6 = -\frac{0.3561999}{10^{10}} N_6^{IV};\quad z_6 = \frac{588.7466}{10^{10}} N_6^{IV2}.$$

Whence $\beta_6 = 133^\circ\ 55'\ 52''.8$; and $\log. N_6^{IV} = 6.9405338$.

$$\begin{aligned} N_6 &= +0.0030841, & N_6^{IV} &= +0.00087204,\\ N_6' &= +0.0017851, & N_6^{V} &= +0.00071215,\\ N_6'' &= +0.0015998, & N_6^{VI} &= -0.0176594,\\ N_6''' &= +0.0011424, & N_6^{VII} &= +0.0018922. \end{aligned}$$

For the root $g_7 = -26''.117473$, we get,

$$x_7 = -\frac{0.3026288}{10^{10}} N_7^{IV};\quad y_7 = +\frac{0.2224873}{10^{10}} N_7^{IV};\quad z_7 = \frac{60.04189}{10^{10}} N_7^{IV}.$$

Whence $\beta_7 = 306^\circ\ 19'\ 21''.6$; and $\log. N_7^{IV} = 7.7962857$.

$$\begin{aligned} N_7 &= -0.0002635, & N_7^{IV} &= +0.00625584,\\ N_7' &= -0.0003035, & N_7^{V} &= -0.0157362,\\ N_7'' &= -0.0027085, & N_7^{VI} &= +0.00068567,\\ N_7''' &= -0.0092412, & N_7^{VII} &= +0.000076911. \end{aligned}$$

22. For an increment of $+\frac{1}{40}m^{V}$, to the assumed mass of *Saturn*, we shall obtain the preliminary computations by merely making all the coefficients positive in equations (307–312). We shall then obtain the following

Fundamental Equations for $\mu^{V} = +\frac{1}{40}$; *or for* $m^{V} = 1 \div 3416.195$.

$$\left.\begin{aligned} A &= g^2 + 38.078720.g + 181.976176;\\ A' &= g^2 + 23.1907166.g + 98.4490593;\\ A'' &= g^2 + 18.7198881.g + 72.8857520;\\ A_1 &= g^2 + 18.6278118.g + 61.6540448;\\ A_2 &= g^2 + 13.3828119.g + 9.178963;\\ A_3 &= g^2 + 26.5724027.g + 10.089397. \end{aligned}\right\} \tag{491}$$

$$\left.\begin{aligned} D &= g^2 + 44.5181269.g + 588.346190;\\ D' &= g^2 + 52.0193531.g + 605.574324;\\ D'' &= g^2 + 32.1854533.g + 256.3822746;\\ D_1 &= g^2 + 44.4628062.g + 176.258864;\\ D_2 &= g^2 + 47.1787023.g + 33.673551;\\ D_3 &= g^2 + 3.4692599.g + 1.72977087. \end{aligned}\right\} \tag{492}$$

$$\left.\begin{aligned} B = \{g + 32.45831\}b;\quad B' &= \{g + 17.5956788\}b;\\ B'' &= \{g + 13.143888\}b. \end{aligned}\right\} \tag{493}$$

$$\left.\begin{aligned} C &= -\{g + 22.263602\}\,[9.4381189]b';\\ C' &= -\{g + 17.6256851\}[9.1138076]b';\\ C'' &= +[0.3976676]b';\\ C''' &= +[0.4411620]b'. \end{aligned}\right\} \tag{494}$$

$$\left.\begin{aligned}
E\ &=-[9.9807377]b'';\\
E'\ &=-\{g+22.2545242\}[8.9723624]b'';\\
E''&=-\{g+17.5731585\}[9.7501125]b'';\\
E'''&=-[1.0016537]b''.
\end{aligned}\right\}\tag{495}$$

$$\left.\begin{aligned}
F\ &=+[8.1679376]b''';\\
F'\ &=-[9.1453586]b''';\\
F''&=-\{g+14.612294\}\ [0.7927855]b''';\\
F'''&=-\{g+34.3936667\}[9.6907241]b'''.
\end{aligned}\right\}\tag{496}$$

$$B_1=\{g+4.5551100\}b_1;\quad \left.\begin{aligned}
B_2&=\{g+\ 0.7176484\}b_1;\\
B_3&=\{g+18.8231590\}b_1.
\end{aligned}\right\}\tag{497}$$

$$\left.\begin{aligned}
C_1&=-\{g+4.1678758\}[9.5433087]b_2;\\
C_2&=-\{g+0.7359221\}[9.4349711]b_2.\\
C_3&=+[0.8751766]b_2;\\
C_4&=+[0.8761888]b_2.
\end{aligned}\right\}\tag{498}$$

$$\left.\begin{aligned}
E_1&=-[8.3317448]b_3;\\
E_2&=-\{g+40.2949304\}[8.7437718]b_3;\\
E_3&=-\{g+\ 0.6538252\}[0.7242832]b_3;\\
E_4&=-[0.9647514]b_3.
\end{aligned}\right\}\tag{499}$$

$$\left.\begin{aligned}
F_1&=+[7.7244692]b_4;\\
F_2&=-[0.3564628]b_4;\\
F_3&=-\{g+\ 2.8154348\}[1.5514854]b_4;\\
F_4&=-\{g+46.44278\}\ \ [9.4626364]b_4.
\end{aligned}\right\}\tag{500}$$

$$\left.\begin{aligned}
&g^4+47.5409132.g^3+774.11613432.g^2\\
&\quad+4966.568399.g+10830.80683
\end{aligned}\right\}=(\chi,\ \chi_1,\ \chi_2,\ \chi_3).\tag{501}$$

$$\left.\begin{aligned}
&g^4+29.7477885.g^3+96.8557913.g^2\\
&\quad+52.577272.g\ \ +\ 0.536670
\end{aligned}\right\}=(\chi_4,\ \chi_5,\ \chi_6,\ \chi_7).\tag{502}$$

The values of b_1, b_2, b_3, and b_4 are given by equations (406); and the values of b, b', b'', and b''' are given by equations (405), by merely multiplying the coefficients of N^v by $1+\mu^v=1.025$.

If we now put equations (501) and (502) equal to nothing, they will give,

$$\begin{aligned}
g\ &=-\ \ 5''.1255241; & g_4&=-\ \ 0''.0104061;\\
g_1&=-\ \ 6\ .5914740; & g_5&=-\ \ 0\ .6688791;\\
g_2&=-17\ .4063959; & g_6&=-\ \ 2\ .9523070;\\
g_3&=-18\ .4175192; & g_7&=-26\ .1161961.
\end{aligned}$$

Substituting these roots in succession in equations (491–502), we shall get the following values:—

For the root g, we get,

$g = -5''.1293390,$

$N' = +0.1224435N$ log. 9.0879358,
$N'' = +0.08766774N$ " 8.9428398,
$N''' = +0.0175016N$ " 8.2430770,
$N^{IV} = -0.000205739N$ " 6.3133160n,
$N^{V} = -0.000261587N$ " 6.4176164n,
$N^{VI} = +0.000236310N$ " 6.3734826,
$N^{VII} = +0.0000062338N$ " 4.7947524.

For the root g_1, we get,

$g_1 = -6''.5971682,$

$N_1' = -0.277326N_1$ log. 9.4429910n,
$N_1'' = -0.223546N_1$ " 9.3493660n,
$N_1''' = -0.0468220N_1$ " 8.6704496n,
$N_1^{IV} = +0.000334045N_1$ " 6.5238055,
$N_1^{V} = +0.000471677N_1$ " 6.6736443,
$N_1^{VI} = -0.000250301N_1$ " 6.3984620n,
$N_1^{VII} = -0.0000148755N_1$ " 5.1724700n.

For the root g_2, we get,

$g_2 = -17''.4073173,$

$N_2' = -\ 5.57006N_2$ log. 0.7458600n,
$N_2'' = +\ 4.57184N_2$ " 0.6609136,
$N_2''' = +33.0320N_2$ " 1.5189348,
$N_2^{IV} = -\ 0.00161379N_2$ " 7.2078461n,
$N_2^{V} = -\ 0.0137070N_2$ " 8.1369422n,
$N_2^{VI} = +\ 0.00136814N_2$ " 7.1361306,
$N_2^{VII} = +\ 0.000156952N_2$ " 6.1957676.

For the root g_3, we get,

$g_3 = -18''.4175195,$

$N_3' = -\ 6.10257N_3$ log. 0.7855130n,
$N_3'' = +\ 6.66495N_3$ " 0.8237970,
$N_3''' = -10.3261N_3$ " 1.0139370n,
$N_3^{IV} = -\ 0.000017665N_3$ " 95.2471266n,
$N_3^{V} = -\ 0.000117075N_3$ " 96.0684623n,
$N_3^{VI} = +\ 0.00000985785N_3$ " 94.9937824,
$N_3^{VII} = +\ 0.00000108146N_3$ " 94.0340085.

For the root g_4, we get,

$g_4 = 0'', \quad N_4 = N_4' = N_4'' = N_4''' = \text{\&c.}$

For the root g_5, we get,

$$g_5=-0''.6675803,$$

$N_5=+1.234453N_5^{IV}$	log. 0.0914746;
$N_5'=+1.132464N_5^{IV}$	" 0.0540246;
$N_5''=+1.109145N_5^{IV}$	" 0.0449882;
$N_5'''=+1.049894N_5^{IV}$	" 0.0211454;
$N_5^{V}=+0.965528N_5^{IV}$	" 9.9847648;
$N_5^{VI}=-0.925706N_5^{IV}$	" 9.9664731n;
$N_5^{VII}=-9.910500N_5^{IV}$	" 0.9960956n.

For the root g_6, we get,

$$g_6=-2''.9514286,$$

$N_6=+\ 3.63816N_6^{IV}$	log. 0.5608818;
$N_6'=+\ 2.08409N_6^{IV}$	" 0.3189167;
$N_6''=+\ 1.86452N_6^{IV}$	" 0.2705668;
$N_6'''=+\ 1.31997N_6^{IV}$	" 0.1205640;
$N_6^{V}=+\ 0.816989N_6^{IV}$	" 9.9122164;
$N_6^{VI}=-20.18151N_6^{IV}$	" 1.3049538n;
$N_6^{VII}=+\ 2.139190N_6^{IV}$	" 0.3302494.

For the root g_7, we get,

$$g_7=-26''.1183694,$$

$N_7=-0.0416526N_7^{IV}$	log. 8.6196418n;
$N_7'=-0.0489363N_7^{IV}$	" 8.6896315n;
$N_7''=-0.426719N_7^{IV}$	" 9.6301420n;
$N_7'''=-1.440405N_7^{IV}$	" 0.1584846n;
$N_7^{V}=-2.42982N_7^{IV}$	" 0.3855740n;
$N_7^{VI}=+0.108637N_7^{IV}$	" 9.0359788;
$N_7^{VII}=+0.012173N_7^{IV}$	" 8.0853954.

23. If we now substitute these values in equations (408) and (409), we shall obtain the following quantities:—

For the root $g=-5''.1293390$, we get,

$$x=+\frac{606572}{10^{20}};\quad y=+\frac{1589168.6}{10^{20}};\quad z=\frac{140213200}{10^{20}};$$

Whence $\beta=20°\ 53'\ 20''.5$; and log. $N=9.0839064$.

$N\ =+0.1213127$;
$N'\ =+0.0148551$;
$N''\ =+0.0106362$;
$N'''=+0.00212364$;
$N^{IV}=-0.00002496$;
$N^{V}\ =-0.000031737$;
$N^{VI}=+0.000028671$;
$N^{VII}=+0.0000007563$.

For the root $g_1 = -6''.5971682$, we get,

$$x_1 = +\frac{702346}{10^{20}}; \quad y_1 = -\frac{650773.9}{10^{20}}; \quad z_1 = \frac{33379780}{10^{20}};$$

Whence $\beta_1 = 132^\circ\ 49'\ 3''.9$; log. $N_1 = 8.4576530$.

$N_1 = +0.0286850;$ $\qquad N_1^{IV} = +0.000009582;$
$N_1' = -0.00795527;$ $\qquad N_1^{V} = +0.000013530;$
$N_1'' = -0.00641252;$ $\qquad N_1^{VI} = -0.000007180;$
$N_1''' = -0.00134306;$ $\qquad N_1^{VII} = -0.0000004267.$

For the root $g_2 = -17''.4073173$, we get,

$$x_2 = -\frac{68779790}{10^{20}}; \quad y_2 = +\frac{28706100}{10^{20}}; \quad z_2 = \frac{4.840416}{10^{10}};$$

Whence $\beta_2 = 292^\circ\ 39'\ 13''.8$; log. $N_2 = 7.1874476$.

$N_2 = +0.0015397;$ $\qquad N_2^{IV} = -0.000002485;$
$N_2' = -0.0085764;$ $\qquad N_2^{V} = -0.000021105;$
$N_2'' = +0.0070528;$ $\qquad N_2^{VI} = +0.0000021066;$
$N_2''' = +0.0508607;$ $\qquad N_2^{VII} = +0.0000002417.$

For the root $g_3 = -18''.4175195$, we get,

$$x_3 = -\frac{67277160}{10^{20}}; \quad y_3 = -\frac{22295350}{10^{20}}; \quad z_3 = \frac{1.9362943}{10^{10}};$$

Whence $\beta_3 = 251^\circ\ 39'\ 34''.1$; log. $N_3 = 97.5635231$.

$N_3 = +0.0036604;$ $\qquad N_3^{IV} = -0.00000006466;$
$N_3' = -0.0223376;$ $\qquad N_3^{V} = -0.00000042853;$
$N_3'' = +0.0243961;$ $\qquad N_3^{VI} = +0.000000036083;$
$N_3''' = -0.0377972;$ $\qquad N_3^{VII} = +0.0000000039585.$

For the root $g_4 = 0''$, we

$$x_4 = +\frac{0.7324877}{10^{10}}; \quad y_4 = -\frac{0.2141555}{10^{10}}; \quad z_4 = \frac{27.41415}{10^{10}};$$

Whence $\beta_4 = 106^\circ\ 17'\ 49''.9$; log. $N_4 = 98.4446362$.

$$N_4 = N_4' = N_4'' = N_4''' = N_4^{IV} = N_4^{V} = N_4^{VI} = N_4^{VII} = +0.0278379$$

For the root $g_5 = -0''.6675803$, we get,

$$x_5 = +\frac{0.1042216}{10^{10}}; \quad y_5 = +\frac{0.2727222}{10^{10}}; \quad z_5 = \frac{245.6575}{10^{10}};$$

Whence $\beta_5 = 20^\circ\ 55'\ 5''.7$; log. $N_5^{IV} = 97.0749906$.

$N_5 = +0.00146716;$ $\qquad N_5^{IV} = +0.00118851;$
$N_5' = +0.0013459;$ $\qquad N_5^{V} = +0.00114755;$
$N_5'' = +0.00131823;$ $\qquad N_5^{VI} = -0.00110001;$
$N_5''' = +0.00124781;$ $\qquad N_5^{VII} = -0.0117785.$

For the root $g_6=-2''.9514286$, we get,

$$x_6=+\frac{0.3712512}{10^{10}};\quad y_6=-\frac{0.3561690}{10^{10}};\quad z_6=\frac{584.6008}{10^{10}};$$

Whence $\beta_6=133^\circ\ 48'\ 43''.9$; log. $N_6^{IV}=96.9445043$.

$$\begin{array}{ll} N_6\ =+0.0032017; & N_6^{IV}\ =+0.000880044; \\ N_6'\ =+0.0018341; & N_6^{V}\ =+0.000718986; \\ N_6''=+0.0016409; & N_6^{VI}=-0.0177606; \\ N_6'''=+0.0011616; & N_6^{VII}=+0.0018826. \end{array}$$

For the root $g_7=-26''.1183694$, we get,

$$x_7=-\frac{0.2996395}{10^{10}};\quad y_7=+\frac{0.2202906}{10^{10}}.\quad z_7=\frac{57.99673}{10^{10}};$$

Whence $\beta_7=306^\circ\ 19'\ 22''.0$; log. $N_7^{IV}=97.8070262$.

$$\begin{array}{ll} N_7\ =-0.00002671; & N_7^{IV}\ =+0.00641248; \\ N_7'\ =-0.00031380; & N_7^{V}\ =-0.0155812; \\ N_7''=-0.0027363; & N_7^{VI}=+0.00069663; \\ N_7'''=-0.0092366; & N_7^{VII}=+0.000078059. \end{array}$$

24. For an increment of $\frac{1}{20}m^{VI}$, to the assumed mass of *Uranus*, we shall obtain the preliminary computations by merely making all the coefficients positive in equations (326-331). We shall then obtain the following

Fundamental Equations for $\mu^{VI}=+\frac{1}{20}$; *or for* $m^{VI}=\frac{1}{23719}$.

$$\left.\begin{array}{l} A\ =g^2+38.068970.g\ +181.871338; \\ A'=g^2+23.1735927.g+\ 98.3308977; \\ A''=g^2+18.7132214.g+\ 72.8344734; \\ A_1=g^2+18.4995010.g+\ 61.4402060; \\ A_2=g^2+13.2095523.g+\ \ 9.147128; \\ A_3=g^2+26.3996602.g+\ 10.055139. \end{array}\right\}\qquad(503)$$

$$\left.\begin{array}{l} D\ =g^2+44.5054401.g+588.063848; \\ D'=g^2+51.9992925.g+604.964956; \\ D''=g^2+32.1623093.g+256.0208612; \\ D_1=g^2+43.9809321.g+175.644437; \\ D_2=g^2+46.5212935.g+\ 33.556164; \\ D_3=g^2+\ 3.4429483.g+\ \ 1.72374142. \end{array}\right\}\qquad(504)$$

$$\left.\begin{array}{ll} B=\{g+32.45043\}b; & B'=\{g+17.5804200\}b; \\ & B''=\{g+13.139086\}b. \end{array}\right\}\qquad(505)$$

$$\left.\begin{array}{l} C\ =-\{g+22.255718\}\ [9.4381189]b'; \\ C'\ =-\{g+17.6104269\}[9.1138076]b'; \\ C''=+[0.3976676]b'; \\ C'''=+[0.4411620]b'. \end{array}\right\}\qquad(506)$$

$$\left.\begin{aligned}E\ &=-[9.9807377]b'';\\ E'&=-\{g+22.2497227\}[8.9723624]b'';\\ E''&=-\{g+17.5578996\}[9.7501125]b'';\\ E''&=-[1.0016537]b''.\end{aligned}\right\}\qquad(507)$$

$$\left.\begin{aligned}F\ &=+[8.1679376]b''';\\ F'&=-[9.1453586]b''';\\ F''&=-\{g+14.604409\}\ [0.7927855]b''';\\ F'''&=-\{g+34.3888650\}[9.6907241]b'''.\end{aligned}\right\}\qquad(508)$$

$$\left.\begin{aligned}B_1=\{g+4.6079205\}b_1;\ B_2&=\{g+\ 0.7255099\}b_2;\\ B_3&=\{g+18.8315378\}b_2.\end{aligned}\right\}\qquad(509)$$

$$\left.\begin{aligned}C_1&=-\{g+4.2013246\}[9.5433087]b_2;\\ C_2&=-\{g+0.7437838\}[9.4349711]b_2;\\ C_3&=+[0.8644527]b_2;\\ C_4&=+[0.8654649]b_2.\end{aligned}\right\}\qquad(510)$$

$$\left.\begin{aligned}E_1&=-[8.3529341]b_3;\\ E_2&=-\{g+39.7796075\}[8.7437718]b_3;\\ E_3&=-\{g+\ 0.6616869\}[0.7242832]b_3;\\ E_4&=-[0.9859407]b_3.\end{aligned}\right\}\qquad(511)$$

$$\left.\begin{aligned}F_1&=+[7.7244692]b_4;\\ F_2&=-[0.3564628]b_4;\\ F_3&=-\{g+\ 2.7812615\}[1.5514854]b_4;\\ F_4&=-\{g+45.77751\}\ \ [9.4626364]b_4.\end{aligned}\right\}\qquad(512)$$

$$\left.\begin{aligned}&g^4+47.5111025.g^3+773.1351541.g^2\\ &\quad+4957.025771g+10802.91193\end{aligned}\right\}=(\chi,\chi_1,\chi_2,\chi_3);\quad(513)$$

$$\left.\begin{aligned}&g^4+29.5535550.g^3+95.6630308.g^2\\ &\quad+52.197094.g\ +\ 0.534229\end{aligned}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7).\quad(514)$$

The values of b_1, b_2, b_3, and b_4 are given by equations (406); and the values of b, b', b'', and b''' are given by equations (405), by merely multiplying the coefficients of N^{VI} by $1+\mu^{VI}=1.05$.

If we now put the equations (513) and (514) equal to nothing, they will give

$$\begin{aligned}g\ &=-\ \ 5''.1224110; & g_4&=-\ \ 0''.0104337;\\ g_1&=-\ \ 6\ .5865615; & g_5&=-\ \ 0\ .6748372;\\ g_2&=-17\ .3929254; & g_6&=-\ \ 2\ .9245347;\\ g_3&=-18\ .4092044; & g_7&=-25\ .9437494.\end{aligned}$$

Substituting these roots in succession in equations (503–514), we shall get the following values:—

For the root g, we get,

$g = -5''.1262710;$
$N' = +0.1227677N$ log. 9.0890840;
$N'' = +0.0879386N$ " 8.9441794;
$N''' = +0.0175667N$ " 8.2446904;
$N^{\mathrm{IV}} = -0.000208367N$ " 6.3188297n;
$N^{\mathrm{V}} = -0.00026463N$ " 6.4226422n;
$N^{\mathrm{VI}} = +0.000232174N$ " 6.3658128;
$N^{\mathrm{VII}} = +0.00000549425N$ " 4.7599088.

For the root g_1, we get,

$g_1 = -6''.5922959;$
$N_1' = -0.276494N_1$ log. 9.4416856n;
$N_1'' = -0.222958N_1$ " 9.3482238n;
$N_1''' = -0.0467327N_1$ " 8.6696208n;
$N_1^{\mathrm{IV}} = +0.000336574N_1$ " 6.5270807;
$N_1^{\mathrm{V}} = +0.000474690N_1$ " 6.6764098;
$N_1^{\mathrm{VI}} = -0.000245839N_1$ " 6.3906513n;
$N_1^{\mathrm{VII}} = -0.0000143346N_1$ " 5.1563868n.

For the root g_2, we get,

$g_2 = -17''.3938608;$
$N_2' = -\ 5.56333N_2$ log. 0.7453350n;
$N_2'' = +\ 4.56392N_2$ " 0.6593380;
$N_2''' = +33.2389N_2$ " 1.5216464;
$N_2^{\mathrm{IV}} = -\ 0.00165952N_2$ " 7.2199837n;
$N_2^{\mathrm{V}} = -\ 0.0140236N_2$ " 8.1468598n;
$N_2^{\mathrm{VI}} = +\ 0.00136629N_2$ " 7.1355420;
$N_2^{\mathrm{VII}} = +\ 0.000156224N_2$ " 6.1937476.

For the root g_3, we get,

$g_3 = -18''.4092048;$
$N_3' = -\ 6.098831N_3$ log. 0.7852466n;
$N_3'' = +\ 6.65618N_3$ " 0.8232250;
$N_3''' = -10.22645N_3$ " 1.0097248n;
$N_3^{\mathrm{IV}} = -\ 0.0000196421N_3$ " 95.2931878n;
$N_3^{\mathrm{V}} = -\ 0.000150200N_3$ " 96.1766688n;
$N_3^{\mathrm{VI}} = +\ 0.0000125074N_3$ " 95.0971660;
$N_3^{\mathrm{VII}} = +\ 0.00000139286N_3$ " 94.1439073.

For the root g_4, we get,

$g_4 = 0'',\ N_4 = N_4' = N_4'' = N_4''' = \&c.$

For the root g_5, we get,

$g_5 = -0''.6735122;$

$N_5 = +1.23718 N_5^{IV}$ log. 0.0924317;
$N_5' = +1.133973 N_5^{IV}$ " 0.0546030;
$N_5'' = +1.110407 N_5^{IV}$ " 0.0454822;
$N_5''' = +1.05043 N_5^{IV}$ " 0.0213676;
$N_5^{V} = +0.964612 N_5^{IV}$ " 9.9843526;
$N_5^{VI} = -0.937483 N_5^{IV}$ " 9.9719633n;
$N_5^{VII} = -9.79928 N_5^{IV}$ " 0.9911943n.

For the root g_6, we get,

$g_6 = -2''.9236175;$

$N_6 = +\ 3.57518 N_6^{IV}$ log. 0.5532976;
$N_6' = +\ 2.06480 N_6^{IV}$ " 0.3148787;
$N_6'' = +\ 1.84969 N_6^{IV}$ " 0.2670992;
$N_6''' = +\ 1.285565 N_6^{IV}$ " 0.1190940;
$N_6^{V} = +\ 0.816008 N_6^{IV}$ " 9.9116944;
$N_6^{VI} = -19.1632 N_6^{IV}$ " 1.2824678n;
$N_6^{VII} = +\ 2.16794 N_6^{IV}$ " 0.3360471.

For the root g_7, we get,

$g_7 = -25''.9459126;$

$N_7 = -0.0420490 N_7^{IV}$ log. 8.6237552n;
$N_7' = -0.0466848 N_7^{IV}$ " 8.6691754n;
$N_7'' = -0.432476 N_7^{IV}$ " 9.6359617n;
$N_7''' = -1.466224 N_7^{IV}$ " 0.1662004n;
$N_7^{V} = -2.491785 N_7^{IV}$ " 0.3965106n;
$N_7^{VI} = +0.1093547 N_7^{IV}$ " 9.0388375;
$N_7^{VII} = +0.0122059 N_7^{IV}$ " 8.0865696.

25. If we now substitute these values in equations (408) and (409), we shall obtain the following quantities:—

For the root $g = -5''.1262710$, we get,

$$x = +\frac{610397}{10^{20}}; \qquad y = +\frac{1587285.2}{10^{20}}; \qquad z = \frac{14044917}{10^{20}}.$$

Whence $\beta = 21^\circ\ 1'\ 11''.9$; log. $N = 9.0833149$.

$N = +0.1211476;$ $N^{IV} = -0.000025246;$
$N' = +0.0148742;$ $N^{V} = -0.000032062;$
$N'' = +0.0106545;$ $N^{VI} = +0.00002813;$
$N''' = +0.00212865;$ $N^{VII} = +0.000000697.$

For the root $g_1=-6''.5922959$, we get,

$$x_1=+\frac{696641}{10^{20}};\quad y_1=-\frac{640902.9}{10^{20}};\quad z_1=\frac{33247150}{10^{20}}.$$

Whence $\beta_1=132^\circ\ 41'\ 2''.3$; log. $N_1=8.4538431$.

$$\begin{array}{ll} N_1\ =+0.0284342; & N_1^{IV}=+0.000009570; \\ N_1'\ =-0.0078621; & N_1^{V}\ =+0.000013498; \\ N_1''=-0.0063398; & N_1^{VI}=-0.0000069904; \\ N_1'''=-0.0013288; & N_1^{VII}=-0.0000004076. \end{array}$$

For the root $g_2=-17''.3938608$, we get,

$$x_2=-\frac{68615050}{10^{20}};\quad y_2=+\frac{28882030}{10^{20}};\quad z_2=\frac{0.4884728}{10^{10}}.$$

Whence $\beta_2=292^\circ\ 49'\ 39''.4$; log. $N_2=97.1830006$.

$$\begin{array}{ll} N_2\ =+0.0015241; & N_2^{IV}=-0.0000025292; \\ N_2'\ =-0.0084788; & N_2^{V}\ =-0.000021373; \\ N_2''=+0.0069557; & N_2^{VI}=+0.0000020823; \\ N_2'''=+0.0050658; & N_2^{VII}=+0.00000023809. \end{array}$$

For the root $g_3=-18''.4092048$, we get,

$$x_3=-\frac{0.006724489}{10^{10}};\quad y_3=-\frac{0.002217478}{10^{10}};\quad z_3=\frac{1.9257107}{10^{10}}.$$

Whence $\beta_3=251^\circ\ 44'\ 58''.0$; log. $N_3=97.5654837$.

$$\begin{array}{ll} N_3\ =+0.0036769; & N_3^{IV}=-0.00000007222; \\ N_3'\ =-0.0224249; & N_3^{V}\ =-0.0000005523; \\ N_3''=+0.0244742; & N_3^{VI}=+0.000000046; \\ N_3'''=-0.0376018; & N_3^{VII}=+0.00000000512. \end{array}$$

For the root $g_4=0''$, we get,

$$x_4=+\frac{0.7265241}{10^{10}};\quad y_4=-\frac{0.2110814}{10^{10}};\quad z_4=\frac{27.31188}{10^{10}}.$$

Whence $\beta_4=106^\circ\ 12'\ 1''.7$; log. $N_4=98.4424954$.

$$N_4=N_4'=N_4''=N_4'''=N_4^{IV}=N_4^{V}=N_4^{VI}=N_4^{VII}=0.0277010.$$

For the root $g_5=-0''.6735122$, we get,

$$x_5=+\frac{0.1020975}{10^{10}};\quad y_5=+\frac{0.2702021}{10^{10}};\quad z_5=\frac{240.6404}{10^{10}}.$$

Whence $\beta_5=20^\circ\ 42'\ 11''.2$; log. $N_5^{IV}=97.0793126$.

$$\begin{array}{ll} N_5\ =+0.0014850; & N_5^{IV}=+0.00120036; \\ N_5'\ =+0.0013612; & N_5^{V}\ =+0.0011579; \\ N_5''=+0.0013329; & N_5^{VI}=-0.0011251; \\ N_5'''=+0.0012609; & N_5^{VII}=-0.0117624. \end{array}$$

For the root $g_6 = -2''.9236175$, we get,

$$x_6 = +\frac{0.3679656}{10^{10}};\quad y_6 = -\frac{0.3547543}{10^{10}};\quad z_6 = \frac{555.2999}{10^{10}}.$$

Whence $\beta_6 = 133^\circ\ 57'\ 9''.9$; log. $N_6^{IV} = 96.9640000$.

$$\begin{array}{ll}
N_6 \ = +0.0032908; & N_6^{IV} = +0.00092045; \\
N_6' \ = +0.0019005; & N_6^{V} \ = +0.00075109; \\
N_6'' = +0.0017026; & N_6^{VI} = -0.01763875; \\
N_6''' = +0.0012109; & N_6^{VII} = +0.00199548.
\end{array}$$

For the root $g_7 = -25''.9459126$, we get,

$$x_7 = -\frac{0.3001025}{10^{10}};\quad y_7 = +\frac{0.2204575}{10^{10}};\quad z_7 = \frac{59.06886}{10^{10}}.$$

Whence $\beta_7 = 306^\circ\ 18'\ 4''.6$; log. $N_7^{IV} = 97.7996217$.

$$\begin{array}{ll}
N_7 \ = -0.0002651; & N_7^{IV} = +0.00630408; \\
N_7' \ = -0.0002943; & N_7^{V} \ = -0.0157084; \\
N_7'' = -0.0027264; & N_7^{VI} = +0.00068938; \\
N_7''' = -0.0092432; & N_7^{VII} = +0.00007695.
\end{array}$$

26. For an increment of $+\frac{1}{10}m^{VII}$, to the assumed mass of *Neptune*, we shall obtain the preliminary computations by merely making all the coefficients positive in equations (345–350). We shall then obtain the following

Fundamental Equations for $\mu^{VII} = +\frac{1}{10}$; *or for* $m^{VII} = 1 \div 17072.73$.

$$\left.\begin{array}{l}
A \ = g^2 + 38.068863.g \ + 181.870184; \\
A' = g^2 + 23.1734071.g + \ 98.3296126; \\
A'' = g^2 + 18.7131480.g + \ 72.8339084; \\
A_1 = g^2 + 18.4537418.g + \ 60.9399039; \\
A_2 = g^2 + 13.2015603.g + \ \ 9.072649; \\
A_3 = g^2 + 26.3912337.g + \ \ 9.975760.
\end{array}\right\} \quad (515)$$

$$\left.\begin{array}{l}
D \ = g^2 + 44.5053009.g + 588.060747; \\
D' = g^2 + 51.9990747.g + 604.958351; \\
D'' = g^2 + 32.1620579.g + 256.0169344; \\
D_1 = g^2 + 43.9488809.g + 174.214515; \\
D_2 = g^2 + 46.5094752.g + \ 33.282984; \\
D_3 = g^2 + \ 3.4725714.g + \ \ 1.70970867.
\end{array}\right\} \quad (516)$$

$$\left.\begin{array}{ll}
B = \{g + 32.45034\}b; & B' = \{g + 17.5802549\}b; \\
 & B'' = \{g + 13.139034\}b.
\end{array}\right\} \quad (517)$$

$$\left.\begin{array}{l}
C \ = -\{g + 22.255630\}\ [9.4381189]b'; \\
C' = -\{g + 17.6102612\}[9.1138076]b'; \\
C'' = +[0.3976676]b'; \\
C''' = +[0.4411620]b'
\end{array}\right\} \quad (518)$$

$$\left.\begin{aligned}E\ &=-[9.9807377]b'';\\ E'&=-\{g+22.2496697\}[8.9723624]b'';\\ E''&=-\{g+17.5577346\}[9.7501125]b'';\\ E'''&=-[1.0016537]b''.\end{aligned}\right\}\tag{519}$$

$$\left.\begin{aligned}F\ &=+[8.1679376]b''';\\ F'&=-[9.1453586]b''';\\ F''&=-\{g+14.604322\}\ [0.7927855]b''';\\ F'''&=-\{g+34.3888122\}[9.6907241]b'''.\end{aligned}\right\}\tag{520}$$

$$\left.\begin{aligned}B_1=\{g+4.5635477\}b_1;\quad B_2&=\{g+\ 0.7189043\}b_1;\\ B_3&=\{g+18.8244977\}b_1.\end{aligned}\right\}\tag{521}$$

$$\left.\begin{aligned}C_1&=-\{g+4.1763135\ \}[9.5433087]b_2;\\ C_2&=-\{g+0.73900559\}[9.4349711]b_2;\\ C_3&=+[0.8644527]b_2;\\ C_4&=+[0.8654649]b_2.\end{aligned}\right\}\tag{522}$$

$$\left.\begin{aligned}E_1&=-[8.3317448]b_3;\\ E_2&=-\{g+39.7725674\ \}[8.7437718]b_3;\\ E_3&=-\{g+\ 0.64869894\}[0.7242832]b_3;\\ E_4&=-[0.9647514]b_3.\end{aligned}\right\}\tag{523}$$

$$\left.\begin{aligned}F_1&=+[7.7658619]b_4;\\ F_2&=-[0.3978555]b_4;\\ F_3&=-\{g+\ 2.8238725\}[1.5514854]b_4;\\ F_4&=-\{g+45.770470\}\ [9.4626364]b_4.\end{aligned}\right\}\tag{524}$$

$$\left.\begin{aligned}g^4+47.5107777.g^3+773.1244605.g^2\\ +4956.921649.g+10802.60734\end{aligned}\right\}=(\chi,\chi_1,\chi_2,\chi_3);\tag{525}$$

$$\left.\begin{aligned}g^4+29.5753988.g^3+96.3534529.g^2\\ +52.082669.g+0.529879\end{aligned}\right\}=(\chi_4,\chi_5,\chi_6,\chi_7).\tag{526}$$

The values of b_1, b_2, b_3, and b_4 are given by equations (406), and the values of b, b', b'', and b''' are given by equations (405), by merely multiplying the coefficients of N^{VII} by $1+\mu^{VII}=1.10$.

If we now put equations (525) and (526) equal to nothing, they will give

$$\begin{aligned}g_1&=-\ 5''.1223768; & g_4&=-\ 0''.01037221;\\ g_2&=-\ 6\ .5865075; & g_5&=-\ 0\ .66492499;\\ g_3&=-17\ .3927801; & g_6&=-\ 2\ .96207581;\\ g_4&=-18\ .4091132; & g_7&=-25\ .93802580.\end{aligned}$$

Substituting these roots in succession in equations (515–526), we get the following values:—

For the root g, we get,

$$\begin{aligned}
g &= -5''.1262327, \\
N' &= +0.122771N && \text{log. } 9.0890969, \\
N'' &= +0.0879417N && \text{“ } 8.9441950, \\
N''' &= +0.0175679N && \text{“ } 8.2447196, \\
N^{IV} &= -0.000207983N && \text{“ } 6.3180273n, \\
N^{V} &= -0.000264402N && \text{“ } 6.4222642n, \\
N^{VI} &= +0.0002360322N && \text{“ } 6.3729713, \\
N^{VII} &= +0.0000061622 && \text{“ } 4.7897356.
\end{aligned}$$

For the root g_1, we get,

$$\begin{aligned}
g_1 &= -6''.5922343, \\
N_1' &= -0.276482N_1 && \text{log. } 9.4416674n, \\
N_1'' &= -0.222950N_1 && \text{“ } 9.3482076n, \\
N_1''' &= -0.0466245N_1 && \text{“ } 8.6696144n, \\
N_1^{IV} &= +0.00033610N_1 && \text{“ } 6.5264690, \\
N_1^{V} &= +0.000474472N_1 && \text{“ } 6.6762012, \\
N_1^{VI} &= -0.000248245N_1 && \text{“ } 6.3948800n, \\
N_1^{VII} &= -0.0000147153N_1 && \text{“ } 5.1677685n.
\end{aligned}$$

For the root g_2, we get,

$$\begin{aligned}
g_2 &= -17''.3937133, \\
N_2' &= -\ 5.56327N_2 && \text{log. } 0.7453298n, \\
N_2'' &= +\ 4.56374N_2 && \text{“ } 0.6593212, \\
N_2''' &= +33.24130N_2 && \text{“ } 1.5216780, \\
N_2^{IV} &= -\ 0.0016539N_2 && \text{“ } 7.2185079n, \\
N_2^{V} &= -\ 0.0140311N_2 && \text{“ } 8.1470920n, \\
N_2^{VI} &= +\ 0.00137024N_2 && \text{“ } 7.1367973, \\
N_2^{VII} &= +\ 0.00015714N_2 && \text{“ } 6.1962851.
\end{aligned}$$

For the root g_3, we get,

$$\begin{aligned}
g_3 &= -18''.4091136, \\
N_3' &= -\ 6.09878N_3 && \text{log. } 0.7852438n, \\
N_3'' &= +\ 6.65608N_3 && \text{“ } 0.8232187, \\
N_3''' &= -10.2254N_3 && \text{“ } 1.0096800n, \\
N_3^{IV} &= -\ 0.0000195968N_3 && \text{“ } 95.2921852n, \\
N_3^{V} &= -\ 0.000150608N_3 && \text{“ } 96.1778487n, \\
N_3^{VI} &= +\ 0.0000125707N_3 && \text{“ } 95.0993576, \\
N_3^{VII} &= +\ 0.00000139394N_3 && \text{“ } 94.1442453.
\end{aligned}$$

For the root g_4, we get,

$$g_4 = 0''; \quad N_4 = N_4' = N_4'' = N_4''' = \&c.$$

For the root g_5, we get,

$g_5 = -0''.6634935,$

$N_5 \ = +1.23293 N_5^{IV}$ log. 0.0909496,

$N_5' \ = +1.13171 N_5^{IV}$ " 0.0537360,

$N_5'' \ = +1.10855 N_5^{IV}$ " 0.0447544,

$N_5''' \ = +1.04961 N_5^{IV}$ " 0.0210270,

$N_5^{V} \ = +0.965270 N_5^{IV}$ " 9.9846490,

$N_5^{VI} = -0.921756 N_5^{IV}$ " 9.9646159n,

$N_5^{VII} = -8.94419 N_5^{IV}$ " 0.9515409n.

For the root g_6, we get,

$g_6 = -2''.9612185,$

$N_6 \ = +\ 3.66750 N_6^{IV}$ log. 0.5643695,

$N_6' \ = +\ 2.09390 N_6^{IV}$ " 0.3209559,

$N_6'' \ = +\ 1.87228 N_6^{IV}$ " 0.2723698,

$N_6''' \ = +\ 1.32235 N_6^{IV}$ " 0.1213462,

$N_6^{V} \ = +\ 0.813392 N_6^{IV}$ " 9.9102996,

$N_6^{VI} = -20.4503 N_6^{IV}$ " 1.3106998n,

$N_6^{VII} = +\ 2.15783 N_6^{IV}$ " 0.3340172.

For the root g_7, we get,

$g_7 = -25''.9401904,$

$N_7 \ = -0.042064 N_7^{IV}$ log. 8.6239109n,

$N_7' \ = -0.0466092 N_7^{IV}$ " 8.6684721n,

$N_7'' \ = -0.432691 N_7^{IV}$ " 9.6361780n,

$N_7''' \ = -1.46720 N_7^{IV}$ " 0.1664891n,

$N_7^{V} \ = -2.49115 N_7^{IV}$ " 0.3963997n,

$N_7^{VI} = +0.109524 N_7^{IV}$ " 9.0395107,

$N_7^{VII} = +0.0122518 N_7^{I}$ " 8.0882005.

27. If we now substitute these values in equations (408) and (409) we shall obtain the following quantities:—

For the root $g = -5''.1262327$, we get,

$$x = +\frac{611711}{10^{20}}; \quad y = +\frac{1586934.1}{10^{20}}; \quad z = \frac{14045156}{10^{20}}.$$

Whence $\beta = 21° \ 4' \ 40''.0$; log. $N = 9.0831058$.

$N \ = +0.1210893$; $N^{IV} = -0.00002519$;

$N' \ = +0.0148664$; $N^{V} \ = -0.00003202$;

$N'' \ = +0.0106498$; $N^{VI} = +0.00002858$;

$N''' \ = +0.0021278$; $N^{VII} = +0.0000007462$.

For the root $g_1=-6''.5922343$, we get,

$$x_1=+\frac{693376}{10^{20}};\quad y_1=-\frac{638974.9}{10^{20}};\quad z_1=\frac{33245220}{10^{20}}.$$

Whence $\beta_1=132^\circ\ 39'\ 44''.3$; log. $N_1=98.4527380$.

$N_1\ =+0.0283621$;
$N_1'\ =-0.0078418$;
$N_1''=-0.0063234$;
$N_1'''=-0.0013254$;
$N_1^{IV}=+0.000009533$;
$N_1^{V}\ =+0.000013457$;
$N_1^{VI}=-0.0000070408$;
$N_1^{VII}=-0.00000041736$.

For the root $g_2=-17''.3937133$, we get,

$$x_2=-\frac{68626660}{10^{20}};\quad y_2=+\frac{28878680}{10^{20}};\quad z_2=\frac{0.4885253}{10^{10}}.$$

Whence $\beta_2=292^\circ\ 49'\ 18''.4$; log. $N_2=97.1830089$.

$N_2\ =+0.0015241$;
$N_2'\ =-0.0084789$;
$N_2''=+0.0069555$;
$N_2'''=+0.0506625$;
$N_2^{IV}=-0.000002521$;
$N_2^{V}\ =-0.000021385$;
$N_2^{VI}=+0.0000020884$;
$N_2^{VII}=+0.0000002395$.

For the root $g_3=-18''.4091136$, we get,

$$x_3=-\frac{67244580}{10^{20}};\quad y_3=-\frac{22173520}{10^{20}};\quad z_3=\frac{1.9255992}{10^{10}}.$$

Whence $\beta_3=251^\circ\ 45'\ 1''.1$; log. $N_3=97.5655046$.

$N_3\ =+0.0036771$;
$N_3'\ =-0.0224258$;
$N_3''=+0.0244750$;
$N_3'''=-0.0375997$;
$N_3^{IV}=-0.00000007206$;
$N_3^{V}\ =-0.0000005538$;
$N_3^{VI}=+0.00000004622$;
$N_3^{VII}=+0.000000005126$.

For the root $g_4=0''$, we get,

$$x_4=+\frac{0.7310062}{10^{10}};\quad y_4=-\frac{0.2158651}{10^{10}};\quad z_4=\frac{27.46919}{10^{10}}$$

Whence $\beta_4=106^\circ\ 27'\ 6''.5$; log. $N_4=98.4432303$;

$N_4=N_4'=N_4''=N_4'''=N_4^{IV}=N_4^{V}=N_4^{VI}=N_4^{VII}=+0.02774790$.

For the root $g_5=-0.6634935$, we get,

$$x_5=+\frac{0.1014674}{10^{10}};\quad y_5=+\frac{0.2722315}{10^{10}};\quad z_5=\frac{222.4744}{10^{10}}.$$

Whence $\beta_5=20^\circ\ 26'\ 30''.0$; log. $N_5^{IV}=97.1159056$.

$N_5\ =+0.0016101$;
$N_5'\ =+0.0014779$;
$N_5''=+0.0014476$;
$N_5'''=+0.0013738$;
$N_5^{IV}=+0.00130589$;
$N_5^{V}\ =+0.00126053$;
$N_5^{VI}=-0.00120371$;
$N_5^{VII}=-0.0116801$.

For the root $g_6 = -2''.9612185$, we get,

$$x_6 = +\frac{0.3722380}{10^{10}}; \quad y_6 = -\frac{0.3654807}{10^{10}}; \quad z_6 = \frac{600.4976}{10^{10}}$$

Whence $\beta_6 = 134^\circ\ 28'\ 30''.7$; log. $N_6^{IV} = 96.9388828$.

$$\begin{array}{ll}
N_6 = +0.0031860; & N_6^{IV} = +0.000868726; \\
N_6' = +0.0018190; & N_6^{V} = +0.00070661; \\
N_6'' = +0.0016265; & N_6^{VI} = -0.0177657; \\
N_6''' = +0.0011488; & N_6^{VII} = +0.00187456.
\end{array}$$

For the root $g_7 = -25''.9401904$, we get,

$$x_7 = -\frac{0.2997138}{10^{10}}; \quad y_7 = +\frac{0.2203005}{10^{10}}; \quad z_7 = \frac{59.04657}{10^{10}}$$

Whence $\beta_7 = 306^\circ\ 19'\ 2''.1$; log. $N_7^{IV} = 97.7993116$.

$$\begin{array}{ll}
N_7 = -0.00026499; & N_7^{IV} = +0.00629958; \\
N_7' = -0.00029362; & N_7^{V} = -0.0156932; \\
N_7'' = -0.00272577; & N_7^{VI} = +0.00068996; \\
N_7''' = -0.00924274; & N_7^{VII} = +0.000077181.
\end{array}$$

28. Having thus obtained the values of all the constants, corresponding to the separate variation of the planetary masses, it now remains to find the coefficients of these variations. This we shall do by taking the difference between the constants corresponding to the assumed masses and those depending on the assumed variation of mass for each planet, and dividing by the assumed variation. By this means we shall obtain the following table—observing that the coefficients of μ, μ', μ'', &c., in the values of N, N', N'', &c., are given in units of the seventh decimal place of these coefficients.

$$g = -5''.126112 + 0''.207190\mu - 1''.49906\mu' - 0''.88154\mu'' - 0''.060606\mu''' - 2''.6952\mu^{IV} \\ - 0''.129080\mu^{V} - 0''.00318\mu^{VI} - 0''.00121\mu^{VII};$$

$$\beta = 21^\circ\ 6'\ 26''.8 + 6752''.6\mu - 35940''\mu' - 71170''\mu'' + 1984''.7\mu''' + 134530''\mu^{IV} \\ + 31452''\mu^{V} - 12596''\mu^{VI} - 1070''\mu^{VII};$$

$$N = +0.121076 + 276274\mu - 568000\mu' - 140640\mu'' + 34384\mu''' + 688000\mu^{IV} \\ + 95880\mu^{V} + 29840^{VI} + 1630\mu^{VII};$$

$$N' = +0.014867 + 127798\mu + 276820\mu' - 52360\mu'' - 6929\mu''' - 273100\mu^{IV} - 4800\mu^{V} \\ + 2840\mu^{VI} - 70\mu^{VII};$$

$$N'' = +0.010650 + 88422\mu + 230840\mu' - 23860\mu'' - 6601\mu''' - 239000\mu^{IV} - 5360\mu^{V} \\ + 1960\mu^{VI} + 20\mu^{VII};$$

$$N''' = +0.002128 + 17510\mu + 51480\mu' + 7640\mu'' - 1066\mu''' - 63400\mu^{IV} - 1680\mu^{V} \\ + 320\mu^{VI} + 0\mu^{VII};$$

$$N^{IV} = -0.0000252 - 255.4\mu - 480\mu' - 14\mu'' + 7.2\mu''' + 580\mu^{IV} + 84\mu^{V} - 32\mu^{VI} - 2\mu^{VII};$$

$$N^{V} = -0.0000320 - 313.4\mu - 648\mu' - 42\mu'' + 7.6\mu''' + 740\mu^{IV} + 104.8\mu^{V} - 24\mu^{VI} - 2\mu^{VII};$$

$$N^{VI} = +0.0000280 + 350.4\mu + 364\mu' - 74\mu'' - 14.0\mu''' - 750\mu^{IV} + 252\mu^{V} + 36\mu^{VI} \\ + 54\mu^{VII};$$

$$N^{VII} = +0.000000775 + 4.28\mu + 24\mu' + 5.2\mu'' + 0.1\mu''' - 5\mu^{IV} - 7.6\mu^{V} - 31.2\mu^{VI} \\ - 2.9\mu^{VII}.$$

$$\begin{aligned}g_1=&-6^{\prime\prime}.592128-0^{\prime\prime}.314940\mu-0^{\prime\prime}.85282\mu^{\prime}-0^{\prime\prime}.78390\mu^{\prime\prime}-0^{\prime\prime}.116744\mu^{\prime\prime\prime}-4^{\prime\prime}.3746\mu^{IV}\\&-0^{\prime\prime}.20160\mu^{V}-0^{\prime\prime}.00336\mu^{VI}-0^{\prime\prime}.00106\mu^{VII};\end{aligned}$$

$$\begin{aligned}\beta_1=&132^\circ\ 40^{\prime}\ 57^{\prime\prime}.8-102434^{\prime\prime}\mu-490268\mu^{\prime}-205068^{\prime\prime}\mu^{\prime\prime}+26729^{\prime\prime}\mu^{\prime\prime\prime}+700270^{\prime\prime}\mu^{IV}\\&+19444^{\prime\prime}\mu^{V}+90^{\prime\prime}\mu^{VI}-735^{\prime\prime}\mu^{VII};\end{aligned}$$

$$\begin{aligned}N_1=&+0.0283520-37374\mu-3360\mu^{\prime}+358300\mu^{\prime\prime}+5235\mu^{\prime\prime\prime}-41500\mu^{IV}+133120\mu^{V}\\&+16400\mu^{VI}+990\mu^{VII};\end{aligned}$$

$$\begin{aligned}N_1^{\prime}=&-0.0078380-8507\mu+222860\mu^{\prime}-75320\mu^{\prime\prime}-8530\mu^{\prime\prime\prime}-100200\mu^{IV}-46800\mu^{V}\\&-4760\mu^{VI}-350\mu^{VII};\end{aligned}$$

$$\begin{aligned}N_1^{\prime\prime}=&-0.0063210-9157\mu+167160\mu^{\prime}-68660\mu^{\prime\prime}-5780\mu^{\prime\prime\prime}-58300\mu^{IV}-36680\mu^{V}\\&-3800\mu^{VI}-260\mu^{VII};\end{aligned}$$

$$\begin{aligned}N_1^{\prime\prime\prime}=&-0.0013250-2162\mu+32900\mu^{\prime}-2360\mu^{\prime\prime}-1437\mu^{\prime\prime\prime}-1400\mu^{IV}-7280\mu^{V}\\&-780\mu^{VI}-50\mu^{VII};\end{aligned}$$

$$N_1^{IV}=+0.00000952+5\mu-250\mu^{\prime}+148\mu^{\prime\prime}+12\mu^{\prime\prime\prime}+67\mu^{IV}+24\mu^{V}+10\mu^{VI}+1\mu^{VII};$$

$$N_1^{V}=+0.00001345+10\mu-346\mu^{\prime}+218\mu^{\prime\prime}+18\mu^{\prime\prime\prime}+80\mu^{IV}+32\mu^{V}+10\mu^{VI}+0\mu^{VII};$$

$$N_1^{VI}=-0.00000696+1\mu+194\mu^{\prime}-96\mu^{\prime\prime}-7\mu^{\prime\prime\prime}+0\mu^{IV}-88\mu^{V}-6\mu^{VI}-8\mu^{VII};$$

$$N_1^{VII}=-0.000000420-0\mu+10.8\mu^{\prime}-6.8\mu^{\prime\prime}-0.6\mu^{\prime\prime\prime}+0\mu^{IV}-2.8\mu^{V}+2.4\mu^{VI}+0.3\mu^{VII}.$$

$$\begin{aligned}g_2=&17^{\prime\prime}.393390-0^{\prime\prime}.028779\mu-1^{\prime\prime}.31820\mu^{\prime}-2^{\prime\prime}.4080\mu^{\prime\prime}+0^{\prime\prime}.065597\mu^{\prime\prime\prime}-12^{\prime\prime}.7389\mu^{IV}\\&-0^{\prime\prime}.55708\mu^{V}-0^{\prime\prime}.00942\mu^{VI}-0^{\prime\prime}.00323\mu^{VII};\end{aligned}$$

$$\begin{aligned}\beta_2=&292^\circ\ 49^{\prime}\ 53^{\prime\prime}.2+12418^{\prime\prime}\mu+333738^{\prime\prime}\mu^{\prime}+270720^{\prime\prime}\mu^{\prime\prime}+23660^{\prime\prime}\mu^{\prime\prime\prime}-711800^{\prime\prime}\mu^{IV}\\&-25592^{\prime\prime}\mu^{V}-276^{\prime\prime}\mu^{VI}-348\mu^{VII};\end{aligned}$$

$$\begin{aligned}N_2=&+0.0015240-1455\mu-67740\mu^{\prime}-50820\mu^{\prime\prime}+4466\mu^{\prime\prime\prime}+139300\mu^{IV}+6280\mu^{V}\\&+0\mu^{VI}+0\mu^{VII};\end{aligned}$$

$$\begin{aligned}N_2^{\prime}=&-0.0084783+8450\mu+454660\mu^{\prime}+283020\mu^{\prime\prime}-24120\mu^{\prime\prime\prime}-882200\mu^{IV}\\&-39240\mu^{V}-10\mu^{VI}-60\mu^{VII};\end{aligned}$$

$$\begin{aligned}N_2^{\prime\prime}=&+0.0069546-9445\mu-392740\mu^{\prime}-348760\mu^{\prime\prime}+14218\mu^{\prime\prime\prime}+909000\mu^{IV}\\&+39280\mu^{V}+220\mu^{VI}+90^{VII};\end{aligned}$$

$$\begin{aligned}N_2^{\prime\prime\prime}=&+0.0506672-13628\mu-648600\mu^{\prime}-421340\mu^{\prime\prime}+79686\mu^{\prime\prime\prime}+1091100\mu^{IV}\\&+77400\mu^{V}-180\mu^{VI}-47\mu^{VII};\end{aligned}$$

$$N_2^{IV}=-0.00000251+1.6\mu+66\mu^{\prime}+62\mu^{\prime\prime}-15.4\mu^{\prime\prime\prime}-140\mu^{IV}+12\mu^{V}-4\mu^{VI}-1\mu^{VII};$$

$$\begin{aligned}N_2^{V}=&-0.00002140+10.1\mu+434\mu^{\prime}+294\mu^{\prime\prime}-120.4\mu^{\prime\prime\prime}-800\mu^{IV}-120\mu^{V}+6\mu^{VI}\\&+2\mu^{VII};\end{aligned}$$

$$N_2^{VI}=+0.00000208-1\mu-44\mu^{\prime}-32\mu^{\prime\prime}+12\mu^{\prime\prime\prime}+70\mu^{IV}+12\mu^{V}+0\mu^{VI}+1\mu^{VII};$$

$$N_2^{VII}=+0.00000024-0.12\mu-5.2\mu^{\prime}-4\mu^{\prime\prime}+1.4\mu^{\prime\prime\prime}+8\mu^{IV}+0.8\mu^{V}-0.4\mu^{VI}-0.1\mu^{VII}.$$

$$\begin{aligned}g_3=&-18^{\prime\prime}.408914-0^{\prime\prime}.087808\mu-5^{\prime\prime}.14616\mu^{\prime}-5^{\prime\prime}.18060\mu^{\prime\prime}-0^{\prime\prime}.317709\mu^{\prime\prime\prime}-7^{\prime\prime}.7278\mu^{IV}\\&-0^{\prime\prime}.34424\mu^{V}-0^{\prime\prime}.00582\mu^{VI}-0^{\prime\prime}.00020\mu^{VII};\end{aligned}$$

$$\begin{aligned}\beta_3=&251^\circ\ 45^{\prime}\ 8^{\prime\prime}.6+4720^{\prime\prime}\mu+157816^{\prime\prime}\mu^{\prime}+123030\mu^{\prime\prime}-17836^{\prime\prime}\mu^{\prime\prime\prime}-331390^{\prime\prime}\mu^{IV}\\&-12580^{\prime\prime}\mu^{V}-212^{\prime\prime}\mu^{VI}-75^{\prime\prime}\mu^{VII};\end{aligned}$$

$$\begin{aligned}N_3=&+0.0036775+1277\mu+82380\mu^{\prime}+65160\mu^{\prime\prime}-2863\mu^{\prime\prime\prime}-172200\mu^{IV}-6880\mu^{V}\\&-120\mu^{VI}-40\mu^{VII};\end{aligned}$$

$$\begin{aligned}N_3^{\prime}=&-0.0224278-7986\mu-290400\mu^{\prime}-468080\mu^{\prime\prime}+11283\mu^{\prime\prime\prime}+925900\mu^{IV}\\&+36080\mu^{V}+580\mu^{VI}+200\mu^{VII};\end{aligned}$$

$$N_3''=+0.0244768+3926\mu+47734\mu'+179920\mu''-2395\mu'''-840900\mu^{IV}-32280\mu^{V}-520\mu^{VI}-180\mu^{VII};$$

$$N_3'''=-0.0375951+27422\mu+1075000\mu'+732200\mu''+100657\mu'''-2002200\mu^{IV}-80840\mu^{V}-1340\mu^{VI}-460\mu^{VII};$$

$$N_3^{IV}=-0.000000072-0.54\mu-28.2\mu'-21.6\mu''+2.70\mu'''+75\mu^{IV}+2.8\mu^{V}+0\mu^{VI}+0\mu^{VII};$$

$$N_3^{V}=-0.000000557-11.32\mu-676.4\mu'-525\mu''+65.7\mu'''+1207\mu^{IV}+51.2\mu^{V}+1\mu^{VI}+0.3\mu^{VII};$$

$$N_3^{VI}=+0.000000046+1.0\mu+57.8\mu'+44.8\mu''-5.6\mu'''-106\mu^{IV}-4\mu^{V}+0\mu^{VI}+0.0\mu^{VII};$$

$$N_3^{VII}=+0.0000000005+0.12\mu+6.6\mu'+5.2\mu''-0.65\mu'''-12\mu^{IV}-0.4\mu^{V}+0\mu^{VI}+0.0\mu^{VII}.$$

$$g_4=0''$$

$$\beta_4=106^\circ\ 14'\ 18''.0-28''.6\mu-138\mu'+0''\mu''-26''.5\mu'''-13320''\mu^{IV}+8476''\mu^{V}-2726''\mu^{VI}+7685''\mu^{VII};$$

$$N_4=+0.0277417+13\mu+140\mu'-200\mu''-13\mu'''-30800\mu^{IV}+38480\mu^{V}-8140\mu^{VI}+620\mu^{VII}.$$

$$g_5=-0''.661666+0''.000020\mu+0''.00004\mu'+0''.00002\mu''+0.000002\mu'''-0''.1701\mu^{IV}-0''.23656\mu^{V}-0''.23692\mu^{VI}-0''.01828\mu^{VII};$$

$$\beta_5=20^\circ\ 31'\ 24''.6+24''.3\mu+298''\mu'-540''\mu''-42''.3\mu'''-70360''\mu^{IV}+56804''\mu^{V}+12932''\mu^{VI}-2946''\mu^{VII};$$

$$N_5=+0.0014778+29\mu-620\mu'-440\mu''-31\mu'''-9400\mu^{IV}-4240\mu^{V}+1440\mu^{VI}+13230\mu^{VII};$$

$$N_5'=+0.0013568+37\mu+100\mu'-260\mu''-25\mu'''-8600\mu^{IV}-4360\mu^{V}+880\mu^{VI}+12110\mu^{VII};$$

$$N_5''=+0.0013291+24\mu+180\mu'-120\mu''-26\mu'''-8400\mu^{IV}-4360\mu^{V}+760\mu^{VI}+11850\mu^{VII};$$

$$N_5'''=+0.0012586+9\mu+80\mu'+40\mu''-1\mu'''-7500\mu^{IV}-4320\mu^{V}+460\mu^{VI}+11520\mu^{VII};$$

$$N_5^{IV}=+0.00119933+5\mu+16\mu'-16\mu''+2\mu'''-6650\mu^{IV}-4328\mu^{V}+206\mu^{VI}+10656\mu^{VII};$$

$$N_5^{V}=+0.00115773+5\mu+20\mu'-10\mu''+2\mu'''-6320\mu^{IV}-4072\mu^{V}+34\mu^{VII}+10280\mu^{VII};$$

$$N_5^{VI}=-0.00112485-4.6\mu+0\mu'+28\mu''-3.5\mu'''-1670\mu^{IV}+9936\mu^{V}-50\mu^{VI}-7886\mu^{VII};$$

$$N_5^{VII}=-0.0117882-55\mu-260\mu'+16\mu''-40\mu'''-19200\mu^{IV}+3880\mu^{V}+5160\mu^{VI}+10810\mu^{VII}$$

$$g_6=-2''.916082+0''.000028\mu+0''.00014\mu'+0''.00004\mu''-0''.000013\mu'''-0''.9004\mu^{IV}-1''.41388\mu^{V}-0''.15070\mu^{VI}-0''.45136\mu^{VII};$$

$$\beta=133^\circ\ 56'\ 10''.8-175''\mu-344''\mu'+310''\mu''-24''.6\mu'''-1800''\mu^{IV}-17876''\mu^{V}+1182''\mu^{VI}+19400''\mu^{VII};$$

$$N_6=+0.0031283+794\mu-13000\mu'-8480\mu''-691u'''-44200\mu^{IV}+29360\mu^{V}+32500\mu^{VI}+5780\mu^{VII};$$

$$N_6'=+0.0018108+596\mu+660\mu'-2820\mu''-341\mu'''-25700\mu^{\mathrm{IV}}+9320\mu^{\mathrm{V}}+17960\mu^{\mathrm{VI}}+820\mu^{\mathrm{VII}};$$

$$N_6''=+0.0016228+378\mu+1220\mu'-1540\mu''-315\mu'''-23000\mu^{\mathrm{IV}}+7240\mu^{\mathrm{V}}+15960\mu^{\mathrm{VI}}+370\mu^{\mathrm{VII}};$$

$$N_6'''=+0.0011557+73\mu+300\mu'+240\mu''-52\mu'''-15300\mu^{\mathrm{IV}}+1560\mu^{\mathrm{V}}+11040\mu^{\mathrm{VI}}-690\mu^{\mathrm{VII}};$$

$$N_6^{\mathrm{IV}}=+0.0008794-4\mu-80\mu'-20\mu''-3\mu'''-7400\mu^{\mathrm{IV}}+240\mu^{\mathrm{V}}+8200\mu^{\mathrm{VI}}-1170\mu^{\mathrm{VII}};$$

$$N_6^{\mathrm{V}}=+0.0007180-4\mu-60\mu'-20\mu''-3\mu'''-5900\mu^{\mathrm{IV}}+400\mu^{\mathrm{V}}+6620\mu^{\mathrm{VI}}-1140\mu^{\mathrm{VII}};$$

$$N_6^{\mathrm{VI}}=-0.0176872+35\mu+1100\mu'+280\mu''+29\mu'''+27800\mu^{\mathrm{IV}}-29360\mu^{\mathrm{V}}+9680\mu^{\mathrm{VI}}-7850\mu^{\mathrm{VII}};$$

$$N_6^{\mathrm{VII}}=+0.0019010-4\mu-600\mu'-40\mu''-9\mu'''-8800\mu^{\mathrm{IV}}-7360\mu^{\mathrm{V}}+18900\mu^{\mathrm{VI}}-2640\mu^{\mathrm{VII}}.$$

$$g_7=-25''.934567-0''.000039\mu-0''.00196\mu'-0''.00430\mu''-0''.00223\mu'''-18''.2906\mu^{\mathrm{IV}}-7''.35208\mu^{\mathrm{V}}-0''.22692\mu^{\mathrm{VI}}-0''.05623\mu^{\mathrm{VII}};$$

$$\beta_7=306^\circ\ 19'\ 21''.2-2''.4\mu-242''\mu'+128''\mu''-65''.6\mu'''+40''\mu^{\mathrm{IV}}+32''\mu^{\mathrm{V}}-1532''\mu^{\mathrm{VI}}-191''\mu^{\mathrm{VII}};$$

$$N_7=-0.0002652-1\mu-300\mu'-740\mu''+73\mu'''+1700\mu^{\mathrm{IV}}-760\mu^{\mathrm{V}}+20\mu^{\mathrm{VI}}+20\mu^{\mathrm{VII}};$$

$$N_7'=-0.0002932+40\mu+5840\mu'+14100\mu''-102\mu'''-10300\mu^{\mathrm{IV}}-8240\mu^{\mathrm{V}}-220\mu^{\mathrm{VI}}-40\mu^{\mathrm{VII}};$$

$$N_7''=-0.0027275-93\mu-12780\mu'-5500\mu''+1614\mu'''+19000\mu^{\mathrm{IV}}-3520\mu^{\mathrm{V}}+220\mu^{\mathrm{VI}}+170\mu^{\mathrm{VII}};$$

$$N_7'''=-0.0092499+12631\mu-2700\mu'-13300\mu''-295\mu'''+8700\mu^{\mathrm{IV}}+5280\mu^{\mathrm{V}}+1340\mu^{\mathrm{VI}}+720\mu^{\mathrm{VII}};$$

$$N_7^{\mathrm{IV}}=+0.00630053+0\mu-80\mu'-40\mu''-10\mu'''-44700\mu^{\mathrm{IV}}+44800\mu^{\mathrm{V}}+720\mu^{\mathrm{VI}}-90\mu^{\mathrm{VII}};$$

$$N_7^{\mathrm{V}}=-0.0156928+3\mu+240\mu'+200\mu''+70\mu'''-43400\mu^{\mathrm{IV}}+44640\mu^{\mathrm{V}}-3120\mu^{\mathrm{VI}}-40\mu^{\mathrm{VII}};$$

$$N_7^{\mathrm{VI}}=+0.0006890-1\mu-20\mu'-20\mu''-5\mu'''-3300\mu^{\mathrm{IV}}+3040\mu^{\mathrm{V}}+80\mu^{\mathrm{VI}}+100\mu^{\mathrm{VII}};$$

$$N_7^{\mathrm{VII}}=+0.00007720+0\mu+0\mu'+0\mu''-0.5\mu'''-290\mu^{\mathrm{IV}}+344\mu^{\mathrm{V}}-50\mu^{\mathrm{VI}}-2\mu^{\mathrm{VII}}.$$

We have thus obtained the system of constants and the coefficients of their variations, corresponding to the ecliptic of 1850, as the plane of reference; and we shall now inquire what modifications are necessary in order to refer the same quantities to the invariable plane of the planetary system.

CHAPTER III.

ON THE POSITIONS AND SECULAR VARIATIONS OF THE ORBITS WHEN REFERRED TO THE INVARIABLE PLANE OF THE PLANETARY SYSTEM.

1. We shall now refer the positions of the orbits to the invariable plane of the planetary system, in order to discover whether there are any laws which control their mutual positions, of a similar nature to those which we have shown to exist relatively to the eccentricities and perihelia. For this purpose it is necessary to first determine the position of the invariable plane with reference to the fixed ecliptic of 1850; and we can then readily refer all the orbits to that plane. The position of this plane is found by the principle, that the sum of the products, formed by multiplying each planetary mass by the projection of the area described by its radius vector, in a given time, is a maximum. If we put γ for the inclination of the invariable plane to the fixed ecliptic of 1850, and Π for the longitude of its ascending node on the same plane, we shall have (*Mécanique Céleste* [1162]),

$$c \tan\gamma \sin\Pi = c''; \quad c \tan\gamma \cos\Pi = c'. \tag{527}$$

But we have

$$\left.\begin{aligned}
c &= m\sqrt{\mu a(1-e^2)}\cos\phi + m'\sqrt{\mu' a'(1-e'^2)}\cos\phi' \\
&\qquad + m''\sqrt{\mu'' a''(1-e''^2)}\cos\phi'' + \&c., \\
c' &= m\sqrt{\mu a(1-e^2)}\sin\phi\cos\theta + m'\sqrt{\mu' a'(1-e'^2)}\sin\phi'\cos\theta' \\
&\qquad + m''\sqrt{\mu'' a''(1-e''^2)}\sin\phi''\cos\theta'' + \&c., \\
c'' &= m\sqrt{\mu a(1-e^2)}\sin\phi\sin\theta + m'\sqrt{\mu' a'(1-e'^2)}\sin\phi'\sin\theta' \\
&\qquad + m''\sqrt{\mu'' a''(1-e''^2)}\sin\phi''\sin\theta'' + \&c.
\end{aligned}\right\} \tag{528}$$

If we denote the sun's mass by unity, we shall have

$$\mu = 1+m, \quad \mu' = 1+m', \quad \mu'' = 1+m'', \ \&c.;$$

but we shall also have

$$\sqrt{\mu a} = na^2, \quad \sqrt{\mu' a'} = n'a'^2, \quad \sqrt{\mu'' a''} = n''a''^2, \ \&c.$$

Substituting these values in equations (528), they will become

$$\left.\begin{aligned}
c &= mna^2\sqrt{1-e^2}\cos\phi + m'n'a'^2\sqrt{1-e'^2}\cos\phi' \\
&\qquad + m''n''a''^2\sqrt{1-e''^2}\cos\phi'' + \&c., \\
c' &= mna^2\sqrt{1-e^2}\sin\phi\cos\theta + m'n'a'^2\sqrt{1-e'^2}\sin\phi'\cos\theta' \\
&\qquad + m''n''a''^2\sqrt{1-e''^2}\sin\phi''\cos\theta'' + \&c., \\
c'' &= mna^2\sqrt{1-e^2}\sin\phi\sin\theta + m'n'a'^2\sqrt{1-e'^2}\sin\phi'\sin\theta' \\
&\qquad + m''n''a''^2\sqrt{1-e''^2}\sin\phi''\sin\theta'' + \&c.
\end{aligned}\right\} \tag{529}$$

Substituting in these equations the values of m, n, a, e, ϕ and θ, given in §§ 5 and 17 of Chapter I, and § 6 of Chapter II, we shall get

$$c=+0.0035274157, \quad c'=-0.00002735230, \quad c''=+0.00009393304. \qquad (530)$$

In finding these quantities n'' has been supposed to equal unity, and the values of n, n', n''', &c. have been multiplied by $\frac{1}{n''}$ in order to preserve the same ratio.

Substituting the values of c, c', and c'', in equations (527), we shall obtain

$$\Pi=106° \, 14' \, 6''.00, \text{ and } \gamma=1° \, 35' \, 19''.376. \qquad (531)$$

If we now denote by ϕ_0, ϕ_0', ϕ_0'', &c., θ_0, θ_0', θ_0'', &c., the respective inclinations and longitudes of ascending nodes of the different planets, on the invariable plane; the values of θ_0, θ_0', θ_0'', &c. being reckoned from the descending node of the fixed ecliptic of 1850, on the invariable plane; we shall have the following equations to determine θ_0, θ_0', &c., ϕ_0, ϕ_0', &c.

$$\left.\begin{aligned} \sin\phi_0 \sin\theta_0 &= \sin\phi \sin(\theta-\Pi), \\ \sin\phi_0 \cos\theta_0 &= \cos\gamma \sin\phi \cos(\theta-\Pi) - \sin\gamma \cos\phi. \end{aligned}\right\} \qquad (532)$$

These equations will give the following elements:—

Mercury, $\phi_0 = 6° \, 20' \, 58''.08$, $\theta_0 = 287° \, 54' \, 5''.12$,
Venus, $\phi_0' = 2 \;\, 11 \;\, 13.57$, $\theta_0' = 307 \;\, 14 \;\, 8.10$,
The Earth, $\phi_0'' = 1 \;\, 35 \;\, 19.376$, $\theta_0'' = 180 \;\, 0 \;\, 0.00$,
Mars, $\phi_0''' = 1 \;\, 40 \;\, 43.70$, $\theta_0''' = 248 \;\, 56 \;\, 21.45$,
Jupiter, $\phi_0^{IV} = 0 \;\, 19 \;\, 59.674$, $\theta_0^{IV} = 210 \;\, 7 \;\, 35.44$,
Saturn, $\phi_0^{V} = 0 \;\, 55 \;\, 30.924$, $\theta_0^{V} = 16 \;\, 34 \;\, 26.66$,
Uranus, $\phi_0^{VI} = 1 \;\, 1 \;\, 45.27$, $\theta_0^{VI} = 204 \;\, 12 \;\, 33.78$,
Neptune, $\phi_0^{VII} = 0 \;\, 43 \;\, 24.845$, $\theta_0^{VII} = 286 \;\, 39 \;\, 55.10$.

2. Now putting

$$\left.\begin{aligned} \tan\phi_0 \sin\theta_0 &= p_0, & \tan\phi_0' \sin\theta_0' &= p_0' \text{ \&c.}, \\ \tan\phi_0 \cos\theta_0 &= q_0, & \tan\phi_0' \cos\theta_0' &= q_0' \text{ \&c.} \end{aligned}\right\} \qquad (533)$$

we shall get the following values

$p_0 = -0.1058879$, $q_0 = +0.0342038$,
$p_0' = -0.0304057$, $q_0' = +0.0231090$,
$p_0'' = 0.$ $q_0'' = -0.0277354$,
$p_0''' = -0.0273512$, $q_0''' = -0.0105324$,
$p_0^{IV} = -0.00273067$, $q_0^{IV} = -0.00503058$,
$p_0^{V} = +0.00460691$, $q_0^{V} = +0.0154792$,
$p_0^{VI} = -0.00736719$, $q_0^{VI} = -0.0163856$,
$p_0^{VII} = +0.0126079$, $q_0^{VII} = +0.000734628$.

If we substitute these values in equations (408) and (409), we shall obtain the values of β, β_1, β_2, &c., N, N_1, N_2, &c., corresponding to the invariable plane. But instead of performing this operation separately for each root, we shall proceed in the following manner.

3. If we neglect the squares of e, e', e'', &c., ϕ, ϕ', ϕ'', &c., γ, we may put

$$\cos\gamma=1;\ \sin\gamma=\tan\gamma=\gamma;\ \sin\phi=\tan\phi;\ \sin\phi'=\tan\phi',\ \text{\&c.};$$

then we shall have

$$\left.\begin{array}{lll} p=\sin\phi\sin\theta, & p'=\sin\phi'\sin\theta', & p''=\sin\phi''\sin\theta''\ \text{\&c.}, \\ q=\sin\phi\cos\theta, & q'=\sin\phi'\cos\theta', & q''=\sin\phi''\cos\theta''\ \text{\&c.} \end{array}\right\}\quad(534)$$

Substituting these values in equations (408) and (409), also putting $t=0$, and remembering that for the root $g_4=0$, we have $N_4=N_4'=N_4''$, &c., we shall have

$$\left.\begin{array}{l} \frac{m}{na}\sin\phi\sin\theta+\frac{m'}{n'a'}\sin\phi'\sin\theta'+\text{\&c.}=\left\{\frac{m}{na}+\frac{m'}{n'a'}+\text{\&c.}\right\}N_4\sin\beta_4; \\ \frac{m}{na}\sin\phi\cos\theta+\frac{m'}{n'a'}\sin\phi'\cos\theta'+\text{\&c.}=\left\{\frac{m}{na}+\frac{m'}{n'a'}+\text{\&c.}\right\}N_4\cos\beta_4. \end{array}\right\}\quad(535)$$

But if we neglect m^2, m'^2, &c., we shall have

$$mna^2=\frac{m}{na},\quad m'n'a'^2=\frac{m'}{n'a'},\quad m''n''a''^2=\frac{m''}{n''a''},\ \text{\&c.},$$

and equations (527) will give, by substituting the values of c, c', c'',

$$\left.\begin{array}{l} \frac{m}{na}\sin\phi\sin\theta+\frac{m'}{n'a'}\sin\phi'\sin\theta'+\text{\&c.}=\left\{\frac{m}{na}+\frac{m'}{n'a'}+\text{\&c.}\right\}\gamma\sin\Pi; \\ \frac{m}{na}\sin\phi\cos\theta+\frac{m'}{n'a'}\sin\phi'\cos\theta'+\text{\&c.}=\left\{\frac{m}{na}+\frac{m'}{n'a'}+\text{\&c.}\right\}\gamma\cos\Pi. \end{array}\right\}\quad(536)$$

Comparing equations (535) and (536) we find $\Pi=\beta_4$, and $\gamma=N_4$. Now substituting $\Pi=\beta_4$, $\gamma=N_4$, in equations (532), they will give

$$\left.\begin{array}{l} \sin\phi_0\sin\theta_0=\sin\phi\sin(\theta-\beta_4)=\sin\phi\sin\theta\cos\beta_4-\sin\phi\cos\theta\sin\beta_4 \\ \qquad\qquad=p\cos\beta_4-q\sin\beta_4=p_0; \\ \sin\phi_0\cos\theta_0=\sin\phi\cos(\theta-\beta_4)-\gamma= \\ \sin\phi\cos\theta\cos\beta_4-\sin\phi\sin\theta\sin\beta_4-\gamma=q\cos\beta_4-p\sin\beta_4-N_4=q_0. \end{array}\right\}\quad(537)$$

And since the relative values of N, N', N'', &c., N_1, N_1', N_1'', &c., are known we may determine their actual values corresponding to the invariable plane, by the analysis of Chapter II, § 5. We shall therefore suppose

$$\left.\begin{array}{l} q_0=\alpha N\cos(gt+\beta^{(0)})+\alpha_1 N_1\cos(g_1t+\beta_1^{(0)})+\alpha_2 N_2\cos(g_2t+\beta_2^{(0)})+\text{\&c.}, \\ q_0'=\alpha N'\cos(gt+\beta^{(0)})+\alpha_1 N_1'\cos(g_1t+\beta_1^{(0)})+\alpha_2 N_2'\cos(g_2t+\beta_2^{(0)})+\text{\&c.}, \\ \text{\&c.}; \\ p_0=\alpha N\sin(gt+\beta^{(0)})+\alpha_1 N_1\sin(g_1t+\beta_1^{(0)})+\alpha_2 N_2\sin(g_2t+\beta_2^{(0)})+\text{\&c.}, \\ p_0'=\alpha N'\sin(gt+\beta^{(0)})+\alpha_1 N_1'\sin(g_1t+\beta_1^{(0)})+\alpha_2 N_2'\sin(g_2t+\beta_2^{(0)})+\text{\&c.}, \\ \text{\&c.}, \end{array}\right\}\quad(538)$$

α, α_1, α_2, &c., being the constant factors which are necessary in order to reduce the numbers already calculated to the corresponding ones for the invariable plane; and $\beta^{(0)}$, $\beta_1^{(0)}$, $\beta_2^{(0)}$, &c., being the constants necessary to satisfy the equations for the given epoch.

Equations (408) and (409) will also become

$$\left.\begin{aligned} &\alpha\left\{Np_0\frac{m}{na}+N'p_0'\frac{m'}{n'a'}+N''p_0''\frac{m''}{n''a''}+\&c.\right\} \\ &=\alpha^2\left\{N^2\frac{m}{na}+N'^2\frac{m'}{n'a'}+\&c.\right\}\sin(gt+\beta^{(0)}); \end{aligned}\right\} \qquad (539)$$

$$\left.\begin{aligned} &\alpha\left\{Nq_0\frac{m}{na}+N'q_0'\frac{m'}{n'a'}+N''q_0'\frac{m''}{n''a''}+\&c.\right\} \\ &=\alpha^2\left\{N^2\frac{m}{na}+N'^2\frac{m'}{n'a'}+\&c.\right\}\cos(gt+\beta^{(0)}); \end{aligned}\right\} \qquad (540)$$

If we substitute in these equations the values of p_0, p_0', &c., q_0, q_0', &c., given by equations (537), they will become

$$\left.\begin{aligned} &\alpha\left\{Np\frac{m}{na}+N'p'\frac{m'}{n'a'}+N''p''\frac{m''}{n'a'}+\&c.\right\}\cos\beta_4 \\ &-\alpha\left\{Nq\frac{m}{na}+N'q'\frac{m'}{n'a'}+N''q''\frac{m''}{n''a''}+\&c.\right\}\sin\beta_4 \\ &=\alpha^2\left\{N^2\frac{m}{na}+N'^2\frac{m'}{n'a'}+N''^2\frac{m''}{n''a''}+\&c.\right\}\sin(gt+\beta^{(0)}); \end{aligned}\right\} \qquad (541)$$

$$\left.\begin{aligned} &\left.\begin{aligned} &\alpha\left\{Nq\frac{m}{na}+N'q'\frac{m'}{n'a'}+N''q''\frac{m''}{n''a''}+\&c.\right\}\cos\beta_4 \\ &-\alpha\left\{Np\frac{m}{na}+N'p'\frac{m'}{n'a'}+N''p''\frac{m''}{n''a''}+\&c.\right\}\sin\beta \\ &-\alpha\left\{N\frac{m}{na}+N'\frac{m}{n'a'}+N''\frac{m''}{n''a''}+\&c.\right\}N_4 \end{aligned}\right\} \\ &=\alpha^2\left\{N^2\frac{m}{na}+N'^2\frac{m'}{n'a'}+N''^2\frac{m''}{n''a''}+\&c.\right\}\cos(gt+\beta^{(0)}). \end{aligned}\right\} \qquad (542)$$

Now according to equations (410), the coefficient of N_4 in this equation is equal to nothing; and if we substitute the values of the coefficients of $\alpha\cos\beta_4$, and $\alpha\sin\beta_4$, which are given by equations (408) and (409), both members of equations (541) and (542) will be divisible by the coefficients of $\alpha\sin(gt+\beta^{(0)})$, and $\alpha\cos(gt+\beta^{(0)})$, and we shall find

$$\left.\begin{aligned} \sin(gt+\beta-\beta_4)&=\alpha\sin(gt+\beta^{(0)}); \\ \cos(gt+\beta-\beta_4)&=\alpha\cos(gt+\beta^{(0)}). \end{aligned}\right\} \qquad (543)$$

Whence we get

$$\tan(gt+\beta-\beta_4)=\tan(gt+\beta^{(0)})$$

Therefore

$$\beta^{(0)}=\beta-\beta_4, \text{ and } \alpha=1. \qquad (544)$$

It therefore follows that in order to apply our numbers to the invariable plane, we have only to diminish the constants β, β_1, β_2, &c., by the longitude of the ascending node of that plane, on the fixed ecliptic of 1850, and neglect the constant term.

Therefore we shall have

$$\left.\begin{aligned} p_0=&N\sin(gt+\beta-\beta_4)+N_1\sin(g_1t+\beta_1-\beta_4)+N_2\sin(g_2t+\beta_2-\beta_4)\\ &+N_3\sin(g_3t+\beta_3-\beta_4)+N_5\sin(g_5t+\beta_5-\beta_4)\\ &+N_6\sin(g_6t+\beta_6-\beta_4)+N_7\sin(g_7t+\beta_7-\beta_4);\\ q_0=&N\cos(gt+\beta-\beta_4)+N_1\cos(g_1t+\beta_1-\beta_4)+N_2\cos(g_2t+\beta_2-\beta_4)\\ &+N_3\cos(g_3t+\beta_3-\beta_4)+N_5\cos(g_5t+\beta_5-\beta_4)\\ &+N_6\cos(g_6t+\beta_6-\beta_4)+N_7\cos(g_7t+\beta_7-\beta_4); \end{aligned}\right\}\quad(545)$$

$$\left.\begin{aligned} p_0'=&N'\sin(gt+\beta-\beta_4)+N_1'\sin(g_1t+\beta_1-\beta_4)+N_2'\sin(g_2t+\beta_2-\beta_4)\\ &+N_3'\sin(g_3t+\beta_3-\beta_4)+\&c.;\\ q_0'=&N'\cos(gt+\beta-\beta_4)+N_1'\cos(g_1t+\beta_1-\beta_4)+N_2'\cos(g_2t+\beta_2-\beta_4)\\ &+N_3'\cos(g_3t+\beta_3-\beta_4)+\&c. \end{aligned}\right\}\quad(546)$$

Substituting for p_0 and q_0 their values given by the first members of equations (537), we shall easily find

$$\left.\begin{aligned} \sin\phi_0\sin(\theta_0-gt-\beta)=&N_1\sin\{(g_1-g)t+\beta_1-\beta\}+N_2\sin\{(g_2-g)t+\beta_2-\beta\}\\ &+N_3\sin\{(g_3-g)t+\beta_3-\beta\}+N_5\sin\{(g_5-g)t+\beta_5-\beta\}\\ &+N_6\sin\{(g_6-g)t+\beta_6-\beta\}+N_7\sin\{(g_7-g)t+\beta_7-\beta\} \end{aligned}\right\}\quad(547)$$

$$\left.\begin{aligned} &\sin\phi_0\cos(\theta_0-gt-\beta)\\ &\quad=N+N_1\cos\{(g_1-g)t+\beta_1-\beta\}+N_2\cos\{(g_2-g)t+\beta_2-\beta\}\\ &\quad\quad+N_3\cos\{(g_3-g)t+\beta_3-\beta\}+N_5\cos\{(g_5-g)t+\beta_5-\beta\}\\ &\quad\quad+N_6\cos\{(g_6-g)t+\beta_6-\beta\}+N_7\cos\{(g_7-g)t+\beta_7-\beta\} \end{aligned}\right\}\quad(548)$$

From these equations it is easy to show that the mean motion of θ_0 is equal to gt when N exceeds the sum of the coefficients of the cosines, all taken positively. We shall also have

$$\left.\begin{aligned} &\textit{maximum}\ \phi_0=N+N_1+N_2+N_3+N_5+N_6+N_7;\\ \text{and }&\textit{minimum}\ \phi_0=N-(N_1+N_2+N_3+N_5+N_6+N_7). \end{aligned}\right\}\quad(549)$$

4. If we now substitute in these equations the values given in Chapter II, § 7, we shall obtain the following *maxima*, *minima*, and *mean motions*.

	Inclination to invariable plane.						Mean motion of nodes in a Julian year.
	Maximum.			Minimum.			
Mercury,	9°	10′	41″	4°	44′	27″	—5″.126076
Venus,	3	16	18	0	0	0	indeterminate
The Earth,	3	6	0	0	0	0	"
Mars,	5	56	2	0	0	0	"
Jupiter,	0	28	56	0	14	23	—25″.934567
Saturn,	1	0	39	0	47	16	—25 .934567
Uranus,	1	7	10	0	54	25	— 2 .916082
Neptune,	0	47	21	0	33	43	— 0 .661666

5. It thus appears that the mean motion of the nodes of *Jupiter* and *Saturn*, on the invariable plane, are exactly the same. This indicates a relation of a permanent character between the positions of the nodes of these two planets, the nature of which we shall now examine.

If we divide equation (547) by (548) we shall obtain an equation similar to (147) and (148). And if in this equation we substitute the numbers corresponding to *Jupiter* and *Saturn*, we find that the mean places of the nodes, on the invariable plane, will be the same if N_7^{IV} and N_7^{V} have the same sign, and that they will differ by 180° if N_7^{IV} and N_7^{V} have different signs. The computed numbers show that the signs of these two quantities are different: it therefore follows that the mean longitudes of the nodes of *Jupiter* and *Saturn*, on the invariable plane of the planetary system, always differ by 180°. We shall find, by the analysis of Chapter I, § 21, that the actual place of *Jupiter's* node may differ from its mean place to the extent of 19° 38′, while that of *Saturn* can deviate from its mean place only to the extent of 7° 7′. It therefore follows that the longitudes of the nodes of these two planets can differ from 180° to the extent of 26° 45′. Their nearest possible approach is therefore 153° 15, while their present distance apart is 166° 27′.

We shall also find that the actual place of *Mercury's* node can differ from its mean place to the extent of 18° 31′; while the nodes of *Uranus* and *Neptune* can respectively deviate from their mean places to the extent of 6° 0′ and 9° 40′.

CHAPTER IV.

ON THE PRECESSION OF THE EQUINOXES AND THE OBLIQUITY OF THE ECLIPTIC.

1. THE analytical formulæ for the precession of the equinoxes and the obliquity of the ecliptic to the equator, referred to a fixed and also to a movable plane, are given by the formulæ [3100, 3101, 3107, and 3110], *Mécanique Céleste.* In order to reduce them to numbers we shall observe that the letter c in the notation of the formulæ corresponds to N'', N_1'', N_2'', &c. in this work. If we denote the mean value of the precession in a Julian year by l, and the mean obliquity of the ecliptic by h, and also put

$$c=\frac{l}{l+g}N'', \quad c_1'=\frac{l}{l+g_1}N_1'', \quad c_2=\frac{l}{l+g_2}N_2'', \text{ \&c.,}$$

$f=l+g$, $f_1=l+g_1$, $f_2=l+g_2$, $f_3=l+g_3$, $f_4=l+g_4=l$, $f_5=l+g_5$, &c., we shall have the following formulæ for determining the precession and obliquity:

$$\left.\begin{aligned}\psi=lt=\zeta+c&\left\{\cot h-\frac{g}{f}\tan h\right\}\sin(ft+\beta)\\+c_1&\left\{\cot h-\frac{g_1}{f_1}\tan h\right\}\sin(f_1t+\beta_1)\\+c_2&\left\{\cot h-\frac{g_2}{f_2}\tan h\right\}\sin(f_2t+\beta_2)\\+c_3&\left\{\cot h-\frac{g_3}{f_3}\tan h\right\}\sin(f_3t+\beta_3)\\+c_4&\left\{\cot h-\frac{g_4}{f_4}\tan h\right\}\sin(f_4t+\beta_4)\\+c_5&\left\{\cot h-\frac{g_5}{f_5}\tan h\right\}\sin(f_5t+\beta_5)\\+c_6&\left\{\cot h-\frac{g_6}{f_6}\tan h\right\}\sin(f_6t+\beta_6)\\+c_7&\left\{\cot h-\frac{g_7}{f_7}\tan h\right\}\sin(f_7t+\beta_7)\end{aligned}\right\}\begin{array}{c}\text{Precession of the}\\\text{Equinoxes on the}\\(550)\\\text{Fixed Ecliptic.}\end{array}$$

$$\left.\begin{aligned}\varepsilon_1=h-c\cos(ft+\beta)-c_1\cos(f_1t+\beta_1)-c_2\cos(f_2t+\beta_2)\\-c_3\cos(f_3t+\beta_3)-c_4\cos(f_4t+\beta_4)-c_5\cos(f_5t+\beta_5)\\-c_6\cos(f_6t+\beta_6)-c_7\cos(f_7t+\beta_7)\end{aligned}\right\}\begin{array}{c}\text{Obliquity of the}\\(551)\\\text{Equator to the}\\\text{Fixed Ecliptic.}\end{array}$$

$$\left.\begin{aligned}
\psi' = lt + \zeta - \frac{g}{f} N'' &\left\{ \cot h + \frac{l}{f} \tan h \right\} \sin (f t + \beta\ ) \\
- \frac{g_1}{f_1} N_1'' &\left\{ \cot h + \frac{l}{f_1} \tan h \right\} \sin (f_1 t + \beta_1) \\
- \frac{g_2}{f_2} N_2'' &\left\{ \cot h + \frac{l}{f_2} \tan h \right\} \sin (f_2 t + \beta_2) \\
- \frac{g_3}{f_3} N_3'' &\left\{ \cot h + \frac{l}{f_3} \tan h \right\} \sin (f_3 t + \beta_3) \\
- \frac{g_5}{f_5} N_5'' &\left\{ \cot h + \frac{l}{f_5} \tan h \right\} \sin (f_5 t + \beta_5) \\
- \frac{g_6}{f_6} N_6'' &\left\{ \cot h + \frac{l}{f_6} \tan h \right\} \sin (f_6 t + \beta_6) \\
- \frac{g_7}{f_7} N_7'' &\left\{ \cot h + \frac{l}{f_7} \tan h \right\} \sin (f_7 t + \beta_7)
\end{aligned}\right\} \begin{gathered}\text{Precession of the}\\ \text{Equinoxes on the}\\ (552)\\ \text{Apparent Ecliptic.}\end{gathered}$$

$$\left.\begin{aligned}
\varepsilon = h + \frac{g}{f} N'' \cos (f t + \beta\ ) &+ \frac{g_1}{f_1} N_1'' \cos (f_1 t + \beta_1) \\
+ \frac{g_2}{f_2} N_2'' \cos (f_2 t + \beta_2) &+ \frac{g_3}{f_3} N_3'' \cos (f_3 t + \beta_3) \\
+ \frac{g_5}{f_5} N_5'' \cos (f_5 t + \beta_5) &+ \frac{g_6}{f_6} N_6'' \cos (f_6 t + \beta_6) \\
&+ \frac{g_7}{f_7} N_7'' \cos (f_7 t + \beta_7)
\end{aligned}\right\} \begin{gathered}\text{Apparent obliquity}\\ (553)\\ \text{of the Ecliptic.}\end{gathered}$$

In equations (550) and (552), ζ is to be determined so that ψ and ψ' shall be equal to nothing when $t=0$. We may determine l and h as follows:—

If we take the differential of equation (552), we shall obtain

$$\left.\begin{aligned}
\frac{d\psi'}{dt} = l - g\ N'' &\left\{ \cot h + \frac{l}{f} \tan h \right\} \cos (f t + \beta\ ) \\
- g_1 N_1'' &\left\{ \cot h + \frac{l}{f_1} \tan h \right\} \cos (f_1 t + \beta_1) \\
- g_2 N_2'' &\left\{ \cot h + \frac{l}{f_2} \tan h \right\} \cos (f_2 t + \beta_2) \\
- g_3 N_3'' &\left\{ \cot h + \frac{l}{f_3} \tan h \right\} \cos (f_3 t + \beta_3) \\
- g_5 N_5'' &\left\{ \cot h + \frac{l}{f_5} \tan h \right\} \cos (f_5 t + \beta_5) \\
- g_6 N_6'' &\left\{ \cot h + \frac{l}{f_6} \tan h \right\} \cos (f_6 t + \beta_6) \\
- g_7 N_7'' &\left\{ \cot h + \frac{l}{f_7} \tan h \right\} \cos (f_7 t + \beta_7)
\end{aligned}\right\} \qquad (554)$$

Now at the epoch of 1850, at which time we have supposed $t=0$, we have $\varepsilon = 23° 27' 31.''0$; and $\frac{d\psi'}{dt} = 50''.23572$ according to the investigations of Bessel.

The first members of equations (553) and (554) are therefore known; and if we substitute in them the values of g, g_1, g_2, &c., N'', N_1'', N_2'', &c., β, β_1, β_2, &c., corresponding to the assumed masses, together with f, f_1, f_2, &c., they will become — the numbers in brackets being logarithms,

$$\left.\begin{aligned} 23^\circ\ 27'\ 31''.0 = h &- \frac{[8.7069903]}{l+g} - \frac{[8.4509982]}{l+g_1} - \frac{[8.6715146]}{l+g_2} \\ &+ \frac{[9.1494996]}{l+g_3} - \frac{[6.9157253]}{l+g_5} + \frac{[7.5163249]}{l+g_6} \\ &+ \frac{[8.6222025]}{l+g_7}. \end{aligned}\right\} \quad (555)$$

$$\left.\begin{aligned} 50''.23572+0''.05933222 \cot h = l + &\left\{ \frac{[8.7069903]}{l+g} + \frac{[8.4509982]}{l+g_1} \right. \\ &+ \frac{[8.6715146]}{l+g_2} - \frac{[9.1494996]}{l+g_3} + \frac{[6.9157253]}{l+g_5} \\ &\left. - \frac{[7.5163249]}{l+g_6} - \frac{[8.6222025]}{l+g_7} \right\} l \tan h. \end{aligned}\right\} \quad (556)$$

If we divide equation (556) by $l \tan h$, and add the quotient to equation (555), we shall get

$$\frac{50''.23572+0''.05933222 \cot h}{l \tan h} + 23^\circ\ 27'\ 31''.0 = h + \cot h. \quad (557)$$

Whence we get

$$l = \frac{50''.23572+0''.05933222 \cot h}{1+(h-84451''.0) \tan h}. \quad (558)$$

The direct determination of h and l from equations (555) and (556) is troublesome, and it is better to solve them by approximation. A few trials will show that $23^\circ\ 17'\ 16'' = 83836''$ is a near approximation to the value of h. If we substitute $h = 23^\circ\ 17'\ 16''$ in equation (558), we shall get $l = 50''.4382997$; and if we substitute this value of l in equation (555) we shall find

$$h = 23^\circ\ 27'\ 31''.0 - 10'\ 14''.4265 = 23^\circ\ 17'\ 16''.5735.$$

Now, substituting this value of h in equation (558), we shall get

$$l = 50''.4382387.$$

Having found h and l, we must substitute them in equations (550–553), and we shall obtain the expressions for the numerical values of the precession and obliquity during all past and future ages.

Adding g, g_1, g_2, &c. to l, we shall get f, f_1, f_2, &c., as follows,

$f = 45''.312168$,
$f_1 = 43\ .846111$,
$f_2 = 33\ .044849$,
$f_3 = 32\ .029325$,
$f_4 = l = 50''.438239$,
$f_5 = 49\ .776573$,
$f_6 = 47\ .522157$,
$f_7 = 24\ .503672$.

We also have

$$\begin{array}{llll@{\qquad\qquad}llll}
\beta\,= & 21^\circ & 6' & 26''.8 & \beta_4= & 106^\circ & 14' & 18''.0 \\
\beta_1= & 132 & 40 & 56.2 & \beta_5= & 20 & 31 & 24.6 \\
\beta_2= & 292 & 49 & 53.2 & \beta_6= & 133 & 56 & 10.8 \\
\beta_3= & 251 & 45 & 8.6 & \beta_7= & 306 & 19 & 21.2
\end{array}$$

Equations (550–553) reduced to numbers become

$$\left.\begin{aligned}
\psi=50''.438239t+8915''.6+\ & 5800''.35 \sin(f\,t+\beta\,) \\
-\ & 3581''.52 \sin(f_1t+\beta_1) \\
+\ & 5583''.09 \sin(f_2t+\beta_2) \\
+\ & 20438''.28 \sin(f_3t+\beta_3) \\
+\ & 13294''.37 \sin(f_4t+\beta_4) \\
+\ & 646''.98 \sin(f_5t+\beta_5) \\
+\ & 834''.76 \sin(f_6t+\beta_6) \\
-\ & 3217''.97 \sin(f_7t+\beta_7)
\end{aligned}\right\}\begin{array}{c}\text{Precession on}\\ \text{fixed Ecliptic}\\ (559)\\ \text{of 1850.}\end{array}$$

$$\left.\begin{aligned}
\varepsilon_1=23^\circ\,17'\,16''.57 & -2445''.33 \cos(f\,t+\beta\,)+1499''.78 \cos(f_1t+\beta_1) \\
& -2189''.55 \cos(f_2t+\beta_2)-7950''.46 \cos(f_3t+\beta_3) \\
& -5722''.14 \cos(f_4t+\beta_4)-\ 277''.79 \cos(f_5t+\beta_5) \\
& -355''.257 \cos(f_6t+\beta_6)+1158''.01 \cos(f_7t+\beta_7)
\end{aligned}\right\}\begin{array}{c}\text{Obliquity of}\\ (560)\\ \text{fixed Ecliptic}\\ \text{of 1850.}\end{array}$$

$$\left.\begin{aligned}
\psi'=50''.438239t+8915''.6 & +696''.462 \sin(f\,t+\beta\,) \\
& -552''.463 \sin(f_1t+\beta_1) \\
& +2250''.29 \sin(f_2t+\beta_2) \\
& +8708''.52 \sin(f_3t+\beta_3) \\
& +10''.0558 \sin(f_5t+\beta_5) \\
& +57''.102\ \sin(f_6t+\beta_6) \\
& -1910''.92 \sin(f_7t+\beta_7)
\end{aligned}\right\}\begin{array}{c}\text{Precession on}\\ (561)\\ \text{apparent Ecliptic.}\end{array}$$

$$\left.\begin{aligned}
\varepsilon=23\ 17'\ 16''.57 & -248''.520 \cos(f\,t+\beta\,)+\ 196''.017 \cos(f_1t+\beta_1) \\
& -755''.057 \cos(f_2t+\beta_2)-2901''.753 \cos(f_3t+\beta_3) \\
& -\ \ 3''.644 \cos(f_5t+\beta_5)-\ \ 20''.539 \cos(f_6t+\beta_6) \\
& \qquad\qquad\qquad\qquad\qquad +\ 595''.433 \cos(f_7t+\beta_7)
\end{aligned}\right\}\begin{array}{c}\text{Obliquity of}\\ (562)\\ \text{apparent}\\ \text{Ecliptic.}\end{array}$$

If we take the differentials of equations (561) and (562), we shall get

$$\left.\begin{aligned}
\frac{d\psi'}{dt}=50''.438239 & +0''.152998 \cos(f\,t+\beta)-0''.117438 \cos(f_1t+\beta_1) \\
& +0''.360530 \cos(f_2t+\beta_2)+1''.352281 \cos(f_3t+\beta_3) \\
& +0''.0024267 \cos(f_5t+\beta_5)+0''.013156 \cos(f_6t+\beta_6) \\
& \qquad\qquad\qquad\qquad\qquad -0''.227011 \cos(f_7t+\beta_7)
\end{aligned}\right\}\ (563)$$

$$\left.\begin{aligned}
\frac{d\varepsilon}{dt}= & +0''.054595\ \sin(g\,t+\beta\,)-0''.041668\ \sin(g_1t+\beta_1) \\
& +0''.120965\ \sin(g_2t+\beta_2)+0''.450592\ \sin(g_3t+\beta_3) \\
& +0''.0008794 \sin(g_5t+\beta_5)+0''.004732\ \sin(g_6t+\beta_6) \\
& \qquad\qquad\qquad\qquad\qquad -0''.0707357 \sin(g_7t+\beta_7)
\end{aligned}\right\}\qquad (564)$$

If we put $t=0$, in equations (563) and (564), we shall get the values of precession and variation of obliquity at the beginning of 1850. These values are

$$\frac{d\psi'}{dt}=50''.23572, \qquad \frac{d\varepsilon}{dt}=0''.489682.$$

If we take the sum of the coefficients of the *sines* in the expression of ψ' equation (561), without regard to their signs, we shall get the maximum quantity by which the true place of the equinox can differ from its mean place. This sum is $14185''.81=3^\circ\ 56'\ 25''.81$; it therefore follows that, if we suppose the equinox to have a uniform yearly motion equal to $50''.438239$, its place, when computed for any epoch, will not differ from the true place by an amount exceeding $3^\circ\ 56'\ 26''$. This remark is of especial importance in regard to the computation of the elements of terrestrial physics during past geological periods.

If we divide the number of seconds in the circumference of the circle by the mean motion of the equinoxes, we shall get $1296000''\div50''.438239=25694.8=$the number of years required for the equinoxes to perform a complete revolution in the heavens.

If we take the sum of the coefficients of the *cosines* in the expression of ε equation (562), without regard to their signs, we shall obtain the maximum quantity by which the obliquity of the ecliptic can differ from its mean value. This sum is $4720''.96=1^\circ\ 18'\ 40''.96$. From this it follows that the obliquity of the ecliptic is always confined within the limits $23^\circ\ 17'\ 16''.57\pm1^\circ\ 18'\ 40''.96$; or, between $24^\circ\ 35'\ 57''.53$ and $21^\circ\ 58'\ 35''.61$. The amount of its oscillations cannot therefore exceed $2^\circ\ 37'\ 22''$.

If we take the sum of the coefficients of $\frac{d\psi'}{dt}$, equation (563), we shall get the maximum quantity by which the annual precession can differ from its mean value. This sum is equal to $2''.225841$; whence it follows that the annual precession is always confined within the limits

$$50''.438239\pm2''.225841.$$

The maximum value of precession in a Julian year is therefore equal to $52''.664080$, and the minimum value of precession during the same time is equal to $48''.212398$. If we divide the difference of these two numbers by the time required for the earth to describe one second of an arc, we shall get the maximum variation of the tropical year, equal to

$$4''.451682\div(3548''.1876)\div86400^s)=108^s.40 \text{ seconds of time.}$$

If we subtract the present value of the precession from its maximum value, we shall get $2''.42836$ for the difference between them. Dividing this number by the time required for the earth to describe one second of arc, we shall get the amount of time by which the present tropical year exceeds the tropical year when it is reduced to its minimum length. The number thus found is $59^s.13$. In like manner we shall find that the tropical year may exceed its present length by

$49^s.27$. The present length of the tropical year is $365^d\ 5^h\ 48^m\ 47^s.26$. Whence we get

$$\begin{array}{rl} \text{maximum length of tropical year} &= 365^d\ 5^h\ 49^m\ 36^s.53; \\ \text{and minimum length of tropical year} &= 365\ \ 5\ \ 47\ \ \ 48.13. \end{array}$$

Lastly, if we take the sum of the coefficients of the *sines* in the formula for $\frac{d\varepsilon}{dt}$, we shall find that the annual variation of the obliquity is always confined within the limits of

$$0'' \text{ and } \pm 0''.744167.$$

CHAPTER V.

TABULAR VALUES OF THE ELEMENTS OF THE PLANETARY ORBITS.

1. In order to complete the subject of the secular inequalities, and render the preceding investigation of greater practical value, we have reduced all the astronomical elements to tables. The tabulated values for the planets embrace a period of 7200 years; commencing 6400 years before 1850, and continuing till A. D. 2650. These are given at intervals of a century during the entire period. The elements of the earth's orbit, together with the precession of the equinoxes in both longitude and right ascension, and the obliquity of the ecliptic to the equator, are given for an interval of 16,000 years; commencing 8000 years before, and ending 8000 years after the year 1850. They are also given at intervals of a century during the first half of that period, and at intervals of four centuries and eight centuries for the remainder of the period. The tabulated values are computed from the constants corresponding to a mass of the *Earth* which is equal to the assumed mass increased by its *tenth* part, or for $m''=\frac{1}{335172}$ of the sun's mass. This value of the *Earth's* mass corresponds to a solar parallax of about 8''.775, which is but little less than the recent determinations of that element; and it is remarkable that this value of the *Earth's* mass is very nearly equal to that which permits the planetary orbits to attain a greater eccentricity than any other mass moving at the same distance. Slight changes in this value of the mass would therefore produce only very inconsiderable changes in the variation of the elements of the planetary orbits.

2. The eccentricities and places of the perihelia have been computed from the values of h, h', h'', &c., l, l', l'', &c., given by equations (C) (page 32), by substituting the values of N, N', N'', &c., N_1, N_1', N_1'', &c., β, β_1, β_2, &c., g, g_1, g_2, &c., given in Art. 31 (pages 64 and 65), by means of the equations

$$\tan \varpi = h \div l,\ \ \tan \varpi' = h' \div l', \text{ \&c.};\ \ e = h \div \sin \varpi = l \div \cos \varpi,\ \ e' \div h' \div \varpi' = l' \div \cos \varpi', \text{ \&c.}$$

In like manner the nodes and inclinations of the orbits on the fixed ecliptic of 1850, have been computed from the values of q, p, q', p', &c., given by equations (F) page 111, by substituting the values of N, N', N'', &c., N_1, N_1', N_1'', &c., β, β_1, β_2, &c., g, g_1, g_2, &c., given in Art. 17, pages 134, &c., by means of the equations

$$\left.\begin{array}{ll}\tan \theta = p \div q,\ \ \tan \theta' = p' \div q', \text{ \&c.};\ & \tan \phi = p \div \sin \theta = q \div \cos \theta, \\ & \tan \phi' = p' \div \sin \theta' = q' \div \cos \theta', \text{ \&c.}\end{array}\right\}$$

3. The inclinations of the orbits of the planets and the longitudes of their ascending nodes on the ecliptic, or variable orbit of the earth, are denoted by ϕ_1, ϕ_1', ϕ_1''', &c., θ_1, θ_1', θ_1''', &c., and have been computed by the equations

$$\tan\theta_1=(p-p'')\div(q-q'');\ \tan\theta_1'=(p'-p'')-(q'-q'');\ \text{\&c.};$$

$$\left.\begin{aligned}\tan\phi_1&=(p-p'')\div\sin\theta_1=(q-q'')\div\cos\theta_1;\\ \tan\phi_1'&=(p'-p'')\div\sin\theta_1'=(q'-q'')\div\cos\theta_1',\ \text{\&c.}\end{aligned}\right\}$$

The longitudes of the perihelia and nodes are in every case counted from the mean equinox of 1850.0.

4. The precession of the equinoxes on the fixed ecliptic of 1850, and the inclination of the equator to the same plane, are denoted by ψ and ε_1; and they are determined by equations (550) and (551). Reducing them to numbers, and transforming so as to dispense with the arbitrary constant quantities, they will become

$$\left.\begin{aligned}
\psi=lt&+[4.0317043]\sin\tfrac{1}{2}ft\cos\tfrac{1}{2}ft+[3.6828039]\sin\tfrac{1}{2}f_1t\cos\tfrac{1}{2}f_1t\\
&-[3.5715890]\sin^2\tfrac{1}{2}ft\qquad\qquad\quad+[3.8013126]\sin^2\tfrac{1}{2}f_1t\\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad+[3.5080433]\sin\tfrac{1}{2}f_2t\cos\tfrac{1}{2}f_2t\\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad+[3.7464212]\sin^2\tfrac{1}{2}f_2t\\
&-[4.0645663]\sin\tfrac{1}{2}f_3t\cos\tfrac{1}{2}f_3t-[3.8713230]\sin\tfrac{1}{2}f_4t\cos\tfrac{1}{2}f_4t\\
&+[4.6281028]\sin^2\tfrac{1}{2}f_3t\qquad\qquad\quad-[4.4070533]\sin^2\tfrac{1}{2}f_4t\\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad+[3.0831687]\sin\tfrac{1}{2}f_5t\cos\tfrac{1}{2}f_5t\\
&\qquad\qquad\qquad\qquad\qquad\qquad\quad-[2.6561496]\sin^2\tfrac{1}{2}f_5t\\
&-[3.0600136]\sin\tfrac{1}{2}f_6t\cos\tfrac{1}{2}f_6t-[3.5903084]\sin\tfrac{1}{2}f_7t\cos\tfrac{1}{2}f_7t\\
&-[3.0760078]\sin^2\tfrac{1}{2}f_6t\qquad\qquad\quad-[3.7238538]\sin^2\tfrac{1}{2}f_7t
\end{aligned}\right\}\quad(565)$$

$$\left.\begin{aligned}
\varepsilon_1=23^\circ\ 27'\ 31''.00&+[3.1962358]\sin\tfrac{1}{2}ft\cos\tfrac{1}{2}ft\\
&+[3.6563511]\sin^2\tfrac{1}{2}ft\\
&-[3.4230485]\sin\tfrac{1}{2}f_1t\cos\tfrac{1}{2}f_1t\\
&+[3.3045398]\sin^2\tfrac{1}{2}f_1t\\
&-[3.3390526]\sin\tfrac{1}{2}f_2t\cos\tfrac{1}{2}f_2t\\
&+[3.1006747]\sin^2\tfrac{1}{2}f_2t\\
&-[4.2160834]\sin\tfrac{1}{2}f_3t\cos\tfrac{1}{2}f_3t\\
&-[3.6525469]\sin^2\tfrac{1}{2}f_3t\\
&+[4.0408748]\sin\tfrac{1}{2}f_4t\cos\tfrac{1}{2}f_4t\\
&-[3.5051445]\sin^2\tfrac{1}{2}f_4t\\
&+[2.2889032]\sin\tfrac{1}{2}f_5t\cos\tfrac{1}{2}f_5t\\
&+[2.7159223]\sin^2\tfrac{1}{2}f_5t\\
&+[2.7049217]\sin\tfrac{1}{2}f_6t\cos\tfrac{1}{2}f_6t\\
&-[2.6889275]\sin^2\tfrac{1}{2}f_6t\\
&+[3.2799396]\sin\tfrac{1}{2}f_7t\cos\tfrac{1}{2}f_7t\\
&-[3.1463942]\sin^2\tfrac{1}{2}f_7t
\end{aligned}\right\}\quad(566)$$

The coefficients in these equations are logarithms of seconds of arc; and f, f_1, f_2, &c. have the following values:—

$$\begin{array}{ll} f=45''.225870 & l=f_4=50''.439525 \\ f_1=43\ .770258 & f_5=49\ .777863 \\ f_2=32\ .812939 & f_6=47\ .523448 \\ f_3=31\ .503029 & f_7=24\ .504515 \end{array}$$

In all cases t denotes Julian years of $365\frac{1}{4}$ days.

5. The general precession of the equinoxes in longitude is very nearly the same as the precession on the apparent ecliptic, which is denoted by ψ', and is given by equation (552). But as the apparent ecliptic is continually shifting its position in space, the motion of precession on such an assumed plane becomes the same as it would be along a warped surface, and very imperfectly represents the general precession at times only a few hundred years from the epoch, although its maximum deviation from the truth can never exceed one-fourth of a degree. But on account of the importance of the subject we shall determine in a rigorous manner the general precession of the equinoxes in both longitude and right ascension. For this purpose we shall consider the spherical triangle formed by the fixed ecliptic of 1850, and the apparent ecliptic and equator of any time t. In this triangle there are known the two angles and the included side; namely, the angle of inclination of the apparent ecliptic to the fixed ecliptic of 1850, which is denoted by ϕ'', and the inclination of the equator to the same plane, which is denoted by ε_1, and the included side which is equal to $\psi+\theta''$. The three remaining parts of the triangle are the distances from the extremities of the known side of the triangle to the point of intersection of the apparent ecliptic and equator, and the angle included by these sides. We shall denote these quantities by $\psi'+\theta''$, ϑ, and ε. ψ' denotes the general precession in longitude, ϑ denotes the planetary precession, which is the distance between the fixed and apparent ecliptics measured on the apparent equator, and ε denotes the apparent obliquity of the ecliptic. The fundamental equations of spherical trigonometry will therefore furnish the following formulæ for the determination of ψ', ϑ, and ε:—

$$\left.\begin{array}{l} \sin\varepsilon\sin\vartheta=-\sin\phi''\sin(\psi+\theta'') \\ \sin\varepsilon\cos\vartheta=\sin\phi''\cos\varepsilon_1\cos(\psi+\theta'')+\cos\phi''\sin\varepsilon_1 \end{array}\right\}\quad(567)$$

$$\left.\begin{array}{l} \sin\varepsilon\sin(\psi'+\theta'')=\sin\varepsilon_1\sin(\psi+\theta'') \\ \sin\varepsilon\cos(\psi'+\theta'')=\sin\varepsilon_1\cos\phi''\cos(\psi+\theta'')+\cos\varepsilon_1\sin\phi'' \end{array}\right\}\quad(568)$$

The negative sign is given to the first equation because a forward motion of the equinox is a diminution of precession.

The equations (567) give the values of ϑ and ε; and either one of the equations (568) will give the value ψ' when ε has been determined. Since ϑ is always a very small arc, it is determinable with all desirable precision by means of its tangent; but this is not the case with ε. This quantity cannot be determined with extreme precision by means of its *sine*, without using logarithms to more than *seven places* of decimals. But as ε never differs greatly from ε_1, we may compute the difference between ε and ε_1 by eliminating ϑ from equations (567), and we shall find,

$$\sin(\varepsilon-\varepsilon_1)=\frac{\sin^2\phi''\cos^2\varepsilon_1-\sin^2\phi''\sin^2\varepsilon_1\cos^2(\psi+\theta'')+2\cos\phi''\sin\phi''\cos\varepsilon_1\sin\varepsilon_1\cos(\psi+\theta'')}{\sin(\varepsilon+\varepsilon_1)}. \quad (569)$$

Although ε appears in the denominator of this equation it is readily determinable from its *sine* with sufficient precision to be used in finding $\sin(\varepsilon-\varepsilon_1)$ with accuracy.

Since ψ' never differs greatly from ψ, we may readily transform equations (568) so as to give the difference of these quantities, and we shall find

$$\left.\begin{aligned}\sin\varepsilon\sin(\psi-\psi')=&\cos\varepsilon_1\sin\phi''\sin(\psi+\theta'')\\&-2\sin^2\tfrac{1}{2}\phi''\sin\varepsilon_1\sin(\psi+\theta'')\cos(\psi+\theta'')\end{aligned}\right\}(570)$$

This equation determines $\psi-\psi'$ with all desirable precision, ε having been previously determined.

6. We shall now consider the spherical triangle formed by the fixed ecliptic of 1850, the fixed equator of 1850, and the apparent equator of any time t. Since the inclination of the equator to the fixed ecliptic of 1850 is given by equation (566), we may suppose this quantity to be known for the given time. Then calling ε_1 the inclination of the fixed equator and ecliptic ε_1', the inclination of the apparent equator to the fixed ecliptic, and ψ the total luni-solar precession during the time t; the two angles and the included side of the proposed triangle will be known, and the three remaining parts may readily be determined in the following manner: Let the distances from the intersection of the fixed and apparent equators to the fixed ecliptic be denoted by $90°-z$ and $90°+z'$, and the angle of intersection of the two equators be denoted by Θ, we shall have the following equations for determining these last named quantities:—

$$\left.\begin{aligned}\sin\Theta\cos z&=\sin\psi\sin\varepsilon_1'\\\sin\Theta\sin z&=\sin\varepsilon_1\cos\varepsilon_1'-\cos\psi\sin\varepsilon_1'\cos\varepsilon_1\\\cos\Theta&=\cos\varepsilon_1\cos\varepsilon_1'+\sin\varepsilon_1\sin\varepsilon_1'\cos\psi\\\sin\Theta\cos z'&=\sin\psi\sin\varepsilon_1\\\sin\Theta\sin z'&=\cos\psi\sin\varepsilon_1\cos\varepsilon_1'-\cos\varepsilon_1\sin\varepsilon_1'\end{aligned}\right\}\quad(571)$$

These equations determine z, z', and Θ rigorously; but since ε_1 is nearly equal to ε_1', they are under a very inconvenient form for accurate computation when ψ is a small angle. They may, however, be readily put under the following form for very accurate and convenient computation:—

$$\left.\begin{aligned}\sin\Theta\cos z&=\sin\psi\sin\varepsilon_1'\\\sin\Theta\sin z&=2\sin^2\tfrac{1}{2}\psi\sin\varepsilon_1'\cos\varepsilon_1+\sin(\varepsilon_1-\varepsilon_1')\\\sin^2\tfrac{1}{2}\Theta&=\sin^2\tfrac{1}{2}(\varepsilon_1-\varepsilon_1')+\sin^2\tfrac{1}{2}\psi\sin\varepsilon_1\sin\varepsilon_1'\\\sin\Theta\cos z'&=\sin\psi\sin\varepsilon_1\\\sin\Theta\sin z'&=2\sin^2\tfrac{1}{2}\psi\sin\varepsilon_1\cos\varepsilon_1'-\sin(\varepsilon_1-\varepsilon_1')\end{aligned}\right\}\quad(572)$$

The sum of the quantities z and z' might very properly be called the luni-solar precession in right ascension; and if to this we add the planetary precession we shall have the general precession in right ascension at any time t, equal to $z+z'+\vartheta$; all of which quantities being taken from the table with the argument t.

7. Tables I—VIII have been computed as explained in §§ 2 and 3. They show the elements of the planetary orbits at the times given in the first column of each table, and seem to require no explanation as to their uses. ψ, and ε_1, in Tables IX and X, have been computed from the formulæ (565) and (566); ϑ, ψ' and ε have been computed from equations (567), (570), and (569), by using the values of θ'', ϕ'', ψ, and ε_1 given in Tables VIII and IX; and lastly, z, z', and Θ have been computed by means of equations (572), by using the values of ψ, ε, ε_1' given in Table IX.

8. Having explained the method of constructing the tables, we will now explain the manner of using Tables IX and X in connection with θ'' and ϕ'', which are given in Table VIII. The quantities θ'', ϕ'', and ψ' are useful in reducing the longitudes of the celestial bodies from the mean equinox of 1850 to the mean equinox of any other date, and *vice versa*. This transformation is effected by means of the following equations, in which λ and β denote the mean longitude and latitude of a celestial body at the epoch of 1850, and λ' and β' denote the same co-ordinates referred to the mean equinox of any time t, before or after that epoch.

$$\left.\begin{aligned} \cos\beta'\cos(\lambda'-\theta''-\psi')&=\cos\beta\cos(\lambda-\theta'') \\ \cos\beta'\sin(\lambda'-\theta''-\psi')&=\cos\beta\cos\phi''\sin(\lambda-\theta'')+\sin\beta\sin\phi'' \\ \sin\beta'&=\sin\beta\cos\phi''-\cos\beta\sin\phi''\sin(\lambda-\theta'') \end{aligned}\right\} \qquad (573)$$

For reducing *to* the equinox of 1850 these equations take the following form:—

$$\left.\begin{aligned} \cos\beta\cos(\lambda-\theta'')&=\cos\beta'\cos(\lambda'-\theta''-\psi') \\ \cos\beta\sin(\lambda-\theta'')&=\cos\beta'\cos\phi''\sin(\lambda'-\theta''-\psi')-\sin\phi''\sin\beta' \\ \sin\beta&=\sin\beta'\cos\phi''+\cos\beta'\sin\phi''\sin(\lambda'-\theta''-\psi') \end{aligned}\right\} \qquad (574)$$

It is sometimes desirable to find the difference in the longitude or latitude of a star arising from the precession of the equinoxes. This difference may be found by the following formulæ, the employment of which is perhaps more laborious than that of the preceding from which they were derived; but they may in ordinary cases be managed by the use of *five-figure* logarithms, whereas equations (573) and (574) require *seven-figure* logarithms to arrive at accurate results.

$$\lambda'=\lambda+\psi'+\Bigg[\text{arc tan}=\left\{\frac{\tan\beta\sin\phi''\cos(\lambda-\theta'')-2\sin^2\tfrac{1}{2}\phi''\cos(\lambda-\theta'')\sin(\lambda-\theta'')}{1+\tan\beta\sin\phi''\sin(\lambda-\theta'')-2\sin^2\tfrac{1}{2}\phi''\sin^2(\lambda-\theta'')}\right\}\Bigg]; \qquad (575)$$

$$\beta'=\beta-2\,\text{arc sin}\left\{\frac{\cos\beta\sin\phi''\sin(\lambda-\theta'')+2\sin\beta\sin^2\tfrac{1}{2}\phi''}{2\cos\tfrac{1}{2}(\beta'+\beta)}\right\}$$

$$\lambda=\lambda'-\psi'-\Bigg[\text{arc tan}=\left\{\frac{\tan\beta'\sin\phi''\cos(\lambda'-\theta''-\psi')+2\sin^2\tfrac{1}{2}\phi''\cos(\lambda'-\theta''-\psi')\sin(\lambda'-\theta''-\psi')}{1-\tan\beta'\sin\phi''\sin(\lambda'-\theta''-\psi')-2\sin^2\tfrac{1}{2}\phi''\sin^2(\lambda'-\theta''-\psi')}\right\}\Bigg] \qquad (576)$$

$$\beta=\beta'+2\,\text{arc sin}\left\{\frac{\cos\beta'\sin\phi''\sin(\lambda'-\theta''-\psi')-2\sin\beta'\sin^2\tfrac{1}{2}\phi''}{2\cos\tfrac{1}{2}(\beta'+\beta)}\right\} \qquad (577)$$

If, in these equations, we neglect quantities of the order $\sin^2 \phi''$, they assume a very convenient form for computation, and at the same time possess sufficient accuracy for computations extending over a period of several hundred years, except for stars situated very near the pole, in which cases some of the preceding equations must be employed if we wish to obtain accurate results.

$$\left.\begin{aligned} \lambda' &= \lambda + \psi' + \phi'' \tan \beta \cos(\lambda - \theta'') \\ \beta' &= \beta - \phi'' \sin(\lambda - \theta''); \end{aligned}\right\} \quad (578)$$

$$\left.\begin{aligned} \lambda &= \lambda' - \psi' - \phi'' \tan \beta' \cos(\lambda' - \theta'' - \psi') \\ \beta &= \beta' + \phi'' \sin(\lambda' - \theta'' - \psi'). \end{aligned}\right\} \quad (579)$$

In these equations ϕ'' is to be expressed in seconds of arc.

9. For the reduction of right ascensions and declinations all the necessary data depending on the motion of the equinox are contained in Table X. The quantities in the table are adapted to computation by the following formulæ, in which α and δ denote the mean right ascension and declination of a star at the epoch of 1850; and α' and δ' denote the same co-ordinates referred to the mean equinox of any time t before or after the epoch.

$$\left.\begin{aligned} \cos\delta' \sin(\alpha' - z' - \vartheta') &= \cos\delta \sin(\alpha + z - \vartheta) \\ \cos\delta' \cos(\alpha' - z' - \vartheta') &= \cos\delta \cos\Theta \cos(\alpha + z - \vartheta) - \sin\delta \sin\Theta \\ \sin\delta' &= \cos\delta \sin\Theta \cos(\alpha + z - \vartheta) + \sin\delta \cos\Theta \end{aligned}\right\} \quad (580)$$

For reducing *to* the mean equinox of 1850 these equations take the following form:—

$$\left.\begin{aligned} \cos\delta \sin(\alpha + z - \vartheta) &= \cos\delta' \sin(\alpha' - z' - \vartheta') \\ \cos\delta \cos(\alpha + z - \vartheta) &= \cos\delta' \cos\Theta \cos(\alpha' - z' - \vartheta') + \sin\Theta \sin\delta' \\ -\sin\delta &= \cos\delta' \sin\Theta \cos(\alpha' - z' - \vartheta') - \cos\Theta \sin\delta' \end{aligned}\right\} \quad (581)$$

The first two of equations (580) will very readily give

$$\alpha' = \alpha + (z + z' + \vartheta' - \vartheta) + \Bigg[\text{arc} \tan = \left\{ \frac{\tan\delta \sin\Theta \sin(\alpha + z - \vartheta) + 2\sin^2 \tfrac{1}{2}\Theta \sin(\alpha + z - \vartheta) \cos(\alpha + z - \vartheta)}{1 - \tan\delta \sin\Theta \cos(\alpha + z - \vartheta) - 2\sin^2 \tfrac{1}{2}\Theta \cos^2(\alpha + z - \vartheta)} \right\} \Bigg] \quad (582)$$

Here the term $z + z' + \vartheta' - \vartheta$ appears as the general precession in right ascension common to all the stars, and the last term of the equation gives the correction depending on the place of each particular star.

In like manner the first two of equations (581) will give

$$\alpha = \alpha' - (z + z' + \vartheta' - \vartheta) - \Bigg[\text{arc} \tan = \left\{ \frac{\tan\delta' \sin\Theta \sin(\alpha' - z' - \vartheta') - 2\sin^2 \tfrac{1}{2}\Theta \sin(\alpha' - z' - \vartheta') \cos(\alpha' - z' - \vartheta')}{1 + \tan\delta' \sin\Theta \cos(\alpha' - z' - \vartheta') - 2\sin^2 \tfrac{1}{2}\Theta \cos^2(\alpha' - z' - \vartheta')} \right\} \Bigg] \quad (583)$$

For stars situated near the pole, equations (580) and (581) are preferable to (582) and (583), because when δ or δ' is equal to 90° the terms depending on tan δ or tan δ' become infinite, and the equations (582) and (583) assume an indeterminate form. But this is not the case with equations (580) and (581); for if $\delta = 90°$ equations (580) will give $\sin\delta' = \cos\Theta$, and then we shall find $\cos(\alpha - z' - \vartheta') = -\sin\Theta \div \cos\delta' = -1$, whence $\alpha' - z' - \vartheta' = 180°$, from which α' is easily determined.

Rigorous expressions for the difference of the declinations depending on the precession are not readily deducible from equations (580) and (581) of sufficient simplicity to possess any advantage in computation over the original formulæ, since Θ is not necessarily a small angle.

10. Having given the necessary formulæ for reducing the position of a star which is given in longitude and latitude, or right ascension and declination, with reference to the equinox of 1850, to that of any other date, and the reverse, we shall now give some examples by way of illustration.

Example I. The mean place of polaris, at the beginning of the year 1850, was, $\alpha=16° \, 15' \, 22''.815$, $\delta=88° \, 30' \, 34''.889$; it is required to find its right ascension and declination at the beginning of 1950, 2050, and 2150, and also the maximum declination of the star.

In 1850 the planetary precession was equal to nothing, therefore ϑ disappears from the second member of equations (580). For $t=+100$, Table X, gives $z=0° \, 38' \, 23''.482$, $z'=0° \, 38' \, 36''.527$, $\Theta=0° \, 33' \, 24''.811$, and $\vartheta=-0° \, 0' \, 12''.433$. Whence $\alpha+z=16° \, 53' \, 46''.297$; and the computation is as follows, using the first and second of equations (580):—

$\alpha+z$		sin 9.4633534
δ		cos 8.4151046
$\alpha+z$		cos 9.9808361
Θ		cos 9.9999795
Θ		sin 7.9876415
δ		—sin 9.9998531n
$\cos\delta\cos(\alpha+z)\cos\Theta$	$=+0.02488400$	log. 8.3959202
$-\sin\delta\sin\Theta$	$=-0.00971616$	" 7.9874946n
$\cos\delta'\cos(\alpha'-z'-\vartheta')$	$=+0.01516784$	" 8.1809237
$\cos\delta'\sin(\alpha'-z'-\vartheta')$	$=\cos\delta\sin(\alpha+z)$	" 7.8784580
$\alpha'-z'-\vartheta'$	$=26° \, 29' \, 21''.70$	tan 9.6975343
$z'+\vartheta'$	$=\quad 38 \;\; 24.10$	
α'	$=27° \;\; 7' \, 45''.80$	
$\alpha'-z'-\vartheta'$		cos 9.9518314
$\cos\delta'\cos(\alpha'-z'-\vartheta')$		log. 8.1809237
δ'	$=89° \;\; 1' \, 44''.277$	cos 8.2290923

Computing in the same way for $t=+200$ and $t=+300$, we shall find the values of α' and δ' as in the following table:—

	α'	δ'
1950	27° 7′ 45″.80	89° 1′ 44″.277
2050	56 51 57.27	89 27 20.654
2150	120 32 36.31	89 28 13.594

The declination evidently attains a maximum value some time between 2050 and 2150. If we compute its place for $t=250$, we shall find $z=1° \, 35' \, 59''.93$, $z'=1° \, 36' \, 26''.63$, $\Theta=1° \, 23' \, 29''.80$, and $\vartheta'=-21''.80$.

Then we find $\alpha'=88^\circ\ 13'\ 39''.20$ and $\delta'=89^\circ\ 32'\ 32''.202$.

Now having the declination for $t=200$, $t=250$, and $t=300$, we very readily find that the maximum will take place when $t=+252.335$, or a little before the middle of the year 2102. Computing the place of the star for this last value of t, we find

$$\alpha'=89^\circ\ 53'\ 2''.78 \text{ and } \delta'=89^\circ\ 32'\ 32''.973.$$

The nearest approach of the pole to the star is therefore equal to $27'\ 27''.027$.

Example II. Let it be required to find the declination of Polaris when $t=-8000$. For the value of t we get $z=-53^\circ\ 0'\ 2''.8$, $z'=-54^\circ\ 34'\ 37''.2$, $\Theta=-39^\circ\ 24'\ 51''.5$, and $\vartheta'=+2^\circ\ 43'\ 48''.3$. $\alpha+z=-36^\circ\ 44'\ 40''.0$.

Therefore the computation will stand as follows:—

$\alpha+z$	sin 9.7768805n
δ	cos 8.4151046
$\alpha+z$	cos 9.9038016
Θ	cos 9.8879407
Θ	sin 9.8027213n
δ	—sin 9.9998531n
$\cos\delta\cos(\alpha+z)\cos\Theta=+0.0161008$	log. 8.2068469
$-\sin\delta\sin\Theta=+0.6347087$	" 9.8025744
$\cos\delta'\cos(\alpha'-z'-\vartheta')=+0.6508095$	" 9.8134539
$\cos\delta'\sin(\alpha'-z'-\vartheta')$	" 8.1919851n
$\alpha'-z'-\vartheta'=358^\circ\ 37'\ 49''.7$	tan 8.3785312n
$z'+\vartheta'=-51\ \ 50\ \ 48.9$	
$\alpha'=306^\circ\ 47'\ 0''.8$	
$\alpha'-z'-\vartheta'$	cos 9.9998759
$\cos\delta'\cos(\alpha'-z'-\vartheta')$	log. 9.8134539
$\delta'=49^\circ\ 23'\ 0$	cos 9.8135780

From this calculation it follows that the present polar star was, 8000 years ago, distant 40° 37′ from the pole.

Example III. In 1850 the place of the star α Cephei was

$$\alpha=318^\circ\ 45',\ \delta=+61^\circ\ 56';$$

required its mean place when $t=+5600$ years.

In this example we find

$$z=+36^\circ\ 55'\ \ z'+\vartheta'=+37^\circ\ 35',\ \Theta=28^\circ\ 44'.$$

Then we get $\alpha'-z'-\vartheta'=249^\circ\ 44'$, whence $\alpha'=287^\circ\ 19'$ and $\delta'=+87^\circ\ 50'$.

It therefore follows that the star α Cephei will be only about 2° distant from the pole in 5600 years; it will therefore constitute the pole star of that period.

The equations for reducing from longitude and latitude to right ascension and declination, and the reverse, are the following:—

$$\left.\begin{aligned}\cos\delta\cos\alpha&=\cos\beta\cos\lambda\\ \cos\delta\sin\alpha&=\cos\varepsilon\cos\beta\sin\lambda-\sin\varepsilon\sin\beta\\ \sin\delta&=\sin\varepsilon\cos\beta\sin\lambda+\cos\varepsilon\sin\beta\end{aligned}\right\}\qquad(584)$$

$$\left.\begin{aligned}\cos\beta\cos\lambda&=\cos\delta\cos\alpha\\ \cos\beta\sin\lambda&=\cos\varepsilon\cos\delta\sin\alpha+\sin\varepsilon\sin\delta\\ \sin\beta&=-\sin\varepsilon\cos\delta\sin\alpha+\cos\varepsilon\sin\delta\end{aligned}\right\}\qquad(585)$$

Example IV. The right ascension and declination of α Tauri (*Aldebaran*) in 1850 was $\alpha=66^\circ\ 49'\ 46''.35$, $\delta=+16^\circ\ 12'\ 11''.0$; required its longitude and latitude for $t=-4900$, or, for the beginning of the year B. C. 3050.

Since $\varepsilon=23^\circ\ 27'\ 31''.0$, in 1850, equations (585) will give the longitude and latitude for the same epoch, as follows:—

$$\lambda=67^\circ\ 41''\ 34'.1,\qquad \beta=-5^\circ\ 28'\ 40''.1.$$

And for $t=-4900$. Tables VIII and IX will give

$$\theta''=5^\circ\ 22'\ 51''.7,\ \phi''=0^\circ\ 41'\ 22''.45,\text{ and }\psi'=-67^\circ\ 40'\ 32''.2.$$

If these quantities be substituted in equations (573), we shall find

$$\lambda'-\theta''-\psi'=62^\circ\ 16'\ 45''.5,\ \sin\beta'=-0.106061;$$

whence we get

$$\lambda'=359^\circ\ 59'\ 5''.0,\text{ and }\beta'=-6^\circ\ 5'\ 17''.9.$$

Therefore the star *Aldebaran*, at the beginning of the year B. C. 3050, was only 55″.0 westward of the vernal equinox, measured on the ecliptic of that date, and coincided with the equinox in the year 3049 B. C.

Example V. The right ascension and declination of *Aldebaran* being, as in the preceding example, at the beginning of 1850, required its right ascension and declination at the beginning of the year B. C. 3050. For $t=-4900$, Table X gives $z=-31^\circ\ 43'\ 26''.4$, $z'=-32^\circ\ 44'\ 31''.8$, $\Theta=-26^\circ\ 14'\ 1''.4$, and $\vartheta'=+1^\circ\ 31'\ 0''0$. If these quantities be substituted in equations (580), we shall find

$$\alpha'-z'-\vartheta'=33^\circ\ 42'\ 2''.8,\text{ and }\sin.\delta'=-0.0969604.$$

Whence we get

$$\alpha'=2^\circ\ 28'\ 31''.0,\text{ and }\delta'=-5^\circ\ 33''\ 51'.0.$$

We might have found these last quantities by means of equations (584) by substituting for λ, β, the values of λ' and β', found in example IV, and using the value of ε corresponding to that epoch, which value is $\varepsilon=24^\circ\ 3'\ 8''.2$.

From this computation we see that, although the star *Aldebaran* was 55″ *westward* of the equinox when measured on the ecliptic, it was nearly $2\frac{1}{2}$ degrees *eastward* of the equinox when measured on the equator, and instead of being in a northern constellation then, as now, it was in reality in a southern constellation.

11. We will now determine the position of the pole of the equator. The longitude of the pole on the fixed ecliptic of 1850 at any time t will evidently be equal

to $90°-\psi$; and the latitude of the pole will also be equal to $90°-\varepsilon_1'$, or to the complement of the obliquity of the ecliptic of 1850, at the given time. If we then put $\lambda=90°-\psi$, $\beta=90°-\varepsilon_1'$, in equations (584), the resulting α and δ will evidently be the right ascension and declination of the pole of the equator for the time t, referred to the equinox and equator of 1850. Calling the right ascension of the pole A, and the declination D, we shall evidently have

$$\left.\begin{aligned}\cos D\cos A&=\sin\varepsilon_1'\sin\psi\\ \cos D\sin A&=-\sin\varepsilon_1\cos\varepsilon_1'+\cos\varepsilon_1\sin\varepsilon_1'\cos\psi\\ \sin D&=\cos\varepsilon_1\cos\varepsilon_1'+\sin\varepsilon_1\sin\varepsilon_1'\cos\psi\end{aligned}\right\}\quad(586)$$

ε_1 denoting the obliquity in 1850, and ε_1' denoting the obliquity of the fixed ecliptic of 1850 *at the time* for which the computation is made. If we compare these equations with equations (571), we find $\sin A=-\sin z$, and $\sin D=\cos\Theta$.

Therefore $A=-z$, and $D=90°-\Theta$.

Now since z and Θ are already tabulated we can enter Table X with the argument t, and take out the right ascension and declination of the pole by mere inspection.

Example VI. What will be the right ascension and declination of pole 5600 years hence, when referred to the equinox and equator of 1850? Entering Table X with the argument $t=+5600$, we find $z=36°\ 55'\ 6''.4$, and $\Theta=28°\ 44'\ 0''.89$. Therefore $A=323°\ 4'\ 53''.6$, and $D=61°\ 15'\ 59''.11$. The mean place of α Cephei in 1850 was $\alpha=318°\ 45'$, and $\delta=61°\ 56'$. This star will therefore be the pole-star of that period.

TABLE I.—*Elements of the Orbit of Mercury.*

t	e	ϖ			θ			ϕ			θ_1			ϕ_1		
—6400	0.205301	65°	13′	40″	54°	0′	37″.5	7°	13′	4″.81	59°	31′	23″.8	6°	36′	57″.1
6300	0.205308	65	22	55	53	53	35.6	7	12	52.31	59	19	30.0	6	37	13.0
6200	0.205314	65	32	11	53	46	33.8	7	12	39.81	59	7	35.5	6	37	48.9
6100	0.205321	65	41	26	53	39	32.1	7	12	27.32	58	55	40.3	6	38	14.6
6000	0.205328	65	50	42	53	32	30.4	7	12	14.84	58	43	44.5	6	38	40.1
5900	0.205335	65	59	57	53	25	28.9	7	12	2.37	58	31	48.0	6	39	5.6
5800	0.205341	66	9	13	53	18	27.4	7	11	49.90	58	19	51.1	6	39	30.9
5700	0.205348	66	18	28	53	11	26.3	7	11	37.44	58	7	53.5	6	39	56.1
5600	0.205354	66	27	44	53	4	25.2	7	11	24.99	57	55	55.2	6	40	21.1
5500	0.205361	66	37	0	52	57	24.7	7	11	12.54	57	43	56.4	6	40	46.1
5400	0.205367	66	46	16	52	50	24.1	7	11	0.10	57	31	57.0	6	41	10.9
5300	0.205373	66	55	32	52	43	23.5	7	10	47.67	57	19	57.0	6	41	35.5
5200	0.205379	67	4	48	52	36	22.9	7	10	35.24	57	7	56.3	6	42	0.0
5100	0.205385	67	14	4	52	29	22.3	7	10	22.83	56	55	55.2	6	42	24.4
5000	0.205391	67	23	20	52	22	21.7	7	10	10.42	56	43	53.5	6	42	48.7
4900	0.205397	67	32	36	52	15	21.1	7	9	58.02	56	31	51.2	6	43	12.8
4800	0.205402	67	41	52	52	8	20.5	7	9	45.64	56	19	48.4	6	43	36.8
4700	0.205408	67	51	8	52	1	20.1	7	9	33.27	56	7	45.1	6	44	0.7
4600	0.205414	68	0	24	51	54	19.8	7	9	20.92	55	55	41.2	6	44	24.4
4500	0.205420	68	9	39	51	47	19.4	7	9	8.58	55	43	36.8	6	44	48.0
4400	0.205425	68	18	55	51	40	19.1	7	8	56.25	55	31	31.9	6	45	11.4
4300	0.205431	68	28	10	51	33	18.9	7	8	43.93	55	19	26.5	6	45	34.8
4200	0.205436	68	37	26	51	26	18.7	7	8	31.62	55	7	20.5	6	45	58.0
4100	0.205442	68	46	41	51	19	18.8	7	8	19.32	54	55	14.0	6	46	21.0
4000	0.205447	68	55	57	51	12	18.9	7	8	7.03	54	43	7.1	6	46	43.9
3900	0.205453	69	5	14	51	5	19.2	7	7	54.75	54	31	0.7	6	47	6.7
3800	0.205458	69	14	30	50	58	19.6	7	7	42.49	54	18	51.8	6	47	29.4
3700	0.205463	69	23	46	50	51	20.1	7	7	30.24	54	6	43.4	6	47	51.9
3600	0.205468	69	33	3	50	44	20.6	7	7	18.01	53	54	34.6	6	48	14.3
3500	0.205473	69	42	19	50	37	21.3	7	7	5.79	53	42	25.3	6	48	36.6
3400	0.205478	69	51	35	50	30	22.0	7	6	53.57	53	30	15.6	6	48	58.7
3300	0.205483	70	0	51	50	23	22.7	7	6	41.37	53	18	5.4	6	49	20.6
3200	0.205487	70	10	7	50	16	23.5	7	6	29.19	53	5	54.8	6	49	42.5
3100	0.205492	70	19	23	50	9	24.3	7	6	17.02	52	53	43.8	6	50	4.2
3000	0.205497	70	28	40	50	2	25.2	7	6	4.87	52	41	32.4	6	50	25.8
2900	0.205501	70	37	56	49	55	26.1	7	5	52.74	52	29	20.5	6	50	47.2
2800	0.205506	70	47	12	49	48	27.1	7	5	40.61	52	17	8.3	6	51	8.5
2700	0.205510	70	56	28	49	41	28.1	7	5	28.49	52	4	55.7	6	51	29.6
2600	0.205515	71	5	44	49	34	29.2	7	5	16.40	51	52	42.7	6	51	50.6
2500	0.205520	71	15	0	49	27	30.3	7	5	4.32	51	40	29.3	6	52	11.5
2400	0.205524	71	24	16	49	20	31.4	7	4	52.25	51	28	15.5	6	52	32.2
2300	0.205529	71	33	33	49	13	32.5	7	4	40.20	51	16	1.4	6	52	52.8
2200	0.205533	71	42	50	49	6	33.7	7	4	28.17	51	3	46.8	6	53	13.3
2100	0.205538	71	52	7	48	59	34.9	7	4	16.17	50	51	31.9	6	53	33.6
2000	0.205542	72	1	24	48	52	36.1	7	4	4.17	50	39	16.5	6	53	53.8
1900	0.205547	72	10	40	48	45	37.4	7	3	52.19	50	27	0.9	6	54	13.8
1800	0.205551	72	19	57	48	38	38.7	7	3	40.23	50	14	44.9	6	54	33.8
1700	0.205555	72	29	14	48	31	40.0	7	3	28.28	50	2	28.5	6	54	53.5
1600	0.205559	72	38	31	48	24	41.4	7	3	16.35	49	50	11.8	6	55	13.2
1500	0.205563	72	47	47	48	17	42.7	7	3	4.44	49	37	54.8	6	55	32.6
1400	0.205567	72	57	4	48	10	44.1	7	2	52.56	49	25	37.5	6	55	52.0
1300	0.205571	73	6	21	48	3	45.5	7	2	40.69	49	13	19.8	6	56	11.2
1200	0.205575	73	15	38	47	56	46.9	7	2	28.84	49	1	1.9	6	56	30.3
1100	0.205579	73	24	55	47	49	48.3	7	2	17.00	48	48	43.7	6	56	49.2
1000	0.205583	73	34	12	47	42	49.7	7	2	5.19	48	36	25.0	6	57	8.0
900	0.205586	73	43	29	47	35	51.1	7	1	53.40	48	24	6.0	6	57	26.7
800	0.205590	73	52	46	47	28	52.5	7	1	41.63	48	11	46.7	6	57	45.2
700	0.205594	74	2	2	47	21	53.9	7	1	29.88	47	59	27.2	6	58	3.6
600	0.205597	74	11	19	47	14	55.2	7	1	18.15	47	47	7.4	6	58	21.8
500	0.205601	74	20	36	47	7	56.6	7	1	6.43	47	34	47.7	6	58	39.9
400	0.205604	74	29	53	47	0	57.9	7	0	54.70	47	22	27.0	6	58	57.9
300	0.205608	74	39	10	46	53	59.3	7	0	43.07	47	10	6.5	6	59	15.7
200	0.205611	74	48	26	46	47	0.6	7	0	31.41	46	57	45.6	6	59	33.3
—100	0.205615	74	57	43	46	40	1.9	7	0	19.77	46	45	24.6	6	59	50.9
0	0.205618	75	7	0	46	33	3.2	7	0	8.16	46	33	3.2	7	0	8.2
+100	0.205621	75	16	16	46	26	4.5	6	59	56.57	46	20	42.0	7	0	25.5
200	0.205624	75	25	33	46	19	5.7	6	59	45.00	46	8	20.4	7	0	42.6
300	0.205627	75	34	50	46	12	7.0	6	59	33.45	45	55	58.6	7	0	59.6
400	0.205630	75	44	7	46	5	8.2	6	59	21.92	45	43	36.6	7	1	16.4
500	0.205633	75	53	24	45	58	9.4	6	59	10.42	45	31	14.4	7	1	33.2
600	0.205636	76	2	41	45	51	10.6	6	58	58.93	45	18	52.0	7	1	49.7
700	0.205639	76	11	58	45	44	11.7	6	58	47.47	45		29.4	7	2	6.1
+800	0.205642	76	21	15	45	37	12.8	6	58	36.02	44	54	6.7	7	2	22.4

TABLE II.—*Elements of the Orbit of Venus.*

t	e'	ϖ'	θ'	ϕ'	θ_1'	ϕ_1'
—6400	0.0104237	124° 30′ 10″	92° 31′ 48″	3° 18′ 33″.9	108° 11′ 5″	3° 19′ 42″.1
6300	0.0103630	124 42 13	92 16 15	3 18 45.6	107 39 55	3 19 45.8
6200	0.0103023	124 50 12	92 0 41	3 18 58.5	107 8 45	3 19 49.4
6100	0.0102417	124 58 6	91 45 6	3 19 10.5	106 37 36	3 19 53.1
6000	0.0101812	125 5 54	91 29 30	3 19 22.2	106 6 27	3 19 56.8
5900	0.0101208	125 13 37	91 13 52	3 19 33.7	105 35 20	3 20 0.5
5800	0.0100606	125 21 15	90 58 14	3 19 45.0	105 4 13	3 20 4.1
5700	0.0100005	125 28 47	90 42 34	3 19 56.0	104 33 7	3 20 7.8
5600	0.0099405	125 36 14	90 26 53	3 20 6.7	104 2 1	3 20 11.5
5500	0.0098806	125 43 35	90 11 10	3 20 17.2	103 30 57	3 20 15.2
5400	0.0098209	125 50 51	89 55 27	3 20 27.4	102 59 53	3 20 18.9
5300	0.0097613	125 58 2	89 39 43	3 20 37.5	102 28 50	3 20 22.6
5200	0.0097019	126 5 7	89 23 58	3 20 47.2	101 57 48	3 20 26.2
5100	0.0096426	126 12 6	89 8 11	3 20 56.8	101 26 46	3 20 29.9
5000	0.0095834	126 19 0	88 52 24	3 21 6.0	100 55 45	3 20 33.6
4900	0.0095244	126 25 48	88 36 35	3 21 15.0	100 24 45	3 20 37.3
4800	0.0094654	126 32 30	88 20 46	3 21 23.8	99 53 46	3 20 41.0
4700	0.0094066	126 39 6	88 4 55	3 21 32.3	99 22 48	3 20 44.7
4600	0.0093480	126 45 36	87 49 3	3 21 40.6	98 51 50	3 20 48.4
4500	0.0092895	126 52 0	87 33 11	3 21 48.6	98 20 53	3 20 52.1
4400	0.0092311	126 58 17	87 17 17	3 21 56.4	97 49 57	3 20 55.8
4300	0.0091729	127 4 28	87 1 22	3 22 4.0	97 19 1	3 20 59.5
4200	0.0091149	127 10 33	86 45 26	3 22 11.2	96 48 7	3 21 3.2
4100	0.0090571	127 16 32	86 29 29	3 22 18.3	96 17 13	3 21 6.8
4000	0.0089994	127 22 24	86 13 31	3 22 25.1	95 46 19	3 21 10.5
3900	0.0089419	127 28 9	85 57 32	3 22 31.6	95 15 27	3 21 14.2
3800	0.0088845	127 33 48	85 41 32	3 22 37.9	94 44 35	3 21 17.9
3700	0.0088273	127 39 20	85 25 31	3 22 44.0	94 13 44	3 21 21.6
3600	0.0087702	127 44 46	85 9 29	3 22 49.8	93 42 54	3 21 25.3
3500	0.0087133	127 50 5	84 53 26	3 22 55.3	93 12 5	3 21 29.0
3400	0.0086566	127 55 17	84 37 22	3 23 0.6	92 41 16	3 21 32.6
3300	0.0086000	128 0 21	84 21 16	3 23 5.7	92 10 28	3 21 36.3
3200	0.0085436	128 5 20	84 5 10	3 23 10.5	91 39 41	3 21 40.0
3100	0.0084873	128 10 11	83 49 3	3 23 15.1	91 8 54	3 21 43.6
3000	0.0084313	128 14 54	83 32 54	3 23 19.4	90 38 9	3 21 47.3
2900	0.0083755	128 19 29	83 16 45	3 23 23.4	90 7 24	3 21 50.9
2800	0.0083198	128 23 57	83 0 34	3 23 27.3	89 36 39	3 21 54.6
2700	0.0082643	128 28 17	82 44 23	3 23 30.8	89 5 56	3 21 58.2
2600	0.0082090	128 32 30	82 28 10	3 23 34.2	88 35 13	3 22 1.9
2500	0.0081539	128 36 35	82 11 57	3 23 37.3	88 4 31	3 22 5.5
2400	0.0080989	128 40 32	81 55 42	3 23 40.1	87 33 49	3 22 9.1
2300	0.0080442	128 44 20	81 39 27	3 23 42.7	87 3 8	3 22 12.8
2200	0.0079896	128 48 0	81 23 10	3 23 45.0	86 32 28	3 22 16.4
2100	0.0079352	128 51 31	81 6 52	3 23 47.1	86 1 49	3 22 20.0
2000	0.0078810	128 54 54	80 50 33	3 23 49.0	85 31 10	3 22 23.6
1900	0.0078271	128 58 8	80 34 14	3 23 50.6	85 0 32	3 22 27.2
1800	0.0077733	129 1 13	80 17 53	3 23 51.9	84 29 55	3 22 30.8
1700	0.0077197	129 4 9	80 1 31	3 23 53.0	83 59 18	3 22 34.4
1600	0.0076663	129 6 56	79 45 8	3 23 53.9	83 28 42	3 22 38.0
1500	0.0076132	129 9 33	79 28 44	3 23 54.5	82 58 7	3 22 41.6
1400	0.0075602	129 12 1	79 12 19	3 23 54.9	82 27 33	3 22 45.2
1300	0.0075074	129 14 19	78 55 53	3 23 55.0	81 56 59	3 22 48.7
1200	0.0074549	129 16 28	78 39 26	3 23 54.9	81 26 26	3 22 52.3
1100	0.0074027	129 18 26	78 22 58	3 23 54.6	80 55 54	3 22 55.8
1000	0.0073506	129 20 14	78 6 30	3 23 53.9	80 25 22	3 22 59.4
900	0.0072987	129 21 52	77 49 59	3 23 53.1	79 54 51	3 23 2.9
800	0.0072470	129 23 21	77 33 29	3 23 52.0	79 24 21	3 23 6.4
700	0.0071956	129 24 39	77 16 57	3 23 50.6	78 53 52	3 23 10.0
600	0.0071444	129 25 46	77 0 24	3 23 49.1	78 23 23	3 23 13.0
500	0.0070934	129 26 44	76 43 50	3 23 47.2	77 52 54	3 23 17.0
400	0.0070427	129 27 31	76 27 14	3 23 45.2	77 22 27	3 23 20.5
300	0.0069922	129 28 7	76 10 38	3 23 42.8	76 52 0	3 23 23.9
200	0.0069419	129 28 33	75 54 0	3 23 40.3	76 21 34	3 23 27.4
—100	0.0068919	129 28 48	75 37 23	3 23 37.5	75 51 8	3 23 30.8
0	0.0068420	129 28 52	75 20 43	3 23 34.4	75 20 42.9	3 23 34.4
+100	0.0067924	129 28 45	75 4 2	3 23 31.1	74 50 18	3 23 37.8
200	0.0067431	129 28 28	74 47 20	3 23 27.6	74 19 55	3 23 41.2
300	0.0066940	129 28 0	74 30 38	3 23 23.8	73 49 32	3 23 44.6
400	0.0066452	129 27 22	74 13 54	3 23 19.7	73 19 9	3 23 48.0
500	0.0065966	129 26 33	73 57 8	3 23 15.4	72 48 48	3 23 51.4
600	0.0065483	129 25 33	73 40 22	3 23 11.0	72 18 26	3 23 54.8
700	0.0065002	129 24 22	73 23 34	3 23 6.2	71 48 6	3 23 58.1
+800	0.0064523	129 23 0	73 6 45	3 23 1.2	71 17 46	3 24 1.5

TABLE III.—*Elements of the Orbit of Mars.*

t	e'''	ϖ'''	θ'''	ϕ'''	θ_1'''	ϕ_1'''
—6400	0.0869446	304° 22′ 34″	63° 53′ 22″	2° 17′ 9″.7	86° 36′ 50″	1° 54′ 44″.0
6300	0.0870448	304 50 33	63 40 30	2 16 50.0	86 2 43	1 54 39.3
6200	0.0871449	305 18 30	63 27 35	2 16 30.2	85 28 32	1 54 34.6
6100	0.0872449	305 46 26	63 14 37	2 16 10.2	84 54 19	1 54 29.9
6000	0.0873449	306 14 19	63 1 36	2 15 50.1	84 20 2	1 54 25.3
5900	0.0874448	306 42 11	62 48 33	2 15 29.8	83 45 41	1 54 20.6
5800	0.0875447	307 10 1	62 35 27	2 15 9.4	83 11 17	1 54 16.1
5700	0.0876445	307 37 49	62 22 18	2 14 48.8	82 36 50	1 54 11.5
5600	0.0877443	308 5 35	62 9 6	2 14 28.1	82 2 19	1 54 7.0
5500	0.0878440	308 33 20	61 55 52	2 14 7.3	81 27 45	1 54 2.4
5400	0.0879437	309 1 2	61 42 35	2 13 46.3	80 53 7	1 53 58.0
5300	0.0880433	309 28 43	61 29 14	2 13 25.1	80 18 26	1 53 53.5
5200	0.0881429	309 56 23	61 15 51	2 13 3.8	79 43 42	1 53 49.1
5100	0.0882424	310 24 0	61 2 24	2 12 42.4	79 8 55	1 53 44.7
5000	0.0883418	310 51 36	60 48 55	2 12 20.8	78 34 4	1 53 40.3
4900	0.0884412	311 19 10	60 35 24	2 11 59.0	77 59 9	1 53 36.0
4800	0.0885405	311 46 43	60 21 49	2 11 37.2	77 24 12	1 53 31.7
4700	0.0886397	312 14 13	60 8 11	2 11 15.1	76 49 11	1 53 27.4
4600	0.0887388	312 41 42	59 54 30	2 10 52.9	76 14 6	1 53 23.2
4500	0.0888378	313 9 10	59 40 47	2 10 30.6	75 38 59	1 53 19.0
4400	0.0889367	313 36 35	59 27 0	2 10 8.1	75 3 48	1 53 14.9
4300	0.0890354	314 3 59	59 13 10	2 9 45.5	74 28 34	1 53 10.7
4200	0.0891341	314 31 21	58 59 17	2 9 22.7	73 53 17	1 53 6.6
4100	0.0892327	314 58 42	58 45 21	2 8 59.8	73 17 56	1 53 2.6
4000	0.0893311	315 20 0	58 31 22	2 8 36.7	72 42 32	1 52 58.5
3900	0.0894294	315 53 18	58 17 20	2 8 13.4	72 7 5	1 52 54.6
3800	0.0895276	316 20 34	58 3 15	2 7 50.0	71 31 35	1 52 50.6
3700	0.0896257	316 47 48	57 49 6	2 7 26.5	70 56 1	1 52 46.7
3600	0.0897236	317 15 0	57 34 54	2 7 2.7	70 20 25	1 52 43.9
3500	0.0898214	317 42 11	57 20 39	2 6 38.9	69 44 44	1 52 39.1
3400	0.0899191	318 9 20	57 6 21	2 6 14.9	69 9 1	1 52 35.4
3300	0.0900166	318 36 28	56 51 59	2 5 50.7	68 33 14	1 52 31.7
3200	0.0901140	319 3 34	56 37 34	2 5 26.3	67 57 24	1 52 28.1
3100	0.0902113	319 30 39	56 23 6	2 5 1.8	67 21 31	1 52 24.5
3000	0.0903084	319 57 42	56 8 34	2 4 37.2	66 45 35	1 52 21.0
2900	0.0904053	320 24 43	55 53 59	2 4 12.4	66 9 36	1 52 17.5
2800	0.0905021	320 51 43	55 39 21	2 3 47.4	65 33 33	1 52 14.0
2700	0.0905987	321 18 41	55 24 39	2 3 22.3	64 57 27	1 52 10.7
2600	0.0906952	321 45 38	55 9 53	2 2 57.0	64 21 18	1 52 7.3
2500	0.0907915	322 12 33	54 55 4	2 2 31.6	63 45 6	1 52 4.0
2400	0.0908876	322 39 26	54 40 12	2 2 6.0	63 8 51	1 52 0.8
2300	0.0909836	323 6 19	54 25 15	2 1 40.2	62 32 32	1 51 57.6
2200	0.0910793	323 33 10	54 10 15	2 1 14.3	61 56 11	1 51 54.5
2100	0.0911748	323 59 59	53 55 12	2 0 48.2	61 19 46	1 51 51.4
2000	0.0912702	324 26 47	53 40 4	2 0 21.9	60 43 18	1 51 48.4
1900	0.0913654	324 53 33	53 24 53	1 59 55.5	60 6 47	1 51 45.5
1800	0.0914603	325 20 18	53 9 38	1 59 28.9	59 30 13	1 51 42.6
1700	0.0915550	325 47 1	52 54 19	1 59 2.2	58 53 35	1 51 39.8
1600	0.0916496	326 13 44	52 38 56	1 58 35.2	58 16 55	1 51 37.1
1500	0.0917440	326 40 24	52 23 29	1 58 8.2	57 40 11	1 51 34.4
1400	0.0918381	327 7 4	52 7 58	1 57 40.9	57 3 25	1 51 31.8
1300	0.0919320	327 33 41	51 52 24	1 57 13.5	56 26 36	1 51 29.2
1200	0.0920257	328 0 18	51 36 46	1 56 46.0	55 49 44	1 51 26.7
1100	0.0921192	328 26 52	51 21 4	1 56 18.3	55 12 50	1 51 24.3
1000	0.0922125	328 53 26	51 5 18	1 55 50.4	54 35 52	1 51 22.0
900	0.0923056	329 19 58	50 49 27	1 55 22.3	53 58 52	1 51 19.7
800	0.0923984	329 46 29	50 33 33	1 54 54.1	53 21 48	1 51 17.5
700	0.0924910	330 12 59	50 17 34	1 54 25.7	52 44 42	1 51 15.3
600	0.0925834	330 39 27	50 1 31	1 53 57.2	52 7 33	1 51 13.2
500	0.0926655	331 5 54	49 45 23	1 53 28.5	51 30 21	1 51 11.2
400	0.0927674	331 32 19	49 29 11	1 52 59.6	50 53 6	1 51 9.3
300	0.0928590	331 58 43	49 12 55	1 52 30.5	50 15 49	1 51 7.5
200	0.0929504	332 25 6	48 56 34	1 52 1.3	49 38 28	1 51 5.7
—100	0.0930415	332 51 28	48 40 8	1 51 31.9	49 1 4	1 51 4.0
0	0.0931324	333 17 47.8	48 23 36.8	1 51 2.30	48 23 36.8	1 51 2.30
+100	0.0932230	333 44 7	48 7 1	1 50 32.6	47 46 6	1 51 0.7
200	0.0933134	334 10 24	47 50 20	1 50 2.6	47 8 33	1 50 59.2
300	0.0934035	334 36 41	47 33 34	1 49 32.5	46 30 57	1 50 57.8
400	0.0934934	335 2 56	47 16 43	1 49 2.2	45 53 18	1 50 56.4
500	0.0935830	335 29 10	46 59 46	1 48 31.7	45 15 36	1 50 55.1
600	0.0936723	335 55 23	46 42 44	1 43 1.1	44 37 50	1 50 53.9
700	0.0937613	336 21 34	46 25 36	1 47 30.3	44 0 2	1 50 52.8
+800	0.0938502	336 47 44	46 8 22	1 46 59.2	43 22 11	1 50 51.7

TABLE IV.—*Elements of the Orbit of Jupiter.*

t	e^{IV}	ϖ^{IV}	θ^{IV}	ϕ^{IV}	θ_1^{IV}	θ_1^{IV}
—6400	0.0393563	3° 10′ 38″	92° 20′ 14″	1° 32′ 14″.9	124° 29′ 0″	1° 41′ 19″.4
6300	0.0394984	3 15 47	92 21 55	1 31 59.0	124 5 5	1 40 58.0
6200	0.0396406	3 21 2	92 23 45	1 31 43.1	123 41 9	1 40 36.7
6100	0.0397830	3 26 25	92 25 42	1 31 27.2	123 17 13	1 40 15.4
6000	0.0399255	3 31 55	92 27 48	1 31 11.4	122 53 17	1 39 54.1
5900	0.0400681	3 37 32	92 30 2	1 30 55.6	122 29 22	1 39 32.7
5800	0.0402109	3 43 16	92 32 24	1 30 39.9	122 5 26	1 39 11.4
5700	0.0403538	3 49 6	92 34 54	1 30 24.3	121 41 31	1 38 50.0
5600	0.0404967	3 55 3	92 37 34	1 30 8.7	121 17 36	1 38 28.7
5500	0.0406397	4 1 7	92 40 21	1 29 53.1	120 53 40	1 38 7.3
5400	0.0407827	4 7 18	92 43 17	1 29 37.6	120 29 45	1 37 46.0
5300	0.0409258	4 13 35	92 46 22	1 29 22.2	120 5 50	1 37 24.6
5200	0.0410689	4 19 58	92 49 36	1 29 6.9	119 41 55	1 37 3.3
5100	0.0412120	4 26 29	92 52 57	1 28 51.6	119 18 0	1 36 41.9
5000	0.0413550	4 33 5	92 56 28	1 28 36.4	118 54 5	1 36 20.6
4900	0.0414980	4 39 48	93 0 7	1 28 21.3	118 30 10	1 35 59.2
4800	0.0416410	4 46 36	93 3 55	1 28 6.3	118 6 15	1 35 37.8
4700	0.0417839	4 53 31	93 7 52	1 27 51.3	117 42 20	1 35 16.5
4600	0.0419267	5 0 32	93 11 57	1 27 36.5	117 18 24	1 34 55.1
4500	0.0420694	5 7 39	93 16 12	1 27 21.7	116 54 29	1 34 33.8
4400	0.0422121	5 14 52	93 20 34	1 27 7.0	116 30 34	1 34 12.4
4300	0.0423546	5 22 10	93 25 6	1 26 52.4	116 6 39	1 33 51.0
4200	0.0424969	5 29 34	93 29 47	1 26 37.9	115 42 43	1 33 29.7
4100	0.0426391	5 37 4	93 34 36	1 26 23.5	115 18 48	1 33 8.3
4000	0.0427812	5 44 40	93 39 34	1 26 9.3	114 54 52	1 32 47.0
3900	0.0429231	5 52 21	93 44 41	1 25 55.1	114 30 56	1 32 25.6
3800	0.0430648	6 0 8	93 49 57	1 25 41.0	114 7 0	1 32 4.2
3700	0.0432064	6 8 0	93 55 22	1 25 27.1	113 43 5	1 31 42.9
3600	0.0433478	6 15 58	94 0 55	1 25 13.2	113 19 9	1 31 21.6
3500	0.0434890	6 24 1	94 6 38	1 24 59.5	112 55 12	1 31 0.2
3400	0.0436300	6 32 10	94 12 29	1 24 46.0	112 31 16	1 30 38.9
3300	0.0437708	6 40 23	94 18 30	1 24 32.5	112 7 20	1 30 17.6
3200	0.0439113	6 48 42	94 24 39	1 24 19.2	111 43 23	1 29 56.3
3100	0.0440516	6 57 6	94 30 57	1 24 6.0	111 19 26	1 29 35.0
3000	0.0441917	7 5 35	94 37 23	1 23 53.0	110 55 29	1 29 13.7
2900	0.0443315	7 14 9	94 43 59	1 23 40.0	110 31 32	1 28 52.4
2800	0.0444711	7 22 48	94 50 43	1 23 27.3	110 7 34	1 28 31.1
2700	0.0446104	7 31 31	94 57 36	1 23 14.6	109 43 37	1 28 9.9
2600	0.0447494	7 40 20	95 4 37	1 23 2.2	109 19 39	1 27 48.6
2500	0.0448882	7 49 13	95 11 47	1 22 49.9	108 55 41	1 27 27.3
2400	0.0450265	7 58 11	95 19 6	1 22 37.7	108 31 42	1 27 6.1
2300	0.0451645	8 7 14	95 26 34	1 22 25.7	108 7 43	1 26 44.8
2200	0.0453023	8 16 22	95 34 9	1 22 13.8	107 43 44	1 26 23.6
2100	0.0454398	8 25 34	95 41 54	1 22 2.2	107 19 45	1 26 2.4
2000	0.0455769	8 34 50	95 49 46	1 21 50.7	106 55 46	1 25 41.2
1900	0.0457137	8 44 11	95 57 47	1 21 39.3	106 31 46	1 25 20.0
1800	0.0458501	8 53 37	96 5 56	1 21 28.2	106 7 46	1 24 58.8
1700	0.0459862	9 3 6	96 14 13	1 21 17.2	105 43 45	1 24 37.7
1600	0.0461220	9 12 41	96 22 38	1 21 6.4	105 19 44	1 24 16.5
1500	0.0462574	9 22 19	96 31 12	1 20 55.7	104 55 43	1 23 55.4
1400	0.0463924	9 32 2	96 39 53	1 20 45.3	104 31 41	1 23 34.3
1300	0.0465271	9 41 48	96 48 42	1 20 35.1	104 7 38	1 23 13.2
1200	0.0466613	9 51 39	96 57 38	1 20 25.0	103 43 35	1 22 52.1
1100	0.0467952	10 1 34	97 6 43	1 20 15.2	103 19 32	1 22 31.0
1000	0.0469286	10 11 32	97 15 54	1 20 5.5	102 55 28	1 22 10.0
900	0.0470616	10 21 35	97 25 14	1 19 56.1	102 32 23	1 21 48.9
800	0.0471942	10 31 42	97 34 40	1 19 46.8	102 7 18	1 21 27.9
700	0.0473264	10 41 53	97 44 14	1 19 37.8	101 43 13	1 21 6.9
600	0.0474581	10 52 7	97 53 54	1 19 28.9	101 19 7	1 20 45.9
500	0.0475894	11 2 25	98 3 42	1 19 20.3	100 55 1	1 20 24.9
400	0.0477202	11 12 48	98 13 36	1 19 11.8	100 30 54	1 20 3.9
300	0.0478505	11 23 14	98 23 38	1 19 3.6	100 6 46	1 19 43.0
200	0.0479804	11 33 43	98 33 46	1 18 55.6	99 42 38	1 19 22.1
—100	0.0481098	11 43 16	98 44 0	1 18 47.8	99 18 30	1 19 1.2
0	0.0482388	11 54 53.1	98 54 20.5	1 18 40.30	98 54 20.5	1 18 40.3
+100	0.0483672	12 5 34	99 4 47	1 18 33.0	98 30 10	1 18 19.4
200	0.0484952	12 16 17	99 15 20	1 18 25.9	98 6 0	1 17 58.6
300	0.0486227	12 27 4	99 25 58	1 18 19.0	97 41 48	1 17 37.8
400	0.0487496	12 37 55	99 36 42	1 18 12.4	97 17 36	1 17 17.0
500	0.0488760	12 48 49	99 47 31	1 18 6.0	96 53 23	1 16 56.2
600	0.0490020	12 59 46	99 58 25	1 17 59.8	96 29 9	1 16 35.4
700	0.0491274	13 10 47	100 9 25	1 17 53.9	96 4 55	1 16 14.7
+800	0.0492523	13 21 51	100 20 29	1 17 48.2	95 40 39	1 15 54.0

TABLE V.—*Elements of the Orbit of Saturn.*

t	e^V	ϖ^V	θ^V	ϕ^V	θ_1^V	ϕ_1^V
—6400	0.0708682	62° 8′ 48″	127° 38′ 42″	2° 8′ 28″.6	144° 49′ 21″	2° 41′ 31″.7
6300	0.0706762	62 34 9	127 25 42	2 8 57.1	144 19 58	2 41 23.4
6200	0.0704829	62 59 31	127 12 37	2 9 25.4	143 50 34	2 41 15.0
6100	0.0702882	63 24 55	126 59 29	2 9 53.4	143 21 8	2 41 6.4
6000	0.0700921	63 50 19	126 46 17	2 10 21.1	142 51 41	2 40 57.8
5900	0.0698947	64 15 45	126 33 2	2 10 48.6	142 22 11	2 40 49.1
5800	0.0696958	64 41 12	126 19 43	2 11 15.8	141 52 40	2 40 40.2
5700	0.0694955	65 6 40	126 6 21	2 11 42.7	141 23 7	2 40 31.3
5600	0.0692939	65 32 9	125 52 55	2 12 9.4	140 53 31	2 40 22.3
5500	0.0690909	65 57 39	125 39 27	2 12 35.8	140 23 54	2 40 13.1
5400	0.0688866	66 23 11	125 25 55	2 13 1.9	139 54 14	2 40 3.9
5300	0.0686810	66 48 44	125 12 20	2 13 27.8	139 24 33	2 39 54.5
5200	0.0684740	67 14 19	124 58 41	2 13 53.4	138 54 50	2 39 45.1
5100	0.0682657	67 39 55	124 45 0	2 14 18·7	138 25 5	2 39 35.5
5000	0.0680560	68 5 32	124 31 16	2 14 43.8	137 55 18	2 39 25.8
4900	0.0678450	68 31 11	124 17 28	2 15 8.6	137 25 29	2 39 16.1
4800	0.0676327	68 56 51	124 3 38	2 15 33.0	136 55 37	2 39 6.2
4700	0.0674191	69 22 32	123 49 45	2 15 57.3	136 25 44	2 38 56.2
4600	0.0672041	69 48 15	123 35 49	2 16 21.2	135 55 49	2 38 46.2
4500	0.0669878	70 14 0	123 21 51	2 16 44.9	135 25 51	2 38 36.0
4400	0.0667702	70 39 46	123 7 49	2 17 8.3	134 55 52	2 38 25.7
4300	0.0665513	71 5 34	122 53 46	2 17 31.4	134 25 50	2 38 15.4
4200	0.0663310	71 31 23	122 39 40	2 17 54.2	133 55 47	2 38 4.9
4100	0.0661095	71 57 14	122 25 31	2 18 16.8	133 25 41	2 37 54.3
4000	0.0658868	72 23 6	122 11 20	2 18 39.0	132 55 33	2 37 43.6
3900	0.0656628	72 49 0	121 57 7	2 19 1.0	132 25 24	2 37 32.8
3800	0.0654375	73 14 56	121 42 51	2 19 22.6	131 55 12	2 37 22.0
3700	0.0652110	73 40 53	121 28 33	2 19 44.0	131 24 58	2 37 11.0
3600	0.0649832	74 6 52	121 14 13	2 20 5.1	130 54 42	2 37 0.0
3500	0.0647542	74 32 53	120 59 50	2 20 25.9	130 24 23	2 36 48.8
3400	0.0645239	74 58 55	120 45 26	2 20 46.4	129 54 3	2 36 37.6
3300	0.0642924	75 25 0	120 31 0	2 21 6.6	129 23 40	2 36 26.2
3200	0.0640597	75 51 6	120 16 31	2 21 26.5	128 53 16	2 36 14.8
3100	0.0638257	76 17 14	120 2 1	2 21 46.1	128 22 48	2 36 3.2
3000	0.0635905	76 43 24	119 47 29	2 22 5.4	127 52 19	2 35 51.6
2900	0.0633541	77 9 36	119 32 54	2 22 24.4	127 21 48	2 35 39.9
2800	0.0631165	77 35 50	119 18 18	2 22 43.2	126 51 14	2 35 28.1
2700	0.0628776	78 2 6	119 3 39	2 23 1.6	126 20 37	2 35 16.2
2600	0.0626375	78 28 24	118 48 59	2 23 19.7	125 49 59	2 35 4.2
2500	0.0623962	78 54 44	118 34 17	2 23 37.5	125 19 18	2 34 52.2
2400	0.0621537	79 21 6	118 19 33	2 23 55.1	124 48 35	2 34 40.0
2300	0.0619100	79 47 31	118 4 48	2 24 12.3	124 17 49	2 34 27.7
2200	0.0616652	80 13 57	117 50 2	2 24 29.2	123 47 2	2 34 15.4
2100	0.0614192	80 40 26	117 35 14	2 24 45.8	123 16 12	2 34 3.0
2000	0.0611720	81 6 57	117 20 24	2 25 2.1	122 45 19	2 33 50.4
1900	0.0609237	81 33 30	117 5 33	2 25 18.1	122 14 24	2 33 37.8
1800	0.0606743	82 0 5	116 50 41	2 25 33.8	121 43 27	2 33 25.1
1700	0.0604237	82 26 43	116 35 47	2 25 49.1	121 12 28	2 33 12.3
1600	0.0601720	82 53 24	116 20 52	2 26 4.2	120 41 26	2 32 59.5
1500	0.0599192	83 20 7	116 5 55	2 26 18.9	120 10 22	2 32 46.5
1400	0·0596652	83 46 52	115 50 57	2 26 33.4	119 39 16	2 32 33.5
1300	0.0594101	84 13 40	115 35 57	2 26 47.5	119 8 7	2 32 20.3
1200	0.0591540	84 40 30	115 20 56	2 27 1.3	118 36 56	2 32 7.1
1100	0.0588968	85 7 23	115 5 54	2 27 14.8	118 5 42	2 31 53.8
1000	0.0586384	85 34 18	114 50 51	2 27 28.0	117 34 26	2 31 40.5
900	0.0583789	86 1 16	114 35 46	2 27 40.8	117 3 7	2 31 27.0
800	0.0581183	86 28 17	114 20 40	2 27 53.4	116 31 45	2 31 13.5
700	0.0578567	86 55 21	114 5 34	2 28 5.6	116 0 21	2 30 59.8
600	0.0575940	87 22 28	113 50 27	2 28 17.5	115 28 55	2 30 46.2
500	0.0573302	87 49 38	113 35 18	2 28 29.2	114 57 26	2 30 32.4
400	0.0570653	88 16 50	113 20 8	2 28 40.4	114 25 54	2 30 18.5
300	0.0567994	88 44 6	113 4 57	2 28 51.4	113 54 19	2 30 4.6
200	0.0565325	89 11 25	112 49 46	2 29 2.0	113 22 42	2 29 50.6
—100	0.0562646	89 38 46	112 34 34	2 29 12.4	112 51 3	2 29 36.5
0	0.0559956	90 6 12.0	112 19 20.6	2 29 22.4	112 19 20.6	2 29 22.4
+100	0.0557257	90 33 40	112 4 7	2 29 32.1	111 47 36	2 29 8.2
200	0.0554547	91 1 12	111 48 52	2 29 41.4	111 15 48	2 28 53.9
300	0.0551827	91 28 47	111 33 36	2 29 50.5	110 43 58	2 28 39.5
400	0.0549097	91 56 25	111 18 19	2 29 59.2	110 12 5	2 28 25.0
500	0.0546358	92 24 7	111 3 2	2 30 7.6	109 40 9	2 28 10.5
600	0.0543609	92 51 52	110 47 44	2 30 15.7	109 8 10	2 27 55.9
700	0.0540850	93 19 41	110 32 25	2 30 23.4	108 36 8	2 27 41.2
+800	0.0538082	93 47 34	110 17 6	2 30 30.9	108 4 4	2 27 26.5

TABLE VI.—*Elements of the Orbit of Uranus.*

t	e^{VI}	ϖ^{VI}	θ^{VI}	ϕ^{VI}	θ_1^{VI}	ϕ_1^{VI}
—6400	0.0480857	165° 1′ 47″	70° 3′ 6″	0° 53′ 14″.98	130° 34′ 2″	0° 54′ 30″.40
6300	0.0480544	165 7 18	70 4 45	0 53 8.46	129 46 15	0 54 13.82
6200	0.0480232	165 12 48	70 6 27	0 53 1.94	128 58 12	0 53 57.49
6100	0.0479920	165 18 18	70 8 10	0 52 55.41	128 9 52	0 53 41.42
6000	0.0479608	165 23 47	70 9 56	0 52 48.89	127 21 17	0 53 25.58
5900	0.0479297	165 29 15	70 11 43	0 52 42.36	126 32 26	0 53 10.00
5800	0.0478987	165 34 43	70 13 33	0 52 35.84	125 43 19	0 52 54.66
5700	0.0478677	165 40 10	70 15 25	0 52 29.31	124 53 57	0 52 39.58
5600	0.0478368	165 45 36	70 17 20	0 52 22.79	124 4 19	0 52 24.75
5500	0.0478059	165 51 2	70 19 16	0 52 16.27	123 14 26	0 52 10.18
5400	0.0477751	165 56 27	70 21 15	0 52 9.75	122 24 18	0 51 55.87
5300	0.0477443	166 1 52	70 23 17	0 52 3.23	121 33 54	0 51 41.83
5200	0.0477136	166 7 16	70 25 20	0 51 56.71	120 43 15	0 51 28.05
5100	0.0476829	166 12 39	70 27 26	0 51 50.19	119 52 21	0 51 14.54
5000	0.0476523	166 18 2	70 29 35	0 51 43.68	119 1 12	0 51 1.29
4900	0.0476217	166 23 24	70 31 46	0 51 37.18	118 9 49	0 50 48.30
4800	0.0475912	166 28 46	70 34 0	0 51 30.68	117 18 11	0 50 35.59
4700	0.0475607	166 34 7	70 36 16	0 51 24.19	116 26 19	0 50 23.16
4600	0.0475303	166 39 27	70 38 34	0 51 17.70	115 34 13	0 50 11.00
4500	0.0474999	166 44 46	70 40 56	0 51 11.21	114 41 54	0 49 59.14
4400	0.0474696	166 50 5	70 43 19	0 51 4.73	113 49 21	0 49 47.56
4300	0.0474394	166 55 24	70 45 46	0 50 58.24	112 56 35	0 49 36.28
4200	0.0474092	167 0 42	70 48 15	0 50 51.75	112 3 36	0 49 25.29
4100	0.0473791	167 5 59	70 50 46	0 50 45.27	111 10 24	0 49 14.60
4000	0.0473490	167 11 15	70 53 20	0 50 38.80	110 17 0	0 49 4.21
3900	0.0473190	167 16 31	70 55 57	0 50 32.34	109 23 24	0 48 54.12
3800	0.0472891	167 21 47	70 58 36	0 50 25.88	108 29 37	0 48 44.34
3700	0.0472592	167 27 1	71 1 18	0 50 14.43	107 35 38	0 48 34.87
3600	0.0472294	167 32 15	71 4 3	0 50 13.00	106 41 28	0 48 25.71
3500	0.0471997	167 37 29	71 6 51	0 50 6.59	105 47 7	0 48 16.86
3400	0.0471700	167 42 42	71 9 41	0 50 0.18	104 52 36	0 48 8.33
3300	0.0471405	167 47 54	71 12 34	0 49 53.78	103 57 56	0 48 0.11
3200	0.0471110	167 53 5	71 15 30	0 49 47.39	103 3 5	0 47 52.20
3100	0.0470816	167 58 16	71 18 29	0 49 41.01	102 8 6	0 47 44.61
3000	0.0470523	168 3 27	71 21 30	0 49 34.63	101 12 58	0 47 37.33
2900	0.0470231	168 8 37	71 24 34	0 49 28.26	100 17 42	0 47 30.38
2800	0.0469939	168 13 46	71 27 41	0 49 21.90	99 22 19	0 47 23.74
2700	0.0469648	168 18 55	71 30 51	0 49 15.56	98 26 48	0 47 17.43
2600	0.0469358	168 24 3	71 34 3	0 49 9.23	97 31 10	0 47 11.44
2500	0.0469069	168 29 10	71 37 18	0 49 2.91	96 35 27	0 47 5.77
2400	0.0468780	168 34 17	71 40 36	0 48 56.60	95 39 37	0 47 0.43
2300	0.0468493	168 39 23	71 43 57	0 48 50.31	94 43 42	0 46 55.42
2200	0.0468206	168 44 29	71 47 21	0 48 44.03	93 47 43	0 46 50.72
2100	0.0467921	168 49 34	71 50 47	0 48 37.77	92 51 39	0 46 46.35
2000	0.0467636	168 54 39	71 54 16	0 48 31.52	91 55 32	0 46 42.32
1900	0.0467352	168 59 43	71 57 48	0 48 25.28	90 59 22	0 46 38.62
1800	0.0467069	169 4 47	72 1 24	0 48 19.05	90 3 9	0 46 35.24
1700	0.0466787	169 9 50	72 5 1	0 48 12.83	89 6 53	0 46 32.19
1600	0.0466506	169 14 52	72 8 42	0 48 6.63	88 10 36	0 46 29.47
1500	0.0466227	169 19 54	72 12 26	0 48 0.45	87 14 18	0 46 27.08
1400	0.0465948	169 24 55	72 16 12	0 47 54.28	86 18 0	0 46 25.01
1300	0.0465670	169 29 56	72 20 2	0 47 48.13	85 21 41	0 46 23.27
1200	0.0465393	169 34 56	72 23 54	0 47 41.99	84 25 25	0 46 21.84
1100	0.0465117	169 39 56	72 27 50	0 47 35.88	83 29 10	0 46 20.75
1000	0.0464842	169 44 55	72 31 48	0 47 29.79	82 32 56	0 46 19.99
900	0.0464568	169 49 54	72 35 49	0 47 23.72	81 36 45	0 46 19.55
800	0.0464295	169 54 52	72 39 54	0 47 17.66	80 40 36	0 46 19.44
700	0.0464023	169 59 49	72 44 1	0 47 11.63	79 44 29	0 46 19.65
600	0.0463752	170 4 46	72 48 11	0 47 5.61	78 48 28	0 46 20.17
500	0.0463482	170 9 43	72 52 24	0 46 59.61	77 52 32	0 46 21.01
400	0.0463213	170 14 39	72 56 40	0 46 53.63	76 56 40	0 46 22.17
300	0.0462945	170 19 34	73 0 59	0 46 47.67	76 0 55	0 46 23.64
200	0.0462679	170 24 29	73 5 21	0 46 41.74	75 5 14	0 46 25.43
—100	0.0462413	170 29 24	73 9 46	0 46 35.83	74 9 41	0 46 27.53
0	0.0462149	170 34 17.7	73 14 13.4	0 46 29.93	73 14 13.4	0 46 29.93
+100	0.0461886	170 39 11	73 18 44	0 46 24.06	72 18 54	0 46 32.64
200	0.0461624	170 44 4	73 23 18	0 46 18.21	71 23 42	0 46 35.65
300	0.0461363	170 48 56	73 27 54	0 46 12.38	70 28 39	0 46 38.96
400	0.0461103	170 53 48	73 32 33	0 46 6.57	69 33 46	0 46 42.57
500	0.0460844	170 58 40	73 37 15	0 46 0.79	68 39 1	0 46 46.47
600	0.0460587	171 3 31	73 42 0	0 45 55.03	67 44 26	0 46 50.67
700	0.0460330	171 8 21	73 46 48	0 45 49.30	66 50 0	0 46 55.16
+800	0.0460074	171 13 11	73 51 39	0 45 43.58	65 55 44	0 46 59.93

TABLE VII.—*Elements of the Orbit of Neptune.*

t	e^{VII}	ϖ^{VII}	θ^{VII}	ϕ^{VII}	θ_1^{VII}	ϕ_1^{VII}
—6400	0.0088883	49° 26′ 1″	130° 29′ 8″	1° 46′ 14″.21	149° 31′ 7″	2° 22′ 12″.02
6300	0.0088924	49 26 35	130 28 47	1 46 14.90	149 12 59	2 21 41.70
6200	0.0088965	49 27 10	130 28 27	1 46 15.59	148 54 52	2 21 11.29
6100	0.0089006	49 27 45	130 28 6	1 46 16.28	148 36 45	2 20 40.78
6000	0.0089047	49 28 21	130 27 45	1 46 16.98	148 18 38	2 20 10.17
5900	0.0089088	49 28 57	130 27 25	1 46 17.67	148 0 30	2 19 39.47
5800	0.0089130	49 29 33	130 27 4	1 46 18.36	147 42 23	2 19 8.66
5700	0.0089172	49 30 10	130 26 44	1 46 19.05	147 24 16	2 18 37.76
5600	0.0089214	49 30 48	130 26 24	1 46 19.74	147 6 8	2 18 6.77
5500	0.0089256	49 31 25	130 26 3	1 46 20.43	146 48 1	2 17 35.69
5400	0.0089299	49 32 4	130 25 43	1 46 21.12	146 29 54	2 17 4.50
5300	0.0089342	49 32 42	130 25 22	1 46 21.81	146 11 46	2 16 33.22
5200	0.0089385	49 33 21	130 25 2	1 46 22.51	145 53 39	2 16 1.85
5100	0.0089428	49 34 1	130 24 42	1 46 23.21	145 35 31	2 15 30.39
5000	0.0089472	49 34 41	130 24 21	1 46 23.91	145 17 24	2 14 58.84
4900	0.0089515	49 35 21	130 24 1	1 46 24.61	144 59 16	2 14 27.20
4800	0.0089559	49 36 2	130 23 41	1 46 25.31	144 41 9	2 13 55.47
4700	0.0089603	49 36 44	130 23 20	1 46 26.01	144 23 1	2 13 23.65
4600	0.0089647	49 37 25	130 23 0	1 46 26.72	144 4 53	2 12 51.75
4500	0.0089691	49 38 8	130 22 40	1 46 27.43	143 46 46	2 12 19.76
4400	0.0089736	49 38 50	130 22 20	1 46 28.14	143 28 38	2 11 47.69
4300	0.0089780	49 39 33	130 22 0	1 46 28.85	143 10 30	2 11 15.53
4200	0.0089824	49 40 17	130 21 39	1 46 29.57	142 52 22	2 10 43.30
4100	0.0089868	49 41 1	130 21 19	1 46 30.28	142 34 14	2 10 10.99
4000	0.0089913	49 41 45	130 20 59	1 46 31.00	142 16 6	2 9 38.60
3900	0.0089957	49 42 30	130 20 39	1 46 31.72	141 57 58	2 9 6.12
3800	0.0090002	49 43 15	130 20 19	1 46 32.44	141 39 49	2 8 33.56
3700	0.0090046	49 44 1	130 19 59	1 46 33.16	141 21 40	2 8 0.92
3600	0.0090091	49 44 47	130 19 39	1 46 33.89	141 3 32	2 7 28.20
3500	0.0090135	49 45 33	130 19 19	1 46 34.62	140 45 23	2 6 55.40
3400	0.0090180	49 46 20	130 18 59	1 46 35.35	140 27 14	2 6 22.51
3300	0.0090225	49 47 7	130 18 39	1 46 36.08	140 9 5	2 5 49.54
3200	0.0090270	49 47 54	130 18 19	1 46 36.81	139 50 56	2 5 16.49
3100	0.0090315	49 48 42	130 17 59	1 46 37.54	139 32 46	2 4 43.37
3000	0.0090361	49 49 30	130 17 39	1 46 38.28	139 14 36	2 4 10.17
2900	0.0090406	49 50 18	130 17 19	1 46 39.01	138 56 27	2 3 36.89
2800	0.0090452	49 51 6	130 17 0	1 46 39.75	138 38 17	2 3 3.53
2700	0.0090497	49 51 56	130 16 40	1 46 40.49	138 20 6	2 2 30.10
2600	0.0090542	49 52 45	130 16 20	1 46 41.23	138 1 56	2 1 56.59
2500	0.0090587	49 53 35	130 16 0	1 46 41.97	137 43 45	2 1 23.01
2400	0.0090633	49 54 26	130 15 40	1 46 42.71	137 25 34	2 0 49.36
2300	0.0090678	49 55 16	130 15 21	1 46 43.45	137 7 23	2 0 15.63
2200	0.0090724	49 56 7	130 15 1	1 46 44.20	136 49 12	1 59 41.83
2100	0.0090769	49 56 59	130 14 41	1 46 44.95	136 31 1	1 59 7.96
2000	0.0090815	49 57 51	130 14 21	1 46 45.70	136 12 49	1 58 34.01
1900	0.0090861	49 58 43	130 14 1	1 46 46.45	135 54 37	1 57 59.99
1800	0.0090907	49 59 36	130 13 42	1 46 47.20	135 36 25	1 57 25.90
1700	0.0090953	50 0 30	130 13 22	1 46 47.95	135 18 13	1 56 51.74
1600	0.0090999	50 1 23	130 13 2	1 46 48.71	135 0 0	1 56 17.51
1500	0.0091045	50 2 17	130 12 42	1 46 49.47	134 41 47	1 55 43.20
1400	0.0091091	50 3 12	130 12 22	1 46 50.23	134 23 33	1 55 8.84
1300	0.0091137	50 4 7	130 12 3	1 46 50.99	134 5 20	1 54 34.40
1200	0.0091184	50 5 2	130 11 43	1 46 51.75	133 47 5	1 53 59.91
1100	0.0091230	50 5 58	130 11 23	1 46 52.51	133 28 51	1 53 25.35
1000	0.0091276	50 6 54	130 11 3	1 46 53.28	133 10 36	1 52 50.72
900	0.0091322	50 7 50	130 10 43	1 46 54.04	132 52 20	1 52 16.04
800	0.0091369	50 8 47	130 10 24	1 46 54.81	132 34 5	1 51 41.29
700	0.0091415	50 9 45	130 10 4	1 46 55.58	132 15 48	1 51 6.48
600	0.0091461	50 10 43	130 9 44	1 46 56.36	131 57 32	1 50 31.60
500	0.0091507	50 11 41	130 9 24	1 46 57.13	131 39 15	1 49 56.66
400	0.0091554	50 12 40	130 9 4	1 46 57.91	131 20 58	1 49 21.66
300	0.0091600	50 13 39	130 8 45	1 46 58.69	131 2 41	1 48 46.60
200	0.0091646	50 14 38	130 8 25	1 46 59.47	130 44 23	1 48 11.47
—100	0.0091692	50 15 38	130 8 5	1 47 0.25	130 26 4	1 47 36.29
0	0.0091739	50 16 38.6	130 7 45.3	1 47 1.04	130 7 45.3	1 47 1.04
+100	0.0091785	50 17 39	130 7 25	1 47 1.82	129 49 26	1 46 25.73
200	0.0091832	50 18 40	130 7 6	1 47 2.61	129 31 6	1 45 50.37
300	0.0091879	50 19 42	130 6 46	1 47 3.40	129 12 46	1 45 14.95
400	0.0091926	50 20 44	130 6 26	1 47 4.19	128 54 25	1 44 39.47
500	0.0091972	50 21 46	130 6 6	1 47 4.98	128 36 3	1 44 3.94
600	0.0092019	50 22 49	130 5 46	1 47 5.77	128 17 41	1 43 28.35
700	0.0092066	50 23 52	130 5 27	1 47 6.56	127 59 18	1 42 52.71
+800	0.0092113	50 24 55	130 5 7	1 47 7.36	127 40 55	1 42 17.01

TABLE VIII.—*Elements of the Orbit of the Earth.*

t	e''	ϖ''	θ''	ϕ''
—8000	0.0192055	75° 23′ 45″.8	13° 12′ 44.3	1° 8′ 9.114
7900	0.0191870	75 41 40.1	12 57 32.4	1 7 17.243
7800	0.0191682	75 59 35.6	12 42 20.6	1 6 25.364
7700	0.0191489	76 17 32.4	12 27 9.0	1 5 33.480
7600	0.0191294	76 35 30.6	12 11 57.5	1 4 41.591
7500	0.0191096	76 53 30.1	11 56 46.2	1 3 49.698
7400	0.0190894	77 11 30.9	11 41 35.1	1 2 57.802
7300	0.0190689	77 29 32.9	11 26 24.2	1 2 5.904
7200	0.0190482	77 47 36.2	11 11 13.4	1 1 14.004
7100	0.0190272	78 5 40.7	10 56 2.8	1 0 22.105
7000	0.0190058	78 23 46.4	10 40 52.3	0 59 30.207
6900	0.0189841	78 41 53.3	10 25 42.1	0 58 38.312
6800	0.0189620	79 0 1.4	10 10 32.0	0 57 46.420
6700	0.0189396	79 18 10.7	9 55 22.0	0 56 54.533
6600	0.0189170	79 36 21.1	9 40 12.3	0 56 2.650
6500	0.0188941	79 54 32.7	9 25 2.7	0 55 10.773
6400	0.0188708	80 12 45.5	9 9 53.3	0 54 18.904
6300	0.0188473	80 30 58.6	8 54 44.0	0 53 27.044
6200	0.0188234	80 49 13.1	8 39 34.9	0 52 35.194
6100	0.0187992	81 7 28.8	8 24 25.9	0 51 43.354
6000	0.0187747	81 25 45.7	8 9 17.2	0 50 51.524
5900	0.0187499	81 44 3.9	7 54 8.6	0 49 59.707
5800	0.0187248	82 2 23.3	7 39 0.1	0 49 7.902
5700	0.0186994	82 20 44.0	7 23 1.9	0 48 16.120
5600	0.0186737	82 39 5.9	7 8 43.8	0 47 24.347
5500	0.0186477	82 57 29.1	6 53 35.8	0 46 32.590
5400	0.0186214	83 15 53.7	6 38 28.0	0 45 40.852
5300	0.0185948	83 34 19.6	6 23 20.4	0 44 49.132
5200	0.0185678	83 52 46.7	6 8 13.0	0 43 57.431
5100	0.0185405	84 11 15.0	5 53 5.7	0 43 5.750
5000	0.0185130	84 29 44.6	5 37 58.6	0 42 14.090
4900	0.0184852	84 48 15.4	5 22 51.7	0 41 22.453
4800	0.0184570	85 6 47.5	5 7 44.9	0 40 30.840
4700	0.0184285	85 25 20.8	4 52 38.3	0 39 39.253
4600	0.0183998	85 43 55.2	4 37 31.8	0 38 47.692
4500	0.0183708	86 2 30.8	4 22 25.5	0 37 56.158
4400	0.0183415	86 21 7.6	4 7 19.4	0 37 4.652
4300	0.0183119	86 39 45.6	3 52 13.4	0 36 13.175
4200	0.0182820	86 58 24.8	3 37 7.6	0 35 21.729
4100	0.0182518	87 17 5.1	3 22 1.9	0 34 30.315
4000	0.0182213	87 35 46.7	3 6 56.4	0 33 38.931
3900	0.0181905	87 54 29.6	2 51 51.1	0 32 47.577
3800	0.0181595	88 13 13.7	2 36 46.0	0 31 56.254
3700	0.0181282	88 31 59.0	2 21 41.0	0 31 4.962
3600	0.0180965	88 50 45.6	2 6 36.2	0 30 13.706
3500	0.0180645	89 9 33.4	1 51 31.6	0 29 22.487
3400	0.0180323	89 28 22.4	1 36 27.1	0 28 31.301
3300	0.0179998	89 47 12.6	1 21 22.8	0 27 40.153
3200	0.0179670	90 6 4.0	1 6 18.6	0 26 49.047
3100	0.0179339	90 24 56.8	0 51 14.6	0 25 57.981
—3000	0.0179005	90 43 51.0	0 36 10.8	0 25 6.958

TABLE VIII.—*Elements of the Orbit of the Earth*—continued.

t	e''	ϖ''	θ''	ϕ''
—3000	0.0179005	90° 43′ 51″.0	0° 36′ 10″.8	0° 25′ 6″.958
2900	0.0178668	91 2 46.5	0 21 7.1	0 24 15.977
2800	0.0178329	91 21 43.4	0 6 3.6	0 23 25.040
2700	0.0177987	91 40 41.6	359 51 0.2	0 22 34.149
2600	0.0177642	91 59 41.1	359 35 57.0	0 21 43.304
2500	0.0177294	92 18 42.0	359 20 53.9	0 20 52.504
2400	0.0176943	92 37 44.2	359 5 51.0	0 20 1.748
2300	0.0176589	92 56 48.0	358 50 48.3	0 19 11.038
2200	0.0176233	93 15 53.3	358 35 45.7	0 18 20.378
2100	0.0175875	93 35 0.0	358 20 43.2	0 17 29.769
2000	0.0175513	93 54 8.3	358 5 41.0	0 16 39.210
1900	0.0175148	94 13 18.0	357 50 38.9	0 15 48.703
1800	0.0174781	94 32 29.3	357 35 36.9	0 14 58.248
1700	0.0174411	94 51 42.0	357 20 35.1	0 14 7.844
1600	0.0174038	95 10 56.3	357 5 33.5	0 13 17.494
1500	0.0173662	95 30 11.6	356 50 32.0	0 12 27.200
1400	0.0173284	95 49 28.1	356 35 30.7	0 11 36.961
1300	0.0172903	96 8 45.6	356 20 29.6	0 10 46.779
1200	0.0172520	96 28 4.1	356 5 28.6	0 9 56.657
1100	0.0172134	96 47 23.9	355 50 27.8	0 9 6.592
1000	0.0171745	97 6 44.9	355 35 27.2	0 8 16.586
900	0.0171353	97 26 7.6	355 20 26.7	0 7 26.642
800	0.0170959	97 45 31.7	355 5 26.4	0 6 36.759
700	0.0170562	98 4 57.5	354 50 25.7	0 5 46.937
600	0.0170163	98 24 24.8	354 35 25.3	0 4 57.178
500	0.0169762	98 43 53.6	354 20 25.4	0 4 7.484
400	0.0169367	99 3 24.0	354 5 25.7	0 3 17.854
300	0.0168949	99 22 55.9	353 50 26.1	0 2 28.290
200	0.0168539	99 42 29.4	353 35 26.6	0 1 38.792
—100	0.0168127	100 2 4.4	353 20 27.4	0 0 49.362
00	0.0167712	100 21 41.0	353 5 28.0	0 0 0.000
+100	0.0167295	100 41 18.9	172 50 28.6	0 0 49.293
200	0.0166875	101 0 58.5	172 35 29.9	0 1 38.516
300	0.0166452	101 20 39.5	172 20 31.2	0 2 27.667
400	0.0166027	101 40 22.0	172 5 32.6	0 3 16.747
500	0.0165599	102 0 6.1	171 50 34.1	0 4 5.754
600	0.0165169	102 19 51.8	171 35 35.8	0 4 54.688
800	0.0164302	102 59 27.8	171 5 39.8	0 6 32.333
1200	0.0162535	104 19 0.7	170 5 48.4	0 9 46.700
1600	0.0160731	105 39 0.3	169 5 59.8	0 12 59.799
2000	0.0158888	106 59 24.9	168 6 13.6	0 16 11.576
2400	0.0157009	108 20 18.9	167 6 29.3	0 19 21.978
3200	0.0153141	111 3 35.3	165 7 6.9	0 25 38.45
4000	0.0149133	113 49 3.6	163 7 51.8	0 31 48.79
4800	0.0144993	116 36 49.1	161 8 46.0	0 37 52.70
5600	0.0140723	119 26 59.1	159 9 46.5	0 43 49.70
6400	0.0136340	122 20 1.0	157 10 53.9	0 49 39.49
7200	0.0131843	125 16 6.1	155 12 7.3	0 55 21.67
+8000	0.0127243	128 15 28.5	153 13 26.4	1 0 55.94

TABLE IX.—*Precession of the Equinoxes and Obliquity of the Ecliptic.*

t	ψ	ε_1	ψ'	ε
—8000	—112° 24′ 57″.5	24° 24′ 51″.3	—109° 55′ 42.2″	24° 15′ 24″.8
7900	111 2 3.0	24 23 13.0	108 34 12.4	24 15 11.8
7800	109 39 3.5	24 21 35.7	107 12 41.6	24 14 57.9
7700	108 15 59.0	24 19 59.4	105 50 9.8	24 14 43.3
7600	106 52 49.6	24 18 24.3	104 29 37.0	24 14 28.0
7500	105 29 35.4	24 16 50.1	103 8 3.3	24 14 11.8
7400	104 6 16.6	24 15 17.2	101 46 28.5	24 13 55.0
7300	102 42 53.1	24 13 45.5	100 24 52.6	24 13 37.4
7200	101 19 25.3	24 12 15.1	99 3 15.7	24 13 19.0
7100	99 55 53.1	24 10 45.8	97 41 37.6	24 12 59.9
7000	98 32 16.8	24 9 18.0	96 19 58.3	24 12 40.1
6900	97 8 36.2	24 7 51.4	94 58 17.9	24 12 19.6
6800	95 44 51.7	24 6 26.3	93 36 36.5	24 11 58.4
6700	94 21 3.2	24 5 2.5	92 14 54.0	24 11 36.4
6600	92 57 11.0	24 3 40.2	90 53 10.2	24 11 13.8
6500	91 33 15.1	24 2 19.3	89 31 25.3	24 10 50.4
6400	90 9 15.8	24 1 0.0	88 9 39.2	24 10 26.4
6300	88 45 12.9	23 59 42.2	86 47 51.7	24 10 1.7
6200	87 21 6.9	23 58 25.9	85 26 3.2	24 9 36.4
6100	85 56 57.6	23 57 11.2	84 4 13.4	24 9 10.3
6000	84 32 45.3	23 55 58.1	82 42 22.3	24 8 43.6
5900	83 8 30.1	23 54 46.5	81 20 30.1	24 8 16.2
5800	81 44 12.1	23 53 36.5	79 58 36.5	24 7 48.2
5700	80 19 51.3	23 52 28.0	78 36 41.6	24 7 19.5
5600	78 55 28.1	23 51 21.2	77 14 45.3	24 6 50.2
5500	77 31 2.3	23 50 16.1	75 52 47.7	24 6 20.3
5400	76 6 34.3	23 49 12.7	74 30 48.7	24 5 49.8
5300	74 42 4.1	23 48 10.9	73 8 48.3	24 5 18.7
5200	73 17 31.9	23 47 10.8	71 46 46.5	24 4 47.0
5100	71 52 57.8	23 46 12.4	70 24 43.3	24 4 14.6
5000	70 28 21.9	23 45 15.8	69 2 38.5	24 3 41.7
4900	69 3 44.2	23 44 20.7	67 40 32.2	24 3 8.2
4800	67 39 5.1	23 43 27.4	66 18 24.5	24 2 34.2
4700	66 14 24.3	23 42 35.6	64 56 15.0	24 1 59.6
4600	64 49 42.4	23 41 45.6	63 34 4.2	24 1 24.5
4500	63 24 59.3	23 40 57.2	62 11 51.9	24 0 48.8
4400	62 0 15.2	23 40 10.6	60 49 38.0	24 0 12.6
4300	60 35 30.1	23 39 25.5	59 27 22.6	23 59 35.8
4200	59 10 44.2	23 38 42.1	58 5 5.4	23 58 58.6
4100	57 45 57.5	23 38 0.2	56 42 46.6	23 58 20.8
4000	56 21 10.3	23 37 20.0	55 20 26.0	23 57 42.6
3900	54 56 22.5	23 36 41.3	53 58 3.7	23 57 3.8
3800	53 31 34.4	23 36 4.3	52 35 39.6	23 56 24.6
3700	52 6 46.0	23 35 29.8	51 12 13.9	23 55 44.9
3600	50 41 57.6	23 34 55.0	49 50 46.4	23 55 4.8
3500	49 17 9.1	23 34 22.6	48 28 17.2	23 54 24.2
3400	47 52 20.7	23 33 51.8	47 5 46.1	23 53 43.2
3300	46 27 32.4	23 33 22.4	45 43 13.2	23 53 1.8
3200	45 2 44.5	23 32 54.5	44 20 38.4	23 52 20.0
3100	43 37 56.8	23 32 28.0	42 58 1.5	23 51 37.7
—3000	— 42 13 9.7	23 32 2.9	— 41 35 22.8	23 50 55.1

TABLE IX.—*Precession of the Equinoxes and Obliquity of the Ecliptic*—continued.

t	ψ		ψ'	ϵ
—3000	—42° 13′ 9.7	23° 32 2.9	—41° 35 22.8	23° 50 55.1
2900	40 48 23.2	23 31 39.2	40 12 42.3	23 50 12.1
2800	39 23 37.5	23 31 16.9	38 49 59.8	23 49 28.7
2700	37 58 52.4	23 30 55.9	37 27 15.3	23 48 45.0
2600	36 34 8.3	23 30 36.1	36 4 29.0	23 48 0.9
2500	35 9 25.1	23 30 17.6	34 41 40.7	23 47 16.5
2400	33 44 42.95	23 30 0.4	33 18 50.41	23 46 31.7
2300	32 20 1.86	23 29 44.3	31 55 57.95	23 45 46.5
2200	30 55 22.04	23 29 29.4	30 33 3.47	23 45 1.4
2100	29 30 43.50	23 29 15.6	29 10 6.89	23 44 15.5
2000	28 6 6.29	23 29 2.86	27 47 8.21	23 43 29.56
1900	26 41 30.59	23 28 51.20	26 24 7.52	23 42 43.32
1800	25 16 56.32	23 28 40.52	25 1 4.64	23 41 56.83
1700	23 52 23.62	23 28 30.80	23 37 59.64	23 41 10.09
1600	22 27 52.57	23 28 22.01	22 14 52.52	23 40 23.23
1500	21 3 23.21	23 28 14.08	20 51 43.23	23 39 36.05
1400	19 38 55.62	23 28 7.00	19 28 31.79	23 38 48.69
1300	18 14 29.87	23 28 0.72	18 5 18.21	23 38 1.17
1200	16 50 6.00	23 27 55.23	16 42 2.47	23 37 13.46
1100	15 25 44.02	23 27 50.47	15 18 44.50	23 36 25.56
1000	14 1 24.01	23 27 46.26	13 55 24.33	23 35 37.46
900	12 37 6.01	23 27 42.72	12 32 1.95	23 34 49.26
800	11 12 50.06	23 27 39.75	11 8 37.37	23 34 0.98
700	9 48 36.20	23 27 37.31	9 45 10.55	23 33 12.55
600	8 24 24.452	23 27 35.352	8 21 41.494	23 32 23.960
500	7 0 14.858	23 27 33.823	6 58 10.204	23 31 35.302
400	5 36 7.442	23 27 32.680	5 34 36.670	23 30 46.562
300	4 12 2.229	23 27 31.874	4 11 0.886	23 29 57.750
200	2 47 59.240	23 27 31.357	2 47 22.850	23 29 8.877
—100	— 1 23 58.492	23 27 31.082	—1 23 42.555	23 28 19.956
0	0 0 0.000	23 27 31.000	0 0 00.000	23 27 31.000
+100	+ 1 23 56.225	23 27 31.063	+1 23 44.818	23 26 42.020
200	2 47 50.175	23 27 31.223	2 47 31.903	23 25 53.027
300	4 11 41.846	23 27 31.432	4 11 21.255	23 25 4.034
400	5 35 31.238	23 27 31.640	5 35 12.875	23 24 15.054
500	6 59 18.351	23 27 31.800	6 59 6.765	23 23 26.097
600	8 23 3.192	23 27 31.863	8 23 2.924	23 23 37.175
800	11 10 26.10	23 27 31.51	11 11 2.05	23 20 59.49
1200	16 44 45.24	23 27 27.53	16 47 27.49	23 17 44.78
1600	22 18 30.18	23 27 16.70	22 24 29.04	23 14 32.43
2000	27 51 43.22	23 26 56.1	28 2 6.45	23 11 22.40
2400	33 24 27.47	23 26 23.0	33 40 19.39	23 8 15.7
3200	44 28 45.5	23 24 29.1	44 58 29.9	23 2 14.7
4000	55 32 2.9	23 21 17.2	56 18 55.9	22 56 34.9
4800	66 35 7.2	23 16 33.7	67 41 31.2	22 51 21.2
5600	77 38 52.4	23 10 10.4	79 6 6.5	22 46 38.8
6400	88 44 16.7	23 2 4.8	90 32 30.1	22 42 32.0
7200	99 52 19.0	22 52 20.6	102 0 25.8	22 39 4.8
+8000	+111 3 56.2	22 41 7.7	+113 29 33.3	22 36 21.0

TABLE X.—*For Precession in Right Ascension and Declination.*

t	*z*	*z′*	*z*+*z′*	Θ	ϑ
—8000	—53° 0′ 2″.8	—54° 34′ 37″.2	—107° 34′ 40″.0	—39° 24′ 51″.5	+2° 43′ 48″.27
7900	52 17 37.9	53 51 58.1	106 9 36.0	39 3 30.2	2 42 13.99
7800	51 35 17.9	53 9 19.7	104 44 37.6	38 41 50.0	2 40 35.30
7700	50 53 3.0	52 26 42.0	103 19 45.0	38 19 51.0	2 38 52.32
7600	50 10 53.0	51 44 5.1	101 54 58.1	37 57 33.4	2 37 5.17
7500	49 28 48.0	51 1 29.0	100 30 17.0	37 34 57.4	2 35 13.97
7400	48 46 48.0	50 18 53.9	99 5 41.9	37 12 3.4	2 33 18.85
7300	48 4 53.0	49 36 20.0	97 41 13.0	36 48 51.6	2 31 19.92
7200	47 23 3.1	48 53 47.4	96 16 50.5	36 25 22.2	2 29 17.35
7100	46 41 18.3	48 11 16.0	94 52 34.3	36 1 35.6	2 27 11.25
7000	45 59 38.6	47 28 46.1	93 28 24.7	35 37 31.8	2 25 1.74
6900	45 18 3.8	46 46 17.6	92 4 21.4	35 13 11.2	2 22 48.98
6800	44 36 34.1	46 3 50.9	90 40 25.0	34 48 34.1	2 20 33.11
6700	43 55 9.3	45 21 25.8	89 16 35.1	34 23 40.6	2 18 14.29
6600	43 13 49.5	44 39 2.5	87 52 52.0	33 58 31.0	2 15 52.63
6500	42 32 34.8	43 56 41.1	86 29 15.9	33 33 5.7	2 13 28.32
6400	41 51 25.0	43 14 21.9	85 5 46.9	33 7 24.9	2 11 1.44
6300	41 10 20.0	42 32 4.7	83 42 24.7	32 41 28.8	2 8 32.18
6200	40 29 19.9	41 49 49.7	82 19 9.6	32 15 17.7	2 6 0.77
6100	39 48 24.7	41 7 37.0	80 56 1.7	31 48 51.7	2 3 27.23
6000	39 7 34.3	40 25 26.7	79 33 1.0	31 22 11.3	2 0 51.77
5900	38 26 48.9	39 43 18.8	78 10 7.7	30 55 16.6	1 58 14.56
5800	37 46 8.2	39 1 13.4	76 47 21.6	30 28 7.9	1 55 35.72
5700	37 5 32.2	38 19 10.4	75 24 42.6	30 0 45.4	1 52 55.44
5600	36 25 0.8	37 37 10.3	74 2 11.1	29 33 9.4	1 50 13.84
5500	35 44 34.0	36 55 12.8	72 39 46.8	29 5 20.0	1 47 31.13
5400	35 4 11.7	36 13 18.3	71 17 30.0	28 37 17.8	1 44 47.44
5300	34 23 53.9	35 31 26.8	69 55 20.7	28 9 2.9	1 42 2.93
5200	33 43 40.5	34 49 38.4	68 33 18.9	27 40 35.6	1 39 17.77
5100	33 3 31.5	34 7 52.9	67 11 24.4	27 11 56.0	1 36 32.12
5000	32 23 26.8	33 26 10.7	65 49 37.5	26 43 4.6	1 33 46.13
4900	31 43 26.4	32 44 31.8	64 27 58.2	26 14 1 4	1 30 59.98
4800	31 3 30.1	32 2 56.0	63 6 26.1	25 44 46.9	1 28 13.80
4700	30 23 37.9	31 21 23.6	61 45 1.5	25 15 21.1	1 25 27.78
4600	29 43 49.7	30 39 54.5	60 23 44.2	24 45 44.4	1 22 42.05
4500	29 4 5.6	29 58 29.1	59 2 34.7	24 15 57.0	1 19 56.77
4400	28 24 25.2	29 17 6.9	57 41 32.1	23 45 59.3	1 17 12.11
4300	27 44 48.8	28 35 48.5	56 20 37.3	23 15 51.4	1 14 28.22
4200	27 5 16.1	27 54 33.6	54 59 49.7	22 45 33.7	1 11 45.26
4100	26 25 47.2	27 13 22.4	53 39 9.6	22 15 6.2	1 9 3.36
4000	25 46 21.8	26 32 14.8	52 18 36.6	21 44 29.4	1 6 22.69
3900	25 7 0.1	25 51 10.7	50 58 10.8	21 13 43.5	1 3 43.39
3800	24 27 41.6	25 10 10.7	49 37 52.3	20 42 48.7	1 1 5.62
3700	23 48 26.2	24 29 14.7	48 17 40.9	20 11 45.3	0 58 29.51
3600	23 9 14.0	23 48 22.5	46 57 36.5	19 40 33.5	0 55 55.21
3500	22 30 5.2	23 7 34.1	45 37 39.3	19 9 13.6	0 53 22.87
3400	21 50 59.3	22 26 49.7	44 17 49.0	18 37 45.9	0 50 52.63
3300	21 11 56.5	21 46 9.1	42 58 5.6	18 6 10.5	0 48 24.62
3200	20 32 56.4	21 5 32.5	41 38 28.9	17 34 27.8	0 45 58.99
3100	19 53 59.1	20 24 59.8	40 18 58.9	17 2 38.0	0 43 35.87
—3000	—19 15 4.4	—19 44 31.1	—38 59 35.5	—16 30 41.3	+0 41 15.38

TABLE X.—*For Precession in Right Ascension and Declination*—continued.

t	*z*	*z′*	*z*+*z′*	Θ	ϑ
—3000	—19° 15 4.4	—19° 44 31.1	—38° 59′ 35″.5	—16° 30′ 41″.3	+0° 41′ 15.″38
2900	18 36 12.5	19 4 6.5	37 40 19.0	15 58 38.1	0 38 57.66
2800	17 57 22.9	18 23 45.9	36 21 8.8	15 26 28.5	0 36 42.82
2700	17 18 35.8	17 43 29.4	35 2 5.2	14 54 12.8	0 34 30.99
2600	16 39 50.8	17 3 16.8	33 43 7.6	14 21 51.3	0 32 22.29
2500	16 1 8.2	16 23 8.2	32 24 16.4	13 49 24.2	0 30 16.84
2400	15 22 27.5	15 43 3.7	31 5 31.2	13 16 51.7	0 28 14.77
2300	14 43 49.0	15 3 3.0	29 46 52.0	12 44 14.2	0 26 16.14
2200	14 5 12.2	14 23 6.4	28 28 18.6	12 11 31.8	0 24 21.04
2100	13 26 37.1	13 43 13.9	27 9 51.0	11 38 44.8	0 22 29.62
2000	12 48 3.7	13 3 25.3	25 51 29.0	11 5 53.5	0 20 42.00
1900	12 9 31.9	12 23 40.6	24 33 12.5	10 32 58.1	0 18 58.24
1800	11 31 1.5	11 43 59.8	23 15 1.3	9 59 58.8	0 17 18.45
1700	10 52 32.5	11 4 22.9	21 56 55.4	9 26 56.0	0 15 42.70
1600	10 14 4.6	10 24 49.7	20 38 54.3	8 53 49.8	0 14 11.08
1500	9 35 38.2	9 45 20.3	19 20 58.5	8 20 40.5	0 12 43.68
1400	8 57 12.6	9 5 54.6	18 3 7.2	7 47 28.4	0 11 20.55
1300	8 18 47.8	8 26 33.0	16 45 20.5	7 14 13.7	0 10 1.77
1200	7 40 23.7	7 47 14.9	15 27 38.6	6 40 56.6	0 8 47.43
1100	7 2 0.5	7 8 0.4	14 10 0.9	6 7 37.5	0 7 37.59
1000	6 23 37.8	6 28 49.4	12 52 27.2	5 34 16.5	0 6 32.30
900	5 45 15.8	5 49 41.9	11 34 57.7	5 0 53.9	0 5 31.61
800	5 6 54.0	5 10 37.8	10 17 31.8	4 27 30.0	0 4 35.58
700	4 28 32.5	4 31 37.3	9 0 9.8	3 54 4.9	0 3 44.26
600	3 50 11.032	3 52 39.776	7 42 50.808	3 20 39.064	0 2 57.696
500	3 11 49.738	3 13 45.575	6 25 35.313	2 47 12.593	0 2 15.920
400	2 33 28.307	2 34 54.553	5 8 22.860	2 13 45.771	0 1 38.971
300	1 55 6.727	1 56 6.585	3 51 13.312	1 40 18.850	0 1 6.879
200	1 16 44.874	1 17 21.582	2 34 6.456	1 6 52.081	0 0 39.673
—100	—0 38 22.631	—0 38 39.455	— 1 17 2.086	—0 33 25.715	+0 0 17.373
0	0 0 0.000	0 0 0.000	0 0 0.000	0 0 0.000	0 0 0.000
+100	+0 38 23.482	+0 38 36.527	+ 1 17 0.009	+0 33 24.811	—0 0 12.433
200	1 16 47.594	1 17 10.547	2 33 58.141	1 6 48.471	—0 0 19.917
300	1 55 12.512	1 55 42.112	3 50 54.624	1 40 10.724	—0 0 22.444
400	2 33 38.370	2 34 11.277	5 7 49.647	2 13 31.327	—0 0 20.013
500	3 12 5.23	3 12 38.10	6 24 43.33	2 46 50.04	—0 0 12.626
600	3 50 33.13	3 51 2.64	7 41 35.77	3 20 6.62	—0 0 0.292
800	5 7 33.36	5 7 46.46	10 15 19.82	4 26 32.15	+0 0 39.178
1200	7 41 52.11	7 40 52.87	15 22 44.99	6 38 46.37	0 2 56.756
1600	10 16 41.14	10 13 38.94	20 30 20.08	8 49 57.76	0 6 30.865
2000	12 52 7.6	12 46 14.2	25 38 21.8	10 59 50.14	0 11 18.674
2400	15 28 17.5	15 18 48.0	30 47 5.5	13 8 7.12	0 17 16.358
3200	20 43 12.7	20 24 34.2	41 7 46.9	17 18 49.10	0 32 21.726
4000	26 2 13.6	25 32 26.2	51 34 39.7	21 19 51.28	0 50 59.337
4800	31 26 0.6	30 43 56.8	62 9 57.4	25 9 0.00	1 12 10.125
5600	36 55 6.4	36 0 38.9	72 55 45.3	28 44 0.89	1 34 44.92
6400	42 29 53.8	41 24 1.0	83 53 54.8	32 2 40.02	1 57 27.37
7200	48 10 33.3	46 55 21.8	95 5 55.1	35 2 45.43	2 18 55.87
+8000	53 57 1.6	52 35 46.6	+106 32 48.1	+37 42 10.22	+2 37 46.67

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.
239

OBSERVATIONS

ON

TERRESTRIAL MAGNETISM

AND ON THE

DEVIATIONS OF THE COMPASSES

OF THE UNITED STATES IRON CLAD MONADNOCK DURING HER CRUISE FROM PHILADELPHIA TO SAN FRANCISCO, IN 1865 AND 1866.

BY

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[ACCEPTED FOR PUBLICATION, SEPTEMBER, 1871.]

INTRODUCTORY NOTE.

THIS paper was originally an official report presented to the Navy Department by Professor Harkness; but, as that department made no use of it, the National Academy of Sciences, in August, 1867, passed a resolution asking for the manuscript. This request was complied with; and, an abstract of the paper having been read to the Academy in April, 1869, it was referred to a commission consisting of the President of the Academy, Professors J. H. C. Coffin, and F. Rogers, in accordance with whose recommendation it is now published by the Smithsonian Institution.

JOSEPH HENRY,
Secretary S. I.

COLLINS, PRINTER,
PHILADELPHIA.

TABLE OF CONTENTS.

SECTION I.

INTRODUCTION.

SECTION II.

DESCRIPTIONS OF STATIONS.

SECTION III.

ASTRONOMICAL OBSERVATIONS.

SECTION IV.

OBSERVATIONS ON TERRESTRIAL MAGNETISM.

Determination of constants peculiar to the portable declinometer, D. 22

SECTION V.

OBSERVATIONS ON THE MAGNETISM OF THE SHIP.

REPORT ON MAGNETIC OBSERVATIONS.

SECTION I.

INTRODUCTION.

On the fifth of October, 1865, I was ordered to the U. S. Iron-clad Monadnock[1] for the purpose of making observations on the action of her compasses during the cruise which she was about to undertake from Philadelphia to San Francisco, by way of the Straits of Magellan. She was then fitting out at the Philadelphia Navy Yard, and the work on her was so far advanced that it was expected she would sail in about two weeks. As the department had not previously intimated its intention of assigning me to this duty, and as everything relating to the number and kind of observations to be made, and the instruments required, was left entirely to my own discretion, it will be seen that the time available for making plans and collecting the necessary apparatus was very limited.

The plan of observation ultimately adopted was that at every port in which we remained for more than twenty-four hours the following operations should be gone through with. 1st. The ship should be swung, and as her head pointed successively to each of the thirty-two true magnetic points, the reading of every compass on board should be recorded for each point. 2d. That at such of the compasses as were so situated as to render it possible, the horizontal force and inclination should be determined. 3d. The position of the dividing line between the north and south polarity should be traced on each turret. 4th. The magnetic declination, inclination, and horizontal force should be determined on shore. While at sea it was intended to observe the declination—and consequently the deviation—and horizontal force daily, by means of the standard compass; but this turned out to be impracticable, because the only place in the ship where it was possible to mount that instrument was on top of the after pilot-house; a situation

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[1] The Monadnock is a double-turreted vessel of the monitor type. During the cruise in question, Lieutenant Commander Francis M. Bunce, U. S. N., was her captain, and she was attached to the squadron commanded by Commodore (now Rear-Admiral) John Rogers, U. S. N., at whose special request I was detailed by the Navy Department to make the observations which are the subject of this paper.

where no binnacle could be put, and where the compass was nearly on a level with the top of the smoke-stack. Thus, while at sea, the position occupied by it was almost constantly enveloped in smoke and gas, rendering it absolutely necessary, whenever we left port, to dismount the instrument in order to preserve it from injury.

Owing to the very short time at my disposal previous to sailing, there was great difficulty in providing proper instruments, but I succeeded in obtaining all that were absolutely necessary. The following is a list of them:

1 Portable Declinometer and stand.
1 Five-inch Altitude and Azimuth Instrument.
1 Dip Circle, with two needles, each three and a half inches long.
1 Pair of eight-inch Bar Magnets.
1 Pair of eleven-inch Bar Magnets.
2 Admiralty Standard Compasses, with stands and deflectors.
1 Burt's Solar Compass and stand.
1 Prismatic Sextant of six inches radius.
1 Mercurial Artificial Horizon.
1 Pocket Chronometer, Fletcher, No. 906.
1 Silver Comparing Watch.
2 Pocket Thermometers.
2 Pocket Compasses.
2 Magnetic Needles, not mounted, each 2.75 inches long, and 0.33 of an inch broad.
1 Fifty feet Chesterman's Patent Tape Line.
1 Case of Drawing Instruments.
1 Gunter's Scale, two feet long.

The portable declinometer belonged to the U. S. Coast Survey, and was kindly lent by Prof. J. E. Hilgard.

The small unmounted magnetic needles were intended to be used for measuring the relative horizontal force on shore and at each of the compasses on board ship. For this purpose it was proposed to vibrate one of them on shore, and then taking it on board ship to the compass at which it was desired to measure the relative horizontal force, to remove the compass card from the centre-point, and putting the small needle in its place, vibrate it again. Unfortunately the small needles were not finished till just before we left Philadelphia, and there was no opportunity of trying them till after we were at sea, when, to my great regret, it was found that the jewels were so small that they would not fit on the centre-point of any compass on board, thus rendering them entirely useless. Under the circumstances, for horizontal force on board ship it was necessary to rely entirely upon measures made with the deflectors belonging to the Admiralty standard compasses—a method certainly not so convenient, and, owing to the constant swinging of the ship when at anchor, probably not so accurate as counting the vibrations of a small needle.

The observations on terrestrial magnetism, and for latitude, time, and true bearings, were all made by myself and recorded by Mr. Corrin F. Smith, who was captain's clerk on the Monadnock, and acted as my assistant when I was observing. My best thanks are due to him for the efficient manner in which he performed his duties, sometimes under circumstances of very considerable physical discomfort.

The reductions and discussions in this report have been made by me, so that I am personally responsible, not only for the general plan of the work, but for every figure contained in it. All the results have been very carefully checked, and it is hoped no material error will be found in them; still, absolute accuracy is scarcely to be expected in any work involving so many figures, the more especially as much of it has been done during moments snatched from other and more pressing professional duties.

The observations naturally divide themselves into three classes: 1st. Those relating to astronomy. 2d. Those relating to terrestrial magnetism. 3d. Those relating to the magnetism of the ship. As that is the order in which they must necessarily be reduced, they will be so treated of in the subsequent sections of this report.

SECTION II.

DESCRIPTIONS OF STATIONS.

UNLESS otherwise stated, the assumed positions of light-houses, forts, etc., have been taken from the English Admiralty Charts, or from the English Admiralty List of Lights, the latest editions obtainable in 1865 being employed. The longitudes are counted from the meridian of Greenwich.

The method used in testing a station for local attraction by means of fore and back sights with a compass, was as follows: The compass was set up at the station, and the bearing of a point distant one hundred yards, or more, was observed. Then the compass was transferred to that point, and the bearing of the station was observed. These two bearings should evidently differ from each other by 180°; if they did not, it was certain that local attraction existed at one or both of the points, and a new station was sought for. This process is almost certain to detect any strictly local magnetic attraction, but it will not suffice to demonstrate the existence of an abnormal state of the magnetic elements extending over a large territory.

PHILADELPHIA, *Pa.* The magnetic observations were made at a spot on the east bank of the Delaware river, about twenty feet from the water's edge. It is nearly southeast from the U. S. Navy Yard, from which it is distant about three-quarters of a mile. The soil is a dark—nearly black—earth, which appears to have been deposited by the river. The approximate position of the station was

Lat.	39° 55′ N.
Long.	5^h 0^m 32^s W.

GOSPORT, *Va.* The magnetic observations were made on a white sandy beach, on the west bank of the Elizabeth river, about thirty feet from the water's edge. From the place where the instruments stood, the flagstaff in the U. S. Navy Yard bore due north by compass, and was distant about half a mile.

Assuming the position of the flagstaff to be lat. 36° 49′ 32″ N., long. 5^h 5^m $9^s.8$ W., as stated by the authorities at the Navy Yard, the position of the spot occupied by the instruments is approximately

Lat.	36° 49′ 0″ N.
Long.	5^h 5^m $9^s.8$ W.

The ship was swung at the compass station in Hampton Roads, on November 1st, 1865, in the usual manner. Her position at the time was lat. 36° 58′ N., long. 76° 20′ W. Joint XII on the after turret was 14.4 inches to port.

ST. THOMAS, *West Indies.* The ship was swung in this harbor, on November 18th, 1865, in the usual manner. Her position at the time was lat. 18° 19′ N., long. 64° 56′ W. Joint XII on the after turret was 14.4 inches to port.

The observations on shore were made in Long Bay, at a spot about thirty feet from the water's edge, on a gravelly beach, to the eastward of the town. From the place where the instruments stood the true bearing of Fort Cowell, at the entrance to the harbor, is S. 34° 50′ W., and it is distant about one mile.

Assuming the position of Fort Christian to be lat. 18° 20′ 27″ N., long. 4^{h} 19^{m} $42^{s}.7$ W., then, according to the English Admiralty Chart, the position of the spot where the instruments were set up is

Lat. 18° 20′ 22″ N.
Long. 4^{h} 19^{m} $40^{s}.6$ W.

ISLE ROYAL, *Salute Islands.* An attempt was made to swing the ship here, on November 30th, 1865, in the usual manner, but it failed on account of the continual rain which shut off the view of the distant azimuth mark. The position of the ship at the time was lat. 5° 17′ N., long. 52° 33′ W. Joint XII on the after turret was 0.6 of an inch to starboard.

The magnetic and astronomical observations on shore were made on the southwest side of the island, at a spot from which the corner made by the southeast and southwest faces of the government coal sheds bears N. 64° W. (true), and is distant one hundred and thirty-two feet. The place was examined carefully for local attraction by taking fore and back sights with a compass, but none could be detected. The position occupied by the instruments is in

Lat. 5° 17′ 29″ N.
Long. 3^{h} 30^{m} $11^{s}.4$ W.

The latitude was determined from a single set of circummeridian altitudes of the sun observed by me, and the longitude was taken from the French chart.

CEARA, *Brazil.* An attempt was made to swing the ship here, on December 19th, 1865, in the usual manner, but although a very favorable opportunity was chosen, she could only be made to turn through ten points. Her position at the time was lat. 3° 44′ S., long. 38° 34′ W. Joint XII on the after turret was 0.6 of an inch to starboard. The wind, current, and sea are so strong here that vessels at anchor in the roads always ride with their heads nearly in the same direction, never swinging more than about three points.

At this place there is no harbor whatever, merely an open roadstead. A heavy surf is constantly running on the beach, and as there are almost no facilities for landing in small boats, getting the instruments on shore involved a good deal of trouble and some risk. However, I succeeded in landing them safely, and obtained a very good set of observations on the white sand beach at a spot about one hundred and fifty feet from the water's edge, and from which the true bearing of the southeast corner of the custom-house on the wharf is N. 53° 19′ W., and its distance two hundred feet. From the same spot the true bearing of

Point Macoripe Light-house is N. 75° 38′ E. The position occupied by the instruments is in

Lat. 3° 43′ 59″ S.
Long. 2^h 34^m 6^s W.

The latitude was deduced from my own observations, and the longitude was taken from the list of geographical positions given in Raper's Navigation.

PERNAMBUCO, *Brazil.* The ship was not swung in this port because there was not room to do it in the position where she took her coal, and as she only remained in the harbor twenty-four hours, there was not time to take up another position in order to swing.

The magnetic and astronomical observations on shore were made on the white sand beach, at a spot from which the true bearing of the salient angle of the southeast bastion of Fort Brum is N. 15° 46′ W., and its distance four hundred and thirty feet.

Assuming the position of the light-house, near to Fort Picao, to be lat. 8° 3′ 42″ S., long. 2^h 19^m $26^s.8$ W., as it is given in the English Admiralty List of Lights, edition of 1866, then, according to the English Admiralty Chart, the position occupied by the instruments is in

Lat. 8° 3′ 37″ S.
Long. 2^h 19^m $28^s.2$ W.

BAHIA, *Brazil.* The ship was swung in this harbor, on December 30th, 1865, in the usual manner. Her position at the time was lat. 12° 59′ S., long. 38° 31′ W. Joint XII on the after turret was 0.6 of an inch to starboard.

The magnetic and astronomical observations of December 27th were made at a spot, one hundred and fifty feet from the water's edge, situated in a cocoanut grove on the beach about half-way between Monserat Point and Fort Victoria. The soil is a coarse white sand. It was not possible to get any bearings which would define the exact position, but the above directions are sufficient to enable any one to find the place very nearly.

Assuming the position of Fort St. Antonio Light to be lat. 13° 0′ 55″ S., long. 2^h 34^m $6^s.9$ W., then, according to the English Admiralty Chart, the position occupied by the instruments is in

Lat. 12° 56′ 55″ S.
Long. 2^h 34^m $0^s.5$ W.

RIO JANEIRO, *Brazil.* The ship was swung in this harbor, on January 10th, 1866, in the usual manner; but, owing to a strong wind which was blowing at the time, it was not possible to get her through more than seventeen points. Her position was lat. 22° 54′ S., long. 43° 9′ W. Joint XII on the after turret was 0.8 of an inch to port.

During the whole week we were at Rio there was not one clear day. Consequently it was extremely difficult to make astronomical observations, and it was only by patiently watching for the sun and seizing the opportunities when it was momentarily visible through breaks in the clouds, that the few sights necessary in order to complete the magnetic observations were obtained.

With a single exception, all the magnetic and astronomical observations were made at a spot from which the true bearing of the entrance on the north face of Fort Caraguata (erroneously spelled Gravata on the English charts) is S. 70° W., and its distance fifty-five feet. There were no guns in the fort at the time. The surrounding country is very hilly, the bare, coarse, granite rocks cropping out everywhere from the hill-sides, but in the more level places they are thinly covered with earth. Assuming the position of Fort Villegagnon to be lat. 22° 54′ 42″ S., long. 2^h 52^m $36^s.0$ W., then, according to the English Admiralty Chart, the position occupied by the instruments is in

Lat. 22° 54′ 5″ S.
Long. 2^h 52^m $30^s.7$ W.

The exception referred to above is some observations of the sun for time, made on January 9th. They were got on Rat Island, the spot where naval officers usually go to rate their chronometers when lying in this harbor. Assuming the position of Fort Villegagnon as above, then, according to the English Admiralty Chart, the position of Rat Island is

Lat. 22° 53′ 45″ S.
Long. 2^h 52^m $37^s.9$ W.

Monte Video, *Uruguay.* The ship was swung in this harbor, on January 24th, 1866, in the usual manner. We first attempted to get her around about 1 P. M., but owing to the force of the wind and tide we only obtained ten points, viz., those from E. by S. to S. S. W. Just at sunset we tried it again, and succeeded in getting the remainder of the circle. It was nearly dark when we finished, but as the distant object used for an azimuth mark shone plainly against the sky, there was sufficient light to see pretty distinctly when it was in range with the sights of the compass.

The readings of part of the circle on the After Ritchie compass were lost, owing to the failure of daylight and delay in procuring a lantern. The officer who usually read the After Azimuth compass was on shore at the time, and the duty of making the observations at that instrument was assigned to another, but it turned out that he did not understand how to read an azimuth compass, and his observations were worthless.

While we were lying at Monte Video the tide was very irregular. Most of the time the ship only swung to it about 90°, but two or three times she swung 180°. At the time we swung her to obtain the deviation of the compasses her position was lat. 34° 55′ S., long. 56° 13′ W., and joint XII on the after turret was 4.5 inches to port.

The greater part of the magnetic observations on shore were made on January 18th, at a station on the ground occupied by Tomkinson's slaughtering establishment. The instruments were set up at a spot where there are four large umbu trees standing in a line. The exact position may be recovered by means of the following true bearings. The corner made by the south and west sides of the dwelling-house bears N. 39° E., and is distant about one hundred feet. The light-house on the Mount, on the west side of the harbor, bears N. 59° 0′ W. The water's edge is distant from the station about four hundred feet. The soil is a thin stratum of very poor earth, covering a greenish-colored slaty rock, which crops out in many places. Assuming the position of the light-house on the Mount to be lat. 34° 53′ 15″ S., long. 3^h 44^m $59^s.0$ W., then, according to the English Admiralty Charts, the position occupied by the instruments is in

Lat. 34° 53′ 39″ S.
Long. 3^h 44^m $55^s.8$ W.

As a check, some magnetic observations were made, on January 19th, at a station from which the true bearing of the light-house on the Mount is N. 89° 41′ W., and the true bearing of the light on the Cathedral is S. 17° 42′ W. Assuming the position of the light-house to be as stated above, and the light on the cathedral to be in lat. 34° 54′ 20″ S., long. 3^h 44^m $50^s.0$ W., as given in the English Admiralty List of Lights in South America, edition of 1865, the geographical position of this station was

Lat. 34° 53′ 16″ S.
Long. 3^h 44^m $48^s.3$ W.

It will be observed that the difference of longitude between the lights on the Mount and on the cathedral, as deduced from the Admiralty List cited above, cannot be made to agree with the positions given on the English Admiralty Chart.

On January 24th some observations for time were made on Rat Island. Assuming the position of the light-house on the Mount to be as stated above, then, according to the English Admiralty Chart, the position of the station on Rat Island was

Lat. 34° 53′ 18″ S.
Long. 3^h 44^m $52^s.9$ W.

Sandy Point, *Straits of Magellan.* The ship was swung in this harbor, on February 10th, 1866, in the usual manner. Her position at the time was lat. 53° 11′ S., long. 70° 55′ W. Joint XII on the after turret was 4.5 inches to port. While we were lying here the ship was perfectly free to swing to the tide, but she generally turned through an arc of only about ninety degrees, namely, from W.N.W. to N.N.E.

The observations on shore were made in the meadow, between the settlement and the beach, at a spot from which the true bearing of the flagstaff was N. 47° 8′ W., and its distance about eight hundred feet. The soil is sandy, and there is no rock anywhere near. The place was examined for local attraction by taking fore and back sights with a compass, but nothing of the kind could be detected.

Assuming the position of the flagstaff to be lat. 53° 10′ 15″ S., long. 4^h 43^m $36^s.0$ W., as given on the English Admiralty Chart, edition of 1861, the position occupied by the instruments is in

Lat. 53° 10′ 20″ S.
Long. 4^h 43^m $35^s.3$ W.

VALPARAISO, *Chile.* The ship was swung in this harbor, on April 4th, 1866, in the usual manner. Her position at the time was lat. 33° 2′ S., long. 71° 38′ W. Joint XII on the after turret was 4.25 inches to port. While we were lying at Valparaiso the ship was perfectly free to swing to the tide, and she turned in all directions.

The observations taken on shore March 2d were made on the south end of the white sand beach at the Estero de Quilpue, at a spot about two hundred and fifty feet from the rocks. Assuming the position of Fort San Antonio to be lat. 33° 1′ 53″ S., long. $4^h\ 46^m\ 46^s.0$ W., then, according to the English Admiralty Chart, the position of this station was approximately

Lat. 33° 1′.4 S.
Long. $4^h\ 46^m\ 31^s$ W.

The observations of March 19th, and all taken subsequently to that date, were made at a spot distant about six hundred and fifty feet, nearly true north, from the most northern of the custom-houses. The instruments were set up, near to the water's edge, on the public road which here runs along under a high bank of rock. The true bearing of the flagstaff at Fort San Antonio, on the top of the hill, was S. 31° 45′ W., and its estimated distance was seven hundred feet. Assuming the position of the fort to be as stated above, the position occupied by the instruments is in

Lat. 33° 1′ 47″ S.
Long. $4^h\ 46^m\ 45^s.7$ W.

Both this station and that of March 2d were carefully tested for local attraction by taking fore and back sights with a compass, but none could be detected.

In adopting $4^h\ 46^m\ 46^s.0$ as the longitude of Fort San Antonio, I have followed Raper, but this value is doubtless too large. Capt. Jas. M. Gilliss, U. S. N., from a series of occultations and moon culminations, observed during the years 1850–51–52, determined the longitude of the Observatory on the hill of Santa Lucia, in Santiago, to be $4^h\ 42^m\ 33^s.8$. Dr. Moesta, from subsequent observations up to the year 1862, corrected this value to $4^h\ 42^m\ 33^s.0$. Capt. Gilliss, by means of the electric telegraph, found the difference of longitude between the Observatory at Santiago and Mr. Mouatt's Observatory at Valparaiso to be $3^m\ 56^s.5$. Hence, adopting Dr. Moesta's value of the longitude of Santiago, we have

$4^h\ 46^m\ 29^s.5$ W.

as the longitude of Mr. Mouatt's Observatory; but I have been unable to find any description of its position, and consequently cannot refer this longitude to Fort San Antonio.

Findlay, in his "Directory to the South Pacific Ocean," edition of 1863, gives for the longitude of Fort San Antonio $4^h\ 46^m\ 28^s.8$, and quotes Dr. Moesta as the authority. The Connaissance des Temps, for the year 1868, on the same authority gives $4^h\ 46^m\ 27^s.5$ for the same position. Which of the two values is nearest correct I am unable to say.

CALLAO, *Peru.* The ship was swung in this harbor, on April 29th, 1866, in the usual manner. Her position at the time was lat. 12° 3′ S., long. 77° 14′ W. Joint

XII on the after turret was 5.5 inches to port. While we were lying at Callao the ship was perfectly free to swing to the tide, but the wind and current were so strong that she did not do so, but always lay with her head pointing in a southerly direction.

The observations taken on shore, April 26th, were made on the northeast side of San Lorenzo Island, about two and a half miles southeast of the light-house. The island is a mass of hills, rising to an elevation of more than a thousand feet, composed of loose friable rock which seems to be of volcanic origin, and which is constantly disintegrating into a fine yellow sand. The place selected for making the observations is at the foot of a gorge where there is a beach, about a quarter of a mile long, of the yellow sand mentioned above. On the beach stand a number of fishermen's huts, and a few steps back, at the foot of the gorge, stands a large, square, two-story house. The spot where the instruments stood is on the southeast end of the beach, a little beyond the fishermen's huts, and just above high-water mark. Assuming the position of the light-house to be lat. 12° 4′ 0″ S., long. 5^h 9^m $18^s.0$ W., the position occupied by the instruments is in

Lat. 12° 5′ 14″ S.
Long. 5^h 9^m $9^s.1$ W.

The place was carefully tested for local attraction by taking fore and back sights with a compass, but none could be detected.

Payta, *Peru.* We remained in this port only from 2^h 30^m P. M. of May 6th, 1866, till 6^h P. M. of May 7th, and there was neither time nor opportunity to swing the ship. However, a complete set of magnetic observations were made on shore at a station on the beach four-tenths of a mile northwest of the large iron building which stands just back from the mole, and is used by the government as a custom-house, etc. As nearly as could be determined from angles carefully measured, and plotted on the English Admiralty Chart, this station is identical with the one occupied by the officers of H. B. M. surveying vessel "Beagle," in the year 1836, when making their observations for determining the position of Payta. According to their determinations it is in

Lat. 5° 5′ 36″ S.
Long. 5^h 24^m $22^s.0$ W.,

the longitude depending upon the position of the northeast bastion at Panama, New Granada, which is taken to be 5^h 18^m $4^s.6$ W.

The instruments were set up, just above high-water mark, on the gray sand beach, about fifty feet back from which the land rises into bluffs, two hundred feet high, composed of a hard yellow earth, alternating with sedimentary rocks. The station was carefully examined for local attraction, by taking fore and back sights with a compass, but none could be detected.

Panama, *New Granada.* The ship was swung in this roadstead, on May 20th, 1866, in the usual manner. Her position at the time was lat. 8° 55′ N., long. 79° 30′ W. Joint XII on the after turret was 5.5 inches to port. While we were lying here the ship was swinging freely in all directions to the wind and tide.

The observations taken on shore, May 14th, were made on the northern side of Flamenco Island, to the westward of a small cocoanut grove, and northeast of the Naval Cemetery. The instruments were set up about ten feet north of the most western of the ruins which are to be found there. The island is rocky, but at this station the rocks are covered with earth. The spot was carefully tested for local attraction by taking fore and back sights with a compass, but none could be detected.

If we assume the position of the northeast bastion at Panama to be lat. 8° 56′ 56″ N., long. 5^h 18^m $4^s.6$ W., as given by Capt. H. Kellet, R. N., then, according to the English Admiralty Chart, the position occupied by the instruments is in

Lat. 8° 54′ 31″ N.
Long. 5^h 18^m $1^s.8$ W.

ACAPULCO, *Mexico.* The ship was swung in this harbor, on June 1st, 1866, in the usual manner. Her position at the time was lat. 16° 50′ N., long. 99° 52′ W. Joint XII on the after turret was 5.5 inches to port. During the three days we were lying at Acapulco the ship was swinging freely to the wind and tide.

At the extreme south end of St. Lucia Bay, in this harbor, are two cocoanut groves, the most western of the two containing the graves of a number of our naval officers. The western end of the eastern grove is the place where the observations taken on shore, on May 30th, were made. The trees come almost close down to high-water mark, and the soil is a gray sand. The instruments were set up about forty feet from high-water mark, at a spot from which the true bearing of the gate of Fort St. Diego is N. 6° 22′ E.

If we assume the position of this gate to be lat. 16° 50′ 56″ N., long. 6^h 39^m $29^s.0$ W., as given on the English Admiralty Chart, then, according to that chart, the position occupied by the instruments is in

Lat. 16° 50′ 3″ N.
Long. 6^h 39^m $29^s.4$ W.

MAGDALENA BAY, *Lower California.* An attempt was made to swing the ship in this bay, on June 9th, 1866, in the usual manner, but owing to a very stiff breeze which was blowing at the time, she could only be turned through fourteen points. Her position was lat. 24° 38′ N., long. 112° 6′ W. Joint XII on the after turret was 5.5 inches to port. During the three days that we lay in this bay the wind was so strong that the ship did not swing to the tide, but rode with her head constantly to the west.

As it is difficult to describe the land-marks here, the most convenient way of giving positions will be to refer them to the English Admiralty Chart, the position formerly occupied by Capt. Sir Edw. Belcher's observatory being taken to be lat. 24° 38′ 18″ N., long. 7^h 28^m $25^s.4$ W., as given on the chart.

On June 8th a landing was effected at a spot on the beach, about a mile south of the position of Capt. Belcher's observatory, for the purpose of making a set of magnetic observations; but, after getting a time sight, it was found that there was a great deal of local attraction, nearly all the stones on the beach being magnetic, and consequently it was useless to attempt anything there. The approximate position of this spot is

Lat. 24° 38′ N.
Long. 7^h 28^m 24^s W.

On June 9th, after going to the extreme northern end of the bay, and pulling a short distance up a creek, a place was found which, upon careful examination by taking fore and back sights with a compass, seemed to be entirely free from all local attraction. The land there is composed of fine white-sand hillocks, which are constantly being shifted by the wind, and are so loose that a man will sink halfway to his knees in walking over them. The only place where the surface was sufficiently solid to admit of the instruments being set up was below high-water mark, where the sand was wet. A complete set of magnetic observations were made there, which, however, were not as satisfactory as could have been wished, owing to the magnets being disturbed by a stiff breeze which shook the instruments, and from which there was no shelter. The position of this station was

Lat. 24° 39′ 36″ N.
Long. 7^h 28^m $26^s.2$ W.

It was on the east side of the creek (on its left-hand bank), at a place where there is a sharp bend in its course, and can easily be found by plotting the position, given above, on the chart.

San Diego Bay, *California*. We were only in this harbor from 11 A.M. of June 15th, 1866, till 11 A.M. of June 16th, and there was no time to swing the ship. However, during the afternoon of the 15th a complete and very satisfactory set of magnetic observations were made on shore at a spot on the beach near the extreme southern end of the slightly rising ground at La Playa. The instruments were set up just above high-water mark, and nearly due east of the U.S. Coast Survey Astronomical Station. The true bearing of the light-house on Point Loma was S. 3° 56′ W., and its distance exactly two statute miles in a direct line. The spot was tested for local attraction by taking fore and back sights with a compass, but none could be detected.

The position of the station, according to the U.S. Coast Survey Chart, was

Lat. 32° 41′ 58″ N.
Long. 7^h 48^m $52^s.6$ W.

San Francisco, *California*. The ship was swung in this harbor, on June 23d, 1866, in the usual manner. Her position at the time was lat. 37° 48′ N., long. 122° 22′ W. Joint XII on the after turret was 5.3 inches to port. While we were lying here the ship was swinging freely to the wind and tide.

The observations taken on shore June 26th were made on the sand beach in a cove on the east side of Yerba Buena Island, the instruments being set up just at high-water mark, and about one hundred and fifty feet north of a long pier which runs out over a mud flat. The place was tested for local attraction by taking fore and back sights with a compass, but none could be detected.

According to the U.S. Coast Survey Chart the position of this station was

Lat. 37° 48′ 46″ N.
Long. 8^h 9^m $22^s.6$ W.

SECTION III.

ASTRONOMICAL OBSERVATIONS.

The observations contained in this section were all made on the sun, and are for the determination of latitude, local time, and true bearings. The instruments used were a prismatic sextant of six inches radius, by Pistor and Martins; a mercurial artificial horizon; and a pocket mean time chronometer, by Fletcher, marked number 906.

The index correction of the sextant was usually obtained by measuring the diameter of the sun, both on and off the arc. For determining the density of the atmosphere thermometers with Fahrenheit scales, and a mercurial barometer graduated to English inches, were employed.

The refractions have been computed by means of **Bessel's** tables, as given in **Loomis'** "Practical Astronomy;" from which book the tabular parts of the reductions to the meridian have also been taken. The necessary fundamental data have been obtained from the American Nautical Almanac.

Observations of circummeridian altitudes of the sun for latitude were made in sets of twelve, so arranged as to eliminate both the sun's semi-diameter, and all errors depending on the roof of the artificial horizon.

Circummeridian Altitudes of the Sun for Latitude, observed at the south front of Fort Christian, St. Thomas, November 17th, 1865.

10h	55m	0s	105°	14′	20″	$2\overline{\odot}$
	55	48		15	20	
	56	14		16	50	
10	57	3		18	0	
11	0	31		21	40	
	1	5		22	20	
	1	33	104	18	10	$2\underline{\odot}$
	2	9		18	20	
	2	46		18	25	
	3	28		18	50	
	3	59		18	55	
	4	29		18	40	

Index correction.

359°	11′	10″	0°	15′	50″
	11	10		16	10
	11	40		16	20
35	11	20.0	0	16	6.7

Correction = + 16′ 16″.7

Ex. ther.	83°
At. ther.	86
Bar.	30.16 inches.

Mean of chronometer times		11h	0m	2s.0
Chronometer slow of local mean time		0	40	47.3
Equation of time	+		14	47.1
Local apparent time		11	55	36.4
Mean of observed double altitudes		104°	48′	19″.2
Index correction	+		16	16.7
Apparent altitude of sun's centre		52	32	18.0
Refraction	—		0	42.1

Parallax	+ 0°	0′	5″.3
Reduction to meridian	+	1	19.4
Sun's declination	—19	6	59.1
Latitude	18°	20′	0″ N.

Circummeridian Altitudes of the Sun for Latitude, observed at Isle Royal, Salute Islands, November 28th, 1865.

10h	13m	57s.	125°	50′	30″	$2\underline{\odot}$
	14	35.5		49	30	
	15	9.5		49	30	
	15	52		49	20	
	16	24.5		48	50	
	17	1.5		48	40	
	17	38	126	52	10	$2\overline{\odot}$
	18	14.5		51	10	
	20	17		48	20	
	21	9		46	10	
	31	46.5		30	0	
	32	30		26	50	

Index correction.

359°	11′	10″	0°	16′	0″
	11	10		16	0
359	11	10	0	16	0

Correction = + 16′ 25″.0

Ex. ther. 91°
At. ther. 85
Bar. 30.13 inches.

Mean of chronometer times	10h	19m	37s.9
Chronometer slow of local mean time	1	30	19.4
Equation of time	+	11	42.6
Local apparent time	12	1	39.9
Mean of observed double altitudes	126°	15′	55″.0
Index correction	+	16	25.0
Apparent altitude of sun's centre	63	16	10.0
Refraction	—	0	27.1
Parallax	+	0	3.9
Reduction to meridian	+	2	35.7
Sun's declination	—21	24	8.5
Latitude	5°	17′	29″ N.

Observations for time were usually made in such a manner as to eliminate both the sun's semi-diameter and all errors which might be produced by the roof of the artificial horizon. For full details of the method see page 33 of the "Reports on Observations of the Total Eclipse of the Sun, August 7, 1869," published by the U. S. Naval Observatory, Washington.

The reduction of the observations for time has been effected by means of the following formulæ:

$$a = \frac{A + \omega}{2} - r + p$$

$$S = \frac{a + d + \phi}{2}$$

$$\sin \tfrac{1}{2} t = \sqrt{\sin (S - a) \cos S \sec \phi \operatorname{cosec} d}$$

$$dt = t + \tau - T$$

T = mean of observed chronometer times.
A = mean of observed double altitudes.
ω = index correction.
r = refraction.
p = parallax.

a = true geocentric altitude of sun's centre.
d = sun's polar distance, measured from the elevated pole.
ϕ = latitude of place where observation is made.
t = hour angle at the pole.
τ = equation of time.
dt = correction of chronometer to reduce the reading of its face to local mean time.

Double Altitudes of the Sun, for Time, observed at the flagstaff in the Navy-yard at Portsmouth, Va., October 29th, 1865.

8h	51m	7s	49°	27′	50″	$2\overline{\odot}$
	51	42		38	20	
	52	22		49	30	
	53	23	50	7	40	
	53	56.5		17	50	
	54	47	50	32	20	
	55	50	49	47	0	$2\underline{\odot}$
	56	25		57	20	
	56	57.5	50	6	20	
	57	59.5		24	50	
	58	32.5		34	30	
	59	13.5		45	50	

Index correction,
= + 15′ 42″

Ex. ther. 50°. At. ther. 92°. Bar. 30.40 inches.
Refraction = — 125″ Sun's declination — 13° 35′ 16″
Parallax = + 8 Latitude + 36 49 32

Mean of observed double altitudes	50°	7′	27″
Local apparent time	9h	6m	40s.8
Equation of time	—	16	10.6
Local mean time	8	50	30.2
Mean of chronometer times	8	55	11.3
Chronometer fast of local mean time	0	4	41.1
Longitude west	5	5	9.8
Chronometer slow of Greenwich mean time	5	0	28.7

Double Altitudes of the Sun for Time, observed at the flagstaff in the Navy-yard at Portsmouth, Va., October 29th, 1865.

3h	11m	55s	40°	10′	10″	$2\underline{\odot}$
	12	54	39	51	20	
	13	32.5		38	30	
	14	9.5	40	30	30	$2\overline{\odot}$
	14	51		17	0	
	15	36.5		2	20	
	16	52.5	39	37	30	
	17	37		23	10	
	18	24.5		8	30	
	19	16.5	37	46	0	$2\underline{\odot}$
	20	2		31	30	
	20	55.5		20	10	

Index correction,
=+15′ 42″

Ex. ther. 55° At. ther. 79 Bar. 30.36 inches.
Refraction = — 170″.1 Sun's declination — 13° 40′ 42″.0
Parallax = + 8.0 Latitude + 36 49 32.

Mean of observed double altitudes	39°	16′	23″.3
Local apparent time	3h	27m	51s.9

Equation of time	— 0h	16m	11s.6
Local mean time	3	11	40.3
Mean of chronometer times	3	16	20.4
Chronometer fast of local mean time	0	4	40.1
Longitude west	5	5	9.8
Chronometer slow of Greenwich mean time	5	0	29.7

Double Altitudes of the Sun for Time, observed at Fort Christian, St. Thomas, West Indies, November 13th, 1865.

9h	0m	42s.5	84°	32′	50″	$2\underline{\odot}$
	1	21.5		46	20	
	2	2		57	30	
	3	2.5	85	16	50	
	4	4		35	10	
	4	54	85	51	0	
	6	0.5	87	15	20	$2\overline{\odot}$
	6	41		28	30	
	7	10		37	0	
	7	54.5		50	20	
	8	21.5		59	20	
	8	48.5	88	7	0	

Index correction.

359°	10′	50″	0°	16′	20″
	11	0		16	10
	11	10		16	40
359	11	0.0	0	16	23.3

Correction = + 16′ 18″.4

Ex. ther. 84° At. ther. 86° Bar. 30.12 inches.
Refraction = — 57″.7 Sun's declination — 18° 5′ 2″.5
Parallax = + 6.2 Latitude + 18 20 27.

Mean of observed double altitudes	86°	26′	25″.8
Local apparent time	10h	1m	20s.0
Equation of time	—	15	31.2
Local mean time	9	45	48.8
Mean of chronometer times	9	5	5.2
Chronometer slow of local mean time	0	40	43.6
Longitude west	4	19	42.7
Chronometer slow of Greenwich mean time	5	0	26.3

Double Altitudes of the Sun for Time, observed at Isle Royal, Salute Islands, November 28th, 1865.

8h	47m	58s	109°	58′	20″	$2\underline{\odot}$
	48	35	110	9	50	
	49	8		20	0	
	49	58		35	30	
	50	31		45	50	
	50	56.5		52	50	
	51	44.5	112	13	0	$2\overline{\odot}$
	52	39.5		30	0	
	53	13.5		40	0	
	53	47		50	0	
	54	19	113	0	0	
	54	53.5		10	0	

Index correction.

359°	11′	0″	0°	15′	50″
	11	0		16	0
	10	50		16	10
359	10	56.7	0	16	0.0

Correction = + 16′ 31″.6

Ex. ther. 93° At. ther. 85° Bar. 30.13 inches.
Refraction = — 36″.3 Sun's declination — 21° 23′ 30″.3
Parallax = + 4.9 Latitude + 5 17 29.

Mean of observed double altitudes	111°	35′	26″.6
Local apparent time	10h	33m	31s.8
Equation of time	—	11	43.8
Local mean time	10	21	48.0
Mean of chronometer times	8	51	28.6

Chronometer slow of local mean time.	1^{h} 30^{m} $19^{\mathrm{s}}.4$
Longitude west	3 30 11.4
Chronometer slow of Greenwich mean time	5 0 30.8

Double Altitudes of the Sun for Time, observed at Ceara, Brazil, December 13*th*, 1865.

1^{h}	15^{m}	$13^{\mathrm{s}}.5$	63°	0′	0″	$2\underline{\odot}$
	15	58.5	62	40	0	
	16	41		20	0	
	17	3.5		10	0	
	17	26	62	0	0	
	18	43	62	30	0	$2\overline{\odot}$
	19	5		20	0	
	19	26.5		10	0	
	19	50	62	0	0	
	20	11.5	61	50	0	

Index correction.

359°	11′	0″	0°	16′	0″
	10	50			10
	10	40			0
359	10	50.0	0	16	3.3

Correction $= +16' \; 33''.3$

Ex. ther. 84° At. ther. 82° Bar. 30.05 inches.

Refraction $= -89''.5$ Sun's declination $-23° \; 12' \; 4''.0$

Parallax $= + \; 7.4$

Mean of observed double altitudes	62° 18′ 0″.0
Mean of chronometer times	1^{h} 17^{m} $57^{\mathrm{s}}.8$
Equation of time	— 5 20.9

Reducing this observation with latitude = — 3° 43′ 15″, we find the chronometer 2^{h} 26^{m} $29^{\mathrm{s}}.6$ slow of local mean time. Reducing it with latitude = — 3° 44′ 15″, we find the chronometer 2^{h} 26^{m} $32^{\mathrm{s}}.0$ slow of local mean time.

Double Altitudes of the Sun for Time, observed at Ceara, Brazil, December 14*th*, 1865.

7^{h}	2^{m}	$0^{\mathrm{s}}.5$	99°	30′	0″	$2\overline{\odot}$
	2	24.5		40	0	
	2	49		50	0	
	3	12.5	100	0	0	
	3	36		10	0	
	6	9	100	10	0	$2\underline{\odot}$
	6	32.5		20	0	
	6	57.5		30	0	
	7	21.5		40	0	
	7	45.5	100	50	0	

Index correction.

359°	10′	30″	0°	16′	10″
		40			20
		40			20
359	10	36.7	0	16	16.7

Correction $= +16' \; 33''.3$

Ex. ther. 81° At. ther. 82° Bar. 30.12 inches.

Refraction $= -45''.9$ Sun's declination $-23° \; 14' \; 46''.2$

Parallax $= + \; 5.6$

Mean of observed double altitudes	100° 10′ 0″.0
Mean of chronometer times	7^{h} 4^{m} $52^{\mathrm{s}}.8$
Equation of time	— 4 59.5

Reducing this observation with latitude = — 3° 43′ 15″, we find the chronometer 2^{h} 26^{m} $33^{\mathrm{s}}.7$ slow of local mean time. Reducing it with latitude = — 3° 44′ 15″, we find the chronometer 2^{h} 26^{m} $30^{\mathrm{s}}.9$ slow of local mean time.

Double Altitudes of the Sun for Time, observed at Ceara, Brazil, December 14th, 1865.

11ʰ 51ᵐ 51ˢ	100° 50′ 0″	$2\overline{\odot}$
52 14.5	40 0	
52 37	30 0	
53 1.5	20 0	
53 26	10 0	
56 0	98 0 0	$2\underline{\odot}$
56 23	97 50 0	
56 48	40 0	
57 11.5	30 0	
57 34	20 0	

Index correction.

359° 10′ 50″	0° 15′ 50″
40	16 20
20	16 0
359 10 36.7	0 16 3.3

Correction = + 16′ 40″.0

Ex. ther. 86° At. ther. 83° Bar. 30.00 inches.

Refraction = — 45″.6 Sun's declination — 23° 15′ 27″.4

Parallax = + 5.6

Mean of observed double altitudes	99° 5′ 0″.0
Mean of chronometer times	11ʰ 54ᵐ 42ˢ.6
Equation of time	— 4 53.7

Reducing this observation with latitude = — 3° 43′ 15″, we find the chronometer 2ʰ 26ᵐ 30ˢ.7 slow of local mean time. Reducing it with latitude = — 3° 44′ 15″, we find the chronometer 2ʰ 26ᵐ 33ˢ.1 slow of local mean time.

In order to determine both the latitude of Ceara and the error of the chronometer from the three observations which have just been given, we proceed as follows:

Comparing the error obtained on the afternoon of December 13th, with that obtained on the afternoon of December 14th, we find that the chronometer was losing 1.17 seconds per day; and this rate is independent of any small change in the adopted value of the latitude.

By means of this rate, reducing all the observed chronometer errors to 2ʰ 26ᵐ P. M. December 14th, and then plotting them according to Sumner's method, we get for the place of observation

Latitude 3° 43′ 59″ S.

and for the chronometer,

Chronometer slow of local mean time	2ʰ 26ᵐ 32ˢ.5
Longitude west	2 34 6
Chronometer slow of Greenwich mean time	5 0 38.5

Double Altitudes of the Sun for Time, observed at Pernambuco, Brazil, December 23d, 1865.

7ʰ 30ᵐ 15ˢ	118° 10′ 0″	$2\overline{\odot}$
30 39.5	20 0	
31 3	30 0	
32 52.5	118 10 0	$2\underline{\odot}$
33 15	20 0	
33 40	30 0	

Index correction.

359° 10′ 50″	0° 16′ 0″
50	16 10
359 10 50.0	0 16 5.0

Correction = + 16′ 32″.5

Ex. ther. 83° At. ther. Bar.

Refraction = — 32″.1 Sun's declination — 23° 26′ 31″

Parallax = + 4.5 Latitude — 8 3 37

Mean of observed double altitudes	118° 20′ 0″.0
Local apparent time	10ʰ 9ᵐ 3ˢ.5
Equation of time	— 0 31.2
Local mean time	10 8 32.3

Mean of chronometer times	7^h 31^m $57^s.5$
Chronometer slow of local mean time	2 36 34.8
Longitude west	2 19 28.2
Chronometer slow of Greenwich mean time	4 56 3.0

Double Altitudes of the Sun for Time, observed at Bahia, Brazil, December 27th, 1865.

6^h 52^m 10^s	98° 30′ 0″	$2\overline{\odot}$
52 31.5	40 0	
52 54.5	50 0	
54 32	98 30 0	$2\underline{\odot}$
54 53.5	40 0	
55 16.5	50 0	

Index correction.

359° 10′ 40″	0° 16′ 10″
50	0
359 10 45.0	0 16 5.0

Correction = +16′ 35″.0

Ex. ther. 88°	At. ther.	Bar.	
Refraction = −45″.9		Sun's declination	−23° 19′ 33″.8
Parallax = + 5.7		Latitude	−12 56 55.

Mean of observed double altitudes	98° 40′ 0″.0
Local apparent time	9^h 14^m $22^s.5$
Equation of time	+ 1 27.3
Local mean time	9 15 49.8
Mean of chronometer times	6 53 43.0
Chronometer slow of local mean time	2 22 6.8
Longitude west	2 34 0.5
Chronometer slow of Greenwich mean time	4 56 7.3

Double Altitudes of the Sun for Time, observed at the Light-house in Fort St. Antonio, Bahia, Brazil, December 29th, 1865.

8^h 14^m $46^s.5$	134° 50′ 0″	$2\overline{\odot}$
15 10	135 0 0	
15 31	10 0	
15 56	20 0	
16 19.5	30 0	
17 17.5	134 50 0	$2\underline{\odot}$
17 44	135 0 0	
18 7	10 0	
18 31.5	20 0	
18 54	30 0	

Index correction.

359° 10′ 50″	0° 16′ 0″
50	10
40	10
359 10 46.7	0 16 6.7

Correction = +16′ 33″.3

Ex. ther. 84°	At. ther.	Bar.	
Refraction = −22″.1		Sun's declination	−23° 13′ 31″.1
Parallax = + 3.3		Latitude	−13 0 55.

Mean of observed double altitudes	135° 10′ 0″.0
Local apparent time	10^h 36^m $25^s.7$
Equation of time	+ 2 27.6
Local mean time	10 38 53.3
Mean of chronometer times	8 16 49.7
Chronometer slow of local mean time	2 22 3.6
Longitude west	2 34 6.9
Chronometer slow of Greenwich mean time	4 56 10.5

Double Altitudes of the Sun for Time, observed at Rio Janeiro, Brazil, January 9th, 1866.

5^{h}	13^{m}	17^{s}	47°	40′	0″	$2\overline{\odot}$
	13	39		50	0	
	14	3.5	48	0	0	
	14	26.5		10	0	
	15	43	47	40	0	$2\underline{\odot}$
	16	8		50	0	
	16	29	48	0	0	
	16	53		10	0	

Index correction.

359°	10′	40″	0°	16′	0″
		30		15	50
359	10	35.0	0	15	55.0

Correction $= +16' \; 45''.0$

Ex. ther. 74° At. ther. 77° Bar. 29.94 inches.

Refraction $= -123''.2$.

Parallax $= + \; 7.9$

Sun's declination $-22° \; 6' \; 24''.6$

Latitude $-22 \; 54 \; 5$.

Mean of observed double altitudes	47°	55′	0″.0
Local apparent time	7^{h}	11^{m}	$19^{s}.5$
Equation of time	+	7	23.8
Local mean time	7	18	43.3
Mean of chronometer times	5	15	4.9
Chronometer slow of local mean time	2	3	38.4
Longitude west	2	52	30.7
Chronometer slow of Greenwich mean time	4	56	9.1

Double Altitudes of the Sun for Time, observed at Rat Island, harbor of Rio Janeiro, January 9th, 1866.

7^{h}	27^{m}	0^{s}	108°	0′	0″	$2\overline{\odot}$
	27	20		10	0	
	27	42.5		20	0	
	28	4.5		30	0	
	28	26.5		40	0	
	29	21	108	0	0	$2\underline{\odot}$
	29	45		10	0	
	30	5		20	0	
	30	26.5		30	0	
	30	48		40	0	

Index correction.

359°	10′	30″	0°	15′	50″
		40			50
		40			50
359	10	36.7	0	15	50.0

Correction $= +16' \; 46''.6$

Ex. ther. 75° At. ther. 77° Bar. 29.94 inches.

Refraction $= -39''.8$

Parallax $= + \; 5.1$

Sun's declination $-22° \; 5' \; 37''.3$

Latitude $-22 \; 53 \; 45$.

Mean of observed double altitudes	108°	20′	0″.0
Local apparent time	9^{h}	25^{m}	$0^{s}.7$
Equation of time	+	7	26.0
Local mean time	9	32	26.7
Mean of chronometer times	7	28	53.9
Chronometer slow of local mean time	2	3	32.8
Longitude west	2	52	37.9
Chronometer slow of Greenwich mean time	4	56	10.7

Double Altitudes of the Sun for Time, observed at Monte Video, Uruguay, January 18*th,* 1866.

4^{h}	0^{m}	$26^{s}.5$	45°	50′	0″	$2\underline{\odot}$
	0	51.5		40	0	
	1	17		30	0	
	2	3.5		10	0	
	3	5.5	45	50	0	$2\overline{\odot}$
	3	30		40	0	
	3	56.5		30	0	
	4	46		10	0	

Index correction.

359°	10′	30″	0°	15′	50″
		40			40
		40			40
359	10	36.7	0	15	43.3

Correction $= +16' \; 50''.0$

Ex. ther. 76° At. ther. 79° Bar. 30.02 inches.
Refraction = − 130″.2 Sun's declination − 20° 26′ 55″.2
Parallax = + 8.0 Latitude − 34 53 39

Mean of observed double altitudes	45° 32′ 30″.0
Local apparent time	5^h 3^m $5^s.2$
Equation of time	+ 10 51.4
Local mean time	5 13 56.6
Mean of chronometer times	4 2 29.6
Chronometer slow of local mean time	1 11 27.0
Longitude west	3 44 55.8
Chronometer slow of Greenwich mean time	4 56 22.8

Double Altitudes of the Sun for Time, observed on Rat Island, harbor of Monte Video, Uruguay, January 24th, 1866.

2^h 29^m $1^s.5$	82° 30′ 0″	$2\underline{\odot}$
29 25.5	20 0	
29 50.5	10 0	
30 13.5	82 0 0	
30 38.5	81 50 0	
31 38.5	82 30 0	$2\overline{\odot}$
32 3	20 0	
32 26	10 0	
32 51	82 0 0	
33 16	81 50 0	

Index correction.

359° 10′ 10″	0° 15′ 40″
10 10	40
10 10	20
359 10 10.0	0 15 33.3

Correction = + 17′ 8″.3

Ex. ther. 74° At. ther. Bar.
Refraction = − 62″.7 Sun's declination − 19° 6′ 33″.8
Parallax = + 6.5 Latitude − 34 53 18

Mean of observed double altitudes	82° 10′ 0″.0
Local apparent time	3^h 30^m $5^s.7$
Equation of time	+ 12 29.2
Local mean time	3 42 34.9
Mean of chronometer times	2 31 8.4
Chronometer slow of local mean time	1 11 26.5
Longitude west	3 44 52.9
Chronometer slow of Greenwich mean time	4 56 19.4

Double Altitudes of the Sun, for Time, observed at Sandy Point, in the Straits of Magellan, February 7th, 1866.

9^h 59^m $24^s.5$	90° 30′ 0″	$2\underline{\odot}$
10 0 11	40 0	
1 1	50 0	
1 49.5	91 0 0	
2 37.5	10 0	
4 39.5	90 30 0	$2\overline{\odot}$
5 27.5	40 0	
6 18.5	50 0	
7 9	91 0 0	
7 58.5	10 0	

Index correction.

359° 10′ 20″	0° 15′ 40″
30	50
35	35
359 10 28.3	0 15 41.7

Correction = + 16′ 55″.0

Ex. ther. 52° At. ther. 70° Bar. 30.04 inches.
Refraction = − 56″.9 Sun's declination − 15° 14′ 15″.6
Parallax = + 6.1 Latitude − 53 10 20

Mean of observed double altitudes	90° 50′ 0″
Local apparent time	10^h 2^m $2^s.2$
Equation of time	+ 14 25.5

Local mean time	10h	16m	27s.7
Mean of chronometer times	10	3	39.6
Chronometer slow of local mean time	0	12	48.1
Longitude west	4	43	35.3
Chronometer slow of Greenwich mean time	4	56	23.4

Double Altitudes of the Sun for Time, observed near Valparaiso, Chile, March 2d, 1866.

3h	50m	15s.5	62°	0′	0″	$2\underline{\odot}$
	50	39.5	61	50	0	
	51	3		40	0	
	51	51.5		20	0	
	52	52	62	0	0	$2\overline{\odot}$
	53	15.5	61	50	0	
	53	39.5		40	0	
	54	30		20	0	

Index correction.

359°	10′	40″	0°	15′	0″
		45			5
		40			10
359	10	41.7	0	15	5.0

Correction = + 17′ 6″.6

Ex. ther. 67° At. ther. Bar.

Refraction = — 92″.4

Parallax = + 7.4

Sun's declination — 7° 1′ 53″

Latitude — 33 1.4

Mean of observed double altitudes	61°	42′	30″.0
Local apparent time	3h	49m	44s.3
Equation of time	+	12	17.9
Local mean time	4	2	2.2
Mean of chronometer times	3	52	15.8
Chronometer slow of local mean time	0	9	46.4
Longitude west	4	46	31
Chronometer slow of Greenwich mean time	4	56	17.4

Double Altitudes of the Sun for Time, observed in Valparaiso, Chile, March 29th, 1866.

2h	36m	55s	73°	30′	0″	$2\underline{\odot}$
	37	40		15	0	
	38	23		0	0	
	40	1.5	73	30	0	$2\overline{\odot}$
	40	45.5		15	0	
	41	28.5		0	0	

Index correction.

359°	10′	20″	0°	14′	50″
		50			45
		45			55
359	10	38.3	0	14	50.0

Correction = + 17′ 15″.8

Ex. ther. 71° At. ther. 69° Bar. 30.23 inches.

Refraction = — 75″.1

Parallax = + 6.9

Sun's declination + 3° 31′ 38″

Latitude — 33 1 47

Mean of observed double altitudes	73°	15′	0″.0
Local apparent time	2h	43m	52s.0
Equation of time	+	4	47.0
Local mean time	2	48	39.0
Mean of chronometer times	2	39	12.2
Chronometer slow of local mean time.	0	9	26.8
Longitude west	4	46	45.7
Chronometer slow of Greenwich mean time	4	56	12.5

Double Altitudes of the Sun for Time, observed in Valparaiso, Chile, April 7th, 1866.

9h	36m	26s.5	77°	30′	0″	$2\overline{\odot}$
	37	16.5		45	0	
	38	9	78	0	0	
	40	1.5	77	30	0	$2\underline{\odot}$
	40	53		45	0	
	41	44.5	78	0	0	

Index correction.

359°	10′	50″	0°	15′	10″
		50			10
		50			10
359	10	50.0	0	15	10.0

Correction = + 17′ 0″.0

Ex. ther. 67° At. ther. 65° Bar. 30.17 inches.
Refraction = — 69″.8 Sun's declination + 6° 53′ 28″.6
Parallax = + 6.7 Latitude — 33 1 47

Mean of observed double altitudes	77°	45′	0″.0
Local apparent time	9h	46m	19s.6
Equation of time	+	2	8.9
Local mean time	9	48	28.5
Mean of chronometer times	9	39	5.2
Chronometer slow of local mean time	0	9	23.3
Longitude west	4	46	45.7
Chronometer slow of Greenwich mean time	4	56	9.0

Double Altitudes of the Sun for Time, observed in Valparaiso, Chile, April 7th, 1866.

9h	43m	15s.5	79°	30′	0″	$2\overline{\odot}$
	44	6.5		45	0	
	45	0.5	80	0	0	
	46	57	79	30	0	$2\underline{\odot}$
	47	49.5		45	0	
	48	44.5	80	0	0	

Index correction

= + 17′ 0″.0

Ex. ther. 67° At. ther. 65° Bar. 30.17 inches.
Refraction = — 67″.3 Sun's declination — 6° 53′ 35″.4
Parallax = + 6.6 Latitude — 33 1 47

Mean of observed double altitudes	79°	45′	0″.0
Local apparent time	9h	53m	14s.0
Equation of time	+	2	8.8
Local mean time	9	55	22.8
Mean of chronometer times	9	45	58.9
Chronometer slow of local mean time	0	9	23.9
Longitude west	4	46	45.7
Chronometer slow of Greenwich mean time	4	56	9.6

Double Altitudes of the Sun for Time, observed in Valparaiso, Chile, April 14*th,* 1866.

3h	50m	20s.5	36°	30′	0″	$2\underline{\odot}$
	51	1.5		15	0	
	51	39		0	0	
	53	7	36	30	0	$2\overline{\odot}$
	53	46		15	0	
	54	24.5		0	0	

Index correction.

359°	10′	40″	0°	14′	50″
		40			45
		45			50
359	10	41.6	0	14	48.3

Correction = + 17′ 15″.0

Ex. ther. 65° At. ther. 66° Bar. 30.13 inches.
Refraction = — 170″.3 Sun's declination + 9° 33′ 33″.6
Parallax = + 8.1 Latitude — 33 1 47

Mean of observed double altitudes	36°	15′	0″.0
Local apparent time	4h	3m	13s.2
Equation of time	+	0	11.6
Local mean time	4	3	24.8
Mean of chronometer times	3	52	23.1
Chronometer slow of local mean time	0	11	1.7
Longitude west	4	46	45.7
Chronometer slow of Greenwich mean time	4	57	47.4

Double Altitudes of the Sun for Time, observed on the Island of San Lorenzo, near Callao, Peru, April 26th, 1866.

Chronometer	Double altitude	
11h 17m 45s	123° 0′ 0″	$2\overline{\odot}$
18 52	15 0	
20 3	30 0	
22 46	123 0 0	$2\underline{\odot}$
24 2	15 0	
25 18	30 0	

Index correction.

359° 11′ 10″	0° 15′ 0″
10	0
359 11 10.0	0 15. 0.0

Correction = + 16′ 55″.0

Ex. ther.	80°	At. ther.		Bar.	
Refraction	= — 29″.2.			Sun's declination	+ 13° 35′ 18″
Parallax	= + 4.0			Latitude	— 12 5 14

Mean of observed double altitudes	123° 15′ 0″.0
Local apparent time	11h 12m 33s.0
Equation of time	— 2 18.8
Local mean time	11 10 14.2
Mean of chronometer times	11 21 27.7
Chronometer fast of local mean time	0 11 13.5
Longitude west	5 9 9.1
Chronometer slow of Greenwich mean time	4 57 55.6

Double Altitudes of the Sun for Time, observed at Payta, Peru, May 7th, 1866.

Chronometer	Double altitude	
8h 40m 44s.5	62° 0′ 0″	$2\overline{\odot}$
41 17.5	15 0	
41 51	30 0	
43 1.5	62 0 0	$2\underline{\odot}$
43 34.5	15 0	
44 7.5	30 0	

Index correction.

359° 11′ 30″	0° 15′ 0″
25	0
25	0
359 11 26.7	0 15 0.0

Correction = + 16′ 46″.6

Ex. ther.	78°	At. ther.	80°	Bar.	30.06 inches.
Refraction	= — 90″.7			Sun's declination	+ 16° 50′ 46″
Parallax	= + 7.3			Latitude	— 5 5 36

Mean of observed double altitudes	62° 15′ 0″.0
Local apparent time	8h 19m 22s.3
Equation of time	— 3 38.1
Local mean time	8 15 44.2
Mean of chronometer times	8 42 26.1
Chronometer fast of local mean time	0 26 41.9
Longitude west	5 24 22.0
Chronometer slow of Greenwich mean time	4 57 40.1

Double Altitudes of the Sun for Time, observed on Flamenco Island, Panama Bay, May 14th, 1866.

Chronometer	Double altitude	
9h 24m 59s	95° 0′ 0″	$2\overline{\odot}$
25 31	15 0	
26 3.5	30 0	
27 12	95 0 0	$2\underline{\odot}$
27 43.5	15 0	
28 15	30 0	

Index correction.

359° 11′ 30″	0° 15′ 10″
20	14 55
20	14 40
359 11 23.3	0 14 55.0

Correction = + 16′ 50″.8

Ex. ther.	85°	At. ther.	85°	Bar.	30.10 inches.
Refraction	= — 49″.5			Sun's declination	+ 18° 39′ 49″
Parallax	= + 5.7			Latitude	+ 8 54 31

Mean of observed double altitudes	95°	15′	0″.0
Local apparent time	9^{h}	10^{m}	$13^{s}.5$
Equation of time	—	3	53.1
Local mean time	9	6	20.4
Mean of chronometer times	9	26	37.3
Chronometer fast of local mean time	0	20	16.9
Longitude west	5	18	1.8
Chronometer slow of Greenwich mean time	4	57	44.9

Double Altitudes of the Sun for Time, observed at Acapulco, Mexico, May 30th, 1866.

10^{h}	25^{m}	36^{s}	89°	0′	0″	$2\overline{\odot}$
	26	5.5		15	0	
	26	38.5		30	0	
	27	49.5	89	0	0	$2\underline{\odot}$
	28	22		15	0	
	28	54		30	0	

Index correction.

359°	11′	10″	0°	15′	0″
		0		14	40
		20		15	0
359	11	10.0	0	14	53.3

Correction = + 16′ 58″.3

Ex. ther. 89° At. ther. 85° Bar. 30.10 inches.
Refraction = — 54″.5 Sun's declination + 21° 48′ 7″
Parallax = + 6.0 Latitude + 16 50 3

Mean of observed double altitudes	89°	15′	0″.0
Local apparent time	8^{h}	48^{m}	$38^{s}.4$
Equation of time	—	2	46.4
Local mean time	8	45	52.0
Mean of chronometer times	10	27	14.2
Chronometer fast of local mean time	1	41	22.2
Longitude west	6	39	29.4
Chronometer slow of Greenwich mean time	4	58	7.2

Double Altitudes of the Sun for Time, observed in Magdalena Bay, Lower California, June 8th, 1866.

5^{h}	20^{m}	49^{s}	100°	45′	0″	$2\underline{\odot}$
	21	23		30	0	
	21	56		15	0	
	23	8.5	100	45	0	$2\overline{\odot}$
	23	41.5		30	0	
	24	5		15	0	

Index correction.

359°	10′	50″	0°	14′	40″
	11	20		14	50
	10	30		15	0
359	10	53.3	0	14	50.0

Correction = + 17′ 8″.4

Ex. ther. 69° At. ther. 70° Bar. 30.02 inches.
Refraction = — 46″.4 Sun's declination + 22° 53′ 42″
Parallax = + 5.4 Latitude + 24 38

Mean of observed double altitudes	100°	30′	0″.0
Local apparent time	2^{h}	53^{m}	$42^{s}.3$
Equation of time	—	1	14.5
Local mean time	2	52	27.8
Mean of chronometer times	5	22	32.2
Chronometer fast of local mean time	2	30	4.4
Longitude west	7	28	24.0
Chronometer slow of Greenwich mean time	4	58	19.6

Double Altitudes of the Sun for Time, observed at La Playa, San Diego Bay, California, June 15th, 1866.

$5^h\ 16^m\ 41^s.$	112° 30′ 0″	$2\underline{\odot}$
17 16	15 0	
17 51.5	0 0	
19 10	112 30 0	$2\overline{\odot}$
19 46	15 0	
20 21.5	0 0	

Index correction.

359° 11′ 30″	0° 14′ 50″
35	30
20	.50
359 11 28.3	0 14 43.3

Correction = + 16′ 54″.2

Ex. ther.	71°	At. ther.	72°	Bar.	30.12 inches.
Refraction	= — 37″.4			Sun's declination	+ 23° 20′ 22″
Parallax	= + 4.7			Latitude	+ 32 41 58

Mean of observed double altitudes	112° 15′ 0″.0
Local apparent time	$2^h\ 27^m\ 47^s.3$
Equation of time	+ 0 11.3
Local mean time	2 27 58.6
Mean of chronometer times	5 18 31.1
Chronometer fast of local mean time	2 50 32.5
Longitude west	7 48 52.6
Chronometer slow of Greenwich mean time	4 58 20.1

Double Altitudes of the Sun for Time, observed on Yerba Buena Island, San Francisco Bay, California, June 26th, 1866.

$4^h\ 16^m\ 40^s.5$	75° 15′ 0″	$2\overline{\odot}$
17 18	30 0	
17 55.5	45 0	
19 18.5	75 15 0	$2\underline{\odot}$
19 54.5	30 0	
20 30	45 0	

Index correction.

359° 11′ 30″	0° 14′ 30″
35	50
25	.50
359 11 30.0	0 14 43.3

Correction = + 16′ 53″.4

Ex. ther.	67°	At. ther.		Bar.	
Refraction	= — 72″.5			Sun's declination	+ 23° 22″ 7″
Parallax	= + 6.6			Latitude	+ 37 48 46

Mean of observed double altitudes	75° 30′ 0″.0
Local apparent time	$8^h\ 2^m\ 58^s.4$
Equation of time	+ 2 29.6
Local mean time	8 5 28.0
Mean of chronometer times	4 18 36.2
Chronometer fast of local mean time	8 13 8.2
Longitude west	8 9 22.6
Chronometer fast of Greenwich mean time	0 3 45.6

The chronometer used in making this observation was T. S. and J. D. Negus' No. 1287.

True bearings were determined by measuring with a sextant the angle between the sun's limb and some well-defined terrestrial object, the time being noted at the instant the angle was observed. If the terrestrial object was much elevated above the horizon its angular altitude was also measured. Knowing the latitude of the place of observation, the local time, and the sun's declination, the sun's zenith distance and true bearing were calculated. Then, having the zenith distance of the sun, the zenith distance of the terrestrial object, and the measured angle between the sun and the terrestrial object, the horizontal angle between them was computed, and applying it to the sun's true bearing the true bearing of the terrestrial object at once became known.

The formulæ employed were as follows. Let

T = mean of observed chronometer times.

dt = correction of chronometer to reduce the reading of its face to local mean time.

τ = equation of time.

t = sun's hour angle, or the apparent time.

Ω = mean of observed angular distances between the sun's limb and the terrestrial object.

ω = index correction of sextant.

s = sun's semi-diameter.

a = apparent zenith distance of sun's centre.

b = zenith distance of terrestrial object.

c = true angular distance between the sun's centre and the terrestrial object.

C = horizontal angle included between the sun's centre and the terrestrial object.

ϕ = latitude of the place of observation.

A = azimuth, or true bearing, of sun's centre.

ζ = true zenith distance of sun's centre.

δ = sun's declination.

r = refraction due to apparent altitude of sun's limb.

B = true bearing of terrestrial object.

Then we have

$$t = T + dt + \tau$$

$$\tan M = \frac{\tan \delta}{\cos t}$$

$$\tan A = \frac{\tan t \cos M}{\sin (\phi - M)}$$

$$\tan \zeta = \frac{\tan (\phi - M)}{\cos A}$$

where A is to be taken greater or less than 180°, according as t is greater or less than 180°.

$$a = \zeta - r$$

$$c = \Omega + \omega + s$$

If b is exactly 90°, we have

$$\cos C = \frac{\cos c}{\sin a}$$

But if b is either greater or less than 90°, we have

$$S = \frac{a + b + c}{2}$$

$$\tan \tfrac{1}{2} C = \sqrt{\frac{\sin (S - a) \sin (S - b)}{\sin S \sin (S - c)}}$$

Finally

$$B = A \pm C$$

In a few instances true bearings were obtained by observing the sun when its apparent elevation above the horizon was equal to its diameter. In that case

$$\zeta = 90^\circ$$

and then

$$\cos A = \frac{\sin \delta}{\cos \phi}$$

in which the azimuth will be north or south of the prime vertical according as the sun's declination is north or south.

Observations of the Sun, made October 31st, 1865, to determine the true bearing of the object used as an azimuth mark in swinging the ship at Hampton Roads, Va.

	10^h 10^m 50^s		127° 20′
	11 45		38
	12 15		45
	14 0		128 4
	14 39		8
T	10 12 42	Ω	127 47
Chronometer fast	0 4 50	ω	+ 16
τ	+ 16 16	s	+ 16
Apparent time	10 24 8	c	128 19
t	23° 58′	ζ	55 59
δ	— 14 16	r	— 1
ϕ	36 58	a	55 58
M	— 15 33	b nearly	90
$\phi - M$	52 31	C	138 26

True bearing of sun	S. 28° 21′ E.
∠ Seminary to sun	138 26
∠ Seminary to Rip Raps	62 44
∠ Rip Raps to tree	114 37
True bearing of tree	S. 10 34 W.

Observations of the Sun, made November 18th, 1865, to determine the true bearing of the object used as an azimuth mark in swinging the ship at St. Thomas, West Indies.

	7^h 0^m 5^s		34° 13′
	2 15		15
	4 45		10
	8 15		12
	9 45		12
T	7 5 1	Ω	34 12
Chronometer slow	0 40 47	ω	+ 16
τ	+ 14 36	s	+ 16
Apparent time	8 0 24	c	34 44
t	59° 54′	ζ	69 48
δ	— 19 19	r	— 2
ϕ	18 20	a	69 46
M	— 34 57	b nearly	90
$\phi - M$	53 17	C	28 52

True bearing of sun	S. 60° 27′ E.
∠ Sun to Peak	28 52
True bearing of Peak	S. 31 35 E.

Observations of the Sun, made November 28th, 1865, *to determine the true bearing of the object used as an azimuth mark in swinging the ship at Isle Royal, Salute Islands.*

	6^{h} 27^{m} 5^{s}		74° 50′
	28 59		46
	31 8		40
T	6 29 4	Ω	74 45
Chronometer slow	1 30 19	ω	+ 17
τ	+ 11 45	s	+ 16
Apparent time	8 11 8	c	75 18
t	57° 13′	ζ	62 4
δ	−21 22	r	− 2
ϕ	5 17	a	62 2
M	−35 52	b nearly	90
$\phi - M$	41 9	C	73 18

True bearing of sun	S. 62° 24′ E.
∠ Sun to Nob	73 18
True bearing of Nob	S. 10 54 W.

Observations of the Sun, made December 12th, 1865, *to determine the true bearing of the object used as an azimuth mark in swinging the ship at Ceara, Brazil.*

	3^{h} 11^{m} 8^{s}		87° 30′
	13 0		22
	14 32		21
T	3 12 53	Ω	87 24
Chronometer slow	2 26 32	ω	+ 16
τ	+ 5 47	s	+ 16
Apparent time	5 45 12	c	87 56
t	86° 18′	ζ	85 4
δ	−23 8	r	− 18
ϕ	− 3 43	a	84 46
M	−81 25	b nearly	90
$\phi - M$	77 42	C	87 56

True bearing of sun	S. 67° 3′ W.
∠ Lantern to sun	87 56
∠ Light-house to Lantern	77 0
True bearing of Light-house	N. 82 7 E.

Observations of the Sun, made December 29th, 1865, *to determine the true bearing of the object used as an azimuth mark in swinging the ship at Bahia, Brazil.*

When the sun's true zenith distance was about 90°, the angle between its nearest limb and a conspicuous tree was measured and found to be 31° 38′, the tree being to the right of the sun.

$\phi = -12° 59'$ $\delta = -23° 12'$

True bearing of sun	S. 66° 9′ W.
∠ Sun to tree	31 38
Sun's semi-diameter	0 16
True bearing of tree	N. 81 57 W.

Observations of the Sun, made January 7th, 1866, *to determine the true bearing of the object used as an azimuth mark in swinging the ship at Rio Janeiro, Brazil.*

	5^h 51^m 30^s		112° 2
	53 45		7
	55 0		12
T	5 53 25	Ω	112 15
Chronometer slow	2 3 32	ω	+ 17
τ	— 6 36	s	—
Apparent time	7 50 21	c	112 32
t	62° 25′	ζ	57 9
δ	— 22 22	r	— 1
ϕ	— 22 54	a	57 8
M	— 41 38	b	85 16
$\phi - M$	18 44	C	120 45

True bearing of sun	S. 77° 21′ E.
∠ Sun to Corcovado	120 45
∠ Corcovado to building	83 8
True bearing of building	N. 53 28 W.

Observations of the Sun, made January 23d, 1866, *to determine the true bearing of the object used as an azimuth mark in swinging the ship at Monte Video, Uruguay.*

Near sunset, when the true zenith distance of the sun was about 90°, the angle between its nearest limb and the Light-house on the Mount, on the west side of the harbor, was measured. The uncorrected reading of the sextant was 69° 40′, and the sun was to the left of the Light-house.

Ω	69° 40′	ϕ	—34° 53′
ω	+ 17	δ	—19 19
s	+ 16		
c	70 13		

True bearing of sun	S. 66° 13′ W.
∠ Sun to Light-house	70 13
∠ Hillock to Light-house	34 18
True bearing of hillock	N. 77 52 W.

Observations of the Sun, made February 9th, 1866, *to determine the true bearing of the object used as an azimuth mark in swinging the ship at Sandy Point, in the Straits of Magellan.*

	9^h 13^m 57^s		119° 15′
	15 19		32
	16 40		42
T	9 15 19	Ω	119 30
Chronometer slow	0 12 48	ω	+ 17
τ	— 14 30	s	+ 16
Apparent time	9 13 37	c	120 3
t	— 41° 36′	ζ	50 32
δ	— 14 37	r	— 1
ϕ	— 53 11	a	50 31
M	— 19 14	b	89 34
$\phi - M$	33 57	C	130 54

True bearing of sun	N. 56° 20′ E.
∠ Mount St. Felipe to sun	130 54
True bearing of Mount St. Felipe	S. 7 14 W.

Observations of the Sun, made April 2d, 1866, to determine the true bearing of the object used as an azimuth mark in swinging the ship at Valparaiso, Chile.

	5^h 10^m 5^s		110° 20′
	11 20		35
	12 10		42
T	5 11 12	Ω	110 32
Chronometer slow	0 9 25	ω	+ 17
τ	— 3 32	*s*	—
Apparent time	5 17 5	*c*	110 49
t	79° 16′	ζ	83 52
δ	5 7	*r*	— 8
φ	— 33 2	*a*	83 44
M	+ 25 40	*b* nearly	90
φ—*M*	— 58 42	*C*	110 56
True bearing of sun			N. 79° 49′ W.
∠ Sun to Point			110 56
True bearing of Point			N. 31 7 E.

Observations of the Sun, made April 27th, 1866, to determine the true bearing of the object used as an azimuth mark in swinging the ship at Callao, Peru.

	7^h 0^m 30^s		100° 50′
	2 20		55
	3 50		101 1
T	7 2 13	Ω	100 55
Chronometer fast	0 11 1	ω	+ 17
τ	+ 2 27	*s*	—
Apparent time	6 53 39	*c*	101 12
t	— 76° 35′	ζ	80 12
δ	13 51	*r*	— 5
φ	— 12 3	*a*	80 7
M	+ 46 44	*b* nearly	90
φ—*M*	— 58 47	*C*	101 21
True bearing of sun			N. 73° 26′ E.
∠ Sun to flagstaff			101 21
∠ Flagstaff to Light-house			88 34
True bearing of Light-house			S. 83 21 W.

Observations of the Sun, made May 13th, 1866, to determine the true bearing of the object used as an azimuth mark in swinging the ship in Panama Bay, New Granada.

	6^h 17^m 3^s		86° 56′
	18 15		58
T	6 17 39	Ω	86 57
Chronometer fast	0 20 17	ω	+ 17
τ	+ 3 53	*s*	—
Apparent time(P.M.)	6 1 15	*c*	87 14
t	90° 19′	ζ	86 54
δ	18 31	*r*	— 14
φ	8 55	*a*	86 40
M	89 3	*b* nearly	90
φ—*M*	— 80 8	*C*	86 14

True bearing of sun	N. 71° 49′ W.
∠ Peak to sun	87 14
True bearing of Peak	S. 20 57 W.

Observations to determine the true bearing of the object used as an azimuth mark in swinging the ship in the harbor of Acapulco, Mexico.

When determining the magnetic declination with the portable declinometer, on May 30th, 1866, an observation of the sun with the theodolite gave N. 6° 22′ E. as the true bearing of the gate of Fort St. Diego from the shore station. We then have

True bearing from station to Fort	N. 6° 22′ E.
∠ Monadnock to Fort	26 54
True bearing from station to Monadnock . . .	N. 20 32 W.
True bearing from Monadnock to station . . .	S. 20° 32′ E.
∠ Clump to station	87 45
True bearing of clump	N. 71 43 E.

Observations of the Sun, made June 9th, 1866, to determine the true bearing of the object used as an azimuth mark in swinging the ship in Magdalena Bay, Lower California.

Owing to a combination of unfortunate circumstances, the only available method of determining a true bearing was by observing with the solar compass, set up on the quarterdeck of the ship. In that way I found

True bearing of Peak . . . . S. 46° 30′ E.

which can only be considered as a near approximation to the truth.

Observations of the Sun, made June 23d, 1866, to determine the true bearing of the object used as an azimuth mark in swinging the ship at San Francisco, California.

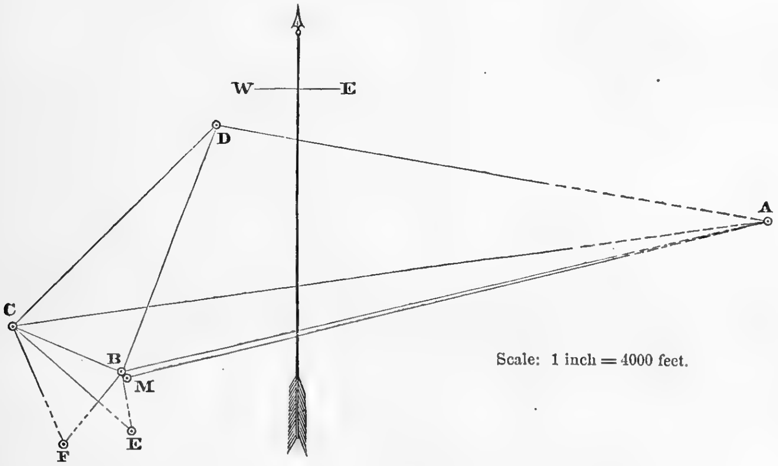
	7^h	5^m	17^s		92°	22′
		6	52			39
		7	55			43
T	7	6	41	Ω	92	35
Chronometer fast	0	3	12	ω	+	17
τ	—	1	51	s		—
Apparent time	7	1	38	c	92	52
t	—	74	35′	ζ	64	8
δ		23	26	r	—	2
ϕ		37	48	a	64	6
M		58	30	b	89	51
$\phi - M$	—	20	42	C	93	16

True bearing of sun	N. 79° 26′ E.
∠ Red Rock to sun	93 16
True bearing of Red Rock	N. 13 50 W.

The following triangulation was made for the purpose of determining the geographical position of some points in and about Ceara, Brazil. The angles were observed on December 14th, 15th, and 16th, 1865. Those between the Powhattan, Monadnock, and Custom-house were not measured simultaneously, and as the two ships were riding at anchor with a considerable amount of chain out, it is probable that they shifted their positions after the angle at the Powhattan was measured, and before the angles at the Monadnock and Custom-house were taken. This will account for the excess of the sum of the three angles over 180°.

In the accompanying sketch the different points are designated as follows:

A = Point Macoripie Light-house.
B = Northeast corner of Custom-house on the wharf.
C = U. S. Iron-clad Monadnock.
D = U. S. Sloop of War Powhattan.
E = most southern of the two steeples on the Church of the Conception.
F = most southern of the two steeples on St. Joseph's Church.
M = Magnetic and Astronomical Station of December 13th and 14th.



The observed angles were as follows:

Angles at B.	*Angles at C.*	*Angles at D.*
D to A = 55° 12′	D to A = 36° 19′	A to B = 101° 35′
D to C = 84 17	D to B = 71 14	B to C = 25 13
F to C = 73 12	B to F = 42 28	A to C = 126 49
E to C = 125 6	B to E = 15 40	
E to F = 52 15		
A to E = 95 6		

From these we obtain the following corrected

Angles at B.	*Angles at C.*	*Angles at D.*
A to E = 95° 11′	D to B = 70° 58	A to B = 101° 36′
E to F = 52 9	D to A = 36 14	B to C = 24 57
F to C = 73 14	A to B = 34 44	
C to D = 84 5	B to E = 15 40	
D to A = 55 21	E to F = 26 48	

The Powhattan fired a salute, and, from the mean of seven observations, the interval between the flash and report, noted at B, was 6.55 seconds. External thermometer 86°. Hence the distance from B to D was 7526 feet.

Distance from B to $M = 200$ feet.

Azimuth from M to $A =$ N. 75° 38′ E.

Angle $AMB = 128° 57′$.

From these data we find the distances between the several points as follows:

$AD = 15814$ feet.
$AC = 21491$ "
$AB = 18826$ "
$AM = 18702$ "

$CE = 4355$ feet.
$BC = 3358$ "
$BF = 2516$ "

$BE = 1443$ feet.
$CD = 7919$ "
$CF = 3568$ "

Angle $BAM = 0° 28′$ | Angle $AMB = 128° 57′$ | Angle $ABM = 50° 35′$

Azimuth from M to $A =$ N. 75° 38′ E.
" " B to $A =$ N. 76 6 E.

Azimuth from B to $E =$ S. 8° 43′ E.
" " B to $F =$ S. 43 26 W.

Assuming the position of M to be

Lat. 3° 43′ 59″.0 S.
Long. 2^h 34^m $6^s.00$ W.

we get finally

Station.	Latitude.	Longitude.
B	3° 43′ 57″.8 S.	2^h 34^m $6^s.11$ W.
E	3 44 12.0	2 34 5.97
F	3 44 15.9	2 34 7.25
A	3 43 13.3	2 33 54.10

For convenience of reference the results of the observations contained in this section, together with the chronometer comparisons made during the cruise, are here collected and appended.

Observed Latitudes.

Name of station.	Latitude.
Fort Christian, St. Thomas	18° 20′ 0″ N.
Isle Royal, Salute Islands	5 17 29 N.
Magnetic Station, Ceara, Brazil	3 43 59 S.
Custom-house, " "	3 43 58 S.
Church of the Conception, Ceara, Brazil	3 44 12 S.
St. Joseph's Church, " "	3 44 16 S.
Point Macoripie Light-house, " "	3 43 13 S.

Errors of Pocket Chronometer, Fletcher, No. 906.

Station.	Date.	Error on Local Mean Time.	Error on Greenwich Mean Time.
Portsmouth, Va.	October 29, 1865	0^h 4^m $41^s.1$ fast	5^h 0^m $28^s.7$ slow
Portsmouth, Va.	“ “ “	4 40.1 “	0 29.7 “
St. Thomas	November 13, “	0 40 43.6 slow	0 26.3 “
Isle Royal	“ 28, “	1 30 19.4 “	0 30.8 “
Ceara	December 14, “	2 26 32.5 “	5 0 38.5 “
Pernambuco	“ 23, “	2 36 34.8 “	4 56 3.0 “
Bahia.	“ 27, “	2 22 6.8 “	56 7.3 “
Bahia.	“ 29, “	2 22 3.6 “	56 10.5 “
Rio Janeiro	January 9, 1866	2 3 38.4 “	56 9.1 “
Rio Janeiro	“ “ “	2 3 32.8 “	56 10.7 “
Monte Video	“ 18, “	1 11 27.0 “	56 22.8 “
Monte Video	“ 24, “	1 11 26.5 “	56 19.4 “
Sandy Point	February 7, “	0 12 48.1 “	56 23.4 “
Valparaiso	March 2, “	0 9 46.4 “	56 17.4 “
Valparaiso	“ 29, “	9 26.8 “	56 12.5 “
Valparaiso	April 7, “	9 23.3 “	56 9.0 “
Valparaiso	“ “ “	0 9 23.9 “	4 56 9.6 “
Valparaiso	“ 14, “	0 11 1.7 “	4 57 47.4 “
Callao	“ 26, “	0 11 13.5 fast	57 55.6 “
Payta.	May 7, “	0 26 41.9 “	57 40.1 “
Panama	“ 14, “	0 20 16.9 “	4 57 44.9 “
Acapulco	“ 30, “	1 41 22.2 “	4 58 7.2 “
Magdalena Bay	June 8, “	2 30 4.4 “	58 19.6 “
San Diego	“ 15, “	2 50 32.5 “	4 58 20.1 “

This chronometer (Fletcher, 906) was habitually carried in my pocket. It was accidentally allowed to run down on the night of December 17th and 18th, 1865, and after remaining stopped twelve hours was wound and compared. Some time between 5^h P. M. of April 13th and 3^h P. M. of April 14th, 1866, it stopped for about 1^m 37^s, but started again of itself. On June 20th, 1866, when its face showed 6^h 45^m P. M. it stopped without any apparent cause, and, as it would not run again, it became useless.

In observing at San Francisco the box chronometer T. S. and J. D. Negus, No. 1287 was used. The observations on June 26th, 1866, showed it to be

8^h 13^m $8^s.2$ fast of local mean time;

and

0^h 3^m $45^s.6$ fast of Greenwich mean time.

Chronometer Comparisons.

Date.		Fletcher, 906.	T. S. and J. D. Negus, 1317.	T. S. and J. D. Negus, 1287.
October 29,	1865	7^{h} 39^{m} $56^{s}.8$ A. M.	12^{h} 44^{m} $0^{s}.0$	
October 29,	"	2 28 56.0 P. M.	7 33 0.0	
October 31,	"	12 8 48.2 "	5 13 0.0	
November 3,	"	4 17 33.0 "	9 22 0.0	
November 13,	"	8 21 4.8 A. M.	1 26 0.0	
November 13,	"	. . . . . .	1 28 0.0	1^{h} 16^{m} $23^{s}.5$
November 17,	"	12 18 46.0 "	5 24 0.0	
November 28,	"	6 55 10.8 "	12 1 0.0	
November 28,	"	6 56 56.8 "	. . . .	11 50 0.0
November 28,	"	2 39 9.8 P. M.	7 45 0.0	
December 14,	"	6 29 23.0 A. M.	11 36 0.0	
December 14,	"	6 30 19.8 "	. . . .	11 25 0.0
December 14,	"	12 43 22.5 P. M.	5 50 0.0	
December 16,	"	8 54 16.0 A. M.	2 1 0.0	
December 16,	"	8 56 15.2 "	. . . .	1 51 0.0
December 18,	"	9 44 42.8 P. M.	2 47 0.0	
December 23,	"	8 7 28.0 A. M.	1 10 0.0	
December 23,	"	8 8 32.5 "	. . . .	12 59 0.0
December 29,	"	6 22 59.2 "	11 26 0.0	
December 29,	"	6 24 9.0 "	. . . .	11 15 0.0
January 9,	1866	6 46 21.8 "	11 50 0.0	
January 9,	"	6 46 43.2 "	. . . .	11 38 0.0
January 24,	"	12 41 4.0 P. M.	5 46 0.0	
January 24,	"	12 41 50.8 "	. . . .	5 34 0.0
April 14,	"	4 16 24.4 "	9 29 0.0	
May 7,	"	11 34 26.4 A. M.	4 49 0.0	
May 14,	"	12 2 49.6 P. M.	5 18 0.0	
May 30,	"	11 55 13.2 A. M.	5 12 0.0	
June 8,	"	6 28 24.8 P. M.	11 46 0.0	
June 15,	"	12 0 46.8 A. M.	5 19 0.0	
June 26,	"	. . . . . .	6 34 0.0 P.M.	6 17 0.2

Table showing the True Bearings of the various objects used as azimuth marks in swinging the U. S. Iron-clad Monadnock during her cruise from Philadelphia to San Francisco in 1865 and 1866.

Station.	True bearing.
Hampton Roads, Va.	S. 10° 34′ W.
St. Thomas	S. 31 35 E.
Isle Royal, Salute Islands	S. 10 54 W.
Ceara	N. 82 7 E.
Bahia	N. 81 57 W.
Rio Janeiro	N. 53 28 W.
Monte Video	N. 77 52 W.
Sandy Point	S. 7 14 W.
Valparaiso	N. 31 7 E.
Callao	S. 83 21 W.
Panama Bay	S. 20 57 W.
Acapulco	N. 71 43 E.
Magdalena Bay	S. 46 30 E.
San Francisco Bay	N. 13 50 W.

SECTION IV.

OBSERVATIONS ON TERRESTRIAL MAGNETISM.

THE observations of magnetic declination and force were made by means of the same instruments—a portable declinometer, and a transit theodolite.

The Declinometer, kindly lent by the U. S. Coast Survey, and marked D. 22, was originally constructed by Jones, of London, but had been altered in many particulars so as to make it more convenient for field use. It was provided with two collimator magnets which were hollow cylinders of steel, each 0.70 of an inch in external diameter, and 0.58 of an inch in internal diameter. One of them, marked C. 32, was 3.92 inches long; while the other, marked S. 8, was 3.25 inches long. Each of these magnets carried in its south end a lens; and in its north end, at the solar focus of the lens just mentioned, a piece of plane glass on which was cut a scale of equal parts containing one hundred and seventy divisions, each division being equal to 0.00255 of an inch. Both magnets were provided with light sliding brass rings which were intended to be used for keeping them horizontal under great changes of magnetic declination, but the slight play which the magnets had in the stirrup was found quite sufficient for that purpose, and the rings were never employed. The same suspension was used during the whole of the observations. It consisted originally of six parallel fibres of unspun silk, each about nine inches long; but at Callao one of the fibres was accidentally broken, and after that the remaining five were used. The torsion circle, which formed part of the suspension apparatus, was 0.88 of an inch in diameter, divided to every three degrees, and read by means of a vernier to single degrees.

The Transit Theodolite, which perhaps might be more correctly called an altitude and azimuth instrument, was provided with a horizontal and a vertical circle, each five inches in diameter, and each reading by means of two opposite verniers to thirty seconds. The telescope had an object-glass with a clear aperture of one inch, and a focal length of about nine inches. It was provided with two eye-pieces; a direct one magnifying about twenty times, which was employed in almost all the observations; and a diagonal one of lower power, which was sometimes used for objects near the zenith. Both these eye-pieces had colored glasses for observing the sun. The system of wires in the focus of the object-glass was a simple rectangular cross, one wire being vertical, the other horizontal.

For the sake of convenience in setting up the instruments, and also for the perfect security which it affords against changes in the angular value of the divisions of the magnet scales depending upon changes in the distance between them and the telescope, a special table was provided, which was mounted upon a tripod stand, and which carried both the declinometer and theodolite in a fixed and invariable position relatively to each other—the object-glass of the telescope being about three inches from the south end of the magnet.

Pocket Chronometer, Fletcher, No. 906, was always used to note time. Its errors have been already given in detail in Section III.

General remarks on the method of using the instruments. When observations were to be made the tripod stand was set up, and the table, having been placed upon it, was approximately levelled by the eye, and set, by means of a pocket compass, so that its longest side was nearly in the magnetic meridian, the end destined to carry the declinometer being to the north. In packing the declinometer for travelling, the glass suspension tube was never unscrewed from the magnet-box, but when the collimator magnet was lifted from the stirrup a cylinder of wood of the same size was at once substituted, and two pieces of wood, provided for the purpose, were slipped in, one from each side of the magnet-box. These pieces of wood completely filled up the box, and at the same time held the wooden cylinder securely between them in such a manner that it could neither break the suspension fibres, nor allow them to twist in the slightest. With this packing, after the suspension fibres were once thoroughly freed from torsion, they remained so, and it was not necessary to examine them whenever the instrument was used, but only at considerable intervals, thus saving much time in the field. The brass carriers for the deflecting magnet having been screwed, one on each end of the wooden bar, and the bar in its turn having been screwed to the bottom of the magnet-box, the declinometer was placed upon the table in such a position that its three levelling screws fitted into the cavities provided for their reception. Then the packing blocks were taken out of the magnet-box, and the wooden cylinder having been removed from the stirrup, the collimator magnet was put in its place, and left free to assume its proper direction. The magnet-box was next levelled. For that purpose the suspension fibres were used as a plumb line, and the box was assumed to be level when they were seen to hang in the axis of the suspension tube throughout its whole length. Finally, the magnet was made to hang nearly level by moving it a little endwise in its stirrup; its scale was placed horizontal, with the figures erect; it was shaded from the direct rays of the sun by covering the glass top of the box; the mirror was screwed to the back of the box and adjusted so as to illuminate the magnet scale properly; and a thermometer was placed inside the magnet-box. The theodolite was next placed in its proper position on the other end of the table and levelled; particular care being taken that the horizontal axis of the telescope was truly level—especially if the altitude of the sun was considerable. The telescope having been turned towards the magnet and adjusted so as to obtain distinct vision of its scale, the horizontal circle was firmly clamped in such a position that the vertical wire in the field of the telescope cut the magnet scale as nearly as possible at the magnetic axis. By means of the vertical circle the optical axis of the telescope was then placed truly level, and the final adjustment of the magnet for horizontality was made by shifting it endwise in its stirrup till the scale was seen in the field of the telescope parallel to, and just in contact with, the horizontal wire.

When making my first observations considerable difficulty was experienced in getting a proper illumination of the magnet scale, but after some practice the following perfectly satisfactory plan was adopted. In cloudy weather the light of a white cloud was reflected into the magnet by means of the concave mirror. In clear weather the light of the blue sky, reflected from the mirror, was not sufficient, and it would not do to throw in the direct rays of the sun because of their heating power, which would certainly have led to the use of a wrong value of the magnetic moment; because the magnet would have been at a higher temperature than that shown by the thermometer in the box. Under these circumstances, in place of the mirror a piece of perfectly white paper was substituted, and the direct rays of the sun being allowed to fall upon it, it afforded a beautiful illumination of the magnet scale.

The copper damper, provided to slip into the magnet-box for the purpose of quieting the vibrations of the magnet, was never used. As the observations were all made in the open air, and as there was frequently wind enough to cause the instruments to vibrate perceptibly, the magnets seldom or never came to a state of absolute rest. Hence, the plan adopted to secure accurate readings of the scales was as follows. A screw-driver was slightly magnetized, and by approaching its south pole for an instant towards the south pole of the vibrating magnet, at a time when the magnet was moving towards the screw-driver, the arc of vibration was readily made quite small. Then, placing my eye to the telescope, I read off, and called out to my assistant, the scale reading at the instant the magnet attained the limit of its excursion in the eastern direction, and again when it attained the limit in a western direction—in other words, the greatest and least readings of the scale were noted. Five complete vibrations were generally observed, thus giving three eastern and three western readings, and the mean of the six was assumed to be the reading which would have been obtained if the magnet had been in a state of perfect rest.

In order to preserve the magnetism of the collimator magnets, they were always packed in a vertical position, with that pole downwards which would be lowest in a dipping needle.

Absolute Declinations were observed as follows: The instruments having been set up and adjusted in the manner already explained, the long magnet, C. 32, was suspended in the magnet-box, the telescope pointed nearly to its magnetic axis, and the horizontal circle of the theodolite firmly clamped. Then, 1°. The horizontal limb of the theodolite was read. 2°. The magnet scale being erect—that is, the figures upon it being right side up—the point upon it cut by the vertical wire of the telescope was observed. 3°. The telescope remaining as before, the magnet scale was inverted—that is, the magnet was turned on its axis through 180°, so that the figures upon its scale were seen inverted—and the point upon it cut by the vertical wire was again noted. 4°. The horizontal circle was unclamped, a colored glass placed upon the eye-piece, and the telescope pointed so that its vertical wire was just in advance of the first limb of the sun. Then the horizontal circle was clamped, the time of transit of the sun's first limb over the vertical wire noted, and the horizontal circle read. 5°. If the observation was made at a time of day when the sun's azimuth was changing tolerably rapidly, the telescope was not moved in azimuth at all, but, the reading of the horizontal circle remaining precisely as before, the sun was followed by moving the telescope in altitude, and the transit of its second limb was waited for and noted. If, however, the sun was changing its altitude much more rapidly than its azimuth then, in order to save time, the horizontal circle was unclamped, the telescope moved till its vertical wire was just in advance of the sun's second limb, the horizontal circle clamped, the time of transit of the sun's second limb over the vertical wire noted, and the horizontal circle read. 6°. The telescope of the theodolite was reversed in its Y's. 7°. The transit of the sun's first limb over the vertical wire was observed, and the horizontal circle read. 8°. The transit of the sun's second limb over the vertical wire was observed, and the horizontal circle read. 9°. The colored glass was removed from the eye-piece of the telescope, and a reading of the magnet scale (which was still inverted) was taken. 10°. The magnet was revolved on its axis through 180°, so as to place the scale erect, and another reading of the scale was taken. 11°. The horizontal circle was read.

Immediately before, and immediately after, going through with the operations just described, the telescope should be pointed to some well-defined distant object, and the reading of the horizontal circle noted. By so doing a check is afforded against any accidental shift of the horizontal circle; and if the same station is occupied at another time, absolute declinations may be determined without again referring to the sun, thus rendering it possible to observe during cloudy weather.

In the instruments under consideration the reading of the horizontal circle of the theodolite increases from left to right; and in both the magnets, C. 32 and S. 8, when the scale is erect an increase of scale reading indicates a motion of the north end of the magnet towards the east.

Let

ρ = reading of magnet, scale erect.

ρ' = reading of magnet, scale inverted.

R' = reading of horizontal circle of theodolite at the time the readings ρ and ρ' were observed.

d = value, in minutes of arc, of one division of the magnet scale.

R'' = reading of horizontal circle of the theodolite at the time of transit of sun's first limb over the vertical wire.

R''' = reading of horizontal circle of the theodolite at the time of transit of sun's second limb over the vertical wire.

α = observed chronometer time of transit of sun's first limb over the vertical wire.

α' = observed chronometer time of transit of sun's second limb over the vertical wire.

dt = correction of chronometer to reduce the reading of its face to local mean time.

τ = equation of time.

t = the sun's hour angle at the pole.
ϕ = latitude of the place of observation; positive when north of the equator.
A = azimuth of sun's centre at the time of its transit over the vertical wire: the azimuth being counted from the south around by the west.
δ = sun's declination; positive when north.

Then we have

$$t = \frac{\alpha + \alpha'}{2} + dt + \tau$$

$$\tan M = \frac{\tan \delta}{\cos t}$$

$$\tan A = \frac{\tan t \cos M}{\sin (\phi - M)}$$

where A is to be taken greater or less than 180° according as t is greater or less than 180°.

$$\text{Magnetic declination} = R' + \frac{d}{2}(\rho - \rho') + A - 180^\circ - \frac{R'' + R'''}{2}$$

in which the declination is east if its sign is positive; west if its sign is negative.

The reading of the magnetic axis of the magnet is

$$\tfrac{1}{2}(\rho + \rho')$$

which we will designate by c. It should be constant. Then, if at any station the magnet has only been observed with its scale erect, if c is known the observation may be reduced by the formula

$$\text{Magnetic declination} = R' + d(\rho - c) + A - 180^\circ - \frac{R'' + R'''}{2}$$

The following example shows fully the form employed in recording and reducing the observations.

Magnetic Declination.

Station, Acapulco, Mexico. *Date,* May 30, 1866. Portable Declinometer, D. 22. Magnet C. 32.
Observer, WM. HARKNESS.

	Circle readings.		Reading of magnet.	
Telescope direct.	Vernier . .	12° 23′ 30″	(1) Scale erect	$78^{d}.0$
			(2) Scale inverted	80.3
			(1) — (2) = Δ	— 2.3
			Transit of sun's	
	Vernier . .	75° 25′ 30″	1st limb	$8^{h}\ 14^{m}\ 28^{s}$
	Vernier . .	74 55 30	2d limb	15 28
	Mean . . .	75 10 30	Mean	8 14 58.0

	Circle readings.		Transit of sun's	
Telescope reversed.	Vernier . .	75° 36′ 0″	1st limb	$8^h\ 17^m\ 29^s$
	Vernier . .	75 6 30	2d limb	18 38
	Mean . . .	75 21 15	Mean	8 18 3.5
			Reading of magnet.	
	Vernier . .	12° 28′ 0″	(1) Scale inverted	$81^d.3$
			(2) Scale erect	77.2
			(2) — (1) = Δ	— 4.1

Value of one division of magnet scale = 2′.349.

The telescope is direct when the vertical circle is on the left-hand side.

These observations were made *before* noon, and time was noted by chronometer *Fletcher*, 906, which was $1^h\ 41^m\ 22^s.2$ *fast* of local mean time.

At the time the azimuth was observed, the reading of the horizontal circle, telescope direct, to distant referring mark was 10° 23′ 30″.

	Telescope direct.	Telescope reversed.
Equation of time	$0^h\ 2^m\ 47^s.1$	$0^h\ 2^m\ 47^s.1$
t (in time)	— 5 23 37.1	— 5 20 31.6
t (in arc)	—80° 54′ 16″	—80° 7′ 54″
δ	+ 21 47 18	+ 21 47 19
Tan δ	9.60177	9.60178
Sec *t*	0.80111	0.76602
Tan *M*	0.40288	0.36780
φ	+ 16° 50′ 3″	+ 16° 50′ 3″
M	+ 68 25 21	+ 66 47 35
(φ — *M*)	— 51 35 18	— 49 57 32
Tan *t*	0.79562	0.75955
Cos *M*	9.56557	9.59556
Cosec (φ — *M*)	0.10592	0.11600
Tan *A*	0.46711	0.47111
Circle reading to magnet . .	12° 23′.5	12° 28.′0
Δ × ½ scale division . . .	— 2.7	— 4.8
Sun's azimuth	251 9.9	251 19.6
Sum	263 30.7	263 42.8
180° + circle reading to sun	255 10.5	255 21.3
Magnetic declination . . .	8 20.2 E.	8 21.5 E.

Observations of Vibrations were made as follows: The instrument having been set up and adjusted in the manner already explained, the long magnet, C. 32, was suspended in the magnet-box; and the telescope having been pointed so that its vertical wire cut the magnet scale approximately at the magnetic axis, the horizontal limb of the theodolite was firmly clamped. Then, 1°. By quickly approaching and withdrawing the magnetised screw-driver the magnet was caused to vibrate horizontally through an arc extending to about twenty scale divisions on each side of the magnetic axis—that is, through a total arc of about 1° 34′. The semi-arc of vibration being only 47′, no correction to the observed time of vibration was ever required on that account. 2°. My assistant having taken the chronometer, I placed my eye to the telescope, and at the instant the 80th division of the scale (which was very near the magnetic axis) crossed the vertical wire I cried "time," and my assistant noted the minute, second, and fraction of a second indicated by the chronometer. Still keeping my eye at the telescope, I counted the transits of the 80th division over the wire, calling the one at which time was noted 0, the next 1, the next 2, and so on up to the 10th, when I again cried "time," and my assistant once more noted the minute, second, and fraction of a second indicated by the chronometer. The difference of these two chronometer times gave a value for the time of ten vibrations of the magnet which was correct within about half a second. However, to guard against mistakes, the process was always repeated a second or third time. 3°. The temperature indicated by the thermometer in the magnet-box was noted; and then putting my eye to the telescope, I read the scale at the instant the magnet attained the eastern extremity, and again when it attained the western extremity, of its arc of vibration. These were the "extreme scale readings." 4°. The chronometer employed was a pocket one, beating five times in two seconds. Taking it in my hand, I commenced counting its beats at some multiple of ten seconds. Then, holding it to my ear and still mentally counting the beats, I put my eye to the telescope and noted the beat, and fraction of a beat, at which the 80th scale division crossed the vertical wire. For example, suppose the beat was taken up at the instant the chronometer indicated $10^h\ 2^m\ 10^s$, and counting the first succeeding beat 1, the next 2, and so on, suppose that the 80th division crossed the wire exactly at the 14th beat. Then, as 14.0 beats are equal to 5.6 seconds, the time of transit of the 80th scale division was $10^h\ 2^m\ 15^s.6$. The time of transit thus obtained was recorded as the 0 vibration. Adding to it the time of making ten vibrations—before determined—the approximate time when the 10th vibration would be completed became known. Taking up the beat of the chronometer at the nearest even ten seconds *before* that time, I put my eye to the telescope and observed the time of transit of the 80th division at the completion of the 10th vibration. In the same manner the time of completing the 20th, 30th, 40th, 50th, 100th, 150th, 160th, 170th, 180th, 190th, and 200th vibration was observed. Subtracting the time of completing the 0 vibration from the 150th, the 10th from the 160th, &c., there result six values of the time of making one hundred and fifty vibrations, from the mean of which a very accurate value of the time of making one vibration is obtained. It will not escape notice that when observing in the manner just described there is no risk of making a mistake of one vibration, because the magnet must, at all subsequent transits, be moving in the same direction as at the first transit, while in order to make a mistake of one vibration it would be necessary that it should be moving in the opposite direction. 5°. The extreme scale readings attained by the magnet at the eastern and western extremities of its arc of vibration were again observed; and then the thermometer in the magnet-box was read. 6°. The necessary observations for determining the coefficient of torsion of the suspension fibres were made. When the instrument was properly adjusted for observation the torsion circle always read 300°. With it remaining at that reading the arc of vibration of the magnet was reduced to four or five scale divisions (by means of the magnetized screw-driver) and then the scale was read. Next the torsion circle was turned *backward* one-quarter of a revolution, so as to make it indicate 210°, and the scale was again read. After that, the torsion circle was turned *forward* half a revolution (passing through the point 300°), so as to make it indicate 30°, and the scale was read. Finally, the torsion circle was turned *backward* one-quarter of a revolution, so as to make it indicate 300°, and the scale was once more read. Subtracting the second scale reading from the first, the second from the third, and the fourth from the third, gave three differences, which were added together and divided by four. The result was the number of scale divisions through which the magnet was deflected by a twist of ninety degrees in the suspension fibres.

Observations of Deflections were made as follows: The instruments having been set up and adjusted in the manner already explained, the short magnet, S. 8, was suspended in the magnet-box, and the telescope having been pointed so that its vertical wire cut the magnet scale approximately at its central division (not necessarily the magnetic axis) the horizontal limb of the theodolite was clamped firmly. Then, 1°. The time was noted. 2°. The thermometer inside the magnet-box was read. 3°. The long magnet C. 32 (which we will now call the deflecting magnet) was placed on the deflecting bar support, with its axis east and west, its centre on a level with and at a distance of two feet to the west of the suspended magnet, and its north end west; the vibrations of the suspended magnet were reduced to four or five scale divisions, by means of the magnetised screw-driver, and then its scale was read. 4°. The deflecting magnet (remaining in the same place on the deflecting bar support as before) was reversed end for end, so as to bring its north end east, and the scale of the suspended magnet was read. 5°. The reversals were repeated twice more, so as to give in all two scale readings with the north end of the deflecting magnet to the west, and two scale readings with it to the east. The mean of the two scale readings obtained with the north end of the deflecting magnet west, were subtracted from the mean of the two scale readings obtained with its north end east. The difference was twice the value of the angle of deflection, as resulting from observations made with the deflecting magnet west of the suspended magnet. 6°. The deflecting magnet was lifted from the deflecting bar support to the west, and placed on that to the east, of the suspended magnet; its distance from the suspended magnet being still two feet, and its north end being to the east, the scale of the suspended magnet was read. 7°. The deflecting magnet (remaining in the same place on the eastern deflecting bar support) was reversed end for end, so as to bring its north end west, and the scale of the suspended magnet was read. 8°. The reversals were repeated twice more, so to give in all two scale readings with the north end of the deflecting magnet to the east, and two scale readings with it to the west. From the mean of the two scale readings obtained with the north end of the deflecting magnet east, the mean of the two scale readings obtained with its north end west were subtracted. The difference was twice the value of the angle of deflection, as resulting from observations made with the deflecting magnet east of the suspended magnet. The mean between this result and that obtained from the observations with the deflecting magnet west of the suspended magnet, was adopted as the true value of twice the angle of deflection, with the deflecting magnet at a distance of two feet from the suspended magnet. 9°. The thermometer inside the magnet-box was read. 10°. The time was noted. 11°. All the observations just described were repeated with the deflecting magnet at a distance of two and a half feet from the suspended magnet. 12°. The torsion of the suspension fibres was determined, precisely as described under the head of "observations of vibrations."

Horizontal Force was calculated from the observations of vibrations and deflections by the following formulæ:

$T_0 =$ observed time of one vibration of the magnet.

$T'' =$ time of vibration, corrected for rate of chronometer and arc of vibration.

$T =$ time of vibration, corrected for rate of chronometer, arc of vibration, torsion force of the suspending thread, temperature, and induction.

$s =$ daily rate of chronometer, + when gaining, — when losing.

$\alpha, \alpha' =$ semiarc of vibration, at the beginning and end of the observation, expressed in parts of radius.

$\frac{H}{F} =$ ratio of the force of torsion of the suspending thread to the magnetic directive force.

$q =$ coefficient of the decrease of the magnetic moment of the magnet produced by an increase of temperature of 1° Fah. (This is not constant for all temperatures, and the correction is more exactly expressed by a formula of the form — correction to $t' = q(t' - t) + q'(t' - t)^2$, where t' is the observed temperature, and t an adopted standard temperature.)

$K =$ moment of inertia of the magnet, including its suspending stirrup and other appendages. (This is constant for the same magnet and suspension, but varies slightly with the temperature, owing to the expansion of the materials.)

$\pi =$ ratio of the circumference of a circle to its diameter $= 3.14159$.

$\mu =$ coefficient of increase in the magnetic moment of the magnet produced by the inducing action of a magnetic force equal to unity of the English system of absolute measurement.

$r_0 =$ apparent distance between the centres of the deflecting and suspended magnets in the observations of deflections.

$r =$ the same distance corrected for error of graduation and temperature. $(r = r_0[1 + 0.00001(t' - 62°)] +$ correction for scale error.)

$d =$ value, in minutes of arc, of one division of the magnet scale.

$u_0 =$ observed angle of deflection, in scale divisions.

u = angle of deflection, corrected for torsion force of the suspending thread.
P = a constant depending upon the distribution of magnetism in the deflecting and suspended magnets.
m = magnetic moment of the deflecting or vibrating magnet.
X = horizontal component of the earth's magnetic force.
$\frac{m'}{X'}$ = value of $\frac{m}{X}$ before the application of the correction $\left(1-\frac{P}{r^2}\right)$

$$\left(1+\frac{H}{F}\right)=\frac{5400+v}{5400}$$

where v = the angle, expressed in minutes of arc, through which the suspended magnet is deflected by a twist of 90° in the suspension thread.

$$T'=T_0\left(1-\frac{s}{86400}\right)\left(1-\frac{\alpha\alpha'}{16}\right)$$

$$T^2=T'^2\left\{1+\frac{H}{F}\right\}\left\{1-(t'-t)q\right\}\left\{1+\mu\frac{X'}{m'}\right\}$$

$$mX=\frac{\pi^2K}{T^2}$$

$$u=du_0\left(1+\frac{H}{F}\right)$$

$$\frac{m'}{X'}=\tfrac{1}{2}r^3\tan u$$

$$\frac{m}{X}=\frac{m'}{X'}\left(1-\frac{P}{r^2}\right)$$

$$m=\sqrt{mX\frac{m}{X}}$$

$$X=\frac{mX}{m}$$

In order to facilitate the finding of log. tan u, in the reduction of observations of deflection, the following table has been prepared. With the argument log. u (u being expressed in minutes of arc) it gives the quantity (log. tan u — log. u), or, in other words, the quantity which it is necessary to add to log. u in order to obtain log. tan u. The arrangement of the table is such that the quantity (log. tan u — log. u) is to be added to the log. u on the same line with it, or to any other log. u less than the one on the line next below. For example, if it were required to find log. tan u corresponding to any log. u from 8.0000 to 1.4340, it would only be necessary to add 6.46373 to the given log. u.

Log. *u*.	Log. tan *u* — Log. *u*.	Log. *u*.	Log. tan *u* — Log. *u*.
8.0000	6.46373	2.1159	6.46394
1.4341	6.46374	2.1261	6.46395
1.5957	6.46375	2.1358	6.46396
1.6874	6.46376	2.1452	6.46397
1.7517	6.46377	2.1541	6.46398
1.8014	6.46378	2.1626	6.46399
1.8414	6.46379	2.1708	6.46400
1.8756	6.46380	2.1787	6.46401
1.9047	6.46381	2.1864	6.46402
1.9310	6.46382	2.1937	6.46403
1.9538	6.46383	2.2008	6.46404
1.9750	6.46384	2.2079	6.46405
1.9934	6.46385	2.2146	6.46406
2.0111	6 46386	2.2209	6.46407
2.0274	6.46387	2.2271	6.46408
2.0426	6.46388	2.2332	6.46409
2.0565	6.46389	2.2393	6.46410
2.0700	6.46390	2.2453	6.46411
2.0824	6.46391	2.2509	6.46412
2.0941	6.46392	2.2565	6.46413
2.1055	6.46393		

The following are specimens of the forms employed in recording and reducing the observations of vibrations and deflections.

Horizontal Intensity.

Observations of Vibrations.

Station, Acapulco, Mexico. *Date,* May 30th, 1866. Magnet C. 32. Inertia ring No.
Chron. Fletcher 906, rate, $1^s.38$ losing on mean time.

Number of vibrations.	Time.	Temp. t'	Extreme scale readings.		Time of 150 vibrations.
0	8^h 32^m $3^s.8$	87°	$57^d.8$	$102^d.2$	
10	8 32 57.0				
20	8 33 50.6				
30	8 34 43.9				
40	8 35 37.0				
50	8 36 30.6				
100	8 40 57.2				
150	8 45 23.4				13^m $19^s.6$
160	8 46 17.2				13 20.2
170	8 47 10.2				13 19.6
180	8 48 3.7				13 19.8
190	8 48 57.0				13 20.0
200	8 49 50.5	91	65.2	95.0	13 19.9
	Means,	89.0			13 19.85

Coefficient of torsion. Value of one scale div. = 2'.349

Tor. cir.	Scale.	Diff's.
300°	$80^d.1$	
		$3^d.4$
30	83.5	
		6.8
210	76.7	
		3.4
300	80.1	
Mean = v = 3.40		

	Log's.
v = 8'.0	
5400' + v'	3.73304
5400 (ar. co.)	6.26761
$1 + \frac{H}{F}$	0.00065

HORIZONTAL INTENSITY.

Calculation.

$$T^2 = T'^2 \left(1 + \frac{H}{F}\right) \left(1 - (t' - t)\, q\right)$$

Observed time of 150 vibrations = $799^s.85$
Time of one vibration = 5.332
Correction for rate = .000
$T' =$ 5.332

			Log's.
q		T'	0.72689
$t' - t$	+ 4.3	T'^2	1.45378
$(t' - t)\, q$		$1 + \frac{H}{F}$	65
$1 - (t' - t) q$		$1 - (t' - t)\, q$	9.99962
$mX = \frac{\pi^2 K}{T^2}$		T^2	1.45405
		$\pi^2 K$	2.17768
		mX	0.72363
		m	9.83487
		$7.740 = X$	0.88876

* Ob's of defl'n. *Date.* May 30th, 1866.

$\frac{{}^*m}{X}$	8.94854
mX	0.72363
m^2	9.67217
m	9.83608

$t = 84^\circ.7$

The chronometer used in this observation was $1^h\ 41^m\ 22^s.2$ fast of local mean time.

HORIZONTAL INTENSITY.

Observations of Deflections.

Station, Acapulco, Mexico. *Date,* May 30th, 1866. Mag. C. 32 deflecting. Mag. S. 8 suspended.
Observer, WM. HARKNESS.

Magnet.	North	Time. A. M. h. m.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	7 22	86°	$53^d.9$	$53^d.9$		$r = 2.0$ ft. log. = 0.30103
	E.			107.0	107.0	$53^d.1$	
	W.			53.9			
	E.			107.0			
East.	E.			107.5			
	W.			53.5	107.6	54.0	
	E.			107.7	53.6		
	W.	7 32	84	53.8			
Means,			85.0		$2u^d$	53.53	

Tors. cir.	Scale.	Diff's.
300°	$80^d.4$	
30	83.6	$3^d.2$
210	76.7	6.9
300	80.4	3.7

Mean = v = 3.45

	Log's
v = 9′.8	
5400′ + v′	3.73318
5400 (ar.co.)	6.26761
$1 + \frac{H}{F}$	0.00079

	Log's.
$2u^d$	1.72876
$\frac{1}{2}^d = 1'4175$	0.15152
$1 + \frac{H}{F}$	79
Sum	1.88107
$\frac{\text{Tan u}}{u'}$	6.46380
Tan u	8.34487
r^3	0.90309
$\frac{1}{2}$	9.69897
$\frac{m'}{X'}$	8.94693
$\frac{m}{X}$	8.94861

HORIZONTAL INTENSITY.

Observations of Deflections.

Station, Acapülco, Mexico. *Date,* May 30th, 1866. Mag. C. 32 deflecting. Mag. S. 8 suspended.
Observer, WM. HARKNESS.

Magnet.	North end.	Time. A. M. h. m.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	7 32	84°	$66^d.9$			r = 2.5 ft. log. = 0.39794
	E.			94.1	$66^d.9$	$27^d.3$	
	W.			66.9	94.2		
	E.			94.2			
East.	E.			94.4			
	W.			66.8	94.4	27.6	
	E.			94.4	66.8		
	W.	7 40	85	66.8			
Means,			84.5		$2u^d$	27.45	

	Log's.
$2u^d$	1.43854
$\frac{1}{2}^d = 1'.4175$	0.15152
$1 + \frac{H}{F}$	79
Sum	1.59085
$\frac{\text{Tan u}}{u'}$	6.46374
Tan u	8.05459
r^3	1.19382
$\frac{1}{2}$	9.69897
$\frac{m'}{X'}$	8.94738
$\frac{m}{X}$	8.94846

The constants, peculiar to the portable declinometer D 22, were obtained as follows:

The Temperature Coefficients of the magnets were furnished by Mr. Chas. A. Schott, of the U. S. Coast Survey. They had been used with the instrument for some years, and I had no opportunity to redetermine them. They are as follows:

For the magnet C 32 $q = 0.00020$
" " " S 8 $q = 0.00027$

In reducing the observations a correction was always applied to the magnetic moment of the magnet C 32 to reduce it to what it would have been if C 32 had had the same temperature as S 8. Hence, the temperature coefficient of C 32 was the only one used, and in order to facilitate its application the following table was computed which furnishes the value of log. $[1 - (t' - t)\,q]$ with the argument $(t' - t)$.

Correction of Magnet C. 32 for Temperature

$(t'-t)$	Log. $[1-(t'-t)\,q]$	$(t'-t)$	Log. $[1-(t'-t)\,q]$
+ 1°	9.99991	— 1°	0.00009
+ 2	9.99983	— 2	0.00017
+ 3	9.99974	— 3	0.00026
+ 4	9.99965	— 4	0.00035
+ 5	9.99957	— 5	0.00043
+ 6	9.99948	— 6	0.00052
+ 7	9.99939	— 7	0.00061
+ 8	9.99930	— 8	0.00069
+ 9	9.99922	— 9	0.00078
+10	9.99913	—10	0.00087

P. P.

0.1	1
0.2	2
0.3	3
0.4	4
0.5	4
0.6	5
0.7	6
0.8	7
0.9	8

The Value of One Division of the Magnet Scale was determined for each magnet in the following manner: The instruments having been set up and adjusted as usual, the magnet was suspended in the magnet-box, and the packing blocks (before described as being used to prevent the suspension fibres from being twisted when the instrument was packed for travelling) were inserted in such a manner as to hold it perfectly steady. Then, the magnet scale being horizontal, the vertical wire of the theodolite telescope was made to coincide with any convenient scale division, and the horizontal circle of the theodolite was read. Next, the vertical wire was made to coincide with some other scale division, and the circle was again read. The difference of the two circle readings, divided by the difference of the two scale readings, gave the angular value of one scale division.

The following are the observations in detail for each magnet:

Magnet C. 32.

Date.	Circle Readings.	Differences.	Scale Readings.	Diff's.	Value of 1 Scale Division.
Nov. 16, 1865	4° 5′ 15″	3° 53′ 30″	$50^{d}.0$	$100^{d}.0$	2′.335
Nov. 16, 1865	0 11 45		150.0		
Nov. 16, 1865	4 6 45	3 55 0	50.0	100.0	2.350
Nov. 16, 1865	0 11 45		150.0		
Nov. 16, 1865	3 7 45	1 57 30	75.0	50.0	2.350
Nov. 16, 1865	1 10 15		125.0		
Nov. 16, 1865	3 7 45	1 57 30	75.0	50.0	2.350
Nov. 16, 1865	1 10 15		125.0		
Jan. 18, 1866	5 36 15	3 55 45	50.0	100.0	2.357
Jan. 18, 1866	1 40 30		150.0		
Jan. 18, 1866	4 37 0	1 57 30	75.0	50.0	2.350
Jan. 18, 1866	2 39 30		125.0		

Hence for the magnet C 32, we have
1 scale division = 2′.349 ± 0′.0020.

Magnet S. 8.

Date.	Circle Readings.	Differences.	Scale Readings.	Diff's.	Value of 1 Scale Division.
Nov. 16, 1865	4° 9′ 45″	4° 43′ 15′	$50^{d}.0$	$100^{d}.0$	2′.833
Nov. 16, 1865	359 26 30		150.0		
Nov. 16, 1865	4 9 45	4 43 15	50.0	100.0	2.832
Nov. 16, 1865	359 26 30		150.0		
Nov. 16, 1865	2 58 45	2 21 45	75.0	50.0	2.835
Nov. 16, 1865	0 37 0		125.0		
Nov. 16, 1865	2 59 0	2 21 30	75.0	50.0	2.830
Nov. 16, 1865	0 37 30		125.0		
Jan. 18, 1866	5 36 30	4 44 15	50.0	100.0	2.842
Jan. 18, 1866	0 52 15		150.0		
Jan. 18, 1866	4 25 30	2 22 0	75.0	50.0	2.840
Jan. 18, 1866	2 3 30		125.0		

Hence, for the magnet S 8, we have
1 scale division = 2′.835 ± 0′.0013.

The Moment of Inertia, and its Temperature Coefficient, of the Magnet C 32, was determined as follows: Let,

K_τ = moment of inertia of the magnet, including its suspending stirrup and other appendages, at the temperature τ.

ΔK = change in the value of K corresponding to a change of temperature of 1° Fah. in the magnet.

K'_τ = moment of inertia of the inertia ring, at the temperature τ.

d_i = internal diameter of the inertia ring, expressed in feet, at the temperature τ_0.

d_e = external diameter of the inertia ring, expressed in feet, at the temperature τ_0.

ε = coefficient of expansion for a change of temperature of 1° Fah. in the metal composing the inertia ring.

W = weight of the inertia ring expressed in grains.

t = time in which the magnet makes one vibration at the temperature τ_0 (corrected for chronometer rate, arc of vibration, and torsion.)

t' = time in which the magnet, loaded with the inertia ring, makes one vibration at the temperature τ_0 (corrected for chronometer rate, arc of vibration, and torsion)

Then

$$K'_\tau = W[1 + 2\varepsilon(\tau - \tau_0)] \left\{ \frac{d_i^2 + d_e^2}{8} \right\}$$

$$K_\tau = K'_{\tau_0} \left(\frac{t^2}{t'^2 - t^2} \right) + \Delta K(\tau - \tau_0)$$

The inertia ring used in making my observations was of bronze. Mr. Joseph Saxton, Assistant Superintendent of the Office of Weights and Measures, very obligingly measured and weighed it, with the following result:

Internal diameter = 2.385 inches = 0.19875 foot
External diameter = 2.947 inches = 0.24558 foot
Weight = 798.72 grains

the temperature of the ring being 74° Fah.

Hence, assuming the coefficient of expansion for an increase of temperature of 1° Fah. in the metal of this ring to be 0.0000105, we find by the formula given above

$$K'_\tau = 9.9601 + (\tau - 50^\circ)\ 0.000209$$

or

$$\text{Log. } K'_\tau = 0.99827 + (\tau - 50^\circ)\ 0.0000091$$

The following table contains all the times of vibration which were observed for the purpose of determining the moment of inertia of the magnet, together with the computation of the corresponding values of log. K from them. The value of t' was always observed either immediately before, or immediately after, the corresponding value of t which was to be used with it. This was done in order to have the temperature in both cases as nearly as possible the same, so that the correction necessary to reduce t' to the same temperature as t was always very small. Then having a sufficient number of values of K, obtained from observations made at widely different temperatures, the value of ΔK was easily found.

Date.	τ	Log. t'^2	Log. t^2	Log. (t'^2-t^2)	Log. $\left(\frac{t^2}{t'^2-t^2}\right)$	Log. K'_τ	Log. K_τ
Oct. 28, 1865	73.0°	1.88210	1.66424	1.47811	0.18613	0.99849	1.18462
Nov. 16, 1865	87.7	1.72767	1.50891	1.32504	0.18385	0.99862	1.18247
Nov. 28, 1865	90.0	1.72835	1.51108	1.32345	0.18763	0.99864	1.18627
Dec. 13, 1865	89.5	1.74459	1.52673	1.34060	0.18613	0.99864	1.18477
Dec. 27, 1865	98.0	1.76681	1.54810	1.36412	0.18398	0.99872	1.18270
Jan. 18, 1866	87.2	1.77770	1.55921	1.37467	0.18458	0.99861	1.18315
March 19, 1866	76.2	1.75849	1.54101	1.35391	0.18710	0.99851	1.18561
April 11, 1866	74.0	1.75824	1.54019	1.35454	0.18565	0.99850	1.18415
May 30, 1866	84.7	1.67351	1.45405	1.27196	0.18209	0.99859	1.18068
Nov. 2, 1866	70.0	1.90424	1.68479	1.50268	0.18211	0.99846	1.18057
Nov. 2, 1866	70.0	1.90391	1.68450	1.50229	0.18221	0.99846	1.18067
Nov. 2, 1866	53.5	1.92843	1.70989	1.52548	0.18441	0.99830	1.18271
	79.5						1.18320

Let K_0 represent the mean of all the logarithms of K in the above table; then

$$K_0 = 1.18320$$

at a temperature of 79°.5. Now, assuming

$$\text{Log. } K_\tau = K_0 + (\tau - 79^\circ.5)\,\Delta K$$

we have

$$0 = K_0 - \text{log. } K_\tau + (\tau - 79^\circ.5)\,\Delta K$$

and each value of log. K_τ, given in the table above, will furnish one equation of condition for the determination of ΔK, as follows: the absolute terms being in units of the fifth place of decimals.

$$\begin{array}{ll} 0 = -142 - 6.5\,\Delta K & 0 = -241 - 3.3\,\Delta K \\ 0 = +73 + 8.2\,\Delta K & 0 = -95 - 5.5\,\Delta K \\ 0 = -307 + 10.5\,\Delta K & 0 = +252 + 5.2\,\Delta K \\ 0 = -157 + 10.0\,\Delta K & 0 = +263 - 9.5\,\Delta K \\ 0 = +50 + 18.5\,\Delta K & 0 = +253 - 9.5\,\Delta K \\ 0 = +5 + 7.7\,\Delta K & 0 = +49 - 26.0\,\Delta K \end{array}$$

From these equations of condition we obtain, by the method of least squares, the normal equation

$$0 = -5856.2 + 1646.0\,\Delta K$$

whence

$$\text{Log. } \Delta K = 0.55119$$

$$\Delta K = +3.56$$

and finally

$$\text{Log. } K_\tau = 1.18320 + (\tau - 79^\circ.5)\,0.0000356 \pm 0.000368$$

or

$$K_\tau = 15.248 + (\tau - 79^\circ.5)\,0.00125 \pm 0.0129$$

Hence we have

$$\pi^2 K_\tau = 150.49 + (\tau - 79^\circ.5)\,0.01234$$

or

$$\text{Log. } \pi^2 K_\tau = 2.17750 + (\tau - 79^\circ.5)\,0.0000356$$

In order to facilitate the reduction of the observations of vibrations, the following table has been computed from the formula last given. It furnishes the value of log. $\pi^2 K_\tau$ to the argument τ.

τ	Log. $\pi^2 K_\tau$	P. P.	
50°	2.17645	1°	4
		2	7
60	2.17681	3	11
		4	14
70	2.17716	5	18
		6	21
80	2.17752	7	25
		8	28
90	2.17787	9	32
100	2.17823		

The Constant P, depending upon the distribution of the magnetism in the magnets C 32 and S 8, was determined by means of the formula

$$P = \frac{A - A'}{\frac{A}{r^2} - \frac{A'}{r'^2}}$$

where

$A =$ value of $\frac{m'}{X'}$ determined from an observation of deflection with the deflecting magnet at the distance r from the suspended magnet.

$A' =$ value of $\frac{m}{X'}$ determined from an observation of deflection with the deflecting magnet at the distance r' from the suspended magnet.

The following table contains all the observed values of A and A', together with the computation of the corresponding values of P. The values of A were obtained from deflections at a distance of 2.0 feet: those of A' from deflections at a distance of 2.5 feet.

Date.	Log. A	Log. A'	Log. $(A-A')$	Log. $\frac{A}{r^2}$	Log. $\frac{A'}{r'^2}$	Log. $\left(\frac{A}{r^2}-\frac{A'}{r'}\right)$	Log. P	P
October 30, 1865	9.1660	9.1669	6.4829*n*	8.5640	8.3711	8.1187	8.3643*n*	—0.0231
November 13, 1865	9.0084	9.0094	6.3881*n*	8.4063	8.2135	7.9608	8.4274*n*	0.0268
November 16, 1865	9.0087	9.0088	5.1491*n*	8.4067	8.2129	7.9629	7.1863*n*	0.0015
November 28, 1865	9.0068	9.0078	6.3989*n*	8.4047	8.2120	7.9591	8.4398*n*	—0.0275
December 13, 1865	9.0234	9.0175	7.1527	8.4213	8.2216	7.9879	9.1649	+0.1462
December 23, 1865	9.0295	9.0317	6.7332*n*	8.4274	8.2358	7.9798	8.7534*n*	—0.0567
December 27, 1865	9.0421	9.0413	6.3230	8.4400	8.2454	7.9978	8.3252	+0.0211
January 6, 1866	9.0628	9.0633	6.0587*n*	8.4608	8.2674	8.0163	8.0424*n*	—0.0110
January 18, 1866	9.0531	9.0536	6.1399*n*	8.4511	8.2578	8.0064	8.1335*n*	0.0136
February 7, 1866	9.0486	9.0495	6.3751*n*	8.4465	8.2536	8.0012	8.3739*n*	0.0237
March 2, 1866	9.0328	9.0339	6.4250*n*	8.4308	8.2380	7.9852	8.4398*n*	—0.0275
March 19, 1866	9.0350	9.0342	6.3106	8.4330	8.2383	7.9907	8.3199	+0.0209
March 29, 1866	9.0347	9 0347	4.8740	8.4326	8.2388	7.9890	6.8850	+0.0008
April 7, 1866	9.0367	9.0373	6.1551*n*	8.4346	8.2414	7.9899	8.1652*n*	—0.0146
April 11, 1866	9.0356	9.0360	5.9295*n*	8.4336	8.2401	7.9893	7.9402*n*	0.0087
April 13, 1866	9.0343	9.0368	6.7852*n*	8.4323	8.2409	7.9842	8.8010*n*	—0.0632
April 26, 1866	8.9902	8.9896	6.1515	8.3882	8.1937	7.9456	8.2059	+0.0161
May 7, 1866	8.9680	8.9704	6.7188*n*	8.3659	8.1745	7.9178	8.8010*n*	—0.0632
May 14, 1866	8.9468	8.9544	7.1930*n*	8.3447	8.1585	7.8872	9.3058*n*	0.2022
May 30, 1866	8.9468	8.9472	5.8890*n*	8.3448	8.1513	7.9004	7.9886*n*	0.0097
June 9, 1866	8.9775	8.9817	6.9669*n*	8.3754	8.1858	7.9241	9.0427*n*	—0.1103
June 15, 1866	9.0376	9.0346	6.8666	8.4355	8.2387	7.9970	8.8697	+0.0741
June 26, 1866	9.0810	9.0826	6.6509*n*	8.4790	8.2868	8.0324	8.6185*n*	—0.0415
November 1, 1866	9.1991	9.1972	6.8414	8.5971	8.4014	8.1568	8.6847	+0.0484

The indiscriminate mean of all the observations gives

$$P = -0.0166 \pm 0.0088$$

But Peirce's criterion for the rejection of doubtful observations throws out those of December 13 and May 14. Accordingly, excluding them, and taking the mean of all the others, there results

$$P = -0.0155 \pm 0.0057$$

and that value I have adopted. Hence, for $r = 2.0$ feet, we have

$$\text{Log.}\left(1 - \frac{P}{r^2}\right) = 0.00168$$

and for $r = 2.5$ feet

$$\text{Log.}\left(1 - \frac{P}{r^2}\right) = 0.00108$$

The Magnetic Moment of the Magnet C 32 was computed as follows: Observations of deflection were always taken at two different distances, viz., at 2.0 feet and at 2.5 feet. In general, the two values of $\frac{m}{X}$ thus obtained differed slightly from each other, and the mean of the two was assumed to be correct. This mean was combined with the value of mX, obtained from a set of vibrations observed on the *same day*, and thus m was determined. In no case was more than one set of observations of deflections taken on any single day, but in a few instances several sets of observations of vibrations were made. Under such circumstances, the mean of all the observed values of mX was combined with the mean of the two values of $\frac{m}{X}$, and thus a single value of m was deduced.

Let

m_τ = observed value of the magnetic moment at the temperature τ.

m = value of m_τ after being multiplied by $[1 + (\tau - 75°.8)\, q]$, or, in other words, after being reduced to the temperature 75°.8 Fah.

m_0 = mean of all the observed values of m.

a = daily decrease in the value of log. m, expressed in units of the fifth decimal place.

d = time in days at which m is taken; d being counted from March 7th, 1866.

The following table contains all the observed values of log. m_τ, together with the computation from them of the final values of the same quantity. The column headed "days" gives the time in days counted from October 24th, 1865.

Date.	τ	Log. m_τ	Log. $[1+(\tau-75°.8)q]$	Log. m	Days.	Concluded Log. m	Concluded Log. m_τ
October 24, 1865	57.5	9.84148	9.99841	9.83989	0	9.83990	9.84149
October 30, 1865	58.7	9.84139	9.99851	9.83990	6	9.83979	9.84128
November 13, 1865	85.5	9.83908	0.00082	9.83990	20	9.83951	9.83869
November 16, 1865	87.7	9.83951	0.00104	9.84055	23	9.83945	9.83841
November 28, 1865	90.0	9.83773	0.00121	9.83894	35	9.83922	9.83801
December 13, 1865	89.5	9.83645	0.00117	9.83762	50	9.83893	9.83776
December 23, 1865	87.2	9.83768	0.00100	9.83868	60	9.83873	9.83773
December 27, 1865	98.0	9.83655	0.00191	9.83846	64	9.83865	9.83674
January 6, 1866	74.2	9.83915	9.99986	9.83901	74	9.83846	9.83860
January 18, 1866	87.2	9.83666	0.00100	9.83766	86	9.83823	9.83723
February 7, 1866	69.5	9.83783	9.99945	9.83728	106	9.83784	9.83839
March 2, 1866	69.7	9.83831	9.99947	9.83778	129	9.83739	9.83792
March 19, 1866	76.2	9.83618	0.00004	9.83622	146	9.83706	9.83702
March 29, 1866	68.2	9.83780	9.99934	9.83714	156	9.83686	9.83752
April 7, 1866	67.0	9.83861	9.99923	9.83784	165	9.83669	9.83746
April 11, 1866	74.0	9.83716	9.99984	9.83700	169	9.83661	9.83677
April 13, 1866	65.7	9.83711	9.99912	9.83623	171	9.83657	9.83745
April 26, 1866	79.2	9.83626	0.00030	9.83656	184	9.83632	9.83602
May 7, 1866	77.0	9.83670	0.00009	9.83679	195	9.83610	9.83601
May 14, 1866	82.2	9.83448	0.00056	9.83504	202	9.83596	9.83540
May 30, 1866	84.7	9.83602	0.00078	9.83680	218	9.83565	9.83487
June 9, 1866	65.0	9.83662	9.99906	9.83568	228	9.83546	9.83640
June 15, 1866	71.0	9.83493	9.99958	9.83451	234	9.83534	9.85576
June 26, 1866	63.0	9.83548	9.99889	9.83437	245	9.83513	9.83624
November 1, 1866	66.2	9.83326	9.99916	9.83242	373	9.83263	9.83347
Means	75.8			9.83729	154		

The mean of the quantities in the column headed τ is 75°.8. Accordingly, adding log. $[1 + (\tau - 75^\circ.8)q]$ to each log. m_τ, we obtain the values of log. m given in the table. Taking the mean of these values, and also the mean of the numbers in the column "days," we find that at 134 days, which corresponds to March 7th, 1866, the value of log. m was 9.83729 = log. m_0. Then, assuming

$$\text{Log. } m = \text{log. } m_0 - \alpha d$$

we have

$$0 = 9.83729 - \text{log. } m - \alpha d$$

and each value of log. m furnishes an equation of condition for the determination of α, as follows.

$$\begin{aligned}
0 &= -260 + 134\alpha \\
0 &= -261 + 128\alpha \\
0 &= -261 + 114\alpha \\
0 &= -326 + 111\alpha \\
0 &= -165 + 99\alpha \\
0 &= -33 + 84\alpha \\
0 &= -139 + 74\alpha \\
0 &= -117 + 70\alpha \\
0 &= -172 + 60\alpha \\
0 &= -37 + 48\alpha \\
0 &= +1 + 28\alpha \\
0 &= -49 + 5\alpha \\
0 &= +107 - 12\alpha
\end{aligned}$$

$$\begin{aligned}
0 &= +15 - 22\alpha \\
0 &= -55 - 31\alpha \\
0 &= +29 - 35\alpha \\
0 &= +106 - 37\alpha \\
0 &= +73 - 50\alpha \\
0 &= +50 - 61\alpha \\
0 &= +225 - 68\alpha \\
0 &= +49 - 84\alpha \\
0 &= +161 - 94\alpha \\
0 &= +278 - 100\alpha \\
0 &= +292 - 111\alpha \\
0 &= +487 - 239\alpha
\end{aligned}$$

By the method of least squares we obtain the normal equation

$$0 = -397497 + 203965\,\alpha$$

Solving, we get

$$\alpha = +1.9488$$

Hence

$$\text{Log. } m = 9.83729 - 0.0000195\,d \pm 0.000090$$

or

$$m = 0.68753 - 0.0000310\,d \pm 0.000144$$

From the first of these expressions the quantities in the column "concluded log. m" were computed.

If, in the expression for log. m, given above, we introduce the correction for temperature, we obtain

$$\text{Log. } m_\tau = 9.83729 - 0.0000195\,d - 0.000087\,(\tau - 75^\circ.8)$$

by means of which the quantities in the column "concluded log. m_τ" were computed.

The probable error of a single observed value of log. m is ± 0.000452, and of a single observed value of m it is ± 0.000719.

Observations of Inclination were all made with a dip circle by Henry Barrow & Co., of London. It was provided with two needles, marked A 1 and A 2, each 3.5 inches long, and having axles 0.016 of an inch in diameter. The distance between the agate planes on which they rested was 0.74 of an inch. By means of two microscopes, one opposite each end of the needle—each of which, assuming distinct vision to be obtained at a distance of ten inches, magnified 18 diameters—the inclination of the needle was referred to, and read off upon a vertical circle six inches in diameter, divided to half degrees, and reading by means of two verniers to single minutes. The pointing of the microscopes to the ends of the needle was effected by means of a clamp and tangent screw. The horizontal circle of the instrument was four inches in diameter, divided to half degrees, and reading by means of one vernier to single minutes. It was provided with a clamp, but no tangent screw.

Readings of the position of the dipping needle were made as follows: In the field of view of each microscope was a plate of glass upon which was engraved three fine parallel lines, the middle one being intended to represent one of the two extremities of a diameter passing through a vertical circle described about the prolongation of the axle of the needle. The north microscope having been turned till the centre line in its field of view coincided with the north end of the needle, the vernier belonging to that microscope was read off, and recorded as the reading of the north end of the needle. Then the south microscope was turned till the centre line in its field of view coincided with the south end of the needle, and the vernier belonging to that microscope was read off, and recorded as the reading of the south end of the needle. In order to distinguish between the two microscopes the letter N was scratched upon one of them, and that one was always, in all positions of the instrument, used to read the north end of the needle.

The instrument having been set up and levelled, before beginning to observe it was necessary to place the plane of the vertical circle in the magnetic meridian. At a few of the earlier stations this was accomplished as follows: The needle was placed on the agate planes, with the side on which the letters were marked facing the microscopes. Then 1°. The microscopes having been turned till they were nearly in a vertical line, the vernier of the lower one was set to 90° 0′, and the vertical circle was moved in azimuth—so that its face (by which is meant the side on which the microscopes were) was south—till the lower end of the needle was bisected by the middle line in the lower microscope; the Y's were raised and lowered gently, and if the bisection of the needle was altered, it was corrected by turning the circle in azimuth. Then the horizontal circle was clamped and read off; and this reading was called A. 2°. The vernier of the upper microscope was set to 90° 0′, and the horizontal circle having been unclamped, the vertical circle was moved in azimuth—its face still remaining south—till the upper end of the needle was bisected by the middle line in the upper microscope; the Y's were raised and lowered gently, and if the bisection of the needle was altered, it was corrected by turning the circle in azimuth. Then the horizontal circle was clamped and read off, and this reading was called B. 3°. The horizontal circle was unclamped, and turned in azimuth 180°, so as to bring the face of the instrument to the north, and then the 1° and 2° processes just described were repeated; thus giving two more readings of the horizontal circle, which were called C and D. Then

$$\frac{A+B+C+D}{4}=E$$

where E is the division of the horizontal circle at which it was necessary to set the vernier in order that the plane of the vertical circle might be *at right angles* to the magnetic meridian. Therefore the vernier was set at $90° + E$, and the plane of the vertical circle coincided with the magnetic meridian. However, it soon became evident that this process consumed too much time, and the following, which is quite as accurate and much more expeditious, was adopted: A fine line was marked permanently upon the top of the instrument parallel to the plane of the vertical circle; then, after the instrument had been levelled, but before the dipping needle had been placed upon the agate planes, a pocket compass, with a needle about one and a half inches long, was placed with its centre upon the fine line, and the vertical circle was turned in azimuth till the compass needle and line were parallel to each other. That being the case, the plane of the vertical circle was known to be in the magnetic meridian, and the horizontal circle was clamped and read off.

The following is the method which was adopted in making observations of dip: 1°. The agate planes, and those parts of the axle of the needle which would rest upon them, were carefully wiped with a piece of chamois leather (I have since seen reason to believe that a piece of cork would have answered the purpose better), and then the instrument was set up, levelled, and the plane of the vertical circle placed in the magnetic meridian by the process before described. 2°. The needle was secured upon a block, provided for the purpose, and magnetised by means of a pair of eight-inch bar magnets, in such a manner that its marked end acquired north polarity. It was considered to be saturated with magnetism when the bar magnets had been drawn from its centre to its extremities six times, the process being performed upon both of its sides, and then it was removed from the block and placed in position upon the agate planes, with its face (by which is meant that side upon which the letters were marked) towards the east. 3°. The plane of the vertical circle being in the magnetic meridian, with the face of the instrument towards the east, and the needle in position upon the agate planes, with its face also towards the east, the north and south ends of the needle were read. Let these readings be designated respectively as ϕ' and ϕ''. 4°. The needle was reversed upon the agate planes, so as to bring its face towards the west, and its north and south ends were read. Let these readings be designated respectively ϕ''' and ϕ^{IV}. 5°. The horizontal circle was unclamped, the vertical circle turned in azimuth 180°, so as to bring its face towards the west, and the horizontal circle again clamped. The face of the needle now being towards the east, its north and south ends were read. Let these readings be designated respectively as ϕ^{V} and ϕ^{VI}. 6°. The needle was reversed upon the agate planes, so as to bring its face towards the west, and its north and south ends were read. Let these readings be designated respectively as ϕ^{VII} and ϕ^{VIII}. 7°. The time was noted, and then the needle, having been removed from the agate planes, was placed upon the block provided for the purpose, and remagnetised in such a manner that its marked end acquired south polarity; after which it was again placed in position upon the agate planes, with its face towards the west, and its north and south ends were read. Let these readings be designated respectively as ψ' and ψ''. 8°. The needle was reversed upon the agate planes, so as to bring its face towards the east, and its north and south ends were read. Let these readings be designated respectively as ψ''' and ψ^{IV}. 9°. The horizontal circle was unclamped, the vertical circle turned in azimuth 180°, so as to bring its face to the east, and the horizontal circle again clamped. The face of the needle now being towards the west, its north and south ends were read. Let these readings be designated respectively as ψ^{V} and ψ^{VI}. 10°. The needle was reversed upon the agate planes, so as to bring its face towards the east, and its north and south ends were read. Let these readings be designated respectively as ψ^{VII} and ψ^{VIII}.

At the first few stations each of the readings $\phi', \phi'', \phi''' \ldots . \phi^{VIII}, \psi', \psi'', \psi''' \ldots . \psi^{VIII}$, was repeated three times, the Y's being raised and lowered again between each repetition; but after some experience I became convinced that the increase of accuracy obtained by three repetitions, over that obtained by a single careful reading, was not sufficient to warrant the greatly increased expenditure of time, and accordingly the repetitions were abandoned.

The needle A 2 proved to be well balanced, and the observations made with it were therefore reduced by the usual formula, namely

$$\frac{\phi' + \phi'' + \phi''' + \phi^{IV} + \phi^{V} + \phi^{VI} + \phi^{VII} + \phi^{VIII}}{8} = \alpha$$

$$\frac{\psi' + \psi'' + \psi''' + \psi^{IV} + \psi^{V} + \psi^{VI} + \psi^{VII} + \psi^{VIII}}{8} = \beta$$

$$\theta = \frac{\alpha + \beta}{2}$$

where θ is the magnetic inclination or dip.

The needle A 1 proved not to be well balanced, which was shown by the great difference between the values of α and β obtained with it in low magnetic latitudes; although they agreed well enough at places where the dip was large. An examination of all the observations showed that in every case

$$\frac{\phi' + \phi'' + \phi^{V} + \phi^{VI}}{4} = \frac{\phi''' + \phi^{IV} + \phi^{VII} + \phi^{VIII}}{4}$$

and

$$\frac{\psi' + \psi'' + \psi^{V} + \psi^{VI}}{4} = \frac{\psi''' + \psi^{IV} + \psi^{VII} + \psi^{VIII}}{4}$$

at least within about one degree. It therefore followed that, although the centre of gravity of the needle did not lie in its axle, it did lie somewhere in the line joining the two extremities of the needle and passing through its axle. In such cases we have

$$\tan \theta = \frac{\tan \alpha + \tan \beta}{2}$$

and by that formula all the observations made with this needle were reduced.

At St. Thomas some observations of dip were made with the plane of the vertical circle out of the magnetic meridian. They were reduced by the formula

$$\tan \theta = \tan \theta' \cos \alpha$$

where θ is the true dip, and θ' the dip observed with the vertical circle in a plane whose azimuth, measured from the magnetic meridian, was α.

The values of the Vertical and Total Force have been computed from the horizontal force and inclination by the formulæ

$$Z = X \tan \theta$$
$$R = X \sec \theta$$

where

$X =$ horizontal component of the earth's magnetic force.
$Z =$ vertical component of the earth's magnetic force.
$R =$ total magnetic intensity.
$\theta =$ magnetic inclination.

All values of force are expressed in English units; namely, in terms of grains, feet, and seconds. If it is desired to have them in metric units, expressed in terms of milligrams, millimeters, and seconds, they must be multiplied by 0.46108.

The observations of magnetic declination, inclination, and force are given in full at the end of this section, but for convenience of reference the following abstract of them is inserted here.

Station.	Date.	Declination.	Inclination.		Log. $\frac{m}{X}$	Log. mX	Temp.	$X =$ Hor. Force
			Needle A. 1.	Needle A. 2.				
Philadelphia, Pa.	Oct. 24, 1865	......	......	......	9.22363	0.45934	57.5	4.148
Gosport, Va.	Oct. 28, 1865	......	......	......	......	0.51303	73.0	4.709
Gosport, Va.	Oct. 30, 1865	2° 37′.8 W.	+69° 21′	+69° 54′	9.16787	0.51492	58.7	4.717
St. Thomas,	Nov. 13, 1865	......	+49 36	+49 32	9.01026	0.66791	85.5	6.749
St. Thomas,	Nov. 16, 1865	0 39.6 E.	+49 39	+49 44	9.01014	0.66888	87.7	6.768
Salute Islands,	Nov. 28, 1865	0 3.8 W.	+34 27	+34 42	9.00868	0.66679	90.0	6.742
Ceara,	Dec. 13, 1865	8 28.8 W.	+21 26	+21 20	9.02178	0.65112	89.5	6.507
Pernambuco,	Dec. 23, 1865	10 59.6 W.	+12 6	+12 10	9.03195	0.64340	87.2	6.392
Bahia,	Dec. 27, 1865	7 56.6 W.	+ 4 31	+ 4 17	9.04305	0.63005	98.0	6.213
Rio Janeiro,	Jan. 6, 1866	......	—11 48	—11 46	9.06444	0.61386	74.2	5.960
Rio Janeiro,	Jan. 9, 1866	2 41.8 W.	......	......	......	0.61205	80.5	5.944
Monte Video,	Jan. 18, 1866	9 16.6 E.	—31 11	—30 58	......	0.61892	87.2	6.049
Monte Video,	Jan. 18, 1866	......	......	—31 8	9.05476	0.61822	87.2	6.039
Monte Video,	Jan. 19, 1866	9 25.0 E.	......	......	......	0.61754	89.5	6.033
Sandy Point,	Feb. 7, 1866	21 52.0 E.	—54 52	—55 2	9.05044	0.62523	69.5	6.121
Valparaiso,	March 2, 1866	15 54.3 E.	—34 50	—35 7	9.03474	0.64188	69.7	6.367
Valparaiso,	March 19, 1866	15 36.6 E.	—35 28	—35 28	9.03599	0.63637	76.2	6.300
Valparaiso,	March 29, 1866	15 54.8 E.	—35 34	—35 27	......	0.64126	68.2	6.364
Valparaiso,	March 29, 1866	......	......	......	9.03607	0.63782	68.2	6.314
Valparaiso,	April 7, 1866	15 49.4 E.	—35 26	—35 23	9.03837	0.63885	67.0	6.330
Valparaiso,	April 11, 1866	15 57.6 E.	—35 29	—35 36	......	0.63697	74.0	6.312
Valparaiso,	April 11, 1866	......	......	......	9.03720	0.63725	74.0	6.317
Valparaiso,	April 13, 1866	15 53.9 E.	—35 40	—35 12	9.03692	0.63730	65.7	6.307
Callao,	April 26, 1866	10 29.6 E.	— 6 28	— 6 29	8.99132	0.68120	79.2	7.001
Payta,	May 7, 1866	8 53.0 E.	+ 5 9	+ 4 47	8.97055	0.70285	77.0	7.359
Panama Bay,	May 14, 1866	5 55.8 E.	+32 5	+31 47	8.95196	0.71700	82.2	7.614
Acapulco,	May 30, 1866	8 20.8 E.	+39 49	+39 58	8.94841	0.72363	84.7	7.740
Acapulco,	May 30, 1866	8 23.6 E.	......	......	......	......	......	......
Magdalena Bay,	June 9, 1866	10 40.5 E.	+48 41	+48 22	......	0.69240	65.0	7.178
Magdalena Bay,	June 9, 1866	......	......	......	8.98098	0.69211	65.0	7.173
San Diego Bay,	June 15, 1866	13 9.4 E.	+57 51	+57 56	9.03746	0.63241	71.0	6.261
San Francisco Bay,	June 26, 1866	16 25.5 E.	+62 13	+62 31	9.08320	0.58777	63.0	5.643
Washington, D. C.	Nov. 1, 1866	2 44.2 W.	+71 51	+72 13	9.19956	0.46695	66.2	4.300
Washington, D. C.	May 6, 1867	......	+71 55	+72 5	......	......	......	......

Taking the means we obtain the final values of the magnetic elements at each station, as follows:

Station.	Latitude.	Longitude West.	Date.	Declination.	No. of Obs.	Inclination.	No. of Obs.	Horizontal Force.	No. of Obs.	Vertical Force.	Total Force.
Philadelphia, Pa. . .	39° 56′ N.	75° 7′	Oct. 24, 1865	° ′		° ′		4.148	1		
Gosport	36 49 N.	76 17	Oct. 29, 1865	2 37.8 W.	1	+69 38	2	4.713	2	12.696	13.542
St. Thomas	18 20 N.	64 55	Nov. 14, 1865	0 39.6 E.	1	+49 38	4	6.758	2	7.950	10.434
Salute Islands	5 17 N.	52 33	Nov. 28, 1865	0 3.8 W.	1	+34 35	2	6.742	1	4.648	8.189
Ceara	3 44 S.	38 31	Dec. 13, 1865	8 28.8 W.	1	+21 23	2	6.507	1	2.548	6.988
Pernambuco	8 4 S.	34 52	Dec. 23, 1865	10 59.6 W.	1	+12 8	2	6.392	1	1.374	6.538
Bahia	12 57 S.	38 30	Dec. 27, 1865	7 56.6 W.	1	+ 4 24	2	6.213	1	0.478	6.231
Rio Janeiro	22 54 S.	43 8	Jan. 8, 1866	2 41.8 W.	1	—11 47	2	5.952	2	1.242	6.080
Monte Video	34 53 S.	56 13	Jan. 18, 1866	9 20.8 E.	2	—31 6	3	6.040	3	3.644	7.054
Sandy Point	53 10 S.	70 54	Feb. 7, 1866	21 52.0 E.	1	—54 57	2	6.121	1	8.725	10.658
Valparaiso	33 2 S.	71 41	March 29, 1866	15 51.1 E.	6	—35 23	12	6.326	8	4.493	7.759
Callao	12 5 S.	77 17	April 26, 1866	10 29.6 E.	1	— 6 28	2	7.001	1	0.794	7.046
Payta	5 6 S.	81 6	May 7, 1866	8 53.0 E.	1	+ 4 58	2	7.359	1	0.640	7.387
Panama Bay	8 54 N.	79 30	May 14, 1866	5 55.8 E.	1	+31 56	2	7.614	1	4.745	8.972
Acapulco	16 50 N.	99 52	May 30, 1866	8 22.2 E.	2	+39 54	2	7.740	1	6.472	10.089
Magdalena Bay . . .	24 40 N.	112 7	June 9, 1866	10 40.5 E.	1	+48 32	2	7.176	2	8.120	10.837
San Diego Bay . . .	32 42 N.	117 13	June 15, 1866	13 9.4 E.	1	+57 54	2	6.261	1	9.981	11.782
San Francisco	37 49 N.	122 21	June 26, 1866	16 25.5 E.	1	+62 22	2	5.643	1	10.779	12.167
Washington	38 54 N.	77 3	Nov. 1 1866	2 44.2 W.	1	+72 2	2	4.300	1	13.260	13.940

OBSERVATIONS OF MAGNETIC DECLINATION.

MAGNETIC DECLINATION.

Gosport, Va. October 30, 1865.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	359° 59′ 15″	(1) Scale erect . . .	$81^{d}.7$
			(2) Scale inverted .	76.5
			(1) — (2) = Δ . . .	+ 5.2
			Transit of Sun's	
	Vernier		1st limb . . .	10^{h} 40^{m} $6^{s}.2$
	Vernier		2d limb . . .	42 27.0
	Mean	162 12 45	Mean	10 41 16.6
Telescope Reversed.	Vernier		1st limb . . .	10^{h} 44^{m} $48^{s}.0$
	Vernier		2d limb . . .	47 8.8
	Mean	163° 34′ 45″	Mean	10 45 58.4
			Reading of Magnet.	
	Vernier		(1) Scale inverted .	$64^{d}.2$
			(2) Scale erect . . .	93.5
			(2) — (1) = Δ . . .	+29.3

	Telescope Direct.	Telescope Reversed.
Equation of time	16^{m} $13^{s}.7$	
t	—16° 47′ 28″	
δ	—13 56 36	
Circle reading to magnet	359° 59′.2	
Δ × ½ scale division	+ 6.1	
Sun's azimuth	339 29.6	
Sum	339 34.9	
180° + circle reading to sun	342 12.7	
Magnetic declination	2 37.8 W.	

These observations were made before noon.
Chronometer 0^{h} 4^{m} $40^{s}.2$ fast of local mean time.

MAGNETIC DECLINATION.

St. Thomas. November 16, 1865.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	359° 59′ 0″	(1) Scale erect (2) Scale inverted	$80^d.3$ 78.2
			(1) — (2) = Δ	+ 2.1
			Transit of Sun's	
	Vernier Vernier		1st limb 2d limb	$9^h\ 10^m\ 35^s.0$ 13 31.0
	Mean	141 24 0	Mean	9 12 3.0
Telescope Reversed.	Vernier Vernier		1st limb 2d limb	$9^h\ 16^m\ 45^s.0$ 19 37.5
	Mean	143° 0′ 45″	Mean	9 18 11.2
			Reading of Magnet.	
	Vernier	359 58 45	(1) Scale inverted (2) Scale erect	$76^d.0$ 82.3
			(2) — (1) = Δ	+ 6.3

	Telescope Direct.	Telescope Reversed.
Equation of time	$15^m\ 0^s.1$	$15^m\ 0^s.0$
t	— 28° 2′ 53″	— 26° 30′ 51″
δ	— 18 51 11	— 18 51 15
Circle reading to magnet	359° 59′.0	359° 58′.7
Δ × ½ scale division	+ 2.5	+ 7.4
Sun's azimuth	321 59.9	323 36.5
Sum	322 1.4	323 42.6
180° + circle reading to sun	321 24.0	323 0.7
Magnetic declination	0 37.4 E.	0 41.9 E.

These observations were made before noon.
Chronometer $0^h\ 40^m\ 45^s.4$ slow of local mean time.

MAGNETIC DECLINATION.

Salute Islands. November 28, 1865.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	0° 11′ 0″	(1) Scale erect (2) Scale inverted	$79^d.2$ 79.3
			(1) — (2) = Δ	— 0.1
			Transit of Sun's	
	Vernier Vernier		1st limb 2d limb	$12^h\ 21^m\ 44^s.0$ 25 37.5
	Mean	228 16 15	Mean	12 23 40.7
Telescope Reversed.	Vernier Vernier		1st limb 2d limb	$12^h\ 29^m\ 11^s.5$ 33 14.0
	Mean	229° 48′ 15″	Mean	12 31 12.7
			Reading of Magnet.	
	Vernier	0 14 30	(1) Scale inverted (2) Scale erect	$81^d.2$ 77.3
			(2) — (1) = Δ	— 3.9

	Telescope Direct.	Telescope Reversed.
Equation of time	$11^m\ 40^s.8$	$11^m\ 40^s.6$
t	+ 31° 25′ 13″	+ 33° 18′ 10″
δ	— 21 25 2	— 21 25 6
Circle reading to magnet	0° 11′.0	0° 14′.5
Δ × ½ scale division	— 0.1	— 4.6
Sun's azimuth	48 0.8	49 35.1
Sum	48 11.7	49 45.0
180° + circle reading to sun	48 16.2	49 48.2
Magnetic declination	0 4.5 W.	0 3.2 W.

These observations were made after noon.
Chronometer $1^h\ 30^m\ 19^s.4$ slow of local mean time.

MAGNETIC DECLINATION.
Ceara, December 13, 1865.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	1° 10′ 30″	(1) Scale erect	$79^{d}.6$
			(2) Scale inverted	78.5
			(1) — (2) = Δ	+ 1.1
			Transit of Sun's	
	Vernier	255 56 45	1st limb	$1^{h}\ 42^{m}\ 0^{s}$
	Vernier	25 15	2d limb	44 0
	Mean	255 41 0	Mean	1 43 0.0
Telescope Reversed.	Vernier	255° 27′ 0″	1st limb	$1^{h}\ 47^{m}\ 0^{s}$
	Vernier	256 5 15	2d limb	48 0
	Mean	255 46 8	Mean	1 47 30.0
			Reading of Magnet.	
	Vernier	1 15 30	(1) Scale inverted	$82^{d}.2$
			(2) Scale erect	76.1
			(2) — (1) = Δ	— 6.1

	Telescope Direct.	Telescope Reversed.
Equation of time	$5^{m}\ 10^{s}.3$	$5^{m}\ 10^{s}.2$
t	+ 63° 40′ 36″	+ 64° 48′ 4″
δ	— 23 12 12	— 23 12 12
Circle reading to magnet	1° 10′.5	1° 15′.5
Δ × ½ scale division	+ 1.3	— 7.2
Sun's azimuth	66 0.4	66 8.9
Sun	67 12.2	67 17.2
180° + circle reading to sun	75 41.0	75 46.1
Magnetic declination	8 28.8 W.	8 28.9 W.

These observations were made after noon.
Chronometer $2^{h}\ 26^{m}\ 32^{s}.1$ slow of local mean time.

MAGNETIC DECLINATION.
Pernambuco, December 23, 1865.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	2° 37′ 0″	(1) Scale erect	$80^{d}.8$
			(2) Magnetic axis	79.11
			(1) — (2) = Δ	+ 1.69
			Transit of Sun's	
	Vernier	135 15 30	1st limb	$7^{h}\ 19^{m}\ 35^{s}$
	Vernier	134 28 45	2d limb	20 55
	Mean	134 52 7	Mean	7 20 15.0
Telescope Reversed.	Vernier		1st limb	
	Vernier		2d limb	
	Mean		Mean	
			Reading of Magnet.	
	Vernier		(1) Scale inverted	
			(2) Scale erect	
			(2) — (1) = Δ	

	Telescope Direct.	Telescope Reversed.
Equation of time	$0^{m}\ 31^{s}.5$	
t	— 30° 39′ 40″	
δ	— 23 26 32	
Circle reading to magnet	2° 37′.0	
Δ × 1 scale division	+ 4.0	
Sun's azimuth	301 11.5	
Sum	303 52.5	
180° + circle reading to sun	314 52.1	
Magnetic declination	10 59.6 W.	

These observations were made before noon, and prior to beginning them the collimation was adjusted.
Chronometer $2^{h}\ 36^{m}\ 34^{s}.8$ slow of local mean time.

MAGNETIC DECLINATION.

Bahia, December 27, 1865.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	3° 32′ 45″	(1) Scale erect	$77^{d}.5$
			(2) Scale inverted	80.7
			(1) — (2) = Δ	— 3.2
			Transit of Sun's	
	Vernier	121 48 45	1st limb	$6^{h}\ 28^{m}\ 19^{s}$
	Vernier	6 15	2d limb	30 28
	Mean	121 27 30	Mean	6 29 23.5
Telescope Reversed.	Vernier	122° 2′ 30″	1st limb	$6^{h}\ 36^{m}\ 38^{s}$
	Vernier	121 17 30	2d limb	37 37
	Mean	121 40 0	Mean	6 37 7.5
			Reading of Magnet.	
	Vernier	3 32 0	(1) Scale inverted	$80^{d}.8$
			(2) Scale erect	77.5
			(2) — (1) = Δ	— 3.3

	Telescope Direct.	Telescope Reversed.
Equation of time	$1^{m}\ 26^{s}.8$	$1^{m}\ 26^{s}.9$
t	— 47° 29′ 0″	— 45° 33′ 9″
δ	— 23 19 36	— 23 19 36
Circle reading to magnet	3° 32′.7	3° 32′.0
Δ × ½ scale division	— 3.8	— 3.9
Sun's azimuth	290 2.3	290 14.9
Sum	293 31.2	293 43.0
180° + circle reading to sun	301 27.5	301 40.0
Magnetic declination	7 56.3 W.	7 57.0 W.

These observations were made before noon.
Chronometer $2^{h}\ 22^{m}\ 6^{s}.8$ slow of local mean time.

MAGNETIC DECLINATION.

Rio Janeiro, January 9, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	2° 36′ 45″	(1) Scale erect	$79^{d}.8$
			(2) Scale inverted	78.2
			(1) — (2) = Δ	+ 1.6
			Transit of Sun's	
	Vernier	111 42 30	1st limb	$4^{h}\ 52^{m}\ 10^{s}$
	Vernier	112 4 15	2d limb	55 8
	Mean	111 53 23	Mean	4 53 39.0
Telescope Reversed.	Vernier	111° 56′ 30″	1st limb	$4^{h}\ 56^{m}\ 50^{s}$
	Vernier	18 0	2d limb	57 50
	Mean	111 37 15	Mean	4 57 20.0
			Reading of Magnet.	
	Vernier	2 37 0	(1) Scale inverted	$78^{d}.3$
			(2) Scale erect	79.8
			(2) — (1) = Δ	+ 1.5

	Telescope Direct.	Telescope Reversed.
Equation of time	$7^{m}\ 23^{s}.4$	$7^{m}\ 23^{s}.4$
t	— 77° 31′ 30″	— 76° 36′ 15″
δ	— 22 6 32	— 22 6 31
Circle reading to magnet	2° 36′.7	2° 37′.0
Δ × ½ scale division	+ 1.9	+ 1.8
Sun's azimuth	286 33.1	286 16.6
Sum	289 11.7	288 55.4
180° + circle reading to sun	291 53.4	291 37.2
Magnetic declination	2 41.7 W.	2 41.8 W.

These observations were made before noon.
Chronometer $2^{h}\ 3^{m}\ 38^{s}.4$ slow of local mean time.

MAGNETIC DECLINATION.

Monte Video, January 18, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	2° 10′ 45″	(1) Scale erect (2) Scale inverted	$79^{d}.1$ 79.0
			(1) − (2) = Δ	+ 0.1
			Transit of Sun's	
	Vernier Vernier		1st limb 2d limb	$4^{h}\ 19^{m}\ 15^{s}$ 23 44
	Mean	250 49 30	Mean	4 21 29.5
Telescope Reversed.	Vernier Vernier		1st limb 2d limb	$4^{h}\ 26^{m}\ 3^{s}$ 30 30
	Mean	249° 55′ 15″	Mean	4 28 16.5
			Reading of Magnet.	
	Vernier	2 10 45	(1) Scale inverted (2) Scale erect	$78^{d}.5$ 79.6
			(2) − (1) = Δ	+ 1.1

	Telescope Direct.	Telescope Reversed.
Equation of time	$10^{m}\ 51^{s}.7$	$10^{m}\ 51^{s}.8$
t	+ 80° 31′ 12″	+ 82° 12′ 55″
δ	− 20 26 47	− 20 26 44
Circle reading to magnet	2° 10′.7	2° 10′.7
Δ × ½ scale division	+ 0.1	+ 1.3
Sun's azimuth	77 53.6	77 1.5
Sum	80 4.4	79 13.5
180° + circle reading to sun	70 49.5	69 55.2
Magnetic declination	9 14.9 E.	9 18.3 E.

These observations were made after noon.
Chronometer $1^{h}\ 11^{m}\ 27^{s}.0$ slow of local mean time.

MAGNETIC DECLINATION.

Monte Video, January 19, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	2° 34′ 52″	(1) Scale erect (2) Scale inverted	$77^{d}.5$ 80.2
			(1) − (2) = Δ	- 2.7
			Transit of Sun's	
	Vernier Vernier		1st limb 2d limb	$3^{h}\ 38^{m}\ 8^{s}$ 42 51
	Mean	256 30 30	Mean	3 40 29.5
Telescope Reversed.	Vernier Vernier		1st limb 2d limb	$3^{h}\ 44^{m}\ 40^{s}$ 49 19
	Mean	255° 36′ 0″	Mean	3 46 59.5
			Reading of Magnet.	
	Vernier	2 34 30	(1) Scale inverted (2) Scale erect	$81^{d}.6$ 76.7
			(2) − (1) = Δ	− 4 9

	Telescope Direct.	Telescope Reversed.
Equation of time	$11^{m}\ 9^{s}.5$	$11^{m}\ 9^{s}.6$
t	+ 70° 13′ 30″	+ 71° 50′ 58″
δ	− 20 14 31	− 20 14 28
Circle reading to magnet	2° 34′.9	2° 34′.5
Δ × ½ scale division	− 3.2	− 5.8
Sun's azimuth	83 23.3	82 32.9
Sum	85 55.0	85 1.6
180° + circle reading to sun	76 30.5	75 36.0
Magnetic declination	9 24.5 E.	9 25.6 E.

These observations were made after noon.
Chronometer $1^{h}\ 11^{m}\ 34^{s}.0$ slow of local mean time.

MAGNETIC DECLINATION.
Sandy Point, February 7, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	2° 54′ 45″	(1) Scale erect (2) Scale inverted	$79^d.8$ 78.5
			(1) − (2) = Δ	+ 1.3
			Transit of Sun's	
	Vernier Vernier		1st limb 2d limb	10^h 53^m 10^s 55 31
	Mean	6 44 15	Mean	10 54 20.5
Telescope Reversed.	Vernier Vernier		1st limb 2d limb	10^h 58^m 53^s 11 1 12
	Mean	4° 38′ 0″	Mean	11 0 2.5
			Reading of Magnet.	
	Vernier	2 54 15	(1) Scale inverted (2) Scale erect	$77^d.9$ 80.9
			(2) − (1) = Δ	+ 3.0

	Telescope Direct.	Telescope Reversed.
Equation of time	14^m $25^s.6$	14^m $25^s.6$
t	− 16° 49′ 15″	− 15° 23′ 45″
δ	− 15 13 36	− 15 13 31
Circle reading to magnet	2° 54′.7	2° 54′.2
Δ × ½ scale division	+ 1.5	+ 3.5
Sun's azimuth	205 38.2	203 34.1
Sum	208 34.4	206 31.8
180° + circle reading to sun	186 44.2	184 38.0
Magnetic declination	21 50.2 E.	21 53.8 E.

These observations were made before noon.
Chronometer 0^h 12^m $48^s.1$ slow of local mean time.
Magnet rendered quite unsteady by the wind.

MAGNETIC DECLINATION.
Valparaiso, March 2, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	4° 13′ 0″	(1) Scale erect (2) Scale inverted	$71^d.1$ 87.3
			(1) − (2) = Δ	− 16.2
			Transit of Sun's	
	Vernier Vernier		1st limb 2d limb	4^h 28^m 50^s 32 51.5
	Mean	264 44 0	Mean	4 30 50.7
Telescope Reversed.	Vernier Vernier		1st limb 2d limb	4^h 35^m $0^s.5$ 39 9.0
	Mean	263° 47′ 30″	Mean	4 37 4.7
			Reading of Magnet.	
	Vernier	4 5 30	(1) Scale inverted (2) Scale erect	$85^d.3$ 73.4
			(2) − (1) = Δ	− 11.9

	Telescope Direct.	Telescope Reversed.
Equation of time	12^m $17^s.4$	12^m $17^s.4$
t	+ 67° 4′ 55″	+ 68° 38′ 25″
δ	− 7 1 15	− 7 1 10
Circle reading to magnet	4° 13′.0	4° 5′.5
Δ × ½ scale division	− 19.0	− 14.0
Sun's azimuth	96 44.8	95 49.9
Sum	100 38.8	99 41.4
180° + circle reading to sun	84 44.0	83 47.5
Magnetic declination	15 54.8 E.	15 53.9 E.

These observations were made after noon.

Chronometer 0^h 9^m $46^s.4$ slow of local mean time.

MAGNETIC DECLINATION.

Valparaiso, March 19, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	$12^{\circ}\ 49'\ 15''$	(1) Scale erect (2) Scale inverted	$82^{d}.4$ 76.2
			(1) — (2) = Δ	+ 6.2
			Transit of Sun's	
	Vernier Vernier		1st limb 2d limb	$3^{h}\ 20^{m}\ 49^{s}$ 24 14.5
	Mean	290 45 36	Mean	3 22 31.7
Telescope Reversed.	Vernier Vernier		1st limb 2d limb	
	Mean		Mean	
			Reading of Magnet.	
	Vernier		(1) Scale inverted (2) Scale erect	
			(2) — (1) = Δ	

	Telescope Direct.	Telescope Reversed.
Equation of time t δ	$7^{m}\ 50^{s}.3$ $+51^{\circ}\ 2'\ 55''$ $-0\ 23\ 16$	
Circle reading to magnet $\Delta \times \frac{1}{2}$ scale division Sun's azimuth	$12^{\circ}\ 49'.2$ + 7.3 113 25.7	
Sum 180° + circle reading to sun	126 22.2 110 45.6	
Magnetic declination	15 36.6 E.	

These observations were made after noon, and have been corrected for error of collimation.
Chronometer $0^{h}\ 9^{m}\ 30^{s}.3$ slow of local mean time.

MAGNETIC DECLINATION.

Valparaiso, March 29, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	$15^{\circ}\ 11'\ 30''$	(1) Scale erect (2) Scale inverted	$80^{d}.2$ 78.1
			(1) — (2) = Δ	+ 2.1
			Transit of Sun's	
	Vernier Vernier		1st limb 2d limb	$2^{h}\ 10^{m}\ 58^{s}$ 13 46
	Mean	311 0 30	Mean	2 12 22.0
Telescope Reversed.	Vernier Vernier		1st limb 2d limb	$2^{h}\ 16^{m}\ 13^{s}$ 19 0
	Mean	$309^{\circ}\ 58'\ 30''$	Mean	2 17 36.5
			Reading of Magnet.	
	Vernier	15 42 30	(1) Scale inverted (2) Scale erect	$85^{d}.8$ 72.3
			(2) — (1) = Δ	— 13.5

	Telescope Direct.	Telescope Reversed.
Equation of time t δ	$4^{m}\ 47^{s}.3$ $+34^{\circ}\ 15'\ 22''$ $+3\ 31\ 11$	$4^{m}\ 47^{s}.3$ $+35^{\circ}\ 34'\ 0''$ $+3\ 31\ 17$
Circle reading to magnet $\Delta \times \frac{1}{2}$ scale division Sun's azimuth	$15^{\circ}\ 11'.5$ + 2.5 131 44.0	$15^{\circ}\ 42'.5$ — 15.9 130 23.9
Sum 180° + circle reading to sun	146 58.0 131 0.5	145 50.5 129 58.5
Magnetic declination	15 57.5 E.	15 52.0 E.

These observations were made after noon.
Chronometer $0^{h}\ 9^{m}\ 26^{s}.8$ slow of local mean time.

MAGNETIC DECLINATION.

Valparaiso, April 7, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	20° 11′ 30″	(1) Scale erect	$79^{d}.4$
			(2) Scale inverted	79.9
			(1) − (2) = Δ	− 0.5
			Transit of Sun's	
	Vernier		1st limb	8^{h} 16^{m} 21^{s}
	Vernier		2d limb	19 35
	Mean	66 36 0	Mean	8 17 58.0
Telescope Reversed.	Vernier		1st limb	8^{h} 21^{m} 38^{s}
	Vernier		2d limb	24 51
	Mean	65° 36′ 0″	Mean	8 23 14.5
			Reading of Magnet.	
	Vernier	20 12 30	(1) Scale inverted	$81^{d}.8$
			(2) Scale erect	76.5
			(2) − (1) = Δ	− 5.3

	Telescope Direct.	Telescope Reversed.
Equation of time	2^{m} $9^{s}.9$	2^{m} $9^{s}.8$
t	− 53° 42′ 4″	− 52° 22′ 55″
δ	+ 6 52 13	+ 6 52 18
Circle reading to magnet	20° 11′.5	20° 12′.5
Δ × ½ scale division	− 0.6	− 6.2
Sun's azimuth	242 16.0	241 17.7
Sum	262 26.9	261 24.0
180° + circle reading to sun	246 36.0	245 36.0
Magnetic declination	15 50.9 E.	15 48.0 E.

These observations were made before noon.
Chronometer 0^{h} 9^{m} $23^{s}.6$ slow of local mean time.

MAGNETIC DECLINATION.

Valparaiso, April 11, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	20° 45′ 0″	(1) Scale erect	$76^{d}.5$
			(2) Scale inverted	81.8
			(1) − (2) = Δ	− 5.3
			Transit of Sun's	
	Vernier		1st limb	2^{h} 28^{m} 53^{s}
	Vernier		2d limb	31 42
	Mean	315 7 30	Mean	2 30 17.5
Telescope Reversed.	Vernier		1st limb	2^{h} 33^{m} 38^{s}
	Vernier		2d limb	36 29
	Mean	314° 3′ 30″	Mean	2 35 3.5
			Reading of Magnet.	
	Vernier	20 44 30	(1) Scale inverted	$81^{d}.8$
			(2) Scale erect	76.6
			(2) − (1) = Δ	− 5.2

	Telescope Direct.	Telescope Reversed.
Equation of time	0^{m} $59^{s}.0$	0^{m} $59^{s}.0$
t	+ 39° 40′ 6″	+ 40° 51′ 36″
δ	+ 8 27 2	+ 8 27 7
Circle reading to magnet	20° 45′.0	20° 44′.5
Δ × ½ scale division	− 6.2	− 6.1
Sun's azimuth	130 26.7	129 22.3
Sum	151 5.5	150 0.7
180° + circle reading to sun	135 7.5	134 3.5
Magnetic declination	15 58.0 E.	15 57.2 E.

These observations were made after noon, and prior to taking them the telescope was adjusted for collimation.
Chronometer 0^{h} 9^{m} $21^{s}.9$ slow of local mean time.

Magnetic Declination.

Valparaiso, April 13, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	20° 37′ 0″	(1) Scale erect	$80^d.4$
			(2) Scale inverted	77.9
			(1) − (2) = Δ	+ 2.5
			Transit of Sun's	
	Vernier	315 6 30	1st limb	$2^h\ 31^m\ 18^s$
	Vernier	314 56 0	2d limb	34 58
	Mean	315 1 15	Mean	2 33 8.0
Telescope Reversed.	Vernier		1st limb	
	Vernier		2d limb	
	Mean		Mean	
			Reading of Magnet.	
	Vernier		(1) Scale inverted	
			(2) Scale erect	
			(2) − (1) = Δ	

	Telescope Direct.	Telescope Reversed.
Equation of time	$0^m\ 27^s.6$	
t	+ 40° 30′ 27″	
δ	+ 9 10 46	
Circle reading to magnet	20° 37′.0	
Δ × ½ scale division	+ 2.9	
Sun's azimuth	130 15.2	
Sum	150 55.1	
180° + circle reading to sun	135 1.2	
Magnetic declination	15 53.9 E.	

These observations were made after noon, through clouds; collimation correct.
Chronometer $0^h\ 9^m\ 21^s.4$ slow of local mean time.

Magnetic Declination.

San Lorenzo Island, April 26, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	23° 13′ 30″	(1) Scale erect	$80^d.7$
			(2) Scale inverted	77.3
			(1) − (2) = Δ	+ 3.4
			Transit of Sun's	
	Vernier		1st limb	$1^h\ 37^m\ 26^s$
	Vernier		2d limb	40 23
	Mean	331 9 0	Mean	1 38 54.5
Telescope Reversed.	Vernier		1st limb	$1^h\ 42^m\ 10^s$
	Vernier		2d limb	45 11
	Mean	329° 38′ 30″	Mean	1 43 40.5
			Reading of Magnet.	
	Vernier	22 26 0	(1) Scale inverted	$57^d.0$
			(2) Scale erect	100.5
			(2) − (1) = Δ	+ 43.5

	Telescope Direct.	Telescope Reversed.
Equation of time	$2^m\ 19^s.8$	$2^m\ 19^s.9$
t	+ 22° 30′ 12″	+ 23° 41′ 43″
δ	+ 13 37 9	+ 13 37 12
Circle reading to magnet	23° 13′.5	22° 26′.0
Δ × ½ scale division	+ 4.0	+ 51.1
Sun's azimuth	138 21.1	136 50.9
Sum	161 38.6	160 8.0
180° + circle reading to sun	151 9.0	149 38.5
Magnetic declination	10 29.6 E.	10 29.5 E.

These observations were made after noon.
Chronometer $0^h\ 11^m\ 13^s.5$ fast of local mean time.

MAGNETIC DECLINATION.

Payta, May 7, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	12° 49′ 0″	(1) Scale erect	$79^{d}.6$
			(2) Scale inverted	78.9
			(1) − (2) = Δ	+ 0.7
			Transit of Sun's	
	Vernier	74 35 30	1st limb	7^{h} 35^{m} 58^{s}
	Vernier	75 5 0	2d limb	37 12
	Mean	74 50 15	Mean	7 36 35.0
Telescope Reversed.	Vernier	74° 25′ 30″	1st limb	7^{h} 39^{m} 10^{s}
	Vernier	51 30	2d limb	40 6
	Mean	74 38 30	Mean	7 39 38.0
			Reading of Magnet.	
	Vernier	12 52 30	(1) Scale inverted	$80^{d}.1$
			(2) Scale erect	78.1
			(2) − (1) = Δ	− 2.0

	Telescope Direct.	Telescope Reversed.
Equation of time	3^{m} $37^{s}.9$	3^{m} $37^{s}.9$
t	−71° 37′ 15″	−70° 51′ 30″
δ	+16 50 1	+16 50 3
Circle reading to magnet	12° 49′.0	12° 52′.5
Δ × ½ scale division	+ 0.8	− 2.3
Sun's azimuth	250 51.6	250 43.1
Sum	263 41.4	263 33.3
180° + circle reading to sun	254 50.2	254 38.5
Magnetic declination	8 51.2 E.	8 54.8 E.

These observations were made before noon.
Chronometer 0^{h} 26^{m} $41^{s}.9$ fast of local mean time.

MAGNETIC DECLINATION.

Flamenco Island, Panama Bay, May 14, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	11° 25′ 30″	(1) Scale erect	$78^{d}.4$
			(2) Scale inverted	79.8
			(1) − (2) = Δ	− 1.4
			Transit of Sun's	
	Vernier	79 18 0	1st limb	7^{h} 58^{m} 1^{s}
	Vernier	78 42 0	2d limb	58 54
	Mean	79 0 0	Mean	7 58 27.5
Telescope Reversed.	Vernier	79° 19′ 0″	1st limb	8^{h} 0^{m} 55^{s}
	Vernier	78 43 0	2d limb	1 58
	Mean	79 1 0	Mean	8 1 26.5
			Reading of Magnet.	
	Vernier	11 33 0	(1) Scale inverted	$82^{d}.8$
			(2) Scale erect	75.2
			(2) − (1) = Δ	− 7.6

	Telescope Direct.	Telescope Reversed.
Equation of time	3^{m} $53^{s}.1$	3^{m} $53^{s}.1$
t	−64° 29′ 4″	−63° 44′ 19″
δ	+18 38 56	+18 38 58
Circle reading to magnet	11° 25′.5	11° 33′.0
Δ × ½ scale division	− 1.6	− 8.9
Sun's azimuth	253 32.1	253 32.6
Sum	264 56.0	264 56.7
180° + circle reading to sun	259 0.0	259 1.0
Magnetic declination	5 56.0 E.	5 55.7 E.

These observations were made before noon.
Chronometer 0^{h} 20^{m} $16^{s}.9$ fast of local mean time.

MAGNETIC DECLINATION.
Acapulco, May 30, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	12° 23′ 30″	(1) Scale erect . . . . . . (2) Scale inverted	$78^d.0$ 80.3
			(1) — (2) = Δ	— 2.3
			Transit of Sun's	
	Vernier . . . . . . Vernier	75 25 30 74 55 30	1st limb . . . . . . 2d limb	$8^h\ 14^m\ 28^s$ 15 28
	Mean	75 10 30	Mean	8 14 58.0
Telescope Reversed.	Vernier . . . . . . Vernier	75° 36′ 0″ 6 30	1st limb . . . . . . 2d limb	$8^h\ 17^m\ 29^s$ 18 38
	Mean	75 21 15	Mean	8 18 3.5
			Reading of Magnet.	
	Vernier	12 28 0	(1) Scale inverted . . . . (2) Scale erect	$81^d.3$ 77.2
			(2) — (1) = Δ	— 4.1

	Telescope Direct.	Telescope Reversed.
Equation of time	$2^m\ 47^s.1$	$2^m\ 47^s.1$
t	—80° 54′ 16″	—80° 7′ 54″
δ	+21 47 18	+21 47 19
Circle reading to magnet	12° 23′.5	12° 28′.0
Δ × ½ scale division	— 2.7	— 4.8
Sun's azimuth	251 9.9	251 19.6
Sum	263 30.7	263 42.8
180° + circle reading to sun	255 10.5	255 21.2
Magnetic declination	8 20.2 E.	8 21.6 E.

These observations were made before noon.
Chronometer $1^h\ 41^m\ 22^s.2$ fast of local mean time.
Circle reading to distant referring mark 10° 23′ 0″.

MAGNETIC DECLINATION.
Acapulco, May 30, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	12° 27′ 30″	(1) Scale erect . . . . . . (2) Magnetic axis	$78^d.0$ 79.11
			(1) — (2) = Δ	— 1.11
			Transit of Sun's	
	Vernier . . . . . . Vernier		1st limb . . . . . . 2d limb	
	Mean		Mean	
Telescope Reversed.	Vernier . . . . . . Vernier		1st limb . . . . . . 2d limb	
	Mean		Mean	
			Reading of Magnet.	
	Vernier	12 27 30	(1) Magnetic axis . . . . (2) Scale erect	$79^d.11$ 78.0
			(2) — (1) = Δ	— 1.11

	Telescope Direct.	Telescope Reversed.
Circle reading to magnet	12° 27′.5	12° 27′.5
Δ × 1 scale division	— 2.6	— 2.6
Azimuth of mark	186 22.4	186 21.4
Sum	198 47.3	198 46.3
180° + circle reading to mark . . .	190 23.0	190 23.0
Magnetic declination	8 24.3 E.	8 23.3 E.

These observations were made at $9^h\ 19^m$ A.M., local mean time.

MAGNETIC DECLINATION.
Magdalena Bay, June 9, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	13° 4′ 0″	(1) Scale erect (2) Magnetic axis	$78^{d}.10$ 79.11
			(1) — (2) = Δ	—1.01
			Transit of Sun's	
	Vernier Vernier	278 39 0 277 50 0	1st limb 2d limb	5^{h} 9^{m} 10^{s} 10 18
	Mean	278 14 30	Mean	5 9 44.0
Telescope Reversed.	Vernier Vernier		1st limb 2d limb	
	Mean		Mean	
			Reading of Magnet.	
	Vernier		(1) Scale inverted (2) Scale erect	
			(2) — (1) = Δ	

	Telescope Direct.	Telescope Reversed.
Equation of time	1^{m} $3^{s}.0$	
t	+ 40° 10′ 39″	
δ	+ 22 58 41	
Circle reading to magnet	13° 4′.0	
Δ × 1 scale division	— 2.4	
Sun's azimuth	95 53.4	
Sum	108 55.0	
180° + circle reading to sun	98 14.5	
Magnetic declination	10 40.5 E.	

These observations were made after noon. Collimation error zero.
Chronometer 2^{h} 30^{m} $4^{s}.4$ fast of local mean time.

MAGNETIC DECLINATION.
San Diego Bay, June 15, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	16° 9′ 30″	(1) Scale erect (2) Scale inverted	$79^{d}.4$ 78.7
			(1) — (2) = Δ	+ 0.7
			Transit of Sun's	
	Vernier Vernier	277 48 30 278 52 30	1st limb 2d limb	6^{h} 42^{m} 20^{s} 45 36
	Mean	278 20 30	Mean	6 43 58.0
Telescope Reversed.	Vernier Vernier	279° 2′ 0″ 278 28 30	1st limb 2d limb	6^{h} 46^{m} 45^{s} 47 43
	Mean	278 45 15	Mean	6 47 14.0
			Reading of Magnet.	
	Vernier	16 33 30	(1) Scale inverted (2) Scale erect	$88^{d}.2$ 69.7
			(2) — (1) = Δ	— 18.5

	Telescope Direct.	Telescope Reversed.
Equation of time	0^{m} $12^{s}.0$	0^{m} $12^{s}.1$
t	+ 58° 18′ 22″	+ 59° 7′ 21″
δ	+ 23 20 30	+ 23 20 31
Circle reading to magnet	16° 9′.5	16° 33′.5
Δ × ½ scale division	+ 0.8	— 21.7
Sun's azimuth	95 19.5	95 43.0
Sum	111 29.8	111 54.8
180° + circle reading to sun	98 20.5	98 45.2
Magnetic declination	13 9.3 E.	13 9.6 E.

These observations were made after noon.
Chronometer 2^{h} 50^{m} $32^{s}.5$ fast of local mean time.

MAGNETIC DECLINATION.

San Francisco Bay, June 26, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	20° 9′ 30″	(1) Scale erect	$79^d.3$
			(2) Scale inverted	78.8
			(1) — (2) = Δ	+ 0.5
			Transit of Sun's	
	Vernier	88 40 0	1st limb	$3^h\ 54^m\ 2^s$
	Vernier	12 30	2d limb	55 18
	Mean	88 26 15	Mean	3 54 40.0
Telescope Reversed.	Vernier	88° 58′ 0″	1st limb	$3^h\ 57^m\ 12^s$
	Vernier	28 0	2d limb	58 9
	Mean	88 43 0	Mean	3 57 40.5
			Reading of Magnet.	
	Vernier	20 28 30	(1) Scale inverted	$89^d.6$
			(2) Scale erect	68.1
			(2) — (1) = Δ	— 21.5

	Telescope Direct.	Telescope Reversed.
Equation of time	$2^m\ 29^s.4$	$2^m\ 29^s.4$
t	— 65° 14′ 24″	— 64° 29′ 17″
δ	+ 23 22 9	+ 23 22 9
Circle reading to magnet	20° 9′.5	20° 28′.5
Δ × ½ scale division	+ 0.6	— 25.3
Sun's azimuth	264 40.7	265 6.2
Sum	284 50.8	285 9.4
180° + circle reading to sun	268 26.2	268 43.0
Magnetic declination	16 24.6 E.	16 26.4 E.

These observations were made before noon.

Chronometer $8^h\ 13^m\ 8^s.2$ fast of local mean time.

MAGNETIC DECLINATION.

U. S. Naval Observatory, Washington, November 1, 1866.

	Circle Readings.		Reading of Magnet.	
Telescope Direct.	Vernier	0° 25′ 0″	(1) Scale erect	$87^d.5$
			(2) Scale inverted	70.5
			(1) — (2) = Δ	+ 17.0
			Transit of Sun's	
	Vernier		1st limb	$7^h\ 9^m\ 6^s.5$
	Vernier		2d limb	11 42.2
	Mean	222 51 0	Mean	7 10 24.4
Telescope Reversed.	Vernier		1st limb	$7^h\ 16^m\ 7^s.5$
	Vernier		2d limb	18 44.0
	Mean	224° 30′ 0″	Mean	7 17 25.8
			Reading of Magnet.	
	Vernier	0 43 30	(1) Scale inverted	$78^d.0$
			(2) Scale erect	79.9
			(2) — (1) = Δ	+ 1.9

	Telescope Direct.	Telescope Reversed.
Equation of time	$16^m\ 18^s.5$	$16^m\ 18^s.5$
t	+ 35° 43′ 47″	+ 37° 29′ 8″
δ	— 14 32 51	— 14 32 55
Circle reading to magnet	0° 25′.0	0° 43′.5
Δ × ½ scale division	+ 20.0	+ 2.2
Sun's azimuth	39 22.2	40 59.7
Sum	40 7.2	41 45.4
180° + circle reading to sun	42 51.0	44 30.0
Magnetic declination	2 43.8 W.	2 44.6 W.

These observations were made after noon, and the readings of the magnet scale were taken two hours before the transits of the sun.

Chronometer $5^h\ 3^m\ 47^s.8$ fast of local mean time.

OBSERVATIONS OF MAGNETIC INCLINATION.

MAGNETIC DIP.
Gosport, October 30, 1865. Needle A. 1.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
70° 39′	70° 22′	71° 3′	70° 44′	109° 58′	109° 55′	109° 25′	109° 46′
70 49	70 30	71 4	70 44	110 10	110 25	109 19	109 37
70 45	70 27	71 5	70 47	110 15	110 30	109 33	109 50
70 44	70 26	71 4	70 45	110 8	110 17	109 26	109 44
70 35		70 55		110 12		109 35	
70 45				109 54			
70 25							

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
111° 52′	112° 4′	112° 13′	112° 29′	68° 49′	68° 24′	68° 45′	68° 26′
111 55	112 8	112 9	112 26	68 47	68 27	68 30	68 13
112 2	112 17	112 9	112 25	68 37	68 17	68 41	68 20
111 56	112 10	112 10	112 27	68 44	68 23	68 39	68 20
112 3		112 18		68 34		68 30	
112 10				68 32			
68 11							

Resulting Dip: + 69° 21′

MAGNETIC DIP.
Gosport, October 30, 1865. Needle A. 2.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
71° 47′	71° 41′	70° 4′	69° 48′	109° 17′	109° 41′	111° 8′	111° 30′
72 5	71 51	70 6	69 52	109 13	109 34	111 8	111 24
71 59	71 46	70 14	69 60	109 0	109 21	110 59	111 18
71 57	71 46	70 8	69 56	109 10	109 32	111 5	111 24
71 51		70 2		109 21		111 15	
70 56				110 18			
70 19							

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
110° 29′	110° 40′	110° 39′	110° 59′	69° 57′	69° 37′	70° 0′	69° 37′
110 33	110 54	110 57	111 12	69 41	69 20	70 7	69 42
110 41	110 23	110 54	111 11	69 40	69 18	70 10	69 45
110 34	110 39	110 50	111 7	69 46	69 25	70 6	69 41
110 36		110 58		69 36		69 54	
110 47				69 45			
69 29							

Resulting Dip: + 69° 54′

NOTE.—It will be observed that at some stations only one end of the needle was read. In such cases the other end of the needle was hidden by the cross-bar which supports the agate planes.

Magnetic Dip.

St. Thomas, November 13, 1865, Needle A. 2.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
53° 37′	53° 18′	52° 54′	52° 30′	127° 23′	127° 37′	128° 17′	128° 36′
53 55	53 35	53 0	52 34	127 30	127 43	128 15	128 35
53 55	53 35	53 5	52 35	127 26	127 40	128 8	128 31
53 49	53 29	53 0	52 33	127 26	127 40	128 13	128 34
53 39		52 46		127 33		128 24	
53 12				127 58			
52 37							

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
127° 11′	127° 30′	127° 59′	128° 17′	53° 24′	53° 13′	52° 35′	52° 8′
127 13	127 32	127 61	128 18	53 35	53 15	52 39	52 11
127 16	127 33	127 54	128 12	53 26	53 7	52 30	52 4
127 13	127 32	127 58	128 16	53 28	53 12	52 35	52 8
127 23		128 7		53 20		52 22	
127 45				52 51			
52 33							

Resulting Dip: + 49° 32′

Azimuth of Dip Circle 26° 16′

Magnetic Dip.

St. Thomas, November 13, 1865, Needle A. 1.

POLARITY OF MARKED END SOUTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
50° 2′	49° 44′	50° 12′	49° 52′	130° 11′	130° 30′	130° 12′	130° 28′
50 35	50 15	50 15	49 45	130 15	130 30	130 6	130 24
50 50	50 24	50 16	49 53	130 11	130 24	130 11	130 26
50 29	50 8	50 14	49 50	130 12	130 28	130 10	130 26
50 18		50 2		130 20		130 18	
50 10				130 19			
49 55							

POLARITY OF MARKED END NORTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
124° 41′	125° 0′	125° 36′	125° 50′	55° 31′	55° 11′	55° 28′	55° 25′
124 22	124 37	125 38	125 51	55 49	55 24	55 21	54 50
124 35	124 52	125 37	125 56	56 0	55 28	55 20	54 54
124 33	124 50	125 37	125 52	55 47	55 21	55 23	55 3
124 42		125 45		55 34		55 13	
125 14				55 24			
55 5							

Resulting Dip: + 49° 36′

Azimuth of Dip Circle 26° 16′

MAGNETIC DIP.

St. Thomas, November 13, 1865. Needle A. 2.

POLARITY OF MARKED END SOUTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
49° 42′	49° 21′	50° 10′	49° 55′	130° 50′	131° 15′	130° 56′	130° 47′
49 44	49 20	51 5	50 42	130 40	131 3	130 37	130 57
40 53	49 25	50 47	50 19	130 46	131 11	130 41	130 66
49 46	49 22	50 40	50 18	130 45	131 10	130 45	130 57
49 34		50 29		130 57		130 51	
50 1				130 54			
49 34							

POLARITY OF MARKED END NORTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
.	N.	S.	N.	S.	N.	S.	N.
129° 32′	129° 59′	130° 4′	130° 30′	49° 4′	49° 1′	50° 26′	50° 29′
129 47	130 11	130 22	130 47	49 25	49 30	50 36	50 45
129 55	130 20	130 8	130 35	49 20	49 28	50 31	50 32
129 45	130 10	130 11	130 37	49 16	49 20	50 31	50 35
129 58		130 24		49 18		50 33	
130 11				49 56			
49 53							

Resulting Dip: + 49° 44′

MAGNETIC DIP.

St. Thomas, November 16, 1865. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
52° 16′	51° 57′	53° 0′	52° 40′	128° 36′	128° 54′	127° 44′	128° 5′
52 26	52 3	53 9	52 47	128 28	128 44	127 45	128 1
52 40	52 8	53 7	52 45	128 30	128 42	127 52	128 7
52 27	52 3	53 5	52 44	128 31	128 47	127 47	128 4
52 15		52 55		128 39		127 55	
52 35				128 17			
52 9							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
133° 43′	134° 0′	133° 12′	133° 28′	47° 34′	47° 14′	47° 50′	47° 30′
133 47	134 0	133 16	133 31	47 28	46 51	47 38	47 20
133 36	133 50	133 24	133 43	47 21	46 53	47 34	47 8
133 42	133 57	133 17	133 34	47 28	46 59	47 41	47 19
133 50		133 . 25		47 14		47 30	
133 37				47 22			
46 52							

Resulting Dip: + 49° 39′

Magnetic Dip.

Salute Islands, November 28, 1865. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
35° 28′ 35 59 35 45	35° 5′ 35 38 35 16	34° 37′ 34 45 34 30	34° 10′ 34 15 33 53	145° 17′ 145 5 145 17	145° 35′ 145 28 145 16	145° 15′ 145 12 145 45	145° 48′ 145 44 146 18
35 44	35 20	34 37	34 6	145 13	145 26	145 24	145 57
35 32		34 22		145 20		145 40	
34 57				145 30			
34 44							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
145° 28′ 145 35 145 40	145° 45′ 145 58 146 6	145° 25′ 145 24 145 25	145° 45′ 145 55 145 55	35° 45′ 35 40 35 41	35° 15′ 35 12 35 15	35° 15′ 34 45 34 40	34° 44′ 34 21 34 9
145 34	145 56	145 25	145 52	35 42	35 14	34 53	34 25
145 45		145 39		35 28		34 39	
145 42				35 3			
34 40							

Resulting Dip: + 34° 42′

Magnetic Dip.

Salute Islands, Nov. 28, 1865. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
38° 57′ 39 0 38 58	38° 35′ 38 25 38 14	38° 50′ 39 25 39 16	38° 15′ 38 50 38 50	142° 5′ 142 26 142 7	142° 23′ 142 45 142 26	141° 40′ 141 31 141 30	142° 2′ 141 53 141 53
38 58	38 25	39 10	38 38	142 13	142 31	141 34	141 56
38 42		38 54		142 22		141 45	
38 48				142 4			
38 22							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
150° 55′ 150 50 150 45		149° 0′ 149 12 149 18	149° 23′ 149 33 149 37	30° 6′ 30 0 30 10	29° 39′ 29 34 29 45	31° 5′ 31 10 31 29	30° 34′ 30 39 30 55
150 50	(151 22)	149 10	149 31	30 5	29 39	31 15	30 43
151 4		149 20		29 52		30 59	
150 12				30 25			
30 7							

Resulting Dip: + 34° 27′

MAGNETIC DIP.

Ceara, December 13, 1865. Needle A. 2.

POLARITY OF MARKED END SOUTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N	S.	N	S.	N.	S.	N.
20° 30′ 20 17 20 30		21° 30′ 21 2 21 15		159° 56′ 160 37 160 30		156° 35′ 157 0 157 0	
20 26		21 16		160 21		156 52	
	20 51				158 36		
			21 7				

POLARITY OF MARKED END NORTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
159° 4′ 158 45 158 40		160° 20′ 160 5 159 23		22° 55′ 22 32 22 28		22° 20′ 22 8 22 26	
158 50		159 56		22° 38′		22 18	
	159 23				22 28		
			21 33				

Resulting Dip: + 21° 20′

MAGNETIC DIP.

Ceara, December 13, 1865. Needle A. 1.

POLARITY OF MARKED END SOUTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
17° 30′ 17 31 17 45		16° 5′ 15 52 15 52		163° 20′ 163 20 163 6		166° 50′ 165 30 166 0	
17 35		15 56		163 15		166 7	
	16 45				164 41		
			16 2				

POLARITY OF MARKED END NORTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
154° 0′ 154 18 154 5		153° 55′ 154 20 154 40		27° 10′ 27 10 27 0		27° 10′ 27 0 27 20	
154 8		154 18		27 7		27 10	
	154 13				27 9		
			26 28				

Resulting Dip: + 21° 26′

Magnetic Dip.

Pernambuco, Dec. 23, 1865. Needle A. 2.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N	S.	N.
12° 20′ 12 15 12 35		12° 30′ 12 10 12 10		169° 45′ 170 10 170 40		167° 5′ 167 10 167 5	
12 23		12 17		170 12		167 7	
12 20				168 40			
11 50							

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
168° 30′ 168 25 169 20		168° 10′ 168 30 167 35		14° 45′ 14 30 14 20		12° 5′ 12 30 12 25	
168 45		168 5		14 32		12 20	
168 25				13 26			
12 30							

Resulting Dip: + 12° 10′

Magnetic Dip.

Pernambuco, Dec. 23, 1865. Needle A. 1.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
18° 30′ 18 35 18 55		18° 0′ 18 0 18 0		162° 10′ 162 40 162 40		163° 5′ 163 20 162 30	
18 40		18 0		162 30		162 58	
18 20				162 44			
17 48							

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
174° 20′ 175 20 175 30		173° 35′ 173 30 173 50		5° 30′ 5 30 5 50		7° 25′ 7 40 7 40	
175 3		173 38		5 37		7 35	
174 20				6 36			
6 8							

Resulting Dip: + 12° 6′

MAGNETIC DIP.

Bahia, December 27, 1865. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
3° 35′	3° 15′	4° 40′	4° 15′	176° 30′	176° 30′	176° 10′	176° 10′
3 45	3 30	5 10	4 45	176 45	176 40	176 5	176 5
3 30	3 35	5 50	5 20	176 45	176 40	176 45	175 50
3 37	3 27	5 13	4 47	176 40	176 37	176 20	176 2
3 32		5 0		176 38		176 11	
4 16				176 24			
3 56							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
174° 45′	174° 50′	176° 30′	176° 30′	4° 30′	4° 10′	5° 25′	5° 0′
174 45	174 40	177 15	177 15	5 5	4 45	5 30	5 5
174 40	174 50	176 0	176 0	5 0	4 40	5 30	5 0
174 43	174 47	176 35	176 35	4 52	4 32	5 28	5 2
174 45		176 35		4 42		5 15	
175 40				4 58			
4 39							

Resulting Dip: +4° 17′

MAGNETIC DIP.

Bahia, December 27, 1865. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
11° 30′		11° 30′		168° 55′		170° 20′	
11 30		11 10		169 10		169 55	
12 0		11 10		169 10		170 25	
11 40		11 17		169 5		170 13	
11 28				169 39			
10 54							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
3° 15′		1° 50′		177° 25′		179° 20′	
3 35		1 55		178 20		179 10	
3 5		1 55		178 5		179 10	
3 18		1 53		177 57		179 13	
—2 35				—178 35			
—2 0							

Resulting Dip: +4° 31′

MAGNETIC DIP.

Rio Janeiro, January 6, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	167° 0′ 166 45 167 10		168° 30′ 169 0 169 15		11° 20′ 12 30 12 30		11° 35′ 11 35 11 30
	166 58		168 55		12 7		11 33
	167 56				11 50		
			11 57				

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	10° 15′ 11 15 11 35		12° 5′ 12 30 12 50		168° 55′ 169 0 169 35		167° 45′ 168 5 168 10
	11 2		12 28		169 10		168 0
	11 45				168 35		
			11 35				

Resulting Dip: — 11° 46′

MAGNETIC DIP.

Rio Janeiro, January 6, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	175° 15′ 175 50 175 45		174° 20′ 174 15 174 0		4° 50′ 5 35 5 0		7° 10′ 7 0 6 35
	175 37		174 12		5 8		6 55
	174 55				6 2		
			5 34				

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	18° 5′ 18 15 18 25		18° 10′ 18 5 18 15		162° 10′ 162 15 161 45		162° 55′ 163 15 163 35
	18 15		18 10		162 3		163 15
	18 12				162 39		
			17 46				

Resulting Dip: — 11° 48′

MAGNETIC DIP.

Monte Video, January 18, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	155° 15′		153° 30′		25° 40′		26° 20′
	155 10		153 40		25 10		27 0
	155 15		153 20		25 40		26 40
	155 13		153 30		25 30		26 40
	154 22				26 5		
			25 52				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	36° 0′		36° 20′		144° 20′		144° 10′
	36 10		36 20		144 20		144 20
	36 10		36 30		144 20		144 30
	36 7		36 23		144 20		144 20
	36 15				144 20		
			35 57				

Resulting Dip: — 31° 11′

MAGNETIC DIP.

Monte Video, January 18, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	149° 10′		149° 20′		32° 10′		31° 0′
	148 50		149 30		31 20		31 0
	149 20		150 10		32 0		31 20
	149 7		149 40		31 50		31 7
	149 23				31 29		
			31 3				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	31° 40′		30° 50′		149° 30′		149° 40′
	32 10		31 0		149 10		150 0
	31 40		30 50		149 50		149 20
	31 50		30 53		149 30		149 40
	31 22				149 35		
			30 54				

Resulting Dip: — 30° 58′

MAGNETIC DIP.

Monte Video, January 18, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	148° 50′		149° 20′		31° 0′		31° 0′
	149 0		148 50		31 10		31 40
	149 30		149 0		31 20		31 40
	149 7		149 3		31 10		31 27
	149 5				31 19		
			31 7				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	32° 0′		31° 0′		149° 10′		149° 10′
	32 0		31 20		149 10		149 30
	31 50		31 40		149 20		149 50
	31 57		31 20		149 13		149 30
	31 39				149 22		
			31 8				

Resulting Dip: —31° 8′

Magnetic Dip.

Sandy Point, February 7, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
128° 10′	128° 0′	128° 15′	128° 10′	52° 35′	52° 30′	52° 45′	52° 45′
128 0	127 45	127 30	127 15	52 45	52 40	52 45	52 50
128 10	128 0	128 20	128 10	52 45	52 40	52 55	52 45
128 7	127 55	128 2	127 52	52 42	52 37	52 48	52 47
128 1		127 57		52 40		52 48	
127 59				52 44			
52 22							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
57° 5′	57° 0′	57° 40′	57° 35′	123° 10′	123° 15′	123° 0′	122° 55′
57 15	57 5	57 40	57 35	123 20	123 10	123 10	123 0
57 15	57 5	57 45	57 40	123 45	123 20	123 10	123 0
57 12	57 3	57 42	57 37	123 25	123 15	123 7	122 58
57 7		57 40		123 20		123 3	
57 24				123 12			
57 6							

Resulting Dip: — 54° 52′

Magnetic Dip.

Sandy Point, February 7, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West	
S.	N.	S.	N.	S.	N.	S.	N.
124° 45′	124° 45′	126° 30′	126° 10′	55° 30′	56° 0′	55° 0′	54° 50′
124 55	124 45	125 45	125 30	55 10	55 5	54 45	55 10
125 15	124 45	125 45	125 40	55 30	56 0	55 0	54 0
124 58	124 45	126 0	125 47	55 23	55 42	54 55	54 40
124 52		125 53		55 33		54 47	
125 22				55 10			
54 54							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
56° 15′	56° 10′	54° 35′	54° 30′	124° 35′	125° 0′	126° 0′	125° 30′
56 15	56 45	54 25	54 40	124 40	124 30	126 15	125 35
56 20	56 15	54 35	54 45	123 55	123 45	125 50	125 45
56 17	56 23	54 32	54 38	124 23	124 25	126 2	125 37
56 20		54 35		124 24		125 50	
55 27				125 7			
55 10							

Resulting Dip: — 55° 2′

MAGNETIC DIP.

Valparaiso, March 2, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	150° 45′ 150 40 150 45		148° 30′ 149 15 149 10		30° 45′ 30 30 29 55		31° 15′ 31 10 31 35
	150 43		148 58		30 23		31 20
	149 50				30 52		
			30 31				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	38° 30′ 38 45 38 50		39° 15′ 39 0 39 30		141° 35′ 142 20 140 55		141° 5′ 141 45 141 15
	38 42		39 15		141 37		141 22
	38 58				141 30		
			38 44				

Resulting Dip: —34° 50′

MAGNETIC DIP.

Valparaiso, March 2, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	145° 15′ 145 25 145 15		145° 30′ 144 40 145 15		35° 20′ 35 35 35 40		35° 0′ 35 0 34 45
	145 18		145 8		35 32		34 55
	145 13				35 14		
			35 0				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	36° 10′ 35 45 36 5		34° 50′ 34 50 35 5		145° 0′ 145 0 144 20		145° 15′ 145 10 145 0
	36 0		34 55		144 47		145 8
	35 27				144 58		
			35 14				

Resulting Dip: —35° 7′

Magnetic Dip.

Valparaiso, March 19, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	144° 50′		144° 15′		35° 50′		34° 45′
	145 0		144 30		36 40		35 5
	144 55		144 20		36 15		35 30
	144 55		144 22		36 15		35 7
	144 39				35 41		
			35 31				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	37° 0′		34° 45′		145° 10′		145° 15′
	37 10		34 40		144 45		145 20
	37 20		34 20		144 30		145 10
	37 10		34 35		144 48		145 15
	35 52				145 1		
			35 25				

Resulting Dip: —35° 28′

Magnetic Dip.

Valparaiso, March 19, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	149° 50′		148° 50′		30° 15′		31° 30′
	150 10		148 30		30 30		31 50
	149 45		148 20		30 30		31 20
	149 55		148 33		30 25		31 33
	149 14				30 59		
			30 52				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	39° 20′		40° 5′		140° 55′		140° 0′
	39 50		40 15		141 10		140 30
	39 30		40 20		140 55		140 45
	39 33		40 13		141 0		140 25
	39 53				140 43		
			39 35				

Resulting Dip: —35° 28′

Magnetic Dip.

Valparaiso, March 29, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	146° 10′		145° 15′		35° 45′		35° 20′
	145 10		145 30		35 40		35 30
	144 50		145 15		35 50		35 30
	145 23		145 20		35 45		35 27
	145 22				35 36		
			35 7				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	36° 40′		35° 30′		144° 15′		144° 40′
	36 40		35 45		145 0		145 15
	37 20		35 45		144 20		145 0
	36 53		35 40		144 32		144 58
	36 17				144 45		
			35 46				

Resulting Dip: —35° 27′

Magnetic Dip.

Valparaiso, March 29, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	149° 30′		149° 15′		30° 10′		31° 45′
	150 0		148 40		30 20		31 50
	150 15		148 40		30 40		31 40
	149 55		148 52		30 23		31 45
	149 24				31 4		
			30 50				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	40° 40′		40° 20′		140° 40′		140° 15′
	40 15		40 20		141 10		140 40
	40 10		40 20		141 10		140 30
	40 22		40 20		141 0		140 28
	40 21				140 44		
			39 48				

Resulting Dip: —35° 34′

MAGNETIC DIP.

Valparaiso, April 7, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	144° 30′		144° 30′		36° 40′		35° 10′
	145 0		145 30		36 15		35 10
	144 30		145 10		35 40		34 40
	144 40		145 3		36 12		35 0
144 51				35 36			
35 22							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	36° 20′		34° 40′		145° 15′		145° 40′
	36 15		35 0		144 15		144 45
	36 40		35 0		144 0		145 15
	36 25		34 53		144 30		145 13
35 39				144 52			
35 24							

Resulting Dip: —35° 23′

MAGNETIC DIP.

Valparaiso, April 7, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	150° 30′		149° 20′		30° 20′		32° 0′
	150 40		149 15		30 25		32 15
	150 0		149 0		31 30		32 10
	150 23		149 12		30 45		32 8
149 47				31 27			
30 50							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	39° 50′		40° 15′		140° 40′		140° 20′
	39 30		40 15		141 45		140 30
	39 40		40 0		141 0		140 30
	39 40		40 10		141 8		140 27
39 55				140 47			
39 34							

Resulting Dip: —35° 26′

MAGNETIC DIP.

Valparaiso, April 11, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	144° 20′		145° 20′		36° 10′		35° 40′
	144 40		145 0		36 0		35 40
	144 50		144 50		35 45		35 40
	144 37		145 3		35 58		35 40
144 50				35 49			
35 30							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	36° 50′		35° 20′		144° 20′		145° 10′
	36 50		36 0		144 30		145 10
	36 50		35 10		144 35		145 0
	36 50		35 30		144 28		145 7
36 10				144 47			
35 42							

Resulting Dip: —35° 36′

MAGNETIC DIP.

Valparaiso, April 11, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	149° 40′		150° 10′		30° 40′		31° 50′
	150 0		148 50		30 20		31 40
	150 20		149 0		30 10		31 40
	150 0		149 20		30 23		31 43
149 40				31 3			
30 42							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	39° 30′		41° 20′		140° 50′		140° 30′
	39 30		40 25		141 10		140 10
	39 40		40 0		140 40		140 10
	39 33		40 35		140 53		140 17
40 4				140 35			
39 45							

Resulting Dip: —35° 29′

MAGNETIC DIP.

Valparaiso, April 13, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	145° 45′ 145 45 145 45		145° 20′ 145 0 145 0		35° 0′ 35 20 35 30		35° 20′ 35 30 35 20
	145 45		145 7		35 17		35 23
145 26				35 20			
34 57							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	36° 10′ 36 0 36 0		34° 50′ 34 50 34 50		144° 30′ 144 30 144 50		144° 30′ 144 30 144 30
	36 3		34 50		144 37		144 30
35 26				144 34			
35 26							

Resulting Dip: —35° 12′

MAGNETIC DIP.

Valparaiso, April 13, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	150° 15′ 150 10 150 10		148° 50′ 148 50 148 50		30° 15′		31° 50′
	150 12		148 50				
149 31				31 2			
30 46							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	40° 5′		40° 30′		140° 40′		139° 50′
40 18				140 15			
40 2							

Resulting Dip: —35° 40′

MAGNETIC DIP.

San Lorenzo Island, April 26, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	172° 20′		175° 35′		8° 10′		6° 20′
	173 57				7 15		
			6 39				

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	6° 15′		7° 40′		175° 10′		173° 30′
	6 58				174 20		
			6 19				

Resulting Dip: —6° 29′

MAGNETIC DIP.

San Lorenzo Island, April 26, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	0° 0′		+1° 15′		0° 35′		1° 50′
	+0 37				—1 13		
			—0 18				

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	13° 10′		12° 40′		167° 30′		168° 20
	12 55				167 55		
			—12 30				

Resulting Dip: —6° 28′

MAGNETIC DIP.

Payta, May 7, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
	4° 30′		4° 20′		176° 45′		175° 0′
4 25				175 52			
4 17							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
	174° 0′		175° 35′		6° 5′		4° 40′
174 48				5 22			
5 17							

Resulting Dip: + 4° 47′

MAGNETIC DIP.

Payta, May 7, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
11° 15′		10° 30′		169° 45′		169° 50′	
10 52				169 48			
+10 32							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
1° 10′		0° 10′		179° 20′		0° 45′	
—0 40				+0 2			
—0 19							

Resulting Dip: + 5° 9′

MAGNETIC DIP.

Flamenco Island, Panama Bay, May 14, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
32° 40′	32° 10′	31° 35′	31° 0′	148° 50′	148° 40′	148° 15′	148° 10′
32 25		31 18		148 45		148 12	
31 52				148 28			
31 42							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
148° 45′	148° 40′	148° 30′	148° 20′	32° 45′	32° 10′	32° 30′	32° 0′
148 42		148 25		32 28		32 15	
148 34				32 22			
31 54							

Resulting Dip: + 31° 48′

MAGNETIC DIP.

Flamenco Island, Panama Bay, May 14, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
36° 20′		36° 40′		144° 25′		144° 25′	
36 30				144 25			
36 2							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
153° 15′		152° 10′		27° 40′		28° 50′	
152 43				28 15			
27 46							

Resulting Dip: + 32° 5′

Magnetic Dip.

Acapulco, May 30, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
41° 15′	40° 45′	40° 10′	39° 30′	139° 40′	139° 45′	140° 10′	140° 10′
41 0		39 50		139 42		140 10	
40 25				139 56			
40 15							
POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
139° 50′	139° 45′	140° 35′	140° 20′	40° 20′	39° 50′	39° 0′	38° 40′
139 47		140 28		40 5		38 50	
140 7				39 28			
39 40							

Resulting Dip: + 39° 58′

Magnetic Dip.

Acapulco, May 30, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
43° 10′	42° 40′	43° 40′	43° 15′	137° 40′	137° 35′	137° 30′	137° 20′
42 55		43 28		137 37		137 25	
43 12				137 31			
42 50							
POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
144° 15′	144° 10′	143° 20′	143° 15′	36° 45′	36° 15′	37° 20′	36° 50′
144 12		143 18		36 30		37 5	
143 45				36 47			
36 31							

Resulting Dip: + 39° 49′

MAGNETIC DIP.

Magdalena Bay, June 9, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
50° 30′	50° 0′	49° 0′	48° 30′	130° 40′	130° 45′	132° 15′	132° 15′
50 15		48 45		130 42		132 15	
49 30				131 29			
49 0							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East	
S.	N.	S.	N.	S.	N.	S.	N.
131° 0′	131° 0′	132° 0′	132° 10′	45° 30′	45° 0′	48° 50′	48° 30′
131 0		132 5		45 15		48 40	
131 32				46 58			
47 43							

Resulting Dip: + 48° 22′

MAGNETIC DIP.

Magdalena Bay, June 9, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
51° 40′	51° 10′	52° 15′	51° 45′	129° 15′	129° 10′	128° 45′	128° 45′
51 25		52 0		129 12		128 45	
51 43				128 58			
51 23							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
135° 0′	134° 50′	134° 30′	134° 30′	46° 0′	45° 40′	46° 30′	46° 0′
134 55		134 30		45 50		46 15	
134 43				46 2			
45 39							

Resulting Dip: + 48° 41′

Magnetic Dip.

San Diego Bay, June 15, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
59° 25′	59° 0′	58° 15′	57° 45′	122° 0′	121° 55′	122° 40′	122° 40′
59 12		58 0		121 57		122 40	
		58 36				122 18	
				58 9			

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
122° 30′	122° 20′	123° 0′	123° 0′	58° 30′	58° 10′	58° 15′	57° 40′
122 25		123 0		58 20		57 57	
		122 42				58 8	
				57 43			

Resulting Dip: + 57° 56′

Magnetic Dip.

San Diego Bay, June 15, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
60° 10′	59° 45′	60° 30′	60° 0′	121° 0′	120° 45′	120° 20′	120° 0′
59 58		60 15		120 52		120 20	
		60 6				120 36	
				59 45			

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
124° 50′	124° 45′	124° 30′	124° 30′	56° 20′	55° 50′	56° 20′	56° 0′
124 47		124 30		56 5		56 10	
		124 38				56 7	
				55 44			

Resulting Dip: + 57° 51′

Magnetic Dip.

San Francisco Bay, June 26, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
63° 30′	63° 10′	62° 50′	62° 20′	117° 20′	117° 10′	117° 40′	117° 30′
63 20		62 35		117 15		117 35	
	62 57				117 25		
			62 46				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
118° 20′	118° 20′	117° 50′	117° 30′	62° 30′	62° 0′	63° 0′	62° 40′
118 20		117 40		62 15		62 50	
	118 0				62 32		
			62 16				

Resulting Dip: + 62° 31′

Magnetic Dip.

San Francisco Bay, June 26, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
63° 0′	63° 40′	64° 40′	64° 15′	116° 40′	116° 30′	116° 20′	116° 10′
63 20		64 27		116 35		116 15	
	63 54				116 25		
			63 45				

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
120° 0′	119° 45′	120° 0′	120° 0′	61° 10′	60° 45′	61° 10′	60° 40′
119 52		120 0		60 57		60 55	
	119 56				60 56		
			60 30				

Resulting Dip: + 62° 13′

MAGNETIC DIP.

U. S. Naval Observatory, Washington, Nov. 1, 1866. Needle A. 1.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
72° 45′	72° 45′	73° 15′	73° 5′	107° 45′	107° 30′	107° 25′	107° 0′
72 45		73 10		107 37		107 13	
72 57				107 25			
72 46							

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
109° 25′	109° 0′	109° 45′	109° 20′	71° 20′	71° 0′	71° 5′	70° 45′
109 12		109 33		71 10		70 55	
109 23				71 2			
70 50							

Resulting Dip: + 71° 51′

MAGNETIC DIP.

U. S. Naval Observatory, Washington, Nov. 1, 1866. Needle A. 2.

POLARITY OF MARKED END NORTH.							
CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
73° 30′	73° 15′	71° 30′	71° 15′	107° 20′	107° 0′	108° 5′	107° 45′
73 22		71 23		107 10		107 55	
72 22				107 33			
72 24							

POLARITY OF MARKED END SOUTH.							
CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
107° 35′	107° 5′	108° 50′	108° 40′	72° 25′	72° 20′	72° 0′	71° 45′
107 20		108 45		72 22		71 53	
108 2				72 8			
72 3							

Resulting Dip: + 72° 13′

MAGNETIC DIP.

U. S. Naval Observatory, Washington, May 6, 1867. Needle A. 1.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
73° 0′	72° 50′	73° 35′	73° 15′	107° 45′	107° 15′	107° 10′	106° 50′
72 55		73 25		107 30		107 0	
73 10				107 15			
72 58							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
110° 0′	109° 30′	109° 40′	109° 15′	71° 25′	71° 10′	71° 10′	70° 50′
109 45		109 28		71 18		71 0	
109 36				71 9			
70 46							

Resulting Dip: + 71° 55′

MAGNETIC DIP.

U. S. Naval Observatory, Washington, May 6, 1867. Needle A. 2.

POLARITY OF MARKED END NORTH.

CIRCLE EAST.				CIRCLE WEST.			
Face East.		Face West.		Face East.		Face West.	
S.	N.	S.	N.	S.	N.	S.	N.
73° 45′	73° 30′	72° 20′	71° 50′	107° 40′	107° 15′	108° 50′	108° 40′
73 38		72 5		107 28		108 45	
72 52				108 6			
72 23							

POLARITY OF MARKED END SOUTH.

CIRCLE WEST.				CIRCLE EAST.			
Face West.		Face East.		Face West.		Face East.	
S.	N.	S.	N.	S.	N.	S.	N.
108° 30′	108° 0′	109° 0′	108° 30′	72° 30′	72° 0′	72° 0′	71° 40′
108 15		108 45		72 15		71 50	
108 30				72 2			
71 46							

Resulting Dip: + 72° 4′

HORIZONTAL INTENSITY. OBSERVATIONS OF VIBRATIONS.

Philadelphia, October 24, 1865.

No.	Time P. M.	No.	Time P. M.	Time of 156 vibrations.
0	$3^h\ 27^m\ 5^s.6$	156	$3^h\ 45^m\ 50^s.8$	$18^m\ 45^s.2$
10	3 28 17.2	166	3 47 2.0	18 44.8
20	3 29 29.6	176	3 48 15.2	18 45.6
30	3 30 42.0	186	3 49 27.2	18 45.2
40	3 31 54.4	196	3 50 39.2	18 44.8
50	3 33 6.4	206	3 51 51.6	18 45.2
			Mean . . .	18 45.13

Extreme scale readings,
At beginning . . . . . 5.0 — 150.0
At end . . . . . . 23.0 — 86.0
Coefficient of torsion $v = 8.12$ div.
Temperature . . . . . 60°.7
Time of one vibration . $7^s.212$

Gosport, October 28, 1865.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	$3^h\ 43^m\ 6^s.4$	150	$4^h\ 0^m\ 3^s.6$	$16^m\ 57^s.2$
10	3 44 14.4	160	4 1 11.6	16 57.2
20	3 45 22.0	170	4 2 19.5	16 57.5
30	3 46 29.6	180	4 3 27.2	16 57.6
40	3 47 37.2	190	4 4 34.9	16 57.7
50	3 48 45.6	200	4 5 42.8	16 57.2
			Mean . . .	16 57.40

Extreme scale readings,
At beginning . . . . . . 69.2 — 88.8
At end . . . . 72.1 — 85.2
Coefficient of torsion, $v = 7.35$ div.
Temperature . . . . . 73°.0
Time of one vibration . $6^s.783$

Gosport, October 28, 1865.

Inertia ring on magnet.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	$4^h\ 39^m\ 7^s.9$	150	$5^h\ 0^m\ 55^s.0$	$21^m\ 47^s.1$
10	4 40 35.1	160	5 2 21.7	21 46.6
20	4 42 2.3	170	5 3 48.8	21 46.5
30	4 43 29.3	180	5 5 16.0	21 46.7
40	4 44 56.4	190	5 6 43.2	21 46.8
50	4 46 23.7	200	5 8 10.1	21 46.4
			Mean . . .	21 46.68

Extreme scale readings,
At beginning . . . . . . 91.0 — 66.5
At end . . . . 88.0 — 69.0
Coefficient of torsion, $v = 8.97$ div.
Temperature . . . . . 70°.0
Time of one vibration . $8^s.711$

Gosport, October 30, 1865.

No.	Time.	No.	Time.	Time of 150 vibrations.
0	$12^h\ 17^m\ 5^s.1$	150	$12^h\ 33^m\ 58^s.8$	$16^m\ 53^s.7$
10	12 18 12.8	160	12 35 7.8	16 55.0
20	12 19 20.7	170	12 36 16.4	16 55.7
30	12 20 28.5	180	12 37 24.0	16 55.5
40	12 21 36.1	190	12 38 29.6	16 53.5
50	12 22 44.0	200	12 39 39.2	16 55.2
			Mean . . .	16 54.77

Extreme scale readings,
At beginning . . . . . . 70.0 — 88.3
At end . . . . 77.0 — 82.0
Temperature . . . . . 60°.0
Time of one vibration . $6^s.765$

St. Thomas, November 13, 1865.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	$2^h\ 23^m\ 6^s.2$	150	$2^h\ 37^m\ 18^s.6$	$14^m\ 12^s.4$
10	2 24 3.2	160	2 38 15.4	14 12.2
20	2 24 59.8	170	2 39 12.2	14 12.4
30	2 25 56.9	180	2 40 8.4	14 11.5
40	2 26	190	2 41 5.7	14
50	2 27 49.0	200	2 42 2.8	14 13.8
			Mean . . .	14 12.46

Extreme scale readings,
At beginning . . . . . . 62.2 — 98.0
At end . . . . 69.8 — 90.2
Coefficient of torsion, $v = 4.10$ div.
Temperature . . . . . 87°.0
Time of one vibration . $5^s.683$

St. Thomas, November 16, 1865.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	$12^h\ 13^m\ 3^s.4$	150	$12^h\ 27^m\ 15^s.1$	$14^m\ 11^s.7$
10	12 14 0.4	160	12 28 12.0	14 11.6
20	12 14 57.2	170	12 29 8.5	14 11.3
30	12 15 54.3	180	12 30 5.4	14 11.1
40	12 16 50.6	190	12 31 2.2	14 11.6
50	12 17 47.8	200	12 31 59.0	14 11.2
			Mean . . .	14 11.42

Extreme scale readings,
At beginning . . . . . . 59.8 — 98.8
At end . . . . 67.2 — 89.5
Coefficient of torsion, $v = 4.25$ div.
Temperature . . . . . 87°.5
Time of one vibration . $5^s.676$

Horizontal Intensity. Observations of Vibrations.

St. Thomas, November 16, 1865.

Inertia ring on magnet.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	$1^{h}\ 0^{m}\ 6^{s}.4$	150	$1^{h}\ 18^{m}\ 20^{s}.5$	$18^{m}\ 14^{s}.1$
10	1 1 18.6	160	1 19 34.1	18 15.5
20	1 2 31.8	170	1 20 46.6	18 14.8
30	1 3 45.1	180	1 21 59.8	18 14.7
40	1 4 58.1	190	1 23 12.9	18 14.8
50	1 6 11.4	200	1 24 26.2	18 14.8
			Mean	18 14.78

Extreme scale readings,
At beginning . . . . . 61.8 — 98.0
At end . . . . 63.5 — 96.2
Coefficient of torsion . . $v = 5.22$ div.
Temperature . . . . . . 86°.0
Time of one vibration . . $7^{s}.299$

Salute Islands, November 28, 1865.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	$9^{h}\ 43^{m}\ 3^{s}.6$	150	$9^{h}\ 57^{m}\ 17^{s}.7$	$14^{m}\ 14^{s}.1$
10	9 44 0.4	160	9 58 14.2	14 13.8
20	9 44 57.4	170	9 59 11.4	14 14.0
30	9 45 54.2	180	10 0 8.6	14 14.4
40	9 46 51.3	190	10 1 5.6	14 14.3
50	9 47 48.3	200	10 2 2.5	14 14.2
			Mean	14 14.13

Extreme scale readings,
At beginning . . . . . 57.5 — 99.8
At end . . . . 71.4 — 86.0
Coefficient of torsion . . $v = 3.72$ div.
Temperature . . . . . . 95°.5
Time of one vibration . . $5^{s}.694$

Salute Islands, November 28, 1865.

Inertia ring on magnet.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	$11^{h}\ 31^{m}\ 9^{s}.5$	150	$11^{h}\ 49^{m}\ 25^{s}.1$	$18^{m}\ 15^{s}.6$
10	11 32 22.5	160	11 50 38.6	18 16.1
20	11 33 35.6	170	11 51 51.6	18 16.0
30	11 34 48.7	180	11 53 4.7	18 16.0
40	11 36 1.4	190	11 54 17.8	18 16.4
50	11 37 14.8	200	11 55 30.3	18 15.5
			Mean	18 15.93

Extreme scale readings,
At beginning . . . . . 54.8 — 105.3
At end . . . . 65.4 — 94.0
Coefficient of torsion . . $v = 5.65$ div.
Temperature . . . . . . 91°.0
Time of one vibration . . $7^{s}.306$

Ceara, December 13, 1865.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	$11^{h}\ 35^{m}\ 8^{s}.3$	150	$11^{h}\ 49^{m}\ 36^{s}.0$	$14^{m}\ 27^{s}.7$
10	11 36 6.2	160	11 50 34.2	14 28.0
20	11 37 4.2	170	11 51 33.4	14 29.2
30	11 38 1.0	180	11 52 31.2	14 30.2
40	11 38 59.1	190	11 53 28.2	14 29.1
50	11 39 57.0	200	11 54 25.6	14 28.6
			Mean	14 28.80

Extreme scale readings,
At beginning . . . . . 59.0 — 101.0
At end . . . . 45.5 — 115.0
Coefficient of torsion . . $v = 5.40$ div.
Temperature . . . . . . 89°.0
Time of one vibration . . $5^{s}.792$

A strong breeze blowing, which made the vibrations somewhat unsteady.

Ceara, December 13, 1865.

Inertia ring on magnet.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	$12^{h}\ 23^{m}\ 14^{s}.1$	150	$12^{h}\ 41^{m}\ 51^{s}.5$	$18^{m}\ 37^{s}.4$
10	12 24 28.8	160	12 43 6.1	18 37.3
20	12 25 43.8	170	12 44 20.0	18 36.2
30	12 26 59.0	180	12 45 33.6	18 34.6
40	12 28 13.6	190	12 46 49.2	18 35.6
50	12 29 28.2	200	12 48 3.8	18 35.6
			Mean	18 36.12

Extreme scale readings,
At beginning . . . . . 104.8 — 58.8
At end . . . . 100.0 — 62.2
Coefficient of torsion . . $v = 7.00$ div.
Temperature . . . . . . 89°.5
Time of one vibration . . $7^{s}.441$

Pernambuco, December 23, 1865.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	$6^{h}\ 50^{m}\ 16^{s}.8$	150	$7^{h}\ 4^{m}\ 54^{s}.4$	$14^{m}\ 37^{s}.6$
10	6 51 15.7	160	7 5 52.6	14 36.9
20	6 52 14.0	170	7 6 51.1	14 37.1
30	6 53 12.6	180	7 7 49.6	14 37.0
40	6 54 10.9	190	7 8 48.0	14 37.1
50	6 55 9.6	200	7 9 46.4	14 36.8
			Mean	14 37.08

Extreme scale readings,
At beginning . . . . . 46.0 — 115.0
At end . . . . 62.0 — 99.0
Coefficient of torsion . . $v = 4.27$ div.
Temperature . . . . . . 90°.5
Time of one vibration . . $5^{s}.847$

HORIZONTAL INTENSITY. OBSERVATIONS OF VIBRATIONS.

Bahia, December 27, 1865.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	7^h 14^m $5^s.6$	150	7^h 28^m $55^s.6$	14^m $50^s.0$
10	7 15 4.9	160	7 29 55.0	14 50.1
20	7 16 4.1	170	7 30 54.4	14 50.3
30	7 17 3.6	180	7 31 53.6	14 50.0
40	7 18 2.9	190	7 32 53.0	14 50.1
50	7 19 2.2	200	7 33 52.2	14 50.0
			Mean	14 50.08

Extreme scale readings,
At beginning . . . . . 92.8 — 63.1
At end . . . . 86.8 — 68.3
Coefficient of torsion . . $v = 4.85$ div.
Temperature . . . . . . 92°.5
Time of one vibration . . $5^s.934$

Bahia, December 27, 1865.

Inertia ring on magnet.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	8^h 3^m $4^s.2$	150	8^h 22^m $9^s.4$	19^m $5^s.2$
10	8 4 20.8	160	8 23 25.8	19 5.0
20	8 5 37.0	170	8 24 42.2	19 5.2
30	8 6 53.4	180	8 25 58.6	19 5.2
40	8 8 9.8	190	8 27 14.8	19 5.0
50	8 9 26.0	200	8 28 30.8	19 4.8
			Mean	19 5.07

Extreme scale readings,
At beginning . . . . . 57.9 — 100.4
At end . . . . 67.9 — 89.2
Coefficient of torsion . . $v = 6.70$ div.
Temperature . . . . . . 97°.5
Time of one vibration . . $7^s.634$

Rio Janeiro, January 6, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	3^h 21^m $6^s.8$	150	3^h 36^m $12^s.5$	15^m $5^s.7$
10	3 22 5.8	160	3 37 12.5	15 6.7
20	3 23 6.6	170	3 38 13.3	15 6.7
30	3 24 7.0	180	3 39 13.6	15 6.6
4	3 25 7.7	190	3 40 14.5	15 6.8
50	3 26 8.1	200	3 41 15.0	15 6.9
			Mean	15 6.57

Extreme scale readings,
At beginning . . . . . 62.1 — 96.3
At end . . . . 70.0 — 89.2
Coefficient of torsion . . $v = 5.10$ div.
Temperature . . . . . . 76°.0
Time of one vibration . . $6^s.044$

Rio Janeiro, January 9, 1866.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	5^h 30^m $11^s.8$	150	5^h 45^m $20^s.2$	15^m $8^s.4$
10	5 31 12.4	160	5 46 21.0	15 8.6
20	5 32 13.0	170	5 47 21.5	15 8.5
30	5 33 13.4	180	5 48 22.1	15 8.7
40	5 34 14.0	190	5 49 22.6	15 8.6
50	5 35 14.6	200	5 50 23.2	15 8.6
			Mean	15 8.57

Extreme scale readings,
At beginning . . . . . 62.2 — 98.1
At end . . . . 69.2 — 91.2
Temperature . . . . . . 80°.5
Time of one vibration . . $6^s.057$

Monte Video, January 18, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	1^h 27^m $8^s.2$	150	1^h 42^m $9^s.4$	15^m $1^s.2$
10	1 28 8.2	160	1 43 9.5	15 1.3
20	1 29 8.3	170	1 44 9.7	15 1.4
30	1 30 8.2	180	1 45 9.7	15 1.5
40	1 31 8.5	190	1 46 9.7	15 1.2
50	1 32 8.5	200	1 47 9.9	15 1.4
			Mean	15 1.33

Extreme scale readings,
At beginning . . . . . 58.4 — 98.3
At end . . . . 66.8 — 90.2
Coefficient of torsion . . $v = 5.10$ div.
Temperature . . . . . . 84°.0
Time of one vibration . . $6^s.009$

Monte Video, January 18, 1866.

Inertia ring on magnet.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	2^h 10^m $3^s.2$	150	2^h 29^m $22^s.9$	19^m $19^s.7$
10	2 11 20.5	160	2 30 40.1	19 19.6
20	2 12 37.8	170	2 31 57.3	19 19.5
30	2 13 55.1	180	2 33 14.6	19 19.5
40	2 15 12.4	190	2 34 31.8	19 19.4
50	2 16 29.8	200	2 35 49.3	19 19.5
			Mean	19 19.53

Extreme scale readings,
At beginning . . . . . 56.9 — 101.0
At end . . . . 65.9 — 91.4
Coefficient of torsion . . $v = 6.25$ div.
Temperature . . . . . . 84°.5
Time of one vibration . . $7^s.730$

Horizontal Intensity. Observations of Vibrations.

Monte Video, January 18, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	2^h 55^m $9^s.3$	150	3^h 10^m $11^s.4$	15^m $2^s.1$
10	2 56 9.2	160	3 11 11.4	15 2.2
20	2 57 9.4	170	3 12 11.5	15 2 1
30	2 58 9.4	180	3 13 11.9	15 2.5
40	2 59 9.4	190	3 14 12.1	15 2.7
50	3 0 9.8	200	3 15 12.1	15 2.3
			Mean	15 2.32

Extreme scale readings,
At beginning . . . . . 58.0 — 100.2
At end . . . . 65.8 — 91.6
Temperature . . . . . . 86°.0
Time of one vibration . . $6^s.015$

Monte Video, January 19, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	3^h 3^m $8^s.8$	150	3^h 18^m $11^s.8$	15^m $3^s.0$
10	3 4 8.9	160	3 19 12.2	15 3.3
20	3 5 9.3	170	3 20 12.6	15 3.3
30	3 6 9.4	180	3 21 12.6	15 3.2
40	3 7 9.7	190	3 22 13.0	15 3.3
50	3 8 10.1	200	3 23 13.3	15 3.2
			Mean	15 3.22

Extreme scale readings,
At beginning . . . . . 56.0 — 102.0
At end . . . . 66.6 — 91.5
Temperature . . . . . . 89°.5
Time of one vibration . . $6^s.021$

Sandy Point, February 7, 1866.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	11^h 37^m $4^s.5$	150	11^h 51^m $58^s.4$	14^m $53^s.9$
10	11 38 4.5	160	11 52 58.4	14 53.9
20	11 39 3.7	170	11 53 58.2	14 54.5
30	11 40 4.1	180	11 54 58.0	14 53.9
40	11 41 3.3	190	11 55 57.8	14 54.5
50	11 42 2.5	200	11 56 57.8	14 55.3
			Mean	14 54.33

Extreme scale readings,
At beginning . . . . . 61.0 — 100.0
At end . . . . 60.5 — 97.5
Coefficient of torsion . . $v = 6.85$ div.
Temperature . . . . . . 71°.5
Time of one vibration . . $5^s.962$
Magnet rendered quite unsteady by the high wind.

Valparaiso, March 2, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	5^h 0^m $3^s.4$	150	5^h 14^m $41^s.0$	14^m $37^s.6$
10	5 1 2.2	160	5 15 39.3	14 37.1
20	5 2 0.6	170	5 16 37.8	14 37.2
30	5 2 59.4	180	5 17 36.6	14 37.2
40	5 3 57.4	190	5 18 35.1	14 37.7
50	5 4 55.7	200	5 19 33.7	14 38.0
			Mean	14 37.47

Extreme scale readings,
At beginning . . . . . 99.8 — 56.8
At end . . . . 97.8 — 57.8
Coefficient of torsion . . $v = 6.17$ div.
Temperature . . . . . . 72°.5
Time of one vibration . . $5^s.850$

Valparaiso, March 19, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	1^h 42^m $6^s.6$	150	1^h 56^m $50^s.2$	14^m $43^s.6$
10	1 43 5.6	160	1 57 48.6	14 43.0
20	1 44 4.2	170	1 58 47.7	14 43.5
30	1 45 3.0	180	1 59 46.3	14 43.3
40	1 46 1.9	190	2 0 44.9	14 43.0
50	1 47 0.8	200	2 1 44.1	14 43.3
			Mean	14 43.28

Extreme scale readings,
At beginning . . . . . 65.0 — 95.8
At end . . . . 61.2 — 96.8
Coefficient of torsion . . $v = 4.75$ div.
Temperature . . . . . . 76°.0
Time of one vibration . . $5^s.889$

Valparaiso, March 19, 1866.

Inertia ring on magnet.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	2^h 32^m $5^s.4$	150	2^h 51^m $0^s.4$	18^m $55^s.0$
10	2 33 21.2	160	2 52 15.8	18 54.6
20	2 34 36.8	170	2 53 30.8	18 54.0
30	2 35 52.5	180	2 54 47.2	18 54.7
40	2 37 8.2	190	2 56 1.2	18 53.0
50	2 38 23.9	200	2 57 15.8	18 51.9
			Mean	18 53.87

Extreme scale readings,
At beginning . . . . . 61.6 — 98.9
At end . . . . 73.3 — 84.0
Coefficient of torsion . . $v = 6.82$ div.
Temperature . . . . . . 73°.0
Time of one vibration . . $7^s.559$

HORIZONTAL INTENSITY. OBSERVATIONS OF VIBRATIONS.

Valparaiso, March 29, 1860.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	12^h 37^m 9^s.0	150	12^h 51^m 47^s.4	14^m 38^s.4
10	12 38 7.4	160	12 52 45.8	14 38.4
20	12 39 5.7	170	12 53 46.2	14 40.5
30	12 40 4.3	180	12 54 44.2	14 39.9
40	12 41 3.4	190	12 55 40.4	14 37.0
50	12 42 2.0	200	12 56 —	14 —
			Mean	14 38.84

Extreme scale readings,
At beginning . . . . 61.3 — 97.2
Temperature . . . . . 76°.0
Time of one vibration . . 5^s.859

Magnet brought to rest by the vibrations of the instrument caused by the wind.

Valparaiso, March 29, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	1^h 28^m 7^s.2	150	1^h 42^m 49^s.0	14^m 41^s.8
10	1 29 5.2	160	1 43 48.0	14 42.8
20	1 30 6.8	170	1 44 46.9	14 40.1
30	1 31 2.4	180	1 45 45.2	14 42.8
40	1 32 0.6	190	1 46 43.8	14 43.2
50	1 32 58.6	200	1 47 43.0	14 44.4
			Mean	14 42.52

Extreme scale readings,
At beginning . . . . 63.0 — 98 8
At end . . . . . . 65.5 — 96.0
Coefficient of torsion . . $v = 3.80$ div.
Temperature . . . . . 75°.5
Time of one vibration . . 5^s.883

Vibrations irregular on account of the wind, which, at one time, almost brought the magnet to rest.

Valparaiso, April 7, 1866.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	10^h 2^m 15^s.6	150	10^h 16^m 55^s.0	14^m 39^s.4
10	10 3 14.2	160	10 17 54.2	14 40.0
20	10 4 13.2	170	10 18 53.6	14 40.4
30	10 5 11.8	180	10 19 53.0	14 41.2
40	10 6 11.2	190	10 20 52.4	14 41.2
50	10 7 9.6	200	10 21 51.2	14 41.6
			Mean	14 40.63

Extreme scale readings,
At beginning . . . . 59.8 — 102.8
At end . . . . . . 56.5 — 106.5
Coefficient of torsion . . $v = 3.92$ div.
Temperature . . . . . 66°.5
Time of one vibration . . 5^s.871

Valparaiso, April 11, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	12^h 15^m 14^s.0	150	12^h 29^m 56^s.6	14^m 42^s.6
10	12 16 13.0	160	12 30 55.4	14 42.4
20	12 17 11.8	170	12 31 54.2	14 42.4
30	12 18 10.4	180	12 32 53.2	14 42.8
40	12 19 9.0	190	12 33 52.0	14 43.0
50	12 20 7.8	200	12 34 51.0	14 43.2
			Mean	14 42.73

Extreme scale readings,
At beginning . . . . 56.0 — 103.0
At end . . . . . . 64.5 — 91.0
Temperature . . . . . 74°.5
Time of one vibration . . 5^s.885

Valparaiso, April 11, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	12^h 37^m 12^s.2	150	12^h 51^m 55^s.0	14^m 42^s.8
10	12 38 11.0	160	12 52 54.0	14 43.0
20	12 39 9.8	170	12 53 52.8	14 43.0
30	12 40 8.6	180	12 54 51.8	14 43.2
40	12 41 7.4	190	12 55 50.6	14 43.2
50	12 42 6.4	200	12 56 49.4	14 43.0
			Mean	14 43.03

Extreme scale readings,
At beginning . . . . 64.5 — 91.0
At end . . . . . . 70.0 — 85.0
Temperature . . . . . 81°.0
Time of one vibration . . 5^s.887

Valparaiso, April 11, 1866.

Inertia ring on magnet.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	1^h 8^m 6^s.6	150	1^h 27^m 2^s.4	18^m 55^s.8
10	1 9 22.2	160	1 28 18.1	18 55.9
20	1 10 37.8	170	1 29 33.8	18 56.0
30	1 11 53.7	180	1 30 49.4	18 55.7
40	1 13 9.4	190	1 32 5.2	18 55.8
50	1 14 25.0	200	1 33 21.0	18 56.0
			Mean	18 55.87

Extreme scale readings,
At beginning . . . . 58.8 — 101.6
At end . . . . . . 67.0 — 93.2
Coefficient of torsion . . $v = 5.50$ div.
Temperature . . . . . 88°.0
Time of one vibration . . 7^s.572

HORIZONTAL INTENSITY. OBSERVATIONS OF VIBRATIONS.

Valparaiso, April 13, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	2^{h} 45^{m} $23^{s}.6$	150	3^{h} 0^{m} $6^{s}.2$	14^{m} $42^{s}.6$
10	2 46 21.8	160	3 1 4.6	14 42.8
20	2 47 21.2	170	3 2 3.6	14 42.4
30	2 48 19.6	180	3 3 2.4	14 42.8
40	2 49 19.0	190	3 4 0.6	14 41.6
50	2 50 17.8	200	3 4 58.6	14 40.8
			Mean . . .	14 42.17

Extreme scale readings,
At beginning . . . 57.8 — 101.5
At end . . . . . . 74.2 — 85.2
Temperature . . . . 66°.5
Time of one vibration . $5^{s}.881$

San Lorenzo Island, April 26, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	12^{h} 40^{m} $6^{s}.9$	150	12^{h} 54^{m} $7^{s}.4$	14^{m} $0^{s}.5$
10	12 41 3.0	160	12 55 3.0	14 0.0
20	12 41 59.0	170	12 55 59.2	14 0.2
30	12 42 55.0	180	12 56 54.9	13 59.9
40	12 43 51.0	190	12 57 50.8	13 59.8
50	12 44 47.1	200	12 58 47.4	14 0.3
			Mean . . .	14 0.08

Extreme scale readings,
At beginning . . . . . 61.2 — 101.1
At end . . . . 71.0 — 89.0
Coefficient of torsion . . . $v = 3.10$ div.
Temperature . . . . . . 89°.0
Time of one vibration . . . $5^{s}.601$

Payta, May 7, 1866.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	9^{h} 21^{m} $9^{s}.8$	150	9^{h} 34^{m} $49^{s}.4$	13^{m} $39^{s}.6$
10	9 22 4.4	160	9 35 44.0	13 39.6
20	9 22 59.2	170	9 36 38.6	13 39.4
30	9 23 53.6	180	9 37 33.2	13 39.6
40	9 24 48.2	190	9 38 27.6	13 39.4
50	9 25 42.8	200	9 39 22.3	13 39.5
			Mean . . .	13 39.52

Extreme scale readings,
At beginning . . . . . 58.2 — 101.8
At end . . . . 67.8 — 92.2
Coefficient of torsion . . . $v = 3.20$ div.
Temperature . . . . . 87°.5
Time of one vibration . . . $5^{s}.463$

Flamenco Island, Panama Bay, May 14, 1866.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	8^{h} 50^{m} $11^{s}.4$	150	9^{h} 3^{m} $37^{s}.8$	13^{m} $26^{s}.4$
10	8 51 5.1	160	9 4 31.4	13 26.3
20	8 51 59.0	170	9 5 25.2	13 26.2
30	8 52 52.8	180	9 6 19.0	13 26.2
40	8 53 46.5	190	9 7 13.0	13 26.5
50	8 54 40.4	200	9 8 6.9	13 26.5
			Mean . . .	13 26.35

Extreme scale readings,
At beginning . . . . . . 58.2 — 101.0
At end . . . . 66.6 — 92.9
Coefficient of torsion . . . $v = 2.78$ div.
Temperature . . . . 92°.0
Time of one vibration . . . $5^{s}.376$

Acapulco, May 30, 1866.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	8^{h} 32^{m} $3^{s}.8$	150	8^{h} 45^{m} $23^{s}.4$	13^{m} $19^{s}.6$
10	8 32 57.0	160	8 46 17.2	13 20.2
20	8 33 50.6	170	8 47 10.2	13 19.6
30	8 34 43.9	180	8 48 3.7	13 19.8
40	8 35 37.0	190	8 48 57.0	13 20.0
50	8 36 30.6	200	8 49 50.5	13 19.9
			Mean . . .	13 19.85

Extreme scale readings,
At beginning . . . . . 57.8 — 102.2
At end . . . . 65.2 — 95.0
Coefficient of torsion . . . $v = 3.40$ div.
Temperature . . . . . . 89°.0
Time of one vibration . . . $5^{s}.332$

Acapulco, May 30, 1866.

Inertia ring on magnet.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	9^{h} 46^{m} $9^{s}.2$	150	10^{h} 3^{m} $19^{s}.5$	17^{m} $10^{s}.3$
10	9 47 17.4	160	10 4 28.2	17 10.8
20	9 48 26.5	170	10 5 37.0	17 10.5
30	9 49 35.2	180	10 6 45.6	17 10.4
40	9 50 43.8	190	10 7 54.4	17 10.6
50	9 51 52.4	200	10 9 3.2	17 10.8
			Mean . . .	17 10.57

Extreme scale readings,
At beginning . . . . . 56.2 — 103.7
At end . . . . 65.1 — 94.8
Coefficient of torsion . . . $v = 4.55$ div.
Temperature . . . . . 99°.5
Time of one vibration . . $6^{s}.870$

HORIZONTAL INTENSITY. OBSERVATIONS OF VIBRATIONS.

Magdalena Bay, June 9, 1866.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	$1^{h}\ 8^{m}\ 5^{s}.4$	150	$1^{h}\ 21^{m}\ 52^{s}.8$	
10	1 8 59.4	160	1 22 49.0	
20	1 9 54.5	170	1 23 44.4	
30	1 10 49.0	180	1 24 40.2	
40	1 11 44.4	190	1 25 36.0	
50	1 12 39.8	200	1 26 30.8	
100	1 17 16.4			

Extreme scale readings,
At beginning . . . . . 55.0 — 101.0
At end . . . . 69.0 — 85.0
Temperature . . . . . . 79°.0
Time of one vibration . . $5^{s}.527$

In this and the following observation the vibrations of the magnet were very irregular on account of a high wind which shook the instrument.

Magdalena Bay, June 9, 1866.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	$1^{h}\ 41^{m}\ 12^{s}.2$	150	$1^{h}\ 55^{m}\ 4^{s}.8$	
10	1 42 7.8	160	1 56 0.4	
20	1 43 3.0	170	1 56 56.0	
30	1 43 59.0	180	1 57 51.4	
40	1 44 54.0	190	1 58 46.4	
50	1 45 48.4	200	1 59 41.6	
100	1 50 25.4			

Extreme scale readings,
At beginning . . . . . 53.5 — 98.5
At end . . . .
Coefficient of torsion . . $v = 4.37$ div.
Temperature . . . . . . 86°.5
Time of one vibration . . $5^{s}.533$

San Diego Bay, June 15, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	$6^{h}\ 11^{m}\ 9^{s}.2$	150	$6^{h}\ 25^{m}\ 58^{s}.2$	$14^{m}\ 49^{s}.0$
10	6 12 8.3	160	6 26 56.6	14 48.3
20	6 13 7.4	170	6 27 55.8	14 48.4
30	6 14 7.0	180	6 28 55.4	14 48.4
40	6 15 6.2	190	6 29 53.8	14 47.6
50	6 16 5.4	200	6 30 53.0	14 47.6
			Mean	14 48.22

Extreme scale readings,
At beginning . . . . . 94.9 — 108.9
At end . . . . 70.0 — 88.0
Coefficient of torsion . . $v = 3.60$ div.
Temperature . . . . . . 79°.0
Time of one vibration . . $5^{s}.921$

San Francisco Bay, June 26, 1866.

No.	Time A. M.	No.	Time A. M.	Time of 150 vibrations.
0	$3^{h}\ 21^{m}\ 22^{s}.7$	150	$3^{h}\ 36^{m}\ 57^{s}.7$	$15^{m}\ 35^{s}.0$
10	3 22 24.7	160	3 38 0.0	15 35.3
20	3 23 27.2	170	3 39 2.5	15 35.3
30	3 24 30.2	180	3 40 4.7	15 34.5
40	3 25 32.0	190	3 41 7.2	15 35.2
50	3 26 34.7	200	3 42 10.0	15 35.3
			Mean	15 35.10

Extreme scale readings,
At beginning . . . . . 57.0 — 102.0
At end . . . . 68.0 — 90.5
Coefficient of torsion . . $v = 4.35$ div.
Temperature . . . . . . 77°.0
Time of one vibration . . $6^{s}.234$

U. S. N. Observatory, Washington, Nov. 1, 1866.

No.	Time P. M.	No.	Time P. M.	Time of 150 vibrations.
0	$5^{h}\ 19^{m}\ 52^{s}.7$	150	$5^{h}\ 37^{m}\ 46^{s}.5$	$17^{m}\ 53^{s}.8$
10	5 21 5.0	160	5 38 58.0	17 53.0
20	5 22 16.0	170	5 40 9.2	17 53.2
30	5 23 27.5	180	5 41 20.7	17 53.2
40	5 24 39.0	190	5 42 31.8	17 52.8
50	5 25 50.7	200	5 43 43.0	17 52.3
			Mean	17 53.05

Extreme scale readings,
At beginning . . . . . 52.5 — 106.0
At end . . . . 66.6 — 95.2
Coefficient of torsion . . $v = 5.80$ div.
Temperature . . . . . . 67°.5
Time of one vibration . . $7^{s}.154$

The following sets of observations of vibrations were made in the basement of the Observatory, where there is much iron, and are to be used only to determine the moment of inertia of the magnet.

Set 1. November 2, 1866.

No.	Time.	No.	Time.	Time of 150 vibrations.
0	$5^{h}\ 37^{m}\ 31^{s}.7$	150	$5^{h}\ 54^{m}\ 53^{s}.8$	$17^{m}\ 22^{s}.1$
10	5 38 41.2	160	5 56 3.2	17 22.0
20	5 39 50.7	170	5 57 12.7	17 22.0
30	5 41 0.2	180	5 58 21.5	17 21.3
40	5 42 9.7	190	5 59 31.2	17 21.5
50	5 43 19.2	200	6 0 40.7	17 21.5
			Mean	17 21.73

Extreme scale readings,
At beginning . . . . . 59.1 — 99.8
At end . . . . 66.9 — 92.2
Temperature . . . . . . 65°.5
Time of one vibration . . $6^{s}.945$

HORIZONTAL INTENSITY. OBSERVATIONS OF VIBRATIONS.

Set No. 2. November 2, 1866.

Inertia ring on magnet.

No.	Time.	No.	Time.	Time of 150 vibrations.
0	$6^{h}\ 17^{m}\ 25^{s}.3$	150	$6^{h}\ 39^{m}\ 46^{s}.8$	$22^{m}\ 21^{s}.5$
10	6 18 55.2	160	6 41 16.2	22 21.0
20	6 20 24.2	170	6 42 45.7	22 21.5
30	6 21 54.0	180	6 44 14.8	22 20.8
40	6 23 23.7	190	6 45 44.2	22 20.5
50	6 24 53.0	200	6 47 13.7	22 20.7
			Mean	22 21.00

Extreme scale readings,
At beginning . . . . . 58.9 — 100.8
At end . . . . 68.3 — 95.5
Coefficient of torsion . . . $v = 7.58$ div.
Temperature . . . . . . 68°.5
Time of one vibration . . $8^{s}.940$

Set No. 3. November 2, 1866.

No.	Time.	No.	Time.	Time of 150 vibrations.
0	$6^{h}\ 57^{m}\ 41^{s}.3$	150	$7^{h}\ 15^{m}\ 3^{s}.2$	$17^{m}\ 21^{s}.9$
10	6 58 50.8	160	7 16 12.8	17 22.0
20	7 0 0.2	170	7 17 22.3	17 22.1
30	7 1 9.8	180	7 18 31.5	17 21.7
40	7 2 19.0	190	7 19 41.0	17 22.0
50	7 3 28.8	200	7 20 50.5	17 21.7
			Mean	17 21.90

Extreme scale readings,
At beginning . . . . . 54.2 — 104.5
At end . . . . 63.2 — 94.9
Temperature . . . . . . 69°.0
Time of one vibration . . $6^{s}.946$

Set No. 4. November 2, 1866.

Inertia ring on magnet.

No.	Time.	No.	Time.	Time of 150 vibrations.
0	$7^{h}\ 26^{m}\ 18^{s}.3$	150	$7^{h}\ 48^{m}\ 39^{s}.0$	$22^{m}\ 20^{s}.7$
10	7 27 47.7	160	7 50 8.5	22 20.8
20	7 29 17.2	170	7 51 37.9	22 20.7
30	7 30 46.7	180	7 53 7.3	22 20.6
40	7 32 16.0	190	7 54 36.7	22 20.7
50	7 33 45.5	200	7 56 5.8	22 20.3
			Mean	22 20.63

Extreme scale readings,
At beginning . . . . . 56.5 — 103.6
At end . . . . 65.1 — 96.3
Temperature . . . . . . 70°.0
Time of one vibration . . $8^{s}.938$

Set No. 5. November 2, 1866.

No.	Time.	No.	Time.	Time of 150 vibrations.
0	$8^{h}\ 7^{m}\ 22^{s}.7$	150	$8^{h}\ 24^{m}\ 44^{s}.2$	$17^{m}\ 21^{s}.5$
10	8 8 32.2	160	8 25 53.7	17 21.5
20	8 9 41.7	170	8 27 3.2	17 21.5
30	8 10 51.2	180	8 28 12.7	17 21.5
40	8 12 0.7	190	8 29 22.0	17 21.3
50	8 13 10.2	200	8 30 31.7	17 21.5
			Mean	17 21.47

Extreme scale readings,
At beginning . . . . . 58.7 — 99.3
At end . . . . 66.5 — 91.2
Coefficient of torsion . . . $v = 6.05$ div.
Temperature . . . . . . 69°.5
Time of one vibration . . $6^{s}.943$

Set No. 6. November 2, 1866.

No.	Time.	No.	Time.	Time of 150 vibrations.
0	$12^{h}\ 31^{m}\ 58^{s}.2$	150	$12^{h}\ 49^{m}\ 51^{s}.2$	$17^{m}\ 53^{s}.0$
10	12 33 9.2	160	12 51 2.5	17 53.3
20	12 34 21.0	170	12 52 14.2	17 53.2
30	12 35 32.7	180	12 53 25.7	17 53.0
40	12 36 44.0	190	12 54 37.2	17 53.2
50	12 37 55.7	200	12 55 48.7	17 53.0
			Mean	17 53.12

Extreme scale readings,
At beginning . . . . . 59.5 — 99.0
At end . . . . 65.5 — 92.0
Temperature . . . . . . 56°.0
Time of one vibration . . $7^{s}.154$

Set No. 7. November 2, 1866.

Inertia ring on magnet.

No.	Time.	No.	Time.	Time of 150 vibrations.
0	$1^{h}\ 3^{m}\ 23^{s}.5$	150	$1^{h}\ 26^{m}\ 22^{s}.7$	$22^{m}\ 59^{s}.2$
10	1 4 55.2	160	1 27 54.2	22 59.0
20	1 6 27.5	170	1 29 26.7	22 59.2
30	1 7 59.2	180	1 30 58.5	22 59.3
40	1 9 31.3	190	1 32 30.2	22 58.9
50	1 11 3.2	200	1 34 2.5	22 59.3
			Mean	22 59.15

Extreme scale readings,
At beginning . . . . . 58.2 — 101.0
At end . . . . 68.0 — 97.2
Temperature . . . . . . 53°.5
Time of one vibration . . $9^{s}.194$

HORIZONTAL INTENSITY. OBSERVATIONS OF VIBRATIONS.

Set No. 8. November 2, 1866.

No.	Time.	No.	Time.	Time of 150 vibrations.
0	1^{h} 40^{m} 19^{s}.2	150	1^{h} 58^{m} 11^{s}.5	17^{m} 52^{s}.3
10	1 41 30.7	160	1 59 23.0	17 52.3
20	1 42 42.2	170	2 0 34.5	17 52.3
30	1 43 53.7	180	2 1 46.0	17 52.3
40	1 45 5.2	190	2 2 57.5	17 52.3
50	1 46 16.7	200	2 4 9.0	17 52.3
			Mean	17 52.30

Extreme scale readings,	
At beginning	60.0 — 101.0
At end	68.0 — 92.8
Temperature	52°.5
Time of one vibration . .	7^{s}.149

HORIZONTAL INTENSITY. OBSERVATIONS OF DEFLECTIONS.

Philadelphia, October 24, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$4^h\ 40^m$	59.°	$141^d.5$			$r = 2.0$ ft.
	E.			41.5	$141^d.5$	$100^d.0$	
	W.			141.4	41.5		
	E.			41.4			
East.	E.			40.5			
	W.			141.8	40.5	101.2	
	E.			40.5	141.7		
	W.	4 58	56.	141.6			
Means			57.5		$2u^d$	100.60	

Gosport, October 30, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$11^h\ 6^m$	59°	$39^d.2$			$r = 2.0$ ft.
	E.			127.7	$39^d.3$	$88^d.2$	
	W.			39.4	127.5		
	E.			127.4			
East.	E.			128.0			
	W.			38.8	127.6	88.7	
	E.			127.3	38.9		
	W.	11 30	59	39.1			
Means			59.0		$2u^d$	88.45	

Coefficient of torsion, $v = 7.82$ div.

Gosport, October 30, 1865.

Magnet.	North end.	Time.	Temp. t	Scale dings.	Alternate Means.	Diff's.	Dist.
West.	W.	$11^h\ 30^m$	59°	$60^d.5$			$r = 2.5$ ft.
	E.			105.7	$60^d.2$	$45^d.3$	
	W.			60.0	105.5		
	E.			105.4			
East.	E.			105.9			
	W.			60.4	105.9	45.5	
	E.			105.9	60.4		
	W.	11 48	58	60.3			
Means			58.5		$2u^d$	45.40	

HORIZONTAL INTENSITY. OBSERVATIONS OF DEFLECTIONS.

St. Thomas, November 13, 1865.

Magnet	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	2^h 5^m	87°.	$46^d.4$ 108.1 46.4 108.1	$46^d.4$ 108.1	$61^d.7$	$r = 2.0$ ft.
East.	E. W. E. W.	2 15	85.	108.3 46.8 108.5 46.9	108.4 46.8	61.6	
Means			86.0		$2u^d$	61.65	

Coefficient of torsion, $v = 4.80$ div.

St. Thomas, November 13, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	2^h 15^m	85.°	$61^d.7$ 93.2 61.6 93.3	$61^d.6$ 93.2	$31^d.6$	$r = 2.5$ ft.
East.	E. W. E. W.	2 35	85.	93.2 61.6 93.3 61.5	93.2 61.5	31.7	
Means			85.0		$2u^d$	31.65	

St. Thomas, November 16, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	12^h 10^m	90.°	$43^d.6$ 105.3 43.7 105.3	$43^d.6$ 105.3	$61^d.7$	$r = 2.0$ ft.
East.	E. W. E. W.	12 20	87.	105.6 43.9 105.5 43.8	105.5 43.8	61.7	
Means			88.5		$2u^d$	61.70	

Coefficient of torsion, $v = 4.55$ div.

St. Thomas, November 16, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	12^h 20^m	87.°	$58^d.7$ 90.4 58.6 90.4	$58^d.6$ 90.4	$31^d.8$	$r = 2.5$ ft.
East.	E. W. E. W.	12 30	87.	90.4 59.1 90.5 58.9	90.4 59.0	31.4	
Means			87.0		$2u^d$	31.60	

Salute Islands, November 28, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	12^h 15^m	91.°	$41^d.1$ 102.5 41.1 102.5	$41^d.1$ 102.5	$61^d.4$	$r = 2.0$ ft.
East.	E. W. E. W.	12 25	90.	102.8 41.3 102.9 41.3	102.8 41.3	61.5	
Means			90.5		$2u^d$	61.45	

Coefficient of torsion, $v = 4.02$ div.

Salute Islands, November 28, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	12^h 25^m	90.°	$56^d.3$ 87.8 56.3 87.8	$56^d.3$ 87.8	$31^d.5$	$r = 2.5$ ft.
East.	E W. E. W.	12 35	89.	88.0 56.4 88.0 56.4	88.0 56.4	31.6	
Means			89.5		$2u^d$	31.55	

Horizontal Intensity. Observations of Deflections.

Ceara, December 13, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$12^{\mathrm{h}}\ 15^{\mathrm{m}}$	89°	$46^{\mathrm{d}}.7$	$46^{\mathrm{d}}.6$	$64^{\mathrm{d}}.0$	$r = 2.0$ ft.
	E.			110.5	110.6		
	W.			46.5			
	E.			110.6			
East.	E.			110.7	110.8	63.5	
	W.			47.2	47.3		
	E.			111.0			
	W.	12 26	90	47.4			
Means			89.5		$2u^{\mathrm{d}}$	63.75	

Coefficient of torsion, $v = 6.72$ div.

Ceara, December 13, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$12^{\mathrm{h}}\ 26^{\mathrm{m}}$	90°	$62^{\mathrm{d}}.7$	$62^{\mathrm{d}}.8$	$32^{\mathrm{d}}.6$	$r = 2.5$ ft.
	E.			95.6	95.4		
	W.			62.8			
	E.			95.2			
East.	E.			95.3	95.5	31.8	
	W.			63.4	63.7		
	E.			95.7			
	W.	12 40	89	64.1			
Means			89.5		$2u^{\mathrm{d}}$	32.20	

Pernambuco, December 23, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$8^{\mathrm{h}}\ 35^{\mathrm{m}}$	85°	$48^{\mathrm{d}}.4$	$48^{\mathrm{d}}.4$	$64^{\mathrm{d}}.8$	$r = 2.0$ ft.
	E.			113.3	113.2		
	W.			48.5			
	E.			113.2			
East.	E.			113.9	114.2	64.6	
	W.			49.5	49.6		
	E.			114.4			
	W.	8 50	88	49.7			
Means			86.5		$2u^{\mathrm{d}}$	64.70	

Coefficient of torsion, $v = 5.10$ div.

Pernambuco, December 23, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$8^{\mathrm{h}}\ 50^{\mathrm{m}}$	88°	$64^{\mathrm{d}}.6$	$64^{\mathrm{d}}.7$	$33^{\mathrm{d}}.4$	$r = 2.5$ ft.
	E.			98.0	98.1		
	W.			64.8			
	E.			98.1			
East.	E.			98.2	98.2	33.2	
	W.			64.9	65.0		
	E.			98.2			
	W.	9 0	88	65.0			
Means			88.0		$2u^{\mathrm{d}}$	33.30	

Bahia, December 27, 1865.

Magne..	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$11^{\mathrm{h}}\ 5^{\mathrm{m}}$	98°	$46^{\mathrm{d}}.5$	$46^{\mathrm{d}}.5$	$65^{\mathrm{d}}.9$	$r = 2.0$ ft.
	E.			112.2	112.4		
	W.			46.6			
	E.			112.7			
East,	E.			113.6	113.7	67.3	
	W.			46.4	46.4		
	E.			113.9			
	W.	11 12	98	46.4			
Means			98.0		$2u^{\mathrm{d}}$	66.60	

Coefficient of torsion, $v = 5.27$ div.

Bahia, December 27, 1865.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternute Means.	Diff's.	Dist.
West.	W.	$11^{\mathrm{h}}\ 12^{\mathrm{m}}$	98°	$62^{\mathrm{d}}.9$	$62^{\mathrm{d}}.8$	$33^{\mathrm{d}}.8$	$r = 2.5$ ft.
	E.			96.6	96.6		
	W.			62.8			
	E.			96.6			
East.	E.			96.9	97.0	34.3	
	W.			62.6	62.7		
	E.			97.1			
	W.	11 20	98	62.8			
Means			98.0		$2u^{\mathrm{d}}$	34.05	

HORIZONTAL INTENSITY. OBSERVATIONS OF DEFLECTIONS.

Rio Janeiro, January 6, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$6^{h}\ 0^{m}$	75°	$39^{d}.1$			$r = 2.0$ ft.
	E.			109.0	$39^{d}.0$	$69^{d}.8$	
	W.			39.0	108.8		
	E.			108.6			
East.	E.			109.4			
	W.			39.4	109.3	69.9	
	E.			109.2	39.4		
	W.	6 10	74	39.3			
Means			74.5		$2u^{d}$	69.85	

Coefficient of torsion, $v = 5.77$ div.

Rio Janeiro, January 6, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$6^{h}\ 10^{m}$	74°	$56^{d}.2$			$r = 2.5$ ft.
	E.			92.0	$56^{d}.2$	$35^{d}.7$	
	W.			56.2	91.9		
	E.			91.8			
East.	E.			92.0			
	W.			56.2	92.1	35.9	
	E.			92.2	56.2		
	W.	6 20	74	56.2			
Means			74.0		$2u^{d}$	35.80	

Monte Video, January 18, 1866.

Magnet.	North end.	Time.	Temp.	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$4^{h}\ 35^{m}$	87°	$37^{d}.2$			$r = 2.0$ ft.
	E.			105.9	$37^{d}.3$	$68^{d}.7$	
	W.			37.4	106.0		
	E.			106.0			
East.	E.			106.0			
	W.			37.7	106.0	68.0	
	E.			105.9	38.0		
	W.	4 45	87	38.3			
Means			87.0		$2u^{d}$	68.35	

Coefficient of torsion, $v = 4.50$ div.

Monte Video, January 18, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$4^{h}\ 45^{m}$	87°	$54^{d}.4$			$r = 2.5$ ft.
	E.			89.5	$54^{d}.4$	$35^{d}.1$	
	W.			54.4	89.5		
	E.			89.5			
East.	E.			89.7			
	W.			54.7	89.6	35.0	
	E.			89.6	54.6		
	W.	4 55	88	54.6			
Means			87.5		$2u^{d}$	35.05	

Sandy Point, February 7, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$12^{h}\ 45^{m}$	72°	$43^{d}.0$			$r = 2.0$ ft.
	E.			110.2	$43^{d}.5$	$66^{d}.8$	
	W.			44.0	110.3		
	E.			110.3			
East.	E.			110.7			
	W.			42.6	110.8	68.2	
	E.			110.9	42.6		
	W.	1 8	69	42.5			
Means			70.5		$2u^{d}$	67.50	

Coefficient of torsion, $v = 8.25$ div.

A high wind blowing which made the magnet very unsteady.

Sandy Point, February 7, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$1^{h}\ 8^{m}$	69°	$58^{d}.8$			$r = 2.5$ ft.
	E.			93.2	$58^{d}.6$	$34^{d}.6$	
	W.			58.3	93.2		
	E.			93.2			
East.	E.			93.4			
	W.			58.9	93.7	34.7	
	E.			94.0	59.0		
	W.	1 23	68	59.1			
Means			68.5		$2u^{d}$	34.65	

HORIZONTAL INTENSITY. OBSERVATIONS OF DEFLECTIONS.

Valparaiso, March 2, 1866.

Magnet.	North end.	Time. P. M.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$5^{h}\ 52^{m}$	71°.	$38^{d}.3$	$38^{d}.1$ 103.4	$65^{d}.3$	$r = 2.0$ ft.
	E.			103.7			
	W.			37.9			
	E.			103.1			
East.	E.			103.3	103.2 38.2	65.0	
	W.			38.7			
	E.			103.2			
	W.	6 3	70.	37.7			
Means			70.5		$2u^{d}$	65.15	

Coefficient of torsion, $v = 6.87$ div.

Valparaiso, March 2, 1866.

Magnet.	North end.	Time. P. M.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$6^{h}\ 3^{m}$	70.°	$53^{d}.8$	$53^{d}.7$ 87.1	$33^{d}.4$	$r = 2.5$ ft.
	E.			87.1			
	W.			53.7			
	E.			87.1			
East.	E.			87.2	87.1 53.6	33.5	
	W.			53.6			
	E.			87.1			
	W.	6 14	68.	53.6			
Means			69.0		$2u^{d}$	33.45	

Valparaiso, March 19, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$1^{h}\ 10^{m}$	75.°	$37^{d}.9$	$37^{d}.8$ 103.6	$65^{d}.8$	$r = 2.0$ ft.
	E.			103.6			
	W.			37.7			
	E.			103.7			
East.	E.			103.7	103.7 38.4	65.3	
	W.			38.4			
	E.			103.7			
	W.	1 20	76.	38.5			
Means			75.5		$2u^{d}$	65.55	

Coefficient of torsion, $v = 4.80$ div.

Valparaiso, March 19, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$1^{h}\ 20^{m}$	76.°	$54^{d}.2$	$54^{d}.1$ 87.7	$33^{d}.6$	$r = 2.5$ ft.
	E.			87.7			
	W.			54.0			
	E.			87.7			
East.	E.			87.8	87.8 54.4	33.4	
	W.			54.3			
	E.			87.8			
	W.	1 35	78.	54.5			
Means			77.0		$2u^{d}$	33.50	

Valparaiso, March 29, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$12^{h}\ 0^{m}$	69.°	$36^{d}.9$	$36^{d}.9$ 102.4	$65^{d}.5$	$r = 2.0$ ft.
	E.			102.1			
	W.			36.9			
	E.			102.6			
	E.			102.8	102.8 37.3	65.5	
	W.			37.2			
	E.			102.8			
	W.	12 13	68.	37.3			
Means			68.5		$2u^{d}$	65.50	

Coefficient of torsion, $v = 4.62$ div.

Valparaiso, March 29, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$12^{h}\ 13^{m}$	68.°	$53^{d}.1$	$53^{d}.0$ 86.6	$33^{d}.6$	$r = 2.5$ ft.
	E.			86.7			
	W.			52.9			
	E.			86.6			
East.	E			86.8	86.8 53.3	33.5	
	W.			53.5			
	E.			86.8			
	W.	12 28	68.	53.2			
Means			68.0		$2u^{d}$	33.55	

Horizontal Intensity. Observations of Deflections.

Valparaiso April 7, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$8^h\ 55^m$	65°	$38^d.2$ 102.9 37.9 103.0	$38^d.0$ 102.9	$64^d.9$	$r = 2.0$ ft.
East.	E. W. E. W.	9 10	67	104.0 37.2 103.9 37.2	103.9 37.2	66.7	
Means			66.0		$2u^d$	65.80	

Coefficient of torsion, $v = 4.68$ div.

Valparaiso, April 7, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$9^h\ 10^m$	67°	$53^d.8$ 87.2 54.0 87.3	$53^d.9$ 87.3	$33^d.4$	$r = 2.5$ ft.
East.	E. W. E. W.	9 25	69	87.7 53.6 87.6 53.4	87.6 53.5	34.1	
Means			68.0		$2u^d$	33.75	

Valparaiso, April 11, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$1^h\ 0^m$	74.°	$39^d.2$ 104.3 39.3 104.4	$39^d.2$ 104.3	$65^d.1$	$r = 2.0$ ft.
East.	E. W. E. W.	1 11	74.	105.2 38.9 105.3 39.2	105.2 39.0	66.2	
Means			74.0		$2u^d$	65.65	

Valparaiso, April 11, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$1^h\ 11^m$	74°	$55^d.2$ 88.4 55.2 88.6	$55^d.2$ 88.5	$33^d.3$	$r = 2.5$ ft.
East.	E. W. E. W.	1 23	74	88.9 54.9 88.9 54.8	88.9 54.9	34.0	
Means			74.0		$2u^d$	33.65	

Valparaiso, April 13, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$1^h\ 55^m$	71°.	$37^d.2$ 102.0 36.9 101.6	$37^d.0$ 101.8	$64^d.8$	$r = 2.0$ ft.
East.	E. W. E. W.	2 7	65.	102.2 36.0 101.7 35.6	101.9 35.8	66.1	
Means			68.0		$2u^d$	65.45	

Valparaiso, April 13, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$2^h\ 7^m$	65°.	$51^d.9$ 84.9 51.5 84.9	$51^d.7$ 84.9	$33^d.2$	$r = 2.5$ ft.
East.	E. W. E. W.	2 20	62.	85.4 51.0 85.0 50.9	85.2 51.0	34.2	
Means			63.5		$2u^d$	33.70	

HORIZONTAL INTENSITY. OBSERVATIONS OF DEFLECTIONS.

San Lorenzo Island, April 26, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$11^h\ 40^m$	79°	$51^d.0$ 109.7 50.9 109.6	$50^d.9$ 109.6	$58^d.7$	$r = 2.0$ ft.
East.	E. W. E. W.	11 52	82	110.4 50.9 110.4 50.7	110.4 50.8	59.6	
Means			80.5		$2u^d$	59.15	

Coefficient of torsion, $v = 4.25$ div.

San Lorenzo Island, April 26, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$11^h\ 52^m$	82°	$65^d.3$ 95.4 65.0 94.9	$65^d.1$ 95.1	$30^d.0$	$r = 2.5$ ft.
East.	E. W. E. W.	12 7	74	95.4 64.8 95.4 65.0	95.4 64.9	30.5	
Means			78.0		$2u^d$	30.25	

Payta, May 7, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$7^h\ 33^m$	77°	$52^d.2$ 107.7 52.0 107.8	$52^d.1$ 107.7	$55^d.6$	$r = 2.0$ ft.
East.	E. W. E. W.	7 46	77	108.4 51.6 108.3 51.6	108.4 51.6	56.8	
Means			77.0		$2u^d$	56.20	

Coefficient of torsion, $v = 3.62$ div.

Payta, May 7, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$7^h\ 46^m$	77°	$65^d.2$ 93.7 65.0 93.6	$65^d.1$ 93.7	$28^d.6$	$r = 2.5$ ft.
East.	E. W. E. W.	7 59	77	94.0 64.7 94.0 64.7	94.0 64.7	29.3	
Means			77.0		$2u^d$	28.95	

Flamenco Island, Panama Bay, May 14, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W. E. W. E.	$7^h\ 55^m$	83°	$50^d.7$ 104.6 51.0 104.7	$50^d.8$ 104.6	$53^d.8$	$r = 2.0$ ft.
East.	E. W. E. W.	8 5	82	105.6 50.4 105.5 50.1	105.5 52.2	53.3	
Means			82.5		$2u^d$	53.55	

Coefficient of torsion, $v = 3.18$ div.

Flamenco Island, Panama Bay, May 14, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternute Means.	Diff's.	Dist.
West.	W. E. W. E.	$8^h\ 5^m$	82°	$64^d.0$ 91.7 64.0 91.6	$64^d.0$ 91.6	$27^d.6$	$r = 2.5$ ft.
East.	E. W. E. W.	8 15	82	92.0 63.8 92.0 63.8	92.0 63.8	28.2	
Means			82.0		$2u^d$	27.90	

Horizontal Intensity. Observations of Deflections.

Acapulco, May 30, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$7^h\ 22^m$	86°	$53^d.9$			$r = 2.0$ ft.
	E.			107.0	$53^d.9$	$53^d.1$	
	W.			53.9	107.0		
	E.			107.0			
East.	E.			107.5			
	W.			53.5	107.6	54.0	
	E.			107.7	53.6		
	W.	7 32	84	53.8			
Means			85.0		$2u^d$	53.55	

Coefficient of torsion, $v = 3.45$ div.

Acapulco, May 30, 1866.

Magnet.	North end.	Time.	Temp. t	Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$7^h\ 32^m$	84°	$66^d.9$			$r = 2.5$ ft.
	E.			94.1	$66^d.9$	$27^d.3$	
	W.			66.9	94.2		
	E.			94.2			
East.	E.			94.4			
	W.			66.8	94.4	27.6	
	E.			94.4	66.8		
	W.	7 40	85	66.8			
Means			84.5		$2u^d$	27.45	

Magdalena Bay, June 9, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$1^h\ 14^m$	65°	$49^d.4$			$r = 2.0$ ft.
	E.			106.6	$49^d.4$	$57^d.3$	
	W.			49.4	106.7		
	E.			106.8			
East.	E.			106.7			
	W.			49.6	107.3	57.6	
	E.			107.9	49.7		
	W.	1 40	65	49.7			
Means			65.0		$2u^d$	57.45	

Assumed coefficient of torsion, $v = 3.87$ div.
Magnet very unsteady, and its readings uncertain on account of a stiff breeze which shook the instrument.

Magdalena Bay, June 9, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$1^h\ 40^m$	65°	$64^d.0$			$r = 2.5$ ft.
	E.			93.1	$63^d.9$	$29^d.7$	
	W.			63.7	93.6		
	E.			94.1			
East.	E.			94.7			
	W.			65.0	95.1	29.7	
	E.			95.4	65.4		
	W.	2 15	65	65.8			
Means			65.0		$2u^d$	29.70	

San Diego Bay, June 15, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$2^h\ 44^m$	72°	$45^d.9$			$r = 2.0$ ft.
	E.			111.3	$46^d.1$	$65^d.2$	
	W.			46.3	111.3		
	E.			111.2			
East.	E.			112.6			
	W.			45.8	112.5	66.7	
	E.			112.5	45.8		
	W.	2 53	71	45.8			
Means			71.5		$2u^d$	65.95	

Coefficient of torsion, $v = 4.28$ div.

San Diego Bay, June 15, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$2^h\ 53^m$	71°	$62^d.2$			$r = 2.5$ ft.
	E.			95.4	$62^d.2$	$33^d.2$	
	W.			62.2	95.4		
	E.			95.4			
East.	E.			95.4			
	W.			61.6	95.6	33.9	
	E.			95.8	61.7		
	W.	3 6	70	61.8			
Means			70.5		$2u^d$	33.55	

HORIZONTAL INTENSITY. OBSERVATIONS OF DEFLECTIONS.

San Francisco Bay, June 26, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
est.	W.	$6^{h}\ 40^{m}$	65.°	$42^{d}.3$			$r = 2.0$ ft.
	E.			114.8	$42^{d}.4$	$72^{d}.5$	
	W.			42.6	114.9		
	E.			115.1			
East.	E.			116.1			
	W.			43.0	116.2	73.2	
	E.			116.3	43.0		
	W.	6 50	62.	43.0			
Means			63.5		$2u^{d}$	72.85	

Coefficient of torsion, $v = 5.30$ div.

San Francisco Bay, June 26, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$6^{h}\ 50^{m}$	62.°	$60^{d}.8$			$r = 2.5$ ft.
	E.			98.0	$60^{d}.8$	$37^{d}.4$	
	W.			60.7	98.2		
	E.			98.4			
East.	E.			98.4			
	W.			61.0	98.4	37.5	
	E.			98.4	60.9		
	W.	6 59	63.	60.9			
Means			62.5		$2u^{d}$	37.45	

U. S. N. Observatory, Washington, Nov. 1, 1866.

Magnet.	North end.	Time.	Temp.	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$1^{h}\ 4^{m}$	66.°	$28^{d}.5$			$r = 2.0$ ft.
	E.			123.6	$28^{d}.5$	$94^{d}.7$	
	W.			28.5	123.2		
	E.			122.8			
East.	E.			124.5			
	W.			29.3	125.0	96.3	
	E.			125.5	28.7		
	W.	1 22	66.	28.1			
Means			66.0		$2u^{d}$	95.50	

Coefficient of torsion, $v = 7.05$ div.

U. S. N. Observatory, Washington, Nov. 1, 1866.

Magnet.	North end.	Time.	Temp. t	Scale Readings.	Alternate Means.	Diff's.	Dist.
West.	W.	$1^{h}\ 22^{m}$	66.°	$52^{d}.5$			$r = 2.5$ ft.
	E.			100.9	$52^{d}.5$	$48^{d}.2$	
	W.			52.6	100.7		
	E.			100.5			
East.	E.			102.0			
	W.			52.6	101.7	49.2	
	E.			101.4	52.5		
	W.	1 44	67.	52.3			
Means			66.5		$2u^{d}$	48.70	

SECTION V.

OBSERVATIONS ON THE MAGNETISM OF THE SHIP.

THE Monadnock is a second rate iron-clad vessel, of the Monitor type, of 1564 tons old or 1091 tons new measurement. On deck her length is 260.5 feet, and her breadth 52.0 feet. She has a wooden hull, but her deck is covered by three layers of iron plates, each one inch thick; and her sides, for a depth of five feet from the deck, are covered by six layers of iron plates, each one inch thick. Thus the deck is protected by three, and the sides by six inches of iron. She is provided with two iron turrets, cylindrical in form, each 22.8 feet in outside diameter, 9.0 feet high, and 11 inches thick. On top of each of them stands an iron pilot-house, 7.7 feet in outside diameter, 6.4 feet high, and 11 inches thick. Each of these pilot-houses is cylindrical in form, and so placed that its axis coincides with the axis of the turret upon which it stands. The sides of the turrets and pilot-houses are not solid, but are composed of iron plates, each one inch thick, placed one upon the other and bolted together till a total thickness of eleven inches is attained. To each of the iron pilot-houses are bolted wooden stanchions, which carry wooden pilot-houses whose floors are about nine and a half feet above the tops of the iron pilot-houses. The centres of the wooden pilot-houses are respectively in the same vertical lines with the centres of the turrets and iron pilot-houses over which they stand. The centres of the turrets coincide with the midships line. The distance from the stern of the vessel to the centre of the after turret is 84.5 feet; from the centre of the after turret to the centre of the forward turret, 99.1; and from the centre of the forward turret to the cut-water, 76.9 feet. Passing forward from the after turret, we come first to the ventilator, which is 6.5 feet in diameter, and 22.8 feet high above the deck; and then to the smoke-stack, which is 9.9 feet in diameter, and 31.0 feet high above the deck, both it and the ventilator being of iron. The distance from the centre of the after turret to the centre of the ventilator is 31.3 feet; from the centre of the ventilator to the centre of the smoke-stack, 16.5 feet; and from the centre of the smoke-stack to the centre of the forward turret, 51.3 feet.

At St. Thomas, before the magnetic observations on board ship were made at that place, a wooden mast 77.7 feet high was placed on the ship in order to enable her to carry some sail. Its centre is 22 feet forward of the centre of the forward turret, and what little iron was used in its construction is so placed that it is not at all probable that it affected the deviation of the compasses in its neighborhood in the slightest.

The following are the designations and positions of the compasses which were used during the cruise:—

The Forward Alidade was a Sands Alidade Compass, and was on top of the forward wooden pilot-house, 33.5 feet above the iron deck.

The Forward Binnacle was a Ritchie Liquid Compass, and was in the binnacle of the forward wooden pilot-house, 27.2 feet above the iron deck.

The Forward Ritchie was a Ritchie Monitor Compass, and was 6.7 feet above the top of the iron pilot-house on the forward turret. It was 22.1 feet above the iron deck.

Of these three compasses, the Forward Alidade and Forward Ritche were placed exactly in the vertical line passing through the centre of the forward turret, and the Forward Binnacle was placed about two feet further forward, but nearly in the same vertical plane.

The Admiralty Standard Compass was on top of the after wooden pilot-house, 37.0 feet above the iron deck.

The After Binnacle was a Ritchie Liquid Compass, and was in the binnacle of the after wooden pilot-house, 27.2 feet above the iron deck.

The After Ritchie was a Ritchie Monitor Compass, and was 6.7 feet above the top of the iron pilot-house on the after turret. It was 22.1 feet above the iron deck.

Of these three compasses, the Admiralty Standard and After Ritchie were placed exactly in the vertical line passing through the centre of the after turret, and the After Binnacle was placed about two feet futher forward, but nearly in the same vertical plane.

The After Azimuth was a common Azimuth Compass which was set up temporarily on the quarter deck every time the ship was swung; small cavities having been cut in the iron surface of the deck for the reception of the feet of the tripod, so as to make sure that the instrument always occupied precisely the same position. It stood 47.5 feet abaft the centre of the after turret, and there were two vertical iron stanchions, each two inches in diameter, 10.3 feet high above the deck, and 12.1 feet distant from the compass, one of them being directly forward and the other directly aft of it. This compass was elevated 4.6 feet above the iron deck; but when observations of magnetic force were made, it was necessary to remove it and substitute an Admiralty Standard Compass, which occupied precisely the same position, except that it was 4.8 feet above the deck. When the dip circle was used it also stood 4.8 feet above the deck.

It will be observed that *all* the compasses stood in the midships line, no matter what their elevation above the deck might be.

All the observations for determining the deviations of the compasses were made by swinging the ship in the following manner: The true azimuth of a well defined distant object was determined by a solar bearing, as explained in Section III, page 26, and the declination of the magnetic needle having been applied to it, its true magnetic azimuth became known; then, supposing the sight vanes of the Admiralty Standard Compass to be kept pointed steadily to that object while the ship was swung, the reading which they would indicate on the azimuth circle attached to the cover of the compass, as the ship's head pointed successively to each of the true magnetic points, was computed by means of the formula

$$R = 180^\circ + A - \zeta$$

where

R = reading of sight vanes on the azimuth circle attached to the cover of the compass.

A = true magnetic azimuth of the distant object; the azimuth being counted from the south around by the west.

ζ = azimuth of the ship's head, counted from the correct magnetic north around by the east.

This having been done, on a tolerably calm day steam was got up in the boilers, and, the vessel riding at a single anchor, slack water was waited for. As soon as the tide ceased to run, the executive officer took the deck; an officer was stationed at each of the compasses; I went to the Admiralty Standard; and a quartermaster was stationed at the ship's bell. Then the helm was put hard-a-starboard, or hard-a-port, depending on the direction in which it was desired to have her head swing, and the engines having been started, one forward and the other backward (the Monadnock was provided with twin screws which were entirely independent of each other), the vessel at once began to turn, without bringing any considerable strain on her cable. Her motion was perfectly under control, and could be made fast or slow at pleasure by merely varying the speed of the engines. I then set the sight vanes of the Admiralty Standard Compass to the reading (on the azimuth circle) of the point at which the ship's head would first arrive, and placing my eye to them I watched for the instant when they pointed to the distant object chosen as an azimuth mark. As the thread of the sight vane approached the object I cautioned the quartermaster to be ready, and at the instant it covered the object I made a signal, by dropping my outstretched arm, and the quartermaster struck a single stroke on the bell. Upon hearing this, every officer at once read off and recorded the heading of the ship, as indicated by the compass at which he was stationed. Then, the engines not having been stopped, I turned the sight vanes forward to the reading of the next point, and the same process was repeated; and so on, till the readings of all the compasses had been observed at each of the thirty-two points, which was generally accomplished in about an hour, or an hour and a half. The difference between any observed reading and the true point to which the vessel's head was directed at the time that reading was made, was of course the deviation of the compass on that point.

The forward iron and wooden pilot-houses were fixed and did not revolve with the turret, so that the lubber lines of the compasses in them always remained in the same position. But with the after iron and wooden pilot-houses the case was different. They were attached to the turret and revolved with it, and by so doing caused the lubber lines of the compasses in them also to revolve. As the turrets were frequently turned, it became necessary to establish marks by which the position of the after one could always be referred to some fixed position, so that a correction could be applied to the readings of the compasses in its pilot-houses to reduce them to what they would have been if their lubber lines had not moved For this purpose, whenever the ship was swung, a fixed line on the under side of the hurricane deck was produced till it touched the after turret, and then the distance from its point of contact with the turret to a joint (marked number XII) on the outside of the turret was measured. This distance, having been converted into degrees and minutes by means of the known diameter of the turret, was the correction to be applied to the position of the lubber lines. The following table gives the measured distance, and its angular equivalent, at every station where the ship was swung; but it must be noticed that these corrections apply *only to the After Binnacle and After Ritchie Compasses.* The lubber line of the Admiralty Standard Compass was always properly adjusted before beginning to observe.

Station.	Joint XII.	Lubber Line.
Hampton Roads	14in.4 port	Assumed correct.
St. Thomas	14.4 "	" "
Salute Islands	0.6 starboard	6° 18′ east.
Ceara	0.6 "	6 18 "
Bahia	0.6 "	6 18 "
Rio Janeiro	0.8 port	5 43 "
Monte Video	4.5 "	4 9 "
Sandy Point	4.5 "	4 9 "
Valparaiso	4.2 "	4 17 "
Callao	5.5 "	3 44 "
Panama	5.5 "	3 44 "
Acapulco	5.5 "	3 44 "
Magdalena Bay	5.5 "	3 44 "
San Francisco	5.3 "	3 49 "

When the ship was being swung, I always read the Admiralty Standard Compass myself. Each of the other compasses was usually read by the officer whose name is set opposite to it in the following table.

Forward Alidade,	Lieutenant M. Miller.
Forward Binnacle,	Lieut. Miller, assisted by a Quartermaster.
Forward Ritchie,	Lieutenant Geo. Smith.
After Binnacle,	Ensign F. Wildes.
After Ritchie,	Master Wm. Barrymore.
After Azimuth,	Mate Jno. Ponte.

My instruments for the measurement of magnetic force restricted me to the method of deflections, and the only compasses on board at which that method could be applied were the Admiralty Standard and the After Azimuth. As the ship was always riding at anchor, and of course swinging a little, when such observations were made, in order to render them as accurate as possible the following plan was adopted.

The deflecting bar was screwed to the movable circle which carried the sight vanes of the Admiralty Standard Compass in such a position as to be at right angles to them. That is, when the sight vanes pointed north and south the deflecting bar pointed east and west. Then, 1°. The sights being directed exactly north and south, as indicated by the compass card, the point, which we will designate by H, cut by them on the northern or southern horizon, as might be most convenient, was noted. 2°. The deflecting magnets were placed in the carriers, one to the east and the other to the west of the compass card, both being at the same distance from the centre of the card, and with their similar poles pointing in the same direction. Then, keeping the sight vanes pointed steadily to the object H, as soon as the compass card ceased to vibrate it was read off by means of the prism attached to the sight vane. Let this reading be designated as A. 3°. Each deflecting magnet was reversed, end for end, in its own carrier, and, the sight vanes being still kept directed to the object H, the card was again read. Let this reading be designated as B. Then the observed angle of deflection is $\frac{A-B}{2}$.

The dip was obtained by removing the Admiralty Standard Compass with which the deflections had been observed, and putting in its place a dip circle; the axle of the dipping needle occupying precisely the same position that had previously been occupied by the pivot of the compass card.

The observations of the deviations of the compasses made during the cruise have been compared with the following theory, which is taken from the English Admiralty Manual of the Deviations of the Compass, edition of 1863.

Let

X, Y, Z, represent the force of the earth's magnetism drawing the north point of the compass needle to the ship's head, to the starboard side and vertically downwards.

X', Y', Z', represent the combined force of the magnetism of the earth and ship in the same directions.

a, b, c, d, e, f, g, h, k, represent constant coefficients depending on the amount and arrangement of the soft iron of the ship.

P, Q, R, represent constant coefficients depending on the amount, arrangement, and independent magnetism of the hard iron of the ship.

$H =$ the horizontal force of the earth.

$H' =$ the horizontal force of the earth and ship.

$\theta =$ the dip.

$\zeta =$ azimuth of the ship's head measured eastward from the correct magnetic north.

$\zeta' =$ azimuth of the ship's head measured from the direction of the disturbed needle.

$\delta = \zeta - \zeta' =$ the deviation of the compass.

Then the whole mathematical theory of the deviations of the compass is comprised in the three following equations:

$$X' = X + aX + bY + cZ + P \tag{1}$$

$$Y' = Y + dX + eY + fZ + Q \tag{2}$$

$$Z' = Z + gX + hY + kZ + R \tag{3}$$

We have also

$$X = H \cos\zeta \qquad Y = -H \sin\zeta \qquad Z = H\tan\theta$$
$$X' = H' \cos\zeta' \qquad Y' = -H' \sin\zeta'$$

Substituting these values in equations (1), (2), and (3), and dividing by H, we have

$$\frac{H'}{H}\cos\zeta' = (1+a)\cos\zeta - b\sin\zeta + c\tan\theta + \frac{P}{H} \tag{4}$$

$$-\frac{H}{H}\sin\zeta' = d\cos\zeta - (1+e)\sin\zeta + f\tan\theta + \frac{Q}{H} \tag{5}$$

$$\frac{Z'}{H} = g\cos\zeta - h\sin\zeta + (1+k)\tan\theta + \frac{R}{H} \tag{6}$$

Equation (6) may be written

$$0 = 1 - \frac{Z'}{Z} + g\frac{\cos\zeta}{\tan\theta} - h\frac{\sin\zeta}{\tan\theta} + k + \frac{R}{Z} \tag{6a}$$

From equations (4) and (5) we obtain the following:

(4) cos ζ — (5) sin ζ gives after some reductions

$$\frac{H'}{H}\cos\delta = 1 + \frac{a+e}{2} + \left(c\tan\theta + \frac{P}{H}\right)\cos\zeta - \left(f\tan\theta + \frac{Q}{H}\right)\sin\zeta$$
$$+ \frac{a-e}{2}\cos 2\zeta - \frac{d+b}{2}\sin 2\zeta \qquad 7)$$

(4) sin ζ + (5) cos ζ gives after some reductions

$$\frac{H'}{H}\sin\delta = \frac{d-b}{2} + \left(c\tan\theta + \frac{P}{H}\right)\sin\zeta + \left(f\tan\theta + \frac{Q}{H}\right)\cos\zeta$$
$$+ \frac{a-e}{2}\sin 2\zeta + \frac{d+b}{2}\cos 2\zeta \tag{8}$$

Now let

$$1 + \frac{a+e}{2} = \lambda \qquad \frac{d-b}{2} = \lambda\mathfrak{A}$$

$$\frac{a-e}{2} = \lambda\mathfrak{D} \qquad \frac{d+b}{2} = \lambda\mathfrak{E}$$

$$c\tan\theta + \frac{P}{H} = \lambda\mathfrak{B} \qquad f\tan\theta + \frac{Q}{H} = \lambda\mathfrak{C}$$

Then from equations (7) and (8) we get the following:

$$\frac{H}{\lambda H}\cos\delta = 1 + \mathfrak{B}\cos\zeta - \mathfrak{C}\sin\zeta + \mathfrak{D}\cos 2\zeta - \mathfrak{E}\sin 2\zeta \tag{9}$$

$$\frac{H'}{\lambda H}\sin\delta = \mathfrak{A} + \mathfrak{B}\sin\zeta + \mathfrak{C}\cos\zeta + \mathfrak{D}\sin 2\zeta + \mathfrak{E}\cos 2\zeta \tag{10}$$

Dividing (10) by (9),

$$\tan\delta = \frac{\mathfrak{A} + \mathfrak{B}\sin\zeta + \mathfrak{C}\cos\zeta + \mathfrak{D}\sin 2\zeta + \mathfrak{E}\cos 2\zeta}{1 + \mathfrak{B}\cos\zeta - \mathfrak{C}\sin\zeta + \mathfrak{D}\cos 2\zeta - \mathfrak{E}\sin 2\zeta} \tag{11}$$

From (11) we easily get

$$\begin{aligned}\sin\delta &= \mathfrak{A}\cos\delta + \mathfrak{B}\sin\zeta' + \mathfrak{C}\cos\zeta' + \mathfrak{D}\sin(\zeta+\zeta') + \mathfrak{E}\cos(\zeta+\zeta') \qquad (12)\\ &= \mathfrak{A}\cos\delta + \mathfrak{B}\sin\zeta' + \mathfrak{C}\cos\zeta' + \mathfrak{D}\sin(2\zeta'+\delta) + \mathfrak{E}\cos(2\zeta'+\delta)\end{aligned}$$

Of the last three equations (11) is used when the deviations are given on the correct magnetic points, (12) when the deviations are given on the compass points affected by deviation.

Equation (12) may be put under the following form, which is sometimes convenient, and which is very nearly exact, viz.:

$$\sin\delta = \frac{1}{1-\mathfrak{D}\cos 2\zeta'}\left\{\mathfrak{A} + \mathfrak{B}\sin\zeta' + \mathfrak{C}\cos\zeta' + \mathfrak{D}\sin 2\zeta' + \mathfrak{E}\cos 2\zeta'\right\} \qquad (12a)$$

By means of the expressions for sin δ we may calculate the values of the coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, if we know the deviations on five points. If we have the deviations on more than five points, we may determine the most probable values of the coefficients by the method of least squares; but the calculation will in general be long and difficult.

If, however, the compass points on which the deviations are given divide the circumference into equal parts, we may determine the exact coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, with great ease, and a sufficient degree of approximation, by determining first the approximate coefficients A, B, C, D, E, and then deducing from them the values of the exact coefficients. For that purpose we proceed as follows:

If the coefficients are less than 20° their squares and products may be neglected, and equation (12) may be put under the form

$$\delta = A + B\sin\zeta' + C\cos\zeta' + D\sin 2\zeta' + E\cos 2\zeta' \qquad (13)$$

Let $\delta_0\ \delta_1\ \delta_2 \ldots \delta_{31}$ be the deviations observed on the 32 points, by compass, $S_1\ S_2\ S_3 \ldots S_7$ the natural sines of the rhumbs or of the angles 11° 15′, 22° 30′.... 78° 45′ respectively, then if the observations have been made on the 32 points we have the following 32 equations from which to determine A, B, C, D, E.

Compass Courses.	Deviation.	A	$+ B \sin \zeta'$	$+ C \cos \zeta'$	$+ D$ and 2ζ	$+ E \cos 2\zeta'$
North	δ_0	A		$+ C$		$+ E$
N. by E.	δ_1	A	$+ B S_1$	$+ C S_7$	$+ D S_2$	$+ E S_6$
N. N. E.	δ_2	A	$+ B S_2$	$+ C S_6$	$+ D S_4$	$+ E S_4$
N. E. by N.	δ_3	A	$+ B S_3$	$+ C S_5$	$+ D S_6$	$+ E S_2$
N. E.	δ_4	A	$+ B S_4$	$+ C S_4$	$+ D$	
N. E. by E.	δ_5	A	$+ B S_5$	$+ C S_3$	$+ D S_6$	$- E S_2$
E. N. E.	δ_6	A	$+ B S_6$	$+ C S_2$	$+ D S_4$	$- E S_4$
E. by N.	δ_7	A	$+ B S_7$	$+ C S_1$	$+ D S_2$	$- E S_6$
East	δ_8	A	$+ B$			$- E$
E. by S.	δ_9	A	$+ B S_7$	$- C S_1$	$- D S_2$	$- E S_6$
E. S. E.	δ_{10}	A	$+ B S_6$	$- C S_2$	$- D S_4$	$- E S_4$
S. E. by E.	δ_{11}	A	$+ B S_5$	$- C S_3$	$- D S_6$	$- E S_2$
S. E.	δ_{12}	A	$+ B S_4$	$- C S_4$	$- D$	
S. E. by S.	δ_{13}	A	$+ B S_3$	$- C S_5$	$- D S_6$	$+ E S_2$
S. S. E.	δ_{14}	A	$+ B S_2$	$- C S_6$	$- D S_4$	$+ E S_4$
S. by E.	δ_{15}	A	$+ B S_1$	$- C S_7$	$- D S_2$	$+ E S_6$
South	δ_{16}	A		$- C$		$+ E$
S. by W.	δ_{17}	A	$- B S_1$	$- C S_7$	$+ D S_2$	$+ E S_6$
S. S. W.	δ_{18}	A	$- B S_2$	$- C S_6$	$+ D S_4$	$+ E S_4$
S. W. by S.	δ_{19}	A	$- B S_3$	$- C S_5$	$+ D S_6$	$+ E S_2$
S. W.	δ_{20}	A	$- B S_4$	$- C S_4$	$+ D$	
S. W. by W.	δ_{21}	A	$- B S_5$	$- C S_3$	$+ D S_6$	$- E S_2$
W. S. W	δ_{22}	A	$- B S_6$	$- C S_2$	$+ D S_4$	$- E S_4$
W. by S.	δ_{23}	A	$- B S_7$	$- C S_1$	$+ D S_2$	$- E S_6$
West	δ_{24}	A	$- B$			$- E$
W. by N.	δ_{25}	A	$- B S_7$	$+ C S_1$	$- D S_2$	$- E S_6$
W. N. W.	δ_{26}	A	$- B S_6$	$+ C S_2$	$- D S_4$	$- E S_4$
N. W. by W.	δ_{27}	A	$- B S_5$	$+ C S_3$	$- D S_6$	$- E S_2$
N. W.	δ_{28}	A	$- B S_4$	$+ C S_4$	$- D$	
N. W. by N.	δ_{29}	A	$- B S_3$	$+ C S_5$	$- D S_6$	$+ E S_2$
N. N. W.	δ_{30}	A	$- B S_2$	$+ C S_6$	$- D S_4$	$+ E S_4$
N. by W.	δ_{31}	A	$- B S_1$	$+ C S_7$	$- D S_2$	$+ E S_6$

By the method of least squares we obtain, from these 32 equations of condition, the five normal equations

$$\begin{aligned}
\delta_0 + \delta_1 + \delta_2 \ldots\ldots\ldots\ldots\ldots\ldots + \delta_{31} &= 32\,A. \\
\delta_1 S_1 + \delta_2 S_2 + \delta_3 S_3 + \&c. \ldots\ldots\ldots &= 16\,B. \\
\delta_0 + \delta_1 S_7 + \delta_2 S_6 + \&c. \ldots\ldots\ldots\ldots &= 16\,C. \\
\delta_1 S_2 + \delta_2 S_4 + \delta_3 S_6 + \&c. \ldots\ldots\ldots &= 16\,D. \\
\delta_0 + \delta_1 S_6 + \delta_2 S_4 + \&c. \ldots\ldots\ldots\ldots &= 16\,E.
\end{aligned}$$

For convenience of computation these equations have been put under the form

$$\begin{aligned}
8\,A = {} & \tfrac{1}{2}\left(\frac{\delta_0 + \delta_{16}}{2} + \frac{\delta_8 + \delta_{24}}{2}\right) \\
& + \tfrac{1}{2}\left(\frac{\delta_1 + \delta_{17}}{2} + \frac{\delta_9 + \delta_{25}}{2}\right) \\
& + \tfrac{1}{2}\left(\frac{\delta_2 + \delta_{18}}{2} + \frac{\delta_{10} + \delta_{26}}{2}\right)
\end{aligned}$$

$$+\tfrac{1}{2}\left(\frac{\delta_3+\delta_{19}}{2}+\frac{\delta_{11}+\delta_{27}}{2}\right)$$

$$+\tfrac{1}{2}\left(\frac{\delta_4+\delta_{20}}{2}+\frac{\delta_{12}+\delta_{28}}{2}\right)$$

$$+\tfrac{1}{2}\left(\frac{\delta_5+\delta_{21}}{2}+\frac{\delta_{13}+\delta_{29}}{2}\right)$$

$$+\tfrac{1}{2}\left(\frac{\delta_6+\delta_{22}}{2}+\frac{\delta_{14}+\delta_{30}}{2}\right)$$

$$+\tfrac{1}{2}\left(\frac{\delta_7+\delta_{23}}{2}+\frac{\delta_{15}+\delta_{31}}{2}\right)$$

$$8B = \frac{\delta_8+\delta_{24}}{2}$$

$$+\frac{\delta_1-\delta_{17}}{2}S_1+\frac{\delta_9-\delta_{25}}{2}S_7$$

$$+\frac{\delta_2-\delta_{18}}{2}S_2+\frac{\delta_{10}-\delta_{26}}{2}S_6$$

$$+\frac{\delta_3-\delta_{19}}{2}S_3+\frac{\delta_{11}-\delta_{27}}{2}S_5$$

$$+\frac{\delta_4-\delta_{20}}{2}S_4+\frac{\delta_{12}-\delta_{28}}{2}S_4$$

$$+\frac{\delta_5-\delta_{21}}{2}S_5+\frac{\delta_{13}-\delta_{29}}{2}S_3$$

$$+\frac{\delta_6-\delta_{22}}{2}S_6+\frac{\delta_{14}-\delta_{30}}{2}S_2$$

$$+\frac{\delta_7-\delta_{23}}{2}S_7+\frac{\delta_{15}-\delta_{31}}{2}S_1$$

$$8C = \frac{\delta_0-\delta_{16}}{2}$$

$$+\frac{\delta_1-\delta_{17}}{2}S_7-\frac{\delta_9-\delta_{25}}{2}S_1$$

$$+\frac{\delta_2-\delta_{18}}{2}S_6-\frac{\delta_{10}-\delta_{26}}{2}S_2$$

$$+\frac{\delta_3-\delta_{19}}{2}S_5-\frac{\delta_{11}-\delta_{27}}{2}S_3$$

$$+\frac{\delta_4-\delta_{20}}{2}S_4-\frac{\delta_{12}-\delta_{28}}{2}S_4$$

$$+\frac{\delta_5-\delta_{21}}{2}S_3-\frac{\delta_{13}-\delta_{29}}{2}S_5$$

$$+\frac{\delta_6-\delta_{22}}{2}S_2-\frac{\delta_{14}-\delta_{30}}{2}S_6$$

$$+\frac{\delta_7-\delta_{23}}{2}S-\frac{\delta_{15}-\delta_{31}}{2}S_7$$

$$
\begin{aligned}
4\,D = {} & \qquad\qquad\qquad\qquad\qquad\qquad + \tfrac{1}{2}\left(\frac{\delta_4+\delta_{20}}{2}-\frac{\delta_{12}+\delta_{28}}{2}\right) \\
& + \tfrac{1}{2}\left(\frac{\delta_1+\delta_{17}}{2}-\frac{\delta_9+\delta_{25}}{2}\right)S_2 + \tfrac{1}{2}\left(\frac{\delta_5+\delta_{21}}{2}-\frac{\delta_{13}+\delta_{29}}{2}\right)S_6 \\
& + \tfrac{1}{2}\left(\frac{\delta_2+\delta_{18}}{2}-\frac{\delta_{10}+\delta_{26}}{2}\right)S_4 + \tfrac{1}{2}\left(\frac{\delta_6+\delta_{22}}{2}-\frac{\delta_{14}+\delta_{30}}{2}\right)S_4 \\
& + \tfrac{1}{2}\left(\frac{\delta_3+\delta_{19}}{2}-\frac{\delta_{11}+\delta_{27}}{2}\right)S_6 + \tfrac{1}{2}\left(\frac{\delta_7+\delta_{23}}{2}-\frac{\delta_{15}+\delta_{31}}{2}\right)S_2
\end{aligned}
$$

$$
\begin{aligned}
4\,E = {} & \tfrac{1}{2}\left(\frac{\delta_0+\delta_{16}}{2}-\frac{\delta_8+\delta_{24}}{2}\right) \\
& + \tfrac{1}{2}\left(\frac{\delta_1+\delta_{17}}{2}-\frac{\delta_9+\delta_{25}}{2}\right)S_6 - \tfrac{1}{2}\left(\frac{\delta_5+\delta_{21}}{2}-\frac{\delta_{13}+\delta_{29}}{2}\right)S_2 \\
& + \tfrac{1}{2}\left(\frac{\delta_2+\delta_{18}}{2}-\frac{\delta_{10}+\delta_{26}}{2}\right)S_4 - \tfrac{1}{2}\left(\frac{\delta_6+\delta_{22}}{2}-\frac{\delta_{14}+\delta_{30}}{2}\right)S_4 \\
& + \tfrac{1}{2}\left(\frac{\delta_3+\delta_{19}}{2}-\frac{\delta_{11}+\delta_{27}}{2}\right)S_2 - \tfrac{1}{2}\left(\frac{\delta_7+\delta_{23}}{2}-\frac{\delta_{15}+\delta_{31}}{2}\right)S_6
\end{aligned}
$$

But the deviations about to be discussed were all observed, not on the compass points, but on the correct magnetic points. Treating them in the manner which has just been described, we obtain the approximate coefficients A_1, B_1, C_1, D_1, E_1, which belong to the correct magnetic points. Then, from equation (11) we get, going to terms of the third order inclusive,

$$
\begin{aligned}
\delta = {} & \mathfrak{A} \\
& + (\mathfrak{B} + \mathfrak{A}\,\mathfrak{C}) \sin\zeta + (\mathfrak{C} - \mathfrak{A}\,\mathfrak{B} \cos\zeta \\
& + \left\{\mathfrak{D} - \frac{\mathfrak{B}^2 - \mathfrak{C}^2}{2}\right\} \sin 2\zeta + \left\{\mathfrak{E} - \mathfrak{B}\,\mathfrak{C} - \mathfrak{A}\,\mathfrak{D}\right\} \cos 2\zeta \\
& + \left\{-\mathfrak{B}\,\mathfrak{D} + \mathfrak{C}\,\mathfrak{E} + \frac{\mathfrak{B}^3}{3} - \mathfrak{B}\,\mathfrak{C}^2\right\} \sin 3\zeta \\
& + \left\{-\mathfrak{B}\,\mathfrak{E} - \mathfrak{C}\,\mathfrak{D} - \frac{\mathfrak{C}^3}{3} + \mathfrak{B}^2\,\mathfrak{C}\right\} \cos 3\zeta \\
& + \left\{-\frac{\mathfrak{D}^2}{2} + (\mathfrak{B}^2 - \mathfrak{C}^2)\,\mathfrak{D}\right\} \sin 4\zeta + \left\{-\mathfrak{D}\,\mathfrak{E} + 2\,\mathfrak{B}\,\mathfrak{C}\,\mathfrak{D}\right\} \cos 4\zeta \\
& + \mathfrak{B}\,\mathfrak{D}^2 \sin 5\zeta + \mathfrak{C}\,\mathfrak{D}^2 \cos 5\zeta \\
& + \tfrac{1}{3}\mathfrak{D}^3 \sin 6\zeta
\end{aligned}
\tag{14}
$$

where δ is expressed in terms of the arc which is equal to radius. If we suppose the complete expression for δ to be

$$
\begin{aligned}
\delta = {} & A_1 + B_1 \sin\zeta + C_1 \cos\zeta + D_1 \sin 2\zeta + E_1 \cos 2\zeta \\
& + F_1 \sin 3\zeta + G_1 \cos 3\zeta + H_1 \sin 4\zeta + K_1 \cos 4\zeta \\
& + L_1 \sin 5\zeta + M_1 \cos 5\zeta + N_1 \sin 6\zeta
\end{aligned}
\tag{15}
$$

Then, comparing equation (14) with equation (15), we find, to terms of the third order inclusive,

$$
\begin{aligned}
\mathfrak{A} &= A_1 \\
\mathfrak{B} &= B_1 - A_1\, C_1 \\
\mathfrak{C} &= C_1 + A_1\, B_1 \\
\mathfrak{D} &= D_1 + \frac{B_1^2 - C_1^2}{2} \\
\mathfrak{E} &= E_1 + B_1\, C_1 + A_1\, D_1 \\
F_1 &= - B_1\, D_1 + C_1\, E_1 - \frac{B_1^3}{6} - \frac{B_1 C_1^2}{2} \\
G_1 &= - C_1\, D_1 + B_1\, E_1 \frac{C_1^3}{6} + \frac{C_1 B_1^2}{2} \\
H_1 &= - \frac{D_1^2}{2} + \frac{D_1\, B_1^2}{2} - \frac{D_1 C_1^2}{2} \\
K_1 &= - D_1\, E_1 + 2\, B_1\, C_1\, D_1 \\
L_1 &= B_1\, D_1^2 \\
M_1 &= C_1\, D_1^2 \\
N_1 &= \tfrac{1}{3}\, D_1^3
\end{aligned}
\qquad (16)
$$

"When the deviation of the compass is small, the several parts of which it is composed are simply added together; these parts are,

1. A, the constant deviation.
2. $B \sin \zeta' + C \cos \zeta'$, the semicircular deviation.
3. $D \sin 2\zeta' + E \cos 2\zeta'$, the quadrantal deviation.

"When the deviation is large, $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, or the angles of which these quantities are the natural sines, may still be considered as the constant and as the several parts of the semicircular and the quadrantal deviation, each of these angles being in fact the maximum deviation which would exist if all the other coefficients were zero; but their effects are no longer combined by simple addition."

Before submitting the observed deviations to comparison with the theory, it is necessary to free them from constant errors. These errors originated in two ways.

1°. When the ship was swung, the variation of the needle at the port where she was lying was seldom accurately known. Hence, in order to obtain the true magnetic azimuth of the object used as an azimuth mark, it was necessary to adopt, for the time being, the best value of the variation which happened to be accessible. In order to facilitate the setting of the sight vanes of the Admiralty Standard Compass while the ship was being swung, the value thus adopted was always so taken that, when the ship's head pointed successively to each of the true magnetic points, the reading of the sight vanes on the azimuth circle attached to the cover of that compass was always either some whole degree or some quarter of a degree. When the declinometer observations were reduced, the true value of the variation of the compass at each port became known, and then it was discovered that in some cases the adopted value was in error by more than three degrees. But an error in the adopted value of the variation produced an error of the same amount in the magnetic azimuth of the distant object used as an azimuth mark, and, therefore, in the pointing of the ship's head to each of the true magnetic points. Bearing in mind that the observed deviations were obtained by simply taking the difference between the heading of the ship and the reading of the compass, it will be apparent that if we apply to each observed deviation the difference between the true and adopted variation of the compass, with its proper sign, we shall obtain the true deviations for the directions in which the ship's head actually pointed at the time the readings of the compasses were made. From these corrected deviations the deviations on the true magnetic points can be found by simple interpolation. Therefore, if we let

m = the true, minus the adopted, magnetic azimuth of the distant object used as an azimuth mark: the azimuths being taken as increasing from the south around by the west.

δ' = the observed deviation of the compass when the ship headed in the direction A.

δ'' = the observed deviation of the compass when the ship headed in the direction $A \mp 11^\circ\ 15'$; the upper sign being taken when m is positive, the lower when m is negative.

δ = the deviation of the compass when the ship heads to the true magnetic point which lies between A and $A \mp 11^\circ\ 15'$; that point being of the same name as A was intended to be when the ship was swung.

Then we shall have with sufficient accuracy

$$\delta = \delta' + m \mp \frac{m\,(\delta' - \delta'')}{11^\circ\ 15'}$$

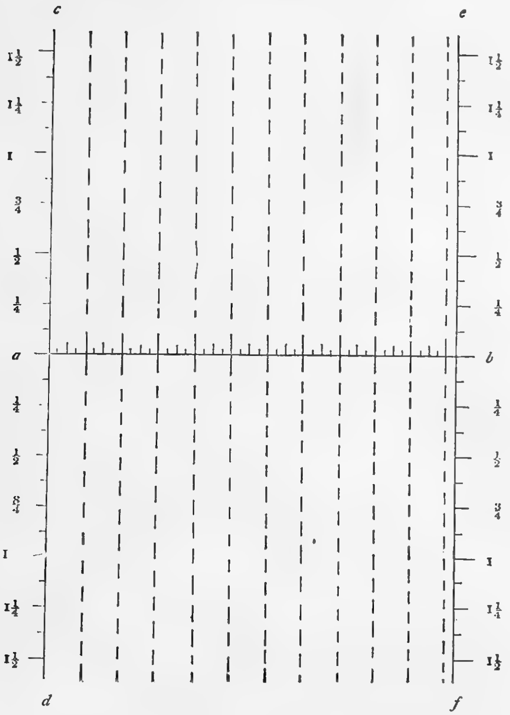
the upper sign being taken when m is positive, the lower when m is negative. By this formula the deviations of the Forward Alidade, Forward Binnacle, Forward Ritchie, Admiralty Standard, and After Azimuth Compasses, on the true magnetic points, have been computed from the observed deviations.

2°. In addition to the correction which has just been explained, the observed deviations of the After Binnacle and After Ritchie Compasses require a further correction on account of the lubber lines of these instruments revolving with the after turret, and thus being frequently out of their true position. This correction, which we will represent by L, is constant, and is equal in amount to the displacement of the lubber line. Its sign is + if the lubber line is to starboard, — if it is to port, of its true position. The deviations of the After Binnacle and After Ritchie Compasses, on the true magnetic points, were therefore computed from the observed deviations by the formula

$$\delta = \delta' + (m + L) \mp \frac{m\,(\delta' - \delta'')}{11^\circ\ 15'}$$

the upper sign being taken when m is positive, the lower when m is negative.

To have computed *numerically* all the values of δ for each compass by means of the expressions just given, would have involved a great amount of labor; it was therefore done graphically as follows:



On a piece of cardboard of suitable size a horizontal line $a\,b$, $5\frac{5}{8}$ inches long, was drawn, and divided into eighths of an inch; each half inch representing one degree, and the whole line representing 11° 15′, or one point of the compass. Touching the extremities of the line $a\,b$, and at right angles to it, were drawn the line $c\,d$ and $e\,f$; and each of them was divided, upward and downward from the line $a\,b$, into points and eights of points;[1] each point occupying the space of $2\frac{13}{16}$ of an inch. Finally, a straight slip of drawing paper was divided on its edge into degrees and sixths of a degree, each degree occupying a space of one-quarter of an inch; and the graduation was numbered from the middle towards each extremity.

Then, to compute the values of δ for any compass at any place, the paper scale was laid down parallel to, and to the right of, $c\,d$, and at a distance from it (measured on the line $a\,b$) equal to m; next, without moving the paper scale at all in the direction $a\,b$, it was slipped up or down, as might be necessary, in the direction parallel to $c\,d$, till the line $a\,b$ cut the division on it which was equal to $(m + L)$; the zero of the scale being above the line $a\,b$ if $(m + L)$ was negative, below it if

---

[1] For computing the deviations of the Admiralty Standard and After Azimuth Compasses the lines $c\,d$ and $e\,f$ were divided into degrees and sixths of a degree, each degree occupying the space of one-quarter of an inch.

$(m + L)$ was positive. Things being thus arranged, a weight was placed on the paper scale to prevent it from moving. Then a ruler being laid so that, while it crossed the line $c\,d$ at a distance from a equal to δ', it also crossed the line $e\,f$ at a distance from b equal to δ'' (the distances δ' and δ'' being taken *above* the line $a\,b$ if they were *positive*, *below* it if they were *negative*), the reading of the point on the paper scale where the ruler crossed its edge was the required value of δ. In that way, without again moving the paper scale, the values of the deviations on each of the thirty-two true magnetic points were computed from the observed values.

The following table contains the constants which were used in computing from the observed deviations the deviations on the true magnetic points. The first column gives the name of the station. The second column, the distance in miles from the ship to the object used as an azimuth mark. The third column, the assumed magnetic azimuth of the object used as an azimuth mark; the azimuth being counted from the south around by the west. The fourth column, the true magnetic azimuth of the same object, found by applying the magnetic declination given in the table on page 61, section IV, to the true azimuth given in the table on page 36, section III. The fifth column, the value of m. The sixth column, the value of L; and the seventh column, the value of $(m + L)$.

Station.	Distance of Object in Miles.	Assumed Magnetic Azimuth.	True Magnetic Azimuth.	m	L	$(m+L)$
Hampton Roads . . .	$6\frac{1}{4}$	9° 15′	13° 12′	+ 3° 57′	0° 0′	+ 3° 57′
St. Thomas	$4\frac{1}{2}$	327 30	327 45	+ 0 15	0 0	+ 0 15
Salute Islands	25	11 0	10 58	— 0 2	+ 6 18	+ 6 16
Ceara	4	268 45	270 36	+ 1 51	+ 6 18	+ 8 9
Bahia	5	103 30	106 0	+ 2 30	+ 6 18	+ 8 48
Rio Janeiro	5	126 30	129 14	+ 2 44	+ 5 43	+ 8 27
Monte Video	5	93 0	92 47	— 0 13	+ 4 9	+ 3 56
Sandy Point	26	345 15	345 22	+ 0 7	+ 4 9	+ 4 16
Valparaiso	$3\frac{1}{2}$	195 15	195 16	+ 0 1	+ 4 17	+ 4 18
Callao	$5\frac{1}{2}$	72 45	72 51	+ 0 6	+ 3 44	+ 3 50
Panama	7	15 0	15 1	+ 0 1	+ 3 44	+ 3 45
Acapulco	4	243 15	243 21	+ 0 6	+ 3 44	+ 3 50
Magdalena Bay	8	303 30	302 50	— 0 40	+ 3 44	+ 3 4
San Francisco	9	150 30	149 45	— 0 45	+ 3 49	+ 3 4

The following tables contain all the deviations of the compasses which were observed during the cruise. In each table the first column contains the assumed magnetic azimuth of the ship's head at the time the reading of the compass, given on the same line in the second column, was taken. The third column contains the observed deviation of the compass for each point, obtained by subtracting the readings in the second column from those in the first column. Hence, a deviation of the north point of the compass to the east is designated by the sign +; a deviation to the west by the sign —. The fourth column contains the deviation of the compass on each of the thirty-two true magnetic points. obtained from the observed deviations in the manner already explained.

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE ADMIRALTY STANDARD COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Hampton Roads, November 1, 1865.

Correction for Object = + 3° 57′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. $\frac{1}{8}$ E.	− $\frac{1}{8}$	° ′	+ 1° 30′
N. by E.	N. by E. $\frac{1}{8}$ E.	− $\frac{1}{8}$		+ 2 30
N. N. E.	N. N. E. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 4 20
N. E. by N.	N. E. by N. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 6 20
N. E.	N. E. $\frac{5}{8}$ N.	+ $\frac{5}{8}$		+ 9 30
N. E. by E.	N. E. by E. $\frac{1}{2}$ N.	+ $\frac{1}{2}$		+ 10 20
E. N. E.	E. N. E. $\frac{1}{2}$ N.	+ $\frac{1}{2}$		+ 9 30
E. by N.	E. by N. $\frac{1}{2}$ N.	+ $\frac{1}{2}$		+ 9 30
EAST.	E. $\frac{5}{8}$ N.	+ $\frac{5}{8}$		+ 10 30
E. by S.	E. by S. $\frac{5}{8}$ E.	+ $\frac{5}{8}$		+ 11 00
E. S. E.	E. S. E. $\frac{1}{2}$ E.	+ $\frac{1}{2}$		+ 10 00
S. E. by E.	S. E. by E. $\frac{1}{2}$ E.	+ $\frac{1}{2}$		+ 9 30
S. E.	S. E. $\frac{3}{8}$ E.	+ $\frac{3}{8}$		+ 8 40
S. E. by S.	S. E. by S. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 7 20
S. S. E.	S. S. E. $\frac{1}{4}$ S.	− $\frac{1}{4}$		+ 3 20
S. by E.	S. by E.	0		+ 3 0
SOUTH.	S. $\frac{1}{8}$ W.	− $\frac{1}{8}$		+ 3 0
S. by W.	S. by W. $\frac{3}{8}$ W.	− $\frac{3}{8}$		+ 0 40
S. S. W.	S. S. W. $\frac{1}{2}$ W.	− $\frac{1}{2}$		− 1 20
S. W. by S.	S. W. by S. $\frac{5}{8}$ W.	− $\frac{5}{8}$		− 2 30
S. W.	S. W. $\frac{3}{4}$ W.	− $\frac{3}{4}$		− 4 0
S. W. by W.	S. W. by W. $\frac{3}{4}$ W.	− $\frac{3}{4}$		− 4 30
W. S. W.	W. by S.	− 1		− 6 20
W. by S.	WEST.	− 1		− 7 20
WEST.	W. by N.	− 1		− 7 20
W. by N.	W. N. W.	− 1		− 7 20
W. N. W.	N. W. by W.	− 1		− 7 20
N. W. by W.	N. W. by W. $\frac{7}{8}$ N.	− $\frac{7}{8}$		− 6 20
N. W.	N. W. $\frac{7}{8}$ N.	− $\frac{7}{8}$		− 5 50
N. W. by N.	N. W. by N. $\frac{5}{8}$ N.	− $\frac{5}{8}$		− 4 0
N. N. W.	N. N. W. $\frac{5}{8}$ N.	− $\frac{5}{8}$		− 3 0
N. by W.	N. by W. $\frac{3}{8}$ N.	− $\frac{3}{8}$		− 1 20
NORTH.		− $\frac{1}{8}$		+ 1 30

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

$$A = +1° \; 37'.4 \qquad B = +9° \; 4'.6 \qquad C = -0° \; 33'.1$$
$$D = +0° \; 29'.2 \qquad E = -0° \; 7'.5$$

Assumed magnetic bearing of tree S. 9° 15′ W. Distant $6\frac{1}{4}$ miles.

St. Thomas, West Indies, November 16, 1865.

Correction for Object = + 0° 16′ Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	− 0° 10′
N. by E.	N. by E. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 10
N. N. E.	N. N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 40
N. E. by N.	N. E. by N. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 0
N. E.	N. E. $\frac{1}{2}$ N.	+ $\frac{1}{2}$		+ 5 20
N. E. by E.	N. E. $\frac{1}{2}$ E.	+ $\frac{1}{2}$		+ 5 20
E. N. E.	N. E. by E. $\frac{1}{2}$ E.	+ $\frac{1}{2}$		+ 5 20
E. by N.	E. N. E. $\frac{3}{8}$ E.	+ $\frac{5}{8}$		+ 6 40
EAST.	E. $\frac{5}{8}$ N.	+ $\frac{5}{8}$		+ 6 40
E. by S.	E. $\frac{1}{4}$ S.	+ $\frac{3}{4}$		+ 8 0
E. S. E.	E. by S. $\frac{1}{2}$ S.	+ $\frac{1}{2}$		+ 5 20
S. E. by E.	E. S. E. $\frac{1}{2}$ S.	+ $\frac{1}{2}$		+ 5 20
S. E.	S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
S. E. by S.	S. E. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 2 30
S. S. E.	S. S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
S. by E.	S. by E.	0		− 0 20
SOUTH.	SOUTH.	0		− 0 20
S. by W.	S. by W. $\frac{1}{8}$ W.	− $\frac{1}{8}$		− 1 40
S. S. W.	S. S. W. $\frac{1}{4}$ W.	− $\frac{1}{4}$		− 3 10
S. W. by S.	S. W. by S. $\frac{1}{2}$ W.	− $\frac{1}{2}$		− 5 50
S. W.	S. W. $\frac{3}{8}$ W.	− $\frac{3}{8}$		− 4 30
S. W. by W.	S. W. by W. $\frac{1}{4}$ W.	− $\frac{1}{4}$		− 3 10
W. S. W.	W. S. W. $\frac{3}{8}$ W.	− $\frac{3}{8}$		− 4 30
W. by S.	W. $\frac{5}{8}$ S.	− $\frac{3}{8}$		− 4 30
WEST.	W. $\frac{3}{8}$ N.	− $\frac{3}{8}$		− 4 30
W. by N.	W. by N. $\frac{3}{8}$ N.	− $\frac{3}{8}$		− 4 30
W. N. W.	W. N. W. $\frac{3}{8}$ N.	− $\frac{3}{8}$		− 4 30
N. W. by W.	N. W. $\frac{5}{8}$ W.	− $\frac{3}{8}$		− 4 30
N. W.	N. W. $\frac{1}{4}$ N.	− $\frac{1}{4}$		− 3 0
N. W. by N.	N. W. by N. $\frac{1}{4}$ N.	− $\frac{1}{4}$		− 3 0
N. N. W.	N. N. W. $\frac{1}{4}$ N.	− $\frac{1}{4}$		− 3 0
N. by W.	N. by W.	0		− 0 20
NORTH.		0		− 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

$$A = +0° \; 14'.6 \qquad B = +5° \; 45'.5 \qquad C = +0° \; 33'.5$$
$$D = +0° \; 3'.2 \qquad E = -0° \; 48'.2$$

Assumed magnetic bearing of Nipple S. 32° 30′ E. Distant $4\frac{1}{2}$ miles.

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE ADMIRALTY STANDARD COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Isle Royal, Salute Islands, November 30, 1865.

Assumed Magnetic Bearing of Object = S. 11° 0′ W.
Correction for Object = — 0° 2′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.				
N. by E.				
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.	S. 5° 20′ W.		+ 5° 40′	+ 5° 40′
E. by S.	S. 5 40 W.		+ 5 20	+ 5 20
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = B = C =
D = E =

Ceara, December 19, 1865.

Assumed Magnetic Bearing of Object = N. 88° 45′ E.
Correction for Object = + 1° 51′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.				
N. by E.	N. 87° 40′ E.		+ 1° 5′	+ 2° 50′
N. N. E.	N. 87 0 E.		+ 1 45	+ 3 30
N. E. by N.	N. 86 0 E.		+ 2 45	+ 4 20
N. E.	N. 84 30 E.		+ 4 15	+ 5 50
N. E. by E.	N. 85 0 E.		+ 3 45	+ 5 40
E. N. E.	N. 85 0 E.		+ 3 45	+ 5 30
E. by N.	N. 85 30 E.		+ 3 15	+ 5 10
EAST.	N. 87 0 E.		+ 1 45	+ 3 50
E. by S.	N. 87 0 E.		+ 1 45	+ 3 30
E. S. E.	N. 88 0 E.		+ 0 45	+ 2 50
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 0° 34′.7 B = + 4° 46′.1 C = + 2° 18′.7
D = + 0° 49′.2 E = — 0° 14′.4

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE ADMIRALTY STANDARD COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Bahia, December 30, 1865.

Assumed Magnetic Bearing of Object = N. 76° 30′ W.
Correction for Object = + 2° 30′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 76° 0′ W.		— 0° 30′	+ 1° 40′
N. by E.	N. 77 40 W.		+ 1 10	+ 3 20
N. N. E.	N. 77 40 W.		+ 1 10	+ 3 40
N. E. by N.	N. 78 40 W.		+ 2 10	+ 4 30
N. E.	N. 78 40 W.		+ 2 10	+ 4 40
N. E. by E.	N. 79 0 W.		+ 2 30	+ 5 0
E. N. E.	N. 79 40 W.		+ 3 10	+ 5 30
E. by N.	N. 79 40 W.		+ 3 10	+ 5 40
EAST.	N. 79 10 W.		+ 2 40	+ 5 20
E. by S.	N. 79 10 W.		+ 2 40	+ 5 10
E. S. E.	N. 78 40 W.		+ 2 10	+ 4 40
S. E. by E.	N. 78 10 W.		+ 1 40	+ 4 20
S. E.	N. 77 0 W.		+ 0 30	+ 3 20
S. E. by S.	N. 76 40 W.		+ 0 10	+ 2 40
S. S. E.	N. 76 10 W.		— 0 20	+ 2 10
S. by E.	N. 76 0 W.		— 0 30	+ 2 0
SOUTH.	N. 75 40 W.		— 0 50	+ 1 40
S. by W.	N. 75 10 W.		— 1 20	+ 1 20
S. S. W.	N. 75 0 W.		— 1 30	+ 1 0
S. W. by S.	N. 74 20 W.		— 2 10	+ 0 30
S. W.	N. 74 0 W.		— 2 30	0 0
S. W. by W.	N. 73 10 W.		— 3 20	— 0 40
W. S. W.	N. 72 40 W.		— 3 50	— 1 10
W. by S.	N. 72 0 W.		— 4 30	— 1 50
WEST.	N. 72 0 W.		— 4 30	— 2 0
W. by N.	N. 71 40 W.		— 4 50	— 2 10
W. N. W.	N. 72 0 W.		— 4 30	— 2 0
N. W. by W.	N. 72 0 W.		— 4 30	— 2 0
N. W.	N. 72 0 W.		— 4 30	— 2 0
N. W. by N.	N. 73 0 W.		— 3 30	— 1 10
N. N. W.	N. 74 0 W.		— 2 30	— 0 10
N. by W.	N. 74 40 W.		— 1 50	+ 0 30
NORTH.				+ 1 40

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 40′.2 B = + 3° 38′.5 C = + 0° 0′.4
D = + 0° 47′.8 E = 0° 0′.0

Rio Janeiro, January 10, 1866.

Assumed Magnetic Bearing of Object = N. 53° 30′ W.
Correction for Object = + 2° 44′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.				
N. by E.				
N. N. E.				
N. E. by N.	N. 56° 0′ W.		+ 2° 30′	+ 4° 50′
N. E.	N. 57 0 W.		+ 3 30	+ 6 0
N. E. by E.	N. 57 20 W.		+ 3 50	+ 6 30
E. N. E.	N. 56 20 W.		+ 2 50	+ 5 50
E. by N.	N. 56 20 W.		+ 2 50	+ 5 30
EAST.	N. 56 0 W.		+ 2 30	+ 5 20
E. by S.	N. 55 40 W.		+ 2 10	+ 5 0
E. S. E.	N. 55 20 W.		+ 1 50	+ 4 40
S. E. by E.	N. 55 0 W.		+ 1 30	+ 4 20
S. E.	N. 54 40 W.		+ 1 10	+ 4 0
S. E. by S.	N. 54 0 W.		+ 0 30	+ 3 20
S. S. E.	N. 54 20 W.		+ 0 50	+ 3 30
S. by E.	N. 53 20 W.		— 0 10	+ 2 50
SOUTH.	N. 53 20 W.		— 0 10	+ 2 30
S. by W.	N. 53 0 W.		— 0 30	+ 2 20
S. S. W.	N. 52 40 W.		— 0 50	+ 2 0
S. W. by S.	N. 52 0 W.		— 1 30	+ 1 30
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 2° 35′.7 B = + 2° 58′.5 C = + 0° 0′.2
D = + 0° 53′.5 E = — 0° 3′.1

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE ADMIRALTY STANDARD COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Monte Video, January 24, 1866.

Assumed Magnetic Bearing of Object = N. 87° 0′ W.
Correction for Object = — 0° 13′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.				(+ 2° 20′)
N. by E.	S. 89° 10′ W.		+ 3° 50′	+ 3 40
N. N. E.	S. 89 20 W.		+ 3 40	+ 3 30
N. E. by N	S. 88 40 W.		+ 4 20	+ 4 10
N. E.	S. 88 20 W.		+ 4 40	+ 4 30
N. E. by E.	S. 87 20 W.		+ 5 40	+ 5 30
E. N. E.	S. 87 40 W.		+ 5 20	+ 5 10
E. by N.	S. 88 30 W.		+ 4 30	+ 4 20
EAST.	S. 88 0 W.		+ 5 0	+ 4 50
E. by S.	S. 88 40 W.		+ 4 20	+ 4 00
E. S. E.	S. 89 10 W.		+ 3 50	+ 3 30
S. E. by E.	N. 89 40 W.		+ 2 40	+ 2 30
S. E.	N. 90 0 W.		+ 3 0	+ 2 50
S. E. by S.	N. 89 40 W.		+ 2 40	+ 2 20
S. S. E.	N. 89 0 W.		+ 2 0	+ 1 40
S. by E.	N. 89 0 W.		+ 2 0	+ 1 40
SOUTH.	N. 88 20 W.		+ 1 20	+ 1 0
S. by W.	N. 88 20 W.		+ 1 20	+ 1 0
S. S. W.	N. 88 0 W.		+ 1 0	+ 0 50
S. W. by S.	S. W. by S. $\frac{1}{4}$ S.	+ $\frac{1}{4}$	+ 2 49	+ 2 30
S. W.	S. W. $\frac{1}{8}$ S.	+ $\frac{1}{8}$	+ 1 24	+ 1 0
S. W. by W.	S. W. by W.	0	0 0	— 0 20
W. S. W.	W. S. W.	0	0 0	— 0 20
W. by S.	W. by S.	0	0 0	— 0 20
WEST.	W. $\frac{1}{8}$ N.	— $\frac{1}{8}$	— 1 24	— 1 40
W. by N.	W. by N. $\frac{1}{4}$ N.	— $\frac{1}{4}$	— 2 49	— 3 10
W. N. W.	W. N. W. $\frac{1}{4}$ N.	— $\frac{1}{4}$	— 2 49	— 3 10
N. W. by W.	N. W. $\frac{3}{4}$ W.	— $\frac{1}{4}$	— 2 49	— 3 0
N. W.	N. W. $\frac{1}{8}$ N.	— $\frac{1}{8}$	— 1 24	— 1 40
N. W. by N.	N. 87° 0′ W.		0 0	— 0 20
N. N. W.	N. 87 0 W.		0 0	— 0 10
N. by W.	N. 88 0 W.		+ 1 0	+ 0 50
NORTH.				(+ 2 20)

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 32′.8   B = + 3° 4′.8   C = + 0° 5′.8
D = + 1° 19′.5   E = + 0° 14′.5

Sandy Point, February 10, 1866.

Assumed Magnetic Bearing of Object = S. 14° 45′ E.
Correction for Object = + 0° 7′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	S. 15° 0′ E.		+ 0° 15′	+ 0° 30′
N. by E.	S. 15 0 E.		+ 0 15	+ 0 30
N. N. E.	S. 15 40 E.		+ 0 55	+ 1 10
N. E. by N.	S. 16 0 E.		+ 1 15	+ 1 30
N. E.	S. 16 0 E.		+ 1 15	+ 1 30
N. E. by E.	S. 16 40 E.		+ 1 55	+ 2 10
E. N. E.	S. 17 0 E.		+ 2 15	+ 2 30
E. by N.	S. 16 40 E.		+ 1 55	+ 2 10
EAST.	S. 15 50 E.		+ 1 5	+ 1 20
E. by S.	S. 16 20 E.		+ 1 35	+ 1 50
E. S. E.	S. 16 0 E.		+ 1 15	+ 1 30
S. E. by E.	S. 16 20 E.		+ 1 35	+ 1 50
S. E.	S. 16 0 E.		+ 1 15	+ 1 30
S. E. by S.	S. 15 20 E.		+ 0 35	+ 0 50
S. S. E.	S. 15 40 E.		+ 0 55	+ 1 10
S. by E.	S. 15 20 E.		+ 0 35	+ 0 50
SOUTH.	S. 15 20 E.		+ 0 35	+ 0 50
S. by W.	S. 16 0 E.		+ 1 15	+ 1 30
S. S. W.	S. 16 0 E.		+ 1 15	+ 1 30
S. W. by S.	S. 15 50 E.		+ 1 5	+ 1 20
S. W.	S. 15 40 E.		+ 0 55	+ 1 10
S. W. by W.	S. 15 30 E.		+ 0 45	+ 1 0
W. S. W.	S. 14 40 E.		— 0 5	+ 0 10
W. by S.	S. 14 0 E.		— 0 45	— 0 30
WEST.	S. 14 10 E.		— 0 35	— 0 20
W. by N.	S. 13 0 E.		— 1 45	— 1 30
W. N. W.	S. 12 50 E.		— 1 55	— 1 40
N. W. by W.	S. 12 40 E.		— 2 5	— 1 50
N. W.	S. 12 40 E.		— 2 5	— 1 50
N. W. by N.	S. 13 0 E.		— 1 45	— 1 30
N. N. W.	S. 13 20 E.		— 1 25	— 1 10
N. by W.	S. 14 0 E.		— 0 45	— 0 30
NORTH.	S. 15 0 E.		+ 0 15	+ 0 30

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 35′.9   B = + 1° 20′.6   C = — 0° 40′.6
D = + 0° 53′.5   E = + 0° 1′.5

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE ADMIRALTY STANDARD COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Valparaiso, April 4, 1866.

Assumed Magnetic Bearing of Object = N. 15° 15′ E.
Correction for Object = + 0° 1′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass
NORTH.	N. 15° 30′ E.		— 0° 15′	— 0° 10′
N. by E.	N. 14 0 E.		+ 1 15	+ 1 20
N. N. E.	N. 14 0 E.		+ 1 15	+ 1 20
N. E. by N.	N. 13 30 E.		+ 1 45	+ 1 50
N. E.	N. 13 0 E.		+ 2 15	+ 2 20
N. E. by E.	N. 13 0 E.		+ 2 15	+ 2 20
E. N. E.	N. 12 40 E.		+ 2 35	+ 2 40
E. by N.	N. 12 40 E.		+ 2 35	+ 2 40
EAST.	N. 13 0 E.		+ 2 15	+ 2 20
E. by S.	N. 14 0 E.		+ 1 15	+ 1 20
E. S. E.	N. 13 40 E.		+ 1 35	+ 1 40
S. E. by E.	N. 14 20 E.		+ 0 55	+ 1 0
S. E.	N. 14 40 E.		+ 0 35	+ 0 40
S. E. by S.	N. 15 0 E.		+ 0 15	+ 0 20
S. S. E.	N. 15 0 E.		+ 0 15	+ 0 20
S. by E.	N. 15 0 E.		+ 0 15	+ 0 20
SOUTH.	N. 14 40 E.		+ 0 35	+ 0 40
S. by W.	N. 14 20 E.		+ 0 55	+ 1 0
S. S. W.	N. 14 40 E.		+ 0 35	+ 0 40
S. W. by S.	N. 14 40 E.		+ 0 35	+ 0 40
S. W.	N. 15 0 E.		+ 0 15	+ 0 20
S. W. by W.	N. 14 30 E.		+ 0 45	+ 0 50
W. S. W.	N. 15 20 E.		— 0 5	0 0
W. by S.	N. 15 40 E.		— 0 25	— 0 20
WEST.	N. 15 30 E.		— 0 15	— 0 10
W. by N.	N. 16 30 E.		— 1 15	— 1 10
W. N. W.	N. 16 40 E.		— 1 25	— 1 20
N. W. by W.	N. 16 40 E.		— 1 25	— 1 20
N. W.	N. 17 0 E.		— 1 45	— 1 40
N. W. by N.	N. 16 30 E.		— 1 15	— 1 10
N. N. W.	N. 15 20 E.		— 0 5	— 0 0
N. by W.	N. 15 40 E.		— 0 25	— 0 20
NORTH.	N. 15 30 E.		— 0 15	— 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 35′.6 B = + 1° 20′.2 C = — 0° 6′.9
D = + 0° 54′.2 E = — 0° 10′.2

Callao, April 29, 1866.

Assumed Magnetic Bearing of Object = S. 72° 45′ W.
Correction for Object = + 0° 6′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	S. 72° 40′ W.		+ 0° 5′	+ 0° 10′
N. by E.	S. 72 40 W.		+ 0 5	+ 0 10
N. N. E.	S. 71 40 W.		+ 1 5	+ 1 10
N. E. by N.	S. 71 0 W.		+ 1 45	+ 1 50
N. E.	S. 70 20 W.		+ 2 25	+ 2 30
N. E. by E.	S. 70 20 W.		+ 2 25	+ 2 30
E. N. E.	S. 70 0 W.		+ 2 45	+ 2 50
E. by N.	S. 70 30 W.		+ 2 15	+ 2 20
EAST.	S. 71 0 W.		+ 1 45	+ 1 50
E. by S.	S. 70 40 W.		+ 2 5	+ 2 10
E. S. E.	S. 71 40 W.		+ 1 5	+ 1 10
S. E. by E.	S. 71 40 W.		+ 1 5	+ 1 10
S. E.	S. 72 0 W.		+ 0 45	+ 0 50
S. E. by S.	S. 72 40 W.		+ 0 5	+ 0 10
S. S. E.	S. 72 40 W.		+ 0 5	+ 0 10
S. by E.	S. 73 0 W.		— 0 15	— 0 10
SOUTH.	S. 73 0 W.		— 0 15	— 0 10
S. by W.	S. 73 0 W.		— 0 15	— 0 10
S. S. W.	S. 73 0 W.		— 0 15	— 0 10
S. W. by S.	S. 73 0 W.		— 0 15	— 0 10
S. W.	S. 74 0 W.		— 1 15	— 1 10
S. W. by W.	S. 74 0 W.		— 1 15	— 1 10
W. S. W.	S. 74 0 W.		— 1 15	— 1 10
W. by S.	S. 75 30 W.		— 2 45	— 2 40
WEST.	S. 76 0 W.		— 3 15	— 3 10
W. by N.	S. 75 40 W.		— 2 55	— 2 50
W. N. W.	S. 76 0 W.		— 3 15	— 3 10
N. W. by W.	S. 75 30 W.		— 2 45	— 2 40
N. W.	S. 75 40 W.		— 2 55	— 2 50
N. W. by N.	S. 75 0 W.		— 2 15	— 2 10
N. N. W.	S. 74 20 W.		— 1 35	— 1 30
N. by W.	S. 73 20 W.		— 0 35	— 0 30
NORTH.	S. 72 40 W.		+ 0 5	+ 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 9′.1 B = + 2° 21′.1 C = — 0° 1′.8
D = + 0° 52′.5 E = + 0° 5′.8

Observations for Determining the Deviations of the Admiralty Standard Compass on the U. S. Iron Clad Monadnock.

Panama, May 20, 1866.

Assumed Magnetic Bearing of Object = S. 15° 0′ W.
Correction for Object = + 0° 1′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	S. 14° 0′ W.		+ 1° 0′	+ 1° 0′
N. by E.	S. 13 30 W.		+ 1 30	+ 1 30
N. N. E.	S. 12 40 W.		+ 2 20	+ 2 20
N. E. by N.	S. 12 20 W.		+ 2 40	+ 2 40
N. E.	S. 11 20 W.		+ 3 40	+ 3 40
N. E. by E.	S. 11 20 W.		+ 3 40	+ 3 40
E. N. E.	S. 11 0 W.		+ 4 0	+ 4 0
E. by N.	S. 11 0 W.		+ 4 0	+ 4 0
EAST.	S. 11 20 W.		+ 3 40	+ 3 40
E. by S.	S. 12 0 W.		+ 3 0	+ 3 0
E. S. E.	S. 12 0 W.		+ 3 0	+ 3 0
S. E. by E.	S. 13 0 W.		+ 2 0	+ 2 0
S. E.	S. 13 30 W.		+ 1 30	+ 1 30
S. E. by S.	S. 14 0 W.		+ 1 0	+ 1 0
S. S. E.	S. 14 0 W.		+ 1 0	+ 1 0
S. by E.	S. 14 20 W.		+ 0 40	+ 0 40
SOUTH.	S. 14 20 W.		+ 0 40	+ 0 40
S. by W.	S. 14 20 W.		+ 0 40	+ 0 40
S. S. W.	S. 14 40 W.		+ 0 20	+ 0 20
S. W. by S.	S. 15 0 W.		0 0	0 0
S. W.	S. 15 40 W.		— 0 40	— 0 40
S. W. by W.	S. 16 20 W.		— 1 20	— 1 20
W. S. W.	S. 16 40 W.		— 1 40	— 1 40
W. by S.	S. 17 50 W.		— 2 50	— 2 50
WEST.	S. 18 0 W.		— 3 0	— 3 0
W. by N.	S. 17 30 W.		— 2 30	— 2 30
W. N. W.	S. 18 0 W.		— 3 0	— 3 0
N. W. by W.	S. 18 0 W.		— 3 0	— 3 0
N. W.	S. 17 10 W.		— 2 10	— 2 10
N. W. by N.	S. 17 0 W.		— 2 0	— 2 0
N. N. W.	S. 16 0 W.		— 1 0	— 1 0
N. by W.	S. 15 20 W.		— 0 20	— 0 20
NORTH.	S. 14 0 W.		+ 1 0	+ 1 0

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 31′.6 B = + 3° 2′.1 C = + 0° 1′.9
D = + 0° 55′.0 E = + 0° 8′.0

Acapulco, June 1, 1866.

Assumed Magnetic Bearing of Object = N. 63° 15′ E.
Correction for Object = + 0° 6′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 63° 40′ E.		— 0° 25′	— 0° 20′
N. by E.	N. 63 0 E.		+ 0 15	+ 0 20
N. N. E.	N. 62 10 E.		+ 1 5	+ 1 10
N. E. by N.	N. 61 30 E.		+ 1 45	+ 1 50
N. E.	N. 61 0 E.		+ 2 15	+ 2 20
N. E. by E.	N. 60 50 E.		+ 2 25	+ 2 30
E. N. E.	N. 60 40 E.		+ 2 35	+ 2 40
E. by N.	N. 60 50 E.		+ 2 25	+ 2 30
EAST.	N. 61 20 E.		+ 1 55	+ 2 0
E. by S.	N. 61 50 E.		+ 1 25	+ 1 30
E. S. E.	N. 62 0 E.		+ 1 15	+ 1 20
S. E. by E.	N. 62 20 E.		+ 0 55	+ 1 0
S. E.	N. 63 20 E.		— 0 5	0 0
S. E. by S.	N. 63 30 E.		— 0 15	— 0 10
S. S. E.	N. 63 40 E.		— 0 25	— 0 20
S. by E.	N. 64 0 E.		— 0 45	— 0 40
SOUTH.	N. 63 50 E.		— 0 35	— 0 30
S. by W.	N. 63 50 E.		— 0 35	— 0 30
S. S. W.	N. 64 10 E.		— 0 55	— 0 50
S. W. by S.	N. 64 30 E.		— 1 15	— 1 10
S. W.	N. 65 0 E.		— 1 45	— 1 40
S. W. by W.	N. 65 30 E.		— 2 15	— 2 10
W. S. W.	N. 66 0 E.		— 2 45	— 2 40
W. by S.	N. 66 40 E.		— 3 25	— 3 20
WEST.	N. 67 0 E.		— 3 45	— 3 40
W. by N.	N. 67 0 E.		— 3 45	— 3 40
W. N. W.	N. 67 20 E.		— 4 5	— 4 0
N. W. by W.	N. 67 0 E.		— 3 45	— 3 40
N. W.	N. 66 40 E.		— 3 25	— 3 20
N. W. by N.	N. 66 10 E.		— 2 55	— 2 50
N. N. W.	N. 65 20 E.		— 2 5	— 2 0
N. by W.	N. 64 40 E.		— 1 25	— 1 20
NORTH.	N. 63 40 E.		— 0 25	— 0 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 0° 36′.9 B = + 2° 45′.4 C = + 0° 5′.5
D = + 0° 56′.8 E = + 0° 8′.0

Observations for Determining the Deviations of the Admiralty Standard Compass on the U. S. Iron Clad Monadnock.

Magdalena Bay, June 9, 1866.

Assumed Magnetic Bearing of Object = S. 56° 30′ E.
Correction for Object = — 0° 41′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	S. 56° 30′ E.		0° 0′	— 0° 40′
N. by E.	S. 56 40 E.		+ 0 10	— 0 30
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.				
E. by S.				
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.	S. 57 0 E.		+ 0 30	— 0 20
S. W. by W.	S. 56 20 E.		— 0 10	— 1 0
W. S. W.	S. 55 30 E.		— 1 0	— 1 50
W. by S.	S. 55 0 E.		— 1 30	— 2 20
WEST.	S. 54 20 E.		— 2 10	— 3 0
W. by N.	S. 53 20 E.		— 3 10	— 3 50
W. N. W.	S. 53 20 E.		— 3 10	— 3 50
N. W. by W.	S. 53 0 E.		— 3 30	— 4 10
N. W.	S. 53 30 E.		— 3 0	— 3 40
N. W. by N.	S. 53 30 E.		— 3 0	— 3 40
N. N. W.	S. 54 30 E.		— 2 0	— 2 40
N. by W.	S. 55 30 E.		— 1 0	— 1 40
NORTH.	S. 56 30 E.		0 0	— 0 40

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 9′.0 B = + 3° 12′.1 C = — 1° 10′.3
D = + 0° 53′.5 E = + 0′.8

San Francisco, June 23, 1866.

Assumed Magnetic Bearing of Object = N. 29° 30′ W.
Correction for Object = — 0° 45′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 28° 20′ W.		— 1° 10′	— 1° 50′
N. by E.	N. 29 30 W.		0 0	— 0 40
N. N. E.	N. 31 0 W.		+ 1 30	+ 0 50
N. E. by N.	N. 32 0 W.		+ 2 30	+ 1 50
N. E.	N. 33 0 W.		+ 3 30	+ 2 50
N. E. by E.	N. 33 30 W.		+ 4 0	+ 3 20
E. N. E.	N. 34 30 W.		+ 5 0	+ 4 10
E. by N.	N. 34 30 W.		+ 5 0	+ 4 10
EAST.	N. 34 15 W.		+ 4 45	+ 4 0
E. by S.	N. 33 30 W.		+ 4 0	+ 3 20
E. S. E.	N. 34 0 W.		+ 4 30	+ 3 40
S. E. by E.	N. 33 30 W.		+ 4 0	+ 3 20
S. E.	N. 33 30 W.		+ 4 0	+ 3 10
S. E. by S.	N. 33 0 W.		+ 3 30	+ 2 40
S. S. E.	N. 32 20 W.		+ 2 50	+ 2 0
S. by E.	N. 31 30 W.		+ 2 0	+ 1 10
SOUTH.	N. 31 0 W.		+ 1 30	+ 0 40
S. by W.	N. 30 0 W.		+ 0 30	— 0 20
S. S. W.	N. 29 40 W.		+ 0 10	— 0 40
S. W. by S.	N. 28 40 W.		— 0 50	— 1 40
S. W.	N. 28 0 W.		— 1 30	— 2 20
S. W. by W.	N. 27 0 W.		— 2 30	— 3 20
W. S. W.	N. 26 20 W.		— 3 10	— 4 0
W. by S.	N. 25 20 W.		— 4 10	— 5 0
WEST.	N. 25 0 W.		— 4 30	— 5 20
W. by N.	N. 24 30 W.		— 5 0	— 5 50
W. N. W.	N. 24 0 W.		— 5 30	— 6 20
N. W. by W.	N. 23 30 W.		— 6 0	— 6 50
N. W.	N. 24 0 W.		— 5 30	— 6 10
N. W. by N.	N. 25 15 W.		— 4 15	— 5 0
N. N. W.	N. 26 0 W.		— 3 30	— 4 10
N. by W.	N. 27 30 W.		— 2 0	— 2 50
NORTH.	N. 28 20 W.		— 1 10	— 1 50

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 0° 39′.6 B = + 4° 53′.2 C = — 1° 15′.4
D = + 0° 51′.2 E = + 0° 5′.8

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER BINNACLE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Hampton Roads, November 1, 1865.

Correction for Object = + 3° 57′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	
NORTH.	N. $\frac{3}{8}$ E.	− $\frac{3}{8}$		− 0° 50′
N. by E.	N. by E. $\frac{1}{8}$ E.	− $\frac{1}{8}$		+ 1 30
N. N. E.	N. N. E.	0		+ 3 30
N. E. by N.	N. E. by N. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 4 50
N. E.	N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 6 20
N. E. by E.	N. E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 6 40
E. N. E.	N. E. by E. $\frac{5}{8}$ E.	+ $\frac{3}{8}$		+ 7 40
E. by N.	E. by N. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 8 10
EAST.	E. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 8 10
E. by S.	E. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 7 10
E. S. E.	E. by S. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 6 40
S. E. by E.	S. E. by E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 5 50
S. E.	S. E.	0		+ 4 30
S. E. by S.	S. E. by S.	0		+ 4 0
S. S. E.	S. by E. $\frac{7}{8}$ E.	− $\frac{1}{8}$		+ 3 0
S. by E.	S. $\frac{3}{4}$ E.	− $\frac{1}{4}$		+ 1 40
SOUTH.	S. $\frac{1}{8}$ W.	− $\frac{1}{8}$		+ 2 0
S. by W.	S. by W. $\frac{1}{4}$ W.	− $\frac{1}{4}$		+ 1 40
S. S. W.	S. S. W. $\frac{3}{8}$ W.	− $\frac{3}{8}$		+ 0 10
S. W. by S.	S. W. $\frac{1}{2}$ S.	− $\frac{1}{2}$		− 1 10
S. W.	S. W. $\frac{1}{2}$ W.	− $\frac{1}{2}$		− 1 40
S. W. by W.	S. W. by W. $\frac{3}{4}$ W.	− $\frac{3}{4}$		− 3 30
W. S. W.	W. by S. $\frac{1}{4}$ S.	− $\frac{3}{4}$		− 4 30
W. by S.	W.	− 1		− 6 20
WEST.	W. by N.	− 1		− 7 20
W. by N.	W. N. W.	− 1		− 7 20
W. N. W.	N. W. $\frac{7}{8}$ W.	− 1$\frac{1}{8}$		− 8 20
N. W. by W.	N. W.	− 1		− 7 50
N. W.	N. W. by N.	− 1		− 7 20
N. W. by N.	N. N. W. $\frac{1}{4}$ W.	− $\frac{3}{4}$		− 5 30
N. N. W.	N. by W. $\frac{1}{4}$ W.	− $\frac{3}{4}$		− 4 30
N. by W.	N. $\frac{1}{2}$ W.	− $\frac{1}{2}$		− 2 40
NORTH.		− $\frac{3}{8}$		− 0 50

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

$A = +0° \ 27'.5 \quad B = +7° \ 16'.8 \quad C = -1° \ 14'.1$

$D = +1° \ 39'.2 \quad E = +0° \ 6'.2$

St. Thomas, November 18, 1865.

Correction for Object = + 0° 16′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	° ′
NORTH.	NORTH.	0		
N. by E.	N. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		
N. N. E.	N. N. E. $\frac{1}{4}$ E.	− $\frac{1}{4}$		
N. E. by N.	N. E. by N.	0		
N. E.	N. E. $\frac{1}{2}$ E.	− $\frac{1}{2}$		
N. E. by E.	N. E. by E. $\frac{1}{4}$ E.	− $\frac{1}{4}$		
E. N. E.	E. N. E.	0		
E. by N.	E. by N.	0		
EAST.	E. $\frac{1}{2}$ N.	+ $\frac{1}{2}$		
E. by S.	E. $\frac{1}{2}$ S.	+ $\frac{1}{2}$		
E. S. E.	E. by S. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		
S. E. by E.	S. E. by E.	0		
S. E.	S. E. $\frac{3}{4}$ S.	− $\frac{3}{4}$		
S. E. by S.	S. E. by S. $\frac{1}{2}$ S.	− $\frac{1}{2}$		
S. S. E.	S. S. E. $\frac{1}{4}$ S.	− $\frac{1}{4}$		
S. by E.	S. by E.	0		
SOUTH.	S. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		
S. by W.	S. by W.	0		
S. S. W.	S. S. W. $\frac{1}{4}$ W.	− $\frac{1}{4}$		
S. W. by S.	S. W. by S. $\frac{1}{4}$ W.	− $\frac{1}{4}$		
S. W.	S. W. $\frac{1}{4}$ W.	− $\frac{1}{4}$		
S. W. by W.	S. W. by W. $\frac{1}{2}$ W.	− $\frac{1}{2}$		
W. S. W.	W. S. W. $\frac{1}{2}$ W.	− $\frac{1}{2}$		
W. by S.	W. by S. $\frac{1}{2}$ W.	− $\frac{1}{2}$		
WEST.	W. $\frac{3}{4}$ N.	− $\frac{3}{4}$		
W. by N.	W. by N. $\frac{1}{2}$ N.	− $\frac{1}{2}$		
W. N. W.	W. N. W.	0		
N. W. by W.	N. W. $\frac{1}{2}$ W.	− $\frac{1}{2}$		
N. W.	N. W. $\frac{1}{2}$ N.	− $\frac{1}{2}$		
N. W. by N.	N. W. by N. $\frac{1}{2}$ N.	− $\frac{1}{2}$		
N. N. W.	N. N. W. $\frac{1}{4}$ N.	− $\frac{1}{4}$		
N. by W.	N. $\frac{3}{4}$ W.	− $\frac{1}{4}$		
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

The officer who usually read this compass was on shore when the ship was swung. He was replaced by another who made the above observations, which, however, are evidently worthless. No use has been made of them.

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER BINNACLE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Isle Royal, Salute Islands, November 30, 1865.

Correction for Object = − 0° 2′. Correction for Lubber Line = + 6° 18′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.			° ′	
N. by E.				
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.	EAST.	0		+ 6° 20′
E. by S.	E. by S.	0		+ 6 20
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign −.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = B = C =
D = E =

Ceara, December 19, 1865.

Correction for Object = + 1° 51′. Correction for Lubber Line = + 6° 18′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.			° ′	
N. by E.	N. by E. $\frac{3}{8}$ E.	− $\frac{3}{8}$		+ 3° 40′
N. N. E.	N. E. by N. $\frac{3}{4}$ N.	− $\frac{1}{4}$		+ 5 10
N. E. by N.	N. E. $\frac{7}{8}$ N.	− $\frac{1}{8}$		+ 6 30
N. E.	N. E.	0		+ 7 50
N. E. by E.	N. E. by E. $\frac{1}{8}$ E.	− $\frac{1}{8}$		+ 7 0
E. N. E.	E. by N. $\frac{7}{8}$ N.	− $\frac{1}{8}$		+ 6 50
E. by N.	E. $\frac{7}{8}$ N.	− $\frac{1}{8}$		+ 6 50
EAST.	E. $\frac{3}{8}$ S.	− $\frac{3}{8}$		+ 4 20
E. by S.	E. by S. $\frac{3}{8}$ S.	− $\frac{3}{8}$		+ 4 0
E. S. E.	S. E. by E. $\frac{1}{2}$ E.	− $\frac{1}{2}$		+ 2 50
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign −.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 21′.5 B = + 4° 34′.9 C = + 2° 4′.8
D = + 2° 3′.4 E = − 0° 16′.5

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER BINNACLE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Bahia, December 30, 1865.

Correction for Object = + 2° 30′. Correction for Lubber Line = + 6° 18′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ½ E.	— ½	° ′	+ 2° 30′
N. by E.	N. by E. ½ E.	— ½		+ 3 10
N. N. E.	N. N. E. ¼ E.	— ¼		+ 5 20
N. E. by N.	N. E. ¾ N.	— ¼		+ 6 0
N. E.	N. E. ¼ E.	— ¼		+ 6 0
N. E. by E.	N. E. by E. ¼ E.	— ¼		+ 6 0
E. N. E.	E. N. E.	0		+ 8 10
E. by N.	E. by N.	0		+ 8 50
EAST.	E. ⅛ S.	— ⅛		+ 7 40
E. by S.	E. by S. ¼ S.	— ¼		+ 6 20
E. S. E.	S. E. by E. ¾ E.	— ¼		+ 6 0
S. E. by E.	S. E. ⅝ E.	— ⅜		+ 4 50
S. E.	S. E. ½ S.	— ½		+ 3 30
S. E. by S.	S. S. E. ½ E.	— ½		+ 3 10
S. S. E.	S. by E. ⅜ E.	— ⅝		+ 2 0
S. by E.	S. ⅜ E.	— ⅝		+ 1 50
SOUTH.	S. ⅝ W.	— ⅝		+ 1 50
S. by W.	S. by W. ⅝ W.	— ⅝		+ 1 50
S. S. W.	S. S. W. ¾ W.	— ¾		+ 0 40
S. W. by S.	S. W. ¼ S.	— ¾		+ 0 20
S. W.	S. W. ⅞ W.	— ⅞		— 0 40
S. W. by W.	W. S. W.	— 1		— 2 10
W. S. W.	W. by S.	— 1		— 2 30
W. by S.	W. ⅛ N.	— 1⅛		— 3 30
WEST.	W. by N. ⅛ N.	— 1⅛		— 3 50
W. by N.	N. W. by W. ¾ W.	— 1¼		— 5 0
W. N. W.	N. W. ¾ W.	— 1¼		— 5 20
N. W. by W.	N. W. ¼ N.	— 1¼		— 5 20
N. W.	N. N. W. ¾ W.	— 1¼		— 5 20
N. W. by N.	N. N. W.	— 1		— 3 10
N. N. W.	N. by W. ⅛ W.	— ⅞		— 1 20
N. by W.	N. ¼ W.	— ¾		0 0
NORTH.		— ½		+ 2 30

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 29′.8 B = + 5° 43′.6 C = — 0° 6′.9
D = + 1° 41′.5 E = + 0° 7′.8

Rio Janeiro, January 10, 1866.

Correction for Object = + 2° 44′ Correction for Lubber Line = + 5° 43′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.			° ′	
N. by E.				
N. N. E.				
N. E. by N.	N. E. ¾ N.	— ¼		+ 5° 20′
N. E.	N. E. ⅛ E.	— ⅛		+ 6 50
N. E. by E.	N. E. by E. ⅛ E.	— ⅛		+ 7 10
E. N. E.	E. by N. ⅞ N.	— ⅛		+ 7 10
E. by N.	E. ¾ N.	— ¼		+ 6 0
EAST.	E. ¼ S.	— ¼		+ 5 40
E. by S.	E. by S. ¼ S.	— ¼		+ 5 40
E. S. E.	S. E. by E. ⅝ E.	— ⅜		+ 4 40
S. E. by E.	S. E. ⅝ E.	— ⅜		+ 4 20
S. E.	S. E. ½ S.	— ½		+ 3 10
S. E. by S.	S. S. E. ½ E.	— ½		+ 2 50
S. S. E.	S. by E. ⅜ E	— ⅝		+ 1 50
S. by E.	S. ⅜ E.	— ⅝		+ 1 30
SOUTH.	S. ⅝ W.	— ⅝		+ 1 30
S. by W.	S. by W. ¾ W.	— ¾		+ 0 20
S. S. W.	S. S. W. ¾ W.	— ¾		+ 0 10
S. W. by S.	S. W. ¼ S.	— ¾		+ 0 10
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 51′.4 B = + 5° 24′.9 C = — 0° 24′.8
D = + 1° 56′.7 E = — 0° 4′.2

Observations for Determining the Deviations of the After Binnacle Compass on the U. S. Iron Clad Monadnock.

Monte Video, January 24, 1866.

Correction for Object = — 0° 13′. Correction for Lubber Line = + 4° 9′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N ⅜ E.	— ⅜	° ′	— 0° 20′
N. by E.	N. by E.	0		+ 4 0
N. N. E.	N. by E. ⅞ E.	+ ⅛		+ 5 20
N. E. by N.	N. E. by N. ¼ N.	+ ¼		+ 6 40
N. E.	N. E. ¼ N.	+ ¼		+ 6 40
N. E. by E.	N. E. ⅝ E.	+ ⅜		+ 8 10
E. N. E.	N. E. by E. ⅝ E.	+ ⅜		+ 8 10
E. by N.	E. by N. ⅜ N.	+ ⅜		+ 8 0
EAST.	E. ¼ N.	+ ¼		+ 6 40
E. by S.	E. ¾ S.	+ ¼		+ 6 40
E. S. E.	E. by S. ⅞ S.	+ ⅛		+ 5 20
S. E. by E.	S. E. by E.	0		+ 4 0
S. E.	S. E. ⅛ S.	— ⅛		+ 2 30
S. E. by S.	S. E. by S. ¼ S.	— ¼		+ 1 0
S. S. E.	S. by E. ¾ E.	— ¼		+ 1 0
S. by E.	S. ⅝ E.	— ⅜		— 0 20
SOUTH.	S. ⅜ W.	— ⅜		— 0 20
S. by W.	S. by W. ⅜ W.	— ⅜		— 0 20
S. S. W.	S. S. W. ½ W.	— ½		— 1 40
S. W. by S.	S. W. ⅝ S.	— ⅜		— 0 20
S. W.	S. W. ½ W.	— ½		— 1 40
S. W. by W.	S. W. by W. ½ W.	— ½		— 1 40
W. S. W.	W. by S. ½ S.	— ½		— 1 40
W. by S.	W. ⅜ S.	— ⅝		— 3 20
WEST.	W. ¾ N.	— ¾		— 4 30
W. by N.	W. by N. ¾ N.	— ¾		— 4 30
W. N. W.	N. W. by W. ¼ W.	— ¾		— 4 30
N. W. by W.	N. W. ¼ W.	— ¾		— 4 30
N. W.	N. W. ¾ N.	— ¾		— 4 30
N. W. by N.	N. N. W. ⅜ W.	— ⅝		— 3 0
N. N. W.	N. by W. ½ W.	— ½		— 1 40
N. by W.	N. ½ W.	— ½		— 1 40
NORTH.		— ⅜		— 0 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 3′.1 B = + 5° 30′.6 C = + 0° 41′.9
D = + 1° 57′.5 E = — 0° 42′.5

Sandy Point, February 10, 1866.

Correction for Object = + 0° 7′. Correction for Lubber Line = + 4° 9′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ½ E.	— ½	° ′	— 1° 20′
N. E.	N. by E. ⅛ E.	— ⅛		+ 2 50
N. N. E.	N. N. E. ⅛ E.	— ⅛		+ 2 50
N. E. by N.	N. E. by N.	0		+ 4 20
N. E.	N. E. ⅛ N.	+ ⅛		+ 5 40
N. E. by E.	N. E. ⅞ E.	+ ⅛		+ 5 40
E. N. E.	N. E. by E. ⅞ E.	+ ⅛		+ 5 40
E. by N.	E. by N. ⅛ N.	+ ⅛		+ 5 40
EAST.	E. ⅛ N.	+ ⅛		+ 5 40
E. by S.	E. by S.	0		+ 4 20
E. S. E.	E. S. E.	0		+ 4 20
S. E. by E.	S. E. ⅞ E.	— ⅛		+ 2 50
S. E.	S. E. ¼ S.	— ¼		+ 1 30
S. E. by S.	S. E. by S. ¼ S.	— ¼		+ 1 30
S. S. E.	S. by E. ⅝ E.	— ⅜		0 0
S. by E.	S. ⅝ E.	— ⅜		0 0
SOUTH.	S. ⅜ W.	— ⅜		0 0
S. by W.	S. by W. ⅜ W.	— ⅜		0 0
S. S. W.	S. S. W. ½ W.	— ½		— 1 20
S. W. by S.	S. W. ½ S.	— ½		— 1 20
S. W.	S. W. ⅝ W.	— ⅝		— 2 50
S. W. by W.	S. W. by W. ⅝ W.	— ⅝		— 2 50
W. S. W.	W. by S. ¼ S.	— ¾		— 4 10
W. by S.	W. ⅛ S.	— ⅞		— 5 40
WEST.	W. ⅞ N.	— ⅞		— 5 40
W. by N.	W. N. W.	— 1		— 7 0
W. N. W.	N. W. by W.	— 1		— 7 0
N. W. by W.	N. W.	— 1		— 7 0
N. W.	N. W. by N.	— 1		— 7 0
N.W. by N.	N. N. W. ⅛ W.	— ⅞		— 5 40
N. N. W.	N. by W. ¼ W.	— ¾		— 4 10
N. by W.	N. ⅜ W.	— ⅝		— 2 50
NORTH.				— 1 20

A deviation of the North Point of the Compass to the East is designated by the sign + a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviations are obtained:

A = — 0° 24′.5 B = + 5° 44′.4 C = — 0° 14′.6
D = + 1° 58′.5 E = + 0° 0′.2

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER BINNACLE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Valparaiso, April 4, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = + 4° 17′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 3/8 E.	− 3/8	° ′	0° 0′
N. by E.	N. by E. 1/4 E.	− 1/4		+ 1 30
N. N. E.	N. N. E. 1/8 E.	− 1/8		+ 2 50
N. E. by. N.	N. E. by N.	0		+ 4 20
N. E.	N. E. 1/8 N.	+ 1/8		+ 5 40
N. E. by E.	N. E. 7/8 E.	+ 1/8		+ 5 40
E. N. E.	N. E. by E. 7/8 E.	+ 1/8		+ 5 40
E. by N.	E. by N.	0		+ 4 20
EAST.	EAST.	0		+ 4 20
E. by S.	E. by S. 1/4 S.	− 1/4		+ 1 30
E. S. E.	S. E. by E. 7/8 E.	− 1/8		+ 2 50
S. E. by E.	S. E. 3/4 E.	− 1/4		+ 1 30
S. E.	S. E. 1/4 S.	− 1/4		+ 1 30
S. E. by S.	S. E. by S. 3/8 S.	− 3/8		0 0
S. S. E.	S. by E. 5/8 E.	− 3/8		0 0
S. by E.	S. 5/8 E.	− 3/8		0 0
SOUTH.	S. 3/8 W.	− 3/8		0 0
S. by W.	S. by W. 3/8 W.	− 3/8		0 0
S. S. W.	S. S. W. 3/8 W.	− 3/8		0 0
S. W. by S.	S. W. 5/8 S.	− 3/8		0 0
S. W.	S. W. 1/2 W.	− 1/2		− 1 20
S. W. by W.	S. W. by W. 1/2 W.	− 1/2		− 1 20
W. S. W.	W. by S. 3/8 S.	− 5/8		− 2 50
W. by S.	W. 3/8 S.	− 5/8		− 2 50
WEST.	W. 3/4 N.	− 3/4		− 4 10
W. by N.	W. by N. 3/4 N.	− 3/4		− 4 10
W. N. W.	N. W. by W. 1/4 W.	− 3/4		− 4 10
N. W. by W.	N. W. 1/8 W.	− 7/8		− 5 40
N. W.	N. W. 3/4 N.	− 3/4		− 4 10
N. W. by N.	N. N. W. 1/4 W.	− 3/4		− 4 10
N. N. W.	N. by W. 3/8 W.	− 5/8		− 2 50
N. by W	N. 1/2 W.	− 1/2		− 1 20
NORTH.		− 3/8		0 0

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

$A = +0° 4'.9 \quad B = +3° 58'.8 \quad C = +0° 7'.9$

$D = +2° 1'.5 \quad E = -0° 0'.2$

Callao, April 29, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = + 3° 44′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 3/8 E.	− 3/8	° ′	− 0° 20′
N. by E.	N. by E. 1/4 E.	− 1/4		+ 1 0
N. N. E.	N. N. E. 1/8 E.	− 1/8		+ 2 30
N. E. by N.	N. E. by N.	0		+ 3 50
N. E.	N. E. 1/8 N.	+ 1/8		+ 5 20
N. E. by E.	N. E. 7/8 E.	+ 1/8		+ 5 20
E. N. E.	E. N. E.	0		+ 3 50
E. by N.	E. by N.	0		+ 3 50
EAST.	EAST.	0		+ 3 50
E. by S.	E. by S. 1/8 S.	− 1/8		+ 2 30
E. S. E.	S. E. by E. 7/8 E.	− 1/8		+ 2 30
S. E. by E.	S. E. 3/4 E.	− 1/4		+ 1 0
S. E.	S. E. 1/4 S.	− 1/4		+ 1 0
S. E. by S.	S. S. E. 5/8 E.	− 3/8		− 0 20
S. S. E.	S. by E. 5/8 E.	− 3/8		− 0 20
S. by E.	S. 5/8 E.	− 3/8		− 0 20
SOUTH.	S. 3/8 W.	− 3/8		− 0 20
S. by W.	S. by W. 3/8 W.	− 3/8		− 0 20
S. S. W.	S. S. W. 3/8 W.	− 3/8		− 0 20
S. W. by S.	S. W. 5/8 S.	− 3/8		− 0 20
S. W.	S. W. 1/2 W.	− 1/2		− 1 50
S. W. by W.	S. W. by W. 1/2 W.	− 1/2		− 1 50
W. S. W.	W. by S. 3/8 S.	− 5/8		− 3 10
W. by S.	W. 3/8 S.	− 5/8		− 3 10
WEST.	W. 3/4 N.	− 3/4		− 4 40
W. by N.	W. by N. 7/8 N.	− 7/8		− 6 0
W. N. W.	N. W. by W. 1/8 W.	− 7/8		− 6 0
N. W. by W.	N. W. 1/8 W.	− 7/8		− 6 0
N. W.	N. W. 7/8 N.	− 7/8		− 6 0
N. W. by N.	N. N. W. 1/4 W.	− 3/4		− 4 40
N. N. W.	N. by W. 3/8 W.	− 5/8		− 3 10
N. by W.	N. 1/2 W.	− 1/2		− 1 50
NORTH.		− 3/8		− 0 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviations are obtained:

$A = -0° 27'.1 \quad B = +4° 12'.5 \quad C = -0° 3'.9$

$D = +2° 7'.5 \quad E = +0° 9'.0$

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER BINNACLE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Panama, May 20, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = + 3° 44′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 3/8 E.	— 3/8	° ′	— 0° 30′
N. by E.	N. by E. 1/4 E.	— 1/4		+ 1 0
N. N. E.	N. E. by N. 7/8 N.	— 1/8		+ 2 20
N. E. by N.	N. E. by N.	0		+ 3 40
N. E.	N. E.	0		+ 3 40
N. E. by E.	N. E. by E.	0		+ 3 40
E. N. E.	N. E. by E. 7/8 E.	+ 1/8		+ 5 10
E. by N.	E. by N.	0		+ 3 40
EAST.	E. 1/8 S.	— 1/8		+ 2 20
E. by S.	E. by S. 1/8 S.	— 1/8		+ 2 20
E. S. E.	S. E. by E. 3/4 E.	— 1/4		+ 1 0
S. E. by E.	S. E. 5/8 E.	— 3/8		— 0 30
S. E.	S. E. 3/8 S.	— 3/8		— 0 30
S. E. by S.	S. E. by S. 5/8 S.	— 5/8		— 3 20
S. S. E.	S. by E. 1/2 E.	— 1/2		— 1 50
S. by E.	S. 1/2 E.	— 1/2		— 1 50
SOUTH.	S. 1/2 W.	— 1/2		— 1 50
S. by W.	S. by W. 3/8 W.	— 3/8		— 0 30
S. S. W.	S. W. by S. 5/8 S.	— 3/8		— 0 30
S. W. by S.	S. W. 5/8 S.	— 3/8		— 0 30
S. W.	S. W. 1/2 W.	— 1/2		— 1 50
S. W. by W.	S. W. by W. 1/2 W.	— 1/2		— 1 50
W. S. W.	W. by S. 1/2 S.	— 1/2		— 1 50
W. by S.	W. 1/2 S.	— 1/2		— 1 50
WEST.	W. 3/8 N.	— 3/8		— 3 20
W. by N.	W. by N. 3/4 N.	— 3/4		— 4 40
W. N. W.	N. W. by W. 1/8 W.	— 7/8		— 6 10
N. W. by W.	N. W. 1/8 W.	— 7/8		— 6 10
N. W.	N. W. 7/8 N.	— 7/8		— 6 10
N. W. by N.	N. W. by N. 3/4 N.	— 3/4		— 4 40
N. N. W.	N. by W. 3/8 W.	— 5/8		— 3 20
N. by W.	N. 1/2 W.	— 1/2		— 1 50
NORTH.		— 3/8		— 0 30

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 0° 50′.0 B = + 3° 19′.5 C = + 0° 22′.0
D = + 2° 32′.7 E = — 0° 18′.0

Acapulco, June 1, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = + 3° 44′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 3/4 E.	— 3/4	° ′	— 4° 40′
N. by E.	N. by E. 1/4 E.	— 1/4		+ 1 0
N. N. E.	N. N. E. 1/4 E.	— 1/4		+ 1 0
N. E. by N.	N. E. 7/8 N.	— 1/8		+ 2 30
N. E.	N. E.	0		+ 3 50
N. E. by E.	N. E. by E.	0		+ 3 50
E. N. E.	E. N. E.	0		+ 3 50
E. by N.	E. 7/8 N.	— 1/8		+ 2 30
EAST.	E. 1/8 S.	— 1/8		+ 2 30
E. by S.	E. by S. 1/4 S.	— 1/4		+ 1 0
E. S. E.	S. E. by E. 3/4 E.	— 1/4		+ 1 0
S. E. by E.	S. E. 5/8 E.	— 3/8		— 0 20
S. E.	S. E. 3/8 S.	— 3/8		— 0 20
S. E. by S.	S. S. E. 1/2 E.	— 1/2		— 1 50
S. S. E.	S. by E. 1/2 E.	— 1/2		— 1 50
S. by E.	S. 1/2 E.	— 1/2		— 1 50
SOUTH.	S. 3/8 W.	— 3/8		— 0 20
S. by W.	S. by W. 3/8 W.	— 3/8		— 0 20
S. S. W.	S. S. W. 3/8 W.	— 3/8		— 0 20
S. W. by S.	S. W. 5/8 S.	— 3/8		— 0 20
S. W.	S. W. 3/8 W.	— 3/8		— 0 20
S. W. by W.	S. W. by W. 1/2 W.	— 1/2		— 1 50
W. S. W.	W. by S. 1/2 S.	— 1/2		— 1 50
W. by S.	W. 3/8 S.	— 5/8		— 3 10
WEST.	W. 3/4 N.	— 3/4		— 4 40
W. by N.	W. by N. 3/4 N.	— 3/4		— 4 40
W. N. W.	N. W. by W. 1/4 W.	— 3/4		— 4 40
N. W. by W.	N. W. 1/8 W.	— 7/8		— 6 0
N. W.	N. W. 3/4 N.	— 3/4		— 4 40
N. W. by N.	N. N. W. 1/4 W.	— 3/4		— 4 40
N. N. W.	N. by W. 1/4 W.	— 3/4		— 4 40
N. by W.	N. 1/2 W.	— 1/2		— 1 50
NORTH.				— 4 40

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 1° 0′.2 B = + 3° 4′.4 C = — 0° 17′.1
D = + 2° 15′.2 E = — 0° 17′.2

Observations for Determining the Deviations of the After Binnacle Compass on the U. S. Iron Clad Monadnock.

Magdalena Bay, June 9, 1866.

Correction for Object = — 0° 41′. Correction for Lubber Line = + 3° 44′.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ½ E.	— ½	° ′	— 2 30′
N. by E.	N. by E. ⅜ E.	— ⅜		— 1 10
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.				
E. by S.				
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.	S. W. ¼ W.	— ¼		+ 0 10
S. W. by W.	S. W. by W. ¼ W.	— ¼		+ 0 10
W. S. W.	W. by S. ⅝ S.	— ⅜		— 1 20
W. by S.	W. ½ S.	— ½		— 2 40
WEST.	W. ½ N.	— ½		— 2 50
W. by N.	W. by N. ¾ N.	— ¾		— 5 20
W. N. W.	N. W. by W. ¼ W.	— ¾		— 5 20
N. W. by W.	N. W. ¼ W.	— ¾		— 5 20
N. W.	N. W. ¾ N.	— ¾		— 5 20
N. W. by N.	N. N. W. ¼ W.	— ¾		— 5 20
N. N. W.	N. by W. ¼ W.	— ¾		— 5 20
N. by W.	N. ⅜ W.	— ⅝		— 3 50
NORTH.		— ½		— 2 30

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 1° 10′.7 B = + 2° 16′.0 C = — 1° 16′.8
D = + 2° 10′.2 E = — 0° 3′.5

San Francisco, June 23, 1866.

Correction for Object = — 0° 45′. Correction for Lubber Line = + 3° 49′.

Assumed Magnetic Direction of Ship's Head.	Bearing of Object by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ½ E.	— ½	° ′	— 2° 30′
N. by E.	N. by E. ⅜ E.	— ⅜		— 1 10
N. N. E.	N. E. by N. ¾ N.	— ¼		+ 0 20
N. E. by N.	N. E. ⅞ N.	— ⅛		+ 1 40
N. E.	N. E. ⅛ E.	— ⅛		+ 1 40
N. E. by E.	N. E. by E. ⅛ E.	— ⅛		+ 1 50
E. N. E.	E. N. E.	0		+ 3 0
E. by N.	E. by N.	0		+ 3 0
EAST.	EAST.	0		+ 3 0
E. by S.	E. by S.	0		+ 3 0
E. S. E.	S. E. by E. ⅞ E.	— ⅛		+ 1 40
S. E. by E.	S. E. ⅞ E.	— ⅛		+ 1 40
S. E.	S. E. ⅛ S.	— ⅛		+ 1 40
S. E. by S.	S. E. by E. ⅛ S.	— ⅛		+ 1 40
S. S. E.	S. by E. ⅞ E.	— ⅛		+ 1 40
S. by E.	S. ⅞ E.	— ⅛		+ 1 40
SOUTH.	S. ⅛ W.	— ⅛		+ 1 40
S. by W.	S. by W. ⅛ W.	— ⅛		+ 1 40
S. S. W.	S. W. by S. ⅞ S.	— ⅛		+ 1 40
S. W. by S.	S. W. ⅞ S.	— ⅛		+ 1 40
S. W.	S. W. ¼ W.	— ¼		+ 0 10
S. W. by W.	S. W. by W. ⅜ W.	— ⅜		— 1 10
W. S. W.	W. by S. ⅝ S.	— ⅜		— 1 20
W. by S.	W. ½ S.	— ½		— 2 50
WEST.	W. ¾ N.	— ¾		— 5 20
W. by N.	W. by N. ¾ N.	— ¾		— 5 20
W. N. W.	N. W. by W. ¼ W.	— ¾		— 5 20
N. W. by W.	N. W. ¼ W.	— ¾		— 5 30
N. W.	N. W. ⅞ N.	— ⅞		— 6 50
N. W. by N.	N. W. by N. ⅞ N.	— ⅞		— 6 40
N. N. W.	N. by W. ¼ W.	— ¾		— 5 20
N. by W.	N. ⅜ W.	— ⅝		— 3 50
NORTH.		— ½		— 2 30

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 0° 35′.2 B = + 3° 28′.2 C = — 2° 13′.9
D = + 1° 47′.5 E = + 0° 10′.2

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER RITCHIE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Hampton Roads, November 1, 1865.

Correction for Object = + 3° 57′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ⅛ W.	+ ⅛	° ′	+ 4° 20′
N. by E.	N. ⅝ E.	+ ⅜		+ 7 10
N. N. E.	N. by E. ¼ E.	+ ¾		+ 10 50
N. E. by N.	N. N. E.	+ 1		+ 14 10
N. E.	N. E. by N.	+ 1		+ 15 10
N. E. by E.	N. E. ¼ N.	+ 1¼		+ 17 0
E. N. E.	N. E. ¾ E.	+ 1¼		+ 18 0
E. by N.	N. E. by E. ½ E.	+ 1½		+ 19 50
EAST.	E. by N. ¼ N.	+ 1¼		+ 19 0
E. by S.	E. ½ N.	+ 1½		+ 19 50
E. S. E.	E. ¾ S.	+ 1¼		+ 19 0
S. E. by E.	E. S. E. ¼ E.	+ 1¼		+ 18 0
S. E.	S. E. by E.	+ 1		+ 16 10
S. E. by S.	S. E.	+ 1		+ 15 10
S. S. E.		+ ¾		+ 13 20
S. by E.	S. by E. ½ E.	+ ½		+ 10 30
SOUTH.	S. ½ E.	+ ½		+ 9 30
S. by W.	S. ¾ W.	+ ¼		+ 7 40
S. S. W.	S. S. W.	0		+ 5 0
S. W. by S.	S. W. ¾ S.	— ¼		+ 2 10
S. W.	S. W. ½ W.	— ½		— 0 40
S. W. by W.	S. W. by W. ½ W.	— ½		— 1 40
W. S. W.	W. by S. ⅜ S.	— ⅜		— 2 40
W. by S.	W. ½ S.	— ½		— 2 10
WEST.	W. ½ N.	— ½		— 1 40
W. by N.	W. by N. ½ N.	— ½		— 1 40
W. N. W.		— ½		— 1 40
N. W. by W.	N. W. ½ W.	— ½		— 1 40
N. W.	N. W. ⅜ N.	— ⅜		— 2 40
N. W. by N.	N. N. W. ½ W.	— ½		— 2 10
N. N. W.	N. by W. ¾ W.	— ¼		+ 0 10
N. by W.	N. ⅞ W.	— ⅛		+ 2 0
NORTH.		+ ⅛		+ 4 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

$A = +7° \ 40'.0 \quad B = +11° \ 26'.5 \quad C = -1° \ 44'.1$

$D = +0° \ 15'.5 \quad E = -0° \ 54'.5$

St. Thomas, November 18, 1865.

Correction for Object = + 0° 16′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ⅜ W.	+ ⅜	° ′	+ 4° 0′
N. by E.	N. ½ E.	+ ½		+ 5 30
N. N. E.	N. by E. ¼ E.	+ ¾		+ 8 10
N. E. by N.	N. N. E.	+ 1		+ 11 0
N. E.	N. E. by N.	+ 1		+ 11 0
N. E. by E.	N. E. ⅛ N.	+ 1⅛		+ 12 20
E. N. E.	N. E. ⅞ E.	+ 1⅛		+ 12 20
E. by N.	N. E. by E. ⅞ E.	+ 1⅛		+ 12 20
EAST.	E. by N.	+ 1		+ 11 0
E. by S.	E. ⅛ N.	+ 1⅛		+ 12 20
E. S. E.	E. by S.	+ 1		+ 11 0
S. E. by E.	S. E. by E. ⅞ E.	+ ⅞		+ 9 30
S. E.	S. E. ⅝ E.	+ ⅝		+ 6 40
S. E. by S.	S. E. ½ S.	+ ½		+ 5 20
S. S. E.	S. E. by S. ⅝ S.	+ ⅜		+ 4 0
S. by E.	S. by E. ¼ E.	+ ¼		+ 2 30
SOUTH.	S. ⅛ E.	+ ⅛		+ 1 10
S. by W.	S. ⅞ W.	+ ⅛		+ 1 10
S. S. W.	S. S. W.	0		— 0 20
S. W. by S.	S. W. by S.	0		— 0 20
S. W.	S. W. ⅛ W.	— ⅛		— 1 40
S. W. by W.	S. W. by W. ⅛ W.	— ⅛		— 1 40
W. S. W.	W. by S. ⅞ S.	— ⅛		— 1 40
W. by S.	W. ¾ S.	— ¼		— 3 10
WEST.	W. ⅜ N.	— ⅜		— 4 30
W. by N.	W. by N. ½ N.	— ½		— 5 50
W. N. W.	N. W. by W. ½ W.	— ½		— 5 50
N. W. by W.	N. W. ½ W.	— ½		— 5 50
N. W.	N. W. ⅜ N.	— ⅜		— 4 30
N. W. by N.	N.W. by N. ¼ N.	— ¼		— 3 10
N. N. W.	N. N. W.	0		— 0 20
N. by W.	N. by W. ⅛ W.	+ ⅛		+ 1 10
NORTH.		+ ⅜		+ 4 0

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

$A = +3° \ 14'.4 \quad B = +8° \ 26'.9 \quad C = +0° \ 40'.4$

$D = +1° \ 54'.2 \quad E = -0° \ 37'.2$

Observations for Determining the Deviations of the After Ritchie Compass on the U. S. Iron Clad Monadnock.

Isle Royal, Salute Islands, November 30, 1865.

Correction for Object = — 0° 2′. Correction for Lubber Line = + 6° 18′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees. ° ′	Corrected Deviation of Compass.
NORTH.				
N. by E.				
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.	E. ½ N.	+ ½		+ 11° 50′
E. by S.	E. ¼ S.	+ ¾		+ 14 40
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = B = C =
D = E =

Ceara, December 19, 1865.

Correction for Object = + 1° 51′ Correction for Lubber Line = + 6° 18′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees. ° ′	Corrected Deviation of Compass.
NORTH.				
N. by E.	N. ½ E.	+ ½		+ 13° 40′
N. N. E.	N. by E. ½ E.	+ ½		+ 13 40
N. E. by N.	N. N. E. ½ E.	+ ½		+ 13 40
N. E.	N. E. by N. ¼ E.	+ ¾		+ 16 10
N. E. by E.	N. E. ¼ E.	+ ¾		+ 16 30
E. N. E.	N. E. by E.	+ 1		+ 19 0
E. by N.	E. by N. ¾ N.	+ ¾		+ 17 0
EAST.	E. ½ N.	+ ½		+ 14 10
E. by S.	E. ¾ S.	+ ¼		+ 11 30
E. S. E.	E. by S. ¾ S.	+ ¼		+ 11 0
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 5° 54′.2 B = + 7° 56′.0 C = + 4° 55′.4
D = + 1° 36′.6 E = — 0° 43′.7

Observations for Determining the Deviations of the After Ritchie Compass on the U. S. Iron Clad Monadnock.

Bahia, December 30, 1865.

Correction for Object = + 2° 30′. Correction for Lubber Line = + 6° 18′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N.	0	° ′	+ 8° 10′
N. by E.	N ¾ E.	+ ¼		+ 11 0
N. N. E.	N. N. E. ¼ N.	+ ¼		+ 11 40
N. E. by N.	N. E. by N. ½ N.	+ ½		+ 13 50
N. E.	N. E. ½ N.	+ ½		+ 14 20
N. E. by E.	N. E. ½ E.	+ ½		+ 14 20
E. N. E.	N. E. by E. ¼ E.	+ ¾		+ 16 30
E. by N.	E. by N. ¾ N.	+ ¾		+ 17 10
EAST.	E. ½ N.	+ ½		+ 15 0
E. by S.	E. ½ S.	+ ½		+ 14 20
E. S. E.	E. by S. ½ S.	+ ½		+ 14 20
S. E. by E.	S. E. by E. ½ E.	+ ½		+ 14 20
S. E.	S. E. ¼ E.	+ ¼		+ 12 10
S. E. by S.	S. E. ¾ S.	+ ¼		+ 11 40
S. S. E.	S. S. E. ¼ E.	+ ¼		+ 11 40
S. by E.	S. by E.	0		+ 9 30
SOUTH.	SOUTH.	0		+ 8 50
S. by W.	S. by W.	0		+ 8 50
S. S. W.	S. S. W.	0		+ 8 50
S. W. by S.	S. W. by S.	0		+ 8 50
S. W.	S. W. ¼ W.	— ¼		+ 6 40
S. W. by W.	S. W. by W. ¼ W.	— ¼		+ 6 0
W. S. W.	W. by S. ½ S.	— ½		+ 3 50
W. by S.	W. ½ S.	— ½		+ 3 10
WEST.	W. ¾ N.	— ¾		+ 1 0
W. by N.		— ¾		+ 0 20
W. N. W.	N. W. by W. ¼ W.	— ¾		+ 0 20
N. W. by W.	N. W. ¼ W.	— ¾		+ 0 20
N. W.	N. W. ¾ N.	— ¾		+ 0 20
N.W. by N.	N. N. W. ½ W.	— ½		+ 2 30
N. N. W.	N. by W. ¾ W.	— ¼		+ 5 20
N. by W.	N. ¾ W.	— ¼		+ 6 0
NORTH.		0		+ 8 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 8° 47′.1 B = + 6° 55′.6 C = — 0° 57′.2
D = + 1° 59′.7 E = + 0° 14′ 2

Rio Janeiro, January 10, 1866.

Correction for Object = + 2° 44′. Correction for Lubber Line = + 5° 43′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.			° ′	
N. by E.				
N. N. E.				
N. E. by N.	N. E. by N. ½ N.	+ ½		+ 14° 0′
N. E.	N. E. ½ N.	+ ½		+ 14 0
N. E. by E.	N. E. ½ E.	+ ½		+ 14 0
E. N. E.	N. E. by E. ¼ E.	+ ¾		+ 16 10
E. by N.	E. by N. ¾ N.	+ ¾		+ 16 50
EAST.	E. ½ N.	+ ½		+ 14 40
E. by S.	E. ¾ S.	+ ¼		+ 12 0
E. S. E.	E. S. E. ⅛ E.	+ ⅛		+ 10 10
S. E. by E.	S. E. by E. ⅛ E.	+ ⅛		+ 9 50
S. E.	S. E. ⅛ E.	+ ⅛		+ 9 50
S. E. by S.	S. E. by S.	0		+ 8 30
S. S. E.	S. S. E.	0		+ 8 30
S. by E.	S. by E.	0		+ 8 30
SOUTH.	SOUTH.	0		+ 8 30
S. by W.	S. by W.	0		+ 8 30
S. S. W.	S. S. W.	0		+ 8 30
S. W. by S.	S. W. by S.	0		+ 8 30
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign + a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviations are obtained:

A = + 9° 39′.0 B = + 3° 46′.6 C = + 1° 9′.8
D = + 1° 50′.1 E = — 0° 7′.4

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER RITCHIE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Monte Video, January 24, 1866.

Correction for Object = — 0° 13′. Correction for Lubber Line = + 4° 9′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	+ 4° 0′
N. by E.	N. ½ E.	+ ½		+ 9 30
N. N. E.	N. by E. ½ E.	+ ½		+ 9 30
N. E. by N.	N. E. by N. ¾ N.	+ ¾		+ 12 20
N. E.	N. E. ¾ N.	+ ¾		+ 12 20
N. E. by E.	N. E. ¼ E.	+ ¾		+ 12 20
E. N. E.	N. E. by E.	+ 1		+ 15 10
E. by N.	E. N. E.	+ 1		+ 15 0
EAST.	E. ½ N.	+ ½		+ 9 20
E. by S.	E. by S.	0		+ 4 0
E. S. E.	E. S. E.	0		+ 3 50
S. E. by E.	S. E. by E. ¼ S.	— ¼		+ 1 0
S. E.	S. E. ¾ S.	— ¾		— 4 30
S. E. by S.	S. E. by S. ½ S.	— ½		— 1 40
S. S. E.	S. S. E.	0		+ 4 0
S. by E.				+ 4 0
SOUTH.	SOUTH.	0		+ 4 0
S. by W.	S. ¾ W.	+ ¼		+ 6 50
S. S. W.	S. by W. ¾ W.	+ ¼		+ 6 50
S. W. by S.	S. W. by S.	0		+ 4 0
S. W.	S. W.	0		+ 4 0
S. W. by W.	S. W. ½ W.	+ ½		+ 9 30
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 6° 32′.8   B = + 0° 50′.3   C = + 3° 10′.9
D = + 2° 1′.8   E = — 0° 5′.5

Sandy Point, February 10, 1866.

Correction for Object = + 0° 7′. Correction for Lubber Line = + 4° 9′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ⅛ W.	+ ⅛	° ′	+ 5° 40′
N. by E.	N. ¾ E.	+ ¼		+ 7 0
N. N. E.	N. by E. ½ E.	+ ½		+ 9 50
N. E. by N.	N. E. by N. ½ N.	+ ½		+ 9 50
N. E.	N. E. ½ N.	+ ½		+ 9 50
N. E. by E.	N. E. ¼ E.	+ ¾		+ 12 40
E. N. E.	N. E. by E. ½ E.	+ ½		+ 9 50
E. by N.	E. by N. ½ N.	+ ½		+ 9 50
EAST.				+ 9 50
E. by S.	E. ½ S.	+ ½		+ 9 50
E. S. E.	E. by S. ¼ S.	+ ¾		+ 12 40
S. E. by E.	E. S. E. ⅛ S.	+ ⅞		+ 14 0
S. E.	S. E. by E. ¼ S.	+ ¾		+ 12 40
S. E. by S.	S. E. ⅛ S.	+ ⅞		+ 14 0
S. S. E.	S. E. by S. ⅛ S.	+ ⅞		+ 14 0
S. by E.	S. by E. ¾ E.	+ ¾		+ 12 40
SOUTH.	S. ½ E.	+ ½		+ 9 50
S. by W.	S. ½ W.	+ ½		+ 9 50
S. S. W.	S. by W. ¼ W.	+ ¾		+ 12 40
S. W. by S.	S. W. by S. ¾ S.	+ ¾		+ 12 40
S. W.	S. W. ½ S.	+ ½		+ 9 50
S. W. by W.	S. W. ¾ W.	+ ¼		+ 7 0
W. S. W.	W. S. W.	0		+ 4 20
W. by S.	W. by S.	0		+ 4 20
WEST.	WEST.	0		+ 4 20
W. by N.	W. by N. ⅛ N.	— ⅛		+ 2 50
W. N. W.	W. N. W. ⅛ N.	— ⅛		+ 2 50
N. W. by W.	N. W. ¾ W.	— ¼		+ 1 20
N. W.	N. W. ¼ N.	— ¼		+ 1 20
N. W. by N.	N. N. W. ¾ W.	— ¼		+ 1 20
N. N. W.	N. N. W. ⅛ N.	— ⅛		+ 2 50
N. by W.	N. by W.	0		+ 4 20
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 8° 18′.4   B = + 4° 3′.2   C = — 3° 25′.6
D = + 1° 14′.5   E = + 0° 58′.5

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER RITCHIE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Valparaiso, April 4, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = + 4° 17′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	+ 4° 20′
N. by E.	N. ¾ E.	+ ¼		+ 7 0
N. N. E.	N. by E. ¾ E.	+ ¼		+ 7 0
N. E. by. N.	N. E. by N. ½ N.	+ ½		+ 9 50
N. E.	N. E. ½ N.	+ ½		+ 9 50
N. E. by E.	N. E. ½ E.	+ ½		+ 9 50
E. N. E.	E. N. E. ¼ N.	+ ¼		+ 7 0
E. by N.	E. by N. ½ N.	+ ½		+ 9 50
EAST.	E. ¼ N.	+ ¼		+ 7 0
E. by S.	E. ¾ S.	+ ¼		+ 7 0
E. S. E.	E. S. E. ¼ E.	+ ¼		+ 7 0
S. E. by E.	S. E. by E. ¼ E.	+ ¼		+ 7 0
S. E.	S. E.	0		+ 4 20
S. E. by S.	S. E. by S.	0		+ 4 20
S. S. E.	S. S. E.	0		+ 4 20
S. by E.	S. by E.	0		+ 4 20
SOUTH.	S. ¼ W.	— ¼		+ 1 30
S. by W.	S. by W.	0		+ 4 20
S. S. W.	S. S. W.	0		+ 4 20
S. W. by S.	S. W. by S.	0		+ 4 20
S. W.	S. W.	0		+ 4 20
S. W. by W.	S. W. by W.	0		+ 4 20
W. S. W.	W. S. W.	0		+ 4 20
W. by S.	W. ¾ S.	— ¼		+ 1 30
WEST.	W. ½ N.	— ½		— 1 20
W. by N.	W. by N. ½ N.	— ½		— 1 20
W. N. W.	N. W. by W. ½ W.	— ½		— 1 20
N. W. by W.	N. W. ½ W.	— ½		— 1 20
N. W.	N. W. ¼ N.	— ¼		+ 1 30
N. W. by N.	N. W. by N. ½ N.	— ½		— 1 20
N. N. W.	N. by W. ¾ W.	— ¼		+ 1 30
N. by W	N. by W.	0		+ 4 20
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 4° 21′.9 B = + 3° 49′.1 C = + 0° 12′.4
D = + 2° 21′.0 E = + 0° 7′.5

Callao, April 29, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = + 3° 44′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ⅛ W.	+ ⅛	° ′	+ 5° 20′
N. by E.	N. ½ E.	+ ½		+ 9 30
N. N. E.	N. by E. ½ E.	+ ½		+ 9 30
N. E. by N.	N. E. by N. ½ N.	+ ½		+ 9 30
N. E.	N. E. ½ N.	+ ½		+ 9 30
N. E. by E.	N. E. ½ E.	+ ½		+ 9 30
E. N. E.	N. E. by E. ½ E.	+ ½		+ 9 30
E. by N.	E. by N. ½ N.	+ ½		+ 9 30
EAST.	E. ½ N.	+ ½		+ 9 30
E. by S.	E. ½ S.	+ ½		+ 9 30
E. S. E.	E. by S. ¾ S.	+ ¼		+ 6 40
S. E. by E.	S. E. by E. ¼ E.	+ ¼		+ 6 40
S. E.	S. E. ½ E.	+ ½		+ 9 30
S. E. by S.	S. E. ¾ S.	+ ¼		+ 6 40
S. S. E.				+ 6 40
S. by E.	S. by E.	0		+ 3 50
SOUTH.	SOUTH.	0		+ 3 50
S. by W.	S. by W.	0		+ 3 50
S. S. W.	S. S. W.	0		+ 3 50
S. W. by S.	S. W. by S.	0		+ 3 50
S. W.	S. W. ¼ W.	— ¼		+ 1 0
S. W. by W.	S. W. by W. ¼ W.	— ¼		+ 1 0
W. S. W.	W. by S. ½ S.	— ½		— 1 50
W. by S.	W. ½ S.	— ½		— 1 50
WEST.	W. ½ N.	— ½		— 1 50
W. by N.	W. by N. ½ N.	— ½		— 1 50
W. N. W.	N. W. by W. ½ W.	— ½		— 1 50
N. W. by W.	N. W. ½ W.	— ½		— 1 50
N. W.	N. W. ½ N.	— ½		— 1 50
N. W. by N.	N. W. by N. ½ N.	— ½		— 1 50
N. N. W.	N. by W. ¾ W.	— ¼		+ 1 0
N. by W.	N. by W.	0		+ 3 50
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviations are obtained:

A = + 4° 19′.4 B = + 5° 50′.1 C = + 0° 14′.1
D = + 1° 30′.5 E = + 0° 52′.0

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER RITCHIE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Panama, May 20, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = + 3° 44′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	+ 3° 50′
N. by E.	N. ¾ E.	+ ¼		+ 6 30
N. N. E.	N. by E. ¾ E.	+ ¼		+ 6 30
N. E. by N.	N. E. by N. ½ N.	+ ½		+ 9 20
N. E.	N. E. ½ N.	+ ½		+ 9 20
N. E. by E.	N. E. ½ E.	+ ½		+ 9 20
E. N. E.	N. E. by E. ½ E.	+ ½		+ 9 20
E. by N.	E. by N. ¾ N.	+ ¾		+ 12 10
EAST.	E. ¾ N.	+ ¾		+ 12 10
E. by S.	E. ½ S.	+ ½		+ 9 20
E. S. E.	E. by S. ½ S.	+ ½		+ 9 20
S. E. by E.	S. E. by E. ½ E.	+ ½		+ 9 20
S. E.	S. E. ⅛ E.	+ ⅛		+ 5 10
S. E. by S.	S. E. ¾ S.	+ ¼		+ 6 30
S. S. E.	S. E. by S. ¾ S.	+ ¼		+ 6 30
S. by E.	S. by E.	0		+ 3 40
SOUTH.	SOUTH.	0		+ 3 50
S. by W.	S. by W.	0		+ 3 40
S. S. W.	S. S. W.	0		+ 3 50
S. W. by S.	S. W. by S.	0		+ 3 40
S. W.	S. W.	0		+ 3 50
S. W. by W.	S. W. by W.	0		+ 3 40
W. S. W.	W. S. W.	0		+ 3 50
W. by S.	W. by S.	0		+ 3 40
WEST.	W. ⅛ N.	— ⅛		+ 2 20
W. by N.	W. by N. ⅛ N.	— ⅛		+ 2 20
W. N. W.	W. N. W. ⅛ N.	— ⅛		+ 2 20
N. W. by W.	N. W. ¾ W.	— ¼		+ 0 50
N. W.	N. W. ⅛ N.	— ⅛		+ 2 20
N. W. by N.	N.W. by N. ¼ N.	— ¼		+ 0 50
N. N. W.	N. by W. ¾ W.	— ¼		+ 0 50
N. by W.	N. ¾ W.	— ¼		+ 0 50
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 5° 20′.6   B = + 4° 3′.1   C = — 0° 10′.2
D = + 1° 17′.0   E = — 1° 33′.0

Acapulco, June 1, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = + 3° 44′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ¼ W.	+ ¼	° ′	+ 6° 40′
N. by E.	N. ¾ E.	+ ¼		+ 6 40
N. N. E.	N. by E. ¾ E.	+ ¼		+ 6 40
N. E. by N.	N. E. by N. ½ N.	+ ½		+ 9 30
N. E.	N. E. ½ N.	+ ½		+ 9 30
N. E. by E.	N. E. ½ E.	+ ½		+ 9 30
E. N. E.	N. E. by E. ½ E.	+ ½		+ 9 30
E. by N.	E. by N. ½ N.	+ ½		+ 9 30
EAST.	E. ¼ N.	+ ¼		+ 6 40
E. by S.	E. ¾ S.	+ ¼		+ 6 40
E. S. E.	E. S. E. ¼ E.	+ ¼		+ 6 40
S. E. by E.	S. E. by E. ¼ E	+ ¼		+ 6 40
S. E.	S. E.	0		+ 3 50
S. E. by S.	S. E. by S.	0		+ 3 50
S. S. E.	S. S. E.	0		+ 3 50
S. by E.	S. by E.	0		+ 3 50
SOUTH.	SOUTH.	0		+ 3 50
S. by W.	S. by W.	0		+ 3 50
S. S. W.	S. S. W.	0		+ 3 50
S. W. by S.	S. W. by S.	0		+ 3 50
S. W.	S. W. ¼ W.	— ¼		+ 1 0
S. W. by W.	S. W. by W. ½ W.	— ½		— 1 50
W. S. W.	W. by S. ½ S.	— ½		— 1 50
W. by S.	W. ½ S.	— ½		— 1 50
WEST.	W. ½ N.	— ½		— 1 50
W. by N.	W. by N. ¼ N.	— ¼		+ 1 0
W. N. W.	N. W. by W. ¾ W.	— ¼		+ 1 0
N. W. by W.	N. W. ½ W.	— ½		— 1 50
N. W.	N. W. ¼ N.	— ¼		+ 1 0
N. W. by N.	N. W. by N. ¼ N.	— ¼		+ 1 0
N. N. W.	N. N. W.	0		+ 3 50
N. by W.	N. by W.	0		+ 3 50
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 4° 0′.6   B = + 4° 29′.1   C = + 1° 12′.8
D = + 1° 12′.2   E = + 0° 47′.0

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER RITCHIE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Magdalena Bay, June 9, 1866.

Correction for Object = − 0° 41′. Correction for Lubber Line = + 3° 44′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	
NORTH.	N. ⅛ E.	− ⅛		+ 1° 40′
N. by E.	N. by E.	0		+ 3 10
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.				
E. by S.				
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.	S. W. ⅛ W.	− ⅛		+ 1 40
S. W. by W.	S. W. by W.	0		+ 3 0
W. S. W.	W. S. W.	0		+ 3 0
W. by S.	W. by S.	0		+ 3 0
WEST.	W. ⅛ N.	− ⅛		+ 1 20
W. by N.	W. by N. ½ N.	− ½		− 2 50
W. N. W.	N. W. by W. ¼ W.	− ¾		− 5 30
N. W. by W.	N. W. ¼ W.	− ¾		− 5 20
N. W.	N. W. ½ N.	− ½		− 2 40
N. W. by N.	N. W. by N. ½ N.	− ½		− 2 40
N. N. W.	N. by W. ½ W.	− ½		− 2 30
N. by W.	N. ⅝ W.	− ⅜		− 1 0
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign −.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 3° 35′.5 B = + 4° 27′.3 C = − 2° 51′.0
D = + 1° 50′.7 E = + 0° 10′.6

San Francisco, June 23, 1866.

Correction for Object = − 0° 45′. Correction for Lubber Line = + 3° 49′.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	
NORTH.	N. ⅛ E.	− ⅛		+ 2° 0′
N. by E.	N. ¾ E.	+ ¼		+ 5 50
N. N. E.	N. by E. ¾ E.	+ ¼		+ 6 10
N. E. by N.	N. N. E. ½ E.	+ ½		+ 8 50
N. E.	N. E. ¾ N.	+ ¾		+ 11 30
N. E. by E.	N. E. ¼ E.	+ ¾		+ 11 30
E. N. E.	N. E. by E. ¼ E.	+ ¾		+ 11 30
E. by N.	E. by N. ¾ N.	+ ¾		+ 11 30
EAST.	E. ¾ N.	+ ¾		+ 11 10
E. by S.	E. ½ S.	+ ½		+ 8 40
E. S. E.	E. by S. ½ S.	+ ½		+ 8 40
S. E. by E.	S. E. by E. ½ E.	+ ½		+ 8 40
S. E.	S. E. ½ E.	+ ½		+ 8 40
S. E. by S.	S. E. ¾ S.	+ ¼		+ 6 0
S. S. E.	S. S. E. ¼ E.	+ ¼		+ 6 0
S. by E.	S. by E. ¼ E	+ ¼		+ 6 0
SOUTH.	S. ¼ E.	+ ¼		+ 6 0
S. by W.	S. ¾ W.	+ ¼		+ 6 0
S. S. W.	S. by W. ¾ W.	+ ¼		+ 6 0
S. W. by S.	S. W. by S. ¼ S.	+ ¼		+ 5 40
S. W.	S. W.	0		+ 2 50
S. W. by W.	S. W. by W. ¼ W.	− ¼		+ 0 20
W. S. W.	W. by S. ¾ S.	− ¼		0 0
W. by S.	W. ½ S.	− ½		− 2 30
WEST.	W. ½ N.	− ½		− 2 50
W. by N.	W. by N. ¾ N.	− ¾		− 5 20
W. N. W.	N. W. by W. ¼ W.	− ¾		− 5 10
N. W. by W.	N. W. ½ W.	− ½		− 2 30
N. W.	N. W. ½ N.	− ½		− 2 30
N. W. by N.	N. W. by N. ½ N.	− ½		− 2 30
N. N. W.	N. N. W. ½ N.	− ½		− 2 20
N. by W.	N. ¾ W.	− ¼		+ 0 20
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign −.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 4° 11′.6 B = + 6° 46′.2 C = − 1° 31′.4
D = + 2° 28′.5 E = + 0° 21′.2

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER AZIMUTH COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Hampton Roads, November 1, 1865.

Correction for Object = + 3° 57′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 3° E.		— 3° 0′	+ 0° 10′
N. by E.	N. 15 E.		— 4 0	+ 0 10
N. N. E.	N. 26 E.		— 3 30	+ 0 10
N. E. by N.	N. 36 E.		— 2 0	+ 1 20
N. E.	N. 48 E.		— 3 0	+ 1 10
N. E. by E.	N. 60 E.		— 4 0	+ 0 10
E. N. E.	N. 74 E.		— 6 30	— 1 50
E. by N.	N. 87 E.		— 8 0	— 3 40
EAST.	S. 79 E.		— 11 0	— 6 0
E. by S.	S. 65 E.		— 14 0	— 9 10
E. S. E.	S. 53 E.		— 14 30	— 10 30
S. E. by E.	S. 42 E.		— 14 0	— 10 20
S. E.	S. 32 E.		— 13 0	— 9 30
S. E. by S.	S. 22 E.		— 12 0	— 8 30
S. S. E.	S. 13 E.		— 9 30	— 6 30
S. by E.	S. 5 E.		— 6 0	— 3 20
SOUTH.	S. 3 W.		— 3 0	— 0 10
S. by W.	S. 12 W.		— 1 0	+ 2 10
S. S. W.	S. 20 W.		+ 2 30	+ 5 10
S. W. by S.	S. 31 W.		+ 3 0	+ 6 40
S. W.	S. 42 W.		+ 3 0	+ 7 0
S. W. by W.	S. 52 W.		+ 4 0	+ 7 30
W. S. W.	S. 64 W.		+ 3 30	+ 7 30
W. by S.	S. 76 W.		+ 3 0	+ 7 0
WEST.	N. 89 W.		— 1 0	+ 4 20
W. by N.	N. 74 W.		— 5 0	+ 0 20
W. N. W.	N. 63 W.		— 4 30	— 0 50
N. W. by W.	N. 50 W.		— 6 0	— 1 40
N. W.	N. 37 W.		— 8 0	— 3 30
N. W. by N.	N. 25 W.		— 9 0	— 4 50
N. N. W.	N. 16 W.		— 6 30	— 3 30
N. by W.	N. 6 W.		— 5 0	— 1 40
NORTH.				+ 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

$$A = -1^\circ\ 5'.0 \quad B = -4^\circ\ 53'.0 \quad C = -0^\circ\ 9'.1$$
$$D = +5^\circ\ 35'.2 \quad E = +0^\circ\ 17'.0$$

St. Thomas, November 18, 1865.

Correction for Object = + 0° 16′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N.		0° 0′	— 0° 10′
N. by E.	N. 9° E.		+ 2 15	+ 2 0
N. N. E.	N. 18 E.		+ 4 30	+ 4 20
N. E. by N.	N. 29 E.		+ 4 45	+ 4 30
N. E.	N. 41 E.		+ 4 0	+ 3 40
N. E. by E.	N. 53 E.		+ 3 15	+ 3 0
E. N. E.	N. 65 E.		+ 2 30	+ 2 10
E. by N.	N. 80 E.		— 1 15	— 1 30
EAST.	S. 86 E.		— 4 0	— 4 20
E. by S.	S. 71 E.		— 7 45	— 8 0
E. S. E.	S. 58 E.		— 9 30	— 9 50
S. E. by E.	S. 45 E.		— 11 15	— 11 30
S. E.	S. 33 E.		— 12 0	— 12 10
S. E. by S.	S. 25 E.		— 8 45	— 9 0
S. S. E.	S. 16 E.		— 6 30	— 6 50
S. by E.	S. 6 E.		— 5 15	— 5 30
SOUTH.	S. 2 W.		— 2 0	— 2 10
S. by W.	S. 11 W.		+ 0 15	0 0
S. S. W.	S. 19 W.		+ 3 30	+ 3 20
S. W. by S.	S. 29 W.		+ 4 45	+ 4 30
S. W.	S. 39 W.		+ 6 0	+ 5 50
S. W. by W.	S. 49 W.		+ 7 15	+ 7 0
W. S.W.	S. 60 W.		+ 7 30	+ 7 10
W. by S.	S. 74 W.		+ 4 45	+ 4 30
WEST.	S. 87 W.		+ 3 0	+ 2 40
W. by N.	N. 77 W.		— 1 45	— 2 10
W. N. W.	N. 64 W.		— 3 30	— 3 50
N. W. by W.	N. 52 W.		— 4 15	— 4 30
N. W.	N. 40 W.		— 5 0	— 5 10
N. W. by N.	N. 29 W.		— 4 45	— 5 0
N. N. W.	N. 20 W.		— 2 30	— 2 50
N. by W.	N. 10 W.		— 1 15	— 1 30
NORTH.				— 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the cofficients of the deviation are obtained:

$$A = -1^\circ\ 17'.5 \quad B = -3^\circ\ 0'.9 \quad C = +1^\circ\ 20'.0$$
$$D = +6^\circ\ 49'.2 \quad E = +0^\circ\ 12'.2$$

Observations for Determining the Deviations of the After Azimuth Compass on the U. S. Iron Clad Monadnock.

Isle Royal, Salute Islands, November 30, 1865.

Correction for Object = — 0° 2′. Correction for Lubber Line = 0

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.				
N. by E.				
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.	S. 70° E.		— 20° 0	— 20° 0′
E. by S.	S. 63 E.		— 15 45	— 15 50
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = B = C =
D = E =

Ceara, December 19, 1865.

Correction for Object = + 1° 51′ Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.				
N. by E.	N. 4° E.		+ 7° 15′	+ 9° 0′
N. N. E.	N. 14 E.		+ 8 30	+ 10 10
N. E. by N.	N. 24 E.		+ 9 45	+ 11 20
N. E.	N. 38 E.		+ 7 0	+ 9 20
N. E. by E.	N. 45 E.		+ 11 15	+ 12 20
E. N. E.	N. 58 E.		+ 9 30	+ 11 40
E. by N.	N. 78 E.		+ 0 45	+ 4 0
EAST.	S. 69 E.		— 21 0	— 15 30
E. by S.	S. 59 E.		— 19 45	— 18 10
E. S. E.	S. 48 E.		— 19 30	— 17 40
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

The compass did not traverse well, and the observations are not sufficiently good to be worth the trouble of reducing.

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = B = C =
D = E =

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER AZIMUTH COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Bahia, December 30, 1865.

Correction for Object = + 2° 30′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 8° E.		— 8° 0′	— 5° 50′
N. by E.	N. 17 E.		— 5 45	— 3 50
N. N. E.	N. 27 E.		— 4 30	— 2 20
N. E. by N.	N. 35 E.		— 1 15	+ 0 30
N. E.	N. 46 E.		— 1 0	+ 1 30
N. E. by E.	N. 58 E.		— 1 45	+ 1 0
E. N. E.	N. 70 E.		— 2 30	+ 0 10
E. by N.	N. 86 E.		— 7 15	— 3 40
EAST.	S. 78 E.		— 12 0	— 8 20
E. by S.	S. 65 E.		— 13 45	— 10 50
E. S. E.	S. 51 E.		— 16 30	— 13 20
S. E. by E.	S. 39 E.		— 17 15	— 14 40
S. E.	S. 29 E.		— 16 0	— 13 50
S. E. by S.	S. 19 E.		— 14 45	— 12 30
S. S. E.	S. 10 E.		— 12 30	— 10 30
S. by E.	S. 2 E.		— 9 15	— 7 30
SOUTH.	S. 6 W.		— 6 0	— 4 20
S. by W.	S. 12 W.		— 0 45	+ 0 30
S. S. W.	S. 22 W.		+ 0 30	+ 2 40
S. W. by S.	S. 31 W.		+ 2 45	+ 4 40
S. W.	S. 40 W.		+ 5 0	+ 7 0
S. W. by W.	S. 50 W.		+ 6 15	+ 8 30
W. S. W.	S. 65 W.		+ 2 30	+ 5 50
W. by S.	S. 78 W.		+ 0 45	+ 3 30
WEST.	N. 89 W.		— 1 0	+ 1 50
W. by N.	N. 75 W.		— 3 45	— 0 40
W. N. W.	N. 62 W.		— 5 30	— 2 40
N. W. by W.	N. 46 W.		— 10 15	— 6 40
N. W.	N. 35 W.		— 10 0	— 7 30
N.W. by N.	N. 23 W.		— 10 45	— 8 10
N. N. W.	N. 11 W.		— 11 30	— 8 50
N. by W.	N. 2 W.		— 9 15	— 7 20
NORTH.			— 8 0	— 5 50

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 3° 36′.9 B = — 4° 28′.6 C = — 0° 19′.5
D = + 7° 22′.0 E = — 1° 5′.5

Rio Janeiro, January 10, 1866.

Correction for Object = + 2° 44′. Correction for Lubber Line = 0°.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.				
N. by E.				
N. N. E.				
N. E. by N.	E. 56° N.		— 0° 15′	+ 2° 40′
N. E.	E. 44 N.		— 1 0	+ 2 0
N. E. by E.	E. 32 N.		— 2 45	+ 0 20
E. N. E.	E. 17 N.		— 5 30	— 2 10
E. by N.	E. 6 N.		— 5 15	— 2 40
EAST.	S. 82 E.		— 8 0	— 4 40
E. by S.	S. 69 E.		— 9 45	— 6 30
E. S. E.	S. 58 E.		— 9 30	— 6 50
S. E. by E.	S. 46 E.		— 10 15	— 7 20
S. E.	S. 36 E.		— 9 0	— 6 40
S. E. by S.	S. 26 E.		— 7 45	— 5 20
S. S. E.	S. 17 E.		— 5 30	— 3 20
S. by E.	S. 8 E.		— 3 15	— 1 10
SOUTH.	S. 2 W.		— 2 0	+ 0 20
S. by W.	S. 10 W.		+ 1 15	+ 3 10
S. S. W.	S. 20 W.		+ 2 30	+ 4 50
S. W. by S.	S. 30 W.		+ 3 45	+ 6 10
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign + a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviations are obtained:

A = — 2° 29′.3 B = — 1° 8′.5 C = — 3° 9′.7
D = + 6° 28′.2 E = — 0° 4′.5

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER AZIMUTH COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Sandy Point, February 10, 1866.

Correction for Object = + 0° 7′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 1° E.		— 1° 0′	— 0° 50′
N. by E.	N. 11 E.		+ 0 15	+ 0 20
N. N. E.	N. 19 E.		+ 3 30	+ 3 40
N. E. by N.	N. 30 E.		+ 3 45	+ 3 50
N. E.	N. 40 E.		+ 5 0	+ 5 10
N. E. by E.	N. 54 E.		+ 2 15	+ 2 20
E. N. E.	N. 63 E.		+ 4 30	+ 4 40
E. by N.	N. 80 E.		— 1 15	— 1 10
EAST.	S. 89 E.		— 1 0	— 0 50
E. by S.	S. 71 E.		— 7 45	— 7 40
E. S. E.	S. 60 E.		— 7 30	— 7 20
S. E. by E.	S. 47 E.		— 9 15	— 9 10
S. E.	S. 36 E.		— 9 0	— 8 50
S. E. by S.	S. 27 E.		— 6 45	— 6 40
S. S. E.	S. 16 E.		— 6 30	— 6 20
S. by E.	S. 8 E.		— 3 15	— 3 10
SOUTH.	S. 1 E.		+ 1 0	+ 1 10
S. by W.	S. 6 W.		+ 5 15	+ 5 20
S. S. W.	S. 17 W.		+ 5 30	+ 5 40
S. W. by S.	S. 26 W.		+ 7 45	+ 7 50
S. W.	S. 37 W.		+ 8 0	+ 8 10
S. W. by W.	S. 47 W.		+ 9 15	+ 9 20
W. S. W.	S. 58 W.		+ 9 30	+ 9 40
W. by S.	S. 74 W.		+ 4 45	+ 4 50
WEST.	S. 86 W.		+ 4 0	+ 4 10
W. by N.	N. 80 W.		+ 1 15	+ 1 20
W. N. W.	N. 66 W.		— 1 30	— 1 20
N. W. by W.	N. 50 W.		— 6 15	— 6 10
N. W.	N. 39 W.		— 6 0	— 5 50
N. W. by N.	N. 28 W.		— 5 45	— 5 40
N. N. W.	N. 18 W.		— 4 30	— 4 20
N. by W.	N. 6 W.		— 5 15	— 5 10
NORTH.				— 0 50

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 0° 5′.6 B = — 2° 57′.8 C = — 0° 47′.2
D = + 7° 10′.2 E = — 0° 25′.5

Valparaiso, April 4, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 2° E.		— 2° 0′	— 2° 0′
N. by E.	N. 10 E.		+ 1 15	+ 1 20
N. N. E.	N. 21 E.		+ 1 30	+ 1 30
N. E. by N.	N. 33 E.		+ 0 45	+ 0 50
N. E.	N. 45 E.		0 0	0 0
N. E. by E.	N. 57 E.		— 0 45	— 0 40
E. N. E.	N. 70 E.		— 2 30	— 2 30
E. by N.	N. 84 E.		— 5 15	— 5 10
EAST.	S. 82 E.		— 8 0	— 8 0
E. by S.	S. 66 E.		— 12 45	— 12 40
E. S. E.	S. 56 E.		— 11 30	— 11 30
S. E. by E.	S. 45 E.		— 11 15	— 11 10
S. E.	S. 33 E.		— 12 0	— 12 0
S. E. by S.	S. 24 E.		— 9 45	— 9 40
S. S. E.	S. 16 E.		— 6 30	— 6 30
S. by E.	S. 6 E.		— 5 15	— 5 10
SOUTH.	S. 2 W.		— 2 0	— 2 0
S. by W.	S. 11 W.		+ 0 15	+ 0 20
S. S. W.	S. 20 W.		+ 2 30	+ 2 30
S. W. by S.	S. 29 W.		+ 4 45	+ 4 50
S. W.	S. 38 W.		+ 7 0	+ 7 0
S. W. by W.	S. 50 W.		+ 6 15	+ 6 20
W. S. W.	S. 60 W.		+ 7 30	+ 7 30
W. by S.	S. 74 W.		+ 4 45	+ 4 50
WEST.	S. 87 W.		+ 3 0	+ 3 0
W. by N.	N. 78 W.		— 0 45	— 0 40
W. N. W.	N. 65 W.		— 2 30	— 2 30
N. W. by W.	N. 52 W.		— 4 15	— 4 10
N. W.	N. 40 W.		— 5 0	— 5 0
N. W. by N.	N. 29 W.		— 4 45	— 4 40
N. N. W.	N. 19 W.		— 3 30	— 3 30
N. by W.	N. 8 W.		— 3 15	— 3 10
NORTH.				— 2 0

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 2° 16′.2 B = — 4° 54′.1 C = + 0° 20′.9
D = + 5° 52′.5 E = + 0° 37′.5

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER AZIMUTH COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Callao, April 29, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 5° E.		— 5° 0′	— 4° 50′
N. by E.	N. 15 E.		— 3 45	— 3 40
N. N. E.	N. 21 E.		+ 1 30	+ 1 40
N. E. by N.	N. 35 E.		— 1 15	— 1 10
N. E.	N. 49 E.		— 4 0	— 3 50
N. E. by E.	N. 59 E.		— 2 45	— 2 40
E. N. E.	N. 69 E.		— 1 30	— 1 20
E. by N.	N. 82 E.		— 3 15	— 3 10
EAST.	S. 84 E.		— 6 0	— 5 50
E. by S.	S. 70 E.		— 8 45	— 8 40
E. S. E.	S. 59 E.		— 8 30	— 8 20
S. F. by E.	S. 45 E.		— 11 15	— 11 10
S. E.	S. 34 E.		— 11 0	— 10 50
S. E. by S.	S. 23 E.		— 10 45	— 10 40
S. S. E.	S. 15 E.		— 7 30	— 7 20
S. by E.	S. 8 E.		— 3 15	— 3 10
SOUTH.	S. 1 W.		— 1 0	— 0 50
S. by W.	S. 12 W.		— 0 45	— 0 40
S. S. W.	S. 22 W.		+ 0 30	+ 0 40
S. W. by S.	S. 31 W.		+ 2 45	+ 2 50
S. W.	S. 43 W.		+ 2 0	+ 2 10
S. W. by W.	S. 52 W.		+ 4 15	+ 4 20
W. S. W.	S. 65 W.		+ 2 30	+ 2 40
W. by S.	S. 80 W.		— 1 15	— 1 10
WEST.	N. 86 W.		— 4 0	— 3 50
W. by N.	N. 72 W.		— 6 45	— 6 40
W. N. W.	N. 62 W.		— 5 30	— 5 20
N. W. by W.	N. 49 W.		— 7 15	— 7 10
N. W.	N. 38 W.		— 7 0	— 6 50
N. W. by N.	N. 26 W.		— 7 45	— 7 40
N. N. W.	N. 15 W.		— 7 30	— 7 20
N. by W.	N. 5 W.		— 6 15	— 6 10
NORTH.				— 4 50

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 3° 56′.2 B = — 2° 0′.6 C = — 0° 49′.6
D = + 5° 6′.5 E = + 0° 35′.7

Panama, May 20, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 1° E.		— 1° 0′	— 1° 0′
N. by E.	N. 11 E.		+ 0 15	+ 0 20
N. N. E.	N. 21 E.		+ 1 30	+ 1 30
N. E. by N.	N. 31 E.		+ 2 45	+ 2 50
N. E.	N. 41 E.		+ 4 0	+ 4 0
N. E. by E.	N. 53 E.		+ 3 15	+ 3 20
E. N. E.	N. 68 E.		— 0 30	— 0 30
E. by N.	N. 81 E.		— 2 15	— 2 10
EAST.	S. 84 E.		— 6 0	— 6 0
E. by S.	S. 69 E.		— 9 45	— 9 40
E. S. E.	S. 56 E.		— 11 30	— 11 30
S. E. by E.	S. 45 E.		— 11 15	— 11 10
S. E.	S. 33 E.		— 12 0	— 12 0
S. E. by S.	S. 23 E.		— 10 45	— 10 40
S. S. E.	S. 13 E.		— 9 30	— 9 30
S. by E.	S. 4 E.		— 7 15	— 7 10
SOUTH.	S. 5 W.		— 5 0	— 5 0
S. by W.	S. 13 W.		— 1 45	— 1 40
S. S. W.	S. 21 W.		+ 1 30	+ 1 30
S. W. by S.	S. 29 W.		+ 4 45	+ 4 50
S. W.	S. 40 W.		+ 5 0	+ 5 0
S. W. by W.	S. 51 W.		+ 5 15	+ 5 20
W. S. W.	S. 62 W.		+ 5 30	+ 5 30
W. by S.	S. 73 W		+ 5 45	+ 5 50
WEST.	S. 88 W.		+ 2 0	+ 2 0
W. by N.	N. 79 W.		+ 0 15	+ 0 20
W. N. W.	N. 65 W.		— 2 30	— 2 30
N. W. by W.	N. 53 W.		— 3 15	— 3 10
N. W.	N. 40 W.		— 5 0	— 5 0
N. W. by N.	N. 29 W.		— 4 45	— 4 40
N. N. W.	N. 19 W.		— 3 30	— 3 30
N. by W.	N. 8 W.		— 3 15	— 3 10
NORTH.				— 1 0

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 2° 6′.9 B = — 3° 47′.2 C = — 1° 44′.6
D = + 6° 21′.2 E = — 0° 34′.0

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE AFTER AZIMUTH COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Acapulco, June 1, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 2° E.		— 2° 0′	— 1° 50′
N. by E.	N. 13 E.		— 1 45	— 1 40
N. N. E.	N. 23 E.		— 0 30	— 0 20
N. E. by N.	N. 33 E.		+ 0 45	+ 0 50
N. E.	N. 45 E.		0 0	+ 0 10
N. E. by E.	N. 57 E.		— 0 45	— 0 40
E. N. E.	N. 70 E.		— 2 30	— 2 20
E. by N.	N. 83 E.		— 4 15	— 4 10
EAST.	S. 82 E.		— 8 0	— 7 50
E. by S.	S. 69 E.		— 9 45	— 9 40
E. S. E.	S. 56 E.		— 11 30	— 11 20
S. E. by E.	S. 45 E.		— 11 15	— 11 10
S. E.	S. 34 E.		— 11 0	— 10 50
S. E. by S.	S. 24 E.		— 9 45	— 9 40
S. S. E.	S. 14 E.		— 8 30	— 8 20
S. by E.	S. 5 E.		— 6 15	— 6 10
SOUTH.	S. 3 W.		— 3 0	— 2 50
S. by W.	S. 11 W.		+ 0 15	+ 0 20
S. S. W.	S. 21 W.		+ 1 30	+ 1 40
S. W. by S.	S. 30 W.		+ 3 45	+ 3 50
S. W.	S. 40 W.		+ 5 0	+ 5 10
S. W. by W.	S. 51 W.		+ 5 15	+ 5 20
W. S. W.	S. 63 W.		+ 4 30	+ 4 40
W. by S.	S. 77 W.		+ 1 45	+ 1 50
WEST.	N. 89 W.		+ 1 0	+ 1 10
W. by N.	N. 76 W.		— 2 45	— 2 40
W. N. W.	N. 62 W.		— 5 30	— 5 20
N. W. by W.	N. 49 W.		— 7 15	— 7 10
N. W.	N. 38 W.		— 7 0	— 6 50
N. W. by N.	N. 28 W.		— 5 45	— 5 40
N. N. W.	N. 17 W.		— 5 30	— 5 20
N. by W.	N. 6 W.		— 5 15	— 5 10
NORTH.				— 1 50

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

$A = -3° \; 11'.2 \qquad B = -3° \; 25'.8 \qquad C = -0° \; 0'.8$

$D = +5° \; 54'.2 \qquad E = +0° \; 23'.8$

Magdalena Bay, June 9, 1866.

Correction for Object = — 0° 41′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 7° E.		— 7° 0′	— 7° 30′
N. by E.	N. 15 E.		— 3 45	— 4 20
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.				
E. by S.				
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.	S. 34 W.		+ 11 0	+ 10 10
S. W. by W.	S. 47 W.		+ 9 15	+ 8 40
W. S. W.	S. 58 W.		+ 9 30	+ 8 40
W. by S.	S. 71 W.		+ 7 45	+ 7 0
WEST.	S. 85 W.		+ 5 0	+ 4 30
W. by N.	N. 87 W.		+ 8 15	+ 7 30
W. N. W.	N. 73 W.		+ 5 30	+ 4 40
N. W. by W.	N. 59 W.		+ 2 45	+ 1 40
N. W.	N. 41 W.		— 4 0	— 4 10
N. W. by N.	N. 38 W.		+ 4 15	+ 3 40
N. N. W.	N. 27 W.		+ 4 30	+ 3 10
N. by W.	N. 4 W.		— 7 15	— 8 0
NORTH.				— 7 30

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

These observations exhibit such discordancies among themselves that they do not seem worth the trouble of reducing.

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD ALIDADE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Hampton Roads, November 1, 1865.

Correction for Object = + 3° 57′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. $\frac{3}{8}$ E.	— $\frac{3}{8}$	° ′	+ 0° 10′
N. by E.	N. by E. $\frac{3}{8}$ E.	— $\frac{3}{8}$		— 0 20
N. N. E.	N. E. by N. $\frac{5}{8}$ N.	— $\frac{3}{8}$		— 0 20
N. E. by N.	N. E. $\frac{3}{4}$ N.	— $\frac{1}{4}$		+ 0 40
N. E.	N. E. $\frac{3}{8}$ E.	— $\frac{3}{8}$		+ 0 10
N. E. by E.	N. E. by E. $\frac{3}{8}$ E.	— $\frac{3}{8}$		— 0 20
E. N. E.	E. by N. $\frac{5}{8}$ N.	— $\frac{3}{8}$		— 0 20
E. by N.	E. $\frac{3}{8}$ N.	— $\frac{3}{8}$		— 0 20
EAST.	E. $\frac{3}{8}$ S.	— $\frac{3}{8}$		— 0 20
E. by S.	E. S. E. $\frac{3}{4}$ E.	— $\frac{1}{4}$		+ 0 40
E. S. E.	S. E. by E. $\frac{5}{8}$ E.	— $\frac{3}{8}$		+ 0 10
S. E. by E.	S. E. $\frac{5}{8}$ E.	— $\frac{3}{8}$		— 0 20
S. E.	S. E. $\frac{1}{4}$ S.	— $\frac{1}{4}$		+ 0 40
S. E. by S.	S. S. E. $\frac{3}{4}$ E.	— $\frac{1}{4}$		+ 1 10
S. S. E.	S. by E. $\frac{7}{8}$ E.	— $\frac{1}{8}$		+ 2 10
S. by E.	S. $\frac{7}{8}$ E.	— $\frac{1}{8}$		+ 2 30
SOUTH.	SOUTH.	0		+ 3 30
S. by W.	S. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 4 50
S. S. W.	S. by W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 6 20
S. W. by S.	S. W. by S. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 6 40
S. W.	S. W. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 6 40
S. W. by W.	S. W. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 5 50
W. S. W.	S. W. by W. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 5 20
W. by S.	W. by S. $\frac{1}{8}$ S.	+ $\frac{1}{8}$		+ 5 20
WEST.	WEST.	0		+ 4 30
W. by N.	W. by N.	0		+ 4 0
W. N. W.	N. W. by W. $\frac{7}{8}$ W.	— $\frac{1}{8}$		+ 3 0
N. W. by W.	N. W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		+ 1 40
N. W.	N. W. $\frac{1}{4}$ N.	— $\frac{1}{4}$		+ 1 10
N. W. by N.	N. N. W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		+ 1 10
N. N. W.	N. by W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		+ 1 10
N. by W.	N. $\frac{3}{4}$ W.	— $\frac{1}{4}$		+ 1 10
NORTH.				+ 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 2° 8′.1 B = — 2° 28′.4 C = — 1° 52′.0
D = + 1° 4′.2 E = 0° 0′.0

St. Thomas, November 18, 1865.

Correction for Object = + 0° 16′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N.	0	° ′	— 0° 10′
N. by E.	N. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
N. N. E.	N. by E. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
N. E. by N.	N. E. by N. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 10
N. E.	N. E. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 10
N. E. by E.	N. E. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
E. N. E.	N. E. by E. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
E. by N.	E. by N.	0		— 0 20
EAST.	E.	0		— 0 20
E. by S.	E. by S.	0		— 0 20
E. S. E.	E. S. E.	0		— 0 20
S. E. by E.	S. E. by E.	0		— 0 20
S. E.	S. E.	0		— 0 20
S. E. by S.	S. E. by S.	0		— 0 20
S. S. E.	S. S. E.	0		— 0 10
S. by E.	S. by E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
SOUTH.	S. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
S. by W.	S. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 30
S. S. W.	S. by W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 30
S. W. by S.	S. W. by S. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 2 30
S. W.	S. W. $\frac{3}{8}$ S.	+ $\frac{3}{8}$		+ 3 50
S. W. by W.	S. W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 30
W. S. W.	S. W. by W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 30
W. by S.	W. S. W. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 1 10
WEST.	W. $\frac{1}{8}$ S.	+ $\frac{1}{8}$		+ 1 10
W. by N.	W. $\frac{7}{8}$ N.	+ $\frac{1}{8}$		+ 1 10
W. N. W.	W. N. W.	0		— 0 20
N. W. by W.	N. W. by W.	0		— 0 20
N. W.	N. W.	0		— 0 20
N. W. by N.	N. W. by N.	0		— 0 20
N. N. W.	N. N. W.	0		— 0 20
N. by W.	N. by W.	0		— 0 20
NORTH.				— 0 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 50′.9 B = — 0° 35′.1 C = — 0° 46′.2
D = + 1° 15′.7 E = + 0° 20′.5

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD ALIDADE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Isle Royal, Salute Islands, November 30, 1865.

Correction for Object = — 0° 2′. Correction for Lubber Line = 0.°

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.			° ′	° ′
N. by E.				
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.	EAST.			
E. by S.				
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = B = C =
D = E =

Ceara, December 19, 1865.

Correction for Object = + 1° 51′. Correction for Lubber Line = 0°.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.				
N. by E.	N. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 3° 0
N. N. E.	N. N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 4 30
N. E. by N.	N. E. by N. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 4 40
N. E.	N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 4 40
N. E. by E.	N. E. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 3 30
E. N. E.	E. N. E. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 3 20
E. by N.	E. by N.	0		+ 2 10
EAST.	EAST.	0		+ 1 50
E. by S.	E. by S.	0		+ 1 50
E. S. E.	S. E. by E. $\frac{7}{8}$ E.	— $\frac{1}{8}$		+ 0 40
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 2° 3′.6 B = + 0° 0′.1 C = + 1° 4′.7
D = + 1° 21′.4 E = + 0° 2′.4

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD ALIDADE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Bahia, December 30, 1865.

Correction for Object = + 2° 30′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	
NORTH.	NORTH.	0		+ 2° 10′
N. by E.	N. by E.	0		+ 2 30
N. N. E.	N. N. E.	0		+ 2 30
N. E. by N.	N. E. by N.	0		+ 2 30
N. E.	N. E.	0		+ 2 30
N. E. by E.	N. E. by E.	0		+ 2 30
E. N. E.	E. N. E.	0		+ 2 30
E. by N.	E. by N.	0		+ 2 30
EAST.	EAST.	0		+ 2 30
E. by S.	E. by S. ⅛ S.	— ⅛		+ 1 30
E. S. E.	S. E. by E. ⅞ E.	— ⅛		+ 1 10
S. E. by E.	S. E. ⅞ E.	— ⅛		+ 1 10
S. E.	S. E. ⅛ S.	— ⅛		+ 1 10
S. E. by S.	S. S. E ⅞ E.	— ⅛		+ 1 10
S. S. E.	S. S. E.	0		+ 2 10
S. by E.	S. by E.	0		+ 2 30
SOUTH.	SOUTH.	0		+ 2 30
S. by W.	S. ⅞ W.	+ ⅛		+ 3 30
S. S. W.	S. by W. ⅞ W.	+ ⅛		+ 3 50
S. W. by S.	S. W. by S. ⅛ S.	+ ⅛		+ 3 50
S. W.	S. W. ⅛ S.	+ ⅛		+ 3 50
S. W. by W.	S. W. ⅞ W.	+ ⅛		+ 3 50
W. S. W.	S. W. by W. ⅞ W.	+ ⅛		+ 3 50
W. by S.	W. by S.	0		+ 2 50
WEST.	WEST.	0		+ 2 30
W. by N.	W. by N. ¼ N.	— ¼		+ 0 20
W. N. W.	N. W. by W. ¾ W.	— ¼		— 0 20
N. W. by W.	N. W. ⅞ W.	— ⅛		+ 0 50
N. W.	N. W. ⅛ N.	— ⅛		+ 1 10
N. W. by N.	N. W. by N. ⅛ N.	— ⅛		+ 1 10
N. N. W.	N. by W. ⅞ W.	— ⅛		+ 1 10
N. by W.	N. ⅞ W.	— ⅛		+ 1 10
NORTH.				+ 2 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 2° 9′.4 B = — 0° 6′.0 C = — 0° 34′.1
D = + 1° 15′.0 E = + 0° 14′.5

Rio Janeiro, January 10, 1866.

Correction for Object = + 2° 44′. Correction for Lubber Line = 0°.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	
NORTH.				
N. by E.				
N. N. E.				
N. E. by N.	N. E. by N. ⅛ N.	+ ⅛		+ 4° 10′
N. E.	N. E. ⅛ N.	+ ⅛		+ 4 10
N. E. by E.	N. E. ⅞ E.	+ ⅛		+ 4 10
E. N. E.	E. N. E.	0		+ 3 0
E. by N.	E. by N.	0		+ 2 40
EAST.	EAST.	0		+ 2 40
E. by S.	E. by S.	0		+ 2 40
E. S. E.	E. S. E.	0		+ 2 40
S. E. by E.	S. E. by E.	0		+ 2 40
S. E.	S. E.	0		+ 2 40
S. E. by S.	S. E. by S.	0		+ 2 40
S. S. E.	S. S. E. ⅛ E.	+ ⅛		+ 3 50
S. by E.		+ ⅛		+ 4 10
SOUTH.	E. ⅛ E.	+ ⅛		+ 4 10
S. by W.	S. ¾ W.	+ ¼		+ 5 10
S. S. W.	S. by W. ¾ W.	+ ¼		+ 5 30
S. W. by S.	S. S. W. ¾ W.	+ ¼		+ 5 30
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the cofficients of the deviation are obtained:

A = + 3° 31′.5 B = — 0° 28′.8 C = — 0° 57′.1
D = + 1° 21′.1 E = + 0° 1′.9

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD ALIDADE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Monte Video, January 24, 1866.

Correction for Object = − 0° 13′ Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	− 0° 10′
N. by E.	N. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
N. N. E.	N. by E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
N. E. by N.	N. E. by N. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 0
N. E.	N. E. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 0
N. E. by E.	N. E. $\frac{5}{8}$ E.	+ $\frac{3}{8}$		+ 4 0
E. N. E.	E. N. E. $\frac{3}{8}$ N..	+ $\frac{3}{8}$		+ 4 0
E. by N.	E. by N. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 0
EAST.	E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 30
E. by S.	E. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 2 30
E. S. E.	E. S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
S. E. by E.	S. E. by E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
S. E.	S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
S. E. by S.	S. E. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 2 30
S. S. E.	S. S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
S. by E.	S. by E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
SOUTH.	S. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
S. by W.	S. $\frac{1}{2}$ W.	+ $\frac{1}{2}$		+ 5 20
S. S. W.	S. by W. $\frac{5}{8}$ W.	+ $\frac{3}{8}$		+ 4 0
S. W. by S.	S. W. by S. $\frac{3}{8}$ S.	+ $\frac{3}{8}$		+ 4 0
S. W.	S. W. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 2 30
S. W. by W.	S. W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 30
W. S. W.	S. W. by W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 30
W. by S.	W. by S. $\frac{1}{8}$ S.	+ $\frac{1}{8}$		+ 1 10
WEST.	W. $\frac{1}{8}$ S.	+ $\frac{1}{8}$		+ 1 10
W. by N.	W. $\frac{7}{8}$ N.	+ $\frac{1}{8}$		+ 1 10
W. N. W.		+ $\frac{1}{8}$		+ 1 10
N. W. by W.	N. W. by W.	0		− 0 10
N. W.	N. W.	0		− 0 10
N.W. by N.	N. W. by N.	0		− 0 10
N. N. W.	N. N. W.	0		− 0 10
N. by W.	N. $\frac{7}{8}$ W.	− $\frac{1}{8}$		− 1 40
NORTH.				− 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 2° 7′.1 B = + 0° 57′.2 C = − 1° 5′.0
D = + 1° 23′.0 E = − 0° 9′.8

Sandy Point, February 10, 1866.

Correction for Object = + 0° 7′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	+ 0° 10′
N. by E.	N. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
N. N. E.	N. by E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 3 0
N. E. by N.	N. N. E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 3 0
N. E.	N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 3 0
N. E. by E.	N. E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 3 0
E. N. E.	N. E. by E. $\frac{5}{8}$ E.	+ $\frac{3}{8}$		+ 4 20
E. by N.	E. by N. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 20
EAST.	E. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 20
E. by S.	E. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
E. S. E.	E. by S. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
S. E. by E.	S. E. by E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 3 0
S. E.	S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 3 0
S. E. by S.	S. E. $\frac{7}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
S. S. E.	S. S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 3 0
S. by E.	S. by E. $\frac{3}{8}$ E.	+ $\frac{3}{8}$		+ 4 20
SOUTH.	S. $\frac{3}{8}$ E.	+ $\frac{3}{8}$		+ 4 20
S. by W.	S. $\frac{5}{8}$ W.	+ $\frac{3}{8}$		+ 4 20
S. S. W.	S. by W. $\frac{5}{8}$ W.	+ $\frac{3}{8}$		+ 4 20
S. W. by S.	S. W. by S. $\frac{1}{2}$ S.	+ $\frac{1}{2}$		+ 5 50
S. W.	S. W. $\frac{3}{8}$ S.	+ $\frac{3}{8}$		+ 4 20
S. W. by W.	S. W $\frac{5}{8}$ W.	+ $\frac{3}{8}$		+ 4 20
W. S. W.	S. W. by W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 3 0
W. by S.	W. by S. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
WEST.	W. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
W. by N.	W. by N.	0		+ 0 10
W. N. W.	W. N. W.	0		+ 0 10
N. W. by W.	N. W. $\frac{7}{8}$ W.	− $\frac{1}{8}$		− 1 20
N. W.	N W. $\frac{1}{8}$ N.	− $\frac{1}{8}$		− 1 20
N. W. by N.	N. W. by N. $\frac{1}{8}$ N.	− $\frac{1}{8}$		− 1 20
N. N. W.	N. N. W. $\frac{1}{8}$ N.	− $\frac{1}{8}$		− 1 20
N. by W.	N. $\frac{7}{8}$ W.	− $\frac{1}{8}$		− 1 20
NORTH.				+ 0 10

A deviation of the North Point of the Compass to the East is designated by the sign + a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviations are obtained:

A = + 2° 25′.6 B = + 0° 58′.5 C = − 1° 54′.4
D = + 1° 47′.0 E = − 0° 20′.2

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD ALIDADE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Valparaiso, April 4, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	0° 0′
N. by E.	N. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
N. N. E.	N. by E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
N. E. by N.	N. E. by N. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 50
N. E.	N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 50
N. E. by E.	N. E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
E. N. E.	N. E. by E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
E. by N.	E. by N. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 50
EAST.	E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 50
E. by S.	E. $\frac{7}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
E. S. E.	E. S. E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. E. by E.	S. E. by E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. E.	S. E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. E. by S.	S. E. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 2 50
S. S. E.	S. S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
S. by E.	S. by E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
SOUTH.	S. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
S. by W.	S. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 50
S. S. W.	S. by W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 50
S. W. by S.	S. W. by S. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 2 50
S. W.	S. W. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 2 50
S. W. by W.	S. W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 50
W. S. W.	S. W. by W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 50
W. by S.	W. by S. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 2 50
WEST.	W. $\frac{1}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
W. by N.	W. $\frac{7}{8}$ N.	+ $\frac{1}{8}$		+ 1 30
W. N. W.	W. N. W.	0		0 0
N. W. by W.	N. W. by W.	0		0 0
N. W.	N. W.	0		0 0
N. W. by N.	N. W. by N.	0		0 0
N. N. W.	N. N. W.	0		0 0
N. by W.	N. by W.	0		0 0
NORTH.				0 0

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 55′.2 B = + 0° 30′.0 C = — 0° 53′.9
D = + 1° 4′.2 E = — 0° 5′.2

Callao, April 29, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. $\frac{1}{8}$ E.	— $\frac{1}{8}$	° ′	— 1° 20′
N. by E.	N. by E.	0		+ 0 10
N. N. E.	N. N. E.	0		+ 0 10
N. E. by N.	N. E. by N.	0		+ 0 10
N. E.	N. E. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 30
N. E. by E.	N. E. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
E. N. E.	N. E. by E. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
E. by N.	E. by N. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 30
EAST.	EAST.	0		+ 0 10
E. by S.	E. $\frac{7}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
E. S. E.	E. S. E.	0		+ 0 10
S. E. by E.	S. E. by E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. E.	S. E.	0		+ 0 10
S. E. by S.	S. E. by S.	0		+ 0 10
S. S. E.	S. S. E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. by E.	S. by E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
SOUTH.	S. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. by W.	S. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 1 30
S. S. W.	S. by W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 3 0
S. W. by S.	S. W. by S. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
S. W.	S. W. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
S. W. by W.	S. W. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 1 30
W. S. W.	S. W. by W. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 1 30
W. by S.	W. by S.	0		+ 0 10
WEST.	WEST.	0		+ 0 10
W. by N.	W. by N. $\frac{1}{8}$ N.	— $\frac{1}{8}$		— 1 20
W. N. W.	N. W. by W. $\frac{7}{8}$ W.	— $\frac{1}{8}$		— 1 20
N. W. by W.	N. W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 40
N. W.	N. W. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 2 40
N. W. by N.	N. N. W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 40
N. N. W.	N. by W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 40
N. by W.	N. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 40
NORTH.				— 1 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 21′.0 B = + 0° 40′.9 C = — 1° 36′.4
D = + 1° 29′.0 E = — 0° 6′.8

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD ALIDADE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Panama, May 20, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	°	0° 0′
N. by E.	N. ⅞ E.	+ ⅛		+ 1 30
N. N. E.	N. by E. ¾ E.	+ ¼		+ 2 50
N. E. by N.	N. E. by N. ¼ N.	+ ¼		+ 2 50
N. E.	N. E. ¼ N.	+ ¼		+ 2 50
N. E. by E.	N. E. ¾ E.	+ ¼		+ 2 50
E. N. E.	N. E. by E. ⅞ E.	+ ⅛		+ 1 30
E. by N.	E. by N. ¼ N.	+ ¼		+ 2 50
EAST.	E. ¼ N.	+ ¼		+ 2 50
E. by S.	E. ⅞ S.	+ ⅛		+ 1 30
E. S. E.	E. by S. ⅞ S.	+ ⅛		+ 1 30
S. E. by E.	S. E. by E. ⅛ E.	+ ⅛		+ 1 30
S. E.	S. E. ¼ E.	+ ¼		+ 2 50
S. E. by S.	S. E. ¾ S.	+ ¼		+ 2 50
S. S. E.	S. S. E. ¼ E.	+ ¼		+ 2 50
S. by E.	S. by E. ¼ E.	+ ¼		+ 2 50
SOUTH.	S. ¼ E.	+ ¼		+ 2 50
S. by W.	S. ⅜ W.	+ ⅜		+ 4 10
S. S. W.	S. by W. ⅝ W.	+ ⅜		+ 4 10
S. W. by S.	S. W. by S. ⅜ S.	+ ⅜		+ 4 10
S. W.	S. W. ⅜ S.	+ ⅜		+ 4 10
S. W. by W.	S. W. ⅝ W.	+ ⅜		+ 4 10
W. S. W.	S. W. by W. ⅝ W.	+ ⅜		+ 4 10
W. by S.	W. by S. ⅜ S.	+ ⅜		+ 4 10
WEST.	W. ¼ S.	+ ¼		+ 2 50
W. by N.	W. ⅞ N.	+ ⅛		+ 1 30
W. N. W.	W. N. W.	0		0 0
N. W. by W.	N. W. by W.	0		0 0
N. W.	N. W.	0		0 0
N. W. by N.	N. W. by N.	0		0 0
N. N. W.	N. N. W.	0		0 0
N. by W.	N. by W.	0		0 0
NORTH.				0 0

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 2° 15′.2 B = + 0° 1′.1 C = — 1° 22′.1
D = + 1° 21′.0 E = — 0° 6′.8

Acapulco, June 1, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	+ 0° 10
N. by E.	N. ⅞ E.	+ ⅛		+ 1 30
N. N. E.	N. by E. ⅞ E.	+ ⅛		+ 1 30
N. E. by N.	N. E. by N. ⅛ N.	+ ⅛		+ 1 30
N. E.	N. E. ⅛ N.	+ ⅛		+ 1 30
N. E. by E.	N. E. ⅞ E.	+ ⅛		+ 1 30
E. N. E.	N. E. by E. ⅞ E.	+ ⅛		+ 1 30
E. by N.	E. by N.	0		+ 0 10
EAST.	E. ⅛ S.	— ⅛		— 1 20
E. by S.	E. by S. ⅛ S.	— ⅛		— 1 20
E. S. E.	S. E. by E. ⅞ E.	— ⅛		— 1 20
S. E. by E.	S. E. ⅞ E.	— ⅛		— 1 20
S. E.	S. E. ⅛ S.	— ⅛		— 1 20
S. E. by S.	S. E. by S. ⅛ S.	— ⅛		— 1 20
S. S. E.	S. S. E.	0		+ 0 10
S. by E.	S. by E. ⅛ E.	+ ⅛		+ 1 30
SOUTH.	S. ⅛ E.	+ ⅛		+ 1 30
S. by W.	S. ¾ W.	+ ¼		+ 3 0
S. S. W.	S. by W. ¾ W.	+ ¼		+ 3 0
S. W. by S.	S. W. by S. ⅜ S.	+ ⅜		+ 4 20
S. W.	S. W. ⅜ S.	+ ⅜		+ 4 20
S. W. by W.	S. W. ⅝ W.	+ ⅜		+ 4 20
W. S. W.	S. W. by W. ⅝ W.	+ ⅜		+ 4 20
W. by S.	W. by S. ¼ S.	+ ¼		+ 3 0
WEST.	W. ¼ S.	+ ¼		+ 3 0
W. by N.	W. ⅞ N.	+ ⅛		+ 1 30
W. N. W.	W. N. W.	0		+ 0 10
N. W. by W.	N. W. by W.	0		+ 0 10
N. W.	N. W.	0		+ 0 10
N. W. by N.	N. W. by N.	0		+ 0 10
N. N. W.	N. N. W.	0		+ 0 10
N. by W.	N. by W.	0		+ 0 10
NORTH.				+ 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 8′.1 B = — 1° 28′.4 C = — 0° 33′.1
D = + 1° 52′.8 E = + 0° 10′.2

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD ALIDADE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Magdalena Bay, June 9, 1866.

Correction for Object = — 0° 41′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	— 0° 40′
N. by E.	N. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 0 50
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.				
E. by S.				
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.	S. W. $\frac{1}{2}$ S.	+ $\frac{1}{2}$		+ 5 0
S. W. by W.	S. W. $\frac{3}{8}$ W.	+ $\frac{5}{8}$		+ 6 10
W. S. W.	S. W. by W. $\frac{5}{8}$ W.	+ $\frac{3}{8}$		+ 3 30
W. by S.	W. by S. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 2 10
WEST.	W. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 2 10
W. by N.	W. $\frac{5}{8}$ N.	+ $\frac{3}{8}$		+ 3 30
W. N. W.	W. by N. $\frac{3}{4}$ N.	+ $\frac{1}{4}$		+ 2 10
N. W. by W.	N. W. by W. $\frac{1}{4}$ W.	+ $\frac{1}{4}$		+ 2 0
N. W.	N. W. $\frac{1}{8}$ W.	+ $\frac{1}{8}$		+ 0 40
N. W. by N.	N. W. $\frac{7}{8}$ N.	+ $\frac{1}{8}$		+ 0 40
N. N. W.	N. N. W.	0		— 0 40
N. by W.	N. by W.	0		— 0 40
NORTH.				— 0 40

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 1° 8′.8 B = — 2° 4′.1 C = — 1° 7′.6
D = + 1° 19′.2 E = 0° 0′.0

San Francisco, June 23, 1866.

Correction for Object = — 0° 45′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. $\frac{1}{8}$ E.	— $\frac{1}{8}$	° ′	— 2° 10′
N. by E.	N. by E. $\frac{1}{8}$ E.	— $\frac{1}{8}$		— 2 10
N. N. E.	N. N. E. $\frac{1}{8}$ E.	— $\frac{1}{8}$		— 2 0
N. E. by N.	N. E. by N.	0		— 0 40
N. E.	N. E.	0		— 0 40
N. E. by E.	N. E. by E.	0		— 0 40
E. N. E.	E. N. E.	0		— 0 50
E. by N.	E. $\frac{7}{8}$ N.	— $\frac{1}{8}$		— 2 10
EAST.	E. $\frac{1}{8}$ S.	— $\frac{1}{8}$		— 2 10
E. by S.	E. by S. $\frac{1}{8}$ S.	— $\frac{1}{8}$		— 2 0
E. S. E.	E. S. E.	0		— 0 40
S. E. by E.	S. E. by E.	0		— 0 40
S. E.	S. E.	0		— 0 40
S. E. by S.	S. E. $\frac{7}{8}$ S.	+ $\frac{1}{8}$		+ 0 50
S. S. E.	S. S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 10
S. by E.	S. by E. $\frac{3}{8}$ E.	+ $\frac{3}{8}$		+ 3 30
SOUTH.	S. $\frac{3}{8}$ E.	+ $\frac{3}{8}$		+ 3 30
S. by W.	S. $\frac{5}{8}$ W.	+ $\frac{3}{8}$		+ 3 30
S. S. W.	S. by W. $\frac{1}{2}$ W.	+ $\frac{1}{2}$		+ 4 50
S. W. by S.	S. W. by S. $\frac{1}{2}$ S.	+ $\frac{1}{2}$		+ 4 50
S. W.	S. W. $\frac{1}{2}$ S.	+ $\frac{1}{2}$		+ 4 50
S. W. by W.	S. W. $\frac{1}{2}$ W.	+ $\frac{1}{2}$		+ 4 50
W. S. W.	S. W. by W. $\frac{5}{8}$ W.	+ $\frac{3}{8}$		+ 3 20
W. by S.	W. by S. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 2 0
WEST.	W. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 2 0
W. by N.	W. $\frac{3}{4}$ N.	+ $\frac{1}{4}$		+ 1 50
W. N. W.	W. N. W.	0		— 0 30
N. W. by W.	N. W. by W. $\frac{1}{4}$ W.	+ $\frac{1}{4}$		+ 1 50
N. W.	N. W.	0		— 0 30
N. W. by N.	N. W. by N.	0		— 0 40
N. N. W.	N. N. W.	0		— 0 50
N. by W.	N. $\frac{7}{8}$ W.	— $\frac{1}{8}$		— 2 10
NORTH.				— 2 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 40′.6 B = — 1° 54′.2 C = — 2° 25′.1
D = + 0° 58′.0 E = + 0° 21′.5

Observations for Determining the Deviations of the Forward Binnacle Compass on the U. S. Iron Clad Monadnock.

Hampton Roads, November 1, 1865.

Correction for Object = + 3° 57′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ½ E.	− ½	° ′	− 1° 40′
N. by E.	N. by E. ½ E.	− ½		− 1 40
N. N. E.	N. E. by N. ½ N.	− ½		− 1 40
N. E. by N.	N. E. ½ N.	− ½		− 1 40
N. E.	N. E. ⅜ E.	− ⅜		− 0 50
N. E. by E.	N. E. by E. ¾ E.	− ¾		− 3 0
E. N. E.	E. by N. ⅛ N.	− ⅞		− 5 30
E. by N.	E. ⅛ N.	− ⅞		− 6 0
EAST.	E. ⅞ S.	− ⅞		− 6 0
E. by S.	E. S. E. ¼ E.	− ¾		− 5 0
E. S. E.	S. E. by E. ¼ E.	− ¾		− 4 30
S. E. by E.	S. E. ⅜ E.	− ⅝		− 3 40
S. E.	S. E. ⅝ S.	− ⅝		− 3 0
S. E. by S.	S. S. E. ½ E.	− ½		− 2 10
S. S. E.	S. by E. ⅝ E.	− ⅜		− 0 50
S. by E.	S. ⅞ E.	− ⅛		+ 1 30
SOUTH.	SOUTH.	0		+ 3 30
S. by W.	S. ⅞ W.	+ ⅛		+ 5 0
S. S. W.	S. by W. ¾ W.	+ ¼		+ 6 20
S. W. by S.	S. W. by S. ⅜ S.	+ ⅜		+ 7 40
S. W.	S. W. ½ S.	+ ½		+ 9 10
S. W. by W.	S. W. ½ W.	+ ½		+ 9 30
W. S. W.	S. W. by W. ½ W.	+ ½		+ 9 30
W. by S.	W. by S. ¼ S.	+ ¼		+ 7 40
WEST.	W. ¼ S.	+ ¼		+ 6 40
W. by N.	W. N. W. ⅞ W.	− ⅛		+ 4 0
W. N. W.	W. N. W.	0		+ 3 30
N. W. by W.	N. W. ¾ W.	− ¼		+ 2 10
N. W.	N. W. ¼ N.	− ¼		+ 1 10
N. W. by N.	N. N. W. ½ W.	− ½		− 0 40
N. N. W.	N. by W. ½ W.	− ½		− 1 40
N. by W.	N. ½ W.	− ½		− 1 40
NORTH.				− 1 40

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign −.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 49′.0 B = − 5° 40′.8 C = − 2° 33′.4
D = + 2° 17′.7 E = + 0° 8′.2

St. Thomas, November 18, 1865.

Correction for Object = + 0° 16′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N.	0	° ′	− 0° 20′
N. by E.	N. by E.	0		− 0 20
N. N. E.	N. N. E.	0		− 0 20
N. E. by N.	N. E. by N.	0		− 0 20
N. E.	N. E.	0		− 0 20
N. E. by E.	N. E. by E.	0		− 0 20
E. N. E.	E. by N. ⅞ N.	− ⅛		− 1 40
E. by N.	E. ⅞ N.	− ⅛		− 1 40
EAST.	E. ¼ S.	− ¼		− 3 10
E. by S.	E. by S. ¼ S.	− ¼		− 3 10
E. S. E.	S. E. by E. ⅝ E.	− ⅜		− 4 30
S. E. by E.	S. E. ¾ E.	− ¼		− 3 10
S. E.	S. E. ¼ S.	− ¼		− 3 10
S. E. by S.	S. E. by S. ¼ S.	− ¼		− 3 10
S. S. E.	S. by E. ¾ E.	− ¼		− 3 10
S. by E.	S. ⅞ E.	− ⅛		− 1 40
SOUTH.	S.	0		− 0 20
S. by W.	S. by W.	0		− 0 20
S. S. W.	S. by W. ⅞ W.	+ ⅛		+ 1 10
S. W. by S.	S. W. by S. ⅛ S.	+ ⅛		+ 1 10
S. W.	S. W. ¼ S.	+ ¼		+ 2 30
S. W. by W.	S. W. ⅝ W.	+ ⅜		+ 4 0
W. S. W.	S. W. by W. ⅝ W.	+ ⅜		+ 4 0
W. by S.	W. by S. ¼ S.	+ ¼		+ 2 30
WEST.	W. ⅛ S.	+ ⅛		+ 1 10
W. by N.	W. by N.	0		− 0 20
W. N. W.	W. N. W.	0		− 0 20
N. W. by W.	N. W. ⅞ W.	− ⅛		− 1 40
N. W.	N. W. ⅛ N.	− ⅛		− 1 40
N. W. by N.	N. W. by N. ⅛ N.	− ⅛		− 1 40
N. N. W.	N. by W. ⅞ W.	− ⅛		− 1 40
N. by W.	N ⅞ W.	− ⅛		− 1 40
NORTH.				− 0 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign −.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = − 0° 44′.4 B = − 1° 56′.2 C = − 0° 12′.4
D = + 1° 59′.5 E = − 0° 7′.2

Observations for Determining the Deviations of the Forward Binnacle Compass on the U. S. Iron Clad Monadnock.

Isle Royal, Salute Islands, November 30, 1865.

Correction for Object = — 0° 2′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			°	° ′
NORTH.				
N. by E.				
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.	EAST.	0		0 0
E. by S.				
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = B = C =
D = E =

Ceara, December 19, 1865.

Correction for Object = + 1° 51′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	
NORTH.				
N. by E.	N. by E.	0		+ 1 50
N. N. E.	N. N. E.	0		+ 1 50
N. E. by N.	N. E. by N.	0		+ 1 50
N. E.	N. E. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 3 0
N. E. by E.	N. E. by E.	0		+ 2 10
E. N. E.	E. N. E.	0		+ 1 50
E. by N.	E. $\frac{7}{8}$ N.	— $\frac{1}{8}$		+ 0 40
EAST.	E. $\frac{1}{4}$ S.	— $\frac{1}{4}$		— 0 40
E. by S.	E. by S. $\frac{3}{8}$ S.	— $\frac{3}{8}$		— 2 10
E. S. E.	S. E. by E. $\frac{5}{8}$ E.	— $\frac{3}{8}$		— 2 20
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 0° 54′.7 B = + 0° 24′.6 C = + 1° 26′.9
D = + 2° 7′.8 E = + 0° 3′.2

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD BINNACLE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Bahia, December 30, 1865.

Correction for Object = + 2° 30′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	+ 1° 50′
N. by E.	N. by E.	0		+ 2 30
N. N. E.	N. N. E.	0		+ 2 30
N. E. by N.	N. E. by N.	0		+ 2 30
N. E.	N. E.	0		+ 2 30
N. E. by E.	N. E. by E.	0		+ 2 30
E. N. E.	E. N. E.	0		+ 2 30
E. by N.	E. by N.	0		+ 2 30
EAST.	E. $\frac{1}{8}$ S.	— $\frac{1}{8}$		+ 1 30
E. by S.	E. by S. $\frac{1}{8}$ S.	— $\frac{1}{8}$		+ 1 10
E. S. E.	S. E. by E. $\frac{3}{4}$ E.	— $\frac{1}{4}$		0 0
S. E. by E.	S. E. $\frac{3}{4}$ E.	— $\frac{1}{4}$		— 0 20
S. E.	S. E. $\frac{1}{4}$ S.	— $\frac{1}{4}$		— 0 20
S. E. by S.	S. E. by S. $\frac{1}{4}$ S.	— $\frac{1}{4}$		— 0 20
S. S. E.	S. by E. $\frac{3}{4}$ E.	— $\frac{1}{4}$		— 0 20
S. by E.	S. $\frac{7}{8}$ E.	— $\frac{1}{8}$		+ 0 50
SOUTH.	SOUTH.	0		+ 2 10
S. by W.	S. by W.	0		+ 2 30
S. S. W.	S. S. W.	0		+ 2 30
S. W. by S.	S. W. by S.	0		+ 2 30
S. W.	S. W.	0		+ 2 30
S. W. by W.	S. W. by W.	0		+ 2 30
W. S. W.	W. S. W.	0		+ 2 30
W. by S.	W. $\frac{7}{8}$ S.	— $\frac{1}{8}$		+ 1 30
WEST.	W. $\frac{1}{4}$ N.	— $\frac{1}{4}$		0. 0
W. by N.	W. by N. $\frac{1}{8}$ N.	— $\frac{1}{8}$		+ 0 50
W. N. W.	W. N. W.	0		+ 2 10
N. W. by W.	N. W. $\frac{1}{2}$ W.	— $\frac{1}{2}$		— 1 50
N. W.	N W. $\frac{1}{2}$ N.	— $\frac{1}{2}$		— 3 10
N. W. by N.	N. W. by N. $\frac{1}{2}$ N.	— $\frac{1}{2}$		— 3 10
N. N. W.	N. N. W. $\frac{1}{2}$ N.	— $\frac{1}{2}$		— 3 10
N. by W.	N. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 1 0
NORTH.				+ 1 50

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 57′.9 B = + 0° 26′.5 C = — 0° 33′.8
D = + 2° 6′.5 E = — 0° 11′.2

Rio Janeiro, January 10, 1866.

Correction for Object = + 2° 44′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.			° ′	
N. by E.				
N. N. E.				
N. E. by N.	N. E. by N.	0		+ 2° 40′
N. E.	N. E.	0		+ 2 40
N. E. by E.	N. E. by E.	0		+ 2 40
E. N. E.	E. N. E.	0		+ 2 40
E. by N.	E. by N.	0		+ 2 40
EAST.	EAST.	0		+ 2 40
E. by S.	E. by S. $\frac{1}{8}$ S.	— $\frac{1}{8}$		+ 1 40
E. S. E.	E. S. E.	0		+ 2 20
S. E. by E.	S. E. $\frac{7}{8}$ E.	— $\frac{1}{8}$		+ 1 40
S. E.	S. E. $\frac{1}{4}$ S.	— $\frac{1}{4}$		+ 0 20
S. E. by S.	S. E. by S. $\frac{1}{8}$ S.	— $\frac{1}{8}$		+ 1 0
S. S. E.	S. by E. $\frac{7}{8}$ E.	— $\frac{1}{8}$		+ 1 20
S. by E.				+ 1 20
SOUTH.	SOUTH.	0		+ 2 20
S. by W.	S. by W. $\frac{1}{8}$ W.	— $\frac{1}{8}$		+ 1 40
S. S. W.	S. S. W. $\frac{1}{8}$ W.	— $\frac{1}{8}$		+ 1 20
S. W. by S.	S. W. $\frac{7}{8}$ S.	— $\frac{1}{8}$		+ 1 20
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N.W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviations are obtained:

A = — 0° 17′.1 B = + 2° 59′.8 C = — 1° 45′.5
D = + 2° 3′.7 E = + 0° 9′.3

Observations for Determining the Deviations of the Forward Binnacle Compass on the U. S. Iron Clad Monadnock.

Monte Video, January 24, 1866.

Correction for Object = — 0° 13′ Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	— 0° 10′
N. by E.	N. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
N. N. E.	N. by E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
N. E. by N.	N. E. by N. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 30
N. E.	N. E. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 0
N. E. by E.	N. E. $\frac{5}{8}$ E.	+ $\frac{3}{8}$		+ 4 0
E. N. E.	E. N. E. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 0
E. by N.	E. by N. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 0
EAST.	E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 30
E. by S.	E. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 2 30
E. S. E.	E. S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 30
S. E. by E.	S. E. by E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
S. E.	S. E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
S. E. by S.	S. E. $\frac{7}{8}$ S.	+ $\frac{1}{8}$		+ 1 10
S. S. E.	S. S. E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
S. by E.	S. by E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 10
SOUTH.	SOUTH.	0		— 0 10
S. by W.	S. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 1 10
S. S. W.	S. by W. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 1 10
S. W. by S.	S. W. by S. $\frac{1}{8}$ S.	+ $\frac{1}{8}$		+ 1 10
S. W.	S. W.	0		— 0 10
S. W. by W.	S. W. by W.	0		— 0 10
W. S. W.	W. S. W.	0		— 0 10
W. by S.	W. $\frac{7}{8}$ S.	— $\frac{1}{8}$		— 1 40
WEST.	W. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 3 0
W. by N.	W. by N. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 3 0
W. N. W.				— 4 30
N. W. by W.	N. W. $\frac{5}{8}$ W.	— $\frac{3}{8}$		— 4 30
N. W.	N. W. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 3 0
N. W. by N.	N. W. by N. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 3 0
N. N. W.	N. by W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 3 0
N. by W.	N. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 3 0
NORTH.				— 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 0° 17′.8 B = + 2° 55′.4 C = — 0° 41′.1
D = + 1° 45′.2 E = — 0° 2′.2

Sandy Point, February 10, 1866.

Correction for Object = + 0° 7′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. $\frac{1}{4}$ E.	— $\frac{1}{4}$	° ′	— 2° 40′
N. by E.	N. by E. $\frac{1}{8}$ E.	— $\frac{1}{8}$		— 1 20
N. N. E.	N. N. E.	0		+ 0 10
N. E. by N.	N. E. by N. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 30
N. E.	N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 3 0
N. E. by E.	N. E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 3 0
E. N. E.	N. E. by E. $\frac{5}{8}$ E.	+ $\frac{3}{8}$		+ 4 20
E. by N.	E. by N. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 20
EAST.	E. $\frac{3}{8}$ N.	+ $\frac{3}{8}$		+ 4 20
E. by S.	E. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
E. S. E.	E. by S. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
S. E. by E.	S. E. by E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 3 0
S. E.	S. E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. E. by S.	S. E. $\frac{5}{8}$ S.	+ $\frac{3}{8}$		+ 4 20
S. S. E.	S. S. E.	0		+ 0 10
S. by E.	S. by E.	0		+ 0 10
SOUTH.	SOUTH.	0		+ 0 10
S. by W.	S. by W.	0		+ 0 10
S. S. W.	S. S. W.	0		+ 0 10
S. W. by S.	S. W. by S.	0		+ 0 10
S. W.	S. W. $\frac{1}{8}$ W.	— $\frac{1}{8}$		— 1 20
S. W. by W.	S. W. by W. $\frac{1}{4}$ W.	— $\frac{1}{4}$		— 2 40
W. S. W.	W. by S. $\frac{3}{4}$ S.	— $\frac{1}{4}$		— 2 40
W. by S.	W. $\frac{1}{2}$ S.	— $\frac{1}{2}$		— 5 30
WEST.	W. $\frac{5}{8}$ N.	— $\frac{5}{8}$		— 7 0
W. by N.	W. by N. $\frac{3}{4}$ N.	— $\frac{3}{4}$		— 8 20
W. N. W.	N. W. by W. $\frac{1}{4}$ W.	— $\frac{3}{4}$		— 8 20
N. W. by W.	N. W. $\frac{1}{4}$ W.	— $\frac{3}{4}$		— 8 20
N. W.	N. W. $\frac{3}{4}$ N.	— $\frac{3}{4}$		— 8 20
N. W. by N.	N. W. by N. $\frac{3}{4}$ N.	— $\frac{3}{4}$		— 8 20
N. N. W.	N. by W. $\frac{3}{8}$ W.	— $\frac{5}{8}$		— 7 0
N. by W.	N. $\frac{1}{2}$ W.	— $\frac{1}{2}$		— 5 30
NORTH.				— 2 40

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 1° 16′.5 B = + 5° 16′.9 C = — 2° 11′.0
D = + 2° 0′.5 E = — 0° 3′.2

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD BINNACLE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Valparaiso, April 4, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. $\frac{1}{8}$ E.	— $\frac{1}{8}$	° ′	— 1° 20′
N. by E.	N. by E.	0		0 0
N. N. E.	N. N. E.	0		0 0
N. E. by N.	N. E. by N. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 30
N. E.	N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 50
N. E. by E.	N. E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
E. N. E.	N. E. by E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
E. by N.	E. by N. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 30
EAST.	E. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 30
E. by S.	E. $\frac{7}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
E. S. E.	E. S. E.	0		0 0
S. E. by E.	S. E. by E.	0		0 0
S. E.	S. E.	0		0 0
S. E. by S.	S. E. by S.	0		0 0
S. S. E.	S. S. E.	0		0 0
S. by E.	S. by E.	0		0 0
SOUTH.	S. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. by W.	S. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 1 30
S. S. W.	S. S. W.	0		0 0
S. W. by S.	S. W. by S.	0		0 0
S. W.	S. W.	0		0 0
S. W. by W.	S. W. by W.	0		0 0
W. S. W.	W. S. W.	0		0 0
W. by S.	W. by S.	0		0 0
WEST.	W. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 2 50
W. by N.	W. by N. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 2 50
W. N. W.	W. N. W. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 2 50
N. W. by W.	N. W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 50
N. W.	N. W. $\frac{3}{8}$ N.	— $\frac{3}{8}$		— 4 10
N. W. by N.	N. W. by N. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 2 50
N. N. W.	N. by W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 50
N. by W.	N. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 50
NORTH.				— 1 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 0° 14′.6 B = + 1° 47′.9 C = — 0° 46′.1
D = + 1° 33′.7 E = — 0° 9′.0

Callao, April 29, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. $\frac{1}{4}$ E.	— $\frac{1}{4}$	° ′	— 2° 40′
N. by E.	N. by E. $\frac{1}{8}$ E.	— $\frac{1}{8}$		— 1 20
N. N. E.	N. N. E. $\frac{1}{8}$ E.	— $\frac{1}{8}$		— 1 20
N. E. by N.	N. E. by N.	0		+ 0 10
N. E.	N. E.	0		+ 0 10
N. E. by E.	N. E. by E.	0		+ 0 10
E. N. E.	E. N. E.	0		+ 0 10
E. by N.	E. by N.	0		+ 0 10
EAST.	EAST.	0		+ 0 10
E. by S.	E. by S.	0		+ 0 10
E. S. E.	S. E. by E. $\frac{7}{8}$ E.	— $\frac{1}{8}$		— 1 20
S. E. by E.	S. E. $\frac{7}{8}$ E.	— $\frac{1}{8}$		— 1 20
S. E.	S. E. $\frac{1}{8}$ S.	— $\frac{1}{8}$		— 1 20
S. E. by S.	S. E. by S.	0		+ 0 10
S. S. E.	S. S. E.	0		+ 0 10
S. by E.	S. by E.	0		+ 0 10
SOUTH.	S. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. by W.	S. $\frac{7}{8}$ W.	+ $\frac{1}{8}$		+ 1 30
S. S. W.	S. by W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 3 0
S. W. by S.	S. W. by S. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
S. W.	S. W. $\frac{1}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
S. W. by W.	S. W. by W.	0		+ 0 10
W. S. W.	W. S. W.	0		+ 0 10
W. by S.	W. $\frac{7}{8}$ S.	— $\frac{1}{8}$		— 1 20
WEST.	W. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 2 40
W. by N.	W. by N. $\frac{3}{8}$ N.	— $\frac{3}{8}$		— 4 10
W. N. W.	N. W. by W. $\frac{5}{8}$ W.	— $\frac{3}{8}$		— 4 10
N. W. by W.	N. W. $\frac{1}{2}$ W.	— $\frac{1}{2}$		— 5 30
N. W.	N. W. $\frac{1}{2}$ N.	— $\frac{1}{2}$		— 5 30
N. W. by N.	N. N. W. $\frac{1}{2}$ W.	— $\frac{1}{2}$		— 5 30
N. N. W.	N. by W. $\frac{1}{2}$ W.	— $\frac{1}{2}$		— 5 30
N. by W.	N. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 40
NORTH.				— 2 40

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 1° 3′.4 B = + 1° 10′.2 C = — 2° 6′.8
D = + 2° 8′.2 E = + 0° 24′.7

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD BINNACLE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Panama, May 20, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 3/8 E.	— 3/8	° ′	— 4° 10′
N. by E.	N. by E. 1/4 E.	— 1/4		— 2 50
N. N. E.	N. E. by N. 3/4 N.	— 1/4		— 2 50
N. E. by N.	N. E. 3/4 N.	— 1/4		— 2 50
N. E.	N. E. 1/4 E.	— 1/4		— 2 50
N. E. by E.	N. E. by E. 1/4 E.	— 1/4		— 2 50
E. N. E.	E. by N. 3/4 N.	— 1/4		— 2 50
E. by N.	E. 3/4 N.	— 1/4		— 2 50
EAST.	E. 1/4 S.	— 1/4		— 2 50
E. by S.	E. by S. 1/4 S.	— 1/4		— 2 50
E. S. E.	S. E. by E. 5/8 E.	— 3/8		— 4 10
S. E. by E.	S. E. 5/8 E.	— 3/8		— 4 10
S. E.	S. E. 3/8 S.	— 3/8		— 4 10
S. E. by S.	S. E. by S. 3/8 S.	— 3/8		— 4 10
S. S. E.	S. by E. 3/4 E.	— 1/4		— 2 50
S. by E.	S. 3/4 E.	— 1/4		— 2 50
SOUTH.	S. 1/8 W.	— 1/8		— 1 20
S. by W.	S. by W.	0		0 0
S. S. W.	S. S. W.	0		0 0
S. W. by S.	S. W. by S.	0		0 0
S. W.	S. W. 1/8 S.	+ 1/8		+ 1 30
S. W. by W.	S. W. 7/8 W.	+ 1/8		+ 1 30
W. S. W.	S. W. by W. 7/8 W.	+ 1/8		+ 1 30
W. by S.	W. by S. 1/8 S.	+ 1/8		+ 1 30
WEST.	W. 1/8 N.	— 1/8		— 1 20
W. by N.	W. by N. 1/4 N.	— 1/4		— 2 50
W. N. W.	N. W. by W. 5/8 W.	— 3/8		— 4 10
N. W. by W.	N. W. 5/8 W.	— 3/8		— 4 10
N. W.	N. W. 1/2 N.	— 1/2		— 5 40
N. W. by N.	N. W. by N. 1/2 N.	— 1/2		— 5 40
N. N. W.	N. by W. 1/2 W.	— 1/2		— 5 40
N. by W.	N. 5/8 W.	— 3/8		— 4 10
NORTH.				— 4 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 2° 31′.9 B = — 1° 1′.5 C = — 1° 33′.0
D = + 2° 6′.5 E = — 0° 23′.5

Acapulco, June 1, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 3/8 E.	— 3/8	° ′	— 4° 10′
N. by E.	N. by E. 1/4 E.	— 1/4		— 2 40
N. N. E.	N. E. by N. 3/4 N.	— 1/4		— 2 40
N. E. by N.	N. E. 3/4 N.	— 1/4		— 2 40
N. E.	N. E. 1/4 E.	— 1/4		— 2 40
N. E. by E.	N. E. by E. 1/4 E.	— 1/4		— 2 40
E. N. E.	E. by N. 5/8 N.	— 3/8		— 4 10
E. by N.	E. 5/8 N.	— 3/8		— 4 10
EAST.	E. 3/8 S.	— 3/8		— 4 10
E. by S.	E. by S. 1/2 S.	— 1/2		— 5 30
E. S. E.	S. E. by E. 1/2 E.	— 1/2		— 5 30
S. E. by E.	S. E. 1/2 E.	— 1/2		— 5 30
S. E.	S. E. 1/2 S.	— 1/2		— 5 30
S. E. by S.	S. E. by S. 3/8 S.	— 3/8		— 4 10
S. S. E.	S. by E. 5/8 E.	— 3/8		— 4 10
S. by E.	S. 3/4 E.	— 1/4		— 2 40
SOUTH.	SOUTH.	0		+ 0 10
S. by W.	S. by W.	0		+ 0 10
S. S. W.	S. by W. 7/8 W.	+ 1/8		+ 1 30
S. W. by S.	S. W. by S. 1/4 S.	+ 1/4		+ 3 0
S. W.	S. W. 1/4 S.	+ 1/4		+ 3 0
S. W. by W.	S. W. 3/4 W.	+ 1/4		+ 3 0
W. S. W.	S. W. by W. 7/8 W.	+ 1/8		+ 1 30
W. by S.	W. by S.	0		+ 0 10
WEST.	W. 1/8 N.	— 1/8		— 1 20
W. by N.	W. by N. 1/8 N.	— 1/8		— 1 20
W. N. W.	N. W. by W. 3/4 W.	— 1/4		— 2 40
N. W. by W.	N. W. 5/8 W.	— 3/8		— 4 10
N. W.	N. W. 1/2 N.	— 1/2		— 5 30
N. W. by N.	N. W. by N. 1/2 N.	— 1/2		— 5 30
N. N. W.	N. by W. 1/2 W.	— 1/2		— 5 30
N. by W.	N. 5/8 W.	— 3/8		— 4 10
NORTH.				— 4 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 2° 31′.2 B = — 2° 2′.4 C = — 1° 41′.1
D = + 2° 39′.2 E = + 0° 10′.7

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD BINNACLE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Magdalena Bay, June 9, 1866.

Correction for Object = — 0° 41′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ½ E.	— ½	° ′	— 6° 10′
N. by E.	N. by E. ⅜ E.	— ⅜		— 4 50
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.				
E. by S.				
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.	S. W. ½ S.	+ ½		+ 5 0
S. W. by W.	S. W. ½ W.	+ ½		+ 4 50
W. S. W.	S. W. by W. ⅝ W.	+ ⅜		+ 3 30
W. by S.	W. by S. ⅜ S.	+ ⅜		+ 3 30
WEST.	W. ¼ S.	+ ¼		+ 2 0
W. by N.	W. by N.	0		— 0 50
W. N. W.	N. W. by W. ⅞ W.	— ⅛		— 2 10
N. W. by W.	N. W. ¾ W.	— ¼		— 3 30
N. W.	N. W. ⅜ N.	— ⅜		— 5 0
N. W. by N.	N. W. by N. ½ N.	— ½		— 6 20
N. N. W.	N. by W. ½ W.	— ½		— 6 20
N. by W.	N. ½ W.	— ½		— 6 20
NORTH.				— 6 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 1° 42′.6 B = — 2° 44′.3 C = — 4° 7′.3
D = + 2° 11′.8 E = — 0° 7′.9

San Francisco, June 23, 1866.

Correction for Object = — 0° 45′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ½ E.	— ½	° ′	— 6° 20
N. by E.	N. by E. ½ E.	— ½		— 6 20
N. N. E.	N. E. by N. ½ N.	— ½		— 6 30
N. E. by N.	N. E. ⅜ N.	— ⅝		— 7 40
N. E.	N. E. ½ E.	— ½		— 6 20
N. E. by E.	N. E. by E. ½ E.	— ½		— 6 30
E. N. E.	E. by N. ⅜ N.	— ⅝		— 7 50
E. by N.	E. ⅜ N.	— ⅝		— 7 50
EAST.	E. ¾ S.	— ¾		— 9 10
E. by S.	E. by S. ¾ S.	— ¾		— 9 10
E. S. E.	S. E. by E. ⅜ E.	— ⅝		— 7 40
S. E. by E.	S. E. ½ E.	— ½		— 6 20
S. E.	S. E. ½ S.	— ½		— 6 10
S. E. by S.	S. E. by S. ¼ S.	— ¼		— 3 30
S. S. E.	S. by E. ¾ E.	— ¼		— 3 20
S. by E.	S. by E.	0		— 0 40
SOUTH.	S. ⅛ E.	+ ⅛		+ 0 50
S. by W.	S. ¾ W.	+ ¼		+ 2 10
S. S. W.	S. by W. ⅝ W.	+ ⅜		+ 3 30
S. W. by S.	S. W. by S. ½ S.	+ ½		+ 4 50
S. W.	S. W. ½ S.	+ ½		+ 4 50
S. W. by W.	S. W. ½ W.	+ ½		+ 4 50
W. S. W.	S. W. by W. ⅝ W.	+ ⅜		+ 3 20
W. by S.	W. by S. ¼ S.	+ ¼		+ 2 0
WEST.	W. ⅛ S.	+ ⅛		+ 0 40
W. by N.	W. by N.	0		— 0 50
W. N. W.	N. W. by W. ⅞ W.	— ⅛		— 2 10
N. W. by W.	N. W. ⅞ W.	— ⅛		— 2 10
N. W.	N. W. ¼ N.	— ¼		— 3 40
N. W. by N.	N. W. by N. ⅜ N.	— ⅜		— 5 0
N. N. W.	N. by W. ½ W.	— ½		— 6 20
N. by W.	N. ½ W.	— ½		— 6 20
NORTH.				— 6 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = — 3° 9′.0 B = — 4° 41′.1 C = — 3° 34′.9
D = + 1° 56′.5 E = + 0° 30′.2

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD RITCHIE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Hampton Roads, November 1, 1865.

Correction for Object = + 3° 57′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ¼ E.	— ¼	° ′	+ 1° 10′
N. by E.	N. by E. ⅛ E.	— ⅛		+ 2 0
N. N. E.	N. N. E.	0		+ 3 30
N. E. by N.	N. E. by N.	0		+ 4 0
N. E.	N. E. ⅛ N.	+ ⅛		+ 4 50
N. E. by E.	N. E. ⅞ E.	+ ⅛		+ 5 20
E. N. E.	N. E. by E. ¾ E.	+ ¼		+ 6 20
E. by N.	E. by N. ⅛ N.	+ ⅛		+ 5 50
EAST.	E. ⅛ N.	+ ⅛		+ 5 20
E. by S.	E. by S.	0		+ 4 30
E. S. E.	E. by S. ⅞ S.	+ ⅛		+ 4 50
S. E. by E.	S. E. by E. ⅛ E.	+ ⅛		+ 5 20
S. E.	S. E. ⅛ E.	+ ⅛		+ 5 20
S. E. by S.	S. E. ¾ S.	+ ¼		+ 6 10
S. S. E.	S. S. E. ¼ E.	+ ¼		+ 6 40
S. by E.	S. by E. ⅜ E.	+ ⅜		+ 7 40
SOUTH.	S. ½ E.	+ ½		+ 9 0
S. by W.	S. ½ W.	+ ½		+ 9 30
S. S. W.	S. by W. ⅝ W.	+ ⅜		+ 8 40
S. W. by S.	S. W. by S. ½ S.	+ ½		+ 9 0
S. W.	S. W. ⅜ S.	+ ⅜		+ 8 40
S. W. by W.	S. W. ¾ W.	+ ¼		+ 7 10
W. S. W.	W. S. W.	0		+ 5 0
W. by S.	W. by S.	0		+ 4 0
WEST.	W. ⅛ N.	— ⅛		+ 3 0
W. by N.	W. by N. ⅜ N.	— ⅜		+ 0 40
W. N. W.	N. W. by W. ¾ W.	— ¼		+ 0 40
N. W. by W.	N. W. ½ W.	— ½		— 0 40
N. W.	N. W. ½ N.	— ½		— 1 40
N. W. by N.	N. W. by N. ½ N.	— ½		— 1 40
N. N. W.	N. by W. ⅝ W.	— ⅜		— 0 50
N. by W.	N. ¾ W.	— ¼		+ 0 40
NORTH.				+ 1 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 4° 22′.5 B = + 1° 19′.2 C = — 3° 37′.2
D = + 2° 17′.2 E = + 0° 27′.5

St. Thomas, November 18, 1865.

Correction for Object = + 0° 16′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ⅛ W.	+ ⅛	° ′	+ 1° 10′
N. by E.	N. by E.	0		— 0 10
N. N. E.	N. by E. ¾ E.	+ ¼		+ 2 40
N. E. by N.	N. E. by N. ½ N.	+ ½		+ 5 20
N. E.	N. E. ½ N.	+ ½		+ 5 20
N. E. by E.	N. E. ½ E.	+ ½		+ 5 20
E. N. E.	N. E. by E. ½ E.	+ ½		+ 5 20
E. by N.	E. by N. ¼ N.	+ ¼		+ 2 30
EAST.	E. ¼ N.	+ ¼		+ 2 30
E. by S.	E. ¾ S.	+ ¼		+ 2 30
E. S. E.	E. S. E.	0		— 0 10
S. E. by E.	S. E. by E. ¼ E.	+ ¼		+ 2 30
S. E.	S. E. ¼ E.	+ ¼		+ 2 30
S. E. by S.	S. E. ¾ S.	+ ¼		+ 2 30
S. S. E.	S. S. E.	0		— 0 10
S. by E.	S. by E.	0		— 0 20
SOUTH.	S.	0		— 0 20
S. by W.	S. ⅞ W.	+ ⅛		+ 1 10
S. S. W.	S. by W. ¾ W.	+ ¼		+ 2 40
S. W. by S.	S. W. by S. ½ S.	+ ½		+ 5 20
S. W.	S. W. ½ S.	+ ½		+ 5 20
S. W. by W.	S. W. ½ W.	+ ½		+ 5 20
W. S. W.	S. W. by W. ¾ W.	+ ¼		+ 2 40
W. by S.	W. ⅞ S.	— ⅛		— 1 40
WEST.	W. ⅛ N.	— ⅛		— 1 40
W. by N.	W. by N.	0		— 0 20
W. N. W.	N. W. by W. ¾ W.	— ¼		— 3 0
N. W. by W.	N. W. ½ W.	— ½		— 5 50
N. W.	N. W. ¼ N.	— ¼		— 3 10
N. W. by N.	N. W. by N. ½ N.	— ½		— 5 50
N. N. W.	N. by W. ¾ W.	— ¼		— 3 0
N. by W.	N. ¾ W.	— ¼		— 3 0
NORTH.				+ 1 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 3′.7 B = + 2° 4′.0 C = — 1° 16′.6
D = + 3° 16′.0 E = — 0° 25′.5

Observations for Determining the Deviations of the Forward Ritchie Compass on the U. S. Iron Clad Monadnock.

Isle Royal, Salute Islands, November 30, 1865.

Correction for Object = —0° 2′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	° ′
NORTH.				
N. by E.				
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.	E. ½ N.	+ ½		+ 5 40
E. by S.				
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = B = C =
D = E =

Ceara, December 19, 1865.

Correction for Object = + 1° 51′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	° ′
NORTH.				
N. by E.	N. ½ E.	+ ½		+ 7 10
N. N. E.	N. by E. ⅜ E.	+ ⅝		+ 8 40
N. E. by N.	N. E. by N. ¾ N.	+ ¾		+ 10 0
N. E.	N. E. ½ N.	+ ½		+ 8 0
N. E. by E.	N. E. ⅝ E.	+ ⅜		+ 6 20
E. N. E.	N. E. by E. ¾ E.	+ ¼		+ 5 0
E. by N.	E. by N ½ N.	+ ½		+ 7 0
EAST.	EAST.	0		+ 2 50
E. by S.	E. by S. ⅛ S.	— ⅛		+ 0 40
E. S. E.	S. E. by E. ⅞ E.	— ⅛		+ 0 30
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 3° 31′.0 B = — 0° 26′.1 C = + 3° 36′.9
D = + 2° 26′.6 E = — 0° 3′.9

Observations for Determining the Deviations of the Forward Ritchie Compass on the U. S. Iron Clad Monadnock.

Bahia, December 30, 1865.

Correction for Object = + 2° 30′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	
NORTH	N. ¼ E.	— ¼		— 0° 20′
N. by E.	N. by E. ¼ E.	— ¼		— 0 20
N. N. E.	N. N. E.	0		+ 1 50
N. E. by N.	N. E. by N. ⅛ N.	+ ⅛		+ 3 30
N. E.	N. E. ½ N.	+ ½		+ 7 10
N. E. by E.	N. E. ½ E.	+ ½		+ 8 10
E. N. E.	N. E. by E. ½ E.	+ ½		+ 8 10
E. by N.	E. by N. ½ N.	+ ½		+ 8 10
EAST.	E. ⅛ N.	+ ⅛		+ 4 50
E. by S.	E. ⅞ S.	+ ⅛		+ 4 0
E. S. E.	E. by S. ⅞ S.	+ ⅛		+ 4 0
S. E. by E.	S. E. by E.	0		+ 2 50
S. E.	S. E.	0		+ 2 30
S. E. by S.	S. E. by S.	0		+ 2 30
S. S. E.	S. S. E.	0		+ 2 30
S. by E.	S. by E. ⅛ E.	+ ⅛		+ 3 30
SOUTH.	S. ⅛ E.	+ ⅛		+ 4 0
S. by W.	S. ¾ W.	+ ¼		+ 5 0
S. S. W.	S. by W. ⅞ W.	+ ⅛		+ 4 10
S. W. by S.	S. W. by S. ⅛ S.	+ ⅛		+ 4 0
S. W.	S. W. ⅛ S.	+ ⅛		+ 4 0
S. W. by W.	S. W. by W.	0		+ 2 50
W. S. W.	W. by S. ⅞ S.	— ⅛		+ 1 30
W. by S.	W. ¾ S.	— ¼		0 0
WEST.	W. ⅝ N.	— ⅝		— 3 40
W. by N.	W. by N. ¾ N.	— ¾		— 5 40
W. N. W.	N. W. by W. ½ W.	— ½		— 3 50
N. W. by W.	N. W. ¾ W.	— ¼		— 1 0
N. W.	N W. ½ N.	— ½		— 2 30
N. W. by N.	N. W. by N. ½ N.	— ½		— 3 10
N. N. W.	N. by W. ¾ W.	— ¼		— 1 0
N. by W.	N. ¾ W.	— ¼		— 0 20
NORTH.				— 0 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 2° 6′.2 B = + 3° 29′.1 C = — 1° 33′.9
D = + 2° 35′.7 E = — 0° 0′.5

Rio Janeiro, January 10, 1866.

Correction for Object = + 2° 44′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
			° ′	
NORTH.				
N. by E.				
N. N. E.				
N. E. by N.	N. E. by N. ½ N.	+ ½		+ 8° 20′
N. E.	N. E. ½ N.	+ ½		+ 8 20
N. E. by E.	N. E. ¾ E.	+ ¼		+ 6 10
E. N. E.	N. E. by E. ½ E.	+ ½		+ 7 40
E. by N.	E. by N. ½ N.	+ ½		+ 8 20
EAST.	E. ⅜ N.	+ ⅜		+ 7 20
E. by S.	E. ⅝ S.	+ ⅜		+ 7 0
E. S. E.	E. by S. ¾ S.	+ ¼		+ 5 50
S. E. by E.	S. E. by E. ¼ E.	+ ¼		+ 5 30
S. E.	S. E. ¼ E.	+ ¼		+ 5 30
S. E. by S.	S. E. ¾ S.	+ ¼		+ 5 30
S. S. E.	S. E. by S. ¾ S.	+ ¼		+ 5 30
S. by E.				+ 4 30
SOUTH.	S. ⅛ E.	+ ⅛		+ 4 10
S. by W.	S. by W.	0		+ 3 0
S. S. W.	S. by W. ⅞ W.	+ ⅛		+ 3 50
S. W. by S.	S. W. by S. ⅛ S.	+ ⅛		+ 4 10
S. W.				
S. W. by W.				
W. S. W.				
W. by S.				
WEST.				
W. by N.				
W. N. W.				
N. W. by W.				
N. W.				
N. W. by N.				
N. N. W.				
N. by W.				
NORTH.				

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviations are obtained:

A = + 3° 14′.0 B = + 4° 23′.5 C = — 1° 10′.4
D = + 2° 10′.5 E = — 0° 0′.1

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD RITCHIE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Monte Video, January 24, 1866.

Correction for Object = − 0° 13′ Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 1/4 W.	+ 1/4	° ′	+ 2° 30′
N. by E.	N. 1/2 E.	+ 1/2		+ 5 20
N. N. E.	N. by E. 3/4 E.	+ 5/8		+ 6 50
N. E. by N.	N. E. by N. 3/4 N.	+ 3/4		+ 8 10
N. E.	N. E. 3/4 N.	+ 3/4		+ 8 10
N. E. by E.	N. E. 1/4 E.	+ 3/4		+ 8 10
E. N. E.	N. E. by E. 1/4 E.	+ 3/4		+ 8 10
E. by N.	E. by N. 7/8 N.	+ 7/8		+ 9 30
EAST.	E. 3/4 N.	+ 3/4		+ 8 10
E. by S.	E. 1/2 S.	+ 1/2		+ 5 20
E. S. E.	E. by S. 1/2 S.	+ 1/2		+ 5 20
S. E. by E.	S. E. by E. 3/8 E.	+ 3/8		+ 4 0
S. E.	S. E. 3/8 E.	+ 3/8		+ 4 0
S. E. by S.	S. E. 5/8 S.	+ 3/8		+ 4 0
S. S. E.	S. S. E. 3/8 E.	+ 3/8		+ 4 0
S. by E.	S. by E. 3/8 E.	+ 3/8		+ 4 0
SOUTH.	S. 1/4 E.	+ 1/4		+ 2 30
S. by W.	S. 3/4 W.	+ 1/4		+ 2 30
S. S. W.	S. by W. 3/4 W.	+ 1/4		+ 2 30
S. W. by S.	S. S. W. 3/4 W.	+ 1/4		+ 2 30
S. W.	S. W 1/4 S.	+ 1/4		+ 2 30
S. W. by W.	S. W. 3/4 W.	+ 1/4		+ 2 30
W. S. W.	S. W. by W. 3/4 W.	+ 1/4		+ 2 30
W. by S.	W. by S. 1/8 S.	+ 1/8		+ 1 10
WEST.	WEST.	0		− 0 10
W. by N.	W. by N.	0		− 0 10
W. N. W.	W. N. W.	0		− 0 10
N. W. by W.	N. W. 3/4 W.	− 1/4		− 3 0
N. W.	N. W. 1/8 N.	− 1/8		− 1 40
N. W. by N.	N. W. by N.	0		− 0 10
N. N. W.	N. N. W.	0		− 0 10
N. by W.	N. bv W.	0		− 0 10
NORTH.				+ 2 30

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 3° 23′.8 B = + 3° 48′.0 C = − 0° 0′.4
D = + 2° 11′.0 E = − 0° 28′.5

Sandy Point, February 10, 1866.

Correction for Object = + 0° 7′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. 1/8 E.	− 1/8	° ′	− 1° 20′
N. by E.	N. by E.	0		+ 0 10
N. N. E.	N. by E. 7/8 E.	+ 1/8		+ 1 30
N. E. by N.				+ 3 0
N. E.	N. E. 1/2 N.	+ 1/2		+ 5 50
N. E. by E.	N. E. 1/2 E.	+ 1/2		+ 5 50
E. N. E.	N. E. by E. 1/2 E.	+ 1/2		+ 5 50
E. by N.	E. by N. 1/2 N.	+ 1/2		+ 5 50
EAST.	E. 1/2 N.	+ 1/2		+ 5 50
E. by S.	E. 1/2 S.	+ 1/2		+ 5 50
E. S. E.	E. by S. 5/8 S.	+ 3/8		+ 4 20
S. E. by E.	S. E. by E. 3/8 E.	+ 3/8		+ 4 20
S. E.	S. E. 1/4 E.	+ 1/4		+ 3 0
S. E. by S.	S. E. 5/8 S.	+ 3/8		+ 4 20
S. S. E.	S. E. by S. 1/2 S.	+ 1/2		+ 5 50
S. by E.	S. by E. 3/8 E.	+ 3/8		+ 4 20
SOUTH.	S. 3/8 E.	+ 3/8		+ 4 20
S. by W.	S. 5/8 W.	+ 3/8		+ 4 20
S. S. W.	S. by W. 3/4 W.	+ 1/4		+ 3 0
S. W. by S.	S. W. by S. 3/8 S.	+ 3/8		+ 4 20
S. W.	S. W. 1/4 S.	+ 1/4		+ 3 0
S. W. by W.	S. W. 7/8 W.	+ 1/8		+ 1 30
W. S. W.	W. S. W. 1/8 S.	+ 1/8		+ 1 30
W. by S.	W. by S.	0		+ 0 10
WEST.	W. 1/8 N.	− 1/8		− 1 20
W. by N.	W. by N. 1/4 N.	− 1/4		− 2 40
W. N. W.	N. W. by W. 1/2 W.	− 1/2		− 5 30
N. W. by W.	N. W. 1/2 W.	− 1/2		− 5 30
N. W.	N. W. 1/2 N.	− 1/2		− 5 30
N. W. by N.	N. W. by N. 3/8 N.	− 3/8		− 4 10
N. N. W.	N. by W. 3/4 W.	− 1/4		− 2 40
N. by W.	N. 3/4 W.	− 1/4		− 2 40
NORTH.				− 1 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 46′.2 B = + 3° 49′.5 C = − 2° 44′.2
D = + 2° 11′.2 E = − 0° 10′.0

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD RITCHIE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Valparaiso, April 4, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	N. ¼ W.	+ ¼	° ′	+ 2° 50′
N. by E.	N. ¾ E.	+ ¼		+ 2 50
N. N. E.	N. by E. ⅝ E.	+ ⅜		+ 4 20
N. E. by N.	N. N. E. ½ E.	+ ½		+ 5 40
N. E.	N. E. ½ N.	+ ½		+ 5 40
N. E. by E.	N. E. ⅜ E.	+ ⅝		+ 7 0
E. N. E.	E. N. E. ⅝ N.	+ ⅝		+ 7 0
E. by N.	E. by N. ¼ N.	+ ¼		+ 2 50
EAST.	E. ¼ N.	+ ¼		+ 2 50
E. by S.	E. ¾ S.	+ ¼		+ 2 50
E. S. E.	E. by S. ⅝ S.	+ ⅜		+ 4 20
S. E. by E.	S. E. by E. ½ E.	+ ½		+ 5 40
S. E.	S. E. ¼ E.	+ ¼		+ 2 50
S. E. by S.	S. E. ¾ S.	+ ¼		+ 2 50
S. S. E.	S. S. E. ⅜ E.	+ ⅜		+ 4 20
S. by E.	S. by E. ½ E.	+ ½		+ 5 40
SOUTH.	S. ½ E.	+ ½		+ 5 40
S. by W.	S. ½ W.	+ ½		+ 5 40
S. S. W.	S. by W. ½ W.	+ ½		+ 5 40
S. W. by S.	S. W. by S. ⅝ S.	+ ⅝		+ 7 0
S. W.	S. W. ½ S.	+ ½		+ 5 40
S. W. by W.	S. W. ⅝ W.	+ ⅜		+ 4 20
W. S. W.	W. S. W. ⅜ S.	+ ⅜		+ 4 20
W. by S.	W. by S. ¼ S.	+ ¼		+ 2 50
WEST.	W. ⅛ S.	+ ⅛		+ 1 30
W. by N.	W. ⅞ N.	+ ⅛		+ 1 30
W. N. W.	W. N. W.	0		0 0
N. W. by W.	N. W. ⅞ W.	— ⅛		— 1 20
N. W.	N. W.	0		0 0
N. W. by N.	N. W. by N.	0		0 0
N. N. W.	N. N. W.	0		0 0
N. by W.	N. by W. ⅛ W.	+ ⅛		+ 1 30
NORTH.				+ 2 50

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 3° 33′.4   B = + 1° 20′.2   C = — 1° 29′.0
D = + 2° 7′.8   E = + 0° 31′.2

Callao, April 29, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	+ 0° 10′
N. by E.	N. ⅞ E.	+ ⅛		+ 1 30
N. N. E.	N. by E. ¾ E.	+ ¼		+ 3 0
N. E. by N.	N. E. by N. ⅜ N.	+ ⅜		+ 4 20
N. E.	N. E. ½ N.	+ ½		+ 5 40
N. E. by E.	N. E. ½ E.	+ ½		+ 5 40
E. N. E.	E. N. E. ½ N.	+ ½		+ 5 40
E. by N.	E. by N. ½ N.	+ ½		+ 5 40
EAST.	E. ⅜ N.	+ ⅜		+ 4 20
E. by S.	E. ⅝ S.	+ ⅜		+ 4 20
E. S. E.	E. by S. ¾ S.	+ ¼		+ 3 0
S. E. by E.	S. E. by E. ¼ E.	+ ¼		+ 3 0
S. E.	S. E. ¼ E.	+ ¼		+ 3 0
S. E. by S.	S. E. ¾ S.	+ ¼		+ 3 0
S. S. E.	S. S. E ¼ E.	+ ¼		+ 3 0
S. by E.	S. by E. ⅜ E.	+ ⅜		+ 4 20
SOUTH.	S. ½ E.	+ ½		+ 5 40
S. by W.	S. ½ W.	+ ½		+ 5 40
S. S. W.	S. by W. ⅝ W.	+ ⅜		+ 4 20
S. W. by S.	S. W. by S. ½ S.	+ ½		+ 5 40
S. W.	S. W. ½ S.	+ ½		+ 5 40
S. W. by W.	S. W. ⅝ W.	+ ⅜		+ 4 20
W. S. W.	W. S. W. ¼ S.	+ ¼		+ 3 0
W. by S.	W. by S. ⅛ S.	+ ⅛		+ 1 30
WEST.	WEST.	0		+ 0 10
W. by N.	W. by N. ⅛ N.	— ⅛		— 1 20
W. N. W.	W. N. W. ¼ N.	— ¼		— 2 40
N. W. by W.	N. W. ¾ W.	— ¼		— 2 40
N. W.	N. W. ¼ N.	— ¼		— 2 40
N. W. by N.	N. W. by N. ⅛ N.	— ⅛		— 1 20
N. N. W.	N. by W. ⅞ W.	— ⅛		— 1 20
N. by W.	N. by W.	0		+ 0 10
NORTH.				+ 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 2° 37′.1   B = + 1° 52′.8   C = — 1° 58′.0
D = + 2° 30′.5   E = + 0° 12′.0

Observations for Determining the Deviations of the Forward Ritchie Compass on the U. S. Iron Clad Monadnock.

Panama, May 20, 1866.

Correction for Object = + 0° 1′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	0° 0′
N. by E.	N. by E.	0		0 0
N. N. E.	N. by E. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
N. E. by N.	N. E. by N. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 30
N. E.	N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 50
N. E. by E.	N. E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
E. N. E.	N. E. by E. $\frac{3}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
E. by N.	E. by N. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 2 50
EAST.	E. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 30
E. by S.	E. by S.	0		0 0
E. S. E.	E. by S. $\frac{7}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
S. E. by E.	S. E. by E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. E.	S. E.	0		0 0
S. E. by S.	S. E. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 2 50
S. S. E.	S. S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
S. by E.	S. by E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 2 50
SOUTH.	S. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. by W.	S. $\frac{5}{8}$ W.	+ $\frac{3}{8}$		+ 4 10
S. S. W.	S. by W. $\frac{3}{4}$ W.	+ $\frac{1}{4}$		+ 2 50
S. W. by S.	S. W. by S. $\frac{3}{8}$ S.	+ $\frac{3}{8}$		+ 4 10
S. W.	S. W. $\frac{1}{2}$ S.	+ $\frac{1}{2}$		+ 5 40
S. W. by W.	S. W. $\frac{1}{2}$ W.	+ $\frac{1}{2}$		+ 5 40
W. S. W.	S. W. by W. $\frac{5}{8}$ W.	+ $\frac{3}{8}$		+ 4 10
W. by S.	W. by S. $\frac{3}{8}$ S.	+ $\frac{3}{8}$		+ 4 10
WEST.	W. $\frac{1}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
W. by N.	W. by N.	0		0 0
W. N. W.	N. W. by W. $\frac{7}{8}$ W.	— $\frac{1}{8}$		— 1 20
N. W. by W.	N. W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 50
N. W.	N. W. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 2 50
N. W. by N.	N. W. by N. $\frac{1}{8}$ N.	— $\frac{1}{8}$		— 1 20
N. N. W.	N. by W. $\frac{7}{8}$ W.	— $\frac{1}{8}$		— 1 20
N. by W.	N. $\frac{7}{8}$ W.	— $\frac{1}{8}$		— 1 20
NORTH.				0 0

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 34′.0 B = + 0° 12′.2 C = — 1° 53′.8
D = + 2° 10′.8 E = — 0° 14′.0

Acapulco, June 1, 1866.

Correction for Object = + 0° 6′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees.	Corrected Deviation of Compass.
NORTH.	NORTH.	0	° ′	+ 0° 10′
N. by E.	N. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
N. N. E.	N. by E. $\frac{7}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
N. E. by N.	N. E. by N. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 3 0
N. E.	N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 3 0
N. E. by E.	N. E. $\frac{5}{8}$ E.	+ $\frac{3}{8}$		+ 4 20
E. N. E.	E. N. E. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 3 0
E. by N.	E. by N. $\frac{1}{4}$ N.	+ $\frac{1}{4}$		+ 3 0
EAST.	E. $\frac{1}{8}$ N.	+ $\frac{1}{8}$		+ 1 30
E. by S.	E. $\frac{7}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
E. S. E.	E. S. E.	0		+ 0 10
S. E. by E.	S. E. by E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. E.	S. E. $\frac{1}{8}$ E.	+ $\frac{1}{8}$		+ 1 30
S. E. by S.	S. E. $\frac{3}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
S. S. E.	S. S. E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 3 0
S. by E.	S. by E. $\frac{1}{4}$ E.	+ $\frac{1}{4}$		+ 3 0
SOUTH.	S. $\frac{3}{8}$ E.	+ $\frac{3}{8}$		+ 4 20
S. by W.	S. $\frac{1}{2}$ W.	+ $\frac{1}{2}$		+ 5 40
S. S. W.	S. by W. $\frac{1}{2}$ W.	+ $\frac{1}{2}$		+ 5 40
S. W. by S.	S. W. by S. $\frac{1}{2}$ S.	+ $\frac{1}{2}$		+ 5 40
S. W.	S. W. $\frac{1}{2}$ S.	+ $\frac{1}{2}$		+ 5 40
S. W. by W.	S. W. $\frac{5}{8}$ W.	+ $\frac{3}{8}$		+ 4 20
W. S. W.	W. S. W. $\frac{1}{4}$ S.	+ $\frac{1}{4}$		+ 3 0
W. by S.	W. by S. $\frac{1}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
WEST.	W. $\frac{1}{8}$ S.	+ $\frac{1}{8}$		+ 1 30
W. by N.	W. by N.	0		+ 0 10
W. N. W.	W. N. W. $\frac{1}{8}$ N.	— $\frac{1}{8}$		— 1 20
N. W. by W.	N. W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 40
N. W.	N. W. $\frac{1}{4}$ N.	— $\frac{1}{4}$		— 2 40
N. W. by N.	N. W. by N. $\frac{1}{8}$ N.	— $\frac{1}{8}$		— 1 20
N. N. W.	N. by W. $\frac{3}{4}$ W.	— $\frac{1}{4}$		— 2 40
N. by W.	N. $\frac{7}{8}$ W.	— $\frac{1}{8}$		— 1 20
NORTH.				+ 0 10

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 52′.8 B = + 0° 38′.2 C = — 2° 11′.8
D = + 2° 24′.2 E = + 0° 26′.2

OBSERVATIONS FOR DETERMINING THE DEVIATIONS OF THE FORWARD RITCHIE COMPASS ON THE U. S. IRON CLAD MONADNOCK.

Magdalena Bay, June 9, 1866.

Correction for Object = — 0° 41′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees. ° ′	Corrected Deviation of Compass.
NORTH.	N. 1/4 E.	— 1/4		— 3° 30′
N. by E.	N. by E. 1/8 E.	— 1/8		— 2 0
N. N. E.				
N. E. by N.				
N. E.				
N. E. by E.				
E. N. E.				
E. by N.				
EAST.				
E. by S.				
E. S. E.				
S. E. by E.				
S. E.				
S. E. by S.				
S. S. E.				
S. by E.				
SOUTH.				
S. by W.				
S. S. W.				
S. W. by S.				
S. W.	S. W. 7/8 S.	+ 7/8		+ 9 0
S. W. by W.	S. W. 1/4 W.	+ 3/4		+ 7 40
W. S. W.	W. S. W. 5/8 S.	+ 5/8		+ 6 20
W. by S.	W. by S. 1/2 S.	+ 1/2		+ 4 50
WEST.	W. 3/8 S.	+ 3/8		+ 3 20
W. by N.	W. by N.	0		— 0 50
W. N. W.	W. N. W. 1/4 N.	— 1/4		— 3 30
N. W. by W.	N. W. 3/4 W.	— 1/4		— 3 30
N. W.	N. W. 1/4 N.	— 1/4		— 3 30
N. W. by N.	N. W. by N. 3/8 N.	— 3/8		— 5 0
N. N. W.	N. by W. 1/2 W.	— 1/2		— 6 10
N. by W.	N. 3/4 W.	— 1/4		— 3 30
NORTH.				— 3 30

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 2° 43′.8 B = + 0° 39′.9 C = — 6° 1′.3
D = + 2° 38′.7 E = — 0° 1′.3

San Francisco, June 23, 1866.

Correction for Object = — 0° 45′. Correction for Lubber Line = 0.

Assumed Magnetic Direction of Ship's Head.	Ship's Head by Compass.	Deviation of Compass in Points.	Deviation of Compass in Degrees. ° ′	Corrected Deviation of Compass.
NORTH.	N. 1/2 E.	— 1/2		— 6° 20′
N. by E.	N. by E. 3/8 E.	— 3/8		— 4 50
N. N. E.	N. N. E. 1/4 E.	— 1/4		— 3 30
N. E. by N.	N. E. 3/4 N.	— 1/4		— 3 30
N. E.	N. E. 1/8 E.	— 1/8		— 2 0
N. E. by E.	N. E. by E.	0		— 0 30
E. N. E.	E. N. E. 1/4 N.	+ 1/4		+ 2 0
E. by N.	E. by N. 1/8 N.	+ 1/8		+ 0 40
EAST.	EAST.	0		— 0 40
E. by S.	E. 7/8 S.	+ 1/8		+ 0 50
E. S. E.	E. by S. 3/4 S.	+ 1/4		+ 2 10
S. E. by E.	S. E. by E. 3/8 E.	+ 3/8		+ 3 40
S. E.	S. E. 1/2 E.	+ 1/2		+ 4 50
S. E. by S.	S. E. 1/2 S.	+ 1/2		+ 4 50
S. S. E.	S. S. E. 1/2 E.	+ 1/2		+ 5 0
S. by E.	S. by E. 5/8 E.	+ 5/8		+ 6 20
SOUTH.	S. 3/4 E.	+ 3/4		+ 7 40
S. by W.	S. 1/4 W.	+ 3/4		+ 7 40
S. S. W.	S. by W. 1/4 W.	+ 3/4		+ 7 40
S. W. by S.	S. W. by S. 7/8 S.	+ 7/8		+ 9 0
S. W.	S. W. 3/4 S.	+ 3/4		+ 7 30
S. W. by W.	S. W. 3/8 W.	+ 5/8		+ 6 20
W. S. W.	W. S. W. 5/8 S.	+ 5/8		+ 6 10
W. by S.	W. by S. 1/2 S.	+ 1/2		+ 4 40
WEST.	W. 1/8 S.	+ 1/8		+ 0 30
W. by N.	W. by N. 1/8 N.	+ 1/8		— 2 0
W. N. W.	W. N. W.	0		— 1 0
N. W. by W.	N. W. 3/4 W.	— 1/4		— 3 30
N. W.	N. W. 1/8 N.	— 1/8		— 2 20
N. W. by N.	N. W. by N. 1/2 N.	— 1/2		— 6 20
N. N. W.	N. by W. 1/2 W.	— 1/2		— 6 40
N. by W.	N. 1/8 W.	— 7/8		— 10 20
NORTH.				— 6 20

A deviation of the North Point of the Compass to the East is designated by the sign +; a deviation to the West by the sign —.

From the observations given above, the following values of the coefficients of the deviation are obtained:

A = + 1° 3′.8 B = — 0° 16′.2 C = — 6° 41′.6
D = + 1° 48′.5 E = — 0° 33′.5

The observations made at stations where the deviations had been determined on all of the thirty-two points were first discussed. For that purpose the values of the coefficients A_1, B_1, C_1, D_1, E_1, for each compass, at each station, were computed from the deviations on the true magnetic points by means of the equations given on pages 126 to 128. A specimen of the form employed in making these computations is appended. It sufficiently explains itself.

ADMIRALTY STANDARD COMPASS. COMPUTATION OF COEFFICIENTS B_1 AND C_1, FROM DEVIATIONS OBSERVED ON 32 POINTS, ON THE U. S. IRON CLAD MONADNOCK.

Bahia, December 30, 1865.

	I.		II.	III. Half Sum of Quantities in Cols. I and II.	IV. Half Sum of Cols. I and II, (changing Signs of Col. II.)	V. Computation of B_1.		VI. Computation of C_1.	
True Magnetic Direction of Ship's Head.	Observed Deviation of Compass.	True Magnetic Direction of Ship's Head.	Observed Deviation of Compass.	Unchanging Part of Deviation.	Semi-circular Deviation.	Multipliers.	Products of Col. IV by Multipliers.	Multipliers.	Products of Col. IV by Multipliers.
NORTH.	+ 1° 40′	SOUTH.	+ 1° 40′	+ 1° 40′	0° 0′	0	0° 0′	1	0° 0
N. by E.	+ 3 20	S. by W.	+ 1 20	+ 2 20	+ 1 0	S_1	+ 0 12	S_7	+ 0 59
N. N. E.	+ 3 40	S. S. W.	+ 1 00	+ 2 20	+ 1 20	S_2	+ 0 31	S_6	+ 1 14
N. E. by N.	+ 4 30	S. W. by S.	+ 0 30	+ 2 30	+ 2 0	S_3	+ 1 7	S_5	+ 1 40
N. E.	+ 4 40	S. W.	0 0	+ 2 20	+ 2 20	S_4	+ 1 39	S_4	+ 1 39
N. E. by E.	+ 5 0	S. W. by W.	— 0 40	+ 2 10	+ 2 50	S_5	+ 2 21	S_3	+ 1 34
E. N. E.	+ 5 30	W. S. W.	— 1 10	+ 2 10	+ 3 20	S_6	+ 3 5	S_2	+ 1 17
E. by N.	+ 5 40	W. by S.	— 1 50	+ 1 55	+ 3 45	S_7	+ 3 41	S_1	+ 0 44
EAST.	+ 5 20	WEST.	— 2 0	+ 1 40	+ 3 40	1	+ 3 40	0	0 0
E. by S.	+ 5 10	W. by N.	— 2 10	+ 1 30	+ 3 40	S_7	+ 3 36	$-S_1$	— 0 43
E. S. E.	+ 4 40	W. N. W.	— 2 0	+ 1 20	+ 3 20	S_6	+ 3 5	$-S_2$	— 1 17
S. E. by E.	+ 4 20	N. W. by W.	— 2 0	+ 1 10	+ 3 10	S_5	+ 2 38	$-S_3$	— 1 46
S. E.	+ 3 20	N. W.	— 2 0	+ 0 40	+ 2 40	S_4	+ 1 53	$-S_4$	— 1 53
S. E. by S.	+ 2 40	N. W. by N.	— 1 10	+ 0 45	+ 1 55	S_3	+ 1 4	$-S_5$	— 1 36
S. S. E.	+ 2 10	N. N. W.	— 0 10	+ 1 0	+ 1 10	S_2	+ 0 27	$-S_6$	— 1 5
S. by E.	+ 2 0	N. by W.	+ 0 30	+ 1 15	+ 0 45	S_1	+ 0 9	$-S_7$	— 0 44
					Sum of + terms = + 29 8			+	9 7
					Sum of — terms = —			—	9 4
					Divisor	8	+ 29 8	8	+ 0 3
							B_1 = + 3 38.5		C_1 = + 0 0.4

N. B.—Easterly deviations are to be entered in this table with the sign +; Westerly deviations with the sign —.

COMPUTATION OF COEFFICIENTS A_1, D_1, E_1, FROM DEVIATIONS OBSERVED ON 32 POINTS.

I.	II.	III.	IV.	V.		VI.	
		Half Sum of Quantities in Cols. I and II. — Constant Part of Deviation.	Half Sum of Cols. I and II, (changing Signs of Col. II.) — Quadrantal Deviation.	Computation of D_1.		Computation of E_1.	
Upper Half of Table A, Col. III.	Lower Half of Table A, Col. III.			Multipliers.	Products of Col. IV by Multipliers.	Multipliers.	Products of Col. IV by Multipliers.
+ 1° 40′	+ 1° 40′	+ 1° 40′	0° 0′	0	0° 0′	1	0° 0′
+ 2 20	+ 1 30	+ 1 55	+ 0 25	S_2	+ 0 10	S_6	+ 0 23
+ 2 20	+ 1 20	+ 1 50	+ 0 30	S_4	+ 0 21	S_4	+ 0 21
+ 2 30	+ 1 10	+ 1 50	+ 0 40	S_6	+ 0 37	S_2	+ 0 15
+ 2 20	+ 0 40	+ 1 30	+ 0 50	1	+ 0 50	0	0 0
+ 2 10	+ 0 45	+ 1 27	+ 0 43	S_6	+ 0 40	— S_2	— 0 16
+ 2 10	+ 1 0	+ 1 35	+ 0 35	S_4	+ 0 25	— S_4	— 0 25
+ 1 55	+ 1 15	+ 1 35	+ 0 20	S_2	+ 0 8	— S_6	— 0 18
Sum of + terms = +		13 22	Sum of + terms = + 3		11	+	59
Sum of — terms = —			Sum of — terms = —			—	59
Divisor 8		+ 13 22	Divisor 4		+ 3 11	4	0 0
		A_1 = + 1 40.2			D_1 = + 0 47.8	E_1 =	0 0.0

NOTE.—S_1 = .195. S_2 = .383. S_3 = .556. S_4 = .707. S_5 = .831. S_6 = .924. S_7 = .981.

The resulting values of the coefficients for each compass, at each station, are given in the following tables:

COEFFICIENTS OF THE DEVIATIONS OF THE ADMIRALTY STANDARD COMPASS.

STATION.	DATE.	A_1	B_1	C_1	D_1	E_1
Hampton Roads	November 1, 1865	+ 1° 37′.4	+ 9° 4′.6	— 0° 33′.1	+ 0° 29′.2	— 0° 7′.5
St. Thomas	November 18, 1865	+ 0 14.6	+ 5 45.5	+ 0 33.5	+ 0 3.2	— 0 48.2
Bahia	December 30, 1865	+ 1 40.2	+ 3 38.5	+ 0 0.4	+ 0 47.8	0 0.0
Monte Video	January 24, 1866	+ 1 32.8	+ 3 4.8	+ 0 5.8	+ 1 19.5	+ 0 14.5
Sandy Point	February 10, 1866	+ 0 35.9	+ 1 20.6	— 0 40.6	+ 0 53.5	+ 0 1.5
Valparaiso	April 4, 1866	+ 0 35.6	+ 1 20.2	— 0 6.9	+ 0 54.2	— 0 10.2
Callao	April 29, 1866	+ 0 9.1	+ 2 21.1	— 0 1.8	+ 0 52.5	+ 0 5.8
Panama	May 20, 1866	+ 0 31.6	+ 3 2.1	+ 0 1.9	+ 0 55.0	+ 0 8.0
Acapulco	June 1, 1866	— 0 36.9	+ 2 45.4	+ 0 5.5	+ 0 56.8	+ 0 8.0
San Francisco	June 23, 1866	— 0 39.6	+ 4 53.2	— 1 15.4	+ 0 51.2	+ 0 5.8

COEFFICIENTS OF THE DEVIATIONS OF THE AFTER BINNACLE COMPASS.

STATION.	DATE.	A_1	B_1	C_1	D_1	E_1
Hampton Roads	November 1, 1865	+ 0° 27′.5	+ 7° 16′.8	— 1° 14′.1	+ 1° 39′.2	+ 0° 6′.2
St. Thomas	November 18, 1865	……	……	……	……	……
Bahia	December 30, 1865	+ 1 29.8	+ 5 43.6	— 0 6.9	+ 1 41.5	+ 0 7.8
Monte Video	January 24, 1866	+ 1 3.1	+ 5 30.6	+ 0 41.9	+ 1 57.5	— 0 42.5
Sandy Point	February 10, 1866	— 0 24.5	+ 5 44.4	— 0 14.6	+ 1 58.5	+ 0 0.2
Valparaiso	April 4, 1866	+ 0 4.9	+ 3 58.8	+ 0 7.9	+ 2 1.5	— 0 0.2
Callao	April 29, 1866	— 0 27.1	+ 4 12.5	— 0 3.9	+ 2 7.5	+ 0 9.0
Panama	May 20, 1866	— 0 50.0	+ 3 19.5	+ 0 22.0	+ 2 32.7	— 0 18.0
Acapulco	June 1, 1866	— 1 0.2	+ 3 4.4	— 0 17.1	+ 2 15.2	— 0 17.2
San Francisco	June 23, 1866	— 0 35.2	+ 3 28.2	— 2 13.9	+ 1 47.5	+ 0 10.2

COEFFICIENTS OF THE DEVIATIONS OF THE AFTER RITCHIE COMPASS.

STATION.	DATE.	A_1	B_1	C_1	D_1	E_1
Hampton Roads . . .	November 1, 1865	+ 7° 40′.0	+ 11° 26′.5	− 1° 44′.1	+ 0° 15′.5	− 0° 54′.5
St. Thomas	November 18, 1865	+ 3 14.4	+ 8 26.9	+ 0 40.4	+ 1 54.2	− 0 37.2
Bahia	December 30, 1865	+ 8 47.1	+ 6 55.6	− 0 57.2	+ 1 59.7	+ 0 14.2
Monte Video	January 24, 1866	......	......	......	......	......
Sandy Point	February 10, 1866	+ 8 18.4	+ 4 3.2	− 3 25.6	+ 1 14.5	+ 0 58.5
Valparaiso	April 4, 1866	+ 4 21.9	+ 3 49.1	+ 0 12.4	+ 2 21.0	+ 0 7.5
Callao	April 29, 1866	+ 4 19.4	+ 5 50.1	+ 0 14.1	+ 1 30.5	+ 0 52.0
Panama	May 20, 1866	+ 5 20.6	+ 4 3.1	− 0 10.2	+ 1 17.0	− 1 33.0
Acapulco	June 1, 1866	+ 4 0.6	+ 4 29.1	+ 1 12.8	+ 1 12.2	+ 0 47.0
San Francisco	June 23, 1866	+ 4 11.6	+ 6 46.2	− 1 31.4	+ 2 28.5	+ 0 21.2

COEFFICIENTS OF THE DEVIATIONS OF THE AFTER AZIMUTH COMPASS.

STATION.	DATE.	A_1	B_1	C_1	D_1	E_1
Hampton Roads . . .	November 1, 1865	− 1° 5′.0	− 4° 53′.0	− 0° 9′.1	+ 5° 35′.2	+ 0° 17′.0
St. Thomas	November 18, 1865	− 1 17.5	− 3 0.9	+ 1 20.0	+ 6 49.2	+ 0 12.2
Bahia	December 30, 1865	− 3 36.9	− 4 28.6	− 0 19.5	+ 7 22.0	− 1 5.5
Monte Video	January 24, 1866	......	......	......	......	......
Sandy Point	February 10, 1866	− 0 5.6	− 2 57.8	− 0 47.2	+ 7 10.2	− 0 25.5
Valparaiso	April 4, 1866	− 2 16.2	− 4 54.1	+ 0 20.9	+ 5 52.5	+ 0 37.5
Callao	April 29, 1866	− 3 56.2	− 2 0.6	− 0 49.6	+ 5 6.5	+ 0 35.7
Panama	May 20, 1866	− 2 6.9	− 3 47.2	+ 1 44.6	+ 6 21.2	− 0 34.0
Acapulco	June 1, 1866	− 3 11.2	− 3 25.8	− 0 0.8	+ 5 54.2	+ 0 23.8
San Francisco	June 23, 1866	......	......	......	......	......

COEFFICIENTS OF THE DEVIATIONS OF THE FORWARD ALIDADE COMPASS.

STATION.	DATE.	A_1	B_1	C_1	D_1	E_1
Hampton Roads . . .	November 1, 1865	+ 2° 8′.1	− 2° 28′.4	− 1° 52′.0	+ 1° 4′.2	0° 0.0
St. Thomas	November 18, 1865	+ 0 50.9	− 0 35.1	− 0 46.2	+ 1 15.7	+ 0 20.5
Bahia	December 30, 1865	+ 2 9.4	− 0 6.0	− 0 34.1	+ 1 15.0	+ 0 14.5
Monte Video	January 24, 1866	+ 2 7.1	+ 0 57.2	− 1 5.0	+ 1 23.0	− 0 9.8
Sandy Point	February 10, 1866	+ 2 25.6	+ 0 58.5	− 1 54.4	+ 1 47.0	− 0 20.2
Valparaiso	April 4, 1866	+ 1 55.2	+ 0 30.0	− 0 53.9	+ 1 4.2	− 0 5.2
Callao	April 29, 1866	+ 0 21.0	+ 0 40.9	− 1 36.4	+ 1 29.0	− 0 6.8
Panama	May 20, 1866	+ 2 15.2	+ 0 1.1	− 1 22.1	+ 1 21.0	− 0 6.8
Acapulco	June 1, 1866	+ 1 8.1	− 1 28.4	− 0 33.1	+ 1 52.8	+ 0 10.2
San Francisco	June 23, 1866	+ 0 40.6	− 1 54.2	− 2 25.1	+ 0 58.0	+ 0 21.5

COEFFICIENTS OF THE DEVIATIONS OF THE FORWARD BINNACLE COMPASS.

STATION.	DATE.	A_1	B_1	C_1	D_1	E_1
Hampton Roads . . .	November 1, 1865	+ 0° 49′.0	− 5° 40′.8	− 2° 33′.4	+ 2° 17′.7	+ 0° 8′.2
St. Thomas	November 18, 1865	− 0 44.4	− 1 56.2	− 0 12.4	+ 1 59.5	− 0 7.2
Bahia	December 30, 1865	+ 0 57.9	+ 0 26.5	− 0 33.8	+ 2 6.5	− 0 11.2
Monte Video	January 24, 1866	+ 0 17.8	+ 2 55.4	− 0 41.1	+ 1 45.2	− 0 2.2
Sandy Point	February 10, 1866	− 1 16.5	+ 5 16.9	− 2 11.0	+ 2 0.5	− 0 3.2
Valparaiso	April 4, 1866	− 0 14.6	+ 1 47.9	− 0 46.1	+ 1 33.7	− 0 9.0
Callao	April 29, 1866	− 1 3.4	+ 1 10.2	− 2 6.8	+ 2 8.2	+ 0 24.7
Panama	May 20, 1866	− 2 31.9	− 1 1.5	− 1 33.0	+ 2 6.5	− 0 23.5
Acapulco	June 1, 1866	− 2 31.2	− 2 2.4	− 1 41.1	+ 2 39.2	+ 0 10.7
San Francisco	June 23, 1866	− 3 9.0	− 4 41.1	− 3 34.9	+ 1 56.5	+ 0 30.2

Coefficients of the Deviations of the Forward Ritchie Compass.

STATION.	DATE.	A_1	B_1	C_1	D_1	E_1
Hampton Roads . . .	November 1, 1865	+ 4° 22′.5	+ 1° 19′.2	− 3° 37′.2	+ 2° 17′.2	+ 0° 27′.5
St. Thomas	November 18, 1865	+ 1 3.7	+ 2 4.0	− 1 16.6	+ 3 16.0	− 0 25.5
Bahia	December 30, 1865	+ 2 6.2	+ 3 29.1	− 1 33.9	+ 2 35.7	− 0 0.5
Monte Video	January 24, 1866	+ 3 23.8	+ 3 48.0	− 0 0.4	+ 2 11.0	− 0 28.5
Sandy Point	February 10, 1866	+ 1 46.2	+ 3 49.5	− 2 44.2	+ 2 11.2	− 0 10.0
Valparaiso	April 4, 1866	+ 3 33.4	+ 1 20.2	− 1 29.0	+ 2 7.8	+ 0 31.2
Callao	April 29, 1866	+ 2 37.1	+ 1 52.8	− 1 58.0	+ 2 30.5	+ 0 12.0
Panama.	May 20, 1866	+ 1 34.0	+ 0 12.2	− 1 53.8	+ 2 10.8	− 0 14.0
Acapulco	June 1, 1866	+ 1 52.8	+ 0 38.2	− 2 11.8	+ 2 24.2	+ 0 26.2
San Francisco	June 23, 1866	+ 1 3.8	− 0 16.2	− 6 41.6	+ 1 48.5	− 0 33.5

In the case of the Admiralty Standard Compass, for some not very evident reason, the variations in the value of the coefficient A_1 are greater than might have been expected. The After Binnacle, Forward Alidade, and Forward Binnacle Compasses were frequently removed from their places, and the fittings were not sufficiently exact to give any certainty of replacing them with their lubber lines always precisely in the same position. This source of error sufficiently accounts for the variations in the values of the A_1s belonging to them. The Forward and After Ritchie Compasses were firmly fixed in their places, and were not removed during the cruise, except at Valparaiso; but the arrangements for reading off their cards were such that an improper position of the eye of the observer might easily introduce a large parallax, which accounts for the changes in the values of the A_1s belonging to them. The After Azimuth Compass was always taken down after each swing, and as there was no fixed mark by which to adjust its lubber line, the changes in the value of its A_1 are not surprising.

It now becomes necessary to determine the probable errors of the values of the coefficients which have just been given. To do this for any compass, at any particular station, the value of δ at each of the thirty-two points must be computed from the coefficients for that station. Comparing the values thus found with the corrected observed values, a series of thirty-two residuals are obtained, from which the probable error of δ for that station is deduced by means of the formula

$$r = 0.6745 \sqrt{\frac{[vv]}{m - \mu}}$$

where r is the probable error of a single observed value of δ; $[vv]$ the sum of the squares of the thirty-two residuals; m the number of the residuals, in this case thirty-two; and μ the number of the coefficients, in the present instance five. Then, letting p_A, p_B, p_C, p_D, p_E, represent respectively the weights, and r_A, r_B, r_C, r_D, r_E, the probable errors, of the values of A_1, B_1, C_1, D_1, E_1, when determined from a set of deviations observed on each of the thirty-two true magnetic points; we have

$$r_A = \frac{r}{\sqrt{p_A}} \qquad r_B = \frac{r}{\sqrt{p_B}}, \qquad \&c.$$

From the normal equations on page 126, we also have,

$$p_A = 32 \qquad p_D = 16$$
$$p_B = 16 \qquad p_E = 16$$
$$p_C = 16$$

It is therefore evident that the probable errors of B_1, C_1, D_1, and E_1, will all be equal to each other.

The probable error of a single observed value of δ has been computed in this way, for each compass, at three stations; namely, Bahia, Sandy Point, and Panama, and the results are given in the following table. The column headed "mean value of r" was obtained by adding together, for each compass, the sum of the squares of the residuals at Bahia, Sandy Point, and Panama; dividing the result by three; and then computing the value of r from the mean value of $[vv]$ thus found. The column headed "$\frac{r}{\sqrt{32}}$" gives the probable error of A_1; and the column headed "$\frac{r}{\sqrt{16}}$" gives the probable error of B_1, C_1, D_1, and E_1, for each compass, when these coefficients have been computed from a set of deviations observed on thirty-two points.

Compass.	Value of r.			Mean value of r.	$\frac{r}{\sqrt{32}}$	$\frac{r}{\sqrt{16}}$
	Bahia.	Sandy Point.	Panama.			
Admiralty Standard	± 9′.8	± 12′.2	± 11′.3	± 11′.1	± 2′.0	± 2′.8
After Binnacle	± 25.8	± 20.1	± 26.2	± 24.2	± 4.3	± 6.1
After Ritchie	± 30.6	± 56.6	± 38.8	± 43.4	± 7.7	± 10.8
After Azimuth	± 39.3	± 51.1	± 32.6	± 41.7	± 7.4	± 10.4
Forward Alidade	± 19.0	± 24.5	± 23.6	± 22.5	± 4.0	± 5.6
Forward Binnacle	± 40.2	± 31.2	± 25.3	± 32.8	± 5.8	± 8.2
Forward Ritchie	± 59.7	± 30.2	± 37.8	± 44.4	± 7.8	± 11.1

As an incidental result, this table shows that for ordinary steering compasses (such as the Forward Alidade, Forward Binnacle, and After Binnacle) when read to the nearest eighth of a point, the probable accidental error of a single reading is about half a degree; for Ritchie Monitor Compasses (such as the Forward and After Ritchie) when read to the nearest eighth of a point, the probable accidental error of a single reading is about three-quarters of a degree; and for Admiralty Standard Compasses, read to the nearest ten minutes, the probable accidental error of a single reading is about eleven minutes.

From the mathematical theory of the deviations of the compass, given in a preceding part of this section, we have

$$\mathfrak{B} = B_1 - A_1 C_1$$

and also

$$\mathfrak{B} = \frac{c}{\lambda} \tan \theta + \frac{P}{\lambda} \times \frac{1}{H}$$

Hence

$$0 = - B_1 + A_1 C_1 + \frac{c}{\lambda} \tan \theta + \frac{P}{\lambda} \times \frac{1}{H}$$

But as P is liable to undergo a slow change, we introduce a term depending upon the time, and the equation becomes

$$0 = -B_1 + A_1 C_1 + \frac{c}{\lambda} \tan\theta + \frac{P}{\lambda} \times \frac{1}{H} + \frac{\Delta P}{\lambda} \times \frac{t}{H} \tag{17}$$

where ΔP is the change of the value of P in one day, and t is the elapsed time in days, counted from November 1st, 1865.

We have further

$$\mathfrak{C} = C_1 + A_1 B_1$$

and also

$$\mathfrak{C} = \frac{f}{\lambda} \tan\theta + \frac{Q}{\lambda} \times \frac{1}{H}$$

Hence

$$0 = -C_1 - A_1 B_1 + \frac{f}{\lambda} \tan\theta + \frac{Q}{\lambda} \times \frac{1}{H}.$$

But as Q is liable to undergo a slow change, we introduce a term depending upon the time, in the same manner as above, and the equation becomes

$$0 = -C_1 - A_1 B_1 + \frac{f}{\lambda} \tan\theta + \frac{Q}{\lambda} \times \frac{1}{H} + \frac{\Delta Q}{\lambda} \times \frac{t}{H} \tag{18}$$

Each observed value of B_1 and C_1 gives two equations of condition; one of the same form as (17), the other of the same form as (18); and from all the equations of condition thus obtained for any compass, the values of A_1, $\frac{c}{\lambda}$, $\frac{P}{\lambda}$, $\frac{\Delta P}{\lambda}$, $\frac{f}{\lambda}$, $\frac{Q}{\lambda}$, and $\frac{\Delta Q}{\lambda}$, for that compass, have been computed by the method of least squares.

The value of A_1 thus found we will designate as the "true A_1" in order to distinguish it from the "apparent A_1" obtained directly from the corrected observed values of the deviations. The value of the true A_1 depends only upon the value of the constants a, b, d, and e, in equations (1) and (2); but the apparent A_1 is made up of the true A_1, together with any errors that may exist in the placing of the lubber line of the compass, or in the determination of the true magnetic bearing of the distant object used as an azimuth mark in swinging the ship.

The equations of condition, formed in the manner just explained; the normal equations derived from them by the method of least squares; and the resulting values of the constants, A_1, $\frac{c}{\lambda}$, $\frac{P}{\lambda}$, $\frac{\Delta P}{\lambda}$, $\frac{f}{\lambda}$, $\frac{Q}{\lambda}$, and $\frac{\Delta Q}{\lambda}$, for each compass are as follows: the values of B_1 and C_1 being expressed in parts of radius.

ADMIRALTY STANDARD COMPASS.

Equations of Condition.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
0 = − 0.158	− 0.010	+ 2.694	+ 0.212				
0 = − 0.100	+ 0.010	+ 1.176	+ 0.148	+ 2.520			
0 = − 0.064	0.000	+ 0.077	+ 0.161	+ 9.516			
0 = − 0.054	+ 0.002	− 0.603	+ 0.166	+ 13.933			
0 = − 0.023	− 0.012	− 1.426	+ 0.164	+ 16.522			
0 = − 0.023	− 0.002	− 0.710	+ 0.158	+ 24.375			
0 = − 0.041	− 0.001	− 0.113	+ 0.143	+ 25.608			
0 = − 0.053	+ 0.001	+ 0.623	+ 0.132	+ 26.316			
0 = − 0.048	+ 0.002	+ 0.836	+ 0.129	+ 27.440			
0 = − 0.085	− 0.022	+ 1.910	+ 0.177	+ 41.519			
0 = + 0.010	− 0.158				+ 2.694	+ 0.212	
0 = − 0.010	− 0.100				+ 1.176	+ 0.148	+ 2.520
0 = 0.000	− 0.064				+ 0.077	+ 0.161	+ 9.516
0 = − 0.002	− 0.054				− 0.603	+ 0.166	+ 13.933
0 = + 0.012	− 0.023				− 1.426	+ 0.164	+ 16.522
0 = + 0.002	− 0.023				− 0.710	+ 0.158	+ 24.375
0 = + 0.001	− 0.041				− 0.113	+ 0.143	+ 25.608
0 = − 0.001	− 0.053				+ 0.623	+ 0.132	+ 26.316
0 = − 0.002	− 0.048				+ 0.836	+ 0.129	+ 27.440
0 = + 0.022	− 0.085				+ 1.910	+ 0.177	+ 41.519

Normal Equations.

0 = 0.000	+ 0.058						
0 = − 0.699	− 0.037	+ 16.294					
0 = − 0.109	− 0.006	+ 0.826	+ 0.258				
0 = − 9.869	− 1.057	+ 70.177	+ 28.825	+ 4983.3			
0 = + 0.037	− 0.699				+ 16.294		
0 = + 0.006	− 0.109				+ 0.826	+ 0.258	
0 = + 1.057	− 9.869				+ 70.177	+ 28.825	+ 4983.3

Hence

$$A_1 = 0.000 \qquad \frac{P}{\lambda} = +0.460 \qquad \frac{f}{\lambda} = -0.0016$$

$$\frac{c}{\lambda} = +0.0240 \qquad \frac{\Delta P}{\lambda} = +0.00102 \qquad \frac{Q}{\lambda} = +0.006$$

$$\frac{\Delta Q}{\lambda} = -0.00023$$

AFTER BINNACLE COMPASS.

Equations of Condition.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
0 = − 0.127	− 0.022	+ 2.694	+ 0.212				
0 = − 0.100	− 0.002	+ 0.077	+ 0.161	+ 9.516			
0 = − 0.096	+ 0.012	− 0.603	+ 0.166	+ 13.933			
0 = − 0.100	− 0.004	− 1.426	+ 0.164	+ 16.522			
0 = − 0.070	+ 0.002	− 0.710	+ 0.158	+ 24.375			
0 = − 0.073	− 0.001	− 0.113	+ 0.143	+ 25.608			
0 = − 0.058	+ 0.006	+ 0.623	+ 0.132	+ 26.316			
0 = − 0.054	− 0.005	+ 0.836	+ 0.129	+ 27.440			
0 = − 0.061	− 0.039	+ 1.910	+ 0.177	+ 41.519			
0 = + 0.022	− 0.127				+ 2.694	+ 0.212	
0 = + 0.002	− 0.100				+ 0.077	+ 0.161	+ 9.516
0 = − 0.012	− 0.096				− 0.603	+ 0.166	+ 13.933
0 = + 0.004	− 0.100				− 1.426	+ 0.164	+ 16.522
0 = − 0.002	− 0.070				− 0.710	+ 0 158	+ 24.375
0 = + 0.001	− 0.073				− 0.113	+ 0.143	+ 25 608
0 = − 0.006	− 0.058				+ 0.623	+ 0.132	+ 26.316
0 = + 0.005	− 0.054				+ 0.836	+ 0.129	+ 27.440
0 = + 0.039	− 0.061				+ 1.910	+ 0.177	+ 41.519

After Binnacle Compass.

Normal Equations.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
0 = 0.000	+ 0.068						
0 = − 0.288	− 0.136	+ 14.910					
0 = − 0.122	− 0.010	+ 0.652	+ 0.236				
0 = − 13.033	− 1.478	+ 67.212	+ 28.451	+ 4977.0			
0 = + 0.136	− 0.288				+ 14.910		
0 = + 0.010	− 0.122				+ 0.652	+ 0.236	
0 = + 1.478	− 13.033				+ 67.212	+ 28.451	+ 4977.0

Hence

$A_1 = -0.010$ $\frac{P}{\lambda} = +0.664$ $\frac{f}{\lambda} = -0.0084$

$\frac{c}{\lambda} = -0.0048$ $\frac{\Delta P}{\lambda} = -0.00112$ $\frac{Q}{\lambda} = +0.002$

$\frac{\Delta Q}{\lambda} = -0.00022$

After Ritchie Compass.

Equations of Condition.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
0 = − 0.200	− 0.030	+ 2.694	+ 0.212				
0 = − 0.148	+ 0.012	+ 1.176	+ 0.148	+ 2.520			
0 = − 0.121	− 0.017	+ 0.077	+ 0.161	+ 9.516			
0 = − 0.071	− 0.060	− 1.426	+ 0.164	+ 16.522			
0 = − 0.067	+ 0.004	− 0.710	+ 0.158	+ 24.375			
0 = − 0.102	+ 0.004	− 0.113	+ 0.143	+ 25.608			
0 = − 0.071	− 0.003	+ 0.623	+ 0.132	+ 26.316			
0 = − 0.078	+ 0.021	+ 0.836	+ 0.129	+ 27.440			
0 = − 0.118	− 0.027	+ 1.910	+ 0.177	+ 41.519			
0 = + 0.030	− 0.200				+ 2.694	+ 0.212	
0 = − 0.012	− 0.148				+ 1.176	+ 0.148	+ 2.520
0 = + 0.017	− 0.121				+ 0.077	+ 0.161	+ 9.516
0 = + 0.060	− 0.071				− 1.426	+ 0.164	+ 16.522
0 = − 0.004	− 0.067				− 0.710	+ 0.158	+ 24.375
0 = − 0.004	− 0.102				− 0.113	+ 0.143	+ 25.608
0 = + 0.003	− 0.071				+ 0.623	+ 0.132	+ 26.316
0 = − 0.021	− 0.078				+ 0.836	+ 0.129	+ 27.440
0 = + 0.027	− 0.118				+ 1.910	+ 0.177	+ 41.519

Normal Equations.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
0 = 0.000	+ 0.127						
0 = − 0.896	− 0.022	+ 15.930					
0 = − 0.161	− 0.018	+ 0.926	+ 0.231				
0 = − 15.837	− 1.525	+ 78.581	+ 26.514	+ 4789.2			
0 = + 0.022	− 0.896				+ 15.930		
0 = + 0.018	− 0.161				+ 0.926	+ 0.231	
0 = + 1.525	− 15.837				+ 78.581	+ 26.514	+ 4789.2

Hence

$A_1 = 0.000$ $\frac{P}{\lambda} = +0.766$ $\frac{f}{\lambda} = +0.0052$

$\frac{c}{\lambda} = +0.0178$ $\frac{\Delta P}{\lambda} = -0.00122$ $\frac{Q}{\lambda} = -0.149$

$\frac{\Delta Q}{\lambda} = +0.00042$

AFTER AZIMUTH COMPASS.

Equations of Condition.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
o = + 0.085	— 0.003	+ 2.694	+ 0.212				
o = + 0.053	+ 0.023	+ 1.176	+ 0.148	+ 2.520			
o = + 0.078	— 0.006	+ 0.077	+ 0.161	+ 9.516			
o = + 0.052	— 0.014	— 1.426	+ 0.164	+ 16.522			
o = + 0.086	+ 0.006	— 0.710	+ 0.158	+ 24.375			
o = + 0.035	— 0.014	— 0.113	+ 0.143	+ 25.608			
o = + 0.066	+ 0.030	+ 0.623	+ 0.132	+ 26.316			
o = + 0.060	0.000	+ 0.836	+ 0.129	+ 27.440			
o = + 0.003	+ 0.085				+ 2.694	+ 0.212	
o = — 0.023	+ 0.053				+ 1.176	+ 0.148	+ 2.520
o = + 0.006	+ 0.078				+ 0.077	+ 0.161	+ 9.516
o = + 0.014	+ 0.052				— 1.426	+ 0.164	+ 16.522
o = — 0.006	+ 0.086				— 0.710	+ 0.158	+ 24.375
o = + 0.014	+ 0.035				— 0.113	+ 0.143	+ 25.608
o = — 0.030	+ 0.066				+ 0.623	+ 0.132	+ 26.316
o = 0.000	+ 0.060				+ 0.836	+ 0.129	+ 27.440

Normal Equations.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
o = 0.000	+ 0.037						
o = + 0.250	+ 0.055	+ 12.282					
o = + 0.082	+ 0.003	+ 0.588	+ 0.200				
o = + 8.100	+ 0.352	— 0.725	+ 19.147	+ 3065.3			
o = — 0.055	+ 0.250				+ 12.282		
o = — 0.003	+ 0.082				+ 0.588	+ 0.200	
o = — 0.352	+ 8.100				— 0.725	+ 19.147	+ 3065.3

Hence

$A_1 = 0.000$ $\frac{P}{\lambda} = -0.373$ $\frac{f}{\lambda} = +0.0066$

$\frac{c}{\lambda} = -0.0026$ $\frac{\Delta P}{\lambda} = -0.00032$ $\frac{Q}{\lambda} = -0.044$

$\frac{\Delta Q}{\lambda} = +0.00039$

FORWARD ALIDADE COMPASS.

Equations of Condition.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
o = + 0.043	— 0.033	+ 2.694	+ 0.212				
o = + 0.010	— 0.013	+ 1.176	+ 0.148	+ 2.520			
o = + 0.002	— 0.010	+ 0.077	+ 0.161	+ 9.516			
o = — 0.017	— 0.019	— 0.603	+ 0.166	+ 13.933			
o = — 0.017	— 0.033	— 1.426	+ 0.164	+ 16.522			
o = — 0.009	— 0.016	— 0.710	+ 0.158	+ 24.375			
o = — 0.012	— 0.028	— 0.113	+ 0.143	+ 25.608			
o = 0.000	— 0.024	+ 0.623	+ 0.132	+ 26.316			
o = + 0.026	— 0.010	+ 0.836	+ 0.129	+ 27.440			
o = + 0.033	— 0.042	+ 1.910	+ 0.177	+ 41.519			
o = + 0.033	+ 0.043				+ 2.694	+ 0.212	
o = + 0.013	+ 0.010				+ 1.176	+ 0.148	+ 2.520
o = + 0.010	+ 0.002				+ 0.077	+ 0.161	+ 9.516
o = + 0.019	— 0.017				— 0.603	+ 0.166	+ 13.933
o = + 0.033	— 0.017				— 1.426	+ 0.164	+ 16.522
o = + 0.016	— 0.009				— 0.710	+ 0.158	+ 24.375
o = + 0.028	— 0.012				— 0.113	+ 0.143	+ 25.608
o = + 0.024	0.000				+ 0.623	+ 0.132	+ 26.316
o = + 0.010	+ 0.026				+ 0.836	+ 0.129	+ 27.440
o = + 0.042	+ 0.033				+ 1.910	+ 0.177	+ 41.519

FORWARD ALIDADE COMPASS.

Normal Equations.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
$0 = \quad 0.000$	$+0.011$						
$0 = +0.255$	-0.135	$+16.294$					
$0 = +0.012$	-0.037	$+0.826$	$+0.258$				
$0 = +1.089$	-4.686	$+70.177$	$+28.825$	$+4983.3$			
$0 = +0.135$	$+0.255$				$+16.294$		
$0 = +0.037$	$+0.012$				$+0.826$	$+0.258$	
$0 = +4.686$	$+1.089$				$+70.177$	$+28.825$	$+4983.3$

Hence

$A_1 = -0.025$ $\qquad \frac{P}{\lambda} = +0.014$ $\qquad \frac{f}{\lambda} = -0.0012$

$\frac{c}{\lambda} = -0.0162$ $\qquad \frac{\Delta P}{\lambda} = -0.00010$ $\qquad \frac{Q}{\lambda} = -0.106$

$\frac{\Delta Q}{\lambda} = -0.00031$

FORWARD BINNACLE COMPASS.

Equations of Condition.

Absolute Terms.	A	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
$0 = +0.099$	-0.045	$+2.694$	$+0.212$				
$0 = +0.034$	-0.004	$+1.176$	$+0.148$	$+2.520$			
$0 = -0.008$	-0.010	$+0.077$	$+0.161$	$+9.516$			
$0 = -0.051$	-0.012	-0.603	$+0.166$	$+13.933$			
$0 = -0.092$	-0.038	-1.426	$+0.164$	$+16.522$			
$0 = -0.031$	-0.013	-0.710	$+0.158$	$+24.375$			
$0 = -0.020$	-0.037	-0.113	$+0.143$	$+25.608$			
$0 = +0.018$	-0.027	$+0.623$	$+0.132$	$+26.316$			
$0 = +0.036$	-0.029	$+0.836$	$+0.129$	$+27.440$			
$0 = +0.082$	-0.062	$+1.910$	$+0.177$	$+41.519$			
$0 = +0.045$	$+0.099$				$+2.694$	$+0.212$	
$0 = +0.004$	$+0.034$				$+1.176$	$+0.148$	$+2.520$
$0 = +0.010$	-0.008				$+0.077$	$+0.161$	$+9.516$
$0 = +0.012$	-0.051				-0.603	$+0.166$	$+13.933$
$0 = +0.038$	-0.092				-1.426	$+0.164$	$+16.522$
$0 = +0.013$	-0.031				-0.710	$+0.158$	$+24.375$
$0 = +0.037$	-0.020				-0.113	$+0.143$	$+25.608$
$0 = +0.027$	$+0.018$				$+0.623$	$+0.132$	$+26.316$
$0 = +0.029$	$+0.036$				$+0.836$	$+0.129$	$+27.440$
$0 = +0.062$	$+0.082$				$+1.910$	$+0.177$	$+41.519$

Normal Equations.

Absolute Terms.	A	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
$0 = \quad 0.000$	$+0.043$						
$0 = +0.690$	-0.211	$+16.294$					
$0 = +0.015$	-0.046	$+0.826$	$+0.258$				
$0 = +1.334$	-6.283	$+70.177$	$+28.825$	$+4983.3$			
$0 = +0.211$	$+0.690$				$+16.294$		
$0 = +0.046$	$+0.015$				$+0.826$	$+0.258$	
$0 = +6.283$	$+1.334$				$+70.177$	$+28.825$	$+4983.3$

Hence

$A_1 = \quad 0.000$ $\qquad \frac{P}{\lambda} = +0.140$ $\qquad \frac{f}{\lambda} = -0.0059$

$\frac{c}{\lambda} = -0.0477$ $\qquad \frac{\Delta P}{\lambda} = -0.00041$ $\qquad \frac{Q}{\lambda} = -0.075$

$\frac{\Delta Q}{\lambda} = -0.00074$

Forward Ritchie Compass.

Equations of Condition.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
0 = − 0.023	− 0.063	+ 2.694	+ 0.212				
0 = − 0.036	− 0.022	+ 1.176	+ 0.148	+ 2.520			
0 = − 0.061	− 0.027	+ 0.077	+ 0.161	+ 9.516			
0 = − 0.066	0.000	− 0.603	+ 0.166	+ 13.933			
0 = − 0.067	− 0.048	− 1.426	+ 0.164	+ 16.522			
0 = − 0.023	− 0.026	− 0.710	+ 0.158	+ 24.375			
0 = − 0.033	− 0.034	− 0.113	+ 0.143	+ 25.608			
0 = − 0.004	− 0.033	+ 0.623	+ 0.132	+ 26.316			
0 = − 0.011	− 0.038	+ 0.836	+ 0.129	+ 27.440			
0 = + 0.005	− 0.117	+ 1.910	+ 0.177	+ 41.519			
0 = + 0.063	− 0.023						
0 = + 0.022	− 0.036				+ 2.694	+ 0.212	
0 = + 0.027	− 0.061				+ 1.176	+ 0.148	+ 2.520
0 = 0.000	− 0.066				+ 0.077	+ 0.161	+ 9.516
0 = + 0.048	− 0.067				− 0.603	+ 0.166	+ 13.933
0 = + 0.026	− 0.023				− 1.426	+ 0.164	+ 16.522
0 = + 0.034	− 0.033				− 0.710	+ 0.158	+ 24.375
0 = + 0.033	− 0.004				− 0.113	+ 0.143	+ 25.608
0 = + 0.038	− 0.011				+ 0.623	+ 0.132	+ 26.316
0 = + 0.117	+ 0.005				+ 0.836	+ 0.129	+ 27.440
					+ 1.910	+ 0.177	+ 41.519

Normal Equations.

Absolute Terms.	A_1	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{\Delta P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$
0 = 0.000	+ 0.042						
0 = + 0.044	− 0.384	+ 16.294					
0 = − 0.052	− 0.068	+ 0.826	+ 0.258				
0 = − 4.306	− 9.388	+ 70.177	+ 28.825	+ 4983.3			
0 = + 0.384	+ 0.044				+ 16.294		
0 = + 0.068	− 0.052				+ 0.826	+ 0.258	
0 = + 9.388	− 4.306				+ 70.177	+ 28.825	+ 4983.3

Hence

$A_1 = 0.000$ $\frac{P}{\lambda} = +0.367$ $\frac{f}{\lambda} = -0.0141$

$\frac{c}{\lambda} = -0.0169$ $\frac{\Delta P}{\lambda} = -0.00102$ $\frac{Q}{\lambda} = -0.083$

$\frac{\Delta Q}{\lambda} = -0.00120$

The value of the true A_1 having thus become known for each compass, the values of the coefficients 𝔅, ℭ, 𝔇, and 𝔈, for each compass, at each station, were next computed by means of the formulæ (16). The results, expressed in parts of radius, are as follows:

Coefficients of the Deviations of the Admiralty Standard Compass.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865	0.000	+ 0.158	− 0.010	+ 0.021	− 0.004
St. Thomas	November 18, 1865	0.000	+ 0.100	+ 0.010	+ 0.006	− 0.013
Bahia	December 30, 1865	0.000	+ 0.064	0.000	+ 0.016	0.000
Monte Video	January 24, 1866	0.000	+ 0.054	+ 0.002	+ 0.024	+ 0.004
Sandy Point	February 10, 1866	0.000	+ 0.023	− 0.012	+ 0.016	0.000
Valparaiso	April 4, 1866	0.000	+ 0.023	− 0.002	+ 0.016	− 0.003
Callao	April 29, 1866	0.000	+ 0.041	0.000	+ 0.016	+ 0.002
Panama	May 20, 1866	0.000	+ 0.053	+ 0.001	+ 0.017	+ 0.002
Acapulco	June 1, 1866	0.000	+ 0.048	+ 0.002	+ 0.018	+ 0.002
San Francisco	June 23, 1866	0.000	+ 0.085	− 0.022	+ 0.018	0.000
Means					+ 0.017	− 0.001

COEFFICIENTS OF THE DEVIATIONS OF THE AFTER BINNACLE.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads	November 1, 1865	— 0.010	+ 0.127	— 0.023	+ 0.037	— 0.001
St. Thomas	November 18, 1865	— 0.010	......	......	......	......
Bahia	December 30, 1865	— 0.010	+ 0.100	— 0.003	+ 0.034	+ 0.002
Monte Video	January 24, 1866	— 0.010	+ 0.096	+ 0.011	+ 0.039	— 0.012
Sandy Point	February 10, 1866	— 0.010	+ 0.100	— 0.005	+ 0.040	— 0.001
Valparaiso	April 4, 1866	— 0.010	+ 0.070	+ 0.002	+ 0.038	0.000
Callao	April 29, 1866	— 0.010	+ 0.073	— 0.002	+ 0.040	+ 0.002
Panama	May 20, 1866	— 0.010	+ 0.058	+ 0.006	+ 0.046	— 0.005
Acapulco	June 1, 1866	— 0.010	+ 0.054	— 0.006	+ 0.041	— 0.006
San Francisco	June 23, 1866	— 0.010	+ 0.060	— 0.040	+ 0.032	0.000
Means					+ 0.038	— 0.002

COEFFICIENTS OF THE DEVIATIONS OF THE AFTER RITCHIE COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads	November 1, 1865	0.000	+ 0.200	— 0.030	+ 0.024	— 0.022
St. Thomas	November 18, 1865	0.000	+ 0.148	+ 0.012	+ 0.044	— 0.009
Bahia	December 30, 1865	0.000	+ 0.121	— 0.017	+ 0.042	+ 0.002
Monte Video	January 24, 1866	......	......	......	......	......
Sandy Point	February 10, 1866	0.000	+ 0.071	— 0.060	+ 0.022	+ 0.013
Valparaiso	April 4, 1866	0.000	+ 0.067	+ 0.004	+ 0.043	+ 0.002
Callao	April 29, 1866	0.000	+ 0.102	+ 0.004	+ 0.032	+ 0.016
Panama	May 20, 1866	0.000	+ 0.071	— 0.003	+ 0.025	— 0.027
Acapulco	June 1, 1866	0.000	+ 0.078	+ 0.021	+ 0.024	+ 0.015
San Francisco	June 23, 1866	0.000	+ 0.118	— 0.027	+ 0.050	+ 0.003
Means					+ 0.034	— 0.001

COEFFICIENTS OF THE DEVIATIONS OF THE AFTER AZIMUTH COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads	November 1, 1865	0.000	— 0.085	— 0.003	+ 0.101	+ 0.005
St. Thomas	November 18, 1865	0.000	— 0.053	+ 0.023	+ 0.120	+ 0.002
Bahia	December 30, 1865	0.000	— 0.078	— 0.006	+ 0.132	— 0.019
Monte Video	January 24, 1866	......	......	......	......	......
Sandy Point	February 10, 1866	0.000	— 0.052	— 0.014	+ 0.126	— 0.007
Valparaiso	April 4, 1866	0.000	— 0.086	+ 0.006	+ 0.106	+ 0.010
Callao	April 29, 1866	0.000	— 0.035	— 0.014	+ 0.090	+ 0.011
Panama	May 20, 1866	0.000	— 0.066	+ 0.030	+ 0.113	— 0.012
Acapulco	June 1, 1866	0.000	— 0.060	0.000	+ 0.105	+ 0.007
San Francisco	June 23, 1866	......	......	......	......	......
Means					+ 0.112	0.000

COEFFICIENTS OF THE DEVIATIONS OF THE FORWARD ALIDADE COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads	November 1, 1865	— 0.025	— 0.044	— 0.032	+ 0.019	+ 0.001
St. Thomas	November 18, 1865	— 0.025	— 0.010	— 0.013	+ 0.022	+ 0.006
Bahia	December 30, 1865	— 0.025	— 0.002	— 0.010	+ 0.022	+ 0.004
Monte Video	January 24, 1866	— 0.025	+ 0.016	— 0.019	+ 0.024	— 0.004
Sandy Point	February 10, 1866	— 0.025	+ 0.017	— 0.034	+ 0.031	— 0.007
Valparaiso	April 4, 1866	— 0.025	+ 0.008	— 0.016	+ 0.019	— 0.002
Callao	April 29, 1866	— 0.025	+ 0.012	— 0.029	+ 0.026	— 0.003
Panama	May 20, 1866	— 0.025	— 0.001	— 0.024	+ 0.023	— 0.003
Acapulco	June 1, 1866	— 0.025	— 0.026	— 0.009	+ 0.033	+ 0.002
San Francisco	June 23, 1866	— 0.025	— 0.034	— 0.041	+ 0.017	+ 0.007
Means					+ 0.024	0.000

COEFFICIENTS OF THE DEVIATIONS OF THE FORWARD BINNACLE COMPASS.

STATION.	DATE.	$\mathfrak{A}$	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{D}$	$\mathfrak{E}$
Hampton Roads . . .	November 1, 1865	0.000	− 0.099	− 0.045	+ 0.044	+ 0.007
St. Thomas	November 18, 1865	0.000	− 0.034	− 0.004	+ 0.035	− 0.002
Bahia	December 30, 1865	0.000	+ 0.008	− 0.010	+ 0.037	− 0.003
Monte Video	January 24, 1866	0.000	+ 0.051	− 0.012	+ 0.032	− 0.001
Sandy Point	February 10, 1866	0.000	+ 0.092	− 0.038	+ 0.039	− 0.004
Valparaiso	April 4, 1866	0.000	+ 0.031	− 0.013	+ 0.028	− 0.003
Callao	April 29, 1866	0.000	+ 0.020	− 0.037	+ 0.037	+ 0.006
Panama.	May 20, 1866	0.000	− 0.018	− 0.027	+ 0.037	− 0.006
Acapulco	June 1, 1866	0.000	− 0.036	− 0.029	+ 0.046	+ 0.004
San Francisco. . . .	June 23, 1866	0.000	− 0.082	− 0.062	+ 0.035	+ 0.014
Means					+ 0.037	+ 0.001

COEFFICIENTS OF THE DEVIATIONS OF THE FORWARD RITCHIE COMPASS.

STATION.	DATE.	$\mathfrak{A}$	$\mathfrak{B}$	$\mathfrak{C}$	$\mathfrak{D}$	$\mathfrak{E}$
Hampton Roads . . .	November 1, 1865	0.000	+ 0.023	− 0.063	+ 0.038	+ 0.006
St. Thomas	November 18, 1865	0.000	+ 0.036	− 0.022	+ 0.057	− 0.008
Bahia	December 30, 1865	0.000	+ 0.061	− 0.027	+ 0.047	− 0.002
Monte Video	January 24, 1866	0.000	+ 0.066	0.000	+ 0.040	− 0.008
Sandy Point	February 10, 1866	0.000	+ 0.067	− 0.048	+ 0.039	− 0.006
Valparaiso	April 4, 1866	0.000	+ 0.023	− 0.026	+ 0.037	+ 0.008
Callao	April 29, 1866	0.000	+ 0.033	− 0.034	+ 0.044	+ 0.002
Panama.	May 20, 1866	0.000	+ 0.004	− 0.033	+ 0.038	− 0.004
Acapulco	June 1, 1866	0.000	+ 0.011	− 0.038	+ 0.041	+ 0.007
San Francisco. . . .	June 23, 1866	0.000	− 0.005	− 0.117	+ 0.025	− 0.009
Means					+ 0.041	− 0.001

The values of the coefficients $\mathfrak{D}$ and $\mathfrak{E}$ for any compass should be constant. Therefore the mean of all the observed values has been assumed as the truth, and is given on the line marked "means" in the case of each compass.

The constants thus far determined furnish the data with which to compute the values of the coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, in any part of the world, for any of the compasses under discussion. For convenience of reference these constants are collected in the following table:

Compass.	$A_1 = \mathfrak{A}$	$\frac{c}{\lambda}$	$\frac{P}{\lambda}$	$\frac{P}{\lambda}$	$\frac{f}{\lambda}$	$\frac{Q}{\lambda}$	$\frac{\Delta Q}{\lambda}$	$\mathfrak{D}$	$\mathfrak{E}$
Admiralty Standard .	0.000	+ 0.0240	+ 0.460	− 0.00102	− 0.0016	+ 0.006	− 0.00023	+ 0.017	− 0.001
After Binnacle . . .	− 0.010	− 0.0048	+ 0.664	− 0.00112	− 0.0084	+ 0.002	− 0.00022	+ 0.038	− 0.002
After Ritchie . . .	0.000	+ 0.0178	+ 0.766	− 0.00122	+ 0.0052	− 0.149	+ 0.00042	+ 0.034	− 0.001
After Azimuth . . .	0.000	− 0.0026	− 0.373	− 0.00032	+ 0.0066	− 0.044	+ 0.00039	+ 0.112	0.000
Forward Alidade . .	− 0.025	− 0.0162	+ 0.014	− 0.00010	− 0.0012	− 0.106	− 0.00031	+ 0.024	0.000
Forward Binnacle . .	0.000	− 0.0477	+ 0.140	− 0.00041	− 0.0059	− 0.075	− 0.00074	+ 0.037	+ 0.001
Forward Ritchie . .	0.000	− 0.0169	+ 0.367	− 0.00102	− 0.0141	− 0.083	− 0.00120	+ 0.041	− 0.001

The values of the coefficients $\mathfrak{A}$, $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, for each compass at each station, were next computed from the quantities given in this table, in the following manner. The coefficients $\mathfrak{A}$, $\mathfrak{D}$, and $\mathfrak{E}$ are constant for each compass, and were taken directly from the table; while the coefficients 𝔅 and ℭ were obtained by means of the formulæ

$$\mathfrak{B}=\frac{c}{\lambda}\tan\theta+\frac{P}{\lambda}\times\frac{1}{H}+\frac{\Delta P}{\lambda}\times\frac{t}{H}$$

$$\mathfrak{C}=\frac{f}{\lambda}\tan\theta+\frac{Q}{\lambda}\times\frac{1}{H}+\frac{\Delta Q}{\lambda}\times\frac{t}{H}$$

where θ is the true magnetic dip; H the earth's magnetic horizontal force, expressed in English units, namely, in feet, grains, and seconds; and t the time in days, counted from November 1st, 1865. The results, expressed in parts of radius, are as follows:

COEFFICIENTS OF THE DEVIATIONS OF THE ADMIRALTY STANDARD COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865	0.000	+ 0.162	− 0.003	+ 0.017	− 0.001
St. Thomas	November 18, 1865	0.000	+ 0.094	− 0.002	+ 0.017	− 0.001
Bahia	December 30, 1865	0.000	+ 0.066	− 0.001	+ 0.017	− 0.001
Monte Video	January 24, 1866	0.000	+ 0.048	− 0.001	+ 0.017	− 0.001
Sandy Point	February 10, 1866	0.000	+ 0.024	0.000	+ 0.017	− 0.001
Valparaiso	April 4, 1866	0.000	+ 0.031	− 0.003	+ 0.017	− 0.001
Callao	April 29, 1866	0.000	+ 0.037	− 0.005	+ 0.017	− 0.001
Panama	May 20, 1866	0.000	+ 0.049	− 0.006	+ 0.017	− 0.001
Acapulco	June 1, 1866	0.000	+ 0.052	− 0.007	+ 0.017	− 0.001
San Francisco	June 23, 1866	0.000	+ 0.085	− 0.011	+ 0.017	− 0.001

COEFFICIENTS OF THE DEVIATIONS OF THE AFTER BINNACLE COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865	− 0.010	+ 0.128	− 0.022	+ 0.038	− 0.002
St. Thomas	November 18, 1865	......	......	......	......	......
Bahia	December 30, 1865	− 0.010	+ 0.096	− 0.002	+ 0.038	− 0.002
Monte Video	January 24, 1866	− 0.010	+ 0.098	+ 0.002	+ 0.038	− 0.002
Sandy Point	February 10, 1866	− 0.010	+ 0.097	+ 0.009	+ 0.038	− 0.002
Valparaiso	April 4, 1866	− 0.010	+ 0.081	+ 0.001	+ 0.038	− 0.002
Callao	April 29, 1866	− 0.010	+ 0.067	− 0.004	+ 0.038	− 0.002
Panama	May 20, 1866	− 0.010	+ 0.055	− 0.011	+ 0.038	− 0.002
Acapulco	June 1, 1866	− 0.010	+ 0.051	− 0.013	+ 0.038	− 0.002
San Francisco	June 23, 1866	− 0.010	+ 0.062	− 0.025	+ 0.038	− 0.002

COEFFICIENTS OF THE DEVIATIONS OF THE AFTER RITCHIE COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865	0.000	+ 0.211	− 0.018	+ 0.034	− 0.001
St. Thomas	November 18, 1865	0.000	+ 0.131	− 0.015	+ 0.034	− 0.001
Bahia	December 30, 1865	0.000	+ 0.113	− 0.020	+ 0.034	− 0.001
Monte Video	January 24, 1866	......	......	......	......	......
Sandy Point	February 10, 1866	0.000	+ 0.080	− 0.025	+ 0.034	− 0.001
Valparaiso	April 4, 1866	0.000	+ 0.079	− 0.017	+ 0.034	− 0.001
Callao	April 29, 1866	0.000	+ 0.076	− 0.011	+ 0.034	− 0.001
Panama	May 20, 1866	0.000	+ 0.080	− 0.005	+ 0.034	− 0.001
Acapulco	June 1, 1866	0.000	+ 0.080	− 0.003	+ 0.034	− 0.001
San Francisco	June 23, 1866	0.000	+ 0.119	+ 0.001	+ 0.034	− 0.001

COEFFICIENTS OF THE DEVIATIONS OF THE AFTER AZIMUTH COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865	0.000	— 0.086	+ 0.008	+ 0.112	0.000
St. Thomas	November 18, 1865	0.000	— 0.059	+ 0.002	+ 0.112	0.000
Bahia	December 30, 1865	0.000	— 0.063	— 0.003	+ 0.112	0.000
Monte Video	January 24, 1866	......	......	......	......	......
Sandy Point	February 10, 1866	0.000	— 0.062	— 0.010	+ 0.112	0.000
Valparaiso	April 4, 1866	0.000	— 0.065	— 0.002	+ 0.112	0.000
Callao	April 29, 1866	0.000	— 0.061	+ 0.003	+ 0.112	0.000
Panama.	May 20, 1866	0.000	— 0.059	+ 0.009	+ 0.112	0.000
Acapulco	June 1, 1866	0.000	— 0.059	+ 0.011	+ 0.112	0.000
San Francisco. . . .	June 23, 1866	......	......	......	......	......

COEFFICIENTS OF THE DEVIATIONS OF THE FORWARD ALIDADE COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865	— 0.025	— 0.041	— 0.026	+ 0.024	0.000
St. Thomas	November 18, 1865	— 0.025	— 0.017	— 0.018	+ 0.024	0.000
Bahia	December 30, 1865	— 0.025	0.000	— 0.020	+ 0.024	0.000
Monte Video	January 24, 1866	— 0.025	+ 0.011	— 0.021	+ 0.024	0.000
Sandy Point	February 10, 1866	— 0.025	+ 0.024	— 0.021	+ 0.024	0.000
Valparaiso	April 4, 1866	— 0.025	+ 0.011	— 0.023	+ 0.024	0.000
Callao	April 29, 1866	— 0.025	+ 0.001	— 0.023	+ 0.024	0.000
Panama.	May 20, 1866	— 0.025	— 0.011	— 0.023	+ 0.024	0.000
Acapulco	June 1, 1866	— 0.025	— 0.014	— 0.023	+ 0.024	0.000
San Francisco. . . .	June 23, 1866	— 0.025	— 0.032	— 0.034	+ 0.024	0.000

COEFFICIENTS OF THE DEVIATIONS OF THE FORWARD BINNACLE COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865	0.000	— 0.099	— 0.032	+ 0.037	+ 0.001
St. Thomas	November 18, 1865	0.000	— 0.036	— 0.020	+ 0.037	+ 0.001
Bahia	December 30, 1865	0.000	+ 0.015	— 0.020	+ 0.037	+ 0.001
Monte Video	January 24, 1866	0.000	+ 0.046	— 0.019	+ 0.037	+ 0.001
Sandy Point	February 10, 1866	0.000	+ 0.084	— 0.016	+ 0.037	+ 0.001
Valparaiso	April 4, 1866	0.000	+ 0.046	— 0.026	+ 0.037	+ 0.001
Callao	April 29, 1866	0.000	+ 0.015	— 0.029	+ 0.037	+ 0.001
Panama.	May 20, 1866	0.000	— 0.022	— 0.033	+ 0.037	+ 0.001
Acapulco	June 1, 1866	0.000	— 0.033	— 0.035	+ 0.037	+ 0.001
San Francisco. . . .	June 23, 1866	0.000	— 0.083	— 0.056	+ 0.037	+ 0.001

COEFFICIENTS OF THE DEVIATIONS OF THE FORWARD RITCHIE COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865	0.000	+ 0.032	— 0.056	+ 0.041	— 0.001
St. Thomas	November 18, 1865	0.000	+ 0.032	— 0.032	+ 0.041	— 0.001
Bahia	December 30, 1865	0.000	+ 0.048	— 0.026	+ 0.041	— 0.001
Monte Video	January 24, 1866	0.000	+ 0.057	— 0.022	+ 0.041	— 0.001
Sandy Point	February 10, 1866	0.000	+ 0.067	— 0.013	+ 0.041	— 0.001
Valparaiso	April 4, 1866	0.000	+ 0.045	— 0.032	+ 0.041	— 0.001
Callao	April 29, 1866	0.000	+ 0.028	— 0.041	+ 0.041	— 0.001
Panama.	May 20, 1866	0.000	+ 0.011	— 0.051	+ 0.041	— 0.001
Acapulco	June 1, 1866	0.000	+ 0.005	— 0.056	+ 0.041	— 0.001
San Francisco. . . .	June 23, 1866	0.000	— 0.010	— 0.092	+ 0.041	— 0.001

Comparing these computed values with the values before found directly from the observations, the following residuals are obtained:

VALUE OF THE COMPUTED MINUS THE OBSERVED COEFFICIENTS OF THE DEVIATIONS OF THE ADMIRALTY STANDARD COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads	November 1, 1865		+ 0.004	+ 0.007	— 0.004	+ 0.003
St. Thomas	November 18, 1865		— 0.006	— 0.012	+ 0.011	+ 0.012
Bahia	December 30, 1865		+ 0.002	— 0.001	+ 0.001	— 0.001
Monte Video	January 24, 1866		— 0.006	— 0.003	— 0.007	— 0.005
Sandy Point	February 10, 1866		+ 0.001	+ 0.012	+ 0.001	— 0.001
Valparaiso	April 4, 1866		+ 0.008	— 0.001	+ 0.001	+ 0.002
Callao	April 29, 1866		— 0.004	— 0.005	+ 0.001	— 0.003
Panama	May 20, 1866		— 0.004	— 0.007	0.000	— 0.003
Acapulco	June 1, 1866		+ 0.004	— 0.009	— 0.001	— 0.003
San Francisco	June 23, 1866		0.000	+ 0.011	— 0.001	— 0.001

VALUE OF THE COMPUTED MINUS THE OBSERVED COEFFICIENTS OF THE DEVIATIONS OF THE AFTER BINNACLE COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads	November 1, 1865		+ 0.001	+ 0.001	+ 0.001	— 0.001
St. Thomas	November 18, 1865		......	......	......	......
Bahia	December 30, 1865		— 0.004	+ 0.001	+ 0.004	— 0.004
Monte Video	January 24, 1866		+ 0.002	— 0.009	— 0.001	+ 0.010
Sandy Point	February 10, 1866		— 0.003	+ 0.014	— 0 002	— 0.001
Valparaiso	April 4, 1866		+ 0.011	— 0.001	0.000	— 0.002
Callao	April 29, 1866		— 0.006	— 0.002	— 0.002	— 0.004
Panama	May 20, 1866		— 0.003	— 0.017	— 0.008	+ 0.003
Acapulco	June 1, 1866		— 0.003	— 0.007	— 0.003	+ 0.004
San Francisco	June 23, 1866		+ 0.002	+ 0.015	+ 0.006	— 0.002

VALUE OF THE COMPUTED MINUS THE OBSERVED COEFFICIENTS OF THE DEVIATIONS OF THE AFTER RITCHIE COMPASS.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads	November 1, 1865		+ 0.011	+ 0.012	+ 0.010	+ 0.021
St. Thomas	November 18, 1865		— 0.017	— 0.027	— 0.010	+ 0.008
Bahia	December 30, 1865		— 0.008	— 0.003	— 0.008	— 0.003
Monte Video	January 24, 1866		......	......	......	......
Sandy Point	February 10, 1866		+ 0.009	+ 0.035	+ 0.012	— 0.014
Valparaiso	April 4, 1866		+ 0.012	— 0.021	— 0.009	— 0.003
Callao	April 29, 1866		— 0.026	— 0.015	+ 0.002	— 0.017
Panama	May 20, 1866		+ 0.009	— 0.002	+ 0.009	+ 0.026
Acapulco	June 1, 1866		+ 0.002	— 0.024	+ 0.010	— 0.016
San Francisco	June 23, 1866		+ 0.001	+ 0.028	— 0.016	— 0.004

Value of the Computed minus the Observed Coefficients of the Deviations of the After Azimuth Compass.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865		— 0.001	+ 0.011	+ 0.011	— 0.005
St. Thomas	November 18, 1865		— 0.006	— 0.021	— 0.008	— 0.002
Bahia	December 30, 1865		+ 0.015	+ 0.003	— 0.020	+ 0.019
Monte Video	January 24, 1866		……	……	……	……
Sandy Point	February 10, 1866		— 0.010	+ 0.004	— 0.014	+ 0.007
Valparaiso	April 4, 1866		+ 0.021	— 0.008	+ 0.006	— 0.010
Callao	April 29, 1866		— 0.026	+ 0.017	+ 0.022	— 0.011
Panama.	May 20, 1866		+ 0.007	— 0.021	— 0.001	+ 0.012
Acapulco	June 1, 1866		+ 0.001	+ 0.011	+ 0.007	— 0.007
San Francisco. . . .	June 23, 1866		……	……	……	……

Value of the Computed minus the Observed Coefficients of the Deviations of the Forward Alidade Compass.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865		+ 0.003	+ 0.006	+ 0.005	— 0.001
St. Thomas	November 18, 1865		— 0.007	— 0.005	+ 0.002	— 0.006
Bahia	December 30, 1865		+ 0.002	— 0.010	+ 0.002	— 0.004
Monte Video	January 24, 1866		— 0.005	— 0.002	0.000	+ 0.004
Sandy Point	February 10, 1866		+ 0.007	+ 0.013	— 0.007	+ 0.007
Valparaiso	April 4, 1866		+ 0.003	— 0.007	+ 0.005	+ 0.002
Callao	April 29, 1866		— 0.011	+ 0.006	— 0.002	+ 0.003
Panama.	May 20, 1866		— 0.010	+ 0.001	+ 0.001	+ 0.003
Acapulco	June 1, 1866		+ 0.012	— 0.014	— 0.009	— 0.002
San Francisco. . . .	June 23, 1866		+ 0.002	+ 0.007	+ 0.007	— 0.007

Value of the Computed minus the Observed Coefficients of the Deviations of the Forward Binnacle Compass.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865		0.000	+ 0.013	— 0.007	— 0.006
St. Thomas	November 18, 1865		— 0.002	— 0.016	+ 0.002	+ 0.003
Bahia	December 30, 1865		+ 0.007	— 0.010	0.000	+ 0.004
Monte Video	January 24, 1866		— 0.005	— 0.007	+ 0.005	+ 0.002
Sandy Point	February 10, 1866		— 0.008	+ 0.022	— 0.002	+ 0.005
Valparaiso	April 4, 1866		+ 0.015	— 0.013	+ 0.009	+ 0.004
Callao	April 29, 1866		— 0.005	+ 0.008	0.000	— 0.005
Panama.	May 20, 1866		— 0.004	— 0.006	0.000	+ 0.007
Acapulco	June 1, 1866		+ 0.003	— 0.006	— 0.009	— 0.003
San Francisco. . . .	June 23, 1866		— 0.001	+ 0.006	+ 0.002	— 0.013

Value of the Computed minus the Observed Coefficients of the Deviations of the Forward Ritchie Compass.

STATION.	DATE.	𝔄	𝔅	ℭ	𝔇	𝔈
Hampton Roads . . .	November 1, 1865		+ 0.009	+ 0.007	+ 0.003	— 0.007
St. Thomas	November 18, 1865		— 0.004	— 0.010	— 0.016	+ 0.007
Bahia	December 30, 1865		— 0.013	+ 0.001	— 0.006	+ 0.001
Monte Video	January 24, 1866		— 0.009	— 0.022	+ 0.001	+ 0.007
Sandy Point	February 10, 1866		0.000	+ 0.035	+ 0.002	+ 0.005
Valparaiso	April 4, 1866		+ 0.022	— 0.006	+ 0.004	— 0.009
Callao	April 29, 1866		— 0.005	— 0.007	— 0.003	— 0.003
Panama.	May 20, 1866		+ 0.007	— 0.018	+ 0.003	+ 0.003
Acapulco	June 1, 1866		— 0.006	— 0.018	0.000	— 0.008
San Francisco. . . .	June 23, 1866		— 0.005	+ 0.025	+ 0.016	+ 0.008

In the following table the columns headed $r_{\mathfrak{B}}$, $r_{\mathfrak{C}}$, $r_{\mathfrak{D}}$, $r_{\mathfrak{E}}$, contain respectively the probable errors of a single observed value of $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, and $\mathfrak{E}$, for each compass, computed from the residuals just given. But as these residuals were got by subtracting the computed from the observed values of the coefficients, and as each observed value was found from a set of deviations observed on all the thirty-two points, it follows that the probable errors here given belong to the coefficients when they have been computed from a set of deviations observed on all the thirty-two points. For convenience of reference we will designate these as the probable errors derived from all the observations of the cruise.

Compass.	$r_{\mathfrak{B}}$	$r_{\mathfrak{C}}$	$r_{\mathfrak{D}}$	$r_{\mathfrak{E}}$	$\frac{r}{\sqrt{16}}$
Admiralty Standard	± 0.0033	± 0.0053	± 0.0032	± 0.0033	± 0.0008
After Binnacle	± 0.0036	± 0.0069	± 0.0026	± 0.0028	± 0.0018
After Ritchie	± 0.0090	± 0.0153	± 0.0072	± 0.0106	± 0.0031
After Azimuth	± 0.0100	± 0.0100	± 0.0094	± 0.0074	± 0.0030
Forward Alidade	± 0.0050	± 0.0059	± 0.0035	± 0.0031	± 0.0016
Forward Binnacle	± 0.0046	± 0.0084	± 0.0036	± 0.0043	± 0.0024
Forward Ritchie	± 0.0070	± 0.0127	± 0.0056	± 0.0047	± 0.0032
Means	± 0.0061	± 0.0092	± 0.0050	± 0.0052	± 0.0023

But we have before found the probable errors of B_1, C_1, D_1, and E_1, when computed from observations made at a single station on each of the thirty-two points, by a totally different process, namely, from the thirty-two observed deviations the values of A_1, B_1, C_1, D_1, and E_1, were computed; next, with the values of A_1, B_1, C_1, D_1, and E_1, thus found, the deviations were computed for each point; then, comparing these computed values of the deviation with the observed values, a series of residuals were obtained from which the probable errors in question (which are given in the table on page 185) were easily got. These we will designate as the probable errors obtained from observations at a single station; and it will be remembered that it was shown that, no matter what their numerical values might be, the probable errors of B_1, C_1, D_1, and E_1 must all be equal to each other. Although the difference between the probable errors of B_1, C_1, D_1, E_1, and those of $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, can never be great, yet, in general, it would not be rigorously correct to assume that they are equal to each other. However, in the case of the compasses under discussion we will make this assumption, for by so doing no error greater than the uncertainty of the probable errors themselves will be introduced. In order to facilitate the comparison of the two sets of probable errors, those of B_1, C_1, D_1, E_1 are given in the table above, in the column headed $\frac{r}{\sqrt{16}}$. This column is identical with the column headed in the same manner in the table on page 185, except that the quantities are here expressed in parts of radius instead of minutes of arc.

Now, comparing the probable errors derived from all the observations of the cruise with those derived from observations at any single station, we see that, taking the mean of the results for all the compasses, $r_{\mathfrak{D}}$ and $r_{\mathfrak{E}}$ are almost identical, as they should be, but they are each more than twice as great as $\frac{r}{\sqrt{16}}$. On the other hand,

$r_{\mathfrak{B}}$ and $r_{\mathfrak{C}}$ are neither equal to each other, nor yet to $r_{\mathfrak{D}}$ and $r_{\mathfrak{E}}$, but are, the one nearly three, and the other four, times as great as $\frac{r}{\sqrt{16}}$. Assuming the theory employed in this discussion to be correct, we should have expected to find $r_{\mathfrak{B}}$, $r_{\mathfrak{C}}$, $r_{\mathfrak{D}}$, $r_{\mathfrak{E}}$ sensibly equal to each other, and all sensibly equal to $\frac{r}{\sqrt{16}}$. Such, however, is not the case; and, as the results for each compass all tend in precisely the same direction as the mean result, a doubt naturally arises whether or not the theory really represents the semi-circular deviation as accurately as it does the quadrantal. As this doubt is founded upon observations which may possibly have been affected by some unknown cause of constant error—as they were all made on a single vessel during a single cruise—perhaps it would not be well to insist upon it too strongly; but at all events, it shows the necessity for further investigation of the subject, and especially the great want of more observations.

The probable errors of the coefficients $\mathfrak{B}$, $\mathfrak{C}$, $\mathfrak{D}$, $\mathfrak{E}$, for each compass, when computed from the values of A_1, $\frac{c}{\lambda}$, $\frac{P}{\lambda}$, $\frac{\Delta P}{\lambda}$, $\frac{f}{\lambda}$, $\frac{Q}{\lambda}$, $\frac{\Delta Q}{\lambda}$, $\mathfrak{D}$, and $\mathfrak{E}$, given in the table on page 193, are as follows:

Compass.	$r^\circ_{\mathfrak{B}}$	$r^\circ_{\mathfrak{C}}$	$r^\circ_{\mathfrak{D}}$	$r^\circ_{\mathfrak{E}}$
Admiralty Standard	± 0.0010	± 0.0017	± 0.0010	± 0.0010
After Binnacle	± 0.0012	± 0.0023	± 0.0009	± 0.0009
After Ritchie	± 0.0030	± 0.0051	± 0.0024	± 0.0035
After Azimuth	± 0.0035	± 0.0035	± 0.0033	± 0.0026
Forward Alidade	± 0.0016	± 0.0019	± 0.0011	± 0.0010
Forward Binnacle	± 0.0014	± 0.0026	± 0.0012	± 0.0014
Forward Ritchie.	± 0.0022	± 0.0040	± 0.0018	± 0.0015

The following table shows, for each compass, the place at which the maximum value of its deviation, δ, was the greatest, together with the point on which that maximum value occurred, and its amount. Also, the place at which the maximum value of its deviation was the least, together with the point on which that maximum occurred, and its amount. These deviations are given on the compass points, and in computing them the true A was used.

Compass and Station.	Point.	δ
Admiralty Standard.		
Hampton Roads	E. by N.	+ 9° 29′
Sandy Point	N. E. by E.	+ 2 3
After Binnacle.		
Hampton Roads	N. W. by W.	— 9 15
Acapulco.	N. W. by W.	— 5 21
After Ritchie.		
Hampton Roads	W. N. W.	— 12 45
Panama	N. W. by W.	— 5 41
After Azimuth.		
Hampton Roads	S. E. by E.	— 10 5
St. Thomas	S. E.	— 8 45

Compass and Station.	Point.	δ
Forward Alidade.		
Bahia	N. W. by N.	− 3° 39′
Sandy Point	N. W.	− 4 34
Forward Binnacle.		
Bahia	N. W.	− 3 31
San Francisco	S. W.	+ 7 43
Forward Ritchie.		
St. Thomas	N. W.	− 4 55
San Francisco	S. W. by S.	+ 6 53

The following table shows, for each compass, the maximum change, $\Delta\delta$, in its deviation, which occurred on any single point, together with the azimuth at which, and the places between which that change occurred.

Compass and Station.	Azimuth.	$\Delta\delta$
Admiralty Standard.		
Hampton Roads and Sandy Point	S. 88° 52′ E.	7° 53′
After Binnacle.		
Hampton Roads and Acapulco	S. 82 43 E.	4 23
After Ritchie.		
Hampton Roads and Panama	S. 84 27 E.	7 28
After Azimuth.		
Hampton Roads and Sandy Point	S. 48 31 E.	1 43
Forward Alidade.		
Hampton Roads and Sandy Point	N. 85 20 E.	3 39
Forward Binnacle.		
Sandy Point and San Francisco	N. 76 17 E.	9 42
Forward Ritchie.		
Sandy Point and San Francisco	N. 43 16 E.	6 18

In order to show the difference between the values of the deviation computed from observations made at a single station, and those computed from all the observations of the cruise, or, in other words, the difference between the theory and the observations, let δ be the deviation of a compass on any point, ζ, at a given station, as computed from values of A_1, B_1, C_1, D_1, E_1, derived from all the observations of that compass made during the cruise; and also let δ' be the deviation of the same compass, on the same point, at the same station, as computed from values of A_1, B_1, C_1, D_1, E_1, derived from observations of that compass made on each of the thirty-two points at the station in question. Then the following table shows, for each compass, the maximum value attained by $\delta - \delta'$ during the cruise, together with the point on which, and the station at which, that maximum occurred.

Compass.	Station.	Point.	$\delta - \delta'$
Admiralty Standard	St. Thomas	S. S. W.	+ 1° 41′
After Binnacle	Panama	S. S. E.	+ 1 14
After Ritchie	Sandy Point	S. by E.	− 2 51
After Azimuth	Callao	S. E. by S.	− 3 4
Forward Alidade	Acapulco	S. E.	+ 1 36
Forward Binnacle	Valparaiso	N. W. by W.	− 1 41
Forward Ritchie	San Francisco	N. N. E.	+ 2 11

As the After Azimuth Compass was a very poor instrument, the descrepancy between theory and observation in the case of its deviations is not surprising. In the case of all the other compasses, except perhaps the Forward and After Ritchie, the agreement of the observed and computed values of the deviations is much more satisfactory; and indeed the differences between them are so small as to be of very little consequence for the ordinary purposes of navigation; still, viewed from a purely scientific stand-point, they are larger than might have been expected.

The hard and soft iron forces involved in the production of the semi-circular deviation were next examined in order to ascertain whether or not their relations to each other were such as to render it possible, in the case of a vessel swung for the first time, to predict from the observed deviations of her standard compass what the deviations would be at any other place. The coefficients of the semi-circular deviation are $\mathfrak{B}$ and $\mathfrak{C}$, and the components of the hard iron force involved in their production are $\frac{P}{\lambda}$ and $\frac{Q}{\lambda}$; while the components of the soft iron force are $\frac{c}{\lambda}$ and $\frac{f}{\lambda}$. As these components act at right angles to each other, the total hard iron force will be

$$\sqrt{\frac{P^2}{\lambda^2}+\frac{Q^2}{\lambda^2}},$$

and if we let α represent the direction in which it acts, measured from the ship's head toward the right hand, we have

$$\tan \alpha = \frac{\frac{Q}{\lambda}}{\frac{P}{\lambda}}$$

In the same way the total soft iron force will be

$$\sqrt{\frac{c^2}{\lambda^2}+\frac{f^2}{\lambda^2}}$$

and to determine its direction we have

$$\tan \alpha' = \frac{\frac{f}{\lambda}}{\frac{c}{\lambda}}$$

By means of these formulæ the following table was computed. It shows the amount and direction of the hard and soft iron forces acting on each compass on November 1, 1865, and June 23, 1866.

Compass.	Hard Iron Force.				Soft Iron Force.	
	November 1, 1865.		June 23, 1866.			
	Amount.	Direction.	Amount.	Direction.	Amount.	Direction.
Admiralty Standard	0.460	000°.8	0.226	348°.0	0.024	356°.1
After Binnacle	0.664	000.2	0.639	353.0	0.010	240.4
After Ritchie	0.780	349.0	0.481	354.0	0.018	16.3
After Azimuth	0.375	186.8	0.449	173.9	0.007	111.2
Forward Alidade	0.107	277.6	0.178	267.3	0.016	184.2
Forward Binnacle . . .	0.159	331.9	0.254	280.1	0.048	187.1
Forward Ritchie	0.376	347.2	0.387	289.1	0.022	219.9

The following table shows the change, in amount and direction, of the hard iron force between November 1, 1865, and June 23, 1866; the ratio of the hard to the soft iron force on each of these dates; and also the mean ratio of the same forces.

Compass.	Change of Hard Iron Force.		Ratio of Hard to Soft Iron Force.		
	Amount.	Direction.	Nov. 1, 1865.	June 23, 1866.	Mean.
Admiralty Standard	— 0.234	— 12°.8	19.2	9.4	14.3
After Binnacle	— 0.025	— 7.2	68.8	66.1	67.4
After Ritchie	— 0.299	+ 5.0	42.1	26.0	34.0
After Azimuth	+ 0.074	— 12.9	52.6	62.8	57.7
Forward Alidade	+ 0.071	— 10.3	6.6	11.0	8.8
Forward Binnacle	+ 0.095	— 51.8	3.3	5.3	4.3
Forward Ritchie	+ 0.011	— 58.1	17.1	17.6	17.3

An examination of the last two tables shows that during the whole cruise the hard iron force was changing in a very remarkable manner, both in amount and direction. In the case of the three compasses mounted above the forward turret, the force was increasing: while in the case of those mounted above the after turret, it was decreasing. In other words, there seems to have been a transfer of hard iron force from aft forward. Now, looking at the change in direction of the force, we see that in every case, excepting only that of the After Ritchie, it took place in such a manner as to correspond to a rotation from right to left. Further, the ratio of the hard to the soft iron force was slowly varying at each compass; and for the different compasses it ranged between 4.3 and 67.4. Finally, there was not a single compass on board at which the direction of the hard and soft iron force coincided; from which it follows that in no case was the ratio of the hard and soft iron forces the same in the coefficient $\mathfrak{B}$ as it was in the coefficient $\mathfrak{C}$. Under these circumstances we are forced to conclude that, so far as can be judged from the observations here given, in the case of a vessel swung for the first time it is impossible to make any reliable estimate of the ratio of the hard to the soft iron force in the coefficients $\mathfrak{B}$ and $\mathfrak{C}$; and, therefore, it is also impossible to make any reliable estimate as to what changes her deviations will undergo upon a change of magnetic latitude. As a further proof of this, we see that the After Azimuth Compass, with a maximum deviation of 10° 5′, changed its deviation during the cruise by only 1° 43′, that is, by about one-sixth of its whole amount; while the Forward Binnacle Compass, with a maximum deviation of only 7° 43′ changed its deviation during the cruise by 9° 42′, that is, by about one and a quarter times its whole amount.

In the beginning of this section it was stated that, at the positions occupied by the Admiralty Standard and After Azimuth Compasses, observations of deflection and dip were made in order to determine the absolute magnetic force; and the details of the method followed in taking these observations were explained. We will now proceed to reduce and discuss the observations themselves, and for that purpose the first thing necessary to be known is the magnetic moment of the deflecting magnets. For its determination we have the observations recorded in the following table, which were all made on shore. The first and second columns of the table give the place where, and the date when, each observation was made. The third and fourth columns give respectively the observed deflections when the north ends of the deflecting magnets were directed towards the west and towards the east; the distance of their centres from the centre of the compass needle being in both cases eleven inches. The fifth column gives the mean of the four observed deflections recorded in the third and fourth columns. The sixth, seventh, and eighth columns contain, in precisely the same manner, the observed deflections, and their mean, when the centres of the deflecting magnets were at a distance of fifteen inches from the centre of the compass needle. Now, let r be the distance, expressed in feet, between the centres of the deflecting magnets and the centre of the compass needle; u, the observed angle of deflection given for each value of r in the column headed "mean"; m, the combined magnetic moment of the two deflecting magnets; and H, the earth's horizontal force at the place where the observation was made, taken from the table on page 61. Then we shall have

$$\tfrac{1}{2}r^3 \tan u = \frac{m}{H}$$

and the ninth column contains the mean of the two values of log. $\frac{m}{H}$ computed respectively from the angles of deflection observed with $r = 11$ inches $= 0.917$ foot, and $r = 15$ inches $= 1.250$ feet. The tenth column contains the value of log. m, found by adding to log. $\frac{m}{H}$ the known value of log. H.

Station.	Date.	Deflections.						Log. $\frac{m}{H}$.	Log. m.
		$r = 11$ inches.			$r =$ 15 inches.				
		West.	East.	Mean.	West.	East.	Mean.		
Gosport	Oct. 30, 1865	19° 30′	22° 40′		14° 30′	17° 30′			
		19 0	22 20	20° 52′	14 20	17 40	*16° 0′	9.1617	9.8344
St. Thomas	Nov. 13, 1865	15 20	14 50		4 20	6 40			
		15 30	14 40	15 5	4 30	6 40	5 32	8.9961	9.8251
Salute Islands . . .	Nov. 28, 1865	14 35	15 0		5 20	5 20			
		14 35	15 5	14 49	4 55	5 20	5 14	8.9799	9.8079
Bahia.	Dec. 27, 1865	15 40	16 10		6 10	5 30			
		16 40	16 10	16 10	5 40	5 30	5 42	9.0184	9.8108
Rio Janeiro	Jan. 6, 1866	17 0	17 0		6 40	6 0			
		17 0	17 10	17 2	6 0	6 0	6 10	9.0476	9.8216
Monte Video . . .	Jan. 18, 1866	16 40	16 40		6 20	5 30			
		17 0	16 40	16 45	6 10	5 30	5 52	9.0328	9.8130
Sandy Point . . .	Feb. 7, 1866	16 30	16 20		5 40	6 40			
		16 40	16 20	16 27	6 0	6 30	6 12	9.0408	9.8270
Valparaiso	March 2, 1866	17 0	15 0		7 20	5 0			
		16 40	14 40	15 50	7 30	5 0	6 12	9.0320	9.8326
Valparaiso	April 7, 1866	14 40	17 40		4 30	7 30			
		14 30	17 30	16 5	4 20	7 40	6 0	9.0284	9.8290
Callao	April 26, 1866	14 30	14 30		5 20	5 10			
		14 30	14 30	14 30	5 10	5 30	5 18	8.9777	9.8222
Panama	May 14, 1866	12 50	13 30		4 30	5 20			
		13 10	13 30	13 15	4 40	5 0	4 52	8.9387	9.8195
Acapulco	May 30, 1866	12 30	12 20		4 40	4 30			
		12 40	12 10	12 25	5 30	4 40	4 50	8.9227	9.8107
San Francisco . . .	June 26, 1866	17 40	17 0		7 0	6 10			
		18 0	16 40	17 20	7 10	6 30	6 42	9.0698	9.8208

* In this observation $r = 12$ inches.

The observed values of log. m show no trace whatever of any change depending upon the time, and therefore the indiscriminate mean of them all has been taken as the truth, and we have

$$\text{Log. } m = 9.8211 \pm 0.0016.$$

The probable error of a single observed value of log. m is ± 0.0058.

The following table contains all the observations which were made at the position occupied by the Admiralty Standard Compass on board the Monadnock, for the determination of absolute force. The first nine columns contain quantities precisely similar to those in the columns headed in the same manner in the table last given. The column headed "Log. H'" gives the logarithm of the combined horizontal force of the earth and ship, obtained by subtracting log. $\frac{m}{H'}$ from the value of log. m given above. The column "θ'" contains the dip, which was observed immediately after the deflections. The column "Log. Z'" contains the logarithm of the combined vertical force of the earth and ship, computed from the quantities in the tenth and eleventh columns by the formula $Z' = H' \tan \theta'$. The columns "Log. $\frac{H'}{H}$," and "Log. $\frac{Z'}{Z}$", explain themselves when it is stated that H represents the horizontal force of the earth; H' the combined horizontal force of the earth and ship; Z the earth's vertical force; and Z' the combined vertical force of the earth and ship. The column "ζ'" contains the azimuth of the ship's head as read off from the compass card at the time the deflections were observed; and the column "ζ" contains the same azimuth, counted from the true magnetic north.

Admiralty Standard Compass.

Station.	Date.	Deflection.						Log. $\frac{m}{H'}$	Log. H'	θ'	Log. Z'	Log. $\frac{H'}{H}$	Log. $\frac{Z'}{Z}$	ζ'	ζ
		$r = 11$ inches.			$r = 15$ inches.										
		West.	East.	Mean.	West.	East.	Mean.								
Hampton Roads	Nov. 1, 1865	22° 20′ 21 20 21 0 23 0	26° 0′ 26 30 26 0 23 40	23° 44′				9.2288	0.5923			9.9196		W. (?)	
St. Thomas . .	Nov. 15, 1865	17 10 17 20	15 40 15 50	16 30	6° 30′ 6 40	7° 10 7 0	6° 50′	9.0628	0.7583			9.9293		East.	
Salute Islands .	Nov. 30, 1865	16 20 16 30	16 0 16 0	16 12	6 20 6 40	5 40 6 0	6 10	9.0361	0.7850	+ 41° 30′	0.7318	9.9570	0.0644	N. 85° E.	N. 89° 41′ E.
Ceara	Dec. 18, 1865	18 0 17 0	16 20 16 0	16 50	7 0 6 30	6 0 6 0	6 22	9.0519	0.7692			9.9566		N. 70 E.	N. 74 30 E.
Bahia	Dec. 29, 1865	16 20 16 0	16 0 16 0	16 5	6 20 6 30	6 0 6 20	6 17	9.0385	0.7826	+ 8 30	9.9571	9.9902	0.2786	N. 35 E.	N. 38 2 E.
Rio Janeiro . .	Jan. 4, 1866	16 30 16 40	16 40 16 40	16 37	6 20 6 20	6 0 6 10	6 12	9.0431	0.7780	— 8 0	9.9258	0.0040	9.8324	N. 4 W.	N. 4 30 W.
Rio Janeiro . .	Jan. 4, 1866	18 40 18 30	18 40 18 40	18 37	7 0 7 0	7 10 7 10	7 5	9.0985	0.7226	— 15 0	0.1506	9.9486	0.0572	S. 2 W.	S. 1 58 W.
Monte Video . .	Jan. 24, 1866	18 0 18 0	18 20 18 20	18 10	7 10 6 40	6 40 7 0	6 52	9.0861	0.7350	— 36 30	0.6042	9.9548	0.0435	S. 20 E.	S. 19 40 E.
Sandy Point . .	Feb. 9, 1866	16 0 16 40	18 20 18 30	17 22	6 20 6 20	7 20 7 30	6 52	9.0757	0.7454	— 60 30	0.9928	9.9592	0.0526	S. 39 W.	S. 39 4 W.
Valparaiso . . .	March 20, 1866	16 40 16 30	17 20 17 10	16 55	6 30 6 20	7 0 6 50	6 40	9.0631	0.7580	— 42 0	0.7124	9.9574	0.0604	S. 20 W.	S. 20 9 W.
Valparaiso . . .	April 4, 1866	17 20 17 20	14 40 14 40	16 0	7 0 7 20	4 40 5 0	6 0	9.0273	0.7938	— 36 15	0.6590	9.9932	0.0070	N. 5 W.	N. 5 34 W.
Callao	April 30, 1866	16 30 15 40	16 0 15 0	15 48	6 20 6 30	5 30 6 0	6 5	9.0274	0.7937	— 7 45	9.9275	9.9492	0.0288	South.	S. 0 13 W.
Panama	May 17, 1866	14 0 14 0	12 40 13 0	13 25	5 20 5 20	5 0 4 30	5 2	8.9489	0.8722	+ 36 45	0.7454	9.9914	0.0699	N. 42 W.	N. 45 6 W.
Acapulco	May 31, 1866	13 30 13 30	14 40 14 40	14 5	5 0 5 0	5 40 5 50	5 22	8.9739	0.8472	+ 45 45	0.8586	9.9592	0.0484	S. 30 E.	S. 29 2 E.
San Francisco .	June 23, 1866	20 0 19 40	20 0 20 20	20 0	6 40 5 30	9 0 10 0	7 48	9.1366	0.6845	+ 67 0	1.0567	9.9335	0.0247	S. 20 E.	S. 18 22 E.

The following table contains, in precisely the same manner, all the observations which were made for the determination of absolute force at the position occupied by the After Azimuth Compass on board the Monadnock.

AFTER AZIMUTH COMPASS.

Station.	Date.	Deflection.						Log. $\frac{m}{H'}$	Log. H'	θ'	Log. Z'	Log. $\frac{H'}{H}$	Log. $\frac{Z'}{Z}$	ζ'	ζ
		$r = 11$ inches.			$r = 15$ inches.										
		West.	East.	Mean.	West.	East.	Mean.								
Hampton Roads	Nov. 1, 1865	25° 20′ 26 0 25 20 23 40	25° 30′ 27 0 28 20 28 20	26° 11′				9.2774	0.5437			9.8710		West (?)	
St. Thomas . .	Nov. 15, 1865	19 10 18 40	19 40 19 20	19 12	6° 40′ 6 20	7° 30′ 7 40	7° 2′	9.1042	0.7169			9.8879		East	
Salute Islands .	Nov. 30, 1865	19 40 19 30	21 40 21 20	20 32	7 20 7 0	8 20 8 30	7 48	9.1428	0.6783	+ 40° 12′	0.6052	9.8503	9.9378		
Ceara	Dec. 18, 1865	25 40 27 0	16 40 18 40	22 0	7 0 8 40	7 20 6 30	7 23	9.1471	0.6740			9.8614			
Bahia	Dec. 29, 1865	21 20 21 10	20 20 20 10	20 45	8 10 8 30	7 10 8 0	7 57	9.1494	0.6717	+ 9 30	9.8953	9.8793	0.2168		
Rio Janeiro . .	Jan. 4, 1866	19 10 19 0	18 10 18 10	18 37	6 40 6 40	6 50 6 40	6 42	9.0864	0.7347	— 6 30	9.7914	9.9607	9.6980	N. 2° E.	N. 2° 4′ E.
Rio Janeiro . .	Jan. 4, 1866	17 20 17 30	16 20 16 0	16 47	5 40 5 50	6 0 6 10	5 55	9.0351	0.7860	— 18 15	0.3042	0.0120	0.2108	S. 10 W.	S. 13 26 W.
Monte Video . .	Jan. 24, 1866	20 20 21 40	17 50 18 30	19 35	8 40 9 40	6 20 5 40	7 35	9.1254	0.6957	— 39 15	0.6079	9.9155	0.0472	S. 45 E.	S. 53 48 E.
Sandy Point . .	Feb. 9, 1866	21 0 21 20	16 50 16 40	18 57	7 0 7 40	9 0 8 50	8 7	9.1327	0.6884	— 59 15	0.9139	9.9022	9.9737	S. 50 W.	S. 59 17 W.
Valparaiso . . .	March 20, 1866	16 20 15 20	16 0 16 50	16 8	7 0 7 30	4 40 4 40	5 58	9.0279	0.7932	— 38 30	0.6938	9.9926	0.0418	S. 10 W.	S. 13 18 W.
Valparaiso . . .	April 4, 1866	19 0 19 40	16 20 16 40	17 55	6 30 7 0	6 0 5 0	6 7	9.0575	0.7636	— 33 15	0.5803	9.9630	9.9283	N. 7 W.	N. 8 23 W.
Callao	April 30, 1866	13 0 13 0	15 0 15 30	14 8	7 0 5 20	5 40 5 40	5 55	8.9960	0.8251	— 10 45	0.1035	9.9806	0.2048	South	S. 0 12 E.
Panama	May 17, 1866	15 20 16 0	15 20 15 30	15 32	6 10 6 0	5 40 5 30	5 50	9.0144	0.8067	+ 40 45	0.7420	9.9259	0.0665	N. 35 W.	N. 38 49 W.
Acapulco	May 31, 1866	18 20 17 30	15 40 16 0	16 52	7 20 6 30	5 20 7 30	6 40	9.0624	0.7587	+ 49 45	0.8310	9.8707	0.0208	N. 75 W.	N. 74 47 W.
San Francisco .	June 23, 1866	19 20 17 30	19 30 19 0	18 50	7 30 7 30	7 0 7 0	7 15	9.1063	0.7148	+ 62 0	0.9891	9.9638	9.9571	S. 50 E.	S. 60 37 E.

From the data already given, the value of λ was next computed by means of the formulæ

$$\sin\delta = \frac{1}{1-\mathfrak{D}\cos 2\zeta'}\Big[\mathfrak{A}+\mathfrak{B}\sin\zeta'+\mathfrak{C}\cos\zeta'+\mathfrak{D}\sin 2\zeta'+\mathfrak{E}\cos 2\zeta'\Big]$$

$$\lambda = \frac{H'}{H}\times\frac{\sin\delta}{\mathfrak{A}+\mathfrak{B}\sin\zeta+\mathfrak{C}\cos\zeta+\mathfrak{D}\sin 2\zeta+\mathfrak{E}\cos 2\zeta}$$

The individual results obtained from the observed values of $\frac{H'}{H}$ are as follows:

Station.	Value of λ	
	Admiralty Standard Compass.	After Azimuth Compass.
Salute Islands	0.918	
Ceara	0.896	
Bahia	0.922	
Rio Janeiro	0.939	0.942
Rio Janeiro	0.904	0.884
Monte Video	0.913	0.814
Sandy Point	0.914	0.821
Valparaiso	0.954	0.848
Valparaiso	0.934	0.886
Callao	0.905	0.820
Panama	0.952	0.861
Acapulco	0.947	0.816
San Francisco	0.914	0.947

Taking the means, for the Admiralty Standard Compass, we have finally

$$\lambda = 0.924 \pm 0.0036$$

and the probable error of a single observed value of λ is ± 0.013. For the After Azimuth compass we have finally

$$\lambda = 0.864 \pm 0.0107$$

and the probable error of a single observed value of λ is ± 0.034.

In order to determine these coefficients which depend upon the value of $\frac{Z'}{Z}$, we have equation (6 a), which is

$$0 = 1 - \frac{Z'}{Z} + g\times\frac{\cos\zeta}{\tan\theta} - h\times\frac{\sin\zeta}{\tan\theta} + k + R\times\frac{1}{Z}$$

But as R is liable to a slow change, a term depending upon the time is introduced, and then we get

$$0 = 1 - \frac{Z'}{Z} + g\times\frac{\cos\zeta}{\tan\theta} - h\times\frac{\sin\zeta}{\tan\theta} + k + R\times\frac{1}{Z} + \Delta R\times\frac{t}{Z} \qquad (6\text{ b})$$

where ΔR is the daily change in the value of R, and t is the time in days, counted from November 1, 1865. Each observed value of $\frac{Z'}{Z}$ furnishes an equation of condition of the same form as (6 b), and from all the equations of condition thus obtained the most probable values of g, h, k, R, and ΔR, can be found by the method of least squares.

The following are the equations of condition, formed in the manner just explained, for the Admiralty Standard Compass.

Absolute Term.	g	h	k	R	ΔR
0 = — 0.160	+ 0.008	— 1.448	+ 1.000	+ 0.215	+ 6.24
0 = — 0.899	+ 10.23	— 8.007	+ 1.000	+ 2.097	+ 125.8
0 = + 0.320	— 4.779	— 0.376	+ 1.000	— 0.806	— 51.61
0 = — 0.141	+ 4.791	— 0.164	+ 1.000	— 0.806	— 51.61
0 = — 0.108	+ 1.561	+ 0.558	+ 1.000	— 0.275	— 23.10
0 = — 0.129	+ 0.545	— 0.442	+ 1.000	— 0.115	— 11.48
0 = — 0.149	+ 1.322	— 0.485	+ 1.000	— 0.223	— 30.76
0 = — 0.016	— 1.401	— 0.140	+ 1.000	— 0.223	— 34.32
0 = — 0.068	+ 8.822	— 0.033	+ 1.000	— 1.263	— 227.3
0 = — 0.175	+ 1.132	+ 1.136	+ 1.000	+ 0.211	+ 41.59
0 = — 0.118	— 1.046	— 0.580	+ 1.000	+ 0.155	+ 32.66
0 = — 0.058	— 0.497	— 0.165	+ 1.000	+ 0.093	+ 21.74

From these equations of condition, the following normal equations have been obtained by the method of least squares.

Absolute Term.	g	h	k	R	$100\,\Delta R$
0 = — 12.462	+ 237.337				
0 = + 7.286	— 79.068	+ 68.794			
0 = — 1.701	+ 20.688	— 10.147	+ 12.000		
0 = — 1.957	+ 9.858	— 16.451	— 0.941	+ 7.605	
0 = — 1.112	— 7.513	— 9.444	— 2.022	+ 6.735	+ 7.892

Solving, we find

$$g = +\,0.04070 \qquad k = +\,0.1006$$
$$h = +\,0.00504 \qquad R = +\,0.1665$$
$$100\Delta R = +\,0.0694$$

Substituting these results in the equations of condition, we find that the probable error of a single observed value of $\frac{Z'}{Z}$ is $\pm\,0.024$, and the probable error of a computed value of $\frac{Z'}{Z}$ is $\pm\,0.007$.

In a precisely similar manner, from the values of $\frac{Z'}{Z}$ observed at the position of the After Azimuth Compass, we obtain the following equations of condition.

Absolute Term.	g	h	k	R	ΔR
0 = + 0.501	— 4.790	+ 0.173	+ 1.000	— 0.806	— 51.61
0 = — 0.625	+ 4.663	— 1.114	+ 1.000	— 0.806	— 51.61
0 = — 0.115	+ 0.979	+ 1.338	+ 1.000	— 0.275	— 23.10
0 = + 0.059	+ 0.358	— 0.603	+ 1.000	— 0.115	— 11.48
0 = — 0.101	+ 1.370	— 0.324	+ 1.000	— 0.223	— 30.76
0 = + 0.152	— 1.393	— 0.205	+ 1.000	— 0.223	— 34.32
0 = — 0.602	+ 8.823	+ 0.031	+ 1.000	— 1.263	— 227.3
0 = — 0.165	+ 1.250	+ 1.006	+ 1.000	+ 0.211	+ 41.59
0 = — 0.049	+ 0.314	+ 1.154	+ 1.000	+ 0.155	+ 32.66
0 = + 0.094	— 0.257	— 0.456	+ 1.000	+ 0.093	+ 21.74

And the resulting normal equations are

Absolute Term.	g	h	k	R	$100\,\Delta R'$
$0 = -11.313$	$+129.164$				
$0 = +0.311$	-3.078	$+6.125$			
$0 = -0.851$	$+11.317$	$+1.000$	$+10.000$		
$0 = +0.840$	-11.053	$+0.888$	-3.253	$+3.161$	
$0 = +1.367$	-19.634	$+1.042$	-3.342	$+4.084$	$+6.305$

Solving, we find

$$g = +0.11398 \qquad k = -0.0509$$
$$h = +0.00981 \qquad R = -0.3918$$
$$100\Delta R = +0.3634$$

Substituting these results in the equations of condition, the probable error of a single observed value of $\frac{Z'}{Z}$ comes out ± 0.030, and the probable error of a computed value of $\frac{Z'}{Z}$ comes out ± 0.010.

For the Admiralty Standard Compass we found $\mathfrak{A} = 0.000$, $\mathfrak{D} = +0.017$, and $\mathfrak{E} = -0.001$. We have also

$$a = \lambda\,(1 + \mathfrak{D}) - 1$$
$$e = \lambda\,(1 - \mathfrak{D}) - 1$$
$$b = \lambda\,(\mathfrak{E} - \mathfrak{A})$$
$$d = \lambda\,(\mathfrak{E} + \mathfrak{A})$$

Hence

$$a = -0.0605 \qquad e = -0.0917$$
$$b = -0.0008 \qquad d = -0.0008$$

For the After Azimuth Compass we found $\mathfrak{A} = 0.000$, $\mathfrak{D} = +0.112$, and $\mathfrak{E} = 0.000$. Hence, in the same manner,

$$a = -0.0396 \qquad e = -0.2324$$
$$b = 0.0000 \qquad d = 0.0000$$

Collecting our results, we have the following final values of the coefficients of the

ADMIRALTY STANDARD COMPASS.

$$\mathfrak{A} = 0.000$$

$$\mathfrak{B} = +0.0240 \tan\theta + 0.460\,\frac{1}{H} - 0.00102\,\frac{t}{H} \pm 0.001$$

$$\mathfrak{C} = -0.0016 \tan\theta + 0.006\,\frac{1}{H} - 0.00023\,\frac{t}{H} \pm 0.002$$

$$\mathfrak{D} = +0.017 \pm 0.001$$

$$\mathfrak{E} = -0.001 \pm 0.001$$

$$\frac{Z'}{Z} = 1 + 0.0407\,\frac{\cos\zeta}{\tan\theta} - 0.0050\,\frac{\sin\zeta}{\tan\theta} + 0.1006 + 0.1665\,\frac{1}{Z} + 0.000694\,\frac{t}{Z} \pm 0.007$$

$$\begin{aligned}
\lambda &= +0.924 \pm 0.004\\
\frac{c}{\lambda} &= +0.0240\\
\frac{P}{\lambda} &= +0.460\\
\frac{\Delta P}{\lambda} &= -0.00102\\
\frac{f}{\lambda} &= -0.0016\\
\frac{Q}{\lambda} &= +0.006\\
\frac{\Delta Q}{\lambda} &= -0.00023
\end{aligned}$$

$$\begin{aligned}
c &= +0.0221\\
P &= +0.425\\
\Delta P &= +0.00094\\
f &= -0.0015\\
Q &= +0.006\\
\Delta Q &= -0.00021\\
a &= -0.0605
\end{aligned}$$

$$\begin{aligned}
b &= -0.0008\\
d &= -0.0008\\
e &= -0.0917\\
g &= +0.0407\\
h &= +0.0050\\
k &= +0.1006\\
R &= +0.166\\
\Delta R &= +0.00069
\end{aligned}$$

Hence, the general equations for the determination of the deviations of this compass are

$$\begin{aligned}
X' &= X - 0.0605\,X - 0.0008\,Y + 0.0221\,Z + 0.425 - 0.00094\,t\\
Y' &= Y - 0.0008\,X - 0.0917\,Y - 0.0015\,Z + 0.006 - 0.00021\,t\\
Z' &= Z + 0.0407\,X + 0.0050\,Y + 0.1006\,Z + 0.166 + 0.00069\,t
\end{aligned}$$

The following are the final values of the coefficients of the

AFTER AZIMUTH COMPASS.

$$\begin{aligned}
\mathfrak{A} &= 0.000\\
\mathfrak{B} &= -0.0026\tan\theta - 0.373\,\frac{1}{H} - 0.00032\,\frac{t}{H} \pm 0.004\\
\mathfrak{C} &= +0.0066\tan\theta - 0.044\,\frac{1}{H} + 0.00039\,\frac{t}{H} \pm 0.004\\
\mathfrak{D} &= +0.112 \pm 0.003\\
\mathfrak{E} &= 0.000 \pm 0.003\\
\frac{Z'}{Z} &= 1 + 0.1140\,\frac{\cos\zeta}{\tan\theta} - 0.0098\,\frac{\sin\zeta}{\tan\theta} - 0.0509 - 0.3918\,\frac{1}{Z} + 0.00363\,\frac{t}{Z} \pm 0.010
\end{aligned}$$

$$\begin{aligned}
\lambda &= +0.864 \pm 0.011\\
\frac{c}{\lambda} &= -0.0026\\
\frac{P}{\lambda} &= -0.373\\
\frac{\Delta P}{\lambda} &= -0.00032\\
\frac{f}{\lambda} &= +0.0066\\
\frac{Q}{\lambda} &= -0.044\\
\frac{\Delta Q}{\lambda} &= +0.00039
\end{aligned}$$

$$\begin{aligned}
c &= -0.0022\\
P &= -0.322\\
\Delta P &= -0.00027\\
f &= +0.0058\\
Q &= -0.038\\
\Delta Q &= +0.00034\\
a &= -0.0396
\end{aligned}$$

$$\begin{aligned}
b &= 0.0000\\
d &= 0.0000\\
e &= -0.2324\\
g &= +0.1140\\
h &= +0.0098\\
k &= -0.0509\\
R &= -0.392\\
\Delta R &= +0.00363
\end{aligned}$$

Hence, the general equations for the determination of the deviations of this compass are

$$\begin{aligned} X' &= X - 0.0396\,X - 0.0000\,Y - 0.0022\,Z - 0.322 - 0.00027\,t \\ Y' &= Y - 0.0000\,X - 0.2324\,Y - 0.0058\,Z - 0.038 + 0.00034\,t \\ Z' &= Z + 0.1140\,X + 0.0098\,Y - 0.0509\,Z - 0.392 + 0.00363\,t \end{aligned}$$

The constants P, Q, R, are the resolved values of the hard iron magnetism of the ship; and in order to show as clearly as possible how it varied during the cruise, at the positions occupied by the two compasses under discussion, the following table is appended. The columns headed "F" contain the values of the total hard iron force, computed by means of the formula

$$F = \sqrt{P^2 + Q^2 + R^2}$$

Date.	Admiralty Standard Compass.				After Azimuth Compass.			
	P.	*Q.*	*R.*	*F.*	*P.*	*Q.*	*R.*	*F.*
November 1, 1865	+0.425	+ 0.006	+0.166	0.456	— 0.322	— 0.038	— 0.392	0.509
June 23, 1866	+0.205	— 0.043	+0.327	0.388	— 0.385	+ 0.042	+ 0.457	0.599

Thus it appears that in the interval between November 1, 1865, and June 23, 1866, the total hard iron force had decreased fifteen per centum at the position of the Admiralty Standard Compass, while it had increased eighteen per centum at the position of the After Azimuth Compass; and in both cases the changes in the direction of the force were very great. On the whole, the so-called permanent and sub-permanent magnetism of the Monadnock seem to have been in a very unstable condition.

There were some places where observations of the deviations of the compasses were obtained on a number of points less than thirty-two, because the ship could not be made to swing completely around. In order to deduce from these observations the corresponding values of the coefficients A_1, B_1, C_1, D_1, E_1, we remark that each observed deviation furnishes an equation of condition of the form

$$0 = -\delta + A_1 + B_1 \sin \zeta + C_1 \cos \zeta + D_1 \sin 2\zeta + E_1 \cos 2\zeta$$

and from all the equations thus obtained the values of the coefficients must be found by the method of least squares. As all the compasses were observed simultaneously; the deviations at each place are given on the same points in the case of each compass. Hence, although the absolute terms in the equations of condition will be different, the numerical coefficients of the unknown quantities A_1, B_1, C_1, D_1, E_1, will be identical for all the compasses at any one station. Advantage has been taken of this circumstance in forming the following table, which gives the equations of condition for all the compasses at Ceara. The absolute terms of the equations of condition belonging to any compass will be found in the column headed with the name of that compass, while the coefficients of the remaining terms of the equations will be found in the columns headed A_1, B_1, C_1, D_1, E_1. For example, the first equation of condition for the Admiralty Standard Compass is

$$0 = -170 + A_1 + 0.195\,B_1 + 0.981\,C_1 + 0.383\,D_1 + 0.924\,E_1.$$

In the same way, the first equation of condition for the After Binnacle Compass is

$$0 = -220 + A_1 + 0.195\,B_1 + 0.981\,C_1 + 0.383\,D_1 + 0.924\,E_1.$$

EQUATIONS OF CONDITION AT CEARA.

Absolute Terms.						Coefficients of the Unknown Quantities.				
Admiralty Standard.	After Binnacle.	After Ritchie.	Forward Alidade.	Forward Binnacle.	Forward Ritchie.	A_1	B_1	C_1	D_1	E_1
— 170′	— 220′	— 820′	— 180′	— 110′	— 430′	+ 1.000	+ 0.195	+ 0.981	+ 0.383	+ 0.924
— 210	— 310	— 820	— 270	— 110	— 520	+ 1.000	+ 0.383	+ 0.924	+ 0.707	+ 0.707
— 260	— 390	— 820	— 280	— 110	— 600	+ 1.000	+ 0.556	+ 0.831	+ 0.924	+ 0.383
— 350	— 470	— 970	— 280	— 180	— 480	+ 1.000	+ 0.707	+ 0.707	+ 1.000	0.000
— 340	— 420	— 990	— 211	— 130	— 380	+ 1.000	+ 0.831	+ 0.556	+ 0.924	— 0.383
— 330	— 410	— 1140	— 200	— 110	— 300	+ 1.000	+ 0.924	+ 0.383	+ 0.707	— 0.707
— 310	— 410	— 1020	— 130	— 40	— 420	+ 1.000	+ 0.981	+ 0.195	+ 0.383	— 0.924
— 230	— 260	— 850	— 110	+ 40	— 170	+ 1.000	+ 1.000	0.000	0.000	— 1.000
— 210	— 240	— 690	— 110	+ 130	— 40	+ 1.000	+ 0.981	— 0.195	— 0.383	— 0.924
— 170	— 170	— 660	— 40	+ 140	— 30	+ 1.000	+ 0.924	— 0.383	— 0.707	— 0.707

From these equations of condition five normal equations were obtained for each compass by the method of least squares; but on attempting to solve them the numerical coefficients of D_1 and E_1 came out so small that no confidence could be placed in the resulting values of these quantities; and moreover, the uncertainty of them vitiated the values of A_1, B_1, and C_1. It was therefore considered best to reject the normal equations in D_1 and E_1, and to employ in their stead the equations

$$0 = -\mathfrak{D} + \mathrm{D}_1 + \tfrac{1}{2}(B_1^2 - C_1^2)$$
$$0 = -\mathfrak{E} + \mathrm{E}_1 + B_1\,C_1 + A_1\,D_1$$

using for $\mathfrak{D}$ and $\mathfrak{E}$ the numerical values already found. The following are the normal equations thus formed, and the resulting values of A_1, B_1, C_1 D_1, and E_1, for each compass. For convenience of computation, the unit of the absolute terms of the normal equations has been changed from minutes of arc to radius.

ADMIRALTY STANDARD COMPASS.

$$0 = -0.7505 + 10.000\,A_1 + 7.482\,B_1 + 3.999\,C_1 + 3.938\,D_1 - 2.631\,E_1$$
$$0 = -0.5789 + 7.482\,A_1 + 6.317\,B_1 + 1.969\,C_1 + 2.334\,D_1 - 3.774\,E_1$$
$$0 = -0.3183 + 3.999\,A_1 + 1.969\,B_1 + 3.685\,C_1 + 3.708\,D_1 + 1.665\,E_1$$
$$0 = -0.0169 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2)$$
$$0 = +0.0009 + E_1 + B_1\,C_1$$

Hence

$$A_1 = -0.0102 = -0^\circ\ 35'.1$$
$$B_1 = +0.0833 = +4\ \ 46.3$$
$$C_1 = +0.0405 = +2\ \ 19.2$$
$$D_1 = +0.0142 = +0\ \ 48.8$$
$$E_1 = -0.0043 = -0\ \ 14.8$$

AFTER BINNACLE COMPASS.

$$0 = -0.9599 + 10.000\,A_1 + 7.482\,B_1 + 3.999\,C_1 + 3.938\,D_1 - 2.631\,E_1$$
$$0 = -0.7253 + 7.482\,A_1 + 6.317\,B_1 + 1.969\,C_1 + 2.334\,D_1 - 3.774\,E_1$$
$$0 = -0.4413 + 3.999\,A_1 + 1.969\,B_1 + 3.685\,C_1 + 3.708\,D_1 + 1.665\,E_1$$
$$0 = -0.0385 + D_1 + \tfrac{1}{2}\,(B_1^2 - C_1^2)$$
$$0 = +0.0018 + E_1 + B_1\,C_1 + 0.0047\,(B_1^2 - C_1^2)$$

Hence

$$A_1 = +0.0062 = +0^\circ\ 21'.3$$
$$B_1 = +0.0801 = +4\quad 35.2$$
$$C_1 = +0.0362 = +2\quad 4.6$$
$$D_1 = +0.0360 = +2\quad 3.6$$
$$E_1 = -0.0048 = -0\quad 16.3$$

AFTER RITCHIE COMPASS.

$$0 = -2.5540 + 10.000\,A_1 + 7.482\,B_1 + 3.999\,C_1 + 3.938\,D_1 - 2.631\,E_1$$
$$0 = -1.9282 + 7.482\,A_1 + 6.317\,B_1 + 1.969\,C_1 + 2.334\,D_1 - 3.774\,E_1$$
$$0 = -1.0844 + 3.999\,A_1 + 1.969\,B_1 + 3.685\,C_1 + 3.708\,D_1 + 1.665\,E_1$$
$$0 = -0.0340 + D_1 + \tfrac{1}{2}\,(B_1^2 - C_1^2)$$
$$0 = +0.0008 + E_1 + B_1\,C_1$$

Hence

$$A_1 = +0.1030 = +5^\circ\ 54'.2$$
$$B_1 = +0.1385 = +7\quad 56.0$$
$$C_1 = +0.0859 = +4\quad 55.4$$
$$D_1 = +0.0281 = +1\quad 36.6$$
$$E_1 = -0.0127 = -0\quad 43.7$$

FORWARD ALIDADE COMPASS.

$$0 = -0.5265 + 10.000\,A_1 + 7.482\,B_1 + 3.999\,C_1 + 3.938\,D_1 - 2.631\,E_1$$
$$0 = -0.3589 + 7.482\,A_1 + 6.317\,B_1 + 1.969\,C_1 + 2.334\,D_1 - 3.774\,E_1$$
$$0 = -0.3022 + 3.999\,A_1 + 1.969\,B_1 + 3.685\,C_1 + 3.708\,D_1 + 1.665\,E_1$$
$$0 = -0.0235 + D_1 + \tfrac{1}{2}\,(B_1^2 - C_1^2)$$
$$0 = -0.0007 + E_1 + B_1\,C_1 + 0.0125\,(B_1^2 - C_1^8)$$

Hence

$$A_1 = +0.0359 = +2^\circ\ 3'.5$$
$$B_1 = +0.0001 = +0\quad 0.2$$
$$C_1 = +0.0188 = +1\quad 4.8$$
$$D_1 = +0.0237 = +1\quad 21.4$$
$$E_1 = +0.0007 = +0\quad 2.4$$

FORWARD BINNACLE COMPASS.

$$0 = -0.1396 + 10.000\,A_1 + 7.482\,B_1 + 3.999\,C_1 + 3.938\,D_1 - 2.631\,E_1$$
$$0 = -0.0593 + 7.482\,A_1 + 6.317\,B_1 + 1.969\,C_1 + 2.334\,D_1 - 3.774\,E_1$$
$$0 = -0.1831 + 3.999\,A_1 + 1.969\,B_1 + 3.685\,C_1 + 3.708\,D_1 + 1.665\,E_1$$
$$0 = -0.0369 + D_1 + \tfrac{1}{2}\,(B_1^2 - C_1^2)$$
$$0 = -0.0011 + E_1 + B_1\,C_1$$

Hence

$$A_1 = -0.0159 = -0^\circ\ 54'.7$$
$$B_1 = +0.0072 = +0\quad 24.6$$
$$C_1 = +0.0253 = +1\quad 26.9$$
$$D_1 = +0.0372 = +2\quad 7.8$$
$$E_1 = +0.0009 = +0\quad 3.2$$

FORWARD RITCHIE COMPASS.

$$0 = -0.9803 + 10.000\,A_1 + 7.482\,B_1 + 3.999\,C_1 + 3.938\,D_1 - 2.631\,E_1$$
$$0 = -0.6394 + 7.482\,A_1 + 6.317\,B_1 + 1.969\,C_1 + 2.334\,D_1 - 3.774\,E_1$$
$$0 = -0.6193 + 3.999\,A_1 + 1.969\,B_1 + 3.685\,C_1 + 3.708\,D_1 + 1.665\,E_1$$
$$0 = -0.0407 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2)$$
$$0 = +0.0013 + E_1 + B_1\,C_1$$

Hence

$$A_1 = +0.0614 = +3^\circ\ 31'.0$$
$$B_1 = -0.0076 = -0\ \ 26.1$$
$$C_1 = +0.0631 = +3\ \ 36.9$$
$$D_1 = +0.0427 = +2\ \ 26.6$$
$$E_1 = -0.0011 = -0\ \ \ 3.9$$

The following are the equations of condition, together with the resulting normal equations, and the values of the coefficients A_1, B_1, C_1, D_1, E_1, as determined for each compass from the observations made at Rio Janeiro.

EQUATIONS OF CONDITION AT RIO JANEIRO.

Absolute Terms.							Coefficients of the Unknown Quantities.				
Admiralty Standard.	After Binnacle.	After Ritchie.	After Azimuth.	Forward Alidade.	Forward Binnacle.	Forward Ritchie.	A_1	B_1	C_1	D_1	E_1
+ 290′	— 320′	— 840′	— 160′	— 250′	— 160′	— 500′	+ 1.000	+ 0.556	+ 0.831	+ 0.924	+ 0.383
+ 360	— 410	— 840	— 120	— 250	— 160	— 500	+ 1.000	+ 0.707	+ 0.707	+ 1.000	0.000
+ 390	— 430	— 840	— 20	— 250	— 160	— 370	+ 1.000	+ 0.831	+ 0.556	+ 0.924	— 0.383
+ 350	— 430	— 970	+ 130	— 180	— 160	— 460	+ 1.000	+ 0.924	+ 0.383	+ 0.707	— 0.707
+ 330	— 360	— 1010	+ 160	— 160	— 160	— 500	+ 1.000	+ 0.981	+ 0.195	+ 0.383	— 0.924
+ 320	— 340	— 880	+ 280	— 160	— 160	— 440	+ 1.000	+ 1.000	0.000	0.000	— 1.000
+ 300	— 340	— 720	+ 390	— 160	— 100	— 420	+ 1.000	+ 0.981	— 0.195	— 0.383	— 0.924
+ 280	— 280	— 610	+ 410	— 160	— 140	— 350	+ 1.000	+ 0.924	— 0.383	— 0.707	— 0.707
+ 260	— 260	— 590	+ 440	— 160	— 100	— 330	+ 1.000	+ 0.831	— 0.556	— 0.924	— 0.383
+ 240	— 190	— 590	+ 400	— 160	— 20	— 330	+ 1.000	+ 0.707	— 0.707	— 1.000	0.000
+ 200	— 170	— 510	+ 320	— 160	— 60	— 330	+ 1.000	+ 0.556	— 0.831	— 0.924	+ 0.383
+ 210	— 110	— 510	+ 200	— 230	— 80	— 330	+ 1.000	+ 0.383	— 0.924	— 0.707	+ 0.707
+ 170	— 90	— 510	+ 70	— 250	— 80	— 270	+ 1.000	+ 0.195	— 0.981	— 0.383	+ 0.924
+ 150	— 90	— 510	— 20	— 250	— 140	— 250	+ 1.000	0.000	— 1.000	0.000	+ 1.000
+ 140	— 20	— 510	— 190	— 310	— 100	— 180	+ 1.000	— 0.195	— 0.981	+ 0.383	+ 0.924
+ 120	— 10	— 510	— 290	— 330	— 80	— 230	+ 1.000	— 0.383	— 0.924	+ 0.707	+ 0.707
+ 90	— 10	— 510	— 310	— 330	— 80	— 250	+ 1.000	— 0.556	— 0.831	+ 0.924	+ 0.383

Normal Equations.

ADMIRALTY STANDARD COMPASS.

$$0 = -1.2217 + 17.000\,A_1 + 8.442\,B_1 - 5.641\,C_1 + 0.924\,D_1 + 0.383\,E_1$$
$$0 = -0.7991 + 8.442\,A_1 + 8.310\,B_1 + 0.462\,C_1 - 1.205\,D_1 - 4.543\,E_1$$
$$0 = +0.1662 - 5.641\,A_1 + 0.462\,B_1 + 8.691\,C_1 + 3.900\,D_1 - 4.438\,E_1$$
$$0 = -0.0169 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2)$$
$$0 = +0.0009 + E_1 + B_1\,C_1$$

Hence

$$A_1 = +0.0453 = +2^\circ\ 35'.7$$
$$B_1 = +0.0519 = +2\ \ 58.5$$
$$C_1 = +0.0001 = +0\ \ \ 0.2$$
$$D_1 = +0.0156 = +0\ \ 53.5$$
$$E_1 = -0.0009 = -0\ \ \ 3.1$$

AFTER BINNACLE COMPASS.

$$
\begin{aligned}
0 &= -1.1228 + 17.000\,A_1 + 8.442\,B_1 - 5.641\,C_1 + 0.924\,D_1 + 0.383\,E_1 \\
0 &= -0.8724 + 8.442\,A_1 + 8.310\,B_1 + 0.462\,C_1 - 1.205\,D_1 - 4.543\,E_1 \\
0 &= -0.0346 - 5.641\,A_1 + 0.462\,B_1 + 8.691\,C_1 + 3.900\,D^1 - 4.438\,E_1 \\
0 &= -0.0385 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2) \\
0 &= +0.0018 + E_1 + B_1\,C_1 + 0.0047\,(B_1^2 - C_1^2)
\end{aligned}
$$

Hence

$$
\begin{aligned}
A_1 &= +0.0148 = +0^\circ\ 50'.8 \\
B_1 &= +0.0947 = +5\ \ 25.4 \\
C_1 &= -0.0073 = -0\ \ 25.2 \\
D_1 &= +0.0340 = +1\ \ 57.1 \\
E_1 &= -0.0012 = -0\ \ \ 4.1
\end{aligned}
$$

AFTER RITCHIE COMPASS.

$$
\begin{aligned}
0 &= -3.3336 + 17.000\,A_1 + 8.442\,B_1 - 5.641\,C_1 + 0.924\,D_1 + 0.383\,E_1 \\
0 &= -1.9499 + 8.442\,A_1 + 8.310\,B_1 + 0.462\,C_1 - 1.205\,D_1 - 4.543\,E_1 \\
0 &= +0.6086 - 5.641\,A_1 + 0.462\,B_1 + 8.691\,C_1 + 3.900\,D_1 - 4.438\,E_1 \\
0 &= -0.0340 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2) \\
0 &= +0.0008 + E_1 + B_1\,C_1
\end{aligned}
$$

Hence

$$
\begin{aligned}
A_1 &= +0.1684 = +9^\circ\ 39'.0 \\
B_1 &= +0.0659 = +3\ \ 46.6 \\
C_1 &= +0.0203 = +1\ \ \ 9.8 \\
D_1 &= +0.0320 = +1\ \ 50.1 \\
E_1 &= -0.0021 = -0\ \ \ 7.4
\end{aligned}
$$

AFTER AZIMUTH COMPASS.

$$
\begin{aligned}
0 &= +0.4916 + 17.000\,A_1 + 8.442\,B_1 - 5.641\,C_1 + 0.924\,D_1 + 0.383\,E_1 \\
0 &= +0.6880 + 8.442\,A_1 + 8.310\,B_1 + 0.462\,C_1 - 1.205\,D_1 - 4.543\,E_1 \\
0 &= -0.2024 - 5.641\,A_1 + 0.462\,B_1 + 8.691\,C_1 + 3.900\,D_1 - 4.438\,E_1 \\
0 &= -0.1116 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2) \\
0 &= +0.0002 + E_1 + B_1\,C_1
\end{aligned}
$$

Hence

$$
\begin{aligned}
A_1 &= -0.0434 = -2^\circ\ 29'.3 \\
B_1 &= -0.0199 = -1\ \ \ 8.5 \\
C_1 &= -0.0552 = -3\ \ \ 9.7 \\
D_1 &= +0.1129 = +6\ \ 28.2 \\
E_1 &= -0.0013 = -0\ \ \ 4.5
\end{aligned}
$$

FORWARD ALIDADE COMPASS.

$$
\begin{aligned}
0 &= -1.0908 + 17.000\,A_1 + 8.442\,B_1 - 5.641\,C_1 + 0.924\,D_1 + 0.383\,E_1 \\
0 &= -0.4111 + 8.442\,A_1 + 8.310\,B_1 + 0.462\,C_1 - 1.205\,D_1 - 4.543\,E_1 \\
0 &= +0.4058 - 5.641\,A_1 + 0.462\,B_1 + 8.691\,C_1 + 3.900\,D_1 - 4.438\,E_1 \\
0 &= -0.0235 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2) \\
0 &= -0.0007 + E_1 + B_1\,C_1 + 0.0125\,(B_1^2 - C_1^2)
\end{aligned}
$$

Hence

$$
\begin{aligned}
A_1 &= +0.0615 = +3^\circ\ 31'.5 \\
B_1 &= -0.0084 = -0\ \ 28.8 \\
C_1 &= -0.0166 = -0\ \ 57.2 \\
D_1 &= +0.0236 = +1\ \ 21.1 \\
E_1 &= +0.0006 = +0\ \ \ 1.9
\end{aligned}
$$

FORWARD BINNACLE COMPASS.

$$
\begin{aligned}
0 &= -0.5643 + 17.000\,A_1 + 8.442\,B_1 - 5.641\,C_1 + 0.924\,D_1 + 0.383\,E_1\\
0 &= -0.3228 + 8.442\,A_1 + 8.310\,B_1 + 0.462\,C_1 - 1.205\,D_1 - 4.543\,E_1\\
0 &= +0.0861 - 5.641\,A_1 + 0.462\,B_1 + 8.691\,C_1 + 3.900\,D_1 - 4.438\,E_1\\
0 &= -0.0369 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2)\\
0 &= -0.0011 + E_1 + B_1\,C_1
\end{aligned}
$$

Hence

$$
\begin{array}{lllll}
A_1 = & -0.0050 = & -0^\circ & 17'.1\\
B_1 = & +0.0523 = & +2 & 59.8\\
C_1 = & -0.0307 = & -1 & 45.5\\
D_1 = & +0.0360 = & +2 & 3.7\\
E_1 = & +0.0027 = & +0 & 9.3
\end{array}
$$

FORWARD RITCHIE COMPASS.

$$
\begin{aligned}
0 &= -1.7570 + 17.000\,A_1 + 8.442\,B_1 - 5.641\,C_1 + 0.924\,D_1 + 0.383\,E_1\\
0 &= -1.0582 + 8.442\,A_1 + 8.310\,B_1 + 0.462\,C_1 - 1.205\,D_1 - 4.543\,E_1\\
0 &= +0.3128 - 5.641\,A_1 + 0.462\,B_1 + 8.691\,C_1 + 3.900\,D_1 - 4.438\,E_1\\
0 &= -0.0407 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2)\\
0 &= +0.0013 + E_1 + B_1\,C_1
\end{aligned}
$$

Hence

$$
\begin{array}{lllll}
A_1 = & +0.0564 = & +3^\circ & 14'.0\\
B_1 = & +0.0766 = & +4 & 23.5\\
C_1 = & -0.0205 = & -1 & 10.4\\
D_1 = & +0.0380 = & +2 & 10.5\\
E_1 = & 0.0000 = & 0 & 0.0
\end{array}
$$

The following are the equations of condition for the determination of the coefficients of the After Ritchie Compass at Monte Video.

$$
\begin{aligned}
0 &= -240' + 1.000\,A_1 \quad 0.000\,B_1 + 1.000\,C_1 \quad 0.000\,D_1 + 1.000\,E_1\\
0 &= -570 + 1.000\,A_1 + 0.195\,B_1 + 0.981\,C_1 + 0.383\,D_1 + 0.924\,E_1\\
0 &= -570 + 1.000\,A_1 + 0.383\,B_1 + 0.924\,C_1 + 0.707\,D_1 + 0.707\,E_1\\
0 &= -740 + 1.000\,A_1 + 0.556\,B_1 + 0.831\,C_1 + 0.924\,D_1 + 0.383\,E_1\\
0 &= -740 + 1.000\,A_1 + 0.707\,B_1 + 0.707\,C_1 + 1.000\,D_1 \quad 0.000\,E_1\\
0 &= -740 + 1.000\,A_1 + 0.831\,B_1 + 0.556\,C_1 + 0.924\,D_1 - 0.383\,E_1\\
0 &= -910 + 1.000\,A_1 + 0.924\,B_1 + 0.383\,C_1 + 0.707\,D_1 - 0.707\,E_1\\
0 &= -900 + 1.000\,A_1 + 0.981\,B_1 + 0.195\,C_1 + 0.383\,D_1 - 0.924\,E_1\\
0 &= -560 + 1.000\,A_1 + 1.000\,B_1 \quad 0.000\,C_1 \quad 0.000\,D_1 - 1.000\,E_1\\
0 &= -240 + 1.000\,A_1 + 0.981\,B_1 - 0.195\,C_1 - 0.383\,D_1 - 0.924\,E_1\\
0 &= -230 + 1.000\,A_1 + 0.924\,B_1 - 0.383\,C_1 - 0.707\,D_1 - 0.707\,E_1\\
0 &= -60 + 1.000\,A_1 + 0.831\,B_1 - 0.556\,C_1 - 0.924\,D_1 - 0.383\,E_1\\
0 &= +270 + 1.000\,A_1 + 0.707\,B_1 - 0.707\,C_1 - 1.000\,D_1 \quad 0.000\,E_1\\
0 &= +100 + 1.000\,A_1 + 0.556\,B_1 - 0.831\,C_1 - 0.924\,D_1 + 0.383\,E_1\\
0 &= -240 + 1.000\,A_1 + 0.383\,B_1 - 0.924\,C_1 - 0.707\,D_1 + 0.707\,E_1\\
0 &= -240 + 1.000\,A_1 + 0.195\,B_1 - 0.981\,C_1 - 0.383\,D_1 + 0.924\,E_1\\
0 &= -240 + 1.000\,A_1 \quad 0.000\,B_1 - 1.000\,C_1 \quad 0.000\,D_1 + 1.000\,E_1\\
0 &= -410 + 1.000\,A_1 - 0.195\,B_1 - 0.981\,C_1 + 0.383\,D_1 + 0.924\,E_1\\
0 &= -410 + 1.000\,A_1 - 0.383\,B_1 - 0.924\,C_1 + 0.707\,D_1 + 0.707\,E_1\\
0 &= -240 + 1.000\,A_1 - 0.556\,B_1 - 0.831\,C_1 + 0.924\,D_1 + 0.383\,E_1\\
0 &= -240 + 1.000\,A_1 - 0.707\,B_1 - 0.707\,C_1 + 1.000\,D_1 \quad 0.000\,E_1\\
0 &= -570 + 1.000\,A_1 - 0.831\,B_1 - 0.556\,C_1 + 0.924\,D_1 - 0.383\,E_1
\end{aligned}
$$

The resulting normal equations are

$$
\begin{aligned}
0 &= -2.5365 + 22.000\,A_1 + 7.482\,B_1 - 3.999\,C_1 + 3.938\,D_1 + 2.631\,E_1 \\
0 &= -1.0294 + 7.482\,A_1 + 9.685\,B_1 + 1.969\,C_1 - 2.334\,D_1 - 3.774\,E_1 \\
0 &= -0.3901 - 3.999\,A_1 + 1.969\,B_1 + 12.316\,C_1 + 3.708\,D_1 - 1.665\,E_1 \\
0 &= -0.0340 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2) \\
0 &= +0.0008 + E_1 + B_1\,C_1
\end{aligned}
$$

Hence

$$
\begin{aligned}
A_1 &= +0.1143 = +6^\circ\ 32'.8 \\
B_1 &= +0.0146 = +0\ \ 50.3 \\
C_1 &= +0.0555 = +3\ \ 10.9 \\
D_1 &= +0.0354 = +2\ \ \ 1.8 \\
E_1 &= -0.0016 = -0\ \ \ 5.5
\end{aligned}
$$

The following are the equations of condition, together with the resulting normal equations, and the values of the coefficients A_1, B_1, C_1, D_1, E_1, as determined for each compass from the observations made in Magdalena Bay.

EQUATIONS OF CONDITION AT MAGDALENA BAY.

Absolute Terms.						Coefficients of the Unknown Quantities.				
Admiralty Standard.	After Binnacle.	After Ritchie.	Forward Alidade.	Forward Binnacle.	Forward Ritchie.	A_1	B_1	C_1	D_1	E_1
+ 20′	— 10′	— 100′	— 300′	— 300′	— 540′	+ 1.000	— 0.707	— 0.707	+ 1.000	0.000
+ 60	— 10	— 180	— 370	— 290	— 460	+ 1.000	— 0.831	— 0.556	+ 0.924	— 0.383
+ 110	+ 80	— 180	— 210	— 210	— 380	+ 1.000	— 0.924	— 0.383	+ 0.707	— 0.707
+ 140	+ 160	— 180	— 130	— 210	— 290	+ 1.000	— 0.981	— 0.195	+ 0.383	— 0.924
+ 180	+ 170	— 80	— 130	— 120	— 200	+ 1.000	— 1.000	0.000	0.000	— 1.000
+ 230	+ 320	+ 170	— 210	+ 50	+ 50	+ 1.000	— 0.981	+ 0.195	— 0.383	— 0.924
+ 230	+ 320	+ 330	— 130	+ 130	+ 210	+ 1.000	— 0.924	+ 0.383	— 0.707	— 0.707
+ 250	+ 320	+ 320	— 120	+ 210	+ 210	+ 1.000	— 0.831	+ 0.556	— 0.924	— 0.383
+ 220	+ 320	+ 160	— 40	+ 300	+ 210	+ 1.000	— 0.707	+ 0.707	— 1.000	0.000
+ 220	+ 320	+ 160	— 40	+ 380	+ 300	+ 1.000	— 0.556	+ 0.831	— 0.924	+ 0.383
+ 160	+ 320	+ 150	+ 40	+ 380	+ 370	+ 1.000	— 0.383	+ 0.924	— 0.707	+ 0.707
+ 100	+ 230	+ 60	+ 40	+ 380	+ 210	+ 1.000	— 0.195	+ 0.981	— 0.383	+ 0.924
+ 40	+ 150	— 100	+ 40	+ 370	+ 210	+ 1.000	0.000	+ 1.000	0.000	+ 1.000
+ 30	+ 70	— 190	— 50	+ 290	+ 120	+ 1.000	+ 0.195	+ 0.981	+ 0.383	+ 0.924

Normal Equations.

ADMIRALTY STANDARD COMPASS.

$$
\begin{aligned}
0 &= +0.5789 + 14.000\,A_1 - 8.825\,B_1 + 4.717\,C_1 - 1.631\,D_1 - 1.090\,E_1 \\
0 &= -0.4310 - 8.825\,A_1 + 7.545\,B_1 - 0.816\,C_1 + 0.934\,D_1 + 4.272\,E_1 \\
0 &= +0.2352 + 4.717\,A_1 - 0.816\,B_1 + 6.456\,C_1 - 4.554\,D_1 + 3.784\,E_1 \\
0 &= -0.0169 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2) \\
0 &= +0.0009 + E_1 + B_1\,C_1
\end{aligned}
$$

Hence

$$
\begin{aligned}
A_1 &= +0.0026 = +0^\circ\ \ 9'.1 \\
B_1 &= +0.0559 = +3\ \ 12.1 \\
C_1 &= -0.0204 = -1\ \ 10.3 \\
D_1 &= +0.0156 = +0\ \ 53.5 \\
E_1 &= +0.0002 = +0\ \ \ 0.8
\end{aligned}
$$

AFTER BINNACLE COMPASS.

$$\begin{aligned}
0 &= +0.8029 + 14.000\,A_1 - 8.825\,B_1 + 4.717\,C_1 - 1.631\,D_1 - 1.090\,E_1\\
0 &= -0.5291 - 8.825\,A_1 + 7.545\,B_1 - 0.816\,C_1 + 0.934\,D_1 + 4.272\,E_1\\
0 &= +0.4497 + 4.717\,A_1 - 0.816\,B_1 + 6.456\,C_1 - 4.554\,D_1 + 3.784\,E_1\\
0 &= -0.0385 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2)\\
0 &= +0.0018 + E_1 + B_1\,C_1 + 0.0047\,(B_1^2 - C_1^2)
\end{aligned}$$

Hence

$$\begin{aligned}
A_1 &= -0.0208 = -1^\circ\ 11'.4\\
B_1 &= +0.0393 = +2\ \ 15.0\\
C_1 &= -0.0222 = -1\ \ 16.2\\
D_1 &= +0.0380 = +2\ \ 10.5\\
E_1 &= -0.0010 = -0\ \ \ 3.3
\end{aligned}$$

AFTER RITCHIE COMPASS.

$$\begin{aligned}
0 &= +0.0989 + 14.000\,A_1 - 8.825\,B_1 + 4.717\,C_1 - 1.631\,D_1 - 1.090\,E_1\\
0 &= -0.1171 - 8.825\,A_1 + 7.545\,B_1 - 0.816\,C_1 + 0.934\,D_1 + 4.272\,E_1\\
0 &= +0.2238 + 4.717\,A_1 - 0.816\,B_1 + 6.456\,C_1 - 4.554\,D_1 + 3.784\,E_1\\
0 &= -0.0340 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2)\\
0 &= +0.0008 + E_1 + B_1\,C_1
\end{aligned}$$

Hence

$$\begin{aligned}
A_1 &= +0.0627 = +3^\circ\ 35'.5\\
B_1 &= +0.0778 = +4\ \ 27.3\\
C_1 &= -0.0497 = -2\ \ 51.0\\
D_1 &= +0.0322 = +1\ \ 50.7\\
E_1 &= +0.0031 = +0\ \ 10.6
\end{aligned}$$

FORWARD ALIDADE COMPASS.

$$\begin{aligned}
0 &= -0.4683 + 14.000\,A_1 - 8.825\,B_1 + 4.717\,C_1 - 1.631\,D_1 - 1.090\,E_1\\
0 &= +0.4115 - 8.825\,A_1 + 7.545\,B_1 - 0.816\,C_1 + 0.934\,D_1 + 4.272\,E_1\\
0 &= +0.1082 + 4.717\,A_1 - 0.816\,B_1 + 6.456\,C_1 - 4.554\,D_1 + 3.784\,E_1\\
0 &= -0.0235 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2)\\
0 &= -0.0007 + E_1 + B_1\,C_1 + 0.0125\,(B_1^2 - C_1^2)
\end{aligned}$$

Hence

$$\begin{aligned}
A_1 &= +0.0200 = +1^\circ\ \ 8'.8\\
B_1 &= -0.0361 = -2\ \ \ 4.1\\
C_1 &= -0.0197 = -1\ \ \ 7.6\\
D_1 &= +0.0230 = +1\ \ 19.2\\
E_1 &= 0.0000 = 0\ \ \ 0.0
\end{aligned}$$

FORWARD BINNACLE COMPASS.

$$\begin{aligned}
0 &= +0.3956 + 14.000\,A_1 - 8.825\,B_1 + 4.717\,C_1 - 1.631\,D_1 - 1.090\,E_1\\
0 &= +0.0125 - 8.825\,A_1 + 7.545\,B_1 - 0.816\,C_1 + 0.934\,D_1 + 4.272\,E_1\\
0 &= +0.7497 + 4.717\,A_1 - 0.816\,B_1 + 6.456\,C_1 - 4.554\,D_1 + 3.784\,E_1\\
0 &= -0.0369 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2)\\
0 &= -0.0011 + E_1 + B_1\,C_1
\end{aligned}$$

Hence

$$\begin{aligned}
A_1 &= -0.0298 = -1^\circ\ 42'.6\\
B_1 &= -0.0478 = -2\ \ 44.3\\
C_1 &= -0.0719 = -4\ \ \ 7.3\\
D_1 &= +0.0384 = +2\ \ 11.8\\
E_1 &= -0.0023 = -0\ \ \ 7.9
\end{aligned}$$

Forward Ritchie Compass.

$$
\begin{aligned}
0 &= +0.0058 + 14.000\,A_1 - 8.825\,B_1 + 4.717\,C_1 - 1.631\,D_1 - 1.090\,E_1\\
0 &= +0.2058 - 8.825\,A_1 + 7.545\,B_1 - 0.816\,C_1 + 0.934\,D_1 + 4.272\,E_1\\
0 &= +0.6749 + 4.717\,A_1 - 0.816\,B_1 + 6.456\,C_1 - 4.554\,D_1 + 3.784\,E_1\\
0 &= -0.0407 + D_1 + \tfrac{1}{2}(B_1^2 - C_1^2)\\
0 &= +0.0013 + E_1 + B_1\,C_1
\end{aligned}
$$

Hence

$$
\begin{aligned}
A_1 &= +0.0477 = +2^\circ\ 43'.8\\
B_1 &= +0.0116 = +0\ \ 39.9\\
C_1 &= -0.1051 = -6\ \ \ 1.3\\
D_1 &= +0.0462 = +2\ \ 38.7\\
E_1 &= -0.0004 = -0\ \ \ 1.3
\end{aligned}
$$

For convenience of reference the values of the coefficients A_1, B_1, C_1, D_1, E_1, obtained at stations where the compasses were not read on all the thirty-two points, have been collected in the following table. No use has been made of them.

Stations and Compasses.	A_1	B_1	C_1	D_1	E_1
Ceara, December 19, 1865.					
Admiralty Standard Compass	− 0° 35′.1	+ 4° 46′.3	+ 2° 19′.2	+ 0° 48′.8	− 0° 14′.8
After Binnacle Compass	+ 0 21.3	+ 4 35.2	+ 2 4.6	+ 2 3.6	− 0 16.3
After Ritchie Compass	+ 5 54.2	+ 7 56.0	+ 4 55.4	+ 1 36.6	− 0 43.7
Forward Alidade Compass	+ 2 3.5	+ 0 0.2	+ 1 4.8	+ 1 21.4	+ 0 2.4
Forward Binnacle Compass	− 0 54.7	+ 0 24.6	+ 1 26.9	+ 2 7.8	+ 0 3 2
Forward Ritchie Compass	+ 3 31.0	− 0 26.1	+ 3 36.9	+ 2 26.6	− 0 3.9
Rio Janeiro, January 10, 1866.					
Admiralty Standard Compass	+ 2 35.7	+ 2 58.5	+ 0 0.2	+ 0 53.5	− 0 3.1
After Binnacle Compass	+ 0 50.8	+ 5 25.4	− 0 25.2	+ 1 57.1	− 0 4.1
After Ritchie Compass	+ 9 39.0	+ 3 46.6	+ 1 9.8	+ 1 50.1	− 0 7.4
After Azimuth Compass	− 2 29.3	− 1 8.5	− 3 9.7	+ 6 28.2	− 0 4.5
Forward Alidade Compass	+ 3 31.5	− 0 28.8	− 0 57.2	+ 1 21.1	+ 0 1.9
Forward Binnacle Compass	− 0 17.1	+ 2 59.8	− 1 45.5	+ 2 3.7	+ 0 9.3
Forward Ritchie Compass	+ 3 14.0	+ 4 23.5	− 1 10.4	+ 2 10.5	0 0.0
Monte Video, January 24, 1866.					
After Ritchie Compass	+ 6 32.8	+ 0 50.3	+ 3 10.9	+ 2 1.8	− 0 5.5
Magdalena Bay, June 9, 1866.					
Admiralty Standard Compass	+ 0 9.1	+ 3 12.1	− 1 10.3	+ 0 53.5	+ 0 0.8
After Binnacle Compass	− 1 11.4	+ 2 15.0	− 1 16.2	+ 2 10.5	− 0 3.3
After Ritchie Compass	+ 3 35.5	+ 4 27.3	− 2 51.0	+ 1 50.7	+ 0 10.6
Forward Alidade Compass	+ 1 8.8	− 2 4.1	− 1 7.6	+ 1 19.2	0 0.0
Forward Binnacle Compass	− 1 42.6	− 2 44.3	− 4 7.3	+ 2 11.8	− 0 7.9
Forward Ritchie Compass	+ 2 43.8	+ 0 39.9	− 6 1.3	+ 2 38.7	− 0 1.3

At a number of the ports visited during the cruise, the line dividing the north from the south polarity, on the exterior of the turrets, was traced out; but as the boundary between the two kinds of magnetism was frequently very badly defined, and the observations were otherwise unsatisfactory; and further, as they throw no light whatever on the theory of the deviations of the compasses, and can only be shown by means of drawings on a rather large scale, it has not been deemed worth while to insert them here.

In conclusion, the results of the observations made during the cruise may be briefly recapitulated as follows:

1°. The latitudes of seven points have been determined.

2°. The magnetic declination, inclination, and horizontal force, have been determined at eighteen places.

3°. The deviations of seven compasses have been observed, and compared with the theory, at ten places so situated as to afford very great changes in the terrestrial magnetic elements. For all these compasses the coefficients depending upon the hard and soft iron have been so far separated from each other as to render it possible to predict the deviations in any part of the world; and for the Admiralty Standard and After Azimuth Compasses every one of the coefficients in Poisson's general equations has been determined separately with a considerable degree of accuracy.

The conclusions drawn from the discussion of the observations are that, in the case of the Monadnock,

a. The agreement between the theoretical and observed deviations is sufficiently exact for the purposes of navigation, but is not entirely satisfactory in a scientific point of view.

b. It is questionable whether the theory really represents the semicircular as well as it does the quadrantal deviation; and to settle this point there is great need of more observations.

c. The so-called permanent and subpermanent magnetism of the ship were undergoing a constant and rapid change such as would correspond to a transfer of magnetism from aft forward; and to a rotation from right to left in the direction of the force.

d. The ratio of the hard to the soft iron force was slowly varying at each compass; and, for the different compasses it ranged between 4.3 and 67.4.

e. There was not a compass on board at which the direction of the hard and soft iron forces coincided; from which it follows that in no case was the ratio of the hard to the soft iron force the same in the coefficient $\mathfrak{B}$ as it was in the coefficient $\mathfrak{C}$.

f. So far as can be judged from the observations discussed in this report, in the case of a vessel swung for the first time, it is impossible to make any reliable estimate of the ratio of the hard to the soft iron force in the coefficients $\mathfrak{B}$ and $\mathfrak{C}$; and therefore, it is also impossible to make any reliable estimate of the changes the deviations of the compasses will undergo upon a change of magnetic latitude.

SMITHSONIAN CONTRIBUTIONS TO KNOWLEDGE.

233

CONVERGING SERIES

EXPRESSING THE

RATIO BETWEEN THE DIAMETER AND THE CIRCUMFERENCE

OF

A CIRCLE.

BY

WILLIAM FERREL.

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CONVERGING SERIES EXPRESSING THE RATIO

BETWEEN THE

DIAMETER AND THE CIRCUMFERENCE OF A CIRCLE.

THE following method of obtaining converging series expressing the value of π, and the series obtained, are thought to be new.*

Let F_x be a function of x, and $\Delta^1, \Delta^2, \Delta^3$, &c., express the different order of finite differences of F_x for the equal intervals of $x = o$, $x = \omega$, $x = 2\omega$, &c. Putting $n = \frac{x}{\omega}$, we have the well-known formula,

$$F_x = Fo + \frac{n}{1}\Delta_{\frac{1}{2}}^1 + \frac{n(n-1)}{1.2}\Delta_0^2 + \frac{(n+1)n(n-1)}{1\ .\ 2\ .\ 3}\Delta_{\frac{1}{2}}^3 + \frac{(n+1)n(n-1)(n-2)}{1\ .\ 2\ .\ 3\ .\ 4}\dots \tag{1}$$

Let us now suppose that F_x is the sine of the arc x, and that $Fo = 0$. In this case Δ_0^2, Δ_0^4, &c., vanish, and when x is infinitely small $F_x = x$. Hence $F_x \frac{\omega}{x} = \omega$ and the preceding equation becomes

$$\omega = \Delta_{\frac{1}{2}}^1 - \frac{1}{2.3}\Delta_{\frac{1}{2}}^3 + \frac{1.2}{3.4.5}\Delta_{\frac{1}{2}}^5 - \frac{1.2.3}{4.5.6.7}\Delta_{\frac{1}{2}}^7 \dots . \tag{2}$$

From the theory of finite differences we have in this case

$$\Delta_{\frac{1}{2}}^{2i+1} = -\left\{-2(1-\cos\omega)\right\}^i \sin\omega \tag{3}$$

If we substitute this in the preceding expression of ω, putting

$$\alpha = 2(1-\cos\omega) \tag{4}$$

we get

$$\omega = \sin\omega\ (1 + \frac{1}{2.3}\alpha + \frac{1.2}{3.4.5}\alpha^2 + \frac{1.2.3}{4.5.6.7}\alpha^3 \dots \tag{5}$$

When the sine of any aliquot part of the circumference of a circle is known, we can readily obtain the cosine of that arc, and from (4) the value of α; and then (5) gives an expression of ω, and consequently of the circumference of the circle. Hence, from the computed series of the continued bisections of any aliquot part of the circumference of which the sine is known, an infinite number of such series may be obtained.

---

* A number of series of this kind may be found in Davies & Peck's Mathematical Dictionary, under the head of the circle. Also in Chauvenet's Plane and Spherical Trigonometry.

First, if we take $\omega = \frac{2}{3}\pi$, or 120°, then $sin\ \omega = \frac{1}{2}\sqrt{3}$ and (4) gives $\alpha = 3$. With these values (5) gives

$$(6) \qquad \pi = \frac{3}{4}\sqrt{3}\left(1 + \frac{1.3}{2.3} + \frac{1.2.3^2}{3.4.5} + \frac{1.2.3.3^3}{4.5.6.7} \dots\right)$$

Secondly, if we take $\omega = \frac{1}{2}\pi$, then we have $sin\ \omega = 1$, and (4) gives $\alpha = 2$. With these values (5) gives

$$(7) \qquad \pi = 2\left(1 + \frac{1}{1.3} + \frac{1.2}{1.3.5} + \frac{1.2.3}{1.3.5.7} + \frac{1.2.3.4}{1.3.5.7.9} \dots\right)$$

Thirdly, if we take $\omega = \frac{1}{3}\pi$, then $sin\ \omega = \frac{1}{2}\sqrt{3}$, and (4 gives $\alpha = 1$. With these values (5) gives

$$(8) \quad \pi = \frac{3}{2}\sqrt{3}\left(1 + \frac{1}{2.3} + \frac{1.2}{3.4.5} + \frac{1.2.3}{4.5.6.7} + \frac{1.2.3.4}{5.6.7.8.9} \dots\right)$$

Fourthly, if we take $\omega = \frac{1}{4}\pi$, then $sin\ \omega = \sqrt{\tfrac{1}{2}}$, and (4) gives $\alpha = 2 - \sqrt{2}$. In this case the value of α not being a rational integral number, the series is not convenient for computation directly as in the preceding cases. But we obtain a series which may be expressed in the following form:—

$$(9) \qquad \pi = P_1 + \frac{1}{2.3}P_2 + \frac{1.2}{3.4.5}P_3 + \frac{1.2.3}{4.5.6.7}P \dots$$

in which

$$P_1 = 4\sqrt{\tfrac{1}{2}}\,,$$

$$P_2 = 4\sqrt{\tfrac{1}{2}}\left(2 - \sqrt{2}\right),$$

and generally after the second

$$P_i = 4P_{i-2} - 2\,P_{i-2}\ .$$

The first two being known, the remainder are easily found by the preceding relation, and then with these (9) gives the value of π as follows:—

$$\begin{array}{rcrr}
2.828427 \times & 1 & = & 2.828427 \\
1.656854 \times & \frac{1}{2.3} & = & .276142 \\
970562 \times & \frac{1.2}{3.4.5} & = & 32352 \\
.568540 \times & \frac{1.2.3}{4.5.6.7} & = & 4061 \\
.333036 \times & \frac{1.2.3.4}{5.6.7.8.9} & = & 529 \\
.125064 \times & \frac{1.2.3.4.5}{6.7.8.9.10.11} & = & 70 \\
.114184 \times & \frac{1.2.3.4.5.6}{7.8.9.10.11.12.13} & = & 10 \\
\hline
 & & \pi = & 3.14159
\end{array}$$

The first column represents the successive value of P_i, in which four times each number, minus twice the preceding, equals the following one; and hence it is easily continued.

In the fifth place, if we take $\omega = \frac{1}{6}\pi$, then $\sin \omega = \frac{1}{2}$, and (4) gives $\alpha = 2 - \sqrt{3}$. In this case α is again not a rational integral number. But in the same manner as in the preceding case, we obtain from (5)

$$\pi = P_1 + \frac{1}{2\,.\,3} P_2 + \frac{1\,.\,2}{3\,.\,4\,.\,5} P_3 + \frac{1\,.\,2\,.\,3}{4\,.\,5\,.\,6\,.\,7} P_4 \ldots \tag{10}$$

in which

$$P_1 = 3\ ,$$
$$P_2 = 3\left(2 - \sqrt{3}\right)\ ,$$

and generally after the second

$$P_i = 4P_{i-1} - P_{i-2}\ .$$

By means of this relation, the first two being known, the others are readily obtained, and with them the value of the whole expression, as follows:—

$$\begin{array}{rcccr}
3.000000000 & \times & 1 & = & 3.000000000 \\
.803847577 & \times & \frac{1}{2\,.\,3} & = & 133974596 \\
.215390308 & \times & \frac{1\,.\,2}{3\,.\,4\,.\,5} & = & 7179677 \\
57713655 & \times & \frac{1\,.\,2\,.\,3}{4\,.\,5\,.\,6\,.\,7} & = & 412240 \\
15464312 & \times & \frac{1\,.\,2\,.\,3\,.\,4}{5\,.\,6\,.\,7\,.\,8\,.\,9} & = & 24547 \\
4143593 & \times & \frac{1\,.\,2\,.\,3\,.\,4\,.\,5}{6\,.\,7\,.\,8\,.\,9\,.\,10\,.\,11} & = & 1495 \\
1110060 & \times & \frac{1\,.\,2\,.\,3\,.\,4\,.\,5\,.\,6}{7\,.\,8\,.\,9\,.\,10\,.\,11\,.\,12\,.\,13} & = & 92 \\
 & & & \pi = & 3.14159265
\end{array}$$

In this case, in the first column representing the successive values of P_i, four times each number, minus the preceding, equals the following one; hence the law of continuance is very simple and convenient.

By continued bisection of the arcs, series of any degree of convergency may be obtained, but the preceding are the only ones found in which α is a rational integral number, or in which the successive values of P_i may be obtained by a simple recurring series.

Each term of the series in (5) is equal to the preceding multiplied into $\frac{i\alpha}{2(2i+1)}$, in which i is the exponent of α, and consequently $(i + 1)$ the number of the term in the series. Hence this is the expression of the ratio of convergency, and when the value of i is considerable, differs but little from $\frac{1}{4}\alpha$, which is the limiting ratio of convergency. The first of the preceding series, on account of the large values of α, converge slowly. In (8) in which $\alpha = 1$, the limiting ratio of convergency is $\frac{1}{4}$. In (9) in which $\alpha = 2 - \sqrt{2}$ the limiting ratio of convergency is $\frac{1}{4}(2 - \sqrt{2}) = \frac{1}{7}$ nearly. In (10) in which $\alpha = 2 - \sqrt{3}$ the limiting ratio is $\frac{1}{4}(2 - \sqrt{3}) = \frac{1}{15}$ nearly. This is much more convergent than the formulæ of Clausen and Dase of Germany, with which they separately and independently computed the value of π to 200 places. It has also the advantage over their formulæ in giving the value of π from the computation of one series only. It is, however, less convergent than the formula of Machin, in which the limiting ratio, in the less convergent of the two series, is $\frac{1}{25}$; but Machin's formula also requires the computation of two series.

All of the preceding series are very interesting on account of the extremely simple laws of their continuance.

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