SMITHSONIAN

MISCELLANEOUS COLLECTIONS

VOL. 51

"EVERY MAN IS A VALUABLE MEMBER OF SOCIETY WHO, BY HIS OBSERVATIONS, RESEARCHES, AND EXPERIMENTS, PROCURES KNOWLEDGE FOR MEN"—SMITHSON

(PUBLICATION 1943)

CITY OF WASHINGTON
PUBLISHED BY THE SMITHSONIAN INSTITUTION
1910

ADVERTISEMENT

The present series, entitled "Smithsonian Miscellaneous Collections," is intended to embrace all the publications issued directly by the Smithsonian Institution in octavo form; those in quarto constituting the "Smithsonian Contributions to Knowledge." The quarto series includes memoirs, embracing the records of extended original investigations and researches, resulting in what are believed to be new truths, and constituting positive additions to the sum of human knowledge. The octavo series is designed to contain reports on the present state of our knowledge of particular branches of science; instructions for collecting and digesting facts and materials for research; lists and synopses of species of the organic and inorganic world; reports of explorations; aids to bibliographical investigations, etc., generally prepared at the express request of the Institution, and at its expense.

In the Smithsonian Contributions to Knowledge, as well as in the present series, each article is separately paged. The actual date of its publication is that given on its special title-page, and not that of the volume in which it is placed. In many cases works have been published and largely distributed, years before their combination into volumes.

CHAS. D. WALCOTT,
Secretary of the Smithsonian Institution

TABLE OF CONTENTS

SMITHSONIAN MISCELLANEOUS COLLECTIONS

PART OF VOLUME LI

THE DEVELOPMENT OF THE AMERICAN ALLIGATOR

(A. mississippiensis)

WITH TWENTY-THREE PLATES

BY

ALBERT M. REESE

Professor of Zoology, West Virginia University

No. 1791

CITY OF WASHINGTON

PUBLISHED BY THE SMITHSONIAN INSTITUTION

1908

WASHINGTON, D. C.
PRESS OF JUDD & DETWEILER, INC.
1908

THE DEVELOPMENT OF THE AMERICAN ALLIGATOR (*A. MISSISSIPPIENSIS*)

By ALBERT M. REESE

(With 23 plates)

Introduction

With the exception of S. F. Clarke's well-known paper, to which frequent reference will be made, practically no work has been done upon the development of the American alligator. This is probably due to the great difficulties experienced in obtaining the necessary embryological material. Clarke, some twenty years ago, made three trips to the swamps of Florida in quest of the desired material. The writer has also spent parts of three summers in the southern swamps—once in the Everglades, once among the smaller swamps and lakes of central Florida, and once in the Okefenokee Swamp. For the first of these expeditions he is indebted to the Elizabeth Thompson Science Fund; but for the more successful trip, when most of the material for this work was collected, he is indebted to the Smithsonian Institution, from which a liberal grant of money to defray the expenses of the expedition was received.

The writer also desires to express his appreciation of the numerous courtesies that he has received from Dr. Samuel F. Clarke, especially for the loan of several excellent series of sections, from which a number of the earlier stages were drawn.

The present paper gives a general outline of the whole process of development of the American alligator (*A. mississippiensis*), it being the intention of the author to take up in detail the more specific points in subsequent researches.

In preparing the material several kinds of fixation were employed, but the ordinary corrosive sublimate-acetic mixture gave about the most satisfactory results. Ten per cent formalin, Parker's mixture of formalin and alcohol, etc., were also used. In all cases the embryos were stained *in toto* with borax carmine, and in most cases the sections were also stained on the slide with Lyon's blue. This double stain gave excellent results. Transverse, sagittal, and horizontal series of sections were made, the youngest embryos being cut into sections five microns thick, the older stages ten microns or more in thickness.

The Egg

Figures 1, 1*a* (Plate I)

The egg (fig. 1) is a perfect ellipse, the relative lengths of whose axes vary considerably in the eggs of different nests and slightly in the eggs of the same nest. Of more than four hundred eggs measured, the longest was 85 mm.; the shortest 65 mm. Of the same eggs, the greatest short diameter was 50 mm.; the least short diameter was 38 mm. The average long diameter of these four hundred eggs was 73.74 mm.; the average short diameter was 42.59 mm. The average variation in the long axis of the eggs of any one nest was 11.32 mm., more than twice the average variation in the short axis, which was 5.14 mm. No relation was noticed between the size and the number of eggs in any one nest. Ten eggs of average size weighed 812 grams—about 81 grams each.

Voeltzkow (18)[1] states that the form of the egg of the Madagascar crocodile is very variable. No two eggs in the same nest are exactly alike, some being elliptical, some "egg-shaped," and some "cylindrical with rounded ends." The average size is 68 mm. by 47 mm., shorter and thicker than the average alligator egg.

When first laid, the eggs are pure white, and are quite slimy for a few hours, but they generally become stained after a time by the damp and decaying vegetation composing the nest in which they are closely packed.

The shell is thicker and of a coarser texture than that of the hen's egg. Being of a calcareous nature, it is easily dissolved in dilute acids.

The shell membrane is in two not very distinct layers, the fibers of which, according to S. F. Clarke, are spirally wound around the egg at right angles to each other. No air-chamber, such as is found in the hen's egg, is found in any stage in the development.

In most—probably all normal—eggs a white band appears around the lesser circumference a short time after being laid. This chalky band, which is shown at about its maximum development in fig. 1*a*, is found, on removal of the shell, to be caused, not by a change in the shell, but by the appearance of an area of chalky substance in the shell membranes. Clarke thinks this change in the membrane is to aid in the passage of gases to and from the developing embryo. Generally this chalky area forms a distinct band entirely around the shorter circumference of the egg, but sometimes extends only partly

[1] The numerical citations throughout the article are to bibliographical references at the end of the paper.

around it. It varies in width from about 15 mm. to 35 mm., being narrowest at its first appearance. Sometimes its borders are quite sharp and even (fig. 1*a*) ; in other cases they are very irregular. If the embryo dies the chalky band is likely to become spotted with dark areas.

The shell and shell membrane of the egg of the Madagascar crocodile are essentially the same as those just described, except that the shell is sometimes pierced by small pores that pass entirely through it. The same chalky band surrounds the median zone of the egg (18).

The white of the egg is chiefly remarkable for its unusual density, being so stiff that the entire egg may be emptied from the shell into the hand and passed from one hand to the other without danger of rupturing either the mass of albumen or the enclosed yolk. The albumen, especially in the immediate neighborhood of the yolk, seems to consist of a number of very thin concentric layers. It varies in color, in different eggs, from a pale yellowish white, its usual color, to a very decided green.

As might be expected, no chalazæ are present.

The yolk is a spherical mass, of a pale yellow color, lying in the center of the white. Its diameter is so great that it lies very close to the shell around the lesser circumference of the egg, so that it is there covered by only a thin layer of white, and care must be taken in removing the shell from this region in order not to rupture the yolk. The yolk substance is quite fluid and is contained in a rather delicate vitelline membrane.

The albumen and yolk of the crocodile's egg, as described by Voeltzkow, differ from those of the alligator only in the color of the albumen, which in the crocodile is *normally* light green (18).

As pointed out by Clarke, the position of the embryo upon the yolk is subject to some variation. During the earliest stages it may occur at the pole of the yolk nearest the side of the egg; later it may generally be found toward the end of the egg; and still later it shifts its position to the side of the egg. It is probable, as Clarke says, that the position at the end of the egg secures better protection by the greater amount of white, at that point, between the yolk and the shell; while the later removal to the side of the egg, when the vascular area and the allantois begin to function, secures a better aëration of the blood of the embryo.

Around the embryo, during the stages that precede the formation of the vascular area, is seen an irregular area of a lighter color and a mottled appearance. This area is bounded by a distinct, narrow, white line, and varies in size from perhaps a square centimeter to one-third the surface of the yolk.

During the earliest stages of development the embryo is very transparent; so that, as there is no fixed place upon the yolk at which it may be expected to occur, it is often very difficult to find. Owing to this transparency, to the extreme delicacy of the embryo, and to the character of the white, the removal of an early embryo from the egg of the alligator is a difficult operation and is accomplished only after some practice.

The Development of the Embryo

As the writer has pointed out elsewhere (13), the embryo of the alligator is often of considerable size when the egg is laid. This makes the obtaining of the earliest stages of development a difficult matter; so that the writer, as has already been said, like S. F. Clarke (5), made three trips to the South in quest of the desired material. Voeltzkow (18) experienced the same difficulty in his work on the crocodile, and made several trips to Africa before he succeeded in obtaining all the desired stages of development.

To obtain the earliest stages, I watched the newly made nests until the eggs were laid, and in this way a number of eggs were obtained within a very few hours after they had been deposited, and all of these eggs contained embryos of a more or less advanced stage of development. Gravid females were then killed, and the eggs removed from the oviducts. These eggs, although removed from a "cold-blooded" animal, generally contained embryos of some size, and only one lot of eggs thus obtained contained undeveloped embryos, which embryos refused to develop further in spite of the most careful treatment. Voeltzkow (18) found, in the same way, that the earlier stages of the crocodile were extremely difficult to handle; so that, in order to obtain the earlier stages, he was reduced to the rather cruel expedient of tying a gravid female and periodically removing the eggs from the oviducts through a slit cut in the body wall.

The older embryos are hardy and bear transportation well, so that it is comparatively easy to obtain the later stages of development.

For the stages up to the formation of the first four or five somites, I am indebted, as I have already said, to Professor Clarke, and, since I have had opportunity to examine only the sections and not the surface views of these stages, I shall quote directly Clarke's paper in the Journal of Morphology (5) in description of these surface views.

STAGE I

FIGURES *2–2f* (PLATES I, II)

The youngest embryo that we have for description is shown in figures 2 and *2a*. Of figure 2 Clarke says:

"The limiting line between the opaque and pellucid areas is clearly marked, and within the latter is a shield-shaped area connected by the narrower region of the primitive streak with the area opaca. The blastopore is already formed near the posterior end of the shield.

"A ventral view of another embryo of the same age (fig. *2a*), seen from the ventral side, shows that the blastopore extends quite through the blastoderm, in an oblique direction downwards and forwards, from the dorsal to the ventral side. The thickened area of the primitive streak is here very prominent. There is, too, the beginning of a curved depression at the anterior end of the shield, the first formation of the head-fold."

Transverse sections of this stage are shown in figures *2b–2f*.

Figure *2b*, through the anterior region of the blastoderm, shows a sharply defined ectoderm (*ec*) which is composed of three or four layers of cells in the median region, while it gradually thins out laterally. Closely underlying this ectoderm is a thin sheet of irregular cells, the entoderm (*en*).

Figure *2c* is about one-fifth of the length of the blastoderm posterior to the preceding and represents approximately the same conditions, except that there is an irregular thickening of the entoderm in the median region (*en*). This thickening apparently marks the anterior limit of the mesoderm, to be discussed shortly.

Figure *2d* represents the condition of the blastoderm throughout about one-third of its length, posterior to the preceding section. The somewhat regular folds in the ectoderm (*ec*) are probably not the medullary folds, but are such artificial folds as might easily be produced in handling the delicate blastoderm. The thickening of the entoderm, noticed in the preceding figure, is here more sharply defined, and as we pass toward the blastopore becomes separated somewhat from the entoderm proper as a middle layer or mesoderm (fig. *2e, mes*). It would thus seem, from a study of these sections, that most of the mesoderm is derived from the entoderm. In fact, all of the mesoderm in front of the blastopore seems to have this origin, for it is not until the anterior edge of the blastopore is reached that there is any connection between the ectoderm and entoderm (fig. *2e*).

Figure *2e* is a section through the region just mentioned, where, medially, the ectoderm, mesoderm, and entoderm form a continuous mass of cells. Laterally the mesoderm (*mes*) is a distinct layer of cells of a fairly characteristic mesodermal type. The notochord is not yet discernible, though a slight condensation of cells in the middle line may indicate its position.

Figure 2*f* is one of the four sections that were cut through the blastopore (*blp*), which is a hole of considerable size that opens, as the figure shows, entirely through the blastoderm. Along the walls of the blastopore the ectoderm and entoderm are, of course, continuous with each other and form a sharply defined boundary to the opening. As we pass laterally from the blastopore the cells become less compact, and are continued on each side as the mesodermal layer (*mes*). In this series the sections posterior to the blastopore were somewhat torn, and so were not drawn; but they probably did not differ materially from those of the corresponding region of the immediately following stages, which are shown in figures 3*m* and 6*i* and will be described in their proper order.

Stage II

Figures 3–3*o* (Plates II, III, IV)

The next stage to be described is shown in surface views in figures 3 and 3*a*. Of this stage Clarke says:

"The head-fold rapidly increases in depth and prominence, as shown in figure 3, which is a ventral view a few hours later [than the preceding stage]. The time cannot be given exactly, as it is found that eggs of the same nest are not equally advanced when laid, and differ in their rate of development. The lighter curve in front of the head-fold is the beginning of the anterior fold of the amnion. The notochord has been rapidly forming, and now shows very distinctly on the ventral side, when viewed by transmitted light. A dorsal view of the same embryo (fig. 3*a*) shows that the medullary or neural groove is appearing, and that it ends abruptly anteriorly near the large transverse head-fold. Posteriorly it terminates at the thickened area in front of the blastopore, which still remains open."

Figures 3*b*–*m* are drawn from transsections of an embryo of about this state of development. For a short distance in front of the beginning of the head-fold, there is a mass of cells of considerable thickness between the ectoderm and entoderm. In figure 3*b* these cells appear as an irregular thickening of the entoderm, while in figure 3*c* they form a continuous mass, uniting the upper and lower germ layers. This condition is seen, though in a much less striking degree, in the following stage of development. As to its significance the writer is not prepared to decide.

Figure 3*d* passes through the head-fold, which in this embryo was probably not so far developed as it was in the embryo shown in figures 3 and 3*a*. Not having seen the embryo, however, before it was sectioned, the writer cannot be certain of this point. The ectoderm and entoderm are here of nearly the same thickness.

Figure 3*e* is a short distance posterior to the preceding. It shows a marked thickening of the ectoderm in the medial region (*ec*), which is continuous posteriorly with the anterior ends of the medullary folds that are just beginning to differentiate (figs. 3*f–h*).

Figure 3*g* passes through the anterior end of the medullary plate or folds (*mf*), whichever they may be called. The ectoderm of the folds is thickened and is considerably elevated above the rest of the blastoderm. There is scarcely any sign, in this region, of a medullary groove. The entoderm (*en*) is considerably thickened in the medial region, this thickening being continuous posteriorly, as in the preceding stage, with the mesoderm.

In figure 3*h*, cut in a plane at some distance posterior to the preceding, the medullary groove (*mg*) is well marked; its bordering folds gradually thin out laterally to the thickness of the ordinary ectoderm. The medial thickening of the entoderm is very marked, but it has not in this region separated into a distinct mesoblastic layer.

Immediately under the medullary groove is a dense mass of cells (*nt*), apparently the anterior end of the notochord in process of formation.

Figure 3*i*, still farther toward the blastopore, shows the medullary groove wider and shallower than in the more anterior sections. The mesoderm (*mes*) is here a layer laterally distinct from the entoderm. In the middle line it is still continuous with the entoderm, and at this place it is the more dense mass of cells that may be recognized as the notochord (*nt*). It is evidently difficult to decide whether this group of cells (*nt*), which will later become a distinct body, the notochord, is derived directly from the entoderm or from the mesoderm, which is itself a derivative of the entoderm. There is here absolutely no line of demarcation between the cells of the notochord and those of the mesoderm and entoderm.

In figure 3*j* the ectoderm (*ec*) is nearly flat, scarcely a sign of the medullary groove appearing. The mesoderm (*mes*) is here a distinct layer, entirely separate from both notochord (*nt*) and entoderm (*en*). The notochord is a clearly defined mass of cells, distinct, as has been said, from the mesoderm, but still closely united with the underlying entoderm, which is much thinner than the ectoderm.

This condition of the notochord, which is found throughout about one-third of the length of the embryo, would give the impression that the notochord is of a distinctly entodermal origin.

In figure 3*k* there is no sign of the medullary groove, though ectoderm (*ec*) is still much thickened in the middle line. The section passes, posterior to the notochord, through the anterior edge of the ventral opening of the blastopore (*blp*). The mesoderm (*mes*) is here again continuous with the entoderm, around the edge of the blastopore, but is distinct from the ectoderm.

Figure 3*l* represents the third section posterior to the preceding. The blastopore, which passes upward and backward through the blastoderm, is seen as an enclosed slit (*blp*). It is surrounded by a distinct layer of compactly arranged cells continuous with the thickened ectoderm (*ec*) above, with the thin entoderm (*en*) below, and laterally with the gradually thinning and scattering mesoderm (*mes*).

Figure 3*m* is the next section posterior to the one just described. It passes through the dorsal opening of the blastopore (*blp*), which appears as a deep, narrow cleft with thick ectodermal borders. The three germ layers are still continuous with each other, though the connection of the entoderm with the other two is slight. The sections posterior to this one will be described in the next stage, where they have essentially the same structure and are better preserved.

Figures 3*n* and 3*o* are sagittal sections of an embryo of about the stage under discussion. In both figures the head-fold is seen as a deep loop of ectoderm and entoderm, while the head-fold of the amnion is seen at *a*.

The beginning of the foregut is seen in figure 3*n* (*fg*), which is the more nearly median of the two sections, figure 3*o* being a short distance to the side of the middle line.

In figure 3*o* the thin entoderm (*en*) is separated from the much thicker ectoderm (*ec*) by a considerable layer of rather loose mesoderm (*mes*). In figure 3*n*, which is almost exactly median in position, there is, of course, no mesoderm to be seen in front of the blastopore, and the entoderm shows a considerable increase in thickness, due to the formation of the notochord (*nt*). The blastopore (*blp*) is the most striking feature of the figure, and is remarkable for its great width in an antero-posterior direction. Its anterior and posterior borders are outlined by sharply defined layers of ectoderm and entoderm. Posterior to the blastopore the lower side of the ectoderm is continuous with a considerable mass of cells, the primitive streak (*ps*).

Stage III

Figures 4, 4*a*, 5, 5*a*, and 6–6*i* (Plates V, VI)

"Figures 4 and 4*a* are of an embryo removed, on June 18, from an egg which had been taken out of an alligator two days before. Figure 4, a dorsal view, is of special interest in that it shows a secondary fold taking place in the head-fold. This grows posteriorly along the median dorsal line, forming a V-shaped process with the apex pointing backward toward the blastopore. There is quite a deep groove between the arms of the V. The head-fold on the ventral side, as seen in figure 4*a*, made from the same embryo as figure 4, grows most rapidly on the mid-line, and also becomes thicker at that place. The medullary folds now begin to form on either side of the medullary groove, ending posteriorly on either side of the blastopore and anteriorly on either side of the point of the V-shaped process in the middle of the head-fold. This is seen in figure 5, which is a dorsal view of an embryo from an egg three days after it was taken out of an alligator. A ventral view of the same embryo (fig. 5*a*) represents the thickened process on the mid-line at its greatest development. For some reason the notochord did not show in this embryo, possibly owing to particles of the yolk material adhering about the mid-line.

"In an embryo a day or two older, the V-shaped fold of the head-fold is seen to have broken through at the apex, and each of the arms thus separated from one another unites with the medullary fold of its respective side. This can be seen in figure 6, which is a dorsal view of part of an embryo a day or two older than the one represented by figures 5 and 5*a*.

"This is so unexpected a method of formation for the anterior part of the medullary folds that I have made use of both figures 4 and 5. They were made from very perfect specimens, and the sections of both of them, and of the specimen from which figure 6 was drawn, proves that the structure is what it is indicated to be in surface appearance. That is, the transverse sections posterior to the V, in the embryos shown in figures 4 and 5, show the medullary groove and the medullary folds; the several sections passing through the apex of the V show neither groove nor folds, but only a median thickening; and in front of the point or apex of the V the successive sections discover a gradually widening groove between the arms, which is also much deeper than the shallow groove found posterior to the V. While I have not seen, and from the nature of the conditions one cannot see, the change actually proceeding from the form of fig. 5 to that of fig. 6, still the explanation given appears to be the only one possible" (5).

A somewhat extended series of transverse sections of an embryo of about this age is represented in figures 6*a–i*.

Figure 6*a* is a section through the head-fold; it passes through the extreme anterior end of the secondary folds (*sf*) that were described, in surface view, above (figs. 5 and 6). The section was not quite at right angles to the long axis of the embryo, so that the fold on the right was cut further toward its anterior end than was the fold on the left. The pushing under of the head causes a forward projection of the secondary folds, so that the fold to the right appears as rounded mass of cells with a small cavity near its center. On the left the plane of the section passes through the posterior limit of the head-fold, and shows the cells of the secondary fold continuous with the dorsal side of the ectoderm (*ec*). As pointed out above by Clarke, the secondary folds are here some distance apart, and gradually approach each other as we proceed toward the posterior. The entoderm (*en*) is here flat and takes no part in the secondary folds.

In figure 6*b*, a short distance back of the one just described, the secondary folds (*sf*) are much larger and are closer together. On the right the section passes through the extreme limit of the head-fold, so that the secondary fold of that side is still a closed circle, with a few scattered cells enclosed. On the left the section is posterior to the head-fold; on this side the secondary fold is seen as a high arch of ectoderm, with a thick mass of entoderm beneath it.

Figure 6*c* represents a section which passes back of the head-fold on both sides. The secondary folds (*sf*) are seen as a pair of ectodermal arches continuous with each other in the middle line of the embryo. The ectoderm of the folds is much thickened and gradually becomes thinner distally. On the right the entoderm shows the same thickening (*en*) that was shown on the left side of the preceding figure. This thickened appearance of the entoderm is due to the fact that the section passes through the anterior limit of a tall fold of that layer, which underlies the similar fold of the ectoderm that has already been described. This secondary fold of the entoderm is seen on the left side of the section. It may be traced through several sections, but soon flattens out posteriorly.

Figure 6*d* is a short distance posterior to the preceding. The secondary folds are here much less pronouncedly arched and the deep groove between them is reduced to a line (*l*). The entoderm (*en*) is no longer markedly arched and is closely adherent, along the median plane, to the ectoderm, where there is seen the thickening (*th*) that has been mentioned by Clarke (see above). Springing from the entoderm on each side of this thickening is a small mass of mesoderm (*mes*).

The section immediately posterior to the one just described is represented in figure 6*e*. The line (*l*) which separated the two secondary folds in the preceding section is no longer present, so that the ectoderm (*ec*) is continuous from side to side, with only a shallow depression (*mg*), which may be considered as the extreme anterior end of the medullary groove. The median thickening (*th*) is cut near its posterior limit and still shows a close fusion of the germ layers. There is no line of demarcation between the gradually flattening secondary folds of the anterior end of the embryo and the just forming medullary folds of the posterior end, so that it is impossible to say whether the thickening of ectoblast in this figure should be called secondary folds or medullary folds. As a matter of fact, the secondary folds become, of course, the anterior ends of the medullary folds. The mesoblast (*mes*) is here of considerable extent, and its entodermal origin is beyond doubt, though not well shown in the figure.

Figure 6*f* is about one-sixth of the length of the embryo posterior to the preceding. The medullary thickening of the ectoderm (*ec*) is still marked and the shallow medullary groove (*mg*) is fairly distinct. The entoderm (*en*) is medially continuous with both mesoderm (*mes*) and notochord (*nt*), though these two tissues are otherwise distinct from each other.

Figure 6*g* is nearly one-third the length of the embryo posterior to the preceding and passes through the posterior third of the embryo. The medullary thickening of the ectoderm (*ec*) is marked, but shows no sign of a medullary groove; in fact, the median line is the most elevated region of the ectoderm. The notochord (*nt*) is larger in cross-section than in the more anterior regions. It is still continuous with the entoderm (*en*) and is fairly closely attached to, though apparently not continuous with, the mesoderm (*mes*) on each side.

Figure 6*h* passes through the blastopore (*blp*). The appearance of the section is almost identical with that of figure 2*f*, already described.

Figure 6*i* is five sections posterior to the preceding and has about the same structure as the corresponding sections in the preceding two stages, where this region of the embryo was injured, and hence not drawn. Continuous with the posterior border of the blastopore (seen in the preceding figure) is the deep furrow, the primitive groove (*pg*). The ectoblast (*ec*) bordering this groove is much thickened and may be called the primitive streak. The lower side of this primitive streak is continuous with the mesoblast (*mes*), while the entoderm (*en*) is here entirely distinct from the mesoderm. It is evident that the mesoderm posterior to the blastopore is proliferated from the lower side of the ectoblast and not from the upper side of the entoblast, as is the case anterior to the blastopore. The primitive groove gradually becomes more and more shallow, as it is followed toward the posterior, until it is no longer discernible; back of this point the primitive streak may be traced for a considerable distance, becoming thinner and thinner until it too disappears, and there remains only the slightly thickened ectoblast underlaid by the thin and irregular layers of mesoblast and entoblast. The primitive streak may be traced for a distance equal to about one-third the distance between the head-fold and the blastopore.

Stage IV

Figures *7a–7h* (Plates VI, VII)

No surface view of this stage was seen by the writer, and hence is not figured. The figures were drawn from one of the series of sections obtained through the courtesy of Prof. S. F. Clarke. This series was marked "3 Urwirbeln," so that the embryo was apparently slightly younger than the youngest stage obtained by myself and represented in figures 8 and 8*a*.

Figure 7*a* represents a section that passed through the head-fold of the amnion (*a*) just in front of the head-fold of the embryo; the amniotic cavity here appears as a large empty space.

Figure 7*b* is several sections posterior to the preceding; it passes through the head-fold of the embryo, but is just back of the head-fold of the amnion. The deep depression of the ectoderm (*ec*) and entoderm (*en*) caused by the head-fold is plainly seen. In this depression lie the ends of the medullary folds, distinct from each other both dorsally and ventrally. Each medullary fold is made up of two parts—a medial, more dense nervous layer (*nl*), and a distal, less dense epidermal layer (*ep*). The section corresponding to this one will be more fully described in connection with the following stage.

Figure 7*c* is some distance posterior to the preceding, though the exact distance could not be determined because of a break in the series at this point. The section passes through the posterior limit of the head-fold. The medullary groove (*mg*) is very deep and comparatively wide; around its sides the germ layers are so closely associated that they may scarcely be distinguished. Ventral to the medullary groove the foregut (*fg*) is seen as a crescentic slit.

Figure 7*d* is about a dozen sections posterior to the one just described and is about one-seventh the length of the embryo from the anterior end. The embryo is much more compressed, in a dorsoventral direction, and the medullary groove (*mg*) is correspondingly more shallow. Where the ectodorm (*ec*) curves over to form the medullary folds it becomes much more compact and somewhat thicker. The notochord (*nt*) is large and distinct, but is still fused with the entoderm (*en*). The mesoderm (*mes*) forms a well-defined layer, entirely distinct from both the notochord and the entoderm. From this region, as we pass caudad, the size of the embryo in cross-section gradually decreases and the medullary groove becomes more shallow.

Figure 7*e* is about one-third of the length of the embryo from the posterior end, and is only a few sections from the caudal end of the medullary groove. The ectoderm (*ec*) is much thinner than in the preceding figure and the medullary groove (*mg*) is much more shallow. The notochord (*nt*) is of about the same diameter as before, but is here quite distinct from the entoderm (*en*) as well as from the mesoderm (*mes*).

Figure 7*f* is seven sections posterior to figure 7*e*. The medullary groove has disappeared and the medullary folds have flattened to form what might be called a medullary plate (at the end of the reference line *ec*), which continues to the anterior border of the blastopore. The notochord (*nt*) is larger in cross-section than in the more anterior regions; it is still distinct from the entoderm.

Figure 7*g* passes through the blastopore and shows essentially the same structure as was described in connection with the corresponding section of stage I (fig. 2*f*).

Figure 7*h* represents the region of the primitive groove (*pg*) and primitive streak (*ps*). The section shows the typical structure for this region—a thick mass of cells is proliferating from the ventral side of the ectoderm (*ec*) and is spreading laterally to form a distinct mesoderm (*mes*). The entoderm (*en*) is entirely distinct from the other layers.

Stage V

Figures 8–8*j* (Plates VII, VIII, IX)

On opening the egg this embryo (figs. 8 and 8*a*) was found to lie on the end of the yolk, near the center of the irregular, lighter area which was mentioned in connection with the description of the egg. The length of the embryo proper is 3 mm. This was the youngest stage found in 1905, and approximates quite closely the condition of the chick embryo after 24 hours' incubation. The dorsal aspect of this embryo, viewed by transmitted light, is shown in figure 8. The medullary folds (*mf*) have bent over until they are in contact, though apparently not fused for a short distance near their anterior ends. As will be described in connection with the sections of this stage, the medullary folds are actually fused for a short distance at this time, though in surface views they appear to be separated from each other. In the Madagascar crocodile (18) the first point of fusion of the medullary folds is in the middle region of the embryo, or perhaps even nearer the posterior than the anterior end of the medullary groove. Throughout the greater part of their length the medullary folds are still widely separated; posteriorly they are merged with the sides of the very distinct primitive streak (*ps*), which seems, owing to its opacity, to extend as a sharp point toward the head. Extending for the greater part of the length of the primitive streak is the primitive groove (*pg*), which, when the embryo is viewed by transmitted light, is a very striking feature at this stage of development and resembles, in a marked way, the same structure in the embryo chick. Clarke (5) figures the blastopore at this stage as a small opening in front of the primitive streak, but does not mention any such condition as above described at any stage of development. Five pairs of somites (*s*) have been formed and may be seen, though but faintly outlined, in both dorsal and ventral views of the embryo; they lie about half way between the extreme ends of the embryo. The head-fold (*h*, fig. 8*a*) shows plainly in a ventral view as a darker, more opaque anterior region, extending for about one-fourth the length of the embryo. The still unfused region of the medullary folds may be seen also in the ventral view at *mg*. The head-fold of the amnion (*a*) forms a very thin, transparent hood over the extreme anterior end of the embryo. The tail-fold of the amnion has not made its appearance, and in fact is not apparent at any stage in the development. This is true also of the Madagascar crocodile. The notochord (*nt*) may be seen in a ventral view as a faint, linear opacity extending along the middle line from the head-fold to the primitive streak.

Two sagittal sections of this stage are shown in figures 8*b* and 8*c*. The embryo from which the sections were made was apparently somewhat crooked, so that it was not possible to get perfect longitudinal sections. For example, in figure 8*b* the plane of the section is almost exactly median in the extreme posterior and middle regions, but is on one side of the middle line elsewhere. This explains the enormous thickening of the ectoblast in the region of the head, where the section passes through one of the medullary folds (*mf*) at its thickest part; and also explains the fact that the ectoblast is thinner in the middle region (*ec*), where the section passes through the medullary groove, than it is farther toward the blastopore, where the section cuts the edge of the medullary folds. The outlines of the middle and extreme posterior regions of the ectoblast are much more irregular and ragged than is shown in the figure. The plane of the section passes through the notochord (*nt*) in the posterior region, but not in the anterior end of the embryo, where a layer of mesoblast (*mes*) is seen. The great size of the blastopore (*blp*) is well shown, as is the beginning of the foregut (*fg*). Comparison of this figure with the more anterior transverse sections and with the dorsal surface view of this stage will make the rather unusual conditions comprehensible.

Figure 8*c* is cut to one side of the median plane, distal to the medullary folds. Being outside of the medullary folds, the ectoderm (*ec*) is thinner and less dense than in figure 8*b*; anteriorly it is pushed down and back as the head-fold, and posteriorly it becomes thin where it forms the dorsal boundary of the primitive streak (*ps*).

The foregut (*fg*), as would be expected, is not so deep as in the median section (8*b*). The most striking feature of the section is the presence of five mesoblastic somites (*s*). Each somite, especially the second, third, and fourth, is made up of a mass of mesoblast whose cells are compactly arranged peripherally, but are scattered in the center, where a small myocoel may be seen.

A series of transverse sections of the embryo shown in figures 8 and 8*a* is represented in figures 8*d*–*j*.

Figure 8*d* is through the anterior end of the embryo; the posterior edge of the amnion is cut only on one side (*a*). The medullary folds (*mf*) are shown as two distinct masses of tissue, separated by a considerable space from each other, both dorsally and ventrally; they are underlaid by the ectoderm of the head-fold, through which the section passes. A mass of yolk (*y*) is shown at one side of the section.

Figure 8*e* represents a section a short distance posterior to the one just described, and passes through the short region where the dorsal edges of the medullary folds have fused with each other. The ventral side of the medullary groove (*mg*) is, as in the preceding section, still unclosed. An epidermal layer of ectoblast (*ep*) is now distinct from the nervous layer (*nl*).

Figure 8*f* is through a region still farther toward the posterior end. Here the medullary groove is *again* open above, and is still open below. A well-marked space is seen between the epidermal (*ep*) and nervous (*nl*) layers of the ectoderm, but no mesoblast is yet to be seen.

Figure 8*g* passes through the middle part of the head-fold, and shows that the medullary folds in this region are fused below, but are widely separated above, where their margins are markedly bent away from the mid-line. Between the epidermal and nervous layers of the ectoderm a considerable mass of mesoderm cells (*mes*) is seen. The curious appearance of the preceding four figures, as well as the first three figures of the next stage, was at first quite puzzling, until a model of the embryo was made from a series of sections. It was then plain, as would have been the case before, except for the unusual depth dorso-ventrally of the head of the embryo, why the medullary canal should at the extreme anterior end be open both dorsally and ventrally, while a few sections caudad it is open only ventrally, and still farther toward the tail it is again open both above and below. These conditions are produced by the bending under of the anterior region of the medullary folds, probably by the formation of the head-fold. It is apparently a process distinct from the ordinary cranial flexure, which occurs later. The fusion of the medullary folds to form a canal begins, as has been already mentioned, near the anterior end, whence it extends both forward and backward. Hence, if the anterior ends of the medullary folds be bent downward and backward, their unfused dorsal edges will come to face ventrally instead of dorsally, and sections through the anterior part of this bent-under region will show the medullary canal open both above and below, as in figure 8*d*, while sections farther caudad pass through the short region where the folds are fused, and we have the appearance represented in figure 8*e*. In figure 8*f* is shown a section passing posterior to the short, fused region of the folds, and we again have the medullary canal open both above and below. Figure 8*g* represents a section through the tip of the bent-under region of the medullary folds, which are here fused below and open above.

Figure 8*h* passes through the posterior part of the head-fold, between the limits of the fold of the ectoderm and that of the entoderm. The medullary groove (*mg*) is here very wide and comparatively shallow; its walls are continued laterally as the gradually thinning ectoderm (*ec*). The enteron (*ent*) is completely enclosed, and forms a large, somewhat compressed, thick-walled cavity. Between the dorsal wall of the enteron and the lower side of the medullary canal lies the notochord (*nt*), a small, cylindrical rod of closely packed cells derived, in this region at least, from the entoderm. In the posterior region of the embryo it is not possible to determine with certainty the origin of the notochord, owing to the close fusion of all three germ layers. Between the wall of the enteron and the lower side of the ectoderm is a considerable mass of mesoderm (*mes*), which here consists of more scattered and angular cells than in the preceding section.

Figure 8*i* shows the appearance of a section through the mesoblastic somites, in one of which a small myocoel (*myc*) is seen. As is seen by the size of the figure, which is drawn under the same magnification as were all the sections of the series, the embryo in this region is much smaller in section than it is toward either end, especially toward the anterior end. The medullary groove (*mg*) is still more shallow than in the more anterior sections, and the ectoderm (*ec*), with which its folds are continuous laterally, is here nearly horizontal. The mesoblast (*mes*) is of a more compact nature than in the preceding section and shows little or no sign of cleavage, although a distinct myocoel may be seen and cleavage is well marked in sections between this one and the preceding.

The notochord (*nt*) has about the same appearance as in the preceding section, but is more distinctly separated from the surrounding cells.

Figure 8*j* is through the posterior end of the embryo; it shows the relation of parts in the region of the primitive streak. Although not visible in surface views, and hence not represented in figure 8, the medullary groove is continued without any line of demarcation into the primitive groove, and the medullary folds into the edges of the primitive streak, so that it is impossible to set any definite boundaries between these structures unless the dorsal opening of the blastopore be taken as the point of division. The medullary groove (*mg*), if it be here so called, is proportionately more shallow than in the preceding figure and is actually much wider. The section passes behind the posterior end of the notochord, so that structure is not seen. Though not so well indicated as might be desired in the figure, the three germ layers are here indistinguishable in the middle line, and in the center of this mass of cells the blastopore (*blp*) or neurenteric canal may be seen as a small vertical slit. As will be more fully described in the following stage, this canal opens dorsally a few sections posterior to the one under discussion and ventrally a few sections farther toward the head.

In all the sections of this stage the ectoderm and entoderm are fairly thick in the region of the embryo proper, but become thinner until reduced to a mere membrane as we pass to more distal regions. Both layers are composed of loosely arranged cells, with scattered nuclei. Where the ectoderm becomes thickened to form the medullary folds, the cells are much more compactly arranged; hence this region stands out in strong contrast to the rest of the ectoderm.

Stage VI

Figures 9*a*–9*m* (Plates IX, X)

The embryo represented by this series of transverse sections is intermediate in development between those represented in surface views by figures 8 and 10. The amnion and head-fold are nearly the same as in figure 8; the medullary folds are intermediate in development, the anterior end not showing so marked an enlargement as shown in figure 10, *v′*. There are six or seven faintly distinguishable somites.

Figure 9*a* represents a section through the anterior part of the head-fold; it shows one unusual condition: the head lies entirely beneath the surface of the yolk. This condition is quite confusing when the section is studied for the first time. The pushing of the head under the yolk is shown at its commencement in figure 11. The process continues until nearly the entire anterior half of the embryo is covered; but when the embryo attains a considerable size it is seen to lie entirely above the yolk, as in the chick. According to Voeltzkow's figures (18), this same condition is found in the crocodile, and Balfour (2) also mentions it in connection with the lizard. The fusion of the medullary folds has made considerable progress, so that the entire anterior end of the canal is enclosed, except in the region where the folds are bent down and back, as in the preceding stage; here the folds are still distinct from each other, leaving the medullary canal open on the ventral side, as shown in figures 9 and 9*b*. In the section under discussion the ectoderm (*ec*) is a very thin membrane on top of a considerable mass of yolk, while no entoderm can be distinguished. The amnion (*a*) completely surrounds the embryo as an irregular membrane of some thickness in which no arrangement into layers can be seen. The epidermal ectoderm (*ep*) is composed of the usual loosely arranged cells, so that it is clearly distinguishable from the compactly arranged cells of the nervous layer (*nl*), from which it is separated by only a line.

In figure 9*b*, which shows a section a short distance posterior to the preceding, the medullary canal (*mc*) is somewhat deeper and is still open ventrally. There is a distinct space between the nervous (*nl*) and epidermal (*ep*) layers of the ectoderm, in which space a few mesoblast cells (*mes*) may be seen. The section is cut just posterior to the edge of the amnion, so that there is now neither amnion nor yolk above the embryo.

Figure 9*c* is about ten sections posterior to figure 9*b*. The section passes through the anterior wall of the bent-under part of the medullary canal (*mc'*), so that the actual canal is shown only on the dorsal side (*mc*), where it is completely closed and begins to assume the shape of the typical embryonic spinal cord. The space between the superficial (*ep*) and nervous (*nl*) layers of the ectoderm is quite extensive and is largely filled by a fairly compact mass of mesoderm (*mes*).

Figure 9*d*, although only five sections posterior to the preceding, shows a marked change in structure. The medullary canal (*mc*) is here of the typical outline for embryos of this age. A large, compact mass of cells (*ent*) appears at first glance to be the same that was noted in the preceding stage at the tip end of the turned-under medullary canal; it is, however, the extreme anterior wall of the enteron, which is in close contact with the above-mentioned tip of the medullary canal. Between this anterior wall of the enteron, of which wall it is really a part, and the medullary canal is the notochord (*nt*). The space surrounding the notochord and enteron is filled with a fairly compact mass of typical, stellate mesoblast cells. The depression of the ectoderm (*ec*) and entoderm (*en*) of the blastoderm caused by the formation of the head-fold is here less marked, and the dorsal side of the embryo in this region is slightly elevated above the level of the blastoderm.

Figure 9*e* represents a section passing through the posterior edge of the head-fold. The epidermal ectoderm is here continuous with the thin layer of superficial ectoderm (*ec*) of the blastoderm, while the entoderm (*en*) of the blastoderm is still continuous beneath the embryo. The thick ectoderm of the embryo is sharply differentiated from the thin layer of ectoderm that extends laterally over the yolk. The pharynx (*ent*) is a large cavity whose wall is thick except at the dorsal side, where it is thin and somewhat depressed, apparently to make room between it and the medullary canal for the notochord (*nt*).

Figure 9*f* is about twenty sections posterior to the preceding section, and passes through the point of separation of the folds of the entoderm (*en*). From this point the entoderm gradually flattens out, leaving the enteron unenclosed. The medullary canal (*mc*) and notochord (*nt*) are about as in the preceding section, but the ectoderm (*ep*) is somewhat thinner and more flattened. The mesoderm (*mes*) on the right side exhibits a distinct cleavage, the resulting body cavity (*bc*) being a large, triangular space.

Figure 9*g*, the twenty-fifth section posterior to that represented in figure 9*f*, shows a marked change in the form of the embryo. While of about the same lateral dimensions, the dorso-ventral diameter of the embryo in this region is less than one-half what it was in the head region. The epidermal ectoderm (*ep*) is now nearly horizontal in position and is not so abruptly separated laterally from the thin lateral sheets of ectoblast. The medullary groove (*mg*) is here a very narrow vertical slit. At this stage the fusion of the medullary folds has taken place over the anterior third of the embryo. For a short distance, represented in about thirty-five sections, the canal is open, as in the figure under discussion; for the next one hundred sections (about one-third the length of the embryo) in the region of the mesoblastic somites the canal is again closed, while throughout the last one-third of its length the canal is widely open dorsally. The enteron is here entirely open ventrally, the entoderm being almost flat and horizontal. The notochord (*nt*) is distinctly outlined and is somewhat flattened in a dorso-ventral direction. The body cavity (*bc*) is well marked, but is separated by a considerable mass of uncleft mesoblast from the notochord and the walls of the medullary groove.

A space of about one hundred sections, or one-third the length of the embryo, intervenes between figures 9*g* and 9*i*. This is the region of the mesoblastic somites, and in this region, as has been above stated, the medullary canal is completely enclosed. It is evident then that the entire anterior two-thirds of the medullary canal is enclosed except for the short region represented in figure 8*g*. Whether or not this short open region between the two longer enclosed regions is a normal condition the material at hand does not show.

Figure 9*h* represents a typical section in the region of the mesoblastic somites just described. It shows the enclosed medullary canal (*mc*), the body cavity (*bc*) on the right, and a mesoblastic somite with its small cavity (*myc*) on the left. The entire section is smaller than the sections anterior or posterior to this region, and seems to be compressed in a dorso-ventral direction, this compression being especially marked in the case of the notochord.

Figure 9*i* is through a region nearly one hundred sections posterior to the preceding, and cuts the embryo, therefore, through the posterior one-fourth of its length. The chief difference between this and the preceding section is in the medullary canal, which is here open and is in the form of a wide groove with an irregular, rounded bottom and vertical sides. The size of the section is considerably greater than in the preceding, the increase being especially noticeable in the notochord (*nt*), which is cut near its posterior end. There is little or no sign of mesoblastic cleavage.

Figure 9*j* is about twenty sections posterior to figure 9*i*. The medullary groove (*mg*) is considerably larger than in the more anterior regions, and its folds are somewhat inclined toward each other, though still wide apart. The notochord and entoderm are fused to form a large, compact mass of tissue close under the ventral wall of the medullary groove. On the ventral side of this mass of cells a groove (*blp*) marks the anterior and ventral opening of the blastopore shown in the next figure. The mesoblast shows no sign of cleavage.

Figure 9*k* shows the medullary groove (*mg*) in about the same position as in the preceding section. The blastopore (*blp*) is here seen as a small cavity in the center of the large mass of cells that was noted in the last figure. The entoderm (*en*) is continuous from side to side, but is not so sharply differentiated from the other germ layers as is represented in the figure.

Figure 9*l* is four sections back of the preceding; the wide, dorsal opening (*blp*) of the blastopore or neurenteric canal into the medullary groove (*mg*) is shown. The blastopore or neurenteric canal, then, is still at this stage a passage that leads entirely through the embryo, the medullary canal being in this region unenclosed above. Ventrally it is seen as a narrow opening through the entoderm; it then passes upward and backward, behind the end of the notochord, as a small but very distinct canal, which may be traced through about ten sections. The enclosed portion of the canal lies, as has been stated (figure 9*k, blp*), in the center of the mass of cells that is fused with or is a part of the floor of the medullary groove.

The above-described neurenteric canal is essentially like that described by Balfour (2) in the *lacertilia*. He does not say, however, and it is not possible to tell from his figures, whether there is a long, gradually diminishing groove posterior to the dorsal opening of the canal, as in the alligator. He says that the medullary folds fuse posteriorly until the medullary canal is enclosed over the opening of the neurenteric canal; also that "the neurenteric canal persists but a very short time after the complete closure of the medullary canal."

In figure 9*m*, for about thirty sections (one-tenth the entire length of the embryo), behind the section represented in the last figure, there is a very gradual change in the embryo, converting the deep groove, *mg* in figure 9*l*, into the shallow slit, *pg*, figure 9*m*.

There is no line of demarcation between the typical medullary groove region of figure 9*l* and the equally typical primitive groove region represented in figure 9*m*. As was noted in the preceding stage, the medullary folds are quite continuous with the folds of the primitive streak, and the medullary groove with the primitive groove; so that, unless we take the dorsal opening of the neurenteric canal as the point of separation, there is no line of division between these structures. The entoderm (*en*) and the lateral regions of the ectoderm (*ec*) and mesoderm (*mes*) in figure 9*m* are about as they were in figure 9*l*, but in the middle line is seen a compact mass of cells forming the primitive streak (*ps*), with the shallow primitive groove (*pg*) on the dorsal side. The cells on each side of the primitive groove and for a short distance below it are compact, as is shown in the figure, but as we pass ventrally and laterally they become looser and more angular to form the lateral sheets of mesoblast (*mes*), very much as is the case in the chick and other forms. For a few sections posterior to the one shown in figure 9*m* the primitive streak may be seen, then it disappears, and only the ectoderm and entoderm remain as thin sheets of tissue above the yolk.

Stage VII

Figures 10 and 10*a* (Plates X, XI)

Although of practically the same size as the preceding, this stage has advanced sufficiently in development to warrant a description.

The medullary folds are apparently still slightly open for the greater part of their length, though they are evidently fused together in the head region, except at the extreme end. Transverse sections, however, of figure 12, in which the medullary folds, from the dorsal aspect, seemed open (*mg*) as in figure 10, have shown that these folds are fused throughout their length.

The first cerebral vesicle (v') is clearly indicated as an enlargement of the anterior end of the nervous system, and a slight enlargement (v'') posterior to the first probably represents the second cerebral vesicle.

There are now eight pairs of somites (*s*).

The head-fold (*h*) now shows in both dorsal and ventral views, appearing in the former, when viewed by transmitted light, as a lighter, circular area on either side of the body, just posterior to the hinder edge of the amnion.

The head-fold of the amnion (*a*) has extended about twice as far backward as it did in the preceding stage.

Owing to the opacity caused by the medullary folds being in contact along the middle line, the notochord is no longer visible in surface views.

The head at this stage begins to push down into the yolk in a strange way that will be described later.

STAGE VIII

FIGURES 11–11*k* (PLATES XI, XII, XIII)

This stage is about one-fourth longer than the preceding. The medullary canal is enclosed throughout its entire length, though it appears in surface view (fig. 11) to be open in the posterior half (*mc*) of the embryo. An enlargement of this apparently open region at the extreme posterior end (*pg*) is probably caused by the remains of the primitive groove or the neurenteric canal, and a slight opacity at the same point may be caused by the primitive streak. The anterior end of the neural tube is bent in a ventral direction (*v'*), as in the preceding stage. The somites (*s*) now number fifteen pairs; they are somewhat irregular in size and shape.

The head-fold is not so striking a feature as in the preceding stage. The head-fold of the amnion (*a*) now covers nearly two-thirds of the embryo. The heart (*ht*) is seen as a dark, rounded object projecting to the right side of the neural canal, just anterior to the first somite. The vitelline blood-vessels are just beginning to form, but are not shown in the figure.

The depression of the anterior region that was noted in the preceding stage has advanced so far that a considerable part of the embryo now projects forward under the blastoderm. In some cases it is almost concealed in a dorsal view; in other cases it may easily be seen through the transparent membranes, especially after clearing.

In opening eggs of this stage one is at first apt to underestimate the size of the embryos, since the anterior part of the embryos cannot be seen until after they are removed from the yolk and are viewed from the ventral side.

The embryo from which the series of transverse sections of this stage was made, while of the same state of development as that shown in figure 11, was more fully covered by the blastoderm than is shown in the surface view in question.

Figure 11*a* passes through the tip of the head. Dorsal to the embryo is the ectoderm and a thick mass of yolk (*y*). The amnion (*a*) is seen as an irregular membrane which entirely surrounds the head. The medullary canal (*mc*) is entirely closed, except at the extreme anterior end, which is bent downward so that the opening is on the ventral side. The nervous (*nl*) and epidermal (*ep*) layers of the ectoderm are in contact throughout, but are clearly distinguishable because of the difference in the compactness of their cells.

In figure 11*b* is represented a section, behind the preceding, which passes through the posterior tip of the turned-under anterior end (*mc'*). Here the medullary canal is closed both above (*mc*) and below (*mc'*). The amnion (*a*) has about the same appearance as in the more anterior section, but there is here a considerable space, filled with mesoblast (*mes*), between the nervous (*nl*) and epidermal (*ep*) layers of ectoderm.

Figure 11*c* is twenty sections, about one-tenth the length of the embryo, posterior to the one last described. The large mass of overhanging yolk (*y*) is still present, as is also the amnion (*a*), though the latter no longer passes entirely around the embryo; only the true amnion could be made out. The thickened walls of the medullary canal have reduced that cavity to a narrow, Y-shaped slit (*mc*). The notochord (*nt*) is very slender in this region, compared to its diameter farther toward the posterior end. The enteron (*ent*) is a large cavity, whose wall is made up of loosely arranged cells except around a median, ventral depression where the cells are more compact. This depression may be traced through ten or fifteen sections and may represent the beginning of the thyroid gland, though this point was not worked out with certainty. Surrounding the notochord and enteron is a loose mass of typical, stellate mesoblast cells (*mes*), which are cleft on either side to form the anterior limit of the body cavity (*bc*). Between the body cavity below and the enteron above, on each side, is a small blood-vessel (*bv*) which when followed caudad is found to open ventrally and medially into the anterior end of the heart.

Figure 11*d* is about a dozen sections posterior to the preceding. The appearance of the overhanging yolk (*y*) of the amnion (*a*) and of the notochord (*nt*) is about as in the more anterior section. The medullary canal (*mc*) is a straight, vertical slit, and the depression in the floor of the pharynx (*ent*) is much more shallow. The body cavity (*bc*) is much larger and extends across the mid-ventral line beneath the heart (*ht*), which is cut through its middle region. The heart may be traced through about twenty sections (one-tenth the length of the embryo); its mesoblastic wall (*mes'*) is thin and irregular, and is lined by a distinct endothelium (*en'*) whose exact origin has not yet been worked out.

Figure 11*e* is just back of the heart, and shows in its place the two vitelline veins (*vv*). The depression in the floor of the enteron (*ent*) is entirely distinct from the one that has been mentioned above, and is simply the posterior limit of the head-fold of the entoderm; the fifth section posterior to this shows where this depression opens ventrally to the yolk sac. The other structures shown in the figure are not markedly different from what was seen in figure 11*d*.

Figure 11*f* is about one-tenth the length of the embryo posterior to figure 11*c*. The chief differences here noticed are in the enteric and cœlomic cavities. The former is no longer enclosed, a dorsal fold in the entoderm being all that remains of the cavity that was seen in the more anterior figures, while the latter is here reduced to a narrow cleft between the somatic and splanchnic mesoblast. A thickening of the mesoblast on either side of the notochord, especially on the left, represents a mesoblastic somite. The medullary canal (*mc*) is more open than in the more anterior sections.

For about one-third of the length of the embryo posterior to figure 11*f* there is a gradual flattening, in a dorso-ventral direction, with loss of the amnion, until the condition represented in figure 11*g* is reached. The most striking feature of this region is the great thickness of the ectoderm (*ec*), which is still made up of scattered, irregular cells. In the middle line, directly over the medullary canal (here a nearly cylindrical tube), is a sort of break in the ectoderm, as though there had not been a complete fusion of the epidermal layer when the nervous layer came together on the closure of the medullary groove. This break in the ectoderm may be followed back to the region of the primitive streak, and will be mentioned again. As has been noted, the medullary canal (*mc*) is nearly circular in cross-section, and is closely underlaid by the notochord (*nt*), which is several times the diameter that it was in more anterior sections. The mesoblast (*mes*) is a comparatively thin layer, intermediate in thickness between the ectoderm and entoderm. It shows laterally a slight separation to form the body cavity.

Figure 11*h* is about ten sections posterior to figure 11*g*, and differs from it chiefly in that the notochord (*nt*) is continuous with the lower side of the medullary canal (*mc*), though still distinct from the underlying entoderm (*en*).

Figure 11*i*, four sections farther from the head, shows the same greatly thickened ectoderm (*ec*) with the same break (*ec′*) in the middle line. The section is posterior to the notochord and passes through the anterior edge of the blastopore or, as it may now perhaps better be called, the neurenteric canal. The cells of the medullary wall are continuous with those of the entoderm. The mesoderm (*mes*) is still distinct from the other germ layers.

Figure 11*j* is the next section posterior to the one just described and differs from it only in showing the actual opening of the neurenteric canal (*nc*) into the medullary canal (*mc*). The medullary canal extends, with gradually diminishing caliber, for about fifteen sections posterior to the point at which the neurenteric canal empties into it. The mesoblast (*mes*) is so closely attached to the lower wall of the neurenteric canal that it seems to be actually continuous with it.

For a considerable distance posterior to the end of the medullary canal we find the structure similar to that shown in figure 11*k*, which is about the twentieth section posterior to figure 11*j*. The break (*ec′*) in the ectoderm is here seen as a compact group of cells which at first glance seem to be continuous with a rounded mass of cells below (*ps*). Examination under greater magnification, however, shows that the two groups of cells are distinct. As the sections are followed back of this region, the upper mass of cells (*ec′*) gradually disappears, and after its disappearance the lower mass (*ps*), which is already continuous with the mesoderm (*mes*) on either side, becomes continuous with the under side of the ectoderm. The mass of cells (*ps*) is apparently the primitive streak, though it is distinct from the ectoderm for a considerable distance posterior to the neurenteric canal. Just what may be the meaning of the thickened ridge of ectoderm (*ec′*) it is difficult to determine.

Stage IX

Figures 12–12*g* (Plates XIII, XIV)

The entire length of the embryo proper is 6.5 mm. from the extreme posterior end to the region of the midbrain (v^2), which now, on account of the cranial flexure, forms the most anterior part of the body. Besides being slightly longer than the preceding stage, the embryo has increased in thickness, especially in the anterior region, where the enlargement of the cerebral cavity is considerable.

Body torsion has begun (fig. 12), so that the anterior third of the embryo now lies on its right side, while the rest of the body is still dorsal side up. The direction of body torsion does not seem to be as definite as it is in the chick, some alligator embryos turning to the right side, others to the left. Clarke has illustrated this fact in his alligator figures. He says (5) that embryos lie "more frequently on the left, but often on the right side."

The head is distinctly retort-shaped, and at the side of the forebrain (v') a small crescentic thickening is the optic vesicle (*e*). The auditory vesicle, though of considerable size, does not show in this surface view. The head-fold (*h*) extends for about one-third the length of the entire embryo, though its exact limit is difficult to determine in surface view. There is no sign of a tail-fold.

About seventeen pairs of somites are present.

The amnion extends over the anterior two-thirds of the embryo.

The above-mentioned increase in the diameter of this embryo over that of the preceding is evident when the first two transverse sections of this stage are compared with the corresponding sections of the earlier stage; in the middle and posterior regions there is not very much difference in size.

Figure 12*a* passes through the region of the forebrain. This end of the embryo lies on its side, as was noted above and as may be recognized from the relative positions of the head and the overlying yolk (*y*). The great size of this and the following figure is due partly to the increase in size mentioned above and partly to the fact that the sections pass through the region of cranial flexure. The present figure (12*a*) represents the brain cavity as large and dumb-bell-shaped, with comparatively thick walls of compactly arranged cells. The ventral end of this cavity (*fb*) is cut anterior to the region of the optic vesicles, while the dorsal end (*mb*) may perhaps be called the midbrain. In the sections that follow this one the two cavities are distinct from each other. The medullary canal, as was stated above, is now completely enclosed, except for the ventral opening of the neurenteric canal, to be presently noticed. Surrounding the brain is a considerable mass of mesoblast (*mes*). It is composed of the typical stellate cells. The ectoderm (*ec*) is made up of the same irregularly and loosely arranged cells that have been seen in earlier stages; it is of unequal thickness in different regions, the thicker parts being at the sides. The amnion (*a*) has the usual appearance, and in this region of course completely surrounds the embryo.

Figure 12*b* is ten sections posterior to the section just described. The width of the embryo is greater in this region, but the dorso-ventral diameter is about the same as in the more anterior section.

The overlying yolk and blastoderm are not shown in any figure of the series except the first. In this figure the forebrain (*fb*) and midbrain (*mb*) are widely separated instead of being connected, as in the preceding figure, where the section passed through the actual bend of the cranial flexure. The anterior and ventral part of the cranial cavity, the forebrain (*fb*), is nearly circular in outline. It exhibits on one side a well-marked optic vesicle (*ov*), which is sufficiently advanced in development to show a rudimentary optic stalk. The outer wall of the optic vesicle is in close contact with the superficial ectoderm, which shows as yet no sign of the formation of a lens vesicle. The plane of the section being probably not quite at right angles to the long axis of the embryo, the optic vesicle of one side only was cut. The wall of this part of the forebrain is of about the same thickness and appearance as in the preceding stage. The other cerebral cavity (*mb*) of this section is probably the hinder part of the midbrain, though it may be the anterior part of the hindbrain; there is no sharp line of demarcation between these regions of the brain. This cavity (*mb*) is much smaller in section than the forebrain; its walls are of about the same thickness.

Ventral to the midbrain is the anterior end of the notochord (*nt*), surrounded by the mesoblast. At various places throughout the mesoblast irregular open spaces may be seen; these are blood-vessels. The ectoderm (*ec*) and amnion (*a*) have about the same appearance as in the preceding figure, though the former seems somewhat thinner.

Figure 12*c* is just back of the bent-under forebrain represented in the preceding figure and in front of the main body of the heart. The plane of the section not being at right angles to the long axis of the body (as was mentioned above), the figure is not bilaterally symmetrical. The neural canal, since the section passes through the auditory vesicles, may here be called the hindbrain (*hb*). It has an almond-shaped cavity, surrounded by a wall of medium thickness. In close contact with the wall of the hindbrain, on each side, is the inner side of the auditory vesicle (*o*), which is seen as a deep, wide-mouthed pit in the superficial ectoderm. On the right side of the section the auditory pit is cut through its middle region; it is simply a thickened and condensed area of the ectoderm which has been invaginated in the usual way. Directly beneath the hindbrain is the notochord (*nt*), on each side of which, in the mesoblast, is the dorsal aorta (*ao*), or rather the continuation of the aorta into the head. Beneath these structures and extending from one side of the section to the other is the pharynx (*ph*); its lining wall is fused on each side with the ectoderm, but there is no actual opening to the exterior. These points of contact (*g*) between entoderm and ectoderm are of course the gill clefts; they are not yet visible from the outside. The roof of the pharynx is flat and comparatively thin, while the floor is thickened and depressed to form a deep, wide pit, traceable through six or eight sections. This pit may be the thyroid gland already noticed in the preceding stage. Below the main cavity of the pharynx and close to each side of the thyroid rudiment just mentioned is a large blood-vessel (*tr*). These two vessels when traced posteriorly are found to be continuous with the anterior end of the heart, and hence may be called the truncus. They were noticed in figure 11*c*, *bv*. The ectoderm surrounding the lower side of the embryo was so thin and indistinct that it could scarcely be distinguished from the mesoderm of that region. The amnion (*a*) is still a continuous envelope entirely surrounding the embryo.

Figure 12*d*, about twenty sections posterior to figure 12*c*, is in the posterior heart region. The spinal cord (*sc*), as might be expected, is smaller than in the more anterior region, but is otherwise not markedly different from what was there seen. The notochord (*nt*) also has the same appearance as before. The enteron (*ent*) shows of course in this region no gill clefts; it is a small, irregular cavity with thicker walls than in the figure just described. The ventro-lateral depression is entirely distinct from the depression that was called the thyroid rudiment in the preceding figure. Dorsal to the enteron are the two dorsal aortæ (*ao*), now smaller and more ventral to the notochord than in the preceding figure. Ventral to the enteron is the large heart (*ht*), projecting below the body cavity, which is no longer enclosed. The mesodermic wall (*mes′*) of the heart is still comparatively thin and is separated by a considerable space from the membranous endocardium (*en′*). The extent and shape of the heart are shown in the surface view of this stage. On the right side of the section the body cavity extends to a point nearly opposite the middle of the spinal cord, considerably dorsal to the notochord, while on the left side the dorsal limit of the body cavity is scarcely level with the lower side of the notochord. Between the dorsal end of the body cavity and the side of the spinal cord, on the left, is a dense mass of mesoblast (*s*), one of the mesoblastic somites. A few sections either anterior or posterior to the one under discussion will show the condition of the two sides reversed—that is, the body cavity will extend to the greater distance on the left and will be interrupted by a mesoblastic somite on the right. It is evident, then, that the upper angle of the body cavity is extended dorsally as a series of narrow pouches between the somites. The mesoblast that lines the body cavity, the splanchnopleure (*sm*), and somatopleure (*so*) is somewhat denser than the general mass of mesoblast, so that these layers are quite distinct, the former (*sm*) extending around the enteron (*ent*) and heart (*ht*), and the latter (*so*) being carried dorsalward as the mesoblastic part of the amnion (*a*). The amnion may be traced through about 130 of the 200 sections into which this embryo was cut.

Figure 12*e* is nearly one-fourth the length of the embryo posterior to figure 12*d*; it is approximately in the middle region. The diameter of the embryo has been gradually decreasing until now it is very much less than in the head region. The section being behind the head-fold the entoderm (*en*) is nearly flat and the enteron is quite unenclosed. The canal of the spinal cord (*sc*) is smaller in proportion to the thickness of its walls, and the notochord (*nt*) is somewhat larger than in the preceding sections. In proportion to its extent, the ectoderm is very thick. Under the notochord the dorsal aortæ (*ao*) are seen as two large, round openings in the mesoblast. On the left side the section passes through the center of a somite and shows a small, round myocoel (*myc*). The mesoblastic layer of the amnion (*so*) is distinct throughout from the ectoblastic layer (*a*).

The most important structures to be here noted are the first rudiments of the Wolffian ducts (*wd*). They are seen in the present section as lateral ridges of mesoblast projecting outward and upward toward the ectoblast, which suddenly becomes thin as it passes over them. These ridges or cords of mesoblast are as yet quite solid. They arise suddenly at about the eightieth section of the series of two hundred and may be traced through about forty sections, or one-fifth of the length of the embryo. Their exact length is difficult to determine because, while their anterior ends are blunt and sharply defined, they taper so gradually posteriorly that it is hard to tell just where they end. They apparently originate anteriorly and gradually extend toward the tail. In a slightly younger embryo the rudimentary Wolffian duct could be seen as a still smaller rod of cells extending posteriorly for a few sections, from the seventy-fifth section of a series of about two hundred. In the particular series under discussion the left rudimentary Wolffian duct was about one-fifth longer than the right one.

Figure 12*f* is just posterior to the head-fold of the amnion, passing, in fact, on the left side through the extreme edge of its lateral fold, which is shown as an upward bend in the ectoblast and somatopleure.

The ectoblast (*ec*) shows the same remarkable thickening that was noted in the corresponding region of the preceding stage. The spinal cord (*sc*), notochord (*nt*), aortæ (*ao*), and entoderm (*en*) need no special mention. The mesoderm seems to be separated by unusually wide spaces from both ectoderm and entoderm, and is made up of rather closely packed cells except around the aortæ, where there seems scarcely enough tissue to hold these vessels in place. The body cavity (*bc*) is large, and a small myocoel (*myc*) is seen on the left.

Figure 12*g* is through the neurenteric canal (*nc*), a distinct opening through the floor of the spinal canal. The section is of course just back of the posterior end of the notochord. The entoderm (*en*) along the margin of the neurenteric canal is naturally continuous with the wall of the spinal cord (*sc*). The ectoderm (*ec*) is thicker than ever, except in the median plane, where it passes over the spinal cord. The mesoblast is more abundant than in the preceding figure, and shows on the left what appears to be a distinct myocoel (*myc*), though in surface view the mesoblastic somites do not extend this far toward the tail.

Stage X

Figures 13–13g (Plates XIV, XV, XVI)

This embryo (fig. 13) is about 5 mm. in length, and hence is slightly smaller than the preceding stage, though somewhat more advanced in development. The medullary canal is still *apparently* unclosed for a short distance at the extreme posterior end; this appearance is probably due to the neurenteric canal (*nc*) and to the thinness of the roof of the medullary canal rather than to any lack of fusion of the medullary folds. The optic vesicle is more distinct than in the preceding stage; a somewhat similar, though smaller, opacity (*o*) marks the position of the ear. There are now about twenty pairs of somites, though it is difficult to determine their exact number on account of the torsion of the body. The amnion is at about the same stage of development as in stage IX. The heart (*ht*) is a large double mass, whose outlines may be dimly seen when the embryo is viewed by transmitted light. The vitelline vessels (*vv*) are still but faintly outlined in the vascular area; the veins and arteries cannot yet be distinguished from each other. The gill clefts, though not visible externally in the embryo drawn, may be seen in sections of this stage as evaginations of the wall of the pharynx.

The transverse sections of this stage are slightly more advanced in development than was the embryo that has just been described in surface view. Only those sections have been figured which show a decided advance in the development of some special structures over their condition in the preceding stage. The sections of the preceding stages were drawn under a magnification of eighty-seven diameters; those of this and the following stage were drawn under a magnification of only forty-one diameters. All of the figures have been reduced one-half in reproduction.

Figure 13*a* is the most anterior section of this series to be described. On account of the cranial flexure, which causes the long axis of the forebrain to lie at right angles to that of the spinal cord, this section cuts the head region longitudinally. The ectoderm (*ec*) is of varying thickness, the thickest areas being on each side of the forebrain; it is more compact than in the earlier stages, and, owing to the low magnification under which it is drawn, it is represented here by a single heavy line. Under this magnification only the nuclei of the mesoderm cells (*mes*) can be seen, so that this tissue is best represented by dots, more closely set in some places than in others. The forebrain is an elongated cavity (*fb*) with thick, dense walls. Attached to each side of the forebrain is an optic vesicle (*ov*), which is considerably larger than in the preceding stage. The connection between the cavity of the forebrain and that of the optic vesicle is not seen in this section; it is a wide passage that may be seen in several sections posterior to the one under discussion. The beginning of the invagination of the optic vesicle to form the optic cup may be seen on both sides, but more plainly on the right. On the right side also is noticed a marked thickening of the ectoderm, which is invaginated to form a small pit, the lens vesicle (*lv*); on the left side the section is just behind the lens vesicle. Above the optic stalk on each side, in the angle between the optic vesicle and the side of the forebrain, is a small blood-vessel (*bv*). Several other blood-vessels may be seen at various places in the mesoblast, four of them near the pharynx being especially noticeable. The hindbrain (*hb*) is wider than, but not so deep as, the forebrain; its walls are very thick laterally, but are thin on the dorsal and ventral sides. The dorsal wall is reduced to a mere membrane, which, with the overlying ectoderm, has been pushed into the brain cavity, as is generally the case with such embryos. Close to the ventral wall of the hindbrain the notochord (*nt*) is seen. The character of the notochord has already begun to change; the cells are becoming rounded and vacuolated, with but few visible nuclei except around the periphery of the notochord. Near the center of the section, close to the ventral end of the forebrain, is the pharynx (*ph*), cut near its anterior limit; it is here a small, irregularly rectangular cavity with a comparatively thin wall. On the left side of the pharynx the first gill cleft (*g*) is indicated as a narrow diverticulum reaching toward the ectoderm. A few sections posterior to this one the first gill cleft is widely open to the exterior. As has been said, in the surface view of this stage above described none of the gill clefts showed; so that in this respect at least the sectioned embryo was more nearly of the state of development of the embryo represented in figure 14, to be described later.

Figure 13*b*, about forty sections posterior to figure 13*a*, passes through the hindbrain in the region of the ears. Being back of the region affected by cranial flexure, this section is of course of much less area than the preceding. The ectoderm shows no unusual features; it is of uniform thickness except where it becomes continuous with the entoderm around the mandibular folds (*md*); there it is somewhat thickened. The most striking feature of the section is the presence of two large auditory vesicles (*o*). The section being not quite at right angles to this part of the embryo, the vesicles are not cut in exactly the same plane; the one on the left is cut through its opening to the exterior, while the one on the right appears as a completely enclosed cavity. In a section a short distance posterior to this one the appearance of the vesicles would be the reverse of what it is here. As may be seen in the figure, the vesicles are large, thick-walled cavities lying close to the lateral walls of the hindbrain. The hindbrain itself has the usual triangular cross-section, with thick lateral walls and a thin, wrinkled dorsal wall. Close to the ventral side of the hindbrain lies the notochord (*nt*), on each side of which, in the angle between the brain and the auditory vesicles, is a small blood-vessel (*bv*). Ventral to these structures and close to the dorsal wall of the pharynx (*ph*) are the two large dorsal aortæ (*ao*). The ventral side of the section passes through the open anterior end of the pharynx (*ph*). On the left is seen the widely open hyomandibular cleft (*g′*), between the main body of the section and the mandibular arch (*md*). On the right side the plane of the section was such that the hyomandibular cleft was not cut through its external opening. In each mandibular fold a large aortic arch (*ar*) is seen, and also a slight condensation of mesoblast, the latter probably being the forerunner of cartilage.

Figure 13*c* passes through the anterior part of the heart about seventy-five sections posterior to figure 13*b*. The embryo in this region is narrow but deep (dorso-ventrally), the depth being largely due to the size of the heart. The ectoderm (*ec*) is considerably thickened on each side of the pharynx (*ph*); this thickened area may be traced for some distance both anteriorly and posteriorly from this point; its significance could not be determined. The spinal cord (*sc*) and notochord (*nt*) need no special description; the former is smaller and the latter larger than in the more anterior sections. The two large blood-vessels (*ac*) near the spinal cord and notochord are probably the anterior cardinal veins. The aortæ are cut by the plane of this section just anterior to their point of fusion into a single vessel. A few blood corpuscles are seen in the right aorta. The enteron (*ent*), cut posterior to the region of the gill clefts, is a large elliptical cavity, with its long axis in a transverse position. Its entodermal wall is comparatively thin and smooth, with the cell nuclei arranged chiefly on the outer side, *i. e.*, away from the cavity of the enteron. The body cavity (*bc*) is here still unenclosed, and its walls, the somatic stalk, are cut off close to the body of the embryo. The heart (*ht*), the most conspicuous feature of this section, is nearly as large in cross-section as all the rest of the embryo. As seen in such a section it is entirely detached from the body of the embryo, and in this particular case has about the shape of the human stomach. The mesoblastic portion of its wall (*mes'*) is of very irregular thickness; it forms a dense layer entirely around the outside, except for the pointed dorsal region, and is especially thick along the ventral margin, where it is thrown into well-marked folds, the heavy muscle columns. Lining the cavity of the heart is the membranous endothelium (*en'*), and between this and the dense outer wall just described is a loose reticular tissue with but few nuclei.

As the series is followed toward the tail the sections diminish in size until, at a point about one-third the embryo length from the posterior end, they are of scarcely one-fourth the area of the sections through the region of the hindbrain.

Figure 13*d* is about one hundred and twenty-five sections posterior to figure 13*c*. Although not so small as the sections that follow it, this section is considerably smaller in area than the one last described. The amnion (*a*), which was not represented in the last three figures, is very evident here. The spinal cord (*sc*) is considerably smaller here than in the preceding figure, while the notochord (*nt*) is not only relatively but actually larger than in the more anterior regions. Beneath the notochord is the aorta (*ao*), now a single large vessel. The mesoblast on each side of the body is here differentiated into a distinct muscle plate (*mp*). These muscle plates have very much the appearance of the thickened ectoderm seen in the younger stages of development. At about its middle region (*i. e.*, at the end of the reference line *ec*) each muscle plate is separated from the overlying ectoderm by an empty space; this space is still more marked in some other series. Ventral to the aorta, and supported by a well marked though still thick mesentery (*ms*), is the intestine. It is a small, nearly cylindrical tube with thick walls; the splanchnic mesoblast which surrounds it is more dense than the general mass of mesoblast; it was somewhat torn in the section and is so represented in the figure. The urinary organs have made considerable progress since the last stage. In the figure under discussion they are seen as a group of tubules on either side of the aorta. The tubule most distant from the middle line, on each side, is the Wolffian duct (*wd*). It extends through the posterior two-thirds of the embryo and varies in diameter at different points; it is usually lined with a single layer of cubical cells which contain large nuclei. The Wolffian bodies (*wt*) are a mass of slightly convoluted tubules that may be traced throughout the greater part of the region through which the Wolffian duct extends. These tubules also vary somewhat in diameter, but they are usually of greater caliber than the duct. No actual nephrostomes are to be seen, though the occasional fusion of a tubule with the peritoneal epithelium, as is seen on the left side of the present figure, may represent such an opening. A detailed description of these structures may be given in a subsequent paper.

Figure 13*e* is about one hundred and forty sections posterior to the section just described. The embryo is here very slender, so that the contrast between this and the first figure (13*a*) of this stage is remarkable. Except in size, this section does not differ greatly from the preceding. The spinal cord, notochord, etc., are smaller than before, but are of about the same relative size. The mesentery (*ms*) in the section drawn was torn across, so that the intestine is not represented. Medial to the Wolffian duct is a tubule (*wt*), which seems to be the same as those which were called Wolffian tubules in the preceding stage, but which may be the beginning of the ureter.

Figure 13*f*, about two hundred and fifty sections posterior to the last, passes through the extreme posterior end of the embryo. The section is nearly circular in outline and is somewhat larger than the preceding. The amnion (*a*) completely encircles the embryo. The ectoderm (*ec*) is of fairly even thickness, and the mesoblast which it encloses is of the usual character. The spinal cord (*sc*) is nearly circular in outline, as is its central canal. The digestive tract (*ent*) is larger in section than it was in more anterior regions; it is nearly circular in cross-section and its walls are made up of several layers of cells, so that it resembles to a considerable degree the spinal cord of the same region. In the narrow space between the spinal cord and the hindgut is seen the notochord (*nt*), somewhat flattened and relatively and actually smaller than in the preceding figure. A few scattered blood-vessels may be seen in the mesoblast at various places.

A sagittal section of an embryo of this stage, drawn under the same magnification as were the transverse sections, is shown in figure 13*g*. The embryo being bent laterally could not be cut by any one plane throughout its entire length, so that only the anterior end is represented in the figure. The amnion (*a*) may be clearly seen except at certain places where it is closely adherent to the superficial ectoderm. Under the low magnification used the superficial ectoderm cannot be distinguished from the ectoderm of the nervous system. The plane of the section being in the anterior end almost exactly median, this part of the central nervous system is seen as the usual retort-shaped cavity, while in the region back of the brain, where the neural canal is narrow, the section passes through the wall of the spinal cord (*sc*) and does not show the neural canal at all. The wall of the forebrain (*fb*) is quite thick, especially at the extreme anterior end; the wall of the midbrain (*mb*), where the marked cranial flexure takes place, is somewhat thinner, and it gradually becomes still thinner as it is followed posteriorly over the hindbrain (*hb*). Between the floors of the fore- and hindbrains, in the acute angle caused by the cranial flexure, is the anterior end of the notochord (*nt*), the only part of that structure that lies in the plane of the section. Ventral and posterior to the notochord is a large cavity, the pharynx (*ph*), whose entoblastic lining can scarcely be distinguished under this magnification from the surrounding tissues. The stomodeal opening being as yet unformed, the pharynx is closed anteriorly; posteriorly also, owing to the plane of the section, the pharynx appears to be closed, since its connection with the yolk stalk is not shown. In the floor of the pharynx, almost under the reference line *ph,* a slight depression marks the position of the first gill cleft. In the mesoblast ventral to the pharynx and near the gill cleft just mentioned, a couple of irregular openings represent the anterior end of the *bulbus arteriosus,* posterior and ventral to which is the heart (*ht*), a large, irregular cavity. The dorsal aorta (*ao*) may be seen as an enlongated opening in the mesoblast, extending in this section from the middle region of the pharynx to the posterior end of the figure where it is somewhat torn. Two of the eighteen or twenty pairs of mesoblastic somites possessed by this embryo are shown at the posterior end of the figure (*s*), where the plane of the section was far enough from the median line to cut them.

Stage XI

Figure 14 (Plate XVI)

Only the anterior region of this embryo is shown in the figure, which is a ventro-lateral view. While there is some change in the general shape and in parts of the head, the reason for figuring this stage is to show the first gill cleft (*g′*), which lies at an acute angle to the long axis of the neck behind the eye (*e*). The cleft is narrow but sharp and distinct in outline; it shows, neither in this nor in the following stages, the branched, Y-shaped outline mentioned by Clarke.

STAGE XII

FIGURES 15–15*f* (PLATES XVI, XVII)

In this stage, also, only the anterior region of the embryo is figured in surface view. The shape of the head is about the same as in the preceding stage, but it is drawn in exact profile. Three gill clefts ($g^{1\text{-}3}$) are now present, and are wide and distinct. The first cleft, as in the preceding stage, lies at an acute angle to the long axis of the pharynx and nearly at right angles to the second cleft. The third cleft sends a wide branch (g^4) toward the posterior, as has been described by Clarke, from which, or in connection with which according to Clarke, the fourth cleft will develop. All three clefts may be distinctly seen to open entirely through the pharyngeal wall. The outlines of the visceral folds, especially of the mandibular, begin to be apparent. The nasal pit (*n*) now shows as a round depression in front of the more definitely outlined eye (*e*). The auditory vesicle (*o*) is so deep beneath the surface that it may be seen only by transmitted light.

Figures 15*a-c* represent transverse sections of an embryo of about this general state of development, except that the gill clefts are not so definitely open as in the surface view.

Figure 15*a*, the most anterior section of the series, passes through the forebrain (*fb*) in the region of the eyes, and through the hindbrain (*hb*) anterior to the auditory vesicles. The forebrain is here a large cavity with a dense wall of a comparatively even thickness. Owing probably to the section not being exactly in the transverse plane, the eyes are cut in different regions, that on the left (*ov*) being cut through its stalk, while that on the right (*oc*) is cut near its middle region and hence does not show any connection with the forebrain. The almost complete obliteration of the cavity of the optic vesicle to form the optic cup by the invagination of the outer wall of the vesicle is shown on the right side of the section (*oc*). The lens vesicle (*lv*) is completely cut off from the superficial ectoderm (*ec*), which is comparatively thin. The hindbrain (*hb*) has the usual shape for that structure. Its ventral wall is dense and thick, while its roof is reduced to the usual thin, wrinkled membrane. Close to the floor of the hindbrain lies the notochord (*nt*), which is large and is distinctly vacuolated. To the right of the hindbrain a large mass of darkly stained cells (*cn*) is one of the cranial nerves, which is connected with the hindbrain a few sections anterior to the one under consideration. The pharynx (*ph*), which is cut near its extreme anterior end, is represented by three irregular cavities near the base of the forebrain. Scattered throughout the mesoblast, which makes up the greater part of the section, are numerous blood-vessels (*bv*).

Figure 15*b* is twenty sections posterior to figure 15*a* and passes through the tip of the bent-under forebrain (*fb*). On the left the section is anterior to the optic vesicle and barely touches the side of the optic stalk, which is seen as a small lump on the ventro-lateral wall of the brain. On the right the connection of the optic vesicle (*ov*) with the forebrain is shown. Dorsal to the optic vesicle just mentioned is a markedly thickened and slightly invaginated region of the ectoderm (*n*) ; this is the nasal pit; on the left side of the figure the thickening is shown, but the section did not pass through the invagination. The hindbrain (*hb*) is somewhat narrower than in the preceding figure, but is otherwise about the same; the origin of a cranial nerve is seen on its left side (*cn*). The notochord (*nt*) has the same appearance as in the preceding section. A number of blood-vessels may be seen, the pair lying nearest the notochord being the aortæ (*ao*), while the two other pairs, on either side of the fore- and hindbrains, are the anterior cardinals (*ac*). The first aortic arches are shown at *ar*. On the left the section passes through the exterior opening of the first gill cleft (*g′*), so that the mandibular fold (*md*) on that side is a distinct circular structure, made of a dense mass of mesoderm surrounded by a rather thick ectoderm. The mesoderm of this fold is especially dense near the center, probably the beginning of the visceral bar. Near the center is also seen the aortic arch that has already been mentioned. On the right the section does not pass through the external opening of the first gill cleft (*g′*) so that the tissue of the mandibular fold is continuous with the rest of the head. It is of course the slight obliquity of the section that causes the pharynx (*ph*) to be completely enclosed on the right, while on the left it is open to the exterior both through the gill cleft and between the mandibular fold and the tip of the head. The superficial ectoderm shown here as a heavy black line varies considerably in thickness, being thickest in the region of the nasal pit already mentioned and thinnest over the roof of the hindbrain. The amnion (*a*) in this, as in the other sections of the series, has the appearance of a thin, very irregular line.

Figure 15*c* is posterior to the region affected by cranial flexure and so shows only one region of the embryo, that of the hindbrain (*hb*), which is here of essentially the same structure as above described. On each side of the hindbrain is a large auditory vesicle (*o*); that on the left is cut through its center and shows the beginning of differentiation, its lower end being thick-walled and rounded, while its upper end is more pointed and has a thin, somewhat wrinkled wall. The notochord (*nt*) is slightly larger than in the more anterior sections. Numerous blood-vessels (*bv, ar*) are seen in the mesoblast. The pharynx (*ph*) is here open ventrally and also through the gill cleft of the left side; on the right side the plane of the section did not pass through the external opening of the cleft. The mesoblast of the visceral folds is much more dense than that of the dorsal region of the section.

Figure 15*d*, as is evident, is a section through the region of the heart, which appears as three irregular cavities (*ht*) with fairly thick mesoblastic walls (*mes'*) lined with endothelium (*en'*). The body wall, though consisting of but little besides the ectoderm (*ec*), completely surrounds the heart, and the pericardial or body cavity thus formed extends dorsally as a narrow space on either side of the foregut, giving the appearance of a rudimentary mesentery, though no especial development of such a structure would naturally be expected in this region of the embryo. The foregut (*ent*) is a moderately large cavity lined with a very distinct entoderm of even thickness. Dorsal to the foregut are three large blood-vessels, a median, and now single, dorsal aorta (*ao*), and a pair of cardinal veins (*cv*). The notochord (*nt*) is small and is flattened against the ventral side of the spinal cord (*sc*), which latter structure needs no special mention. The muscle plates (*mp*) are considerably elongated, so that they now extend ventrally to a point slightly below the upper angles of the body cavity.

Figure 15*e* is through the middle region of the embryo, and, owing to the curvature of the body, is not an exact dorso-ventral section; this accounts, in part at least, for the unusual diameter in a dorso-ventral direction of the aorta (*ao*), which is very large in proportion to the other structures. The posterior cardinal vein is shown on the left, but not on the right. The relative sizes of the spinal cord (*sc*) and notochord (*nt*) are very different from what was seen in the preceding figure. In this section the spinal cord is considerably smaller than in the preceding, while the notochord is very much larger; in fact the notochord here seems abnormally large when compared to corresponding sections of other series. It is true, however, that while the spinal cord has been diminishing in diameter the notochord has been increasing. The spinal cord, notochord, and dorsal aorta are all so large that they are flattened against each other, the pushing in of the ventral side of the spinal cord being even more marked than is shown in the figure. On either side of the spinal cord a large spinal ganglion (*sg*) is seen, closely wedged in between the spinal cord and the adjacent muscle plate (*mp*). As in the preceding stage, there is a marked space between the muscle plate and the adjacent ectoderm (*ec*). The somatic mesoblast at the upper angle of the unenclosed body cavity is thickened on each side and somewhat bulged out by the Wolffian body to form what might be termed a Wolffian ridge (*wr*). In the mid-ventral line is the considerably developed mesentery (*ms*), from which the intestine has been torn. The Wolffian bodies now consist, on each side, of a group of five or six tubules (*wt*) of various sizes, near which in a more ventro-lateral position, close to the upper angle of the body cavity, is the more distinct Wolffian duct (*wd*). The allantois is fairly large by this time, and may be seen in the most posterior sections as an irregular, thick-walled outgrowth from the hindgut.

A horizontal section through the anterior end of an embryo of this age is shown in figure 15*f*. While enclosed of course in the same membranous amnion (*a*), the pharyngeal region of the section is separated by a considerable space from the more anterior region where the section passes through the forebrain (*fb*) and eyes. The spinal cord (*sc*), notochord (*nt*), muscle plates (*mp*), aortæ (*ao*), and anterior cardinal veins (*ac*) need no special description. The appearance of the pharynx (*ph*), with its gill clefts and folds, is quite similar to that of the corresponding structures in the chick. None of the four clefts (g^{1-4}) show, in the plane at which the section was cut, any connection with the exterior; in fact the fourth cleft (g^4) would scarcely be recognized as a cleft if seen in this section alone. One or two of the more anterior clefts are open to the exterior. Three pairs of aortic arches are seen, and each visceral fold has a central condensation of mesoblast.

Stage XIII

Figures 16–16*g* (Plates XVII, XVIII)

The embryo (fig. 16) now lies on one side, body torsion being complete. The curvature of the body is so marked that the exact length is difficult to determine. The eye (*e*) and ear (*o*) have about the same superficial appearance as in the preceding stage. The nose is not shown in this figure. About thirty somites are present; the exact number cannot be determined in surface view. The amnion is complete, though not shown in the figure, and the tail (*t*) is well formed. The umbilical stalk was torn in the removal of the embryo, so that it is not shown in the figure. The dim outline of the now convoluted heart may be seen if the "cleared" embryo be viewed by transmitted light; it is not shown in the figure. The allantois (*al*) is a rounded sac of considerable size just anterior to the tail. Four gill clefts (g^{1-4}) are now present; the most posterior one is more faint than is represented in the figure, and it could not be definitely determined from a surface view whether or not it opened to the exterior. The mandibular fold (*md*) is now fairly well outlined, but there is as yet no sign of the maxillary process.

Figure 16*a* is the most anterior of a series of transverse sections made of an embryo of the approximate age of the surface view just described; it passes through the tip of the forebrain (*fb*) and shows the nasal pit (*n*) of the right side. The great thickening of ectoderm in the region of the nasal invagination is represented by a solid line. Owing to the obliquity of the section, the left nasal pit was not cut. The mesoblast is quite dense and contains two or three small blood-vessels near the roof of the brain. The plane of this section, owing to the cranial and body flexure, cut the embryo also in the region of the pharynx; this part of the section was, as a matter of convenience, omitted from the drawing.

Figure 16*b* is in reality more anterior in position considering the entire embryo, than the preceding; but the region of the embryo represented is most posterior, so that it is described at this point. The greatly elongated outline of the brain is due to its being cut through the region of flexure, so that the forebrain (*fb*) or, perhaps, midbrain, is shown at one end and the hindbrain (*hb*) at the other. The walls of these cavities are somewhat wrinkled and irregular and their constituent cells are beginning to show slight differentiation, though this is not shown in the figure. On the left side are seen a couple of darkly stained masses; one is the origin of a cranial nerve (*cn*); and the other is one of the auditory vesicles (*o*), which is still more irregular in outline than it was in the preceding stage. The only blood-vessels to be seen are a few very small ones that lie close to the wall of the brain. The ectoderm is quite thin at all points.

Figure 16*c*, the largest section of this series, passes through the forebrain in the region of the eyes and through the gill clefts. The forebrain (*fb*) exhibits on the left a marked thickening of its wall (*ch*), the edge of the cerebral hemisphere of that side, which is just beginning to develop; on its right side the lower part of the forebrain is connected by a well-marked optic stalk (*os*) with the optic cup (*oc*), in whose opening lies the lens vesicle (*lv*), now reduced to a crescentic slit by the thickening of its posterior wall. The mesoblast is more dense in those parts of the section adjacent to the pharynx than in the more distant regions, and the ectoderm thickens in a marked way as it approaches the borders of the pharynx and gill clefts. Only a few small blood-vessels (*bv*) are to be seen in the region of the forebrain.

Parts of three pairs of clefts (*g*) are shown in the figure: one pair opens widely on either side, so that there is a large area of the section that is distinct from the two still larger portions and contains a small, thick-walled cavity (*g*) on the right side; this cavity is a gill cleft that is cut through neither its outer nor its pharyngeal opening.

No structures other than this small portion of a gill cleft and a few blood-vessels are to be seen in this middle region of the section. In the more posterior part of the section, in which the notochord (*nt*) is located, a pair of curved clefts may be seen, opening entirely through the wall on the left, but closed on the right (*g*). One distinct pair of aortic arches is shown (*ar*), and also the dorsal aortæ (*ao*), which are of very unequal size. The spinal cord (*sc*) and muscle plates need no special description.

Figure 16*d* is in the region of the heart (*ht*) and lungs (*lu*). The former is an irregular cavity whose walls, especially on the ventral side (*mes'*) are becoming very thick and much folded. Although thin, the body wall completely surrounds the heart, as would be expected, since this was true of the preceding stage. The lung rudiments (*lu*) and the foregut from which they have arisen have the same appearance as in the chick; they consist of three small, thick-walled tubes so arranged as to form a nearly equilateral triangle. They are surrounded by a swollen, rounded mass of mesoblast which almost completely fills the surrounding portion of the body cavity (*bc*). The pleural sides of these crescentic portions of the body (or pleural) cavity—that is, the boundary of the mass of mesoblast just mentioned—is lined with a thickened layer of cells, shown by the solid black lines in the figure. The lung rudiments may be traced through about fifty sections of this series, or about one-twelfth of the entire series. At the dorsal angle of the part of the body cavity (*bc*) just described, near the dorsal aorta (*ao*), are two dark, granular masses (*ge*), which, under a higher magnification than is here used, are seen to consist of a small group of blood-vessels filled with corpuscles; although several sections in front of the anterior limits of the kidneys, these are evidently glomeruli. They may be traced, though diminishing in size, far toward the tail, in close connection with the Wolffian bodies. At intervals they are connected by narrow channels with the dorsal aorta; no such connection was present in the section drawn. The notochord (*nt*), spinal cord (*sc*), muscle plates (*mp*), and spinal ganglia (*sg*) need no special mention. The mesoblast is beginning to condense in the neighborhood of the notochord, and the ectoderm is slightly thickened laterally and dorsally.

Figure 16*e* is in the region of the liver and the Wolffian bodies; it also shows the tip of the ventricular end of the heart. The liver (*li*) is a large irregular mass, of a blotchy appearance under this magnification, lying between the heart (*vn*) and the intestine (*i*). Under greater magnification it is seen to be made up of indefinite strings of cells; and its still wide opening into the intestine may be seen in more posterior sections. The intestine (*i*), which in this section might be called the stomach, is a fairly large cavity with the usual thick entodermic walls; it is supported by a comparatively narrow mesentery. The body cavity on the side next this mesentery has the same thick lining that was noted in the region of the lungs. The convolutions of the thick peritoneal lining may easily be mistaken in places for parts of the enteron. The Wolffian bodies may be seen as two groups of tubules (*wt*) in their usual location. The heart is cut through the ventricle (*vn*), as has been said. The section being at right angles to the long axes of the villi-like growths of the myocardium, the depressions between these mesoblastic cords are seen as a number of small irregular areas, each one lined with its endocardium. The incompleteness of the body wall below the heart is apparently due to an artificial break and not to a lack of fusion. The only point that need be mentioned in connection with the structures of the dorsal part of the section is that the distinctness of the myocoel (*myc*) on the right side is somewhat exaggerated.

Figure 16*f* is in the middle region of the embryo, where both spanchnopleure and somatopleure are unfused. Owing chiefly to the unclosed condition of the midgut (*i*) and to the increase in length of the mesentery (*ms*), the section is quite deep dorso-ventrally. The continuation of the amnion (*a*) with the somatopleure is of course here evident.

The most striking feature of the section is the marked projection of the Wolffian ridges, though no local enlargements of these ridges indicate the rudiments of the limbs. A large mass of Wolffian tubules (*wt*) is seen projecting into the upper part of the body cavity on each side; close to each of these masses is the posterior cardinal vein (*pc*), and between them is the large aorta (*ao*). The other structures are about as in the preceding section.

Figure 16*g* represents a sagittal section of the anterior half of the body of an embryo of this or possibly a slightly younger stage of development. The three regions of the brain are clearly indicated, as well as the cavity of the spinal cord (*sc*). The roof of the hindbrain has been made too thick in the figure; it should be represented by a mere line. A little mesoblast is to be seen at places between the roof of the brain and the superficial ectoderm. A slight invagination of the epithelium (*p*), between the floor of the brain and the anterior end of the notochord, probably represents the beginning of the hypophysis. No indication of the pineal body is yet to be seen. Extending from the region of the hypophysis to the posterior end of the section is the notochord (*nt*); it is much vacuolated and gradually increases in thickness toward the posterior, though its outline is quite irregular; except at the extreme anterior end and at one or two other places, it lies in close contact with the floor of the neural tube. Directly under the notochord lies, in the posterior half of the figure, the large dorsal aorta (*ao*). The pharynx (*ph*), opening between the end of the forebrain and the thick mandibular fold (across which opening the amnion (*a*) of course extends), is a funnel-shaped space which passes out of the plane of the section toward the posterior end of the figure. Its thick endodermal lining extends to the mandibular fold on the ventral side, while on the dorsal side it gradually thins out and becomes continuous with the thin ectoderm that extends over the forebrain. Just back of the mandibular fold is the bulbus (*b*), and back of that is the edge of the ventricle (*vn*). Posterior and dorsal to the ventricle the liver (*li*) is seen as an irregular mass of cells, and dorsal to the liver one of the Wolffian bodies (*wt*) is cut through its extreme edge.

Stage XIV

Figures 17–17*g* (Plates XVIII, XIX)

Body flexure has increased until now the forebrain and tail are almost in contact (fig. 17). The eye has developed somewhat; the ear vesicle, which is not shown in the figure, is small and seems to lie nearer the ventral side; the nasal pit is much larger and is crescentic in shape. The hyomandibular cleft (*g′*) still persists as a small crescentic slit, while the next three clefts are now represented merely by superficial grooves separated by distinct ridges, the visceral folds. No indication of a fifth cleft is seen. The maxillary process (*mx*) grows ventralward under the forebrain and is already longer than the manibular arch (*md*).

The chief advance in development over the preceding stage, besides the formation of the maxillary process, is in the appearance of the appendages (*aa* and *pa*); they have the characteristic shape of the rudimentary vertebrate appendage, though the anterior pair seem to point in an unusual direction at this stage and to be slightly more developed than the posterior. The curious, anteriorly directed heart (*ht*) is, perhaps, somewhat abnormal. The umbilical stalk (*u*) is comparatively narrow and, like the allantois, was cut off close to the body.

Transverse sections of an embryo of this stage are represented in figures 17*a-g*, drawn under a lower magnification than were any of the preceding figures.

Figure 17*a* is in the region of the pharynx, and passes through the forebrain (*fb*) and posterior part of the hindbrain (*hb*). In the thick walls of both of these structures histological differentiation has begun, so that even under low power an inner granular and an outer clear zone may be distinguished. Under greater magnification the presence of short fibers may be made out among the cells. The cerebral hemispheres (*ch*) are well-marked structures, their asymmetry being of course due to the obliquity of the section. Only one eye is cut by the plane of the section, and this one shows no connection with the forebrain. The outer wall of the optic cup (*oc*) is so thin that under this magnification it can scarcely be seen as a dark line surrounding the retinal wall. The lens (*ln*) is now a solid mass, of the usual type for vertebrate embryos, its front or outer wall being a scarcely discernible line. The hindbrain (*hb*) has the usual form for that region and does not differ particularly from what was noted in earlier stages except in the histological differentiation that has already been mentioned. As with the eye, it is only on the right side that the auditory vesicle (*o*) is shown. It shows some differentiation, but not so much as would be seen were it cut in another region. In the center of the section the pharynx (*ph*) forms an irregular cavity connected with the exterior on the left by a gill cleft (*g*) and by another slit which is simply the anterior margin of the stomodaeum. On the right neither of these openings are in the plane of the figure, though the gill cleft (hyomandibular), which lies close to the auditory vesicle, is almost an open passage. A few small blood-vessels are scattered through the section; one of these (*bv*), lying between the notochord (*nt*) and the floor of the brain, is noticeable from its being very closely packed with corpuscles, so that at first glance, under low magnification, it looks more like a nerve than a blood-vessel.

Figure 17*b* is also through the pharyngeal region, a short distance behind the preceding section. The growth of the cerebral hemispheres (*ch*) is better shown than in the preceding figure, as is also the general form of the optic cup (*oc*). On the left the nasal cavity (*n*) is seen as an elongated slit with thick walls; it is cut near, but not through, its opening to the exterior. The same gill cleft (*g*) that was seen in the preceding figure is seen here as a narrow, transverse cleft, open at both ends. Between the notochord (*nt*) and the spinal cord (*sc*) is the same, though now double, blood-filled vessel (*bv*) that was seen in the preceding section. The other blood-vessels are larger here than in the more anterior region. There is a faint condensation of mesoblast in the neighborhood of the notochord, and a more marked condensation (*mp*) farther toward each side is the curiously shaped muscle plate.

Figure 17*c* is through the heart region, and that organ is cut through the opening from the lower or ventricular into the upper or auricular chamber. The thickening of the wall of the ventricle, which was noticed in the preceding stage, has increased to such an extent that there is now a marked difference in the thickness of the ventricular and auricular walls. As in the preceding stage, the body wall is torn, probably in handling, so that it appears to be incomplete around the ventral side of the heart. Dorsal to the heart two small circular holes (*cnt*) with thick walls are the œsophagus and trachea, cut anterior to the point of bifurcation of the latter into the bronchial or lung rudiments. On either side of these structures is an elongated blood-vessel (*ac*), the anterior cardinal vein, its elongation being due to the fact that it is cut at the place where it turns downward to empty into the heart. Dorsal to the œsophagus are the aortæ (*ao*), which are here cut just at the point where the two vessels unite to form one; the next section, posterior to the one under discussion, shows an unpaired aorta. The notochord (*nt*) and spinal cord (*sc*) need no description, except to note that the latter shows active histological differentiation, numerous mitotic figures being seen under higher magnification, especially in the cells that line the spinal canal. On the right of the cord the edge of a spinal ganglion (*sg*) is seen, in connection with which in other sections are seen the clearly defined nerve roots. The condensation of mesoblast around the notochord is quite evident, and in close contact with this medial condensation are two very characteristic, S-shaped muscle plates (*mp*), which extend from the level of the dorsal side of the spinal cord to the upper limits of the cardinal veins. In some sections the muscle plates even yet show slight remains of the myocoel at the dorsal end.

Figure 17*d* is in the region of the posterior end of the heart (*ht*), which is cut through the tip of the ventricle, and the anterior end of the liver (*li*), which has the appearance of a mass of darkly stained cords or strands of cells surrounding a large blood-vessel (*mv*). This blood-vessel may be called the *meatus venosus,* though it is not separated by any line of demarcation from the auricle. A few sections anterior to this region the *meatus venosus* opens dorsally into a large vessel on each side (*dc*), which at first glance seems a part of the body cavity, but which is in reality the *ductus Cuvieri,* formed by the union of the anterior and posterior cardinal veins. An irregular, crescentic cleft (*bc*), lying medial and parallel to each of the Cuvierian vessels, is the body cavity. In the upper angle of this cavity is a granular mass, the glomerulus, that of the left side being accompanied by the extreme anterior end of the Wolffian duct. In the rounded mass of mesoblast, between the cleft-like regions of the body cavity, the lung rudiments (*lu*) and the œsophagus (*oe*) are seen as three small, circular openings; that of the œsophagus is somewhat smaller than the other two. The notochord (*nt*), spinal cord (*sc*), and muscle plates (*mp*) have almost the same appearance as in the preceding section. A spinal ganglion (*sg*) is seen on each side of the spinal cord; the one on the left shows a well-defined spinal nerve (*sn*), which may be traced ventrally as far as the end of the muscle plate, along whose medial side it courses. The ventral nerve root is plainly seen; the dorsal root, in this section, less plainly. The amnion (*a*) and abdominal wall are, as in the preceding figure, torn in the region of the ventricle.

Figure 17*e* is a short distance posterior to the figure just described. The liver is cut through its middle region and forms a large, darkly staining, reticular mass on the left side of the figure. The digestive tract is seen at two places to the right of the liver; the smaller and more ventral of these openings (*i*) may be called the intestine, while the larger is evidently the stomach (*i'*). The body wall is here unfused and becomes suddenly thinner as it passes upward into the amnion (*a*). The Wolffian tubules (*wt*) form a very conspicuous mass on either side of the mesentery, in close connection with the posterior cardinal veins (*pc*). In the mesoblast between the dorsal aorta (*ao*) and the notochord are two small, irregular, darkly stained masses (*sy*). These are shown in the preceding two figures, but were not mentioned in the description. They may be traced through a great part of the length of the embryo back of the head region; at intervals corresponding in length to the distance between the spinal ganglia they are enlarged, while between these enlargements they are very small in cross-section. At certain points a small blood-vessel is given off by the dorsal aorta to the immediate neighborhood of each of these small areas. Although they show no connection with the central nervous system, these structures appear to be the rudiments of the sympathetic nerves.

Figure 17*f* is in the region just in front of the hind legs. The abdominal walls are here unfused, and into the unenclosed body cavity projects the intestine (*i*), supported by a narrow mesentery and surrounded by a comparatively thick mass of mesoblast. The Wolffian body and duct form a mass of considerable size on each side of the mesentery. The Wolffian body is cut near its posterior end and consists of smaller tubules than in the more anterior regions. The Wolffian ducts (*wd*), on the other hand, are very large and are much more clearly distinguishable from the Wolffian tubules than in the preceding sections. The Wolffian ridges (*wr*) are very marked projections on the sides of the body, and in a region further caudad become especially developed as the posterior appendages, to be described in connection with the following section. Both spinal ganglia are shown in this figure (*sg*), and in connection with the left ganglion the spinal nerve (*sn*), extending ventrally as far as the level of the Wolffian duct. The sympathetic nerve rudiments do not extend so far caudad as the plane of this section. The dorsal end of each muscle plate (*mp*) is seen, in this and other sections, to be slightly enlarged to form a round knob; this knob contains a distinct cavity (not shown in the figure), the myocoel.

In figure 17*g*, owing to the curvature of the body, the plane of the section passes through the body at three places: through the region of the heart and lungs (fig. 17*d*), through the region of the posterior appendages, and through the tail. In fact, the plane of the section represented by each of the preceding figures cut the embryo in more than one region, but for the sake of simplicity only one region was represented in each figure. In the figure under discussion only the leg and tail regions have been drawn, though the latter region (*t*), being cut through one of its curves, is seen as an elongated body with a section of the spinal cord, notochord, etc., at each end. Both regions shown in the figure are enclosed in the same fold (*a*) of the amnion. Of the structures in the dorsal side of the larger or more anterior part of this figure nothing need be said. The most striking feature of the section is the presence of the large posterior leg rudiments (*pa*). As was noted in the preceding figure, they are, as usual, merely local enlargements or projections of the mesoblast (covered, of course, with ectoblast) of the Wolffian ridge. They are, as shown in this section and in the surface view of this stage (fig. 17), bluntly pointed projections from the sides of the body. The anterior appendage seems to be slightly more developed than the posterior, as was noted in describing the surface view of the embryo. The digestive tract is cut through its extreme posterior end, in the region that may be termed the cloaca (*cl*), for into it at this point the Wolffian ducts open (*wdo*). As the narrow cloacal chamber is followed toward the tail, it becomes still smaller in diameter, and the ventral depression or cleft seen in this figure gradually becomes deeper until its walls are continuous with the ectoderm that covers the ventral projection of mesoderm between the hind legs; no actual opening to the exterior is present, however. There is a space of about twenty-five or thirty sections (in a series of eight hundred) between the posterior ends of the Wolffian bodies and the cloacal openings of the Wolffian ducts. The body cavity (*bc*) and the posterior cardinal veins (*pc*) are very small in this region, as might be expected.

Stage XV

Figure 18 (Plate XIX)

Only the head of this embryo is represented, as the general state of development is about the same as in the preceding stage.

The chief object in making the figure is to show the five gill clefts (g^{1-5}). The fifth cleft, though small and probably not open to the exterior, is quite distinct in this embryo. The writer would feel more doubt of its being a true, though rudimentary, gill cleft had not Clarke (5) found a fifth pair of clefts in his material. The nasal pit has advanced in development somewhat and shows the beginning of the groove that connects it with the mouth. In this figure the crescentic hyomandibular cleft shows its connection with the groove between the mandibular and the hyoid folds.

Stage XVI

Figure 19 (Plate XIX)

This embryo is only slightly more developed than the preceding. Body flexure is so great that the forebrain and tail nearly touch. Only the anterior three gill clefts are visible. The maxillary process (*mx*) is longer and more narrow; the mandibular fold has not changed appreciably. The nasal pit (*n*) is now connected by a distinct groove with the stomodaeum. The appendages have increased in size, the posterior (*pa*) being the longer. The anterior appendage (*aa*) is distinctly broadened to form the manus, while no sign of the pes is to be seen at the extremity of the posterior appendage. The heart (*ht*) is still very prominent. The stalk of the umbilicus (*u*), which is quite narrow, projects from the ventral wall in the region between the heart and the hind legs. The tail is of considerable length and is closely coiled.

Stage XVII

Figures 20–20*j* (Plates XX, XXI)

The superficial changes noted in this stage chiefly concern the head, which has increased considerably in length (fig. 20). The curvature of the body is slightly more marked, and the tail is more tightly coiled at the end. There are still signs of three gill clefts. The maxillary process (*mx*) is long and narrow, while the mandibular arch (*md*) is still short and broad. The fronto-nasal region has greatly increased and has the acquiline profile noted by Clarke. The nasal groove has disappeared, and there remains the small opening (*n*) at the side of the fronto-nasal region, near the end of the still separate maxillary process. The umbilicus is in about the same condition as in the preceding stage, but the heart is less prominent. The outline of the manus (*ma*) is more definite, and the extremity of the posterior appendage is distinctly flattened out to form the rudimentary pes (*pe*). The position of the elbow-joint in the anterior appendage is seen at the end of the reference line *aa*.

Typical transverse sections of this stage are shown in figures 20*a*-*j*.

Figure 20*a* is a section through the middle region of the head, cutting the hindbrain on one side and the forebrain on the other. The walls of the brain show rather more histological differentiation than was seen in the preceding sections, though this cannot be shown under the low magnification used. The hindbrain (*hb*), which is cut near its anterior border, exhibits the usual membranous dorsal and thick ventral walls. The forebrain is here seen as three distinct cavities—a median third ventricle (*vt*), with a thick ventral wall, and a thin dorsal wall extended to form a large pineal body (*epi*), and two lateral ventricles (*ch*), the cavities of the cerebral hemispheres, whose walls are quite thick except on the side next the third ventricle. The sections of this series being slightly oblique, the eye is here cut on the right side only, where it is seen as a large, semicircular cavity (*e*) with thick, dense walls. The mesoblast, in which several blood-vessels (*bv*) are seen, exhibits three distinct areas—a median, lighter zone, with a more dense area on either side. The significance of this variation in the density of the mesoblast is not apparent.

Figure 20*b* is only a few sections posterior to the section just described. It is drawn chiefly to show the appearance of the forebrain, the other structures being about as in the preceding figure, except that both eyes (*e*) are here represented. The section passes through the wide opening between the third (*tv*) and the lateral ventricles (*ch*) and cuts the anterior edge of the pineal body (*epi*). The pineal body is very large and is directed backward instead of forward, as is usually the case among the lower vertebrates (if the alligator may be so classed). It is shown in figure 17*a* of a preceding stage and will be again shown in a sagittal section to be described later. The same areas of more dense and less dense mesoblast noted in the preceding figure are seen here.

Figure 20*c*, though still in the head region, shows several features that were not seen in the preceding figures. On the left of the hindbrain (*hb*) the auditory vesicle (*o*), which is now considerably more advanced than in earlier figures, is seen as a larger, flask-shaped cavity and a smaller, round one. Between the larger cavity and the hindbrain is the root of a cranial nerve (*cn*), apparently the eighth, since in another section it comes in close contact with the wall of the larger part of the auditory vesicle just mentioned. On the right side, ventral to the hindbrain, another and much larger nerve (*cn*) is seen. Nearly in the center of the figure is seen a small, irregular, thick-walled cavity (*p*), this is the pituitary body, and its connection with the roof of the pharynx may easily be made out in another section. The mesoblast in this region of the sections contains numerous large and small blood-vessels and exhibits certain denser areas which probably represent the beginnings of the cranial cartilages. No sign of the forebrain is seen (the plane of the section passing in front of that region), except that the tip of the wall of one of the cerebral hemispheres (*ch*) is cut. The left nasal chamber (*n*) is shown: it will be noted again in the following section. The eye on the right side shows no remarkable features; its lens (*ln*) is large and lies well back of the lips of the optic cup, which may now be called the iris (*ir*). A thin layer of mesoblast has pushed in between the lens and the superficial ectoderm to form the cornea, and the outer wall of the optic cup is now distinctly pigmented. The inner wall of the optic cup is beginning to differentiate into the retinal elements. The eye on the left side is cut farther from its central region and has a very different appearance from the eye just described. This unusual appearance is due to the fact that the section passed through the choroid fissure, which is very large and seems to be formed by the pushing in of the walls of the cup and not by a mere cleft in these walls. This fissure is hardly noticeable in the stage preceding the present, and in a stage slightly older it has disappeared; so that it would seem to be a very transient structure. It apparently is formed at about the time that the optic stalk, as such, disappears. It is in the region of the choroid fissure, if not through it, that the optic nerve (*on*) enters the eye. Through the fissure also enters a vascular tuft of mesoblast (*pt*) which may be seen projecting into the optic cup after the disappearance of the fissure. This loop of blood-vessels is doubtless the pecten.

Figure 20*d* represents a section through the hindbrain (*hb*), pharynx (*ph*), and tip of the snout. On either side of the hindbrain are a convoluted auditory vesicle (*o*), and several blood-vessels and nerves, while ventral to it is seen the anterior end of the notochord (*nt*), around which the mesoblast is somewhat more dense than elsewhere. The pharynx (*ph*) sends out toward the surface a narrow gill cleft (*g′*) in the neighborhood of each auditory vesicle. These clefts connect with the exterior by very narrow slits, not seen in the plane of this section. The opposite end of the pharynx, as seen in this figure, opens on the left (*pn*) into the nasal chamber. The nasal cavity on the right is cut in such a plane that it shows neither its external nor its pharyngeal opening. The nasal passages are here fairly long and nearly straight chambers; their lining epithelium is quite thick in the middle region, but becomes thinner where it merges into the epithelium of the pharynx at one end, and into the superficial epithelium at the other end. The unusual appearance of the eye (*e*), on the right side of the figure, is due to the fact that the plane of the section cut tangentially through the extreme edge of the eye in the region of the choroid fissure.

Figure 20*e* is only a short distance posterior to the preceding. On the left side the pharynx (*ph*) is connected with the exterior through the stomodeaum, and on the right the hyomandibular cleft (*g′*) is cut almost through its opening to the exterior. The auditory vesicle (*o*) on the right is cut near its middle region, while that on the left is barely touched by the plane of the section. The notochord (*nt*), with its condensed area of mesoblast, is somewhat larger than in the preceding section. The nasal canal on the right (*n*) is cut through neither anterior nor posterior opening, while on the left side the canal shows the anterior opening (*an*).

Figure 20*f*, which is in the region of the posterior part of the pharynx and the anterior part of the heart, shows some rather unusual conditions.

The spinal cord (*sc*) and notochord (*nt*), with the faintly outlined condensations of mesoblast in their region, need no special description. The pharynx (*ph*) is here reduced to an irregular, transversely elongated cavity, the lateral angles of which are connected on each side with the exterior through a tortuous and almost closed gill cleft (*g*), which must be followed through many sections before its inner and outer openings may be determined. Dorsal to the pharynx numerous blood-vessels (*bv*), both large and small, may be seen, while ventral to it is noticed a faint condensation of mesoblast (*la*), in the form of an inverted T, the anlage of the laryngeal structures. The ventral portion of the figure is made up of a nearly circular, thin-walled cavity, the pericardium (*pr*). Most of the pericardial cavity is occupied in this section by the thick-walled ventricle (*vn*), above which is the bulbus (*b*) and the tip of the auricle (*au*). The bulbus is nearly circular in outline, though its cavity is very irregular. A few sections anterior to this, the opening of the bulbus into the ventricle is seen.

In figure 20*g* the section represented is only a short distance posterior to the one represented by figure 20*f*. The mesoblastic structures in the neighborhood of the spinal cord (*sc*) and notochord (*nt*) will be described in connection with the next figure, where they are more clearly defined. The œsophagus (*oe*)—or posterior end of the pharynx, whichever it may be called—is here a crescentic slit, with its convex side upward; ventrally it opens by a narrow glottis into the trachea (*ta*). The trachea is surrounded by the same condensed area of mesoblast (*la*) that was mentioned in connection with the preceding figure, but the condensation is here more marked. From the bulbus (*b*) an aortic arch (*ar*) extends upward for a short distance on the right side, while to the left of the œsophagus an aortic arch (*ar*) is cut through the upper part of its course. Ventral to the bulbus the ventricle (*vn*) and two auricles (*au*) are seen surrounded by the pericardial wall.

Figure 20*h* is in the region of the liver (*li*), which has about the same position in relation to the auricles (*au*) that was occupied by the ventricle in the last figure. The auricles are connected with each other by a wide passage. The trachea (*ta*) and the œsophagus (*oe*) are entirely distinct from each other; the former is a small, nearly circular hole, while the lumen of the latter is obliterated and its walls form a solid, bow-shaped mass of cells. Since there is a narrow space between this mass of cells and the surrounding mesoblast, it might be thought that the lumen of the œsophagus had been closed by the simple shrinkage of its walls; higher magnification, however, fails to show any sign of a collapsed lumen. It is doubtless the problematic and temporary closure of the œsophagus that is noticed in other forms. On each side of the œsophagus, in close relation with the anterior cardinal vein (*ac*), is noticed a nerve (*cn*) cut through a ganglionic enlargement. When traced forward these nerves are seen to arise from the region of the medulla, and when followed caudad they are found to be distributed chiefly to the tissues surrounding the newly formed bronchi; they are doubtless the tenth cranial nerves. On the right side of the figure the close connection of this nerve with the near-by gill cleft is seen. Above the paired aortæ (*ao*) the sympathetic nerves (*sy*) will be noticed. The mesoblast surrounding the spinal cord (*sc*) and notochord (*nt*) is distinctly condensed (more so than the figure shows) to form what may be called the centrum (*c*) and neural arch (*na*) of the vertebræ. The arch, owing to the slight obliquity of the section, shows here only on one side. The spinal cord is not yet completely enclosed by the neural arches. The muscle plates (*mp*) are in close connection with the rudiments of the vertebræ just mentioned. The spinal cord (*sc*) is here differentiated into three areas—a dense, deeply stained area immediately around the neurocoel; a less dense area of cells surrounding the inner area and extending ventralward as a rounded projection on each side; and an outer layer, with few or no nuclei, surrounding the inner two layers except on the dorsal side.

In figure 20*i* the size and complexity of the figure are due, it will be easily understood, to the fact that the plane of the section passed through the curve of the body, thus practically cutting the embryo in two regions—an anterior, where the lungs (*lu*) and liver (*li*) are seen, and a posterior, where the Wolffian bodies (*wt*) are present. The spinal cord and the surrounding structures have almost the same characteristics at both ends of the figure, except that the primitive spinal column is rather more distinct in the posterior end of the section. The posterior cardinal veins (*pc*), Wolffian ducts (*wd*), and Wolffian bodies (*wt*) are also prominent structures of this end of the figure, the last being made up of a great number of tubules. The extreme anterior ends of the Wolffian bodies are seen in the other half of the section in the upper angles of the body cavity, dorsal to the lung rudiments (*lu*). Filling most of the body cavity (*bc*) and making up the greater part of the middle of the figure are the liver (*li*), now a very large organ; the stomach (*i'*), also quite large; the pancreas (*pan*), a small body lying near the stomach; and the lungs (*lu*), which here consist of several thick-walled tubes, surrounded by lobes of mesoblast. The other features of the figure need no special mention.

Figure 20*j* is through the base of the posterior appendages (*pa*), in which the cartilages are already being outlined by condensations of mesoblast. The intestine (*i*) is cut in two regions—at a more anterior point, where it is seen as a small, circular hole surrounded by mesoblast and hung by a narrow mesentery, and through the cloacal region, the larger and more ventral cavity, into which the Wolffian ducts (*wd*) open a short distance caudad to this section. The blood-vessels present a rather curious appearance. A short distance anterior to this point the aorta has divided into three, or it might be said that it has given off two, large branches. These two branches, one on either side near the posterior cardinal vein, pass toward the ventral side of the embryo on each side of the cloaca and end at about the region represented by the present figure. The small portion of the aorta that remains after the giving off of the two branches just described continues, as the caudal artery (*ca*), into the tail; it is a small vessel just under the notochord, and gives off small, paired branches at regular intervals toward the vertebral region. The posterior cardinal veins (*pc*), posterior to the openings of the Wolffian ducts into the cloaca, unite to form a large caudal vein lying just ventral to the caudal artery.

Stage XVIII

Figure 21 (Plate XXII)

This embryo, as may be seen, for example, by the form of the appendages, is slightly further developed than the one represented in figure 20. The figure is from a photograph of a living embryo as it lay in the egg, a portion of the shell and shell membranes having been removed. The embryo, which lies on its left side, is rather faintly outlined because of the overlying allantois. The allantois has been increasing rapidly in size, and is here so large that it extends beneath the cut edges of the shell at all points except in the region in front of the head of the embryo, where its border may be seen. Its blood-vessels, especially the one that crosses the head just back of the eye, are clearly shown in the figure, and in the living specimen, when filled with the bright red blood, they form a most beautiful demonstration. As in the chick, the allantois lies close beneath the shell membranes and is easily torn in removing them.

Stage XIX

Figure 22 (Plate XXII)

Figure 22 is a photograph of a somewhat older embryo, removed from the egg and freed of the fœtal membranes. The appendages show the position of both elbow and knee joints, and in the paddle-shaped manus and pes the digits may be faintly seen. The tail is very long and is spirally coiled, the outer spiral being in contact with the frontal region of the head. The jaws are completely formed, the upper projecting far beyond the lower. The elliptical outline of the eyes is noticeable, but the lids are still too little developed to be seen in this figure. The surface of the embryo is still smooth and white.

Stage XX

Figures 23–23*b* (Plate XXII)

In this surface view (fig. 23) several changes are seen, though no very great advance in development has taken place. The outlines of the digits (five in the manus and four in the pes) are now well defined; they even project slightly beyond the general outline of the paddle-shaped part. The tail has begun to straighten out, and it now extends across the front of the face. The lower jaw has increased in length, but is still shorter than the upper. The eyelids, especially the upper, are beginning to be discernible in surface view. Though still without pigment, the surface of the body is beginning to show by faint transverse lines the development of scales; these lines are most evident in this figure in the middle region of the tail, just before it crosses the nose.

A sagittal section of the entire embryo (except the tail) of this age is shown in figure 23*a*. In the head region the section is nearly median, while the posterior part of the body is cut slightly to one side of the middle line. At the tip of the now well-developed snout is seen one of the nostrils (*an*), cut through the edge; its connection with the complicated nasal chamber (*n*) is not here seen, nor is the connection of the nasal chamber with the posterior nares (*pn*). The pharynx (*ph*) is anteriorly connected with the exterior through the mouth (*m*) and the nares, while posteriorly it opens into the œsophagus (*oe*); the trachea (*ta*), though distinct from the œsophagus, does not yet open into the pharynx. In the lower jaw two masses of cartilage are seen, one near the symphysis (*mk*) and one near the wall of the trachea, doubtless the rudiment of the hyoid. The deep groove back of the Meckel's cartilage (*mk*) marks the tip of the developing tongue, which here forms the thick mass on the floor of the mouth cavity. Dorsal to the pharynx a mass of cartilage (*se*) is developing in the sphen-ethmoid region. This being a median section, the ventricles of the fore- (*fb*), mid- (*mb*), and hindbrain (*hb*) are seen as large cavities, while the cerebral hemispheres (*ch*) appear nearly solid, only a small portion of a lateral ventricle showing. The pineal gland (*epi*) is cut a little to one side of the middle and so does not show its connection with the brain. At the base of the brain the infundibulum (*in*) is seen as an elongated cavity whose ventral wall is in close contact with a group of small, darkly staining alveoli (*p*), the pituitary body. Extending posteriorly from the pituitary body is a gradually thickening mass of cartilage (*bp*), which surrounds the anterior end of the notochord (*nt*) and may be called the basilar plate. In its anterior region, where the section is nearly median, the spinal column shows its canal, with the enclosed spinal cord, while toward the posterior end of the figure the vertebræ are cut to one side of the middle line, and hence show the neural arches (*na*) with the alternating spinal ganglia (*sg*). Near the posterior end of the figure the pelvic girdle (*pl*) is seen. The largest organ of the embryo, as seen in this section, is the heart, of which the ventricle (*vn*) seems to be closely surrounded, both in front and behind, by the auricles (*au*). The liver (*li*) is the large, reticular mass back of the heart. Dorsal and anterior to the liver is the lung (*lu*), now of considerable size and development. The enteron is cut in several places (*oe, i*) and its walls are beginning to show some differentiation, though this cannot be seen under the magnification here used. One of the Wolffian bodies is seen as a huge mass of tubules (*wt*) extending from the pelvic region, where the mass is greatest, to the region of the lungs. The Wolffian tubules stain darkly and the whole structure forms a very striking feature of the section. Dorsal to the posterior end of the Wolffian body is a small, oval mass of very fine tubules (*k*), which do not stain so darkly as do the Wolffian tubules; this mass is apparently the beginning of the permanent kidney, the metanephros. Its tubules, though their origin has not been determined, seem to be entirely distinct from the tubules of the Wolffian body.

A single vertical section through the anterior part of the head of an embryo of this age has been represented in figure 23*b*. On the right side the plane of the section cut through the lens of the eye (*ln*); on the left side the section was anterior to the lens. The upper (*ul*) and lower (*ll*) eyelids are more evident here than in the surface view. Owing to the hardness of the lens, its supporting structures were torn away in sectioning. The vitreous humor is not represented in the figure. The superior (*ur*) and inferior (*lr*) recti muscles are well shown on the right side; they are attached to the median part of a Y-shaped mass of cartilage (*se*), which may be termed the sphenethmoidal cartilage. Between the branches of this Y-shaped cartilage the anterior ends of the cerebral hemispheres (*ch*)—better called, perhaps, the olfactory lobes—are seen. Between the lower end of the sphenethmoidal cartilage and a dorsally evaginated part of the pharynx are two small openings (*pn*); when traced forward these tubes are found to open into the convoluted nasal chamber, while a short distance posterior to the plane of this figure they unite with each other and open almost immediately into the pharynx. The rather complicated structures of the nasal passages of the alligator have been described by the writer in another paper (12). In the lower jaw the cartilage (*mk*) is seen on either side and several bands of muscle are developing in the mesoblast. Two deep grooves give form to what may be called the rudimentary tongue (*tn*). In both jaws one or two tooth rudiments (*to*) may be distinguished as small invaginations of ectoderm.

Stage XXI

Figure 24 (Plate XXII)

In this stage the curvature of the body and tail is less marked than was seen in the last surface view. The body has increased greatly in size, so that the size of the head is relatively not so great. The size of the eye in relation to that of the head is much diminished also. The five anterior and four posterior digits are well formed, and their claws are of considerable size, though of course not present on all the digits. The outlines of scales may be traced from the tip of the tail to the skull; they are especially prominent along the dorsal profile. The skin is just beginning to show traces of pigment, which is, however, not shown in the photograph. The umbilical stalk is seen projecting with a loop of the intestine from the abdominal wall; this is shown more clearly in the next stage. The embryo now begins to exhibit some of the external characteristics of the adult alligator.

Stage XXII

Figure 25 (Plate XXIII)

This embryo needs no particular description. It has reached in its external appearance practically the adult condition, although there is still considerable yolk (not shown in the figure) to be absorbed, and the embryo would not have hatched for many days. Pigmentation, begun in the last stage, is now complete. The umbilical stalk is clearly seen projecting from a large opening in the body wall. The long loop of the intestine that extends down into the yolk sac is here evident, and it is hard to understand how it can all be drawn up into the body cavity when the umbilical stalk is withdrawn. No sharp shell-tooth at the tip of the snout, such as is described by Voeltzkow (18) in the crocodile, is here seen.

Stage XXIII

Figure 26 (Plate XXIII)

This figure shows the relative sizes of the just-hatched alligator and the egg from which it came. It also shows the position of the young alligator in the egg, half of the shell having been removed for that purpose. The blotchy appearance of the unopened egg is due chiefly to stains produced by the decayed vegetation of the nest. At hatching the young alligator is about 20 cm. long, nearly three times the length of the egg; but the tail is so compressed that, though it makes up about half of the length of the animal, it takes up very little room in the egg.

Summary

Owing to the fact that the embryo may undergo considerable development before the egg is laid, and also to the unusual difficulty of removing the very young embryos, the earlier stages of development are very difficult to obtain.

The mesoderm seems to be derived chiefly by proliferation from the entoderm, in which way all of that anterior to the blastopore arises. Posterior to the blastopore the mesoderm is proliferated from the lower side of the ectoderm in the usual way. No distinction can be made between the mesoderm derived from the ectoderm and that derived from the entoderm.

The ectoderm shows during the earlier stages a very great increase in thickness along the median longitudinal axis of the embryo.

The notochord is apparently of entodermal origin, though in the posterior regions, where the germ layers are continuous with each other, it is difficult to decide with certainty.

The medullary folds have a curious origin, difficult to explain without the use of figures. They are continuous posteriorly with the primitive streak, so that it is impossible to tell where the medullary groove ends and the primitive groove begins, unless the dorsal opening of the blastopore be taken as the dividing point.

The amnion develops rapidly, and entirely from the anterior end.

The blastopore or neurenteric canal is a very distinct feature of all the earlier stages up to about the time of closure of the medullary canal.

Preceding the ordinary cranial flexure there is a sort of temporary bending of the head region, due apparently to the formation of the head-fold.

During the earlier stages of development the anterior end of the embryo is pushed under the surface of the blastoderm, and is hence not seen from above.

Body torsion is not so definite in direction as in the chick, some embryos lying on the right side, others on the left.

Of the gill clefts, three clearly open to the exterior and probably a fourth also. A probable fifth cleft was seen in sections and in one surface view.

The first trace of the urinary system is seen as a dorsally projecting, solid ridge of mesoblast in the middle region of the embryo, which ridge soon becomes hollowed out to form the Wolffian duct.

The origin of the pituitary and pineal bodies is clearly seen; the latter projects backward.

No connection can be seen between the first rudiments of the sympathetic nerves and the central nervous system.

The lumen of the œsophagus is for a time obliterated as in other forms.

The choroid fissure is a very transitory but well-marked feature of the eye.

REFERENCES

1. ANDERSON, A.: An Account of the Eggs and Young of the Gavial (*G. gangeticus*). Proc. Zoöl. Soc., 1875.
2. BALFOUR, F. M.: The Early Development of the Lacertilia. Quar. Jour. Mic. Soc., 1879, vol. XIX, pp. 421–430.
3. BALFOUR, F. M.: Comparative Embryology, vol. II.
4. BRONN, H. G.: Klassen des Thier-Reichs (vols. on reptiles).
5. CLARKE, S. F.: The Habits and Embryology of the American Alligator. Jour. Morph., vol. V, pp. 182–214.
6. DENDY, ARTHUR: Outlines of the Development of Tuatara (*Sphenodon punctatus*). Quar. Jour. Mic. Soc., vol. XXXXII, 1899, pp. 1–87.
7. EISLER, P.: Zur Kentniss der Histologie des Alligatormagens. Archiv. f. Mik. Anat., vol. XXXIV, pp. 1–10, 1889.
8. HOFFMANN, C. K.: Beiträge zur Entwicklungsgeschichte der Reptilien. Zeit. f. wiss. Zoöl., vol. XXXX, 1884, pp. 214–246.
9. HOFFMANN, C. K.: Weitere Untersuchungen zur Entwicklungsgeschichte der Reptilien. Morph. Jahrb., vol. XI, 1886, pp. 176–219.
10. PARKER, W. K.: On the Structure and Development of the Skull in the Crocodile. London, 1883. Zoöl. Soc. London, 1883, vol. XI, pp. 263–310.
11. RATHKE, H.: Untersuchungen über die Entwicklung und den Körperbau der Krokodile. Braunschweig, 1866.
12. REESE, A. M.: The Nasal Passages of the Florida Alligator. Proc. Phila. Acad. Nat. Sc., 1901.
13. REESE, A. M.: The Breedings Habits of the Florida Alligator. Smithsonian Misc. Coll. (Quarterly Issue), vol. XLVIII, pp. 381–387, 1907.
14. ROESE, C.: Über die Zahnleiste und die Eischweile der Sauropsiden. Anat. Anz., 1892, vol. VII, pp. 248–264.
15. STRAHL, H.: Beiträge zur Entwicklung von Lacerta agilis. Archiv. f. Anat. u. Physiol., 1882, pp. 242–278.
16. STRAHL, H.: Beiträge zur Entwicklung der Reptilien. *Ibid.*, 1883, pp. 1–43.
17. STRAHL, H.: Über frühe Entwicklungsstadien von Lacerta agilis. Zoöl. Anz., vol. VI, 1883, pp. 347–350.
18. VOELTZKOW, ALFRED: Biologie und Entwicklung der äusseren Körperform von Crocodilus madagascariensis Grand. Abhandl. Senckenberg. Naturf. Gesell., vol. XXVI, pt. 1, pp. 1–149, 1889.
19. WIEDERSHEIM, R.: Comparative Anatomy.

LETTERING FOR ALL FIGURES

a, head-fold of amnion.
aa, anterior appendage.
ac, anterior cardinal vein.
al, allantois.
an, anterior nares.
ao, aorta.
aop, area opaca.
ap, area pellucida.
ar, aortic arch.
au, auricle.
b, bulbus arteriosus.
bc, body cavity.
blp. blastopore.
bp, basilar plate.
bv, blood vessel.
c, centrum of vertebra.
ca, caudal artery.
ch, cerebral hemisphere.
cl, cloaca.
cn, cranial nerve.
cp, posterior choroid plexus.
cv, cardinal veins.
dc, ductus Cuvieri.
e, eye.
ec, ectoderm.
ec', thickening of ectoderm.
en, entoderm.
en', endocardium.
ent, enteron.
ep, epidermal layer of ectoderm.
epi, pineal body.
es, embryonic shield.
f, fronto-nasal process.
fb, forebrain.
fg, foregut.
g^{1-5}, gill clefts.
gf^{1-6}, gill folds.
gl, glomerulus.
h, head-fold.
hb, hindbrain.
ht, heart.
i, intestine.
i', stomach.
in, infundibulum.
ir, iris.
it, iter.
k, kidney (metanephros).
l, remains of groove between secondary folds.
la, larynx (cartilages of).
li, liver.
ll, lower lid of eye.
ln, lens.
lr, inferior rectus muscle of eye.
lu, lungs.
lv, lens vesicle.
m, mouth.
ma, manus.
mb, midbrain.
mc, medullary canal.
mc', tip end of medullary canal.
md, mandibular fold.
mes, mesoderm.
mes', myocardium.
mf, medullary fold.
mg, medullary groove.
mk, Meckel's cartilage.
mp, muscle plate.
ms, mesentery.
mv, meatus venosus.
mx, maxillary fold.
myc, myocoel.
n, nasal invagination or cavity.
na, neural arch of vertebra.
nc, neurenteric canal.
nl, nervous layer of ectoderm.
nt, notochord.
o, ear vesicle.
oc, optic cup.
oe, œsophagus.
on, optic nerve.
os, optic stalk.
ov, optic vesicle.
p, pituitary body.
pa, posterior appendage.
pan, pancreas.
pc, posterior cardinal vein.
pe, pes.
pg, primitive groove.
ph, pharynx.
pl, pelvis.
pn, posterior nares.
pr, pericardial cavity.
ps, primitive streak.
pt, pecten.
rt, retina.
s, somites.
sc, spinal cord.
se, sphenethmoid cartilage.
sf, secondary fold.

sg, spinal ganglion.
sm, splanchnic mesoblast.
sn, spinal nerve.
so, somatic mesoblast.
st, stomodæum.
sy, sympathetic nervous system.
t, tail.
ta, trachea.
tg, thyroid gland.
th, thickening and posterior limit of *sf*.
tn, tongue.
to, tooth anlage.
tr, truncus arteriosus.
tv, third ventricle of brain.
tv', third ventricle of brain.
u, umbilical stalk.
ul, upper lid of eye.
ur, superior rectus muscle of eye.
v'-''-''', first, second, and third cerebral vesicles.
va, vascular area.
vm, vitelline membrane.
vn, ventricle of heart.
vv, vitelline blood-vessels.
wd, Wolffian duct.
wdo, opening of Wolffian duct.
wr, Wolffian ridge.
wt, Wolffian tubules.
y, yolk.

EXPLANATION OF FIGURES 1–26 ON PLATES I–XXIII

All of the figures, with the exception of the photographs and those copied by permission from S. F. Clarke, were drawn under a *camera lucida*.

The magnification of each figure, except those from Clarke, is indicated below.

The photographs were made by the author, and were enlarged for reproduction by the photographic department of the Smithsonian Institution. The other surface views were made, under the author's direction, by Miss C. M. Reese.

With the exception of Stage III, all of the figures of any one stage are given the same number, followed where necessary by a distinguishing letter, so that it is possible to tell at a glance which section and surface views belong together. The transverse sections are all cut in series from anterior to posterior.

FIGURE 1. Surface view of egg. × 2/3.

1*a*. Egg with part of the shell removed to show the chalky band in the shell membrane. × 2/3.

FIGURES 2 and 2*a*. Dorsal and ventral views respectively of the blastoderm before the formation of the notochord, medullary folds, etc. After Clarke.

2*b*–2*f*. Transverse sections of an embryo of the age represented in figures 2 and 2*a*. × 43.

3 and 3*a*. Ventral and dorsal views respectively of an embryo a few days older than that represented in figures 2 and 2*a*. After Clarke.

3*b*–3*m*. Transverse sections of an embryo of the age shown in figures 3 and 3*a*. × 43.

FIGURES 3*n* and 3*o*. Two sagittal section of an embryo of the same stage as figures 3 and 3*a*. × 43.

4 and 4*a*. Dorsal and ventral views respectively of a slightly older embryo than the one shown in figures 3 and 3*a*. Figure 4*a* shows only the head region. After Clarke.

5 and 5*a*. Dorsal and ventral views respectively of an embryo of

almost the same age as the preceding, to show the further development of the medullary folds. After Clarke.

FIGURE 6. Dorsal view of an embryo only a day or two older than the preceding. After Clarke.

FIGURES *6a–6i*. A series of transverse sections of this stage. × 43.

FIGURES *7a–7h*. A series of transverse sections of an embryo slightly older than the one shown in figures 4–6. × 43. (No surface view of this stage is figured.)

8 and *8a*. Dorsal and ventral views respectively of an embryo with five pairs of mesoblastic somites. × 20. (Drawn by transmitted light.)

8b and *8c*. Two sagittal sections of an embryo of this stage. × 43.

FIGURES *8d–8j*. A series of transverse sections of the embryo represented in figures 8 and *8a*. × 43.

9a–9m. A series of transverse sections of an embryo somewhat more advanced in development than the one represented in the last series. × 43.

FIGURES 10 and *10a*. Dorsal and ventral views respectively of an embryo with eight pairs of mesoblastic somites. × 20. (Drawn chiefly by transmitted light.)

FIGURE 11. Dorsal view of an embryo with fourteen pairs of mesoblastic somites. The area pellucida and the developing vascular area are shown, the latter having a mottled appearance. The pushing of the head under the blastoderm is also shown. × 20. (Drawn chiefly by transmitted light.)

FIGURES *11a–11k*. A series of transverse sections of an embryo of this stage. × 43.

FIGURE 12. Dorsal view of an embryo with about seventeen pairs of mesoblastic somites. Part of the area pellucida is represented. (Both transmitted and reflected light were used in making the drawing.) × 13.

FIGURES *12a–12g*. A series of transverse sections of an embryo of this stage. × 43.

FIGURE 13. Surface view of an embryo with about twenty pairs of mesoblastic somites. × (about) 15. (Drawn with both reflected and transmitted light.)

FIGURES 13–*13f*. A series of transverse sections of an embryo slightly more developed than the one shown in figure 13. × 20.

FIGURE *13g*. A sagittal section of an embryo of about the age of the one represented in figure 13. × 20.

14. Head of an embryo with one pair of gill clefts; ventro-lateral view. × 13.

15. Profile view of the head of an embryo with three pairs of gill clefts. × 13.

FIGURES *15a–15e*. A series of transverse sections of an embryo of about the age of the one represented in figure 15. × 20.

FIGURE *15f*. A horizontal section through the anterior region of an embryo of the age of that shown in figure 15. × 20.

16. Surface view in profile of an embryo with four pairs of gill clefts. × (about) 12.

FIGURES 16*a*–16*f*. A series of transverse sections of an embryo of the approximate age of the one represented in figure 16. × 20.

FIGURE 16*g*. A sagittal section of an embryo of the age (possibly slightly younger) of the one represented in figure 16. × 20.

17. Surface view in profile of an embryo at the time of origin of the limbs. × (about) 5.

FIGURES 17*a*–17*g*. A series of transverse sections of an embryo of the age of the one represented in figure 17. × 7.

FIGURE 18. Surface view in profile of the head of an embryo slightly larger than, though of about the same state of development as, the one represented in figure 17. Reproduced here chiefly to show the gill clefts. × (about) 3.

19. Surface view of an embryo somewhat more developed than the one just described. × (about) 3.

FIGURE 20. Surface view of an embryo older than the one represented in figure 19; with well-developed manus and pes. × (about) 5.

FIGURES 20*a*–20*j*. A series of transverse sections of an embryo of the age of the one represented in figure 20. × 7.

FIGURE 21. A photograph of a living embryo in the egg, showing the allantois, yolk mass, etc. The embryo is somewhat more developed than the one shown in figure 20. × 2/3.

22. A photograph of a still larger embryo, removed from the shell and freed from the fetal membranes. × (about) 1.

23. A photograph of a still more advanced embryo, in which the digits are quite evident and the scales are beginning to show. × (about) 1.

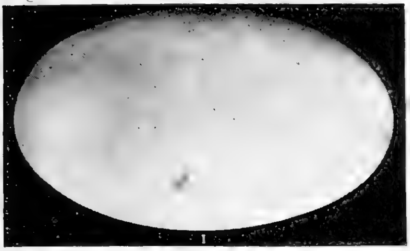
23*a*. A sagittal section of an embryo of the age of the one represented in figure 23; the tail has not been shown in this figure. × (about) 3.

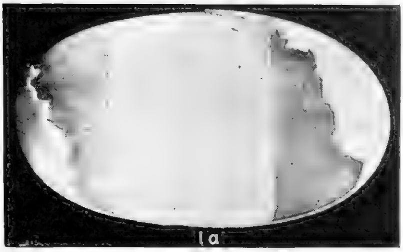
23*b*. A vertical section through the head of an embryo of about the size (perhaps slightly smaller) of the one shown in figure 23. × (about) 3.

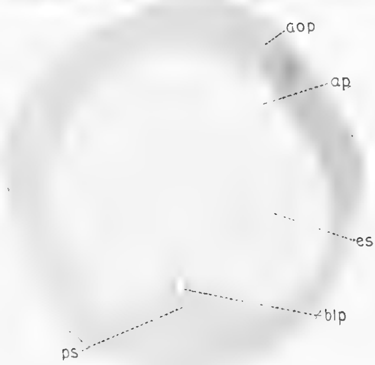
24. A photograph of an older embryo in which the pigmentation of the scales is evident, though not shown in the figure. × (about) 1.

25. A photograph of an embryo in which the pigmentation and the development of the body form are practically complete. The allantois, unabsorbed yolk, etc., have been removed. × (about) ¾.

26. A photograph of a just-hatched alligator, of an alligator egg, and of a young alligator in the egg just before hatching. × (about) 3/7.





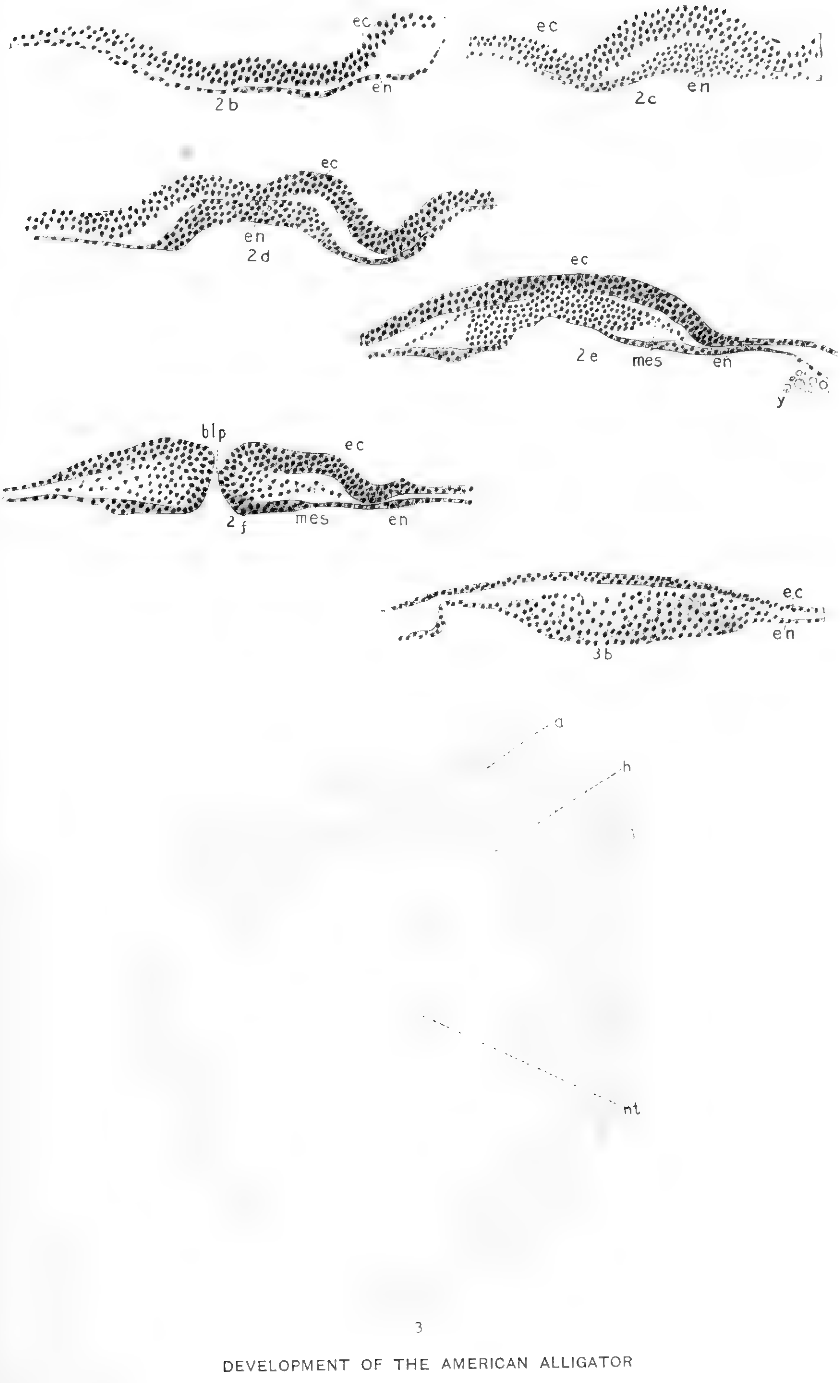


2

2 a

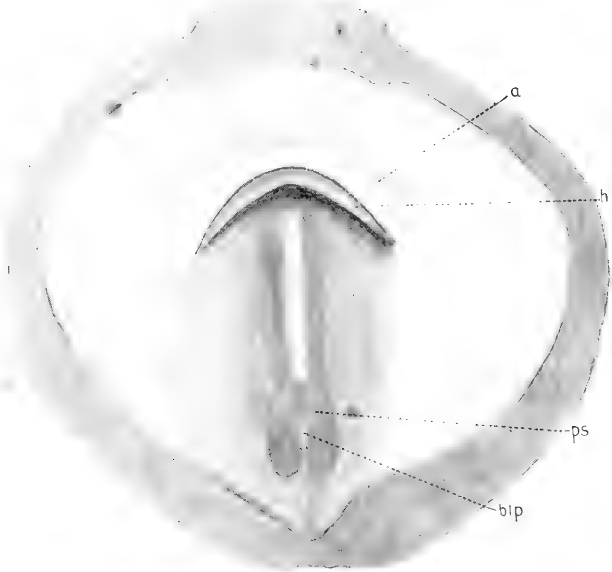
DEVELOPMENT OF THE AMERICAN ALLIGATOR

1, 1a.—The Egg. *2, 2a.*—Stage I



DEVELOPMENT OF THE AMERICAN ALLIGATOR

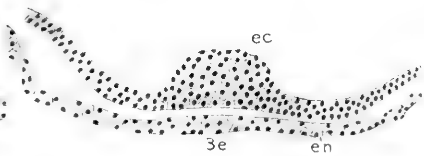
2b–2f.—Stage I. *3, 3b.*—Stage II



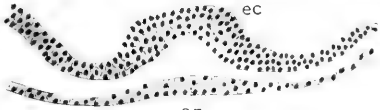
3a

3c

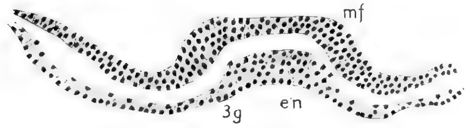
3d



3e



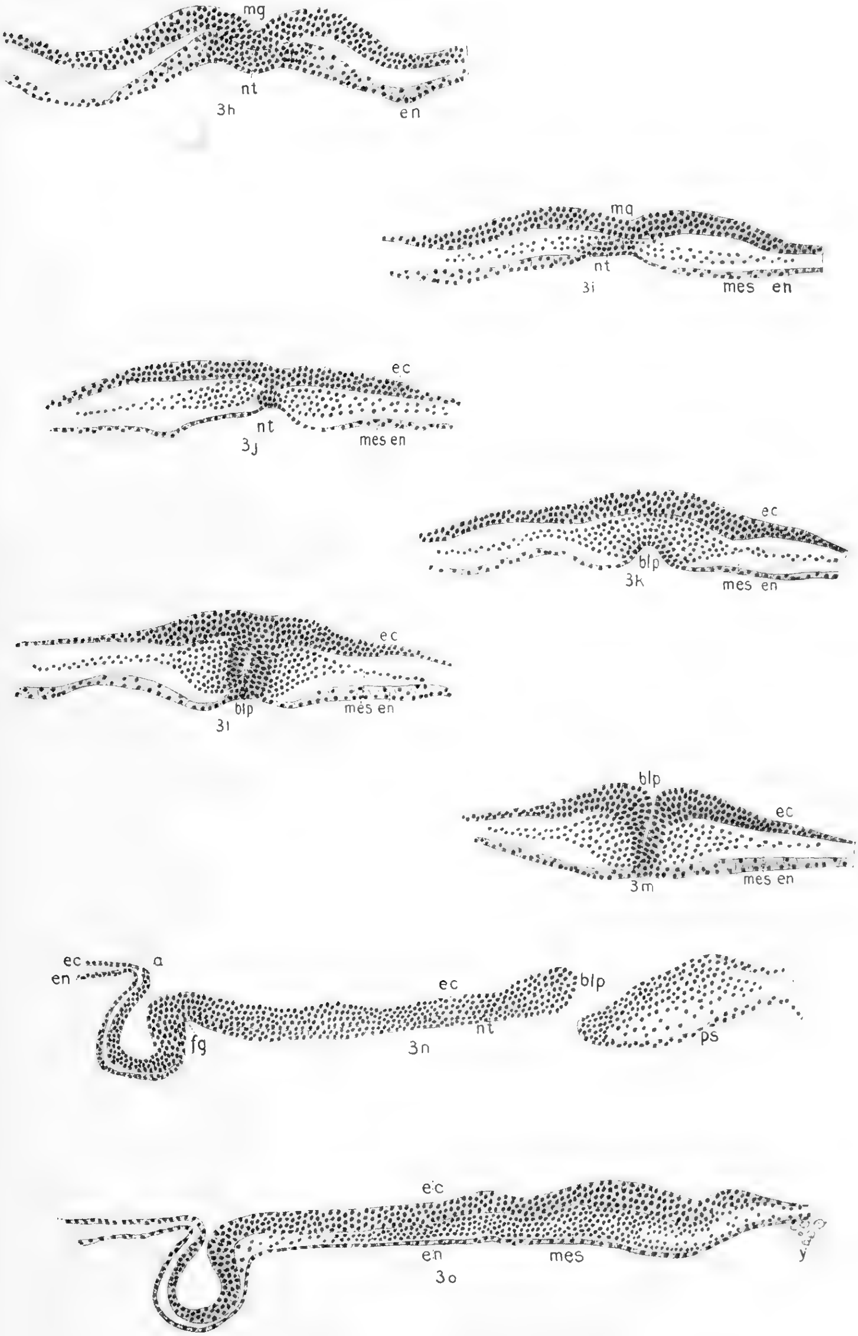
3f



3g

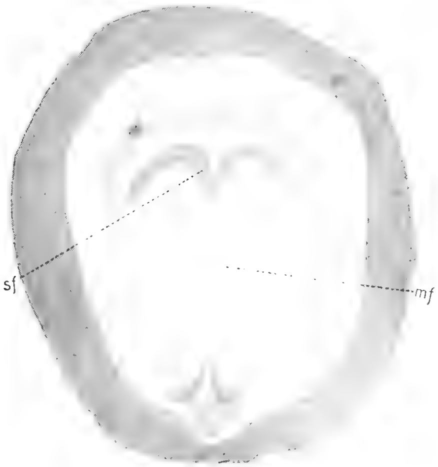
DEVELOPMENT OF THE AMERICAN ALLIGATOR

3a, *3g*.—Stage II

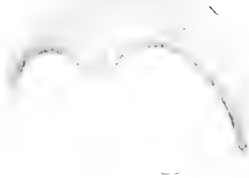


DEVELOPMENT OF THE AMERICAN ALLIGATOR

3h–3o.—Stage II



4



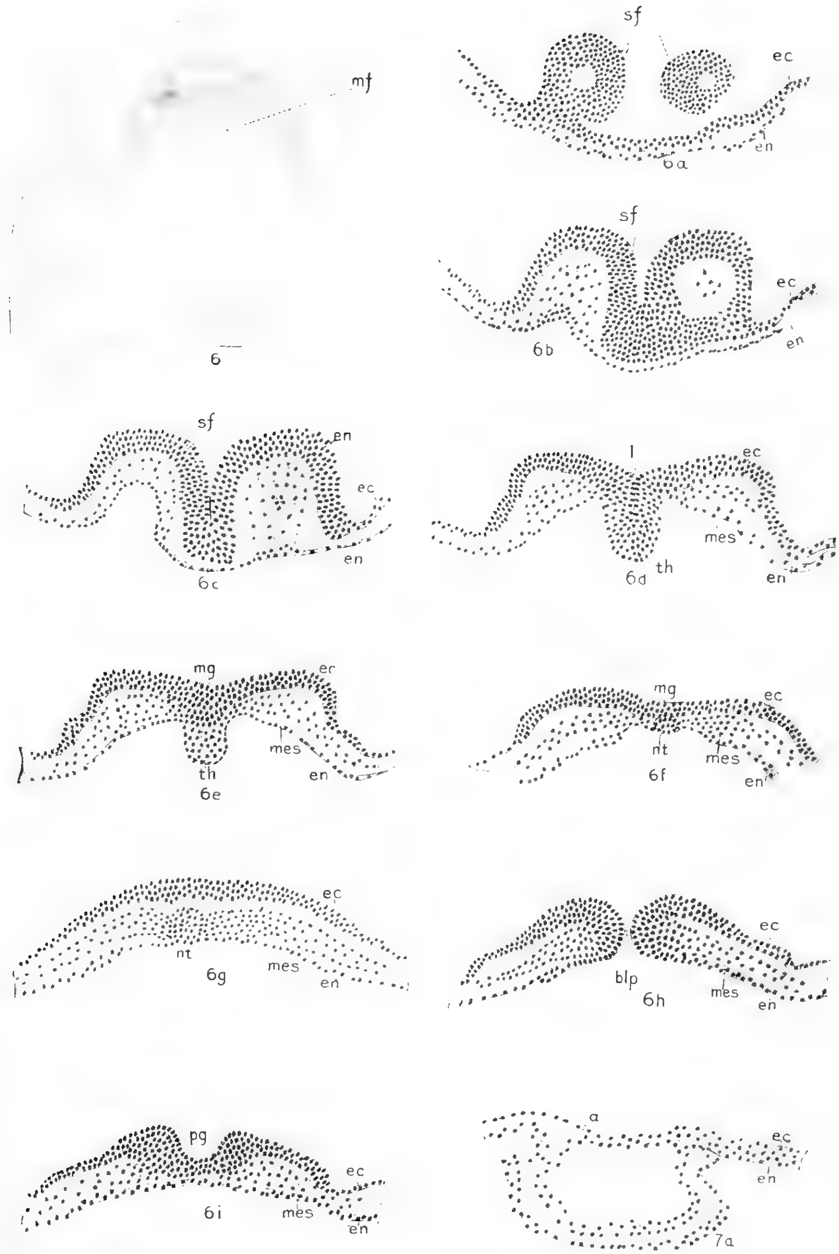
4a

5

5a

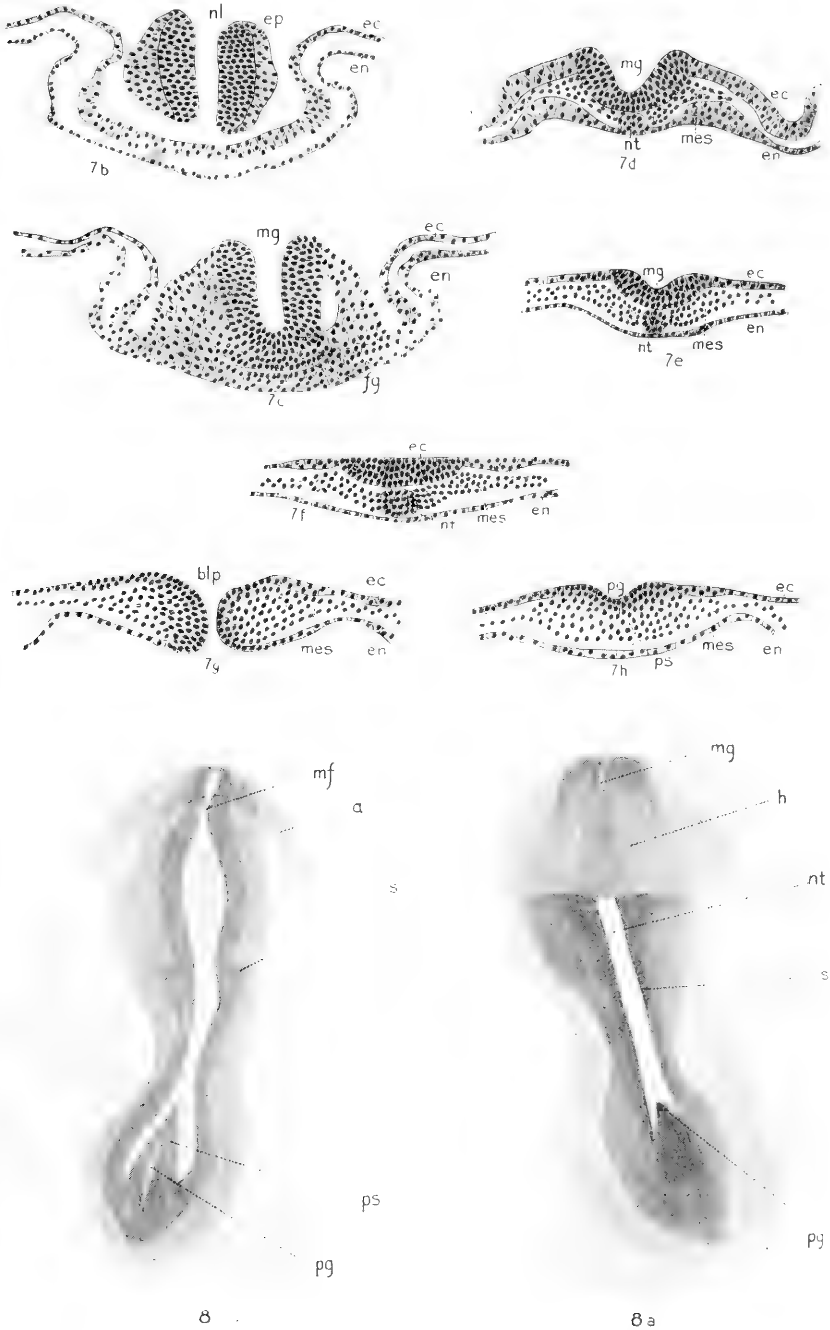
DEVELOPMENT OF THE AMERICAN ALLIGATOR

Stage III



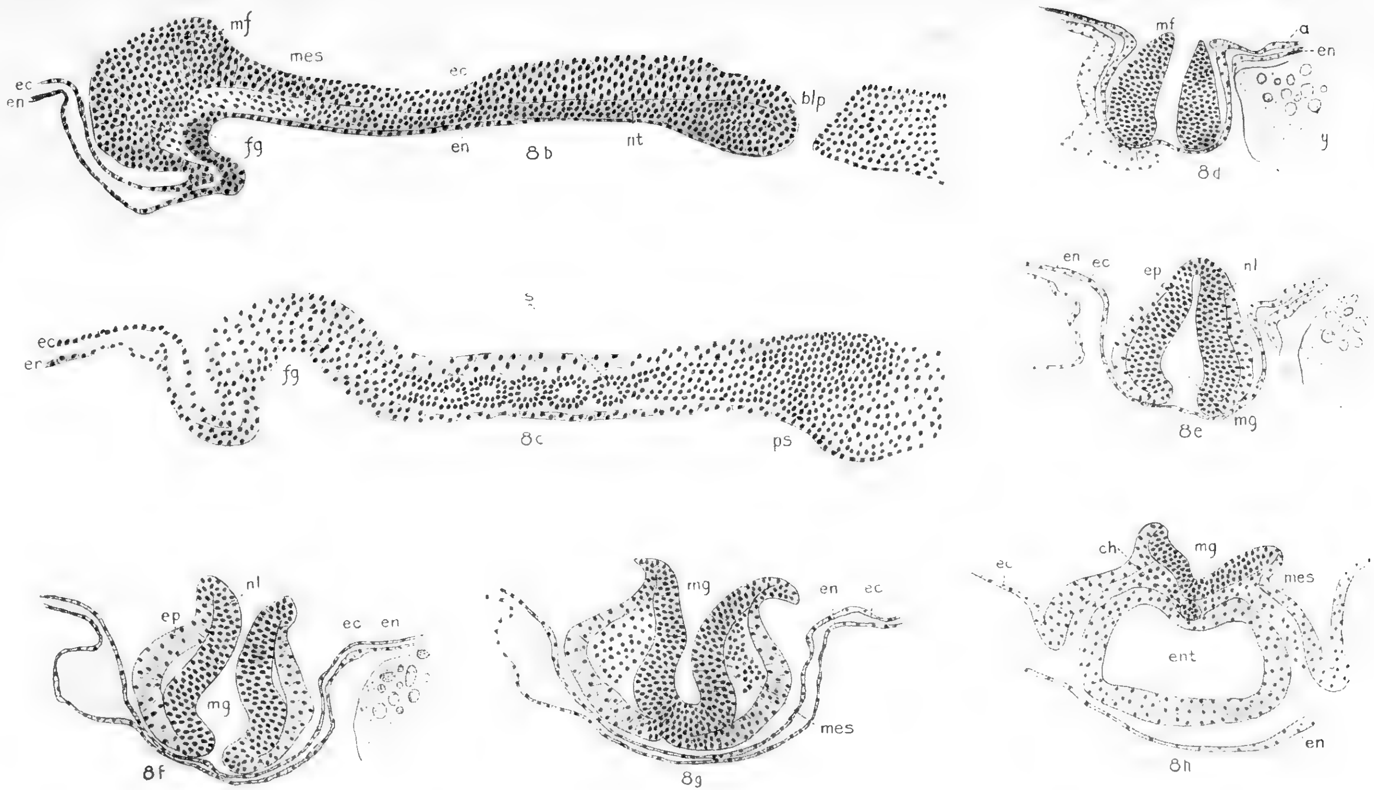
DEVELOPMENT OF THE AMERICAN ALLIGATOR

6-6i.—Stage III. *7a.*—Stage IV



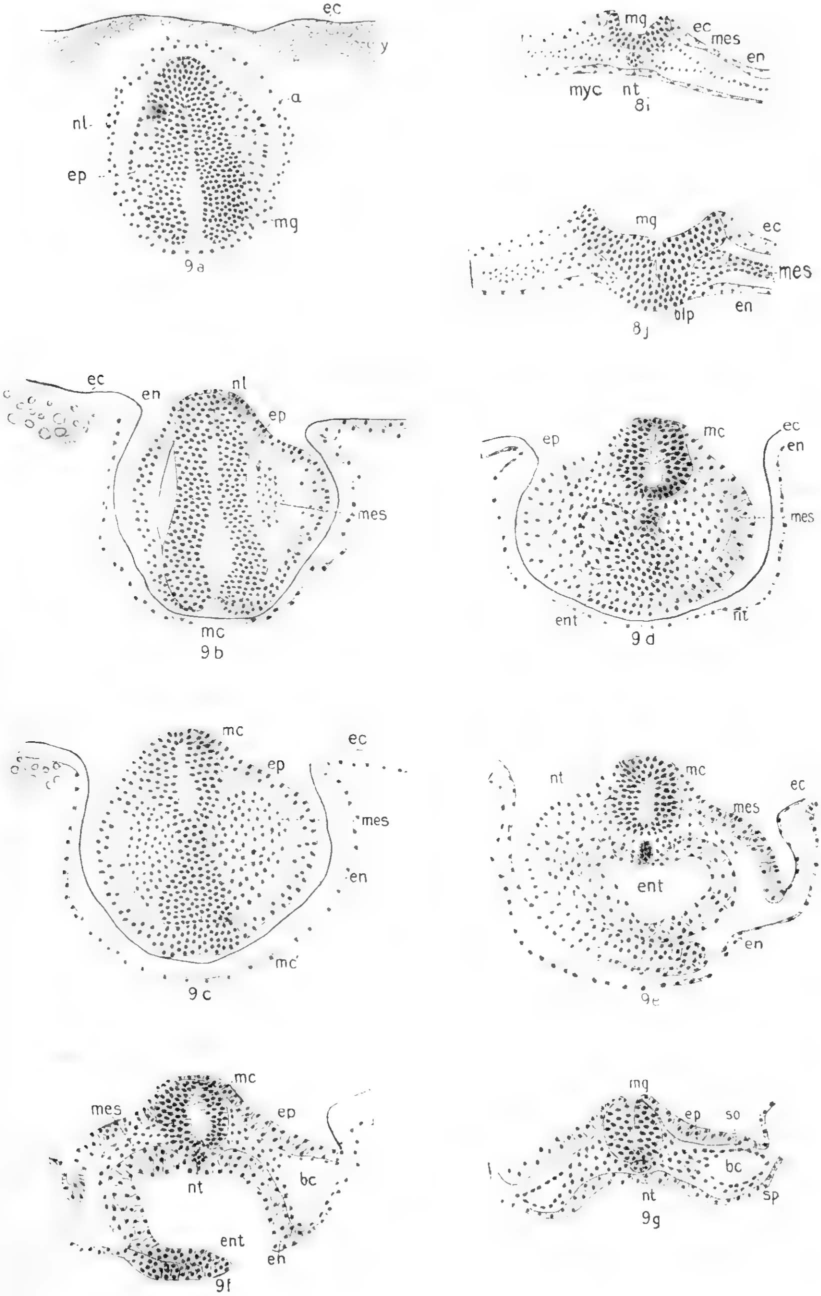
DEVELOPMENT OF THE AMERICAN ALLIGATOR

7b–7h.—Stage IV. *8, 8a.*—Stage V



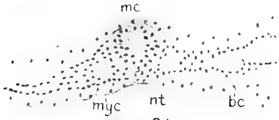
DEVELOPMENT OF THE AMERICAN ALLIGATOR

Stage V

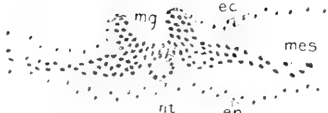


DEVELOPMENT OF THE AMERICAN ALLIGATOR

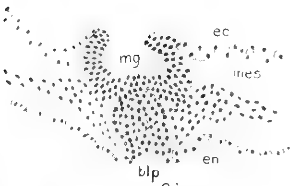
8i, 8j.—Stage V. *9a-9g.*—Stage VI



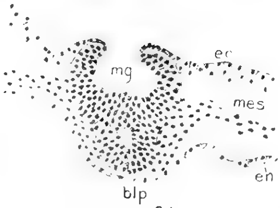
9 h



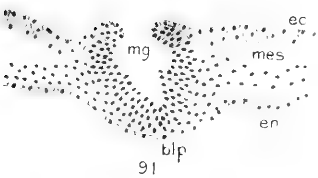
9 i



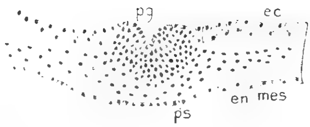
9 j



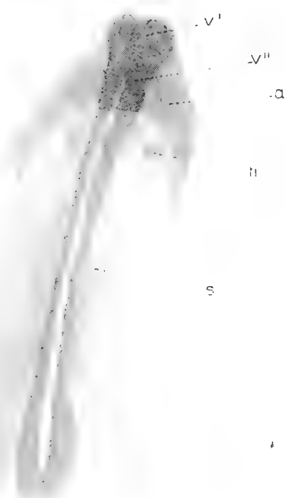
9 k



9 l



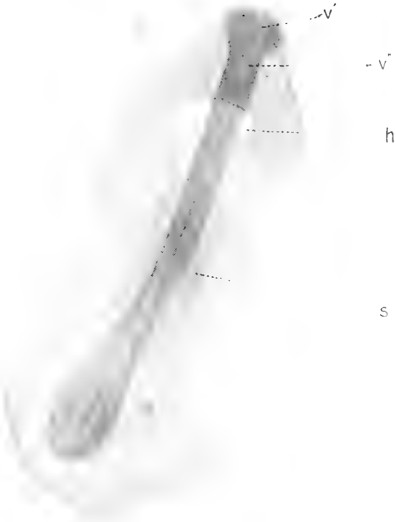
9 m



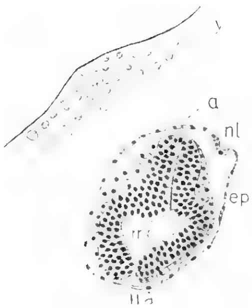
10

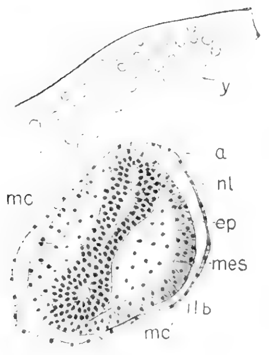
DEVELOPMENT OF THE AMERICAN ALLIGATOR

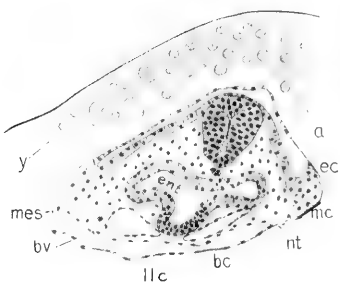
9h-9m.—Stage VI. *10.*—Stage VII

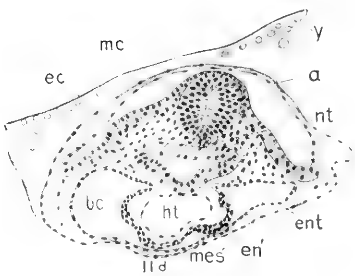


10 a





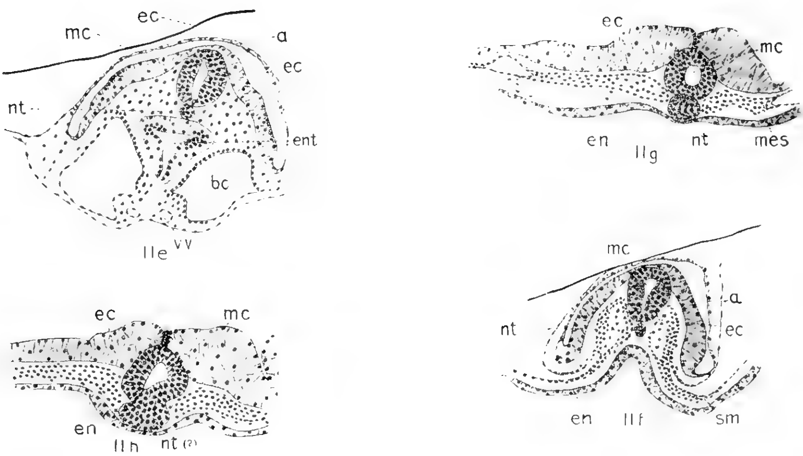




DEVELOPMENT OF THE AMERICAN ALLIGATOR

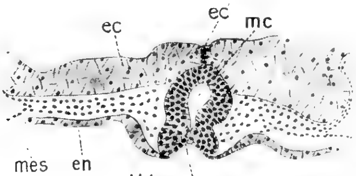
10a.—Stage VII. *11a-11d.*—Stage VIII

11

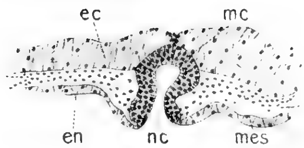


DEVELOPMENT OF THE AMERICAN ALLIGATOR

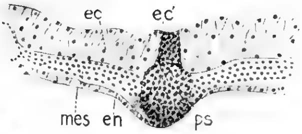
Stage VIII



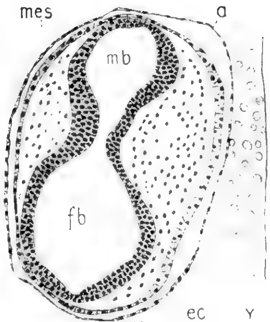
11 i



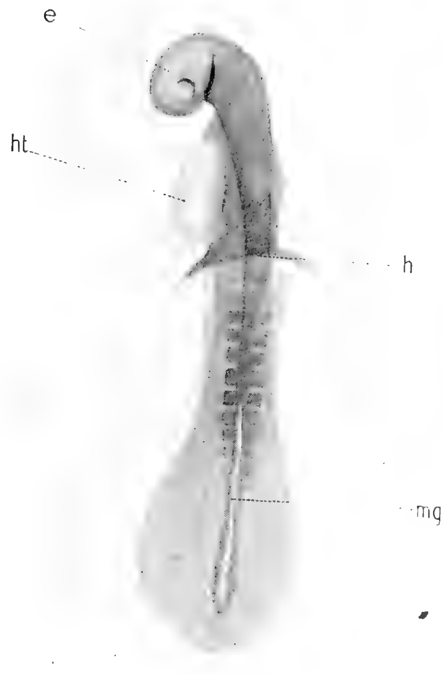
11 j



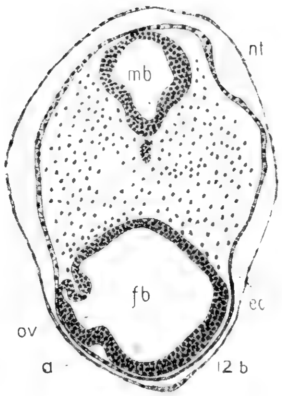
11 k



12 a



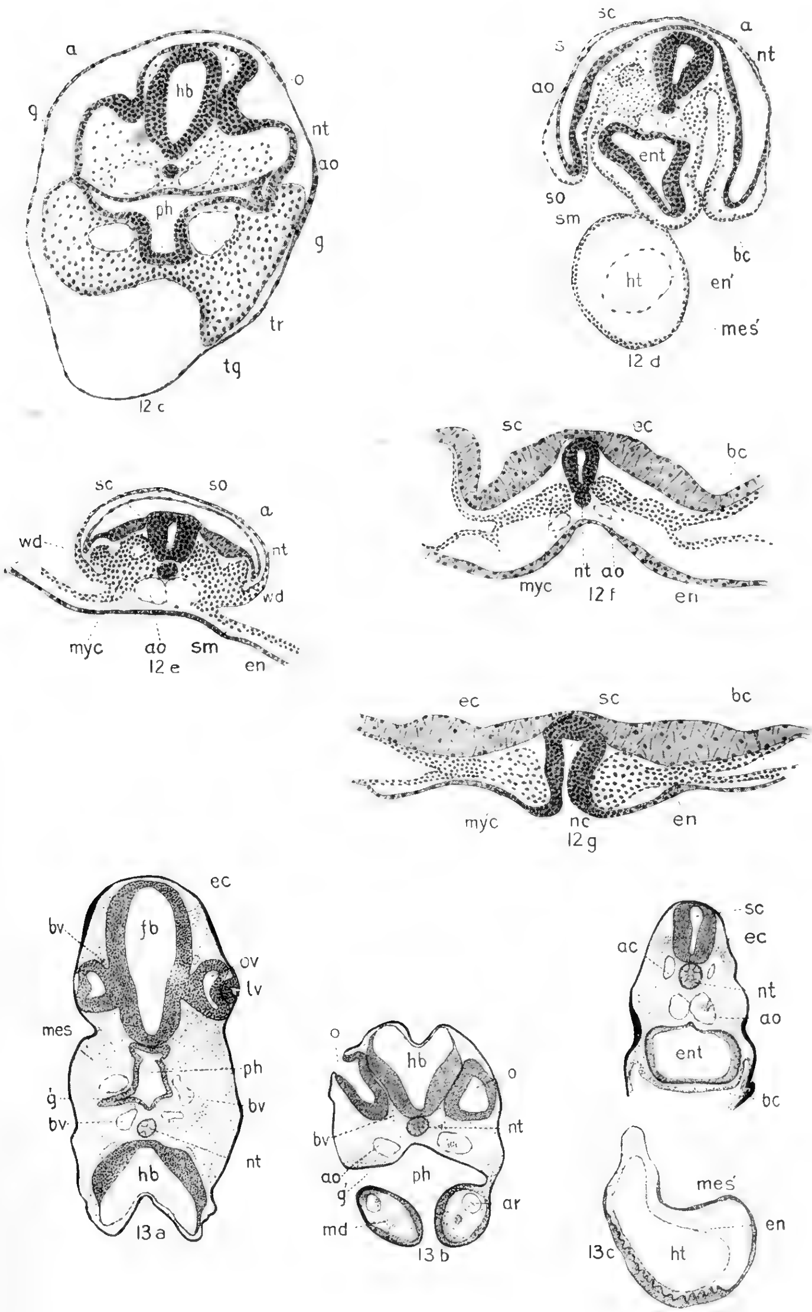
12



12 b

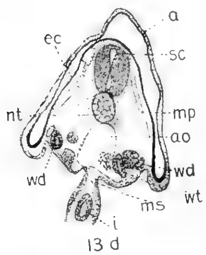
DEVELOPMENT OF THE AMERICAN ALLIGATOR

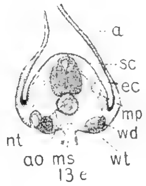
11i–11k.—Stage VIII. *12–12b.*—Stage IX

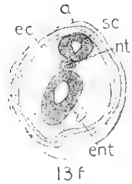


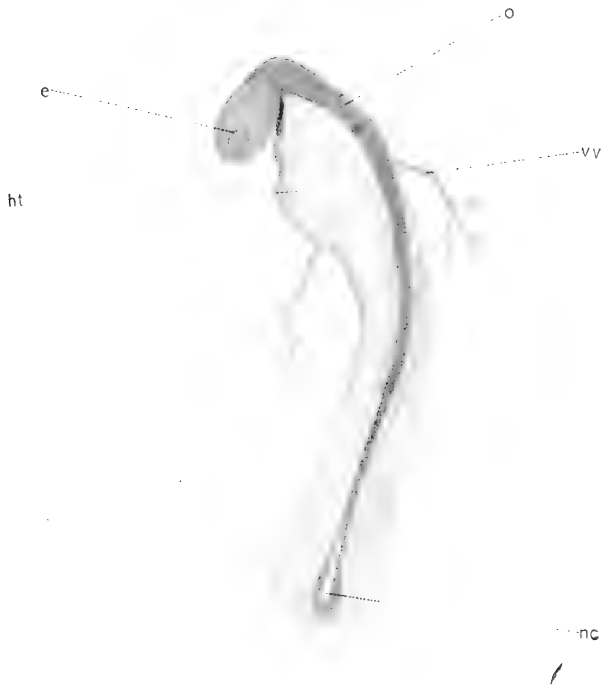
DEVELOPMENT OF THE AMERICAN ALLIGATOR

12c–12g.—Stage IX. *13a–13c*.—Stage X





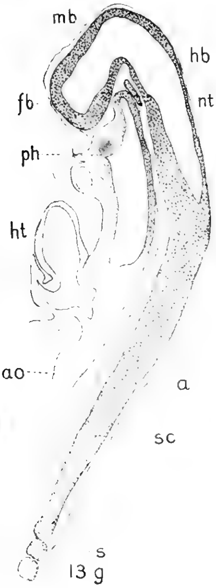




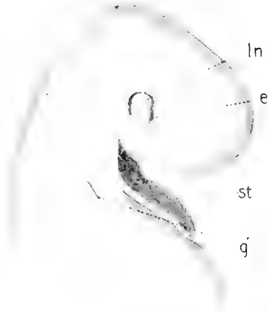
13

DEVELOPMENT OF THE AMERICAN ALLIGATOR

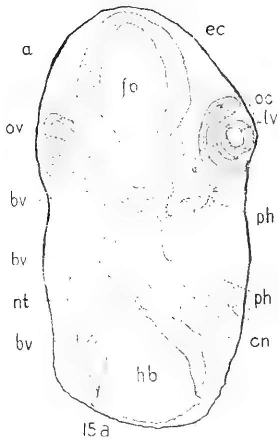
Stage X



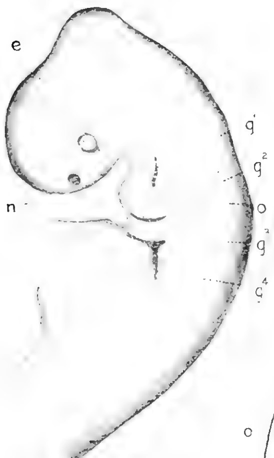
13 g



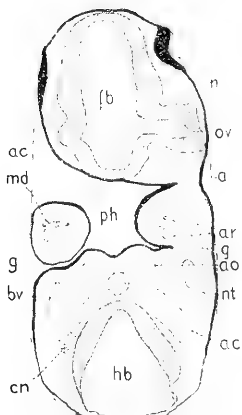
14



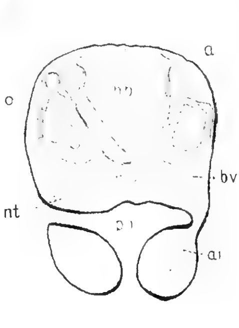
15 a



15



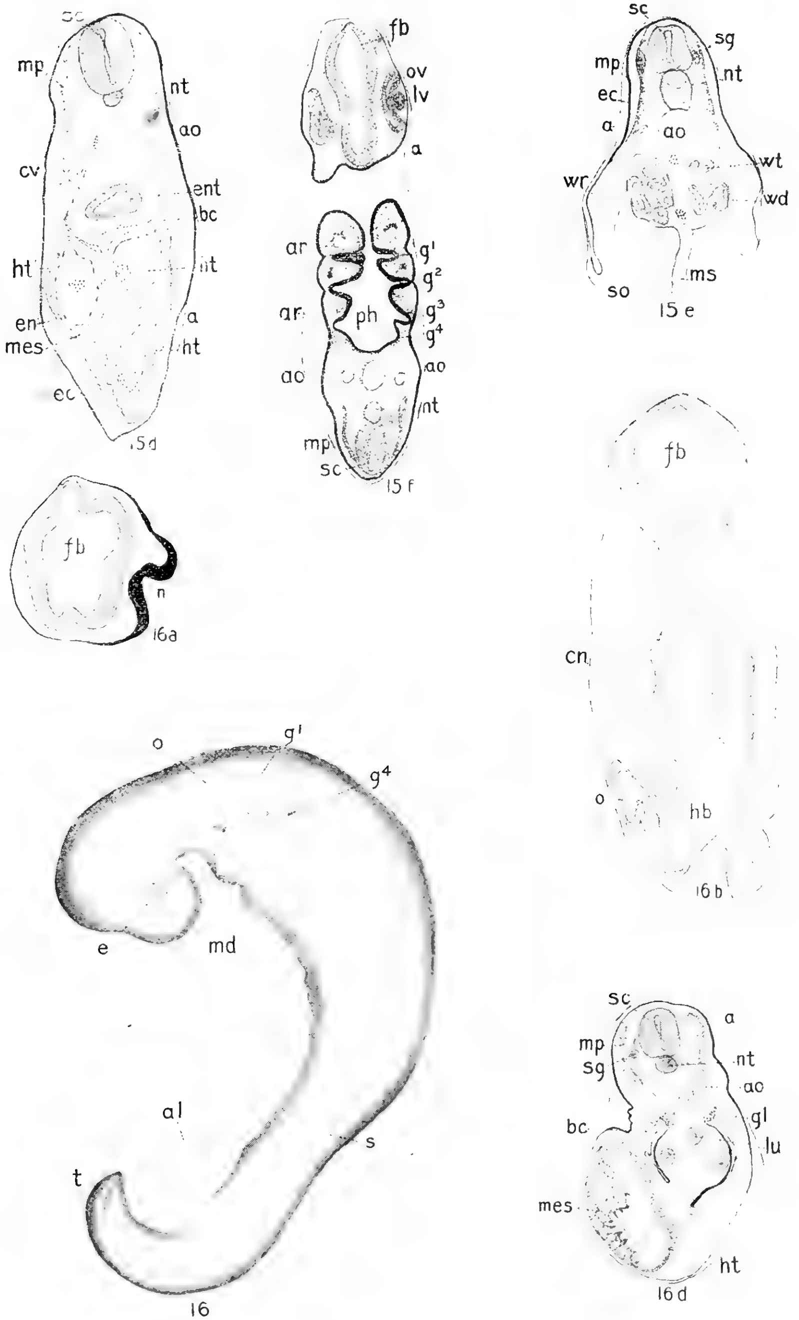
15 b



15 c

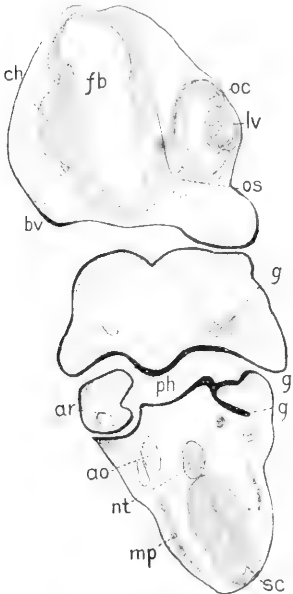
DEVELOPMENT OF THE AMERICAN ALLIGATOR

13g.—Stage X. *14.*—Stage XI. *15-15c.*—Stage XII

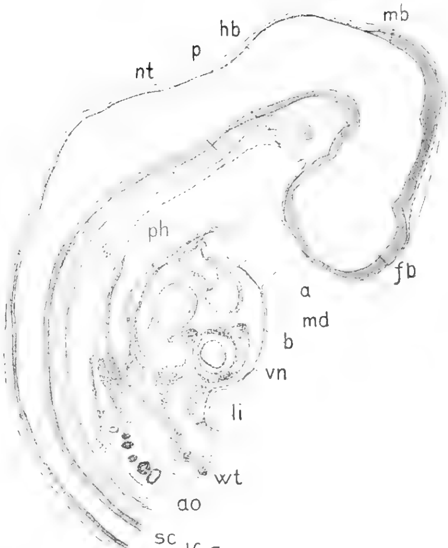


DEVELOPMENT OF THE AMERICAN ALLIGATOR

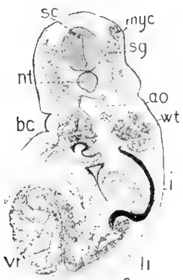
15d-15f.—Stage XII. *16-16d.*—Stage XIII



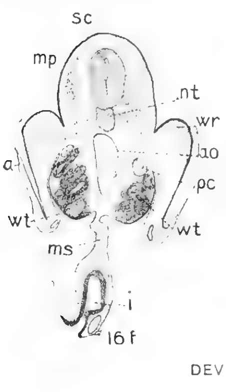
16 c



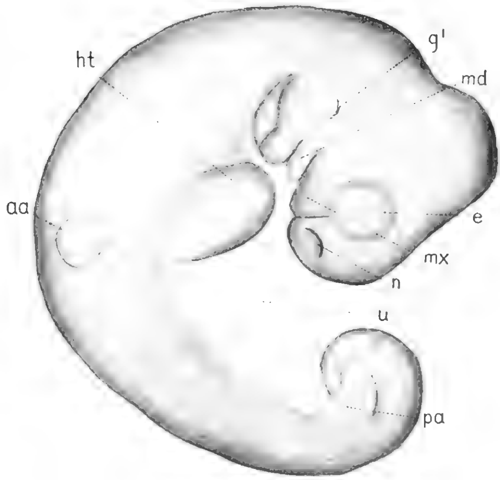
16 g



16 e



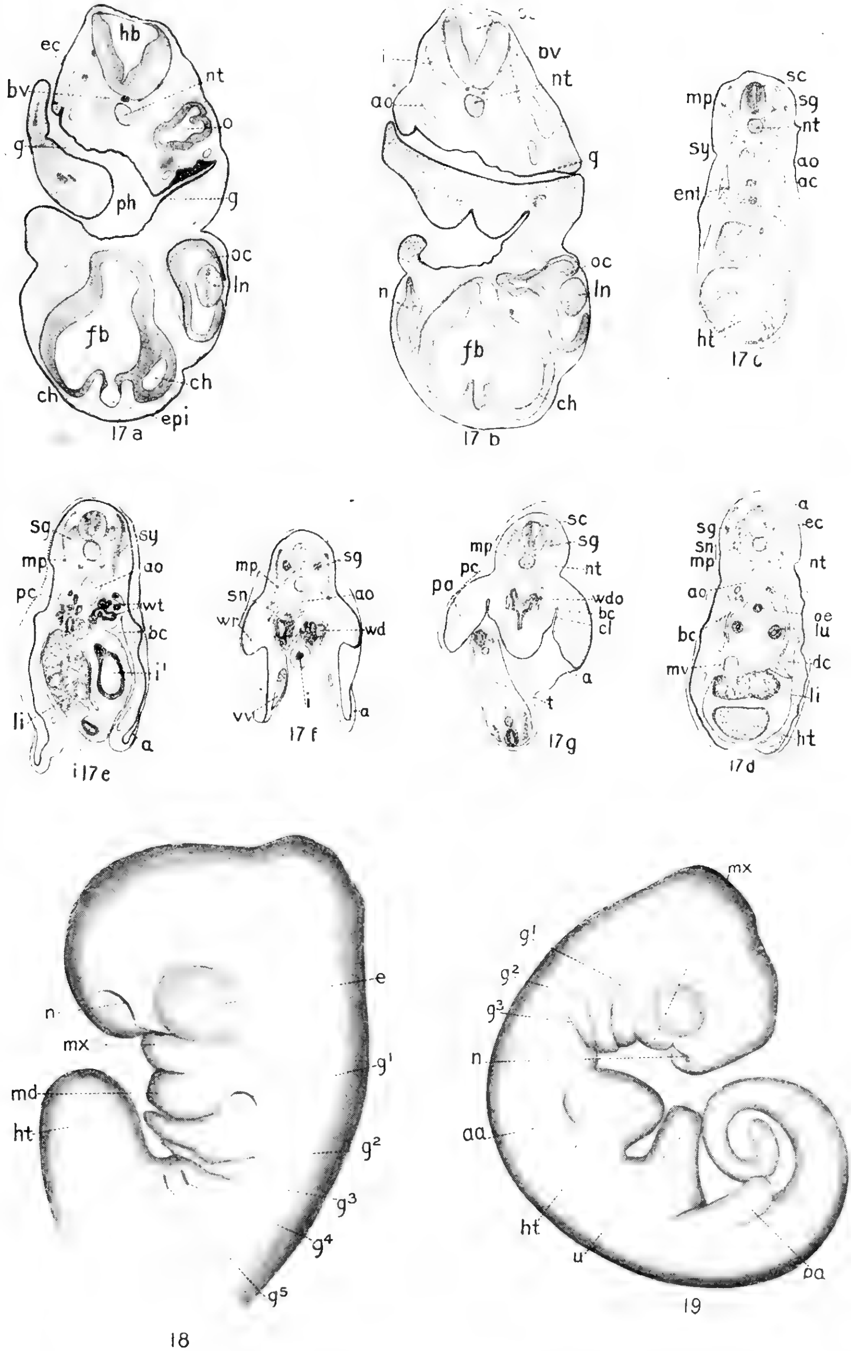
16 f



17

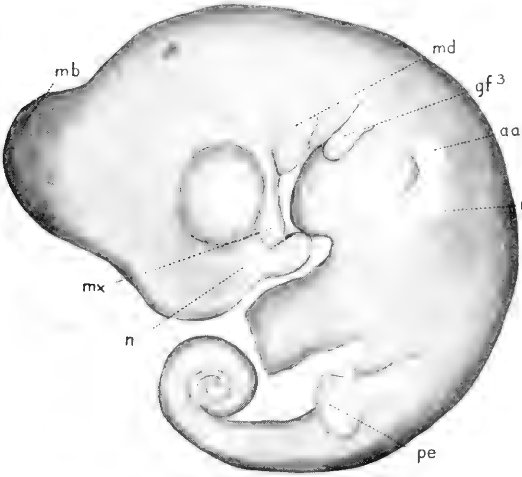
DEVELOPMENT OF THE AMERICAN ALLIGATOR

16c–16g.—Stage XIII. *17.*—Stage XIV

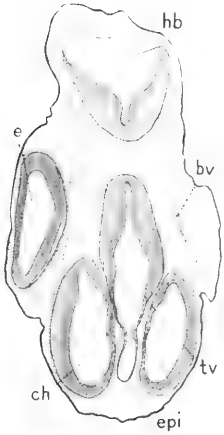


DEVELOPMENT OF THE AMERICAN ALLIGATOR

17a–17g.—Stage XIV. *18.*—Stage XV. *19.*—Stage XVI

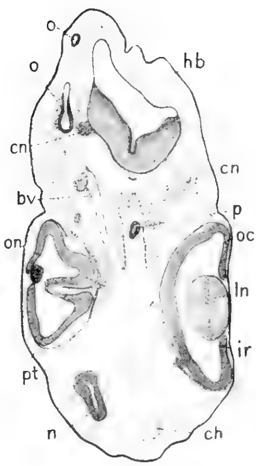


20

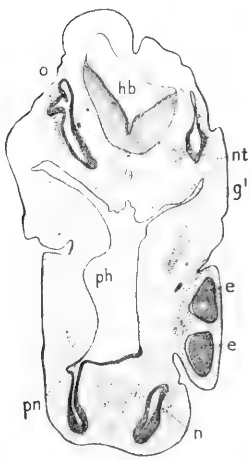


20 a

20 b



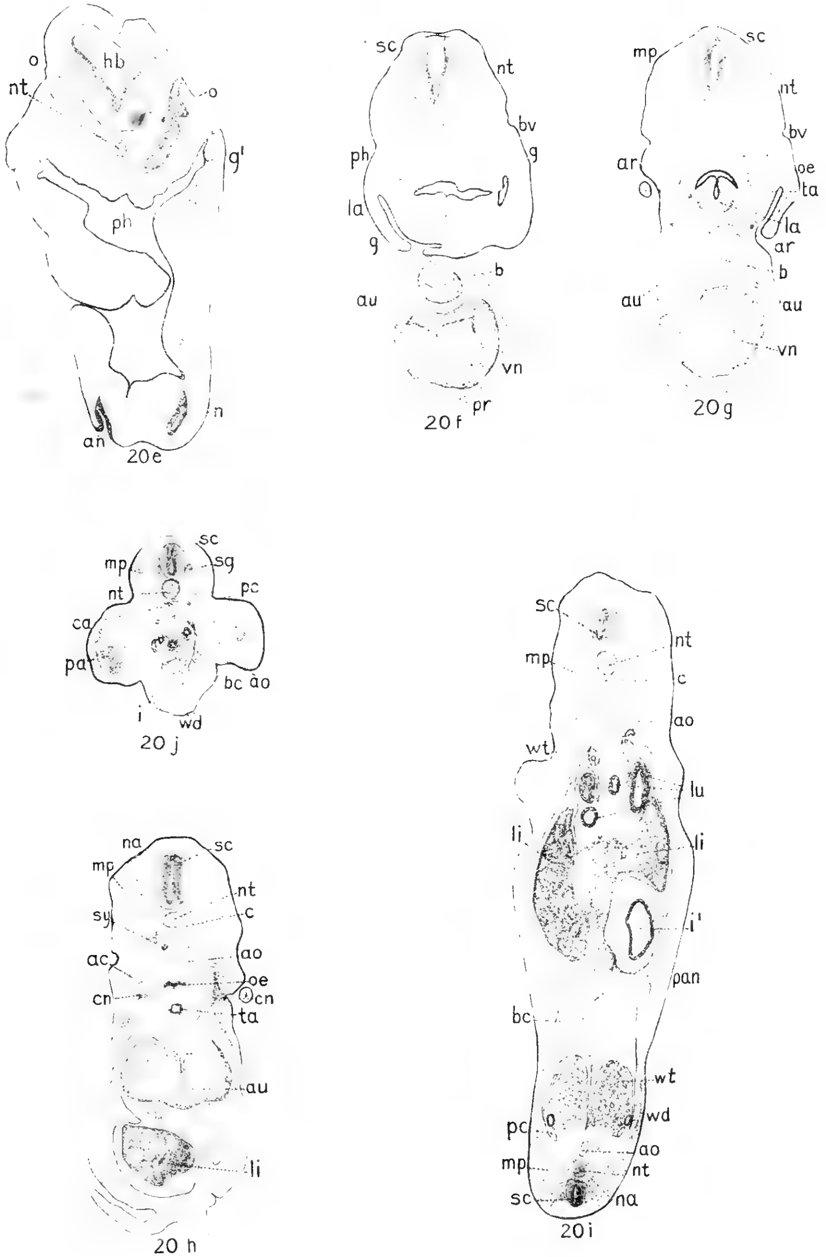
20 c



20 d

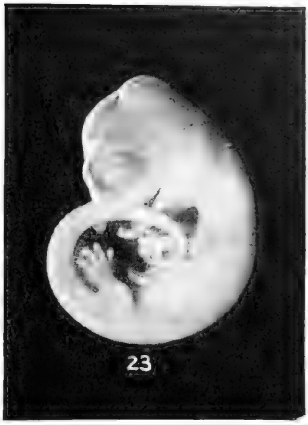
DEVELOPMENT OF THE AMERICAN ALLIGATOR

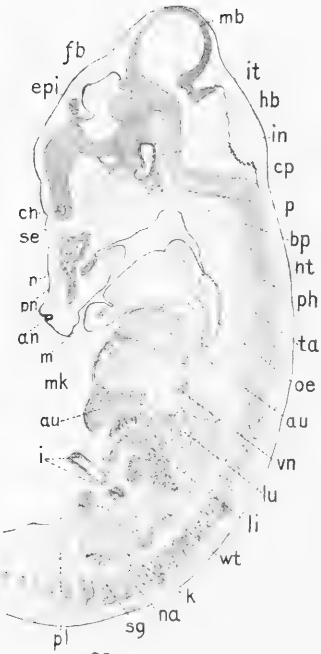
Stage XVII



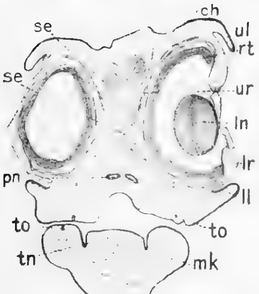
DEVELOPMENT OF THE AMERICAN ALLIGATOR

Stage XVII





23 a



23 b

DEVELOPMENT OF THE AMERICAN ALLIGATOR

21.—Stage XVIII. *22*—Stage XIX. *23-23b.*—Stage XX. *24.*—Stage XXI

25

26

DEVELOPMENT OF THE AMERICAN ALLIGATOR

25.—Stage XXII Alligator embryo. 26.—Stage XXIII Alligator just hatched and relative size of egg.

SMITHSONIAN MISCELLANEOUS COLLECTIONS

PART OF VOLUME LI

THE TAXONOMY OF THE MUSCOIDEAN FLIES, INCLUDING DESCRIPTIONS OF NEW GENERA AND SPECIES

BY

CHARLES H. T. TOWNSEND, B. Sc.

In Charge of Collections of Muscoidean Flies, U. S. National Museum

No. 1803

CITY OF WASHINGTON
PUBLISHED BY THE SMITHSONIAN INSTITUTION
1908

Press of Judd and Detweiler
Washington D C

THE TAXONOMY OF THE MUSCOIDEAN FLIES, INCLUDING DESCRIPTIONS OF NEW GENERA AND SPECIES

By CHARLES H. T. TOWNSEND, B. Sc.

History

When we review the history of the classification of any highly specialized group of insects, provided it has attained a considerable degree of popularity among systematists, we find it to exhibit a well-marked series of oscillations between the two extremes commonly known as bunching and splitting. This is especially true of the dipterous superfamily Muscoidea.[1]

The systematists of the eighteenth and nineteenth centuries, according to the work they did on this superfamily, mark alternate periods of action and reaction which fall conveniently into five historical epochs.

Linné, Fabricius, and Latreille must be considered the pioneers. The system they established was followed by their immediate contemporaries. Very few others concern us here, but Geoffroy erected the genus *Stomoxys,* and Scopoli, Rossi, and Panzer did some work on the superfamily. As a natural result of approaching a quite new subject, these early workers did not always grasp the real value of characters. Largely because of the comparative dearth of material in those initial days of systematic work, they did not clearly discern anatomical values, and hence did not recognize many characters whose worth has since been well established.

Meigen introduced a new epoch in 1804, and considerably increased the number of genera by splitting up the original ones established by his predecessors. Collections had become richer in mate-

[1] It is to be noted that the superfamily Muscoidea, as herein restricted, includes but a portion of the forms to which the name was applied by its author, Mr. D. W. Coquillett. As now restricted, it includes practically the old calyptrate Muscidæ minus the Anthomyiidæ, or the same group as that treated by Brauer and von Bergenstamm—Muscaria Schizometopa, exclusive Anthomyiidæ. The Muscoidea is here divided into five families, as follows: (1) Œstridæ, (2) Macronychiidæ (being a part of the old Dexiidæ), (3) Tachinidæ (including the old Gymnosomatidæ, Phaniidæ, Ocypteridæ, Sarcophagidæ, and most of Dexiidæ as subfamilies), (4) Muscidæ, and (5) Phasiidæ (including *Rutilia* and its allies).

rial by this time, and Meigen's attention was naturally drawn to the discovery of further characters that could be used in classification. He was indorsed and followed by his contemporaries, Olivier, Fallen, Say, Wiedemann, who adopted his genera without proposing new ones, except that the last-named author erected the single genus *Glossina* for the tse-tse flies. Duméril erected the genus *Echinomyia,* and Le Peletier de Saint-Fargeau the single genus *Prosena.* Meigen's best work was in genera. His descriptions of species were in many cases faulty. On the whole, however, he is clearly to be looked upon as an epoch-maker.

The first really intuitive student of the superfamily was Robineau-Desvoidy who, in 1830, introduced the third epoch and very greatly increased the number of genera, besides defining more or less natural taxonomic divisions for their reception. It must be understood that very considerable accumulations of material from the Americas, both North and South, had reached Europe during the early part of the nineteenth century, besides much material from the African, Oriental, and Australasian regions. To most of this Robineau-Desvoidy had access. Notable among the accumulations were the rich collection of the Count Dejean, which had been added to constantly by Latreille, and the quite extensive material secured from all parts by the Museum of the Jardin du Roi in Paris. Palisot de Beauvois, Saint-Hilaire, Bosc, and many others collected in the Americas, and various representatives of the Jardin du Roi in other parts of the world. Besides these, many European entomologists sprang up who began to do much more thorough collecting at home. Thus a comparatively great wealth of material in the Muscoidea was brought together from all parts of the world, both at home and abroad, which stimulated Robineau-Desvoidy to a detailed study of characters in this superfamily. His "Essai sur les Myodaires" remains to this day a monument to his very considerable grasp of Muscoidean relationships. His posthumous work (1863) can not be considered as affecting in any way the status of the "Essai."

Macquart, almost contemporaneous with Robineau-Desvoidy, but possessed of less discernment, bunched the latter's genera to a very considerable extent. However, it must be pointed out in defense of Macquart that he was eminently a general dipterist, while Robineau-Desvoidy was preëminently a specialist in the Myodaria.

Zetterstedt erected only two genera in the superfamily, and practically employed Meigen's genera for all of his work. Perty, Bouché, Guérin, and Bremi each erected a single genus in the superfamily.

Robineau-Desvoidy's system, founded largely on habits, was in a degree faulty and insecure. Attention should be called to the fact, apparently long since lost sight of, that Robineau-Desvoidy originated the idea of including the Œstridæ with his Calypterata (although renounced in his posthumous work), and the Conopidæ with the Myodaria (Conopidæ not included at all in posthumous work). The founding of the now obsolete division Calypterata is also to be accredited to him, though it is to be noted that he did not include the Anthomyiidæ therewith. The latter family was included in that division by subsequent authors. In this connection, see Osten-Sacken for statement that the term "Acalypterata" was interpolated in Robineau-Desvoidy's posthumous work by the editors (Berl. Ent. Zeit., 1896, pp. 329, 335-6).

Róndani marked a fourth epoch beginning about 1850. He revised in large part the work of Robineau-Desvoidy, still further increased the number of genera, was altogether a very close student of relationships, and possessed a remarkably clear insight into the affinities of the Muscoidea, in which he was essentially a specialist. His system was followed to some extent by his more immediate contemporaries, but Schiner, with a fine grasp of dipterous characters in general and little conception of the needs of the Muscoidea, was especially active in bunching his genera.

Schiner was a splendid general dipterist, but the method of treatment adapted to other groups of Diptera fails when the attempt is made to apply it to the Muscoidea. That is where Schiner, Macquart, and all the other conservatists fell. And it is to be noted that these conservatists were always general dipterists. They tried to apply the same system throughout the Diptera, but the Muscoidea need a distinct method of treatment, as will appear further on in this paper under that heading. Even such conscientious students as van der Wulp, Loew, Osten-Sacken, Williston, and others, who followed Schiner largely, but were somewhat less conservative than he, nevertheless fell far short of reaching a requisite degree of radicalism in their views as to a proper treatment of this superfamily.

Others who entered the ranks during this fourth epoch, Walker, Bigot, Bellardi, Jaennicke, Thomson, Meade, von Roeder, Kowarz, Mik, followed Schiner more or less, adopting Róndani and Robineau-Desvoidy at times on certain points, and gradually increased the stock of genera as seemed warranted along more or less conservative lines.

Robineau-Desvoidy had divided the Muscoidea into many smaller groups which he called stirpes, corresponding more or less in value to our present subfamilies. These were not recognized by Róndani, who grouped all into six stirpes. Neither Robineau-Desvoidy nor Róndani were really adopted by Schiner, who recognized eight stirpes, mainly founded, however, on certain of Robineau-Desvoidy's. Schiner thus largely adopted Robineau-Desvoidy's stirpes in those divisions which he did recognize, but bunched his genera along with those of Róndani, Robineau-Desvoidy's reviser. The eight taxonomic divisions adopted by Schiner generally obtained throughout the epoch.

Róndani's system, unlike Robineau-Desvoidy's, took little note of habits, and, while less detailed, was more secure from being founded primarily on external anatomical characters. But these characters were liable to misinterpretation in certain cases.

Brauer and von Bergenstamm inaugurated the present and fifth epoch in 1889, which is destined to hold out for a greater degree of radicalism than its predecessors. They approached the subject largely in a new way, greatly lessening the difficulties of classification in the superfamily by recognizing a large number of sections which correspond to the subfamilies and tribes of the present paper. At the same time, they greatly multiplied the number of genera, whereby they were able to present comparatively concise diagnoses of these, as well as of their sections.

They adopted Robineau-Desvoidy's plan of grouping the forms into many small divisions, but they did not feel bound, as did he, to adhere to any definite scheme of life habits for indicating taxonomic limitations. In the main their divisions were made on quite original lines. However, many of Robineau-Desvoidy's old stirpes are still recognizable, now more or less revised, restricted or enlarged, and they must be considered as the original foundation of our present subfamilies and tribes. Brauer and von Bergenstamm's characters were better chosen and represent a more exhaustive study of the subject, as would naturally follow from their having enjoyed the greatly superior advantages derived from marked increase in biologic progress since the time of Robineau-Desvoidy and Róndani, and access to the greatly enriched collections of material drawn from all parts of the globe.

Until quite recently Brauer and von Bergenstamm's system has been followed rather indifferently—in some cases enlarged upon, in some revised—by students of the group contemporaneous with them and continuing in the work since their time. The general trend of sentiment now, however, is strongly in their favor, recognizing, as it does, the necessity of a subdivision of the superfamily into many subfamilies, tribes, and genera, so as to allow of more careful and concise diagnoses. While it is true that a middle course between the two extremes of conservatism and radicalism is usually the best one to follow, the present superfamily furnishes a notable exception to the rule in that it can not be successfully treated on other lines than what are to be considered as quite radical compared with the treatment accorded to other superfamilies in the order.

In this historical review, Robineau-Desvoidy, Róndani, and Brauer stand forth prominently as the greatest students of the Muscoidea that the world has produced. Each had a deeper insight into the peculiar relationships and affinities of the superfamily and a closer grasp of the subject as a comprehensive whole than any of his predecessors or contemporaries.

The following is a tabular arrangement of the five epochs, with the respective students who belong to each, including the approximate periods during which they were more or less active in work on the superfamily. The asterisk indicates those authors who established one or more genera. The plus sign indicates work continued to the present time:

EPOCH I (prior to 1804).

Redi, 1671–1712 (general insects).
Réaumur, 1738–1740.
Scopoli, 1760–1763.
*Linné, 1761–1766.
Poda, 1761.
*Geoffroy, 1762 (one genus—*Stomoxys*).
*Fabricius, J. C., 1775–1805.
De Geer, 1776.
Schranck, 1781–1803.
Herbst, 1789–1801 (general insects).
Rossi, 1790.
*Latreille, 1792–1805 (*Trichopoda, Bucentes, Hypoderma, Ocyptera, Œdemagena*).
Panzer, 1793–1809.
Baumhauer, 1800.
Illiger, 1801–1807 (general insects).

EPOCH II (1804–1830).

*Meigen, 1804–1830.
Schoenher, 1806–1817 (general insects).
Gyllenhal, 1808–1829 (general insects).
Dufour, 1809–1833.
Olivier, 1811.
Germar, 1813–1821 (general insects).
Fallen, 1814–1825.
*Clark, 1815 (one genus—*Cuterebra*).
Lamarck, 1815–1822 (general invertebrates).
*Leach, 1817 (one genus—*Gastrophilus*).
Say, 1817–1832.
*Duméril, 1819 (one genus—*Echinomyia*).
*Wiedemann, 1821–1830 (one genus—*Glossina*).
*Le Peletier de Saint-Fargeau, 1825 (one genus—*Prosena*).

EPOCH III (1830–1850).

*Robineau-Desvoidy, 1830–1863.
*Perty, 1830–1834 (one genus—*Diaugia*).
Haliday, 1832.
*Macquart, 1834–1855.
*Bouché, 1835–1847 (one genus—*Compsilura*).
*Guérin, 1835–1850 (one genus—*Formosia*).
*Zetterstedt, 1838–1855 (*Wahlbergia, Cinochira, Gymnopeza*).
*Bremi, 1846 (one genus—*Amsteinia*).

EPOCH IV (1850–1889).

*Róndani, 1850–1865.
*Walker, 1850–1866 (*Doleschalla, Schizotachina, Hammaxia, Saralba, Toroca, Zambesa*).
*Egger, 1856 (*Zelleria, Halidaya, Frauenfeldia, Microphthalma*).
*Doleschall, 1856 (*Spiroglossa, Megistogaster*).
*Brauer, 1858–1889.
*Bigot, 1859–1893.
Bellardi, 1859–1862.
*Meinert, 1860–1880 (one genus—*Philornis*, larva).
*Loew, H., 1861–1872 (*Stegosoma, Blæsoxipha, Euthera, Himantostoma, Phylloteles*).
*Schiner, 1862–1868.
*Jaennicke, 1867 (one genus—*Archytas*).
*van der Wulp, 1867–1903.
*Thomson, 1868 (*Glaurocara, Tricharæa*).
*Osten-Sacken, 1877–1902 (one genus—*Urodexia*).
*Pokorny, 1880–1896 (*Parastauferia, Sarromyia, Steringomyia, Trigonospila*).
*Meade, 1881–1899.
*von Roeder, 1881–1896.
*Kowarz, 1882–1894 (*Ctenocnemis, Mikia*).
*Mik, 1882–1901 (*Crossocosmia, Zygobothria, Microtachina, Microtricha*).
*Williston, 1886 + (*Melanophrys, Acroglossa, Talarocera, Dichocera, Melanodexia*).

EPOCH V (1889 +).

*Brauer, 1889–1899.
*von Bergenstamm, 1889–1894 (co-author with Brauer).
*Portschinsky, 1890–1902.
*Schnabl, 1890–1902.
*Giglio-Tos, 1891–1897.
*Wachtl, 1891–1895.
*Townsend, 1891 +
*Girschner, 1893–1901.
*Meunier, 1892 +
*Strobl, 1892 +
Bezzi, 1892 +
*Pandellé, 1894 +
Becker, 1894–1901.
Snow, 1895.

Corti, 1895–1897.
*Austen, 1895 +
*Coquillett, 1895 +
*Hough, 1898 +
Kertész, 1899 +
Robertson, 1901 +
*Bischof, 1901 +
*Grimshaw, 1901 +
*Hendel, 1901 +
*Hutton, 1901 +
Villeneuve, 1902 +
Wainwright, 1902 +
*Speiser, 1903 +
*Johnson, 1903 +

Treatment

Speaking of the Muscoidea, Dr. Williston has said: "Species, genera, and even families, show such slight plastic or colorational differences that only the most patient study will define their limits. At the present time there is a decided tendency to base the classification of even the higher groups upon apparently trivial characters. Most naturalists have long since abandoned the idea that genera, or even families, represent anything but the conveniences of classification, and the recent writers on this family are probably right in seizing upon any characters that will satisfactorily group the vast number of species irrespective of their relative values. But it is very probable that, in the proposal of so many genera in such rapid succession, many characters have been employed which future research will show to be entirely inadequate. We yet know very little about individual variations in this family, or the real value of many of the characters now used. The absence or presence of a bristle may be found to represent a group of species, but we should first learn how constant the character is in species. * * * Seriously, is not the stock of Tachinid genera sufficiently large for the present? Would it not be advisable to study species more before making every trivial character the basis of a new genus?"—Insect Life, vol. v (1892-3), pp. 238-40.

These words, from the leading authority on American dipterology, written some fifteen years ago and shortly after the appearance of the first two instalments of Brauer and von Bergenstamm's work, may advantageously be taken as a text for some pertinent considerations at this time.

While the great multitude of forms in the Muscoidea seems at first sight chaotic and formidable, the student soon perceives that standing forth from the general mass there occur certain well-marked generic types, such as *Œstrus, Cuterebra, Dexia, Macronychia, Phasia, Trichopoda, Meigenia, Masicera, Phorocera, Tachina, Gonia, Belvosia, Plagia, Thryptocera, Phania, Œstrophasia, Miltogramma, Pyrrhosia, Ocyptera, Gymnosoma, Echinomyia, Hystricia, Dejeania, Sarcophaga, Calliphora, Musca, Stomoxys, Glossina,* and at least a hundred others. These types correspond in value to the more settled genera of the older superfamilies, where intermediate forms are largely lacking. In the present superfamily, however, it is quickly seen that massed in between these many typical forms are numerous intermediate ones, which collectively vary in all directions and combine certain of the characters of the various types. These intermediates are the bridges for the passage of genera, so to speak—the inevitable precursors and resultants in the process of the evolution of genera. The same holds good of species. Numerous intergrades are found to group naturally around and between the various species. That these intermediates and intergrades are present is due to the fact that the Muscoidea are now—at the present day, geologically speaking—in their period of greatest prolificacy, a period characterized by a condition of multiform development. After the lapse of a great space of time, many of these intermediate forms will have dropped out of the struggle, leaving a residue more or less well defined from each other and thus much more amenable to taxonomic treatment. This is now the case with the older dipterous superfamilies, which have long since passed their period of greatest prolificacy.

It should be explained that the term "intermediates" is used to designate forms of generic rank or higher, and "intergrades" to designate those which are only of specific rank. The further term "intergradants" may be employed to designate individuals which connect species, but upon which it is not practicable to bestow names.

The Muscoidea are of very recent evolution—in fact, their evolution is still going on. Here are species, genera, and families in the making. The whole superfamily is one enormous assemblage of thousands upon thousands of forms distinguishable from each other by only slight differences and exhibiting characters which intergrade in all directions. That such a multitude of closely similar forms is exceedingly difficult to classify goes without saying. These forms can not be classified in the ordinary way, but demand special treatment adapted to the conditions.

The key to the whole situation, when it comes to methods of taxonomic treatment in this superfamily, is that we have here the task of defining not only the numerous well-marked types corresponding to the existing forms in the older and less specialized dipterous superfamilies, but also a great mass of the intermediates, intergrades, and intergradants that have resulted during the long-continued process of the evolution of these types.

Brauer and von Bergenstamm recognized these conditions in the Muscoidea and treated the superfamily accordingly. As being highly apropos of this subject, the following remarks are quoted from the translation of these authors' Introduction (published in *Psyche*, vol. VI, pp. 313-16, and 329-32), the whole of which can be studied with much profit:

"It is a fundamental principle in the development of the whole dipterous stock that, from the lowest (*Orthorrhapha nematocera*) to the most differentiated or highest (*Cyclorrhapha schizometopa*), the actual value of the genus, and of the systematic series generally, becomes less and less. This proposition seems applicable to all groups of animals—in all cases the most recent forms are more closely related and more difficult to characterize than older ones. . . . The cause lies in the numerous intermediate forms occurring in a group of animals which has just reached its period of greatest prolificness."

As the same authors point out farther along in their Introduction, it is absolutely futile to attempt a classification of these flies along any other lines than a separation into many comparatively restricted categories. The authors are also correct in maintaining that the classification of all animals must be based on the entire development—not on the adult alone. The characters of the imago are most important for genera and species; those of the earlier stages are most important for families and higher categories, even up through orders and classes. In studying early stages, it may be pointed out that some characters will occasionally serve for generic separation, but much judgment must be exercised in deciding which characters are of value for this purpose, since conspicuous ones may in some cases possess less than generic value. Such are those of special adaptation to peculiar conditions of life.

The fact should be recognized, as suggested in the opening text to this chapter and emphasized in the quotation just given, that generic values are not necessarily uniform throughout the organic world. It is fallacious to attempt to set a standard whereby plant and animal genera, or animal genera alone, shall be gauged by a certain fixed measure of difference. This holds good even in different superfamilies of the same order or suborder of insects. The demands of the group in hand must be considered in each case. A superfamily in the multiform stage of development, contingent upon its being still in process of evolution, demands a less generic value than an older and well established superfamily whose forms have become fixed through a long period of conformity to their environment. If this be not conceded, it becomes impossible to treat the younger superfamilies by any satisfactory system.

It will be alleged by some that such plan will result in multiplying genera unduly. There is, however, no doubt that the course adopted is warranted by the conditions. This conclusion has been reached after full and mature deliberation. The only possibility of successfully systematizing the superfamily, so that its myriads of forms can be designated definitely by name, lies in the recognition of genera founded upon comparatively slight characters—slight compared with those recognized as the standard in the older and less specialized superfamilies. The differences between genera are less pronounced in the more specialized than in the less specialized groups. All are genera, and of equal value systematically; but, as already pointed out, they can not be measured by a standard gauge.

The writer has always contended that a proper treatment of the Muscoidea demands the definition of smaller categories and more carefully restricted genera (see *Psyche,* vol. VI, p. 313, Sept., 1892). As the characters of the early stages are investigated, more light will be thrown on higher divisions in the superfamily. Such a vast assemblage of closely related forms is not amenable to separation, in the adults, into divisions conceived on lines of mathematical precision. Any system of classification must become more or less artificial if it attempts, in the presence of intermediates and the absence of a knowledge of early-stage characters, to mark off precise lines of division between categories of higher value. When the intermediates are lacking, or largely lacking, it becomes a comparatively easy matter to fix the lines of demarcation, and the system appears extremely natural simply through the absence of the immense mass of intermediate forms that at one time existed. But when these numerous intermediates and intergrades are extensively present, any attempt to apply an arbitrary system of classification to the group can not but result in disaster. A system can be thoroughly natural only in so far as it indicates natural types of families, subfamilies, tribes, and genera, and groups the intermediates and intergrades around them. Properly conceived and executed, such a system is the only natural one, since it must accord with the facts as known. At the same time the fact must not be lost sight of that taxonomy is at best merely a means to an end, and does not exist in nature. It is artificial in its original conception, because it is practically intended to ignore numerous steps in the development of life—steps that have been lost during the evolution of forms now existing, and which, if still present, would make a taxonomic system simply impossible.

Taking these points into consideration, there is evidently but one course open. Draw lines of demarcation between the best marked types, and let the others, with their respective coteries of intermediate forms, fall in whatever divisions a preponderance of their characters in each case indicates. Definitions of characters for the higher divisions can not be exact, because the forms themselves in nature do not fall into well defined divisions.

Such a system as outlined would recognize typical forms as genera and species, and would then intercalate necessary additional genera and species for the convenient reception of the intermediate forms, which group around the typical ones and connect them with each other. The one great difficulty here will be to arrive at the true relationships of the intermediate forms, for their affinities are often so complex that it is very hard to decide with what genus or species they are most closely related. The real truth will ultimately be attained only after many years of continued research into their ontogeny, combined with an exhaustive study of the geological history of the superfamily.

What have been called typical forms, both genera and species, it is proposed to term *typic*. The additional genera and species to be intercalated between the typical ones it is proposed to term *atypic*. We will thus have a system of *typic genera* and *atypic genera* for the reception of typical genera and intermediates respectively, and *typic species* and *atypic species* for the accommodation of the typical species and intergrades respectively. This scheme accords with the facts, which do not conveniently admit of the employment of subgenera and subspecies. The latter concepts are here inapplicable on account of the nature and intricate relationships of the forms. To include subgenera, the genera would have to be too loosely characterized. Furthermore, this scheme preserves the binomial nomenclature, which is highly desirable. It can be designated in each case whether a genus is typic or atypic, if this is found desirable.

All the more primary divisions—those above the subfamilies, up to the very subordinal divisions themselves—can at present be only imperfectly characterized and defined. Here is where aid will be derived from early stage characters, when these become known. Even the Cyclorrhapha and the Orthorrhapha[1] can not be sharply

[1] The writer is aware that Osten-Sacken claims there is a clearer line of separation between the Nemocera and Brachycera than between the Orthorrhapha and Cyclorrhapha, but this is outside our subject.

differentiated from each other in the adults on account of intermediate forms. Less and still less grows the clearness of limitation as we descend through the series, sections, subsections and superfamilies to the families. Limitations clear a little in the families, but it is not until we get to the subfamilies and tribes that we can, from a study of the adults, begin to draw moderately well marked lines and set fairly concise limits. A moderate degree of conciseness is possible here only because we are now concerned with divisions sufficiently low in the taxonomic scale to allow the exclusion of refractory and disturbing elements, and if necessary put them alone by themselves. Many subfamilies and tribes are seen to stand out as natural groups of genera.

At first sight it would appear advisable to ignore the higher divisions, and drop at once to the very considerable number of subfamilies and tribes necessary to the system outlined. But it evidently serves a better purpose to recognize these higher categories, however much their boundaries may be obscured by connectant forms. They are certainly present, and their existence should not be lost sight of. Therefore they should be retained in any taxonomic system as indicating steps in the evolution of these flies. They may be kept somewhat in the background, with the caution that they can not be clearly and concisely defined until the ontogeny of the intermediate forms is known.

Many genera stand more or less apart and do not fall actually into any of the subfamilies. Very restricted groups of such genera, which may be termed refractory on account of either their complex relationships or their apparent neutrality with reference to the various subfamilies, will best be treated directly as tribes, without reference to any particular subfamily.

Some few genera will prove to be quite isolated, and yet not entitled to subfamily or tribal rank. A final system should aim at the definition of as many well-marked subfamilies and tribes as possible to concisely characterize, and the consequent reduction of the number of these isolated forms. A comprehensive table can thus be prepared, including the subfamilies, the non-referable tribes, and the non-referable genera in one synoptic treatment, which will be convenient for general use. Separate tables can follow defining the genera within each subfamily and non-referable tribe. No attempt should be made to force refractory genera into any subfamily or tribe where they do not fall naturally, or any tribe into any subfamily where it does not clearly belong, or to antagonize natural affinities in any way, or to combine refractory forms in one heterogeneous tribe or subfamily. The refractory elements should rather be left to stand alone.

In such manner as the above will it be possible to work out a serviceable system of classification, which will indicate, so far as may be, the true relationships, and at the same time preserve approximately the relative values of taxonomic divisions in the Cyclorrhapha.

A very important point remains to be noticed: What is a species in this superfamily? The preceding remarks on intermediate forms apply especially to the higher divisions, but are also largely true of genera and species. The difficulties as to genera can be practically overcome by the erection of a sufficient number to accommodate all the intermediates. But who can tell what is a species in nature, and especially what is a species in the Muscoidea? It is clear that we must have a definition that will answer to the term. In large assemblages of insects, where intergrades and intergradants have not been lost, there is no such thing as a species in the generally accepted sense. No sharp specific distinctions can be drawn in such cases. The term is a necessary conception in taxonomy, however, and it is to be noted that the only reason for its employment is the necessity for being able to distinguish between assemblages of individuals that are unlike. Therefore it seems clear that the only safe course to pursue is to give a name to every assemblage that can be distinguished from other assemblages.

It is proposed to use the term "species" in a well-restricted sense. Typic species are already explained. The term atypic species will be used for recognizable assemblages of individuals grouping around typic species. The term "forms" may be used interchangeably as referring to either or both.

When two atypic species are connected by intergradant individuals, the former should be given names and the latter referred to as intergradants between the two atypic species. A few words of descriptive matter will serve to fix practically the exact taxonomic position of these intergradants. Such a course will afford students of bionomics an opportunity to attain some degree of definiteness in their investigations. As the names now stand in the Aldrich Catalogue, this element of definiteness is totally lacking. Many distinct forms are bunched under one name on almost every page. Absolute exactness is impracticable in this phase of nature, where variation through pressure of environment is constantly at work in the evolution of new forms. But a reasonable degree of definiteness is possible of attainment. So long as we can refer by name to recognizable forms, we may be certain that we are not going wrong. Such forms should not be bunched merely because it is difficult to distinguish them. If it is possible to separate them, they should be separated.

The conviction is constantly growing among biologists that we really do not comprehend species. Multitudes of insect forms have been confused under one specific name since systematic entomology began. The scientific concept of the invertebrate species is gradually growing less vague and more restricted. There is practically no doubt that in most groups of insects, the Coccidæ excepted, there are many times more forms that will eventually be termed "species" than have heretofore been recognized. Every year new results obtained from a study of the early stages of insects force this conviction upon us. (The Coccidæ probably form an exception. Mr. J. G. Sanders is authority for the statement that the species have been largely split on characters pertaining to different ages of the same stage.) Without doubt, bunching is infinitely more harmful to a system of classification than splitting. Splitting, even if injudiciously done, does not give rise to actual error, but bunching produces all kinds of error in the bionomic literature, which errors, moreover, are irremediable except through a restudy of the specimens originally referred to. It goes without saying, however, that forms can be properly separated only on constant structural characters pertaining to the same age or stage of development, and on color, form, and size only when such are known to be constant. A plea is herewith entered for judicious splitting,[1] up to the limit of practicability. A reasonable degree of conciseness in the designation of forms of insects is absolutely unattainable by any other means.

A word is not out of place here bearing upon the causes of variation which give rise to vast multitudes of forms during the period of greatest prolificacy of a group in any order of life.

Mr. W. L. Tower, in his paper on *Leptinotarsa* (Carnegie Institution of Washington, Publication No. 48), has demonstrated that variation is not inherent in the germ plasm, but is invariably induced by external stimuli acting thereon. The demonstration consisted of several experiments in which the stimuli were directly applied to pregnant females of *Leptinotarsa,* so as to reach the germ plasm within the contained ova. This one point is by far the most important contribution to science that the author makes in the whole of his long and highly instructive paper. All variations are directly caused by the action of external stimuli—such as heat, humidity,

[1] This term is adopted in a serious sense because it is both apt and expressive. Splitting can be accomplished only along lines of formation or natural cleavage, and this is true of the proper division of taxonomic groups.

atmospheric pressure, food, etc.—in other words, by the pressure of environment, which means all stimuli taken together and acting together.

It is thus seen that climatic or meteorologic conditions are potent factors in the evolution of forms of life, and that as a rule one form does not inhabit two widely different life zones or areas. Few, if any, forms inhabit both temperate and tropical regions, or both humid and arid regions. The external stimuli natural to the different zones and areas result in the modification of forms coming within the sphere of their influence, and the consequent production of new forms. Thus the progeny of individuals of one and the same form, spreading gradually through areas where they become subjected to new sets of stimuli, are gradually differentiated into distinct forms through the pressure of environment. Dr. Merriam's exposition of this law in his address before Section F of the American Association for the Advancement of Science, at its 55th meeting (Proc. Am. Ass. Adv. Sci., 1906, pp. 387-9), is an admirable one, and can be studied with much profit. His observations, as there given, agree perfectly with the results of the writer's studies in Diptera. For instance, the arid and humid regions of North America will be found to possess very few species in common. These very different life areas are divided and subdivided by temperature as we go north or south, or ascend above sea level, and again and again subdivided by various climatic and other environmental factors. The result is many separate life areas, more or less restricted, each of which exhibits a distinct stamp of environment. Intergradations occur along the peripheries of the ranges of closely related species, when such lie contiguous, as they often do. These intergradants must not be confused with the normal specimens of the form as exhibited throughout the more central portions of the area of range. That the intergradants occur between two such forms does not invalidate the distinctness of the forms themselves.

It may safely be stated as a theorem in bionomics that, given an arid area and a humid area contiguous to each other, both originally stocked with individuals of the same form—whether of Diptera or any other order of life—the descendants of this form will not remain identical in the two areas throughout any considerable period of time. The theorem may be enlarged to include temperate and tropical contiguous areas, and many divisions and subdivisions of these and of the arid and humid areas as well. The resultant differentiation is brought about by dynamic variation, incited by the respective sets of external stimuli acting on the germ plasm of the ova con-

2

tained within pregnant females of the form, already referred to as demonstrated by Tower.

A careful study of these factors and of the results produced by them demonstrates the fallacy of the idea that forms from the north Atlantic coast region of the United States and the south Gulf coast region of Mexico are identical. In other words, forms originally described from Vera Cruz are not to be identified in Massachusetts material. Likewise, forms from arid regions are not to be identified in humid region material. Furthermore, European species are not to be identified in American material, except in the few cases of forms that have been imported through the agency of man. There exist today practically no Muscoidean forms common originally to Europe and North America. The Muscoidea did not originate from circumpolar stock. The forms that immigrated to northern America from Eurasia during the warm periods that existed in the subarctic region in interglacial times have long since given rise to new forms, and no longer persist in their original state.

There are certain more or less cosmopolitan flies, such as *Musca domestica, Stomoxys calcitrans, Lucilia cæsar, Calliphora erythrocephala,* and others, which find their natural environment in the wake of man. These are not so amenable to the above factors, but even they show some effects of their agency. A considerable number of such species doubtless accompanied primitive man in his wanderings through various parts of the earth. Other species are of comparatively recent dispersion through commercial agencies. Both classes have been involuntarily spread by man. The detection of the second class calls for extremely careful study and fine powers of perception. Still another and very recent class has been purposely spread by man for economic ends.

A word may be said as to the difficulty of distinguishing between many of the distinct but closely similar forms that occur in the Muscoidea. While many of these forms that closely resemble each other do so by virtue of their close relationship through common origin, it is evident that others of more diverse origin have developed a close resemblance through counterfeitism[1] attained by means of natural

[1] The writer herewith proposes a change from the use of the words *mimic, model,* and *mimicry*. The terms "mimic" and "model" have nothing, except usage and priority, to commend them. "Mimic" is exceptionally faulty, and does not nearly convey the intended meaning. In the strict sense of the word a mimic is one who, by sound or action, imitates another. The word does not imply any idea of form, color, or size. The word "counterfeit," however, embodies the full concept. Again, "model" does not carry the idea of size, and in an art sense only partially that of form; moreover, it is not necessarily

selection and the pressure of environment. This may be termed convergent evolution. Somewhat similar are cases of parallelism, or recurring types of structure in nowise related to one another, which are to be explained by use and adaptation to external conditions. A thorough study of larval and puparium characters will determine such cases beyond a doubt, but in many instances an intimate knowledge of the adults will enable one to separate these forms quite accurately. Parallel series in the adult of forms of common origin will usually show their distinctness very readily to the experienced eye without a lens. In this way the writer has often made a preliminary arrangement of much material, which subsequent study demonstrated to be correctly separated into distinct forms, many of them so closely resembling each other that they were extremely liable to be confused. A very serviceable guide in distinguishing between forms of common origin is the character and color of the pollen, which is present to a greater or less extent in all the forms. This, strange to say, is extremely constant throughout series of individuals of the same form and the same sex. In those forms which possess golden pollen on the head, the male as a rule has the golden shade more pronounced and extensive than the female. The color of the pollen of thorax, and especially that of abdomen, is very constant in both sexes. A slight difference in the shade of color of the abdominal pollen, such as that between a silvery cinereous and an ashy cinereous, will frequently serve to correctly separate closely related but distinct forms which might otherwise be confused. It is almost needless to say that reference is here made only to fresh and well-preserved specimens. Greased specimens must be restored before attempting to place them.

Illustrative of convergent evolution and parallelism, in which adults of two or more forms closely resemble each other through causes other

---

imitated, often has no relation to color, and may even be a miniature or other representation and not the original at all. "Pattern," on the contrary, means the original, to be imitated as to form, size, and color, strictly speaking, and is the term used in mechanics in the exact sense of our concept.

By using these terms—counterfeit and pattern—we can adhere strictly to the significance of our diction. We would thus speak of an edible *counterfeit* (species) of an inedible *pattern* (species), which latter has been unconsciously and involuntarily adopted by the former as a subject for imitation, impelled thereto by certain accruing advantages. Both words express the sense exactly, and both can be used without change as either nouns or adjectives. Derivatively, instead of the objectionable term "mimicry," we have the very suggestive and thoroughly appropriate name *counterfeitism* to apply to a subject of rapidly growing importance. It would seem that neither priority nor usage have any claim to consideration in a case of this kind.

than those implied in close relationship, the following is an excellent case in point among the Coleoptera. Mr. W. Dwight Pierce makes the statement that three species of *Anthonomus* (*A. nigrinus, æneolus,* and *albopilosus*), which breed in the flower-buds of *Solanum* spp. (*S. carolinense, eleaginifolium, rostratum,* and *torreyi*) in Texas, resemble each other so closely in the adult that they are often confused by experienced coleopterists. Yet Mr. Pierce, who has studied the early stages of these species, has found that the anal characters of the pupæ serve to readily distinguish them. *A. æneolus* and *nigrinus* belong in the same group, are distinguished in the pupa by a slight difference in the proportions of the posterior terminal structures of the anal segment, and in the adult only by color. But *A. albopilosus* belongs in a distinct group, is inseparable in the adult except by leg characters, and markedly different in anal characters in the pupa. *A. albopilosus* is thus a case of convergence toward *æneolus* and *nigrinus*, which two are closely related forms. It should also be mentioned that *albopilosus* has been found recently breeding in great numbers in buds of *Croton* spp. Dr. Chittenden is authority, however, for its former breeding in *Solanum* spp.

The reasons for such convergent evolution or parallelism are often difficult to ascertain and are outside our subject. This case is introduced from the Coleoptera merely as paralleling certain very similar ones in the Muscoidea. For example, the species *Achætoneura datanarum, A. promiscua,* and *Parexorista futilis* seem to form a group similar to the above species of *Anthonomus*. The first two are closely related, and the third furnishes a case of convergent evolution in their direction. All three forms are entirely cinereous pollinose, have the anal segment brassy, and the parafrontals and parafacials golden pollinose. (*Achætoneura frenchi* has a different facies, but has been confused with the first two.)

Similar groups will be found in the genera *Tachina, Masicera, Phorocera,* etc. Another group is probably exemplified in *Myiophasia* spp., *Phasioclista metallica, Ennyomma clistoides,* and certain other species.

Such conditions as the above explain why specimens of tachinids looking strongly alike and bred from the same caterpillar, perhaps issuing on the same date, are at times found to belong to different forms and ever to different genera. In such of these cases as are due to convergent evolution and parallelism, the larvæ and puparia will be found to exhibit better differential characters than the adults. No work connected with the taxonomy of the Muscoidea could more solidly advance our knowledge of the subject than the careful and painstaking study and rearing of the early stages. It is a most promising and inviting field, and one whose problems are intimately woven with subjects of broad biologic significance.

It may be pointed out that the well-known promiscuity of oviposition with reference to hosts in the Muscoidea is another evidence, and a necessary result, of the geologically recent evolution of the superfamily. The Microhymenoptera are of far more remote evolution, as evidenced by the fact that each genus is restricted to a group of hosts. Microhymenopterous parasites bred from host larvæ belonging to different families may safely be pronounced offhand to belong to different genera. This demonstrates a fixed habit of oviposition that has endured through a long period of time. No such fixed habit is to be found among those Muscoidea parasitic upon lepidopterous larvæ, or among any of the superfamily except the Œstridæ.

It has been alleged that much of the so-called synonymy in this superfamily, as it stands in the Aldrich Catalogue, is due to a misguided erection of species on stunted specimens developed from underfed larvæ, through a lack of acquaintance with the breeding habits of the species. It is well known to all students of the Muscoidea that the females sometimes, if not frequently, carry the act of oviposition to an extreme, ovipositing upon larvæ that are already overstocked with eggs. This has been observed and recorded in a number of instances. It has been observed at the Gipsy Moth Laboratory of the Bureau of Entomology in Massachusetts that tachinids would oviposit at times upon larvæ covered with eggs, while masses of unstocked larvæ were abundant close by. Some of the unmolested larvæ were dissected and found unparasitized. This, moreover, was in the open, outside the breeding cages. However puzzling this may seem, it is certainly unsafe to draw conclusions as to habits from observations made in the gipsy moth area, since the equilibrium of the various forms is in a state of extreme unrest. This is due not only to the enormous increase of comparatively newly introduced host elements in the fauna, but also to the more recent introductions of new parasitic species, both tachinid and microhymenopterous. These agencies have so disturbed the balance between species that the resultant conditions have become highly artificial. Similar conditions could hardly arise except through man's interference. Had the gipsy and browntail moths and their parasites spread into Massachusetts from a contiguous area, the change of equilibrium between them and the resident fauna would have taken place more gradually, and the balance between species would not have been so suddenly upset. It is not at all likely that tachinids oviposit so heedlessly as above observed, provided they are subjected to thoroughly normal conditions.

As far as the recognition of stunted and underdeveloped individuals of a form is concerned, there is rarely any difficulty provided one is familiar with the characters. The stunted specimens always exhibit practically the same characters, and if there is any exception the true status of a specimen is quite recognizable.

Characters

The following outline of the construction and development of the head capsule in *Calliphora,* principally drawn from Lowne (Anat. Blowfly, pp. 114-16), forms a fitting introduction to a consideration of characters, inasmuch as those of the head take precedence over all others in the taxonomy of the Muscoidea.

The Metacephalon comprises the segmented *post-oral* portion of the head.

The Paracephalon, which is formed of the two *paracephala,* or two *lateral procephalic* lobes of the nymph, comprises the *pre-oral* portion of the head.

The paracephala bear the compound eyes and antennæ.

They are united in front and below and form the epistoma and labrum.

The portion of the *facial paracephalon* behind the epistoma shows three distinct parts. These are two bladder-like swellings, the *anterior* and *posterior cephaloceles,* and the *antennal ridge* between them. The last is developed by a process from each of the two lateral procephalic lobes.

The anterior and posterior cephaloceles correspond with the thin portion of the blastoderm which intervenes between the two lateral lobes or paracephala.

The posterior cephalocele is the forehead (Vorderkopf) of the German embryologists. It bears the ocelli, and the front is developed from it.

The anterior cephalocele develops into the facial region.

Behind the front there are two plates which extend forward from the metacephalon; these form the *epicephalon* (*parafrontal-occipital ridge*).

That portion of the procephalic lobe which lies in front of the antennal ridge unites with its fellow, and curves downward and backward over the mouth to form the prefacial region.

When the posterior cephalocele is closed by plates of chitin, these are the triangular median epifrontal, and the two frontals (frontalia or frontal vitta).

The frontal sac or *ptilinum* consists of a great part of the posterior cephalocele withdrawn into the interior of the head between the frontals and the antennal ridge.

The *lunula* is thus an anterior chitinized portion of this sac or ptilinum.

The anterior cephalocele is the vesicle of the olfactory lobes.

The posterior cephalocele is the vesicle of the cerebral hemispheres and their median ventricle.

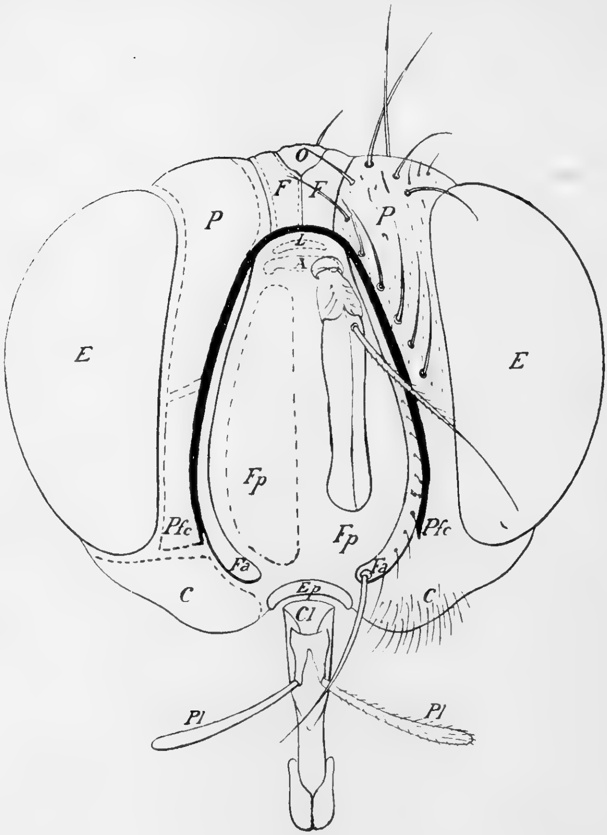
In the nymph the median parts of the head capsule lie in a deep cleft between the two lateral lobes or paracephala, and in close proximity to the ganglia with which they correspond, so that the head appears to be open on the median line. Sections show this to be a deep infolding of the inner edges of the paracephala (Lowne).

The two paracephala (two lateral procephalic lobes), having united on the median line, become the paracephalon of the imago.

The paracephalon is opened transversely by a horseshoe-shaped suture running up from the cheek border on each side and passing between the antennal ridge and the frontals, bridged by a widely distensible membranous tissue (the ptilinum), on the forward median portion of which is the lunula somite. This suture ends on each side at the *cheek groove,* which is formed in the integument by the mechanical strain on it when the suture is opened to thrust forth the ptilinum. The suture may be properly called the *paracephalic suture,* but the writer prefers to employ the term *ptilinal suture.*

The following is a detailed statement of the external anatomical parts to be studied in the superfamily Muscoidea, arranged primarily in the order of their importance, and severally in the order of their relative position. The characters of the superfamily are to be found in the various features exhibited by these anatomical parts, and are pointed out so far as possible under each head. The parts preceded by 1) afford characters of family, subfamily, tribal, and partly generic value, and those preceded by 2) characters of mainly generic value. The terminology is made to conform so far as possible to that already in use. New terms are introduced only in such cases as demand their use for reasons of clearness, conciseness, and permanence, and for such few parts as had no name and afford characters of taxonomic value.

The figure here introduced is diagrammatic and intended to show the main sclerites of the front aspect of the head, the characters afforded by which take rank over all others for taxonomic use within this superfamily.



Front view of head of a Muscoidean fly (half in diagram), much enlarged.

(Original, from drawing prepared by the Bureau of Entomology.)

The heavy black line indicates the *ptilinal suture.* O = *Ocellar plate.* FF = *Frontalia.* PP = *Parafrontals.* Pfc Pfc = *Parafacials.* CC = *Cheeks.* EE = Compound *eyes.* L = *Lunula* (postfront of larval insects). A = *Antennal ridge* (mesofront of larval insects). Fp Fp = *Mesofacial plate* (plus facialia equals prefront of larval insects). Fa Fa = *Facialia.* (Parts from lunula to facialia both inclusive taken together constitute the homologue of the *front* of larval insects.) Ep = *Epistoma.* Cl = *Clypeus.* Pl Pl = *Palpi.*

EXTERNAL ANATOMICAL PARTS AND CHARACTERS

(Head)

1) Ptilinal suture (through which is protruded the *ptilinum* of Robineau-Desvoidy) evenly rounded and widened above, narrowed above, subangular at top; its sides parallel, divergent, convergent; its termini high or low where they join the cheek grooves; position of its termini with relation to lower eye-border, epistoma and vibrissal angles.

[Before *ptilinal suture*]

1) Ptilinal area (area enclosed by ptilinal suture = *facial depression* of descriptions plus *antennal somite* plus *lunula; front* of Berlese) of what form, width above and below compared with adjacent parts of parafacials and parafrontals.

1) Facial plate (*clypeus* of Brauer and von Bergenstamm; *face, facial plate, mesofacial plate* of Lowne plus *epistoma; facial depression* of authors, *prefront* of Berlese, *transverse impression of face* of Hough—in each case minus *facialia* and plus *epistoma*) produced and swollen in middle like the bridge of the nose, merely swollen nose-like below, tube-like, projecting forward in profile below, flat, even, elongate, reaching almost to lower margin of head, extending obliquely downward and posteriorly, reaching straight down between vibissal angles, widened below same; shortened in front view, ending high above lower margin of head; widened below, oval, triangular, comparative width above and below, narrowed high or low by the facialia or by the vibrissal angles.

1) Mesofacial plate (do. of Lowne; *facial plate* minus *epistoma*).

1) Fossæ of facial plate (*foveæ* plus *foveal sinuses*) long, short, wide, narrow, deep, shallow, curved, straight.

2) Foveæ (*foveæ* of Robineau-Desvoidy; *antennal grooves* of descriptions; simply depressions in the *facial plate*) deep, shallow, elongate, short, double, single, and confluent.

1) Foveal sinuses (more or less linear grooves which in certain cases form outlets of the *foveæ* anteriorly) linear, widened, deep, faint, convergent, divergent, etc.

2) Facial carina (*keel* of descriptions) present, absent, developed only above, weak, strong, high, sharp, knife-like, thin, thick, flattened, rounded, widened, canaliculate or furrowed on its median line, or simple.

1) Facialia (*facialia* of Robineau-Desvoidy and Osten-Sacken; *facial ridges* of descriptions; *facial edges of paracephalon* of Lowne; *Vibrissenleisten* of Brauer and von Bergenstamm; *vibrissal ridges* of Hough) parallel, gradually convergent below, short, long, bare, ciliate, narrow, sharp, widened, flattened, divergent, or absent.

1) Facial bristles (those on *facialia; Vibrissen* of Brauer and von Bergenstamm) ascending less than half way on facialia, or half way, or to point opposite lowest frontals, or nearly or quite to base of antennæ; in one or two rows, bushy, in irregular position, short, weak, long, represented by many rows of fine hairs, normal with hairs among the bristles, only one or two above vibrissæ, or wholly absent.

1) Vibrissal angles (*Vibrissenecken* of Brauer and von Bergenstamm; angles or corners where the *facialia* and *peristomalia* meet) pronounced, weak, high above the lower margin of head, set low, rounded, sharp, or absent.

2) Vibrissal papillæ (*Vibrissenwülste* of Brauer and von Bergenstamm; sometimes present at *vibrissal angles*) prominent, pronounced, flattened, weak, inconspicuous, or absent.

1) Vibrissæ (the two longest or strongest bristles, one at each *vibrissal angle; Vibrissen* of Brauer and von Bergenstamm) approximated, widely separated; their insertion on, close to, well removed from the oral margin, or on, close to the under margin of the head, or on the upper edge of the oral margin when this is turned up and broadened, or on or near end of facial plate,

on a level with uppermost front edge of oral margin, or above or below same.

1) Peristomalia (*lateralia* of Robineau-Desvoidy; peristomal ridges, the ridges on lower edges of *peristoma* or *cheeks*, extending to *vibrissæ*) with one or many rows of bristles, extending how far up; parallel above oral margin, divergent, convergent; parallel, divergent, convergent posteriorly below oral margin; effect on epistoma.

2) Peristomal bristles (those on *peristomalia*) strong, weak, in one or more rows, or few and with row of hairs.

1) Epistoma (*epistoma* of Rob.-Desv.; *Mundrand* of Br. and v. Berg.; the portion of *facial plate* below *vibrissal angles* and enclosed between the *peristomalia*, its point of junction with the *mesofacial plate* being indicated by the *vibrissal angles*) projecting nose-like, prominent in profile, retreating, set back or removed, produced downward or anteriorly, turned up, drawn out tube-like, transversely cut off, broad, narrow, thin; thickened, widened on edge, callous or indurated, projecting forward and downward below vibrissæ; drawn up in middle to form anterior part of narrow oral slit, its sides thereby becoming nearly parallel; square, or curved in front outline. [The characters of the epistoma are usually best included in those of facial plate, of which it forms a part.]

Oral margin (the anterior edge of the *oral cavity*, being the lower edge of *epistoma*).

1) Oral cavity covered over transversely in front with an oblique posteriorly-extending skin or membrane developed probably from the clypeus, open, elongate, short, wide, narrow, deep, shallow, slit-like, or closed.

1) Clypeus (*clypeus* of Rob.-Desv., Lowne, and Berlese; the anterior or dorsal plate of the cephalopharyngeal skeleton, or fulcrum, of the rostrum) distinct, rectangular, triangular, developed into a plate closing oral cavity, or vestigial.

1) Mouth parts normal, vestigial, immovably fixed at base of shallow oral cavity, hidden in a narrow deep oral slit, or wanting.

2) Proboscis short, fleshy; not longer than head height, shorter or longer than same; very elongate, bristle-like, twice geniculate, once geniculate, slender and horny, large, stout, vestigial, or absent.

2) Labella well developed, large, broad, small, vestigial.

2) Palpi absent, vestigial, filiform, club-shaped, strongly elongate, normal.

1) Longitudinal axis of head at oral margin longer than that at insertion of antennæ, or the two equal, or the former shorter.

1) Facial profile advancing thereby, or more or less straight or concave, or receding or convex.

1) Facio-peristomal profile angular, rounded, strongly or gently convex.

1) Antennæ (arising from *antennal ridge* of Lowne; from *antennale* or *2d somite of front* of Berlese) inserted above, on, or below a line drawn through middle of eyes; above or below middle of extreme head height, widely separated or closely approximated.

1) Second antennal joint strongly elongate compared with first, longer than shortened third joint, normal, with or without strong bristles on front edge.

2) Third antennal joint entire, fissiform in one or both sexes, elongate, narrowed, widened, enlarged, with curved point on front apical corner, normal.

1)2) Arista bare, microscopically pubescent, hairy, pectinate, partly or wholly plumose, geniculate, flattened, thickened in what part of its length; first and second joints elongate, short, strongly elongate; or only second joint strongly elongate, its length compared with its width or with the third joint.

1) Lunula (*postfront* of Berlese) enlarged in middle inferiorly and superiorly into a more or less diamond-shaped or rounded plate, like an extension of the facial plate into a secondary one; elongated below between the antennæ into a keel-like prolongation, widely separating the antennæ, or normal.

Note.—The lunula reaches its greatest development in the *Syrphoidea.*

(N. B.—Mesofacial plate [= 2 *mesofacials* of Lowne + *carina* if present, since latter is formed by inner edges of the two mesofacials] + 2 facialia + antennal ridge + lunula = homologue of front of larval insects [= ptilinal area]).

[Behind *ptilinal suture*]

2) Eyes absolutely bare; thinly microscopically hairy, sometimes distinctly so, sometimes indistinctly so; thickly pubescent, sometimes more so in male, less so in female; reaching as low as vibrissæ, or lower, or only to middle of face, or very short. [N. B.—In comparisons last mentioned, hold head in full profile with plane of posterior aspect of occiput perpendicular.]

2) Vertex wide, narrow, comparative width in sexes.

2) Vertical bristles present or absent, or present only in female; proclinate, reclinate, divergent, convergent.

2) Postvertical bristles (+ *postocellar bristles* = *lesser ocellar bristles* of Hough) large or small, separated or approximated, how many pairs.

1)2) Front prominent in profile; flattened, or only anteriorly so; bulging, narrow, wide, widened anteriorly, conically produced, of equal width, or not so.

1) Ocellar plate (*stemmata* of Rob.-Desv.; *epifrontal* of Lowne) triangular, rounded, large, small.

1) Ocelli separated, approximated.

1)2) Ocellar bristles (*Ocellenborsten* of Brauer and v. Berg.; *greater ocellar bristles* of Hough) strong, weak, proclinate, reclinate, divergent, vestigial, or absent.

1)2) Postocellar bristles (a second or posterior pair sometimes present on ocellar plate just behind the two posterior ocelli; + *postvertical bristles* = *lesser ocellar bristles* of Hough) present, absent, or represented by fine hairs only.

2) Preocellar bristles (do. of Hough; small pair on frontalia in front of anterior ocellus) present or absent.

2) Frontalia (*frontalia* of Rob.-Desv.; *frontals, mesofrontals* of Lowne; *frontal vitta* of descriptions) polished, opaque, wide, narrow, long, short, equilateral; widened or narrowed anteriorly or posteriorly, or in middle; square in front, notched in front or behind.

2) Parafrontals (*optica frontis* of Rob.-Desv.; *parafrontals* of Lowne; *sides of front* of descriptions; *geno-vertical plates* of Hough) swollen, dilated, bare except for frontal and fronto-orbital bristles, hairy, bristly, short, long, wide, narrow, equilateral; widened before or behind, or both; prolonged anteriorly.

1) Frontal bristles (those inserted on the inner edges of the *parafrontals,* always convergent, often extending posteriorly only to point about half way between *ptilinal suture* and *vertex; transfrontal bristles* of Hough) in a single row, in two or more rows; descending below base of antennæ, continuation

below represented by row on parafacials descending nearly as low as oral margin, or about half way down, or less than half way, or not descending below base of antennæ; or represented only by one or more rows of weak bristly hairs on parafrontals.

2) Upper fronto-orbital bristles (those on posterior portion of *parafrontals* immediately in front of the *vertical bristles* and often appearing as a continuation of frontal rows posteriorly, always reclinate; *ascending frontal bristles* of Hough) in line with frontal bristles, or with middle fronto-orbital bristles; position, direction, number; or absent.

2) Middle fronto-orbital bristles (*Orbitalborsten* of Brauer and v. Berg.; *fronto-orbital* of Osten-Sacken; *orbital bristles* of descriptions; they are usually a little nearer the orbit than the preceding, and always proclinate) present in both sexes, or in female only, strong, weak, divergent, convergent; one, two, three, or a row, or represented only by weak hairs; or absent in both sexes.

2) Lower fronto-orbital bristles (*lower fronto-orbital* of Osten-Sacken and Williston; occurring occasionally in the Acalypterata, but rarely in the Muscoidea) present or absent, number.

1)2) Parafacials (*optica faciei* of Rob.-Desv.; *Wangen, genæ* of Brauer and v. Berg.; *sides of face* of descriptions; *genæ* of Hough) widened above, or not so; bare, hairy, bristled; widened below and narrowed above, more or less swollen, very wide, very narrow, elongate, short; or narrowed, shortened, or abbreviated below.

2) Facio-orbital bristles (those on *parafacials*) present or absent, number, position, direction.

1) Cheeks (*peristoma* of Rob.-Desv.; *Backen, peristoma* of Br. and v. Berg.; *buccæ* of Hough) wide, narrow, very narrow; width equaling or exceeding eye height, or equaling what proportion of eye height; naked, hairy, bristly, or so only below or behind. [N. B.—Br. and v. Berg. give apparent height (not width) of cheeks as seen in profile, with eyes included. Their actual greatest width (distance from peristomal margin to eye) should be compared with eye height, as seen in front view.]

1) Cheek margins (portions bordering on *parafacials* and *ptilinal area*) ascending, encroaching on face, more or less circumscribing the facial plate.

2) Cheek grooves (*mediana* of Rob.-Desv.) present, well defined, curved, wide, deep, shallow, position, vestigial.

2) Cheek bristles (strong bristles which sometimes occur on *cheeks* near lower border, slightly outside of *peristomalia*) present or absent, number, direction, position.

2) Posterior orbits (bare space between posterior eye margin and row of hairs fringing *occiput*) widened below, narrowed above, of even width, wide, or narrow.

2) Lower margin of head (lower border as seen in profile) straight, bulged downward or outward posteriorly, long, short.

2) Occiput (all the portion of the head behind the plane which defines the limit of the *posterior orbits,* as marked by the fringe-like row of small bristles or hairs bordering same and called by Hough and others *cilia of posterior orbit*) evenly swollen, flat; flat above and swollen below, bulging the cheek profile posteriorly.

1)2) Parafrontal-occipital ridge (ridge-like sclerite formed by what seems a continuation of *parafrontals* over *vertex* on *occiput* and which bifurcates above *great central foramen; cerebrale* of Rob.-Desv.; *epicephalon* of Lowne).

1)2) OCCIPITO-CENTRAL BRISTLE (do. of Hough; small bristle on *parafrontal-occipital ridge* just below *inner vertical bristle* before bifurcation of ridge) present or absent, character of.

1)2) OCCIPITO-LATERAL BRISTLE (do. of Hough; small bristle on *occiput* just below *outer vertical bristle*) present or absent.

2) OCCIPITAL AREA (the characteristic hairy area of occiput which sometimes invades the *cheeks* posteriorly) invading cheeks, or restricted to occiput.

2) LONGITUDINAL DIAMETER OF OCCIPUT (shows its degree of swelling at any specified point) above or below compared with eye width in profile.

2) BEARD (pilosity arising and depending from lower portion of *occiput*, and in certain cases clearly defining a portion of *cheeks* invaded by *occipital area*) long, short, thick, thin.

(THORAX)

1)2) STERNOPLEURAL BRISTLES one, two, three, or more, in what arrangement.

1) HYPOPLEURAL BRISTLES strong, weak, or represented only by hairs.

1) PTEROPLEURAL BRISTLES strong, weak, or hair-like.

1) MESOPLEURAL BRISTLES very strong, or normal.

1) PROPLEURAL BRISTLES strong, weak, number, direction.

1) NOTOPLEURAL BRISTLES (*posthumeral* of Osten-Sacken) strong, weak, number.

2) POSTSUTURAL BRISTLES (*dorsocentral* of Girschner behind suture; *outer dorsocentral* of Osten-Sacken behind suture) strong, weak, relative strength, number, position.

2) DORSOCENTRAL BRISTLES (*dorsocentral* of Girschner before suture; *outer dorsocentral* of Osten-Sacken before suture) strong, weak, relative strength, number.

1)2) ACROSTICHAL BRISTLES (2 middle rows both before and behind suture) strong, weak, number, position.

2) HUMERAL BRISTLES strong, weak, number, direction.

1) INTRAHUMERAL BRISTLES (*posthumeral* of Girschner) present or absent, number, position.

1) PRESUTURAL BRISTLES (+ *posthumeral* of Girschner = *intrahumeral* of Osten-Sacken) strong, weak, position in relation to preceding.

1) INTRAÄLAR BRISTLES strong, weak, whether one in front of suture.

2) SUPRAÄLAR BRISTLES (+ *postalar* = *supraälar* of Osten-Sacken) strong, weak, number.

2) POSTALAR BRISTLES strong, weak, number.

1)2) SCUTELLAR BRISTLES strong or weak, comparative strength of the various pairs, number of lateral pairs; a weaker apical pair present or absent, erect, suberect, directed posteriorly, decussate, or divergent; discal pairs present or absent.

(WINGS)

2) WINGS broad, long, narrow, short; costal margin swollen or dilated in male, or in both sexes, or normal.

2) COSTAL SPINE distinct, strong, weak, double, or absent.

2) LONGITUDINAL VEINS bristly, to what extent, or bare.

1)2) Fourth longitudinal vein incomplete, straight, not forked, reaching neither the wing margin nor the third vein, normal, ending at or before wing-tip, angular or rounded at bend, bowed or not beyond bend, bend approximated to or removed from hind margin of wing; last section forming petiole of apical cell when latter is petiolate, or third vein in such case forming petiole; or forked and main vein represented beyond apical crossvein by only a short stump, or by a mere wrinkle or fold in the wing-integument, or by a long stump.

2) Apical crossvein (this term should be employed only when the fourth vein is furcate, or shows indication of previous furcation in a stump, fold or wrinkle) bent in, straight, oblique, long, short, absent.

1) Fifth longitudinal vein bent up to fourth vein, not forked; or furcate, giving off posterior crossvein; represented beyond latter by a short stump, or a long one, or only by a wrinkle, or partly by stump and wrinkle, or continuous to wingborder.

1)2) Posterior crossvein (term not to be employed in the few cases where fifth vein shows no sign of furcation) oblique, in line with apical crossvein or with last section of fourth vein, or still more oblique than latter, or normal; nearer to bend of fourth vein (or to origin of apical crossvein) than to small crossvein, or nearer to latter, or about in middle between the two; trisinuate, bisinuate, singly curved, straight.

2) Small crossvein on, before, or behind middle of discal cell; short, long, straight, oblique, direction.

1)2) Apical cell (*first posterior* of descriptions) ending near wingtip, or far before; open, closed in margin, or long or short petiolate, or extremely short petiolate; wide, narrow, short, elongate, tapering equilaterally at apex.

2) Tegulæ large, small, relative size of two scales; deeply smoky or infuscate, or white, or yellow; bare, pubescent, or hairy.

(Abdomen)

1)2) Abdomen (shape of whole) linear, cylindrical in one or both sexes, widened on some portion, conical or oval in both sexes, swollen, convex dorsally, concave ventrally, flattened in one or both sexes, or laterally compressed.

1) Abdominal segments apparently four, or how many visible from above; how many actually present, which ones shortened, and relative development of their respective *dorsal* and *ventral plates*. [See notes on *Gymnosoma*, *Trichopoda*, *Rhachoëpalpus*, etc., under head of Descriptions. In many cases, at least, there are more segments in the Muscoidean abdomen than have heretofore been recognized, an undeveloped basal segment being quite hidden from view, and only visible with difficulty on the sides below. Its dorsal and ventral plates are easily seen on detaching the abdomen. In order to avoid confusion, the old terms "first," "second," "third," and "fourth" segments are retained as referring to those apparent from above in the undetached abdomen.]

2) Abdominal macrochætæ present or absent, bristle-like, true, very strong, thorn-like, discal and marginal, or only marginal; discal present on second and third segments (counting *apparent* segments from above), or only on third and fourth, or only on fourth; marginal present on all, or absent on first, or absent on both first and second segments.

1) Ventral membrane (membrane connecting the *ventral* and *dorsal plates* of the *abdominal segments*) visible, concealed by the sides of the dorsal sclerites or plates, or apparently absent.

1) Ventral plates free, or not so; or that of second segment in both sexes with its edges upon and covering the edges of the corresponding dorsal plate, the other ventral plates free, or this true of only one sex; how many ventral plates, last one in male deeply or weakly Y-cleft or V-cleft, or entire.

2) Ventral carina present in female, absent, rudimentary, more or less developed, emargination of plates of same, or latter entire.

2) Ovipositor elongate, short, tapering, stout, furnished with terminal hooks, appressed, exserted; directed downward, or forward, or posteriorly; integumental, membranaceous, or horny.

2) Hypopygium prominently exserted, elongate, appressed, directed downward, short, rounded, bulb-like, tube-like, of what formation and characteristics.

(Legs)

1)2) Legs strongly elongate, only moderately so, short, or only one pair elongate, relative length of pairs; bristly, bare, shaggy-haired, with or without macrochætæ.

2) Hind femora ciliate or not so, character and position of the cilia.

2) Hind tibiæ completely and densely feather-barb-ciliate, only comb-ciliate, subciliate, with some longer bristles; cilia flattened and widened, scale-like, bristle-like, or of what character.

2) Middle tibiæ with or without strong bristles or macrochætæ on outer side, or on any portion.

2) Tarsi slender, swollen, compressed, short, elongate, relative length of pairs in each sex; last joint or more of which pairs oval, thickened, swollen, or compressed, in one or both sexes.

2) Metatarsi short, elongate, comparative length with relation to other tarsal joints of same pair, comparative length of pairs, slender, stout.

2) Front tarsi widened in female, or widened and flattened, or only flattened, in one or both sexes.

2) Claws and pulvilli elongate in male, or in both sexes, or short in both, or only anterior ones elongate in male; claws stout, slender, curved, shape and character; pulvilli of what shape and character.

While the foregoing enumeration of anatomical parts affording characters of taxonomic value in the superfamily is not necessarily complete, it is believed that it brings out practically all the characters requisite to a proper separation of the forms in the adult.

Of all these characters, those of the head take first rank. For this reason much space has been devoted to their consideration—in fact, nearly twice as much as to all the other characters together. It is conceded that the Schizophora are the most specialized insects, the most highly developed from the standpoint of ontogeny, as evidenced by their remarkable and practically complete reorganization of larval parts within the nymph. Everything points to the Muscoidea as the most highly organized Schizophora, and this is emphasized by their acute sensory development. It is therefore naturally to be expected that certain non-functional parts of the head, which is the chief seat of the specially developed senses, should afford the most important characters for taxonomic use.

Here, and practically here alone in the Muscoidean anatomy, are to be found certain useful atavic characters pertaining to organs not of any functional importance in the economy at present, but possessing phylogenetic significance as indicating origin and relationships. These are of especial value for the separation of families and subfamilies. It has long been recognized that rudimentary organs in recent forms bear a significant relation to those of their allied predecessors. Such are physiologically non-functional now, and appear in more or less developed condition only in the embryo, but were functional throughout life in the early fossil forms. They have been lost through disuse, involving a process of degeneration or retrogressive development. If, then, these organs present sufficient variation, their rudimentary presence is of much importance to us in the preparation of a natural taxonomic system. Atavic characters, to be of use, must be exhibited by parts which vary sufficiently to offer conveniently distinguishing marks. To be of use in the separation of higher, or family, divisions, the parts must present just enough variation to offer distinctive characters that will hold throughout considerable aggregations of forms.

Such are the characters afforded by the *facial plate* in its lower extent, and by the *facialia, vibrissal angles,* and *peristomalia.* The parts in question present sufficient variation to afford distinguishing characters. These are all atavic, and possess in consequence a high phylogenetic significance. They are connected with the portions of the head whose development in the nymph is not influenced by the coincident development of functional parts. While the development of the highly sensory third antennal joint affects in a degree the upper portion of the facial plate and determines the character of the foveæ, its influence does not extend below the vibrissal angles.

Atavic characters are afforded by the wing veins in a remarkable degree, but the general plan of venation is too uniform to afford us good family characters. They can be used in higher and lower divisions. It may further be noted that, since the wings are so highly functional in a mechanical (not sensory) way, the characters derived from lesser variations in venation would in any event be secondary in importance to the head characters just mentioned. It must be borne in mind that the wings are of great functional importance, and the veins bear the mechanical strain incident upon their use, while the special head characters above pointed out, whose importance as affording distinctions for higher divisions has been dwelt upon, are in nowise connected with any present function, either mechanical or sensory, in the economy of the adult insect. The type of venation furnishes atavic characters of value in separating higher divisions. The bristles or hairs of certain thoracic plates likewise furnish atavic characters of high value here.

Atavic characters also occur to a limited extent in the abdomen, chiefly in the atrophied basal segment, which can be clearly made out only by detaching the parts. These are also too uniform to be of use for the separation of the larger divisions, so far as we yet know. But their comparative study offers promising results.

Practically all the other portions of the Muscoidean anatomy are preëminently functional, even including the halteres, tegulæ, etc., and the parts of the head other than those enumerated above. The frontalia and lunula may be practically non-functional, but they likewise do not present sufficient variation to offer any useful characters for family separation. The second antennal joint is probably not functional, although in the Nemocera it is the seat of the so-called "Johnston's organ," whose function is supposed to be auditory. This organ does not appear to be developed in the Cyclorrhapha. Practically the only character afforded by the second antennal joint, however, is that of relative length compared with the first joint, and this is at best available only for subfamily and generic separation. The arista is doubtless functional. A consideration of certain characters of functional parts, and especially of the physiological functions of certain of these parts whose characters have in the past been largely used in taxonomy, is now taken up.

Antennæ proper.—The first and second antennal joints are practically non-functional. The third joint is highly functional, and hence does not afford reliable taxonomic characters for higher divisions than species, and within certain limits for genera. The relative length of third joint to second affords no valid character, and especially gives a wrong impression in those forms having the second joint elongate. The first joint is almost universally short, but the second is often more or less elongated, and in some cases strongly so. The relative length of second joint to first affords a good generic character. The third joint affords excellent specific characters, so far as its relative length and size go, with proper recognition of sexual variations. Its shape may furnish characters of generic, or even of tribal, value.

The olfactory sense is very highly developed in the Muscoidea. Blow flies will come for miles to decaying, and even to fresh, meat shortly after its exposure to the air. Most other members of the superfamily possess this high olfactory sense, though in some it is developed in a varying degree. The sense of smell in these flies is located in the third joint of the antennæ, which contains numerous olfactory pits communicating with the main nerve trunk by means of minute nerve-ends.

According to Gustav Hauser (Zeitschr. f. Wissens. Zoöl., XXXIV, pp. 367-403, 1880), who studied over sixty species of Diptera in this connection, the Muscoidea and other cyclorrhaphous Diptera, and also the Brachycera, have the olfactory pits without exception confined to the third antennal joint. Their number varies greatly in different forms of Cyclorrhapha. Certain syrphids, as *Helophilus florens*, have only one pit on each disk of the third joint, while *Echinomyia grossa* has two hundred. In certain forms the pits are compound, containing from ten to one hundred olfactory hairs arising from the coalescence of the several original pits. No compound pits occur in the Tipulidæ, but only simple ones with a single olfactory hair, such as are found in the brachycerous (s. str.) forms only. The latter have also compound pits, containing from two to ten nerve-terminations.

The olfactory pits are sac-like invaginations of the external chitinous integument, and are of various shapes in different forms of diptera. They are always open externally, and never closed by a membrane. In the Cyclorrhapha, and the Muscoidea especially, the pits differ but little in the various forms. Hauser (l. c.) figures and describes at length those of *Muscina stabulans* as generally typical of not only the Cyclorrhapha, but the Brachycera s. str. as well. He gives a figure of the third antennal joint in longitudinal section showing simple and compound pits, the pits themselves being shown in both transverse and longitudinal section and from above. The main nerve trunk, accompanied by the much smaller tracheal trunk, passes through the second antennal joint entire and without division, but on entering the third joint gives off a very small branch to the arista, to which also runs a small branch of the trachea. The bulk of the nerve trunk continues undivided and undiminished into the mass of the third antennal joint, where it branches in all directions, but especially apically and inferiorly (opposite the edge bearing insertion of arista), the main trachea following it with less branching. This centralization of nerve-branches, nerve-ends, and olfactory pits in the apical and ventral tracts of the third antennal joint—that is to say, outside the aristal area—bears out the conclusion that the arista was originally terminal and that the highly functional extra-aristal area of the joint has simply grown away from it as fast as more space was required by the advancing development of the olfactory sense.

It has been conclusively proved by the experiments of Hauser and others that the sense of olfaction is located exclusively in the antennæ in *Sarcophaga, Calliphora,* and *Cynomyia,* and not at all in the palpi. This has also been demonstrated in many Hymenoptera, Lepidoptera, Orthoptera, and Staphylinidæ; but in certain Hemiptera experimented with it was found that the loss of their antennæ did not affect in any way their sense of smell. Certain Coleoptera were only partially affected by the excision of their antennæ.

The olfactory organs of the Muscoidea consist of (1) a thick nerve trunk arising from the brain and passing into the antennæ; (2) a sensitive apparatus at the end, consisting of rod-like modified hypodermis cells, connecting with the nerve-fibre terminations; (3) a supporting and accessory structure consisting entirely of pits. The same is true of the other Diptera, the Lepidoptera, Orthoptera, and probably the Hemiptera; but in the Neuropteroid orders, the Coleoptera, and the Hymenoptera, the accessory structure consists of peg-like projecting epidermal invaginations filled with a serous fluid. Both pegs and pits occur, however, in the Coleoptera and Hymenoptera, while only tactile hairs were found by Hauser in *Pyrrhocoris* of the Hemiptera, though Lespès has recorded the presence of pits in that order.

It should be mentioned here that another sense, capable of distinguishing between various degrees of atmospheric pressure, is believed to reside in certain sensory structures, like the sensillum placodeum, found in the antennal joints of bees and wasps. It is evident that insects have some means of perception, through certain sense-organs, of approaching changes in meteorologic conditions.

Arista.—The arista is the persistent rudiment in the Cyclorrhapha of the terminal antennal joints still to be found in many of the lower groups of Orthorrhapha. In the development of the third antennal joint of the Muscoidea as a special olfactory sense organ, the arista has become dorsal or basal, being left to occupy a position to one side during the extraordinary development of the joint away from it. It is invariably situated close to the base of the front edge of the joint. Its persistent retention in this position indicates that it is to some extent functional.

It is a rule in nature, which carries no exception, that there is a reason for everything that exists. Therefore there is some cogent reason for the pubescence, plumosity, and nudity of the arista, as well as for its presence. The arista has become subordinated to the third joint, but retained as an accessory. It therefore must be functional. The point is to discover its function, which must be the key to the explanation of its varying degrees of pubescence and plumosity. The joint is mainly olfactory, and certainly highly sensory. As such it is highly important to the insect. The arista is directed forward, outward, and downward from its insertion on the anterior basal edge of the joint. This would indicate that it is primarily functional as a tactile sensory organ for the protection of the highly functional third joint. Such an indispensable organ in the economy of the insect as the third antennal joint would naturally demand the presence of some tactile sense organ extended before its exposed surfaces, to serve as a warning against contact with foreign objects. In other words, the arista has taken to itself the original function of the antenna, on account of the latter being practically turned into an olfactory sense organ. The bristles of the facial and frontal areas protect the other parts of the head from injurious contacts.

What light does this function of the arista throw on the question of its nudity, pubescence, or plumosity? Simply that the separate hairs have a tactile function, pointing in all directions from which danger may come. It is to be noted that the plumosity is always stronger on the upper or outer than on the under or inner side. Those forms which have the basal joints of the arista elongated lack the plumosity. This elongation of the basal joints indicates an increased freedom of movement of the arista. When bare of plumosity the arista either is long and tapering, indicating a somewhat restricted movement in the comparatively short basal joints, or it is short, stout, and geniculate, with greatly elongated basal joints, indicating much freedom of movement. The nudity of the arista may be generally taken to indicate greater freedom of movement in its basal joints, and its shortening, when combined with geniculation, still further increase of movement. In any case, the function of the organ is seen to be a tactile one, intended to guard the highly sensory olfactory pits and nerve-ends located in the third antennal joint.

Those forms which have the arista more or less atrophied doubtless have the third antennal joint less highly olfactory and more tactile in function.

From this functional nature of the arista we can only conclude, in accordance with the general and almost invariable rule, that it possesses little value for the definition of subfamilies and higher groups, but that its characters may well be employed in the separation of tribes, genera, and species.

Eyes.—The organs of vision are with little doubt more highly developed in the Muscoidea than in any other superfamily of non-aërial insects. These flies possess, on the whole, a distinctively terrestrial life-habit, in contradistinction to an aërial one. The relatively small percentage of achætophorous and subachætophorous forms, and even the few of these possessing the aërial or hovering habit, maintain practically the same type of eye-structure, extensive holopticism of the type obtaining in the Bombyliidæ and Tabanidæ being present in none of them. Partial holopticism is present in very few, and there is a considerable approach to this condition in certain others, but dichopticism is practically the rule. In no other group of insects of a generally terrestrial life-habit is there so relatively large an area of the head occupied by visual surface.

This and other facts further argue for an average higher development in the Muscoidea of the visual sense *per se* than in any other equally extensive group of insects, or perhaps in any other group whatever. The Odonata, Hymenoptera, Lepidoptera, brachycerous and nemocerous Diptera, and some other insects which equal or surpass them in relative visual area of the head, do so by virtue of the correlative evolution of visual surface and aërial life-habit. But their eye structure is less highly developed. While the number of facets in general in the Muscoidea is not nearly so great as in Odonata, certain Lepidoptera, and even Coleoptera, their eye is of a higher order of organization. The Muscoidea possess what is called the pseudocone eye, which is the most highly evolved type of the facetted eye.

It is generally conceded that insects possess what may be termed microvision. Their ability to perceive certain minutiæ approaches that of the human eye supplemented with the microscope. The presence of this microvisual sense in insects is the cause of the marvelous beauty of coloring and sculpture exhibited by their external parts, and which is revealed to us in detail only by the use of a lens. In other words, the facetted insect eye gains impressions from light rays by which the unaided vertebrate eye is unaffected. Most birds, especially the condor and other birds of prey, and some mammals, as the big-horn sheep of our western mountains, have a specially developed far-sight, approaching in a degree the power of the human eye aided by the telescope. Contrasted with this is the extreme nearsight of insects, which do not see in general more than a few feet, and which see best at very close range.

Johannes Müller's mosaic theory of insect vision, which gained such wide credence, especially as modified by Huxley, really seems untenable and quite at variance with well-known facts. It presupposes a very imperfect vision, which can not be the case. Lowne's dioptric theory, which indicates a perfect microvision, with sharpness and clearness of sight, would appear to be the correct one. Yet subsequent investigators, notably both Hickson (1885) and Hewitt (1907), hold that Lowne's interpretation of the functions of the compound eye structures is incorrect. However this may be, it seems certain that insects possess a clear and perfect vision.

Mouth parts.—Kraepelin has recorded taste-pits, with hairs or pegs arising from them, on the proboscis of *Musca* (Zeitsch. f. Wissens. Zoöl., XXXIX, 1883).

The palpi are probably not generally gustatory in function in the superfamily. In certain of the forms they are with little doubt practically non-functional, and some forms have in consequence more or less completely lost them. In others their very considerable, sometimes extreme development, indicates some function, which may be either gustatory or tactile. In certain insects they are olfactory in function, but probably not in the Muscoidea. They furnish characters of not more than generic value.

Wings.—The venational characters are in the main quite constant. The wings themselves are highly functional, but this does not necessarily imply that the style of venation is functional. However, as already pointed out, the plan of venation is so comparatively uniform in the superfamily that it yields no characters for separation of families. The venational characters are of very great importance in separating this superfamily from the Anthomyioidea, but do not become again available for taxonomic use in the Muscoidea until we descend to tribes and genera.

It is reasonable to attach high importance to the main features of the dipterous wing venation, since the wing system of Diptera is a very highly specialized type. The hind pair has undergone atrophy, its rudiments being diverted to another function, and the entire flight function, at least so far as propelling power goes, has been concentrated in the front pair. As a natural consequence of this high wing specialization, the venation is a practically non-functional system of long standing, extending over a sufficient period of time to allow its systemic features to become well fixed and quite constant. There are a few minor venational characters that can not be relied upon in certain restricted groups. The last section of fourth vein (or apical crossvein) may vary in degree of curvature, but not in kind. The hind crossvein may vary in strength of sinuosity, but the double curve is never entirely lost in the same form.

There is some ambiguity involved in the term "apical crossvein," as it has been used in the past. In certain genera it is impossible to decide its true limits. The use of the term should therefore be restricted to those cases in which its entire course is exactly defined. The apical crossvein has resulted from a bifurcation of the fourth vein at its point of flexure. In those genera showing what has been called a "stump, or a wrinkle, at bend of fourth vein," the point of bifurcation, and therefore the true origin of the apical crossvein, is apparent. In such cases the term should be used. It should not be used in those other genera in which no point of bifurcation is indicated, this having not arisen during their development. In such cases the term "fourth vein" is correctly applicable to the whole, whereas the term "apical crossvein" can not be so applied, especially in *Hyalomyia, Phorantha, Alophora, Beskia, Sciasma, Stomatodexia,* and many other genera. The latter class of genera represents a lesser specialization than the class showing a stump or wrinkle, and therefore is an older type, and indicates a more ancient assemblage of forms. Following Williston, the last three sections of fourth vein, when latter exhibits no furcation, but is more or less angularly bent, may very appropriately be termed the antepenultimate, penultimate, and ultimate.

Halteres and tegulæ.—The sense of audition is acutely developed in insects, at least in the majority of the forms, as evidenced by the sounds they produce. There is nothing to indicate that the Muscoidea are in any way an exception. Air waves which produce no effect whatever upon our ears doubtless register impressions upon the auditory nerve-ends of Diptera. Auditory organs are located near or at the base of the wings in Diptera, Coleoptera, Lepidoptera, Neuroptera, Orthoptera, and Hemiptera. They are less perfect in the Lepidoptera, Neuroptera, and Orthoptera, and only faintly represented in the Hemiptera, in which four orders other chordonotal structures have succeeded and more or less supplanted them. In the Culicidæ, and perhaps in other of the nemocerous groups, the antennal hairs are auditory. This has been established in the male mosquito.

In the Diptera the nerve supplied to the halter is next in size to the optic nerve, the latter being the largest nerve in the body. At the base of the halter is a number of vesicles arranged in four groups, to each of which groups the nerve sends a branch. These vesicles are perforated and contain a minute hair, and the vesicles of the upper groups are protected by chitinous hoods.

Sharp (Cambridge Nat. Hist., VI, p. 448) says of the halteres: "They possess groups of papillæ on their exterior surface, with a chordotonal organ inside the base. Each halter is provided with four muscles at the base, and can, like the wings, execute most rapid vibrations. Seeing that they are the homologues of wings, it is remarkable that in no Diptera are they replaced by wings, or by structures intermediate between these two kinds of organs." This is because they have taken on special functions.

E. Weinland (Zeitschr. f. Wissens. Zoöl., LI, pp. 55-166) has concluded from his studies of the halteres that these organs are functional in determining the direction of flight. They can be used to steer a course in the vertical plane as well as in other directions. He also concluded that the chordotonal structures in the base of the halteres allow the perception of the steering movements of these organs. But it is highly probable that the great nerve trunk supplied to the halter is not primarily subservient to this dirigible function, but rather to that of audition, at least in the higher families. In the Nemocera the halteres may be mainly dirigible or equilibratory in function, since the auditory organs are located in the antennæ. In the Cyclorrhapha, however, it seems safe to assume that their function is primarily auditory. As Lowne suggests, the halteres are doubtless microphones of a most efficient nature, capable of perceiving sound waves of such low intensity that they do not affect the vertebrate ear. They possess a function of coördination, similar to that of the semicircular canals of vertebrates, and thus are organs combining the functions of equilibration and audition.

The tegulæ of the Schizometopa and some other Diptera are very likely functional in collecting sound-waves, increasing the perceptive power of the chordotonal organs of the halteres, thus being analogues of the external cartilaginous ear-lobes of the mammalia. They also doubtless serve secondarily as a protection to the highly sensory halteres. It seems safe to assume that in those dipterous groups having no tegulæ the halteres perform chiefly a function of equilibration, but that in those groups furnished with tegulæ the halteres are mainly organs of audition. In other words, the presence of well-developed tegulæ indicates the presence of a highly developed auditory sense in the halteres. Mere protection to the latter would not demand such structures as the tegulæ, while it can not be denied that they are admirably adapted to such a function as the collection of sound-waves.

Whatever may be finally determined as to their functions, it is certain that the halteres are highly specialized organs. The tegulæ, without doubt accessory to them, are by inference equally functional and of coincident evolution with some function pertaining to them. The latter, therefore, can not be accepted as affording characters of value for the separation of large groups, but are rather of decidedly inferior rank in this respect to the veins of the wings. They occur in other groups entirely outside of and removed from the Schizophora, and even from the Cyclorrhapha. Their presence in the Anthomyioidea is therefore not necessarily to be construed as indicating a close relationship between that superfamily and the Muscoidea.

The Acroceridæ, for instance, are to be noted as an extra-cyclorrhaphous group which has developed very large tegulæ, wholly concealing the halteres and probably accessory to a highly developed auditory sense in the latter. It seems to be chiefly groups containing a large percentage of endoparasitic forms which are provided with tegulæ, and it is possible that a greatly increased auditory perception is necessary to these forms as an aid to them in the search for and ultimate detection of their hosts.

The validity of the time-honored separation of the Calypterata and Acalypterata on the characters of the comparative presence or absence of tegulæ alone may well be open to serious doubt. The unflexed fourth vein, which from its doubtless far greater age should be a much more valid character, would indicate a closer relationship of the Anthomyioidea with the Acalypterata than with the Muscoidea. Yet this does not appear to be the proper and natural grouping. It rather seems preferable to adopt Brauer's names Schizometopa and Holometopa as founded on characters of greater value than either those afforded by relative development of tegulæ or those of wing venation, and to recognize therefore the Anthomyioidea as a superfamily of the Schizometopa. While the result is mainly the same, the divisions become founded on valid rather than on mutable characters. The tegulæ have developed, though not uniformly, in the Schizometopa. They have also developed to a certain extent in some of the Holometopa. This fact demonstrates their unfitness for taxonomic use in these divisions. There is a distinction between the characters of a functional organ and the character of the presence or absence of such organ. Moreover, it may be noted that Robineau-Desvoidy's division Calypteratæ was applied by him to the superfamily Muscoidea of the present paper in the main sense, as is further brought out under the head of Synopses.

Abdomen.—The number of abdominal sclerites should be of subfamily significance at least, and the form of the abdomen is almost invariably of generic value.

Macrochætæ and bristles.—Chætophorousness in the Diptera finds the climax of its development in the tachinoid stock of the Muscoidea. While chætophorous characters are, evolutionally, of recent origin, yet the arrangement of the macrochætæ of the head, thorax, abdomen, and legs becomes highly important in separating tribes, genera, and species. The characters to be derived from the macrochætæ of the head rank even higher and serve for the separation of subfamilies in certain cases. In one or two groups, the Gymnosomatinæ and Phasiinæ, the peculiar chætotactic characters of the head are correlated with an absence of macrochætæ on the abdomen, while in certain other groups, as the Hystriciinæ, a different type of them is correlated with a true spinose development of the abdominal macrochætæ. The cephalic bristles are uniformly present in the superfamily, though sometimes weakly developed, whether the abdominal ones are present or absent. The same is usually true of those of the thorax and scutellum. The function of the macrochætæ and bristles of the abdomen is doubtless tactile. They are capable of movement in life.

In most insects the antennæ, and to a less extent the palpi, are the main seat of the tactile sense. The cyclorrhaphous Diptera, however, have the antennæ so modified as to preclude this function. It is probable that the vibrissæ are functionally tactile, and the frontal and vertical bristles as well. The vibrissæ project straight out in front near the ends of the ptilinal suture, and naturally serve as anterior tactile organs for the protection of the lower portion of the head. Likewise the frontal bristles serve as anterior and superior cephalic, and the vertical bristles as superior and posterior cephalic tactile organs. The fact that the vertical bristles are almost invariably stronger and longer than the frontal bristles strengthens this view. The inner vertical bristles correspond in development to the vibrissæ.

The macrochætæ of the thorax, scutellum, and abdomen serve as lateral and dorsal tactile organs, those of anal and preanal segments always being the strongest of the abdomen and those of scutellum the strongest of the thorax. The scutellar are doubtless the main dorsal tactile organs, and the anal the main posterior ones. The abdominal macrochætæ, when dense and of spinose character, possibly serve also as a defense against insectivorous animals, as in *Dejeania, Paradejeania, Bombyliomyia, Hystricia, Hystrichodexia,* and others.

The macrochætæ, especially those of the abdomen, constitute the most recent form of specialization in the Myodaria, and are especially characteristic of the Muscoidea. As such, and considering further their probable functional character as tactile sense organs, those of the abdomen at least can not be expected to furnish valid characters for the separation of higher categories in these flies than species, genera, and at most tribes.

The macrochætæ of the head, thorax, and scutellum appear to be of far longer standing than those of the abdomen. With the exception of most of the Œstridæ, they are present not only in all Muscoidea, many of which lack abdominal macrochætæ, but also in practically all of the Myodaria except the Œstridæ already named and Conopidæ, which two families stand well apart from the other Myodaria. The bristles of certain of the thoracic plates are here used as main atavic characters for separating the Muscoidea from the Anthomyioidea, as will appear later on under Synopses, accessory supporting atavic characters being furnished by the type of venation.

An extra-tactile function is no doubt discharged by certain of the cephalic bristles in the Muscoidea. The orbital bristles (middle fronto-orbital especially) of the females, which are usually wanting, or of less number, in the males, have probably arisen in those forms where present for the purpose of enabling the males to recognize the opposite sex. They are especially conspicuous in profile, when the strongly proclinate middle fronto-orbital are prominently contrasted with the reclinate upper fronto-orbital bristles. A front view would reveal the female in the wider front in most of the forms. The fact that in some forms the males as well have the orbital bristles does not militate against this view, but is explained by a transference of the female character to the male through heredity. The breast nipples of male mammals furnish an example of such hereditary transfer of a female character to the male, with absolutely no functional cause.

The bristles of the facialia and the frontal bristles possibly serve for the recognition of forms among themselves. They are most developed in the more inconspicuously colored forms, which run closely together in general habitus. Further confirmatory evidence is found in the fact that conspicuously colored and otherwise striking species often have the cephalic bristles but little developed. It is to be noted, however, that certain of the latter lack abdominal macrochætæ as well. An absence of abdominal bristles is usually correlated with a weakness of cephalic bristles, doubtless due in these cases to the marked development of an aërial life-habit.

Secondary Sexual Characters.—These should be accorded generic rank when they can be correlated with equally constant characters in the opposite sex. The secondary sexual characters in the Muscoidea are to be found in the comparative width of front, presence or absence of orbital bristles, size and length of third antennal joint, sometimes form of latter, varying degrees of holopticism or dichopticism, comparative length of claws, ventral carinæ, and certain anal processes of abdomen; also often in the shade of coloration and distribution of pollen, especially on the parafrontals and parafacials, less often on the thorax, and sometimes in the distribution of ground color and even of the pollen of the abdomen.

Synopses

It seems desirable to state at the outset that the subject of taxonomic divisions is approached in this paper entirely without prejudice. The main lines of interest in all departments of biology lie in problems of descent, distribution, and bionomics, and the only desirable point as regards classification is to secure a correct delimitation of forms so that they can be definitely referred to by name.

This paper also distinctly disclaims any attempt or intention to present a taxonomic system that is entirely original, likewise any attempt to follow any particular author or authors—in either case to the exclusion of any useful and valid characters already pointed out by previous authors. This is not intended to be a revolutionary scheme of classification in any sense, nor one that will upset any previously conceived ideas of recognized taxonomic value. Rather have all available characters been used that could be brought together for a clear definition of the various divisions in each case, those of value being adopted wherever they were to be found, whether old, recent, or newly worked out. As a matter of fact, the present paper is naturally based largely on Brauer's extensive and careful work, but the latter is not by any means followed blindly or undeviatingly, and points are at the same time drawn from Robineau-Desvoidy and Róndani. Brauer based his work to a very considerable extent upon the work of the latter authors, and Róndani drew many valuable ideas from the work of Robineau-Desvoidy, whose reviser he became. As already pointed out, these three students are the ones to whom we owe most for our present knowledge of the Muscoidea, and of these Brauer naturally accomplished the most, since he enjoyed the greatest advantages. Any one who will conscientiously study this superfamily can not fail of the conviction that Brauer and von Bergenstamm's work, while not by any means perfect, is by far the best that has ever been produced on these flies. The object of the writer of the present paper has uniformly been to sift the entire subject, retaining the good, discarding the useless, and adding such ideas of value as it has been possible to develop independently.

The following tabular arrangement of taxonomic divisions is intended to convey at a glance an idea of the system of classification adopted:

Order Diptera.
Suborder Cyclorrhapha.
Series Schizophora.
Section Myodaria.
Subsection Schizometopa.
Superfamily Muscoidea.
Family Tachinidæ.
Subfamily Tachininæ.
Tribe Tachinini.
Genus Tachina.
Species larvarum.

The suborder Cyclorrhapha is without doubt one of the most natural divisions of the Diptera, and yet its line of demarcation from the Orthorrhapha is obscured by intermediate forms. For details on the limitation of the suborders of Diptera the student is referred to the works of Brauer, Osten-Sacken, and Williston.

As to the limits of the series Schizophora, and the final conclusions to be reached regarding the forms which naturally belong within its boundaries, a word may be said with special reference to the Pupipara. It seems quite evident that some, at least, of the latter are simply degradedly specialized Schizophora. There are strong points of resemblance, both in structure and in reproduction, between *Ornithomyia* and *Glossina*. The venation is fundamentally of the same plan. In *Ornithomyia* the hind crossvein has been lost. The apical crossvein is absent, and probably never was present. In *Trichobius* the apical crossvein is not present, but the posterior one is, and there is even a second posterior crossvein which has been developed between the fifth and sixth veins. *Trichobius* has lost all but a trace of the auxiliary vein. All the winged Pupipara show a venation which indicates evolution from a Myodarian prototype. Many of them seem quite closely allied structurally with the Myodaria, and it is also to be noted that we have as yet no proof of any pupiparous habit in either the Streblidæ or the Nycteribidæ. In fact, it is highly improbable that such exists. Kolenati, as long ago as 1863, stated that the larvæ of Streblidæ live in bats' excrement. If this is true, it is probable that the Nycteribidæ also have a coprophagous larval habit. Müggenburg has investigated the morphology of the Nycteribidæ, and asserts that they possess no trace of a ptilinum. On the other hand, he asserts that in *Braula* a ptilinum exists, and that the mouth parts are essentially similar to those of the Hippoboscidæ. It is probable that the Streblidæ and Nycteribidæ are derived from an extra-myodarian cyclorrhaphous stock. Müggenburg states that the Hippoboscidæ and *Braula* are descended from genuine muscid stock, and that the Nycteribidæ are probably derived from some other stock within the Cyclorrhapha. He strongly indorses Brauer's judgment of the Pupipara as being nearly related to the Myodaria.

Robineau-Desvoidy's name Myodaria is adopted for the Eumyidæ of Brauer, the Muscidæ s. lat. of authors. In the sense in which it is here used, it includes both the Œstridæ and the Conopidæ. Both Robineau-Desvoidy and Brauer were correct in their views on the inclusion of these families in the section. It seems that Brauer did not study Robineau-Desvoidy's *Essai* sufficiently to know that the latter author had, in 1830, included the Œstridæ with his Calypteratæ. Brauer claimed that the idea was original with him, and probably arrived at his conclusions on both the Œstridæ and the Conopidæ quite independently (see *Psyche,* vol. 6, p. 259. The author was unaware at that time of the above facts). Brauer claimed to have studied Robineau-Desvoidy's posthumous work exhaustively, and probably neglected the *Essai.* In the former the Œstridæ are separated entirely from the Myodaria, which would explain the above oversight on Brauer's part.

Reference has already been made to the advisability of employing the subsection name Schizometopa. Robineau-Desvoidy, when he wrote his *Essai,* had practically the same idea of the limits of the superfamily as those here arrived at quite independently. He excluded the Anthomyiidæ from his Calypteratæ, which division thus coincides in the main with the present superfamily Muscoidea, as here restricted. Latreille originally applied the name Creophilæ to these flies, and Macquart and Westwood used this name. The division Calypteratæ of Robineau-Desvoidy was later made to include the Anthomyiidæ on account of the presence of tegulæ in that family. As has been already pointed out, the tegulæ do not afford characters of sufficiently high value to be applied to these divisions. Therefore, for several very cogent reasons, which are self-evident, it becomes not only advisable, but necessary, to drop both Creophilæ and Calypterata as subsection names. The superfamily name Muscoidea covers the field to which they were originally applied, and the name Schizometopa designates the subsection.

The failure heretofore, chiefly on the part of Schiner and his followers, to properly define the grand divisions of the Myodaria, and especially the families of Muscoidea, has been due to the attempted application, in a case demanding primary, constant, and approximately well-defined characters, of two secondary and gradating characters—namely, the presence or absence of tégulæ and aristal pubescence. These two characters are unserviceable, both because they intergradate to such an extent as to preclude the drawing of any natural lines of separation, and, further, because the parts exhibiting them are so functional that they afford characters of only secondary value or less. It was inevitable that a system founded on such characters could not stand, for the natural boundaries do not exist where it was endeavored to set them.

In the present paper the Muscoidea and Anthomyioidea are separated in such a manner, on atavic chaetotactic and venational characters, as to throw a few forms heretofore classed with the old Muscidæ s. str. into the Anthomyioidea, which arrangement is believed to represent their relationships more truly.

It is also believed that the five families into which the Muscoidea are divided will ultimately be found to closely correspond in value with the families now recognized in the other divisions of the Cyclorrhapha.

Professor J. H. Comstock published a very able paper in the Wilder Quarter Century Book, setting forth certain suggestions as to taxonomic work. The idea is there elaborated that, in order to determine the proper taxonomic system for a given group of insects, the forms should be arranged independently on each one of their many characters in turn, and the final mean between all these separate arrangements should then be determined. This mean would indicate the correct taxonomic system. It is understood, of course, that the varying values of the various characters should be taken into consideration in such a procedure. It has been the aim to present a systematic arrangement in this paper to agree quite closely with the results that might be obtained from such a final average between characters in this superfamily.

In the following synoptic treatment of taxonomic categories a plan is followed which has been devised and perfected by Dr. A. D. Hopkins, to whom thanks are due for an exposition of it. This synoptic plan possesses decided advantages over any scheme of the kind yet devised, and is really a perfected system on the lines of that used by Brauer and von Bergenstamm, and some other European systematists. The present synopses are carried down to families only, and do not exhibit the plan in detail. It will be a labor of years to perfect the arrangement of the forty or more subfamilies, the numerous tribes, the two or three hundred American typic genera and five hundred or more additional atypic genera in this superfamily, to say nothing of the multitude of typic and atypic species. The synoptic plan referred to is carried out in detail by employing the following system of characters in turn: I, II, III, etc.; A, B, C, etc.; a1, a2, a3, etc.; b1, b2, b3, etc.; c1, c2, c3, etc.

Order DIPTERA

Lunula absent.................................Suborder ORTHORRHAPHA
Lunula present................................Suborder CYCLORRHAPHA

Suborder CYCLORRHAPHA

Ptilinal suture absent.................................Series ASCHIZA
Ptilinal suture present..............................Series SCHIZOPHORA

Series SCHIZOPHORA

Head closely united to thorax or folding back into dorsal groove on same.
Section PUPIPARA
Head separated from thorax by a free neck...............Section MYODARIA

Section MYODARIA

Front in both sexes of equal width, or if wider in female the greater width is due to a widening of the frontalia and the tegulæ are absent; tegulæ never well developed (includes Conopidæ).............Subsection HOLOMETOPA

Front in male narrower than in female, the wider front of female never due to a widening of the frontalia, tegulæ never absent; if the front is not wider in female the tegulæ are well developed..Subsection SCHIZOMETOPA

Subsection SCHIZOMETOPA

Hypopleural and pteropleural bristles and hairs always absent, fourth longitudinal vein lying partly in the hind margin of the wing behind middle of extreme wing-tip, proboscis never adapted for bloodsucking, if three sternopleural bristles present their formula is 1:2.
Superfamily ANTHOMYIOIDEA

Either hypopleural or pteropleural bristles or hairs always present, fourth longitudinal vein rarely continuous with hind margin of wing behind middle of extreme wing-tip except when proboscis is adapted for bloodsucking,[1] if three sternopleural bristles present their formula is either 2:1 or 1:1:1 ....Superfamily MUSCOIDEA

Superfamily MUSCOIDEA

Facial plate strongly produced below vibrissal angles like the bridge of the nose, the produced portion convex laterally and not flattened, the vibrissæ separated by this bulging and situated high above the oral margin; the mesofacial plate and epistoma completely fused into one piece.
(PHASIID Stem) Family PHASIIDÆ

Facial plate not so produced, at most projecting nose-like below with flattened slope, or if latter is somewhat convex (Gymnosomatinæ) the vibrissæ are inserted quite near oral margin.
I

I

Facial plate always receding below vibrissal angles and oral margin never prominent, thus giving the facio-peristomal profile an evenly and gently convex outline; vibrissal angles situated at or above the lower two-thirds point between oral margin and base of antennæ, always very much higher above median oral margin than length of second antennal joint, at least twice as high, the mesofacial plate in consequence greatly shortened, never widely produced downward, if not completely cut off by vibrissal angles then at least very strongly constricted thereby, the peristomalia either approximated and forming parallel lines for a considerable distance or bowed outwardly and more or less widely separated so as to enclose the epistoma as a more or less distinct sclerite of the facial plate between them; antennæ almost always very short..(ŒSTRID-MACRONYCHIID Stem)
A

[1] *Mesembrina* and *Eumesembrina* are the only exceptions known to the writer, aside from the bloodsucking forms.

Facial plate below vibrissal angles never receding conspicuously, the oral margin always more or less prominent, the facio-peristomal profile in consequence never evenly and gently convex; vibrissal angles approximated to oral margin and never placed much higher above its median portion than length of second antennal joint, distinctly below two-thirds point of face, the mesofacial plate elongate, never very strongly constricted, if constricted at all the constriction is close to oral margin; antennæ usually long ....(TACHINID-MUSCID Stem)
B

A

Vibrissæ and macrochaetæ absent; mouthparts wanting or rudimentary, nonfunctional ....Family ŒSTRIDÆ
Vibrissæ and macrochætæ present, mouthparts functional.
Family MACRONYCHIIDÆ

B

Macrochætæ developed, or if not (Gymnosomatinæ only) then the more or less red abdomen highly swollen or inflated and covered with very short, fine, black, bristly hairs; oviposítor never *Musca*-like.
Family TACHINIDÆ

Macrochætæ not developed, or if so (*Reinwardtia* only) then no fronto-orbital bristles present and ovipositor integumental, long and *Musca*-like; abdomen never swollen or inflated......................Family MUSCIDÆ

The series Aschiza (Becher and Brauer) includes the Phoridæ, Pipunculidæ, Platypezidæ, and Syrphidæ. The series Schizophora (Becher and Brauer) includes all the rest of the Cyclorrhapha.

The section name Pupipara might well be replaced with Nymphipara (Réaumur), which has priority. The section Myodaria (Robineau-Desvoidy) corresponds to the Eumyidæ of Brauer, and to the Muscoidea of Coquillett plus the Conopidæ.

The subsection Holometopa (Brauer) includes the Malacosomæ, Palomydæ, Phytomydæ, etc., of Robineau-Desvoidy, and corresponds in the main to the Acalypteratæ of authors plus the Conopidæ. The subsection Schizometopa (Brauer) corresponds in the main to the Calypteratæ of authors, not of Robineau-Desvoidy.

The superfamily Anthomyioidea (Townsend) corresponds to the Mesomydæ of Robineau-Desvoidy; and to the Anthomyiden of Girschner minus most of the Muscinen of Girschner. The superfamily Muscoidea (Townsend) corresponds to the Creophilæ of Latreille, Westwood, Macquart; to the Calypteratæ of Robineau-Desvoidy; to the Muscaria Schizometopa (exclusive Anthomyiidæ) of Brauer and von Bergenstamm; and to the Tachiniden of Girschner plus most of the Muscinen of Girschner.

For detailed characters defining the suborders, series, and subsections, see the works of Brauer, Becher, Williston, and Girschner.

From the section Myodaria inclusive down to the families, and in some cases the subfamilies, the divisions are particularly difficult of exact definition, from adult characters alone, on account of the numerous intermediates. A study of the characters of the early stages is needed to determine beyond question the location of certain intermediate forms.

The family Macronychiidæ includes those forms approaching the Œstridæ in the character of the facial and peristomal development of the head, and which have heretofore been classed partly with the true tachinids and partly with the true dexiids. It corresponds practically to the group Macronychiidæ of Brauer and von Bergenstamm, but it should be noted that *Megaprosopus,* and not *Macronychia,* is the real type of the family.

The old family Dexiidæ can not be maintained. With the exception of the few just mentioned as included in the Macronychiidæ, its forms all fall in the Tachinidæ, of which they constitute several subfamilies and tribes.

Concerning the three types to be distinguished in the Muscoidea, it may be pointed out that the most generalized type seems to be the Phasiid. The primeval stock was the possessor of a Phasiid-like facial-plate development, in all probability, more or less after the Syrphoidean style. From this stock sprang the three present stems.

Phasiid—Facial plate of the primeval type practically preserved, the mesofacial plate and epistoma becoming solidly anastomosed into one piece, retaining the characteristic bridge-of-the-nose production below. Both antennæ and mouthparts, especially the latter, well developed.

Tachinid-Muscid—Mesofacial plate much increased and epistoma more or less reduced from the preceding, losing the bridge-of-the-nose production, but retaining a more or less prominent oral margin, the mesofacial plate gaining a length and width sufficient to accommodate the greatly developed antennæ. The epistomal development is largely retained to accommodate the very functional mouthparts.

Œstrid-Macronychiid—Mesofacial plate much reduced and epistoma (except in Hypodermatinæ) greatly narrowed and rounded off, losing the prominent oral margin entirely. Antennæ and mouthparts approaching atrophy from disuse.

The following detailed notes on the connectant forms appearing to lie more or less between the superfamilies Muscoidea and Anthomyioidea will be useful for comparison with the synoptic table just given. The former superfamily includes the bulk of the old Muscidæ, the Sarcophagidæ, Dexiidæ, Tachinidæ, et al. (Phasiidæ, Gymnosomatidæ, Ocypteridæ, Phaniidæ), and the Œstridæ; the latter superfamily includes the Anthomyiidæ as herein accepted. The forms which are here referred to the latter, and upon which there has in the past been any question as to position, are:

Anthomyioidea

Myiospila meditabunda et spp.—No hypopleural nor pteropleural bristles or hairs. Sternopleural bristles 1. 0. 2. A weaker sternopleural bristle below first one, so as to appear 2. 0. 2. Venation like *Stomoxys.*

Muscina stabulans et spp.—No hypopleural nor pteropleural bristles or hairs. Sternopleural bristles 1. 0. 2. Venation like *Stomoxys.*

Muscina cæsia (det. Coquillett).—No hypopleural nor pteropleural bristles or hairs. Sternopleural bristles 1. 0. 2. Venation typical anthomyiid.

Cyrtoneura podagrica, gluta, et spp.

Pararicia pascuorum et spp.

Clinopera frontina et spp.—No hypopleural nor pteropleural bristles or hairs. Sternopleural bristles 1. 0. 2.

These forms have heretofore been classed in the old Muscidæ s. str. by Williston, van der Wulp, and Brauer. It is believed they should be excluded from the Muscoidea on the general averages of their characters.

Muscoidea

The following forms here included in the Muscoidea were referred by Girschner to his Anthomyiden:

Musca domestica, corvina, et spp.—No hypopleural hairs. Distinct pteropleural hairs. Sternopleural bristles 1. 0. 2, but in some the last two bristles are separated so as to appear almost like 1. 1. 1. Typical Muscoidean venation.

Stomoxys calcitrans et spp.—Hypopleural hairs, also pteropleural hairs. Sternopleural bristles 0. 0. 1. Fourth vein bent, arcuate, partly continuous with hind border. Proboscis adapted for bloodsucking.

Lyperosia irritans et spp.—No hypopleural bristles. Pteropleural hairs present. Sternopleural bristles none, 0. 0. 0. Aberrant venation; fourth vein hardly bent, yet apical cell narrowly open at wing-tip; third vein bulged upward, convex in front or above. Proboscis adapted for bloodsucking.

Hæmatobia stimulans et spp.—No specimens for study.

Graphomyia maculata, americana (det. Coquillett), et spp.—Hypopleural hairs present. No pteropleural bristles or hairs. Sternopleural bristles 0. 0. 2. Fourth vein arcuate at bend.

Synthesiomyia brasiliana et spp.—Hypopleural hairs strong, quite bristly No pteropleural hairs. Sternopleural bristles 1. 0. 2. Fourth vein arcuate at bend.

Glossina longipalpis et spp.—No hypopleural hairs or bristles. Distinct black pteropleural bristles, with yellowish hairs also. Sternopleural bristles 1. 0. 2. Venation aberrant, in Œstrid direction; apical crossvein continuous with posterior crossvein, fourth vein deeply arcuate before small crossvein so that latter appears continuous with the section of fourth vein following it.

Morellia violacea (det. Coquillett, Brazil), *micans* Macquart (det. Coquillett, Maine), et spp.—No hypopleural hairs. Pteropleural hairs present, bristly hairs in *micans.* Sternopleural bristles 1. 0. 2. Fourth vein arcuate at bend.

Mesembrina mystacea et spp.—No hypopleural hairs or pile. Pteropleural black pile present. Sternopleural bristles 1. 0. 1, but often hard to distinguish from the black hairs or pile. Venation like *Stomoxys,* also like *Myiospila,* fourth vein partly continuous with hind border. This and the five following genera have the inner side of middle tibiæ furnished with one or more strong bristles.

Metamesembrina (gen. nov.) *meridiana* Linné (det. Brauer and von Bergenstamm, Alaska).—No hypopleural hairs or bristles. Pteropleural bristly hairs present. Sternopleural bristles 0. 0. 1. Fourth vein reaching front margin of wing before tip and arcuate at bend.

Eumesembrina (gen. nov.) *latreillei* Robineau-Desvoidy, et spp.—No hypopleural hairs. Pteropleural hairs present. Sternopleural bristles 1. 0. 2. Venation as in *Mesembrina,* but fourth vein more continuous with hind margin.

Dasyphora pratorum et spp.—No specimens. Venation of *Lucilia* (acc. Brauer and von Bergenstamm).

Pyrellia cadaverina (1 spm. det. Brauer and von Bergenstamm), *serena* Meigen (det. Coquillett), et spp.—No hypopleural hairs. Pteropleural hairs present, bristly and short in *cadaverina.* Sternopleural bristles 1. 0. 3 (sometimes 4); in the single specimen of *cadaverina* 1. 0. 2 on one side and 1. 0. 4 on the other, but probably normally 1. 0. 3. Fourth vein arcuate at bend.

Pseudopyrellia cornicina et spp.—No hypopleural hairs. Pteropleural hairs present. Sternopleural bristles 1. 0. 2, but the hind pair with anterior bristle placed nearly as high as the posterior one. Fourth vein arcuate at bend.

Phasiophana obsoleta et spp.

Cyrtoneura sp. (det. Brauer, N. C. and Cala.).—No hypopleural hairs. Pteropleural bristles present. Sternopleural bristles 1. 0. 2. Fourth vein arcuate at bend, apical cell narrowly open. *Morellia micans* (det. Coquillett) and *hortorum* have nearly these characters, and it is likely that the present North Carolina and California specimens belong to *Morellia.*

Auchmeromyia spp.—This genus evidently belongs here. It probably has either hypopleural or pteropleural hairs or bristles.

Ochromyia jejuna J. C. Fabricius (N. W. India) et sp. (Amboyna).—Hypopleural bristles present. No pteropleural bristles, but yellowish pile present. Sternopleural bristles 1. 0. 1. Venation typical.

It will be at once seen from the above notes that the characters of the presence of one or other or absence of both the hypopleural and pteropleural bristles or hairs are the final determining test in the separation of the two superfamilies.

Metamesembrina, Graphomyia, and *Synthesiomyia* do not have the fourth vein continuous in any part of its extent with the hind margin of wing, but all show a more or less distinct posterior inclination of fourth vein where it joins the wing margin, this being less distinct in *Synthesiomyia.* The genera with this venation might be considered by some students to form an aberrant group of the Anthomyioidea, exhibiting a transition toward the Muscoidean type of venation; but, considered from all points of view, their relationships are mainly with the Muscoidea.

Synthesiomyia has strong hypopleural hairs, which can hardly be considered true bristles, yet they serve as a character of equal value. It has also a bare arista. It lacks the pteropleural hairs and bristles.

Musca, Glossina, Pseudopyrellia, Pyrellia, Morellia, and *Dasyphora* (?) have the Muscoidean type of venation strongly marked (except *Pyrellia*), but possess no hypopleural bristles. *Glossina* and *Musca,* however, possess distinct pteropleural bristles like the other Muscoidea, while *Pseudopyrellia, Pyrellia, Morellia,* and *Dasyphora* (?) possess a tuft of more or less bristly hairs in their place, directly beneath the wing bases. *Morellia hortorum* has pteropleural bristles approaching those of *Glossina* and *Musca* in strength, and is doubtless not a true *Morellia,* which has only a tuft of pteropleural hairs. All these genera are more or less intermediate, but they can be distinguished by the above characters.

Some doubt may arise with *Myiospila,* etc., which belong in the Anthomyioidea. They have neither hypopleural nor pteropleural hairs, which will always distinguish them, and it may be seen that the fourth vein is continuous with wing margin behind the middle point of the rather widened apex of wing.

In connection with the characters given for the Muscoidea in the table, it is to be noted that the fourth vein is incomplete in certain genera, as *Roeselia, Phytomyptera, Thrixion, Gastrophilus, Syllegoptera, Euryceromyia, Dichætoneura,* etc.

Finally it may be pointed out that certain species of the old genus *Cyrtoneura,* referred to *Pararicia* by Brauer and von Bergenstamm, and belonging to the Anthomyioidea, show the gentle removal of the fourth vein from the wing margin which is characteristic of the forms whose position has been heretofore misunderstood. These forms were considered by some authors as belonging to the old Muscidæ s. str., and by others as belonging to the Anthomyiidæ, but the characters pointed out by Girschner serve to reveal their true position. They are distinctly to be considered as a genealogical group descended from forms with a wholly straight (as far as wing margin) fourth vein. The extensive removal of the fourth vein from the wing margin in *Pyrellia, Mesembrina, et al.* must be considered as a further step in the development of the venation toward the Muscoidean type. The Muscoidea are without question more specialized than the Anthomyioidea; and since the form normal in the latter exhibits the type of venation universal in the Holometopa (excepting the Conopidæ), the last named subsection is less specialized than the Schizometopa. The Conopidæ stand evidently to one side as a large group rather closely related to both the Schizometopa and the Holometopa, but with a preponderance of affinities for the latter. They doubtless represent a branch which sprung from the proto-Myodarian stem during its period of multiform development. They should be considered as one of the primary divisions of the Holometopa, probably equal in taxonomic rank to all of the other Holometopa taken together. They stand in practically the same relation to the Holometopa as do the Œstridæ to the Schizometopa, the Œstridæ also being a group apart from the other Schizometopa and of older origin. Moreover, the Œstridæ is a polyphyletic group showing affinities with various subfamilies and tribes of Muscoidea, but owing to its present preponderance of characters due to mode of life it is best treated as a family. For similar reasons the Conopidæ are also best treated as of family rank.

While on the subject of the relationships and extreme specialization of the Schizophora in general and the Muscoidea in particular, it becomes highly significant to note that the Muscoidean stock has originated three separate and distinct types of parasitism on mammals, all having the same end in view—that of nourishing their larvæ at the expense of Mammalia—but each of the three attaining this result in radically opposite ways.

Cuterebra and its allies attain this end by their well-known subcutaneous larval endoparasitism, in which the larva does all the feeding, the imago taking no nourishment whatever, this peculiarity being developed even to the extent of the adult mouthparts having become atrophied and nonfunctional.

Glossina secures the same result by a supracutaneous imaginal ectoparasitism, in which the adult does all the feeding, by actual mechanical blood-letting, and retains and nourishes the larva within the oviduct until it is fully grown, when it is extruded and becomes a pupa almost immediately and absolutely without feeding. This is the exact antithesis of the preceding.

But the Muscoidea must be credited with developing yet a third, and still more remarkable method, because wholly unique and unparalleled among dipterous larvæ of this description, of living at the expense of mammals. *Auchmeromyia* produces a bloodsucking larva, and thus furnishes a case of supracutaneous larval ectoparasitism, since the larva sucks blood externally by mechanical means. This is the so-called Congo floor-maggot, which has recently attracted some attention in the literature. It possesses an extended range on the West African coast and has also been reported from Uganda. The maggot-like larva pierces the skin of sleeping persons with its small but sharp jaws, and sucks their blood. It is an unique habit, because the larva is a footless maggot with extremely small jaws and no means of attaching itself to the skin of its host other than by its mouthparts. It can not cling during the act of piercing by any structure except its mouth-hooklets. The acquirement of such a habit has been possible through the fact that the natives of the region inhabited by it have from time immemorial slept on mats spread upon the earthen floors of their dwellings. The larvæ probably originally fed on fermenting juices and liquids, as evidenced by the fact that they are especially common beneath the urine-stained mats which have been occupied by sleeping children. The flies are attracted by sour-smelling liquids, and doubtless oviposit beneath the sleeping-mats of young children.

The peculiar mode of reproduction of *Glossina* is carried even farther by the Hippoboscid genera of mammal ectoparasites (*Lipoptena, Melophagus, Hippobosca, Ortholfersia*). The larva in these forms is retained and nourished within the oviduct of the female until full grown, but upon being extruded is incapable of movement. The *Glossina* larva upon extrusion is capable of only sufficient movement to find a suitable place for pupation, whereupon its integument undergoes chitinization to form the pupal envelope. The Hippoboscid larva upon extrusion forthwith undergoes this process of external chitinization. The Hippoboscid female therefore extrudes the larva in a situation and position suitable for it to remain during its pupal period. It is thus evident that some relationship exists between *Glossina* and the Hippoboscidæ, doubtless to the extent of a not very remote common origin. The Hippoboscidæ are probably an offshoot from the old muscid stock on the one hand, and the Œstridæ are likely an earlier offshoot in a quite opposite direction from several stems of the same stock.

The Œstrid habit of parasitism seems the oldest, the *Glossina* and Hippoboscid habit next, while the *Auchmeromyia* mode is evidently very recent.

DESCRIPTIONS

But little need be said in preface to the following descriptions of genera and species. In addition to the treatment of new forms, there is given considerable supplementary descriptive matter on forms already described.

As a basis of operations in determining the North American Muscoidea, the recent Smithsonian Catalogue of North American Diptera, by Professor Aldrich, will be found quite indispensable. Its value lies in its references to descriptions. It will be necessary to use it with much caution so far as the synonymy is concerned. It should also be pointed out that the sequence of genera there employed is unnatural and misleading. This is not the fault of the cataloguer, but is due to the present unsatisfactory state of the literature of the North American forms.

The sequence of subfamilies and tribes here adopted is as nearly a natural one as is possible of attainment in the present state of our knowledge. No doubt further study will modify this arrangement in certain details.

It is to be noted that the tribes which appear in center heads are independent of the subfamilies preceding them, except those in italics under the families Muscidæ and Phasiidæ.

Family MACRONYCHIIDÆ

Tribe Trixodini

Genus Trixodes Coquillett

Trixodes Coquillett clearly exhibits in its weakly developed mouthparts, peculiar facial plate, and weak macrochætæ a close affinity with the Œstridæ. The type species is *obesa* Coquillett, described from a specimen collected by the writer in the Sierra Madre of Chihuahua. A second specimen was collected by the writer on the West Fork of the Gila, in New Mexico.

Subfamily Megaprosopinæ

Genus Microphthalma Egger.

Microphthalma trifasciata Say.—*Tachina disjuncta* Wiedemann may be a small specimen of this species.

The genus *Microphthalma* is distinct from *Dexiosoma,* from which it differs in its relatively small eyes, almost bare and much shortened arista, more compressed third antennal joint, and almost bare parafacials. The antennæ are inserted on eye middle.

M. michiganensis Townsend is a large northern form, distinct from *trifasciata* or *disjuncta* in its red face and cheeks, third antennal joint hardly longer than second, facial profile more flattened, silvery pollen of abdominal segments general and not restricted into basal fasciæ.

Tribe NEOPHYTOINI

NEOPHYTO, gen. nov.

The genus is like *Megaprosopus* in the formation of the facial plate, epistoma, and facial ridges, but the vibrissæ are distinct from the peristomal bristles below them, and the parafacials have an oblique well-marked row of thinly set bristles (not thickly placed as in *Macronychia*). Frontal bristles not strong. Peristomalia quite closely approximated. Cheeks more than one-half as wide as eye height, sometimes appearing almost as wide in female. Front prominent, facial profile strongly receding and slightly convex. Antennæ inserted distinctly below middle of eyes. Apical cell closed in margin considerably before wing tip; fourth vein bent at angle, without stump but with slight wrinkle, hind crossvein in middle between small and apical crossveins. Male without, female with two middle fronto-orbital bristles. Type, *Phyto setosa* Coquillett.

Neophyto anomala, sp. nov.

Syn. *Phyto clesides* COQUILLETT (non Walker).

Length, 6 to 8 mm. Grayish cinereous. Facial plate narrow, oval, acute below, the vibrissal angles but little more approximated than the peristomalia below them, the facial profile strongly convex. Face, parafacials and parafrontals silvery; palpi, cheeks, frontalia and antennæ light reddish brown, third antennal joint brown. Antennæ very short, third joint no longer than second, second about three times as long as the very short first joint. Male front much narrower than eyes, female front wider than eyes. Male cheeks two-thirds eye height in width, female cheeks fully equal to eye height. Mesoscutum cinereous, with three dusky vittæ in male, almost obsolete in female. Abdomen dusky cinereous, with anterior portions of second and third segments and most of anal segment silvery-cinereous. In the female especially the dusky portion is more variable, appearing in some only on narrow hind margins of second and third segments. Discal macrochætæ on all the segments except the first. Male abdomen long-subconical, female abdomen oval and flattened. Legs blackish. Wings clear, veins brown. A strong costal spine present, apical cell sometimes extremely short petiolate. Tegulæ white.

Missouri to Louisiana.

Type.—Cat. No. 11,646, U. S. N. M. (Missouri, Riley Coll.).

Family TACHINIDÆ

Tribe Miltogrammini

Genus Senotainia Macquart

Senotainia rubriventris Macquart.—The writer retains *Senotainia,* of which this species is the type, as distinct from *Miltogramma* in having a more evenly rounded facio-frontal profile, narrower cheeks, bare parafacials, distinct vibrissæ, and longer antennæ.

Miltogramma and *Senotainia,* with certain other forms yet to be described, constitute a tribe by themselves. The writer can not follow Brauer and von Bergenstamm in grouping *Metopia, Araba, Hilarella,* etc., with them. The latter genera have the vibrissal angles close to oral margin.

Tribe Myiophasiini

Genus Myiophasia Brauer and von Bergenstamm

Myiophasia of Brauer and von Bergenstamm has the eyes bare; cheeks in female more than one-third eye height in width, in male scarcely one-fourth eye height; both sexes with short but strong claws, the front claws of male being the only ones that are somewhat longer than last tarsal joint; no macrochætæ on first and second abdominal segments; arista thickened only at extreme base, second joint short.

It is hardly possible that the Uruguayan and United States forms that have been referred to this species are identical. Several other well-marked forms have been confused here. *M. ænea* has the apical cell distinctly though narrowly open

Myiophasia setigera, sp. nov.

Differs from *M. ænea* in having a median marginal pair of macrochætæ on second abdominal segment in both sexes. Male with rows of hairs on parafacials, female with same rows somewhat less developed.

Texas, New Mexico, Nevada, Oregon.

Type.—Cat. No. 11,647, U. S. N. M. (Male, Beulah, New Mexico, 8,000 feet, August, Cockerell.)

A female specimen received from the Cotton Boll Weevil Laboratory (Hunter) was collected on an acorn of *Quercus alba* at Ruston, Louisiana, October 31, and was apparently ovipositing on a weevil larva within.

A female specimen from New Mexico (Santa Fé, Cockerell) has a pair of small macrochætæ on anterior border of second and third abdominal segments, and a submarginal posterior pair on third segment. It may be a distinct form.

These forms are placed in *Myiophasia* tentatively, and may need to be removed on further study.

Genus Phasioclista Townsend

The genus *Phasioclista* Townsend also has the eyes bare, but the cheeks are almost or quite one-half eye height in width in both sexes; male claws long, all being distinctly longer than last tarsal joint; female claws very short; arista bulbous at base, indistinctly jointed; first and second abdominal segments without macrochætæ, apical cell closed or sometimes very narrowly open, hind crossvein nearly straight.

Myiophasia differs from *Phasioclista* in having a loosely set, oblique, fringe-like row of bristly hairs on parafacials, in addition to the shorter irregularly arranged hairs above them; the cheeks are not so wide, as above pointed out, a double costal spine is present, and the antennæ reach almost to insertion of vibrissæ.

Whether the specimens with apical cell open and closed represent different forms of *Phasioclista* is still a question, but the fact is recorded in *Psyche* (June, 1893, p. 467) that specimens bred from different hosts differed in this character. A specimen bred from *Leucania unipuncta* had the apical cell open, and another bred from *Sphenophorus parvulus* had same closed. The radical difference between a parasitic habit involving a lepidopterous larval host with soft skin, and one affecting an adult coleopterous host, would easily imply the distinctness of these forms.

Phasioclista metallica Townsend.—Both sexes have perfectly bare eyes. Female with more or less suggestion of pollen on mesoscutum in front. No macrochætæ on first two abdominal segments. Male with rows of hairs on parafacials, female practically without.

Florida, Georgia.

Genus Ennyomma Townsend

Ennyomma, at least in the male, has the eyes thickly pubescent; arista distinctly three-jointed, not so bulbous at base as in *Phasioclista;* second abdominal segment with marginal macrochætæ;

apical cell open, sometimes narrowly so; hind crossvein strongly sinuate. The genus may be at once distinguished from both *Myiophasia* and *Phasioclista* by its thickly hairy eyes. *M. robusta* Coquillett belongs to *Ennyomma.*

Ennyomma robusta Coquillett.—Eyes thickly pubescent (at least in male). Last two abdominal segments and anterior border of second segment thickly pollinose. Large species.

California, Mexico.

Ennyomma globosa Townsend.—Eyes thickly pubescent in male, bare in female. Male with purplish shining mesoscutum, showing no pollen. Female showing pollen at least anteriorly and on humeri. Small species. The species was described in the male only, and referred to *Loewia.* Numerous male specimens agree perfectly with the description. The female is without macrochætæ on first two abdominal segments, the male having them as in the description.

White Mountains, New Hampshire; Maryland, Georgia, Florida, Louisiana, Missouri, Sierra Madre of Chihuahua, Mexico City, Nicaragua.

Two specimens, male and female, bred from *Anthonomus grandis,* Alexandria, Louisiana (Hunter, No. 1326, W. 6).

Tribe EUMEGAPARIINI

EUMEGAPARIA, gen. nov.

This genus may be considered intermediate between *Megaparia* and *Dexia,* but must be classed with the Tachinidæ in the neighborhood of the Dexiinæ. The oral margin is only slightly prominent and the facio-peristomal profile approaches that of the Megaprosopinæ, but the oral margin is nevertheless sufficiently prominent to destroy the evenly convex outline characteristic of the Megaprosopine profile. The antennæ are short and the mouthparts much reduced, the proboscis being very short. The mesofacial plate, however, is of good width and length; the vibrissal angles are widely separated and only feebly convergent, about as high above oral margin as length of second antennal joint. Ptilinal suture terminating well above vibrissal angles. Claws of male very long. Type, *Megaparia flavcola* Coquillett (No. 6236, U. S. N. M.), Colorado.

Subfamily DEXIINÆ

Genus Ptilodexia Brauer and von Bergenstamm

Clinoneura and *Ptilodexia.*—*Ptilodexia* has parafacials hairy, more than one pair of discal macrochætæ on middle abdominal segments, and male claws very long. *Clinoneura* has parafacials bare, only one pair of discal macrochætæ on middle abdominal segments. The species described by Robineau-Desvoidy as *Estheria tibialis* is neither a *Ptilodexia* nor a *Clinoneura*, since it has the apical cell petiolate.

Ptilodexia has cheeks (male) about, or slightly over, one-half eye height; antennæ inserted low, so as to give a long frontal profile; vibrissæ inserted high above oral margin; no strong or other reclinate vertical bristles; second antennal joint elongate and third shortened.

DOLICHOCODIA, gen. nov.

Near *Myiocera*, from which it differs as follows: Head conspicuously elongated anteriorly, apical cell open. Antennæ inserted on or above middle of eyes; proboscis slender and horny, with long filiform palpi which are but slightly thickened apically and bear very long bristles; parafacials wider; long axis of head at antennal insertion fully equal to that at epistoma; head longer than high. Type, *Myiocera bivittata* Coquillett, described from specimens collected by the writer on the Rio Ruidoso, in the White Mountains of New Mexico.

EUCHÆTOGYNE, gen. nov.

Like *Chætogyne*, but proboscis rather stout and only a little longer than head height; hind tibiæ completely ciliate on outer edge, with no bristles among the cilia. It agrees with *Chætogyne* in having the carina wide, flattened on its edge and conspicuously furrowed on median line. Type, *Hystrichodexia roederi* Williston (Kansas Univ. Quarterly, II, pp. 77-78), described from Arizona (1 male). For purposes of comparison, the following characters are given for certain allied genera:

Hystrichodexia has proboscis shorter than head height.

Paraprosena has carina narrow and thin.

Chætogyne has proboscis very long and slender, hind tibiæ with long macrochætæ among the cilia.

Phorostoma has only a weak rudimentary facial carina.

Euchætogyne roederi Williston.—Three males in U. S. N. M.; two collected by the writer in Meadow Valley, Sierra Madre of western Chihuahua, head of Rio Piedras Verdes, about 7,300 feet, August 30 and September 2; and one labeled "Mexico, 400, *Phorostoma*."

Williston says in his description: "Third, fourth, and fifth segments opaque golden yellow." The so-called *fifth* segment shows very narrowly, being the base of the hypopygium. It is in reality the sixth segment, since there is a very abbreviated basal segment present. What appears to be the fifth segment is only the portion of the fourth behind the transverse row of submarginal macrochætæ. The scutellum shows practically no yellowish on apex. The third segment (called second heretofore) has, in addition to the six approximated macrochætæ on hind border in middle, three or four (usually four) approximated lateral ones on each side. The second segment (so-called first) has one lateral macrochæta on each side. The apical decussate pair of scutellar macrochætæ is quite as strong and long as any of the others of scutellum. The narrow linear yellow of hind margin of third segment is continued in a slight anterior prolongation on the median line in the two Sierra Madre specimens. In addition to the two large silvery spots of third segment of venter, there are two smaller ones on the second and fourth ventral segments in the above specimens.

Genus Myxodexia Brauer and von Bergenstamm

Syn. *Tropidomyia* BRAUER & VON BERGENSTAMM (preocc.).
Neotropidomyia TOWNSEND, nom. nov. (Dec., 1891), Trans. Am. Ent. Soc., XVIII, p. 382.

The type of this genus is *M. macronychia* Brauer and von Bergenstamm, of Syria.

Subfamily TRIXINÆ

EUCLYTIA, gen. nov.

This genus is herewith proposed for the species *Clytia flava* Townsend (Tr. Am. Ent. Soc., XVIII, pp. 372-373). It may be known by the two rows of weak frontal bristles on each side of frontalia, the outer row weaker and somewhat irregular. The epistoma is but slightly prominent. Specimens in U. S. N. M. have been referred by Brauer and von Bergenstamm to *Redtenbacheria,* but the species certainly can not be included in that genus.

It is distinct from the old genus *Clytia,* now to be known as *Clytiomyia,* of which the European *C. helvola* is to be taken as the type. *Clistomorpha* also is a very different genus. Both *Clistomorpha* and *Clytiomyia* belong in the Phasiidæ.

Tribe PHASIOPTERYGINI

Genus Phasiopteryx Brauer and von Bergenstamm

Phasiopteryx bilimeki Brauer and von Bergenstamm.—The remarks on this species in Ann. and Mag. Nat. Hist., XIX, pp. 33-34, indicate differences between *Phasiopteryx* and *Neoptera,* the significance of which did not appeal to the writer at the time. It seems quite certain that several forms are confused here. The specimens that the writer has seen of related forms in the Œstrophasiinæ incline him to the belief that large series of material will demonstrate the distinctness of *Neoptera* and *Phasiopteryx*. It must be remembered that only a fraction of the neotropical fauna is yet known.

Besides the differences, pointed out below, between *Œstrophasia* and *Phasiopteryx,* the following may also be noted: *Œstrophasia* and *Cenosoma* have the facial plate flat or subcarinate; antennæ inserted distinctly below middle of extreme head height, almost as low as lower margin of eyes; arista very short and bare, and third antennal joint only about as long as second. *Phasiopteryx* has the facial plate more strongly, often quite strongly, carinate; antennæ inserted but little below middle of eyes, distinctly above middle of extreme head height; arista very long, very distinctly but finely and thinly hairy (looks bare in some specimens, apparently from the fine hairs being lost or rubbed off), and the third antennal joint always twice as long as second.

Subfamily Œstrophasiinæ

Genus Œstrophasia Brauer and von Bergenstamm

Œstrophasia clausa Brauer and von Bergenstamm.—This is a northern species. The specimens from Cuautla, Mexico, referred here by Giglio-Tos, doubtless represent another form. Cuautla is thoroughly tropical, and *clausa* is a transition and boreal form.

The ultimate section of fourth vein in *Cenosoma signifera* and *calva* is normally rather deeply bowed in, but not so in *Œ. setosa* and *clausa,* both of which have the apical cell very short petiolate, while *setosa* has third vein bristly nearly to small crossvein.

The antennæ of *Œstrophasia* and *Cenosoma* are widely separated by a characteristic median enlargement of the lunula in both sexes of all the species. This is absent in *Phasiopteryx,* which has the antennæ closely approximated.

Genus Euœstrophasia Townsend

Euœstrophasia aperta Brauer and von Bergenstamm.—This South American form seems generically distinct from the species of *Œstrophasia* in its open first posterior cell, as pointed out in Trans. Am. Ent. Soc., XIX (1892), p. 133.

Genus Cenosoma van der Wulp

Cenosoma signifera van der Wulp.—It is likely that this tropical species will prove generically distinct from both *Œstrophasia* and *Euœstrophasia,* when sufficient material is studied. Two species are probably confused in the catalogue under the name of *signifera.* The Canadian and New England specimens are probably a northern form distinct from the tropical one. *Œ. calva* may be considered congeneric with *signifera.*

Subfamily Paramacronychiinæ

Genus Pachyophthalmus Brauer and von Bergenstamm

Pachyophthalmus aurifrons Townsend.—This species is quite distinct from the European *signatus* Meigen, which probably does not occur in America. It differs from *signatus* in the golden pollinose sides of front and face, third antennal joint about the length of second, hind crossvein very slightly bowed, front quite strongly produced, etc. *P. signatus* has pollen of front and face silvery white with blackish reflections but without golden, third antennal joint about twice as long as second, hind crossvein strongly bowed, front scarcely protruded, etc. Both *aurifrons* Townsend and *floridensis* Townsend are best assigned to this genus.

Genus Sarcomacronychia Townsend

Sarcomacronychia unica Townsend.—This species, *S. sarcophagoides,* and *S. trypoxylonis* are to be considered as three valid forms. The genus *Pachyophthalmus* differs from *Sarcomacronychia* in having the ptilinal area wider in comparison with parafacials, being three-fifths to almost three-fourths width of face; cheeks as wide as one-sixth to one-eighth eye height, or less; eyes descending but little lower than vibrissæ, as seen in profile. *Sarcomacronychia* has facial plate very small and restricted, being two-fifths to one-third width of face, parafacials proportionately wider, often nearly as wide as facial plate itself, but sometimes appearing narrow in profile; width of cheeks from little less than one-fourth to about one-fifth eye height; eyes descending far below vibrissæ, and even below epistoma, nearly as low as lateral oral margins, as seen in profile. *Pachyophthalmus* has the vibrissæ inserted but little above epistoma, and the antennæ are inserted below middle of eyes. *Sarcomacronychia* has vibrissæ inserted much farther above epistoma, and the antennæ are inserted on eye middle.

Tribe Melanophryonini

Genus Atropharista Townsend

The affinities of *Melanophrys* and *Atropharista* are uncertain. One can hardly agree with Brauer and von Bergenstamm's reference of them to the Paramacronychiinæ. They seem rather to belong in the Tachinidæ.

Atropharista jurinoides Townsend.—The writer has previously considered this genus synonymous with *Melanophrys,* but it appears after all to be distinct. *Melanophrys* has the second antennal joint short, the third joint being three to five times as long as second, according to sex. *Atropharista* has second antennal joint elongate, the third joint same length or a little longer, probably never twice as long even in the male.

The species *jurinoides* is distinct from Walker's *Tachina insolita,* if any reliance is to be placed on the description of the latter. *T. insolita* is described as having the third antennal joint fully twice as long as second, third aristal joint very stout, and a white oblique stripe on each side of head, presumably (from the connection) opposite the antennæ. The last character agrees with *Mel. flavipennis,* but the other characters only partially agree. None of them seems to agree with *Atropharista,* as the second antennal joint does not appear to be elongate in *insolita.*

A. jurinoides differs from both in having a broad, elongate silvery crescent bordering the orbit on each side of the head, partly on the parafrontals and partly on the parafacials. It is quite certain that the elongate second antennal joint will prove *Atropharista* to be a valid genus, as genera will ultimately go in this superfamily.

Subfamily Phytoinæ

EUPHYTO, gen. nov.

Differs from *Phyto* (Robineau-Desvoidy) Brauer and von Bergenstamm in having parafacials absolutely naked, tegulæ small and rounded, cheeks not widened posteriorly, apical cell quite long petiolate, hind crossvein in middle between small crossvein and bend of fourth vein, and no discal macrochætæ on abdomen.

Differs from *Stevenia* Robineau-Desvoidy in parafacials being wide, same width above and below, their width being equal to that of cheeks, which are over one-third eye height. Cheeks bare, same as parafacials.

Type, *Leucostoma subopaca* Coquillett.

Tribe METOPIINI

Genus Hilarella Rondani

The cheeks in this genus are about one-fourth eye height, parafacials with a row of bristles to lower eye margin, arista pubescent or hairy, front sharply produced in profile. *Opsidia* is much closer to *Hilarella* than is *Eumacronychia*.

Tribe EUMACRONYCHIINI

Genus Eumacronychia Townsend

Eumacronychia decens Townsend.—This species is the type of the genus, which has cheeks about one-half eye height in width, parafacials bare of bristles, frontal bristles stopping at base of antennæ.

Genus Gymnoprosopa Townsend

Gymnoprosopa polita, argentifrons, and *clarifrons* are perfectly distinct, in spite of the note in the catalogue. They may be recognized by the descriptions.

SPHENOMETOPA, gen. nov.

This genus is proposed for *Araba nebulosa* Coquillett. The specimens from which this species was described were collected in the vicinity of Meadow Valley, six or eight miles south of Colonia Garcia, in the Sierra Madre of western Chihuahua, on the head of the Rio Piedras Verdes, in the pine zone, about 7,000 to 7,500 feet (Townsend). The form is not referable to *Araba*.

The genus may be known by the front being conspicuously narrowed anteriorly, the parafacials very narrow and bare, the vibrissæ quite distinct, and the front not produced conically like *Metopia* and *Araba*. It comes near to *Metopodia* in the latter character. The wings are slightly clouded on the veins.

Subfamily PSEUDODEXIINÆ

EUCALODEXIA, gen. nov.

This genus is proposed for *Homodexia flavipes* Bigot. It may be recognized from the characters pointed out by Brauer (Sitzungsber. Kais. Akad. Wiss., CVII, 1, p. 515), who failed to give it a name.

Genus Atrophopoda Townsend and allies

The following new genera are here proposed:

DIAPHOROPEZA, gen. nov.

Type, *Atrophopoda braueri* Williston.

ŒDEMAPEZA, gen. nov.

Type, *Atroph. townsendi* Williston.

CATEMOPHRYS, gen. nov.

Type, *Vanderwulpia sequens* Townsend.

BRAUERIMYIA, nom. gen. nov.

Type, *Wulpia* Brauer and von Bergenstamm (1892), preocc. by Bigot in Dipt. (1886). The genus is a valid one. The new name is proposed as a tribute to the memory of Friedrich Brauer, the one student who has most advanced our knowledge of the Muscoidean flies.

Below is a table of these and certain allied genera. All of them except *Vanderwulpia, Brauerimyia,* and *Catemophrys* have the parafacial bristles continuous with frontal row. This character, however, does not seem to indicate close relationship in all cases, as it is probable that *Ceratomyiella, Metachæta, Dichocera,* and *Atrophopalpus,* all here included, belong in other subfamilies from the rest. *Hypertrophocera* and certain other genera not included in the table possess this character.

Frontal bristles stopping short at insertion of antennæ, apical cell ending well before wing-tip, stump of vein at bend of fourth, hind crossvein nearer bend of fourth, cheeks about one-third eye height, eyes bare, arista pubescent basally, abdomen slender and rather conical, macrochætæ only marginal.. 2

Frontal bristles descending to middle of second antennal joint, apical cell ending well before wing-tip closed or extremely short petiolate, a black wrinkle but no stump at bend of fourth, hind crossvein nearer to bend, cheeks about one-half eye height, eyes bare, arista pubescent basally, abdomen elongate, macrochætæ only marginal.
Type, *Vander. sequens*......................*Catemophrys,* gen. nov.

Frontal bristles descending on parafacials to lower border of eyes..... 3

2. No costal spine, apical cell long petiolate, parafacials bare.
Type, *atrophopodoides* ....*Vanderwulpia*

A costal spine present, apical cell narrowly open or closed in margin, parafacials distinctly very short pilose.
Type, *Wulpia aperta*..................................*Brauerimyia*

3. Palpi atrophied, minute, apical cell closed in border at wing-tip, hind crossvein nearer bend, eyes almost bare.
Type, *angusticornis* ....*Atrophopalpus*

Palpi normal 4

4. Eyes bare, a costal spine, cheeks not over one-fourth eye height; apical cell ending well before wing-tip, long petiolate; claws of both sexes atrophied and tarsal joints compressed and swollen, arista pubescent

in female, hind crossvein is middle between small crossvein and bend of fourth vein, macrochætæ only marginal.

Type, *townsendi*............................*Œdemapeza*, gen. nov.

Eyes bare, a costal spine, cheeks one-half eye height 5

Eyes hairy 6

5. Apical cell long petiolate, ending well before wing-tip; hind crossvein in middle between small crossvein and bend of fourth vein.

Type, *atra* ....*Metachæta*

Apical cell very short petiolate, ending but slightly before wing-tip; hind crossvein a little nearer to bend of fourth vein.

Type, *conica* ....*Ceratomyiella*

6. Apical cell open and ending well before wing-tip, bend of fourth vein with long stump, male antennæ with third joint lyriform cleft.

Type, *lyrata* ....*Dichocera*

Apical cell ending well before wing-tip, a costal spine, hind crossvein much nearer to bend of fourth vein, macrochætæ only marginal (except on anal segment).. 7

Apical cell ending at or but slightly before wing-tip, eyes thinly hairy.. 8

7. Apical cell closed in border (or narrowly open, or very short petiolate), eyes thinly hairy, cheeks fully one-half eye height.

Type, *mexicana* ....*Microchira*

Apical cell moderately long petiolate, eyes thickly hairy, cheeks nearly one-half eye height.

Type, *magnicornis* ....*Lachnomma*

8. Apical cell narrowly open, a costal spine, hind crossvein nearer to bend of fourth vein, cheeks not over one-fourth eye height, eyes very sparsely hairy, all the male claws and pulvilli much elongated, female claws atrophied and tarsal joints compressed and swollen, macrochætæ only marginal.

Type, *braueri*..............................*Diaphoropeza*, gen. nov.

Apical cell closed in border (or very narrowly open or very short petiolate), double costal spine, hind crossvein much nearer to bend of fourth vein, cheeks fully one-half eye height, macrochætæ discal on last segment only, claws and pulvilli of both sexes atrophied (?) and tarsal joints swollen.

Type, *singularis* ....*Atrophopoda*

Apical cell open, cheeks nearly one-half eye height, hind crossvein near middle, no discal macrochætæ (acc. v. d. Wulp) or present on last segment only (acc. B. & v. B.), male claws and pulvilli of anterior tarsi elongated.

Type, *validinervis* ....*Paradidyma*

NOTE TO TABLE.—The group of *Pseudomintho, Olivieria,* etc., has the front tarsi of female plump and swollen, with very small claws. The group of *Mintho, Actinochæta,* and *Euantha* has the last tarsal joint of all the feet in both sexes swollen, and claws very short. But the frontal bristles do not descend half way to vibrissæ in any of these forms, and they are thus easily to be distinguished from the above genera in the table, having somewhat similar feet.

Cholomyia inæquipes Bigot.—One specimen bred at the Cotton Boll Weevil Laboratory, Dallas Texas, from *Conotrachelus elegans,* issued May 29 (Hunter).

NEAPORIA, nom. gen. nov.

This name is proposed for *Aporia* (Macquart) Brauer and von Bergenstamm, which is preoccupied. The type of the genus is *quadrimaculata* Macquart, of South America. The species *limacodis* Townsend seems to belong here also. The latter is distinct from *Dexia pristis* Walker in its practically bare arista. *D. pristis,* so far as can be judged from the description, is not a *Macquartia* s. str. Mr. E. E. Austen has referred it to *Aporia* (Ann. Mag. N. H., ser. 7, vol. XIX, p. 344).

RONDANIMYIA, nom. gen. nov.

This name is proposed in honor of Camillo Róndani for his genus *Gymnopsis* (Dipt. Ital. Pr., III, 1859, pp. 90-91), which is preoccupied by Rafael in Pisces (1815). The type is *Macq. chalconata* (Wiedemann, Meigen, Zetterstedt) Róndani, of Europe. Brauer and von Bergenstamm retain the species in *Macquartia,* but it seems preferable to maintain it separately on the characters pointed out by Róndani.

METHYPOSTENA, gen. nov.

This genus is proposed for the type of *Hypostena barbata* Coquillett, which can be referred to neither *Hypostena, Tachinophyto.* nor *Pseudomyothyria.* There are no bristles on the third longitudinal vein, the small crossvein is almost opposite to the end of the first vein, hind crossvein is in middle between small crossvein and bend of fourth vein, apical cell ends in exact wing-tip. The wings are narrow, their width being much less than one-half their length. The parafacials are narrowed below to a mere line next the lower border of eyes, the facial profile is very oblique and receding, the lower margin of head short, the arista strongly curved.

Subfamily PYRRHOSÜNÆ

Genus Leskia Robineau-Desvoidy

Syn. *Pyrrhosia* pt. (Róndani) Brauer and von Bergenstamm.

Type, *aurea* Fallen.

Genus Pyrrhosia Róndani (restricted)

Syn. *Myobia* (Schiner) Brauer and von Bergenstamm.

Type, *inanis* Fallen.

Genus Anthoica Róndani

Syn. *Myobia* Robineau-Desvoidy (preocc.—non Schiner, Brauer and von Bergenstamm).

Type, *atra* Róndani.

It is clear that the name *Leskia* Robineau-Desvoidy can not be properly substituted for *Myobia* Robineau-Desvoidy (preocc.), inasmuch as the species referred to *Leskia* by that author are not typical *Myobia* in his sense.

Róndani proposed the name *Anthoica* for this very purpose, and it must therefore be employed. *Leskia* should be recognized as distinct.

Genus Aphria Robineau-Desvoidy

Aphria ocypterata Townsend.—One female, Massachusetts (No. 142, Riley Coll.). Length, 7 mm. Agrees with original description. The stump of fifth longitudinal vein does not quite reach margin of wing. The hind crossvein is nearly in middle between the small crossvein and bend of fourth vein, the bend being quite rounded. The third antennal joint is distinctly and evenly rounded on both apical corners.

Aphria occidentale, sp. nov.

One female, Colorado (No. 120, Riley Coll.); one female, Beulah, N. Mex., August (Cockerell); one male, Roswell, N. Mex., August (Cockerell).

Length of female, 7½ to 8 mm.; of male, 9 mm. Differs from *ocypterata* in being more robust, larger, the abdomen more broadly red on sides, the red extending length of first segment and half or more length of third segment; third antennal joint in both sexes distinctly angular on front apical corner, rounded on posterior apical corner, widened in male; stump of fifth vein extending to margin of wing; hind crossvein more noticeably approximated to bend of fourth vein, which bend is abrupt.

The greater size, the character of third antennal joint, and the more widely red abdomen will at once distinguish the species.

Type.—Cat. No. 10,900, U. S. N. M. (Colorado, Coll. Riley).

Aphria georgiana, sp. nov.

Two females, Georgia (Riley Coll.), (= ? *Ocyptera triquetra* Olivier et ? *Ervia triquetra* Robineau-Desvoidy).

Length, 10 mm. This is a distinct species from both of the preceding. It is not so typical of *Aphria* as are the other species, being much larger and wider-bodied. Frontal bristles descend but slightly below insertion of antennæ, hardly more than to base of second antennal joint. The third vein is spined only one-half or three-fourths way to small crossvein, hind crossvein is nearly in middle between small crossvein and bend of fourth vein, stump of fifth vein extends to wing border; first and second antennal joints and base of third reddish yellow, arista and rest of third antennal joint black or brownish; third antennal joint rounded on posterior apical corner, subangular on anterior apical corner. Palpi brownish yellow, or with a reddish tinge. Front fully one-half width of head, frontalia brownish yellow; face and front silvery white, parafrontals slightly cinereous. Thorax, scutellum, and pleuræ quite thickly silvery pruinose over the black ground color. Abdomen obscure light brownish red, obscurely blackish on median line, broadening on hind portions of second and third segments and nearly covering fourth segment; anterior borders of second to fourth segments broadly silvery pruinose, but more faintly so than thorax; legs blackish; wings clear, slightly tawny at base. Tegulæ white, very slightly tinged with yellowish.

Type.—Cat. No. 10,901, U. S. N. M.

PHOSOCEPHALA, gen. nov.[1]

Form rather *Lucilia*-like, narrow, abdomen round-oval, head yellow, wings slightly smoky, palpi absent, thorax and abdomen metallic.

Head and thorax about same width, abdomen slightly wider. Front (female) not prominent in profile, distinctly more than one-half width of head, flattened anteriorly, steeply sloping on anterior two-thirds, ocelli marking summit, vertex lower; parafrontals wide, not swollen, clothed with some fine black hairs; vertex not narrowed; frontal bristles descending in a single row about to middle of second antennal joint, the four front pairs decussate and widely divergent below; two strong reclinate frontal bristles next behind these and between them a pair of weak bristles also directed backward, the outer one outward; two reclinate vertical bristles of equal strength on each side, the outer one directed also outward, these

[1] This genus and several others were purposely described in detail in order to furnish a forcible illustration of the length of a full generic description in these flies, mentioning all the characters, such as would be necessary to enable the student to absolutely place the form in its proper tribe or subfamily without reference to the specimen. Such a description is far too long for practical use, and demonstrates the inadvisability of attempting systematic work in this superfamily without a great amount of previous study and access to a large central collection where all types are to be permanently preserved. Especial attention is here called to the fact that all these characters require to be studied and compared in order to determine the final location of a genus of these flies.

being strongest of all; postvertical bristles small, of same size as the black border row of occiput; ocellar bristles strong, proclinate, divergent; postocellar bristles represented only by weak hairs; two strong proclinate orbital bristles; lunula normal; frontalia differentiated only by being bare of hairs; facial plate elongate, ovate, widened below, about as high as parafacials, greatest width just above vibrissal angles and taking up three-fifths the facial width at that point, quite flat, slightly advancing below, reaching quite to lower margin of head; facial carina absent, antennal grooves hardly developed at all; facialia divergent inferiorly to point just above vibrissæ, then feebly convergent; facial bristles about two above vibrissæ; vibrissæ quite widely separated, inserted just a little above the oral margin; vibrissal angles only moderately pronounced, rather rounded, situated moderately close to oral margin; parafacials not quite twice as wide at base of antennæ as on lower orbits, flattened anteriorly, with some fine black hairs next lower eye-margins; epistoma moderately prominent, narrowed, showing a cut-off flattened edge below vibrissæ; mouthparts normal, proboscis when extended about as long as head height, moderately fleshy, only once bent, labella moderately developed; palpi entirely absent, showing no trace; oral cavity moderately narrow and elongate; peristomalia with a row of seven or eight black bristles, which are continued around edges of occiput; longitudinal axis of head at oral margin practically same as that at insertion of antennæ, the facial profile being slightly concave, and profile of parafacials straight but obliquely receding; antennæ inserted about on a line drawn through middle of eyes and about on upper three-fifths of head height, closely approximated; second antennal joint slightly elongate, fully twice as long as first joint; arista bare, moderately long and slender, a little thickened on basal one-third, basal joints short and indistinct; third antennal joint about twice the length of second, moderately wide and of equal width, rounded on both apical corners; eyes bare, not large, set rather high, not extending as low as vibrissæ, about twice as long as wide; cheeks about as wide as one-half of eye height, clothed with very fine light hairs, cheek grooves absent; lower margin of head nearly straight, but rounded behind; occiput slightly swollen on lower two-thirds.

Sternopleural bristles 3, the middle one inserted lower than the others and about equally distant from them; hypopleural bristles moderately strong, about 5 in number; 2 pteropleural bristles, the posterior one very strong, curved, reclinate; mesopleural bristles in a posterior fringe of 7; propleural bristles 3, curved upward and forward; notopleural bristles 2, strong, curved, reclinate; postsutural bristles 4, the posterior one on each side reaching beyond hind border of scutellum, the others much less strong; 3 dorsocentral bristles; 4 short acrostichal bristles before suture, one strong one next scutellum (if more behind suture, the pin has destroyed them); 6 humeral bristles, moderately short; 3 intrahumeral bristles; 1 presutural bristle nearly in line with last; 3 intra-alar bristles, one in front of suture; 3 strong reclinate supra-alar bristles; 1 strong postalar bristle reaching middle of second abdominal segment, 2 weak ones below it; scutellar bristles consisting of 3 strong and 2 weak pairs, an apical decussate weak pair, a weak, more separated subdiscal pair in front of last, a strong subapical pair reaching almost to base of third abdominal segment, a shorter pair outside these, and the strongest macrochætæ of entire body being a lateral pair inserted on border in front of last, and which reach nearly to base of third segment; some other bristly hairs on scutellum appearing more or less like weak macrochætæ.

Wings not large, rather narrow, extending about length of anal segment beyond end of abdomen, normal; costal spine distinct but short; third longitudinal vein with about five bristles at base; other veins not spined; fourth vein ending in wing-tip, straight to bend, which is sudden (but hardly angular) and very obtuse, last section straight, the whole vein so gently bent as to distinctly narrow the apical cell, bend without stump or wrinkle and slightly more removed from hind margin than any part of the vein beyond it; fifth vein running half way from hind crossvein to wing border, rest being wrinkle; apical cell closed in margin, hind crossvein distinctly trisinuate, a little nearer to bend of fourth vein than to small crossvein, but not greatly removed from middle, the axis of its anterior half at almost a right angle to fourth vein; small crossvein slightly before middle of discal cell.

Abdomen of 4 segments, broad-oval, almost round, strongly convex above, subflattened below, first segment shortened; macrochætæ weak, only marginal except on last segment, first segment without any, second segment with a weak median pair and a weak lateral one, third segment with a marginal row of 8, anal segment with some marginal ones and a row of 6 subdiscal; ovipositor withdrawn inside the subcircular anal orifice on ventral side of last segment.

Legs short (only the hind pair present), femora with short black bristles; hind tibiæ not ciliate, with sharp bristles on posterior side and some shorter ones on front side; tarsi not stout, moderately slender, short, about same length as tibiæ, metatarsi fully as long as the other joints taken together; claws and pulvilli short, a little shorter than last tarsal joint. Type, the following species:

Phosocephala metallica, sp. nov.

One female, Tucurrique, Costa Rica, collected by Messrs. Schild and Burgdorf.

Length of body, 8 mm.; of wing, 6 mm. Head entirely pale yellowish, face and cheeks with a faint silvery bloom; parafrontals, frontalia, and two basal antennal joints unicolorous with a faint brownish tinge; third antennal joint, arista and proboscis pale yellowish brown; eyes dark purplish brown. Thorax, scutellum, and abdomen shining metallic dark purplish, the abdomen hardly more of a purplish black, humeri yellowish; presutural part of mesonotum deep golden pruinose, through which run only two linear vittæ, the pruinose covering thickest on sides and in front, extending backward behind suture very faintly on sides of mesonotum; scutellum faintly silvery pruinose; metanotum faintly silvery, pleuræ silvery gray; abdomen very faintly silvery, not obscuring the metallic sheen, most noticeable on bases of segments, particularly second segment, least so on anal segment. Wings distinctly smoky throughout, a little more so on costal border, extreme base of costa narrowly yellowish. Tegulæ appearing almost white in some lights, but with a smoky yellowish tinge, much whiter than the wings, halteres pale yellowish. Legs brownish yellow, tarsi hardly darker, but appearing blackish from the many short black bristles, coxæ lighter yellowish.

Type.—Cat. No. 10,902, U. S. N. M.

Paranaphora diademoides, gen. nov. et sp. nov.

This new genus and species are proposed for *Ervia triquetra* of Mr. Coquillett's Revision of the Tachinidæ (1897), page 66. The species is not to be identified with Olivier's *Ocyptera triquetra,* which is probably an *Aphria.* It does not fit Robineau-Desvoidy's *Ervia triquetra,* nor does it belong to his genus *Ervia.* The species looks some like *Stomatodexia diadema,* from which it may be at once known by the bare arista, the very elongate second antennal joint, and the atrophied palpi.

PARANAPHORA, gen. nov.

The salient characters of the genus are the elongate second antennal joint and the atrophied palpi, as just mentioned. Front at vertex one-third width of head in female, one-fourth in male. Palpi extremely small, cylindrical, like a minute grass seed, with a long, delicate apical hair. Apical cell narrowly open a little before wing-tip, sometimes almost closed in margin. Bend of fourth vein angular, with slight wrinkle, sometimes with slight stump. Hind crossvein much nearer to bend of fourth vein than to small crossvein, the latter on middle of discal cell. First vein ending well beyond small crossvein. A long costal spine present. Macrochætæ only marginal, except some submarginal on last segment.

Second antennal joint about four times as long as first, about equal to third. Frontal bristles descending only two below base of antennæ. Arista and eyes bare. Front prominent; parafacials moderately wide, about one-half width of facial plate. Face receding, epistoma slightly prominent; facialia bare, except two or three short bristles above vibrissæ in male, but practically absent in female. Vibrissæ strong and inserted a little above oral margin. Cheeks about one-half eye height. Antennæ inserted above line drawn through middle of eyes. Occiput swollen inferiorly. Male without, female with two orbital bristles.

Scutellum with a very short apical decussate pair of bristles, and two strong lateral pairs with a weaker bristle between them. Abdomen composed of five segments, first short, second shorter than those following. Male abdomen elongate-conical, last segment laterally compressed; female abdomen ovate with apex conical. Legs rather long, tarsi of male very elongate; male claws very long. Type, the following species:

Paranaphora diademoides, sp. nov.

Five females, four males, Mississippi, Louisiana, Texas.

Length, 7 to 12 mm. Head, thorax, and scutellum of male golden pollinose, most thickly so on thorax and scutellum. Same parts of female brassy gray, extending over abdomen. Antennæ of male reddish yellow, those of female brownish yellow. Frontalia reddish brown. Palpi minute, pale yellowish. Mesoscutum with a median pair of linear vittæ interrupted at suture and obliterated shortly behind same; a lateral triangular blackish marking just in front of suture outside these, and a longer, narrower, posteriorly pointed one corresponding to it behind suture. Abdomen of male reddish yellow with base, median line and broad hind borders of last three segments brown, a golden bloom over the lighter portions. Female with narrow hind borders of last three segments brown, with brassy gray bloom, the second segment faintly yellowish. First segment without macrochætæ; second with anterior and posterior lateral, and a median marginal pair; third with two lateral pairs and a median pair; fourth with 8; anal segment of male with about 8 marginal and 6 or more submarginal, those of female less in number and more nearly apical. Legs reddish yellow, tarsi brownish, hind tibiæ brownish, and sometimes the femora less so. Tibiæ of female all reddish or yellowish. Wings distinctly yellow along narrow front border, the submarginal cell clear. Tegulæ slightly tinged with yellowish, mostly on borders.

Type.—Cat. No. 10,903, U. S. N. M. (Mississippi, H. E. Weed).

It is possible that further study will show the distinctness of some of the above specimens.

PARAFISCHERIA, gen. nov.

This genus is proposed for the type of *Demoticus venatoris* Coquillett, which is not a *Demoticus.* (Latter genus has the epistoma not at all produced, and furthermore has a short and fleshy proboscis.) The present genus approaches both *Demoticus, Rhinotachina,* and *Fischeria,* but agrees with neither, though it is clearly more closely related to the latter, as shown by its produced epistoma.

There are orbital bristles in the male, all the claws of the male are short, and the epistoma is strongly produced noselike (as in *Fischeria);* second aristal joint is short but distinct, and arista is short-pubescent; macrochætæ discal and marginal, though only weak discal ones (if any) are present on third abdominal segment. The proboscis is elongate and horny (also as in *Fischeria*), the portion below geniculation equal to head height (also equal to lower margin of head). Cheeks wide, fully one-half of eye height, hind crossvein nearer to bend of fourth vein than to small crossvein. Washington State (O. B. Johnson).

NEOFISCHERIA, gen. nov.

This genus is founded on the specimen from Philadelphia, Pa., mentioned on page 120 of Mr. Coquillett's "Revision" as *Demoticus venatoris.* It is related to *Parafischeria,* from which it differs as follows:

Male: Discal macrochætæ well developed on last three abdominal segments, consisting of a transverse discal row on last segment and a single discal pair on intermediate segments; basal segment with a lateral marginal, but no median marginal; next segment with both; last two segments with a marginal row. Hind crossvein nearly in middle between small crossvein and bend of fourth vein; no orbital (middle fronto-orbital) bristles in male, and male claws elongate. Cheeks about one-third eye height, proboscis elongate. Front tarsi (male) much longer than front tibiæ. Type, the following species:

Neofischeria flava, sp. nov.

One male, Philadelphia, Pa. Coll. Coquillett.

Length, 11 mm. General color yellowish. Antennæ reddish yellow; arista and third antennal joint, except base, blackish. Head, thorax, and scutellum dark in ground color, thickly light golden pollinose, the face more silvery. Palpi light reddish yellow. Abdomen and legs light reddish yellow, the tarsi quite blackish; abdomen thickly light golden pollinose, under which shows faintly a broad median stripe that widens on next to last segment into a triangular marking spreading along hind border, anal segment tinged with blackish only on the broad median line. Venter tinged with darker apically. Wings clear, very slightly yellowish at base; tegulæ tinged with yellowish; halteres yellowish, including stalks. Pulvilli rather smoky; claws brownish, with black tips.

Type.—Cat. No. 10,904, U. S. N. M.

EUDEMOTICUS, nom. gen. nov.

This name is proposed for *Plagiopsis* Brauer and von Bergenstamm (1889), which is preoccupied in Hemiptera by Bergroth (1883). Type, *Demoticus soror* Egger, of Europe.

APACHEMYIA, gen. nov.

This genus is proposed for *Demoticus pallidus* Coquillett.

Only marginal macrochætæ, front tarsi much longer than front tibiæ; proboscis only moderately elongate, horny, with large labella, cheeks fully one-half eye height, male without orbital bristles. Claws of male elongate. Hind crossvein nearer to bend of fourth vein than to small crossvein, apical cell narrowly open before wing-tip, bend of fourth vein without wrinkle. Frontal bristles descending to middle of second antennal joint, latter being more than twice the length of the somewhat elongate first joint, second aristal joint short, some fine hairs on parafrontals outside the frontal bristles, facialia bare, epistoma strongly produced. Palpi well developed, elongate, a little thickened apically, slightly curved.

Represented in U. S. N. M. by two male specimens collected on the Rio Ruidoso, White Mountains, New Mexico (Townsend), on flowers of *Rhus glabra,* 6,500 to 6,700 feet, July 25 and 29, and by type of *D. pallidus,* male, Denver, Colo. The species is large and robust.

All three specimens may be considered as *A. pallida* Coq.

EUPHASIA, nom. gen. nov.

This name is proposed for the Australian *Neophasia* Brauer and von Bergenstamm, which is preoccupied in Lepidoptera.

Genus Drepanoglossa Townsend

The genus *Drepanoglossa* (type, *lucens* Townsend) has the cheeks one-third or more of eye height. *Epigrimyia* is distinct in having extremely narrow cheeks and parafacials; the eyes long and extending low, fully to insertion of vibrissæ; proboscis shorter, parafacials hardly widened above, front not prominent, epistoma strongly produced below, face perpendicular, and tarsal joints short.

Drepanoglossa amydriæ, sp. nov.

Three specimens, bred from masses of pupæ of a tortricid, *Amydria* sp., sent by Prof. A. L. Herrera, Cuernavaca, Mexico.

Length, 6 to 7 mm. Differs from *lucens* in whole coloration being darker; wings slightly infuscate, with a faint yellow tinge in the marginal cell; the mesoscutum cinereous pollinose with a faint tinge of brassy; abdominal segments, except anal, with a narrow hind margin of brown. Proboscis black on apical half.

Type.—Cat. No. 10,905, U. S. N. M.

Drepanoglossa lucens Townsend.—This species has the wings perfectly clear, the mesoscutum pale flesh tint with silvery-white pollen, the abdomen pale clear yellowish except median line and more or less of anal segment, no dark hind margin on first segment and only faint ones on middle segments.

Tribe Epigrimyiini

Tribe Epigrimyiini is close to Phaniinæ, but best retained as a separate tribe not actually coming within that subfamily. It includes *Epigrimyia* only. The genus *Drepanoglossa* clearly falls within the subfamily Pyrrhosiinæ.

Tribe Leucostomini

Genus Leucostoma Meigen

Leucostoma nigricornis Townsend.—The species *nigricornis* and *senilis* are distinct, and may be recognized by the characters given in the descriptions. *L. nigricornis* is essentially a southwestern and western species, and *senilis* an eastern and northeastern species. The former has the antennæ more uniformly blackish, the second and third joints equal in length; the latter has them more rufous, the third joint being distinctly longer than the second as a rule. *L. nigricornis* has the sides of abdomen somewhat reddish at base, and the femora and tibiæ more or less so as well. Both species belong in *Leucostoma*. The genus *Phyto* has the cheeks and parafacials much widened, the cheeks about one-half eye height.

The species *atra* Townsend and *neomexicana* Townsend are likewise distinct forms, and do not belong to *Phyto*. It is doubtful if they can be properly referred to *Leucostoma*.

Leucostoma analis Meigen.—This species should not be recorded from America, as van der Wulp was presumably in error in his determination of it in Mexican material.

Subfamily PHANIINÆ

Genus Hemyda Robineau-Desvoidy

Hemyda aurata Robineau-Desvoidy.—The males of this species have the yellow of third segment wider than the females, but only slightly so, and nowhere nearly approaching in that respect the form described below from New Mexico. There are eight specimens of this form in the U. S. N. M., from Missouri, Kansas, Illinois, and Wisconsin, one of them being labeled "attracted to light, July, 1876" (Riley Coll.).

It is interesting to note that one of the above specimens, from Milwaukee, Wis., has the small crossvein of both wings practically absent; frontal bristles long, numerous, and thickly placed, and vibrissæ distinct, as in several others of the specimens.

Hemyda sp.—One male, Rio Ruidoso, White Mountains, New Mexico (Townsend), about 6,500 feet, August 1, on flowers of *Monarda stricta*. Length, 12 mm. Differs from all the above specimens by having the yellow of third segment taking up anterior two-thirds of length of segment except a median triangular prolongation anteriorly of the black of hind portion, which stops well before anterior edge of segment. The femora have only a faint trace of the black of *aurata* in a tinge of brown before apex. The yellow of second abdominal segment is more extended forward also. The specimen shows only microscopic vibrissæ, invisible save with a high-power lens.

This specimen probably represents a distinct form, but it is not deemed wise to name it as such without first securing a considerable series of specimens to substantiate its claim to distinctness.

Hemyda (Ancylogaster) armata Bigot.—It is highly probable that this is a good species. It may even be a good genus. Bigot states that the second antennal joint is longer than the third. If it develops that the second joint in Bigot's type is strongly elongate, more so than is *aurata,* that is to say more elongate as compared with the first joint (not the third), then it is likely that *Ancylogaster* should be retained.

Genus Penthosia van der Wulp

Penthosia satanica Bigot.—In this species the fourth longitudinal vein is slightly rounded at bend, and often bears a very short stump which can not be considered as the prolongation of the fourth vein beyond the apical crossvein, since no wrinkle is present in its absence. It always points straight away from the bend, like the stem from the arms of a Y, and is to be regarded perhaps as indicating an original sharp bend of the vein back upon itself for a short distance, the two approximated parts having later coalesced, finally disappearing more or less completely. The writer knows of no other tachinid which exhibits this peculiarity in the same degree.

Genus Cercomyia Brauer and von Bergenstamm

Synonyms are *Uromyia* Meigen (preocc.) and *Neouromyia* Townsend, nom. gen. nov. (Trans. Am. Ent. Soc., December, 1891, p. 382).

Subfamily GYMNOSOMATINÆ

Genus Gymnosoma Meigen

The following description of the external anatomy of the male abdomen will be of interest as throwing light on the taxonomic position of the genus.

The male of *Gym. fuliginosa* Robineau-Desvoidy has six abdominal segments besides the genitalia. The first segment is very short, and its width is equal to about one-half the greatest width of abdomen. It consists below of a small, much shortened, subquadrate, basal ventral plate, wide in front and somewhat incurvate on front edge where it joins metathorax, rapidly narrowed posteriorly, its hind margin much shorter than its front margin. The second ventral plate is a smaller replica of the first, its front edge being the same length as the posterior edge of first, its sides converging posteriorly on same lines, its posterior edge being correspondingly shortened. The first and second ventral plates together thus appear much like a right-angled triangle in outline, with the hypothenuse representing the front margin and the right angle cut off truncate to represent the hind margin.

The first segment consists above and on sides of a strip-like dorsal plate evenly depressed between its lateral edges, which are tucked-up rounded folds of the plate, the latter ending ventrally on each side in a short, pointed strip that does not meet the ventral plate, the ventral membrane intervening between them. A small spiracle, smaller than any of the others of abdomen, is present in the extreme point of the first dorsal plate on each side where it joins the ventral membrane, and each one of the other five dorsal plates has a similar but larger spiracle on its inner edge, these being in each case quite well removed from the lateral margin where it joins ventral membrane.

The third ventral plate is nearly rectangular, a little broader than long, about as wide as mean width of second plate. The fourth ventral plate is considerably broader than the third and much shorter, thus looking like a narrow transverse strip set in the ventral membrane. The fifth ventral plate is much wider than fourth, about same length, and its median portion (about middle one-third) appears to be crowded under the fourth plate by the walls of the sheath-like genital opening, partially retracted within which lies the hypopygium. Thus only the lateral one-third of the fifth plate is visible on each side, and these two portions form the narrow visible strips of the curved plate, bordering the edge of the genital opening on each side, and each pointed at its outer extremity.

The sixth abdominal segment is not apparent from a dorsal view. It is a shortened anal segment that has been pushed over and crowded beneath the extremity of the abdomen. It lies just under the posterior edge of the abdomen, is rather crescent-shaped, subsemicircular on posterior (appearing anterior owing to inverted position) edge where it encloses the basal segment of the hypopygium, slightly squared on anterior lateral corners. It little more than half surrounds the orifice of the genital cavity, and bears a spiracle on each side at some distance before the pointed end of its tapering lateral portion. The basal sclerite or plate of the hypopygium bears another spiracle, which is one of the largest in the abdomen, on its basal edge, near the spiracle of the sixth segment and appearing as if it belonged to that segment. This basal plate of the hypopygium represents another abdominal segment, and it should be considered as forming a seventh segment of the abdomen rather than the base of the hypopygium.

The ventral membrane is widely apparent and extensive, the ventral plates all lying free within it so far as contact with the dorsal plates is concerned. The area in which the ventral membrane, with the enclosed plates, is visible occupies more than one-third the width of the ventral aspect of abdomen.

The plates, both ventral and dorsal, are at once distinguished throughout their extent from the membrane by being clothed with bristly hairs.

The above description was drawn from a specimen collected by F. C. Pratt, at Poolesville, Maryland, July 9. The abdomen was detached and put under the microscope.

ŒDEMASOMA, gen. nov.

This form (male) agrees with the description of *Wahlbergia brevipennis* H. Loew, except that the fourth vein is bent at a rounded angle, and hind crossvein is not strongly oblique. The hind crossvein is straight, almost at right angles to the fourth vein, hardly nearer to bend of latter than to small crossvein, and at right angles to fifth vein. The petiole of apical cell is slightly longer than small crossvein, but not twice as long—about one and one-fourth times as long. The abdomen is swollen and strongly convex above, wider than the thorax, exactly oval in outline from above, the wider end forward, absolutely without macrochætæ. Palpi are extremely small, almost atrophied, very slender and quite short. Antennæ as long as face, second joint almost as long as third. No orbital bristles. Wings very short and narrow. The claws are about as long as last tarsal joint. Type, the following species:

Œdemasoma nuda, sp. nov.

One male, Ormsby County, Nevada, July 6, C. F. Baker, Coll.

Length, 6 mm.; of wing, 4 mm. Face, parafacials and parafrontals from above silvery white pruinose, blackish from in front, the silvery extending on cheeks. Frontalia silvery white pruinose, with a faint brassy tinge or a golden reflection. Abdomen densely covered with moderately short and fine brown or black hairs, and entirely without bristles, wholly yellowish red or brownish red. The mesoscutum is silvery pollinose in front of suture, but it does not show well in some lights. Tegulæ white. Palpi pale reddish brownish in color. All the rest of insect is black, except the clear wings, which are yellowish at base. Otherwise agrees with Loew's description of *Wahlbergia brevipennis*.

This form apparently belongs in the neighborhood of *Gymnosoma*, indicated by the absence of macrochætæ and the possession of a swollen abdomen. *Wahlb. brevipennis* H. Loew is this genus, but a different species. Loew's specimen is a female from Nebraska, length $4\frac{2}{3}$ mm., of wing 3 mm. The writer has examined the type in Cambridge. The hind crossvein does not form a right angle with fifth vein, the petiole of apical cell is fully twice as long as small crossvein, the head is black and shining except face, and the mesoscutum does not show silvery before suture.

Type.—Cat. No. 10,898, U. S. N. M.

Subfamily OCYPTERINÆ

Genus Ocyptera Latreille

Ocyptera euchenor Walker.—While it seems probable that this form and *epytus* Walker are the same, there can be no certainty in the matter until the types are compared. Probably *O. carolinæ* Robineau-Desvoidy is distinct. Some of Bigot's species may also prove distinct. It seems probable that *carolinæ* is a southern form, and that *euchenor* is the more northern large form, having the cheeks and parafacials narrow, and the eyes elongate, descending low. Further study may also show the distinctness of *dosiades*.

Genus Beskia Brauer and von Bergenstamm

Beskia cornuta Brauer and von Bergenstamm and allies.—*B. cornuta* is the South American form. The type is from Brazil. The figure of the head given by the authors (fig. 276, Musc. Schiz., 1) is not typical of Southern States specimens in U. S. N. M. There is a marked difference in the third antennal joint. Williston's figure of his St. Vincent specimen shows the third antennal joint same as the Brazilian. *Beskia* and *Ocypterosipho* may be separated on this character.

Genus Ocypterosipho Townsend

Our species may be known as *Ocypterosipho ælops* Walker. Although Walker says "palpi black," and does not mention the slender and elongate proboscis, Mr. E. E. Austen's statement that *ælops* belongs here (Ann. Mag. N. H., Ser. 7, vol. 19, p. 345) must be accepted. This is the Georgia and Southern States form, and has the third longitudinal vein bristly to small crossvein (Georgia, Louisiana, and Texas specimens in U. S. N. M.). Santo Domingo specimens agree with those from the Southern States in having the third antennal joint strongly convex on under border and concave on upper, presenting a curved outline like that of a pruning-knife blade with cutting edge upward, the anterior distal corner of the joint being produced in profile into a sharply pointed prolongation.

Van der Wulp's figure (in the Biol. C. A. Dipt., II, pl. 13, fig. 12) of Mexican specimens gives somewhat the same impression. Two specimens from Mexico (one Tehuantepec, Sumichrast) show this character markedly, the third antennal joint not being truncate at tip as in the figures given by Brauer and von Bergenstamm and by Williston. It therefore seems evident that not only is *willistoni* a good species, but the genus *Ocypterosipho* may be retained, *O. willistoni* being the West Indian and Central American form, while *O. ælops* is the more northern form occurring in our Southern States. It is to be noted that St. Vincent belongs to the South American fauna, while Santo Domingo belongs to the Central American, which includes parts of Mexico and the Southern United States.

ICHNEUMONOPS, gen. nov.

Bearing much superficial resemblance to *Ocyptera,* but differing radically in the structure of the basal portion of the abdomen, and in head characters as well. Elongate and narrowed in form. Head, thorax, and abdomen of almost equal width, but the head distinctly wider than the thorax, the abdomen constricted basally into a pedicel formed principally by the base of second segment, which shows more constriction than any other part.

No vibrissæ that can be differentiated from the bristles of peristomalia. Second antennal joint rather elongate, about three times as long as first; third joint elongate, narrowed, about two and one-half times as long as second. Arista not distinctly jointed. Front at vertex narrower than eye width, but about equal to latter at base of antennæ. One row of weak frontal bristles extending only to base of antennæ. One pair of weak ocellar bristles, slightly proclinate. One pair of vertical bristles longer than frontal bristles, directed well backward. No orbital bristles (male). Cheeks about two-fifths of eye height. Face receding, facial profile straight, epistoma prominent. Facialia bare, not divergent below, ptilinal area about as wide as eye width, parafacials about half as wide above, but narrower below. Facial plate elongate, not narrowed below, produced on lower edge. Antennæ inserted above eye middle, rather above three-fourths of head height. Eyes bare, descending about three-fourths way to lower margin of head, which is long. Seen from in front, the space between lower angles of eyes is more than twice that between upper angles, the frontofacial area evenly widening from middle of front to cheeks. Proboscis below geniculation hardly as long as antennæ, labella well developed. Palpi extremely small and short, atrophied. Occiput convex, swollen on lower three-fourths.

Three postsutural bristles, no acrostichal bristles either before or behind suture. Only one sternopleural bristle (possibly one has been lost anteriorly on both sides).

Scutellum with a weak apical pair of bristles that are strongly decussate. Also a much longer marginal subapical pair, and a weak marginal pair behind last.

Abdomen strikingly *Ichneumon*-like in outline, consisting of five segments visible from above. Basal segment narrow, but little wider than base of scutellum, narrowed behind. Postbasal or second segment still narrower on basal portion, the greatest constriction being at about anterior three-fourths of the segment where the abdomen is narrower than scutellum. The second segment gradually widens posteriorly from the point of its greatest constriction, until on hind border it is twice its anterior width. The third segment widens posteriorly at not quite same angle, the fourth or preanal segment narrowing posteriorly about as rapidly as the third segment narrows anteriorly. Anal segment still narrowing posteriorly and evenly rounded on apex. The basal and anal segments are about same length, the second segment nearly twice as long. The third and fourth segments are equal and each is a little over twice as long as anal.

Second, third, and fourth segments with a median marginal pair of macrochætæ quite removed from posterior border of segment, also a lateral marginal one on each side. Anal segment with only an outer pair on each side near hind margin. Second segment with a lateral one in middle on each side.

Ventral plates not visible, except that the basal plate shows plainly with adjacent ventral membrane rather widely surrounding it.

Wings elongate and narrow, reaching about to end of abdomen. No costal spine. Small crossvein nearly opposite end of first vein, distinctly beyond middle of discal cell. Hind crossvein in middle between apical crossvein and small crossvein, strongly bisinuate. Apical crossvein still more strongly bisinuate, quite S-shaped. Fourth vein produced beyond apical crossvein in a short stump. Petiole of apical cell half as long as apical crossvein, reaching anterior wing border well before wing-tip. No veins spined, not even third vein at base.

Tegulæ of moderate size, inner portion subsemicircular in outline, so transparent (except narrow borders) that the halteres beneath are almost as clearly seen through them as through glass.

Legs moderately long, but quite normal. Claws and pulvilli elongate, but not longer than the last tarsal joint, which is itself elongate. Type, the following species:

Ichneumonops mirabilis, sp. nov.

One male, Beulah, N. Mex., August 17. Prof. T. D. A. Cockerell, Coll.

Length, 10 mm.; of wing, about 7 mm. Wholly dull black, abdomen very slightly shining. Antennæ light yellowish brownish on second joint and base of third. Face and parafrontals slightly silvery, extending on occipital orbits. Thorax and scutellum thinly silvery pollinose. Abdomen still more thinly silvery pollinose, the narrow hind margins of first three segments pale brownish with a yellowish tinge, that of first segment twice as broad as those of the others. Legs largely brownish reddish on femora, and especially on tibiæ. Wings on costal half deeply smoky, tinged with yellowish, including basal cells. Tegulæ and portion of wing behind fifth and sixth veins clear hyaline; discal and apical cells faintly clouded, the latter more so.

Type.—Cat. No. 10,899, U. S. N. M.

Tribe Coronimyiini

Genus Coronimyia Townsend

Coronimyia and *Epigrimyia* are distinct genera, belonging to and representing distinct tribes. *Coronimyia* has the arista short and geniculate, with very long second joint. *Epigrimyia* has the arista elongate, with basal joints short.

EUCORONIMYIA, nom. gen. nov.

This name is proposed for the genus *Isoglossa* Coquillett (Can. Ent., 1895, pp. 125-126), which is preoccupied by Casey in Coleoptera (Annals New York Acad. Sci., 1893, p. 304). The characters are sufficient to retain the genus.

Genus Olenochæta Townsend

Olenochæta kansensis Townsend.—This form, *Pseudogermaria georgiæ* Brauer and von Bergenstamm, and *Distichona varia* van der Wulp are all generically distinct.

Genus Chætoglossa Townsend

Chætoglossa nigripalpis Townsend.—This species differs from *violæ* Townsend in having black palpi and discal macrochætæ on third abdominal segment. It is also twice the size of *violæ*. By some error the words "palpi black" were omitted from the description (Tr. Am. Ent. Soc., XIX, p. 126). *C. violæ* has palpi light orange, and third abdominal segment is without discal macrochætæ.

Subfamily THRYPTOCERATINÆ

Genus Ceratomyiella Townsend

Ceratomyiella conica Townsend.—This genus may be known by the apical cell ending but slightly before wing-tip, usually if not always short petiolate; bend of fourth vein not sharply angular, third vein bristly not quite to small crossvein, fifth vein not at all bristly, and costal spine very small. The face is so elongate and retreating in profile below eyes as to bring the insertion of vibrissæ nearly or quite into the transverse plane of the hind margins of eyes; the cheeks are one-third to one-half eye height in width (nearly one-half in *C. conica*).

Chætoplagia has the apical cell narrowly open or closed in border. *Metachæta* greatly resembles *Ceratomyiella* in facial characters. The facial profile is very receding and elongate below, so as to bring the insertion of vibrissæ close to or nearly into the transverse plane of hind border of eyes (as viewed in full profile).

ACRONARISTA, gen. nov.

Allied to *Schizotachina* Walker, from which it is at once distinguished by the remarkable characters of the third antennal joint. This is biramose in the female, being split into an anterior and a posterior ramus, the two rami almost meeting apically and showing in profile like an imperfect zero. The inner or under ramus is a little widened apically; otherwise the profile width of both is practically the same throughout, even including the base of the joint where the rami join. The arista is inserted in the anterior edge of the upper ramus well before its apex, but much nearer the apex than the base, being at a distance from the apex equal to one-third the length of the joint. The second aristal joint is only about twice as long as wide, the first about as long as wide, and the third about three times as long as second. Front equilateral, about one and one-half times as wide as one eye, two middle fronto-orbital bristles in female, facial plate very wide, parafacials reduced to a mere line, facialia practically bare. Apical cell closed in margin near wing-tip, last section of fourth vein bent in; hind crossvein distinctly nearer to small crossvein than to bend of fourth vein, but not nearly so approximated to small crossvein as in *Schizotachina*. Third vein with a few bristles at base only, costal spine present. Type, the following species:

Acronarista mirabilis, sp. nov.

One female, Palm Beach, Florida. Dr. H. G. Dyar, collector.

Length, 4 mm. Blackish, with gray pollen. Antennæ reddish brown, becoming more or less reddish yellow at base. Face, front, thorax, and scutellum silvery gray pollinose. Abdomen blackish, narrow anterior margin of second and third segments and all of fourth segment silvery gray pollinose. Legs quite blackish. Tegulæ whitish. Wings faintly tawny at base.

Type.—Cat. No. 11,685, U. S. N. M.

It is strongly probable that the male of *Acronarista* has the third antennal joint much more elaborate in structure than that above described for the female, and it will be very interesting to look for the male in South Florida material.

In *Talarocera* female the location of the arista approaches in a measure that of *Acronarista* female, but is not nearly so apical. *Acronarista* female seems to be a farther development of *Talarocera* female in this regard, in that the ramus of third antennal joint bearing the arista has become elongated and enlarged into almost the counterpart of the other ramus, the elongation taking place at the base of the ramus, and thus making the arista subapical thereto. It is probable that in the male of *Acronarista* the arista will be found to be apical to one of many rami, as in the male of *Talarocera*. However, this would indicate no near relationship, since *Talarocera* is a large form belonging to the Hystriciinæ.

LIXOPHAGA, gen. nov.

Differs from *Gymnostylia* by having macrochætæ of abdomen only marginal; parafacials and parafrontals bare except for the frontal and orbital bristles. Male cheeks hardly one-fourth eye height; no orbital bristles in male, but a row of six or seven minute bristles between frontalia and eye margin on the parafrontals. Apical cell closed in margin just before wing-tip. Hind crossvein in middle between small crossvein and bend of fourth vein, the latter rounded and bent at an obtuse angle. Front about one-third head width, widening on anterior portion. Face fully one-half head width. Type, the following species.

Lixophaga parva, sp. nov.

One male bred from *Lixus scrobicollis,* Hunter No. 219, Dallas, Texas, issued August 15, 1907.

Length, 3.5 mm. Face, cheeks, parafacials, and parafrontals silvery, the parafrontals tinged with cinereous. Frontalia, antennæ, and legs blackish. Third antennal joint about three and one-half times as long as second. Thorax silvery pollinose with tinge of cinereous above; four narrow linear black vittæ, the outer ones interrupted at suture, the inner ones abbreviated just behind suture. Scutellum silvery pollinose. Abdomen blackish, the second to fourth segments thickly silvery-cinereous pollinose leaving a median vitta and the hind margins blackish or brown, the vitta not so marked on anal segment. The pollen of abdomen is flecked with numerous small dots marking insertion of bristly hairs. Macrochætæ in a median marginal pair and a lateral one on first two segments, weaker than the marginal rows on third and anal segments.

Type.—Cat. No. 11,648, U. S. N. M.

Subfamily BAUMHAUERIINÆ

Genus Euthyprosopa Townsend

Euthyprosopa petiolata Townsend.—There are two pairs of ocellar bristles in this genus, the posterior pair being about same length as frontal bristles. The anterior pair is strongly proclinate, almost appressed; the posterior pair is slightly reclinate, suberect, and inserted between the two posterior ocelli.

Subfamily PLAGIINÆ

Genus Plagia Meigen

Plagia aurifrons Townsend.—This species is from the northeastern United States, and is not conspecific with the Mexican *americana* van der Wulp.

Genus Plagiprospherysa Townsend

Plagiprospherysa valida Townsend.—It is possible that the Presidio specimens referred by van der Wulp to his species *parvipalpis* may be conspecific with this species, but the others are likely to prove distinct.

Genus Heteropterina Macquart

Heteropterina nasoni Coquillett.—This form seems, from an examination of the type, to be quite typical of the genus *Heteropterina.* The cheeks are very narrow, not over one-tenth eye height, and the few fine hairs of the normal row on parafacials are almost imperceptible with an ordinary low-power lens, but they are present.

Subfamily PHOROCERATINÆ

Genus Achætoneura Brauer and von Bergenstamm

Achætoneura.—This genus is characterized by having the second antennal joint but little longer than the first, and thus is easily distinguished from *Tachina* s. str., to which it otherwise bears a strong resemblance. Type is *hesperus* Brauer and von Bergenstamm, of North America. *T. aletiæ* Riley belongs to this genus.

HEMIARGYRA, gen. nov.

Form *Pollenia*-like, eyes pilose, facialia and hind tibiæ ciliate. Ptilinal suture bent at a rounded angle in middle superiorly, its ends divergent inferiorly, making the ptilinal area almost triangular in shape and about one-third head width below. Facial plate elongate, not narrowed below, depressed, but not produced anteriorly on lower portion, fossæ running full length of plate; foveæ shallow, but marking length of third antennal joint; a low, sharp, narrow carina fading out before reaching inferior end of foveæ. Facialia sharp-edged, narrow in front outline from being set on edge, with a row of strong bristles running fully half way up and marking the edge, which is nearly straight in outline save for a slight curve inward at lower end, and is closely approximated to suture until it begins to curve. Vibrissal angles quite distinctly removed from oral margin, very faintly pronounced, but slightly more approximated than the rows of bristles above them, the vibrissæ strong and decussate. Peristomalia subparallel in epistomal region, divergent posteriorly, with a row of black bristles extending to beard. Epistoma not prominent, not showing in profile. Proboscis very short and fleshy, part below geniculation hardly as long as eye width, but about as long as the palpi; labella large, with long hairs; palpi rather elongate, moderately slender, but thickened on apical half. Axis of head at insertion of vibrissæ distinctly less than that at insertion of antennæ, facial profile very gently receding, but nearly straight. Antennæ inserted about on eye middle, at about three-fifths of head height; second joint rather short, but about twice as long as first; third joint elongate, not wider than second, sides nearly parallel, subtruncate at apex. Arista thickened on hardly more than basal one-third, finely short-hairy, basal joints distinct but short. Eyes thickly pilose, extending not quite to vibrissal angles, inner outline appearing slightly bulged on middle by reason of a faint incurvature below. Front (female) not prominent in profile, at vertex (seen from in front) about one-fourth of head width, gradually widening anteriorly to distinctly more than one-third head width at base of antennæ. No ocellar bristles, but some long fine hairs on and in front of ocellar area. Frontal bristles in a single row on each side close to frontalia, but widely divergent at an angle anteriorly, the foremost two being out of line with main row and the only ones inserted below first antennal joint. Vertical bristles consisting of a moderately strong inner and a very weak outer one, the latter but slightly longer than the row forming occipital fringe. Two strong, lightly reclinate upper fronto-orbital bristles inserted close to frontalia, in profile showing same strength and curvature as inner vertical, all three being same distance apart in profile. Two strong middle fronto-orbital bristles, strongly curved and proclinate, outside line of preceding, the posterior one being inserted midway in profile between the two upper fronto-orbital bristles, the anterior one about half way between foremost frontal bristle and vertex. Parafacials wide, only gently narrowed below, fully two-thirds as wide on lower portion as opposite base of antennæ, bare. Width of cheeks equal to about one-fourth eye height, cheek grooves well marked. Lower margin of head arcuate, evenly bulged and rounded. Occiput considerably swollen below, behind eyes.

Two sternopleural bristles, strong, formula 1:0:1; hypopleural bristles in a curved row, long but slender; one moderately strong pteropleural bristle with some fine hairs; three postsutural bristles, supra-alar bristles stronger. Scutellar bristles in three strong marginal pairs and a weak apical decussate pair; subapical pair longest, reaching nearly to base of preanal segment; a widely separated discal pair about as strong as the apical.

Wings decidedly longer than abdomen, rather broad, with very small but distinct costal spine. No veins spined, except a few bristles at base of third vein. Fourth vein bent roundly at an obtuse angle, ultimate section slightly and evenly crooked, no wrinkle or stump at bend. Hind crossvein bisinuate, slightly more than one-half as far from bend as from small crossvein, which is on middle of discal cell and about half way between ends of auxiliary and first veins. Apical cell well open, ending on front border well before wing-tip. Tegulæ very large, antitegulæ one-third as long.

Abdomen broadly oval, rounded anally, only four segments visible from above, with almost equally short marginal and discal macrochætæ. Ventral plates not visible. Anal segment with a ventral median cleft, within which is the retracted ovipositor.

Legs not elongate, normal, the hind tibiæ quite thickly ciliate with a slightly stronger bristle near middle. Claws and pulvilli (female) short, not as long as last tarsal joint. Type, the following species:

Hemiargyra nigra, sp. nov.

One female, San Carlos, Costa Rica, collected by Schild and Burgdorf.

Length, 8.5 mm.; of wing, 8 mm. Blackish, or brownish black. Palpi reddish yellow, quite thickly black-hairy, blackish at base. Space from anterior fronto-orbital bristle to cheek grooves conspicuously silvery white pruinose as seen from above, covering whole area of parafacials and anterior half of parafrontals, but appearing dead black when seen from below. Facial plate silvery from above, blackish from below. Epistoma yellowish. Third antennal joint three and one-half times as long as second. Cheeks brownish, clothed with black hairs; cheek grooves and edge of parafacials bordering suture slightly golden in some lights, brownish or reddish in others, the golden continued on occipital orbits. Frontalia, thorax, scutellum, and basal abdominal segment soft black with slight brownish tinge, apex of scutellum silvery. Two middle segments of abdomen heavily golden silvery pollinose seen from in front, behind, or above, but nearly lost when seen from side; the coating showing broadly on venter, anteriorly on each segment at least. Sides of middle segments slightly reddish under the pollen. Anal segment black, with some golden pollen on sides and base. Macrochætæ as follows: A median and lateral marginal pair on basal and postbasal segments, a median discal pair on postbasal and preanal segments, the latter with a marginal row, anal segment with only bristly and fine hairs. Wings slightly infuscate along the veins, chiefly on costal half, rest subhyaline. Tegulæ smoky, with smoky yellowish borders widening in oblique lights. Pulvilli whitish, with a slight smoky yellowish tinge. Halteres rufous, knobs fuscous.

Type.—Cat. No. 10,907, U. S. N. M.

POLIOPHRYS, gen. nov.

Ptilinal suture rounded subangular in middle, its ends divergent below, giving ptilinal area an oval outline that is quite narrowed above and fully one-third head width below. Facial plate elongate, not narrowed below, depressed, produced anteriorly on lower portion; fossæ running full length of plate, foveæ deep and marking length of third antennal joint; a distinct narrow carina between the foveæ, with a linear median furrow on its edge. Facialia wide, but with rather sharp edge, latter gently curved in outline, well inside suture and furnished with bristles extending more than half way up. Vibrissal angles quite distinctly removed from oral margin, not sharp, conspicuously more approximated than the rows of facial bristles above them, the vibrissæ strong and decussate. Peristomalia almost straight, nearly parallel, with bristles extending to beard. Epistoma prominent, but not showing greatly in profile owing to strong depression of facial plate. Proboscis short, fleshy, part below geniculation about as long as eye width (in front view), labella well developed; palpi elongate, rather slender, more attenuate on basal one-third. Axis of head at insertion of vibrissæ very noticeably less than that at insertion of antennæ, facial profile gently receding, but quite straight. Antennæ inserted above eye middle, at about three-fourths of head height; second joint about twice as long as first, with a pair of bristles on lower front edge; third joint elongate, wider than second, sides nearly parallel. Arista bare, thickened on more than basal one-half, conspicuously jointed; first joint slightly elongate; second joint elongate, two or three times as long as first, and one-fourth to one-fifth as long as third joint. Eyes thickly pilose, not extending as low as vibrissal angles, inner outline S-shaped in front view in male, straight in female. Front not strongly prominent in profile, but flattened and sloping straight to base of antennæ; at vertex (seen from in front) one-third head width in both sexes, in male suddenly swelling in lateral outline anteriorly, in female gradually widening anteriorly, almost one-half head width at base of antennæ. A strong pair of proclinate, divergent ocellar bristles. Frontal bristles in two rows, the inner rows strongly curved and widely divergent below, the outer rows nearly straight; the lowermost bristle on each side sometimes in line with both rows so as to appear (male) as belonging to either, but belonging (as shown in female) to inner row, which descends strongly to point somewhat below insertion of arista. Short fine hairs on parafrontals, long hairs on and in front of ocellar area. The usual strong inner and weaker outer vertical bristles, both reclinate, latter also divergent; two lightly reclinate upper fronto-orbital bristles, the frontal bristles extending only about half way back from base of antennæ to vertex. No middle fronto-orbital bristles in male, two strong, decidedly proclinate ones in female nearly in line with the posterior one of the upper fronto-orbital bristles. Parafacials wide, narrower below than opposite antennal insertion, least width about equal to length of second antennal joint, greatest width bordering parafrontals and not twice as much. Facio-orbital bristles in a median row of about five or six, not so strong as frontal or facial bristles, some fine hairs outside them. Width of cheeks equal to one-third eye height, cheek grooves faint. Lower margin of head nearly straight, about two-thirds as long as axis of head at antennal insertion. Occiput swollen below, behind eyes.

Four sternopleural bristles, formula 2 : 1 : 1; hypopleural bristles in a curved row, strong; pteropleural bristles several, one strong; four postsutural bristles. Scutellar bristles strongly developed; apical pair decussate, suberect, weaker than the other marginal ones, a weak discal pair in front of them; three strong marginal pairs, the subapical longest and reaching nearly to middle of preanal segment (male), or only to base of same (female).

Wings a little longer than abdomen, moderately broad, with very small but distinct costal spine. No veins spined, except third vein with two or three bristles at base. Fourth vein bent at nearly a right angle, with very slight (almost imperceptible) wrinkle at bend, latter not sharp, apical crossvein well bowed in near origin, hind crossvein slightly bisinuate and approximated to bend of fourth vein, small crossvein half way between end of auxiliary and end of first vein. Apical cell widely open, ending far before wing-tip. Tegulæ large, antitegulæ overlapping them for one-third of their length.

Abdomen rather broadly oval, quite pointed at apex, only four segments visible above, with strong marginal and short, weak discal macrochætæ. Ventral plates not visible. Anal segment in both sexes with a median ventral slit for protrusion of genitalia, which are retracted.

Legs not elongate, middle tibiæ with three strong bristles in middle, hind tibiæ weakly ciliate with a long bristle in middle of ciliated edge; tarsi normal; male claws and pulvilli elongate, longer than last tarsal joint; female claws about as long as last tarsal joint. Type, *Poliophrys sierricola* sp. nov.

This genus is proposed for what Mr. Coquillett has identified as *Gædiopsis mexicana,* represented by four male specimens from Organ Mountains, New Mexico, about 5,300 feet, September 4–5 (Townsend), on flowers of *Lippia wrightii;* and two specimens, male and female, from Sierra Madre of western Chihuahua, head of Rio Piedras Verdes, about 7,000 feet, July 19 (Townsend), on flowers of *Rhus glabra.* The Sierra Madre specimens are distinct from the Organ Mountains species, and probably represent two species. The male from the Sierra Madre, *P. sierricola,* is made the type, and the Organ Mountains species is called *P. organensis.*

The genus differs from *Phrissopolia* chiefly in having the eyes pilose. The genus *Gædiopsis* differs from *Chætogædia* chiefly in having the eyes hairy. *G. setosa* Coquillett belongs close to if not in the genus *Poliophrys.*

Poliophrys sierricola, sp. nov.

One male, Sierra Madre, Chihuahua, collected by Townsend.

Length, 9 mm.; of wing, 7½ mm. Blackish, clothed with silvery cinereous. Parafrontals with a golden tinge (male), which extends very faintly on parafacials and cheeks. Facial plate silvery. Frontalia seen from in front silvery, seen from behind brownish. Antennæ brownish, third joint blackish and three times as long as second, arista blackish. Palpi yellowish. Thorax silvery pollinose, with four moderately wide vittæ, which are blackish seen from behind, but salmon colored seen from in front. Scutellum light brownish reddish, blackish at base, silvery. Abdomen silvery, slightly marmorate above, with a faint golden tinge which is strong on anal segment, where it is about the same as on parafrontals. Sides of abdomen faintly reddish under the pollen. Wings clear, tegulæ whitish. Legs blackish, femora silvery; tibiæ reddish or brownish yellow, except at ends; pulvilli slightly smoky.

Type.—Cat. No. 10,908, U. S. N. M.

A female from same locality differs only in second antennal joint being clear reddish yellow, the third joint little more than two and one-half times as long as second, and the sexual characters given above under the genus. It may prove to be a distinct species, but more material is needed to make sure of this.

Poliophrys organensis, sp. nov.

Four males, Organ Mountains, New Mexico, Townsend, Coll.

Length, 8 to 10 mm.; of wing, 6 to 7.5 mm. Front fully one-third head width at vertex, cheeks more than one-third eye height in width. Differs further from *P. sierricola* in having hardly a tinge of golden to the pollinose covering of head, which is silvery white throughout face and with only a slight tinge of golden on parafrontals not extending on facial plate at all. Antennæ black, third joint nearly four times as long as second (male). Abdomen more noticeably reddish on sides; anal segment less distinctly golden, about the same as other segments.

Type.—Cat. No. 10,909, U. S. N. M.

PHRISSOPOLIA, gen. nov.

This genus is proposed for *Prospherysa crebra* van der Wulp, which was included in *Chætogædia* by Brauer and von Bergenstamm. It is characterized by a double row of frontal bristles, the outer row nearly or quite as strong as the other, and especially by a row of strong bristles on parafacials close to orbit, the facio-orbital bristles, of same strength as frontal bristles, and, except for their downward curve, appearing like a continuation of latter to lower eye border. The second aristal joint is long, the third much shorter than in *Chætogædia,* and the whole arista is widened and flattened, usually geniculate or subgeniculate. Eyes bare.

Phrissopolia desertorum, sp. nov.

Las Cruces, New Mexico, Cockerell, No. 4,952. Specimens from Beulah, New Mexico (Cockerell), and Santa Clara County, California, may also be referred to this species.

Length, 9 to 10 mm. The species differs from van der Wulp's description of *crebra* as follows: All the tibiæ rufous (male) or yellowish (female). Face, including parafacials, silvery roseate white in male without yellowish tinge, which is confined to front; in female with a faint yellowish white tinge spreading over face. Third antennal joint of male four times as long as second, of female three times as long. Arista thickened nearly to end in both sexes, only the apical one-third or one-fourth appearing slender from certain viewpoints due to flattening; second joint very distinct, elongate, fully one-fourth as long as last joint, the articulation geniculate in some cases. Hind tibiæ rather weakly ciliate, with a long bristle in middle. Wings faintly yellowish tinged at base.

Type.—Cat. No. 10,910, U. S. N. M. (Las Cruces, N. Mex.).

Genus Chætogædia Brauer and von Bergenstamm

Chætogædia acroglossoides Townsend.—This is a good species. It is neither a *Frontina* nor a *Baumhaueria,* but is apparently to be referred to *Chætogædia. Frontina* has the parafacials bare, and the second aristal joint is not elongate. *Baumhaueria* has the front greatly produced, the parafacials hairy and of exaggerated width, much wider than the eyes, and the cheeks as wide as eye height.

The identification of this species with *Baumh. analis* van der Wulp is quite out of the question, if the description agrees with the type. The second antennal joint is elongate, the third is not over four times as long in male and less than three times as long in female as second joint. The description was of the female.

Chætogædia vilis van der Wulp, the type of the genus, has the frontal bristles in two rows, the outer row usually weaker, and the parafacials are clothed only with fine bristly hairs.

Genus Gædiopsis Brauer and von Bergenstamm

Gædiopsis cockerelli Coquillett.—This species appears to be correctly referred to the genus *Gædiopsis.* The material from which it was described was all collected by the writer in the White Mountain region of New Mexico (not New Hampshire, as given in the Catalogue), at about 8,000 feet, on the head of Eagle Creek, a stream which takes its rise on the upper slopes of the peak known as Sierra Blanca (altitude, 10,050 feet).

TREPOPHRYS, gen. nov.

Head in profile almost half round. Antennæ inserted about at eye middle. Front flattened, rounded in profile, showing just the same width beyond eye margins as do parafacials. Eyes bare, reaching quite to vibrissæ. Cheeks very narrow, not over one-tenth of eye height. Front about one-third of head width, or slightly less, the inner outline of eyes but slightly divergent below base of antennæ. Parafrontals a little wider than frontalia, parafacials gradually narrowing from base of antennæ until they become almost linear at lower eye margin. Ptilinal suture inverted V-shaped, the median angle a little rounded. Ptilinal area elongate, about one-third head width below. Facial plate elongate, not narrowed below, depressed, with a distinct and sharp but low median carina full length, not produced at lower margin. Facialia edge-like, bristly more than half way up, vibrissal angles hardly perceptible. Vibrissæ inserted close to oral margin, well developed.

Frontal bristles in a single row close to frontalia and extending back to ocelli, all curved inward, more or less decussate, descending in front to insertion of arista. The usual strong inner and weak outer vertical bristles. Two upper reclinate fronto-orbital bristles set well forward, almost far enough forward to occupy the usual place of insertion of the middle or proclinate ones. These two fronto-orbital bristles are of exactly the same strength, length, curvature, and direction as the inner vertical bristle, and look like two replicas of it in profile. They are also quite in line with it, and the three in profile are seen to be an equal distance apart. Two proclinate middle fronto-orbital bristles in female, outside the upper ones; none in male.

Proboscis short and fleshy, palpi slender and normal. Second antennal joint about twice as long as first; arista indistinctly jointed and minutely pubescent, slightly thickened on basal one-third. Third antennal joint about two and one-half times as long as second. Occiput slightly swollen behind on lower one-fourth, the lower margin of head short, long axis of head at vibrissæ but little over one-half that at base of antennæ.

Three sternopleural bristles, 1. 1. 1, the middle one weakest, the posterior one strongest. Three postsutural bristles. Scutellar bristles in five pairs, one decussate apical, three lateral, of which anterior and posterior are longer and stronger, and one separated discal pair.

Abdomen above of four visible segments, macrochætæ only marginal.

Wings reaching well beyond end of abdomen, apical cell narrowly open just before and almost in wing-tip, ultimate section of fourth vein bowed in about the middle so as to attenuate the terminal portion of apical cell. Hind crossvein slightly curved, not quite in middle between small crossvein and bend of fourth vein, distinctly nearer latter. None of the veins spined, the small crossvein slightly or distinctly before middle of discal cell.

Legs normal, hind tibiæ weakly ciliate, with a bristle or two among the cilia, claws and pulvilli very short. Type, *T. cinerea,* n. sp.

Comes near *Pseudochæta* Coquillett, with which it agrees in the arrangement of the upper and middle fronto-orbital bristles, and from which it differs as follows:

Pseudochæta	*Trepophrys*
Antennæ inserted distinctly above eye middle in both sexes, but especially so in the male.	Antennæ inserted practically on eye middle in both sexes.
Two sternopleural bristles.	Three sternopleural bristles.
Four postsutural bristles.	Three postsutural bristles.
Apical cell ending not far but very distinctly before wing-tip.	Apical cell ending slightly before, almost in, wing-tip.
[1] Hind crossvein almost in middle, male; nearer bend of fourth vein, female.	[1] Hind crossvein distinctly nearer bend of fourth vein, male; almost in middle, female.
[1] Small crossvein nearly in middle of discal cell, male; before middle, female.	[1] Small crossvein well before middle of discal cell, male; nearly in middle, female.
Wings very short and broad, hardly more than twice as long as wide.	Wings elongated beyond end of abdomen, at least two and one-half times as long as broad.
Head distinctly widened.	Head hardly wider than thorax.

Trepophrys cinerea, sp. nov.

Three specimens bred from masses of pupæ of a tortricid, *Amydria* sp., sent by Prof. A. L. Herrera, from Cuernavaca, Mexico.

Length, 4.5 to nearly 6 mm. Blackish, parafrontals and parafacials golden, extending on occipital orbits; frontalia, antennæ, and face blackish, latter with a slight silvery reflection. Cheeks slightly golden. Pleuræ very faintly silvery. Dorsum of thorax and abdo-

[1] These characters represent the average of the specimens.

men cinereous pollinose, with a distinct golden tinge most noticeable on scutellum. First abdominal segment, narrow hind margins of second and third, and apical half of anal blackish, the black surface of anal segment shining. Wings clear, tegulæ whitish with a tawny tinge. Legs black.

Type.—Cat. No. 10,911, U. S. N. M.

Subfamily MASICERATINÆ

Genus Exorista Meigen and allies

The genus *Exorista,* as restricted, has the cheeks wide, one-third to one-half eye height; second antennal joint somewhat elongate, and abdominal macrochætæ discal and marginal.

Parexorista differs from *Exorista* in cheeks being not over one-fourth eye height, second antennal joint not elongate, second aristal joint usually elongate; abdominal macrochætæ usually only marginal, but long discal bristles present, those on third segment approaching macrochætæ in character.

The genus *Carcelia* Robineau-Desvoidy may be known by having no long discal bristles on abdominal segments, all being short and of even length. Macrochætæ only marginal. Cheeks not over one-fourth eye height. Type, *gnava* Meigen.

Nemorilla has cheeks not over one-fourth eye height, second antennal joint elongate, second aristal joint short, macrochætæ discal and marginal, hind tibiæ weakly ciliate.

EUSISYROPA, gen. nov.

Proposed for *Exorista blanda* Osten-Sacken. Differs from *Parexorista* in possessing regularly arranged discal macrochætæ on abdominal segments, without the erect and usually long bristly hairs of that genus, and especially in the peculiar form of the abdomen in both sexes. The latter is high, somewhat arched, slightly wedge-shaped ventrally in female, and obliquely truncate downward and forward at apex in profile. The female has these abdominal characters more marked, but the male also possesses them in a hardly less degree. The female has the venter quite distinctly carinate.

Eusisyropa blanda Osten-Sacken.—This species has the legs and second antennal joint more or less deeply blackish or brownish, with usually only faint suggestions of yellowish or reddish. The palpi have a reddish tinge. Both sexes have the parafrontals with a slight golden tinge, and anal segment very distinctly golden. There are two sternopleural bristles only.

New Jersey, New York, Massachusetts, and south to District of Columbia.

This species has been bred from *Cymatophora pampinaria* Guenée, one specimen issuing from a larva collected on cranberry at Cotuit, Massachusetts, by J. B. Smith (Riley Notes, Bureau of Entomology); also from *Hyphantria textor*, at Washington, D. C. (No. 78^{03} Riley Notes).

One female specimen was bred at the Gipsy Moth Parasite Laboratory, North Saugus, Massachusetts, issued July 29, 1907, which may have come from native *Euproctis chrysorrhœa.*

Eusisyropa boarmiæ Coquillett.—This is a Florida and Southern States form closely allied to *blanda.* It reaches Arkansas and Missouri. It has light reddish yellow legs and second antennal joint, these being quite concolorous with the reddish yellow palpi, and possesses a small third sternopleural bristle.

The type specimen was bred from a larva of *Aletia argillacea,* received from Oxford, Mississippi, issued November 14, 1882 (No. 468 L°, Riley Notes, Bureau of Entomology). The species has not been bred from *Cymatophora* (*Boarmia*), the *Boarmia*-bred specimen mentioned by Coquillett being *E. blanda.*

Genus Argyrophylax Brauer and von Bergenstamm and allies

The following is a table of *Argyrophylax* and the forms closely related to it:

1. Ocellar bristles wanting (type, *albincisa* Wd.) .... *Argyrophylax* B. B.
   Ocellar bristles present 2
2. Apical pair of scutellar bristles much stronger than those next to them, hind tibial cilia dense (type, *pupiphaga* Rdi.) .... *Sturmia* R. D.
   Apical pair of scutellar bristles much weaker than those next to them .. 3
3. Third abdominal segment of male with two shining or pilose black spots on ventral surface, a distinct longer bristle in cilia of hind tibiæ near middle (type, *bimaculata* Htg.) .... *Zygobothria* Mik.
   Third abdominal segment of male without such spots, hind tibial cilia dense and without longer bristle (type, *scutellata* Rdi.) ... *Blepharipa* Rdi.

Argyrophylax piperi, nom. sp. nov.

This name is proposed for *Sturmia schizuræ* Coquillett, which is an *Argyrophylax.* The specific name is preoccupied by *Argyr. schizuræ* Townsend.

Pullman, Washington State (Piper). Bred from *Schizura ipomœa.*

Length, 10.5 mm. Much larger than type of *A. schizuræ* Townsend, with which it at first seemed identical. Agrees with description of *A. schizuræ* Townsend except as follows: The facial plate shows no appreciable tinge of golden, fourth vein is quite abruptly bent, first abdominal segment has one lateral marginal macrochæta, second segment has a lateral marginal pair of macrochætæ, pulvilli are smoky blackish, and size is larger.

Genus Zygobothria Mik.

Zygobothria nidicola, sp. nov.

Male.—Fifteen specimens. Thirteen bred at the Gipsy Moth Parasite Laboratory, North Saugus, Massachusetts, as follows: Four bred by E. S. G. Titus, in 1906, from *Euproctis chrysorrhœa* imported from Germany (Erfurt, Munich, and Fuhlsdorf, received from Marie Ruhl); nine bred by W. F. Fiske, in 1907, from hibernated larvæ of *Euproctis chrysorrhœa* from imported nests received from Vienna and other parts of Nieder-Oesterreich, and from summer importations of larvæ of same species from South Tirol and Carniola. Two bred at Simferopol, Russia, by S. Mokschetsky, from *Euproctis chrysorrhœa,* June 7, 1905, and July, 1907.

Length, 7 to 9 mm. Eyes very faintly hairy, appearing bare. Antennæ blackish, third joint more or less reddish or lighter colored at base, palpi light yellow. Face and front silvery, parafrontals darker in some lights, but not golden. No middle fronto-orbital bristles. Frontalia slightly, if any, wider than one parafrontal. Parafrontals with fine hairs outside the row of frontal bristles. Front anteriorly hardly as wide as one eye, half as wide as one eye at vertex. Facialia with some bristles extending about one-third way up. Arista thickened on less than proximal half, first two aristal joints short. Cheeks hardly one-third eye height. Thoracic dorsum thinly silvery pollinose. Scutellum testaceous except extreme base, apical pair of bristles weak, almost erect, decussate. Abdomen with more or less red on sides, first segment and narrow hind borders of second and third segments shining black, rest thickly cinereous pollinose leaving a more or less distinct median line. A median marginal pair of macrochætæ on first and second segments, also three lateral marginal ones on each side of same segments, third segment with a marginal row of twelve or fourteen. Anal segment with only bristly hairs. Legs wholly black, claws and pulvilli very elongate, hind tibiæ thickly ciliate, but with a longer bristle near middle. Tegulæ white. Four sternopleural and four postsutural bristles, a few specimens showing a fifth weaker sternopleural bristle.

A specimen of this series sent to Dr. K. Kertész, at Budapest, was returned by him as *Argyrophylax galii* Brauer and von Bergenstamm. As *galii* has the male vertex one and one-third times the eye width and female vertex twice the eye width, this can not be that species. Two specimens sent to Dr. A. Handlirsch, at Vienna, were returned as unknown to him, and indicated with a query as American.

Female.—Fourteen specimens. Twelve bred at the Gipsy Moth Parasite Laboratory, North Saugus, Massachusetts, as follows: Five bred by E. S. G. Titus, in 1906, from *Euproctis chrysorrhœa* imported from Germany (Baden and Dresden, received from Marie Ruhl and Schopfer, respectively); seven bred by W. F. Fiske, in 1907, from summer importations of *Euproctis chrysorrhœa* from Germany. Two bred at Simferopol, Russia, by S. Mokschetsky, from *Euproctis chrysorrhœa,* June 10, 1905, and July, 1907.

Length, 7 to 8 mm. Differs from the male as follows: Thickly yellowish-cinereous pollinose all over, including front and first abdominal segment. Face more silvery. Thoracic vittæ fine, outer ones broken at suture and somewhat widened. Scutellum yellowish on margin. Middle fronto-orbital bristles two in number. Front from more than one-third to about two-fifths width of head, hardly narrowed from facial width. Abdominal macrochætæ same as male, but the second segment rarely has four median marginal macrochætæ more or less well developed from the long marginal hairs on each side of the original pair. Hind tibiæ sparsely but distinctly ciliate, a long bristle near middle. Four sternopleural and four postsutural bristles.

A specimen of this series sent to Dr. K. Kertész was returned undetermined; another sent to Dr. A. Handlirsch was returned as unknown to him, and probably American. The two sexes were not suggested by either Kertész or Handlirsch as belonging together, but it seems highly probable that they are the same species. Both are positively European, as conclusively demonstrated not only by the breeding records of the Gipsy Moth Parasite Laboratory, but also by Mr. Mokschetsky's breeding of both at Simferopol, Russia.

Types.—Cat. No. 11,803, U. S. N. M. (2 types: male from Nieder-Oesterreich, issued July 29, 1907; female from Central Europe, issued July 10, 1907).

Mr. W. F. Fiske has bred this species (male specimens) from cages containing hibernated larvæ of *Euproctis chrysorrhœa* under circumstances indicating that the female tachinids oviposit in the *Euproctis* nests in the fall, the tachinid larvæ remaining through the winter in the nests and issuing from the host larvæ or pupæ in the summer. This is a remarkable habit of oviposition among tachinids, and credit is due to Mr. Fiske for the discovery of it.

Genus Comatacta Coquillett

Comatacta nautlana, sp. nov.

The material upon which the genus *Comatacta* Coquillett was founded was collected at San Rafael, near Jicaltepec, Veracruz (Townsend). The specimens were erroneously identified with *Brachycoma pallidula* van der Wulp (Can. Ent., 1902, pp. 199-200), which is to be considered the type of the genus *Comatacta.*

The present species differs from van der Wulp's description of *pallidula* as follows: Facial plate silvery like parafacials; frontalia honey yellow, parafrontals silvery with a golden shade. Frontal bristles descending but one or two below base of antennæ. Beard very short, grayish. Antennæ reaching two-thirds to three-fourths way to oral margin. Arista rufous, concolorous with antennæ. Anal segment hardly at all rufous.

Type.—Cat. No. 10,906, U. S. N. M.

PARADEXODES, gen. nov.

Wings longer than *Dexodes* (type *spectabilis* Meigen), abdomen very bristly like *Dexodes,* with many discal macrochætæ and erect hairs, apical pair of scutellar bristles weak but long and markedly divaricate. Eyes bare; male front narrow, about or nearly equaling eye width anteriorly, vertex about or more than one-half eye width. Frontal bristles in male closely placed, descending three to four below base of antennæ. Ocellar bristles present. Vibrissæ close to oral margin, facialia with a number of bristles above vibrissæ. Parafacials quite narrowed below, widening above. Antennæ inserted on eye-middle. Second antennal joint nearly three times as long as first; third joint narrow, two or more times as long as second, equilateral in profile, subtruncate at tip. Second aristal joint short but distinct, arista thickened on proximal fourth. Legs rather long, male claws and pulvilli elongate. Wings long and narrowed in male, apical cell open well before wing-tip, hind crossvein approximated to bend of fourth vein, small crossvein on middle of discal cell. Abdomen elongate, conico-cylindrical in male. Type, the following species:

Paradexodes aurifrons, sp. nov.

One male, North Saugus, Massachusetts (Gipsy Moth Laboratory, bred in Cage E, 14 July, 1906. No. 698, E. S. G. Titus. Host unknown).

Length, 10 mm. Blackish, gray pollinose. Entire face and front, even including cheeks and orbit, deep golden pollinose. Frontalia quite black, nearly as wide as one parafrontal. Antennæ blackish. Palpi reddish yellow. Thorax and scutellum very thinly pollinose, humeri thickly so. Abdomen quite uniformly pollinose except first segment, thickly bristly and hairy. Legs entirely black, femora pollinose on under side. Claws long and black, pulvilli smoky. Tegulæ whitish, with narrow yellowish edge.

Type.—Cat. No. 11,686, U. S. N. M.

Paradexodes albifacies, sp. nov.

One male, White Mountains, New Hampshire, Morrison. This specimen is figured in Dr. Howard's Insect Book, pl. 22, fig. 7, as *Hypostena variabilis* Coquillett, but is not congeneric with the type of that species.

Length, 9.5 mm. Face and front, cheeks and orbits silvery white. Frontalia reddish brown, wider than one parafrontal. Antennæ reddish brown. Palpi yellow. Thorax, scutellum, and abdomen shining black, very thinly bluish silvery pollinose, most thickly so on humeri and bases of last three abdominal segments. Legs blackish brown, femora silvery beneath, pulvilli yellowish white, claws pale reddish brown. Tegulæ white, with narrow yellowish edge.

Type.—Cat. No. 11,687, U. S. N. M.

Genus Ceromasia Róndani

Ceromasia aurifrons, sp. nov.

Three females and one male, New Hampshire (2 females and the male from Canobie Lake, Dimmock).

Length, 7.5 to 10 mm. Differs from the European *C. florum* Meigen (determined by Brauer and von Bergenstamm) by having whole of parafrontals, parafacials, and orbits deep golden in both sexes, even the cheeks showing golden in fresh specimens; the pollen of thorax and abdomen whitish gray, without the brassy tinge of florum; anal segment in both sexes with a noticeable tinge of golden, and scutellum testaceous only on apical half.

Type.—Cat. No. 11,649, U. S. N. M. (female).

Two males of this species were bred at the Gipsy Moth Parasite Laboratory, North Saugus, Massachusetts, by E. S. G. Titus, from unidentified lepidopterous larvæ.

Ceromasia auricaudata, sp. nov.

Two females and one male, Harrison, Idaho (male and female), and Pullman, Washington (female, July 16).

Length, 7 to 9 mm. Differs from *C. aurifrons* Townsend by having the anal segment wholly deep golden, same shade as parafrontals, etc.; humeri with a faint, abdomen with a more distinct golden tinge, scutellum hardly more narrowly testaceous, and thorax more distinctly vittate.

Type.—Cat. No. 11,650, U. S. N. M. (female, Harrison, Idaho).

EUDEXODES, gen. nov.

This genus is proposed for *Dexodes eggeri* Brauer and von Bergenstamm, of Europe. The characters of the facial plate throw the species into a different tribe (if not subfamily) from *Dexodes,* of which the type is *spectabilis* Meigen.

Subfamily Willistoniinæ

Genus Belvosia Robineau-Desvoidy and allies

Dr. Williston published a plate of *Belvosia* and allies in Insect Life, vol. v (1893), facing p. 238, exhibiting the difficulties to be encountered in separating the forms. By studying this plate, it will be seen that there is a correlation between length of second antennal joint and bristles on the facialia, also between former and distance of vibrissæ from oral margin.

The more elongate the second antennal joint is, the less bristles there are on the facialia. Conversely, the shorter the second joint, the more strongly are the facialia ciliate. In all cases, the distance of the vibrissæ above the oral margin is about equal to the length of the second antennal joint.

The forms having facialia not ciliate have the second antennal joint long, vibrissæ inserted far above oral margin, and fourth vein angular at bend. Those having facialia ciliate have the second joint much shorter and vibrissæ inserted only a little above oral margin; they fall into two categories by the character of the bend of fourth vein. We thus have the following table:

1. Facialia not ciliate, fourth vein angular at bend; second antennal joint strongly elongate, nearly as long as third joint; vibrissæ inserted high above oral margin, male claws normally very elongate, female claws less elongate. (Sto. Domingo, Brazil, California, New Mexico, Mexico, Jamaica.) .... *Belvosia bicincta* Robineau-Desvoidy
   Facialia ciliate 2
2. Fourth vein bent at a sharp angle, with or without stump, but often V-shaped and with stump; claws of male normally elongate, of female not; second antennal joint not strongly elongate, vibrissæ inserted normally above oral margin. (Brazil.) ... *Willistonia esuriens* J. C. Fabricius

Fourth vein rounded at bend, second antennal joint short or only appreciably elongate; vibrissæ inserted close on or only appreciably above oral margin, about as far above as length of second antennal joint; male claws usually not elongate, but longer than the short female claws..................*Latreillimyia,* nom. nov. (*Latreillia* preocc.) 3

3. Vibrissæ inserted close on the oral margin.......................... 4
Vibrissæ inserted appreciably above the oral margin, male claws strongly elongate. (Brazil, Pennsylvania.)...........*L.* (aberrant form)

4. Second antennal joint appreciably elongate, female claws rather short. (Minnesota.).................................*L.* (intermediate form)
Second antennal joint short, not at all elongate. (Brazil, Mexico, New York.)..................................*Latreillimyia* (typical forms)
bifasciata J. C. Fabricius, *leucopyga* (van der Wulp) Williston.

The character of the ciliate facialia is more important than the venational character and the same holds good of the vibrissal character and the elongation of second antennal joint. As already pointed out in a previous section of this paper, the relative length of the second and third antennal joints will not hold for generic separation, since the length and size of the third joint in these flies is largely a sexual character. But the actual length of second joint taken independently furnishes a good character. Only a few genera have the second joint elongate. It may be compared in length with the first joint. The first two joints do not vary sexually.

Brauer and von Bergenstamm state that the claws of male are elongate in *Willistonia* and short in *Latreillimyia.* It is doubtful how far these characters can be relied upon, since they are also sexual. The same authors also give as a character of *Willistonia* a stump at the angular bend of fourth vein and the angle more approximated to hind margin of wing. These may hold good, especially the latter, but are not necessary for the separation of the forms at present known to us. Further material will probably call for their use.

The writer pointed out in 1892 (Trans. Am. Ent. Soc., XIX, p. 89) that *bifasciata* has the facialia strongly ciliate and *bicincta* has not; that five specimens of *bicincta* from New Mexico had the third antennal joint scarcely longer than the second, which means that the second joint was strongly elongate; that three specimens from New York were easily referable to *bifasciata,* and one from Jamaica to *bicincta;* and that, while the parafacials are bare in both species, the whole anterior aspect of head is altogether more bristly in *bifasciata,* which possesses also greater hairiness of cheeks.

The elimination of *Belvosia,* argued for by Brauer and von Bergenstamm, is not permissible under the rules of the International Code. Its maintenance fortunately does not conflict with the genus *Willistonia,* since *bicincta* differs generically from *esuriens.*

Belvosia and *Latreillimyia* show no ventral plates or ventral membrane.

LATREILLIMYIA, nom. gen. nov.

This name is proposed for *Latreillia* Robineau-Desvoidy (1830), which is preoccupied by Roux in Crustacea (1827).

GONIOMIMA, gen. nov.

This genus is proposed for *Belvosia luteola* Coquillett. Bears a striking resemblance to *Gonia*. Second antennal joint short, third joint very long and narrow; arista long and flattened whole length, in front view appearing as a mere line, but in lateral view showing itself to be uniformly widened nearly to apex; frontal bristles in one main inner row bordering frontalia, with a row of weaker bristles outside, and orbital bristles (female) outside these; second aristal joint very short, front not widened and swollen, facialia ciliate almost to base of antennæ. Abdomen appearing conical from above, but laterally appressed on apical portion, fully as thick dorsoventrally for its whole length as its greatest width, which is at base. The body and wing characters agree perfectly with *Gonia*, but the head characters are totally different, and it is the latter which place the genus in the Willistoniinæ.

The genus appears to come near *Thelymorpha* Brauer and von Bergenstamm, but is at once distinguished by having no discal macrochætæ on abdomen. The head is almost the same, and the abdomen is described as very similar.

TRIACHORA, gen. nov.

This genus is proposed for *Latreillia unifasciata* Robineau-Desvoidy, of which *Exorista flavicauda* Riley is a synonym. Differs from *Latreillimyia* in having three rows of frontal bristles on each side of frontalia, besides the fronto-orbital bristles of female. The arista is flattened, and the antennal characters are similar to those of *Goniomima*. The main or strongest row of frontal bristles, of the three rows on each side, is in the middle, the inner row being decidedly weaker, and the outer row but little weaker than the main or middle row.

Genus Rileymyia Townsend

(Ent. News, 1893, p. 277)

This name was proposed by the writer in 1893 for *Rileya* Brauer and von Bergenstamm, which is preoccupied in Hymenoptera. The type of the genus is *Blepharipeza fulvipes* Bigot, according to Brauer

(Sitzungsber. Math.-Naturwiss. Cl. k. Akad. Wiss., CVI, I, p. 348), who says that *R. americana* Brauer and von Bergenstamm is a synonym of Bigot's species. *B. adusta* H. Loew is also typical of the genus, which may be distinguished from *Blepharipeza* by the absence of apical scutellar bristles and thornlike macrochætæ.

Rileymyia albifacies Bigot.—Brauer (l. c.) says this is a synonym of *fulvipes* Bigot, but in view of the widely removed type localities it would seem that the point needs verification. *R. albifacies* was founded on a specimen from Brazil, while *fulvipes* is from Washington State. *R. americana* is from California.

Subfamily MEIGENIINÆ

Genus Viviania Róndani

It must be noted that this genus is characterized quite fully by Róndani on p. 53, vol. IV, of Dipt. Ital. Prod., where the imperfectly erected *Biomyia* (l. c., vol. I, p. 72) is given as a partial synonym. *Biomyia* does not cover the same forms, so far as any one knows, and its one-line characterization entitles it to no notice in the face of its author's subsequent rejection of it. It is therefore quite out of the question to attempt to use it, especially since we have no definition of it.

Viviania mutabilis Coquillett, etc.—*Biomyia mutabilis* is a *Viviania*. So also is *B. aurigera* Coquillett. *B. genalis* Coquillett does not belong anywhere near this genus.

Viviania lachnosternæ, sp. nov.

One female, Urbana, Ill. (No. 36,817, Forbes). "Supposed to have bred in *Lachnosterna* adults."

Length, 10 mm.; of wing, 8 mm. Gray-cinereous, more or less silvery. All three antennal joints and arista wholly reddish yellow, the frontalia same color posteriorly, but darker in front. Parafrontals blackish, thinly silvery. Parafacials more distinctly silvery. Ptilinal area blackish, the lower portion of facial plate broadly yellowish on oral margin. Palpi light reddish yellow. Mesoscutum with five vittæ, the middle one narrow behind and obsolete in front of suture, the outer ones more or less triangularly widened, shorter and more triangular in front of than behind suture. Wings wholly hyaline, tegulæ white. Legs wholly blackish, femora silvery below, pulvilli smoky. Abdomen black, thickly cinereous pollinose.

Type.—Cat. No. 10,913, U. S. N. M.

Genus Tachinomyia Townsend

Tachinomyia robusta Townsend.—The genus differs from *Tachina* in the vibrissæ being inserted higher above oral margin, cheeks one-half eye height in width, and abdomen very elongate.

Genus Emphanopteryx Townsend

Emphanopteryx eumyothyroides Townsend.—This genus differs from *Cryptomeigenia* by having the abdomen large; claws and pulvilli of female elongate, those of male very long and strong; arista finely pubescent, strong subdiscal and discal macrochætæ (at least on second segment), fourth vein rather angular at bend and usually represented beyond apical crossvein by a short stump.

Subfamily Tachininæ

Genus Tachina Meigen

Tachina clisiocampæ Townsend.—*Achætoneura fernaldi* Williston is very probably a synonym. The strongly marked wrinkle at angular bend of fourth vein, the elongate second antennal joint which is about three times as long as first, the frontal bristles descending low on parafacials, and the strongly ciliate facial ridges, whose bristles ascend at least to opposite the lowest frontal bristles, make the species typical of *Tachina* s. str. The third antennal joint is about twice as long as second. *Achætoneura* has the second antennal joint hardly longer than the first, the third joint thus being easily five or six times as long as the second.

This species can not be identified with *T. mella* Walker, if the description of latter is to be depended on, since it states that the second antennal joint is ferruginous apically, third joint three times as long as second, arista much longer than third antennal joint, second aristal joint moderately long, large ferruginous spot on each side of second segment. The venational characters also do not agree. In *clisiocampæ* there is at most in either sex only a very faint tinge of reddish, hardly perceptible in fact, on sides of second abdominal segment. The antennæ are wholly blackish, the third joint, even in male, hardly more than twice as long as second, arista but little longer than third antennal joint, apical crossvein well bowed in, hind crossvein quite strongly sinuate.

Tachina orgyiarum, nom. sp. nov.

This name is proposed for *T. orgyiæ* Townsend, which is preoccupied by *T. orgyiæ* Le Baron. Both species belong in the genus *Tachina,* as here restricted.

Tachina utilis, sp. nov.

Length, 6 to 8 mm. Differs from *T. larvarum* in its much smaller size, vertex of male not wider than one eye, thoracic dorsum not so thickly pollinose, abdomen more shining, with pollinose bands not so distinct, and anal segment not thickly hairy. The male in some specimens shows signs of reddish on sides of abdomen.

The anal stigmata of puparium also show important differences, the median slit being much abbreviated.

Germany, Bavaria, and Carniola. Bred at the Gipsy Moth Parasite Laboratory, North Saugus, Massachusetts, by E. S. G. Titus, in 1906, and W. F. Fiske, in 1907, from both *Euproctis chrysorrhœa* and *Porthetria dispar* larvæ received as summer importations from above localities; and also bred by W. F. Fiske, in 1907, from native larvæ of both species collected in field colonies near Boston (Oak Island and Woburn), Massachusetts, where European specimens of this tachinid had been previously liberated, showing that this species has gained a foothold.

Type.—Cat. No. 11,804, U. S. N. M. (male, length 6 mm.; Dresden, Germany, from *Euproctis* larvæ collected and shipped by Schopfer).

This type specimen was submitted to Dr. K. Kertész, and by him determined as *Tachina glossatorum* Róndani. It can not be that species, which is described by Róndani as having the second aristal joint four times as long as wide, and belongs to the genus *Microtachina* established on that character. *Tachina*, including the present species *utilis*, has the second aristal joint no longer than wide.

Genus Euphorocera Townsend

Euphorocera slossonæ, sp. nov.

One female, Franconia, N. H. (Mrs. A. T. Slosson). Syn. *E. cinerea* Coquillett (non van der Wulp), Rev. Tach., p. 102.

Differs from van der Wulp's description of *Phorocera cinerea* (Biol. C. A., Dipt., II, pp. 81-82) as follows: Frontalia as broad or broader than the parafrontals. Lowest frontal bristles not close to the eyes. Face very distinctly yellowish. Second antennal joint two and one-half times as long as first, the third joint a little more than twice as long as second. Arista thickened on basal third only. Palpi somewhat swollen, evenly clothed with black hairs. No trace of dorsal stripe on second and third abdominal segments. Two discal macrochætæ on second segment as well as on third. Anal segment only moderately beset with bristles. Small crossvein slightly before the middle of discal cell. Fourth vein bent at an obtuse angle. Posterior crossvein gently bisinuate.

Type.—Cat. No. 10,912, U. S. N. M.

Subfamily Echinomyiinæ

Genus Varichæta Speiser

The name *Varichæta* has been proposed by Speiser for *Erigone* Robineau-Desvoidy (1830), which is preoccupied by Savigny in Arachnida (1827). The type species is *V. radicum* Fallen.

Varichæta aldrichi Townsend.—This species, described under *Hystricia,* belongs in the genus *Erigone* (Robineau-Desvoidy) Brauer and von Bergenstamm, and must thus be known as *Varichæta aldrichi.* It is quite distinct from *V. radicum.* The latter has only three postsutural macrochætæ, while *aldrichi* has four or five. There are also differences in the abdominal macrochætæ.

Genus Elachipalpus Róndani

This genus is characterized by Róndani as possessing palpi, though small; and having apical cell appendiculate by reason of the continuation of fourth vein beyond apical crossvein. The type cited for it by Róndani is *Micropalpus longirostris* Macquart, from the Cape of Good Hope. The species is figured by Macquart as having a proboscis like *Spanipalpus,* but with distinct filiform palpi, and venation like *Spanipalpus* and *Deopalpus,* except that, instead of a wrinkle, there is a distinct stump representing fourth vein beyond apical crossvein. Brauer and von Bergenstamm indicate *E. longirostris* Róndani as type of *Elachipalpus,* but throw doubt on Róndani's *longirostris* being the same as *Micropalpus longirostris* Macquart. However this may be, it is certain that the American species *ruficauda* van der Wulp and *macrocera* Wiedemann do not belong to *Elachipalpus,* since they have absolutely no palpi, the proboscis is much shorter, and the venation markedly different. The new genus *Copecrypta* is therefore proposed for *Schineria ruficauda* (van der Wulp) Williston. The species was referred to *Cuphocera* by Williston.

COPECRYPTA, gen. nov.

Distinguished by a characteristic narrowing of the apical cell at the end, the ultimate section of fourth vein being crookedly bowed in and for the last one-third or one-fourth of its extent parallel with the third vein and very closely approximated to it, thus forming a narrow handle-like tip to the apical cell. The proboscis beyond geniculation is shorter than head height. Palpi absent. Two orbital bristles in female, none in male. Some extra bristles outside the frontal row, but these do not form a definite second row except anteriorly in some males. No ocellar bristles. Claws of female short, those of male as long as last tarsal joint.

The genus differs from *Trichophora* by having the abdomen elongate, subconical or subcylindrical, reaching nearly to end of wings. *Trichophora* has abdomen much shorter than wings and rounded.

SPANIPALPUS, gen. nov.

This genus is proposed for *Trichophora miscelli* Coquillett. It differs from *Copecrypta* in possessing a strong pair of ocellar bristles; proboscis long and slender, much longer than head height; abdomen considerably widened (female). Male not known. Female with two strong orbital bristles; only one row of frontal bristles; inner pair of vertical bristles very long and strongly curved, decussate, reclinate. Apical crossvein normal, not crooked, evenly bowed in near origin; apical cell widely attenuate on terminal portion, widely open. A distinct wrinkle at origin of apical crossvein.

DEOPALPUS, gen. nov.

Differs from *Spanipalpus* only as follows: No ocellar bristles. Two very definite rows of frontal bristles on each side of frontalia. No orbital bristles (male), claws of male not elongate. Parafacials, parafrontals, and cheeks evenly and thinly pilose with rather long fine black hairs. Parafrontals not metallic or blackish, silvery white. Venation and proboscis like *Spanipalpus*. Abdomen about like *Copecrypta*. The head bristles, like those of all the rest of the body, are strong. The inner frontal rows are decussate, extending only half way back between base of antennæ and vertex. The outer row on each side is composed of lightly reclinate bristles of nearly equal strength, nearly as strong as the vertical bristles. Both rows descend well below base of antennæ, the outer row slightly lower than the inner and to base of third antennal joint. Two facio-orbital bristles as strong as the frontal bristles. Facial plate strongly produced below. Second antennal joint elongate, about as long as third. Second aristal joint strongly elongate, slightly geniculate. Cheeks nearly equal to eye height. Type, the following species:

Deopalpus hirsutus, sp. nov.

One male, Meadow Valley, head of Rio Piedras Verdes, about 7,300 feet, Sierra Madre of western Chihuahua, July 29 (Townsend).

Length, 9.5 mm. Bears considerable superficial resemblance to *Copecrypta ruficauda,* but may be distinguished therefrom by the generic characters above given. Head entirely silvery white, frontalia showing very faintly pale brownish, first two antennal joints light brownish yellow, third joint hardly darker, but with anterior terminal border and arista blackish. Proboscis black, shining. Thorax cinereous pollinose, with two interrupted heavy outer dark vittæ, and two narrow inner vittæ stopping a little behind suture. Scutellum tawny yellowish, darker at base, silvery, with two very strong pairs of lateral macrochætæ reaching beyond base of third abdominal segment, a moderately strong but shorter apical decussate pair, and two lateral weak pairs besides discal bristles. Abdomen faintly blackish on dorsum, pale reddish or brownish yellow on sides, anal segment wholly reddish. All of abdomen more or less thickly silvery pollinose, showing most on basal half or more of last three segments. Macrochætæ as follows: One lateral marginal on first segment; one lateral marginal, and one median marginal pair on second segment; eight strong marginal in a row on third segment; anal segment with about twenty in marginal, submarginal, and discal rows. Legs black, tibiæ reddish, especially hind ones, pulvilli only faintly smoky. Wings clear, tegulæ white, third vein bristly to small crossvein.

Type.—Cat. No. 10,914, U. S. N. M.

EUPELETERIA, gen. nov.

Erected for *Echinomyia fera* Linné, *magnicornis* Zetterstedt, *præceps* Meigen, etc. Differs from *Peleteria* Robineau-Desvoidy in lacking the two or three facio-orbital bristles (macrochætæ on parafacials next orbit and separated from descending frontal bristles). Differs from *Echinomyia* Duméril, as restricted, by having abdominal macrochætæ not closely set and thorn-like. Body *Peleteria*-like, not *Jurinia*-like.

EUFABRICIA, gen. nov.

Second antennal joint strongly elongate, fully four times as long as first, much longer than third; third joint strongly convex on front border in profile. Second aristal joint elongate, fully four times as long as wide. Palpi widened and flattened on distal one-third or so, somewhat spatulate. No ocellar bristles. Parafacials wide, front not specially prominent in profile. Cheeks about two-thirds eye height in width. Anterior tarsi of female not more widened than those of other legs.

Differs from *Fabricia,* to which it is most nearly related, in the absence of ocellar bristles, and form of palpi and third antennal joint.

Type, the following species (to be figured in the forthcoming new edition of Dr. S. W. Williston's Manual of Diptera, fig. 157).

Eufabricia flavicans, sp. nov.

One female, Brazil, H. H. Smith, Coll. Received from Dr. S. W. Williston.

Length, 14 mm. General yellowish or rufous yellowish in ground color. Head silvery whitish, frontalia and first two antennal joints reddish yellow, third joint and arista light brown. Palpi yellow. Parafrontals with a faint tinge of brassy yellow. Thorax and scutellum brassy yellow pollinose. Abdomen rufous yellow, first segment brown on depressed median portion, other segments more tinged with rufous on median line, third segment wholly so tinged. Narrow anterior margin of second and third and all of anal segment yellowish silvery pollinose. A median marginal pair of macrochætæ on second segment, a marginal one on each side of first and second segments, a median marginal pair and three lateral marginal ones on third segment (eight marginal in all), anal segment with a discal and marginal row. Legs blackish or brown, the tibiæ more or less rufous, hind tibiæ especially so. Claws reddish or yellowish brown, tips darker, pulvilli yellowish. Wing bases broadly yellow, tegulæ whitish.

Type.—Cat. No. 11,805, U. S. N. M.

Subfamily Hystricinæ

Genus Dejeania Robineau-Desvoidy and allies

Dejeania vexatrix Osten-Sacken and *Paradejeania rutilioides* Jaennicke.—Speaking of *Dejeania vexatrix,* Osten-Sacken said: "It is very remarkable that *Dejeania,* a South American and Mexican genus, should occur so commonly at high altitudes in the Rocky Mountains among alpine forms, and it would be worth the while to investigate on what insect (probably Lepidopterous) it preys as a parasite" (Western Diptera, p. 343). At the close of his paper (l. c., p. 354), he again referred to the same matter, and included a reference to *P. rutilioides,* not, however, mentioning it by name.

These instances of a tropical group of tachinids developing boreal forms is paralleled in birds by the parrot genus *Rhynchopsitta,* peculiar to the pine region of the Sierra Madre of western Chihuahua. The tropical bird group of parrots has here developed a sub-boreal genus peculiar to the pine region, and which both passes the winter and nests there. Likewise a species of trogon occurs, belonging to the monotypic genus *Euptilotis,* also peculiar to the same region and breeding there.

Paradejeania may be considered as more or less of a boreal offshoot from *Dejeania,* and *D. vexatrix* Osten-Sacken is a boreal and distinct form from the tropical *corpulenta* Wiedemann. Osten-Sacken was mistaken in taking Wiedemann's type to be the same as *vexatrix.*

PTEROTOPEZA, nom. gen. nov.

This name is proposed for *Chætoprocta* Brauer and von Bergenstamm (1891), which is preoccupied by Nicéville in Lepidoptera (1890). Type is *Blepharipeza tarsalis* Schiner, of South America.

Genus Gymnochæta Robineau-Desvoidy

Gymnochæta alcedo H. Loew.—This species is not typical of the genus *Gymnochæta.* The type of the genus is *viridis* Fallen, which has second antennal joint elongate, second aristal joint elongate, antennæ inserted a little below middle of eyes, and cheeks one-half eye height.

EUJURINIA, gen. nov.

This genus is proposed for *Hystricia pollinosa* van der Wulp. Antennæ, frontal bristles, arista, and palpi like *Jurinia,* but resembling *Hystricia* in having the eyes hairy and the cheeks not so wide. It differs from *Jurinella* in the narrower cheeks and wider parafacials, and from *Pseudohystricia* in the first of these characters and the less produced front.

The cheeks of *Jurinia* are nearly equal to eye height, and the eyes are bare. *Hystricia* has the third antennal joint truncate at tip, second joint not so elongate, but attenuated at origin; frontal bristles weaker, straighter, descending lower, and all directed forward; no macrochætæ on lower border of cheeks, second aristal joint not strongly elongate.

A female specimen in U. S. N. M., collected by the writer July 3, at San Rafael, near Jicaltepec, Veracruz, is apparently to be identified as *Eujurinia pollinosa,* although van der Wulp says "arista indistinctly jointed," which, it seems, must be an error, and not intended by the author. The first two aristal joints in above specimen are elongate and distinct. Also there are some fine strong bristles on under side of middle femora. Length, 16 mm.

RHACHOËPALPUS, gen. nov.

This genus is proposed for *Saundersia testacea* van der Wulp. Mr. van der Wulp has remarked on the striking resemblance which this species bears to *Paradejeania rutilioides.*

Rhachoëpalpus olivaceus, sp. nov.

Two specimens, male and female, collected on the head of Rio Piedras Verdes, about 7,000 feet, Sierra Madre of western Chihuahua (Townsend), one on flowers of *Rhus glabra,* July 15, the other August 16.

Length of male, 18.5 mm.; of female, 19 mm. Thorax with an olive green tinge. Frontalia with much the same tinge, but darker. Second antennal joint with a strong bristle on front border near distal end, sometimes a pair of them. Third joint only a little longer than second, hardly one and one-half times as long, same size in both sexes. Arista thickened on rather more than basal half, distinctly jointed, second joint elongate. Scutellum with at least four rows of spines. The male shows a median dorsal stripe on abdomen, widened in front on second segment, where it is marked by an area of spines, narrower on third segment, and narrowest, but still distinct, on anal segment. This stripe shows only on anal segment in female, but the area of spines is present on second segment. The anal segment in both sexes is gently emarginate in middle on hind border, presenting a double curve like a pair of buttocks. Wings evenly infuscated. Color of scutellum is same in both sexes—quite yellowish. Abdomen of male is of a distinctly more reddish shade, female abdomen being of nearly same shade as scutellum, if anything, slightly lighter. Claws of female are yellow, with black tips. Front of female is wider, with three proclinate fronto-orbital bristles on one side and only two on the other. Front tarsi of female not dilated.

Type.—Cat. No. 10,915, U. S. N. M.

Rh. olivaceus bears the same striking resemblance to *Paradejeania* that *Rh. testaceus* does; perhaps even more so, since in the latter there seems to be no posterior emargination of anal segment on the median line. Mr. van der Wulp's figure shows none, and his text mentions none.

Rhachoëpalpus shows broad ventral plates in both sexes, but ventral membrane in female is not visible. There are five abdominal segments, the first very short and barely discernible from the side. The female shows ventral plates, bearing thick bunches of spines, corresponding to second to fifth dorsal plates, the lateral edges of latter overlapping sides of former, and a sixth ventral plate, or sclerite appearing as such, at base of ovipositor. The latter bears only hairs. The male with second and third ventral plates bearing thick bunches of spines as in female, but fourth with only hairs and free, the ventral membrane showing widely on sides of fourth only; fifth ventral plate much narrower, longer than wide, bare, not free; what seems a sixth ventral plate in female represented in male by a paired process articulating with the hypopygium.

EUËPALPUS, gen. nov.

Differs from *Epalpus* in having third antennal joint elongate and convex in profile on anterior edge, front and epistoma much less prominent, face less deeply concave in profile. Eyes absolutely bare. Parafacials very wide, black-hairy. Cheeks about equal to eye height. Second aristal joint hardly twice as long as wide.

Differs from *Xanthozona* (type, *melanopyga* Wiedemann) in having no discal macrochætæ on abdomen.

Type, the following species (to be figured in the forthcoming new edition of Dr. S. W. Williston's Manual of Diptera, fig. 156).

Euëpalpus flavicauda, sp. nov.

One female, Brazil, April, H. H. Smith, Coll. Received from Dr. S. W. Williston.

Length, 15 mm. Black; face, cheeks, and beard silvery white. Frontalia and parafrontals blackish, quite concolorous. Antennæ and arista brown. Thoracic scutum metallic black with greenish tinge, thinly silvery pollinose, more thickly so on anterior edge, humeri, and pleuræ. Scutellum and abdomen metallic brown with a hardly purplish tinge, the anal segment with a conspicuous subtriangular (from above) yellow area defined by the discal row of macrochætæ and extending under so as to narrowly surround the genital opening. A comb of median marginal thorn-like macrochætæ on ventral segments, and discal row on ventral side of anal segment. A single lateral marginal macrochæta on first and second segments, two median marginal pairs on second, a marginal row of ten on third, the discal row on anal; and only a row of weak bristles on posterior edge of anal, appearing like bristly hairs compared with the other macrochætæ. Legs brown or blackish; claws and pulvilli rufous yellow, tips black. Wings entirely and evenly infuscate, tegulæ decidedly smoky.

Type.—Cat. No. 11,806, U. S. N. M.

XANTHOZONA, gen. nov.

This genus is proposed for *Tachina melanopyga* Wiedemann. Two female specimens in U. S. N. M., Campinas, Brazil (A. Hempel), and Sao Paulo, Brazil (Ad. Lutz), labeled "parasitic on *Brassolis astyra*."

The ventral plates (female) only narrowly showing, overlapped by edges of corresponding dorsal plates, exposed portion being wider behind and narrowed anteriorly owing to the posteriorly rounded-off shape of edges of dorsal plates overlapping them, the posterior ones showing more widely than anterior ones, all widening successively from anterior to anal segments.

Family MUSCIDÆ

Subfamily CALLIPHORINÆ

Genus Calliphora Robineau-Desvoidy

Girschner and Hough have paved the way for a clearer understanding of *Calliphora* and its allies, and the genera as established by them are accepted in this paper, with the addition of two new ones.

Calliphora texensis, sp. nov.

Two males, three females, Paris, Texas. A. A. Girault, Coll.

Length, 9 to 11 mm. Differs from *C. coloradensis* Hough in the third posterior intra-alar bristle being absent and without a trace. The male front at vertex is about one-fifth of head width, and narrows very noticeably in front of vertex in an even curve, widening at same curve on anterior portion. The male parafrontals and parafacials are conspicuously pale brassy. The female parafrontals are obscure brownish, the parafacials light russet and unicolorous with facialia and facial plate, which are also this color in male. In one female the anterior reddish portion of buccæ (hairy part of cheeks) looks almost black in some lights, but the reddish tinge can be distinctly seen, and the specimen should be included with this species. The color of abdomen varies from metallic green to purplish blue.

Type.—Cat. No. 10,883, U. S. N. M.

Calliphora rubrifrons, sp. nov.

Two females, one male, Stickeen River, British Columbia, H. F. Wickham, Coll.; two females, one male, Kaslo, British Columbia, H. G. Dyar, Coll.

Length of female, 9.5 to 12.5 mm.; of male, 8.5 to 9.5 mm. Buccæ black, beard black. The two Stickeen River females and one of those from Kaslo, being the three largest specimens, show the buccæ with a good reddish tinge on anterior half, the two males and the other Kaslo female not. Third posterior intra-alar bristle absent. Frontalia bright orange red on anterior portion, in the Stickeen River male more of a yellowish red, in the Kaslo male a brownish yellow. Parafacials, facialia, epistoma, palpi, and apex of second antennal joint with base of third joint nearly the same color as the frontalia anteriorly, but sometimes a lighter shade of same color. Female front over one-third of head width, male front about one-twentieth of head width. Thorax faintly silvery white dusted, most thickly so on front border. Abdomen metallic green to blue, distinctly silvery pollinose in certain lights. Wings clear, with more or less distinct flecks of black on humeral, small, and basal cross-veins, origin of third vein, and apex of auxiliary. Alulæ well tinged with smoky, appearing quite black if resting against thorax or base of wing, tegulæ blackish with narrow white margins.

Type.—Cat. No. 10,884, U. S. N. M. (Stickeen River, British Columbia).

Calliphora popoffana, sp. nov.

One female, Popoff Island, Alaska, July 16, 1899. Harriman Expedition. T. Kincaid, Coll.

Length, 10.5 mm. Buccæ black, beard black. Front and face black, with a faint silvery white pollen distinctly to be seen in certain lights, even on facial plate, and especially on the broad frontalia and on the parafacials. Palpi light reddish yellow, facialia and epistoma darker reddish yellow, second antennal joint reddish, rest of antennæ black. Front distinctly more than one-third head width. No trace of third posterior intra-alar bristle. Wings quite clear, even at base, tegulæ white. Abdomen metallic green. Legs black. The plumosity of the arista is much shorter than in the other species.

Type.—Cat. No. 10,885, U. S. N. M.

A male from Bear Lake, British Columbia, 7,000 feet, R. P. Currie, Coll., measures 7 mm., and may be this species. The front is about one-eighth head width. The parafacials and narrow parafrontals are strongly silvery white; also facial plate. The frontalia are brownish. Tegulæ blackish. Wings with two smoky streaks on costal half. Abdomen metallic blue, silvery white dusted. The antennæ are paler on basal half of third joint. Otherwise it agrees with the female just described. The plumosity of the arista is quite normal, and this, taken with the blackish tegulæ and wing streaks, would point to it as a distinct form.

Calliphora irazuana, sp. nov.

One female, Irazu, Costa Rica, Schild and Burgdorf.

Length, 11.5 mm. Buccæ black, beard black. Third posterior intra-alar bristle wholly absent. Parafrontals black, with a soft brassy brown pollen on front half. Parafacials dark dragon's-blood red, facial plate blackish. Palpi reddish yellow, antennæ blackish, inner basal portions of third joint paler. Front equilateral, one-third head width. Thorax and scutellum black, faintly silvery on front and lateral edges. Tegulæ and wing bases blackish. Abdomen purplish blue. Legs wholly soft black, as are also the pleuræ, with hardly a trace of silvery.

Type.—Cat. No. 10,886, U. S. N. M.

EUCALLIPHORA, gen. nov.

Proposed for *Calliphora latifrons* Hough. Differs from *Calliphora* in possessing two strong pairs of ocellar bristles. This is a character of considerable importance in the Muscoidea, especially in the higher groups, and may well form a generic distinction here.

Eucalliphora latifrons Hough.—A large series of this interesting species, consisting of some sixty specimens, was brought from Kaslo, British Columbia, by Messrs. Dyar, Caudell, and Currie. The character of the second pair of ocellar bristles is constant in all.

There are also two females in the U. S. N. M., collected by H. S. Barber, Las Vegas Hot Springs, N. Mex., and Fieldbrook, Cal., which both belong to this genus and are apparently this species.

Genus Lucilia Robineau-Desvoidy

There are several species of this genus, notably *sericata* (Meigen) Hough and *sylvarum* (Meigen) Hough, which have a well-developed second pair of ocellar bristles. The latter are remarkably strongly developed in these two species, and were it not for the presence of certain intermediate forms, like *pilatei* Hough, and especially *oculata*, n. sp., they would constitute a well-marked new genus separable on this character. But in *pilatei* the second pair in the male is hardly to be differentiated in strength from some of the other pairs of divergent ocellar hairs, and in *oculata* the male shows no second pair, though the females of both possess the character quite distinctly. As genera are mere matters of convenience, and these forms do not otherwise differ in points of generic value, the character in question can not be used here for the erection of a separate genus. This is only another illustration of the fact that a character of value for the separation of certain forms may be valueless for this purpose in certain other forms closely allied to the first. In all the species there are several widely divergent pairs of weak ocellar hairs behind the first or regular pair of ocellar bristles. In the forms which have a second pair of ocellar bristles well developed, this second pair is always inserted just behind the two posterior ocelli, and not inside the ocellar triangle. In other words, it is only the pair of hairs inserted just behind the two posterior ocelli that ever develop into a second strong pair of bristles. *L. cæsar* is typical of the forms in which this pair of hairs is not developed in either sex, but it is to be noted that some of the bristly hairs within the ocellar triangle in this species often seem strong enough to be considered additional pairs of ocellar bristles.

Fourteen species of *Lucilia* are here recognized, occurring in material in U. S. N. M. They may be separated as follows:

Table of Lucilia spp.

1. Only one postacrostichal bristle present................*morrilli,* sp. nov.
   Two postacrostichal bristles present.................................. 7
   Three postacrostichal bristles present................................ 2
2. Palpi more or less yellowish.. 4
   Palpi wholly black or blackish...................................... 3
3. Second pair of ocellar bristles developed.......................*sylvarum*
   Second pair not developed..........................*nigripalpis,* sp. nov.
4. Palpi wholly yellow.. 5
   Palpi infuscate at tip... 6
5. Second pair ocellar bristles developed.........................*sericata*
   Second pair not developed..........................*angustifrons,* sp. nov.
6. Second pair developed..................................*giraulti,* sp. nov.
   Second pair not developed...............................*barberi,* sp. nov.
7. Abdomen unicolorous.. 8
   Basal segment of abdomen black or blackish........................... 9
8. Second pair developed..................................*unicolor,* sp. nov.
   Second pair not developed..*cæsar*
9. Whole body purplish, except basal abdominal segment, second abdominal segment with a marginal row of bristles.........*purpurea,* sp. nov.
   Whole body not so.. 10
10. Second and third abdominal segments with a purplish or blackish margin 11
    Second and third segments unicolorous............................... 12
11. Buccæ yellow wholly or partly......................................*pilatei*
    Buccæ black, not at all yellow.........................*australis,* sp. nov.
12. Eyes normal, face black.................................*infuscata,* sp. nov.
    Eyes flattened anteriorly with large front aspect, face brownish yellow. *oculata,* sp. nov.

Lucilia morrilli, sp. nov.

Six males, nine females, Texas, New Mexico, Arizona, California, British Columbia, and Missouri.

Only one postacrostichal bristle. Male front one-seventh of head width, female front fully two-fifths of head width. Whole of abdomen, thorax, parafrontals, and cheeks, including occiput, strongly metallic green. Face and frontalia black, silvery. Palpi black. Tegulæ white. No macrochætæ on abdomen. No second pair of ocellar bristles.

Type.—Cat. No. 10,887, U. S. N. M. (Victoria, Texas—Morrill).

Lucilia sylvarum (Meigen) Hough.—One female, Prussia.

Three postacrostichal bristles. Male front very narrow, female front one-third head width. Palpi black. Two stout marginal macrochætæ on second abdominal segment. Second pair of ocellar bristles well developed.

Lucilia nigripalpis, sp. nov.

Two females, Cuyahoga County, Ohio. W. V. Warner.

Differs from *infuscata* only by having three postacrostichal bristles; palpi quite blackish, faintly paler basally; antennæ, face, buccæ, and front all more deeply black; tegulæ white. A trace of purplish on hind margins of second and third abdominal segments, especially on second. Second segment with a marginal row of weak macrochætæ. No second pair of ocellar bristles.

Type.—Cat. No. 10,888, U. S. N. M.

Lucilia sericata (Meigen) Hough.—Two males, six females, eastern United States, Alabama, Hidalgo (Mexico), Kadiak Island (Alaska).

Three postacrostichal bristles. Male front one-eighth to one-sixth head width, female front two-fifths head width. Palpi yellow. Abdomen unicolorous, tegulæ white. A strong second pair of ocellar bristles in both sexes.

Lucilia angustifrons, sp. nov.

One male, England (Brunetti).

Same as *cæsar,* but having three postacrostichal bristles. Front linear, eyes almost contiguous. Palpi yellow. A female, having front one-third head width, from Kaslo, British Columbia (Caudell), seems to be this form. No second pair of ocellar bristles.

Type.—Cat. No. 10,889, U. S. N. M.

Lucilia giraulti, sp. nov.

One male, Paris, Texas. A. A. Girault, Coll.

Three postacrostichal bristles. Male front one-eighth head width. Abdomen like *pilatei,* but no dark hind margins to second and third segments. Buccæ and whole face and front black, palpi yellowish but infuscate apically. Tegulæ nearly white. No strong macrochætæ except marginal row on third segment. Of the three postacrostichal bristles, the front one is well behind the front postsutural bristle, and the middle one is a little behind the middle postsutural. A second pair of ocellar bristles present.

Type.—Cat. No. 10,890, U. S. N. M.

Lucilia barberi, sp. nov.

Six males, Arizona (H. S. Barber), California (Coquillett), Guanajuato (Mexico), Alabama, West Virginia, and District of Columbia.

Three postacrostichal bristles. Differs from *giraulti* practically only in the second pair of ocellar bristles not being developed appreciably longer than the ocellar hairs, and the three postacrostichal bristles being even with the three postsutural bristles. Palpi yellowish, infuscate at tip. Buccæ, face, and front blackish, facialia reddish, epistoma yellowish. Tegulæ white. Basal abdominal segment black. An even row of ten marginal macrochætæ on third segment above, and three on each side below. No dark margins to second and third segments. Male front one-eighth head width.

Type.—Cat. No. 10,891, U. S. N. M. (Williams, Arizona).

Lucilia unicolor, sp. nov.

Five females, New Mexico, Mexico, and British Columbia.

This form corresponds to *cæsar,* differing therefrom in having the second pair of ocellar bristles distinctly developed. Two postacrostichal bristles. Female front a little less than one-third head width. Palpi yellow. Abdomen unicolorous. Tegulæ white.

Type.—Cat. No. 10,892, U. S. N. M. (Mesilla, N. Mex.—Cockerell).

Lucilia cæsar Linné.—Numerous specimens of both sexes, England, eastern United States, and British Columbia.

Two postacrostichal bristles. Male front linear, female front one-third head width. Abdomen unicolorous. Tegulæ white. Palpi yellow. Second pair of ocellar bristles not developed, or only very weakly so.

Lucilia purpurea, sp. nov.

One female, Fort Wrangel, Alaska, Wickham; one male, Kadiak, Alaska, Kincaid (Harriman Expedition).

Two postacrostichal bristles. Male front one-twelfth of head width, female front one-third head width. Palpi yellow. Basal abdominal segment blackish. Whole body purplish, strongly violet tinged, especially in the female. Tegulæ of female white, of male smoky. Buccæ, face, and front blackish, epistoma paler. Second abdominal segment with a marginal row of bristles or macrochætæ, but not as strong as those of marginal row of third segment. No second pair of ocellar bristles.

Type.—Cat. No. 10,893, U. S. N. M. (Fort Wrangel, Alaska).

Lucilia pilatei Hough.—Two males, two females, Florida, Porto Rico, Guatemala, and Peru. A neotropical species.

Two postacrostichal bristles. Male front one-eighth head width, female front one-fourth head width. Palpi yellow. Abdomen as in *australis,* only the purplish or black margins of segments often more marked. Buccæ of female yellow, of male gray with yellow anteriorly. A second pair of ocellar bristles in female more or less hair-like, but distinctly larger and thicker than the other hairs of ocellar area; in male very weak, not appreciably stronger than the other ocellar hairs.

The purplish black hind margins of second and third abdominal segments are characteristic of this and one or two other species, added to which is the blackish basal segment. The latter in some females shows a little metallic green on sides, but the general opaque black of its dorsum is the distinguishing character. Also the hind margins of second and third segments are only faintly purplish in some specimens, but distinct traces are present in all. The white tegulæ are characteristic of this species, and serve to separate the males of *pilatei* from the males of similar species having a black basal abdominal segment.

Lucilia australis, sp. nov.

Two females, Tennessee, Texas (Girault); one male, Popoff Island, Alaska (Kincaid, Harriman Expedition). The male is provisionally referred here.

Two postacrostichal bristles. Male front one-twelfth head width, female front one-fourth head width. Palpi infuscate yellow. Basal abdominal segment black above, conspicuously so, the purplish or darker hind margins of second and third segments also showing.

Distinguished from *pilatei* by the buccæ being black, silvery gray pollinose, not at all yellow. Second pair of ocellar bristles present in female, not developed in male.

Type.—Cat. No. 10,894, U. S. N. M. (Tennessee, Coll. Riley).

Lucilia infuscata, sp. nov.

Nine males, six females, Massachusetts, New Hampshire, Ohio, Missouri, New Mexico, Arizona, and British Columbia.

Two postacrostichal bristles. Male front very narrow, female front two-sevenths of head width. Basal abdominal segment black or purplish black in female, but no dark margins to second and third segments. Male tegulæ infuscate, female tegulæ more nearly white. Palpi yellowish. Buccæ, face, and front black. No second pair of ocellar bristles. The female can be told from female *cæsar* only by narrower front and darker basal segment.

Type.—Cat. No. 10,895, U. S. N. M. (Organ Mountains, New Mex., on flowers of *Lippia wrightii*—Townsend).

Lucilia oculata, sp. nov.

Six males, two females, District of Columbia, Kentucky, North Carolina, Mississippi, Kansas, and Cuba.

Two postacrostichal bristles. Male front linear, eyes nearly contiguous and approximated more anteriorly than in *infuscata,* with larger front aspect than in that species. Female front one-fourth of head width. Tegulæ nearly white, only very faintly tinged with yellowish. Antennæ and face brownish yellow instead of black. Basal abdominal segment quite black. Male shows no second pair of ocellar bristles, but female has them developed. Otherwise like *infuscata.*

Type.—Cat. No. 10,896, U. S. N. M. (Cumberland Gap, Ky.—G. Dimmock).

PROTOPHORMIA, gen. nov.

Hough characterizes *Phormia* as having the mesonotum "somewhat flattened caudad the transverse suture," as in *Protocalliphora.* This is a mistake. *P. regina,* which is the type of *Phormia,* does not show this flattening at all. The species *terrænovæ* is not a *Phormia,* but differs in possessing the same conspicuous flattening seen in *Protocalliphora.* The new genus *Protophormia* is herewith proposed for its reception. The characters given by Hough for *Phormia* (Ent. News, x, p. 66) all apply to *P. regina* except the character of the flattened thorax. This flattening carries with it a more or less complete abortion of the postacrostichal bristles except the hindmost one of each row.

Subfamily MUSCINÆ

Tribe *Mesembrinini*

Two new genera are here proposed in this tribe, and the genus *Mesembrina* is restricted as follows:

Genus Mesembrina Meigen

Type of the genus, *M. mystacea* Linné. Densely pilose flies. Subalar pile present, representing the pteropleural bristles. Sternopleural bristles 1. 0. 1. Fourth longitudinal vein very deeply and roundly bent far before reaching margin of wing, which latter point is same distance behind that termination of third vein is before extreme wing-tip, the portion between bend and margin being fully three times that in margin. Apical cell much narrowed, its mouth width not over one-third its greatest width. Small crossvein distinctly before middle of discal cell.

METAMESEMBRINA, gen. nov.

Proposed for *Mes. meridiana* Linné. Hairy, not pilose, flies. Subalar bristly hairs present, representing pteropleural bristles. Sternopleural bristles 0. 0. 1. Fourth longitudinal vein reaching front margin of wing before tip, arcuate at bend.

EUMESEMBRINA, gen. nov.

Proposed for *Mes. latreillei* Robineau-Desvoidy. Hairy, not pilose, flies. Pteropleural hairs present. Sternopleural bristles 1. 0. 2. Fourth vein very slightly and roundly bent a little before reaching hind margin of wing, the portion between bend and margin about equal to the portion in margin. Apical cell very widely open, its mouth width equal to about three-fourths its greatest width. Small crossvein distinctly beyond middle of discal cell.

Eumesembrina latreillei Robineau-Desvoidy.—Two specimens, White Mountains, New Hampshire, Morrison; one, Colorado; two, Washington State; two, Kaslo Creek, British Columbia, June 18, R. P. Currie and A. N. Caudell. All show face and parafacials silvery white from above. Antennæ reddish yellow to brownish. Palpi reddish or brownish red.

Eumesembrina alascensis, sp. nov.

Four specimens.—Kukak Bay, July 4; Kadiak, July 20; Saldovia, July 21; Juneau, July 25. All Alaska. Collected by T. Kincaid (Harriman Expedition).

These specimens are more hairy, more bristly on thorax and scutellum, and on peristomalia. They also usually show less silvery on face and parafacials, and the antennæ are quite black. Palpi black. The Kukak Bay and Kadiak specimens show no silvery on the soft blackish facial plate, and the parafacials are tan-colored without a sign of silvery. The other two specimens show some silvery, not only on facial plate, but also on the more or less tan-colored parafacials.

Type.—Cat. No. 10,897, U. S. N. M. (Kukak Bay, Alaska).

The two Washington State and two British Columbia specimens mentioned under *latreillei* are certainly distinctly to be referred to that species, which is the eastern form, and which is thus seen to range from the Atlantic to the Pacific. *Eumes. alascensis* doubtless represents rather a boreal form.

Family PHASIIDÆ

Tribe ANUROGYNINI

Genus Hyalomyodes Townsend

Hyalomyodes weedii Townsend.—This species seems distinct from *Hyalomyia triangulifera* H. Loew, but needs further study. The writer has examined the type of the latter in Cambridge.

Hyalomyodes triangulifera H. Loew.—Ten specimens from the White Mountains of New Hampshire, one from Massachusetts, and one from Maryland agree perfectly with the description of *H. weedii* Townsend. They also agree with Loew's description, but an examination of the type in Cambridge seemed to indicate differences. The front, frontalia, and parafacials are wider in the male, and the claws are elongate. Humeri grayish.

Hyalomyodes robusta, sp. nov.

Two males, North Fork of Rio Ruidoso, White Mountains, New Mexico, about 8,200 feet, on flowers of *Solidago trinervata,* August 17, Townsend.

Differs from *triangulifera* in being more robust, and first abdominal segment with pollinose fascia same as second and third. The thorax is also more conspicuously pollinose. Hind crossvein quite straight, in one specimen much nearer to small crossvein than to bend of fourth vein, in both distinctly nearer. The pollen of median portion of thorax and abdomen has a brassy tinge, that on sides being silvery-whitish. Macrochætæ not so well developed, considerably weaker. Parafacials wide in both specimens. Length, 5 mm.

Type.—Cat. No. 11,651, U. S. N. M.

Hyalomyodes californica, sp. nov.

Two specimens, male and female, Santa Clara county, California (C. F. Baker).

Almost like *triangulifera,* but distinguished by humeri being more golden, extending back in a lateral stripe.

Type.—Cat. No. 11,652, U. S. N. M. (female).

Tribe CLISTOMORPHINI

Genus Clistomorpha Townsend

A synonym of *Clistomorpha* is *Clytiomyia* Coquillett (non Róndani). This genus is very distinct from *Clytiomyia* Róndani (*Clytia* Robineau-Desvoidy). *C. hyalomoides* Townsend is distinct from *C. didyma* H. Loew (described as *Xysta*). The writer recognized the fact of the two being congeneric nearly fifteen years ago, from drawings of the type furnished by Mr. Samuel Henshaw, and has since examined the type of *didyma* in the Cambridge Museum.

Clistomorpha didyma H. Loew.—The apical cell is very short-petiolate, and the hind crossvein is curved and in middle between the small crossvein and bend of fourth vein.

Illinois.

Clistomorpha hyalomoides Townsend.—The apical cell is practically closed in the margin. The hind crossvein is in middle and straight.

New York.

Clistomorpha atrata Coquillett.—The apical cell is closed in margin, or almost narrowly open. The hind crossvein is sinuate and nearer to bend of fourth vein than to small crossvein.

Idaho, Washington State.

Genus Himantostoma H. Loew

Himantostoma sugens H. Loew.—This genus belongs in this tribe, as shown by an examination of the type in Cambridge.

Subfamily PHASIINÆ

Tribe *Alophorini*

The following table will serve to separate the genera of this tribe:

1. Front above antennæ thickly beset on both sides with small bristles..... 2
   Front above antennæ naked, only one row of frontal bristles on each side 3

2. First longitudinal vein elongate, small crossvein placed opposite end of auxiliary vein, fourth vein very obtusely bowed, apical cell sharp-angled at extremity and short petiolate........................*Alophora*
   First longitudinal vein not elongate, small crossvein placed opposite end of same, fourth vein roundly bowed, apical cell usually long petiolate. *Hyalomyia*
3. Second longitudinal vein ending opposite the junction of third and fourth veins, wings of male usually much widened............*Phorantha*
   Second longitudinal vein elongated beyond junction of third and fourth veins, wings of male not widened, apical cell very long petiolate, fourth vein roundly bowed.................................*Paralophora*

Genus Alophora Robineau-Desvoidy

Alophora sp.—A large species from Texas. The female shows ventral plates overlapped by dorsal plates. The male shows ventral plates free, at least those of second, third, and fourth segments, with membrane widely exposed on each side.

Genus Phorantha Róndani

The genus *Alophora* has the front prominent in profile above insertion of antennæ. *Phorantha* has front flattened, and with greater slope so as to present in profile an almost perfectly straight line from insertion of antennæ to vertex.

Probably all, or nearly all, of the various forms of the *Alophorini* that have been described are distinct and entitled to recognition. We know practically nothing of the early stages or the mating of the adults, and it is premature to attempt to outline the synonymy in the absence of such knowledge.

Tribe *Cistogasterini*

Genus Gymnoclytia Brauer and von Bergenstamm

The genus *Gymnoclytia* is distinct from *Cistogaster*. The peduncle of apical cell is continuous with fourth vein in *Gymnoclytia*, but with third vein in *Cistogaster*.

Gymnoclytia has ventral membrane (female) very widely visible and ventral plates free, much as in *Gymnosoma*.

Gymnoclytia occidua Walker.—Male.—Thorax brassy or golden pollinose, with two straight narrow median vittæ extending from front margin to behind suture, and two irregularly widened vittæ obsolete before and interrupted at suture. Abdomen more or less ferruginous, sometimes entirely so, but usually with a longitudinal fuscous stripe in connection with a median pollinose vitta, and more or less brown on third and fourth segments with grayish pollen.

Female.—Thorax silvery-whitish pollinose, with two heavy shining black vittæ, sides of front silvery-white pollinose becoming blackish posteriorly, abdomen black with silvery pollen in median vitta and two or three fasciæ.

New Hampshire, District of Columbia, North Carolina, Georgia, and Texas.

Gymnoclytia occidentale, sp. nov.

Male.—Thorax deep brassy to old-gold pollinose, with same vittæ as in *occidua*. Abdomen like *occidua* except that pollen is golden, the ground color bright ferruginous and markings varying from none to the usual ones strongly marked.

Female.—Colored almost like the male of *occidua*. Thorax brassy pollinose, with two broad heavy brown vittæ extending from anterior margin almost to scutellum, and two very narrow linear vittæ between them. Abdomen the same as in the male, pollen being golden, but no specimens occur with abdomen entirely ferruginous, the usual markings being pronounced in all.

Colorado and New Mexico to California.

Type.—Cat. No. 11,653, U. S. N. M. (female, Beulah, New Mexico, Cockerell, July, 1902).

Gymnoclytia immaculata Macquart.—Male.—Fuscous stripe of abdomen wanting, median pollinose vitta more or less distinct. Abdomen yellowish, the third and fourth segments with lateral pollinose reflections.

Female.—Thorax shining black, without pollinose markings except the humeri, sides of front shining black, abdomen without distinct pollinose vitta or crossbands, apical cell quite long petiolate (as in the males of the preceding species). Abdomen distinctly red on the sides, especially anteriorly.

This form and *Gym. occidua* Walker are distinct. See Robertson's and the writer's notes in T. A. E. S., XXII (1895), pp. 66-67, and Ann. and Mag. N. H., XX, pp. 283-284.

Gymnoclytia ferruginosa van der Wulp.—Male.—Thorax deep golden or old-gold pollinose, with the same stripes as *occidua* more or less apparent. Abdomen ferruginous, fuscous stripe hardly apparent, but pollinose stripe present, and third and fourth segments more or less pollinose, pollen being golden.

Female.—Sides of front faintly golden-silvery, thorax shining black, with three faintly golden pollinose vittæ. Abdomen shining black, with median pollinose vitta and third and fourth segments more or less pollinose, pollen being grayish with a hardly brassy tinge.

Veracruz and Nicaragua.

Tribe *Xanthomelanodini*

Genus Xanthomelanodes Townsend

Syn. *Xanthomelana* VAN DER WULP preocc.

The name used by van der Wulp was applied by Bonaparte to a genus of birds in 1850.

Xanthomelanodes arcuata Say.—Only a single vibrissa on each side.

Male.—Front and face deeply golden, especially parafrontals. Abdomen usually with a well-defined black median vitta, last segment and last half of penultimate segment black.

Female.—Front and face silvery-white. Abdomen all black except yellow on sides of second and third segments, only covering anterior half of third segment, but some specimens show less black.

New Hampshire, Kansas, Veracruz.

Xanthomelanodes atripennis Say.—One vibrissa on each side.

Male.—Front golden. Abdomen golden, with only some brownish shading for the median vitta. Wings quite smoky on inner border.

Dixie Landing, Virginia (Townsend).

Xanthomelanodes californica, sp. nov.

Two vibrissæ on each side.

Male.—Front and face almost silvery, with only a faint suggestion of golden, in some specimens quite silvery-white. Abdomen ferruginous, more or less dusky, the brown markings not well defined as a rule, consisting of a broken median stripe and the usual dark markings of last two segments.

Female.—Face and parafrontals silvery-white. Abdomen nearly same as in *arcuata*.

Colorado, Nevada, California.

Type.—Cat. No. 11,654, U. S. N. M. (male, Los Angeles county, California, Coquillett).

Tribe *Trichopodini*

The following is a table of the genera of this tribe:

1. Hind tibiæ without flattened cilia, with only a row of short appressed bristles and one or two stronger bristles among them. 2
   Hind tibiæ ciliate with the usual flattened cilia. 3

2. Wings infuscate on less than costal half, gray or hyaline on other portion .... *Acaulona*
   Wings almost wholly infuscate, but much more faintly so on inner half, the infuscation rather graduated into almost hyaline on inner border. *Euacaulona*
3. Wings wholly infuscate.. 4
   Wings more or less widely hyaline on inner border, the hyaline abruptly defined 6
4. Hind femora ciliate with closely appressed flattened bristles on one or opposite edges.. *Galactomyia* (males)
   Hind femora not ciliate at all.. 5
5. Apical cell open.. *Homogenia*
   Apical cell closed.. *Euomogenia*
6. Wings hyaline on more than inner half, abdomen subcylindrical in both sexes and largely translucent in both........................... *Pennapoda*
   Wings with inner hyaline border almost as wide as the infuscate costal half, abdomen subcylindric in both sexes and wholly opaque in both .... *Polistomyia*
   Wings with hyaline border about one-third width of wing...*Eutrichopoda*
   Wings with hyaline border very narrow, not over one-fifth of wing width 7
7. Wholly black form........................... *Galactomyia lanipes* (female)
   Partly reddish forms.. 8
8. Hind femora ciliate distally on inner edge with closely placed bristles; abdomen cylindrical, reddish or orange with apical half or at least anal segment black........................ *Galactomyia radiata* (female)
   Hind femora not at all ciliate, smaller forms with abdomen more or less flattened and almost wholly light reddish or yellowish in both sexes .... *Trichopoda*

Genus Acaulona van der Wulp

Acaulona costata van der Wulp.—One female, Tehuantepec, Sumichrast; one male, Frontera, Tabasco, February 9, Townsend. The Tehuantepec specimen is of much lighter coloration than Veracruz and Tabasco specimens. Tegulæ are yellowish in this genus.

Acaulona tehuantepeca, sp. nov.

One female, Tehuantepec, Sumichrast. Labeled "17. *Homogenia* sp." Length, 7 mm.

Differs from *A. costata* in having the apical cell subfuscous, the abdomen with a median blackish vitta and more or less wholly blackish on apical half, and the hind tibiæ weakly subciliate in a row of short, closely approximated bristles. The form is intermediate between *Acaulona, Euacaulona,* and *Homogenia,* but nearest to *Acaulona.*

Type.—Cat. No. 10,878, U. S. N. M.

EUACAULONA, gen. nov.

Differs from *Acaulona* in having somewhat more than costal half of wings pronounced fuscous, the rest of wing not being clearly hyaline, but more or less so, the fuscous rather gradually fading out on inner border. There are also two distinct grayish or milky vittæ on wings (male), one between the first and second veins, one between the third and fourth veins, besides a short one in front of the auxiliary vein. Tegulæ brownish or fuscous, paler in middle.

The front at vertex is nearly as wide as either eye, and gradually widens anteriorly to almost width of both eyes as viewed from in front, the face in same view being fully three-fifths width of head. The frontalia are very wide, of equal width, as wide as front at vertex.

Apical cell closed in margin. Hind tibiæ not ciliate, bearing only a row of short appressed bristles with one or two stronger bristles among them. Claws of male elongate. Abdomen of male flattened. Type, the following species:

Euacaulona sumichrasti, sp. nov.

One male, Tehuantepec, Sumichrast. Length, 9.5 mm.

Blackish, the venter and basal half of femora, also base of hind tibiæ, yellow; the usual golden yellow markings on prothorax along and in front of suture, also extending posteriorly on the sides and along the scutellar suture. Frontalia black, the narrow parafrontals and all of face and cheeks golden yellow. Thorax, scutellum, and abdomen above brown or blackish.

Type.—Cat. No. 10,879, U. S. N. M.

Genus Homogenia van der Wulp

Syn. *Trichopododes* TOWNSEND.

This genus has the wings wholly infuscate, those of male with considerable luteous and more or less of a milky bloom (*latipennis* and *nigroscutellata*); female not known. Hind femora not ciliate at all, hind tibiæ only weakly ciliate. Apical cell open. Tegulæ yellowish. Type, *H. latipennis* van der Wulp. The species *rufipes* van der Wulp evidently does not belong with the other two described under the genus, and will have to be separated generically.

H. latipennis van der Wulp.—One male, Tehuantepec, Sumichrast. Labeled "*Trichopoda luteipennis* Wd." This specimen agrees with van der Wulp's description except that there is no trace of a black median abdominal vitta, and abdomen is only a little dusky on anal segment as seen through the golden silvery bloom.

H. nigroscutellata van der Wulp.—One male, Cacao, Trece Aguas, Alta Vera Paz, Guatemala, April 18. Barber and Schwarz, collectors. This specimen agrees well with van der Wulp's description except that abdomen is widely blackish on median portion, with only narrow lateral borders yellow. The scutellum has golden pollen on dorsum.

EUOMOGENIA, gen. nov.

Differs from *Euacaulona* in the wings (male) being wholly infuscate, uniformly so, the same milky vittæ being present; and in the hind tibiæ being ciliate with moderately developed cilia. Front like *Homogenia,* very broad. Apical cell closed in border. Tegulæ blackish. Type, the following species:

Euomogenia lacteata, sp. nov.

One male, Frontera, Tabasco, March 3, Townsend.

Length, 9.5 mm. Blackish, the usual silvery golden markings on mesoscutum, including the sides back to scutellar suture and along latter. Scutellum somewhat silvery golden on dorsum. Abdomen wholly fuscous, with a reddish tinge showing through the fuscous. The broad frontalia velvety blackish, narrow parafrontals and whole of face and cheeks golden. Antennæ brownish. Palpi yellow, dark on tips. Basal half of femora yellow, least extensive on front pair, most extensive on hind pair, base of hind tibiæ yellow, rest of legs black, claws and pulvilli yellow, tips of claws black. Wings blackish, with the milky or golden grayish vittæ described for *Euacaulona.*

Type.—Cat. No. 10,880, U. S. N. M.

Genus Pennapoda Townsend

This was described as a subgenus, in Ann. and Mag. N. H., xx, p. 282. It is here raised to generic rank. Type, *Trich. phasiana* Townsend, loc. cit., male and female. The species *Phania simillima* Wiedemann and *Trich. subalipes* Townsend may belong to this genus. There are no specimens in U. S. N. M. for examination.

POLISTOMYIA, gen. nov.

This genus is proposed for the *Trich. trifasciata* H. Loew group. The abdomen is subcylindrical in both sexes, slightly more widened on apical portion in male. Apical cell closed and quite long petiolate. Wings with but little more than costal half colored, the inner portion clear. Abdomen in both sexes wholly opaque, brown or black in ground color, more or less golden pollinose, never with translucent portions. Scutellum always yellow. Both sexes have yellow on the wings. Hind femora not at all ciliate. Tegulæ white or yellowish. Parasitic in Acridiidæ (*Dissosteira*), so far as known.

The male has frontalia suddenly narrowed, presenting a curved outline on each side, closely followed by the frontal row of bristles, the width on posterior half being only one-half the width at base of antennæ. Claws strongly elongate in male, hypopygium exserted and tucked up under the end of abdomen.

The female has the frontalia but little narrowed behind, being evenly narrowed from anterior to posterior end, the sides and frontal row of bristles being quite straight. Claws somewhat elongate in female, even slightly longer than last tarsal joint, but very markedly less elongate than in male. Anal end of abdomen truncate, the ovipositor more or less withdrawn within anal segments, its apex usually showing.

Type, *T. trifasciata* H. Loew.

The other species belonging here are *histrio* Walker, *indivisa* Townsend, probably *umbra* Walker and *plumipes* J. C. Fabricius; also the following new species. The writer formerly suggested these (except *plumipes*) as varieties of one species, but now considers them valid forms differing in marked characters. They form a group apart by themselves, distinctly contrasted with the other members of the Trichopodini.

Polistomyia subdivisa, sp. nov.

One female, St. Helena, Napa County, Cal., bred by A. Koebele from a locust (*Dissosteira venusta* Stål); issued August 30, 1887.

Length, 6.33 mm. Segments three and four of abdomen golden pollinose, segment three with a median vitta and median hind margin brown, segment four wholly pollinose with a trace of vitta, segment two with a large yellow spot on each side, and segment one with a similar smaller spot on each side.

Type.—Cat. No. 10,881, U. S. N. M.

Two female specimens from Las Cruces, New Mexico, collected by the writer, August 25 and September 2, on flowers of *Solidago arizonica,* are larger, measuring 7 to 8 mm., show no median vitta on third and fourth segments, and only a faint vitta on second segment, which bears a fascia rather than separated spots. They very likely represent another form, but more material is needed from California and New Mexico before separating them as distinct. They occupy an intermediate position between *trifasciata* and *subdivisa.*

T. plumipes (J. C. Fabricius) Wiedemann is probably a *Polistomyia,* as indicated by the yellow scutellum, broad, clear inner margin of wings, and the cylindrical abdomen. The latter is described as black, which would indicate a form without fasciæ or pollinose markings, since it is hardly possible that such could have been so far lost as to leave no trace. We thus have the following forms of this genus, to be separated as below:

Polistomyia plumipes—No pollinose fasciæ on abdomen. Continuous black surface.
histrio—Two pollinose fasciæ, interrupted.
trifasciata—Three fasciæ, broadly interrupted.
subdivisa—Four fasciæ, two broadly interrupted and two faintly so.
umbra—Continuous pollinose surface, interrupted by median vitta.
indivisa—Continuous pollinose surface.

EUTRICHOPODA, gen. nov.

Differs from *Trichopoda* in the apical cell being moderately long petiolate, and the wings with inner border broadly hyaline, the latter being nearly or about one-third of wing breadth. Hind tibiæ ciliate, hind femora without cilia. Abdomen cylindrical in female, probably flattened in male. Tegulæ pale or whitish yellow. Type, the following species:

Eutrichopoda nigra, sp. nov.

Syn: *Trich. lanipes* VAN DER WULP (non J. C. Fabricius, Wiedemann), Biol. C. A. Dipt., II, pp. 434–5.

One female, Tehuantepec, Sumichrast.

Length, 9 mm.; of wing, 8 mm. Black. Parafrontals silvery white, with only a faint tinge of golden, which tinge is lost in view from above and behind. Face wholly silvery white, including parafacials. Transverse suture of mesoscutum marked by a golden yellow linear fascia, with two golden lines running to front border of thorax, humeri broadly golden. Scutellum is of the same dull black as the abdomen, with hardly a brownish tinge. Tegulæ saturated with a faint yellow tinge. Femora almost as black as rest of legs, with a faint brownish tinge. The mesoscutum behind suture is faintly purplish or bluish shining. The wings have no yellowish tinge in the black, and the inner hyaline border is hardly one-half as wide as the black portion.

Type.—Cat. No. 10,882, U. S. N. M.

This form comes nearer agreeing with Wiedemann's description of *plumipes* than anything that has turned up since Bosc's time. It differs therefrom only as above described, and principally in the black scutellum.

Mr. van der Wulp (l. c.) has described this species from what he records as one male and four females, but says nothing as to whether the apical cell is petiolate or closed in the margin, nor does he mention the shape of the abdomen in the sexes. It seems quite certain that his specimens are this species, and it is likely that all five of them have the apical cell moderately long petiolate.

Genus Trichopoda Latreille

This genus, as here restricted, has the wings with inner margin narrowly hyaline, hind femora not ciliate at all; only male with yellow in wings, no milky radiations, apical cell very short petiolate, and tegulæ yellowish. Type, *T. pennipes* J. C. Fabricius. Parasitic in Heteroptera (*Anasa, Leptoglossus*), so far as known.

GALACTOMYIA, gen. nov.

This genus is proposed for *Trich. radiata* H. Loew. *Trich. lanipes* J. C. Fabricius (description is of female; *T. formosa* Wiedemann is the male) also belongs in this genus.

The males have the abdomen flattened; the wings infuscate to inner margin, milky radiate on a yellow or fuscous background, the milky radiations conspicuous and the yellow less pronounced. Hind femora strongly ciliate on posterior half, with flattened bristles.

The females have the abdomen cylindrical; the wings wholly black except narrow inner border, without yellow coloring, the internal border abruptly limpid. Hind femora at least short-ciliate distally, though bristles may not be flattened. *G. radiata* female has the abdomen reddish, with at most the apical half black. *G. lanipes* female is to be distinguished by its wholly black coloration, aside from the usual yellow of head, thorax, claws, and pulvilli.

It is yet uncertain what species can be referred to this genus besides *radiata* and *lanipes* (syn. *formosa* Wiedemann). As to the distinctness of these two species, Loew pointed out in his description of *radiata* (male) that it has the palpi reddish yellow, abdomen purple black, and bases of femora yellow. *G. lanipes* (male) has palpi black, abdomen obscure rufous, and femora wholly black.

The males of *Galactomyia* have ventral membrane widely visible, and all the ventral plates free. There are six abdominal segments, the first extremely short and not visible above unless abdomen is detached, but visible on sides below; the second to sixth segments visible above, and a seventh wedged between the sides of ventral aspect of sixth, rounded in outline and forming the base of the hypopygium. This seventh segment occupies the position of a ventral plate to sixth segment and belongs to dorsum, being a dorsal plate. Five ventral plates, corresponding to first to fifth segments; first plate rather crescent shaped, much shorter (antero-posteriorly) and wider than second to fourth; second plate long-oval, third and fourth long-elliptical; fifth subquadrate and widened behind, about as wide as first. Immediately behind the fifth plate is the hypopygium, and behind latter is the seventh segment, with the lateral ends of sixth dorsal plate enclosing it on the sides.

In the female of *G. lanipes* there are seven ventral plates visible, the first three free, with ventral membrane showing on each side, fourth plate showing ventral membrane only around anterior edge and corners, fourth and fifth plates overlapped on sides by lateral edges of corresponding dorsal plates, sixth and seventh plates overlapping the corresponding dorsal plates, but sixth overlapped basally by fifth, and seventh by sixth, as is to a less extent fifth by fourth. Seven segments visible on sides and below, the first shortened, the sixth and seventh retracted with only their narrow posterior edges showing, the sixth being retracted within fifth and seventh within sixth. The seventh segment does not show at all dorsally, though the sixth shows equally widely dorsally and ventrally, and sixth and seventh show equally widely ventrally.

Galactomyia lanipes J. C. Fabricius.—As the description of *lanipes* is earlier than that of *formosa*, the species must be known by the former name. Mr. C. W. Johnson, of the Boston Society of Natural History, has a pair of this species taken in copula by Mr. P. Laurent, at Miami, Florida, March 26, 1901. This pair is mentioned in *Ent. News*, November, 1901, page 294. The capture of these specimens in copula confirms Brauer and von Bergenstamm's statement as to the sexes of this species. Both specimens have the palpi black, and the femora wholly black. The male has the abdomen obscure rufous, the female wholly black. The hind femora are conspicuously flattened-ciliate distally in the male, but only short-bristly-ciliate in the female. Apical cell closed practically in margin, tegulæ blackish. The female is the form described by Fabricius and Wiedemann as *lanipes*. The male is the form described by Wiedemann as *formosa*.

Carolina, Florida, Texas.

A small female from Costa Rica (Schild and Burgdorf) differs only in its smaller size and in having the apical cell rather more than short-petiolate. More material is needed to demonstrate its distinctness.

Galactomyia tropicalis female.—This is a large robust form, with hind femora distinctly ciliate near tip. Body wholly black. Palpi lighter colored, bases of femora reddish. Apical cell closed in margin. Male not known. Closely allied to *lanipes*. (Mexico, Costa Rica.)

Galactomyia radiata H. Loew.—Mr. C. W. Johnson has males from New Jersey, Pennsylvania, and New York. He also has a female specimen collected by him at Delaware Water Gap, New Jersey, July 10, 1898, which is doubtless the female of this species. It has the palpi yellow and bases of femora yellow. The hind femora are short-bristly-ciliate distally. The abdomen is reddish yellow, except anal segment, which is wholly shining black including narrow posterior border of preanal segment. A female specimen in the U. S. N. M., and others that the writer has collected in the District of Columbia, agree with this specimen in Mr. Johnson's collection and are no doubt females of *radiata*.

The writer wishes to especially thank Mr. Johnson for kindly placing his private collection at his disposal, and for many other favors.

Subfamily AMENIINÆ

Genus Amenia Robineau-Desvoidy

Amenia leonina J. C. Fabricius (det. Coquillett).—Australia. Both sexes show broad ventral plates overlapped by sides of dorsal plates.

Subfamily AMPHIBOLIINÆ

Genus Amphibolia Macquart

Amphibolia fulvipes Guérin (det. Coquillett).—Australian genus. This species shows in both sexes posterior triangular views of ventral plates where the rounded-off posterior corners of dorsal plates fail to cover them from view. The male shows a very large paired plate-like hypopygial process similar to that of *Rutilia*.

Subfamily RUTILIINÆ

Genus Rutilia Robineau-Desvoidy

Rutilia spp.—The species are all Australian. An examination of specimens of both sexes of several species in U. S. N. M. reveals the following characters: Neither sex shows any ventral plates, but the males show a paired plate-like process widened on apex, occupying the position of a ventral plate to the hypopygial segment, and the long fifth or anal segment is shortened to a mere margin on venter by reason of the hypopygial cavity being pushed strongly forward. There are five abdominal segments, the first being rudimentary and greatly shortened.

SMITHSONIAN MISCELLANEOUS COLLECTIONS

PART OF VOLUME LI

SMITHSONIAN EXPLORATION IN ALASKA IN 1907 IN SEARCH OF PLEISTOCENE FOSSIL VERTEBRATES

WITH THIRTEEN PLATES

BY

CHARLES W. GILMORE

Of the United States National Museum

No. 1807

CITY OF WASHINGTON

PUBLISHED BY THE SMITHSONIAN INSTITUTION

1908

WASHINGTON, D. C.
PRESS OF JUDD & DETWEILER, INC.
1908

SMITHSONIAN EXPLORATION IN ALASKA IN 1907 IN SEARCH OF PLEISTOCENE FOSSIL VERTEBRATES. SECOND EXPEDITION

By CHARLES W. GILMORE

TABLE OF CONTENTS

I. Introduction

Since the discovery of extinct vertebrate remains in Alaska by Otto von Kotzebue, in 1815, while on "A Voyage of Discovery into the South Sea and Beering Straits," much interest has been manifest regarding the occurrence and cause of extinction of the Pleistocene fauna of this northern country; and, although various expeditions have collected specimens and much has been written concerning them, it was not until 1904, when the first Smithsonian expedition was organized, that the subject was taken up in a systematic manner. This expedition was conducted by Mr. A. G. Maddren, whose report has now been some years before the public.[1] It was planned, at that time, to carry on the exploration for two or more consecutive seasons, but it was not until 1907 that the present writer was detailed to continue the work so well begun three years previous. The report herewith presented gives the results of this second trip, undertaken, as was the first, under a grant made by the Secretary of the Smithsonian Institution at the suggestion of Dr. George P. Merrill, Head Curator, Department of Geology, U. S. National Museum.

The writer's instructions were, in part, as follows:

"You are hereby authorized to proceed to Alaska, on or about May 22, 1907, for the purpose of exploring the regions herein described, with a view to securing remains of large extinct vertebrate animals and investigating the causes which have led to their extinction.

"While it is expected that you will exercise your best judgment in carrying out the details of your itinerary, it is suggested that on leaving the city of Washington you proceed to Seattle, securing at that point the necessary outfit, excepting provisions, and arranging for the services of a competent assistant.

"On leaving Seattle you will go by way of Skagway, Alaska, to White Horse, and thence down the Yukon River to Rampart, where the first stop will be made and the area explored, from which certain bison skulls now in the Museum collections have been obtained. You will then proceed to Fort Gibbon, exploring the territory in the direction of the Nowi River—the so-called "Bone Yard" region—and from this point either by steamer or canoe, to Hall Rapids, investigating the areas on both sides of the Yukon as far as Andreafski.

"Should the explorations so far outlined not yield results warranting your delay, it will then be advisable for you to proceed, provided the season be not too far advanced, by the most expeditious route to Kotzebue Sound, and make similar investigations in the areas drained by the Buckland River.

"Should you at any point discover material of such importance as to justify the making of immediate excavations, you are authorized to undertake such work, though bearing in mind that it may be advisable to first make a reconnaissance of the entire field, leaving the work of actual excavation until the following year. This is a matter, however, which must be left to your discretion.

"It is expected that the explorations herein authorized will probably consume not more than four months of the present year."

[1] Smithsonian Exploration in Alaska in 1904 in Search of Mammoth and Other Fossil Remains. Smithsonian Misc. Coll., vol XLIX, pp. 1–117.

II. Itinerary

In compliance with the above instructions the writer left Washington, D. C., May 22, for Seattle, Washington. At this place a canoe and the necessary camp equipment were purchased and shipped to Rampart, Alaska, where the first active field work was to be done. Some time prior to leaving Washington the services of Mr. Benno Alexander were engaged. His several seasons' experience with various scientific expeditions in the different parts of Alaska made him a very desirable companion and an efficient assistant.

The party consisted of Mr. Alexander and the writer, the plan being, as explained in the instructions, to employ such help from time to time as might be necessary.

On May 30 we took passage on the steamer *Jefferson,* arriving at Skagway, Alaska, June 4. It was learned upon our arrival there that all accommodations on the first boat down the Yukon had been engaged and that it would be best to remain in Skagway until the next boat, which was scheduled to sail from White Horse June 12. On June 10 we left Skagway over the White Pass and Yukon Railroad for White Horse, Northwest Territory, Canada, the terminus of the railway and head of steamboat navigation on the Yukon River. Here passage was secured on the river boat *White Horse,* which sailed June 12 and arrived in Dawson, Yukon Territory, Canada, June 14. This being a transfer point between the upper and lower river boats, we were again delayed because of inadequate accommodations, and it was not until June 22 that we left Dawson on the steamer *Sarah* for Rampart.

The delay at Dawson was profitably spent, however, in examining fossils in the possession of citizens of that place; in making inquiries concerning the occurrence of the fossils found in the Klondike region, and in visiting some of the localities on Bonanza Creek from which many of the specimens examined had been obtained. Scattered remains of Pleistocene mammals are commonly found in the diggings of this region, but the result of diligent inquiry regarding the finding of complete or partial skeletons in the mining operations conducted here were not encouraging. In only one instance were we told of the finding of an accumulation of bones such as would lead one to believe that an entire skeleton or any considerable part of the skeleton of a single individual had ever been found. The single case mentioned was that of the remains of a mammoth (*Elephas primigenius*) disinterred while sinking a shaft on Quartz Creek in March, 1904. The skull and tusks were recovered intact (see pl. VII), but, according to our informant, although surrounded by a mass of other bones, no attempt had been made to preserve them.

We arrived at Fort Yukon, Alaska, the farthest point north in our journey, at midnight June 23, and Rampart (see pl. 1, fig. 1), the limit of steamer travel, was reached the evening of June 24. While here, the area drained by Little Minook Creek, Junior, where scattered mammal remains had been found, was visited. We were shown a few specimens taken out by miners, but the character of their occurrence here did not justify a continued search; so, after overhauling our camp outfit and laying in a supply of provisions, we loaded our canoe, and on the evening of June 28, left Rampart (see pl. 1, fig. 2) for our trip down the Yukon.[1] For thirty or forty miles below Rampart the Yukon flows between walls of the older rocks with a current of from five to six miles an hour, accelerating somewhat as the rapids are reached, near the lower end of what is known as the Lower Ramparts. The first alluvial deposits encountered of any considerable thickness after passing the rapids were on the right-hand bank some twelve miles above the mouth of the Tanana River. Imbedded in these were myriads of small land shells representing the living forms, *Euconulus trochiformis* Mtg. and *Succinea grosvenori* Lea, as determined by Dr. W. H. Dall. No vertebrate remains were found.

Fort Gibbon, a military post at the junction of the Tanana and Yukon rivers, was reached the evening of June 30. Here inquiry was made regarding localities on the lower river points and particularly relating to the Palisades, better known locally as the "bone yard,"[2] some thirty-five miles below. We were informed that scattered fossil remains were also to be found along the Tanana River and its tributaries; but, as the information was somewhat indefinite as to exact localities, it was decided not to investigate the reports at this time.

The first exposures of the elevated Yukon[3] silts were observed twenty miles below Fort Gibbon, where the bluffs are undermined by the river for a half mile or more, and although a careful examination was made for the presence of vertebrate fossils, none were found either in the face of the cliff or in the talus at its base. This point marks the beginning of the escarpment of which the Palisades, some

---

[1] The day we left Rampart a small tusk of the mammoth was brought in by some miners from Ray River, a locality from which Pleistocene mammals had not been previously reported.

[2] So named because of the great number of fossil bones found here.

[3] Spurr, J. E.: 18th Ann. Rept. U. S. Geological Survey, pt. III, p. 200.

Fig. 1.--RAMPART, ON YUKON RIVER, WHERE THE CANOE TRIP COMMENCED

Fig. 2.--EXPEDITION ABOUT TO LEAVE RAMPART

fifteen miles farther down, are a part. Covered with a dense vegetation, this level-topped bluff or "plateau terrace," as called by Russell,[1] extends along the left side of the river, only separated from it by a heavily timbered flood-plain at its base. The Palisades were reached July 3, and two days were spent in the studying of this historic locality. Some scattering fossil remains were found, of which a more detailed account will be given later.

The evening of July 5 camp was pitched some five miles below the Palisades, at the mouth of "Wasikakat" River, a small tributary flowing into the Yukon from the south. This stream, which enters the river through a low alluvial flat, was ascended some distance in the expectation of reaching a place where it had dissected the higher silts of the Palisade escarpment, but we were obliged to turn back because of its small size and the consequent difficulty in navigating it.

The mouth of the Nowitna River[2] was reached June 7. Inquiry concerning the occurrence of bones along this stream elicited the information from an intelligent Indian, who visited the headwaters of this stream occasionally on hunting excursions, that he had seen "big horns and other big bones" on the river bars, and a white trapper also told us of having picked up the "shank bone" of some large animal along the stream.

The information was stimulating, for it had been planned before leaving Washington that this stream should constitute one of the principal areas of search. Before leaving Fort Gibbon, three weeks provisions had been purchased in the expectation of the supply being sufficient for us to reach the headwaters of this stream, the length of which, as given by Dall,[3] is one hundred miles. We ascended the stream for nine days, and at the farthest point reached, estimated to be at least one hundred and seventy to one hundred and eighty miles from the Yukon, found it to be a considerable stream still (see pl. VI, fig. 1). It may be explained, however, that in a straight line the distance covered might not be half of this estimate. Trappers who have ascended its entire course estimate its total length as being two hundred and seventy-five to three hundred miles. The Nowitna enters the Yukon from the southwest, about seventy-five miles below the mouth of the Tanana. It rises on the eastern flank of the Kaiyuh Mountains, and we were told its headwaters are con-

---

[1] Russell, I. C.: Geological Society of America, vol. 1, 1890, p. 146.

[2] By a recent decision of the United States Geographic Board, this stream, which has been successively designated *Nowekaket, Nowikakat,* and *Nowi,* now becomes the *Nowitna.*

[3] Dall, W. H.: Alaska and Its Resources, 1870, pp. 87–282.

nected by portages with those of the Innoko and Kuskokwim rivers. There are no settlers living on this stream, although deserted winter cabins of the lonely trapper were passed several times on our journey. The stream flows by a tortuous, meandering course through a low alluvial valley covered with a dense growth of alder, willow, poplar, birch, and spruce. Its course forms a series of curves alternately sweeping from right to left, the channel being confined between banks of unconsolidated alluvium and silt from twelve to fifteen feet in height. It presents the typical effects of meandering erosion so well described by Maddren[1] in his description of the lower reaches of the Porcupine, *i. e.*, "cutting away the banks on the concave side and depositing the material removed lower down on the opposite side as bars" (see pl. VI, fig. 1). Often the water has cut in under the bank, which extends out over the stream like a great shelf. The trees growing on these undermined banks frequently lean far over and dip their tops in the water before being carried away. Large blocks of the bank, with its superincumbent vegetation, cave off into the stream, where they remain standing half submerged for long periods. Another feature of the undermined banks is the mantle of moss that hangs down from the top like a curtain (see pl. II, fig. 2), as if to hide the destruction the waters have wrought. This blanket is composed of the tenacious and closely woven moss and rootlets which everywhere cover the ground throughout these lowlands.

The banks are not sufficiently high to prevent their overflow by the spring floods, and the quantity of drift materials lodged in the growth on top of the banks indicates the great volume of water that flows down during the spring break up. Lanes through the dense undergrowth indicate recently abandoned watercourses, many of which hold ponds and sloughs. The erosional effects of ice are also seen in the scarred and abraded tree trunks and the deep gouges and gashes along the higher banks.

The bars on the lower part of the stream are low and frequently covered with stranded trees and other drift materials, but on the upper reaches where the bends are more abrupt, they are fairly clear of drift and furnish a good path for the "trackers." On some of the upper river bars the interstratified sands and gravels have been piled in great heaps nearly as high as the inclosing banks. In ascending the stream, the first two days good progress was made with the paddle against the clear but sluggish current, but on the third day, to facilitate our movements against the rapidly increasing current, a

[1] Maddren, A. G.: Smithsonian Misc. Coll., vol. XLIX, No. 1584, 1905, pp. 9–11.

Fig. 1.--CHARACTERISTIC CUT BANKS OF THE LOW FLOOD-PLAINS DEPOSITS OF THE NOWITNA RIVER

Fig. 2.--CUT BANKS ON THE YUKAKAKAT RIVER, SHOWING THE CHARACTERISTIC CURTAIN OF MOSS AND TURF

"cache" was made of all articles in the outfit not absolutely needed. Many times, in order to get over the swift places, "tracking" was resorted to, and a little later it was nearly all tracking and wading, as we alternately crossed from bar to bar at the bends in the river. Nearly every bar searched yielded something—either fragments or one or more complete elements of skeletons representing the mammoth, horse, bison, and other extinct forms.

The first of the older series of rocks encountered was some seventy to seventy-five miles above the mouth, where the stream has cut the end of a low-lying ridge on the right bank. This outcrop is composed of a mass of badly shattered schistose rock. Some fifteen miles farther up, the river again touches the end of a spur of this same ridge, exposing rocks of a similar nature.

Elevated beds of silt of perhaps fifty feet in height were observed twice in the ascent, but appeared local in character, and no fossils were found in them.

The ridge paralleling the right bank extended along the river to the most distant point reached by us and as far beyond as the eye could reach. It rises above the level of the stream from three hundred to five hundred feet, and is covered with a dense growth of trees.

The "Suletna,"[1] the first important tributary, enters the Nowitna from the west ninety miles above its mouth.

The ascent of the stream was continued until July 16, when an inventory of the remaining supplies showed only enough provisions to last until we should reach the Yukon again. On this account we were obliged to turn back. While the specimens found at the farthest point reached were not more abundant or better preserved than those collected farther downstream, it was hoped we could reach the very headwaters, to learn, if possible, the source of all the scattered bones found along its course, and it was with reluctance that we abandoned the search.

The Yukon was reached on the 19th of July, and "Mouse Point," a small trading post, the same day. After a short stop here our journey was continued to Kokrines, an Indian settlement where the Northern Commercial Company maintains a trading post. Some little time was spent here in overhauling our outfit, laying in supplies, packing fossils for shipment, etc.

An exposure of elevated silts on the right bank of the Yukon, some three miles above Melozi, a United States telegraph station, was

[1] The name by which this tributary is known to the Indians and trappers of this region.

the next area visited. Here members of the Tenth United States Infantry had unearthed the almost complete jaws of a mammoth shown the writer while at Fort Gibbon. On our visit, however, nothing was found.

We had been told by Indians, who are in a position to be best informed concerning these out-of-the-way places, that large bones were to be found on the Yukakakat River,[1] a tributary entering the Yukon some seventeen miles above the settlement of Louden.

The mouth of the Yukakakat was reached on July 23. The exploration of the stream occupied the best part of a week, but was without especial incident. The farthest point reached was estimated to be ninety miles from the mouth, and although the current on the upper reaches was swift, it was free from serious rapids and usually had along its shores bars sufficiently broad to give good "tracking."

The sluggish current of the first few miles of its meandering course flows through a low alluvial flat, heavily wooded and very similar in character to that part of the Nowitna. Farther up, however, the course of the stream is flanked by low ranges of hills which gradually converge and thus confine its wanderings to a shorter radius. On either side of the stream back of the low hills mountains were observed rising from one to two thousand feet in elevation.

In many places the growth on the banks was very sparse, and consisted principally of scattering clumps of alders, willow, and birch interspersed with a few stunted spruce trees. Here and there back from these low banks were many shallow lakes that furnish splendid breeding grounds for the geese and ducks which abound there On the uppermost part of the stream reached by us the shores were more heavily timbered and there were long straight stretches of river flowing between banks from ten to twelve feet in height, which in most cases were covered with undergrowth and a tall luxuriant growth of grass extending nearly down to the water's edge. At the bends the undermining of the concave side presented features similar to those observed on the Nowitna River.

The first elevated silts of any importance observed were some sixty miles upstream from the Yukon, where the river makes a right-angled bend. At a comparatively recent date the river at this point has changed its course, and at the time of our visit was not cutting the bluffs (see pl. III). It could undermine them only at an extremely high stage of water. These cliffs have almost perpendicular faces and are from eighty to one hundred feet in height, com-

[1] This stream appears to be known in Alaska as the *Yukakakat,* although Dall has indicated it on a map compiled by him in 1875 as the *Soonkakat.*

EXPOSURE OF ELEVATED SILTS ON THE YUKAKAKAT RIVER, 60 MILES FROM ITS MOUTH

posed mostly of fine light-colored, unstratified silts. Some sixty feet down from the top is a layer of coarse gravel conformable with the silt, which may represent the Palisade conglomerate of Spurr.[1]

This terrace at irregular intervals has been dissected somewhat by the drainage from above (see pl. III). In many places along the cut banks of the stream the silt was underlaid by a stratum of rather fine reddish-colored gravel. A section of these flood-plain deposits, when no complications occur, presents the following divisions in their natural order and approximate thickness:

Layer of peat	18 inches to 2 feet
Layer of fine silt	8 feet to 10 feet
Fine reddish gravel	4 feet to —

Dall[2] noted the occurrence of a similar fine reddish gravel in the deposits of Eschscholtz Bay.

A few scattered bones were collected on the bars below the deposits of elevated silts just described, but although continued search was made for two days upstream from this point, no fossils were found. Even though no indication of vertebrate remains were seen in the silts, the writer is inclined to the opinion that the few fragmentary specimens picked up on the bars below may have been washed out of these bluffs and carried downstream by the river during a flood stage. This idea is strengthened somewhat from the fact that no mammal remains were found in the lower cut banks or alluvial deposits of either this stream or the Nowitna, although persistent and continued search was made, and from our own experience and that of others we do know they occur in the elevated lacustral phase of the silts.

The absence of fossil evidence on the last two days of our ascent and the fact that little had been found previously showed that this stream did not cut an extensive deposit of Pleistocene mammal remains, and it appeared to be a waste of time to continue our search; so we returned by the same route we had ascended, reaching the Yukon on July 30.

A short distance above Louden we met Mr. R. A. Motschman, who, being thoroughly familiar with the region, told us of several localities where fossils had been found. The most important of these was an exposure on the Klalishkakat River, a locality visited by Mr. Arthur J. Collier, of the U. S. Geological Survey, some five years previous. At the time of Mr. Collier's visit a large tusk was

[1] Spurr, J. E.: 18th Ann. Rept. U. S. Geological Survey, 1896–97, p. 199.

[2] 17th Ann. Rept. U. S. Geological Survey, pt. 1, 1895–96, p. 852.

protruding from the bank, a picture of which is shown on plate II, fig. 2, in Mr. Maddren's account of his trip in 1904.

This small stream enters a branch of the Yukon from the south three miles below the settlement of Louden. At the time of our visit there was a high stage of water, and it was with some difficulty that we made the comparatively short distance upstream to the point where the river cuts the elevated silts. That portion of the bluff where Collier had photographed the tusk in place had been undermined and washed away. Scattered fragments of fossil ivory found by us on the bars below probably tell the story of its disappearance. A few fragmentary bones were found, some imbedded in the undisturbed silt and others in the talus at its base.

Eight miles below Louden, on the right bank of the Yukon, occurs a typical exposure of the Yukon silts. The bluffs extend for a distance of perhaps two miles and present faces from two hundred to two hundred fifty feet in height, equal to those of the Palisade escarpment, which they resemble in all their stratigraphic detail. Mr. Motschman told us of finding fossils here, but not even a fragment was secured at the time of our visit.

Here, it was observed that the wind is quite a factor in the erosion of these bluffs. The fine silt dries rapidly, and as it commences to sift down the precipitous face it is caught by the currents of air and carried away. From a distance this silt-laden air, as it poured up over the crest of the bluff, reminded one of an ever-ascending volume of smoke. In places large drifts had accumulated like so much wind-drifted snow.

Nulato, an important Indian village, was reached on August 2, and Kaltag on August 5. Here the Government telegraph line that extends down the river leaves the bank of the Yukon, ascends Kaltag River to near its head, crosses the divide to Unalaklik River, and descends that stream to Norton Sound, a total distance of one hundred miles.

Inquiry here concerning localities on the Kaltag River failed to elicit information of enough importance to warrant investigation; so canoe travel was resumed to Anvik, some two hundred miles below Nulato. Many stops were made to examine silt deposits, but in only two places were fossils found. Some five or six miles above Hall's Rapids, on the right bank, bones of the mammoth and bison were collected at the foot of the silt bluffs, and again above the old station of Greyling, some twenty-five miles above Anvik, where the silts are exposed for two or three miles by the cutting of the river. Here, during the summer of 1907, a fine pair of lower jaws of *Elephas* were picked up by Mr. W. C. Chase, of Anvik, and presented by him through the writer to the Smithsonian Institution. The Rev. J. W. Chapman, of the same place, also had specimens in his possession from this locality.

It was planned before reaching Anvik to explore the area drained by the Anvik River, as some years previous, while visiting this place, Mr. A. H. Brooks, of the U. S. Geological Survey, had been shown fossils by the Indians said to have been collected along the banks of this stream. Inquiry here among both the white and native inhabitants, many of whom are thoroughly familiar with the river and the country drained by it, developed the fact that, so far as they knew, no fossils had ever been found in the region. Nevertheless, we ascended this stream some distance to fully satisfy ourselves as to the conditions prevailing there, but nothing in the nature of a fossil vertebrate was found. It appears quite probable that the specimens shown Mr. Brooks came from the deposits near Greyling.

Upon our return to Anvik we were delayed some few days by continued rains from resuming our journey down the Yukon. At Holy Cross, a Catholic mission, fifty miles below Anvik, we were told of the occurrence of large bones in the banks of one of the sloughs leading to the portage to the Kuskokwim River. Difficulty in securing the services of a competent guide deterred us from making an investigation of this locality, which was some distance off from the Yukon.

Russian Mission was reached August 25, and Andreafski, where our canoe trip ended, on August 29. The almost incessant rains, accompanied by winds, during the last ten days of canoe travel were the most annoying feature of the whole trip. On several occasions it became necessary to go ashore and wait for the wind to abate, for fear of being swamped by the high waves encountered.

In the two months spent upon the Yukon and its tributaries, after leaving Rampart, we traveled by canoe alone nearly fourteen hundred miles.

At Andreafski passage was secured on the river boat *D. R. Campbell*, for St. Michael, which was reached September 1.

Here it was learned fossils were occasionally found on the mainland shore across the bay, and this area was investigated, but no success was met with.

Nome was reached by the local steamer *Yale* on September 7.

The autumn season being too far advanced to undertake an exploration of the Eschscholtz Bay and Buckland River localities, we took passage on the ocean steamer *Northwestern* from Nome September 20, and Seattle, Washington, was reached on September 29.

We were not successful in finding that which was most desired. a fairly complete skeleton of a mammoth, but the expedition was by no means barren of results, as will be noted later.

III. Occurrence of Fossils

The scattered remains of Pleistocene animals occur throughout the unglaciated region of Alaska and adjacent Canadian territory in three quite distinct deposits: First, in the black muck accumulated in gulches and the valleys of the smaller streams; second, in the fine elevated clays of the *Yukon silts* and *Kowak clays;* and, third, in the more recent fluvial and alluvial deposits. The specimens as found have been disinterred either through the erosive agency of the streams or by the work of the miner in the operations conducted in search of gold.

Although so generally distributed, there have been reported, so far as known to the writer, but two well-authenticated occurrences of accumulations of bones under such conditions as to suggest an original entombment. While the writer was shown bones protruding from the face of the undisturbed beds in the Klondike region (see pl. IV, fig. 1), and in other instances collected specimens actually imbedded in the elevated silts along the Yukon River, they were in all cases disarticulated and scattered, and there was no evidence of an association of any of the parts found.

Diligent inquiry was made among miners, trappers, and other residents of Alaska, met along the route traveled, concerning what they knew of the occurrence of fossil specimens. While nearly all were familiar with the fragmental and scattered parts, very little information was elicited of an accumulation of bones that would lead one to believe a skeleton or even a part of a skeleton had ever been found together in any one place.

While the scattered depositions occur as separate bones, skulls, teeth, tusks, horns, etc., throughout the formations mentioned, the condition of the specimens found varies greatly. Some are in such a good state of preservation they certainly could not have traveled far from the original place of interment, while on the other hand many bones are broken, abraded, and water-worn, and show unmistakable evidence of having been carried considerable distances. Bones representing these several phases were often found commingled and occupying relatively the same positions, whether it be in the muck, on a river bar, or imbedded in the undisturbed silt deposits.

Fig. 1.--TUSK OF ELEPHAS PROTRUDING FROM THE FACE OF THE UNDISTURBED MUCK IN FOX GULCH, BONANZA CREEK

Fig. 2.--SKULLS OF ELEPHAS PRIMIGENIUS AND BISON FROM THE MUCK OF FOX GULCH, BONANZA CREEK, NEAR DAWSON, YUKON TERRITORY, CANADA

The best-preserved specimens coming under the observation of the writer were those seen at Fox Gulch, on Bonanza Creek, in Yukon Territory, Canada, some twelve miles distant from the city of Dawson. On account of the excellent state of preservation of many of the specimens found here and the fact that they occur in what may be considered as an approach to a primary deposition, a somewhat detailed description of this locality will be given.

Bonanza Creek Localities

Bonanza Creek empties into the Klondike River about a mile and a quarter above Dawson. The valley is trough-like in character and follows a sinuous line bending from right to left. The present valley, according to McConnell,[1] has been cut down through the floor of an older valley. At irregular intervals the sides of the valley have been dissected by gulches. Magnet and Fox Gulches (see pl. IX (X)), on the left-hand side, are the most important from the standpoint of vertebrate fossils. Gold has been found in both, and at the time of our visit hydraulic operations were being carried on here by the Yukon Consolidated Gold Fields Company. In the prosecution of this work the content of the entire gulch to bed-rock was being sluiced down (see pl. VI, fig. 2), the talus spreading out fan-like into the creek bed below.

In the talus from Magnet Gulch representative parts of the mammoth, horse, bison, and moose were picked up.

At Fox Gulch we were shown many fine skulls and other skeletal parts of *Elephas, Bison, Equus,* and *Alce.* On the bank near the working face was a complete skull of the mammoth beside two bison skulls, recently uncovered (see pl. IV, fig. 2), and protruding from the face of the undisturbed muck was a large tusk (see pl. IV, fig. 1) and the skull of another bison. These had been uncovered the morning of our visit.

Fox Gulch is a short, deep gulch that has been cut down through the quartz drift and "White Channel" deposits and deep into the present bed-rock. The bed-rock is covered with a thin layer of rather coarse gravel on top of which is a thick layer of muck (see fig. 1). The gold occurs in the gravel underlying this muck, and in order to reach it the mass of superincumbent material is washed down by the powerful streams of water from the nozzles of the "jacks."

[1] McConnell, R. G.: Preliminary Report on the Klondike Gold Fields, Yukon District, Canada. Geol. Surv. Canada, No. 687, 1900, p. 21.

The working face as we saw it varied from twenty to forty feet in height. It is in the bottom part of the muck that the fossils are found. Those seen in place were from twelve to eighteen inches above the layer of gravel, and upon inquiry it was learned that all of the specimens taken out here had come from approximately the same horizon.

The muck and gravel, which rest unconformably upon the underlying rocks, is solidly frozen, but thaws rapidly under the heat of the summer sun, and large pieces were continually dropping during our examination of the face. This thawed material emitted the disagreeable odor of decomposing organic matter, a phenomenon observed by many others, particularly Dall,[1] who attributed it to

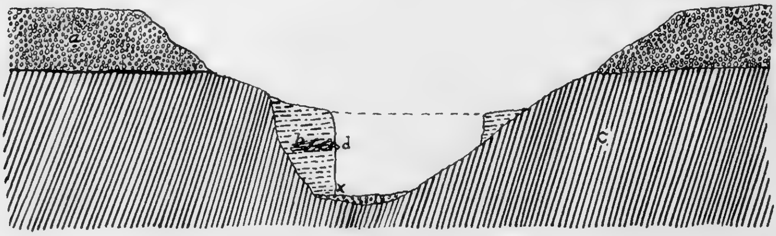


Fig. 1.—Cross-section of Fox Gulch, Bonanza Creek, Yukon Territory, Canada. *a.* "White channel" gravels and quartz drift; *b.* Muck; *c.* Bed rock; *d.* Layer of logs, limbs, etc.; *x.* Level where fossils occur.

decaying animal flesh or to dung of the mammoth or other herbivorous animals. The present writer agrees with Mr. Maddren,[2] who attributes it to the gases from decaying vegetable matter, of which the deposits are largely composed.

Interbedded with the muck in Fox Gulch was a layer of wood, represented by many fair-sized sticks (see (*d*), fig. 1), their ends in many instances being much rounded and water-worn.

Many of the fossils found here were beautifully preserved. For example, several of the bison skulls had the external horn, the entire dentition, and the frail, delicate bones of the anterior portion of the face remaining intact. The conditions are unusual, for, as a rule, only the horn cores and the heavier and stronger parts are found, and it is upon such fragmentary specimens that the descriptions of most of the extinct species of bison of this continent are based. Stranger still, however, is the fact that here no parts of these animals are found articulated or even so associated that skeletons might be assembled. All of the material is dismembered and scattered.

[1] Dall, W. H.: 17th Ann. Rept. U. S. Geol. Surv., pt. 1, 1895–96, p. 853.
[2] Maddren, A. G.: Loc. cit., pp. 64–65.

The preservation of the horn sheaths, as in the cases of the bison skulls, and the completeness of many of the skulls and other elements show they have not been subjected to the rough usage incident to their removal from one place to another; nor after death could they have long lain on top of the ground exposed to the vicissitudes of the elements. The external horn would in such case be the first to disappear, as all know who have visited our western plains and have noted the almost total disappearance of the horn sheaths from the buffalo skulls scattered about. Their destruction, even in a dry climate, has been accomplished in a comparatively few years.

Little Minook Creek Junior

This small stream enters Big Minook Creek from the right some six miles distant from the town of Rampart. Here, as in the Klondike region, the fossils occur in the lower part of the muck, which covers everything from two to twenty-five feet in depth. Specimens would be uncovered here only through the agency of mining, as the volume of water in the creek is not sufficient to cut away the banks.

While sinking a shaft on claim No. 21, operated by Messrs. Bowen and Coole, a skull of *Bison crassicornis* (No. 5727, U. S. National Museum) associated with bones of *Elephas* was taken out twenty feet below the surface.

Some years previous to our visit, we were told, the tusks of a large "*mastodon*" (*mammoth*) were found in a shaft sunk on a claim above No. 21.

Little Minook Creek

This creek is also a tributary of Big Minook Creek, and here, as in other localities, the fossils found occur in the lower layers of the muck.

In the vertebrate fossil collection of the U. S. National Museum is a portion of the skull of *Bison alleni* (No. 2383, see plate XI) from this locality having the entire horn sheaths preserved.

Mr. J. B. Duncan, of Rampart, presented to the Smithsonian Institution, through the writer, a skull of *Bison crassicornis* (No. 5726, U. S. National Museum, see plate X) from one of the claims on this creek, and Mr. C. B. Allen, of the same place, presented the Institution with the calcaneum of *Elephas* from claim No. 1.

Palisades

The Palisades, or "Bone Yard," on the left bank of the Yukon, thirty-five miles below Fort Gibbon (see pl. V, fig. 1), has long been famous as a locality for vertebrate remains. This escarpment has been described by Russell,[1] Spurr,[2] and later by Collier[3] and Maddren.[4]

The bluff region extends for a mile or more down and around an almost right-angled bend in the left-hand channel of the river (see fig. 2). The bluffs, from one hundred and fifty to two hundred feet in height, and solidly frozen, are composed principally of an extremely fine silt, greenish gray in color and showing no traces of stratification. Their almost perpendicular faces are being continually undermined (see pl. v, fig. 2) by the swift current causing large masses to break off, many times with a startling report and subsequent splash, as they fall into the water below. Often during the two days' stay here the report sounded so like the firing of a gun that we were startled by the sharpness of it.

Near the lower end of these exposures the bluffs have been elevated somewhat, exposing the gravels which underlie them. These last have been called by Spurr the Palisades conglomerate, and it has been suggested they may be of Pliocene age. The top of the bluffs extend back from the river as a level, densely wooded tableland. In several places small watercourses have dissected this table, forming deep gorges. Near their mouths, where they enter the Yukon, their levels are but little elevated above its high-water stage.

At the up-river end of the bluffs we found numerous bones of the mammoth in the débris from a recent slide, and a short distance farther down (2 on map fig. 2) the scattered elements of a bison were found securely imbedded in a huge block of silt not long since displaced from its original position higher in the face of the cliff. The sacrum, part of the pelvis, two dorsals, rib, and scapula were the parts recovered. The scapula (shoulder-blade) was quite complete (see fig. 4), which, on account of its frail nature, appeared rather remarkable, the heavier and stronger bones being broken and abraded before their interment here.

The small streams mentioned previously as dissecting the bluffs were followed inland for considerable distances, and although their banks in many places presented very clean-cut exposures of the silt, no evidence of the presence of fossil remains was found. However,

---

[1] Russell, I. C.: Notes on Surface Geology of Alaska. Bull. Geol. Soc. Am., vol. 1, 1890, p. 122.

[2] Spurr, J. E.: Geology of the Yukon Gold District. 18th Ann. Rept. U. S. Geol. Surv., pt. 3, 1898, pp. 200–221.

[3] Collier, A. J.: Bull. No. 218, U. S. Geol. Surv., 1903, pp. 18 and 43.

[4] Maddren, A. G.: Loc. cit., pp. 17–18.

Fig. 1.--PALISADES ON THE YUKON RIVER, 35 MILES BELOW FORT GIBBON, VIEWED FROM A DISTANCE UP THE RIVER

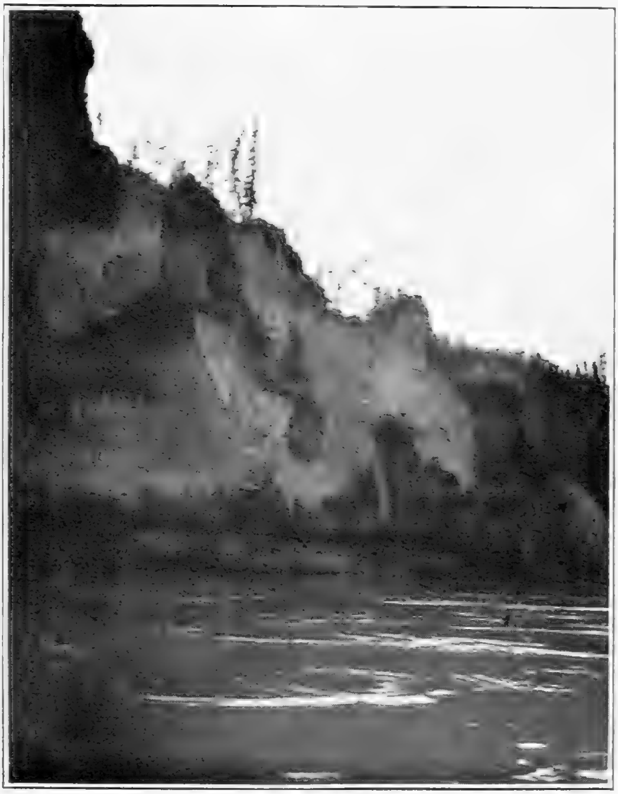


Fig. 2.--ELEVATED SILT BLUFFS AT THE PALISADES, SHOWING BLOCKS OF FROZEN SILTS AS THEY ARE UNDERMINED AND SUBSIDE INTO THE RIVER

among the débris of driftwood and other vegetable material accumulated at their mouths many disassociated bones were recovered (see (+), fig. 2). The concentrating action of the water in carrying away the fine silt and leaving the heavier objects behind would account for their abundance here.

In drifting down along the base of the cliffs in the canoe, the skull of an *Ovibos* sp. nov. (No. 5728, U. S. National Museum) was found on a narrow shelf just above the point (3 on map, see fig. 2) where the underlying gravels first appear. That the skull came

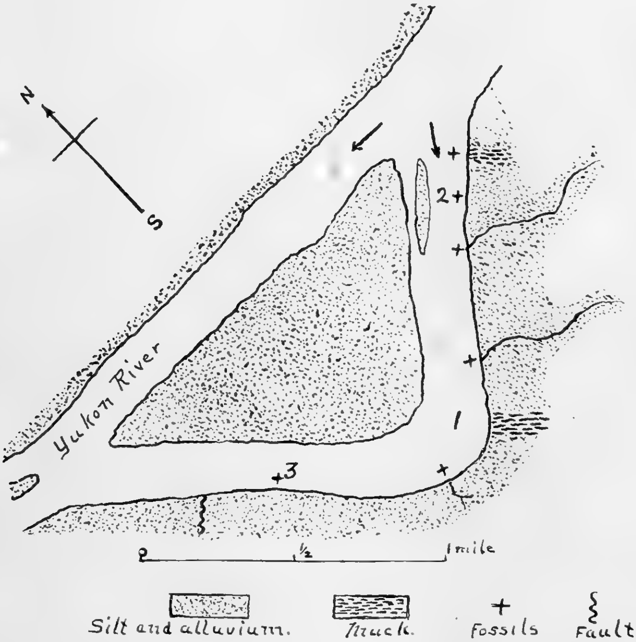


Fig. 2.—Sketch Map in Vicinity of "Palisades."
1. Where section was taken, shown in Fig. 3; 2. Bison bones (Fig. 5); 3. Musk ox skull, No. 5728, U. S. National Museum.

down from the cliff above there can be no doubt, for it lay on a pile of talus accumulated since the last high stage of water. The high-water marks were still plainly evident on either side and above the heaps of detritus. Moreover, the cranial and other cavities of the skull were filled with the fine silt composing the bluff. This skull was in fairly good condition, having, beside two of the molars, some of the bones of the anterior part of the face in a good state of preservation. The worn and abraded appearance of most of the fossils here indicates that they are drift and not in a place of primary entombment.

Maddren[1] makes the observation, "There is a little ice on top of these bluffs, but nothing like the extensive development exposed in the Old Crow Basin." Ice was also observed here by the writer, which on first sight appeared to represent the typical ice-bed deposits of many other localities. Upon closer examination, however, it was found to be a superficial layer on the face of the exposure and not a continuous ice-sheet interstratified with the muck and humus. The formation of this layer of superficial ice appears to be of interest and it may explain the presence of apparent ice-beds in some other places. Moreover, it does show that caution should be exercised in pronouncing all ice on the face of a cliff as being a section of a continuous bed.

At (1) on the map (fig. 2), a deep depression or basin in the top of the silt has been filled with alluvium and mucky material. The brow of the escarpment here, three or four hundred feet back from the edge of the stream, was estimated to be one hundred and fifty to one hundred and seventy-five feet above the level of the river. The Yukon, having cut laterally into the center of this basin, has left the remaining muck resting on a slope of silt inclined toward the river (see cross-section, fig. 3). By the undermining of the face of the cliff, one block after another of this frozen muck has broken away from its original position in the face of the escarpment and moved riverward. In most instances this movement has been so gradual that the blocks retain their upright positions and carry with them the superincumbent turf and vegetation undisturbed. The thawing of their faces and subsequent wasting away has allowed the turf to bend down without breaking, thus affording protection against further disintegration. The final destruction of the blocks, as they eventually fall into the stream (where several were seen half submerged), has resulted in leaving a basin-like area of an acre or more in extent devoid of its former covering of from thirty to forty feet of muck, except that here and there are masses recently detached from the walls of the basin. The inclosing walls or faces of the basin are perpendicular and from twenty to thirty feet in height. From three to four feet below the top of the walls was a layer of ice. Upon first sight it had every indication of being a section of a continuous bed. Some of the detached blocks standing in the center of the basin showed ice on both front and back faces. The top of this ice was straight, but the lower margins were irregular when not covered by the detritus at the foot. The face of the ice was also irregularly melted, due to the more exposed position of some parts.

[1]Maddren: Loc. cit., p. 18.

Upon ascending to the top of the escarpment at the point most remote from the river, it was found that a mass of frozen muck, estimated to be two hundred feet long and fifteen to twenty feet in thickness, with a vertical face of twenty to thirty feet, had moved outward at its center for fully fifty feet, but had not yet become detached at its ends. The crevasse formed by this displacement was filled by water to such a depth that the bottom could not be found with a long pole. Back of the crevasse, in the surface of the bluff, were numerous parallel cracks varying from six to eighteen inches in width and many feet in length. These had water standing in them nearly to the top of the ground. The conditions observed here appeared to the writer to explain the presence of the ice on the

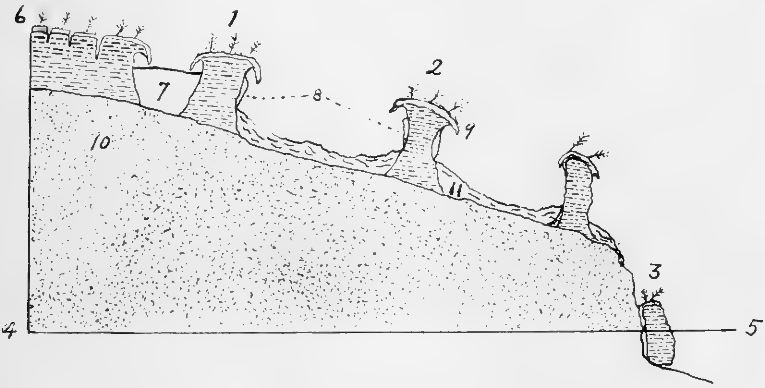


FIG. 3.—Cross-section of "Palisades" Escarpment, showing Formation of Superficial Ice.

1–2–3. Blocks of frozen silt; 4–5. Water level of the Yukon; 4–6. 150–170 feet; 7. Crevasse filled with water; 8. Ice on faces; 9. Overhanging turf; 10. Lacustrian silts; 11. Detritus (thawed muck).

faces below. With the advent of winter, assisted by the already frozen ground, the water in the crevasses becomes frozen solid. A subsequent outward movement of the blocks would leave the ice clinging to the face of either the cliff, or the block, or both, and under the influence of the rays of the summer sun would rapidly smooth the broken and ragged edges. On the faces of blocks 1 and 2 (see fig. 3) such layers of ice were observed, and where protected by the wet mantle of overhanging turf and moss were thawing very slowly. In places the ice was so thin the writer with a few strokes of his pick was able to penetrate it and into the frozen muck wall behind. Sections of the ice, protected by curtains of turf and falling débris, would persist for considerable periods. In places it had melted away, leaving its mould in the face of the cliff.

Under the influence of the summer sun the blocks were gradually disintegrating. Large pieces were continually falling as thawing progressed, and all along the bases of the face and around the blocks were small piles of talus of the mucky material.

The same pungent, disagreeable odor of decaying organic matter was noticed here as in the deposits of Bonanza and Minook creeks. The stench was so strong it could be easily detected on the river a considerable distance away. In many places on the wet muck banks a rusty red fungus-like plant grew in extensive patches.

The writer does not wish to be understood that the observations recorded here apply to all ice deposits, but as a local phase it may explain the occurrence of many so-called "ice-beds." It may also help to explain the position of the mammoth found frozen in the cliff along the Berezovka River in Siberia in 1901. From the position in which the carcass was found it would appear as though he had fallen into a crevasse from which there was no escape. The description[1] of the locality is not so unlike the conditions observed here.

Nowitna River

The exploration of this stream added but little information concerning the occurrence or derivation of the fossils found along its course.

After the first day of our ascent of this stream nearly every bar yielded some fossil evidence, either in the shape of a tooth, limb bone, vertebra, or scattered fragments. The specimens found were in various stages of preservation; many broken, others entire, some badly water-worn, and a few as perfect as the day they performed their functions in the skeleton itself. Some elements, which on account of their frail nature should by the very character of their structure have been broken and abraded, were found complete.

In examining the bars we soon came to know that the up-river ends, where the materials composing them were coarsest, was the most favorable part for finding the scattered bones. The remains without exception were all found below the high-water level of the flood stages of the river, and were without question brought down from some source or sources of deposition, either by the water itself or by floating ice.

A close examination was made of the low-cut banks and elevated silts, but not in a single instance were fossils actually found in place.

The conditions on this stream differed somewhat from those found

---

[1] Herz, O. F.: Frozen Mammoth in Siberia. Ann. Rept. Smithsonian Inst., 1903, pp. 611–625.

Fig. 1.--TYPICAL SANDBAR ON THE NOWITNA RIVER, 180 MILES FROM ITS MOUTH

Fig.2 .--LOOKING UP FOX GULCH, BONANZA CREEK, NEAR DAWSON, CANADA

Sluice box in the center, and the muck filling of the gulch not yet sluiced out may be seen in the background on the right

by Mr. Maddren on the Porcupine and Old Crow rivers, from the fact the fossils did not become more abundant on the bars as we went upstream. On some bars many fossils would be found, while others would yield only a single specimen. The varying degrees of preservation exhibited by the specimens points to the conclusion that the source of supply is diverse and not one large deposit. The writer is inclined to the opinion that the fossils found on the bars have been washed out of the silt banks along the stream and transported to their present resting places largely by the action of the water.

The finding of abundant remains on the bars of a stream that is cutting elevated silts does not necessarily lead to the conclusion that all of the specimens found there have come from the headwaters of that stream, for we know that scattered bones occur in the silt deposits, and it appears that the bones brought down from far upstream may be augmented in numbers by those washed out of the silts along its course.

The following list gives the fauna of this area as represented by the scattered bones collected:

Elephas primigenius.
Bison.
Equus.
Ursus.
Alce.
Castor.

Yukakakat River

Although fewer fossils were collected along this stream, the prevailing conditions as to their occurrence were found to be similar in most respects to those observed on the Nowitna River.

The following forms were recognized:

Elephas primigenius.
Bison.

Klalishkakat River

A locality on this stream some three miles inland from the Yukon was visited. Here the bluffs present nearly perpendicular faces from sixty to eighty feet in height, the lower parts of which are composed of reddish cross-bedded gravels, varying from fine to very coarse and unconformable with the overlying silt. The silt shows no traces of stratification and is solidly frozen. Back from the bluff is a level tableland, bordered on all sides, except that adjacent to the river, by low hills. It was at this locality that Mr. Collier in 1902 photographed a tusk protruding from the face a few feet below the top of the escarpment.

The bases of the bluffs are washed by the stream, and during stages of high water are undermined, causing large masses to break off. The tusk seen by Collier five years previous had disappeared, but a recent slide had exposed the distal end of a femur of *Elephas* in place about three feet above the underlying stratum of gravel. Other broken fragments were found in the loose clay of the talus along the foot of the bluffs. The silt varies in thickness from thirty to thirty-five feet, and broken and abraded fossil remains occur, scattered throughout. The conditions here are not favorable for the securing of good specimens. Bones of *Elephas* and *Bison* were collected.

Discussion

After a review of the conditions prevailing in localities where fossils have been found in Alaska and contiguous territory, the writer feels inclined to dissent somewhat from the views expressed by Maddren regarding the most promising collecting grounds.

Mr. Maddren[1] has advanced the opinion in the following statement that the old lake shores offer the greatest inducements:

> "That the fluvio-glacial Pleistocene lakes of Alaska were subject to annual winter freezing, at least at various stages of their existence, there appears no doubt, because scattered apparently indiscriminately through the clays, at varying depths and considerable distances from the former shore lines of these basins, are some mammal remains. Their positions can only be accounted for by supposing they were carried out on the waters of the lakes from the adjacent shores or tributary streams by ice during spring breakups and freshets, there to be dropped by its melting to their present positions interbedded in the silts. There appears no other logical way of explaining the presence of these bones in the lacustrine areas." . . .
>
> "The main point is that the remains occur in the silts as scattered depositions.
>
> "The animals from which they were derived probably died about the shores of these lakes, and it is these Pleistocene lake shores we must examine carefully if we are to obtain anything like complete remains of the mammals inhabiting the region at that time."

There appears to be one objection to this hypothesis as applied to these fine-silt deposits. If the great number of isolated mammalian bones scattered through it were carried out from the shores and tributary streams by ice, it is hard to understand how they could be

[1] Maddren, A. G.: Smithsonian Misc. Coll., vol. XLIX, No. 1584, 1905, p. 26.

selected for distribution in deposits from which all other large fragments of detrital materials are absent.

It might be explained, however, on the supposition that the bones have been carried out from muck deposits in which there is no heavy detrital material. In that event many of these deposits might be considered older than are the silts; or, the presence of interbedded layers of lignite at the "Palisades" and in the silts of Cooleen basin (which would indicate a local drainage or elevation of these beds at one time) might furnish the necessary conditions for the accumulation of animal remains, followed by subsidence and further deposition.

Up to this time the best-preserved remains have been found in the deposits of muck accumulated in gulches and the valleys of the smaller streams. Typical examples of the occurrence of this muck may be seen on Little Minook Creek, near Rampart, Alaska, and Bonanza and other creeks, near Dawson, in Canadian territory. Only a single skull of bison with the horn sheaths preserved is recorded as coming from the silt, while they are of common occurrence in the muck. Their presence here may be accounted for on the supposition that the animals became mired in the bogs before they became solidly frozen as they are now. This naturally raises the question: If mired down in such a place, why is it that the remains should be so universally scattered? The writer suggests that they may have been separated by the creeping of the muck or peat—a phenomenon familiar to all students of deposits of this nature. By such creeping the muck may have moved considerable distances, particularly where the floor is inclined, as in many of the gulches. From the fact that most of the bones occur in the lower layers of the muck, no matter what the depth of the deposits may be, it is apparent that their specific gravity has caused them to sink to their present resting places. Thus it would not be necessary for the extermination of the fauna to have taken place at one time, as might be inferred by their occurrence at one level.

It was from the muck forty-two feet below the surface that the skull and tusks, surrounded by other bones of the skeleton of *Elephas primigenius* shown in plate VII, was obtained. Mr. A. H. Brooks, of the U. S. Geological Survey, tells me of seeing a portion of a skeleton of *Elephas* from Woodchopper Creek, Alaska, probably taken from a similar deposit.

The two instances just cited undoubtedly represent places of primary entombment, and the manner of their occurrence appears to approximate the conditions found in the bogs and swamps in the

Eastern States, from which many of the best skeletons of the *Mastodon* have been obtained.

From the evidence reviewed the writer believes that the deposits of muck represent the most likely places from which to secure remains of this extinct fauna.

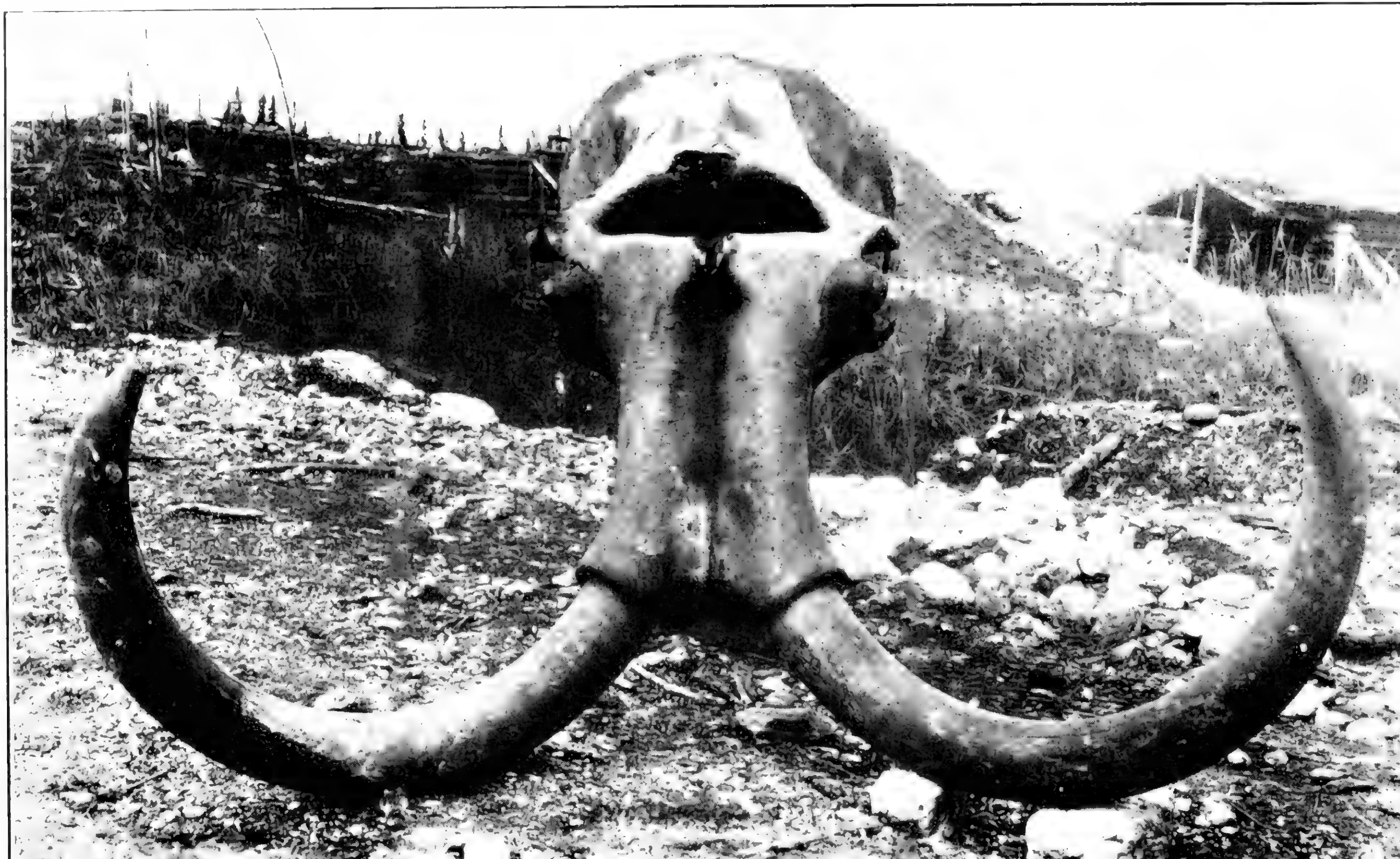
The writer takes this opportunity to express his appreciation for the assistance given him by Mr. A. H. Brooks and Mr. A. G. Maddren, of the U. S. Geological Survey. Many services were rendered by residents of Alaska along the route traveled, and favors were extended by agents and officials of the Northern Commercial Company. Mr. J. B. Duncan, of Rampart; Mr. Frank Haslund, of Kokrines, and Mr. Frederick, of Andreafski, were especially kind in many ways. My thanks are also due Mr. J. W. Gidley of the National Museum, for help in the identification of specimens.

IV. The Pleistocene Fauna of Alaska.

Although a number of species have been described from the Pleistocene deposits of Alaska, they have for the most part been based on fragmentary, and therefore rather unsatisfactory, specimens. In many cases the principal osteological and dental characters are not known, and on that account it is not always possible to compare them intelligently with related forms.

Only a few of the large number of localities where fossils have been found furnish well-defined specimens, capable of specific determination, and while these vertebrates are interesting from the standpoint of their general geographical distribution, they are of comparatively little aid in the interpretation of the local deposits. The forms have been entombed under such exceptional conditions as to raise some question regarding the exact age of the deposits in which they are many times found, although they could not have antedated Pleistocene time. A glance at the list of determinable species is sufficient to show at once that they represent a typical Pleistocene fauna, some of which, as the moose, caribou, musk-ox, sheep, bear, and beaver, have persisted down to the present day.

To aid the student, there is given here a list of the various genera and species thus far reported as occurring in Alaska, followed by a brief review of each, with a reference to the original description; the condition and present location of the type specimen (if known, and when based upon fossil remains) upon which these were founded, and in some cases figures of representative specimens from Alaska. Some additional information has been derived from a study of specimens in the vertebrate paleontological collection of the



SKULL AND TUSKS OF ELEPHAS PRIMIGENIUS FOUND 42 FEET BELOW THE SURFACE, IN THE MUCK, ON QUARTZ CREEK, NEAR DAWSON, YUKON TERRITORY, CANADA, IN MARCH, 1904

U. S. National Museum, collected in Alaska by Lieutenant Hooper, Dr. W. H. Dall, E. W. Nelson, L. M. Turner, A. G. Maddren, and others.

ELEPHAS PRIMIGENIUS Blumenbach

THE NORTHERN MAMMOTH

Elephas primigenius BLUMENBACH, Handb. Naturg., 1st French ed., vol. II, 1803, p. 407.

DESCRIPTION.—"Jaw broad and rounded; profile in front of tooth row almost vertical; enamel folds narrow and compressed; rather more than two folds to the inch, or twenty-four in ten inches; enamel itself thin."[1]

REMARKS.—This species is, geographically, the most widely distributed of extinct elephants. It has been reported as ranging from Florida, Texas, and Mexico on the south and northward into Canada and Alaska. It is also found in Great Britain and nearly all Europe and northern Asia.

Its remains are particularly abundant in parts of Alaska and Siberia. As yet no complete specimens have been found in Alaska, although several good skulls and nearly all parts of the skeleton are known from scattered but well preserved bones. Neither have specimens been found in the flesh, as is so often reported through the columns of the newspapers and even by some of the magazines.

The size of the mammoth has been so grossly overestimated by the general public that a few comparisons may help to correct some of these false impressions. The largest mounted specimen known is the skeleton in the collection of the Chicago Academy of Sciences, obtained in 1878 from Spokane County, in the State of Washington. The height of this animal when alive has been estimated to be thirteen feet. The African elephant "Jumbo" was eleven feet high, and there have been other elephants recorded as measuring twelve feet in height; so, as this would indicate, there is not so much difference in size between the mammoth and living elephants as is often supposed.

Mr. Lucas says:

"Tusks offer convenient terms of comparison, and those of a fully grown mammoth are from eight to ten feet in length, those of the famous St. Petersburg specimen and those of the huge specimen in

---

[1] Lucas, F. A.: Systematic Paleontology of the Pleistocene Deposits of Maryland. Maryland Geol. Surv., December, 1906, p. 163.

The above characters are given by Mr. F. A. Lucas as distinguishing this species from all other elephants.

Chicago measuring respectively nine feet three inches and nine feet eight inches. . . . Compared with these we have the big tusk that used to stand on Fulton Street, New York, just an inch under nine feet long and weighing one hundred eighty-four pounds."

In a footnote[1] he gives the measurement of the left tusk of an African elephant that is ten feet three and one-half inches in length along the outer curve, twenty-four and one-quarter inches in circumference, and weighing two hundred and thirty-nine pounds.

The longest tusk reported from Alaska is twelve feet ten inches in length. During the summer of 1907 the writer measured a tusk at Fort Gibbon that was ten feet seven inches long and the greatest circumference was twenty-one inches. This specimen was broken at both ends.

The tusks belonging to the skull shown in plate VII are seven feet six inches in length.

The tusks of the mammoth, as a rule, were more curved and of greater length than of the living forms, although there is a great variety of shapes and sizes.

Economic Importance of Mammoth Ivory.—It appears that the mammoth remains found in Alaska are not in as fresh a state of preservation as those found in Siberia, where for a good many years their tusks have constituted an important article of export. Dr. Middendorf, who visited Siberia about the year 1840, estimated the annual output of this fossil ivory to be one hundred and ten thousand pounds and representing at least one hundred individuals.[2] From their great abundance, Dr. R. Lydekker[3] has suggested that tusks were probably developed in both sexes.

It is seldom, if at all, that tusks are found in Alaska sufficiently well preserved to compete on the market with those of the African and Indian elephant, as is the case with the Siberian ivory; usually they are found to be discolored and either badly checked or exfoliated. A curio dealer in Nome, however, told the writer, "A few years ago a man would not take a tusk as a gift, but of late the best ones had acquired a commercial value, being cut into curios for the tourist trade."

In the "curio" stores at Skagway we were shown some of the articles manufactured for the trade from this ivory, consisting of

[1] Lucas, F. A.: Annual Report Smithsonian Institution, 1899, p. 355.

[2] This estimate appears rather low, as the average tusk would hardly weigh two hundred and fifty pounds, or five hundred pounds for the pair, which would give over two hundred individuals.

[3] Lydekker, R.: Annual Report Smithsonian Institution, 1899 (pp. 361–366), p. 362.

sawed sections polished for paper-weights, on which were etched representative scenes and animals of Alaska. The life restoration of the mammoth with its long hair and curved tusks appeared to be a favorite subject. In one instance a miniature of the mammoth had been carved from it. This carving and etching is done by the Indians and Eskimo, many of whom become quite adept at this line of work. Similar objects were observed in the curio stores at Nome. The Skagway dealers obtain most of their tusks from the Klondike region, while the Nome dealers procure the ivory used by them from the Eschscholtz Bay, Buckland, and Kobuk River localities.

In 1854 Sir John Richardson said:

"Eskimos are in the habit of employing the soundest tusks for the formation of various utensils; and the American fossil ivory has for at least a century, and for a longer period of unknown duration, been an article of traffic with the Tchutche of the opposite shores of Beering Straits; so that we can venture upon no calculation of the multitudes of mammoths which have found graves in several icy cemeteries of the American coast of Beering Sea."

Dr. W. H. Dall[1] tells of obtaining "in 1880 a deep ladle as large as a child's head, carved, handle and all, out of a solid tusk of mammoth ivory by those people," referring here to the Eskimo.

The writer also saw pieces of tusks fashioned into sled runners, having holes at intervals by which they were lashed to the wooden framework above. On the Yukon it was observed the Indians sometimes used sections of tusks as weights for sinking their salmon nets.

An account of this fossil ivory would not be complete without a mention of the blue phosphate of iron sometimes formed by the decomposition of the tusks and used by the Alaskan Eskimo as a pigment.

Sir John Richardson was the first to make note, in 1854, of this phosphate[2] (Vivianite) occurring between the plates of the exfoliated tusks. The writer saw this blue stain on many of the tusks examined by him, and it was particularly noticeable on those just recently removed from the ground. The same iron phosphate was found in the metacarpal bones of the bison collected on the Nowitna River.

---

[1] Dall, W. H.: Seventeenth Annual Report, U. S. Geol. Surv., pt. 1, p. 857.

[2] In this connection it is interesting to quote from Warren's report on *Mastodon giganteus:* "On burning the bone, the ash which remains is of a beautiful blue color, owing to the presence of phosphate of iron, which appears to have been formed from the iron which had penetrated into the bone from the marl surrounding the skeleton."

ELEPHAS COLUMBI Falconer

Elephas columbi FALCONER, H., 1857, Quart. Jour. Geol. Soc. of London, XII, p. 319.

The only reported occurrence of *E. columbi* is given by Dall,[1] who mentions that tusks, teeth, and bones of *E. primigenius* and *E. columbi* were collected by Wossnessenski near Topanika Creek, Norton Sound. We quite agree with Maddren[2] that "the identification needs verification before it is assigned to Alaska."

MAMMUT AMERICANUM (Kerr)

THE AMERICAN MASTODON

Elephas americanum, KERR, R., 1792 Anim. Kingdom, p. 116.

DESCRIPTION.—It may be readily distinguished from *Elephas primigenius* by the character of the teeth, which bear simple tent-like ridges (see plate VIII). By its low massive build and shape of the skull and the tooth characters just reviewed, it may be told apart from the mammoth by the most casual observer.

REMARKS.—This animal also has a wide distribution. Its remains have been found from New York to Florida and west to Texas and Washington. It extended north into Canada, and recently two teeth have been found in the Klondike region near Dawson. The writer refers here to a Mastodon molar secured by Dr. T. W. Tyrrell on Gold Run Creek in 1902, and through him presented to Mr. W. H. Osgood,[3] of the U. S. Biological Survey, and now in the vertebrate paleontological collection of the U. S. National Museum (see pl. VIII, fig. 1). A second occurrence of this species in this region was noted by the writer in the summer of 1907—a tooth collected during the spring of 1906 on Sulphur Creek, near Dawson (see map, plate IX), and now in the possession of Mr. Joseph Nichlas, of that city. This specimen is reproduced here from a photograph (pl. VIII, fig. 2). It is of interest to note the occurrence of the mastodon in this region and in both places associated with remains of the mammoth.

In 1904 Mr. M. T. Obalski[4] mentions the occurrence of the mas-

[1] Dall, W. H.: Seventeenth Annual Report U. S. Geological Survey, 1896, p. 856.

[2] Maddren, A. G.: Smith. Misc. Coll., vol. XLIX, No. 1584, 1905, p. 7.

[3] Osgood, W. H.: Proc. Biol. Soc. of Washington, November, 1905, vol. XVIII.

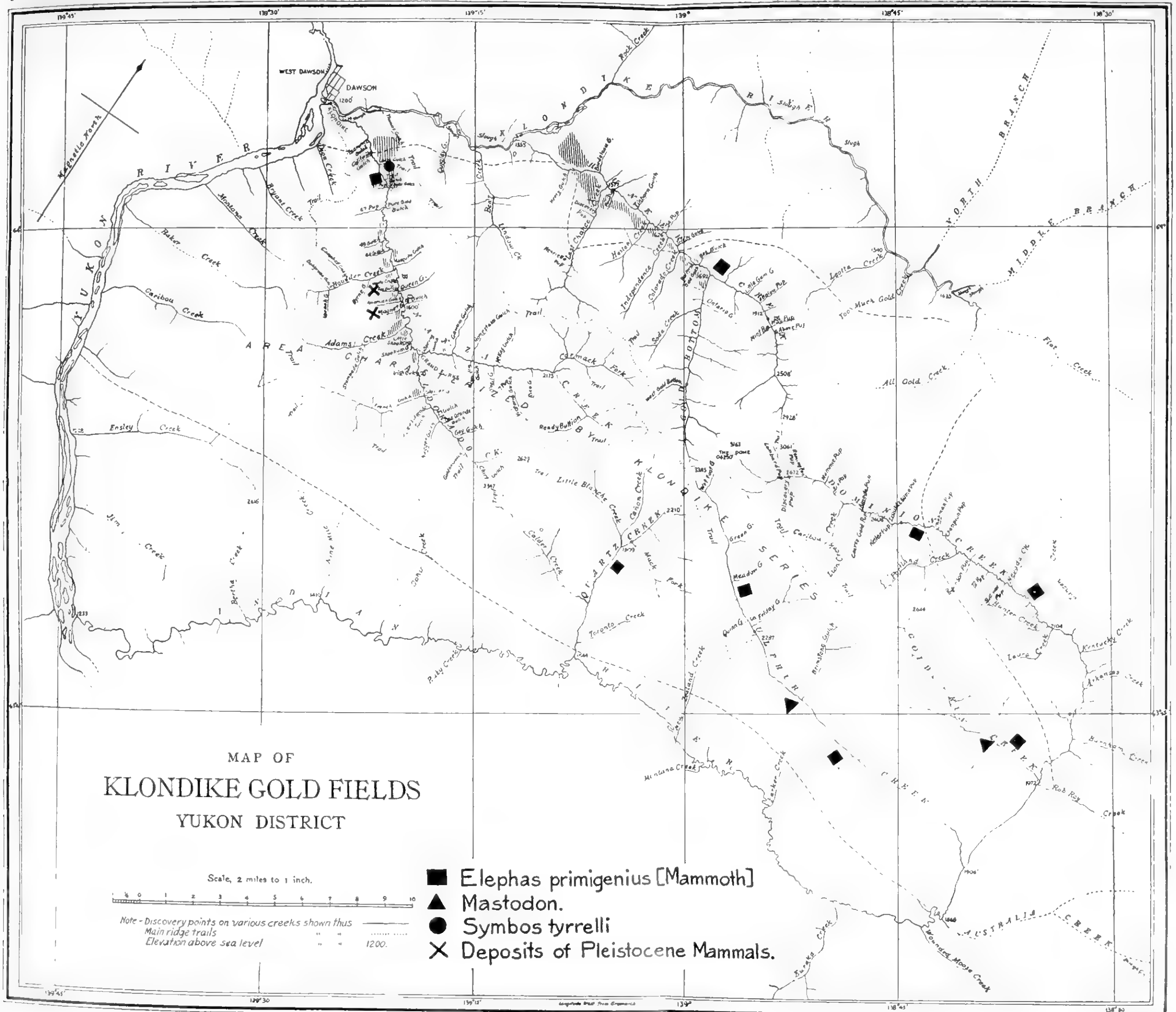
[4] Obalski, M. T.: Les grandes Fossiles dans le Yukon et l'Alaska. Bull. de la Musée d'Hist. Nat., Paris, 1904, No. 5, pp. 214–217.

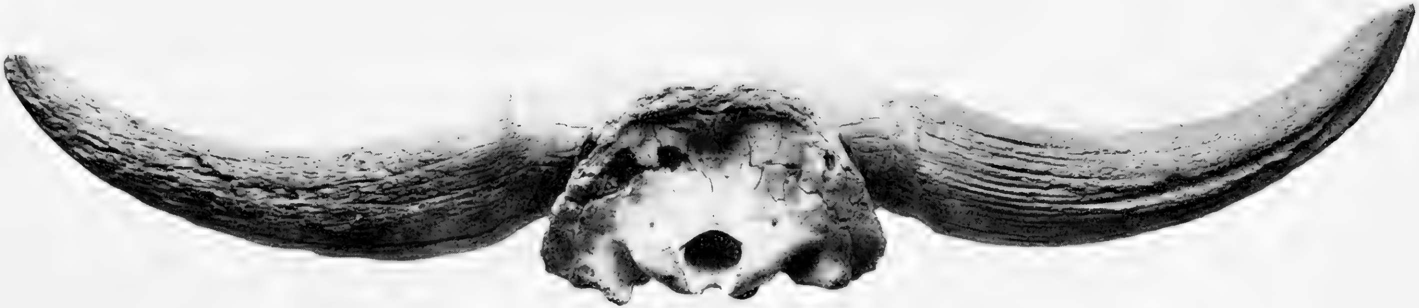
Fig. 1.--UPPER MOLAR OF MASTODON (No. 5102, U. S. National Museum) FOUND ON GOLD RUN CREEK, NEAR DAWSON, YUKON TERRITORY, CANADA, IN 1902

About 2/5 natural size

Fig. 2.--MOLAR OF MASTODON FOUND ON SULPHUR CREEK, NEAR DAWSON, YUKON TERRITORY, CANADA, IN 1906

In the possession of Mr. Joseph Nichlas, of the city of Dawson. About ½ natural size


MAP OF
KLONDIKE GOLD FIELDS
YUKON DISTRICT
Scale, 2 miles to 1 inch.
Note - Discovery points on various creeks shown thus
Main ridge trails
Elevation above sea level 1200.
Elephas primigenius [Mammoth]
Mastodon.
Symbos tyrrelli
Deposits of Pleistocene Mammals.
DAWSON
WEST DAWSON
YUKON RIVER
KLONDIKE RIVER
Magnetic North



BISON CRASSICORNIS Richardson

(Cat. No. 5726, U. S. National Museum.) Posterior view of cranium from Little Minook Creek, near Rampart, Alaska
Greatest width 46½ inches

todon in the placer gravels of the Klondike region. Maddren[1] attributes this to an error. While it may have been an error in this particular instance, it is likely to be a very common one, for throughout this entire region all of the tusks, teeth, and big bones are usually referred to by the people as those of the mastodon.

So far as the writer knows, there have been no authentic cases recorded of the occurrence of mastodon in Alaska. From the fact that its remains do occur in the Klondike region, there appears no logical reason why it should not be found in Alaskan territory as well. It is on that account that the brief review is appended here.

Through the kindness of Mr. R. G. McConnell, of the Canadian Geological Survey, the writer is enabled to present a map (see plate IX) of the Klondike district on which has been indicated the localities where mastodon and mammoth remains have been found. With three exceptions, the localities indicated are based upon specimens seen by the writer.

EQUUS sp. undet.

Scattered remains of *Equus* are commonly associated with the other Pleistocene fossils found in Alaska. These bones have been considered by various authorities as representing the extinct species *Equus fossilis* and *E. fraternus,* and by some referred to the living form *E. caballus.* On account of the very fragmentary nature of the specimens upon which these determinations have been made, in all cases the identifications are open to question, and until better material is found the species should be considered undeterminable.

Remains of horses have been found in the following localities:

Eschscholtz Bay, Seward Peninsula, on the Kobuk and Buckland Rivers; "Palisades," on the Yukon; Nowitna River, Old Crow River, and in many places in the Klondike district.

BISON CRASSICORNIS Richardson

Bison crassicornis RICHARDSON, Zoöl. Voy. of H. M. S. *Herald,* 1852–54, pp. 40–60, pls. IX, XI, fig. 6; pl. XII, figs. 1–4; pl. XIII, figs. 1–2, pl. XV, figs. 1–4.

TYPE.—Poorly preserved skull in the British Museum, from Eschscholtz Bay, Alaska.

DESCRIPTION.—"Horns long; length of horn core along upper curve very much greater than circumference at base; horn cores slightly flattened on superior face; transverse diameter much greater than vertical; curve of horn regular, the tip not abruptly reflected nor pointing decidedly backward; horn cores raking decidedly backward."

[1] Maddren, A. G.: Smithsonian Misc. Coll., vol. XLIX, No. 1584, 1905, p. 7.

REMARKS.—This species heretofore has not been known outside of Alaska, but, as might have been anticipated, skulls of this species were observed at Fox Gulch, Bonanza Creek, Yukon Territory, Canada, by the writer during the summer of 1907. In the foreground of plate IV, fig. 1, may be seen a portion of the skull and horn cores

FIG. 4.—Scapula of *Bison crassicornis* (?) (Cat. No. 5941) from "Palisades," on the Yukon River. (See 2, Fig. 2.) Greatest length, 22½ inches.

of this species. Remains of *B. crassicornis* have been collected from the following localities: On the tundra back of Point Barrow, Elephant Point, Eschscholtz Bay, Little Minook Creek, Little Minook Creek Junior, and Bonanza Creek, Yukon Territory, Canada.

BISON ALLENI Marsh

(Cat. No. 2383, U. S. National Museum.) Posterior view of cranium, with horn sheaths preserved, from Little Minook Creek, near Rampart, Alaska
Greatest width 45 inches

This is the largest of the extinct bisons found in the deposits of this region, and a scapula (see fig. 4) collected by the writer at the "Palisades" on the Yukon River may, on account of its size, pertain to this species. Plate X represents a typical skull of this form collected on Little Minook Creek and presented to the Smithsonian Institution through the writer by Mr. J. B. Duncan, of Rampart, Alaska.

BISON ALLENI Marsh

Bison alleni MARSH, Amer. Jour. of Science, vol. XIV, 1877, p. 252.

TYPE.—Horn core, No. 911, Museum of Yale College, New Haven, Connecticut, from Blue River, near Manhattan, Kansas.

DESCRIPTION.[1]—"Horn cores long, slender, much curved, slightly flattened above at base; transverse diameter considerably greater than vertical; length along upper curve much greater than circumference at base. *Bison alleni* is distinguished from *B. crassicornis* by the much greater curvature of the horn cores, these being also more flattened and more elliptical in section in *crassicornis*.

REMARKS.—This species is represented in the U. S. National Museum paleontological collection, by a skull, No. 2383, from Little Minook Creek, near Rampart, Alaska.[2] It (see pl. XI) was found in the frozen muck twenty-five feet below the surface, and is of more than usual interest on account of the excellent state of preservation of the horn sheaths and from its being the first of this species found in Alaska. This species is also reported as occurring in Idaho.

A skull of *B. alleni* from the Porcupine River is now in the Grand Rapids Museum, of Grand Rapids, Michigan.

BISON OCCIDENTALIS Lucas

Bison occidentalis LUCAS, *Science,* November 11, 1898, p. 678.

TYPE.—Portion of skull with horn cores, No. 4157, U. S. National Museum, from Fort Yukon, Alaska, collected by Sir John Richardson.

DESCRIPTION.—"Horn cores moderate; circumference at base equal to or slightly greater than length along upper curve; subcircular in section, regularly curved upward and backward."

---

[1] The descriptions of the *Bison* from Alaska is taken from Mr. F. A. Lucas' article, "The Fossil Bison of North America." Proc. U. S. National Museum, vol. XX, 1899, pp. 755–771.

[2] This specimen was presented to the Museum by Messrs. McLain and Ballou, of Rampart, through the efforts of Gen. Timothy Wilcox, U. S. A., of Washington, D. C.

REMARKS.—A second specimen, No. 2643 (see pl. XII), in the U. S. National Museum collections, was collected by Mr. A. G. Maddren on the Old Crow River in 1904. A fairly complete skeleton[1] of this species from Gove County, Kansas, is now in the University of Kansas Museum. This species has also been reported as occurring on the Tatlo River and St. Michaels, Alaska. The writer doubts very much the authenticity of this last locality. Mr. Lucas says: "It is the species most nearly resembling the existing bison, with which it was probably for a time contemporaneous." In that event *B. crassicornis* was also a contemporary, as the writer recognized skulls of *B. occidentalis* and *B. crassicornis* at Fox Gulch, on Bonanza Creek, coming from the same layers in the deposits there.

BISON PRISCUS (?)

A skull collected at Eschscholtz Bay, Alaska, was provisionally referred[2] to this species by Sir John Richardson. In a more recent paper,[3] however, Mr. F. A. Lucas has considered this specimen (No. 24,589, British Museum) as representing an immature individual or "spike horn" of *B. crassicornis.*

Horn cores collected by Maddren on the Old Crow River and by the writer at the "Palisades" on the Yukon River appear to resemble the figure (see pl. XIII, fig. 3) given by Richardson in his report.

SYMBOS TYRRELLI Osgood

Scaphoceros tyrrelli OSGOOD, Smithsonian Miscellaneous Collections, vol. XLVIII, No. 1585, 1905, pp. 173–183, pl. XXXVII, fig. 2; pl. XXXVIII, fig. 2; pl. XXXIX, fig. 1; pl. XL, fig. 2.
Symbos tyrrelli OSGOOD.

TYPE.—Fairly complete skull, No. 2555, U. S. National Museum, from Lovett Gulch, Bonanza Creek, Klondike District, Yukon Territory, Canada (see map, plate IX).

DESCRIPTION—*Generic characters.*[4]—"Similar to *Ovibos,* but horn cores much smaller, less compressed at base, and more divergent at tips; crown of skull between bases of horn cores surmounted by a prominent exostosis with an anterior bounding rim and a deep median excavation; orbits much less produced laterally than in

---

[1] Stewart, Alban: Kansas University Quart., July, 1897, Sec. A, pp. 127–135. Described as *B. antiquus,* but referred later by Lucas to *B. occidentalis.*

[2] Richardson, Sir John: Zoölogy of Voyage of H. M. S. *Herald,* 1852–54, pl. VII, fig. 1, p. 34.

[3] Lucas, F. A.: The Fossil Bison of North America. Proc. U. S. Nat. Mus., vol. XXI, 1899, p. 762.

[4] Generic and specific characters as given by Osgood.

BISON OCCIDENTALIS Lucas

(Cat. No. 2643, U. S. National Museum.) Posterior view of cranium from Old Crow River, Canada
Greatest width 33 + inches

Ovibos; facial part of skull nearly as wide as cranial; basioccipital without a high median ridge; teeth very large and relatively broad; m^1 and m^2 quadrate in transverse view."

Specific characters.—"Size smaller than in *S. cavifrons* (Leidy); horn cores much smaller and shorter; exostosis less extensive, but more deeply excavated; depth of brain case and surmounting bony mass decidedly less."

Remarks.—The only reported occurrence of this species in Alaska is a horn core, No. 2378, U. S. National Museum, presented by Rev. J. W. Chapman through Dr. Arthur Hollick. The label with the horn gives the locality as Anvik, on the Yukon River, but it is unlikely the specimen was collected in the immediate vicinity of that place. It is more probable that it comes from some of the silt deposits along the Yukon twenty-five or thirty miles above Anvik.

SYMBOS CAVIFRONS (Leidy)

Hay[1] cites the occurrence of *O.*[2] *cavifrons* in Alaska, due to the fact that he includes Richardson's indeterminate species, *Ovibos maximus,* under this head.

This species, therefore, is not known to occur in Alaska.

OVIBOS MAXIMUS Richardson

Ovibos maximus Richardson, Zoöl. Voy. of H. M. S. *Herald,* 1852–54, pp. 25–28, pl. xi, figs. 2, 3, and 4.

Type.—An imperfect cervical vertebra, the axis or *dentata* (No. $\frac{90}{2}$, Haslar Museum), from Eschscholtz Bay, Alaska.

Remarks.—From the very fragmentary nature of the type this species appears indeterminable.

OVIBOS MOSCHATUS (?) Zimmerman

This is a recent species found at present in northern North America and Greenland. At present this animal is not known to range west of the McKenzie River, but Pleistocene remains which have not been distinguished from this species are found in Alaska. As in the case of other remains referred to living species, more complete material may show an extinct species separable from the living form.

This appears more probable since a skull, collected by the writer at the Palisades, on the Yukon, in 1907, is being described by Mr. J. W. Gidley as the type of a new species, and it may be that all the remains formerly considered *O. moschatus* should be referred to this species.

---

[1] Hay, O. P.: Bulletin No. 179, U. S. Geological Survey, p. 688.

[2] Osgood now includes *Ovibos cavifrons* under *Symbos.*

Buckland, because of the preservation of a horn sheath on a skull of *Ovibos* submitted to him from Eschscholtz Bay, considered it of recent origin, but now that *Bison* (see pl. x) skulls are known distinct from living species having the horn thus preserved, this argument would apply equally to the case in question.

OVIS, sp. undet.

A list of species occurring in the Eschscholtz Bay deposits is given by Seeman in his "Narrative of the Voyage of H. M. S. *Herald* in 1853, in which *Ovis montana* is mentioned as being found there.

This list was compiled from a report[1] made by Sir John Richardson, but a careful perusal of his report failed to reveal any mention of fossil remains, although he does describe the recent skeleton of *Ovis montana.*

It is probably by mistake that this species was included in Seeman's list, although sheep remains will undoubtedly be found, as Mr. W. H. Osgood, of the U. S. Biological Survey, has fragmentary remains of *Ovis* in his possession from the Klondike district, Yukon Territory, Canada. At present, however, the writer does not know of an authentic record of their occurrence in Alaska.

ALCE, sp. undet.

Like *Rangifer,* scattered remains of the moose are known from several widely separated localities in Alaska and adjacent territory. These bones have usually been referred to as representing the living form *Alce americanus,* but it appears the identifications have been based upon such scanty material that the assignment to this species is open to question. When better specimens are known, characters of sufficient importance to distinguish it from the living species will probably be found.

Remains of *Alce* are known from the deposits of Eschscholtz Bay, on the Old Crow and Nowitna rivers, and fragmentary antlers were found in the muck of Magnet and Fox gulches on Bonanza Creek near Dawson.

RANGIFER, sp. undet.

Fragmentary remains representative of this genus are commonly found with the bones of other Pleistocene animals in Alaska. These scattered and fragmentary parts have been referred by various writers to the living species, *R. caribou* and *R. tarandus.* It appears

[1] Zoölogical Voyage of H. M. S. *Herald,* 1852–54.

more likely, however, if referable at all to a living form, it would be *R. articos,* the barren-ground caribou and now living in these regions.

As mentioned by Richardson, Zoölogy of the Voyage of H. M. S. *Herald,* 1854, p. 20, fragmentary remains have been found at Eschscholtz Bay, and the writer collected fragments of antlers on Little Minook Creek Junior and on the Nowitna River.

So far, remains have not been found sufficiently complete upon which an accurate specific determination could be based.

URSUS, sp. undet.

The finding of a scapula and astragulus of *Ursus* associated with the remains of other Pleistocene animals on the Nowitna River during the summer of 1907 verifies a former record of the occurrence of the bear in the Pleistocene of Alaska.

The scapula, although incomplete, indicates an animal about the size of the black bear (*Ursus americanus*), an inhabitant of these regions at the present time.

Bones[1] of *Ursus* have also been found associated with mammoth remains in a cave on St. Paul Island of the Pribilof group.

CASTOR, sp. undet.

Among the vertebrate remains collected on the Nowitna River in 1907 were the left pelvic bones (No. 5942, U. S. National Museum) of a beaver. This appears to be the first occurrence recorded of the finding of bones of *Castor,* although Mr. E. W. Nelson,[2] who visited Eschscholtz Bay in 1881 with the U. S. S. *Corwin,* observed a beaver's nest imbedded in the cliffs at that place, and noted that many of the sticks composing it had been gnawed and others still retained the tooth-marks made by that animal.

The remains found, however, are too fragmentary to admit of specific determination.

Summary

From the preceding review of the extinct vertebrates reported as occurring in the Pleistocene deposits of Alaska, it will be seen that the identification of several of the forms has been based upon such scanty and fragmentary material that their determination is

[1] These remains, collected by the party with Dr. D. S. Jordan in 1897, are now in the paleontological collection of the U. S. National Museum.

[2] Maddren, A. G.: Smithsonian Misc. Coll., vol. XLIX, No. 1584, 1905, pp. 112–113.

open to question. This observation is particularly applicable to those so long regarded as being identical with living species. The writer believes that when more perfect material is available it will be found, probably in all instances, to be quite distinct from the living forms. That this is in some instances the case is shown by the discovery this past summer of a skull of *Ovibos* sufficiently complete to show characters of enough importance to warrant its separation from the living form *O. moschatus,* to which nearly all musk-ox material found in this region previously had been referred.

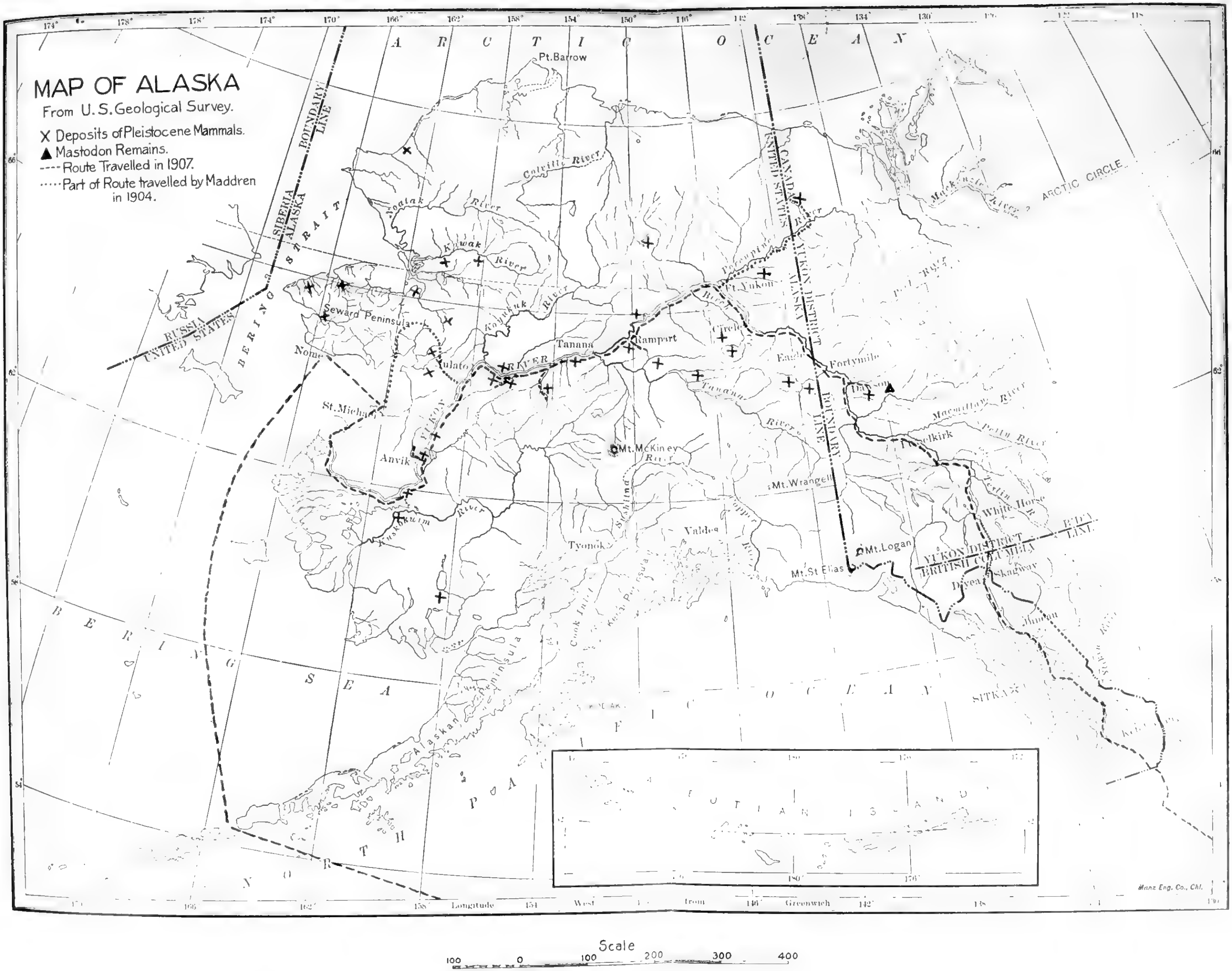
More persistent collecting, aided by improved methods, will undoubtedly increase the faunal list and widen the geographical distribution of the known forms.

Now that *Mastodon* and *Ovis* remains have been found in Canadian territory and at a comparatively short distance from the international boundary, there appears no logical reason why both of these animals should not have lived in Alaska at one time.

While in some cases we are unable to adequately define many of the species, still a very good idea of the fauna as a whole is obtained. Its close relationships in many instances with living animals furnishes an interesting link in the development of mammalian life of this continent.

The following list, based upon material sufficiently complete for fairly accurate determinations, represents the Pleistocene fauna of Alaska as we know it today:

Elephas primigenius Blumenbach.
Equus, sp. undet.
Alce, sp. undet.
Rangifer, sp. undet.
Ovibos, sp. nov.
Symbos tyrrelli Osgood.
Bison crassicornis Richardson.
Bison occidentalis Lucas.
Bison alleni Marsh.
Ursus, sp. undet.
Castor, sp. undet.


MAP OF ALASKA
From U. S. Geological Survey.
X Deposits of Pleistocene Mammals.
▲ Mastodon Remains.
----Route Travelled in 1907.
·····Part of Route travelled by Maddren in 1904.
ARCTIC OCEAN
BERING SEA
PACIFIC OCEAN
NORTH
Pt. Barrow
Colville River
Noatak River
Kowak River
Koyukuk River
Porcupine River
Seward Peninsula
Nome
St. Michael
Anvik
Nulato
Tanana
Rampart
Ft. Yukon
Circle
Eagle
Fortymile
Dawson
Selkirk
Macmillan River
Pelly River
White Horse
Mt. McKinley
Mt. Wrangell
Mt. Logan
Mt. St. Elias
Valdez
Tyonok
Cook Inlet
Kenai Peninsula
Alaskan Peninsula
Kuskokwim River
Tanana River
Sushitna
Dyea
Skagway
Juneau
SITKA
BERING STRAIT
SIBERIA ALASKA BOUNDARY LINE
RUSSIA UNITED STATES
CANADA UNITED STATES
YUKON DISTRICT ALASKA BOUNDARY LINE
YUKON DISTRICT BRITISH COLUMBIA
ARCTIC CIRCLE
Mackenzie River
ALEUTIAN ISLANDS
Longitude West from Greenwich
Manz Eng. Co., Chi.
Scale
100 0 100 200 300 400

SMITHSONIAN MISCELLANEOUS COLLECTIONS
VOLUME 51, NUMBER 4

Hodgkins Fund

THE MECHANICS OF THE EARTH'S ATMOSPHERE

A COLLECTION OF TRANSLATIONS

BY

CLEVELAND ABBE

THIRD COLLECTION

(Publication 1869)

CITY OF WASHINGTON
PUBLISHED BY THE SMITHSONIAN INSTITUTION
1910

CONTENTS

THE MECHANICS OF THE EARTH'S ATMOSPHERE

A THIRD COLLECTION OF TRANSLATIONS

BY CLEVELAND ABBE

INTRODUCTION

In order to introduce English-speaking students of meteorology to the rapidly increasing literature bearing on the fundamental mechanical problems of that science, I have been encouraged to publish numerous translations either in the "Monthly Weather Review" of the U. S. Weather Bureau or in the technical journals. Others are collected in the "Short Memoirs on Meteorological Subjects," Smithsonian Report for 1877, pp. 376-478, and in "The Mechanics of the Earth's Atmosphere," Smithsonian Miscellaneous Collections, 1891. As our knowledge of the subject progresses and we perceive new difficulties arising, so also we learn to conquer those older ones that were the ultima thule of the past generation. Step by step man is penetrating the complex maze of forces that push our atmosphere hither and thither. Its internal mechanism is so complex that superficial students content themselves with empirical rules or search for cosmical relations of minor importance: the very ablest investigators have as yet solved only the simpler problems relating to idealized conditions that rarely occur in nature.

In this third collection of translations bearing on the mechanics of the earth's atmosphere I have ventured to begin with that elementary but classic memoir by Hadley which gave occasion to the Berlin Academy in 1746 to offer a prize for a mathematical discussion of the motions of the atmosphere. The prize was awarded to d'Alembert; subsequently Musschenbroek, deLuc, Euler, Bernoulli, Lambert, von Lindenau (in 1806) and Brandes (in 1822) successively contributed to the elucidation of this subject. But it was Poisson who, in 1837, first deduced correctly the influence of the earth's rotation on moving solids, and Tracy who in 1843, applied similar views to the rotation of storms. Poisson's works and ideas were generally known to the scholars of France as shown by the prolonged discussion, 1850–1860, of the Foucault pendulum and gyroscope phenomena. There are indications that Babinet and others at that time applied his results to the mechanics of the atmosphere. But the modern study of this subject is properly traceable to the influence of Prof. William Ferrel in America and Prof. William Thomson in England, both of whom coöperated to put our knowledge of the subject on a firmer basis than was before possible. Meanwhile a profound Russian scholar, Braschmann, and the equally profound German scholar, Erman, were independently working over the same ground, though their publications have been scarcely noticed by technical meteorologists. The neglect of Erman's work in dynamic meteorology seems remarkable, but has been atoned for by the enthusiastic activity of Sprung and his successors at Hamburg and at Berlin.

The works of Espy, 1840, on the Philosophy of Storms; Thomson, 1862, on the Convectional Equilibrium in the Atmosphere; Peslin, 1868, on the Thermodynamics of Moist Air; Ferrel, 1857, on The Motions of Solids and Fluids, and his subsequent important memoirs; together with Sprung's Lehrbuch, 1885, mark the transition from ancient to modern meteorology.

The modern sounding balloon has assured us of the intimate connection between the lowest stratum of air and that which is 20 miles above us; but the conditions above this latter level are doubtless of equally great importance to our surface climatology and these can be made known to us only by the study of meteors, auroras, spectrum lines, and refractions. I have, therefore, included a memoir by Kerber on the limit of the earth's atmosphere, that avoids some of the difficulties attending every application to the outer atmosphere of our knowledge of the kinetic theory of gases.

All students will gladly welcome the translation by Waldo of the memoir by Guldberg and Mohn, first published in two parts, 1876 and 1880; it was revised by the authors in 1883 at the personal request of Prof. Frank Waldo who expected its prompt publication, and to him we owe the privilege of including in the present collection this new edition of that classic paper.

The series of papers by Von Bezold were revised by himself in 1906, for publication in his collected memoirs and as thus revised they are now reproduced by permission of his heirs and publishers.

The study of strictly adiabatic changes that was so greatly facilitated by the Hertzian diagram published in the preceding collection of translations is now advantageously replaced by the diagrams of Neuhoff, 1900, which adapt themselves to any atmospherical condition.

In conclusion, the two memoirs by Margules (XXIII, 1901, and XXIV, 1904) introduce us to the great problems of the future, that is, the thermal transformations of energy persistently going on in the atmosphere. Margules has been the first to find methods of studying and solving these problems. It only remains for future students to combine the equations of thermodynamics with those of hydrodynamics so as to further elucidate the details of the phenomena as to time and place—a result that we may hope will eventually be attained by the analysis of fields of force that is now being perfected by Bjerknes of Christiania.

CLEVELAND ABBE.

Washington, D. C.
November, 1908.

I

CONCERNING THE CAUSE OF THE GENERAL TRADE WINDS

BY GEO. HADLEY, ESQ., F. R. S.

[*Phil. Trans. Vol. XXXIX, London, 1735-36, p. 58*]

I think the causes of the General Trade Winds have not been fully explained by any of those who have wrote on that subject, for want of more particularly and distinctly considering the share the diurnal motion of the earth has in the production of them. For although this has been mentioned by some amongst the causes of those winds, yet they have not proceeded to show how it contributes to their production; or else have applied it to the explication of these phenomena, upon such principles as will appear upon examination not to be sufficient.

That the action of the sun is the original cause of these winds, I think all are agreed; and that it does it by causing a greater rarefaction of the air in those parts upon which its rays falling perpendicularly, or nearly so, produce a greater degree of heat there than in other places; by which means the air there becoming specifically lighter than the rest round about, the cooler air will by its greater density and gravity, remove it out of its place to succeed into it itself, and make it rise upward. But it seems, this rarefaction will have no other effect than to cause air to rush in from all parts into the part where 'tis most rarefied, especially from the north and south, where the air is coolest, and not more from the east than the west, as is commonly supposed: so that, setting aside the diurnal motion of the earth, the tendency of the air would be from every side towards that part where the sun's action is most intense at the time, and so a NW. wind be produced in the morning, and a NE. in the afternoon, by turns, on this side of the parallel of the sun's declination, and a SW. and SE. on the other.

That the perpetual motion of the air towards the west, cannot be derived merely from the action of the sun upon it, appears more evidently from this: If the earth be supposed at rest, that motion of the air will be communicated to the superficial parts, and by little and little produce a revolution of the whole the same way, except there be the same quantity of motion given the air in a contrary direction in other parts at the same time, which is hard to suppose. But if the globe of the earth had before a revolution towards the east, this by the same means must be continually retarded. And if this motion of the air be supposed to arise from any action of the parts of it on one another, the consequence will be the same. For this reason it seems necessary to show how these phenomena of the Trade Winds may be caused, without the production of any real general motion of the air westwards. This will readily be done by taking in the consideration of the diurnal motion of the earth. For, let us suppose the air in every part to keep an equal pace with the earth in its diurnal motion; in which case there will be no relative motion of the surface of the earth and air, and consequently no wind, then by the action of the sun on the parts about the equator, and the rarefaction of the air proceeding therefrom, let the air be drawn down thither from the N. and S. parts. The parallels are each of them bigger than the other, as they approach to the equator and the equator is bigger than the tropics, nearly in the proportion of 1000 to 917, and consequently their difference in circuit about 2083 miles, and the surface of the earth at the equator moves so much faster than the surface of the earth with its air at the tropics. From which it follows, that the air, as it moves from the tropics towards the equator, having a less velocity than the parts of the earth it arrives at, will have a relative motion contrary to that of the diurnal motion of the earth in those parts, which being combined with the motion towards the equator, a NE. wind will be produced on this side of the equator and a SE. on the other. These, as the air comes nearer to the equator, will become stronger, and more easterly, and be due east at the equator itself, according to experience, by reason of the concourse of both currents from the N. and S. where its velocity will be at the rate of 2083 miles in the space of one revolution of the earth or natural day, and above one mile and one-third in a minute of time; which is greater than the velocity of the wind is supposed to be in the greatest storm, which according to Doctor Derham's observations, is not above one mile in a minute. But it is to be considered, that before the air from the tropics can arrive at the equator, it must have gained some motion eastward from the surface of the earth or sea, whereby its relative motion will be diminished, and in several successive circulations, may be supposed to be reduced to the strength it is found to be of.

Thus I think the NE. winds on this side of the equator, and the SE. on the other side, are fully accounted for. The same principle as necessarily extends to the production of the west trade-winds without the tropics; the air rarefied by the heat of the sun about the equatorial parts, being removed to make room for the air from the cooler parts, must rise upwards from the earth, and as it is a fluid, will then spread itself abroad over the other air, and so its motion in the upper regions must be to the N. and S. from the equator. Being got up at a distance from the surface of the earth, it will soon lose great part of its heat, and thereby acquire density and gravity sufficient to make it approach its surface again, which may be supposed to be by that time 'tis arrived at those parts beyond the tropics where the westerly winds are found. Being supposed at first to have the velocity of the surface of the earth at the equator, it will have a greater velocity than the parts it now arrives at; and thereby become a westerly wind, with strength proportionable to the difference of velocity, which in several revolutions will be reduced to a certain degree, as is said before, of the easterly winds, at the equator. And thus the air will continue to circulate, and gain and lose velocity by turns from the surface of the earth or sea, as it approaches to or recedes from the equator. I do not think it necessary to apply these principles to solve the phenomena of the variations of these winds at different times of the year, and different parts of the earth; and to do it would draw this paper into greater length than I propose.

From whatever has been said it follows:

First, That without the assistance of the diurnal motion of the earth, navigation, especially easterly and westerly, would be very tedious, and to make the whole circuit of the earth perhaps impracticable.

Secondly, That the NE. and SE. winds within the tropics must be compensated by as much NW. and SW. in other parts, and generally all winds from any one quarter must be compensated by a contrary wind somewhere or other; otherwise some change must be produced in the motion of the earth round its axis.

II

ON THE MOTION OF PROJECTILES IN THE AIR, TAKING INTO CONSIDERATION THE ROTATION OF THE EARTH

BY M. [S. D.] POISSON[1]

In this memoir the projectile will be considered as an isolated and material point, that is to say, as a body whose mass is collected at the center of gravity, and the problem will be to ascertain the influence of the rotation of the earth on its motion. I shall present shortly another memoir to the Academy, in which we shall take into consideration the form and the dimensions of the moving body, and the object of that will be to determine, principally in what relates to the projectiles used in artillery, the influence that their own rotation can produce on their motion of translation.

Up to the present time the theory of the resistance which fluids in general, and the air in particular, offer to the motion of the bodies that traverse them, has received only a very imperfect development. We compare this force to a continual succession of shocks of the moving body against the particles of the fluid, which disappear and are annihilated, so to speak, when they have been struck by the body and have carried away small quantities of motion, proportional to their own masses and its velocity. Newton, to whom we owe this theory, had concluded that, ignoring the rotation of the moving body, the resistance of the air for a sphere, for example, is equal to the weight of a cylinder of this fluid having for its base the great circle of the sphere and for height the "full-height" due to its velocity. But the experiments made on the fall of bodies in the air soon showed him the inaccuracy of this result, and led him to reduce by one-half this measure of resistance; subsequently, it has been found that this

---

[1] Memoire sur le mouvement des projectiles dans l'air en ayant égard la rotations de la terre. By [S. D.] Poisson. Read before the Academy of Sciences, Paris, November, 1837. Published in the Journal de l'Ecole Royale Polytechnique, Vol. XVI, Cahier 26, Paris 1838, pp. 1–68. Translated by Profs. Frank Waldo and Cleveland Abbe.

reduction is too great, and Borda has concluded from his own observations that the measure of the resistance must be only diminished to three-fifths of its theoretical value. From the theory of Newton as modified by experiment, the retarding force relative to the unit of mass for a sphere moving through the air has for its expression the square of the velocity of the sphere divided by its diameter and by the ratio of its density to that of the fluid, and multiplied by a numerical coefficient concerning which the writers on ballistics do not agree. According to Lombard,[2] and relying on the experiments of Borda, this coefficient should be equal to about nine-fortieths. But the true law of the resistance as a function of the velocity is far more complex; for motions which are either very rapid or very slow the coefficient seems to deviate considerably from being proportional to the square of the velocity; in the case of very great velocities it increases at a much greater ratio, and on the contrary when it is a question of small velocities, such as the very small vibrations of the seconds pendulum[3] this coefficient is proportional to the simple velocity.

In order to determine directly and without any hypothesis the law of the resistance that a body meets with in moving through a fluid, it will be necessary to consider at the same time both the motion of the body and that which the moving body communicates to the fluid; as the result of this double motion the fluid exerts at each instant a certain pressure at each point of the moving body and normal to its surface; this pressure is different from that which occurs in the state of rest and produces the resistance, properly so-called, that the moving body experiences, and to which it will be necessary also to add the force tangential to the surface of the body arising from the friction of this body against the layer of fluid in contact with it. In fact, this is what I have been able to do in my Memoir on the simultaneous Motions of the Pendulum and of the surrounding Air,[4] and which has led me to deduce from theory the new correction which M. Bessel has confirmed by experiment on the length of the seconds pendulum. Hereafter I shall try to extend that analysis to the case of the progressive motion of projectiles in the air and to determine, if it is possible for me to do so, the pressure that the fluid displaced by them exerts on their surfaces by its compression on one side and expansion on the other, or the resistance that they

[2] Treatise on the motion of projectiles, p. 99.
[3] Additions to the Connaissance des Temps for the year 1834, p. 18.
[4] Memoirs of the Academy of Sciences, Vol. XI [Paris].

meet with considered from the point of view that I shall indicate. I do not need to say that the exact and general knowledge of this law will be important in many questions, for example, in the problem of ballistics. But for the object which I have in view in this present memoir I can admit the ordinary law of the resistance proportional to the square of the velocity as being sufficiently accurate.

It is Newton, also, who has given the first example of the determination of the motion of a heavy body in a resisting medium. He solved the problem when the motion is vertical by assuming the resistance proportional either to the velocity or to its square, but when the projectile is projected into the atmosphere in any direction whatever he confined himself to considering the case of a resisting force proportional to the simple velocity, observing nevertheless that this case is not that of nature. The two equations that Newton was obliged to integrate in order to determine the horizontal and vertical components of the velocity at any instant, are linear of the first order and with constant coefficients; and the two unknown quantities are so separated in them that these two equations are solved independently of each other, and their solution really implies only a simple direct integration. This is no longer true in the case of a resistance proportional to the square of the velocity; the two unknown quantities enter at the same time into each of the equations of motion, which are no longer linear, and it is only by a special combination that we succeed in separating the variables therein and in reducing them to quadratures, which we consider as the complete solution of the problem.

This was done by John Bernoulli, who published it in the Acta Eruditorum, Leipzig, May 1719, pp. 216–226, more than thirty years after the solution by Newton, and at an epoch when the integral calculus had already made great progress. However, Euler, at the beginning of his memoir on this subject,[5] expresses his surprise at seeing that Newton, "who has well solved other problems more difficult," should stop with the case of the resistance proportional to the simple velocity, and not consider the case of nature. We know, however, that the question of the trajectory in a medium resisting in proportion to the square of the velocity was proposed as a challenge to the geometers of the continent by an Englishman named Keil, who believed the problem insol-

[5] Memoirs of the Academy of Berlin; year 1753.

uble because his illustrious countrymen had not solved it. Now the numerical calculation of the integrals which express the time and the two coördinates of the moving body, in functions of a fourth variable, is effected as simply as the question allows, and enables the approximations to be carried as far as we wish. We can see an example in the "Exercises du calcul Integral" of Legendre[6] in which these coördinates are calculated to within less than a hundred-thousandth part of their values.

Independently of the centrifugal force arising from the rotation of the earth (which influences the motions of heavy bodies by diminishing the force of gravity by a quantity that varies with the latitude), this rotation also produces in these motions certain deviations that it is interesting to understand, either in themselves or in order to know to what extent they can influence the trajectory of the projectiles, and whether it is necessary to consider them in the practice of artillery.

Many physicists have measured, with as much precision as has been possible, the small distances by which bodies that fall from a considerable height deviate from the foot of the vertical. Laplace and Gauss submitted this question to the calculus, but in integrating the equations of this almost exactly vertical motion they have left out of consideration the resistance of the air, which can, however, sometimes have a very great influence on the result. I have therefore thought it would be useful to go over this problem entirely and to extend the solution to the general case in which the projectile is projected into the atmosphere with any velocity and in any direction whatever.

To this end I have in the first place formed the differential equations of the absolute motion in space by referring the coördinates of the moving body to fixed axes; then I have deduced from these the equations of apparent motion such as we observe near the surface of the earth, referred to fixed axes at the surface which participates as well as we ourselves in the rotation of the earth. These differential equations are very complicated, but by taking the second of time for the unit of time, the angular velocity of the diurnal motion becomes a very small fraction, which permits (us) to reduce them to a more simple form. From these we deduce some general consequences, enumerated as follows:

(1) The motion of the earth prevents a liquid contained in a vase and turning with a constant velocity about a vertical axis

---

[6] Vol. I, p. 336.

from assuming the rigorously permanent figure of a paraboloid of revolution as it would do if the earth were immovable.

(2) If the body moves along a given curve that is attached firmly to the surface of the earth, the differential equation of its motion does not contain the velocity of the rotation of the earth and consequently this motion is the same as if the earth were at rest. Thus, for any given value of gravity resulting from the figure and the rotation of the terrestrial spheroid, the oscillations of the pendulum are the same in all azimuths around the vertical; a result that was important to demonstrate, considering the degree of precision that we now attain in the determination of the length of the seconds pendulum at different places on the earth. But the diurnal rotation and the direction of the plane of oscillation have a slight influence on the variable tension that the wire experiences during the oscillations and which is not rigorously the same in all azimuths.

(3) Finally, when a projectile is sent into the air in any direction whatever the rotation of the earth neither increases nor diminishes the distance that it attains at any instant from a plane through the point of departure and parallel to the equator.

Before seeking the integrals of the equations of apparent motion in the general case of an initial velocity having any magnitude and any direction whatever, I have considered the simpler special cases.

The first case is that where the moving body starts from a point situated at a given height above the ground without imparting to it any initial velocity whatever and is left to the action of gravity, so that it commences to fall vertically. The velocity [of the eastward motion] at the point of departure, due to the rotation of the earth in which it participates, being greater than that which belongs to the foot of the vertical, we perceive that the moving body when it has reached the earth must have departed from the foot of the vertical line, to the eastward or in the direction of the true motion of the earth, but mathematics alone can give the measure of this distance, especially when we consider the resistance of the air; one can see that the deviation takes place toward the east and that it is nothing in the direction of the meridian. In order to compare with experience the formula which expresses the amount of deviation, I have chosen the observations of this phenomenon which were made in 1833 by Professor Reich in the mines of Saxony. The height of the fall was 158.5 meters and M. Reich concluded for the mean of 106 experiments that there was a deviation to the east of 28.33^{mm}. He also found very nearly six seconds for the duration of the fall. By means of this latter datum I have been able to calculate without any hypothesis the coefficient of resistance of the air which the moving body must have experienced, and the formula gives 27.5^{mm} for the deviation; which differs from the experiments by less than a millimeter. In a vacuum this deviation would not have exceeded by a tenth of a millimeter that which occurred in the air; so that in this case the resistance of the air has had only an inappreciable influence.

When the projectile starts from the surface of the earth and is thrown vertically from below upwards with a given velocity, we conceive that during the time of its ascent it must be departing from the vertical toward the west, or in a direction contrary to the rotation of the earth. It would seem that afterwards during its fall it should approach this line and return again very nearly to its point of departure, but this is in fact not the case. When it has arrived at the highest point of its trajectory and has lost all its vertical velocity, the projectile by deviating towards the west has also acquired a horizontal velocity in the same direction, by virtue of which it continues to deviate in this direction, at least during part of its fall. The analytical difficulty which this second case presents is to reconcile, so to speak, the two successive motions, ascending and descending, of the projectile, which are expressed by very different formulæ when we take account of the resistance of the air. In order to apply to an example the formula expressing the total deviation of the moving body when it has fallen back to the earth, I have assumed that this body is a spherical ball fired vertically from an infantry gun, with a velocity of about 400 meters per second. The amount of this deviation varies much with the resistance of the air; by giving successively to the coefficient of this resistance different values which have to each other the ratio of four to three, we find deviation toward the west in both cases but of about one and three decimeters respectively. In a vacuum this deviation would be about fifty-five meters, so that by the greater of these two resistances it is reduced to the fifteenth part of this value.

I have also examined in particular the case where the initial velocity of the projectile is nearly horizontal, which corresponds to firing at a target. In my present memoir will be found the formulæ that relate to this and which express all the circumstances according as the firing is directed toward any given point of the horizon. Here I shall only stop to say that the initial velocity being always about 400 meters and the distance of the target, placed point blank, being equal to 200 meters, then the horizontal and vertical deviations of the ball, due to the motion of the earth, would amount to scarcely half a centimeter, that is to say, they have no sensible influence on the precision of this shooting and it is unnecessary to consider them in practice. These deviations are equally unimportant in firing a cannon, and in all motions which take place in a nearly horizontal direction.

In general the effects that the motion of the earth produces on the motion of a projectile are: first, to increase, either positively or negatively, the interval of time that the moving body takes to go from its point of departure to the point where it falls on the earth; second, to increase the distance of this latter point from the former, which we call the horizontal range. The signs of these increments depend on the direction of the vertical plane in which the projectile is thrown; there is augmentation in one direction and diminution in another; their values are expressed by double integrals, whose numerical calculation would be very laborious.

In addition to this the diurnal motion causes the moving body to leave the vertical plane in which it was initially projected. This gives place to a horizontal deviation, whose value is composed of two distinct parts, expressed also by double integrals. One of these partial deviations is independent of the direction of the vertical plane; it is always toward the right of an observer stationed at the point of departure and facing the trajectory. In our latitude we can consider it as being the principal effect of the rotation of the globe, and happily we can obtain for it limiting values that are easier to calculate than its own value, and which may, if we wish, be deduced numerically by means of the length of the range and the duration of the movement as given by observation, with an accuracy sufficient to appreciate the amount of the deviation. Applying, for example, these limits to such firing of shells as takes place in actual artillery practice, that is to say, at an angle of elevation of 45°, with an initial velocity of 120 meters per second, which gives a range of about 1200 meters, for a projectile of 27 centimeters in diameter, and 51 kilograms in weight (the shell of 10 inches and 104 pounds old French measure); we find that the deviation of the point of impact will be between 90 and 120 centimeters when we aim in a vertical plane, tangent to the parallel of latitude at the point of departure. The deviation will be toward the south if we fire toward the east, and toward the north if we fire toward the west. Calling it a meter and observing that such a deviation in a distance of 1200 meters corresponds to an angle of about three minutes of arc, it follows that in order to be more sure of hitting the mark it will be necessary to aim in a vertical plane to the left of the given plane, and making with that an angle of three minutes. The consideration of this result may influence the accuracy of the aim and the chance of striking the target in exercises where the gunner must seek great precision. The horizontal deviation will be a little less and will be toward the east when we fire toward the north; it will be a little more and toward the west when we fire toward the south. In the firing of a shell at long range, for example, at a distance of about 4,000 meters from the mark, which supposes an initial velocity of a little more than one-third of 800 meters, at the elevation angle of 45°, and for a projectile weighing 90 kilograms and a third of a meter in diameter, the limits of deviation, firing either to the east or to the west, will be very nearly 5 meters and 10 meters, respectively. Estimating then its average amount at 7 or 8 meters, we see that in sieges some buildings and persons have been reached because of the deviation of a shell by the motion of the earth and others have not been from the same reason.

These numbers, and those that we have before given, relate to a mean latitude; they vary with the latitude of the place of the experiment. At the equator when the firing takes place in its plane, the horizontal deviation vanishes while the increase in the duration of the trajectory and in the length of the range attain their maximum values. In high latitudes, on the contrary, it is the deviation which approaches its maximum and the increase of duration which diminishes. At the pole, the horizontal deviation, which is the same at this point for all vertical planes of firing, would exceed by very nearly one-half that which takes place in our latitude. Everywhere the increments of the range and of the time are nothing when the initial velocity is directed in the plane of the meridian.

[The preceding text is followed by a detailed mathematical analysis that need not be reproduced here].

III

ON THE ROTARY ACTION OF STORMS

BY CHARLES TRACY[1]

The investigations of Mr. Redfield and Colonel Reid have accumulated a vast amount of evidence in favor of the propositions they maintain. The tendency of this evidence is to demonstrate, that in the large storms which affect extensive districts, and also in the violent tornadoes which devastate a brief path, there are two motions, the rotary and the progressive; and that the rotary is by far the most violent, and has an uniform direction of revolution, being from right to left if the storm is in the northern hemisphere, and the reverse if it is in the southern hemisphere. That is to say, on our side of the equator the rotation is about the center through the points of the compass, in the order of N. W. S. E., or contrary to the movement of the hands of a watch lying on its back; and south of the equator the rotation is through the points in the order of N. E. S. W., or conformable to that of the hands of a watch.

These propositions, although authorized by induction, have encountered doubts or gained a feeble faith in many minds, for the want of a good cause to assign for the production of the alleged phenomena. Hence the occurrence of rotary storms, and the uniformity of direction of revolution, have been too readily attributed to mere accident; and the notion that a whirlwind, once started by mere chance, contains the elements of growth and stability of motion, has been too easily admitted. An active whirlwind, great or small, undergoes a constant change of substance. As the central portions waste into the ascending column, supplies from the adjacent tranquil air must be drawn into the vortex and set in motion; and if the fresh air is neutral to the circular movement and must acquire velocity from the whirling mass itself, then since "action and reaction are equal and in opposite directions,"

[1] Reprinted from the American Journal of Science and Arts, Vol. XLV, October, 1843, pp. 65–72. Read before the Utica Natural History Society. (Dated Utica, N. Y., February 27, 1843.)

the whirling mass itself must lose just so much velocity as the fresh supply gains. By such a process the forces of the whirlwind would be rapidly exhausted, and its existence must speedily cease. A stable source of momentum, adapted to originate and sustain the uniform rotary movement, is still required; and it is now proposed to develop such a source of momentum in the forces generated by the earth's diurnal revolution.

The velocity of the earth's surface in the daily revolution being at the equator more than one thousand miles an hour, in latitude 60° half as much, at the pole nothing, and varying in intermediate places as their perpendicular distances from the earth's axis, and the atmosphere near the ground everywhere taking in part or wholly the motion of the surface it rests on, important consequences upon aerial currents must follow. A body of air set in motion from the equator northward maintains the equatorial eastward velocity, and when it passes over regions of slower rotation deviates eastward from the meridian, and ultimately describes over the earth's surface a curved line bearing towards the east. A current of air from latitude 45° north, having a due south direction, soon reaches regions moving faster to the east, falls behind them and describes a curve to the west. Winds oblique to the meridian are similarly affected. These familiar matters are referred to here, and illustrated by fig. 1, to elucidate what follows.

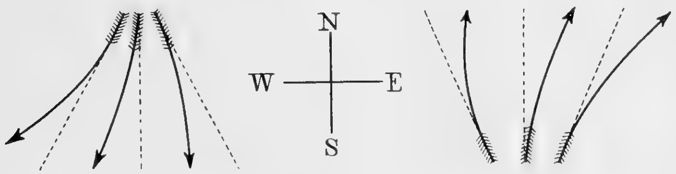


FIG. 1.

The influence of the figure and revolution of the earth upon east and west winds, must also be considered. A parallel of latitude, being a lesser circle of the globe, and at all points equally distant from the pole, necessarily describes upon the earth's surface a curved line. But a direct course, due east at the commencement follows a great circle and parting from the parallel reaches a lower latitude. The due east course continued in a right line describes a tangent to the curve of the latitude. The velocity of the earth's surface at any place, by virtue of the diurnal revolution, has for its direction the line of that tangent; and when the air reposing over any spot is transferred to a region of diverse motion, the *direction*, as well as the *degree*, of its previous force is to be taken from that of the soil on which it previously rested. Hence a wind from the west, if in our hemisphere, will soon be found pursuing a southeasterly course, and crossing successive parallels of latitude.

The labors of Mr. Espy have been directed to the hypothesis of a central ascending column of rarefied air, and centripetal currents from every side rushing towards its base. Without pursuing his reasoning, it will be safe to assume that his collection of facts established the existence of a qualified central tendency of the air, in both the general storms and the smaller tornadoes. He presents a theory to account for such motion, which it is not necessary now to examine. Dr. Hare has proposed another method of accounting for tornadoes—a truly brilliant suggestion—of which it is only to be remarked, at present, that it proceeds on the assumption of a rush of air from all quarters to a central point. It has been attested also, that at large clearing fires in calm weather, creating centripetal currents, the whirlwind and mimic tornado have been produced. In accounting for the whirlwind motion, therefore, the central tendency of the air will be presupposed.

In the case of a large fire kindled in an open plain on a calm day, a small circle about the fire is first acted on by the abatement of pressure on the side next the fire, and thus receives an impulse toward the common center. As this moves in, the next outer circle loses support and begins to move. Each particle of air is moved at first by an impulse towards the center, and during its approach to the central region it receives fresh impulses of the same direction; and if it comes from some distance its velocity is in this way accelerated, until it reaches the space where the horizontal is broken by the upward motion. It is obvious that particles propelled by such impulses would seek the common center in the lines of its radii, and their horizontal forces would be neutralized by impact, if no cause for deviation was at hand. But the great law of deflection which affects the course of the winds applies to the movements of these particles. The particles which seek the center from the northern points are deflected west, while those from southern points are deflected east. The whole rush of air from the northern side of the center, coming like a breeze bears west of the center, while an equal breeze from the southern side bears east of the center. The consequence is that the central body of air, including the fire, is acted upon by two forces which combine to make it turn round to the left. These forces are aided by the deviation of the currents from the easterly and westerly parts of the circle. The breeze from the west extreme inclines to the tangent of the parallel of latitude at its original place of repose, and therefore strikes south of the center into which the impulse it receives would otherwise carry it. The air from the east side also inclines toward the tangent of the parallel of latitude there, which is, obliquely to the north from the radius, and therefore is deflected northwards and strikes north of the center. The breezes from all quarters thus coöperate to produce the result; and all their forces are constant; and act with precision and at great advantage to cause and maintain a whirlwind. A diagram presenting the lines of approach of the particles or streams of air, will explain this result. The black lines in fig. 2 show the deviating currents, from the cardinal points alone, when the area affected by the fire is so small as to require no perceptible curve in those lines.

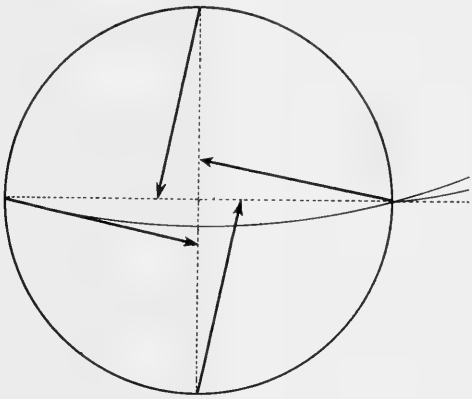


FIG. 2.

Upon the same principle, the tornado, the typhoon, and the widespread storm of the Atlantic, if their currents move toward a central spot, must have a rotary character. The circular motion in the outer portions may be slight, but it is stronger near the center. In every such case the incoming air may be regarded as a succession of rings taken off the surrounding atmosphere and moving slowly at first, but swifter as they proceed towards the center. Each such ring is affected by the law of deviation during its passage. The particles are veering from the radii, in its northern quarter westward, in its southern quarter eastward, in its eastern quarter northward, and in its western quarter southward, and hence the ring begins to revolve when far from the center, turns more and more as it draws near it, and finally as it gathers about the central spot all its forces are resolved into a simple whirl. Ring after ring succeeds, and the whirling action is permanent.

The deflecting power thus applied is not small. The rotary motion of the earth varies as the cosine of the latitude, and the differences of velocity for any differences of latitude are easily computed. The following are samples; being differences of velocity for 1° or 69½ miles of latitude.

	Miles per hour.
Between lat. 2° and 3° diff. of velocity	0.79
Between lat. 3° and 4° diff. of velocity	1.11
Between lat. 10° and 11° diff. of velocity	3.31
Between lat. 23° and 24° diff. of velocity	7.25
Between lat. 42° and 43° diff. of velocity	12.28

The differences of velocity for one mile, or 51.84″ of latitude are as follows:

Latitude.	Difference of velocity for 1 mile north. Feet per minute.
10°	4
23°	9
42°	15.4
43°	15.7
45°	16.3

The deflection of easterly and westerly breezes by reason of the spherical form of the earth, also, can be computed; and it is obviously no less important than the deflection produced in meridional winds. The angle between the courses north and east, at any point, is a right angle; and if two points in the same latitude are taken, it is evident that the obliquity of the north courses from the two points equals the obliquity of the east courses from the same points.

These results show that in the northern states a fire large enough to affect the atmosphere over a few acres may possess the essential force for generating a whirlwind, and may produce it in fact if the day be calm. A large storm, covering the whole country with its centripetal currents, must produce a vortex about the center, which will combine the principal energies of the storm. The tornado and water-spout must revolve with terrific violence.

The necessary condition, centripetal motion, may arise whenever a central spot subjected to intense heat is surrounded by a cool atmosphere. This state of things, on a small scale, may occur on a summer day, upon a ploughed field surrounded by extensive pastures; upon a black and charred clearing in the midst of a cool forest; or at a large clearing fire. Upon a great scale—if an island beneath a tropical sun received upon rocks and sands the intense radiance of a succession of clear, calm, and hot days, and consequent sea breezes from the deep and cool ocean pressed in upon all its shores with the violence of a high wind, it should not cause surprise if these various breezes combined to generate a vast whirlwind; nor if the lofty revolving column should at last leave the place of its origin and traverse the sea, as a hurricane. The cause which first excited the centripetal tendencies of the storm, might be renewed as the upper currents of the atmosphere bore it over other heated spots; and the law of deflection will inevitably transform the central into circular motion. The destructive storms of our sea-coast may have such an origin among the islands of the West Indies, from which they appear to proceed.

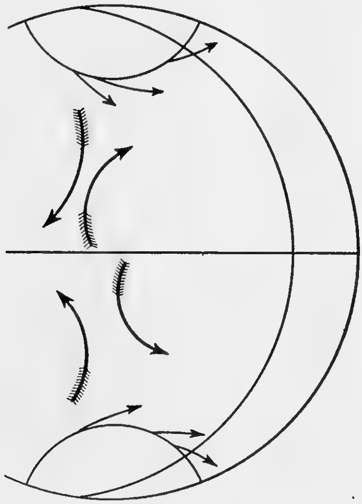


FIG. 3.

In the southern hemisphere the same law of deflection produces contrary results. There the wind which first moves north bends to the west, and the wind which moves south at first turns towards the east, that from the east turns south, and that from the west turns north. Fig. 3 represents these effects. Hence south of the equator storms revolve from left to right, or conformably to the movement of watch hands. Fig. 4 exhibits the rotary action of a storm in the northern hemisphere; fig. 5 the same in the southern hemisphere.

The relative motions of the parts of a small circular space on the earth's surface, by reason of the diurnal revolution, are precisely what they would be if the same circular space revolved upon an axis passing through its center parallel to the axis of the globe. If such space be regarded as a plane revolving about such supposed axis, then the relative motions of its parts are the same as if the plane revolved about its center upon an axis perpendicular to the plane itself; with this modification, that an entire revolution on the axis perpendicular to the plane would not be accomplished in twenty-four hours. Such plane daily performs such part of a full revolution about such perpendicular axis, as the sine of the latitude of its center is of radius. The plane itself—the field over which a storm or tornado or a water-spout is forming—is in the condition of a whirling table. Hence the tendency to rotary action in every quarter of the storm is equal and all the forces which propel the air towards the center coöperate in harmony to cause the revolution.

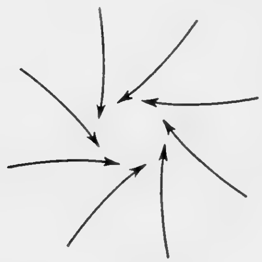


FIG. 4.

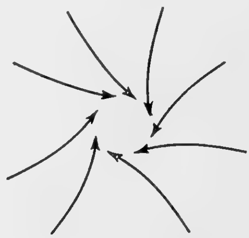


FIG. 5.

Water discharging from a broad basin through a central orifice, is subject to the same law. It forms a vortex which in our hemisphere turns to the left, or against the sun, and in the southern hemisphere must turn to the right or contrary to the sun there.

These rotations of the atmosphere and of water, being from west to east about lines inclined to parallelism with the earth's axis, are singularly coincident in direction with the rotation of the globe, and harmonize with the general mechanism of the heavens.

IV

THE INFLUENCE OF THE DIURNAL ROTATION OF THE EARTH ON CONSTRAINED HORIZONTAL MOTIONS, EITHER UNIFORM OR VARIABLE

AN ABSTRACT BY PROFESSOR A. ERMAN OF A MEMOIR BY PROFESSOR N. BRASCHMANN, WITH ERMAN'S NOTES THEREON

[*Translated from Archiv für Wissenschaftliche Kunde von Russland Vol. XXI, 1862, pp. 52–96 and 325–332*[1]]

The first volume of Braschmann's Theoretical Mechanics, Moscow, 1859, combines to such a high degree condensed and precise presentation with abundance and variety of problems that the best interests of the mathematical study of motion demands its broadest possible dissemination. . . . .

As an example of the treatment of the problems enumerated above, we choose first the theory of relative motion of a free or restricted mass point and then the application of this theory to the so-called Foucault experiment and to other physical problems arising from the rotation of the earth, which first began to be considered and empirically studied in recent years. . . . .

The second practical application that Braschmann makes[2] of his general formulæ for relative motion consists (1) in the remark that every mass moving on the earth's surface along a restricted path exerts a horizontal pressure on the boundary of its path

---

[1] The early publications of William Ferrel relative to the motions of the atmosphere, beginning with 1856, were made accessible by reprints in Professional Papers, Nos. VIII and XII, of the U. S. Signal Service. In these, as in most other memoirs on the subject, the motions of the atmosphere were assumed to be uniform in velocity, but in 1854–1862 Braschmann and Erman gave a notable enlargement of our ideas. Since that date two elaborate memoirs by Dr. Joseph Finger of Vienna (I, 1877; II, 1880) have given further details of the motions of bodies on a rotating spheroid.

C. A.

[2] See his article in Bull. Imp. Acad. Sci. St. Petersburg, Feb. 3, 1860.

which tends toward the right-hand side in northern latitudes but to the left-hand in southern latitudes:

(2) In the determination of the amount of this pressure.

The following is Braschmann's treatment of this problem as given in his Memoir of Feb. 3, 1860.

Equations (1) [general equations of motion, here omitted] determine also the accelerating force that the current of a river exerts on its right-hand bank.

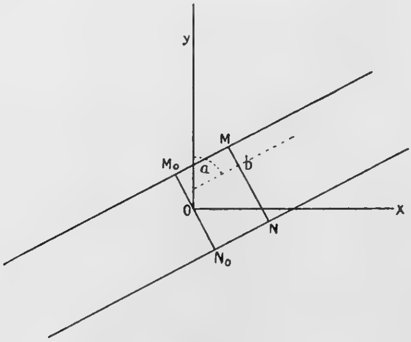


FIG. 1.

Let $M\ N$, fig. 1, be a section of the river, b any point in this section at which the average velocity of the steady stream of water is v, that is to say, the uniform or steady velocity is the result of all other influences combined with that of the acceleration due to the rotation of the earth. Designate by x positive towards the east and y positive towards the north the rectangular coördinates of a point in the horizontal plane through b, neglecting the very small fraction ω^2, we then obtain from equation (1) the following:

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= +2\ \omega \sin\lambda.\ \frac{dy}{dt} \\ \frac{d^2y}{dt^2} &= -2\ \omega \sin\lambda.\ \frac{dx}{dt} \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (2)$$

Let a be the angle between the direction of the current and the positive axes of y, then we have the following integrals[3]

[3] For $x = o$ and $y = o$ or at the origin of coördinates, the steady velocity v and therefore its projections on the x and y axes are

$$\frac{dx}{dt} = v \sin a \qquad\qquad \frac{dy}{dt} = v \cos a$$

$$\frac{dx}{dt} = A + 2\omega \sin \lambda . y = v \sin a + 2\ \omega \sin \lambda . y.$$

$$\frac{dy}{dt} = B - 2\ \omega \sin \lambda . x = v \cos a - 2\ \omega \sin \lambda . x.$$

By substituting these values in equations (2) and neglecting ω^2 we have

$$\frac{d^2x}{dt^2} = +2\ \omega v \cos a \sin \lambda$$

$$\frac{d^2y}{dt^2} = -2\ \omega v \sin a \sin \lambda$$

These equations determine the accelerative force

$$F = \sqrt{\left(\frac{d^2x}{dt^2}\right)^2 + \left(\frac{d^2y}{dt^2}\right)^2} = 2\ \omega v \sin \lambda$$

This shows that the amount of this force is independent of th direction of the current of the stream and moreover that it is directed steadily toward the right-hand bank.

If u is the angle made by this accelerative force with the axis of y or the north line counting it positively around to the right, then

$$\frac{d^2x}{dt^2} = F \sin u$$

$$\frac{d^2y}{dt^2} = F \cos u$$

but

$$F = +2\ \omega v \sin \lambda$$

hence

$$\sin u = +\cos a$$

$$\cos u = -\sin a$$

$$u = a + 90°$$

that is to say, when λ is positive the direction of F is 90° farther toward the right than the direction of the current.

When λ is negative we have

$$\sin u = -\cos a$$

$$\cos u = +\sin a$$

$$u = a + 270°$$

that is to say, in southern latitudes the pressure F will be against the left hand and will be zero at the equator itself.

In order to determine the pressure mF exerted upon the right bank by the whole section MN of the river, let A be the area of this section and ρ the density of the water, then

$$mF = A v \rho . 2 \omega v . \sin \lambda$$

This mF relates of course to the pressure exerted by the mass of water that flows through the section MN in a unit time or by the mass $Av\rho$.

In the preceding paragraphs we have assumed a constant or steady and uniform velocity. It seems unnecessary to say that this condition is rarely fulfilled in practical cases, or that the demonstrated result does not hold good in such cases. But since there is a distinction to be made between a theorem that is not yet demonstrated and one that is clearly shown to be erroneous it will be of interest to see from the following modifications of Braschmann's analysis as summarized in the Comptes Rendus, Paris 1861, how in the case of non-uniform motions along paths restricted to the earth's surface, the direction of the horizontal pressure depends on the variations of the respective components of its velocity.

[The original memoir in the Paris Comptes Rendus contains numerous typographical errors that are corrected by Erman in his Archiv Wiss. Kunde Russland.]

Let X, Y, Z, be the projections (or rectangular components) of the accelerating force and P_x, P_y, P_z the projections of the pressure exerted on a unit surface, at a point whose coördinates relative to a fixed system of rectangular axes are $x_1\ y_1\ z_1$; we have then

$$\left.\begin{aligned} X - \frac{d^2x_1}{dt^2} &= P_x \\ Y - \frac{d^2y_1}{dt^2} &= P_y \\ Z - \frac{d^2z_1}{dt^2} &= P_2 \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

If both the axes and the origin of coördinates are all assumed to be in motion then we have to substitute herein the values of the acceleration at the point $x_1\ y_1\ z_1$. Let x, y, z be the coördinates of this point referred to the moving axes at the time t. Let $\omega_1\ \omega_2\ \omega_3$ be the angular velocities of this point about these axes at the same instant. Let ω be the resultant of these angular velocities and let α, β, γ be the coördinates of the origin of the movable system of axes with reference to the fixed system; we have then the following expression for the accelerations:

$$\left.\begin{aligned}
\frac{d^2x_1}{dt^2} &= \frac{d^2x}{dt^2} + 2\left(\omega_2\frac{dz}{dt} - \omega_3\frac{dy}{dt}\right) + z\,\frac{d\omega_2}{dt} - y\cdot\frac{d\omega_3}{dt} \\
&\quad + \omega_1\,(\omega_1 x + \omega_2 y + \omega_3 z) - \omega^2\,x + \frac{d^2\alpha}{dt^2}. \\
\frac{d^2y_1}{dt^2} &= \frac{d^2y}{dt^2} + 2\left(\omega_3\frac{dx}{dt} - \omega_1\frac{dz}{dt}\right) + x\,\frac{d\omega_3}{dt} - z\,\frac{d\omega_1}{dt} \\
&\quad + \omega_2\,(\omega_1 x + \omega_2 y + \omega_3 z) - \omega^2 y + \frac{d^2\beta}{dt^2} \\
\frac{d^2z_1}{dt^2} &= \frac{d^2z}{dt^2} + 2\left(\omega_1\frac{dy}{dt} - \omega_2\frac{dx}{dt}\right) + y\,\frac{d\omega_1}{dt} - x\,\frac{d\omega_2}{dt} \\
&\quad + \omega_3\,(\omega_1 x + \omega_2 y + \omega_3 z) - \omega^2 z + \frac{d^2\gamma}{dt^2}
\end{aligned}\right\} \quad \cdot \quad (2)$$

[The detailed demonstration of these expressions was given by Braschmann in the Bulletin of the St. Petersburg Academy for 1851 and is repeated in Vol. I, Chap. IV of his Treatise on Theoretical Mechanics and at pp. 74–88 of Erman's Archiv XXI, 1862; its equivalent is found in many recent treatises on mechanics under "constrained motion."]

If the last named initial point or origin of the movable system of coördinates be at the point O on the earth's surface and movable with it about the earth's axis then it has a constant velocity of rotation about this axis. Hence the accelerations of its motion are zero and therefore we have

$$\left.\begin{aligned}
\frac{d^2\alpha}{dt^2} &= 0 \\
\frac{d^2\beta}{dt^2} &= 0 \\
\frac{d^2\gamma}{dt^2} &= 0
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Imagine the three coördinate axes movable with the earth to be so drawn through this point that the xy plane is horizontal, x being positive eastward along the tangent to the small circle of latitude and y positive northward along the tangent to the meridian but z positive downward toward the center of the earth considered as a sphere. Let Ol be the direction of the positive half of the momentary axis of rotation or in our case a line drawn through O parallel to that half of the earth's axis that extends from its center to the south pole; then in general

$$\omega_1 = \omega_2 \cos (lx) \qquad \omega_2 = - \omega \cos (ly) \qquad \omega_3 = \omega \cos (lz)$$

and in our special case

$$\omega_1 = \omega \qquad \omega_2 = - \omega \cos \lambda \qquad \omega_3 = \omega \sin \lambda$$

where λ is the latitude of the place of observation (O) and ω is the angular velocity corresponding to the diurnal rotation of the earth. Since in this case of steady rotation

$$\frac{d\omega_1}{dt} = 0 \qquad \frac{d\omega_2}{dt} = 0 \qquad \frac{d\omega_3}{dt} = 0$$

and considering the conditions expressed in equation (3), therefore in the present problem the general equations (2) become

$$\left.\begin{aligned}
\frac{d^2x_1}{dt^2} &= \frac{d^2x}{dt^2} - 2\,\omega\left(\cos\lambda\frac{dz}{dt} + \sin\lambda\frac{dy}{dt}\right) - \omega^2 . x \\
\frac{d^2y_1}{dt^2} &= \frac{d^2y}{dt^2} - 2\,\omega\sin\lambda\frac{dx}{dt} - \omega^2\cos\lambda\,(-\cos\lambda . y + \sin\lambda . z) - \omega^2 y \\
\frac{d^2z_1}{dt^2} &= \frac{d^2z}{dt^2} - 2\,\omega\cos\lambda\frac{dx}{dt} - \omega^2\sin\lambda\,(-\cos\lambda . y + \sin\lambda . z) - \omega^2 z
\end{aligned}\right\} \quad (2_1)$$

The terms in ω^2 may be omitted because they are very small; but for the same reason the term $2\,\omega\cos\lambda\frac{dz}{dt}$ must be omitted in all those cases in which the gradient of the surface (the rails or the river), and hence also $\frac{dz}{dt}$, is small or zero.

Let a be the angle between the direction of the motion of the point x, y, z and the direction of the positive axis of y and let v be the velocity of this point, then

$$\frac{dx}{dt} = v \sin a$$

$$\frac{dy}{dt} = v \cos a$$

Let v_x v_y v_z be the components of the momentary velocity along the axes x y z and hence their differential quotients with respect to t will be the momentary accelerations in these directions and identical with

$$\frac{d^2x}{dt^2} \qquad \frac{d^2y}{dt^2} \qquad \frac{d^2z}{dt^2}$$

Equation (2_1) now becomes

$$\left.\begin{aligned} \frac{d^2x_1}{dt^2} &= \frac{dv_x}{dt} - 2\,\omega \sin \lambda . v . \cos a \\ \frac{d^2y_1}{dt^2} &= \frac{dv_y}{dt} + 2\,\omega \sin \lambda . v . \sin a \\ \frac{d^2z_1}{dt^2} &= \frac{dv_z}{dt} + 2\,\omega \cos \lambda . v . \sin a \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Now gravity and friction are the accelerative forces acting on a point in contact with the sides of the track or path of constraint. The projections of gravity on the horizontal plane are equal to zero and equally so the projections of the lateral friction on the direction of the lateral pressure disappear. Hence if we designate the acceleration of gravity by g we have the forces

$$X = 0$$

$$Y = 0$$

$$Z = g$$

and equation (4) gives the pressure

$$\left.\begin{aligned} P_x &= \quad - \frac{dv_x}{dt} + 2\,\omega \sin \lambda . v . \cos a \\ P_y &= \quad - \frac{dv_y}{dt} - 2\,\omega \sin \lambda . v . \sin a \\ P_z &= g - \frac{dv_z}{dt} - 2\,\omega \cos \lambda . v . \sin a \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If the horizontal motion is uniform then

$$\frac{dv_x}{dt} = 0 \text{ and } \frac{dv_y}{dt} = 0$$

whence

$$\left.\begin{aligned} P_x &= +2\,\omega \sin\lambda\, v \cos a \\ P_y &= -2\,\omega \sin\lambda\, v \sin a \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \quad (6)$$

whence

$$P = 2\,\omega\, v \sin\lambda$$

where P is the whole pressure exerted in a horizontal direction.

If we substitute in equation (6) successively all values of a between $a = 0$ and $a = 2\pi$ we soon perceive that the pressure P is always perpendicular to the path of the moving mass and if λ is positive the pressure is always directed to the quadrant on the right-hand side of the direction of motion.

If the motion is uniform along the axis of x only, then for positive λ the direction of the pressure P will still be always toward the right-hand of the direction of progress of the mass so long as the value of a lies between 0 and π, i. e., so long as the progress is in a direction between east and south and west.

But if the motion is uniform along the axis of y only, then for positive λ the pressure of P will be directed toward the right of the direction of progress for any value of this latter direction that lies between $a = \pi$ and $a = 2\pi$.

If the motion is not uniform along the axis of x or the axis of y then the direction of the pressure P may be either toward the left or the right of the direction of motion depending on the current values of $\frac{dv_x}{dt}$ and $\frac{dv_y}{dt}$ to an extent and manner easily determined from equation (5).

V

ON THE STEADY MOTIONS OR THE AVERAGE CONDITION OF THE EARTH'S ATMOSPHERE

BY PROF. DR. ADOLPH ERMAN[1]

In the third book of his Mécanique Céleste, Laplace has demonstrated that the atmosphere of a rotating planet is at rest relative to any point of the solid nucleus of this planet and that at the same time any pressure and any density can occur within any level surface of such an elastic fluid, i. e., within any surface that at any point is normal to the resultant of gravity and centrifugal force. In this he assumes that a uniform temperature prevails throughout the elastic fluid.

The surface of the sea is such a level surface and apparently on the strength of the above demonstration by Laplace most physicists assume that the product of gravity by the height of the barometric column[2] which measures the pressure of the air must necessarily be the same everywhere at sea level. They grant that temporary disturbances of the atmospheric equilibrium are accompanied by temporary interruptions of this uniformity of atmospheric pressure, but imagine that these two exceptions are only periodical (viz: variations about a mean condition) which mean must be primarily a condition of rest relative to the earth, and secondarily must be that uniform mean reduced barometric height that one should find from measurements taken during one or many whole years at different points on the earth's surface.

The falsity of every portion of these assumptions was shown many years ago and should have been evident a priori still earlier.

I have found the mean reduced barometric readings for different localities at sea level extremely different. Among others, for instance, the pressure at the polar limits of the two trade wind belts is from two to three Paris lines (0.18 to 0.27 English inches) greater

---

[1] Translated from the Astronomische Nachrichten, No. 1680, February, 1868, Vol. LXX, cols. 369–378.

[2] For brevity I will call this product the reduced barometric height or the pressure of the air.

than at the equator; again, on the Sea of Okhotsk and at Cape Horn it is six Paris lines (0.53 inch) smaller than at latitude 23.5°. At any point in the interior of a continent whose altitude above sea level is known by geometrical measurements (e. g., railroad levellings), the observed mean air pressure can be reduced to the value appropriate to the sea level vertically below it and thus gradually an empirical expression can be found for the pressure at sea level and, therefore, for the atmospheric pressure in general, as a function of the longitude, latitude, and altitude above sea level, or the distance from the center of the earth. Individual contributions that I have made to this subject leave no doubt that above the solid land, as also above the ocean, the mean air pressure at any lev layer differs very much according to the latitude and longitude.

The first of the above stated two fundamental assumptions (v that the atmosphere is in a state of rest relative to the globe also decidedly negatived by ordinary observation. In one p tion of the atmosphere, lying between the parallels of +25° an −25° the air is at every minute and, therefore, on the averag of all time, in that state of strong steady flow that we call the trade wind; in other words, therefore, the average or permanent condition of 0.4225 of the total mass of the atmosphere is a regular flow tha is certainly not to be ignored. In the remaining portions of th atmosphere, however, the movements are less steady as to time. But when the successive motions of the air, during one or m whole years at any place are combined into one resultant, the general this resultant differs from zero and in such a way tha direction and velocity depend upon the coördinates of the loca and are independent of the years or number of years for which computation was made.

Therefore, after eliminating the influence of periodic and accid tal circumstances and in direct opposition to the above-gi assumptions of the physicists, we find that the earth's atmospher shows the following phenomena:

(1) A current whose direction and velocity are independent of the time and which, therefore, at every place depend only on the coördinates of locality.

(2) At any level surface (or one that is perpendicular to the resultant of the explicit forces) the atmosphere is under a pressure that varies with the coördinates of the points of this surface, but is constant as regards the time.

These two observed facts contradict the results of the analysis of Laplace only because in place of a certain assumption of his analysis precisely the opposite holds good in the earth's atmosphere. The temperature which Laplace assumed to be uniform throughout the fluid is in the earth's atmosphere extremely unequal and, indeed, not only so in respect to the periodical portion of its expression depending on the time, but also as to the other permanent portion, which we ordinarily call the mean temperature of the place. These mean temperatures, which are invariable as to time, are, as is well known, a function of the coördinates of the location to which they belong and, not only the analytical form of this function, but also the constants that enter it are already known with considerable approximation.[3] In accordance with these re-ults of experience, it is certainly worth while to investigate the ollowing problem:

What is the nature of the movement and how great is the pressure reduced barometric reading at any point of an atmosphere for which oth the resultant of gravity and centrifugal force and, also, the temperature are expressed as functions of the coördinates of locality and are independent of the time?

The remarks that follow seem to me to prove that this problem can be solved.

If at any point of a liquid, or an elastic fluid or gas, at the time t and with reference to three rectangular axes, we have the coördi-tes x, y, z; the explicit forces X, Y, Z; the velocities of motion u, w; and if at this same point x, y, z, we have the density ρ, the ssure p and the temperature τ, then by combining the conditions quilibrium of this fluid with the general theorem for the move-nt of any system, remembering[4] that

$$u = \frac{\partial x}{\partial t}; \quad v = \frac{\partial y}{\partial t}; \quad w = \frac{\partial z}{\partial t};$$

we obtain

$$\left.\begin{aligned}
\frac{1}{\rho}\frac{dp}{dx} &= X - \frac{\partial u}{\partial t} - u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - w\frac{\partial u}{\partial z} \\
\frac{1}{\rho}\frac{dp}{dy} &= Y = \frac{\partial v}{\partial t} - u\frac{\partial v}{\partial x} - v\frac{dv}{\partial y} - w\frac{\partial v}{\partial z} \\
\frac{1}{\rho}\frac{dp}{dz} &= Z - \frac{\partial w}{\partial t} - u\frac{\partial w}{\partial x} - v\frac{\partial w}{\partial y} - w\frac{\partial w}{\partial z}
\end{aligned}\right\} \quad \ldots\ldots \quad (1)$$

ee for example Erman's Memoirs in the Archiv Wiss. Kunde, Russland.
he translator has taken the liberty of substituting ∂ for partial differ- and d for total differential instead of Erman's notation.—C. A.

and, as the condition that the mass of each particle of fluid shall remain invariable, we have

$$o = \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} \quad . \ . \ . \ . \ . \quad \text{(II)}$$

In these equations the variations with the time, when x, y, z remain constant, are expressed by the differential with regard to t.

Since now in our present case of the earth's atmosphere, we assume that at every point the pressure, temperature, motion and, therefore, the density are invariable as to time, therefore, we may in equation (II) substitute

$$\frac{\partial \rho}{\partial t} = 0,$$

and also in equations (I)

$$\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = \frac{\partial w}{\partial t} = 0$$

Furthermore, we have

$$\rho = \frac{p \,.\, \delta}{P\,(1 + k\tau)} \quad \text{or} \quad \frac{P\,(1 + k\tau)}{\delta} = \frac{p}{\rho}$$

where P is that value of the pressure (or the reduced barometric reading that measures the pressure) for which the atmospheric air at the temperature o° C. is δ times heavier than the mercury of the barometric column and k is the coefficient of expansion of the air for one unit of the thermometric scale that is used to measure the temperature or τ.

Since τ is assumed to be independent of the time it can be expressed as

$$\tau = f\,(x, y, z)$$

where f indicates a known function, which, as above stated, is now approximately known; therefore we may also write

$$\frac{p}{\rho} = \frac{P}{\delta} + \frac{Pk}{\delta} f\,(x, y, z) = F\,(x, y, z)$$

where $F\,(x, y, z)$ again indicates a known function which for brevity we indicate by F.

Therefore, we now have

$$\rho = \frac{p}{F\,(x, y, z)} = \frac{p}{F.} \text{ or } \frac{1}{\rho} = \frac{F.}{p}$$

and the left-hand portions of the equations (I) become respectively

$$\frac{1}{\rho}\cdot\frac{\partial p}{\partial x} = \frac{F}{p}\cdot\frac{\partial p}{\partial x} = F.\frac{\partial \log p}{\partial x} = X$$

$$\frac{1}{\rho}\cdot\frac{\partial p}{\partial y} = \frac{F}{p}\cdot\frac{\partial p}{\partial y} = F.\frac{\partial \log p}{\partial y} = Y$$

$$\frac{1}{\rho}\cdot\frac{\partial p}{\partial z} = \frac{F}{p}\cdot\frac{\partial p}{\partial z} = F.\frac{\partial \log p}{\partial z} = Z$$

where log indicates the natural logarithm.

Since it is known that the components x, y, z, of the resultant of gravity and centrifugal force, as also the components u, v, w of the velocity of a fluid particle, depend only on x, y, z or on the variable coördinates by which we express the location of any point on the sphere, therefore, as is well known, there are two functions of x, y, z, which I will designate by V and φ respectively[5] which are determined by the relations

$$X = \frac{\partial V}{\partial x};\ Y = \frac{\partial V}{\partial y};\ Z = \frac{\partial V}{\partial z}$$

and

$$u = \frac{\partial \varphi}{\partial x};\ v = \frac{\partial \varphi}{\partial y};\ w = \frac{\partial \varphi}{\partial z}$$

If these values are substituted in equation (I) and the sum is taken after multiplying the first, second, and third respectively by dx, dy, dz, and if we recall that

[5] These functions are now generally called the force potential, V, and the velocity potential, φ, or the potential function for the external forces and the potential function for the resulting velocity. If such functions actually exist there can be no discontinuous whirls, and if the whirls exist then there can be no such functions.—C. A.

$$\frac{\partial u}{\partial x} \cdot u \cdot dx + \frac{\partial v}{\partial x} \cdot u \cdot dy + \frac{\partial w}{\partial x} \cdot u \cdot dz =$$

$$= \frac{\partial \varphi}{\partial x} \left\{ \frac{\partial \left(\frac{\partial \varphi}{\partial x} \right)}{\partial x} \cdot dx + \frac{\partial \left(\frac{\partial \varphi}{\partial y} \right)}{\partial x} \cdot dy + \frac{\partial \left(\frac{\partial \varphi}{\partial z} \right)}{\partial x} \cdot dz \right\}$$

$$= \frac{\partial \varphi}{\partial x} \left\{ \frac{\partial \left(\frac{\partial \varphi}{\partial x} \right)}{\partial x} \cdot dx + \frac{\partial \left(\frac{\partial \varphi}{\partial x} \right)}{\partial y} \cdot dy + \frac{\partial \left(\frac{\partial \varphi}{\partial x} \right)}{\partial z} \cdot dz \right\}$$

$$= \frac{\partial \varphi}{\partial x} \cdot d \left(\frac{\partial \varphi}{\partial x} \right)$$

and recall that

$$\frac{\partial \varphi}{\partial y} \cdot d \left(\frac{\partial \varphi}{\partial y} \right) \text{ and } \frac{\partial \varphi}{\partial z} \cdot d \left(\frac{\partial \varphi}{\partial z} \right)$$

result in a similar manner from the six remaining terms of the right-hand side of the summation, then there results

$$\frac{1}{\rho} dp = F \cdot d \log p = dV - \tfrac{1}{2} d \left\{ \left(\frac{\partial \varphi}{\partial x} \right)^2 + \left(\frac{\partial \varphi}{\partial y} \right)^2 + \left(\frac{\partial \varphi}{\partial z} \right)^2 \right\} \quad \text{(A)}$$

$$= F \cdot d \log \rho + d F.$$

Again, from equation (II), after substituting the values of u, v, w and dividing by ρ, there results[6]

$$o = \left\{ \frac{\partial \log \rho}{\partial x} \cdot \frac{\partial \varphi}{\partial x} + \frac{\partial \log \rho}{\partial y} \cdot \frac{\partial \varphi}{\partial y} + \frac{\partial \log \rho}{\partial z} \cdot \frac{\partial \varphi}{\partial z} \right\}$$

$$+ \left\{ \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} \right\} \quad \text{(B)}$$

The equation (A) can be more perspicaciously replaced by the following

---

[6] Because ρ is assumed not to vary with t, therefore $\frac{\partial \log \varphi}{\partial t} = 0$, and this term drops out.—C. Abbe.

$$\left.\begin{aligned}
\frac{\partial \log \rho}{\partial x} &= \frac{1}{F}\cdot\frac{\partial V}{\partial x} - \frac{\partial \log F}{\partial x} - \frac{1}{2F}\cdot\frac{\partial\left\{\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right\}}{\partial x}\\
\frac{\partial \log \rho}{\partial y} &= \frac{1}{F}\cdot\frac{\partial V}{\partial y} - \frac{\partial \log F}{\partial y} - \frac{1}{2F}\cdot\frac{\partial\left\{\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right\}}{\partial y}\\
\frac{\partial \log \rho}{\partial z} &= \frac{1}{F}\cdot\frac{\partial V}{\partial z} - \frac{\partial \log F}{\partial z} - \frac{1}{2F}\cdot\frac{\partial\left\{\left(\frac{\partial\varphi}{\partial x}\right)^2+\left(\frac{\partial\varphi}{\partial y}\right)^2+\left(\frac{\partial\varphi}{\partial z}\right)^2\right\}}{\partial z}
\end{aligned}\right\} \quad (A^*)$$

and we can, therefore, by simple substitution of A* in B construct an equation in which, in addition to the partial differential quotients of the first and second order of the function $\varphi = \varphi(x\ y\ z)$ there enter only the known functions $F = F\ (x, y, z)$ and $V = V\ (x, y, z)$ and their first differential coefficients.

If x, y and z are replaced by the angular coördinates of any point of the atmosphere so that

$$\begin{aligned}
x &= r \cos\lambda \sqrt{1-\mu^2} &&= r\cos\beta\cos\lambda\\
y &= r \sin\lambda \sqrt{1-\mu^2} &&= r\cos\beta\sin\lambda\\
z &= r\,\mu &&= r\sin\beta
\end{aligned}$$

where r is the distance from the center of the earth to any point in the atmosphere; λ is the longitude of this point; β is the latitude and μ is the sine of the latitude, then, as is well known, we have[7]

$$V = (\gamma R^2)\cdot\frac{1}{r} + \left(\frac{q}{3R}\right)\cdot r^2 - \frac{q}{2R}\left(\mu^2 - \tfrac{1}{3}\right) r^2$$

where for the surface of the earth and at the equator, we have

$r = R$

γ = acceleration due to the attraction of the earth

q = acceleration due to centrifugal force $= \dfrac{\gamma}{289}$.

---

[7] This expression assumes that gravity and centrifugal force are the only external forces and thus ignores viscosity or internal friction and the resistance of the earth's surface and the attractions of sun and moon.

This expression for V, as well as those that must obtain for F, for φ and for ρ respectively, when we write these out as functions of μ, λ and r, all possess the properties that Laplace has demonstrated for all functions of this kind that have definite real values for constant r and for all values of λ, from 0° to 360° and of μ from -1 to $+1$. That is to say, since this latter condition (the having a definite real value) is evidently fulfilled in the earth's atmosphere as to temperature τ, density ρ, and the function φ whose differential coefficients are the component velocities, therefore, in accordance with the Laplacian theorem just referred to, we may similarly assume for V, for F, for φ, and for ρ, respectively such expressions as the function

$$\alpha P^o + \beta P' + \gamma P'' \ldots . + \nu P^{(n)} + \ldots .$$

in which, as we pass from one to another of these four functions, V, F, φ and ρ, there occur:

(1) Those coefficients α, β, γ . . . υ which only vary with r (2) the constant numbers that enter into P^0, P^1, P^2, P^n, as defined in the next following paragraph.

In general P^n is defined by the partial differential equation

$$0 = \frac{\partial \left\{ (1-\mu^n) \dfrac{\partial P^{(n)}}{\partial \mu} \right\}}{\partial \mu} + \frac{\left(\dfrac{\partial^2 P^{(n)}}{\partial \lambda^2} \right)}{1-\mu^2} + n(n+1) P^{(n)}.$$

and from this it follows explicitly that

$$P^{(n)} = B_n^o \, X^{(n)} + (A'_n \sin \lambda + B'_n \cos \lambda) \times \frac{(1-\mu^2)^{\frac{1}{2}}}{n} \cdot \frac{\partial X^{(n)}}{\partial \mu}$$

$$+ \ldots . (A_n^i \sin i\lambda + B_n^i \cos i\lambda) \frac{(1-\mu^2)^{i/2}}{n(n-1) \ldots . (n-i+1)} \cdot \frac{\partial^i X^{(2)}}{\partial \mu^i} + \ldots .$$

where B_n^0, A_n', B_n' are constant numbers and

$$X^{(n)} = \mu^n - \frac{n(n-1)}{2(2n-1)} \mu^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2 . 4 . (2n-1)(2n-3)} \mu^{n-4} - \text{etc.}$$

Since the development of each function, V, F, φ, ρ, in the form

$$aP^o + \beta P' + \text{etc.},$$

is only possible in one manner and since it always gives a converging series, therefore, each of the functions occurring in equation (B) consists of a limited and, in fact, probably a small number of terms,

$$\alpha P^o, \beta P', \gamma P'', \text{etc.},$$

which altogether constitute a series progressing according to the whole powers of

$$\mu \text{ and } (1-\mu^2)^{\frac{1}{2}}$$

whose terms are multiplied into the sines and cosines of multiples of λ. Furthermore, since the terms of this nature, in equation (B), resulting from the development of V and F contain respectively only a well-known function of r, while, on the other hand, the terms arising from the differential quotients of φ contain the functions α, β, γ of this same form and the constants A, B, C—which are the only unknown quantities of the problem—therefore, these latter must be determined by equating to zero each of the sums of known and unknown terms that in equation (B) are multiplied by

$$\mu^q(1-\mu^2)^{\frac{1}{2}} \sin i\lambda \text{ or } \mu^q(1-\mu^2)^{\frac{1}{2}} \cos i\lambda .$$

In order to practically execute the determination of the velocity function φ, for a given temperature function F, we can easily convert that form of the latter equation which results from the combination of equations (A^*) and (B) into the equivalent differential equation in r, μ, and λ whose specialization then leads directly to the desired end.

The two following relations between any two functions, ϕ and ϕ', of the coördinates x, y, z are easily demonstrated

$$\frac{\partial\phi}{\partial x}\cdot\frac{\partial\phi'}{\partial x}+\frac{\partial\phi}{\partial y}\cdot\frac{\partial\phi'}{\partial y}+\frac{\partial\phi}{\partial z}\cdot\frac{\partial\phi'}{\partial z}=\frac{\partial\phi}{\partial r}\cdot\frac{\partial\phi'}{\partial r}+\frac{\partial\phi}{\partial\mu}\cdot\frac{\partial\phi'}{\partial\mu}\cdot\frac{1-\mu^2}{r^2}$$
$$+\frac{\partial\phi}{\partial\lambda}\cdot\frac{\partial\phi'}{\partial\lambda}\frac{1}{(1-\mu^2)r^2}.$$

and

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}=\frac{\partial\left\{(1-\mu^2)\frac{\partial\phi}{\partial\mu}\right\}}{\partial\mu}+\frac{\left(\frac{\partial^2\phi}{\partial\lambda^2}\right)}{1-\mu^2}+r\,\frac{\partial^2(r\phi)}{\partial r^2}$$

whence it follows that equation (B) may be written thus:

$$\left.\begin{aligned}
0 &= \left(\frac{\partial V}{\partial r} - \frac{\partial F}{\partial r}\right)\frac{\partial \varphi}{\partial r} + \left(\frac{dV}{d\mu} - \frac{\partial F}{\partial \mu}\right)\frac{1-\mu^2}{r^2}\cdot\frac{\partial \varphi}{\partial \mu} + \left(\frac{\partial V}{\partial \lambda} - \frac{\partial F}{\partial \lambda}\right)\frac{1}{(1-\mu^2)r^2}\cdot\frac{\partial \varphi}{\partial \lambda} \\
&+ \frac{1}{2}\cdot\frac{\partial\left\{\left(\frac{\partial \varphi}{\partial r}\right)^2 + \left(\frac{\partial \varphi}{\partial r}\right)^2\frac{1-\mu^2}{r^2} + \left(\frac{\partial \varphi}{\partial \lambda}\right)^2\frac{1}{(1-\mu^2)r^2}\right\}}{\partial r}\cdot\frac{\partial \varphi}{\partial r} + \\
&+ \frac{1}{2}\cdot\frac{\partial\left\{\left(\frac{\partial \varphi}{\partial r}\right)^2 + \left(\frac{\partial \varphi}{\partial \mu}\right)^2\frac{1-\mu^2}{r^2} + \left(\frac{\partial \varphi}{\partial \lambda}\right)^2\frac{1}{(1-\mu^2)r^2}\right\}}{\partial \mu}\cdot\frac{1-\mu^2}{r^2}\cdot\frac{\partial \varphi}{\partial \mu} \\
&+ \frac{1}{2}\,\frac{\partial\left\{\left(\frac{\partial \varphi}{\partial r}\right)^2 + \left(\frac{\partial \varphi}{\partial \mu}\right)^2\frac{1-\mu^2}{r^2} + \left(\frac{\partial \varphi}{\partial \lambda}\right)^2\frac{1}{(1-\mu^2)r^2}\right\}}{\partial \lambda}\cdot\frac{(1-\mu^2)}{r^2}\cdot\frac{\partial \varphi}{\partial \lambda} \\
&+ F\left\{\frac{\partial\left\{\left(1-\mu^2\right)\frac{\partial \varphi}{\partial \mu}\right\}}{\partial \mu} + \frac{\left(\frac{\partial^2 \varphi}{\partial \lambda^2}\right)}{1-\mu^2} + r\,\frac{\partial^2 (r\varphi)}{\partial r^2}\right\}
\end{aligned}\right\}\ \text{(C)}$$

where, finally, V, F, and φ are each to be replaced by a converging series of the form

$$aP^o + \beta P' \; . \; . \; . \; . \; + \nu P^{(n)}$$

and where the series for V and F will contain known functions of r and known constants, but the series for φ will contain similar terms whose functions and constants are to be determined.

In order to obtain an approximate idea of the practical solution we may take the above given value of V,

$$V = (\gamma R^2)\ \frac{1}{r} + \left(\frac{q}{3R}\right)r^2 - \left(\frac{q}{2R}\right)r^2\ (\mu^2 - \tfrac{1}{3}),$$

which agrees with the form of the converging series when we put

$$a = (\gamma R^2)\frac{1}{r} + \left(\frac{q}{3R}\right)r^2$$

$$\beta = \delta = \varepsilon = \, . \, . = 0$$

$$\gamma = -\left(\frac{q}{2R}\right)r^2$$

and further take for the function of F the following.

$$F = a + br + c\ (\mu^2 - \tfrac{1}{3})$$

which agrees with the form of the converging series by putting

$$\alpha = (a + br)$$
$$\beta = \delta = \varepsilon = \dots \quad = 0$$
$$\gamma = + c$$

In agreement with these assumptions determine the functions of r that I will in general indicate by

$$f^{(n)}_{(m)}$$

and the constants

$$A^{(n)}_{(m)}, \quad B^{(n)}_{(m)}, \text{ etc.}$$

that enter into the following general expression for φ,

$$\varphi = f_o^{\,o} + f_1^{\,o}(\mu) + f_1'(A_1' \sin\lambda + B_1' \cos\lambda)(1-\mu^2)^{\frac{1}{2}}$$
$$+ f_2^{\,o}(\mu^2 - \tfrac{1}{3}) + f_2'(1-\mu^2)^{\frac{1}{2}}. \quad \mu\,.(A_2' \sin\lambda + B_2' \cos\lambda)(1-\mu^2)$$
$$+ f_2^{\,2}(A_2^{\,2} \sin 2\lambda + B_2^{\,2} \cos 2\lambda)$$
$$+ f_3^{\,o}\left(\mu^3 - \frac{3}{2}\cdot\frac{2}{5}\cdot\mu\right) + f_3'\left(1-\mu^2\right)^{\frac{1}{2}}\left(\mu^2 - \frac{1}{5}\right) \times$$
$$\times (A_3' \sin\lambda + B_3' \cos\lambda) + \dots$$

From equation (C) it is evident that in this special case the terms in φ that are multiplied by functions of λ must disappear and that therefore also the direction and velocity of the steady wind must be as independent of the longitude of the place as is the assumed distribution of temperature.

If we have thus carried out the determination of φ, then, from equation (A) there follows

$$\log p = \int \frac{dv}{F} - \int \frac{d\left\{\left(\frac{\partial\varphi}{\partial r}\right)^2 + \left(\frac{\partial\varphi}{\partial\mu}\right)^2 \cdot \frac{1-\mu^2}{r^2} + \left(\frac{\partial\varphi}{\partial\lambda}\right)^2 \frac{1}{(1-\mu^2)\,r^2}\right\}}{2F}$$
$$+ \text{Constant.}$$

and since the quantities under the integration sign can also be developed in series of the kind above considered, then, for all points of the atmosphere, we shall know

$$\log p = \log b + \tfrac{1}{2}\log\left\{\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2\right\}$$

where b designates the mean barometric pressure, as soon as we have determined the constant of integration by observation of the barometer at only one point, for which also the force of gravity,

$$\left\{ \left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2 \right\}^{\frac{1}{2}}$$

shall have been given.

If the mean barometric pressures computed in this manner be compared with the observed pressures, we get a sharp control over the theory.

It is only when in this way it shall have been shown that the observed steady components of the motions of the atmosphere and of the barometric pressures are not properly represented, that we shall be justified in assuming that the friction of the particles of air against each other and against the earth's surface exert a sensible influence on the phenomena. In this case, and in so far as the friction is assumed to be proportional to the square of the velocity and uniform throughout the atmosphere, we shall have to replace equation (A) by the following:

$$Fd \log \varphi = dV - dF - \tfrac{1}{2} d \left\{ \left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2 \right\}$$
$$- \; C \left\{ \left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2 + \left(\frac{\partial \varphi}{\partial z}\right)^2 \right\}.$$

By the introduction of the fourth term in the right-hand side of this equation and of the undetermined constant, C, nothing of importance is changed in the development above considered.

VI

THE LIMIT OF THE ATMOSPHERE OF THE EARTH

BY DR. A. KERBER

(Dated Chemnitz, February, 1881)

[*Annalen der Physik und Chemie, New series XIV, whole series 250. Leipzig, 1881, pp. 117–128*]

I. THE CARDINAL POINTS OF THE ATMOSPHERE

For small zenith distances the atmosphere constitutes an optical system of refracting media separated by centered spherical surfaces of small aperture, so that the theory of such optical systems developed by Gauss and Mobius[1] can be applied to it. The important matter is the determination of the "Cardinal points" by this theory. It is well known that the cardinal points are:

(1) The principal foci f and f' in the first medium A and the last medium A' (fig. 1). At either of these points are united the rays that come through the opposite medium parallel to the optical axis ff'.

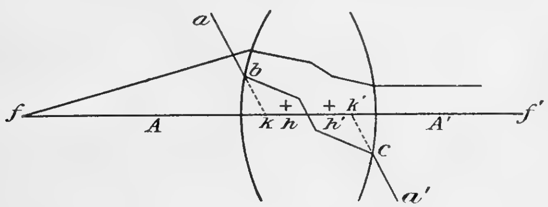


FIG. 1.

(2) The nodal points k and k' having the property that every ray (ab) passing through the first medium in such a direction as would pass through k, when it reaches the second medium goes onward in a direction $c\ a'$ passing through k' parallel to its initial direction ab.

---

[1] Gauss: Dioptrische Untersuchungen, Goettingen, 1840. Compare also Helmholtz, Physikalische Optik. Braunschweig, 1861. In the present article I refer to the Physik of Mousson, which is widely used.

(3) The principal points h and h' for which object and image are identical.

As in all complicated systems whose constitution is not known, so here, the location of these cardinal points can only be found experimentally[2] based on the astronomical determinations of the refraction of light by the atmosphere.

Let m in fig. 2 be the center of the earth, c the location of the observer; cc' a small arc of a great circle of the earth; bb' the intersection of the circle with the boundary of the atmosphere; a a fixed star; abc a pencil of rays from a toward c which is refracted at c toward the direction cf' so that the apparent zenith distance ζ determines the astronomical refraction ρ; am the axis of this optical system whose first medium is the vacuum for which the refractive index is $n = 1$, and whose last medium is the lowest stratum of air whose refractive index is n', and which is assumed to have no limit.

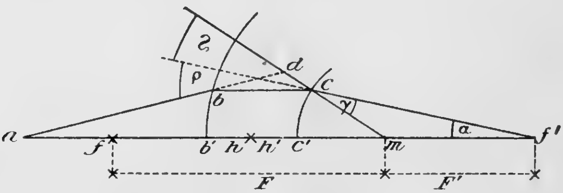


FIG. 2.

The nodal points of the atmosphere coincide at the center of the earth. For, because of the concentric boundaries of the refracting media a pencil of light passes through the atmosphere in a curve whose tangents are never parallel to each other, so that only one ray, moving in the medium n' toward m, can proceed farther in the same direction.

The location of the second focal point f' is easily found when we consider the ray starting from a as originally parallel to the axis. It is deflected from its initial direction by the amount of astronomical refraction ρ and in the last medium finally proceeds toward the second focus of the system or f'. The corresponding focal distance F' is found from the triangle $c.\ m.\ f'$; in this triangle we have $c\ m = R$, $\gamma = \zeta$ and since in this case ab is parallel to af' therefore $\alpha' = \rho$ hence

$$F' = \frac{\zeta}{\rho} R. \qquad (1)$$

[2] Mousson: Physik, 2d Edition, section 731; 3d Edition, section 810.

The ratio of the astronomical refraction to the apparent zenith distance is $\frac{\rho}{\zeta} = 57.3''$ according to the current tables of refraction for the average condition of the atmosphere,[3] so that we can write

$$\frac{\rho}{\zeta} = 57.3'' (1 + \Delta) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

where $57.3''\Delta$ represents any possible error of observation that we include in our computations in order to show the degree of accuracy of the results. Hence we have

$$\left.\begin{aligned} F' &= \frac{R}{57.3''}(1 + \Delta) \\ &= \text{about } 22\,918\,400 \text{ kilometers for } R = 6366.7 \text{ kilometers.} \end{aligned}\right\} \quad (3)$$

The first focal distance of the atmosphere follows from the relation according to which the distance of the two focal points from the respective nodal points is inversely proportional to the ratio of the refractive indices of the respective media.[4] Hence we have

$$F = n' F' = \frac{n' R}{57.3''}(1 + \Delta) = \text{about } 22\,924\,900 \quad . \; . \; . \quad (4)$$

where we assume the coefficient of refraction for the average condition of the atmosphere ($b = 0.752$ meters, $t = 9.3°$ Cent.) to be $n' = 1.000282$ according to Ketteler's determination.

All celestial bodies except the moon are farther removed from us than the focal distance F. Hence the atmosphere produces as a whole and at the focus inverted real images of these. The convergent pencil of rays coming through the atmosphere enters the eye so that it also unites the pencil into an inverted image within the focus φ of the whole eye (or it falls short of the retina) and thus there forms on the retina under all circumstances for a perfect far-sighted eye a circle of diffuse light, no sharp image, and thus the spread of the stellar image over the retina finds its physiological explanation.

On the other hand, the atmosphere produces a correct upright virtual image of the moon because she is within the front focal dis-

---

[3] Bruhns: Astronomische Strahlen-brechung, p. 19.

[4] Mousson: 2d Edition, section 731; 3d Edition, section 809 (4).

tance and the eye receives a diverging pencil of rays. Thus the inverted image produced by the two optical systems combined, the eye and the atmosphere, lies inside φ counting from the retina forward; but on account of the great distance of the objects, always so far from the retina that it requires an extraordinary power of accommodation to produce a sharply defined image on the retina. The muscular tension thus excited explains the apparent floating of the moon in the atmosphere. But even with this extraordinary accommodation the image of the moon will only just attain the nerves of the retina, and since this partial touching also occurs in ordinary vision for objects that are at a definite terrestrial distance that we may call D, therefore the eye locates the moon also at the same distance D because it produces the same sensation or excitement on the retina, whereby is explained the apparent floating of the moon at a relatively nearby point in the atmosphere.[5]

In relation to the location of the principal point, h, its distance from the corresponding focus is equal to the distance of the opposite focus from its nodal point.[6]
Therefore we get from fig. 2

$$fh = F' \text{ and } f'h' = F$$

whence

$$mh = fm - fh = F - F'$$

and also

$$mh' = f'h' - f'm = F - F'$$

consequently

$$mh = mh' = F - F'$$

and thence by substituting from equations (3) and (4)

$$mh = mh' = \frac{(n' - 1)\ R}{57.3''}(1 - \Delta) = \text{about } 6463 \text{ kilometers} \quad . \quad (5)$$

Therefore the two principal points, like the nodal points, coincide in one point h, that is 6463 km. distant from the center of the earth or $96.3 = 6463 - 6366.7$ above the surface of the earth at c (fig. 2 or fig. 3). Therefore according to the definition of the principal points the object and the image coincide at the point h, that is to

[5] We determine D experimentally by measuring the distance at which an intense flame is sharply seen.

[6] Mousson: 2d Edition, section 731 (2); 3d Edition, section 809 (2).

say, all rays that in vacuo are directed towards h, diverge after their passage through the atmosphere from this same point h. If, for instance we imagine a planetary nebula between any fixed star and the earth and the star-like image of the nebula located at the point h (see fig. 3), which is now to be considered as the luminous

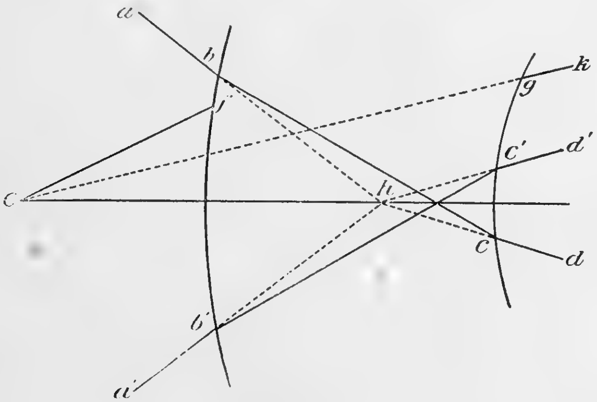


FIG. 3.

object seen through our atmosphere, then will the rays ab converging toward h be so refracted by the atmosphere, that on their entrance into the last medium n' they will appear to diverge from h in the direction cd, and hence an identical virtual image should be seen at that same point (h) at which a real star-like image must have existed if there had been no atmosphere.

II. FIRST APPROXIMATION TO THE HEIGHT OF THE ATMOSPHERE

By reason of the nature of the curve of the beam of light $abcd$, which has its concave side toward the center of the earth, it is evident that the principal point must lie within the atmosphere. For if e, fig. 3, were the principal point, then a pencil of rays that in vacuum may have the direction ef must necessarily on its entrance into the medium n' go on farther in the direction egk, if an identical image of the object is to be formed at e; but this is impossible because ef and gk cannot be tangents to the same curve at f and g. Hence therefore it follows that $H > ch$, that is to say, according to the note on equation (5),

$$H > 96.3 \text{ kilometers. . . . . (6)}$$

But a more accurate determination results at once from the following consideration. Since for small zenith distances the curved path of a pencil of light is very nearly an arc of a circle of very large radius of curvature[7] therefore the tangents (to these curves) *bh* and *ch*, fig. 3, can be considered as equal and because for small zenith distances the tangents can be exchanged for the distances of the principal points from the limits of the atmosphere, therefore we have approximately

$$H = 2\ ch$$

or from equation (5)

$$H = 2R\left[\frac{n' - 1}{57.3''}(1 - \varDelta) - 1\right] = \text{about } 192.6 \text{ kilometers} \quad (7)$$

Hence the determination of *H* from the observations of the twilight arc as given by Alhazen (leading to the value of 79 kilometers) is far too small, and in fact it follows from Fresnel's formula for the intensity of reflected unpolarized light that the argumentation by Alhazen by no means excludes the existence of still higher strata of air.

According to Fresnel's formula, if the incident light has the intensity unity, and *e* and *b* indicate the angles of incidence and refraction, then the intensity of the reflected light is

$$J'' = \frac{1}{2}\frac{\sin^2(e-b)}{\sin^2(e+b)} \cdot \left[1 + \frac{\cos^2(e+b)}{\cos^2(e-b)}\right]$$

Since in one case the reflection takes place at the thinner layer therefore the refractive index is $\frac{n}{n + \delta n}$ and we have

$$\sin b = (1 + \delta n) \sin e$$

$$\cos b = \cos e - \sin e\ tg\ e\ .\ \delta n$$

and by substituting these values we obtain

$$J'' = \frac{1}{2}\left[\frac{\delta n}{2 \cos^2 e - \delta n}\right]^2 \times \left[1 + (\sin^2 e - \cos^2 e)^2\right]$$

or approximately, since *e* is not far from 90°

$$J'' = \left(\frac{\delta n}{2 \cos^2 e}\right)^2 \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (8)$$

[7] Bruhns: Astronomische Strahlen-brechung, p. 66.

Now let mc in fig. 4 be the radius of the earth and ac the horizon of the place c, and s the sun. If the sunlight is reflected from the layer of the air at a whose radius is $R + h$ toward the horizon at c then

$$\sin e = 1 - \frac{h}{R}$$

$$\cos^2 e = \frac{2\,h}{R}$$

and the intensity of the beam of light seen at c will according to equation (8) be

$$J'' = \left(\frac{\delta n\ R}{4h}\right)^2$$

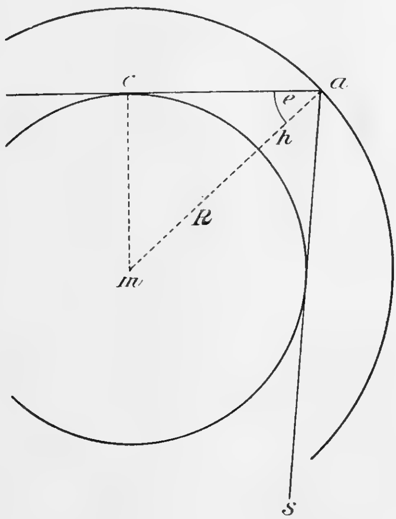


FIG. 4.

If now the twilight ends, or the stars of the feeblest intensity J_0 become visible to the naked eye, when the sunlight is reflected from the layer of air at an altitude of 79 kilometers, then the only conclusion that should be drawn is that at this altitude the intensity of the reflection

$$\left(\frac{\delta n\ R}{4 \times 79}\right)^2$$

has become less than J_0. But whether there are still higher reflecting layers of air and a still further diminution of the intensity of the twilight is beyond our power of direct observation; however, the possibility cannot be gainsaid so long as $J_0 > 0$.

The fact that the limit of the atmosphere really is considerably higher than 79 kilometers, assuming that $\frac{\rho}{\zeta} = 57.3''$ is correct, is shown by the well-known differential equation for astronomical refractions,

$$\delta \rho = \frac{\frac{R}{R+h} n' . \sin \zeta \, . \, n'}{\sqrt{n^2 - \left(\frac{R}{R+h}\right)^2 n'^2 \sin^2 \zeta}} \cdot \frac{\delta n}{n} \quad . \quad . \quad . \qquad (9)$$

which for small zenith distances reduces to the expression

$$\delta \rho > \frac{R}{R+h} n' \zeta \frac{\delta n}{n^2}$$

If for h we substitute its maximum value H so that the right-hand side of this expression becomes still smaller, then

$$\delta \rho > \frac{R}{R+H} n' \zeta \frac{\delta n}{n^2}$$

and after integration between the limits $n = 1$ for the highest layer and $n = n'$ at the earth's surface we have

$$\rho > \frac{R}{R+h} (n' - 1) \zeta$$

whence

$$H > \left[\frac{\zeta}{\rho} \left(n' - 1 \right) - 1 \right] R$$

in which

$$\frac{\zeta}{\rho} = \frac{1}{57.3''}, \qquad n' - 1 = 0.0002820, \qquad R = 6366.7 \text{ kilometers}$$

whence

$$H > 96.3 \text{ kilometers} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

which may be compared with the value in (6) above given.

III. SYMMETRIC POINTS OF THE ATMOSPHERE AND THE ZENITH POINT OF THE RAYS

For brevity I speak of the two conjugate points a and a' in fig. 5 as symmetrical when the angles of divergence of the rays from the axis are equal to each other so that

$$\alpha = \alpha' \quad \ldots\ldots\ldots \quad (11)$$

Let the distances of these points from the center of the earth be D and D'. According to the theory of Gauss the ratio of the dimen-

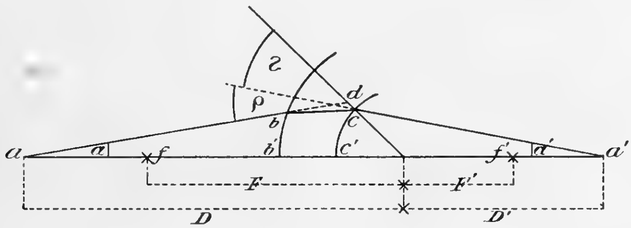


FIG. 5.

sions of the object and its image, or D/D', multiplied by the ratio of the tangents of the respective angles of divergence $\frac{tg\,\alpha}{tg\,\alpha'}$ is equal to the inverse ratio of the indices of refraction of the first and last media[8] whence

$$\frac{D}{D'} \cdot \frac{tg\,\alpha}{tg\,\alpha'} = n'$$

or by equation (11)

$$\frac{D}{D'} = n'$$

and by equation (4)

$$\frac{D}{D'} = \frac{F}{F'} \quad \text{or} \quad \frac{F}{D} = \frac{F'}{D'} \qquad (12)$$

A second equation between D and D' results from the relation developed by Gauss between two pairs of conjugate points[9] for instance, the nodal point m and the symmetrical points a and a' of fig. 5. If we divide the distances F and F' of any pair of conjugate points

[8] Mousson: 2d Edition, section 730 (7); 3d Edition, section 808 (19).
[9] Mousson: 2d Edition, section 730 (4); 3d Edition, section 808 (16).

m, from the focal points, by the distances D and D' of the two pairs from each other (see fig. 5), then the sum of the quotients is unity and we have therefore

$$\frac{F}{D} + \frac{F'}{D'} = 1 \quad \ldots \ldots \ldots \quad (13)$$

hence from equation (12)

$$\frac{2F}{D} = 1, \quad D = 2F, \quad D' = 2F'.$$

From these we at once find the distances of the points of symmetry from the (upper or lower) boundaries of the atmosphere, namely,

$$a\,b' = 2F - R - H$$
$$a'\,c' = 2F' + R$$

and substituting

$$F = 22\,924\,900 \text{ km.} \qquad R = 6366.7 \text{ km.}$$
$$F' = 22\,918\,400 \text{ km.} \qquad H = 200 \text{ km.}$$

there results

$$a\,b' = 45\,843\,233 \text{ km.} \quad \ldots \ldots \ldots \quad (14)$$
$$a'\,c' = 45\,843\,167 \text{ km.}$$

Hence the points of symmetry are at approximately equal distances from the boundaries of the atmosphere, and since $\alpha = \alpha'$ the points of entrance and exit, b and c, of the corresponding pencil of rays

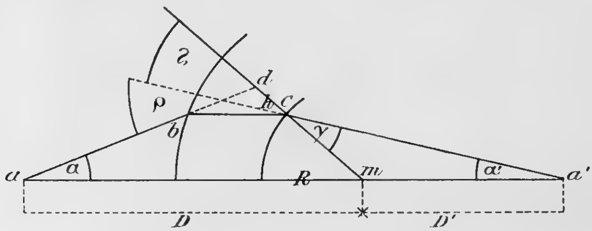


FIG. 6.

are at equal distances from the axis so that bc is parallel to am. By the zenith point of the pencil I understand the point of intersection d (fig. 6) of its initial direction with the prolonged radius of the place of observation.

Let a and a' be the conjugate points; D and D' their distances from the center of the earth; α and α' the angles of divergence of the corresponding pencils; then in the neighborhood of the zenith and according to the theorem just stated we have

$$\frac{D}{D'} \cdot \frac{\alpha}{\alpha'} = n' \quad \ldots \ldots \ldots \ldots \quad (16)$$

But from the triangle mca' since $\gamma = \zeta$ there results

$$\frac{D'}{R} = \frac{\zeta}{\alpha'} \text{ or } D' \alpha' = R \zeta$$

substituting this in equation (16) gives us

$$\frac{D}{R} \cdot \frac{\alpha}{\zeta} = n' \quad \ldots \ldots \ldots \ldots \quad (17)$$

Furthermore from the triangle mad, designating the altitude of the zenith point by h we have

$$\frac{a}{\zeta + \rho} = \frac{R + h}{D}$$

or

$$D \alpha = (R + h)(\zeta + \rho)$$

so that equation (17) becomes

$$\left(1 + \frac{h}{R}\right)\left(1 + \frac{\rho}{\zeta}\right) = n' \quad \ldots \ldots \quad (18)$$

If in this we substitute for $\frac{\rho}{\zeta}$ its value from equation (2) we get

$$h = [(n' - 1) - 57.3'' \, n' \, (1 + \Delta)] \, R = \text{about } 0.027 \text{ kilometers.} \quad (19)$$

For zenith distances up to 1° the altitude of the zenith point is independent of ρ and ζ; or, for the same locality, R, and the same condition of the atmosphere, n', the zenith point has an invariable position.

IV. SECOND APPROXIMATE VALUE OF H; ACCURACY OF THE RESULT

By reason of equation (15) the triangles *bcd* and *amd*, fig. 5, are similar to each other, wherefore for the point of symmetry *a* we have

$$bc = \frac{am}{dm}\, cd \text{ or } H = \frac{2\,h}{R+h}\,.\,F$$

and substituting the values from equations (4) and (19) we get

$$H = 2\,R\left[\frac{n'-1}{57.3''}(1-\Delta) - n'\right] = \text{about } 189.0 \text{ km.} \quad . \quad . \quad . \quad (20)$$

as compared with 192.6 in equation (7).

The agreement of these two values shows that the error made by equating the distances in equation (14) is without important influence on the result of the computations; that therefore in fact the distances of the points of symmetry from the boundaries of the atmosphere are nearly equal to each other when $\frac{\rho}{\zeta} = 57.3''$ and $n' - 1 = 0.000282$.

But of the two formulæ (7) and (20) for H the first is more exact because the curvature of the beam of light through the zenith departs but infinitesimally from a circular arc.

As to the numerical determination, $H = 192.6$, the assumption that the ratio of the refraction to the zenith distance (57.3″) is correct to within 0.001 part of itself makes $\Delta < 0.001$ and this would lead to an error of a few kilometers in the determination of the height of the atmosphere.[10]

V. CONCLUSIONS

On the basis of the preceding determinations it seems natural to attempt a new development of the differential equation (9) for astronomical refraction.

The law of diminution of refractive power with altitude may with great probability be

$$h = \left(\frac{n'-n}{n'-1}\right)^m .\, H$$

[10] I have recently found that the numerical determinations of $\frac{\rho}{\zeta}$, $n = 1$, and H really do need important corrections.

which is deduced in a manner similar to that of Bunsen's law of absorption. After substituting this value then the solution of the differential equation offers no difficulty and the equation of the curve of the pencil of light is easy to find.

As to the constant exponent *m*, that is best found from one accurate determination of the refraction. I will hereafter check the value of *m* thus found against the diminution of temperature with the altitude, since I hope to be put in possession of the necessary observational material through the kindness of a physicist, a relative, who expects to remain several years in the tropics.

In accord with my previous efforts I also believe that I shall succeed in obtaining from the observation of the twilight colors material for the direct demonstration of the diminution of the refracting power and the determination of the constants.

Let *cm* in fig. 7 be the earth's radius and *ca* the horizon of the observer at *c*. At t' and t'' hours after sunset the sun is at s' and

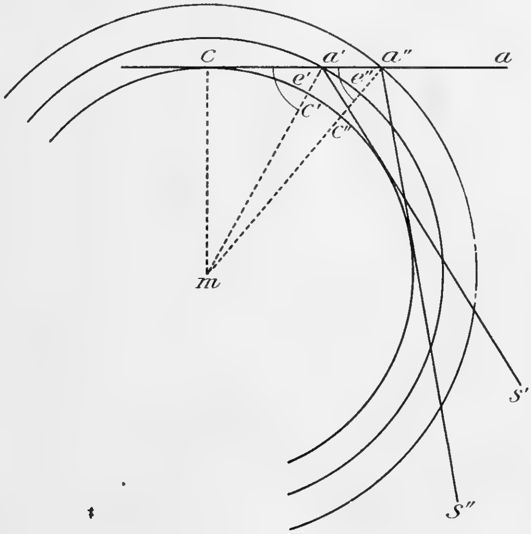


FIG. 7.

s''; the altitudes of the reflecting strata of air are $a'\,c' = h'$ and $a''\,c'' = h''$; the corresponding angles of incidence and reflection are e' and e''. We easily find

$$e' = \left(\frac{1}{2} - \frac{t'}{24}\right)\pi \qquad e'' = \left(\frac{1}{2} - \frac{t''}{24}\right)\pi$$

$$h' = (1 - \sin e')\,R \qquad h'' = (1 - \sin e'')\,R$$

and according to equation (8) the intensity of the reflections from these layers h' and h'' whose indices of refraction are n' and n'' will be given by

$$J' = \left(\frac{\delta n'}{2 \cos^2 e'}\right)^2 \text{ and } J'' = \left(\frac{\delta n''}{2 \cos^2 e''}\right)^2$$

If by means of a good photometer we measure the intensities J' and J'', then the diminution of the refracting power between the two neighboring layers at the altitudes $h,'$ $h,''$ h''' can be computed from the equations

$$\begin{aligned} \delta n' &= 2 \cos^2 e' \sqrt{J'}, \\ \delta n'' &\quad 2 \cos^2 e'' \sqrt{J''}, \end{aligned}$$

etc.

VII

ON THE PATHS OF PARTICLES MOVING FREELY ON THE ROTATING SURFACE OF THE EARTH AND THEIR SIGNIFICANCE IN METEOROLOGY

BY DR. A. SPRUNG

(Dated Hamburg, June, 1881)

[*Published (August 1881) in Wiedemann's Annalen der Physik und Chemie New series, Vol. XIV, 1881, pp. 128–149.*

Translated by Thomas Russell and C. Abbe]

Although the view expressed by Hadley in the year 1735 as to the influence of the rotation of the earth on the currents of the atmosphere has become very well known, especially through Dove's writings, and has been treated of in all manuals of meteorology and physics, still the actual construction of the path of a particle of air has in general only seldom been carried out according to Hadley's principle. There are, however, in this very "Annalen" three articles[1] in which the problem of rigorously calculating the paths of the winds is proposed either under the definitely expressed or readily recognized assumption that the particles of air are to be considered as freely moving points or elementary particles of mass. The question treated in these articles is therefore a mechanical problem that can be formulated exactly, namely, the free motion (motion due to its inertia) of a material particle which is constrained to remain on a rotating surface. Since the year 1858 a number of mathematicians have busied themselves with this problem, and about the year 1861 a general theorem was enunciated by Coriolis[2] by which every problem of relative motion can be reduced to one of absolute motion. From these analytical investigations it evidently follows that the Hadlerian principle gives only very imperfect expression to the influence of the rotation of the earth on motions parallel to its sur-

[1] Von Baeyer: Pogg. Ann., 104, p. 377, 1858.
Ohlert: Pogg. Ann., 110, p. 234, 1860.
Mousson: Pogg. Ann., 129, 652, 1866.

[2] Coriolis: Journ. de l'Ecole Polytechnique. XV, p. 142.

face, so that the calculations based thereon in the above-mentioned three articles must necessarily lead to incorrect results.

The oldest of these, however (by von Baeyer), appears of special interest as in it reflections are made which betray a tendency to abandon the Hadlerian motion. On p. 380 von Baeyer says: "a particle of air put in motion at a definite angle to the meridian on the surface of our spheroid of rotation at rest and continuing its way in the direction given to it without any hindrance or disturbance under the general influence of gravity would describe a shortest line. . . . Let us now imagine the terrestrial spheroid put in motion from its condition of rest, then the particle of air when set in motion in the direction α will already have this motion of rotation, it can therefore no longer describe a shortest line but its path will be the development of the shortest line on the spheroidal surface according to the circumstances of the rotation pertaining to it." It is to be regretted that these fruitful ideas were completely set aside in the course of the mathematical discussion in favor of the Hadlerian theory. When I first became acquainted, this year, with von Baeyer's article the above lines reminded me forcibly of a process I had made use of in the year 1879 to set forth the origin of the relative paths in simpler cases of a rotating system such as the earth presents and to base on it also a derivation of the equations of relative motion.[3]

My treatment was simply as follows: If a plane disk revolves uniformly, then will a body or material particle influenced by no forces whatever or by those whose direction is perpendicular to the disk, progress uniformly in a right line (that is when considered absolutely) and the relative path on the disk will be the continuous series of points which come in contact successively with the body. This conception, which evidently agrees essentially with that of General von Baeyer, can be extended to the rotating system of the spheroid which is under discussion, but at the same time it is evident at a glance that the first part of the above quotation from von Baeyer's article contains an inaccuracy. The course of the point on the surface of a spheroid at rest can be a shortest or geodetic line described with constant velocity, only when the attractive force of the earth is everywhere perpendicular to the surface; in reality, however, it is the force of apparent gravity, that is to say, the result-

[3] Sprung: Studies concerning the wind and its relations to air pressure, Part I; On the mechanics of the motions of the air. From the archives of the Deutschen Seewarte (German Marine Observatory), No. 1, 1879. Zeitschrift der Oesterreichisches Gesellschaft für Meteorologie, XV, 1880.

ant of the attractive force of the earth and the centrifugal force that is perpendicular to the earth's surface; the latter is moreover a level surface only by virtue of its rotation. If the earth should come to rest without a change of form, then the body would move parallel to the surface under the influence of a horizontal force directed towards the pole, which force is a component of the force of attraction and whose magnitude can be easily given. If we denote by φ the latitude of a point on the earth's surface (which here and in what follows will be considered in an entirely general manner as a body of revolution) by r the distance from the axis, and by ω, the angular velocity of rotation of the earth, then the acceleration[4] of the horizontal poleward directed component of the force of attraction is equal to the expression

$$r\,\omega^2 \sin\varphi$$

which represents the horizontal component of the centrifugal force directed toward the equator in the case of a point at relative rest on the rotating surface of the earth; for in fact the condition that these two horizontal forces are in equilibrium determines the form of the surface of the rotating earth.

At the pole and at the equator this force has the value zero, it reaches its maximum at $\varphi = 45°$; over the zone from the pole to 45° latitude it acts in a manner similar to the action of the component of the force of gravity in the case of the spherical pendulum under the influence of which for angles of elevation between 0° and 90° the pendulum makes its vibrations. Hence in general the free absolute motion of a body which does not take part in the rotation of the earth's surface, but glides on it without friction will consist of uninterrupted oscillations around the pole as a center; if the original motion began in the direction of a meridian, then the body would never leave it; if the body be started in the direction of a specific parallel of latitude (dependent on its velocity), then it would forever keep moving along this parallel with constant velocity, etc.

It cannot be doubted that it is allowable in our consideration of the relative motion due to inertia on the rotating surface of the earth to begin in the above indicated manner with the consideration of the absolute motion; for since we do not consider the influence of the rotating surface as any other than that of a rigid opposing shell it can therefore be considered as infinitely thin and as closely

[4] In what follows, the expression "force" will for brevity always be used for the accelerating force.

enclosing the similarly formed body of the earth at rest within it, while the physical forces at work (force of attraction) are perfectly independent of the condition of motion of the mass of the earth It therefore appears profitable to approach the above stated problem of the absolute motion under the influence of the force $r\,\omega^2 \sin\varphi$ directed towards the pole and to treat it at least approximately just as the problem of the oscillations of a pendulum is solved particularly in the case of infinitely small amplitudes.

In the vicinity of the pole $\sin\varphi$ changes only very slowly, but r very rapidly; no great error will then be committed if we give to $\sin\varphi$ the limiting value 1 at the pole and neglect the corresponding component of the motion parallel to the earth's axis, that is to say, the motion is to be considered as taking place in a plane; the error is thus purely geometrical and easily estimated inasmuch as the forces arising from the special form of the surface are already taken into account. Adopting the coördinates x and y in a tangent plane and the origin at the pole the differential equations of motion are therefore as follows:

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= -r\omega^2\frac{x}{r} = -x\omega^2 \\ \frac{d^2y}{dt^2} &= -r\omega^2\frac{y}{r} = -y\omega^2 \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (1)$$

(Strictly speaking these equations apply to the absolute motion of a liquid particle parallel to the level bottom of a circular vessel revolving with the velocity ω, in which the liquid is subjected to a force perpendicular to the bottom surface—in so far as this absolute motion can be considered as entirely unimpeded.)

Equations (1) agree perfectly with those on which is founded the theory of oscillations in an elastic medium; they can (for example by the substitution of x or $y = e^{xt}$) be integrated separately and lead to the final equations[5]

$$\left.\begin{aligned} x &= a\sin\omega t \\ y &= b\cos\omega t \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (2)$$

$$\left.\begin{aligned} U_x &= a\omega\cos\omega t \\ U_y &= -b\omega\sin\omega t \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (3)$$

---

[5] Compare, for example, the following treatises on physics: Wüllner: 3d Edition, I, pp. 443 and 450; Mousson: 2d Edition, II, p. 531; Müller: 7th Edition, I, pp. 278 and 281. There will also be found given in these places the geometrical representation to be spoken of presently.

from which it is apparent that the point moves in an ellipse whose semi-axes are a and b; for from (2) we get the equation of the ellipse

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

If we put

$$\omega = \frac{2\,\pi}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

then T denotes the entire time of revolution of the point in the ellipse, for when $t = T$ then both the coördinates and the components of the velocity U_x and U_y attain the same values they had at the time $t = 0$. Since ω denotes the angular velocity of the rotating surface, equation (4) shows that for the absolute motion (elliptical) the time of revolution agrees with that of the rotating surface.

The absolute motion of the point can now be easily constructed. Let us choose for example the time of revolution of the surface (and of the material particle) as $T = 24$ seconds (compare fig. 1, page 62) and divide the circumference of a circle constructed on the diameter $2a$ into 24 equal parts and from the points of division let fall perpendiculars on the diameter, which in this case can be done by joining the points in pairs by straight lines as the points of division are symmetrically distributed with respect to $2a$. The 12 diameters which join the 24 points of division divide at the same time into 24 equal parts a circle constructed on the small axis $2b$; from the points of division of this small circumference let fall normals to the diameter $2b$, which is perpendicular to $2a$, and prolong them on both sides to the larger circle. If the time t is reckoned from the moment at which the body is at the extremity of the radius b, then the points of intersection of the two systems of normals which are marked 0, 1, 2, 3 . . . lie on the elliptical path characterized by equations (2), (3) and (4) for $T = 24$, from which the corresponding relative motion is derived in the following manner.

A rotation of the surface about an angle ω, 2ω, 3ω . . . corresponds to the absolute motion of the body up to the points 1, 2, 3, . . . ; evidently therefore we can find the positions at the moment $t = 0$ of those points I, II, III . . . of the rotating surface which will come in contact with the body after 1, 2, 3 . . . seconds, by going toward them in a direction contrary to the motion of rotation of the surface along the concentric circles through

1, 2, 3 . . . by the arcs $\tau_1\omega, 2\tau_2\omega, 3\tau_3\omega$ . . . respectively. From this follows in a striking manner the important result that the relative "inertia path" consists of a circle which will be described twice in the absolute time of revolution T and in such a way that the direction of the rotation is opposite to that of the rotation of the surface.

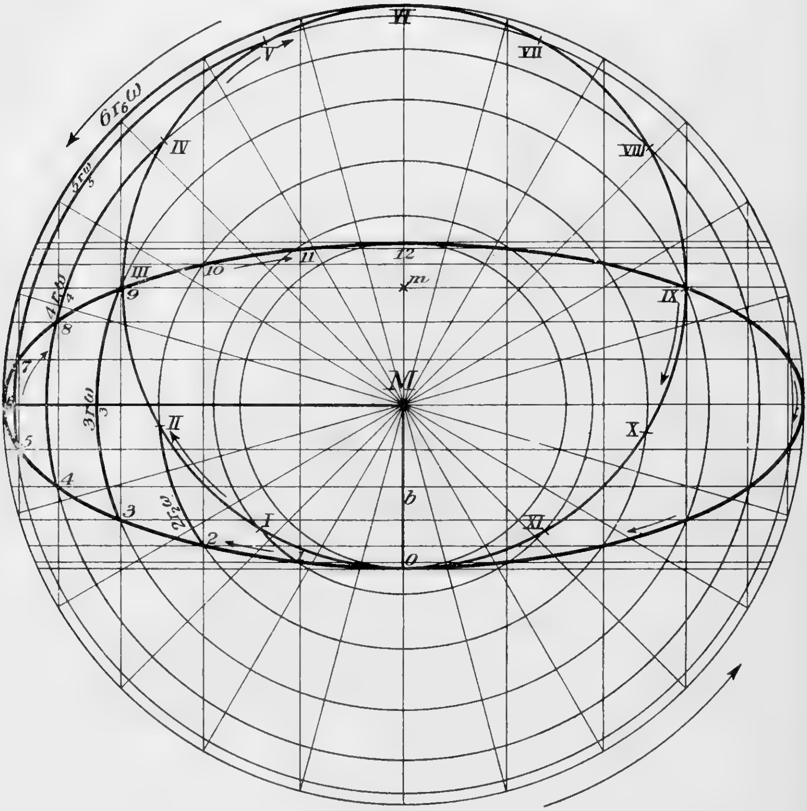


FIG. 1

Introducing[6] different modifications regarding the form of the elliptical path and the direction in which the body traverses t, we can exhibit clearly in a direct geometrical manner the mutual dependence of the absolute and relative motions and also, for exam-

[6] Fig. 1, for example, differs from fig. 2 only in the circumstance that the ellipse is traversed in the opposite direction; the relative velocity and circle of inertia are in consequence about $2\frac{1}{2}$ times as great as in fig. 1.

ple, convince ourselves that for the same relative velocity v the circle has always the same magnitude whether it passes through the point of rotation M or at a greater distance from it.[7] There is no tendency on the part of the moving body to remain in a circle of latitude or to move parallel to the latitude circles as assumed in the theories of Hadley and Dove; for every azimuth of the motion the

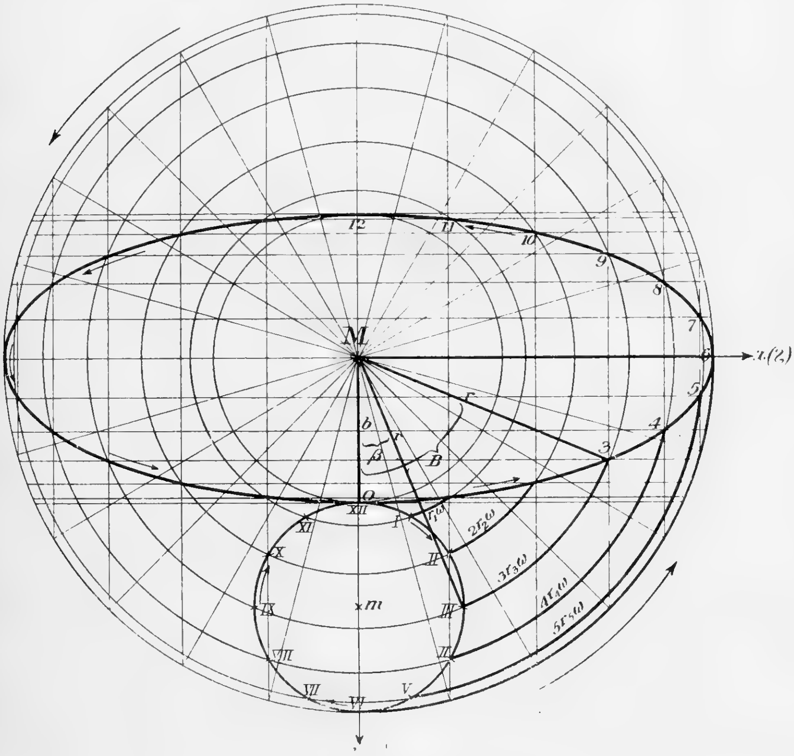


FIG. 2

tendency is precisely the same, that is, to deviate toward the right from the momentary current direction of motion.

Let us now try to follow the above construction analytically.

[7] By the aid of a ball of chalk rolling about on a rotating parabolic-shaped blackboard an autographic representation of these inertia paths can be reproduced; the unavoidable friction will only be manifest in this, that the curves (approximately circular) become gradually narrower and narrower. I recommend the following as suitable dimensions for this apparatus: diameter of the parabolic shell, 120 cm.; depth in the middle, 10 cm. In this case the time of revolution, T, must be 2.7 seconds.

The relative motion will be referred to the coördinates (ζ, η), moving with the rotating system and which at the instant $t = 0$ (as the figure shows) coincide with x, y. Denote by B the angular distance from the y-axis at the time t in the case of the absolute motion, and by β the corresponding difference from the η-axis in the case of relative motion, evidently then

$$\beta = B - \omega t \quad \ldots \ldots \ldots \quad (5)$$

Further if r denotes the radius vector at the time t:

$$\left.\begin{aligned} x &= r \sin B \\ y &= r \cos B \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (6)$$

$$\left.\begin{aligned} \xi &= r \sin \beta \\ \eta &= r \cos \beta \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (7)$$

From (5) we derive

$$\left.\begin{aligned} r \sin \beta &= r \sin B \cos \omega t - r \cos B \sin \omega t \\ r \cos \beta &= r \cos B \cos \omega t + r \sin B \sin \omega t \end{aligned}\right\} \quad \ldots \ldots \quad (8)$$

By substituting x, y from (2) in (6); $r \sin B$ and $r \cos B$ from (6) in (8); and finally $r \sin \beta$ and $r \cos \beta$ from (8) in (7); we get

$$\xi = (a - b) \sin \omega t \cos \omega t$$

$$\eta = b \cos^2 \omega t + a \sin^2 \omega t = a - (a - b) \cos^2 \omega t.$$

But for this can be written, by application of known goniometrical formulæ:

$$\left.\begin{aligned} \xi &= \frac{a - b}{2} \sin 2 \omega t, \\ \eta &= - \frac{a - b}{2} \cos 2 \omega t + \frac{a + b}{2} \end{aligned}\right\} \quad \ldots \ldots \quad (9)$$

$\frac{1}{2}(a - b)$ is the radius of the desired circle; and $\frac{1}{2}(a + b)$ is the distance of its center m from the center of rotation M on the η-axis.

Instead now of starting with the construction of the absolute motion, as here done, we can also follow the reverse process, assuming the arbitrary relative velocity v_0 in an arbitrary direction as being given for any distance b of the body from the center of revolution M. For simplicity it will however be for the present assumed that v_0 is perpendicular to the radius vector b, and that v_0 is reckoned positive in the direction of rotation of the system (from west to east). Hence $v_0 + b\omega$ is the absolute west-easterly velocity of the body at the time $t = 0$, for which according to (3) we have the expression $(U_x)_0 = a\omega$; we have therefore the relation:

$$a\omega = v_o + b\omega \qquad \text{or} \qquad a - b = \frac{v_o}{\omega} \quad . \quad . \quad . \quad . \quad . (10)$$

by using which relations the equations (9) finally pass into the following form:

$$\left.\begin{aligned} \xi &= \frac{v_o}{2\,\omega} \sin 2\,\omega t, \\ \eta &= -\frac{v_o}{2\,\omega} \cos 2\,\omega t + \left(b + \frac{v_o}{2\,\omega}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (11)$$

Therefore we have:

$$\left.\begin{aligned} \rho &= \frac{v_o}{2\,\omega} \text{ the radius of the circle of inertia,} \\ \eta_m &= b + \frac{v_o}{2\,\omega} \text{ the } \eta\text{-coördinate of its center} \end{aligned}\right\} \quad . \quad . \quad . (11')$$

These equations give us all desired information concerning the relative motion due to the inertia of the body. We first derive

$$\frac{d\xi}{dt} = v_o \cos 2\,\omega t;$$

$$\frac{d\eta}{dt} = v_o \sin 2\,\omega t.$$

But from these we have

$$v = \sqrt{\left(\frac{d\xi}{dt}\right)^2 + \left(\frac{d\eta}{dt}\right)^2} = v_o,$$

that is to say, the relative motion due to inertia is a uniform one and only distinguishable from the absolute motion due to inertia in free space, by this, that its path is not a straight line but curved. For the direction of rotation of the system assumed in our figure, which agrees with that of the northern hemisphere, the center of curvature always lies on the right-hand side of the path, since the coördinate η_m of the center of the circle will be $< b$ as soon as v_0

becomes negative, that is to say, when the original relative velocity is directed from east to west.

For $v_0 = 0$, we have $\eta_m = b$, $\rho = 0$ and $a = b$, that is to say, the point remains at rest relatively while its absolute path is a circle.

For $v_0 = -b\omega$, we have $\eta_m = +\frac{1}{2}b$ and $a = 0$; the absolute path consists of a pendulous oscillation in a straight line. When a changes sign, that is to say, for still smaller values of the velocity v_0, the ellipses are described in the opposite direction.

For $v_0 = -2b\omega$, we have $\eta_m = 0$, $\rho = \mathrm{b}$, and $a = -b$; the center m of the relative inertia circle coincides with the center of rotation M, the absolute path is again a circle as for $v_0 = 0$, but the direction of the rotation is opposite to what it was before.

The angular velocity 2ω *of the relative inertia motion is twice* as great as that of the rotation of the system, the relative path will therefore be traversed twice during the time T of one revolution of the system. The figures 1 and 2 also show directly, as has already been indicated above, a time of rotation of 12 seconds, when 24 seconds is assumed for the whole system. As now our investigation may be applied with a high degree of approximation to the region surrounding the north pole we attain at once the interesting result that a body confined to the earth's surface, but otherwise free to move, will deviate from its original direction twice as much as does the plane of the Foucault pendulum.

The value of ω is 0.00007992 m, so that the length of the radius ρ becomes about 69 km. when the velocity is $v = 10$ m. or that of a fresh breeze; at this velocity therefore the body only passes a little way beyond the space between two successive parallels of latitude.[8]

Now on a plane that is not in rotation a path can be produced, similar to the path of inertia found for the relative motion, by introducing a physical force $A = \frac{v^2}{\rho}$ always acting from left to right, perpendicular to the momentary direction of motion, But if the value of ρ given by (11′) is substituted in this expression, then we have

$$A = 2v\omega \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The motion on the rotating region near the pole can therefore be treated as an absolute one, if in addition to all the forces customarily taken into consideration there is introduced this other force A, which in modern meteorology is called the "deflecting force of the earth's rotation."

[8] Namely, the difference between 138 and 111 km.

A current of air in the neighborhood of the north pole can only flow in a straight line no matter in what direction, when a force to the amount of $2v\omega$ in the opposite direction to this deflecting force or directed from right to left renders this departure from the path of inertia possible; in this case therefore the barometric or elastic pressure in the current of air must increase from the left towards the right. In the same manner in a straight channel, no matter in what direction it trends, the moving liquid should stand a little higher on the right than on the left, and in fact independently of the nature of the liquid it results that we must have[9]

$$\frac{H - H_0}{L} = \frac{2v\omega}{g}$$

where $H - H_0$ denotes the difference of height between the two shores of the stream, and L the width of the stream assumed to flow everywhere with uniform velocity.

The special case of motion due to inertia in the region of the pole has been treated so fully in the foregoing text because the construction of the paths in that region can be made on the basis of certain

[9] It will be proven presently that the above expression corresponds to the following equation for any latitude φ:

$$g\,(H - H_0) = 2v\omega \sin \varphi . L$$

For example, if for $\varphi = 50°$, $2\omega \sin \varphi$ has the value 0.0001117; for a width of river $L = 100$ meters and a velocity $v = 10$ meters there results $H - H_0 = 0.0114$ meter. Hence in every horizontal layer the pressure of the water is by about 1.14 cm. of a column of water, greater toward the right than toward the left. This amount certainly appears to be inconsiderable, but geologists are accustomed to see very insignificant but constant causes produce great results. Hence it has been attempted by the axial rotation of the earth to account for such gradual displacement of river beds as is seen in the frequently recurring and notable phenomenon that the right side of a river is frequently closely bordered by a range of hills while on the left side a tolerably broad strip of entirely flat land stretches along the course of the river, as, for example, on the lower part of the courses of the Elbe, Weser, Thames, Seine and Gironde and also on the Danube, Volga, and other rivers of southern Russia where this is especially noticeable. But recently this explanation, proposed by von Baer, has been freely contradicted.

The relation between the direction of the wind and its force and the distribution of atmospheric pressure which has found empirical expression in Buys-Ballot's law can be easily derived from the above text. Further details on this subject can be found in the works of Guldberg and Mohn in the Zeitschrift der österreichisches Gesellschaft für Meteorologie, 1877, Vol. XII, pp. 49, 177, 257 and 273; also Sprung. Ann. d. Hydr. u. maritime Meteorol., VIII, Jahrg. 1880, p. 603, and Beiblätter, V, 1881, p. 237.

well-known theorems of physics. For other geographical latitudes the problem becomes considerably more difficult; however, even here with the aid of our conception of the process we are enabled to directly attain some results.

The condition of relative rest of a body on a horizontal plane in any latitude φ, consists, absolutely considered, of a circular oscillation (diurnal rotation about the axis of the earth) under the influence of a poleward-directed component $r\omega^2 \sin \varphi$ of the force of attraction of the earth which neutralizes the local equatorial tendency. The forces required in the absolute motion are evidently the same whether the circle of latitude is traversed from west to east or east to west with the velocity $r\omega$. In the latter case, however, the relative motion of the body is an east-west one with the relative velocity $v = 2r\omega$; the latter motion is therefore, just as in the case of relative rest, a special case of the relative inertia motion. Only in two very special cases of the relative velocity is it possible for a free body to remain on a circle of latitude, whilst it was formerly assumed that the final results of every deviation due to the rotation of the earth consists in a motion parallel to the circle of latitude.

It can easily be seen that the horizontal radius of curvature of the small circle of latitude (whose curvature must always be determined by comparison with the geodetic line which is a great circle on the sphere) is equal to the slope $\frac{r}{\sin \varphi}$ of that cone which is tangent to the earth's surface at the latitude φ. The "deflecting force of the earth's rotation" is therefore in this case $(2r\omega)^2 \left(\frac{\sin \varphi}{r}\right)$ and by using the above relation $v = 2r\omega$ this can be written $A = 2v\omega \sin \varphi$. The "deflecting force" at the latitude φ (at least for the velocity $v = 2r\omega$) is then smaller than at the pole, where the value is $2v\omega$; its direction is the same as there, perpendicular to the path, towards the right in the northern, towards the left in the southern hemisphere, and the influence of the earth's rotation is thus represented perfectly, because the relative motion due to inertia here under consideration is a uniform one.

For the completion and generalization of this result the general problem of absolute motion under the influence of the force $r\omega^2 \sin \varphi$ directed poleward will be here treated briefly.

Denote by V the absolute velocity parallel to the surface of the rotating body, by θ the azimuth of the absolute motion (counted positive from the north around by the east towards the south) and by ds the differential of the path, then will the principle of living force, vis viva or mechanical energy be represented by the following equation:

$$d\left(\tfrac{1}{2}\ V^2\right) = r\omega^2 \sin\varphi\ ds \cos\theta$$

But since, as is evident at once geometrically, $-\dfrac{dr}{ds\cos\theta} = \sin\varphi$, then the same equation can be written:

$$d\left(\tfrac{1}{2}V^2\right) = -\ r\omega^2\ dr$$

from the integration of which results

$$V^2 = D - r^2\,\omega^2 \quad \ldots\ldots\ldots \quad (13)$$

where D is a constant. Again the principle of the conservation of areas gives

$$V\sin\theta = \frac{C}{r} \quad \ldots\ldots\ldots \quad (14)$$

In these two equations the general problem is contained, and to a certain extent already solved. From (14) there is first derived

$$V^2\cos^2\theta = V^2 - \frac{C^2}{r^2}$$

and by introducing the value of V^2 from (13)

$$V\cos\theta = \sqrt{D - r^2\omega^2 - \frac{C^2}{r^2}} \quad \ldots\ldots \quad (15)$$

The expressions (14) and (15) contain the west-easterly and south-northerly components of the absolute velocity as functions of the distance r from the axis; we have only to subtract from these the velocity of the surface of the earth at the place in question to obtain the corresponding components $v\sin\theta$ and $v\cos\theta$ of the relative velocity; in this way we obtain

$$\left.\begin{aligned} v\sin\theta &= \frac{C}{r} - r\omega \\ v\cos\theta &= \sqrt{D - r^2\omega^2 - \frac{C^2}{r^2}} \end{aligned}\right\} \quad \ldots\ldots \quad (16)$$

By squaring and adding these equations the following is obtained:

$$v^2 = D - 2\,C\omega = v_o^{\,2} \quad \ldots\ldots\ldots \quad (17)$$

Hence the velocity of the relative motion due to inertia is constant at any latitude whatever on the earth's surface, just as found before for the region of the pole.

By a general theorem applicable to all rotating bodies, the radius ρ of the geodetic curvature of a curve running in any direction whatever on the surface of a rotating body can be expressed by

$$\rho = \frac{r \cos \theta \, ds}{d \, (r \sin \theta)} .$$

If in

$$d \, (r \sin \theta) = r \cos \theta \, d \, \theta + \sin \theta \, dr$$

we substitute the two values derived from the first of equations (16), having regard to (17):

$$\cos \theta \, d \, \theta = - \frac{1}{v} \left(\frac{C}{r^2} + \omega \right) dr$$

and

$$\sin \theta = \frac{1}{v} \left(\frac{C}{r} - r\omega \right)$$

then we have

$$\rho = - \frac{v \cos \theta \, ds}{2 \, \omega dr} ,$$

or, since

$$\frac{\cos \theta \, ds}{- \, dr} = \frac{1}{\sin \varphi}$$

$$\rho = \frac{v}{2 \, \omega \sin \varphi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

This value of the radius of curvature ρ of the relative path due to inertia corresponds perfectly to the value $\rho = \frac{1}{2} . \frac{v}{\omega}$ found above (compare equation (11′) for the region of the pole, and shows that the path is less slightly curved the more nearly the equator is approached. For the equator itself ($\varphi = 0$) the path becomes the geodetic line or great circle itself. In the southern hemisphere φ is negative and therefore the radius of curvature has the opposite sign from that in the northern hemisphere. The center of curvature in the one case lies on the right side and in the other on the left side of the "inertia path." The proof of this statement is easily deduced by a closer consideration of the expression for sin θ in the first of equations (16); if, for example, we introduce the condition that sin $\theta = 0$ for $r = r_0$, and write

$$\sin \theta = \frac{\omega}{v}\left(\frac{r_0^2}{r} - r\right)$$

If now we consider two places on the earth's surface at the same distance r_0 from the axis, one of which is in the northern hemisphere, the other in the southern, then sin θ is in both cases $= 0$, that is to say, the motion is to be a purely south-northerly one. In the farther course of these motions, however, r becomes smaller in the northern hemisphere and therefore sin $\theta > 0$; on the contrary in the southern hemisphere r will become greater and therefore sin $\theta < 0$; the body therefore deviates from the meridian towards the right in the northern hemisphere but towards the left in the southern hemisphere.

If the motion is followed still farther (in the northern hemisphere for example) then we have

for the value $\theta = 90°$ the distance $r_1 = -\frac{v}{2\omega} + \sqrt{r_0^2 + \left(\frac{v}{2\omega}\right)^2}$

" " " $\theta = 270°$ " " $r_2 = +\frac{v}{2\omega} + \sqrt{r_0^2 + \left(\frac{v}{2\omega}\right)^2}$

the value of θ becomes 360° again for $r_0 = r$ and therefore in the same geographical latitude in which $\theta = 0$. But it can be easily seen that in this case the body has not returned to the meridian of the starting place but to one lying farther west; for since the curvature of the path continually diminishes while θ passes through its values from 90° to 270°, therefore the southernmost point of the path must lie farther westward than the preceding northernmost point. The motion is therefore enclosed between two definite parallels of latitude and carries the body in many nearly circular convolutions gradually toward the west. Presumably this progression is connected with a peculiarity of the corresponding absolute motion which the latter has in common with a peculiarity of the spherical pendulum; in this latter case it is known that the successive temporary highest positions show a regular advance in a determinate direction on a horizontal circle.

The correct representation of the relative (or absolute) path in the form of an equation between the geographical coördinates φ and

λ presupposes that the form of the rotating body is known [i. e. the slope of the surface of revolution] therefore that r is given as a definite function of the latitude or $r = F. (\varphi)$: for example, in the case of a sphere $r = R \cos \varphi$: in the case of a spheroid

$$r = \frac{R \cos \varphi}{\sqrt{1 - \varepsilon^2 \sin^2 \varphi}}$$

Since $v \sin \theta = r \left(\frac{d\lambda}{dt}\right)$, and $v \cos \theta = -\frac{1}{\sin \varphi} \left(\frac{dr}{d\varphi}\right) \left(\frac{d\varphi}{dt}\right)$ therefore

equations (16) would become

$$\left.\begin{aligned} \frac{d\lambda}{dt} &= \frac{C - r^2\omega}{r^2} \\ \frac{d\varphi}{dt} &= -\sin\varphi \frac{1}{\frac{dr}{d\varphi}} \sqrt{D - r^2\omega^2 - \frac{C^2}{r^2}} \end{aligned}\right\} \quad . \; . \; . \; . \quad (19)$$

and from this by the elimination of dt is derived the definite integral:

$$\lambda = -\int_{\varphi_0}^{\varphi} \frac{\left(C - r^2\omega\right)\frac{dr}{d\varphi}}{r \sin\varphi \sqrt{Dr^2 - r^4\omega^2 - C^2}} \, . \, d\varphi \quad \Bigg\} \quad . \; . \quad (20)$$

in which the constants D and C from (16) can be expressed by the values of v, θ, and r or from (14) and (15) by the values of V, Θ and r for the initial circumstances of the motion as follows:

$$\left.\begin{aligned} D &= v^2 + 2\, r_0^2\omega^2 + 2\, vr_0\omega \sin\theta_0 \quad (= V_0^2 + r_0^2\omega^2) \\ C &= r_0^2\omega + vr_0 \sin\theta_0 \qquad\qquad\qquad (= V_0 r_0 \sin\Theta_0) \end{aligned}\right\} \quad . \quad (21)$$

The solution of this problem leading to elliptic integrals does not seem to be worth the while, because in the first place the function $r = F. (\varphi)$ for the earth can not be given with entire certainty, and in the second place the careful determination of the path has only a subordinate interest in meteorology, since the notion, formerly entertained, that the particles of air actually follow the "inertia path" has been completely refuted by the synoptic weather charts that show the simultaneous conditions of the atmosphere over large regions. It may even be asserted that in fact the direction of curvature that pertains to the inertia path is not even the more frequent:

in fact there are many more curved wind paths that are cyclonal than anticyclonal.

On the contrary it is of the greatest interest to know that the tendency towards change of direction by the rotation of the earth is far greater and more general than was formerly supposed. The "deflecting force" acting from left to right, corresponding to equation (12) can by substituting the value of ρ from (18) be written

$$A = 2\, v\omega \sin\varphi \qquad (22)$$

in which φ is to be taken positive for the northern hemisphere and negative for the southern. Therefore, for horizontal motions on the rotating surface of the earth the dynamical differential equations in a rectangular system of coördinates, for which the positive y-axis extends from the positive direction of the x-axis towards the left, are as follows:

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= X + 2\,\omega \sin\varphi \frac{dy}{dt}, \\ \frac{d^2y}{dt^2} &= Y - 2\,\omega \sin\varphi \frac{dx}{dt} \end{aligned}\right\} \qquad (23)$$

In fluid motions the given forces generally consist of pressures, so that X and Y are to be replaced respectively by

$$-\frac{1}{\sigma}\left(\frac{dp}{dx}\right) \text{ and } -\frac{1}{\sigma}\left(\frac{dp}{dy}\right) \text{ (where } \sigma \text{ is the density).}$$

With reference to the application of these equations and their necessary extension to motion in any direction, reference may be made to the theoretical investigations in the domain of meteorology[10] whose number has lately increased to a most encouraging extent. Since, however, the vertical forces can be readily ascertained a short discussion of these may properly follow here as a conclusion to the preceding presentation.

For motions parallel to the earth's surface the vertical pressure N directed downwards, evidently has the value

$$N = G - \frac{V^2}{R} = G - \frac{V^2 \cos^2\theta}{R_1} - \frac{V^2 \sin^2\theta}{R_2}$$

[10] W. Ferrel: Meteorological researches; Reports of the Superintendent of the U. S. Coast and Geodetic Survey for 1875 and 1878.

C. M. Guldberg et H. Mohn: Etudes sur les mouvements de l'atmosphère; programme de l'Université de Christiania pour 1876 et 1880.

J. Finger: Wien, Sitzungsberichte, Jahrg. 1877, 1880.

in which G is the vertical component of the accelerating force of attraction and R the radius of curvature of that normal section which is tangent to the direction of the *absolute* motion; R_1 and R_2, however, denote the radii of curvature of the principal normal sections of the rotating body, respectively parallel to the meridian and to the circle of latitude for which we have in fact:

$$\frac{1}{R_1} = -\sin\varphi\,\frac{d\varphi}{dr}$$

$$\frac{1}{R_2} = \frac{\cos\varphi}{r}$$

For Θ must be introduced the azimuth θ of the relative motion; from (14), (15) and (16) we have

$$V\cos\Theta = v\cos\theta$$

$$V\sin\Theta = v\sin\theta + r\omega$$

From all this we obtain

$$N = G - \left(\frac{v^2\cos^2\theta}{R_1} + \frac{v^2\sin^2\theta}{R_2}\right) - \frac{r^2\omega^2}{R_2} - \frac{2\,rv\omega\sin\theta}{R_2},$$

or, by introducing in the last two terms the preceding expression for R_2:

$$N = G - r\omega^2\cos\varphi - 2\,v\omega\cos\varphi\sin\theta - \frac{v^2}{R'} \quad \ldots\ldots\ldots\ldots (24)$$

in which R' denotes the radius of curvature of the normal-section parallel to the direction of the relative motion. The first two terms represent the local acceleration g of the force of gravity[11]

$$G - r\omega^2\cos\varphi = g \quad \ldots\ldots\ldots (25)$$

If the motion of the body has a vertical component, then the same equation (24) will apply if the v therein is made to denote the velocity of the horizontal projection of the motion, and θ its azimuth. The magnitude of the force N will be changed slightly by the vertical component of the motion only when this latter motion is not uniform and in fact the change corresponds to $\frac{d^2h}{dt^2}$ which is

[11] The apparent force of gravity or the vertical component of the attraction of the earth minus the vertical component of the centrifugal force.

C. A.

the expression for the vertical acceleration, The entire system of vertical forces must be considered as a modified force of gravity g, and therefore it must be introduced, instead of g alone, in the differential equation $dp = -\sigma\, g\, dh$ of the barohypsometric formula if it is desired to take the state of motion of the atmosphere into account in the derivation of the hypsometric formula. According to the equation for the gaseous condition the density σ is dependent on the pressure p and the absolute temperature T in the following manner:

$$p = \sigma KT$$

where K is the gas constant for atmospheric air.

If the change with altitude in the composition of the air is left out of account and the decrease of the temperature upwards, in accordance with the usual custom, is assumed to be constant so that $T = T_0 - \varepsilon h$ then there results finally the following equation

$$K\frac{dp}{p} = -\frac{dh}{T_0 - \varepsilon h}\left(g - 2\,v\omega\cos\varphi\sin\theta - \frac{v^2}{R'} + h\frac{d^2h}{dt^2}\right) \text{.. (26)}$$

(Strictly speaking g is also a function of the height h and the geographical latitude φ.) This equation is of great importance in meteorology inasmuch as it gives us the means of determining the horizontal distribution of pressure at any altitude h, in case this distribution is known for any other altitude (for example, at the mean level of the ocean where $h = 0$), and provided sufficient initial points are given for the estimation of the condition of the atmosphere as to temperature and motion.

For the purpose of illustration and investigating briefly the magnitude of the influence of the horizontal motion of the air on the vertical distribution of pressure, it will be assumed that the temperature everywhere $= T_0$, therefore $\varepsilon = 0$; since also $\frac{d^2h}{dt^2} = 0$, therefore by integration there results:

$$T_0 Kl\frac{p_0}{p} = \left(h - h_0\right)\left(g - 2\,v\omega\cos\varphi\sin\theta - \frac{v^2}{R'}\right).$$

For an atmosphere at rest we should have

$$T_0\, Kl\,\frac{p_0}{p} = (h - h_0)\, g \text{ . . . . (27)}$$

If now it is assumed that p (the atmospheric pressure at the upper level) has the same value in both equations, then by subtraction there results:

$$T_0\, Kl\, \frac{\overline{p}_0}{p} = \left(h - h_0\right)\left(2\, v\omega \cos\varphi \sin\theta + \frac{v^2}{R'}\right).$$

If in this we replace $h - h_0$ by its value from (27) and the ratio $\frac{p_0}{p}$, etc., by the ratio $\frac{B_0}{B}$ etc., from the recorded barometric readings we have finally:

$$\frac{\overline{B}_0}{B_0} = \left(\frac{\overline{B}_0}{B}\right)^{\frac{1}{g}\left(2\, v\omega \cos\varphi \sin\theta + \frac{v^2}{R'}\right)} \quad . \quad . \quad . \quad . \quad . \quad (28)$$

For example, let $B = 620^{mm}$ denote the reading of the barometer on the Schneekopfe, $B_0 = 748^{mm}$ the corresponding reading of the barometer at Breslau, the difference of level being about 1450^{m}); also let $v = 30^{m}$, (the velocity of a violent wind storm), and $\varphi = 51°$; then by computation the exponent on the right of equation (28) is found to be

$$0.000\ 280\ 8 \sin\theta + 0.000\ 014\ 4.$$

The extreme values of this exponent and the corresponding values of B_0 at the level of Breslau are as follows:

	Exponent.	B_0^{mm}
for $\theta = 90°$ (west wind);	0.000 295 2	747.958
for $\theta = 270°$ (east wind);	−0.000 266 4	748.037

From this it follows that under the same circumstances in other respects, the pressure on the lower side of a stratum of air 1450^{m} thick, moving with a horizontal velocity of 30 m. p. s. and having an equal pressure at the upper side in the two cases will with an east wind be about 0.079^{mm} higher than with a west wind. If the term $\frac{v^2}{R'}$ had been neglected then for the west wind there would have resulted $B_0 = 747.960$ and the difference between the west and east wind would have been 0.080^{mm}. The influence of this term is therefore very inconsiderable.

Moreover the whole effect of the horizontal movements of the air must be called very insignificant because a change of pressure of 0.08^{mm} can scarcely be observed with our barometers.

Since the horizontal forces conditioned on the axial rotation of the earth are of the same order as the vertical ones just considered (equation 22) and become equal to them at the latitude 45°, therefore the question arises, how comes it that the first are of such great importance in meteorology; the reason lies simply in this that much greater dimensions come into play in horizontal directions. It frequently happens indeed that the whole region between the Alps and southern Scandinavia is occupied by one and the same current of air, in which the difference of pressure on the two sides of the current (measured in a horizontal direction perpendicular to the isobars) amounts to 30 or 40^{mm}. From this there results a "Gradient" of 2.5 to 3.3^{mm} (for the unit length of one equatorial degree or 111 km.), whereas for a distance of $1\frac{1}{2}$ km. (corresponding to the vertical distance above considered between Breslau and Schneekopfe) there results a proportional difference of pressure of 0.039^{mm}, a quantity that is no longer measurable with our barometers. Therefore if at about latitude 50° a parallelopiped of air extending from west to east of 1.5 km. height and breadth and previously at rest were set into a condition of stormy motion then the simultaneous difference of pressure for the surfaces lying opposite each other in a horizontal as well as a vertical direction, must change by about 0.04^{mm}. Inversely the production and maintenance of such an insignificant difference of pressure would suffice to gradually bring about these same stormy motions; but the fundamental fact is that the horizontal difference of pressure suffices for this purpose and we should conceive the processes as going on toward completion in the following order:

At the start the motion of the air takes place in the direction of the gradient, but this is departed from more and more with increasing velocity and diminishing acceleration of the wind, until the direction of its motion when it has become uniform finally stands perpendicular to the direction of the gradient or at least approaches this perpendicular direction to a certain degree, because of the action of frictional resistances which render necessary the introduction of a component of the gradient parallel to the motion of the air and directed forward with it. The change of the vertical difference of pressure is to be considered as primarily a consequence of the circumstances of the motion since the effect of the latter can here be conceived of as a mere diminution of the force of gravity.[12]

---

[12] See the exponent of equation (28) where the influence of velocity opposes that of gravity.—ABBE.

Another important question is that of the relation between the vertical distribution of pressure and the motion of the air in the vertical direction. If there are no motive forces present except the difference of pressure, then equation (26) is to be applied in this case and for $v = 0$ it becomes:

$$T_0 K \frac{dp}{p} = - dh \left(g + \frac{d^2 h}{dt^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

It will be assumed that $\frac{d^2h}{dt^2} = b$ is a constant quantity. Proceeding in a method entirely similar to the preceding we finally get

$$\frac{\overline{B}_0}{B} = \left(\frac{\overline{B}_0}{B}\right)^{-\frac{b}{g}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

where $\overline{B}_0$ is the value of the barometer-reading B at the lower level under the condition of uniform motion throughout the whole mass. If, for example, it is asked how great b will become in the case of the values used above for $\overline{B}_0$ and B when the relation $\overline{B}_0 \times 748.1 = B_0 \times 748.0$ exists (when therefore the difference of pressure is about 0.1^{mm} greater in the condition of accelerated motion than in that of rest or uniform motion), the result is, there will be an upward directed acceleration of $b = 0.007$ meter per second.

If the air has simultaneously an east-west component of velocity to the amount of 25 to 30 m.p.s., then the diminution of the pressure from below upwards will become about $0.10 + 0.04 = 0.14^{mm}$ greater than in the condition of rest.

If in any manner whatever an increase of the vertical difference of pressure of 0.1^{mm} should be brought about and maintained, for example, by an upward directed removal of air at any altitude, then an ascension of the air must take place and by integration of the above equation $\frac{d^2h}{dt^2} = b$ for uniformly accelerated motion, the velocity which a particle of air attains in passing over a distance $h - h_0$ of $1\frac{1}{2}$ km., can be deduced. As b is assumed to be constant there results:

$$\frac{dh}{dt} = \sqrt{2\, b\, (h - h_0)}$$

For the above value of $b = 0.007$ and for $h - h_0 = 1500^{m}$ we obtain $dh = 4.58^{m}$ per second. In this vertical motion, as is known, there appears again a horizontal component of motion in consequence of the rotation of the earth, in case such motion is not prevented by differences of pressure. In an ascending motion the tendency to deviate toward the west is represented by the expression $2\left(\frac{dh}{dt}\right)\omega\cos\varphi$ as can be very readily proved by the aid of the principle of the preservation of areas.

VIII

THE THEORY OF THE FORMATION OF PRECIPITATION ON MOUNTAIN SLOPES

BY PROF. F. POCKELS

School of Technology, Dresden, Germany

[*Translated from Ann. d. Physik, (4) Vol. IV, pp. 459–480. 1901*]

Reprinted from the Monthly Weather Review for April, 1901

It is a well known principle of climatology that the side of a mountain range which is turned toward the prevailing wind has in general a greater precipitation than the plain on the windward side, and a still greater in comparison with the leeward side of the mountain range. There has been no doubt as to the explanation of this phenomenon since it has been recognized that the principal cause of the condensation of the aqueous vapor is the adiabatic cooling of the rising mass of air; for a current of air impinging against rising ground must, in order to pass over it, necessarily rise. So far as the author knows, however, no attempt has yet been made to investigate this process quantitatively, except perhaps, for the stratum of air immediately contiguous to the earth, whose ascension being equal to that of the surface itself, is thereby known directly. Such a quantitative treatment will be attempted in the following article. Even although this is only possible under special assumptions which represent nature with the closest approximation, it will, however, always offer a practical basis for estimating the purely mechanical influence exerted by the configuration of the surface of the earth on the formation of rain.

I

In order to find the standard vertical components of the velocity of the air currents that determine the condensation, we must, first of all, solve the hydrodynamic problem of the movement of the air over a rigid surface of a given shape. In this connection we must make a series of simplifying assumptions, as follows:

1. The current must be steady; 2, it must be continuous and free from whirls; 3, it must flow everywhere parallel to a definite

vertical plane, and consequently depend only on the vertical coördinate (y), and one horizontal coördinate (x); 4, the internal friction, as well as the external (or that due to the earth's surface), may be neglected; 5, at great heights there must prevail a purely horizontal current of constant velocity (a). As to the configuration of the ground, we must, corresponding to proposition 3, assume that the profile curves are identical in all vertical planes that are parallel to the plane of $x\,y$; 6, and further, we assume the surface profile to be *periodic*, that is to say, the surface of the earth is formed of similar parallel waves of mountains without, however, determining in advance the special equation of the profile curves.

If we designate by u and v the horizontal and vertical components of velocity and by ε the density, then, in consequence of assumptions 1 and 3, there follows the condition

$$\frac{\partial\,(\varepsilon\,u)}{\partial x} + \frac{\partial\,(\varepsilon\,v)}{\partial y} = 0$$

and in consequence of 2 there must exist a velocity potential, φ, which, according to 3, can only depend upon x and y, so that

$$u = \frac{\partial\varphi}{\partial x};\quad v = \frac{\partial\varphi}{\partial y},\ \text{and}\ \frac{\partial}{\partial x}\left(\varepsilon\frac{\partial\varphi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\varepsilon\frac{\partial\varphi}{\partial y}\right) = 0.$$

If we consider that the density of the air (excluding large differences of temperature at the same level) changes much more slowly in a horizontal than in a vertical direction, then we can regard ε as a function of y only, and obtain for φ the differential equation—

$$\varepsilon\,\Delta\,\varphi = -\frac{\partial\varepsilon}{\partial y}\frac{\partial\varphi}{\partial y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The law of the diminution of density with altitude will, strictly speaking, be different for each particular case, because the vertical diminution of temperature in a rising current of air, which determines the rate of diminution of density, depends upon the condensation. But it is allowable, as a close approximation and as is usually done in barometric hypsometry, to assume the law of diminution of pressure which obtains, strictly speaking, for a constant temperature only, and which, as is well known, reads as follows:

$$\text{nat}\log\frac{p_0}{p} = q\,y,$$

where q is a constant and has very nearly the value of 1/8000 if y, the difference in altitude, be expressed in meters. In this case the following also holds good:

$$\log \frac{\varepsilon_0}{\varepsilon} = q\, y,$$

and, consequently,

$$-\frac{1}{\varepsilon} \cdot \frac{\partial \varepsilon}{\partial y} = q;$$

hence the differential equation for φ becomes

$$\Delta \varphi = q \cdot \frac{\partial \varphi}{\partial y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

A solution of this differential equation that satisfies the assumptions 5 and 6, is given by the expression

$$\varphi = a\,(x - b \cos m\, x.\, e^{-ny}) \quad \ldots\ldots\ldots\ldots (3)$$

in which the following relation exists between the constants m and n,

$$\left.\begin{aligned} m^2 - n^2 &= qn; \\ n &= -\frac{q}{2} + r, \text{ where } r = \sqrt{m^2 + q^2/4}. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (4)$$

In order to ascertain what profile or configuration of the ground corresponds to the current determined by this velocity potential, we must look for the lines of flow; for one of these must certainly agree with the profile curve. The differential equation of the stream lines reads as follows:

$$dy : dx = \frac{\partial \varphi}{\partial y} : \frac{\partial \varphi}{\partial x} = abn \cos mx\,.\, e^{-ny} : a\,(1 + bm \sin mx\,.\, e^{-ny}).$$

The integration of this equation gives

$$e^{-ny}.\, \sin mx = -\frac{m}{bqn} + B\, e^{qy} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

wherein B represents the parameter of the stream lines.

If we assume that the curve of the profile of the surface passes through the points $x = 0$ and $y = 0$, then for these values $B =$

$m/b\,q\,n$, and if its ordinates are designated by η, its equation becomes

$$b\frac{qn}{m}\sin mx\,.\,e^{-n\eta}=e^{q\eta}-1$$

or

$$b\frac{n}{m}\sin mx.e^{-r\eta}=\frac{e^{\frac{q}{2}\eta}-e^{-\frac{q}{2}\eta}}{q}.$$

As long as η remains so small that for both the highest and lowest points of the profile of the surface of the earth $(q\,\eta/2)^2$ is negligible in comparison with unity—which is practically always the case for the mountains that come under our consideration—we can write

$$\eta=b\frac{n}{m}\sin mx.e^{-r\eta};\quad\left[\begin{array}{l} n=-\dfrac{q}{2}+r,\\ r=\sqrt{m^2+q^2/4}\end{array}\right]\quad\ldots\ldots\ (5')$$

In these expressions b and m appear as parameters that can be chosen at will, the first of which determines the altitudes and the second the horizontal distances between the mountain ridges; we have, namely, $m=2\pi/\lambda$, if λ denotes the wave length, that is to say the distance between two corresponding points, as for example the summits of neighboring mountain ranges.

It is easy to show that the stream line determined by the velocity potential (3) for the configuration of the ground given by the transcendental equation (5′) is the only one compatible with the general conditions 1 to 5. Moreover, since a potential current is determined single valued, for the interior, by the value of $\frac{\theta\varphi}{\theta n}$ along the boundary of a closed region, therefore, our solution in case it gives horizontal velocities that are constant, or slowly diminish with the altitude above the center of the valley, is also applicable to the specially interesting practical case in which only one single mountain range rises above an extended plain and is struck perpendicularly by a uniform horizontal current of air. To what extent this holds good must be established in each special case.

The horizontal and the vertically upward velocity components corresponding to our solution are:

$$u=a\,(1+bm\sin mx\,.e^{-ny})\quad\ldots\ldots\ (6)$$

$$v=abn\cos mx\,.e^{-ny}\quad\ldots\ldots\ (7)$$

It would now be desirable, in order to be able to handle the cases actually occurring in nature, to adapt our solution to some form of the earth's surface arbitrarily chosen. The first thought would be to attempt this by the superposition of a series of velocity potentials of the form of equation (3) having different constants m and b, or in other words to write

$$\varphi = \sum \varphi_h = a\,\{x - \sum_h b_h \cos m_h x . e^{-n_h y}\} \quad . \quad . \quad . \quad . \quad (8)$$

but we find that this solution only corresponds to a superposition of the profile curves, that is to say, it gives

$$\eta = \sum \eta_h = \sum b_h \frac{n_h}{m_h} \sin m_h x . e^{-r_h \eta} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

only when we can put the exponential functions $e^{-n_h y}$ and $c^{-r_h \eta}$ both equal to unity. In this case η is at once transformed into the simple trigonometrical series

$$\eta = \sum_h b_h \frac{n_h}{m_h} \sin m_h x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9')$$

and therefore, by putting $m_h = h\ m_1$ we can develop any arbitrary function, $\eta = f\ (x)$, into a series, proceeding for any value of x greater than zero and less than $\lambda/2$. But the condition that $e^{\pm hm\eta}$ is equal to unity for any large value of the quantity h will not be fulfilled for any arbitrary form of the profile curve if its maximum altitude is assumed to be very small in comparison with the wave length λ. Therefore, we must limit ourselves to an approximate representation of the desired profile curve by a definite number of terms of the series that enters equations (9) or (9′). Especially can we in this way never attain the rigid solution for a ground profile that has sharp angles. However, the neglected higher terms of the series have a proportionately smaller influence on the vertical velocity at great altitudes and, therefore, on the resulting precipitation, in proportion as their serial number h is larger.

II

As a first example, we choose a form of profile to correspond as closely as possible to a plane, broad, valley and a plateau-like mountain range, because, in this case, we may expect nearly the same conditions on the slope of the mountain as if it were struck by a uniform horizontal current of air. A profile curve of this kind, which rises steadily between the values x greater than $-\frac{\lambda}{12}$, and less than $+\frac{\lambda}{12}$ and falls also with uniform gradient between the limits $x = 5/12\lambda$ and $x = 7/12\lambda$, and in the intermediate region describes a horizontal straight line at the distance $+H$ from the axis of $x\eta$ is obtained by means of the Fourier series

$$\eta = \frac{24\,H}{\pi^2} \sum_h \frac{1}{h^2} \sin \frac{h\pi}{6} \,.\, \sin \frac{2\,h\pi}{\lambda} x,$$

where h has all positive uneven numbers. In order to represent a profile curve of the given form approximately, we take the first three terms of the series, and therefore have

$$\eta = C\,\{\tfrac{1}{2}\sin m_1\,x + \tfrac{1}{9}\sin 3\,m_1\,x + \tfrac{1}{50}\sin 5\,m_1\,x\} \quad . \; . \; . \quad (10)$$

The numerical values of the parameters are:

$$\lambda = 60000 \text{ meters, hence } m_1 = \frac{2\,\pi}{\lambda} = 0.1047 \times 10^{-3}$$

and

$$C = 1100 \text{ meters.}$$

The coefficients b_h, in the expressions (8) and (9) therefore, have the following values:

$$b_1 = 881, \quad b_3 = 148.3, \quad b_5 = 24.8$$

The profile given by equation (10) is shown in fig. 1, where the vertical scale is magnified five times. We perceive that the ascending gradient is nearly all confined to the interval between

$$x \text{ greater than } -\frac{\lambda}{12} \text{ and less than } +\frac{\lambda}{12}$$

where, moreover, it is quite uniform, and further that the surface of the valley is raised a little in the center, and the surface of the plateau mountain is depressed by the same amount. The difference in altitude between the center of the valley and the center of the mountain, which according to the adopted numerical values should be 900 meters, is, therefore, not the absolute maximum difference but is about 18 meters less. The profile curve here considered corresponds indeed, according to what has been above said only approximately to the velocity potential.

$$\left.\begin{aligned}\varphi = a\ \{x - b_1 \cos m_1 x . e^{-n_1 y} - b_3 \cos 3\, m_1\, x . e^{-n_3 y} \\ - b_5 \cos 5\, m_1\, x . e^{-n_5 y}\},\end{aligned}\right\} \quad . . (11)$$

as determined by the above coefficients, b_h, but we can easily demonstrate that in the present example the differences could scarcely be observed in fig. 1.

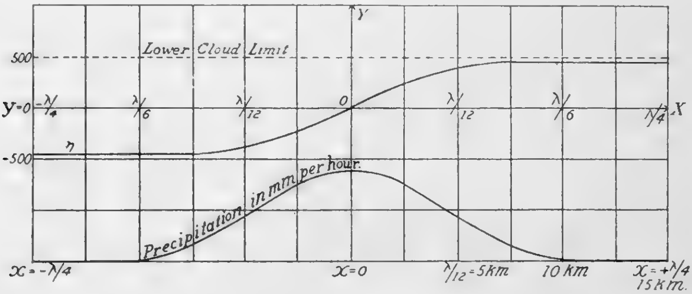


FIG. 1

From the preceding value of φ we derive the following values for the components of the velocities of the current:

$$\left.\begin{aligned} u &= a\left\{1 + \sum b_h\, m_h\, e^{-n\ y} \sin m_h\, x\right\} \\ &= a\left\{1 + \frac{2\pi}{\lambda}\left(b_1 e^{-n_1 y} \sin m_1\, x + 3\, b_3\, e^{-n_3 y} \sin 3\, m_1\, x\right.\right. \\ &\qquad \left.\left. + 5\, b_5 e^{-n_5 y} \sin 5\, m_1\, x\right)\right\}, \end{aligned}\right\} \quad . . . (12)$$

$$\left.\begin{aligned} v &= a \times \sum b_h\, n_h\, e^{-n_h y} \cos m_h\, x \\ &= a \times 0.1152\left\{\tfrac{1}{2}\, e^{-n_1 y} \cos m_1\, x + \tfrac{1}{3}\, e^{-n_3 y} \cos 3\, m_1\, x\right. \\ &\qquad \left. + \tfrac{1}{10}\, e^{-n_5 y} \cos 5\, m_1\, x\right\}. \end{aligned}\right\} \quad . . (13)$$

These equations show that when $x = 0$, that is to say above the center of the slope of the mountain, u is a constant $= a$ at all altitudes; above the valley where x is less than 0, u is smaller than a; and above the mountain, or plateau, where x is greater than 0, u is larger than a; the constant a can also be considered the mean horizontal velocity at any given altitude.

For different altitudes H above the center of the valley we have the following values:

$H = 450 + y$:	0	450	2000	5000
$\frac{u-a}{a}$:	−0.068	−0.0676	−0.0675	−0.0646

Therefore, up to the altitude of 5000 meters, the horizontal velocity is sensibly constant and the vertical velocity 0; and, according to what is said in reference to equation (5′) our solution holds good for the case when the profile is continued as a horizontal straight line indefinitely toward the negative side from the point $x = -\lambda/4$, and above this there flows a truly horizontal current of air whose velocity is sensibly constant, namely, 0.93 a up to an altitude of 5000 meters and increases in the strata above that until it attains the value a.

Above the mountain, as at the point where $x = +\lambda/4$, the velocities, u, are greater than a by nearly as much as they are smaller above the valley.

The distribution of the vertical velocity component which determines the condensation of aqueous vapor is a more complicated matter. In order to represent it, let the values of v/a for different values of the coördinates x and y be as given in the following table:

y	x				
	0	$\pm\frac{\lambda}{12}$	$\pm\frac{\lambda}{8}$	$\pm\frac{\lambda}{6}$	$\pm\frac{\lambda}{4}$
500	0.099	0.0406	0.0129	−0.0012	0
1,530	0.0842	0.04075	0.0149	+0.00226	0
2,440	0.0740	0.0400	0.0182	0.0064	0
3,460	0.0651	0.0387	0.0206	0.0093	0
4,530	0.0575	0.0370	0.0217	0.0108	0

Therefore, whereas there is a steady decrease of v with altitude above the center of the slope of the mountain, on the other hand these vertical velocities increase with the altitude in the neighborhood of the foot of the mountain as well as on the plateau at the point $x = \pm \lambda/8$ up to a maximum at some very great altitude. (The isolated negative value that occurs for $x = \lambda/6$ and $y = 500$ is explained by the above-mentioned slight depression of the summit of the plateau mountain.)

In order now, to investigate the condensation of aqueous vapor that occurs in consequence of the ascending currents of air forced upward by the upward slope of the ground, we first make the assumption that the ascending mass of air experiences an adiabatic change of condition and that adiabatic equilibrium prevailed already in the horizontal current of air advancing toward the slope of the mountain. In this case the air will be everywhere saturated at a certain altitude that can be computed from the temperature and humidity of the air at the surface of the valley. In a unit of time the quantity of air, $v\varepsilon$, flows in a vertical direction through a space having a unit of horizontal surface and an altitude dy. If this element of space lies above the lower limit of the clouds, then in this quantity of air there will be as much aqueous vapor condensed as the difference between what it can contain in the state of saturation at the altitude $y + dy$ and what it can contain at the altitude y. Therefore this quantity is

$$v\varepsilon . \frac{-\partial F}{\partial y} dy,$$

where $F(y)$ is the specific humidity of saturated air at the altitude y.

Still assuming a stationary condition, we have—

$$W = -\int_{y_0}^{y'} v\varepsilon F'(y)\, dy, \tag{14}$$

as representing the total quantity of aqueous vapor condensed in a unit of time in a stratum of cloud above the unit of basal area between the altitudes y_0 and y'.

This would also be equal to the quantity of precipitation falling from that layer of cloud on to the unit of horizontal base in case the products of condensation simply fell vertically without being carried along by the horizontal current of air. We will make this assumption, since as yet we have no clue by which to frame a computation of the horizontal transportation of the falling particles of precipitation. It is, however, easy to foresee that the horizontal transportation would be of importance, especially for the slowly-falling particles of water or ice in the upper strata of clouds, and that on the other hand, the larger drops that carry down with themselves the water condensed in the lower strata of clouds will fall at a relatively slight horizontal distance. But now, as the numerical computation shows, the lower cloud strata contribute relatively far more to the condensation than the upper clouds; therefore, the influence of the horizontal transport will not be so very large, at least with moderate winds. Moreover, this influence does not affect the total quantity of precipitation caused by the flow up the mountain side, but only its distribution on the mountain slope and it consists essentially in a transfer of the location of maximum precipitation toward the mountain. In this sense, therefore, we have to expect a departure of the actual distribution of precipitation from that which is theoretically given by the computation of W as a function of x, according to equation (14). This departure will, under otherwise similar circumstances, be considerably larger in the case of snowfall than in the case of rain.

As concerns the upper limit y', which is to be assumed in the integration of equation (14) in order to obtain the total quantity of precipitation falling upon a unit of surface, we have to substitute for y' that altitude at which condensation actually ceases in the ascending current of air. Theoretically, if from the beginning adiabatic equilibrium prevails up to any given altitude, then the condensation brought about by the rising of the earth's surface must also extend indefinitely high, even to the limit of the atmosphere, since the vertical component of velocity diminishes asymptotically toward zero. But practically, our solution of the problem of flow probably no longer holds good for very high strata, and certainly the assumption of adiabatic equilibrium does not hold good; but even if the latter were the case, if therefore, the ascending current carried masses of air from the surface of the earth up to any given altitude, still, in consequence of the increasing weight of the particles of precipitation carried up by the ascending current on the one hand, and the increasing insolation on the other hand, an upper limit of cloud must be formed.[2]

We will therefore assume as given some such upper limit of clouds at a definite altitude, and for simplicity will assume this to be the same everywhere. The value of this altitude, y', is the upper limit of the integral (14). However, the altitude assumed for y' if it is large, namely, many thousands of meters, can have only a slight

[2] W. von Bezold: Sitzb. Ber. Akad. Wiss., Berlin, p. 518, 1888, and p. 303, 1891.

influence on the value of W, since both—$F'(y)$ and $v\varepsilon$ rapidly diminish with the altitude.

For the numerical computation of W, it is advantageous to first bring the expression (14) by partial integration into the following form:

$$W(x) = \Big[v\,\varepsilon\, F(y) \Big]_{y'}^{y_0} + \int_{y_0}^{y'} F(y)\,\frac{\partial \varepsilon v}{\partial y}\,dy. \quad . \quad . \quad . \quad (14a)$$

In this expression v is given by equation (13) as a function of y and x. $F(y)$, or the saturation value of the specific moisture at the altitude y, as well as the corresponding values of the pressure and temperature necessary for the computation of ε are most easily obtained with the help of the graphic diagram for the adiabatic changes of condition of moist air first given by H. Hertz, since a simple analytical expression for these quantities cannot be given. In using the Hertzian table[3] we have to remember that y is not the absolute altitude but the altitude above the axis of x in our system of coördinates, therefore, in order to obtain the altitude above sea level, it is still to be increased by the quantity $-\eta\left(x = -\frac{\lambda}{4}\right)$ and also by the altitude of the valley above the sea. The integral in equation (14a) can be evaluated with sufficient accuracy by dividing the integral from y_0 to y' into parts $y_0 \ldots y_1$, $y_1 \ldots y_2$, $y_{h-1} \ldots y_h$ (where $y_h = y'$), and for each of these introducing an average value F_{mk} whereby we obtain equation (15).

$$\int_{y_0}^{y_h} F(y)\,\frac{\partial(\varepsilon v)}{\partial y}\,dy = \sum_0^h{}_k\, F_{m_k} \Big[(\varepsilon v)_k - (\varepsilon v)_{k-1} \Big] \quad . \quad . \quad (15)$$

In order to execute the complete computation of W for a special example, we will assume that the current of air which strikes the mountain having the profile shown in fig. 1 has a pressure of 760 millimeters, temperature 20°, and specific humidity, 9.0,[4] at the bottom of the valley. Hence, according to our assumption of adiabatic equilibrium it follows that the lower limit of the clouds will lie at an altitude of 950 meters above the bottom of the valley, and, therefore, 50 meters above the center of the mountain, if $y_0 = 500$;

[3] H. Hertz: Met. Zeit., Vol. I, pp. 421–431, 1884, or the preceding collection of translations, 1891, p. 198.

[4] That is, 9.0 grams of water per kilogram of air.

the specific humidity is at this cloud level, $F(y)' = 9.0$, and the temperature is 11° C. We will further assume that the upper limit of the clouds is at an altitude of about 5000 meters, or $y' = 4530$ meters, where the temperature has sunk to $-13.6°$ and the specific humidity to $F(y) = 2.5$. At the altitude of 3000 meters the temperature 0° C. is attained. The application of the Hertzian tables assumes that for temperatures below 0° C. the product of condensation is ice; whether this is really true is at least questionable for moderately low temperatures, but the assumption that water below the freezing point is precipitated will not change the results very much. Since corresponding to the assumed stationary condition, we have to assume that all condensed water immediately falls from the clouds; therefore, in our computation we have to omit the hail stage of Hertz, in which the water that is carried along with the cloud is frozen.[5]

For the computation of the integral according to equation (15) the cloud is divided into four layers whose mutual boundaries or limits occur at $y_1 = 1530$, again $y_2 = 2440$, and $y_3 = 3460$ meters; for these altitudes we have $\varepsilon = 1.00$ and 0.912 and 0.816, and corresponding to these $F(y) = 6.9$ and 5.35 and 3.8.

We thus find the following values for W/a:

$x =$	0	$\pm\frac{\lambda}{12}$	$\pm\frac{\lambda}{8}$	$\pm\frac{\lambda}{6}$	
$\frac{W}{a} =$	0.475	0.241	0.0985	0.0081	grams per second per square meter.

From this table we obtain the depth of the precipitation in millimeters per hour by multiplying by 3.6; the result is shown in the lower curve of fig. 1. The values of the precipitation for a mean horizontal velocity of the current of 1 meter per second are as follows:

$x =$	0	$\pm\frac{\lambda}{24}$	$\pm\frac{\lambda}{12}$	$\pm\frac{\lambda}{8}$	$\pm\frac{\lambda}{6}$	$\pm\frac{\lambda}{4}$
$W' =$	1.71	1.47	0.867	0.355	0.029	0

Hence, the precipitation is heaviest above the middle of this slope of the mountain, where for the very moderate wind velocity of 7 meters per second, it attains the very considerable rate of 12 milli-

[5] The influence upon the adiabatics of condensation, whether we assume, as in the Hertzian table, all condensed water to be carried with it or to immediately fall away, is of no importance in the present problem.

meters per hour. In this connection it is, indeed, to be remembered that we have assumed exceptionally favorable conditions for the precipitation in that we have assumed the onflowing air to have been already fully saturated throughout the whole 4000 meters in depth of the layer between y_0 and $y.'$

The comparison of the curve of precipitation with the curve of profile in fig. 1 shows that although the maximum of precipitation coincides with the maximum gradient of the slope of the mountain, yet the depth of precipitation diminishes more slowly toward the plane of the valley and the plateau of the mountain than does the slope of the earth's surface; thus, for instance, the latter slope at the point where $x = \pm \lambda/12$, and which is given by $\partial\eta/\partial x$, amounts only to 1/20 of the maximum slope, while the precipitation at this point is more than 1/5 of its maximum value. Therefore, under the conditions here assumed, the effect of a mountain slope in producing precipitation makes itself felt in the plain lying in front of the foot of the slope. All of which agrees with actual experience.[6] The fact that in reality the maximum precipitation appears to be pushed more toward the ridge of the mountain is certainly partly explained, as well as suggested, by the horizontal transportation of the products of condensation in the clouds, but also in part by the departure of the real distribution of temperature and moisture from that here assumed. (See Section IV, page 95.)

The determination of the total quantity of precipitation caused by the mountain slope will be attained if we integrate the value of W as determined by equation (14) as a function of x between the limits $x = -\lambda/4$ and $x = +\lambda/4$. The result is, therefore,

$$G = \int_{-\frac{\lambda}{4}}^{+\frac{\lambda}{4}} W(x)\,dx = -\int_{y_0}^{y'} \varepsilon F'(y) \int_{-\frac{\lambda}{4}}^{+\frac{\lambda}{4}} v\,dx. \quad \ldots (16)$$

In this equation, according to equation (13) we have:

$$\int_{-\frac{\lambda}{4}}^{+\frac{\lambda}{4}} v\,dx = a \times 1100 \left\{ e^{-n_1 y} - \frac{2}{9} e^{-n_3 y} + \frac{1}{25} e^{-n_5 y} \right\}.$$

[6] Hann: Climatology, 2d Edition, Vol. I, p. 295; also Assmann, Einfluss der Gebirge auf das Klimat von Mittel Deutschland, p. 373, 1886.

For our present example we find $G = 5100a$ grams per second over a strip 1 meter wide and about 22 kilometers long. Hence, there follows for the average precipitation for the whole mountain slope

$$W_m' = 0.833a \text{ millimeters per hour.}$$

III

In the example we have just discussed the lower limit of the clouds was higher than the summit of the mountain. If the reverse is the case, then, for that portion of the mountain slope that is immersed in the clouds we must take η instead of y_0 as the lower limit of the integral in the formulæ (14) to (16): therefore, the theoretical distribution of precipitation would no longer be symmetrical with respect to the zero point on the axis of abscissas. As an example of this case we will consider the flow of air above the ground profile that is represented by the simple equation

$$\eta = C \sin m\,x.e^{-r\eta}$$

As to the constants we will adopt the following:

$$C = 1000 \text{ meters}, \qquad \lambda = 24000 \text{ meters};$$
$$\text{hence } m = 0.262 \times 10^{-3}, \qquad r = 0.269 \times 10^{-3},$$

and for the vertical coördinate η we find from equation (5)

$$\text{for } x = -\frac{\lambda}{4} \quad -\frac{\lambda}{6} \quad -\frac{\lambda}{12} \quad 0 + \frac{\lambda}{12} \quad +\frac{\lambda}{6} \quad +\frac{\lambda}{4}$$
$$\eta = -1495 \quad -1194 \quad -585 \quad 0 + 444 \quad +715 \quad +805 \text{ meters.}$$

The resulting curve is shown in fig. 2. The altitude of the summit of the mountain above the plain of the valley amounts to 2300 meters. The valley may be 100 meters above sea level; the atmospheric pressure in the valley is assumed at 750 millimeters, the temperature 23°, and the specific humidity 10 grams of water per kilogram of air. From the Hertzian table we find the lower cloud limit at the altitude of 1220 meters, that is to say at $y = -375$. The upper limit of the clouds is assumed at $y' = 2400$ and, therefore, at 4000 meters above sea level. Therefore, for that portion of the clouds lying below the summit of the mountain, which is limited to the negative values of the abscissas up to $x = -1.35$ kilometers approximately, since according to equation (7)

$$v = Cam \cos m\,x.e^{-ny}$$

we have:

$$W = -\int_{y_0}^{y'} \varepsilon\, v\, F'\, d\,y = -\,a\, C\, m \cos m\, x \int_{y_0}^{y'} \varepsilon\, F'\,(y)\, e^{-ny}\, d\,y$$

$$= a \cos m\, x \times 1.09.$$

Therefore, the depth of the precipitation will here be represented by a simple cosine curve and, in general, corresponds to the slope of the mountain, which is computed from equation (5′) by the expression:

$$\frac{d\,\eta}{d\,x} = \frac{C\,m\ \cos m\,x\,.\,e^{-r\eta}}{1 + Cr\ \sin m\,x\,.\,e^{-r}}$$

For the region lying above the lower cloud limit y_0 the value of $W(x)$ cannot be represented by a simple function of x. We find

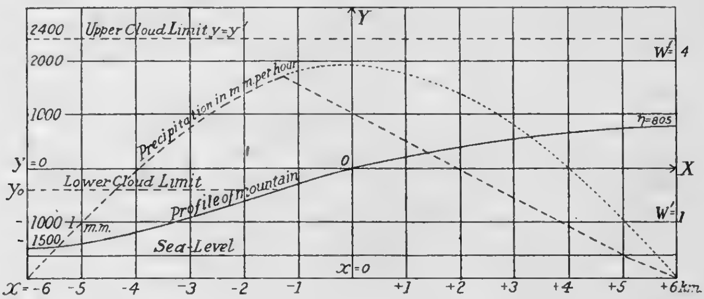


FIG. 2

the precipitation in millimeters per hour for a horizontal velocity $a = 1$, as follows:

For x	=	−6	−5	−4	−3	−2	Below the cloud.
W'	=	0	1.01	1.96	2.78	3.40	
For x	=	−1	0	+2	+4	+6	In the cloud.
W'	=	3.50	2.94	1.95	0.88	0	

The distribution of precipitation, as given by these figures is shown in fig. 2 by the curve of dashes. The curve of dots represents the symmetrical line that would obtain if the mountain were not immersed in the clouds. The location of maximum precipitation is 3.93 for $x = 0$ and is 3.68 for $x = -1.3$.

The total quantity of precipitation is computed by the formula:

$$G = -\, a\, C \sin m x \int_{y_0}^{y'} \varepsilon\, F'(y)\, e^{-ny}\, d y$$

and is approximately equal to 22730; this is distributed over a horizontal strip 12000 meters in length, and therefore, for a uniform distribution for $a = 1$ the precipitation averages 1.9 millimeters.

From the preceding expression for G, it is plain that for any given altitude of the mountain summit G will be smaller the shorter and steeper the slope becomes, that is to say, the smaller the value of λ is, since the exponent ny increases with diminishing values of λ. In the present case the horizontal velocity of the wind is given by the expression:

$$u = \frac{\partial \varphi}{\partial x} = a\left(1 + C\frac{m^2}{n}\sin m x . e^{-ny}\right)$$
$$= a\,(1 + 0.332 \sin m x . e^{-ny});$$

which attains its minimum, $= 0.547a$, at the bottom of the valley, and its maximum, $1.283\ a$, at the summit of the mountain, and has a for the mean value of all the horizontal planes. Above the center of the valley, it increases gradually with altitude, asymptotically approaching its limiting value, a; for example, at the level $y = 0$, it is equal to $0.668a$, and at the level $y = 2400$ it is already equal to $0.80a$. Therefore, if the stream under consideration proceeds from a point $x = -\lambda/4$, as a purely horizontal current of air flowing over a plain, then its velocity must diminish with the altitude in the ratio e^{-ny}. This would, of itself, be a plausible assumption, but there would then be a vortex motion for each horizontal current of air, which cannot, strictly speaking, continue steadily in the above assumed potential motion.

IV

The assumptions hitherto made by us, namely, that the distribution of temperature in the current of air that impinges upon the mountain side already corresponds to the condition of indifferent equilibrium, that is to say that it is the same as would occur in an ascending current of air under adiabatic changes of condition, is in general not actually fulfilled. The scientific balloon ascensions at Berlin have recently given us reliable conclusions as to the real conditions of temperature and moisture in the free atmosphere up to altitudes of 8000 meters. The mean values of the temperature and moisture at successive levels, 500 meters apart, which von Bezold has deduced[7] from the observations of Berson and Süring show that the mean vertical diminution of temperature is slower than the adiabatic, and that, in general, the moisture does not attain the saturation value. In a horizontal current of air, in which these average conditions prevail, the air will, therefore, never be saturated, and, consequently, our assumption of the existence of a constant lower limit to the clouds is not allowable. Moreover, it is no longer the vertical component alone that controls the condensation that shall occur at any given point in the current of air ascending above the mountain slope, as was assumed in the derivation of formula (14). We must rather, in the computation of W, consider that the quantity of water condensed in a unit of space under steady stationary conditions is equal to the excess of the quantity of water vapor flowing into the space above that simultaneously flowing out. For one cubic meter and one second this excess is:

$$-\left(\frac{\partial(\varepsilon u F)}{\partial x}+\frac{\partial(\varepsilon v F)}{\partial y}\right),$$

or since because of the equation of continuity we have approximately

$$\frac{\partial \varepsilon u}{\partial x}+\frac{\partial \varepsilon v}{\partial y}=0,$$

therefore,[8]

$$-\varepsilon\left(u\frac{\partial F}{\partial x}+v\frac{\partial F}{\partial y}\right),$$

and hence,

$$W=-\int_{y^0}^{y'}\varepsilon\left(u\frac{\partial F}{\partial x}+v\frac{\partial F}{\partial y}\right)dy \quad . \; . \; . \; . \; . \; . \; (17)$$

where y^0 and y' indicate the altitudes of the limits of the clouds above the point under consideration. The evaluation of the integral still demands not only a complete knowledge of the stream, but

[7] W. von Bezold: Theoretische Betrachtungen, etc. Theoretical considerations relative to the results of the scientific balloon ascensions of the German Association for the Promotion of Aeronautics at Berlin. Brunswick, 1900, pp. 18–21. (See No. XIV of this present collection).

[8] In so far, namely, as the quantity of the aqueous vapor condensed in a unit of volume is inappreciably small in comparison with the total quantity of moist air flowing through this space.

also the determination of the cloudy region, that is to say, that region in which the atmosphere is saturated and the distribution of temperature therein, since the latter first gives us the value of F. To this end we have to follow the adiabatic change of condition of the air along each curve of flow, starting with the given temperature and humidity, in the vertical above the center of the valley where $x = -\lambda/4$, where the current is truly horizontal.

By connecting together those points in the individual stream lines at which saturation is just attained we find, first, the contour of the cloudy region.

Since the form of the cloud is also of interest in and of itself[9] therefore its determination will be carried through as a part of our second example, in that above the center of the valley, where $x = -\lambda/4$ first for the summer, then for the winter, we make some assumption as to the mean distribution of temperature in accordance with von Bezold's collected data, on page 21 of his memoir above quoted. In accordance with this, we have:

	For y =	−1500	−600	+400	+1400	+2400 meters.
		Valley above sea level 100 m.				Height above sea level, 4000 m.
Summer	t =	17.7°	11.0°	5.3°	+0.9°	− 5.0°
	F =	8.2	6.69	4.59	3.03	2.60*
Winter	t =	0.2°	0.6°	−5.1°	−10.8°	−14.6°
	F =	2.92	2.17	1.64	1.19	0.86

In place of the value of F, designated by a star, we will take that value (2.2) that results from the smoothing out of the protuberant corners which the curve for F (see von Bezold, fig. 11,) shows at the altitude of 4000 meters.

According to equation 5 the lines of flow have for their expression

$$e^{-ny} \sin mx = -\frac{m}{bqn} + B e^{qy},$$

or if y_0 is the value of y when $x = 0$, and $y - y_0 = \eta$, there results,

$$e^{-n\eta} e^{-ny_0} \sin mx = \frac{m}{bqn} (e^{q\eta} - 1),$$

$$\frac{bn}{m} e^{-ny_0} e^{-r\eta} \sin mx = \frac{1}{q} \left(e^{\frac{q\eta}{2}} - e^{-\frac{q\eta}{2}} \right).$$

[9] It seems, for example, quite possible to argue from the observed boundary of the clouds inversely to the percentage of moisture in the current of air flowing toward the mountain slope.

With the same degree of approximation as before the right-hand side of this equation can be put equal to η; therefore the equation takes the following form:

$$\eta = b\frac{n}{m}\sin m x . e^{-r n}\, e^{-n y_0} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

which differs from equation (5′) of the profile curve of the ground only through the factor which is constant for each line of flow, which factor causes the amplitude of the waves to steadily diminish upward.

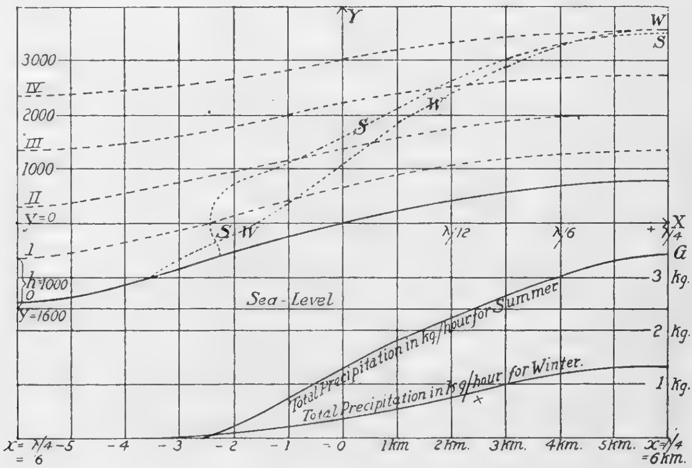


FIG. 3

If, now, the lines of flow are made through a definite point y'_h for the vertical and $x = -\lambda/4$, then for this point we determine the appropriate value η' from the transcendental equation:

$$\eta' = -b\frac{n}{m} e^{-r n}\; e^{-n\,(y'_h - \eta)} \quad . \; . \; . \; . \; . \; . \; . \quad (19)$$

and then substitute $y^0_h = y'_h - \eta'$ in equation 18.

In this way we have computed the four lines of flow whose initial and lowest points are at the altitude above sea level of 1000, 2000, 3000, and 4000 meters, and which are drawn as curves I, II, III, IV, in fig. 3. The highest points of these curves are at the altitudes 2940, 3610, 4333, 5100 meters, respectively.

If now, by means of the Hertzian table, we determine the altitudes at which condensation begins at the base curve 0 and for the curves I, II, III, IV, then assuming the above given values[10] of t and F we find the following results:

	0	I	II	III	IV
For the summer	930	1570	2730	4060	(5125)
For the winter	600	2070	3100	4130	5100

In the summer, according to this table, condensation will not take place on the stream line IV, since its summit lies at the altitude of 5100 meters; the summit of the clouds will, therefore, lie a little below this. In the winter, the summit of line IV accidentally agrees with the summit of the cloud. In the construction of the cloud limit, introduced as a dotted line in fig. 3, and indicated by S for summer and W for winter, we have also used the lines of flow midway between 0 and I, and I and II, respectively.[11]

We can now, with the help of the Hertzian table, easily find the quantity of water condensed in every kilogram of moist air as it progresses along any one of the lines of flow that we have constructed, either in its totality or as it passes successive vertical lines: we thus attain the following values of the total condensation:

Curve	0	I	II	III
For the summer	2.85	2.42	1.22	0.26 grams.
For the winter	1.5	0.74	0.34	0.14 grams.

Let $g_x(h)$ be the quantity condensed up to the abscissa x when moving along that line of flow whose initial point is at the altitude h, and let H be the initial altitude of that line of flow which at the given abscissa x intersects the upper cloud limit; moreover, let u'

[10] From the above numbers it follows that an elevation of any kind of less than 500 meters will not give occasion for condensation under average atmospheric conditions, neither in summer nor in winter. In the summer, for a mountain altitude of between 600 and 800 meters, a cloud will form between the altitudes 1000 and 3000 meters, but will not touch the mountain; it is only for greater mountain heights that the cloud will rest on the mountain.

[11] In an analogous way for the first example, where we have assumed a plateau-like mountain of 900 meters altitude, we find a region of cloud which for the average summer conditions begins at 40 meters below the summit of the plateau and reaches up to over 3000 meters; but in winter, on the other hand, it begins at 500 meters above the valley and rises up only about 700 meters above the mountain top; therefore, in this season it covers the mountain like a flat cap.

be the horizontal velocity of flow and ε' the density of the air at the altitude h above the bottom of the valley, therefore, for the point whose abscissa $= -\lambda/4$; then will the total quantity condensed per second above the base area one meter broad from the beginning of the clouds to the point x, expressed in grams, be as follows:

$$G_x = \int_0^H \varepsilon' u' g_x (h)\, d h. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The quantity of air, $\varepsilon\, u$ kilograms, flows in one second through a strip of the vertical plane at $x = -\lambda/4$, having a unit width and the height $d\,h$; but an equal quantity must flow out per second through the vertical whose abscissa is x, and since the condition is steady, it therefore behaves as though the quantity of air, $\varepsilon\, u$, had moved in one second along the line of flow from $-\lambda/4$ up to x; but in this the quantity of water $\varepsilon\, u\, g_x(h)$ is separated from the air according to our definition of g.

If we have computed G as a function of x, according to formula (20), then, finally, we have

$$W = \frac{\partial G}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

as the quantity of water, expressed in grams, per horizontal square meter per second, that falls at the place x. In this way the determination of W is executed more conveniently than through the direct formula (17). By assuming the average conditions for the summer in the above example for $a = 1$, we find that the integral (20), if we compute it as approximately equal to the sum of the intervals between the individual current curves of flow as constructed, gives the following:

$$G_{x=0} = 1352, \quad G_{x=\lambda/8} = 2680, \quad G_{x=\lambda/4} = 3460 \text{ grams.}$$

This last number indicates the total precipitation falling on a strip one meter wide in one second on the side of the slope that faces the wind. According to the course of the curve SS, as shown in fig. 3, the precipitation begins, first, in the neighborhood of $x = -0.108\lambda$ and therefore is distributed along a strip of the ground surface, whose length is 0.358λ, or 8600 meters; from this we compute the average precipitation per hour, as follows:

$$\frac{3.6 \times 3460}{8600} = 1.45 \text{ mm. depth}$$
$$= 1.45 \text{ kg. mass}$$

Similarly, we find for winter:

$$G_{x=0} = 380, \quad G_{x=\lambda/6} = 770, \quad G_{x=\lambda/4} = 1264;$$

the total precipitation is distributed over a strip 9400 meters long, so that the average precipitation is 0.485 millimeters per hour.

From the above three values of $G(x)$ we can graphically construct the course of this function approximately by considering that the tangent to the curve for G is horizontal at its initial point and when $x = +\lambda/4$.

The tangent to the slope of the curve is found by considering its measure W'. Thus we recognize in our case that the maximum of the precipitation in summer is attained between $x = 0$ and $x = -1$, but in winter between $x = 0$ and $x = +2$ kilometers and amounts to $a \times 2.2$ millimeters, or $a \times 0.75$ millimeters per hour, respectively, for a wind velocity of a meters at some very great altitude; furthermore, after passing the summit of the mountain the precipitation diminishes more slowly than was found under our previous assumption of a constant thickness of clouds. In reality, on account of the conveyance of the water or ice with the cloud, which we still neglect as before, the maximum of precipitation is pushed still more toward the summit of the mountain. Moreover, since one part of the cloud floats over the summit and is there dissipated in the sinking or descending currents of air, the precipitation will stretch a little beyond the summit, but its total quantity will be less than the computed.

The results of the preceding analysis, namely, (1) that there exists a zone of maximum precipitation on the windward slope of a mountain and (2) that the inclination of the surface of the earth is more important than its absolute elevation, in determining the quantity of precipitation, are confirmed by observations, at least for the higher mountains.[12]

[12] See Hann: "Klimatologie," Vol. I, p. 298.

SUPPLEMENTARY REMARKS BY PROFESSOR POCKELS ON THE THEORY OF THE FORMATION OF RAIN ON MOUNTAIN SLOPES

(Reprinted from Monthly Weather Review for July, 1901)

(1) Assuming the average vertical distribution of temperature and moisture for each of the four seasons of the year as it is deduced by von Bezold from the scientific balloon ascensions published by Berson and Assmann in their "Ergebnissen." "The results of scientific balloon voyages," there result the following minimum elevations required in order that condensation may begin in a mass of air that was originally at the absolute altitude H above sea level.

H.	SPRING-TIME.	SUMMER.	AUTUMN.	WINTER.
meters.	*meters.*	*meters.*	*meters.*	*meters.*
0	725	850	405	400
500	485	710	615	760
1,000	855	570	600	1,070
1,500	890	680	835	1,140
2,000	920	730	1,180	1,100
3,000	830	1,060	1,208	1,130
4,000	700	1,125	1,240	1,100

The smallest number in each column is also the smallest altitude that a mountain ridge must possess in order to cause the formation of clouds under the assumed conditions, but it is only in the case of a very broad mountain ridge that such small altitude will suffice. We see that in the autumn and winter a mountain of about 400 meters in height will suffice to produce a formation of cloud in contact with the summit of the mountain whereas, in spring and summer the mountain must be higher (namely about 500 or 570 meters respectively), and when the air passes over this mountain the formation of cloud will begin in the layer lying at 500 or 1000 meters above its summit. These numbers at present serve only as examples; in practice, however, they suggest that as soon as we observe the formation of cloud above a mountain of less altitude than the above given tabular minimum altitude, we may conclude somewhat as to the average moisture at that altitude at that time. We may also remark that on account of the increasing flatness of the lines of flow as the altitude increases, the above given minimum altitudes must be exceeded by so much the more in proption as the width of the summit ridge is smaller, and the altitude of the layer in which the condensation begins is higher.

(2) The method developed by me for computing the condensation that occurs on any given mountain slope cannot be applied to computing the mean value of the precipitation for any given interval of time, by introducing into the computation the mean values of the temperature and moisture for this interval. We should in this way find too small a precipitation. Thus, for example, the altitude of the mountains might not suffice to cause any condensation at all for the average condition of the air, but could cause it on those occasions when the moisture exceeds its average value, wherefore the average value of the rainfall for the interval of time under consideration would be different from zero. As the variation of the moisture from its average value may cause rainfalls where otherwise there would be none, so also, with the currents of air mechanically forced to ascend mountain ranges, and whose effect is superposed upon that of the general circulation of the air in cyclonic areas; for it can happen that neither one of these two causes may alone suffice to form rain, but that both together do. This explains why elevations of the surface of the earth of from 100 to 200 meters increase the annual mean value of the total precipitation, as for instance, as shown by the charts in Assmann's memoir of 1886, "Einfluss, etc.," "On the influence of mountains on the climate of central Germany."

(3) The examples given in my article show that in so far as condensation in general takes place on the slopes of mountains, its intensity (therefore also, the density of the precipitation when falling vertically) is in general greatest where the slope of the mountain is steepest. If now we consider that in the course of all the various conditions of the atmosphere that may occur in a long interval of time, the first condensation occurs most frequently above the upper portion of the slope, then it follows that the average density of precipitation computed for a long interval of time, must increase, not only with the inclination of the slope, but also with the absolute altitude of the locality under consideration. To this case corresponds the formula for the annual quantity of precipitation expressed in millimeters deduced by Dr. R. Huber in his "Untersuchungen, etc. Investigation of the distribution of precipitation in the canton of Basle," namely:

$$N = 793 + 0.414h + 381.6 \tan \alpha$$

where h is the altitude in meters, and α indicates the gradient angle. (See A. Riggenbach, Verhandlungen der Naturforschenden Gesellschaft. Basel, 1895. Vol. X, p. 425).

(4) From a comparison of the effects of different broad mountain ranges of the same altitude, it results (see page 474 of my article, or page 95 of this translation from the *Monthly Weather Review*) that the smaller, and therefore steeper, mountains always cause a smaller total condensation than the broader and narrower mountain summits. Notwithstanding this, the density of precipitation on the slope of the smaller is generally larger than on the slope of the larger mountains because the smaller total precipitation is distributed over a ground surface that is relatively much smaller yet. In reality, however, this only obtains so long as the quantity of water remaining suspended in the cloud is only a small fraction of the total condensation; in the case of very narrow mountain ridges it will be more apt to happen that a considerable fraction passes on over and beyond the summit and is subsequently again evaporated [and therefore does not appear as rainfall].

(5) I regret to notice that in the first two figures of my original memoir, as also in the translation, the legend inscribed on the curves representing the distribution of precipitation reads "precipitation in millimeters per second," instead of "per hour," as is correctly stated in the text; the necessary correction should be made. [Corrections have been made in the present volume.]

(6) A precise test of this theory cannot at present be carried out because we have not sufficient observations of the condition of the upper strata and of ground along the slope of a given mountain range.

IX

RESEARCHES RELATIVE TO THE INFLUENCE OF THE DIURNAL ROTATION OF THE EARTH ON ATMOSPHERIC DISTURBANCES

BY M. GORODENSKY

Memoirs of the Imperial Academy of Sciences of Saint Petersburg, 1904, Volume XV, section 9]

[The original memoir above quoted is published in the Russian language; the brief abstract, in French, communicated by the author to the "Annales" of the Met. Soc. of France, Vol. LIII, pp. 113–120, May 1904, has been folowed in the present translation]

In order to study in detail the causes of the origin of atmospheric disturbances and their more important properties it is first necessary to resolve the fundamental problem: *given a certain system of mechanical forces, how will it act on the air in the different strata of the terrestrial atmosphere?* We propose to seek the possible solution for the special case of the strata situated in the immediate neighborhood of the ground.

It is easy to determine the action of any force in the midst of an ideal gas, whose particles move among themselves without friction, as in a vacuum and according to the law of gravitation only. In order to pass from such a gas to the atmosphere it is necessary to know not only the special properties of the air itself but also the value of the friction. The influence of this friction is appreciable in two ways: (1) it diminishes the velocity of the progressive movement of the air, (2) it enfeebles the action of the force perpendicular to this current of air, and to the same extent diminishes the angular velocity of the atmospheric particles. The evaluation of this normal friction, that is to say, the effect directed along the line normal to the direction of motion is the subject that we propose to study in this memoir.

Fortunately there is at our disposition a very convenient agent that one can utilize to this end. This is a well-known constant force and one which is always perpendicular to the trajectory of any mass that is moving on a horizontal plane: it is the action of the diurnal rotation of the earth.

Let us imagine an elementary small mass projected horizontally on a perfectly polished plane surface with an initial velocity v_0. It continues along its route with this same velocity tracing a curved line whose radius of curvature, as is well known, is given by the equation

$$r = \frac{v_0}{\frac{4\pi}{T} \sin \alpha} \quad \ldots\ldots\ldots \quad (1)$$

where T is the duration of a complete day, i. e., one rotation of the earth and α the latitude of any place on the earth traversed by the center of the mass. The center of curvature of the path is always on the left-hand of the direction of motion where the mass is in the southern hemisphere and on the right-hand in the northern hemisphere.

Assume the notation

$$K = \frac{4\pi}{T} \sin \alpha$$

which expresses the angular velocity of the moving point. Its value can be calculated from observation of the wind between two stations in the following manner:

At the station A take an observation of the velocity of the wind blowing towards the station B. The distance between the stations and the velocity of the wind being known we obtain by simple division the interval of time required by the particle of air to reach the station B. By observing at this moment the direction of the wind (at B) we find a difference between the two observed directions, which difference should give us the required value K. This value generally differs greatly from that calculated by the theoretical formula because of the many accidental conditions, among which there is however one force that constantly and continuously influences the movements of the atmosphere. This is the internal friction (or viscosity) of the air and also the friction between the air and the surface of the terrestrial globe. If the number of observations employed by us is sufficiently large, as well as the length of the period of time and the number of stations collated, then all anomalies neutralize each other and one obtains a resulting mean value for K as diminished by friction only, or $k = \mu K$
Now it is the coefficient μ that is the desired characteristic of the air near the ground.

The preceding expresses only the general scheme of the proposed method, for in fact we still have to surmount numerous difficulties, the more important of which are the following:

(1) The terrestrial surfaces separating the stations A and B should be as flat and smooth as possible, not having any high obstructions, in order that the air may pass freely from one station to the other. For this reason we have not utilized observations of refined anemometers and anemographs which are located frequently in large cities and have felt obliged to rely on the observations of stations of the second class in the meteorological system of Russia.

(2) We do not generally find at station A a current of air flowing exactly towards station B but inclined to that direction by the angle β so that for the length of the path described by the wind between the two stations it suffices to take the distance

$$S = A\,B \cos\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In fact this can only be an approximation since the trajectories of the atmospheric particles are curves and not straight lines and the value of S is larger rather than smaller than that indicated by the equation (3).

(3) The instrument by which at Russian stations we ordinarily measure the direction and the velocity of the wind is a wind vane placed at the summit of a mast and furnished with a suspended plate of steel which by its departure from a vertical position indicates the velocity of the wind. Now the iron cross-piece of this mast showing the cardinal points, *N.*, *S.*, *E.*, *W.*, is often oriented inaccurately and the wooden mast that carries it often acquires after awhile a twist introducing an angular error amounting to many degrees. The observations made by such a primitive apparatus cannot be very exact, so that one must utilize very many of them in order to eliminate these errors.

(4) At stations of the second class the observations are made at 7 a.m., 1 p.m. and 9 p.m., and consequently we do not generally find at the station B any observations for the moment of time that we have found by our calculation. It remains then only to make a proportional interpolation between the two observations that come nearest to this moment.

As the detailed exposition of the method now proposed cannot be given within the limit of this abstract we must here confine ourselves to giving the results of our calculation:

The mean value of μ for 3762 cases is found to be

$$\mu = 0.026 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

But this value has neither practical nor scientific interest because it corresponds to the whole scale of different velocities of the wind from 5 meters per second up to 20. This itself explains the importance of the so-called mean error.

By grouping the numbers in such a way that each group corresponds to a certain velocity we have formed the following table:

v Meters per second.	$\Sigma\mu$	n Number of cases.
5	18.0	1461
6	51.9	1232
7	5.7	800
8	17.4	698
9	4.9	386
10	25.2	444
11	0.3	24
12	20.4	296
13	−3.8	14
14	17.5	194
15	3.3	18
16	5.9	39
17	−3.7	52
18	6.4	24
19	——	0
20	17.7	185

The negative values show the cases where the wind deviated to the left, in spite of the theory, and not to the right of the rectilinear direction.

After having submitted this table to a detailed examination, which I need not repeat here, we have obtained the following values of the coefficient μ corresponding to four different values of v.

v	μ
5.8	0.022
8.8	0.031
12.0	0.051
15.9	0.092

Representing these figures graphically by orthogonal coördinates we obtain a very regular curve, a sort of parabola, whose natural prolongation crosses the axis of μ at some distance from the origin. The equation of such a parabola, as we well know, is

$$\mu = c\, v^2 + c' \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

where c and c' are constant parameters. The introduction of the parameter c' is explained by the law of Dove, according to which the weather vane at any meteorological station in Europe generally turns in a direction contrary to the motion of the hands of a watch when an area of low pressure is passing by the station. In fact by employing the observations of stations for which Dove's law holds good we obtain a coefficient greater than it ought to be by a quantity independent of the velocity of the wind. This interesting phenomenon is shown with perfect clearness in the graphical representation.

As the function $\mu\, v$ characterizes the friction of the air in a direction perpendicular to the current, one ought to be able to determine this function theoretically, if we knew a similar function for the direction parallel to the current, since the two coefficients ought to depend directly on each other.

During the progressive motion of masses of air a certain friction is developed whose reaction, tending to reduce the linear velocity of the movement, is perpendicular to this velocity, according to the simple law of Guldberg and Mohn $f = \eta\, v$ where f is the reaction of the friction, v is the velocity of this wind and η is a coefficient that depends only on the pysical state of the air and the surface of the earth.

This being recognized, we have studied a regular stationary cyclone of large extent and without any progressive movement, from a purely mechanical point of view. After having examined a portion of this whirlwind somewhat distant from its axis we have obtained the following expression for the function $(v\mu)$ viz:

$$\frac{1}{\mu} = \frac{1}{\varepsilon\, v^2} + 1 \qquad \text{. . . . . (7)}$$

where

$$\varepsilon = \frac{\sin\alpha \cos\alpha}{J\,(\eta \sin\alpha - K\cos\alpha)} \qquad \text{. . . . . . . (8)}$$

The letters introduced into these formulæ have the following signification: α is the angle formed by the direction of the current of air with the radius vector R drawn to the axis of the whirlwind and is counted positively starting in the direction of the motion of the hands of a watch from some initial radius vector; J indicates the product $v\, R$ which I have called the *expression of the intensity of any atmospheric disturbance;* K is given, as already stated, by

$$K = \frac{4\,\pi}{T} \sin\alpha \qquad \text{. . . . . (9)}$$

The values of K and η being absolutely independent of v, as also are the values of J and α, for the given disturbance, as has been demonstrated in our study, therefore the coefficient ε is also independent of v. But equation (7) expresses a certain property of the atmosphere, whatever may be the special phenomena by means of which it has been determined. It results from this that the value ε in formula (8) is absolutely constant for any physical state of the atmosphere and that it cannot vary with α nor with J, but only with the coefficient η; the values of ε and η characterize the friction, both of them, but in different directions only.

This argumentation may seem to be erroneous, as I have already had occasion to convince myself on hearing the opinions of several experts to the effect that any such process of investigation seems to them doubtful and untrustworthy.

In place of defending my logic or my honesty against the incredulous, I allow myself here to show in brief some properties of equation (8) which result from the assumption as to the invariability of the coefficient ε.

We can consider equation (8) as the expression of the connection that must exist between the angle α on the one hand and the values J K and η on the other. Let us examine each of these connections separately by means of the special partial derivatives:

α considered as a function of J.—This function has two different branches:

(1) At the moment when a disturbance originates $J = 0$, the air is put into gyratory motion in the direction of the hands of a watch in the southern hemisphere, for which $\alpha = \dfrac{3\pi}{2}$ and in the inverse direction in the northern hemisphere for which $\alpha = \frac{1}{2}\pi$. In proportion as the velocity of rotation increases the current of air deviates towards the center, that is to say, to the right hand in the southern hemisphere (and J also increases as α increases), but to the left-hand in the northern hemisphere, where J increases as α decreases. We thus have a cyclone properly so-called, and particularly so far as regards its lower portion.

(2) At the commencement of the phenomenon the air expands outward from the center along the radius for which $\alpha_0 = \pi$. Then as the disturbance develops the currents of air commence to deflect in the direction of the movement of the hands of a watch in the northern hemisphere, where α increases, and in the inverse direction in the southern hemisphere, where α decreases. This is the lower portion of the anticyclone.

For these two cases there is a certain limit that the angle α can only attain when J becomes infinitely large, which is determined by the equation (10),

$$\tan \alpha = \frac{K}{\eta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

K as a function of α.—This function also has two branches.

When a disturbance takes place in the equatorial regions the air flows along the gradients, that is to say, towards the center (for which $\alpha = 0$ or away from the center in the opposite direction (for which $\alpha = \pi$). The first case corresponds to an area of low pressure and the second to an area of high pressure. If the center of the disturbance moves towards the north, the currents of air will deflect to the right (or α will increase with K). If the center moves towards the south, the current will deviate towards the left and α decreases with K.

η *as a function of* α.—It may be remarked that the analysis of this function can only be of a general character because in the form of equation (8) it occurs as a function of ε of unknown form. If we rely upon equations (1), (2) and (7) we find without difficulty that ε is infinitely large when $\eta = 0$; ε is zero when η is infinitely large; finally the derivative of ε with regard to η is always very small or nearly equal to zero. Moreover it is evident that if η has positive values, different from zero, then ε also has values greater than zero.

A discussion of equation (8) shows moreover that the product $\varepsilon\eta$ is positive for $\eta = 0$ and for $\eta =$ infinity. These peculiarities lead us to adopt $\varepsilon = \frac{n}{\eta}$ as the value of the function ε, in which n is a constant.

The vitality or duration of any atmospheric disturbance depends directly on the magnitude of the angle α between the wind and the gradient; in proportion as α increases the duration increases also; in proportion as the wind deviates from the gradient it is more and more difficult to reestablish static equilibrium.

This being understood, let us examine some interesting mechanical phenomena that we may draw from the preceding analysis.

(1) When an anticyclone continues to develop its vitality: (α) increases steadily, whence it results that disturbances of this character ought to have a very considerable stability not requiring help from outside.

(2) On the contrary, when a cyclone is developing, its vitality is decreasing so that a fully formed cyclone carries within itself the beginnings of its destruction, hence the extreme instability

of cyclonic formations, which can only persist by the aid of such exterior sources as furnish the necessary energy.

(3) The movement of a disturbance towards the pole increases its vitality, and vice versa.

(4) As the friction increases, the vitality of a disturbance diminishes. Since friction [internal friction or viscosity] is greater in moist air than in dry air it follows that a disturbance should lose vitality when approaching moist vapors, and vice versa.

Ordinarily barometric maxima follow this rule quite closely, but the minima seems to behave contrariwise and very persistently so. This fact shows again that the cyclone of the temperate zone is essentially a thermodynamic disturbance while the anticyclone is a mechanical disturbance. Thus we explain the profound difference that exists in all respects between these two kinds of whirlwinds which are so similar in appearance.

(5) The intensity of any disturbance, or the product $v\ R$, certainly increases in proportion to the distance from its center, for the atmospheric currents become more and more nearly horizontal: hence follows the following very interesting theorem:

The vitality of a cyclone diminishes in passing from its center towards its boundary which causes an excessive sensitiveness at the latter; when the cyclone is extensive with a very deep depression its exterior isobars vary incessantly. On the contrary the anticyclone has permanent and firm contours and its center of high pressure moves hither and thither without exerting any influence whatever on the boundaries of the whirl.

Because of this difference the collision between these two classes of disturbances acts destructively upon only one of the two, that is, the cyclone, which eventually is destroyed or modified.

From the preceding we see that equation (8) gives us a fairly probable as well as general representation of the characteristics and motions that belong to atmospheric disturbances, excepting only one of the most important of the movements, that is, the progressive motion of the whirlwind itself. The direction and velocity of this movement are determined principally by the diurnal rotation of the earth, which action becomes stronger in proportion as the height of the whirl is greater. Now we are not yet able to study this action because the law according to which the friction of the air varies with altitude is at present wholly unknown. However, we hope that the current exploration of the atmosphere with kites and sounding balloons will not fail to clear up this question, which is as interesting from a purely scientific point of view as it is important for the practical forecasting of the weather.

X

THE RELATION BETWEEN WIND VELOCITY AT ONE THOUSAND METERS ALTITUDE AND THE SURFACE PRESSURE DISTRIBUTION

BY E. GOLD, M.A.

Fellow of St. John's College, Cambridge

[*Communicated to the Royal Society, London, by Dr. W. N. Shaw, F.R.S.' February 25, and Read March 5, 1908. Printed in Proc. Roy. Soc.* Vol., *80, May 25, 1908*]

For the steady horizontal motion of air along a path whose radius of curvature is r, we may write directly the equation

$$\frac{(\omega r \sin \lambda \pm v)^2}{r} = \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{(\omega r \sin \lambda)^2}{r},$$

expressing the fact that the part of the centrifugal force arising from the motion of the wind is balanced by the effective gradient of pressure.

In the equation p is atmospheric pressure, ρ density, v velocity of moving air, λ is latitude, and ω is the angular velocity of the earth about its axis.

If $\partial p/\partial r$ be negative, it is clear that v and $\omega\ r \sin \lambda$ must have opposite signs: or, for motion in a path concave towards the higher pressure, the air must rotate in a clockwise direction, the well-known result for anticyclonic motion. Further, the maximum numerical value of

$$\frac{1}{\rho}\frac{\partial p}{\partial r} \text{ is } \frac{(\omega r \sin \lambda)^2}{2}$$

and the corresponding maximum value for v is $\omega\ r \sin \lambda$. Therefore, in anticyclonic regions there are limiting values which the gradient and the velocity cannot exceed. This limiting value of v for latitude 50° and r = 100 miles is approximately 20 miles per hour.

At the surface of the earth, owing to friction and eddies, the mean direction of the motion of the air is nearly always inclined to the isobars; but over the sea the inclination is very much less, and it seemed probable that in the upper regions of the atmosphere, if the motion were steady, the air would in general move tangentially to the isobars, and its velocity would agree with that calculated from the equation given above.

The question, however, arises as to whether the pressure is likely to continue steady long enough for a condition to supervene in which the equation is applicable. We can get an idea of the time that would elapse before air, starting from rest, would reach a state of steady motion, by considering the motion of a particle on the earth's surface (1) under a constant force in a constant direction, corresponding to straight isobars; (2) under a constant radial force corresponding to cyclonic and anticyclonic conditions. The particle would begin to move at right angles to the isobars in the direction of the force, but as its velocity increased it would be deflected by the effect of the earth's rotation until it moved perpendicularly to the force.

The equations of motion of a particle, referred to axes fixed relatively to the earth and having an origin on the surface in latitude λ, are

$$\frac{d^2x}{dt^2} - 2\omega\cos\lambda\frac{dz}{dt} - 2\omega\sin\lambda\frac{dy}{dt} = X,$$

$$\frac{d^2y}{dt^2} + 2\omega\sin\lambda\frac{dx}{dt} = Y,$$

$$\frac{d^2z}{dt^2} + 2\omega\cos\lambda\frac{dx}{dt} = Z,$$

where the axis of z is vertical and the axes of x and y are west and south respectively.

If there is no vertical motion we may write the first two equations

$$\frac{d^2x}{dt^2} - a\frac{dy}{dt} = X, \qquad \frac{d^2y}{dt^2} + a\frac{dx}{dt} = Y,$$

and the form of the equations and the value of a are unaltered by changing to other axes in the same plane. Let us take the y axis to be in the direction of the constant force b. Then

$$\frac{d^2x}{dt^2} - a\frac{dy}{dt} = 0, \qquad \frac{d^2y}{dt^2} + a\frac{dx}{dt} = b,$$

whence

$$x = \frac{b}{a^2}(at - \sin at), \qquad y = \frac{b}{a^2}(1 - \cos at),$$

if the particle start from rest. The motion is therefore oscillatory, and the particle moves in a series of cycloidal-like curves, fig. 1. The times to the successive intersections with $y = b/a^2$ are $\pi/2a$, $3\pi/2a$, etc. For latitude 50° these are about 4 and 12 hours. They are independent of b. If there is damping the motion will be as in fig 2. If the motion is resisted by a force $k\,v$ proportional to the velocity, the path will be inclined to the x-axis. Fig. 3 gives the path for the particular case $k = a$ and for a period of time equal to $2\pi/a$ or 16 hours.

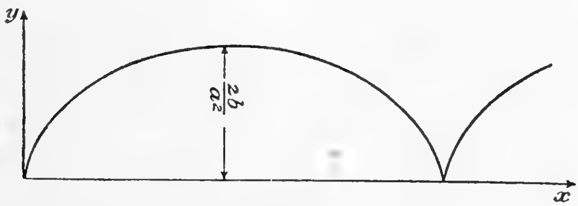


FIG. 1

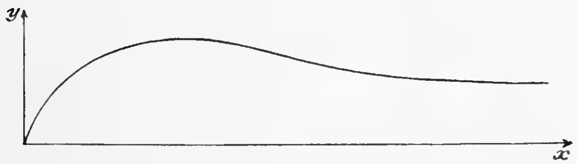


FIG. 2

FIG. 3

In the case of a constant radial force we have for the motion

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = R + ar\frac{d\theta}{dt},$$

$$r\frac{d^2\theta}{dt^2} + 2\frac{dr}{dt}\frac{d\theta}{dt} + a\frac{dr}{pt} = 0,$$

whence

$$r^2 \frac{d\theta}{dt} + \tfrac{1}{2} ar^2 = B.$$

If the particle start from the center,

$$B = 0 \text{ and } \frac{d\theta}{dt} = -\tfrac{1}{2}a,$$

and we obtain

$$r = \frac{4R}{a^2}(1 - \cos \tfrac{1}{2} at) = \frac{4R}{a^2}(1 - \cos \theta).$$

The particle therefore describes a cardioid, but if there is damping the motion will come to be along the circle $r = 4R/a^2$.

The time to reach the circle is π/a, or about 8 hours for latitude 50°.

These times are not large meteorologically, and we may therefore expect the relation between air velocity and pressure gradient to be that corresponding to steady motion so long as there are no irregularities to produce turbulent motion.

For application to wind velocities in the upper air we require to know the upper-air isobars. If we have air in which the horizontal layers are isothermal, then from the equations

$$dp = -g\rho\, dz, \qquad p = gk\rho T,$$

it follows that

$$\log \frac{p_0}{p_z} = \int_0^z \frac{dz}{kT}.$$

We have, therefore, if p_0 and $p_0 + dp_0$ are surface isobars and p_z and $p_z + dp_z$ the corresponding upper isobars,

$$\frac{dp_z}{p_z} = \frac{dp_0}{p_0}, \text{ so that } \frac{dp_z}{\rho_z} = \frac{dp_0}{\rho_0}\frac{T_z}{T_0}.$$

Therefore the velocity calculated from the surface isobars will apply to the upper air, except for the factor, T_z/T_0. For $z = 1000$ meters the effect of this factor is to diminish the velocity by about 2 per cent.

If the conditions are not isothermal, but such that the isotherms and isobars intersect at an angle ψ, the upper isobars will have a different direction from the surface isobars, and the value of the upper gradient will also be changed (see fig. 4).

The pressure at a height z above B the point of intersection of p_0, T_0, is $p_0 e^{-z/kT_m}$, and above A, the point of intersection of $p_0 + dp_0$, $T_0 - dT_0$, is[1]

$$(p_0 + dp_0)e^{-z/k(T_m - dT_m)}.$$

If we assume the vertical temperature gradient to be the same over all the region considered, dT will be the same for every element of the above integral, and we can put $dT_m = dT_0$.

If these two pressures at height z are equal, we must have

$$p_0 e^{-z/kT_m} = (p_0 + dp_0)\, e^{-z/k(T_m - dT_0)},$$

or

$$\frac{dp_0}{p_0} = \frac{z}{k}\frac{dT_0}{T_m^2} \quad \text{or} \quad \frac{dp_0}{\rho_0} = gz\frac{T_0 dT_0}{T_m^2}.$$

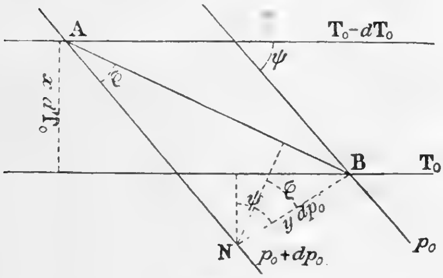


FIG. 4

In this case AB is the direction of the upper isobar and its inclination ϕ to the lower isobar is given by

$$\tan\phi = \frac{ydp_0}{xdT_0 \operatorname{cosec}\psi + ydy_0 \cot\psi}$$

where $x\,dT_0$ and $y\,dp_0$ are the distances between the isotherms and isobars.

Substituting for dT_0 and dividing out by dp_0 we get

$$\cot\phi = \cot\psi + \frac{x}{y}\frac{T_m^2}{g\rho_0 z T_0}\operatorname{cosec}\psi.$$

Taking y and x for millimeter isobars and 1° C. isotherms and putting z = 1000 meters and $T_m^2/T_0 = 2T_m - T_0 = 270°$ C. say, we find

$$\cot\phi = \cot\psi + 2{\cdot}8\frac{x}{y}\operatorname{cosec}\psi.$$

[1]In this and subsequent formulas the reader will understand that the erms following the / belong to the exponents of e.

To obtain the upper pressure gradient, we consider the upper isobars over B and N. The difference of temperature between B and N is assumed $y/x \,.\, dp_0 \cos \Psi = dt$.

Therefore the upper pressure difference is

$$(p_0 + dp_0)\, e^{-z}/k\,(T_m + dt) - p_0 e^{-z}/kT_m = e^{-z}/kT_m \left[dp_0 + \frac{p_0 z dt}{kT_m^2} \right]$$

$$= p_0^{-z}/\,kT_m \left[1 + \frac{p_0 z \,.\, y \cos \psi}{xkT_m^2} \right]$$

The distance between these isobars is $ydp_0 \cos \phi$ and the upper gradient is consequently

$$\frac{1}{y \cos \phi}\, e^{-z}/kT_m \left[1 + \frac{p_0 z y \cos \psi}{xkT_m^2} \right]$$

$$= \frac{1}{y \cos \phi}\, e^{-z}/kT_m \left[1 + \frac{g\rho_0 z T_0}{T_m^2}\, \frac{y \cos \psi}{x} \right],$$

and the ratio

$$\frac{1}{\rho_z}\frac{\partial p_z}{\partial r} \Big/ \frac{1}{\rho_0}\frac{\partial p_0}{\partial r} \text{ is } \sec \phi \left[1 + \frac{g\rho_0 z T_0}{T_m^2}\, \frac{y \cos \psi}{x} \right] \frac{T_z}{T_0},$$

which is

$$g\,\rho\, z \operatorname{cosec} \phi .\, \frac{y \sin \psi}{x},$$

taking T_z/T_0 to be unity, namely,

$$\operatorname{cosec} \phi \,.\, \frac{1}{\cot \phi - \cot \psi} \text{ or } \frac{\sin \psi}{\sin (\psi - \phi)} .$$

In the special cases, $\psi = 0$ or $180°$, the ratios are

$$\left(1 \pm \frac{g\rho_0 z}{T} . \frac{y}{x} \right) \text{ or } \left(1 \pm \frac{y}{2.8\, x} \right), \text{ for } z = 1000 \text{ meters.}$$

If $x = 2y$, which would represent a possible case, the increase or decrease would be about 18 per cent.

For $\phi = \frac{1}{2}\pi$ the rotation would in the same circumstances be about 10°.

During the year 1905 a series of observations in the upper air was made at Berlin and Lindenberg, near the time of the general 8 a.m.

morning observations. It was therefore possible to compare the wind velocities observed with those calculated from measurements of the gradient by the use of the formula at the beginning of this paper, the motion being assumed tangential to the isobars.

For purposes of calculation the formula may be written

$$v\,(1 \pm 0.00108\,v \cot \psi \operatorname{cosec} \lambda) = \frac{709 \operatorname{cosec} \lambda}{x} \frac{T}{T_0} \frac{B_0}{B},$$

where ψ is the angular radius of the small circle, on the earth's surface, osculating the path, v is in meters per second, x is the distance in kilometers between millimeter isobars, T, B are the temperature and pressure, and T_0, B_0 the corresponding values for air at 0° C. and 760^{mm}.

If the motion is along straight lines, $\cot \psi = 0$, and the values of v for $B = B_0$, $T = T_0$, are as follows if $x = 50$ kilometers:

Latitude......	30°	40°	50°	60°	70°
v	28.4	22.1	18.5	16.4	15.1

If v_0 represent the velocity when $\cot \psi = 0$, we can most easily express the solutions of the equation for different values of ψ, x, λ, by taking as independent variables, ψ, v_0, λ.

Taking, as an example of the dependence on ψ, $\lambda = 50°$, $v_0 = 40$ meters per second, we obtain the following values for v in meters per second in the case of cyclonic motion:

ψ	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
v	17	21	24	26	28	29	30	31	31	32

For anticyclonic motion the gradient corresponding to $v_0 = 40$ meters per second is above the maximum, and we take for two examples $v_0 = 12$, and $v_0 = 30$ meters per second.

The values of v are then as follows for the two cases:

	ψ	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°
For $v_0 = 12$	$v =$	–	–	–	20	16	15	14	14	14	13 m.p.s.
For $v_0 = 30$	$v =$	–	–	–	–	–	–	–	–	–	50 "

Where no value is inserted for v, the gradient corresponding to the given value of v_0 is above the maximum for the corresponding value of ψ.

To show the dependence on λ, we take $\psi = 3°$ and put $v_0 = 40$ meters per second for cyclonic motion, and $v_0 = 10.5$ meters per second for anticyclonic motion. The following table gives the values of v for different latitudes in the three cases:

	λ	30°	40°	50°	60°	70°	
For $v_0 = 40$	$v =$	21	23	24	25	26	m.p.s.
For $v_0 = 10$	$v =$	—	—	—	17	15	"
For $v_0 = 5$	$v =$	7.1	6.3	6.0	5.8	5.7	"

By the use of tables giving values of v_0 for different values of x, T, B, and of v for different values of λ, v_0, ψ, each wind observation at 1000 meters altitude was compared with the value deduced from the surface isobars. The temperature correction was not applied.

The following table gives the result of the comparisons:

	Theoretical velocity v. M.P.S.	Velocity v_1 at 1000 meters. M.P.S.	Percentage difference between v and v_1.	Mean deviation in points of v_1 from v (+ when v_1 veers from v).	Mean surface wind. M.P.S.	Mean deviation in points of wind v_1 from surface wind.	Surface wind ÷ v cosine (deviation).	Wind at 1000 meters ÷ v_1 cosine (deviation).	Component wind ⊥ to isobars at surface ÷ component at 1000 meters.
Berlin:									
January	15.7	15.1	4.0	−0.6	5.0	3.3	0.45	0.97	1.9
February	12.0	11.6	3.5	−0.6	4.0	2.7	0.41	0.98	1.7
March	8.1	6.6	23.0	−0.4	2.6	1.7	0.35	0.82	1.9
Lindenberg:									
April	10.2	8.8	16.0	−1.0	4.8	1.7	0.54	0.90	1.4
May	8.3	6.5	28.0	−1.0	4.5	2.3	0.65	0.80	1.9
June	6.8	6.4	6.0	−0.5	3.9	1.1	0.60	0.95	1.9
July	8.4	8.2	2.4	−0.9	4.5	1.2	0.58	1.0	1.3
August	8.9	8.0	11.0	−0.9	5.1	1.6	0.64	0.92	1.6
September	10.3	10.5	−2.0	−0.8	5.4	2.2	0.63	1.03	1.6
October	12.1	11.9	2.0	−0.5	6.3	2.7	0.64	0.99	2.7
November	10.6	10.1	5.0	−0.9	5.9	3.1	0.78	0.97	2.3
December	11.0	10.3	7.0	+0.3	5.5	3.9	0.65	0.94	−5.−
Summer	8.8	8.1	8.5	−0.8	4.7	1.7	—	0.93	—
Winter	11.6	10.9	6.5	−0.45	4.9	[illegible].9	—	0.95	—
Year	10.2	9.5	7.5	−0.6	4.8	2.3	—	0.94	—

The upper wind coincides in direction very nearly with the isobars at the surface, and the wind velocity observed agrees well with that calculated from the pressure distribution. The differences are not greater than possible errors of observation, except in spring.

It is known that the upper wind always veers from the surface wind, and the numbers in Column 7 show that in 1905 the veering was considerably greater in winter than in summer.

If the effect of the earth's surface were the same as if a frictional force opposed the motion, the relation between the wind and gradient of pressure would be as above, except that the effective gradient would be the maximum gradient multiplied by the cosine of α, the angle between the path and the isobars. The corresponding velocity would be approximately $v \cos \alpha$, except in cases of considerable curvature. In the majority of the observations the curvature was small, and we should therefore expect the surface wind to be nearly $v \cos \alpha$, so that the numbers in Column 8 would be nearly unity. This is far from being the case; but the change of the station of observation from Berlin to Lindenberg is accompanied by a corresponding change in the ratio of the surface wind velocity to $v \cos \alpha$.

This suggests that the effect of the surface, apart from the purely frictional effect, is to reduce the velocity in a given direction in a constant ratio depending on the locality, and that departures in the observed velocities from those corresponding to this ratio are to be associated with unsteady meteorological conditions.

The last column[1] gives approximately the ratio of the volume of air crossing the isobars at the surface to the volume crossing at 1000 meters.

The ratio appears to be nearly constant; the change in December is probably due to the exceptional conditions which prevailed during part of the month, when the air was considerably warmer at 1000 meters altitude than at the surface.

[1]Or, the wind component, perpendicular to the isobars at sea level divided by the analogous component at 1000 meters.

XI

STUDIES ON THE MOVEMENTS OF THE ATMOSPHERE

BY C. M. GULDBERG AND H. MOHN[1]

PART I

(Christiania, 1876, revised 1883)[2]

PREFACE

Meteorological phenomena being very complicated, we shall attain final success in their mathematical study only by treating simple cases which are analogous to those of nature. The equilibrium and the movement of the air form a part of the mechanics of fluids that is as yet very little developed because there exists too few observations for the verification of the numerical calculations. Encouraged by the fine results obtained by M. M. Peslin, Reye, Colding, Ferrel and Hann in this new application of analysis to meteorology, we have applied the principles of mechanics to the movements of the atmosphere, and have arrived at some results which we think are not without importance for the development of meteorological science. In the first place we have found that one of the first things to do in order to insure the success of meteorology is the creation of meteorological stations at high altitudes; either on mountains or in balloons, and supplied if possible with self-registering instruments.

The winds or the horizontal currents of air at the surface of the earth are intimately connected with the vertical currents; but the origin and the displacement of these latter depend not only on the physical state of the air at the surface of the earth, but also on the

---

[1] Etudes sur les Mouvements de L'Atmosphère. Par C. M. Guldberg et H. Mohn. Première Partie, Christiania, 1876. Deuxième Partie, Christiania, 1880. [Revised by the Authors in 1883–'85.]

[2] By personal interview with the authors, and correspondence during the years 1883 to 1886, Prof. Frank Waldo secured from Professors Guldberg and Mohn a revision of the original French edition of this Memoir with permission to publish a translation for the use of American students. The delay in publication has given me opportunity for a slight revision of Prof. Waldo's translation.—C. Abbe.

physical state of the air of the upper strata. Moreover the velocity of the wind and its direction are both eminently under the influence of the surface of the earth, while their values at a certain height would probably present the regularity that must obtain in order to be able to predict the progress of meteorological phenomena.

In studying horizontal currents under simple hypotheses, we have introduced the friction due to the surface of the earth, and we have applied our theory to the winds crossing over the equator, and to whirlwinds.[3]

The numerical calculations accord with the phenomena of nature within such limits as correspond to the established hypotheses. It follows that the exact observation of the velocity of the wind will be of great importance to meteorology. We hope that these results drawn from the mechanics of the atmosphere will show the necessity of more extended meteorological observations especially in the tropical regions and in the higher strata of the atmosphere, and that true progress in meteorology is founded on the development of the mechanics of the atmosphere.

Chapter I

THE ATMOSPHERE

§1. *Pressure, virtual temperature*

In studying the equilibrium and the movements of the atmosphere, it suffices to consider the air as a mixture of dry air and of aqueous vapor. The other gases forming the elements of the atmosphere, of which carbonic acid gas is the most important, are found only in such small quantities that their action may be neglected.

The quantity of aqueous vapor in the atmosphere is so small that we can accept the law of Mariotte and Gay-Lussac for moist air within the range of temperatures that occur on the earth. It is necessary, however, to consider the cases in which the vapor condenses and passes into the liquid state or the solid state.

We use the notation

p the pressure in kilograms on a square meter.

ρ the density or mass of a cubic meter.[4]

τ the temperature in degrees centigrade.

---

[3] M. C. M. Guldberg had already, in 1872, developed a part of this theory in the Norwegian Polytechnic Journal (Polyteknisk Tideskrift), p. 73, 1872. —Editor.

[4] In the absolute system here used this mass is the weight divided by gravity.—Editor.

The law of Mariotte and Gay-Lussac as applied to a kilogram of gas is written

$$p = a\rho\,(273^\circ + \tau) \quad \ldots\ldots\ldots \quad (1)$$

where $273^\circ + \tau$ is the absolute temperature.

Here a designates a constant which depends on the nature of the gas; for dry air we have

$$a = 287.09$$

In applying this law to a mixture containing $1 - q$ kilograms of dry air and q kilograms of the vapor of water, expressing the tension of the vapor of water by f and its relative density by $1/\varepsilon$ we find

$$p - f = a\,(1 - q)\,\rho\,(273 + \tau).$$

$$f = \varepsilon\, a q\, \rho\,(273 + \tau).$$

$$p = a\,(1 + (\varepsilon - 1)\,q)\,\rho\,(273 + \tau) \quad \ldots\ldots \quad (2)$$

$$\frac{f}{p} = \frac{\varepsilon q}{1 + (\varepsilon - 1)\,q} \quad \ldots\ldots\ldots \quad (3)$$

$$q = \frac{\dfrac{1}{\varepsilon} \cdot \dfrac{f}{p}}{1 - \dfrac{\varepsilon - 1}{\varepsilon} \cdot \dfrac{f}{p}} \quad \ldots\ldots\ldots \quad (4)$$

By substituting this value of q in equation (2) and putting

$$T = \frac{273 + \tau}{1 - \dfrac{\varepsilon - 1}{\varepsilon} \cdot \dfrac{f}{p}} \quad \ldots\ldots\ldots \quad (5)$$

we have for moist air

$$p = a\rho T \quad \ldots\ldots\ldots \quad (6)$$

We call the quantity T the *virtual temperature;* for dry air the virtual temperature is the same as the absolute temperature.

If we consider a mixture which contains 1 kilogram of dry air and x kilograms of vapor of water, we shall have

$$x = \frac{q}{1 - q} = \frac{1}{\varepsilon} \cdot \frac{f}{p - f} \quad \ldots\ldots\ldots \quad (7)$$

$$\frac{f}{p} = \frac{\varepsilon x}{1 + \varepsilon x} \quad \ldots\ldots\ldots \quad (8)$$

If we have

$$\frac{1}{\varepsilon} = 0.623 \qquad \frac{\varepsilon - 1}{\varepsilon} = 0.377$$

Then for air at 760^{mm} we have the values in the following table:

TEMPERATURE	TENSION OF THE VAPOR OF WATER	T	x
	mm		
$-30°C$	0.386	243.05	0.000317
-20	0.927	253.12	0.000761
-10	2.093	263.27	0.001720
0	4.600	273.62	0.003794
10	9.165	284.30	0.007605
20	17.391	295.55	0.014590
30	31.548	307.81	0.026981

§2. *Height of the atmosphere—mean pressure*

We can adopt either one of two hypotheses concerning the height of the atmosphere. We can suppose that the atmosphere is limited; in this case the temperature of the exterior stratum must necessarily be absolute zero, for at this temperature the tension of a gas is equal to zero. The other hypothesis is that the atmosphere extends indefinitely into space and that space is filled with a gas whose tension is extremely feeble. For meteorology it matters little which hypothesis is chosen, because in both cases the tension of the air at very great heights will be insensible. Suppose 760^{mm} be the pressure at the surface of the earth and suppose the temperature of the atmosphere constant and equal to zero centigrade, we shall find the pressure at the height of 200 000 meters equal to $0.000\,000\,01^{mm}$.

If the atmosphere does not contain the vapor of water its mass will be invariable; if we suppose, moreover, that gravity does not vary with elevation, the weight of this mass will be constant and by calculating the mean pressure on the entire surface of the earth it will be found to remain always the same. Considering the presence of the vapor of water whose quantity varies from time to time, we shall see that the mass of the atmosphere does not remain constant and that, consequently, the mean pressure varies with the seasons.

We have assumed that gravity is constant. In truth it varies with the altitude and consequently the pressure of the atmosphere depends on the law of the distribution of the mass in a vertical direction. This distribution is a function of the temperature, and consequently even the pressure of a dry atmosphere will vary with the temperature in such manner that the pressure will diminish in proportion as the temperature increases. However, the variation of gravity for atmospheric strata at slight elevations is so slight that its action can be neglected in meteorology.

§3. *Temperature of the atmosphere*

Temperature depends on many considerations and there has not yet been found any function that expresses the temperature in terms of the coördinates of position and the time.

The heat of the sun and of space, the absorption of the earth and of space, the radiation, the conductibility and the movement of the air, all affect the temperature. Hitherto we have sought to determine the temperature at the surface of the earth as a function of the time. We shall see that the variation of the temperature with the height is of the greatest importance in meteorology. The observations of this phenomenon are not numerous and it seems not to follow simple laws. But one can at least recognize that at slight elevations where the action of the sun is most energetic the layers of air experience equal variations of temperature, while it is very probable that in somewhat elevated strata the variation is slight, and that whatever may be the temperature at the surface of the earth, we shall always arrive at the same temperature at a certain height which will however oscillate slightly.

We shall apply some approximate formulas. The most simple hypothesis is that the temperature decreases proportionally to the height; then we have

$$\tau = \tau_0 - \alpha z;$$

where z is the height; α, a constant and τ_0 the temperature at the surface of the earth. In some problems it will be more convenient to introduce the above described virtual temperature and write

$$T = T_0 - \alpha z.$$

These two formulæ apply only to small heights; if we wish to calculate the variation of temperature for the greatest height, we can divide the whole elevation into layers and apply the formula to each stratum with different values of α.

§ 4. *Variation of pressure with altitude*

According to the theory of the equilibrium of fluids the increase of pressure per unit of length is equal to the force which acts on the unit of volume. Let g designate the force of gravity per unit of mass and z the height, we shall have:

$$\frac{dp}{dz} = - g\rho \quad \ldots\ldots\ldots\ldots \quad (1)$$

Introducing the value of ρ given by equation (6) of § 1 we shall have

$$\frac{dp}{p} = - \frac{g dz}{aT} \quad \ldots\ldots\ldots\ldots \quad (2)$$

(1) *The virtual temperature remains constant.*

In this case designating by p_0 the pressure at the surface of the earth we shall find by integration.

$$p = p_0 e^{-\frac{gz}{Ta}} \quad \ldots\ldots\ldots\ldots \quad (3)$$

in which e is the base of the system of Napierian logarithms.

(2) *The virtual temperature decreases proportionally to the height.*

By introducing $T = T_0 - \alpha z$ in equation (2) we shall find, by writing $m = \frac{g}{\alpha\alpha}$

$$T = T_0 - \frac{gz}{am} \quad \ldots\ldots\ldots\ldots \quad (4)$$

$$z = \frac{a}{g} m (T_0 - T) \quad \ldots\ldots\ldots\ldots \quad (5)$$

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^m \quad \ldots\ldots\ldots\ldots \quad (6)$$

$$\frac{p}{p_0} = \left(1 - \frac{gz}{amT_0}\right)^m \quad \ldots\ldots\ldots\ldots \quad (7)$$

As to the variation of the pressure of the vapor of water we can adopt various hypotheses. We shall consider only the following formula:

$$\frac{f}{f_0} = \left(\frac{p}{p_0}\right)^\beta \quad \ldots\ldots\ldots\ldots \quad (8)$$

In which β is a constant whose value depends on circumstances. Knowing p and f we shall find the temperature τ by formula (5) of §1, namely

$$\tau = T\left(1 - \frac{\varepsilon - 1}{\varepsilon} \cdot \frac{f}{p}\right) - 273 \quad \ldots\ldots \quad (9)$$

Applications

$m =$	3.441	4.0	5	10
Variation of temperature per 100^{m}	0°.994	0°.854	0°.683	0°.342
Height for a variation of 1° C.	101^{m}	117^{m}	146^{m}	293^{m}

$z =$	1000^{m}	4000^{m}	10000^{m}	20000^{m}
	$T_0 = 273$	$m = 3.5$		
$\frac{p}{p_0} =$	0.88038	0.58259	0.21250	0.01233
	$T_0 = 273$	$m = 5$		
$\frac{p}{p_0} =$	0.88094	0.59013	0.23681	0.03106
	$T_0 = 273$	$m = 10$		
$\frac{p}{p_0} =$	0.88166	0.59841	0.26266	0.05609

Dr. Julius Hann has published a series of observations on the tension of aqueous vapor at different altitudes (see the Zeitschrift der Oesterreichischen Gesellschaft für Meteorologie, 1874, page 195). In this case applying formula (8) we shall assume

$$\tau_0 = 20^{0},\ f_0 = 10^{mm},\ m = 10 \text{ and } \beta = 3$$

The values of $\frac{f}{f_0}$ calculated for these constants are found in the third line of the following table.

Assuming T = constant = 273° and $\beta = 3$ we find the values written in the fourth line. The observed values are found in the second horizontal line.

Altitude, English feet	1000	4000	8000	12000	16000	20000	24000	28000
Observed f/f_0	0.87	0.64	0.42	0.27	0.18	0.13	—	—
Computed f/f_0	0.90	0.65	0.42	0.27	0.17	0.11	0.07	0.04
Computed f/f_0	0.89	0.63	0.40	0.25	0.16	0.10	0.06	0.04

§ 5. *Expansion and contraction of the air*

The pressure and temperature of a mass of air that experiences any transformations whatsoever depend on the quantity of heat which it has gained or lost. We will first consider the case in which the air experiences a series of transformations without gaining or losing heat at any moment. The equation between the pressure and the volume represents a line that has been called the *adiabatic line.*

In the study of meteorology it is also necessary to find the equation between the pressure and the temperature. It is necessary to distinguish between several cases. The air can be dry or moist, and the aqueous vapor water can remain without condensation or it can pass into the liquid state or into the solid state.

Representing by U the internal energy of a mixture; by V its volume and by A the mechanical equivalent of heat, we have

$$0 = d\,U + A\,p\,d\,V \qquad (1)$$

(1) *Dry air.*

Applying equation (1) to dry air we shall find from the mechanical theory of heat

$$\frac{p}{p_0} = \left(\frac{273 + \tau}{273 + \tau_0}\right)^m \qquad (2)$$

$$m = \frac{c\,g}{A\,a} \qquad (3)$$

where c represents the specific heat of dry air at constant pressure, whence we have,

$$m = 3.441$$

(2) *Moist air without condensation.*

Supposing we have one kilogram of dry air and x kilograms of aqueous vapor we shall find

$$m = 3.441 \left(\frac{1 + 2.023\,.x}{1 + \varepsilon\,x}\right) \qquad (4)$$

where 2.023 is the ratio between the specific heat of the vapor of water and that of dry air. These formulas apply only in so far as the air is not saturated with the vapor of water. At the moment when the air becomes saturated, the decrease of temperature is accompanied by a condensation of vapor and it is necessary to distinguish between the three cases. We will assume that the condensed vapor remains suspended in the mass of air under consideration.

(3) *The vapor of water is partially transformed into water.*

We will consider a mixture consisting of 1 kilogram of dry air, x kilograms of vapor of water, and y kilograms of water. Expressing by U', U'', and U''' the energies of dry air, of the vapor of water, and of water respectively we have the total energy of the mixture

$$U = U' + xU'' + yU'''$$

The sum of x and y remains constant and writing

$$x + y = \xi$$

we shall find

$$d\,U = dU' + \xi\, d\,U''' + d(x(U'' - U'''))$$

Designating by v' and v'' the specific volumes of the dry air and of the vapor of water and neglecting the volume of the water, we have the volume of the mixture

$$V = v' + x\,v''$$

We can then write

$$p\,d\,V = (p - f)\,d\,v' + fd\,(x\,v'')$$

Expressing the latent heat of vaporization by l, and the specific heats of dry air and of water by c and c' we have approximately, neglecting the volume of water:

$$l = (U'' - U''') + A\,f\,v''$$

$$(273 + \tau)\,d\left[\frac{x\,l}{273 + \tau}\right] = d\,[x\,(U'' - U''')] + A\,f\,d\,(x\,v'')$$

$$c\,d\,\tau = d\,U' + \frac{A\,a}{g}\,d\,\tau$$

$$c'\,d\,\tau = d\,U'''$$

$$(p - f)\,v' = \frac{a}{g}\,(273 + \tau)$$

$$l = 606.5 - 0.696\,\tau$$

Substituting the values of $d\,U$ and of $p\,d\,V$ in equation (1) and introducing the values given by the equations mentioned, we shall find

$$0 = c_v d\,\tau + \xi\,c'\,d\,\tau + (273 + \tau)\,d\left(\frac{x\,l}{273 + \tau}\right)$$
$$- \frac{A\,a}{g}\,(273 + \tau)\,\frac{d\,(p - f)}{p - f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Expressing the initial values by the subscript index o, we shall by integration and introducing numerical values find

$$\log\left(\frac{p_0 - f_0}{p - f}\right) = 3.341\,[1 + 4210\,\xi]\,\log\left[\frac{273 + \tau_0}{273 + \tau}\right]$$
$$+ 6.291\left[\frac{x_0\,l_0}{273 + \tau_0} - \frac{x\,l}{273 + \tau}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From equation (7) of §1 we have

$$x = \frac{1}{\varepsilon}\cdot\frac{f}{p - f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Equations (5) and (6) apply in general, so long as the tempera ture remains above zero. The temperature being at zero the water is changed into ice. However, we can imagine the possibility of the vapor of water being changed into water at temperatures above zero. We know this phenomenon in physics; it is not water only, but several salts which present the phenomenon of super-saturation. This passage from the state of vapor to the liquid state at temperatures above the point of congealing involves a state of unstable equilibrium and the introduction of a crystal of ice makes the whole mass pass suddenly into a solid state. It is probable that this state of unstable equilibrium is intimately connected with the formation of hail. In ordinary cases congelation commences at the temperature zero and we will now consider the passage from the liquid state to the solid state at zero.

(4) *Congelation at* o°.

During this stage the temperature remains constant, the water is transformed for the most part into ice, but a part of the water is vaporized because according to equation (7), section (1), any diminution of the pressure produced by dilatation demands a greater quantity of vapor of water for the same vapor tension.

Consider a mixture containing 1 kilogram of dry air, x kilograms of the vapor of water, y kilograms of water and z kilograms of ice. The sum $x + y + z$ remains constant and we put

$$x + y + z = \xi$$

Denoting by U'''' the energy or specific heat of the ice and by L the heat of fusion of the ice, we have

$$\begin{aligned} U &= U' + x\,U'' + y\,U''' + z\,U'''' \\ &= U' + \xi\,U'' + x\,(U'' - U''') - z\,(U''' - U'''') \end{aligned}$$

The temperature remaining constant, we have

$$d\,U = (U'' - U''')\,d\,x - (U''' - U'''')\,d\,z$$

We can neglect the volumes of the water and the ice and put

$$\begin{aligned} V &= x\,v'' \\ p\,d\,V &= (p - f)\,v''\,d\,x + f\,v''\,d\,x \end{aligned}$$

From the mechanical theory of heat we have approximately

$$\begin{aligned} l &= U'' - U''' + A\,f\,v'' \\ L &= U''' - U'''' \end{aligned}$$

By the aid of equation (1) we find

$$0 = l\,d\,x - L\,d\,z + A\,(p - f)\,v''\,d\,x$$

Introducing from equation (7) the value of $p - f$ and observing that at the temperature of zero we have

$$f\,v'' = \frac{\varepsilon\,a}{g}\,273$$

we shall find

$$0 = l\,d\,x - L\,d\,z + \frac{A\,a}{g}\,273\,\frac{d\,x}{x} \quad \ldots\ldots\ldots \quad (8)$$

At the commencement we have

$$x = x_0,\ y = y_0,\ z = 0$$

and when all the water has disappeared (by congelation) we have

$$x = x;\ y = 0;\ z = x_0 + y_0 - x$$

By integration and substituting $l = 606.5$ and $L = 79.06$ we have

$$\log\frac{x}{x_0} = 1.822\,y_0 - 15.80\,(x - x_0) \quad \ldots\ldots \quad (9)$$

Having determined x, we shall find p by equation (7).

When all the water is transformed into ice and vapor, we have a mixture of vapor of water and of ice and from this moment onward the vapor of water is transformed directly into ice by the lowering of the temperature.

(5) *The aqueous vapor is partially transformed into ice.*

For this stage we will apply the formulas given in case (3) substituting $l + L$ for l and the specific heat of ice ($c'' = 0.5$) for the specific heat of water.

$$\log\left(\frac{p_0 - f_0}{p - f}\right) = 3.441\,(1 + 2.105\,\xi)\log\left(\frac{273 + \tau_0}{273 + \tau}\right)$$

$$+ 6.291\left(\frac{x_0\,(l_0 + L)}{273 + \tau_0} - \frac{x\,(l + L)}{273 + \tau}\right) \quad . \; . \; . \; . \quad (10)$$

We have supposed that the water and the ice remain suspended in the air and take part in the thermal phenomena during the three periods in which the vapors of water are condensed. If we wish to consider the case in which the water and the ice after their formation separate from the mass of air, it will be necessary to consider the term $c + \xi\, c'$ or $c + \xi\, c''$ as variable, We can in this case give to ξ a mean value and consider it as constant, since its value is very small. In this case the period of freezing at 0° disappears.

M. Peslin[1] has developed similar formulæ, but he has not considered the variation with temperature of the latent heat of vaporization. This causes the difference between his formulæ and ours.

We shall apply our formulæ to the case in which a mass of air rises in the atmosphere with a velocity so small that it can be neglected. Designating the height by h we can write the equation of equilibrium from §4,

$$V\,d\,p = -(1 + \xi)\,d\,h$$

Combining this equation with equation (1) we find

$$0 = d\,U + A\,d\,(p\,V) + A(1 + \xi)\,d\,h \quad . \; . \; . \; . \quad (11)$$

Applying this formula to moist air we find the formulæ of section 4. When we consider the cases in which the vapor is condensed, we distinguish the following:

[1] The bulletin hebdomadaire de l'association scientifique de France, No. 67.

(6) *The vapor of water is transformed partially into water.* Approximately we have

$$pV = (p - f)\, v' + f\, v''\, x = \frac{a\,(273 + \tau)}{g} + x\, f\, v''$$

By the substitution of this equation in equation (11) and by the aid of the formulas of case No. 3. we find

$$\left.\begin{aligned} 0 &= (c + \xi\, c')\, d\,\tau + d\,(x\, l) + A\,(l + \xi)\, d\, h \\ 0 &= c + \xi\, c')\,(\tau\ - \tau_0) + x\, l - x_0\, l_0 + A\,(1 + \xi)\, h \end{aligned}\right\} \quad . \quad . \quad (12)$$

(7) *Congelation at* 0°.

In this case we find

$$\left.\begin{aligned} 0 &= l\, d\, x - L\, d\, z + A(1 + \xi)\, d\, h \\ 0 &= (l + L)\,(x - x_0) - L\, y_0 + A(1 + \xi) h \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (13)$$

(8) *The vapor of water is partially transformed into ice.*

We find

$$0 = (c + \xi\, c'')\,(\tau - \tau_0) + x(l + L) - x_0(l_0 + L) + A(1 + \xi) h \quad (14)$$

All these formulas that we have developed, pertain to the case in which the air experiences transformations without gaining or losing heat. Let us suppose that the air receives some heat and that the heat absorbed is proportional to the variation of the temperature. When the quantity of absorbed heat is small, the temperature decreases during the expansion and in place of equation (1) we write

$$-\, b\, d\,\tau = d\, U + A\, p\, d\, V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Here b expresses a constant which depends on exterior circumstances. Applying this equation to the dry air we find the law given by formula (2) and m is given by the equation

$$m = 3.441\left(1 + \frac{b}{c}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

We see that we can easily apply equation (15) to other cases in which the vapor is condensed, but we refrain from the development of the formulæ because there are no observations wherewith to check the results. Besides it is more convenient to apply the formula

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^m \quad \ldots\ldots\ldots\ldots \quad (17)$$

and to attribute to m suitable values, which vary with the height of the layer of air. We shall consider then formula (17) as the general formula, when the air experiences a series of transformations. By the aid of formula (6) from § 1 we shall find

$$\frac{p}{p_0} = \left(\frac{\rho}{\rho_0}\right)^{\frac{m}{m-1}} \quad \ldots\ldots\ldots\ldots \quad (18)$$

consequently we can write

$$\frac{dp}{\rho} = m\,d\left(\frac{p}{\rho}\right) = \frac{1}{\rho_0}\left(\frac{p_0}{p}\right)^{\frac{m-1}{m}} dp \quad \ldots\ldots \quad (19)$$

By integration we find

$$\int_{p_0}^{p} \frac{dp}{\rho} = m\left(\frac{p}{\rho} - \frac{p}{\rho_0}\right) = m\,a\,T_0\left[\left(\frac{p}{p_0}\right)^{\frac{1}{m}} - 1\right] \quad . \quad (20)$$

We shall make use later of formulæ (19) and (20).

Applications

Let us apply our formula to a mass of air that rises slowly in the atmosphere.

Assume

$$p_0 = 760^{\text{mm}};\ f_0 = 15^{\text{mm}};\ \tau_0 = 20^\circ$$

(1) *When the air is not saturated.*

By the aid of formula (7) of § 1 we find

$$x = 0.0125.$$

Substituting this value of x in § 5 formula (4) we find

$$m = 3.46.$$

So long as the air is not saturated, the value of x remains constant and consequently the ratio $\frac{f}{p}$ becomes constant. We have then

$$\frac{p}{p_0} = \frac{f}{f_0} = \left(\frac{273 + \tau}{273 + \tau_0}\right)^m$$

Attributing to τ different values we find the point of saturation by comparison with the following table, which contains the values of τ corresponding to the maximum tensions of aqueous vapor.

We can adopt $\frac{1}{m} = 0.29$ approximately and calculate a table of the quantity v determined by the formula

$$v = \frac{273 + \tau}{f^{\frac{1}{m}}}$$

We shall find

$$v = 133.6;\ f = 14.4^{mm};\ \tau = 17°;\ p = 733.4^{mm};\ h = 306^{m}$$

Table

τ	f max.	v	τ	f max.	v
−30°	0.39mm	319.3	0°	4.60mm	175.4
−25	0.61	286.2	5	6.53	161.3
−20	0.93	259.2	10	9.17	148.8
−15	1.40	234.0	15	12.70	137.8
−10	2.09	212.4	20	17.39	127.9
− 5	3.11	192.9	25	23.55	119.2
0	4.60	175.4	30	31.55	111.4

(2) *The air is saturated above 0°.*

By substituting in equation (6) the value of x from equation (7) and assuming

$$\xi = 0.0125 = x_0;\ \tau_0 = 17°;\ \tau = 0°;\ f_0\,0 = 14.4^{mm},$$
$$p_0 = 733.4^{m} \text{ and } f = 4.60^{mm}$$

we shall find

$$\log (p - f) = 2.6005 + \frac{40.05}{p - f}$$

and

$$p = 487.2^{mm};\ x = 0.00594$$

Formula (12) gives $h = 3384$ meters.

If we had used formula (17) we should have given m the value 6.36.

(3) *Water freezing to ice at 0°.*

By substituting in equation (9) the values

$\xi = 0.0125,\ x_0 = 0.00594,\ y_0 = \xi - x_0 = 0.00656,\ f = f_0 = 4.6^{mm}$ we shall find

$$x = 0.00607 \text{ kg. and from formula (7) } p = 476.5^{mm}$$

Formula (13) gives $h = 178$ meters.

(4) *The air is saturated below* 0°.

By substituting in formula (10)

$\tau_0 = 0, \ \tau = -20°, \ f_0 = 4.6^{mm}, \ f = 0.93^{mm}, \ p_0 = 476.5^{mm}$
$\xi = 0.0125, \ x_0 = 0.00607$

we shall find

$$\log (p - f) = 2.4613 + \frac{10.08}{p - f}$$

and

$$p = 312.5^{\text{ mm}}, \ x = 0.00186$$

Formula (14) gives $h = 3239$ meters.

If we had used formula (17) we should have given to m the value 5.36.

Resulting Values

ALTITUDE h	PRESSURE p	TEMPERATURE τ	TENSION f	VAPOR x
0^m	760.0^m	20°	15.0	0.01250
306	733.4	17	14.4	0.01250
3690	487.2	0	4.6	0.00594
3868	476.5	0	4.6	0.00607
7107	312.5	−20	0.9	0.00186

§6. *Causes of the movement of the air*

When the air is in equilibrium, the active forces are the attraction of the earth and the centrifugal force produced by the rotation of the earth. These two forces have a resultant which we call the weight, which varies with the latitude and the altitude. In meteorology we consider only the strata of air at a slight elevation and we generally consider the weight constant and express its value for the unit of mass by g.

In an atmosphere in equilibrium the weight is normal to the level surfaces, and the surface of the earth is itself a level surface. At the same time the density of the air and consequently its temperature vary only from one level surface to the other; we see then that in the state of equilibrium the level surfaces are surfaces of equal pressure and isothermal surfaces. We can consider the earth as approximately spherical, and the level surfaces as spheres concentric with the earth. Then the temperature can vary only with the altitude. In truth the temperature will never be uniform on the earth and equilibrium has therefore no place in nature.

When the atmosphere is in equilibrium the law of variation of the temperature with the altitude has no influence on the equilibrium, but the stability of the equilibrium does depend on that law. It is necessary to distinguish between stability with reference to an ascending movement and to a descending movement. In giving to a particle of air an ascending motion the temperature of the particle of air can change more rapidly or more slowly than the variation of the temperature of the surrounding air. If the temperature of the ascending particle decreases more rapidly than the temperature of the atmosphere, the particle will acquire a specific weight greater than the surrounding air, and consequently it will descend when the impressed motion is consumed, and we call the equilibrium *stable*. If the temperature of the particle of air decreases more slowly than that of the atmosphere, the particle will attain a specific weight less than the surrounding air and it will continue its ascending movement; then the equilibrium is *unstable*.

By impressing upon a particle of air a descending velocity we see in the same way that the equilibrium is stable if the temperature of the particle is increased more rapidly than that of the surrounding air and that it is unstable if the temperature of the particle of air increases more slowly than that of the surrounding air.

The stability of the atmosphere depends consequently on the law of the variation of the temperature of the atmosphere with the height.

Let us suppose that in a calm atmosphere the virtual temperature decreases proportionally to the altitude according to the formulæ of §4; by impressing a slight velocity upon a particle of air we can calculate approximately the variation of its virtual temperature from formula (17) of §5. Let m be the coefficient of the particle of air and m' that of the calm atmosphere, we see[5] that the equilibrium is stable for an ascending movement when $m < m'$.

The general cause of the disturbances of the equilibrium of the atmosphere is the heat from the sun. The sun communicates heat to the atmosphere both directly and by the intervention of the surface of the earth indirectly. This quantity of heat represents

[5] Let T and T' be the virtual temperatures of the particle of air and of the calm atmosphere respectively we have

$$T = T_0 - \frac{gz}{am} \qquad T' = T_0 - \frac{gz}{am'}.$$

an active force which can produce the movements of the particles of air. The action of the heat of the sun is presented under two different forms; on the one hand it produces the changes of the temperature of the atmosphere and on the other hand by the evaporation of water it produces changes in the mass of the atmosphere. The direct action of these phenomena is to produce changes in the pressure of the air accompanied by movements of the particles of air which give rise to the currents of air.

The currents of air, which can have any direction whatever, tend always to destroy the perturbations and to produce a new state of equilibrium. We can imagine permanent currents in the atmosphere; let us suppose that a continuous heating takes place at one point and that a cooling takes place at another, we see that there will arise two currents, one carrying warm air and the other cold air.

The heat set free in the atmosphere in any way whatever, produces currents of air. We notice the currents of air during a forest fire, and during the eruption of a volcano. In the last case the vapors and the shower of ashes set free the heat. We take this occasion to remark that during the eruptions of volcanoes and during earthquakes it is probable that masses in the interior of the earth change their positions. If the masses are great enough to influence local gravitation we can explain the formation of the currents of air, by supposing that the displacement of the masses in the interior of the earth produces a sudden change in the force of gravity. This change will be accompanied by a rapid change in the pressure of the air which will produce currents of air.

Chapter II

PERMANENT AND HORIZONTAL CURRENTS OF AIR

§7. *Isobars and gradients*

During the movement of the air certain new forces come in play and the level surfaces of § 6 are no longer surfaces of equal pressure. Let us consider a surface of equal pressure, or an isobaric surface, during such movement; this surface cuts the level surfaces into lines that are called isobaric. We shall occupy ourselves here principally with the isobars at the surface of the earth.

In considering the variation per unit of length of the pressure at any point we perceive that the variation along the isobar is nothing and that the variation has its maximum value along the normal to the isobar.

We shall find the variation of the pressure in any other direction whatever by projecting the maximum variation upon that direction, which projection is geometrically represented by the chord of a circle whose diameter is the maximum variation. See the fig. No. 1, in which $I\,I$ represents the isobar, $O\,N$ the maximum variation, and $O\,P$ the variation of the pressure in the direction $O\,P$.

In meteorology we call the gradient the variation of the pressure normal to the isobar and expressed in millimeters of mercury per degree of a great circle.

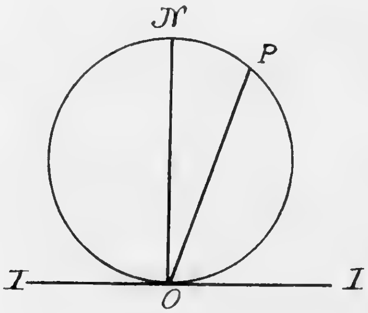


FIG. 1

Let G be the gradient, $d\,p$ infinitely slight increase of pressure, $d\,n$ an infinitely slight increase of the normal, μ a constant, we have

$$\left.\begin{aligned} \frac{d\,p}{d\,n} &= \mu\,G \\ \mu &= \frac{10333}{760}\cdot\frac{90}{10\,000\,000} = 0.00012237\,. \end{aligned}\right\} \quad \ldots \quad (1)$$

The direction of the gradient is shown by that point of the compass toward which the pressure is least. From the theory of fluids it is evident that the quantity $\frac{\mu}{\rho}G$ represents the force produced by the variation of the pressure acting on the unit of mass. This force which acts in the direction toward which the pressure diminishes, must be added to the exterior forces. We shall see later that the gradient is a small magnitude which even in cyclones does not exceed 100^{mm}.

Terrestrial gravity g is equivalent at the surface of the earth to a gradient of 10570 mm.

§8. *Forces which act during the motion*

During the motion of the air there are two new forces that come into action, namely, the action of the rotation of the earth and the friction between the molecules of air both between themselves and on the surface of the earth. The action of the rotation of the earth produces properly speaking two forces, the centrifugal force, which with the attraction of the earth produces the resultant g and the force called the composite centrifugal. This latter force which we shall call the deflecting force is perpendicular to the trajectory of the particle of air, and is directed to the right in the northern hemisphere and to the left in the southern hemisphere.

Expressing by v the velocity of the air, by ω the angular velocity of the earth and by θ the latitude, we have the deflecting force

$$= 2\,\omega\,v\sin\theta \quad \ldots\ldots\ldots\ldots \quad (1)$$

The velocity is expressed in meters per second and

$$\omega = \frac{2\pi}{86164} = 0.00007292$$

The deflecting force of the rotation of the earth is found by considering the movement of a point relative to the earth, which is supposed to be at rest. If we do not introduce this force in all the dynamic problems that introduce movements relative to the earth, it is because this deflecting force is very feeble and the trajectories do not extend to considerable distances. On the contrary the currents of air travel over large parts of the surface of the earth, and the forces which produce them are very feeble. We may then anticipate that the deflecting force of the rotation of the earth plays an important part in the problems of meteorology. Let us add that this force being perpendicular to the trajectory has no influence on the velocity of the current, but tends only to change its direction.

On the contrary, friction is a force that tends to diminish the velocity. The complete theory of the friction between the molecules of air is very complicated and will be developed in the second part of these studies.

For the present we admit that friction is a tangential force and opposed to the motion. As to its magnitude we will suppose that it is proportional to the velocity, and expressing by k the coefficient of the friction, we write

$$\textit{the force of friction} = k\,v \quad \ldots\ldots\ldots \quad (2)$$

The complete theory shows that the value of k depends on the height of the current. When the height of the current increases, the value of k diminishes, which conforms with what we know of the coefficient of friction of water in open channels. For very broad channels the coefficient of friction is in the inverse ratio of the height of the current.

In studying the movement of a particle of air it is necessary to add to the exterior forces the tangential forces and the centrifugal force produced by the motion. Expressing by s the distance traveled over and by R the radius of curvature of the trajectory, we have

$$\textit{the tangential force} = \frac{v\,d\,v}{d\,s} \quad \ldots\ldots\ldots \quad (3)$$

$$\textit{the centrifugal force} = \frac{v^2}{R} \quad \ldots\ldots\ldots \quad (4)$$

Let us add that the horizontal currents move along the surface of the earth which is normal to gravity. Consequently we neglect the action of the gravity in the following problems and the acting forces will be (1) *the gradient force*, (2) the *deflective force* of the rotation of the earth, (3) the force of *friction*, (4) the *tangential* force of the motion and (5) the *centrifugal* force of the motion.

§9. *Horizontal rectilinear and uniform motion*

When the motion is uniform and rectilinear, the tangential force and the centrifugal force disappear, and equilibrium is established between the force of the gradient, the force of friction and the deflecting force of the rotation of the earth.

Expressing by α the angle between the gradient and the trajectory and resolving the forces along the trajectory $A\,B$, fig. 2, and perpendicularly to its direction we have

$$\frac{\mu}{\rho}\,G\cos\alpha = k\,v \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$\frac{\mu}{\rho}\,G\sin\alpha = 2\,\omega\sin\theta\,v \quad \ldots\ldots\ldots \quad (2)$$

By division we obtain

$$\tan\alpha = \frac{2\,\omega\sin\theta}{k} \quad \ldots\ldots\ldots\ldots \quad (3)$$

In the northern hemisphere the latitude θ is positive and in the southern hemisphere θ is negative. The angle α has the same sign as θ and consequently the wind is deflected to the right in the northern hemisphere and to the left in the southern hemisphere.

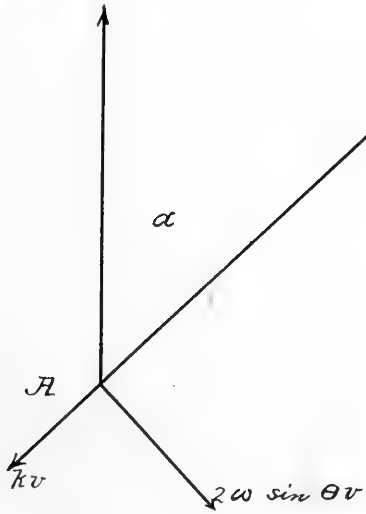


FIG. 2

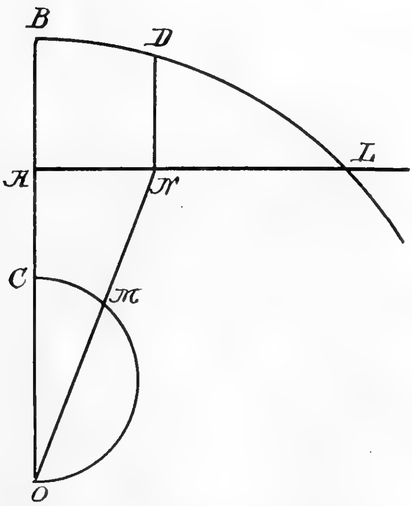


FIG. 3

The ratio between the velocity and the gradient is found by equation (1)

$$\frac{v}{G} = \frac{\mu \cos \alpha}{\rho k} \quad \ldots \ldots \ldots \ldots \quad (4)$$

By formulæ (3) and (4) we can determine the direction and the velocity of the wind: by graphical construction we find their values in the following manner.

Let $O\ B$, fig. 3, be the direction of the gradient and let two distances $O\ B$ and $O\ A$ be in the ratio.

$$O\,B : O\,A = 2\,\omega : k$$

Describe a circle with the radius $O\ B$ and erect $A\ L$ perpendicular to $O\ A$: making the arc $B\ D$ equal to the latitude and draw $D\ N$ parallel to $B\ O$; we shall have the angle $A\ O\ N = \alpha$. Describe a semicircle with the diameter $O\ C = \frac{\mu\,G}{\rho\,k}$ and the chord $O\ M$ will be equal to the velocity v.

We shall call the angle α, determined by equation (3), the normal angle of inclination.

Table of the normal angle of inclination α

LATITUDE	COEFFICIENT OF FRICTION K =					
	0.00002	0.00004	0.00006	0.00008	0.00010	0.00012
0°	0.0°	0.0°	0.0°	0.0°	0.0°	0.0°
5	32.4	17.6	12.0	9.0	7.3	6.0
10	51.7	32.3	22.9	17.6	14.2	11.9
15	62.1	43.3	32.2	25.3	20.7	17.5
20	68.2	51.3	39.7	32.0	26.5	22.6
30	74.7	61.2	50.6	42.4	36.1	31.3
40	78.0	66.9	57.4	49.5	43.2	38.0
50	79.8	70.3	61.8	54.4	48.2	43.0
60	81.0	72.4	64.6	57.7	51.6	46.5
70	81.7	73.7	66.4	59.7	53.9	48.8
80	82.1	74.4	67.3	60.9	55.2	50.1
90	82.2	74.7	67.6	61.3	55.6	50.6

The relation between the velocity and the gradient depends on the density of the air. Supposing the temperature to be 0° and the pressure 760^{mm} the density ρ is

$$\frac{\delta}{g_{45°}} = \frac{1.29305}{9.8089} = 0.13184$$

and we find the following values for the latitude 45°:

k	α	$\frac{v}{G}$	$\frac{G}{v}$
0.00002	79° 2′	8.84	0.113
0.00004	68 48	8.39	0.119
0.00006	59 49	7.78	0.129
0.00008	52 12	7.11	0.141
0.00010	45 53	6.46	0.155
0.00012	40 41	5.87	0.170

where

v = velocity in meters per second,
G = gradient in millimeters of mercury per degree of a great circle on the earth's surface.

Supposing the temperature to be 20° and the pressure 740^{mm} the value of $\frac{v}{G}$ will be increased in the ratio of 1 to 1.102. Supposing the temperature to be − 10° and the pressure 770^{mm} the value of $\frac{v}{G}$ will be diminished in the ratio of 1 to 0.951.

By the aid of the last table we have calculated the following table for the latitude of 45°, the velocities being always expressed in meters per second. The scale of wind force is that hitherto employed in several meteorological systems of Europe; the numbers have the following signification:

0 = calm; 1 = feeble; 2 = moderate; 3 = quite strong; 4 = strong; 5 = very strong or storm; 6 = a hurricane. The values of the coefficient of friction are the extreme limits which we have found by preliminary calculations, for the ocean and for the irregular surface of the earth.

WIND FORCE SCALE 0 — 6	VELOCITY M.P.S.	GRADIENT MM. PER DEGREE G. C.	
		k = 0.00002	k = 0.00012
0 = calm........	0— 1	0—0.1	0—0.2
1 = feeble.......	1— 4	0.1—0.5	0.2—0.7
2 = moderate....	4— 7	0.5—0.8	0.7—1.2
3 = quite strong	7—11	0.8—1.2	1.2—1.9
4 = strong.......	11—17	1.2—1.9	1.9—2.9
5 = storm	17—28	1.9—3.1	2.9—4.8
6 = hurricane....	28—50	3.1—5.5	4.8—8.5

§10. *Horizontal currents of air with rectilinear isobars—the latitude is supposed to be constant*

When a permanent current of air flows over rectilinear isobars the mass of air that flows perpendicularly to the isobars in the unit of time must be constant. Expressing by ψ the angle between the gradient and the tangent to the trajectory then the equation of continuity becomes

$$v \cos \psi = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Decomposing the five forces (see §8) in the directions of the tangent and the normal we shall have

$$\frac{\mu}{\rho} G \cos \psi = k v + v \frac{dv}{ds} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{\mu}{\rho} G \sin \psi = 2 \, \omega \, v \sin \theta + \frac{v^2}{R} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

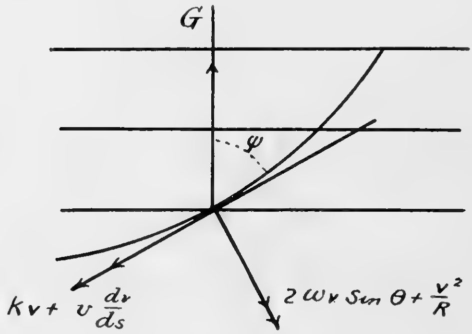


FIG 4

Differentiating equation (1) we have,

$$d \, v = \tan \psi \, d \, \psi$$

and introducing the value of the radius of curvature

$$\frac{1}{R} = - \frac{d \psi}{d \, s}$$

we shall have

$$\frac{\mu}{\rho} G \cos \psi = v \left(k + v \tan \psi \frac{d \, \psi}{d \, s}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{\mu}{\rho} G \sin \psi = v \left(2 \, \omega \sin \theta - v \frac{d \, \psi}{d \, s}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

These two equations are transformed into the following

$$\frac{\mu}{\rho} G = k v \cos \psi \left(1 + \frac{2 \omega \sin \theta}{k} \tan \psi \right) \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$0 = k \sin \psi - 2 \omega \sin \theta \cos \psi + \frac{v}{\cos \psi} \frac{d \psi}{d s} \quad . \quad . \quad . \quad . \quad (7)$$

introducing

$$\cos \psi \, d s = d x$$

where x is the distance along the gradient and s along the trajectory, equation (7) can be written

$$\frac{v \cos \psi}{k} \cdot \frac{d (\tan \psi)}{d x} = \tan \psi + \frac{2 \omega \sin \theta}{k} \quad . \quad . \quad . \quad . \quad (8)$$

The general integral of this equation is

$$\tan \psi = \frac{2 \omega \sin \theta}{k} + C e^{-\frac{kx}{v \cos \psi}}$$

where C is the arbitrary constant. In nature it is necessary to place $C = 0$ because the angle ψ does not increase to infinity with increasing values of x.

Thus we have (as in §9, eq. 3)

$$\tan \psi = \frac{2 \omega \sin \theta}{k} = \tan \alpha$$

Substituting this value of ψ in equation (1) we see that the velocity becomes constant and consequently according to equation (2) the gradient likewise becomes constant, provided that we suppose the density ρ constant, and in this case the isobars are equidistant. If we wish to consider the variation of the density ρ we introduce

$$d p = - \mu G d x$$

and by the aid of formula (20) from §5 we can calculate the pressure p. In general we can introduce a mean value of the density and consider it as constant.

§11. *Influence of the variation of latitude on horizontal currents of air with rectilinear isobars*

We consider only the case in which the gradient coincides with a meridian. The latitude θ is expressed by the following equation

$$\theta = \theta_0 + \lambda x \quad \text{. . . . . (1)}$$

$$\lambda = \pm \frac{9}{10^6} \cdot \frac{\pi}{180} \quad \text{. . . . . (2)}$$

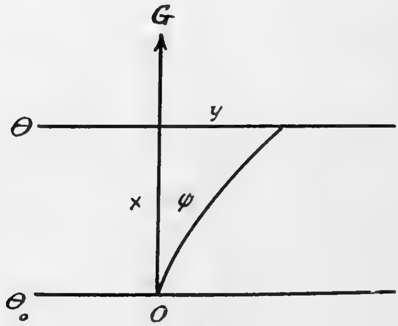


FIG. 5

The coefficient λ is positive when the gradient is directed toward the north and negative when it is directed toward the south.

The equations developed in §10 now hold good by considering θ as variable.

Equation (8) of §10 becomes

$$\frac{d\ (\tan \psi)}{dx} = \frac{2\ \omega \sin\ \theta - k \tan \psi}{v \cos\ \psi} \quad \text{. . . . . . . (3)}$$

For the sake of abbreviation we write

$$\text{tau}\ \varepsilon = \lambda\ \frac{v \cos\ \psi}{k} \quad \text{. . . . . (4)}$$

Placing the arbitrary constant equal to zero, as we have done in §10, we find that the integral of (3) is

$$\tan\ \psi = \frac{2\ \omega}{k} \cos \varepsilon \sin\ (\theta - \varepsilon) \quad \text{. . . . . . . (5)}$$

Substituting this value of tan ψ in equation (6) of §10 we shall have

$$\frac{\mu}{\rho} G = k v \cos \psi \left(1 + \left(\frac{2 \omega}{k} \right)^2 \cos \varepsilon \sin \theta \sin (\theta - \varepsilon) \right) . \quad (6)$$

In order to obtain the equation of the trajectory we introduce

$$d y = \tan \psi \, d x$$

Substituting herein the value of tan ψ and integrating, assuming $y = 0$ for $x = 0$, we get

$$y = \frac{4 \omega}{k} \cdot \frac{\cos \varepsilon}{\lambda} \, sin \left(\frac{\theta - \theta_0}{2} \right) \sin \left(\frac{\theta - \theta_0}{2} - \varepsilon \right) \quad . \quad . \ (7)$$

Introducing $\mu G = -\dfrac{dp}{dx}$ into equation (6) we shall by integration find the pressure p. Finally by graphically constructing the curve of the gradient we shall easily determine the curve of the pressure.

According to equation (5) the angle of inclination ψ depends on the quantity $\varepsilon : \varepsilon$ being so small that cos ε does not differ sensibly from unity, we conclude that ψ approaches the normal angle α, when the latitude θ has a large enough numerical value. As for the winds that cross the equator, θ has small numerical values, and the angle ψ can be very different from the normal angle of inclination. It is necessary to distinguish two cases:

(1) *The gradient is directed towards the north.*

In this case λ and ε are positive; the angle ψ is negative for southern values of θ and for northern values until $\theta = \varepsilon$. When θ is greater than ε, the angle ψ becomes positive and approaches more and more to the normal value α. We see then that the winds that come from the south have turned to the left even after crossing the equator, that the deviation is nothing at the latitude ε and that beyond this point the deviation is to the right.

We recognize this law of deviation in nature in the trade winds of the Atlantic and of the Indian Oceans during the summer.

(2) *The gradient is directed towards the south.*

The value of λ and that of ε become negative. In this case the angle ψ remains positive north of the equator and also south of the equator to latitude $\theta = \varepsilon$; then ψ becomes negative and approaches more and more nearly the normal value α.

We see then that the winds that come from the north have deviated to the right even after crossing the equator and until $\theta = \varepsilon$;

at this latitude the deviation is nothing, then it turns to the left. In nature, the monsoon called the west monsoon in the Indian Ocean follows this law during the winter.

We will apply our formulæ to numerical examples that we can compare with the charts of general winds. Among these we mention especially the excellent charts published by the Meteorological Office at London, under the title "Monthly Charts of Meteorological Data for Square 3: published by authority of the Meteorological Committee." These are in fact those charts that have led us to establish the theory of winds crossing the equator presented in this paragraph. We have developed the preceding formulæ by supposing that the gradient coincides with the meridian; approximately we may apply them to the cases in which the angle between the meridian and the gradient is small. In the first example that we shall compute we assume this angle to be 20° as we see it in fig. 6.

Applications

(1) *The gradient is northerly* (*see fig.* 6).

Let

$$\theta_0 = 0^\circ,$$
$$v \cos \psi = 10^m,$$
$$k = 0.00002,$$
$$\tau = 20^\circ;$$

we shall find

$$\varepsilon = 4^\circ\ 13'.2$$

and the following values:

θ	ψ	v	G	ν
	°	m.	mm.	
− 5°	− 50.2	15.6	0.35	4.4°
0	− 29.6	11.5	0.20	0
5	3.7	10.0	0.21	− 1.3
10	34.9	2.2	0.38	0.6

(2) *The gradient is southerly* (*see fig.* 7).

Let

$$\theta_0 = 0^\circ,$$
$$v \cos \psi = 5^m,$$
$$k = 0.00002,$$
$$\tau = 20^\circ;$$

we shall find

$$\varepsilon = -2^\circ\ 15'$$

and the following values:

θ	ψ	v	G	y
		m	mm	
5°	42°5	6.8	0.16	3.0
0	15.9	5.2	0.10	0.
− 5	− 19.2	5.3	0.12	0.2
− 10	− 44.4	7.0	0.22	3.5

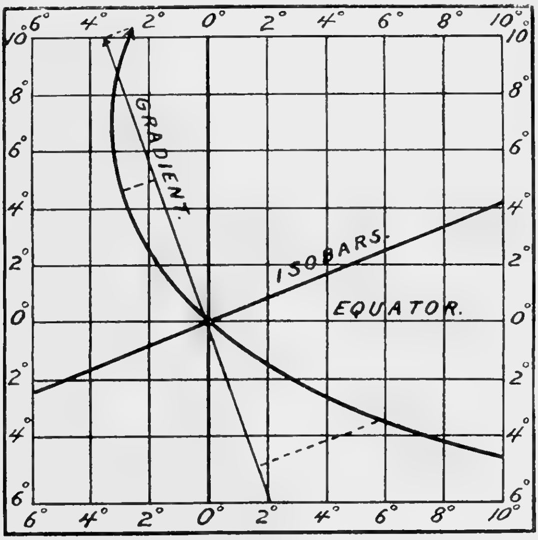


FIG. 6

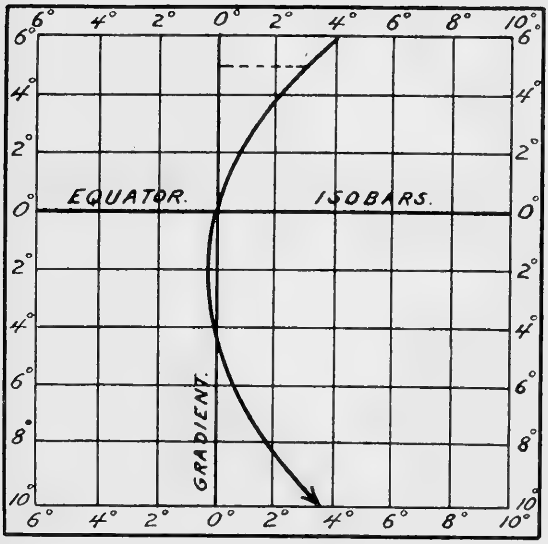


FIG. 7

We shall see that the law of inclination (eq. 5) is quite conformable to the observations; however the observed velocities and gradients do not follow our formulæ always, which is very easily explained when we note that in nature the currents of air near the equatorial calms have an ascending movement that diminishes the horizontal velocity and the magnitude of the gradient. We could easily introduce this influence into the formulæ, but we shall not profitably extend our researches any further since we shall treat the problem in a more general manner in the second part of these "Studies."

§12. *Horizontal currents of air with circular isobars around a barometric minimum*

We shall consider the latitude as constant[6] and the isobars as concentric circles. The system being symmetrical with respect to the center of the isobars therefore the quantity of air that enters per unit of time must remain constant. Designating by ψ the angle between the direction of the wind and the radius, which latter is the direction of the gradient, the component of the velocity in the direction of the radius will be $v \cos \psi$. Let r be the radius and h the altitude of the horizontal current, the section of the current will be $2\pi\, r\, h$, and remarking that h remains constant the equation of continuity will give

$$vr \cos \psi = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The acting forces are the same as in §10 and the equations (2) and (3) hold good by substituting

$$\frac{1}{R} = \frac{\sin \psi}{r} + \cos \psi \frac{d\,\psi}{d\,r}$$

$$\cos \psi\, d\,s = -\,d\,r$$

By the aid of equation (1) we shall find

$$\frac{\mu}{\rho} G \cos \psi = v \left(k + \frac{v \cos \psi}{r} - \frac{v \sin \psi\, d\,\psi}{d\,r} \right)$$

$$\frac{\mu}{\rho} G \sin \psi = v \left(2\,\omega \sin \theta + \frac{v \sin \psi}{r} + \frac{v \cos \psi\, d\,\psi}{d\,r} \right)$$

[6] That is, uniform over the whole barometric depression.—EDITOR.

These two equations are transformed into

$$\frac{\mu}{\rho} G = v \left(k \cos \psi + 2 \omega \sin \theta \sin \psi + \frac{v}{r} \right) \quad . \quad . \quad . \quad . \quad . (2)$$

$$0 = k \sin \psi - 2 \omega \sin \theta \cos \psi - v \frac{d \psi}{d r} \quad . \quad . \quad . \quad . \quad (3)$$

The last equation can be written

$$\frac{d (\tan \psi)}{r\, d r} = \frac{k}{v\, r \cos \psi} \left(\tan \psi - \frac{2 \omega}{k} \sin \theta \right)$$

By integration making the arbitrary constant equal to zero for the same reason as in §10 we obtain

$$\tan \psi = \frac{2 \omega}{k} \sin \theta \tan \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

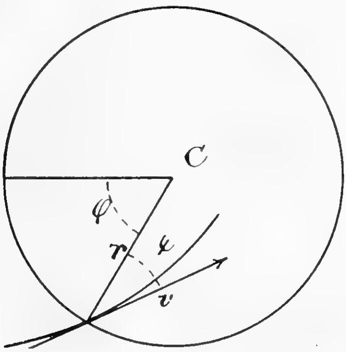


FIG. 8

The angle of inclination has therefore the normal value and the trajectory is a logarithmic spiral. Designating by φ the angle between the radius and some fixed direction, the equation of the trajectory will be

$$\log \text{nat}\, r = - \varphi \cot \alpha + C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Let r_0 and v_0 be the values of r and of v for any point whatever, we can transform equations (1) and (2) into the following:

$$v\, r = v_0\, r_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\left. \begin{aligned} \frac{\mu}{\rho} G &= \frac{kv}{\cos \alpha} + \frac{v^2}{r} \\ \frac{\mu}{\rho} G &= \frac{kv_0\, r_0}{\cos \alpha} \cdot \frac{1}{r} + \frac{v_0^2\, r_0^2}{r^3} \end{aligned} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

By introducing a mean value of the density and by expressing the distance r in degrees of the meridian, we can write

$$G = \frac{a}{r} + \frac{a'}{r^3} \quad \ldots\ldots\ldots\ldots (8)$$

in which a and a' are constants. Then the increase $d\,b$ of the pressure in millimeters is equal to $G\,d\,r$ and we shall find

$$b - b_0 = \frac{a}{M} \log . \frac{r}{r_0} - \frac{a'}{2}\left(\frac{1}{r^2} - \frac{1}{r_0^{\,2}}\right) \quad \ldots\ldots (9)$$

The equations that we have developed demand that the altitude of the current of air remains constant since we assume its horizontality. We can then, therefore, only apply the equations to the exterior parts of a whirl about a barometric depression, for in the interior of the whirlwind the currents have an ascending movement so rapid that we cannot neglect it.

Applications

(1) *Whirlwind having a great velocity* (see fig. 9).
Let the latitude $= 20°$,

$k = 0.00002$,
$\tau = 20°$,
$\frac{\mu}{\rho} = 0.001006$ (for a mean pressure of 753^{mm})
$v_0 = 50^m$,
$r_0 = 0°.3$.

Expressing r in degrees of the meridian we have

$$G = \frac{0.8}{r} + \frac{2.014}{r^3}\,;\ b - b_0 = 1.842 \log \frac{r}{r_0} + 11.19 - \frac{1.007}{r^2}$$

$$\varphi = 328\,\frac{r_0}{r};\ \alpha = 68°\,5.8'$$

$r = 0°.3$	$0.°4$	$0°.5$	$0°.6$	$0°.8$	$1°.0$	$2°.0$
$v = 50^m$	37.5^m	30^m	25^m	18.75^m	15^m	7.5^m
$G = 77.3^{mm}$	33.5^{mm}	17.7^{mm}	10.6^{mm}	4.9^{mm}	2.8^{mm}	0.6^{mm}
$b - b_0 = 0^{mm}$	5.1^{mm}	7.6^{mm}	8.9^{mm}	10.4^{mm}	11.1	14.3^{mm}
$-\varphi = 0°$	$41°$	$73°$	$99°$	$140°$	$172°$	$340°$

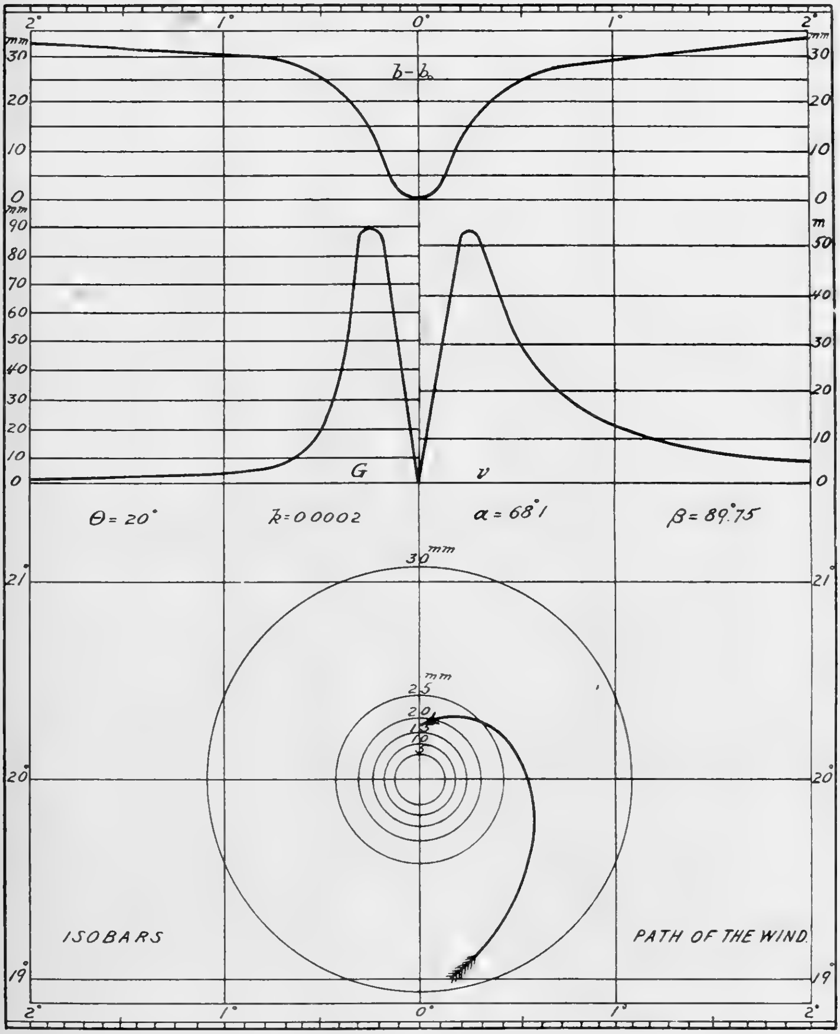


FIG. 9. WHIRLWIND AROUND A BAROMETRIC MINIMUM

(2) *Whirlwind having an average velocity* (see fig. 10.)

Let the latitude $= 60°$,

$$k = 0.00012$$
$$v_0 = 15^m,$$
$$r_0 = 7°,$$
$$\frac{\mu}{\rho} = 0.0009404 \ (\tau = 0°, \ b = 750^{mm}).$$

We shall find

$$\alpha = 46° \, 23.3'; \ G = \frac{19.43}{r} + \frac{105.5}{r^3};$$

$$b - b_0 = 44.73 \log \frac{r}{r_0} + 1.076 - \frac{52.75}{r^2}; \ \varphi = 138.5 \log . \frac{r_0}{r}.$$

$r =$	7°	8°	9°	10°	12°	15°	20°
$v =$	15^m	13.1^m	11.7^m	10.5^m	8.7	7.0^m	5.2^m
$G =$	3.08^{mm}	2.64^{mm}	2.30^{mm}	2.05^{mm}	1.68^{mm}	1.33^{mm}	0.98^{mm}
$b - b_0 =$	0^{mm}	2.8^{mm}	5.3^{mm}	7.5^{mm}	11.2^{mm}	15.6^{mm}	21.3^{mm}
$-\varphi =$	0°	8°	15°	21°	32°	46°	63°

§13. *Horizontal currents of air with circular isobars around a barometric maximum*

By making the same hypothesis as in § 12, we shall find the same equations, but it is necessary to write

$$\psi = 180° + \alpha \quad \ldots\ldots\ldots\ldots \quad (1)$$

and to change the sign of the gradient, on the supposition that the pressure diminishes with distance from the center. We shall then have

$$v\, r = v_0\, r_0$$

$$\frac{\mu}{\rho} G = \frac{k\, v_0\, r_0}{\cos \alpha} \cdot \frac{1}{r} - \frac{v_0^2\, r_0^2}{r^3} \quad \ldots\ldots\ldots \quad (2)$$

where

$$G = \frac{a}{r} - \frac{a'}{r^3} \quad \ldots\ldots\ldots\ldots \quad (3)$$

$$b_0 - b = \frac{a}{M} \log \frac{r}{r_0} + \frac{a'}{2}\left(\frac{1}{r^2} - \frac{1}{r_0^2}\right) \quad \ldots\ldots \quad (4)$$

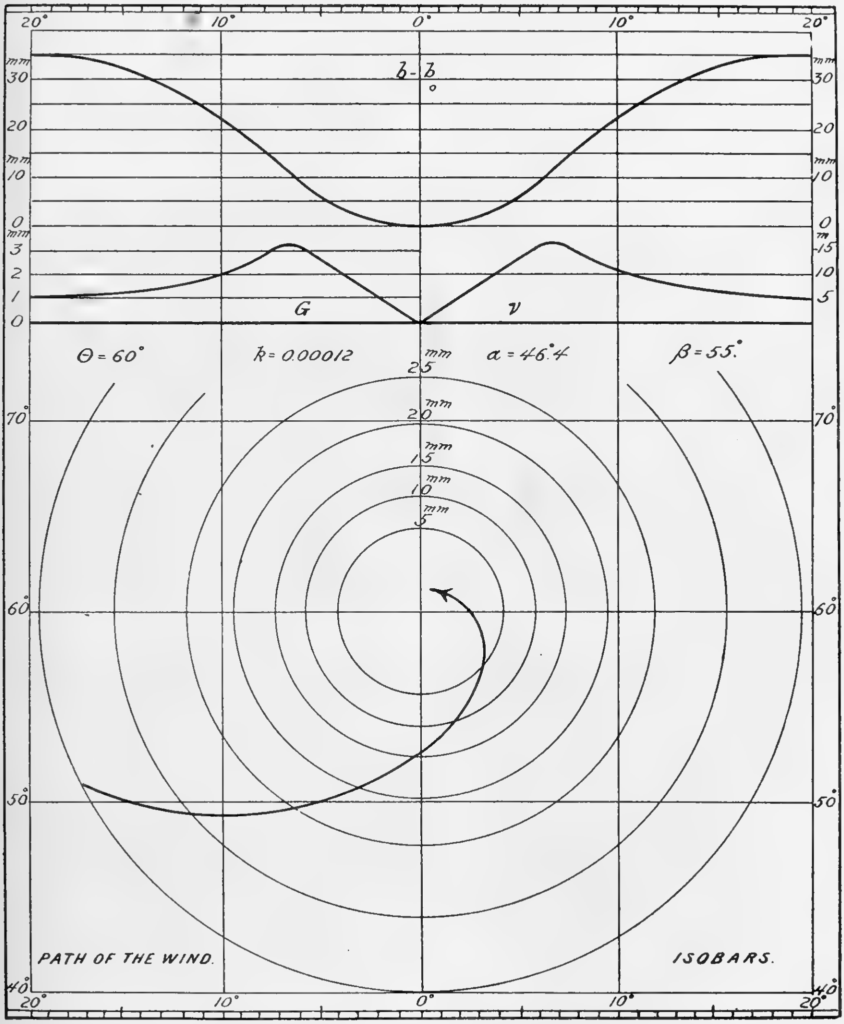


FIG. 10. WHIRLWIND AROUND A BAROMETRIC MINIMUM

The hypothesis that the pressure diminishes with the distance from the center requires that

$$v_0 < \frac{k\, r_0}{\cos\,\alpha}.$$

In nature the wind about barometric maxima always has a slight velocity and the pressure diminishes with distance from the center, However, we can apply the formulæ only to the exterior parts of the whirlwind, because in the interior part the currents are not horizontal, but have a vertical descending velocity which influences the horizontal movement.

Applications

Whirlwind about a barometric maximum (see fig. 11).

Let the latitude = 45°,

$$k = 0.00012,$$

$$\frac{\mu}{\rho} = 0.0009281\ (\tau = 0°,\ b = 760^{mm}),\ v_0 = 4^{m},\ r_0 = 5°$$

we find

$$\alpha = 40°\,35'.9;\ G = \frac{3.406}{r} - \frac{3.879}{r^3}$$

$$b_0 - b = 7.841 \log \frac{r}{r_0} - 0.0776 + \frac{1.94}{r^2};\ \varphi = 113 \log . \frac{r_0}{r}$$

r =	5°	6°	8°	10°	15°	20°
v =	4^{m}	3.3^{m}	2.5^{m}	2.0^{m}	1.3^{m}	1.0^{m}
G =	0.65^{mm}	0.55^{mm}	0.42^{mm}	0.34^{mm}	0.23^{mm}	0.17^{mm}
$b_0 - b$ =	0^{mm}	0.57^{mm}	1.57^{mm}	2.34^{mm}	3.73^{mm}	4.72^{mm}
$-\varphi$ =	0	9°	23°	34°	54°	68°

§14. *Currents of air in the interior part of an atmospheric whirl*

In § 12 and § 13 we have considered currents of air flowing at a constant elevation approaching the center of the isobars or moving away from it. In nature the elevation of the currents does not remain invariable; in atmospheric whirls around a barometric minimum the currents have an ascending movement that increases toward the center, and in whirls about a barometric maximum the currents have a descending movement that diminishes with the distance from the center. We shall treat the general problem in the second part of these studies, but at present we will consider a

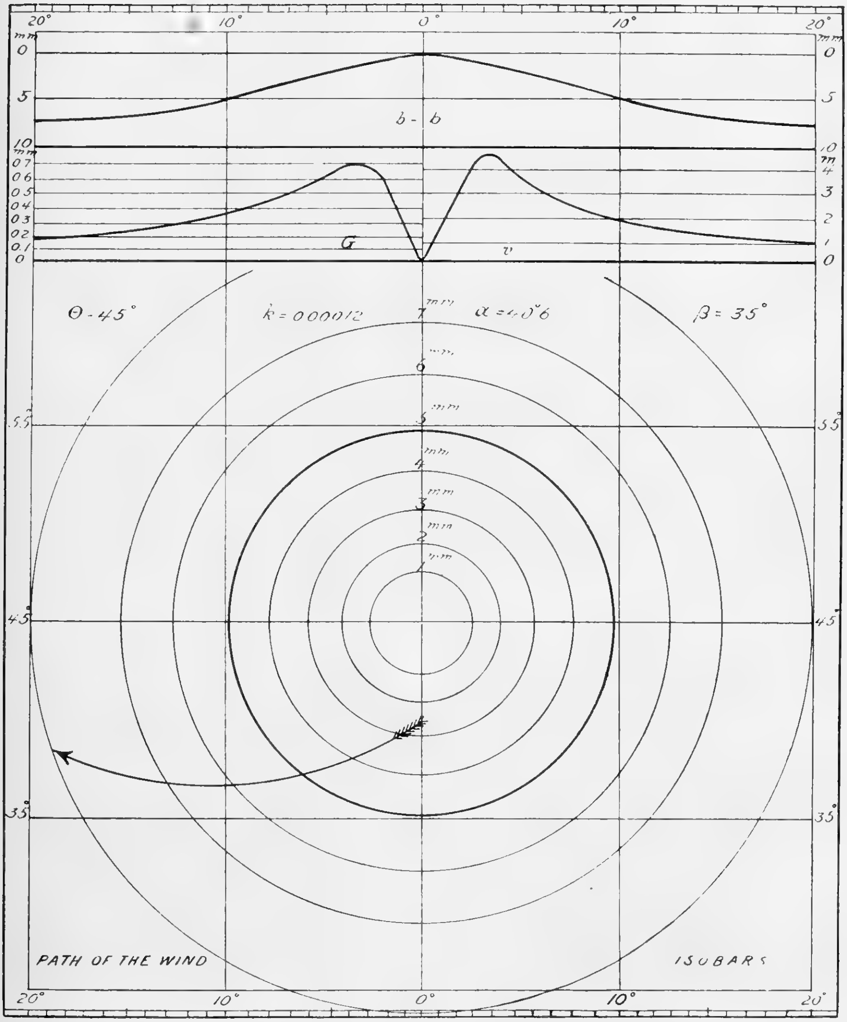


FIG. 11. WHIRLWIND AROUND A BAROMETRIC MAXIMUM

special case, namely, the central part of the whirl. For a system of circular isobars the equation of continuity can be written

$$2\pi\, r\, h\, v = \text{constant}$$

Supposing the height to be variable and a function of r, we can write

$$v\ r\ \cos\ \psi = f\ (r)$$

Assuming the following hypothesis:

$$f\ (r)\ = c\ r^2$$

where c is a constant, then the equation of continuity takes the form

$$v\ \cos\ \psi = cr$$

Introducing this value of v in equations (2) and (3) of §10 and by the aid of the formulæ of §12, we shall find

$$\frac{\mu}{\rho}\, G \cos\ \psi = v\left(k - v \sin \psi \frac{d\,\psi}{dr} - c\right)\ \ldots\ldots\ldots\ (2)$$

$$\frac{\mu}{\rho}\, G \sin\ \psi = v\left(2\,\omega \sin \theta + \frac{v\ \sin\ \psi}{r} + v \cos \psi \frac{d\,\psi}{dr}\right)\ \ldots\ (3)$$

Eliminating G we shall have

$$0 = k \sin \psi - 2\,\omega\ \sin \theta\ \cos\ \psi - 2\ c\ \sin\ \psi - v \frac{d\,\psi}{dr}\ \ldots\ (4)$$

This equation can be written

$$c\, r\, \frac{d\,(\text{tang}\,\psi)}{dr} = (k - 2\,c)\ \text{tang}\,\psi - 2\,\omega\ \sin \theta$$

By integration placing the arbitrary constant equal to zero we find

$$\text{tang}\,\psi = \frac{2\,\omega\ \sin\ \theta}{k - 2\,c}\ \ldots\ldots\ldots\ldots\ (5)$$

The angle of inclination is therefore constant and the trajectory is a logarithmic spiral, but the angle of inclination has a value different from the normal value α of § 9, eq. 3. We express this value by β, and introducing the value

$$\tan\,\alpha = \frac{2\omega}{k}\sin\,\theta$$

we shall have

$$\text{tang } \beta = \frac{\text{tang } \alpha}{1 - \frac{2c}{k}} \quad \ldots\ldots\ldots\ldots (6)$$

Equations (1) and (2) may be written

$$v = \frac{c}{\cos \beta} . r \quad \ldots\ldots\ldots\ldots (7)$$

$$\frac{\mu}{\rho} G = \frac{(k - c)\, c}{\cos^2 \beta} r \quad \ldots\ldots\ldots\ldots (8)$$

Attributing to ρ a mean value, we can write

$$G = G_1\, r. \quad \ldots\ldots\ldots\ldots (9)$$

in which G_1 denotes a constant magnitude and r is expressed in degrees of the meridian.

Then we have

$$b - b_0 = 1/2\; G_1\, r^2. \quad \ldots\ldots\ldots\ldots (10)$$

in which b_0 is the pressure in millimeters at the center. Thus the curve of pressure becomes a parabola.

The preceding formulæ apply to whirls around a barometric minimum; by making c negative and substituting $(180° + \beta)$ for β we shall have the following formulæ which apply to a whirl about a barometric maximum:

$$\text{tang } \beta = \frac{\text{tang } \alpha}{1 + \frac{2c}{k}} \quad \ldots\ldots\ldots\ldots (11)$$

$$\frac{\mu}{\rho} G = \frac{(k + c)\, c}{\cos^2 \beta} . r \quad \ldots\ldots\ldots\ldots (12)$$

Applications

(1) *Whirlwind of great velocity about a barometric minimum* (see fig. 9).

Consider the central part of the whirlwind No. 1 in §12.

Let

$$\beta = 89° \; 45'; \frac{\mu}{\rho} = 0.00103; \; (\tau = 20°,\; b = 735^{\text{mm}})$$

we shall find

$$v = 252\ r;\ G = 566.5\ r;\ b - b_0 = 283\ r^2$$

We apply these formulæ to the central part situated between the center and $r = 0.14°$ (where the current is one of ascension). By constructing the curves of v and G we shall also find by graphic interpolation their values for the region between $r = 0.14°$ and $r = 0.3°$ (where the transition to horizontal motion occurs). The results are given in the following table:

r =	0°	0.1°	0.2°	0.3°
v =	0^m	25.2^m	50.4^m	50.0^m
G =	0^{mm}	56.6^{mm}	113.3^{mm}	77.3^{mm}
$b - b_0$ =	0^{mm}	2.8^{mm}	10.0^{mm}	18.6^{mm}

(2) *Whirlwind of mean velocity about a barometric minimum* (see fig. 10).

Consider the central part of the whirlwind No. 2 in § 12.

Let

$$\beta = 55°;\ \mu = 0.0009596\ (\tau = 0,\ b = 735^{mm})$$

we shall find

$$v = 3.082\ r;\ G = 0.583\ r;\ b - b_0 = 0.2915\ r^2$$

We apply these formulæ to the central part (ascension) situated between the center and $r = 5°$, but we find by graphic interpolation the values for the part situated between $r = 5°$ and $r = 7°$, as follows:

r =	0°	1°	2°	3°	4°	5°	6°	7°
v =	0^m	3.1^m	6.2^m	9.2^m	12.3^m	15.4^m	16^m	15^m
G =	0^{mm}	0.58^{mm}	1.17^{mm}	1.75^{mm}	2.33^{mm}	2.91^{mm}	3.20^{mm}	3.08^{mm}
$b - b_0$ =	0^{mm}	0.29^{mm}	1.17^{mm}	2.62^{mm}	4.66^{mm}	7.28^{mm}	10.38^{mm}	13.58^{mm}
φ		132°	75°	42°	18°	0°	....	....

(3) *Whirlwind about a barometric maximum* (see fig. 11).

Consider the central part of the whirlwind of §13.

Let

$$\beta = 35°;$$

$$\frac{\mu}{\rho} = 0.0009281$$

we shall find

$$v = 1.823\ r;\ G = 0.32\ r;\ b_0 - b = 0.16\ r^2$$

Applying these formulæ to the central (descending) region situated between the center and $r = 1.5°$ and by graphic interpolation beyond, we find the following values:

r	$=0°$	$1°$	$2°$	$3°$	$4°$	$5°$
v	$=0^m$	1.8^m	3.6^m	4.6^m	4.7^m	4.0^m
G	$=0^{mm}$	0.32^{mm}	0.62^{mm}	0.68^{mm}	0.69^{mm}	0.65^{mm}
$b_0 - b$	$=0^{mm}$	0.16^{mm}	0.62^{mm}	1.25^{mm}	1.93^{mm}	2.60^{mm}

By these formulæ and examples we see that for a given latitude and a given coefficient of friction the whole system of a whirlwind is determined by the maximum velocity and the distance from the center of the movement of the point where that velocity is found.

CHAPTER III

THE PERMANENT VERTICAL CURRENTS OF AIR

§15. *Rectilinear movement*

We consider a particle of air moving in the direction of the vertical axis of z which we suppose positive upward. We neglect the action of the rotation of the earth and the viscosity or resistance between the molecules of air. Then we have three forces, namely: the force produced by the variation of the pressure

$$\frac{1}{\rho}\frac{dp}{dz};$$

the force of the weight g, and the tangential force

$$w\frac{dw}{dz}$$

where w represents the vertical velocity.

The equilibrium between these three forces is given by

$$\frac{1}{\rho}\frac{dp}{dz} = -g - w \, . \frac{dw}{dz} \quad \ldots\ldots\ldots\ldots (1)$$

Expressing by ρ_0 and w_0 the values of ρ and of w at any point, the equation of continuity becomes

$$\rho \, w = \rho_0 \, w_0 \quad \ldots\ldots\ldots\ldots (2)$$

This equation demands that the section of the current be of constant area. The above equations hold good for ascending movement, but by writing $-w$ in place of w they apply to a descending movement. To integrate equation (1) it is necessary to know the density as a function of the pressure. We will assume the relation given by equation (18) of § 5, and then from equation (20) we shall have

$$\frac{w^2 - w_0^2}{2g} = -z + \frac{m\alpha T_0}{g}\left[1 - \left(\frac{p}{p_0}\right)^{\frac{1}{m}}\right] \quad \ldots\ldots (3)$$

$$\frac{w}{w_0} = \left(\frac{p_0}{p}\right)^{\frac{m-1}{m}} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

Here we have supposed $p = p_0$ and $w = w_0$ when $z = 0$.
Differentiating equation (18) of §5 we have

$$\frac{d\rho}{\rho} = \frac{m-1}{m} \cdot \frac{dp}{p}.$$

If we differentiate equation (2) and eliminate $d\rho$ and dp by the aid of equation (1) we shall find

$$\frac{dw}{dz} = \frac{g\,w}{\frac{m}{m-1} \cdot \frac{p}{\rho} - w^2} \quad \ldots\ldots\ldots\ldots (5)$$

We conclude from the last equation that the velocity w can never exceed a certain limit w_m determined by the equation

$$w_m = \sqrt{\frac{m}{m-1} \cdot \frac{p}{\rho}} \quad \ldots\ldots\ldots\ldots (6)$$

Introducing the value of $\frac{p}{\rho}$ expressed in terms of w and w_0 and noting that $\frac{p_0}{\rho_0}$ is equal to $\alpha\, T_0$ we find

$$\frac{w_m}{w} = \left[\frac{m}{m-1} \cdot \frac{\alpha T_0}{w_0^2}\right]^{\frac{m-1}{2m-1}} \quad \ldots\ldots (7)$$

If we introduce this value into equations (4) and (3) we shall determine the maximum value of the height z, that a vertical current cannot exceed, for a given value of the initial velocity w_0. We see by equation (4) that the velocity increases while the pressure decreases, and equation (3) shows that the pressure p diminishes for increasing values of w. In a vertical ascending current the pressure is therefore less at the same height than in the surrounding atmosphere.

If we wish to apply equation (3) to a vertical current whose section varies, it is necessary to employ the following equation of continuity:

$$\frac{w}{w_0} = \frac{F_0}{F}\left(\frac{p_0}{p}\right)^{\frac{m-1}{m}} \qquad \text{(4 }^{bis}\text{)}$$

However this formula applies only in case the section of the current varies very slowly. It shows that the velocity diminishes as the section increases, or vice versa, as at the commencement and at the end of the currents.

Applications

(1) *Calculation of the altitude.*

Let

$$m = 5,$$
$$T_0 = 293°,$$

we shall find

$\frac{p}{p_0}$	$w_0 = 0$	$w_0 = 1^m$	$w_0 = 10^m$	$w_0 = 20^m$	$w_0 = 30^m$	$w_0 = 50^m$
0.80	1874^m	1874^m	1872^m	1865^m	1854^m	1819^m
0.60	4169	4169	4163	4143	4111	4008
0.40	7180	7180	7163	7112	7027	6755
0.20	11800	11799	11738	11553	11244	10230
0.10	15820	15818	15622	15028	14037	....
0.05	19440	19434	18807	16908	....	....
0.01	25810	25729	....	....	....	....

(2) *Maximnm altitude for a given initial velocity.*

w_0	w_m	z_m	$\frac{p}{p_0}$	T_m	$\frac{\rho_0}{u} w_0 \left(\frac{dw}{dz}\right)_0$
1^m	170.5^m	29530^m	0.001623	81.0	0.10^{mm}
10	220.3	20620	0.02095	135.2	10.06
20	238.0	16920	0.04525	157.8	40.35
30	248.9	14530	0.07103	172.6	91.24
50	263.4	11180	0.1253	193.4	257.4
100	284.5	6247	0.2707	225.6	1111.
200	307.3	1588	0.5845	263.2	6488.

The values of T_m represent the virtual temperatures at the height z_m. In the last column we have written numbers that represent the tangential force at the surface of the earth, expressed in millimeters per degree of the meridian. One should compare this force with the gradients for horizontal movements in order to get a clear idea of the force that acts in the vertical ascending movements.

§16. *Conditions of the existence of ascending and descending currents of air*

If the vertical currents preserve a steady motion, the pressure within the currents and in the surrounding atmosphere must satisfy certain conditions which we shall now consider.

Ascending currents

In ascending currents (fig. 12) the air enters along the surface of the earth and consequently the pressure p'_0 of the atmosphere must be greater than pressure p_0 of the lowest part of the current; this necessitates the existence of a barometric minimum at the surface of

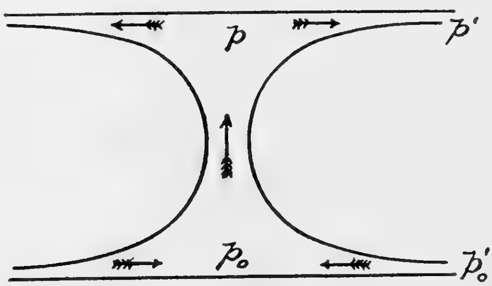


FIG. 12

the earth. In the higher strata where the air flows out from the vertical current, the pressure p in the current must remain greater than the pressure p' of the surrounding atmosphere and consequently we shall find a barometric maximum at a certain elevation.

We remark that great velocities may perhaps modify the phenomena and that the air can flow outward even from a barometric minimum[7] but it is probable that in nature we shall always find barometric maxima in such cases, because the velocities are slight at the boundaries of the currents.

[7] By substituting $180° + \alpha$ for α in the equations of §12, we shall have the formulæ that belong to currents flowing from the center. This hypothesis requires that $v > \frac{k\,r}{\cos\alpha}$.

In order to study the conditions that obtain on the inside and on the outside of the current between the pressures p_0 and p_0' at the surface of the earth and between the pressures p and p' on the inside and outside of the current at a given altitude, where the vertical velocity of the current is equal to zero, we shall assume that the virtual temperatures T in the current and T' in the surrounding atmosphere decrease proportionally to the altitude. Let m be the coefficient that belongs to the current and m' that belonging to the atmosphere, we have from §4

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^m = \left(1 - \frac{gz}{amT_0}\right)^m \quad \ldots\ldots\ldots (1)$$

$$\frac{p'}{p_0'} = \left(\frac{T'}{T_0'}\right)^{m'} = \left(1 - \frac{gz}{am'T_0'}\right)^{m'} \quad \ldots\ldots (2)$$

By writing

$$\frac{\left(1 - \dfrac{gz}{amT_0}\right)^m}{\left(1 - \dfrac{gz}{am'T_0'}\right)^{m'}} = f(z) = \left(\frac{T}{T_0}\right)^m \cdot \left(\frac{T_0'}{T'}\right)^{m'} \quad \ldots (3)$$

we shall have

$$\frac{p}{p'} = \frac{p_0}{p_0'} . f(z) \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

If the movement is ascending it is necessary to have simultaneously

$$\frac{p_0}{p_0'} < 1 \text{ and } \frac{p}{p'} > 1$$

If the movement is descending it is necessary to have simultaneously

$$\frac{p_0}{p_0'} < 1 \text{ and } \frac{p}{p'} > 1$$

Differentiating $f(z)$ we shall find

$$\frac{df(z)}{dz} = f'(z) = \frac{g}{a} f(z) \frac{T_0 - T_0' - \dfrac{g}{a} . \dfrac{m' - m}{m'm} z}{T_0 T_0' \left(1 - \dfrac{gz}{amT_0}\right)\left(1 - \dfrac{gz}{am'T_0'}\right)} \quad (5)$$

If $f'(z)$ is positive, $f(z)$ increases with the altitude and remains larger than unity: the condition of an ascending current is then fulfilled.

We shall distinguish three cases:

(1) $T_0 > T'_0$. At the surface of the earth where $z = 0$ we shall have $f'(z)$ positive. We conclude then that an ascending current can always exist when the virtual temperature of the current is greater than that of the surrounding atmosphere.

If $m < m'$ or $m = m'$ then $f(z)$ increases continuously with z and the intensity of the current increases with the altitude.

If $m < m'$ then $f(z)$ increases at first and reaches a maximum at an altitude given by

$$h = \frac{a}{g} m\, m' \frac{T_0 - T_0'}{m' - m} \quad \ldots\ldots\ldots\ldots (6)$$

and at the same time, we have

$$T = T'$$

(2) $T_0 = T_0'$. It is necessary that $m > m'$ so that $f(z)$ can be positive. This case includes the unstable equilibrium of the atmosphere.

(3) $T_0 < T'_0$. At the surface of the earth $f(z)$ is negative. If $m > m'$ then $f'(z) = 0$ at an altitude h determined by equation (6) and $f'(z)$ becomes positive for greater altitudes. We conclude then that $f(z)$ at first decreases with the altitude and reaches a minimum at the altitude h and then decreases with the altitude. It is therefore possible that an ascending current can occur even when the virtual temperature of the surrounding atmosphere is higher than that of the current. The altitude of the current must be greater than h and the virtual temperature of the current must decrease with the height more slowly than that of the atmosphere.

Descending currents

In descending currents (fig. 13) the air enters at the height z and flows outward along the surface of the earth where a barometric maximum occurs. Therefore the conditions of the descending motions are $p'_0 > p_0$ and $p > p'$. We will count the altitude z from the top downward and write

$$T = T_0 + \frac{gz}{a\,m} \text{ and } T' = T_0' + \frac{gz}{a\,m'}$$

We may write

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^m = \left(1 + \frac{gz}{am\,T_0}\right)^m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\frac{p'}{p_0'} = \left(\frac{T'}{T_0'}\right)^{m'} = \left(1 + \frac{gz}{am'T_0'}\right)^{m'} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Assuming

$$f(z) = \frac{\left(1 + \frac{gz}{am\,T_0}\right)^m}{\left(1 + \frac{gz}{am'\,T_0'}\right)^{m'}} = \left(\frac{T}{T_0}\right)^m \left(\frac{T_0'}{T'}\right)^{m'} \quad . \quad . \quad . \quad (9)$$

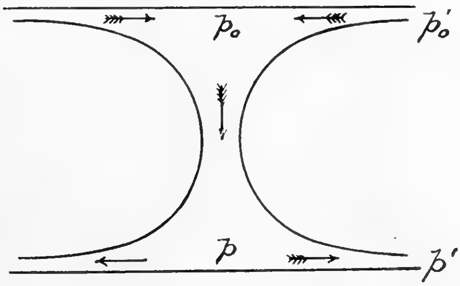


FIG. 13

we shall have

$$\frac{p}{p'} = \frac{p_0}{p_0'} \cdot f(z)$$

and the conditions for descending motion are written

$$\frac{p}{p'} > 1 \text{ and } \frac{p_0}{p_0'} < 1$$

and consequently we must have

$$f(z) > 1$$

By differentiating $f(z)$ we shall find

$$\frac{df(z)}{dz} = f'(z) = \frac{g}{a} \cdot f(z) \cdot \frac{T_0' - T_0 + \frac{g}{a} \cdot \frac{m - m'}{m\,m'} \cdot z}{T\,T'} \quad . \quad (10)$$

We shall distinguish three cases:

(1) $T_0 < T'_0$. For $z = 0$ we have $f'(z)$ positive and we conclude then that a descending current can always exist since the virtual

temperature of the current is colder than that of the surrounding atmosphere.

If $m > m'$ or $m = m'$, $f(z)$ increases with z and the intensity of the current increases with the altitude.

If $m < m'$ we have $f'(z) = 0$ for a height h determined by the equation

$$h = \frac{a}{g} m\, m' \frac{T_0' - T_0}{m' - m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

For this value of z, $f(z)$ reaches a maximum and at the same time we have

$$T = T'$$

If the altitude of the current is greater than h the virtual temperature of the current at the surface of the earth is higher than that of the surrounding calm atmosphere.

(2) $T_0 = T_0'$. The descending movement requires that $m > m'$ and this case includes the unstable equilibrium of the atmosphere.

(3) $T_0 > T'_0$. In this case $f'(z)$ is negative up to the upper stratum where the descending current must begin and consequently $f(z)$ decreases.

If $m > m'$ then $f'(z)$ becomes zero for $z = h$, as given by equation (11) and $f(z)$ attains a minimum for that value. For values of z greater than h, $f(z)$ increases and it is then possible that a descending current can even occur when the virtual temperature of the current is higher than that of the surrounding upper layers of atmosphere. The altitude of the current must be greater than h and the virtual temperature of the descending current must increase more slowly than that of the atmosphere.

§17. *Horizontal velocity produced by a vertical current*

In nature the ascending currents produce horizontal velocities along the surface of the earth, which can attain very considerable values and which are dangerous to the obstacles they meet in their way. As to the descending currents we nearly always find that in nature the resulting horizontal velocities along the surface of the earth are slight, but it is probable that the horizontal velocities at a certain altitude where the air enters the descending current, have considerable values.

Let us consider an ascending current and let v_0 be the maximum horizontal velocity, p_0 the minimum pressure in the current at the surface of the earth; let p_0' be the pressure in the calm atmosphere at a point so distant that we can neglect the velocity, we have approximately for the equation of the living force or energy, assuming the density of the air to be constant,

$$\tfrac{1}{2} v_0^2 = \frac{p_0' - p_0}{\rho} - F \quad \text{. . . . . (1)}$$

in which F expresses the energy consumed in overcoming friction or the work of the friction along the surface of the earth. We shall find

$$p_0' - p_0 = \rho(\tfrac{1}{2} v_0^2 + F) \quad \text{. . . . . . . (2)}$$

The work of friction [or done in overcoming friction] depends on the path traversed by the particles of air and on the variation of the velocity. There are whirlwinds where the work F is very small and others where it is very great. It is especially the dimension of the whirlwind that determines the work of friction. In every case we see that the horizontal velocity depends principally on the barometric depression $p_0' - p_0$, which we can consider as the measure of the force of the current. Let us denote by D the barometric depression at the surface of the earth. For ascending currents by introducing $f(z)$ we shall have

$$D = p_0' - p_0 = p_0'\left(1 - \frac{p}{p'} \cdot \frac{1}{f(z)}\right) \quad \text{. . . . . (3)}$$

The depression cannot exceed the value given by this equation after substituting $p = p'$.

Let us assume the pressure in the calm atmosphere equal to 760^{mm} and designate by D_{m} the maximum value of the depression expressed in millimeters, we have

$$D_m = 760\left(1 - \frac{1}{f(z)}\right) \quad \text{. . . . . (4)}$$

For descending currents, we find in the same way,

$$p - p' = p'\left(\frac{p_0}{p_0'} f(z) - 1\right) \quad \text{. . . . . . (5)}$$

$$D_{\mathrm{m}} = 760\ (f(z) - 1) \quad \text{. . . . . (6)}$$

Here D denotes the excess of pressure in the center of the whirlwind over the pressure of the calm atmosphere and D_{m} its maximum value.

By the aid of these formulæ and the equations of § 16 we have calculated the following tables.

Table I. Ascending currents

T_0	T_0'	m	m'	z	h	T	T'	D_m
293°	273°	7	6	5000m	....	268.6	244.5	38.9mm
293	283	6	5	5000	....	264.5	248.8	22.6
300	293	7	7	2000	....	290.2	285.2	3.7
283	273	5	7	5123	5123	248.0	248.0	9.3
283	273	5	7	7000	5123	235.2	238.8	7.7
300	300	6	5	2000	....	288.6	286.3	0.4
300	300	6	5	5000	....	271.5	265.8	4.9
280	283	6	5	7000	2635	240.2	235.2	3.3

Table II. Descending currents

T_0	T_0'	m	m'	z	h	T	T'	D_m
253	263	10	7	2000m	....	259.8°	272.8°	8.7mm
250	260	6	6	3000	....	267.1	277.1	11.2
253	263	5	7	5122	5122	288.0	288.0	10.6
253	263	5	7	6000	5122	294.0	292.3	9.9
260	260	6	5	5000	....	288.5	294.2	4.7
243	240	6	5	6000	2635	277.2	281.0	0.4

In whirlwinds of small dimensions we can neglect the action of the rotation of the earth. Assuming the altitude l of the horizontal current to be very small it is necessary to attribute to the coefficient of friction a large value and consequently the air enters into the current almost radially. In this case denoting by r the radius of the whirl, the equation of continuity gives

$$2\pi\, r\, l\, v_0 = \pi\, r^2\, w_0$$

and supposing $v_0 = w_0$ we shall have $r = 2\, l$.

In this case the radius of the whirlwind will be equally small, as is proved by observations of whirls of smoke, the whirls of dust over roads, and whirls of sand over deserts. In order to calculate the horizontal velocity we neglect the work of friction, because the distance traversed is very short, then we shall find

$$v_0 = \sqrt{2\frac{p_0' - p_0}{\rho}}$$

If we suppose the altitude z to be less than 1000^m, we can develop $f(z)$ as determined by equation (3) of §16 in a series and introducing therein this value of $f(z)$ into equation (4) and placing $p = p'$ we shall have

$$v_0 = \sqrt{2\, g\, z.\ \frac{T_0 - T_0'}{T_0'}}$$

We see therefore that for small altitudes the coefficients m and m' do not appear in the formulæ, that is to say the latent heat of the vapor of water plays no part in this case. The formula is identical with that for the velocity of the air in a chimney (fig. 14) when we neglect the friction.

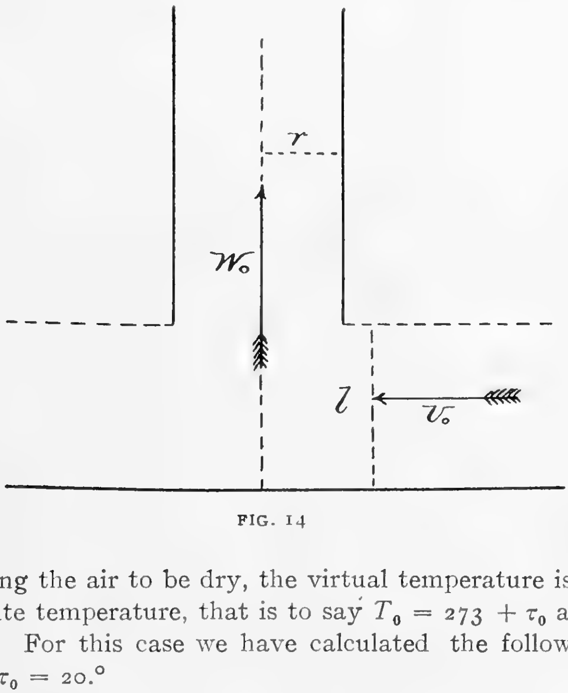


FIG. 14

Supposing the air to be dry, the virtual temperature is equal to the absolute temperature, that is to say $T_0 = 273 + \tau_0$ and $T_0' = 273 + \tau_0'$. For this case we have calculated the following table assuming $\tau_0 = 20.°$

Table III. Horizontal velocity in small whirlwinds. z = the altitude of the vertical current

τ_0	$z = 10^m$	$z = 50^m$	$z = 100^m$	$z = 200^m$
25°	1.6^m	4.1^m	5.2^m	8.2^m
30	2.6	5.8	8.2	11.6
40	3.7	8.2	11.6	16.4
50	4.5	10.0	14.2	20.0
100	7.3	16.4	23.1	32.7
200	11.0	24.6	34.5	49.1

When the whirlwinds have great dimensions, we cannot neglect the work of friction. Assuming that the trajectories of the particles of air are logarithmic spirals, we can calculate the barometric depression as we have done in paragraphs 12 and 14 (see figures 9, 10, 11) where the whirlwind of great velocity shows a barometric depression equal to 32.9^{mm} for a radius equal to 2 degrees of a great circle and with a maximum velocity equal to 50 meters per second. The whirlwind of average velocity shows a barometric depression equal to 34.9^{mm} for a radius equal to 20 degrees and with a maximum velocity equal to 16 m. p. s. In the last case the work of the friction is much greater than in the first, because the distance traversed is ten times longer.

Considering table I, we shall see that barometric depressions can be produced by different states of the atmosphere. The two whirlwinds, in which the barometric depressions do not sensibly differ, are distinguished by their maximum velocities, and it is necessary to seek the explanation of this difference in the lengths of the radii of the vertical currents that produce the horizontal velocities. The whirlwind of great velocity belongs to a vertical current whose radius is probably several tenths of a degree, but whose initial vertical velocity is very great. The other whirlwind of average velocity belongs to a vertical current whose radius extends over several degrees and whose initial vertical velocity is not great.

The length of the radius of the vertical current, which we can assume proportional to the distance from the center to the point where the velocity attains its maximum value, plays an important part in the theory of whirlwinds. Comparing two whirlwinds having the same barometric depression, that which has the shorter radius has the greater velocity and consequently is the most violent. Comparing two whirlwinds with the same maximum velocity, that which has the shorter radius has the greater gradient and the smaller depression.

The physical cause that determines the length of the radius depends on the difference in the condition of the ascending air and of the surrounding atmosphere.

PART II

(Christiania, 1880, revised 1885)

CHAPTER IV

ON MOTION IN GENERAL

§18. *Isobaric surfaces.—Vertical gradient*

In meteorology we speak of isobaric surfaces as surfaces of equal pressure or isopiestic surfaces or simply isopiesics. If the air is in equilibrium we can consider the isobaric surfaces approximately as spheres concentric with the earth, and, for a small part of the surface of the earth, we shall treat these surfaces as horizontal planes. If the air is in motion the isobaric surfaces differ from horizontal planes. In order to fix our ideas we consider a horizontal current of air whose isobaric lines at the surface of the earth are concentric circles. Let the values of the pressure for the different distances from the center be as follows:

r in degrees of a great circle =	0	4.5	6.5	8	9.8	11.8	14.5	17.8
b in millimeters of mercury =	725	730	735	740	745	750	755	760

The diminution of the pressure Δb for a change of altitude Δz can be approximately calculated by the formula (see §4, eq. 2).

$$\frac{\Delta b}{b_0} = -\frac{g \Delta z}{a T} = -\frac{\Delta z}{8200}$$

Giving successively to b_0 the values 725, 730. . . . . we calculate the values of Δz for any value of Δb, and we can construct a vertical section of the isobaric surfaces (fig. 15). When the air is in equilibrium a vertical section of the isobaric surfaces will present a series of straight horizontal lines (fig. 16*a*): supposing that we have a vertical current, the vertical section of the isobaric surfaces would also present a series of straight horizontal lines, but different from those of the series in fig. 16*a*. We shall call these lines of intersection *vertical isobars*, and if we wish to introduce the term *vertical gradient* we should be obliged to establish a definition similar to that of the term horizontal gradient. Assume the vertical gradient equal to the difference of the pressures shown by two isobars divided by their distance, we shall always and even in a state of equilibrium find a vertical gradient whose value will exceed 10.000^{mm} provided that we use the millimeter and the degree of meridian as our units. It is evident that from this definition we do not get a clear idea of the force that acts during the vertical motion and which must be represented by the vertical gradient.

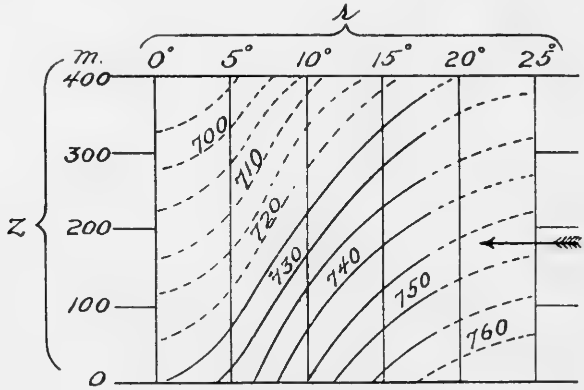


FIG. 15

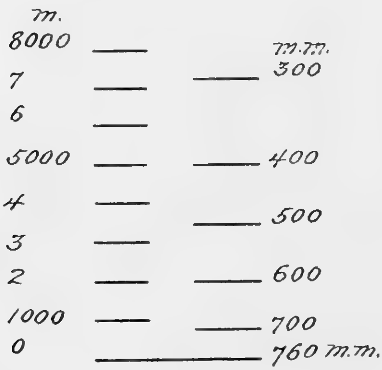


FIG. 16*a*

Let the pressure be p at the height z and q the weight of the column of air below z and we get

$$\Pi = p + q. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If the air is in equilibrium, the values of Π will be equal to the pressure p_0 at the surface of the earth and consequently Π will be constant and independent of the height z. If the air has a vertical motion the value of Π will be different from p_0 and will vary with the height z. We shall call Π *the pressure reduced to the surface of the earth.* We call the horizontal lines that correspond to the values of Π, the *reduced vertical isobars.* We call the difference of two values of Π expressed in millimeters divided by their distance expressed in degrees of the meridian *the vertical gradient.* Denoting the vertical gradient by H, the coefficient of reduction by μ (see §7) we have

$$-\mu H = \frac{d\Pi}{dz} = \frac{dp}{dz} + \frac{dq}{dz} = \frac{dp}{dz} + g\rho \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The sign minus is taken, because we consider the vertical gradient positive upward in the direction in which the pressure Π diminishes. As regards the rectilinear motion we find from §15

$$\mu H = \rho \, . \, w \frac{dw}{dz} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Introducing $d\,\Pi$ we find by integration

$$\Pi = p_0 - \rho_0 \, w_0 \, (w - w_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If in the formulæ of §15 we substitute

$$T = 290°; m = 6; p_0 = 760^{mm}; w_0 = 20^{m}$$

we shall find the results contained in the following table:

PRESSURE p	ALTITUDE z	VELOCITY w	VERTICAL GRADIENT H	REDUCED PRESSURE Π
760^{mm}	0^{m}	20.0^{m}	40.0^{mm}	760.00^{mm}
700	684	21.4	43.5	759.84
600	1954	24.9	50.8	756.21
500	3413	28.3	61.0	758.48
400	5135	34.2	76.7	757.42
300	7335	43.4	103.1	755.73
200	9992	60.8	158.7	752.55
100	14031	108.4	354.2	743.86

By constructing the curve of Π as a function of z we shall find the reduced vertical isobars as we see in fig. 16*b*. The difference between p_0 and Π we shall call the *vertical depression.* In the same way that the horizontal gradient G produces a horizontal force $\frac{\mu}{\rho} G$ (see §7) so does the vertical gradient H produce a vertical force $\frac{\mu}{\rho} H$, which must be added to the exterior forces. This vertical force includes also the force of gravity.

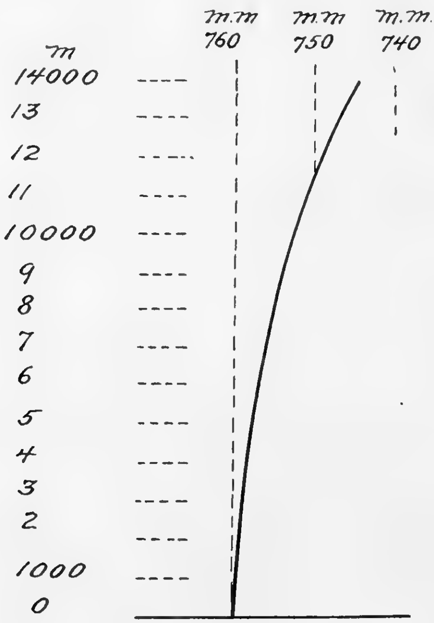


FIG. 16*b*

§19. *Equations of motion*

In order to study the general motion of the air we take three rectangular axes of which the axes $O\,X$ and $O\,Y$ are horizontal and the axis $O\,Z$ vertical and ascending. Denoting by u, v and w the components of the velocity parallel to the axes and by x, y and z the components of the forces referred to the unit of mass and by ρ the density, the equations of hydrodynamics are written

$$\left.\begin{aligned} X - \frac{1}{\rho}\frac{dp}{dx} &= \frac{du}{dt} \\ Y - \frac{1}{\rho}\frac{dp}{dy} &= \frac{dv}{dt} \\ Z - \frac{1}{\rho}\frac{dp}{dz} &= \frac{dw}{dt} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Writing

$$\varDelta = \frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} \quad \ldots\ldots\ldots\ldots (2)$$

the equation of continuity assumes the form

$$\frac{d\rho}{dt} + \rho\,\varDelta = 0 \quad \ldots\ldots\ldots\ldots (3)$$

In the preceding equations

$$\frac{du}{dt},\ \frac{dv}{dt} \text{ and } \frac{dw}{dt}$$

represent the components of the whole force; the forces produced by the variation of the pressure are represented by

$$-\frac{1}{\rho}\frac{dp}{dx} \text{ and } -\frac{1}{\rho}\frac{dp}{dy}$$

which are the components of the horizontal force $\frac{\mu}{\rho} G$

and by

$$-\frac{1}{\rho}\frac{dp}{dz}$$

which is included in the vertical force $\frac{\mu}{\rho} H$.

The components X, Y, and Z are the components of the exterior and interior forces.

The exterior forces are the two following:

Gravity. This is the resultant of the attraction of the globe and of the centrifugal force produced by the rotation of the earth. The direction of gravity is normal to the surface of the earth and is represented by the axis $O\,Z$.

Gravity has therefore only one component $-g$, and by introducing the vertical gradient H, we have

$$\frac{\mu}{\rho} H = -g - \frac{1}{\rho}\frac{dp}{dz}.$$

We consider the force of gravity as constant, because the winds that we are studying are located in the lower strata of the atmosphere.

The deflecting force of the rotation of the earth. This compound centrifugal force is the force that we must add to the exterior forces in order to be able to treat a problem of relative motion as if one had to do with absolute motion. Denoting the angle between the axis $O\ X$ and the direction north by a and the components of the deflecting force by X_0, Y_0 and Z_0 we have

$$\left.\begin{aligned} X_0 &= 2\,\omega \sin\theta\, v - 2\,\omega \cos\theta \sin a\, w \\ Y_0 &= 2\,\omega \sin\theta\, u - 2\,\omega \cos\theta \cos a\, w \\ Z_0 &= 2\,\omega \cos\theta \sin a\, u + 2\,\omega \cos\theta \cos a\, v \end{aligned}\right\} \quad . \; . \; . \; . \; (4)$$

Here θ denotes the latitude considered as positive in the northern hemisphere and negative in the southern hemisphere and ω denotes the angular velocity of the earth per second of mean time.

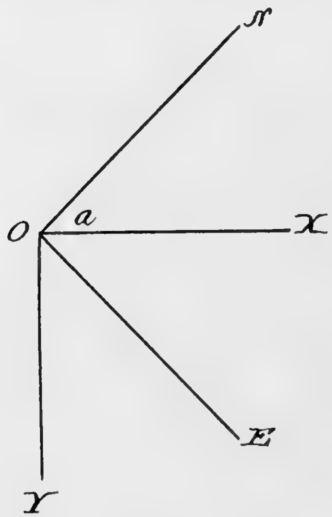


FIG. 17

The interior forces are the components of the *internal friction* or viscosity produced by the difference between the velocities of the different adjacent strata of air. The surface of the earth offers a resistance to the currents of air, the effect of which, in diminishing the velocity of the lower strata, is shown by the variation of velocity between the different strata. The particles of air having a greater velocity increase the motion of the particles having a less velocity and, inversely, the particles having less velocity retard the motion of the particles having greater velocity. The resistance of the surface of the earth, therefore, transfers its influence through all the strata of air and influences both the direction and the velocity of the motion. We shall in the following chapter consider some special cases of interior friction. However, the absence of observations on the variation of the velocity with the altitude prevents the application of the exact theory to the winds in general. We shall consider friction as an exterior force acting along the surface of the earth. Denoting the components of the friction by X_1 and Y_1, we write (see §7):

$$\left.\begin{aligned} X_1 &= -k\,u \\ Y_1 &= -k\,v \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (5)$$

in which k denotes the coefficient of friction.

By introducing the preceding values of the components of the exterior and interior forces and noticing that the velocities and the density are functions of the four variables x, y, z and t, the equations of motion are written as follows:

$$\left.\begin{aligned} \frac{1}{\rho}\frac{dp}{dx} &= X_0 + X_1 - \frac{du}{dt} - u\frac{du}{dx} - v\frac{du}{dy} - w\frac{du}{dz} \\ \frac{1}{\rho}\frac{dp}{dy} &= Y_0 + Y_1 - \frac{dv}{dt} - u\frac{dv}{dx} - v\frac{dv}{dy} - w\frac{dv}{dz} \\ \frac{1}{\rho}\frac{dp}{dz} &= Z_0 - g - \frac{dw}{dt} - u\frac{dw}{dx} - v\frac{dw}{dy} - w\frac{dw}{dz} \end{aligned}\right\} \quad \ldots \quad (6)$$

$$\frac{d\rho}{dt} + u\frac{d\rho}{dx} + v\frac{d\rho}{dy} + w\frac{d\rho}{dz} + \rho\,\Delta = 0 \quad \ldots\ldots \quad (7)$$

The trajectory of a particle of air is determined by the equations

$$\left.\begin{aligned} \frac{dx}{dt} &= u \\ \frac{dy}{dt} &= v \\ \frac{dz}{dt} &= w \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (8)$$

§20. *Classification of the systems of wind*

Each disturbance of the equilibrium of the atmosphere produces a motion of the air or what we in general call a system of winds. Considering the forces which act during the motion, we divide the systems of wind into two classes. The systems of the *first order* are those which extend only over quite a limited part of the surface of the earth, and which at the same time exhibit variations of velocity so great that we can neglect friction and the deflecting force of the rotation of the earth. For example we mention tornadoes, waterspouts, whirlwinds of smoke, etc. The systems of wind of the *second order* are those in which all the acting forces have some importance. As examples we mention cyclones, the trade winds, the sea breezes and land breezes.

Considering the motion of the air in the systems of wind we distinguish between the *permanent* systems and the *variable* systems. In a *permanent system* the pressure and the velocity at any place are independent of time and vary only from one place to another. In

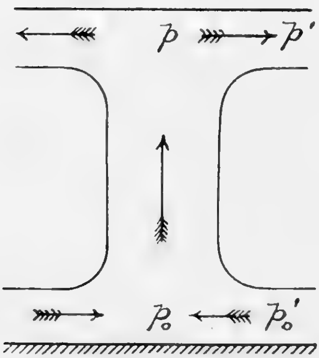


FIG. 18

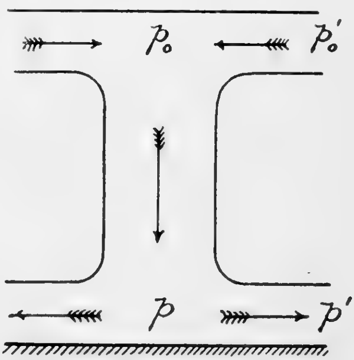


FIG. 19

nature we never find a permanent system, but we may consider the systems of wind which remain nearly invariable for quite a long time as permanent. As examples we mention the trade winds, an immovable anticyclone or cyclone, with constant pressure at the center. The variable systems of wind are divided into movable systems and *immovable* or fixed systems. In the variable fixed systems the minimum or the maximum barometer does not change position with the surface of the earth, but its value varies with the time.

In our following studies we shall consider four simple systems of wind.

(1) *System of ascending parallel winds.*

This system (see fig. 18) has rectilinear isobars, a barometric minimum at the surface of the earth and a barometric maximum in the higher strata. The air flows inward along the surface of the earth from both sides toward the barometric minimum and the horizontal current is gradually changed into a vertical *ascending* current. At a certain altitude the vertical current is changed into a horizontal current which flows outward from the barometric maximum. Denoting by p_0 the pressure at the lower barometric minimum and by p the pressure at the upper maximum and by p_0' and p' the corresponding pressures in the exterior atmosphere, we shall have for the depression D_0 which belongs to the horizontal current along the surface of the earth,

$$D_0 = p_0' - p_0 \qquad (1)$$

The excess of the pressure D in the upper barometric maximum is given by the formula

$$D = p - p' \qquad (2)$$

Let q and q' be the weights of the columns of air of the vertical current and of the calm atmosphere, respectively, and let Π be the reduced pressure (see §18) we have

$$\Pi = p + q \qquad (3)$$
$$p_0' = p' + q' \qquad (4)$$
$$E = p_0 - \Pi \qquad (5)$$

in which we call E the *vertical depression* of an ascending current (see §18).

From these equations we obtain

$$D_0 + E + D = q' - q \qquad (6)$$

This last equation shows us that the difference in weight of the columns of air produces the motion of the three currents.

(2) *System of descending parallel winds.*

This system (see fig. 19) has rectilinear isobars and a barometric maximum at the surface of the earth and a barometric minimum in the upper strata. The air flows in from the two sides toward the barometric minimum and the horizontal current in the upper strata changes little by little into a *descending* vertical current. The vertical current then changes into a horizontal current which flows out from the barometric maximum at the surface of the earth.

Denoting by p_0 the pressure at the barometric minimum and by p the pressure at the maximum and by p_0' and p' the corresponding pressures in the exterior calm atmosphere, we shall have the depression

$$D_0 = p_0' - p_0. \qquad (7)$$

which relates to the horizontal current in the upper stratum. The excess of the pressure D in the barometric maximum is given by the formula

$$D = p - p'. \qquad (8)$$

Let q and q' be the weights of the columns of air of the vertical current and of the calm atmosphere respectively and let Π be the reduced pressure, we have

$$\Pi = p_0 + q \qquad (9)$$

$$p' = p_0' + q' \qquad (10)$$

writing

$$E = \Pi - p. \qquad (11)$$

we shall find

$$D_0 + E + D = q - q'. \qquad (12)$$

The quantity E represents the vertical depression in the descending current, and this last equation shows us that the difference between the weights of the columns of air produces the motion of the three currents.

(3) *System of cyclonic winds.*

This system has circular isobars around a barometric minimum at the surface of the earth; in the upper strata it has a barometric maximum. The air flows in along the surface of the earth from all sides and the horizontal currents are changed little by little into vertical ascending currents. At a certain height the vertical motion is changed into a horizontal motion and in the upper strata the air flows out from the barometric maximum. By introducing the same notation as we have employed in the first system of parallels, equations (1) to (6) hold good also for cyclones.

(4) *System of anti-cyclonic winds.*

This system has isobars circular around a barometric maximum at the surface of the earth: it has a barometric minimum in the upper strata. The upper air flows inward toward the barometric minimum and the horizontal currents in the upper strata are changed little by little into *descending* vertical currents. The vertical motion then changes into a horizontal motion and the air flows out from the barometric maximum at the surface of the earth. Equations (7) to (12) hold good for anti-cyclones.

Each of these four systems of wind has its calm space at the surface of the earth which represents the interior part where the motion of the air is nearly vertical and where consequently we do not feel any wind. In the upper strata we must also find calm spaces where the vertical motion changes into the horizontal motion or vice versa.

These four systems which we have called simple systems, transport masses of air, either from the surface of the earth to the upper strata, or from the upper strata to the surface of the earth. When we consider the case in which two or several simple systems exist simultaneously so that their motions encroach upon each other and the masses of air pass from above to below and inversely, we have a composite system of winds of which nature offers an infinite number of examples.

Chapter V

INTERNAL FRICTION

§21. *Horizontal currents of air of small extent*

We shall at first consider horizontal currents so small that we can neglect the effect of the rotation of the earth; we also assume the density to be constant. Let $A\ B$ and $C\ D$ (fig. 20) be two horizontal planes that enclose the mass of air; assume that the plane $C\ D$ is fixed and that the plane $A\ B$ moves with a uniform velocity V. The motion of the air will therefore proceed in horizontal strata of

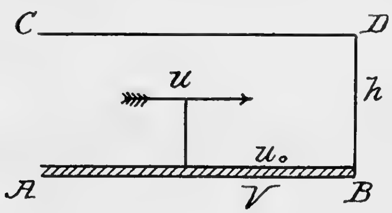


FIG. 20

different velocities; along $A\ B$ the velocity of the air may be u_0. and along $C\ D$ the velocity may be zero. Admitting the hypothesis that the internal friction or the viscosity is proportional to the difference between the velocities of any two strata, we conclude that the velocity decreases proportionally to the distance z from the plane $A\ B$. Let h be the distance of the two planes, the increase of velocity per unit of length will be $\frac{\mu_0}{h}$ and we shall find the velocity

u at the distance z by the formula

$$u = \frac{u_0}{h}\left(h - z\right) = u_0 - \frac{u_0}{h} z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The internal friction per unit of surface which we denote by F will be equal to a coefficient K multiplied by the rate of increase of velocity and consequently

$$F = K \frac{u_0}{h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The plane $A\ B$ moves with the velocity V and the air along $A\ B$ moves with the velocity u_0; the resistance between the air and the plane $A\ B$ is proportional to the difference $V - u_0$ and to the coefficient of friction f between the air and the plane; consequently we can write

$$F = f\,(V - u_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From these equations we find

$$u_0 = \frac{fV}{f + \frac{K}{h}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In the preceding case the pressure has been supposed to be constant. We shall now consider the case where the horizontal current of air has a gradient; the horizontal velocity u depends solely on the distance z and the vertical velocity is zero. The increase of the horizontal velocity per unit of vertical distance is $\frac{du}{dz}$ and the internal friction is equal to $K \frac{du}{dz}$. Considering a parallelopipedon whose thickness is dz and whose face is a unit of area, the result of the frictions of the two faces will be $d(K \frac{du}{dz})$ and the mass of the element will be ρdz. The force per unit of mass resulting from the internal friction will therefore be $\frac{K}{\rho} \frac{d^2u}{dz^2}$ and this force acts in the same direction as the force of the gradient. The equation of equilibrium is

$$\frac{\mu}{\rho} G = - \frac{K}{\rho} \frac{d^2u}{dz^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The vertical motion being zero, the vertical gradient H will disappear and consequently the pressure is independent of the altitude z. We conclude therefore that the horizontal gradient G is independent of z and constant. Integrating equation (5) we shall find

$$K\frac{du}{dz} = C - \mu\, G z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In order to determine the constant C we notice that the internal friction disappears at a certain value of z which we will denote by h; at the same time the velocity has its maximum value U. It is evident that the friction is equal to zero in the stratum whose velocity is a maximum because the velocity decreases equally on each side of this stratum and consequently the difference between the velocities of the two strata located symmetrically is equal to zero. By choosing the origin of coördinates at this distance h from the surface of the earth (see fig. 21) we shall have the constant C equal to zero and by integrating equation (6) we shall find

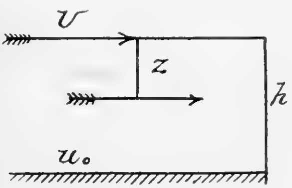


FIG. 21

$$u = U - \frac{\mu\, G}{2K} z^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Let the velocity of the air at the surface of the earth be u_0, then for $z = h$ we shall have

$$u_0 = U - \frac{\mu}{2} \cdot \frac{G}{K} \cdot h^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The upper limit of a free horizontal current is found by placing $u = 0$ and let the corresponding value of z be H, we have

$$0 = U - \frac{\mu}{2} \cdot \frac{G}{K} \cdot H^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

From equation (6) we conclude that the friction at the distance z, is equal to $\mu\, G\, z$; at the surface of the earth the internal friction is equal to $\mu G h$; the friction between the surface of the earth and the air is equal to $f u_0$ and consequently we find

$$\mu G h = f u_0$$

or

$$\frac{\mu}{\rho} . G = \frac{f u_0}{\rho h} = k u_0 \quad \ldots \ldots \ldots \ldots (10)$$

Here k denotes the coefficient of ordinary friction which we have introduced in our previous problems and we have

$$k = \frac{f}{\rho h} \quad \ldots \ldots \ldots \ldots \ldots (11)$$

This equation shows that the coefficient of friction k is inversely proportional to the depth of the current measured from the surface of the earth to the stratum of maximum velocity.

By experiments on the viscosity of the air, *Clerk Maxwell* found the value of K at 0° C. equal to 0.001878. Introducing this value in equation (7) we shall have

$$u = U - 0.0033\, G.z^2. \quad \ldots \ldots \ldots (12)$$

Experiments on the motion of liquids show that inequalities of depth produce little vortices which play an important part in the law of velocity. We are led to adopt the following formula:

$$u^2 = U^2 - 0.04\, G.z^{\frac{3}{2}}. \quad \ldots \ldots \ldots \ldots (13)$$

The value 0.04 is taken from experiments on the motion of water in straight channels.

§22. *Horizontal currents of air of large extent*

We shall consider a horizontal current of air that moves over so large a part of the surface of the earth that we cannot neglect the effect of the rotation of the earth For horizontal motion the deflecting force of the rotation of the earth is normal to the trajectory of the wind and its value is expressed by $2\, \omega \sin \theta\, U$, where ω denotes the angular velocity of the earth, θ the latitude and U the horizontal velocity of the wind.

Assuming that the motion of the current of air is uniform, then the velocity and the gradient will be constant; the acting forces will be the deflecting force of the rotation of the earth, the force of the gradient and the friction. In the special case where the current of air moves along a surface without friction, equilibrium will exist between the deflecting force of the rotation of the earth and the force of the gradient; consequently the two forces must be opposite and their directions must be along the same straight line.

We shall then have

$$\frac{\mu}{\rho} G = 2\,\omega\, U \sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The deflecting force of the rotation of the earth being normal to the path of the wind, we conclude that in the case where the friction is zero, the current is normal to the gradient, that is to say, the wind moves along the isobar. The ratio between the velocity of the wind and the gradient is expressed by

$$\frac{U}{G} = \frac{\mu}{\rho} \Big/ 2\,\omega \sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let the pressure be 760^{mm}, the temperature $0°$ C., and the tension of the vapor of water 0, we shall have

$$\frac{U}{G} = \frac{6.304}{\sin\theta}$$

whence the following values

θ =	10°	20°	30°	40°	50°	60°	70°
$\frac{U}{G}$ =	36.6	18.6	12.7	9.9	8.31	7.31	6.77

We have supposed that the force of friction, at the surface of the earth, is opposite to the motion of the particle of air. In this case its path will form an acute angle with the direction of the gradient. Since friction has its greatest value at the surface of the earth and diminishes with altitude, the velocity of the air and at the same time the angle of inclination ψ in §11 must increase with the height, which observations also show to be the case.

In the stratum that separates the lower current from the higher current, (in the systems of wind that we considered in the preceding chapter) the gradient must be zero and consequently the velocity of the air nothing. Thus the velocity of the air increases with the altitude in the part near the earth while it diminishes toward zero in the region near the stratum that is intermediate between the two currents. The velocity of the air must consequently attain its maximum at a certain height.

As the resultant force of the internal friction does not necessarily act in the direction opposite to the motion therefore the direction of the motion of the air in the stratum of maximum velocity remains uncertain. Probably it does not sensibly deviate from a direction perpendicular to the gradient.

How are the velocity and the direction of the motion related in the different strata of a current? This is a problem hardly solvable in the present state of our knowledge of the laws of friction and in the absence of precise observations of the gradients and the motions of the upper strata of the atmosphere.

§23. *A rotary current of air*

We shall now consider a mass of air revolving about a vertical axis in consequence of the motion of the surrounding air. The exterior air moves in circular trajectories and with constant velocity and by internal friction produces a rotation of the interior mass of air. We have therefore a mass of air within a cylindrical boundary whose velocity is given and which turns about a vertical axis by reason of internal friction. The tangential velocity U is a function of the distance r from the axis; the isobars are concentric circles; the gradient is directed along the radius. The acting forces are the force of the gradient, the centrifugal force and the deflecting force of the rotation of the earth, all of which act in the direction of the radius, and finally the force of the internal friction which acts in the direction of the tangent. We neglect the friction at the surface of the earth, so that the velocity is independent of the altitude. The resultant of the internal frictions acts tangentially on each element and should be equal to zero, because there exists no tangential force with which to establish equilibrium; the result is, that the internal friction along a cylindrical surface must be constant. Let the mass of rotating air be divided into cylindrical portions which rotate with different velocities. The internal friction is due to the differences of the velocities U, but the radius r varies at the same time and with it the frictional surface; it is necessary therefore, to make the friction proportional to the variation of the product of the velocity and the frictional surface, divided by the increase of the volume. We shall find then

$$\frac{d\,(r\,U)}{r\,d\,r} = a = \text{constant} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

By integration we find

$$r\,U = \tfrac{1}{2}\,a\,r^2 + b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where a and b denote two constants that we can determine in the following manner:

Let the given velocity of the exterior air be U_1 at the distance r_1 and assume the velocity of the interior mass equal to zero at the distance r_0, we find

$$\tfrac{1}{2}\,a = \frac{r_1\,U_1}{r_1^2 - r_0^2}\,;\; b = -\,\frac{r_0^2\,r_1\,U_1}{r_1^2 - r_0^2}$$

and

$$U = \frac{r_1}{r} \cdot \frac{r^2 - r_0^2}{r_1^2 - r_0^2} \cdot U_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

It is quite probable that in nature the radius r_0 is equal to zero and we shall then have

$$U = \frac{r}{r_1} \cdot U_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Hence, *the current of air rotates with a constant angular velocity* (see §14).

In order to determine the gradient and the pressure, we distinguish two cases in the northern hemisphere.

(1) *Rotation contrary to the sun.*

In the cyclones of the northern hemisphere the rotation takes place contrary to the apparent diurnal motions of the sun, the gradient is directed toward the center, the centrifugal force and the deflecting force of the rotation of the earth are directed outward. We have then

$$\frac{\mu}{\rho}\,G = \frac{U^2}{r} + 2\,\omega \sin\theta \,.\, U \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

By writing $\mu\,G = \dfrac{dp}{dr}$ and introducing the value of U given in equation (4) we find by integration, p_0 being the pressure at the center where $U = 0$,

$$\frac{p - p_0}{\rho} = \tfrac{1}{2}\,(U^2 + 2\,\omega \sin\theta \,.\, U\,r) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

(2) *Rotation with the sun.*

In the anti-cyclones of the northern hemisphere the rotation takes place with the apparent diurnal motion of the sun; the gradient and the centrifugal force are directed outward and the deflecting force of the rotation of the earth is directed toward the center. We shall have then

$$\frac{\mu}{\rho} G = 2\,\omega \sin \theta \,.\, U - \frac{U^2}{r} \quad \ldots\ldots\ldots \quad (7)$$

and

$$\frac{p_0 - p}{\rho} = \tfrac{1}{2}\,(2\,\omega \sin \theta \,.\, U r - U^2) \quad \ldots\ldots \quad (8)$$

Equation (7) demands that the angular velocity $\frac{U}{r}$ be less than $2\,\omega \sin \theta$ because the gradient must be positive.

Chapter VI

PERMANENT SYSTEMS OF WIND

§24. *Permanent wind-systems of the first order*

In nature tornadoes and waterspouts represent examples of cyclones of the first order, but meteorological observations of these phenomena being very scarce do not suffice to show us the changes of pressure and velocity which take place in them. We cannot as yet by mathematical analysis construct a complete system of wind. However, we shall consider some simple cases which show analogies with the systems of nature and from them we shall seek to deduce applications.

In the general equations of § 19 we may neglect the components X_0, Y_0, Z_0, X_1 and Y_1 and we shall consider the density as constant

The equations assume the form

$$\left.\begin{aligned} \frac{1}{\rho} \cdot \frac{dp}{dx} &= -u\frac{du}{dx} - v\frac{du}{dy} - w\frac{du}{dz} \\ \frac{1}{\rho} \cdot \frac{dp}{dy} &= -u\frac{dv}{dx} - v\frac{dv}{dy} - w\frac{dv}{dz} \\ g + \frac{1}{\rho} \cdot \frac{dp}{dz} &= -u\frac{dw}{dx} - v\frac{dw}{dy} - w\frac{dw}{dz} \end{aligned}\right\} \quad \ldots\ldots \quad (1)$$

$$0 = \frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} \quad \ldots\ldots\ldots \quad (2)$$

Considering the special case, where we have

$$\frac{dw}{dy} = \frac{dv}{dz}, \frac{du}{dz} = \frac{dw}{dx}, \frac{dv}{dx} = \frac{du}{dy}$$

equations (1) are reduced to a single one. Denoting the absolute velocity by V, we have

$$V^2 = u^2 + v^2 + w^2$$

and

$$\frac{1}{\rho} d p = - V d V - g d z. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and consequently

$$p = p_0 + \tfrac{1}{2} \rho (V_0^2 - V^2) + g \rho (z_0 - z). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where p_0 denotes the pressure for $V = V_0$ and $z = z_0$.

We designate the distance from the origin of coördinates by R and its horizontal projection by r, the horizontal velocity by U, and consequently we have

$$x^2 + y^2 + z^2 = r^2 + z^2 = R^2$$
$$U^2 = u^2 + v^2$$

First example. The trajectories are straight lines directed toward a fixed center.

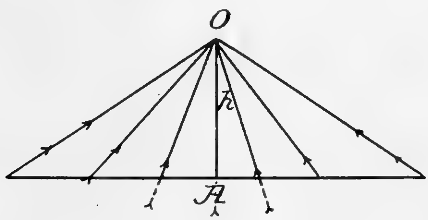


FIG. 22

We take the fixed center (see fig. 22) at the origin of coördinates and put the equations of the trajectories under the form

$$\frac{x}{x_0} = \frac{y}{y_0} = \frac{z}{z_0} = \frac{R}{R_0} = \left(1 + \frac{3 a t}{R_0^3}\right)^{\frac{1}{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Calculation shows that these equations satisfy the conditions which we have introduced above. For the time $t = 0$, we have $R = R_0$, from which we conclude that all the particles of air are found at first on the surface of a sphere whose radius is R_0. Differentiating x,

y and z with respect to t and introducing u, v and w (see §19) equation (8) and eliminating the arbitrary constants, we have

$$\frac{u}{x} = \frac{v}{y} = \frac{w}{z} = \frac{U}{r} = \frac{V}{R} = \frac{a}{R^3} \quad \ldots \ldots \ldots \quad (6)$$

For $a < 0$, the air flows in to the center and for $a > 0$ the air flows away from the center.

In any horizontal plane (z = constant) the gradient G is found by equation (3), by noticing that

$$V = \frac{a}{R^2}; \qquad R^2 = r^2 + z^2$$

$$G = \frac{1}{\mu} = \frac{dp}{dr} = -\frac{\rho}{u} : \frac{VdV}{dr} = \frac{\rho}{\mu} \cdot \frac{2\,a^2 r}{R^6} \quad \ldots \quad (7)$$

Let us consider a horizontal plane at the distance $z = -h = z_0$, and study the phenomena along this plane, which can represent the surface of the earth.

Denoting the absolute velocity at the point A by V_0, we shall have

$$V_0 = \frac{a}{h^2}$$

Writing

$$r = \xi\, h$$

we shall find

$$R = h\sqrt{1 + \xi^2}$$

$$U = \frac{\xi}{(1 + \xi^2)^{\frac{3}{2}}} \cdot V_0$$

$$G = \frac{2\,\rho}{\mu} \cdot \frac{V_0^2}{h} \cdot \frac{\xi}{(1 + \xi^2)^3}$$

$$p - p_0 = \tfrac{1}{2}\,\rho\,(V_0^{\,2} - V^2)$$

Denoting by p_0' the pressure at a point so distant that we can consider the velocity V as zero [$V = \frac{a}{R^2}$ from equation (6)] we place

$$D_0 = p_0' - p_0 = \tfrac{1}{2}\,\rho\, V_0^2$$

and

$$p - p_0 = \frac{2\,\xi^2 + \xi^4}{(1 + \xi^2)^2} \cdot D_0$$

We easily see that all these formulæ depend on only two constants or parameters, namely the altitude h and the maximum velocity V_0. We can change the last parameter and consider the depression D_0 as the second parameter. Thus the function of U shows that the horizontal velocity has a maximum value U_0 for $\xi = \sqrt{\frac{1}{2}}$; the distance r_0 from point A to the point where U has its maximum value, is

$$r_0 = h\sqrt{\tfrac{1}{2}} \text{ and } U_0 = V_0\sqrt{\tfrac{4}{27}}$$

The gradient G has its maximum value G_m for

$$\xi = \sqrt{\tfrac{1}{5}}$$

whence

$$G_m = \frac{\rho}{\mu} \cdot \frac{25\sqrt{5}}{216} \cdot \frac{2\,V_0^2}{h}$$

We shall now choose D_0, expressed in millimeters of height of mercury, and r_0 expressed in degrees of the meridian, as the parameters of the system and are thus able to establish the following formulæ, by introducing a mean value of ρ (0.1318 at the temperature 0° and the pressure of 760mm and for dry air):

The maximum horizontal velocity	$= U_0 = \sqrt{30.6\ D_0}$
The maximum horizontal gradient	$= G_m = 0.715\ D_0/r_0$
The distance from G_m to the point A	$= r_m = 0.63\ r_0$
The height of the absolute center O	$= h = 1.41\ r_0$
The absolute maximum velocity	$= V_0 = 2.6\ U_0$

By the aid of the preceding formulæ we have calculated the following table, in which D denotes the barometric difference:

ξ	0.5	1	2	3	4
$r : r_0$	0.71	1.41	2.83	4.24	5.66
$U : U_0$	0.93	0.92	0.46	0.25	0.15
$G : G_m$	0.99	0.48	0.06	0.01	0.003
$D : D_0$	0.36	0.75	0.96	0.99	0.9965

In fig. 23 we have constructed, from this table, the curve of velocity, the curve of gradient and the curve of pressure that determines the system of isobars. We can compare our system of wind to the lower half of a cyclonic system in nature; probably in nature the maximum gradient occurs at the same point as the maximum velocity. The depression D_0 which depends on the physical state of the air, determines the maximum velocity; the maximum gradient depends on the depression and the distance r_0 which in nature probably represents the radius of the vertical current. The radius r_0 depends on the height of the vertical current. Finally, it is necessary to remark that in our example the vertical velocity is very great; the velocity V_0 at the point A represents the vertical velocity at this point. But on the other hand, in natural systems of wind the motion of the air differs much from this motion in our case, because the surface of the earth compels the particles of air to follow trajectories of a different form.

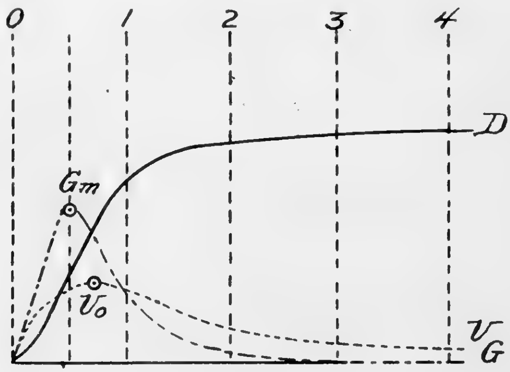


FIG. 23

Second example. The trajectories are parallel to a vertical plane and pass through a horizontal line.

We take the plane $X\ Z$ (see fig. 22) parallel to the trajectories and the axis $O\ Y$ as the horizontal line. The ordinate y disappears and we write

$$u = U, x = r \text{ and } R^2 = x^2 + z^2$$

We write the equations of the trajectories under the form

$$\frac{x}{x_0} = \frac{z}{z_0} = \frac{R}{R_0} = \sqrt{1 + \frac{2\,a\,t}{R_0^2}} \quad \ldots\ldots \quad (8)$$

Placing $t = 0$, we have $R = R_0$; consequently the particles of air are found at first at the surface of a cylinder whose radius is R_0. Differentiating with respect to t and eliminating the constants we shall have

$$\frac{u}{x} = \frac{w}{z} = \frac{V}{R} = \frac{a}{R^2} \quad \ldots\ldots \quad (9)$$

For $a < 0$, the air flows towards the axis and for $a > 0$, the air flows away from the axis. The gradient G in a horizontal plane is found by the formula

$$G = \frac{1}{\mu} \cdot \frac{dp}{dx} = -\frac{\rho}{\mu} \cdot \frac{V\,dV}{dx} = \frac{\rho}{\mu} \cdot \frac{a^2 x}{R^4} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Consider a horizontal plane at the distance $z = z_0 = -h$, and study the phenomena on this plane.

Writing

$$V_0 = \frac{a}{h} \text{ and } \xi = \frac{x}{h} \text{ and } D_0 = \tfrac{1}{2}\rho V_0^2$$

we shall have

$$u = \frac{\xi}{1+\xi^2} \cdot V_0 \,;\, G = \frac{\rho}{\mu}\,\frac{V_0^2}{h} \cdot \frac{\xi}{(1+\xi^2)^2}$$

$$p - p_0 = \tfrac{1}{2}\rho\,(V_0^2 - V^2) = \frac{\xi^2}{1+\xi^2} \cdot D_0$$

The horizontal velocity has a maximum u_0 when $\xi = 1$ and $u_0 = \frac{1}{2} V_0$. The horizontal gradient has a maximum

$$G_m \text{ when } \xi = \sqrt{\tfrac{1}{3}}$$

and

$$G_m = \frac{3\sqrt{3}}{16}\,\frac{\rho}{\mu}\,\frac{V_0^2}{h}$$

Choosing as the parameters D_0 expressed in millimeters and the distance of the axis $x_0 = h$, where the horizontal velocity has its maximum, as expressed in degrees of a great circle, we shall find

The maximum horizontal velocity	$U_0 = \sqrt{51.5\,D_0}$
The maximum horizontal gradient	$G_m = 0.65\,\frac{D_0}{x_0}$
The distance from G_m to the vertical axis	$x^m = 0.58x_0$
The height of the horizontal axis	$h = x_0$
The absolute maximum velocity	$V_0 = 2\,U_0$

By the aid of the preceding formulæ we have calculated the following table in which D denotes the barometric difference:

ξ	0.5	1	2	3	4
$U : U_0$	0.80	1.	0.80	0.60	0.47
$G : G_m$	0.99	0.77	0.25	0.09	0.04
$D : D_0$	0.20	0.50	0.80	0.90	0.94

From this table one can construct the curves of velocity, gradient, and pressure, and the system of isobars and we shall find a system of curves analogous to those of fig. 23.

We can compare this last wind system with the lower half of a system of parallels of the first order in nature and we can make the same remarks as on the first example.

§25. *System of parallel winds of the second order*

Mathematically speaking the systems of parallels have an infinite length. In nature the length is limited, but we can neglect the disturbances produced by the lateral limits. Along the surface of the earth the system of parallel ascending winds presents two horizontal currents, which flow from both sides toward the barometric minimum or trough situated along a straight line. We distinguish two halves on each side of the barometric minimum and each half has its internal part whose breadth may be r_0, and its exterior part. The horizontal current moves in the exterior part approximately at a constant altitude, and in the lower part at an increasing altitude. Consequently the horizontal velocity has its maximum value U_0 at the distance r_0 from the barometric minimum.

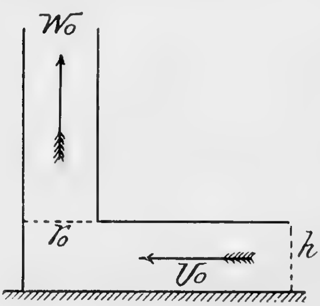


FIG. 24

Denoting the height of the external current by h (see fig. 24) and the angle between the maximum velocity and the gradient by ψ_0, the quantity of air which enters per unit of length is represented by $U_0 \cos \psi_0 h$. In the interior the current changes little by little into a vertical current whose velocity we may indicate by w_0 and consequently we have the condition

$$w_0 r_0 = U_0 \cos \psi_0 h \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

It is probable that in nature the ratio h / r_0 is so small that we can neglect the vertical velocity and the vertical barometric depression that results from it. We shall therefore consider only the horizontal currents with either constant or variable velocity.

Constant Latitude

We have already in §10 discussed the systems of parallel winds with rectilinear isobars and constant velocity. We now write

$$U \cos \psi = c_0 + c\,x \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (2)$$

in which the distance x is measured along the gradient.

Differentiating this equation and introducing the value of U and of $d\,U$ in place of v and $d\,v$ in equations (2) and (3) of §10, we shall have

$$\frac{\mu}{\rho} G \cos \psi = U\left(k + c + U \sin \psi \frac{d\psi}{dx}\right) \quad . \; . \; . \; . \; (3)$$

$$\frac{\mu}{\rho} G \sin \psi = U\left(2\,\omega \sin \theta - U \cos \psi \frac{d\psi}{dx}\right) \quad . \; . \; . \; (4)$$

If we eliminate G from these equations, we shall have the equation

$$0 = (k + c) \sin \psi - 2\,\omega \sin \theta \cos \psi + U \frac{d\psi}{dx}$$

in place of which we can write

$$\frac{U}{\cos \psi} \cdot \frac{d\psi}{dx} = U \cos \psi \frac{d\,(\text{tang}\,\psi)}{dx} = 2\,\omega \sin \theta - (k + c) \tan \psi \quad (5)$$

We see that we can satisfy this equation by placing the last term equal to zero. Then we have

$$\text{tang}\,\psi = \frac{2\,\omega \sin \theta}{k + c}$$

The angle of inclination ψ becomes constant, and the first term of equation (5) also becomes zero. Equations (3) and (4) become

$$\left.\begin{aligned} \frac{\mu}{\rho} G \cos \psi &= (k + c)\; U \\ \frac{\mu}{\rho} G \sin \psi &= 2\,\omega \sin \theta . U \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \; . \; . \; (6)$$

The normal angle of inclination being expressed by the formula

$$\text{tang } \alpha = \frac{2\, \omega \sin \theta}{k}$$

we shall have

$$\text{tang } \psi = \frac{2\, \omega \sin \theta}{k + c} = \frac{\text{tang } \alpha}{1 + \frac{c}{k}} \quad \ldots\ldots\ldots (7)$$

and

$$\frac{U}{G} = \frac{\mu}{\rho} \frac{\cos \psi}{k + c} = \frac{\mu}{\rho} \frac{\sin \psi}{2\, \omega \sin \theta} \quad \ldots\ldots\ldots (8)$$

From these equations we conclude: That *the angle of inclination Ψ is constant for a wind of variable velocity and rectilinear isobars, but that it differs from the normal angle α.* The ratio between the velocity and the gradient remains constant and is expressed by the same function of the latitude and of the angle of inclination as for winds of constant velocity.

The gradient increases proportionally to the velocity and consequently to the distance x. It follows that the depression between two isobars is found by multiplying the distance by the mean of the corresponding gradient.

When $c > 0$ the wind blows with increasing velocity and the angle of inclination is less than the normal angle. When $c < 0$ the wind blows with decreasing velocity and the angle of inclination is greater than the normal angle.

If we consider a station situated on the seashore and note that the coefficient of friction is greater on the land than on the ocean, we must expect that the ocean winds at such a station will have an angle of inclination greater than the inclination for the land winds.

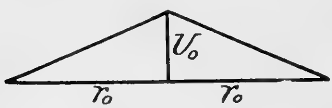


FIG. 25

Let us consider a system of parallels in which we can represent the curve of velocities approximately by two straight lines (see fig. 25). The curve of the gradient will be also represented by two straight lines and placing the maximum gradient equal to G_0 we shall have

$$D_0 = G_0\, r_0 \quad \ldots\ldots\ldots\ldots (9)$$

In nature the velocity is represented by a curve, and at the point where $U = U_0$, the variation of the velocity is zero. Consequently the angle of inclination is equal to the normal angle α for the maximum velocity. Choosing D_0 and r_0 as the parameters of the system we shall have

$$\left.\begin{aligned} &\text{The maximum gradient } G_0 = \frac{D_0}{r_0} \\ &\text{The maximum velocity } U_0 = \frac{\mu}{\rho}\,\frac{\cos\alpha}{k}\cdot\frac{D_0}{r_0} \end{aligned}\right\} \quad . \; . \quad (10)$$

In the neighborhood of the equator with $\rho = 0.1199$, $\alpha = 0$ and $k = 0.00002$, we have $U_0 = 51\,\frac{D_0}{r_0}$; assuming that the system of parallels has a breadth of $2r_0 = 20°$ and that the total depression is 2^{mm} we shall find $G_0 = 0.2^{mm}$ and $U_0 = 10$ meters.

Variable Latitude

We employ the same notation as in §11, and we consider only the case in which the gradient coincides with the meridian. Take the origin at the equator, and write the latitude

$$\theta = \lambda x$$

where

$$\lambda = \pm\frac{9}{10^6}\cdot\frac{\pi}{180}$$

Here the sign plus indicates that the gradient is directed toward the north and the sign minus that the gradient is directed toward the south. Assuming that the velocity is expressed by equation (2), we again find equations (3) and (4).

Eliminating G from these equations we shall have

$$U \cos\psi \frac{d\,(\text{tang}\,\psi)}{d\,x} = 2\,\omega \sin\theta - (k + c)\,\text{tang}\,\psi \quad . \; . \quad (11)$$

Introducing θ in place of $\sin\theta$, we see that equation (11) is satisfied by

$$\text{tang}\,\psi = \frac{2\,\omega}{k + 2\,c}(\theta - \varepsilon) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

$$\varepsilon = \frac{\lambda\,c_0}{k + c} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

For, by differentiating (12) we have

$$\frac{d\,(\text{tang}\,\psi)}{dx} = \frac{2\,\omega}{k + 2\,c} \cdot \frac{d\,\theta}{dx} = \frac{2\,\omega\,\lambda}{k + 2\,c}.$$

Substituting this value and the value of $U \cos \psi$ from equation (2) we have

$$(c_0 + c\,x)\,\frac{2\,\omega\,\lambda}{k + 2\,c} = 2\,\omega\theta - (k + c)\,\frac{2\,\omega}{k + 2\,c}\,(\theta - \varepsilon)$$

$$\lambda\,c_0 + \lambda\,c\,x = \lambda c_0 + c\,\theta = (k + 2\,c)\,\theta - (k + c) \cdot (\theta - \varepsilon)$$

$$\lambda\,c_0 = (k + c)\,\varepsilon$$

Whence

$$\varepsilon = \frac{\lambda\,c_0}{k + c}$$

By eliminating $\dfrac{d\psi}{dx}$ from equations (3) and (4) we find the gradient

$$G = \frac{\rho}{\mu} \cdot U \cos \psi\,(k + c + 2\,\omega\,\theta\,\text{tang}\,\psi) \quad . \quad . \quad . \quad . \quad (14)$$

The radius of curvature of the trajectory is found by the equation

$$R = \frac{ds}{d\psi} = \frac{dx}{\cos \psi\,d\,\psi}$$

From equation (12) we deduce

$$\frac{d\psi}{\cos^2 \psi\,d\,x} = \frac{2\,\omega}{k + 2\,c} \cdot \frac{d\,\theta}{dx} = \frac{2\,\omega\,\lambda}{k + 2\,c}$$

and we thus have

$$R = \frac{k + 2\,c}{2\,\omega} \cdot \frac{1}{\lambda\,\cos^3 \psi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

It is evident that at the point of maximum velocity c changes its sign, and that in nature at this point the equation $c = 0$ must be true.

In a system of parallels we have two horizontal currents, one along the surface of the earth and the other in the upper strata, For the last we can employ the same equations as for the first, but for want of observations the coefficient of friction remains unknown for the upper strata. Neglecting the vertical depression E, the sum of the two horizontal depressions, D_0 at the surface of the earth and D in the upper stratum, is equal to the difference of the weights of the columns of air [see §20, eq. (12)]. We can approximately calculate this difference by equation (4) of § 17. To fix our ideas we assume that the air at the point A has the virtual temperature $T_0 = 298°$, and the coefficient $m = 6$, if there is an ascending current. At the point B the calm air has the virtual temperature $T_0' = 294°$ and the coefficient $m' = 7$. If we assume that the air moves from B to A and there ascends, we shall find by the formula (6) of §16, that the ascending current extends up to a height of 4918^{m} and that the difference of weight of the columns of air is 3.1^{mm}. Assume $D_0 = 2^{mm}$ and $D = 1.1^{mm}$. If the extent of the system of wind $B\ A$ is 20° we shall find by the formula (10), $U_0 = 10^{m}$. The time the current requires to move from B to A is expressed in hours.

$$\frac{10^6}{9} \cdot \frac{20}{\frac{1}{2}\, U_0} \cdot \frac{1}{3600} = 123.3 \text{ hours}$$

If now the air from B can in 123 hours attain the physical state belonging to the air at A, then this system of wind is realized and, as we see, the parameters of the system are determined by the physical state of the air and of the surface of the earth. If we assume the distance $B\ A = 16°$, we shall have $U_0 = 12.5^{m}$ and the time equals 103 hours, but in this case the air from B will arrive at A with temperature lower than the temperature at A, and consequently the depression will diminish and the system of wind cannot be permanent.

§26. *Cyclonic system of the second order*

We have already in § 12 and § 13 studied the cyclones of the second order in respect to the motion along the surface of the earth. We have assumed that the horizontal current has a constant height h in the exterior portions and that the horizontal velocity increases in this part toward the center and attains its maximum value U_0 at the distance r_0 from the center of the isobars. Then the current enters into the interior portion, where its velocity decreases at the same time that the motion is changed little by little into a vertical motion. Let the mean vertical velocity be w_0 and the angle between the gradient and the maximum velocity be ψ_0; the condition that the same mass of air passes from the horizontal current to the vertical current is expressed by the formula

$$\pi r_0^2 w_0 = 2 \pi r_0 h U_0 \cos \psi_0$$

whence

$$w_0 = \frac{2h}{r_0} U_0 \cos \psi_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In cyclones of the second order the ratio h/r_0 is probably so small that we can neglect the vertical velocity and the vertical depression E which belongs to the accelerated motion of the vertical current. The rotation of the cyclonic motion is determined by the deflecting force T (fig. 26) of the rotation of the earth. We have assumed that the cyclonic system has a barometric minimum at the surface of the earth and a barometric maximum in the upper strata. It follows that the rotation in the upper strata is opposed to that at the surface of the earth. The *intermediate vertical current* which joins the two horizontal currents is consequently *rectilinear*. The phenomena are inverse to those of the anti-cyclones. However, the little that we know about the motions of the cirrus clouds seems to indicate that the axes of rotation of the lower current and of the upper current do not lie in the same vertical line.

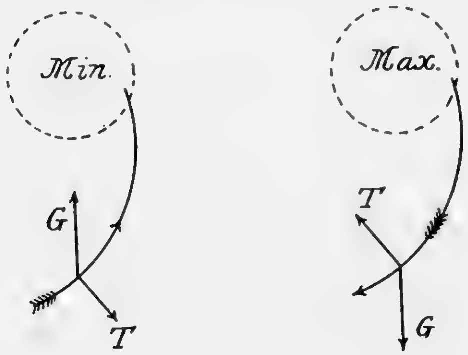


FIG. 26

As parameters of the cyclonic system we can choose the depression D_0 and the radius r_0 which depend on the physical state of the air. We can approximately establish the following relations between the maximum velocity U_0 and the gradient G_0. Assume that the gradient curve (fig. 27) is composed of a straight line and of a curve whose equation is

FIG. 27

$$G = \frac{a}{r} + \frac{a'}{r^3}$$

From §12 we have

$$a = \frac{\rho}{\mu} U_0 r_0 \frac{k}{\cos \alpha}$$

and

$$a' = \frac{\rho}{\mu} U_0^2 r_0^2 \frac{9}{10^6}$$

In the interior portion the depression is equal to

$$\int_{r=0}^{r=r_0} dp = \int_{r=0}^{r=r_0} \frac{G_0}{r_0} r\,dr = \tfrac{1}{2} G_0 r_0$$

and in the exterior portion, the depression is equal to

$$\int_{r=r_0}^{r=r} dp = \int_{r=r_0}^{r=r} \left(\frac{a}{r} dr + \frac{a'}{r^3} dr \right) =$$

$$= a \log.\text{nat}.\left(\frac{r}{r_0}\right) + \tfrac{1}{2} \cdot \frac{a'}{r_0^2} \left(1 - \left(\frac{r_0}{r}\right)^2\right)$$

Neglecting the term $\left(\frac{r_0}{r}\right)^2$ and noticing that

$$G_0 = \frac{a}{r_0} + \frac{a'}{r_0^3}$$

we get the total depression between the center and the point whose radius is r,

$$D_0 = a\left(\tfrac{1}{2} + \log.\text{nat}.\left(\frac{r}{r_0}\right)\right) + \frac{a'}{r_0^2}$$

If the radius r denotes the radius of action of the system, we can determine it by giving to U in the equation

$$\frac{r}{r_0} = \frac{U_0}{U}$$

a conventionally slight value. In violent cyclones we assume $U = 5$ meters per second. It seems that the ratio r/r_0 falls between 2 and 10.

The time that a particle of air requires to pass through the exterior portion, that is to say, to go from a point at the distance r to a point at the distance r_0, is found by the formula

$$t = \int_{r}^{r_0} \frac{d\,s}{v} = \int_{r_0}^{r} \frac{d\,r}{U \cos\psi_0} = \frac{1}{U_0\, r_0 \cos\psi_0} \int_{r_0}^{r} r\, d\,r =$$

$$= \frac{r_0^2}{2\, U_0 \cos\psi_0\, r_0} \left(\left(\frac{r}{r_0}\right)^2 - 1 \right).$$

By the aid of the preceding formulæ we have calculated the following tables. In the sixth column h is expressed in kilometers and w_0 in meters. We compute the value of U_0 by successive trials.

Cyclone of the temperate zone

$\theta = 60°$ $K = 0.00004$ $\psi_0 = 72.4°$

D_0	r_0	U_0	G_0	r	$w_0 : h$	t
10mm	1°	21.0m	7.0mm	4.2°	0.11m	40 *hours*
20	1	31.3	14.0	6.3	0.17	62
30	1	39.5	21.0	7.9	0.21	79
10	2	15.8	3.5	6.3	0.04	58
20	2	24.4	6.3	9.8	0.07	96
30	2	31.2	9.1	12.5	0.08	124
40	2	36.8	11.9	14.7	0.10	148
10	4	11.0	2.0	8.8	0.015	71
20	4	17.2	3.2	13.8	0.023	129
30	4	22.2	4.4	17.8	0.030	172
40	4	26.7	5.5	21.4	0.036	211
50	4	30.8	6.7	24.6	0.042	245

Cyclone of the temperate zone

$\theta = 30°$ $K = 0.00002$ $\psi_0 = 74.7°$

D_0	r_0	U_0	G_0	r	$w_0 : h$	t
10mm	0.1°	31m	96mm	0.6°	1.5m	7 *hours*
15	0.1	38	144	0.8	1.8	9
20	0.1	44.5	193	0.9	2.1	10
25	0.1	50	245	1.0	2.4	12
10	0.4	29	22	2.3	0.34	26
15	0.4	35.5	33	2.8	0.42	32
20	0.4	41	44	3.3	0.49	38
25	0.4	46.5	56	3.7	0.55	43
30	0.4	51	67	4.1	0.61	47

CHAPTER VII

VARIABLE SYSTEMS OF WIND

§27. *Variation of pressure in a stationary system of wind*

In permanent systems of wind the air that flows inward and produces the vertical current, is always homogeneous. In variable systems of wind the flowing air is heterogeneous and consequently the motion of the system varies with the time. The intensity of the systems of wind depends especially on the vertical current and we call the air, which enters into a system of winds and has such a physical state that it can produce or sustain a vertical current either ascending or descending, the *alimentary air.* We call the air which enters a system of wind, but which cannot sustain a vertical current the *supplementary* air. Let us consider a vertical ascending current whose height is h and in the first place suppose the motion to be permanent. Denoting the vertical velocities by w and w_1 (see fig. 28) and the densities by ρ and ρ_1, at the influx and the efflux of the current respectively, the equation of continuity assumes the form

FIG. 28

$$0 = \rho\, w - \rho_1\, w_1$$

Suppose that after a certain time the inflowing air acquires the density ρ' and the temperature τ' and that the vertical velocities remain unaltered in the first moments, the equation of continuity takes the form

$$d\,(\rho h) = h d\rho = (\rho'\, w - \rho_1\, w_1)\, dt$$

Eliminating $\rho_1\, w_1$, we find

$$\frac{d\rho}{dt} = \frac{w}{h}\,(\rho' - \rho) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

The change of density produces a change of pressure and assuming approximately

$$\frac{d\rho}{\rho} = \frac{dp}{p}\,;\ \frac{\rho'}{\rho} = \frac{273 + \tau}{273 + \tau'}\,;\ \frac{p}{\rho} = a\,(273 + \tau')$$

we shall have

$$\frac{1}{\rho} \cdot \frac{dp}{dt} = a\,\frac{w}{h}\,(\tau - \tau') \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

We conclude from equation (2) that *the pressure diminishes when the incoming air is warmer, and the pressure increases when the incoming air is colder.* Applying this result to nature we infer that the supplementary air is colder than the alimentary air.

Denoting by δ the change of the pressure per hour and in millimeters and expressing h in kilometers we shall have

$$\delta = \frac{3600.760}{10333} \cdot \frac{a\rho}{1000} \cdot \frac{w}{h} (\tau - \tau')$$

and for an average value of ρ (0.1318) we have

$$\delta = 10\frac{w}{h} (\tau - \tau') \quad \ldots\ldots\ldots\ldots \quad (3)$$

Let us consider a stationary cyclone whose pressure at the center varies; δ represents the variation of the horizontal depression D_0. In order to introduce the relation given by equation (1) we notice that this can be written

$$\frac{d\rho}{\delta t} = \frac{w}{dz} (\rho' - \rho)$$

In passing from these infinitesimal values of altitude to the finite differences, it is necessary to consider the whole height h of the horizontal current, because in the latter we do not know the variations of velocity with the height and when w expresses the vertical velocity in the ascending current at the height h, that is to say, at the level where the motion commences to be purely ascensional, we can introduce the relation given by equation (1) of §26, and expressing r_0 in degrees of a great circle we shall have

$$\delta = 0.18 \frac{U_0 \cos \psi_0}{r_0} (\tau - \tau') \quad \ldots\ldots\ldots \quad (4)$$

By the aid of equation (3) we can easily calculate the variation of the pressure in the cyclones given in the tables of §26.

Equation (2) applies only for the first few moments. If the vertical current is continually being supplied by heterogeneous air, the change of pressure must depend also on the humidity of the air. According to § 5 moist air has during ascension a mean temperature higher than dry air. If we consider τ and τ' as approximately mean temperatures, we arrive at the conclusion that the supplementary air is colder and dryer than the alimentary air. If then the air flowing into a stationary cyclone changes its physical state and becomes colder and dryer, the horizontal barometric depression diminishes little by little and the cyclone is destroyed after a certain time.

§28. *Instantaneous systems of wind*

Let us consider a column of air of the height l that has been heated so that the pressure p at the upper end (see fig. 29) exceeds the pressure p' of the surrounding air. The air commences to leave the upper end of the column and at the same time air enters at the lower end, but the density of the supplementary air filling the column up to the height z has a value ρ different from the value ρ' of the air of the calm atmosphere and consequently the weight of the column diminishes so that the pressure p_0 at the surface of the earth decreases and produces a depression $p_0' - p_0$. The pressure p_0 diminishes at the same time that the vertical velocity w of the current increases up to a limit that corresponds to the maximum value of the vertical velocity, and after this moment the steady motion goes on. As an approximation we can neglect the variation of density due to gravity and consider the force that maintains the ascending motion as equal to $\frac{p_0 - p}{\rho l}$. The equation of motion assumes the form

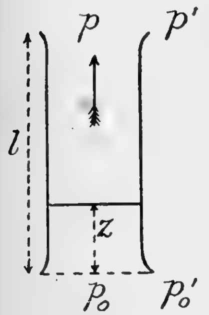


FIG. 29

$$\frac{dw}{dt} = \frac{p_0 - p}{pl} - g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The difference $p_0 - p$ is equal to the weight of the column of air z having the density ρ and of the column $l - z$ having the density ρ'; consequently we have

$$p_0 - p = g \, \rho \, z + g \, \rho' \, (l - z)$$

Introducing this value in eq. (1) we have

$$\frac{dw}{dt} = g \cdot \frac{(\rho' - \rho)}{\rho} \cdot \frac{l - z}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From equation (1) we conclude that the vertical velocity increases up to the moment when the pressure has attained the value

$$p_0 = p + g \, \rho \, l$$

At this moment the column is filled with air of the density ρ and $z = l$ and the motion is steady. Assuming approximately $p' = p$ we shall find

$$p'_0 = p + g\,\rho'\,l$$

and consequently

$$p'_0 - p_0 = g\,l\,(\rho' - \rho) \qquad (3)$$

Introducing $w = \frac{dz}{dt}$ equation (2) will by integration give

$$z = l\left(1 - \cos\frac{w_0}{l}\,t\right) \qquad (4)$$

where

$$w_0 = \sqrt{\frac{p_0' - p_0}{\rho}} \qquad (5)$$

is the maximum velocity.

The duration of the current up to the moment when the steady motion commences, may be t_0 and we shall have

$$t_0 = \frac{\pi\,l}{2\,w_0} \qquad (6)$$

Denoting by τ and τ' the mean temperatures of the column of the current and of the column of the calm atmosphere, respectively, we can assume approximately

$$\frac{\rho'}{\rho} = \frac{273 + \tau}{273 + \tau'}$$

Equation (5) is now written in the form

$$w_0 = \sqrt{\frac{g\,l\,(\tau - \tau')}{273 + \tau'}} \qquad (7)$$

Let

$$l = 1000^{\mathrm{m}},$$
$$\tau - \tau' = 6^\circ,$$
$$273 + \tau' = 290^\circ;$$

we shall find

$w_0 = 14.2^{\mathrm{m}}$ and the duration $t_0 = 110$ sec.

The steady motion continues as long as the alimentary air remains unaltered. Suppose that after a certain interval of time t, the air flowing along the surface of the earth enters at the lower end of the column with the density ρ', then the column will little by little be filled with air at this density, at the same time that the velocity decreases to zero, and the pressure p_0 increases to p_0', and the motion ceases altogether.

The duration of the steady motion depends on the quantity of air that can supply the current. If for example the system can be regarded as a radial cyclone, then denoting by r_0 the radius of the vertical current, by r the radius of the alimentary air, by h its height, and by t_1 the duration we shall have

$$\pi\, r_0^2\, w_0\, t_1 = \pi\, r^2\, h$$

or

$$t_1 = \left(\frac{r}{r_0}\right)^2 \frac{h}{w_0} \quad \ldots\ldots\ldots\ldots \quad (8)$$

If, on the contrary, the alimentary air can be regarded as a stratum whose length is very great compared with the breadth, we can imagine that the system of wind consists of a series of instantaneous systems, such that the cyclone moves along the mean or central line of the alimentary stratum (tornadoes, hailstorms). Let the breadth be L and the velocity of propagation be W, we shall have

$$L\, h\, W = \pi\, r_0^2\, w_0$$

and consequently

$$W = \frac{\pi\, r_0^2\, w_0}{L h} \quad \ldots\ldots\ldots\ldots \quad (9)$$

The time t that the cyclone consumes in passing any point, is given by equation

$$t = \frac{2\, r_0}{W} \quad \ldots\ldots\ldots\ldots \quad (10)$$

Let

$$r_0 = 200^{\text{m}},$$
$$h = 100^{\text{m}},$$
$$L = 1200^{\text{m}},$$

we find

$$W = 14.9^{\text{m}} \text{ and}$$
$$t = 27 \text{ seconds.}$$

§29. *Ocean wind and land wind*

We consider the ocean winds and the land winds as variable systems of parallel winds of the second order. The ocean winds belong to an ascending system of parallels and the land winds to a descending system of parallels. During the day the land becomes much warmer than the sea and consequently the pressure p at the upper end of the column of air (see fig. 30) increases and exceeds the pressure p'; the air at the upper end leaves the column and at the same time the pressure p_0 diminishes, because the weight of the column diminishes, and thus produces a horizontal current which is the ocean wind. Approximately we can neglect the time necessary to fill the column of air from the ocean and we can consider the depression $p_0' - p_0$ as a function of the temperature.

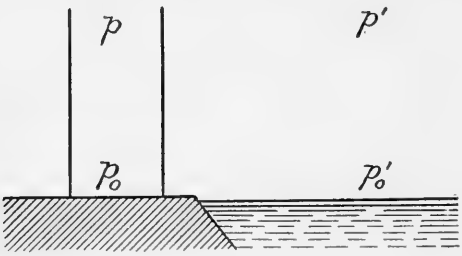


FIG. 30

During the night the land becomes much cooler than the ocean, the pressure p diminishes at the same time that the weight of the column of air increases and a descending system of parallels obtains with a depression $p_0 - p_0'$.

The barometric depression which depends on the unequal heating of the ocean and the land is a function of the time and of the place, and must be determined by observations. This depression produces a horizontal current which commences with a velocity equal to zero; the depression gradually increases, the velocity increases and the current extends more and more up to the moment when the depression attains its maximum value. Then the depression and the velocity of the current decrease simultaneously up to the moment when the current ceases.

Consider the horizontal current at any time and denote its maximum velocity which occurs near the coast, by U_0, its length along the gradient by x, and the depression in millimeters by D_0; it is evident that D_0 is a function of U_0, of x and of the time.

We approximately assume

$$\frac{10\,333}{760}\,D_0 = \rho\,U_0^2$$

and introducing a mean value of ρ we shall have

$$D_0 = \frac{U_0^2}{103} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If we suppose that the curve of the gradient can be represented by two straight lines and if we express x in degrees of a great circle we have

$$D_0 = \tfrac{1}{2} G_0 x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The gradient G_0 is determined for the velocity U_0 by the known formulæ

$$\frac{G_0}{U_0} = \frac{\rho}{\mu} \cdot \frac{k}{\cos \alpha}$$

$$\operatorname{tang} \alpha = \frac{2 \omega \sin \theta}{k}$$

consequently we have

$$x = \frac{2\, U_0 \cos \alpha}{k} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For example let

$$\theta = 30^\circ,$$
$$k = 0.00004,$$

we have

$$\alpha = 61.25^\circ$$
$$G_0 : U_0 = 0.09.$$

If

$$D_0 = 0.5^{\mathrm{mm}},$$

we shall have

$$U_0 = \text{about } 7^{\mathrm{m}},$$
$$G_0 = 0.63$$
$$x = 1.6^\circ$$

§30. *Movable systems of wind*

When the barometric minimum or maximum changes its position along the surface of the earth, the system of wind is called *movable*. The movement of the barometric minimum or maximum is accompanied by a movement of the ascending or descending vertical current, and the cause of this is due to the heterogeneity of the air that enters the barometric minimum either at the surface of the earth or in the upper strata. The alimentary air on entering, produces a new vertical current at the same time that the supplementary air suppresses the existing current, and consequently the vertical current moves in advance of the barometric minimum and causes its change of position. When the barometric minimum is situated in the upper strata its movement is accompanied by the movement of the barometric maximum at the surface of the earth, and inversely.

In any movable system of wind the pressure at any point whatever varies with the time and this variation of pressure is closely connected with the velocity of propagation of the barometric minimum or of the central calm region.

Let x and y be the coördinates of any point, whatever; ξ and η the coördinates of the movable origin which represents the barometric minimum; we can generally express the pressure as a function of the location and the time, or

$$p = f\,(x - \xi, \eta - y, t)$$

Differentiating we shall have

$$\frac{dp}{dt} = \frac{dp}{d\xi} \cdot \frac{d\xi}{dt} + \frac{dp}{d\eta} \cdot \frac{d\eta}{dp} + \left[\frac{dp}{dt}\right] \quad . \; . \; . \; . \; . \quad (1)$$

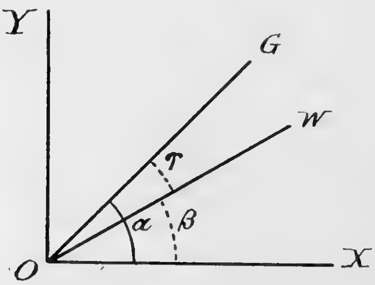


FIG. 31

Denoting the velocity of propagation the [movement of the minimum] by W and its angle with the axis $O\ X$ by β (see fig. 31) the gradient by G and the angle of the direction of the gradient with the axis of X by α, we have

$$W \cos \beta = \frac{d\xi}{dt}; \; W \sin \beta = \frac{d\eta}{dt};$$

$$\mu G \cos \alpha = -\frac{dp}{dx} = \frac{dp}{d\xi};$$

$$\mu G \sin \alpha = -\frac{dp}{dy} = \frac{dp}{d\eta}.$$

Substituting these values we shall have

$$\frac{dp}{dt} = \mu G W \cos (\alpha - \beta) + \left[\frac{dp}{dt}\right]$$

Denote the angle between G and W by γ and let δ be the total variation of the pressure at any point (expressed in millimeters per hour) and δ' the variation of the pressure, if the system is stationary, we have

$$\left.\begin{aligned} \frac{dp}{dt} &= \frac{10333}{760} \cdot \frac{1}{3600} \delta \\ \left[\frac{dp}{dt}\right] &= \frac{10333}{760} \cdot \frac{1}{3600} \delta' \end{aligned}\right\} \quad \ldots \ldots \quad (2)$$

Substituting these values we shall have

$$\delta = \delta_0 + 0.0324 \, G \, W \cos \gamma \ldots \ldots \quad (3)$$

If the pressure at the movable origin is invariable we have

$$\delta_0 = 0$$

and consequently

$$\delta = 0.0324 \, G \, W \cos \gamma \ldots \ldots \quad (4)$$

At the front of a cyclone the angle $\gamma > \pi$ and consequently δ is negative and the pressure decreases; at the rear $\gamma < \pi$ and the pressure increases.

Example. Let us consider a movable cyclone whose central pressure is constant and whose velocity of propagation is so small that we can consider the motion of the system as a geometrical movement of the isobars; finally we suppose that the radius of action is so great and the maximum velocity so slight that we can apply the same ratio between the gradient and the velocity as in rectilinear motion. At the station A (see fig. 32) we observe the velocity $U_1 = 12^{m}$ and the variation of pressure $\delta_1 = -0.5^{mm}$; at the station B we observe $U_2 = 8^{m}$ and $\delta_2 = -0.4^{mm}$. Assume the mean latitude equal to 60° and the coefficient of friction $k = 0.00006$, we shall have (see §9) the normal angle $\alpha = 64°.6$ and the normal ratio $G : U = 0.15$. From equation (4) by substituting $G_1 = 1.8^{mm}$ and $G_2 = 1.2^{mm}$, we shall find $W_1 \cos \gamma_1 = -8.6^{m}$

and $W_2 \cos \gamma_2 = 10.3^{m}$.

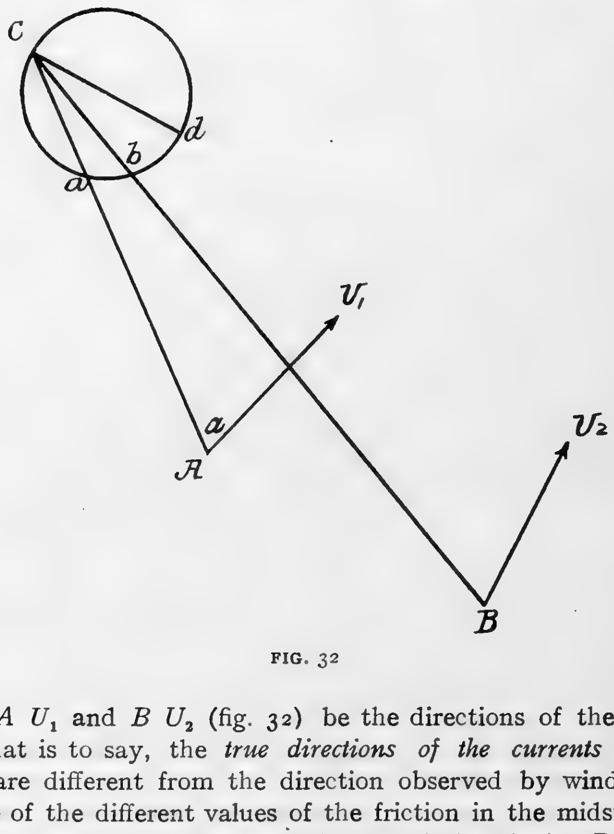


FIG. 32

Let $A\ U_1$ and $B\ U_2$ (fig. 32) be the directions of the velocities, that is to say, the *true directions of the currents of air*, which are different from the direction observed by wind vanes, because of the different values of the friction in the midst of the current of air and at the surface of the earth (see §34). Draw the angles $U_1\ A\ C = U_2\ B\ C = \alpha$ then the point of intersection C is the movable origin or the location of the barometric minimum. Lay off $C\ a = W \cos \gamma_1$ and $C\ b = W \cos \gamma_2$ and construct a circle through the three points a, b and C then the diameter $C\ d$ represents the velocity of propagation W both in direction and in extent.

§31. *Velocity of propagation of a cyclone*

As we shall now explain, the movement of the barometric minmum is due to the heterogeneity of the air. At the front of a cyclone the alimentary air whose temperature is τ_1 enters and produces a lowering of the pressure that we denote by δ_1. At the rear of the cyclone the supplementary air (see §24) whose temperature is τ_2, enters and produces an increase of pressure whose value is δ_2. The air of the central part of the cyclone has the temperature τ and we have $\tau_1 > \tau > \tau_2$. In accord with equation (4) of §27 we write

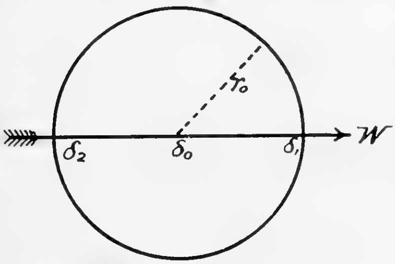


FIG. 33

$$\delta_1 = 0.18 \frac{U_0 \cos \psi_0}{r_0} \left(\tau - \tau_1 \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . (1)$$

$$\delta_2 = 0.18 \frac{U_0 \cos \psi_0}{r_0} \left(\tau - \tau_2 \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . (2)$$

Designating the variation of the pressure at the center by δ_2 and assuming that this variation occurs at all points, we can substitute successively $\gamma = \pi$ and $\gamma = o$ in equation (3) of §30, then we shall have

$$\delta_1 = \delta_5 - 0.0324\, G_0\, W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . (3)$$

$$\delta_2 = \delta_0 + 0.0324\, G_0\, W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . (4)$$

Eliminating between these four equations we shall find

$$\delta_0 = 0.18 \frac{U_0 \cos \psi_0}{r_0} \left(\tau - \frac{\tau_1 + \tau_2}{2} \right) \quad . \quad . \quad . \quad . \quad . \quad . (5)$$

$$W = 2.78 \frac{U_0}{G_0} \cos \psi_0 . \frac{\tau_1 - \tau_2}{r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . (6)$$

If we have $\tau = \frac{1}{2}(\tau_1 + \tau_2)$, the pressure at the center remains constant; if τ is greater than the mean of τ_1 and τ_2, the pressure at the center increases; if τ is less than this mean, the pressure at the center diminishes.

For example assume

$$U_0 = 30^{\mathrm{m}},$$
$$\psi_0 = 72.4°,$$
$$r_0 = 4°,$$
$$G_0 = 6.5^{\mathrm{mm}},$$
$$\tau_1 - \tau_2 = 10°;$$

we have W about 10^{m}.

For

$$U_0 = 50^{\mathrm{mm}},$$
$$\psi_0 = 74.7°,$$
$$r_0 = 0°.1,$$
$$G_0 = 246^{\mathrm{mm}},$$
$$\tau_1 - \tau_2 = 2°$$

we have W about 3^{m}.

In cyclones of the temperate zones the radius r_0 is generally so great that we can approximately calculate the ratio $U_0 : G_0$ by the formulæ deduced for rectilinear isobars. In this case the quantity $2.78 \frac{U}{G_0} \cos \psi_0$ depends on the latitude θ and on the coefficient of friction k. Denoting this quantity by B we have

$$W = B \frac{\tau_1 - \tau_2}{r_0}$$

The value of B is given in the following table:

θ		$k=0.00002$	$k=0.00004$	$k=0.00006$	$k=0.00008$	$k=0.00010$
50°	B	4.0	7.3	9.6	10.9	11.5
60°	B	3.2	5.9	7.9	9.2	10.0

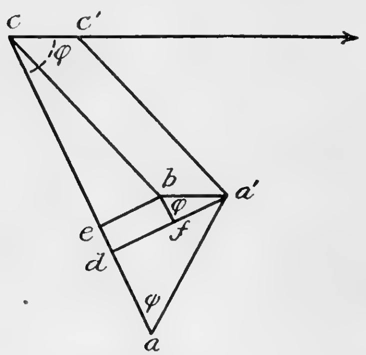


FIG. 34

As to the direction of the propagation of a barometric minimum that depends on the trajectory of the alimentary air. Let $c\,c'$ in fig. 34 be the direction of the velocity of propagation W; the barometric minimum moves from c to c' in an infinitely short time $d\,t$; at the same time a particle of alimentary air moves from a to a' with the velocity U. Draw $c\,b$ parallel to $c'\,a'$; $a'\,b$ parallel to $c\,c'$, $a'd$ and $b\,e$ perpendicular to $a\,c$ and $b\,f$ parallel to $a\,c$.

Make

$$c\,c' = d\,\sigma,\ a\,a' = d\,s,\ c\,a = r,\ c'\,a' = r + d\,r;$$
the angles $c'\,c\,a = \varphi$ and $c\,a\,a' = \psi$.

We have

$$\begin{aligned} a\,c\,b &= -\,d\,\varphi, \\ e\,a &= -\,d\,r \\ a\,d &= a\,e - b\,f \\ a'\,d &= b\,e + a'\,f \end{aligned}$$

By substituting the values of these quantities we shall find

$$\left.\begin{aligned} \cos\psi.\,d\,s &= -\,d\,r - \cos\varphi\,.d\,\sigma \\ \sin\psi.d\,s &= -\,r\,d\,\varphi + \sin\varphi.\,d\,\sigma \end{aligned}\right\} \quad \ldots\ldots\ldots (7)$$

From these equations by substituting

$$d\,s = Udt \text{ and } d\,\sigma = Wdt$$

we find

$$\frac{dr}{rd\varphi} = \frac{U\cos\psi + W\cos\varphi}{U\sin\psi - W\sin\varphi}$$

and consequently

$$U\sin\psi.d\,r - U\,r\cos\psi.\,d\,\varphi = W\,d\,(r\sin\varphi) \quad \ldots\ldots (8)$$

Supposing that $U\,r\cos\psi$ is constant and that the angle ψ is constant and equal to α as in permanent cyclones, then by integration and determining the arbitrary constants so that $\varphi = 0$ for $r = r_0$ we shall have

$$U_0\,r_0\cos\alpha\left[\text{tang}\,\alpha\,\text{log.nat.}\frac{r}{r_0} - \varphi\right] = W\,r\sin\varphi \quad . \quad (9)$$

By this equation we can determine the angle φ that alimentary air must describe in order to reach the interior limit of the cyclone.

The equations that we have developed apply also to the upper strata of an anti-cyclone where the barometric minimum occurs.

First example.

$$\frac{r}{r_0} = 6; \quad \varphi = 20°$$

By equation (9) we calculate the following values:

$\alpha =$	40°	50°	60°	70°
$\dfrac{W}{U_0} =$	0.44	0.57	0.68	0.77

This case is that of cyclones that move nearly parallel to the alimentary stratum and where the alimentary air describes a very small angle in order to reach the interior region.

In the northern hemisphere the wind deviates to the right and turns around the center against the sun and consequently the cy-

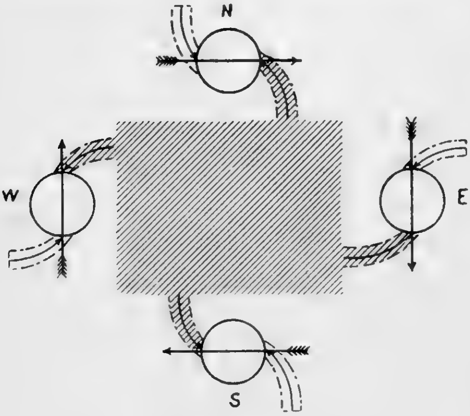


FIG. 35

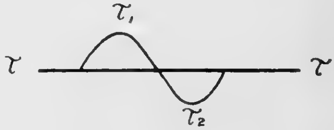


FIG. 36

clone moves around the alimentary stratum with the sun (see fig. 35). In the southern hemisphere the inverse phenomenon occurs (see fig. 37).

When the cyclone passes any point, the temperature increases at first, but during the passage of the center it lowers (see figs. 36 and 38).

Since in general the mean or normal isotherms do not deviate much from the direction of the parallels of latitude of the terrestrial globe, we must expect the cyclone to be formed on the south of the supplementary air and on the north of the alimentary air and also that it move in general from west to east.

Second example. Assume $\frac{r_0}{r} = 10$ and $\varphi = 200°$. By equation (9) we shall find

α	$= 40°$	$45°$	$50°$	$55°$
$\frac{W}{U_0}$	$= 0.35$	0.25	0.41	0.03

In this case the cyclone also moves nearly parallel to the alimentary stratum, but the alimentary air describes about half a revolution around the cyclone before reaching its interior region.

Therefore in the northern hemisphere the cyclone moves around the alimentary stratum against the sun and inversely in the southern hemisphere (see fig. 37). When the cyclone passes by any point, the temperature is lowered at first and then increases, and during the passage of the center it is lowered again to finally increase (see fig. 38).

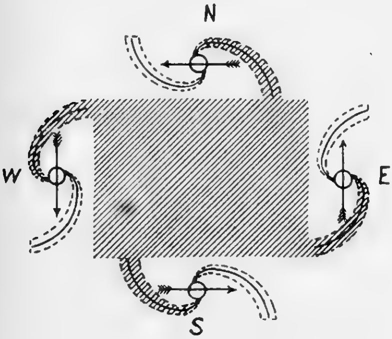


FIG. 37

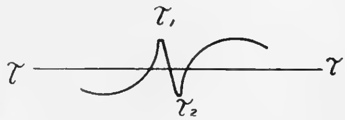


FIG. 38

The cyclones of the inter-tropical regions, at least certain of them described by the meteorologists of the East Indies, seem to belong to the last class. However, the thermometric and hygrometric observations in the cyclones of low latitudes are unfortunately still too rare for it to be possible to determine the position of the alimentary stratum and the extent of the arc traversed by the alimentary air before it commences to ascend in the anterior portion of the interior circle.

§32. *Isobars of a variable cyclone*

We shall distinguish three cases:

(1) *Stationary cyclone.*

The isobars of a stationary cyclone are concentric circles that change their size at the same time that the barometric minimum varies. Consequently, the curves of equal variation of pressure are also concentric circles. The variation of the pressure δ_0 is a function of the distance r. We can approximately determine this variation by calculating two cyclones whose parameters are different. For example, suppose that the radius r_0 be the same in each and that the maximum velocity U_0 diminishes during a certain time. By the formulæ of § 13 and § 14 we have calculated the following tables, assuming $\theta = 50°$ and $k = 0.00010$:

	FIRST CYCLONE			SECOND CYCLONE		
r	U	G	$b - b_0$	U	G	$b - b_0$
0°	0.0^m	0.00^{mm}	0.00^{mm}	0.0^m	0.00^{mm}	0.00^{mm}
2	6.0	0.97	0.97	5.6	0.90	0.90
4	12.0	1.93	3.87	11.2	1.78	3.59
6	18.0	2.90	8.70	16.8	2.69	8.06
7	20.0	3.30	11.80	19.0	3.17	11.00
8	18.7	3.16	15.03	17.7	3.00	14.10
10	15.0	2.55	20.78	14.2	2.41	19.52
12	12.5	2.07	25.38	11.8	1.96	23.86
14	10.7	1.75	29.19	10.1	1.65	27.45
16	9.4	1.52	32.44	8.9	1.43	30.52
18	8.3	1.34	35.29	7.9	1.26	33.21
20	7.5	1.20	37.87	7.1	1.13	35.60

Assume the radius of action of these cyclones to be about 20° and that at this distance the absolute pressure is 760^{mm}. The pressure b_0 at the center is then in the first cyclone 722.13^{mm} and in the second 724.40^{mm}, and the increase of the pressure at the center is 2.27^{mm} at the same time that the maximum velocity has diminished 1^m from 20.0 to 19.0. Adding b_0 to $b - b_0$ we shall find the pressure b and consequently we calculate the increase of pressure at each distance r. Assuming that the change has taken place in 4 hours, we find the hourly variation δ_0 by dividing the increments by 4 as follows;

r	δ_0	r	δ_0	r	δ_0	r	δ_0
0°	0.57^{mm}	6°	0.41^{mm}	10°	0.25^{mm}	16°	0.09^{mm}
2	0.55	7	0.37	12	0.19	18	0.05
4°	0.50	8	0.33	14	0.13	20	0.00

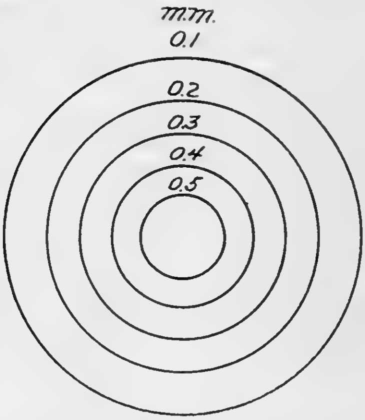


FIG. 39

By constructing a curve representing $\delta_0 = f\ (r)$ we easily determine the distance r for successive values of δ_0 and can construct the curve of equal variation (see fig. 39).

(2) *Moving cyclone with pressure constant at its center.*

When the pressure at the moving center remains constant, the variation of the pressure at any fixed station is determined by equation (4) of §30 and we have

$$\delta \doteq 0.0324\, G\, W \cos \gamma$$

Assume that W, the velocity of propagation of the center, is constant. For the value $\gamma = \pi/2$ we have $\delta = 0$, that is to say, *the curve of no variation is a straight line that passes through the center, and is perpendicular to the direction of propagation of the center.*

Assuming $\gamma = 0$ and $\gamma = \pi$, and also $G = G_0$ we obtain the maximum value of δ, which consequently falls at two points at the distance r_0 from the center along the trajectory of the cyclone.

The curves of equal variation are determined in general by the equation

$$G \cos \gamma = \text{constant.}$$

We can easily construct these curves by the aid of the curve of the gradient.

In the interior portion, we have the equation

$$G = G_1 r$$

and the curves of equal variation assume the form

$$r \cos \gamma = \text{constant.}$$

These curves are straight lines perpendicular to the direction of propagation.

By using the values of G given in the preceding table we have constructed, for every 0.2^{mm}, the curves of fig. 40, assuming $W = 10^m$.

If we wish to construct curves of equal variation of pressure for any date whatever, we can construct two systems of isobars appropriate to the given date, and then determine graphically the curves of equal variation. It is evident that by choosing two appropriate dates so far apart that the distance between the centers exceeds the diameter of action, $2\,r$, the curves of equal variation and the isobars themselves become identical and the maximum variations are the centers of the two systems of isobars.

(3) *Moving cyclone with variable pressure at the center.*

When the pressure at the center varies, the variation of pressure is determined by equation (3) of §30 and we have

$$\delta = \delta_0 + 0.0324\, G\, W \cos \gamma$$

The variation δ_0 which is a function of r, is determined as we have shown in the first case where the system is stationary. Assuming $W = 10^m$ and introducing the values of G and of δ_0 given in the preceding table for the first case, we shall find δ as follows:

r	$\gamma = 0°$	$= 30°$	$= 60°$	$= 90°$	$= 120°$	$= 150°$	$= 180°$
2	0.86^{mm}	0.82^{mm}	0.71^{mm}	0.55^{mm}	$+0.39^{mm}$	$+0.28^{mm}$	$+0.24^{mm}$
4	1.13	1.04	0.81	0.50	+0.19	−0.04	−0.12
6	1.35	1.22	0.88	0.41	−0.06	−0.40	−0.53
7	1.44	1.29	0.90	0.37	−0.16	−0.55	−0.70
8	1.35	1.22	0.84	0.33	−0.18	−0.56	−0.69
10	1.08	0.97	0.66	0.25	−0.16	−0.47	−0.58
12	0.85	0.77	0.53	0.19	−0.15	−0.39	−0.48
14	0.70	0.62	0.41	0.13	−0.15	−0.36	−0.44
16	0.58	0.52	0.34	0.09	−0.14	−0.34	−0.40
18	0.48	0.43	0.27	0.05	−0.17	−0.33	−0.38
20	0.39	0.34	0.19	0.00	−0.19	−0.34	−0.39

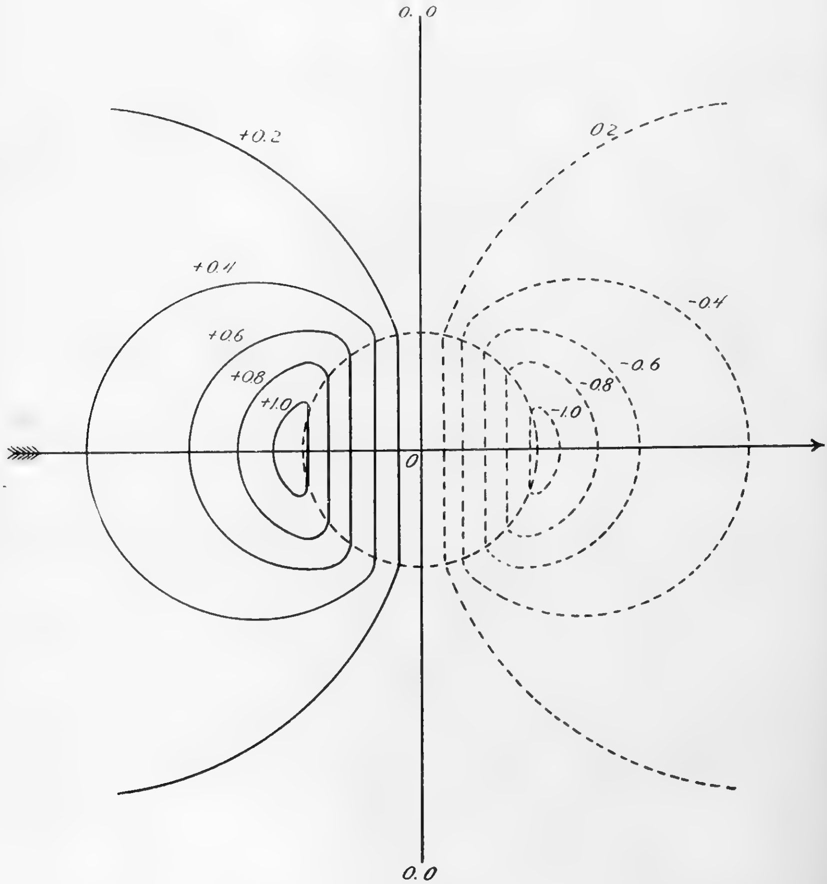


FIG. 40

By the aid of this table we have constructed for every 0.2^{mm} the curves of equal variation shown in fig. 41.

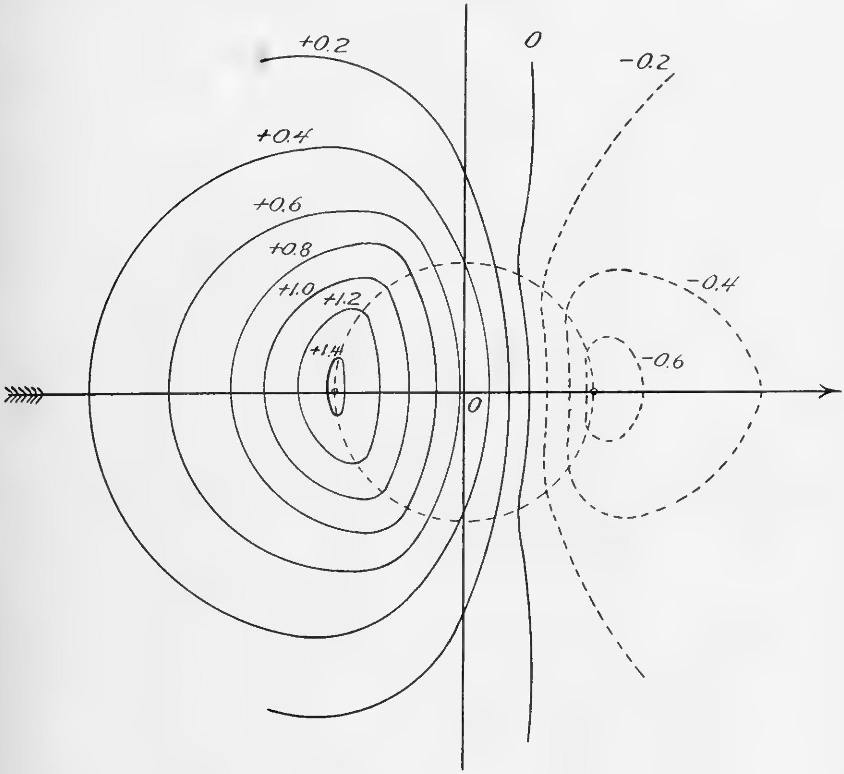


FIG. 41

We can also determine the curves of equal variation by constructing two systems of isobars for given dates.

§33. *Isotherms of systems of wind*

Permanent systems of wind demand a *uniform temperature* for the air that enters into the barometric minimum. Assuming that the temperature of the air varies with the pressure when nearest the surface of the earth, it is evident that a permanent cyclone must have circular isotherms around its center. Moreover, the system of isotherms must itself be permanent.

If we consider a variable and stationary cyclone, the isotherms must be circular around the barometric minimum but they can vary with the time, so that the curves of equal variation of temperature may be concentric circles around the center of the cyclone.

In the general case where the isotherms have at first any situation whatever, the system of wind is mobile, and the trajectory of the barometric minimum depends on the situation of the isotherms before the motion commences.

The isotherms at the surface of the earth belong to particles of air that move without vertical velocity. The trajectories of those particles of air that remain always at the surface of the earth assume the form

$$\left.\begin{aligned} x &= x_0 + f(t) \\ y &= y_0 + g(t) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

by taking the axis of X and of Y at the surface of the earth, and designating the time by t.

Let the equation of the isotherms for the time $t = 0$, be

$$F\ (x_0\ y_0) = 0 \quad \ldots\ldots\ldots\ldots \quad (2)$$

If we assume that the particles of air maintain their temperature during motion, we determine the equation of the isotherms at any moment by eliminating x_0 and y_0 between equations (1) and (2).

If, on the contrary, the temperature of a particle of air varies, either because the pressure changes its value or because the surface of the earth causes a heating or a cooling, we shall be obliged to consider its temperature as dependent on the time while the particle is moving. The problem will be very complicated but its solution can be effected approximately by the graphic method by constructing the trajectories of the particles of air and thus following up the variations of temperature due to the pressure and to the surface of the earth.

First example. Let the trajectories be straight lines parallel to $a\ b$ (fig. 42) and the velocity be constant. Their equations become

$$x = x_0 + at;\ y = y_0 + bt.$$

Assume that the isotherms for the initial time, $t = 0$ are straight lines parallel to the axis $O\ Y$; their equations become

$$x_0 = f\ (\tau)$$

Eliminating x_0 we shall have

$$x = f\ (\tau)\ +\ at$$

which equation represents a series of straight lines parallel to the axis $O\,Y$. We conclude therefore that any isotherm, as $m\,n$, moves parallel to this axis.

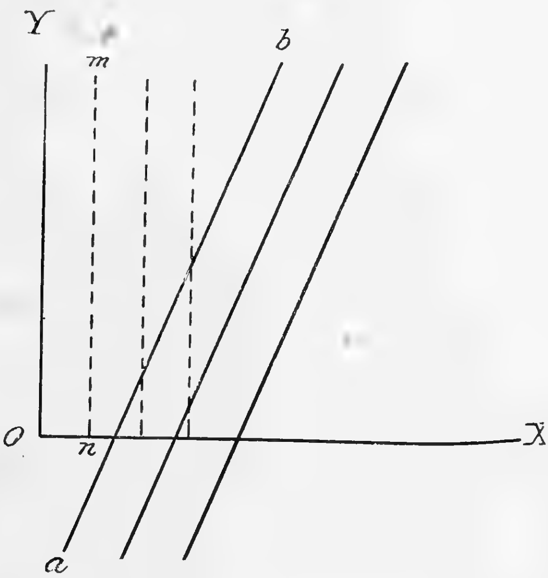


FIG. 42

Second example. Let the trajectories be logarithmic spirals represented by the equations

$$r^2 = r_0^2 - 2at$$

$$\varphi = \varphi_0 - \text{tang}\,\alpha\,\text{log.nat.}\,\frac{r}{r_0}$$

in which we have

$$r\frac{dr}{dt} = -a = r\,U\cos\psi$$

Assume that the isotherms for the initial time $t = 0$ be straight lines parallel to the axis $O\,X$. The equation of an isotherm $a\,b$ (fig. 43) assumes the form

$$r_0 \sin\varphi_0 = f\,(\tau)$$

Eliminating r_0 and φ_0 we shall find

$$\sin\left(\varphi + \tfrac{1}{2}\tan\alpha\,\text{log nat}\,\frac{r^2}{r^2 + 2\,at}\right) = \frac{f\,(\tau)}{\sqrt{r^2 + 2\,at}}$$

Let us consider the cyclone of §32, in which we have $\alpha = 48°$. The value of a is $U\,r \cos \alpha$, and by introducing hours and degrees of the great circle we shall have

$$a = 150 .\ \cos 48° . \frac{60 \times 60 \times 9}{10^6} = 3.25$$

For the isotherm ab we have $f(\tau) = 12°$ and we have calculated its position at the end of 2, 4 and 6 hours (see the dotted lines in fig. 43).

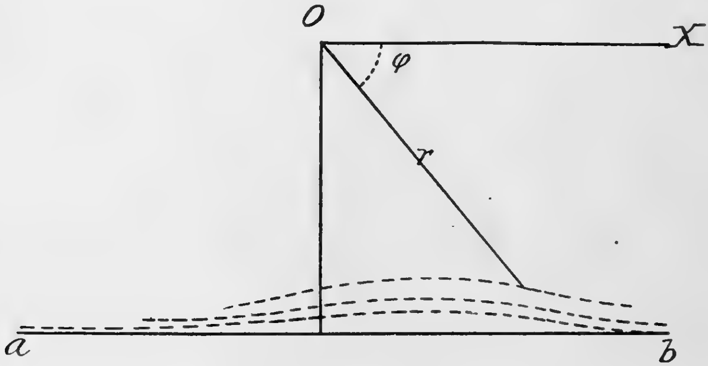


FIG. 43

Instead of determining the movement of the isotherms we can study the variation of temperature and constant curves of equal variation of r. In a short time the center of the cyclone passes from O' to O'' (fig. 44) and we will consider the mean position O. A particle of air describes the distance $a\,b = d\,s$ and we assume that it maintains its temperature constant. Then at the point b the temperature will be changed and the increase of the temperature $d\ \tau$ will be equal to the difference of the temperature between the isotherms which pass through a and through b.

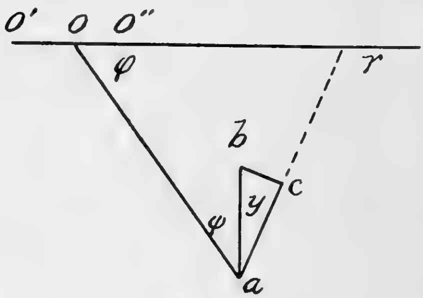


FIG. 44

Let us call the variation of the temperature per degree of a great circle measured perpendicularly to the isotherm in the direction toward which the temperature diminishes, *the thermometric gradient.*

Let $a\,c$ be the direction of the thermometric gradient J, and draw $b\,c$ perpendicular to $a\,c$: the temperature is the same at b and at c and the increase of temperature from a to b is $J\,\frac{ac}{1^\circ}$. Introducing $a\,c = d\,s \cos y$ and the value of 1° we have

$$d\tau = \frac{9}{10^6}\,J \cos y\,d\,s$$

Denoting the variation of the temperature per hour by i, we have

$$i = 3600\,\frac{d\tau}{dt}$$

Since

$$U = \frac{ds}{dt},$$

we find

$$i = 0.0324\;J\;U \cos y$$

The angle y depends on the angle γ between the thermometric gradient and the axis, and we have

$$\gamma = \varphi + \psi + y$$

Consequently we shall find

$$i = 0.0324\;J\;U \cos\left(\varphi + \psi - \gamma\right)\;.\;.\;.\;.\;.\;.\;.\quad (3)$$

By the aid of this equation we can construct curves of equal variation of temperature. Supposing γ to be constant we shall have for $i = 0$,

$$\varphi = \frac{\pi}{2} - \psi + \gamma = \text{constant.}$$

Hence, *the curve of no variation is a straight line which passes through the center of the cyclone.*

It is evident that there exists some relation between the velocity of propagation W and the thermometric gradient J. According to equation (6) of §31 we have

$$W = 2.78\,\frac{U_0}{G_0}\cos\psi_0\,\frac{\tau_1 - \tau_2}{r_0}$$

The mean value of the thermometric gradient (see fig. 20) is approximately:

$$J = \frac{\tau_1 - \tau_2}{2\, r_0} \quad \ldots\ldots\ldots\ldots \quad (4)$$

and consequently

$$W = 5.56\, \frac{U_0 \cos \psi_0}{G_0} \cdot J \quad \ldots\ldots\ldots \quad (5)$$

We have then in the examples following equation (6) of section 31

$$W : J = 2\, B.$$

Third example. Let the isotherms be straight parallel lines. Let us consider a cyclone and distinguish the interior region where we have $U = U_1 r$ and the exterior region where we have $U r = m$. Substituting these values in equation (3) we shall find for the interior region

$$r \cos\, (\varphi + \psi - \gamma) = \frac{i}{0.0324\ U_1\, J} = c$$

which represents (see fig. 45) a straight line $a\, b$ at the distance c from the origin, and the angle between the axis $O\, X$ and the perpendicular c is $(\psi - \gamma)$.

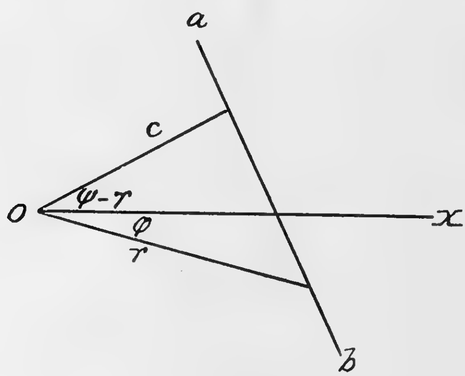


FIG. 45

For the exterior region we shall find

$$r = \frac{0.0324\ m\ J}{i} \cos\, (\varphi + \psi - \gamma)$$

which represents (see fig. 46) a circle through the origin and whose diameter

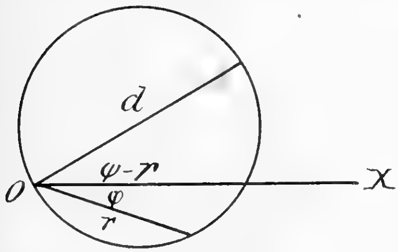


FIG. 46

$$d = \frac{0.0324\, m\, J}{i}$$

forms the angle $(\psi - \gamma)$ with the axis $O\,X$. Considering the cyclone of §32, we have $\psi = 48°$, $U_1 = 3^m$, $m = 150$. Assuming $J = 1$ and $\delta = 60°$, we shall have $\psi - \gamma = -12°$, which signifies that the perpendicular c and the diameter d are below the axis $O\,X$. Substituting $i = 0.2, 0.4$, etc., we shall find the curves of fig. 47, in which $a\,b$ represents the direction of the isotherms.

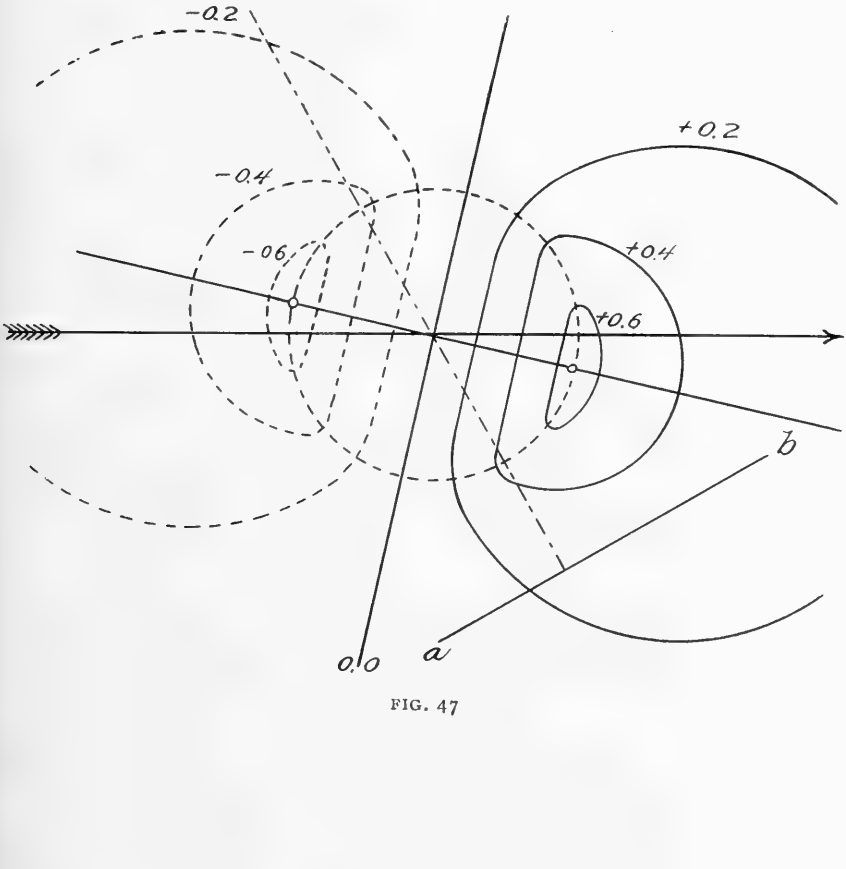


FIG. 47

CHAPTER VIII

SYSTEMS OF WIND IN NATURE

§34. *Influence of the surface of the earth*

In nature the systems of wind show many deviations from the ideal systems that we have considered. It is especially the surface of the earth with its irregularities that produces the greatest disturbances. To take an extreme case, let us consider a valley, that is to say an uncovered channel in the crust of the earth, it is evident that the wind follows the direction of the channel, whatever may be the direction of the gradient in the strata above the valley.

At meteorological stations situated at the surface of the earth, we must expect that the angle ψ between the gradient and the direction of the wind will generally differ from the theoretical angle, because the value of the friction depends on the irregularities of the surrounding land. We must therefore add a local correction $\Delta\ \psi$ which is determined by observations. We believe that the determination of this correction is of great importance for the prediction of the movement of systems of wind.

When a system of wind extends over a great part of the surface of the earth, the variation of the latitude produces disturbances of the normal angle, and a disturbance of this angle acts also on the system of isobars.

When the surface of the earth over which the system of wind occurs offers irregularities, the coefficient of friction varies. The variation of the coefficient of friction from one point to another produces disturbances of the angle between the gradient and the wind and consequently a deformation of the system of isobars.

Example. Let us consider a cyclone of which one half is over the land and the other half over the ocean. The equation of continuity is independent of the coefficient of the friction and of the latitude, and for the exterior portion where the current is considered as horizontal it is necessary to have

$$U_1\, r_1 \cos \alpha_1 = U_2\, r_2 \cos \alpha_2$$

Assume that the radius of the ascending part be 7° and calculate the different curves as we have shown them in §12 and § 14; and we shall find for the two portions of the cyclone;

	OVER THE LAND	OVER THE OCEAN
Mean latitude..............	50°	60°
Coefficient of friction........	0.00010	0.00005
Normal angle..............	48° 10′	68° 24′
Maximum velocity...........	20^{m}	30.7^{m}
Maximum gradient..........	3.30^{mm}	5.34^{mm}

$b - b_0$	r (land)	r (ocean)
5^{mm}	4.6°	3.2°
10	6.4	4.3
15	8.0	5.4
20	9.7	6.3
25	11.7	7.2
30	14.4	8.2
35	17.7	9.2

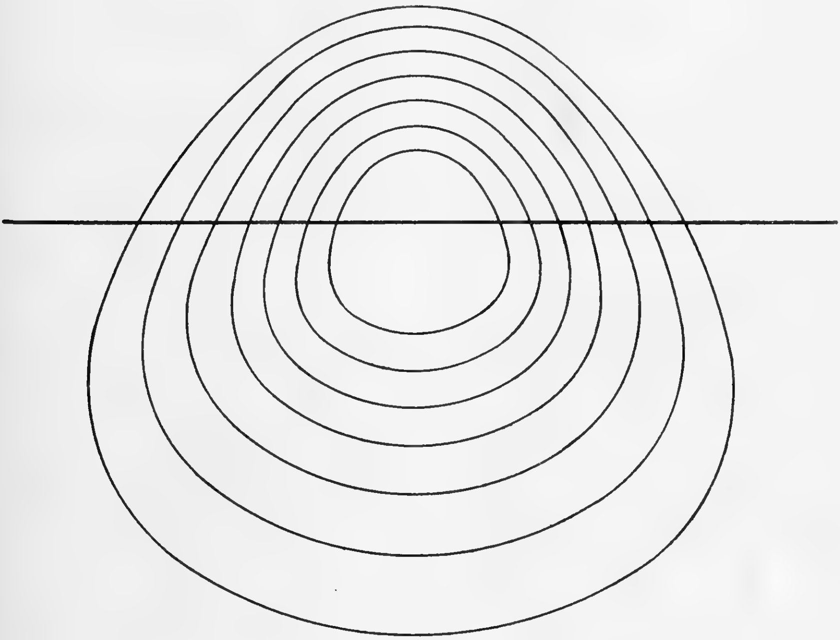


FIG. 48

By the aid of these values taken from the curves of pressure we have constructed the system of isobars shown in fig. 48.

§35. *Influence of the movement of the system of wind*

In the preceding example we have considered the movement of a system of wind under the hypothesis that the system keeps its form unaltered while moving. But this hypothesis is not correct. A moving cyclone does not transport its system of isobars in the same way that a system of circles is geometrically transported. We shall consider the general case in which the cyclone is movable as to location and variable as to the pressure at the center. We can, therefore, by the aid of the following equations determine directly the variations of the pressure δ_0 and δ that we have already deduced less precisely in another way in §§30 and 32.

Exterior portion

Consider a horizontal current; the equations of motion (see §19) assume the form

$$\frac{1}{\rho} \cdot \frac{dp}{dx} = -2\,\omega \sin\theta \,.\, v - k\,u - \frac{du}{dt} - u\frac{du}{dx} - v\frac{du}{dy} \quad . \quad . \quad (1)$$

$$\frac{1}{\rho} \cdot \frac{dp}{dy} = 2\,\omega \sin\theta \,.\, u - k\,v - \frac{dv}{dt} - u\frac{dv}{dx} - v\frac{dv}{dy} \quad . \quad . \quad . \quad (2)$$

Let ξ and η be the coördinates of the moving origin and assume that the velocities u and v are expressible in the form

$$u = \frac{M\,(x - \xi) + N\,(y - \eta)}{r^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

$$v = \frac{M\,(y - \eta) - N\,(x - \xi)}{r^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

where

$$r^2 = (x - \xi)^2 + (y - \eta)^2$$

Here M and N like ξ and η are functions of the time t, and we designate their derivatives with respect to t by $M,'$ $N,'$ ξ' and η'

We easily see that the condition

$$\frac{d^2p}{dy\,dx} = \frac{d^2p}{dx\,dy}$$

will be satisfied when we have

$$\frac{du}{dy} = \frac{dv}{dx}$$

at the same time that the equation of continuity

$$\frac{du}{dx} + \frac{dv}{dy} = 0$$

is fulfilled.

The absolute horizontal velocity U is determined by

$$U^2 = u^2 + v^2 = \frac{M^2 + N^2}{r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Writing

$$u = \frac{dF}{dx}, \quad v = \frac{dF}{dy}$$

we shall have

$$F_1 = M \text{ log. nat. } r + N \text{ arc} \left(\text{tang} = \frac{x - \xi}{y - \eta} \right)$$

Writing

$$u = \frac{dF_1}{dy}, \quad v = \frac{dF_1}{dx}$$

we shall have

$$F_1 = N \text{ log. nat. } r - M \text{ arc} \left(\text{tang} = \frac{x - \xi}{y - \eta} \right)$$

Equations (1) and (2) assume the form

$$\frac{1}{\rho}\frac{dp}{dx} = 2\,\omega \sin\theta \frac{dF_1}{dx} - k\frac{dF}{dx} - \frac{d^2F}{dx\,dt} - \tfrac{1}{2} \cdot \frac{dU^2}{dx}$$

$$\frac{1}{\rho}\frac{dp}{dy} = 2\,\omega \sin\theta \frac{dF_1}{dy} - k\frac{dF}{dy} - \frac{d^2F}{dy\,dt} - \tfrac{1}{2} \cdot \frac{dU^2}{dy}$$

consequently we have

$$\frac{p}{\rho} = 2\,\omega \sin\theta\, F_1 - kF - \frac{dF}{dt} - \tfrac{1}{2}\,U^2 + C \quad . \quad . \quad . \quad . \quad (6)$$

Writing

$$x - \xi = r \sin \varphi$$
$$y - \eta = r \cos \varphi$$

and substituting the values of F and F_1, we shall find

$$\left.\begin{aligned}\frac{p}{\rho} &= [2\,\omega \sin\theta\,.\,N - kM - M']\log\text{nat}\,r \\ &\quad - [2\,\omega\sin\theta\,.\,M + kN + N']\,\varphi \\ &\quad + \frac{M\xi' - N\eta'}{r}\sin\varphi + \frac{M\eta' + N\xi'}{r}\cos\varphi \\ &\quad - \tfrac{1}{2}U^2 + C\end{aligned}\right\}\quad \ldots \; (7)$$

The condition that the isobars are fixed curves requires that

$$2\,\omega\sin\theta\,M + k\,N + N' = 0 \quad \ldots\ldots\ldots \; (8)$$

Equation (7) shows that the isobars are not circles: they are curves dependent on ξ' and η' which are the components of the velocity of propagation. The gradient no longer coincides with the radius r and the angle Ψ between the velocity and the gradient differs from the normal angle α.

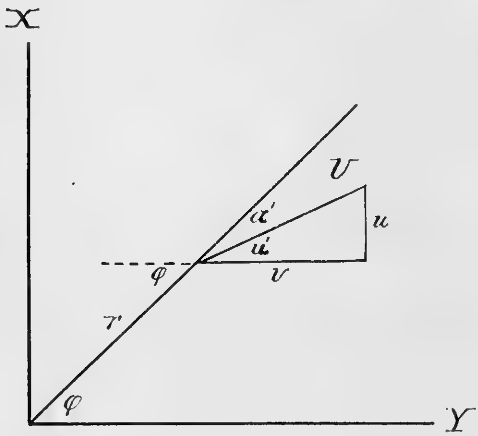


FIG. 49

Let α' be the angle between the velocity U and the radius r (see fig. 49) by combining the equations (3) and (4) with the preceding equations we obtain

$$\text{tang}\,\alpha' = \text{tang}\,(\varphi - i) = \frac{\text{tang}\,\varphi - \dfrac{u}{v}}{1 + \dfrac{u}{v}\,.\,\text{tang}\,\varphi} = -\frac{N}{M} \quad \ldots \; (9)$$

Determining M and N by equations (5) and (9) we find

$$\left.\begin{aligned} M &= -\,U\,r\cos\alpha' \\ N &= U\,r\sin\alpha'\end{aligned}\right\}\quad \ldots\ldots\ldots \; (10)$$

By equation (8) we shall find

$$\operatorname{tang} \alpha' = \operatorname{tang} \alpha + \frac{N'}{kM} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The last equation shows us that for a stationary but variable cyclone the angle between the gradient and the wind differs from the normal angle α. Suppose that we have $U r = 150$, $\alpha = 48°$ and $k = 0.00010$, and consequently in the constant cyclone $N = U r \sin \alpha = 111.5$. In one hour N may increase to 115.1, then we find $N' = 0.001$ and $\alpha' = 44.°7$. $U r$ is increased by 13.7 and the maximum velocity by about 1.5^{m}.

Assuming $\eta' = 0$ and $\xi' = W$ that is to say that the cyclone is propagated with the velocity W and in a constant direction, and considering the special case in which M and N are independent of the time and $M' = 0$, $N' = 0$, the equations (5) and (7) show that then U and p are independent of the time and consequently that the pressure is constant at the center during the propagation, and we have from equation (11) $\alpha' = \alpha$ and

$$\frac{p}{\rho} = \frac{k U r}{\cos \alpha} \log \text{nat}\, r - \tfrac{1}{2} U^2 - \frac{W}{r} U r \sin (\varphi - \alpha) + C \quad . \quad (12)$$

Assuming that at the distance R the pressure P remains constant and that the velocity U can be neglected, the equation is written

$$\frac{P - p}{\rho} = \frac{k U r}{\cos \alpha} \log \text{nat} \frac{R}{r} + \tfrac{1}{2} U^2 + \frac{W}{r} U r \sin (\varphi - d) \quad . \quad (13)$$

where the influence of the velocity of propagation is represented by the term

$$\frac{W}{r} U r \sin (\varphi - \alpha)$$

which term has its maximum value for

$$\varphi = 90° + \alpha$$

and consequently the axis of the isobar makes the angle α with the velocity of propagation. In the direction $o\,b$, perpendicular to the axis $o\,a$ (fig. 50) this term is zero and the pressure has the same value in this direction as in the stationary cyclone.

In order to determine the angle ε between the radius r and the gradient, we notice that the gradient is normal to the isobar and consequently

$$\operatorname{tang} \varepsilon = \frac{dr}{rd\varphi} = - \frac{\left(\frac{dp}{d\varphi}\right)}{r\frac{dp}{dr}} \quad \ldots \ldots \ldots \quad (14)$$

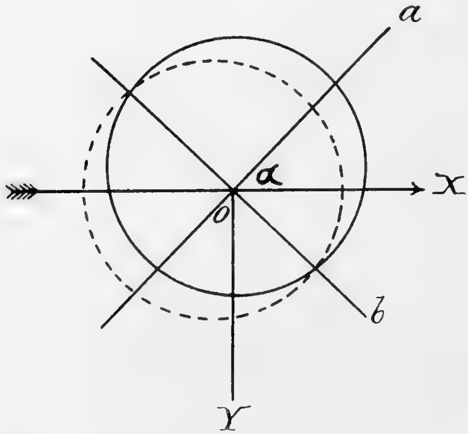


FIG. 50

Assuming that N and M are independent of the time, we shall find by differentiating equation (12)

$$\operatorname{tang} \varepsilon = \frac{\frac{W}{r} U r \cos(\varphi - \alpha)}{\frac{k U r}{\cos \alpha} + U^2 + \frac{W}{r} U r \sin(\varphi - \alpha)} \quad \ldots \ldots \quad (15)$$

Along the axis of the isobar where we have $\varphi = \alpha + 90°$ or $\varphi = \alpha - 90°$, ε becomes zero. The maximum value of ε occurs for $\varphi = \alpha$ and we find

$$\operatorname{tang} \varepsilon_m = \frac{W}{\frac{k r}{\cos \alpha} + U} \quad \ldots \ldots \ldots \ldots \quad (16)$$

We conclude therefore that the angle ψ between the wind and the gradient has its minimum value ($\alpha - \varepsilon_m$) along $o\,b$ (fig. 50) at the anterior limit and its maximum value ($\alpha + \varepsilon_m$) in the opposite direction.

Interior portion

We have assumed that in the interior portion (see § 14) the velocity and the gradient diminish proportionally to the radius and that the angle β between the direction of the wind and that of the gradient is connected with the normal angle α by the equation

$$\operatorname{tang} \alpha = \operatorname{tang} \beta \left(1 - \frac{2U}{kr} \cos \beta\right)$$

We make

$$U_1 = \frac{U}{r}$$

$$G_1 = \frac{G}{r} = \frac{k}{\cos \beta} \cdot \frac{U}{r} - \left(\frac{U}{r}\right)^2$$

Consider a variable cyclone and assume that the velocities u and v have the form

$$\left.\begin{aligned} u &= M(x - \xi) + N(y - \eta) \\ v &= M(y - \eta) - N(x - \xi) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

It is evident that the conditions under which equations (1) and (2) are integrable are satisfied when we have

$$\frac{d^2 p}{dx\, dy} = 0$$

Introducing u and v into equations (1) and (2) we shall find the condition

$$2\,\omega \sin \theta . M + kN + 2MN + N' = 0 \quad . \quad . \quad . \quad . \quad . \quad (18)$$

Making

$$S = 2\,\omega \sin \theta\, N - kM - M^2 + N^2 - M' \quad . \quad . \quad . \quad . \quad (19)$$

we shall have

$$\left.\begin{aligned} \frac{1}{\rho} \cdot \frac{dp}{dx} &= S(x - \xi) + M\xi' + N\eta' \\ \frac{1}{\rho} \frac{dp}{dy} &= S(y - \eta) + N\eta' - N\xi' \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (20)$$

By integration we have

$$\rho = 2\,S\,[(x - \xi)^2 + (y - \eta)^2] + (M\,\xi' + N\,\eta')(x - \xi) + (M\,\eta' - N\,\xi')\,(y - \eta) + C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Making

$$l^2 = \left[x - \xi + \frac{M\,\xi' + N\,\eta'}{S}\right]^2 + \left[y - \eta + \frac{M\,\eta' - N\,\xi'}{S}\right]^2 \quad . \quad (22)$$

we shall have

$$\frac{p}{\rho} = \tfrac{1}{2}\,S\,l^2 + C$$

and

$$\frac{p - p_0}{\rho} = \tfrac{1}{2}\,S\,l^2 \cdot \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Hence the isobars are circles about a center different from the moving origin.

Let the angle between U and r be β,′ we have

$$\text{tang}\,\beta' = -\frac{N}{M} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

By the aid of the formula

$$U^2 = u^2 + v^2 = (M^2 + N^2)\,r^2$$

we shall have

$$\left.\begin{aligned} M &= -\frac{U}{r}\cos\beta' \\ N &= \frac{U}{r}\sin\beta' \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

By aid of equation (18) we shall find

$$\text{tang}\,\beta' = \text{tang}\,\alpha + \frac{2\,U}{k\,r}\sin\beta' - \frac{N'\,r}{k\,U\cos\beta'} \quad . \quad . \quad . \quad (26)$$

We conclude therefore that also in a stationary but variable cyclone the angle between the gradient and the wind differs from the angle β belonging to the permanent cyclone.

Let us consider the special case in which M and N are independent of the time and make

$$\eta = 0 \text{ and } \xi' = W$$

Then by placing $N' = 0$ and by comparing the formula for $\tan \beta'$ with the first formula for tangent α we have $\beta' = \beta$ and by eliminating $2\omega \sin \theta$ between the equations (18) and (19) we shall find

$$S = \frac{k}{\cos \beta} \cdot \frac{U}{r} - \left(\frac{U}{r}\right)^2 = \frac{\mu}{\rho} \cdot G_1 \quad . \; . \; . \; . \; . \quad (27)$$

$$l^2 = \left(x - \xi - \frac{U_1 W}{S} \cos \beta\right)^2 + \left(y - \eta - \frac{U_1 W}{S} \sin \beta\right)^2 \quad (28)$$

and by introducing the barometric height b in place of the pressure p, in equation (23) we have

$$b - b_0 = \tfrac{1}{2} G_1 l^2 . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (29)$$

We infer from these equations that the system of isobars belonging to the stationary cyclone has been transferred from the origin O (see fig. 51) to the center A, whose distance is

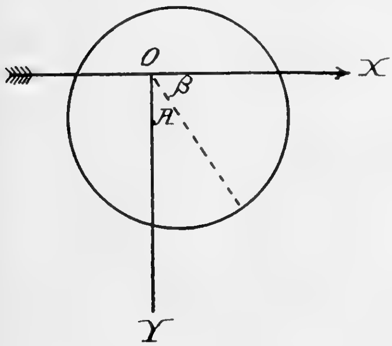


FIG. 51

$$\frac{\rho}{\mu} \cdot \frac{U}{G} \cdot W$$

and which falls on the right line $A\ O$ making the angle β with the direction of propagation of the center of the cyclone.

Example. Assume that the cyclone has a velocity of propagation $W = 15^{\text{m}}$ and that we have for its exterior portion, $U\,r = 150$ and $\alpha = 48°$ and for its interior portion $U_1 = 3^{\text{m}}$, $G_1 = 0.483^{\text{mm}}$, $\beta = 57°.5$ we shall find

$$O\ A = 0°.85$$

By constructing the isobars for the exterior part and for the interior part we shall by interpolation find the isobars for the intermediate region.

The isobars in fig. 52 are constructed according to the following table:

b	VALUE OF r		
	$\varphi = \alpha$	$\varphi = \alpha + 90$	$\varphi = \alpha - 90°$
760	20.0°	20.8°	19.2°
755	16.3	17.2	15.4
750	13.3	14.2	12.2
745	10.8	11.7	10.0
740	8.9	9.7	8.1
735	7.9	....	....
730	6.3	....	....
725	4.5	....	....

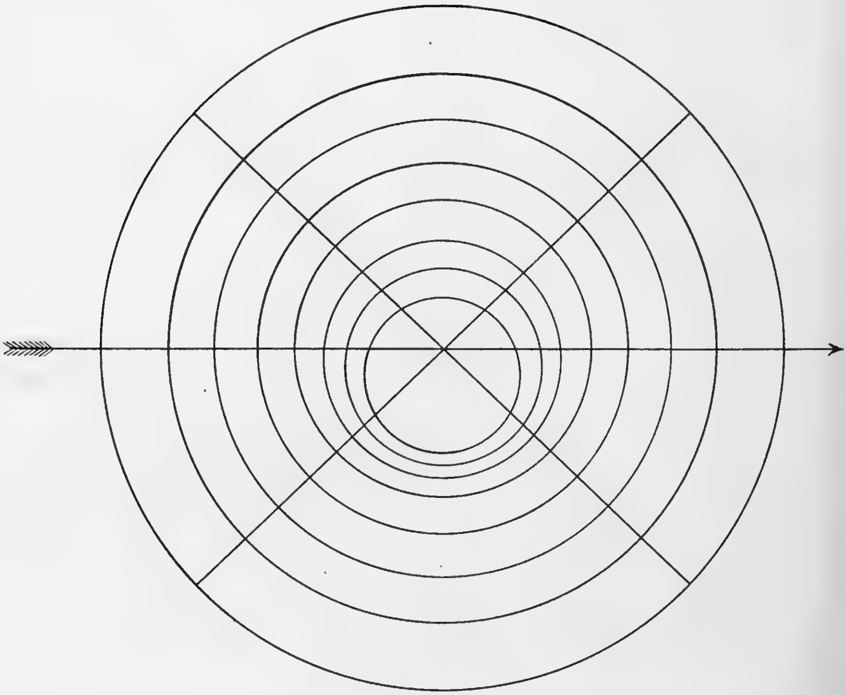


FIG. 52

The maximum value of the angle ε between the gradient and the radius is determined by equation (16). Assuming $r = 9°$, $U = 16.7^m$, $k = 0.00010$, we shall have ε about 5° and consequently the angle of inclination ψ varies from 43° at the anterior edge to 53° at the posterior edge of the progressing cyclone.

§36. *Influence of the rotation of the earth on the vertical currents*

As we have shown in §19, the force produced by the rotation of the earth depends also on the vertical velocity. Let us consider in the first place a steady vertical current and assume the horizontal velocities u and v to be zero. The equations of motion assume the following form, denoting by a the angle between the axis $O\ X$ and the meridian (see fig. 53):

$$\frac{1}{\rho} \cdot \frac{dp}{dx} = -2\ \omega \cos\theta \sin a\ w$$

$$\frac{1}{\rho} \cdot \frac{dp}{dy} = -2\ \omega \cos\theta \cos a\ w$$

$$\frac{1}{\rho} \cdot \frac{dp}{dz} = -g - w\frac{dw}{dz}$$

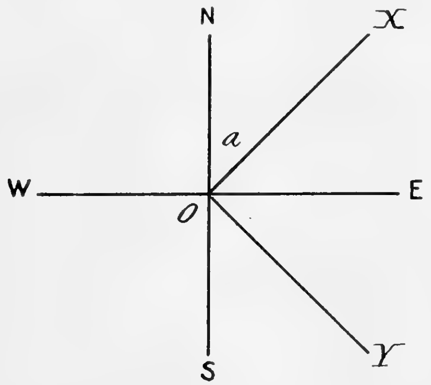


FIG. 53

It follows that the vertical motion goes on in the same way as we have developed in §15, but that there exists a *horizontal gradient.* Denoting this gradient by G we have

$$\frac{\mu}{\rho} \cdot G = 2\ \omega \cos\theta\ w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and this gradient is directed *toward the east,* when w is positive which is true for ascending currents. For descending currents w is negative and the gradient is directed toward the west. The gradient produced by the vertical current has its maximum value at the equator where $\theta = 0$ and it disappears at the poles where $\theta = 90°$. For a mean value of ρ we have

$$G = 0.16 \cos \theta \, w$$

Let us consider an *inclined current* and seek the conditions under which the horizontal gradient produced by the vertical velocity is zero. Suppose that the inclined current lies in the plane $Z\,O\,X$. We assume the component $v = 0$.

By substituting

$$\frac{dp}{dx} = \frac{dp}{dy} = 0$$

the equations of motion take the following form:

$$0 = -2\,\omega \cos \theta \sin a\, w - u\frac{du}{dx} - w\frac{dw}{dz} \quad . \quad . \quad . \quad . \quad (2)$$

$$0 = 2\,\omega \sin \theta\, u - 2\,\omega \cos \theta \cos a\, w \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{1}{\rho}\cdot\frac{dp}{dz} = -g + 2\,\omega \cos \theta \sin a\, u - u\frac{dw}{dx} - w\frac{dw}{dz} \quad . \quad . \quad (4)$$

From equation (3) we obtain

$$\frac{u}{w} = \text{cotg } \theta \cos a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The horizontal gradient being zero, it is necessary that p be a function of z only and consequently from equation (4) we shall obtain

$$\frac{du}{dx} = 0 \qquad \frac{dw}{dx} = 0$$

Eliminating u between the equations (2) and (5) we shall find

$$\frac{dw}{dz} = -2\,\omega \sin \theta \text{ tang } a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Supposing that the density ρ is a function of z only, the equation of continuity is put under the form

$$\rho \, w = \text{constant}$$

From the last equation we obtain

$$\frac{dw}{dz} = -\frac{w}{\rho}\frac{d\rho}{dz}$$

Approximately we can write (see §4)

$$\frac{1}{\rho}\cdot\frac{d\rho}{dz} = -\frac{g}{a\,T}$$

and consequently we shall find

$$\operatorname{tang} a = -\frac{g}{a\,T}\cdot\frac{w}{2\,\omega\sin\theta} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (7)$$

For the ascending currents the value of w is positive and the angle a falls between 270° and 360°, that is to say, the direction of the axis $O\,X$ is between the west and the north.

By eliminating w between equations (5) and (7) we shall find

$$u = -\frac{a\,T}{g}2\,\omega\cos\theta\sin a \quad . \; . \; . \; . \; . \; . \; . \; . \; (8)$$

The maximum value of u occurs for $\theta = 0°$ and then we have $a = 270°$; for $T = 273°$ we shall find $u = 1.2^{m}$.

For example let $\theta = 45,°$ $T = 273°$ and $w = 1$ meter, we shall find $a = 309°.5$ and $u = 0.64^{m}$.

The inclination of the ascending current to the vertical being i, we have

$$\operatorname{tang}\; i = \frac{u}{w} = 0.64,$$

whence $i = 32°\ 37'$.

If the height of the current was 10000^{m}, the center of the barometric maximum in the upper strata would be moved 0.64 kilometer, or about 0.06 degree of a great circle from the vertical passing through the barometric minimum at the surface of the earth. We come therefore to the conclusion that in cyclones the current axis is inclined backward by the action of the rotation of the earth, but so little that we can neglect the effect of this inclination.

§37. *Influence of simultaneous systems of wind*

In nature we generally find that various systems of wind exist simultaneously. The simultaneity of two or of many barometric maxima or minima produces certain disturbances in the systems of isobars of each system of wind and especially do the isobars deviate from the normal form along the passage from one system to another.

We shall illustrate the passage of the wind from one system to another by an example which offers an analogy with certain cases of nature. Consider a horizontal motion and assume that the velocities along the axes $O\,X$, $O\,Y$ and $O\,Z$ assume the form

$$u = M\,y,\ v = M\,x, \text{ and } w = 0 \quad \ldots \ldots \ldots \quad (1)$$

Substituting these values in equations (1) and (2) of §35, we have

$$\frac{1}{\rho} \cdot \frac{dp}{dx} = -2\,\omega \sin\theta \cdot M\,x - k\,M\,y - M^2\,x \ldots \ldots (2)$$

$$\frac{1}{\rho} \cdot \frac{dp}{dy} = 2\,\omega \sin\theta \cdot M\,y - k\,M\,x - M^2\,y \ldots \ldots (3)$$

By integration we shall find

$$\frac{p - p_0}{\rho} = -\tfrac{1}{2}\,x^2\,(2\,\omega \sin\theta \cdot M + M^2) -$$

$$- k\,M\,x\,y + \tfrac{1}{2}\,y^2\,(2\,\omega \sin\theta\,.\,M - M^2)$$

and by introducing

$$\text{tang}\,\alpha = \frac{2\,\omega \sin\theta}{k}$$

we shall have

$$\frac{p - p_0}{\rho} = \frac{k\,M}{2}\left[y^2\left(\text{tang}\,\alpha - \frac{M}{k}\right) - 2\,x\,y - x^2\left(\text{tang}\,\alpha + \frac{M}{k}\right)\right] \ldots (4)$$

If $\tan\alpha > \frac{M}{k}$, then the isobars represented by equation (4) are *hyperbolas.*

The asymptotes are represented by the equation

$$\frac{y}{x} = \frac{1 \pm \sqrt{1 + \tan^2 \alpha - \left(\frac{M}{k}\right)^2}}{\tan \alpha - \frac{M}{k}} \quad \ldots \ldots \quad (5)$$

The trajectories of the motions of the air are determined by

$$\frac{dx}{dt} = My \text{ and } \frac{dy}{dt} = Mx$$

and consequently

$$x\,d\,x - y\,d\,y = 0.$$

By integration we shall have

$$x^2 - y^2 = \text{constant} \quad \ldots \ldots \ldots \quad (6)$$

We conclude therefore that the trajectories are equilateral hyperbolas.

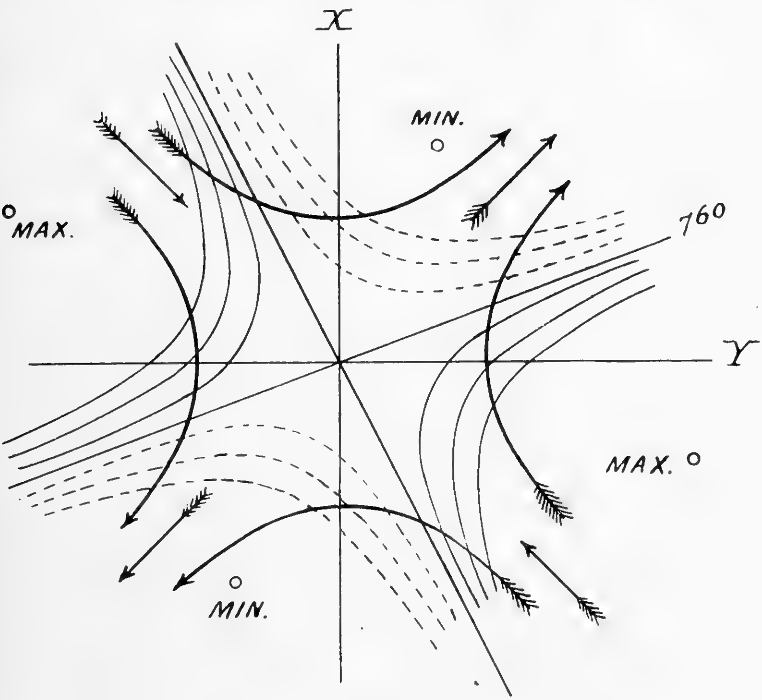


FIG. 54

The absolute velocity U is found by the equations (1) and we have

$$U = Mr \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Assuming $\alpha = 45°$; $\frac{U}{r} = 1$; $k = 0.00006$ we shall find for the asymptotes of the isobars

$$\frac{y}{x} = 2.83 \text{ and } \frac{y}{x} = -0.478$$

We easily construct the hyperbolas by the aid of the asymptotes and of one point in the hyperbola. By introducing a mean value of ρ and expressing the pressure in millimeters of mercury we shall find by equation (4)

$$\text{for } x = 0: \quad y^2 = 36.37 \quad (b - b_0)$$

$$\text{for } y = 0: \quad x^2 = 26.88 \quad (b_0 - b)$$

By giving to the difference $b - b_0$ the successive values 1^{mm}; 2^{mm}; 3^{mm}; etc., we shall locate the points by which we can construct the hyperbolic isobars, as shown in fig. 54.

To construct a trajectory of the wind we start from any point whatever.

XII

ON THE THERMODYNAMICS OF THE ATMOSPHERE

BY PROF. DR. WM. VON BEZOLD

[*Fourth Memoir, Sitzungsberichte of the Berlin Academy. 1892, pp. 279–309. Translated from the Gesammelte Abhandlungen von Wm. v. Bezold, Braunschweig, 1906, pp. 184–215*]

SUPERSATURATION AND SUBCOOLING. FORMATION OF THUNDER-STORMS

In the third memoir[1] on the Thermodynamics of the Atmosphere, which was devoted to the investigation of the mixture of masses of moist air as well as the formation of fog and clouds, I have shown what the consequences must be when condensation suddenly occurs in air supersaturated with vapor.

On that occasion I remarked that I considered it very probable that such supersaturations, whose possibility is demonstrated by laboratory experiments, occur also in the free atmosphere and that they certainly may be the cause of cloudbursts.

At that time it appeared to me important to restrict myself to a simple suggestion, as I was not able to adduce any proof of the correctness of this idea. Meanwhile it has become clear to me that there exists still another unstable condition for the water contained in the atmosphere, that is, the "Subcooling," whose sudden dissipation must result in phenomena similar to those of "Supersaturation." Now the subcooling of fog and cloud has been often observed. In regard to this I recall the investigations that Assmann made on the Brocken,[2] as well as the results of the very interesting balloon voyage made fromBerlin, June 19, 1889, by Moedebeck and Gross, and described in an excellent manner by the latter.[3]

The above mentioned observations give convincing evidence that at temperatures below the freezing point there occur clouds that contain no ice but are true water-clouds, but from which there

---

[1] See p. 272 of the preceding collection of translations. Smithsonian Miscellaneous Collections, Vol. XXXIV.

[2] Met. Zeit., Vol. II, 1885, pp. 41–47.

[3] Zeit. f. Luft-schiffahrt, 1889, Vol. VIII, pp. 249–262.

precipitates ice of the same peculiar structure as is observed in *Glatt eis* and which gives rise to the formation of the so-called *Anraum* or ice storm.

If now we try to reason out how the sudden cessation of the state of subcooling or the supersaturation ought to become manifest, we find that it must be followed by a phenomenon that has long since been recognized as a regular accompaniment of thunderstorms, namely, a sudden rise in the atmospheric pressure. This rise, with the subsequent less prominent fall, must show exactly the same peculiarities in continuous barograms that are prominent in the so-called "Gewitter nase" or "thunder nose."

Moreover, by the more accurate prosecution of the study we have arrived at ideas about the constitution of thunder clouds and of the processes going on therein that appear likely to throw new light on the whole theory of the formation of thunderstorms.

But in order to understand these questions it is first necessary to investigate from a purely theoretical point of view the consequences of a sudden disruption of the condition of supersaturated vapor or subcooled water.

SUPERSATURATION

The influence that the dissipation of any existing state of supersaturation must exert on the thermal condition of the atmosphere, under the assumption of constant pressure, is explained in the preceding third memoir,* although only as a pure hypothesis in the course of the investigation of the mixture of masses of air having different temperatures and humidities.

It was shown that in such a case there occurs a rise of temperature whose extent can be most easily determined by graphic methods.

If we know the pressure prevailing in the supersaturated air, we then represent the normal quantity of water y' corresponding to saturation or the quantity contained in a kilogram of the mixture by the ordinate of a curve $F'F'$ in a rectangular system of coördinates whose abscissæ represent the temperatures. (See fig. 30.)

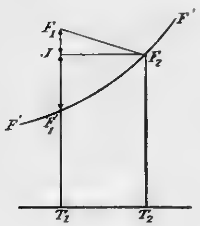


FIG. 30

If now, $y_1 = F_1T_1$ or the quantity of vapor contained in the mixture of air and vapor, exceeds the normal quantity required for saturation by $y_1 - y_1' = F_1 T_1 - F_1' T_1$ then the temperature t_2 that will

*See the preceding collection of translations, p. 272.—C. A.

prevail after the dissipation of the supersaturation will be found by drawing through F_1 a straight line making an angle $\alpha = tg^{-1}\frac{1}{2.5}$ if the temperatures are above freezing, but $\alpha^* = tg^{-1}\frac{1}{2.9}$ if the temperatures are below freezing.

The abscissa $O\,T_2$ of the intersection F_2 of this straight line with the curve of saturation $F'\,F'$ will be the desired temperature t_2.

The graphic construction just given may also be applied with a slight but very important modification to the case now under consideration.

In the investigation above mentioned it was assumed that the pressure remains constant since the assumption of supersaturation served only as a numerical artifice, when the actual process must go on gradually, and therefore the expansion due to the rise in temperature can also follow quietly. Therefore the adopted value of the thermal capacity of air was that for constant pressure.

Now, on the other hand, emphasis is laid on the assumption that the disruption takes place so rapidly that the volume is to be considered as constant at first, so that the rise in temperature must make itself felt as a change in pressure.

Of course an equilibrium must eventually be attained, the air must expand until its pressure comes into equilibrium with that of the surrounding atmosphere, which process must again cause a cooling.

Therefore, whereas corresponding to the problem previously discussed, we had

$$tg\,\alpha = \frac{1000\,c}{r}$$

for which we may now more appropriately write

$$tg\,\alpha_p = \frac{1000\,c_p}{r}$$

where the subscript p indicates that we treat of constant pressure, now, on the other hand, for the present case we introduce the angle α_v for which we have the equation

$$tg\,\alpha_v = \frac{1000\,c_v}{r}$$

where c_v is the specific heat of moist air under constant volume.

For the value of this quantity we obtain

$$c_v = 0.1685 + 0.000175\, y'$$

by considerations quite analogous to those that Hann has adopted in his determination of the value of c_p.

At the temperatures with which we have to do y' seldom exceeds the value 10 and then only slightly, and since r varies about the value 600 therefore the quantity

$$\cot \alpha_v = \frac{r}{1000\, c_v}$$

can be considered as constant and without incurring important error can be taken as

$$\cot \alpha_v = 3.5,$$

a value that is generally a little too small, but is nearer the truth than 3.6, which is considerably too large.

For temperatures below 0°C, we must use, instead of α_v, a value α^*_v determined by the equation

$$\cot \alpha^*_v = \frac{r + l}{1000\, c_v} = 4.0$$

where l is the latent heat of melting ice.

If we introduce the angle α_v into the construction of fig. 30, or its equivalent value into the formula if we prefer numerical computation, then we can utilize the same method as above described.

In this method T_2 is the end of the abscissa belonging to the temperature t_2 and $y_1 - y_2 = F_1 J$ is the quantity of precipitated water.

We see from this that only a part of the excess of vapor over the amount required for saturation is precipitated while the remainder remains vapor in spite of the discontinuance of supersaturation, since the air now needs a greater quantity of water for its saturation in consequence of the rise in temperature.

But a rise in pressure accompanies this rise in temperature because of the unchanged volume, at least during the first instant, which is given by the equation

$$\frac{\beta_2}{\beta_1} = \frac{273 - t_2}{273 - t_1}$$

[4] Zeit. d. Oest. Gesell. für Met., 1874, p. 374.

where β_1 and β_2 represent the barometric pressure respectively before and after the cessation of supersaturation.

But the objection may be raised that the curve of saturation quantity is drawn under the assumption of a constant initial pressure β_1, since such a constant pressure is everywhere assumed in the diagrams of the previous memoir.

But this assumption is, however, even now justifiable in so far as concerns the curve $F' F'$. For a knowledge of the initial pressure β_1 is only necessary in order to be able to determine the volume that a kilogram of air occupies at a given temperature; since this volume experiences no change during the extraordinary short time that we have under consideration while the change in temperature makes itself apparent by the increase in pressure, therefore the assumption is perfectly unobjectionable.

In order to judge as to the probability of the occurrence of such supersaturation as would suffice to explain the observed changes of pressure in thunderstorms it will be best to consider the supersaturation as a consequence of adiabatic expansion without the accompanying condensation.

Adiabatic expansion, in the absence of dust or ions that can act as nuclei for condensation, is the only process by which the occurrence of supersaturation in the atmosphere is conceivable. On the other hand adiabatic expansion does play the most prominent part in the formation of thunderstorms.

In considering the present problem we can profitably make use of the method of presentation used in the first memoir[5] but whereas in that memoir it is assumed that we imagine one kilogram of dry air to be mixed with x kilograms of aqueous vapor, we shall now assume that one kilogram of the *mixture* contains y grams of water.

It is easy to see that within the limits of the ordinary values of x or y the diagram drawn for one method of consideration also serves for the other, since only slight changes will be needed which in fact can only be appreciable when the diagrams are drawn with unusual accuracy.

Especially is this possible so long as we assume that the precipitated water is carried along with the air; if we drop this assumption then minor modifications must enter into the treatment of the problem, but these are not important in the present memoir.

In general it is easy to see that the following equation will hold good:

$$1 + x = \frac{1000}{1000 - y}$$

[5] See No. XV, p. 212 of the 1891 series of these translations.—Ed.

If we assume that the initial condition of the air as it ascends from the ground is represented by the point a_0 of the diagram (fig. 31), and that S_0 S_0 is the corresponding saturation-curve, then supersaturation will occur when the adiabat a_0 a_1 intersects this curve without being broken at the point of intersection, that is to say, when the cooling takes place according to the same law after passing the point of saturation, as it did before in the dry stage.

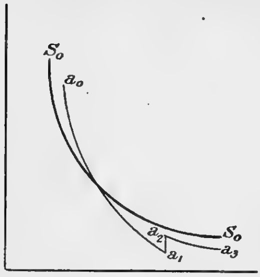


FIG. 31

But we attain supersaturation or the quantity

$$\bar{y}_1 = y_1 - y_1'$$

when we seek the curve of saturation belonging to a_1 and with it the value y_1'. Moreover, we can apply to the supersaturated air the conception of relative humidity and put

$$R = 100 \frac{y^1}{y_1'},$$

where, of course $R > 100$.

If now by reason of any sudden paroxysm the supersaturation should suddenly cease, then an increase in temperature would occur, and since the expansion of the mass of air can only take place gradually there must also be an increase of pressure, that is to say, the curve representing the adiabatic condition must rise suddenly from a_1 to a_2. In this process the point a_2 always lies below the saturation curve, since a part of the water falls away from the mass when it attains the final condition represented by a_2, and therefore less vapor is present in a kilogram of the mixture than at the original beginning of the expansion.

After the cessation of the sudden rise in temperature (which carries the pressure from β_1 up to β_2) there will, of course, again begin the process of expansion, but this will now be along the adiabat of the rain-stage or the snow-stage. The graphic method just described may be used to locate the position of the point a_2 for the purpose of determining the conditions corresponding thereto.

The position of a_2 is found by considering the line drawn vertically through a_1 as the fundamental line, since the pressures increase with temperature linearly in this direction. We may therefore identify the points a_1 and a_2 directly with the points T_1 and T_2 of fig. 30.

In order to obtain an idea of the magnitude of the changes here considered it is advantageous to choose a definite case and determine the magnitude of the sudden rise in pressure when we assume that the dissipation of supersaturation takes place at different stages of progress successively far apart.

In the computations I assume that air having a temperature of 25° C. at sea-level contains such a quantity of vapor as would cause it to cool to its dew-point by adiabatic expansion when rising to an elevation of about 800 meters. I make this specific assumption because in the Alps where the mountains offer excellent level marks, I have often observed thunderstorm clouds whose lower surfaces had about that altitude above the valleys.

With this assumption we obtain the group of values shown in the accompanying table.

h	β_1	t_1	y_1	y_1'	y_1-y_1'	R_1	t_2	y_2'	β_2	$\beta_2-\beta_1$
0	760	25.0	12.7	19.4	−6.7	66	....	....	....	......
834	690	16.6	12.7	12.7	0.0	100	....	....	....	......
957	680	15.5	12.7	12.1	0.6	105	16.2	12.6	631.6	1.6
1083	670	14.2	12.7	11.2	1.5	113	15.6	12.2	673.3	3.3
1339	650	11.7	12.7	9.8	2.9	130	14.6	11.8	656.6	6.6
2000	600	5.1	12.7	6.8	5.9	187	11.8	10.5	614.5	14.5

In this table all quantities having the subscript 1 belong to the condition of supersaturation before its dissipation; those having the subscript 2 belong to the condition immediately after supersaturation is broken up. It need hardly be repeated that h is the altitude in meters above sea-level, β the barometric pressure in millimeters of mercury, t the temperature Centigrade, y_1 the number of grams of water contained in a kilogram of the mixture, and y_2 the grams of water needed to produce saturation at the temperatures t_1 and t_2 respectively. Finally, R expresses the relative humidity just before the dissipation of the supersaturation and is therefore the measure of the extent of the supersaturation. The corresponding value R_2 after the breaking up is always 100 and is therefore omitted from the table.

In these computations I have made use of the graphic method given by H. Hertz,[6] which, although it does not allow of any great accuracy, yet is so remarkably convenient and gives the result so

[6] Met. Zeit., 1884, Vol. I, pp. 421–431. See No. XIV of these translations.

quickly that I prefer to recommend it in all cases such as this where a general orientation is all that is needed.[7]

The altitude is the only datum not taken from Hertz's diagram of adiabats, but has been directly computed since this quantity as given by the diagram cannot be absolutely relied on if we wish to avoid large error. But when the object is to attain a general idea of the magnitude of the quantities in question I think it justifiable to be content with approximations.

From this example we perceive that an ascending current of air needs to pass only a very little beyond the saturation point in order to develop a supersaturation whose dissipation completely suffices to produce a rise of pressure of such magnitude as is observed in thunderstorms.

For instance, under the above assumed conditions the rising air has only to pass 120 meters above the altitude for saturation in order to produce a rise of pressure of 1.6 mm. when the change occurs and in fact 75 meters will produce a sudden rise of 1 mm. in pressure. Hence we utilize no risky hypotheses in explaining by means of supersaturation the sudden rise in barometric pressure observed during thunderstorms.

In this explanation we must not attribute too much importance to the fact that hitherto we have not yet been able to prove the existence of such supersaturation in the free atmosphere. For independent of the fact that our ordinary apparatus for measuring humidity, such as the psychrometer and the hair hygrometer, are not available as indicators of supersaturation, we note that the greater frequency of fog in the neighborhood of large manufacturing cities indicates that the point of saturation is not infrequently attained, and even exceeded, in the free atmosphere without the attendant condensation because of the absence of the necessary fog nuclei. Moreover, the fact recently demonstrated by Hellmann[8] that the rainfall from thunderstorms west of Berlin are heavier than from those over the city or east of it, argues for the occurrence of supersaturation at places where there is a dearth of nuclei and especially during thunderstorms.

Furthermore, in the powerful movements and in the peculiar

[7] The new adiabatic tables and diagrams by O. Neuhoff as published in the Abhandlungen of the Preuss. Met. Institute, Vol. I, No. 6, Berlin. 1900, can now be more conveniently used for such computations. See No. XXI of this collection of translations.

[8] Jahresbericht des Berliner Zweigvereins der Deutsche Met. Gesell. für 1891, p. 21, Berlin, 1892.

puffing up and pushing forth of new heads from the cumulus clouds I perceive evidence that within the cloud itself there must be present a source of power and that we have not to do with the simple results of a quickly ascending current of air.

The observations made by Moedebeck and Gross in the interior of a cumulus cloud during the above-mentioned balloon voyage, according to which, the balloon was set into great oscillations and the drops of water whirled in confusion past each other, show that within the clouds powerful movements take place independent of the general movement of the air.

Precisely such sources of power must exist within the cloud in connection with the dissipation of supersaturation.

On the other hand, I cannot conceal the fact that the very low temperatures observed within the clouds are not easily reconciled with any such assumption, although the deprivation of the sunshine from the interior of the cloud must bring it about that particles of water which have condensed in the upper portion of the cloud must fall with lower temperatures into the lower layer of the cloud, and we thus perceive that we have to do with very complicated processes that must produce remarkably different temperatures in different portions of the same cloud.

Moreover, the measurements of temperature made under such difficult circumstances can only be completely conclusive when they are made with the perfected apparatus that has lately been described by Assmann.[9]

SUBCOOLING

If the above given considerations have to a certain extent the character of theoretical speculations, since we have not yet experimentally demonstrated the existence in the free atmosphere of true supersaturation, this is not true of the investigation to be presented in the present chapter, on clouds containing subcooled particles of water. Such clouds frequently occur and can exist for a long time if the subcooling does not exceed certain limits. If, however, we consider clouds of very great size, in whose highest parts the temperature must be remarkably low, then a small external stimulus will suffice to dissipate the subcooling and cause a sudden rise of temperature and pressure.

Let $t_1 < 0$ be the temperature of the mixture of dry air, vapor, and subcooled water; y_1 grams the total quantity of water, partly

[9] Das aspirations-psychrometer. Abhandlungen d. Preussischen Met. Institute, Vol. I, No. 5, Berlin, 1892.

liquid and partly vapor, contained in a kilogram of the mixture, but of which y'_1 is vapor; then the quantity of dry air in a kilogram of the mixture is

$$1 - 0.001\ y_1.$$

If now the liquid water suddenly becomes ice then a rise of temperature must occur because of the setting free of the latent heat of melting, but the freezing point cannot thereby be exceeded, since no further freezing of the remaining liquid masses would be possible above this temperature.

In the case of very slight subcooling the dew-point is simply attained, whereas for greater subcooling and not too great a quantity of subcooled water, i. e., for moderate degrees of mechanical subcooling, the temperatures remain below the freezing point.

But no equilibrium can be thus attained by the simple conversion of water into ice, for in consideration of the higher final temperature and since the air is to remain saturated, a part of the existing water must be converted into vapor and hence the final temperature will not be so high as if this evaporation did not take place.

It seems to me probable that these processes take place not exactly simultaneously and not with the same rapidity, but rather that the freezing takes place suddenly, whereas the evaporation takes place subsequently and gradually.

Indeed, it is possible that at the first instant the particles of water all attain the freezing point, but that so long as the subcooling is not extraordinarily large, only a part of the water can freeze and that afterwards the surrounding air, as well as the evaporation, have a further cooling influence.

However this may be, it is certainly advantageous analytically to assume that first there is established a temperature equilibrium t_2 between the frozen water particles and the air and that then the temperature is reduced to t_3 by evaporation.

The following consideration serves for this determination of the temperature t_2 depending on the simple conversion of subcooled water into ice and the corresponding warming of the air.

In the freezing of $\bar{y}_1 = y_1 - y_1'$ grams of water $80\,\bar{y}_1$, units of heat (in small calories) are set free.

This quantity of heat serves first to warm $1000 - \bar{y}_1$ grams of moist air from t_1 up to t_2, that is to say, by $t_2 - t_1 = t$ degrees, and furthermore to warm y_1 grams of ice from t_1 to t_2 degrees.

Consequently

$$80\,\bar{y}_1 = (1000 - \bar{y}_1)\,c_v t + 0.51\,\bar{y}_1 t$$

since 0.51 is the specific heat of ice,

Since c_v lies between the limits 0.1693 and 0.1701 under the conditions here considered we can substitute 0.17 for this value and obtain

$$80\,\bar{y}_1 = (1000 - \bar{y}_1) \times 0.17\,t + 0.51\,\bar{y}_1 t$$

or

$$80\,\bar{y}_1 = (170 + 0.34\,\bar{y}_1)\,t$$

whence

$$t = \frac{80}{170 + 0.34\,\bar{y}_1}\bar{y}_1 = \frac{8}{17}\,\frac{1}{1 + 0.002\,\bar{y}_1}\bar{y}_1.$$

By developing into a series the fraction containing $\bar{y}_1$ in the denominator and retaining only the first two terms we obtain

$$t = 0.4706\,\bar{y} + 0.00094\,\bar{y}_1^2$$

Since 10 is a high value for $\bar{y}_1$ that is not likely to be exceeded, and since the second term attains only the value 0.1° C. when $\bar{y}_1 = 10$, therefore we may ordinarily confine ourselves to the use of the first term of this equation.

The formula just given enables us to compute the temperature t that prevails immediately after the sudden freezing occurs throughout the whole of a subcooled mass, assuming that t_2 is below freezing, that is to say, that $t_2 < 0$.

If this assumption does not hold good, then the computed value loses its significance and the value 0 must be substituted for it, no matter how large $\bar{y}_1$ may be.

In this case only a part $y_1{}^*$ of the subcooled water can be frozen, which part will evidently be given by the equation

$$t_1 = 0.4706\ y_1{}^*$$

where for simplicity we have omitted the second correction term.

We have now still to consider how to determine the final temperature t_3 as it must result when the air, which was no longer saturated at the moment of freezing, becomes again saturated with aqueous vapor. For although as above remarked, the final condition is not of importance with reference to the sudden rise in pressure which is at present our first consideration, since the pressure must eventually come to a complete equilibrium, still I hold it to be advantageous to supplement the investigation in this direction also.

We thus obtain a correct idea as to how large the error would be if the assumption as to the sudden freezing of the whole mass and the subsequent gradual evaporation should prove not to be appropriate.

I have therefore in the example to be communicated later determined the value β_3 of the final pressure as it would result from the assumption that the necessary evaporation goes on directly hand in hand with the sudden freezing.

In this determination we again advantageously make use of the graphic method.

In fig. 32 again as before in fig. 30 let $F_1 T_1 = y_1$ and $F_1' T_1 = y_1'$: then we find T_2 by drawing through F_1 a straight line that makes the angle $\beta = \text{arc tg } 0.47 = 25° 11'$ with the axis of ordinates and find the point of intersection F_2 of this line with a horizontal line through F_1'. Then T_2 is the end of the abscissa representing t_2 and consequently $T_1 T_2 = t$.

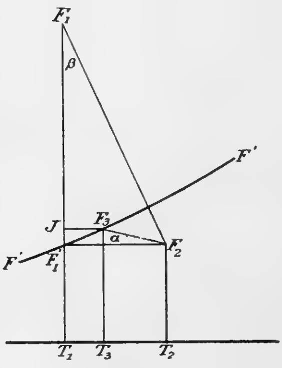


FIG. 32

We now draw through F_2 a straight line making an angle[10] $\alpha^*_v = \text{arc tg } \frac{1}{4}$ with the horizontal axis of abscissæ (owing to want of room, this angle is only marked with α in fig. 32).

The abscissa of the intersection F_3 of this line with the curve $F'F'$ of the quantity of vapor needed for saturation corresponds to the temperature t_3 while the ordinate itself $F_3 T_3$ is equal to y_3.

In the preceding it is assumed that $t_2 < 0$ and that the whole process goes on so rapidly that the volume can be regarded as constant.

If the graphic construction or the numerical computation should give a value $t_2 > 0$, then this argument loses its significance.

But in this case the temperature t_2 simply rises to 0° C. Hence only a part of the subcooled water is converted into ice, while another part remains fluid and a third part becomes vapor.

Since it is not now of importance to determine the magnitudes of these three parts we may abstain from their evaluation, as well as from the investigation of the special cases where the subsequent

[10] See p. 252 of this translation.

evaporation again depresses the temperature t_3 below the freezing point, since this would demand considerable space out of proportion to its importance.

In order now, as in the preceding case, to obtain a definite idea as to how large are the changes of pressure that may be brought about by the disruption of the supersaturation, the above given example is again worked out numerically in the following table, but under the assumption that there is no supersaturation and that only various degrees of subcooling occur.

Table 2

h	β_1	y_1	y_1'	$y_1 - y_1'$	t_1	t_2	β_2	$\beta_2 - \beta_1$	t_3	β_3	$\beta_3 - \beta_1$
m.	mm.	grms.	grms.	grms.	C.	°C.	mm.	mm.	C°.	mm.	mm.
0	760	12.7	19.4	−6.7	25.0	......	......	......	......	......	......
834	690	12.7	12.7	0.0	16.6	......	......	......	......	......	......
2011	600	12.7	10.2	+2.5	11.0	......	......	......	......	......	......
3512	500	12.7	7.1	5.6	3.0	......	......	......	......	......	......
4063	467	12.7	6.1	6.6	0.0	......	......	......	......	......	......
4360	450	12.7	5.5	7.2	−1.9	0.0	453.2	3.2	−1.3	451.0	1.0
4721	430	12.7	4.9	7.8	−4.0	−0.3	435.9	5.9	−2.5	432.4	2.4
5288	400	12.7	4.1	8.6	−7.4	−3.4	406.0	6.0	− 5.7	402.6	2.6
6319	350	12.7	2.7	10.0	−14.3	−9.6	356.4	6.4	−11.6	353.7	3.7
7474	300	12.7	1.5	11.2	−22.7	−17.5	306.2	6.2	−14.6	303.7	3.7

This table shows that under the adopted assumptions of an initial temperature of 25°, an initial pressure of 760^{mm}, a relative humidity of 66 per cent and adiabatic expansion, condensation occurs at the altitude of 834 meters and the freezing point is reached at the altitude of 4063 meters.

If now the water formed by condensation is carried 300 meters further without freezing (being thereby subcooled 1.9° C.) and if freezing then suddenly occurs, then the (local) pressure suddenly increases 3.2^{mm}.

If the sudden freezing first occurs at the altitude 4721 meters, or for a subcooling of 4.0°C., then the change in pressure is 5.9^{mm}; for still later freezings this change increases but slightly and in fact eventually diminishes.

The reason for this latter diminution lies in the fact that in the formula

$$\beta_2 - \beta_1 = \beta_1 \frac{t_2 - t_1}{273 + t_1}$$

the diminution of β_1 with increasing altitude is at higher altitudes not fully compensated (or offset) by a corresponding increase of the factor

$$\frac{t_2 - t_2}{273 + t_1}$$

so that $\beta_2 - \beta_1$ must attain a maximum which in this special example is to be found at an altitude of about 6500 meters. In this connection I must especially point out the fact that the numbers given in the table for the quantity of water carried upward do not suggest anything improbable. If we compute the volume that a kilogram of air occupies at different altitudes we find that under the above given assumption the quantity of fluid water in a cubic meter amounts at most to 5 grams, that is to say, 5 mg. per liter, a quantity that can certainly be easily floated in a rapidly rising current.

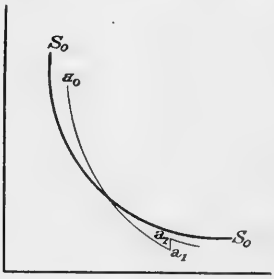


FIG. 33

The diagram (fig. 33), for the process here considered, differs from that presented in fig. 31, which latter related to supersaturation proper. Whereas in that the adiabat $a_0\,a_1$ simply intersected the saturation curve $S_0\,S_0$, in the present case it shows a sharp bend at this intersection, since it changes from the adiabat of the dry stage to that of the rain stage. On the other hand, in the former case of supersaturation the expansion simply continued along the adiabat of the dry stage even after passing the point of saturation.

THUNDERSTORMS

From the above given considerations and developments it results that both supersaturation of air with aqueous vapor and subcooling of the water already condensed must, when these conditions are suddenly dispelled, cause a rapid local rise in atmospheric pressure which in general will last only a short time (except in so far as special circumstances yet to be mentioned do not diminish the restoration of the pressure) and cause a true jump or spring or so-called step up in pressure or a "knick" in the barogram instead of an oscillation at the locality.

But such oscillations and jumps, as already mentioned, are almost always an accompaniment of thunderstorms and it only remains to investigate whether the processes that come into play in thunderstorms are such as would lead us to expect supersaturation or sub-cooling.

This study leads us unavoidably to the consideration of thunder-storm phenomena in general and makes it necessary here to say something on this subject. I must, however, first of all, remark with reference to the well-known classification into cyclonic thunder-storms and heat thunderstorms, that it seems to me that often this classification is not made in an appropriate manner.

Frequently all thunderstorms that are in any way connected with the forerunner of a barometric depression, or are located on the edge of the "low" when this is a very flat or feeble depression, are designated as cyclonic thunderstorms when they may be perfectly characteristic heat thunderstorms. On the other hand, the term heat thunderstorms is often applied only to isolated local thunder-storms, whereas, in my opinion, the majority of all thunderstorms observed in interior regions, with very few exceptions, are decidedly heat thunderstorms.

In fact one might say that the circumstance that the division into these two classes allows of such different points of view, indicates that this classification is not a natural one. But this is by no means the case. On the other hand, I consider this classification as of the highest importance and if the appropriate definitions have not yet been made so clear and sharp as is desirable, then this is, I think, to be ascribed to the circumstance that the first attempts to make this distinction were undertaken in a country where both groups frequently occur and where they merge into each other more than is the case elsewhere.

So far as I know, this classification originated with Mohn and it is precisely in Scandinavia that one has opportunity to observe true cyclonic thunderstorms more frequently than in Germany where they are confined almost entirely to the coasts and, as above stated, only in a few exceptional cases occur in the interior.

For this reason therefore one is tempted in Scandinavia to consider as cyclonic thunderstorms many that I should call heat thunder-storms, but which are not so typically developed as those observed in the interior of the continent.

In fact Mohn and Hildebrandsson in their admirable memoir[11] on the thunderstorms of the Scandinavian peninsula, expressly say: "However it is in Sweden impossible to find a well-defined boundary between these two classes of thunderstorms."

---

[11] Les Orages dans la Péninsule Scandinave. Upsala, 1888, p. 3.

On the other hand, in the interior of Germany the cyclonic thunderstorms are among the greatest rarities, but the heat thunderstorms are so typically developed that one often recognizes their characteristic peculiarities reproduced in special Scandinavian storms that Mohn and Hildebrandsson have considered as cyclonic thunderstorms.

For instance, I should consider the thunderstorm of August 6, 1881, at least after its penetration into the interior of Scandinavia, as undoubtedly a heat thunderstorm.

It is precisely the possibility of such fundamental differences in our views that appears to demand that the path to a better understanding be indicated and the definitions be made more exact than has hitherto been the case.

As to this matter the most important indications are given us by the great differences in the diurnal and annual periods of the thunderstorms on the coasts and in the interior of continents, as shown in the above-mentioned observations in Scandinavia, as well as in material gathered elsewhere.

Whereas in the continental interior the maximum frequency of thunderstorms occurs in the afternoon hours and in fact only a little later than the maximum of temperature, while a very feebly indicated secondary maximum in the early morning hours can only be demonstrated with much labor; on the other hand, in the neighborhood of the oceans the nocturnal thunderstorms are much more frequent. Thus, for instance, in the coast region of Schleswig-Holstein the absolute maximum of destructive lightning occurs between midnight and 3 a.m.[12] Similarly in the maritime regions the winter thunderstorms are much more frequent than in the interior, and it is the winter storms that happen so frequently in the night time, so that in fact these determine the peculiarities of the diurnal period in that region.

In this respect the oceanic exposure of Norway as compared with Sweden makes itself felt to a most remarkable extent; whereas in Norway in the ten years 1871 to 1880 there were 235 January thunderstorms as contrasted with 1811 July storms, on the other hand, during these same years there were in Sweden 14 January and 4419 July thunderstorms.[13] But in the true coast region of Norway the numbers for January and July were in fact 198 and 646, respec-

[12] G. Hellmann, Zeitschrift des Preuss. Statistischen Bureaus, Vol. 26, 1886, p. 179.

[13] H. Mohn and H. H. Hildebrandsson. Les Orages dans la Péninsule Scandinave. Upsala, 1888, p. 39.

tively, so that in this region the secondary maximum for January in the annual curve of thunder storms attains a very great significance.

This comparison plainly shows that we have to do with two fundamentally different groups of thunderstorm phenomena which certainly must owe their origin to very different causes. Indeed it is the unequal relative frequency of occurrence of cyclonic and heat thunderstorms in the two regions to which we would ascribe these peculiarities in the diurnal and annual periods of the coastal and interior regions, as Hellmann has already clearly stated[14] in the year 1885, when he wrote:

> The cyclonic thunderstorms occur most frequently in the colder parts of the year and the day; the heat thunderstorms occur most frequently in the warmer parts of the year and the day.

In the same place, Hellmann also emphasizes the following:

> That the winter thunderstorms, or those of the cold season from October to March, occur always in connection with cyclonic storms and frequently at night time; that they often occur rapidly over long stretches of country, but individually in rather more interrupted succession and in rather narrower extent of territory than the average thunderstorm of the warm season; that they are of shorter duration but generally accompanied by some strokes of lightning which on account of the low altitude of the clouds from which they emanate produce conflagrations more frequently than in the summer season.

Whereas in these sentences, which I heartily indorse, it is expressly stated that the cyclonic thunderstorms, even when their paths are very long, have only a small extent, we also find elsewhere[15] the remark that the cyclonic thunderstorms are the larger while the heat thunderstorms, by contrast, seem as rather local phenomena.

On the other hand, it seems to me that most of the thunderstorms occurring in Central Europe, many of whose fronts extend from the German coasts to the Alps, must be classed as heat thunderstorms.

The one feature common to all thunderstorms is the presence of a strong ascending current of air as the fundamental condition for the formation of the great clouds that never fail in any thunderstorm, but the process by which this ascending current comes about is quite different in the two kinds of thunderstorms.

I will now attempt to give such definitions of the two groups as will, as far as possible, prevent confusion:

---

[14] G. Hellmann, Meteorologische Zeitschrift, II, 1885, p. 445.
[15] Sohneke: Meteorologische Zeitschrift, V, 1888, p. 413.

(*a*) *Cyclonic Thunderstorms.*—Cyclonic thunderstorms accompany the central parts of the deeper, well-developed barometric depressions. They are phenomena characteristic of a rapidly ascending current of air such as is brought about in cyclones by the greater disturbances of the atmospheric equilibrium. Hence they occur during disturbed, cloudy weather and especially in the neighborhood of the paths of barometric depressions and wherever these develop into specially well-marked lows, or on the oceans and on the coasts. During these cyclonic thunderstorms the general motion of the atmosphere is cyclonic. This cyclonic motion itself goes on in a horizontal direction with a slightly upward component around a vertical axis or at least one that is inclined so as to intersect the earth's surface. The annual and diurnal periods of these thunderstorms follow those of the cyclonic storms themselves.

The cause of these thunderstorms is fundamentally the same as that of the cyclones in general and can therefore at the present time not be given with any more certainty than that of the cyclones themselves, which according to the most recent researches are now no longer to be explained as due to temperature and moisture conditions alone, but to no little extent are consequences of the general circulation of the atmosphere.

The question whether there are other special circumstances on which depend the presence or absence of thunderstorms as companions of the cyclones must be cleared up by further investigations.

(*b*) *Heat Thunderstorms.*—While, as just stated, the cyclonic thunderstorms occur during disturbed, stormy weather and decided cyclonic movements of the atmosphere; on the other hand, the heat thunderstorm demands quiet air for its formation without decided cyclonic or anticyclonic movement and unrestricted powerful insolation. These occur neither in the central parts of the barometric depressions nor in those of barometric highs, but in the border regions between these two.

The regions in which heat thunderstorms originate when there is sufficient insolation are the areas of slight depression with scarcely recognizable centers[16] extending in advance of a large barometric depression such as are designated as thunderstorm

[16] These partial depressions or "pockets" are often so imperfectly developed that their shapes on the charts of isobars are entirely changed by slight changes in the method of reduction of pressure to sea-level, so that one can hardly speak of their centers.

These slight depressions are most clearly revealed when we make use of pockets on our charts of isobars, like shallow furrows or troughs between two areas of maxima, ridges or tongues of high pressure between two low areas and especially between shallow depressions of wide extent.

In other words, heat thunderstorms originate in regions above which there is neither a decided ascending nor descending current, so that at the earth's surface there is opportunity for such an overheating of the air as would bring about unstable equilibrium in this part of the atmosphere.

In this connection, in general, too much importance has, in my opinion, been given to the depressions present in the distribution of atmospheric pressure just described, and thus the difference between the heat thunderstorm and cyclonic thunderstorm has been effaced, whereas these depressions are so infrequently developed in the case of heat thunderstorms that they form a sort of intermediary condition between barometric maxima and minima.

We may therefore just as properly consider the protruding arm of the area of maximum pressure which is necessary to the formation of the so-called "thunderstorm pocket," or the ridge or tongue of high pressure that most emphatically favors the formation of thunderstorms, as the important feature and give less attention to the accompanying barometric depression.

The only important consideration is that there be the possibility of an unusual rise in the temperature of the lowest layer of air so that the potential temperature below may be higher than above, i. e., so that unstable equilibrium may occur.

But this is only possible when in the preparatory stage there is neither a decided ascending current (such as occurs in areas of low pressure where the lower air being warmed is carried along and moreover by reason of the cloud necessarily formed by the ascent checks the superheating) nor a strong descending current (such as is present in the interior of an anticyclone causing a steady outflow of the lower layer or the rapid dissolution of individual local ascending clouds).

---

my method of partial isobars, which however assumes a very accurate reduction of atmospheric pressure to sea-level,

Compare M. v. Rohr (Die gewitter, etc.): "The thunderstorm of December 11, 1891, in connection with the simultaneous weather." Publications of the Royal Preuss. Meteorological Institute. Results of thunderstorm observations in the year 1891. Berlin, 1895, pp. xi–xxxv.

Also

W. Wundt (Barometrische Theiledepression-en, etc.): "Barometric pockets and their wave-like repetition." Memoirs of the Royal Prussian Meteorological Institute, Vol. II, No. 4, Berlin, 1904. Note added 1905.

That the superheating of the lowest layer of air and consequently the accompanying unstable equilibrium within it, is a regular precursor of heat thunderstorms, can certainly be considered as a fact, even now, although the further establishment of its certainty is very desirable by means of observations such as can be furnished in necessary completeness only by self-registering instruments.

Thus from the observations at Freiburg in Bavaria and on the Höchenschwand, 719 meters higher, Sohnke[17] has shown that in general the difference of temperature between these two stations before the thunderstorm breaks, exceeds its normal value, while in three cases (1881, June 3 and June 26, and 1882, July 22) it passed the limit of unstable equilibrium; indeed on June 3, 1881, it exceeded this limit by a very considerable amount, since on that day the temperature diminished at the rate of 1.53° C. per 100 meters.[18]

Similarly, in the memoir by Assmann[19] on The Thunderstorms in Central Germany, we find on p. 68 a collection of observations of temperature on the Inselsberg and at various lower stations made at special moments before the outbreak of thunderstorms, including a series of cases in which the diminution of temperature with altitude exceeds 1° C. per 100 meters and where consequently the limit of unstable equilibrium is exceeded.

An excessive superheating of the lower strata of air was found in the case of the thunderstorm of March 29, 1888, minutely studied by Assmann,[20] on which day there were observed gradients as high as 2.26° C. per 100 meters.

---

[17] Sohnke (Der Ursprung, etc.): The origin of the electricity of thunderstorms, etc. Jena, 1885, pp. 69 et seq.

[18] But I cannot agree with Sohnke when he thinks that from these observations he is warranted in drawing the conclusion that on such days as these the isothermal surface of 0° C. lies especially low in the atmosphere. In the majority of the cases quoted by him the temperature is above its normal value even in the higher levels of the atmosphere, and although in the lower levels the departure from normal is still greater than in the upper, yet this does not warrant us in applying the corresponding temperature gradient to higher levels. We are no more justified in doing this than we are in arguing as to the temperature at very high levels from the vertical gradients observed in the anticyclones of the winter season. Moreover I don't understand why Sohnke attaches such great importance to the special low position of this surface before a thunderstorm, since in support of his theory we need only to consider where this surface is within the thunder cloud itself, a matter that we can only determine at present by computation based on the assumption of adiabatic expansion.

[19] Assmann: Die Gewitter in Mittel Deutschland. Halle a. S. 1885.

[20] See von Bezold (Ergebnisse, etc.). Results of Met. Obs. in Prussia for the year 1888, p. lvii. Berlin, 1891.

If now the conditions above described are fulfilled; if there be no decided ascending or descending movement of the air, while the sun heats the ground very hot, then unstable equilibrium will be produced at different portions of the earth's surface and especially where this heating is favored by the character of the ground.

Thus, during the summer months at least, at any given moment the heating effect may be about the same all along a line inclined to the meridian and trending NNW. and SSE. since points on such a line will have experienced equal durations of insolation.

Consequently, and independently of the influence of the general distribution of pressure, an approximately equal and simultaneous superheating of the lower air and hence unstable equilibrium will occur on any given day along such a nearly meridional line.

Thus, then, at first there will be a series of centers arranged along this line, where favored by local peculiarities the lower air will rise; on account of the increase of buoyancy due to the condensation, this ascending current will rise higher and higher until it is no longer able to raise and carry up the condensed mass of water, when it falls and we say the thunderstorm has broken.

As will be shown later on, this fall generally begins at altitudes at which the temperature is below freezing and in most cases therefore the precipitation at the higher levels consists of hail or sleet, which, however, seldom reaches to the ground but melts during the fall and therefore reduces the temperature of the lowest stratum in the well-known manner.

Hence, after the thunderstorm begins there is a sudden fall of temperature, the surfaces of equal barometric pressure press closer together within the region of precipitation, while the air rising at the front or eastern edge of the thunderstorm and adjoining the still warmer parts of the atmosphere overflows toward the cooled side and causes a rise of pressure there.

On the other hand, in the lower stratum, the air flows with great force eastward out of the region of precipitation, the air resting in front of it is disturbed and, if not already so, is now forced to a rapid ascent without necessarily being itself in a state of unstable equilibrium.

Thus the thunderstorm renews itself continually on its front edge and if the original superheating was great enough, and the atmosphere in general sufficiently quiet, to allow the individual thunderstorms developed along the above-mentioned meridional line to unite into one large band, then the thunderstorm front thus originating rolls farther eastward as a great whirl with horizontal axis until the declining of the sun and the accompanying cooling of the lower air gradually enfeeble the conditions that favor the process and thus during the night time bring about a gradual extinction of the thunderstorm.

As a special indication of the existence of well-developed whirls and horizontal axes, must be mentioned the fact that the wind then blows nearly perpendicularly to the isobars, constituting an apparent exception to the basic law of the winds and for which Moller[21] has given an explanation.

The preceding explanation does not exclude the existence of individual small vortex whirls around a vertical axis, since it may be that the thunderstorm roll is not continuous or that irregularities in special places develop such vortices. But it seems to me that such details are not sufficient to justify designating such storms as vortex or cyclonic thunderstorms. The description here given corresponds substantially with the presentation of the subject of the propagation of thunderstorms as given by Koeppen[22] ten years ago in his "Untersuchungen, etc.," "Investigations Relative to the Thunderstorm of August 9, 1881."

I must return, however, to this subject again because I recognize the characteristics of "heat thunderstorms" in this method of origination and propagation, which, indeed, has been subsequently confirmed in the case of many great thunderstorms both by the investigations of Ciro Ferrari[23] as also by the investigations carried on in Bavaria and the neighboring States and later also in North Germany.

I have also intentionally attached small importance to the presence of areas of depression or barometric lows, but more rather to the fact that there exists a region in which neither the cyclonic nor the anticyclonic character is especially well marked.

I have therefore entirely omitted to consider the circulation of the upper atmosphere as dependent on the general distribution of pressure over large areas and have made this substantially provisional study under the assumption that the general circulation of the atmosphere does not come into consideration.

It seems important to me to make it clear that under this assumption we must expect a propagation of the thunderstorms from west

[21] Zeitschrift d. Oest. Gesell. für Met. 1884, XIX, pp. 80–84.

[22] Ann. d. Hydrographie, 1882, X, pp. 595 and 714. Compare also Sprung, Lehrbuch, etc.: "Treatise on Meteorology," pp. 294 et seq.

[23] Annali dell Ufficio Centrale di Meteorologia, 1883, Vol. V, part 1, and 1884, Vol. VII.

to east, whereas it is a well-known fact that under the influence of the general circulation of the atmosphere[24] the opposite direction of motion can sometimes occur.

The thunderstorms that come from the east are, however, always relatively infrequent and moreover in comparison with those from the west are feebly developed and have a smaller progressive velocity.[25]

I find the reason for this in the consideration that without the coöperation of the general circulation of the air the direction of propagation must always be from west to east, and that therefore in those cases where the region in question lies under the influence of a barometric depression located to the southward so that the general atmospheric movement is from east to west, there are two opposing factors that disturb the vigorous and typical development of the phenomenon.

Finally, I may add that it seems to me appropriate to designate as "Front gewitter" those thunderstorms that present the just described *band* advancing perpendicularly to its length or "broadside on," whereas the individual scattered thunderstorms, such as frequently occur under otherwise similar conditions I call "Erratic thunderstorms," just as was done by Fron.

The fact that in individual years, and often in a series of consecutive years, "Front" thunderstorms occur, whereas in other years only "Erratic" storms occur, seems to me to be a question that is in the highest degree worthy of a thorough study and one to which I would therefore call especial attention.

Moreover, it is very clear that the occurrence of Front storms must depend largely on the configuration of the country and on the features of the soil.

Therefore in accord with this idea such storms attain to a greater development in the interior of France and Germany than in the Scandinavian Peninsula or in Italy, where the plains of Sweden and the Po satisfy the required conditions to a high degree.

It is also apparent that the sloping surfaces of meridional mountain ranges, as in the Vosges, the Black Forest, and the Bohemian Forest, must especially favor the formation of Front storms.

After these general remarks on the theory of the formation of

---

[24] Compare von Bezold and Lang (Beob. etc.): Observations at the meteorological stations in the Kingdom of Bavaria. Annual volume for 1880, pp. xviii–xx.

[25] C. Lang, in Lang and Erk: Observations at the meteorological stations in the Kingdom of Bavaria. Annual volume for 1888, pp. xxxvii–xlix.

thunderstorms (which one may consider as a departure) I will now return to the principal question, i. e., what rôle the supersaturation and subcooling may play in thunderstorms.

In this investigation I have in mind only the heat thunderstorms, since the observational material available to me for true cyclonic thunderstorms is too scanty and especially since I do not know whether in these latter anyone has observed the peculiar variation of pressure that can be almost invariably recognized in the barograms during heat storms.

As has already been stated, it is at present still difficult to decide to what extent true supersaturations occur, since as yet we have no sure foundation of experience on this point.

On the other hand, it seems to me that the above-mentioned observations lately published by Hellmann on the behavior of the thunderstorm rains that pass from the west eastward over Berlin, indicate that supersaturations do play a part in thunderstorms. For the great clouds of dust and smoke that always exist over that city must hinder the formation of the conditions of supersaturation.

Again the fact demonstrated by me many years ago that buildings within populous cities are much less frequently injured by lightning than those in the surrounding country, may indicate that the severity of the thunderstorms experiences a diminution which may be referred back to similar causes. However, I readily acknowledge that such considerations have only slight force as a demonstration.

It is otherwise with the peculiar movements and uprisings that the thunder clouds show even when they have not attained to elevations at which we may expect subcooling. The shapes of the clouds and especially their changes in appearance do not at all correspond to those that we should expect from currents that are steadily ascending and are accompanied by condensations that are only the result of expansion. As already remarked, by close observation of the clouds we can scarcely avoid the thought that in their interior there are forces in action that cause the peculiar expansions and projections of the individual cumulus-heads. One can scarcely suggest any other forces than the heating that must occur within the clouds when there is a sudden release from the supersaturated or subcooled conditions.

This assumption of the occurrence of supersaturation also receives an important support when we study the variations of pressure during thunderstorms in cases where the clouds attain only slight altitudes, as is not infrequently the case in the Riesengebirge, according to the observations of Reichmann.[26] It is therefore desirable that special attention be given to the occurrence of low-lying thunderstorm clouds of slight power.

In reference to subcooling we are in a more favorable position than in reference to supersaturation. Here we have to do with demonstrated facts and it is only necessary to give precision to our ideas as to the formation of clouds by adiabatic expansion and especially the formation of thunderstorm clouds.

When an active ascending current exists condensation will occur when the dew-point is attained, in so far as the necessary nuclei are present. If now the expansion continues, more and more water will collect on these nuclei while presumably the number of fog particles is not increased.

Consequently the individual fog particles become larger and larger, and by this means as well as by the union of many into one, they develop into small drops that may be visible to the unaided eye.

But in a sufficiently active ascending current these droplets will by no means sink but be carried up to great altitudes, so long as their magnitudes do not exceed a certain limit, which of course depends on the velocity of the ascending current and on the density of the air.

If this process did not proceed in this manner and if the water particles at once fell down as rain, then a progressive increase in the size of rainless clouds would be impossible.

Moreover, the clouds could never attain that appearance that we are accustomed to see in the cumulus clouds which reminds us of compact masses, but they could only produce the impression of streaks of fog or mist, which would be thinner and more transparent as the altitude increased without having any sharp boundary on the upper side.

Since all this is not the case in nature we must assume that the fog particles formed in the lower part of the cloud are at least to some extent carried up to the upper boundary, after which, by falling through the lower layers of the cloud, they grow larger.

If in this ascension the particles pass through the isotherm of 0° C. or freezing, still it does not follow that they will freeze into ice, but it is quite possible that they may retain their fluid condition while being carried up into regions where the temperature is far below freezing.

Now the ascending current of air has by no means ceased at that

---

[26] Met. Zeit., 1886, III, pp. 249 et seq. and 1887, IV, pp. 164 et seq.

altitude at which the water particles reverse their motion and begin to fall in consequence of the increased size to which they have attained by reason of the progressive condensation attending their long path.

Rather will this current continue far above the possible boundary of the cloud not only in the case of the summer cumuli proper, but in all cases where we have to do with the ascension of individual masses of air and their penetration into upper strata.

But in fact in the case of the heat thunderstorms with their perpetually renewed whirl about a horizontal axis we have to do in a certain sense with a steady process even if, as such, it is moving forward.

In this current of air flowing outward from a "thunderhead" condensation must again occur in consequence of the progressive cooling.[27] But in this case, on the one hand, the quantity of aqueous vapor coming into consideration is slight; on the other hand, the precipitation will be directly in the form of crystals of ice or snow because of the temperature prevailing at these altitudes.

In this process which is comparable with sublimation, the supersaturation or the subcooling can no longer play any important rôle even when that might otherwise be possible because of the small quantity of water remaining present. Hence in these clouds there do not exist the protruding and expanding heads, but in accord with the steadily ascending current they develop rather in the screen-like form of a layer of cirrus.

Doubts have indeed been expressed as to whether the screen of cirrus that accompanies the thunderstorm cloud really consists always of ice or snow, since the characteristic optical phenomena are not always observable in it. Hence it seems to me to be important to point out that between ice clouds at comparatively low altitudes, such as correspond to the layer of cirrus, and those in the highest layers of the atmosphere, there may be very considerable differences that may exert an influence on the optical behavior.[28] Thus, at very low temperatures ice needles easily form, whereas at temperatures that lie near the freezing point the precipitation takes place in the form of starlike crystals of snow or indeed masses of snow, which latter must be less conducive to the development of the well-known optical phenomena.

---

[27] Compare pp. 237 et seq. of the preceding collection of translations.

[28] According to recent investigations it must be recognized as very probable that the ions play an important part in the formation of the cirrus *screen* (overflow or false cirrus or cirrus veil or cirrus screen) as indeed in the formation of cirri in general. (Note added in 1905. W. v. B.)

From these remarks on the cirrus overflow we now again turn our attention to the thunderstorm cumulus itself.

When this cumulus penetrates upward into regions where the temperature sinks considerably below freezing the subcooling of the cloudy element (vapor) must finally attain its limit and some exterior shock will alone be needed to initiate a sudden freezing. But, as was shown in the first part of this memoir, a warming and sudden increase of pressure must go hand in hand with this freezing. Of course an expansion must follow this rise of pressure and thus we may explain the fact that new cumuli of considerable size often suddenly burst forth from the thunderstorm cumulus.

Thus, on the 6th June, 1889, on the Summit of the Säntis, Assmann took some photographs of an increasing thundercloud, from which in a very short time there broke out of the cloud a typical cumulus dome which afterwards developed into a cirrus of mushroom shape with broad cirrus screen.

This change of form corresponded completely with the above given formation of the cirrus screen.

When in consequence of such a dissipation of the subcooled condition as above assumed, and as appears to make itself known in the variations of pressure, the now frozen masses are thrown far above the altitude that they would have attained in the subcooled condition, then, after the extinguishment of the upward impulse there must occur a sinking downward, at least of the heavier masses, while the current that blows through the cloud, and is the cause of the whole phenomenon, still continues and thus gives occasion to the formation of the cirrus screen.[29]

According to the above-mentioned investigations of Assmann on the Brocken, as well as in accord with more recent observations by him[30], such subcooled fog particles never form crystals of ice or snow, but only make lumps of ice without internal structure.

But sleet is formed by their combination. The assumption of a sudden freezing of subcooled fog particles or very small droplets therefore explains the formation of both sleet and hail without any difficulty.

At first the frozen subcooled droplets unite into a little pellet of sleet, since they, by falling and striking other subcooled droplets, probably bring these also to freezing and at the same moment congeal into one mass. When these pellets drop into lower strata in

---

[29] Compare also Möller: Met. Zeit.. Vol. II, 1890, pp. 220–222.

[30] Met. Zeit., VI, 1889, pp. 339–342.

which the particles of water have a temperature more nearly that of the freezing point, they cover themselves with a coating of clear ice, upon which during a second ascent, such as frequently occurs in the commotion within a thunder cloud, there is deposited a second layer of subcooled particles, after which the hailstone, now become heavier, again sinks and is again covered with clear ice.

In this way is formed the cloudy milky nucleus with its surrounding concentric layers such as we find in hailstones.

It seems quite natural that a regelation occurs when the hailstones already formed strike violently together and thus grow together into the irregular forms that are frequent among hailstones.

Thus it is that from the assumption that subcooled water particles play an important rôle in thunder clouds, there follow easily and naturally the series of phenomena that actually do accompany a thunderstorm.

Still there remains one great difficulty to be overcome in that it is not easy to get any clear idea of the process of dissipation of the subcooled condition.

According to observations that have been frequently made, among which I need only mention those of Assmann, Moedebeck, and Gross, the freezing of the subcooled water at individual points of the cloud does not spread throughout the whole cloud at once. Whereas on the Brocken all subcooled microscopic droplets immediately freeze when they strike a solid body and gradually inclose this body in a sheet of ice, and whereas in the oft-cited case of the balloon voyage of 1891, June 19, all the rigging of the balloon became rapidly covered with ice, still the fog or cloud as such remained unchanged.

It is not easy to understand how this freezing spreads in a short time throughout the larger part of a cloud, and yet this must be the case if in fact the sudden rise in pressure, as already described, with all its consequences is to occur.

Whether this is brought about by the crystals of ice that fall from the cirrus screen and by contact with the subcooled water particles cause this sudden freezing, or whether some electrical process here comes into play, are still open questions.

On the other hand, I must consider the heavy showers characteristic of thunderstorms as a proof that in these phenomena the above-described dissipation does play a part.

Moreover, it is not improbable that many thunderstorm rains begin in the upper regions as sleet or hail and only become rain in the lower strata of the atmosphere.

At least sleet and hail are certainly observed at elevated stations more frequently than in the low lands.

Equally do the large drops that not infrequently occur in thunderstorm showers suggest that in such cases we have to do with melted hail or sleet. I consider this assumption especially reasonable since I have often had occasion to observe that the occurrence of a heavy fall of hail is announced by the immediately preceding fall of very large drops of rain.

In such cases I have observed drops of such a size as can only exist momentarily and can indeed only be explained as being melted hailstones.

I therefore consider it probable that sleet and hail play a greater part in thunderstorms than we have generally assumed, and that their relatively rare occurrence at the earth's surface is to be explained by the fact that they frequently arrive in a melted condition.[31]

The above-given presentation of the processes going on within the thunder cloud appears to greatly favor the hypothesis framed by Sohnke as to the electricity of the thunderstorms.

On the other hand, I would say that for myself at least I do not, on this account, accept the Sohnke theory. For, on the one hand, it is difficult for me to understand how a permanent separation of the positive and negative electricities can be brought about by the mutual friction of the falling sleet or hail, since at temperatures below freezing the water particles must immediately freeze together with ice particles, whereas at temperatures above freezing the surfaces of the hailstones are already covered with water and hence there can only be friction of water on water.

On the other hand, even if it be possible to overcome this objection, still I cannot agree with the reasoning by which Sohnke refers even the normal electricity of the atmosphere back to the same source. Especially does it seem to me exceedingly improper to attribute such great importance, even on perfectly clear days, to

[31] In these earlier investigations the processes that occur in thunderstorms are considered only from purely thermodynamic points of view and the nuclei of condensation are only those that have long been recognized. According to recent investigations it is probable that electrons or perhaps also the cathode rays proceeding from the sun play a part in the condensation. Moreover in the formation of hail electric processes seem to play an important part. (See W. Trabert: Die Bildung des Hagels. Met. Zeit., 1889, XVI, pp. 433–447.) But since all these questions are far from being settled, therefore I have not attempted to rewrite the whole memoir from these new points of view. (Note added in 1905. W. v. B.)

the isothermal surface of 0° C. or to consider it as the carrier of the positive electricity, when there is nothing else present in this layer except atmospheric air and gaseous matter and no condensed aqueous vapor.

I could indeed imagine that the cirrus clouds are the carriers of the positive electricity, but since these clouds are wholly absent on many clear days while on others they float at very great altitudes, therefore this assumption would not suffice to explain the diurnal and annual periods in atmospheric electricity, whose cause Sohnke thinks he has found in the oscillations of the altitude of the isothermal surface of 0° C.

Moreover, if this surface possesses any such importance then its entrance into the earth, that is to say, the fall of the air temperature at the earth's surface below the freezing point, must cause a considerable diminution of the potential gradient, if indeed it does not cause a complete reversal, whereas, on the other hand it is precisely during very cold and dry winter weather that this potential gradient has especially high values.

But these are questions that really do not belong here. If I have discussed them, I have done it for fear lest any one should imagine in the views that I have presented a new support for a theory that, in my opinion, has found too ready acceptance by many meteorologists and which we ought to view with critical eye, although I do not deny that it has some value and is worthy of closer study.

The expositions contained in the preceding memoir may be summarized in the following theorems:

(1) If supersaturated vapor or subcooled water is present in the atmosphere then the sudden dissipation of such a condition must produce a quick variation of pressure that must make itself visible by a rapid rise and subsequent fall in the barometer.

(2) If cooling precipitations fall quickly after this dissipation then the barometric fall will be diminished or even entirely prevented and a jump in pressure or "step up" rather than an oscillation, will take place by reason of the contraction of the surfaces of equal pressure due to the cooling and the consequent inflow of air from above.[32]

---

[32] On the theory of the origin of barometric jumps compare the following memoirs by Dr. Max Margules: Vergleichung der Barogramme, etc. Met. Zeit., XIV, 1897, pp. 241–253. Einige Barogramme und Thermogramme, etc. Met. Zeit., XV, 1898, p. 1–16. (Note added in 1905. W. v B.)

(3) Such variations of atmospheric pressure and barometric steps occur frequently during thunderstorms and of magnitudes such as without difficulty may be referred back to supersaturation or subcooling.

(4) It is also a fact that during thunderstorms the conditions that favor the existence of such unstable conditions are fulfilled, especially should subcooling occur very frequently in the higher portions of thunder clouds.

(5) Since the dissipation of such conditions must produce sudden warming in individual locations, therefore by such processes we must explain the peculiar changes of form that we observe in the thunderstorm cumuli and which we cannot consider as a simple consequence of a steadily ascending current even when this ascent occurs in connection with vortex motions.

(6) The formation of sleet and hail may without difficulty be referred back to subcooling.

XIII

ON THE THERMODYNAMICS OF THE ATMOSPHERE

BY PROF. DR. WM. VON BEZOLD

Fifth Communication[1]

(Sitzb. Berlin Academy 1900, pp. 356–372. Translated from the Gesammelte Abhandlungen von Wm. v. Bezold, Braunschweig 1906, pp. 216–220)

THE CLIMATOLOGICAL IMPORTANCE OF THE THEORY OF ASCENDING AND DESCENDING CURRENTS OF AIR

In the second of this series of memoirs[2] I have submitted to more precise consideration the idea of "potential temperature" first introduced by H. von Helmholtz under the designation "thermal content," and have therein deduced a theorem that has great similarity with the second fundamental theorem of the mechanical theory of heat.

From the circumstance demonstrated in the first article of this series, that the changes of saturated moist air, when heat is neither added nor abstracted and when the precipitated water or ice falls from it, are not as a whole reversible but only in their smallest portions, it resulted that in such changes of condition the potential temperatures never diminish but can only increase.

I then drew various consequences from this theorem that are of fundamental importance not only in the consideration of individual processes but also in understanding the most important prominent facts in the general averages.

Thus, in the first place the average diminution of temperature with altitude finds its explanation in this theorem, and secondly in studying the average temperatures for whole circles of latitude, the results deduced from this theorem stand clearly forth.

---

[1] The substance of the present memoir (which could only be established by correct observations after the conclusion of the work then being done on the "Results of the Scientific Balloon Ascensions from Berlin") had been previously communicated to the Berlin Academy at its session of the 5th of May, 1898. (Note added in 1905. W. v. B.)

[2] See Mechanics of the Earth's Atmosphere, 1891, p. 243. Smithsonian Miscellaneous Collections Vol. XXXIV.—C. A.

The climatological aspect of that second memoir which is now to be considered seems to have remained quite unnoticed and equally so the various conclusions as to the "exchange of heat,"[3] which I would also consider of fundamental importance for Climatology.

I will therefore here develop more fully the results of a climatological character that were only indicated in the first mentioned memoir and as preliminary will more clearly illustrate the point of departure.

We must recall the fact that the expansion of saturated moist air without adding or abstracting heat should only be called *adiabatic* when the precipitated water remains floating in the air. As soon as it wholly or partly falls away as precipitation then this term is no longer strictly applicable since the whole expenditure of the internal energy is not converted into external work.

In this case the falling particles of water or ice, since they still have the temperature of the mixture and not of absolute zero, withdraw from the mixture energy that has not yet been expended in the work of expansion.

Therefore I have called such processes "pseudo-adiabatic."

Since the quantities of energy that are lost by the falling away of the condensation are very small, therefore the formulæ for the adiabats and the pseudo-adiabats differ from each other only very slightly. Therefore in the computations and in the graphic presentations we may consider them as identical, that is to say, we may use the formulæ and curves for the adiabats instead of those for the pseudo-adiabats.

On the other hand, an incisive difference is manifest as soon as the expansion changes to compression or when the ascending current becomes a descending one.

In this case it makes a very great difference whether the water condensed during the ascent is carried along with the air or falls away from it. If it is carried along, that is, if the expansion truly follows the adiabat, then the compression follows exactly the same law, so that the change of condition is truly reversible; but if the expansion is "pseudo-adiabatic" it follows a law entirely different from that which obtained during the expansion.

Since the water or ice which is formed scarcely ever falls away completely immediately after its formation, for in that case precipitation would fall from a clear sky, therefore this departure from the adiabatic law does not immediately follow the passage from expan-

[3] See my memoir of 1892, Sitzb. Berlin Acad., pp. 1139–1178. or No. XV, pp. 316–356 of the Gesammelte Abhandlungen (or No. XIX of this collection of translations).

sion to compression but only after the last of the water and ice is completely evaporated. Therefore the pseudo-adiabatic change of condition is in its minutest features always reversible but in its entirety is not so. I have therefore designated such processes as "partially reversible" or "pseudo-reversible."

But this theorem that the so-called adiabatic changes of condition of moist air in the free atmosphere are *not* completely reversible, is one of the most important for all theoretical meteorology and climatology. By it there become explicable not only the foehn phenomena, whose study, as is well known, formed the starting point for all incisive investigations, but also the contrast in the character of the weather in the regions of high and low atmospheric pressure; the difference in the conditions on the windward and leeward sides of mountain ranges; the distribution of cloudiness and precipitation in general; and finally, as above mentioned, the law of the average diminution of temperature with altitude, at least in its principal features, together with the relatively slight diminution of the average temperature of whole small circles of latitudes from the equator up to the "horse" latitudes.

It is therefore worth while to first examine most carefully the verbal expression of this theorem and then to deduce more rigorously and elucidate more thoroughly than I did in my first memoir[4] the conclusions that result from it.

The shortest and most rigid statement of the theorem is given when we introduce the idea of potential temperature. I have elsewhere done this and expressed the theorem in two ways as follows:

"In the adiabatic change of condition of moist air, the potential temperature remains unchanged so long as the dry stage is not passed, but it increases when condensation begins and in proportion to the quantity of water that is abstracted."

Or otherwise it is expressed as follows when specially applied to atmospheric processes:

"Adiabatic changes in the free atmosphere and when there is no evaporation, leave the potential temperature either unchanged or higher."

In both of these equivalent methods of expression I have at that time made it understood that I assumed the adiabatic and pseudo-adiabatic processes as equivalent, as we may safely do both in the computations and the graphic presentations.

---

[4] See Sitzb. Berlin, 1888, pp. 1189–1206, or pp. 243–257 of the preceding collection of translations.

But since this is not allowable from a strictly theoretical point of view, therefore I would prefer to substitute the following method of enunciation:

"An adiabatic change of condition of moist air leaves the potential temperature unchanged, but a pseudo-adiabatic change raises the potential temperature. The rise increases with the amount of water that is abstracted."

Of course these theorems are only applicable so long as we exclude cases of mixture with air having a different temperature and different absolute humidity, as also cases where water is added from any source whatever.

The precipitation or abstraction of water during adiabatic changes of condition is also excluded because the very definition of "adiabatic change" implies that the mass under consideration, that is, the mixture of air and water remains the same quantitatively notwithstanding the change of condition as to aggregation.

On the other hand, under "pseudo-adiabatic changes" are included all those in which the condensations that are formed either wholly or partially fall away so that the quantity of water mixed with the given quantity or unit mass of dry air is diminished by the abstraction or precipitation of the condensation. An increase of this mass of water by addition from outside is also excluded by this definition.

In consideration of adherence to this latter definition of our terms, the following theorem may be expressed:

"An adiabatic change of condition may be either an expansion or compression. A pseudo-adiabatic change is only possible with expansion."

Since the ascension of a mass of air is always attended by expansion, and is pesudo-adiabatic after the beginning of the formation of precipitation, if this expansion takes place without addition or abstraction of heat, therefore by this process the potential temperature of the upper layers of air is increased.

Since the potential temperature remains constant in descending currents so long as no heat is added or taken away, therefore the vertical movements of the air without change of heat should be alone sufficient to make the average diminution of temperature with altitude smaller than would result if the air contained no aqueous vapor. Therefore, independent of all processes of gain or loss of heat the simple ascent and descent of currents of air suffice to explain why the temperature diminishes with altitude and in fact slower than 1° (or more exactly than 0.99° C.) per 100 meters.

Consequently for the reason thus explained the mean gradient of temperature with altitude is smaller in the actual atmosphere than for the convective equilibrium of pure dry air, and therefore on the average the earth's atmosphere is in stable equilibrium.

This fact was already recognized by Lord Kelvin.[5]

In the second memoir I have expressed this theorem in the following form:

"In general the potential temperature of the atmosphere increases with the altitude."

Evidently it will now be of interest to investigate more accurately what the diminution of temperature with altitude would be were it dependent only on vertical circulation without addition or abstraction of heat.

When one has first clearly understood this question then he can obtain from observational data an idea as to how far these processes actually affect the atmosphere and what rôle other circumstances play such as hitherto have been almost exclusively considered.

[In the original memoir there follows an investigation into "the average distribution of temperature in a vertical direction." In order to avoid repetition this portion is not reprinted, since this subject is treated in fuller manner in the following memoir.[6] Moreover, my succeeding remarks on the "influence of complete convection on the mean temperature of the circles of latitude" are not reprinted, since these are contained partly in No. VI of this collection[7] and partly in No. XVI of this collection.[8] Note added 1905, W. v. B.]

---

[5] On the convective equilibrium of the atmosphere, dated January 21, 1862, published in the Memoirs Manchester Phil. Soc., 1865, (3) II, pp. 125–132.

[6] No. XIV of these translations.

[7] See Mechanics of the Earth's Atmosphere, 1891, p. 254.

[8] See No. XX of present collection of translations.

XIV

THEORETICAL CONSIDERATIONS RELATIVE TO THE RESULTS OF THE SCIENTIFIC BALLOON ASCENSIONS OF THE GERMAN ASSOCIATION AT BERLIN FOR THE PROMOTION OF AËRONAUTICS

BY PROF. DR. WM. VON BEZOLD

[From Wissenschaftliche Luftfahrten, or The Scientific Balloon Voyages carried out by "The German Association at Berlin for the Promotion of Aeronautics. Published by R. Assmann and A. Berson by the coöperation of O. Baschin, W. von Bezold, R. Boernstein, H. Gross, V. V. Kremser, H. Stade and R. Süring." Brunswick, 1900, Vol. III, pp. 283–313]

[*Translated from the Gesammelte Abhandlungen von Wm. v. Bezold, Braunschweig, 1906, pp. 221-264*]

The authors of this publication have asked me to express in a final chapter the most important results from a theoretical point of view and thus unite the individual portions by one common thought.

Willingly as I would respond to this desire, yet I find myself in a difficult position. Notwithstanding the fact that I have steadily followed the whole undertaking from the first step in the work, still I cannot seriously begin this crowning chapter until the individual memoirs united in these three volumes are available to me; and thus there remains to me no other choice than either to limit myself to a superficial review or to delay the appearance of the whole work.

I have decided to pursue the first of these alternatives in recognition of the conviction that weeks and months would not suffice for the completion of my work.

Therefore in a first section of this chapter I confine myself to an exposition of the importance that pertains to observations in balloons from the present point of view of our science. In a second section I will attempt to develop the ideas as to the processes going on in vertical columns of air to which we are led by purely theoretical considerations and by which much may be anticipated that may be subsequently confirmed by the observations. In the third section the average vertical distribution of the meteorological elements will be presented as resulting from the data given by the ascensions.

In the expectation that this work will not be used by technical meteorologists alone I have labored to write as far as possible in popular style in order to give a large circle of readers a glimpse of the kind of problems that have to be solved by scientific ballooning and thus to vividly present to them the full meaning of the results that are laid before the world in these volumes.

Under these conditions, I must of course in the first two sections repeat many well-known matters along with others that have not yet been clearly understood. But as I have tried to do this in a uniform and, as I hope, in a novel way, therefore these considerations may not be without interest even for the specialist. To the latter the summaries presented in the third section will be welcome.

Unfortunately, in the short time allowed me I must omit from this review the many works published elsewhere, some of them very recently, especially the beautiful investigations of Messrs. Teisserenc de Bort, A. L. Rotch, H. Hergesell, H. C. Frankenfield, H. C. Clayton, F. Erk, et al., but must restrict myself exclusively to the discussion of the materials submitted in these present volumes.

I must expressly state that the omission of these highly important works arises in no wise from any low estimate of them, but is simply demanded by the necessity of speedily finishing my work.

(1.) THE IMPORTANCE OF SCIENTIFIC BALLOONING

The importance that attaches to the investigations of the atmosphere by means of balloons, an undertaking that has been made possible through the grace of His Majesty the Emperor, as well as the results attained thereby, can only be reached when we take the broadest view of the present condition and the ultimate object of meteorological investigations.

The oldest scientific balloon voyages were made at a time when men had scarcely begun to systematically study the meteorological processes going on in the lowest stratum of air. Therefore the aëronaut found himself in a position similar to that of an explorer who is the first to enter a country hitherto wholly unknown and the results that he brings back from his journey must be recognized as an addition to our scientific data, but can only in a very limited degree contribute to our deeper knowledge.

Moreover, during the whole long interval of time in which meteorology was regarded principally as a statistico-geographical study we could not possibly recognize the true importance of the exploration of the higher strata of the atmosphere.

The observations in balloons first attained their true importance when we began to investigate the causal connection of the atmospheric processes and to trace the latter back to fundamental physical laws.

When we enter on this problem we must consider the atmosphere as a whole. We can no longer be satisfied with observations that are made in the lowest stratum of air, but have to strive for data from the upper strata and representing all conditions.

Observations made under ordinary conditions will suffice to give us a picture of the average distribution of temperature, precipitation, wind, etc., as well as of their variations, that is to say, they suffice as a basis for climatological studies. By means of such observations, with the help of the graphic weather charts, the phenomena of the weather as they follow one after the other, and the connection of the processes that occur together can be made out to very considerable extent. But the explanation of the phenomena is impossible so long as the study is confined to the lowest stratum of air.

The fundamental generalization that the areas of low barometric pressure are accompanied by cloudiness and precipitation, whereas in the high pressure areas clear, dry weather prevails, becomes intelligible when we consider that in the first case we have to do with ascending air but in the second case with descending currents.

From that moment when we recognized what fundamentally different rôles the ascending and descending currents play and what incisive importance the vertical movements of the air have for meteorology and climatology, it must have been recognized as a problem of the highest importance to follow along the path of these currents and numerically determine their behavior above and below by exact observations.

The recognition of the importance of these questions prompted at first the establishment of mountain observatories, and the observations collected there have contributed not a little to advance our understanding of atmospheric processes and to the attainment of new points of view.

The changes that rising and falling currents of air experience as to temperature and moisture can be followed strictly mathematically by means of the formulæ of the mechanical theory of heat, under the assumption that neither mixture with other masses of air nor increase or diminution of heat occurs. In fact such considerations enable us to explain a series of phenomena.

But whether these assumptions actually apply, and to what extent absorption or emission of heat or mixture with other air having other temperatures and other moisture content, are to be considered, are questions that can only be determined by observations in balloons.

Moreover the other fundamental question, in what manner the ascent and descent of air takes place in the areas of high pressure and low pressure can only be explained in this way. For there can be no doubt that these movements are by no means so simple as are represented in the sketches in the text-books, but that horizontal and vertical motions are combined together in the most varied and intricate manner and that mixtures etc., take place.

Those questions also that belong to pure dynamics, in distinction from the just mentioned thermodynamic questions, can only be specifically considered with the help of research by means of balloons.

The great difficulty of this problem forbids its general treatment as a whole; we must therefore consider the individual portions separately and afterwards attempt to establish the connection of these.

Among the results attained for which we have to thank the present series of balloon voyages (by the Berlin branch of the German association for promotion of aeronautics) the first place must be given to the elucidation that they have given us as to the warming and cooling of the atmosphere and the general distribution of temperature and moisture in the vertical direction.

(2.) THE VERTICAL DISTRIBUTION OF TEMPERATURE FROM A THEORETICAL POINT OF VIEW

In a memoir recently published (No. IX, pp. 216–228, or No. XIII of these translations) but whose principal contents I communicated to the Berlin Academy of Sciences on May 5, 1898, I tried to state in a purely theoretical manner the influence of adiabatic ascending and descending currents of air on the average distribution of heat in the atmosphere.

I started with the assumption that radiation inward or outward can only be influential at the earth's surface and at the upper surface of the clouds, and that a gain or loss of heat in the free cloudless atmosphere by absorption or emission can only play a subordinate part and may be neglected in our final approximations.

It seems that these assumptions do actually suffice to explain at least the prominent features of the vertical gradient of temperature, although further elaborations are needed in respect to many peculiarities.

At the same time these considerations led to views relative to the exchange of heat in the atmosphere that had indeed been indicated by several investigators, especially by Lord Kelvin and by H. von

Helmholtz and had been rather thoroughly developed by W. M. Davis, but that differed often and sometimes to an important extent from the views generally adopted.

The Results of Balloon Voyages published in the present work[1] (whose complete reductions by A. Berson and R. Süring have been accessible to me only within a very short time) allow us to check by experience the theoretical conclusions given in the above-mentioned memoir of May 1898, and to show to what extent they need to be corrected and supplemented.

Since it is remarkably difficult to perceive the full significance of the formulæ that represent the thermodynamic changes of ascending and descending currents, therefore it is helpful to present them graphically.

The first to apply a graphic method to these problems was H. Hertz. But the diagram prepared by him had for its object only the replacing of numerical computation by a simpler, less laborious operation. Some years afterward I attempted to follow the processes themselves as such by means of appropriate diagrams.

Since in the present work an extensive use will be made of this method in order to present the condition of the strata of air through which the balloon passes, it seems appropriate to say a few words relative to these diagrams in general.

In designing such diagrams one may have in mind many points of view. If, for instance, we deal only with purely theoretical investigations then it is most appropriate to consider only the fact that the condition of a given quantity of atmospheric air is completely determined when we know the pressure, the temperature, and the quantity and form of its moisture. This last item is necessary because particles of water or ice so long as they float in the air are to be counted as constituents of the atmosphere.

Since the above-mentioned data determine also the volume occupied by the unit mass of atmosphere or the so-called specific volume, therefore this latter quantity may be adopted as the independent variable instead of either one of the others, and we may characterize the condition of the atmosphere by the pressure, specific volume and water-content. Hence in the graphic presentation, we choose pressure and volume as the coordinates as has long been customary in the mechanical theory of heat.

This method which I have hitherto used exclusively offers the great advantage that for any given change of condition we can take

[1] Wissenschaftliche Luftfahrten, 3 volumes. Berlin. 1900.

directly from the diagram not only the work done but also, with but little trouble, the increase or diminution of heat.

The altitude above the earth's surface does not enter into the formulæ and graphic diagrams used in this method of treatment, a circumstance that is of great importance for the complete understanding of meteorological processes. From this we can conclude that such changes of temperature as accompany the ascent or descent of the air are not to be referred back to the work of elevation, but are conditioned only by the changes of atmospheric pressure associated with the change of altitude. If this view had been properly considered earlier, one would never have accepted the erroneous idea that the cooling in ascending currents is a consequence of the work done in elevating the air.

Notwithstanding all this, it does not seem advisable to make use of this method in the present memoir since it demands an abstraction too great to suit the new ideas that are immediately pressing for attention.

If we wish to consider the conditions in a vertical column of air then the altitude of any point above the surface of the earth is the determining item that seems to us especially characteristic. Even if we know that the atmospheric pressure diminishes with the altitude, still this pressure makes no such direct impression on our senses as does the altitude.

If we introduce the altitude as one of the coördinates then we more appropriately choose it as the ordinate, while the other element whose relation to the altitude is to be presented should be laid off as abscissa.

It is evident that this method of presentation can be applied not only to the temperature but to all meteorological elements that have any relation whatever to the altitude, such as the pressure, moisture, electric potential, etc.

If the curve of temperature is plotted in this way, then we obtain the diagram that W. M. Davis has used (with only a change imposed by the English system of measures), and from which he has drawn a system of consequences, to which I also had arrived somewhat earlier by another method and which I have developed quite recently in the above-mentioned memoir (of May 5, 1898) from a different point of view.

In the present memoir the last mentioned style of presentation is always used and the metric units of 1° C. and 100 meters of altitude are represented by the same equal distances measured along the axes of abscissæ and ordinates, respectively. This offers the great advantage that the adiabats of dry air appear almost exactly as straight lines inclined to the axes at an angle of 45°. On the other hand we must not forget that by this choice of coördinates it is only the temperature that is represented in its dependence on the altitude and not the thermodynamic condition properly so called, to whose determination a knowledge of the pressure or the specific volume is also necessary.

But whatever method of presentation we may choose they all have this one point in common, i. e., that each condition characterized by the corresponding variables corresponds to a point on the coördinate plane. If now we imagine a given quantity of air (the unit of mass is best) passing successively through different conditions then the points corresponding to these conditions arrange themselves in a connected series and form a continuous curve.

In this way we obtain "curves of change of condition," e. g., as above, "curves of *temperature change*."

Since in this study one must know in what direction the changes of condition follow each other, it is therefore necessary to indicate this by an arrow along the curve. If, for instance, the changes follow the altitude, then the arrow shows whether we have to do with an ascending or descending current.

But we may just as well apply these graphic methods to represent the *condition* prevailing at a given moment, or the average of a longer interval, along any given line. If, for instance, we plot as ordinates the altitude and as abscissa the temperature prevailing throughout a vertical column of air at a definite moment of time, then the curve becomes a representation of the distribution of temperature prevailing at this moment, or if we add similar curves for the moisture and the pressure it becomes a plot of the total thermal condition along the given vertical.

In this case I call these "curves of condition" as opposed to the above-mentioned "curves of change of condition," or if we consider only the temperature they become "curves of temperature" instead of ' curves of change of temperature." Since the curves of condition do not relate to conditions that follow each other consecutively but that prevail simultaneously, therefore of course no arrow is needed.

I do not know that any one has yet dwelt on the sharp difference between these two kinds of curves, although both kinds have been made use of. Thus, for instance, the curve for the dependence of the mean temperature on the altitude given on p. 90 in the well-known Lehrbuch of Sprung and constructed from Glaisher's observations, is a "curve of condition," whereas the curves that I have drawn in my memoirs on thermodynamics are "curves of changes of conditions."

The curves of dependence of temperature on altitude that are always used in the description of individual balloon voyages are in most cases, strictly speaking, neither curves of condition of the vertical column of air nor yet curves of change of condition; they are rather representative of the successive conditions that the balloon has met during its flight.

If the ascent is very rapid (and the greater number relate specially to the ascent), then the curve can be considered as approximately the curve of condition along the vertical; but if the balloon floats in equilibrium without any ascensive power and in company with the air surrounding it then the diagram does actually present the curve of change of condition. If the balloon has attained an altitude where the diurnal period is very small and if the horizontal path is not too long the curve will to a high degree of approximation present the condition of the vertical column.

These approximations may be pushed further especially when we have observations at the points on the earth's surface immediately below the balloon, which allow of a reduction of the individual observations to a definite moment of time. The extent of the error that may be made by the use or neglect of these reductions can be seen exactly from the curves in dotted lines connecting the values at the earth's surface with those observed simultaneously in the balloons. Especially instructive in this respect are the curves in Vol. II (of the Wissenschaftlichen Luftfahrten) for ascensions No. 12, p. 136; No. 18, p. 188; No. 19, p. 202; No. 25, p. 274; No. 32, p. 332, and others, which will be easily recognized by examining the dotted straight lines.

Where the results of many voyages are united in an average value the curves that represent the connection between the values and the altitude can be considered as approximately the mean curve of condition for the vertical column of air above the lowlands of Northern Germany. The small systematic error that might be expected for the reason that the lower parts of the curves belong largely to the late morning hours while the upper parts belong to the midday or early afternoon, is of insignificant magnitude.

Now, before I proceed to speak in detail of the curves of results published in this work (Wiss. Luftfahrten, Berlin, 1900) it seems proper to apply the just-mentioned difference between the curves of condition and curves of changes of condition to a definite and more general question to the elucidation of which they are peculiarly appropriate, that is, the so-called convective equilibrium.

The mechanics of elastic fluids offers disproportionately far greater difficulties than that of liquids. A drop of oil that is placed at the bottom of a vessel full of water rises in the water without experiencing any change in its volume or its temperature. On the other hand, a particle of air enlarges its volume in proportion as it approaches the surface, it also cools and consequently its volume does not change to the same extent as when the temperature remains unchanged.

Still more complicated is the case when a mass of air that has been locally warmed rises in an atmosphere whose temperature and density themselves change with altitude and when, moreover, aqueous vapor is mixed with the air which condenses at a definite temperature.

In order to deal with this question one must make the simplifying assumption that the mass of air after being once warmed over the region receives or loses no heat thereafter, that is to say, the expansion during ascent is adiabatic. Under this assumption the temperatures through which the ascending air will pass can be computed and corresponding to these the curve of the change of condition or the so-called "adiabat" can be drawn.

This adiabat is a straight line inclined 45° to the axes of the coördinates so long as the saturation temperature is not attained. At this point the line experiences a sharp bend and thence rises more or less steadily as a flat curve convex to the right and above. It rises steeply so long as the temperatures are high, that is to say, so long as large quantities of water are present whose latent heat of condensation is able to perform much work; it approaches more and more the adiabat of the dry stage the lower its temperature, that is, the less the water contained in the saturated air.

Fig. 34 shows three adiabats that I take from the memoir of O. Neuhoff[2] now in press and to be published in the "Abhandlungen" or Memoirs of the Prussian Meteorological Institute. These curves correspond to masses of air that ascend from the earth's surface with temperatures of $-10°$ C., $+10°$ C. and $+30°$ C. respectively and a barometric pressure of 760^{mm}. and relative humidity of 62 per cent.

If in this way we have attained an idea of the course of such curves

[2] O. Neuhoff: Adiabatische, etc. Adiabatic changes of condition of moist air and their determination, numerically and graphically. Memoirs Preuss. Meteorolog. Institute, vol. I. No. 6, Berlin, 1900. [See No. xxi, p. 436 et seq.]

then it is not difficult to study the conditions of equilibrium of a vertical column of air when the curve of condition is known. Let such a curve be represented by $Z Z$ in fig. 35.

If we now assume that a particle of air (a) for any reason whatever experiences a slight rise in temperature then it will pass over into the condition represented by the location of the particle a_2. But since the atmospheric pressure must be the same at places of equal altitude therefore it must be specifically lighter than its surroundings, and must rise higher. If this rise occurs without further addition or subtraction of heat, then it must cool according to the adiabatic law, i.e., the corresponding curve of change of condition will, in so far as no condensation occurs, be a straight line inclined 45° to the axis and cutting the curve of condition at a point lying above a. When the ascending air attains this altitude it has attained again the temperature of its surroundings and there is no reason apparent that should cause a further ascent. The equilibrium temporarily disturbed is now again restored.

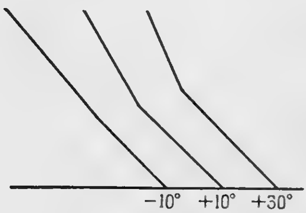


FIG. 34

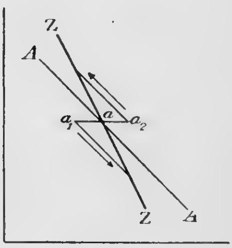


FIG. 35

When a particle at a experiences a cooling the inverse phenomena occur: instead of an ascending straight line from a_1 it follows the descending line from a_1 and the particle warms up until at some lower altitude it attains the temperature of its surroundings and thus again the movement comes to an end.

We thus perceive that "when the adiabat, whose direction is shown by the straight line A A, ascends less steeply than the curve of condition Z Z the equilibrium is stable."

When the adiabat ascends more steeply than the curves of condition, as is shown in fig. 36, that is to say, when the temperature along the vertical column diminishes with altitude more rapidly than in air that is ascending adiabatically, then the phenomena are quite different from the preceding case.

In this case no special warming or cooling is needed in order to make a particle rise or sink with increasing speed, but an initiation of motion in either direction suffices to cause the motion to continue with increasing acceleration. If the particle receives a push upward or in the direction from a toward b_1 along the adiabat, then in consequence of the smaller cooling, the temperature difference between the particle and the surrounding air when it arrives at b_1 will steadily increase in proportion as the motion continues. Conversely for a descending particle the temperature, as shown by the line $a\ b_2$ representing the change of condition, will continually depart more and more from that of the surrounding air, so that this movement also progresses with increasing acceleration.

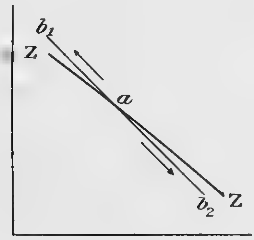


FIG. 36

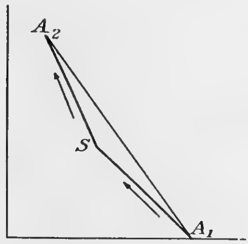


FIG. 37

The condition corresponding to this state of affairs is unstable and can exist only temporarily. The condition of indifferent or neutral equilibrium occurs when the change of temperature with altitude exactly follows the adiabatic line, but since this condition changes over into the unstable condition with the slightest change of temperature, therefore it rarely occurs in nature but immediately becomes unstable.

In the neighborhood of the earth's surface, where the atmosphere is generally in the dry stage, a temperature gradient of 1° per 100 meters constitutes the limit, that is never or very rarely exceeded. But the problem changes entirely when saturation occurs, since then a comparatively much smaller gradient of temperature suffices to produce unstable equilibrium.

For in this case, as above remarked, the adiabat has a form such as is shown in fig. 37 by the complex line $A_1\ S\ A_2$ broken at the point S'.

If then we compute the temperature gradient from observations at two points, one of which lies somewhat above, but the other below the limit of condensation, e. g., from the temperatures observed at A_1 and A_2, fig. 37, then, as is evident from the figure, we obtain a value of the gradient that is less than 1° C. per 100 meters no matter whether neutral or even unstable equilibrium prevails. If, for example, air having a temperature of 20° C. and a relative humidity of 67 per cent rises from sea level, then under adiabatic expansion the condensation will begin at the altitude 770 meters.

Then at 1500 meters altitude a temperature of 8.6° C. will prevail. The difference therefore would amount to 11.4° for 1500 meters; the rate of decrease would, however, be only 0.76° per 100 meters. Notwithstanding this we should make a grave error if in this case we should assume a condition of stable equilibrium. Therefore this circumstance must never be lost sight of in judging as to the stability of atmospheric equilibrium, for instance, from observations on mountains and in valleys, where we generally must rely on observations at two points only.

This difficulty generally disappears in the deductions from balloon observations, since in most cases these give us complete curves of condition for long distances. But this point deserves consideration often in case of rapid ascensions, where not infrequently the first observation in the balloon can be made only after passing the limit of condensation.

It may be incidentally mentioned that Reye even in his time, although in different form, has shown what importance attaches, for the whirlwind storms, to this "knick" in the adiabatic curve, or the steeper ascent in the condensation stage. Of course, these remarks of Reye hold good, with appropriate changes, for the thunderstorm phenomena of our regions.

The question as to the "critical" gradient of temperature (a term that we may properly apply to the limiting value characteristic of unstable equilibrium) becomes especially complicated in one particular case that I will now explain more fully.

Assume that the curve of condition has some such appearance as shown by the line $Z\,Z$ in fig. 38 and that the limit of condensation lies at S. Then it can happen that the adiabat of the condensation stage has the course $S\,A_2$ while that of the dry stage is represented by $A_1\,S$. Under these conditions stable equilibrium prevails below the limit of condensation but unstable equilibrium above that limit.

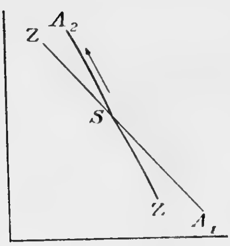


FIG. 38

This case cannot easily occur in masses of air that ascend as broad currents, for an ascent along the curve $Z\,S$ can only happen when the ascending air after leaving the ground continually receives so much heat, either by radiation or by mixture with other air, which of course must bring with it sufficient aqueous vapor, that a part of the work of expansion is done by it so that adiabatic cooling is avoided. But after passing the condensation limit and thereby entering upon unstable equilibrium, the ascent goes on with increasing velocity, hence either the whole condition must be different or else the movement must come to a stop in consequence of the subsidence of the mass of air cooled by ascent.

On the other hand, similar processes are certainly possible under anticyclonic weather conditions where in general the temperatures diminish along a curve such as is represented by Z Z, but where in between the descending masses of air others are ascending, whose temperatures follow a different law, and thus may be explained the occurrence of individual cumulus clouds that attain considerable altitudes.

If stratifications are present then, after the introduction of such movements in individual cumulus clouds, masses of air may possibly be drawn from the horizontal stratum that corresponds to its base, which masses fill up the gap that would be left by the ascension of the cloud.

Finally it is also possible that counter currents may be caused by the sinking of air in the neighborhood of the cumulus cloud. Of course these must be quite dry when they reach the lower level and thus cause dissolution or evaporation of the cloud. Numerous observations made during the balloon voyages, showing relatively great dryness when in gaps between the clouds or in the neighborhood of cumuli, seem to be in harmony with this conclusion.

At least the case represented by fig. 38, as plausible, seems to be worthy of consideration.

According to what has already been said, one easily recognizes that the play of ascending and descending currents of air will suffice to bring about a diminution of temperature with altitude. The problem is to ascertain to what extent the diminution actually observed in scientific balloon voyages is really explained as to direction and quantity by the above-given cause.

If the diminution of temperature with altitude is simply a consequence of ascending and descending currents, then must the average temperatures of different altitudes be the average of those that correspond to the ascending or descending branches of the different currents that move in the same vertical.

Under this assumption the course of the temperature is as shown in general by fig. 39.

If moist air ascends without increase or loss of heat, then the change of condition is shown by the broken line $a\ b\ c$; if now it sinks from some altitude that cannot be shown in the diagram and after all the water has fallen from it, then the increase of temperature with descent follows $c\ d$, or the adiabat of the dry stage. The nick at b lies lower in proportion as the air is moister when the ascent begins and it is sharper, or the first part of the curve $b\ c$ rises more steeply in proportion as the initial temperature is higher.

If air alternately and for equal intervals of time rises according to the law $a\ b\ c$ and descends according to the adiabat $c\ d$ of the dry stage, then we obtain the average curve of condition by halving the horizontal lines between the two curves and joining all the half-way points. If t_1 and t_2 are the temperatures corresponding to the points T_1 and T_2, then the mean temperature t_m for the altitude h is given by

$$t_m = \tfrac{1}{2}\,(t_1 + t_2)$$

and the curve of average condition is represented by the median line $c\ m$.

Since now in general the currents ascending above any place will have very various initial temperatures and humidities, therefore the average of all must give curves whose $a\ c$ branches correspond

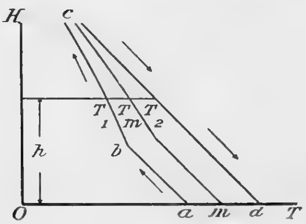


FIG. 39

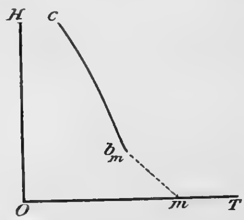


FIG. 40

only in general to the form of the line $a\ c$ but individually show great diversity. On the other hand the average curves corresponding to the descending branch will, under the adopted assumption of adiabatic descent, run parallel to the line $d\ c$ but cut the axis of abscissæ at very different places.

Hence as an average curve of condition of the vertical column there results a curve that must have approximately the course shown by $m\ b_m\ c$ in fig. 40. The lower part of this curve is dotted for a reason that will be explained at once.

In the views that have just been elucidated we had to consider that at very great altitudes all the adiabats of saturated air become approximately asymptotic to the adiabats of the dry stage, since, for the remarkably slight moisture contents that correspond to the highest parts of the curves, the latent heat of condensation no longer suffices to perform any appreciable fraction of the work of expansion.

Moreover, at the greatest altitudes the emission and absorption can play only a very subordinate rôle on account of the extraordinary rarity of the air, so that the changes in these strata take place nearly adiabatically.

We thus come to the very important result that at great altitudes the temperature curves more and more nearly approximate to the adiabats of the dry stage and therefore the vertical temperature gradient must tend toward the value of 1° C. fall per 100 meters[3].

The course of the temperature curve constructed as formerly according to the numbers deduced from Glaisher's observations must therefore from purely theoretical grounds seem very doubtful, at least in the highest portions. The same is of course true of the formulæ of Hann (for moisture) and Mendelieff (for temperature) which rest on these same observations.

It may be said certainly that one of the most important attainments of the undertaking described in this present publication (Ergebnisse Wiss. Luftfahrten, etc.) is the fact that, as regards the diminution of temperature in the highest strata of the atmosphere, there has been established a complete accord between theory and experience.

---

[3] Here it may be especially stated that all these remarks as to rising and falling currents are only first theoretical approximations. In fact it will frequently happen that masses of air that have ascended over very warm places and with a relatively large humidity arrive overhead with such high temperatures that it is impossible for them to descend at neighboriug localities. Under such conditions abnormally warm air overhead must spread horizontally and flow away to great distances above the lower strata of air. Such phenomena as give rise to very slight vertical temperature gradients and may in fact lead to temperature inversions have been frequently observed in recent years even at moderate altitudes, as was, for instance, the case in September, 1900, when such a warm layer extended from the Alps to the North Sea (see W. Brückmann "Die, etc." The Temperature inversions in summer anticyclones. Inaugural Dissertation. Berlin, 1904), whereas it occurs as a regular phenomenon in the much higher strata and such cases are certainly referable back to the air flowing out from the tropics (see Assmann "Ueber, etc." "On the existence of a warm current of air at the altitude of 10 to 15 kilometers." Sitz. Ber. d. Berlin Akad. 1902, pp. 495–504). It cannot be too often stated that the considerations here set forth are only crude approximations. (Note added in 1905. W. v. B.)

The fact that meanwhile the same results have been confirmed by observations made elsewhere, can only be incidentally mentioned, since in this present summary, as already stated, for want of time I must confine myself to the material submitted in this present publication.

When heretofore one refused to entertain the idea that such large temperature gradients existed in the highest strata, the reason lay in the consideration that the temperature could not diminish to infinity. But we must not forget that the term temperature of the air becomes less applicable in proportion to the increasing distance from the earth, and that for the extremest rarefaction the ordinary considerations must be replaced by an entirely different series of ideas.

The median portion of the curve shown in fig. 40, which is based simply on the consideration of currents ascending and descending adiabatically, already shows in its general course a certain agreement with facts that will hereafter be more exactly described, but with certain appreciable limitations.

For instance, it follows from considerations based on the above-given assumptions that the diminution of temperature with altitude in the median atmosphere strata must be less than in adiabatically ascending and descending dry air, and especially so in those layers in which the condensation is most frequent and most considerable, namely, between the altitudes 1000 and 4000 meters. This is in fact the case qualitatively; but the diminution actually observed is much smaller than would result from the above-described method of formation of averages.

For saturated air at temperatures between + 26° C. and − 30° C. ascending adiabatically the temperature gradient at 1000 meters altitude varies between −0.37° and −0.88°. Under the assumption that such ascending currents interchange during equal intervals of time with adiabatically descending currents whose gradient is always − 0.99° there should result average gradients that lie between − 0.68 and − 0.93.

For adiabatically ascending air that leaves sea-level with the temperature + 10° C. and attains its dew-point at altitude 1000 meters, the gradient at this elevation is − 0.59 and the average of this value and that of adiabatically descending air is − 0.79 or around − 0.80° C. per 100 meters.

But for this altitude the observations give an average value of − 0.50, or 0.58 if we exclude those cases in which large temperature reversals were observed, or values that are far smaller than those above computed. In fact in these strata the average curve of condition rises much more steeply than should be the case under the assumption above made.

This is remarkable inasmuch as thus the problem is to a certain extent reversed as compared with the older views.

Hitherto it has been believed that we could only explain the higher temperature of the lower strata by the suspicious hypothesis of the non-transmissibility of dark rays through the atmosphere, but now we must seek the reasons why the diminution of temperature with altitude is not far greater than observation shows it to be, at least in the lower and middle strata.

For the condensation, which contributes in an important degree to diminish this gradient, still does not suffice to fully explain the observed diminution.

The ideal curve sketched in fig. 40 departs from observation even still further, in its lowest portion. Whereas according to the general scheme one should expect larger gradients in the lowest stratum than in the median strata, yet in fact according to the numbers deduced from the balloon voyages for the lowest three kilometers there is a nearly constant and rather small value for the gradient.

It was to be expected that our scheme would fail in this portion, since near the earth's surface, where radiation and absorption come into play to such a large extent, it is only seldom that pure adiabatic processes can occur.

Especially at times of excessive outward radiation must the ideal scheme be disturbed, since at such times the lowest strata become relatively cold, whereas the opposite is the case at times of excessive inward radiation (or insolation) and thereby an approximation to unstable equilibrium or even that condition itself may easily occur.

Actually, however, it is only the general average whose course departs so far from the ideal, on the other hand, the average values for the summer, which will be considered in the third section of this memoir, do, especially in the lowest portion, approximate far more closely to the ideal.

To a much higher degree do various individual cuves resemble the form of the theoretical scheme. In this respect I recall the curves of condition given in volume 2 (of the Ergebnisse) for the voyages, No. 1, p. 10; No. 4, p. 38; No. 10, p. 106; No. 11, p. 121; No. 60, p. 553; No. 67–70, p. 579, and No. 72, p. 601.

Hence the course of the lowest portion of the temperature curve can be very well explained by the overpowering influence of the radiation, as will be hereafter more precisely set forth. On the other hand, one other most remarkable point offers greater difficulties. For we find that not only does the average curve for the anticyclonic day show in its course a great similarity with the adiabats of saturated air, but this also holds good for the individual curves, some of which indeed agree altogether with such adiabats.

For the average curve there is indeed not such close agreement, but even for it at least the differential gradients follow almost exactly the same law as does the adiabat of a saturated or ascending mass of air that left the ocean level with a temperature of 18° C.

For the sake of comparison this adiabat is shown by dashes in the diagram of annual means for various elements, in fig. 43 of the third section of this memoir. We see at once that it needs only a horizontal displacement of 8° toward the left in order to make this adiabat coincide with the curve t_m.

This comparison is of great interest because it shows strikingly how far the observed change of average temperature along the vertical departs from the average of the adiabats of the dry stage and the condensation stage, and how the temperature diminution with altitude, as actually shown by observation, is not only in the lower, but also in the median strata, much smaller than if it were exclusively the consequence of the play of ascending and descending currents.

The above-quoted cases in which the gradient of temperature follows the type theoretically developed, point out the way already indicated to unravel these peculiar relations. They occur always at times of the day and year when the insolation is in excess or at least begins to acquire the greatest importance.

It is probable that this approach to the adiabat of the dry stage or to unstable equilibrium would be much more frequently observed if ascensions had been more frequently made in the midday hours, whereas for reasons easily understood the later morning hours must preferably be chosen.[4]

At any rate these cases present the proof of the fact that it truly is the insolation and radiation processes at the earth's surface that strongly influence the course of the temperature curve in the lower strata.

---

[4] The most recent kite ascensions have shown that a temperature gradient of over 1° per 100 meters frequently occurs in the lower strata in the morning hours in the warmer season of the year; e. g., compare R. Assmann and A. Berson; Ergebnisse, etc. (Results of the labors at the Aeronautical Observatory in the years 1900 to 1904. Publications of the Royal Meteorological Institute, Berlin, 1902, 1904, 1905. (Note added 1905. W. v. B.)

In fact, from the consideration of the average curve with its unexpected slight gradients in the lower portion, that is to say, with its surprising steep ascents, there results at first the most astonishing fact that in the general average the influence of the soil makes itself felt in a relative cooling of the lower strata of air.

This result is in direct contradiction of the older views. Formerly, as already stated, we felt that we must accept some very special assumption in order to explain why the lowest strata of the atmosphere are warmer than the upper, but today we confront the question, why the difference in the temperature is not much larger than it is.

I have already treated this subject theoretically in the above-mentioned memoir,[5] which was published a few weeks ago, and now I will only attempt to briefly repeat the most important points in a less abstract form.

The explanation is found in one circumstance, namely, the great difference of the influences that the outward and inward radiation at the earth's surface exert on the atmosphere; a subject to which Lord Kelvin and H. v. Helmholtz have occasionally referred and which W. M. Davis afterwards treated more thoroughly both in his memoirs on whirlwind storms and also in his admirable "Elementary Meteorology."

Although the cooling and warming of the surface of the globe stand in a simple antithesis to each other, still the processes by which these influences are transmitted to the air are fundamentally different; they are processes for which many years ago I introduced the term "limited reversible" or "pseudo-reversible."

This distinction impressed me still earlier as I investigated the processes in adiabatically ascending and descending currents from a very general point of view. If we consider a saturated ascending current of air we find that the law of diminution of temperature (ignoring the hail stage) remains exactly the same down to an exceedingly small difference, no matter whether the water that is formed falls from the current or is carried along with it. The formulæ that we apply to the so-called reversible changes of condition apply perfectly to this case. In fact the changes in question remain reversible even to the smallest portions if the water falls away as rain or snow, for the separation never proceeds so rapidly that the precipitation disappears immediately from the neighborhood of (or association with) the mass of air in which it originated.

---

[5] See No. IX of my collected memoirs, (or No. XIII of this collection of translations).

If therefore the ascent of the air suddenly becomes a descent, then, at least in the first moments, re-evaporation must take place, that is to say, the processes attending the immediately preceding moments will be exactly repeated in the reverse order.

If all precipitation were carried up with the ascending current of air, as it is in the case of clouds from which no rain has as yet fallen, then in general by reversing the movement the air would arrive at its starting point in the same condition in which it had left the earth. If, on the other hand, water has actually fallen from the cloud, then by reversing the movement the air enters sooner into the dry stage and its warming follows a very different law from that of its cooling.

The process is therefore reversible in its smallest details but not in the larger nor as a whole, and it is precisely to this peculiarity that, as is well known, we owe the Foehn phenomena, the differences of the weather in the areas of high pressure and low pressure, the peculiarities of the windward side and leeward side of mountain ranges, etc.

We meet analogous conditions in the warming and cooling of the atmosphere in contact with a terrestrial surface that is subject to insolation and radiation.

A limit to the warming of the lowest layer of air is soon set by the occurrence of unstable equilibrium, but, on the other hand, its cooling can proceed as long as the radiation continues and so long as no rapid renewal of the air is produced either by the drainage away of the cooled air or by the wind. It is well known that the very low temperatures that are observed in valleys are thus produced and especially on *tundras* in winter and to a less extent also in other seasons during very clear calm nights. The same is true of the so-called inversions of temperature that were first observed in mountainous regions and which for a long time were supposed to be principally confined to such regions.

Scientific balloon voyages have shown that these inversions occur regularly at times of overpowering radiation and gentle atmospheric motion.

Moreover, balloon voyages furnish us far more complete pictures of the temperature inversions than do simultaneous observations at a summit station and at a neighboring valley station, in which latter case such an inversion can pass entirely unnoticed when the location of the (stratum of) highest temperature lies only slightly above the level of the lower station, so that the temperature of the upper station is still lower than that of the lower station. Thus, for example, on the dates and the times when the balloon ascensions Nos. 16, 17, 30, 37, and 51 were made [6] the very pronounced inversions would have been entirely overlooked if one had at hand observations at only one station in the lower plain and one at an altitude of 1000 meters.

Again, the full extent of the inversion within comparatively small elevations can only be observed by means of balloons. For example, during balloon voyage No. 22, on January 12, 1894, the following temperatures were observed; at the ground − 6° C.; at 400 meters, + 6.5° C.; giving a temperature gradient of + 3.2°: but on February 24, 1891 (ascension "G," fig. 41) there had been observed at 230 meters − 2°, whereas at 340 meters this had risen to + 9°, corresponding to a gradient of + 10.0° C. per 100 meters. By experiment with the captive balloon "Meteor" on October 9, 1891, there was observed at about 5 h. 27 m. p.m. an increase of temperature of 2° between 1.5 meters and 8 meters above the ground corresponding to a gradient of 25.0° C.

A very interesting diagram is formed by extending downward the main branch of various curves of this type. as is done in fig. 41, for the curves of the voyages "G," Nos. 22, 54, and 55, and then placing near them the curves for cases that correspond to the heating of the soil such as are typically represented in voyages Nos. 72 and 69*b*—70.

The numerals for the voyages are entered near the respective lines; the curve marked "G" refers to an ascension made by Captain Gross on February 24, 1891, and which does not belong to the general series. The sides of the small squares correspond to 200 meters altitude and 2° difference of temperature. The whole number of degrees inscribed on the bottom line indicate the scale numbers for the observed temperatures on the respective curves. From this diagram we see at a glance how differently the warming and cooling of the earth's surface affects the air and we comprehend how seriously the cooling influence must affect the average values.

Explanation of fig. 41

NO.	DATE			DURATION UP TO MAXIMUM ALTITUDE	Reference Wissenschaftliche Luftfahrten.	
	Day	Month	Year			
G	24	II	1891	....	vol. I	p. 105–106
22	12	I	1894	9:15 a.m. to 1:06 p.m.........	II	225–236
54	18	II	1897	10:10 a.m. to 4:18 p.m.......	II	500–516
55	18	II	1897	9:40 a.m. to 2:15 p.m.......		
69b	8	VI	1898	Descent 1 to 3:08 p.m.......	II	566–588
70	8	VI	1898	11:55 a.m. to 6:15 p.m.......		
72	15	IX	1898	2:05 p.m. to 3:05 p.m.	II	594–604

[6] See the Wiss. Luftfahrten, Vol. II, pp. 167, 176, 305, 371, 457, respectively.

In general the conditions here presented have led to the following results:

(*a*) "The warming and cooling of the atmosphere are determined principally by the radiation processes at the earth's surface and to a less degree by the analogous processes at the upper boundary surfaces of the clouds."

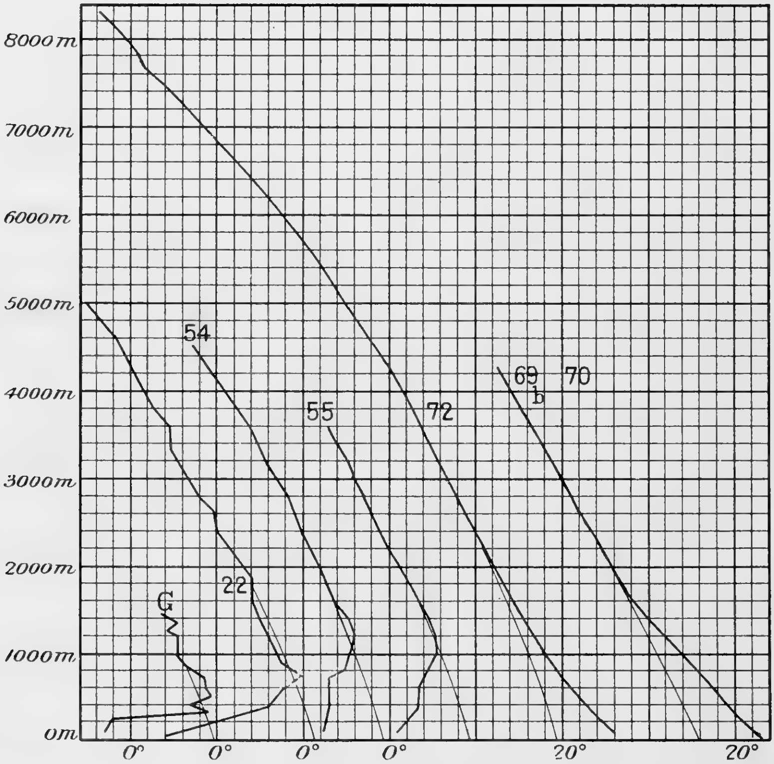


FIG. 41

(*b*) "The latter processes at the cloud surface will on account of the large evaporation probably be more like those above extensive surfaces of water, especially above the ocean, for which, as yet, no observations are available."

(*c*) "Of these two processes the warming cannot make itself felt in the lowest stratum so decidedly as does the cooling, since the warmed air rises and so much the more rapidly in proportion as the diminution of temperature with altitude approaches the limiting value for unstable equilibrium. After the beginning of condensation this limiting value is smaller than in the dry stage."

(*d*) "There is no limit of this kind for the cooling, so that the increase of temperature with altitude in the lowest stratum at the time of the so-called inversion of temperature can attain values that may amount to many times the greatest possible diminution of temperature for the same differences of altitude. On the twenty-fourth of February, 1891, there was observed a positive gradient of 10°, whereas for the negative gradient − 1.0° constitutes the limit that can scarcely be exceeded."

(*e*) "This difference in the processes of warming and cooling brings about a lowering of the average temperature of the lower strata or a steeper ascent in its lowest portion of the curve of condition for temperature."

(*f*) "Similar considerations must obtain with reference to the absorption and emission by the atmosphere itself which may be very considerable in the lowest strata as shown by the growth of ground-fog from below upward. Here, then, these processes must also contribute to diminish the rate of diminution of temperature with altitude."

(*g*) "Finally it must not be forgotten that at the season of excessive transfer of heat (to the atmosphere) above the surface of water or wet soil, the evaporation also contributes to depress the temperature of the lowest stratum."

(*h*) "The masses of air ascending from the ground carry upward with them the heat acquired below (allowing for that which is used in expansion) and that too not only the heat shown thermometrically as they leave the ground, but also that which had been used to evaporate the accompanying water. The heat used for this latter purpose becomes appreciable in the strata in which condensation takes place, where it diminishes the temperature gradient and that too in proportion as the loss of the precipitation is greater; but as sensible heat this first becomes evident in the descending current of air and thus gives rise to that form of transfer of heat that I have called 'complex convection.'"[7]

(*i*) "Finally, at the greatest altitudes where absorption and emission disappear and almost no aqueous vapor is present, the adiabatic ascent and descent of dry air is the only cause of the change of temperature with altitude."

---

[7] See von Bezold's 2d memoir translated in Mechanics of the Earth's Atmosphere, 1891, p. 255.—C. A.

(*j*) "The curve of condition for temperature must consequently in the highest strata asymptotically approach a straight line that cuts the axes at an angle of 45°."

(3.) THE OBSERVED MEAN ANNUAL AND SEASONAL VERTICAL DISTRIBUTION OF THE METEOROLOGICAL ELEMENTS

Now that we have in the second section endeavored to explain the mean distribution of temperature in a vertical column of air at least as to its principal features, we will exhibit both numerically and graphically the mean temperature and moisture as deduced by Besson and Süring (from observations in balloons). These quantities are indicated by t_m and y_m; and then for completeness are added under β_m barometric readings corresponding to the different altitudes when the atmospheric pressure at sea-level is 762^{mm} and the temperatures are such as given by Besson's computation: on the other hand under β_a are given the pressures that would correspond to dry air adiabatically rising or falling, and leaving the sea-level or arriving there from above with the temperature 10°.4.

The numbers under β_m give the average vertical distribution of pressure for the North German plains just as these under t_m give the average temperature distribution. Of course these numbers do not represent any greater degree of accuracy than can be expected from the relatively small number of observations submitted in the "Ergebnisse"[8] but still they afford a very instructive picture.

As representative of the moisture I have chosen the specific moisture for a reason that will be explained immediately. The appropriate numbers are found in the column y_m. Moreover, under y_s are found the values of the specific moisture that represent the condition of saturation for the corresponding pressures β_m and temperature t_m. The quotient y_m/y_s, whose value can be at once approximately seen from the graphic diagram, fig. 43, when multiplied by 100 gives the relative humidity in per cent.

Finally the numbers given under Y_m represent the total quantity of water in kilograms corresponding to the observed values of y_m and contained in a vertical column of air 1 meter square extending from the ground up to the respective altitudes. On account of the slight accuracy naturally attaching to these numbers they are only given for each full thousand meters.

With these remarks the following table needs no further explanations:

[8] The Ergebnisse: Results of Scientific Balloon Ascensions.

Table 1. The vertical distribution of pressure, temperature, and moisture for successive altitudes

h	t_m	y_m	y_s	β_m	β_a	Y_m
m	°c	*gram*	*gram*	*mm*	*mm*	*kg.*
20	+10.3	5.86	7.68	760	760	—
500	7.9	5.33	6.97	717	717	—
1000	5.4	4.54	6.24	675	673	6.34
1500	2.9	3.61	5.56	635	632	—
2000	+ 0.4	3.08	4.95	597	593	10.14
2500	− 2.3	2.66	4.32	560	555	—
3000	− 5.0	2.23	3.76	526	519	12.60
3500	− 7.6	1.88	3.29	494	485	—
4000	−10.3	1.68	2.83	463	452	14.23
4500	−13.5	1.57	2.35	434	421	—
5000	−16.7	1.18	1.92	406	391	15.38
5500	−20.1	0.81	1.54	380	363	—
6000	−23.6	0.67	1.21	355	336	15.99
6500	−27.0	0.57	0.94	331	311	—
7000	−30.4	0.30	0.73	309	288	16.30
7500	−34.0	0.26	0.54	288	265	—
8000	−37.6	0.22	0.42	267	244	16.42
8500	−41.6	—	0.29	249	224	—
9000	−45.6	—	0.20	231	205	—
9500	(−49.6)	—	0.14	(214)	187	—
10000	(−53.6)	—	0.09	(198)	171	—

Figs. 42 and 43 give graphic presentations of the numbers contained in table 1, the curves being designated by the same letters as those that stand at the heads of the respective columns of table 1.

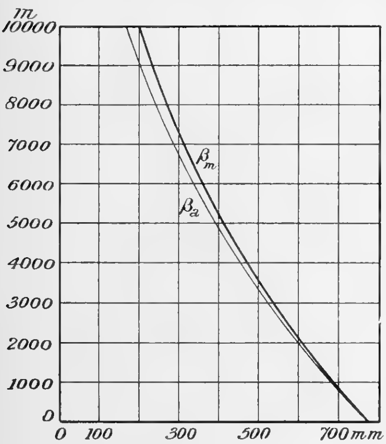


FIG. 42

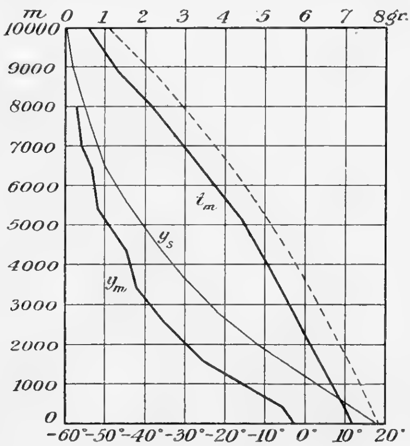


FIG. 43

The axis of ordinates is the zero line for both the pressure and the specific moisture; the value of the specific moisture in grams per kilogram of moist air is shown by the numbers at the top of fig. 43; the pressures and temperatures are given at the bottom. These curves are curves of condition ($Z\ Z$) in the sense explained in section (2) and are specially appropriate to set forth in clear light the advantages of this method of representation.

The curves t_m of temperature and y_m of average moisture directly represent the results of observation. The values of β_m and y_s are then obtained by computation with the help of the temperatures t_m the first of them being computed step by step.

To this latter circumstance is also to be attributed the fact that irregularities in the course of the temperature curve must also produce others in the curves for β_m and y_m.

The only results of computation exclusively are the barometer readings β_a computed for various altitudes on the assumption of a linear temperature gradient of 1° per 100 meters.

The values of β_a are added because it is not uninteresting to present both numerically and graphically the law of diminution of pressure for the impassable limiting case of the unstable equilibrium of dry air, and thus bring vividly to the eye the fact that convective equilibrium establishes a limit not only for the rate of diminution of temperature but also for that of pressure.

Moreover, the dependence of the distribution of atmospheric pressure on that of temperature finds a very instructive presentation by the comparison of the curves for β_m and for β_a.

Finally, in fig. 43, at the extreme right hand the adiabat for saturated ascending air is added as a line of dashes and as it results according to the tables constructed by Otto Neuhoff[9] and supplemented by him for the highest altitudes, and *assuming that the ascent began* at the ground at the temperature 18° C.

At the first glance we see this curve has almost the same course as that for the average temperatures and that by pushing it toward the left it can be made to nearly cover that. It may, indeed, be asked whether in this peculiarity there be not concealed a deeper connection, at any rate the fact is so interesting that it should not be passed by unnoticed.

The value of the mode of presentation here used is especially evident when we apply it not only to the general mean, but also to average values for short intervals of time and when we set them beside each other.

---

[9] O. Neuhoff: Adiabatic changes of condition for moist air and their determination numerically and graphically. Abhandlungen der Königl. Preuss. Met. Institute, Vol. I, No. 6. Berlin, 1900. (See No. XXI of this collection of translations.)

In such cases the curves give us pictures of the changes during the year that have great similarity with those curves that I used previously for the presentation of the movements of heat within the soil and which I have called "Tauto-chrones."[10] In order that these latter should exactly correspond with the former we must use the pressure instead of the altitude as ordinate, that is to say, the tables as well as the diagrams must proceed by increase of pressure and not by increase of altitude.

But first the method of presentation hitherto used will be applied to the respective seasons individually. Therefore I first of all combine the values deduced by Besson[11] and Süring[12] in one table, No. 2, which are then also presented in figs. 44 and 45.

In the temperature curves (see fig. 44) for each season (S = summer; W = winter; F = spring; H = autumn) the influence of the ground is evident in the same way as in the curves for individual days that were collected together in fig. 41.

Table 2. The average conditions as to temperature and moisture for each season

ALTITUDE h	TEMPERATURE (t)				MOISTURE (y)			
	W.	F.	S.	H.	W.	F.	S.	H.
m	°C.	°C.	°C.	°C.	*gram*	*gram*	*gram*	*gram*
0	0.3	8.7	18.4	9.3	3.00	4.71	8.38	5.71
500	—	—	—	—	2.61	4.49	7.47	4.83
1000	−0.6	2.5	11.0	5.4	2.17	3.67	6.69	4.40
1500	—	—	—	—	1.88	2.72	5.35	3.53
2000	−5.1	−2.1	5.3	1.6	1.64	2.41	4.59	2.68
2500	—	—	—	—	1.36	2.26	3.82	2.43
3000	−10.8	−3.6	0.9	−2.6	1.19	1.71	3.03	2.17
3500	—	—	—	—	0.98	1.42	2.61	2.03
4000	−14.6	−1.5	−5.0	−7.7	0.06	1.33	2.60	1.59
4500	—	—	—	—	0.88	1.10	1.84	1.72
5000	—	—	—	—	0.68	0.78	1.63	1.30
5500	—	—	—	—	—	0.60	1.59	0.88
6000	—	—	—	—	—	0.65	—	0.66

The summer curve shows decidedly the character proper to the season of prevailing insolation whereas in the winter curve the influence of the cooling of the ground is very evident; the part played

[10] See figs. 56 and 57 of Memoir XV. "The Heat Exchange", Sitzungsberichte, Berlin, 1892. (See No. XIX of this collection of translations.)

[11] Wissenschaftliche Luftfahrten, Vol. III, pp. 93–95.

[12] Wissenschaftliche Luftfahrten, Vol. III, p. 166.

by the ground is also beautifully seen in the changes from winter to spring and from summer to autumn.

It is also evident that especially in the summer curve between 3000 and 4000 meters peculiarities may be recognized similar to those in the lowest thousand meters, although on a much feebler scale. Perhaps we may in this perceive an indication of the circumstance that similar, although perhaps feebler, processes take place at the upper boundary surface of thick clouds as at the surface of the earth.

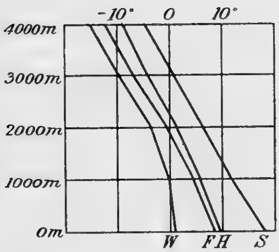


FIG. 44

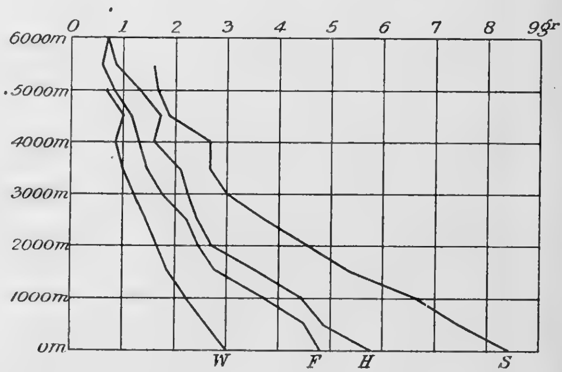


FIG. 45

The curves in fig. 45 representing the change of the specific humidity show great similarity to the temperature curves, as indeed could but be expected. The irregularities shown by them are not surprising. We would rather wonder that the curves are not still more irregular when we consider the difficulties that we encounter in determining the humidity and how small are the psychrometric differences in the upper strata on which these determinations are based. We may therefore rather regard these curves as a welcome proof of the excellence of the observations.

Reference has already been made to the fact that the curves of temperature condition here used have great similarity to those that I have previously used in order to study the movements of heat in the ground and which can be equally well applied to the corresponding processes in lakes or the ocean.

In that memoir[13] I have drawn the depths vertically downward from the earth's surface as ordinates and the corresponding tem-

[13] See Berlin Sitzungsberichte, 1892, pp. 1139–1178; or Memoir No. XIX of this collection of translations.

peratures horizontally as abscissæ. But the curves themselves representing the temperature conditions at a given moment of time I called "tauto-chrones." If we assume that the physical peculiarities of the soil are uniform throughout the whole stratum under consideration or at least, and more properly, that the calorimetric values for equal volumes or the so-called volume-capacity is uniform, then the quantities of heat received or given out between any two moments of time are proportional to the areas included between the tauto-chrones that belong to these two moments.

This same theorem would be true for the curves of condition as to temperature and moisture of the atmosphere corresponding to different moments or intervals of time, if the air had everywhere a uniform density. But, as is well known, this is not the case in the atmosphere; however, by appropriate choice of coördinates we can obtain curves of condition for which, as in the tauto-chrone, the area between two neighboring curves is proportional to the quantities of heat that must be received or given out in the passage from one condition to the other assuming that the masses of air remain the same and that the change of temperature is a simple consequence of gain or loss of heat. Similarly, the curves constructed in a corresponding manner for the specific humidity give the increase or loss of water or, since we can in this case start from the condition of absolute dryness, they give the total moisture contained in a given section of the vertical column of atmosphere.

We obtain such curves of condition if we construct a diagram in which the pressures diminish as ordinates from below upward and the temperatures or the specific humidities diminish backward as abscissæ. If we imagine a prism cut from the atmosphere erected vertically on a base of one square meter, and that the barometric pressures β_1 and β_2 prevail at the altitudes h_1 and h_2 then between these two altitudes there is included a mass of $13.6\ (\beta_1 - \beta_2)$ kilograms of air.

Hence in the prism between h and $h + dh$ the air mass is $13.6\, d\beta$ if the barometric pressure is β at the altitude h. If we further assume that at first the air in this prism has everywhere the uniform temperature 0° C. and that it is to be brought to the temperature t corresponding to that of its temporary condition, then the infinitely thin layer between h and $h + dh$ is to receive the quantity of heat $d\,Q = 13.6\ c_p\, t d\beta$ where c_p is the thermal capacity or specific heat under constant pressure. But the whole mass of air under consideration at whose lower and upper boundary surfaces the barometric pressures are β_1 and β_2 must receive the quantity of heat

$$Q = 13.6\, c_p \int_{\beta_2}^{\beta_1} t\, d\beta$$

in order to warm it from 0° C. up to the temperature corresponding to its temporary condition—where β_1 is the lower pressure at its upper boundary and β_2 the higher pressure at its lower boundary surface.

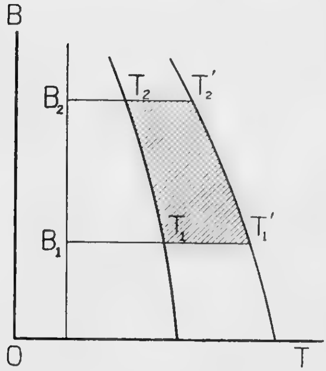


FIG. 46

Assume that the straight line $B_1 B_2$ in fig. 46 corresponds to the zero of the temperature scale and that β_1 and β_2 are the ordinates belonging to B_1 and B_2, and furthermore let the curves through the points $T_1 T_2$ and $T_1' T_2'$ be two curves of condition representing the course of the temperatures t' and t then will

$$\int_{\beta_2}^{\beta_1} t d\beta$$

be the surface bounded by the straight lines $B_1 B_2$, $B_1 T_1$, $B_2 T_2$ and the portion $T_1 T_2$ of the curve of condition where the points T_1 and T_2 correspond to the temperatures t_1 and t_2.

We can thus convert the above given equation into the form

$$Q = 13.6\, c_p\, F$$

where the surface $B_1 T_1 T_2 B_2$ is represented by F.

Now imagine another condition for which we have the temperatures t_1' and t_2' corresponding to the same pressures β_1 and β_2 as before, then the quantity of heat Q' that is now to be added in order to bring that portion of the column of air which at first had the temperature 0° C. up to the temperature condition represented by the second curve will be given by the equation $Q' = 13.6\, c_p\, F'$ where F' is the area of the figure $B_1 T_1' T_2' B_2$.

Finally the quantity of heat that is needed to convert the portion of the air column between the pressures β_1 and β_2 from the condition of the temperature t over into that of the temperature t' is

$$Q_1 - Q = 13.6\, c_p\, (F' - F) = 13.6\, c_p\, F^*$$

where the area $T_1\, T_1'\, T_2'\, T_2$ is represented by F^*.

In the method of graphic presentation here chosen, where equal lengths of ordinates correspond to equal differences of pressure, the curves of condition are actually therefore tauto-chrones, and the surfaces bounded by two horizontal lines and the portions of two curves of condition intercepted between them, give us a measure of the quantities of heat that have to be given to the corresponding portion of the column of air in order to convert it from one condition to the other under constant pressure.

These considerations are applicable not only to the temperature but equally well to the humidity. If y is the specific humidity, i. e., the quantity of water contained in a kilogram of moist air, then the quantity of water Y contained in the vertical prism erected on a base of one square meter and at whose limiting end surfaces the pressures B_1 and B_2 prevail, will be

$$Y = 13.6 \int_{\beta_2}^{\beta_1} y\, d\beta = 13.6\, F$$

if the specific humidities are laid off as abscissæ.

If the diagram be drawn in such manner that the zero of abscissæ corresponds to the zero of specific humidity then the total quantity of water in the vertical column is proportional to the surface bounded by the two axes and by the curve of condition for specific humidity.

Hence the last mentioned method of presentation, in which equal differences of atmospheric pressure correspond to equal lengths offers specific advantages.

If we would represent numerically the dependence of any quantity whatever on the atmospheric pressure, we must of course also proceed by equal differences of pressure instead of equal differences of altitude as was done above.

But in this method of presentation one must never forget that the portions of the vertical column corresponding to equal differences of pressure have very various altitudes corresponding to the absolute values of the pressure and to the temperatures. Thus, for example, the layer above considered between the pressures β_1 and β_2 has different altitudes for the two conditions represented by the curves of condition T_1 T_2 and T_1' T_2' and in fact the altitude will be smaller for the first case than for the second.

A clear idea of the operation of the method here used is obtained from the following consideration.

Assume that the so-called standard pressure of 760^{mm} prevails at sea-level and that we ascend step by step in altitude so that for each step the pressure diminishes 76 ^{mm} then each layer contains one-tenth of the atmosphere that is present above the given locality. For 760^{mm} of the mercurial barometer there is a pressure of 10,333 kilograms per square meter. Each of these ten sections into which we have in imagination divided the prism above one square meter contains therefore 1033 kilograms of air or more than a metric ton. If now the average specific humidity in such a section is 4, that is if a kilogram of air contains 4 grams of water, then there are 4.13 kilograms of water in that section. In a corresponding manner we find that for each such section of the column of air there are needed $1033 \times 0.2375 = 245.3$ or in round numbers 245 large calories (kilogram-calories) in order to raise the temperature of the corresponding air by 1° C.

But these sections, each of which contains one-tenth of the total column of superincumbent atmosphere, have remarkably different altitudes. For instance, whereas the lowest sections, according to the mean temperature deduced from balloon observations will reach from an initial elevation of 20 meters up to 890 m. the second section will extend from 890 to 1850 meters.

In the above-described method of representation we imagine these layers of differing altitudes all brought to the same thickness exactly as if the air in each were compressed to the same density and thus formed a so-called homogenous atmosphere.

Conversely, the strata which actually have equal altitudes in the natural atmosphere would in such an ideal case occupy very unequal volumes since the higher strata would be crowded together more and more.

In order to keep the relations clearly in sight, lines of altitude have been introduced in all the diagrams drawn on this system. These lines of altitude are based on the average distribution of temperature deduced from the balloon voyages and therefore correspond strictly only to the conditions presented by the curve t_m in figures 43 and 47.

Notwithstanding this last-mentioned limitation with reference to the applicability of these lines of altitude, still in general they furnish an excellent summary view of the distribution of mass throughout the atmosphere and lead to the agreeable conviction that the observations already given by balloon voyages as to conditions prevailing in the atmosphere apply to a very considerable fraction of the whole atmosphere.

They also show what an important part the lower and best known strata plays with reference to the economy of heat in the atmosphere. In all ascensions above 3300 meters and for the average distribution of temperature, we have already surmounted one-third of the whole atmosphere and in the ascensions of sounding balloons above 8400 meters we have passed through two-thirds of the atmosphere.

Furthermore, one sees quite vividly how rapidly the quantities of heat diminish with the altitude, which fact comes in play when equal changes of temperature occur in high and low strata, and we see how erroneous it is to replace the clearly defined expression "Temperature of the air" by that of "Atmospheric heat."

In order to warm by 1° C. a definite given volume of air or a layer of air of definite thickness at the surface of the earth, there is needed twice as much heat as under average conditions is needed at the altitude of 5000 meters. Equal oscillations of temperature in the upper and lower strata which one naturally assumes to be of equal mass, i. e., of equal altitude, will therefore have much less importance in the former than in the latter. Moreover, the expression "Distribution of heat over the surface of the globe," which is generally used and therefore not easily abolished, is, strictly speaking, not correct, since we do not really mean the distribution of heat, which depends not only on the density but also on the moisture of the air, but we have in mind only the distribution of the temperature at a few meters altitude above the ground.

The diagram, fig. 47, is now easily understood, especially when we add that the full horizontal lines refer to the scale of barometric pressures given on the left, while the dash lines refer to the scale ot altitudes given on the right hand of the diagram.

If we consider the vertical line belonging to the temperature $-60°$C. as the axis of ordinates then the abscissæ of the curve of condition t_m are directly proportional to the quantities of heat that must be given to a unit mass of air having the pressure given at the left-hand in order to warm it from the temperature $-60°$ C. up to that of t_m.

In a corresponding way the abscissæ of the curve y_m give the grams of water contained in a kilogram of air of the corresponding stratum; the abscissæ of the curve y_s give the grams of water that would be contained in this kilogram of air if it were perfectly saturated. The number of grams is inscribed at the top of the diagram, fig. 47

The numbers used in the construction of this figure are also given in table 3.

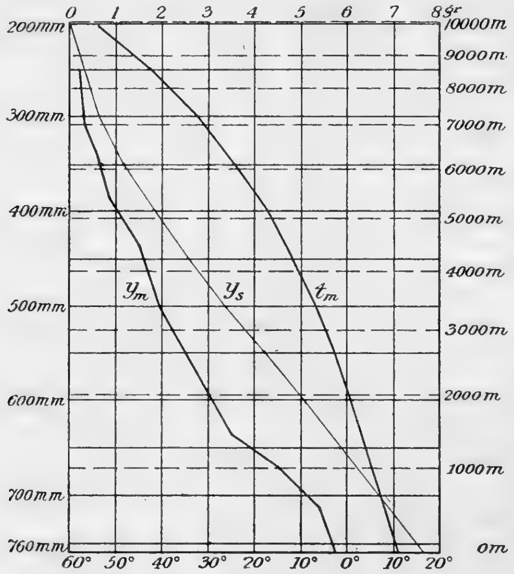


FIG. 47

Along with the quantities previously considered this table also gives, as in table 1, under the column Y_m the average quantity of water in kilograms contained in the whole column of air of one square meter section from the ground up to the altitude of the given pressure, while in the last column under Y_s is given the number of grams of water that would be contained therein if the air were fully saturated for the average distribution of temperature (as shown by the curve t_m).

The numbers collected in table 3 show clearly how the aqueous vapor is distributed throughout the atmosphere above Germany.

Table 3. The average vertical distribution of temperature and moisture for successive pressures

β_m	h_m	t_m	y_m	y_s	Y_m	Y_s
mm	m	°C.	grams	grams		
760	20	10.3	5.77	7.68	0.0	0.0
750	130	9.8	5.65	7.53	—	—
700	700	6.9	5.09	6.63	4.45	5.91
650	1300	3.9	4.08	5.80	—	—
600	1950	0.6	3.17	4.97	9.95	13.69
550	2650	−3.1	2.49	4.13	—	—
500	3400	−7.1	1.92	3.36	13.38	19.68
450	4210	−11.4	1.61	2.67	—	—
400	5110	−17.1	1.04	1.88	15.51	23.20
350	6103	−23.9	0.63	1.19	—	—
300	7210	−31.6	0.25	0.68	16.39	24.87
250	8490	−41.7	—	—	—	—
200	9850	−54.1	—	—	—	—

By a little extrapolation we find that the column of air resting on one square meter contains on the average only 16.5 kilograms of water, and that even for complete saturation under the average distribution of temperature this quantity can at the most rise only a little more than 2.5 kilograms. Therefore in the neighborhood of Berlin the whole atmosphere contains on a average only 1.6 per thousand (or one-sixth of one per cent) of water, an amount which can only be increased to 2.5 per thousand for complete saturation under the average distribution of temperature.

A graphic interpolation shows at once that on the average we find one-half of this total quantity of water in the stratum between the surface of the ocean and 1600 meters altitude, so that, for instance, at the summit of the Schneekoppe we already have one-half of the total aqueous vapor of the atmosphere below us.

If, on the other hand, we consider that 1 kilogram of water spread over 1 square meter of ground covers it exactly 1^{mm} deep, we can get a standard for determining how rapidly the ascending air must be renewed over such a surface in order to furnish the quantities of precipitation actually observed and which may still be very considerable even at altitudes of 1600 meters.

However, we must not forget that we can obtain from the psychrometric measurements only the water that is present as vapor; how large the quantities of water may be that are present in condensed form in the clouds has up to the present time completely eluded our observations.

The considerations just set forth enable us not only to follow the average-content of the atmosphere for each season but also the increase and loss of heat during great intervals of time. To this end the curves of average condition as to temperature and moisture as they are already given in fig. 44 and fig. 45 are transformed into the new system of coördinates, and thus we obtain figures 48 and 49. From these diagrams by measuring the areas of the surfaces we obtain the numbers given in the following tables 4*a* and 4*b* just as previously we had done for the values Y_m and Y_s in table 3.

As we can deal only with rough approximations, therefore the following table 4 is arranged only for large intervals.

Table 4. Average kilograms of water contained in a vertical column of atmosphere, one square meter section, for each season

(4*a*) FOR INTERVALS OF PRESSURE

PRESSURE	WINTER	SPRING	SUMMER	AUTUMN
mm				
760–700	2.25	3.68	6.33	4.17
700–600	2.72	4.93	8.05	5.12
600–500	1.76	2.70	4.81	3.19
500–400	1.28	1.61	2.90	2.21
760–400	8.01	12.92	22.09	14.69

(4*b*) FOR INTERVALS OF ALTITUDE

ALTITUDE	WINTER	SPRING	SUMMER	AUTUMN
mm				
0–1000	2.96	5.54	8.42	5.70
1000–2000	2.07	3.19	6.10	3.81
2000–3000	1.23	1.88	3.32	2.11
3000–4000	0.84	1.30	2.41	1.70
4000–5000	0.66	0.88	1.51	1.26
0–5000	7.76	12.77	21.76	14.58

The numbers here given are of such nature that they have no claim to great accuracy, since in the passage from the numbers given by Berson and Suring for different altitudes, as presented above in table 2, errors slipped in because the relations between pressure and altitude change for different temperatures.

For the same reason also in the diagrams, figures 48 and 49, drawn for different pressures there should, strictly speaking, be drawn different scales of altitude for the different seasons of the year. Since, however, the application of more accurate methods of computation would have caused a disproportionate amount of labor and since the gain in reliability would still be small, therefore, and for simplicity, we may be satisfied with the scale of altitudes based on the temperature deduced from the totality of all the observations, as we find it under t_m in table 1.

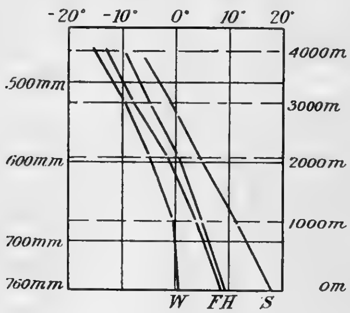


FIG. 48

We obtain a good idea of the magnitude of the error due to this simplification by means of fig. 48, in whose construction only the values up to 4000 meters could be utilized. We see that the corresponding curves end at different ordinates since the same altitudes correspond to smaller values of the atmospheric pressure when temperatures are low but to higher pressures when the temperatures are high.

In a perfectly similar way to that by which we have just given the quantity of water we compute in the following table 5 the differences of the quantities of heat that are contained, on the average of the respective seasons, in the several sections of the column of air resting on one-square meter of ground.

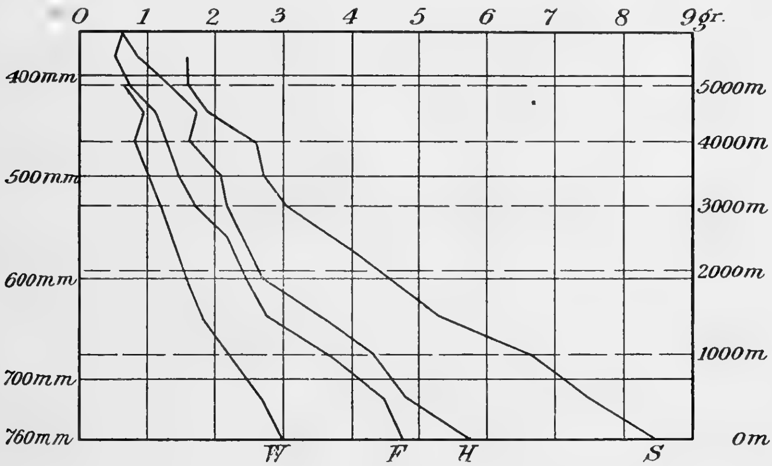


FIG. 49

Of course in this summary the tens have but little significance, exactly as in the above-mentioned table, where the second decimal figure is a pure result of computation, and is retained only for the sake of an easier check.

Before I come to the table itself I must first explain the word "Thermal-content" or "Wärme gehalt" used in the title of this table.

By "Thermal-content" I understand the quantity of heat that must be communicated to a given quantity of air to bring it under constant pressure from any arbitrary initial temperature up to any given final temperature. The same expression was, as is well known, some time since applied by H. v. Helmholtz to this conception for which, with his approval, I afterward substituted the term "potential temperature." Since, however, this last term has meanwhile become generally accepted, and since I have not been able to find another appropriate term for the idea now under consideration, therefore I think it allowable to apply the term "thermal content" in this place in a different sense.

With this understanding I submit the following table 5. In this table W, F, S, H, represent Winter, Spring, Summer, and Autumn, respectively, and therefore the numbers in the column under F-W represent the number of kilogram-calories that need to be given to the unit-mass of air at the respective pressures or altitudes in order to bring it under constant pressure from the temperature of winter to that of spring.

Table 5. Seasonal differences in thermal content of the atmosphere expressed in calories, for vertical columns having 1 square meter sectional area

(*a*) FOR SUCCESSIVE PRESSURES

PRESSURE	F-W	S-F	S-H	H-W
mm				
760–700	1290	1700	1530	1460
700–600	1610	2260	1910	1960
600–500	780	2030	1390	1420
760–500	3680	5990	6830	4840

(*b*) FOR SUCCESSIVE ALTITUDES

ALTITUDE h	F-W	S-F	S-H	H-W
m				
0–1000	2010	2360	2110	2260
1000–2000	960	1710	1400	1270
2000–3000	580	1510	1040	1050
3000–4000	440	1330	810	960
0–4000	3990	6910	5360	5540

The subsequent columns are to be understood in a similar way but with this difference, that the numbers in the two last columns indicate the quantities of heat that are to be abstracted from the respective sections of the column of air to bring about the transition from summer to autumn and from autumn to winter.

Of course it is understood that all these numbers are fundamentally only algebraic sums, since in reality the passage from one season to the next by no means implies continuous progressive addition or abstraction of heat but processes that frequently alter within short intervals of time and which are here represented only by this final outcome.

These two latter tables 5*a* and 5*b* have been especially introduced because from them and especially from the data by altitudes in table 5*b* we see clearly how rapidly the quantities of heat interchanged in given strata, diminish with altitude.

Hence the influence of the highest strata on the economy of heat in the atmosphere is of subordinate importance even when the annual range of temperature is larger than we formerly suspected. Even when ranges at great altitudes are as large as they are at the ground still the quantities of heat exchanged in strata of equal thickness are proportional only to the density of the air present therein. This is another reason why it is not recommended to speak of the heat of the air (luftwärme) instead of the temperature of the air.

Now that we have thus illustrated the vertical distribution of the meteorological elements from very various points of view, there still remains the important question to what extent the results attained by these ascensions are appropriate to remove the doubt that exists as to the interchange of air between the cyclone and anticyclone or as to the ultimate origin of these two groups of atmospheric whirl-winds.

It follows from the memoir of Berson[14] that this important question cannot yet be finally answered even from the results of balloon voyages. But in order that we may at least approach the problem somewhat more nearly I have requested Berson to prepare for me a new analysis of the results with reference to the temperatures in the cyclones and anticyclones for the summer and winter half years separately. I submit this as table 6 but include in brackets those numbers that result from *one* voyage only.

Table 6. Vertical distribution of temperature during cyclonal and anticyclonal weather

ALTITUDE	WINTER		SUMMER	
	Cyclone	Anti-cyclone	Cyclone	Anti-cyclone
m	°C.	°C.	°C.	°C.
ground	+3.0	+1.5	+15.7	+20.6
1000	−2.2	+1.3	+9.1	13.6
2000	−8.0	−2.0	+3.0	7.7
3000	−15.1	−6.7	−0.8	2.1
4000	−20.8	−10.9	−7.0	−3.3
5000	−27.5	−16.0	−15.3	−9.1
6000	−34.0	−25.8		(−17.2)
7000	(−44.4)	(−30.2)	....	(−22.0)
8000	(−48.5)	(−37.9)	....	(−30.7)

[14] Berson: Wissenschaftliche Luftfahrten, Vol. III, p. 103 et seq.

From this table, as well as from the figures 50 and 51 belonging therewith, in which the curves belonging to the cyclonal and anticyclonal weather conditions are indicated by C and A, we see at once that in winter as well as in summer the temperatures in the anticyclones are higher than in the cyclones at the same altitude; the only exception is in the very lowest portion of the winter curves. Thus from these voyages we deduce the same result that Hann drew from the observations made in the Alps.

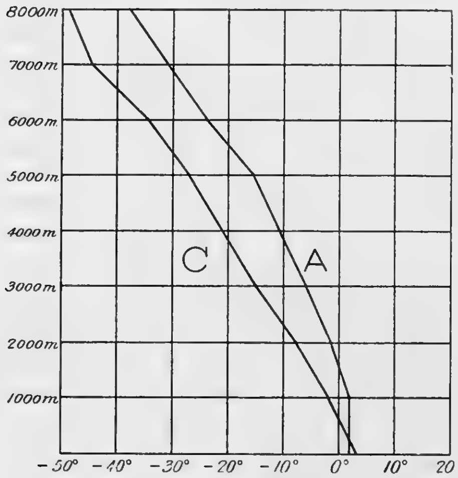


FIG. 50

Thus the temperatures observed in cyclonal and anticyclonal regions up to 8000 meters altitude do not suffice to explain the origin or existence of the ascending and descending movement that is demanded by the so-called convection theory.

This is true, indeed, only for Central Europe, but since the cyclones generally arrive on the European coasts already well developed, and since also, on the other hand, for reasons easily understood, the balloon voyages have given us very few observations from the highest and central parts of the cyclones; therefore the question is far from being definitely decided.

Moreover, the fact that in the majority of cases the cyclones lose their intensity after entering the continent and are finally dissipated in the interior of Northern Asia, indicates that observations over Central Europe cannot possibly suffice to answer the fundamental question. At least it will require a much broader base of observations than is here given to answer this question.

Whether and to what extent a thorough discussion of the simultaneous voyages undertaken at a great variety of places in Europe, or a profound comparative study of the observations made in America and Europe with balloons and kites, will suffice, may be left to the future. Personally I incline to the conclusion that the nature of the cyclones and anticyclones will be understood only after we begin to study them from broader points of view in connection with the general circulation of the atmosphere.

For the present one must be satisfied with theoretical considerations concerning this aspect of the question since it will certainly be a long time before observations are at hand from the oceans and the Tropics similar to those for Central Europe that are presented by the ascensions described in the present publication.

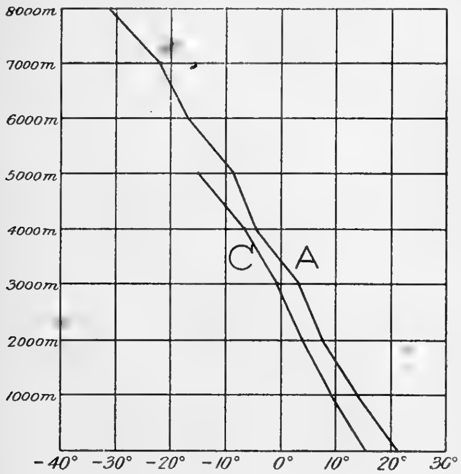


FIG. 51

At any rate, however, through these ascensions the theoretical foundations have acquired such strength that we may safely venture to build further thereon until the conclusions drawn therefrom once again find confirmation, just as now the theoretical work that has been prosecuted for years is to a certain degree confirmed by the labors whose results are recorded in this work.

XV

ON THE REDUCTION OF THE HUMIDITY DATA OBTAINED IN BALLOON ASCENSIONS

BY PROF. DR. W. VON BEZOLD

[*Zeitschrift für Luftschiffahrt und Physik der Atmosphäre. Vol. 13, pp. 1–9, 1884. Translated from Gesammelte Abhandlungen, W. von Bezold, 1906, pp. 264–273*]

Ordinarily we use the vapor pressure, the absolute humidity and the relative humidity as the measure of the moisture in the atmosphere.

The determination of these three quantities, or any two of them, suffices in general to define the condition as to moisture. This is especially true when one has a definite portion of the atmosphere under consideration no matter whether one wishes to present its condition as to moisture at a given moment, or to present the chronological changes of condition, especially as in climatological investigations.

It is different when the problem is to follow a mass of air in its path through the atmosphere and to take into consideration the increase or decrease of the amount of water.

In order to handle these latter problems a knowledge of the above-mentioned quantities does not suffice, at least not directly, rather must we from these deduce still other quantities if we would attain a correct idea.

If, for instance, we consider a quantity of air with given constant mixing ratio of aqueous vapor and dry air, and we wish to investigate the changes that this undergoes as it rises higher in the atmosphere, then in spite of the constant mixing ratio both the vapor pressure and also the absolute humidity will in general vary.

The very important circumstance that during these processes the composition of the air has not experienced any changes cannot be deduced from the data ordinarily employed.

Inversely, the relative humidity can remain constant, whereas in fact water is steadily being precipitated, as, for instance, in case an ascending current of air has exceeded the limit of saturation.

We were therefore forced long since, in our theoretical investigations to introduce two other quantities by the use of which not only did such investigations first become possible, but which are also adopted to give a deeper insight into the condition of the atmosphere as to moisture. These quantities are on the one hand the quantity of vapor contained in the unit mass of moist air, which can be conveniently called the "specific humidity;"[1] and, on the other hand, the quantity of moisture mixed with the unit mass of dry air or briefly "the mixing ratio."

How important the knowledge of these quantities may be in the discussion of the observed numerical data gained by balloon ascensions is shown by the simple consideration that these must remain constant so long as the balloon preserves a course in companionship with the air that surrounds it, no matter how complex the changes may be that this air experiences as to its pressure and temperature and consequently also as to its absolute and relative humidity.

So also do these quantities experience no change so long as the balloon remains within an ascending or descending current provided that no other air, with a different vapor constant, becomes mixed with it.

Hence also, conversely, any change in these quantities becomes in one sense a measure of the admixture of foreign masses of air, a process whose study is of the highest importance.

This much by way of introduction. We will now first consider the relations that exist between these two quantities and those others that are ordinarily used as characteristic of the moisture in the atmosphere.

To this end I shall use the following notation:

ε the vapor pressure in millimeters of mercury.

ε' the maximum possible vapor pressure for the temperature t.

f the absolute humidity or the number of grams of water vapor in a cubic meter.

R the relative humidity.

x the mixing ratio or the mass of vapor mixed with a unit mass of dry air expressed as a fraction of this latter unit.

---

[1] In the mechanical theory of heat, as is well known, the fraction of water, that is in the form of vapor, in a mixture of saturated vapor and water is called the "specific quantity of vapor." By using the name above proposed we give expression to the analogy of the two quantities (vapor and humidity), whereas on the other hand we prevent any confusion in the two different ideas.

y the specific humidity or the quantity of vapor in a unit mass of moist air expressed in fractional parts of this unit.

β the pressure to which this mixture is subjected expressed in millimeters of mercury.

β_0 the pressure of the standard atmosphere or 760mm.

α the coefficient of expansion of air = 0.00367 = 1/273.

t the temperature of the mixture.

Recalling that 1 cubic meter of dry air at 0° C. and under a pressure of 760mm weighs 1293 grams, and that the density of aqueous vapor is 0.623 times that of dry air at the same temperature and pressure, then we have the following equations:

$$f = 0.623 \times 1293 \frac{\varepsilon}{760} \times \frac{1}{1 + \alpha t} = 1.060 \frac{\varepsilon}{1 + \alpha t} \quad . \; . \quad (1)$$

$$R = 100 \frac{\varepsilon}{\varepsilon'}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

$$x = 0.623 \frac{\varepsilon}{\beta - \varepsilon} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

$$y = \frac{x}{1 + x} = 0.623 \frac{\varepsilon}{\beta - 0.377\,\varepsilon} \quad . \; . \; . \; . \; . \; . \quad (4)$$

Since the quantities x and y are always among the hundredths therefore in many cases it will be advisable to multiply the value by 1000, that is to say, we use the number of grams of vapor that are mixed with a kilogram of dry air, or that are contained in a kilogram of moist air respectively.

If we indicate these values by x_g and y_g then we have

$$x_g = 623 \frac{\varepsilon}{\beta - \varepsilon} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3a)$$

and

$$y_g = 623 \frac{\varepsilon}{\beta - 0.377\,\varepsilon} = 1000 \frac{x_g}{1000 + x_g} \quad . \; . \; . \; . \quad (4a)$$

In the headings of the tables that are to follow, as also in the diagrams, I will, for the sake of clearness, omit the subscript index g and will by x and y understand quantities 1000 times as large as those given by the preceding definition.

Since the quantities x and y are in general very small and scarcely ever exceed the value 0.03 but are generally much smaller, therefore they are never very different from each other and for a rough approximation may be considered briefly as equal to each other.

Now in order to put the meaning of the specific humidity and the mixing ratio in a very clear light it seems appropriate to investigate how the other quantities vary when the former is constant.

To this end write equation (3) in the form

$$\varepsilon = \beta \frac{x}{x + 0.623}$$

and substitute this in equation (1), then we obtain

$$f = 1.060 \frac{\beta}{1 + \alpha t} \cdot \frac{x}{x + 0.623}$$

This equation shows that with constant pressure but variable temperature, the absolute humidity experiences changes even when the composition of the air remains constant.

We get the best idea of the matter if we assume that we change the mixture from an initial condition, in which the appropriate quantities have the subscript index 1, over into another for which we use the subscript 2.

If then we assume

$$t_2 = t_1$$

$$\beta_2 = \beta_1$$

$$t_2 > t_1$$

it results that

$$\varepsilon_2 = \varepsilon_1$$

but

$$f_2 < f_1 \text{ and } R_2 < R_1$$

If, on the other hand, we imagine that the volume remains unchanged, then according to the well-known law of Mariotte-Gay-Lussac we have the relation

$$\frac{\beta_2}{\beta_1} = \frac{273 + t_2}{273 + t_1}$$

or

$$\frac{\beta_2}{1 + \alpha t_2} = \frac{\beta_1}{1 + \alpha t_1}$$

and then for $x_2 = x_1$ and $t_2 > t_1$, we have

$$f_2 = f_1 \text{ but } \varepsilon_2 > \varepsilon_1 \text{ and } R_2 < R_1$$

that is to say, the warming of a quantity of air having a given invariable composition and inclosed within a non-expansible vessel, causes a rise of vapor pressure and a diminution of relative humidity while the absolute humidity remains unchanged.

This example is very instructive because it allows us to recognize very clearly the difference between vapor pressure and absolute humidity, whereas otherwise not infrequently one tends to consider these two ideas as equivalent.

The reason for this latter error lies in the fact that the quotient

$$\frac{1.060}{1 + \alpha t}$$

occurring in equation (1) is equal to unity when $t = 16.38°$ C. and varies less than 2 per cent from unity between the values $t = 10°$ and $t = 22°$. Since in the metric system the numbers for vapor pressure and for absolute humidity are nearly the same for the temperatures that most frequently occur, therefore in ordinary language the difference between these two ideas is frequently entirely overlooked.

These analyses may suffice to help us clearly recognize the meaning of the specific moisture and the mixing ratio.

I will now pursue the example further and show how the average distribution of aqueous vapor in a vertical column of air is expressed by the use of this idea.[2]

I assume the atmospheric pressure at sea-level to be 760^{mm}, the temperature 9.0° C. and the vapor pressure 6.5^{mm}, which correspond to annual average conditions in the neighborhood of Berlin.

As to the diminution of temperature with altitude I have used the numbers deduced by A. Berson.[3] The value of the vapor pressure for different altitudes is computed by the formula given by R. Süring

$$\varepsilon = \varepsilon_0 \, 10^{-\frac{h}{6}\left(1 + \frac{h}{20}\right)}$$

[2] The following portion of this memoir has been revised, taking into consideration the results of recent balloon voyages. [1905. W. v. B.]

[3] Wissenschaftlichen Luftfahrten., Vol. III, p. 63.

where ε_0 is the vapor pressure at sea-level and h is the altitude in meters.[4]

This being premised I obtain the following table 1 and arrange the columns in the order in which they are derived from each other, i. e., first the altitude and the accompanying temperature, then the air pressure and vapor pressure, then values of x and x/x_0 deduced from these, where x_0 is the value at the surface of the ground, expressed in grams of water per kilogram of dry air, and finally the computed values of the absolute and relative humidity.

TABLE 1

h	t	β	$\beta/760$	ε	x	x/x_0	f	R
m	° *C*	*mm*		*grams*	*grams*		*grams*	
0	+ 9.0	760	1.00	6.50	5.38	1.00	6.66	76
1000	+ 4.0	673	0.89	4.34	4.04	0.75	4.53	72
2000	− 1.0	595	0.78	2.79	2.94	0.55	2.96	66
3000	− 6.4	524	0.69	1.73	2.06	0.38	1.88	61
4000	−11.7	461	0.61	1.03	1.40	0.26	1.14	55
5000	−18.1	404	0.53	0.59	0.91	0.87	0.67	53
6000	−25.0	353	0.46	0.32	0.57	0.11	0.37	52
7000	−31.8	307	0.40	0.17	0.37	0.07	0.22	53
8000	−39.0	266	0.35	0.09	0.21	0.04	0.11	60

In order to make the course of these numbers perfectly clear the quantities β, ε, x and R are represented by curves in fig. 52, where the altitudes are ordinates, and the corresponding values of the other quantities are abscissæ.

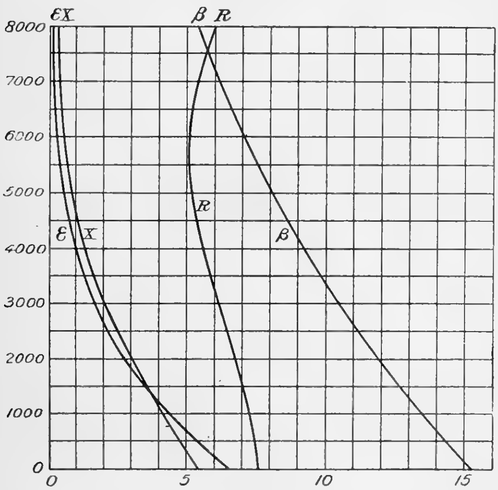


FIG. 52

In explanation of this diagram it need only be added that the distance between two consecutive vertical lines is taken as unity in plotting the quantities ε and x but is taken as 10 in plotting relative humidity and as 50 in plotting barometric pressure.

The numbers, as also the accompanying diagram, now show clearly

[4] Süring: Wiss. Luft., Vol. III, p. 160.

the diminution of humidity in respect to the mixing ratio. Whereas according to Süring's formula, the vapor pressure (ε) at an altitude of scarcely 1000 meters is reduced to one-half of that observed at the earth's surface, we must rise to 2000 meters before finding the mixing ratio (x) or the specific humidity (y) diminished to the same extent. But at an altitude of 6000 meters the vapor pressure is only 1/20 of that in the lowest stratum, whereas the mixing ratio and the specific humidity are somewhat less than $\frac{1}{5}$ of those in the lowest stratum.

The slight increase of the relative humidity (R) in the highest stratum, as expressed by the numbers (52 and 60), can hardly have any general importance for we should not overlook the fact that it needs only very slight changes in the course of the temperatures to greatly change the values of the relative humidity and the course of the corresponding curve.

I omit any further remarks that are suggested by figure 52 and turn rather to another example that shall relate to the behavior of ascending air without experiencing any mixture with other air or any decrease or diminution of heat.

Let this air at its start have the altitude of 0 meter, pressure 760^{mm}, temperature 25° C., vapor pressure 9.25^{mm}. If this air ascend adiabatically, then at the altitude 1800 meters condensation will begin and at 3070 meters it will have attained the snow stage.

In detail we have the series of values given in table 2.

TABLE 2

h	β	ε	x	t	R
m	*mm*	*grams*	*grams*	°*C.*	*per cent*
0	760	9.25	7.69	25	39
1000	677	8.24	7.69	15	65
1800	615	7.49	7.69	7	100
3070	526	6.60	5.51	0	150
6840	322	0.61	1.18	−25	100

Fig. 53, which needs no further elucidation, shows the peculiar course of these values.

On the other hand this diagram shows clearly how important it is to take into account the mixing ratio, or if one prefers it the specific humidity, together with the vapor pressure and the relative humidity, in the reduction of the humidity data for various altitudes.

This is especially important if we keep in mind the fact that the change of x for any given change of altitude gives directly the quantity of water that is precipitated in the ascent of the air through this vertical distance.

Hence from the above given course of the average values of this quantity (x) we may conclude, as to the values precipitated in the individual strata of the atmosphere and thence, in connection with the precipitation measured at the earth's surface, may conclude something as to the average intensity of the vertical circulation of the air.

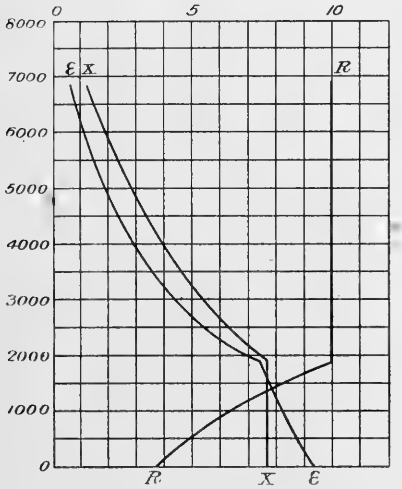


FIG. 53

I have already remarked that from the constancy of the mixing ratio within any part of the atmosphere we can with some confidence draw the conclusion that in that particular portion no mixture of various kinds of air has taken place.

Conversely, rapid changes in the mixing ratio as we pass through different strata of air show that there are present masses of air having different origins.

The study of the mixing ratio acquires increased importance, in view of the circumstance that the frequent occurrence of the Helmholtzian billow clouds forces us to the conclusion that very frequently strata of air of quite different temperatures and humidities are flowing over each other, since the surest indications of the stratification of the atmosphere are found in the numerical value of this quantity.

These remarks may suffice to demonstrate how desirable it is that in the discussion of the results of balloon voyages the mixing ratio should be regularly taken into consideration.

Perhaps it may indeed be worth the trouble also to include it in investigations into the humidity condition at the earth's surface itself, since for equal vapor pressure the value of the mixing ratio varies very nearly as the reciprocal of the barometric pressure. So that the distribution of humidity at the earth's surface will in many cases, by utilizing this element have quite a different aspect from that obtained directly from the vapor pressure alone.

XVI

ON THE CHANGES OF TEMPERATURE IN ASCENDING AND DESCENDING CURRENTS OF AIR

BY PROF. DR. WM. VON BEZOLD

[*Meteorologische Zeitschrift, 1898, XV, 441–448. Translated from Gesammelte Abhandlungen, W. von Bezold, 1906, pp. 274–283*]

When I published my first memoir "On the Thermodynamics of the Atmosphere"[1] I expressed the hope that by the elucidations therein given we had finally set aside the view that slipped into the excellent work of Guldberg and Mohn, according to which the cooling of ascending air depends on the work done in lifting it. But since A. Schmidt of Stuttgart, not only in the year 1890[2] but also more recently[3] has come forward again as defender of the idea that the work done in lifting plays a part in the cooling of ascending currents of air, therefore I think that I ought not to delay to repeat and supplement in more thorough manner my earlier explanation of this matter. I will, however, pursue a course directly opposite to that adopted by Schmidt.

Whereas Schmidt takes the kinetic theory of gases into consideration and thereby unnecessarily obscures such a simple question, I will attempt to treat it in a manner as elementary as possible.

This course seems to me so much the more advisable since in fact the principle of Archimedes as also the theorems based on experience relative to the thermal behavior of gases suffice for the investigation, whereas the introduction of the kinetic theory of gases has only the result of causing an unnecessary and therefore injurious complication of the present question.

If we are to consider the subject of the work done in the ascent of a mass of air, we must first clearly understand the conditions attending a given volume of air within the atmosphere. Let us assume that the volume under consideration encloses the mass m,

[1] Sitzb. Berlin Akad., 1888. [See pp. 212–242 of the previous collection of translations.—C. A.]

[2] On the cause of the diminution of temperature with altitude. Tübingen, 1890.

[3] Illustrierte Aeronautische Mittheilungen, 1898, II, pp. 12–15.

then its weight must be $P = mg$ if we consider the mass as enclosed within an enclosure that has no weight and as located within a vacuous space such as the receiver of an air pump and at a place where the acceleration of gravity is g. Under these suppositions the full amount of work required to raise this weight through the vertical distance h would be mgh or that which Schmidt considers necessary under the conditions existing in the atmosphere. But this volume of air thus imagined cut out of the atmosphere, is in fact surrounded by air. Consequently its weight in the atmosphere is

$$P' = gm - gm' = g(m - m')$$

where m' is the mass of the air displaced by it. But since the barometric pressure within the enclosed mass is the same as that in its immediate neighborhood therefore

$$m' = \frac{1 + \alpha t}{1 + \alpha t'} m = \frac{a + t}{a + t'} m,$$

where t is the temperature of the mass and t' that of the surrounding air and where

$$a = \frac{1}{\alpha} = 273$$

Hence we find

$$P' = gm = \left(\frac{t' - t}{a + t'}\right)$$

This latter value can be either positive or negative or zero according as t is smaller or larger or equal to t'. Hence no work is done in lifting the enclosed air unless its mass is colder than that of the surrounding air. If it is warmer then it has a buoyancy and it rises of itself through the surrounding cool air which flows in to fill its place, and thus the center of gravity of the whole system sinks exactly as the theory of equilibrium demands.

But under all circumstances the absolute value of P' is much smaller than mg, so that for a mass of air rising through the altitude h (and whose ascent indeed never occurs alone in the atmosphere but only in connection with other air descending at some other place, thus forming a connected whole) it is never allowable to introduce mgh into the computations as representing the work done.

But even when work is really done in lifting, which can only occur when ascending air is cooler than the descending, as in the case of cyclones with cold centers, still the work is exceedingly slight in comparison with the work of expansion done by the ascending air, as will be seen by the formula to be developed hereafter. It is evident that such processes are in general only possible when actual changes, that is, motions of the atmosphere sufficient to overcome the existing gradients, are produced by the buoyancy of other portions of the atmosphere.

If, however, we ignore such special cases and consider only the normal interchange of air between cyclones and anticylones under the assumption that a steady state of motion has been established and omitting the vortex motions due to the rotation of the earth, then we may imagine a number of stream lines united to form a closed ring and we have a process analogous to that in the closed system of a hot-water heating system.

If now we study the process within such a ring, assuming for the sake of simplicity that it has only a slight vertical range so that equal differences of altitude correspond to equal differences of pressure then we may consider it as represented by the scheme outlined in fig. 54.

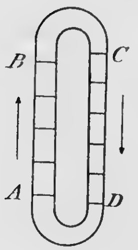


FIG. 54

In this figure let A B represent the ascending and C D the descending branch, and so choose the connecting pieces B C and A D that the masses contained in them are in equilibrium with each other (that is to say that equal masses are contained in those portions on the right and left hand of the central line) (which of course requires that C shall stand a little higher than B, and D somewhat lower than A) then the excess of pressure at D is equal to the difference of the weights of the fluid columns A B and C D. If now we further assume that the horizontal lines or divisions indicated in this figure correspond to equal differences of pressure, then equal masses of fluid are contained between two successive sections (if the areas of the sections of the tubes are uniform), and the weights of the two vertical columns are proportional to the total number of the sections.

But the excess of pressure corresponding to the difference of the weights of these columns is the same at each cross-section of this closed system.

Now, when vertical motion is set up and a steady condition of motion is established, the ascent of any mass in A B is always at the expense of the sinking of equal mass in D C, since the masses that flow through a unit section in a unit time must be the same for every section. In consideration of the different densities in the ascending and descending branches this leads of course to the conclusion that either the sections of the two branches have a definite ratio to each other or else the velocities at different places must be different.

The former of these assumptions is the simpler, whereas changes of velocity would materially burden our course of reasoning unless indeed they are neglected entirely. In consideration of this fact the scheme assumes that there is a variation in sectional areas of the two branches. But under all conditions the existing surplus of pressure after initiating the motion or after attaining a stationary condition serves only to overcome the friction, and if that did not exist it would, after equilibration of the temperatures, maintain the movement forever.

Exactly the same conditions exist in the interchange of air between ascending and descending currents, as soon as the movements are started and the steady condition is established. Here also the rise in the ascending branch takes place at the expense of the mass that is sinking in the descending branch and there can be no thought of a work of elevation performed by other forces such as heat introduced from without or by the loss of internal energy, i. e., cooling.

But these preceding remarks obtain only for the steady condition. If the motion is to be first initiated then either the center of gravity must be raised on one side by warming or on the other hand it must be lowered by cooling. In the first case actual work must be performed to raise the mass and this is to be added to the work done by expansion; in the second case the energy that is necessary to set in motion the whole mass that enters into circulation must be obtained from the descent of the center of gravity of the cooling side of the whole system.

In this process, however, it is absolutely necessary that the heat be "taken in" by the warming side or else "given up" by the cooling side unless mechanical acceleration be given to the mass by other masses of air not belonging to this system, as, for instance, by friction or pressure or suction.

If we ignore these last mentioned influences and confine attention to those processes in which only heat comes in play, we get an excellent insight into the behavior of the phenomena by the presentation given at pp. 107–110 of Sprung's Lehrbuch der Meteorologie and the experiment there described.

If now we apply similar considerations to the rise and fall of the surfaces of equal pressure, such as occur in the production of land winds and sea-breezes or mountain and valley winds, then we at once see that here also in case of warming, work is actually done by lifting, or in case of cooling there is a disturbance of equilibrium by the sinking of the center of gravity, which then brings about a motion of the mass of air.

If the above-given analysis does not yet suffice to make the subject perfectly clear, then perhaps the following considerations may succeed in doing so.

Assume that we have three cylindrical vessels, two of them filled with mercury to the height of 760^{mm}, the third filled to the same height with water. Let the external air pressure also amount to 760^{mm} mercury. Now assume further, that at the base of the first vessel, filled with mercury, there is a piece of iron that is at the beginning held down but by some appropriate arrangement may be suddenly left free. When set free, the iron rises until it swims on the surface of the mercury. But now this surface itself stands somewhat lower since the floating iron protrudes partly above it and the center of gravity of the whole system is now somewhat lower.

No one will imagine that the iron cools by rising, but will rather at once perceive that its rising is at the cost of the sinking of the mercury.

At the bottom of the second vessel imagine a mass of air enclosed in a small bell glass whose mouth opens downward. This air is therefore under a pressure of two atmospheres. Turn the bell glass over by appropriate mechanism so that its mouth opens upward and the air rises through the mercury to its upper surface. In this process the air expands and consequently cools. Assuming that the ascent proceeds so rapidly that there can be no interchange of heat between air and mercury or that the process is adiabatic, then the amount of this cooling can be easily computed.

The formula for the computation of the final temperature t_2 is

$$\frac{273 + t_2}{273 + t_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}$$

where t_1 is the initial temperature; p_1 the initial pressure; and p_2 the final pressure and κ the well-known constant 1.41. Under the assumptions above made we have

$$\frac{p_2}{p_1} = \frac{1}{2}$$

If now $t_1 = 0°$ then we find $t_2 = -50.2°$ or a cooling of about 50° C.

If now we repeat the experiment last described in the third vessel filled with water up to 760^{mm}, then the air in the bell glass has the initial pressure

$$760 + \frac{760^{mm}}{13.6}$$

of mercury; corresponding to this we have

$$\frac{p_2}{p_1} = \frac{13.6}{14.6}$$

and if t_1 is 0° C. then $t_2 = -5.6°$. In this case therefore the cooling scarcely amounts to 6° C.

Since we can assume that the piece of iron has the same mass as the quantity of air used in the other experiments, therefore in all these three cases we have allowed equal masses to rise through equal altitudes; and yet in one case no cooling takes place; in the second case a cooling of about 50° C; and in the third case one of scarcely 6° C. Thus the cooling is not to be attributed to the work done in lifting but exclusively to the work done by expansion.

By these considerations it must have been put beyond all doubt that in the ascent of masses of air in the atmosphere the work of lifting does not come into consideration during steady motion and only to a very slight degree during the process of the establishment of such motions.

In all investigations hitherto made relative to ascending and descending currents the steady motion has been assumed or implied, therefore nothing need be said as to the work done in lifting.

Now that we consider the error of basing the theory of cooling of ascending masses of air on the work of lifting to have been fully demonstrated, we have still to solve the question how it was possible from this assumption to arrive at the same numerical values as by the exclusive consideration of the work of expansion.

This most surprising fact is easily explained by the consideration of the following well-known formulæ.[4]

For any given change of condition of the unit mass of dry air let the quantity of heat communicated to it be Q, the specific pressure p, the specific volume v, the specific heat under constant pressure c_p, the reciprocal of the mechanical equivalent of heat A, we have then the equation

$$dQ = c_p\,dt - Av\,dp$$

[4] See J. Hann. Zeit. Oesterreich. Gesell. f. Met. 1874. IX, p. 321. Or the translation published in the "Short Memoirs" Annual Report Smithsonian Institution 1877.

but for adiabatic change

$$o = c_p\,dt - Avdp$$

or

$$c_p\,dt = Avdp. \qquad (1)$$

But the diminution of atmospheric pressure with altitude, or the baro-hypsometric formula follows from the equation

$$dp = -\rho\,dh \qquad (2)$$

where ρ is the weight of the air contained in a unit volume, and h is the altitude, and where we assume that at the point under consideration the force of gravity has its normal value or that the column of air is located at latitude 45° and that the change of gravity with altitude may be neglected.

Since now according to the definition here adopted the unit of weight is the weight of the unit mass, therefore ρ is the mass of the air contained in the unit of volume, hence

$$v = \frac{1}{\rho}$$

Hence we can write the equation (2) in the form

$$dh = -v\,d\,p \qquad (2a)$$

and now by combination with equation (1) we obtain for the diminution of temperature with altitude the well-known formula

$$dt = -\frac{A}{c_p}\,dh \qquad (3)$$

or after substituting the numerical values

$$dt = -\frac{1}{424 \times 0.2375}\,dh = -0.0099\,dh.$$

If now, on the other hand, we ask as to the work necessary to lift the unit mass, then under the assumption above made, that the weight of the unit mass is the unit of weight,[5] we have the equation

$$L = h \text{ or } d\,L = dh$$

[5] In the metric system. the mass of a kilogram is the unit of mass, its weight under normal conditions is the unit of weight and the kilogram-meter is the unit of work.

The performance of this amount of work requires the consumption of a quantity of heat expressed by.

$$dQ = AdL = Adh$$

If this quantity of heat is to be drawn from a body whose specific heat is c then we have

$$dQ = -cdt$$

or

$$dt = -\frac{A}{c}dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which is exactly as found above, if we take the heat from the air and if the abstraction of the heat can go on under constant pressure so that c can be put equal to c_p.

But all this is not possible under the conditions that prevail in a steady ascending current in the atmosphere. The specific mass of air is not enclosed in an envelope that has no weight, in a vacuous space, but it floats in its surrounding atmosphere.

But even if the above-mentioned condition were fulfilled still the mass would not rise and thereby cool any more than a mass of iron would rise from the earth without the application of exterior forces and would thereby cool 2.09° per rise of 100 meters, as results if we substitute for c the value 0.113 as the specific heat of iron.

On the other hand, the mass of iron will certainly rise and that too without cooling when it forms one member in an endless chain that glides frictionless over a roller and to which there has once been given a velocity, no matter how small.

"It is therefore a purely arbitrary arithmetical operation when in the formula (4) we substitute for c the value c_p as the specific heat of air under constant pressure and thus bring about an apparent agreement with the values deduced from the formulæ (1) and (2)."

The fact that in this treatment the introduction of the specific heat under constant pressure rests on no secure basis, is evident also from the fact that Schmidt himself thought that instead of this, one must substitute the value c_v or the specific heat for constant volume instead of the specific heat for constant pressure as used by Guldberg and Mohn.

The only logical conclusion that we ought to draw from this simultaneous consideration of the work of expansion and the work of lifting is that of a clearer understanding of the results attained by the first-named process.

We may say "The cooling that a mass of air experiences when rising in a steady current without increase or diminution of heat, is precisely the same as that which it would experience if under constant pressure a quantity of heat were abstracted equivalent to the work that would be performed by raising an equal weight through an equal altitude,"

This theorem is analogous to that which refers to the circulation of a particle of air under the influence of a gradient and which can be expressed as follows:

"The acceleration which a particle of air experiences when the atmospheric equilibrium is disturbed in a horizontal direction, is the same as that which a heavy mass would experience if it could glide without friction on the rigid surface of equal pressure."

Both these theorems are simple developments or illustrations of the formula deduced from purely physical considerations.

In order to avoid any misunderstanding I repeat that the work done in lifting can be neglected only during steady motion.

So long as this condition is not yet attained, as, for example, in the above-described cases, it cannot be neglected although in fact in general it is only a small portion of the work done by expansion. Certainly, however, one can imagine processes in which the work of lifting becomes quite important.

If, for example, we assume that a partially vacuous tube extends above the atmosphere while at its bottom there is air within an enclosure and we now by opening a slot let this air enter into the tube, then after equilibrium is attained the center of gravity of the whole mass will lie much higher than before and then of course the work done in lifting must be considered in addition to the work done by expansion, as the former, like the latter, will be done at the expense of the internal energy, that is to say by cooling.[6]

If we are to investigate such cases then we cannot apply the ordinary formulæ of the mechanical theory of heat, but must rather add to these equations another term expressing the work of lifting.

We must indeed never forget that all the ordinary formulæ of this theory are based on the assumption that the work needed to raise

[6] This example is not strictly appropriate, but rather in this special case according to the well-known experiments of Joule, the work of expansion will wholly disappear, or at best is to be considered as a small quantity of high order wherever the work of lifting comes completely into consideration. Hence cases may be imagined in nature in which the work of lifting cannot be neglected; but these are always irreversible processes that must be especially investigated in each individual case. (Note added in 1905. W. v. B.)

the center of gravity so far as this comes into the problem, as also the energy needed to increase the progressive motion of the whole mass so far as it occurs, are negligible in comparison with the work done otherwise.

Hence therefore the two ways that have been used to compute the cooling of ascending masses of air, and which apparently lead to the same results, are by no means to be considered equivalent. They would in fact both be false because of neglect of the supplementary term above mentioned, if the work of lifting is to be introduced.

But in this case all investigations in this field, beginning with Kelvin, Reye, and Hann, as also those of Guldberg and Mohn down to the latest works on the dynamics of the atmosphere would fall with one blow.

The fundamental importance of the whole question has alone moved me also, independent of the wish of the editors of the Meteorologische Zeitschrift, and quite contrary to my general habit, to treat this simple question with so many repetitions of well-known things, in such breadth and detail, that I feel as though I ought to apologize to those familiar with the subject.

I hope that I have been successful in finally dissipating any doubts that may still linger here and there and in proving that under ordinary conditions there can be no work done by lifting in the ascending atmospheric currents but that in these cases the work of expansion alone comes into consideration.[7]

---

[7] By later publications of A. Schmidt (see Gerland's Beiträge zur Geo.-physik, 1899, IV, pp. 1–25; 1903, V, pp. 389–400) the contradictions between his views and mine have been considerably diminished. (Added 1905. W. v. B.)

XVII

ON THE THEORY OF CYCLONES

BY PROF. DR. WM. VON BEZOLD

[*Sitzungsberichte of the Berlin Academy, 1890, pp. 1295–1317. Translated from Gesammelte Abhandlungen, 1906, pp. 284–305*]

If one follows up the meteorological literature of recent years he will not deny that a complete reversal has gradually taken place in the fundamental views as to atmospheric movements.

Whereas under the domination of the old trade wind theory nearly all these movements were considered as consequences of the interchange of air going on between the poles and the equator and nearly all individual processes were sought to be explained from this point of view, we now go to the opposite extreme since the establishment of the so-called "Modern Meteorology."

Since by means of the daily weather charts we have learned the importance that attaches to the areas of high and low atmospheric pressure, we now imagine that the old point of view may at the most still have some value only in explaining the processes in the tropical zones, but that in higher latitudes only local warming and cooling, as also the condition of the moisture, are the controlling feature in the formation of cyclones and anticyclones, and therefore of all the weather phenomena.

Previously we considered the low pressure in the interior of a cyclone as only a consequence of the whirling motion produced by the coöperation of the equatorial and the polar currents. Subsequently, on the other hand, we thought of the whirling motion as exclusively the consequence of the low pressure which itself had its origin in the local conditions just mentioned. We need not go into detail to show how much of truth there was in this newer view and how much our knowledge was advanced by it, but it cannot be denied that we went too far when we thought that in it we had the key to the explanation of all weather phenomena.

Absorbed by the many results offered by the study of individual phenomena from the new point of view we have almost entirely lost sight of the general circulation. However, individual investigators have made distinguished exceptions to this and William Ferrel has prosecuted fundamental work not only in the theory of the general circulation but also especially in the theory of the dynamics of the earth's atmosphere. But independently of the fact that his views were first made known to a larger circle of students (in Germany) by the "Lehrbuch" of Sprung, even Ferrel considered this general circulation as a series of phenomena complete within itself, while for him, as also for the majority of modern meteorologists, the cyclones and anticylones are independent structures whose theory he sought to develop in a corresponding way and independent of that of the general circulation.

On the other hand, already in 1879 Hann[1] had expressed himself in favor of a more general view of the problem and in a short article under the title ("Einige" etc) "Remarks on the theory of the general atmospheric currents" had developed views that correspond in general with those toward which the most recent researches are tending.

But this memoir appears to have attracted but little consideration, and I must confess that only recently I was by Hann himself referred back to this memoir, since it had previously escaped my notice, as is easily explained since the publication occurred at a time when I had first begun to occupy myself with meteorology and had to first make myself familiar with the details of the accepted current theories.

The merit of attracting the attention of [German] meteorologists in general to the treatment of this problem from more general points of view belongs undoubtedly to Werner von Siemens,[2] whose memoir ("Ueber, etc") on the "Conservation of energy in the atmosphere" gave a powerful stimulus to this study quite independent of what we may think as to some details of the views therein developed.

From this time onward various memoirs have appeared which either directly had as an object the investigation of the general circulation of the atmosphere, or else attempted to show the unsatisfactory nature of the theory of cyclones and anticyclones as developed too narrowly.

Of these we mention, first, H. von Helmholtz, who in a memoir. ("Ueber, etc") on "The movements of the atmosphere"[3] demonstrated that "by the action of continuous forces there can be formed surfaces of discontinuity," and that "the anticyclonic movement

[1] Hann: Zeit. d. Oest. Ges. f. met. 1879, XIV, pp. 33–41.

[2] W. v. Siemens: Sitzb. d. Berliner Akad., f. 1886, pp. 261–275.

[3] Sitzb. d. Berliner Akad. f. 1888, p. 663. [See p. 93 of the previous collection of translations. C. A.]

of the lower stratum and the great and steadily-growing cyclone of the upper stratum which are to be expected at the pole, break up into a great number of irregular wandering cyclones and anticyclones with a prevalence of the former."

Thus at least the way is indicated by which we have to seek the connection between the general circulation and the individual phenomena such as cyclones and anticyclones which had hitherto been considered as quite independent existences.

We need only recall the investigations of Moeller, Oberbeck and others, which also relate to the general circulation of the atmosphere.

While theoretical researches thus pressed forward toward a more general comprehensive treatment of all movements of the atmosphere, Hann undertook[4] to give a basis of fact, deduced from the observations made at elevated stations, to the doubts that he had previously expressed as to the incompleteness of the current views.

He demonstrated that in very various cases the temperatures in the interior of cyclones and anticyclones up to considerable altitudes are such that it is impossible to explain the existence of these as due to the specific weight of the central column of air, and that one is inevitably led to explain them as the result of the influence of the general circulation.

Therefore the theories hitherto accepted as to the origin and movement of cyclones and anticyclones undoubtedly need important modifications and it will be important to explain how the above-mentioned local causes, or the specific gravity of the column of air due to them, coöperates with the general circulation to bring about the phenomena actually observed.

It is comparatively easy to recognize this coöperation in the arrangement of the mean annual and monthly isotherms of the globe as I will briefly sketch it in the following paragraph.

The difference of temperature between the equatorial and the polar regions causes a flow of air in the upper regions of the equatorial zone towards the pole. This upper current will by reason of the deflecting force of the diurnal rotation of the earth be converted first into one from the southwest in the northern hemisphere but

[4] Hann: On the relations between the variations of atmospheric pressure and temperature on the summits of mountains. Met., Zeit. 1888, V, pp. 7–17. The Maximum pressure of November, 1889. Denkschriften d. Vienna Akad., LVII, pp. 401–424, 1890. Remarks on the temperature in cyclones and anticyclones. Met. Zeit., 1890, VII, pp. 328–344.

from the northwest in the southern hemisphere, and then gradually into a nearly true west wind. At the same time its velocity increases as it advances into higher latitudes, according to the theorem of the conservation of areas. At and beyond a certain latitude the centrifugal forces thus developed overpower the influence of the temperature which would cause a steady rise of atmospheric pressure toward the poles, so that this pressure which at first increased with distance from the equator now diminishes from this latitude onward very nearly up to the pole itself. Thus arise two belts of high pressure which the averages show as nearly continuous but with easily recognized separate nuclei, but which the individual charts show as broken up into many parts.

These two belts of high pressure are regions of descending currents as is recognized by the clouds.

Moreover, here the movements of the air are feeble since the kinetic energy is materially diminished by reason of the enormous change in section that the air currents experience in their transition from horizontal to vertical motion.

The trade winds blow on the equatorial sides of the two belts of high pressure except at the point of interruption introduced into the whole system by the monsoons; on the polar sides, at least at great altitudes, the conditions are fulfilled that according to Helmholtz must give occasion for whirlwinds to originate.

Thus in these regions cyclone follows after cyclone separated from each other only by ridges of high pressure as they are carried eastward in the great whirl that surrounds the pole. But the anticyclones are portions of the ring of high pressure and the temperature conditions are an important consideration in determining the locations of their central portions in so far that they always seek relatively cold regions and therefore in summer and in low latitudes rest on the sea, but in winter and in high latitudes on the continents.

To these conditions is to be ascribed the fact that the ring of high pressure in the southern hemisphere shows very closely the form that is to be expected according to this theory, whereas that of the northern hemisphere appears greatly modified.

Especially does the influence of the great Asiatic continent make itself felt to such an extent that the nucleus of the great Siberian anticyclone is pushed about 25° north from the latitude at which the average atmospheric pressure for a whole circle of latitude attains its maximum value. Whereas this value both in the annual mean and also in the extreme months falls nearly on the 35th degree of latitude,[5] on the other hand, the center of the Siberian anticyclone in January is in the latitude of about 60° north.[6]

If we develop further the idea suggested in these few lines, we perceive how easily and simply the average distribution of pressure at the earth's surface can be summarized.

The application of analogous methods of consideration to individual cases and the explanation of definite single phenomena by the coöperation of the general circulation with local conditions may for years to come well form one of the most important subjects for investigations.

A complete and rigorous solution of such questions will indeed offer very great difficulties and it cannot yet be foreseen when that will be successful. Hence at first we must satisfy ourselves with considering especially simple cases from the point of view just explained.

But first it appears to be important to establish simple criteria showing whether the temperature and moisture conditions alone suffice to explain the facts of very definite phenomena or, still better, those of any given cases of cyclones or anticyclones, or whether and to what extent we have to consider the coöperation of motions whose causes lie outside the given whirlwind or at least outside the portions immediately considered.

The object of the following lines is to make a contribution in this direction, so that in general it has the same object as the abovementioned investigations of Hann. But while in the latter the main feature consisted in the discussion of data of observation where the temperature conditions especially were considered; on the other hand, here theoretical considerations will be carried out and especially will the atmospheric pressure and the wind be considered.

The question as to the influence of the general atmospheric circulation on the processes within a cyclone, always assuming a stationary or steady condition may be formulated as follows:

"Does the actually existing distribution of pressure and temperature suffice to completely explain the simultaneous observed motions or does it not?"

Or, in other words,

---

[5] Sprung: Lehrbuch der Meteorologie; Hamburg, 1885, p. 193. In the following pages I shall frequently cite this work instead of the original memoirs, since frequently the latter can be obtained only with difficulty and since the references will be found in the work of Sprung.

[6] Hann: Atlas of Meteorology. Plate No. VII, Gotha, 1887.

"Are the movements observed within a cyclone exclusively the consequence of the presence of the lighter air at its center or, conversely, is the latter wholly or partly the consequence of these movements, in which case these latter must, of course, result from exterior causes?"

If we consider only a portion of the whirl, then an affirmative reply to this last question only shows that the cause of the motion must lie outside the portion under consideration without forcing us to seek it outside the whole whirl.

Unfortunately, even the simple question whether in any portion of the whirl the observed movements are wholly explained by the distribution of pressure cannot be answered in its generality because one must still make some more or less arbitrary assumptions as to the coefficients of friction and as to the influence of neighboring strata.

On the other hand, this question can be at once answered in the negative if the so-called angle of deflection (of the wind from the gradient) is equal to or greater than 90°, that is to say, if the direction of the wind agrees with the isobar or has a component against the gradient.

For under such conditions work is being done that can not be due to the gradient force present in the cyclone or in the portion of the cyclone under consideration, since in the first case the gradient force is perpendicular to the direction in which the work (which consists of overcoming the friction) is being done; whilst in the second case there must exist a component of force that is directed oppositely to the only one that can arise from the distribution of pressure.

Of these two cases, the first is easily accessible to mathematical treatment, and therefore the following remarks apply to it, that is to say, this investigation is confined to cyclones with circular isobars and winds whose direction coincide with such isobars, or according to Sprung's notation[7] to cyclones of circular symmetrical form and having angles of deflection of 90°.

Perhaps such cyclones might be designated as "centered cyclones" or in general such whirls as "centered whirls" by analogy with "centered optical systems."

Now it might appear as if by the limitation of our consideration to such centered whirls we have pushed the specialization of the problem to the furthest limit and thus rendered the results quite valueless. But this thought is not quite justified, for, on the one

[7] Sprung: Lehrbuch, p. 208.

hand, the synoptic charts show that in strongly developed cyclones the winds very frequently favor the isobars, i. e., the direction agrees with that of the tangent to the isobar; and, on the other hand, we are not yet able in theoretical investigations to free ourselves from the simplifying assumption of circular isobars.

On the other hand, I can but think that the investigation of this simplest case should suffice to considerably further our understanding of the cyclonal and anticyclonal motions and illuminate many points in reference to which an incorrect view has often been maintained.

Moreover, the centered whirl or the centered portion of such has a special interest in that it represents the limiting case between whirls with centripetal and with centrifugal motion or between the corresponding portions of such a whirl.

It is now necessary first to express exactly the fundamental condition for the existence of the centered whirl, which is very easily done.

Three forces are acting on every particle of the whirl; the centrifugal force p_c arising from the rotation about the axis of the whirl; the deflecting force p_i of the earth's rotation, which we can also represent as a centrifugal force directed toward the center of curvature of the inertia curve; finally, the gradient force Γ which is the force arising from the difference of the atmospheric pressure.

In a centered whirl, in which each particle describes a circle, these three forces all act in the direction of the radius of this circle and it is only the directions of each that differ according as they have to do with a gradient directed inward or outward, i. e., with cyclonal or anticyclonal rotations and distributions of pressure.

The fundamental condition for the maintenance of a centered whirl is therefore

$$p_c + p_i + \Gamma = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where the summation is algebraic and we must first give each quantity its correct sign.

If we consider the absolute values of the quantities p_c; p_i and Γ as known and give each its proper sign, then we have to distinguish four cases:

(A) Cyclonal rotation with gradients directed inward, or, as we may appropriately say with cyclonal distribution of pressure. In this case, which we see presented in the lower strata of the ordinary cyclones, p_c and p_i have the same signs, but Γ the opposite sign and thus the equation reads

$$p_c + p_i - \Gamma = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(B) Cyclonal rotation with gradients directed outward, i. e., with anticyclonal distribution of atmospheric pressure. These conditions are met within the upper strata of cyclones with warm centers. Here the gradient is directed outward but the curvature of the orbits of the particles of air must be cyclonal up to very considerable altitudes, since the movement of rotation that the mass of air has under the ordinary conditions brought up from the lower strata cannot immediately disappear.[8]

But under these conditions the whirl cannot be centered, since the equation appropriate to this case, viz:

$$p_c + p_i + \Gamma = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (3)$$

cannot be fulfilled; hence perfect equilibrium must prevail, that is to say, each of these three quantities must be equal to zero.

(C) Anticyclonal rotation with gradients directed outward hence with anticyclonal distribution of pressure. These are the conditions that we usually meet with in the lower part of the anticyclone.

In this case the equation of condition for the centered whirl is

$$p_c - p_i + \Gamma = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (4)$$

Although theoretically this is not impossible yet this equation may still be practically meaningless, since the relations in the lower part of the anticyclone are always such that an agreement of wind direction with the isobars is not imaginable. Thus there remains at best only the very highest portions of the cyclones with warm centers, in which, indeed, anticyclonal distribution of pressure must prevail and where perhaps anticyclonal movements of the atmosphere can also be present, provided that this system extends so far upward that the moment of rotation in cyclonal direction as brought up from below is already completely consumed in overcoming the resistances.

But since we have no basis of facts for the investigation of this question it will be better to lay it entirely aside.

---

[8] The scheme of atmospheric motions in the upper portion of a cyclone deduced by Clement Ley (Quarterly Journal Met. Soc. London, 1877, III, p. 437) from observations of the cirrus clouds corresponds to this presentation of our second case. We attain the same result if we think of a cyclonal movement in which the paths of the air particles are more and more straightened out by the forces p_c, p_i and Γ all acting in the same direction until finally the paths are curved in the opposite or anticyclonal direction while the velocity of the outflow increases and at the same time the whole system is carried eastward in the great whirl of the polar region.

(D) Anticyclonic rotation with gradients directed inward, viz: with cyclonal distribution of pressure. In this case the condition as to centering is

$$p_c - p_i - \Gamma = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . (5)$$

As to this equation also it is doubtful whether it has any practical importance. In general, in the lower strata of the atmosphere we meet only with the above-mentioned cases (A) and (C).

We usually assume that in the upper portion of the anticyclone there is a cyclonal distribution of pressure, that is to say, a gradient directed inward,[9] since we consider this necessary in order to explain the inflow of air from above. But the presence of such a distribution of pressure in the upper half of an anticyclone has, so far as known to me, never been shown by any facts; on the contrary, thermodynamic considerations make it in the highest degree improbable that the low temperatures observed at the lower surface of the so-called cyclones with cold centers extend to any considerable altitudes. But if this latter is not the case then also the assumed change in the curvature of the surface of equal pressure (which should generally pass from convex above, at great altitudes, to concave above, at lower altiudes) will not exist. Consequently the flow in the upper regions toward the anticyclone is not to be explained as the result of a gradient directed inward, but rather dynamically as a heaping up phenomenon due to obstruction.

However, if in individual cases the assumption of a cyclonal distribution of pressure in the highest part of an anticyclone should be correct, as has hitherto ordinarily been assumed, still by reason of the slight moment of inertia that is ordinarily present in an anticyclonic whirl there is no reason why the direction of the rotation should remain the same over extensive regions, as in the case of cyclones where the pressure distribution is of the opposite type.

These considerations show that of the four cases of centered whirls that can be imagined, only the first mentioned has any practical importance in meteorology and the following lines are therefore devoted to its consideration.

Therefore we consider here only a whirl with barometric gradients directed inward, circular isobars, and cyclonal motion of the air, under the special assumption that the directions of the wind agree everywhere with those of the tangents of the isobars.

Under these conditions the equation

$$p_c + p_i - \Gamma = 0$$

[9] Sprung: Lehrbuch, p. 211, fig. 39.

must be satisfied and the problem consists essentially in the discussion of this equation.

Consider a special isobar and let its radius be r_c, the radius of curvature of the inertia curve r_i, and the velocity of the wind along the isobar v, then for a particle of air whose mass is m moving along the isobar in the prescribed manner we have

$$p_c = m\frac{v^2}{r_c}$$

and

$$p_i = m\frac{v^2}{r_i}$$

Let the whole process go on at the geographic latitude φ and for simplicity assume that this latitude is the same for all points of the cyclone, which of course cannot be the case but will cause no great error if we assume an average value for φ; finally, let T be the length of the siderial day expressed in seconds of mean solar time, then we have[10]

$$r_i = \frac{v\,T}{4\,\pi \sin\varphi}$$

or

$$p_i = \frac{4\,\pi\,m\,v}{T}\,.\sin\varphi$$

whence

$$\Gamma = m\left(\frac{v^2}{r_c} + \frac{4\,\pi}{T}\,v\sin\varphi\right)$$

Put $k = 0.0001458 = \frac{4\,\pi}{T}$ and $\Gamma = m\,\gamma$, where γ is the acceleration communicated to the mass m by the gradient force Γ, then the preceding equation assumes the simpler form

$$\gamma = \frac{v^2}{r_c} + vk\sin\varphi \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (6)$$

But for the acceleration γ we also have the equation

$$\gamma = \frac{G}{111\,111}\cdot\frac{13.6}{\rho}\,g = 0.000\,122\,37\,G\frac{g}{\rho} \quad . \; . \; . \; . \quad (7)$$

[10] Sprung: Lehrbuch, p. 24.

where G is the gradient, i.e., the difference of barometric pressure between two points distant from each other by one degree of the meridian, or 111,111 meters on the spherical surface of the earth, and in the direction of the maximum change of pressure.

But we also have

$$\gamma = \frac{h}{l} g = g \tan \alpha \quad \ldots \ldots \ldots \quad (8)$$

where h is the altitude by which the surface of equal pressure (isobaric surface) drawn through the point under consideration, rises or falls in the course of the horizontal distance l, and where α is the angle that the surface of equal pressure at this point makes with the horizon.

Therefore the acceleration that is given to the air by the gradient force is equal to that which a heavy point experiences when it glides without friction along an imaginary rigid isobaric surface, at least in so far as α is small enough to allow us to consider sine α as equal to tangent α, which is always the case in our problem.[11] The acceleration due to gliding down such a surface is $g \sin \alpha$, whereas the force $g \tan \alpha$ acting on the point horizontally, is required to prevent the gliding downward.

If we substitute in (7) and (8) the value of γ given in equation (6) and write r instead of r_c since the quantity r_i no longer occurs in the problem, then the equation of condition for the centered whirl finally assumes the form

$$0.000\,122\,37\, g \frac{G}{\rho} = \frac{v^2}{r} + v\,k \sin \varphi \quad \ldots \ldots \quad (9a)$$

or if β is the barometric pressure

$$g \frac{13.6}{\rho} \frac{d\beta}{dr} = \frac{v^2}{r} + v\,k \sin \varphi \quad \ldots \ldots \ldots \quad (9b)$$

or finally

$$g \tan \alpha = \frac{v^2}{r} + v\,k \sin \varphi \quad \ldots \ldots \ldots \quad (9c)$$

[11] In my original memoir the first deduction and formulation of this theorem was attributed to Moeller, but it is due to Hann, who first gave it in his memoir "On the relations between wind velocity and differences of pressure according to the theories of Ferrel and Colding," Zeit. d. Oest. Gesell. f. Met. 1875, X, pp. 81–88, 97–106. Compare also my memoir of 1901, XVIII of this collection of translations. (Note added 1906. W. v. B.) [My translation of Hann's memoir will be found in my "Short Memoirs," Smith. Rept. 1877.—C. A.]

which equation can be still further simplified in special cases since we can consider φ to be constant and put

$$k \sin \varphi = K.$$

The first form of this equation could also have been deduced directly from the fundamental equations of Guldberg and Mohn, giving attention of course to the algebraic signs[12] here adopted.

The second form is more convenient for application to special cases drawn from the synoptic weather charts especially when in place of tan α we introduce the value

$$\frac{h}{l} \text{ or } \frac{dr}{dh}$$

For definite values of pressure and temperature the heights h can be taken directly from tables which give the heights of columns of air corresponding to a pressure of 1^{mm}, such as Table V of Mohn's "Grundzüge," whilst the distances of the isobars corresponding to differences of pressure of 1^{mm} are measured directly on the weather chart.

Suppose, for instance, we wish to determine the inclinations of the isobaric surfaces for northern England, for points between Shields and Bradford, from the weather chart of October 14, 1881, as published in Sprung's Lehrbuch, Plate VII; we first find for h the value 11.4 meters, for the pressure 730^{mm} and the temperature 10° C. then prevailing; for the distance between the isobars 725^{mm} and 753^{mm} we find 180 kilometers and therefore l = 18 kilometers or 18,000 meters for the distance between 729 and 730 or between 730 and 731^{mm}, whence follows

$$tg\ \alpha = \frac{11.4}{18000} = tg\ 0° \ 1' \ 36''.$$

This example is interesting in that it shows very clearly how remarkably slight in general is the inclination of the isobaric surfaces, since even for the great atmospheric disturbances that prevailed in that region on that day one must go northward 18 kilometers in order to experience a change of pressure equal to that found by rising vertically only 12 meters.

If now we seek to draw some general conclusions from equation (9) then we perceive, first of all, that it is essential to the exist-

[12] Sprung: Lehrbuch, p. 119, equation (5).

ence of a centered cyclonic whirl that very precise relations must exist between the wind velocity and the distribution of atmospheric pressure.

Hence in all cases where the wind circles about a center in the strict sense of the word there must be a very exact distribution of pressure that renders possible the continuation of such a whirl, and inversely for every symmetrical circular distribution of pressure there must be corresponding definite velocities belonging to it.

The entire omission of the friction in this theorem implies the assumption that this is overcome by forces that do not appear in this calculation, as, for instance, the difference of velocity in neighboring strata which on its part must of course be maintained by causes that are outside the region under consideration. In no case can these resistances within a centered whirl be overcome by the forces arising from the distribution of pressure, and this is a fundamental point for the following discussion.

The questions that interest the meteorologist with reference to the centered whirl are the following:

(1) Are there really any cyclones that show, at least at the earth's surface itself, such a distribution of pressure and wind as must exist in the centered cyclone?

(2) Can these conditions be satisfied simultaneously in layers of great vertical extent, under the conditions prevailing in our atmosphere, or is it improbable that a cyclone that appears as a centered whirl at the earth's surface may also possess the same peculiarity at greater or even only moderate altitudes?

(3) When the equation of condition (9*a*) is not satisfied but when departures therefrom are present in any given direction, what conclusions can be drawn from that fact?

Let us consider the formula

$$g\ tg\ \alpha = \frac{v^2}{r_c} + v k \sin \varphi$$

from the point of view proposed in the first of these three questions, after writing it in the simpler form

$$g\ tg\ \alpha = \frac{v^2}{r} + vK$$

since r_i no longer occurs in the following discussion and since we may always limit the investigation to some one definite value of φ. We note, first, that for diminishing values of r, i. e., with approach toward the center, the inclination of the isobaric surface, or the gradient, must increase, except in so far as a compensation does not occur by reason of a simultaneous diminution of velocity. This increase of inclination or gradient must obtain to a still larger extent when the velocity v also increases with approach toward the center. In immediate proximity to the center, even with a uniform velocity for the inner and the outer rings the gradient becomes infinite, which of course is impossible. On the other hand, the increase of the centrifugal force due to the diminution of r can be counteracted or even overcompensated by a corresponding diminution of the velocity so that in the immediate neighborhood of the center the gradient again diminishes precisely as has been frequently observed. We see from what has just been said that at least so far as we limit ourselves to a purely qualitative consideration of the subject the relations here expressed as the condition for the existence of the centered cyclone are in reality frequently met with, and that therefore the existence of centered cyclones is by no means improbable.

Even when we study the matter more closely and numerically we come to the same conclusion and find that cyclones which are at least approximately centered at their base, can scarcely be said to be rare.

In order to acquire a starting point, I have computed the wind velocities that would be necessary in order that a cyclone should be centered when the pressure distribution is such as Sprung[13] found for the average of four well-developed cyclones.

The velocities that I found for the respective distances from the center are

Distance Kilometers	Velocity Meters per second
100	10.8
200	20.7
300	21.4
400	23.0
600	18.0
800	13.3
1000	10.4

and these numbers are not contradictory to the wind velocities read off from the synoptic charts for the respective days.

This is still more easily seen if we assume that the cyclone is located at latitude 45° and that in the portions under consideration the prevailing temperature is 10° C. and pressure 730^{mm}; or 15° C. and 745^{mm}, etc.; in such cases $0.00012237 \frac{g}{\rho} = 0.001$ and $0.0001458 \sin \varphi = 0.0001031$ or nearly 0.0001.

[13] Sprung: Lehrbuch, p. 150.

Under these assumptions the equation (9*a*) assumes the very simple form

$$0.001\, G = \frac{v^2}{r} + 0.0001\, v$$

or, if we indicate by β_1 and β_2 the pressures at the two ends of a line whose length is one degree of a great circle, lying in the direction of the gradient, this becomes

$$G = \beta_1 - \beta_2 = 1000 \frac{v^2}{r} + 0.01\, v \quad \ldots\ldots (10)$$

From this we at once conclude that in centered whirls there must be gradients of 2^{mm} or 11^{mm} or 101^{mm} per degree of a great circle, at distances of 100 kilometers, 10 kilometers, and 1 kilometer respectively from the center, where there is a wind velocity of 10 meters per second at these points.

Gradients like 101^{mm} per degree never occur or at least only over very limited regions, and 10 meters per second is not a strong wind, but from this paragraph we see very clearly how powerfully centrifugal force comes in play even in moderate winds in the neighborhood of the center, and how remarkably large the gradients must be (nearly four-fold when $v = 20$ meters per second) if centrifugal movements are not to replace centripetal.

But in ordinary cyclones, beginning at a definite and often considerable distance, the wind velocity diminishes with approach to the center and so also does the magnitude of the gradient, so that even in this portion the whirl may remain centered, as was already shown in the above case of an accurately defined example of the typical cyclone.

However, the idea seems not to be excluded that even in these latter, descending currents may replace the ascending, whenever at moderate altitudes centritugal movements replace the centripetal, and in so far as the masses of air required to supply these cannot be obtained from below. At least the diminution of the cloudiness in the neighborhood of the center, which is often recognized as the "eye of the storm," speaks very decidedly in favor of this conclusion.

On a subsequent page we will explain how these conditions adjust themselves in the tornadoes proper and in the waterspouts.

In the second place, we will inquire whether it is probable that a cyclone that is centered at its base will also possess this peculiarity at greater altitudes.

Of course this question would be at once answered affirmatively if it were allowable to assume that the wind movement and the barometric gradient were uniform above every point of a large portion of the base of such a cyclone.

But since this is only true in exceptional cases and in layers of moderate thickness, therefore the question is to be modified to the inquiry whether changes of the two elements in question, such that the condition of centering still remains fulfilled, are conceivable.

We most easily attain a summary view of these relations by the following consideration:

Let z be the altitude of a point above the horizontal base, then in the case of a symmetrical circular form for the whole whirl, we have the equations

$$\beta = f(r, z)$$

and

$$v = \Phi(r, z)$$

Making use of equation (9*b*) and recalling that

$$\rho = \rho_0 \frac{\beta}{\beta_0} \frac{273}{T}$$

where $\rho_0 = 1.293$, $\beta = 760$, and T is the absolute temperature, we can put this equation in the form

$$\frac{g\beta_0 13.6}{\rho_0 273} \cdot \frac{T}{\beta} \cdot \frac{d\beta}{dr} = \frac{v^2}{r} + vK$$

or, if T is constant for each horizontal plane or for every value of z considered as constant, this assumes a still simpler form

$$\frac{K'}{\beta} \frac{d\beta}{dr} = \frac{v^2}{r} + vK$$

If now we consider the relation above assumed according to which

$$v = \Phi(r, z)$$

then, instead of the total differential quotient $\frac{d\beta}{dr}$ we have the partial differential quotient and get

$$\frac{K'}{\beta} \frac{\partial\beta}{\partial r} = \frac{[\Phi(r,z)]^2}{r} + \Phi(r, z)K \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This equation shows us that for every given symmetric circular distribution of pressure there is a system of velocities, for which the whirl becomes a centered whirl, and that, conversely, for every system of uniform circular motions about one and the same axis, provided these merge continuously into each other, there is a definite distribution of pressure for which these motions are permanent or steady, that is to say, they correspond to the conditions of the centered whirl: but of course this is true only in so far as we can neglect the frictional resistances.

First assume the distribution of pressure as given and imagine the isobaric surfaces to become suddenly rigid and that velocities such as are indicated by equation (11) are communicated to heavy points movable without friction along these surfaces, then these will all remain on the horizontal circles and move forward as before in similar manner, since in this case there arises from the resistances of the surface a force directed inward that holds in equilibrium the force $g\ tg\ \alpha$ directed outward.

In this case the acceleration in the direction of the slope that is communicated by gravity to a heavy point resting on the surface is $g \sin \alpha$, whereas the component of the outwardly directed horizontal force $g\ tg\ \alpha$ that raises the point upward along this surface is $g \tan \alpha \cos \alpha$, that is to say it is $g \sin \alpha$ also.

Whenever the velocities at any point whatever, or along any horizontal circle whatever, become greater or smaller than those required by equation (11) then the point will rise or fall respectively.

Therefore these velocities given by this equation for any particular distribution of pressure are called the "critical velocities," whereas the isobaric surfaces given by this equation (11) for any particular set of velocities will be called "critical surfaces."

But the gradient corresponding to this critical distribution of pressure will be known as the "critical gradient," in distinction from the "effective gradient" ordinarily present, so that the fundamental condition for the existence of a centered whirl may be expressed thus: "In centered whirls the isobaric surfaces must coincide with the critical surfaces and the effective gradients must equal the critical gradients."

By the help of this theorem we at once see that it is not at all probable that a cyclone that is centered at the earth's surface should also possess the same peculiarity at great altitudes.

The distance between two isobaric surfaces is in general throughout their whole extent subject to only moderate variations, since it is simply proportional to the absolute temperatures prevailing at the various locations.

Hence in the cyclone, on account of the diminution of temperature with altitude (even when the temperature at the earth's surface does not diminish with distance from this axis, as is the case in cyclones with warm centers), the isobaric surfaces will gradually approach each other with increasing distance from the axis, but this approach will always be relatively slight.

It is entirely different with the critical surfaces; these rise very considerably as one proceeds outward while the velocities increase with the altitudes.

Since in general the second term of the equation comes only slightly into consideration, therefore the inclination of the isobaric surfaces increases very nearly with the square of the wind velocity.

If, therefore, in any cyclone the isobaric surfaces are such as is shown by the full lines in fig. 55, and this frequently corresponds to the actual conditions, and if this cyclone is centered for the horizontal section *A A*, then it will not be so above or below this section as the velocities increase with altitude.

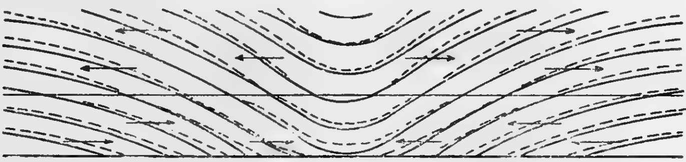
A  A

FIG. 55. ISOBARIC SURFACES AND CRITICAL SURFACES IN A CYCLONE

Under this assumption the critical surfaces are arranged as is suggested by the dotted lines in fig. 55, and therefore agree with the isobaric surfaces only at the section *A A*, since there they have the same common tangents.

"Above this section the centrifugal forces are greater than the gradient forces directed toward the axis and therefore movements must occur opposed to the gradients."

Hence the surfaces of this section *A A*, which is not necessarily a plane but is here thought of as such for simplicity only, is the boundary between a region of centripetal and centrifugal movements.

At the same time we easily see that such a reversal in the direction of the movement must occur without change in the sign of the gradients when the motion at the surface of the earth becomes approximately circular. For since the velocities of the wind increase rapidly with the altitude as shown by experience, while the inclination of the surfaces of pressure generally shows a diminution, therefore such a movement as that above considered, which has a feeble centripetal component at the earth's surface, must change into a centered whirl as it extends into higher strata and eventually in fact into a centrifugal whirl.

Therefore the second of the questions above formulated is to be answered negatively and to the effect that it is most improbable that cyclones should remain centered through any considerable vertical extent. We are rather to expect centrifugal movements in the upper portions of such cyclones even if they must proceed against the gradients.

The preceding section, which was devoted strictly to answering the second of the three questions above formulated, contains also the reply to the third. This latter refers to the conclusions that can be drawn from the non-fulfillment of the conditions that apply to the centered whirl.

By the introduction of the idea of the critical surfaces these conditions can be expressed in the following simple form: "In centered whirls the critical surfaces and the surfaces of equal pressure must coincide."

According to what precedes, the inclination of the isobaric surface measures the magnitude of the effective gradient toward the axis, but the inclination of the critical surface measures the component of the force directed from the axis arising from the centrifugal force and the rotation of the earth.

If therefore at any given location the critical surface has an inclination *less* than that of the isobaric surface, then we have to do with a resultant directed inward or centripetal, but if the critical surface is *more* strongly inclined than the isobaric surface then the resultant is outward or centrifugal.

But in this matter we must recall that even for symmetrical circular isobars the critical surfaces are surfaces of rotation only when the atmospheric motions proceed in circular whirls whose planes are perpendicular to the axis and whose centers also lie in this axis.

But under these conditions the centered condition is unstable unless the isobaric surfaces and the critical surfaces have the same inclination at every point and coincide throughout the whole region under consideration.

Notwithstanding this instability the study of this case, which is of course only to be thought of as a transition stage, has some interest in that, as already mentioned, the observed movements actually do come extraordinarily near to being circular, whereas, on the other hand, the generalization of the problem offers very considerable difficulties.

Nearly circular movements are to be observed, for instance, in the case of tornadoes and waterspouts.

If now we apply the considerations just introduced, to these latter cases, we find that the critical surfaces have extraordinarily large inclination near the axes and hence there must be present enormous [counteracting barometric] gradients if these circular movements are not to turn into centrifugal movements. For instance, from the approximate formula (10) for $r = 10$ and $v = 30$ (or for a wind velocity of 30 meters per second at a distance of 10 meters from the axis) we find a gradient of 90,000, i. e., a diminution of pressure of 0.81 ^{mm} for 1 meter of approach to the axis.

Under the given assumptions, the inclination of the critical surface will be about 84°.

If then such motions are maintained at the cost of the energy that is gained at other points, then great rarefactions of the air must take place in the neighborhood of the axis that can be computed when definite assumptions are made as to the diminution of the velocity with the distance from the axis.

Such computations have already been made for the tornado by William Ferrel[14] which he considered as a simple centered whirl and for which he represented in a diagram the form of the isobaric surfaces which, under the assumption of the given velocities, are the same as our critical surfaces.

It would therefore scarcely be necessary here again to touch on this point, but that it seems to me that in one respect different conclusions are to be drawn from these studies than those drawn by that investigator.

The enormous gradients that must prevail in a very thin mantle enclosing the axis of a tornado, if there are to be no centrifugal movements, make it very improbable that any air penetrates this mantle from without and moves inward to the axis, or that any centripetal movements occur.

In order to bring about such movements the isobaric surfaces actually existing must be more strongly inclined than the critical surfaces, or, which comes to the same thing, the effective gradients must be still greater than the critical gradients, which themselves already have such extraordinary large values.

But if there be no continuous flow of air inward toward the axis

[14] Sprung: Lehrbuch, p. 224.

then the existence of an ascending current in this axis itself is not conceivable.

On the contrary, I would rather hold it to be probable that in the axial tube no important vertical movement takes place, but that this is essentially a moving rarified space, whereby new particles of air are being drawn into the motion and thus subjected to the rarefaction.

The assumption of an ascending current in the axial tube is also by no means necessary, since Ferrel has proved that the rarefaction of the air produced by the centrifugal force, when no addition of heat comes in, is sufficient to explain the condensation and the origin of the filmy cloud. This is quite natural when the trunk of the cloud axis is first recognized as an appendix hanging below a cloud and appears to gradually descend lower, since in the earlier stages of the development of this phenomenon where the friction at the earth's surface does not come into consideration, considerable velocities will occur, which might cause rarefaction of the air and hence the condensation. Moreover, the air must be most nearly saturated directly beneath the cloud and hence it requires only very slight rarefaction of the air to bring about condensation.

It is only when the velocities in the lower strata of the atmosphere have acquired a corresponding increase, that the rarefaction proceeds to such a degree that the cloudy film extends down to the earth.

But we are not to conclude from this that the cause of the whole phenomenon is to be sought in the upper regions; rather is it to be expected that in even the cases where the process is initiated by overheating of the lowest strata of air and the unstable equilibrium produced thereby, still the larger velocities will be attained at great altitudes sooner than below.

For since after breaking up the unstable equilibrium the accelerating forces increase with altitude, therefore not only does the ascending current (which is to be thought of not as exactly in the axis of the subsequent tornado but as extending over a large area) itself acquire steadily increasing velocities, but this is also true especially of the currents of air streaming inward from all sides, since with increasing height slighter resistances oppose its motion.

In general, the fact that the axial cloud appears to descend from above justifies no conclusion whatever as to whether the true cause of its origin is to be sought for above or below.

Neither are we to draw any conclusion from the apparent descent of the cloud-axis as to any descending movements in its interior.

On the contrary, the existence of the cloud proves that in any such case these descending motions, which in themselves are not improbable, cannot be very important since otherwise adiabatic compression must occur and thus cloud formation would be impossible.

The matter is somewhat different in the case of large cyclones, for there it is quite conceivable that in the beginning of these or in case of their rapid development in the median strata of the atmosphere (which may either be due to the general circulation or be a result of local expansion of the air), air may be drawn in as easily from above as below.

By simple modifications of the above-given figure we can also obtain systems of critical and isobaric surfaces in which the down-draft must extend down to the earth's surface so that a natural explanation is found for the so-called "eye of the storm," as also for the remarkable dryness observed in the interior of a cyclone, as, for example, in the hurricane[15] of 1882, October 28, at Manila.[16]

The investigations which I have here carried out started with the consideration of the centered whirl. Notwithstanding the limitation to this very special case they seem sufficient to remove the characterization of abnormal or inexplicable from the peculiar relations that Hann has shown to exist in cyclones with cold centers and anticyclones with warm centers.

Not the less are they appropriate to lead us back to a correct appreciation of the views defended by Faye as to the descending currents in the interior of cyclones and within certain limits prepare the way for a reconciliation between this idea and that which has become almost universal.

[15] Sprung: Lehrbuch, p. 24.

[16] Similar phenomena may also be reproduced in rotating liquids. In these we can even develop whirls in which ascending movements occur in the mantle but descending in the axis. See v. Bezold. Sitzb. Berlin Acad., 1887, pp. 261–277.

XVIII

ON THE REPRESENTATION OF THE DISTRIBUTION OF ATMOSPHERIC PRESSURE BY SURFACES OF EQUAL PRESSURE AND BY ISOBARS

BY PROF. DR. WM. VON BEZOLD

[*Archives Néerlandaises des Sciences exactes et naturelles, Series II, Tome VI, pp. 563–574, 1901. Translated from Gesammelte Abhandlungen von W. v. Bezold, Berlin, 1906, pp. 306–315*]

In order to obtain a clear idea of the distribution of pressure in the atmosphere we imagine surfaces of equal pressure extending through the atmosphere for a series of pressures differing from each other successively by a given constant difference. As such constant we most frequently adopt 5 millimeters, that is to say, we consider surfaces for which the pressures are given in millimeters by the equation

$$\beta = 760 \pm 5\, n$$

so that the constant difference is $\Delta\,\beta = 5$. In special cases we may also choose $\Delta\,\beta = 2.5$ or $\Delta\,\beta = 1.0^{mm}$.

If now we seek the "traces" or intersections of these surfaces with any other surface of known location and form, then we obtain lines of equal pressure or isobars in the most general sense of this term.

As such surfaces of known location and form we choose either "level surfaces of gravity" in which case the traces are isobars in the ordinary sense, or we seek the intersections with a vertical surface in which case we speak, but not quite correctly, of the representation by means of "baric surfaces."

Both these methods of presentation have their special advantages and disadvantages which I will elucidate more clearly in the following lines.

If it be not possible to avoid restating many well-known points, still I imagine such reconsideration by no means superfluous, since it would appear that many of those who daily make use of either of the two methods of presentation, in spite of the publications of

R. v. Miller-Hauenfels[1], Nils Ekholm,[2] and V. Bjerknes,[3] still do not appreciate the different peculiarities of these surfaces and lines as perfectly as is desirable.

Especially is this true as to researches on the processes in the upper strata of the atmosphere, researches that have acquired increased interest in recent times.

One of the principal questions that comes up in the consideration of an irregular distribution of atmospheric pressure is as to the accelerations that the particles of air experience in consequence of this irregular distribution. Most important is the acceleration in a horizontal direction, that is to say, along the level surfaces of gravity. Since the component perpendicular to this direction is in general very small, therefore it will here be ignored. The acceleration is greatest in the direction of the greatest change of pressure, that is to say, in that of the greatest so-called gradient.

This acceleration I will call the gradient acceleration and will designate it by the letter γ.

In the following paragraphs we will more exactly investigate how useful the two above-mentioned methods of presentation are in the determination of this quantity. Especially will be considered to what extent even a casual consideration of them allows of an orientation in this direction.

To this end it is necessary to discuss more minutely the well-known formulæ for this acceleration. Ordinarily one makes use of the equation

$$\gamma = \frac{G}{111\,111} \cdot \frac{13.6}{\rho} \cdot g \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1a)$$

where G is the so-called gradient, or the difference in millimeters between the barometric readings that prevail at the ends of a straight line 111,111 meters long whose direction coincides with that of the greatest change of pressure; ρ is the density or the mass in kilograms of air contained in a cubic meter; and g is the local acceleration of gravity expressed in meters per second. The negative sign that should be prefixed to this and the following formulæ I omit since I consider only the numerical or absolute value of γ.

Since we cannot assume that the change of pressure is actually

---

[1] Theoretische Meteorologie. Vienna, 1883.

[2] Bihang till, K. Svenska Vet. Akad. Handlingar., Bd. XVI. Abt. 1, No. 5 Stockholm, 1891.

[3] K. Svenska Vet. Akad. Handlingar, Bd. XXXI, No. 4, Stockholm, 1898.

uniform throughout such a long distance, it would be more correct to put this equation in the form

$$\gamma = \frac{d\beta}{dl} \cdot \frac{13.6}{\rho} \cdot g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1b)$$

where $d\,\beta$ expresses the change of pressure for the elementary distance $d\,l$ in the direction of the greatest barometric change. Hence the value of G should be deduced from the equation[4]

$$\frac{d\beta}{dl} = \frac{G}{111\,111}$$

Finally for many points of view it is very advantageous to write

$$\frac{d\beta}{dl} = \frac{\varDelta\beta}{l}$$

where $\varDelta\,\beta$ indicates some definite change of pressure and l the distance in the direction of the greatest gradient to which one must go until a difference of pressure $\varDelta\,\beta$ is attained, assuming a uniform barometric gradient.

Thus the formula assumes the form

$$\gamma = \frac{\varDelta\beta}{l} \cdot \frac{13.6}{\rho} \cdot g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1c)$$

Instead of these three formulæ which I will speak of collectively as the formulæ (1) since they are in fact only different forms of the same fundamental formula, one may also use the following very different form

$$\gamma = g\,tg\,\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where α is the angle that a surface of constant pressure makes with the horizon, and always in the direction of the steepest gradient.

The formulæ (1) and (2) are closely associated with the two above-mentioned geometrical methods of presentation that we used to express the distribution of atmospheric pressure.

Since we can easily lay off a distance of 111, 111 meters or 111 kilometers on any map, no matter what its style of projection may be, therefore the formula (1a) is specially appropriate to such investigations as those based on the ordinary synoptic charts.

For this reason also in using formula (1a) one should not, as is often done, speak of the length of one degree of the equator, since

[4] Compare C. M. Guldberg and H. Mohn: Études sur les Mouvements de l'Atmosphere, p. 18. Christiania, 1876. [Supra XI. p. 146.]

many charts contain no portion of the equator, but should speak of a degree of the meridian, or a degree of latitude, since on every chart a portion of the meridian appears or can be easily drawn. The degree measured on the meridian, therefore, under all circumstances corresponds closely to a length of 111 kilometers.

Therefore by comparing any distance on any chart with a degree of latitude we can express it in fractions or multiples of 111, 111 kilometers.

Since the formulæ (1) collectively contain certain lengths (or distances on the earth's surface) therefore they are specially convenient for studies based on the synoptic weather map. On the other hand, they suffer from the defect that in contrast with formula (2) they contain two variables, i. e., G and ρ or l and ρ, or strictly speaking three, since ρ itself depends on pressure and temperature.

In the ordinary discussions we consider only one variable G, since ρ is assumed to be constant.

But this is only a crude approximation for in fact

$$\rho = \rho_0 \frac{\beta}{760} \cdot \frac{273}{273 + t} \text{ or } \rho = \rho_0 \frac{\beta}{\beta_0} \frac{T_0}{T}$$

where for simplicity $\beta_0 = 760$, $T_0 = 273$, $T = 273 + t$ or the absolute temperature, and $\rho_0 = 1.293$, the value that ρ assumes corresponding to the normal pressure β_0 and the temperature T_0.

If we substitute this value in the formulæ (1) and recall that

$$\frac{13.6}{\rho_0} \frac{\beta_0}{T_0} \text{ or } \frac{13.596}{\rho_0} \cdot \frac{\beta_0}{T_0} = 29.272$$

is simply the gas constant for dry air occurring in the law of Mariotte-Gay-Lussac and which is ordinarily represented by the letter R, then we have

$$\gamma = \frac{G}{111\,111} \frac{T}{\beta} R.g. \quad \dots\dots\dots \quad (1\alpha)$$

$$\gamma = \frac{d\beta}{dl} \frac{T}{\beta} R.g \quad \dots\dots\dots \quad (1\beta)$$

$$\gamma = \frac{\Delta\beta}{l} \frac{T}{\beta} R.g \quad \dots\dots\dots \quad (1\gamma)$$

These formulæ contain altogether three independent variables, i. e., β, T and G, – or β, T and $\frac{d\beta}{dl}$, – or β, T and l – instead of the one independent variable that is implied in the ordinary approximate consideration of this subject.

Hence follow the following important consequences, which are not always sufficiently kept in mind:

"For equal values of the gradient G the gradient-acceleration γ is inversely proportional to the density of the air, hence it increases with increasing temperature and with diminishing pressure."

Of course at the surface of the earth this difference in the density of the air is not generally of much importance, especially so long as we consider only a small part of the surface. But if we do not thus limit ourselves then its influence can in extreme cases certainly amount to more than 30 per cent.

Assume, for instance, that at some place on the earth's surface the temperature is 37° C. and the barometric pressure 710^{mm}, such as can happen in whirlwind storms, but that at another place we have − 33° C. and 780^{mm}, then the values of the quotient $\frac{T}{\beta}$ at the two localities will have the ratio 44/31, so that for equal gradients the gradient-acceleration in the neighborhood of the highest pressure will amount to only 70 per cent of that within the low pressure.

Even for smaller regions, such as those covered by our ordinary weather charts, this influence can be considerable.

For instance, assume that in the center of a depression of 715^{mm} the temperature is 12° C., but in that of a maximum of 775^{mm} on the same weather map the temperature is − 33° C., then the ratio of the two values of $\frac{T}{\beta}$ is as 100/77 and for equal gradients the accelerations at the two localities would have the same ratio.

Since now the examples here chosen, although slightly exaggerated, reproduce the conditions that are ordinarily observed in low areas with warm centers and high areas with cold centers, therefore we perceive that in general "the gradient-accelerations" near barometric maxima would be greater than would be expected from the gradient itself. It is only in the case of depressions with cold centers that a partial compensation of the rarefaction of the air due to the low pressure is brought about by the low temperature. Therefore the lesser density of the air, such as generally occurs in the lows, contributes still further to increase the velocity of the wind in this region independent of the closeness of the isobars, i. e., independent of the strong gradients.

"Since now in front of the cyclones, where the air flows in from the equatorial side, the temperature averages higher than in the rear, so also in general for equal atmospheric pressure, i. e., along any one isobar, the density of the air in front of the depression is less than in the rear and correspondingly the gradient-acceleration is greater than would correspond to the average gradient."

However, even for equidistant successive isobars in the front of a depression one must expect greater accelerations and correspondingly greater wind velocities than in the rear.

Hence the isobaric charts directly allow a conclusion as to the gradients in that according to formula (1c) these are always proportional to the reciprocal of the distance between neighboring isobars, but not any conclusion, or at least only a crude approximation, as to the gradient-acceleration, which still depends to a large degree on the density of the air.

Therefore the isobars can in no wise be compared with altitude lines or isohypsen. For whilst we can from the reciprocal of the horizontal distance of the isohypsen conclude directly as to the gravity gradient, i. e., the tangent of the angle of inclination, and thence as to the acceleration which a heavy point experiences when it moves without friction on the given surface, we cannot do this from the isobars. Such a conclusion would only be allowable when the density of the air is uniform over the whole area under consideration, i. e., when $\frac{T}{\beta}$ is constant.

But even in summer, when it is generally cooler at the base of the cyclone than in the anticyclone, this condition is only seldom satisfied. For instance, in a region of maximum pressure 775^{mm} and another of minimum 745^{mm} the temperatures must be respectively $27°$ and $16°$ C. if the densities are to be the same in both. But more than this there is also the condition, which is almost never satisfied, that the temperature be constant along each isobar.

If the temperature were uniform over the whole region covered by a chart of isobars, then certainly we would be in a position to draw a system of lines whose reciprocal distance would certainly be proportional to the gradient-acceleration. The formula (1β) can be written

$$\gamma = \frac{d\beta}{\beta}\,\frac{T}{dl} \cdot Rg$$

or

$$\gamma = \frac{d \log \beta}{dl}\, T R g.$$

From this formula by passing to differences we can deduce the following:

$$\gamma = \frac{\Delta \log \beta}{l}\, T R g.$$

If in such a case of general uniform temperature, we draw a system of lines in such a manner that the logarithms of the atmospheric pressure proceed by equal differences, then to a high degree of approximation, the accelerations will be proportional to the distances apart of the neighboring lines. The lines would then also be truly comparable with isohypsen, at least so long as the inclination of the surface in question is so slight that we may consider the sine and tangent of the inclination angle as equivalent.

From the preceding considerations it follows, that any conclusions as to the acceleration effective at various points can only be drawn with great care, even in the case of the ordinary charts of isobars at the earth's surface.

"The gradient is always inversely proportional to the distance apart of the neighboring isobars; the gradient-acceleration is in general greater in proportion as the pressure is smaller. If therefore we would adhere to the meaning ordinarily attached to the chart of isobars, we must think of the isobars in the neighborhood of the barometric pressure as being closer together than they really are."

If we draw isobaric charts for higher levels we incur danger of drawing still more erroneous conclusions, unless we give special attention. For instance, the chart for the altitude 5500 meters[5] should have its isobars drawn for every 2.5 millimeters, since only then will its appearance justify general conclusions such as those suggested by the chart drawn for the sea-level (on which the isobars are drawn for every five millimeters).

If from this point of view we consider the isobaric charts for altitudes 5000 and 10,000 meters communicated by H. Hergesell to the Met. Zeit. for January, 1900, we are surprised to find how enormous the gradient acceleration is at these altitudes on these dates.

Indeed Hergesell himself intended to indicate this point in that on page 27 he said that at an altitude of 5000 meters the same difference of atmospheric pressure corresponds to about twice the gradient, whereas he evidently should have said that the same gradients at sea-level and at this altitude produce a gradient acceleration at the highest level twice that at the sea-level.

It is evident from what precedes that the representation of the distribution of atmospheric pressure by isobars on a level surface of gravity has the important advantage of being easy to prepare and of allowing of a comprehensive view of an unlimited extent of area

---

[5] See the table No. 1, Memoir XIV, p. 309 of this collection of translations.

of any such surface, but that as a basis for theoretical considerations, especially of acceleration, it must always be used circumspectly.

It is quite otherwise with the presentation of atmospheric pressure by a vertical section, which, as before stated, has been designated by the inappropriate name "presentation by surfaces of pressure." This method is difficult to put into practice since the pressure surfaces can be determined only by a roundabout process, but it offers many advantages from a theoretical point of view.

It is because of this peculiarity that the "vertical sections of isobars," as they should be called, are relatively seldom used and even then almost never applied to specific cases, but only schematically for purely theoretical considerations.

So far as I know, the first one to make use of this method was Julius Hann, who according to his own statement explained it in 1875 in his university lectures and also deduced the law of the acceleration experienced by a particle of air at any point on an isobaric surface.

The first more detailed publication in reference to this point is found in Hann's memoir on "Mountain and Valley Winds."[6] Subsequently H. Januschke[7] in 1882 and L. Teisserenc de Bort[8] in 1884 made use of this method of presentation. It would lead us away too far to develop the general formula for such an isobaric surface, although this could easily be done with the help of the well-known barometric formula, but of course with the uncertainty inherent in this formula relative to the vertical distribution of temperature and moisture.

On the other hand, certain properties of this surface may at least be mentioned here.

If there are given two surfaces of constant pressure, one of which corresponds to the atmospheric pressure β, the other to $\beta + \Delta\beta$, then the vertical distance between these surfaces at any point is given by the equation

$$\Delta h = 13.6 \frac{\Delta \beta}{\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

since $\rho \Delta h$ is the mass of the air contained in a vertical cylinder or prism standing on the unit of surface and $13.6 \Delta \beta$ is the corresponding mass of mercury.

---

[6] J. Hann: Zeit. d. Oest. Gesel. f. Met. 1879, XIV, p. 444. Compare also my note 11, Memoir XVII, p. 354 of this collection of translations.

[7] H. Januschke: Zeit. d. Oest. Gesel. f. Met. 1882, XVII, p. 136.

[8] L. T. de Bort: Annales du Bureau Central. Année 1882, pp. 73–80. Paris, 1884.

Therefore, if we have given a system of surfaces of constant pressure corresponding to the pressures β; $\beta + \Delta\beta$; $\beta + 2\Delta\beta$, etc., then the vertical distance between the neighboring surfaces at any point is inversely proportional to the prevailing density of the air at this point.

This latter theorem corresponds to that for the isobars according to which the gradient is obtained from the reciprocal of the distance between neighboring isobars.

If now for ρ we substitute in the above expression

$$\rho_0 \frac{\beta}{\beta_0} \frac{T_0}{T}$$

then equation (3) becomes

$$\Delta h = \frac{13.6}{\rho_0} \cdot \frac{\beta_0}{\beta} \frac{T}{T_0} \Delta\beta.$$

Hence it follows that

"The vertical distance between two definite isobaric surfaces β and $\beta + \Delta\beta$ at any point is proportional to the absolute temperature prevailing at that point."

Finally, if in equation (1c) in place of the expression

$$13.6 \frac{\Delta\beta}{\rho}$$

we substitute the difference Δh from equation (3) we obtain

$$\gamma = \frac{\Delta h}{l} g.$$

Now since l is the distance between the pressure surfaces β and $\beta + \Delta\beta$ measured horizontally from a point on the surface β, whereas Δh is the distance of the same surfaces measured vertically, therefore $\frac{\Delta h}{l}$ is the tangent of the angle of inclination of the surfaces or tg α. From this consideration we obtain the well-known equation (2) above given or $l = g \ tg\ \alpha$. This equation is distinguished by its simplicity from all of those to which the number (1) was given in the preceding paragraph.

Translated into words this equation (2) would read

"The gradient acceleration is proportional to the inclination of the isobaric surfaces."

Or if we recall that the angle α is always very small so that we can consider sine and tangent as equivalent, we have:

"The acceleration experienced by a particle of air at a given point on an isobaric surface is equal to that which a heavy mass would experience if it could slide without friction on a similar rigid isobaric surface."

This theorem holds good in general without reference to the density of the air, that is to say, without reference to the absolute value of the pressure or the temperature. These two quantities or the equivalent density of the air have already exerted their influence on the form of the isobaric surfaces and therefore do not need to be further considered in the final result.

"Therefore the presentation by the isobaric surfaces allows of direct conclusions as to the gradient-acceleration, and the density of the air at different points of the space under consideration, and, as to the course of the temperatures between neighboring isobaric surfaces."

Since these conclusions are all rigorous, therefore the unprejudiced consideration of any such presentation suffices to answer the individual questions, whereas in the use of the ordinary charts of isobars one must always proceed with caution and must consider attendant circumstances.

Unfortunately this great advantage of the presentation by vertical surfaces suffers from one defect, that the construction of the isobaric surfaces or their intersections with a vertical surface, offers the greatest difficulties practically, so that, as above remarked, it is generally only employed for schematic considerations.

XIX

THE INTERCHANGE OF HEAT AT THE SURFACE OF THE EARTH AND IN THE ATMOSPHERE

BY PROF. DR. WM. VON BEZOLD

[*Sitz. Ber. of the Berlin Academy of Sciences for 1892, pp. 1139–1178. Translated from Gesammelte Verhandlungen, Berlin, 1906, pp. 316–356*]

(I.) INTRODUCTION.

The "distribution of heat at the earth's surface" or, more correctly, "the distribution of temperature in the lowest strata of the atmosphere" has been the object of many exhaustive investigations since the days of Alexander von Humboldt.

It is especially Dove, Wild, and Hann who have gradually completed the idea that was sketched out in a few lines by Humboldt and have worked out its details for a large portion of the surface of the earth.

In this way we have learned at least in general of those influences that, together with the predominant radiation from the sun, determine the distribution of heat, and thus give the lines of equal temperature (isotherms) the exact form that we find in the charts drawn by the above-mentioned investigators.

But in general these studies are confined to purely qualitative considerations. One is satisfied to state the general trend of the influence of the distribution of land and water, and of the currents of air and ocean. Hitherto only to the most modest extent have attempts been made to determine the numerical or quantitative extent of these influences, or to consider together the general economy of the heat in the atmosphere and on the earth's surface.

In this respect a section of Samuel Haughton's Physical Geography must be first mentioned.[1] To a certain extent the works of Zenker[2] belong to this subject. We also meet with attempts in this

[1] Samuel Haughton: Six lectures on physical geography. Dublin and London, 1880.

[2] Zenker: The distribution of heat at the earth's surface. Berlin, 1888; also Met. Zeit., 1892, IX, pp. 336–344, 380–394.

direction by Woeikof[3] and similarly in a recent highly interesting memoir by W. Trabert.[4]

Attention has hitherto to a large degree been given to only one side of this problem, namely, the theory of insolation or the radiation from the sun to the earth and that of the radiation from the earth into space—a chapter on which, as is well-known, there is an extensive literature.

But although it must be allowed that exact determinations of these two elements are among the most important points of the whole question, still we ought not to forget that it is precisely here that we meet the greatest difficulties unless we restrict ourselves to purely theoretical considerations as J. H. Lambert, L. W. Meech, and Christian Wiener have done.

Recently O. Chwolson[5] has clearly shown how important are the difficulties and how large the corresponding uncertainty that still attends this field of work in spite of all the thought that has been bestowed upon it.

That the degree of accuracy that has hitherto been attained in the determination of the intensity of the solar radiation is still quite moderate is seen moreover from the simple fact that we cannot yet recognize the change from perigee to apogee, although it must amount to one-fifteenth or 7 per cent of the total amount.

In consideration of the difficulties offered by the solution of this apparently simple question, and in view of the uncertainty that still exists with reference to the most important constants, it might indeed seem premature to attempt to extend the investigation over the far more complicated process through which the heat furnished by the sun has to pass, from its entrance into the atmosphere until its exit therefrom into celestial space.

But still this labor must be undertaken. We must attempt to determine, at least approximately, what fraction of the heat which comes into play at any part of the earth's surface or of the atmosphere, in a given time, is furnished by direct insolation and lost by direct radiation; how much is brought hither and removed hence by simple or complex convection; how much is used in evaporating water or melting ice; how much is stored in the ground only to be

---

[3] Woeikof: The climates of the Globe. Jena, 1887.

[4] Trabert: The diurnal variation of temperature and sunshine on the summit of Sonnblick. Memoirs of the Vienna Acad. Math. Classe, Vol. LIX, 1892.

[5] Chwolson: On the present condition of actinometry. Wild Repertorium, 1892, XV, No. 1.

subsequently given up, etc. If ever these questions can be solved even with only a rough approximation this will be considered a great advance in our knowledge.

Then first will the many items that go to make up the general problem be separately treated, and only thus will be attained that point of view that must even now be kept in mind in assembling the observations, if indeed we are ever to succeed in more completely attaining the desired end.

The present memoir and others to follow later will contain an effort in this direction.

After some introductory considerations I will first present a number of quite general theorems and then develop the individual chapters.

With reference to the order in which these individual investigations will follow each other, I shall not bind myself to any prearranged sequence, but rather let it depend on my success in bringing each of the appropriate problems to a definite conclusion.

I will endeavor to give the statement of the general theorems with the greatest rigor, whereas in treating the individual problems I must often be satisfied with first approximations, as I do not consider it proper to compute with five decimals when we can scarcely be sure of the whole units, or to develop elegant formulæ for a problem as to whose fundamental character we are still seeking for light.

But before I consider the problem proper it appears appropriate first to undertake a rapid survey of the whole field and to attempt by use of the most important well-known constants to obtain at least a superficial idea as to the weight with which the processes that are hereafter to be more accurately studied enter into the computation, since only thus can we learn what points must be considered as of first importance and what may be neglected so long as we cannot attain a high grade of accuracy.

We most easily attain such a general view of the problem when we seek for the quantities of heat that are necessary in order to bring about certain effects at the earth's surface, and when we compare these with the quantities that suffice to melt a layer of ice of definite thickness or to evaporate a layer of water of definite depth—a method of presentation that has already been frequently used, especially by Haughton.

I choose the large calory or the kilogram calory, as the unit of heat the minute as the unit of time, and the meter as the unit of length, unless some other is expressly mentioned.

With these preliminaries we find the following as the number of units of heat required.

	Kg. calories.
Required to warm $(1^m)^3$ water 1° C.	1000
to warm $(1^m)^3$ earth 1° C.	300–600*
to evaporate 1^{mm} depth per $(1^m)^2$ water	600†
to melt 1^{mm} depth per $(1^m)^2$ area = 1 cubic decimeter ice	76
to warm by 1° C. the total atmosphere resting on $(1^m)^2$ of ground	2454‡
to warm by 1° C. $(1^m)^3$ air at 0° C. under pressure of 760^{mm}	0.307

* See p. 414.

† Temperatures between 0° C. and 30° C. principally occur in the case of evaporation of water at the earth's surface. For these temperatures, according to Regnault, the heat of evaporation lies between 606.5 and 585.6, hence 600 can be adopted.

‡ Under the assumption that the pressure at the earth's surface is 760 mm.

Although this is a very elementary tabulation, yet it gives some important indications. First, we see that the difference in the capacities of water and earth for heat, which is frequently adduced as a principal basis for the explanation of the difference between the continental and oceanic climates, is greatly diminished when we compare together not equal masses but, what is far more correct, equal volumes, that is to say, when we consider the capacities per unit volume instead of per unit weight. But above all it shows what an enormous importance attaches to evaporation in the economy of nature, and how it is that this, together with the mobility of water, assumes the first place in the questions now at issue, a fact moreover that Dove laid stress on in his memoir on monthly isotherms,[6] whereas subsequently and notwithstanding this we often find the importance of the difference of the thermal capacities greatly exaggerated.

The powerful influence of evaporation is still more evident when, by means of the figures above given, we come to clearly comprehend that the evaporation of a depth of one millimeter of rainfall demands as much heat as the melting of a layer of ice about eight times as thick, and that this same amount of heat would suffice to warm up the ground by 1° C. to a depth of one or two meters, or to warm up by one-fourth of a degree Centigrade the whole column of air standing on the same area of ground and reaching up to the extreme limit of the atmosphere.

Dove: Memoirs of the Berlin Acad., 1848, p. 219.

In continuation of these views it is not difficult to form an idea as to the extent to which the total kinetic energy actually present in the atmosphere, e. g. in the shape of wind, can in an extreme case affect these investigations.

Assume that one kilogram of air is moving with the velocity v so that the corresponding kinetic energy is

$$\frac{v^2}{2} = gh,$$

where gh is the amount of work corresponding to this energy. If this work is converted into heat it becomes

$$\frac{h}{424}$$

calories, an amount of heat that suffices to raise the temperature of one kilogram of air under constant pressure by

$$\frac{h}{424 \times 0.2375} \text{ degrees Centigrade or about } \frac{h}{100} \text{ degrees.}$$

This latter number expresses the rise in temperature that the air would experience if the wind could be suddenly brought to a stop and it be then allowed to expand until equilibrium be attained.

If v have values of 10, 20, or 30 meters per second then this warming would correspond to 0.05°, 0.20°, and 0.45°, respectively.

But we estimate it too high when we assume that the mean velocity of the whole atmosphere is 20 meters per second (10 meters would be too high for the lowest stratum) and yet even so the sudden conversion of the translatory motion of the whole atmosphere into heat would cause a rise in temperature of 0.20° C.

But this rise of temperature corresponds to an amount of heat that would not suffice to evaporate a layer of water even one millimeter in depth. The potential energy that we observe in the form of differences of atmospheric pressure, i. e., in superposed surfaces of equal pressure, is of course of the same order as the actual energy of translatory motion evolved from them, and thus we see that the quantities of heat present in this form of energy are very small in comparison with those that are absorbed and evolved in the change of condition of water especially in its evaporation and condensation.

Hence in the determination of the total energy of any portion of the atmosphere, the first matter to be considered is the quantity of aqueous vapor contained therein.

In order to give the estimates here set forth their full value, it is necessary to compare the consumption of heat in the processes above enumerated with the quantity furnished by the sun within the given time.

Unfortunately here we find ourselves in a very difficult position, since the solar constant or the number of gram-calories that one square centimeter of surface at the outer limit of the atmosphere receives in one minute when the rays of the sun fall perpendicularly on it, has not yet been determined with certainty.

The values that have been obtained for this constant, which I will designate by s vary between the limits 1.763 and 4.0.[7] But since the greater number lie between 2 and 3, therefore, in order to at least have a definite proposition, I will here use the value $s = 2.5$, or if the equivalent constant be expressed in square meters and kilograms and minutes as units, then $S = 25$ (kilogram-calories).

Under this assumption the whole earth receives $25 \pi r^2$ units of heat in one minute, where r expresses the radius of the globe including its atmosphere. This quantity of heat is distributed over the hemisphere illumined by the sun, that is to say, over a spherical surface whose area is $2 \pi r^2$ and hence on the average the sun gives each square meter of the hemisphere on which it shines 12.5 calories per minute or $12.5 \times 60 \times 12 = 9000$ calories per day since, ignoring the eccentricity of the earth's orbit, the average length of the day light is 12 hours for every point of the earth.

This amount of heat would suffice to melt a layer of ice 11.84 cm. in thickness, or to evaporate a layer of water 15 mm. deep per day; or 550 centimeters of water or 43 meters of ice per year.

If by anticipation (see page 414) we add that the quantity of heat entering and leaving the soil during one year can in an extreme case evaporate a layer of 40 mm., and if in general we express all this data for the whole earth uniformly by the depth of evaporated water or melted ice we obtain the following table:

	CAN EVAPORATE WATER.	CAN MELT ICE
	cm.	*cm.*
The sunshine in one average day	1.5	12.0
The sunshine in one whole year	550.0	4325.0
The heat received and lost annually in the soil is less than	4.0	31.6
The heat that warms the atmosphere by 1°	0.4	3.2
The kinetic energy of the atmosphere is less than	0.08	0.6

[7] Chwolson: On the present state of actinometry, pp. 10 to 14.

If we compare the value of the evaporation here given as the equivalent of the total insolation with the observed rainfall of the globe we come to the conclusion that either the value $s = 2.5$ is still far too high or that out of the total insolation falling on the boundary of the atmosphere only a much smaller portion arrives in the lower stratum than one might expect from the measures of absorption made on very clear days.[8]

This fractional portion could be easily estimated if the average precipitation were known for the whole earth; since, as just stated, the re-evaporation of the fallen precipitation represents the principal work that the sun's heat has to perform.

Unfortunately we are not able to give even an approximately reliable value of this precipitation, since observations of the quantity of rain are almost wholly wanting for the greater part of the globe, namely for the ocean.

If the average precipitation be 55 centimeters and if $s = 2.5$, then the heat consumed in the evaporation of this amount of water would be one-tenth of the total furnished by the sun and we should therefore have to assume that the heat which reaches the lower strata of the atmosphere amounts to not much more than one-tenth. If the average precipitation were 110 centimeters, which certainly seems to be too high an estimate, then we should conclude that about one-fifth of the total solar radiation reaches the lower strata.

In any case the quantity of heat reaching the surface of the earth is a much smaller fraction of the total insolation than has been given by the measurements hitherto made on perfectly cloudless days.

Of course a very considerable fraction of the incident radiation is absorbed by the clouds and certainly a still larger part of it is reflected by their upper surfaces, and thus a quantity of radiant energy is rejected at the very threshold.

"It needs but a single glance from the summit of a mountain down on the ocean of cloud lying below one and illumined by the sun to convince one that the diffuse reflection from that surface is incomparably greater than the similar reflection from the ground or from a water surface and that therefore this must play a very important part in the thermal economy of the earth."

How strong this reflection is (and to it I have moreover often referred, although it seems to have received but little attention) may

[8] See Angot-Pernter: Met. Zeit., III, 1886, pp. 540–546.

be seen from the observations in balloons that R. Assmann will soon publish.[9]

It is very important to devise methods that will enable us to at least approximately measure the reflection from the surfaces of the earth and the clouds. But these are questions that will be fully considered hereafter. At present we are concerned only to obtain a general view as to the most important of the quantities under consideration. This object seems now to have been attained and I will proceed to the closer study of the matter.

Before I treat closely the individual problems that offer themselves, I will establish a number of theorems that will serve as guides for all that follows. These theorems are of such simple nature that they might seem almost self-evident and can easily be expressed in words. But I will also put them in the shape of formulæ, although these latter become more complex than would be expected from the simple verbal expressions. Nevertheless, I find it expedient to give them such a form. By this means we not only attain accuracy of expression and thus remove every chance of misunderstanding, but we can from the formulæ deduce a number of individual items that would otherwise be overlooked.

(II.) GENERAL THEOREMS

The next following theorems are all founded on the assumption that we may consider the thermal condition of the earth as a stationary one, or more correctly, one that is periodically stationary.

These therefore assume that there are average values of all the quantities considered which remain the same within small limits of error when we have deduced them from a sufficiently long series of years of observation without regard to the exact length of the series or to the year with which it began.

These theorems depend upon the justifiable assumption that the earth, at least within the time covered by our observations, has neither become warmer nor colder and that the succession of seasons goes on as uniformly as ever at every point on the earth's surface.

Hence, all the quantities that we consider except the times, the dimensions, the coördinates, etc., represent average values as obtained from series of observations that are long enough to allow of applying to them the fundamental laws of averages, and yet not so long

[9] R. Assmann and A. Berson: Wissenschaftliche Luftfahrten. Volumes II and III. Braunschweig, 1900.

as to render necessary the consideration of those changes that have gone on during the immense periods of geology.

Consequently the equations that will be deduced in the following pages should, strictly speaking, include a quantity $\pm \varepsilon$, where ε refers to the uncertainty peculiar to the nature of their average values; but for simplicity this will be omitted.

Before I proceed to give the promised theorems, I present first the notations that will be employed as follows:

t, the time in minutes counted from the beginning of the year.

$T = 525949$, the duration of a mean solar year in minutes.

q' the quantity of heat that, at the time t, enters a unit surface in a unit of time, at any given point of the earth's surface or of the atmosphere, or which in a certain sense may be said to pass through the unit surface.

q'' the quantity of heat that passes outward through the elementary unit surface, or that flows through it in the opposite direction.

$q'_{t_{1,2}}$ and $q''_{t_{1,2}}$ the quantities of heat that enter or leave the unit surface in the intervals of time between t_1 and t_2 or briefly q'_τ and q''_τ when the interval of time $t_{1,2}$ between t_1 and t_2 is indicated by τ.

$\bar{q}$ the quantity of heat entering the unit surface in the unit of time at the upper boundary of the atmosphere.

$\bar{q}_{t_{1,2}} = \bar{q}_\tau$ and $\bar{Q}$ the corresponding quantities for the interval $t_{1,2}$ and for the whole year.

$\bar{\bar{q}}$, $\bar{\bar{q}}_{t_{1,2}} = \bar{\bar{q}}_\tau$ and $\bar{\bar{Q}}$ the corresponding quantities of heat leaving the unit surface at the upper limit of the atmosphere.

$\mathfrak{q}'$ and $\mathfrak{q}''$ the quantities of heat entering and leaving a closed surface of definite extent in a unit of time at the moment t.

$\bar{\mathfrak{q}}$ and $\bar{\bar{\mathfrak{q}}}$ the corresponding values for the boundary of the atmosphere, that is to say, for a spherical surface enclosing the whole atmosphere and earth.

$\mathfrak{q}'_{t_{1,2}}$, $\mathfrak{q}''_{t_{1,2}}$, $\bar{\mathfrak{q}}_{t_{1,2}}$, $\bar{\bar{\mathfrak{q}}}_{t_{1,2}}$ or briefly $\mathfrak{q}'_\tau$, $\mathfrak{q}''_\tau$, $\bar{\mathfrak{q}}_\tau$, $\bar{\bar{\mathfrak{q}}}_\tau$ the corresponding values for the interval from t_1 to t_2.

$\mathfrak{Q}'$ $\mathfrak{Q}''$ $\bar{\mathfrak{Q}}$ and $\bar{\bar{\mathfrak{Q}}}$ the corresponding values for the whole year, i. e., for $t_2 - t_1 = T$.

$\mathfrak{q}'_a$, $\mathfrak{q}'_b$, $\mathfrak{q}''_a$, $\mathfrak{q}''_b$, $\bar{\mathfrak{q}}_a$, $\bar{\mathfrak{q}}_b$, $\bar{\bar{\mathfrak{q}}}_a$, etc., the corresponding quantities for definite portions a, b, of the above-named surface or for the boundary of the atmosphere for the unit of time.

$\mathfrak{q}'_{a,\tau}$ $\mathfrak{q}'_{b,\tau}$, etc., the corresponding quantities for the interval $t_{1,2}$.

$\mathfrak{Q}'_a$, $\mathfrak{Q}''_a$, $\bar{\mathfrak{Q}}_a$, $\bar{\bar{\mathfrak{Q}}}_a$ the quantities of heat respectively entering and leaving an area a of the surface of the earth or of the boundary of the atmosphere within a whole year.

$\mathfrak{u}$ the total energy contained in an enclosed portion of the surface of the earth or the atmosphere at the time t.

$\mathfrak{u}_1$ the corresponding quantity for the time t_1.

r the radius of a sphere centered at the earth's center and enclosing the whole atmosphere, or a quantity exceeding the greatest radius of the globe by 100 kilometers.

$d\sigma$ an element of a surface.

β the geographic latitude.

λ the geographic longitude.

S the solar constant or kilogram calories of heat received by the earth from the sun at the earth's mean distance, per minute per square meter.

If we consider this system of notation we shall perceive that the following points of view have been kept in mind:

The quantities relating to the unit of surface have been designated by Roman letters, those relating to larger areas and the boundaries of the whole atmosphere by German letters.

The quantities relating to the unit of time are indicated by small letters; those relating to other intervals except the whole year are indicated by the same letters but with special index. For all quantities that relate to a whole year the capital letters are used.

The quantities of heat are considered as absolute quantities and letters indicating added heat have one accent while those indicating subtracted or lost heat have two accents. These accents are placed above and to the right when the heat passes through surfaces that are within the boundary surface of the atmosphere, but are horizontal lines placed above the letters when the heat passes through this boundary itself.

We now proceed to establish the following theorems:

I. "The quantities of heat received by insolation and lost by radiation by the whole earth in the course of a whole year are on the average equal to each other."

For if these quantities were not equal there would occur either a progressive warming or progressive cooling, which has not been the case during the time accessible to more accurate investigation.

Translated into algebra this theorem assumes the simple form

$$\bar{\mathfrak{Q}} = \bar{\bar{\mathfrak{Q}}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Here according to the definitions above given, we have

$$\overline{\mathfrak{Q}} = \int \overline{Q}\, d\sigma$$

where the integral is to be taken over the whole surface of the sphere whose radius is r, hence

$$\overline{\mathfrak{Q}} = r^2 \int_0^{2\pi} d\lambda \int_{-\pi/2}^{+\pi/2} \overline{Q} \cos\beta\, d\beta \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since the quantity of heat $\overline{Q}$ that comes by insolation to a unit surface at any given point of the boundary surface of the atmosphere in the course of a year is a function of the geographical latitude only, this equation becomes

$$\overline{\mathfrak{Q}} = 2\pi r^2 \int_{-\pi/2}^{+\pi/2} \overline{Q} \cos\beta\, d\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Moreover, as Lambert has demonstrated, for this function of the latitude $\overline{Q} = \varphi(\beta)$, we have

$$\varphi(\beta) = \varphi(-\beta)$$

that is to say, points at the same latitude in the northern and southern hemispheres receive from the sun by radiation equal sum-totals of heat in the course of a year.

Therefore we can write equation (3) in the form

$$\overline{\mathfrak{Q}} = 4\pi r^2 \int_0^{\pi/2} \varphi(\beta) \cos\beta\, d\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The value of the function $\varphi(\beta)$ is known from the investigations of Meech[10] and Wiener[11] and is only uncertain to the extent of the uncertainty of the solar constant that enters it as a coefficient.

Moreover, as is well known, we also obtain the value $\overline{\mathfrak{Q}}$ in the simplest way from the consideration that the sum total of the radiation coming to the whole earth within a given interval of time is equal to that which falls in that time on the great circle of the globe perpendicular to the line connecting the sun and the earth.

[10] Meech: On the relative intensity of the heat and light of the sun. Smithsonian Contributions, IX, Washington, 1857.

[11] Wiener: Zeit. Oest. Gesell für Met. 1879, XIV, p. 113. [See Angot Réchérches Theoriques. Paris, 1885.—C. A.]

Hence we have

$$\overline{\mathfrak{Q}} = \pi r^2 TS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where S indicates the solar constant as determined for the mean distance between the earth and the sun.

For $\overline{\overline{\mathfrak{Q}}}$ we may also establish similar but not nearly so simple formulæ. We have the formula

$$\overline{\overline{\mathfrak{Q}}} = r^2 \int_0^{2\pi} d\lambda \int_{-\pi/2}^{+\pi/2} \overline{\overline{Q}} \cos \beta \, d\beta \quad . \quad . \quad . \quad . \quad . \quad (6)$$

but the quantity $\overline{\overline{Q}}$ is not like $\overline{Q}$ a function of the geographic latitude only, but also of the longitude, in so far as we take into consideration the individual peculiarity as to outward radiation of each element of the boundary surfaces (referring especially to the lower portion of the atmosphere and the adjacent earth's surface).

The total insolation that an element at the outer boundary of the atmosphere receives in the course of a year depends only on the geographical latitude; the quantity that is returned to space by radiation through this element varies from point to point of the earth.

Hence $\overline{\overline{Q}} = \psi\,(\beta, \lambda)$ and the function ψ is never a simple one and is scarcely expressible empirically.

The formula

$$\overline{\overline{\mathfrak{Q}}} = r^2 \int_0^{2\pi} \int_{-\pi/2}^{+\pi/2} \psi\,(\beta, \lambda) \cos \beta \, d\beta \quad . \quad . \quad . \quad . \quad (7)$$

is therefore not susceptible of further simplification or modification; but on the basis of the above-given considerations and by the help of equations (4) and (5) the final result, viz:

$$\overline{\overline{\mathfrak{Q}}} = \overline{\mathfrak{Q}} = \pi r^2 TS$$

may be given directly.

Hence the quantities expressing the gain and loss or insolation and radiation show a very great difference in that one is expressible by rigorous mathematical formula but the other is not, unless we can express the latter in terms of the former by theorems relative to the equality of the quantities of gain and loss.

The difference between these two classes of quantities would be still more striking if the so-called solar constant really were properly so-called, i. e., if the intensity of solar radiation actually remained invariable.

Under this latter assumption all quantities relating to insolation would be mathematically rigorous in contrast to those relating to terrestrial radiation which can only be thought of as general average values.

However, the variations observed on the sun's surface make it highly improbable that the intensity of solar radiation can be invariable, and therefore we must certainly also assume that the values relative to insolation always have only the character of average values.

II. "The quantities of heat that are given or lost by a definite portion of the earth's surface or the atmosphere in all the various possible ways in the course of a year are on the average equal to each other."

This theorem is, like Theorem No. I, a direct consequence of the assumption that the sun and the earth are in a certain sense in a stationary condition, that is to say, that we are in general justified in speaking of average values of the various quantities that come into consideration.

III. "The quantities of heat that individual portions of the earth's surface or of the atmosphere gain by insolation or lose by radiation, in the course of a year, are in general not equal to each other, but there are portions of the earth where the insolation is in excess and other portions where the radiation is in excess."

The correctness of this theorem follows from the simple fact that warm air and warm water flow continually from equatorial regions poleward, while conversely cold air and cold water or ice flow from polar regions toward the equatorial.

Hence the equatorial belt continuously loses heat by convection (and also certainly in the form of the kinetic energy of translatory motion) which must be replaced by excess of insolation if the mean temperature is to remain constant, whereas the reverse holds good for the polar regions.

We can therefore subdivide the whole earth into three zones, viz: one equatorial in which the insolation exceeds the radiation, and two polar, in which the radiation exceeds the insolation.

These zones I will designate as "insolation zone" and "radiation zone," respectively.

The lines that separate these zones from each other at the boundary surface of the atmosphere may be called "lines of equal insolation and radiation" or "lines of radiation equilibrium" or briefly "neutral lines."

There are two such neutral lines where the radiation outward and inward balance each other, one of which belongs to the northern and the other to the southern hemisphere. But it is not incredible that, besides these, other similar closed lines may exist that must be regarded as boundaries for smaller regions, like islands.

If we give the positive sign to the heat radiated from the sun to the earth and the negative sign to that radiated by earth to space, then the algebraic sum of the quantities of heat exchanged through the boundary surface of the atmosphere is positive within the equatorial zone but negative in the two polar zones. We can, therefore, as to annual averages, think of the whole exchange of heat within the atmosphere and at the earth's surface as schematically represented by a current of heat that enters into the equatorial zone through the boundary surfaces of the atmosphere and after splitting into two branches departs in the polar zones.

The location of the neutral lines for the balance of radiation and the determination of the intensities of these ideal streams, i. e., the quantities of heat that are interchanged in this way, forms an important problem in that chapter of the physics of the atmosphere that we have now under consideration.

In fact we have not to do with such a simple single flow of heat but with double currents, since warm masses are simultaneously carried poleward and cold masses equatorward, whose sum total represents the simple current of the above scheme. Therefore the considerations to be here set forth have a certain analogy with those by which one passes from the assumption of a double current over to that of a single current as in the case of the double and single current theories in electricity.

The theorems just stated may be algebraically expressed in the following forms:

$$\left.\begin{array}{lr} \overline{Q} > \overline{\overline{Q}} \;\ldots\ldots\ldots & \text{in the equatorial zone} \\ \overline{Q} < \overline{\overline{Q}} \;\ldots\ldots\ldots\ldots & \text{in the polar zones} \\ \overline{Q} = \overline{\overline{Q}} \;\ldots\ldots & \text{along the two neutral lines} \end{array}\right\} \ldots\ldots (8)$$

which latter may be represented by the equations

$$\Phi\,(+\beta\lambda) = 0 \quad \text{and} \quad \Psi\,(-\beta\lambda) = 0$$

where β expresses the absolute value of the latitude, and we reckon northern latitudes as positive and southern latitudes as negative.

I may here remark that so far as I can see as yet the values of β vary about a mean that is to be found between 35° and 40°.

If now we designate the total quantities of heat received and lost in the equatorial zone by radiation during the whole year by $\overline{\mathfrak{Q}}_a$ and $\overline{\overline{\mathfrak{Q}}}_a$, respectively, and the similar quantities for the two polar zones by $\overline{\mathfrak{Q}}_p$ and $\overline{\overline{\mathfrak{Q}}}_p$ respectively, then we have

$$\overline{\mathfrak{Q}} = \overline{\mathfrak{Q}}_a + \overline{\mathfrak{Q}}_p$$

and

$$\overline{\overline{\mathfrak{Q}}} = \overline{\overline{\mathfrak{Q}}}_a + \overline{\overline{\mathfrak{Q}}}_p$$

whence recalling that by assumption $\overline{\mathfrak{Q}} = \overline{\overline{\mathfrak{Q}}}$, we obtain

$$\overline{\mathfrak{Q}}_a - \overline{\overline{\mathfrak{Q}}}_a = \overline{\overline{\mathfrak{Q}}}_p - \overline{\mathfrak{Q}}_p \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

that is to say, the excess of insolation received in the equatorial zone is counterbalanced by an exactly equal excess of radiation outward lost from the two polar zones and this counterbalance is effected by the convection into the polar zones of the excess attained in the equatorial zone.

Hence the difference $\overline{\mathfrak{Q}} - \overline{\overline{\mathfrak{Q}}}_a$ is equal to the quantity of heat that by convection (in the broadest sense of the word including the kinetic energy of moving masses of air) flows through the two neutral sections from the equatorial zone into the two polar zones.

Moreover, the quotient

$$\frac{\overline{\mathfrak{Q}}_a - \overline{\overline{\mathfrak{Q}}}_a}{T} = \frac{\overline{\overline{\mathfrak{Q}}}_p - \overline{\mathfrak{Q}}_p}{T}$$

expresses the average intensity of the currents of heat entering into the equatorial zone and flowing toward the poles, as we can imagine them in our scheme replacing the counterbalancing interchanges that are actually occurring within the atmosphere.

This quotient will therefore be designated by J_a. So that we have

$$\frac{\overline{\mathfrak{Q}}_a - \overline{\overline{\mathfrak{Q}}}_a}{T} = J_a \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

On the other hand, we divide the quantities representing the two polar caps into two parts relating to the northern and southern hemispheres, respectively.

If we give the index n to the quantity relating to the northern hemisphere and s to that relating to the southern hemisphere, then we have the following formulæ:

$$\overline{\mathfrak{Q}}_p = \overline{\mathfrak{Q}}_n + \overline{\mathfrak{Q}}_s$$
$$\overline{\overline{\mathfrak{Q}}}_p = \overline{\overline{\mathfrak{Q}}}_n + \overline{\overline{\mathfrak{Q}}}_s$$

and

$$J_a = J_p = J_n + J_s \quad \ldots\ldots\ldots (11)$$

as also

$$J_n = \frac{\overline{\overline{\mathfrak{Q}}}_n - \overline{\mathfrak{Q}}_n}{T}$$

and

$$J_s = \frac{\overline{\overline{\mathfrak{Q}}}_s - \overline{\mathfrak{Q}}_s}{T}$$

whence there follow

$$\left.\begin{aligned} \overline{\overline{\mathfrak{Q}}}_a &= \mathfrak{Q}_a - J_a T \\ \overline{\overline{\mathfrak{Q}}}_p &= \mathfrak{Q}_p + J_a T \\ \overline{\overline{\mathfrak{Q}}}_n &= \mathfrak{Q}_n + J_n T \\ \overline{\overline{\mathfrak{Q}}}_s &= \mathfrak{Q}_s + J_s T \end{aligned}\right\} \quad \ldots\ldots\ldots (12)$$

Since now, on the basis of a definite value of the solar constant, one can at least approximately compute all the quantities relating to insolation, when we know the location of the neutral lines, and since also the approximate determination of the intensities J_n and J_s of the two branches of the schematic flow of heat is not a matter of insuperable difficulty, therefore there is also a possibility of finding the quantity of heat radiated outward in the two radiation zones, including that reflected outward from the highest regions of the atmosphere.

These formulæ teach us that we may hope for information as to the interchange of heat in the highest inaccessible regions of the atmosphere as soon as we are successful in determining with sufficient accuracy the solar constant and also the intensity of the flow of heat through the two neutral sections.

Even this latter problem seems not insoluble, at least within certain limits, since in this flow the lower accessible strata principally comes into consideration.

Theorems similar to those just expressed for the whole year can also be established for shorter intervals of time, some of which may here be given.

IV. "The quantities of heat that are within special portions of a year given to and lost by special portions of the earth's surface or the atmosphere are in general not equal."

The proof of this theorem consists in the simple fact that the thermal condition of the earth's surface and the atmosphere is subject to periodical variations, that is to say, this is simply an expression of the fact that there occur times of excess of insolation and other times of excess of radiation.

By use of the notation, introduced above, this theorem takes the following form:

$$\mathfrak{q}'_{a,\tau} \gtreqless \mathfrak{q}''_{a,\tau} \quad \ldots \ldots \ldots \ldots (13)$$

or also

$$\int q_{\tau} d\sigma = \mathfrak{q}_{a,\tau} = \mathfrak{q}'_{a,\tau} - \mathfrak{q}''_{a,\tau} \gtreqless 0$$

where the integral is to be taken over the whole closed area a and where the omission of the accents over q_{τ} and $\mathfrak{q}_{a,\tau}$ indicates that the quantities are to be considered as algebraic and can therefore have correspondingly different signs.

If we consider not a closed surface but only a definite portion of the boundary surface of the atmosphere then we have the expression

$$\bar{\mathfrak{q}}_{a,\tau} \gtreqless \bar{\bar{\mathfrak{q}}}_{a,\tau} \quad \ldots \ldots \ldots \ldots (14)$$

In this expression we consider the case in which the inequality $>$ or $<$ becomes an equality $=$, as an exceptional case and in general we have to use the sign $\lessgtr$.

If $\mathfrak{q}_{a,\tau} > 0$ then this expresses the fact that there is an excess of heat added over that abstracted; if $\mathfrak{q}_{a,\tau} < 0$ then this expresses the deficiency that has been experienced by the masses contained within the volume a during the interval τ from t_1 to t_2.

We can also write

$$\mathfrak{q}'_{a,\tau} - \mathfrak{q}''_{a,\tau} = \mathfrak{u}_{a_l, t_2} - \mathfrak{u}_{a_1, t_1} \quad \ldots \ldots \ldots (15)$$

When this difference has a positive sign it indicates an increase of energy within the volume under consideration; a negative sign indicates a diminution.

This increase may consist in an increase of temperature, an increase of the quantity of vapor, a conversion of ice into water, a development or an increase of pressure differences (potential energy) or of motions (kinetic energy).

Frequently such an increase of energy is also called incidentally a storage of heat.

On the other hand, if $\mathfrak{q}_{a,\tau}$ is negative this teaches that the total energy has diminished during the interval of time under consideration, which must indicate either a fall of temperature, or condensation or freezing of the water, a diminution of pressure gradient or the diminution of the existing motions. If we have to do with changes in bodies that are stationary or scarcely movable, like water frozen into ice, or like the solid earth, then we could include also the storage of heat or cold.

For $t_2 - t_1 = T$ we have $\mathfrak{q}_{a,T}, = 0$ or $\mathfrak{u}_{a,t_2} = \mathfrak{u}_{a,t_1}$, since in accordance with the assumption that lies at the base of this whole investigation the thermal and kinetic condition of the earth at the close of a year always returns to its same initial condition no matter what moment of time t_1 we choose as our starting point.

Since therefore the total increase of heat within a year is zero, whilst it has finite values during the separate seasons, therefore for each point of the earth the whole year is divided into periods of excess of insolation and excess of radiation or, briefly, seasons of warming and cooling.

In the passage from one such period of one kind over to one of the opposite kind the differential quotient $\frac{d\mathfrak{q}}{dt}$ changes its sign, and $\mathfrak{q}$ itself at this moment of time therefore attains a maximum or minimum value.

Such extreme values occur within every day, but the absolute maxima and the minima in general only once during each year except twice at the equator.

If we ignore the daily extremes, and at least for regions outside the tropics, we can by appropriate choice of dates for the beginning of the year, divide the year into two halves such that for one we have an increase of heat and for the other a decrease.

These halves will in general be unequal, since the inflow and the outflow of heat follow very different laws.

If therefore $t_1 = 0$ be so chosen that $\mathfrak{u}_0$ is the absolute minimum and if we remove the secondary maxima and minima by some appropriate method of elimination, and if furthermore u_m is the absolute maximum of u, and t_m the moment of time at which this maximum is attained, then

$$\frac{dq_a}{dt} > 0 \text{ when } t \gtrless \begin{matrix} 0 \\ t_m \end{matrix}$$

and

$$\frac{dq_a}{dt} < 0 \text{ when } t \gtrless \begin{matrix} t_m \\ T \end{matrix}$$

moreover

$$q_{a,\tau_1} = u_{a,t_m} - u_{a,o}$$

and

$$q_{a,\tau_2} = u_{a,0} - u_{a,t_m}$$

if by τ_1 we understand the interval of time from o to t_m and by τ_2 the interval of time from t_m to T.

Hence also follows

$$q_{a,\tau_1} = -q_{a,\tau_2} \quad \ldots\ldots\ldots\ldots (16)$$

that is to say, the sum total of the heat received by a given portion of the earth or the atmosphere in the half year when the gain exceeds the loss, is exactly equal to that which is given up during the half year when the loss exceeds the gain.

Moreover equation (16) also holds good if the year is divided into any two perfectly arbitrary portions provided only that $\tau_1 + \tau_2 = T$; in every case the heat that is gained in one portion must again be lost in the other portion; but in any such arbitrary division q_{a,τ_1} cannot have a maximum. If, however, this value is a maximum then this must be designated as "the annual heat exchange for the portion of air or earth under consideration."

Hence it follows that "The annual heat exchange for any portion of the earth or atmosphere or both is equal to the difference between the maximum and the minimum quantities of energy contained in such portion."

For shorter periods, such as the diurnal periods, this theorem needs a slight modification, since in general for any such period not so much is taken away during its season of diminishing heat as is gained during its season of increasing heat; but less during one half of the year and more during the other half.

If therefore again q_{τ_1} indicates the heat added during the interval from any moment of one secondary minimum of total energy to the same moment of the next following secondary maximum, and q_{τ_2} the heat lost from the latter maximum to the moment of the next following secondary minimum, then we have

$$q_{\tau_1} \gtrless - q_{\tau_2}$$

where, however, the difference between the two quantities q_{τ_1} and q_{τ_2} is always small.
Consequently the amount of the daily exchange is

$$\tfrac{1}{2}\,(q_{\tau_1} + q_{\tau_2})$$

These considerations lead naturally to the determination of the total energy contained in any portion of the atmosphere or the earth.

For the present purpose it is important to choose such portion of the atmosphere appropriately and to bring it in connection with a corresponding limited part of the earth's surface.

By the term "total energy of any portion of the earth's surface" I will therefore understand the total energy in that portion of the earth's crust and the atmosphere resting on it, cut out by a straight line that, starting from the center of the earth, passes around that part of the earth's surface while its upper end is at the limit of the atmosphere, but its lower end is limited by a surface parallel to the earth's surface but so chosen that no annual period of temperature is observable therein.

On the other hand by the expression "total energy of a definite place on the earth's surface" I understand the energy within a truncated cone whose apex is at the center of the earth while its conical surface cuts a unit of area from the surface of the earth, and whose upper and lower surfaces are defined respectively by the upper boundary of the atmosphere and by a surface lying deep enough below and parallel to the earth's surface.

The determination of the total energy for the different points of the earth's surface, both as to its average value and also as to its variation with time constitutes an important problem in the theory of the thermal economy of the earth.

The amplitude of the variation of the total energy, that is to say, the difference of its extreme values, affords a measure of the thermal exchange both for the annual period and also, with a small modification, for the diurnal period.

The dates of these extremes lead to a division of the year into seasons of warming and cooling, which division opens up other points of view than the division of the year on a purely astronomical basis.

The geographical distribution of the total energy over the surface of the earth gives for the first time an idea of the actual distribution of heat at the earth's surface, whereas hitherto we have given this name to what is actually only the distribution of temperature in the lowest stratum of air.

But we must first come to an understanding as to a zero point if we wish to compare together the energies of different points of the earth's surface, although this is a matter that need not at all be considered in the investigation of the annual or the diurnal changes at any special points. But this is a point on which I will dwell in a later communication.

At present I need only remark that the approximate computation of the total energy in the sense just established ought not to encounter insurmountable difficulties.

In fact the portion belonging to the solid earth's crust is determined with relative ease, as will be shown in the second part of the present communication.

The extraordinary importance of the solution of this problem may be seen from the remarks that will be made when we speak of the remarkable variations that the so-called average temperature of the whole earth (i. e., of the lowest stratum of air over the whole earth) experiences in the course of the annual period, whereby it will be found that it is not allowable to reason from this directly to variations of the total energy of the whole earth. Similarly the importance of this question will appear in considering the peculiar behavior that the polar regions show at the time of the maximum altitude of the sun.

V. "The quantities of heat that enter and leave through the outer limit of the whole atmosphere in the course of a definite portion of the year are not necessarily equal to each other."

If the earth's surface and the atmosphere were perfectly homogeneous, at least throughout each surrounding layer concentric with the earth's center, and if moreover the earth's orbit about the sun were circular, then the above-mentioned equality would necessarily exist, but since these conditions are not fulfilled, and since the regions of excess of insolation and excess of terrestrial radiation change their locations [on the globe] in the course of the year and in fact at regions whose surfaces have entirely different properties, therefore there is really no reason whatever for such equality.

Hence there probably are, even for the whole earth, some portions of the year during which the increase of heat is in excess and other portions in which the loss of heat is in excess; in other words, "The total energy of the whole earth is probably subject to periodic variations within the year."

The fact that the average temperature of the lowest stratum of air over the whole earth is higher during the summer half-year of the northern hemisphere than during the winter half-year seems to agree with this idea.

But of course we must not forget that this *temperature* is by no means a measure of the total energy. On the contrary it is very probable that the changes of total energy of the whole surface of the earth including the atmosphere are not so large, by far, as one would expect from the change of the mean temperature of the lowest stratum of air.

For since the masses of water that in the course of a year are frozen to ice and afterward again melted are apparently much larger in the southern hemisphere than in the northern, and since the same is certainly true of the quantities of water that are evaporated and condensed—therefore a larger portion of the added energy is applied to the work of melting and evaporating during the summer of the southern hemisphere than during the summer of the northern hemisphere. Therefore even in the case of equal values of the total energy [of the two halves of the year] the mean temperature of the whole earth must be lower in the half-year containing the northern winter than in that containing the northern summer, since in the winter half-year of the northern hemisphere the larger portion of the added heat falls in [southern] regions in which the consumption of energy in [producing] changes of condition [snow and ice to water and vapor] is far greater than it ever can be in the northern hemisphere.

But only detail investigation can decide to what extent the compensation just indicated takes place or, in other words, whether and to what extent the total energy of the whole earth has an annual period.

Of course in this matter, as indeed throughout the whole of this class of investigations, one must be content with estimates, but certainly the problem itself is a striking example of the special question to which we are led by these general considerations.

In order to put this theorem into algebraic form it suffices to take a more general view of the equations (14) and (15) above given and slightly alter them.

We have indeed only to drop the index a that was introduced to show that we are considering a definite portion of the earth or atmosphere, and to add the horizontal line above in order to emphasize the fact that we are considering the boundary surface of the atmosphere, thus transposing the equation into the following:

$$\overline{\mathfrak{q}_T} \gtreqless \overline{\overline{\mathfrak{q}_T}} \text{ and } \overline{\mathfrak{q}_T} - \overline{\overline{\mathfrak{q}_T}} = \mathfrak{u}_{t_2} - \mathfrak{u}_{t_1} \quad . \; . \; . \; . \; . \; (17)$$

but must always remember that in the present case, where we are dealing with the whole earth, the difference $\mathfrak{u}_{t_2} - \mathfrak{u}_{t_1}$ which I will designate by $\mathfrak{u}_\varepsilon$ is always small in comparison with the quantities $\overline{\mathfrak{q}_T}$ and $\overline{\overline{\mathfrak{q}_T}}$.

Of course the equation (16) also holds good in this case after corresponding modifications; therefore if we put

$$\mathfrak{u}_{t_1} - \mathfrak{u}_{t_0} = \mathfrak{u}_{\tau_1} \text{ and } \mathfrak{u}_{t_2} - \mathfrak{u}_{t_1} = \mathfrak{u}_{\tau_2}$$

and

$$\tau_1 + \tau_2 = T$$

we must have

$$\mathfrak{u}_{\tau_1} = -\,\mathfrak{u}_{\tau_2}$$

that is to say, if we divide the year into two arbitrary intervals then the increase of the energy that the whole earth has gained in one of these intervals is equal to the loss that it has experienced in the other interval.

If now we imagine the beginning of the year so chosen and the division into intervals so devised that in one interval $\mathfrak{u}$ is always above and in the other always below the annual mean, then the year falls into two halves that are in general unequal, one of which we may call the warmer and the other the colder half respectively.

Since the insolation at different portions of the earth's surface shows remarkably great differences at all times of the year, being indeed zero at some places and times, whereas the radiation outward is always effective, and since, on the other hand, the total energy of the whole earth is only subject to small periodic changes within the year so that $\mathfrak{u}_\varepsilon$ is always small as compared with $\overline{\mathfrak{q}}$ and $\mathfrak{q}$, therefore we can so subdivide the whole boundary of the atmosphere that in one part the insolation is in excess but in the other part the radiation.

If we indicate by the index a the quantities that relate to the region where insolation is in excess, and by tbe index p the other region then the equation (17) can be rewritten as

$$\bar{q}_{a,\tau} - \bar{\bar{q}}_{a,\tau} = \bar{\bar{q}}_{p\tau} - \bar{q}_{p\tau} \pm u_{\varepsilon} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where u_{ε} is small with respect to the differences that enter both sides of this equation.

This theorem can be expressed as follows:

"At any moment of the year the earth's surface is divided into regions having insolation in excess and others having radiation in excess."

If we ignore the diurnal period, the equatorial zone always belongs to the region of excess of insolation but the limiting neutral lines are subject to important variations in the course of the year.

The proof of this theorem consists simply in this, that at all seasons of the year warm currents flow poleward from the tropics, whereas insolation and radiation remain apparently uniform in the tropics through the whole year, and therefore these currents can be maintained to only a very slight extent by any energy that may be stored up.

The polar regions belong to the region of excess of insolation although only during a small part of the year, since during midsummer of either hemisphere the corresponding polar region receives during a limited interval of time more heat from the sun than the regions of lower latitude or than those of the opposite hemisphere.

Hence, while the two neutral lines depart in the same direction from the average location that they occupy at the time of the equinoxes (in the northern spring time both move northward but in the southern spring southward), therefore the polar region enclosed by one of these lines diminishes steadily until it entirely disappears at midsummer.

If then the convection of heat from lower latitudes continues during the midsummers of the respective hemispheres, this can only be explained by the fact that the whole of the heat gained by excess of insolation and by convection is consumed in covering the loss of energy that the polar region had suffered during the winter half-year, and which had resulted in the formation of enormous masses of ice and the diminution of the vapor contained in the atmosphere.

Hence the equations of the neutral lines for any given moment of the year take the form

$$\Phi\ (+\beta, \lambda, t) = 0 \text{ and } \Psi\ (-\beta, \lambda, t) = 0$$

where for definite values of t the one or the other of these equations becomes useless since the line to which it refers vanishes entirely.

If the surface of the earth were perfectly homogenous and the earth's orbit circular the following equation between the functions ϕ and ψ would hold good

$$\psi\ (-\beta, \lambda, t) = \phi\ (-\beta, \lambda, t \pm \frac{T}{2})$$

that is to say, under this assumption the neutral line on either hemisphere would at any given moment have exactly the same location that it would occupy on the other hemisphere a half-year earlier or later. Also each hemisphere would belong to its region of excess of insolation for exactly the same intervals of time as the other.

Now an interchange of heat by convection takes place between the region of excess of insolation and that of excess of radiation just as in the annual average.

But the equations for this convection current are much more complex for short intervals of time than for the annual average, since in this latter case all quantities that refer to the storage of energy fall out, whereas for shorter intervals they play an important rôle.

In order to understand this we do best to subdivide the energy $\mathfrak{u}_\varepsilon$ in equation (18) into two portions $\mathfrak{u}_a$ and $\mathfrak{u}_p$, one of which relates to the insolation region and the other to the radiation region.

Thus that equation takes the form

$$\bar{\mathfrak{q}}_a - \bar{\bar{\mathfrak{q}}}_a - \mathfrak{u}_a = \bar{\bar{\mathfrak{q}}}_p - \bar{\mathfrak{q}}_p + \mathfrak{u}_p \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where the left-hand side of the equation represents the remnant of heat that remains after subtracting the radiated heat and the stored-up heat from the heat received by insolation in the insolation region.

Evidently this remnant must flow as a convection current to the radiation region.

The average [thermal] intensity of this current is

$$J_a = \frac{\bar{\mathfrak{q}}_a - \bar{\bar{\mathfrak{q}}}_a - \mathfrak{u}_a}{\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and this is the current that [arriving] in the radiation region partially replaces the loss due to the excess of radiation whilst the remaining excess of radiation is represented by actual loss of energy, i. e., by cooling, formation of ice, etc.

This formula differs in important respects from the equation (10) as before established for the whole year. Whereas in that the convection current depended only on the difference between the insolation and the radiation, here there is also considered the quantities of energy that are [in any way] received or lost in the region and within the given period of time.

Therefore it may theoretically be conceivable that the influence of the difference of radiation may be entirely balanced or even overcompensated by the storage of energy.

However, this is not now the case on our earth for the whole region of excess of insolation, since a flow of heat toward the winter half of the globe [by convection] is always taking place and, on the other hand, this makes itself felt in most incisive manner in the polar regions at the time when the sun has his highest altitude.

It is well known that even in midsummer warm currents flow from lower latitudes toward the poles, whereas cold air and cold water flow thence away, except where foehn-like phenomena make an exception in special localities.

Hence the convection current poleward continues even during that season of the year in which the pole receives more heat than any other point on the surface of the earth or of the boundary of the atmosphere.

Now imagine any line surrounding the pole over which this current is flowing, and let it serve as a dividing line between a polar portion and the remaining portion of the insolation region, which latter may therefore be designated as the equatorial region, and distinguish by the index $\mathfrak{p}$ all the quantities relating to the polar region, then for the intensity of the current $J_{\mathfrak{p}}$ we have the equation

$$J_{\mathfrak{p}} = \frac{\bar{\mathfrak{q}}_{\mathfrak{p}} - \bar{\bar{\mathfrak{q}}}_{\mathfrak{p}} - \mathfrak{u}_{\mathfrak{p}}}{\tau} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (21)$$

Since now the current flows toward the pole therefore $J_{\mathfrak{p}}$ must have the same sign as would belong to it if $\mathfrak{q}_{\mathfrak{p}}$ and $\mathfrak{u}_{\mathfrak{p}}$ were both equal to zero, that is to say, as if only radiation were effective within the dividing line. Therefore $J_{\mathfrak{p}}$ must be negative.

But since at the time of the summer solstice we must have

$$\bar{q}_{\mathfrak{p}} - \bar{\bar{q}}_{\mathfrak{p}} > 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

therefore we must also have

$$u_{\mathfrak{p}} > \bar{q}_{\mathfrak{p}} - \bar{\bar{q}}_{\mathfrak{p}}$$

Hence the excess of insolation, large as it is in the polar region at this season, still does not suffice to supply the demand for heat to increase the energy, i. e., that needed for melting the ice and evaporating the water.

It is not difficult to deduce other theorems from these and thus increase their number.

But these will suffice, since we have only intended to attain a general point of view in connection with which various individual investigations are to be conducted, and since the preceding theorems suffice we will proceed to their application.

The general views thus set forth show that there are essentially three points to which attention must be given in investigations concerning the economy of heat:

(1) Insolation and radiation, including reflection.

(2) Increase and diminution of energy over individual portions of the surface of the earth and in the atmosphere.

(3) Convection, or the transportation of heat by air and water.

The first of these subjects has already been studied by many and will therefore not here be made the subject of new investigations.

On the other hand, attention will be given to the two other headings which it seems to me offer less difficulty than the first, although as yet less attention has been given to them.

(III.) THE THERMAL EXCHANGE IN THE GROUND

During the warmer portions of the day and the year the ground absorbs heat which it again gives up during the colder portions. It therefore plays the part of an accumulator or reservoir which during special times stores up the energy that must be consumed again at other times.

In this case the energy occurs in its simplest form and therefore this investigation offers by far the least difficulties of all that refer to heat exchange.

If the ground contains no water (which, however, can only be the case approximately in rocks and in the desert) or if the water content remains unchanged, while at the same time its temperature does not fall below the freezing point, then the total stored up energy is present only in the form of heat that can be measured thermometrically.

If the ground contains water and if the temperature passes the freezing point in either one direction or the other, then the relations become more complicated, but still problems relating even to these cases are much simpler than most of the others that occur in reference to the subject here treated.

Moreover, as will be seen later, this matter [of freezing] does not at all come into consideration, at least in lower and middle latitudes, in the determination of the quantity of heat taken in and given out during an annual period and independent of the diurnal exchange.

To attain our present object the important matter is the solution of the two following questions:

(1) How great is the difference between the quantities of heat taken in and given out by a unit area of surface during a given interval of time, that is to say, how great is the increase or decrease of energy experienced by the earth beneath that unit of surface during this interval of time?

(2) How great is the difference between the maximum and minimum values of the energy present in this portion of earth during a given interval of time?

The reply to the first of these questions will give the energy stored in the earth during a given interval of time or the quantity present therein at any moment.

The reply to the second question gives us a measure of the efficiency of the ground as a regulator of heat, provided we choose the interval to be studied so that it includes a complete period of thermal-exchange, such as a whole day or a whole year.

The reply to these two questions is extremely simple, as will be shown immediately, since it only assumes a knowledge of the temperatures at different depths and of the capacity for heat of the unit volume of earth, the so-called volume capacity[12] whereas the conductivity of the earth as well as the radiation or emission at the surface do not come into play.

Moreover, in the solution of the second question, it suffices if we know the earth temperature for that day or season at which the temperature gradient in the highest layer of earth is zero.

Easy as it would be to answer these questions, and important as

---

[12] As distinguished from the specific capacity which relates to the unit mass.
C. A.

they are from the meteorological point of view, still the material for reply offered by a superabundance of observations of earth temperatures is extremely scarce, since in only very few cases has the volume capacity of the appropriate earth been directly determined and therefore the essential datum is missing.

The above questions will now be first answered theoretically and then an attempt be made to see how far the formula can be converted into numbers; also for simplicity it will first be assumed either that the temperatures remain always above the freezing point or that the earth is wholly free from water.

This being assumed the first of the two questions, i. e., the increase of energy contained in the ground within a given interval of time t_2 is answered by the following consideration:

Let C be the thermal capacity of the unit volume, h the distance of any point from the surface of the earth, reckoned positive downward, θ_1 the temperature of the earth at this point at the moment t_1, θ_2 the corresponding temperature at the moment t_2; imagine a prism cut from the ground beneath the unit surface, then an infinitely thin horizontal element of this prism having the thickness dh receives in the given interval of time the quantity of heat represented by

$$C\ (\theta_2 - \theta_1)\ dh$$

The quantity of heat received by the whole prism to the depth H, that is to say, the change of the energy within the prism results from the equation

$$u_2 - u_1 = \int_0^H C\ (\theta_2 - \theta_1)\ dh$$

or if C is constant

$$u_2 - u_1 = C \int_0^H (\theta_2 - \theta_1)\ dh \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

In this equation θ_1 and θ_2 are functions of h such that with increasing values of h they very rapidly approach toward equality, so that if great accuracy be not required the difference $\theta_2 - \theta_1$ may be assumed to be 0 when $H = 10$ meters even when t_1 and t_2 differ greatly. If we consider only a short interval of time, such as 24 hours, then we may assume that this limit is reached when $H = 1$ and can put $\theta_1 = \theta_2$ for this depth.

It we write equation (23) in the form

$$u_2 = C\int_0^H \theta_2\,dh - C\int_0^H \theta_1\,dh + u_1$$

and choose t_1 as the initial point for counting the time so that $t_1 = 0$ we may write

$$u_2 = C\int_0^H \theta_2\,dh + K$$

or briefly, by omitting the index

$$u = C\int_0^H \theta\,dh + K \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where K is a constant whose value depends on what we adopt as the zero of energy. Theoretically it would be most correct to adopt for this the absolute zero, but frequently it will prove advantageous to start from the zero point of the ordinary thermometer scale. Of course by doing this one may in certain cases obtain negative values for the energy, but this will not be objectionable so long as we are clear as to the meaning of this result.

The last given equation can also be written in the form

$$u = CH.\frac{1}{H}\int_0^H \theta\,dh + K \quad = CH\ \Theta + K$$

where we put

$$\frac{1}{H}\int_0^H \theta\,dh = \Theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

But this value Θ is simply the average temperature of a prism cut out of the ground to the depth H beneath the unit of surface, while CH is the so-called water equivalent of this prism if we adopt the expression used in calorimetry.

If now we designate by Θ_1 and Θ_2 the values of Θ corresponding to the times t_1 and t_2 we obtain

$$u_2 - u_1 = CH\ (\Theta_2 - \Theta_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

This equation may be interpreted thus:

The change in a given interval of time of the amount of energy contained in the ground beneath a unit of surface is equal to the change in the average temperature of the ground from the surface down to the depth at which the variations are inappreciable, multiplied by the water value of a prism cut from this ground from the unit surface down to the same depth.

Hence the energy stored in the earth attains its extreme values simultaneously with those of the average temperature of the ground if in determining these latter we consider the temperatures down to the depths at which the variations become inappreciable.

The equation (23) allows of a very simple geometrical presentation (see fig. 56).

Let the depths h be shown as ordinates counted positively downward, and the temperatures θ as abscissæ, then will the distribution of temperature in the ground at the moment t and down to the depth h be represented by the curve A_1 B_1 as in fig. 56.

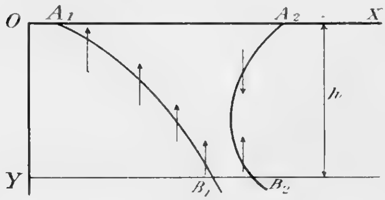


FIG. 56

If now the distribution of temperature is different at the moment t_2 and if it be represented by the curve A_2B_2 then the area A_1 $B_1B_2A_2$ which is included between the two curves and the axis of abscissæ and the line parallel to this at the depth h, and which may be designated by f, becomes at once a measure of the added quantity of heat, since

$$f = \int_0^h (\theta_2 - \theta_1)\, dh$$

At the same time this method of presentation gives immediately information as to the direction in which the movement of the heat is taking place *in* the different strata of the ground *at* the given times, since the course of the lines allows us to recognize immediately whether the temperatures increase or diminish downward. The direction of the flow of heat is shown by arrows in fig. 56.

Because of the great advantages offered by the consideration of this curve, I will give it a special name, the "tautochrone," since each such curve presents the temperatures that prevail at the given time at the different depths.

Incidentally it may be remarked that from these curves we can construct a remarkably instructive picture.

Assume that we have at different depths perfectly similar accurate thermometers, whose scale degrees have exactly the same length, and that these are placed horizontally, imbedded in the ground so that all the zero points lie vertically one above the other, then the curve connecting the ends of all the mercurial columns is the tautochrone for any given moment of time. See fig. 57.

Since the phase of the oscillations going on daily and annually in each layer changes from one stratum to the next, therefore the curves A_1 B_1 and A_2 B_2 intersect each other at special depths and generally speaking an infinite number of times; but since these curves steadily approach each other as the depth increases and almost coincide at very moderate depths, therefore we shall not often need to consider more than two such intersections.

Of course, in the computation of the total heat received and expended, the areas on each side of such intersecting points must be given opposite signs, as is shown by the signs inscribed in fig. 57.

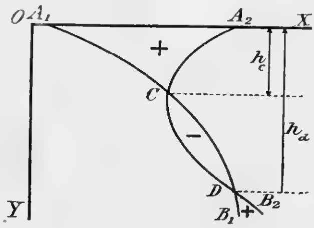


FIG. 57

But the consideration of these curves becomes especially valuable in that they allow us to recognize at once when the quantity of heat contained in the ground below any given horizontal plane attains a maximum or minimum.

Of course this is only the case when no heat passes through the plane in question neither in one direction nor the other, i. e., when the temperature gradient in this plane is zero or

$$\frac{d\theta}{dh} = 0$$

Therefore at this place the tangent to the curve representing the temperature is a vertical line.

If, therefore, we know only the daily mean temperatures for the upper layers of the earth we can then directly find the two days of the year on which the heat contained in the ground attains a maximum or a minimum by seeking for those days on which the above-mentioned condition is fulfilled, i. e., when the temperature curve is perpendicular to the earth's surface.

If then we also know for these days the distribution of temperature in the strata below, then the area between the tautochrones for these two dates gives directly a measure of the difference between the greatest and the smallest quantities of heat contained in the ground during the annual period; of course it is assumed that the volume capacity of the ground is known.

But this difference is the quantity of heat that is exchanged through the earth's surface within one year, omitting of course that which is exchanged within the diurnal periods and of which the remnant left over at the close of each day alone enters into this present computation.

"The consideration just expressed has led to the surprising result that for the determination of the annual exchange of heat it suffices to know the distribution of temperature in the ground at those dates of the year when the increase of heat changes to a decrease and vice versa."

In temperate latitudes these dates agree approximately with the equinoxes.

Of course the exchange of heat within the diurnal period can be determined in a perfectly analogous way.

We find the heat exchange within the diurnal period by selecting from the tautochrones for individual hours those two that are perpendicular to the earth's surface and then determine the area included between them or from the corresponding integral.

A determination of the moments of time at which this occurs, i. e., of the hours of the day at which the energy in the ground attains its maximum and minimum values is, of course, only possible where hourly observations for the upper strata of the ground are available or at least those for quite short intervals of time.

In general we can at present only state that the changes from increasing heat to diminishing heat occur some little time after sunrise and a rather longer time before sunset. At Pavlovsk[13] this occurs at the following hours: in December about 11 a.m. and shortly before 1 p.m.; in January after 11 a.m. and before 2 p.m.; in June after 5 a.m. and about 5:30 p.m.; in July about 5 a.m. and before 6 p.m., as shown by the fact that at these hours the difference between the temperature at depths of 0.01 meter and 0.02 meter changes its sign.

At Nukuss[14] these changes occur in January at about 8 a.m. and 4:30 p.m. but in July at 6 a.m. and 6 p.m.

---

[13] Leyst: "Bodentemperaturen," in Wild Repertorium für Meteorologie. Band XIII, No. 7, 1890.

[14] H. Wild: Ueber die Bodentemperatur in St. Petersburg und Nukuss. Wild Repertorium für Meteorologie. Band VI, No. 4, 1878.

Unfortunately these moments in the diurnal period can scarcely ever be determined very accurately since it is precisely in the upper strata of ground that most disturbances occur.

But circumstances are still more unfavorable to the determination of the quantity of heat that is exchanged during the diurnal period, since the volume capacity of the ground is subjected to continual variations and especially in these upper strata on account of their varying content of water.

Therefore in determining the annual exchange we do well to first leave the uppermost strata quite out of consideration and confine ourself to the determination of the quantity of heat that is exchanged through a plane lying somewhat below the surface, e. g., at a depth of 0.5 meter, and then correct the error thus incurred by an addition that will, however, intrinsically be less trustworthy than the other numbers.

All the views hitherto set forth rest on the assumption that we have to do either with a perfectly dry soil or else that the temperatures θ_1 and θ_2 on the Centigrade scale have the same sign.

This latter condition is always fulfilled in evaluating the annual exchange so long as we confine the investigation to regions where the ground is free from frost or ice at the time of the equinoxes.

If we wish to free ourselves from the above-mentioned restrictions and include also those cases in which $\theta_1 < 0$ and $\theta_2 > 0$ and where also the soil contains water, then we obtain the corresponding formulæ from the following considerations:

Let c be the volume capacity of the perfectly dry porous earth, x the water contained in a unit volume expressed as a fraction of the unit of mass, then for the volume capacity C of the saturated ground we have

$$C = c + x \text{ for } \theta > 0 \text{ Centigrade}$$

but for that of the frozen ground

$$C = c + 0.5\,x \text{ for } \theta < 0 \text{ Centigrade}$$

Moreover, the thawing out of a unit volume of frozen ground at 0° C. requires heat to the extent of $80x$ calories.

Now assume that in its initial condition at the time t_1 the ground is frozen to the depth H_1 and that corresponding to this we have $\theta_1 < 0$ for $h > H$ and $\theta_1 > 0$ for $h > H_1$—but that at the time t_2 the ground is completely free from ice and therefore $\theta_2 > 0$, then instead of equation (23) we have the following more complicated one:

$$\left.\begin{aligned} u_2 - u_1 = &- \int_0^{H_1} \left(c + \frac{x}{2}\right) \theta_1 \, dh + 80\,x\,H_1 + \int_0^{H_1} (c + x)\,\theta_2\, dh \\ &+ \int_{H_1}^{H} (c + x)\ (\theta_2 - \theta_1)\ dh = c \int_0^{H_1} (\theta_2 - \theta_1)\ dh\ . \\ &+ x \int_0^{H_1} \left(\theta_2 - \frac{\theta_1}{2}\right) dh + (c + x) \int_{H_1}^{H} (\theta_2 - \theta_1)\, dh + 80\,x\,H_1, \end{aligned}\right\} \quad . \ (27)$$

where, however, we have still always to remember that θ_2 is always positive, whereas in the first two integrals θ_1 occurs with the negative sign.

We can of course also represent this formula geometrically, but as the presentation thus obtained is by far not so simple and clear as above, where the temperatures are either wholly above or wholly below the freezing point, therefore we refrain from reproducing them here.

From these two expositions we see how very easy it is to determine the quantity of heat exchanged through the earth's surface if only one knows the course of the temperature at different depths as well as the volume capacity of the ground and for temperatures below freezing, as also the water content.

By so much the more it is to be regretted that there is such a remarkably small number[15] of series of observations of earth temperatures for which the volume-capacity of the ground is known from direct experimental determinations.

In a subsequent communication I will attempt to show how far the available observations can be utilized in order to determine numerically from them the annual and perhaps also in some cases the diurnal heat exchange in the ground for different places and under climatic conditions as various as possible.

---

[15] Such values for the actual soil in which the temperature observations were made, I have only been able to find in the memoir by Lord Kelvin (William Thomson "On the Reduction of Underground Temperature." Edinburgh. Trans. Vol. XXII; Part II, pp. 405–427. 1860) in which the determinations made by Forbes are discussed. The values there given are: trap rock of Calton Hill, 0.5283; sand from the observation station in the garden, 0.3006; sandstone of Craigleith, 0.4623.

At present I will restrict myself to communicating the tautochrones for Munich and for Nukuss.

Singer has deduced ten day means[16] from the observations of Prof. J. von Lamont at Munich extending over a period of 25 years, and therefore this series is especially appropriate for the determination of the dates on which the heat content of the ground is a maximum or a minimum and thence the determination of the annual heat exchange.

Unfortunately this series does not include observations at slight depths, so that the values for the upper layer of 1.29 meters must be extrapolated. I have performed this extrapolation graphically

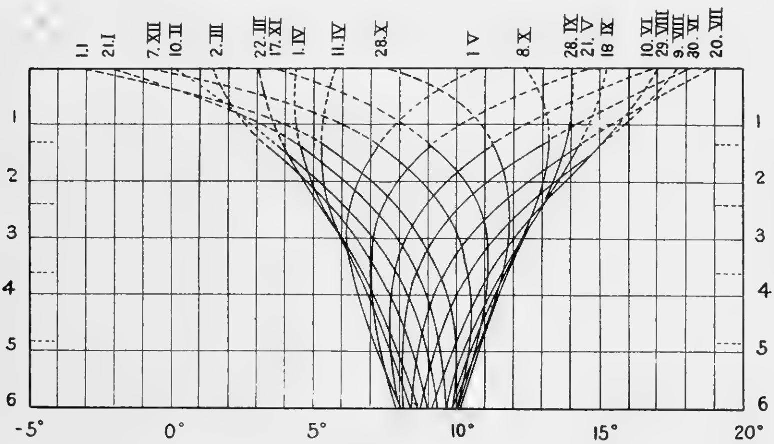


FIG. 58

but only in a rough way by the utilization of the observed temperatures of the air; but since the temperatures of the highest strata are subject to important disturbances, as will be shown from the observations at Nukuss, therefore it seemed unwise to expend much time and labor in attaining a result that would eventually not have the assurance of great accuracy.

Hence also in order to enable one to at once recognize this uncertainty in the diagram the extrapolated portions of the curves have been drawn in dashes.

Moreover, in fig. 58, the tautochrones have been drawn for intervals of 20 days only, although in the memoir of Singer the data are

[16]Lang and Erk. Met. Beobachtungen im Königreich Bayern im Jahr. 1889. Anhang, p. 10.

given for every ten days, since otherwise the diagram would be too crowded with lines.

However, I thought I must include the curves for April 1 and September 28, although they would remain unused in connection with the twenty-day interval beginning with January 1st, since these are the dates which among those contained in Singer's table seem to come nearest to the dates of the minimum and maximum heat content of the ground.

It is, moreover, quite possible that this condition is more precisely fulfilled on the 21st March and 22d September. I therefore intentionally adhere strictly to the data available without undertaking further numerical or graphical operations in order to avoid giving the appearance of a greater accuracy than I can truly assume it to have.

The great symmetry shown by both the curves, and easily recognized in fig. 58 by the crowding of the lines at the above given dates is remarkable, in consideration of the non-artificial and direct method of utilization of the data.

A special explanation of fig. 58 is hardly necessary since the scale of temperatures (Centigrade) is given below on the lower line and the scale of depths in meters on either end. The short dotted lines at either end give the depths in meters at which the thermometers were placed; therefore the intersection of the corresponding prolonged horizontal lines with the curves gives the points that were deduced from the observations.

The dates for which the curves are drawn are given at the top in Arabic numerals for the days and Roman numerals for the months.

As a contrast to the curve for Munich we give in fig. 59 the tautochrone for Nukuss.

This offers a special interest because this station situated on the Amu Darja represents a region of remarkably great insolation and radiation with very slight amount of precipitation.

Moreover the series of observations is one of the very few that give the material needed for the determination of the changes of temperature in the very highest strata.

However, the temperatures of these upper strata are in fact deduced from only one year of observation, whilst the numbers for the greater depths are the means of three years.

This fact is remarkably shown in fig. 59 which is constructed directly from the data published by Wild[17] without further interpolation or smoothing, in that the curves show the greatest irregularities in their upper portions.

This irregularity is easily explained, since on account of the many disturbances that we encounter just beneath the surface of the ground, we can only expect fairly reliable mean values from many years of observations made at short intervals of time.

Since we have only monthly averages for Nukuss we must regard it as purely accidental that among the tautochrones constructed from these values there are any that are exactly perpendicular to the surface of the earth and correspond therefore to the limiting values of the energy stored up in the ground. Such cases would in fact assume that the times at which these extremes occurred fell quite near the middle of the two months respectively.

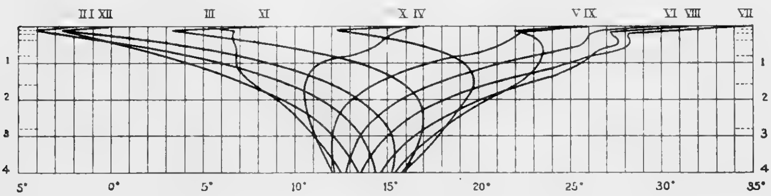


FIG. 59

If, however, we consider the values for the very highest strata as too uncertain and fix our attention first on the curves from 0.4 meter downward we find then March and September as the months of least and greatest heat stored in the ground. But it looks as if the September curve does not correspond to the full maximum, although in August the maximum is not yet attained.

This seems to suggest that in Nukuss the increase of heat comes to an end and the decrease begins before the autumnal equinox (September 21), if we may be justified in drawing such a conclusion from averages that represent so few years of observation.

If now on the basis of these considerations we actually compute the annual heat exchange for Munich and Nukuss by taking as a base the earth temperatures of April 1st and September 28th at Munich but the monthly means for March and September at Nukuss, we find the following approximate maximum values of $u_2 - u_1$

For Munich 36 C_m

For Nukuss 48 C_n

[17] H. Wild: Ueber die Bodentemperatur in St. Petersburg und Nukuss. Wild Repertorium, VI, No. 4, 1878, pp. 45, 46.

where C_m and C_n indicate the quantities of heat that are required at Munich and Nukuss respectively to warm up the unit volume of the respective soils by 1° C.

Unfortunately we are not able to say anything as to the values of these quantities except that they can scarcely be smaller than 300 or greater than 600.[18]

However, the numbers 35 and 48 as just given for Munich and Nukuss are still affected with great uncertainty since the data for Munich begins first at 1.29 meters depth whilst those for Nukuss end at 4 meters, so that in one case we must extrapolate upward, in the other case downward. In fact this extrapolation downward is necessary in both cases although to a less extent in the series for Munich.

As values of the diurnal exchange we obtained for Nukuss 0.56 C_n in January and 1.5 C_n in July, but again of course only a crude approximation.

However, these numbers suffice to show the general magnitude of the part that the solid ground plays as a reservoir of heat or a regulator of temperature.

Thus, in order to deal with round numbers assume briefly that $C_n = C_m = 500$, then would the quantity of heat exchanged within the annual period suffice to evaporate a layer of water 30 millimeters deep at Munich and 40 millimeters deep at Nukuss.

Therefore, compared with the depth of the total rainfall which amounts to 800^{mm} in Munich but only 85^{mm} in Nukuss, the result is that the quantity of heat absorbed into the ground during the warm season at Munich but given out again during the cold season is scarcely 1/26th part of that required to re-evaporate the annual precipitation, and even in Nukuss, the driest region of the whole Euro-Asian continent, it is not one-half.

On the other hand, the quantity of heat exchanged at Nukuss within the diurnal period is much larger than required for the evaporation of the average daily rainfall at that place.

Of course it must not be forgotten that the heat used in evaporation goes partly to maintain the temperature of the upper strata, so that the quantities of heat exchanged within the ground must on this account appear somewhat smaller than they really are.

[18] See von Liebenberg: Ueber den gegenwärtigen Stand der Bodenphysik. Wollny, Forschungen., Vol. I, 1878, p. 3. And further C. Lang: Wärmekapazität der Boden-konstituanten. Forschungen I, p. 109. Compare also Ad. Schmidt: Schriften d. physik-oekonomische Gesellschaft zur Königsberg in Preussen, XXXII, 1891, p. 123.

The investigations carried out in the previous section of this memoir have led to the following results:

"The quantities of heat exchanged in [i. e., entering and leaving] the solid earth are in general small as compared with those that are required to evaporate the [local] precipitation.

"In middle latitudes the knowledge of the earth temperatures in spring and autumn, in connection with the knowledge of the capacity for heat of the unit volume of the local soil, suffices for the determination of the quantities of heat interchanged in the ground in the course of the annual period."

In these theorems, however, the following points must be considered:

The earth temperature must be determined at least by decades, still better by pentades during these seasons, if not for the whole year:

The observations should extend to at least 6 meters in depth beginning at 5 centimeters befow the surface:

The temperature should be determined for at least three points within the upper meter arranged so that the successive distances of the thermometers diminish with approach toward the surface.

"Hourly observations of those thermometers on which the diurnal period exerts an influence, at least during the hours after sunrise and before sunset, are needed for the determination of the diurnal exchange of heat."

Continuous registrations for these strata are still more desirable, but these will attain their full importance only when it at the same time becomes possible even at large intervals of time to make continuous determinations of the heat capacity of the unit volume in these strata or at least to keep informed of their water content.

In general all measurements of earth temperatures first acquire their true value when the thermal capacity of the unit volume of the local soil with its average moisture is directly determined.

It is very desirable indeed that for all places for which we already have observations of earth temperatures such determinations be made as supplementary thereto.[19]

---

[19] Relative to the exchange of heat in the ground compare also the following memoirs by J. Schubert: Der jährliche Gang der Luft und Boden-temperatur im Freien und im Waldungen und der Wärme-austausch im Erdboden. Berlin, 1900. As also Der Wärme-austausch im festen Erdboden, im Gewässern und in der Atmosphäre. Berlin 1906. [Note added 1906, W. v. B.]

XX

ON CLIMATOLOGICAL AVERAGES FOR COMPLETE SMALL CIRCLES OF LATITUDE

BY PROF. DR. W. VON BEZOLD

[*Sitzungs Berichte der Berliner Akademie für 1901, pp. 1330–1343. Translated from Gesammelte Abhandlungen, Berlin, 1906, pp. 357–370*]

In a previous memoir[1] I have referred to the fact that in the tabular or graphic presentation of average values for the complete parallel-circles it is not advantageous to choose the geographical latitude as argument or as abscissa. By this method of presentation which has hitherto been exclusively employed we obtain a picture in which the polar regions are relatively too prominent.

It is well known that the zones included between small circles having the same difference of latitude correspond to very different areas according as they lie in high or low latitudes.

In a table progressing by equal angular differences the numbers between the equator and latitude 30° occupy only one-half as much space as those relating to the higher latitudes 30–90°, whereas that part of the earth's surface between thirty degrees and the pole is not larger than the zone between the equator and 30°.

A table arranged in steps of 10° gives to the zone from 0° to 10° only the same space as that given to the polar cap between 80° and 90°, whereas the former occupies more than eleven times the area of the latter.

Similarly a graphical presentation in which the geographic latitudes are chosen as abscissæ produces a wholly distorted image from which one can obtain a correct idea only after careful meditation. But the matter is entirely changed when we introduce the sine of the geographic latitude as argument or as abscissa. When this is done then the same tabular differences, i.e., equal differences, of the arguments or equal distances on the axis of abscissæ, correspond to zones of equal areas on the surface of the earth and the separate values or ordinates appear to have the weight that

[1] No. XIII of this collection of translations.

naturally belongs to them, independent of course of the uncertainty that may affect individual numbers.

"Correct average values can now be deduced at once by simple mechanical quadratures from the data of the table or from the ordinates."

In the above-quoted memoir I have already referred to these properties of the method hitherto employed and of the one here recommended, except as to this last-mentioned point.

This idea will now be further developed and applied to various meteorological elements, and it will be shown how simply the connection between the corresponding average values can be perceived and what special considerations come to light almost spontaneously.

This much being premised I now give the annual average values of insolation, temperature and pressure of the air, cloudiness and precipitation, first in tabular and then in graphic form arranged according to the sine of the geographic latitude.

As fundamental data I use the average values given in the ordinary way [for degrees of latitude]; for the insolation I use the values computed by Meech;[2] for the temperature of the air I use those given by Spitaler and Batchelder;[3] for the atmospheric pressure, the numbers given by W. Ferrel; for the amount of precipitation, the figures given by John Murray, and finally for the cloudiness, those given by Svante Arrhenius deduced from the charts of Teisserenc de Bort; all of which are found collected in Hann's "Klimatologie," p. 217 [or Ward's translation, p. 100].

From these values by very careful graphic interpolation the values were deduced that correspond to the series of values 0.05, 0.10 . . . . . 0.95 of the sine of the latitude.

The values thus obtained are found collected in table 1 and represented by curves in fig 60. On the other hand, fig. 61 gives curves whose ordinates are the arithmetical averages of the pairs of values belonging to equal north and south latitudes. These latter averages I call "holospherical" to avoid any misunderstanding, while the two values belonging to each definite circle of latitude I call "hemispherical." I will refer to this point in a subsequent paragraph.

---

[2] See Hann: Handbuch der Klimatologie; 2d edition, vol 1, p. 103. [See also L. W. Meech: "On the Relative Intensity of the Heat and Light of the Sun." Smithsonian Contributions, IX, Washington 1857, or R. de C. Ward's translation of Hann's Handbook of Climatology, 1903, p. 100].

[3] Hann: Handbuch, p. 200 [or Ward's translation].

Table I. Fundamental table of mean values for each circle of latitude *

Geog. Lat.		Insolation		TEMPERATURE: Spitaler		TEMPERATURE: Batchelder		Pressure		Precipitation		Cloudiness	
Sin φ	φ	D Days	ΔD	t °C	Δt	t °C	Δt	β mm	$\Delta\beta$	m cm.	Δm	n %	Δn
1.00	90° 0′	151.6	17.6	-20.0	8.7	-20.0	8.5	—		—		—	
0.95	71 48	169.2	20.6	-11.3	6.7	-11.5	6.6	758.7	-0.3	35	—	—	—
0.90	64 9	189.8	25.5	- 4.6	5.0	- 4.9	5.1	58.4	+0.4	40	+ 5	60	+1
0.85	58 13	215.3	21.7	0.4	3.1	0.2	3.2	58.8	+1.1	50	+10	61	-1
0.80	53 8	237.0	18.6	3.5	3.3	3.4	3.8	59.9	+1.0	58	+ 8	60	-3
0.75	48 35	255.6	16.6	6.8	3.8	7.2	3.4	60.9	+0.7	58	0	57	-4
0.70	44 26	272.2	14.5	10.6	3.1	10.6	2.9	61.6	+0.4	55	- 3	53	-3
0.65	40 38	286.7	12.7	13.7	2.4	13.5	2.5	62.0	0.0	53	- 2	50	-3
0.60	36 52	299.4	11.6	16.1	2.2	16.0	2.2	**62.0**	-0.1	53	0	47	-3
0.55	33 22	311.0	10.0	18.3	2.0	18.2	2.0	61.9	-0.2	55	+ 2	44	-2
0.50	30 0	321.0	9.1	20.3	1.9	20.2	1.7	61.7	-0.6	60	+ 5	42	-3
0.45	26 45	330.1	7.5	22.2	1.7	21.9	1.5	61.1	-0.8	64	+ 4	39	-1
0.40	23 34	337.6	6.7	23.9	1.5	23.4	1.3	60.3	-1.0	69	+ 5	38	+1
0.35	20 30	344.3	5.5	25.4	0.6	24.7	1.1	59.3	-0.6	80	+11	39	+3
0.30	17 28	349.8	5.0	26.0	0.2	25.8	0.7	58.7	-0.6	109	+29	42	+3
0.25	14 29	354.8	3.8	26.2	0..2	26.5	0.5	58.1	-0.2	159	+50	45	+3
0.20	11 32	358.6	3.3	26.4	0.0	27.0	0.1	57.9	0.0	185	+26	48	+4
0.15	8 38	361.9	1.7	**26.4**	-0.1	**27.1**	-0.1	57.9	0.0	195	+10	52	+2
0.10	5 44	363.6	1.2	26.3	-0.2	27.0	-0.2	57.9	+0.1	196	+ 1	54	+3
0.05	2 52	364.8	0.4	26.1	-0.2	26.8	-0.2	58.0	0.0	**196**	0	57	+1
0.00	0 0	**365.24**	- 0.4	25.9	-0.2	26.6	-0.2	58.0	+0.3	195	- 1	58	+2
-0.05	- 2 52	364.8	- 1.2	25.7	-0.3	26.4	-0.2	58.3	+0.2	191	- 4	**60**	0
-0.10	- 5 44	363.6	- 1.7	25.4	-0.2	26.2	-0.3	58.5	+0.3	186	- 5	60	-2
-0.15	- 8 38	361.9	- 3.3	25.2	-0.4	25.9	-0.4	58.8	+0.7	178	- 8	58	-2
-0.20	-11 32	358.6	- 3.8	24.8	-0.6	25.5	-0.6	59.5	+0.7	162	**-16**	56	-3
-0.25	-14 29	354.8	- 5.0	24.2	-0.8	24.9	-0.7	60.2	+0.8	130	-32	53	-2
-0.30	-17 28	349.8	- 5.5	23.4	-0.7	24.2	-1.0	61.0	+0.8	97	-33	51	-3
-0.35	-20 30	344.3	- 6.7	22.7	-1.2	23.2	-1.4	61.8	+0.8	71	-26	48	-2
-0.40	-23 34	337.6	- 7.5	21.5	-1.2	21.8	-1.6	62.6	+0.6	65	+ 6	46	-1
-0.45	-26 45	330.1	- 9.1	20.3	-1.8	20.2	-1.9	63.2	+0.3	65	0	45	+1
-0.50	-30 00	321.0	-10.0	18.5	-2.3	18.3	-2.0	**63.5**	-0.5	66	+ 1	46	+3
-0.55	-33 22	311.0	-11.6	16.2	-2.3	16.3	-2.1	63.0	-1.3	72	+ 6	49	+4
-0.60	-36 52	299.4	-12.7	13.9	-2.4	14.2	-2.4	61.7	-1.8	83	+11	53	+4
-0.65	-40 38	286.7	-14.5	11.5	-2.3	11.8	-2.6	59.9	-2.7	97	+14	57	+4
-0.70	-44 26	272.2	-16.6	9.2	-2.5	9.2	-2.9	57.2	-3.2	110	+13	61	+4
-0.75	-48 35	255.6	-18.6	6.7	-2.5	6.3	-3.1	54.0	-4.0	116	+ 6	65	+4
-0.80	-53 8	237.0	-21.7	4.2	-3.0	3.2	-3.3	50.0	-5.5	113	- 3	69	+5
-0.85	-58 13	215.3	-25.5	1.2	—	-0.1	—	44.5	-4.2	105	- 8	74	—
-0.90	-64 9	189.8	-20.6	—	—	—	—	40.3	—	95	-10	—	—
-0.95	-71 48	169.2	-15.6	—	—	—		—	—	—	—	—	—
-1.00	-90	151.6		—		—				—	—	—	

*The numbers here given under *D* and *t* often differ somewhat from those given in the above quoted former edition of this memoir. The reason for this lies in the fact that the interpolations have been executed much more carefully for this table than in the memoir XIII above referred to, where only an approximate statement was desired.

The tables and curves relating to the individual elements in these two methods of presentation are so arranged as to bring out most clearly the relations between their relative progress. In order to make this also prominent in the tables the more important extreme values are set in heavy-faced type.

In the diagram also the ordinates of the first three elements which are in their nature positive have been plotted positively upward, while the two others are plotted positively downward [as shown by the scales at the sides of the diagrams]. Finally the scale of ordinates is so chosen that the relationship between the neighboring curves strikes the eye at once.

No further explanations of table I seem necessary. It need only be added that the insolation D is expressed in multiples of that for an average day at the equator; the barometric pressure (β) is in millimeters; the temperature of the air (t) in Centigrade degrees; the cloudiness (n) is in percentages and the rainfall or equivalent depth of melted snow (m) is centimeters.

The sines of the geographic latitude are used as abscissæ, as already mentioned in the introduction. The semicircle at the base of the figure can be considered as one-half of an orthogonal projection of a diminutive globe whose axis $S.\ N.$ lies horizontally and on which parallels of latitude are drawn for each 10°.

Hence it suffices to prolong any ordinate of this figure down to the periphery of this semicircle in order to at once perceive the latitude to which it belongs. This figure also elucidates the method by which the interpolation was carried out.

The numbers on the side of the diagram give the values of the ordinates for the individual curves. Their significance is easy to understand by means of the attached letters and symbols, so that the connection between these numbers and the accompanying curves is seen at once.

Moreover, the portion of the network projecting above the coördinates and leading up to the inscribed numerals is drawn in like the corresponding curves, but rather feebler.

If now we consider these curves we at once derive the very comforting impression that our knowledge of the distribution of the most important meteorological elements is far more complete than would be suspected from the ordinary method of presentation.

That portion of the polar regions for which the averages of temperature and pressure for whole circles of latitude can only be formed by bold extrapolations amounts to scarcely one-tenth of the whole surface of the earth, and even for cloudiness and precipitation this is true to very nearly the same extent, at least as concerns the principal features of this distribution in latitude.

Moreover, we see from the tables and especially from the diagrams that the distribution of the most important meteorological elements, which of course depends primarily on the insolation, is modified by the distribution of atmospheric pressure, and this is the most important matter.

The curve of average temperatures has the greatest similarity with that of the theoretical sums of insolation, provided the scale of ordinates be properly chosen.

But whereas the insolation by its very nature is accurately symmetrical on both sides of the equator with a maximum at that circle, the maximum of the temperature curve is pushed into the northern hemisphere. At the same time a second symmetrically located but much feebler maximum in the southern hemisphere is indicated by the change in the differences, i. e., in the differential quotients.

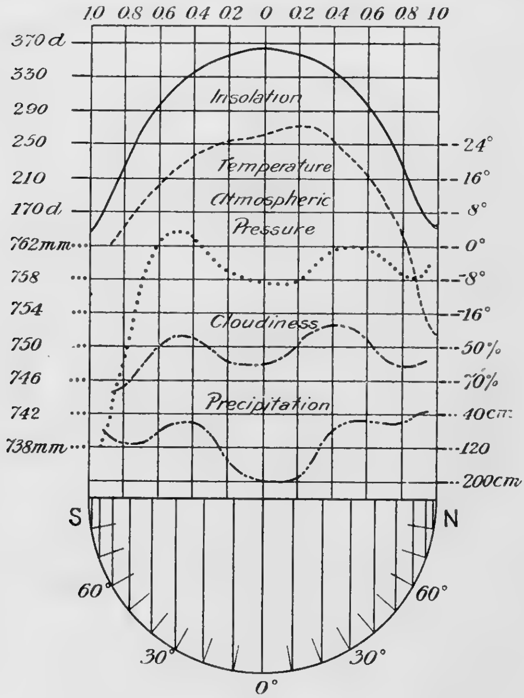


FIG. 60. HEMISPHERIC AVERAGES.

This peculiarity in the curve of average temperatures for whole circles of latitude is still more striking when we seek to approximately eliminate the influence of the unequal distribution of water and land in the two hemispheres by uniting into one arithmetical mean the two values corresponding to equal north and south latitudes.

By this process which has already been once used by my advice, by E. Sella,[4] we obtain average values that, as already remarked, may be designated as "holospheric" in contradistinction to the ordinary "hemispheric," which apply only to the latitude circles of either hemisphere alone.

[4] See Met. Zeit. XIII, pp. 161–166, 1876.

Doing this and applying a similar method to the other elements we obtain the curves given in fig. 61.

In this figure it is still more clearly seen or at least suggested, that the maximum of the insolation curve is broken into two separate maxima in the temperature curve. This separation becomes more striking if we imagine the temperature curve as formed of two superposed systems, one of which, the simpler curve, has only one maximum at the equator; the second superposed on that would therefore show two clearly separated maxima.

If we pass to the next curve in fig. 60, that for atmospheric pressure, we perceive easily the two well-known maxima first pointed out by Ferrel. The difference between this present method of presentation and the ordinary method such as we find, for instance, in the "Lehrbuch" of Sprung consists essentially in the fact that the maxima are separated farther apart and that the regions of low pressure in the higher latitudes are compressed to smaller spaces. However, the ordinary method of arrangement of the tables and graphic presentation has one advantage, not to be underestimated especially in the investigation of the average distribution of pressure, since the differences in respect to atmospheric pressure for equal increase of latitude are simply proportional to the gradients toward the pole.

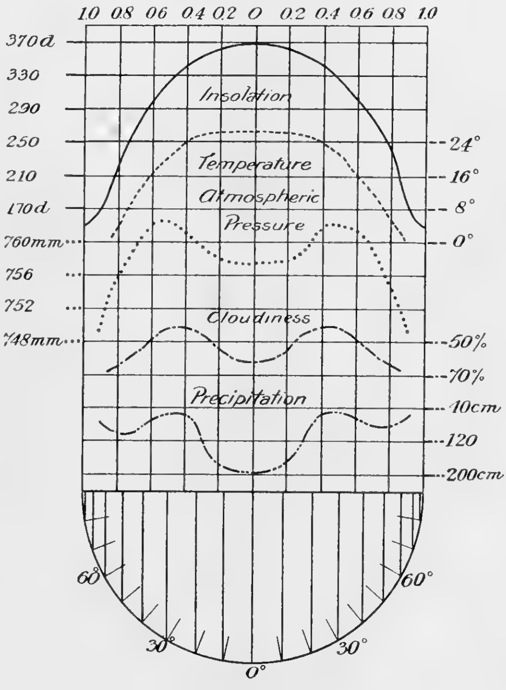


FIG. 61. HOLOSPHERIC AVERAGES.

In the present method of presentation the inclination of the curves to the axis of abscissæ gives directly and most appropriately an idea of the magnitude of the gradients.

In the diagram fig. 60, as already stated, the two maxima of pressure are separated farther from each other than in the older method, that is to say, we perceive that the zone between these two circles of maximum pressure covers more than one-half of the whole surface of the earth, whereas in the temperature curve the zone between the maxima covers only about three-tenths of the earth's surface.

Proceeding further in our study of fig. 60 we next consider the curves of mean cloudiness and of precipitation. As these two elements diminish with increasing pressure, therefore, as already mentioned, for the sake of comparability of the curves the ordinates are drawn positive downward.

This method of collating the different elements reveals the connection existing between them in truly surprising manner and clearly brings out the great importance of Ferrel's zones of atmospheric pressure.

Indeed I might go so far as to recommend the considerations here established as the starting point for lectures on Climatology, and as a method of passing gradually from the so-called "solar climate" to the actual local conditions by some scheme that keeps the average values of whole latitude circles plainly in view.

By always choosing the sine of the latitude as the argument, as has been done here, we at once attain correct ideas as to the weight that is to be attached to the individual values in considering the economy of nature, but independent of course of any uncertainty that may attach to them by virtue of the method of determination and which will gradually diminish.

Before I go further and draw certain conclusions relative to the temperature curve, I first give at the end of this section the table in accordance with which fig. 61 is drawn, to which I will only add that the numbers interpolated from Spitaler and Batchelder are indicated by *S* and *B*.

On examining these numbers one is surprised that the averages of precipitation and cloudiness, resting on rather feeble observational bases, when combined into holospherical averages show a remarkably regular progression.

After these general remarks which refer only to the presentation in general as also to the average values for the whole year, still further conclusions will be drawn from the whole diagram.

As already remarked, first of all we see the great similarity that exists between the curves of insolation and temperature as drawn on the scale here adopted, and which at once shows that these two quantities can be connected by one empirical formula at least throughout a considerable portion of their courses.

Table II. Holospherical averages

Sin φ	Insolation. Days.	Atmospheric Pressure. mm.	TEMPERATURE.		Precipitation cm.	Cloudiness. per ct.
			$S.$ ° C.	$B.$ ° C.		
0.9	189.8	749.3	—	—	67	—
0.85	215.5	52.6	0.8	0.1	77	67
0.8	237.0	54.9	3.8	3.3	85	63
0.75	255.6	57.4	6.7	6.7	87	61
0.7	272.2	59.4	9.9	9.9	82	57
0.65	286.7	60.9	12.6	12.6	75	53
0.6	299.4	61.8	15.0	15.1	68	50
0.55	311.0	62.4	17.2	17.7	63	46
0.5	321.0	62.6	19.4	19.5	63	44
0.45	330.1	62.1	21.4	21.0	64	42
0.4	337.6	61.4	22.7	22.6	67	42
0.35	344.3	60.5	24.0	23.9	75	44
0.3	349.8	59.8	24.7	25.0	98	46
0.25	354.8	59.2	25.2	25.8	144	49
0.2	358.6	58.9	25.6	26.2	173	52
0.15	361.9	58.4	25.7	26.5	186	55
0.1	363.6	58.2	25.9	26.6	191	57
0.05	364.8	58.1	25.9	26.6	193	58
0.0	365.2	58.0	25.9	26.6	195	58

In fact one can with surprising accuracy compute the average temperature t for any latitude circle from the number D of any thermal days corresponding to that circle by the formula

$$t = \frac{D}{5.2} - 42.5$$

The following table III shows how far this approximation holds good, as it gives not only the computed values of t but also under S those given by Spitaler and under B those given by Batchelder, united into their respective holospheric averages. The adjoining columns give the differences between these averages and the computed t.

Table III. Comparison of holospheric values deduced from observations and those computed by the formula

φ	t	$S.$	$t-S.$	$B.$	$t-B.$	$S.-B.$
0°	27.7	25.9	1.8	26.6	1.1	−0.7
10	26.8	25.7	1.1	26.4	0.4	−0.7
20	23.9	24.1	−0.2	24.1	−0.2	0.0
30	19.2	19.4	−0.2	19.2	0.0	+0.2
40	13.0	12.9	0.1	13.0	0.0	−0.1
50	5.5	5.7	−0.2	5.5	0.0	+0.2
60	−2.5	−0.3	−2.2	−1.1	−1.4	+0.8

In this table, I first use the ordinary method of presentation in which one proceeds by equal differences of latitude in order to simplify the comparison between the values computed by this formula and those found by Spitaler and Batchelder. Later I will give the similar table with the sine as the argument.

From latitudes 20° to 50°, or 0.6 of the whole surface of the earth, this table shows a surprising agreement between the computed values and those deduced by Spitaler and Batchelder from observations and here united into general averages for the two hemispheres or into holospheric averages. The error is nowhere more than 0.2° C. within the given latitudes.

It is only in the equatorial zone and in the higher latitudes that the differences become larger and that too for explanable reasons, so that thereby the empirical formula acquires higher interest.

In the equatorial zone the computed temperatures are higher than the observed. This is undoubtedly a consequence of the larger cloudiness, since this depresses the temperatures in lower latitudes, as also a consequence of the above mentioned influence[5] of the complex convection whereby heat is carried from this zone into the two surrounding belts, so that the temperatures in the equatorial zone proper must be lower, but those in the two neighboring zones higher than would be suspected from the insolation conditions.

Since in higher latitudes the cloudiness hinders the terrestrial radiation, therefore to this circumstance we must ascribe the fact that the temperatures beyond 50° of latitude are higher than would be expected from this formula.

Moreover, the last column of table III containing the differences between the values deduced from Spitaler and Batchelder for similar latitudes shows that these differences are of about the same size as the departures of the numbers computed by the formula from those derived by these authors from observations, excepting for the above-explained systematic differences in the equatorial zone.

Hence the formula reproduces the actual existing conditions with surprising accuracy.

The result of this study may therefore be expressed as follows:

"A change of 5.2 thermal days in passing from one parallel of latitude to another corresponds to a change of 1° C. in the mean temperature of the whole circle of latitude."

[5] See Mechanics of the Earth's Atmosphere, 1891, p. 243, and this present collection of translations, No. XIII.—C. A.

If we compare this result of the formula, not as above with the holospheric value $t = \frac{1}{2}(t_{+\varphi} + t_{-\varphi})$, but directly with the numbers given by Spitaler and Batchelder for the individual latitude circles, then the departures between computation and observation are of course somewhat larger but still they are always within moderate limits.

We see this from the accompanying table IV, which also provides an interesting survey of the different relations of the two hemispheres.

Table IV. Comparison between the hemispheric values deduced from observations and the computed values

φ.	t	$S.$	$t-S.$	$B.$	$t-B.$	$S-B.$
90° N.	−13.4	−20.0	6.6	−20.0	6.6	0.0
80	−12.4	−16.5	4.1	−16.9	4.5	+0.4
70	− 9.2	−9.9	0.7	−10.2	1.0	+0.3
60	− 2.5	−0.8	−1.7	− 1.2	−1.3	+0.4
50	5.5	5.6	−0.1	5.8	−0.3	−0.2
40	13.0	14.0	−1.0	13.9	−0.9	+0.1
30	19.2	20.3	−1.1	20.2	−1.0	+0.1
20	23.9	25.6	−1.7	24.9	−1.0	+0.7
10	26.8	26.4	0.4	27.1	−0.3	−0.7
0	27.7	25.9	1.8	26.6	1.1	−0.7
−10	26.8	25.0	1.8	25.7	1.1	−0.7
−20	23.9	22.7	1.2	23.3	0.6	−0.6
−30	19.2	18.5	0.7	18.3	0.9	+0.2
−40	13.0	11.8	1.2	12.2	0.8	−0.4
−50	5.5	5.9	−0.4	5.3	0.2	+0.6
−60° S.	− 2.5	0.2	−2.7	−1.1	−1.4	+1.3

If now we use the sine as the argument we obtain the following tables V and VI.

Table V. Comparison between the holospheric values deduced from observations and the computed values

Sin φ	t	$S.$	$t-S.$	$B.$	$t-B.$	$S.-B.$
0.0	27.7	25.9	1.8	26.6	1.1	−0.7
0.1	27.4	25.85	1.55	26.6	0.8	−0.7
0.2	26.5	25.6	0.9	26.2	0.3	−0.6
0.3	24.8	24.7	0.1	25.0	−0.2	−0.3
0.4	22.4	22.7	−0.3	22.6	−0.2	0.1
0.5	19.2	19.4	−0.2	19.5	−0.3	−0.1
0.6	15.1	15.0	0.1	15.1	0.0	−0.1
0.7	9.8	9.9	−0.1	9.9	−0.1	0.0
0.8	3.1	3.8	−0.7	3.3	−0.2	0.5
0.9	− 6.0	—	—	—	—	—
1.0	−13.4	—	—	—	—	—

If we compare the numbers in table V with those before given then the departures between the values deduced from observation and those computed by the formulæ give a similar picture. But the positive differences in the lower latitudes stand out more prominently in correspondence to the greater surface that the equatorial regions occupy so that the latter receive their proper weight only in this method of collation.

Similar remarks apply to table VI which now allows us to recognize the differences of the temperatures of the two hemispheres in a manner corresponding to the true importance of the individual zones.

The two tables V and VI show in admirable manner the systematic departures from the formula that are caused by the dissimilar distribution of water and land over the two hemispheres.

The last columns of these tables are also worthy of notice as they also show that there are differences between the values deduced by Spitaler and Batchelder that, so far as concerns magnitude, are of the same order as the differences between the computation and observation of holospheric means. We thus perceive how perfectly the formula is adopted to represent the average distribution of temperature.

Table VI. Comparison between the hemispheric values deduced from observation and the computed values

Sin φ	t	$S.$	$t-S.$	$S.$	$t-B.$	$S.-B.$
1.0	−13.4	−20.0	6.6	−20.0	6.6	0.0
0.9	− 6.0	− 4.6	−1.4	− 4.9	−1.1	0.3
0.8	3.1	3.5	−0.4	3.4	−0.3	0.1
0.7	9.8	10.6	−0.8	10.6	−0.8	0.0
0.6	15.1	16.1	−1.0	16.0	−0.9	0.1
0.5	19.2	20.3	−1.1	20.2	−1.0	0.1
0.4	22.4	23.9	−0.9	23.4	−1.0	0.5
0.3	24.8	26.0	−1.2	25.8	−1.0	0.2
0.2	26.5	26.4	0.1	27.0	−0.5	−0.6
0.1	27.4	26.3	1.1	27.0	0.4	−0.7
0.0	27.7	25.9	1.8	26.6	1.1	−0.7
−0.1	27.4	25.4	2.0	26.2	1.2	−0.8
−0.2	26.5	24.8	1.7	25.5	1.0	−0.7
−0.3	24.8	23.4	1.4	24.2	0.6	−0.8
−0.4	22.4	21.5	0.9	21.8	0.6	−0.3
−0.5	19.2	18.7	0.5	18.3	0.9	0.4
−0.6	15.1	13.9	1.2	14.2	0.9	−0.3
−0.7	9.8	9.2	0.6	9.2	0.6	0.0
−0.8	3.1	4.2	−1.1	3.2	−0.1	+1.0

These last columns also show that the departures between the numbers found by the two authors progress quite regularly. From

53° to 17° north latitude the numbers given by Batchelder are almost invariably smaller than those by Spitaler but are larger in the equatorial zone and in the southern hemisphere.

Especially do we recognize the value of the method of presentation here developed when we apply it not only to annual averages but to specific small periods of time.

Thus, for instance, the curves given by Wiener[6] and which have been copied in educational works[7] showing the distribution of insolation on March 20, April 12, May 5, and June 21, present a very different picture after being redrawn as shown in fig. 62.

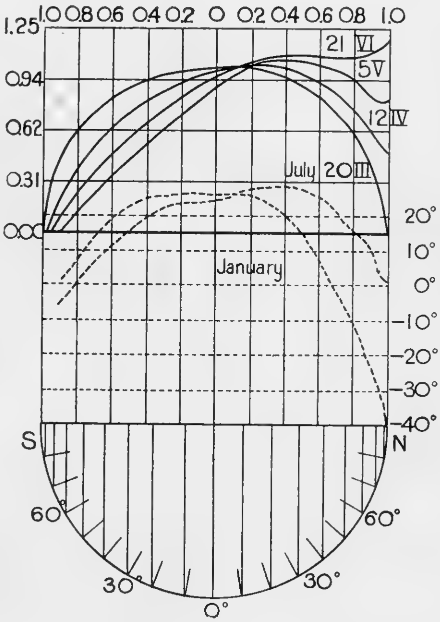


FIG. 62. DISTRIBUTION OF INSOLATION.

In this figure the scale of ordinates is chosen, as done by Hann, so that the sum total of the solar radiation received on the 20th March by a point on the equator [at the upper surface of the atmosphere] or the so-called "Thermal day" is taken as the unit.

If now we examine this figure in which we have also added below for comparison, the temperatures for January and July as given by Spitaler we see that the remarkably large sum total of insolation that comes to the polar regions during the summer solstice takes up far less space in this diagram than in the older method of presentation, in other words, that portion of the circumpolar region that receives such a relatively large insolation is only a very small fraction of the surface of the earth.

It is easily understood what a great advantage this method has in that the total insolation coming to any zone on a given day, is

[6] Zeit. d. Oesterr. Gesell. f. Met., XIV, plate 1, fig. 3, 1879.

[7] See, for example, Hann. Handbuch der Klimatologie; 2d edition; Vol. I, p. 97.

always represented by the area of the portion bounded by the corresponding portion of the curve, the initial and final ordinates and the intermediate portion of the axis of abscissæ.

Similarly the whole of the surface that lies below the curve for any given day is proportional to the sum total of the insolation that the whole earth receives on that date and inversely proportional to the square of the distance of the earth from the sun.

If now by means of a planimeter or by means of mechanical quadratures, using the tables in which the argument is the sine, we convert these areas into rectangles, then the vertical sides of these rectangles represent the average sum total of insolation for the whole earth, and of course similar results may be obtained for all elements that can be plotted in corresponding manner.

By the application of this method to the numbers under D in table I we obtain 299.3 or in round numbers 300 thermal days as the mean amount of insolation received annually.

If now we imagine the energy given by the sun annually to the whole earth's surface as distributed uniformly over this surface, then on the average every element of the surface would receive as much as an equal surface element at the equator receives in 300 average equatorial days.

We can thus also determine with great ease the latitudes that receive annually this average sum total of insolation. We have only to seek in the preceding tables the places at which $D = 300$. This value we find at $\sin \varphi = 0.6$ or if we interpolate more accurately sine $\varphi = 0.604$ which corresponds to $\varphi = \pm 37° \; 9'$. Hence those points on the earth's surface (or the upper limit of the atmosphere) lying between the parallels of 37° 9′ north and south receive more than the average insolation and those lying poleward of them receive less.

For this reason we may appropriately designate these two parallels as the insolation normals or the "Median lines of insolation."

Since the sines are proportional to the surfaces of the corresponding zones, it follows directly that 0.604 or 6/10 or 3/5 of the earth's surface receives more and 2/5 receives less than the average quantity of radiation coming annually from the sun.

In similar manner we can take from these same tables the "temperature-normal" or the "median line of the temperatures" by seeking that circle of latitude whose temperature is the same as the average temperature of the whole earth's surface, which is 15° C.

In the northern hemisphere we find this latitude at $\sin \varphi = 0.62$

or $\varphi = 38° 18'$, and in the southern hemisphere at $\sin \varphi = 0.57$ or $\varphi = 35° 00'$—so that here also within a zone that covers 0.6 of the whole surface of the earth there prevail temperatures that lie above the average, whereas outside of this zone or over the polar segments, that both together cover 0.4 of the total surface, the temperatures are below the general average.

These statements may suffice to give an idea of the great advantages offered by the method of presentation here employed.

The important conclusions that we can draw on this basis with reference to the thermal economy at the earth's surface and in the atmosphere will be reserved for a future memoir.

XXI

ADIABATIC CHANGES OF CONDITION OF MOIST AIR AND THEIR DETERMINATION BY NUMERICAL AND GRAPHICAL METHODS

BY DR. OTTO NEUHOFF

[*Memoirs of the Royal Prussian Meteorological Institution, Vol. I, No. 6. Berlin, 1900, pp. 271–306*]

§ 1. INTRODUCTION

The investigation of the adiabatic changes in the condition of moist air, that is to say, those changes that a mass of air experiences when it is expanding as it rises, or when it is being compressed as it sinks, without addition or diminution of its internal heat, has achieved a very prominent importance in modern meteorology. The old view as to the formation of precipitation, in which we attributed the principal influence to the mixture of masses of air having different temperatures, and also the idea that the heat of condensation of water raises the temperature of the place above which the condensation occurs, became untenable after the more accurate study of the foehn winds in the Alps by von Helmholtz (1862) and Hann (1866) had led to very different results. It was by the application of the principles of the mechanical theory of heat to the processes in the atmosphere that we attained to the laws of the changes of temperature in ascending or descending air, and these latter were thus established by the most prominent philosophers, such as Lord Kelvin,[1] Reye,[2] and Peslin.[3]

In the publications of Hann,[4] and Guldberg and Mohn,[5] the

[1] W. Thomson: On the convective equilibrium in the atmosphere. Mem. Manch. Soc. (3) II, 125–131.

[2] Reye: Vertikale Luftströme in der Atmosphäre. Zeitschr. f. Math., 1864, IX, S. 250–276.

[3] Peslin: Bull. hebd. de l'Assoc. scient. de France, 1868, III.

[4] Hann: Die Gesetze der Temperaturänderung in aufsteigenden Luftströmungen und einige der wichtigsten Folgerungen aus denselben. Meteorol. Zeitschr., 1874, S. 321–29, 337–46.

[5] Guldberg and Mohn: Études sur les mouvements de l'atmosphère. Christiania, 1876 and 1880. Ueber die Temperaturänderung in vertikaler Richtung der Atmosphäre. Meteorol. Zeitschr., 1878, S. 113–124.

behavior of expanding moist air was developed mathematically more fully and brought into convenient arithmetical solution.

At the same time Hertz[6] constructed his very practical diagram which made it possible to determine graphically, by the use of curves, the changes in condition of moist air, avoiding any complicated numerical computations.

Although this diagram served as a very convenient help in computations, von Bezold[7] introduced into meteorology not only the more exact mathematical development of the changes of condition of moist air but also the graphic method of presentation by Clapeyron[8] in order to represent graphically the thermodynamic processes going on in the atmosphere independent of any of the limitations introduced by any assumptions.

Von Bezold also first called attention to the fact that the processes going on in the atmosphere are often not reversible except in a very limited sense and that we have not always to do with strictly adiabatic changes but with those that can be designated as pseudo-adiabatic.

W. M. Davis[9] afterwards sought by diagrams to explain the changes of temperature and the associated processes in the atmosphere in a manner similar to that of von Bezold but by the application of another system of coördinates, in that he used a horizontal line as the scale of temperatures but a vertical line as the scale of altitudes. This latter method of presentation had been occasionally used to graphically present the results of Glaisher's balloon voyages.[10]

In recent times this method has been used by von Bezold for a great variety of cases where it is important to represent the dependence of any meteorological element on the altitude. In order to represent the course of the temperature a diagram or network of squares is used, in which the horizontal side corresponds to a change of temperature of 1° C, and each vertical side to a hundred meters in altitude.

Unfortunately, an extensive application of the theoretical results

---

[6] Hertz: Graphische Methode zur Bestimmung der adiabatischen Zustandsänderungen feuchter Luft. Meteorol. Zeitschr., 1884, S. 421–31.

[7] von Bezold: Zur Thermodynamik der Atmosphäre. Sitzungsber. der Berl. Akad., 1888.

[8] Clapeyron: Ueber die bewegende Kraft der Wärme. Pogg. Ann. 1843, S. 446–51, 566–86.

[9] W. M. Davis: Ferrel's convectional theory of tornadoes. American Meteorol. Journal, 1889, S. 344.

[10] Sprung: Lehrbuch der Meteorologie. Hamburg, 1885, S. 90.

to the atmospheric processes has hitherto been greatly hindered by the fact that memoirs on this subject are distributed through the greatest variety of periodicals, most of which are very difficult of access. On the other hand, the want of tables and diagrams is felt all the more since even the diagram of Hertz is at the present time no longer easily accessible. In the handbook of Zeuner[11] moreover, in which the changes of condition of moist air are fully considered this subject is treated only as a side issue.

The present publication has for its object to respond to the existing needs in the most perfect manner possible. In this memoir the equations for the determination of changes of condition in moist air are given under rigorous physical assumptions and with an exact mathematical treatment of the problems.

It has been possible to so arrange the results that the processes during the different periods can be expressed by one single, simple, and general formula, whose application to special cases by means of the accompanying tables demands only the smallest amount of time and trouble.

Moreover, a new adiabatic diagram for the diminution of temperature with altitude has been constructed which makes it possible to accomplish a graphic determination of the adiabatic changes of condition of moist air and that too by use of the above-mentioned extremely convenient net work of squares. In this diagram the pressure is represented by straight lines crossing the page on a slant and is read off by the scales on either side of the diagram. By reason of the diversity in the style of treatment of the systems of lines and by the use of a red tint for the network of squares, the use of the table is made easier and its perspicacity is increased.

The accompanying tables, 1 to 6, thus serve as auxiliary for the computation whereas table 7 considered as a table of adiabatics gives in one view for every 2° C. the associated values of the pressure, temperature, altitude and change of temperatuure per 100 meters, of saturated air, ascending adiabatically.

The utility of the tables, their accuracy, and method of use are illustrated by practical examples.

At this place I would express my sincere thanks to my highly honored master, von Bezold, for the stimulus that he has given me in the accomplishment of this work as well as for his kind and active interest in it.

---

[11] Zeuner: Technische Thermodynamik, Bd. II. Leipzig, 1890.

§2. IN GENERAL

The elements that represent the condition of a mass of air at any given moment are temperature (t), pressure (p) and moisture; the latter being considered as to quantity and form of aggregation. If now the mass of air is by any influence whatever forced to rise or sink, then the most important questions are as to the changes that it will experience and as to the altitudes at which these occur.

The problem is simplest when we assume that the mass of air neither gives up heat to its surroundings nor receives heat from them. Under these circumstances we have to do with adiabatic changes of condition, and the equations that express the relation between the different variables for this condition are called adiabatic equations. If we represent adiabatic conditions by curves, then we obtain adiabatic curves, or, for brevity, "adiabats."

If a mass of air is carried upwards to greater heights it comes under lower pressure since the pressure in the atmosphere diminishes with the altitude. Consequently its volume increases and the mass becomes specifically lighter. But by the increase of volume and the overcoming of the external pressure a work of expansion is performed; the quantity of heat necessary to perform this work, if the change is an adiabatic one must be drawn from the internal energy of the air. But in the case of gases, this internal energy is determined only by the temperature. Consequently the result of adiabatic expansion is a lowering of temperature.

In this case for dry air the diminution of temperature for a given diminution of pressure can be expressed by a simple law which reads

$$p/p_0 = (T/T_0)^m$$

or

$$\log p - m \log T = \log p_0 - m \log T_0 = \text{constant} \quad . \quad . \quad (1)$$

This law was derived by Poisson by an elementary course of reasoning but entirely in harmony with the developments of thermodynamics, hence this equation is known as Poisson's equation. In this equation p and p_0 express the atmospheric pressure and T and T_0 the absolute temperature of the free air; the exponent is

$$m = \frac{k}{k - 1}$$

where

$$k = \frac{c_p}{c_v}$$

is the ratio of the specific heat of dry air under constant pressure to the specific heat of dry air under constant volume; $c_p = 0.2375$; $c_v = 0.1685$ therefore

$$k = 1.41 \text{ and } m = \frac{k}{k-1} = 3.441^*$$

The equation of Poisson is here written in the form usually used in thermodynamics where the "specific pressure p" in kilograms per square meter corresponds to the weight of a column of mercury having the height b and the sectional area of one square meter, therefore we have $p = 13.6\, b$ or, still better,

$$p = 13.596\, b \frac{g}{g_{45}}$$

where g is the acceleration of gravity at the location in question and g_{45} corresponds to the acceleration of gravity at sea-level and 45° latitude.

Since in this equation there occurs only the ratio p/p_0 of the pressures therefore instead of specific pressure we may introduce the heights of the corresponding columns of mercury, which is always done in the following memoir.[12]

If now the air contains aqueous vapor, then for adiabatic expansion we must take into consideration the condensation of the aqueous vapor at the different stages of expansion according as the result of the condensation and precipitation is liquid (i. e., rain) or solid (i. e., ice and snow), and these stages are best characterized by the terms introduced by Hertz as the *dry stage*, the *rain stage*, the *hail stage*, and the *snow stage*.

According to this system the first stage is that in which the cooling of moist air by virtue of adiabatic expansion proceeds without saturating the air with aqueous vapor. So long as this condition holds good there is no precipitation of water, wherefore this stage is called the dry stage.

The second stage begins as soon as saturation occurs in consequence of diminishing temperature. If the expansion is pushed further and the cooling goes on with it, then the aqueous vapor is

---

* Lummer and Pringsheim make $k = 1.4025$ for dry air.—C. A.

[12] It may here be remarked that the notation here used generally agrees with that of Guldberg and Mohn in their "Études."

partly condensed and falls as rain or is suspended in the air as water, and this is the rain stage.

This stage continues until the temperature has fallen to 0° C.; now the precipitated water freezes while at the same time partial evaporation takes place while the temperature remains constant. As soon as all the water is frozen this isotherm of freezing, this hail stage or third period, is past and the fourth or snow stage begins as the lowering of temperature proceeds further.

These processes bring about different final results according as the condensed water falls away from the cooling air, either immediately or at some subsequent time. But for mathematical study we may assume that the mass of air rising up to a certain height carries with it its condensed aqueous vapor and therefore remains unchanged as to its total constituents.

If the inverse process takes place when the air is sinking, then in consequence of the increase of temperature the condensed water evaporates again and the mass of air returns gradually to its former condition at its initial elevation above sea-level or its initial pressure; in this case we have perfectly reversible changes of condition.

But it is otherwise if the condensed aqueous vapor falls away from the air as precipitation. The quantity of vapor remaining in the air at the end of the ascension will, because of the increasing temperature that accompanies its ascent, depart further and further from its point of saturation. This process is now in descending air entirely different from that which took place in ascending air, wherefore von Bezold expresses the changes of condition when we take into consideration the loss of the precipitation as "limited reversible."

If we have mathematically and numerically considered the first case, that of the unchanged constituents of a mass of air, then the further modifications necessary for the second case, that of the entire or partial loss of the precipitation, are easy to understand and to apply numerically. But the latter is only possible when we start, not from the ordinary assumption of thermodynamics which considers a unit weight 1 kilogram of moist air as the basis of the computation, but when we separately consider one kilogram of dry air and x kilograms of aqueous vapor. Under the assumption that the condensed aqueous vapor is to be assumed constant then the weight $(1 + x)$ *kg.* of the moist air during the whole ascension will also be constant. The quantity x that is mixed with the weight of 1 kilogram of dry air or the quantity of aqueous vapor that is contained in $(1 + x)$ *kg.* of moist air is designated "*the mix-*

ing ratio" by von Bezold, while the quantity of aqueous vapor contained in 1 kilogram of moist air is "*the specific moisture.*"

If the precipitation separates from the moist air at a definite altitude then the weight of x kilograms of aqueous vapor diminishes to a smaller quantity, ξ, which is then to be introduced into the subsequent computations as the appropriate quantity of moisture.

If we have a mixture of gases then the total pressure of the mixture is equal to the sum of the partial pressures of the components, while the volume of the mixture is the same as the volume of each separate gas, since each gas expands as though the others were not present.

In computations relative to these changes of condition moist atmospheric air is to be considered as a mixture of air and aqueous vapor so long as the condition of saturation is not attained.

The pressure of the saturated aqueous vapor is a function of the temperature only; its values have been determined experimentally by Regnault and expressed in numerical tables. Since the specific weight of aqueous vapor is 0.804 therefore the relative weight of this vapor with respect to the air is

$$E = \frac{0.804}{1.293} = 0.622^*$$

If R represents the gas constant for dry air, then the constant for aqueous vapor is $R_1 = R/\varepsilon$.

We shall now proceed to consider the individual stages of adiabatic expansion of moist air.

§3. THE DRY STAGE AND THE SATURATION POINT

Let the volume of the mass of moist air containing 1 kilogram of dry air and x kilograms of aqueous vapor be V cubic meters; the general temperature of the mass of gas be $T = 273° + t$ on the absolute Centigrade scale; the partial pressure of the dry air be p' and the partial pressure of the aqueous vapor p'', therefore the total pressure of both is $p = p' + p''$; then according to the equa-

*The value of $R = 29.272$. $R_1 = 47.061$. The ratio $k = 1.4025$ for dry air, and other physical constants, especially the revised values of vapor pressure for water and ice, will be found in the last edition of the Smithsonian Meteorological Tables.—C. A.

tion of condition for gases and for each component of the mixture we have the equations $V\,p' = RT$ and

$$Vp'' = x\frac{R}{\varepsilon}T$$

respectively. Hence by addition we obtain

$$Vp = \left(1 + \frac{x}{\varepsilon}\right)R.T \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

as the equation of condition for the mixture and by division we obtain

$$x = \varepsilon\frac{p''}{p'}$$

for the mixing ratio, which is proportional to the ratio of the partial pressures of the two components, so that if x is constant then this ratio must also be constant. It is often advantageous to introduce the barometric pressure p into the equation of condition and in this case R is specially designated by an index letter β.

If in this last equation we substitute for the pressures whose ratio enters therein the heights of the mercurial columns, then the mixing ratio is expressed in grams by the equation

$$x = 622\,\frac{e}{p - e}$$

Let e_m be the pressure for saturated aqueous vapor then we have

$$x_m = 622\,\frac{e_m}{p - e_m}$$

by the use of which expression the quantity of aqueous vapor contained in $1 + x$ kilograms of saturated moist air can be determined. The quantity x_m required for saturation is a function of the barometric pressure p and also of the temperature, since the pressure for saturation $e_m = f(t)$ is dependent on the temperature alone.

The quantity x in grams of the aqueous vapor contained in $(1 + x)$ kilograms of saturated air is computed according to this formula for the temperatures $+ 30°$ C. to $- 30°$ C. ordinarily occurring in meteorology and for different barometric pressures p, and is given in table 1, columns 4 to 9. This table contains also in the second column the values of the vapor pressure e_m of saturated aqueous vapor corresponding to the different temperatures according to the data given by Guldberg and Mohn in their "Etudes," p. 15.

The table for x here given differs from that previously devised by von Bezold,[13] in that the latter determined in grams the quantity of moisture in the form of vapor contained in a kilogram of saturated air and which therefore corresponded to the idea of the specific moisture of saturated air.

The quantity

$$\left(1 + \frac{x}{\varepsilon}\right) R$$

or $(1 + 1.608\ x)\ R = f\ (x)$ can appropriately be called the *mixing constant* as it is dependent on the mixing ratio, x. In column 3 of table 4 are given the auxiliary quantities needed for the computation of the mixing constant corresponding to values of the mixing ratio from 0 up to 30 grams; we have only to substitute R_β for R, so that, for example, for dry air $R_\beta = 2.1528$ and for $x = 12.5$ grams we have $R'_\beta = 2.1960$.

If the quantity of heat $d\,Q$ is given to any gas then its change of condition is expressed by the thermal equation

$$dQ = c_v\, dT + A p dv$$

or since we have

$$c_v = c_p - AR \text{ and } pv = RT$$

hence

$$dQ = c_p\, dT - A v dp = c_p\, dT - ART\,\frac{dp}{p}.$$

We make use of this latter form of equation in order to obtain the equations for pressure and temperature, and therefore for any change of condition of the mixture we obtain the following equations for the changes in the individual components, the dry air and the aqueous vapor, which we distinguish by means of the superscript indices prime and double prime.

$$dQ' = c'_p\, dT - ART\,\frac{dp'}{p'}$$

$$dQ'' = x c''_p\, dT - x\, A\,\frac{R}{\varepsilon}\, T\,\frac{dp''}{p''}$$

---

[13] von Bezold: Zur Thermodynamik der Atmosphäre. Sitsb. Berlin Akad, 1890, p. 239 or 390, or Mechanics of the Earth's Atmosphere, 1891, p. 287.

Hence as the equation of change for the mixture there follows:

$$dQ = (c_p' + xc_p'')\, dT - A\left(1 + \frac{x}{\varepsilon}\right) RT \frac{dp}{p} \quad . \; . \; . \; . (3)$$

where $c_p' = 0.2375$ indicates the specific heat of air and $c_p'' = 0.4805$ the specific heat of aqueous vapor, both under constant pressure.

In the case of adiabatic changes the quantity $dQ = 0$, hence after separating the variables we obtain as the differential equation of the adiabat

$$0 = (c_p' + x\, c_p'')\frac{dT}{T} - A\left(1 + \frac{x}{\varepsilon}\right) R \frac{dp}{p}$$

$$= \frac{c_p'}{AR}\left(\frac{1 + x\frac{c_p''}{c_p'}}{1 + \frac{x}{\varepsilon}}\right)\frac{dT}{T} - \frac{dp}{p}$$

For brevity put

$$\frac{c_p'}{AR}\left(\frac{1 + x\frac{c_p''}{c_p'}}{1 + x/\varepsilon}\right) = m_{\mathrm{I}}$$

then by integration we obtain

$$\log p/p_0 = m_{\mathrm{I}} \log T/T_0$$

and also

$$\begin{aligned} \log p - m_{\mathrm{I}} \log T &= \text{constant} \quad . \; . \; . \; . \; . \; . \; . \; (4) \\ &= \log p_0 - m_{\mathrm{I}} \log T_0 \end{aligned}$$

as the equation of the adiabat for the dry stage.

This equation is identical with Poisson's except that the factor m has various values that depend upon the mixing ratio x of the air. After substituting the numerical values we obtain

$$m_{\mathrm{I}} = 3.441\left(\frac{1 + 2.023\, x}{1 + 1.608\, x}\right)$$

In order to establish the numerical value of this factor and to understand its influence on the result of our computation for different quantities of moistures table 3 has been computed progressing from 0 to 30 gram by gram with x as the argument giving in column 4 the value m_1 for the different mixing ratios, for instance, for $x = 12.5$ grams the humidity factor is $m_1 = 3.459$. The limits of this table are for $x = 0$ grams or dry air $m_1 = 3.44$ and for $x = 30$ grams $m_1 = 3.48$.

Equation 4 holds good for the adiabats of the dry stage only up to the point when the air attains a condition of saturation. Any further diminution of temperature then causes a condensation of the aqueous vapor. This saturation point is therefore the bginning of the rain stage and its determination is therefore necessary before proceeding to any further computation. The expansion proceeds not only by reason of the diminution of air pressure but also by reason of the diminution of vapor pressure. But since so long as the air is not saturated the weight of the vapor that is present, or x, remains constant, therefore during the dry stage

$$x = \varepsilon \frac{e}{p - e} = \varepsilon \frac{e/p}{1 - e/p}$$

is to be considered constant, hence also the ratio e/p is unchanged. Further in equation 4 in place of the total pressure (p) the vapor pressure (e) can be substituted (or the dry air pressure is zero) whence

$$\log e/e_0 = m_1 \log T/T_0$$

or

$$\log e - m_1 \log T = \text{constant} \quad \ldots\ldots\ldots (5)$$
$$= \log e_0 - m_1 \log T_0$$

The fundamental condition for the existence of the dry stage consists in the fact that the actual pressure of the aqueous vapor e is smaller than the pressure e_m that belongs to air[14] of the same temperature saturated with aqueous vapor. This last equation (5) therefore holds good up to the point when $e = e_m$. At the moment of saturation we have equation

$$\log e_m - m_1 \log T_s = \log e_0 - m_1 \log T_0 = \text{constant} = S. \quad . \ (6)$$

where the subscript index $_0$ designates the initial condition and T_s

[14] So in the original but possibly it would be more exact to say "that belongs to it when in air, etc."—C. A.

indicates the temperature of saturation. This value of S can now be obtained directly from a table such as table 3 that contains the appropriate values of S computed according to this last equation (6) for all the temperatures likely to come into consideration, i. e., for each degree from $+30$ to -30 and for the values of $m_1 = 3.44$ to 3.48. For any value of S computed from the initial values t_0 and e_0 for which also $m_1 \log T_0$ can be taken out of table 3, we seek in the table of corresponding temperatures that one which represents the temperature of saturation and which can be obtained by interpolation to the nearest tenth of a degree.

For example, let us now consider a mass of air expanding adiabatically from the initial pressure $p_0 = 760^{mm}$ and the initial temperature $t_0 = 20°$ C. and the relative humidity 86 per cent. According to table 1 for $t_0 = 20°$ the vapor pressure for saturated vapor is $e = 17.4^{mm}$ and the quantity of moisture at saturation is $x_m = 14.6$ grams, hence for relative humidity 86 per cent the vapor pressure will be $e_0 = 15.0^{mm}$. The mixing ratio or the weight of aqueous vapor present in $(1 + x)$ kilograms of moist air is obtained from the expression

$$x = 622 \frac{e_0}{p - e_0}$$

and is 12.5 grams. We may approximately write $x/x_m = e_0/e_m$, whence also follows for x the value 12.5 grams. According to table 4, column 4, there corresponds to this mixing ratio the humidity factor for the dry stage $m_1 = 3.46$. If this mass of air expands adiabatically to the point of saturation then for the determination of the saturation temperature according to table 3, column 5, we form the product $m_1 \log T_0 = 8.5355$, whence for $e_0 = 15.0^{mm}$ there results $S = 7.3594$. With this value of S we enter table 3 under $m_1 = 3.46$ in column 11 and find the corresponding temperature of saturation $t_s = 17.0°$ C. The corresponding vapor pressure is $e_m = 14.4^{mm}$.

The corresponding pressure is obtained in millimeters at once from the equation (4) and is $p = 733.2^{mm}$, and with this we obtain $p' = p - e = 718.8^{mm}$. If we wish to know the volume (V) of the mixture, or $(1 + x)$ kilograms of air, we shall obtain it from the equation of condition. Thus from table 4, column 3, for $x = 12.5$ grams we obtain $R_\beta = 2.196$. Therefore for the initial condition $p = 760^{mm}$ and $T = 293°$ we have $V = 0.847$ cubic meters and the specific volume $v = V/1.0125 = 0.837$ cbm. In the saturated condition $V = 0.869$ cubic meters and $v = 0.858$ cubic meters. Under diminishing pressure the air becomes specifically lighter, hence for the same mass the volume becomes larger.

If we wish to form an equation for the direct determination of t_s it is necessary to represent e_m as a function of t. For this purpose we can make use of the empirical formula of Magnus

$$\log e_m = \log 4.525 + \frac{7.45\,t}{235 + t}$$

Substitute this value of e_m in the equation (6) and we obtain for the variables t or T the relation

$$\frac{7.45\,t}{235 + t} = m \log T - (m \log T_0 - \log e_0 + \log 4.525)$$

but from this equation t can only be obtained in an indirect way by successive trials.

The upper limit of the first stage is determined by the saturation point. When the expansion continues further, the values just computed become the initial values for the second or rain stage.

In the dry stage the behavior of moist air differs from the behavior of dry air only by reason of the value of the factor m which can be taken from table 4 as a function of the quantity of vapor (x) that is present. The departures of the value m_1 from that for dry air $m = 3.44$ are only slight.

§4. THE RAIN STAGE

After the attainment of the saturation point the condensation of aqueous vapor begins and during the further expansion of the air it continues to be saturated. In order to obtain the relation between pressure and temperature in this stage we form the thermo-dynamic equations. First we have

$$dQ' = c_p\,dT - ART\frac{dp'}{p'}$$

in which dQ' expresses the quantity of heat that is necessary for the expansion of 1 kilogram of air.

The total quantity of moisture at the beginning of the condensation, consisting of vapor (x grams) and water (y grams) we will call

ξ, so that $\xi = x + y$ for this stage. On the assumption that all of the water remains in the air we have $\xi = x_{\text{I}}$ or the same quantity of vapor that existed in the dry stage. On the other hand, if we take account of the loss of the precipitation by its fall from the cloud then ξ will have a smaller value on the average.

The heat necessary for any change of condition involving condensation when $\zeta = (x + y)$ kilograms of moisture are present is

$$d\,Q'' = \xi\, c\, d\,T + T\, d\left(\frac{x\,r}{T}\right)$$

where c is the specific heat of water or on the average 1.013 according to Clausius and r is the latent heat of evaporation of water which latent heat being set free from the condensing vapor does a part of the work of expansion as the air ascends and thus diminishes the rate of fall of temperature with altitude. r is a function of the temperature and according to Regnault can be expressed by the empiric formula $r = 606.5—0695t$. The total quantity of heat required for the change in condition of the mixture of dry air and vapor is equal to the sum of these two quantities or

$$d\,Q = (c_p + \xi\, c)\, d\,T + T\, d\left(\frac{x\,r}{T}\right) - A\,R\,T\,\frac{dp'}{p'}$$

In adiabatic expansions we have $dQ = o$ whence we obtain the following as the differential equation of the adiabatic for this case

$$0 = (c_p + \xi\, c)\,\frac{d\,T}{T} + d\left(\frac{x\,r}{T}\right) - A\,R\,\frac{dp'}{p'}$$

If we indicate the initial conditions by the subscript index o we obtain by integration and a simple transformation

$$\log\frac{p'}{p'_0} = \frac{c_p + \xi\, c}{A\,R}\log\frac{T}{T_0} + \frac{M}{A\,R}\left(\frac{x\,r}{T} - \frac{x_0\,r_0}{T_0}\right) = m_{\text{II}}\log\frac{T}{T_0} + \frac{M}{A\,R}\left(\frac{x\,r}{T} - \frac{x_0\,r_0}{T_0}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; (7)$$

where we have introduced the notation

$$m_{\text{II}} = \frac{c_p + \xi\, c}{A\,R} = \frac{c_p}{A\,R}\left(1 + \frac{c}{c_p}\,\xi\right) = 3.441\ (1 + 4.265\ \xi)$$

The value of the humidity factor m_{II} of the rain stage is given in table 4, column 5, for quantities of moisture between 0 and 30 grams. [M is the modulus of the system of logarithms.]

For example, we find for ξ = 0 gr., 11 gr. and 30 grams respectively, the following values m_{II} = 3.44; 3.60 and 3.88. In consequence of the high specific heat of water the differences are considerably larger than those of m_I for the dry stage.

For the variable quantity of vapor (x) we may again also substitute the partial pressure by introducing the relations

$$x = \varepsilon \frac{p''}{p'} = \varepsilon \frac{e_m}{p'}$$

and

$$V p'' = x \frac{R}{\varepsilon} T$$

where $e_m = f(t)$ is to be taken from the table of vapor tension for saturated aqueous vapor as given in column 2, table 1. We have here assumed that the volume of the saturated aqueous vapor follows the law of the equation for elastic gases; this assumption, for the low temperatures we have to deal with, is far more correct than if we should put $V = xu$ in strict accordance with the law of thermodynamics where u is the specific volume of aqueous vapor, which must first be obtained for specific temperatures from the equation given by Clapeyron,[15]

$$\frac{r}{u} = A\,T \frac{dp}{dt}$$

The volume of any water that may be present is negligible and is indeed so slight relative to that of the vapor that numerically speaking it cannot come into consideration.

According to the above reasoning equation (7) now becomes (8), where the small quantities r and e depend on temperature only.

$$\log \frac{p'}{p_0'} = m_{II} \log \frac{T}{T_0} + \frac{\varepsilon M}{A R}\left(\frac{e_m}{p'}\frac{r}{T} - \frac{e_0}{p_0'}\frac{r_0}{T_0}\right) \quad . \quad . \quad . \quad (8)$$

---

[15] Vergl. Zeuner: Technische Thermodynamik., Bd. II, 1890, p. 333.

If now we put

$$\frac{r}{T} \cdot \frac{M}{AR} \varepsilon\, e_m = a$$

and

$$\frac{r_0}{T_0} \cdot \frac{M}{A\,R} \varepsilon\, e_0 = a_0$$

then we obtain the equation for the adiabat in the following simple form

$$\log \frac{p'}{p_0'} = m_{\text{II}} - \log \frac{T}{T_0} + \frac{a}{p'} - \frac{a_0}{p_0'}$$

which may be written

$$\log p' - \frac{a}{p'} - m_{\text{II}} \log T = \log p_0' - \frac{a_0}{p_0'} - m_{\text{II}} \log T_0 - \text{constant} \quad (9)$$

The factor a which may be designated *the condensation factor* is a function of the temperature only and therefore behaves analogously to the vapor tension e_m of aqueous vapor. The law of variation with temperature followed by a is also analogous to that for e_m. The graphic representation of this quantity shows a curve analogous to the curve of elastic pressure for aqueous vapor.

In order to determine from equation (9) the value of p' expressed as a height of a column of mercury we must also express the pressure e_m by its corresponding height in mercury.

The values of the coefficient a are given for the corresponding temperatures for each degree from 0° to 30° in table 5, column 5, and can be taken from this by using the argument t. Intermediate values can easily be determined by linear interpolation. For instance, for $t = 17°$, 10°, and 0° we have the corresponding values $a = 115.71$, 75.98, and 39.99.

The volume (V) of the mixture is obtained from the equation of condition for dry air $Vp' = RT$. The quantity of vapor present x is obtained from the corresponding equation

$$x = \varepsilon \frac{e_m}{p'}$$

If in equation (9) by the use of the last-mentioned relation we express the atmospheric pressure p' in terms of x then we may also form the adiabatic equation for the diminution of the quantity of vapor with temperature. We thus obtain

$$\log \frac{e}{e_0} \cdot \frac{x_0}{x} = m_{II} \log \frac{T}{T_0} + \frac{M}{A R} \cdot \frac{r}{T} x - \frac{M}{A R} \frac{r_0}{T_0} x_0$$

If we substitute

$$\frac{M}{A R} \cdot \frac{r}{T} = \alpha$$

and collect together the constant terms there results

$$\log x + \alpha x + m \log T - \log e_m = \text{constant}$$

The coefficient α which is contained as a factor in the coefficient a can be taken from the same table 5, column 5, as a function of t.

The quantity of moisture ξ which enters the value of m_{II} in equation (7) is the sum of the vapor x and the water y, therefore the difference $\xi - x = y$ gives the quantity of the condensed water.

The rain stage attains its upper boundary when the temperature has fallen to 0° C. If now the liquid water has remained in the air then it enters the isothermal hail stage in which the water freezes to ice.

As an example we will now follow the mass of air, hitherto considered during its further expansion within the rain stage.

For the point of saturation, which is the initial point of the rain stage, we had found $t = 17.0°$; $p = 733.2^{mm}$; $e = 1.44^{mm}$; $p' = 718.8^{mm}$; $\xi = x_1 = 12.5$ grams.

During the rain stage the connection between pressure and temperature is given by the equation of the adiabat (9): from the table 4, column 5, we have in this case $m_{II} = 3.62$.

If now the expansion goes on further until the temperature cools to 10° C. then $a = 76.0$ and equation (9) in this case becomes

$$\log p' - \frac{76.0}{p'} = 2.6573$$

From this we obtain the partial atmospheric pressure $p' = 606.3^{mm}$ and $p = 615.5$, since $e = 9.2^{mm}$ when $t = 10°$.

The amount of the vapor still present is 9.4 grams, hence the vapor that has been condensed to water is $y = \xi - x = 3.1$ grams.

The volume now amounts to $V = 1.005$ cbm. and the specific volume $v = 0.993$ cbm. For further expansion to 0° C. or to the end of the rain stage we should obtain the values $a = 40.0$ and $m_{II} = 3.62$.

By making use of equation (9) we obtain

$$\log p' - \frac{40.0}{p'} = 2.6008$$

whence $p' = 482.6^{mm}$ and thence $p = 487.2^{mm}$.

The quantity of vapor present is $x = 5.9$ grams and the quantity of water is $y = 6.6$ grams. The volumes are $V = 1.218$ cbm. and $v = 1.203$ cbm.

§5. HAIL STAGE

If fluid water is still present in the air during the freezing stage which begins when the temperature falls to 0° C. then the temperature will remain constant and therefore also the quantities that depend upon the temperature, namely, $e = 4.6^{mm}$ and $r = 606.50 - 0.695\,t$.

The total moisture ξ which will be considered as unchanged and which consists of vapor (x), water (y) and ice (z), is therefore still $\xi = x + y + z$.

The quantity of heat that is needed for isothermal expansion is now made up of the quantity that is needed for the expansion of the air added to that which is needed for the evaporation and freezing of water.

If r_e is the latent heat of melting ice then the thermal equation is

$$dQ = A\,R\,T_0 \frac{dV}{V} + r\,dx - r_e\,dz$$

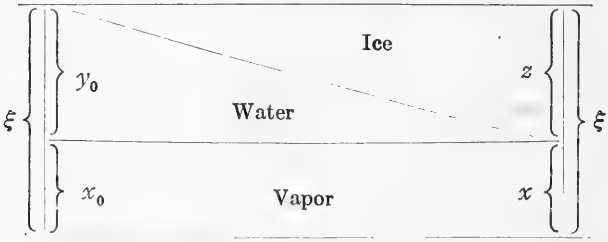
In the case of adiabatic expansion $dQ = 0$ and by integration there results

$$0 = \frac{A\,R\,T_0}{M} \log \frac{V}{V_0} + r\,(x - x_0) - r_e\,(z - z_0)$$

For the freezing period, which when it occurs is generally of very short duration, we will compute only the condition of the moist air at the end of this stage.

At the beginning of this hail stage no ice is present, therefore $z_0 = 0$ and at the end of the period all the water is frozen, wherefore $y = 0$ and therefore $z = \xi - x$, showing that the initial quantity of vapor (x_0) has been slightly increased to a new value x. The connection between these quantities is best understood by the use of the scheme given by von Bezold in his lectures:

Von Bezold's scheme for the hail stage



Beginning	End
$z_0 = 0$	$y = 0$
$\xi = x_0 + y_0$	$\xi = x + z$

For the final result we have therefore

$$0 = \frac{A R T_0}{M} \log \frac{V}{V_0} + (r + r_e)\, x - r\, x_0 - r_e\, \xi\,.$$

In consideration of the relations

$$\frac{V}{V_0} = \frac{p_0'}{p} \text{ and } x = \varepsilon \frac{e_m}{p'}$$

this equation becomes

$$\log p' - \frac{1}{p'} \frac{M}{A R} \frac{r + r_e}{T_0} \varepsilon e = \log p_0' - \frac{1}{p_0'} \frac{M r}{A R T_o} \varepsilon e - \frac{M}{A R} \frac{r_e}{T_o} \xi$$

In this equation the numerical values for the temperature 0° C. are as follows:

$$a_{IV} = \frac{M}{A R} \cdot \frac{r + r_e}{T_0} \varepsilon e = 45.20$$

$$a_{II} = \frac{M}{A R} \cdot \frac{r}{T_o} \varepsilon e \qquad = 39.99$$

$$\alpha_e = \frac{M}{A R} \frac{r_e}{T_0} \qquad = 1.82$$

and therefore for the determination of the partial pressure of the air at the end of the hail stage the above equation becomes

$$\log p' - \frac{45.20}{p'} = \log p_0' - \frac{40.0}{p_0'} - 1.82\, \xi$$

Instead of the temperature term we have another in this equation, namely, $-1.82\,\xi$ which contains the total quantity of moisture ξ. At the end of this stage the quantity of vapor (x), which is not constant but increased, can be obtained from the expression

$$x = \varepsilon \frac{e}{p'}$$

The equation for the change in the quantity of vapor can also be written directly, viz:

$$\log x + 15.80\, x = \log x_0 + 13.98\, x_0 + 1.82\, \xi$$

After all the water is frozen the snow stage begins.

If at the beginning of the hail stage the partial atmospheric pressure $p'_0 = 482.6^{mm}$ is equal to the barometric pressure $p = 487.2^{mm}$ diminished by the vapor pressure $e = 4.6^{mm}$, as in our previous example, and if the quantity of moisture present is $\xi = 12.57$ grams of which the vapor is $x = 5.9$ grams and the water is $y = 6.6$, then we obtain from equation (10) for the final pressure at the end of the hail stage the values $p' = 471.8^{mm}$ whence $p = 476.4^{mm}$.

At the end of the hail stage there is present $x = 6.1$ grams of vapor and $z = 6.4$ grams of ice, while 0.2 gram of the water that was originally present has been evaporated.

The new volumes are $V = 1.246$ cbm. and $v = 0.231$ cbm.

§6. SNOW STAGE

The behavior of the mass of air during its further adiabatic expansion is now similar to that which prevailed during the rain stage. The only difference consists in the fact that in place of the specific heat of water we introduce the specific heat of ice $c_e = 0.5$ and we also add the latent heat of liquefaction for ice $r_e = 79.24$ to the latent heat of evaporation (r) for water, so that instead of r we have to substitute $r + r_e$.

In other respects the process is the same as before; as the air ascends and cools a further condensation of aqueous vapor occurs but now it condenses directly to snow.

Therefore the differential equation of the adiabat of the snow stage becomes

$$0 = (c_p + \xi\, c_e)\, d\,T + T d \left(\frac{x}{T}(r + r_e)\right) - A\,R\,T \frac{dp'}{p'} \cdots (10)$$

In this equation ξ again indicates the quantity of moisture in the air which is now composed of vapor (x), ice (z) and snow (s) so that ξ has the same meaning as (x_1) in the first stage, since we assume that all the water has remained suspended in the air after condensation. If this is not the case then ξ has a smaller value in proportion to the quantity of precipitation that has fallen away from the air.

The integration of this equation gives us

$$\log \frac{p'}{p'_0} = \frac{(c_p + \xi\, c_e)}{AR} \log \frac{T}{T_0} + \frac{M}{AR}\left(\frac{x\,(r + r_e)}{T} - \frac{x_0\,(r_0 + r_e)}{T_0}\right)$$

If we adopt the notation

$$\frac{c_p + \xi\, c_e}{AR} = \frac{c_p}{AR}\left(1 + \frac{c_e}{c_p}\xi\right) = 3.441\,(1 + 2.105\,\xi) = m_{IV}$$

and consider that

$$x = \varepsilon\,\frac{e_m}{p'}$$

there results

$$\log \frac{p'}{p'_0} = \frac{M}{AR}\cdot\frac{r + r_e}{T}\cdot\frac{\varepsilon\, e}{p'} - \frac{M}{AR},\frac{r_0 + r}{T}\,\frac{\varepsilon\, e_0}{p'_0} + m_{IV}\,.\log \frac{T}{T_0}$$

If we substitute

$$\frac{M}{AR}\cdot\frac{r + r_e}{T}\,\varepsilon\, e = a$$

we then obtain the adiabatic equation for the snow stage in the same form as in previous cases, viz:

$$\log \frac{p'}{p'_0} = m_{IV} \log \frac{T}{T_0} + \frac{a}{p'} - \frac{a_0}{p'_0}$$

which also may be written

$$\log p' - \frac{a}{p'} - m_{IV} \log T = \text{constant}\ \ldots\ldots (11)$$

The humidity factor $m_{IV} = f\,(\xi)$ is determined by the quantity of moisture or the mixing ratio ξ and is computed as given in table 4, column 6, for values of ξ from 0 up to 30 grams. Thus, for example, we find for $\xi = 12.5$ grams; $m_{IV} = 3.53$ and for $\xi = 30$ grams; $m_{IV} = 3.658$.

On the other hand, for the rain stage and $\xi = 30$ grams we had $m_{II} = 3.881$ and for the dry stage with $\xi = 30$ grams we had $m_I = 3.482$.

Therefore in the snow stage the influence of ξ on m is less than in the rain stage. The coefficient a is a function of the temperature only; its value is given in table 6, column 6, for each degree of temperature from 0° C. to − 30° C. Thus, for instance, in the snow stage for $t = 0°$ and for $t = -20°$, respectively, we have $a = 45.20$ and $a = 10.06$.

Thus the equation of an adiabatic line is easily and quickly written to suit any special case. The solution of this equation must be made by trial, which is, however, quite simple, when we start with a correct estimate of the first approximate values.

From the preceding it is evident without further explanation how we obtain the total pressure p and the quantity of vapor x as well as the volume of the mass of air.

The diminution of the quantity of vapor with temperature is deduced from the following equation found by the combination of equation (11) with the value $x = \varepsilon \frac{e}{p'}$

$$\log x + \alpha x + m_{IV} \log T - \log e = \text{constant}$$

where

$$\alpha = \frac{M}{AR} \frac{r + r_e}{T}$$

is to be taken from column α_{IV} of table 6, column 4, as a function of the temperature.

The mass of air continues in the snow stage as long as no further expansion takes place. As the initial pressure of the snow stage occurs at the level where temperature is 0° C. it is the same as the final pressure at the end of the hail stage for which $e = 4.6$ and $p'_0 = 471.8^{mm}$. If the expansion goes on until the temperature becomes − 20° C. then for $\xi = 12.5$ grams we obtain from table 4 the value $m_{IV} = 3.53$. But since from table 6 for $t = 0$ we have $a = 45.2$ and for $t = -20$ we have $a = 10.06$, therefore from equation (11) we obtain the final pressure $p' = 311.5$ and consequently since $e = 0.9^{mm}$ we have the total pressure $p = 312.4$.

The quantities are: vapor, $x = 1.8$ grams; ice, $z = 6.4$ grams, and snow, $s = 4.3$ grams.

The volumes resulting are $V = 1.749$ cbm. and $v = 1.727$ cbm. The mass of air cooled during expansion to $-20°$ C. in this example has the same initial value as was used in the example computed by Guldberg and Mohn. The final pressure of 292 millimeters as computed by them in their "Etudes" is incorrect, as indeed Hertz has already shown.[16] [17]

§7. THE GENERAL EQUATION FOR THE ADIABATS OF MOIST AIR

Considering all the preceding results connectedly, we perceive that the law of variability of temperature and pressure in ascending and descending currents of moist air for the whole series of stages can be represented with mathematical exactness by the general adiabatic equation

$$\log p' - \frac{a}{p'} - m \log T = \text{constant}$$

which without further change is to be used in this form for the computation. This equation differs from the original Poisson equation only in form by the corrective term $-a/p'$.

In all previous attempts to make use of Poisson's equation for moist air during the stages of condensation, as for instance by a variation in the factor m and by the introduction of the so-called virtual temperature, T_v on the absolute scale

$$T_v = \frac{273 + t}{1 - 0.378 \dfrac{e}{p}}$$

we have been led to very complex and obscure formulæ. For example, Guldberg and Mohn[18] found for the rain stage the following formula

$$m = \frac{1}{AR} \frac{c_{p'} + x\left(c + \dfrac{dr}{dt} + r\dfrac{de}{dt}\right)}{1 + \dfrac{1}{AR} \cdot \dfrac{1}{273 + t} \cdot x}$$

---

[16] Hertz: Postscript to his previous article on a graphic method. Met. Zeit., 1884, p. 475.

[17] Note. This error does not occur in the text of the translation as given in this present collection which indeed was entirely revised by Guldberg and Mohn in 1885, so as to constitute a new edition. C. A.

[18] Guldberg and Mohn: Ueber die Temperaturänderung in vertikaler Richtung in der Atmosphäre. Met. Zeit., 1878, S. 113.

Sprung, in his *Lehrbuch*, has followed their course of reasoning but in place of m he has introduced the quantities

$$\bar{\varepsilon} = 3.44\left(1 + 0.258\frac{e}{p}\right) \text{ in the dry stage}$$

$$\bar{\bar{\varepsilon}} = \frac{c_p + 0.622\frac{r}{p}\cdot\frac{de}{dt}}{AR + 0.622\frac{r}{p}\frac{e}{T}} \text{ in the rain stage}$$

In the formula now developed we have to distinguish merely a dry stage and a condensation stage which are separated from each other by the temperature of saturation, which may equally well be above or below 0° C.

If the condensation stage begins at temperatures above 0° C. then, when the air has cooled to the latter temperature and if the condensed water remains in the air, the process of condensation is interrupted by the isotherm of the hail stage, which must be considered as a special case by itself since its course is conditioned only on the presence of liquid water in the air.

This also can be at least approximately expressed in the general form, when with the initial pressure at the end of the second or rain stage we use the factor $a_{II} = 40.0$ and with the final pressure at the beginning of the fourth or snow stage we use the factor $a_{IV} = 45.20$. In place of the temperature term we have to substitute the term

$$\alpha_e \xi = 1.82 \xi$$

which contains the quantity of moisture so that the isothermal adiabat now reads

$$\left.\begin{aligned} &\log p' - \frac{a_{IV}}{p'} = \log p'_0 - \frac{a_{II}}{p'_0} - \alpha_e \xi \\ &\text{or} \\ &\log p' - \frac{45.2}{p'} = \log p'_0 - \frac{40.0}{p'_0} - 1.82 \xi \end{aligned}\right\} \quad \ldots (12)$$

The exact formulæ for the computation of the final pressure from the associated temperature and the given initial condition assuming adiabatic expansion of moist air are therefore as follows:

I. The dry stage. Adiabat for dry air.

$$\log p - m \log T = \text{constant.}$$

II. The condensation stage. Adiabat for saturated air.

$$\log p' - \frac{a}{p'} - m \log T = \text{constant.}$$

In this way the exact determination of the adiabatic changes of moist air is made dependent on the evaluation of one simple formula.

§8. PSEUDO-ADIABATS

The adiabatic change of condition in moist air assumes that the condensed water remains suspended in the air during the expansion and that the mass of air retains its original total constituents unchanged, and that there is therefore no diminution in its total energy by reason of any removal of the results of precipitation. But when the mass of the water due to condensation becomes considerable it will partly or entirely separate from the air.

That change of condition which results from the separation of the precipitation but without addition or subtraction of heat is called *pseudo-adiabatic* by von Bezold and is accurately studied by him mathematically.[19] The differential equation of the adiabat of the rain stage is, as already given

$$0 = (c_p + \xi c)\, d\, T - ART \frac{dp'}{p'} + T\, d \left(\frac{xr}{T} \right)$$

but for the case in which the water formed by condensation is immediately separated, this equation changes to the following equation for the pseudo-adiabat:

$$0 = c_p d\, T + x\, c\, d\, T - ART \frac{dp'}{p'} + T\, d \left(\frac{xr}{T} \right) \quad . \quad . \quad . \quad (13)$$

In the second member of this equation we can substitute $\xi - y$ for x or even $x_1 - y$, that is to say, the original quantity of vapor diminished by the quantity of water that is formed, and we thus obtain

$$0 = (c_p + \xi c)\, d\, T - y\, c\, d\, T - ART \frac{dp'}{p'} + T\, d \left(\frac{xr}{T} \right)$$

[19] Von Bezold. Zur Thermodynamik der Atmosphäre. Sitzungsber. der Berl. Akad., 1888. Translated in Mechanics of the Earth's Atmosphere, 1891, § 15, p. 212.

From this equation by integration we obtain

$$\frac{c_p + \xi c}{AR} \log \frac{T_2}{T_1} - \frac{M}{AR} c \int_{T_1}^{T_2} y \frac{dT}{T} - \log \frac{p_2'}{p_1'} + \frac{M}{AR}\left(\frac{x_2 r_2}{T_2} - \frac{x_1 r_1}{T_1}\right) = 0.$$

By substitution of

$$x = \frac{\varepsilon e_m}{p'}$$

and by the abbreviated notation*

$$\frac{M \varepsilon e_m r}{ART} = a \text{ and } \frac{c_p + \xi c}{AR} = m$$

we finally obtain

$$\log p_2' - \frac{a_2}{p_2'} = \log p_1' - \frac{a_1}{p_1} - m \log \frac{T_1}{T_2} - \frac{M}{AR} c \int_{T_1}^{T_2} y \frac{dT}{T},$$

which is the rigorous equation of the pseudo-adiabat where $m = f(\xi)$ is a constant and corresponds to the initial quantity of moisture.

The last term in this equation

$$\frac{M}{AR} c \int_{T_1}^{T_2} y \frac{dT}{T}$$

is a corrective term that is not integrable since y is also variable with the temperature. If now y is assumed constant for any small range of temperature, which is true in proportion as the limiting temperatures T_1 and T_2 are near together then the integration would give

$$\frac{M}{AR} c \int_{T_1}^{T_2} y \frac{dT}{T} = \frac{y}{AR} c \log \frac{T_2}{T_1} = -\frac{y}{AR} c \log \frac{T_1}{T_2}$$

This value combined with

$$-m \log \frac{T_1}{T_2}$$

gives

$$-\left(m - \frac{yc}{AR}\right) \log \frac{T_1}{T_2} = -\frac{c_p}{AR}\left(1 + (\xi - y)\frac{c}{c_p}\right)$$

*a and m are essentially the same as in the previous pages.—C. A.

Since moreover $\xi - y = x$ is the quantity of vapor that is temporarily present, therefore for the pseudo-adiabat we have

$$m_1 = \frac{c_p + xc}{AR} \text{ or } m = f(x)$$

that is to say, m is now to be considered as a function of the quantity of moisture x that is actually present and that can be assumed as constant for small intervals and is to be taken from table 4, column 5 or column 6.

Therefore whereas for the adiabat the value $m = f(\xi) = f(x_1)$ was a function of the mixing ratio which remained constant during the whole process, now, in the computation of the pseudo-adiabat, m is to be taken as variable during the process of change of condition; but in the computation of every new condition developing from the previous one (m) must be taken as a constant and considered as a factor of $\log \frac{T_1}{T_2}$ and in fact equal to that value which corresponds to the average quantity of vapor x whose numerical value as a function of the temperature and the approximate pressure we take from table 3. Hence we see at once that which the practical use of the adiabatic equation (9) has already demonstrated that by reason of the separation of the precipitation the quantity of moisture (ξ) originally mixed with the air gives a smaller value for m than is shown in table 4. But still m as a factor of $\log \frac{T_1}{T_2}$ remains constant within the limits of the change of condition to be computed. It would be an error to introduce for T_1 a value m_1 and for T_2 a value m_2 since in this case the equation would lose its applicability.

If the moist air is cooled below 0° C. then the precipitation assumes the form of snow. Then, besides the latent heat of evaporation r we have also to consider the latent heat of liquefaction of the water r_e which enters into the factor a in table 6, and also in place of the specific heat of water c, there occurs c_e the specific heat of ice which enters into the factor m, table 4, column 6, for the snow stage.

The hail stage can only occur when liquid water is present in the air; for pseudo-adiabatic changes of condition the hail stage is entirely omitted [since the water drops have fallen down] and the final pressure of the rain stage for 0° C. holds good as the initial pressure of the snow stage.

In order now to show how the computation of the pseudo-adiabat differs from that of the adiabat, as to its results, and in order to distinguish as to the admissibility of one or the other boundary limits on the basis of an accurate computation we have computed the adiabat of 20 C. and 760^{mm}, for saturated ascending air.

Table A. Computation of the pseudo-adiabat for saturated ascending air for the initial temperature 20° C. and pressure 760^{mm}

$t°$	x Table 2	m_{II} and m_{IV}	log T Table 3	log $\frac{T_1}{T_2}$ Table 3	m log $\frac{T_1}{T_2}$	a_{II} and a_{IV}	log $p' - \frac{a}{p'}$	p'	em	p
C.	gr.						N.	mm.	mm.	mm.
+20	14	3.64	2.4669	0.0030	0.0109	137.6	2.6858	743	17	760
18	13	.63	.4639	30	109	122.7	.6749	706	15	721
16	12	.62	.4609	30	109	109.1	.6640	671	14	685
14	11	.60	.4579	31	112	96.9	.6531	639	12	651
12	10	.59	.4548	30	108	85.9	.6419	608	10	618
10	9	.57	.4518	31	111	76.0	.6311	579	9	588
8	9	.57	.4487	31	111	67.1	.6200	552	8	560
6	8	.56	.4456	31	110	59.1	.6089	527	7	534
4	7	.54	.4425	32	113	52.0	.5979	503	6	509
2	6	3.53	.4393	31	109	45.7	.5866	481	5	486
+0			2.4362			40.0	2.5757	460	5	465
−0	6	3.48	2.4362	32	111	45.2	2.5645	460	5	465
−2	5	.48	.4330	32	111	39.0	.5534	438	4	442
−4	5	.48	.4298	33	115	33.6	.5423	419	3	422
−6	4	.47	.4265	32	111	29.0	.5308	401	3	404
−8	4	.47	.4233	33	115	25.0	.5197	385	2	387
−10	3	.46	.4200	34	118	21.5	.5082	369	2	371
−12	3	.46	.4166	33	114	18.5	.4964	354	2	356
−14	3	.46	.4133	34	118	16.0	.4850	340	2	342
−16	2	3.45	.4099	34	117	13.7	.4732	328	1	329
−18			2.4065			11.7	2.4615	315	1	316

The preceding table (A) shows at once how this kind of computation is best executed; each new condition is computed from the data given by the preceding one.

Column 1 contains the temperatures, column 2 the quantities of moisture (x) for the average temperature of the interval after the condensation water has fallen away. The values for this column are taken from table 1, and in doing so, approximately estimated values of the pressure are used. For each value of x we seek the corresponding value of m in table 4 and enter it in the 3d column of table A. We have here to distinguish between temperatures above 0° C. for which we use the value m_{II} of the rain stage and temperatures below 0° C. for which we use the value m_{IV} of the snow stage. Columns 4 and 5 give the values of log T and the differences of $\log \frac{T_1}{T_2}$ that is to say the difference between each two successive values. These differences are to be multiplied by the corresponding values of m and the product is written in column 6. For the initial condition $t = 20°$ and $p = 760^{mm}$ we now compute

$$\log p'_0 - \frac{a_0}{p'_0}$$

The values of $a = f(t)$ are to be taken from tables 5 and 6 and placed in column 7 of table A.

We distinguish between $+0°$ C. as the final temperature of the rain stage and $-0°$ C. as the initial temperature of the snow stage since the values of a are different for these two cases.

If $p_0 = 760$ and $e_m = 17^{mm}$, then $p'_0 = 743^{mm}$. Since $a = 137.6$, therefore

$$\log p'_0 - \frac{a_0}{p'_0} = 2.6858 \text{ for } t = 20°$$

and again

$$\log p' - \frac{a}{p'} = 2.6749 \text{ for } t = 18°$$

This latter value is now the next initial value so that we obtain the value for each successive

$$\log p' - \frac{a}{p'}$$

by subtracting

$$m \log \frac{T_1}{T_2}$$

from the preceding value. But this is true only up to the temperature 0° C. or the end of the rain stage. With the temperature 0° C. we enter at once into the snow stage, that is to say, we compute with the values a and m that correspond to the snow stage. The final pressure of the rain stage as it is found from the equation

$$\log p' - \frac{40.0}{p'} = 2.5757 \text{ for } +0° \text{C}$$

viz: $p' = 460$ is the initial pressure of the snow stage.

Now for $-$ 0° C. we first compute

$$\log p' - \frac{45.2}{p'} = 2.5645$$

and each corresponding subsequent expression is deduced from the preceding by the addition of

$$- m \log \frac{T_1}{T_2}$$

There follows then the solution of all the equations

$$\log p' - \frac{a}{p'} = N$$

(the numbers given in column 8) by numerical trials The app oximate values can easily be estimated from the progressively diminishing differences of pressure, so that with the first or second trial we shall hit upon the right value and for the computation of the quotient a/p' we use Crelle or other practical multiplication table. The values of p' resulting from the solution of the equations N are found in column 9 of table A; adding to each p' the saturation vapor pressure e_m as given in table 2 corresponding to the temperature we obtain the values of the total pressure p as given in column 11. The values of the pressure are only given to the nearest whole millimeter, which is quite sufficient for present consideration.

We will now compare the final result with that given by the rigorous adiabat for the initial temperature 20° and pressure 760.

The principal difference in the computation itself lies in the fact that m remains constant for the rigorous adiabat during the whole course of each stage and that each new condition can be derived directly from the original initial condition itself.

If we assume as the initial quantity of moisture $\xi = x_1 = 14$ grams and consider this as remaining constant, then from table 4 we have for the rain stage $m = 3.64$. For the final temperature +0° C. we obtain

$$\log p' - \frac{40.0}{p'} = 2.5741$$

whence follows for the adiabat $p' = 458.5^{mm}$ for 0° C., whereas for the pseudo-adiabat we had $p' = 460$ in table A, wherefore the difference is only 1.5. If a hail stage occurs then, for $p' = 458$ and $\xi = 14$ grams we obtain

$$\log p' - \frac{45.2}{p'} = \log p'_0 - \frac{40.0}{p'} - 1.82\,\xi = 2.5486$$

whence we find $p' = 447^{mm}$ or an isothermal diminution of pressure of 11^{mm} and the final $p = 452^{mm}$.

With $p' = 447^{mm}$ as the initial pressure of the snow stage and $m_{IV} = 3.54$ there results for the final temperature $-18°$ C. and

$$\log p' - \frac{11.7}{p'} = \log p'_0 - \frac{45.2}{p'} - m \log \frac{T_0}{T} = 2.4434$$

In this case we find $p' = 303$ and $p = 304$.

The final results for the adiabat as compared with those for the pseudo-adiabat are given in the following table:

Table B

	Pseudo-adiabat p	Adiabat p
+20	760 mm	760mm
0	465	463
− 0	465	452
−18	316	304.

The values of the pressure for the pseudo-adiabat are always higher than those for the adiabat for the same temperatures. If we ignore the isothermal diminution of pressure during the hail stage, which only occurs in special cases, then the departure at the end is only 1 millimeter and is therefore so slight that it can come into consideration only in very rigorous investigations. In the computation of changes of condition of ascending currents of air it is practically almost indifferent whether we compute by the adiabat or pseudo-adiabat formula, that is to say, whether we assume that the condensed water remains suspended in the air or falls away as precipitation. The only characteristic difference is the omission of the isotherm of the hail stage in the pseudo-adiabatic changes of condition.

Since in the actual atmospheric processes neither one nor the other boundary limit is strictly fulfilled and it is therefore almost indifferent which of the two isotherms we take in the computation of the adiabats, it is perfectly practicable to introduce into the rain stage the value $m_{II} = 3.60$ as a constant moisture factor corresponding to an average quantity of moisture of about 8 or 10 grams; in this case the slight departures for values above or below this have but little importance.

If we desire to compute the final pressure at 0° C. of the adiabat of 20° and 760^{mm} assuming $m = 3.60$ then we have the expression

$$\log p' - \frac{40.0}{p'} = 2.5753$$

whence we find $p' = 460^{mm}$, or the same value as that which we found for the pseudo-adiabat.

If now we seek to find how large the difference will be in the value of m when we entirely neglect the influence of the moisture, that is to say, when we assume for the rain stage the value $m = 3.44$, or the same as for dry air, then in this same example we obtain the same equation

$$\log p' - \frac{40.0}{p'} = 2.5802$$

whence $p' = 464^{mm}$ and the departure as compared with the previous value is only 4^{mm} in excess.

Therefore the extreme result of the entire neglect of a quantity of moisture amounting to 10 grams in comparison with the weight of air, in the factor m, amounts to only a difference of pressure of 4^{mm} in the expansion and cooling of the air from the temperature of 20° C. down to 0° C. Therefore for small variations in moisture on either side of 10 grams when we use $m_{II} = 3.60$ as a constant for the rain stage we introduce departures that scarcely come into consideration.

In many cases when the quantity of moisture in the air is small it suffices to assume the factor $m = 3.44$ as a constant even for the rain stage, especially when we do not keep the higher value in memory or wish to avoid using the tables.

In the snow stage the original quantity of moisture present will scarcely ever need to be used. It is always so slight that here also we may assume an average quantity of moisture of about 2 or 3 grams and a corresponding value of $m = 3.46$, and since it scarcely matters in the computation whether we use 3.46, or 3.44 therefore we adopt for the snow stage $m = 3.44$ or the value which holds good for dry air.

For instance, in the snow stage for $t = -0°$ C.; $p_0 = 760^{mm}$, $m = 3.48$ (for $\xi = 5$ grams) and for a diminution of temperature in consequence of expansion down to $-20°$ C., we find the value $p' = 527^{mm}$, whereas if we had used $m = 3.44$ we should have had $p' = 528$. The departure therefore in this extreme case amounts to only 1^{mm}.

Still less important is the influence of the neglect of the weight of the moisture with respect to that of the air in the factor m in the dry stage, where m has the value 3.46 even for a moisture content of 15 grams. In the case of a large moisture content the saturation point is very quickly attained as the cooling proceeds; if the temperature is much reduced before cooling produces saturation, then the moisture content must be small. For the dry stage we are perfectly justified in adopting in our computations the value $m = 3.44$, such as holds good for absolutely dry air if we do not desire to obtain the values of pressure accurate to within 0.1^{mm}.

§9. ADIABATIC EXPANSION OF ASCENDING AIR

The passage of a mass of air from a given initial condition p_0 and t_0 by adiabatic expansion or compression into another condition p and t occurs in the atmosphere principally and on the largest scale through a change in the altitude of the mass.

Assuming that the air is dry and that we have a uniform distribution of temperature at 0° C. through the whole column of air we arrive, by integration of the equation

$$-dh = vdp \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

at the well-known formula

$$h = 18401 \log \frac{p_2}{p_1} = K \log \frac{p_2}{p_1}$$

which latter enables us to determine the difference of altitude of two atmospheric layers from their difference of pressure, approximately, it is true, but in a very simple way.

K is ordinarily designated the barometric constant and the signification of the remaining letters in this formula may from the preceding paragraph be considered as well known. In order to determine the altitudes Hertz has made use of this formula in the construction of the scale of altitudes given in his adiabatic diagram.[20]

[20] Hertz: Met. Zeit., 1884.

Hann has also determined the altitudes given in his small table [21] from this formula, in which the initial level of 0 meters corresponds to the atmospheric pressure 760 millimeters. If we desire to proceed more exactly, by considering the temperatures in the determination of the altitudes, then T must be expressed as a function of the altitude. But since the law of the diminution of temperature with altitude is not a general one, therefore we ordinarily assume that in each special case there is to be introduced a value for T that is equal to the arithmetical mean of the two temperatures for the upper and lower levels respectively.

In this method we assume that the temperature is a linear function of the altitude, an assumption that is more or less proper but occasionally may be entirely false.

The adiabatic diagram shows that with adiabatic changes of condition this assumption is perfectly justified during the dry stage, since then the line which indicates the diminution of temperature with altitude is perfectly straight. To a limited extent this is also true up to differences of temperature of 10° in the case of the adiabats of the condensation stage; at least the curvature of the line is in this case so feeble that the departure from the linear average temperature can have scarcely any influence on the computation of the differences of altitude. We come nearer to the truth in proportion as the changes of condition are closer together or in proportion as the interval of temperature for the two conditions is smaller. We have thus acquired a simple means for computing the total difference of altitude step by step in the condensation stage from corresponding values of temperature and pressure by the summation of small differences of altitude as in mechanical quadrature.

In the determination of the altitudes the quantity of moisture that is present in moist air is of less influence. The barometric formula of Koeppen takes account of the average moisture conditions, viz:

$$h = (18432 + q\,\tau) \log \frac{p_0}{p} = K_t \log \frac{p_0}{p}$$

in which τ is the average temperature of the upper and lower levels, q is a factor that has the value 72 when τ is above 0° C., but in other cases has the value 69.

---

[21] Hann: Die Gesetze der Temperatur-Aenderung in aufsteigenden Luftströmungen und einge der wichtigsten Folgerungen aus denselben. Met. Zeit., 1874, pp. 321–329, 337–347.

Since we shall make use of this formula therefore the value of the barometric constant K_t has been computed for each degree between the temperatures $+30°$ and $-30°$; these are given in table 2. In continuation of the example already treated a tabular view of the method of computation of the altitudes is given in detail in the following table C:

Table C

t	τ	p	$\log p$	$\Delta \log p$	K_t	Δh	h
°C	°C	mm				m	m
20	—	760	2.8808	—	—	—	0
—	18.5	—	—	0.0156	19764	308	—
17	—	733.2	2.8652	—	—	—	308
—	13.5	—	—	0.0760	19404	1474	—
10	—	615.5	2.7892	—	—	—	1782
—	5	—	—	0.1015	18792	1902	—
+ 0	—	487.2	2.6877	—	—	—	3689
—	—	—	—	0.0097	18432	179	—
− 0	—	476.4	2.6780	—	—	—	3868
—	−10	—	—	0.1833	17742	3252	—
−20	—	312.4	2.4947	—	—	—	7120

The initial level of 0 meters altitude is here taken to be that at which the temperature is 20° and the pressure 760^{mm}.

Instead of determining the change of altitude for adiabatic expansion from an initial to a final condition by applying the barometric formula we may also combine the equation $-dh = vdp$ with the adiabatic equation $f(p, t)$ and thus directly attain an adiabatic hypsometric formula. This process has been applied by Guldberg and Mohn as also by Hann following the precedent of Peslin in order to deduce a formula for the diminution of temperature with altitude.

The combination of the adiabatic thermal equation for dry air

$$0 = c_p{}' \, dT - A\, vdp$$

with the formula (14) gives the adiabatic hypsometric formula

$$0 = c_p{}' \, dT + A \, dh$$

or the equation

$$-dh = \frac{c_p{}'}{A} dT = C\, dT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

In this formula $C = 100.7$ meters or the change of altitude for a diminution of 1° C., and the vertical temperature gradient or the diminution of temperature for 100 meters ascent is 0.993° C.

For $(1 + x)$ kilograms of moist air where x is the quantity of moisture we have for adiabatic expansion in the dry stage

$$0 = (c_p' + xc_p'')\, dT - A\ (1 + x)\ vdp = C_p\, dT - A\ (1 + x)\ vdp$$

where we have put

$$C_p = c_p' + x\, c_p''$$

By combination of this equation with the formula (14) we obtain as the adiabatic hypsometric formula

$$-\,dh = \frac{C_p}{(1 + x)\ A}\ dT = \frac{c_p' + x\ c_p''}{(1 + x)\ A}\ dT = C_1\, dT \quad . \ . (16)$$

According to this formula we may compute the differences of altitude of dry as well as of moist air in the dry stage, without knowing the pressure and temperature, but only the difference of temperature for the two levels. The coefficient C_1 is, however, variable with the mixing ratio (x) for moist air, as is shown in the following table D. It is only for small altitudes and for high temperatures

Table D

x	Cp	$\frac{Cp}{A}$	C_1	$\Delta = \frac{100}{C_1}$
gr.			m	
0	0.2375	100.7	100.7	0.993
5	2400	101.7	101.2	988
10	2422	102.6	101.7	983
15	2447	103.9	102.2	978
20	2471	104.7	102.7	973
25	2500	106.0	103.4	968

that larger quantities of moisture occur than are contained in this table, and in such cases the point of saturation is generally attained very soon. For high altitudes the mixing ratio amounts to only 0.001 or 0.003, that is to say, only 1 to 3 grams of aqueous vapor are mixed with a kilogram of dry air, so that on the average $C_1 =$ 101 meters or Δt can be adopted as being 0.990° C. per 100 meters in the dry stage.

For the condensation stage we have the adiabatic thermal equation

$$0 = (c_p + \xi\, c)\ dT + T\, d\left(\frac{xr}{T}\right) - ART\, \frac{dp'}{p'}$$

In place of the second term in this equation and since

$$T\,d\left(\frac{xr}{T}\right) = d\,(xr) - \frac{xr}{T}\,d\,T$$

we may by the application of Clapeyron's equation obtain

$$\frac{xr}{T}\,d\,T = AV\,dp''$$

and since $dp = dp' + dp''$ therefore in this case the thermal equation becomes

$$0 = (c_p + \xi\,c)\,d\,T + d\,(xr) - AV\,dp \quad . \quad . \quad . \quad . \quad (17)$$

This form may also be used as the initial theorem and thence inversely the equation first given may be deduced from it.

By the combination of equation (17) with the formula

$$-\,(1 + \xi)\,d\,h = Vdp$$

we now obtain as the adiabatic hypsometric formula for the condensation stage

$$0 = (c_p + \xi\,c)\,d\,T + d\,(xr) + A\,(1 + \xi)\,dh$$

or

$$-\,dh = \frac{c_p + \xi\,c}{A\,(1 + \xi)}\,d\,T + \frac{1}{A\,(1 + \xi)}\,d(xr)$$

and by abbreviating

$$\frac{c_p + \xi\,c}{A\,(1 + \xi)} = C_2$$

and neglecting ξ in the second term in the denominator we obtain the equation

$$-\,dh = C_2\,d\,T + \frac{1}{A}\,d(xr) \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

where C_2 varies with the quantity of moisture ξ. In the rain stage $c = 1.01$ is the specific heat of water and in the snow stage $c = 0.5$ is the specific heat of ice. We have for example the values given in table E.

Table E

ξ	$c_p + \xi c$	$\frac{c_p + \xi c}{A}$	C_2	
gr.				
0	0.2375	100.7	100.7	Rain Stage
5	0.2425	102.8	102.3	Rain Stage
10	0.2475	104.9	103.9	Rain Stage
15	0.2925	107.0	105.5	Rain Stage
20	0.2975	109.1	107.1	Rain Stage
0	0.2375	100.7	100.7	Snow Stage
5	0.2400	101.2	100.7	Snow Stage
10	0.2425	101.8	101.8	Snow Stage

In general it is sufficient to adopt for C_2 in the rain stage an average quantity of moisture of 8 grams and corresponding to this use the value $C_2 = 103$, whereas for the snow stage, in which only a small quantity of moisture comes into consideration, we may put $C_2 = 101$, the same as in the dry stage.

In order to actually compute the altitudes according to the adiabatic altitude formula for the condensation stage a knowledge of the quantity of aqueous vapor (x) at the final condition is necessary and this must be obtained by the method already described. But it is not possible to deduce in a simpler form the expressions for

$$\frac{dx}{dt} \text{ or } \frac{dx}{dh}$$

and hence also

$$\frac{dt}{dh}$$

for the condensation stage, since these values vary with pressure or altitude and temperature. We can only establish more or less complicated approximate formulae such as Hann has used for the computation of his table for the value

$$\frac{dt}{dh}$$

and such as Sprung has given in his *Lehrbuch*. In general the adiabatic hypsometric formula (18) presents in the simplest way mathematically the law of the diminution of temperature with altitude for the adiabatic expansion of air. The second term corresponds to the change of elevation in consequence of the condensation of aqueous vapor and becomes 0 in the dry stage.

The computation of the example graphically worked out by Hertz in his table will now be given, in order, on the one hand, to discuss the accuracy of this table and, on the other hand, to show the new method of computation and the use of the tables.

We consider a mass of air that is expanding adiabatically as it rises and for whose initial condition we have $p = 750^{mm}$, $t = 27°$, and the relative humidity 50 per cent. According to the table 1 the temperature 27° corresponds to a vapor pressure $e_m = 26.5^{mm}$ and to a total quantity of vapor at saturation $x = 22.8$ grams. Hence for 50 per cent relative humidity we have the existing vapor pressure $e = 13.5$ and the mixing ratio $x = 11.4$ grams. Corresponding to these figures there results from table 4

$$m_{\mathrm{I}} = 3.46 \text{ in the dry stage,}$$
$$m_{\mathrm{II}} = 3.64 \text{ in the rain stage,}$$
$$m_{\mathrm{IV}} = 3.52 \text{ in the snow stage,}$$

in which we assume that the condensed water remains suspended in the air.

According to equation (6) S is found to be 7.4470, which value according to table 3 corresponds to the temperature of saturation $t_s = 13.2°$. The corresponding pressure as given by equation (4) is $p = 637.5$ and the partial pressure for $e = 11.3$ is $p' = 626.2^{mm}$. From the barometric formula there follows the difference of altitude $H = 1403$ meters.

We further compute the final pressure for three values $t = 10°$ C, $t = 0°$ C, and $t = -20°$ C, by the general adiabatic equation (11) since the temperature of saturation represents the beginning or initial condition of the condensation stage. If we consider the hail stage, then in place of

$$m \log \frac{T_0}{T}$$

we have the value $-1.82\,\xi$; the number corresponding to this is made conspicuous by bold-face type in column 3 of the following table F.

Table F

$t°$	$m \log T$	$m \log \frac{T_0}{T}$	a	$\log p' - \frac{a}{p'}$	p'	e_m	p	x	Δh	h
C					mm	mm	mm	gr	m	m
17	—	—	—	—	—	—	750	—	1403	—
13.3	8.9424	—	92.37	2.6498	626.2	11.3	637.5	11.4	—	1403
10	8.9246	0.0178	76.0	2.6320	579.5	9.2	588.7	9.9	2605	—
+ 0	8.8677	0.0747	40.0	2.5751	459.6	4.6	464.2	6.2	—	4008
—	—	—	—	—	—	—	—	—	135	—
− 0	8.5998	0.0207	45.2	2.5544	451.9	4.6	456.5	6.3	—	4143
—	—	—	—	—	—	—	—	—	3298	—
− 10	8.4830	0.1168	10.06	2.4384	296.6	0.9	297.5	1.9	—	7441

The values for a given in table F are taken from tables 5 and 6 with the argument t. The whole computation is best made in a systematic tabular form as in Table F. For comparison with other methods we give in Table G the corresponding results deduced from the diagram of Hertz.

Table G

$t°$	p	x	h
C	mm	grams	m
27°	750	11.0	0
13.3	640	11.0	1270
0	472	6.1	3750
− 0	463	6.2	3900
− 20	305	2.0	7200

In these tables the values of the pressure show differences of nearly 8 millimeters in the condensation stage; the differences in the altitudes exceed 200 meters and to this extent the results of the Hertzian diagram are uncertain. This error has been introduced by the neglect of the vapor pressure in his construction of the adiabats of the rain stage and snow stage; but this can be eliminated by a subsequent correction that can be applied to the values deduced graphically by subtracting from the final value the difference of the vapor pressures corresponding to the appropriate temperatures. We thus obtain 295^{mm} instead of 305, and in this way the value obtained from the Hertzian diagram becomes too small by only 2.5^{mm}.

§10. ADIABATIC TABLES

Tables 1, 3, 4, 5, and 6, given in the Appendix, serve as auxiliary for the computation of the adiabatic changes of condition of moist air. Table No. 2 offers a numerical auxiliary for the computation of the barometric formula of Köppen. Table 7 contains a collection of the associated values of temperature, pressure, altitude, and temperature gradient for each two degrees of temperature for air expanding adiabatically during the condensation stage.

The initial pressure for all the cases of ascending air saturated at temperatures 30°, 28°, 26° C., etc., has been assumed as 760 millimeters for an initial altitude of 0 meters or mean sea-level.

In the general adiabatic equation and for temperatures above 0° C. or the rain stage, we have taken $m = 3.60$ corresponding to an average quantity of moisture of 8 or 10 grams per kilogram of dry air; but for temperatures below 0° C., or in the snow stage we have taken $m = 3.44$. The snow stage immediately adjoins the rain stage, that is to say, the isotherm of the hail stage is not considered, therefore the table corresponds to the limiting case of pseudo-adiabatic ascent in which all the water that is precipitated separates from the ascending air.

The difference of altitude has always been computed according to the hypsometric formula of Koeppen for temperature intervals of 2° from the respective pressures and average temperatures. The total altitude of the ascension has then been computed by summation. The temperature gradient, or the diminution of temperature per 100 meters has been computed for each condition from the difference of altitude per 2° of diminution of temperature.

Intermediate values can be taken from the table by interpolation and we thus obtain, for example, the following small table H for the temperature gradients per 100 meters of altitude of saturated ascending air.

Table H

h	INITIAL TEMPERATURES						
	30°	20°	10°	0°	−10°	−20°	−30°
m							
0	0.37°	0.44°	0.54°	0.62°	0.75°	0.86°	0.91°
1000	0.37	0.46	0.56	0.68	0.82	0.90	—
2000	0.38	0.49	0.56	0.75	0.87	—	—
3000	0.40	0.51	0.65	0.82	0.89	—	—
4000	0.42	0.57	0.73	0.88	—	—	—
5000	0.43	0.59	0.80	—	—	—	—
6000	0.45	0.63	0.84	—	—	—	—
7000	0.48	0.72	—	—	—	—	—

For an adiabat whose initial temperature is $t = 10°$ the freezing point occurs between the altitudes 1000 meters and 2000 meters, This table H shows most plainly the slower rate of diminution of temperature, that occurs at these altitudes. These adiabatic tables give the values for the graphic presentations to which we shall now turn our attention.

§11. THE GRAPHIC PRESENTATION

It seems to be of importance to represent by curves the results thus far obtained on account of the great advantage that the graphic presentation of formulæ has in meteorological studies, since by the application of this method not only the connections between temperature, pressure, and moisture in the atmosphere are more easily perceived but also because laborious computations may thus be avoided.

We can express the relations between pressure and temperature for adiabatic changes of condition of moist air by a system of coördinates whose horizontal axis presents the data as to temperature and whose vertical axis presents those of pressure. It is unimportant what units of length are used in these diagrams, but in our case we use a square network in which the individual sides of the squares represent 1° C. and 10^{mm} of pressure, respectively. In the present case the temperatures extend over a range of from + 30° to − 30° and the pressures from 300^{mm} to 760^{mm}.

If we consider air expanding adiabatically in the dry stage then this condition is expressed by the equation (1) between the variables p and t.

If we compute the pressure for successive diminutions of temperature of 4° C. each and enter the corresponding values of p and t in the coördinate network and connect the individual points by a continuous curve which can be assumed as running approximately in straight lines between the plotted points, we then obtain the adiabatic curve of the dry stage for the initial condition 30° C. and 760^{mm}, which we will call the dry adiabat for 30° and 760^{mm}.

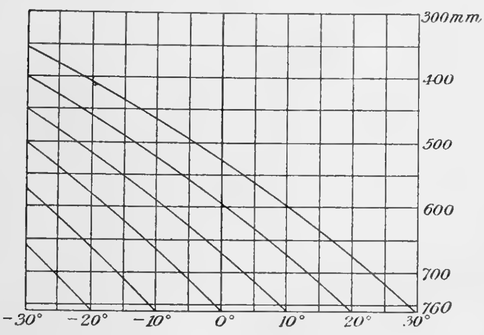


FIG. 1. DRY STAGE

In a similar way we compute for each 10° the adiabats for 20°,

760^{mm}; $10°$, 760^{mm}; etc., and plot the results and thus obtain fig. 1, which represents all these adiabats of the dry stage.

The values of p and t for the dry stage, needed for the construction of this figure are contained in the following table I.

Table I. Table for constructing the adiabats of the dry stage

$t°$	p					
° C	mm	mm	mm	mm	mm	mm
30	760	—	—	—	—	—
26	726	—	—	—	—	—
22	693	—	—	—	—	—
20	—	760	—	—	—	—
18	662	742	—	—	—	—
14	631	708	—	—	—	—
10	601	674	760	—	—	—
6	572	642	723	—	—	—
2	545	611	688	—	—	—
0	—	—	—	760	—	—
− 2	518	581	655	741	—	—
− 6	492	552	622	704	—	—
− 10	467	524	591	668	760	—
− 14	443	497	560	634	721	—
− 18	420	471	531	601	683	
− 20	—	—	—	—	—	760
− 22	398	446	503	569	647	740
− 26	376	422	476	538	612	700
− 30	336	399	450	509	579	662

Adiabats of the dry stage are segments of hyperbolic curves. This system of curves enables us without any further labor to deduce the associated values of pressure and temperature for any given intermediate conditions with sufficient accuracy.

In a similar way the adiabats of the condensation stage are drawn by means of values that are given in table 7, page 490.

Since the factor (m) varies for different mixing ratios in the adiabat of the condensation stage, therefore each mixing ratio corresponds to a special system of curves. But the error due to the introduction of an average value $m = 3.60$ in the rain stage (corresponding to an average quantity of moisture ξ of 8 or 10 grams) is so small in comparison with the use of other values that it can not be expressed graphically, therefore the construction of one single diagram is sufficient.

In the snow stage $m = 3.44$ is to be adopted since here in general only a slight quantity of moisture can occur.

The curves given in fig. 2 present the changes of condition for pseudoadiabatic expansion under the assumption that none of the water that is present at the freezing temperature = 0° C. has frozen; these adiabats of the snow stage therefore join directly on to those of the rain stage. In consequence of the sudden introduction of the latent heat of liquefaction (r) at + 0° C. the curves do not proceed continuously but have a small nick at the 0° line.

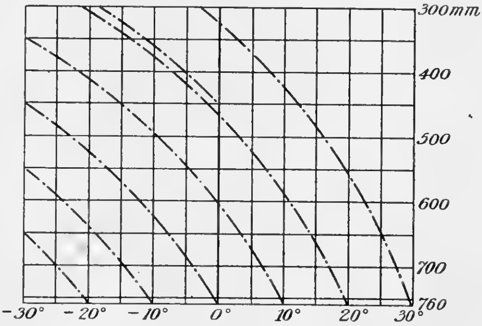


FIG. 2. CONDENSATION STAGE

If at temperature 0° C. in consequence of the freezing of water there should occur an isothermal fall of pressure, for instance, of 10^{mm}, then this would be graphically indicated by a parallel change in the adiabat of the snow stage, as is indicated by the fine line in fig. 2 drawn above the adiabat for 20° and 760^{mm}.

The adiabats of the condensation stage are more steeply inclined than those of the dry stage but at low temperatures closely approximate to the latter.

In the determination of the adiabatic changes of condition for moist air the determination of the point of saturation or the transition from the adiabats of the dry stage, to those for the condensation stage, is important.

The point of saturation depends upon the mixing ratio (x): this is determined in grams for saturated air from the equation

$$x_m = 622 \frac{e_m}{p - e_m}$$

In this equation e_m is a function of the temperature and therefore the quantity of moisture for saturation x_m is a function of the pressure and temperature.

If x is constant we obtain from the preceding equation the curve of constant quantity of saturation for the varying values of p and t. The associated values of p and t for the individual curves of saturation, which latter we can for brevity call "gram-lines" or lines of equal quantity of moisture, are obtained for individual gram-lines from table I, page 486. Fig. 3 is constructed by the values of table J.

Table J

p	1 gr.	2 gr.	3 gr.	5 gr.	7 gr.	10 gr.	15 gr.	20 gr.	25 gr.
mm	° *C*	° *C*	° *C*	° *C*	° *C*	° *C*	° *C*	° *C*	° *C*
760	−16.7	− 8.0	− 3.0	3.9	8.8	14.1	20.5	25.1	28.7
700	−17.6	− 9.1	− 4.0	2.7	7.6	13.0	19.2	23.6	27.2
600	−19.5	−11.0	− 6.0	0.5	5.3	10.5	16.7	21.3	24.7
500	−21.7	−13.4	− 8.3	−1.8	2.7	7.9	13.8	18.3	—
400	−24.4	−16.0	−11.0	−4.7	−0.4	4.5	10.4	—	—
300	−27.6	−19.6	−14.7	−8.3	−4.0	0.4	—	—	—

The curves of gram-lines are feebly curved and run nearly parallel to each other. The distances between them for equal differences of weight increase approximately logarithmically corresponding to the curve $e_m = f(t)$.

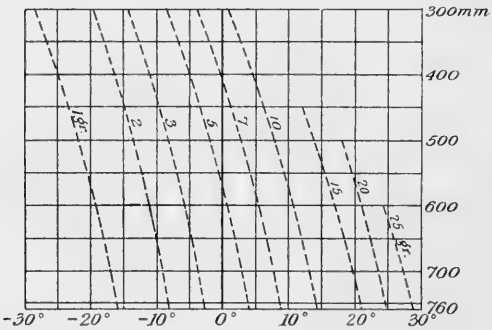


FIG. 3. GRAM-LINES OF SATURATION

These saturation curves may also be transferred to another network of coördinates whose vertical axis, as before, shows the pressure while on the horizontal axis the number of grams is shown. In this case therefore p and x are the variable coördinates in the equation

$$e = f(t) = \frac{xp}{622 + x} = \text{constant},$$

which equation leads us to the construction of the isothermal curves of saturation shown in fig. 4.

By using pressure and temperature as coördinates the adiabats of the dry stage and of the condensation stage as well as the gram-lines may all be combined in one diagram, by the use of which it becomes possible to determine all the adiabatic changes of moist air in successive series. Such a system of curves is shown in fig. 5.

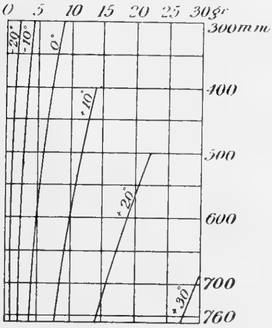


FIG. 4. ISOTHERMS OF SATURATION

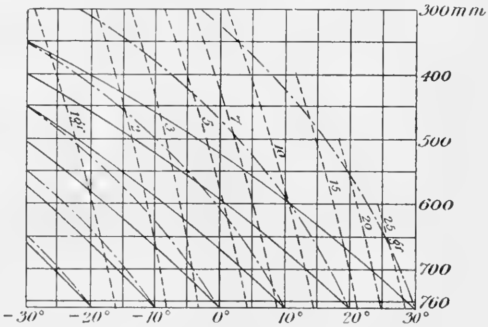


FIG. 5. DIAGRAM OF ADIABATS

Every point of the saturation curve that corresponds to a definite condition p, t shows how many grams of aqueous vapor are contained in $(1 + x)$ kilograms of saturated air. For instance, at 30° temperature and 760^{mm} pressure we have the gram line for 27 grams. If this air is still in the dry stage and if the mixing ratio is 10 grams, then from the ratio

$$\frac{10 \times 100}{27}$$

we obtain the relative humidity, 37 per cent.

Conversely if for 20° and 760^{mm} of pressure we have 20 per cent as the relative humidity then, since the saturation curve at this point is 15 grams, we find the mixing ratio to be

$$\frac{15 \times 20}{100} = 3 \text{ grams.}$$

The expansion continues along the adiabat of the dry stage until the point of saturation is reached, that is to say, until the adiabat of the dry stage intersects the gram line that corresponds to the mixing ratio.

Thus for expansion from 30° and 760^{mm} upward and with 10 grams as a mixing ratio, we find the saturation point at the intersection of the dry stage adiabat with the 10 gram line, which occurs at 11° C. and 610^{mm} pressure.

Any further expansion must now follow along the adiabat of the condensation stage which reaches the freezing point at a pressure of 465^{mm} and attains a temperature of − 20° C. at 350^{mm}.

The intersection of the condensation adiabat with the gram line will therefore show at any moment how many grams of aqueous vapor are still present, for instance, 3 grams at − 13° C. Since the original quantity of moisture was 10 grams therefore 7 grams have been condensed either to water or snow. At 0° C. we find 6 grams of aqueous vapor remaining, therefore 4 grams have been condensed to water, which would now at once freeze if it were to remain floating in the air.

For the mixing ratio of 3 grams and for initial condition of 20° and 760^{mm} the point of saturation is at −7° C. and 545^{mm} which is the intersection of this dry adiabat with the 3-gram line.

The adiabats of the dry stage become straight lines if instead of the coördinates p and t we introduce log p and log t as the variables putting $\log p = X$ and $\log T = Y$ so that $X - mY =$ constant which is the equation of a straight line.

Hertz used this principle in the construction of his table,[22] in which along the horizontal line the values of log p are set off and along the vertical line the values log t. The numbers themselves are written along side as indices. This usage has materially facilitated the construction of the table. It is only necessary to know any two points of an adiabat in order to draw not only this one but all the others that run parallel to it and are distant from it only by the differences of the constants.

The value of the constant corresponds to

$$\int \frac{dQ}{T}$$

which expression is called "entropy" by Clausius and "Wärmegewicht" by Zeuner.

In the condensation stage we have the formula

$$\log p' - \frac{a}{p'} - m \log T - \text{constant} = \int_{p_0', T_0}^{p', T} \frac{dQ}{T}$$

[22] Hertz: Graphische Methode usw. Met. Zeit., 1884, S. 426. See previous collection of translations, 1891, p. 210. C. A.

If we neglect the vapor pressure and therefore adopt p' as the total pressure then we can construct this curve in the same network as the curves of equal "entropy." Inversely therefore, from given values of p and t we may compute the entropy and allow the individual curves to grow one from the other by the same differences of entropy. In this way the curves of the Hertzian table have been represented. The objection to this arrangement, independent of the error introduced by the neglect of the vapor pressure in the condensation stage, consists in the difficult reproduction of the logarithmic network in comparison with the advantage of having straight lines for the curves of the first or dry stage.

The important question as to the altitude at which the individual changes of condition occur can also now be easily answered graphically by constructing level curves or isohypsen or lines of equal altitude in the adiabat table of fig. 5; that is to say, we seek the points on the adiabats corresponding to definite altitudes and connect these points of equal altitude by curves.

In doing this we must first consider the two stages separately. We locate the initial level of 0 meters in the pressure line of 760 millimeters.

In the dry stage we now obtain the difference of the altitudes in the simplest manner by the adiabatic hypsometric formula $-dh = CdT$. If we take approximately $C = 100$ then the temperature diminution of 10° C. along any adiabat corresponds to a difference of level of 1000 meters. Therefore the intersection of any adiabat with two temperature lines 10° apart gives points in the altitude line corresponding to 1000 meters, 2000 meters, etc. If now we combine the points having equal altitude we obtain the level lines and these also are represented as straight lines. This construction is carried out in fig. 6. The altitude lines approach each other as the temperatures increase and at the same time are inclined to each other.

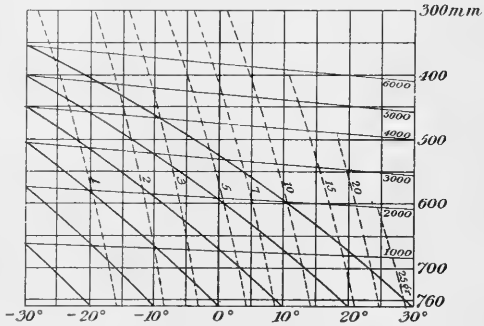


FIG. 6. ALTITUDE LINES FOR THE DRY STAGE

In the condensation stage the altitudes at which certain conditions are attained when the initial level is at 760 millimeters, are determined, first, from the final condition p, t, either step by step from the equation (14) or from the adiabatic formula (18), in which case x must first be computed for each final condition. In table 7 the altitudes are computed for the adiabats of saturated air, whence we find by interpolation the conditions at any other definite altitude such as 1000 or 2000 meters as given in table K.

Table K. Table for h in the condensation stage

h	INITIAL CONDITIONS											
	760	30°	760	20°	760	10°	760	0°	760	−10°	760	−20°
m	*mm*	°*C*	*mm*	°*C*	*mm*	°*C*	*mm*	°*C*	*mm*	°*C*	*mm*	°*C*
0	760	30.0	760	20.0	760	10.0	760	0.0	760	−10.0	760	−20.0
1000	679	26.3	677	15.5	673	4.5	670	− 6.4	667	−17.8	663	−28.8
2000	605	22.5	600	10.8	594	− 1.4	589	−13.6	584	−26.3	—	—
3000	539	18.7	532	5.9	524	− 7.6	515	−21.5	—	—	—	—
4000	479	14.8	470	0.5	460	−14.5	449	−29.8	—	—	—	—
5000	424	10.6	415	− 4.8	402	−22.1	—	—	—	—	—	—
6000	377	6.2	364	−10.8	349	−30.5	—	—	—	—	—	—
7000	334	1.7	320	−17.5	—	—	—	—	—	—	—	—

These data of table K are marked on the individual adiabats and the points of equal altitude are connected with each other; we thus find that the altitude lines are also represented as straight lines as shown in fig. 7; at least the departures therefrom are so slight that they are not shown graphically.

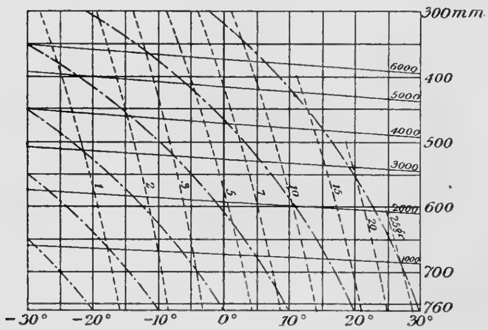


FIG. 7. ALTITUDE LINES FOR THE CONDENSATION STAGE

By comparison of the level lines for the two stages in figs. 6 and 7 we find a slight departure only at great altitudes; thus on the temperature line −30° for the line 6000 meters the departures are only 60 meters.

Therefore both tables can be united in a single one which can also contain the curves of saturation, and in doing this we either smooth out the differences between the two systems of altitudes and choose an average system or we decide to use either one of the two by preference. The small error of 1 per cent in the total altitude will therefore be considered negligible.

Such a table of adiabats and altitudes constructed on a large scale enables us to solve in the most convenient manner the most important questions as to the altitudes at which certain atmospheric conditions occur under adiabatic expansion and as to the conditions that must be present at certain altitudes.

§ 12. THE ADIABATIC DIAGRAM

The adiabatic diagram facing page 494 is a very practical form for most problems. The basis of this diagram is a square network and the units of length are 1° for temperature, and 100 meters for difference of altitude. The diagram covers a range of temperatures from + 30° to − 30° C. and of altitudes from 0 to 7000 meters. In order to explain its construction more conveniently the following fig. (8) is introduced which allows the individual items to be more easily perceived.

According to equation (15) the adiabats of the dry stage are straight lines and exact diagonals if we put $C = 100$. If $C = 101$ then there is a slight departure therefrom. Ordinarily, however, this difference can be neglected and we can assume that the adiabats are, or run parallel to, the diagonals of the respective small squares.

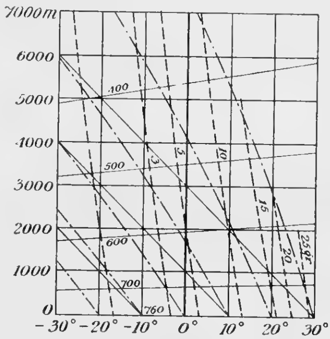


FIG. 8. DIAGRAM OF ADIABATS

The diagram (see fig. 8) contains diagonal lines only for each 10° to 10°, since the interpolation for other values is very easy.

The adiabats of the condensation stage are constructed point by point with the proper temperatures and altitudes.

The curves are drawn at distances of every 2° of temperature, so that on the one hand the appearance of the diagram is not confused and on the other hand interpolations are not made too difficult. Every fifth curve is emphasized by heavy lines. For the more convenient distinction of the adiabats of the condensation stage from the adiabats of the dry stage, which are drawn across the diagram, the first mentioned are indicated by dot and dash curves.

On the isotherm of 0° there is shown a small bend or nick in the adiabatic curves corresponding to the transition out of the rain stage into the snow stage.

As we have already in the construction of the pressure and temperature diagram as previously explained, constructed lines of equal altitude from the adiabats, so now the lines of equal pressure are drawn in order to be able to read off the pressure changes from this diagram, taking as the basis the initial altitude 0 meter and pressure 760^{mm}. The isobars run in straight lines, inclined downward toward the lower temperatures and separated farther and farther from each other as the altitudes increase.

But in this diagram there is a special system of pressure lines for the dry adiabats and another for the condensation adiabats. The departure of one from the other is first noticeable at considerable altitudes and remains always very slight, amounting to only 6^{mm} even at 6000 meters of altitude. In this diagram the pressure lines for the condensation stage are drawn as given by the temperatures corresponding to the adiabats that have been constructed. The value for any other given pressure is easily interpolated from table 7; see, for example table L.

Table L

p	°*C*					
mm	°	°	°	°	°	°
760	30.0	20.0	10.0	0.0	−10.0	−20.0
700	27.3	16.9	6.2	− 4.1	−14.8	−25.3
600	22.2	10.8	− 1.0	−12.5	−24.4	—
500	16.1	3.2	− 9.9	−23.3	—	—
400	8.3	−6.5	−22.5	—	—	—

If we desire exact values of the pressure for the dry adiabats at great altitudes we can obtain them from equation (1).

In order not to overburden the diagram, the pressure lines are only drawn through for each difference of 100^{mm} of pressure and the remaining lines at intervals of 10^{mm} are indicated as to their beginning and ending on the two sides of the diagram by short dashes. By means of these dashes, laying a straight edge across the diagram or, perhaps, by ruling the lines in another color, such as blue, these dashes may be utilized.

For the determination of the point of transition from the dry stage into the condensation stage and to determine the quantity of moisture at any point of the diagram, the curves of constan quantity of moisture needed for saturation are introduced for the corresponding values of pressure and temperature. The slight difference between the pressure lines in the two stages does not come into consideration.

The gram-lines are represented by dashes for each 5 grams. The use of the diagram is now intelligible after the explanations of the preceding section.

This diagram of adiabats possesses not only the advantage of being easily reproduceable but also the advantage that adiabatic changes of condition can be graphically compared directly with those that are produced by change of either temperature or altitude alone.

A special diagram for the hail stage which was given by Hertz can be omitted entirely by introducing the altitudes in place of the pressures. In the equation for the isotherm of 0° C. of the hail stage, the final pressure varies with the total quantity of moisture ξ. If now we choose as coördinate $\log p'$ and ξ then the adiabats for different values of ξ can be drawn as straight lines because of their short length, and of the relatively small quantity of water coming into consideration and can all be considered as running parallel to each other. These lines all begin together at the saturation curve of 0° C. or the dotted line in fig. 9 which indicates that the quantity of moisture needed for saturation at 0° C. must be subtracted from the total quantity of moisture ξ that is present; the remainder is the quantity of water present.

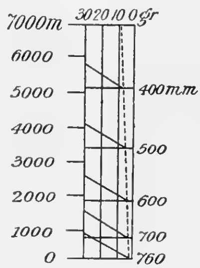


FIG. 9. HAIL STAGE

If now we introduce the altitudes in place of the pressures corresponding to the formula

$$h = 18432 \log \frac{p_0}{p}$$

for the constant temperature 0° C. then the altitude lines will run parallel to the pressure lines and at equal distances from each other for equal intervals of pressure, and we obtain the following simple result:

The isothermal change of altitude at 0° C. is proportional to the quantity of water present,

and we find empirically the formula

$$h = 27\ y,$$

where h is the change of altitude in meters; y is the quantity of frozen water in grams; therefore 1 gram of freezing water corresponds to a change of altitude of 27 meters.

The evaporation of water can be assumed proportional to the quantity of water present; the amount is very slight and is only 0.1 gram for 5 grams of water; this would introduce an error of three meters in the altitude when there is an isothermal ascent of 135 meters but the empirical value has considered this fact.

If the condensation adiabat reaches 0° C. then the gram-line passing this point gives the quantity of aqueous vapor still present, for instance, 6 grams at 4000 meters. If the original quantity of moisture be 10 grams then at the altitude of 2000 meters condensation occurs and 4 grams of water are condensed. If these grams remain suspended in the air and freeze while the temperature remains constant at 0° then the hail stage would represent a change of altitude of 108 meters or a jump of this amount at 0° C. line; for further expansion one must then follow the new adiabat.

It is easy to memorize the number 30 meters as the change of altitude for each gram of water; this results approximately from the formula

$$\frac{r_e}{A} = \frac{424 \times 80}{1000}$$

The initial level of the adiabat diagram is taken at 760^{mm} of pressure. If we have some other initial pressure, as, for instance, 730^{mm} and a temperature + 20° then must the altitudes be moved up a corresponding 250 meters, but we easily recognize the fact that in the use of the diagram where we only care for altitudes and temperatures it is indifferent whether at the initial level a pressure of 760^{mm} prevails or any other pressure.

According to table 7, page 490, for the condensation adiabat we have the data given in table M.

Table M

T	p	ALTITUDE
°	*mm*	*m*
30	760	0
20	561	2667
10	419	5137
0	320	7339

With these data we compute the corresponding values for the initial pressures 750 and 770 as shown in tables N and O.

Table N. For initial pressure 750^{mm}

t°	$m \log T$ $(m=3.60)$	$m \log \frac{T_0}{T}$	a from Tab. 5	$\log p' - \frac{a}{p'}$	p'	e_m	p	Δh	h
C					mm	mm	mm	m	m
30	8.9330	—	238.6	2.5238	718	32	750		0
								2674	
20	8.8808	0.0522	137.6	2.4716	536	17	553		2674
								2483	
10	8.8265	0.1065	76.0	2.4173	403.5	9	412.5		5157
								2202	
0	8.7703	0.1627	40.0	2.3611	310	5	315		7359

Table O. For initial pressure 750^{mm}

t°	$\log p' - \frac{a}{p'}$	p'	e_m	p	Δh	h
C		mm	mm	mm	m	m
30	2.5448	738	32	770		0
					2654	
20	2.4926	552	17	569		2654
					2442	
10	2.4383	417.5	9	426.5		5096
					2194	
0	2.3821	321	5	326		7290

Therefore for air that is saturated at its initial temperature of 30° and is ascending adiabatically the final temperature 0° will be attained at the following altitudes:

7339 meters altitude for an initial pressure of 760^{mm}
7359 " " " " " 750^{mm}
7290 " " " " " 770^{mm}

This indicates a difference in altitude of from 20 to 51 meters or, for the same final altitude an uncertainty in the final temperature of 0.1° or 0.2°.

If we neglect this difference then the diagram gives the diminution of temperature with altitude for adiabatic expansion as independent of the initial pressure.

When pressures are to be read off from the diagram this difference is to be taken account of, that is to say, the initial point is to be graphically transferred to the pressure line of the initial pressure. This must always be done in fact for low pressures; for instance, if we should locate 700^{mm} initial pressure on the pressure line that is marked 700^{mm} we must then diminish all the altitudes by this difference with respect to the initial level.

Of the many applications, that the diagram allows, we mention especially the processes in connection with the occurrence of the foehn wind. Whereas in considering ascending air it is almost indifferent whether we consider the condensed water to be floating in the air or to fall away from the air, on the other hand it makes an important difference in the case of the compression that accompanies descending air.

If, for instance, the mass of air again sinks after it has passed over the ridge of a mountain range, while the total quantity of moisture ξ grams remains unchanged so that no precipitation falls away from the air, then in the course of the descent of the air exactly the reverse process takes place, so that graphically represented the air may be said to pass backward along the same adiabats, first the adiabat of the condensation stage until all the water is again evaporated or until the saturation curve, ξ grams, is attained and then down the adiabat of the dry stage.

But it is otherwise if the precipitation has fallen away from the air. In this case during the descent of the air no water can be evaporated and the air follows backward along the adiabat of the dry stage only.

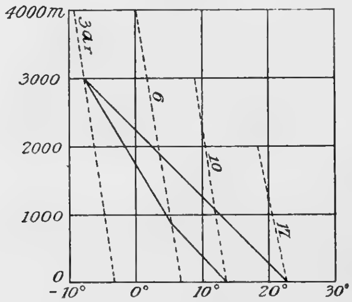


FIG. 10. FOEHN WIND

An example for the foehn wind is given in fig. 10.

At the initial level the temperature was 14° C. and the relative humidity was 60 per cent. For this we find the saturation curve for 10 grams and hence

$$10 \times 60 \div 100 = 6 \text{ grams}$$

is the mixing ratio. If now the air expands adiabatically then as shown by the intersection of the diagonal for the dry adiabat with the 6-gram line of saturation the air will attain a temperature of 5° at 900 meters altitude. If further expansion takes place the air follows the condensation adiabat, it attains the freezing point at 1750 meters altitude where we find the quantity for saturation 4.6

grams and it attains the ridge of the mountain range at 3000 meters altitude with the temperature $-7^\circ.5$ and the quantity of saturation 3 grams.

If now the precipitation all falls away so that at the summit of the ridge the total quantity of moisture or mixing ratio is equal to the quantity of vapor or 3 grams, and if now in consequence of the sinking, adiabatic compression takes place, then the air follows the adiabat of the dry stage in its descent and attains the temperature of $22^\circ.5$ when it reaches its initial level. At this point the saturation curve is 17 grams hence the relative humidity is only 12 per cent.

These and similar questions which hitherto have been solved only to a very limited extent and by a crude approximate computation with the assistance of the small table published by Hann[23] can now be quickly answered by the use of this diagram. In this way the graphic presentation allows of the solution of problems that can be solved numerically only by very tedious interpolations.

---

[23] Hann: Die Gesetze der Temperaturanderung in aufsteigenden Luftströmen, usw. Met. Zeit., 1874, p. 328.

Table I. Pressure e_m of saturated aqueous vapor: x weight of water in $(1+x)$ kilogram of saturated air

Table 2. Koeppen's barometric constant K_t $18432+q\tau$

t^0	e_m	εe_m	x p 760 mm	x p 700 mm	x p 600 mm	x p 500 mm	x p 400 mm	x p 300 mm	K_t	t^0
°C	mm	mm	gram	gram	grams	grams	grams	grams		C°
30°	31.55	19.624	26.94	29.39					20 592	30
29	29.78	18.523	25.37	27.65					20 520	29
28	28.10	17.478	23.88	26.01					20 448	28
27	26.51	16.489	22.48	24.48					20 376	27
26	24.99	15.544	21.15	23.03					20 304	26
25	23.55	14.648	19.89	21.65	25.43				20 232	25
24	22.18	13.796	18.69	20.46	23.87				20 160	24
23	20.89	12.994	17.58	19.14	22.44				20 088	23
22	19.66	12.229	16.52	17.99	21.08				20 016	22
21	18.50	11.507	15.52	16.90	19.77				19 944	21
20	17.39	10.816	14.57	15.83	18.55	22.39			19 872	20
19	16.35	10.170	13.68	14.87	17.41	21.01			19 800	19
18	15.36	9.554	12.82	13.95	16.33	19.70			19 728	18
17	14.42	8.969	12.03	13.08	15.31	18.45			19 656	17
16	13.54	8.422	11.28	12.27	14.35	17.33			19 584	16
15	12.70	7.899	10.57	11.50	13.46	16.22	20.41		19 512	15
14	11.91	7.408	9.90	10.77	12.60	15.18	19.09		19 440	14
13	11.16	6.942	9.27	10.08	11.79	14.20	17.85		19 368	13
12	10.46	6.506	8.67	9.44	11.03	13.28	16.68		19 296	12
11	9.79	6.089	8.12	8.83	10.32	12.43	15.61		19 224	11
10	9.17	5.704	7.60	8.26	9.65	11.62	14.59		19 152	10
9	8.57	5.331	7.11	7.72	9.02	10.86	13.63		19 080	9
8	8.02	4.988	6.63	7.21	8.43	10.14	12.73		19 008	8
7	7.49	4.659	6.19	6.73	7.86	9.45	11.86		18 936	7
6	7.00	4.354	5.78	6.28	7.34	8.83	11.08		18 864	6
5	6.53	4.062	5.39	5.86	6.85	8.22	10.34	13.86	18 792	5
4	6.10	3.794	5.03	5.47	6.39	7.68	9.63	12.90	18 720	4
3	5.69	3.539	4.69	5.10	5.96	7.16	8.98	12.04	18 648	3
2	5.30	3.297	4.37	4.74	5.54	6.66	8.35	11.18	18 576	2
1	4.94	3.073	4.07	4.42	5.17	6.21	7.78	10.42	18 504	1
0	4.60	2.861	3.79	4.11	4.81	5.78	7.24	9.68	18 432	0
−1	4.25	2.643	3.50	3.80	4.43	5.33	6.68	8.93	18 363	−1
−2	3.93	2.444	3.23	3.51	4.10	4.93	6.17	8.26	18 294	−2
−3	3.64	2.264	2.99	3.26	3.80	4.57	5.72	7.65	18 225	−3
−4	3.36	2.090	2.76	3.00	3.50	4.21	5.27	7.04	18 156	−4
−5	3.11	1.934	2.55	2.78	3.24	3.89	4.87	6.51	18 087	−5
−6	2.87	1.785	2.36	2.56	2.99	3.59	4.49	6.01	18 018	−6
−7	2.65	1.648	2.18	2.37	2.76	3.32	4.15	5.55	17 949	−7
−8	2.45	1.524	2.01	2.18	2.55	3.06	3.83	5.12	17 880	−8
−9	2.27	1.412	1.86	2.02	2.36	2.84	3.55	4.74	17 811	−9
−10	2.09	1.300	1.72	1.86	2.17	2.61	3.27	4.36	17 742	−10
−11	1.93	1.200	1.58	1.72	2.01	2.41	3.02	4.03	17 673	−11
−12	1.78	1.107	1.46	1.59	1.85	2.22	2.78	3.72	17 604	−12
−13	1.65	1.026	1.35	1.47	1.72	2.06	2.58	3.44	17 535	−13
−14	1.52	0.945	1.25	1.35	1.58	1.90	2.37	3.17	17 466	−14
−15	1.40	0.871	1.15	1.25	1.45	1.75	2.19	2.91	17 397	−15
−16	1.29	0.802	1.06	1.15	1.34	1.61	2.01	2.68	17 328	−16
−17	1.19	0.740	0.98	1.06	1.24	1.48	1.85	2.48	17 259	−17
−18	1.09	0.678	0.89	0.97	1.14	1.36	1.70	2.28	17 190	−18
−19	1.01	0.628	0.83	0.90	1.05	1.26	1.57	2.10	17 121	−19
−20	0.93	0.578	0.76	0.83	0.96	1.16	1.45	1.93	17 052	−20
−21	0.85	0.529	0.70	0.76	0.88	1.06	1.33	1.77	16 983	−21
−22	0.78	0.485	0.64	0.69	0.81	0.97	1.22	1.62	16 914	−22
−23	0.72	0.448	0.59	0.64	0.75	0.89	1.12	1.50	16 845	−23
−24	0.66	0.411	0.54	0.59	0.69	0.82	1.03	1.38	16 776	−24
−25	0.61	0.379	0.50	0.54	0.63	0.76	0.95	1.27	16 707	−25
−26	0.56	0.348	0.46	0.49	0.58	0.70	0.87	1.17	16 638	−26
−27	0.51	0.317	0.42	0.45	0.53	0.64	0.79	1.06	16 569	−27
−28	0.46	0.286	0.39	0.41	0.48	0.58	0.72	0.96	16 500	−28
−29	0.42	0.261	0.35	0.37	0.44	0.53	0.66	0.87	16 431	−29
−30	0.39	0.243	0.32	0.35	0.41	0.49	0.61	0.81	16 362	−30

Table 3. For the determination of the temperature of saturation from
$S = m_1 \log T - \log e$

$t°$ °C	$\log T$	$m_1 \log T$ $m_1 =$ 3.44	$m_1 =$ 3.45	$m_1 =$ 3.46	$m_1 =$ 3.47	$m_1 =$ 3.48	$\log e_m$	S $m_1 =$ 3.44	$m_1 =$ 3.45	$m_1 =$ 3.46	$m_1 =$ 3.47	$m_1 =$ 3.48
30	2.4814	8.5360	8.5608	8.5856	8.6104	8.6352	1.4990	7.0370	7.0618	7.0866	7.1114	7.1362
29	.4800	.5312	.5560	.5808	.6056	.6304	.4739	.0573	.0821	.1069	.1317	.1565
28	.4786	.5263	.5511	.5759	.6007	.6255	.4487	.0776	.1024	.1272	.1520	.1768
27	.4771	.5212	.5460	.5708	.5956	.6203	.4234	.0978	.1226	.1474	.1722	.1969
26	.4757	.5164	.5411	.5659	.5906	.6154	.3978	.1186	.1433	.1681	.1928	.2176
25	.4742	.5113	.5360	.5608	.5855	.6102	.3720	.1393	.1640	.1888	.2135	.2382
24	.4728	.5064	.5311	.5559	.5806	.6053	.3460	.1604	.1851	.2099	.2346	.2593
23	.4713	.5013	.5260	.5507	.5754	.6001	.3199	.1814	.2061	.2308	.2555	.2802
22	.4698	.4961	.5208	.5455	.5702	.5949	.2936	.2025	.2272	.2519	.2766	.3013
21	.4684	.4913	.5160	.5407	.5654	.5901	.2672	.2241	.2488	.2735	.2982	.3229
20	.4669	.4862	.5109	.5355	.5602	.5848	.2403	.2459	.2706	.2952	.3199	.3445
19	.4654	.4810	.5056	.5303	.5549	.5796	.2135	.2675	.2921	.3168	.3414	.3661
18	.4639	.4759	.5005	.5252	.5498	.5744	.1864	.2895	.3141	.3388	.3634	.3880
17	.4624	.4707	.4953	.5199	.5446	.5692	.1590	.3117	.3363	.3609	.3856	.4102
16	.4609	.4655	.4901	.5147	.5393	.5640	.1316	.3339	.3585	.3831	.4077	.4324
15	.4594	.4604	.4850	.5096	5342	.5588	.1038	.3566	.3812	.4058	.4304	.4550
14	.4579	.4552	.4798	.5043	.5289	.5535	.0759	.3793	.4039	.4284	.4530	.4776
13	.4564	.4501	.4747	.4992	.5238	.5483	.0477	.4024	.4270	.4515	.4761	.5006
12	.4548	.4445	.4690	.4936	.5181	.5427	1.0195	.4250	.4495	.4741	.4986	.5232
11	.4533	.4393	.4638	.4884	.5129	.5375	0.9908	.4485	.4730	.4976	.5221	.5467
10	.4518	.4342	.4587	.4832	.5077	.5322	0.9624	.4718	.4963	.5208	.5453	.5698
9	.4503	.4290	.4535	.4780	.5025	.5270	0.9330	.4960	.5205	.5450	.5695	.5940
8	.4487	.4236	.4481	.4726	.4971	.5215	0.9042	.5194	.5439	.5684	.5929	.6173
7	.4472	.4184	.4429	.4674	.4919	.5163	0.8745	.5439	.5684	.5929	.6174	.6418
6	.4456	.4128	.4372	.4617	.4861	.5106	0.8451	.5677	.5921	.6166	.6410	.6655
5	.4440	.4074	.4318	.4563	.4807	.5051	0.8149	.5925	.6169	.6414	.6658	.6902
4	.4425	.4022	.4266	.4510	.4755	.4999	0.7853	.6169	.6413	.6657	.6902	.7146
3	.4409	.3967	.4211	.4455	.4699	.4944	0.7551	.6416	.6660	.6904	.7148	.7393
2	.4393	.3912	.4156	.4400	.4644	.4887	0.7243	.6669	.6913	.7157	.7401	.7644
1	.4378	.3860	.4104	.4347	.4591	.4835	0.6937	.6923	.7167	.7410	.7654	.7898
0	.4362	.3806	.4049	.4293	.4536	.4780	0.6628	.7178	.7421	.7665	.7908	.8152
−1	.4346	.3750	.3993	.4237	.4480	.4724	0.6284	.7466	.7709	.7953	.8196	.8440
−2	.4330	.3695	.3938	.4181	.4424	.4668	0.5944	.7751	.7994	.8237	.8480	.8724
−3	.4314	.3640	.3883	.4126	.4369	.4613	0.5611	.8029	.8272	.8515	.8758	.9002
−4	.4298	.3585	.3828	.4071	.4314	.4557	0.5263	.8322	.8565	.8808	.9051	.9294
−5	.4281	.3526	.3769	.4012	.4255	.4497	0.4928	.8598	.8841	.9084	.9327	.9569
−6	.4265	.3472	.3715	.3957	.4200	.4442	0.4579	.8893	.9136	.9378	.9621	7.9863
−7	.4249	.3417	.3659	.3902	.4144	.4387	0.4233	.9184	.9426	.9669	7.9911	8.0154
−8	.4233	.3361	.3603	.3846	.4088	.4331	0.3892	.9469	.9711	7.9954	8.0196	.0439
−9	.4216	.3303	.3545	.3787	.4029	.4271	0.3560	7.9743	7.9985	8.0227	.0469	.0711
−10	.4200	.3248	.3490	.3732	.3974	.4216	0.3202	8.0046	8.0288	.0530	.0772	.1014
−11	.4183	.3189	.3431	.3673	.3915	.4157	0.2856	.0333	.0575	.0817	.1059	.1301
−12	.4166	.3131	.3373	.3614	.3856	.4097	0.2504	.0627	.0869	.1110	.1352	.1593
−13	.4150	.3076	.3318	.3559	.3801	.4042	0.2175	.0901	.1143	.1384	.1626	.1867
−14	.4133	.3017	.3258	.3499	.3741	.3983	0.1818	.1199	.1440	.1681	.1923	.2165
−15	.4116	.2959	.3200	.3441	.3682	.3923	0.1461	.1498	.1739	.1980	.2221	.2462
−16	.4099	.2901	.3142	.3383	.3624	.3865	0.1106	.1795	.2036	.2277	.2518	.2759
−17	.4082	.2842	.3083	.3324	.3565	.3806	0.0756	.2086	.2327	.2568	.2809	.3050
−18	.4065	.2784	.3025	.3265	.3506	.3746	0.0374	.2410	.2651	.2891	.3132	.3372
−19	.4048	.2725	.2965	.3206	.3446	.3687	0.0043	.2682	.2922	.3163	.3403	.3644
−20	.4031	.2666	.2906	.3146	.3387	.3627	9.9685	.2981	.3221	.3461	.3702	.3942
−21	.4014	.2608	.2848	.3088	.3328	.3569	.9294	.3314	.3554	.3794	.4034	.4275
−22	.3997	.2550	.2790	.3030	.3270	.3510	.8921	.3629	.3869	.4109	.4349	.4589
−23	.3979	.2488	.2728	.2968	.3208	.3447	.8573	.3915	.4155	.4395	.4635	.4874
−24	.3962	.2429	.2669	.2908	.3148	.3388	.8195	.4234	.4474	.4713	.4953	.5193
−25	.3945	.2371	.2610	.2850	.3089	.3329	.7853	.4518	.4757	.4997	.5236	.5476
−26	.3927	.2309	.2548	.2787	.3026	.3266	.7482	.4827	.5066	.5305	.5544	.5784
−27	.3909	.2247	.2486	.2725	.2964	.3204	.7076	.5171	.5410	.5649	.5888	.6128
−28	.3892	.2189	.2428	.2667	.2906	.3144	.6628	.5561	.5800	.6039	.6278	.6516
−29	.3874	.2127	.2366	.2605	.2843	.3082	.6233	.5894	.6133	.6372	.6610	.6849
−30	2.3856	8.2064	8.2303	8.2541	8.2780	8.3018	9.5911	8.6153	8.6392	8.6630	8.6869	8.7107

Table 4. The humidity factors for moist air containing x grams per (1 + x) kg

x	C_p	R_m	m_{I}	m_{II}	m_{IV}
gr.					
0	0.2375	2.1528	3.441	3.441	3.441
1	.2380	.1563	.442	.456	.448
2	.2384	.1597	.443	.470	.455
3	.2389	.1631	.445	.485	.463
4	.2394	.1666	.446	.500	.470
5	.2399	.1700	.448	.514	.477
6	.2404	.1735	.449	.529	.484
7	.2408	.1770	.450	.544	.492
8	.2413	.1805	.452	.558	.499
9	.2418	.1840	.453	.573	.506
10	.2422	.1874	.455	.588	.514
11	.2427	.1909	.456	.602	.521
12	.2432	.1943	.458	.617	.528
13	.2437	.1978	.459	.632	.535
14	.2442	.2012	.461	.646	.542
15	.2447	.2047	.462	.661	.550
16	.2452	.2082	.463	.676	.557
17	.2457	.2116	.465	.690	.564
18	.2461	.2151	.466	.705	.571
19	.2466	.2185	.468	.720	.579
20	.2471	.2220	.469	.734	.586
21	.2475	.2255	.470	.749	.593
22	.2480	.2289	.471	.764	.600
23	.2485	.2324	.473	.778	.607
24	.2490	.2358	.474	.793	.615
25	.2495	.2393	.475	.808	.622
26	.2500	.2428	.477	.822	.629
27	.2505	.2462	.478	.837	.636
28	.2510	.2497	.480	.852	.644
29	.2515	.2531	.481	.867	.651
30	0.2519	2.2565	3.482	3.881	3.658

$$C_p = c_p' \,(1 + 2.023\,x)$$

$$R_m = R_\beta \,(1 + 1.608\,x)$$

$$m_{\mathrm{I}} = \frac{c_p' + x\,c_p''}{AR\left(1 + \frac{x}{\varepsilon}\right)} = 3.441 \left(\frac{1 + 2.023\,x}{1 + 1.608\,x}\right)$$

$$m_{\mathrm{II}} = \frac{c_p' + \xi c}{A\,R} = 3.441\,(1 + 4.265\,\xi)$$

$$m_{\mathrm{IV}} = \frac{c_p' + \xi c_e}{A\,R} = 3.441\,(1 + 2.105\,\xi)$$

$$\alpha_{\mathrm{II}} = \frac{r}{T} \cdot \frac{M}{A\,R}$$

$$\alpha_{\mathrm{IV}} = \frac{r + r_c}{T} \cdot \frac{M}{A\,R}$$

$$a_{\mathrm{II}} = \frac{r}{T} \cdot \frac{M}{A\,R}\,\varepsilon\, e_m = \alpha_{\mathrm{II}}\,\varepsilon\, e_m$$

$$a_{\mathrm{IV}} = \frac{r + r_c}{T}\,\frac{M}{A\,R}\,\varepsilon\, e \quad = \alpha_{\mathrm{IV}}\,\varepsilon\, e_m$$

Table 5. For the computation of α_{II} and a_{II} in the rain stage

$t°$	r	$\frac{r}{T}$	α_{II}	a_{II}
°C				
30	585.650	1.933	12.16	238.60
29	586.345	.942	12.22	226.26
28	587.040	.950	12.27	214.45
27	587.735	.959	12.32	203.19
26	588.430	.968	12.38	192.43
25	589.125	.977	12.43	182.18
24	589.820	.986	12.49	172.40
23	590.515	1.995	12.54	163.06
22	591.210	2.004	12.60	154.17
21	591.905	.013	12.66	145.71
20	592.600	.023	12.72	137.63
19	593.295	.032	12.78	129.96
18	593.990	.041	12.84	122.66
17	594.685	.051	12.90	115.71
16	595.380	.060	12.96	109.12
15	596.075	.070	13.02	102.85
14	596.770	.079	13.08	96.89
13	597.465	.089	13.14	91.24
12	598.160	.099	13.20	85.88
11	598.855	.109	13.26	80.80
10	599.550	.119	13.33	75.98
9	600.245	.129	13.39	71.42
8	600.940	.139	13.45	67.09
7	601.635	.149	13.52	62.99
6	602.330	.159	13.58	59.12
5	603.025	.169	13.64	55.46
4	603.720	.180	13.71	52.00
3	604.415	.190	13.77	48.73
2	605.110	.200	13.84	45.65
1	605.805	.211	13.91	42.74
0	606.500	2.222	13.98	39.99

Table 6. For the computation of α_{IV} and a_{IV} in the snow stage

$t°$	$r + r_e$	$\frac{r + r_e}{T}$	α_{IV}	a_{IV}
°C				
−30	706.350	2.907	18.28	4.44
−29	705.655	.892	18.19	4.75
−28	704.960	.877	18.10	5.18
−27	704.265	.863	18.01	5.71
−26	703.570	.848	17.92	6.24
−25	702.875	.834	17.83	6.77
−24	702.180	.820	17.74	7.28
−23	701.485	.806	17.65	7.91
−22	700.790	.792	17.56	8.52
−21	700.095	.778	17.48	9.24
−20	699.400	.764	17.39	10.06
−19	698.705	.751	17.31	10.87
−18	698.010	.737	17.22	11.68
−17	697.315	.724	17.14	12.68
−16	696.620	.711	17.05	13.68
−15	695.925	.697	16.96	14.78
−14	695.230	.684	16.88	15.97
−13	694.535	.671	16.80	17.25
−12	693.840	.658	16.72	18.52
−11	693.145	.646	16.64	19.98
−10	692.450	.633	16.56	21.53
− 9	691.755	.620	16.48	23.28
− 8	691.060	.608	16.40	25.00
− 7	690.365	.595	16.32	26.91
− 6	689.670	.583	16.25	29.01
− 5	688.975	.571	16.17	31.28
− 4	688.280	.559	16.10	33.64
− 3	687.585	.547	16.02	36.27
− 2	686.890	.535	15.95	38.98
− 1	686.195	.523	15.87	41.95
0	685.500	2.511	15.80	45.20

Table 7. The associated values of temperature, pressure, altitude and temperature gradient for adiabatic expansion of ascending saturated air

t^0	p		h		Δt^0 for 100 m	p		h		Δt^0 for 100 m	p		h		Δt^0 for 100 m
^0C	mm	Δ	m	Δ		mm	Δ	m	Δ		mm	Δ	m	Δ	
30	760	45	0	544	0.37										
28	715	42	544	538	0.37	760	44	0	528	0.38					
26	673	40	1082	536	0.37	716	41	528	518	0.39	760	43	0	512	0.39
24	633	37	1618	525	0.38	675	38	1046	506	0.39	717	40	512	500	0.40
22	596	35	2143	524	0.38	637	37	1552	503	0.40	677	38	1012	500	0.40
20	561	33	2667	523	0.39	600	34	2055	501	0.40	639	35	1512	485	0.41
18	528	30	3190	499	0.40	566	31	2556	495	0.40	604	32	1997	472	0.42
16	498	28	3689	490	0.41	535	30	3051	490	0.41	572	31	2469	464	0.43
14	470	27	4179	478	0.42	505	28	3541	480	0.42	541	29	2933	463	0.44
12	443	24	4657	470	0.43	477	25	4021	467	0.43	512	26	3396	439	0.46
10	419	22	5127	467	0.43	452	24	4488	454	0.44	486	25	3835	437	0.46
8	397	22	5994	463	0.44	428	22	4942	434	0.46	461	23	4272	420	0.48
6	375	20	6057	447	0.45	406	20	5376	411	0.48	438	22	4692	420	0.48
4	355	18	6504	421	0.47	386	20	5787	410	0.49	416	20	5112	400	0.50
2	337	17	6925	414	0.48	366	18	6197	405	0.49	396	19	5512	393	0.51
0	320	18	7339	463	0.43	348	19	6602	448	0.45	377	20	5905	434	0.46
− 2	302		7802			329	17	7050	419	0.48	357	18	6339	410	0.49
− 4						312		7469			339	16	6749	380	0.52
− 6											323	15	7129	369	0.54
− 8											308		7498		
−10															
−12															
−14															
−16															
−18															
−20															

t^0	p	Δ	h	Δ	Δt^0 for 100 m	p	Δ	h	Δ	Δt^0 for 100 m	p	Δ	h	Δ	Δt^0 for 100 m
18	760	37	0	430	0.47										
16	723	34	430	428	0.47	760	37	0	423	0.47					
14	687	34	858	426	0.47	723	35	423	417	0.48	760	36	0	409	0.49
12	653	31	1284	405	0.49	688	32	840	398	0.50	724	33	409	388	0.52
10	622	29	1689	395	0.51	656	30	1238	388	0.52	691	31	797	382	0.53
8	593	27	2084	384	0.52	626	28	1626	377	0.53	660	29	1179	370	0.54
6	566	26	2468	383	0.52	598	26	2003	363	0.54	631	28	1549	369	0.54
4	540	24	2851	368	0.54	572	25	2366	362	0.56	603	26	1918	356	0.56
2	516	22	3219	352	0.57	547	23	2728	346	0.58	577	23	2274	327	0.61
0	494	23	3571	380	0.52	524	24	3074	373	0.54	554	25	2601	367	0.54
− 2	471	21	3951	361	0.55	500	22	3447	357	0.56	529	23	2968	351	0.57
− 4	450	19	4312	338	0.58	478	20	3804	335	0.59	506	20	3319	326	0.60
− 6	431	18	4650	332	0.60	458	19	4139	330	0.61	486	20	3645	319	0.63
− 8	413	17	4982	326	0.62	439	18	4469	324	0.63	466	18	3964	304	0.66
−10	396	16	5308	316	0.64	421	16	4793	298	0.66	448	17	4268	297	0.68
−12	380	14	5624	296	0.67	405	15	5091	297	0.68	431	16	4565	288	0.69
−14	366	14	5920	286	0.70	390	15	5388	287	0.70	415	15	4853	287	0.70
−16	352	13	5206	281	0.72	375	13	5675	272	0.73	400	15	5140	278	0.73
−18	339	12	6487	267	0.75	362	13	5947	264	0.76	385	13	5418	257	0.76
−20	327	11	6754	259	0.77	349	12	6211	258	0.78	372	12	5675	256	0.78
−22	316	11	7013	253	0.79	337	11	6469	249	0.80	360	12	5931	248	0.80
−24	305		7266			326	11	6718	243	0.82	348	12	6179	246	0.82
−26						315		6961			336	11	6425	241	0.83
−28											325	11	6666	239	0.84
−30											314		6905		

Table 7 (Continued)

p	Δ	h	Δ	Δt^0 for 100 m	p	Δ	h	Δ	Δt^0 for 100 m	p	Δ	h	Δ	Δt^0 for 100 m	t^0
mm		*m*			*mm*		*m*			*mm*		*m*			^0C
															30
															28
															26
760	42	0	496	0.40											24
718	39	496	483	0.41	760	41	0	481	0.42						22
679	36	979	469	0.43	719	38	481	465	0.43	760	39	0	453	0.44	20
643	34	1448	464	0.43	681	35	946	452	0.44	721	36	453	437	0.46	18
609	32	1912	457	0.44	646	33	1398	443	0.45	685	34	890	437	0.46	16
577	30	2369	449	0.45	613	31	1841	438	0.46	651	33	1327	431	0.47	14
547	27	2818	426	0.47	582	29	2279	427	0.47	618	30	1758	415	0.48	12
520	26	3244	424	0.47	553	27	2706	414	0.48	588	28	2173	405	0.49	10
494	25	3668	423	0.48	526	25	3120	401	0.50	560	26	2578	392	0.51	8
469	23	4091	412	0.49	501	24	3521	400	0.50	534	25	2970	391	0.51	6
446	21	4503	390	0.51	477	22	3921	382	0.52	509	23	3361	371	0.54	4
425	20	4893	387	0.52	455	20	4303	361	0.55	486	21	3732	353	0.57	2
405	21	5280	430	0.46	435	22	4664	413	0.48	465	23	4085	406	0.49	0
384	19	5710	401	0.50	413	19	5077	374	0.53	442	20	4491	366	0.54	− 2
365	17	6111	375	0.53	394	18	5451	367	0.55	422	18	4857	342	0.58	− 4
348	15	6486	345	0.56	376	16	5818	339	0.59	404	17	5199	335	0.60	− 6
333	14	6831	344	0.58	360	15	6157	329	0.61	387	16	5534	326	0.61	− 8
319	14	7175	331	0.60	345	14	6486	318	0.63	371	15	5860	316	0.63	−10
305		7506			331	13	6804	315	0.64	356	14	6176	307	0.65	−12
					318	13	7119	305	0.66	342	13	6483	292	0.68	−14
					305		7424			329	12	6775	278	0.72	−16
										317		7053			−18
															−20

p	Δ	h	Δ	Δt^0 for 100 m	p	Δ	h	Δ	Δt^0 for 100 m	p	Δ	h	Δ	Δt^0 for 100 m	t^0
															18
															16
															14
760	34	0	382	0.52											12
726	32	382	372	0.54	760	33	0	369	0.54						10
694	30	754	365	0.55	727	31	369	359	0.56	760	32	0	354	0.56	8
664	29	1119	363	0.55	696	30	728	358	0.56	728	30	354	345	0.58	6
635	27	1482	352	0.57	666	28	1086	348	0.58	698	29	699	342	0.58	4
608	24	1834	324	0.62	638	25	1434	320	0.62	669	26	1041	318	0.63	2
584	26	2158	364	0.55	613	27	1754	360	0.56	643	27	1359	342	0.58	0
558	23	2522	331	0.58	586	24	2114	332	0.60	616	26	1701	340	0.60	− 2
535	22	2853	331	0.60	562	22	2446	313	0.64	590	23	2041	313	0.64	− 4
513	21	3184	324	0.63	540	21	2759	309	0.65	567	21	2354	297	0.67	− 6
492	18	3508	297	0.67	519	20	3068	304	0.66	546	20	2651	294	0.68	− 8
474	18	3805	288	0.70	499	18	3372	282	0.69	526	20	2945	288	0.70	−10
456	16	4093	280	0.72	481	17	3654	281	0.71	506	18	3233	277	0.72	−12
440	16	4373	272	0.74	464	17	3935	275	0.73	488	17	3510	268	0.75	−14
424	15	4645	271	0.75	447	15	4210	264	0.76	471	16	3778	259	0.77	−16
409	14	4916	258	0.78	432	15	4474	255	0.78	455	15	4037	251	0.79	−18
395	13	5174	254	0.79	417	14	4729	252	0.79	440	14	4288	248	0.81	−20
382	13	5428	253	0.80	403	13	4981	251	0.80	426	14	4536	246	0.82	−22
369	12	5681	247	0.82	390	13	5232	247	0.81	412	14	4782	244	0.82	−24
357	12	5928	246	0.83	377	13	5479	240	0.83	398	13	5026	239	0.83	−26
345	12	6174	239	0.84	364	12	5719	239	0.84	385	13	5265	239	0.84	−28
333		6413			352		5958			372		5504			−30

Table 7 (Continued)

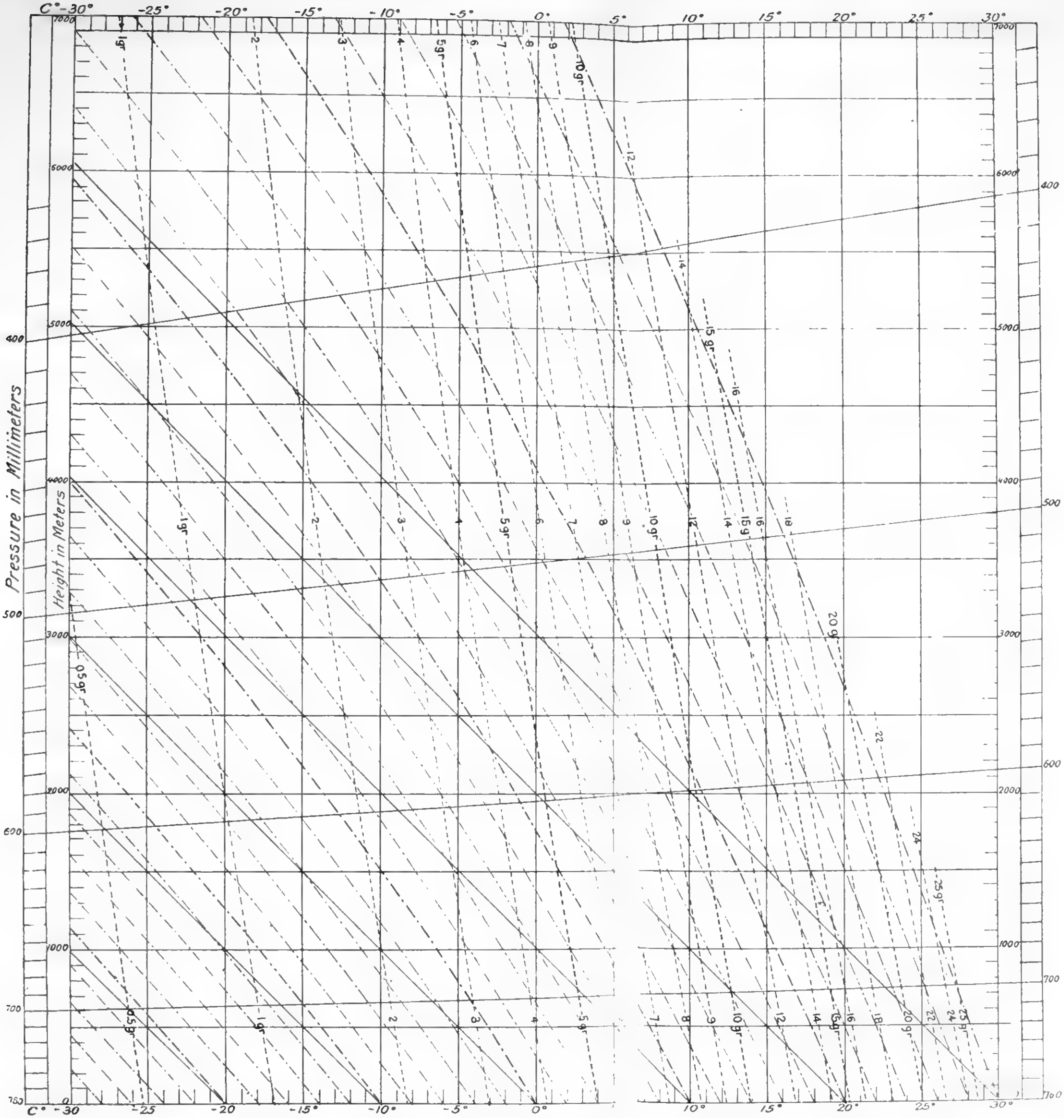
t^0	p		h		Δt^0 for 100 m	p		h		Δt^0 for 100 m	p		h		Δt^0 for 100 m
C^0	mm	Δ	m	Δ		mm	Δ	m	Δ		mm	Δ	m	Δ	
10															
8															
6	760	31	0	341	0.59										
4	729	30	341	340	0.59	760	30	0	326	0.61					
2	699	27	681	316	0.63	730	28	326	315	0.64	760	28	0	302	0.66
0	672	28	997	340	0.59	702	29	641	336	0.60	732	30	302	334	0.60
− 2	644	26	1337	326	0.61	673	27	977	324	0.62	702	27	636	310	0.63
− 4	618	23	1663	307	0.64	646	24	1301	307	0.65	675	26	946	309	0.65
− 6	595	23	1970	303	0.66	622	24	1608	297	0.67	649	24	1255	292	0.68
− 8	572	22	2273	299	0.68	598	21	1905	279	0.71	625	22	1547	278	0.72
−10	550	19	2572	277	0.71	577	20	2184	276	0.73	603	21	1825	272	0.73
−12	531	19	2849	271	0.74	557	20	2460	273	0.74	582	20	2097	268	0.75
−14	512	18	3120	270	0.75	537	19	2733	270	0.75	562	19	2365	259	0.76
−16	494	17	3390	262	0.77	518	17	3003	255	0.78	543	18	2624	252	0.78
−18	477	15	3652	249	0.80	501	17	3258	250	0.80	525	17	2876	251	0.80
−20	462	15	3901	246	0.81	484	16	3508	248	0.81	508	17	3127	245	0.82
−22	447	15	4147	242	0.83	468	15	3756	240	0.83	491	16	3372	242	0.83
−24	432	14	4389	239	0.84	453	14	3996	239	0.84	475	15	3614	239	0.84
−26	418	14	4628	238	0.85	439	14	4235	234	0.86	460	15	3853	232	0.86
−28	404	13	4866	233	0.86	425	14	4469	227	0.88	445	14	4085	228	0.88
−30	391		5099			411		4696			431		4313		
− 6	760	27	0	282	0.71										
− 8	733	25	282	276	0.73	760	26	0	269	0.74					
−10	708	25	558	269	0.75	734	25	269	265	0.76	760	26	0	267	0.75
−12	683	23	827	268	0.77	709	23	534	257	0.78	734	24	267	261	0.77
−14	660	22	1095	250	0.79	686	23	791	252	0.79	710	24	528	253	0.79
−16	638	21	1345	249	0.81	663	22	1043	252	0.81	686	22	781	243	0.82
−18	617	19	1594	238	0.83	641	20	1295	241	0.83	664	21	1024	240	0.83
−20	598	19	1832	237	0.84	621	20	1536	236	0.85	643	20	1264	239	0.84
−22	579	18	2069	236	0.85	601	19	1772	236	0.85	623	19	1503	233	0.86
−24	561	18	2305	233	0.86	582	18	2008	234	0.86	604	19	1736	230	0.87
−26	543	18	2538	231	0.87	564	18	2242	227	0.88	585	19	1966	230	0.87
−28	525	16	2769	228	0.89	546	17	2469	225	0.90	566	18	2196	227	0.88
−30	509		2997			529		2694			548		2423		
−18	760	24	0	238	0.84										
−20	736	22	238	233	0.86	760	24	0	236	0.86					
−22	714	22	471	229	0.87	736	22	236	232	0.87	760	24	0	234	0.87
−24	692	21	700	229	0.87	714	22	468	228	0.87	736	22	234	230	0.87
−26	671	21	929	224	0.89	692	22	696	227	0.89	714	22	464	221	0.89
−28	650	21	1153	224	0.90	670	21	923	222	0.90	692	22	685	220	0.91
−30	629		1377			649		1145			670		905		

Table 7 (Concluded)

p mm	Δ	h m	Δ	Δt^0 for 100 m	p mm	Δ	h m	Δ	Δt^0 for 100 m	p mm	Δ	h m	Δ	Δt^0 for 100 m	t^0 C^0
															10
															8
															6
															4
															2
760	30	0	322	0.62											0
730	29	322	320	0.63	760	29	0	308	0.65						− 2
701	26	642	297	0.66	731	27	308	295	0.67	760	28	0	295	0.68	− 4
675	25	939	294	0.68	704	26	603	294	0.68	732	26	295	282	0.71	− 6
650	23	1233	278	0.72	678	24	897	278	0.72	706	25	577	278	0.72	− 8
627	22	1511	274	0.73	654	23	1175	276	0.73	681	23	855	266	0.75	−10
605	21	1785	270	0.75	631	21	1451	264	0.76	658	22	1121	265	0.76	−12
584	20	2055	263	0.76	610	19	1715	258	0.77	636	22	1386	258	0.78	−14
564	19	2318	257	0.78	589	19	1973	252	0.79	614	20	1644	249	0.80	−16
545	17	2575	247	0.81	570	19	2225	246	0.81	594	19	1893	242	0.82	−18
528	17	2822	245	0.82	551	18	2471	245	0.82	575	18	2135	241	0.83	−20
511	17	3067	241	0.83	533	17	2716	240	0.83	557	18	2376	238	0.84	−22
494	16	3308	239	0.84	516	16	2956	236	0.85	539	17	2614	234	0.85	−24
478	16	3547	236	0.86	500	16	3192	234	0.86	522	17	2848	232	0.86	−26
462	15	3783	235	0.88	484	16	3426	229	0.88	505	16	3080	230	0.88	−28
447		4018			468		3655			489		3310			−30

p mm	Δ	h m	Δ	Δt^0 for 100 m	p mm	Δ	h m	Δ	Δt^0 for 100 m	p mm	Δ	h m	Δ	Δt^0 for 100 m	t^0
															− 6
															− 8
															−10
760	24	0	261	0.77											−12
736	25	261	247	0.80	760	25	0	252	0.80						−14
711	23	508	244	0.82	735	24	252	249	0.82	760	24	0	240	0.83	−16
688	21	752	242	0.83	711	22	501	235	0.84	736	23	240	236	0.84	−18
667	21	994	236	0.85	689	21	736	234	0.85	713	22	476	235	0.85	−20
646	20	1230	235	0.86	668	21	970	234	0.86	691	21	711	232	0.86	−22
626	20	1465	231	0.87	647	20	1204	228	0.87	670	21	943	231	0.87	−24
606	20	1696	229	0.88	627	20	1432	227	0.88	649	21	1174	226	0.88	−26
586	18	1925	215	0.89	607	19	1659	227	0.89	628	19	1400	220	0.89	−28
568		2140			588		1886			609		1620			−30

p mm	Δ	h m	Δ	Δt^0 for 100 m	p mm	Δ	h m	Δ	Δt^0 for 100 m	p mm	Δ	h m	Δ	Δt^0 for 100 m	t^0
															−18
															−20
															−22
760	23	0	229	0.88	760	24	0	230	0.90						−24
737	23	229	224	0.90	736	23	230	227	0.91	760	24	0	228	0.91	−26
714	22	453	222	0.91	713		457			736		228			−28
692		675													−30



ADIABATIC DIAGRAM (Neuhoff).

XXII

THE RELATION BETWEEN "POTENTIAL TEMPERATURE" AND "ENTROPY"[1]

BY L. A. BAUER

[*Reprinted from the Physical Review, Vol. XXVI, No. 2, February, 1908*]

In 1888 the late Professor von Helmholtz incidentally introduced the term "wærmegehalt" in connection with his investigation,[2] "On Atmospheric Motions." According to him the "wærmegehalt" or the actual heat contained in a given mass of air is to be measured by the absolute temperature which the mass would assume if it were brought adiabatically to the normal or standard pressure. It remained for the late Professor von Bezold, however, to perceive the full significance of this term and to reveal its important bearing in the discussion of meteorological phenomena.

As the quantity really involved in this new term is not a quantity of heat, von Bezold suggested that the term be replaced by the evidently more appropriate one of "potential temperature."[3] This met with von Helmholtz's approval.

With the aid of this happy idea of "potential temperature" von Bezold was enabled to draw in a simple and beautiful manner a number of important conclusions governing thermodynamic phenomena taking place in the atmosphere. Thus, for example, he found that:

"Strict adiabatic changes of state in the atmosphere leave the potential temperature unchanged, whereas pseudo-adiabatic ones invariably increase the same, the increase being in proportion to the amount of aqueous evaporation."

[1] Presented before the Philosophical Society of Washington, March 16, 1907.

[2] Sitzungsberichte Berliner Akademie, 1888, Vol. XLVI, p. 652, "Ueber atmosphærische Bewegungen," see translation in Abbe's Mechanics of the Earth's Atmosphere, Washington, 1891, p. 83. The symbol θ is used to denote the "Wærmegehalt."

[3] Sitzb. Berliner Akad., 1888, Vol. XLVI, p. 1189, "Zur Thermodynamik der Atmosphære;" also in von Bezold's "Gesammelte Abhandlungen," Vieweg und Sohn, Braunschweig, 1906, p. 128. A translation will be found in Abbe's Mechanics, etc., p. 243.

Von Bezold called attention to the fact that this law bears a striking resemblance to the well-known theorem of Clausius, now commonly known as the second law of thermodynamics, viz: "that the entropy strives towards a maximum;" but, he says, "it is not identical with it."

The purpose of this paper is to examine into the precise relationship between the two functions "potential temperature" and "entropy" and to see whether any use can be made advantageously of the former in the treatment of certain thermodynamic problems as well as to ascertain wherein the potential temperature law fails to give full expression of the second law of thermodynamics. To my knowledge no application has as yet been made of the new term in treatises on thermodynamics. The substance of this paper was communicated to the American Association for the Advancement of Science at the Springfield meeting in 1895, but publication pending opportunity for further elaboration was deferred.

The "potential temperature" of a body is defined as the absolute temperature assumed when the body is brought adiabatically to standard pressure.

Defining the thermodynamic state per *unit of mass* of a body by the three variables, T, the absolute temperature, v, the volume per unit of mass, p, the pressure supposed uniform, the following characteristic equation subsists between them: $T = f(v, p)$.

If the body be brought now adiabatically to standard pressure p_0, then the temperature assumed at the end of the process is the so-called *potential temperature* as above defined and is designated by the symbol θ. Hence,

$$\theta = f(v, p_0) \quad \ldots \ldots \ldots \quad (1)$$

For a perfect gas, since $kT = pv$, k being a constant for any particular gas,

$$\theta = \frac{p_0}{k} . v = k_0 . v \quad \ldots \ldots \ldots \quad (2)$$

or the potential temperature for any particular gas is directly proportional to the volume and, hence, as von Bezold showed, the potential temperature readily admits of a graphical representation on the usual pv diagram, being simply proportional to the v abscissæ of points of intersection of the line of standard pressure, $p = p_0$, with the adiabats.

Hence, were it possible to express the entropy function for perfect gases directly in terms of potential temperature, we should likewise have for certain cases an easy graphical representation of the entropy function.

In the *pv* diagram, fig. 1, let aa' and bb' represent portions of two adiabats, and $o'a'b'$ be the line of standard pressure $p = p_0$.

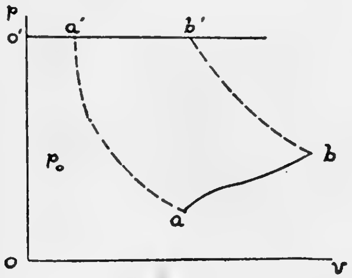


FIG. 1.

Suppose the initial thermodynamic state of the body experimented upon be represented by the point a and some process ab be carried out. According to definition, the potential temperature, θ_a, in the state a will be the temperature at the point along the adiabat aa' where it is intersected by the line of standard pressure. But according to equation (2) the temperature at this point, a', is proportional to the volume, i.e., to $o'a'$. Similarly the potential temperature in the state b will be proportional to the abscissa $o'b'$. Hence if measured on the same scale, $o'a'$ and $o'b'$ will represent directly for the same substance the respective potential temperatures. It is thus easy to represent graphically at any stage of the process ab the corresponding potential temperature.

If it is desired to determine the numerical value of the potential temperature, this can be done with the aid of the equation of the adiabat thus:

$$\theta_a = \theta_{a'} = T_a\left(\frac{v_a}{v_{a'}}\right)^{\varepsilon - 1} = T_a\left(\frac{p_0}{p_a}\right)^{\frac{\varepsilon-1}{\varepsilon}}$$

or

$$\theta_a = \left(\frac{p_0 v_a}{k}\right)^{\frac{\varepsilon-1}{\varepsilon}} T_a^{\frac{1}{\varepsilon}} = k' v_a^{\frac{\varepsilon-1}{\varepsilon}} T_a^{\frac{1}{\varepsilon}} \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

where $\varepsilon = 1.41$

For a perfect gas, the entropy, s, *per unit of mass* may be expressed by the following equation:[4]

[4] See, *e. g.*, Planck's Thermodynamics.

$$s = \int - \frac{dh}{T} = c_v \log T + k \log v + \text{Const.}, \quad . \quad . \quad . \quad (4)$$

c_p and c_v are, respectively, the specific heats at constant pressure and at constant volume; k is a constant for any particular gas. Utilizing equation (3) and remembering that

$$\varepsilon = \frac{c_p}{c_v} \text{ and } k = (c_p - c_v)$$

we get

$$s = c_p \log \theta + (\varepsilon - 1) \log \frac{k}{p_0} + \text{Const.}$$

or

$$s = c_p \log \theta + \text{Const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This gives us the relation sought between potential temperature and entropy. Since c_p is invariably a positive quantity, it follows at once that for any process the potential temperature varies in precisely the same direction as the entropy. If the entropy is increased as it invariably is for irreversible processes in accordance with the second law of thermodynamics, then is the potential temperature likewise increased. When the entropy remains constant, as for reversible processes, e.g., a strict adiabatic process, then the potential temperature likewise remains constant. In other words as far as perfect gases are concerned it is possible to express the entropy function in its simplest form by means of a quantity—the potential temperature—not only readily interpretable but also easy of direct graphical representation.

Owing to the intimate relationship between entropy and potential temperature the term "entropic temperature" might appear as being possibly a more suggestive one for von Helmholtz's "wærmegehalt" than that of "potential temperature," but it may hardly seem advisable now since von Bezold's extensive use of the latter term to recommend a change.

Cyclical process.—By turning back to the diagram, it will be seen that the change in potential temperature in going from a to b is precisely the same as from a' to b', i. e., the same as for a simple expansion process under constant pressure. Hence, in carrying out the cyclical process $abb'\,a'a$, it will readily be seen that the sum total of the potential temperature changes is zero, just as in the case of the sum total of the entropy changes.

We have in general:

$$s_b - s_a = c_p\,(\log \theta_b - \log \theta_a) = c_p\,(\log \theta_{b'} - \log \theta_{a'}) \quad . \; . \; (6)$$

or the entropy change in passing from a to b by any process whatsoever—reversible or irreversible—can be measured ideally by the temperature changes incurred in allowing the body to expand under standard pressure between the initial and final adiabats.

For other substances.—If the substance acted upon be not a perfect gas we have:

$$\int_a^b ds = s_b - s_a = \int_{a'}^{b'} -\frac{dh}{T} = \int_{a'}^{b'} c_p \frac{d\theta}{\theta} = \int_a^b c_p \frac{d\theta}{\theta} \quad . \; . \; . \; (7)$$

Here c_p is not a constant as in the case of a perfect gas, but varies with temperature and may even be discontinuous, hence it is impossible, in general, to carry out the integration of the right-hand member. This we know, however, that c_p is invariably positive, i.e., heat must always be *supplied* to a substance to raise its temperature under a constant pressure. Since

$$ds = c_p \frac{d\theta}{\theta} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (8)$$

it follows that the sign of ds is the same as that of $d\theta$, so that whenever the entropy increases, the potential temperature does likewise. This, while true for cases treated, is not so, in general, as previously explained.

In the foregoing paragraphs the law of potential temperature has been deduced from that of entropy; however, an independent deduction can readily be made if desired.

For example, we may build up the law of potential temperature in precisely the same manner as in the case of the entropy law by taking typical examples of natural processes and showing that nature unaided invariably tends to increase the potential temperature.

Thus take the well-known case of the sudden expansion of a perfect gas without the performance of external work. It is very easy to show on the pv diagram, since the adiabat is a steeper curve than the isotherm, that the potential temperature in the final stage is greater than in the initial stage.

So again with the case of heat conduction. Suppose we have the same mass of the same perfect gas enclosed in each of two vessels a and b of the same size and enclosed in a non-conducting vessel. The temperature of a is greater than b. Connect now a

and b thermally, whereupon in accordance with nature's law heat will flow from the hotter body to the colder until the two are of the same temperature. It will be found that here again the potential temperature of the entire system at the end of the process is greater than at the beginning. This may be proven most readily thus: For a perfect gas we have from (3), when the volume remains constant;

$$\theta = k'' \, T^{1/\varepsilon}$$

hence

$$d\theta = \frac{k''}{\varepsilon} \cdot \frac{1}{T^{\mu}} \, dT$$

where $\mu = 1 - 1/\varepsilon$ = positive quantity, since $1/\varepsilon < 1$. Consequently, *the change in potential temperature for a given change in absolute temperature, the volume remaining constant, decreases with absolute temperature.* Hence, although the two bodies, a and b, under the conditions imposed, change in absolute temperature by the same amount, the first losing, the second gaining, because of the law just stated, the potential temperature of the colder body, b, suffers a greater increase than the decrease in potential temperature experienced by the warmer body, a, which was to be proven.

So also for imperfect gases the law of increase of potential temperature for natural processes can be established independently of the entropy principle. It is merely necessary to show that in general the adiabat is steeper than the isotherm or that the change in potential temperature varies inversely with the absolute temperature, when the volume remains constant.

Thus far it has appeared as though the potential temperature law might suffice equally as well as the entropy law. However, in all thermodynamic problems where the element of mass enters, the former law necessarily fails to give as complete a representation of the second law of thermodynamics as the entropy law. The entropy function is not alone a function of pressure and volume but also of mass, whereas the potential temperature is independent of the latter. Equation (8) shows likewise that the substitution of the obviously more convenient function—potential temperature—for entropy cannot be made in general. There are doubtless, however, a number of thermodynamic problems, as was shown by von Bezold, as also in this paper, where the application of the potential temperature law may be found convenient. The main purpose of this paper, as above stated, has been to show the precise relationship between the two functions.

XXIII

THE MECHANICAL EQUIVALENT OF ANY GIVEN DISTRIBUTION OF ATMOSPHERIC PRESSURE, AND THE MAINTENANCE OF A GIVEN DIFFERENCE IN PRESSURE

BY MAX MARGULES

[Read at the meeting of the Imperial Academy of Sciences, Vienna, July 11, 1901, commemorating the Jubilee of the k.k. Central Institute for Meteorology and Terrestrial Magnetism; translated from the Jubilee volume]

In this memoir some minor studies connected with the problem of the cyclone have been collected together as a contribution to this memorial volume of the Central Institute for Meteorology.

In Part I there is determined the work that must be done in order to transfer air from any prescribed condition of equilibrium over to any other distribution of mass. In a closed atmospheric system this work is to be considered as potential energy. The comparison of the kinetic energy of a simple vortex with its potential energy teaches that the kinetic is by far the greater.

In Part II the discussion relates to the well-known scheme of circulation for columns of air of unequal temperatures. The calculation of the additional heat necessary for the maintenance of any given horizontal difference of pressure and its useful effect—or coefficient of efficiency—still remained to be accomplished as is now done in this part.

In the concluding Part III will be found a general calculation as to the loss of energy in moving air. The internal friction can have only an inappreciable influence on large systematic atmospheric currents. Even the complex small movements that pervade general atmospheric currents consume less of the kinetic energy of the wind than the lowest stratum gives up in starting and maintaining the waves of the ocean, or in concussion against the obstacles offered by the solid ground.

PART I. THE WORK EQUIVALENT TO A GIVEN DISTRIBUTION OF PRESSURE

(1.) IN GENERAL FOR ANY GAS

Given air in a definite volume k on which no outer forces are acting.

The initial condition is uniform constant density μ_0 and uniform constant pressure p_0. The final condition is μ and p. During the transition we have μ_t and p_t.

For a small change of condition the elementary mass δm performs a work of expansion expressed by

$$\delta\, m p_t d\left(\frac{1}{\mu_t}\right) = -\,\delta\, m\, \frac{p_t}{\mu_t^2}\, d\mu_t$$

and from beginning to end the total work is

$$\delta\, a = -\,\delta\, m \int_{\mu_0}^{\mu} \frac{p_t}{\mu_t^2}\, d\mu_t .$$

The air which when brought to the density μ is contained in the elementary volume dk, has the mass μdk, therefore the work of expansion done by the whole mass is

$$a = -\int \mu\, dk \int_{\mu_0}^{\mu} \frac{p_t}{\mu_t^2}\, d\mu_t$$

If the relation between pressure and density is independent of the path followed by the particle of air, if for instance it is arranged that the transition or change of position of the particle of air shall take place under constant temperature (isothermal), or that it shall take place without increase of heat and without exchange of heat (adiabatically), then the value of a will be determined by the initial and final conditions.

For the final distribution of pressure p the gas has a store of energy A that is equal and opposite to a. It is demonstrable by means of the aerodynamic equations that this represents the potential energy of the pressural forces for the given distribution of mass, μ, or

$$A = \int \mu\, dk \int_{\mu_0}^{\mu} \frac{p_t}{\mu_t^2}\, d\mu_t . \quad \text{. . . . . (I)}$$

(a.) Isothermal change of pressure

Let R = gas constant, T = absolute temperature and in the equation for elastic gases,

$$p = R\,T\,\mu$$

let T be constant and use the relation

$$\int \mu\, dk = \int \mu_0\, dk$$

then it follows that

$$A = RT \int dk\, \mu \log\left(\frac{\mu}{\mu_0}\right) = \int dk\, p \log\left(\frac{p}{p_0}\right) \quad . \quad . \quad (\mathrm{I}a)$$

(b.) Adiabatic change of pressure

For this case we have

$$\frac{p_t}{p_0} = \left(\frac{\mu_t}{\mu_0}\right)^{\gamma}$$

where γ is the ratio of the specific heat of a gas under constant pressure to that under constant volume, whence

$$A = \frac{1}{\gamma - 1} \int \left(p - p_0 \frac{\mu}{\mu_0}\right) dk$$

under the condition that the mass of air within the volume k remains unchanged, this becomes

$$A = \frac{1}{\gamma - 1} \int (p - p_0)\, dk \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{I}b)$$

Relatively small changes of pressure

The expressions (I*a*) and I*b*) seem to imply that the elementary volumes for which pressure and density are above the average value, give a positive addition to the integral, but that those for which these are below the average give negative contributions. But this is not correct.

If we put

$$\mu = \mu_0\,(1 + \sigma) \text{ and } p = p_0\,(1 + \varepsilon)$$

Then for an isothermal change we have

$$\varepsilon = \sigma \qquad (a)$$

For an adiabatic change we have

$$\varepsilon = \gamma\sigma + \frac{\gamma(\gamma - 1)}{2!}\sigma^2 + \dots \qquad (b)$$

And always

$$\int \sigma \, d k = 0$$

Hence we obtain for isothermal change

$$A = p_0 \int dk \left(\frac{\sigma^2}{1.2} - \frac{\sigma^3}{2.3} + \frac{\sigma^4}{3.4} \cdots \right) = p_0 \int dk \left(\frac{\varepsilon^2}{2} \cdots \right) \quad (\text{I}a^*)$$

and for adiabatic change

$$A = \gamma \, p_0 \int dk \left(\frac{\sigma^2}{2} - \frac{(2-\gamma)}{2.3}\sigma^3 + \frac{(2-\gamma)(3-\gamma)}{2.3.4}\sigma^4 \dots \right)$$

$$= \frac{p_0}{\gamma} \int dk \left(\frac{\varepsilon^2}{2} - \dots \right) \qquad (\text{I}b^*)$$

These forms, like the fundamental equation (I), show that the contribution of each elementary volume whose density deviates from the average value, is positive. The contribution of a volume in the low-pressure region is indeed somewhat larger than that of the same volume in a high-pressure region having the same absolute value of σ.

For very small changes of pressure the first term of the development is sufficient. This solution was first given by Lord Rayleigh (see Vol. II, page 22, of his Theory of Sound, German edition (Brunswick, 1880). For equal values of σ the potential energy of any distribution of pressure is γ times greater under adiabatic conditions than it is under isothermal; but with equal values of ε it is only $1/\gamma$ times as large.

The work stored in a very large volume of gas, when only a small portion of it is disturbed

Let k indicate the volume that suffers a disturbance of its equilibrium; k', the remaining far greater volume whose density is not appreciably changed by transfer of any mass to or from k; σ and

σ' the relative change of density in k and k'; we now have

$$\int \sigma\, d\, k + \int \sigma' dk' = 0$$

and for the limiting case

$$\int \sigma'^2\, dk' = 0$$

which latter equation and similar ones for the higher powers of σ' hold good for an infinite volume k' of gas.

Therefore the expressions (Ia*) and (Ib*) remain unchanged if the integrals are extended only over the disturbed portion.

In order to formulate expressions for the work, or potential energy of a closed system, we note that the share contributed by the volume k' to the potential energy is given for isothermal conditions by

$$A' = \text{Lim}\, RT \int dk' \mu' \log\left(\frac{\mu'}{\mu_0}\right) = RT\mu_0 \int \sigma'\, dk$$

$$= -RT \int (\mu - \mu_0)\, dk$$

or for adiabatic conditions by

$$A' = \frac{1}{\gamma - 1} \int (p' - p_0)\, dk = -\frac{p_0 \gamma}{\gamma - 1} \int \frac{\mu - \mu_0}{\mu_0} dk.$$

If again A indicates the potential energy of the whole mass of gas, then we have:
for isothermal conditions:

$$A = RT \int dk \left(\mu \log \frac{\mu}{\mu_0} + \mu_0 - \mu\right) = \int dk \left(p \log \frac{p}{p_0} + p_0 - p\right) \ldots (\text{I}a')$$

or for adiabatic conditions:

$$A = \frac{p_0}{\gamma - 1} \int \left[\frac{p}{p_0} - 1 - \gamma\left(\frac{\mu}{\mu_0} - 1\right)\right] dk \quad . \; . \; . \quad (\text{I}b')$$

The integrals are to be extended over the disturbed portion, or indeed over the whole volume, since the terms that are added to the previous expressions (Ia) or (Ib) contribute nothing more to the result that pertains to the volume $k + k'$.

(2.) THE ATMOSPHERE OF THE EARTH

If external forces act on the air then in a condition of equilibrium the pressure varies from place to place. In studying perturbations the potential energy of these external forces comes into consideration in addition to the potential energy of the change of distribution of pressure. Still in many cases the expressions above deduced can easily be applied.

We will designate as an atmosphere, any mass of air on which the force of gravity is acting. For brevity we assume the acceleration of gravity, g, to be constant, and the ground to be a smooth plane and the initial temperature to be a function of the altitude only. If this atmosphere be divided into individual layers of indefinitely small thickness dz then under simple assumptions we can carry out the analysis for each layer with the formulæ that apply to a mass of gas of constant initial density.

If the condition of equilibrium is disturbed in only a relatively small portion of the whole mass, then it will be assumed that the excess or deficit of gas in each horizontal layer of this disturbed cylinder comes from or has flowed into the undisturbed portion of this same layer: hence the potential of the gravitational force remains unchanged. The potential energy of the distribution of pressure is given by equation (I) or the formulæ derived therefrom. The elementary volume dk is to be replaced by the product of the elementary area dS and the altitude dz.

If we assume that the equilibrium is disturbed in the horizontal direction only, and that on the other hand the vertical equilibrium remains unchanged and that the hypsometric formula is still applicable, then the integration with reference to or along the vertical direction is easily executed.

(*a*.) *Isothermal solution*

Under isothermal conditions and according to equation (Ia') the store of work for any layer is

$$dA = dz \int \left(p \log \frac{p}{p_0} + p_0 - p \right) dS$$

where the integral extends over a surface that includes all the disturbed portion.

If now we assume the temperature of the atmosphere to be constant and designate by P the pressure at the base over the elementary surface dS then we have

$$p = Pe^{-\frac{gz}{RT}}$$

$$A = \int_0^\infty e^{-\frac{gz}{RT}} dz \int \left(P \log \frac{P}{P_0} + P_0 - P\right) dS.$$

The integral with respect to z between zero and infinity is the so-called height of a homogeneous atmosphere having the temperature T. If T is a function of the altitude still nothing is changed in this expression for the volume of A except that in RT/g in the value of the integral with respect to z, the T now indicates the mean temperature of any horizontal layer. If we write

$$P = P_0 (1 + \varepsilon)$$

we obtain the following formula which is more convenient for numerical computation:

$$A = \frac{RT}{g} P_0 \int \left(\frac{\varepsilon^2}{1.2} - \frac{\varepsilon^3}{2.3} + \frac{\varepsilon^4}{3.4} - \;.\;.\;.\right) dS \quad . \quad . \quad \text{(II}a\text{)}$$

Hence the potential energy of the assumed distribution of pressure in the atmosphere is equal to that in a layer of air on which no outside forces are acting, whose altitude is RT/g and in which the distribution of pressure in all upper strata is the same as that at the base of the atmosphere.

If M is the mass above the surface S in the undisturbed condition; $[\varepsilon^2]$ the average value of ε^2 in S, if moreover the first term of the above series is very large relative to the others, then we have the following approximate formula:

$$A = MRT \frac{[\varepsilon^2]}{2}$$

Example. The following example will serve for a preliminary estimate of the potential energy of the distribution of pressure in a cyclone:

Let the area of disturbed pressure be a circle of radius ρ; the pressure in the center at the base be P_0 (1 $-$ c) increasing linearly from that point to the circumference, so that

$$\varepsilon = -c\left(1 - \frac{r}{\rho}\right)$$

For the elementary surface dS in equation (II*a*) we have to substitute $2\pi r dr$ and making use of the relation

$$\int_0^\rho \left(1 - \frac{r}{\rho}\right)^n r dr = \frac{\rho^2}{(n+1)(n+2)}$$

we obtain for this special case

$$\frac{A}{\pi\rho^2} = \frac{RT}{g} . P_0 . \frac{2\,c^2}{4!}\left(1 + \frac{1!\,4!}{5!}c + \frac{2!\,4!}{6!}c^2 + \ldots\right)$$

as the average value of the potential energy of the system for the unit of horizontal surface.

If the barometric pressure in the center at the base is 745^{mm} [in the disturbed region,] and 760^{mm} throughout the undisturbed region we then have

$$C = \frac{15}{760} \text{ and } P_0 = 10333 \times 9.806\ kg^{+1}\ m^{-1}\ \sec^{-2};$$

for the temperature 0° C. = 273° absolute we have

$$\frac{RT}{g} = 8000 \text{ meters nearly}$$

whence

$$\frac{A}{\pi\rho^2} = 26210\ kg\ \sec^{-2} = 6.3 \text{ Kilogram-calories } m^{-2}$$

Assuming the radius of the cyclone to be 5 degrees of a great circle or $\rho = 555{,}500$ meters, then the whole work needed to produce this diminution of atmospheric density is equal to 6.1×10^{12} calories. This work will not appear so large when expressed in other terms; the equivalent amount of heat would raise the temperature of the whole volume of the cyclone (under constant pressure) by only about 0.0026° C. or approximately by the $\frac{1}{400}$ part of a degree Centigrade.

With the above assumed linear formula for ε and the values of c and ρ we have at sea level a constant gradient of pressure of 3^{mm} mercury per degree of a great circle. The average value of $A/\pi\rho^2$ is independent of ρ and nearly proportional to the square of the difference between the normal pressure and that in the center of the cyclone. A barometric reading of 730^{mm} instead of 745^{mm} at the center would increase the above computed value of $A/\pi\rho^2$ fourfold.

(*b*.) *Adiabatic solution*

If we define the average temperature (T) of the disturbed strata by the equation

$$\frac{R[T]}{g} = \int_0^\infty e^{-\int_0^z g\,dz/RT}\,dz$$

then under adiabatic conditions in the atmosphere we have

$$\left.\begin{aligned} A &= \frac{1}{\gamma}\cdot\frac{R[T]}{g}P_0\int\left(\frac{\varepsilon^2}{2}+\ldots\right)dS. \\ &= \gamma\cdot\frac{R[T]}{g}P_0\int\left(\frac{\sigma^2}{2}+\ldots\right)dS. \end{aligned}\right\} \quad \ldots\ldots (\mathrm{II}b)$$

If we confine our attention to the first term of this series then it is true that, as before in (Ia*) and (Ib*) so now for the atmosphere for equal value of T and for adiabatic changes of conditions, with equal value of σ A is γ times greater than for isothermal changes, but with equal values of ε, A is $1/\gamma$ times as large as (i. e., smaller than) for isothermal changes.

(3.) STATIONARY WHIRLS IN THE ATMOSPHERE

Let a distribution of pressure of the kind assumed in the preceding article be produced in the atmosphere by a stationary whirl; we wish to know the ratio between the kinetic energy of the moving mass and the potential energy of the difference of pressure.

Let the atmospheric particles describe circles around the vertical axis of the cylinder whose radius is ρ; the hypsometric formula applies as before. Such a motion is possible in frictionless air, that is to say, the assumptions are compatible with the aërodynamic equations, provided the velocity is constant along the vertical.

(*a*.) *Whirl in a quiet atmosphere*

For the change of pressure in the horizontal direction we have the relation

$$\frac{G^2}{r} = \frac{1}{\mu}\frac{\partial p}{\partial r} \quad \ldots\ldots\ldots\ldots (\alpha)$$

where G is the velocity of a particle and r the radius of its circular path. For the kinetic energy of the whirl we have

$$K = \int_0^\infty dz \int_0^\rho \frac{\mu G^2}{2} \, 2\pi r dr$$

$$= \pi \int_0^\infty dz \int_0^\rho \frac{\partial p}{\partial r} r^2 dr$$

$$= \pi \frac{RT}{g} \cdot P_0 \int_0^\rho \frac{d\varepsilon}{dr} r^2 dr$$

where the notation is similar to that of the preceding pages.
The assumptions of a very large volume k' of quiet air in comparison with the volume of the whirl k and no change in pressure for the outer region where $r \overline{\overline{>}} \rho$, are also retained.
Thus by partial integration we obtain

$$K = -2\pi \frac{RT}{g} P_0 \int_0^\rho \varepsilon r dr$$

Only negative values of ε are possible.
Using the value of $\varepsilon = -c(1 - r/\rho)$ adopted in the preceding example we find:

$$K = \pi \rho^2 . P_0 . \frac{RT}{g} . \frac{c}{3}$$

If M is the mass of the air in a cylinder of radius ρ under pressure 760^{mm} and the temperature T, then for K, the kinetic energy of the whirl, and for A, the potential energy of the distribution of pressure produced by the whirl, we have

$$K = MRT \frac{c}{3}$$

$$A = MRT \frac{c^2}{12}$$

$$\frac{K}{A} = \frac{4}{c}$$

For $c = 15/760$ we find K two hundred times larger than A. For $c = 30/760$ we have K still 100 times A.

(b.) Whirl on a revolving horizontal plane

For the case of relative motions above a small area on the surface of the earth we simplify the equations by the assumption of a constant polar distance and thus attain an approximation to terrestrial conditions that suffices for certain special cases.

We consider the substratum of the atmosphere as a rotating plane. In order that the atmospheric pressure in the condition of relative rest may be a function of the altitude alone, we must also assume a force directed toward the axis of rotation, opposite and equal to the centrifugal force. The location of the foot of this axis may then remain arbitrary.

Now the distribution of pressure in a stationary cylindrical whirl with vertical axis is given by the equation

$$\frac{1}{\mu}\frac{\partial p}{\partial v} = \frac{G^2}{r} + jG \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\beta)$$

where G designates the velocity of the air relative to the rotating surface of the earth and is positive when the direction of rotation of the whirl agrees with that of the earth [as in cyclones]. The constant factor of the deflecting force due to the earth's rotation is

$$j = 2\,\nu \sin \varphi$$

where ν is the angular velocity of rotation of the earth and φ is the geographic latitude of the place.

The potential energy of the distribution of pressure is not changed by the rotation. But for positive G, the kinetic energy of the relative motion is smaller than it would have been for an equal pressure gradient and $j = 0$. Therefore the ratio K/A is also smaller.

If G/jr is very large then in the equation (β) the first term on the right-hand side exceeds the second. In the case of cyclones in low latitudes whose horizontal extent is relatively small, we can estimate the value of K/A approximately according to the example just given, where $K/A = 4/c$.

At latitude 15° and for $r = 100$ km. we have $jr = 3.8$ meters per second. If now G is five times larger than this, then the right hand of equation (β) becomes ($25\,jr + jr$) so that for such cases the neglect of the second term will incur an error of 4 per cent or less. In middle latitudes (40° to 50°) the two terms are in general of the same order of magnitude.

If in equation (β) we substitute

$$\frac{1}{\mu}\frac{\partial p}{\partial r} = RT\frac{\partial \varepsilon}{\partial r}$$

then for positive values of G we have

$$G^2 = RT\left\{ r\frac{\partial \varepsilon}{\partial r} - \frac{j^2 r^2}{2\,RT}\left(\sqrt{1 + \frac{4\,RT}{j^2 r^2}\,r\,\frac{\partial \varepsilon}{\partial r}} - 1\right)\right\}.$$

Example. For this example, and for the sake of abbreviating the computation, we here assume another law for the distribution of pressure, i. e.:

$$\varepsilon = -c\left(1 - \frac{r^2}{\rho^2}\right)$$

(for the region over which $r \overline{\overline{<}} \rho$)

We thus obtain

$$K = M\,.\,R\,.\,T\,\frac{c}{2}\left\{1 - \frac{j^2 s^2}{4\,RT}\cdot\frac{1}{c}\left(\sqrt{1 + \frac{8\,RT}{j^2 \rho^2}\,c} - 1\right)\right\}.$$

If we assume the following special numerical values

$$\log j = 6.01337 - 10; \qquad RT = 287.273\ m^2\ \sec^{-2};$$

$$c = \frac{30}{760}\ ; \quad \frac{4\,RT}{j^2 \rho^2}\,c = 1$$

corresponding to a whirl of 1080 km. radius, with the barometric pressure 730^{mm} at the center, in latitude 45°, then we find

$$K = M\,.RT\,\frac{c}{2}\,0.268.$$

If the earth be not rotating (that is to say if $j = 0$) and the other conditions be retained, we should only be able to maintain this same distribution of pressure by assuming a whirl whose kinetic energy is

$$K = MRT\,\frac{c}{2}$$

For both these cases, under the newly assumed law for ε we find the potential energy of the pressure distribution to be

$$A = M \cdot RT \cdot \frac{c^2}{6}$$

hence the ratio A/K is 1/20 for the rotating earth, but 1/76 for the non-rotating earth.

Although this ratio is thus seen to be appreciably increased

$$\text{from } 1/76 = 0.013 \text{ to } 1/20 = 0.050$$

by the action of the so-called deflecting force of the earth's rotation, still even in whirls of the middle latitudes the potential energy of the distribution of pressure is far less than the kinetic energy of the relative motion.

(4.) PROGRESSIVE WHIRLS IN THE ATMOSPHERE

One may ask whether the progressive cyclones are more properly compared with single waves or with revolving whirls. Cyclones have one feature in common with waves, i. e., the partial interchange of the moving masses. On the other hand the dust fall progressing over broad areas with some individual cyclones (notably from Sicily to the North Sea on March 10 and 11, 1901) is a proof that a large portion of the atmosphere remains constrained to move in true whirls.

As regards the ratio K/A of the kinetic to the potential energies, cyclones are to be considered as whirls rather than as waves. According to Lord Rayleigh, "Theory of Sound," this ratio is unity for progressive plane waves of air. In our present case we find it from the following rough estimate:

Let M be the moving mass of air, $[G^2]$ the average of the squares of the velocity, $[\varepsilon^2]$ the average of the squares of the relative diminutions of pressure, then we have

$$K = M\frac{[G^2]}{2}\,;\ A = MRT\frac{[\varepsilon^2]}{2}\,;\ \frac{K}{A} = \frac{[G^2]}{RT\,[\varepsilon^2]},$$

where $\sqrt{RT}$ is the speed of propagation of isothermal waves or the so-called Newtonian velocity of sound and is 280 meters per second for temperature 0° C.

For a cyclone wherein the maximum absolute value of ε is $3/76$ we may estimate the average value $[\varepsilon^2]$ as being at the very highest, only $3/76^2$ as in the last example, since the smaller values of ε cover by far the larger surfaces. In order that K should equal A the value of $[G^2]$ must be $(6.4)^2$, hence the average value of the velocity must be not more than 6.4 meters per second.

Observations give even for the lowest layer a greater average velocity than this for the lowest barometric pressure of 730^{mm}. If the radius of the cyclone is 10° of a great circle and the average gradient 3^{mm} mercury, then the mean wind velocity for the lower stratum of air (about 20 meters above the ground) at median latitudes is to be estimated at 12 meters per second, but that of all the higher strata at, at least 18m/sec., and therefore K is at least 8 times larger than A.

(5.) RELATIONS BETWEEN PRESSURE AND WORK

The equation (I) has been based on the conception of work done by the expansion of the gas and is provisionally spoken of as the potential energy of the distribution of pressure within the closed volume k. If this be correct then the work done by the pressural forces in any elementary portion of time must be equal to the change in $-A$.

The work done by the pressural forces on the small mass δm during dt is given by the expression

$$-\frac{\delta m}{\mu} \cdot \frac{\partial p}{\partial s} G\, dt = -G \frac{\partial p}{\partial s} dk.dt$$

where ds is an element of the path of the moving mass and G is the velocity in the direction ds. This is equivalent to

$$-\left(u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} \right) dk.dt$$

where u, v, w are the component velocities along the axes of x, y, z. We have to prove that

$$\frac{\partial A}{\partial t} = \int \left(u \frac{\partial p}{\partial x} + v \frac{\partial p}{\partial y} + w \frac{\partial p}{\partial z} \right) dk.$$

In equation (I) substitute

$$\int_{\mu_0}^{\mu} \frac{p}{\mu^2}\, d\mu = F\,(\mu) - F\,(\mu_0)$$

then with the equation

$$\int \mu\, dk = \int \mu_0\, dk$$

which expresses the constancy of the mass of the gas enclosed within the volume k we obtain the following:

$$A = \int \mu\, F\,(\mu)\, dk = \text{Constant}; \quad \frac{dF}{d\mu} = \frac{p}{\mu^2}$$

$$\frac{\partial A}{\partial t} = \int \frac{\partial \mu}{\partial t}\left(F + \frac{p}{\mu}\right) dk$$

The equation of continuity of a mass of gas

$$\frac{\partial \mu}{\partial t} + \frac{\partial\,(\mu\, u)}{\partial x} + \frac{\partial\,(\mu\, v)}{\partial y} + \frac{\partial\,(\mu\, w)}{\partial z} = 0$$

combined with the preceding gives us

$$\frac{\partial A}{\partial t} = \int \frac{p}{\mu}\frac{\partial \mu}{\partial t}\, dk - \int F\left[\frac{\partial\,(\mu\, u)}{\partial x} + \frac{\partial\,(\mu\, v)}{\partial y} + \frac{\partial\,(\mu\, w)}{\partial z}\right] dk.$$

By well-known transformation, the second integral on the right hand becomes

$$-\int \mu\left(u\frac{\partial F}{\partial x} + v\frac{\partial F}{\partial y} + w\frac{\partial F}{\partial z}\right) dk - \int \mu F\,(u \cos Nx + v \cos Ny + w \cos Nz)\; dO$$

where O is the surface of the volume k and N is the normal to that surface directed inward.

The last portion of this expression vanishes when no gas passes inward or outward through O.

Under this latter condition we have

$$\frac{\partial A}{\partial t} = \int \frac{p}{\mu}\left(\frac{\partial \mu}{\partial t} + u\frac{\partial \mu}{\partial x} + \ldots\right) dk$$

$$= -\int p\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dk$$

$$= \int \left(u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z}\right) dk.$$

The first and second of these equations state that $-\delta A \delta t$ is the sum of the works of expansion done by all the elementary masses within the volume k in a unit of time; the third equation states that this is also the work done in the same unit of time by the pressural forces. In general these two quantities of work differ for each individual elementary mass; it was therefore important to demonstrate that starting with the work of expansion we arrive finally at a correct expression for the potential energy of the distribution of pressure.

We can also deduce the value of A by another route, i. e., by computing the work done by the pressural forces during the passage from the initial to the final stage. We thus arrive at the expression

$$A = \int dk \int_{\mu_0}^{\mu} d\mu \int_{p_0}^{p} \frac{dp}{p}$$

whose identity with equation (I) can easily be proved: both of these, by partial integration under the assumption of the constancy of the mass of gas within the volume k, lead to the following:

$$A = \int \mu dk \int_{p_0}^{p} \frac{dp}{\mu} - \int dk\ (p - p_0)$$

Now the simple assumptions that have been made the foundation of the preceding computation of A do not obtain in the atmosphere. If we assume the atmosphere to pass adiabatically from any initial condition in which we happen to find it over into a condition of equilibrium, then this will not be possible unless some masses of air pass from some one horizontal layer over into another layer. If an interchange of heat take place, then, except in the case of constant temperatures the whole work done by the pressural forces cannot be expressly or exactly stated, for it depends on the path along which the transfer takes place. Generally the question is as to the changes of A with time and these can be computed provided that the serial succession of conditions is known. But the total potential energy can only be given under certain assumptions; an estimate of its value can however be obtained by means of the formulæ here deduced.

APPENDIX TO PART I

(6.) THE EQUATION OF ENERGY OF A FRICTIONLESS MOVING MASS OF AIR

Retaining the notation of the last section we have for a unit mass of air the following equation for the living force or kinetic energy:

$$\frac{1}{2}\frac{d\,(G^2)}{dt} = G\frac{dG}{dt} = -G\frac{\partial V}{\partial s} - \frac{G}{\mu}\frac{\partial p}{\partial s}$$

where V or the potential of the exterior force is so chosen that the negative derivative with reference to any coördinate expresses the force acting on the unit mass, in the dissection of that coördinate.

We will also introduce the Eulerian Symbol

$$\frac{d}{dt} = \frac{\partial}{\partial t} + G\frac{\partial}{\partial s}$$

$$= \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

which expresses the variation with time of the variables associated with the elementary mass. If V is only a function of the location then the equation of energy becomes

$$\frac{1}{2}\frac{dG^2}{dt} = -\frac{dV}{dt} - \frac{1}{\mu}\left(\frac{dp}{dt} - \frac{\partial p}{\partial t}\right) \tag{III}$$

This holds good also for movements relative to the earth.

(*a.*) *Horizontal motion in a steady field of pressure*

When gravity is the only exterior force and the distribution of pressure remains unchanged, the equations of condition for horizontal movements are:

$$\frac{dV}{dt} = 0 \text{ and } \frac{\partial p}{\partial t} = 0$$

whence

$$\frac{1}{2}\frac{dG^2}{dt} = -\frac{1}{\mu}\frac{dp}{dt} = -\frac{RT}{p}\frac{dp}{dt}.$$

(a) Under isothermal conditions, therefore, the integral becomes

$$\frac{1}{2}(G^2 - G_0^2) = RT \log \frac{p_0}{p}$$

where G_0, p_0 and G, p are associated sets of values.

(b) For movements with adiabatic changes (see I*b*) we have the equation

$$C_p \frac{dT}{dt} - \frac{RT}{p} \cdot \frac{dp}{dt} = 0$$

Here it should be noted that the second term with its negative sign represents the work of the pressural forces on the unit mass in unit time in a *steady field*, and for subsequent use we also note that the whole work done on the unit mass in moving it from p_0 to p or from T_0 to T is equal to

$$C_p\ (T_0 - T)$$

This is quite independent of whether the motion takes place with or without friction. In the case of frictionless motion we have therefore,

$$\frac{1}{2}(G^2 - G_0^2) = C_p\ (T_0 - T) = C_p\ T_0\left(1 - \left(\frac{p}{p_0}\right)^{R/C_p}\right)$$

(c) If $(p_0 - p)$ is small relative to p_0 then the preceding equations (*a*) and (*b*) give alike the same approximate formula for the increase of the kinetic energy of the unit mass

$$\frac{1}{2}(G^2 - G_0^2) = RT_0 \frac{p_0 - p}{p_0}$$

$$= \frac{p_0 - p}{\mu_0}$$

The following small table gives the velocity that a mass of air acquires in passing horizontally and without friction through a stationary field from rest and the pressure corresponding to 760^{mm} mercury to a pressure that is less by 1 . . . . 30^{mm}. It is computed[1] from this last equation, assuming $RT_0 = 287.273\ m^2 \sec^{-2}$.

Computed G, assuming no friction and small changes of pressure.

$p_0 = 760$ and $G_0 = 0$	
$p_0 - p$ mm.	G
1	14.4
2	20.3
5	32.1
10	45.4
20	64.2
30	78.6

From this table we may perceive to what linear distance a horizontal current of air, in a steady field, can flow, frictionless, against a given gradient. If its initial velocity is 20 meters per second then this sinks to zero as soon as the pressure rises by 2/760 of the initial pressure. With an initial velocity three times as great, or a living force nine times as great, it can overcome a difference of pressure of 18^{mm} mercury in the lower horizontal layer.

(*b.*) *Horizontal motion in a variable field of pressure*

Equation III now becomes

$$\frac{1}{2}\frac{dG^2}{dt} = -\frac{1}{\mu}\frac{dp}{dt} + \frac{1}{\mu}\frac{\partial p}{\partial t}$$

[1] Compare p. 416 of the recently published Lehrbuch by Hann, where these same results are deduced in another way.—ABBE.

For relatively small changes of pressure this gives us

$$\frac{1}{2}(G^2 - G_0^2) = RT\,\frac{p_0 - p}{p_0} + \frac{RT}{p_0}\int \frac{\partial p}{\partial t}\,dt$$

If an air mass is flowing toward a place of lower pressure and at the same time the pressure at every point of its path is changing with the time, then the increase of the living force of the moving mass is no longer determined simply by its initial and final pressure. If the pressure rises with the time then the increase in kinetic energy is greater than it would be in a steady field, but if the pressure falls, then the increase is less.

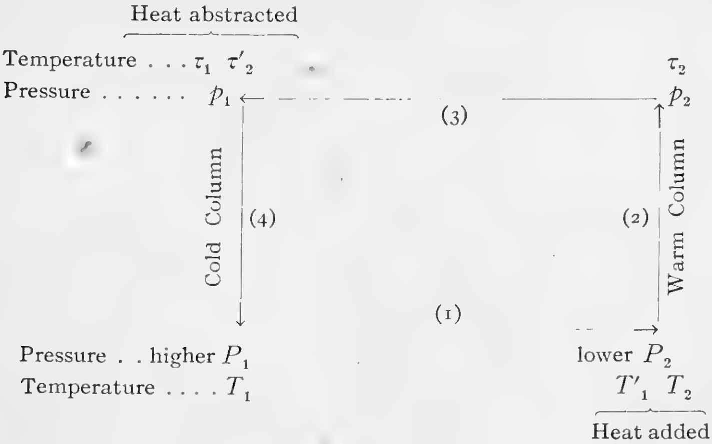
Assume that the barometric pressure in the moving mass under consideration falls 10^{mm} during ten hours, but that in the field through which the moving mass describes its path it rises 10^{mm}, then in this case the increase of the kinetic energy of the moving mass is twice as great as in a steady field. But if the pressure in the field surrounding the path had fallen 10^{mm} instead of rising during these ten hours, then the moving mass would not have needed to move at all and the increase of kinetic energy over that of the stationary field would have been zero.

PART II. ON THE MAINTENANCE OF A DIFFERENCE OF PRESSURE BY THE ADDITION OF HEAT

(7.) STEADY CIRCULATION IN A DRY ATMOSPHERE

During movements of the air out of regions of higher pressure into regions of lower pressure, work is being expended continuously by the pressural forces drawing from a previously accumulated supply. The potential energy of the system must exhaust itself and the differences of pressure at any level must disappear, unless there be compensation from some source. Movements against the gradient could indeed reconvert kinetic energy into potential energy, but then the process would develop some sort of wave action and even then the loss by friction must be replaced.

So far as the study of energy is concerned, one can imagine a scheme for a steady circulation between regions of differing pressures as explained in the following diagram and text.



(1) At the lower level the air flows from the higher pressure P_1, to the lower P_2 and at the same time receives an increase of heat. For the sake of the analysis we assume that the adiabatic change of condition prevails in the passage from P_1 to P_2 and that there has therefore been a cooling from T_1 to T'_2 but that then an addition of heat under the pressure P_2 suddenly takes place, producing a rise from T'_2 to T_2.

(2) An adiabatic ascent at the location of lower pressure P_2 and above it a vertical equilibrium prevails or a condition inappreciably different therefrom, so that the pressure falls to p_2 and the temperature falls to τ_2.

(3) A horizontal movement along the upper level from p_2 to p_1 together with cooling by radiation or conduction; this process will for convenience in analysis be decomposed into adiabatic change of condition from $p_2\,\tau_2$ to $p_1\,\tau'_2$ then abstraction of heat and cooling from τ'_2 to τ_1 under the pressure p_1 to such an extent that, ...

(4) When the air descends adiabatically it arrives at the original temperature T_1 and the pressure P_1. Here also we assume that the equilibrium is maintained between gravity and the vertical diminution of pressure.

In both the two vertical portions of this path the change of temperature with altitude is at the adiabatic rate $-\ g/C_p$ per unit of length (for dry air $C_p = 987\ m^2\ sec^{-2}\ Centigrade^{-1}$. On any given level the difference of temperature between the two vertical columns is constant. The difference of pressure will diminish with the altitude and become zero at a certain altitude and above that it will increase but with the opposite sign.

We choose an appropriate altitude for the column of air such that the horizontal motion of the upper layer takes place in the direction of the gradient—though this is not really important.

The potential energy would diminish by the amount of work done by the pressural forces, if there were no addition of heat. The quantity of heat converted into work supplies that potential energy which is converted into kinetic energy by motion in the direction of the gradient. This increase of kinetic energy is consumed by friction.

For each kilogram of air that makes this complete circulation, and during the interval of time occupied in so doing, we have the quantity of heat added at P_2, or below

$$(T_1' - T_2)\ Q = C_p$$

The quantity of heat abstracted at p_1 or above

$$(\tau - \tau_2)\ Q' = C_p$$

The quantity of heat converted into work

$$Q - Q' = q = C_p\,[T_2 - T_1' - (\tau_2' - \tau_1)\,]$$

Now we have

$$\tau_2 = T_2 - \frac{g}{C_p} h$$

$$\tau_1 = T_1 - \frac{g}{C_p} h$$

Whence

$$T_2 - \tau_2 = T_1 - \tau_1$$

or

$$T_2 + \tau_1 = T_1 + \tau_2$$

and

$$\left.\begin{aligned} q &= C_p\ (T_1 - T_1' + \tau_2 - \tau_1') \\ &= C_p\ T_1\left[1 - \left(\frac{P_2}{P_1}\right)^{R/C_p}\right] - C_p\,\tau_2\left[1 - \left(\frac{p_1}{p_2}\right)^{R/C_p}\right] \end{aligned}\right\} \quad . \ . \ \text{(IV)}$$

As stated in §6 the work done on 1 kilogram of air by the pressural forces in the stationary field along the path $P_1\,P_2$ is $C_p\ (T_1 - T_1')$ and $C_p\ (\tau_2' - \tau_2')$ along the path $p_2\,p_1$. According to the assumed conditions of our problem there is no active effective force and therefore no work done along the remaining portions $P_2\, p_2$ and $p_1\, P_1$ of the whole circuit. Hence the quantity of heat q is the equivalent of the total work done along the horizontal portions of this circulation.

Since by introducing the equation for adiabatic change of condition we obtain

$$T_2\, \tau_1 = T_1'\, \tau_2' = T_1\, T_2 \left(\frac{p_1}{p_2} \right)^{R/C_p}$$

therefore for 1 kilogram the total change of entropy or the sum of the changes during one circulation is

$$\int_{T'_1}^{T_2} \frac{C_p\, dT}{T} + \int_{\tau_2'}^{\tau} \frac{C_p\, dT}{T} = C_p \log \frac{T_2\, \tau_1}{T_1'\, \tau_2'} = 0$$

This cycle is reversible. If during the upper path the air is forced against the gradient, then we have to add as much heat as was withdrawn in the above-described direct cycle, and similarly in the lower path we must abstract heat instead of adding it. The work done against the pressural force is converted into heat and the difference of pressure remains unchanged.

The useful effect or efficiency of the heat added in the former direct cycle is very nearly equal to

$$1 - \frac{\tau_2' + \tau_1}{T_1' + T_2}$$

and it increases with the difference of level of the upper and lower horizontal paths.

(8.) STEADY CIRCULATION IN MOIST ATMOSPHERE

A cycle with additions of heat varied so as to imitate the process in moist air may be constructed in the following manner:

In the preceding scheme let aqueous vapor be added at P_2 and let the mixture of air and vapor ascend so high that at p_2 the vapor has nearly disappeared. The water condensed in each elementary portion of the path $P_2\, p_2$ is assumed to fall away immediately and be collected again at P_2. In this case using one cycle the addition of heat to one kilogram of dry air is not only the quantity needed to raise its temperature from T'_1 to T_2 but also the additional quantity needed for the evaporation of the water taken up by that kilogram of air.

In the case of adiabatic change of condition along the path $P_2 p_2$ the condensation of vapor causes the vertical temperature decrease to be smaller than in dry air. The ratio of the masses of saturated vapor and dry air in any given volume is nearly 0.01 at 15° C. If we neglect small quantities of this order of magnitude as compared with unity then we have

$$ldx = C_p [(d\tau) - dt]$$

where dx is the mass of vapor condensed per kilogram of dry air along the path dz; ldx is the corresponding amount of heat of evaporation; $-d\tau$ the corresponding change of temperature in saturated moist air and $-(d\tau)$ the corresponding change of temperature in dry air.

The complete treatment of a cycle for moist air requires a great display of formulæ. But we can with dry air imitate all that is important and thus the process becomes more perspicuous.

Instead of adding the heat required for evaporation at the point P_2 on the lower path, where it would be used to warm the air ascending along the path $P_2 p_2$ (that is to say, in diminishing the vertical diminution of temperature)—we arrange a graduated series of sources of heat along the path $P_2 p_2$ so as to cause a prescribed temperature to prevail along this whole path so that the change $-dt$ occurs in the distance dz. If the change of temperature is to be $-(d\tau)$ for adiabatic conditions then the quantity of heat to be added will be $dQ = C_p[(d\tau) - d\tau]$ where we write dQ instead of ldx.

We now imagine the following cycle:

(1) Movement along the path $P_1 P_2$ and heat added at P_2 as in previous case.

(2) Heat added along the path $P_2 p_2$ in such a manner that the temperature at every altitude has a prescribed value. Equilibrium between gravity and the vertical diminution of pressure.

(3) and (4) as before.

The conditions of the second step in this cycle are for the added heat

$$dQ = C_p dT - \frac{RT}{p} dp$$

and for the vertical change of pressure under static equilibrium

$$\frac{RT}{p}\frac{dp}{dz} = -g$$

whence this second step requires that

$$\frac{dQ}{dz} = C_p\left(\frac{dT}{dz} + \frac{g}{C_p}\right).$$

But in addition to the quantity of heat $C_p\ (T_2 - T'_1)$ added at P_2 there is still to be added

$$C_p\left(\tau_2 - T_2 + \frac{g}{C_p}h\right)$$

along the path $P_2\ p_2$; and as before there is to be subtracted at p_1 the quantity

$$C_p\ (\tau_2' - \tau_1),$$

the quantity converted into work is

$$q = C_p\left(\tau_2 - T_1' + \frac{g}{C_p}h + \tau_1 - \tau_2'\right) \quad . \quad \quad . \ . \ \text{(IV)}$$

$$= C_p\ (T_1 - T_1' + \tau_2 - \tau_2') \quad .$$

But this expression, which is of the same form and meaning as in the process for dry air (see IV of §7), has now a different numerical value. If $P_1\,P_2\,T_1\,T_2$ and h remain the same in the two cases but the column $P_2\,p_2$ is warmer now than before, then will p_2 be larger, but p_1 remains unchanged and a larger value of $\tau_2 - \tau_2'$ corresponds to the larger difference $p_2 - p_1$. In the second case more heat is converted into work than in the first case; this increase will be used to maintain the greater difference of pressure that now exists in the upper layer.

The following numerical data are made the basis of an example of this second method:

$$h = 6000^{\mathrm{m}} \qquad g = 9.8\,\frac{m}{\sec^2} \text{ for the average altitude}$$

$$\frac{R}{C_p} = \frac{0.41}{1.41} \qquad R = 287\,\frac{m^2}{\sec^2 \text{ Cent.}^\circ}$$

$$C_p = 987\,\frac{m^2}{\sec^2 \text{ Cent.}^\circ} = 0.2375\,\frac{\text{Calories}}{kg\ C.^\circ}$$

$$P_1 = 770^{\mathrm{mm}} \text{ mercury} \qquad P_2 = 740^{\mathrm{mm}} \text{ mercury}$$

$$T_1 = 285^\circ\ C. \qquad T_2 = 288^\circ\ C.$$

$$\left(\frac{dT}{dz}\right)_1 = -\frac{g}{C_p} = -0.009929\,\frac{C.^\circ}{m} \qquad \left(\frac{dT}{dz}\right)_2 = -0.006$$

$$\tau_1 = 225.4^\circ \qquad \tau_2 = 252^\circ.2$$

We assume the vertical temperature gradient in the column $P_2\,p_2$ to be constant: it is very nearly equal to its average value for moist air saturated at the temperature 15° C. at the sea level. Using the hypsometric equation for linear vertical temperature gradient

$$\frac{p}{P} = \left(\frac{\tau}{T}\right)^{-g/R\frac{dT}{dz}}$$

we compute

$$p_1 = 343.62^{\mathrm{mm}} \qquad p_2 = 345.92^{\mathrm{mm}} \text{ mercury}$$

and using the analogous equation for adiabatic temperature gradient

$$\frac{T}{T'} = \left(\frac{P}{P'}\right)^{R/C_p}$$

we compute

$$T_1' = 281^\circ.73\ C. \qquad \tau_2' = 251^\circ.51\ C.$$

The quantity of heat to be communicated to a kilogram of air along the vertical path $P_2\,p_2$ is $23.6\ C_p = 5.6$ calories. Air saturated at 15° C. and 740$^{\mathrm{mm}}$ contains 0.01086 *kg* vapor for each kilogram of dry air; assuming the latent heat of evaporation to be 595 therefore the heat in this quantity of vapor is equal to 6.46 calories and can do the corresponding amount of work.

		Calories
(1) The heat added to a kilogram of air during a complete cycle is	$29.87\ C_p$	$= 7.09$
(2) The heat withdrawn during the cycle	$26.11\ C_p$	$= 6.20$
(3) The heat converted into work	$3.76\ C_p$	$= 0.89$
(4) The efficiency of the added heat $3.76/29.87 = 0.126$		

We will now, for comparison, compute an example for dry air by the first process: the h, P_1, P_2, T_2 remains as before, but T_1 is so chosen that the average difference of temperature of the two vertical paths is nearly the same as before in the example for above, i. e., 15° C. or

$$\tfrac{1}{2}(T_1 + \tau) - \tfrac{1}{2}(T_2 + \tau_2) = 15^\circ$$

Assumed data for $h = 6000^{\mathrm{m}}$

$P_1 = 770^{\mathrm{mm}}$	$T_1 = 273$	$\left(\frac{dT}{dz}\right)_1 = -g/C_p$	$\tau_1 = 213.4$
$P_2 = 740^{\mathrm{mm}}$	$T_2 = 288$	$\left(\frac{dT}{dz}\right)_2 = -g/C_p$	$\tau_2 = 228.4$

Computed data

$$p_1 = 330.07^{\mathrm{mm}} \qquad T_1' = 269.87$$

$$p_2 = 333.38 \qquad \tau_1' = 227.74$$

		Calories
Added heat per kilogram	$18.13\ C_p$	$= 4.31$
Abstracted heat per kilogram	$14.34\ C_p$	$= 3.41$
Heat converted into work	$3.79 C_p$	$= 0.90$
Efficiency $3.79/18.13 = 0.21$		

In these examples of these two processes, for equal differences of temperature between the two vertical columns of air we have almost equal quantities of heat (0.89 and 0.90 calories) converted into work; but in the second example for dry air the efficiency is greater than in the first (for moist air).

The quantity of heat converted into work in the unit of time is nq/θ, where n is a factor depending on the sectional area of the circulating stream and θ is the length of time required for one cycle. If other conditions are the same then θ increases with the length of the path, but q is independent of it. The longer the length of path the less heat must be expended in maintaining an equal difference of pressure in the two vertical columns.

(9.) COMPARISON OF THE PRECEDING SYSTEM WITH NATURE

The preceding scheme was formerly a favorite one when it was assumed that the anticyclones of winter are cold; but it has become obsolete since Hann has shown that this is only true for the lowest calm layer and that on the other hand the higher strata of air in anticyclones are very warm.

In an area of high pressure the columns of air not only have a temperature that is high for the season, but also one that is higher than anywhere in the surrounding region of lower pressure. In the lower portion the air flows steadily away in the direction of the gradient: while the anticyclone as a whole remains stationary and often for a week or more. We therefore must necessarily assume that in the upper layer there is an inflow [toward the anticylone].

Under this assumption the upper inflow can only take place *against* the gradient. The differences of pressure do not disappear with elevation, but become relatively larger in the upper level provided the whole column in the area of high pressure is warmer than the surrounding air. If we assume a circulation to exist under these conditions then we cannot assume any heat to be converted into work. The pressural forces do the work below the lower layer, but for the inflow in the upper layer work must be expended, and more than we gain in the lower layer. Hence this system or cycle cannot be considered as in any manner similar to that hitherto considered but must maintain itself by drawing directly from a store of kinetic energy that feeds the upper inflow. Like all other movements on the earth, this kinetic energy can only have its ultimate source in heat: but in order to get an idea of the whole process we must consider the conditions prevailing over a very much larger region and for that purpose must devise some scheme that shall include the conversion of heat into work.[2]

---

[2] Compare Ekholm, Met. Zeit. 1891, p. 366.

PART III. FRICTION

(10.) INTERNAL FRICTION OR VISCOSITY

There are many obstacles to the analytical treatment of great currents of air; one of these is the difficulty of introducing the influence of friction in a proper manner into the equations of motion. This influence certainly is very large: the unequal warmings of the air are continually giving rise to new differences of pressure and new motions, but there is no corresponding steady increase in the mechanical energy. Hence for large intervals of time the whole increase of energy is consumed by friction. An argument for this conclusion can also be based on the motions of individual masses of air. Thus, near the ground and in by far the most numerous cases we find a component of the wind in the direction of the pressure gradient; hence the motion of the wind is thereby accelerated. The same peculiarity or a motion perpendicular to the resultant force also occurs in the upper layer, or at least the study of the winds during balloon voyages has as yet established nothing as to winds contrary to the barometric gradient. It is scarcely to be doubted that such cases do occur, but they appear not to be very frequent. There is no other reason for the diminution of velocity except movement against the active forces and friction. If the first of these very rarely occurs then in general it must be true that the friction prevents the steady acceleration of the moving mass of air.

And yet the influence of the internal friction or viscosity of the air on movements that occupy a large volume is certainly *very* slight. This has been demonstrated many times and in various ways by Helmholtz. Perhaps it will not be superfluous to estimate the consumption of energy by viscosity by using the following equation deduced by Stokes:

$$\frac{\partial K}{\partial t} = \kappa \int \left\{ \begin{array}{c} 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] \\ + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2 \\ - \frac{2}{3}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right]^2 \end{array} \right\} dk.$$

This equation gives the quantity of energy consumed by viscosity in the unit time within the space k.

The coefficient of viscosity of the air may with abundant accuracy for the purpose of our estimate be assumed to be

$$\kappa = 0.00002 \; kg \; m^{-1} \sec^{-1}$$

In order to study a movement that takes place with a very large amount of internal friction, or in order to greatly overestimate the influence of this friction it will be assumed that each component of the velocity, along the direction of each of the three rectangular axes, increases or diminishes by 10 meters per second per kilometer of distance traveled, so that

$$\left.\begin{cases} \dfrac{\partial u}{\partial x} = \dfrac{\partial u}{\partial y} = \dfrac{\partial u}{\partial z} \\[2ex] \dfrac{\partial w}{\partial x} = \dfrac{\partial w}{\partial y} = \dfrac{\partial w}{\partial z} \\[2ex] \dfrac{\partial v}{\partial x} = \dfrac{\partial v}{\partial y} = \dfrac{\partial v}{\partial z} \end{cases}\right\} = \frac{10 \; m \sec^{-1}}{1000 \; m}$$

$$\left(\frac{\partial u}{\partial x}\right)^2 = \left(\frac{\partial v}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial z}\right) = \ldots 0.0001 \sec^{-2}$$

Moreover it will be assumed that the last or negative term with the factor $\frac{2}{3}$ in the Stokes equation as given above is to be omitted. Under these assumptions Stokes equation becomes

$$\begin{aligned} \frac{\partial K}{\partial t} &= \kappa \int 18 \left(\frac{\partial u}{\partial x}\right)^2 dk \\ &= \kappa \int 0.0018 \, dk \\ &= 0.0018 \, \kappa k \end{aligned}$$

Hence, for a column of the atmosphere standing on a square meter whose height we will for further exaggeration estimate at 100,000 meters and whose volume is therefore $10^5 m^3$ we have

$$\frac{\partial K}{\partial t} = (2 \times 10^{-5}) \; (10^5) \; (18 \times 10^{-4})$$

$$= 36 \times 10^{-4} \frac{kg \; m^2}{sec^3}$$

Since the work corresponding to one kilogram-meter is 9.8, expressed in these same units therefore it will require at least

$$\frac{9.8}{0.0036} \text{ seconds} = 7.6 \text{ hours}$$

for an amount of kinetic energy equivalent to one kilogram-meter existent in a square-meter column of this hypothetical atmosphere to be consumed by internal friction.

Now the kinetic energy of this column of atmosphere, assuming each part of it to have a velocity of 10 meters per second is equal to

$$8000 \times 1.293 \times 50 \; kg \; m^2 \; sec^{-2}$$

or more than 50,000 kilogram-meters. Thus we see that the kinetic energy in the atmosphere would last a very long time if it were to be consumed only by such internal friction or viscosity as is effective in strictly parallel or lamellar motions.

(11.) RESISTANCES AT THE SURFACE OF THE EARTH

Other much greater obstacles to motion must be present. The roughness of the surface of the earth, the irregularity of the motion, hence also the numerous small whirls that originate and disappear in the large currents, and even surfaces of discontinuity must be taken into consideration. Possibly the first of these is sufficient so that the loss of energy may not be caused by exterior friction proper, but by impact and the sudden transmission of the energy of the lowest layer of air to solid and fluid bodies. So long as differences of pressure exist the lowest stratum of air will be continuously accelerated and for this purpose the energy will be furnished by the upper strata. If now the whole system receives no energy from without, while on the other hand the nth part of the total supply on hand (E) is consumed per unit of time (either by maintenance of waves, or by carrying up of dust and water vapor by pushing or overturning branches, trees, houses, etc.), therefore, we have

$$\frac{\partial E}{\partial t} = -\frac{E}{n}; \qquad\qquad E = E_0 \, e^{-1/n}$$

The interval of time required to reduce the kinetic energy to $\frac{1}{4}$ of the original supply, or to reduce the velocities to $\frac{1}{2}$ of their original values is[3]

$$t = -n \lg \frac{1}{4} = \frac{n \operatorname{Log} 4}{\operatorname{Log} e} = 1.386\, n$$

If $n = 10^5$ then the one-hundred-thousandth part of the supply of kinetic energy will be consumed per second and the velocity will fall to one-half of its original value in 138,600 seconds or 38.5 hours. This might be fairly approximate as to its order of magnitude for cyclones that are on the wane—for we are only endeavoring to get a rough idea of the magnitude of the forces in action.

The mass of the lowest layer 8 meters deep of the atmosphere is 1/1000 of the whole. If we assume its average velocity to be about 1/3 and its energy 1/10 of that of an equal mass of the outer layer then the lowest layer has 1/10000 part of the total energy of the whole moving atmosphere. If the tenth part of this is given up per second to the fixed obstacles that project from the ground [i.e., the smooth oblate spheroid of the geodesists] or to the maintenance of oceanic waves, then this would suffice to withdraw from a cyclone three-fourths of its initial energy in 38 hours.

If in addition to these there are yet other obstacles to motion still it is probable that these altogether would not cause so great a loss of energy as the pseudo-friction of this lowest stratum.

[3] The notation is lg – Napierian; Log – Briggian. e = Napierian base.

XXIV

ON THE ENERGY OF STORMS

BY MAX MARGULES

[*Dated November, 1904. Translated from the appendix to the annual volume for 1903 of the Imperial Central Institute for Meteorology. Vienna, 1905*]

GENERAL SUMMARY

When the velocity of the wind is 30 *m./sec.*, the living force or kinetic energy of one kilogram of air is 450 $kg.\ m.^2\ sec.^{-2}$ or nearly equivalent to 0.1 calorie. This amount, which is not large, in comparison with the energy corresponding to the quantities of heat that 1 *kg.* of air at the surface of the ground receives and loses during one day, does appear very large when we compare it with the energy of a wind of average velocity, such as 5*m./sec.*

It is not probable that such a very large proportion of the heat communicated to the air is converted into kinetic energy during stormy weather. We shall now seek for conditions of the atmosphere by virtue of which a sufficient supply of potential energy is stored up to maintain the storm; we shall allow ourselves to be led by experience and will start with the relations between the mechanical and the thermal forms of energy in a gaseous mass.

§(1) A mass of air extending from the ground upward and bounded by a vertical wall (and sometimes even the whole atmosphere) will serve as our closed atmospheric system. For any such system the equation of energy[1] is

$$\delta\overline{K} + \delta\overline{P} + \delta\overline{A} + (\overline{R}) = 0$$

where $\delta\overline{K}$ is the increase in the kinetic energy of the system;

$\delta\overline{P}$ is the corresponding change in the potential energy of position, considering gravitation as the only external force;

—$\delta\overline{A}$ is the work done by the pressural forces;

---

[1] I have taken the liberty of substituting capitals with the superscript dash for the German type used by Margules.—Abbe.

—$\overline{(R)}$ is the work done by the frictional forces, or $+\ (\overline{R})$ is the loss of kinetic energy by the action of friction and other resistances.

Of these quantities the first two depend only on the initial stage and final stage, the other two depend on the nature of the motion.

In a closed system the work of the pressural forces [or $-\delta\overline{A}$] is equal to the whole work done by the expansion of the air. Let (Q) be the increase of heat, and $\delta\overline{I}$ the increase of internal energy then

$$(Q) = \delta\overline{I} - \delta\overline{A}$$

For motions that occur without any general increase of heat (but in which internal exchanges of heat or even external additions and withdrawals that balance each other are allowable), the value of $\delta\overline{A}$ has also this same property, since $\delta\overline{I}$ depends only on the final stage.

The general equation of energy for a closed system as deduced from the preceding considerations,

$$(Q) = \delta\,(\overline{K} + \overline{P} + \overline{I}) + (\overline{R})$$

tells us that that part of the added heat that does not serve to increase the internal energy, represents the increase in kinetic energy and in the potential energy of position and in the consumption of energy in overcoming friction. If there be no increase of heat then the increase of mechanical energy takes place at the expense of the internal energy already present.

§(2) By the help of this last equation we will first seek for a closed dry air system that without any increase of heat can develop such great kinetic energy as we observe in storms.

Let the air be initially at rest but not in equilibrium. It starts in motion and tends to attain a condition of stable equilibrium. In general we know the characteristics of this final condition: every horizontal layer is a surface of equal pressure and equal temperature, the entropy (or the potential temperature) increases with the altitude. In order to completely determine this final stage we will assume that every part of the mass behaves adiabatically, or isentropically, during the motion. We now construct the final stage by the following process: we seek first the masses having the least entropy at the initial stage; these will form the lowest stratum, the other masses will arrange themselves proceeding upward, i the order of the increasing values of initial entropy. Equal masses of equal entropy can be interchanged at will.

The available kinetic energy of the system, including the loss by friction, is determined by the relation

$$\delta\overline{K} + (\overline{R}) = -\delta(\overline{P} + \overline{I}) = (\overline{P} + \overline{I})_a - (\overline{P} + \overline{I})_e$$

where the index a belongs to the initial and e to the final stage. $(\overline{P} + \overline{I})_e$ is the smallest value that the sum total of potential and internal energy can have under adiabatic condition.

If there be no friction then this equation is to be understood as follows: The largest amount of kinetic energy will be attained when all the masses pass through their respective appropriate positions of equilibrium simultaneously; if this does not occur then the kinetic energy attainable will be less than this which we will designate as "available energy." During the pendulous oscillation of the masses, a part of the potential and of the internal energy is still latent.[2]

If there be friction then the kinetic energy increases and becomes a larger fraction of the total amount of available energy in proportion as the influence of the friction is smaller; after the masses of air have approached their final positions this fraction diminishes and becomes zero when the final stage is attained. The condition of isentropic change is not precisely fulfilled since the friction produces heat. In our analysis we assume that this heat is again withdrawn from the air-mass. This limitation is of slight importance in the case of atmospheric motions.

§(3) Of all the different kinds of storms those best known to me are the gusts of wind (the *boe-en*) which are accompanied by rapid increase of atmospheric pressure and rapid fall of temperature. These were first investigated by Koeppen. Masses of air of unequal temperatures at identical levels are separated by a sharp boundary that advances with the storm wind toward the warmer side. The difference of temperature, which is often 10° C. at the surface of the ground, continues up to an altitude of nearly 2000 meters. The pressures at greater altitudes, not far from the boundary, are equal over the warm and cold regions but at the ground they are greater in the cold region.

Based on this general experience I have formulated the following problem: Let the mass of air in the lower part of a closed system, A, B, C, D, be initially divided by a screen into two parts

[2] Helmholtz describes the free (freie) energy, F, of any system, the total (gesammt) energy, u, and the latent (gebundene) energy, $u—F$.—EDITOR.

(see fig. 1*a*). Let the cold air be in the left hand chamber 1 but the warm air in the right hand 2; each mass of air to be at rest and in either stable or neutral equilibrium; the whole mass of air in the enclosure above these two chambers takes no part in the following processes, we can replace it by a movable heavy piston. What amount of kinetic energy becomes available when we remove the screen and let the masses 1 and 2 move adiabatically?

If in the initial stage the entropy of a kilogram of the highest layer of the mass (1) is smaller than that of the lowest layer of mass (2), then in the final stage the whole cold mass of air will spread out below or at (1′), and the warm mass will be spread out above or at (2′) as shown in fig. 1*b*.

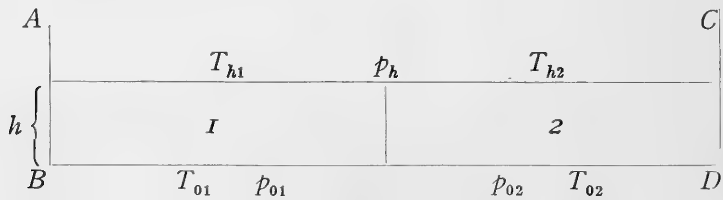


FIG. 1*a*.

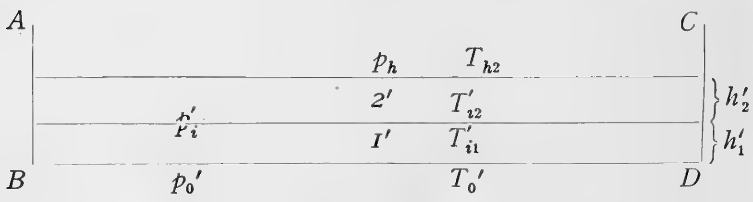


FIG. 1*b*.

If the equilibrium was stable on both sides of the screen then in every part the serial order of the strata is retained after the overturning.

If the equilibrium in (1) and (2) was neutral in the initial stage then it will remain so in the final stage (1′) and (2′).

Let T be the absolute temperature of an elementary mass dm in the initial stage: T' that of the same mass in the final stage; C_p the specific heat of air under constant pressure; C_v that for constant volume, $C_p - C_v = R$, then we find

$$\delta \bar{P} = R \int (T' - T)\, dm \qquad \delta \bar{I} = C_v \int (T' - T)\, dm$$

whence the available kinetic energy of the system is

$$- \delta\, (\bar{P} + \bar{I}) = C_p \int (T - T')\, dm$$

where $\delta \overline{P}$ is the change in the potential energy of position for the whole system including that of the piston. $\delta \overline{P}$ and $\delta \overline{I}$ are the analogs of the exterior work and the change of internal energy of a small mass of air under constant pressure. These equations hold good in general for any change of location of the masses of air when the pressure remains constant in the superposed movable level surface.[3] The integrals are to be extended over all the masses lying below this surface.

The quotient of the available kinetic energy by the mass below the piston gives the average energy $\frac{1}{2}V^2$ per unit of mass. When the above assumed overturning of the masses takes place and the volumes of the two chambers are equal, the corresponding value of V is given by

$$V = \tfrac{1}{2}\sqrt{g\,h\,\tau}$$

where g = acceleration of gravity; h = altitude of the chambers; T_1 and T_2 are the initial average temperatures: $\tau = (T_2 - T_1)/T_1$. Assuming $T_1 = 273°$, $T_2 = 283°$ and h = 2000, 3000, 6000 meters successively, we find V = 13, 16, 23 meters per second respectively.

In the case of cold and warm rooms in dwellings having the same temperatures as above but h = 5 meters we find V = 0.67 *m./sec.*

But this computation tells us nothing as to how the available kinetic energy is distributed within the masses. We see, however, that with chambers 2000 meters high and a difference of temperature of 10° C. a storm velocity cannot prevail throughout the whole mass but only in about one-fourth part of it, and in even a still smaller fraction of the whole mass if we abstract a large amount for the loss of the energy due to friction. In boe-en or gusts, strong winds occur only on the cold side and close to the boundary.[4]

The ratio of R to C_v is that of 1 to 2.5 and by this ratio is the work done by the force of gravity less than the diminution of internal energy when the change of location takes place under constant pressure. When the overturning takes place under constant volume we have the same value of V, and so also when it takes place with a very slow change of pressure in the level surfaces near the boundary; but in these cases the ratio $\delta \overline{P}/\delta \overline{I}$ is changed. We have the same available kinetic energy if we fill each chamber with an incompressible fluid whose mass is the same as that of the air in this chamber, but in this case the energy arises from the work of gravity only.

[3] Corresponding to the piston.
[4] So also in northers, blizzards, chinooks, boras, purgas, etc.—C. A

§(4) Let us now consider another system. In the chambers (1) and (2) of fig. 1*a* let two masses of air be enclosed whose respective entropies are the same at the same altitudes: therefore the greater pressure will be on the side of the warmer air; a difference of pressure of 10^{mm} mercury at the base implies a difference of temperature of 1° C. only. Let each of the two masses be initially in stable equilibrium.

If now the screen be removed and the total volume be unchanged then the horizontal strata that were initially at the same altitudes unite to form one new stratum. The vertical succession remains unchanged and so also the altitude of the center of gravity. The change of internal energy as computed for the unit of mass depends only on the initial difference of pressure, not on the height of the chambers. We may therefore replace this system by a thin horizontal stratum that initially contained air in two equal divisions at pressures of 765^{mm} and 755^{mm} mercury respectively. The available kinetic energy is that appropriate to a velocity of 1.5 *m/sec.* for the whole mass: that which is available for each unit of mass is far smaller than in those systems that with the same distribution of pressure at the base have horizontal differences of entropy.

It has already been shown by me, in my treatise, "On the work represented by any distribution of atmospheric pressure"[5] that the energy stored up in such a distribution of pressure as is observed in our lower strata during a stormy period would not suffice to develop the observed kinetic energy of the storm, if the masses of air were only pushed horizontally out of their positions of equilibrium.

A great velocity of a mass of air over a broad area under the influence of a horizontal pressure gradient can only arise when this gradient is maintained by some outside source of energy; otherwise it would disappear before any portion of the mass of air had attained the velocity of a moderate wind. Dry air possesses such a store of energy when horizontal differences of entropy of ordinary amount exist at any level; and not only when there is a sharp boundary between warm and cold air but also when there is a steady continuous horizontal gradient of entropy.

§(5) The available kinetic energy of a system in which masses of unequal entropy are superposed in unstable equilibrium can be computed from the energy equation. Here we find a store of

[5] Jubilee volume of the Central Institute for Meteorology, p. 329, Denkschriften, Imp. Acad. Science. Vol. XXIII. Vienna, 1901. See No. XXIII of this collection of translations.

energy sufficient for the development of storms; hitherto it has only been assumed that storms start from such beginnings. The existence of unstable conditions before a storm has never been demonstrated. Prohaska never found such cases in his numerous studies of thunder-storms. Even in so-called calms there is still enough motion to disturb a condition of unstable equilibrium. The forces that are thus set free are greater than those corresponding to the largest horizontal pressure gradients that have been observed in the atmosphere. The accelerating force acting on a foreign particle of air whose temperature is T_1 when surrounded by air whose temperature is T, is

$$g\,\frac{(T_1 - T)}{T}$$

and therefore, for $T = 273°$ and $T_1 = 274°$ this becomes $\frac{g}{273}$, whereas the force represented by a barometric gradient of 1^{mm} mercury per degree of a great circle, at the base of the atmosphere is $\frac{g}{1000}$. The vertical distances in our atmosphere are small. Unstable conditions can scarcely exist for any length of time over extensive areas; they would disappear very quickly; their existence has not been demonstrated nor are they probable. Where there are adjacent masses of air with very large differences of temperature on the same level, as in the boe-en gusts, then cold air may intrude upon the warm region, and warm air flow over into the cold region, but in this case the storm wind velocity arises not only by reason of the barometric horizontal pressure but also directly by the action of gravity.

§(6) Many seek for the source of energy of a storm in the latent heat of condensation evolved by the formation of clouds. I will now compute the available kinetic energy for the following initial condition:

In chamber 1 of fig. 1*a* let there be dry air in neutral equilibrium, but in chamber 2 an ideal fictitious gas that has the property of expanding only when heat is added [and not by any diminution of pressure] but otherwise behaves like dry air. This latter gas replaces the moisture-saturated air [of nature] and the heat added during its expansion corresponds to the latent heat of condensation; for neutral equilibrium the vertical diminution of temperature in this fictitious gas is smaller than that in the dry air. The initial condition is to be so chosen that after the removal of the screen the mass 1 spreads out below and the mass 2 above; we may then omit the assumption of neutral equilibrium for the result will hold good also when stable equilibrium exists initially in each chamber.

The equation of energy is now to be applied in the form

$$d\overline{K} + (\overline{R}) = (Q) - \delta(\overline{P} + \overline{I})$$

where (Q) replaces the latent heat of condensation. The mass 2 cools by expansion less than dry air, wherefore it contributes a smaller portion to the $-\delta(\overline{P} + \overline{I})$. The difference is exactly made up by the added heat (Q). It follows that the latent heat of condensation contributes nothing to the energy of the storm. The available kinetic energy remains unchanged, if in the initial stage we substitute dry air of the same temperature for the fictitious gas. We must replace the moisture-saturated air with dry air of equal density, wherefore the latter having the same pressure must have a somewhat higher temperature.

§(7) The diagram, fig. 1, is not intended to give a complete idea of the phenomena in boe-en; it only contains that which we consider as the condition for the origin of the storm.

The length of the chamber must be measured by hundreds of kilometers but the altitude by very few kilometers. It is indifferent whether the separating surface (or screen) is initially vertical or inclined, if only the wedge-shaped volume is small relative to that of the whole chamber If the [inclined] boundary surface be so laid that the colder mass of air 1 extends as a sharp wedge toward the ground, we have everywhere steady distribution of pressure. For an angle of 10° between the boundary surface and the horizon and for a chamber of 3000 meters altitude the length of the wedge is 17 kilometers; the diminution of pressure at the ground extends over this distance. If the boundary advances at the rate of 85 kilometers per hour, then in 12 minutes the barometer at any place will rise by the amount that corresponds to the difference in the weights of unit columns of cold and warm air. If the length of the chamber were 500 kilometers, the result would not be notably different but the computation would be more troublesome. Even an inclination of 1° for the boundary surface with a distribution of the fall of pressure over a distance of 170 kilometers would allow of the existence of a great amount of available kinetic energy.

I have as yet no definite idea as to how a condition involving the presence of a great store of potential energy arises without the immediate occurrence of an unloading or diminution of the potential by virtue of some movement of the air; in the present memoir we simply assume an initial stage generalized from actual observations.

The restriction of the mass of air to a closed system, the assumption of a well-defined boundary between the cold and the warm air, the introduction of level surfaces of equal pressure, are all analytical auxiliaries that we employ in order to ascertain the value of the available kinetic energy of a perfectly definite system.

§(8) If we are to give a broad interpretation to the results of our analyses, we must omit numerical details. The phenomena of motion in the great storm areas that we call cyclones are less intelligible than those of the boe-en. But these also, at least in median and higher latitudes, consist of warm and cold masses of air lying adjacent to each other horizontally; cold air often spreads out over the earth in the lower strata behind the passing storm. It is therefore not unlikely that these storms are fed by the potential energy of an initial stage similar to that which we have adopted in the preceding lines.

The opinion that the energy of cyclones arises from the overturning of masses of unequal temperature has been expressed in recent years by Prof. F. H. Bigelow. In one of his memoirs[6] we find the following sentence:

"The cyclone is not formed from the energy of the latent heat of condensation, however much this may strengthen its intensity; it is not an eddy in the eastward drift, but is caused by the counterflow and overflow of currents of air of different temperatures."

Long ago W. Blasius had vainly labored to introduce a similar view as to the origin of storms. In his books (Storms, Philadelphia, 1875; and "Stürme and Moderne Meteorologie," Braunschweig, 1893) this idea is found mixed up with other views that are, I believe, less reasonable; however, they deserve careful consideration. From giving too much attention to the isobars, it has been generally assumed that the air must ascend only in the central part of a region of low pressure and that the motions are nearly symmetrical about this. Ferrel as well as Guldberg and Mohn have built on this assumption.

Our analysis gives us only a general idea as to the source of the energy of storms; a working model of the cyclone with symmetrical distribution of temperature has not yet been constructed.

---

[6] "Studies on the meteorological effects of the solar and terrestrial physical processes. Weather Bureau publication, No. 290, Washington, 1903, p. 37; separate print from the Monthly Weather Review, Feb., 1903, p. 84, column 2.

It is stated of tropical whirlwinds that ordinarily within their areas there is observed at the earth's suface no great difference of temperature and that the cyclonic distribution of pressure only extends upward to altitudes of a few kilometers. Hence it may be concluded that near the ground the central part of the storm area is the warmest. Helmholtz, in his address on "Whirlwinds and Thunderstorms," assumes that the storm begins with an unstable condition above a small area; according to the computations of Reye, the vertical distribution of temperature around this central region may be stable; the lower strata are pushed upward through this central hole; the layers that at first were above then sink lower. This process is dependent materially on the vapor content; and on another occasion we will investigate how the available energy is to be computed for this case. In order to judge whether this idea will apply to tropical cyclones we must know accurately the conditions, especially the temperature both inside and outside the relatively small area of the storm.

W. M. Davis has shown[7] that tornadoes originate only in the neighborhood of the boundary between cold and warm masses of air, a circumstance that had scarcely been considered before and the knowledge of which can be very useful in finding the source of energy of these enigmatical storms.

ADOPTED NOTATION

§(9) The following notation and the numerical values of the more important constants will be used:

t, the time.

$x\,y\,z$, the rectangular coördinates of a point referred to a system of axes that rotate with the earth, z being vertical and positive upwards

c, the velocity at this point relative to the origin of these axes.

$u\,v\,w$, the corresponding components of the velocity c.

k, the volume.

m, the mass.

$\frac{d}{dt}$, the symbol for the total differential coefficient of a function with regard to the time or

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

[7] Hann: Lehrbuch der meteorologie, Braunschweig, 1901, p. 705.

∂, round d, the symbol for partial differentiation of any function or variable.

W, the potential of the attraction of gravitation combined with the centrifugal force of the diurnal rotation,

$g = \frac{\partial W}{\partial z}$ = the local apparent force of gravity, assumed to be constant in these present applications and to be 9.805966 $m/sec.^2$

A_x, A_y, A_z, rectangular components of the deflecting force due to the rotation of the earth, or the general terms of the equations of motion that must satisfy the condition

$$u A_x + v A_y + w A_z = 0.$$

p, elastic pressure within the air.

μ, density of the air.

T, absolute temperature of the air in Centigrade degrees.

T^*, average temperature of large mass of air.

$a = \frac{\partial T}{\partial z}$ = rate of change of temperature of atmosphere with altitude.

R, the constant of the equation of elasticity for gas.

$p = RT\mu$, the equation of elasticity for gas.

C_p, specific heat of air under constant pressure.

C_v, specific heat of air under constant volume.

$$C_p - C_v = R = \kappa\, C_p\,; \quad \kappa = \frac{R}{C_p}\,; \; \gamma = \frac{C_p}{C_v}$$

For adiabatic change from $p_0 \mu_0 T_0$ to $p \mu T$ we have

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{1/\kappa} = \left(\frac{\mu}{\mu_0}\right)^{\gamma}$$

For dry air the numerical values are

$$\gamma = 1.41\,;\, \kappa = \frac{0.41}{1.41}$$

$$R = 287.026\, \frac{met^2}{sec^2\ C^0} = 0.0689\, \frac{Cal}{kg\ C^0};$$

$$C_p = 987.09\, \frac{met^2}{sec^2\ C^0} = 0.237\, \frac{Cal}{kg\ C^0}$$

$$\frac{g}{C_p} = 0.009934\,23\, \frac{C^0}{met^2}$$

$-\overline{\mathbf{R}}$, work done by frictional forces or $+\overline{\mathbf{R}}$ the kinetic energy lost by the whole system through friction.

$\overline{\mathbf{R}}_x$, $\overline{\mathbf{R}}_y$, $\overline{\mathbf{R}}_z$. the corresponding rectilinear components.

Q, quantity of heat added to unit mass of air per unit time.

S, entropy of the unit mass of air.

(Q), quantity of heat added to the whole closed system in whole time.

V, the average velocity of motion throughout any closed system for the whole time.

$\overline{\mathbf{K}}$, kinetic energy of the whole mass of air in the system.

$\delta\overline{\mathbf{K}}$, increase of kinetic energy of the system.

$\overline{\mathbf{T}}$, potential temperature of air whose actual temperature is T at pressure p.

$\overline{\mathbf{P}}$, potential energy of the system due to its position and the action of gravity.

$\delta\mathbf{P}$, change in potential energy of the system accompanying the increase of kinetic energy, $\delta\overline{K}$.

$\overline{\mathbf{I}}$, internal energy of the system.

$\delta\overline{\mathbf{I}}$, change of internal energy of the system when pressural forces do external work.

$\delta\overline{\mathbf{A}}$, the work done by pressural forces.

For the notation and values of these quantities in moist air see Chapter IV later.

Chapter I

THE EQUATIONS OF ENERGY OF A MOVING PARTICLE AND OF THE WHOLE MASS OF AIR IN A CLOSED SYSTEM

§(10) One of the equations that holds good for the motion of the air relative to the system of coördinates rotating with the earth is

$$\frac{du}{dt} = -\frac{\partial W}{\partial x} - \frac{1}{\mu}\frac{\partial p}{\partial x} + \overline{\mathbf{R}}_x + A_x$$

and the other two are analogous. From these three there results as the equation of energy of a definite particle of air of unit mass moving with the velocity c,

$$\frac{d}{dt}\left(\frac{c^3}{2} + W\right) + \frac{1}{\mu}\left(\frac{dp}{dt} - \frac{\partial p}{\partial t}\right) - \overline{\mathbf{R}}\, c \cos(\overline{\mathbf{R}}, c) = 0 \quad . \quad . \quad (1)$$

The increase of the total kinetic and potential energy of the particle is equal to the work done by the pressural forces and the frictional forces. The deflecting force of the rotation of the earth being normal to the path does no work; this is also true of the other portions of the term A. Equation (1) differs from the equation of energy for absolute motion only in respect to the meaning of W which contains a term depending on the rotation of the earth in addition to the potential of the attraction.

We combine the dynamical relation (1) with the thermal relation

$$\frac{dQ}{dt} = C_p \frac{dT}{dt} - \frac{1}{\mu}\frac{dp}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which applies when the quantity of heat dQ is imparted to the particle of air while describing its path ds during the element of time dt. It is here assumed that just as in air at rest, so here the imparted heat dQ serves only to increase the internal energy by the quantity $C_v dT$ and to perform the work of expansion $pd\left(\frac{1}{\mu}\right)$. Consequently

$$\frac{dQ}{dt} = C_p \frac{dT}{dt} + \frac{d}{dt}\left(\frac{c^2}{2} + W\right) - \frac{1}{\mu}\frac{p}{t} - \overline{\mathbf{R}}\, c \cos\left(\overline{\mathbf{R}}\, c\right) \quad . \quad . \quad (3)$$

We must include in dQ not only the heat communicated from without but also that portion of the heat due to friction that belongs to this small elementary unit mass of air.

§(11) The equations (1) and (3) with the factor μdk when integrated over the space k, assumed to be filled with air, give the relations for the total energy of the whole mass within that space.

In this integration we make use of the equation of continuity

$$\frac{\partial \mu}{\partial t} + \frac{\partial (\mu u)}{\partial x} + \frac{\partial (\mu v)}{\partial y} + \frac{\partial \mu w}{\partial z} = 0$$

If F indicates a quantity that is considered as attached to a definite elementary mass and can be expressed by a continuous function of the place and time, then by using a well-known transformation we have

$$\int \frac{dF}{dt}\mu dk = \int \frac{\partial F}{\partial t}\mu\, dk + \int \left(\mu u \frac{\partial F}{\partial x} + \mu v \frac{\partial F}{\partial y} + \mu w \frac{\partial F}{\partial z}\right) dk$$

$$= \frac{\partial}{\partial t}\int \mu F dk - \int \mu F c \cos(n, c)\, dO$$

where n is the direction of the normal directed inward to the surface O of the space k.

In a similar way we obtain

$$\int \left(\frac{dF}{dt} - \frac{\partial F}{\partial t} \right) dk = \int \frac{F}{\mu} \frac{d\mu}{dt} dk - \int F c \cos(n, c)\, dO$$

The surface integrals disappear in our case when the mass of air is bounded by fixed walls; and so also when we extend the space integral or the mass integral over the whole atmosphere, or over a mass of air that is bounded by the ground and fixed vertical walls but is open above; assuming that both the pressure and also the product of pressure by vertical velocity diminish to zero as the altitude increases.

The equation for the energy of the whole mass of a closed system as obtained from equation (1) is as follows:

$$\frac{\partial}{\partial t} \int \left(\mu \frac{c^2}{2} + \mu W \right) dk + \int \frac{p}{\mu} \frac{d\mu}{dt} dk - \int \overline{\mathbf{R}} c \cos(\overline{\mathbf{R}}, c)\, \mu dk = 0\,. \quad (4)$$

We would remark that here in closed systems the work done by the pressural forces is equal to the sum of the work done by the expansions of all the elementary masses or

$$-\int \frac{1}{\mu} \left(\frac{dp}{dt} - \frac{\partial p}{\partial t} \right) \mu dk = \int p \frac{d}{dt} \left(\frac{1}{\mu} \right) \mu dk$$

In a similar way there follows from equation (2)

$$\left.\begin{aligned} \int \frac{dQ}{dt} \mu dk &= C_p \frac{\partial}{\partial t} \int T \mu dk - \int \frac{p}{\mu} \frac{d\mu}{dt} dk - \int \frac{\partial p}{\partial t} dk \\ &= C_v \frac{\partial}{\partial t} \int T \mu dk - \int \frac{p}{\mu} \frac{d\mu}{dt} dk \end{aligned}\right\} . \quad (5)$$

From equations (4) and (5) there results the following equation (6) which we call the equation of energy for a mass of air in a closed system

$$\left.\begin{aligned} \int \frac{dQ}{dt} \mu\, dk = C_v \frac{\partial}{\partial t} \int T \mu dk + \frac{\partial}{\partial t} \int \left(\mu \frac{c^2}{2} + \mu W \right) dk - \\ - \int \overline{\mathbf{R}} c \cos(\overline{\mathbf{R}}\, c)\, \mu dk \end{aligned}\right\} . \quad (6)$$

These expressions become more perspicacious by the introduction of the following abbreviating symbols:

$\overline{\mathbf{K}} = \int \frac{\mu c^2}{2} dk$ = the kinetic energy of the whole mass of air in the closed system.

$\overline{\mathbf{P}} = \int \mu W dk$ = the potential energy of position.

$\overline{\mathbf{I}} = C_v \int T\mu dk = \frac{C_v}{R} \int p dk$ = the internal energy.

The changes in the values of these quantities in the time t are indicated by $\delta\overline{\mathbf{K}}$, $\delta\overline{\mathbf{P}}$, $\delta\overline{\mathbf{I}}$ and are completely determined by the initial and final conditions.

The three following quantities depend on the path that each elementary mass pursues; these time integrals extend over the same interval t.

$- \partial\overline{\mathbf{A}} = - \int dt \int \frac{p\, d\mu}{\mu\, dt} dk$ . . . $\Big\{$ the work of the pressural forces, or the work of expansion in the time t.

$- (\overline{\mathbf{R}}) = \int dt \int \overline{\mathbf{R}} c \cos (\overline{\mathbf{R}}\, c)\, \mu dk$ $\Big\{$ the work of the frictional forces or loss of energy, $\overline{\mathbf{R}}$, by friction, in the time t.

$(Q) = \int dt \int \frac{dQ}{dt} \mu dk$ . . . . . $\Big\{$ the quantity of heat communicated to the closed system, in the time t.

We therefore write the equations for the kinetic energy, thermal equilibrium, and total energy of the whole mass, respectively as follows:

Kinetic energy...... $\partial (\overline{\mathbf{K}} + \overline{\mathbf{P}} + \overline{\mathbf{A}}) + (\overline{\mathbf{R}}) = 0$ . . (4*)

Thermal energy..... $(Q) = \partial\, \overline{\mathbf{I}} - \partial\, \overline{\mathbf{A}}$ . . . . (5*)

Total energy .... $(Q) = \partial (\overline{\mathbf{K}} + \overline{\mathbf{P}} + \overline{\mathbf{I}}) + (\overline{\mathbf{R}})$. (6*)

§(12) In the case of friction there can be no steady motion without a corresponding continual addition of heat. $\overline{\mathbf{K}}$, $\overline{\mathbf{P}}$, $\overline{\mathbf{I}}$ remain unchanged in steady motion and the equations reduce to $(Q) = (\overline{\mathbf{R}}) = - \delta\overline{\mathbf{A}}$. The additions of heat (necessarily consisting of positive and negative portions), the loss of energy by friction and the work of the pressural forces are equal to each other.

$(Q) = 0$. Atmospheric motions are primarily dependent on the heat communicated from without. But it is possible that for certain general movements of the air, it is not the absorption of heat during their occurrence that is the determining factor but the temporary distribution of pressure, temperature and velocity throughout the mass of air. In the following pages we will treat these latter movements as though there were no addition of heat but only an effort to attain equilibrium as the result of some given initial condition.

It is a characteristic of every motion performed without exchange of heat that the work done by the pressural forces is determined by the initial and final stages alone. It is well known that the quantity $\overline{\mathbf{A}}$ has the significance of a potential energy under certain conditions such as, when the mass is kept at constant temperature, or when every elementary particle of mass behaves adiabatically. This last condition is a special case of $(Q) = 0$. In this case an interchange of heat within the interior of the mass is allowable, or such additions and subtractions of heat as balance each other, or sum up zero, during the whole time t under consideration. If we confine ourselves to the case that $(Q) = 0$ during every element of time then the changes of $\overline{\mathbf{A}}$ and $\overline{\mathbf{I}}$ are continuously equal to each other. In the case of motion without increase of heat, the quantity $\overline{\mathbf{A}}$ that I have in other places[8] called the potential energy of the distribution of pressure becomes identical with the internal energy of the mass.

If $(Q) = 0$ then

$$\partial\overline{\mathbf{K}} + (\overline{\mathbf{R}}) = -\,\partial\,(\overline{\mathbf{P}} + \overline{\mathbf{I}}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6^{**})$$

In this case $\overline{\mathbf{P}} + \overline{\mathbf{I}}$ is to be considered as the total potential energy of the system; in the case of gaseous motions in a closed system this expression has the same meaning as $\overline{\mathbf{P}}$ alone in the case of motions of rigid bodies or incompressible fluids under the influence of gravity. One easily recognizes the meaning of $\overline{\mathbf{I}}$ in the expression for the potential energy if one considers the case of masses of air pushed horizontally (by compression or expansion) out of the condi-

[8] Max Margules: Ueber den Arbeitswert einer Luftdruckvertheilung und ueber die Erhaltung der Druckunterschiede. Jubiläum des KK Central Anstalt, Vienna. 1901. See No. XXIII of this collection of translations

tion of equilibrium while $\overline{\mathbf{P}}$ remains unchanged. If the displacement of each element of mass proceeds adiabatically, or if there be an internal interchange of heat, then $\overline{\mathbf{I}}$ must increase; after this if the mass is left to itself it strives toward the position of minimum $\overline{\mathbf{I}}$.

The problem that we would solve by means of equation (6**) is as follows:

A mass of air in a closed system is at the beginning at rest and has a given initial internal distribution of temperature and pressure. It is set in motion by its tendency toward a condition of stable equilibrium. If there were no friction the individual portions of the mass would oscillate about their positions of equilibrium. In the presence of friction the final condition is attained by the gradual consumption of the kinetic energy. We seek the maximum values of $\delta\overline{\mathbf{K}} + (\mathbf{R})$ which we designate as the available kinetic energy of the system. Since the initial values of $\overline{\mathbf{P}}$ and $\overline{\mathbf{I}}$ are known our problem is to compute their values for the final stage.

No special assumption will be introduced as to the frictional forces; in fact for turbulent motions there is no assumption that can satisfy all demands. Hence the frictional force will not be treated as a part of the pressural force, as is usual nowadays in the treatment of steady motions. The condition $(Q) = 0$ demands that the heat generated in the system by friction be immediately withdrawn. But this is not of great importance in atmospheric motions. Even in cases where it is assumed that the addition of heat has no influence on the motion there are more abundant sources of heat than this friction. For the present study the important point is that the heat due to friction shall not give rise to new or additional kinetic energy [during the interval of time under consideration].

Chapter II

APPLICATION OF THE EQUATION OF ENERGY TO THE OVERTURNING OF STRATA IN A COLUMN OF AIR

§(13) The analyses to be executed in this second section will not afford much that is new, they serve only as preparatory to the following sections.

Consider a column of air of unit area section, in which p and T

are functions of the altitude z only. The condition of hydrostatic equilibrium is

$$\left.\begin{aligned} g + \frac{1}{\mu}\frac{\partial p}{\partial z} = 0 \\ \text{or} \qquad \\ \frac{1}{p}\frac{\partial p}{\partial z} = -\frac{g}{RT} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (\alpha)$$

The analysis is simplified by the assumption that g is independent of z whence the potential equation becomes

$$W = gz \quad \ldots\ldots\ldots\ldots \quad (\beta)$$

Let us distinguish between a lower part of the column having a mass M extending to the altitude h and a higher mass M_h. Let overturnings of the strata take place in M whereby h is changed but the pressure

$$p_h = gM_h \quad \ldots\ldots\ldots\ldots \quad (\gamma)$$

on the upper boundary of M remains constant.

Let Z be the altitude of the center of gravity of M_h then during all changes $Z - h$ remains unchanged, since M_h rises and falls like a solid piston.

The potential energy of position for the whole column of air is

$$\overline{\mathbf{P}} = \int_0^\infty gz\mu dz = \int_0^h gz\mu dz + gM_h z$$

By utilizing the relation (α) we have

$$\int_0^h gz\mu dz = -\int_{p_0}^{p_h} zdp = \int_0^h pdz - hp_h$$

$$\overline{\mathbf{P}} = \int_0^h pdz + (Z - h)\,p_h = R\int Tdm + \text{Constant} \quad . \quad (1)$$

where the integral in the last part of the equation is to be extended over the whole of the lower mass M.

The internal energy of the column is

$$\overline{\mathbf{I}} = C_v \int Tdm + \text{Constant} \quad \ldots\ldots\ldots \quad (2)$$

Overturning of the strata can occur spontaneously whenever the center of gravity sinks thereby; the available kinetic energy in the initial stage is

$$\partial \overline{\mathbf{K}} + (\overline{\mathbf{R}}) = -\delta\,(\overline{\mathbf{P}} + \overline{\mathbf{I}})$$

and for $(Q) = 0$ this becomes

$$-C_p \int \delta\, T\, dm,$$

which is in the ratio C_p/R or $1.41/0.41 = 3.44$ larger than the work done by gravity; the principal part of the kinetic energy is derived from the internal energy.

Let T and T' be the temperatures of the elementary mass dm and T^* and $T^{*\prime}$ the average temperatures of the whole mass M at the initial and final stages, respectively, then we have

$$\delta \overline{\mathbf{K}} + (\overline{\mathbf{R}}) = C_p \int (\mathrm{T} - \mathrm{T}')\, dm = C_p\,(T^* - T^{*\prime})\, M \quad . \quad (3)$$

Under the condition here given, kinetic energy is available; that is to say, when by adiabatic overturning of the layers the average temperature of the whole mass of air sinks, then the condition is not stable.

§(14) *Computation of the available kinetic energy in the case of any change of position of a layer.*

Let the thin layer m_1 (see fig. 2) that initially lay beneath M_2 be adiabatically brought to lie above M_2. In this case nothing changes in the lower mass M_0 since the upper mass M_2 acts like a piston of constant weight. Let the layers of the mass M_2 retain the same consecutive order. The kinetic energy that is available when the mass m_1 ascends in small particles and spreads out over M_2, is now to be computed from equation (3).

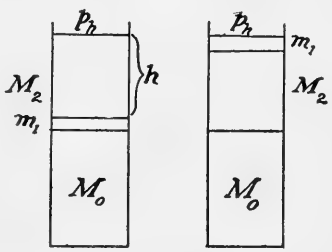


FIG. 2.

Let p_1 and T_1 be the initial pressure and temperature of m_1; p_h and T'_1 the same quantities at the end. Then

$$T'_1 = T_1 \left(\frac{p_h}{p_1}\right)^{\kappa}; \quad p_1 = p_h + g\, M_2$$

Let p_2, T_2 and p'_2, T'_2 be the corresponding pairs of values for a stratum dM_2, then from the adiabatic condition and from

$p'_2 = p_2 + gm_1$ and from the assumption that gm_1 is small relative to p_2 there follow

$$T'_2 = T_2\left(\frac{p'_2}{p_2}\right)^{\kappa} = T_2\left(1 + \frac{gm_1}{p_2}\right)^{\kappa} = T_2\left(1 + \kappa\frac{gm_1}{p_2}\right)$$

$$C_p\,(T_2 - T'_2) = -\,gm_1\frac{RT_2}{p_2} = -\frac{gm_1}{\mu_2}$$

The contribution of the whole mass M_2 to the right-hand side of equation (3) is

$$C_p\int(T_2 - T'_2)\,dM_2 = -\,gm_1\int\frac{dM_2}{\mu_2} = -\,gm_1\int dz = -\,gm_1\,h$$

where h is the altitude of the mass M_2 at the initial stage.

The overturning of the masses is brought about by the slightest initial impulse, and kinetic energy develops when the equation

$$-\,\delta\,(\overline{\mathbf{P}} + \overline{\mathbf{I}}) = m_1\left\{C_p\,T_1\left[1 - \left(\frac{p_h}{p_1}\right)^{\kappa}\right] - gh\right\} \quad . \quad .(4)$$

has a positive value.

---

§(15) *Continuous distribution of temperature.* We will first assume that the separation between m_1 and M_2 was only made for the convenience of computation and that in the initial stage the temperature is a continuous function of the altitude throughout these masses. The value of h must differ from that appropriate to the final stable position, but it may be chosen small enough to allow us to assume T_2 expressible as a linear function, viz.,

$$T = T_2 = T_1 - az \qquad\qquad a = -\frac{\partial T}{\partial z}.$$

With this function of the temperature the equation α gives us

$$\frac{p_h}{p_1} = \left(\frac{T_1 - ah}{T_1}\right)^{g/Ra}$$

whence assuming ah to be small relative to T_1 we get

$$\left(\frac{p_h}{p_1}\right)^{\kappa} = \left(1 - \frac{ah}{T_1}\right)^{g/C_pa} = 1 - \frac{gh}{C_p\,T_1} + \frac{g\,(g - C_pa)}{C_p^2\,T_1^2}\cdot\frac{h^2}{2}$$

By substituting this in equation (4), the linear term in h disappears and there remains

$$-\delta(\overline{\mathbf{P}} + \overline{\mathbf{I}}) = \frac{gm_1}{2T_1}h^2\left(a - \frac{g}{C_p}\right) \quad . \; . \; . \; . \; . \; (4a)$$

This value is positive and gives the kinetic energy made available by the overturning of m_1 provided

$$a > \frac{g}{C_p} \text{ or } -\frac{\partial T}{\partial z} > \frac{g}{C_p}.$$

In this case the equilibrium of the column of air is unstable.

If $a < g/C_p$ then with every adiabatic overturning of any layer there is associated an increase of the total potential energy $\overline{\mathbf{P}} + \overline{\mathbf{I}}$ and the equilibrium of the column of air is stable. In the limiting case $a = g/C_p$ and the equilibrium is neutral.

---

§(16) *Discontinuous distribution of temperature.* If at the boundary between m_1 and M_2 the temperature passes suddenly from T_1 to T_2 it will suffice to assume M_2 to be of an order of magnitude similar to m_1 whence we will now indicate it by m_2. The interchange of positions of m_1 and m_2 gives us

$$-\delta(\overline{\mathbf{P}} + \overline{\mathbf{I}}) = C_p\{m_1(T_1 - T'_1) + m_2(T_2 - T'_2)\}$$

$$T'_1 = T_1\left(1 - \frac{gm_2}{p_1}\right)^\kappa$$

$$T'_2 = T_2\left(1 + \frac{gm_1}{p_2}\right)^\kappa$$

hence when p is the pressure at the boundary and gm is small in comparison with p we have

$$-\partial(\overline{\mathbf{P}} + \overline{\mathbf{I}}) = m_1 m_2 \frac{gR}{p}(T_1 - T_2) \quad . \; . \; . \; . \; (4b)$$

which corresponds to unstable equilibrium when the warmer mass lies below.

---

§(17) *The entropy as the criterion of stable equilibrium in a column of air of uniform constant constitution.*

The condition for stable equilibrium can be expressed very simply when we make use of the entropy. If P, Θ, S_0 are respectively the pressure, temperature, and entropy for a normal condition of the air, then for any other pair of values, p and T, the entropy for a unit mass is

$$S = S_0 + C_p \log \frac{T}{\theta} - R \log \frac{p}{P}$$

For a column at rest we have from equation (α)

$$\frac{\partial S}{\partial z} = \frac{C_p}{T}\frac{\partial T}{\partial z} - \frac{R}{p}\frac{\partial p}{\partial z} = \frac{C_p}{T}\left(\frac{\partial T}{\partial z} + \frac{g}{C_p}\right)$$

whence it follows that stable equilibrium exists when S increases with the altitude, but unstable when S diminishes with increasing altitude.

This result also holds good for sudden changes of temperature at special localities; in such cases $S_2 - S_1 = C_p \log \frac{T_2}{T_1}$; the entropy increases with the altitude when $T_2 > T_1$ and the warmer layer lies above the colder.

---

§(18) *Potential temperature.* Helmholtz and von Bezold* define the potential temperature $\overline{\mathbf{T}}$ of a mass whose actual temperature is T and pressure p as being that temperature which the mass will attain when brought adiabatically to the normal pressure P. Hence we have

$$\overline{\mathbf{T}} = T\left(\frac{P}{p}\right)^{\kappa} \qquad dS = C_p \frac{d\overline{\mathbf{T}}}{\overline{\mathbf{T}}} \qquad S = C_p \log \overline{\mathbf{T}} + \text{Constant.}$$

The equilibrium is stable if the layers are arranged to succeed each other upward in the order of increasing potential temperature or increasing entropy.

---

§(19) *Buoyancy of an elementary mass of air that has a temperature different from that of its surroundings.*

In a mass of air at rest where T is a function of the altitude only, we will introduce a foreign particle of air m, which has the temperature θ_0 at the altitude z_0 but always the same pressure p as that of

*See the article by Dr. L. A. Bauer, reprinted as No. XXII of this collection.—C A.

the surrounding air at the same altitude. Under adiabatic conditions its temperature in all positions is given by the equation

$$\theta = \theta_0 \left(\frac{p}{p_0} \right)^\kappa$$

whence

$$\frac{1}{\theta} \frac{d\theta}{dz} = \frac{\kappa}{p} \frac{dp}{dz}$$

Let us assume that during the motion of the particle m the pressure in the larger mass remains unchanged and that the equation (α) holds good so that we have

$$\frac{d\theta}{dz} = -\frac{g}{C_p} \frac{\theta}{T}.$$

If $[\mu]$ is the density of m but μ that of the surrounding air then there is acting upward on m the accelerating force or buoyancy

$$g\left(\frac{\mu}{[\mu]} - 1 \right)$$

or, since the pressures are the same,

$$g\left(\frac{\theta}{T} - 1 \right)$$

If we consider m as an elementary mass moving without friction and express its vertical ordinate by z we then have the equation of motion

$$\frac{d^2 z}{dt^2} = g\left(\frac{\theta}{T} - 1 \right) = -C_p \frac{d\theta}{dz} - g$$

Hence follows the equation of energy, assuming that m has no initial velocity at z_0

$$\frac{m}{2}\left(\frac{dz}{dt} \right)^2 = m \left\{ C_p (\theta_0 - \theta) - g(z - z_0) \right\}$$
$$= m \left\{ C_p \theta_0 \left[1 - \left(\frac{p}{p_0} \right)^\kappa \right] - g(z - z_0) \right\}$$

This latter expression is identical with the right-hand side of equation 4 section 14, substituting m_1 for m, T_1 for θ_0 and h for $(z-z_0)$.

The deduction of the amount of the available kinetic energy for the whole system is rigorous as there given, but is inaccurate in the present article. The mass of air does not remain at rest when m moves through it; also, even if the energy of every particle of the mass is small, still the sum total may be of the same order of magnitude as the kinetic energy of m.

With this proviso I will quote some figures computed by these last equations in order to show how large the velocity may be that is produced by this force of buoyancy when the initial difference of temperature amounts to 10°.

When the acceleration is 0 the greatest absolute value of the velocity w will be attained at the altitude ζ, where $\theta = T$. Let the temperature of the larger mass of atmosphere at the initial position z_0 be $T_0 = 273°$ C. and in general $T = T_0 - a(z - z_0)$; when $a = \frac{\partial T}{\partial z} = 0$ we obtain

$$\left\{\begin{array}{llll} \text{for } \theta_0 = T_0 + 10°, & \zeta - z_0 = & 988.6 \text{ meters}, & w = \ \ 18.8\ m./sec. \\ \text{for } \theta_0 = T_0 - 10°, & \zeta - z_0 = & -1025 \text{ meters}, & w = -19.3\ m./sec. \end{array}\right\}$$

Therefore the values of $\zeta - z_0$ and w are larger in proportion as the rate of vertical diminution of temperature throughout the mass of atmosphere is larger; thus when $a = \frac{1}{2}\frac{g}{C_p} = \frac{\partial T}{\partial z}$ we have

$$\left\{\begin{array}{llll} \text{for } \theta_0 = T_0 + 10°, & \zeta - z_0 = & 1942 \text{ meters}, & w = \ \ 26.4\ m./sec. \\ \text{for } \theta_0 = T_0 - 10°, & \zeta - z_0 = & -2090 \text{ meters}, & w = -27.4\ m./sec. \end{array}\right\}$$

Similar computations for moist air are given by Reye in his "*Wirbelstürme*," Hanover, 1872, p. 227, etc.

---

§(20) *Computation of* $\int Tdm$ *for linear vertical diminution of temperature in the column of atmosphere.*

Assuming $T = T_0 - az$ then for the state of rest [or hydrostatic equilibrium] the pressure is given by the integration of the equation (α), i. e.,

$$p = p_0 \left(\frac{T}{T_0}\right)^{g/Ra}$$

whence we deduce the following equation (3), that will often be used hereafter, for the integral of the product of the temperature by the mass in a vertical column of atmosphere whose section has the unit area and for which $p = RT\mu$

$$\left.\begin{aligned}\int_0^z T\mu\,dz &= \frac{1}{R}\int_0^z p\,dz \\ &= \frac{1}{Ra}\,p_0\,T_0^{-g/Ra}\int_T^{T_0} T^{-g/Ra}\,dT \\ &= \frac{1}{g+Ra}\,(p_0 T_0 - pT)\end{aligned}\right\} \quad . \; . \; . \; . \quad (5)$$

For neutral equilibrium throughout the atmospheric column we have $a = g/C_p$ whence in this case

$$\int_0^z T\mu\,dz = \frac{1}{g}\cdot\frac{1}{1+\kappa}\cdot(p_0 T_0 - pT) \quad . \; . \; . \; . \quad (5a)$$

§(21) *Overturning of upper strata in the column; computation of the available kinetic energy in this case.*

We will apply equation (3) to the following problem. In one column (see fig. 3) are at first two masses, 1 and 2, each in neutral equilibrium; the entropy of the lower mass 2 is higher than that of 1; between these two masses as a whole there exists initially unstable equilibrium, after the overturning and in the final stable condition the whole mass 1, which now becomes 1′, rests below and 2 becomes 2′ on top. In this overturning, the altitudes of both strata are changed and also the location of their surface of separation i; the pressure on this surface i will be designated p_i for the initial and p'_i for the final stage; the temperature of each mass changes but the condition of neutral equilibrium remains true of 1′ and 2′ individually provided it be true as we assume that the transition from 1 to 1′ and 2 to 2′ is performed isentropically. The pressures p_0 at the base and p_h at the upper boundary remain unchanged.

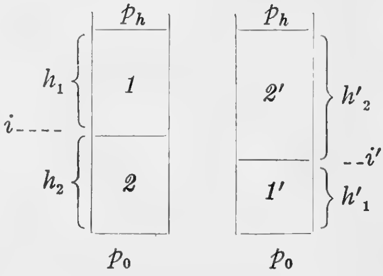


FIG. 3

At the boundary surface i there is initially a sudden transition from the temperature T_{i2} to the smaller value T_{i1}.

For this initial condition we have given the data

$$p_0, \; T_{02}, \; h_2, \; T_{i1}, \; h_1$$

whence we know

$$T_{i2} = T_{02} - \frac{g\,h_2}{C_p} \qquad T_{h1} = T_{i1} - \frac{g\,h_1}{C_p}$$

$$p_i = p_0 \left(\frac{T_{i2}}{T_{02}}\right)^{\frac{1}{\kappa}} \qquad p_h = p_i \left(\frac{T_{h1}}{T_{i1}}\right)^{\frac{1}{\kappa}}$$

With these values by the help of equations (1), (2) and (5*a*) we compute the following equation except the arbitrary constant

$$(\overline{\mathbf{P}} + \overline{\mathbf{I}})_a = \frac{C_p}{g} \cdot \frac{2}{1+\kappa} \left(p_0 T_{02} - p_i T_{i2} + p_i T_{i1} - p_h T_{h1}\right) + \text{Constant.}$$

For the final condition $p'_i = p_h + (p_o - p_i)$ whence the temperatures of the two masses at the boundaries become

$$T'_{01} = T_{h_1}\left(\frac{p_0}{p_h}\right)^{\kappa} \qquad T'_{i_1} = T_{h_1}\left(\frac{p'_i}{p_h}\right)^{\kappa}$$

$$T'_{i_2} = T_{02}\left(\frac{p'_i}{p_0}\right)^{\kappa} \qquad T'_{h_2} = T_{02}\left(\frac{p_h}{p_0}\right)^{\kappa}$$

and

$$(\overline{\mathbf{P}} + \overline{\mathbf{I}})_e = \frac{C_p}{g} \cdot \frac{1}{1+\kappa} \left(p_0 T'_{01} - p'_i T'_{i_1} + p'_i T'_{i_2} - p_h T'_{h_2}\right) + \text{Constant.}$$

We find the heights or depths of the masses 1′ and 2′ from

$$h'_1 = \frac{C_p}{g}\left(T'_{01} - T'_{i_1}\right); \qquad h'_2 = \frac{C_p}{g}\left(T'_{i_2} - T'_{h_2}\right)$$

The sum of the two masses is $\frac{p_0 - p_h}{g}$ wherefore the available kinetic energy of a unit mass is

$$\tfrac{1}{2} V^2 = g \, \frac{(\overline{\mathbf{P}} + \overline{\mathbf{I}})_a - (\overline{\mathbf{P}} + \overline{\mathbf{I}})_e}{p_0 - p_h}$$

Numerical example. Let $h_1 = h_2 = 2000$ meters and let the other data for this problem be the numbers indicated by a star (*) as follows (the pressures being in millimeters of mercury and the temperatures absolute Centigrade):

Initial stage

$p_0 = 760^*$ $\quad p_i = 591.690^{mm}$ $\quad p_h = 450.222^{mm}$

$T_{02} = 283^*$ $\quad T_{i2} = 263.1315^\circ$ $\quad T_{i1} = 260.1315^\circ$ $\quad T_{hi} = 240.2630^\circ$

$h_1^* = 2000$ meters $\quad h_2^* = 2000$ meters

Final stage

$p_0 = 760^{mm}$ $\quad p_i' = 618.532^{mm}$ $\quad p_h = 450.222^{mm}$

$T'_{01} = 279.773^\circ$ $\quad T'_{i1} = 263.509^\circ$ | $T'_{i2} = 266.548^\circ$ $\quad T'_{h2} = 243.034^\circ$

$h'_1 = 1637.17$ meters | $h'_2 = 2366.97$ meters

$$(\overline{\mathbf{P}} + \overline{\mathbf{I}})_a = \frac{C_p}{g} \cdot \frac{1}{1+\kappa} \cdot p_h \cdot 233\,.\,5132 + \text{Constant}$$

$$(\overline{\mathbf{P}} + \overline{\mathbf{I}})_e = \frac{C_p}{g} \cdot \frac{1}{1+\kappa} \cdot p_h \cdot 233\,.\,4140 + \text{Constant}$$

$$\tfrac{1}{2} V^2 = \frac{p_h}{p_0 - p_h} \cdot \frac{1}{1+\kappa} \cdot C_p\, 0.0992^\circ$$

whence $\quad V = 14.85$ meters per second.

This computation is laborious because the conclusion depends on a small difference between large quantities. We may find V by a shorter method without previously computing a number of other quantities as in the next following article.

---

§(22) *Approximate method of computing V.* We first assume that each of the masses 1 and 2 is initially in stable equilibrium but that the greatest entropy in 1 is smaller than the least entropy in 2, so that as before in the final stage the whole of 1′ comes to lie below 2′; the serial order of the strata in each of the masses will be retained provided the overturning proceeds isentropically.

Let p_1 and p'_1 be the pressures for the same layer in 1 and 1′, or in the initial and final stages of mass 1 respectively, and similarly T_1 and T'_1 for the corresponding temperatures. Let p_2; p'_2; T_2;

T'_2; be the analogous quantities for any given layer in the mass 2 and 2'. We thus have

$$p'_1 = p_1 + p_0 - p_i \qquad\qquad p'_2 = p_2 - (p_i - p_h)$$

$$T'_1 = T_1\left(\frac{p'_1}{p_1}\right)^\kappa = T_1\left(1 + \frac{p_0 - p_i}{p_1}\right)^\kappa$$

$$= T_1\left(1 + \kappa\frac{p_0 - p_i}{p_1}\right) \text{ approximately.}$$

With this last approximate value, which holds good in proportion as $p_0 - p_h$ is small relative to p_h, we obtain for mass 1

$$T_1 - T'_1 = -\frac{p_0 - p_i}{C_p\,\mu_1}$$

$$C_p \int (T_1 - T'_1)\,dm_1 = -(p_0 - p_i)\,h_1.$$

Similarly for the other mass 2 we obtain

$$T_2 - T'_2 = \frac{p_i - p_h}{C_p\,\mu_2}$$

$$C_p \int (T_2 - T'_2)\,dm_2 = (p_i - p_h)\,h_2.$$

Hence when h_1 and h_2 are small enough to allow of this approximate method we have

$$C_p \int (T - T')\,dm = (p_i - p_h)\,h_2 - (p_0 - p_i)\,h_1$$

From stable equilibrium we now pass on to the limiting case of neutral equilibrium within the masses 1 and 2; we have

$$p_h = p_i\left(1 - \frac{gh_1}{C_pT_{i1}}\right)^{1/\kappa} \quad\text{and}\quad p_0 = p_i\left(1 + \frac{gh_2}{C_pT_{i2}}\right)^{1/\kappa}$$

whence approximately

$$p_i - p_p = p_i\frac{gh_1}{RT_{i1}} \quad\text{and}\quad p_0 - p_i = p_i\frac{gh_2}{RT_{i2}}$$

$$C_p \int (T - T')\,dm = h_1\,h_2\,p_i\,\frac{g}{R}\cdot\frac{T_{i2} - T_{i1}}{T_{i1}\,T_{i2}} = \tfrac{1}{2}MV^2$$

$$M = \frac{p_0 - p_h}{g} = \frac{p_i}{R}\cdot\frac{h_1T_{i2} + h_2T_{i1}}{T_{i1}\,T_{i2}}$$

$$V^2 = 2g\cdot\frac{h_1\,h_2\,(T_{i2} - T_{i1})}{h_1T_{i2} + h_2T_{i1}}$$

If $h_2 = h_1$ and $T_i = \frac{1}{2}(T_{i1} + T_{i2})$ this last equation becomes

$$V^2 = gh_1 \frac{T_{i2} - T_{i1}}{T_i}$$

If this approximate formula be applied to the preceding example as computed in §21, we have

$$\left.\begin{array}{l} h_1 = 2000 \text{ meters} \\ T_{i2} - T_{i1} = 3^\circ \\ T_i = 261.62^\circ \end{array}\right\} \text{whence } V = 14.82 \text{ meters per second.}$$

Chapter III

METHODS OF COMPUTING THE AVAILABLE KINETIC ENERGY FOR A MASS OF AIR THAT PASSES ADIABATICALLY FROM ANY GIVEN INITIAL STAGE INTO THAT OF EQUILIBRIUM

§(23) Let there be given a mass of dry air, bounded by the horizontal ground plane and vertical walls, that is, initially at rest but not in equilibrium. We assume an initial condition of rest in order to be able to make use of the equation (α) of the preceding chapter, in our computation of the potential energy. We assume furthermore that that portion of the mass that is above the level surface h is already initially in equilibrium, and that during the changes that occur in the lower portion of the mass, this upper part acts like a piston of constant weight $p_h B$, where B is the area of the base or of any horizontal section.

Using a notation analogous to that of §13 we now have as before the following expression for the potential energy corresponding to the position of the column of air above the surface element dB

$$d\overline{\mathbf{P}} = dB \left\{ \int_0^h pdz + (Z - h)\, p_h \right\}$$

Whence follows the $\overline{\mathbf{P}}$ for the whole mass from the ground up to the level surface at the altitude h

$$\overline{\mathbf{P}} = \int B \int_0^h pd\,B\,dz + B\,(Z - h)\,p_h = \int pdk + \text{Constant}$$
$$= R \int T dm + \text{Constant.}$$

Similarly for the initial stage we obtain

$$(\overline{\mathbf{P}} + \overline{\mathbf{I}})_a = C_p \int T dm + \text{Constant}$$

The constant on the right hand is the same in both the initial and final stages.

In the case of an adiabatic passage from the initial over to the final stage the temperature T of the elementary mass dm becomes T' and the total potential energy of the whole mass becomes

$$(\overline{\mathbf{P}} + \overline{\mathbf{I}})_e = C_p \int T' dm + \text{Constant}$$

Hence the available kinetic energy in the initial stage is

$$-\delta\,(\overline{\mathbf{P}} + \overline{\mathbf{I}}) = C_p \int (T - T')\,dm\,.$$

---

§(24) *First analysis. The initial stage.* The mass of air in the chamber 1 of our fig. 1 is in neutral equilibrium, and S_1 is the entropy of the unit of mass. Similarly in the chamber 2 the air has the same volume but a higher entropy, S_2, and is in neutral equilibrium. The problem is wholly analogous to that treated previously, only the mass having the smaller entropy now lies alongside the other and not above it.

The given data of the present problem are: The temperatures T_{h1} and T_{h2} at the altitude h and the values B, h, p_h. From these we find the following initial temperature and pressures at the bases of the two masses.

$$T_{01} = T_{h1}\left(1 + \frac{gh}{C_p T_{h1}}\right)$$

$$T_{02} = T_{h2}\left(1 + \frac{gh}{C_p T_{h2}}\right)$$

$$p_{01} = p_h\left(\frac{T_{01}}{T_{h1}}\right)^{1/\kappa} = p_h\left(1 + \frac{gh}{C_p T_{h1}}\right)^{1/\kappa}$$

$$p_{02} = p\left(\frac{T_{02}}{T_{h2}}\right)^{1/\kappa} = p_h\left(1 + \frac{gh}{C_p T_{h2}}\right)^{1/\kappa}$$

From these and equation (5*a*) of the preceding chapter, section 20, there results

$$(\overline{\mathbf{P}} + \overline{\mathbf{I}})_d = C_p \cdot \frac{1}{g} \cdot \frac{1}{1+\kappa} \cdot \frac{B}{2} \cdot \left\{ T_{01}\, p_{01} - T_{h1} p_h + T_{02}\, p_{02} - \right.$$

$$\left. - T_{h2} p_h \right\} + \text{Constant.}$$

The final stage. The mass 2′ of higher entropy now occupies the upper part of the whole volume of the trough, the mass 1 having lower entropy is separted from it by the level suface i; at this level the temperature changes suddenly from T'_{i1} to T'_{i2}. Each of the two masses is individually in neutral equilibrium. At the upper surface of $2'$ the pressure is p_h, consequently the temperature is T_{h2}. Since the entropies S_1 and S_2 remain unchanged therefore the values of the pressures at i and at the base (p'_2 and p'_0) are to be computed from the weight of the mass and thus we completely know the final stage, as follows:

Location.	Pressure.	Temperature.
At the upper boundary 2′	p_h	T_{h2}
At the surface of separation i between the masses 2′ and 1′	$p'_i = p_h + \frac{1}{2}(p_{02} - p_h)$	$\begin{cases} T'_{i2} = T_{h2}\left(\dfrac{p'_i}{p_h}\right)^{\kappa} \\ T'_{i1} = T_{h1}\left(\dfrac{p'_i}{p_h}\right)^{\kappa} \end{cases}$
At the base level we have...............	$p'_0 = p_h + \frac{1}{2}(p_{02} - p_h) + \frac{1}{2}(p_{01} - p_h)$	$T'_0 = T_{h1}\left(\dfrac{p'_0}{p_h}\right)^{\kappa}$

We thus obtain all the quantities that enter into the expression for the total energy of the final stage

$$(\overline{\mathbf{P}} + \overline{\mathbf{I}})_e = C_p \cdot \frac{1}{g} \cdot \frac{1}{1+\kappa} \cdot B \left\{ T'_0 p'_0 - T'_{i1} p'_i + T'_{i2} p'_i - \right.$$
$$\left. - T_{h2} p_h \right\} + \text{Constant}$$

except only the arbitrary constant which will itself disappear when the difference is taken.

We assume that the available kinetic energy, viz.,

$$-\delta(\overline{\mathbf{P}} + \overline{\mathbf{I}}) = (\overline{\mathbf{P}} + \overline{\mathbf{I}})_a - (\overline{\mathbf{P}} + \overline{\mathbf{I}})_e,$$

belongs specifically to the mass M below the piston, and that this therefore may be written $\frac{1}{2} MV^2$.

In frictionless motion if the final stage be attained simultaneously by all the masses, then $\frac{1}{2}V^2$ is their average kinetic energy. Since $M = B\left(\dfrac{p'_0 - p_h}{g}\right)$ therefore V is independent of the area of the base.

Finally we compute the heights of the strata $1'$ and $2'$ from the temperatures by the formulæ

$$h'_1 = \frac{C_p}{g}(T'_0 - T'_{i1}) \qquad h'_2 = \frac{C_p}{g}(T'_{i2} - T_{h2})$$

Example 1. Given $h = 3000$ meters; $p_h = 510^{\mathrm{mm}}$ mercury

$$T_{h1} = 243° \qquad T_{h2} = 248°$$

These data and the quantities computed from them for the initial and final stages are arranged in the following tabular form:

Initial stage

$$h = 3000 \text{ meters}$$

$$T_{h1} = 243° \qquad p_h = 510^{\mathrm{mm}} \qquad T_{h2} = 248°$$

$$T_{01} = 272.8027° \quad p_{01} = 759.1980 \mid p_{02} = 753.4621 \quad T_{02} = 277.8027°$$

$$(\overline{\mathrm{P}} + \overline{\mathrm{I}})_a = \frac{C_p}{g} \cdot \frac{1}{1+\kappa} B \,.\, p_h \times 162.7598 + \text{Constant}$$

Final stage

$$p_h = 510^{\mathrm{mm}} \ldots \left\{ \begin{aligned} T_{h2} &= 248° \\ T'_{i2} &= 263.9266 \end{aligned} \right\} h'_2 = 1603.2 \text{ meters.}$$

$$p'_i = 631.7310$$

$$p'_0 = 756.3300 \quad \left\{ \begin{aligned} T'_{i1} &= 258.6055 \\ T'_{01} &= 272.5026 \end{aligned} \right\} h'_1 = 1398.9 \text{ meters.}$$

$$(\overline{\mathrm{P}} + \overline{\mathrm{I}}) = \frac{C_p}{g} \cdot \frac{1}{1+\kappa} \,.\, B \,.\, p_h \times 162.7127 + \text{Constant}$$

$$\tfrac{1}{2} V^2 = C_p \frac{1.41}{1.82} \times \frac{0.0471}{0.4830}$$

$$V = 12.2 \textit{ meters per second.}$$

Example 2 $\qquad h = 3000^{\mathrm{m}} \quad p_h = 510^{\mathrm{mm}}$

$T_{h1} = 243° \quad T_{h2} = 253° \quad$ whence $V = 17.3\ m/sec.$

Example 3 $\qquad h = 6000^{\mathrm{m}} \quad p_h = 325^{\mathrm{mm}}$

$T_{h1} = 213° \quad T_{h2} = 218° \quad$ whence $V = 18.3\ m/sec.$

Example 4 $\qquad h = 6000^{\mathrm{m}} \quad p_h = 325^{\mathrm{mm}}$

$T_{h1} = 213° \quad T_{h2} = 223° \quad$ whence $V = 25.8\ m/sec.$

---

§(25) *Second analysis:* approximate method for the case when the masses 1 and 2 are each initially in stable equilibrium and the entropy of the highest layer of 1 is smaller than that of the lowest layer of 2.

Since it is again assumed that every particle of the mass behaves adiabatically, therefore the layers in 1 retain their relative positions when they become 1′; similarly for the layers of 2 when they become 2′. We will designate the areas of the floors of the chambers by B_1 and B_2 so that $B_1 + B_2 = B$.

If p'_1 is the pressure in the final stage of any layer of 1′ which had the pressure p_1 when it was in the initial stage, then we have the relation

$$p'_1 = p_h + \frac{B_1}{B}(p_1 - p_h) + \frac{B_2}{B}(p_{02} - p_h) = p_1 + \frac{B_2}{B}(p_{02} - p_1)$$

Similarly when p'_2 and p_2 refer to another equal mass in the chambers 2′ and 2 in fig. 1 we have

$$p'_2 = p_h + \frac{B_2}{B}(p_2 - p_h) = p_2 - \frac{B_1}{B}(p_2 - p_h).$$

Hence the temperatures T'_1 and T'_2 of these masses in the final stage are to be computed from their initial temperatures T_1 and T_2 by the following equations:

$$T'_1 = T_1\left(\frac{p'_1}{p_1}\right)^\kappa = T_1\left(1 + \frac{B_2}{B}\cdot\frac{p_{02} - p_1}{p_1}\right)^\kappa$$

$$= T_1\left(1 + \kappa\frac{B_2}{B}\cdot\frac{p_{02} - p_1}{p_1}\right) \text{approximately.}$$

$$T'_2 + T_2\left(\frac{p'_2}{p_2}\right)^\kappa = T_2\left(1 - \frac{B_1}{B}\cdot\frac{p_2 - p_h}{p_2}\right)^\kappa$$

$$= T_2\left(1 - \kappa\frac{B_1}{B}\cdot\frac{p_2 - p_h}{p_2}\right) \text{approximately.}$$

The approximate formulæ hold good for every p_1 and p_2 when $p_{01} - p_h$ is small relative to p_h.

In the computation of the integrals that occur in the expression for the available kinetic energy we take layers of 1 and 2 as the elementary masses and therefore by making use of the equation (α) § 13 we have

$$dm_1 = -B_1\frac{dp_1}{g} \qquad dm_2 = -B_2\frac{dp_2}{g}$$

Substituting hereafter only approximate values we obtain

$$\int (T_1 - T_1')\, dm_1 = -\frac{\kappa}{g}\cdot\frac{B_1 B_2}{B}\int_{p_h}^{p_{01}} T_1\frac{p_{02}-p_1}{p_1}dp_1$$

$$\int (T_2 - T_2')\, dm_2 = +\frac{\kappa}{g}\cdot\frac{B_1 B_2}{B}\int_{p_h}^{p_{02}} T_2\frac{p_2-p_h}{p_1}dp_2$$

We may remark that although in the development of the binomials we previously retained only the terms of the first degree in $\dfrac{p_0 - p_h}{p_h}$ yet on account of the limits of these definite integrals, they are accurate to terms of the second degree.

From equation (α) there results

$$\int_{p_h}^{p_{01}}\frac{T_1}{p_1}dp_1 = \int_{p_h}^{p_{02}}\frac{T_2}{p_2}dp_2 = \int_0^h\frac{g}{R}dz = \frac{gh}{R}$$

If we introduce the average temperatures T_1^* and T_2^* of the masses 1 and 2 as determined by the equations

$$T_1^*\,(p_{01} - p_h) = \int_{p_h}^{p_{01}} T_1\,dp_1; \qquad T_2^*\,(p_{02} - p_h) = \int_{p_h}^{p_{02}} T_2\,dp_2$$

we thus obtain for the above integrals

$$\int (T_1 - T_1')\, dm_1 = \frac{\kappa}{g}\cdot\frac{B_1 B_2}{B}\left\{T_1^*\,(p_{01}-p_h) - \frac{gh}{R}\,p_{02}\right\}$$

$$\int (T_2 - T_2')\, dm_2 = \frac{\kappa}{g}\cdot\frac{B_1 B_2}{B}\left\{T_2^*\,(p_{02}-p_h) - \frac{gh}{R}\,p_h\right\}$$

These expressions may be still further simplified if we introduce other average temperatures $\odot_1$ and $\odot_2$ which for distinction we will call barometric temperatures as determined by the barohypsometric relations*

$$p_{01} = p_h e^{\frac{gh}{R\odot_1}} = p_h\left(1 + \frac{gh}{R\odot_1} + \frac{1}{2}\left(\frac{gh}{R\odot_1}\right)^2\right) \text{ approximately.}$$

$$p_{02} = p_h e^{\frac{gh}{R\odot_2}} = p_h\left(1 + \frac{gh}{R\odot_2} + \frac{1}{2}\left(\frac{gh}{R\odot_2}\right)^2\right) \text{ approximately.}$$

In the serial development of these exponentials we have stopped at the terms of second degree in accordance with a preceding remark. With these values our integrals become†

$$\int (T_1 - T_1')\, d\,m_1 = \frac{\kappa}{g} \cdot \frac{B_1 B_2}{B} \cdot p_h \cdot \frac{gh}{R} \times$$
$$\times \left\{\frac{T_1^*}{\odot_1} - 1 - \frac{gh}{R}\left(\frac{1}{\odot_2} - \frac{T_1^*}{2\,\odot_1^2}\right)\right\}$$

$$\int (T_2 - T_2')\, d\,m_2 = \frac{\kappa}{g} \cdot \frac{B_1 B_2}{B} \cdot p_h \cdot \frac{gh}{R} \cdot \times$$
$$\times \left\{\frac{T_2^*}{\odot_2} - 1 + \frac{gh}{R}\frac{T_2^*}{2\odot_2^2}\right\}$$

Now in the actual applications to our atmosphere the quantities $T^* - \odot$ are always very small relative to $\frac{gh}{R}$. For the value $h = 3000$

---

* The e is the base of the napierian logarithms.

† T = temperature of air at any time or place.

T^* = true average temperature of a mass of air as determined by the mass-relation,

$$T^* = \frac{\int_{p_h}^{p_o} T\, dp}{p_o - p_h}$$

$\odot$ = Approximate average temperature of mass of air as determined by the barohypsometric relation

$$p_o = p_h\, e^{gh/R\odot}$$

whence

$$\log \frac{p_o}{p_h} = \text{Modulus} \times \frac{gh}{R\odot}$$

$$\odot = \text{Mod.} \frac{gh}{R} \frac{1}{\log \dfrac{p_o}{p_h}}$$

meters this quantity is 102° and the difference between the true average temperature T^* and the barometric average temperature $\odot$ can only attain 2° Centigrade in extreme cases. For smaller values of h the $T^* - \odot$ is also smaller. We can therefore here substitute T^* for the corresponding $\odot$ and obtain

$$C_p \int (T_1 - T_1') \, dm_1 = - \frac{R}{g} \cdot \frac{B_1 B_2}{B} \cdot p_h \left(\frac{gh}{R}\right)^2 \left(\frac{1}{T_2^*} - \frac{1}{2 T_1^*}\right)$$

$$C_p \int (T_2 - T_2') \, dm_2 = - \frac{R}{g} \frac{B_1 B_2}{B} p_h \left(\frac{gh}{R}\right)^2 \left(\frac{1}{2 T_2^*}\right)$$

Finally by writing

$$(T^*)^2 = T_1^* T_2^*,$$

$$\frac{T_2^* - T_1^*}{T^*} = \tau,$$

$$B p_h \frac{gh}{RT^*} = gM \text{ (approximately)}$$

we obtain the following expression for the available kinetic energy,

$$C_p \int (T - T') \, dm = \frac{MV^2}{2} = \frac{M}{2} \cdot \frac{B_1 B_2}{B^2} gh\tau.$$

V is independent of the constants R and C_p that characterize the physical properties of the gas. For a given value of B this expression for the available kinetic energy is greatest when $B_1 = B_2 = \frac{1}{2} B$; wherefore when the chambers 1 and 2 have equal volumes then the velocity is

$$V = \frac{1}{2} \sqrt{gh\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I)$$

This approximate method suffices completely for the cases above given as examples of our first method of computation (see section 24). Since in the limiting case neutral equilibrium becomes stable equilibrium we may also apply this method of computation to those examples also. In the computation of T we may substitute $\frac{1}{2} (T_0 + T_h)$ for the average temperature. The values of V computed by the approximate formula for those four examples now become respectively

Example	*Velocity m. p. s.*
(1)	12.3
(2)	17.4
(3)	18.6
(4)	26.3

The assumptions of the approximate computation do not hold good for $h = 6000$ meters; but still the final formula gives very good approximate values; this is also true for systems having still greater altitudes.

---

§(26) If the overturning of the masses 1 and 2 takes place under constant volume the computation proceeds in a perfectly similar manner. These masses considered by themselves now form the closed system to which equation (6**) of section 20 is to be applied. The diminution of $\overline{\mathbf{P}}$ will be greater than before and by so much smaller will be the diminution of $\overline{\mathbf{I}}$, so that we obtain the same value of V as for the overturning under constant pressure.

We may also state the problem thus: During the overturning the pressure at the upper boundary surface changes but has the same value throughout all parts of this surface. If the change proceeds so slowly that no appreciable amount of kinetic energy is thereby produced, then we again obtain the same equation (I) as just now deduced for the velocity V. This is not a case of a closed system. In place of the equation (6**) we now have

$$\delta\,\overline{\mathbf{K}} + (\overline{\mathbf{R}}) = -\,\delta\,(\overline{\mathbf{P}} + \overline{\mathbf{I}}) - B\int^{\prime} p_h\,dh$$

where the first term on the right hand refers only to the masses 1 and 2 and can be written in the form

$$C_p\int(T - T')\,d\,m + B\,(h'\,p'_h - h\,p_h)$$

whereas the second term on the right results from the motion of the movable piston. When p_h is constant the sum of these right-hand terms becomes $C_p \int (T - T')dm$. The case of constant volume corresponds to $h' = h$.

We will now leave the two chambers system and compute the available kinetic energy in a special case of continuous distribution of temperature.

---

§(27) *Third analysis.* The initial conditions are a continuous horizontal distribution of temperature and a vertical diminution of temperature corresponding to that of neutral equilibrium.

We assume that the trough is a parallelopipedon of air having a unit breadth and a length l along the horizontal axis of x, and that in the initial stage the temperature of the lowest surface is a continuous function of the distance x from the left-hand end of the trough; therefore the temperature at the point (x, z) is expressed by the equation

$$T = f\,(x) - \frac{gz}{C_p}$$

If the $f\,(x)$ increases from the left toward the right then the entropy will also do so, since the pressure p_h is again assumed to be constant at the level h; hence the entropy is only a function of the distance x or the length of the trough and this is equally true of the pressure p_{0x} at the base level.

If the mass of air under the pressure p_h passes over adiabatically into the condition of equilibrium, then a horizontal layer is formed from each vertical column of the initial condition and the masses will now succeed each other from below upward in the same order as they were before arranged from the left toward the right. A mass originally in a vertical column at x having an initial p and T has in the final stage a p' and T' such that

$$p' = p_h + \frac{1}{l}\int_{l-x}^{l} (p_{0x} - p_h)\,dx$$

$$= p - \left(p - p_h - \frac{1}{l} \cdot \int_{l-x}^{l} (p_{0x} - p_h)\,dx \right)$$

In this last equation we consider the member in the parenthesis as small relative to p, which is true when the difference between any two values of the pressure within the mass is small as compared with the total pressure p_h. Under this assumption we introduce an approximate computation analogous to that of the last section § (25),

$$T - T' = T - T\left(\frac{p'}{p}\right)^{\kappa} = \kappa\left[T - \frac{p_h}{R\mu} - \frac{1}{lR\mu}\int_{l-x}^{l} (p_{0x} - p_h)\,dx\right].$$

We first seek for the mass-integral of $(T - T')$ throughout the whole mass of the unit column above the point x on the axis of abscissæ; in accord with the previous definition we put T_x^* for the average temperature of this column, then we have

$$T_x^* \frac{p_{0x} - p_h}{g} = T_x^* \int_0^h \mu dz = \int_0^h T \mu dz$$

whence we obtain the mass-integral of $(T - T')$ as follows:

$$\left.\begin{aligned}\int_0^h (T - T')\,\mu dz = \frac{\kappa}{g}\,.\ \Big\{ T_x^* (p_{0x} - p_h) - \\ - \frac{gh}{R}\left[p_h + \frac{1}{l}\int_{l-x}^{l} (p_{0x} - p_h)\,dx \right]\Big\}\end{aligned}\right\} \ldots (A)$$

This expression multiplied by the factor $C_p dx$ and integrated throughout the whole length l gives the available kinetic energy of the whole system. But since we take the true average temperature T^* instead of the barometric average temperature therefore we first substitute

$$p_{0x} - p_h = p_h \left[\frac{gh}{RT_x^*} + \frac{1}{2}\left(\frac{gh}{RT_x^*}\right)^2 \right] \quad \ldots\ldots (B)$$

and remark that in the first member, on the right hand side of the integral (A), both terms of (B) as the serial development of $p_{ox} - p_h$ are to be used, but in the last member of (A) only the linear term in h or the first term of (B) need be considered, if we desire to go only as far in the result as terms of the order

$$T^* \left(\frac{gh}{RT^*} \right)^2$$

For T_x^* we choose a linear function of the length in which τ is small relative to unity so that

$$T_x^* = T_0^* \left(1 + \tau \frac{x}{l} \right) \;;\; \frac{1}{T_x^*} = \frac{1}{T_0^*}\left(1 - \tau \frac{x}{l} \right)$$

This gives for the last term in the integral (A)

$$\frac{1}{l}\int_{l-x}^{l} (p_{0x} - p_h)\,dx = p_h\,\frac{gh}{RT_0^*}\left(\frac{x}{l} - \tau\,\frac{2\,lx - x^2}{l^2} \right)$$

and for the complete integral (A)

$$\int_0^h (T - T')\,\mu\,dz = \frac{\kappa}{g} p_h\, T_0^* \left(\frac{gh}{RT_0^*} \right)^2 \left\{ \frac{1}{2} - \frac{x}{l} + \frac{\tau}{2}\cdot\frac{x}{l} - \frac{\tau x^2}{2 l^2} \right\}$$

whence the available energy

$$\left.\begin{aligned} C_p \int (T - T')\, dm &= C_p \int_0^l dx \int_0^h (T - T')\, \mu\, dz \\ &= \frac{M}{2} V^2 = l p_h \frac{gh}{RT_0^*} \,.\, h \,.\, \frac{\tau}{12} \end{aligned}\right\} \quad . \; . \; (C)$$

Using the approximate value

$$M = l\, p_h \frac{gh}{R\, T_0^*}$$

we obtain

$$V = \sqrt{\frac{gh\tau}{6}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(II)}$$

In this case the available kinetic energy for the unit of mass is smaller in the ratio of 2 to 3 than in the system previously considered, if we substitute T_0^* and T_1^* therein instead of T_1^* and T_2^*. In this first approximation, this energy depends only on the altitude and on the maximum horizontal difference of temperature and is independent of the length of the trough and therefore also of the horizontal temperature gradient.

The result is principally determined by the assumption that the vertical diminution of temperature is that corresponding to neutral equilibrium, as is shown in the following section.

---

§(28) *To find the location of the surfaces of equal entropy in a mass of air when the pressure is constant throughout any level surface and the temperature is a function of the length and a linear function of the altitude.*

In the expression for the entropy

$$S = \text{Constant} + C_p \log T - R \log p,$$

we put

$$T = T_h + \frac{g}{n C_p}(h - z) \qquad \text{and} \qquad p = p_h \left(\frac{T}{T_h}\right)^{\frac{n C_p}{R}}$$

where T_h is a function of x only and p_h is constant. The system of curves of equal entropy in the xz plane is determined by the equation

$$F(x, z) = n \log T_h - (n - 1) \log T = \text{Constant}$$

The angle α that is included between the direction of the curve in the vertical plane of xz and the horizon is given by the equation

$$\tan \alpha = \frac{\partial F}{\partial x} \Big/ \frac{\partial F}{\partial z} = \frac{n}{n-1}\left(\frac{h-z}{T_h} + \frac{C_p}{g}\right)\frac{\partial T_h}{\partial x}$$

For $n = 1$ the angle α is a right angle. If $n > 1$ with stable equilibrium in each isolated column, and if x increases toward the right hand, then the curves of equal entropy trend downward toward the right. With increasing values of n the inclinations of the surfaces of equal entropy to the level surfaces diminish very rapidly.

If the horizontal increase of temperature is 1° C. for 20 kilometers or

$$\frac{\partial T_h}{\partial x} = 0.00005° \text{ Centigrade per meter}$$

we obtain the following respective sets of values:

n	Vertical Temp. Gradient per 100m	α_h for $z = h$	α_0 for $z = 0$, $h = 6000m$, $T_h = 250°$
1.00	0.993	90° 0′	90° 0′
1.01	0.984	26° 57′	32° 12′
1.10	0.903	3° 10′	3° 55′
1.50	0.662	0° 52′	1° 4′
2.00	0.497	0° 34′	0° 43′

Even for very large horizontal temperature gradients, the surfaces of equal entropy are but slightly inclined to the level surfaces when the vertical diminution of temperature in dry air is less than 0.9° per 100 meters. If now from such an initial condition the masses pass adiabatically to the final stage then there is evidently available a smaller amount of kinetic energy than in the cases where the entropy surfaces are at first vertical.

---

§(29) We now return to the two-chamber system.

To find the final stage of two masses of air under constant pressure with initial linear vertical diminution of temperature.

If the vertical gradient of temperature within the masses 1 and 2 (fig. 4) is smaller than that for neutral equilibrium then the entropy increases with the altitude and it can happen that the entropy at the altitude c_1 in the cooler mass 1 is as large as that at the base of the mass 2, i. e., at the altitude $(h - c_2)$; the upper layers of 1 may one after the other serially have respectively the same entropy as the layers in 2 up to the altitude $(h - c_2)$. In the final stage the lower part of 1 will become spread out at the base; over it will lie strata consisting of 1 and 2 mixed; above this will rest that portion of 2 that initially lay between $(h - c_2)$ and h. At the boundaries between these three layers the temperature changes are continuous.

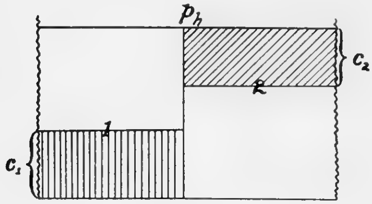


FIG. 4.

If the temperatures diminish linearly as in the equations

$$T_1 = T_{h1} + \frac{g}{n_1 C_p}(h - z) \qquad T_2 = T_{h2} + \frac{g}{n_2 C_p}(h - z)$$

then we have to seek the altitude at which the entropies are equal or $S_1 = S_2$ for the same value of p_h in the equations

$$S_1 = K + C_p [n_1 \log T_{h1} - (n_1 - 1) \log T_1]$$
$$S_2 = K + C_p [n_2 \log T_{h2} - (n_2 - 1) \log T_2]$$

Let us assume $n_1 = n_2 = n$; let θ_1 be the temperature of the mass 1 at the altitude c_1 and θ_2 the temperature of the mass 2 at the altitude $h - c_2$ then we have

$$\log \frac{T_{02}}{\theta_1} = \log \frac{\theta_2}{T_{1h}} = \frac{n}{n-1} \log \frac{T_{h2}}{T_{h1}}$$

$$c_1 = \frac{C_p}{n g}(T_{01} - \theta_1) \qquad c_2 = \frac{C_p}{n g}(\theta_2 - T_{h2})$$

Hence for $n = 2$ and a vertical temperature gradient of about 0.5° per 100 meters and for $h = 3000$ meters $T_{h1} = 263°$, $T_{h2} = 273°$, we find $c_1 = 2154$ meters and $c_2 = 2090$ meters.

In this case the greater part of the masses 1 and 2 remain unmixed, the available kinetic energy will not be much smaller than if in the final stage the whole of mass 1 lies below and the whole of 2 above.

Again, for $h = 6000$ meters $T_{h1} = 248°$, $T_{h2} = 258°$, we find $c_1 =$ 2390 meters and $c_2 = 2096$ meters.

In this case the descent of the mass 1, and the ascent of the other mass 2, are much smaller than for the same values of h in the first and second analyses §(24) and §(25); hence also the available kinetic energy cannot be evaluated by using the equations there deduced.

---

§(30) *Two masses of air having constant temperatures.* In order to be able to express more accurately the influence of the formation of three strata on $-\delta(\bar{\mathbf{P}} + \bar{\mathbf{I}})$ I have computed an example for the case of constant T_1 and T_2. In this case we have

$$S_1 = K + C_p \log T_1 - g\frac{h-z}{T_1}$$

$$S_2 = K + C_p \log T_2 - g\frac{h-z}{T_2}$$

$$\frac{gh}{T_2} - \frac{g(h-c_1)}{T_1} = \frac{gc_2}{T_2} = C_p \log\frac{T_2}{T_1}$$

In the final stage the temperature of the median layer is constant, viz.,

$$(T') = 2^{-\kappa}\left(T_1^{1/\kappa} + T_2^{1/\kappa}\right)^{\kappa}$$

For $h = 3000$ meters, $T_1 = 258°$, $T_2 = 268°$, we find $c_1 = 1099.5$ meters $c_2 = 1025.9$ meters, $(T') = 263.12°$.

The available kinetic energy is to be found as in the first computation but by a much longer route. The velocity for the average kinetic energy and for two chambers of equal volume is $V = 9.5$ meters per second or only half as much as for the more complete overflow from 2 and underflow from 1 as computed by equation (I) of section 25.

FOURTH ANALYSIS. EQUALIZATION OF THE PRESSURES IN A HORIZONTAL LAYER THAT RETAINS THE CONSTANT VOLUME

§(31) *Computation of the available kinetic energy.*

Initial stage. Assume a thin horizontal layer, bounded by rigid walls, divided by a screen into two chambers having volumes k_1 and k_2, whose masses are under the pressures p_1 and p_2 respectively. After removal of the dividing screen the pressure throughout the whole volume $k = k_1 + k_2$ becomes p'; if the expansion of one mass and the compression of the other takes place adiabatically then the new volumes k'_1 and k'_2 will be

$$k'_1 = k_1 \left(\frac{p_1}{p'}\right)^{1/\gamma} \qquad k'_2 = k_2 \left(\frac{p_2}{p'}\right)^{1/\gamma}.$$

From the condition that $k'_1 + k'_2 = k$ we find

$$p' = \left(\frac{k_1}{k} p_1^{\ 1/\gamma} + \frac{k_2}{k} p_2^{\ 1/\gamma}\right)^{\gamma}$$

The center of gravity of the whole mass remains at the same altitude wherefore $\delta \overline{\mathbf{P}} = 0$.

The available kinetic energy is therefore

$$- \delta \overline{\mathbf{I}} = (\overline{\mathbf{I}}_a) - (\overline{\mathbf{I}}_e) = C_v \int (T - T') \, \mu dk = \frac{C_v}{R} \int (p - p') \, dk$$

$$= \frac{1}{\gamma - 1} (k_1 p_1 + k_2 p_2 - k p')$$

If the difference $p_1 - p_2$ is small as compared with the pressure in one of the chambers so that $p_1 - p_2 = \varepsilon p_1$ then

$$p_2 = p_1 (1 - \varepsilon)$$

$$p' = p_1 \left(1 - \frac{k_2}{k} \varepsilon - \frac{k_1 k_2}{k^2} \cdot \frac{\gamma - 1}{\gamma} \cdot \frac{\varepsilon^2}{2}\right)$$

$$- \partial \overline{\mathbf{I}} = \frac{1}{\gamma} \cdot \frac{k_1 k_2}{k^2} \cdot k p_1 \cdot \frac{\varepsilon^2}{2} \cdot = \tfrac{1}{2} m (V)^2$$

Therefore if T^* is the average temperature of the whole mass, then the average available kinetic energy for the unit mass $\frac{p_1 k}{RT}$ is given by

$$\frac{[V^2]}{2} = \frac{(V)^2}{2} = \frac{k_1 k_2}{k^2} \cdot \frac{RT^*}{\gamma} \cdot \frac{\varepsilon^2}{2}$$

and when the other circumstances are the same this has its maximum when

$$k_1 = k_2 = \frac{1}{2} k.$$

If then the volumes of the chambers 1 and 2 are equal, we have

$$(V) = \frac{1}{2}\varepsilon . \sqrt{\frac{C_v}{C_p} RT^*} = \frac{1}{2} \frac{p_1 - p_2}{p_1} \sqrt{\frac{C_v}{C_p} R . T^*} \quad . \; . \; . (III)$$

If $p_1 = 765$ mm mercury, $p_2 = 755$mm mercury, $T = 273°$ then this equation gives $(V) = 1.55$ meters per second.

We will compare this value of (V) with the V that was deduced previously in §(3) for two masses that initially lay in a deep trough alongside of each other (see fig. 1). If h is the altitude [of the two chambers] and T_1^* T_2^* are the average temperatures and if the greatest entropy in mass (1) is smaller than that of the lowest layer of mass (2), then for chambers of equal volume the second analysis gave the velocity $V = \frac{1}{2} \sqrt{gh\tau}$ (see the expression (I) §25). This may be brought into a form similar to that given above in (III) if we put

$$\frac{gh}{RT_1^*} = \log \frac{p_{01}}{p_h} \qquad \frac{gh}{RT_2^*} = \log \frac{p_{02}}{p_h}$$

$$gh\tau = gh \frac{T_2^* - T_1^*}{T^*} = \text{approximately } RT^* \frac{p_{01} - p_{02}}{p_{01}}$$

whence

$$V = \frac{1}{2} \sqrt{\frac{p_{01} - p_{02}}{p_{01}}} \sqrt{RT^*} \quad . \; . \; . \; . \; . \; . \; . \; (I^*)$$

If for p_{01} p_{02} and T^* we assume the same values as those just given $p_1 = 765$, $p_2 = 755$, $T = 273°$ there results $V = 16$ meters per second, or a velocity ten times larger than from equation (III), and therefore a hundred times the kinetic energy per unit mass.

---

§(32) We can also add the following problem:

Let the chambers 1 and 2 of fig. 1 be limited above by a rigid partition and contain masses of air that have equal entropies but different pressures at the same altitudes, consequently there must be a higher temperature on the side of the higher pressure. The difference of pressure at any level is in the same direction at all altitudes and is nearly proportional to the average pressure at that level. The initial stages of 1 and 2 are respectively that of stable equilibrium. After removing the separating vertical partition the adjacent layers on the same level unite. The altitude of the center of gravity remains unchanged. The resulting (V) has the same form (eq. III) as for the thin horizontal layer considered above, provided we now let T indicate the average temperature of the whole mass and let p_1 and p_2 indicate the initial values of the pressures at the base.

Again let there be resting in these chambers masses whose distribution of entropies is such as was assumed in the second analysis, see section 25; this equation (I) or (I*) applies to their overturning even in this present case of constant volume. For the same differences of pressure at the base, and for smaller differences above, and when $p_{01} - p_{02}$ is a small fraction, as is the case in our atmosphere, equation (I) gives a much larger value of the living force than equation (III).

It seems now to have been abundantly demonstrated that the available kinetic energy of such a system is not dependent materially on the horizontal differences of pressure but on the distribution of entropy and the buoyancy dependent thereon.

---

§(33) *Appendix to the fourth analysis. Study of Joule's experiment relative to the mutual independence of the internal energy and the volume of a gas.*

In the first chamber of fig. 1 let the horizontal layer of gas be under the pressure p_1 but let the other chamber be empty or $p_2 = 0$; we now have the same arrangement as in Joule's experiment, if we put $k_1 = \frac{1}{2}k$.

For this case the first equations of our fourth analysis, § 31, become

$$p' = 2^{-\gamma}\, p_1$$

$$-\delta\,\bar{\mathrm{I}} = \frac{C_v}{R}\left(\frac{k}{2}\,p_1 - k\,.\,2^{-\gamma}p_1\right)$$

$$= \frac{C_v}{R}\,\frac{kp_1}{2}\,(1 - 2^{1-\gamma})$$

If T is the initial temperature in chamber one, then the mass of the gas is

$$M = \frac{k}{2}\,\frac{p_1}{RT}$$

whence

$$-\frac{\delta\,\bar{\mathrm{I}}}{M} = \frac{(V)^2}{2} = C_v T\,(1 - 2^{1-\gamma})$$

In the case of motion without friction we have for air

$$(V) = \sqrt{2\,C_v T\,(1 - 2^{-0.41})} = 0.7034\,\sqrt{C_v T}.$$

For the temperature $T = 273°$ this equation gives $(V) = 307 m./sec.$

If we assume that the kinetic energy is confined to that half of the mass that flows out of the first chamber into the second, we find the velocity 435 *m./sec.* These values are of the same magnitude as the molecular velocities. In small vessels the kinetic energy is very soon converted (by friction) into heat and is added to the internal energy of the system This added heat that we fail to notice in our atmospheric movements is decisive for the result of the laboratory experiment. Probably the experiment of Joule would have given a very different conclusion if he could have performed it in vessels of very large horizontal extent. In that case one should have observed a diminution of the internal energy at whose cost arose the kinetic energy of the systematic motion appropriate to the extent of the vessel. The previous portion of this article has been suggested by the relation of our problem to Joule's experiment but the following suggestion which has perhaps already been made elsewhere may be added. In the case when the volumes of the gas chamber and the vacuum chamber are unequal and with the assumptions that the motion is frictionless and that the change of condition is adiabatic and that at certain moments the pressure p' is uniform throughout the whole space, we have

$$p' = \left(\frac{k_1}{k}\right)^{\gamma} p_1$$

$$-\delta\bar{\mathrm{I}} = \frac{M\,(V)^2}{2} = M\,C_v T\left[1 - \left(\frac{k_1}{k}\right)^{\gamma - 1}\right]$$

If the vacuum chamber is very large relatively to the gas chamber so that $k_1/k = 0$ approximately and since $\gamma - 1 > 0$ therefore this equation gives

$$\frac{1}{2}\,(V)^2 = C_v T$$

On the other hand, from the kinetic theory of gases, if (u^2) is the mean of the squares of the velocities of the molecules of gas in the non-systematic free motions of molecules [or (u^2) is the square of the mean free path] we have

$$\frac{1}{2}\,(u^2) = \frac{3}{2} R T$$

Hence if $C_v = \frac{3}{2}R$ or $\gamma = \frac{5}{3}$ we have $(V) = (u)$. This is the value of γ that is found for non-atomic gases but for other gases γ is less than $\frac{5}{3}$ and (V) is greater than (u).

---

§(34) *Comparison of the problem above treated with analogous analyses for incompressible liquids.*

We will now consider a system of liquids each of which retains its constant density during its change of position. The available kinetic energy is to be computed by the equation

$$\partial \overline{\mathbf{K}} + (\overline{\mathbf{R}}) = - \partial \overline{\mathbf{P}}$$

The potential energy of position for a unit column of a homogeneous body between the altitudes λ_{I} and λ_{II} is given by

$$[\mathbf{P}] = \int_{z_{\mathrm{I}}}^{z_{\mathrm{II}}} g z \mu d z = g \mu \frac{z_{\mathrm{II}}^2 - z_{\mathrm{I}}^2}{2}$$

(A) Let the chamber 1 (fig. 1) contain liquid of the constant density μ_1, the chamber 2 a liquid of smaller density μ_2. So long as the screen separates the two chambers there is neutral equilibrium on both sides of it. The volumes 1 and 2 are in this case assumed constant and therefore equal to 1′ and 2′ respectively, B being the area of either base. We now have for the initial and final stages

$$\overline{\mathbf{P}}_{1a} = \frac{B}{2} g \mu_1 \frac{h^2}{2} \qquad \overline{\mathbf{P}}_{2a} = \frac{B}{2} g \mu_2 \frac{h^2}{2}$$

$$\overline{\mathbf{P}}_{1e} = B g \mu_1 \frac{h^2}{8} \qquad \overline{\mathbf{P}}_{2e} = B g \mu_2 \frac{3h^2}{8}$$

After the removal of the vertical screen the available kinetic energy becomes

$$- \partial \overline{\mathbf{P}} = \frac{M V^2}{2} = g B h^2 \frac{\mu_1 - \mu_2}{8}$$

and the mass is

$$M = B \frac{\mu_1 + \mu_2}{2}$$

Let

$$\mu_1 = \mu \left(1 + \frac{\sigma}{2}\right) \qquad \mu_2 = \mu \left(1 - \frac{\sigma}{2}\right)$$

$$p_{01} = g h \mu_1 \qquad p_{02} = g h \mu_2 \qquad p_0 = g h \mu$$

then analogous to equation (I) see §25, or (I*) see §31, we have

$$V = \frac{1}{2}\sqrt{gh\sigma} \qquad \text{or} \qquad V = \frac{1}{2}\sqrt{gh \cdot \frac{p_{01} - p_{02}}{p_0}}$$

(B) Following the initial condition assumed in the third analysis we now assume that a parallelopipedon having the height h, length l and breadth unity, is filled with liquid whose density is a function of the length x only. We also assume that the density diminishes continuously from $x = 0$ up to $x = l$.

We thus have for the initial stage

$$\overline{\mathbf{P}}_a = \int_0^l dx \int_0^h gz\mu dz = \frac{gh^2}{2}\int_0^l \mu dx.$$

In the final stage the liquid that was initially in a vertical column at x above the elementary strip dx with density μ becomes a horizontal layer at the altitude ζ with the thickness $d\zeta$.

Since $hdx = ld\zeta$ and $hx = l\zeta$ therefore for the final stage we have

$$\overline{\mathbf{P}}_e = l\int_0^h g\mu\zeta d\zeta = \frac{gh^2}{l}\int_0^l \mu x dx$$

If now we put

$$\mu = \mu_0\left(1 - \sigma\frac{x}{l}\right)$$

there results

$$M = lh\mu_0\,(1 - \tfrac{1}{2}\sigma)$$

$$\overline{\mathbf{P}}_a = gh^2 l\mu_0\left(\frac{1}{2} - \frac{\sigma}{4}\right) \qquad \overline{\mathbf{P}}_e = gh^2 l\mu_0\left(\frac{1}{2} - \frac{\sigma}{3}\right)$$

$$-\delta\,\overline{\mathbf{P}} = \overline{\mathbf{P}}_a - \overline{\mathbf{P}}_e = gh^2 l\mu_0\frac{\sigma}{12} = \frac{MV^2}{2}$$

and when σ is a small fraction then, and in agreement with (II) (§27) we have

$$V = \sqrt{\frac{gh\sigma}{6}}$$

(C) Let us now arrange the liquid in the trough in parallel inclined layers of equal density and assume for the initial stage

$$\mu = \mu_0\left(1 - \sigma_x\frac{x}{l} - \sigma_z\frac{z}{h}\right) = \mu_0\,(1 - \varphi).$$

The computation of the available kinetic energy for this case supplements the computation for air whose entropy is a function of length and altitude. The angular inclination to the horizon of the layers of equal density is

$$\alpha = \text{arc tan}\left(\frac{\sigma_x}{\sigma_z} \cdot \frac{h}{l}\right).$$

If $\sigma_x > \sigma_z$ then the layer that starts from the edge where $x = 0$ and $z = h$ will intersect the bottom of the trough.

The analysis must be executed separately for the three regions in which $\varphi = \sigma_x \frac{x}{l} + \sigma_z \frac{z}{h}$ is between 0 and σ_z; or between σ_z and σ_x; or between σ_x and $\sigma_x + \sigma_z$. The first and last of these regions are triangles in the xz plane, the second is a parallelogram. The evaluation of $\overline{\mathbf{P}}_a$ and $\overline{\mathbf{P}}_e$ for the separate regions is rather tedious. Eventually we find

$$V^2\left(1 - \frac{\sigma_x}{2} - \frac{\sigma_z}{2}\right) = gh\left\{\frac{\sigma_x - \sigma_z}{6} + \frac{1}{12} \cdot \frac{\sigma_z^2}{\sigma_x} \cdot \left(1 - \frac{1}{5} \cdot \frac{\sigma_z}{\sigma_x}\right)\right\}$$

If $\sigma_x < \sigma_z$ then the above described layer (that starts from the edge for which $x = u$, $z = h$) will intersect the opposite vertical wall of the trough. For this case we have

$$V^2\left(1 - \frac{\sigma_x}{2} - \frac{\sigma_z}{2}\right) = gh \cdot \frac{1}{12} \cdot \frac{\sigma_z^2}{\sigma_x} \cdot \left(1 - \frac{1}{5} \cdot \frac{\sigma_x}{\sigma_z}\right).$$

In this case also the available kinetic energy is independent of the length of the trough.

If $\sigma_z = 0$ [then $\alpha = 90°$ and the layers are vertical columns as in case B and] the former of these two equations becomes $V^2\left(1 - \frac{\sigma_x}{2}\right) = gh\,\frac{\sigma_x}{6}$ which is identical with the value of V computed in case B when σ_x is a small fraction.

If $\sigma_z = \sigma_x$ then the layers are parallel to the diagonal surface of the trough. For this case the two equations agree in giving

$$V^2\,(1 - \sigma) = \frac{gh\,\sigma}{15};$$

therefore, if σ is a small fraction, the available kinetic energy is about one-fourth of that which we found for the same value of σ in case (A) or two-fifths of the analogous quantity in case (B).

(D) Finally, in order to imitate with incompressible liquid the case treated in the fourth analysis we return to the two chambers; we assume their basal areas to be equal; the chamber 1 to be filled with liquid with density μ to the height $h + \frac{\eta}{2}$ and the chamber 2 filled with the same liquid to the height $h - \frac{\eta}{2}$, so that in the final stage the fluid extends to the altitude h throughout the whole trough. We now have

$$\overline{\mathbf{P}}_a = \frac{B}{2} g\mu\left(h^2 + \frac{\eta^2}{4}\right) \qquad \overline{\mathbf{P}}_e = \frac{B}{2} g\mu h^2$$

$$p_{01} = g\mu\left(h + \frac{\eta}{2}\right) \qquad p_{02} = g\mu\left(h - \frac{\eta}{2}\right) \qquad p_0 = g\mu h.$$

$$(V)^2 = \frac{g\eta^2}{4h} \qquad (V) = \frac{1}{2}\frac{p_{01} - p_{02}}{p_0}\sqrt{gh}$$

This last expression is the analogue of equation (III) of the fourth analysis, §31. For equal values of p_{01} and p_{02} and when their difference is small relative to p_0 then in this case D, the velocity (V), is much smaller than the V in case (A).

Chapter IV

THE EQUATION OF ENERGY FOR MOIST AIR IN WHICH CONDENSATION OCCURS IN CONNECTION WITH THE CHANGE OF LOCATION

In order to investigate the influence of the latent heat of condensation on the available kinetic energy a fictitious gas is introduced.

§(35) We may consider the equation (6*) deduced previously for an ideal gas of constant composition as applicable to any closed system. The significance of $\overline{\mathbf{I}}$ depends on the nature of the system. We will apply the equation of energy to an atmosphere composed of air, water, and vapor, assuming thereby that the vapor is an ideal gas up to its point of condensation. In this case we can deduce the equation of energy by a special method if we change the equation of continuity so as to include the processes of condensation and evaporation.

We adopt the equation (6*) of §11 as an axiom

$$(Q) = \delta\,(\overline{\mathbf{K}} + \overline{\mathbf{P}} + \overline{\mathbf{I}}) + (\overline{\mathbf{R}})$$

Of the many processes that occur in moist air we will base our analysis on those only that we consider important for our problem, which is to determine the available kinetic energy of any initial stage; all other considerations we temporarily omit.

The condensation of vapor into small drops causes the capillary energy to be included in $\bar{\mathbf{I}}$, and the pressure of the saturated vapor to depend on the size of the drops, and the water floating in the air to contribute slightly by its weight to the pressure of the strata beneath. We free ourselves from these influences which are of minor importance in our present problem, by assuming that in our system the condensed water immediately falls to the ground [or at least separates from the air under consideration].

Let α, β, γ be the subscript indices referring to the air, vapor, and water, respectively, so that the mass (m) of moist air is composed of three portions: m_α; m_β; m_γ respectively; the specific heats are

$$C_{v\alpha}\ C_{v\beta}\ C_{v\gamma} : C_{p\alpha}\ C_{p\beta}\ C_{p\gamma} : \text{moreover } C_{p\beta} - C_{v\beta} = R_\beta, \text{ etc.}$$

L = latent heat of evaporation of a kilogram of water at the temperature T; and L' at the temperature T'.

$L - R_\beta T$ = internal latent heat of evaporation.

C = specific heat of water, assumed to be constant.

CT = internal energy of a kilogram of water.

$CT + L - R_\beta T$ = internal energy of a kilogram of vapor.

Assuming that aqueous vapor follows the laws of ideal gases we have

$$L = L_0 - (C - C_{p\beta})\ T$$

$$L - R_\beta T = L_0 - (C - C_{v\beta})\ T$$

$$L_0 = \text{Constant}$$

For the temperature T the internal energy $d\bar{\mathbf{I}}$ of the elementary mass dm which is composed of dm_α, dm_β and dm_γ is given by the expression

$$d\bar{\mathbf{I}} = C_{v\alpha}\ T\, dm_\alpha + C\ T\,(dm_\beta + dm_\gamma) + (L - R_\beta\, T)\, dm_\beta$$

$$= C_{v\alpha}\, T\, dm_\alpha + C_{v\beta}\, T\, dm_\beta + C\ T.dm_\gamma + L_0\, dm_\beta$$

We will first assume that during any change in the system the constituents of any elementary mass remain the same, so that in the final stage all have again a common temperature as in the initial stage; therefore dm_α remains unchanged and $dm'_\beta + dm'_\gamma = dm_\beta + dm_\gamma$ where the superscript primes refer to the final stage.

§(35*a*) Let us consider a system that in its initial stage contains no liquid water. Its internal energy will be

$$\overline{\mathbf{I}_a} = C_{v\alpha}\int T\, d\, m_\alpha + C_{v\beta}\int T\, d\, m_\beta + L_0\int d\, m_\beta$$

For the final stage this becomes

$$\overline{\mathbf{I}_e} = C_{v\alpha}\int T'\, d\, m_\alpha + C_{v\beta}\int T'\, d\, m'_\beta + C\int T'\, d\, m'_\gamma + L_0\int d\, m'_\beta$$

Since we assume $dm_\gamma = 0$ therefore $dm'_\beta = d\, m_\beta - d\, m'_\gamma$ and

$$\overline{\mathbf{I}}_e = C_{v\alpha}\int T'\, d\, m_\alpha + C_{v\beta}\int T'\, d\, m_\beta + L_0\int d\, m_\beta -$$
$$- \int [L_0 - (C - C_{v\beta})\, T'\,]\, d\, m'_\gamma.$$

The excess of the internal energy in the initial stage over that in the final stage is

$$-\delta\, \overline{\mathbf{I}} = \overline{\mathbf{I}}_a - \overline{\mathbf{I}}_e = C_{v_\alpha}\int (T - T')\, d\, m_\alpha + C_{v_\beta}\int (T - T')\, d\, m_\beta +$$
$$+ \int (L' - R_\beta T')\, d\, m'_\gamma.$$

We assume further that the density of the water is infinitely great in comparison with the density of the air (or that the volume of the water is zero). Under this assumption $\overline{\mathbf{I}}_e$ and $\delta\overline{\mathbf{I}}$ retain their values when the condensed water falls to the ground.

If the system is, as before in figure 1, composed of a lower portion of the mass of atmosphere, extending up to the altitude h, and an upper portion that acts only as a piston of constant weight Bp_h, where B is the area of the base; if also the air is at rest in both its initial and its final stages, then for the potential energy of position we have the following expressions, in which the volume and mass integrals are to be extended over the lower portion only. This latter is true also for the mass-integrals in $\overline{\mathbf{I}}$. The partial pressures are p_α and p_β.

$$\overline{\mathbf{P}}_a = \int dB \int_0^h p\, dz + \text{Constant} = \int (p_\alpha + p_\beta)\, dk + \text{Constant}$$
$$= R_\alpha \int T d\, m_\alpha + R_\beta \int T\, d\, m_\beta + \text{Constant}$$

$$\overline{\mathbf{P}}_e = R_\alpha \int T' d\, m_\alpha + R_\beta \int T' d\, m'_\beta + \text{Constant}$$
$$= R_\alpha \int T' d\, m_\alpha + R_\beta \int T' d\, m_\beta - R_\beta \int T' d\, m_\gamma + \text{Constant}$$

The water that in the final stage is lying on the ground contributes nothing to the $\overline{\mathbf{P}}_e$. The excess of the potential energy of the initial stage over the final stage is

$$-\delta\overline{\mathbf{P}} = \overline{\mathbf{P}}_a - \overline{\mathbf{P}}_e = R_\alpha\int(T - T')\,d\,m_\alpha + R_\beta\int(T - T')\,d\,m_\beta + \\ + R_B\int T'dm'_\gamma$$

If we assume that each portion of the system passes adiabatically over to its final stage, then we have $(Q) = 0$ and the available kinetic energy is

$$\delta\,\overline{\mathbf{K}} + (\overline{\mathbf{R}}) = -\delta(\overline{\mathbf{P}} + \overline{\mathbf{I}}) = C_{p\alpha}\int(T - T')\,d\,m_\alpha + \\ + C_{p\beta}\int(T - T')\,d\,m_\beta + \int L'\,d\,m'_\gamma \qquad (F)$$

For moist air the final stage of stable equilibrium must be determined under conditions similar to those required for a gas of constant composition, i. e., that the strata be so arranged that the entropy increases upward. Assuming that we know the final location of each mass, then we can determine the pressure p' that the element dm' experiences in its final stage: since we also know the pressure p and temperature T for this element dm in its initial stage, therefore T' and thence L' and dm'_γ are to be computed from the well-known equations for the change of condition of moist air.

§(35*b*) We will now substitute a mixture of dry gases α and β whose composition has a local variability in place of the moist air [whose composition was uniform]. Assume that for each element of the mass the ratio $\frac{dm_\alpha}{dm_\beta}$ remains unchanged during the overturning.

The specific heat is determined by the relation

$$C_p\,d\,m = C_{p\alpha}\,d\,m_\alpha + C_{p\beta}dm_\beta$$

The change of the total potential energy due to the overturning process under constant pressure is

$$-\delta\,(\overline{\mathbf{P}} + \overline{\mathbf{I}}) = \int C_p\,(T - T')\,d\,m$$

$$= C_{p\alpha}\int(T - T')\,d\,m_\alpha + C_{p\beta}\int(T - T')\,d\,m_\beta$$

For the element dm the values of $(T - T')$ are the same as before for moist air. But in order that this be possible, we must add heat during the overturning and the total quantity for the whole mass will be (Q). Hence according to (6*) the available kinetic energy will be

$$\delta \overline{\mathbf{K}} + (\overline{\mathbf{R}}) = (Q) + C_{p\alpha} \int (T - T')\, dm_\alpha + C_{p\beta} \int (T - T')\, dm_\beta$$

and the flow of heat is determined by the equation (F)

$$(Q) = \int L' dm'_\gamma \qquad (F^*)$$

It is to be noted that $L'dm'_\gamma$ is not exactly the latent heat of condensation that the element dm receives during its whole path, but the quantity of heat evolved by the condensation of dm'_γ at the temperature of the final stage of dm. This is in accordance with the assumption that was made in the computation of $\mathbf{I}_e$ where it was assumed that condensed water is carried along with the air to its final stage.

We will therefore now investigate the influence of the latent heat of condensation on the available kinetic energy, by means of another system that is more perspicacious than moist air. Since the local difference of composition is of slight influence in this problem we will replace the moist air by a homogenous gas that can expand with increase of heat.

---

§(36) *Equations for a fictitious gas that receives increase of heat by its own expansion.*

The fictitious gas that we will introduce instead of moist air behaves when it is compressed, like dry air and obeys in general the equation of elasticity $p = RT\mu$. But with every diminution of pressure there is connected an addition of heat so that for expansion

$$\frac{dT}{T} = \lambda \frac{dp}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where λ differs from the κ that holds good during compression. The quantity of heat imparted by this law of expansion is

$$dQ = C_p\, dT - \frac{RT}{p}\, dp = -\,(R - \lambda\, C_p)\, T \frac{dp}{p} \quad . \quad . \quad . \quad (2)$$

We assume that dQ is positive when dp is negative

therefore $$R - \lambda C_p \quad \text{is positive}$$

and also that $$\lambda < \kappa \quad \ldots\ldots \quad (3)$$

and that cooling accompanies expansion as λ is positive. . . (4)
and finally assume that λ is constant . . . . (5)

If T_0 and p_0 belong to the initial stage of a mass then by the expansion or transition to a smaller pressure p we have

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^{\lambda} \quad \ldots\ldots \quad (1a)$$

$$[Q]_{p_0}^{p} = C_n \frac{\kappa - \lambda}{\lambda} (T_0 - T) \quad \ldots\ldots \quad (2a)$$

Consider a vertical column filled with this gas at rest: Let p and T at the altitude z have the same values that a particle would have when ascending [adiabatically] from the base (p_0 and T_0) to this altitude.

The distribution of temperature in this column is now given by the above equation (1) by the condition of equilibrium (α) [§13], and by the equation of condition for elastic gases $p = RT\mu$. Combining these we obtain

$$\frac{1}{p}\frac{\partial p}{\partial z} = -\frac{g}{RT} = \frac{1}{\lambda} \cdot \frac{1}{T}\frac{\partial T}{\partial z}$$

whence

$$-\frac{\partial T}{\partial z} = \frac{g\lambda}{R} = \frac{\lambda}{\kappa} \cdot \frac{g}{C_p} \quad \ldots\ldots \quad (6)$$

With this fictitious gas, and for any given distribution of temperature in the vertical column, we can carry through a process similar to that considered in our preceding second chapter (§§ 13–22) and find that for ascending particles (or diminishing pressure) the diminution of temperature just given in equation (6) belongs to the condition of neutral equilibrium, and that a more rapid diminution of temperature corresponds to unstable equilibrium. In the first case the vertical temperature gradient is smaller than that for neutral equilibrium of dry air.

§(37) We now pass to the computation of the available kinetic energy of an extended horizontal system (fig. 1).

First analysis. Initial stage. Chamber 1 contains dry air, chamber 2 the fictitious gas; both of these are in neutral equilibrium, and both under the same pressure p_h at the altitude h. The temperatures are to be so chosen that, after the overturn, in the final stage, 2′ lies wholly above 1′ and p_h remains unchanged. Therefore if the serial sequence of the elementary layers be unchanged, every layer of chamber 2 expands except the highest one which retains its original pressure. The layers of 1′ and 2′ are individually in the condition of neutral equilibrium. As regards 2′ this result follows from the application of equation (1*a*), §36.

Let (Q) be the quantity of heat added to the mass 2 by its expansion, then for the available kinetic energy of the whole system we have $\delta\overline{\mathbf{K}} + \overline{\mathbf{R}} = (Q) - \delta(\overline{\mathbf{P}} + \overline{\mathbf{I}}) = m\dfrac{(V)^2}{2}$

$$= (Q) + C_p\left\{\int (T_1 - T'_1)\,dm_1 + \int (T_2 - T'_2)\,dm_2\right\}$$

The given data are h, p_h, T_{h1}, T_{h2}, λ, and β; whence for the initial stage we find

$$T_{01} = T_{h1}\left(1 + \kappa\frac{gh}{RT_{h1}}\right) \qquad p_{01} = p_h\left(1 + \kappa\frac{gh}{RT_{h1}}\right)^{\frac{1}{\kappa}}$$

$$T_{02} = T_{h2}\left(1 + \lambda\frac{gh}{RT_{h2}}\right) \qquad p_{02} = p_h\left(1 + \lambda\frac{gh}{RT_{h2}}\right)^{\frac{1}{\lambda}}$$

For the final stage we have

Locality	*Pressure*	*Temperature*
At the upper boundary of 2′	p_h	T_{h2}
At the boundary i between the masses 2′ and 1′	$p'_i = \frac{1}{2}(p_{02} + p_h)$	$\begin{cases} T'_{h2} = T_{h2}\left(\dfrac{p'_i}{p_h}\right)^{\lambda} \\ T'_{h1} = T_{h1}\left(\dfrac{p'_i}{p_h}\right)^{\kappa} \end{cases}$
At the base	$p'_0 = \frac{1}{2}(p_{01} + p_{02})$	$T'_0 = T_{h1}\left(\dfrac{p'_0}{p_h}\right)^{\kappa}$

By reason of equation (2a) we have

$$(Q) - C_p \frac{\kappa - \lambda}{\lambda} \int (T_2 - T_2') \, dm_2$$

whence

$$\delta \mathbf{K} + (\mathbf{R}) = C_p \int (T_1 - T_1') \, dm_1 + \frac{\kappa}{\lambda} \int (T_2 - T_2') \, dm_2$$

In equation (5) in chapter II, §20, for a, the vertical gradient of temperature, substitute the values g/C_p for the masses 1 and 1′ and $\lambda g/(\kappa C_p)$ (see eq. 6 §36) for the masses 2 and 2′ and we obtain

$$\int T_1 \, dm_1 = \frac{B}{2g} \cdot \frac{1}{1 + \kappa} \cdot (p_{01} T_{01} - p_h T_{h1})$$

$$\int T_1' \, dm_1 = \frac{B}{g} \cdot \frac{1}{1 + \kappa} \cdot (p_0' T_0' - p_i' T_{i1}')$$

$$\int T_2 \, dm_2 = \frac{B}{2g} \cdot \frac{1}{1 + \lambda} : (p_{02} T_{02} - p_h T_{h2})$$

$$\int T_2' \, dm_2 = \frac{B}{g} \cdot \frac{1}{1 + \lambda} \cdot (p_i' T_{i2}' - p_h T_{h2})$$

§(38) Example. In order to make the fictitious gas similar to moist air we compute the values for the initial condition in a different order of succession than that above given.

Initial stage. For mass 2: assume $T_{02} = 303°$, $p_{02} = 760$, $p_h = 500^{mm}$ mercury and seek first the value of T_{h2} for saturated moist air, that is to say, the temperature that such air attains when it expands from 303° and 760^{mm} to 500^{mm} mercury. This value lies between 289° and 290°.

We adopt $T_{h2} = 290°$ and with it by equation (1a) compute $\lambda = 0.1047307$ and further the vertical gradient of temperature in mass 2 or $a_2 = \dfrac{\lambda}{\kappa} \dfrac{g}{C_p} = 0.0035780$ degree Centigrade per meter which is nearly 3.6° per 1000 meters.

Finally we compute

$$h = \frac{T_{02} - T_{h2}}{a_2} = 3633.29 \text{ meters}$$

Initial stage. For mass 1 we adopt the same temperature at the base T_{01} as for mass 2, i. e., 303°, whence for the same altitude $h = 3633.29$ meters we find T_{h1} and p_{01} as follows:

$$\left.\begin{aligned} T_{01} &= 303^\circ \\ h &= 3633.29 \end{aligned}\right\} T_{h1} = 266^\circ.9061 \qquad p_{01}' = 773.4023^{\text{mm}} \text{ mercury.}$$

Final stage. For the final stage we compute the following values

$$\begin{array}{lll} p_i' = 630^{\text{mm}} & T_{i2}' = 297.1047^\circ & T_{i1}' = 285.4593^\circ \\ p_c' = 766.7011^{\text{mm}} & & T_0' = 302.2342^\circ \\ h_1' = 1688.60 \text{ meters.} & & h_{i2}' = 1985.65 \text{ meters.} \end{array}$$

From these we now obtain

$$\int (T_1 - T_1')\, dm_1 = -2.2312\, p_h \frac{B}{g}$$

$$\int (T_2 - T_2')\, dm_2 = +2.3323\, p_h \frac{B}{g}$$

$$M \frac{V^2}{2} = C_p\, 0.1011\, p_h \cdot \frac{B}{g}$$

$$M = 0.5334\, p_h \frac{B}{g}$$

$$V = 19.3 \text{ meters per second.}$$

The true average temperatures of the masses 1 and 2 in the initial stage are $T^*_1 = 285.9^\circ$ and $T^*_2 = 296.9^\circ$.

If instead of the fictitious gas we had assumed dry air of the same average temperature in chamber 2 and if its entropy had been such that in the final stage 2′ it lay uppermost spread over 1′ then we should have computed the corresponding $V = 18.5$ meters per second from the approximate formula (I) §(25). Therefore the available energy is not much larger for the fictitious gas in chamber 2 than for dry air of the same temperature.

With the above given equation for (Q) (see 2*a*, §36), we compute the quantity of heat communicated, per unit area of the base of the whole trough, to the mass 2 during its expansion

$$\frac{(Q)}{B} = 2409 \frac{\text{Calories}}{M^2}$$

If this is due to precipitation of aqueous vapor then it corresponds to the condensation of 4 kilograms of water per square meter or to a depth of rainfall of 4^{mm} over the whole area.

If instead of the fictitious gas in chamber 2 we had assumed saturated moist air then we could have found the approximate quantity of water condensed and falling away from it as follows:

By the overturning the lowest layer of 2 changes from $T_{02} = 303°$ and $p_{02} = 760^{mm}$ to $T'_{i2} = 297$ and $p'_i = 630^{mm}$. In this lowest layer there is initially 0.02696 kg. vapor associated with each kilogram of dry air but in the final stage this is reduced to 0.02271; therefore in this layer there condenses 0.00425 kg. water out of every 1.027 kg. of mixed air and saturated vapor. The highest layer of mass 2 does not expand and contributes no water; we may assume that the whole mass 2 contributes on the average 0.0021 kg. water per kg. of its own mass so that the unit column gives up

$$\frac{260}{760} \times 10333.0 \times 0.0021 = 7.4 \text{ kg. of water}$$

Because of the spread of the mass 2 over the whole trough this water is distributed over double the original base giving 3.7^{mm} depth rainfall in close agreement with the value above given.

---

§(39) *Second analysis. Approximate method for the computation of the available kinetic energy when the chamber 1 is filled with dry air and chamber 2 with the fictitious gas.*

In this case all the considerations that were made in the analogous second analysis, §25, chapter III, are to be repeated excepting only that λ is used instead of κ in chamber 2.

Initially the masses 1 and 2 are each in stable equilibrium within itself; the succession of layers within each mass remains unchanged in the overturning. Hence we obtain p'_1 and p'_2 and, assuming equal volumes for the two chambers, we find the following approximate values:

$$T'_1 = T_1\left(1 + \frac{\kappa}{2} \cdot \frac{p_{02} - p_1}{p_1}\right)$$

$$T'_2 = T_2\left(1 - \frac{\lambda}{2} \cdot \frac{p_2 - p_h}{p_2}\right)$$

$$\delta\overline{\mathbf{K}}+(\overline{\mathbf{R}})=C_p\left\{\int(T_1-T_1')\,dm_1+\frac{\kappa}{\lambda}\int(T_2-T_2')\,dm_2.\right\}$$

$$=C_p\frac{B\kappa}{4g}\left\{\mathrm{T}_1\,(p_{01}-p_h)+\mathrm{T}_2\,(p_{02}-p_h)-\frac{gh}{R}(p_{02}+p_h)\right\}$$

This is the same value as before, and now the same considerations as in §25 lead to the approximate formula (I) of that article for the velocity V. We arrive at the same result as if instead of the fictitious gas in chamber 2 we had used dry air of the same average temperature. The reason why this happens is shown by the course of the analysis. The change of $\overline{\mathbf{P}}+\overline{\mathbf{I}}$ for the unit mass remains the same no matter whether chamber 2 is filled with dry air or with the gas; the amount that the mass 1 contributes to the available kinetic energy remains unchanged.

The contribution of mass 2 by simple change of location only, or the change of $\overline{\mathbf{P}}+\overline{\mathbf{I}}$ for this mass, $\left(\text{which is } C_p\int(T_2-T'_2)\,dm_2\right)$, is in the new case (for the fictitious gas) smaller than for dry air since the gas cools less by expansion and therefore T'_2 is larger.

But on account of the development of heat associated with the expansion (which we have introduced as equivalent to the latent heat of condensation) there is (Q) to be added to the expression $\delta\,\overline{\mathbf{K}}+(\overline{\mathbf{R}})$ and for the fictitious gas the expression

$$(Q)+C_p\int(T_2-T_2')\,dm_2$$

is as large as the second term alone would be for dry air.

Thus it is that the addition of heat by virtue of the expansion causes no increase in the available kinetic energy, but serves only to warm the expanding mass or to diminish its cooling.

---

§(40) *The difference between the fictitious gas and the moist air.*

In order to simplify the analysis and investigate separately the influence of the latent heat of condensation we have given the fictitious gas that has replaced moist air the properties of dry air and have only introduced the condition that it shall expand when heat is added. In order that it might more nearly resemble moist air we should have also assumed its density smaller and its R to be variable with its condition.

The fictitious gas cannot replace moist air in every respect. The proof that the condensation has no influence on the available kinetic energy, does not exclude the possibility that the proximity of masses of relatively dry air and moist air should favor the origination of a storm.

Since the available kinetic energy of the system is derived from the buoyancy, therefore any diminution of density by reason of the vapor content of a mass of air operates like an increase of temperature. A particle of the fictitious gas remains in stable equilibrium in dry air, if it has the same temperature as the surrounding air and if the vertical temperature gradient in that air is smaller than $\lambda\, g/R$. A particle of saturated moist air, having the same exponential factor λ [would not be in stable or neutral equilibrium but unstable and] would rise because of its smaller density.

In the example of the first analysis §38, if instead of the fictitious gas we had used saturated moist air of the same temperature and pressure, then for the altitudes o and h its densities would have been respectively equal to that of dry air at $I_{02}=307.8°$ and $p_{02} = 760^{mm}$ mercury and that of dry air at $T_{0h}= 293.2°$ and $p_{oh}=5co^{mm}$ mercury. This 307.8° corresponds to a difference of $T_{02} - T_{01}= 1.8°$ from the value $T_{01} = 303.0°$ as there adopted and a difference cf 3.2° from the value $T_{h2}=290°$ there computed for the altitude h or a difference of 4.0° from the average temperature of the whole mass of 2 [when it is composed of saturated air instead of the fictitous gas]. Therefore for these two cases the available kinetic energy is in the ratio 15/11 and the velocities V are larger than in the fictitious gas in the ratio 1.17/1. Or, if we require the velocity V to be the same as before, then we need a smaller difference of temperature between dry and moist air, viz., about 8° instead of 11°. All this relates to the unusual high temperatures of our examples; for lower temperatures the influence of the moisture on the density will be still smaller.

---

§(41) The kinetic energy of a mass of air is derived from its internal energy and from the work done by the force of gravity. In the case of a continuous distribution of density the importance of gravity in the production of great velocities can be concealed, whence we derive the very common belief that the horizontal gradient of pressure produces the storm. But it is now demonstrated that, even when the distribution of pressure at the base is as observed in storms still the horizontal movements of the masses have a potential energy that is only a small fraction of the observed kinetic energy. So far as I can see the source of storms is to be sought only in the potential energy of position. A system in which the masses are disturbed vertically from equilibrium can contain the necessary potential energy. Hence, therefore, the storm winds develop by reason of the velocity due to descent and that due to buoyancy, notwithstanding the fact that these evade attention because of the large horizontal and small vertical dimensions of the storm area. The horizontal distribution of pressure appears as a translation of the driving power of the storm; by means of it a portion of the mass can attain greater velocity than by simple ascent in the coldest or descent in the warmest portion of the storm area. Here we come into the presence of problems that cannot be solved by a simple consideration of the energy alone.

XXV

THE THEORY OF THE MOVEMENT OF THE AIR IN STATIONARY ANTICYCLONES WITH CONCENTRIC CIRCULAR ISOBARS

BY DR. F. POCKELS

[*Translated from the Meteorologische Zeitschrift, January, 1893. Vol. X, pp. 9–19*]

The solution of the hydrodynamic differential equations for the movement of the air in stationary cyclones with circular isobars as attained by Oberbeck under certain simplifying assumptions[1] does not allow of application to the case of the analogous anticyclone. This circumstance led Oberbeck to the conclusion that there is really an important difference between anticyclones and cyclones, in that the former must be a phenomenon dependent on the latter and must originally cover ring-like regions adjoining the cyclones. But on the other hand the synoptic weather charts show us that quite frequently well rounded anticyclones continue to exist for a long period of time although no well defined cyclones are present; consequently a mathematical presentation of the movement of the air in such anticyclones founded on the principles of hydrodynamics must be possible, quite independent of the reason for the existence of these anticyclones. In the following memoir I will attempt to give such a presentation of this problem as may be of interest as a supplement to Oberbeck's investigation, notwithstanding the fact that it is based on certain special assumptions.

The hypotheses under which I shall treat this problem are the same as those that were adopted by Oberbeck, viz.,

(1) That the system of winds is a stationary system, that is to say, the movement at any place is independent of the time.

(2) That the air is an incompressible fluid (this is allowable because of the insignificance of the ordinary differences of pressure) and that the temperature is constant.

(3) That the portion of the earth's surface under consideration is a plane surface and that the geographic latitude has a constant

[1] A. Oberbeck: Annalen der Physik und Chemie, 1882, N. F., XVII, pp. 108–148.

average value: (this is all the more allowable in proportion as the average latitude is nearer the pole).

(4) We shall study only the movement over the earth's surface, of a layer of air of moderate depth assuming that it experiences a frictional resistance opposed to the direction of its movement and proportional to the velocity of the current.

(5) That a descending current of air prevails over a surface bounded by a circle of radius R and that its descending velocity is directly proportional to the altitude above the earth's surface and is the same, for any given altitude, over the whole of this region.

(6) That purely horizontal motions prevail outside the circle whose radius is R.

From this last assumption combined with the second and third it follows at once, that the atmospheric pressure, the velocity of the current and the angle that the current makes with the radius-vector depend only on the distance, r, from the center of the descending current. Designate by V_n the radial component of the velocity V positive inward; by V_t the velocity component tangential to the isobar counted positive in case the rotation is directed contrary to the movement of the hands of a watch; let p be the atmospheric pressure expressed in absolute units of force, ρ the average density of the air, k the coefficient of friction, and λ the product of the angular velocity of rotation of the earth multiplied by the sine of the geographic latitude; then in the case under consideration of concentric circular isobars, the hydrodynamic equations of motion can be written[2]

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{dV_n}{dt} + \frac{V_t^2}{r} + \lambda V_t + kV_n \quad . \quad . \quad . \quad . \quad . \quad \text{(Ia)}$$

$$0 = \frac{dV_t}{dt} - \frac{V_t V_n}{r} - \lambda V_n + kV_t \quad . \quad . \quad . \quad . \quad . \quad \text{(Ib)}$$

which equations moreover can be deduced directly from very simple considerations. Because of the assumption of a stationary condition, the total acceleration is identical with the acceleration depending on a change of location, that is to say, a change in the distance from the center of the whole system of winds, consequently in our problem we have

$$\frac{dV_n}{dt} = \frac{dV_n}{dr} \cdot \frac{dr}{dt} \text{ and } \frac{dV_t}{dt} = \frac{dV_t}{dr} \cdot \frac{dr}{dt}$$

[2] See Sprung's Lehrbuch der Meteorologie, Hamburg, 1885, p. 135.

Further, if we consider that $\frac{dr}{dt}$ is nothing else but the negative of the velocity component V_n then the equations of motion take the form

$$\frac{1}{\rho}\frac{dp}{dr} = -\frac{dV_n}{dr} \cdot V_n + \frac{V_t^2}{r} + \lambda V_t + k V_n \quad . \; . \; . \quad \text{(Ia')}$$

$$0 = -\frac{dV_t}{dr} \cdot V_n - \frac{V_t V_n}{r} - \lambda V_n + k V_t \quad . \; . \quad \text{(Ib')}$$

To this we add the condition of continuity [as to mass] which requires that [if the density is to remain constant then] as much air flows out of any elementary volume in a unit of time by reason of the horizontal current as flows into it [in the same unit of time] by reason of the vertical current. Now the mass of the horizontal outflow of air for an elementary prism whose altitude is unity, whose base is bounded by two circular arcs of the length $d\varphi$ and by the radii r and $r + dr$, is equal to

$$-\frac{d(\rho r V_n)}{dr} \cdot dr d\varphi.$$

On the other hand by reason of a descending current whose velocity increases by the quantity w within a unit distance, or is itself equal to w at the unit altitude (compare assumption No. 5) the mass of air that flows into the unit volume is equal to $\rho w r dr d\varphi$. Therefore we must have

$$-\frac{d\,(\rho r V_n)}{dr} = r \rho w$$

or if ρ is considered to be constant then

$$\frac{d\,(r V_n)}{dr} = -rw \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{(Ic)}$$

In this equation w might be considered as a known function of r but as already stated in assumption (5), we will make a special hypothesis, i. e., that this quantity has a constant value γ within the region for which $r < R$ but that outside of this region it has the value zero so that the condition (Ic) consists of two, one for each region, i. e.,

$$\frac{d\,(r V_n)}{dr} = \dot{r}\,\gamma \text{ for } r < R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(Ic}')$$

$$\frac{d\,(r V_n)}{dr} = 0 \text{ for } r > R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(Ic}'')$$

The continuity condition (Ic) alone determines V_n as a function of r; if then we substitute this function in (Ib′) we shall obtain a differential equation for V_t; finally the equation (Ia′) serves for the determination of the distribution of pressure after we have substituted therein the functions found for V_n and V_t. The complete solution therefore demands the integration of three ordinary linear differential equations of the first order for the inner region and three others for the outer region; in this solution the integration constants are to be so determined that both V_n and V_t as also p are continuous at the boundary between the two regions, i. e., for $r = R$, since at the passage from the inner to the outer region both the velocity and direction of the current of air, as also the pressure of the air, must change continuously.

In the manner thus indicated we first find

$$V_n = -\frac{1}{2}\gamma\, r \text{ for } r < R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where the constant of integration must be zero because otherwise V_n in the center of the system of winds would become infinite; furthermore

$$V_n = \frac{\text{constant}}{r} \text{ for } r > R,$$

and the constant of integration is $-\frac{1}{2}\gamma R^2$ so that V_n shall remain continuous when $r = R$, hence

$$V_n = \tfrac{1}{2}\,\gamma\,\frac{R^2}{r} \text{ for } r > R$$

In the case of a cyclone the sign of γ is to be taken oppositely from that in this equation for the anticyclone. Now the differential equation (Ib′) becomes

for $r < R$ $$0 = \frac{1}{2}\gamma\left(\frac{d V_t}{dr}\, r + V_t\right) + k V_t + \frac{1}{2}\gamma\,\lambda\, r \qquad \text{(IIa)}$$

for $r > R$ $$0 = \frac{1}{2}\gamma\, R^2\left(\frac{d V_t}{dr}\cdot\frac{1}{r} + \frac{V_t}{r^2}\right) + k V_t + \frac{1}{2}\gamma\,\lambda\, R^2\cdot\frac{1}{r} \qquad \text{(IIb)}$$

The first of these equations is satisfied by the function

$$-\frac{\lambda\gamma}{2\,(k+\gamma)}\cdot r$$

the second equation is satisfied by

$$-\frac{1}{2}\frac{\gamma\,\lambda}{k}\cdot\frac{R^2}{r}$$

as is easily seen without further comment. In order to obtain the general integral, we have still to add the integral, multiplied by an arbitrary constant, of the differential equation that arises from the omission of the terms that do not contain V_1 in equations (IIa) or (IIb); this homogenous differential equation reads

$$\frac{d\,(r\,V_t)}{dr}+\frac{2\,k}{\gamma}\cdot\frac{r\,V_t}{r}=0 \text{ for } r<R$$

whose integral is

$$r\,V_t = C'\,r$$

and again

$$\frac{d\,(r\,V_t)}{dr}=\frac{2\,k}{\gamma\,R^2}\cdot r\,.\,(r\,V_t)=0 \text{ for } r>R$$

whose integral is

$$r\,V_t = C''\,e^{\left(-\frac{k}{\gamma}\frac{r^2}{R^2}\right)}$$

Hence the complete solution for V_t reads as follows:

$$V_t=-\frac{\lambda\gamma}{2\,(k+\gamma)}r+C'\,r^{-\frac{2\,k+\gamma}{\gamma}} \text{ for } r<R \quad . \quad . \quad . \quad (2)$$

$$V_t=-\frac{1}{2}\cdot\frac{\gamma\lambda}{k}\cdot\frac{R^2}{r}+C''\frac{1}{r}\,e^{-\frac{k}{\gamma}\cdot\frac{r^2}{R^2}} \text{ for } r>R \quad . \quad . \quad (2')$$

The integration constants C' and C'' are now to be determined in harmony with the conditions of continuity. In the case of a cyclone for which we have negative values of γ (that is to say, an ascending current in the interior region) we must put $C''=0$, since otherwise V_t would become infinitely large at an infinite distance; we thus arrive at Oberbeck's solution.[3] In the case to be considered by us of an anticyclone, or a positive γ, C' is to be put zero in order that V_t be not infinite when $r = 0$; on the other hand the exponential function in the expression (2') is to be retained since its exponent is negative and it therefore disappears when $r = \alpha$. It is because he omitted the exponential function in his general solution that Oberbeck could not apply his solution to the anticyclones.

In order that the expressions for V_t namely (2) with $C' = 0$ and (2') may be equal to each other when $r = R$, as is required by the continuity of the velocity, we must have

$$-\frac{\lambda\gamma}{2\,(k+\gamma)}R = -\frac{\lambda\gamma}{2k}R + \frac{C''}{R}e^{-k/\gamma}$$

whence the following value results

$$C'' = \frac{\gamma^2\lambda\,R^2}{2\,k\,(k+\gamma)}\cdot e^{k/\gamma}$$

Finally, as the complete solution for the velocity components we obtain

$$\left.\begin{aligned} V_n &= -\frac{1}{2}\gamma\,r \\ V_t &= -\frac{1}{2}\,\frac{\gamma\lambda}{k+\gamma}\,r \end{aligned}\right\} \text{ for } r < R \quad \ldots\ldots (3)$$

$$\left.\begin{aligned} V_n &= -\frac{1}{2}\gamma\frac{R^2}{r} \\ V_t &= -\frac{1}{2}\cdot\frac{\gamma\lambda}{k}\,\frac{R^2}{r}\left\{1 = \frac{\gamma}{k+\gamma}\cdot e^{-\frac{k}{\gamma}\left(\frac{r^2}{R^2}-1\right)}\right\} \end{aligned}\right\} \text{ for } r > R \quad (3')$$

The exponent

$$-\frac{k}{\gamma}\left(\frac{r^2}{R^2}-1\right)$$

[3] Oberbeck (Annalen, 1882, XVII, p. 143) subjects the quantity $-\gamma = c$ to the condition $c < k$ in order to obtain an infinitely large value of the velocity at the center and a deflection of the direction of the wind to the left instead of to the right of the gradient. But this is attained by the condition $c < 2k$; for the velocity (logarithmic) becomes infinitely large when $c = 2k$ and the angular deviation remains in general always between the limits $+ tg^{-1}\lambda/k$ and $+\pi/2$, which latter value is attained at the center as soon as $c \geq k$.

is negative throughout the whole exterior region and the factor in brackets in the expression for V_t in (3′) is therefore positive; hence V_t is everywhere finite and negative whatever may be the absolute value of γ and therefore the intensity of the descending current in the inner region of the anticyclone need not be subject to any limitation whatever. Hence from the expressions (3) and (3′) there result at once the following values of the absolute wind velocity, $V = \sqrt{V_n^2 + V_t^2}$ and of the tangent of the angle of deviation, $\tan\psi = V_t/V_n$ namely,
in the inner region $(r<R)$

$$V = \tfrac{1}{2}\gamma r\sqrt{1 + \frac{\lambda^2}{(k+\gamma)^2}} \quad \ldots \ldots \quad (4)$$

$$\tan\psi = \frac{\lambda}{k+\gamma} \quad \ldots \ldots \quad (5)$$

in the outer region $(r>R)$

$$V = \tfrac{1}{2}\gamma\frac{R^2}{r}\sqrt{1 + \left\{1 - \frac{\gamma}{k+\gamma}\cdot e^{-\frac{k}{\gamma}\left(\frac{r^2}{R^2}-1\right)}\right\}\frac{\lambda^2}{k^2}} \quad \ldots \quad (4')$$

$$tg\,\psi = \frac{\lambda}{k}\left\{1 - \frac{\gamma}{k+\gamma}e^{-\frac{k}{\gamma}\left(\frac{r^2}{R^2}-1\right)}\right\} \quad \ldots \ldots \quad (5')$$

Hence the velocity of the wind in the inner regions is proportional to the distance r from the center, but in the outer region at a sufficiently great distance from the boundary circle it is inversely proportional to that distance. The maximum wind occurs in the outer region in the neighborhood of the boundary between it and the inner region or even at this boundary itself depending on the values of λ, k and γ.

In accord with the expression (5) the angle of deviation is constant in the interior region and smaller than the "normal" value which is given by $tg\,\psi = \frac{\lambda}{k}$

On the other hand in the outer region in accord with equation (5′) the angle increases with distance and rapidly approaches this "normal" value. (It should be remarked that conversely, for *cyclones* there is in the outer region a constant angle of deviation,

i. e., the "normal"[4] while in the inner ascending region the angle of deflection from the gradient increases as we proceed inward.) Hence in the interior region of an anticyclone of the special structure here assumed the paths of the winds are logarithmic spirals.

The distribution of barometric pressure remains now to be determined by substituting the value of V_n and V_t above determined in the equation (Ia′). This latter equation is thus transformed into the following:

for $r < R\frac{x}{\rho}$

$$\frac{1}{\rho}\frac{dp}{dr} = -\frac{\gamma\,(2\,k+\gamma)}{4}\left(1+\frac{\lambda^2}{(k+\gamma)^2}\right)r \quad . \quad . \quad . \quad . \quad (6)$$

for $r > R$

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{\gamma^2 R^4}{4\,r^3} - \tfrac{1}{2}\,\gamma k\frac{R^2}{r} -$$

$$-\frac{\gamma\lambda^2}{2k}\cdot\frac{R^2}{r}\left\{1-\frac{\gamma}{2k}\cdot\frac{R^2}{r^2}\left(1-\frac{\gamma}{k+\gamma}e^{-\frac{k}{\gamma}\left(\frac{r^2}{R^2}-1\right)}\right)\times\right.$$

$$\left.\times\left(1-\frac{\gamma}{k+\gamma}e^{-\frac{k}{\gamma}\left(\frac{r^2}{R^2}-1\right)}\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6')$$

These expressions need only to be multiplied by a constant factor $\frac{\rho}{\mu}$ in order to obtain the gradient G or the change of barometric pressure per 111 kilometers in the direction of the radius vector r: to this end the constant μ has the numerical value $\frac{10\,333 \times 9.81}{760 \times 111\,000}$ $= 0.0012$ and ρ has the value 1.293 where the units are a kilogram of mass, meter of length and second of mean solar time.

Equation (6) shows that in the interior region of the anticyclone the gradient is simply proportional to r as we found to be also the case with the velocity of the wind. In the exterior region of the anticyclone according to equation (6′) the law is much more complex

[4] This term was introduced by Guldberg and Mohn in 1876. See No. XI of this present volume, pp. 143–146.—C. A.

but at a moderate distance from the boundary circle it assumes the following simpler very approximate form

$$G = \frac{\rho}{\mu} \cdot \frac{\gamma}{2} R^2 \left(1 + \frac{\lambda^2}{k^2}\right) \left(\frac{\gamma}{2} \cdot \frac{R^2}{r^3} - \frac{k}{r}\right)$$

where the second negative term is the important one as it ought to be since the atmospheric pressure diminishes outwardly. This last given law for the gradients differs only by the sign of γ from that which was deduced by Oberbeck for the outer region of a cyclone.

Finally, in order to express the barometric pressure as a function of the distance r from the center we have to integrate the differential equations (6) and (6′) which only requires simple quadrature. If P represents the pressure on the boundary circle whose radius is R we obtain

for $r < R$

$$p = P + \rho \frac{\gamma (2k + \gamma)}{8} \left(1 + \frac{\lambda^2}{(k + \gamma)^2}\right) (R^2 - r^2) \quad . \quad . \quad . \quad (7)$$

On the other hand for the exterior region, we find the following complex expression

for $r > R$

$$p = P - \tfrac{1}{2} \rho \left(\frac{\gamma R^2}{2}\right)^2 \left(1 + \frac{\lambda^2}{k^2}\right) \left(\frac{1}{r^2} - \frac{1}{R^2}\right) -$$

$$- \rho \frac{\gamma}{2} k R^2 \left(1 + \frac{\lambda^2}{k^2}\right) \log \text{nat} \frac{r}{R} +$$

$$+ \rho \frac{(\gamma \lambda R)^2}{2k (k + \gamma)} \times \left[e^{k/\gamma}\right] \left\{ A + B \right\} -$$

$$- \rho \left(\frac{\gamma \lambda R}{2 (k + \gamma)}\right)^2 \frac{2\gamma}{k} \left[e^{2k/\gamma}\right] \left\{ A' - B' \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7')$$

where

$$A = \left[\frac{e^{-x}}{x}\right]_{x = k/\gamma}^{x = k r^2/\gamma R^2} \qquad B = \int_{k/\gamma}^{k r^2/\gamma R^2} \frac{e^{-x} dx}{x}$$

$$A' = \left[\frac{e^{-x}}{x}\right]_{x = 2k/\gamma}^{x = 2k r^2/\gamma R^2} \qquad B' = \int_{2k/\gamma}^{2k r^2/\gamma R^2} \frac{e^{-x} dx}{x}$$

The integral

$$\int \frac{e^{-x} dx}{x}$$

that occurs in equation (7′) cannot be presented in definite form except as the converging series

$$\int \frac{e^{-x}dx}{x} = \log x - x + \frac{1}{2} \cdot \frac{x^2}{1.2} - \frac{1}{3}\,\frac{x^3}{1.2.3} + \frac{1}{4}\,\frac{x^4}{1.2.3.4} \text{ etc.}$$

which series is, however, for large values of x rather inconvenient for computing*. In such cases and when great accuracy is not important we compute only the gradients G_0 and G for R and r and $G_1, G_2 \ldots G_n$ for a series of intermediate values $r_1 \ldots r_n$ and then from these compute the pressure in millimeters of mercury for the given r according to the formula

$$b = B - \frac{G_0 + G_1}{2}(r_1 - R) -$$

$$- \frac{G_1 + G_2}{2}(r_2 - r_1) \ldots - \frac{G_n + G}{2}(r - r_n) \quad \ldots \quad (8)$$

where B represents the barometric reading at the distance $r = R$ and the values of r are expressed in units of $111/km$ [or degrees of a great circle].

The objection might be urged that according to equation (7) the difference $(P - p)$ becomes infinite for an infinitely large value of r (logarithm of r). But it must be noted that for very large distances the assumptions made by us become in part inapplicable (for example the geographic latitude can no longer be considered as constant) and this too quite independent of the fact that in the actual atmosphere the neighboring cyclones or other anticyclones affect the distribution of wind and barometric pressure. Therefore we need only expect that our results will apply up to moderate distances from the center of the anticyclone, which may perhaps be slightly larger than the radius of the inner region.

In order to show that within these limits the theoretical results as concerns wind-force and pressure-difference correspond to those actually occuring, I have computed in detail the following example:

The constants λ and k are considered as given previously for that portion of the earth's surface over which the anticyclonal system of winds prevail; on the other hand the parameters γ and R which also occur in our final formulæ remain adjustable in order to repre-

---

*Smithsonian Mathematical Tables. Washington, 1909. Table IV, pp. 225-262.—C. A.

sent anticyclones of different intensity and extent. In the succeeding example we shall adopt the following values:

$\lambda = 0.00012$, which value applies exactly to the geographic latitude $55\frac{1}{2}°$.

$k = 0.00008$, a value of the coefficient of friction that has been deduced from the average angle of deflection as determined for North America and approximately for Norway.

$\gamma = 0.00004$.

$R = 400{,}000$.

The meter, second, and kilogram, are adopted as units. The assumed value of γ is such that for $r = R$ the formula (4) gives a velocity of about 11 meters per second corresponding nearly to a wind of force 5 on the Beaufort Scale.

Since the velocity of the descending current of air according to our hypothesis is equal to γ multiplied by the elevation above the earth's surface, therefore the assumption that $\gamma = 0.00004$ is equivalent to the statement that this velocity is 4 centimeters per second at the altitude 1000 meters, or 12 centimeters per second at the altitude 3000 meters, at least in so far as that hypothesis is applicable to such a great altitude. Therefore we have to deal with very small vertical velocities that are not at all improbable. In his numerical example for a cyclone Oberbeck assumes that the vertical component of the velocity is 2.4 times as large as this.

Under the above given assumptions our theory gives the following results for the wind velocity V (in meters per second), the angle of deflection ψ, gradient G and difference of pressure $(b - B)$ in millimeters of mercury, for a series of different distances r expressed in kilometers from the center.

Interior region.

r	0	100	200	300	400	kilometers
V	0	2.82	5.65	8.47	11.3	meters
ψ	(45°)	45°	45°	45°	45°	
G	0	0.43	0.86	1.29	1.72	millimeters
$b - B$	3.1	2.9	2.32	1.36	0	millimeters

Exterior region

r	400	450	500	600	700	800	900	1000	kilometers
V	11.3	11.12	10.67	9.57	8.22	7.20	6.42	5.78	meters
ψ	45°	50° 21′	53° 14′	55° 36′	56° 10′	56° 18′	56° 19′	56° 19′	
G	1.38	1.425	1.413	1.305	1.17	1.05	0.95	0.86	millimeters
$B - b$	0	0.63	1.27	2.49	3.61	4.61	5.51	6.32	millimeters

Therefore at 1000 kilometers from the center the barometric pressure is $3.1 + 6.32 = 9.42^{mm}$ lower than at the center.

At the boundary between the inner and outer regions the gradient is discontinuous, since it falls suddenly from 1.72 to 1.38; but that this must be so is evident from equations (Ia′) since

$$\frac{dV_n}{dr}$$

has different values when $r = R$ outward and inward because of the conditions as to continuity.

In the exterior region and as the distance from the boundary circle increases, the angle of deflection rapidly approximates to the normal value which in our case is $58° \ 19' = tg^{-1} \ 3/2$.

In general the winds and pressures computed in our example correspond very well to those that are actually observed in barometric maxima especially in those of the warmer season of the year and where the lowest stratum of air does not play too prominent a part.

Perhaps we should have come still nearer to the actual conditions of nature if we had assumed the intensity of the descending current of air in the interior region of the anticyclone, not constant, but diminishing toward the boundary so that at the boundary between the inner and outer regions a continuous transition exists for the vertical velocity component and consequently for the gradient. A simple assumption of this kind for the vertical velocity at the altitude unity above the earth's surface is

$$w = \gamma \left\{ 1 - \left(\frac{r}{R} \right)^n \right\}$$

where n may be any positive number, for in accord with this theorem w attains its greatest value γ at the center of the anticyclone but disappears at the boundary circle whose radius is R.

For this value of w the equation of continuity (Ic′) becomes

$$\frac{d(rV_n)}{dr} = -\gamma r \left\{ 1 - \left(\frac{r}{R} \right)^n \right\}$$

and by integration

$$V_n = -\frac{1}{2}\gamma r + \frac{\gamma}{n+2} \frac{r^{n+1}}{R^n}$$

where again the constant of integration is unnecessary.[5]

[5] It is zero as in equation (1).—C. A.

The differential equation (Ib′) for the tangential component V_t for the inner region now becomes

$$\frac{1}{r} \cdot \frac{d\,(r\,V_t)}{d\,r} - \frac{k}{\gamma}\,\frac{r\,V_t}{\dfrac{r^{n+2}}{(n+2)\,R^n} - \dfrac{r^2}{2}} + \lambda = 0$$

The general integral of this equation is the sum of a function that satisfies the complete differential equation, added to the product of an arbitrary constant multiplied by a second function (V'_t) that satisfies the same differential equation after omitting the term λ. For this latter function we find

$$V'_t = C \cdot \frac{1}{r} \cdot \left\{ \frac{n+2}{2} \cdot \frac{R}{r} - 1 \right\}^{2\,k/\gamma}$$

Since this expression becomes infinite when γ is positive and $r = 0$ therefore we must have $C = 0$. On the other hand this constant must be retained in the case of the cyclone where γ has a negative value whose absolute value however can not exceed $2k$.

Hence our solution for V_t is only attained through the integral of the complete differential equation for which the following expression is found

$$V_t = -\frac{\lambda}{2\,r}\left(\frac{n+2}{2}\,\frac{R^2}{r^2} - 1\right)^{k/\gamma} \int \frac{r^{2\,k/\gamma}\,d\,(r^2)}{\left(\dfrac{n+2}{2}\,R^2 - r^2\right)^{k/\gamma}}$$

which does not allow of presentation in definitive terms for all values of $\frac{k}{\gamma}$. A simplification is possible when $\frac{k}{\gamma}$ is a simple rational fraction. I give the following result for the special case when $\gamma = k$.

$$V_t = +\frac{\lambda}{2\,r}\left(\frac{n+2}{2} \cdot \frac{R^2}{r^2} - 1\right) \times$$
$$\times \left(r^2 + \frac{n+2}{2} R^2 \log \text{nat} \left\{ 1 - \frac{2}{n+2} \cdot \frac{r^2}{R^2} \right\} \right).$$

The last factor in this expression (and therefore also V_t itself) is negative and for $r = 0$ becomes infinitely small in such a way that V_t also itself becomes zero.

The preceding relatively simple result was attained without special limitation as to the exponent n in the law for the vertical velocity, and no further difficulties arise in computing from it the distribution of pressure in the inner region of the anticyclone and in so determining the constants of our previous solution for the exterior region whose form remains the same that V_n, V_t and p and now also $\frac{dp}{dr}$ pass continuously over at the boundary circle.

I will not now go more precisely into this computation but will only communicate some results for the special case of $n = 2$. In this case the velocity of the descending air is

$$w = \gamma\left(1 - \frac{r^2}{R^2}\right)$$

Since now we put $\gamma = k$ therefore, by retaining the values of k and R adopted in the previous example we obtain for the center, $r = 0$, a descending velocity twice as large as then; on the other hand the total mass of the descending air for the whole interior region remains the same and is proportional to $\int_0^R wrdr$. We have also

$$V_n = -\frac{\gamma}{2}r\left(1 - \frac{1}{2}\frac{r^2}{R^2}\right)$$

$$V_t = \frac{\lambda}{2r}\left(2\frac{R^2}{r^2} - 1\right)\left\{r^2 + 2R^2 \log \text{nat}\left(1 - \frac{r^2}{2R^2}\right)\right\}$$

$$tg\,\psi = -2\frac{\lambda}{k}\left(\frac{R}{r}\right)^2\left\{1 + 2\left(\frac{R}{r}\right)^2 \log \text{nat}\left(1 - \frac{2r^2}{R^2}\right)\right\}$$

Therefore in the interior region also the angle of deflection is no longer constant: let the "normal" angle of deflection be

$$\alpha = tg^{-1}\lambda/k$$

then for our two special values of r we have

$$\text{for } r = 0 \qquad tg\,\psi_0 = \frac{1}{2}\,tg\,\alpha$$

$$\text{for } r = R \qquad tg\,\psi_R = 0.7726\,tg\,\alpha$$

whence by introducing the numerical values above given (i. e., $\lambda = 0.00012$ and $k = 0.00008$) we find the angles $\psi_0 = 36°\ 32'$ and $\psi_R = 49°\ 13'$ respectively.

The wind-velocity at the boundary of the interior region is

$$V_R = \frac{\gamma}{4} \frac{R}{\cos \varphi} = 12.16 \text{ meter per second}$$

which is rather larger than for the preceding solution as tabulated above.

In case $\gamma = k$ and $n = 2$ the equation (Ia′) becomes

$$\frac{1}{\rho} \frac{dp}{dr} = -\frac{3}{4} \gamma^2 r \left(1 - \frac{1}{2} \frac{r^2}{R^2}\right) + \frac{1}{2} \lambda^2 r \left(2 \frac{R^2}{r^2} - 1\right) \times$$

$$\times \left[1 + 2 \frac{R^2}{r^2} \log \left(1 - \frac{1}{2} \frac{r^2}{R^2}\right)\right] \left[\frac{3}{2} + \frac{R^2}{r^2} \log \left(1 - \frac{1}{2} \frac{r^2}{R^2}\right)\right]$$

and for $r = R$ this gives the special value of the gradient at that distance

$$G_R = \frac{\rho}{\mu} \frac{1}{\rho} \frac{dp}{dr} = 1.484^{\text{mm}}$$

per 111 kilometers or one degree of a great circle which now holds good for both sides of the boundary circle. In the *interior region* the gradient now diminishes with distance from the center more slowly than in the previous case when w is constant; therefore in spite of the above given smaller value of G the total difference of pressure between the center and the boundary circle still remains about as large as before. In the *exterior region* the distribution of pressure and wind remains very similar to that of the example previously computed, it is indeed plausible that this depends principally on the total intensities of the descending currents (which are the same in our two examples) and only slightly on the horizontal distribution of the descending velocities.

Perhaps it would be of interest to compare the solutions that we have obtained in the manner above explained for cyclones under various special assumptions as to γ and n, with cases actually observed, in order to ascertain what law as to the ascending current best corresponds with the facts. In the case of *anticyclones* the data presented in the daily synoptic weather chart is not so well adapted to such a comparison with theory because the differences of pressure and the wind force are generally too slight to be obtained from them with sufficient accuracy.

INDEX

A.

D.

E.

F.

G.

H.

N.

O.

P.

R.

S.

W.