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SPECIFIC GRAVITY AND TREE WEIGHT OF SINGLE-TREE SAMPLES OF GRAND FIR

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SPECIFIC GRAVITY AND TREE WEIGHT OF SINGLE-TREE SAMPLES OF GRAND FIR[1]

Albert R. Stage

SYNOPSIS

Analysis of data from 108 grand fir trees sampled in northern Idaho provided the following prediction equations for specific gravity of 20-year periodic increment on the tree bole (S_{20}), average specific gravity of the tree bole (S_{ent}), and the dry weight of the tree bole (W):

$$S_{20} = 0.5413 - 0.08838(1/SC_{20}) + 0.9907(1/H)$$

$$S_{ent} = 0.5579 - 0.05614(1/SC_{20}) - 0.03299(1/SC_{ent})$$

$$W_{(cwt)} = 0.4422 + 0.001305(D^{2} \cdot H \cdot SC_{20}) - 0.1990(C)$$

where: H = total tree height in feet

D = d.b.h.

C = crown class

SC_{20} = specific gravity of the outer 20 rings of increment core taken at breast-height.

SC_{ent} = specific gravity of increment core to pith taken at breast-height.

Trends of specific gravity of the increment at different heights in the tree were shown to be a function of distance from the apex, crown length, and crown class.

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[1] Fieldwork for this study was conducted with the cooperation of Potlatch Forests, Inc., Inland Empire Paper Company, and the Forest Products Laboratory of the U.S. Forest Service.

INTRODUCTION

Growth and yield data for pulping species are most meaningful when expressed in units of dry fiber weight. Because grand fir (Abies grandis Lindl.) is one of the better pulping species in the Inland Empire,[2] the growth and yield study of this species conducted by the author was designed to provide data on specific gravity at breast height on each yield plot. In this report, the prediction equations for converting the breast-height specific gravity to estimates for the entire stem are derived. Three dependent variables are included:

1. Average specific gravity of the last 20 years of growth.

2. Average specific gravity of the stem.

3. Tree weight in pounds of dry matter.

Each of these variables pertains to the tree stem from a stump height of 1 foot to a top diameter of 3.6 inches inside bark.

COLLECTION OF TREE DATA

Field-sampling procedures.--Single-tree samples of grand fir were selected in two steps. In the first step, breast-high cores were taken from trees near the yield study plots. Each tree in this preliminary sample was numbered, and its crown class and d.b.h. were recorded. Specific gravity of the core was determined by the procedure described below. From this preliminary sample, trees were selected to cover uniformly as wide a range of specific gravities as possible within crown and diameter classes.

From the 108 trees so selected, one additional core to the pith and two cores extending through the outer 20 annual rings were taken from the three unsampled quadrants at the breast-height position. The trees were then felled, and seven disks cut from the stem at equal intervals starting at the breast-height position and extending upward to the point where the diameter inside bark was 3.6 inches. For trees taller than 100 feet, disks were cut from the top of each 16.3-foot log, at breast height, and at the point where diameter inside bark becomes less than 3.6 inches. The diameter inside bark of each disk was recorded. The total height, height to base of the live crown, and age at breast height were recorded for each tree.

Distribution of the sampled trees by diameter and crown class is shown in table 1.

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[2] As used in this paper, the term "Inland Empire" includes northern Idaho and northeastern Washington.

Table 1.--Distribution of tree samples for specific gravity analysis

D.b.h. (in.)	Crown class				Total
	Dominant	Codominant	Intermediate	Suppressed	
	Number				
4		2	2	2	6
6	2	2	4	5	13
8	1	6	8	5	20
10	1	6	3	2	12
12	4	3	4		11
14	8	4	1		13
16	3	4	2		9
18	3	3			6
20	3				3
22	3	3			6
24	2				2
26	2				2
28	2				2
30+	3				3
Total	37	33	24	14	108

Determination of core specific gravity.--Specific gravity at breast height was obtained from cores taken with a standard increment borer. The cutting diameter of the borer was calibrated by a taper gage periodically during the fieldwork. The average diameter was 0.171 inch. The length of the core from end of last growing season to the pith was measured to ±0.01 inch immediately upon extraction. The last 20 annual rings were then severed and their length recorded. The cores were inserted in numbered paper straws for transportation to the laboratory. After drying for 25 hours at 105° C., the cores were weighed on a precision balance. Specific gravity (grams of dry wood fiber per cubic centimeter of green volume) was computed from the following formula:

$$\text{Sp. gr.} = 0.0777\ W/D^2L$$

where W = ovendry weight (grams)

D = diameter of increment borer (inches)

L = length of core (inches).

The mean specific gravity of the cores to pith was 0.388 with a standard deviation of ±0.030 per tree. The standard deviation of the separate cores about the mean for the tree was ±0.022. Hence, the standard error of the mean of the two cores to pith for a single tree was ±0.016.

The mean specific gravity of the 20-year increment was 0.397 with a standard deviation of ±0.048 per tree. The standard deviation of the four separate cores about the tree mean was ±0.054. The mean of the four cores for the 20-year increment specific gravity thus has a standard error of ±0.027.

Determination of bole weight and specific gravity.--Pie-shaped segments of the disks were split along the 20th ring from the cambium to separate the 20-year increment from the entire segment. After soaking the segments to regain lost moisture, their volumes were obtained by water displacement; then the segments were dried in several steps to an ovendry condition and weighed. From these data the total volume and increment volume were computed in cubic feet by summing the Smalian formula for volume of a parabolic frustum for each log. The volume of the section from a stump height of 1 foot to breast-height point was computed as $(0.03)(\text{dia. inside bark at b.h.})^2$ and assumed to have a specific gravity equal to that of the breast-height section.

The weight of each bole was computed by multiplying each end-area in the volume summation by its corresponding specific gravity, then multiplying the total by 62.4 pounds per cubic foot. The bole specific gravity was obtained as an intermediate step by dividing the above total by the total volume. Analogous calculations were used to determine the specific gravity of the outer 20-year increment.

VARIATION IN SPECIFIC GRAVITY AS A FUNCTION OF HEIGHT

The way in which specific gravity of a given annual increment changes with height must be considered in deriving formulas to convert breast-height data to total-tree estimates. Much has been written concerning variation in specific gravity (Goggans 1961). However, three papers provide special insight into the causes and nature of the variation of specific gravity with height.

1. Larson's (1960) investigations showed that the transition from large diameter to small diameter cells was related to the cessation of terminal elongation, and that the vertical extent and magnitude of the change in tracheid development depended upon the intensity of the apical stimulus. This theory suggests that variations in tracheid development should be related to distance from apical meristem, rather than to height above the ground level. Although cell diameter is only one factor affecting specific gravity, the correlation of secondary wall thickening with cell diameter permits a similar argument to be applied to specific gravity.

2. The strength of this approach is demonstrated by data cited by Richardson (1961). Of the three sequence types described by Duff and Nolan (1953), the type I or oblique sequence showed the best concordance of data from various portions of the tree bole. In this sequence, a single annual increment is traced from the apex to the base. Points are identified by the number of internodes from the apex at the time of deposition. For Corsican pine, Richardson reported that gravity decreased to a minimum at about the fifth internode from the apex, then increased to about the 20th, and subsequently leveled off.

3. Smith and Wilsie (1961), studying the effect of summer water deficits on tracheid development in the oblique series in loblolly pine, found that the slope of the linear regression of specific gravity on internodes from the apex was closely related to the moisture stress during the year of formation.

These three papers indicate that:

1. Factors such as crown development and vigor may affect the trend of upper tree specific gravity by supplying additional sources of auxin from elongating laterals.

2. Much of the variation in specific gravity at breast height may be simply the result of sampling at different distances from the apex.

3. Regression coefficients depend on the past climate, especially for estimating specific gravity of increment over the bole surface.

An analytical expression for this function can be derived from the following assumptions:

1. One component of specific gravity at distance (T) from the apex is inversely related to the concentration of auxin per unit area of cambium (C) (perhaps through the mediation of other substances present in proportion to the auxin transported from the apex).

2. A second additive component represents effects proportional to the distance from the apex. An example of this latter type is the effect of wind stresses acting through a moment-arm proportional to the length of the stem (T) above the point of measurement. Thus

$$S = a + b/C + cT \qquad (1)$$

3. The tree bole is approximately a paraboloid. For D_T = diameter at distance T from apex:

$$D_T = gT^{\frac{1}{2}}. \qquad (2)$$

Since concentration would vary inversely proportional to cambial area, for a given vertical dimension, it is inversely proportional to the horizontal dimension. Hence

$$C = k/D = \frac{k}{g}\ T^{-\frac{1}{2}} \qquad (3)$$

because circumference is proportional to diameter.

Taking differentials of (1) and (3)

$$\partial S = -b\ C^{-2}\partial C + c\ \partial T \qquad (4)$$

$$\partial C = \frac{-k}{2g}\ T^{-3/2}\partial T \qquad (5)$$

and, substituting (3) and (5) in (4):

$$\partial S = -b\ (\frac{g^2}{k^2}\ T)\ (\frac{-k}{2g}\ T^{-3/2})\ \partial T + c\partial T$$

$$\partial S = \frac{bg}{2k}\ T^{-\frac{1}{2}}\ \partial T + c\partial T. \qquad (6)$$

Integrating:

$$\int \partial S = \int (\frac{bg}{2k}\ T^{-\frac{1}{2}} + c)\ \partial T$$

gives

$$S = \frac{bg}{4k}\ T^{\frac{1}{2}} + cT + a_o \qquad (7)$$

or, combining coefficients:

$$S = a_0 + a_1 T + a_2 T^{\frac{1}{2}}. \qquad (8)$$

This equation, for appropriate coefficients a_1 and a_2, has an analytical form similar to that described by Richardson (1961).

ANALYSIS OF DATA

The above analysis applies to specific gravity of a single annual increment. A similar functional form should also apply to the average specific gravity of a uniform number of annual increments. Because the cores taken from the trees on the growth study plots were measured for the specific gravity of the outer 20 years' increment, this period was used for the analysis of the individual tree data.

The specific gravity of the outer 20 annual rings of each disk was used to estimate the three coefficients in (8) for each tree.

These three coefficients formed the components of the dependent vector of a multiple linear regression on the four independent variables: d.b.h., crown length, age, and crown class of the sample tree. A test statistic for the predictive value of each of the independent variables was computed. This statistic has the F-distribution with 3 and 103-q degrees of freedom where q is the number of independent variables included in the multivariate regression. In the first part of table 2, this statistic is indicated for each variable separately.

After removing the effect of crown length, crown class still accounted for a significant part of the remaining generalized variance, but d.b.h. did not.

The regression equations for the coefficients of (8), finally expressed in terms of crown length (L) and crown class (C), were:

$$a_0 = 0.377 - 0.000971\,(L) + 0.0628\,(C)$$

$$a_1 = 0.000552 - 0.0000883\,(L) + 0.00229\,(C)$$

$$a_2 = 0.0474 + 0.000678\,(L) - 0.0223\,(C).$$

Curves predicted from these regressions are shown in figure 1 for two typical crown lengths in each crown class.

ESTIMATING INCREMENT SPECIFIC GRAVITY FROM BREAST-HEIGHT CORE DATA

The average specific gravity of the layer of wood produced in a 20-year period is the weighted average of the ordinates of the curves in figure 1, where the weights are the cross-sectional area of the increment at the corresponding distance from the apex. The specific gravity of the outer 20 rings of the core taken at breast height provided the principal index to relative position of the curve for a particular tree.

Table 2.--Value of variables for predicting parameters of trend of specific gravity with distance from apex

Variable	F-statistic	Degrees of freedom
Crown class	17.0**	3,102
Crown length	16.6**	3,102
D.b.h.	14.3**	3,102
Age	2.4	3,102
	Independent of crown length:	
Crown class	6.24**	3,101
D.b.h.	1.60	3,101

** Significant at the 1% level.

The 20-year sheath of increment studied here was laid down in the years 1940 to 1959 inclusive. Application of the predictors derived below should be restricted to trees subject to moisture regimes similar to those experienced in the Inland Empire during that period.

The analysis of the previous section showed that this curve changed slope with crown length and crown class, and particular attention was paid to exploring the possible functional form in which these variables might enter the prediction equation for increment specific gravity. Tree height, (H), as a measure of the distance from the apex at which the specific-gravity sample was taken, was also expected to be an important variable. In addition, age, volume growth percent, d.b.h., and diameter growth rate were also considered because other workers had hypothesized that these variables are related to specific gravity. Three separate regressions were computed using 18 to 28 variables representing various transformations and interactions of the variables in the above list. In each regression, two terms were sufficient to explain all but an insignificant portion of the explainable variance.[3]

Breast-height core specific gravity (SC_{20}) was the most important factor. Its reciprocal was consistently the best functional form of this factor. The best prediction equation of the set solved was:

$$S_{20} = 0.5413 - 0.08838\ (1/SC_{20}) + 0.9907\ (1/H).$$

The coefficient of determination was $R^2 = 0.771$ and the standard error of estimate was ±0.0146.

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[3] In this paper, explainable variance is taken to be (Variance of Y) (Max (R^2)) where the Max (R^2) is the coefficient of determination for the multiple regression on the entire set of independent variables.

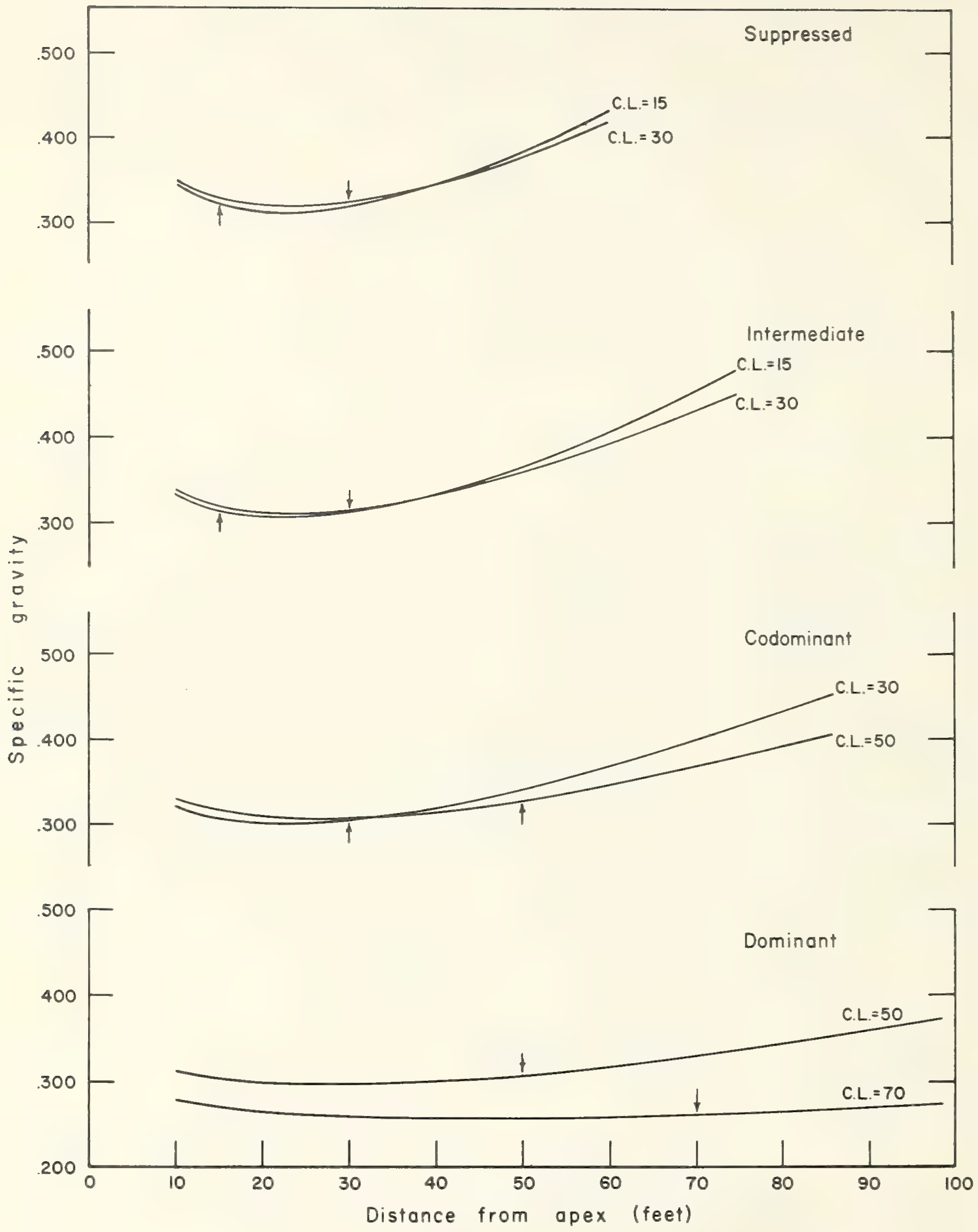

Figure 1.--Specific gravity of the outer 20 annual rings as a function of distance from apex of the tree for Abies grandis. Separate curves are drawn for two typical crown lengths in each of four crown classes. Arrows indicate base of live crown.

Although the coefficient of the reciprocal of height was very significantly different from zero, dropping this variable only reduced the coefficient of determination to $R^2 = 0.741$. The simpler prediction equation was:

$$S_{20} = 0.5592 - 0.09022\ (1/SC_{20})$$

with a standard error of estimate of ±0.0153.

Zobel et al. (1960) reported one of the few studies in which the outer sheath of wood was treated separately. For the regression of outer bole on outer breast-height specific gravity, he found standard errors of ±0.016 and ±0.011 for slash and loblolly pines, respectively. His breast-height samples consisted of 1-inch disks.

ESTIMATING BOLE SPECIFIC GRAVITY FROM BREAST-HEIGHT CORE DATA

The average specific gravity of the bole to 3.6" i.b. (S_{ent}) was the dependent variable of a regression on 15 independent variables derived from the same factors studied in relation to increment specific gravity. In addition, the specific gravity of the entire core from cambium to pith (SC_{ent}) and its reciprocal were added. Of the 17 independent variables, two variables, the reciprocals of SC_{20} and SC_{ent} were sufficient to explain most of the explainable variance. The prediction equations were:

$$S_{ent} = 0.5579 - 0.05614\ (1/SC_{20}) - 0.03299\ (1/SC_{ent})$$

with $R^2 = 0.696$ and standard error of estimate of ±0.0145, and

$$S_{ent} = 0.5175 - 0.0738\ (1/SC_{20})$$

with $R^2 = 0.666$ and standard error of estimate of ±0.0151.

The ratio of crown length to total height explained an additional 1 percent of the total variation. Although statistically significant, the contribution of this additional variable is too small to be of value for prediction purposes.

It is interesting to note that the correlation of the outer 20 years of the core with average bole specific gravity was 0.78 while the correlation of the entire core with average bole was 0.71. Apparently the central portion of the bole follows a less well-defined trend of specific gravity with height in the tree than does the sheath of increment. Zobel et al. (1960) found a similarly small change in correlation when the juvenile wood was ignored.

Wahlgren and Fassnacht (1959) obtained standard errors of from ±0.021 to ±0.0285 for four species of southern pine when the independent variable was the reciprocal of specific gravity of a single breast-height core. Gilmore et al. (1961), also working with southern pines, found standard errors of ±0.017 when predicting tree specific gravity from a single breast-height core could be reduced to ±0.015 when the prediction was based on the product of two core values, breast height and stump height. Judging from the within-tree variance of specific gravity at breast height found in the present study, the advantage of Gilmore's second core may be chiefly a matter of obtaining more samples, rather than a virtue of position. Whether the same argument holds for adding SC_{ent} to the predictions for grand fir is not clear, because in this case the SC_{ent} sample includes two of the four SC_{20} cores.

Zobel et al. (1960), using disks at breast height rather than cores, arrived at a standard error of ±0.016 for slash pine. They estimated specific gravity at breast height without the sampling error inherent in the increment core methods (but still having some measurement error). From Wahlgren and Fassnacht's data, it appears that their standard errors in prediction would have been reduced from about ±0.025 to ±0.015 by replacing the core by the surrounding disk.

Apparently, several cores to pith are adequate to provide a means of predicting tree specific gravity with a standard error comparable to that obtainable from an entire disk taken at breast height.

ESTIMATING DRY WEIGHT OF THE TREE BOLE

Dry weight of standing trees can be estimated by procedures analogous to those used to estimate the cubic contents of such trees. One approach might be to use the equations of the previous section to estimate specific gravity and a conventional volume table to estimate the cubic-foot contents of the bole. Multiplying the product of these two factors by 62.4 lbs./cu.ft. would give an estimate of tree dry weight. However, such a procedure would make it very difficult to assess the possible effect of crown characters acting through specific gravity and form class on the final estimate of weight. A more direct procedure is to use dry weight of the tree bole in hundreds of pounds (W) as the dependent variable in a multiple regression.

The variance of dry weight increases with tree size. Hence, a set of weights inversely proportional to the variance was needed to improve the estimates for the smaller tree sizes. In previous analyses, the variance of specific gravity had appeared to be independent of tree size. Thus, most of the change in dry-weight variance must be the result of the increase in variance in cubic feet. Accordingly, a weighting function derived in the course of calculating a cubic-foot volume table for grand fir was used to stabilize the variance of dry weight.

Independent variables were selected by combining breast-height core specific gravity with the combinations of d.b.h. (D) and height (H), commonly used in volume equations. The specific gravity at breast height was based only on the data for the outer 20 rings of the core. This portion of the core was used because it corresponds to the data collected on the grand fir growth and yield plots. The analysis of the previous section showed that adding the data for the entire core would have resulted in only a small increase in precision of the estimate.

In addition, crown length (L) and crown class (C) were introduced to account for changes in form class and specific gravity associated with crown characters.

Four significant variables explained 98 percent of the variance in dry weight, but the combined variable $D^2 \cdot H \cdot SC_{20}$ alone accounted for 97 percent. The most useful additional variable was crown class. Together, these two variables resulted in a prediction equation having a coefficient of determination of 0.978 and a standard error of estimate of ±27.2 lbs. (for an observation of unit weight).

The two prediction equations were:

$$W_{(cwt)} = -0.2807 + 0.001345(D^2 \cdot H \cdot SC_{20})$$

$$s.e. = \pm 0.307$$

and

$$W_{(cwt)} = 0.4422 + 0.001305(D^2 \cdot H \cdot SC_{20}) - 0.1990(C)$$

$$s.e. = \pm 0.272.$$

The units of the variables are:

D = diameter at breast height in inches

H = total tree height in feet

SC_{20} = specific gravity of the outer 20 rings of an increment core taken at breast height.

C = crown class coded as:

Dominant	= 1	Intermediate	= 3
Codominant	= 2	Suppressed	= 4

The average of the absolute percentage deviations was 11.3 percent of the tree weight. By comparison, a similar cubic-foot volume table with arguments of height and diameter prepared by the author had an average absolute percentage deviation of 9.8 percent.

LITERATURE CITED

Duff, G. H., and Norah J. Nolan
1953. Growth and morphogenesis in the Canadian Forest Species. I. The controls of cambial and apical activity in Pinus resinosa Alt. Canad. Jour. Bot. 31: 471-513.

Gilmore, A. R., G. E. Metcalf, and W. R. Boggess
1961. Specific gravity of shortleaf pine and loblolly pine in Southern Illinois. Jour. Forestry 59: 894-896.

Goggans, James F.
1961. The interplay of environment and heredity as factors controlling wood properties in conifers with special emphasis on their effects on specific gravity. North Carolina State College, School of Forestry, Tech. Rpt. No. 11. 56 pp.

Larson, Philip R.
1960. A physiological consideration of the springwood-summerwood transition in red pine. Forest Sci. 6: 110-122.

Richardson, S. D.
1961. A biological basis for sampling in studies of wood properties. Tappi 44: 170-173.

Smith, Diana M., and Mary C. Wilsie
1961. Some anatomical responses of loblolly pine to soil-water deficiencies. Tappi 44: 179-185.

Wahlgren, Harold E., and Donald L. Fassnacht
1959. Estimating tree specific gravity from a single increment core. U.S. Dept. Agr. Forest Service, Forest Products Lab. Rpt. No. 2146. 24 pp.

Zobel, Bruce, Faye Henson, and Charles Well
1960. Estimation of certain wood properties of loblolly and slash pine trees from breast height sampling. Forest Sci. 6: 155-166.

(FOR YOUR CARD FILE)

Stage, Albert R.
1963. Specific gravity and tree weight of single-tree samples of grand fir. U.S. Dept. Agr., Forest Serv., Intermountain Forest and Range Expt. Sta., Ogden, Utah. 11 pp., illus. (U.S. Forest Serv. Res. Paper INT-4)

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