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The SPM Energy Flux Method for Predicting Longshore Transport Rate

by

Cyril Galvin and Charles R. Schweppe

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This report explains in detail the energy flux method in Section 4.532 of the Shore Protection Manual (SPM). Appendix A describes the derivation of four energy flux factors. Appendix B explains how the significant wave height enters these equations. Appendix C identifies the data that led to the prediction of longshore transport rate from the energy flux factor. The importance of the correct formulation of breaker speed, and its effect on estimates of breaker angle are demonstrated. The report describes the steps used to arrive at the energy flux method, but it does not critically analyze those steps.

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PREFACE

This report is published to provide coastal engineers with detailed explanations of three frequently asked questions concerning the energy flux method described in Section 4.532 of the Shore Protection Manual (SPM). The questions involve the derivaton of equations for the energy flux factor, the use of significant wave height in these equations, and the data used to relate energy flux factors to longshore transport rate. These questions require more elaborate explanations than is possible in the SPM. The work was carried out under the coastal processes research program of the U.S. Army, Coastal Engineering Research Center (CERC).

The report was written by Dr. Cyril Galvin, formerly Chief, Coastal Processes Branch, and Charles R. Schweppe, formerly a student trainee in Coastal Processes Branch, under the general supervision of R.P. Savage, Chief, Research Division, CERC. The authors wish to thank C.M. McClennan for advice on the derivation, D.L. Harris for help with the significant wave height, T.L. Walton for pointing out the importance of the breaker speed estimate, and P. Vitale, principal investigator of the longshore transport prediction work unit.

Comments on this publication are invited.

Approved for publication in accordance with Public Law 166, 79th Congress, approved 31 July 1945, as supplemented by Public Law 172, 88th Congress, approved 7 November 1963.

TED E. BISHOP
Colonel, Corps of Engineers
Commander and Director

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CONVERSION FACTORS, U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U.S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	by	To obtain
inches	25.4	millimeters
	2.54	centimeters
square inches	6.452	square centimeters
cubic inches	16.39	cubic centimeters
feet	30.48	centimeters
	0.3048	meters
square feet	0.0929	square meters
cubic feet	0.0283	cubic meters
yards	0.9144	meters
square yards	0.836	square meters
cubic yards	0.7646	cubic meters
miles	1.6093	kilometers
square miles	259.0	hectares
knots	1.852	kilometers per hour
acres	0.4047	hectares
foot-pounds	1.3558	newton meters
millibars	1.0197×10^{-3}	kilograms per square centimeter
ounces	28.35	grams
pounds	453.6	grams
	0.4536	kilograms
ton, long	1.0160	metric tons
ton, short	0.9072	metric tons
degrees (angle)	0.01745	radians
Fahrenheit degrees	5/9	Celsius degrees or Kelvins[1]

[1]To obtain Celsius (C) temperature readings from Fahrenheit (F) readings, use formula: C = (5/9) (F -32).

To obtain Kelvin (K) readings, use formula: K = (5/9) (F -32) + 273.15.

SYMBOLS AND DEFINITIONS

$A(w)$	spectrum function of the amplitude of η (see eq. B-5)
a_t	vertical distance from breaker wave trough to MWL (see Fig. A-3)
$B(t)$	envelope function (see eq. B-6 and Fig. B-2)
b	crest length between orthogonals (see Fig. A-2)
b	subscript for condition at the breaker
C	wave velocity
d	water depth
$\overline{E}$	wave energy density (see eq. B-1) with subscripts
g	acceleration of gravity
H	wave height
H'_o	deepwater height assuming no refraction
H_p	mean value of the highest fraction of wave heights above a height H which has a probability, p, of occurrence (see eqs. B-8 and B-10)
H_{rms}	root-mean-square height (eq. B-3)
H_s	significant wave height (eq. B-13)
h	water depth measured from wave crest to bottom (Fig. A-3)
h^*	generalized water depth under a breaking wave (eq. 2)
i	subscript for conditions at point i (Fig. A-2)
K_r	refraction coefficient (eq. A-11)
K_s	shoaling coefficient (eq. A-12)
L	wavelength
ℓn	natural log symbol
N	number of waves
n	ratio of group velocity to individual wave velocity (eq. A-17)
o	subscript for conditions in deep water (except eq. B-6)
P	energy flux, per unit length of shoreline, defined by equation (A-6)
$P(H)$	probability function of wave height

P^*	energy flux per unit length of wave crest (eq. A-1)
P_ℓ	part of the energy flux in waves defined by equation (A-7)
$P_{\ell s}$	approximate evaluation of P_ℓ for conditions in the surf zone
p	fraction of wave heights, in a train of N waves, whose height exceeds H (eq. B-8)
Q	volume rate of longshore transport
s	distance in the longshore direction between adjacent orthogonals (Fig. A-2)
T	wave period
t	time, or conversion factor, seconds per year
w	weight density of water
α	angle between wave crest and shoreline (Fig. A-2)
β	ratio of breaker depth to breaker height (Fig. A-3)
η	departure of water surface from MWL (Fig. B-2)
ρ	mass density of water
σ	ratio a_t/H_b (Fig. A-3)
ω, ω_o	wave frequency or central wave frequency (eq. B-6)

THE SPM ENERGY FLUX METHOD FOR PREDICTING LONGSHORE TRANSPORT RATE

by
Cyril Galvin and Charles R. Schweppe

I. INTRODUCTION

The Shore Protection Manual (SPM) (U.S. Army, Corps of Engineers, Coastal Engineering Research Center, 1977) describes procedures for estimating quantities important in coastal engineering design. Among the most important of these is the estimation of longshore transport rate. Aside from relying solely on judgment or historical data, the principal way to estimate longshore transport rate is by use of the energy flux method, described in Section 4.532 of the SPM. Since the SPM provides design guidance rather than explanation, the derivations of the energy flux equations are only briefly indicated in the manual.

This report supplements the SPM with a documented development of the energy flux method as it is presented on pages 4-96 to 4-102 of the SPM. The documentation is mainly found in the three appendixes to this report: Appendix A, Derivation of the Longshore Energy Flux Factor; Appendix B, Distinction Between Significant and Root-Mean-Square Wave Heights in Predicting Longshore Transport Rates; and Appendix C, Field and Laboratory Data Appearing in the SPM Energy Flux Discussion. The subject matter of these appendixes are the three most commonly questioned aspects of the SPM presentation on the energy flux method. Each appendix gives an independent explanation of one topic without necessary cross-reference to other parts of the report.

This report only describes what was done to arrive at the energy flux method already published in the SPM. The assumptions necessary to arrive at the formulation in the SPM are described, but not reviewed, although these assumptions were closely examined by reviewers when Chapter 4 of the SPM was under preparation (May 1972 to August 1973). The current issue of the SPM is the third edition (1977). Section 4.532 on the energy flux method is the same in all three editions, with the exception of an error on the ordinate scale of Figure 4-36 which was corrected after the first (1973) printing. Galvin and Vitale (1977) compared the energy flux method documented in the SPM with its predecessor, TR-4, Shore Protection, Planning and Design (U.S. Army, Corps of Engineers, 1966).

II. DISCUSSION OF THE THREE APPENDIXES

1. The Equations (App. A).

The energy flux method relates longshore transport rate, Q, to wave conditions by use of the longshore energy flux factor, $P_{\ell s}$. Two equations are needed: an equation that converts wave conditions into $P_{\ell s}$, and an equation that predicts Q from $P_{\ell s}$. In the SPM, four theoretically equivalent equations for longshore energy flux, P_ℓ, are developed from small-amplitude, linear theory. From these four equations, four design equations for the longshore energy flux factor in the surf zone, $P_{\ell s}$, are derived. Although each equation for P_ℓ is equivalent to any of the three other equations for P_ℓ, no two of the four $P_{\ell s}$ equations are exactly equivalent because each $P_{\ell s}$ equation was derived from a different P_ℓ equation using a different set of approximations.

The four equations for P_ℓ, equations 4-31 to 4-34, and the four equations for $P_{\ell s}$, equations 4-35 to 4-38, are given in Tables 4-7 and 4-8, respectively, in the SPM, and they are reproduced in Appendix A.

Appendix A derives the eight equations and identifies the assumptions used. These derivations involve only simple algebra and the assumptions are standard. However, the exact formulation of the assumptions does affect the value of coefficients in the equations.

2. Wave Height (App. B).

The energy flux factor, $P_{\ell s}$, depends sensitively on wave height. In one equation, it is an H^2 dependence; in another one, H^3; and in two equations, $H^{5/2}$. In all, the height enters $P_{\ell s}$ primarily as the energy density, $\bar{E}$, which depends on H^2. Since, in nature, height varies from one wave to the next, the average energy density of a group of waves will not be determined from the average height, but rather from a height which produces the average value of H^2. This is the root-mean-square (rms) height, H_{rms}.

The height commonly used in coastal engineering work is neither the average height nor the H_{rms}, but the significant height, H_s. Appendix B explains the relations between H_{rms} and H_s, and how the energy flux factor is calibrated for use with H_s.

3. The Data (App. C).

To obtain an equation to predict Q from $P_{\ell s}$, it is necessary to have sufficient data for wave conditions to compute $P_{\ell s}$ and to measure values of Q at the time the wave conditions are measured. Appendix C identifies the sources of data which led to the relation between Q and $P_{\ell s}$ shown in Figure 4-37 and equation 4-40 of the SPM.

III. WAVE SPEED AND BREAKER ANGLE

This report primarily provides documentation for the energy flux method as given in Section 4.532 of the SPM. It does not critically evaluate the method. However, this section does examine the effect of assumptions about wave speed and breaker angle on the computed values of Q to enable the user to form a judgment of the overall accuracy.

The energy flux is proportional to group velocity. The group velocity for linear waves in shallow water is equal to wave speed. For energy flux entering the surf zone, the appropriate wave speed is the speed of the breaker. Breaker characteristics have long been assumed to be described by solitary wave theory. Solitary theory can be used to locate the breaker point using the rule-of-thumb

$$d_b = 1.28\ H_b \tag{1}$$

and to estimate the breaker speed using an equation of the form,

$$C_b^* = \sqrt{gh^*} \tag{2}$$

where h^* is a depth. If h^* is set equal to the mean depth, then the breaker speed is that of linear theory

$$C_b = \sqrt{gd_b}$$

or using equation (1)

$$= 6.42 \sqrt{H_b} \text{ (feet per second)} \tag{3}$$

If h^* is set equal to the depth under the crest of the breaking wave, and the depth is properly corrected for the depression of the trough below mean water level, the result is

$$C_b = 8.02 \sqrt{H_b} \text{ (feet per second)} \tag{4}$$

If h^* is set equal to $H_b + d_b$ without correcting for depression of trough below mean water level, the speed will be

$$C_b = 8.57 \sqrt{H_b} \text{ (feet per second)} \tag{5}$$

Since wave speed enters as a linear multiplier in the equation for energy flux, the estimated energy flux will vary directly as the estimated wave speed. The field data plotted on Figure 4-37, from which the relation between Q and $P_{\ell s}$ is obtained, include estimates of $P_{\ell s}$ using equations (3) and (5). The nine data points from Watts (1953) and Caldwell (1956) have energy flux values computed with equation (3). The 14 data points from Komar (1969) include energy flux estimates based on both equations (3) and (5). Equation (4) is used in the derivations for all $P_{\ell s}$ equations, as shown in Appendix A.

Since the $P_{\ell s}$ equations used in the SPM depend on equation (4) for wave speed, it is evident that wave speed has not been treated consistently throughout the analysis. The $P_{\ell s}$ computed with the equations in the SPM (depending on eq. 4 for wave speed) will be 25 percent too high for the plotted relation between Q and $P_{\ell s}$ for the points of Watts (1953) and Caldwell (1956) on Figure 4-37.

The error between the SPM $P_{\ell s}$ equations and the 14 data points of Komar (1969) is more difficult to evaluate. The depth used by Komar is the 20-minute time average at the wave gage (P.D. Komar, Oregon State University, personal communication, 1978). This depth was often significantly deeper than the breaker depth, as can be judged from the fact that the average crest angle at the wave gage was about 57 percent greater than the average breaker angle in Komar's data (App. IV of Komar, 1969). Thus, the use of the depth at the gage raises the speed above the linear theory estimate of equation (3) toward the value of equation (4). Since it is not easy to evaluate this effect, and since the relation between $P_{\ell s}$ and Q is heavily dependent on the 14 plotted points of Komar, the error due to using equation (4) for breaker speed in calculating $P_{\ell s}$ is concluded to be less than 25 percent and may be negligible.

Moreover, the Komar (1969) data for energy flux probably include an overestimate of the breaker angle. This is because breaker angle, α_b, was computed from the angle at the sensor, α, by

$$\sin \alpha_b = \frac{C_b}{C} \sin \alpha$$

where C_b was effectively equation (5) and C was equation (3). Since equation (5) probably overestimates wave speed (by about 7 percent) and equation (3) underestimates it (by about 10 percent), the resulting breaker angle probably should be multiplied by (1 - 0.10)/(1 + 0.07) or 0.84 to be theoretically correct.

The net result of the variable estimates of wave speed and breaker angle is to suggest that equation (4) is a logical compromise, and this is what is used in the SPM equations.

IV. SUMMARY

This report describes the energy flux method of estimating longshore transport rate and provides detailed explanations of the three most frequently asked questions about this method (see Apps. A, B, and C). The following general conclusions result from this study.

1. Energy flux may be estimated by four separate methods, depending on the available field data. The results show that the energy flux factor, $P_{\ell s}$, is proportional to any one of the following groups of wave variables (App. A):

   (a) $H_b^{5/2} \sin 2\alpha_b$

   (b) $H_o^{5/2} (\cos \alpha_o)^{1/4} \sin 2\alpha_o$

   (c) $H^2 T \sin \alpha_b \cos \alpha_o$

   (d) $\left(\frac{H_b^3}{T}\right) \sin \alpha_o$

2. The wave height used in these equations is the significant wave height (App. B).

3. Longshore transport rate, Q, is directly proportional to the energy flux factor, $P_{\ell s}$. The indicated proportionality constant is 7,500 for traditional units (Q in cubic yards per year and $P_{\ell s}$ in foot-pounds per second per foot). Appendix C describes the data used to derive this constant.

4. There is uncertainty in the proportionality constant because of variations in field data, in the equation for breaker speed, and in measurement of breaker angle. An intuitive estimate of the uncertainty in the constant is ± 40 percent.

5. The energy flux method is expected to be improved with further knowledge.

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APPENDIX A

DERIVATION OF LONGSHORE ENERGY FLUX FACTOR

This appendix derives four formulas for longshore energy flux, P_ℓ, and four formulas for corresponding approximations, the longshore energy flux factors, $P_{\ell s}$. These eight formulas are given in the SPM in Tables 4-7 and 4-8, based on assumptions summarized in Table 4-9. For convenience, Tables 4-7 and 4-8 are reproduced here as Figure A-1 (p. 4-97 of the SPM).

The basic longshore energy flux derivation is given in Galvin and Vitale (1977), who made use of earlier work by Walton (1972). The derivation also benefits from Longuet-Higgins' (1970) work on conservation of momentum flux in the surf zone. Equations well known from linear wave theory are presented without derivation but are keyed to other chapters in the SPM, or to the development in Wiegel (1964) where more detailed derivation is provided.

1. Equations for P_ℓ.

The derivation for P_ℓ proceeds as follows: Assume a coast with contours that are parallel to a straight shoreline (Fig. A-2). Waves approaching this coast are assumed to be described by linear small-amplitude theory. In general, a wave crest that makes an angle α_o with the shoreline when in deep water will refract to make an angle α_i at some shallower depth (Fig. A-2), where α_i is related to α_o by Snell's law. In what follows, the subscript o refers to deepwater conditions, and the symbols are those used in the SPM.

The path of a wave passing through point i is shown on Figure A-2 as the dashed orthogonal labeled "wave path." The flux of energy in the direction of wave travel *per unit length of wave crest* at point i is given by

$$P^* = C_g \overline{E}$$

$$= n\, C\, \overline{E} \qquad \text{(A-1)}$$

where C_g is the wave group velocity, C is the wave phase velocity, $n = C_g/C$, and $\overline{E}$ is the energy density, the total average energy per unit area of sea surface, and is defined as (eq. 2-39 in the SPM)

$$\overline{E} = \frac{wH^2}{8} \qquad \text{(A-2)}$$

where w is the weight density of water, 64.0 pounds per cubic foot for seawater, and H is the wave height. (This wave height is the height of a uniform periodic wave. How it relates to the significant wave height and to wave heights characterizing wave height distributions is discussed in App. B.)

From equation (A-1), the energy flux in the direction of wave travel, at point i, for crest length, b_i, is (see eq. 7 in Galvin and Vitale, 1976)

$$P^*\, b_i = \overline{E}\, C_g\, b_i \qquad \text{(A-3)}$$

Table 4-7. Longshore Energy Flux, P_ℓ, for a Single Periodic Wave in Any Specified Depth. (Four Equivalent Expressions from Small-Amplitude Theory)

Equation	P_ℓ (energy/time/distance)	Data Required (any consistent units)
4–31	$2C_g\ (\frac{1}{4}\ \overline{E} \sin 2\alpha)$	d, T, H, α_o
4–32	$C\ (\frac{1}{4}\ \overline{E}_o \sin 2\alpha_o)$	d, T, H_o, α_o
4–33	$K_R^2 C_o\ (\frac{1}{4}\ \overline{E}_o \sin 2\alpha)$	T, H_o, α_o, α
4–34	$(2C)\ (K_R^2\ C_o)^{-1}\ C_g\ (\frac{1}{4}\ \overline{E} \sin 2\alpha_o)$	$d, T, H, \alpha_o, \alpha$

no subscript indicates a variable at the specified depth where small-amplitude theory is valid

C_g = group velocity (see assumption 1b, Table 4-9)

C_o = deepwater

d = water depth

H = significant wave height

T = wave period

α = angle between wave crest and shoreline

K_R = refraction coefficient $\sqrt{\frac{\cos \alpha_o}{\cos \alpha}}$

Table 4-8. Approximate Formulas for Computing Longshore Energy Flux Factor, $P_{\ell s}$, Entering the Surf Zone

Equation	$P_{\ell s}$ (ft.-lbs./sec./ft. of beach front)	Data Required (ft.-sec. units)
4–35	$32.1\ H_b^{5/2} \sin 2\alpha_b$	H_b, α_b
4–36	$18.3\ H_o^{5/2}\ (\cos \alpha_o)^{1/4} \sin 2\alpha_o$	H_o, α_o
4–37	$20.5\ T\ H_o^2 \sin \alpha_b \cos \alpha_o$	$T, H_o, \alpha_o, \alpha_b$
4–38	$100.6\ (H_b^3/T) \sin \alpha_o$	T, H_b, α_o

H_o = deepwater

H_b = breaker position

H = significant wave height

T = wave period

α = angle between wave crest and shoreline

See Table 4-7 for equivalent small amplitude equations and Table 4-9 for assumptions used in deriving $P_{\ell s}$ from P_ℓ.

4-97

Figure A-1. Page 4-97 of the SPM, showing four formulas for longshore energy flux, P_ℓ, and four formulas for longshore energy flux factors, $P_{\ell s}$.

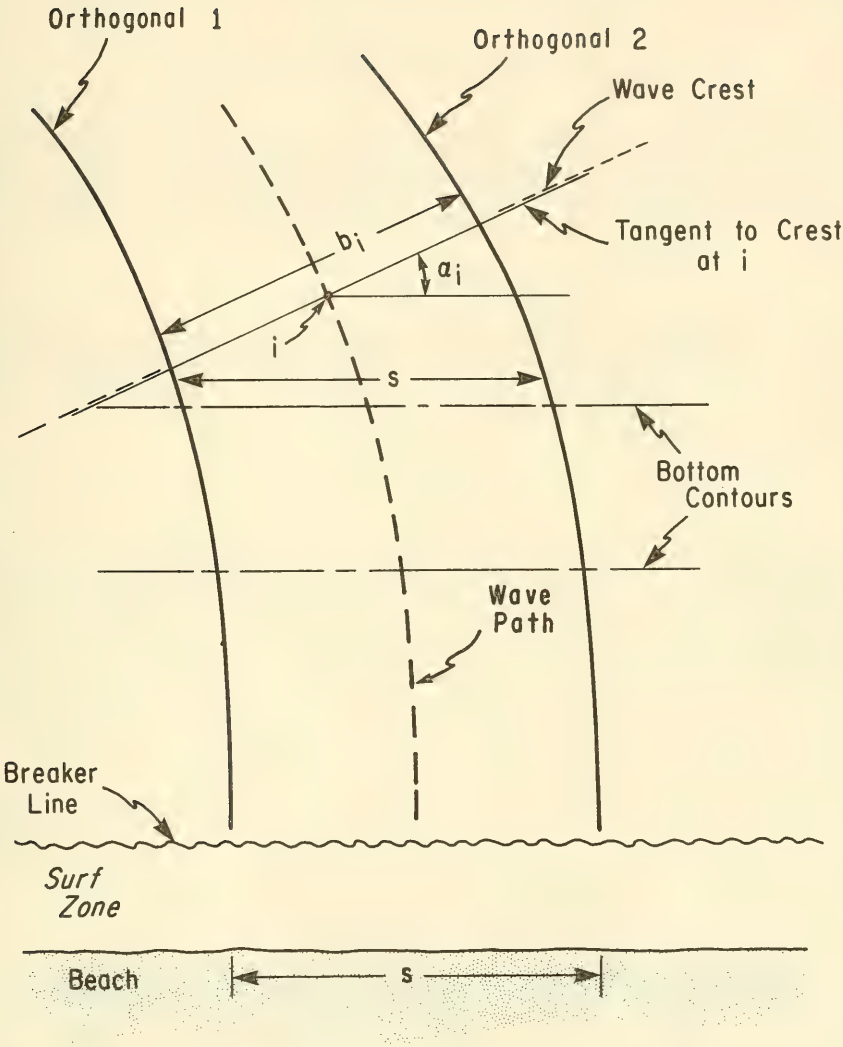

Figure A-2. Definitions for conservation of energy flux for shoaling wave (after Walton, 1972).

In the small-amplitude theory assumed, energy is not dissipated and does not cross wave orthogonals. Therefore, the energy flux in the direction of wave travel must remain constant between orthogonals, i.e., between deep water and point i.

$$P_o^* b_o = P_i^* b_i = \text{total wave power between orthogonals} \tag{A-4}$$

From the geometry of Figure A-2, it is obvious that b_i changes with position on the wave path. However, the distance between adjacent orthogonals, s, measured in the longshore direction, does not change because orthogonal 2 must be identical to orthogonal 1 in all respects except being displaced along the coast by a distance s. Therefore, at any point, i, in the wave path

$$b_i = s \cos \alpha_i \tag{A-5}$$

For the straight parallel contours assumed, the distance s is arbitrary. Therefore, divide both sides of equation (A-4) by s, and set s equal to the unit of distance. In the SPM, this unit is 1 foot; in metric units, it would be 1 meter. This procedure defines a total wave power, per unit length of shoreline, indicated by P

$$P = P^* \cos \alpha_i \tag{A-6}$$

In the Appendix, i indicates any point on the path where equations (A-1) to (A-5) are valid. With this understanding, the subscript i is dropped in the derivation below. Since P has both a magnitude and a direction at any point, a longshore component, P_ℓ, can be defined. Using equations (A-1) and (A-6), this longshore component of energy flux can be written

$$P_\ell = (\bar{E} C_g \cos \alpha) \sin \alpha \tag{A-7}$$

By use of the identity

$$\sin 2\alpha = 2 \cos \alpha \sin \alpha \tag{A-8}$$

equation (A-7) can be written as the first of the four alternate forms for P_ℓ given in Table 4-7 of the SPM, i.e., equation (4-31) in the SPM:

$$P_\ell = 2\, C_g \left(\frac{1}{4} \bar{E} \sin 2\alpha \right) \tag{A-9}$$

The equation is given in this form so that the term in parentheses corresponds to the longshore force term used in Longuet-Higgins (1970). The energy density, $\bar{E}$, is proportional to the local height squared, H^2, (eq. A-2). The local height can be related to the deepwater height, H_o, by equations (7.8) and (7.9) in Wiegel (1964)

$$H = K_R K_s H_o \tag{A-10}$$

where K_R is the refraction coefficient

$$K_R = \left(\frac{\cos \alpha_o}{\cos \alpha}\right)^{1/2} \tag{A-11}$$

and K_s is the shoaling coefficient

$$K_s = \left(\frac{C_{go}}{C_g}\right)^{1/2} \tag{A-12}$$

K_s for later use may also be approximated by the breaker height index

$$K_s \sim \frac{H_b}{H_o} \tag{A-13}$$

where H_b is the wave height at the point of breaking. Thus,

$$\begin{aligned} \overline{E} &= K_R^2 \, K_s^2 \, \overline{E}_o \\ &= \left(\frac{\cos \alpha_o}{\cos \alpha}\right)\left(\frac{C_{go}}{C_g}\right) \overline{E}_o \end{aligned} \tag{A-14}$$

so P_ℓ becomes

$$P_\ell = K_R^2 \, K_s^2 \, C_g \, \overline{E}_o \sin \alpha \cos \alpha \tag{A-15}$$

By substituting equations (A-11) and (A-12) into equation (A-15) and canceling like terms,

$$P_\ell = C_{go} \, \overline{E}_o \cos \alpha_o \sin \alpha \tag{A-16}$$

Equation 2.68 of Wiegel (1964) gives that

$$n = \frac{C_g}{C} = \frac{1}{2}\left[1 + \frac{4\pi \frac{d}{L}}{\sinh\left(4\pi \frac{d}{L}\right)}\right] \tag{A-17}$$

where d is the water depth. To a good approximation, n is equal to 0.5 in deep water, i.e., $d/L > 0.5$, and n is equal to 1.0 in shallow water, i.e., $d/L < 0.04$. (Exact values of n at these limits are 0.5117 at d/L equal to 0.5, and 0.9795 at d/L equal to 0.04.) Equation (A-16) can be further modified using Snell's law (where C is the local wave speed given by equation (2-3) in the SPM),

$$\sin \alpha = \frac{C \sin \alpha_o}{C_o} \tag{A-18}$$

and equations (A-8) and (A-17) to get the second of the equivalent forms for P_ℓ given in Table 4-7 of the SPM, i.e., equation (4-32) in the SPM:

$$P_\ell = C\left(\frac{1}{4}\,\overline{E}_o \sin 2\alpha_o\right) \tag{A-19}$$

The third equivalent form for P_ℓ is equation (4-33) in Table 4-7 of the SPM. It is obtained from equation (A-15) by using equations (A-12) and (A-8),

$$P_\ell = K_R^2\, C_o\left(\frac{1}{4}\,\overline{E}_o \sin 2\alpha\right) \tag{A-20}$$

The fourth and last of the equivalent forms of P_ℓ is equation (4-34) in Table 4-7 of the SPM. It is obtained by substituting equations (A-8), (A-11), and (A-18) into equation (A-9),

$$P_\ell = \left(\frac{2}{K_R^2}\right)\left(\frac{C}{C_o}\right) C_g\left(\frac{1}{4}\,\overline{E} \sin 2\alpha_o\right) \tag{A-21}$$

2. Equations for $P_{\ell s}$.

Up to this point, all results are for small-amplitude linear theory. However, the assumed relation between longshore transport and energy flux in the surf zone requires that P_ℓ be evaluated at the breaker line where small-amplitude theory is less valid. To indicate approximations for waves entering the surf zone, the symbol $P_{\ell s}$ will be used in place of P_ℓ. This approximation is called the energy flux factor, $P_{\ell s}$, in the SPM, and like P_ℓ, it is measured in units of energy per second per unit length of shoreline; e.g., foot-pounds per second per foot. One expression for $P_{\ell s}$ will be derived from each of the four equivalent expressions for P_ℓ (eqs. A-9, A-19, A-20, and A-21) to obtain the four equations in Table 4-8 of the SPM.

The energy density appears in all four equations for P_ℓ. In foot-pound-second units, and for saltwater (w = 64 pounds per cubic foot), the energy density is, from equation (A-2),

$$\overline{E} = \frac{wH^2}{8}$$

$$\simeq 8\,H^2 \text{ (foot-pounds per square foot)} \tag{A-22}$$

In shallow water, group velocity equals wave speed, and near breaking, wave speed depends on depth measured from the crest elevation, as in solitary wave theory (Section 2.27 in the SPM).

The equation for wave speed near breaking, C_b, is an approximation. Several approximations are possible, but the solitary wave approximation used in obtaining the SPM equations for $P_{\ell s}$ is as follows (symbols defined in Fig. A-3). The first approximation for the speed of the solitary wave is

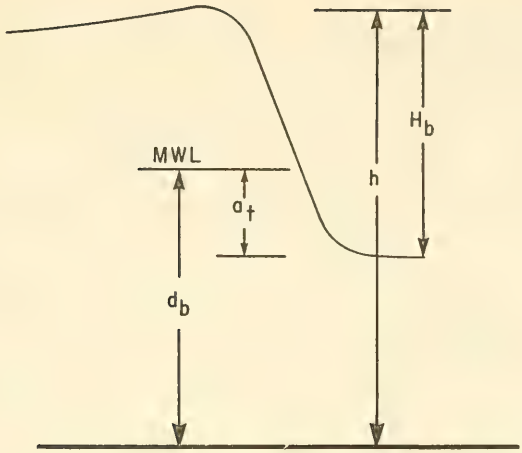

Figure A-3. Definition of terms for breaker speed equation.

$$C_b = \sqrt{gh} \tag{A-23}$$

where

$$h = H_b + d_b - a_t \tag{A-24}$$

By dividing with H_b, the wave speed becomes

$$C_b = K\sqrt{H_b} \tag{A-25}$$

where K is a dimensional term depending on two ratios and g (see eq. 11 in Galvin, 1969)

$$K = \sqrt{g(1 + \beta - \sigma)} \tag{A-26}$$

The depth-to-height ratio, β, was assumed equivalent to the usual coastal engineering rule-of-thumb ($\beta = 1.28$). The relative depression of the trough, σ, was known to vary from 0.15 to 0.40, and a standard value of 0.28 was used for σ giving a wave speed of (eq. 4)

$$C_b = 8.02\sqrt{H_b} \text{ (feet per second)}$$

It should be noted that there is recent evidence that both β and σ should have somewhat lower values than those used to obtain equation (4) but the net result of the reductions does not significantly change the coefficient in equation (4).

A third approximation is the replacement

$$\alpha = \alpha_b \tag{A-27}$$

where the subscript b refers to breaker zone conditions.

Substituting the approximations represented by equations (4), (A-22), and (A-27) into equation (A-9) yields $P_{\ell s}$ approximations for each of the four equations for P_ℓ in Table 4-7.

Equation (4-31) for P_ℓ in Table 4-7 of the SPM is approximated by

$$P_{\ell s} \approx 2(8.02\ H_b^{1/2})\ \frac{1}{4}\ (8\ H_b^2)\ \sin 2\alpha_b$$

$$= 32.1\ H_b^{5/2}\ \sin 2\alpha_b \text{ (foot-pounds per second per foot)} \qquad \text{(A-28)}$$

which involves only height and direction at the breaker. This is equation (4-35) of Table 4-8 in the SPM.

In the same way, equation (4-32) for P_ℓ (eq. A-19) can be reduced to

$$P_{\ell s} = 16.0\ H_b^{1/2}\ H_o^2\ \sin 2\alpha_o \text{ (foot-pounds per second per foot)} \qquad \text{(A-29)}$$

H_b is put in terms of H_o using equations (A-10) and (A-11) to get

$$H_b^{1/2} = \left(\frac{\cos\ \alpha_o}{\cos\ \alpha_b}\right)^{1/4} (K_s H_o)^{1/2} \qquad \text{(A-30)}$$

In this equation, $\cos\ \alpha_b$ equals 1.0 to a good approximation. For example, even if α_b has a high value of 20° (exceeded less than 5 percent of the time on straight beaches), $(\cos\ 20°)^{1/4} = 0.98$.

So, using this approximation and substituting equation (A-30) into equation (A-29),

$$P_{\ell s} = 16.0\ H_o^{5/2}\ (\cos\ \alpha_o)^{1/4}\ K_s^{1/2}\ \sin 2\alpha_o \text{ (foot-pounds per second per foot)} \qquad \text{(A-31)}$$

The shoaling coefficient, K_s, is approximated by the breaker height index. H_b/H_o', obtained from the experimental work of Iversen (1952). Related data are in Figure 2-65 of the SPM. H_o' is the unrefracted deepwater water height. If the slope and period are known, a steepness can be computed, and H_b/H_o' obtained from these experimental results. However, the period in offshore wave statistics correlates poorly with the littoral wave period (Harris, 1972). A reasonable approximation without using the period is obtained by observing that H_b/H_o' ranges from 0.95 to 1.7 for plunging and spilling waves, a center range of about 1.3 for expectable slopes and moderately steep waves. Therefore, K_s is assumed here to be 1.3. Since Q depends on the square root of K_s, this assumed value will usually give Q to within 10 percent of the value from Iversen's data when steepness and slope information is available.

This approximation reduces equation (A-31) to a convenient approximation involving only deepwater height and direction, which is equation (4-36) in the SPM:

$$P_{\ell s} = 18.3\ H_o^{5/2}\ (\cos\ \alpha_o)^{1/4}\ \sin 2\alpha_o \text{ (foot-pounds per second per foot)} \qquad \text{(A-32)}$$

The deepwater wave velocity, C_o, can be approximated by (see eq. 2.37 in Wiegel, 1964)

$$C_o = \frac{gT}{2\pi} = 5.12\ T \text{ (feet per second)} \tag{A-33}$$

Using this along with equations (A-8), (A-11), and (A-22), the third expression for $P_{\ell s}$ can be obtained from equation (A-20):

$$P_{\ell s} \approx \left(\frac{\cos\ \alpha_o}{\cos\ \alpha_b}\right) (5.12\ T)\ \frac{1}{4}\ (8\ H_o^2)(2\ \cos\ \alpha_b\ \sin\ \alpha_b)$$

$$= 20.5\ T\ H_o^2\ \cos\ \alpha_o\ \sin\ \alpha_b \text{ (foot-pounds per second per foot)} \tag{A-34}$$

This is equation (4-37) in the SPM.

The final equation for P_ℓ, equation (A-21), reduces to

$$P_{\ell s} \approx 2(8.02\ H_b^{1/2}) \left(\frac{\cos\ \alpha_o}{\cos\ \alpha}\ 5.12\ T\right)^{-1} (8.02\ H_b^{1/2})\ (8\ H_b^2) \left(\frac{1}{2}\ \cos\ \alpha_o\ \sin\ \alpha_o\right)$$

$$= 100.5 \left(\frac{H_b^3}{T}\right) \cos\ \alpha_b\ \sin\ \alpha_o \text{ (foot-pounds per second per foot)} \tag{A-35}$$

by use of equations (4), (A-8), (A-11), (A-22), and (A-33). The $\cos\ \alpha_b$ term in many cases, is close enough to 1.0 to be ignored. Therefore, equation (A-35) can be further reduced to:

$$P_{\ell s} = 100.5 \left(\frac{H_b^3}{T}\right) \sin\ \alpha_o \text{ (foot-pounds per second per foot)} \tag{A-36}$$

This is equation (4-38) in the SPM.

The four approximate equations for $P_{\ell s}$ (eqs. A-28, A-32, A-34, and A-36) are given in Table 4-8 of the SPM. Unlike the four exact equations (P_ℓ in Table 4-7), these approximations are not equivalent, due to the different assumptions made in deriving them. Trial solutions of all four approximations suggest that they give values of $P_{\ell s}$ that agree within a factor of 2. Figures 4-36 and 4-37 in the SPM show that variation of $P_{\ell s}$ by a factor of 2 is within the scatter of the data defining the dependence of Q on $P_{\ell s}$.

All results in this appendix, including the $P_{\ell s}$ approximations, are for a coast with straight, parallel (but not necessarily evenly spaced) contours. All results assume no friction loss, which may become an important omission for waves traveling long distances over shallow, flat slopes (Bretschneider and Reid, 1954; Walton, 1972). Additional assumptions used in the derivations are summarized in Table 4-9 of the SPM. The essentially approximate basis for the longshore transport prediction must be recognized when the energy flux method is used. The more that the prototype conditions agree with the conditions

assumed in these derivations, the greater the confidence in the resulting estimate.

3. Summary.

This appendix provides the derivation for eight equations given in the SPM. These eight equations are the four equivalent equations for the longshore energy flux, P_ℓ (Table 4-7), and the four nonequivalent equations for the longshore energy flux factor, $P_{\ell s}$ (Table 4-8) given in the SPM. The correspondence between equation numbers in this appendix and the SPM is as follows:

P_ℓ in SPM Table 4-7		$P_{\ell s}$ in SPM Table 4-8	
SPM	This Appendix	SPM	This Appendix
4-31	A-9	4-35	A-28
4-32	A-19	4-36	A-32
4-33	A-20	4-37	A-34
4-34	A-21	4-38	A-36

Tables 4-7 and 4-8 of the SPM are reproduced as Figure A-1 on page 16 of this Appendix.

APPENDIX B

DISTINCTION BETWEEN SIGNIFICANT AND ROOT-MEAN-SQUARE WAVE HEIGHTS IN PREDICTING LONGSHORE TRANSPORT RATES

The four equivalent expressions for longshore energy flux, P_ℓ, given in Table 4-7 of the SPM are developed from small-amplitude linear theory (see App. A), and each expression for P_ℓ contains the wave energy density, $\overline{E}$, as a linear factor. The energy density per wave is

$$\overline{E} = \frac{w}{8} H^2 \tag{B-1}$$

where w is the weight density of seawater, and H is the wave height measured from crest to trough (see Fig. B-1). For a group of waves, N in number, each with wave height H_i, for $i = 1$ to N, the average energy density is given by

$$(\overline{E})_{avg} = \frac{1}{N} \sum_{i=1}^{N} \frac{w}{8} H_i^2$$

$$= \frac{w}{8} (H_{rms})^2 \tag{B-2}$$

where H_{rms} is the root-mean-square (rms) wave height given by

$$(H_{rms})^2 = \frac{1}{N} \sum_{i=1}^{N} H_i^2 \tag{B-3}$$

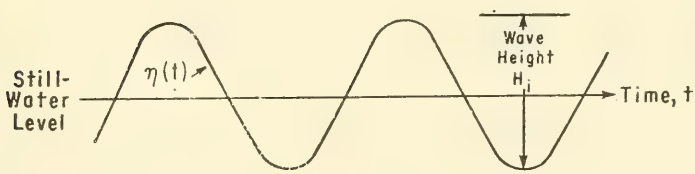

Figure B-1. Definition of wave height and amplitude for simple sinusoidal wave function.

Thus, the rms wave height is the proper wave height to use in evaluating $\overline{E}$ and hence P_ℓ. In conditions where a uniform train of periodic waves exists, approximated in some laboratory experiments, all of the H_i are equal. Thus, by equation (B-3), H_i is identical to H_{rms}. In such cases H_i is interchangeable with H_{rms} in equation (B-2). In natural conditions, however, such as ocean waves approaching a shore, the wave heights are usually not uniform. In such cases, the entire distribution of wave heights to determine H_{rms} must be considered or some other type of average wave height computed. It has been the general practice of coastal engineers to use an average wave height called the significant wave height, H_s, assumed equivalent to the mean of the highest

one-third of the wave heights in the distribution. This value was selected since it appeared to approximate the results of wave heights as reported by experienced observers making visual estimates of wave heights in the ocean (Sverdrup and Munk, 1947). In addition, since from equation (B-1), $\overline{E}$ is proportional to H^2, most of the energy is carried by the higher waves. So H_s is a better representative wave height than the mean wave height in most coastal engineering problems.

However, the significant wave height, H_s, does not equal the rms wave height, H_{rms}, so H_s cannot be used to evaluate $\overline{E}$ or P_ℓ directly. Thus, the relation between H_s and H_{rms} must be determined. This can be done for restrictive conditions following Longuet-Higgens' (1952) work (see also Kinsman, 1965 (Sec. 3.4) and Harris, 1973).

To develop a relation between H_s and H_{rms}, it is assumed that the total wave energy density, $\overline{E}$, as given in equation (B-1), depends on the potential and kinetic energy densities, $\overline{E}_P$ and $\overline{E}_K$, respectively, both given by

$$\overline{E}_P = \overline{E}_K = \frac{w}{16} H^2 \tag{B-4}$$

where $H = H_{rms}$. For this to hold, the following conditions must occur:

(a) The waves are linear and have small amplitudes.

(b) All waves of a single frequency arrive from the same direction.

(Multidirectional waves will not seriously affect the result as long as waves at a given frequency do not come from more than one direction.)

The wave heights will form a Rayleigh distribution if the following conditions are also assumed:

(a) The wave spectrum contains a single, narrow band of frequencies.

(b) The wave energy comes from a large number of different sources of random phase.

Let $\eta(t)$, the departure of the water surface from the mean sea level with respect to time, t, be expressed as the Fourier integral

$$\eta(t) = \int_{-\infty}^{\infty} A(\omega)\, e^{i\omega t}\, d\omega \tag{B-5}$$

where $A(\omega)$ is the spectrum function (possibly complex though only the real part of the integral in eq. B-5 is taken) of the amplitude of $\eta(t)$ with respect to the frequency ω. At any time, twice the amplitude equals the wave height, H_i, measured from trough to crest (see Fig. B-1).

If the spectrum function, $A(\omega)$, is appreciable only within a single, narrow frequency band of wavelength $2\pi/\omega_o$ around frequency ω_o, then

$$\eta(t) = e^{i\omega_o t} \int_{-\infty}^{\infty} A(\omega)\, e^{i(\omega-\omega_o)\, t}\, d\omega$$

$$= e^{i\omega_o t}\, B(t) \qquad \text{(B-6)}$$

Here, the factor $e^{i\omega_o t}$ represents a carrier wave of wavelength $2\pi/\omega_o$ and the integral $B(t)$ represents an envelope function around the carrier wave (see Fig. B-2). The values of the amplitudes at the maximums and minimums of $\eta(t)$ are approximately equal to the values of the envelope function at those points. Thus, the probability distribution of the amplitudes, and hence of the wave heights, is the same as the Rayleigh probability distribution of the values of $B(t)$. Therefore, the probability, $P(H)$, that the wave height H is between H and $H + dH$ is given by

$$P(H)\, dH = -d\left[e^{-(H/H_{rms})^2}\right]$$

$$= \left(\frac{2H}{H_{rms}^2}\right) e^{-(H/H_{rms})^2}\, dH \qquad \text{(B-7)}$$

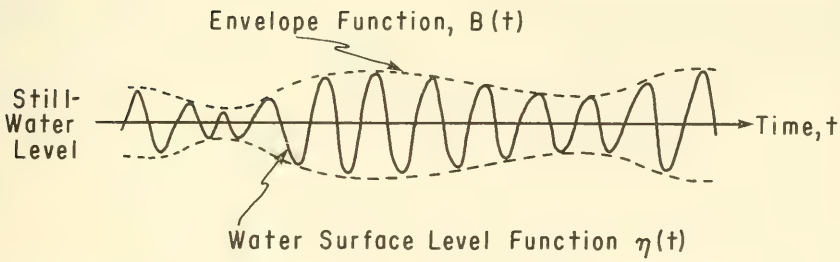

Figure B-2. Definition of envelope wave function for $\eta(t)$ with single narrow frequency band.

For N waves the fraction, p, of the wave heights which are larger than a given wave height, H, is equal to the probability that a wave height will exceed H and is given by

$$p = \int_H^{\infty} P(H)\, dH$$

$$= e^{-(H/H_{rms})^2} \qquad \text{(B-8)}$$

Thus,

$$H = H_{rms} \left[\ell n \left(\frac{1}{p} \right) \right]^{1/2} \tag{B-9}$$

The mean value, H_p, of the wave heights larger than H is, using equations (B-7) and (B-8),

$$H_p = \frac{1}{p} \int_H^\infty HP(H) \; dH$$

$$= -e^{(H/H_{rms})^2} \int_H^\infty H d\left[e^{-(H/H_{rms})^2} \right] \tag{B-10}$$

Integrating by parts, substituting equation (B-9), and dividing through by H_{rms} give

$$\frac{H_p}{H_{rms}} = \frac{H}{H_{rms}} + \left(\frac{1}{H_{rms}} \right) e^{(H/H_{rms})^2} \int_H^\infty e^{-(H/H_{rms})^2} \; dH$$

$$= \left[\ell n \left(\frac{1}{p} \right) \right]^{1/2} + \frac{1}{p} \int_a^\infty e^{-x^2} \; dx$$

$$= \left[\ell n \left(\frac{1}{p} \right) \right]^{1/2} + \frac{1\sqrt{\pi}}{2p} \left[1 - \frac{2}{\sqrt{\pi}} \int_o^a e^{-x^2} \; dx \right] \tag{B-11}$$

where $a = [\ell n(1/p)]^{1/2}$ and $x = H/H_{rms}$. The ratio of the significant wave height, H_s, to the rms height is found by letting $p = 1/3$ (from the definition of H_s), which gives

$$\frac{H_s}{H_{rms}} = 1.416$$

$$\simeq \sqrt{2} \tag{B-12}$$

since $\sqrt{2} = 1.414$. Thus,

$$H_s^2 \simeq 2 \; H_{rms}^2 \tag{B-13}$$

From equation (B-2), $\overline{E} \propto H^2$ and from equation (A-9) of Appendix A, $P_\ell \propto \overline{E}$. This means that if the significant wave height is used to compute $\overline{E}$ or P_ℓ, the result will be approximately twice what it should be using the rms wave height.

Since coastal engineers are more familiar with H_s than H_{rms}, all longshore transport predictions in the SPM are designed so that H_s is used when the equation requires a wave height. (To emphasize that the resulting $P_{\ell s}$ is approximately twice the theoretical longshore energy flux, $P_{\ell s}$ is called the "longshore energy flux *factor*" in the SPM.) The design equations in the SPM

have been "calibrated" for use with H_s by plotting the field data in Figure 4-37 of the SPM using H_s to calculate $P_{\ell s}$. The resulting line in Figure 4-37 yields equation 4-40 of the SPM:

$$Q = (7.5 \times 10^3)\ P_{\ell s} \tag{B-14}$$

The 23 field data points used to determine equation (B-14) are shown in Figure 4-37 of the SPM. Nine data points of Watts (1953) and Caldwell (1956) (one of Caldwell's data points is missing from Fig. 4-37 since Q and $P_{\ell s}$ had opposite signs; see App. C) were reported in terms of significant wave height. Thus, the values for $P_{\ell s}$ are taken directly from these reports. Similarly, the one data point of Moore and Cole (1960) (not shown in Fig. 4-37 since it plots off the scale) is assumed to be in terms of significant wave height, although this is not stated in their report. The 14 data points of Komar (1969), however, are reported in terms of rms wave height and so his values for $P_{\ell s}$ are multiplied by a factor of 2 before being used in Figure 4-37.

In Figure 4-36 of the SPM, 161 data points from 5 laboratory studies (listed in App. C) are shown in addition to the 25 field data points. In the laboratory studies a train of (relatively) uniform waves was used in each test, so that the wave height measured is approximately equal by definition to the rms wave height. Since the use of the laboratory wave heights gives the theoretical longshore energy flux based on H_{rms}, they were multiplied by a factor of 2 before being used in Figure 4-36 to make them consistent with the energy flux factors plotted for the field data.

Because it is doubtful that the numerous assumptions relating H_s to H_{rms} are all valid at the same time, the basic relation between Q and $P_{\ell s}$ (eq. B-14) might be considered an empirical relation, calibrated for field use with significant wave height data. The engineering application depends on how well the equations for $P_{\ell s}$ predict Q when used in equation (B-14).

APPENDIX C

FIELD AND LABORATORY DATA IN THE SPM ENERGY FLUX DISCUSSION

Figure 4-36 in the SPM is a plot of longshore transport rate, Q, versus longshore energy flux factor, $P_{\ell s}$, for both field and laboratory data. A similar plot of Q versus $P_{\ell s}$ (Fig. 4-37 of the SPM) presents only the field data. The empirical relation,

$$Q = (7.5 \times 10^3)\ P_{\ell s} \tag{C-1}$$

where Q is measured in cubic yards per year and $P_{\ell s}$ is measured in foot-pounds per second per foot of beach front, is a visual fit of the field data in Figure 4-37. It is given as equation (4-40) in the SPM.

The field data are taken from Watts (1953), Caldwell (1956), Moore and Cole (1960), and Komar (1969). The laboratory data are taken from Krumbein (1944), Saville (1950), Shay and Johnson (1951), Sauvage and Vincent (1954), and Fairchild (1970). These data are described and listed in Das (1971) and the derivation of empirical relations of the form of equation (C-1) is summarized in Das (1972).

The field data include 25 data points, although only 23 of them are shown on Figure 4-37 of the SPM. Four data points come from Watts (1953). The longshore transport rate, Q, was measured by surveying the amount of sand pumped into a detention basin by a bypassing plant on the jetty at south Lake Worth inlet in Florida. Surveys were made after every 6 hours of pumping or daily, whichever was more often. Four monthly totals are used. The equivalent longshore energy flux factor, $P_{\ell s}$ (denoted as E_T in Watts' paper), was computed from linear theory

$$P_{\ell s} = \frac{w}{8} LH^2 \left[1 - M\left(\frac{H}{L}\right)^2\right] n \frac{t}{T} \sin\alpha \cos\alpha \tag{C-2}$$

where

w = weight density of seawater

L = wavelength

H = wave height

M = a function of d/L where d is the water depth

T = wave period

α = the angle between the wave crests and the shoreline

t = conversion of seconds to days

n = solution given by equation (2.68) of Wiegel (1964),

$$n = \frac{1}{2}\left[1 + \frac{4\pi \frac{d}{L}}{\sinh\left(\frac{4\pi d}{L}\right)}\right] \tag{C-3}$$

Substituting the value for w, taking the shallow-water approximation that the factor $[1 - M(H/L)^2] \simeq 1$, and evaluating L from linear theory by

$$L = \frac{g}{2\pi} T^2 \tanh\left(\frac{2\pi d}{L}\right)$$

$$L = 5.12\ T^2 \tanh\left(\frac{2\pi d}{L}\right) \qquad \text{(C-4)}$$

equation (C-2) becomes

$$P_{\ell s} = 41\ TH^2\ n \tanh\left(\frac{2\pi d}{L}\right) t \sin\alpha \cos\alpha \quad \text{(foot-pounds per day per foot)} \qquad \text{(C-5)}$$

Note that Watts (1953) computes $P_{\ell s}$ in units of foot-pounds per day per foot of beach front, which is converted in the SPM to units of foot-pounds per second per foot of beach front.

Significant wave height and period were taken from the analysis of the wave records (a 12-minute record every 4 hours) of a pressure gage installed at the seaward end of the Palm Beach pier, located 17.7 kilometers (11 miles) north of south Lake Worth inlet, in approximately 5.2 meters (17 feet) of water. Wave direction was obtained from twice daily observations using an engineer's transit with sighting bar and auxiliary sights from an elevated point 5.6 kilometers (3.5 miles) north of the inlet. No mention is made of how the quantities d and L are evaluated.

There are six data points listed in Caldwell (1956). One of these six data points is not plotted in Figure 4-37 of the SPM. That point represents a condition where the measured longshore transport and the computed longshore energy flux are in opposite directions. The longshore transport rate, Q, was measured by comparing successive sets of surveys of the beach out to the 6.1-meter (20 feet) depth contour at 152.4-meter (500 feet) intervals along the 3.4-kilometer (11,000 feet) study area immediately south of the jetties at Anaheim Bay, California. The longshore component of wave energy flux, $P_{\ell s}$, was computed from equation (C-5) and, as before, converted to units of foot-pounds per second per foot of beach front for use in the SPM. Significant wave height and period were taken from analysis of the wave records of a step-resistance wave gage, supplemented by hindcasting when necessary, and checked by a float-type wave gage. The gages were installed on the seaward end of the Huntington Beach pier, located about 10 kilometers (6 miles) south of Anaheim Bay. Wave direction was obtained from wave hindcasting and wave refraction analysis using synoptic weather charts. Again, no mention is made of how d and L are evaluated.

One data point is taken from an observation in Moore and Cole (1960). The longshore transport rate, Q, was measured by comparing two surveys marking the growth of a spit across the outlet to Tasaychek Lagoon, Alaska. The longshore energy flux, $P_{\ell s}$, was computed by Saville (1962) using the following equation (T. Saville, CERC, personal communication, 1974)

$$P_{\ell s} = \frac{wLH^2}{8} n \frac{t}{T} \sin\alpha \cos\alpha \qquad \text{(C-6)}$$

No mention is made in Moore and Cole (1960) of how or where the wave height, period, and angle between wave crest and beach are measured. Saville assumes that they are deepwater values and the SPM assumes that the wave height is significant wave height. The computed value for $P_{\ell s}$ is larger by nearly an order of magnitude than in the other field data and hence does not plot on the scale of Figure 4-37 of the SPM. It would plot below the line in Figure 4-37 given by equation (C-1).

Fourteen data points are taken from Komar (1969), 10 from work at El Moreno Beach, Mexico, and 4 from Silver Strand Beach, California. The longshore transport rate, Q, was calculated from the movement of fluorescent tracer sand, determined from the amount and location of the tracer sand in regularly spaced core samples taken on the beach about 3 to 4 hours after injection of the tracer. Wave energy flux and wave direction were measured by an array of digital wave sensors (wave staffs and pressure transducers) in the nearshore region. Integrating the energy densities under the spectra peak of the records of these wave sensors gives the mean square elevation of the water surface, $\langle\eta^2\rangle$. The energy of the wave train is then

$$\overline{E} = \rho g \langle\eta^2\rangle \tag{C-7}$$

The wave period of the wave train is at the point of maximum energy density for the particular spectra peak being analyzed. By knowing the water depth, group velocity can be found. The wave energy flux per unit crest length, $\overline{E}C_g$, may now be calculated. Comparing wave records of various gages of the array produces the wave angle, α, which gives

$$\frac{\text{wave energy flux}}{\text{unit beach length}} = \overline{E}C_g \cos\alpha$$

Assuming no energy dissipation until breaking, the energy flux per unit beach length at the breaker zone is

$$(\overline{E}C_g \cos\alpha)_b = \overline{E}C_g \cos\alpha \tag{C-8}$$

Equation (C-8) is then multiplied by the $\sin\alpha_b$, where α_b is measured in the surf zone or calculated from α using Snell's law

$$\sin\alpha_b = \frac{C_b}{C}\sin\alpha \tag{C-9}$$

to give the energy flux factor

$$P_{\ell s} = (\overline{E}C_g \cos\alpha)_b \sin\alpha_b \tag{C-10}$$

In (C-9), C_b is evaluated from phase speed near breaking assumed to be (Komar, 1969)

$$C_b = (2.28\ gH_b)^{1/2} \tag{C-11}$$

Note that Komar uses rms wave height to compute values for $P_{\ell s}$. To make his values consistent with the other field data points in Figure 4-37 of the SPM,

which are computed from significant wave height, Komar's values for $P_{\ell s}$ are multiplied by a factor of 2 before being used in the SPM (see App. B).

Two additional sources of field data are included in Figure 6 of Das (1972), a plot of Q versus $P_{\ell s}$ similar to Figure 4-36 of the SPM. These sources are 5 data points from Johnson (1952) and 14 data points from Thornton (1969). Neither source is included in Figures 4-36 and 4-37 of the SPM nor are they used in the derivation of the empirical relation between Q and $P_{\ell s}$, equation (C-1). Johnson's data do not include wave direction, but the corresponding points must plot above the line given by equation (C-1) in Figure 4-37 for reasons given in Galvin and Vitale (1977). Thornton's data are from bedload traps between the inner and outer bar, and represent minimum values of Q.

The design prediction (eq. C-1) is based solely on field data, but Figure 4-36 does show laboratory data for comparison. Das (1971) lists 177 laboratory data points, and of these, 161 are shown in Figure 4-36 of the SPM. In the two data points from Price and Tomlinson (1968), crushed coal with a specific gravity of 1.35 (as opposed to 2.65 for the quartz that makes up most beach sand) was used. In 14 of the 17 data points from Sauvage and Vincent (1954), sediments with specific gravities of 1.1 and 1.4 were used. Thus, these 16 lightweight data points were not included in Figure 4-36.

The longshore energy flux, $P_{\ell s}$, for the laboratory data was computed, using equation (32) of Das (1972)

$$P_{\ell s} = \frac{1}{2} \frac{\rho g}{8} H_o^2 L_o N K_R^2 \sin \alpha_b \cos \alpha_b \qquad \text{(C-12)}$$

H_o is the deepwater wave height, L_o is the deepwater wavelength, and N is the number of waves per day, given by

$$N = \frac{(86{,}400 \text{ seconds per day})}{T} \qquad \text{(C-13)}$$

where the period, T, is measured in seconds, and K_R is the refraction coefficient. The leading factor of 1/2 in equation (C-12) is the deepwater approximation for n as given in equation (C-3).

The value for H_o comes from

$$H_o = \frac{H'_o}{K_R}$$

$$= \frac{H}{\left(\frac{H}{H'_o}\right) K_R} \qquad \text{(C-14)}$$

where H'_o is the deepwater wave height unaffected by refraction, the ratio H/H'_o is obtained from Wiegel's tables (SPM, App. C, Table C-1), and the

refraction coefficient, K_R, is given by equation (7-12) of Wiegel (1964)

$$K_R = \left(\frac{\cos \alpha_o}{\cos \alpha}\right)^{1/2} \tag{C-15}$$

The depth, d, the period, T, and the angle α are measured in the laboratory studies. The deepwater approximation for the wavelength, $L_o = 5.12\ T^2$, obtains d/L, and hence L, from d/L_o and Wiegel's tables. Then, α_o can be calculated from

$$\sin \alpha_o = \frac{L_o}{L} \sin \alpha \tag{C-16}$$

and α_b can be calculated from

$$\sin \alpha_b = \frac{L_b}{L_o} \sin \alpha_o \tag{C-17}$$

so that equation (C-15) can be evaluated to give the refraction coefficient, K_R, at the breaker position.

To summarize, 23 field data points were used to derive equation (C-1), the relation between Q and $P_{\ell s}$. The values for $P_{\ell s}$ for the nine data points taken from Watts (1953) and Caldwell (1956) were computed from equation (C-2), using values of significant wave height, period, and direction, as estimated by Watts (1953) and Caldwell (1956). The values of $P_{\ell s}$ for the 14 data points taken from Komar (1969) were computed from equation (C-10) whose input comes from the mean square water elevation, $\langle \eta^2 \rangle$, obtained from the wave sensor records, the use of linear theory, and equations (C-11) and (C-9) to get the α_b. It is again emphasized that only field data were used to compute equation (C-1), and the equations for $P_{\ell s}$ require the use of the observed significant height.

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