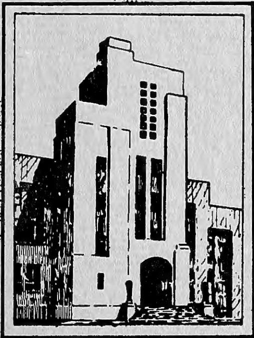

NAVY DEPARTMENT

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STATISTICS ON WAVE HEIGHTS AND PERIODS FOR THE NORTH ATLANTIC OCEAN

by

R.L. Brooks, CDR, USN, and N.H. Jasper, Dr. Eng.

STRUCTURAL MECHANICS LABORATORY

RESEARCH AND DEVELOPMENT REPORT

September 1957

Report 1091

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ABSTRACT

This report gives the frequency distributions of wave heights and wave periods obtained from weather ships stationed in the North Atlantic, together with an evaluation of the reliability of the visual wave-height estimates comprising the basic data from which the distributions are derived. Visual estimates are compared with values determined from stereophotographs. An additional check is provided by wave-meter measurements. It is shown that a log-normal distribution is applicable to the frequency distribution of wave heights experienced over a typical year and that this distribution is a useful guide to the determination of the incidence of a particular sea state at a given location.

INTRODUCTION

The David Taylor Model Basin is conducting a long-range research project[1] to evaluate present methods of ship structural design and to suggest improvements based on a realistic knowledge of the loads, stresses, and motions which ships experience in service. Instrumentation has been developed which measures the responses of ships to wave action in terms of stress, roll and pitch angle, and the corresponding accelerations. A large amount of data has been collected during voyages in the North Atlantic of aircraft carriers, destroyers, destroyer escorts, seaplane tenders, tankers, and dry-cargo ships. Typical of these studies is that conducted on the USCGC UNIMAK.[2,3]

Since the stresses and motions of ships are induced by wave action, these studies have included, as an important component, the problem of defining the surface variation of the sea, i.e., the waves.

It is the purpose of this report to present the frequency distributions of wave heights and wave periods obtained from weather ships stationed in the North Atlantic together with an evaluation of the reliability of the visual wave-height estimates which comprise the basic data. Observations of wave heights and wave periods over a period of about six years have been made available by the U.S. Weather Bureau. In order to gain some idea of the reliability of the visual estimates made by observers, such as the Weather Bureau personnel, a special effort was made by the Taylor Model Basin to obtain stereophotographs of the sea surface at the same time that visual estimates were made. Such comparative data were gathered during extensive sea operations of the USS VALLEY FORGE (CVS 45) and the USCGC UNIMAK (WAVP 379) in 1955. A comparison of the visual estimates with values determined from the stereograms is given in this report. An additional check on the validity of the Weather Bureau data on wave heights is made by comparing them with wave measurements obtained by Darbyshire[4,5] at Ocean Stations I and J.

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[1]References are listed on page 51.

The statistical presentation of wave data given here will show at a glance the probability of exceeding any given sea condition (as specified in terms of a characteristic* wave height) in an average year, for ten representative locations in the Atlantic Ocean. Such information can be utilized in the solution of design and operational problems connected with the strength, speed, and motion of ships at sea and in planning model tests of seaworthiness.

SOURCES OF DATA

WEATHER BUREAU DATA

Wave heights and periods from the Weather Bureau records are presented in Tables 1 and 2. These data are visual estimates and were made every three hours by trained weather observers in accordance with instructions prescribed by the U.S. Weather Bureau.[6] Only one quantitative value for wave height and one for wave period were reported each time the sea was observed. These data cover a period of about six years and were made from weather ships at ten ocean stations, located as shown in Figure 1.

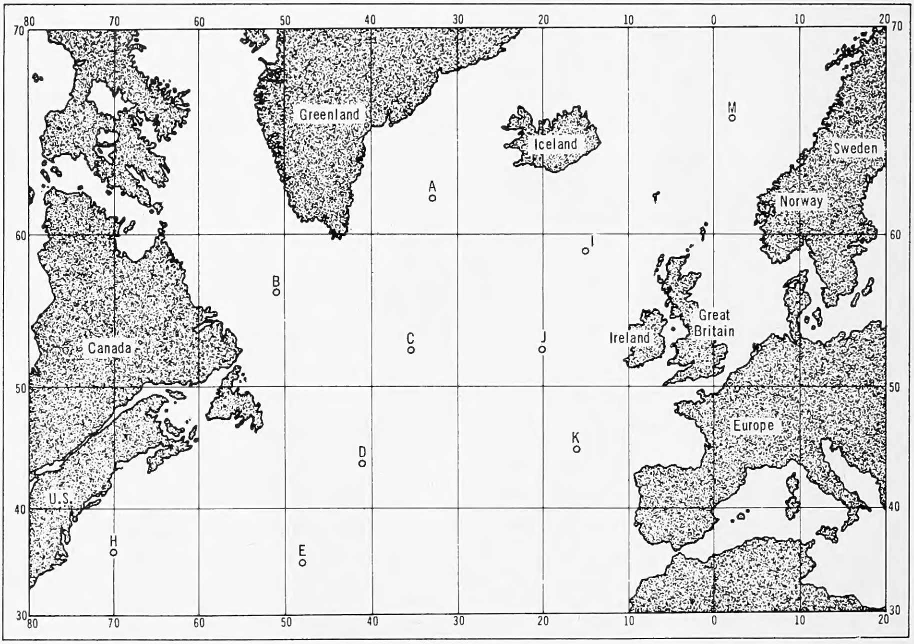

Figure 1 - Locations of Ocean Stations

*The "characteristic" wave height is the average height of the larger well-defined waves. See the discussion under "Sources of Data" for a more specific definition. The term "characteristic" height should be differentiated from "significant" height. The latter term has a precise mathematical definition, the former does not.

TABLE 1

Frequency Distribution of Characteristic Wave Heights Reported by U.S. Weather Bureau

Ocean Station	Period of Record	Wave Heights,* feet										Total Number of Observations
		<1.0 15.6-17.2	1.0-2.5 17.2-18.9	2.5-4.1 18.9-20.5	4.1-5.7 20.5-22.1	5.7-7.4 22.1-23.8	7.4-9.0 23.8-25.4	9.0-10.7 25.4-27.1	10.7-12.3 27.1-28.7	12.3-13.9 28.7-30.3	13.9-15.6 >30.3	
A	1/49- 6/54	103	891	2106	2403	2111	1468	1118	684	382	528	
		186	175	151	134	80	47	66	53	41	164	12,891
B	1/49-12/54	144	939	2601	2507	2484	1800	1469	1005	460	634	
		237	264	286	166	113	125	120	62	30	101	15,547
C	1/49-12/54	105	860	2479	2964	2999	2125	1602	1191	556	600	
		248	226	235	168	94	119	83	76	17	110	16,857
D	1/49-12/54	96	797	2861	3452	2887	1911	1471	1011	496	622	
		281	217	231	124	74	81	55	35	37	65	16,804
E	1/49-12/54	280	2232	4629	3371	2569	1335	926	559	237	260	
		108	85	71	38	29	33	6	2		7	16,777
H	1/49- 6/54	255	1863	3446	2720	2082	1310	1057	654	295	360	
		136	114	112	59	34	43	33	11	5	18	14,607
I	1/47- 6/53	243	37	1256	2295	1806	1508	901	943	565	209	
		667	64	206	209	31	27	187	9	6	105	11,274
J	1/47- 6/53	272	38	1409	2271	1982	1524	877	1028	763	270	
		684	55	189	196	54	55	196	27	21	105	12,016
K	1/49-12/53	144	743	1816	2146	2261	1552	955	401	506	168	
		116	93	89	29	44	16	26	16	26	35	11,182
M	1/49-12/53	89	1632	3644	3357	2466	1485	876	467	217	65	
		15	3	6	1	1	-	-	-	-	-	14,324

*Two wave height ranges are shown at the top of each column. Opposite each weather station, two entries appear in each column. The top entries apply to the top wave height ranges, and the bottom entries apply to the bottom wave height ranges.

TABLE 2

Frequency Distribution of Characteristic Wave Periods Reported by U.S. Weather Bureau

Ocean Station	Period of Record	Wave Periods, seconds										Total Number of Observations
		< 5	5-7	7-9	9-11	11-13	13-15	15-17	17-19	19-21	>21	
A	1/49- 6/54	2592	5331	2928	1037	274	107	49	15	4	5	12,342
B	1/49- 6/55	3655	6896	3918	1159	297	106	19	1	4	5	16,060
C	1/49- 6/55	3192	7346	4721	1620	387	106	26	4	17	52	17,471
D	1/49- 6/55	3478	8174	4006	1230	307	83	14	1	3	14	17,310
E	1/49- 6/55	5573	7091	3009	934	151	42	20	1	13	62	16,896
H	5/49- 6/54	4864	6113	2103	450	58	21	1	3	2	32	13,647
I	1/49-12/51 1/53-12/54	1044	4651	5099	1079	181	34	5	-	2	47	12,142
J	1/49-12/51 1/53-12/54	802	3866	5431	1289	178	17	3	6	1	-	11,593
K	6/49-12/53	391	2219	4754	3528	814	100	43	5	46	6	11,906
M	1/49-12/53	1826	5477	4374	2051	382	69	3	1	4	1	14,188

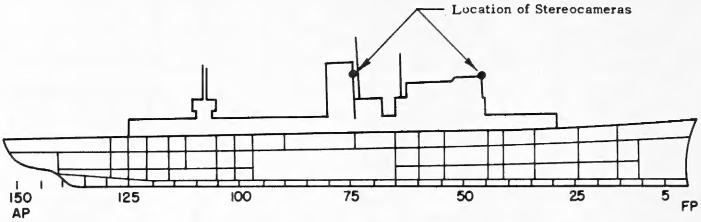

Figure 2 - Inboard Profile of USCGC UNIMAK Showing Location of Stereocameras

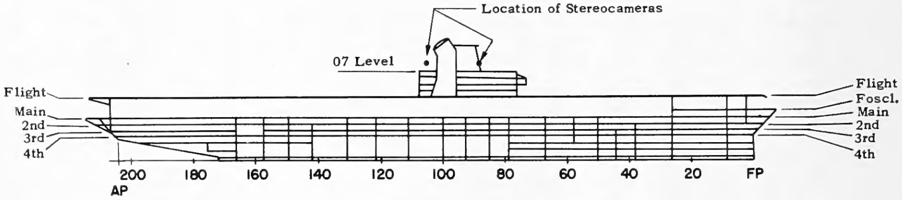

Figure 3 - Inboard Profile of USS VALLEY FORGE (CVS 45) Showing Location of Stereocameras

Basic Characteristics

Midship Section Moment of Inertia	27,502 ft^4
Midship Section Modulus	129,210 ft $in.^2$
Block Coefficient	0.585
Prismatic Coefficient	0.597
Midship Section Area Coefficient	0.980
Waterplane coefficient	0.743

STEREOPHOTOGRAPHS OF SEA SURFACE

In order to evaluate the accuracy of the Weather Bureau wave estimates and their usefulness in the statistical analyses of ship strength, stereocameras were installed on the USCGC UNIMAK (WAVP 379) and the USS VALLEY FORGE (CVS 45). Figure 2 shows the camera locations on the weather ship, and Figure 3 shows those on the aircraft carrier. Table 3 gives identification and installation data for the cameras.

Visual estimates of the sea surface were made by trained observers at the same times that stereophotographs were taken. The UNIMAK estimates were made by regularly assigned Weather Bureau observers. The VALLEY FORGE estimates were made by trained Navy aerological personnel; two or more of them made independent estimates twice each hour during most observation periods. By special arrangement, five or six of the Navy aerological personnel made independent wave estimates three times each day. Subsequently, the VALLEY

TABLE 3

Stereocamera Data

Ship	USS VALLEY FORGE (CVS 45)		USCGC UNIMAK (WAVP 379)	
Type of Camera	Aircraft Precision Mapping Navy Type CA 8		Aerial K 24	
Manufacturer	Fairchild		Eastman Kodak	
Manufacturer Serial Number	292	295	35,540	109,994
Instruction Manual	AN-10-10AC-63 1 Aug 1947 Revised 1 Jun 1953		10-10AB-1 over AP2315A 30 Jul 1943 Revised 30 Oct 1943	
Position Installed on Ship	Forward	Aft	Forward	Aft
Frame	89	105	47	73
Height above Baseline	111 ft 7 in.	111 ft 7 in.	56 ft	55 ft
Angle of View, Horizontal	73° 40′		39° 20′	
Separation	63 ft 11¼ in.		51 ft 9 in.	
Equivalent Focal Length	153.57 mm	152.83 mm	179.66 mm	180.03 mm
Calibrated Focal Length	153.59 mm	152.86 mm	---------	

FORGE estimates were averaged and plotted against time of observation (see Figure 4), and a smooth curve was fitted to these points.

In making their observations, the Weather Bureau personnel on the UNIMAK followed the instructions of Reference 6, from which the following sentences are quoted as they appear in the Ninth edition, pages 57 and 58: "Waves in the same system usually occur in a sequence of a few, large, well-formed waves followed by an interval in which only small and poorly formed waves appear, then another series of large, well-formed waves. To obtain uniform wave data from all ships, observers will record only the larger, well-formed waves, and omit entirely the low and poorly formed waves. . . . The wave height as recorded. . . is the average of the estimated heights of the larger, well-formed waves."

The Navy observers were guided by Reference 7 which says: "In view of the considerable variation in height between waves observed in a 7-minute period, reference is conveniently made to the significant wave height. This wave height is the *average of the higher, well-defined waves present during the observation*. Statistically, significant waves are defined as the average of the 1/3 highest waves observed in a given time. As the height is the most important wave characteristic from the operational point of view, care should be taken to observe and report it accurately."* It is apparent that the estimating procedures specified by

*Italics added.

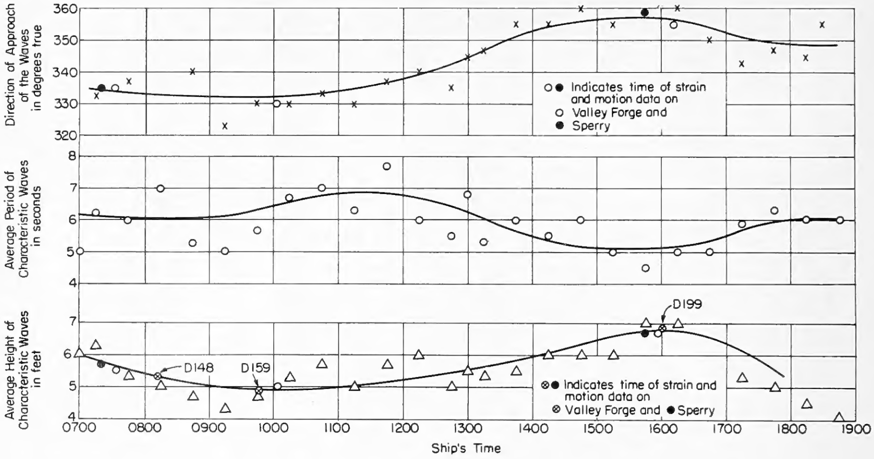

Figure 4 - Illustrative Example of Procedure Used to Average Visual Observations

Note plotted points are averages of values estimated by seven aerologists on USS VALLEY FORGE for 12 October 1955.

both the Weather Bureau and the Hydrographic Office for observers are essentially the same.

Throughout this report the term "*characteristic height*" will be used to denote *the average height of the higher, well-defined waves*. The statement in italics will serve as a definition of characteristic wave height. The term "significant wave height" will be used only in its statistical sense. The "*characteristic wave period*" *is the average period of the higher well-defined waves*. The characteristic values may be estimated by shipboard observers, or they may be obtained by more precise measurements from stereograms or wave records.

SEA SURFACE PROFILES

Sea surface profiles were developed from the stereophotographs by means of planigraphs at the U.S. Naval Photographic Interpretation Center and at the U.S. Navy Hydrographic Office; see Appendix A. Then all profiles were analyzed at the Hydrographic Office by statistical methods as outlined in Appendix A. These analyses resulted in histograms of wave heights and wave lengths as well as in "significant" wave heights. The significant wave heights are listed in Table 4. Only waves of length greater than a certain "cutoff" length were included in the histograms. The cutoff length is a function of the wind conditions that generated the sea. See Appendix A for an explanation of how the cutoff length was determined.

The sea surface profiles were later used to determine the characteristic wave height by following the same general procedure as that used by the shipboard observers in making their estimates; thus these two independent determinations of characteristic wave height should be comparable.

TABLE 4

Correlation of Wave Data

Ship	Stereophotographs: Date	Stereophotographs: Time of Photo	Stereophotographs: Position in deg Time: Lat, Long	Stereophotographs: Stereophoto and Sea Surface Profile Number	Visual Observations Made Aboard Ship: Number of Estimators	Visual Observations Made Aboard Ship: Characteristic Wave Height, ft (Average of Larger Waves): Sea	Visual Observations Made Aboard Ship: Characteristic Wave Height, ft (Average of Larger Waves): Swell	DTMB Stereoanalysis: Number of Largest Waves N: Sea	DTMB Stereoanalysis: Number of Largest Waves N: Swell	DTMB Stereoanalysis: Characteristic Wave Height, ft (Average of Larger Waves): Sea	DTMB Stereoanalysis: Characteristic Wave Height, ft (Average of Larger Waves): Swell	USN Hydrographic Office Stereoanalysis: Number of Waves in Sample	USN Hydrographic Office Stereoanalysis: Cutoff Wave Length ft	USN Hydrographic Office Stereoanalysis: Significant Wave Height ft	USN Hydrographic Office Stereoanalysis: E in (ft)^2 Estimated from Wave Height*	USN Hydrographic Office Stereoanalysis: E in (ft)^2 Estimated from Wind Data*
CVS45	11 Sep 55	1435 P	1200: 40 N, 58 W	A-0119	3	5.0		12	12	5.4	6.5	200	23	3.6	6.6	13.6
	12 Sep 55	1701 P	2000: 40 N, 51 W	A-0200	4	4.8		11	7	4.4	6.9	197	18	4.2	8.3	9.2
	20 Sep 55	1113 N	1200: 45 N, 17 W	B-0163	4		9.3	11	15	6.1	11.6	201	42	7.2	25.6	20.0
	20 Sep 55	1202 N	1200: 45 N, 17 W	B-0165	3		8.0	12	6	9.6	10.0	199	33	6.8	23.6	32.0
	15 Sep 55	0925 O	0800: 43 N, 37 W	B-0218	2	7.2		25	12	6.3	5.9	200	32	4.1	14.3	24.0
	22 Sep 55	1434 Z	1200: 48 N, 8 W	C-0233	3		10.0	11	7	5.9	11.2	198	42	6.1	17.0	28.0
	28 Sep 55	1715 Z	2000: 42 N, 13 W	C-0418	4		9.0	11	14	5.5	9.0	200		5.3	12.5	24.0
	30 Sep 55	1100 Z	1200: 40 N, 15 W	C-0452	3		8.0	12	21	6.6	8.6	182	30	5.9	17.2	32.0
	30 Sep 55	1417 Z	1200: 40 N, 15 W	C-0466	3		7.7	12	12	8.1	8.2	161	34	6.0	17.7	24.0
	30 Sep 55	1515 Z	1200: 40 N, 15 W	C-0470	3		7.8	12	15	9.1	7.5	209		4.4	8.9	24.0
	1 Oct 55	1245 Z	1200: 43 N, 13 W	C-0495	6		9.0	12	16	5.8	7.0	196	31	4.2	10.7	28.0
	1 Oct 55	1415 Z	1200: 43 N, 13 W	C-0500	3		8.8	10	11	6.3	8.0	110	34	6.0	14.3	32.0
	2 Oct 55	1300 Z	1200: 39 N, 13 W	D-0017	3		4.4	12	8	2.8	4.5	177	20	3.4	5.1	9.2
	11 Oct 55	0948 Z	0800: 37 N, 15 W	D-0085	5		6.2	15	11	4.2	5.4	208	21	3.7	6.1	10.4
	11 Oct 55	1616 Z	2000: 37 N, 18 W	D-0122	7		8.0	16	8	6.2	6.5	241	20	4.8	9.9	12.0
	12 Oct 55	0812 Z	0800: 36 N, 21 W	D-0148	6		5.5	14	5	8.5	5.4	223		6.7	19.6	18.0
	12 Oct 55	0946 Z	0800: 36 N, 21 W	D-0159	5		5.0	13	10	7.1	6.7	184	20	5.6	13.6	9.2
	12 Oct 55	1600 Z	2000: 35 N, 22 W	D-0199	3		6.9	10	14	6.5	6.3	190		4.3	8.9	16.0
	13 Oct 55	1046 N	0800: 35 N, 26 W	E-0017	5		4.0	10	8	2.2	3.5	184		2.1	2.1	5.2
	13 Oct 55	1215 N	1200: 35 N, 27 W	E-0021	7		3.7	12	7	5.6	4.5	200		3.3	5.3	5.2
	13 Oct 55	1646 N	2000: 35 N, 29 W	E-0034	7		2.3	13		5.0		148	30	4.5	8.9	5.2
	14 Oct 55	1315 O	1200: 34 N, 34 W	E-0062	3		6.1	10	5	7.2	7.1	200	26	5.2	11.8	16.0
	14 Oct 55	1345 O	1200: 34 N, 34 W	E-0064	3		6.1	15	12	6.9	6.9	199	26	5.9	15.8	16.0
	14 Oct 55	1415 O	1200: 34 N, 34 W	E-0068	4	4.6	6.0					180	33	6.0	17.0	16.0
	14 Oct 55	1445 O	1200: 34 N, 34 W	E-0071	4		6.0	10	14	6.9	5.9	178	28	3.4	9.3	18.0
	14 Oct 55	1515 O	2000: 34 N, 35 W	E-0074	3		5.9	17	12	7.3	6.5	200		4.9	11.3	18.0
	16 Oct 55	0645 P	0800: 34 N, 45 W	E-0145	1		10.3	13	9	8.0	10.4	148	42	5.4	18.4	24.0
	17 Oct 55	0945 P	0800: 35 N, 57 W	F-0222	4		7.3	16	10	4.1	5.7	200		5.2	7.1	20.0
	18 Oct 55	1245 P	1200: 35 N, 59 W	F-0289	4		6.1	15		5.6		200	26	3.3	6.8	16.0
	19 Oct 55	1015 Q	1200: 35 N, 67 W	F-0344	4	4.8		12	9	5.8	5.1	176	25	5.0	11.6	13.6
	19 Oct 55	1445 Q	1200: 35 N, 67 W	F-0362	4		6.6	11	3	9.2	6.3	166	30	5.9	15.9	20.0
	19 Oct 55	1551 Q	2000: 35 N, 68 W	F-0365	1	7.7		13	4	8.2	11.0	150	26	6.0	15.3	20.0
	20 Oct 55	1715 Q	2000: 35 N, 71 W	G-0049	5		3.7	15	7	5.0	5.0	163	23	4.4	9.1	8.0
	22 Oct 55	0715 R	0800: 37 N, 71 W	G-0113	4		5.9	14	11	5.7	5.1	200	26	4.8	10.4	20.0
	22 Oct 55	1215 R	1200: 36 N, 72 W	G-0127	6		6.6	15	10	5.5	6.7	63	40	6.1	19.8	16.8
	22 Oct 55	1617 R	2000: 37 N, 73 W	G-0150	2		6.5	14	8	4.2	7.1	200		4.1	8.1	18.4
	15 Nov 55	0750 R	0800: 37 N, 73 W	H-0047	2		3.7	12	9	3.6	3.9	200		3.2	5.0	5.2
	15 Nov 55	0950 R	0800: 37 N, 73 W	H-0060	2		2.7	14	10	3.6	3.7	181	20	3.3	4.6	9.2
	15 Nov 55	1055 R	1200: 38 N, 73 W	H-0078	4		2.7	14		3.8		162	20	2.5	3.0	9.2
	15 Nov 55	1115 R	1200: 38 N, 73 W	H-0087	1		2.7	14	10	2.9	3.7	200		2.8	3.5	9.2
	15 Nov 55	1155 R	1200: 38 N, 73 W	H-0100	3		2.8	14	12	3.5	4.5	200	20	3.4	5.2	9.2
	16 Nov 55	0715 R	0800: 37 N, 72 W	H-0155	6		3.6	14	15	6.0	5.3	188	20	5.1	12.0	9.2
	16 Nov 55	1255 R	1200: 37 N, 72 W	H-0201	2		7.0	16	11	5.1	5.3	169	30	5.2	11.9	32.0
	16 Nov 55	1545 R	2000: 37 N, 72 W	I-0014	3		7.5	16	13	6.4	7.9	148	32	6.9	22.0	30.0
	17 Nov 55	0815 R	0800: 37 N, 74 W	I-0052	5		7.1	20	13	3.8	6.0	200	--	4.6	9.7	18.4
	17 Nov 55	1345 R	1200: 37 N, 74 W	I-0073	2		4.3	17	12	4.3	6.0	200	--	4.6	10.3	10.4
	9 Dec 55	1530 R	1200: 36 N, 70 W	J-0003	4	10.5	9.9	16	11	7.3	9.3	139	34	6.8	21.5	24.0
	10 Dec 55	0945 R	0800: 36 N, 72 W	J-0035	3	12.5	12.7	18	9	15.3	12.2	144	38	8.1	29.4	40.0
	10 Dec 55	1045 R	1200: 36 N, 72 W	J-0040	3	12.0	12.1	13		16.8		150	40	10.9	58.8	48.0
WAVP379	16 Oct 54	1629 Z	1500: 44 N, 41 W	U- 359	1	13.0		14		11.6		146	--	9.4		
	22 Oct 54	1318 Z	1200: 44 N, 41 W	U- 415	1	5.0		11		7.1		31	--	5.5		
	16 Jan 55	1240 Z	1200: 57 N, 51 W	U- 505	1	14.0		10		14.7		91	--	8.1		
	16 Jan 55	1628 Z	1500: 57 N, 51 W	U- 517	2	19.5		10		21.6		87	--	9.0		
	28 Jan 55	1516 Z	1500: 57 N, 51 W	U- 525	2	5.5		20		6.5		90 } 156	--	5.1 } 6.2		
	28 Jan 55	1536 Z	1500: 57 N, 51 W	U- 526	2	5.5		8		8.4		66	--	6.9		
	31 Jan 55	1226 Z	1200: 57 N, 51 W	U- 537	1	18.0		7		15.8		37	--	8.2		
	1 Feb 55	1220 Z	1200: 57 N, 51 W	U- 540	1	16.0		12		17.3		111	--	7.4		
	1 Feb 55	1221 Z	1200: 57 N, 51 W	U- 541	1	16.0		8		20.7		33 } 258	--	12.0 } 7.9		
	1 Feb 55	1222 Z	1200: 57 N, 51 W	U- 542	1	16.0		12		17.4		114	--	7.2		
	2 Feb 55	1218 Z	1200: 57 N, 51 W	U- 548	1	18.0		9		19.1		34	--	8.4		

* E is the mean square value of wave heights.

The data from the sea surface profiles worked up by the Hydrographic Office were used further to obtain the mean square of the wave height, E. These quantities are also listed in Table 4. In order to check the utility of the theoretical method of Reference 8 for the prediction of wave heights, the Hydrographic Office also computed the mean square of the wave height E on the basis of the distribution of wind velocities for the sea area and sea surface profile in question; these values are listed in the last column of Table 4. Graphs of these various quantities are presented and discussed in Appendix B, Figures 34 through 36.

STATISTICAL BACKGROUND

Wave heights and wave periods estimated from the Weather Bureau data for ten ocean stations are presented in the form of their distribution functions. For example, all wave heights reported by the shipboard observers are considered to be members of a statistical "population" of wave heights. The distribution function of wave heights indicates the relative probability of encountering a wave of a given height as a function of that height. Figure 5 illustrates this distribution function. The area under the curve to a value x_i is the integral of the function up to x_i and is equal to the fraction of all members of the population of wave heights which have a height less than x_i. Mathematically

$$P(x) = \int_0^x p\,dx \qquad \text{and} \qquad P(x \to \infty) = \int_0^\infty p\,dx = 1$$

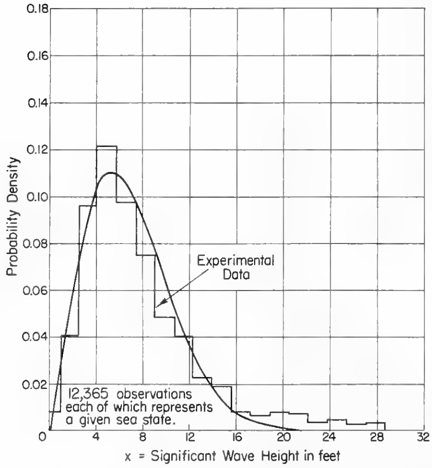

Figure 5 - Distribution Function

where p is the probability density and P is a function of x designated as the cumulative distribution function of x. $P(x)$ is numerically equal to the probability that a value chosen at random from the population is less than x.

A detailed discussion of the statistical methods utilized in this report is given in Reference 9. Only a few of the major concepts will be described here. The distribution applicable to a given sea condition is here called a "short-term" distribution, whereas the distribution applicable when the sea conditions are allowed to vary over a wide range, such as over a year's time, is called a "long-term" distribution. Thus the long-term distribution is the result of a summation of a number of short-term distributions. Oceanographers have held that the short-term* distribution of wave heights x is approximately of the Rayleigh type (a narrow power spectrum is assumed) for which,

$$P(x) = \frac{2x}{E_i} e^{-x^2/E_i}$$

where E_i is the mean square of all the individual wave heights x corresponding to sea condition i. Note that numerically the value of E computed for wave height will be four times the value of E computed for wave amplitude because wave height is taken equal to twice the wave amplitude. See References 8 and 10 for a discussion of the distribution of wave heights in terms of the power spectrum concept.

In Reference 9, it was suggested that the long-term distribution of wave heights and wave periods is of the log-normal type, that is, that the logarithms of these heights and periods are approximated by a normal distribution. Thus

$$p(\log x)\, d(\log x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(\log x - u)^2/2\sigma^2}\, d(\log x)$$

$$P(x) = P(\log x) = \int_{\log x = -\infty}^{\log x} p(\log x)\, d(\log x)$$

where $p(\log x)$ is the probability density of the variate, $\log x$,

u is the mean value of $\log x$, and

σ^2 is the variance of $\log x$.

Then the parameters u and σ define this distribution completely.

In this report log-normal distributions are fitted to the characteristic wave heights and periods reported by the Weather Bureau. The resultant graphs represent long-term distributions and give the probability with which a given value of the variate x will or will not be exceeded in an average year.**

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*The short-term distribution is approximately valid if measurements are taken over a relatively short period of time, of the order of one hour, during which interval the sea conditions do not change appreciably. It can be shown that this distribution is the same as that representing the wave heights, in the area under consideration, at one instant of time.

**Although the distributions given here are for a six-year period, study of the distributions for the individual years making up the six-year period indicates that a single year gives a typical sample of the distribution obtained for many years. Therefore, the six-year distribution may be considered valid for an average year.

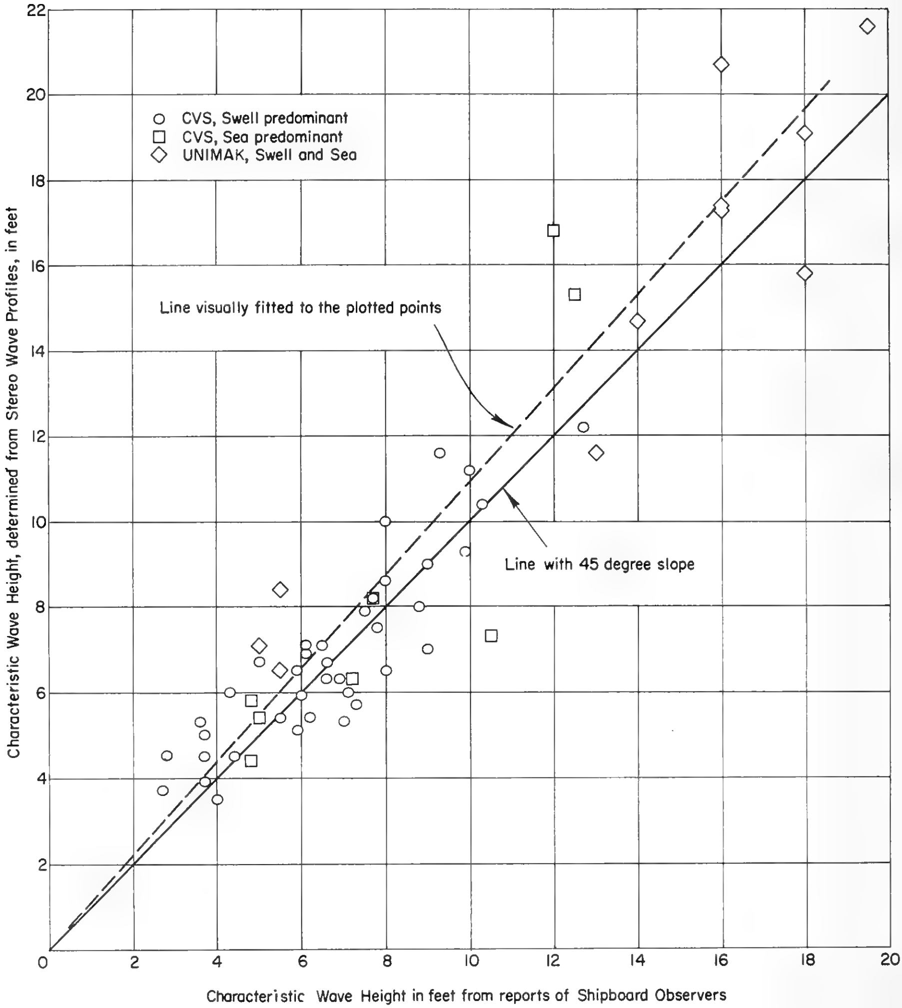

Figure 6 - Scatter Diagram for Comparison of Characteristic Wave Heights Determined Independently from the Wave Profile Analysis and by Shipboard Observers

The plotted data are taken from Table 4, Columns 7 and 9.

EVALUATION OF RELIABILITY OF VISUAL ESTIMATES OF WAVE HEIGHT

The U.S. Naval Photographic Interpretation Center and the U.S. Navy Hydrographic Office utilized the stereophotographs from the UNIMAK and the VALLEY FORGE to produce the sea surface profiles illustrated in Appendix A. From these profiles, wave height data were determined as discussed in the section on "Sea Surface Profiles" and tabulated in Table 4. Note that this table includes waves associated with sea conditions in which seas predominated, others in which swells predominated, and still others in which both seas and swells were present.

The characteristic wave heights obtained by the shipboard observers are compared with those derived from the stereograms for the same sea condition in Figure 6, where values from Columns 7 and 9 of Table 4 are plotted as abscissas and ordinates, respectively. If exact agreement existed between visual estimates and the results of photogrammetric analysis, then all points would lie on a straight line with a 45-deg slope. The points plotted in Figure 6 scatter fairly well about a straight line which has a slope somewhat greater than 45 deg. The average deviation of the points from the line is expected to decrease as the number of points is increased. It should be noted that each stereophotograph covers a limited field of view compared with the field of view of the shipboard observer; see Figure 7 for ocean areas included in the camera perspectives for both ships.

It is considered that Figure 6 shows good correlation between the visual estimates and quantitative height determinations made from the stereophotographs. Individual estimates may not be accurate, but when the number of estimates is large the correlation is good.

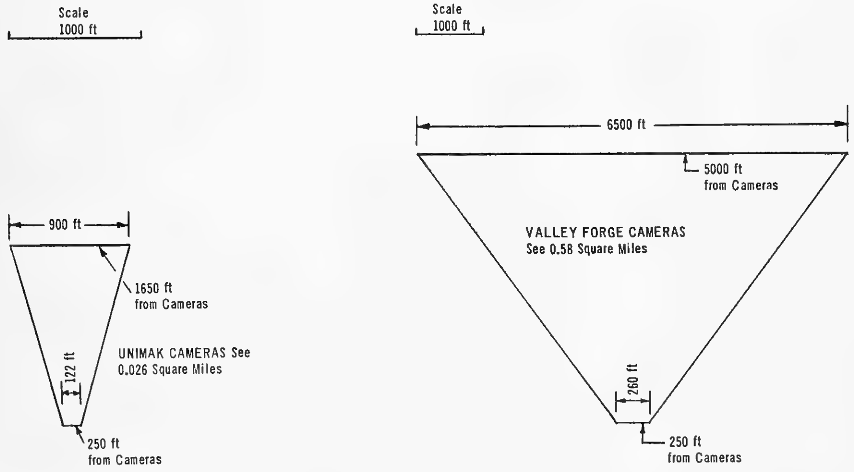

Figure 7 - Ocean Perspectives Seen by Stereocameras on UNIMAK and VALLEY FORGE

Areas indicated are fixed by properties of cameras, film, stereoplanigraphs, and camera separation.

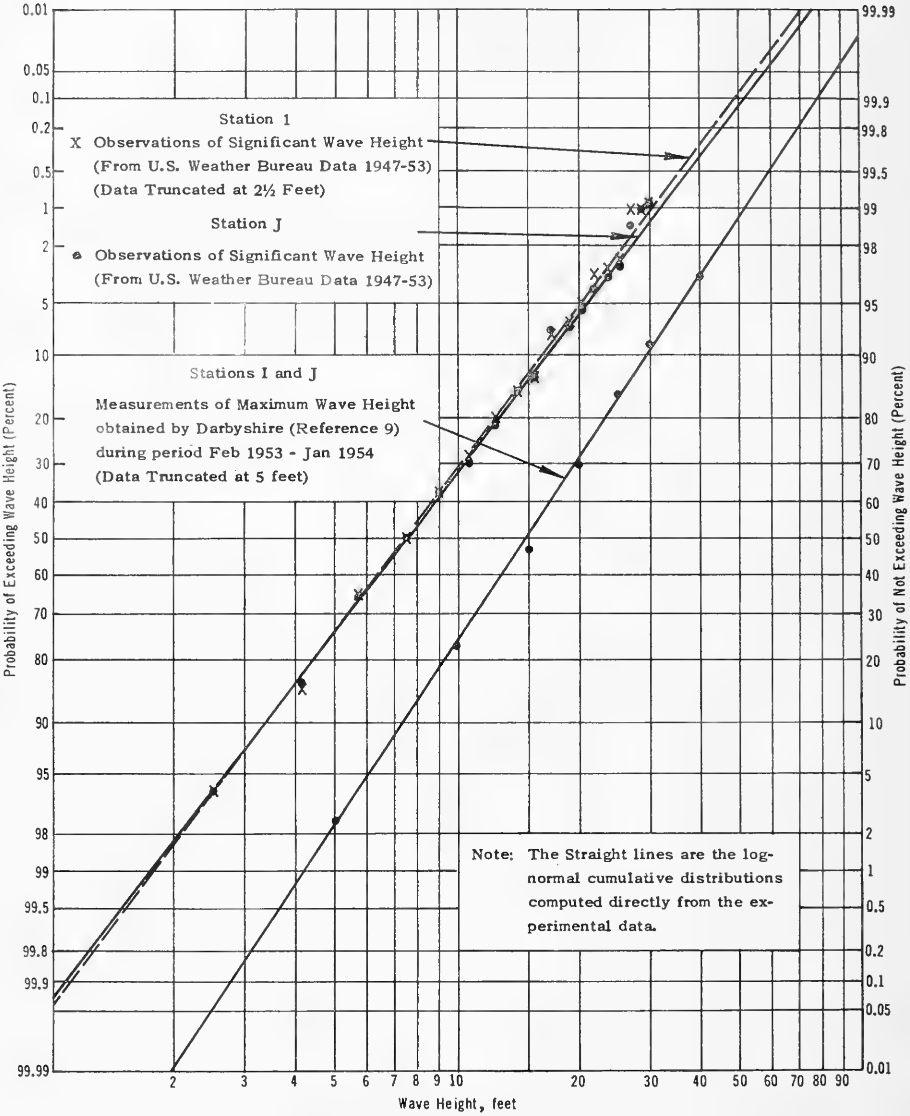

Figure 8 - Comparison of Wave Height Distributions Derived from Visual Observations and from Measurements of Wave Heights at Atlantic Ocean Stations I and J

The distribution fitted to the Darbyshire data corresponds to a standard deviation of 0.57 for $\log_e$ (maximum wave height) and a median value of the maximum wave height equal to 15 feet.

The UNIMAK data and the VALLEY FORGE data do not indicate divergent trends, that is, the methods[6,7] of wave estimation used by the observers on the UNIMAK and the VALLEY FORGE give approximately the same characteristic wave height.

The U.S. Weather Bureau data, on which the long-term distributions given in this report are based, comprise between 11,000 and 18,000 separate observations for each ocean station. It is concluded that the errors associated with the visual observations are fairly well averaged out when such a large number of observations are utilized to define the distribution and that the reported characteristic wave heights are therefore proportional to the severity of the sea.

Further evidence to support the validity of the Weather Bureau data can be drawn from an analysis of measurements of wave height recently made by J. Darbyshire[4] by means of a wave meter installed on a weather ship. These measurements were made over a period of about one year, February 1953 to January 1954, at North Atlantic Weather Stations I and J; see Figure 8. Darbyshire reported the maximum wave height for each 3-hr period for which visual wave observations were made while the ship was at sea. The visual observations made by weather observers are reported as the "characteristic" wave height. According to Appendix B the characteristic height is proportional to the significant height. It is of interest to compare the visual observations with the measurements obtained with the wave meter. If the hypothesis is accepted that the short-term distribution of wave height follows the Rayleigh distribution, then the maximum significant and characteristic wave height for any given sea condition are related by a constant factor. Thus the long-term distributions of maximum and characteristic wave heights should be of the same type, log-normal in this case, and should differ only in their mean values. The U.S. Weather Bureau data indicates that the standard deviation* of $\log_e$ (characteristic wave height) is 0.622 at Station J and 0.612 at Station I as compared with a value of 0.57 for $\log_e$ (maximum wave height) for the measurements at Stations I and J reported by Darbyshire; see Figure 8. A log-normal distribution has been fitted to the wave-meter data on the assumption that the distribution of maximum wave heights is log-normal. The experimental data indicate excellent agreement with the fitted distribution, well within the accuracy of the measurements. The latter fact, together with the good agreement between the standard deviations of characteristic (visual estimates) and maximum (measurements) wave heights, supports the hypothesis that the distribution of wave heights may be approximated by Rayleigh and log-normal distributions for the short and long term, respectively. In a recent article[5] Darbyshire tests the applicability of the long-term log-normal distribution to extensive data on maximum wave heights obtained by use of the British wave meter. He concludes that the logarithmic law appears to be a useful guide to determine the incidence of a particular wave state at a given location.

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*The numerical values given here apply for wave heights measured in feet.

It is concluded on the basis of the foregoing discussion that the visual estimates by Weather Bureau personnel of sea state, reported as a "characteristic" wave height, may be used with confidence in establishing distribution patterns such as are given in the following section.

DISTRIBUTION PATTERNS OF WAVE HEIGHTS AND WAVE PERIODS FROM ANALYSIS OF U.S. WEATHER BUREAU DATA

Cumulative long-term distribution patterns of the characteristic wave heights and periods are given in Figures 9 through 30 for the ten ocean stations shown in Figure 1. For each station the odd-numbered figure gives the wave height distribution and the even-numbered figure gives the corresponding wave period distribution. Methods for fitting a log-normal distribution to the data are given in Reference 11. In Appendix C a sample calculation illustrates the method used for deriving Figures 9 through 30 from the data (Tables 1 and 2) furnished by the Weather Bureau.

The rather good fit of the computed lines to the plotted data, in Figures 9 through 30, suggests that a log-normal distribution is a good approximation to the distribution pattern of characteristic wave heights and periods for values above the truncation point.*

Distribution patterns for wave length can be derived from the data for wave periods by applying an approximate conversion**

$$\text{Wave Length} = 5.1\,(\text{Wave Period})^2$$

This conversion has been made for all the weather stations. It is apparent that the distribution of wave lengths will be log-normal if that for the periods is log-normal, since the conversion involves only a change in mean value and slope from the distribution of the periods. See Figure 16 for an illustration of the conversion to wave length.

In Table 5 mean values and variances are given for the wave height and period data reported from each ocean station. Also the latitudes, longitudes, and observation periods over which the data were collected are shown.

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*The truncation point is that value of wave height or period below which no observations are available or are utilized. In this report only wave periods above 5 sec and wave heights above 2.5 ft were used.

**This conversion is applicable to gravity waves in deep water. The numerical value of the factor, 5.1 in this case, does not affect the type of distribution; it only changes the value of the median.

TABLE 5

Statistics for Log-Normal Distributions Computed from Wave Observations Made in North Atlantic Ocean

Ocean Station	Latitude deg, min	Longitude deg, min	Characteristic Wave Heights					Characteristic Wave Periods				
			Dates of Observations	Number of Observations	Median Value of Characteristic Wave Height ft	Mean Value of Logarithm of Characteristic Wave Height	Variance (σ^2) of Logarithm of Characteristic Wave Height	Period of Records	Number of Observations	Mean Value of Characteristic Wave Period sec	Mean Value of Logarithm of Characteristic Wave Period	Variance (σ^2) of Logarithm of Characteristic Wave Period
A	62°00′N	33°00′W	1/49- 6/54	12,891	6.34	1.847	0.4524	1/49- 6/54	12,342	6.30	1.840	0.0935
B	56°39′N	51°00′W	1/49-12/54	15,547	6.59	1.886	0.4434	1/49- 6/55	16,060	6.40	1.857	0.0748
C	52°45′N	35°30′W	1/49-12/54	16,857	6.75	1.910	0.3763	1/49- 6/55	17,471	6.48	1.869	0.0873
D	44°00′N	41°00′W	1/49-12/54	16,804	6.26	1.834	0.3843	1/49- 6/55	17,310	6.24	1.831	0.0800
E	35°00′N	48°00′W	1/49-12/54	16,777	4.56	1.516	0.3765	1/49- 6/55	16,896	5.86	1.768	0.0939
H	36°00′N	70°00′W	1/49- 6/54	14,607	5.08	1.625	0.4237	5/49- 6/54	13,647	5.92	1.779	0.0643
I	61°00′N	15°20′W	1/47- 6/53	11,274	7.36	1.996	0.3747	1/49-12/51 1/53-12/54	12,142	7.02	1.948	0.0546
J	52°30′N	20°00′W	1/47- 6/53	12,016	7.40	2.002	0.3863	1/49-12/51 1/53-12/54	11,593	7.29	1.986	0.0423
K	45°00′N	16°00′W	1/49-12/53	11,182	6.20	1.824	0.3033	6/49-12/53	11,906	8.30	2.117	0.0548
M	66°00′N	02°00′W	1/49-12/53	14,324	4.99	1.608	0.2344	1/49-12/53	14,188	6.99	1.946	0.0676

Note: The statistical computations are based on truncated data. The truncation point is 2.5 ft for wave heights and 5 sec for wave periods.

SUMMARY

Frequency distribution patterns of wave heights and wave periods may be approximated by a one-parameter type of distribution function when the environmental conditions are steady, whereas they will tend to follow the two-parameter logarithmically normal distribution when the environmental conditions are allowed to vary over a wide range. It should be emphasized that the log-normal distributions in Figures 9 through 30 are influenced much more by the usual sea conditions than by the rare occurrences of very high or very low seas. Thus one should expect greater deviations from the fitted line for very small and very large wave heights and wave periods than for those heights and periods which occur more frequently. It is concluded that the long-term distributions of wave height and wave period may be approximated by the log-normal distribution.

Reasonably accurate visual estimates of wave height can be obtained from trained observers, provided a number of independent estimates are averaged. A single estimate may be considerably in error.

ACKNOWLEDGMENTS

This work would not have been possible without the cooperation of many persons on board the VALLEY FORGE and UNIMAK, on the Coast Guard and Navy staffs, in the Weather Bureau, at the Hydrographic Office, at the Naval Photographic Interpretation Center, and at the Taylor Model Basin.

Special thanks are due to Dr. R.W. James of the Hydrographic Office and Mr. W. Marks of the TMB staff who assisted in the oceanographic planning and analysis. The camera installations were made by Mr. C.E. Lemich of the Taylor Model Basin. Mr. B.M. Wigle and Mr. R.J. Dominic, also the Taylor Model Basin staff, sorted and matched the many photographs and assisted in analyzing the stereograms and in developing the plotted figures. The stereograms were developed by Mr. John Davis and Mr. John Boyle of the Hydrographic Office and Photographic Interpretation Center, respectively. Much encouragement has been given to the overall program, of which this study is a part, by Mr. John Vasta of the Bureau of Ships.

The paper was reviewed by Dr. E.H. Kennard, Mr. R.T. McGoldrick, and Miss M.C. Crook of the Taylor Model Basin staff. The authors wish to express their sincere appreciation to the persons mentioned here and to the many others whose assistance made this work possible.

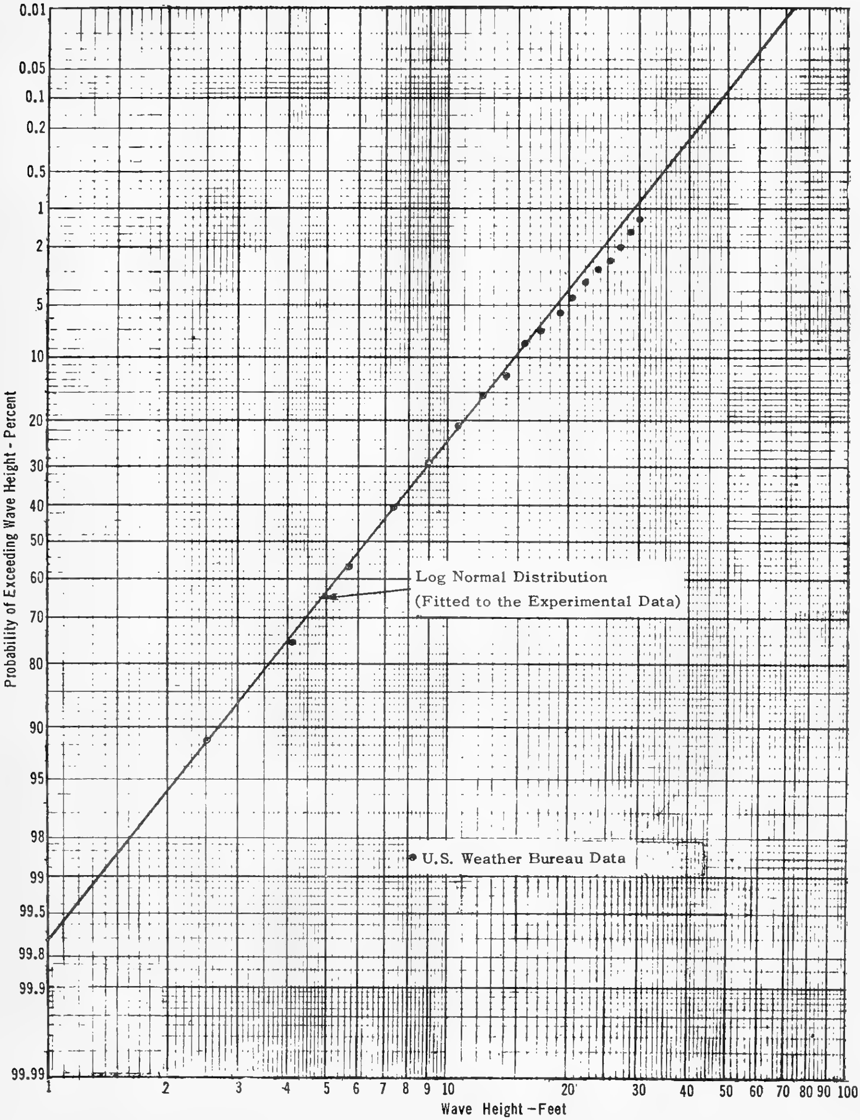


Figure 9 - Distribution of Characteristic Wave Heights
at Station A

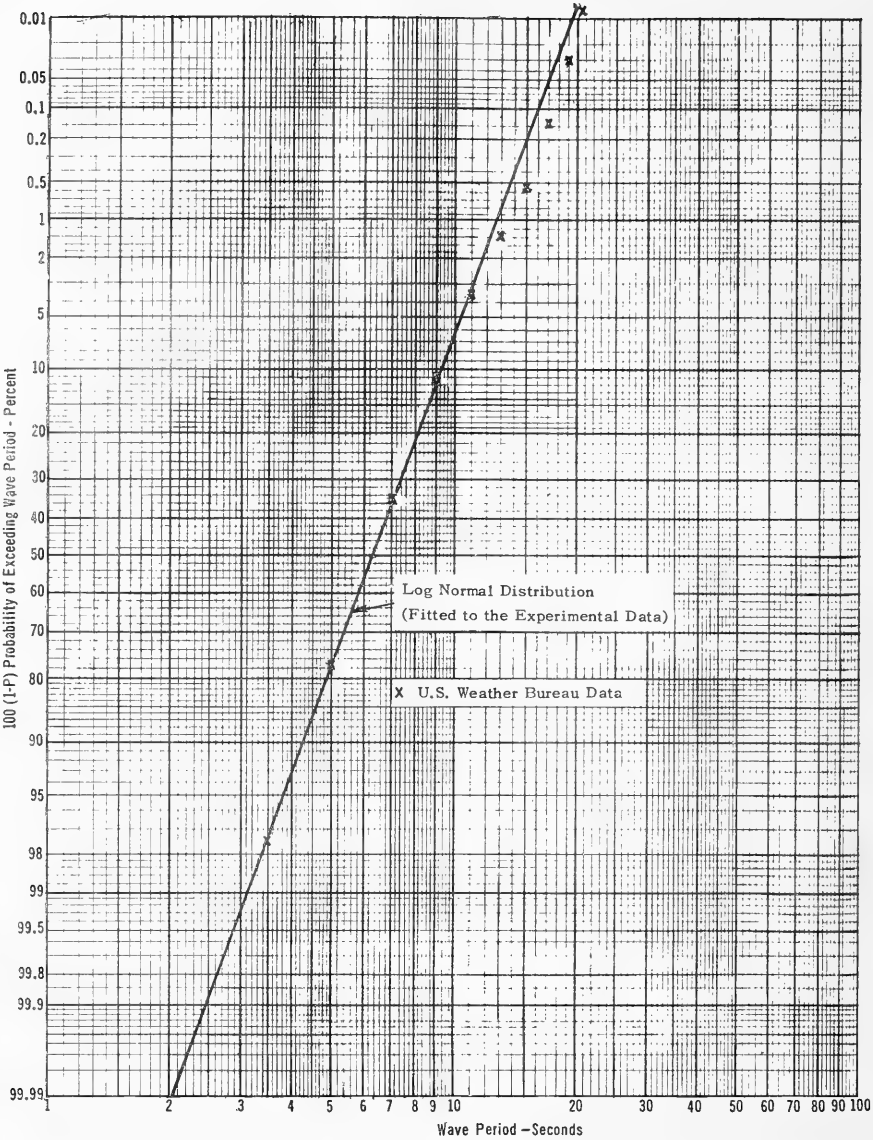


Figure 10 - Distribution of Characteristic Wave Periods at Station A

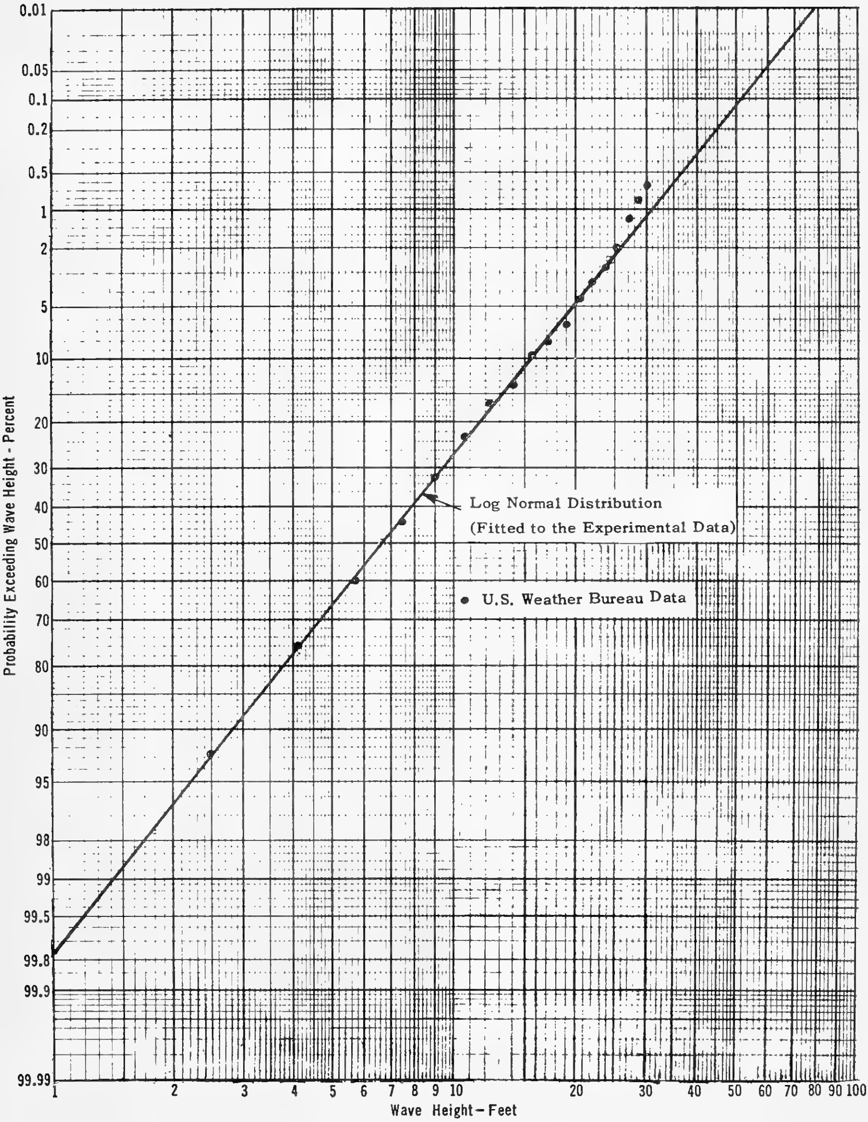


Figure 11 - Distribution of Characteristic Wave Heights at Station B

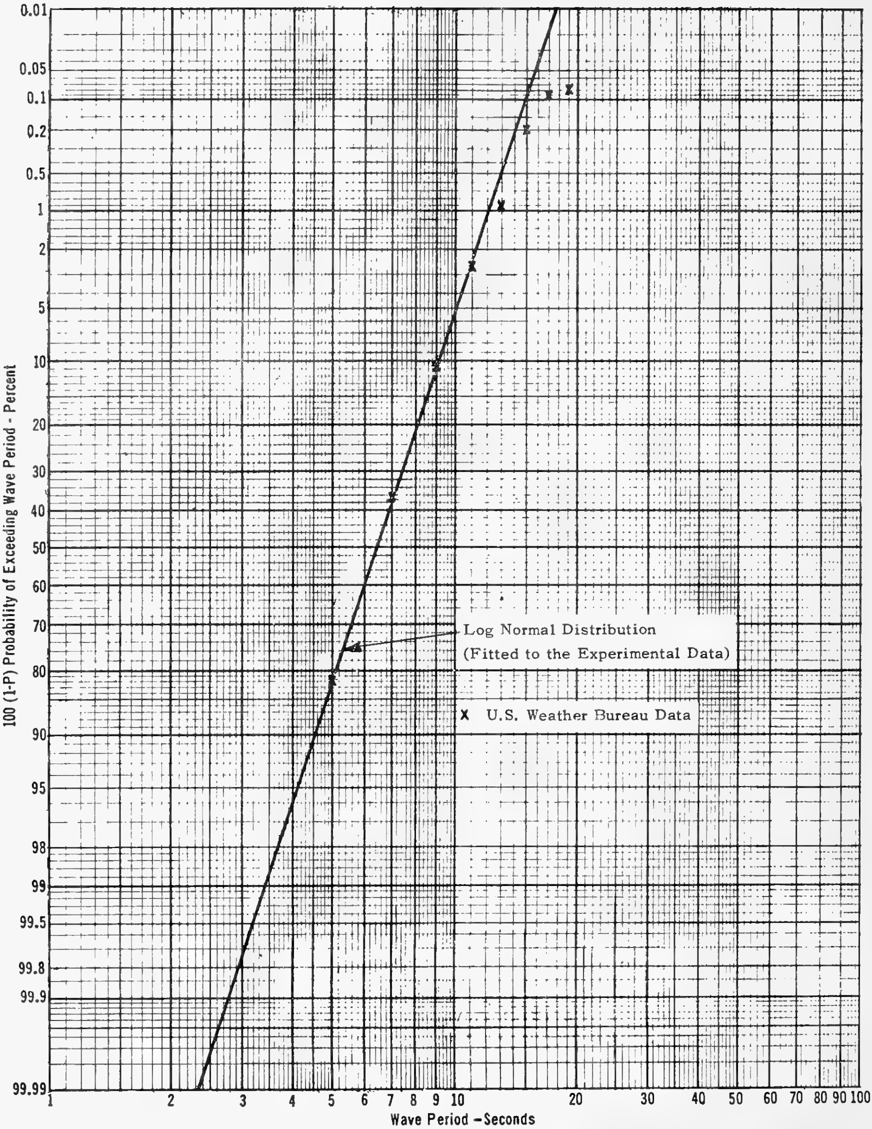


Figure 12 - Distribution of Characteristic Wave Periods at Station B

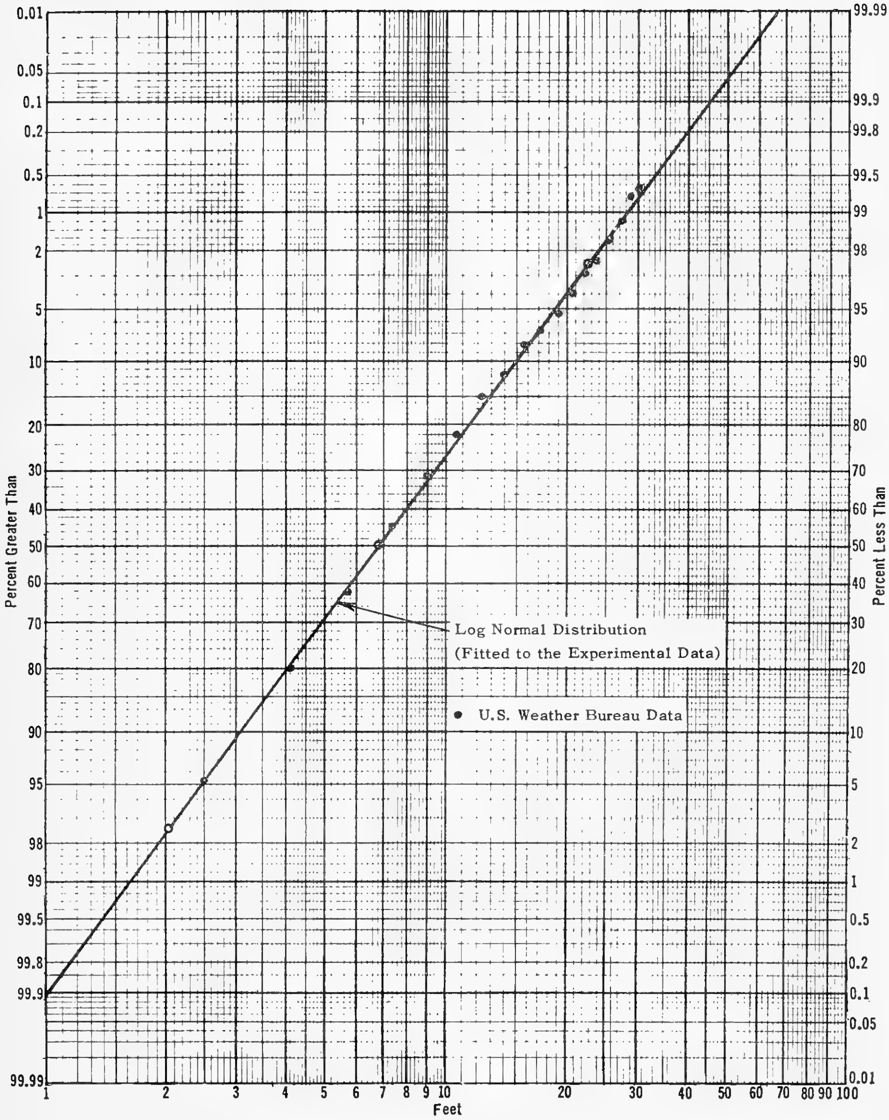


Figure 13 - Distribution of Characteristic Wave Heights at Station C

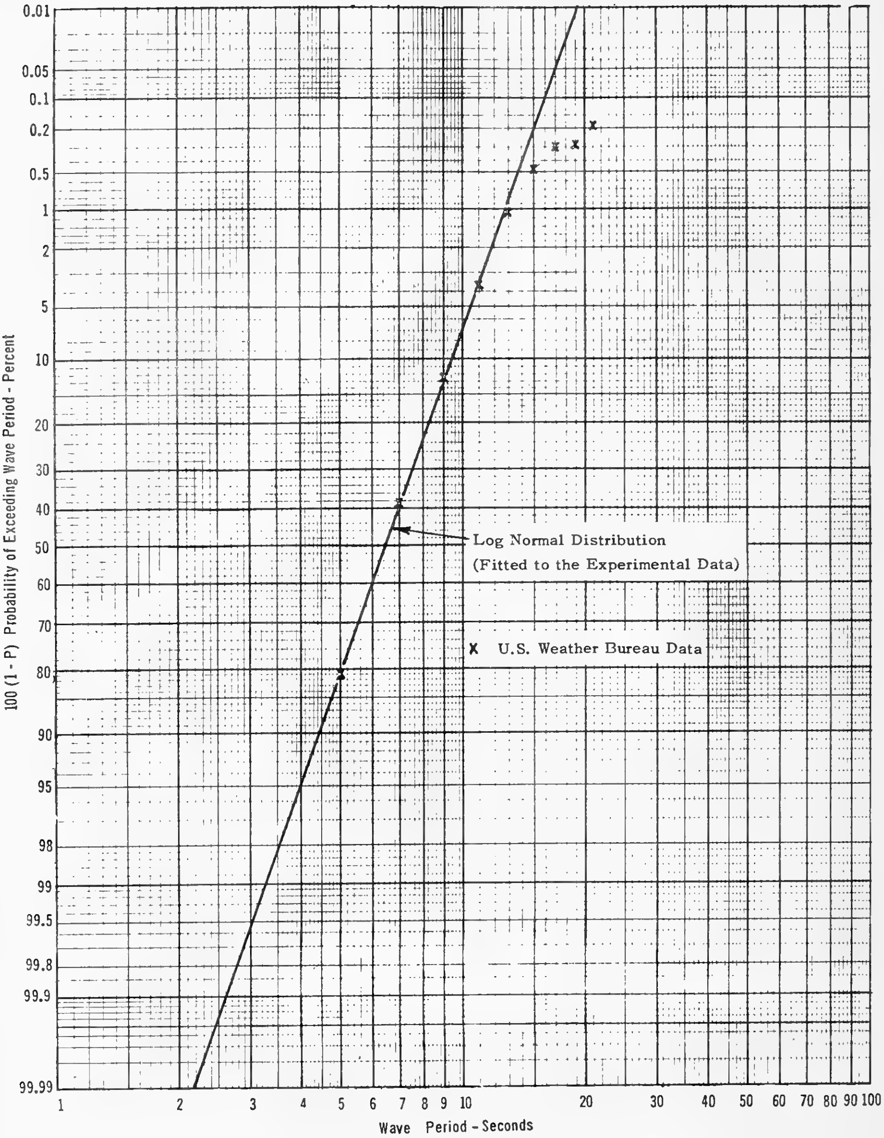


Figure 14 - Distribution of Characteristic Wave Periods at Station C

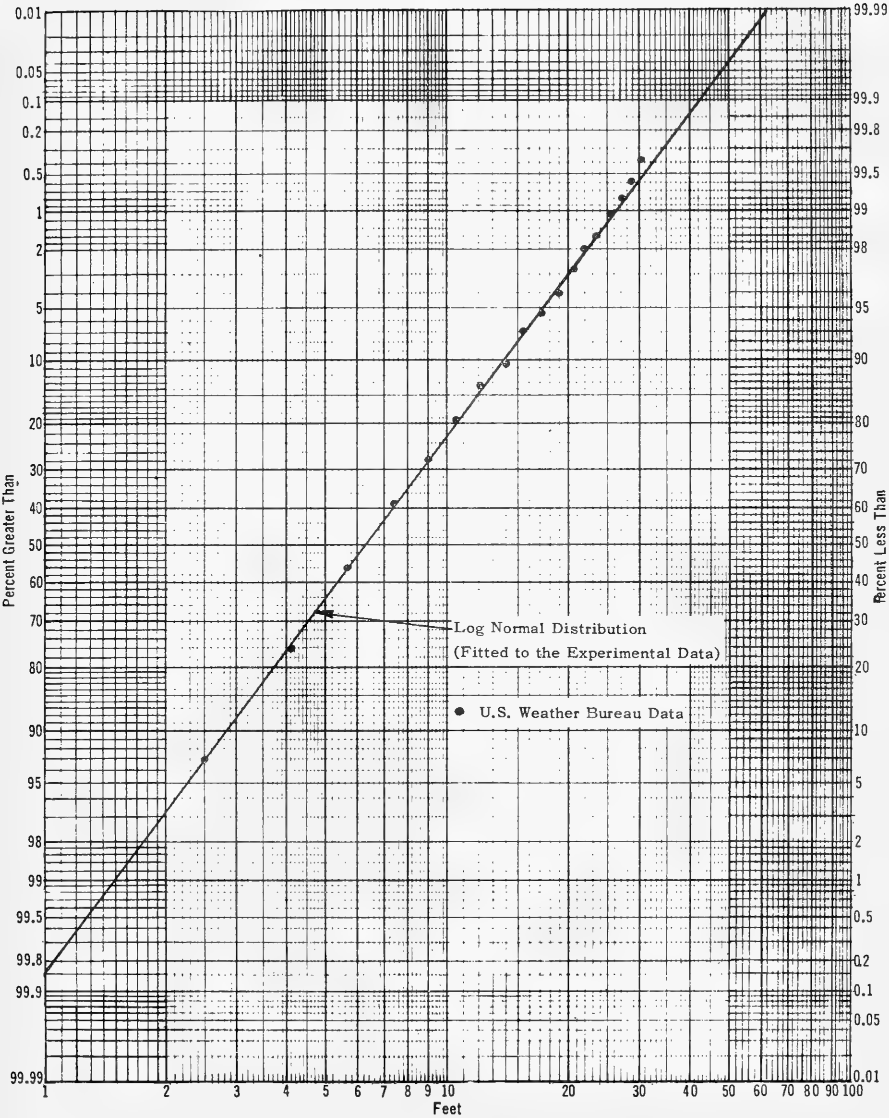


Figure 15 - Distribution of Characteristic Wave Heights at Station D

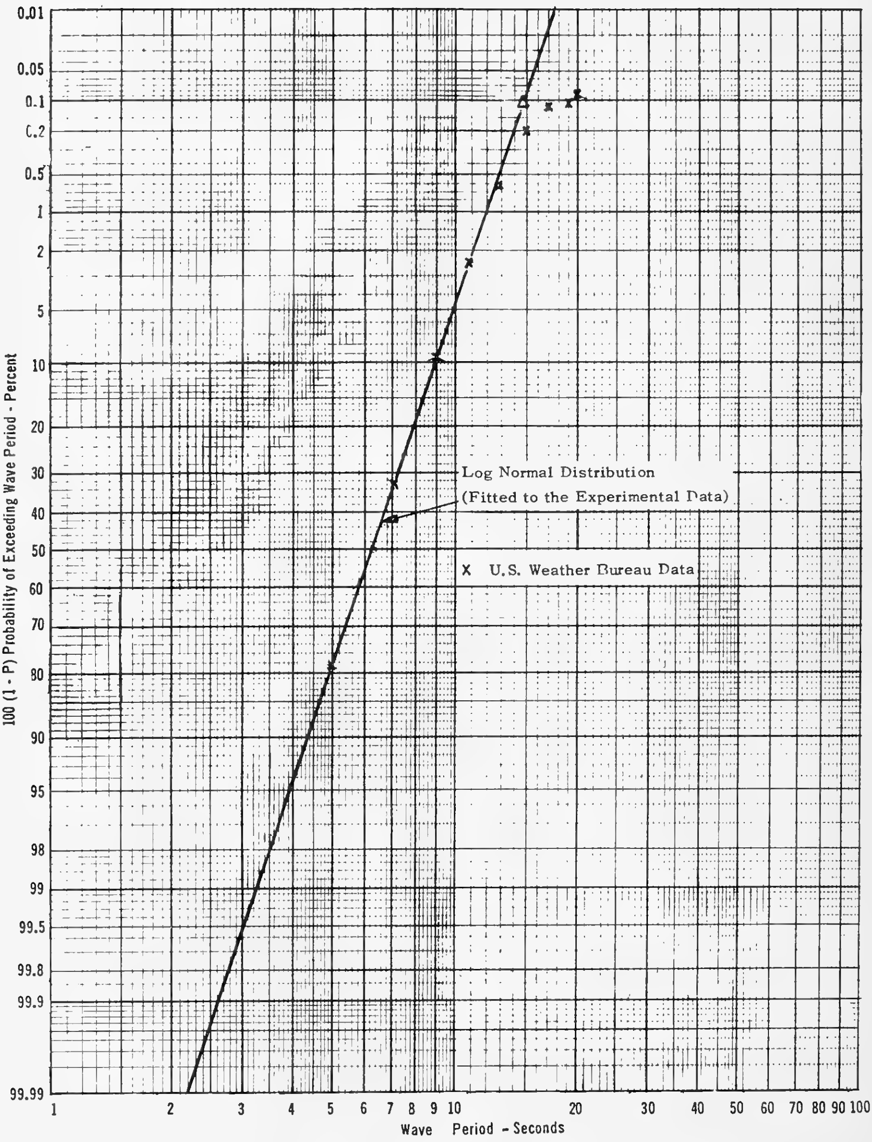


Figure 16 - Distribution of Characteristic Wave Periods at Station D

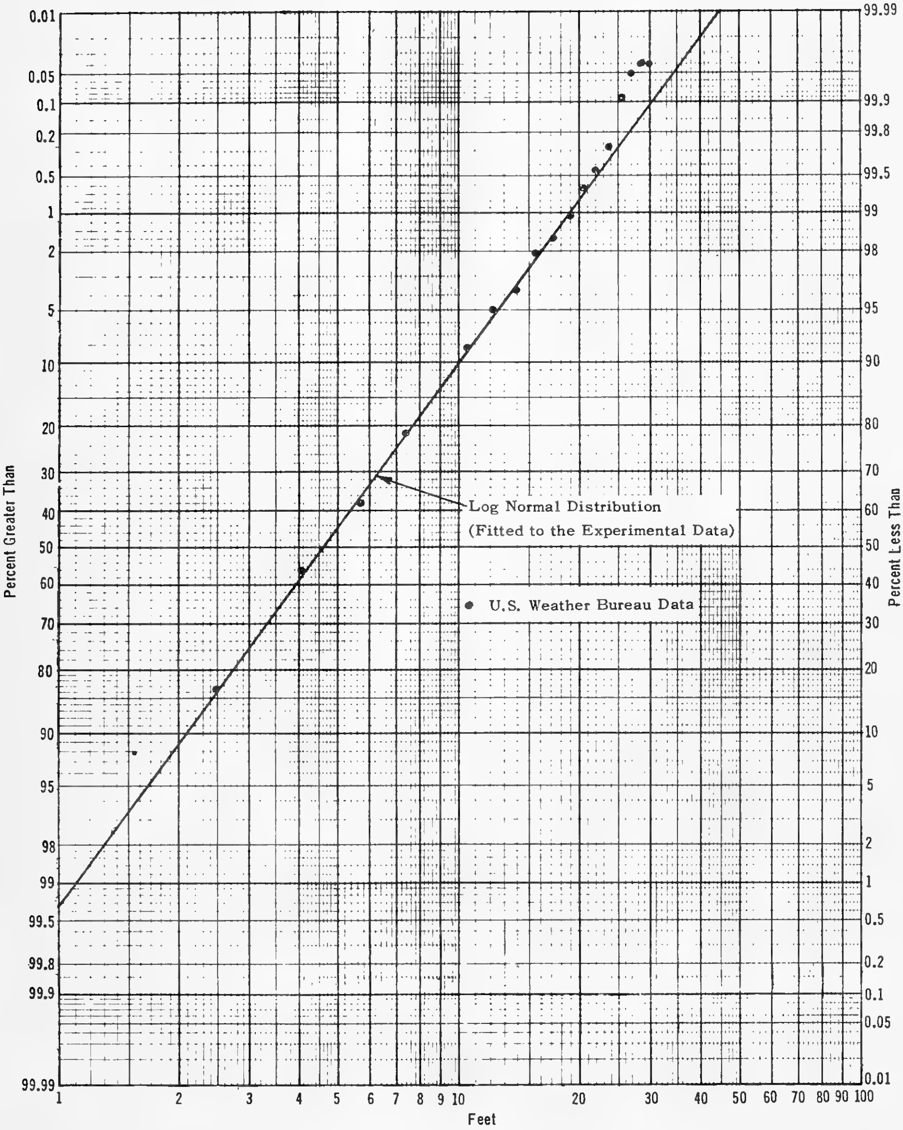


Figure 17 - Distribution of Characteristic Wave Heights at Station E

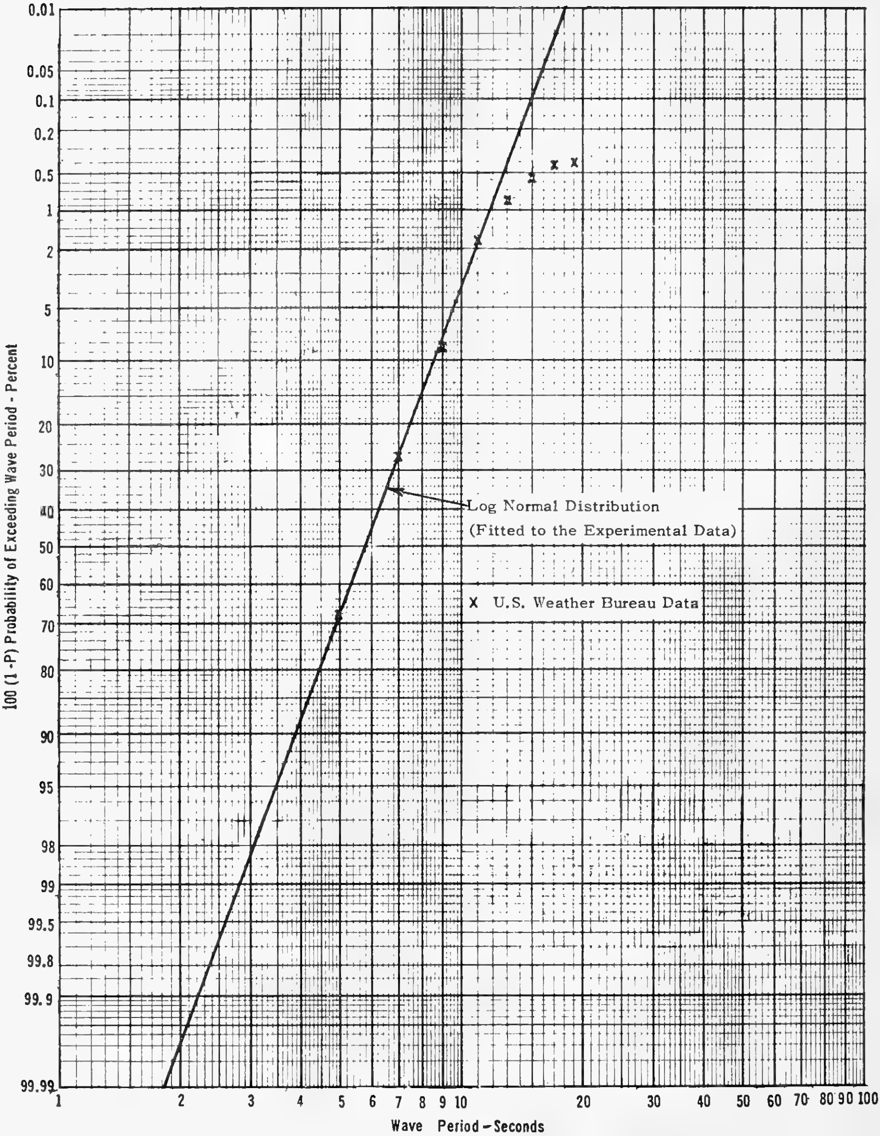


Figure 18 - Distribution of Characteristic Wave Periods at Station E

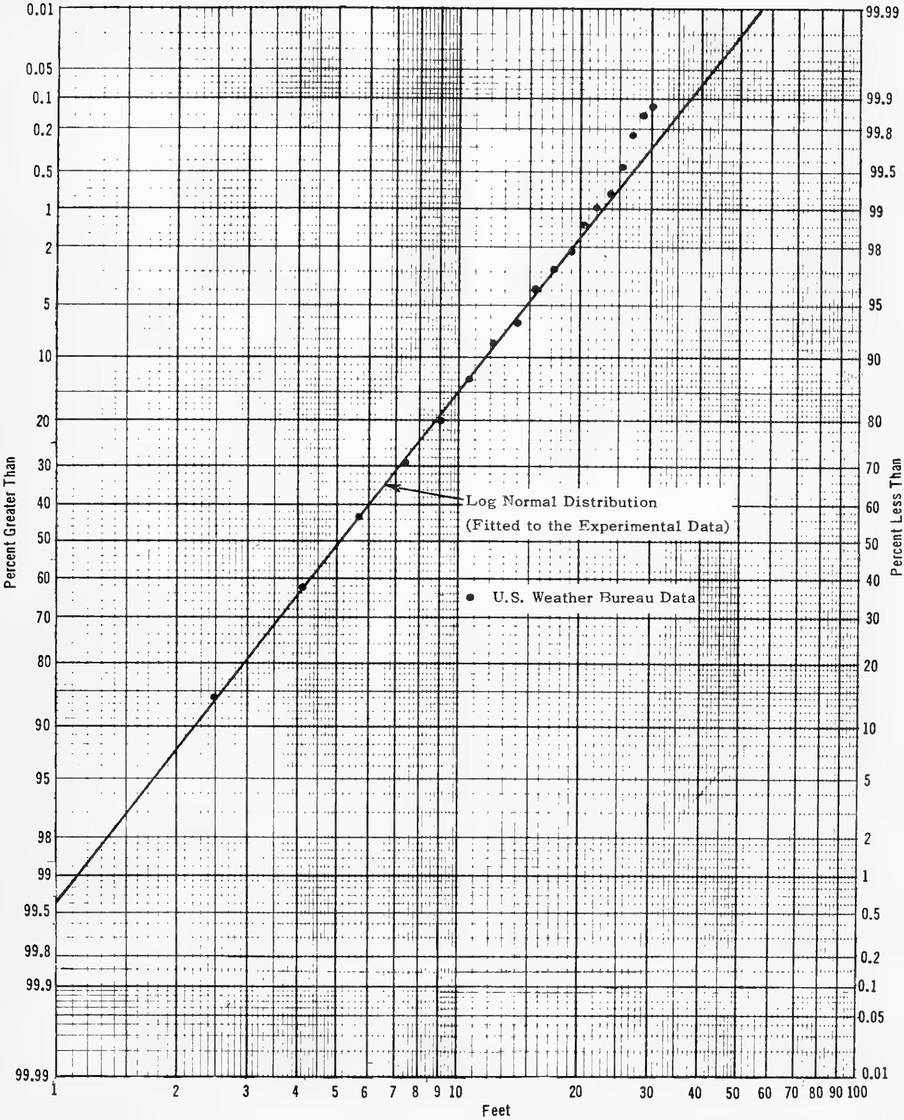


Figure 19 - Distribution of Characteristic Wave Heights
at Station H

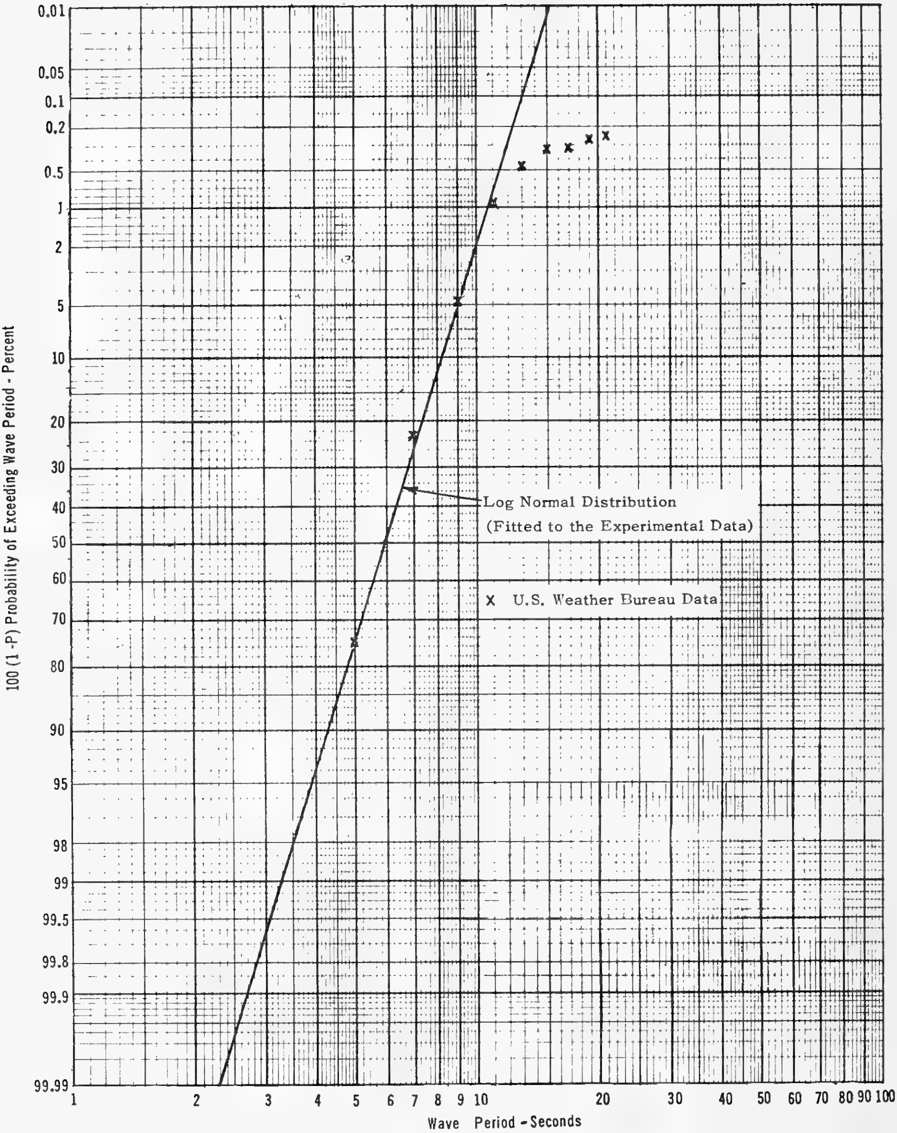


Figure 20 - Distribution of Characteristic Wave Periods at Station H

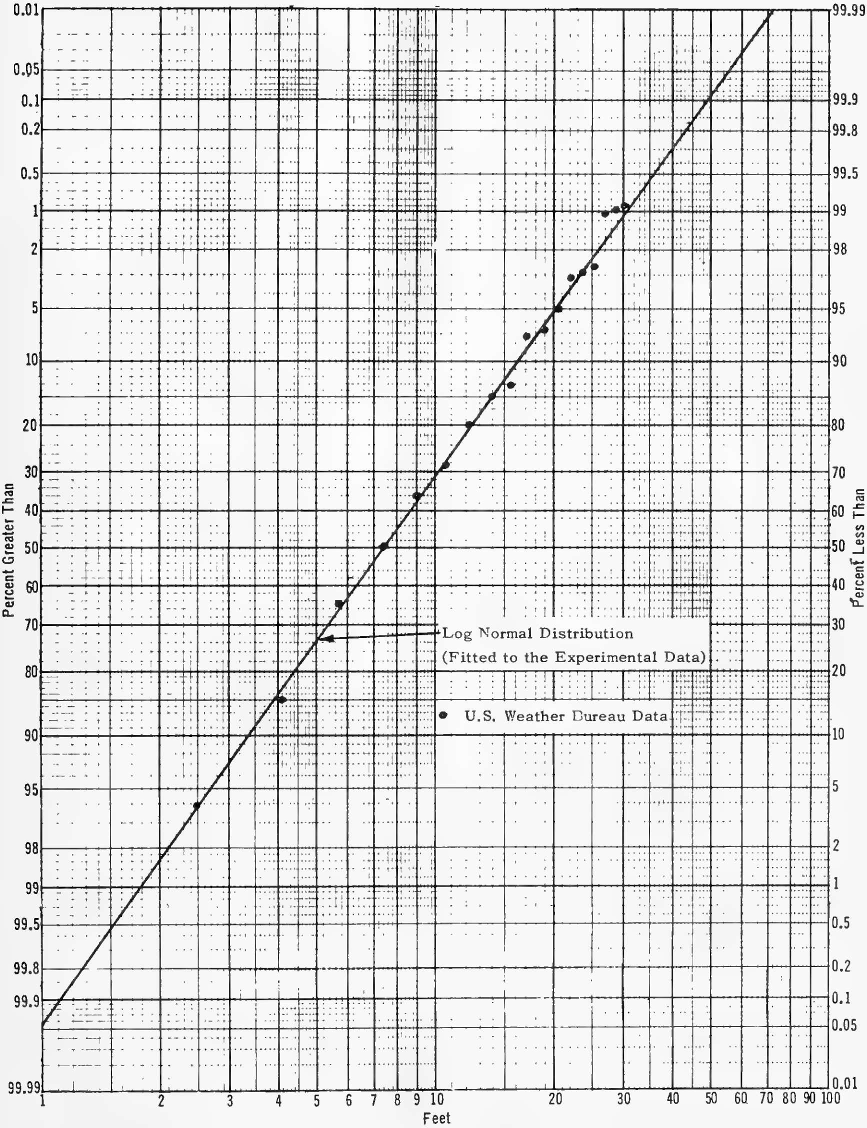


Figure 21 - Distribution of Characteristic Wave Heights at Station I

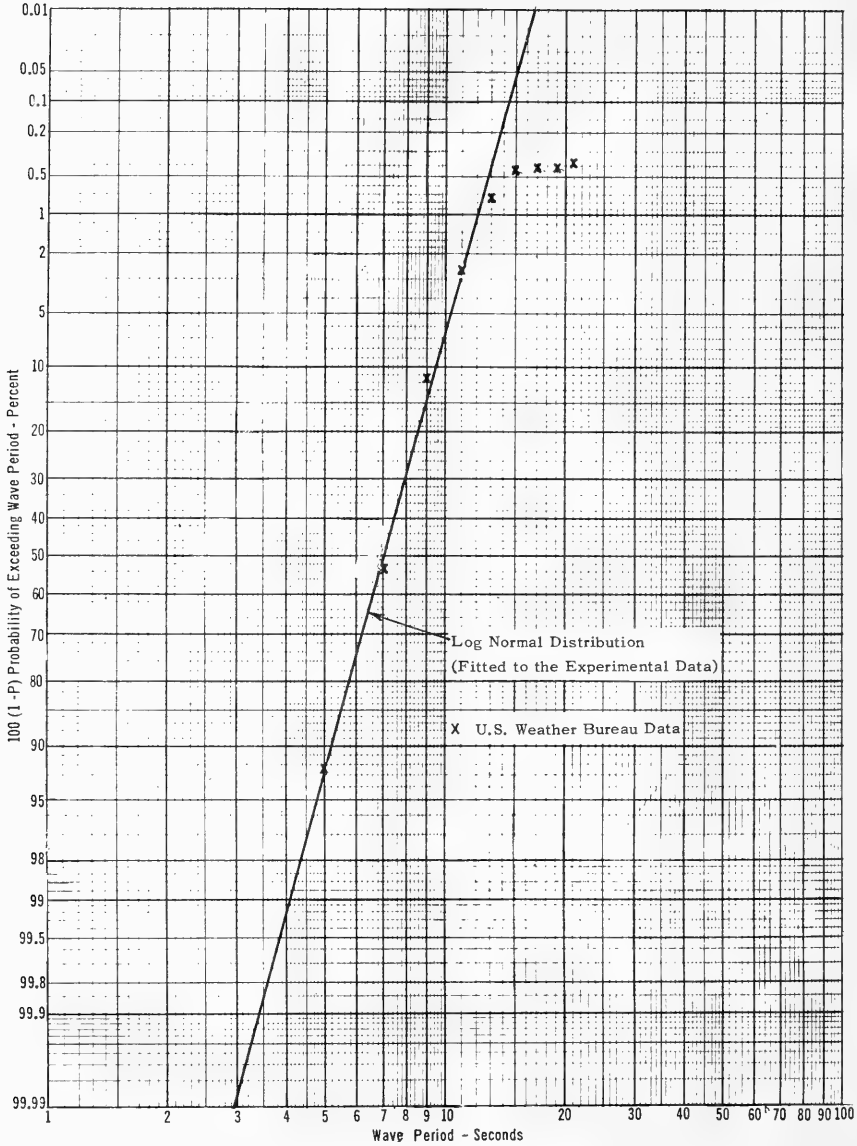


Figure 22 - Distribution of Characteristic Wave Periods at Station I

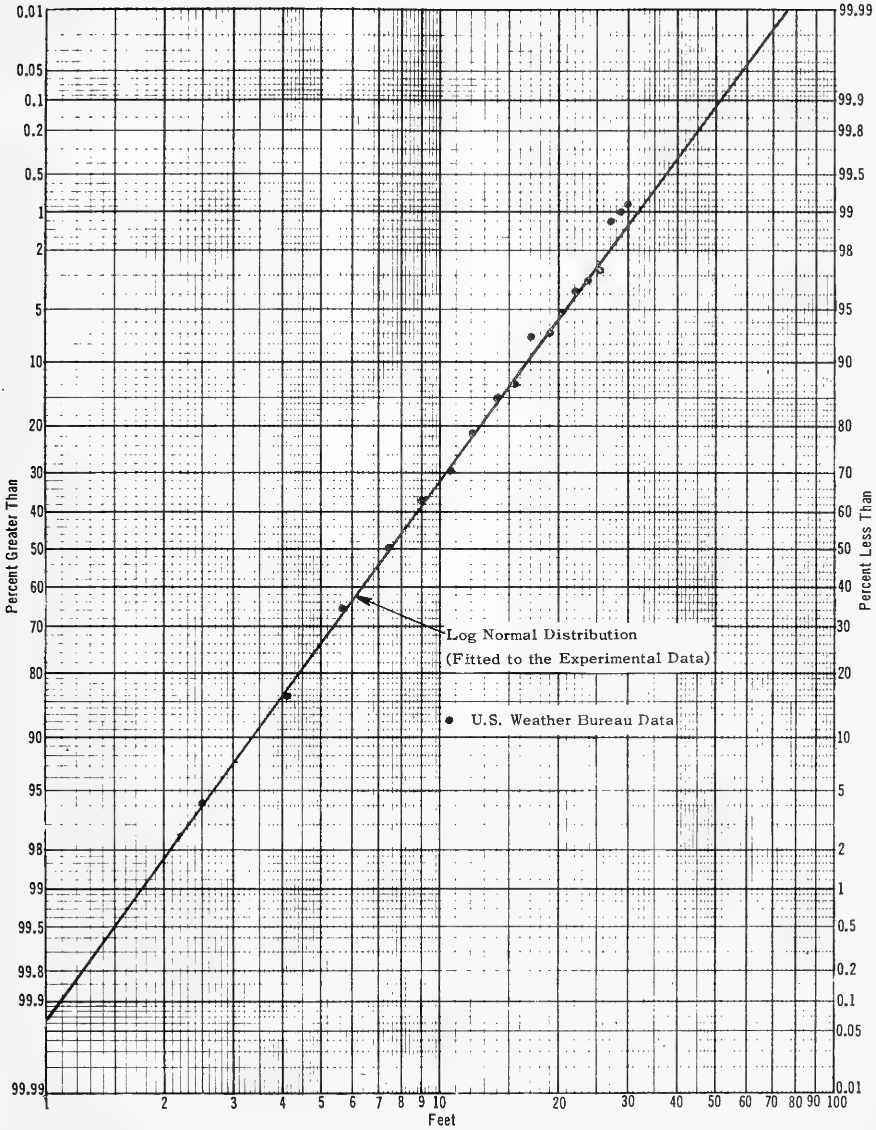


Figure 23 - Distribution of Characteristic Wave Heights at Station J

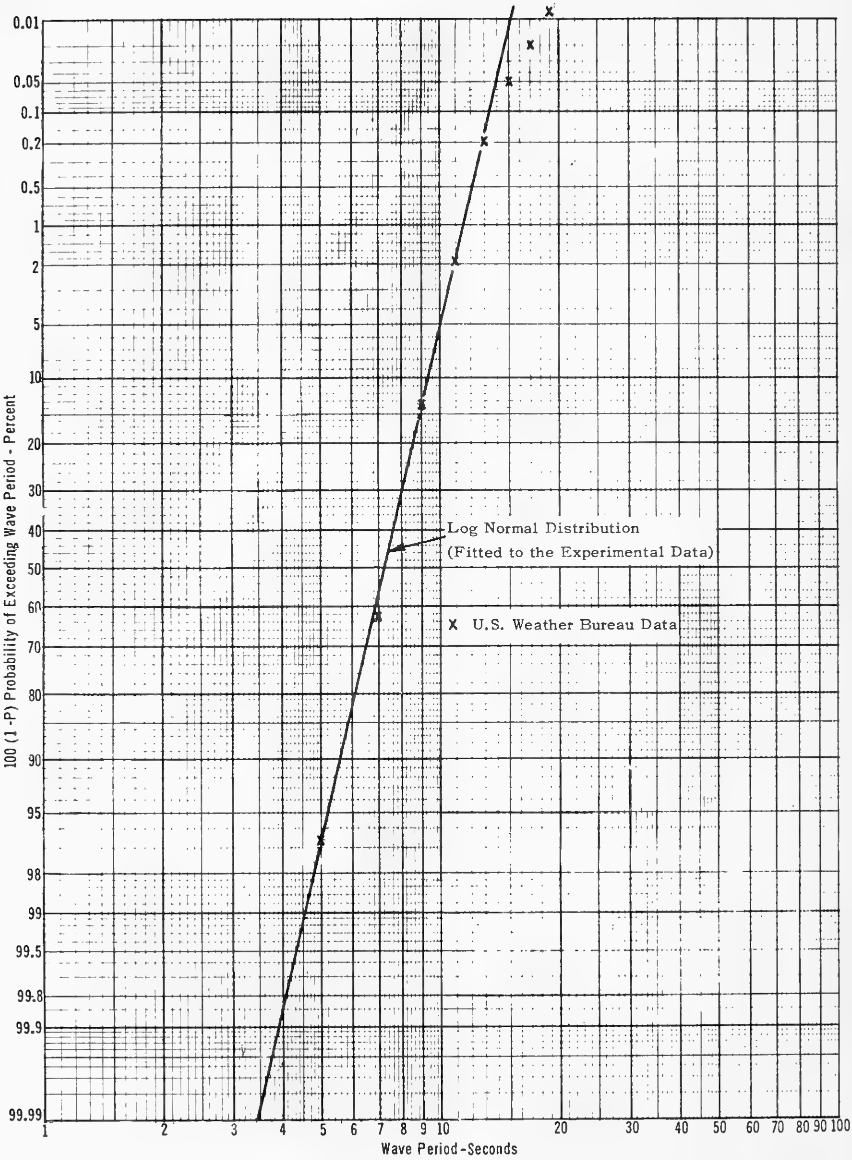


Figure 24 - Distribution of Characteristic Wave Periods at Station J

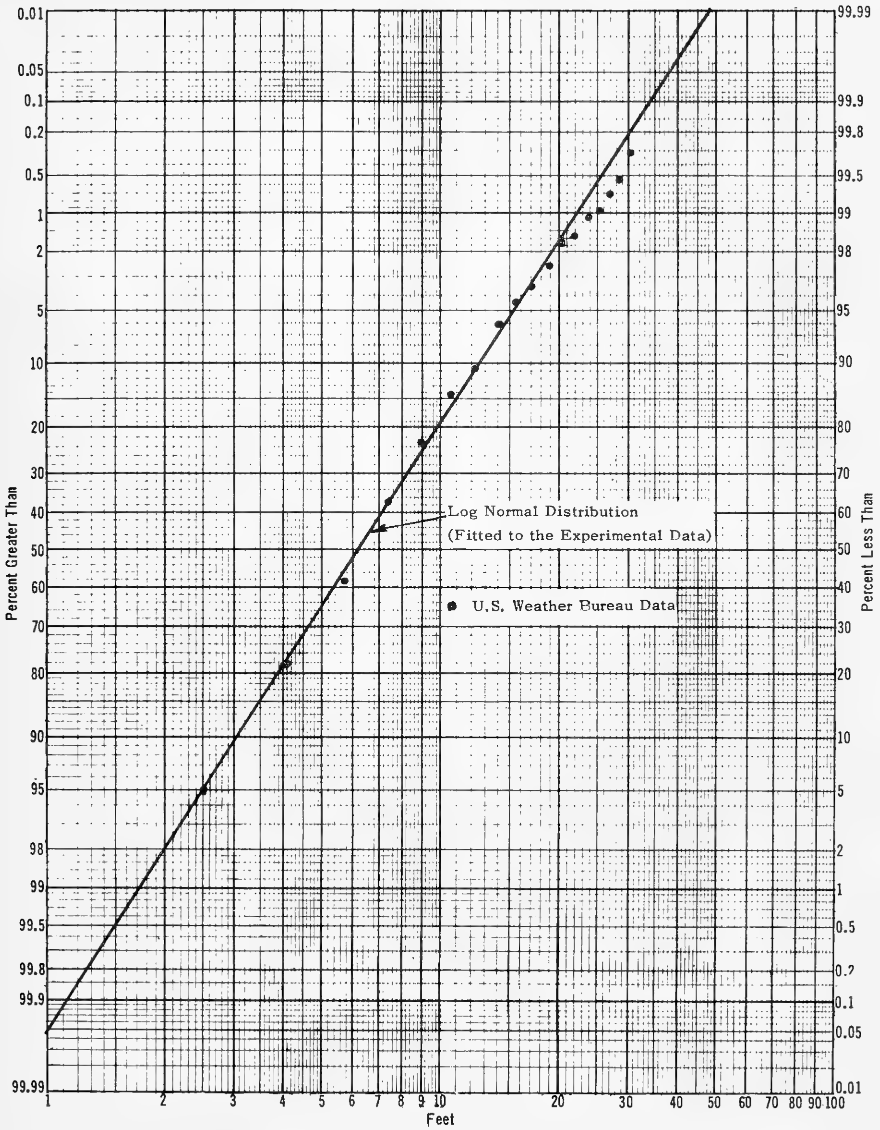


Figure 25 - Distribution of Characteristic Wave Heights at Station K

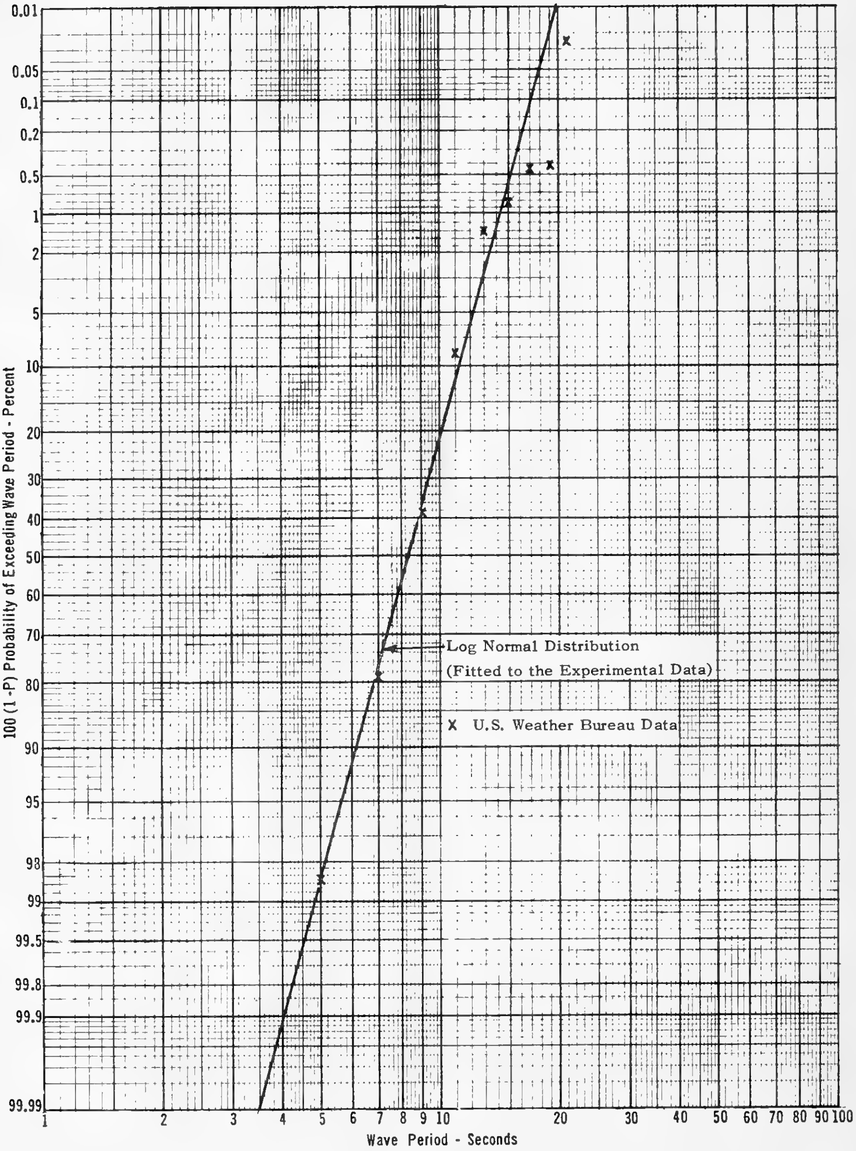


Figure 26 - Distribution of Characteristic Wave Periods at Station K

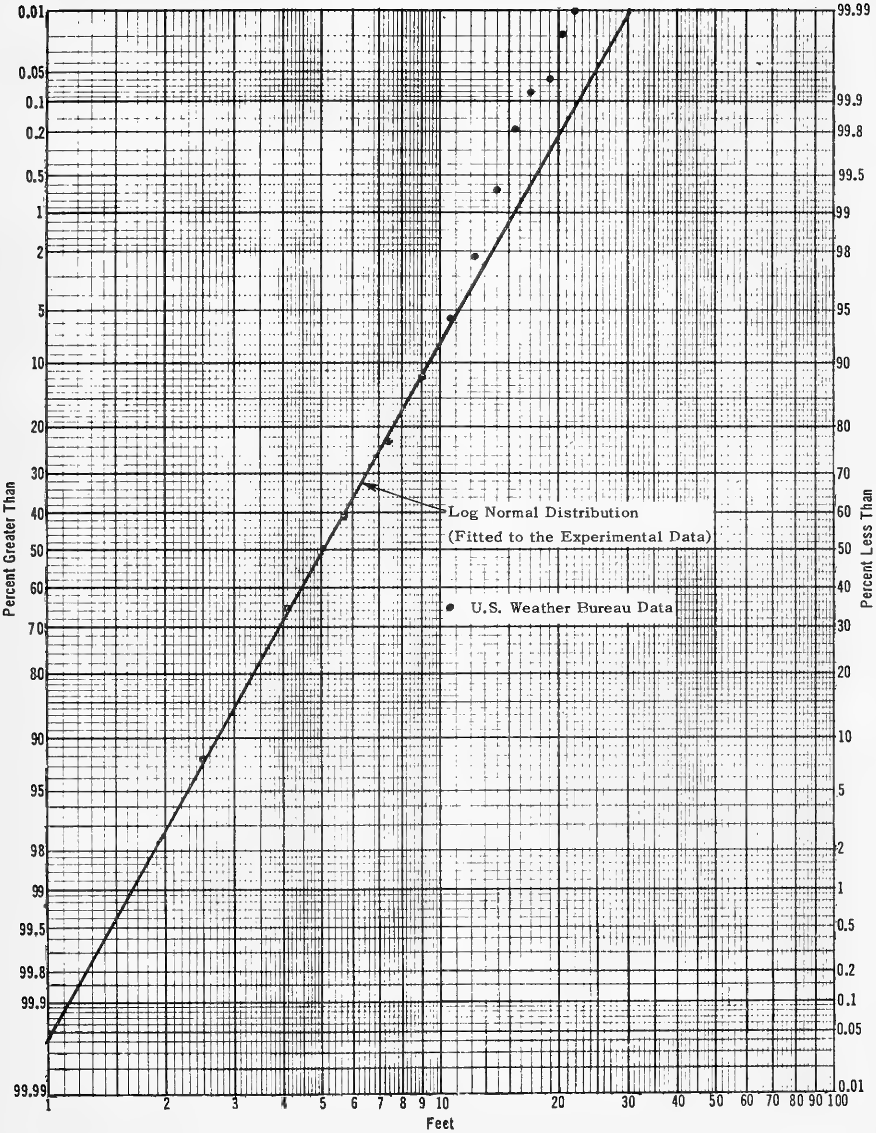


Figure 27 - Distribution of Characteristic Wave Heights
at Station M

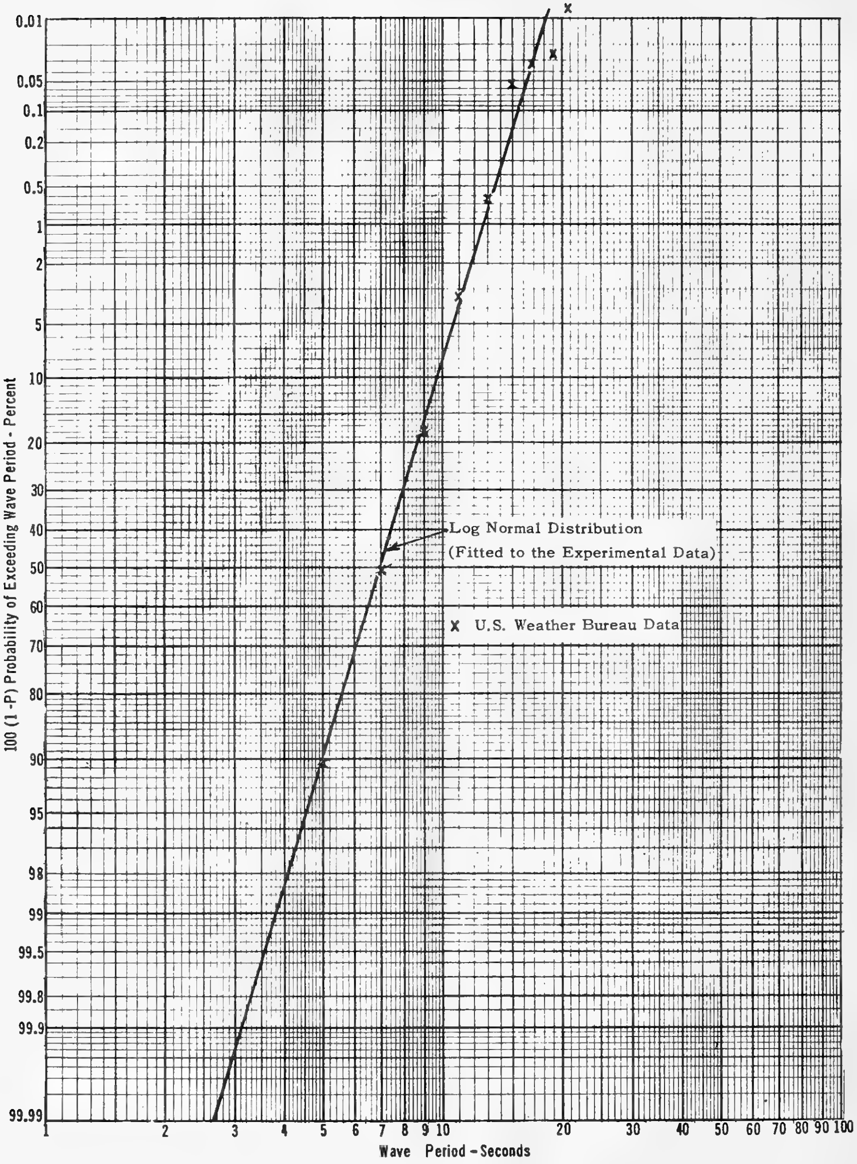


Figure 28 - Distribution of Characteristic Wave Periods at Station M

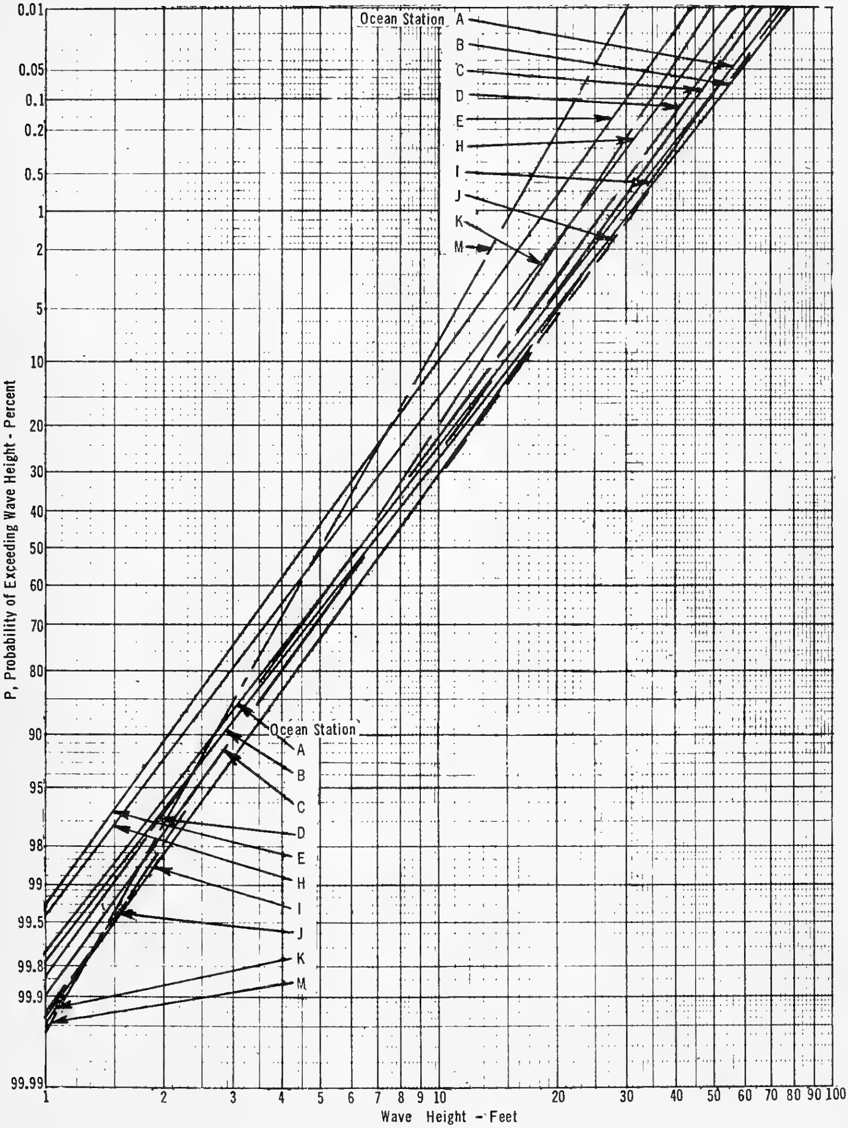


Figure 29 - Distribution of Characteristic Wave Heights
for Ten Ocean Stations

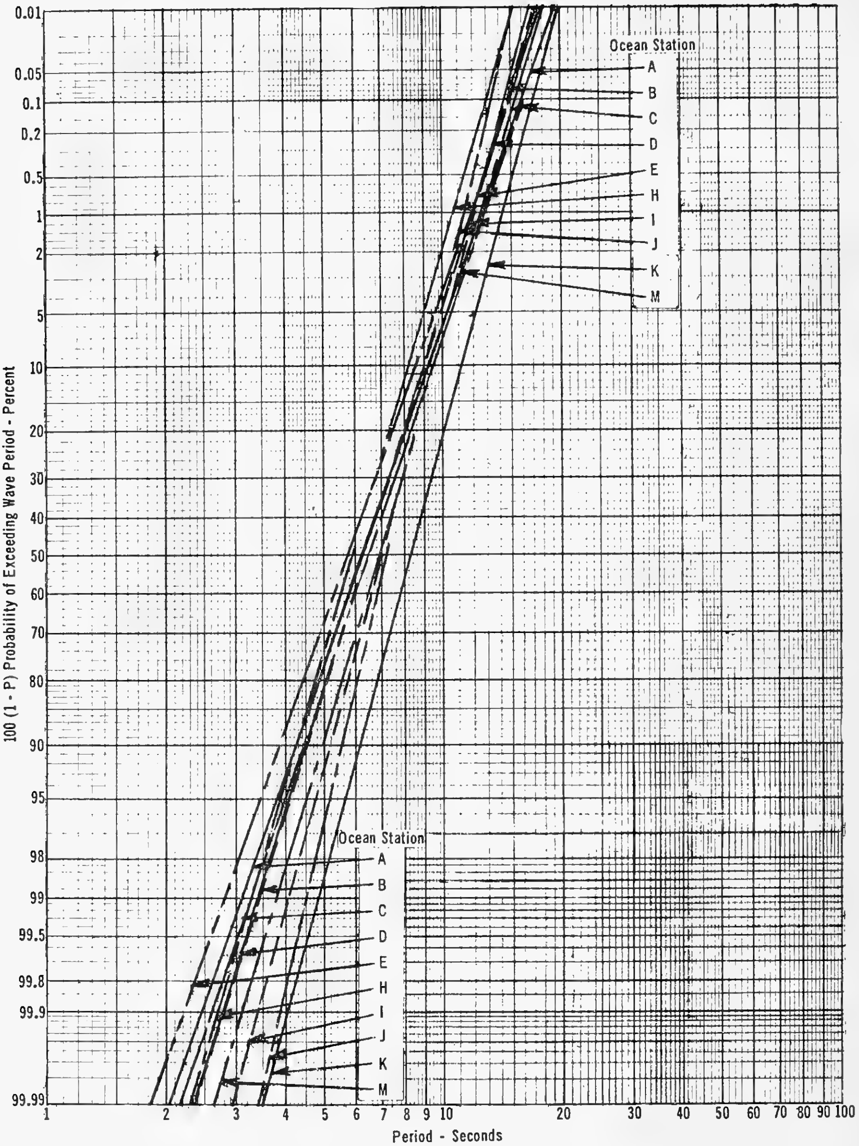


Figure 30 - Distributions of Characteristic Wave Periods for Ten Ocean Stations

APPENDIX A

PROCEDURE FOR ANALYSIS OF STEREOPHOTOGRAPHS

SELECTION OF PHOTOGRAPHS

Many hundreds of stereophotographs were taken during the sea trials of the VALLEY FORGE and the UNIMAK. Since a photogrammetric analysis of each photograph is time-consuming and expensive, a limited number were selected for analysis. The purpose of the stereo analysis was (1) to provide quantitative data against which the visual estimates of trained observers could be checked and (2) to provide a quantitative measure of the sea conditions for correlation with simultaneous measurements of the ship's response to the sea.

Of the many stereophotographs available, sixty were selected for analysis; see Table 4. The selection was made to satisfy both the requirements just stated and to cover as wide a range of sea conditions as practicable. The accuracy of a wave profile varies with the distance from the camera to the profile. The average accuracy is about ± 0.5 ft at a distance of 2000 ft and is better than this at shorter distances.

ANALYSIS OF STEREOPHOTOGRAPHS

Each of the selected pairs of stereophotographs was converted into sea surface profiles by photogrammetric specialists at the Naval Photographic Interpretation Center and Navy Hydrographic Office. The Wild A5 Audograph and Zeiss Stereo Planigraph Model C5 were used by the respective agencies, and vertical mapping techniques were adapted to this horizontal application. Next the sea surface profiles were analyzed by the Oceanographic Division of the Hydrographic Office. The procedures devised for this analysis are given in the following sections.

PROFILE DETERMINATION FROM STEREOPHOTOGRAPHS

Sea surface profiles were determined from the stereophotographs by the following procedure:

1. Draw the first profile at a distance not less than 250 ft from the camera stations.

2. Draw successive profiles at increments of 125 ft.

3. Draw as many profiles as possible. The profiles should be approximately 1 in. apart on the manuscript.

4. Maintain the horizontal scale constant. Give horizontal scale factor.

5. Exaggerate the vertical scale (wave heights) as much as possible. Give the vertical scale factor.

6. Use number at the left of the profile to indicate the distance from camera station in feet.

7. Use number at the right of the profile to indicate the estimated accuracy of the vertical distance in inches.

8. Label doubtful profiles with "?" at the left.

9. Leave the masked portions of the profiles blank.

10. Indicate the dimensions of the sea surface over which the profiles have been drawn.

PROCEDURE FOR SELECTION OF PROFILES TO BE ANALYZED

Each set of profiles should be analyzed separately. The procedure is as follows:

1. Record the date and time of observation and any other pertinent information.

2. Label the most distant profile with the number 1 and the succeeding profiles 2, 3, 4 . . . n. Profile 1 is the first useful piece of information.

3. Compare Profile 2 with Profile 1. If there is a distinct similarity in shape between them for a distance greater than one-half the length of Profile 2 (for a noninterrupted distance), then discard Profile 2. Next compare Profile 3 with Profile 1 and test for acceptability in the same manner. Some Profile k is eventually examined which has the property that nowhere does half of its length coincide in shape with Profile 1. Profile k is the second usable piece of information.

4. Compare Profile k + 1 with Profile k in the manner described, and continue the process until all profiles have been exhausted. The net result is p usable profiles, where $p \leq n$.

PROCEDURE FOR DATA REDUCTION FROM WAVE PROFILES

All the profiles from one pair of stereophotographs comprise a set. All the sets which pertain to the same sea state comprise a group. Analyze each set separately.

On the first profile draw a horizontal line which, as nearly as possible, divides the wave record in half. This line may be thought of as mean sea level.

Definition of Large Wave Lengths

Beginning from the left, the horizontal line is crossed by portions of the profile which have alternately negative and positive slopes below the line and vice versa above the line. This creates bounded areas which possess minima and maxima.

1. Above the horizontal line mark with an x the greatest maximum in each enclosed area, below the line mark with an x the lowest minimum in each enclosed area.

2. Record and number the horizontal distances between successive minima in every other column of a data sheet. These will be the lengths of the predominant waves.

3. Label the columns "relative wave length."

4. Record and number the vertical distance between each minimum and the succeeding maximum on another data sheet, in every other column, and label these columns "relative wave height." All wave lengths less than a specified magnitude will not be considered, nor will the wave heights associated with these lengths.

5. Record the wave lengths and wave heights of all waves corresponding to lengths larger than the specified minimum value. The wave lengths will be taken to be the distance between successive minima of the wave surface. The corresponding wave height is taken to be the distance between a minimum and the succeeding maximum.

6. Multiply each number by a given constant (scale factor) and record it in the blank columns which are then labeled "absolute wave length" or "absolute wave height."

Definition of Small Wave Lengths

1. Start from the left of the record again and measure the horizontal distances between *all* the successive minima, regardless of whether the profile is bounded by mean sea level or not. Measure the associated heights.

2. Observe the rules set forth in Steps 4 and 5 for the definition of largest wave lengths.

PROCEDURE FOR DETERMINING THE LENGTH CUTOFF VALUE

1. For the time and geographic location at which the stereophotographs were taken: determine the wind field, including velocity, duration, and fetch (distance over which wind has blown).

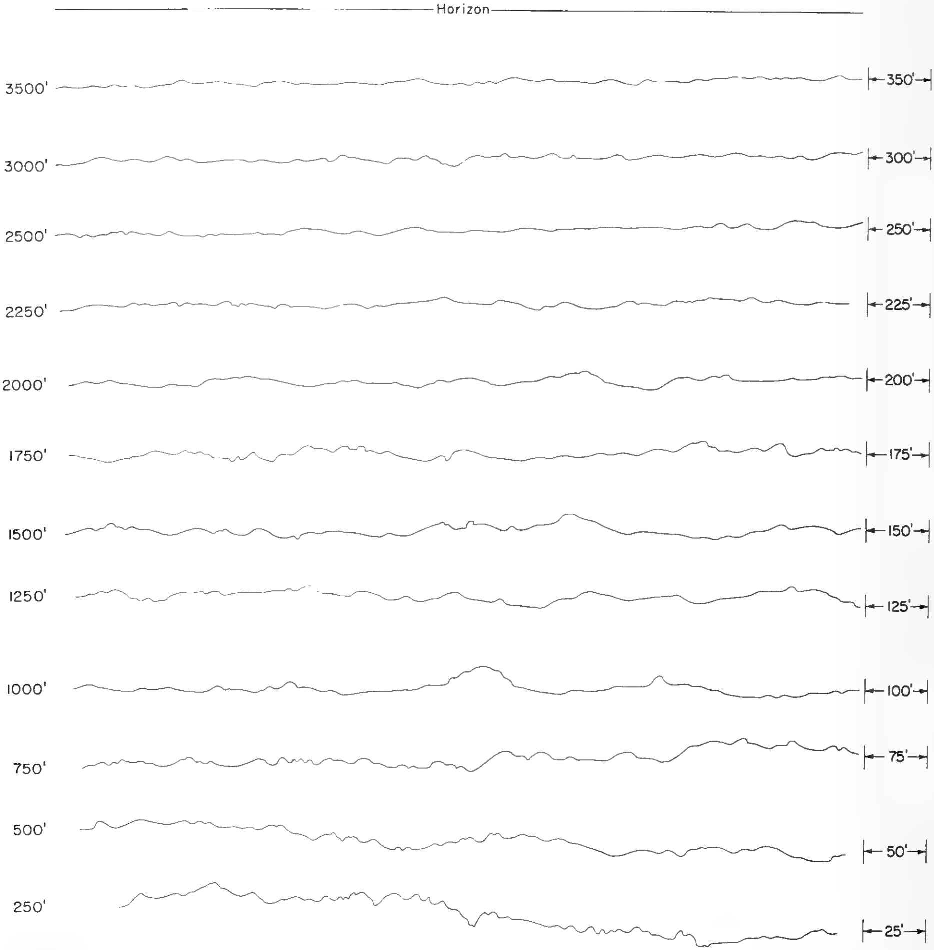
2. With the wind data enter Pierson's Co-Cumulative Duration Spectra[8] and determine the energy value E.

3. With 3 percent of the E value determined in Step 2, enter the Co-Cumulative Fetch Spectra of Reference 8 at the wind velocity determined in Step 1, and obtain a frequency cutoff value f.

4. Using the f value obtained in Step 3, determine the length cutoff value in feet by means of the following conversion formula:

$$L = \frac{3.41}{f^2}$$

5. In measuring heights and lengths from the sea surface profiles do not record any values for waves whose length is less than the value L determined from Step 4 above.

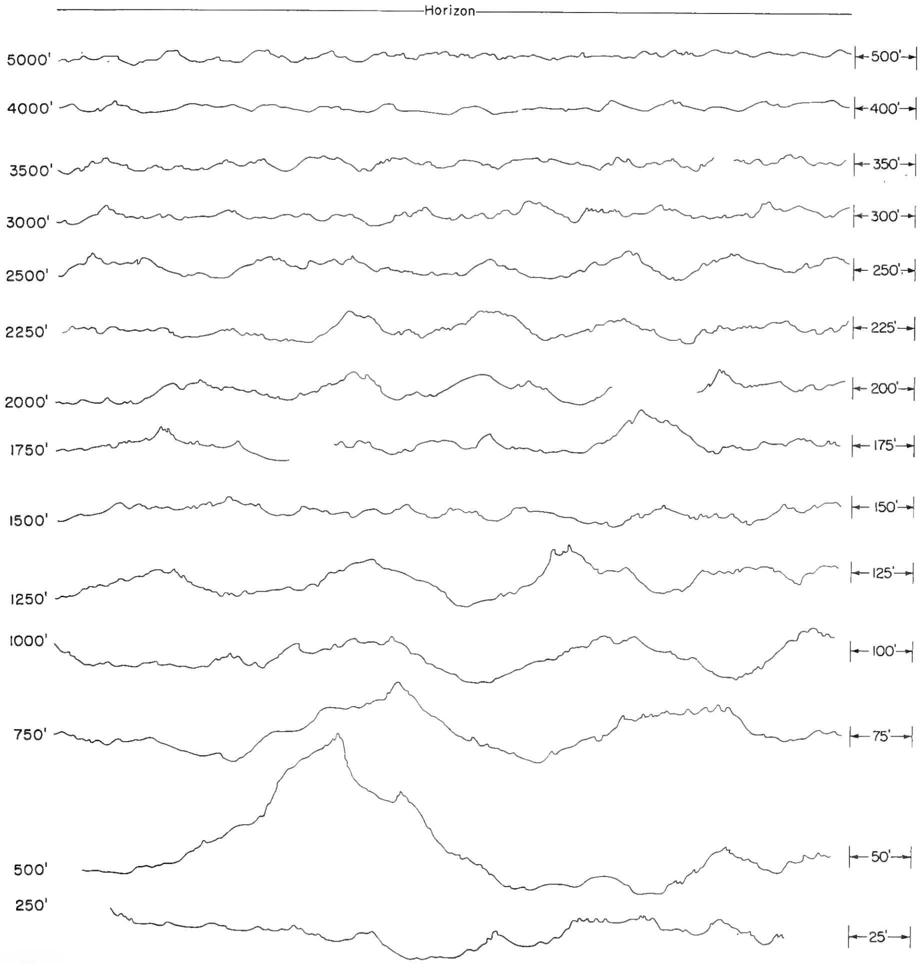


Prepared by the Photogrammetry Branch, Division of Chart Construction U.S. Navy Hydrographic Office for the David Taylor Model Basin using stereophotogrammetric methods (stereoplanigraph).

Model No E5 - 0071
Compilation Date 13 Feb 1956

Numbers at left of profiles represent distances from camera stations in feet. Numbers at right of profiles represent the horizontal scale.
Vertical scale is exaggerated 5 times the horizontal scale.
Profiles are arbitarily spaced 3/4 inch apart to facilitate calculations.

Figure 31 - Wave Profile E-0071



Prepared by the Photogrammetry Branch, Division of Chart Construction, U.S. Navy Hydrographic Office for the David W. Taylor Model Basin using stereophotogrammetric methods (stereoplanigraph).

Model No J10 - 0040
Compilation Date 1 Feb 1956

Numbers at left of profiles represent distances from camera stations in feet. Numbers at right of profiles represent the horizontal scale.
Vertical scale is exaggerated 5 times the horizontal scale.
Profiles are arbitarily spaced 3/4 inch apart to facilitate calculations.

Figure 32 - Wave Profile J-0040

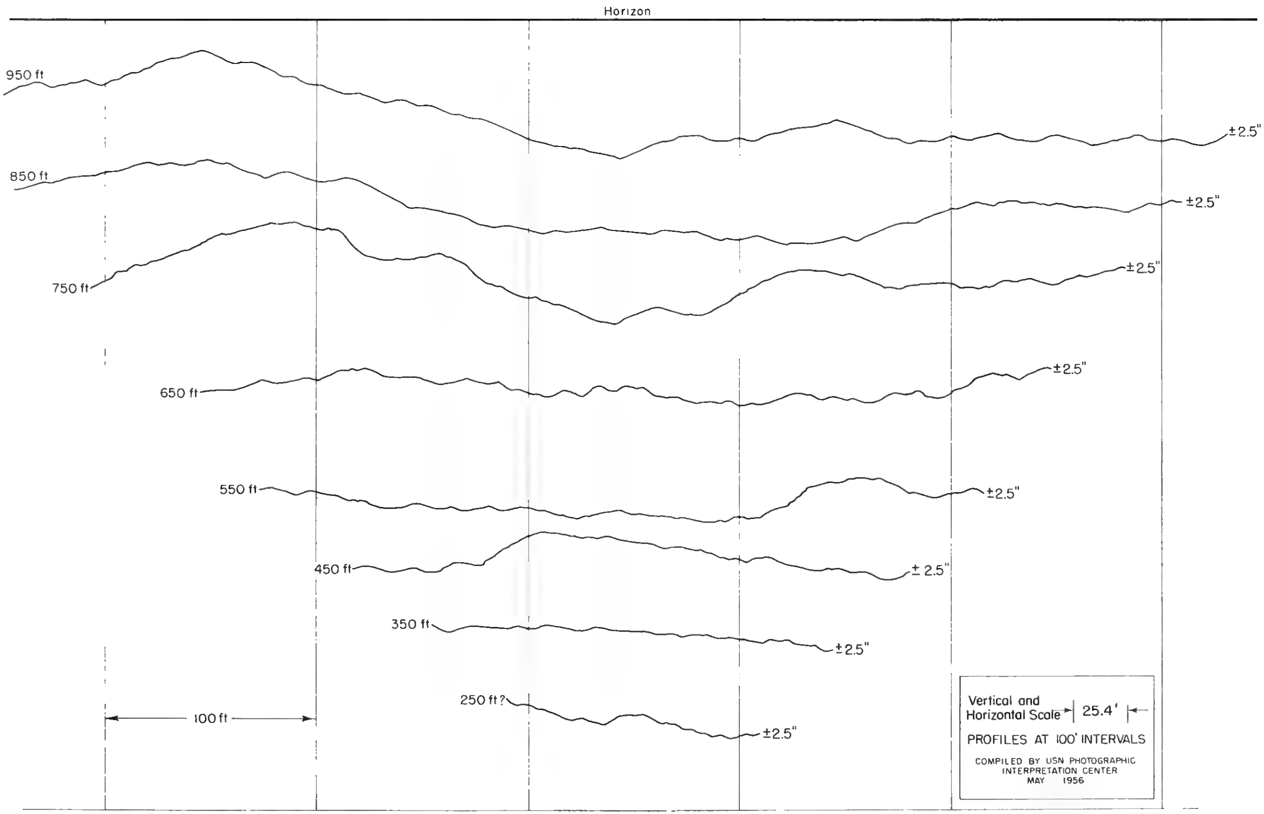


Figure 33 - Wave Profile U-541

APPENDIX B

COMPARISON OF WAVE STATISTICS

It is of interest to utilize the wave data obtained during the sea tests of the VALLEY FORGE and the UNIMAK to gain some insight into the validity of a few of the assumptions often made in the forecasting and analysis of ocean waves. For this purpose the following items were computed:

(a) The characteristic wave height was determined from the wave profiles.

(b) The wave profiles were analyzed according to the method outlined in Appendix A to obtain the frequency distribution of individual wave heights and of the corresponding wave lengths above a certain cutoff length.*

(c) The mean value of the squares of all individual wave heights corresponding to waves longer than the cutoff length was determined from the data obtained under Item (b). This value is denoted by the symbol E.

(d) The average value of the upper third of the waves having the largest magnitudes (significant wave height) was determined from the data obtained under Item (b).

(e) The average value of the characteristic wave heights determined by the shipboard observers was tabulated.

(f) The Hydrographic Office computed** a theoretical value of E on the basis of the distribution of wind velocities that generated the sea. The method of Reference 8 was used, according to which E is proportional to the area under the power spectrum of the sea. These values are also given in Table 4.

In Figure 6, Item (a) is plotted against Item (e). In Figure 34, Item (c) is plotted against Item (f). In Figure 35, Item (a) is plotted against the square root of Item (f). In Figure 36, Item (d) is plotted against Item (e).

Figure 6 indicates that trained shipboard observers can, on the average, estimate the heights of the predominant waves reasonably well.

The value of E determined from the wind data should agree with the E obtained from the wave profiles provided the theory of Reference 7 is valid, a narrow sea spectrum exists, swell is a negligible factor, and the stereophotograph covered a representative area of the ocean. One may expect considerable deviations from these assumptions; for example, the sea surface profiles sometimes indicate considerable deviation from a narrow spectrum as well as the

---

*The cutoff length is that wave length below which lies three percent of the area under the power spectrum. For a detailed description of the power spectrum concept and a method for computing E from wind data, see Reference 7. The numerical values of E given in this report are four times those of Reference 7, since we are dealing with crest-to-trough wave heights rather than with wave amplitudes.

**This computation does not take account of swell that may have been present in the wave system.

presence of swell. Nevertheless Figure 34 suggests a linear relationship between the E's determined by two independent methods. It is concluded that the wind data may be used to determine the sea state, at least qualitatively.

Figure 35 suggests a linear relation between the characteristic wave heights h, as determined by the method of References 5 and 6, and the square root of E, except for very severe sea conditions.

Figure 36 would be expected to indicate a linear relationship between the significant wave height and the visual shipboard estimates of the characteristic wave height since the latter is presumably proportional to E in accordance with the indications of Figures 34 and 35. Figure 36 does not contradict such a linear relationship. The scatter of values is most likely due to errors in the determination of the significant wave height, inasmuch as Figure 6 shows that the visual shipboard estimates are reasonably correct. The computed value of the significant wave height, Item (d), is very much a function of the cutoff length. The UNIMAK stereophotographs did not furnish sufficient data, in the opinion of the Hydrographic Office oceanographers, to permit an evaluation of the significant wave height; and therefore these data were not available for the plots.

From an overall point of view, consideration of Figures 6, 34, 35, and 36 suggests that

1. The methods of Reference 8 may be applied to make a rough estimate of wind waves.

2. Trained observers can, on the average, make reasonably accurate observations of the heights of the larger, well-formed waves that are present in a given sea.

3. The characteristic wave height reported by trained observers is proportional* to the square root of the statistic E, corresponding to the sea state considered, except for severe sea states.

4. The so-called "significant" wave height is not particularly significant since it is difficult to compute, although it is statistically well defined. The average height of the predominant wave heights,** as reported by observers, is physically more meaningful and is more easily reproduced on repetitive estimates than is an estimate of the significant wave height.

---

*The empirical relationship between the characteristic wave height h reported by the observers and the statistic $\sqrt{E}$, for the data plotted in Figures 34 and 35, is approximately as follows:

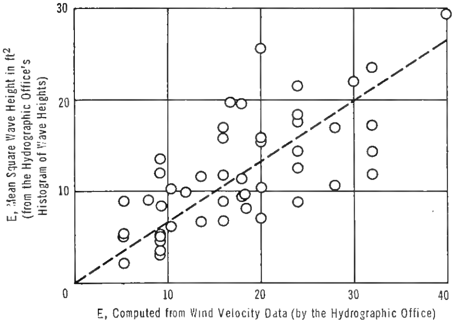
$$h \approx 1.53 \sqrt{E} \quad \text{when } E \text{ is derived from wind data, and}$$

$$h \approx 1.88 \sqrt{E} \quad \text{when } E \text{ is derived from the wave data.}$$

**Here designated by the term "significant wave height."

Figure 34 - Scatter Diagram for Comparison of Values E Derived from Wind and Wave Data

Each plotted point corresponds to the analysis of one stereophotograph. The computation of E from the wind data neglects the presence of decaying swell. The values plotted are taken from the last two columns of Table 4.



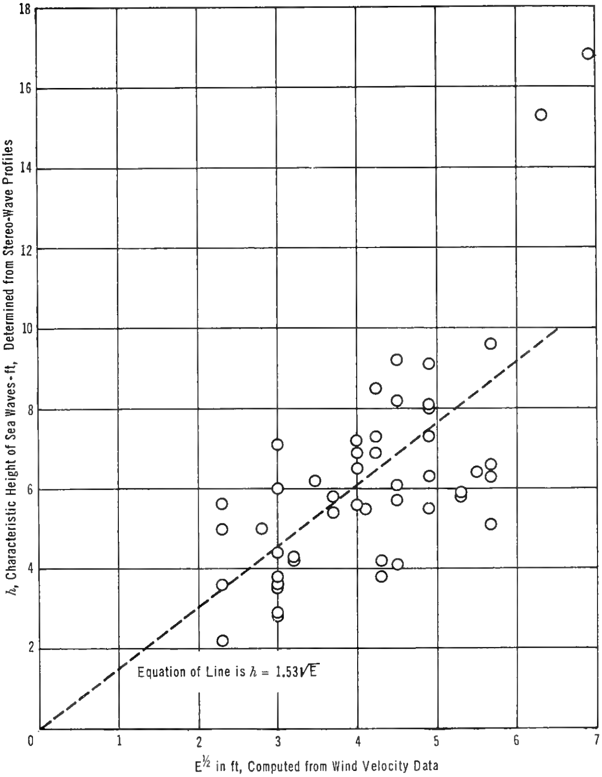


Figure 35 - Scatter Diagram Showing a Plot of Characteristic Wave Height Against the Statistic $E^{1/2}$

The plotted data are taken from Table 4, Columns 9 and 14.

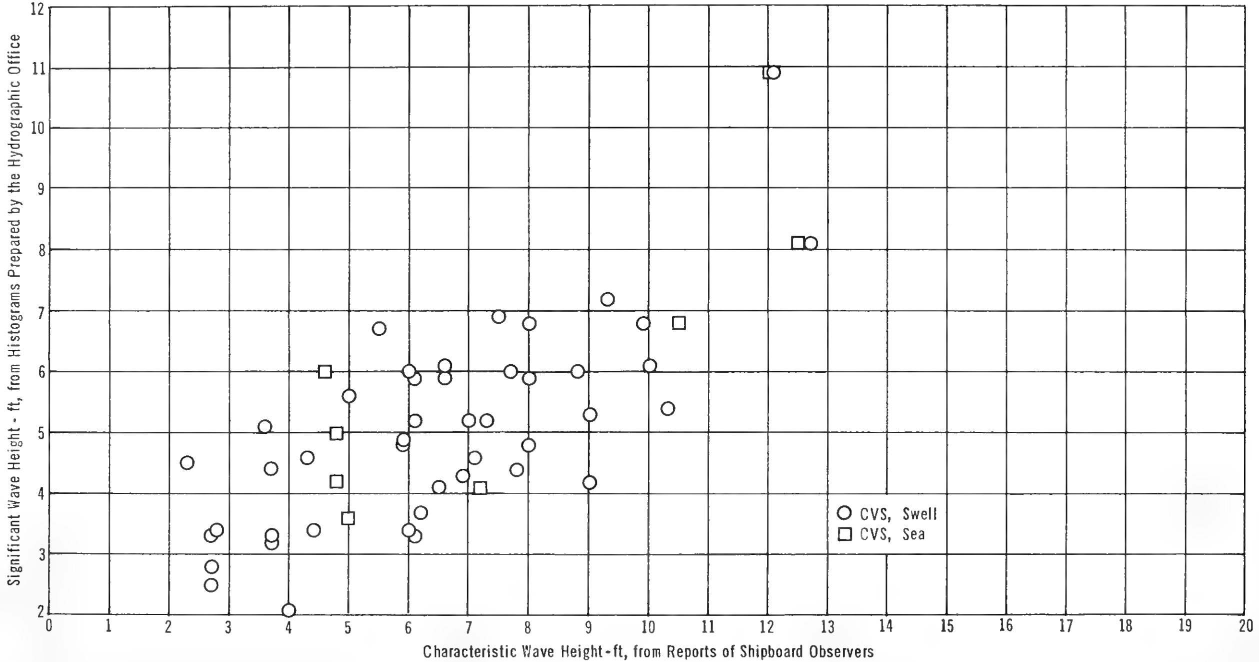


Figure 36 - Scatter Diagram Showing a Plot of Significant Wave Height Against Characteristic Wave Height

The plotted data are taken from Table 4, Columns 7 and 12.

APPENDIX C

SAMPLE CALCULATION

In fitting a log-normal distribution to the data on wave heights and wave periods given in Tables 1 and 2, a difficulty arises due to the fact that the lower limit of the lowest class is zero and inasmuch as the logarithm of zero is minus infinity, it is not possible to assign a mean value of the logarithm of the height or period for the lowest class. One way of circumventing this difficulty would be to use the logarithm of the algebraic mean of the class limits. A less arbitrary solution, used here, is to omit the relatively few values falling into the lowest class and treat the remaining truncated data by the standard statistical method[11] for fitting a truncated normal distribution. In the statistical sense used here, a truncation means that only values larger than a specified lower limit are used. To fit a log-normal distribution to the truncated data requires the calculation of the mean value and the standard deviation from the truncated data.

The method and tables of Reference 11 are applied as indicated below. In the calculations, the symbols used are σ for standard deviation and y and z for parameters needed to enter Table IX of Reference 11, z being an estimate of the point of truncation.

Following the procedure outlined on page 29 of Reference 10, we have from Table 6:

$$y = \frac{(\Sigma N\omega^2)(\Sigma N)}{2(\Sigma N\omega)^2} = \frac{(459.864)(12{,}362)}{2(2071.605)^2}$$

$$y = 0.6623$$

and from Table IX, $z = -1.293$ at $y = 0.6623$ and $g(z) = 0.6736$.

$$\sigma \approx s = \frac{\Sigma N\omega}{\Sigma N}\, g(z) = \frac{2071.6\,(0.6736)}{12{,}362} = 0.1129$$

From Table II of Reference 11, at $z = -1.293$, we obtain

Theoretical percent of truncation = 9.80

Mean value of $\omega = \bar{\omega} = -z\,s$ $= 1.293\,(0.1129) = 0.1460$

Mean value of $h = h_{50} = h_T + \bar{\omega}$ $= 0.6990 + 0.1460 = 0.8450$

The value of h corresponding to $P = 0.975 = h_{97.5} = h_{50} + 1.96\,(0.1129) = 1.0663$

The value of h corresponding to $P = 0.025 = h_{2.5} = h_{50} - 1.96\,(0.1129) = 0.6237$

Taking the antilogarithms, we have

The period corresponding to $P = 0.975$ = antilog (1.0663) = 11.7 sec

The period corresponding to $P = 0.500$ = antilog (0.8450) = 6.99 sec

The period corresponding to $P = 0.025$ = antilog (0.6237) = 4.21 sec

Any two of these three sets of values (P, x) determine the straight line (log-normal distribution) plotted in Figure 28.

The values of the mean and variance listed in Table 5 are in terms of natural logarithms. Using the conversion $\log_e x = 2.3026 \log_{10} x$, we have

Standard Deviation	$(\log_e x)$	$= 2.3026\ (0.1129) = 0.260$
Variance	$(\log_e x)$	$= (0.260)^2 = 0.0676$
Mean Value	$(\log_e x)$	$= 2.3026\ (0.8450) = 1.946$

TABLE 6

Long-Term Distribution of Estimated Wave Periods at Station M

Mean value and standard deviation were calculated from data given in Table 2. The data are truncated at a wave period of 5 sec.

Wave Period sec x	$\text{Log}_{10}\ x$ at End of Class Interval	$\text{Log}_{10}\ x$ at Center of Class Interval h	$(\text{Log}_{10}\ x) - h_T$ Measured from Point of Truncation $\omega = h - h_T$	$\omega^2 = (h - h_T)^2$	N Number of Observations Falling within the Class	Percent of Variations Falling within Class Interval	$N\omega^2$	$N\omega$	Cumulative Percent
0						9.80			9.80
5	0.6990*								
		0.7721	0.0731	0.0053	5,477	39.96	29.028	400.369	49.76
7	0.8451								
		0.8997	0.2007	0.0403	4,374	31.92	176.272	877.862	81.68
9	0.9542								
		0.9978	0.2988	0.0893	2,051	14.97	183.154	612.839	96.65
11	1.0414								
		1.0777	0.3787	0.1434	382	2.79	54.779	144.663	99.44
13	1.1139								
		1.1450	0.4460	0.1989	69	0.503	13.724	30.774	99.943
15	1.1761								
		1.2033	0.5043	0.2543	3	0.022	0.763	1.513	99.965
17	1.2304								
		1.2546	0.5556	0.3087	1	0.007	0.309	0.556	99.972
19	1.2788								
		1.3005	0.6015	0.3618	4	0.029	1.447	2.406	100.001
21	1.3222								
		1.3222	0.6232	0.3884	1	0.007	0.388	0.623	100.008
>21	1.3222								
					12,362		459.864	2071.605	

*This value is the point of truncation h_T.

Note: This data is the basis for Figure 28.

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STATISTICS ON WAVE HEIGHTS AND PERIODS FOR THE NORTH ATLANTIC OCEAN, by R.L. Brooks and N.H. Jasper. Sep 1957. ii, 53p. figs., tables, refs. (Research and development report) UNCLASSIFIED

This report gives the frequency distributions of wave heights and wave periods obtained from weather ships stationed in the North Atlantic, together with an evaluation of the reliability of the visual wave-height estimates comprising the basic data from which the distributions are derived. Visual estimates are compared with values determined from stereophotographs. An additional check is provided by wavemeter measurements. It is shown that a log-normal distribution is applicable to the frequency distribution of wave heights experienced over a typical year and that this distribution is a useful guide to the determination of the incidence of a particular sea state at a given location.

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2. Ocean waves – Frequency measurement – Statistical data
3. Ocean waves – Atlantic Ocean (North)
I. Brooks, R.L.
II. Jasper, Norman H.
III. NS 731-037

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