REPORT NO. 1370863

PROJECT TRIDENT

TECHNICAL REPORT

STRESS ANALYSIS OF SHIP-SUSPENDED HEAVILY LOADED CABLES FOR DEEP UNDERWATER EMPLACEMENTS

ARTHUR D. LITTLE, INC.

35 ACORN PARK CAMBRIDGE, MASSACHUSETTS

DEPARTMENT OF THE NAVY

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NObsr-81564 S-7001-0307

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PREFACE

This is one of a series of Technical Reports being issued by Arthur D. Little, Inc., under contract NObsr-81564 with the Bureau of Ships as part of the TRIDENT Project.

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I. SUMMARY

A. PURPOSE AND SCOPE

This report relates to the operational aspect of lowering heavy array structures to the bottom of the deep ocean. Because of the weight of the array and the depth to which it will be lowered, the lowering cable can be expected to be under a high stress. Ocean currents, surface waves, and motions of the lowering vessel will also contribute to the stress placed on the lowering cable. Therefore, a critical design problem may exist for the lowering cable.

The objective of this report is to develop a reasonable theory on which to base the ultimate design of the lowering cable. The study is limited to the case of a single cable lowering the array vertically. Each factor contributing to the stress on the cable is considered separately, and the conditions under which the analyses are valid and of practical significance are given. From the results of this theory some general conclusions are drawn as to the feasibility of the operation with regard to dynamically optimum array configurations, stability of the lowering platform in a rough sea and degree of roughness of the sea (sea state). The results of the study are presented in plot form involving a non-dimensional maximum (dynamic) stress versus a non-dimensional frequency for various mass and drag parameters.

Other methods of operation, which are not treated in this report, but which will be considered in later investigations, are:

(1) Cancel out most of the dynamic inputs to the cable by an auxiliary mechanical system on the vessel. There are two crucial problems here: the horsepower of the driver of such a system and the rapidity with which the overall system can respond.

(2) Introduce considerable damping along the cable, which will have the effect of flattening out resonant peaks. This can be effected by attaching bluff bodies to the cable at certain intervals. The drag of the water on them will give the desired friction. This idea seems quite attractive, since these bodies can also provide buoyancy to reduce the rather high static stresses in the cable, but its plausibility must be investigated quantitatively.

(3) Use more than one cable. However, it should be borne in mind that each cable must actually be designed to withstand much more than an equal share of the total load because of some adverse conditions that can readily occur. Also, the cables can be tangled, a situation which can lead to serious problems.

B. CONCLUSIONS AND RECOMMENDATIONS

The static stresses due to the weight of the array and cable can be taken care of by practicable cables. For a maximum depth of 20,000 feet and 50- and 100-ton steel arrays, the maximum static stress in a steel cable of three square inches metallic area is 100,000 and 130,000 psi, respectively, when no buoyancy is introduced anywhere. For the same depth and a 15-ton aluminum array (which in water weighs about 10 tons), the maximum static stress in a steel cable of one square inch metallic area is 100,000 psi. The ultimate tensile strength of such cables is about 220,000 psi; therefore, these cables can handle these stresses with a factor of safety of about two. However, the above static stresses must be reduced, because higher factors of safety (about 3) are usually required for such an operation, and in addition, there are other stresses (dynamic) in the cable.

Static stresses can be reduced by increasing the metallic area of the cable or by making the array and cable more buoyant. Since a large portion of the static loading is due to the weight of the cable (the maximum length of cable which can hold itself without any factor of safety is only 62,000 ft), the most effective method is to introduce buoyancy in the cable. An equivalent way of effecting this is by using a tapered cable.

Nylon type ropes are desirable in this respect, because they are much lighter than steel ropes while their ultimate strength is just about as high as that of the strongest steel ropes. The velocity of sound of nylon ropes is also much smaller than that of steel ropes. With respect to the dynamic stresses that can be induced by a rough sea, these two factors (small weight and velocity of sound) render nylon ropes undesirable. Of course, the small modulus of elasticity, characteristic of nylon ropes, is a favorable factor and can probably compensate for the undesirable effects of the above two factors. In any case, in this operation there are some environmental hazards which make steel ropes preferable to nylon ropes.

The effects (stress and array offset) of reasonable ocean currents are negligible, provided that the vertical forces due to gravity minus buoyancy are much larger than the horizontal drag forces due to the currents. This will be the case in practice, because buoyancy must not be used to the extent of

canceling the gravity forces completely. It is highly desirable to have the cable in an appreciable static tension so that compressive dynamic stresses will not buckle the cable. Buckling of the cable is not in itself dangerous, but the formation of kinks, which can occur when the cable buckles appreciably, is. Some consideration of this problem is given in Section VI. Perhaps, the problem should be pursued further.

The motions of the vessel in a rough sea (heave, roll, etc.) will induce dynamic stresses in the cable. Due to its great length, the cable cannot be considered nonflexible for the kind of inputs that exist in a realistic sea. In fact, for sinusoidal inputs, resonance can occur at frequencies as small as 1.5 rad/sec. These frequencies correspond to periods as large as four seconds, which are included in the spectrum of a disturbed sea.

In order to obtain an accurate value of the maximum dynamic stress induced in the cable by a regular (sinusoidal) surface wave, the dynamics of the vessel, cable, and array must be analyzed simultaneously. The parameters necessary for such an analysis are too many, and the problem can become formidably long and difficult. However, it can be safely assumed that the dynamic loading of the vessel by the cable and array is negligible, except possibly when the cable is attached to the vessel by a boom extended from the side. Thus, the problem can be simplified by considering the dynamics of the vessel and of the cable-array system separately.

A theory is developed which takes into account the propagation of longitudinal elastic waves in the cable and the complete dynamics of the array in water. It is shown that friction on the cable by the surrounding water is small and can be neglected, even near resonance, since friction on the array is much larger. The drag on the array must be taken as quadratic because of the large Reynolds number involved. This is the only nonlinear term in the theory. It is linearized to a quasi-quadratic form. Thus, a formula is derived for the maximum dynamic stress due to a sinusoidal input. It gives this stress in a parametric form as a function of the input frequency and amplitude, characteristics of the cable, and of the weight and shape of the array. This formula is plotted in a dimensionless form with two parameters.

It is shown that resonant frequencies as small as 1.5 rad/sec can induce very large stresses for input amplitudes of one or two feet; furthermore, that dynamically heavy arrays increase the dynamic stresses a lot, especially near resonance. Entraining or displacement of large amounts of water increases the dynamic mass of the array. Therefore, the array should be a trussed open structure. Also, the velocity of sound in the cable should be as large as possible. This makes the resonant frequencies large so that they lie on the cut-off edge of the spectrum of a given sea state.

A 15-ton trussed aluminum linear array, whose length is about 600 feet and whose cross dimensions are a few feet, and a 1-5/8-inch diameter 6 x 37 Special Flexible Hoisting wire rope (USS) are selected as a numerical example. A curve is computed giving, for each frequency, the maximum safe input amplitude for any length of the cable up to 20,000 feet. This curve shows that, as the frequency increases, the maximum safe input amplitude decreases sharply. At a frequency of 1 rad/sec (six-second period), the maximum safe amplitude is 20 feet, while at a frequency of 2 rad/sec (three-second period), it is only 1.7 feet. This shows that the vessel must be rather stable if this operation is to be carried out successfully in a moderately rough sea. Also, it indicates that ways in which the cable can be attached to the vessel are limited.

Two obvious ways of attaching the cable to the vessel are: (1) from the center of gravity, and (2) from the side through a boom. In the first, heaving motions of the vessel in beam seas will be critical, while in the second, rolling motions in beam seas will be the most critical. Amplitude response curves for heaving and rolling of Cuss I (see footnote of page 27 for a description of this vessel) in beam regular (sinusoidal) waves are plotted.

The problem now is that a realistic sea is highly irregular, with a continuous probabilistic spectrum of amplitudes and frequencies. To overcome this problem, we adopt the procedure, used by many other investigators, of representing an irregular sea by a regular one. In this way, we can draw some conclusions as to the chances of the success of the operation, although much depends on the definition of the regular sea representing an irregular one of a specified state.

It is shown that, by attaching the above cable to the center of gravity of Cuss I, the chances for the success of the operation are quite good in seas excited by a wind velocity up to 25 knots (beginning of sea state 6). It is also shown that, when the cable is attached to the side of Cuss I through a boom, the chances of success can be acceptable only in very small wind velocities.

The above results lead one to the idea of using either an unconventional vessel or a lowering platform detached from the main vessel, in order to achieve higher stability. The use of a very stable platform for lowering arrays into deep water will be investigated further and discussed in a later report.

II. STATIC STRESSES DUE TO THE WEIGHT OF THE ARRAY AND CABLE

The distribution along the cable of the static tensile stress, σ, due to the weight of the array and cable is linear and is given by:

$$\sigma = \frac{1}{S} \left[(L - x)(w - b) + W_a - B_a \right] \qquad (1)$$

where:

x	=	Space coordinate measured as indicated in Figure 1.
L	=	Length of the cable.
S	=	Metallic cross-sectional area of the cable.
w	=	Weight of the cable per unit length.
b	=	Buoyancy force on the cable per unit length.
W_a	=	Weight of the array.
B_a	=	Buoyancy on the array.

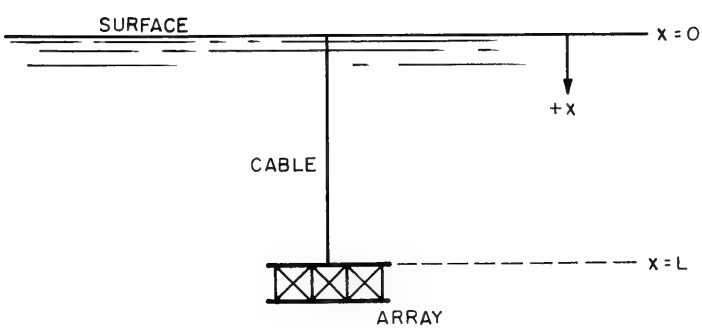

FIGURE 1 DEFINITION OF SPACE COORDINATES

This stress is maximum at the top of the cable and minimum at the array. For a steel cable with $S=3\ in^2$ and $L=20,000$ ft, and a 50-ton array (without any buoyancy), the maximum and minimum static stresses are 103,000 and 33,300 psi, respectively. For a 100-ton array these values become 130,000 and 66,700 psi. This cable could be a steel wire rope of the Galvanized Bridge Strand type with a diameter of 2.25 inches. The ultimate tensile strength of this rope is about 220,000 psi. Therefore, this rope can take the above stresses safely. This type of rope has a ratio of ultimate tensile strength to weight per unit length equal to about 62,000 ft, which is equal to the maximum length of rope that can hold itself.

On the other hand if buoyancy is utilized, the static stresses can be decreased considerably. In this respect, nylon type ropes are preferable, since they are almost weightless in water while their breaking strength is just about as high as that of the strongest steel ropes.

III. THE EFFECT OF OCEAN CURRENTS

Due to the frictional forces on the cable, a current will bend the cable as shown in Figure 2. (Such frictional forces will also act on the array and vessel if they are within the current.) Of interest here are three quantities: the additional stresses in the cable, the offset (D) from the target, and the horizontal force (F) which must be exerted by the vessel in order to remain stationary against the current.

This problem can be solved very accurately by balancing the components of the various forces acting on an infinitesimal element of the cable. The results for all pertinent variables can be expressed in terms of integrals which have been tabulated (David Taylor Model Basin Report 687 and its Supplement). However, if the net vertical force (gravity minus buoyancy) is much larger than the horizontal frictional forces due to the current, bending of the cable will be very small, and the following simple analysis can be applied.

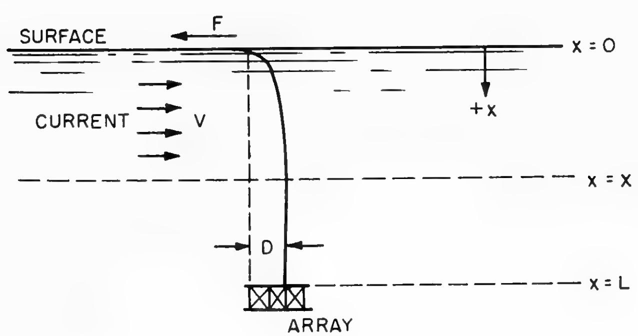

FIGURE 2 BENDING OF THE CABLE BY A HORIZONTAL CURRENT

Consider a uniform current of velocity V and width X (see Figure 2). The horizontal force, F, which is equal to the total drag of the current on the cable, is given by:

$$F = \alpha a \rho V^2 X \qquad (2)$$

where:

α = Drag coefficient. It is a function of the Reynolds number, $\frac{2aV}{\nu}$, where ν is the kinematic viscosity of water.

a = Radius of the cable.

ρ = Density of water.

The stress at the top of the cable (which is the maximum stress) will be altered only slightly by the current. The offset, D, is given approximately by:

$$D = \frac{\alpha a \rho V^2 X^2}{L(w-b) + W_a - B_a} \qquad (3)$$

which is an overestimate of the actual D.

As a numerical example, let us consider a current with V = 1 ft/sec and X = 5,000 feet.* The cable described in Section II will be used. The drag of this current on the cable is 0.225 lb/ft, which shows that the above approximation is valid. The horizontal force, F, is equal to 1,100 pounds. For the 50-ton array without any buoyancy in the cable and array and for L = 20,000, 10,000 and 5,000 feet, D is equal to 18, 28, and 37 feet, respectively. These offsets are, indeed, very small. Even for lighter cables and arrays the offsets will be small as compared to the length of the cable.

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*A current with this velocity and width is about the largest that will be found in the Atlantic.

When the array is in the current, an additional horizontal force, due to the drag on the array, must be included. The magnitude of this force will vary for arrays of different shapes. Assume a linear array composed of three horizontal hollow cylinders each 600 feet long. Two of the cylinders are assumed to have diameters of two feet, and the third a diameter of one foot. For normal incidence, the drag of the current on the array is equal to 1.8 tons. Since this force is small, compared to the weight of the array, its contribution to the stress will be small. The offset and the horizontal force, F, will increase accordingly.

IV. THE EFFECT OF MOTIONS OF THE SEA SURFACE

A. FORMULATION OF THE PROBLEM

A realistic wavy sea will cause the vessel to move in the vertical direction, and, thus, generate waves in the cable which will interact with the array. This Section deals with the analysis of the dynamic stresses induced in the cable by such waves.

This problem can become as complicated as one wishes to make it. Here we will aim at a reasonable linear analysis. Therefore, we must, first of all, assume an elastic cable. Metallic cables remain elastic as long as the total stress does not exceed their elastic limit. For the cable considered in Section II, the elastic limit is about 1.5×10^5 psi. The maximum static stress calculated for the cable is below this elastic limit, and therefore, this cable is still elastic. Nylon type ropes are more plastic than elastic, unless they have been prestressed sufficiently. On the other hand, the dynamic stress wave propagating in the cable can be a compressive one. Then, if this compression exceeds the static tension in the cable, the cable will buckle, and the propagation of stress in the cable becomes a nonlinear problem. Buckling is treated briefly in Section VI. Since the present linear analysis assumes that the cable does not buckle, any results which indicate that the cable should have buckled are not valid.

Consider the cable and the array in a vertical position. It has been shown in Section III that the deflection of the cable by a reasonable horizontal current is small under conditions stated there; therefore, even in the presence of such a current, the cable can be considered to be vertical for our present purpose. As the upper end of the cable moves vertically, waves will be propagating in the cable. If $u(x,t)$ is the dynamic displacement at time t of an element of the cable Δx, which in the absence of waves is located at x (see Figure 1), the governing equation for the propagation of waves is:

$$\rho_c S \frac{\partial^2 u}{\partial t^2} = S E \frac{\partial^2 u}{\partial x^2} - K \frac{\partial u}{\partial t} \tag{4}$$

where:

ρ_c = Density of the cable

S = Metallic cross-sectional area of cable

E = Modulus of elasticity of cable

K = Constant of friction on the cable by the surrounding water. It should be kept in mind that K may not be actually a constant.

In an actual operation, these waves can appear while the array is being lowered at some rate, i.e., L in general changes with time. However, if we assume that the rate of lowering is small as compared to the velocity of the dynamic displacement due to the waves, then L can be considered constant in time and the following dimensionless variables and parameters can be defined:

$$x' = \frac{x}{L}, \quad t' = \frac{tc}{L} \tag{5}$$

$$c^2 = \frac{E}{\rho_c}, \quad \beta_c = \frac{KL}{\rho_c c S} \tag{6}$$

where c is the "velocity of sound" in the cable.

Equation 4 then reduces to:

$$\frac{\partial^2 u}{\partial t'^2} + \beta_c \frac{\partial u}{\partial t'} = \frac{\partial^2 u}{\partial x'^2} \tag{7}$$

In order to solve Equation 7, one needs two boundary conditions (most likely one at each end of the cable) and, if transient solutions are sought, initial conditions as well. One of the boundary conditions, which we will apply here, is the specification of u at $x' = 0$ for all times. Practically speaking, we should specify the motion of the ocean surface for all times. But, then the dynamics of the vessel and the way in which the cable is attached to the vessel must be considered simultaneously with the dynamics of the cable and array, which is a very complex problem. A discussion of the dynamics of the vessel is presented in Section V; here we will assume that we know u at $x' = 0$ for all times.

The other boundary condition is obtained at the array (x = L), where we must have:

$$\left[M_a \frac{\partial^2 u}{\partial t^2} + ES \frac{\partial u}{\partial x} + \frac{1}{2} \alpha \rho A \left| \frac{\partial u}{\partial t} \right| \frac{\partial u}{\partial t} \right]_{x=L} = 0 \tag{8}$$

where:

M_a = Dynamic mass of the array. In general, it will be composed of three parts: the actual mass of the materials of the array, an apparent mass due to the motion of the array in water, and the mass of any water trapped in the array and having to move with it.

α = Drag coefficient of the array. This is a function of the Reynolds number, and for certain array configurations such as cylindrical or spherical, its value can be found in the literature.

A = Area of the array projected in the direction of motion.

Notice that a quadratic form has been taken for the hydrodynamic drag. (The two vertical bars in the first velocity factor mean "absolute value of," and they are necessary since the drag must always oppose the motion). This is absolutely necessary because the Reynolds numbers involved are high.

Let us define the following two parameters:

$$\mu = \frac{\rho_c SL}{M_a} \quad , \quad B = \frac{\alpha \rho A}{2 M_a} \tag{9}$$

Notice that μ is the ratio of the mass of the cable to the mass of the array. Equation 8 can then be put in the following form:

$$\left[\frac{\partial^2 u}{\partial t'^2} + \mu \frac{\partial u}{\partial x'} + B \left| \frac{\partial u}{\partial t'} \right| \frac{\partial u}{\partial t'} \right]_{x' = 1} = 0 \tag{10}$$

B. THE STEADY STATE SOLUTION FOR SINUSOIDAL INPUTS

We will take u at $x' = 0$ as sinusoidal, of amplitude U_0, and of angular frequency ω and try to find the steady state solution of the problem.

In Appendix A it is shown that, under conditions which are valid here, the friction on the cable is proportional to the velocity, and an expression for the friction constant (β_c) is derived. In Appendix B it is shown how the drag term in Equation 10 can be linearized and how good such an approximation is. Once this linearization is introduced, the entire problem is linear and therefore, for sinusoidal inputs we will have sinusoidal waves.

It must be emphasized, once and for all, that the approximation of the nonlinear quadratic drag on the array, developed in Appendix B, is not linear but "quasi-quadratic." It replaces the drag, which is actually proportional to the square of the instantaneous velocity, not by a term proportional to the instantaneous velocity but by a term which varies in time as the velocity and whose amplitude is proportional to the square of the amplitude of the velocity.

In what follows a capital letter will be used to denote the complex amplitude (magnitude and phase) of the respective instantaneous sinusoidal variable. For instance, $U(x')$ means the complex amplitude (magnitude and phase) as a function of x' of the instantaneous sinusoidal dynamic displacement $u(x', t')$.

Let us define a normalized displacement amplitude U' as being equal to U divided by $|U_0|$. If U_1' is the value of U' at the array ($x' = 1$), then the solution for U' as a function of x' is:

$$U = U_1 \cos \omega' y' + C \sin \omega' y' \tag{11}$$

where C is a complex constant and

$$y' = 1 - x' = 1 - \frac{x}{L}, \quad \omega' = \frac{\omega L}{c} \tag{12}$$

This solution satisfies Equation 7 if the friction of the water on the cable is neglected. As is shown in Appendix A, this is valid for the frequencies of interest here. Only at very sharp resonant peaks can the friction on the cable be important, and even then its role can be insignificant if damping of a greater order of magnitude exists somewhere else in the system (as, for example, in the array).

The two unknown constants U_1' and C will be determined by the two boundary conditions. By defining the origin of the time (t') properly, we can take U_1' as being real and positive. Then, according to the linearization introduced in Appendix B, substituting Equation 11 in Equation 10 we obtain:

$$- (\omega')^2 U_1' - \omega' \mu C + i \beta (\omega')^2 (U_1')^2 = 0 \qquad (13)$$

where:

$$\beta = \frac{4 \alpha \rho A \; |U_o|}{3 \pi M_a} \qquad (14)$$

This parameter is the ratio of the drag to the inertial force of the array. Therefore:

$$C = \frac{\omega'}{\mu} U_1' \; (- 1 + i \beta U_1') \qquad (15)$$

Then Equation 11 reduces to:

$$U' = U_1' \sec \phi \cos (\omega' y' + \phi) + i \beta (U_1')^2 \tan \phi \sin \omega' y' \qquad (16)$$

where:

$$\tan \phi = \frac{\omega'}{\mu} , \quad 0 \leq \phi \leq \frac{\pi}{2} \qquad (17)$$

Finally, the unknown real positive constant U_1' will be determined by requiring that $|U'|$ at $y' = 1$ be equal to 1. This gives:

$$(U_1')^2 = \frac{\cos^2 (\omega' + \phi)}{2 \beta^2 \sin^2 \phi \sin^2 \omega'} \left[\left(1 + \frac{\beta^2 \sin^2 \omega' \sin^2 2\phi}{\cos^4 (\omega' + \phi)} \right)^{\frac{1}{2}} - 1 \right] \qquad (18)$$

If we denote the amplitude of the dynamic stress by Σ and define a normalized stress amplitude, Σ', as being equal to $\frac{L \Sigma}{|U_o| E}$, then the distribution of Σ' is given by:

$$\Sigma' = \omega' U_1' \sec \phi \sin (\omega' y' + \phi) - i \omega \beta (U_1')^2 \tan \phi \cos \omega' y' \qquad (19)$$

Equations 16, 18, and 19 determine completely the amplitudes of the normalized displacement and stress waves as functions of y', ω' and the two parameters μ and β.

1. Summary of Formulae for the Magnitudes of the Amplitudes of the Array Displacement, Maximum Stress, and Stress at the Two Ends of the Cable

Equation 19 shows that the distribution of Σ' along the cable is sinusoidal. Therefore, depending on the frequency and length of the cable, Σ' can have absolute maxima and minima. These extreme values of Σ', as well as their locations, can be computed analytically. Denoting the magnitude of the maximum of Σ' as $|\Sigma'_{max}|$ the location(s) of this maximum as y'_{max}, the magnitudes of Σ' at $x' = 0$ and $x' = 1$ as $|\Sigma'_o|$ and $|\Sigma'_1|$, respectively, one can show that:

$$\left|\Sigma'_{max}\right|^2 = (\omega')^2 \, (U'_1)^2 \left[1 + \tan\phi \, (\tan\Psi + \sec\Psi)\right] \tag{20}$$

$$2\,\omega' y'_{max} = \frac{n\pi}{2} - \Psi, \qquad n = 1, 5, 9 \ldots \tag{21}$$

$$\left|\Sigma'_o\right|^2 = (\omega')^2 \, (U'_1)^2 \left[1 + 2\tan\phi \cos^2\omega' \, (\tan\Psi + \tan\omega')\right] \tag{22}$$

$$\left|\Sigma'_1\right|^2 = (\omega')^2 \, (U'_1)^2 \left[1 + 2\tan\phi \tan\Psi\right] \tag{23}$$

where:

$$(U'_1)^2 = \frac{\cos^2(\omega' + \phi)}{2\beta^2 \sin^2\phi \sin^2\omega'} \left[\left(1 + \frac{\beta^2 \sin^2\omega' \sin^2 2\phi}{\cos^4(\omega' + \phi)}\right)^{\frac{1}{2}} - 1\right] \tag{24}$$

$$\phi = \arc\tan\frac{\omega'}{\mu}, \qquad 0 \leq \phi \leq \frac{\pi}{2} \tag{25}$$

$$\Psi = \arc\tan\left[\frac{1}{2}\beta^2 (U'_1)^2 \tan\phi - \cot 2\phi\right], \qquad -\frac{\pi}{2} \leq \Psi \leq \frac{\pi}{2} \tag{26}$$

For convenience, we repeat here the following definitions:

$$\omega' = \frac{\omega L}{c}, \quad U' = \left|\frac{U}{U_o}\right|, \quad \Sigma' = \left|\frac{L\Sigma}{U_o E}\right| \tag{27}$$

$$\mu = \frac{\rho_c SL}{M_a}, \quad \beta = \frac{4\alpha\rho A\left|U_o\right|}{3\pi M_a} \tag{28}$$

If the values of y'_{max}, as given by Equation 21, are outside the range 0 to 1 for all of the indicated n's, the maximum stress, as given by Equation 20 does not occur in the cable. In that case, the maximum and minimum stresses in the cable are the stresses at the top and the bottom of the cable, respectively. In order to have $0 \leq y_{max} \leq 1$, the condition that $2\omega' + \Psi \geq \frac{\pi}{2}$ must be fulfilled.

2. Special Limiting Cases

Case 1

$\omega' = m\pi$, where $m = 1, 2, 3 \ldots,$ and $\mu \ll 1$ so that $\phi \cong \frac{\pi}{2}$.

Then $U'_1 = 1$ and:

$$\left|\Sigma'_{max}\right| = \left|\Sigma'_o\right| = \left|\Sigma'_1\right| = \frac{\pi^2 m^2}{\mu}(1 + \beta^2)^{\frac{1}{2}} \tag{29}$$

Case 2

$L \rightarrow 0$ so that ω' and $\mu \rightarrow 0$ but ϕ is a finite number other than zero. Then everywhere in the cable U and Σ are constant, and they are given by $U = U_o$ and:

$$\Sigma = \frac{1}{S}\omega^2 U_o M_a (1 + \beta^2)^{\frac{1}{2}} \tag{30}$$

C. PARAMETRIC ANALYSIS OF THE MAGNITUDE OF THE MAXIMUM STRESS FOR SINUSOIDAL INPUTS

The magnitude of the normalized amplitude of the maximum stress, as given by Equation 20, is obviously a function of only three variables, namely ω', u, and β. It is plotted in Figure 3 versus ω' with u and β as parameters.* As it is stated at the end of Section IVB1 the maximum stress as given by Equation 20 occurs in the cable if $2\omega' + \Psi \geq \frac{\pi}{2}$. When this condition cannot be fulfilled, which can happen only for small values of ω', the stress at the top of the cable (Equation 22) is used in Figure 3, because it is greater than the stress anywhere else in the cable. Figure 3 then shows the dependence of the maximum stress on the input characteristics (ω and U_0), the characteristics of the cable (c, E, and S), the length of the cable (L), and the weight and shape of the array.

Let us see whether Figure 3 is compatible with the well known results of simpler systems. For $\beta = 0$, there is no damping in the system. In this case the resonant frequencies of the system are the roots of the equation $\tan \omega' = \mu/\omega'$. The smallest of these roots lies between 0 and $\pi/2$, the next between π and $3\pi/2$, and so on. At these resonances, the reflections of the oncoming stress waves by the hanging mass have the proper phase so that their individual contributions result in an infinite total stress. As the hanging mass is decreased indefinitely ($u \to \infty$), the resonant frequencies approach the values $\pi/2$, $3\pi/2$, etc., and we have the case of the free end spring. As the hanging mass is increased indefinitely ($u \to 0$), the resonant frequencies approach the values π, 2π, etc., and we have the case of the fixed end spring. On the other hand, when $\beta \neq 0$, energy is dissipated by the hanging mass, and the amplitudes of the reflected waves are diminished. Resonance can still occur but with finite amplitude. The actual value of β (amount of damping) should have a small effect on the values of the resonant frequencies and a very profound effect on the amplitudes at resonance. One could expect that the amplitude at resonance will be decreased as the damping is increased.

Figure 3 shows quite clearly all these expected trends, except one; that is, for the resonance occurring near $\omega' = \pi$ (and 2π, 3π, etc., as well), the amplitude increases when the damping is increased beyond a certain value.

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*In computing these curves, the values of $|\Sigma'_0|$ and $|\Sigma'_1|$ have also been recorded. These values can be of use in the design of the joints at the top and bottom of the cable.

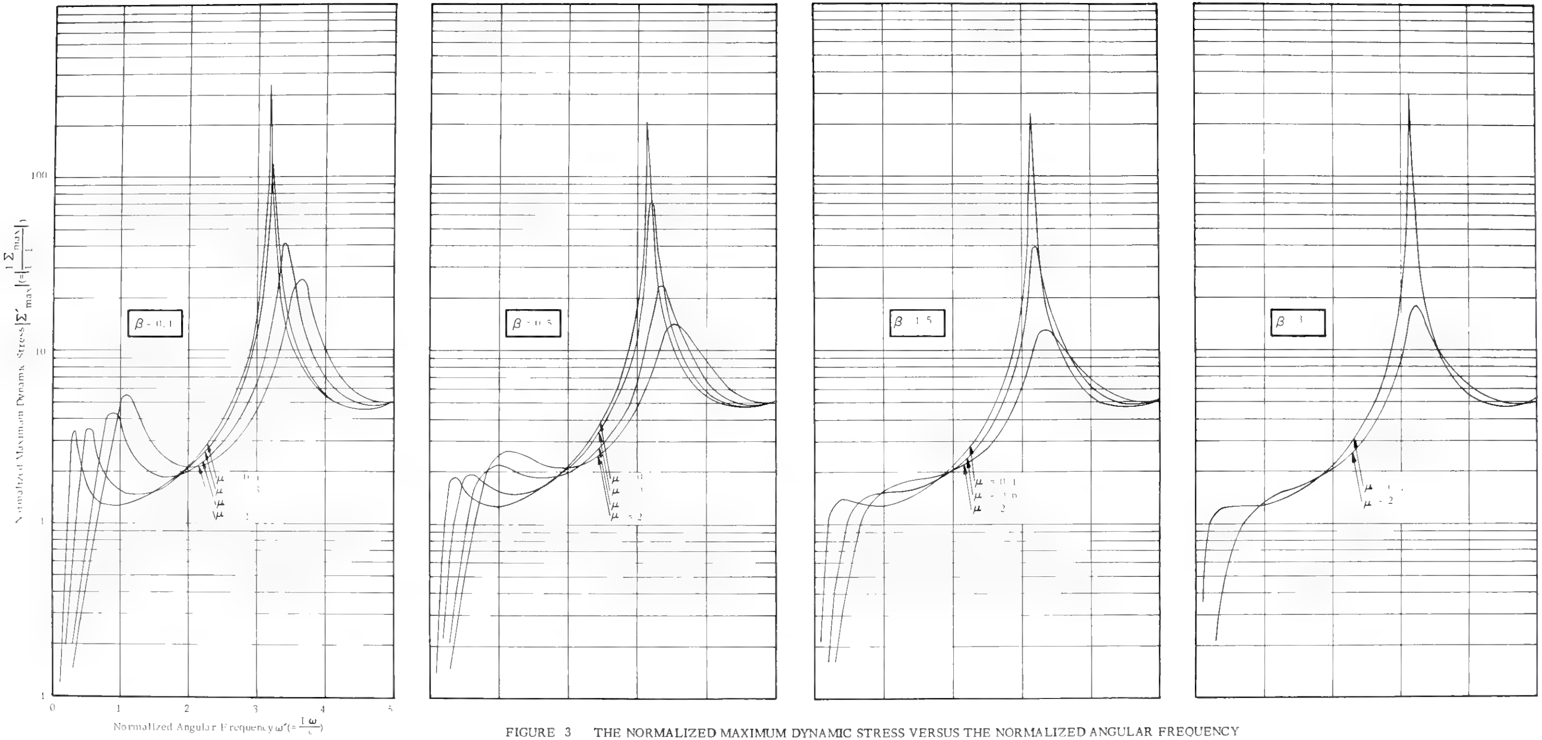


FIGURE 3 THE NORMALIZED MAXIMUM DYNAMIC STRESS VERSUS THE NORMALIZED ANGULAR FREQUENCY FOR VARIOUS VALUES OF THE PARAMETERS $\beta\left(=\frac{4\alpha\,\rho A|U_0|}{3\pi M_a}\right)$ and $\mu\left(=\frac{WL}{M_a}\right)$

I. SUMMARY

A. PURPOSE AND SCOPE

This report relates to the operational aspect of lowering heavy array structures to the bottom of the deep ocean. Because of the weight of the array and the depth to which it will be lowered, the lowering cable can be expected to be under a high stress. Ocean currents, surface waves, and motions of the lowering vessel will also contribute to the stress placed on the lowering cable. Therefore, a critical design problem may exist for the lowering cable.

The objective of this report is to develop a reasonable theory on which to base the ultimate design of the lowering cable. The study is limited to the case of a single cable lowering the array vertically. Each factor contributing to the stress on the cable is considered separately, and the conditions under which the analyses are valid and of practical significance are given. From the results of this theory some general conclusions are drawn as to the feasibility of the operation with regard to dynamically optimum array configurations, stability of the lowering platform in a rough sea and degree of roughness of the sea (sea state). The results of the study are presented in plot form involving a non-dimensional maximum (dynamic) stress versus a non-dimensional frequency for various mass and drag parameters.

Other methods of operation, which are not treated in this report, but which will be considered in later investigations, are:

(1) Cancel out most of the dynamic inputs to the cable by an auxiliary mechanical system on the vessel. There are two crucial problems here: the horsepower of the driver of such a system and the rapidity with which the overall system can respond.

(2) Introduce considerable damping along the cable, which will have the effect of flattening out resonant peaks. This can be effected by attaching bluff bodies to the cable at certain intervals. The drag of the water on them will give the desired friction. This idea seems quite attractive, since these bodies can also provide buoyancy to reduce the rather high static stresses in the cable, but its plausibility must be investigated quantitatively.

III. THE EFFECT OF OCEAN CURRENTS

Due to the frictional forces on the cable, a current will bend the cable as shown in Figure 2. (Such frictional forces will also act on the array and vessel if they are within the current.) Of interest here are three quantities: the additional stresses in the cable, the offset (D) from the target, and the horizontal force (F) which must be exerted by the vessel in order to remain stationary against the current.

This problem can be solved very accurately by balancing the components of the various forces acting on an infinitesimal element of the cable. The results for all pertinent variables can be expressed in terms of integrals which have been tabulated (David Taylor Model Basin Report 687 and its Supplement). However, if the net vertical force (gravity minus buoyancy) is much larger than the horizontal frictional forces due to the current, bending of the cable will be very small, and the following simple analysis can be applied.

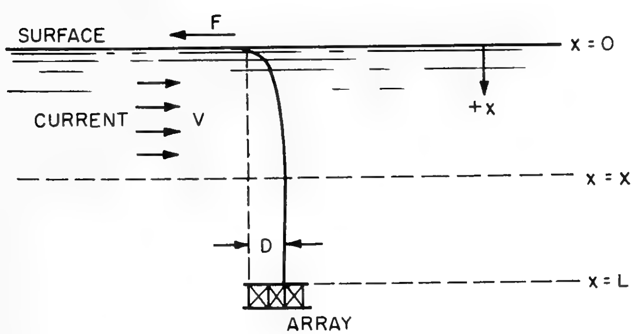


FIGURE 2 BENDING OF THE CABLE BY A HORIZONTAL CURRENT

This should be expected, because when the damping of the hanging mass becomes very large, the mass cannot move very much, and we have again the fixed end spring with resonant frequencies at π, 2π, 3π, etc., and infinite amplitudes. To put it another way, if a very large amount of energy is dissipated by the hanging mass, the energy propagating along the spring must be very large, since the spring is the only link between source and load.

Two remarks are in order here. First, as Figure 3 shows, there are resonances (some of them highly sharp depending on the values of μ) at $\omega' \cong \pi$ or $\omega = \frac{\pi c}{L}$. For $c \cong 10{,}000$ ft/sec and $L \leq 20{,}000$ ft, this corresponds to frequencies as small as 1.5 rad/sec or periods as high as four seconds. Since such periods can be found easily in a realistic sea, it is not correct to consider the cable nonflexible when an accurate estimation of the stresses is desired, unless such frequencies are filtered out by the vessel very effectively. Also, for small values of β, there are resonances at values of ω' much smaller than π, corresponding to periods much larger than four seconds--which are quite frequent and prominent in a rough sea. Second, an idealized system, such as the free end or fixed end flexible cable, will give portions of the curves of Figure 3 rather accurately. However, this does not render the present analysis, which includes the dynamics of the array, superfluous. Our objective here is not to display a rough picture of the phenomena, which could be done by the free end or fixed end cable, but rather to find as accurately as possible the dependence of the maximum stress on the various quantities. In terms of a realistic and sound design of the cable, which hinges on the magnitude of the maximum stress, simple models--such as the nonflexible and the free end or fixed end (flexible) cables--are useless, and a complicated model, like the present one, is indispensable.

In order to demonstrate how these curves can be used, let us consider a cable and array as specified below:

$L = 20{,}000$ ft	$M_a = 600{,}000$ lb
$E = 20 \times 10^6$ psi	$A = 3{,}000$ ft^2
$S = 3$ in^2	$\alpha = 1.2$
$w = 10.6$ lb/ft	
$c = 13{,}600$ ft/sec	

The array described at the end of Section III has been taken here. This array has a weight of 50 tons. Because it is hollow with rather thin walls, approximately (600) (62) (2.25π) $\cong$ 250,000 pounds of water are trapped in it and have to move with it. Also, the apparent additional mass of the array in water is about 250,000 pounds. Hence, the total dynamic mass of the array, M_a, is about 600,000 pounds. The value of A is equal to (600) (5) = 3,000 ft^2. The value of 1.2 for α has been obtained from the literature. Therefore, we have:

$$\mu = \frac{\rho_c S L}{M_a} = \frac{w L}{M_a} = 0.353$$

$$\beta = \frac{4 \alpha \rho A |U_o|}{3 \pi M_a} = 0.16 \ |U_o|$$

$$\omega' = \frac{\omega L}{c} = 1.47 \ \omega$$

$$|\Sigma_{max}| = \frac{E}{L} \ |U_o \ \Sigma'_{max}| = 1,000 \ |U_o \ \Sigma'_{max}|$$

Now, if we take $|U_o|$ = 1 ft and ω = 1.26 rad/sec (which corresponds to a period of 5 sec), then β = 0.16 and ω' = 1.85. Then, from Figure 3, $|\Sigma'_{max}|$ = 1.9 and therefore, $|\Sigma_{max}|$ = 1,900 psi. Also, for the same ω, and $|U_o|$ = 10 ft, $|\Sigma_{max}|$ = 20,000 psi. Similarly, for $|U_o|$ = 19 ft and ω = 0.63 (which corresponds to a period of 10 sec), $|\Sigma_{max}|$ = 23,700 psi. The static stress distribution is linear, and for this cable and array without any buoyancy, the maximum static stress (at the top of the cable) is 103,000 psi while its minimum value (at the bottom of the cable) is 33,300 psi. Stresses due to currents of the type considered in Section III are much smaller than these static stresses. Even without determining the locations of the above dynamic stresses, it is obvious that this cable will be able to withstand the maximum total stress with a factor of safety of about 2. If a larger factor of safety is desired, the metallic cross-sectional area of the cable can be increased and/or buoyancy in the cable and array can be utilized in order to decrease the large static stresses. However, if the static stresses are decreased below the dynamic stresses, the cable will buckle (see Section VI).

On the other hand, as Figure 3 shows, certain resonant frequencies can induce very large dynamic stresses. The frequencies of these resonances are close to $\omega' = \pi$, and their peaks are very high for small values of μ. For the above cable and $U_o = 1$ ft, we have (at resonance) $\omega' = 3.2$ and $|\Sigma'_{max}| \cong 90$. Therefore, $\Sigma_{max} = 90{,}000$ psi and $\omega = 2.2$ rad/sec, corresponding to a period of about three seconds. If we take $|U_o| = 3$ ft, then $|\Sigma_{max}| = 195{,}000$ psi at $\omega = 2.2$ rad/sec. Clearly then, resonant frequencies with rather small amplitudes can break the present cable easily. Depending on the state of the sea, the above resonant frequency with the corresponding amplitudes can be found in a realistic sea. However, the input amplitude to the cable at this frequency can be much less than the amplitude of the sea, depending on the stability of the vessel (or lowering platform) and the method used to attach the cable to the vessel. These aspects will be treated in the following Section, but the following two general conclusions must be emphasized here.

First, since the resonant frequency is given by $\omega = \frac{\pi c}{L}$, it is desirable to have as high values of c as possible so that this frequency can fall in the region of the input spectrum which is characterized by small amplitudes.

Second, it is highly desirable to have a large value of μ. This means that the dynamic mass of the array must be small. The present cylindrical array is poorly designed with regard to this aspect, because of the large quantities of displaced and entrained water.

A linear array made of aluminum trusses can serve the same purpose as the 50-ton cylindrical array. The aluminum array does not entrain any water, it displaces a very small amount of water, and it weighs only about 15 tons. It appears that a reasonable value for the dynamic mass of this array in water is about 20 tons. (Compare this with the 300-ton dynamic mass of the 50-ton cylindrical array.) Let us consider with this array a 6 x 37 Special Flexible Hoisting wire rope (USS) with a diameter of 1-5/8 inches, which has the following characteristics:

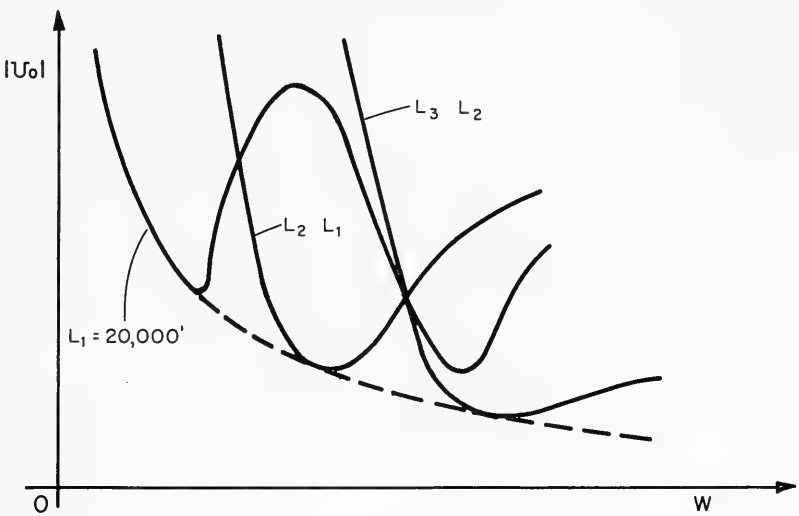
$E = 11 \times 10^6$ psi $\qquad S = 1.1\ \text{in}^2$

$c = 10{,}000$ ft/sec $\qquad \omega = 4.1$ lb/ft

Notice that, for $L = 20{,}000$ ft, μ is now equal to 2.

The only unknown factor in the system is the drag coefficient β for the array. For the present array, we cannot obtain a value of β from the literature. For the cylindrical array, however, we found that $\beta = 0.16|U_0|$. Since the present array has the same dimensions as the cylindrical array, values of β a few times smaller than that for the cylindrical array seem to be reasonable.

If we plot the value of $|U_0|$ versus the value of ω which will give a specified maximum dynamic stress for various values of L, we will have curves like those shown in the following diagram. The envelope (dashed line) drawn through



the first minimum of each of these curves has the following significance. Input characteristics $|U_0|$ and ω corresponding to a point which lies on or below this envelope will never produce a maximum dynamic stress greater than the specified one for any value of L up to 20,000 feet. Then, if the specified maximum dynamic stress is the design stress for dynamic loads of a particular cable, this envelope is the boundary of safe (points lying below it) and unsafe (points lying above it) input characteristics.

Such curves are plotted in Figure 4 (dashed line) for the aluminum array and above specified Special Flexible Hoisting wire rope. The maximum tolerable dynamic stress is 32,000 psi, about one-sixth of the ultimate strength of this cable. The values of β assumed are stated on the curves. Notice that more friction moves the curve to the right. Assuming that $\beta = 0.05 \, |U_o|$, we see that at $\omega = 2$ rad/sec the maximum dynamic stress in the cable will not exceed 32,000 psi as long as $|U_o|$ is equal to or less than 1.7 feet and so on for other frequencies. If $\beta = 0.1 |U_o|$, then for the same frequency the maximum allowed $|U_o|$ is 3 feet.

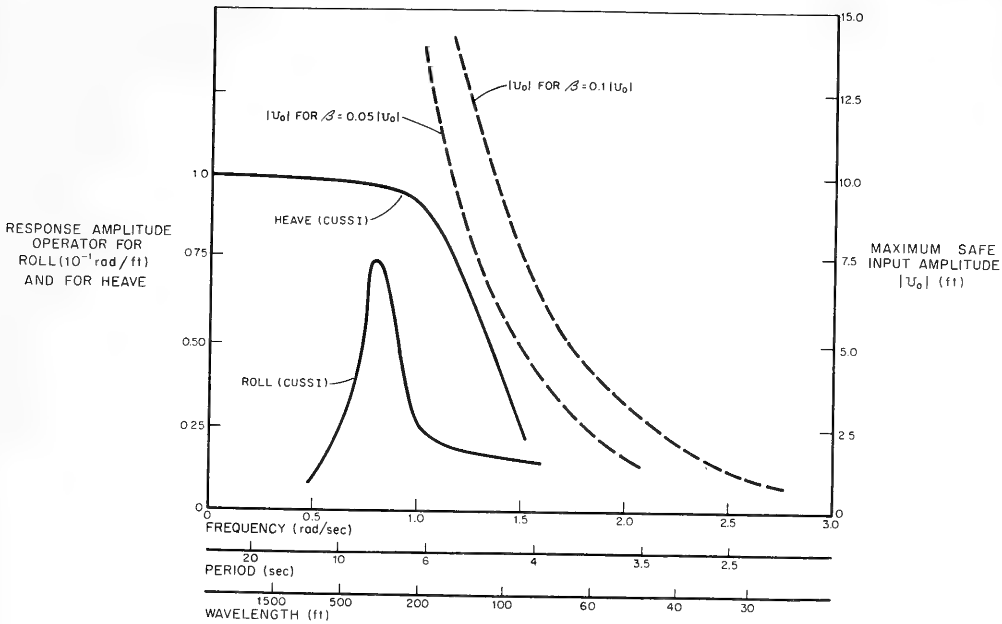


FIGURE 4 RESPONSE AMPLITUDE OPERATORS FOR CUSS I AND MAXIMUM SAFE INPUT AMPLITUDE TO THE CABLE AS FUNCTIONS OF FREQUENCY. (CABLE IS A SPECIAL FLEXIBLE HOISTING WIRE ROPE OF USS.)

V. THE DYNAMICS OF THE VESSEL AND SPECTRAL CHARACTERISTICS OF A REALISTIC SEA

It has been already pointed out that, in calculating the dynamic stresses induced in the cable by a disturbed sea, the dynamics of the vessel and the way in which the cable is attached to it must be considered simultaneously with the dynamics of the cable and array. Besides the elastic force exerted by the cable, other forces acting on the vessel are inertia, dynamic buoyancy due to the waves of the sea, viscous damping, and damping due to the "radiation" of surface waves by the vessel. Then, in an analysis of the entire system, similar to the analysis carried out in Section IV for the system without the vessel, at least three additional parameters must be introduced. This will be a very complicated problem.

We will assume that the dynamic loading of the vessel by the cable and array is negligible and, therefore, the motions of the vessel during the operation will not be influenced by the cable and array. Because of the relative dynamic masses of the vessel and the cable and array, this condition is well satisfied in practice, except possibly when the array is lowered through a long boom from the side of a vessel which is rather unstable in rolling.

In Section IV, we have shown that near resonant frequencies, which can be as small as 1.5 rad/sec, the amplitude of the input to the cable should be about 2 feet or less. This definitely shows that the vessel must be very stable at such frequencies, and it limits the variety of ways in which the cable can be attached to the vessel.

Let us first examine the spectral characteristics of a realistic sea where such an operation will be carried out. A realistic sea has a continuous probabilistic distribution of all possible amplitudes and frequencies. Depending on the weather conditions (mostly the speed of the wind) and the degree of development of the sea (duration of the wind and extent of the fetch), amplitudes above a certain value and frequencies outside a certain range are not very probable, while amplitudes and frequencies within a certain range are most probable. It appears that separate probability data for amplitude or frequency exist in available literature, but data for the joint probability for a specified amplitude and frequency do not exist; i.e., we do not really know what is the probability for the occurrence of a certain amplitude with a certain frequency in a given sea state. Tables I and II show some important characteristics of a fully arisen sea (infinite fetch and wind duration) at various wind speeds. These data have been obtained from "Observing and Forecasting Ocean Waves," H.O. Pub. No. 603, U.S. Navy Hydrographic Office. The significant range of frequencies is defined as the range outside which it is highly improbable to find a frequency at the indicated wind speed.

TABLE I

AMPLITUDE CHARACTERISTICS OF FULLY DEVELOPED SEA

Wind Speed (knots)	Most Frequent Amplitude (ft)	Average Amplitude (ft)	Average of 30% of Highest Amplitudes (ft)	Average of 10% of Highest Amplitudes (ft)
20	2	2.5	4	5
30	5.4	6.8	11	14
40	11	14	22	28

TABLE II

FREQUENCY CHARACTERISTICS OF FULLY DEVELOPED SEA

Wind Speed (knots)	Most Frequent Frequency (rad/sec)	Average Frequency (rad/sec)	Significant Range of Frequencies	
			Lowest (rad/sec)	Highest (rad/sec)
20	1.4	1.1	.56	2.1
30	0.9	.73	.38	1.3
40	0.53	.55	.29	.97

Tables I and II show that frequencies as small as 1.5 rad/sec (the smallest resonant frequencies of the cable) are more likely to be found at smaller wind speeds, but the amplitudes are much smaller at smaller wind speeds. Obviously, as the wind speed decreases, the chances for the success of the operation increase. The largest wind speed that is safe will depend on the stability of the vessel and the manner of attaching the cable to it.

Two obvious ways of attaching the cable to the vessel are from the center of gravity, and from the side through a boom. In the first, heaving motions of the vessel in beam seas will be critical, while in the second, rolling motions in beam seas will be the most critical.

In Figure 4, the heave and roll response amplitude operators for Cuss I* in unidirectional sinusoidal (regular) deep ocean beam waves are plotted versus the frequency ω. Scales for the period (equal to $2\pi/\omega$) and the wavelength (equal to $2\pi g/\omega^2$) are provided. For waves approaching the vessel from other directions, heaving and rolling are less. These curves were obtained from "The Motions of a Moored Construction-Type Barge in Irregular Waves and Their Influence on Construction Operation," Contract NBy-32206, U.S. Naval Civil Engineering Laboratory.

In the case of attaching the cable to the center of gravity of Cuss I, some meaningful conclusions as to the safe sea state can be drawn from Tables I and II and Figure 4. Assuming that we can represent an irregular sea by a regular one with amplitude equal to the average amplitude of the irregular sea and frequency equal to the most frequent frequency of the irregular sea, then, when the wind velocity is 20 knots, the input to the cable is $\omega = 1.4$ rad/sec and $|U_0| = (0.42)(2.5) = 1.05$ ft. This input can be tolerated by the cable. According to this representation of an irregular sea, a wind speed of 30 or even 40 knots can be tolerated by this cable. Since the input amplitudes that the cable can tolerate are small for high frequencies, perhaps a fairer representation of an irregular sea by a regular one is by the average of the 10% of highest amplitudes and the most frequent frequency or the highest frequency of

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*Cuss I, originally a nonpropelled freight barge, is 260 feet long and has a beam of about 50 feet and a draft of about 11 feet at 3,000 tons displacement. It has been converted to a sea drilling vessel, and in March 1961, it was used successfully by Project Mohole to drill in almost 12,000 feet of water at a site 40 miles east of Guadalupe Island, Mexico. Cuss I is used for our preliminary computations, because it is the only vessel, of the type that could be used in the present operation, for which we can obtain the amplitude response curves from existing literature.

the significant range of frequencies. Under these conditions, a 20-knot sea is quite safe, while a 30-knot sea is marginally safe. Of course, in the final analysis the reasonableness of our argument depends on the value of the friction coefficient β for the array. We only guessed its value here. Assuming that we have not made a gross overestimate of β, we can conclude that the chances for the success of this operation are quite good for wind velocities up to 25 knots.

If the cable is attached to the end of a boom extended from the side of Cuss I, a rather quiet sea is required for a successful operation. Since Cuss I has a beam of about 50 feet, a boom at least 30 feet long is required. Heaving and rolling can occur in phase, and, as seen from Tables I and II and Figure 4, only in a sea excited by a wind velocity of much less than 20 knots are the chances for the success of the operation acceptable. Indeed, it will take a vessel having much greater stability in rolling than Cuss I to carry this operation successfully in a sea excited by a wind speed of 20 knots.

VI. BUCKLING

We assume that the cable buckles as soon as the total stress at some point becomes compressive. Since, without any dynamic input, the cable is under tension due to gravity, not all kinds of inputs will make the cable buckle. In order to find whether a given input will cause buckling, step by step we must trace in the cable the propagation of the input, as well as its interaction with and reflection by the array and vessel. Here we will investigate only one aspect of this complex problem, namely, buckling within the time $T = \frac{L}{c}$ after the onset of the input.

In the absence of any input, the static elongation of the cable, u_s, is given by

$$u_s = \frac{L}{SE} \left[\frac{L}{2} (w-b) + W_a - B_a \right] \tag{31}$$

Now the simplest input dynamic displacement, u_o, which satisfies the condition that at time $t = 0$ the vessel is stationary is $u_o = \frac{a}{2} t^2$, where a is an acceleration. Therefore, if buckling is to occur within the time T, u_o must become greater than u_s in time equal to or less than T, i.e.:

$$a > \frac{2g}{Lw} \left[\frac{L}{2} (w-b) + W_a - B_a \right] \tag{32}$$

where g is the acceleration of gravity. Thus, the acceleration a necessary for buckling decreases with increasing L and with decreasing net weight of the array. For $L = 20,000$ ft, $w = 10$ lb/ft and $W_a = 100$ tons (and no buoyancy anywhere), $a > 3g$. For $W_a = 0$, $a > g$ regardless of L and w. Therefore, it takes tremendous input accelerations to buckle the cable in this manner.

Suppose, though, that the cable has buckled. Then, depending on the degree of buckling, the cable may or may not form kinks. When the vessel now moves upwards and stretches the cable, these kinks will make the failure of the cable easier. In order to determine the degree of buckling, let us assume that the array falls freely in the sea after buckling.

The forces acting on the array are inertia, gravity, buoyancy, and friction. The downward displacement of the array, $u_1(t)$, must obey the following differential equation:

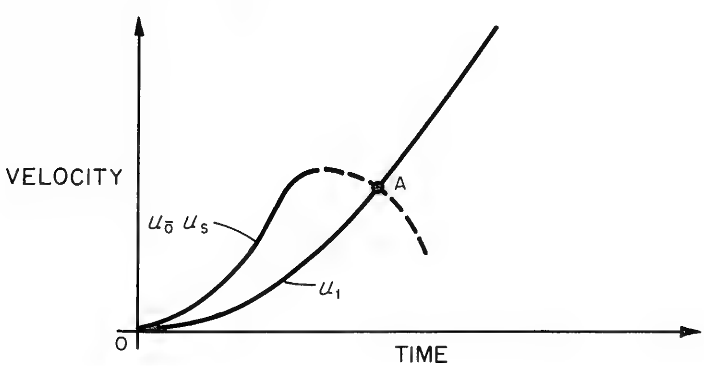
$$M_a \frac{d^2 u_1}{dt^2} = W_a + g\rho V - B_a - \tfrac{1}{2}\alpha\rho A\left(\frac{du_1}{dt}\right)^2 \qquad (33)$$

The second term in the right side of the above equation accounts for the gravity on a volume V of water trapped in the array and moving with it. For the cylindrical array discussed thus far, $g\rho V$ will be approximately equal to B_a, and the solution of this equation satisfying the conditions that at $t = 0$, $u_1 = \frac{du_1}{dt} = 0$ is:

$$u_1 = V_o t_1 \ln \cosh \frac{t}{t_1} \qquad (34)$$

where $V_o = \left[(2W_a)/(\alpha\rho A)\right]^{\frac{1}{2}}$ is the ultimate velocity of the array and $t_1 = 1.41 M_a (\alpha\rho A W_a)^{-\frac{1}{2}}$. For the 50-ton cylindrical array, $V_o = 5.3$ ft/sec and $t_1 = 1$ sec.

To determine the degree of buckling, u_1 and $u_o - u_s$ must be plotted as indicated below.



The fact that the cable has buckled means that the curve $u_o - u_s$ must lie above the curve u_1 for some time. Of course, after some time of downward motion, the vessel will move upwards, as indicated by the dashed line. The difference between $u_o - u_s$ and u_1 is a measure of the degree of buckling, and can be used in some way to determine whether the cable will form kinks. At the time corresponding to point A (or at some previous time, if the cable has formed kinks), the cable will be again under tension. The failure of the cable by kinks should, perhaps, be investigated.

APPENDIX A

FRICTION ON THE CABLE

In order to derive an expression for the friction on the cable by the surrounding water, we will consider the mathematical model of an infinite straight cable surrounded by a viscous fluid of infinite extent. Furthermore, we will assume that the cable moves longitudinally like a rigid body with a specified velocity. This rigid-body approximation is valid as long as the distance along the cable required for an appreciable change in the velocity is much greater than the radius of the cable. This requirement is easily met in the present case.

The governing equations of the motion of the fluid reduce to:

$$\frac{\partial w}{\partial t} = \nu \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} w \tag{A-1}$$

where w is the velocity of the fluid along the cable, ν is the kinematic viscosity of the fluid, and r is the radial coordinate.

If the velocity of the cable is sinusoidal of amplitude W_o and frequency ω, then the solution for the amplitude of the velocity of the fluid W is:

$$W = W_o \frac{H_o^{(1)}\left(r\sqrt{-i\frac{\omega}{\nu}}\right)}{H_o^{(1)}\left(a\sqrt{-i\frac{\omega}{\nu}}\right)} \tag{A-2}$$

where a is the radius of the cable and $H_o^{(1)}$ is the Hankel function giving outgoing waves.

Therefore, the amplitude of the force F exerted by the fluid on the cable per unit length is given by:

$$F = -2\pi a \rho \nu W_o \sqrt{-i\frac{\omega}{\nu}} \frac{H_1^{(1)}\left(a\sqrt{-i\frac{\omega}{\nu}}\right)}{H_o^{(1)}\left(a\sqrt{-i\frac{\omega}{\nu}}\right)} \tag{A-3}$$

where ρ is the density of the fluid.

Now, for water $\nu = 1.1 \times 10^{-5}\,\mathrm{ft}^2/\mathrm{sec}$. Therefore, for a and ω even as small as 1 inch and 0.16 rad/sec, respectively, $a\sqrt{\frac{\omega}{\nu}} \gg 1$ and the Hankel functions can be expanded asymptotically. Then

$$\frac{H_1^{(1)}\left(a\sqrt{-i\frac{\omega}{\nu}}\right)}{H_0^{(1)}\left(a\sqrt{-i\frac{\omega}{\nu}}\right)} \longrightarrow i \qquad \text{(A-4)}$$

Therefore,

$$F = -2\pi\, a\rho\,\sqrt{\omega\nu}\; W_0\, e^{+i\frac{\pi}{4}} \qquad \text{(A-5)}$$

This equation shows that the viscous force is out of phase with the velocity of the cable.

Thus, it has been shown that the viscous damping on the cable is proportional to the velocity of the cable. The coefficient β_c used in Section IV-A is obviously given by:

$$\beta_c = 2\pi\, a\rho\sqrt{\omega\nu}\; e^{+i\frac{\pi}{4}}\; \frac{L}{\rho_c S c} = 2\,\frac{\rho}{\rho_c}\left(\frac{\nu L}{c a^2}\,\omega'\right)^{\frac{1}{2}} \qquad \text{(A-6)}$$

Now for a metallic cable $\rho_c \cong 500\ \mathrm{lb/ft}^3$ and $c \cong 12{,}000$ ft/sec. Thus, with the values of ρ and ν for water and L and a equal to 20,000 feet and 1.125 inches, $|\beta_c| = 0.013\ \omega'^{\frac{1}{2}}$. For smaller L, $|\beta_c|$ is even smaller, and it is hardly possible to have a of a smaller order of magnitude than 1.125 inches. The inertial term in Equation 7 is of order ω'^2, while the friction term is of order $\omega'\,|\beta_c| = 0.013\ \omega'^{3/2}$. Therefore, for $\omega' \geq 0.02$ the friction term can be neglected as compared to the inertial term (except at very sharp resonances).

APPENDIX B

THE DRAG ON THE ARRAY

The problem formulated in Section IV is linear except for the term of the drag on the array. This term makes the entire problem nonlinear, and enormously more difficult than a linear problem. It is, therefore, expedient to linearize this term.

For sinusoidal inputs, one can take the drag on the array, D_a, as being proportional to the velocity of the array (not the square of the velocity) and then determine the constant of proportionality experimentally. For velocities and arrays of the type involved in this problem, this constant of proportionality will be found to be a function not only of the frequency but also of the amplitude of the velocity of the array. However, such an experiment is not practical in the present case; no effective scaling, according to the principle of similarity, of the variables is possible, because of the enormous dimensions of the arrays and the appreciable velocities involved. We therefore propose the following analytical linearization.

The drag D_a on the array is given by (see Equation 8):

$$D_a = \frac{1}{2}\alpha\rho A\left|\frac{\partial u}{\partial t}\right|\frac{\partial u}{\partial t} \tag{B-1}$$

For sinusoidal inputs, it is expected that the motion of the array will be periodic. If the $|\partial u/\partial t|$ is replaced by a constant, then D_a and the entire problem become linear, and therefore the displacement of the array will be sinusoidal of amplitude U_1. We set the constant which replaced $|\partial u/\partial t|$ equal to $\frac{8}{3\pi}\omega U_1$. This selection results in the same amount of dissipation of energy by the array when u is taken as sinusoidal in both factors in Equation B-1 and when only the second factor is taken as sinusoidal. Thus, with U_1 a real positive number, the amplitude of the drag D_a will be given by:

$$D_a = \frac{4}{3\pi}\alpha\rho\omega^2 A U_1^2 \tag{B-2}$$

To estimate the error involved in the above approximation, we consider the array alone and a sinusoidal force acting on it (in the actual problem this force is the force exerted by the cable). If we denote the velocity of the array by v, then the dynamics of the array will be governed by the following differential equation:

$$\frac{dv}{dt} = F \cos \omega t - p|v|v \tag{B-3}$$

where F and p are constants. Then, according to the above approximation:

$$v = V \cos(\omega t - \phi) \tag{B-4}$$

where:

$$V^2 = 2\left(\frac{3\pi\omega}{16p}\right)^2 \left\{ -1 + \left[1 + \left(\frac{16pF}{3\pi\omega^2}\right)^2\right]^{\frac{1}{2}} \right\} \tag{B-5}$$

and ϕ can also be computed. (In obtaining the above result, $|v|$ in Equation B-3 was replaced by $\frac{8}{3\pi}V$.) This solution can be considered as a first iteration toward the exact solution. Now we can compute a correction v' through a second iteration, by solving the equation:

$$\frac{dv'}{dt} + \frac{8pV}{3\pi} v' = p\left(\frac{8pV}{3\pi} - |v|\right) v \tag{B-6}$$

where v and V are as given by Equations B-4 and B-5. By expanding the term in the right hand side of Equation B-6 into a Fourier series, we find that v' is given by a Fourier series, the frequencies of the components being 3ω, 5ω, 7ω, ...(i.e., v' has no fundamental component!). This result is due to the above choice of the constant replacing $|v|$ in the first iteration. The amplitude B_3 of the leading component of v' of frequency 3ω is given by:

$$B_3 = 0.2\,V\left(1 + 12.5\,\frac{\omega^2}{p^2V^2}\right)^{-\frac{1}{2}} \tag{B-7}$$

This shows that the correction v' is not more than 20% of v, and, therefore, the error involved in the above linearization of the drag is at most of the same order.

LIST OF SYMBOLS

a	Radius of cylindrical cable, except in Section VI where it stands for a constant acceleration.
A	Area of the array projected in the direction of motion.
b	Buoyancy force on the cable per unit length.
B_a	Buoyancy force on the array.
c	Velocity of sound in cable.
D	Horizontal offset of the array from the target due to a horizontal current.
D_a	Hydrodynamic drag on the array.
E	Young's modulus of elasticity.
F	In Section III total horizontal force of the current on the cable.
g	Acceleration of gravity.
$H_o^{(1)}$, $H_1^{(1)}$	Hankel functions of orders 0 and 1, respectively, giving outgoing waves.
L	Length of the cable.
M_a	Dynamic mass of the array. (See page 12.)
r	Radial coordinate measured from the axis of the cable.
S	Metallic cross-sectional area of the cable.
t	Time coordinate.
t'	Normalized time coordinate ($= ct/L$).
t_1	Characteristic time of the array in free fall in water (see Equation 34).
T	Time ($= L/c$).

u	Dynamic displacement of an element of the cable due to dynamic (time varying) inputs.
u_o	Dynamic displacement of the top of the cable.
u_1	Dynamic displacement of the array.
u_s	Static elongation of the cable due to its own weight and the weight of the array.
U	Complex amplitude of u for sinusoidal variation with time.
U'	Normalized U $(= U/\lvert U_o \rvert)$.
U_o	Value of U at the top of the cable.
$\lvert U_o \rvert$	Magnitude of U_o.
U_1	Value of U at the array.
U'_1	Value of U' at the array.
v	Velocity of the array.
v'	Correction term for v.
V_o	Terminal velocity of the array in free fall in water.
w	Weight of the cable per unit length, except in Appendix A where it stands for the instantaneous velocity of the fluid along the cable.
W	Complex amplitude of the velocity w for sinusoidal variation with time.
W_o	Value of W at the cable.
W_a	Weight of the array.
x	Space coordinate measured along the axis of the cable.
x'	Normalized x $(=x/L)$.
X	Width of current.

y' Normalized space coordinate $(= 1 - x')$.

y'_{max} Value (s) of y' at which the amplitude of the dynamic stress in the cable for sinusoidal variation with time becomes maximum.

α Drag coefficient.

β Dimensionless drag coefficient for the array, equal to the ratio of the drag to the inertial force.

β_c Dimensionless friction coefficient for the cable.

μ Parameter equal to the ratio of the total mass of the cable to the dynamic mass of the array.

ν Kinematic viscosity.

ρ Density of water.

ρ_c Density of cable.

σ Tensile stress in cable due to static loading.

Σ Complex amplitude of the dynamic stress in the cable for sinusoidal variation with time.

Σ' Normalized Σ $(= L\Sigma / E|U_o|)$

Σ'_o Value of Σ' at the top of the cable.

Σ'_1 Value of Σ' at the array.

Σ'_{max} Maximum value of Σ' .

ϕ Angle $(= \arctan \frac{\omega'}{\mu})$.

Ψ Angle (see Equation 26).

ω Angular frequency (radians/sec).

ω' Normalized ω $(= \omega L/c)$.

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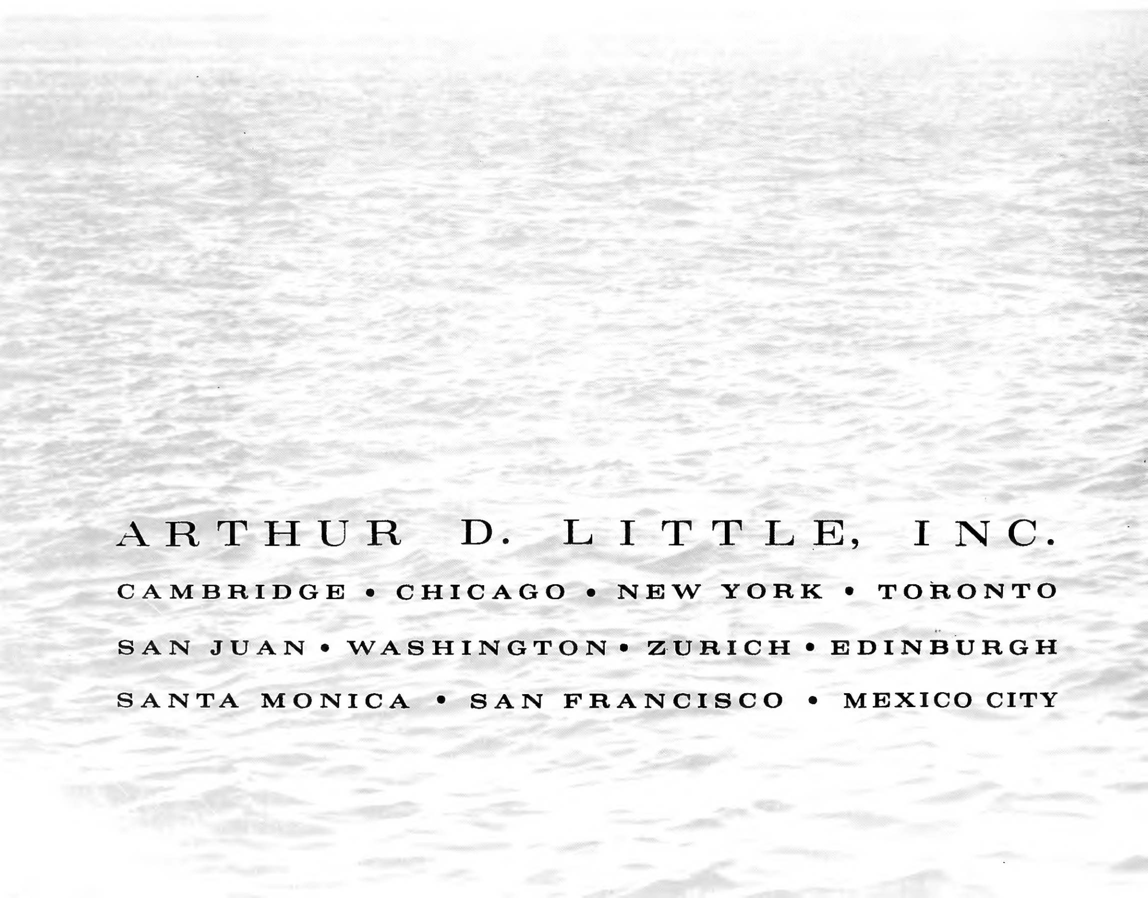

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