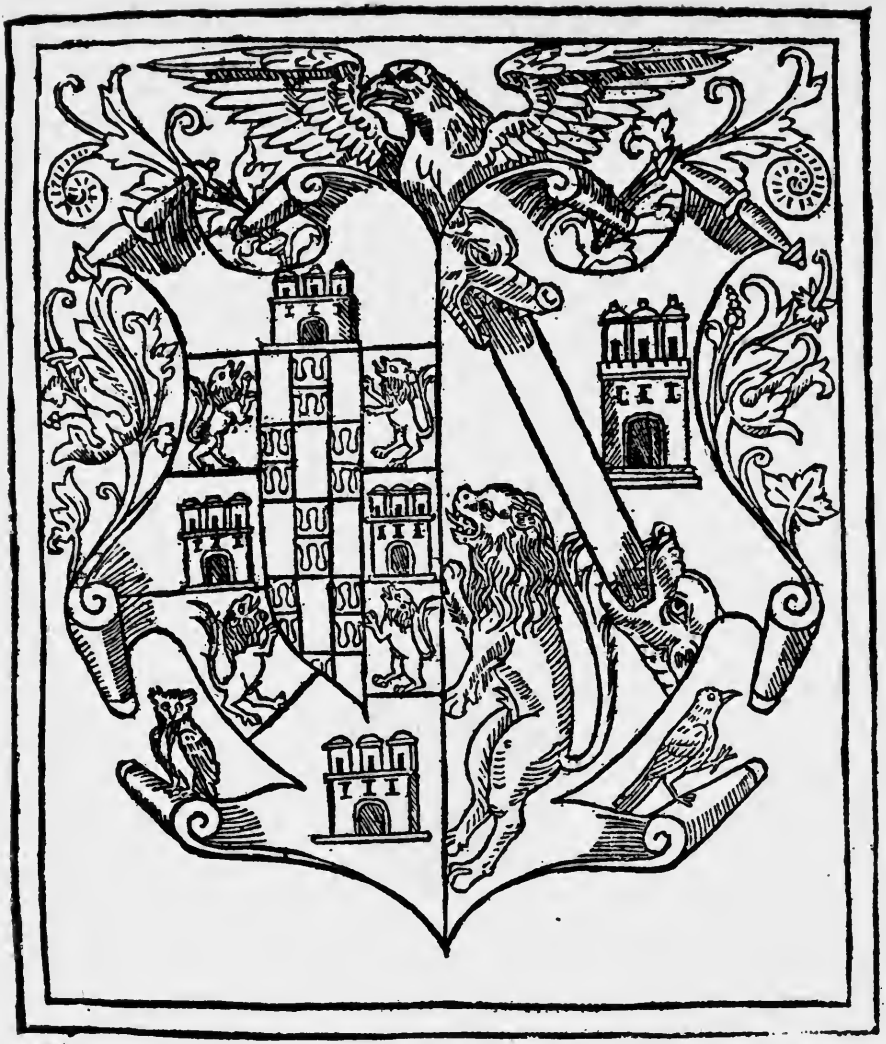
Diez, Juan. Sumario Compendioso; earliest mathematical work of the New World, by D. E. Smith. 1921. Facsimile text & trans. $5.00

(1952)

First work on mathematics published in New World.

Roland Hussey

THE SUMARIO COMPENDIOSO



Sumario cõpẽdioso delas quẽtas de plata y oro q̃ en los reynos del Piru son necessarias a los mercãderes: y todo genero de tratantes. Cõ algunas reglas tocantes al Arithmetica.

Fecho por Juan Diez freyle.

THE SUMARIO COMPENDIOSO OF BROTHER JUAN DIEZ

THE EARLIEST MATHEMATICAL WORK OF THE NEW WORLD

BY

DAVID EUGENE SMITH

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PREFACE

If the student of the history of education were asked to name the earliest work on mathematics published by an American press, he might, after a little investigation, mention the anonymous arithmetic that was printed in Boston in the year 1729. It is now known that this was the work of that Isaac Greenwood who held for some years the chair of mathematics in what was then Harvard College. If he should search the records still farther back, he might come upon the American reprint of Hodder's well-known English arithmetic, the first textbook on the subject, so far as known, to appear in our language on this side the Atlantic. If he should look to the early Puritans in New England for books of a mathematical nature, or to the Dutch settlers in New Amsterdam, he would look in vain; for, so far as known, all the colonists in what is now the United States were content to depend upon European textbooks to supply the needs of the relatively few schools that they maintained in the seventeenth century.

The earliest mathematical work to appear in the New World, however, antedated Hodder and Greenwood by more than a century and a half. It was published long before the Puritans had any idea of migrating to another continent, and fifty years before Henry Hudson discovered the river that bears his name. Of this work there remain perhaps only four copies, and it is desirable, not alone because of its rarity but because of its importance in the history of education on our continent, that a record of the text should be made generally accessible.

In making the translation the original methods of expression have been followed in many cases in which smoother diction would have suggested greater freedom of rendition. The reason has been, in general, the desire to bring such expressions as "8 per cent," "8 per 100," "one *cosa* and $\frac{1}{2}$ a *cosa*," and "four and $\frac{1}{2}$" into sharp contrast with the corresponding ones of the present time, and to show the great advance in symbolism and in methods of solution and proof. Such minor errors as were common in a century of careless proofreading have usually been corrected without comment. Aside from this, a certain freedom of translation has been assumed for the evident purpose of aiding the reader to follow the spirit of the text.

The editor wishes to express his indebtedness to Señorita Carolina Marcial Dorado for her scholarly assistance in the translation of the Spanish text.

DAVID EUGENE SMITH

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INTRODUCTION

I

THE MEXICO OF THE PERIOD

In order to understand the *Sumario Compendioso* it is necessary to consider briefly the political and social situation in Mexico in the middle of the sixteenth century. Cortés entered the ancient city of Tenochtitlan, later known as Mexico, in the year 1519, but its capture and destruction occurred two years later, in 1521. Thus, in the very year that Luther was attacking certain ancient privileges in the Old World, the representatives of other ancient privileges were attacking and destroying a worthy civilization in the newly discovered continent.

The rebuilding of the city began at once, and the new capital soon entered upon an era of great prosperity, disturbed, however, by the failure of Cortés to show the power as a civil leader that he had shown in his military capacity. The first viceroy of New Spain, which included the present Mexico, was a man of remarkable genius and of prophetic vision, — Don Antonio de Mendoza. He assumed his office in 1535, and for fifteen years administered the affairs of the colony with such success as to win for himself the name of "the good viceroy." He founded schools, established a mint, ameliorated the condition of the natives, and encouraged the development of the arts. In his efforts at improving the condition of the people he was ably assisted by Juan de Zumárraga, the first Bishop of Mexico. Among the various activities of these leaders was the arrangement made with the printing establishment of Juan Cromberger of Seville whereby a branch should be set up in the capital of New Spain.

Mendoza became viceroy of Peru in 1550 and died in Lima in 1552. Upon leaving Mexico he was succeeded by Don Luís de Velasco, a member of an illustrious Castilian family and one who labored faithfully for the betterment of the people intrusted to his charge. One of the first steps taken by him was to found, in 1551, the *Real y Pontificia Universidad de la Ciudad de Mexico*, and he was at all times interested in the success of the press and in the work of its manager, Juan Pablos, as the name appears in the books of the period. Don Luís died in 1564, sincerely mourned by the people, and was laid to rest in the Monastery of Santo Domingo, in the city in which he had exercised his benevolent authority.

In the same year that Mendoza left Mexico for Peru, Zumárraga passed away, his death being a genuine loss to the State as well as the Church. In the following year Alonso de Montufar* was nominated as his successor and was

* The spelling is substantially as given in the facsimile on page 62. The first name usually appears as Alphonso or Alfonso.

consecrated in 1553. For sixteen years he presided with great success over the Church in New Spain, and five years after the death of Don Luís de Velasco he too was buried within the precincts consecrated to the memory of Santo Domingo. It was in his time and with his sanction that the *Sumario Compendioso* was issued from the press, and well did he deserve the praise accorded to him by a contemporary writer in the words

"Clarissimo, et omnibus animi bonis ornatissimo Sacr. Theolog. Mag. Fr. Alphonso á Montufare, Archipraesuli Mexicano."

Tempora mutantur, nos et mutamur in illis. Such words of praise, expressed in the most sonorous of tongues and in words that seem exaggerated to our ears, belong to the past. In our rapid, unsettled, materialistic life we seem to take pride in our neglect of dignity of eulogy, excusing ourselves by condemning the past as insincere in its praise. Who that reads the story of this early period, however, can say that such descriptions of the characters and accomplishments of those who carried the Cross to the New World were exaggerated, or that they failed to express the genuine sentiments of the people to whose spiritual needs these brothers of the holy orders so conscientiously ministered?

II

PRINTING ESTABLISHED IN MEXICO

The idea of setting up a press in Mexico seems to have been considered as early as 1534, even before Mendoza became viceroy, doubtless at the suggestion of Juan de Zumárraga; but it was not until 1536 that the plan was carried out. Juan Cromberger then sent over as his representative Juan Pablos, a Lombard printer, and so the "casa de Juan Cromberger" was established, prepared to spread the doctrines of the Church to the salvation of the souls of the unbelievers. Cromberger himself never went to Mexico, but his name appears either on the *portadas* or in the colophons of all the early books. From and after 1545, however, the name is no longer seen, Cromberger having died in 1540.

It was this John Paul who printed the *Sumario Compendioso*, in 1556, and in order that the significance of the work may be the better appreciated it is appropriate to mention the following books, known to have been printed by him before that year:

c. 1537. *Escala Espiritual para llegar al cielo*, possibly in 1537, but there is no copy extant.
1539. *Breve y más compendiosa doctrina Christiana en lengua Mexicana y Castellana.*
1540. *Manual de Adultos.*
1541. *Relacion d'l espātable terremoto.*
1543. *Doctrina breue.*
1544. *Tripartito del Christianissimo y consolatorio doctor Juan Gerson.* This contains the earliest woodcut printed in Mexico.
1544. *Cōpēdio breue . . . ꝓcessiones . . . por Dionisio Richel.*
c. 1544. A second edition of the preceding work.
1544. *Dotrina xp̃iana.*
1545–1546. *Doctrina cristiana.*
1546. *Doctrina xp̃iana.*
1546. *Doctrina cristiana.*
1546. *Cancionero Spiritual.* The first book to bear the name of Juan Pablos as the printer, — "Juā pablos Lōbardo."
1547. *Regla christiana breue.*
c. 1547. *Doctrina cristiana en lengua mexicana.*
1548. *Doctrina Cristiana.*
1548. *Ordenāças y copilacion de leyes.*
1548. *Doctrina Cristiana en Lengua Huasteca.*
1550. *Doctrina cristiana.*
1553. *Doctrina cristiana.*
1554. *Recognitio, Svmmularum.*
1554. *Dialecta resolutio cum textu Aristotelis.*
1554. *Diálogos*, by Cervantes (Francisco Salazar).
1555. *Vn vocabulario en la lengua Castellana y Mexicana.*

In 1556 five books were published, among them the *Sumario Compendioso.* It thus appears that not only was this the first book on mathematics, but it was the first textbook of any kind, except for religious instruction, to be published outside of Europe.

In his *Bibliografía Mexicana del Siglo XVI,* Icazbalceta speaks of a copy in the library of the Convento de la Merced and of one in the Ramírez sale. There is also one in the Biblioteca Nacional at Madrid, from which three folios are missing, and it is this copy that has been used in the preparation of the present work, the missing portion containing parts of tables not included in this edition. There is also a copy in the British Museum.

The author of the *Sumario* was Juan Diez, a native of the Spanish province of Galicia, a companion of Cortés in the conquest of New Spain, and the editor of the works of Juan de Avila, known as "the apostle of Andalusia," and of the *Itinerario* of the Spanish fleet to Yucatan in 1518. He is sometimes confused with Juan Diaz, a contemporary theologian and author. In a letter written to Charles V in 1533 he is mentioned as a "clérigo anciano y honrado," so that he must have been advanced in years when the *Sumario* appeared. That this was the case is also apparent from a record of the expedition of 1518 in which it is stated that "triximus vn clerigo que dezia joan diaz," doubtless a young and adventurous apostle, full of zeal and desire to make known the gospel in the New World.

The other four books appearing from this press in the year 1556 are as follows:

Costituciones del Arzobispado y Provincia de la muy insigne y muy leal ciudad de Tenuxtitlan México de la Nueva España;

Costitutiones Fratruum Heremitarum Sancti patris nostri Augustini Hiponensis Episcopi et doctoris Ecclesiae;

Speculum Conjugiorum;

Catecismo y Doctrina Cristiana en Idioma Utlateco.

Not again in the sixteenth century did the Mexican printers publish any work on mathematics, except for a brief *Instrucción Nautica* which appeared in 1587. The press was generally true to its early purpose to issue only books relating to the conversion of the native inhabitants to the way of the Cross.

III

GENERAL DESCRIPTION OF THE BOOK

The *Sumario Compendioso* consists of one hundred and three folios, generally numbered. After the dedication (folios i, *v*, and ij, *r*) there is an elaborate set of tables, including those relating to the purchase price of various grades of silver (folio iij, *v*), to per cents (folio xlix, *r*), to the purchase price of gold (folio lvij, *v*), to assays (folio [lxxxj, *r*]), and to monetary affairs of various kinds.

The mathematical text (folio xcj, *v*) consists of twenty-four pages besides the colophon (folio ciij, *v*). Of these pages, eighteen relate chiefly to arithmetic and six to algebra.

The signatures are *a* (j, · · ·, iiij [· · ·, viij]), and similarly for b, · · ·, i, k, l, m, n (j, · · ·, iiij [, · · ·, vij]).

Folio i, *r* consists of the arms as shown in the facsimile, and the title: ¶ Sumario cõpẽdioso delas quẽtas / de plata y oro q̃ en los reynos del Piru son necessarias a los mercaderes: y todo genero de tratantes. Cõ algunas reglas tocantes al Arithmetica. Fecho por Juan Diez freyle.

Folios i, *v* and ij, *r* contain the dedication to the viceroy, beginning: ¶ Al Illustrissimo Señor Don Luys / de Velasco Visorrey y gouernador d'la nueua España. / &c. Juan diez freyle: que perpetua felicidad le dessea.

As to himself the author says:

"Por quãto Juã diez freyle estãte pre / sente enesta ciudad de Mexico me a becho relaciõ q̃l cõ ci ã cu y / dado trabajo & industria a cõpuesto vn libro de quẽtas de plata & / oro cõ algunas reglas t' uera del ordinario: tocãtes al arismetica: el q̃l es de / mucha vtilidad & puecho pa en los reynos del Piru a causa d'las muchas / variedades q̃ enel ay enlas leyes de plata & oro & otras cosas q̃ alla le vsã / lasq̃les todas estan en el dicho libro muy copiosamẽte puestas."

He therefore undertook the work for the purpose of assisting those who were engaged in the buying of the gold and silver which was already being taken from the mines of Peru and Mexico for the further enriching of the moneyed class and the rulers of Spain. The author felt that he could best serve this purpose by preparing such a set of tables as should relieve these merchants as far as possible from any necessity for computation. For this he had very good precedent, not so much in Spain as in Italy. In 1503 Anton Bartholomeo di Paxi had published in Venice a *Tariffa de pexi e mesvre* containing numerous tables relating to weight, value, and the like, and intended for the Venetian merchants engaged in foreign trade; in 1535 Giovanni Mariani had published a *Tariffa perpetva* in the same city and intended for a similar purpose among the merchants of all of Northern

Italy; and besides these, various other works of a similar nature had already been issued with the intention of relieving merchants from the extensive calculations imposed upon them by the complex systems of measures then in use.

Apparently prompted by the further demand for a brief treatment of arithmetic which should be suited to the needs of apprentices in the counting houses of the New World, the author devotes eighteen pages to the subject of computation and presents it in a manner not unworthy of the European writers of the period.

The most interesting feature of the work, however, is neither the tables nor the arithmetic; it consists of six pages devoted to algebra, chiefly relating to the quadratic equation.

The reason for this interest will be appreciated the more when we consider the state of algebra in Europe in the middle of the sixteenth century. Puzzle problems involving numbers, such as would now be solved by algebra, were known to the Egyptians in the second millennium B.C.; but no treatise upon the theory of equations is known before about A.D. 275, when Diophantus wrote his great work. It is not until the beginning of the ninth century that the word *algebra* appears in its present sense, having first been used by al-Khowârizmî in a treatise written in Bagdad in the time of the caliphs.

In the Middle Ages there appeared a number of algebraists of ability, notably Leonardo Fibonacci of Pisa, who lived early in the thirteenth century; and little by little these scholars added to the store of material which had already accumulated in the works of the later Greeks and the Orientals.

With the advent of printing from movable types, in the second half of the fifteenth century, there was awakened a new interest in mathematics, and particularly in the field of algebra. The Greek and oriental writers had solved the quadratic equation, but the equations of the third and fourth degrees still awaited solution, and a better symbolism was in urgent demand.

The middle of the sixteenth century saw the solution of the cubic and biquadratic equations by the Italian algebraists, and saw numerous efforts made at devising a convenient symbolism.

It was at this time that Juan Diez wrote. There had already appeared the notable algebra of Cardan (the *Ars Magna* of 1545), the Germans had published two treatises of merit, and there had appeared in 1514, from the pen of Gillis Vander Hoecke, a Dutch mathematician, a work of some consequence; but the number of treatises printed before 1556 was small, and these were far from being popular. It is therefore of considerable interest to know that an obscure writer in Mexico should have produced even six pages on the subject at this early period in the development of printed scientific literature.

IV

NATURE OF THE TABLES

The general nature of the tables may be seen from the facsimile on page 10. The abbreviations used are as follows:

p̃s is used for *peso* and *pesos*, originally a certain weight of metal, like pound, libra, and lira. The word comes from the Latin *pensum*, from *pendere*, "to hang." From the same root we have such words as *poise*, which also appears in *avoirdupois*, and such physical terms as *pendant* and *pendulum*. The Castilian *libra*, which found its way into Mexico, was about 1.014 avoirdupois pounds.

t is used for *tomin* and *tomines*. The tomin was the eighth part of a *peso*, and this was the same as the later *real*. The *peso* was therefore a "piece of eight." The tomin was also $\frac{1}{8}$ of a peso of weight, or $\frac{1}{3}$ of a drachm. The name comes from the Arabic *tomn*, "an eighth part."

mr̃os is used for *maravedi* and *maravedis*, a word derived from the name of the Moorish dynasty, *Murābitīn*, during which the coin was first struck. In these tables 56 *maravedis* make 1 *tomin*, but in tables of a later period the *real* (*tomin*) is given as equivalent to 34 *maravedis de plata Mexicanos*. In some of the tables of Juan Diez the *tomin* is taken as $56\frac{1}{4}$ *maravedis* (fol. lvij), and there are several other slight variations of this kind in the tabular work. The *maravedi* is also used as a weight, as on page 10.

on is used for *onça*, our ounce, from the Latin *uncia*, "a twelfth part," the ounce being the twelfth part of a Roman and early Spanish pound. From the same root we have our inch, the twelfth part of a foot. The uncial script of the Middle Ages received its name from the same source.

gr̃os is used for *grano* and *granos*, our grain as a unit of weight, one twelfth of a *tomin*; — " 12. granos q̃ tien vn tomin."

U is often used for 1000. Thus, we have ijU for 2000, iijU for 3000, and so on. The name for U is *cuento*, given in the tables as *cuẽto*, a word derived from *contar*, "to reckon." This use of U was common in Spain in the sixteenth century and has an interesting history. The symbol may be seen in the last two lines of the facsimile on page 10, where 300 maravedis correspond to jU pesos, that is, to 1000 pesos. The U in this sense is of uncertain origin. It appears a century earlier as ⌊J and may possibly have come from one of the several Roman symbols for a thousand. Among the curious variants are D with the vertical bar duplicated, and a symbol resembling the late Greek character for 900. In the sixteenth century the Portuguese used for the same purpose a symbol, the *cifrao*, which somewhat resembled our present dollar sign.

Una q̄rta	p̄s	t.rlvj.mſs.	rlvj.mſos	cliij.p̄s	ij.t.rrrvij ij.
media on	p̄s	j.t.rrrvij. ij	rlvij mſos	clvj.p̄s	v.t.rviij.iij.
j.on	p̄s	iq̄.t.rviij. iij	rlviij mſos	clx. p̄s	
ij.on	p̄s	vj.t.rrrvij. ij	rlx. mſos	clxiij.p̄s	ij.t.rrrvij.ij.
iij.on	j.p̄s	ij t.	l. mſos	clxvj.p̄s	v.t.rviij.iij.
iiij.on	j.p̄s	v.t.rviij. iij	l j mſos	clxx. p̄s	
v.on	ij p̄s	t.rrrvij. ij	l ij.mſos	clxxiij p̄s	ij t.rrrvij.ij
vj.on	ij.p̄siiij.t.		l iij.mſos	clxxvj.p̄s	v.t.rviij.iij.
vij.on	ij.p̄svij.t.rviij.	iij.	l iiij. mſos	clxxx. p̄s	
j.mſos	iij p̄s	ij.t rrrvij ij	lv.mſos	clxxiij.p̄s	ij.t.rrrvij.ij.
ij.mſos	vj p̄s	v.t.rviij. iij.	lvj.mſos	clxxvj.p̄s	v.t.rviij. iij.
iij.mſos	x. p̄s	t.	lvij.mſos	cxc. p̄s	
iiij.mſos	xiij.p̄s	ij.t.rrrvij. ij	lviij mſos	cxciij.p̄s	ij.t.rrrvij.ij
v.mſos	xvj.p̄s	v.t.rviij. iij.	l ix.mſos	cxcvj.p̄s	v.t.rviij.iij.
vj.mſos	xx. p̄s	t.	lx. mſos	cc. p̄s	
vij.mſos	xxiij.p̄s	ij.t.rrrvij. ij	lx j.mſos	cc iij.p̄s	ij t.rrrvij.ij.
viij mſos	xxvj.p̄s	v.t rviij. iij.	lxij.mſos	cc vj.p̄s	v.t.rviij. iij.
ix.mſos	xxx. p̄s	t.	lxiij.mſos	ccx. p̄s	
x mſos	xxxiij. p̄s	ij t.rrrvij. ij	lxiiij mſos	ccxiij.p̄s	ij t.rrrvij ij.
xx.mſos	lx. vj.p̄s	v.t.rviij. iij.	lxv. mſos	ccxvj.p̄s	v.t.rviij. iij.
xxx.mſos	c. p̄s	t.	lxx. mſos	ccxxxiij.p̄s	ij.t.rrrvij. ij.
xl. mſos	cxxxiij p̄s	ij.t.rrrvij .ij.	lxxx mſos	cclxvj.p̄s	v.t.rviij. iij.
xl j.mſos	cxxxvj p̄s	v.t.rviij. iij	xc. mſos	ccc. p̄s	
xlij.mſos	cxl. p̄s		c.mſos	cccxxxiij p̄s	ij.t.rrrvij.ij
xliij mſos	cxliij.p̄s	ij.t.rrrvij. ij	cc.mſos	dclxvj.p̄s	v.t.rviij. iij
xliiij mſos	cxlvj.p̄s	v.t.rviij. iij.	ccc.mſos	j.D p̄s	
xlv. mſos	cl. p̄s		cccc mſos	j.Dcccxxxiij p̄s	ij.t.rrrvij.ij

The approximate nature of the tables may be seen from the page here shown in facsimile. In the third line the value of 1 ounce is given as 0 pesos 3 tomines $18\frac{3}{4}$ maravedis. The half ounce should then be worth half of this, or 1 tomin $37\frac{1}{2}$ maravedis, as stated. The quarter ounce should then be worth half this amount, or $46\frac{7}{8}$ maravedis, whereas it is given in the table as only 46 maravedis. Similar instances of a lack of exactness are found throughout the tables, — a fact that would hardly have been considered significant in the somewhat crude financial transactions of the period.

The Roman numerals are used in all the tables, as was the custom among many bankers in various parts of Europe until the close of the seventeenth century. Where the chief commercial and financial operations consisted in additions and subtractions, these numerals were nearly as convenient for purposes of practical computation as the Hindu-Arabic symbols in use to-day.

The tables extend to "dos mil. cccc. de ley." There is a table of per cents extending to 30%. In this there are such entries as

iij . por . ciento. c . p̃s iij. p̃s

that is, 3% of 100 pesos is 3 pesos.

In general it may be said that the tables give the value of various numbers of ounces of silver in pesos, tomines, and maravedis.

The terms *pesos*, *tomines*, *maravedis*, and *varas* seem more acceptable in the translated text than any English words, and hence have been used. The more familiar marks, grains, ounces, crowns, and ducats have been given in English.

The tables are no longer of any importance, but as a matter of interest a single page is here shown in facsimile. Only the mathematical text has any historic significance, and it is this that appears in the translation.

THE TEXT

WITH TRANSLATION AND NOTES

Reglas ordinarias.

YEl que baſtantemente tengo pueſto por donde ſin hazer quenta ſe pueda ſaber el valor de qualquier varra o tejo de plata o oro por diferente ley y peſo que tenga y el valor delos yntereſes que ſe acoſtumbran a dar por qualquier plata o oro haſta treynta por ciento. y aſſi miſmo el valor de qualeſquier peſos de plata corriente comprados de enſayado razonando el ynteres de ocho a veynte por ciento juntamente con todo lo mas neceſario dela nueua España cõ las reduciones de peſos ducados y coronas, de aqui adelante pondre algunas reglas delas neceſarias en los reynos del Peru juntamente con algunas quiſtiones para curioſos entre las quales van algunas del arte mayor reſeruadas al algebra: las quales conlo demas ſino fuere tal como conuiene recebid la voluntad y ſea caritatiuamente emendado de lafalta que tuuiere.

Common Rules

NOW that I have sufficiently explained,* without doing the actual computing, how the value can be found of any ingot or bar of silver or gold of whatever standard or weight, and how to find the amount of commission up to thirty per cent which it is customary to give for any gold or silver; and, in the same way, how to ascertain the value of divers weights of silver currency bought as assayed, reckoning the commission from eight to twenty per cent, together with all else that is necessary in regard to the reduction of pesos, ducats, and crowns in New Spain, — from here on, I shall set forth some of the necessary rules which are used in the kingdom of Peru, together with certain problems for those who are interested, among which are certain parts of the *arte mayor* † pertaining to algebra. If these with the rest do not entirely meet with the approval of the reader, may he accept my good intentions, and, in as kindly a spirit as possible, excuse the mistakes which I may have made.

* In the tables, which make up the greater part of the book.

† The *arte mayor* was a term commonly used in the sixteenth century for algebra. It appears in the Latin of the period as *ars magna* and in the Italian as *l' arte maggiore*. Cardan, for example, called his great work on algebra by the name of *Ars Magna*, the work appearing at Nürnberg only eleven years before the *Sumario Compendioso* was published.

The name was occasionally combined with the ancient title, "The Science of Dark Things," used by Ahmes, an Egyptian mathematician of c. 1550 B.C., and with the Arabic title *al-jabr w'al muqâbalah*, used by al-Khowârizmî, c. 820. An illustration of this is seen in the title of Gosselin's treatise, *De Arte Magna, seu de occulta parte numerorum quae et Algebra et Almucabala vulgo dicitur, libri IV*, which appeared in Paris in 1577.

The use of *l' arte maggiore* for higher arithmetic (algebra) as distinguished from *l' arte minore* for elementary arithmetic may have been suggested by the seven *arti maggiori* and the fourteen *arti minori* of the merchants of medieval Florence.

The Italians also called the science by the name *Regola de la cosa*, the reason being that the unknown quantity was called the *cosa*, as stated on page 51. Because of this fact the German algebraist Rudolff (1525) called his treatise *Die Coss*, and English writers of the same century spoke of algebra as the "cossike arte."

The Arabic title given above means "restoration and equation," and hence *algebra* came also to mean "restoration to health." It is for this reason that, in *Don Quixote*, they sent for "*un algebrista* who attended to the luckless Samson."

¶Capitulo primero por el qual se da a entender la regla para hazer de plata corriente ensayada.

Entendido tengo que pocas vezes sera necessario hazer quenta que passe delos veynte por ciento que esta escripto: pero dexado esto a parte dare aqui la regla para que con saber partir la haga quien quiera: y es que ala cantidad q̃ quieres saber quanto es de ensayado\aña diras adelante dos zeros o cifras como estas,00, y despues ajusta con ciento el interesse q̃ vas por lo ensayado, por lo qual parte aquello a q̃ añadiste las dos cifras, y lo que saliere ala particion seran los pesos ensayados a q̃ se buelue lo corriente. Y nota q̃ lo que sobrare enla particion son pesos, y q̃ los has de multiplicar por. 8. tomines que tiene vn peso y lo produzido has de partir por el partidor de antes y el aduenimiento sera tomines: y ansi mesmo si algo sobrare son tomines y has los de multiplicar por. 12. granos q̃ tiene vn tomin y partir los por el mesmo partidor: y el aduenimiento sera granos los quales pon con los pesos y tomines de las particiones de antes\ y aquello sera lo que vale la plata corriente buelta en ensayada. Y sea te auiso que si en lo corriente ouiere tomines que por cada vn tomin põdras en lugar delas dos cifras. 12. y medio y porque mejor lo entiendas pondre aqui vn exemplo.

¶Primero Exemplo.

¶Digo que yo tengo. 4321. ps. 6. tomines de plata corriente los quales quiero cõprar de plata ensayada o de oro que melo dã a. 24 por ciento de yntereſe para lo qual tengo dicho que ala cãtidad corriente has de añadir dos cifras y si ouiere tomines por cada vno en lugar delas cifras. 12. y medio porq̃ las cifras por si no valẽ nada

Common Rules

¶ Chapter I, in which is explained the rule for finding the value of assayed silver.

I UNDERSTAND that sometimes it will be necessary to make calculations above the prescribed twenty per cent; but aside from this I shall now give a rule which anyone who knows division can follow, namely: to the amount of money with which you wish to buy the assayed silver annex two zeros or ciphers (00); then compute on a basis of a hundred the commission which you give for the assaying, and divide by this the number with the two ciphers annexed; the result of the division will be the assayed pesos which the currency will buy. It should be noticed that the remainder left from the division represents pesos and must be multiplied by 8, the number of tomines in a peso; this product must be divided by the same divisor as before, the quotient being the number of tomines. In the same way if there is again a remainder, it represents tomines and must be multiplied by 12, the number of grains in a tomin, and if we divide this by the same divisor, the quotient will be the number of grains. Now put these with the pesos and tomines and the result will be the value of silver currency in assayed form. Let me also say that if you wish the currency in tomines, put in place of the ciphers 12 and a half for each tomin, and in order that you may better understand I give an example.*

¶ First example

¶ Suppose that I have 4321 pesos 6 tomines in silver currency with which I wish to buy as much assayed silver or gold as they will give me at 24 per cent commission.† Now to the amount of currency you must annex two ciphers; and if you wish tomines, for each one put in place of the ciphers 12 and a half, because the ciphers themselves are not of any value when considered alone,

* The author here makes an approach to the decimal fraction. A tomin is $\frac{1}{8}$ of a peso, and hence 4321 pesos 6 tomines is equal to 4321 pesos plus $6 \times 0.12\frac{1}{2}$ pesos. It follows that 4321 pesos 6 tomines is equal to $(432{,}100 + 6 \times 12\frac{1}{2})$ hundredths of a peso. If, now, we wish to divide 4321 pesos 6 tomines by 1.24, we may avoid decimal fractions by dividing 432,175 by 124. This is what the author does in the illustrative problem which follows.

† The word *ynteres* (interest) is used by the author to mean any kind of percentage.

y puestas allí delãte siruen de aumẽtar en tal manera alas de a tras que al vno hazen valer ciento y assi poniendo el valor de vn tomin o dos o quales quier tomines siruen por si y por las dos cifras por quanto tienen dos granos que son vnidad y dezena: y assi mesmo aumentã al vno q̃ sea. 100. Y nota que esto no es otra cosa que multiplicar por.100. Y que se pone assi por mas breuedad: pues tornando a nuestro exmplo ya veys que lo corriente es. 4321. pesos.6. tomines por los quales pon adelante en lugar delas cifras 75.ques su valor delos seis tomines a doze y medio cada vno y vienen a ser. 432175. los quales parte por ciento y veynte y quatro que son el valor delos. 100. y el ynterese y venirte a ala particion. 3485. pesos. y sobran. 35. los quales haz tomines\ que es multiplicando los por.8. y son. 280. que partidos por.124. vienen.2.to. y sobrã.32.los quales haz granos que es multiplicando los por.12 que tiene vn tomin y son.384. que partidos por ciento y veynte y quatro te vendran.3. granos que juntos con lo de mas son. 3485.p̃s.2.to.3.granos.y esto es enlo que liquidamente se tornan los. 4321. pesos. 6.tomines de corriente comprados de ensayado o de oro a.24.por ciento y si quieres ver si es verdad añade les su yntereſle alos.24.por.100.que son,836.pesos.3. tomines.9.g̃ros y venirte a verissimo.

4321.p̃s.6.tomines.corriente 000
432175. particion 121
124. partidor 206
3485.p̃s:2.to. 3.g̃ros.ensayado. 06293
836.p̃s.3.to.9.g̃ros.yntereſe. 17055
delo ensayado. 432175 |3485.p̃s.2.to.3.g̃ros.

32 sobra 1 1244444 35 sobra. 3
por.12.g̃ros. 022 1222 por.8.to. 042
son.384 384 |3 11 son. 280 280 |2
124 124

but are annexed merely to raise the number in such a way that one becomes a hundred; and substituting the value of as many tomines as you wish, they serve, for themselves and for the two ciphers, to advance the number two places, through units and tens, thus raising one to 100. Observe also that this is nothing more than multiplying by 100 and is stated in this way for brevity. Now, using our example, we observe that the amount is 4321 pesos 6 tomines. In place of the two ciphers to be affixed, substitute 75, the value of six tomines, each being twelve and a half hundredths, and the result is 432,175. Divide this by one hundred and twenty-four, the value of the 100 plus the commission, and the result of the division is 3485 pesos, with a remainder of 35. This remainder is reduced to tomines by multiplying by 8, the result being 280. If we divide this 280 by 124, we have 2 tomines with a remainder of 32. Multiplying this 32 by 12, the number of grains in a tomin, we have 384, which divided by one hundred and twenty-four gives 3 grains.* The entire result now is 3485 pesos 2 tomines 3 grains, the amount of assayed silver or gold purchased with 4321 pesos 6 tomines of currency at 24 per cent. If you wish to prove this to be true, add the commission at 24 per 100, which is 836 pesos 3 tomines 9 grains, and you will see that it checks.

4321 pesos 6 tomines	currency		000		
432175	dividend		121		
124	divisor		206		
3485 pesos 2 tomines 3 grains	assayed		06293		
836 pesos 3 tomines 9 grains			17055		
commission for assaying			432175 | 124444	3485 pesos 2 tomines 3 grains †	
32 remainder	1		1222	35 remainder	3
by 12 grains	022		11	by 8 tomines	042
are 384	384 | 3			are 280	280 | 2
	124				124

* The result should be $3\frac{3}{31}$, but the fraction is rejected. This shows again how difficult it was to perform such operations to a high degree of precision without the aid of decimal fractions. Indeed, the use of such denominations as pesos, tomines, and maravedis was due solely to the necessity experienced by the ancients for avoiding fractions. For example, 6 tomines is merely a substitute for $\frac{3}{4}$ of a peso, or 0.75 of a peso.

† The method of division here shown is about the last stage of the medieval galley method which had been in use in Europe for a long time. By that method the figures were canceled out as soon as they had served their purpose. Evidently, however, the Mexican press had no canceled figures in their fonts, and hence they do not appear in this text.

¶ Declaracion dela regla pasada.

¶ Nota que la regla pasada es verisimamaente la regla de tres y q̃ assi como esta se funda por plata se puede fundar por otras muchas vias que bien podria dezir: yo tengo.2000.botijas de vino que las 1000.son .25.por.100.mayores que las otras.1000.compran melas todas con quelas chicas de por la medida delas grandes. Demando en quantas se bolueran las.1000.chicas medidas por la medida delas grandes. Para esta y las semejantes as de fundar la regla desta manera. Si.125.delas chicas se tornan en.100.delas grãdes en que se tornarã.1000.delas chicas:multiplica las.1000.por 100.seran.100000.q̃ es lo mismo como vees que añadir adelante las dos cifras:pues parte.100000.por.125.y venir te han.800.y en tantas se tornaran las.1000.botijas chicas medidas por la medida delas grandes:la prueua es que les eches su interesse a.25.por 100.y venir te han.200.como veis figurado

¶ 1000.botijas grandes.25.por.100.
mas que.10000.chicas.

¶ multiplicacion	1000	00
¶ multiplicador	100	024
¶ lo produzido	100000	100000 /800
¶ su partidor	125	12555
¶ aduenimiento	800	122
¶ enteres	280	1

¶ Hazer de pesos ducados y de ducados pesos muy en breue.

¶ Si quisieres saber,tantos pesos quãtos ducados son,saca el quinto delos pesos y sumalo conellos mesmos y el remaniente sera lo q desseas saber.

Common Rules

¶ Explanation of the former rule

¶ Observe that the preceding rule is really the Rule of Three,* and in the same way that this applies to problems relating to silver it can be applied to as many other processes or kinds of problems as one may wish. For example, I have 2000 jugs of wine, 1000 are 25 per 100† larger than the other 1000. Buy them all from me so that you will get the small ones at a price proportional to the price of the large ones. First find how many of the small jugs are equivalent to 1000 of the large jugs. For this and for like problems the rule can be applied in this way: If 125 of the small ones are equal to 100 of the large ones, to how many large ones are the 1000 small ones equal? Multiply the 1000 by 100 and the result, 100,000, is the same as if you had annexed two ciphers; now divide by 125 and the quotient is 800, the equivalent of 1000 small jugs. To prove this, increase the number 800 by 25 per 100 and the result is 1000, as you see worked out below.

¶ 1000 large jugs, 25 per 100 more than 1000 small ones.

¶ Multiplicand	1000	00	
¶ Multiplier	100	024	
¶ Product	100000	100000	800
¶ Divisor	125	12555	
¶ Quotient	800	122	
¶ Percentage ‡	200	1	

¶ A short method of changing pesos into ducats and ducats into pesos

¶ If you wish to find how many ducats there are in a certain number of pesos, take one fifth of the number of pesos, add it to the number of pesos, and the result is what you wish to find.

* The Rule of Three was the most popular of all the medieval commercial rules. It came to Europe, through the Arabs, from the Hindu arithmeticians. In the Middle Ages it went by such names as the *Regula de Tribus*, *Regula Rerum Trium*, *Regula Aurea*, and *Regula Mercatorum*.

† The Latin form was 25 *per centum*, whence 25 *por ciento*, 25 p^{o}c, 25 p$^{o}_{o}$, 25$^{o}_{o}$, and 25%.

‡ It will be observed that there are two inaccuracies in the original edition, namely, 10000 appears for 1000, and 280 for 200, "ynteres."

¶Exemplo:

¶445.p̃s el quinto es.89.que sumados juntos montan.534.y tantos ducados son los.445.p̃s.

¶265.p̃s.el quinto es.53.sumados cõlos.265. son.318.como veys

¶Si quisieres saber vna cantidad de ducados quantos pesos son saca el sesmo de los ducados,y lo que restare sera lo que buscas.

¶Exemplo.

¶534.ducados el sesmo es.89.restados de.534.quedan.445.y tãtos pesos son los.534.ducados.

¶318.ducados:el sesmo es.53.restados de.318.quedan.264.

¶Reduzir pesos a maranedis sin multiplicar.

¶Si quisieres saber vna orden muy bueua para saber vna cãtidad de pesos quantos marauedis son por muy mas facil y breue manera q̃ multiplicar:baz los pesos millares y saca el diezmo,y delo que restare la mitad y aquello sera lo que desseas saber.

¶Exemplo.

¶Toma.456,p̃s.baz los millares son.456000. el diezmo delos quales es.45600.(como veys figurado) q̃ restados del principal quedã,410400. la mitad es.205200. mr̃s. y tanto montan los 456.pesos a razon de.450.marauedis el peso.

456.p̃s.	son	456000.
	El diezmo es	45600.
	Restan	410400.
	La mitad es.	205200.marauedis.

¶Semejante ala passada en mayor cantidad.

¶Toma.34568.pesos.4.tomines y baz los millares y por el medio peso pon.500,y seran .34568500.el diezmo es.3456850.restados del principal quedan.31111650.la mitad es.15555825.marauedis como veys por la figura.

ℂ Example

ℂ One fifth of 445 pesos is equal to 89, which added to 445 gives 534, the number of ducats in 445 pesos.
ℂ One fifth of 265 pesos is 53. Adding this to 265 gives 318, as you see.

ℂ If you wish to find out how many pesos there are in a number of ducats, subtract one sixth of the number of ducats from the number of ducats.

ℂ Example

ℂ One sixth of 534 ducats is equal to 89, which subtracted from 534 is equal to 445, and this is the number of pesos in 534 ducats.
ℂ One sixth of 318 ducats is equal to 53, which subtracted from 318 gives 265.

ℂ To reduce pesos to maravedis without multiplying

ℂ If you wish a good rule for finding the number of maravedis in a certain number of pesos by a much easier and shorter method than that of multiplying, raise the number of pesos to thousands, find one tenth, subtract it from the number of thousands, and then find one half of the remainder; this gives the required number.

ℂ Example

ℂ Raise 456 pesos to thousands and you have 456,000; one tenth of this is equal to 45,600 (as you see by the work); this subtracted from the principal leaves 410,400, the half of which is 205,200 maravedis, and this is the number of maravedis in 456 pesos at the rate of 450 maravedis to a peso.

456 pesos are	456000.
The tenth is	45600.
The remainder	410400.
The half is	205200. maravedis.

ℂ Similar to the above, using larger figures

ℂ Take 34,568 pesos 4 tomines and raise them to thousands, and for the half peso put 500, and the result will be 34,568,500. One tenth of this is 3,456,850; and this subtracted from the principal leaves 31,111,650. One half of this is 15,555,825 maravedis, as you see by the work.

3456 8.ps.4.to.

¶Yo esta tengo por mejor ẽ ser breue y cierta como veras si lo vsas.

	34568500.
¶El diezmo.	3456850.
¶Restan.	31111650.
¶La mitades.	15555825.mrs.

¶Si quisieres saber tantos pesos quantas coronas son sin hazerlo por marauedis, multiplica los pesos por nueue y parte por siete, y el aduenimiento seran coronas.

¶Exemplo.

¶56.ps.multiplica por.9.son.504.parte por.7.vienẽ.72.coronas.

¶Para hazer ỏlas coronas pesos, multiplica por .7. y parte por.9

¶Exemplo.

¶63.coronas, multiplica por.7. son. 441. parte por.9 vienen. 49 que son pesos por la suma: tambien se puede hazer desta manera, a los.56.pesos añade sus dos setenes que son.16.y seran las.72.coronas, la otra ỏlas.63.coronas resta sus dos nouenes que son.14.que dan los.49.pesos como veys.

¶Para hazer de ducados coronas y de coronas ducados, aunque esta puesto por quenta, multiplica los ducados por. 15. y parte por 14. y pa hazer de coronas ducados, multiplica por.14. y parte por quinze, y los vltimos aduenimientos seran lo que buscas,

¶Exemplo.

¶42. ducados, multiplica por..15.son. 630. parte por.14. y venir te han.45.y tantas coronas son los.42.ducados.

¶Exemplo.

¶60.coronas, multiplica por.14.son.840. parte por.15.vienẽ.56. y tantos ducados son las.60.coronas: tambien lo puedes hazer como lo passado: y es que alos.42.ducados ajustes su catorzauo, que es tres: con que son las.45.coronas: y assi mismo alas .o. coronas

34568 pesos 4 tomines

¶ I consider this the shortest and best and surest method, as you will see if you use it.

	34568500.
¶ The tenth	3456850.
¶ Remainder	31111650.
¶ The half	15555825. maravedis.

¶ If you wish to find how many crowns there are in a certain number of pesos, without reducing to maravedis, multiply the pesos by nine and divide by seven, and the quotient will be crowns.

¶ Example

¶ 56 pesos multiplied by 9 is 504. Divide by 7 and the result is 72 crowns.

¶ To reduce crowns to pesos, multiply by 7 and divide by 9.

¶ Example

¶ 63 crowns multiplied by 7 is equal to 441. Divide this by 9 and the result is 49, which result is pesos. These two rules may be worked out in this way: For the first rule, to 56 pesos add its two sevenths, which is 16, and the result will be 72 crowns. For the second rule, from 63 crowns subtract its two ninths, which is 14, and the remainder is 49 pesos as you see.

¶ To reduce any number of ducats to crowns, multiply the ducats by 15 and divide by 14; and to reduce crowns to ducats multiply by 14 and divide by 15. The quotients will be the desired numbers.

¶ Example

¶ 42 ducats multiplied by 15 is equal to 630; divide this by 14 and the result is 45, and so many crowns are the 42 ducats.

¶ Example

¶ 60 crowns multiplied by 14 is equal to 840; divide this by 15 and the result is 56, and so many ducats are the 60 crowns.

Also you may reduce the first of the above sums, involving the 42 ducats, in this way: add to 42 its one fourteenth, which is three, giving 45 crowns. Proceeding in a similar manner with the second, from the 60 crowns, since they are

porque vienen a menos, restales su quinzauo que es, 4. quedã los 56. ducados.

¶ Auiso de memoria:

¶ Si quisieres saber de memoria muy facil y verissimamente vna cantidad de marauedis quantos pesos son, dobla los millares y despues añadeles su diezmo hasta que no aya dezenas, y ẽla suma por cada vnidad toma. 50. marauedis: y porq̃ mejor lo entiendas nota este exemplo porque me parece te bastara.

¶ Exemplo.

¶ Toma. 120000. marauedis, dobla los millares son. 240. el diezmo es. 24. y de. 24 dos, q̃ todos son. 266. pesos, y delos seys. 300. marauedis, y tanto valen los. 120000. marauedis.

¶ Auiso para saber lo que se deue de quinto de qualquier plata corriente que se fuere a quintar.

¶ Si fueres a quintar alguna plata corriente y quisieres saber por la pluma o de cabeça lo que te han de lleuar de quinto, toma el quarto delo que fueres a quintar razonando el marco a. 4. p̃s. y despues toma vno por. 100. delo mesmo y sumalo concillo, y la suma sera lo que deues del quinto y derechos del vno por ciento: y para mas satisfacion tuya pondre vna figura.

¶ Exemplo.

¶ Toma. 4575. p̃s ỗa vna raya por debaxo y luego saca el quarto es. 1143. pesos. 6. tomines y despues pon debaxo vno por ciento ỗ lo que fuiste a quintar̃ que son. 45. pesos. 6. tomines por quanto los 75. que te sobraron son tres quartos de. 100. y assi los tres quartos de vn peso son. 6. tomines: lo qual suma conlos de mas y montaran. 1189 pesos. 4 tomines: y tanto es lo que te han de lleuar de quinto y derechos como veys figurado.

to be reduced, subtract the fifteenth part, which is 4, leaving 56 ducats.

ℂ A rule to be memorized

ℂ If you wish to memorize an easy and sure way for finding the number of pesos in a number of maravedis, double the number of thousands, then to the result add its tenth, and so on until there are no more tens. Then for every unit in the sum take 50 maravedis. In order that you may better understand this rule, consider the following example which seems to me to be sufficient.

ℂ Example

ℂ Take 120,000 maravedis, double the thousands, and the result is 240; the tenth of this number is 24, and the tenth of 24 is two, and the sum total is 266 pesos. In units column there is 6, and for each of these units take 50 maravedis and the result is 300 maravedis. Thus we have 266 pesos 300 maravedis, which is the value of 120,000 maravedis.

ℂ A rule for finding the tax on any amount of silver currency to be assessed

ℂ If you are to compute the tax on any amount of silver currency and wish to know how much they will demand, take one fourth of the amount to be assessed, reckoning the mark at 4 pesos; then take one per cent of the original amount, add the one fourth and one per 100, and the sum will be the tax and the one per cent fee demanded. For a better understanding I shall set forth an example.

ℂ Example

ℂ Take 4575 pesos, draw a line underneath, below this write one fourth of the number, or 1143 pesos 6 tomines, and below this write one per cent of what you took to be assessed, or 45 pesos 6 tomines. The 75 which is left over is three fourths of 100, and the three fourths of a peso is 6 tomines. Add these together and the sum, 1189 pesos 4 tomines, is what they will demand of your money, including the fee, as you will see in the work below.*

* That is, $25\% + 1\% = \frac{1}{4} + 1\frac{1}{00}$.

¶ Mas de tener auiso q̃ si lo hizieres por mar cos, que has de tomar por cada marco, 1, ps, y por cada onça vn, to, y por cada quarta tres granos, y mas el vno por ciento como tẽgo dicho, Y nota que todo lo del vno por ciento es los derechos del marcador, y no mas, porq̃ aunq̃ a el le viene el q̃rto mas de derecho que aq̃ se le da su magestad lo lleua de menos: y la causa es que le paga el vno por ciento delo que le viene de quinto como tu delo q̃ lleuas a quintar o por mejor dezir se lo quinta de balde.

	4575, pesos
¶ El q̃rto.	1143, pesos, 6, to,
1 por, 100,	45, pesos, 6, to,
¶ Son,	1189, pesos, 4, to,

¶ Muy muchas maneras ay de multiplicar entre las quales yo tẽgo esta por la mejor y de mas verdad y certidumbre lo vno por no se lleuar nada ó memoria lo otro porque para la regla de tres no es necessario mudar las letras para auer de multiplicar.

¶ Exemplo.

¶ Digo que multipliques, 879, por, 758. lo qual has de poner enla manera como ves figurado y dar entre la multiplicacion y multiplicador vna raya como esta. \. y dar debaxo otra raya como aqui ves. 875\978, y luego cõel, 8, primera letra de mano yzquierda que es multiplicador multiplica todas las dela multiplicacion que son las de adelante dela raya poniendo las letras enesta manera, 8, vezes, 9 son, 72, el siete que es dezena debaxo del, 8, y el, 2, que es vnidad vn grado adelante que sera debaxo del, 7, y sino ouiera vnidad auias ó poner la dezena en su mesmo lugar debaxo dela letra que tomaste por multiplicador y por la vnidad enel grado adelante vn zero como este. o. y sino ouiera dezena no auias de poner nada: pero la vnidad en su lugar vn grado adelante de do auia ó estar la dezena y luego di. 8, vezes, 7. 56, el, 5, debaxo del, 2, y el, 6, vn grado adelãte del 2, y luego di. 8, vezes, 8, 64, el, 6, debaxo del otro, 6, y el, 4, vn grado adelante, que es debaxo dela primera letra de la multiplicacion: ago

¶ Let me advise that if you wish to do this by marks of weight, for each mark take 1 peso; for each ounce, one tomin; for each quarta, three grains, and besides the one per cent of which I have spoken.

	4575 pesos
¶ The fourth	1143 pesos 6 tomines
1 per 100	45 pesos 6 tomines
¶ Makes	1189 pesos 4 tomines

Observe that the entire one per cent is the fee for the weigher or assayer, and no more, because, although there is due him one fourth more as a fee than is given him by his majesty, he receives less than this. The reason is that he pays to his majesty one per cent of that which is due himself of the tax, as you do on what you take to be taxed; or better expressed, it is taxed gratis.

¶ There are many ways of multiplying,* among which I consider the following to be best and most accurate. For one reason, no memory work is required; and for another, according to the Rule of Three it is not necessary to move the figures in order to know how to multiply.

¶ Example

¶ To multiply 879 by 758, we must place the figures in order you see set forth, and draw between multiplicand and multiplier a line like this \, and then draw underneath them another line, thus: 875\978. Then, using the figure 8, the first one on the left-hand side, which is the multiplier, multiply all the figures in the multiplicand which are beyond the line in the following manner: 8 times 9 is 72; place the 7, which is tens, under 8, and the 2, which is units, under the next figure, which is 7. If you have no units, you must put the tens in their place under the first figure which you take as the multiplier, and in the place of units you must write a zero, like this: 0. If you have no tens, you must not put down anything, but must put the units in the column next to the one in which there would have been tens. Then observe that 8 times the 7 of the multiplicand is 56, and so we place the 5 under 2, and place 6 in the next column after 2. Similarly, 8 times 8 is 64, and we place the 6 under the other 6 and place 4 in the next column under the first figure of the multiplicand.

* The method most commonly used in the sixteenth century was not the one given in the example, but one of the two known by the Italian names *gelosia* and *bericuocolo*.

ra vera el,8,y tomo el,7,de delãte conel qual ví,7,vezes nueue,63 el,6,debaxo del,5,enla meſma orden de el,7;y el tres vn grado adelãte debaxo del,6,y luego ví,7,vezes ſiete,49,el quatro debaxo del,3,y el,9,debaxo del,4,de adelante luego ví,7,vezes,8,56,el cinco debaxo del nueue y el,9,debaxo del,7,ſegũ la letra dela multiplicació:agora vera el,7,y toma el,5,que es poſtrero multiplicador con el ql ví,5,vezes,9,45,el,4,debaxo del otro,4,enla orden,ví,5,que tomaſte por multiplicador y el,5,debaxo del,5,ó delante y luego ví,5,vezes,7 35,el,3,debaxo del,5,y el,5,debaxo del,6,y luego ví,5,vezes,8,40,el,4,debaxo del,5,y el zero debaxo dela poſtrera letra y luego ſuma.

875\978	
726	460
56	95
63	54
4	5
4	3
855	750

¶ Regla para cobrar de ſu mageſtad por el quinto.

¶ Muchos creo ay o a auido q̃ les deue ſu mageſtad dineros y para los auer de cobrar les es neceſſario buſcar oro o plata por quitar para que por el quinto lo cobren los quales por no ſaber lo que an ó lleuar a quintar para cobrar lo q̃ ſe les deue o lleuan demas o de menos delo qual lleuando de mas les es perjuyzio en que la plata y oro que lleuan les queſta intereſſe y lleuando de menos les es neſceſſario boluer otra vez a quintar de que tengo entendido algunos reciben peſadumbre y para los tales con ayuda de dios dare aqui vna regla como facil y ſin ningun yerro ſepan lo que hã de lleuar a quintar para cobrar al juſto lo que les deue.

¶ Regla.

¶ Pon lo que ſe te deue por ſuma ala qual añade dos zeros como eſtos.00.y ſi ouiere tomines pon por cada tomin en lugar delos zeros doze y medio como tengo dicho enla regla de tornar plata corriente a enſayada y deſpues parte aquella cãtidad por veynte y ſeys y lo que ala particion ſalicre ſera lo que deſſeas ſaber como aq̃ veys

Now leave the 8 and take 7 for the multiplier, thus: 7 times 9 is 63, so we place the 6 under 5 in the same column as the 7, and place the 3 in the next column under the 6. Then 7 times 7 is 49, and we place the 4 under the 3, and the 9 under the 4 in the column of the first figure of the multiplicand. Then 7 times 8 is 56, and we place the 5 under the 9, and the 6 under the 7 in the column of the second figure of the multiplicand. Now leave the 7 and take 5, the last figure of the multiplier. Then 5 times 9 is 45, and we place the 4 under the other 4 in the column of 5, the multiplier, and put the 5 under the other 5. Then 5 times 7 is 35, and we place the 3 under the 5, and the 5 under the 6. Then 5 times 8 is 40, and we place the 4 under the 5, and the zero under the last figure of the multiplicand, and then we add.

875\978
726 460
56 95
63 54
4 5
4 3
855 750

ℂ Rule for collecting what is due from his majesty

ℂ I believe there are and have been many to whom his majesty owes money. For those who have to collect, it is necessary for them to bring gold or silver to be assessed so that they may collect what is due. These people, because they do not know how much they have to take to be assessed in order to collect what is due them, take too much or too little. Taking too much is hurtful because commission is charged on the extra gold or silver that they take. Taking too little makes it necessary to assess again, from which, I understand, some unpleasantness results. For such as they, with the help of God, I will state here an easy rule, without any error, by which they may know how much to take to be assessed so as to collect justly what is due them.

ℂ Rule

ℂ Put down the sum that is due you; to it annex two zeros like these: oo. If you would like tomines, put for each tomin twelve and a half in place of the zeros, as I have stated in the rule for converting currency into assayed silver. Divide the sum by twenty-six, and the quotient will be what you wish to know, as you will now see.

¶Exemplo.

¶Digo que te deuan,1144,pesos alos quales añade los dos zeros y montarã,114400,lo qual pte por 26,y viene ala partición,4400.ps que a,4,pesos el marco son, 1100, marcos y tantos has d̃ lleuar a quitar para cobrar los,1144,pesos la prueua sera que saques el $\frac{1}{4}$ y despues vno por ciento como te tengo dicho enla segunda regla antes desta.

00,
12
0300
114400|4400
26666
222

¶Prueua, 1100,mrcos
¶Son. 4400,ps,
¶El q̃rto 1100,ps,
¶1.por.100. 44,ps,
¶Son. 1144,ps,

De aqui adelãte pondre algunas preguntas que aunque no son necessarias para lo que eneste reyno se vsa \ los que son aficionados ala quenta se holgaran con ellas porque aunque no son sutiles para los que algo saben los que lo desean saber con ellas tendran principios para mas subir.

¶Primera pregunta.

¶Fui a quintar cierto oro no se lo que pague del quinto porq̃ me lo quitaron de vna libraça, pero se que sacados quintos y derechos me costo este oro sin interesse,1584,pesos: demando quanto es lo que me quitaron enla libraça y que es lo que agora vale este oro. Regla saca el $\frac{1}{4}$ de,1584,es,396,suma los cõ,1584,son 1980,los quales parte por,99,el aduenimiento es veynte ajustados a,1980,son,2000,y tanto vale agora el oro,suma,396,con,20 son,416,y tanto se quito dela libraça verissimo.

¶Segunda pregunta.

¶Semejante ala passada fui a quintar vn pedaço de oro no se lo q̃ pesaua antes d quintar pero se que me quitaron del,416,pesos de derechos. Demãdo ques lo que me ha de quedar. Regla multiplica 416,por,5,son, 2080, parte los por,26,vienen,80,restados de, 2080, quedan, 2000, delos quales saca el $\frac{1}{5}$ es,400, restados

Common Rules

ℂ Example

ℂ Suppose that there is due you 1144 pesos. Annex to this number two zeros, making 114400; divide this number by 26 and the result is 4400 pesos which, at 4 pesos to the mark, make 1100 marks, the amount you ought to take to collect the 1144 pesos. To prove this, find $\frac{1}{4}$ of 4400, and then one per cent of 4400, as I have stated in the second rule preceding this one, and then add.

From now on I shall propose certain questions in which, although not necessary for what is used in this kingdom, those that like arithmetic will delight. Although they are not difficult for those who know something of mathematics, those who desire to know more will find in them the beginnings for further advance.

```
                    00
                    12
                   0300
                114400 | 4400
                 26666
                   222
```

ℂ Proof	1100 marks
ℂ Are	4400 pesos
ℂ The fourth	1100 pesos
ℂ 1 per 100	44 pesos
ℂ They are	1144 pesos

ℂ First problem

ℂ I took a certain amount of gold to have taxed. I do not know what tax I paid, because it was paid from a bill of exchange, but I know that, deducting the tax and the fees, this gold cost me without commission 1584 pesos. I want to know how much they took from me in the bill of exchange and what the gold is now worth.

Rule: Take $\frac{1}{4}$ of 1584, which is 396; add this to 1584 and the result is 1980. Now dividing 1980 by 99 the quotient is 20, and this added to 1980 gives 2000, the present value of the gold. Adding 396 to 20 gives 416, and this is what was taken from the bill of exchange.*

ℂ Second problem

ℂ Similar to the above, I went to assess a piece of gold. I do not know what it weighed before being assessed but I know that they took 416 pesos as the fee. I want to know what should be left for me.

Rule: Multiplying 416 by 5 we have 2080; dividing this by 26 there results 80. This subtracted from 2080 gives 2000, from which find $\frac{1}{5}$, which is 400;

* We have 25% of 1584 = 396, tax; 125% of 1584 = 1980, value of gold less 1%. Hence the value is 2000. Then 2000 − 1980 = 20, fee, and 396 + 20 = 416, total payment.

de,2000,quedan,1600,delos quales saca vno por cẽto son16,restados ỏ,1600,quedã.1584,y tanto es lo q̃ agora vale el oro. Mas breue ajusta dos ceros como estos,00,adelãte de,416,son,41600 parte por,26,vienen,1600,delos quales saca vno por,100,viene 16,restados de,1600 queda n 1584,por la suma.

¶Tercera pregunta.

¶E comprado diez varas de terciopelo menos,20,pesos por,34, pesos y mas vna vara ỏ terciopelo,ỏmãdo a como costo la vara. Regla suma los pesos,20,y,34,son,54,que sera tu particion resta de las,10,varas la vna que dize y mas,quedan,9,varas por las quales pte los,54,viene,6,y tanto es el precio de cada vara Prueua,10 varas a,6,pesos hazen,60,pesos menos,20,pesos quedan,40,dize que costaron,34,pesos y mas vna vara que son,6,con que valẽ los mesmos,40,pesos.

¶Quarta pregunta.

¶E comprado,12,varas delo dicho menos,30,pesos por,98, ps menos quatro varas demãdo a como costo la vara, Nota esta q̃ es muy breue y verissima suma los pesos,30,y,98,son,128,suma las varas,12 y,4,son,16,por los quales parte,128,venir te an,8,y tãto es el precio de cada vara. Prueua,12,varas a,8,son,96,menos 30,es,66,dize costarõ,98,menos,4,varas que son,32,pesos que como veys,quedan los dichos,66.

¶Quinta pregunta.

¶E comprado,9,varas delo dicho por tanto mas de,40,pesos, quanto,13,varas al mesmo precio valen menos de.70.demãdo a como costo la vara. Regla haz como ẽla passada suma los pesos,40,y 70,son,110,suma las varas,9,y,13,son,22,por los quales parte los 110,el aduenimiento es,5,y tãto es el precio de cada vara. Prueua, 9,varas a,5,pesos son,45.pesos que son,5,mas de 40,y,13,varas a,5,son.65,que son,5.pesos menos de,70,como veys

this subtracted from 2000 leaves 1600, of which find one per cent, which is 16; and this subtracted from 1600 gives 1584, the present value of the gold. Briefly, annex two zeros to 416, making 41,600; then divide by 26, giving 1600. Take one per 100 of this, or 16, and 1600 minus this gives 1584, the required sum.*

¶ Third problem

¶ I bought 10 varas of velvet at 20 pesos less than cost, for 34 pesos plus a vara of velvet. How much did it cost a vara?

Rule: Add 20 pesos to 34 pesos, making 54 pesos, which will be your dividend. Subtract one from 10 varas, leaving 9. Divide 54 by 9, giving 6, the price per vara.

Proof: 10 varas at 6 pesos is 60 pesos. This minus 20 pesos is 40. You paid 34 pesos plus a vara costing 6 pesos, and this gives the result, 40 pesos.†

¶ Fourth problem

¶ I bought 12 varas of velvet at 30 pesos less than cost, for 98 pesos minus 4 varas. How much was the cost per vara? The following is a short method: add the 30 pesos and the 98 pesos, making 128; add the number of varas, 12 and 4, making 16; divide 128 by 16, giving 8, the price per vara.

Proof: 12 varas at 8 pesos are 96 pesos; this less 30 pesos is 66 pesos. You paid 98 pesos minus 4 varas, or 32, and this leaves 66.‡

¶ Fifth problem

¶ I bought 9 varas of velvet for as much more than 40 pesos as 13 varas at the same price is less than 70 pesos. How much did a vara cost?

Rule: Add the pesos, 40 and 70, making 110. Add the varas, 9 and 13, making 22. Dividing 110 by 22 the quotient is 5, the price of each vara.

Proof: 9 varas at 5 pesos are 45 pesos, which is 5 more than 40 pesos; and 13 varas at 5 pesos are 65, which is 5 pesos less than 70, as you see.§

* The second method amounts to this:

$416 = 1\% + 25\%$ of amount assessed

$= 26\%$ of amount assessed.

$\frac{100}{26} \times 416 = 1600$, amount assessed.

1% of 1600 = 16, fee.

Therefore 1584 is left after the fee is paid.

† $10x - 20 = 34 + x$,

$x = 6$.

‡ $12x - 30 = 98 - 4x$,

$x = 8$.

§ $9x - 40 = 70 - 13x$,

$x = 5$.

¶9. vaſas por tanto mas de.40.ps. quãto.13. varas alo miſmo. ſon menos. de.70.ps.

	13
40	9
70	22
110	00
	110/5.ps
	22

¶Quiſtiones por los numeros q̃drados

¶Para auer de hazer qualquier pregunta que te fuere demandada de numeros quadrados, es neceſſario que ſepas que es numero q̃drado y porque ſe llama quadrado, y que es numero cubo y porque ſe llama cubo.

¶Numeros quadrados ſe llaman y ſon aquellos que nacen dela multiplicacion o ſon produzidos de algun numero en otro ſemejante como.4,9,16,ꝛc.q̃el,4,nace del,2,multiplicado por ſi meſmo diziendo,2,vezes,2,ſon,4,y el,9.nace del,3, por el meſmo conſiguiente porque,3,vezes,3,ſon,9, delos quales numeros los lineales como el,2,o el,3,ſon las rayzes.

¶Numeros cubicos ſe llaman y ſon aquellos que ſon contenidos de 3,numeros yguales lineales dela qual multiplicacion ſon dichos ſer procreados aſſi como,8,27,64,125,ꝛc.porque,8, nace de,2,y del pues enel produzido es a ſaber, 2, vezes,2,4,y, 2,vezes,4,ſon, 8, y,3,vezes,3,9,y,3,vezes,9,27.ꝛc.

Primera quiſtion.

¶Da me vn tal numero que bajuſtandole, 15, baga numero quadrado y reſtando del,4,ſea lo meſmo,regla ſuma,15,y,4,ſon,19,ajuſtales, 1, ſon, 20, toma la mitad q̃ es, 10,quadralos en ſi dizien-

Square Numbers

		13
¶ 9 varas for as much more than 40 pesos	40	9
as 13 varas at the same price is less than	70	22
70 pesos.*	110	00
		110|5 pesos
		22

¶ Problems relating to square numbers

¶ In order to know how to solve any problem that is given to you relating to square numbers it is necessary to know what a square number is and why it is called a square, and also what a cube number is and why it is called a cube.

¶ A square number is a number that is derived by the multiplication of a number by itself, as is the case with 4, 9, 16, &c. The 4 comes from multiplying 2 by itself, as when we say that 2 times 2 is 4; and the 9 is the product of 3 multiplied by itself, because 3 times 3 is 9. Of such numbers the lineals like the 2 or the 3 are called the roots.†

¶ A cube number is a number that contains the three identical numbers multiplied together, as is the case with 8, 27, 64, 125, &c.; for 8 is the product of 2 times 2 times 2; similarly, 3 times 3 times 3 are 27, &c.

¶ First problem

¶ Give me a number which, increased by 15, is a square number; and decreased by 4 is also a square number.‡

Rule for solving: Add 15 and 4, making 19; then add 1 to this result, making 20. Now take the half of this number 20, which is 10, and then square this result,

* The work here given shows the cumbersome method used in solving the equation

$$9x - 40 = 70 - 13x.$$

† In this work the word *root*, taken by itself, signifies square root.

‡ The rule depends on the fact that $\left(\frac{a+b+1}{2}\right)^2 - a - b$ (or $+ a$) is a square.

do,10,vezes,10,son,100,delos quales resta los,15,que se an de ajustar quedan,85,y este es el numero demandado, de el qual si restas los,4,qdā,81,su rayz es.9.y assi mesmo si le ajustas.los.15,son cierto rayz delos q̄les es,10,porque,10,vezes,10,son cierto y esto valla.

¶Segunda quistion.

¶Dame vn numero que ajustandole,8,sea quadrado y restando dl 8.quede quadrado, toma medio de ocho es,4,quadrale es,16,ajustale,1,es,17,y este es el numero demandando, al qual si ajustas,8, haze,25,que su rayz es,5,y si le restas,8,quedan,9,que su rayz es,3 porque,3,vezes,3,son nueue como veys.

¶En mayor cantidad, da me vn numero que ajustādole,20,sea numero quadrado y restando del,20,quede quadrado,toma mitad de 20,es,10, quadra es,100,ajusta vno haze,101,y este es el vn numero q̄ si le ajustas,20,es quadrado y si le restas,20,queda quadrado.

¶Quistion tercera.

¶Tiene vno dos reatas muy buenas van le por ellas.8.p̄s,no las quiere dar,viene vno cōprasela por varas en esta manera que le va por cada vara de cada vna tantos tomines quantas varas tuviere aquella pieça dadas y hecha la quenta no hallan que valen mas que los.8.pesos que daua el primero,demādo que varas tenia cada vna por si para la qual regla es menester que busques dos numeros quadrados tales,que juntos en vno no sean mas que vno quiero dezir.q̄ hagan numero quadrado los quales numeros son,3,y 4,que multiplicados cada vno por si mesmo diziendo,3,vezes,3 9.4,vezes,4 16,y sumados el vno cō el otro son,25,y su rayz es,5,y luego vi por regla de,3,si cinco rayz de,25,son venidos de,8,que es el valor de-

thus: 10 times 10 is 100. From this subtract 15, and we have 85, and this is the number required, that is, the one from which if you subtract 4 you have 81, the root of which is 9. The same thing happens if you add the 15, the result being a hundred, the root of which is 10; for 10 times 10 is 100, which checks.

ℂ Second problem

ℂ Required a number which increased by 8 is a square, and decreased by 8 is also a square. Take half of eight, which is 4; square it, making 16; add 1, making 17, and this is the number which increased by 8 is 25, the root of which is 5; and which decreased by 8 is 9, the root of which is 3; for 3 times 3 is 9, as you see.*

ℂ Using larger numbers, required a number which increased by 20 is a square number, and decreased by 20 is also a square number. Take half of 20, namely 10; square it, making 100; add one, making 101, and this is a number which increased by 20 is a square, and decreased by 20 is also a square.

ℂ Third problem

ℂ A man has two very good ropes for which he can get 8 pesos, but he refuses the offer. Someone offers to buy them by the vara in such a way that for every vara in each rope he gets as many tomines as there are varas in that rope. When the computation is made, they find that the money is no more than the 8 pesos which the first one offered. How many varas are there in each rope?

To solve this it is necessary to find two numbers whose squares added together make a number which is no greater than that of the one given. These numbers are 3 and 4. Multiply each number by itself and we have 3 times 3 which is 9, and 4 times 4 which is 16; and these added together are 25, and the root of this is 5. Then by the Rule of Three, as five, the square root of 25, is to 8, the value

* This depends on the fact that

$$\frac{x^2}{4} + 1 + x = \left(\frac{x+2}{2}\right)^2, \text{ a square,}$$

and

$$\frac{x^2}{4} + 1 - x = \left(\frac{x-2}{2}\right)^2, \text{ also a square,}$$

the rule simply giving the expression $\frac{x^2}{4} + 1$.

lo q̃ se vendio de ado vendran,3,y,4,que fueron los numeros halla dos multiplica,8,por,3, son,24, parte por cinco vienen quatro y $\frac{4}{5}$ y estas son las varas dela vna, luego di,8,vezes,4,son,32,par te por,5,vienen,6, $\frac{2}{5}$ y estas son las varas dela otra que sumadas entrambas tienen,11, varas y vn quinto y vendidas cada vna por si dādo por cada vara de cada.1. tantos tomines quātas varas tiene vienen a valer los,8,pes,la prueua,multiplica,6 $\frac{2}{5}$,por,6,$\frac{2}{3}$ vienen 40,y $\frac{24}{15}$ y multiplica,4,y,4,quintos por,4,y $\frac{4}{5}$,viene,23,y $\frac{1}{15}$ q̃ sumados son, 64,tomines y partidos por,8,tomines que tiene vn peso son,8,pesos.

¶Quistion quarta.

¶Si te fuesse pedida vna quistion en tal manera que te dixessen,da me vn numero quadrado y tal que quitando del vna cantidad cier ta quede quadrado y ajuntando se la sea quadrado, para auer de ab soluer vna tal quistion es necessario que sepas que cosa es numero congruo y que cosa es numero congruente.

¶El numero cōgruo se llama y es vn tal numero ques abto a dar y re cebir otro numero el qual se llama congruente en tal manera que dā dole o recibiendole siempre sea quadrado, y para que mejor y mas claramēte lo entiendes pondre aqui bajo los numeros congruos y congruentes que me parecē seran necessarios, y assi mesmo pōdre vn exemplo,por el q̃l si bien le notas podras declarar todas las quis tiones q̃ por esta via te fue en demandadas siēdo tal el numero de mādado q̃ se halle numero cōgruente q̃ partido por el el aduenimiē to sea numero quadrado, porq̃ no lo siendo a otro secreto, el qual dexo por no ser prolixo.

at which it was sold, so we have 3 and 4 to the numbers to be found. Multiply 8 by 3 and we have 24; divide by five and we have four and $\frac{4}{5}$, and this is the number of varas in one piece. Then 8 times 4 is 32; divide by 5 and we have 6, $\frac{2}{5}$, and this is the number of varas in the other piece. These added together give 11 varas and a fifth. If we pay for each vara of each rope as many tomines as there are varas, the value comes to 8 pesos.

Proof: Multiply $6\frac{2}{5}$ by $6\frac{2}{5}$ and we have 40 and $\frac{24}{5}$. Multiply 4 and 4 fifths by 4 and $\frac{4}{5}$ and we have 23 and $\frac{1}{25}$, which added together gives 64 tomines. This divided by 8, the tomines in a peso, gives 8 pesos.*

ℭ Fourth problem

ℭ Suppose that you were given this problem: Find a square number such that if we take from it a certain number, there remains a square; and if we add to it the same number it is also a square. In order to solve such a problem it is necessary to know the nature of a congruous number and of a congruent number.

ℭ A congruous number is such a square number that, subtracting from or adding to it another number, called a congruent number, it will still be a square. So that you may better and more clearly understand I set forth below the congruous and congruent numbers which I think necessary, and I also give an example by which, through careful examination, you will be able to solve all the problems of this kind that may be proposed. The number required must be such that when the congruent is divided by it the quotient will be a square; if it is not, there is another secret way, but this I will not give lest I be too prolix.†

* We have $x^2 + y^2 = 8^2$. If we take $v = 3$ and $w = 4$, we have $v^2 + w^2 = 5^2$. Hence the author assumes that $8 : 5 = x : 3$, and that $8 : 5 = y : 4$.

† This congruent number is what Leonardo Fibonacci (1225) called a *congruum*, a number of the form $4xy(x + y)(x - y)$. He gives the problem: "To find a number which, being added to or subtracted from a square number, leaves a square number," and uses the identities

$$(x^2 + y^2)^2 - 4xy(x^2 - y^2) = (y^2 + 2xy - x^2)^2,$$
$$(x^2 + y^2)^2 + 4xy(x^2 - y^2) = (x^2 + 2xy - y^2)^2.$$

Quiſtiones por los numeros.

¶ Cõgruos.	¶ Cõgruentes.	¶ Exemplo.
.25. Re. y da.	24.	¶ Da me vn numero q̃drado tal
100. Re y da.	96.	que ajuſtãdole, 6, haga numero q̃-
169. Re. y da.	120.	drado y reſtando del, 6, quede nu-
225. Re. y da.	216.	mero quadrado, para lo qual has
289. Re. y da.	240.	de buſcar vn tal numero congruẽ
400. Re. y da.	384.	te que partiendole por, 6, vẽga nu
625. Re. y da.	336 y, 600.	mero quadrado el qual como a fue
676. Re. y da.	480.	ra veys el p̃mero es, 24, pues par
841. Re. y da.	840.	te, 24, por, 6, viene, 4, que es qua
900. Re. y da.	864.	drado (y ſu rayz es dos) y luego to
1156. Re. y da.	960.	ma el numero congruo quadrado
1225. Re. y da.	1176.	correſpõdiente deſte numero con
1212. Re. y da.	1080.	gruente que es, 25, partale por los
1681. Re. y da.	720.	4, que es el aduciniiento de el p̃-
2025. Re. y da.	1944.	mero vienen, 6, $\frac{1}{4}$ y aqueſte es
2500. Re. y da.	1344 2400.	el numero demãdado que ſi le aju
2602. Re. y da.	2160.	ſtas ſeys haze, 12, $\frac{1}{4}$ que es nu-
2704. Re. y da.	1920.	mero quadrado y ſu rayz es, 3, $\frac{1}{2}$
2809. Re. y da.	2520.	y ſi reſtas del, 6, queda $\frac{1}{4}$ y ſu
3025. Re. y da.	2905.	rayz es $\frac{1}{2}$ porque media vez me
3364. Re. y da.	3360.	dia es $\frac{1}{4}$ y el meſmo es quadra-
3600. Re. y da.	3456.	do que ſu rayz es dos y medio.
3221. Re. y da.	1320.	Jta in aliis.
4225. Re. y da.	2026.	

Questions Relating to Numbers

¶ Congruous	¶ Congruents
25	24
100	96
169	120
225	216
289	240
400	384
625	336\& 600
676	480
841	840
900	864
1156	960
1225	1176
1212	1080
1681	720
2025	1944
2500	1344 2400
2602	2160
2704	1920
2809	2520
3025	2905
3364	3360
3600	3456
3221	1320
4225	2026

¶ Example

¶ Find a square number which being increased by 6 will still be a square, and which being decreased by 6 will also be a square. To solve, you must find a congruent number which being divided by 6 the quotient will be a square. The first number, as you see, is 24; this divided by 6 gives 4, which is a square (and its root is two). Now take the congruous number corresponding to 24, which is 25. Divide it by 4, which is the quotient of the first one, 24, divided by 6, and we have 6, $\frac{1}{4}$, and this is the required number. Add 6 to it and you have 12, $\frac{1}{4}$, which is a square number, the root being 3, $\frac{1}{2}$. If you subtract 6 you have $\frac{1}{4}$ and the root is $\frac{1}{2}$, since a half times a half is $\frac{1}{4}$ and the same number, $6\frac{1}{4}$, is a square of which the root is two and a half.

And so with others.*

* In the list read 1521 for 1212, 2601 for 2602, 3721 for 3221, 2925 for 2905, and 2016 for 2026. In the problem, since

$$25 \pm 24 = x^2,$$

we have

$$\frac{25}{4} \pm \frac{24}{4} = \frac{x^2}{4} = y^2.$$

¶ Quistion quinta.

¶ Si quisieres hallar o te fuere demandado que busques tres numeros quadrados o mas y tales que juntos en vno hagan numero quadrado: toma el primer numero quadrado inpar es, 9, del qual q̃ta vno quedan, 8, toma la mitad, quadra son, 16, y esto es el segundo ajusta, 9, y, 16, son, 25, quita vno quedan, 24, toma la mitad es, 12, quadra, son, 144, y este es tercero: si lo quieres ver suma, 9. y, 16, y 144, son, 169, rayz delos quales es, 13, como veys: y nota q̃ por esta via lo podras hazer in infinitum.

¶ Sesta quistion.

¶ Digo que me des vn tal numero quadrado que quitandole o ajustandole sus tres rayzes hagan numero quadrado. Regla ten a mientes aun numero congruente y aun al numero suyo congruo quadrado correspondiente y el tal numero congruente parte le por tantas vnidades quantas son las rayzes que manda ajustar o quitar y por el aduenimiento parte el numero suyo congruo quadrado correspondiente y el vltimo aduenimiento quadra lo en si mesmo y lo produzido, sera el numero demandado como veras por este exemplo.

¶ Exemplo.

¶ Toma, 24, primer numero congruente parte le por, 3, que sõ las rayzes demandadas viene, 8 por los quales parte, 25 que es su cõgruo quadrado correspondiente vienen, 3, y $\frac{1}{8}$ lo qual quadra en si mesmo lo produto es, 9, y $\frac{49}{64}$ y este es el numero demandado q̃ ajustãdole o quitandole sus, 3, rayzes sera quadrado como veys ajustale, 9, y $\frac{24}{64}$ que son las tres rayzes monta, 19 y $\frac{9}{64}$ que es numero quadrado y su rayz es, 4 y $\frac{3}{8}$ y si le quitas sus, 3, rayzes que dan $\frac{73}{64}$ que es numero quadrado y su rayz es. 5. o chabos.

¶ Septima quistion.

¶ Nota esta, 2, vezes, 2, son, 4, y, 3 vezes, 3 son, 9, sumados juntos hazen, 13, pues dame otros. 2. numeros que ni sean, 2, ni, 3, y q̃ quadrados en si mesmos y lo produto sumado jũto seã los mesmos, 13.

Square Numbers

¶ Fifth problem

¶ If you wish to find three or more square numbers which added together make a square, take the first odd square number 9; subtract one, which gives 8; take the half, its square being 16, and this is the second number. The sum of 9 and 16 is 25; subtract one and we have 24; the half of 24 is 12; the square of 12 is 144, and this is the third number. If you wish a proof, the sum of 9 and 16 and 144 is 169, of which the root is 13, as you see. In this way, you can solve any number of problems.*

¶ Sixth problem

¶ Find such a square number that if you subtract from it or add to it thrice its root, you have a square number. Keep in mind some congruent number and its corresponding square congruous number. Divide the congruent number by the number by which you are to multiply its roots when you add or subtract them. Divide this quotient into the corresponding congruous square number and square the quotient. The result will be the required number, as you will see by the following example.

¶ Example

¶ Take 24, the first congruent number. Divide it by 3, the number by which you are to multiply the roots, and the quotient is 8. Divide this 8 into 25, the corresponding square congruous number, and the quotient is 3 and $\frac{1}{8}$. The square of this is 9 and $\frac{4}{6}\frac{9}{4}$, the required number which, added to or subtracted from thrice its root, will be a square as you see. Add 9 and $\frac{2}{6}\frac{4}{4}$, which is thrice the root, and you have 19 and $\frac{9}{6}\frac{}{4}$, which is a square whose root is 4 and $\frac{3}{8}$. If you subtract thrice the root, you have $\frac{2}{6}\frac{5}{4}$, a square number whose root is 5 eighths.†

¶ Seventh problem

¶ Observe this: 2 times 2 is 4, and 3 times 3 is 9; these numbers added together make 13. Now find two other numbers, neither 2 nor 3, which squared in the same way and added together will give the same result 13.

* Take as the first odd number $2n + 1$. Following the directions, the first square is $4n^2 + 4n + 1$, the second is $4n^4 + 8n^3 + 4n^2$, and the third is the square of $2n^4 + 4n^3 + 4n^2 + 2n$.

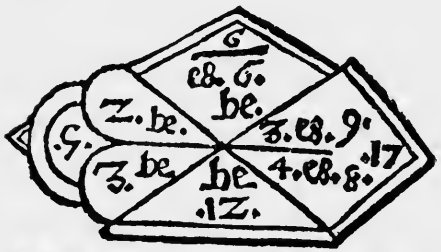
† Since $25 - 24 = 1$,

we have, multiplying by $\frac{2}{6}\frac{5}{4}$, $9\frac{4}{6}\frac{9}{4} - 9\frac{3}{8} = \frac{2}{6}\frac{5}{4}$, a square, and similarly for $25 + 24 = 49$.

Quiſtiones.por los numeros.

Regla.

¶ Buſca. 2. numeros. quadrados que hagan numero quadrado que tenga raiz diſcreta los primeros ſon.3.y.4.que ſus quadrados ſon 9.y.16 y juntos azen.25.q̃ es quadrado y ſu raiz es.5. Pues nota que tienes. 5. numeros. que ſon. 2.y.3. Primeros.y.3.y.4. q̃ ſon los propueſtos y.5.que es ſu raiz pon los como veys figurado. y luego multiplica en cruz diziendo.3. vezes.3. ſon.9.y 2. vezes.4.ſon.8. ponlos ala mano derecha el vno debajo del otro y luego buelue a dez ir por arriba.2. vezes.3. ſon.6. por abajo.3.vezes. 4.12.reſta el menor del mayor.que es.6.de.12.quedan.6. los quales parte por.5 raiz delos numeros propueſtos: el aduenimiento es.$1\frac{1}{5}$ y eſte es el vn numero demãdo y luego ſuma.9.y.8.que ſon lo produto delos que primero multiplicaſte ſon.17. los quales parte por el.5.y el aduenimiento es.$3\frac{2}{5}$ y eſte es el ſegundo numero demãdado,la prueua delo q̃l es q̃ quadres.$1\frac{1}{5}$ en ſimiſmo es.$1\frac{11}{25}$ y.$3\frac{2}{5}$ en ſi meſmos ſon.11.$\frac{14}{25}$ que ſumados juntos como veys ſon los meſmos.13.



02 1
17 | 3 $\frac{2}{5}$ 6 | 1 $\frac{1}{5}$
5 5

¶ Es vno que tiene cinco peſas con las quales puede peſar deſde vn tomin haſta quince peſos y mas. Demando que es lo que peſa cada vna Poſternario. La primera,1,to. La ſegunnda,3,to. La tercera,1,p̃s,1,to. La quarta,3,p̃s,3,to, laquinta.10,p̃s.1,tomin

Questions Relating to Numbers

ℂ Rule

ℂ Find 2 numbers the sum of the squares of which will make a square number which has an integral root. The first numbers are 3 and 4, for their squares are 9 and 16, and these added together make 25, the root of which is 5. Observe that you have 5 numbers; the first are 2 and 3; the next are 3 and 4, the proposed numbers; and there is also 5, which is their root. Place these numbers as you see in the figure below. Then use cross multiplication, saying "3 times 3 is 9, and 2 times 4 is 8." Place these numbers at the right-hand side, one under the other. Then multiply again at the top, 2 times 3 is 6; and underneath, 3 times 4 is 12. Now subtract the less from the greater, that is, 6 from 12, and there remains 6. Divide this by 5, the root of the assumed numbers, and the quotient is $1\frac{1}{5}$, one of the numbers required. Now add 8 and 9, the products of the first multiplication, and the sum is 17. Divide this by 5 and the quotient is $3\frac{2}{5}$, and this is the second required number.

Proof: The square of $1\frac{1}{5}$ is $1\frac{1}{2}\frac{1}{5}$; the square of $3\frac{2}{5}$ is $11\frac{1}{2}\frac{4}{5}$; and these added together, as you see, make 13.*

$$\begin{array}{rcccccl} & & 6 & & & & \\ 5 & \frac{2}{3} & 6 & 3 & \frac{9}{8} & 17 & \\ & & 12 & & & & \end{array} \qquad \begin{array}{rl} & 02 \\ 17 & 3\frac{2}{5} \\ & 5 \end{array} \qquad \begin{array}{rl} & 1 \\ 6 & 1\frac{1}{5} \\ & 5 \end{array}$$

ℂ A man has five weights with which he can weigh from 1 tomin to fifteen or more pesos. What is the weight of each? The first, 1 tomin; the second, 3 tomines; the third, 1 peso 1 tomin; the fourth, 3 pesos 3 tomines; the fifth, 10 pesos 1 tomin.†

* The equation $u^2 + v^2 = 13$ is indeterminate. It is given by Diophantus (II, 9), a late Greek algebraist of *c.* A. D. 275. In Sir Thomas Little Heath's edition of Diophantus, second edition, page 145, Euler's general solution is given as well as the special solution leading to Diophantus's results. In the latter solution $(x + 2)^2 + (2x - 3)^2 = 13$, whence $x = \frac{8}{5}$ and the two numbers are $1\frac{8}{5}$ and $\frac{1}{5}$. This solution is rather more simple than the one in the text.

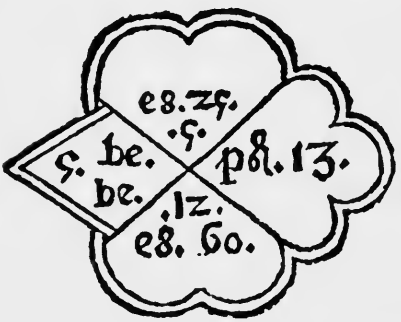
† This is the well-known Problem of the Weights. Expressed in tomines the weights are 1, 3, 9, 27, 81, a geometric progression. This solution requires that the weights be placed on either or both pans of the scales. It is evidently inserted to fill the page, having no close connection with the problems which immediately precede or follow.

¶Octaua quiſtion.

¶3.vezes,3,ſon 9,y,4,vezes,4,ſon,16,ſumadosſõ,25,que es nu mero quadrado y ſu rayz es,5,da me otros,dos. numeros,q̃ ni ſeã 3,ni,4,y que quadrados ẽ ſi meſmos y lo produzido ſumado ſea,25

¶Regla.

¶Buſca,2,numeros que juntos los quadrados en vno hagan nu mero quadrado que tenga rayz diſcreta:toma,5,y doze que ſus qua drados ſon,25,y,144,que ſon,169,rayz delos quales es,13,pues nota que enla paſſada a eſta ſemejante tuuiſte,5,numeros y aqui tie nes,4,la cauſa es que,3,y,4,primeros numeros tienen rayz diſcre ta que es,5,el quales el vno delos.4,y ſirue por el,3,y el,4,de qui ẽ es rayz y los otros ſõ ,5.y.12, numeros hallados o preſupueſtos y ſu rayz que es,13,los quales pon enla manera que veys figurado y con el,5,rayz,de,3,y.4,primeros numeros mul.el,5.y el,12,nume ros propueſtos diziendo,5,vezes,5,ſon,25,y,5,vezes doze ſon,60 parte entramos produtos por,13,que es la rayz delos numeros pro pueſtos y el aduenimiento ſeran los numeros que buſcas:parte,25, por,13,vienen,1 $\frac{12}{13}$ que es el vno parte,60,por,13 viene.4, $\frac{8}{13}$ que es el otro los quales ſi los multiplicas cada vno por ſi meſmo y lo produzido ſumas ſeran los,13,que demandas como veys niul. en ſi.1$\frac{12}{13}$.es.3. $\frac{113}{169}$. multiplica en ſi,4, $\frac{8}{13}$ es.21. $\frac{51}{169}$ ſuma los y ſon.25.como veys por la figura.



partidor. 12 25 13 | 1 $\frac{12}{13}$ 0 28 60 | 4 $\frac{8}{13}$ 13

Square Numbers

ℂ Eighth problem

ℂ 3 times 3 is 9, 4 times 4 is 16, and the sum of 9 and 16 is 25, which is a square number having 5 for its root. Find two other numbers the sum of whose squares is 25.

ℂ Rule

ℂ Find 2 numbers whose squares added together will make a square having an integral root. Take 5 and twelve whose squares are 25 and 144; these added together make 169, the root of which is 13. Observe that in the previous problem similar to this one you had 5 numbers. Here you have 4. The reason is that 3 and 4, the first numbers, have an integral root which is 5, which is one of the 4 numbers here, and serves for the 3 and 4 of which it is the root; the others are 5 and 12, the assumed or presupposed numbers, and their root is 13. Put them down as you see below; then taking the root derived from 3 and 4, the first numbers, say "5 times 5 is 25, and 5 times 12 is 60"; divide each product by 13, which is the root of the sum of the squares of the proposed numbers, and the quotients will be the required numbers. Dividing 25 by 13, the result is $1\frac{12}{13}$, which is the one part; dividing 60 by 13, the result is 4, $\frac{8}{13}$; square each quotient and add the results and the sum will be 25 as required. That is, $1\frac{12}{13}$ squared is 3. $\frac{118}{169}$; 4, $\frac{8}{13}$ squared is 21. $\frac{51}{169}$; and their sum is 25, as you see by the work.*

	25					0	
	5			12		28	
5		13	Divisor	25	$1\frac{12}{13}$	60	$4\frac{8}{13}$
	12			13		13	
	60						

* This is simply a variant of the seventh problem. Euler's general solution, referred to on page 47, applies to the equation $x^2 + y^2 = f^2 + g^2$. If $f = 3$ and $g = 4$, as in this case, the solution is

$$x = \frac{8pq + 3(q^2 - p^2)}{p^2 + q^2}, \quad y = \frac{6pq + 4(p^2 - q^2)}{p^2 + q^2},$$

where p and q may have any values whatsoever.

Notables. Quistiones.

Quistiones del arte mayor tocantes al algebra.

Primera quistion.

Da me vn numero quadrado que restando del,15,y $\frac{3}{4}$ quede su propria rayz.

Regla.

Digo que el numero sea vna cosa demediala es media cosa multiplica la en si haze $\frac{1}{4}$ dezenso ajuntale a,15,y $\frac{1}{4}$ haze,16, cuya rayz quadrada y mas el medio dela cosa es rayz del numero demãdado Prueua quadra rayz quadrada de.16.y mas el medio dla cosa q̃ es: q̃tro y medio haze.20,y $\frac{1}{4}$ que es el numero quadrado demandado resta del,15,y $\frac{1}{4}$ quedan,4,y $\frac{1}{2}$ que es la rayz del propio.

Segunda quistion.

Es vno que se fleta en vn nauio y pregunta al maestre que es lo que ha de dar de flete el maestre dize que no le ha de lleuar mas q̃ alos otros boluiendo el passajero a replicar quanto sería el maestre responde que han de ser tantos pesos que multiplicados por si y ayuntando los alo produto el remanente sera,1260.demando quanto demanda el maestre.

Regla.

Digo que el flete sea vna cosa de p̃s. la mitad es media cosa quadra la en si haze $\frac{1}{4}$ dezenso ayunta lo a.1260.haze̅,1260,y vn q̃r to rayz delos quales menos medio dela cosa es el numero demãdado del flete:reduce,1260,y $\frac{1}{4}$ a quartos son $\frac{5041}{4}$ la rayz es.71,medios resta el medio dela cosa que es medio quedan,70,medios que son,35,p̃s y tanto es lo q̃ demãda del flete: Prueua multiplica.35. en si haze̅,1225,ayunta los cõ,35,son,1260,q̃ es numero demãdado

Tercero quistion.

Vno vende cabrasno se las que son mas de que llego vn mercha̅te y le pregunta quantas abra el vendedor responde son tantas que si las multiplicays en si y lo produto quadruplays el vltimo produzido sera,90000,demando quantas cabras tenía.

Noteworthy Problems

¶ Problems of the Arte Mayor, relating to algebra

¶ First problem

¶ Find a square from which if $15\frac{3}{4}$ is subtracted the result is its own root.

¶ Rule

¶ Let the number be *cosa* (x). The square of half a *cosa* is equal to $\frac{1}{4}$ of a *zenso* (x^2). Adding 15 and $\frac{3}{4}$ to $\frac{1}{4}$ makes 16, of which the root is 4, and this plus $\frac{1}{2}$ is the root of the required number.*

Proof: Square the square root of 16 plus half a *cosa*, which is four and a half, giving 20 and $\frac{1}{4}$, which is the square number required. From $20\frac{1}{4}$ subtract 15 and $\frac{3}{4}$ and you have 4 and $\frac{1}{2}$, which is the root of the number itself.

¶ Second problem

¶ A man takes passage in a ship and asks the master what he has to pay. The master says that it will not be any more than for the others. When the passenger again asks how much it will be, the master replies: "It will be the number of pesos which, multiplied by itself and added to the number, will give 1260." Required to know how much the master asked.†

¶ Rule

¶ Let the price be a *cosa* of pesos. Then half of a *cosa* squared makes $\frac{1}{4}$ of a *zenso*, and this added to 1260 makes 1260 and a quarter, the root of which less $\frac{1}{2}$ of a *cosa* is the number required. Reduce 1260 and $\frac{1}{4}$ to fourths; this is equal to $\frac{5041}{4}$, the root of which is 71 halves; subtract from it half of a *cosa* and there remains 70 halves, which is equal to 35 pesos, and this is what was asked for the passage.

Proof: Multiply 35 by itself and you have 1225; adding to it 35, you have 1260, the required number.

¶ Third problem

¶ A man is selling goats. The number is unknown except that it is stated that a merchant asked how many there were and the seller replied: "There are so many that, the number being squared and the product quadrupled, the result will be 90,000." Required to know how many goats he had.‡

* $x^2 - 15\frac{3}{4} = x$, $x^2 - x + \frac{1}{4} = 16$, $x = 4\frac{1}{2}$, the negative root being neglected. *Cosa* (thing) was the unknown (x), and *zenso* (Latin *census*) was our x^2.

† $x^2 + x = 1260$, whence $x = 35$.

‡ $4x^2 = 90{,}000$, whence $x = 150$.

¶Regla.

¶Digo que tenga vna cosa de cabras multiplica en si haze vn zenso multiplica el zenso por,4,que es quadruplo haze 4,zensos yguales a.900 00.cabras q̃ es numero parte numero por censo el aduenimiento es,22250.rayz delos quales son las cabras q̃ tenia. Prueua toma,150, rayz de.22250.multiplica en si hazen,22250.multiplica los por,4,que es quadruplallo son.90000.

¶Quarta quistion.

¶Uno va por vn camino pregunta a otro que leguas auia hasta vna cierta parte el otro le responde ay tantas leguas que si las multiplicays en si y lo produto partis por,5,el aduenimiento sera.80.demandado que leguas abra enlo que dize.

¶Regla.

¶Digo que aya vna cosa de legua quadrala en si haze vn cẽso parte le por.5. el aduenimiento es $\frac{1}{5}$ de censo yguala.80.leguas parte numero por censo que es.80.por $\frac{1}{5}$ el aduenimiento es.400.cuya rayz son las leguas que ay:pues multiplica rayz de.400.en si q̃ es.20,y lo produzido parte por.5.el aduenimiento sera.80.numero demandado.

¶Quinta quistion.

¶Uno compra ropa dela tierra en tripla proporcion de tal suerte q̃ multiplicando el triple por el quarto del su triple que son las pieças de ropa que compro lo produto sera,48.pesos demando que pieças de ropa compro.

¶Regla.

¶Digo que compro vna cosa de pieças de ropa por tres cosas de ps que es en tripla proporcion de ropa multiplica vn quarto de cosa de pieça de ropa por,3.cosas de pesos es $\frac{1}{4}$ de censo yguales a 48.pesos que es numero parte numero por cẽso que es.48.por $\frac{3}{4}$ el aduenimiento es,64.rayz delos quales son las pieças de ropa q̃

On Algebra

ℂ Rule

ℂ Let a *cosa* represent the number of goats. Squaring this we have a *zenso*; multiplying the *zenso* by 4, which is a quadruple, makes 4 *zensos*, which is equal to 90,000 goats. Divide 90,000 by the number of *zensos*, and the quotient is 22,500 [not 22,250 as given], the root of which is the number of goats he had.

Proof: Square 150, the root of 22,500, and you have 22,500; multiply this by 4, which is quadrupling it, and you have 90,000.

ℂ Fourth problem

ℂ A man traveling on a road asks another how many leagues it is to a certain place. The other replies: "There are so many leagues that, squaring the number and dividing the product by 5, the quotient will be 80." Required to know the number of leagues.*

ℂ Rule

ℂ Let a *cosa* represent the number of leagues. This squared makes a *zenso*; and this divided by 5 equals $\frac{1}{5}$ of a *zenso*, which is equal to 80. Divide 80 by $\frac{1}{5}$ and the quotient is 400, whose root is the number of leagues required.

Proof: Multiply the root of 400, which is 20, by itself. Then divide the product by 5 and the quotient is 80, the number required.

ℂ Fifth problem

ℂ A man buys a number of pieces of clothing for three sums of pesos which are in triple proportion, so that multiplying the triple of the first by $\frac{1}{4}$ of the number tripled,† which is the number of pieces of clothing, the product will be 48 pesos. Required to know how many pieces of clothing he bought.‡

ℂ Rule

ℂ Let a *cosa* of pieces of clothing be bought for three *cosas* of pesos in triple proportion to the pieces of clothing. Multiply a quarter of the number of pieces of clothing by 3 *cosas* of pesos and you have $\frac{3}{4}$ of a *zenso*, equal to 48 pesos. Divide 48 by $\frac{3}{4}$ and the quotient is 64, the root of which is the number of pieces of clothing

* $\frac{1}{5}x^2 = 80$, whence $x = 20$. † That is, $3x \times \frac{1}{4}x$. ‡ $\frac{3}{4}x^2 = 48$, $x = 8$.

exemplo costaron le el triple que es rayz de,676, Prueua multiplica rayz de,676, que es,24,por.2. que es el $\frac{1}{4}$ del su triple de rayz de,64,el aduenimiento es,48,numero demandado.

¶Sexta quistion.

¶Uno tiene yeguas y vacas en quincupla proporcion de tal suerte que si multiplicas las yeguas en si y las vacas en si y lo produto sumas seran,1694,demãdo q̃ntas sõ las yeguas y quantas las vacas.

¶Regla.

¶Digo que tenga vna cosa de yeguas y,5,cosas de vacas multiplica vna cosa en si haze vn censo mul.5.cosas en si hazen,25,cẽsos ayunta lo. son.26.censos yguales a,1664,yeguas y vacas numero demandado parte numero por censo que es.1664,por,26,el aduenimiento es,64,cuya rayz son las yeguas y el quincuplo las vacas q̃s rayz de.1600, Prueua toma.rayz de,64.es,8, quincupla los hazen,40,que es rayz de,1600, suma el quadrado de,8, q̃ son las yeguas con el quadrado de 40,que son las vacas hazẽ,1664, q̃ fue lo demandado.

¶Setima quistion.

¶Uno tiene tres joyas ẽ q̃drupla proporciõ de valor de tal manera que multiplicando lo que vale la primera por el valor dela segunda y lo produzido por el valor dela tercera el vltimo produto sera, 1728,demando que es el valor de cada joya,

¶Regla.

¶Digo que la primera valga vna cosa y la segũda,4,cosas y la tercera,16,que como veys estan en quadrupla proporcion mul vna cosa por,4,cosas es,4,censos multiplica los por,16 cosas dela tercera hazes,64,cubos yguales a,1728,que es numero parte numero por cubo el aduenimiento es, 27,cuya rayz cuba que es,3,es el valor dela primera y la segunda vale raiz cuba de.1728.que es,12,y la

that he bought. They cost him three times this, which is the root of 676 [576].

Proof: Multiply the root of 676 [576], which is 24, by 2, which is $\frac{1}{4}$ of its cube, that is, of 8, the square root of 64. The result is 48, the number required.

ℂ Sixth problem

ℂ A man has mares and cows in quintuple proportion, in such a way that if you square the number of mares and square the number of cows, the products added will be 1664. Required the number of mares and the number of cows.*

ℂ Rule

ℂ Let there be a *cosa* of mares and 5 *cosas* of cows. Squaring the first makes a *zenso*, and 5 *cosas* squared makes 25 *zensos*, and the sum is 1664 mares and cows, the required number. Divide this number by the number of *zensos*, that is, divide 1664 by 26. The quotient is 64, whose root is the number of mares, and the quintuple, or square root of 1600, is the number of cows.

Proof: Take the square root of 64; it is 8. Quintuple it and you have 40, which is the square root of 1600. Add the square of 8, the number of mares, to the square of 40, the number of cows, and you have 1664, which was required.

ℂ Seventh problem

ℂ A man has jewels in quadruple proportion of value such that, multiplying the value of the first by the value of the second and the product by the value of the third, the last product will be 1728. Required the value of each jewel.†

ℂ Rule

ℂ Let the value of the first be one *cosa*; that of the second, 4 *cosas*; and that of the third, 16 *cosas*, which you see are in quadruple proportion. Multiply one *cosa* by 4 *cosas* and this is equal to 4 *zensos*. Multiply this by 16 *cosas* and the result is 64 cubes, and this is equal to 1728. Divide this number (that is, 1728) by the cube (that is, by 64) and the quotient is 27, whose cube root is 3, the value of the first jewel. The second one is worth the cube root of 1728, or 12; and the

* $x^2 + 25x^2 = 1664$ (not 1694 as in the original), whence $x = 8$.

† Take x, $4x$, $16x$. Then $64x^3 = 1728$, whence $x = 3$.

tercera vale,48,que es rayz cuba de,2304. Prueua mul.el valor de la primera que es,3,por el de la segunda que es doze y el aduenimiẽto por la tercera, que es.48,lo produto delas multiplicacionesera ,1728.

¶ Octaua quistion.

¶ Vno tiene hijos y hijas en proporcion sis que altera de tal arte q̃ mul.los hijos por las hijas y lo produto por la mitad delos hijos el vltimo produzido sera,162,demando quantos son los hijos y quãtas las hijas.

¶ Regla.

¶ Digo que los hijos sean vna cosa y las hijas vna cosa y media que es en sis que altera proporcion mul.vna cosa por vna cosa y media es vn censo y medio el qual multiplica por media cosa que es mitad delos hijos hazes $\frac{1}{4}$ de cubo yguales a,162,hijos y hijas que es numero parte numero por cubo que es,162.por $\frac{1}{4}$ el aduenimiento es.216.rayz cuba delos quales son los hijos y las hijas rayz cuba de,729, Prueua,mul,rayz cuba de,216,que es,6,por rayz cuba de.729 que es,9,hazen,54,los quales mul por medio de seys que son los hijos lo produto es,162,que es el numero demandado.

¶ Nouena quistion.

¶ Vno ha de hazer dos pagamentos cquadupla proporció de meses en tal manera:q̃ quadrando el su cuadruplo y lo que saliere mull. por el quaduplo y lo produzido cubicando el vltimo produzido sea. 32768.demando a quantos meses son los pagamentos.

¶ Regla.

third is worth 48, the cube (*sic*) root of 2304.

Proof: Multiply the value of the first, which is 3, by that of the second, which is twelve, and this product by the value of the third, which is 48, and the product of the multiplications is 1728.

ℂ Eighth problem

ℂ A man has a certain number of sons and daughters in *altera* proportion such that multiplying the number of sons by the number of daughters and the product by half of the number of sons, the last product will be 162. Required the number of sons and the number of daughters.*

ℂ Rule

ℂ Let the number of sons be a *cosa*, and the number of daughters be a *cosa* and a half, which numbers are in *altera* proportion. Multiply one *cosa* by a *cosa* and a half and the result is a *zenso* and a half, which multiplied by half a *cosa*, which is half the number of sons, makes $\frac{3}{4}$ of a cube which is equal to 162, the number of sons and daughters. Divide this number by the number of cubes, that is, divide 162 by $\frac{3}{4}$, and the quotient is 216, the cube root of which is the number of sons, and the cube root of 729 is the number of daughters.

Proof: Multiply the cube root of 216, which is 6, by the cube root of 729, which is 9, and the result is 54; multiply this by half of six, which is the number of sons, and the result is 162, the number required.

ℂ Ninth problem

ℂ A man has two payments to make, in quadruple proportion of months, so that squaring the first, multiplying the product by the quadruple, and cubing this product, the result will be 32,768. Required to know how the payments were made.†

* By the ancient Greek theory of proportion (ratio), $\frac{3}{2}x$ and x are in *altera* proportion. The word *proportion* was commonly used for ratio in the sixteenth century, and the same is still the case outside the school. Take x and $\frac{3}{2}x$. Then $\frac{3}{4}x^3 = 162$, whence $x = 6$.

† $64x^9 = 32{,}768$, whence $x = 2$.

Notables Quistiones.

Regla.

Digo que los.2.pagamentos sean vna cosa y 4.cosas que es cuadrupla proporcion quadra el su quadruplo es vn censo multiplica le por el quadruplo es. 4. cubos cubicalos ases.64.cubos de cubos yguales a. 32768. que es numero: parte numero por cubos de cubos que es. 32768. por. 64.el aduenimiento es.512.cuya rayz cuba de rayz cuba seran los meses del primer pagamento y rayz cubica de rayz q̃drada de.262144.serã el segundo pagamẽto. Prueua:rayz cubica de.512.es.8.y rayz cubica de.8.es.2.que es el primer plazo:y rayz q̃drada de.262144.es.512.y rayz cubica de.512. es. 8. que es el segundo plazo:quadra el. 2.q̃ es su quadruplo de.8. es.4.el qual multiplica por el quadruplo es.32. cubicalos el vltimo produzido es.32768.numero demandado.

Dezena quistion.

En hombre tiene dos hijos en proporciõ sis que quarta ỏ hedad en tal manera que,mul.el su quadruplo dela hedad del menor por el su quincuplo dela hedad del mayor y lo que saliere quadruplando y dello produzido sacãdo su rayz y cubicãdo la mitad el vltimo produzido sera.125.años:demando que hedad tiene cada vno.

Regla.

Digo que el menor aya vna cosa de años y el mayor aya.vna cosa y $\frac{1}{4}$ de cosa de años que es sis que quarta proporciõ,mul.el su quadruplo del menor por el su quincuplo del mayor que es $\frac{1}{4}$ por $\frac{1}{4}$ lo produto es $\frac{1}{16}$ de censo quadrupla $\frac{1}{16}$ es $\frac{1}{4}$ de censo su rayz es media cosa cubica su mitad que es vn q̃rto de cosa el vltimo produzido es $\frac{1}{64}$ de cubo yguales a.125.que es lo que se busca pte numero por cubo que es.125.por $\frac{1}{54}$ el aduenimiento es. 8000, cuya rayz cubica son los años del menor y rayz quadrada de.625. son los del mayor. Prueua toma rayz cuba de.8000.hedad del menor es.20.mul.el su quadruplo que es.5 por el su quincuplo de.25. rayz quadrada de.625.que es.5.lo produto es.25.quadruplalos ha

Noteworthy Problems

ℂ Rule

ℂ Let the 2 payments be represented respectively by one *cosa* and 4 *cosas*, which are in quadruple proportion. Square the first, making a *zenso*; multiply this by the quadruple, giving 4 cubes; cube this, making 64 cubes of cubes equal to 32,768. Divide 32,768 by 64 and the quotient is 512, of which the cube root of the cube root is the number of months for the first payment. The cube root of the square root of 262,144 will be the number of months for the second payment.

Proof: The cube root of 512 is 8, and the cube root of 8 is 2, which is the number of months for the first payment. The root of 262,144 is 512, and the cube root of 512 is 8, which is the second number of months. Square the 2, which is a fourth of 8, and this is equal to 4, which, multiplied by the quadruple 8, is equal to 32. Cube 32, and the result is 32,768, the required number.

ℂ Tenth problem

ℂ A man has two sons whose ages are in such a *quarta* proportion that, multiplying one fourth of the age of the younger by one fifth of the age of the elder, and quadrupling the result, and cubing half the root, the final product will be 125 years. What is the age of each? *

ℂ Rule

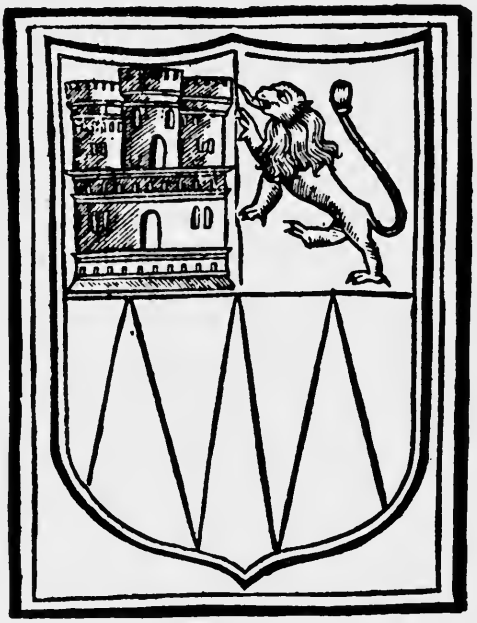
ℂ Let the age of the younger be a *cosa* of years and that of the elder be a *cosa* and $\frac{1}{4}$ of a *cosa* of years, which is a *quarta* proportion. Multiply a fourth of the younger by a fifth of the elder, which is $\frac{1}{4}$ by $\frac{1}{4}$. The product is $\frac{1}{16}$ of a *zenso*. Now quadruple $\frac{1}{16}$, and this is equal to $\frac{1}{4}$ of a *zenso*, of which the root is half a *cosa*. Cube half of this and the product, $\frac{1}{64}$ of a cube, is equal to 125. Divide 125 by $\frac{1}{64}$ and we have 8000, whose cube root is the number of years in the age of the younger, and the square root of 625 is the number of years in the age of the elder.

Proof: The cube root of 8000 is 20, the number of years in the age of the younger; multiply a fourth of it, or 5, by a fifth of 25, the square root of 625, which is 5, and the product is 25; and this multiplied by 4 (that is, 25 being quadrupled)

* He means to multiply by $\frac{1}{4}$ instead of 4, and $\frac{1}{5}$ instead of 5. That is, the age of the younger is x; of the elder, $\frac{5}{4}x$. Then $\frac{x}{4}\cdot\frac{1}{5}\cdot\frac{5x}{4}=\frac{x^2}{16}$; $4\times\frac{x^2}{16}=\frac{x^2}{4}$; $\sqrt{\frac{x^2}{4}}=\frac{x}{2}$; $\frac{1}{2}\cdot\frac{x}{2}=\frac{x}{4}$; $\frac{x^3}{64}=125$, whence $x=20$.

zen ciento, toma la mitad de su rayz es 5. cubicalos el vltimo produzido es. 125. numero demandado lo qual nota.

¶ No he querido ser en esto mas largo lo vno por euitar prolixidad y lo otro porque como siempre he dicho mi yntento nunca fue otro que poner las cosas necessarias en el comun destos reygnos: y assi vereys como en lo de mas he sido breue e suplico os, sea tomado e ser uicio y recebida la voluntad como de quien dessea seruir.



makes a hundred. Take half of the root of 100, which is 5 ; cube this, and you have 125, the number required, which note carefully.

¶ It has not been my desire to extend this work, one reason being that I would avoid becoming tiresome, and the other reason being, as I have always said, that I have wished merely to set down the things which are necessary and familiar in this kingdom. You will therefore see that I have always written succinctly, and so I beg that this book may be judged and received merely as the work of one who seeks to be of service to his fellows.

Fin dela obra.

¶ Alabanza y gloria de nro señor Jesu Christo y de la bendita y gloriosa virgen santa Maria su madre y señora nra. En la qual se acaba el presente tratado Intitulado Sumario compendioso de cuentas de plata y oro necessarias en los reynos del Piru. El qual fue impresso en la muy grande y insigne y muy leal ciudad de Mexico, en casa de Juan pablos Bressano: con licencia del muy Illustrissimo señor Don Luys de Velasco, Visorrey y governador desta Nueua españa. E assi mismo cõ licẽcia del muy Illustre y reuerẽdissimo. S. dõ. fray Alõso de Mõtufar arçobispo de mexico: por ãnto fue visto y examinado, y se hallo ser ,puecho so imprimirse, Acabose de imprimir: a veynte y nueue dias del mes de Mayo. Año del nacimiẽto de nro Señor Jesu Christo, de. 1556 años

End of the work

¶ To the honor and glory of our Lord Jesus Christ and the blessed and glorious Virgin Holy Mary, His mother and our Lady. This is the end of this treatise entitled Sumario cõpendioso of the computations of gold and silver necessary in the Kingdoms of Peru. This was published in the magnificent, famous, and most loyal City of Mexico, in the house of Juan Pablos Bressano; with the permission of the illustrious señor Don Luys de Velasco, Viceroy and Governor of New Spain and, also, with the permission of the most illustrious and reverend señor, Brother Alonso de Montufar, archbishop of Mexico, by whom this has been seen and examined and found worthy to be printed. The printing was finished on the twenty-ninth day of the month of May in the year of our Lord Jesus Christ 1556

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