RESEARCH DIVISION

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DEPARTMENT OF METEOROLOGY

A SURVEY OF SELECTED SOLUTIONS TO PROBLEMS OF REFRACTION AND DIFFRACTION OF LIGHT WAVES AND SOUND WAVES AND THE ANALOGIES OF THESE PROBLEMS TO WATER WAVES

Prepared for
BEACH EROSION BOARD
Corps of Engineers, Department of the Army
5201 Little Falls Road, N. W.
Washington 16, D. C.
Contract No. W49-055-eng-1

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A SURVEY OF SELECTED SOLUTIONS TO PROBLEMS OF REFRACTION AND DIFFRACTION OF LIGHT WAVES AND SOUND WAVES AND THE ANALOGIES OF THESE PROBLEMS TO WATER WAVES

By

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Preliminary Distribution

Prepared under a contract sponsored by the Beach Erosion Board, Corps of Engineers, Department of the Army, Washington 16, D.C.

New York University
College of Engineering
Department of Meteorology

October 1951

PREFACE

The present paper by Mr. Marks is one of a number of technical reports on investigations dealing with the forecasting of characteristics of ocean surface waves on the coast of New Jersey. This project is conducted by the Department of Meteorology under the sponsorship of the Beach Erosion Board, Corps of Engineers, Department of the Army. The project is administered by Dr. H. K. Work, Director of Research, College of Engineering, New York University.

During the work on this project it became clear that some of the problems of wave propagation arising due to the complicated bottom topography off the New Jersey shore could best be studied by means of experiments in a ripple tank. Experimentation in such a tank allows the verification of results obtained by mathematical analysis. Such experimental verifications are particularly desirable when for reasons of mathematical expediency somewhat unrealistic assumptions have to be made to permit a theoretical solution.

The cases considered in Mr. Marks' study do not include any problems in which questions of the applicability of the mathematical theory arise. Nevertheless, the following paper forms an important part of the experimental work with the ripple tank since it demonstrates the feasibility of small-scale experimental studies which contribute to our knowledge of surface wave propagation in shallow water and under the influence of structures erected for the protection of beaches.

25 October 1951

B. Haurwitz
Chairman
Department of Meteorology

TABLE OF CONTENTS

	Page
Preface	iii
Abstract	1
1. Introduction	3
2. Experimental technique	3
Chapter 1. Discussion of the equation of wave motion	8
3. Fundamental concepts	8
4. The equation of non-dispersive wave motion	9
Chapter 2. Refraction	10
5. Basis of modern refraction theory	10
6. Snell's Law	11
7. Refraction of light by a vertical glass cylinder	17
8. Atmospheric refraction of sound waves caused by a uniform temperature variation	22
9. Reflection by a plane rigid surface	26
10. Reflection at an air interface	30
11. The phenomenon of total reflection	34
Chapter 3. Diffraction	41
12. Beginning of modern diffraction theory	41
13. Diffraction of light by a black half-plane	42
14. Diffraction by an infinite slit	46
15. Diffraction of light by two noncoplanar parallel straight edges	51
Resumé and Conclusions	60
Acknowledgments	61
Bibliography	62

LIST OF ILLUSTRATIONS

		Page
Figure 1.	Photograph of the ripple tank	5
Figure 2.	Schematic diagram of the ripple tank . . .	7
Figure 3.	Refraction at a plane surface	13
Figure 4.	Photograph illustration Snell's Law. Waves enter shallow water from deep water after crossing abrupt transition zone	16
Figure 5.	Photograph showing refraction of light by a vertical glass cylinder	18
Figure 6.	Projection of the wave front on the plane passing through the two caustics when the plane wave is incident normal to the glass cylinder	19
Figure 7.	Photograph showing refraction of waves by a submerged level disc at constant depth. .	20
Figure 8.	Projection of the wave front on the plane passing through the two caustics, when the plane water wave travels over a submerged shoal of constant depth	21
Figure 9.	Refraction of sound waves in the atmosphere due to a variation of temperature	24
Figure 10.	Photograph showing refraction of water waves by a uniformly sloping beach	25
Figure 11.	Reflection by a rigid plane	27
Figure 12.	Photograph showing reflection of water waves by a rigid breakwater. Angles of incidence are approximately 60° and 45° . .	29
Figure 13.	Incident (x), reflected (y) and refracted (z) rays at an air interface	32
Figure 14.	Photograph showing incident, refracted and reflected waves at the boundary of the step model	35
Figure 15.	Incident and totally reflected wave. The observer (P) is in the second medium . . .	37

List of Illustrations (Cont.)

Page

Figure 16. Photograph showing total reflection of water waves at the boundary of the step model 39

Figure 17. Photograph showing an infinite screen (jetty) placed at the zone of transition between deep and shallow water 40

Figure 18. Diffraction of plane monochromatic light by a black half-plane 43

Figure 19. Photographs showing diffraction by a rigid breakwater. Angles of incidence are approximately 55° and 40° 45

Figure 20. Diffraction by an infinite slit (sound) . . . 48

Figure 21. Diffraction by an infinite slit 49

Figure 22. Photograph showing diffraction by a breakwater. The width of the gap is small compared with the wave length 52

Figure 23. Photograph showing diffraction by a breakwater. The width of the gap is large compared with the wave length 53

Figure 24. Photograph showing diffraction by a breakwater. The width of the gap is very large compared with the wave length 54

Figure 25. Arrangement of source, straight edges and plane of observation 55

Figure 26. Curve of light intensity for coplanar straight edges. $d = 0$; $\delta = 0.48$ cm.. 57

Figure 27. Experimental curve of light intensity (solid line) with corresponding theoretical curve (broken line) for $d = 40$ cm. and $\delta = 0.44$ cm. 57

Figure 28. Experimental curve of light intensity (solid line) with corresponding theoretical curve (broken line) for $d = 80$ cm. and $\delta = 0.56$ cm. 57

Figure 29. Photograph showing increase in dissymmetry of the diffraction pattern as d is increased. 58

Figure 30. Photograph showing increase in dissymmetry of the diffraction pattern as d is increased still further 59

A SURVEY OF SELECTED SOLUTIONS TO PROBLEMS OF REFRACTION AND DIFFRACTION OF LIGHT WAVES AND SOUND WAVES AND THE ANALOGIES OF THESE PROBLEMS TO WATER WAVES

Abstract

Several refraction and diffraction problems in light waves and sound waves have been selected for study in the N.Y.U. ripple tank. The analogies to these problems are demonstrated by testing plastic analogue models of the system under investigation in the ripple tank. If it is assumed that the initial and boundary conditions of the particular problem have been satisfied by the nature of the experiment, and it is found that photographs of the systems undergoing experimentation clearly demonstrate the physical interpretation of the mathematical solution, then we may conclude that the problem has an analogy in water waves.

The selection of the various problems depends on the practicability of experimentation and upon the photographic technique employed. The ripple tank, mentioned above, has been designed in such a manner that it propagates plane waves. Therefore, one of the conditions imposed on the selection of the problem is that the source be of such a nature that it propagates plane waves at a finite distance from the system. A possible source is a plane in three dimensions, which reduces to a line source in two dimensions. Another possible source is a point source originating a great distance from the system, so that the waves may be considered as plane waves in the region of the system. Since our problems

must be studied in two dimensions, it is necessary that the systems involved in the examples of wave diffraction be uniform with respect to an axis normal to the water surface.

The mathematical complexities of the problems vary from the trivial (non-mathematical, elementary discussions) to treatments involving several transformations.

Most of the problems presented here are the products of individual authors. Those readers who are interested in the rigid mathematical derivation of any particular solution are referred to the source of the material by the footnote at the beginning of each section or the bibliography at the end of the paper.

1. Introduction

That light waves and sound waves have an analogy in water waves can be seen from the two properties common to all waves: (1) energy is propagated to distant points, (2) the disturbance travels through the medium in such a way that the fluid as a whole is not given any permanent displacement. We shall see that whether the transporting medium is air or water, these two properties are common to the wave motion.

Since all types of wave motion obey the equation of wave motion, the solution of each problem differs only in the imposition of the initial and boundary conditions.

The ripple tank used in these experiments is essentially the same one used by Rix (1949), although the dimensions are somewhat larger.

> "We have built a tank 3 feet by 2 feet in which we produce ripples in tap water with a mechanically driven vibrator consisting of a strip of plate glass. The wave field is observed directly or photographed by means of stroboscopic illumination, the wave crests acting as lenses to focus the transmitted light in a plane at a convenient height above the water surface. Refraction, interference and diffraction effects are produced with the help of obstacles made of glass or plastic."

The ripple tank has proved its usefulness by facilitating the study of wave propagation. Its superiority over other methods of studying certain types of wave motion is seen by the fact that the actual progress of the water waves can be both directly observed and easily photographed.

2. Experimental technique

The ripple tank used in testing the models is an improvement over the one in which earlier refraction studies, for the Beach Erosion Board, were made (Pierson, 1950).

The tank is made of durable plexi-glass and is mounted about three feet from the floor in a sturdy iron framework. Its dimensions are: 4 feet long, 3 feet wide, 3 inches deep. Figure 1 shows a photograph of the ripple tank.

An improved audio-oscillator has been installed which is capable of generating frequencies from one-half cycles per second up to 8,000 cycles per second, and which controls a speaker magnet that causes a long cylindrical rod to vibrate at one end of the tank, producing parallel waves. It has been found that for the most part, a frequency of eight cycles per second is most effective in producing the desired results.

In order to simulate wave motion in the ripple tank, we must make some adjustments in the medium of propagation. The velocity of a periodic wave of small but finite amplitude is given by

$$c^2 = \left(\frac{g\lambda}{2\pi} + \frac{2\pi}{\lambda}\frac{\mathcal{T}}{\rho}\right)\tanh\frac{2\pi h}{\lambda} \qquad (2.1)$$

where λ = wave length

g = gravitational force

c = wave velocity

$\mathcal{T}$ = surface tension

ρ = density

h = depth

Suppose a liquid to have no surface tension; then

$$\frac{2\pi}{\lambda}\frac{\mathcal{T}}{\rho} = 0 \qquad (2.2)$$

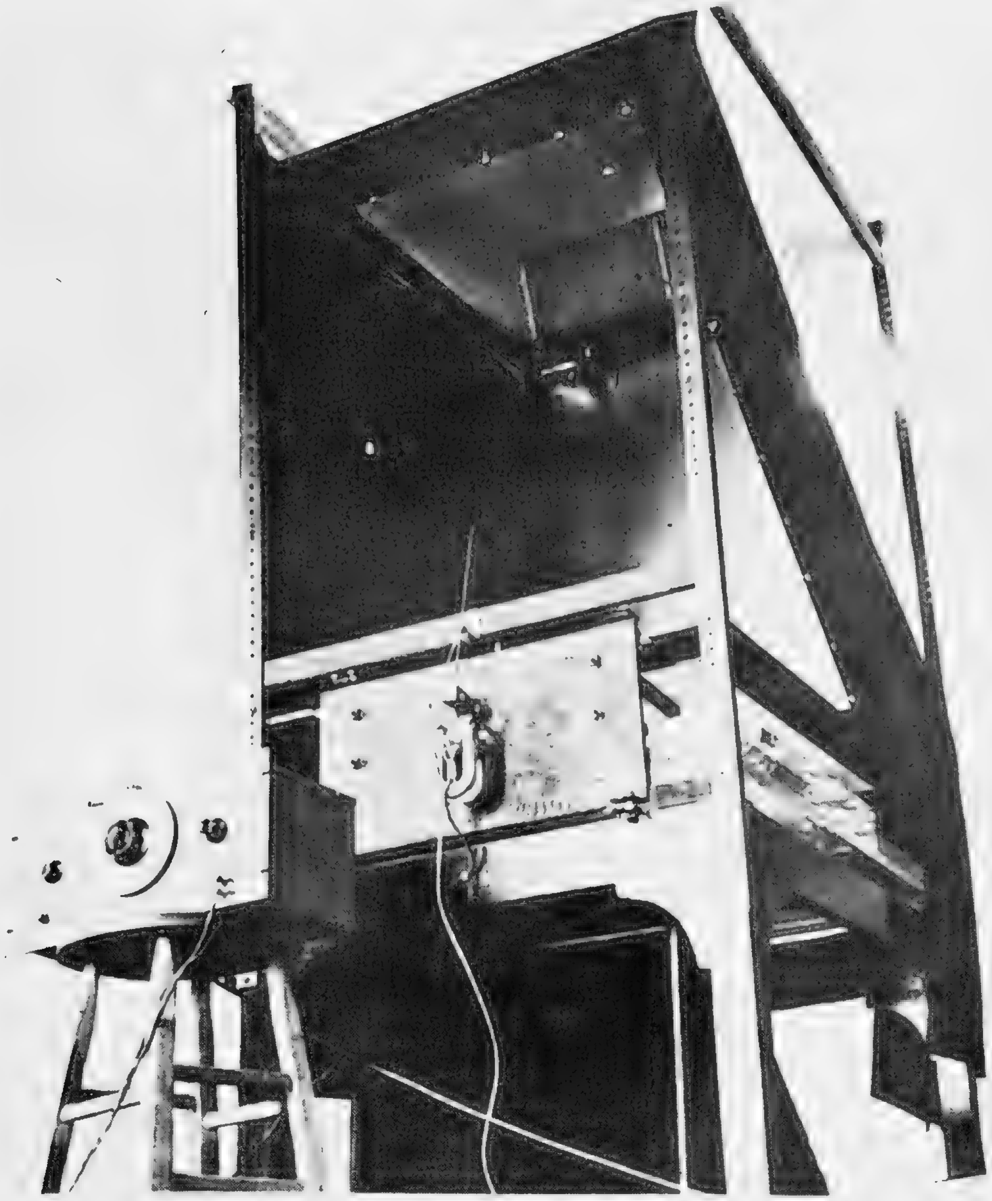

FIGURE I. PHOTOGRAPH OF THE RIPPLE TANK.

and (2.1) reduces to

$$c^2 = \frac{g\lambda}{2\pi} \tanh \frac{2\pi h}{\lambda} \qquad (2.3)$$

which is the equation for the velocity of any wave. Hence, if the surface tension can be reduced to such a magnitude that $\frac{2\pi}{\lambda}\frac{\tau}{\rho}$ is so small compared to $g\lambda/2\pi$ that it can be neglected, then the ripples would behave like waves.

In order to obtain this situation, we have used Tergitol Wetting Agent 7 to reduce the surface tension to the level where (2.3) can be applied to a good degree of accuracy. The wetting agent causes the surface tension/density ratio to be reduced to about .317 times that of water, which, for our purpose, is sufficient.

The photographic technique formerly employed, whereby the "freezing" of the field by a strobolume enabled contact prints to be made, has been discarded in favor of direct camera photography. The advantages of this method are twofold: (1) it is not necessary to operate under conditions of absolute darkness, and (2) the whole wave field can be photographed. (In contact photography, only the portion of the field covered by the photo-sensitive paper can be "captured.")

Instead of the strobe light used in earlier experiments, the point source of light is now furnished by the bulb of a common slide projector. The light is focussed by the wave crests on a screen of ground glass, at a suitable height above the water surface, so that the wave pattern is reflected as light and dark bands in the mirror mounted above the ground glass screen. A schematic

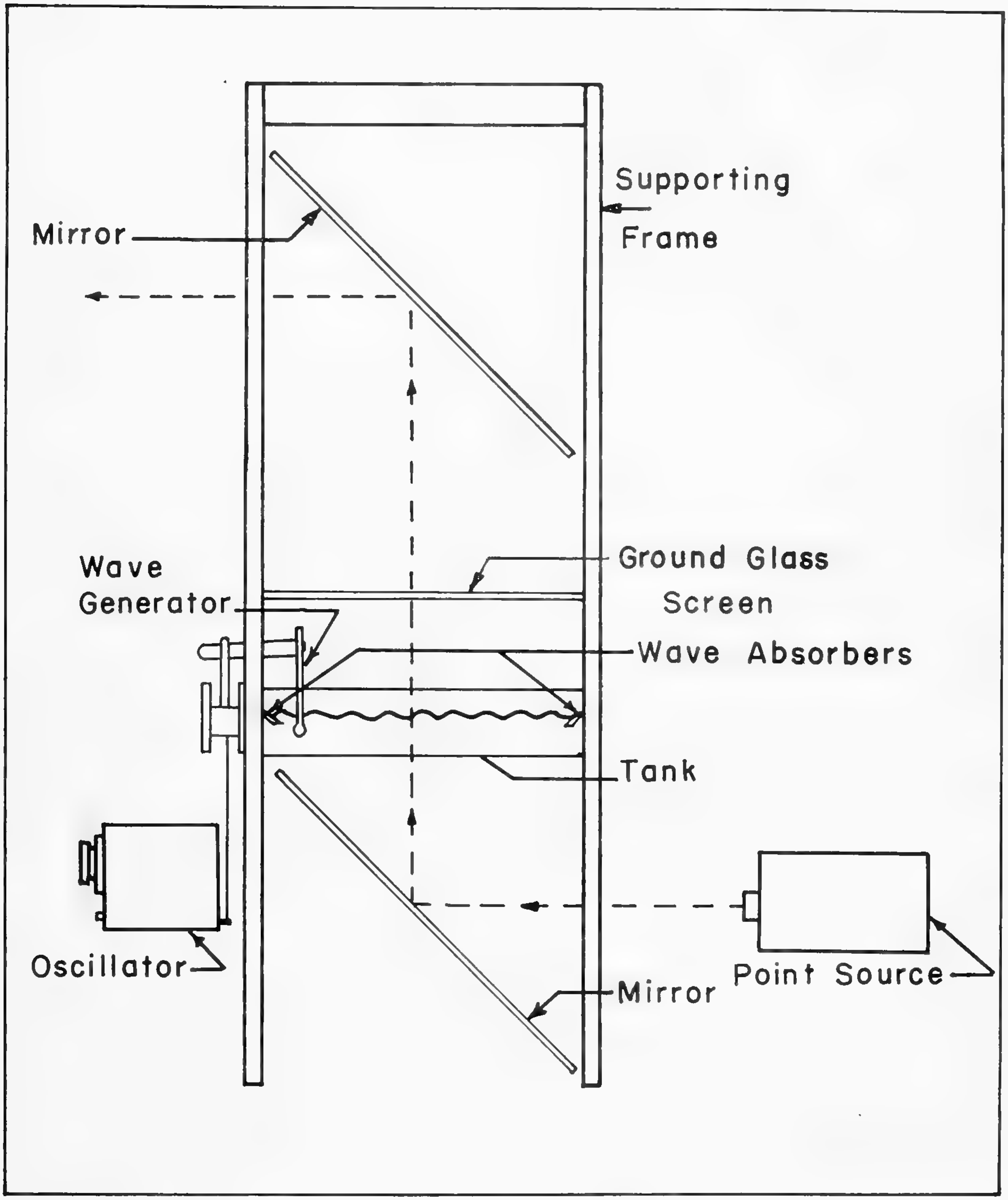

FIGURE 2. SCHEMATIC DIAGRAM OF THE RIPPLE TANK.

diagram of the ripple tank is featured in figure 2.

A Graphic-View camera was used in all the ripple tank studies. After a series of experiments, it was determined that an exposure of 1/100 seconds and an aperture opening of F4.5 was best suited for freezing the motion. In processing the pictures, the wave crests appear as dark bands on the negative and as bright bands on the positive, or actual photograph.

Chapter I. Discussion of the Equation of Wave Motion

3. Fundamental concepts

In a non-dispersive medium, that is, a medium in which the velocity of the waves is independent of the period, consider a disturbance φ, which is propagated along the x-axis with a velocity c. When $t = 0$, φ is some function of x, $f(x)$, which is the wave profile of the disturbance (φ) plotted against x. At $t = 0$, the curve obtained will be $\varphi = f(x)$. If we suppose that the disturbance is propagated without change of shape, then at some later time, t, the curve will be identical with that at $t = 0$, except that the profile has moved a distance ct in the positive direction of the x-axis. If a new origin is taken at $x = ct$ and if distances measured from this origin are called X, so that $x = X + ct$, then the equation of the wave profile referred to the new origin would be

$$\varphi = f(X) \tag{3.1}$$

Referred to the fixed origin, this means that

$$\varphi = f(x - ct) \tag{3.2}$$

This is the most general expression for a wave moving with constant velocity c, along positive direction x. If the wave is travelling in a negative direction, (3.2) becomes

$$\varphi = f(x + ct) \ . \tag{3.3}$$

4. The equation of non-dispersive wave motion

If the disturbance is constant over all points of a plane drawn perpendicular to the direction of propagation of a wave, it is called a plane wave, and the plane is called a wave front. The wave front moves perpendicular to itself with the velocity of propagation c. Let us consider a plane wave in two dimensions. If $x:y = l:m$ is the direction of propagation, where l and m are the direction cosines of the normal to each wave front, then the equation of the wave front in two dimensions is

$$lx + my = \text{const}, \tag{4.1}$$

so that at any moment t, φ is constant for all x and y which satisfy (4.1). It is clear then, that

$$\varphi = f(lx + my - ct) \tag{4.2}$$

is a function which fulfills all these requirements and therefore represents a plane wave travelling with velocity c in the direction $l:m$.

Since l and m are direction cosines, $l^2 + m^2 = 1$ and it can be verified that φ satisfies the differential equation

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} \tag{4.3}$$

This is the two dimensional equation of non-dispersive wave motion and represents all types of wave motion in which the velocity is constant. The expressions (3.1), (3.2), (3.3) and (4.2) are all particular solutions of this equation.

Since the equation of wave motion is linear, φ_1 and φ_2 are any two solutions of (4.3) and $A_1\varphi_1 + A_2\varphi_2$ is also a solution, A_1 and A_2 being arbitrary constants. This illustrates the principle of superposition, which states that, when all the relevant equations are linear, we may superpose any number of individual solutions to form new functions which are themselves solutions.

In a dispersive medium, only periodic motions of one discrete period can be considered for simple problems. This is so, because the superposition of several different periods involves a change of shape of the wave form with the distance traveled. Therefore (4.3) is valid only when we assume constant depth and one constant period.

Chapter 2. Refraction

5. Bases of modern refraction theory

Wave velocity is a function of depth (h) and period (T), that is

$$c = c[h(x,y),T]. \qquad (5.1)$$

For this to hold in the known refraction problems which were tested in the ripple tank, it was necessary to maintain the period constant.

In discontinuous, non-dispersive media, the general wave equation is satisfied, the wave equation for each medium taking the form

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = \frac{1}{C_1{}^2}\frac{\partial^2\varphi}{\partial t^2} \tag{5.2}$$

and
$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} = \frac{1}{C_2{}^2}\frac{\partial^2\varphi}{\partial t^2} \tag{5.3}$$

where C_1 is the velocity in medium I and C_2 is the velocity in medium II.

However, if we are dealing with a medium of constantly varying index of refraction, the velocity does not change abruptly, so that at every point, $C = C(x,y)$, and the fundamental equation to be solved is

$$(C^2\varphi_x)_x + (C^2\varphi_y)_y = \varphi_{tt} \tag{5.4}$$

If we assume that φ is the real part of $\varphi'(x,y)e^{i2\pi t/T}$, then (5.4) becomes

$$[C^2\varphi'(x,y)_x]_x + [C^2\varphi'(x,y)_y]_y + \frac{4\pi^2}{T^2}\varphi' = 0 \tag{5.5}$$

Equation (5.5) accounts for all solutions in which both the period of the motion is constant and the disturbance is sinusoidal as a function of time, at the source.

It is worth noting that in the refraction problems that will be studied here, the refracting objects are all large compared with the wave length.

6. Snell's law[1]

The most elementary consideration in refraction problems is

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1. Barton, E. H. (1908): Sound, (London, Macmillan and Co. Ltd.), pp. 76-78.

that phenomenon which occurs at the plane surface separating two media of different indices of refraction. The treatment here is after Barton (1908), and was chosen because it follows most closely the method used by Huygens.

In figure 3, AB is a line representing the transition plane between the two media. The velocity of the waves in the second medium has a constant ratio to the velocity of the waves in the first medium. If v is the velocity in medium I and v' is the velocity in medium II, then we may say that

$$\frac{v}{v'} = R \tag{6.1}$$

where R is a constant called the index of refraction. CA may be said to be the wave front at the time when A is incident upon the plane AB. If there were just one medium, the incident wave front would advance unimpeded to BG. But, since there is a second medium present in which the advance of the waves is retarded, some other place, NB, is reached instead of GB. If NB is to be obtained by Huygens' principle, choose some points H,H,H, on the wave front CA. Straight lines normal to the wave front cross the transition zone at K,K,K, and in the absence of a second medium would proceed to M,M,M on BG. Since there is a second medium, this is not the case. Upon reaching K, on the zone of transition, a spherical wavelet can be assumed to originate from K and this wavelet spreads into the second medium. At a time t, the radius of the wavelet is v't. Now, in order to obtain the refracted wave front NB it is necessary to: (1) describe from A an arc of radius AN, where

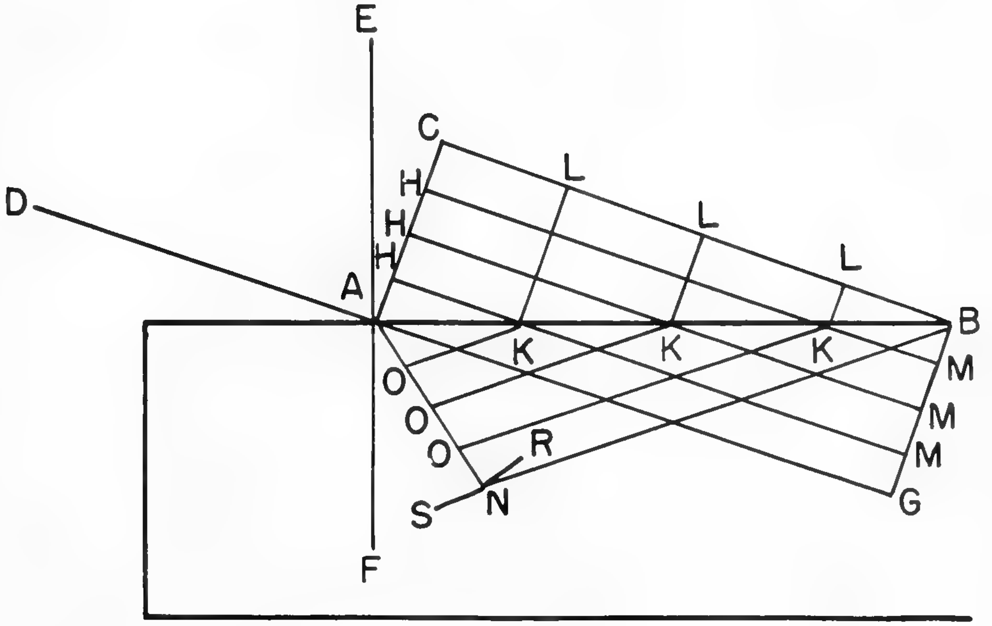

Figure 3. Refraction at a plane surface.

-After Barton

$$AN:AG = v':v, \tag{6.2}$$

and (2) at each point K describe an arc with radius v'/v times the corresponding length KM. Therefore, the line NB is obtained as the new wave front after refraction and this is the envelope of the spherical wavelets originating at K,K,K.

The angles of incidence and refraction are seen from figure 3 to be CAB and NBA, respectively, and they are located in the two right-angled triangles whose common hypotenuse is AB. Thus, by taking the sines of the angles, we have by construction

$$\frac{\sin CAB}{\sin NBA} = \frac{CB/AB}{NA/AB} = \frac{v}{v'} \tag{6.3}$$

or

$$\frac{\sin i}{\sin r} = \frac{v}{v'} = R \tag{6.4}$$

where i and r denote the angles of incidence and refraction, respectively.

Barton concludes, "This consequence of Huygens' principle that the ratio of the sines is constant, is seen to be in accord with the well-known optical laws of refraction. The same laws are valid for acoustics also."

To illustrate Snell's law for ocean waves, we have constructed a rectangular plastic "step" model which when submerged in the ripple tank is one and one half inches from the bottom, in water one and three quarters inches deep. We may now say that for certain wave lengths part of the tank contains shallow water and part of the tank contains deep water, the zone of transition being a straight line, in two dimensions.

Plane waves are generated in deep water and travel into shallow water crossing the zone of transition at a particular angle of incidence (fig. 4). Upon entering shallow water at normal incidence, the wave crests are not deflected, but it is observed that the wave length has decreased appreciably. The expression for the velocity of a plane wave in water of any depth is

$$v = \frac{L}{T} \tag{6.5}$$

where T is the period and L is the wave length. From (6.5), the velocity varies directly as the wave length. Therefore, as the wave length decreases upon entering shallow water, the velocity decreases proportionally, the period remaining constant.

Measurement of the angles of incidence and refraction, and of the wave velocities in the deep and shallow water (by means of the wave length), will verify Snell's law, when these data are substituted in (6.4).

Another proof of Snell's law and a discussion of the behavior of the refracted and reflected waves has been given by Coulson (1943).

Suppose that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is less than one. Then the angle of refraction is greater than the angle of incidence. However, the limiting value of sin i and sin r is one, therefore when $i = 90°$, $\sin i = 1$, and sin r must be greater than 1, to maintain the condition that $\sin i/\sin r < 1$. This, of course, is impossible. Since it is necessary that the ratio of the sines be constant in (6.4), failure to satisfy the condition above usually occurs at some angle

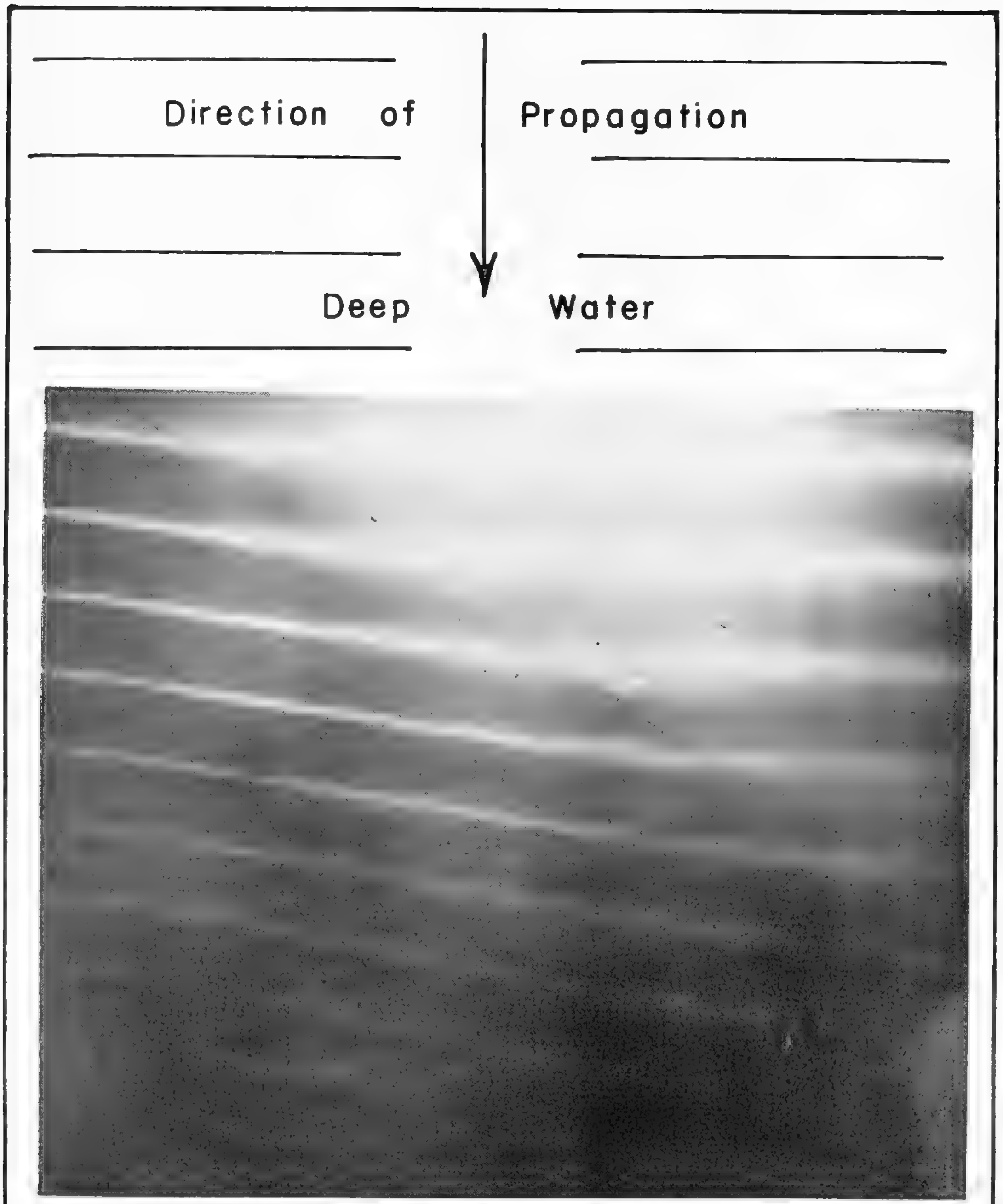

Fig.4 Photograph illustrating Snell's law. Waves enter shallow water from deep water after crossing abrupt transition zone.

of incidence less than 90°. This particular angle is called the critical angle and its value depends on the two media comprising the system. When the critical angle has been passed, the incident wave, instead of refracting into the second medium, rebounds into the first medium. This phenomenon is called total reflection and the angle which the reflected wave makes with the normal to the boundary of the two media is called the angle of reflection.

We now have a whole new family of problems available for investigation and some of these will be studied in future sections.

7. Refraction of light by a vertical glass cylinder[2]

A geometrical optics solution has been obtained by Chinmayanandum (1918) for the intensity of illumination along an internal ray for any angle of incidence, θ. Using monochromatic light of wave length 5.5×10^{-5} cm. emanating from a horizontal collimator slit, and incident on a glass cylinder of radius 0.0184 cm., Chinmayanandum made the photograph shown in figure 5, and from the available data he constructed the diagram in figure 6.

To study the analogous problem in the ripple tank, it was necessary to set a plastic disc in the water. The disc, 8 inches in diameter and 1 inch high, was submerged in the tank which was filled with water one and one-eighth inches deep. The photographic result is shown in figure 7 and the projection of the wave front after refraction is shown in figure 8.

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2. Chinmayanandum, T.K. (1918): Diffraction of light by an obliquely held cylinder. Physical Review, vol. 12, pp. 314-324.

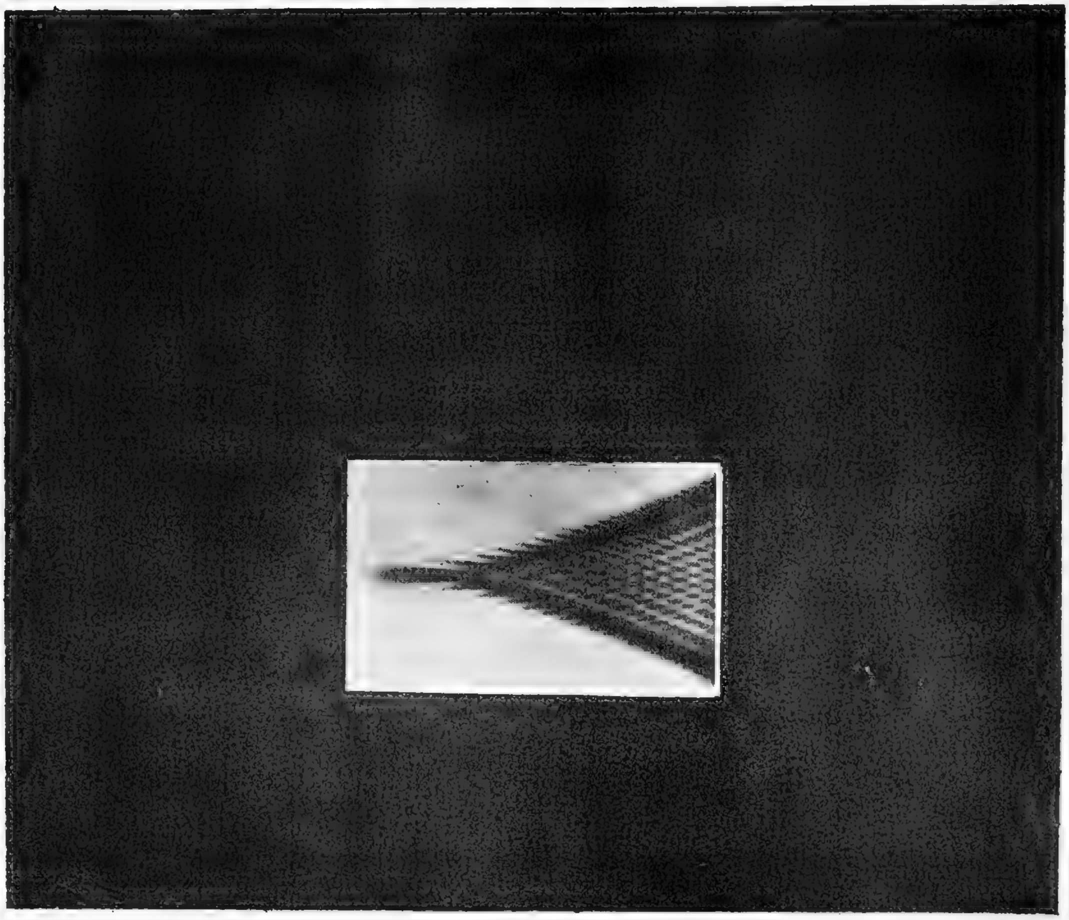

Fig. 5. Photograph showing refraction of light by a vertical glass cylinder. - After Chinmayandum

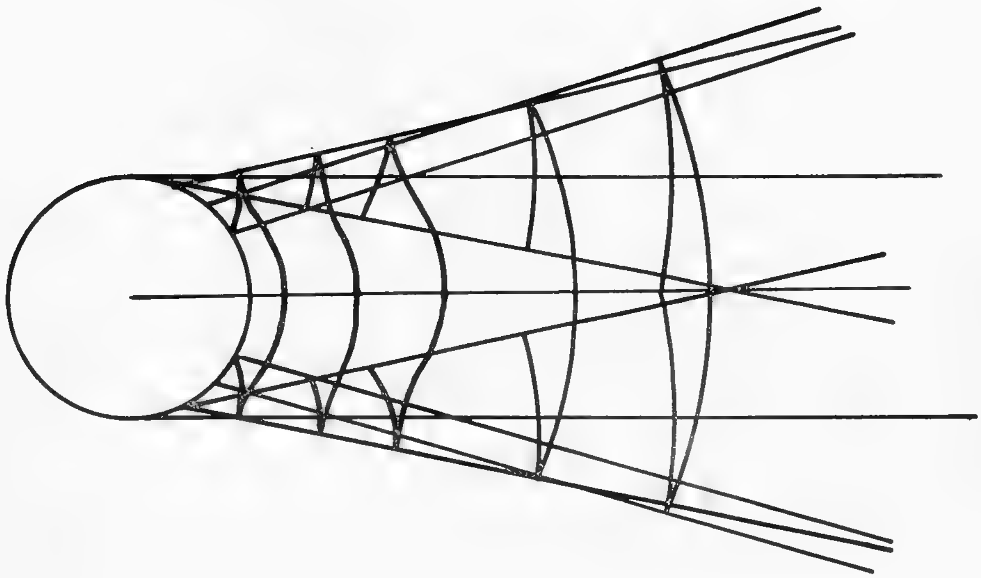

Fig.6. Projection of the wave front on the plane passing through the two caustics when the plane wave is incident normal to the glass cylinder.

—After Chinmayanandum

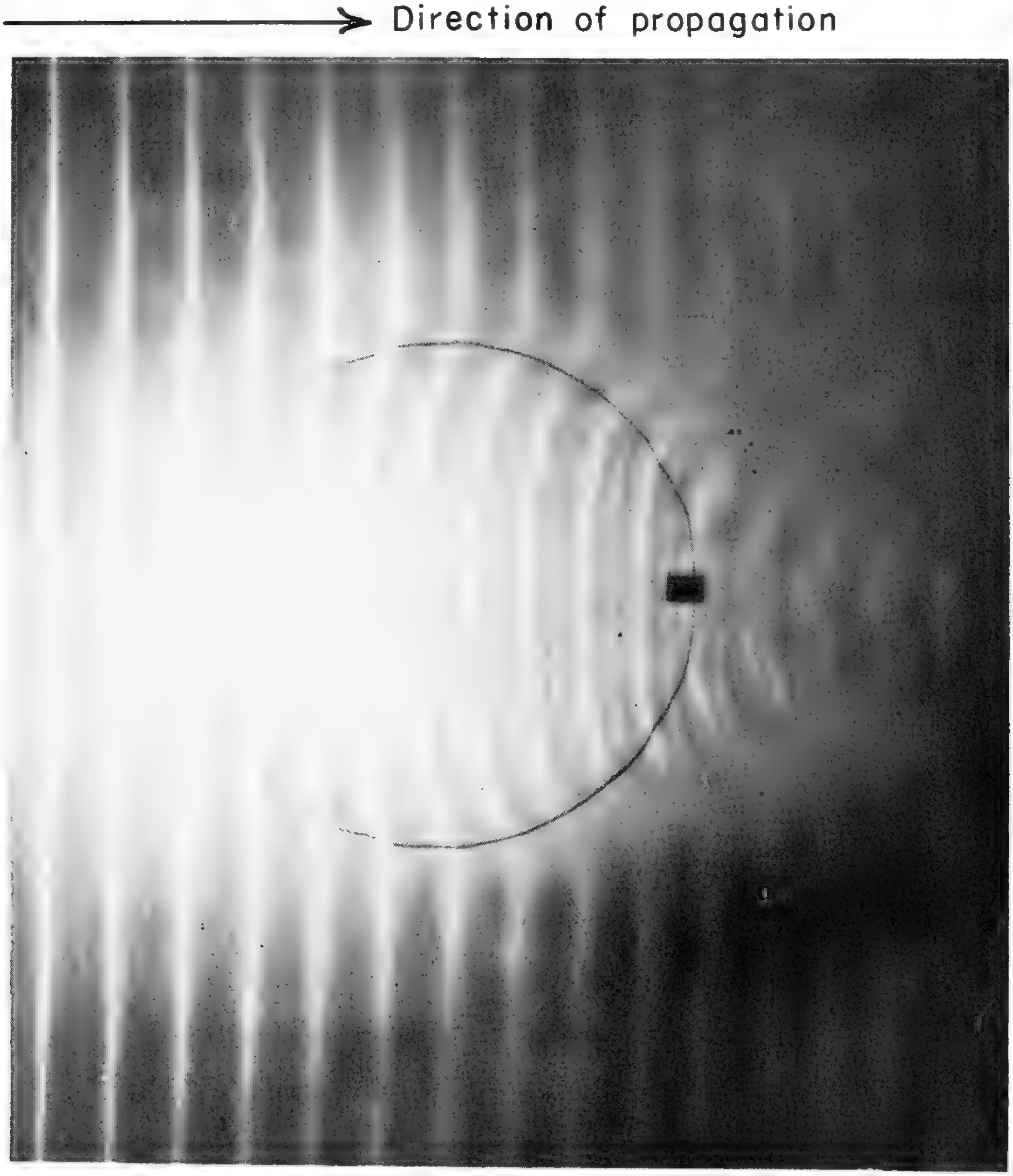

Fig. 7 Refraction of waves by a submerged level disc at constant depth.

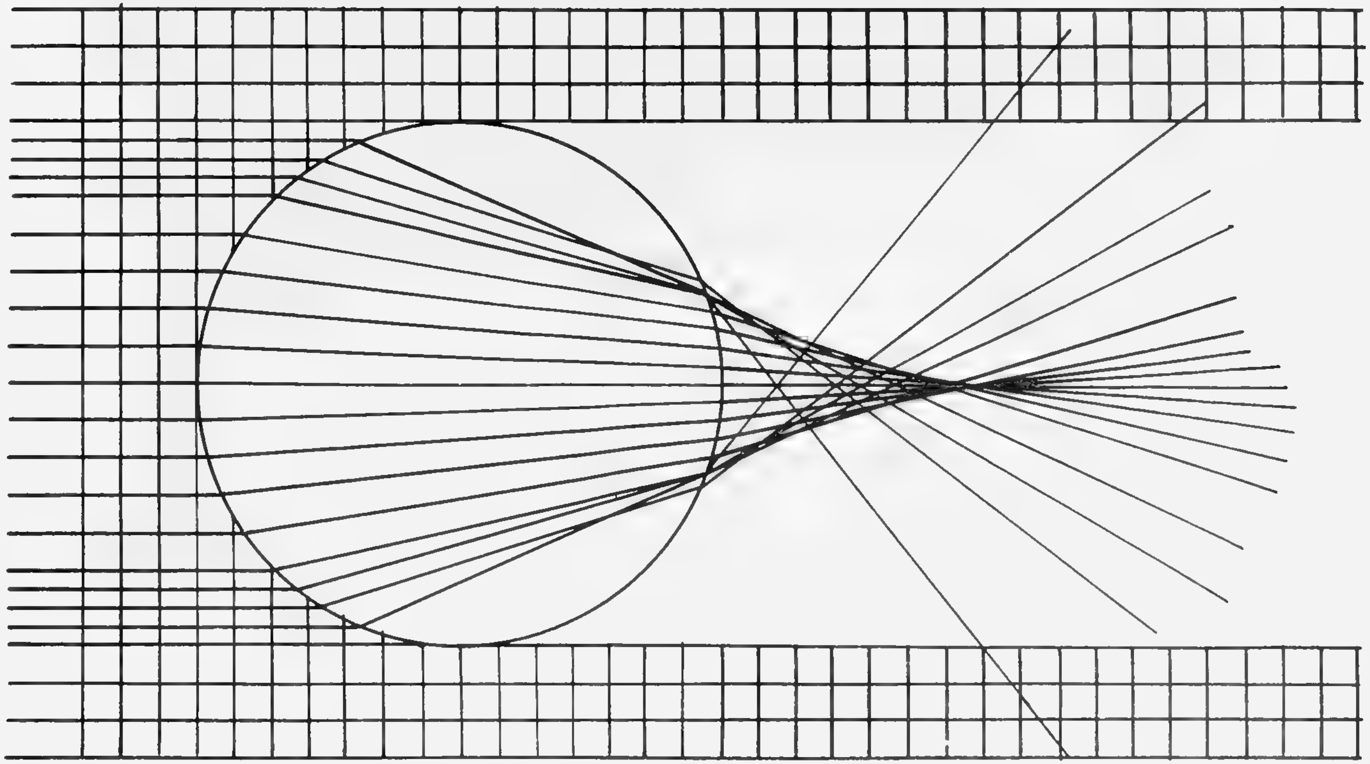

Fig.8 - Projection of the wave front on the plane passing through the two caustics, when the plane water wave travels over a submerged shoal of constant depth.

It will be noticed that the respective photographs and the figures constructed by

$$\frac{\sin i}{\sin r} = \frac{v_1}{v_2} \tag{7.1}$$

where i and r are the angles of incidence and refraction, respectively, and v_1 and v_2 are the wave velocities in the two media, do not agree. This is due to the fact that it was not possible to get the water over the disc shallow enough for proper results. When the water was made shallow enough, the waves would dissipate as they passed over the disc. However, the picture taken (fig. 7) does show refraction by a submerged circular shoal.

Figure 8 shows the appropriate ray-wave crest diagram for the two relative wave speeds involved, and it compares quite well with the actual photograph (fig. 7). If the depth were shallower, the rays would come to a focus inside the circle (Pierson, 1950).

8. Atmospheric refraction of sound waves caused by a uniform temperature variation

A practical application of refraction in the atmosphere is the deviation of sound waves from a straight path due to the variation of temperature with height.

The velocity of sound in gases is given by the expression

$$v = \sqrt{\frac{\gamma p}{\rho}} \tag{8.1}$$

where p is the pressure of the gas, ρ is the density of the medium, and γ is the ratio of the specific heat of the gas at constant pressure to the specific heat at constant volume ($\gamma = 1.40$ for air).

From the equation of state

$$\frac{p}{\rho} = RT \tag{8.2}$$

where R is the gas constant and T is the absolute temperature, substitute (8.2) in (8.1). The result is

$$v = \sqrt{\gamma RT} \ . \tag{8.3}$$

Since $\sqrt{\gamma R}$ is constant, the velocity of sound in air is proportional to the square root of its absolute temperature.

Suppose a sound wave is propagated upwards through an atmosphere of continually decreasing temperature. Then the velocity will decrease proportionally and in a situation similar to that shown in figure 9, the sound waves will be refracted.

Although air temperatures decrease with height there is nothing constant about the rate of decrease. On clear days, the temperature lapse rate during the day is greater than on cloudy days, but towards sunset the temperature becomes approximately constant.

At a given place a number of lapse rates have been observed and statistically averaged. This average value for the lapse rate is known as the "normal lapse rate" and is approximately 6°C per kilometer.

The analogous model tested in the ripple tank was made in the form of a sloping uniform surface, the waves being propagated in the deeper water to simulate the higher temperature at the ground where the sound waves were produced (fig. 10).

As the waves proceed into the shallow water they are noticeably refracted and at the portion of the model which is above water

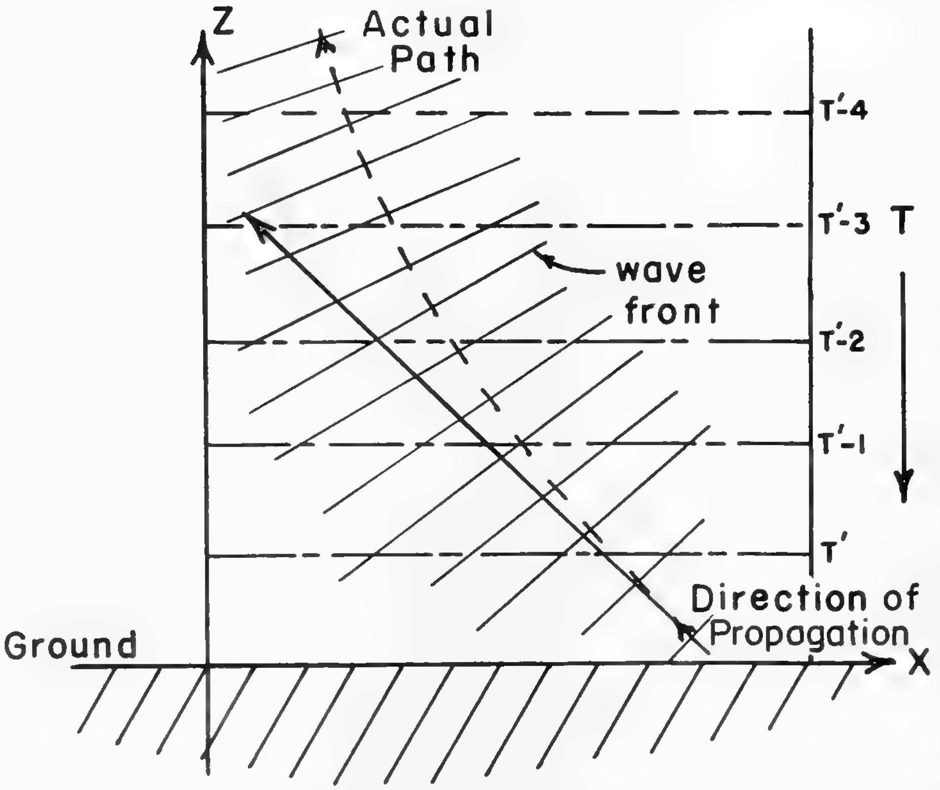

Fig 9. Refraction of sound waves in the atmosphere due to a variation of temperature.

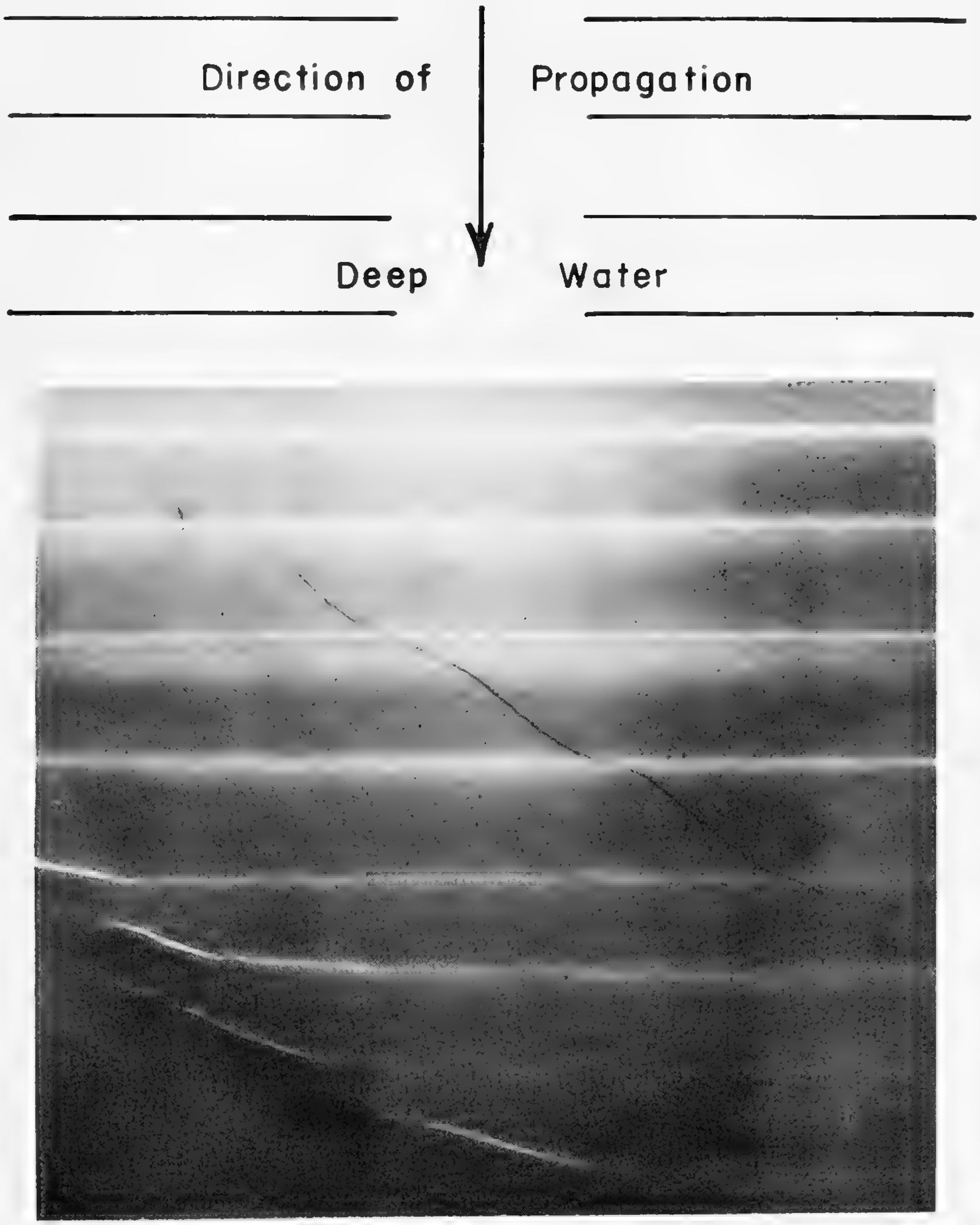

Fig 10. Photograph showing refraction of water waves by a uniformly sloping beach.

level the wave crests are in a position nearly parallel to the "shore."

It must be kept in mind that the linear slope of the model is not necessarily in the same ratio as the normal lapse rate. However, this does not destroy the analogy.

9. Reflection by a plane rigid surface[3]

Suppose an incident sound wave is made to impinge normally on a plane surface which is absolutely unyielding with respect to the slight variations of atmospheric pressure due to the incident wave. The wave, upon striking the "wall," will be unable to proceed in the line of propagation, and the wall will react as if it, and not the air, were vibrating with the given period and amplitude, resulting in a reflected wave along the line of direction. Hence, sound waves incident upon a plane rigid medium and light waves incident on a totally reflecting surface, are completely reflected, the incident and reflected waves having the same phase at the rigid boundary.

The more general case, that of a plane harmonic wave incident obliquely on a rigid wall has been discussed mathematically (Rayleigh, 1945), with the assumption that the incident and reflected waves have equal amplitudes and wave lengths. This being the case, everything will be the same in planes perpendicular to the lines of intersection of the two wave fronts, and therefore the problem may be considered as a two-dimensional one. Let the angle between the incident and reflected rays be 2α (fig. 11), and let the rays proceed with a velocity, a, in a direction perpendicular

3. Rayleigh, Lord, (Strutt, J. W.), 1945: The Theory of Sound. New York, Dover Publication, Volume II, pp. 75-77.

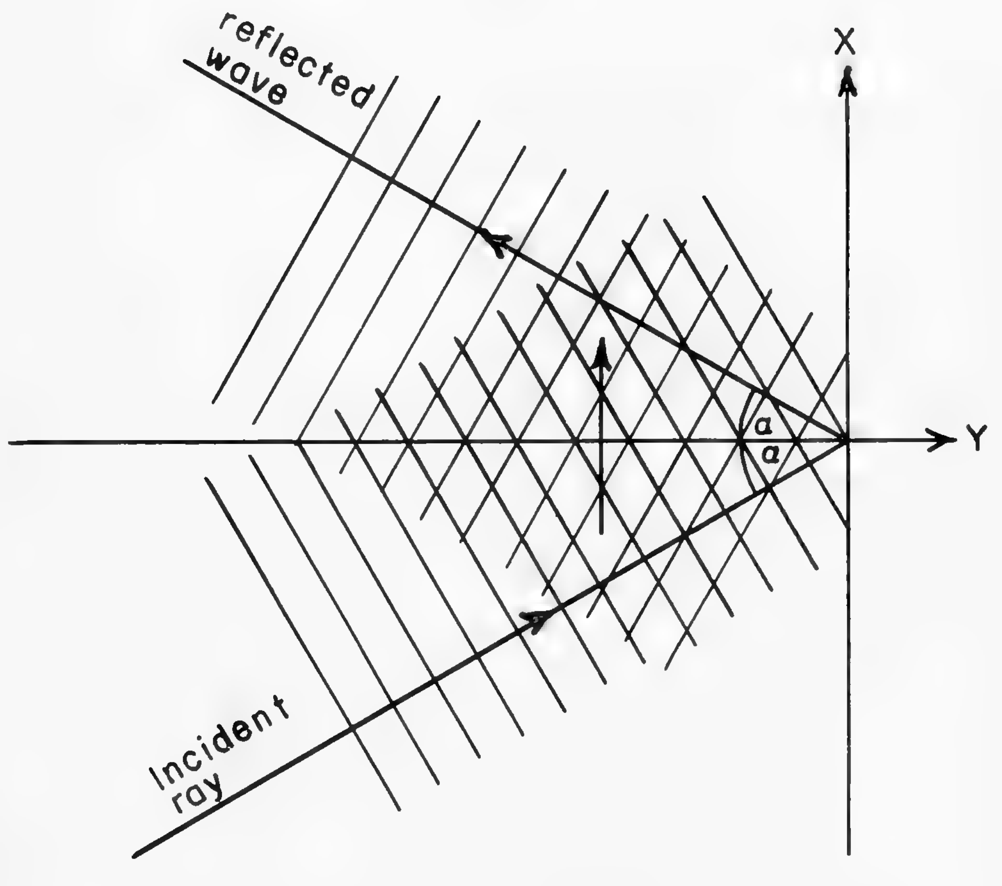

Fig. 11. Reflection by a rigid plane.

to the wave fronts. As the reflected waves cross the incident waves, a pattern is set up consisting of equal parallelograms advancing in the direction of one set of diagonals.

At the corners of each parallelogram, the two trains of waves are superposed giving maximum condensation, and likewise the center of each parallelogram is a point of maximum rarefaction. Hence, in each diagonal, there is a series of maxima and minima condensations advancing with a velocity $a/\cos\alpha$. There are parallel lines of zero condensation between each adjacent pair of lines in maxima and minima.

Rayleigh says, "It is especially remarkable that, if the wave pattern were visible (like the corresponding water wave-pattern to which the whole of the preceding argument is applicable), it would appear to move forward without change of type in a direction different from that of either component train, and with a velocity different from that with which both component trains move." This phenomenon has been observed in the N.Y.U. ripple tank and a photograph showing one of the stages is presented here (fig. 12). If the conditions were the same as shown in figure 11, the wave pattern would appear to move in the direction of the arrow, that is, in the positive x direction.

Since the angle between the incident and reflected rays is 2α, the angle between either ray and the normal to the reflecting plane is α. Then, the condensations may be written as

$$\cos \frac{2\pi}{\lambda} (at - x \cos\alpha - y \sin\alpha) \qquad (9.1)$$

where λ is the wave length, and

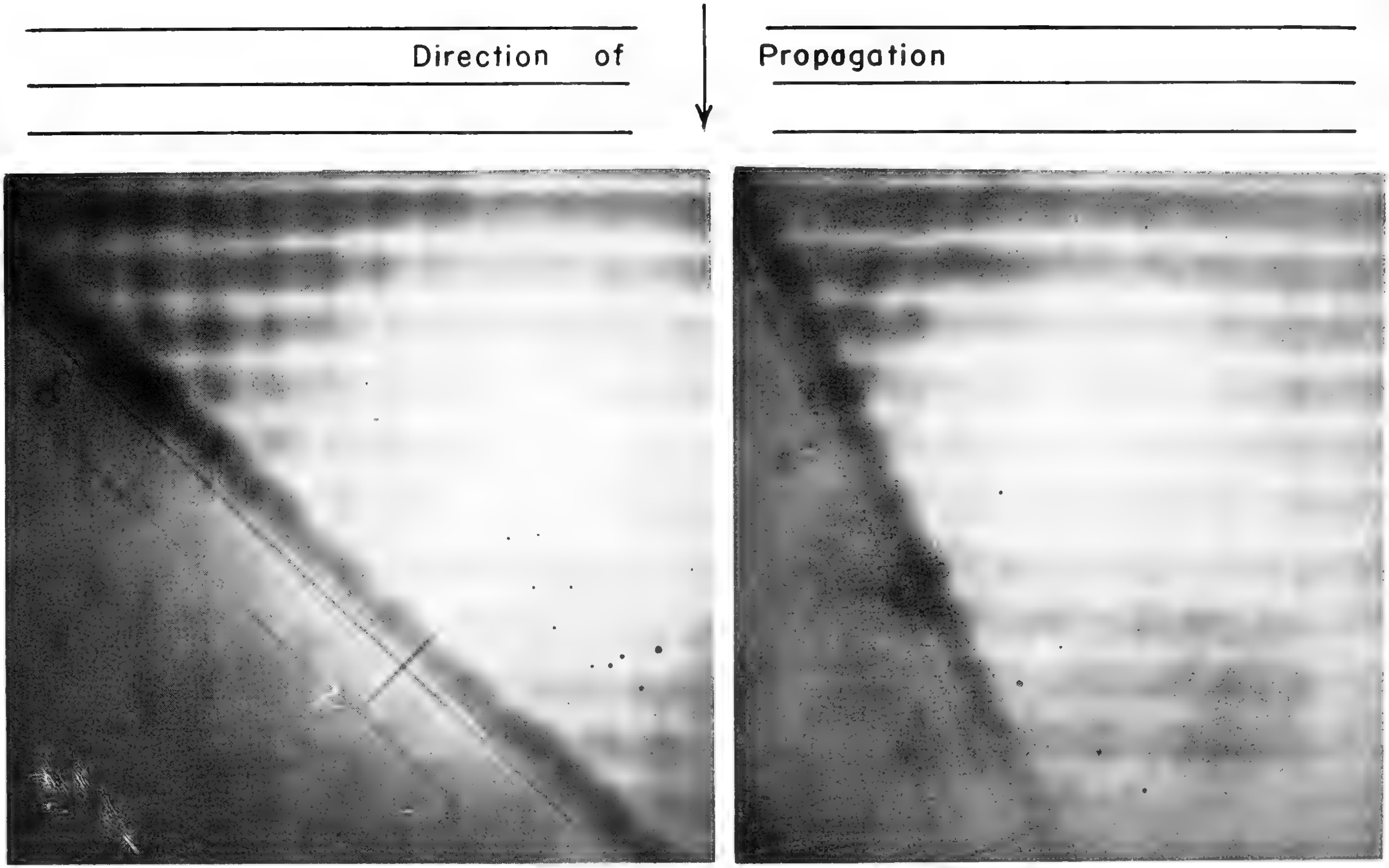

Fig.12. Photograph showing reflection of water waves by a rigid breakwater. Angles of incidence are approximately 60° and 45°.

$$\cos \frac{2\pi}{\lambda} (at - x \cos\alpha + y \sin\alpha) \tag{9.2}$$

respectively, and the expression for the resultant is

$$s = \cos \frac{2\pi}{\lambda} (at - x \cos\alpha - y \sin\alpha) + \cos \frac{2\pi}{\lambda} (at - x \cos\alpha + y \sin\alpha)$$

$$= 2 \cos \frac{2\pi}{\lambda} (at - x \cos \alpha) \cos \frac{2\pi}{\lambda} (y \sin \alpha). \tag{9.3}$$

Since the expression on the right side of (9.3) is an even function of y, s is symmetrical with respect to the x-axis, so that there is no motion across that axis. Under these conditions, a rigid wall placed along the x-axis would in no way impede the motion. Therefore, the resultant ray (s) satisfies the boundary condition that there be no motion across the x-axis.

From the expression (9.3) it is seen that the resultant wave (s) on the xy plane advances parallel to the x-axis unchanged in type and with a constant velocity $a/\cos\alpha$.

If, of course, $\alpha = \frac{1}{2}\pi$ which is the case for normal incidence, then (9.3) reduces to

$$s = 2 \cos (\frac{2\pi}{\lambda} at) \cos (\frac{2\pi}{\lambda} y), \tag{9.4}$$

and we have standing waves.

10. Reflection at an air interface[4]

Let two adjacent air masses, I and II be separated by the boundary M. If they have the same pressure, but different densities due to differences in temperature and/or humidity, then a wave proceeding from air mass I to air mass II will undergo a change in

4. Humphreys, W. J. (1929): Physics of the Air. Second edition, McGraw-Hill Book Company, Inc., New York and London. pp. 407-410.

velocity. If also it is assumed that none of the energy of the incident wave is dissipated, then it may be concluded that the energy is contained in the refracted and reflected waves. Since the laws for reflection and refraction are the same for both light waves and sound waves, we will discuss the general case of a wave incident at the air interface described above.

Consider the incident, reflected and refracted waves as shown in figure 13. Let the amplitudes of these waves be a, b and c respectively; let the wave velocity in medium I be v and in medium II, v'; and let the densities of the respective media be ρ and ρ'. Now, since the energy of the incident wave is divided between the reflected and refracted waves without loss, and since, in each case, it can be shown that the energy of the wave is proportional to the volume affected and to the squares of the respective amplitudes, then the energy may be expressed as

$$v\rho a^2 = v\rho b^2 + v'\rho' c^2 \frac{\cos r}{\cos i} \tag{10.1}$$

where i and r are the angles of incidence and refraction, respectively. From Snell's law, we know that

$$\frac{v}{v'} = \frac{\sin i}{\sin r}\ , \tag{10.2}$$

hence (10.1) becomes

$$\rho(a^2 - b^2)\sin 2i = \rho' c^2 \sin 2r. \tag{10.3}$$

As previously seen

$$\left(\frac{\rho'}{\rho}\right)^{1/2} = \frac{v}{v'} = \frac{\sin i}{\sin r}\ ,$$

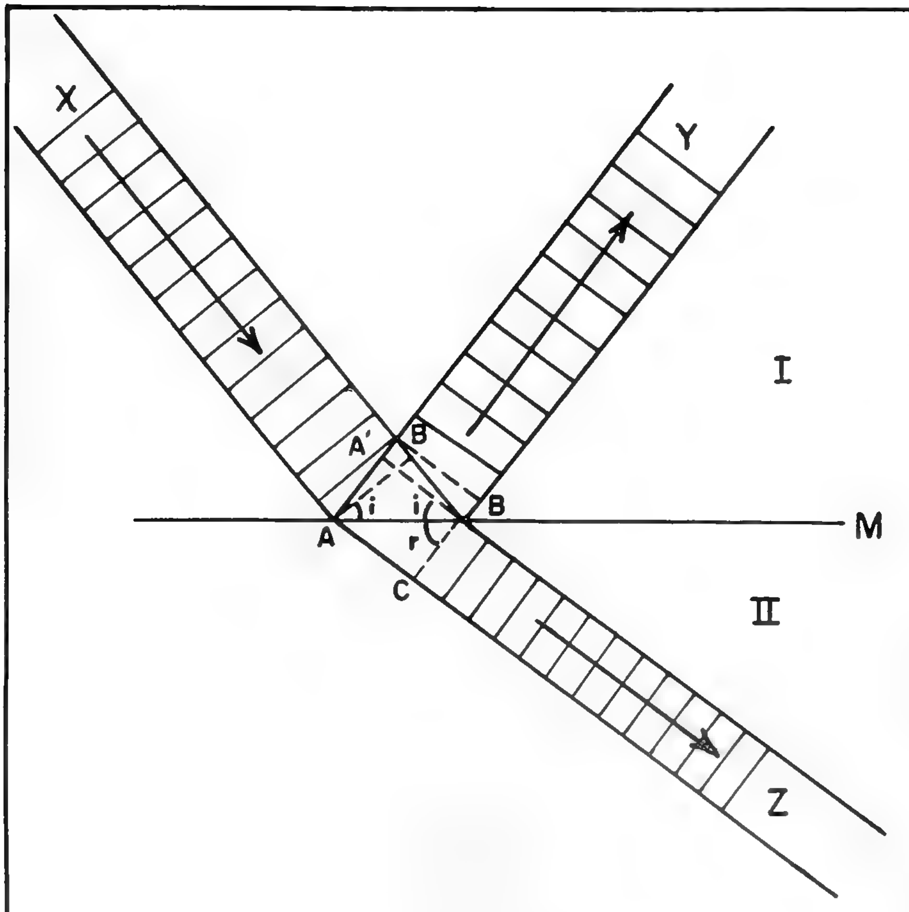

Fig.13. Incident (x), reflected (y), and refrdcted (z) rays at an air interface.

therefore, (10.3) becomes

$$a^2 - b^2 = c^2 \tan i \cot r. \tag{10.4}$$

If there is no slipping between the media, then the algebraic sum of the amplitudes of the incident wave and the reflected wave equals the amplitude of the refracted wave, or

$$a + b = c \tag{10.5}$$

Substituting (10.5) in (10.4), it is seen that

$$a - b = c \tan i \cot r, \tag{10.6}$$

so that

$$b = - a \frac{\sin(i - r)}{\sin(i + r)} = - a \frac{\left[\left(\frac{\rho'}{\rho}\right)^{1/2} - \frac{\cos i}{\cos r}\right]}{\left[\left(\frac{\rho'}{\rho}\right)^{1/2} + \frac{\cos i}{\cos r}\right]} \tag{10.7}$$

and

$$c = \frac{2a \cos i \sin r}{\sin(i + r)} = \frac{2a \cos i}{\left(\frac{\rho'}{\rho}\right)^{1/2} \cos r + \cos i} \tag{10.8}$$

Suppose a sound wave impinges normally on a cloud in which the density of the air at the same pressure is 1% less, (a possible ratio owing to differences in temperature and humidity), the energies in the incident, reflected and refracted waves will be to each other as the ratios

$$v\rho a^2 : v\rho b^2 : v'\rho' c^2 = 160{,}000 : 1 : 159{,}999. \tag{10.9}$$

If therefore, the angle of incidence is made larger and gradually approaches 90°, then the reflection becomes greater, but is still small for any angle up to 90°.

To show this phenomenon in the ripple tank, we have taken the boundary M to be a plane in three dimensions (reducing to a line in two dimensions), the waves impinging on it at increasing angles of incidence. The model used here (step model) is the same as the one used in section 6 to illustrate Snell's law. When the angle of incidence is 0° then the refracted waves are parallel to the incident waves and the change in velocity manifests itself as a change in wave length (in the refracting medium).

When the angle of incidence is increased, reflection is observed at the boundary and it is seen that the crests of the refracted waves are fainter than the crests of the incident waves, indicating that the energy which would have gone into the second medium if there had been no discontinuity, is now contained in the reflected waves (figure 14). The greater the angle of incidence, the greater the reflection.

11. The phenomenon of total reflection[5]

Suppose that the second medium is not unyielding to the incident wave front and that total reflection is accomplished by merely reaching and passing the critical angle. It is of interest to investigate what happens to the "superficial" wave in the second medium.

Christian Huygens showed that the absence of a refracted wave and the increased intensity of reflection for angles of incidence exceeding the critical angle follow from his principle as simple consequences. Therefore, from the standpoint of wave theory, Huygens' principle is the natural starting point for the study of

5. Raman, C.V. (1927): Huygens' principle and the phenomenon of total reflection, Optical Society of London, Transactions, vol. 28, pp. 149-160.

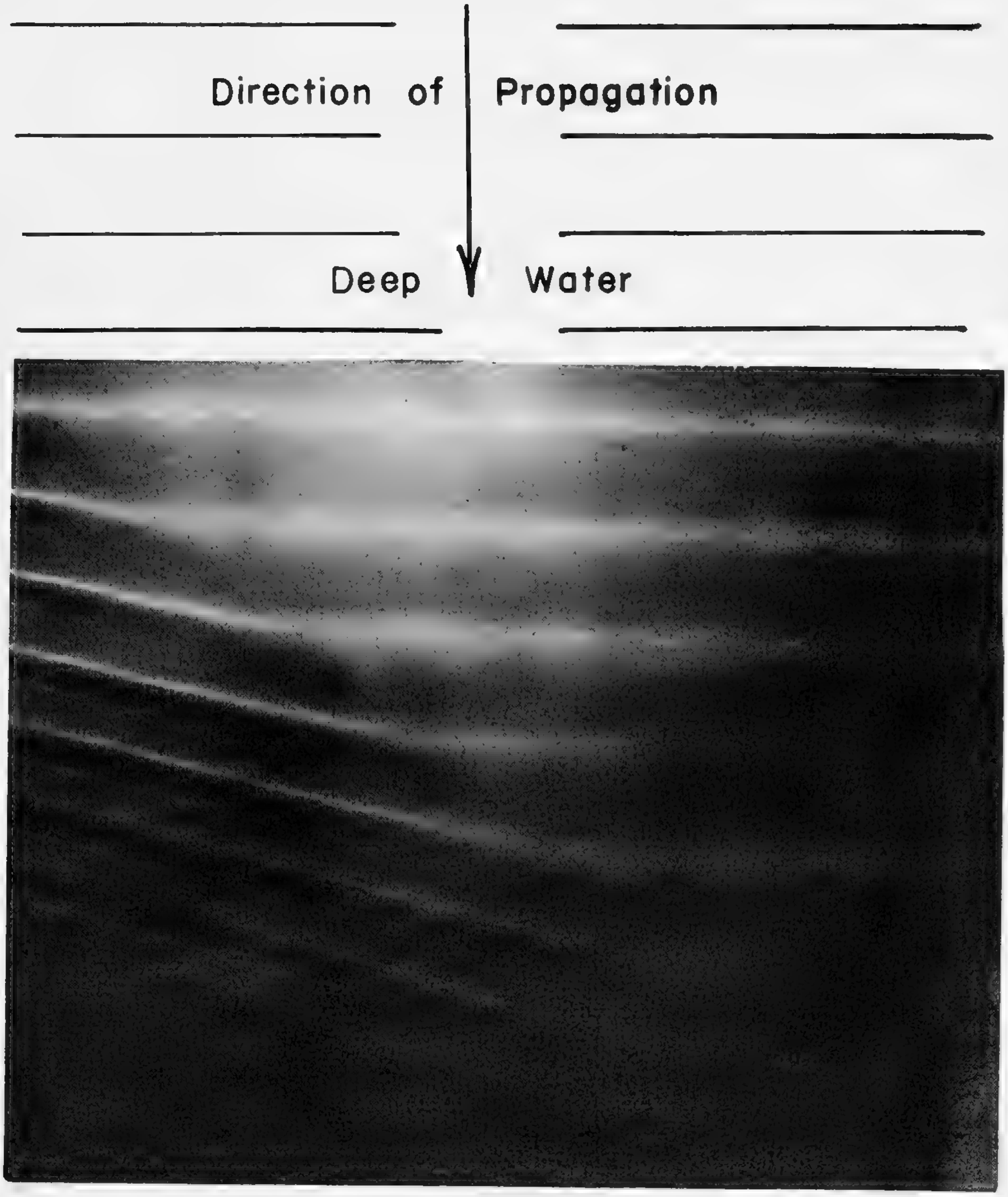

Fig. 14 Photograph showing incident, refracted, and reflected waves at the boundary of the step model.

total reflection.

The usual treatment given the problem of total reflection is based on the formulae obtained by Fresnel. It is necessary that proper mathematical interpretation be given the angle of refraction, which becomes imaginary when the angle of incidence exceeds the critical angle. The method of approach adopted by Raman (1927) is shown in figure 15. Suppose the plane of the paper is the plane of incidence (xz plane), and the origin of coordinates O is taken to be on the surface at which total reflection occurs (xy plane). According to Huygens' principle, the disturbance in the second medium at a point P (coordinates x,z) may be regarded as the superposition of an infinite number of wavelets radiated from elements of the bounding surface and the disturbance is determined by evaluating the integral which expresses the result of such superposition.

If a train of light waves (or sound waves) of period T is incident on the boundary between two media, where the velocities of light (or sound) are respectively V_1 and V_2, the refractive index of the second medium relative to the first medium is proportional to the wave velocities in the two media (V_1/V_2), and is called μ. The disturbance in the first medium due to the incident waves is expressed as

$$\eta_1 = A \cos \left[\frac{2\pi t}{T} - \frac{2\pi(x \sin\theta + z \cos\theta)}{V_1 T} - \frac{1}{2}\,\delta\right] \qquad (11.1)$$

where θ is the angle of incidence, t is the time, A is the amplitude of the incident wave and 1/2 δ is the phase difference between the disturbance incident on any element of surface and the secondary wavelet starting out from it.

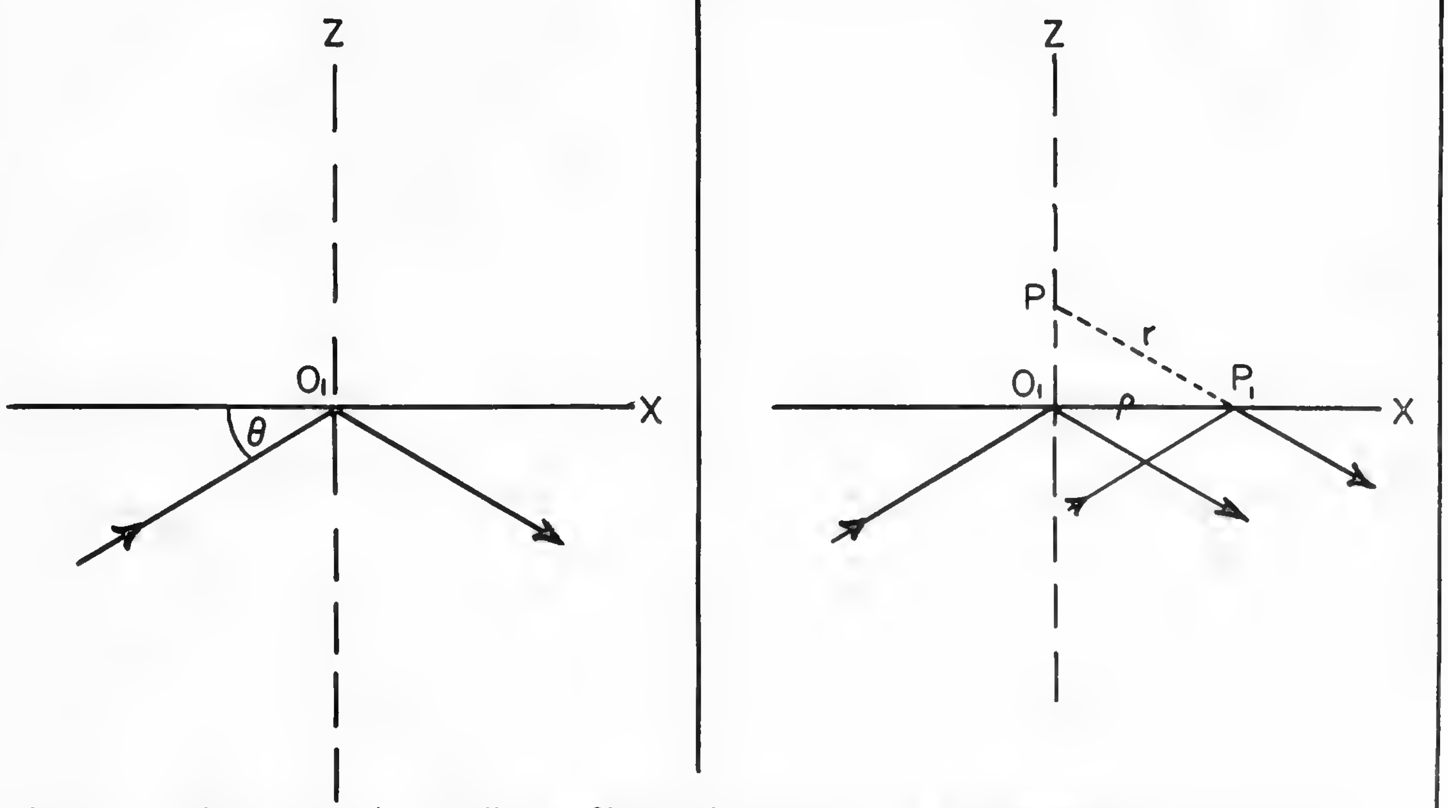

Fig.15 Incident and totally reflected wave. The observer (P) is in the second medium.

–After Raman

It is found that the disturbance in the second medium has the form

$$\eta_2 = \sigma A \cdot e^{-\frac{2\pi}{TV_2}\sqrt{\frac{\sin^2\theta}{\mu^2} - 1}\, z} \cos \frac{2\pi}{T}\left(\frac{t - x \sin\theta}{\mu V_2}\right). \qquad (11.2)$$

It is seen from (11.2) that the superficial wave has an amplitude σA at the surface along the x-axis, which decreases exponentially with z. Furthermore, the energy of the superficial wave is propagated parallel to the surface along the x-axis, thus the energy-flux across any element of area of the surface must be zero. Since there is no energy-flux across the boundary, it follows that the amplitudes of the incident and reflected waves must be equal. Hence, we have total reflection.

If a rigid plane is placed at the boundary between the two media then the phenomenon which occurs is the same as demonstrated in figure 16.

That this is the case can be seen by placing a plane strip of plastic at the boundary of the step model in an earlier experiment. If the strip of plastic extends above the water, then we have the case of total reflection by a rigid plane (figure 17). Comparison of figure 17 with figure 16 bears out the theory already discussed, although in figure 16, there is some leakage of energy across the boundary, due to a poor deep water to shallow water ratio.

The laws of reflection which are valid for light waves and sound waves have now been shown by theory and experimentation (sections 9, 10, and 11), to hold also for water waves, and so

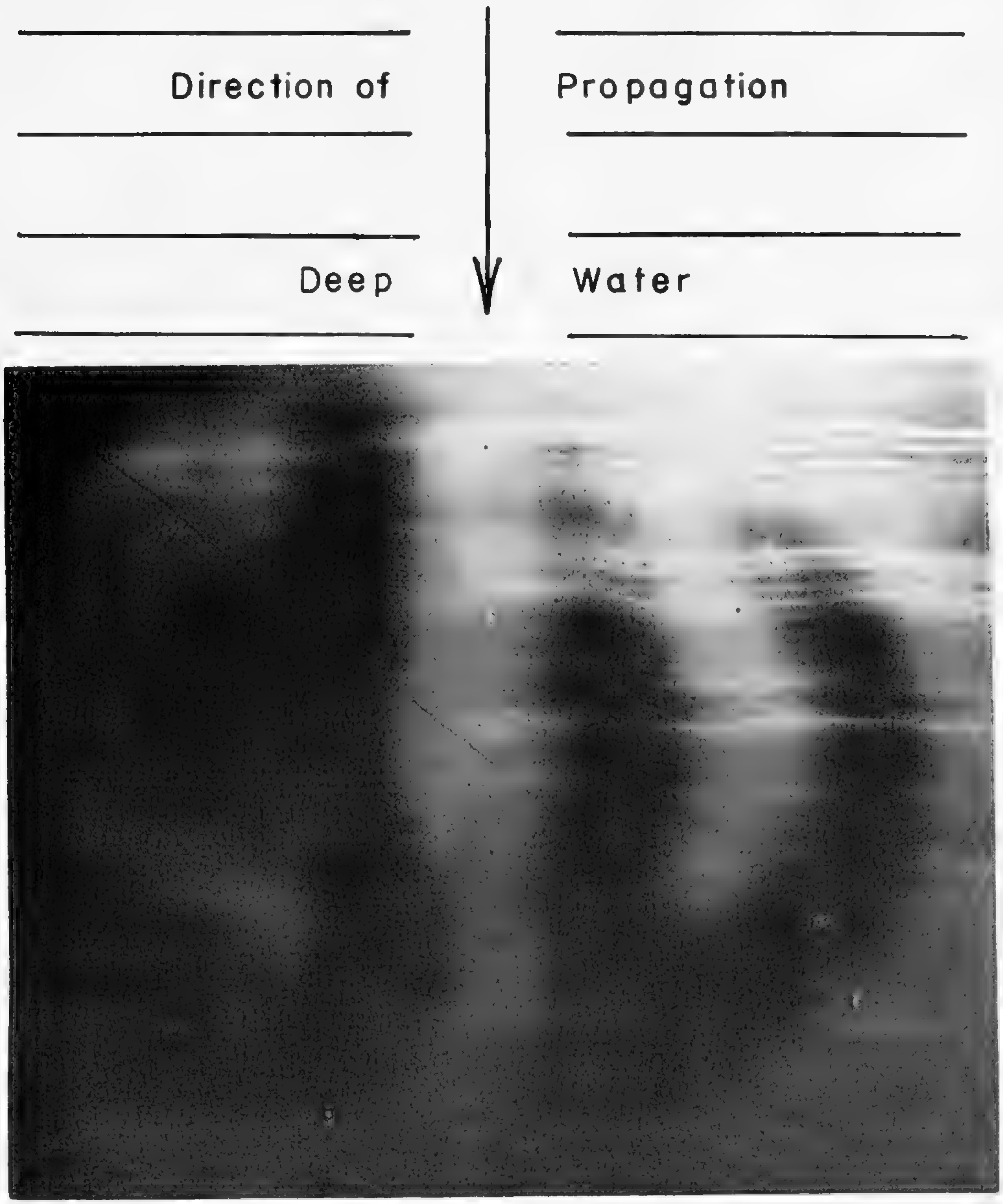

Fig.16 Photograph showing total reflection of water waves at the boundary of the step model.

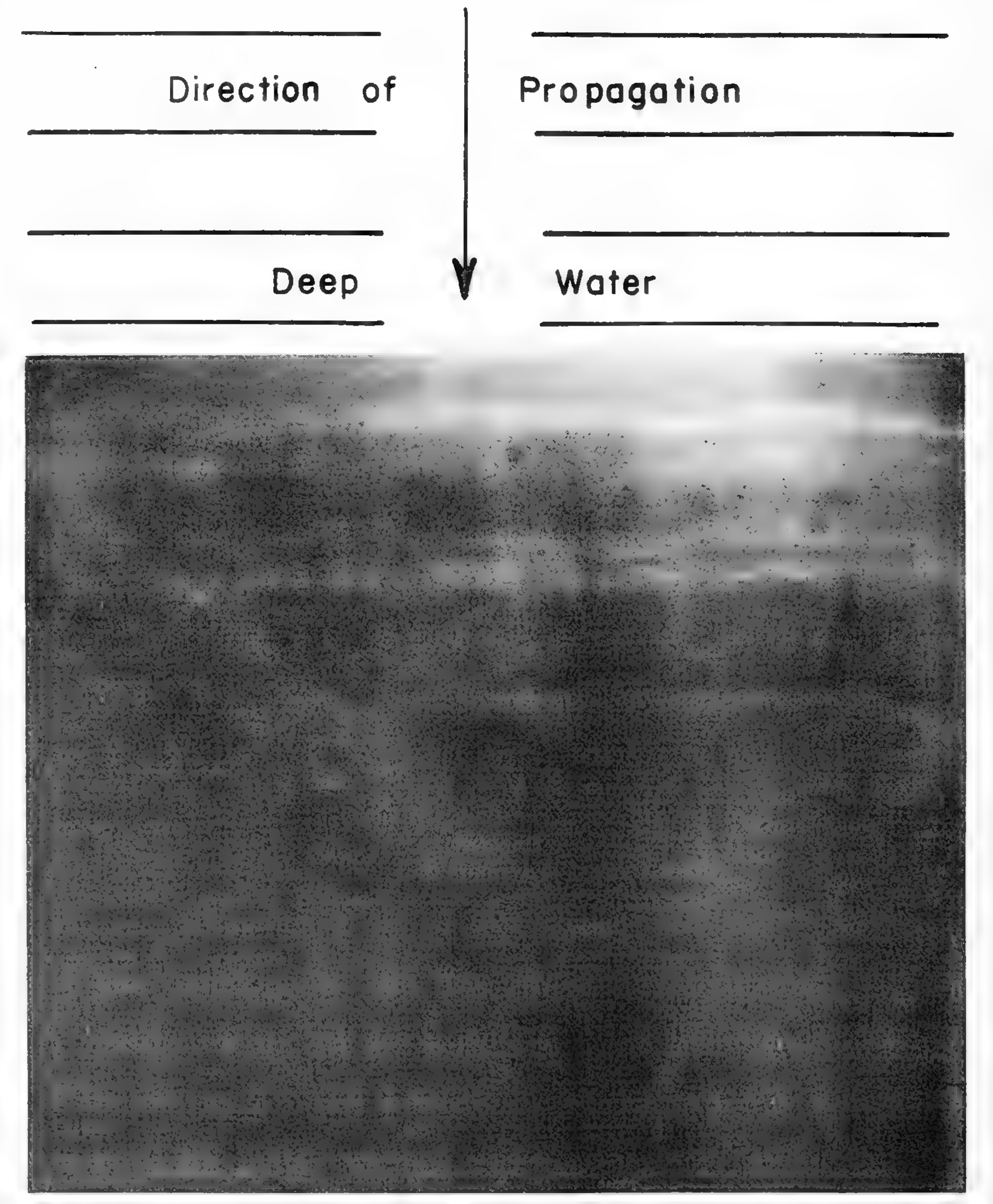

Fig. 17 Photograph showing an infinite screen(jetty) placed at the zone of transition between deep and shallow water.

in concluding this chapter, we will write them here.

(1) The normal to any point on the reflecting surface, and the directions of both the incident wave front and the reflected wave front at that point all lie in the same common plane.

(2) If the reflecting surface is of such a nature that its every diameter is large compared with the wave length, then the reflection is said to be regular, i.e., the angle of reflection is equal to the angle of incidence.

(3) Surfaces whose diameters are smaller than the wave length do not regularly reflect the sound waves, but merely scatter or diffuse them.

Chapter 3. Diffraction

12. Beginning of modern diffraction theory

In accordance with geometrical optics, rays of light are straight lines wherever the index of refraction is constant. With this observation in mind, it was thought that light which emanates from a point source a great distance from an opaque screen should give a sharply defined shadow, which is called the geometrical shadow. Actually, it is observed that the light is propagated up to the screen as if the screen were absent, but once beyond the screen, light enters the geometrical shadow. This phenomenon which violates the laws of geometrical optics is known as diffraction.

Christian Huygens was the first pioneer in diffraction and those that followed him used his principle as a springbroad to more complex theory. It was Fresnel who first discovered the real cause of diffraction. In order to obtain satisfactory solutions, Fresnel had to make somewhat arbitrary assumptions on the nature of the secondary waves. Most of the difficulties of Fresnel's theory were overcome by Kirchhoff, who used Helmholtz's formulation of Huygens' principle for monochromatic phenomena.

13. Diffraction of light by a black half-plane[6]

The first type of diffraction problem to be considered is one in which plane monochromatic light is incident on a black screen of infinite extent and bounded by a straight edge (figure 18).

For the portion of the y-axis covered by the screen (S), a physical optics solution has been obtained in the form

$$U(P) = -\frac{1}{4\pi}\int_{-\infty}^{\infty} e^{iK\rho\cosh\tau - iKct}\,\frac{\sin\varphi}{\cosh\tau + \cos\varphi}\,d\tau \qquad (13.1)$$

U = wave profile

K = constant

ρ = distance from the edge of the screen

τ = variable of integration

c = velocity of light

t = time.

This expression holds only when P, the point of observation, is in the geometrical shadow. If it isn't, the additional term $\mathcal{E}U(P)$, which is the wave function according to geometrical optics, must be added to (13.1), where $\mathcal{E} = 0$ or 1 depending on whether P is or is not in the geometrical shadow. If we write the expression

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6. Baker, B. B. and E.T. Copson, 1950: Mathematical Theory of Huygens' Principle, Oxford at the Clarendon Press, pp. 84-92.

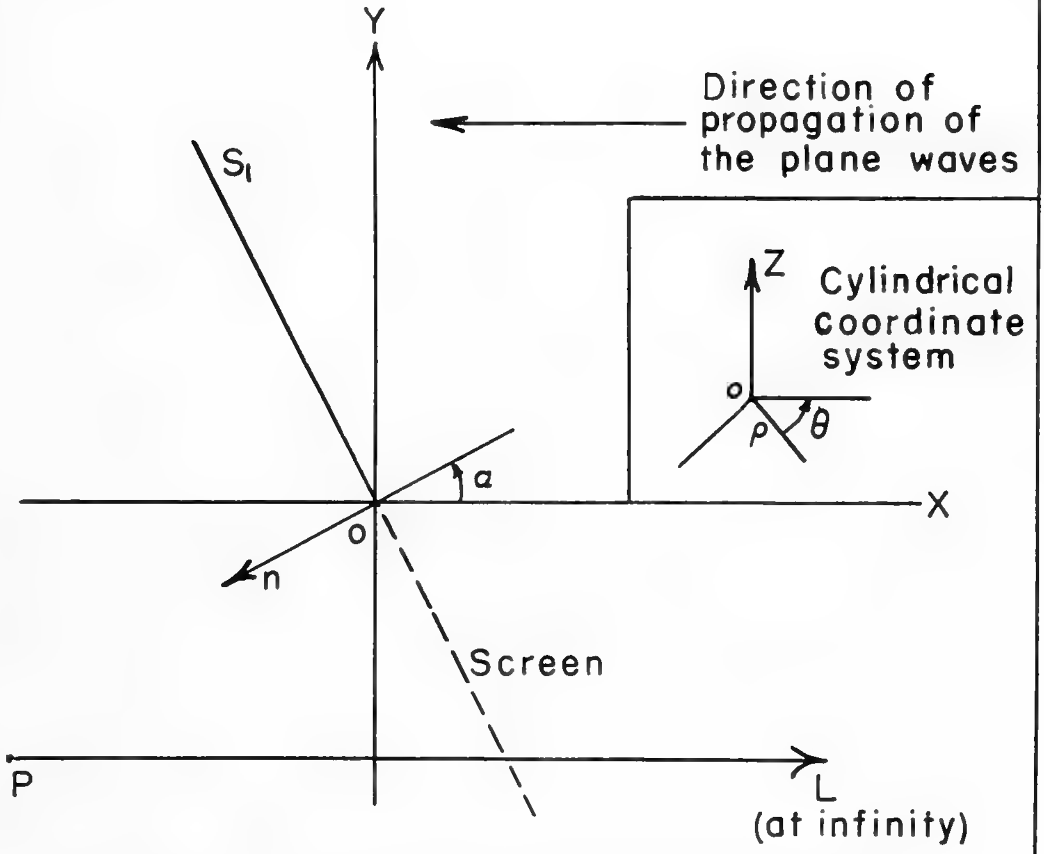

Fig. 18 - Diffraction of plane monochromatic light by a black half plane.

$\mathcal{E}U(P)$ as u^* and call the right hand side of (13.1) u^B (the effect of diffraction), the solution may be written

$$U = u^* + u^B \tag{13.2}$$

In the geometrical shadow, $U = u^B$, so that the intensity of illumination is measured by

$$|U|^2 = |u^B|^2 \sim \frac{1}{8\pi K\rho} \tan^2 \frac{1}{2}\varphi \tag{13.3}$$

when ρ is large compared with the wave length. The intensity of the incident light having measure unity, the illumination in the geometrical shadow is very feeble, but there is nowhere absolute darkness since $\tan^2 \frac{1}{2}\varphi$ never is actually zero in the shadow. According to (13.1), when ρ is very large compared with the wave length, the diffracted wave is a non-isotropic cylindrical wave propagated outwards from the edge of the screen. Diffraction by a black half plane is actually an edge effect, because neither u^* nor u^B depends on the angle of incidence and so U is the same for all black screens having the same edge and the same shadow.

The model tested in the ripple tank is a simple arrangement consisting of a vertical strip of plastic bounded by a straight edge and supported in such a manner that it protrudes above the surface of the water, the plane of the strip being normal to the surface. The depth of the water is constant everywhere so that the wave front is only affected by the "wall" and the bounding straight edge.

As was mentioned before, there is nowhere absolute darkness in the geometrical shadow, since $\tan^2 \frac{1}{2}\varphi$ is never zero there (13.3). This phenomenon may be seen in the prepared photographs (fig. 19).

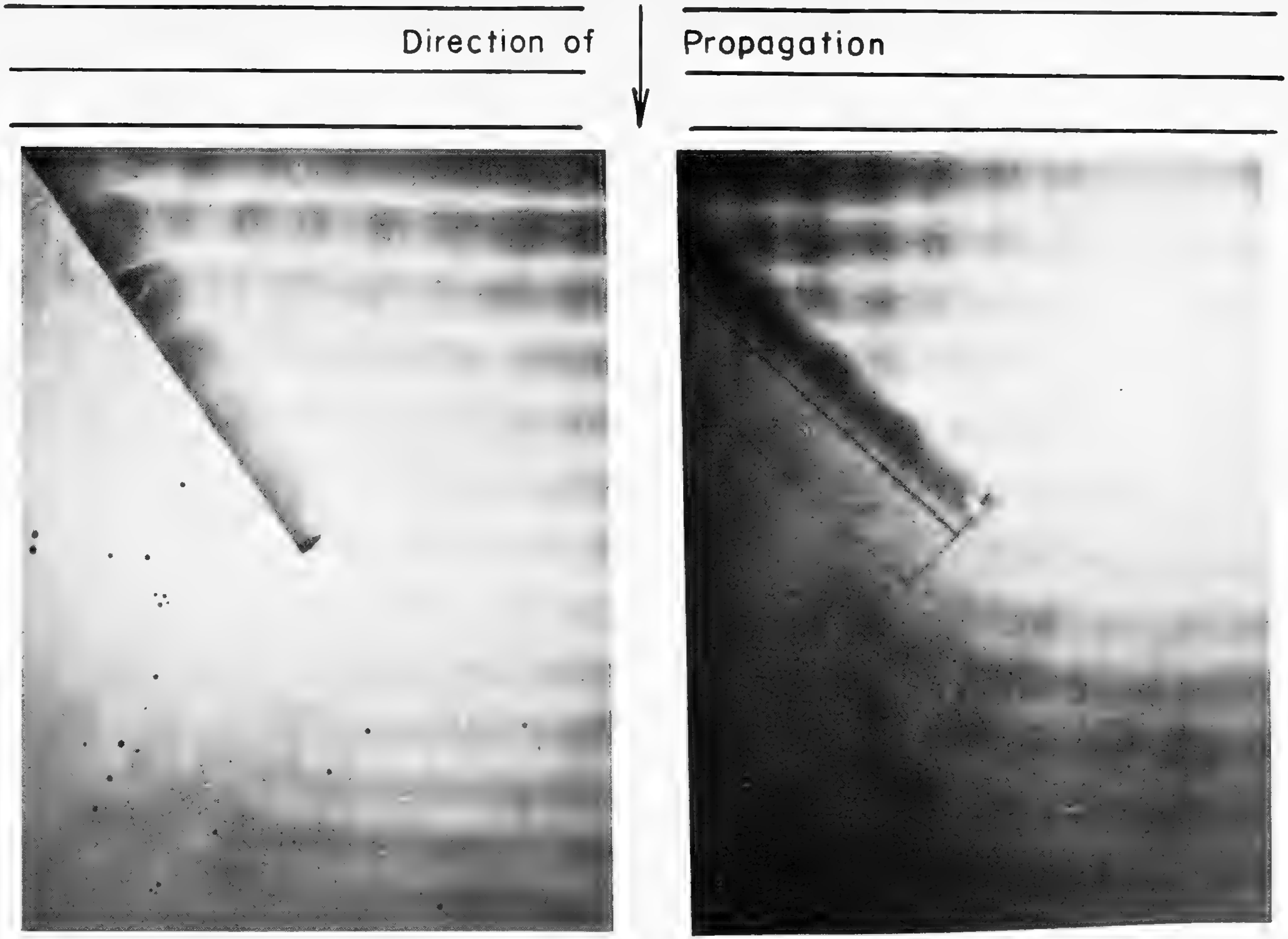

Fig.19. Photographs showing diffraction by a rigid breakwater. Angles of incidence are about 55° and 40°.

That this particular case of diffraction is purely an edge effect can also be seen from the two views, in which different angles of incidence were used.

The analogous problem, that of diffraction of sound waves by a semi-infinite screen, has been studied by many investigators. Although a somewhat different approach was used by Lamb (1906), his results were the same, because it was shown (Friedlander, 1946) that although the solution differed in form from the one obtained by Baker and Copson (1950), the two solutions are related by a suitable transformation. It has further been shown (Sivian and O'Neill, 1931-1932) by means of the solution obtained by Lamb, that essentially the same situation holds for any angle of incidence for which the plane of the wave front is parallel to the edge of the screen. This means that diffraction by a straight edge is an edge effect, which is in complete agreement with Baker and Copson and with our ripple tank investigation (fig. 19).

14. Diffraction by an infinite slit[7]

> "Partially opened doors and windows in a house or other building afford apertures which may be small compared with the wave length of many common sounds. Thus the openings may be a few inches wide, and the wave length of the speaking voice of a man may be eight feet or more. Hence such sounds spread in all directions beyond those openings, as is well known, instead of proceeding as straight lines and giving sharp sound 'shadows' as in the case of light through the same opening."
>
> Barton.

Consider the incidence of a wave upon an infinite slit of very small width compared with the wave length. If the slit

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7. Drude, P. K. L. (1922): Theory of Optics. Longmans, Green and Co., pp. 198-200. Translated by C. Riborg Mann and R. A. Millikan.

is parallel to the z-axis and the incident light travels in the xy plane, then the slit itself may be regarded as a single point on the incident wave front, and according to Huygens' principle, the slit becomes the origin of a spherical wavelet, and the only source of wave activity beyond the slit (fig. 20).

In the case of a plane wave front which advances along parallel lines, the wave front upon passing through the slit becomes spherical and the direction along which it advances becomes radial, and therefore diverging.

In the subject of physical optics, many such phenomena due to the spreading of light waves which pass through small apertures are known and are studied under the general name of diffraction. In all such cases, the openings in question bear about the same relation to the wave length of light as the openings in the analogous acoustical phenomena bear to the wave length of sound.

Consider the emitted light to be lying in a plane which passes through the source Q and is perpendicular to the edges of a slit. This plane is the xz plane (fig. 21). Let the x coordinates of the slit be x_1 and x_2. If the point P_o, where the intensity is to be calculated, is in the geometrical shadow of one of the screens which bounds the slit on either side, then x_1 and x_2 are both positive or both negative. But if the line connecting Q with P_o passes through the open slit, then the signs of x_1 and x_2 are opposite.

The intensity of illumination (J) at some point P_o a distance ρ_o from the plane of the slit was calculated and is given by the expression

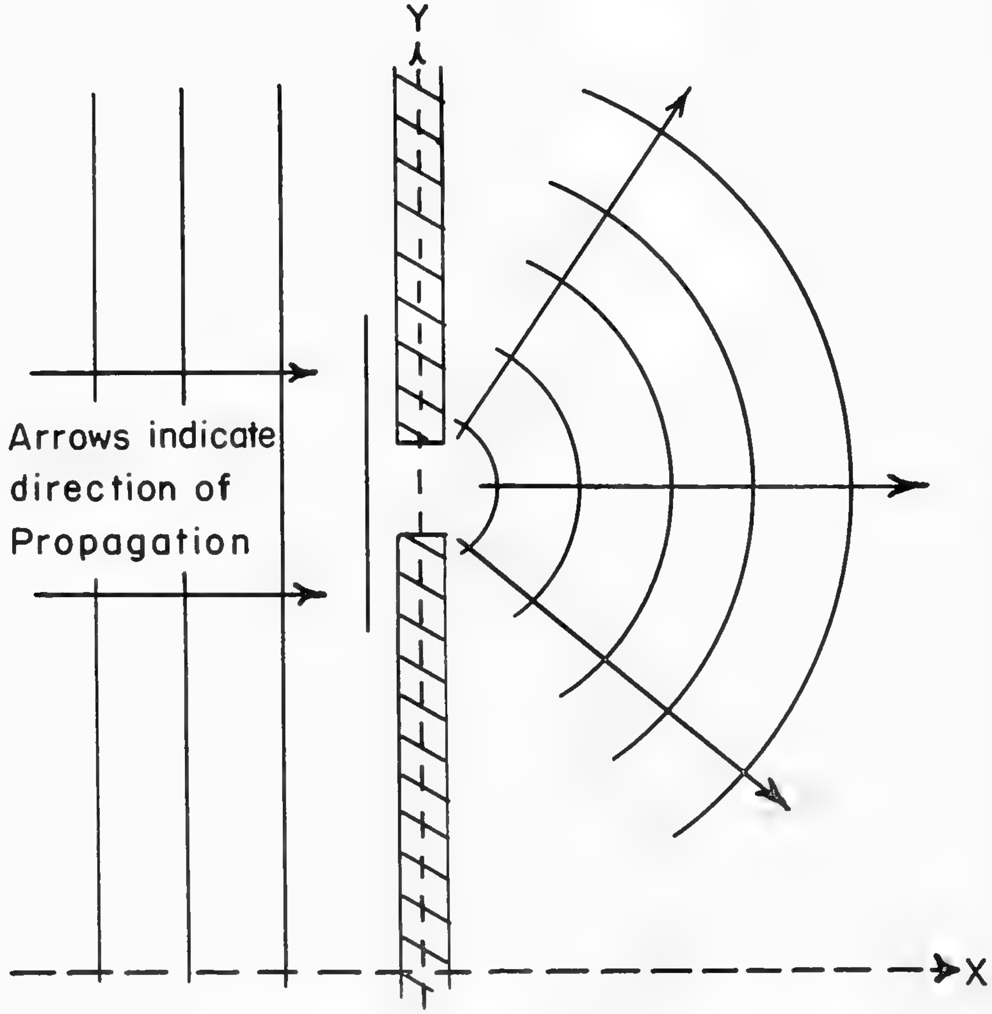

Fig. 20. Diffraction by an infinite slit (sound).

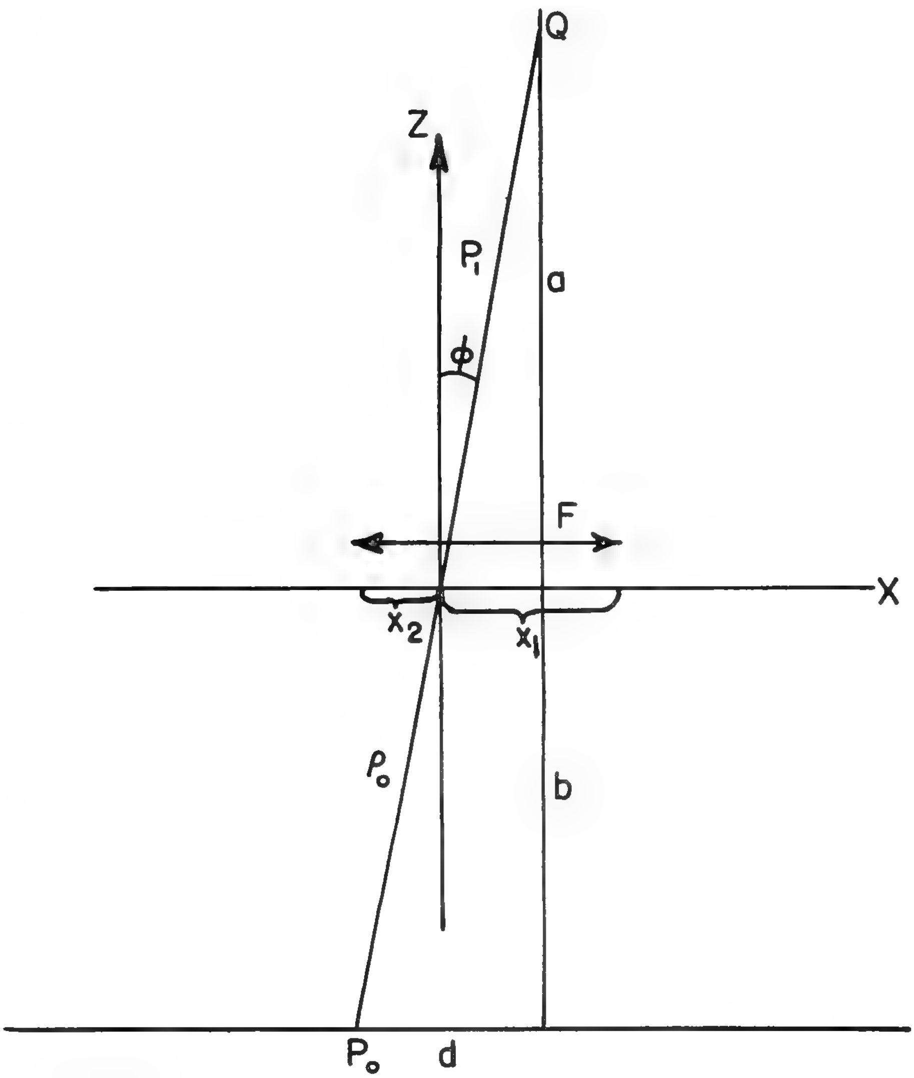

Figure 21 - Diffraction by an infinite slit
-After Drude

$$J = \frac{A^2}{2(\rho_o + \rho_1)^2} (v_1,v_2)^2, \ldots.. \quad (14.1)$$

where

A = amplitude

ρ_o, ρ_1 = distances shown in figure 21

v_1, v_2 = values corresponding to the limits of integration x_1 and x_2.

In order to investigate the distribution of light in a plane which lies a distance, b, behind the screen, it is necessary to take the width of the slit into account. In the case where the width of the slit is small compared to the wave length, the geometrical shadow cannot be even approximately located, for the light is distributed almost evenly over a large region and there is nowhere a sharp shadow formed. In the case where the width of the slit is large compared with the wave length, the effect will be simple diffraction at each of the edges.

In the entrance to harbors or bays, breakwater gaps are frequently encountered which are physically analogous to the problem just discussed. In order to study the effect of ocean waves on the breakwater gap (finite slit), two strips of plastic, bounded by straight edges, were placed in the ripple tank so that they were alined in the same plane. The distance between the two edges was then varied and the phenomenon observed and photographed.

It has been found (Penney and Price, 1944) that in the case of the breakwater gap, the wave pattern is essentially the same whether the barrier is cushioned or rigid. Therefore our rigid barrier will suffice to demonstrate diffraction effects behind the breakwater gap.

When the gap width is less than the wave length, the gap acts as the source of a spherical wavelet and the diffracted waves spread into the geometrical shadow, even though very faintly (figure 22). Upon increasing the gap width, the diffracted waves also extend far into the geometrical shadow, and the effect of diffraction is greater than before, since more energy is admitted by the breakwater gap (fig. 23). However, when the gap is made very large compared with the wave length, the phenomenon becomes an edge effect, so that the part of the wave passing through the center of the gap progresses without being disturbed for a very great distance (fig. 24).

15. Diffraction of light by two noncoplanar parallel straight edges[8]

Heirtzler (1949) used the Kirchoff diffraction formula to determine the intensity of illumination as a function of distance along some plane of observation, for the case of diffraction by two noncoplanar parallel straight edges. The method is essentially the same as that employed by Drude (1922) in the case of a slit.

It is seen from figure 25 that the opening lies between the source Q and the point of observation P. The projection of the line QP on the xz plane coincides with the x-axis and Q'; the projection of Q on the xy plane is located midway between the two edges which have coordinates x' and x". The expression for the intensity of illumination is then found to be

8. Heirtzler, J. R., 1949: Diffraction by two noncoplanar parallel straight edges, American Journal of Physics, v. 17, no. 7, pp. 419-422.

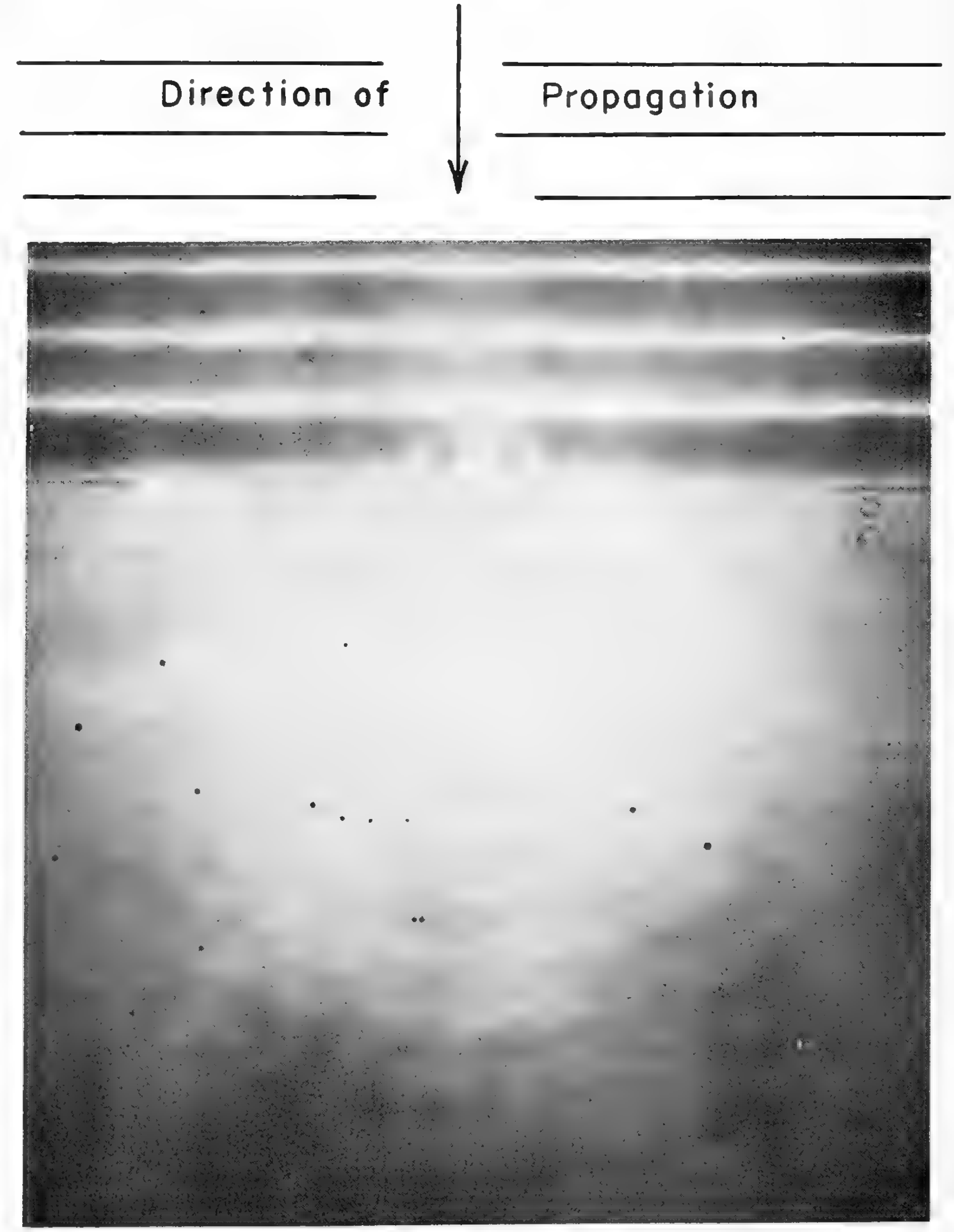

Fig. 22. Photograph showing diffraction by a breakwater. The width of the gap is small compared with the wave length.

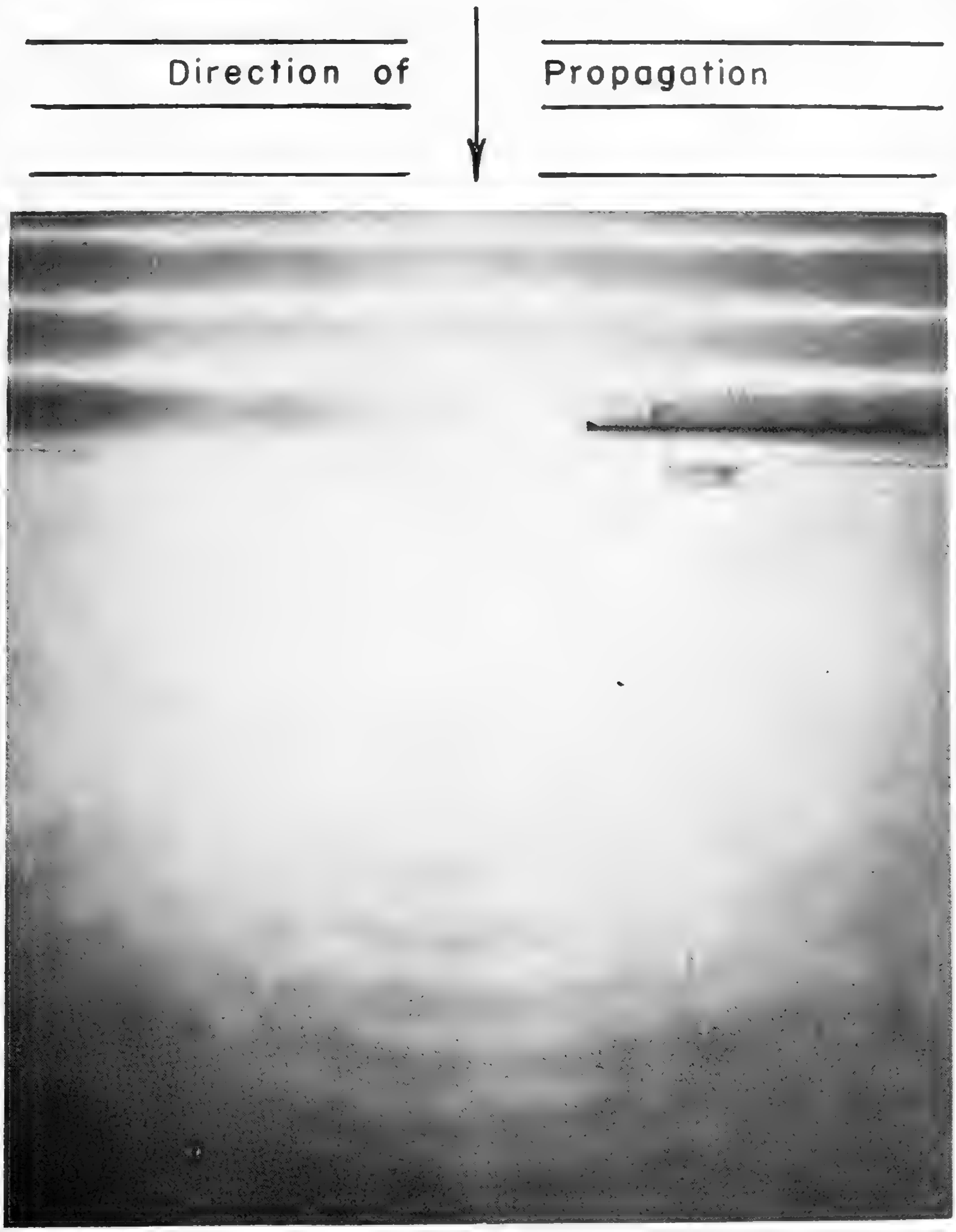

Fig. 23. Photograph showing diffraction by a breakwater gap. The gap is large compared with the wave length.

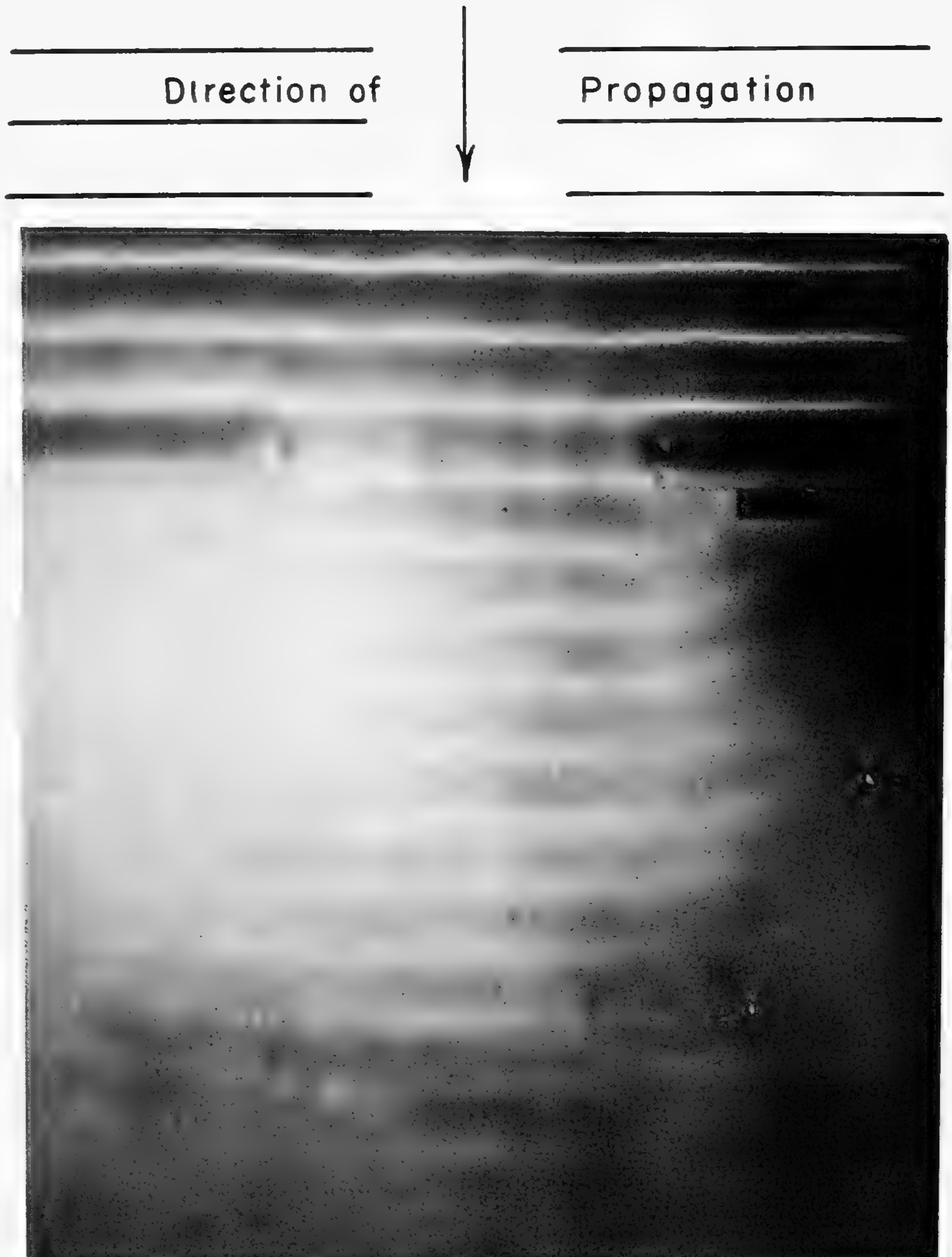

Fig. 24. Photograph showing diffraction by a breakwater gap. The width of the gap is very large compared to the wave length.

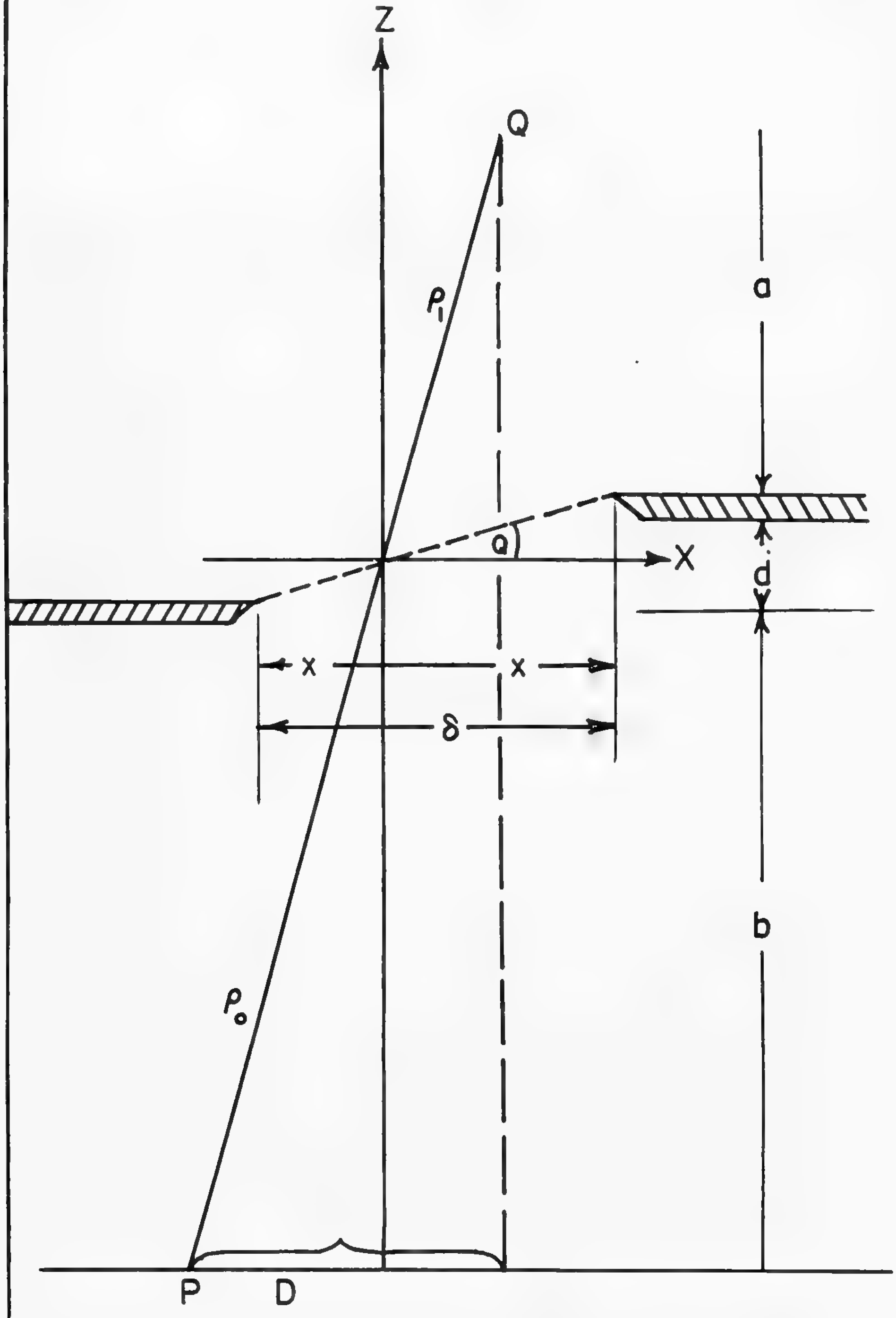

Fig. 25. Arrangement of source, straight edges and plane of observation.
-After Heirtzler

$$J = \frac{A}{2(\rho_o + \rho_1)^2} \left[\left(\int_{v''}^{v'} \cos \frac{\pi v^2}{2} dv \right)^2 + \left(\int_{v''}^{v'} \sin \frac{\pi v^2}{2} dv \right)^2 \right] \quad (15.1)$$

where A is a constant, and

$$v' = x' \cos\varphi \left[\frac{2}{\lambda} \left(\frac{1}{\rho_o} + \frac{1}{\rho_1}\right)\right]^{1/2} E \quad (15.2)$$

$$v'' = x'' \cos\varphi \left[\frac{2}{\lambda} \left(\frac{1}{\rho_o} + \frac{1}{\rho_1}\right)\right]^{1/2} E \quad (15.3)$$

where λ is the wave length of light and

$$E = 1 - \frac{\tan\varphi}{\delta} \quad (15.4)$$

The significance of the symbols ρ_o, ρ_1, a, d and δ is evident from figure 25, and φ is the angle that ρ_1 makes with the z-axis.

Note that the equations (15.1), (15.2) and (15.3) are the same as those used by Drude (1922) except for the factor E in equations (15.2) and (15.3). As d approaches zero, the factor E approaches 1.

Heirtzler obtained a geometrical optics solution for the intensity of illumination along a plane of observation (15.2) and made several graphs of J(intensity of illumination) against D(the distance from the observer to the point on the normal to the plane of observation through the source).

When d = 0, that is the straight edges lie in the same plane, it is seen from Heitzler's graph (fig. 26) that the theoretical diffraction pattern is symmetrical. This agrees with several photographs which have just been shown (figs. 22, 23, 24). As d is made larger, it is seen that the theoretical diffraction curves become more and more dissymmetrical (figs. 27, 28). By

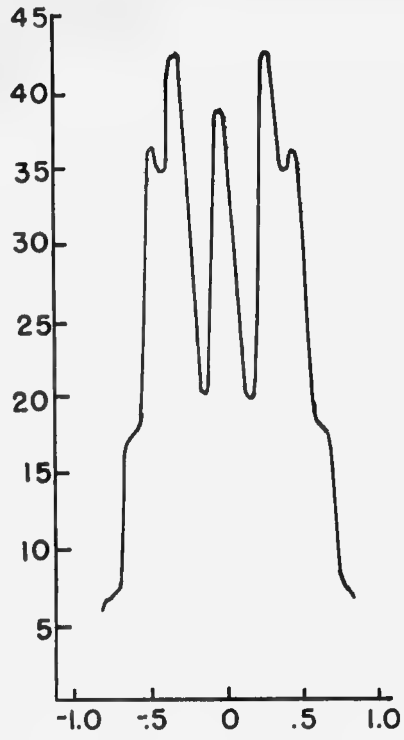

Fig.26 Curve of light intensity for coplanar straight edges. a = 0 δ = 0.48cm.

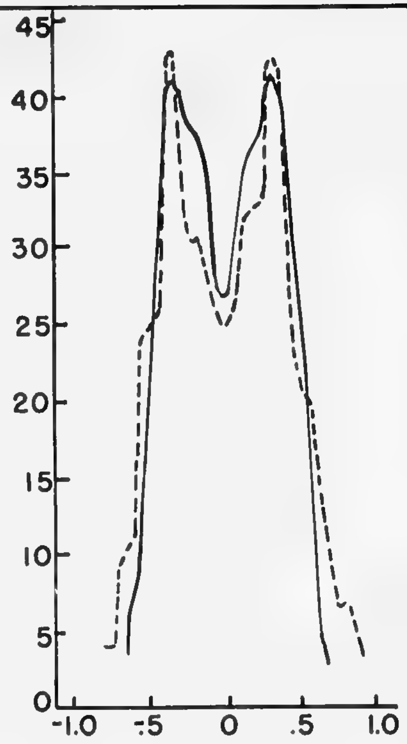

Fig.27 Experimental curve of light intensity (solid line) with corresponding theoretical curve (broken line) for d = 40cm. and δ = 0.44 cm.

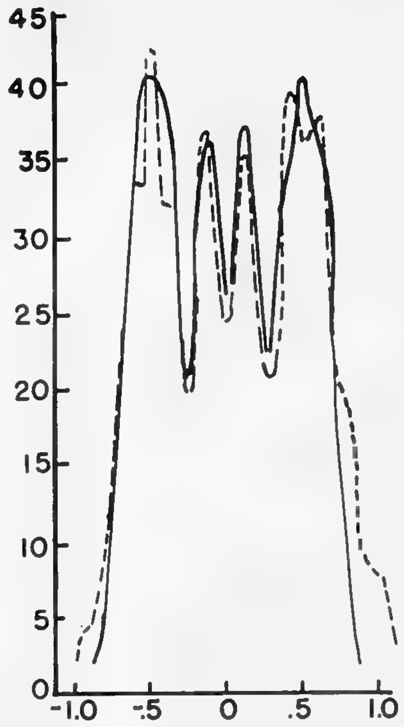

Fig.28 Experimental curve of light intensity (solid line) with corresponding theoretical curve (broken line) for d=80cm δ = 0.56 cm.

—After Heirtzler

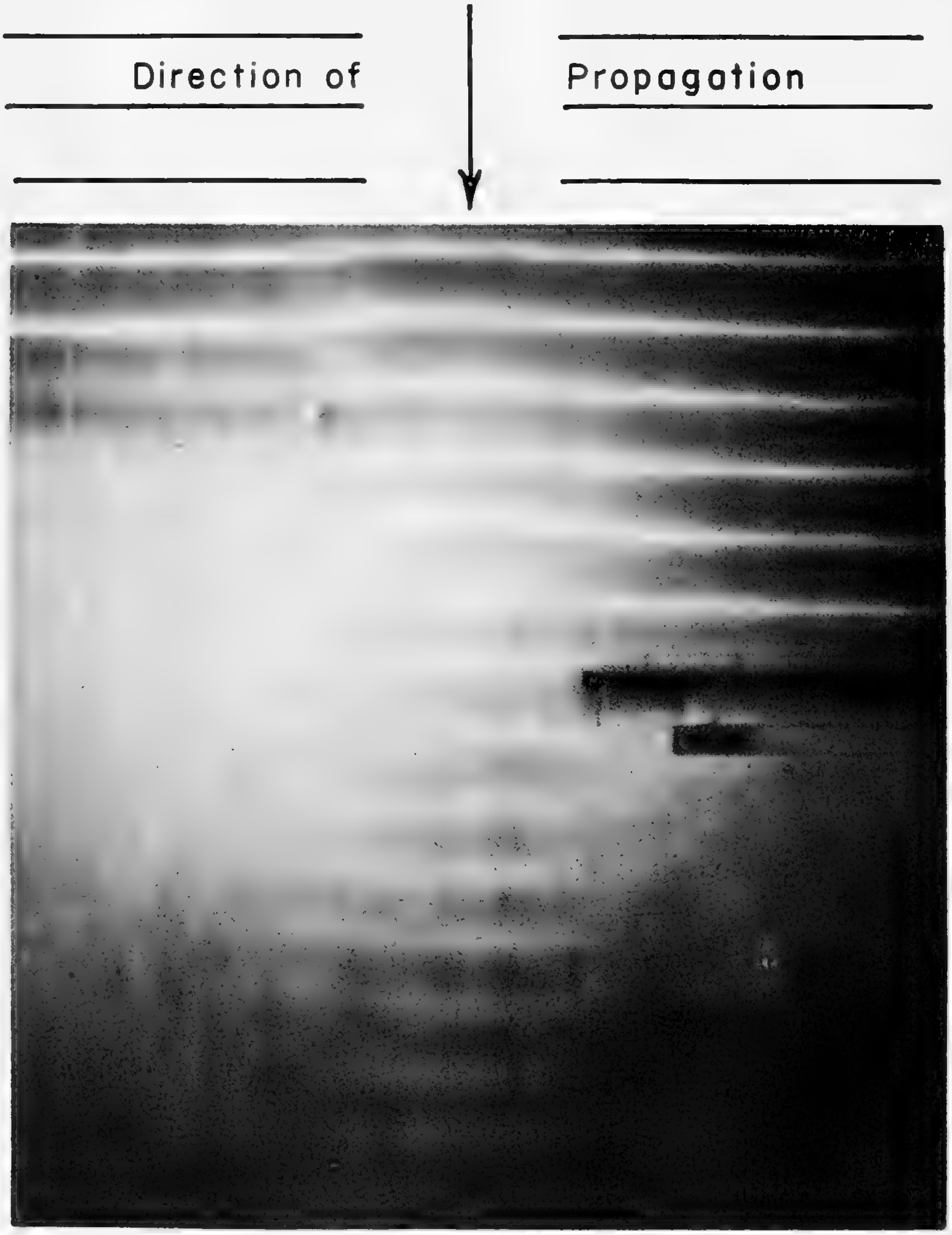

Fig.29. Photograph showing increase in dissymmetry of the diffraction pattern as d is increased.

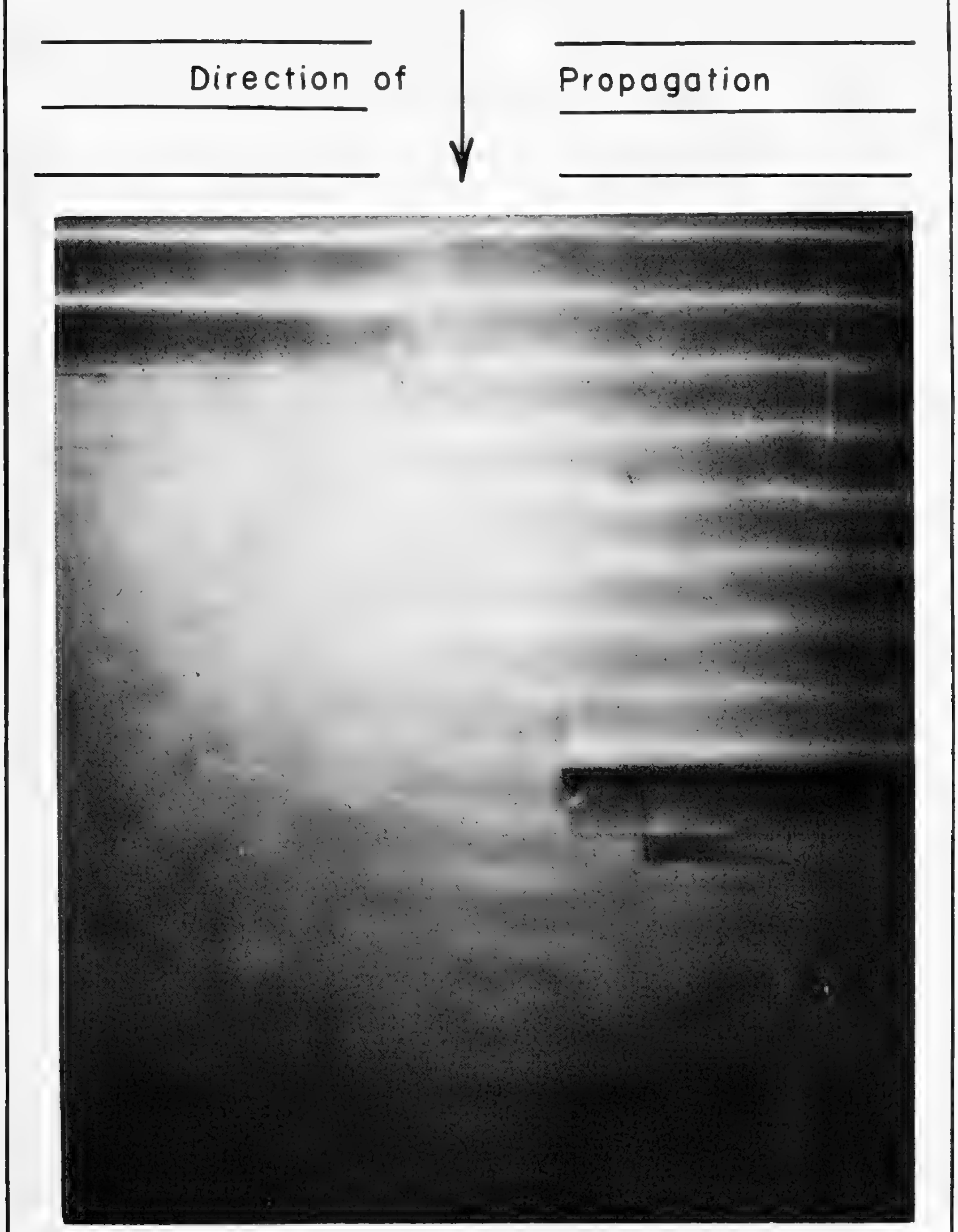

Fig. 30 Photograph showing increase in dissymmetry of the diffraction pattern as d is increased still further.

moving one side of the breakwater gap system in the ripple tank, results were achieved (figs. 29, 30) which bear out the theoretical computations.

Resumé and conclusions

Several relatively simple problems in refraction and diffraction have been covered by this paper. The reason for the relative simplicity of these problems is the very nature of the ripple tank itself, which has already been discussed.

The formulation of the method of solution of problems in refraction of ocean waves by complex bottom topography has been presented by Eckart (1951), but actual formal solutions have not yet been found.

Also of great interest is the theoretical work which has been done in wave motion involving superposition of periods (Pierson, 1951). Since the ripple tank is now only capable of producing waves of one discrete period, it has been necessary to forego discussion of these problems.

However small the scope of this paper, it has been shown that problems in the refraction and diffraction of light waves and sound waves are analogous to problems in the refraction and diffraction of water waves. The analogy has provided a means for testing theoretical results and a way to observe waves in motion.

A more exhaustive search of the literature would undoubtedly yield many more solutions to problems in wave propagation. A catalogue of these problems would make a useful reference for studies in ocean wave refraction and diffraction. Quantitative tests of the solutions to these problems, made in a test basin, such as the coast model test basin of the Beach Erosion Board (1949), would yield much useful information.

Acknowledgments

I wish to express my sincere thanks to Dr. Willard J. Pierson, Jr. of the Department of Meteorology, New York University, who suggested the theme of this paper, offered useful advice during the frequent discussions I had with him, supervised the photography, and aided in correcting the preliminary draft.

I would also like to thank Dr. Bernhard Haurwitz, chairman of the Department of Meteorology, New York University, whose valuable suggestions greatly aided me in preparing the final draft.

My thanks also to the personnel of the New York University Engineering library. They went to a great deal of trouble to secure some of the papers I desired and in general aided me in every way possible.

The ripple tank used in these experiments was built at N. Y. U. for the Beach Erosion Board, under the direction of Dr. Pierson and Dr. Rao.

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