A SURVEY OF THEORETICAL [illegible] OF THE ANTARCTIC CIRCUMPOLAR CURRENT

by

Jeffrey E. Callahan

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of
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Abstract

Five theoretical studies of the Antarctic Circumpolar Current are critically reviewed. The structures of the models, including significant assumptions and approximations, are discussed. Theoretical results are compared with observed features of the Circumpolar Current. Progress in the effort to understand the dynamics of the Current is summarized, and suggestions are made for future work related to this problem.

Acknowledgments

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Introduction

The Antarctic Circumpolar Current, the only ocean current which circles the earth, is the principal agent of water exchange among the world's oceans. Calculations of mass transport through Drake Passage indicate that the Circumpolar Current is also the strongest of the world's ocean currents. In spite of these unique and interesting features, the Circumpolar Current has received relatively little attention from oceanographers, either in theoretical models or in field studies. As a consequence, the dynamics of the Current are not yet clearly understood, and its role in the general circulation of the ocean has not been properly evaluated.

This essay is a survey of theoretical papers dealing with the Circumpolar Current. Contributions by Munk & Palmén (1951), Hidaka & Tsuchiya (1953), Stommel (1957; 1962), and Gill (1968) are critically reviewed. Throughout the study the emphasis is on the physical structure of models. Are the models realistic analytical representations of natural conditions? Are the assumptions and approximations reasonable? How meaningful are the results?.

Two reasons may be given for conducting this survey. One is to find out how much has been learned about the dynamics of the Circumpolar Current: what dynamical features have been revealed by past models? The complementary purpose is to identify unanswered questions concerning the Current, that is, to suggest directions for future work.

Descriptive Features of the Circumpolar Current

In order to provide a basis for evaluating the models to be studied, a summary of major features of the Southern Ocean and of the Circumpolar Current will first be given. Much of the information contained in this section has been taken from the descriptive accounts by Deacon (1937a; 1963) and Sverdrup et al (1942, Chapter XV).

Southern Ocean is the name given to the great body of water which surrounds the Antarctic continent. To the north it merges with the Atlantic, Pacific, and Indian oceans. The absence of natural boundaries makes it difficult to delineate the northern limit; 40°S may be used as an arbitrary boundary.

With the exception of regions where major submarine ridges are found, average depth is about 4000 m. The greatest depression is the South Sandwich Trench with a maximum sounding of roughly 8300 m.

During part of the year a large portion of the Southern Ocean is covered with ice. In October, at the end of the austral winter, pack ice extends to 55°S - 57°S everywhere except in the Pacific, where it extends only to 63°S. By the end of summer (March) the edge of the pack has retreated almost to the Antarctic coast (Mackintosh and Herdman, 1940).

Knowledge of the wind field over the Southern Ocean is somewhat limited. Except in the Drake Passage area, almost all data come from ships. Ship data are variable in quality and uneven in time/space distribution. The available data indicate that the mean wind field is characterized by strong westerlies between about 40°S and 60°S, with weaker and more variable

easterlies south of that latitude (von Arx, 1957; Vowinckel, 1957). The westerlies do exhibit polar asymmetry with respect to speed, as is shown in figure 1.

This essay is concerned primarily with the segment of the Southern Ocean known as the Circumpolar Current. Here, too, a certain amount of arbitrariness is required to define the subject. The interior boundary falls at about 60°S in the Atlantic and Indian sectors, somewhat further south in the Pacific. A countercurrent, driven by prevailing easterlies, flows between the Circumpolar Current and the Antarctic continent.

Surface current charts depict a general eastward motion, known as the West Wind Drift, from 60°S to roughly 40°S. However, the true Circumpolar Current covers only part of this zone. Approximate limits are shown in figure 2, in which transport relative to the 3000-decibar level is plotted. The center of the Current, say the transport line marked "2", is found at 50°S in the Atlantic and Indian Ocean sectors, but it swings south to 60°S in the Pacific. Large-scale meanders may be observed in several places.

Several oceanographic expeditions, notably the Discovery Investigations (Great Britain), Ob cruises (U.S.S.R.), and Eltanin cruises (U.S.A.), have made observations in the Southern Ocean. Climatic conditions make work in this area difficult, and data is sparse over large sections of the Ocean. As a result, water motions within the Circumpolar Current can be described only in broad outline.

The mean velocity field at the naviface has been deduced in large part from ship drift reports. Taljaard (1957) plotted

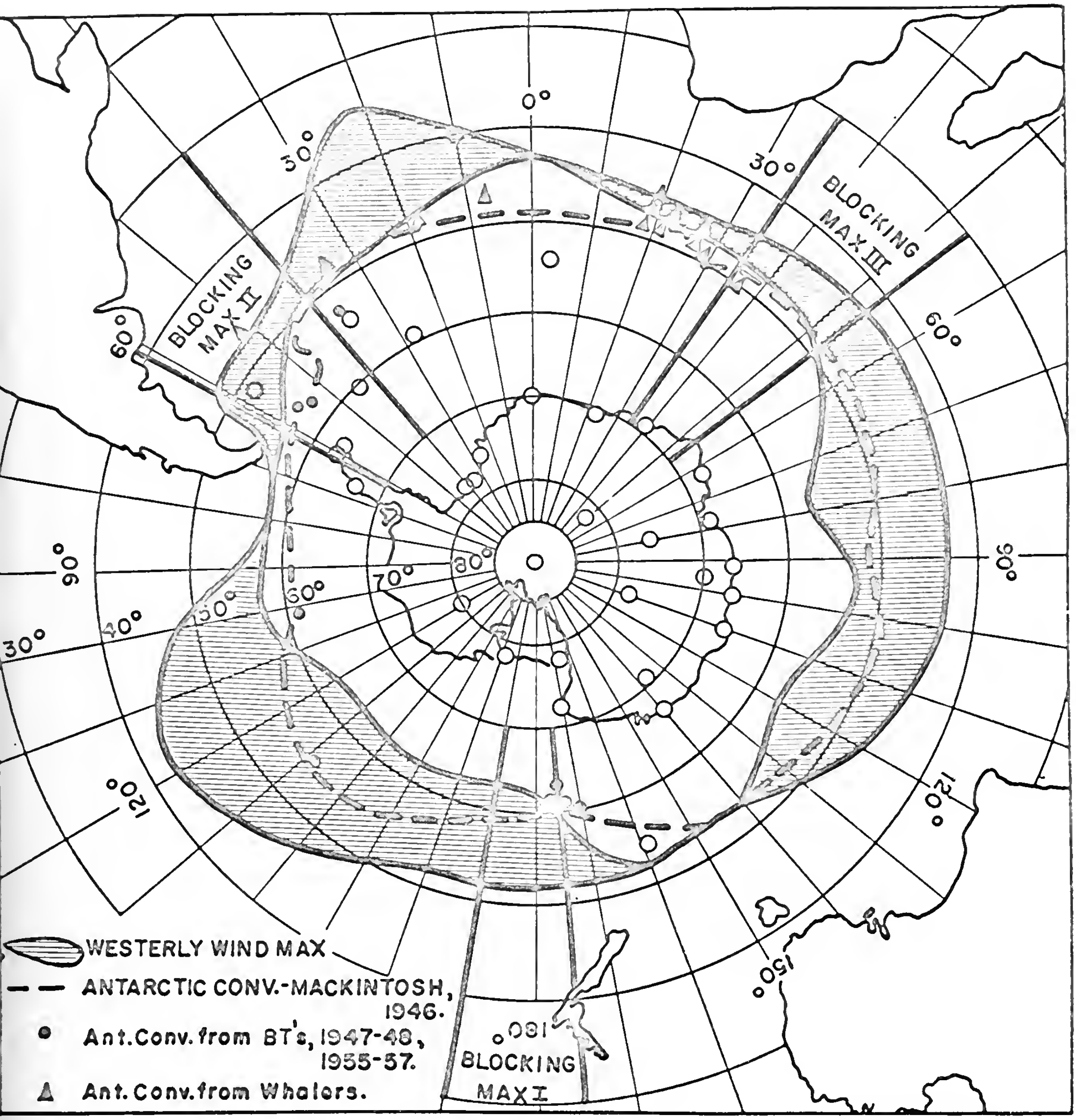

Figure 1. Zone of maximum westerly winds over the Southern Ocean, computed from sea-level barometric pressure data. (From Wexler, 1959.)

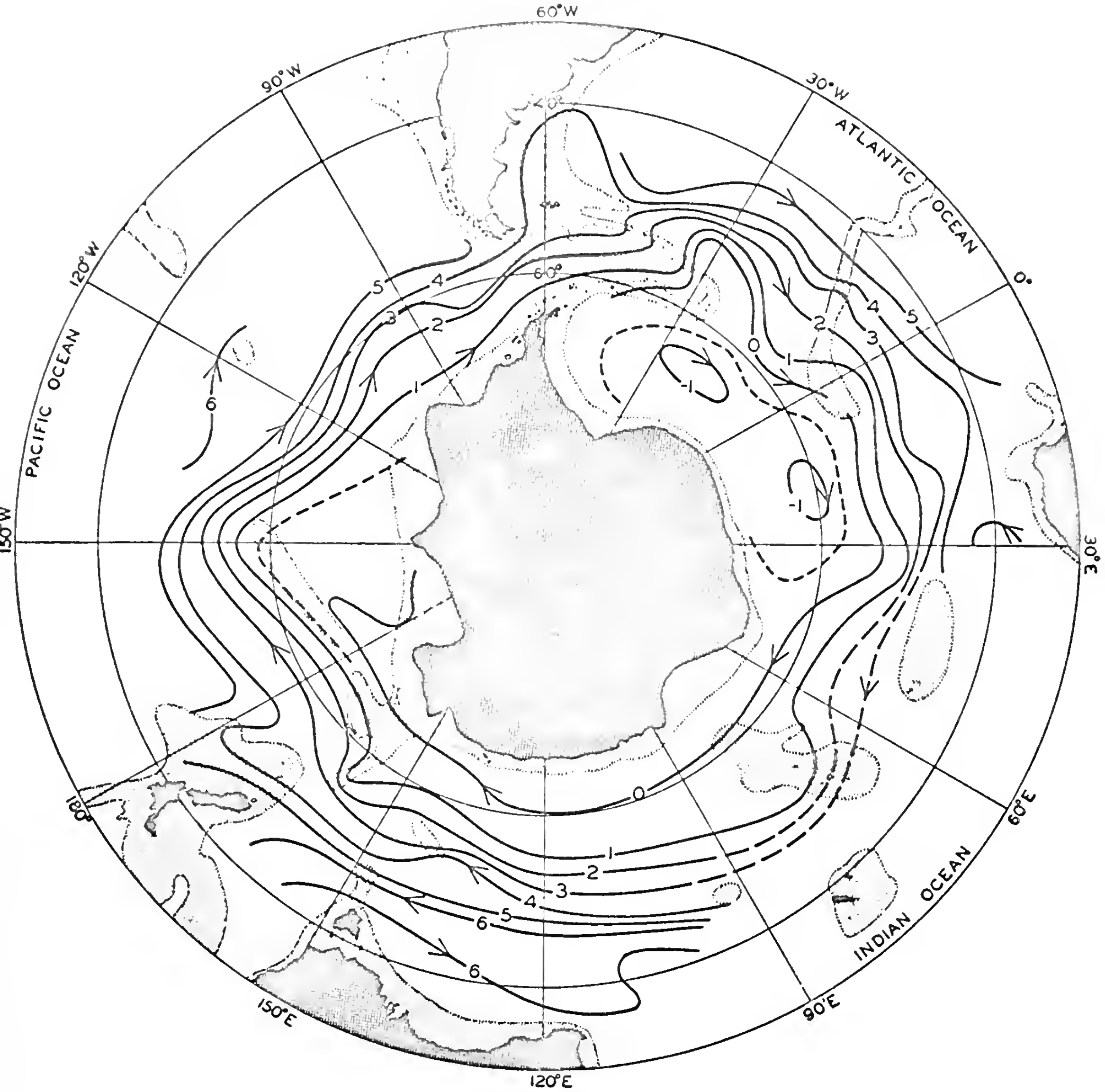

Figure 2. Computed mass transport of the Circumpolar Current relative to the 3000-decibar surface. Transport between two lines is about 20 x 10^{12} g sec^{-1}. Light shading covers areas with depth less than 3000 m. (From Sverdrup et al, 1942, p. 615.)

surface currents around Antarctica using data from several sources, including the British Admiralty "Antarctic Pilot" and U.S. Naval Hydrographic Office "Sailing Directions for Antarctica". If the area near Drake Passage (where currents are sometimes greater than 1 knot) is excluded, the drift is generally toward the east at 0.2 to 0.6 knots (12 to 36 cm sec^{-1}). Deacon (1963) states that the average surface current in the West Wind Drift is 8 miles per day (15-17 cm sec^{-1}). This figure is based on drift bottle measurements.

Geostrophic computations based on hydrographic data provide an indication of subsurface velocities in the Current. An example for a section between Antarctica and Australia is shown in figure 3. Sections such as these give the impression that the Circumpolar Current is significantly deeper than most wind-driven currents. It must be emphasized that geostrophic profiles depend heavily on choice of reference level, and any uncertainty associated with this choice introduces uncertainty in the results.

The importance of the reference level chosen for geostrophic computations is illustrated in a summary of Drake Passage mass transport calculations published by Gordon (1967). Gordon includes the work of seven previous authors and his own transport figures, too. The calculated transports range from 85 to 218 x 10^{12}g sec^{-1}, discounting Ostapoff's extremely low values. The lower values are those computed using the 3000-decibar surface as a reference level, while the higher values are generally those based on Defant's method for determining the level of no motion.

Gordon himself estimates the transport at 218 x 10^{12}g sec^{-1} using a set of seven Ob stations evenly spaced across the Passage.

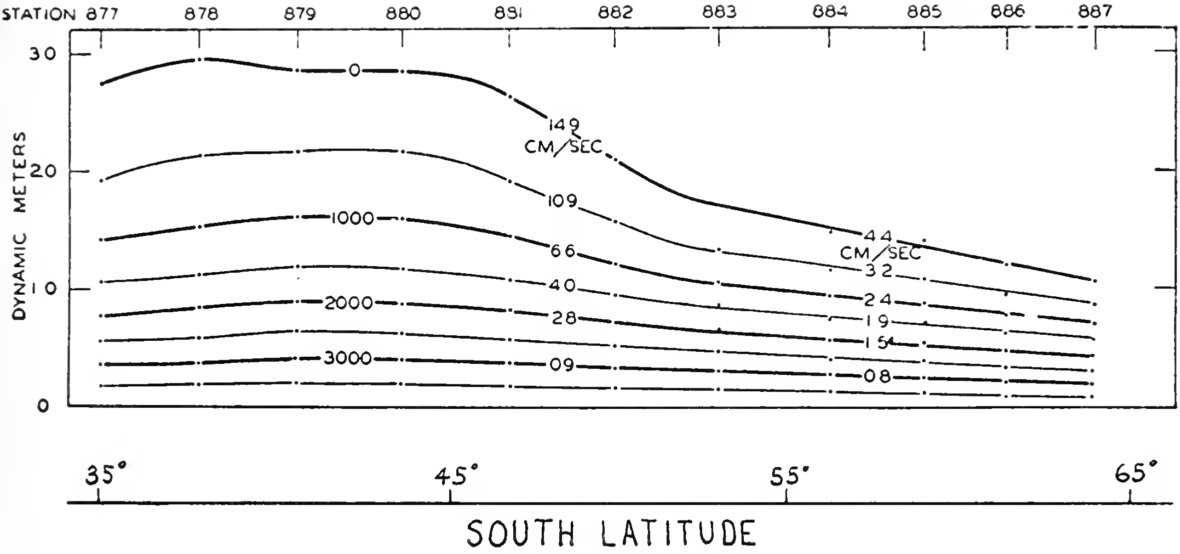

Figure 3. Horizontal geostrophic velocity, relative to 4000 decibars, normal to a section from Cape Leeuwin, Australia, to the Antarctic continent. (From Sverdrup et al, 1942, p. 614.)

He obtains a reference level using the "equivalent-baratropic assumption," namely, that the mean density between the reference level and the naviface is constant. Starting from an assumed level of no motion in the southern end of Drake Passage, he finds that the reference level slopes down sharply toward the north. In fact, the reference level lies below the ocean floor in the northern two thirds of the Passage.

If Ostapoff's calculations are excluded, only two of the remaining nine listed by Gordon fall below $100 \times 10^{12} g\ sec^{-1}$. Thus, although there is a considerable spread in computed mass transport values, it is probable that the transport of the Circumpolar Current is greater than $100 \times 10^{12} g\ sec^{-1}$.

Geostrophic mass transport calculations made for several sections across the Current indicate a large longitudinal variation in transport. This variation is evident in figure 2 and also in the following transport estimates by Kort (1962):

Section	Transport ($10^{12} g\ sec^{-1}$)
Drake Passage	150
Antarctica - South Africa	190
Antarctica - Tasmania	180

While zonal flow is the most conspicuous mean water motion (at least at the naviface), it is not the only significant one taking place. Analysis of water types and the distributions of temperature, salinity, and oxygen content indicate the presence of a well developed meridional circulation. Estimates of meridional transport are even rougher than those of zonal

transport, but they show that the former may be significant, at least in certain parts of the Southern Ocean. In the Atlantic, for example, Sverdrup et al (1942, p 629) estimate from Meteor data that 35 x 10^{12} g sec^{-1} flows south across 30°S, balanced by an equal flow to the north. Note that meridional transport of this order would be required to account for the longitudinal variations in zonal transport found by Kort.

Deacon's (1937a) interpretation of the meridional circulation is given in figure 4. South of the 50th parallel it is basically a three-layer system. Between about 200 m and 1500 m there is a southward flowing deep current of relatively warm (~ 2C), saline (~ 34.7‰) water. The major source of this deep water is thought to be the area southeast of Greenland, where surface water cools during winter, sinks, and spreads south. The supply of deep water is augmented by mixing with intermediate and bottom water. Deep water is also called circumpolar water because of its uniform distribution around the continent (Sverdrup et al, 1942, p 607).

Wedging beneath the deep layer is Antarctic bottom water, which is nearly as saline as, and 1 C to 2 C colder than, deep water. Bottom water is probably formed from a mixture of deep water and extremely cold surface water sliding down the Antarctic continental shelf, mostly in the Weddell Sea (Fofonoff, 1956). From there bottom water spreads north along the western trough of the Atlantic Ocean, where it can be detected well into the North Atlantic, and around the Antarctic continent. The rate of formation of bottom water is probably greatest during winter.

The upper few hundred meters of the Antarctic zone consist of cold, poorly saline surface water. This layer is influenced

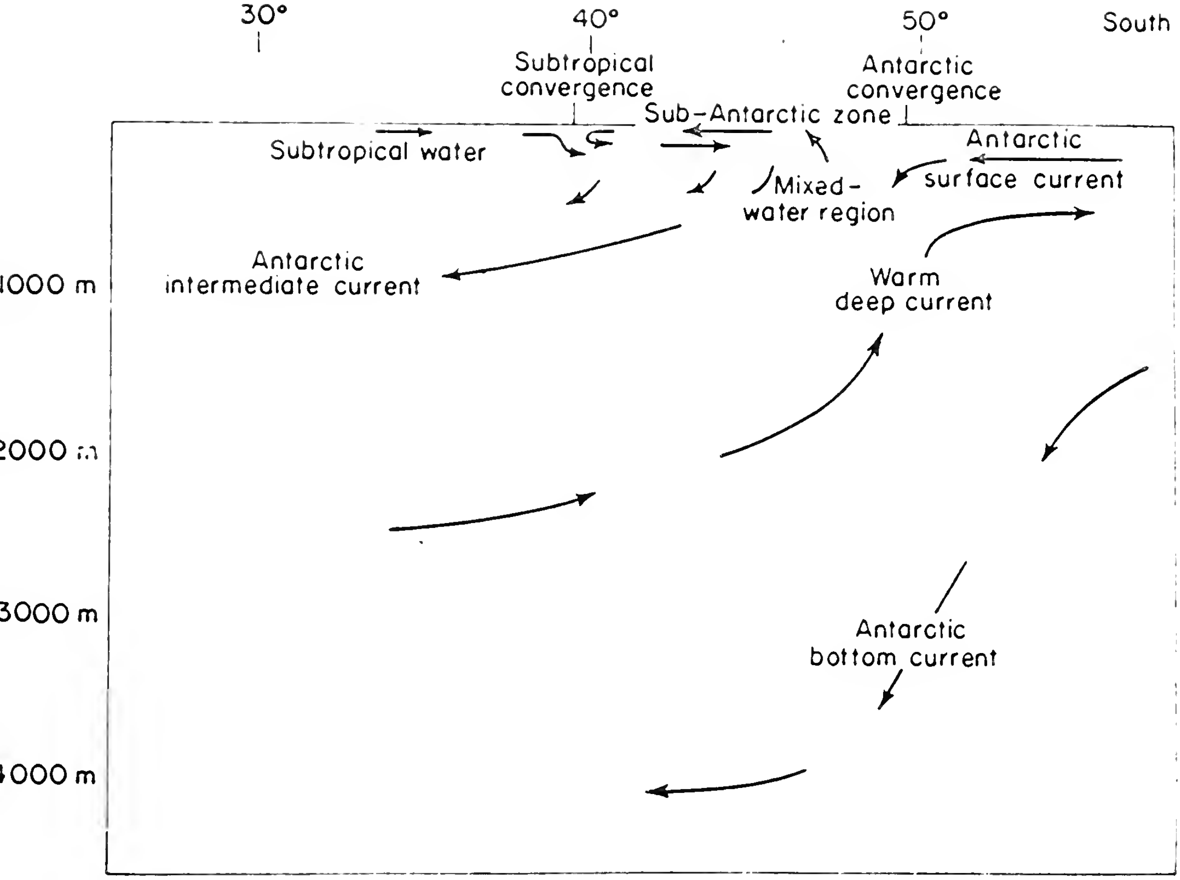

Figure 4. Schematic representation of circulation in a meridional section across the Southern Ocean. (From Deacon, 1963.)

strongly by seasonal fluctuations in air temperature and ice conditions. It is uniform in winter, but patches of water with anomalous properties are found there in summer.

Before concluding this section, mention will be made of the prominent feature known as the Antarctic polar front or Antarctic convergence. The front has been observed in all sectors of the Southern Ocean. At the naviface, the polar front is manifested by a relatively sharp north-south gradient in water temperature, occasionally as much as 2 C in five miles. A few authors, for example, Wexler (1959) have attempted to identify the front on the basis of subsurface features in the water column, but these methods have not met with general acceptance. According to Mackintosh (1946) the polar front is found near 50°S in the Atlantic and Indian sectors, while in the Pacific it lies closer to 60°S. Mackintosh found that the mean monthly position of the front varies only slightly with season. Somewhat larger variations with time scales of several days do occur.

Channel Flow Models

Among the earliest theoretical discussions of the Antarctic Circumpolar Current are those of Munk & Palmén (1951) and Hidaka & Tsuchiya (1953). These papers differ little in their fundamental concept of the current as an axi-symmetric channel flow. Wind stress is balanced primarily by frictional forces. It is found that this simple balance will not yield realistic transport figures unless unusually large values are taken for the eddy viscosity coefficients.

Munk & Palmén start with a simple analytical model. The equations are written in cylindrical coordinates (see Appendix).

It is assumed that the current is a steady, purely zonal flow of uniform depth H. Nonlinear terms are neglected. Under these restrictions only the tangential component of the horizontal equations of motion remains,

$$(1) \qquad A_h \left(\nabla_h^2 u - \frac{u}{r^2} \right) + A_v \frac{\partial^2 u}{\partial z^2} = 0.$$

If (1) is integrated from the bottom to the naviface and if bottom friction is neglected, then

$$A_h \left(\nabla_h^2 M - \frac{M}{r^2} \right) + \tau = 0,$$

where M = net zonal mass transport (per unit width)

$$\equiv \int_{-H}^{0} \rho u \, dz$$

and τ = zonal wind stress.

Because of the axial symmetry,

$$\nabla_h^2 = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right).$$

Thus,

$$\tau + A_h \left(\frac{d^2 M}{dr^2} + \frac{1}{r} \frac{dM}{dr} - \frac{M}{r^2} \right) = 0$$

or

$$(2) \qquad \tau + A_h \frac{d}{dr} \left(\frac{dM}{dr} + \frac{M}{r} \right) = 0.$$

Boundary conditions are M = 0 (i) at the coast of Antarctica $(r = r_0)$ and (ii) at some other latitude circle to the north $(r = r_1)$. The appropriate solution of (2) is

$$M = \frac{\tau}{3 A_h} r (r - r_0) \left(\frac{r + r_0}{r_1 + r_0} \cdot \frac{r_1^2}{r^2} - 1 \right).$$

Integrating M from r_0 to r_1 gives the net zonal transport,

$$T = \frac{\tau}{18A_h}\left(r_1^3 - r_0^3 - \frac{6r_1^2 r_0^2}{r_1 + r_0} \ln \frac{r_1}{r_0}\right).$$

In order to make a quantitative estimate of the transport Munk & Palmen set $\tau = 2$ dynes cm^{-2}, $A_h = 10^8$ cm^2 sec^{-1}, and place the boundaries at latitudes 70°S and 45°S. The computed transport is over 10^{16} g sec^{-1}. If the boundaries are moved to 65°S and 55°S, the approximate limits of the Drake Passage, computed transport is still more than 10^{15} g sec^{-1}.

It was shown earlier that the geostrophic transport through Drake Passage is between 10^{14} g sec^{-1} and 2×10^{14} g sec^{-1}. Thus the theoretical transport is at least an order of magnitude greater than that computed from hydrographic data. It is unlikely that the computed value is in error by an order of magnitude. Munk & Palmén offer two ways to bring the theoretical result into line with the observed.

The first and most direct way is to increase the eddy coefficient A_h by one or more orders of magnitude. However, other studies of large-scale oceanic flows indicate that A_h ranges between 10^7 and 10^8. There is no reason to suppose that it should be considerably larger in this region.

Instead, Munk & Palmén favor introducing bottom stress. This source of retarding action was explicitly neglected in the original model.

Bottom stress could manifest itself as "skin friction." Sverdrup et al (1942, p 479) give the following formula, based on Prandtl's mixing length theory, for the mean horizontal velocity

within the turbulent boundary layer over a rough bottom:

$$u_b = 2.5\sqrt{\frac{\tau}{\rho}}\ \ln\left(\frac{z + z_0}{z_0}\right).$$

The roughness length z_0 is given as 2 cm from measurements made by Revelle and Fleming in San Diego Harbor*. If the wind stress of 2 dyne cm^{-2} were balanced entirely by bottom friction, u_b would be 14 cm sec^{-1} only 1 m above the sea floor. Since this is approximately equal to the maximum surface velocities of the Circumpolar Current, Munk & Palmén consider skin friction an unlikely retarding mechanism.

Alternatively, bottom stress might be caused by the so-called mountain effect. This term refers to the retarding effect of a pressure drop across a submerged barrier, e.g., a submarine ridge, imbedded in the flow. Assume there is a pressure difference ΔP_b across each ridge over which the current flows. Then the average bottom stress is

$$\tau_b = \frac{1}{C}\,\Delta P_b\,\Delta h\,,$$

where C = distance around a latitude circle

and Δh = sum of heights of ridges.

The Circumpolar Current crosses four major ridge systems in its path around the Antarctic continent. East of Drake Passage lies the Scotia Ridge, including the South Sandwich and South Orkney island groups, having a height of 4 km; at 75°E is the Kerguelen-Gaussberg Ridge, h = 3 km; at 165°E is the Macquarie Ridge, h = 2 km; and at 150°W is the Pacific-Antarctic ridge,

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*One must wonder if a roughness parameter measured at the bottom of San Diego Harbor is appropriate to the bottom of the Southern Ocean.

h = 1 km. The combined height of the ridges is 10 km; at 60°S C = 18,000 km. Setting $\tau_b = 2$ dyne cm^{-2}, Munk & Palmén find $\Delta p_b = 4000$ dyne cm^{-2}, which is equivalent to 4 dynamic centimeters. Deacon (1937b) calculated cross-current dynamic height differences on the order of 1 m at the surface of the Southern Ocean. Pressure gradients over ridges amounting to just a few percent of those found at the surface would be sufficient to balance the wind stress.

Use of bottom stress, whatever the mechanism, requires that the Current penetrate to the sea floor over at least part of its path. What reason is there to expect the Circumpolar Current to be unusually deep? Munk & Palmén reject the possibility that momentum is transmitted from the surface to extreme depths by vertical turbulent exchange. Such a mechanism would imply a vertical eddy coefficient greater than 10^4 cm^2 sec^{-1}, which is one to two orders of magnitude larger than commonly used values.

A different mechanism is revealed by considering the balance of angular momentum in the current. Under steady conditions the absolute angular momentum of the Current about the earth's axis is constant, and any changes in angular momentum caused by stresses or transport phenomena must cancel.

Somewhat artificially Munk & Palmén divide the meridional circulation into two layers. They suppose that angular momentum is exported in the top layer while it is imported to the Current in the bottom layer. Next they hypothesize that all the angular momentum produced by the wind torque over the Current is advected across the northern boundary of the Current in the upper layer. If the northward mass flux is Q then

$$Q(R\cos 45^\circ)^2\Omega = \int_{45^\circ S}^{70^\circ S} \tau(2\pi R\cos\phi)(R\cos\phi)\,R\,d\phi,$$

where Ω = earth's angular velocity

R = earth's radius

and ϕ = latitude.

The flux Q is found to be about 30 x 10^{12} g $\sec^{-1}$, roughly the amount of water transported north in the Peru and Benguela currents.

Water to replace that lost in the upper layer comes from three sources: precipitation, runoff from the Antarctic continent, and southward transport of deep water. Precise data measuring the first two sources are not available, but rough estimates show that they provide at most a few percent of the required amount. Virtually all the water lost in the upper layer is replaced by water from the lower layer.

Southward flowing deep water comes from a region of higher absolute angular momentum than water in the Circumpolar Current. In order to preserve that angular momentum it must develop an eastward drift as it moves toward the pole. The momentum excess of deep water relative to "local" water is balanced by the loss of angular momentum associated with the mountain effect. Thus the total angular momentum of the Current is conserved and, at the same time, the entire water column acquires an eastward drift.*

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*This argument has a certain qualitative appeal, but it also has at least one serious quantitative defect. The eastward velocity which a particle of water must acquire in order to compensate for the loss of angular momentum as it moves south is much larger than the maximum velocities in the Current. For example, a particle moved without friction from 45°S to 46°S would develop an eastward velocity of 800 cm $\sec^{-1}$ relative to the earth.

Like Munk & Palmén, Hidaka & Tsuchiya (1953) model the Circumpolar Current as a steady flow between solid boundaries at 45°S and 70°S. Density is assumed constant, and nonlinear terms are neglected. Hidaka & Tsuchiya write their equations in terms of the horizontal velocity components instead of mass transport.

With the x, y, z-axes positive eastward, northward, and vertically downward, the horizontal equations of motion are

$$A_v \frac{\partial^2 u}{\partial z^2} + A_h \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + f v = \frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$A_v \frac{\partial^2 v}{\partial z^2} + A_h \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - f u = \frac{1}{\rho} \frac{\partial p}{\partial y} ,$$

where u, v are the x, y-components of velocity

and f = Coriolis parameter = 2Ω sin latitude.

It is postulated that a uniform wind is blowing in the x-direction only. Therefore u, v, p, and the surface elevation ζ are independent of x. Using the hydrostatic equation and letting W = u + iv, the two equations above may be written as

$$A_v \frac{\partial^2 W}{\partial z^2} + A_h \frac{\partial^2 W}{\partial y^2} - i f W = i g \frac{\partial \zeta}{\partial y} . \tag{3}$$

Continuity is

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

where w = vertical velocity component.

Boundary conditions are

$$W = 0 \quad \begin{cases} \text{at } z = H \text{ (the bottom)} \\ \text{at } y = \pm \ell \ (45°\text{S}, \ 70°\text{S}) \end{cases}$$

and $$A_v \frac{\partial W}{\partial z} + \tau = 0 \qquad \text{at } z = 0 \text{ (naviface).}$$

Hidaka & Tsuchiya assume a Fourier series solution of W,

$$(4) \qquad W(y,z) = \sum_{s=1}^{\infty} \omega_s(y) \cos \frac{(2s-1)\pi z}{2H} ,$$

where $$\omega_s(y) = \frac{2}{H} \int_0^H W(\lambda) \cos \frac{(2s-1)\pi\lambda}{2H} \, d\lambda .$$

By substituting (4) in (3) and expressing $\omega_s(y)$, $\zeta(y)$ and $\tau(y)$ also as Fourier series, it is found that

$$W(y,z) = \sum_{m,s} D_{ms} \sin \frac{m\pi(\ell+y)}{2\ell} \cos \frac{(2s-1)\pi z}{2H} .$$

The quantity D_{ms} is a complex coefficient whose value depends on A_v, A_h, and τ_m, the Fourier coefficients of $\tau(y)$.

Having derived expressions for the velocity field and surface elevation, Hidaka & Tsuchiya compute u, v, ζ, and the total mass transport T. Water depth is set equal to 4 km, and wind stress is set equal to 2 dyne cm^{-2}. The unknowns are evaluated for two values of A_h: 10^8 cm^2 sec^{-1} and 10^{10} cm^2 sec^{-1}. In both cases $A_v = 2 \times 10^3$ cm^2 sec^{-1}. Results are given in table 1.

Table 1

Numerical results from Hidaka & Tsuchiya (1953)

A_h ($cm^2\ sec^{-1}$)	u_{max} ($cm\ sec^{-1}$)	v_{max} ($cm\ sec^{-1}$)	T ($g\ sec^{-1}$)	$\Delta\zeta$ across Current (m)
10^8	100	3	8.1×10^{15}	25
10^{10}	14	2	9.3×10^{14}	3

Values computed with $A_h = 10^{10}\ cm^2\ sec^{-1}$ are more realistic than those with $A_h = 10^8\ cm^2\ sec^{-1}$. Transport and surface slope are rather large, but the maximum velocities are comparable to observed values. Vertical plots of the velocity components reveal that the meridional component has a nonzero value only in the upper few hundred meters. The zonal component stays near its maximum value from the naviface almost to the bottom. At the naviface the velocity has a parabolic distribution between the latitudinal boundaries, with a maximum at 58°S.

Hidaka & Tsuchiya conclude that the gross features of the Circumpolar Current can be explained without including submarine topography effects if unusually high levels of lateral turbulence exist in the Southern Ocean.

Asymmetric Models

Henry Stommel was the first to suggest that the Circumpolar Current can not be treated analytically as a zonal, axisymmetric flow.* Stommel (1957) observes, " . . . if one plots the minimum depth for each complete latitude circle, it is found that the latitude circles that pass through Drake Passage are blocked by the island arc somewhat to the east . . . . It is seen that nowhere in the Antarctic Water-ring is there a latitude with a deeper threshold than 1000 m. The Antarctic Circumpolar Current therefore cannot be purely zonal."

Stommel concludes that, because of the partial barrier across the Passage, the Southern Ocean is more nearly an enclosed basin than a uniform channel. With this hypothesis it is reasonable to treat the Current in the manner of Sverdrup (1947). That is, over most of its extent only wind stress, Coriolis, and pressure-gradient terms play an important part in the equations of motion. Near Drake Passage, Stommel expects boundary currents similar to those found along meridional boundaries in other ocean current systems, and he predicts that any instability or higher-order processes associated with the Current occur here.

In this paper Stommel does not develop an analytical model to test his notions about the dynamics of the Current. He only presents a qualitative interpretation of how such a current might evolve. Several schematic representations taken from his paper are shown in figure 5.

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*Sverdrup <u>et al</u> (1942) did note that the Current " . . . is <u>locally</u> deflected from its course, partly by the distribution of land and sea and partly by the submarine topography." (p 615)

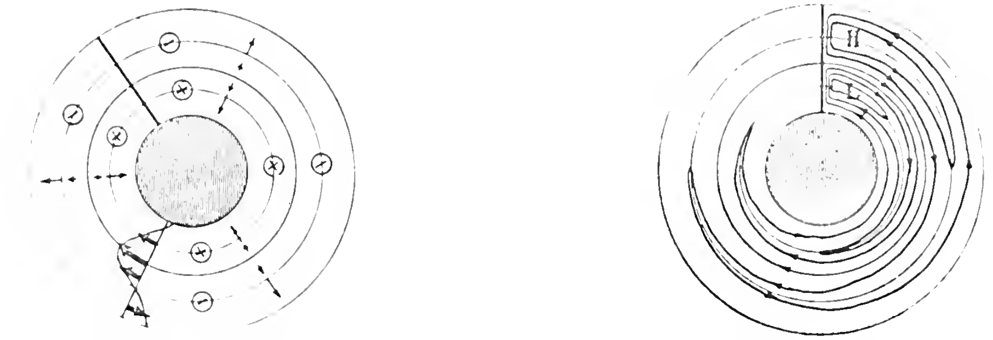


Fig. (a). The schematic Southern Ocean. Antarctica is the solid black circle. The meridional barrier extending northward from Antarctica is represented by the solid heavy black vertical line. The schematic wind system (purely zonal) is depicted by the heavy arrows on the lower left. The concentric circles are latitude circles. Latitudes of Ekman convergence and sinking at the surface are indicated by minus signs, latitudes of Ekman divergence and upwelling are indicated by plus signs. The direction of the required meridional geostrophic flow is indicated by light radial arrows.

(b). Transport lines of the solution for the model depicted in Fig. (a). The western boundary currents are to be interpreted schematically.

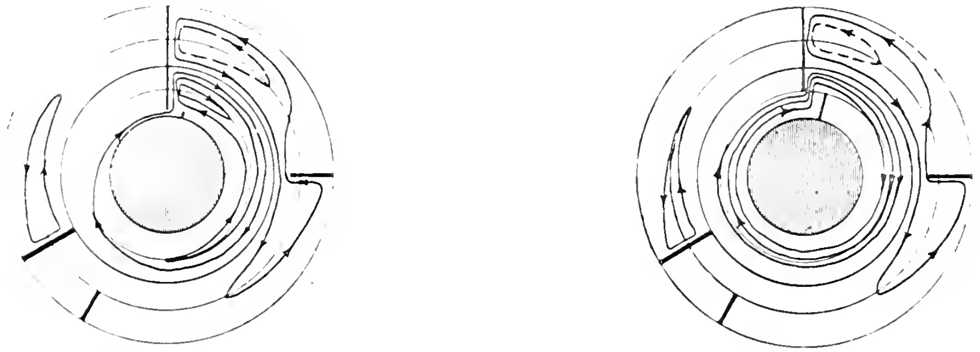


Fig. (c). Modification of the transport field produced by introduction of other meridional barriers corresponding to Africa, Australia, and New Zealand, and by breaking the American-Antarctic barrier so as to admit a very constricted Davis Straits.

Fig. (d). Hypothetical form of the solution that results from rupturing the American-Antarctic barrier in such a way as to permit water to flow through, but to obstruct all latitude circles.

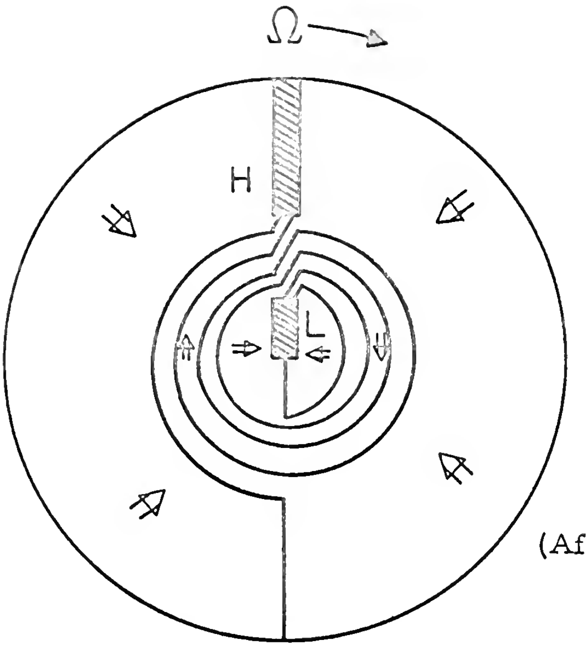
Figure 5. Hypothetical circulation patterns based on assumption of Sverdrup-like solution. (From Stommel, 1957.)

One difficult aspect of Stommel's approach is visualizing the role of Drake Passage in the flow regime. Stommel suggests that higher-order processes take place there, but he does not indicate what these might be. He regards the island arc to the east as a partial barrier to zonal flow, but the analytical representation of the barrier is not readily apparent.

In an attempt to gain some insight into the problem without directly confronting these analytical obstacles Stommel (1962) proposes a laboratory model of the Antarctic Circumpolar Current. The model consists of a rotating cylinder with a single radial (meridional) barrier. The meridional barrier is so constructed that it can be changed from a solid boundary to a porous boundary of variable flow resistance R. Poleward convergent flow is superposed on solid-body rotation by a source-sink combination which has the net effect of a distributed sink over the water surface. The distributed sink models the divergent Ekman drift caused by westerly winds over the Southern Ocean.

Recalling the results of earlier experiments with rotating cylindrical models (Stommel et al, 1958; Faller, 1960), Stommel predicts the effect of the barrier on the flow regime. When the barrier is solid ($R = \infty$), poleward geostrophic flow dominates. With $R < \infty$ a zonal component is introduced in the geostrophic flow, giving it a spiral form, and non-geostrophic radial flow develops in the bottom Ekman layer. At $R = 0$ the geostrophic flow is purely zonal. Its magnitude is just great enough to drive the radial Ekman flow required by continuity.

As an analogue to the Circumpolar Current, Stommel envisions the following configuration:



(After Stommel, 1962)

The black portions of the wall are solid, the gap is porous. In this case flow is poleward everywhere except in the narrow ring which passes through the porous gap. In the gap itself flow is zonal and nongeostrophic; in the remainder of the ring flow is quasi-zonal and geostrophic. Several isobars have been drawn, and regions of relative high and low pressure are marked. The arrows indicate direction of flow.

This model is much too simple to duplicate actual conditions in the Circumpolar Current. The purpose is to study the influence of the Passage on the Current. Unfortunately Stommel has yet to perform the experiment (Stommel, personal communication), so his idea remains untested.

Stommel's discussions of the Circumpolar Current have stimulated further theoretical work. Two models based on his

suggestion of a Sverdrup-like solution are those by Wyrtki (1960) and Gill (1968). The basic equations used in both studies are almost identical. Only Gill's model will be discussed in detail, because he deals directly with the effect of Drake Passage on the primary longitudinal flow. Wyrtki devotes most of his paper to the transverse water motions and their influence on the Antarctic convergence.

The analysis is carried out in rectangular coordinates, shown in figure 6 (upper); the model is also shown in polar projection. Line $y = 0$ is the coast of Antarctica. South America is represented by the solid boundary $x = 0$, L. The gap from $y = B$ to $y = 0$ is Drake Passage.

The dynamical equations are patterned after those of Stommel (1948); pressure gradient, Coriolis, and vertical friction are the dominant forces. Bottom friction is assumed to be a linear function of velocity,* and the β-plane approximation is made.

The equations are integrated over depth, which is assumed uniform. The assumption of incompressible flow allows Gill to introduce a transport function Ψ, yielding the governing equation

$$\delta\left(\Psi_{xx} + \Psi_{yy}\right) + \Psi_x = Y_x - X_y \quad ,$$

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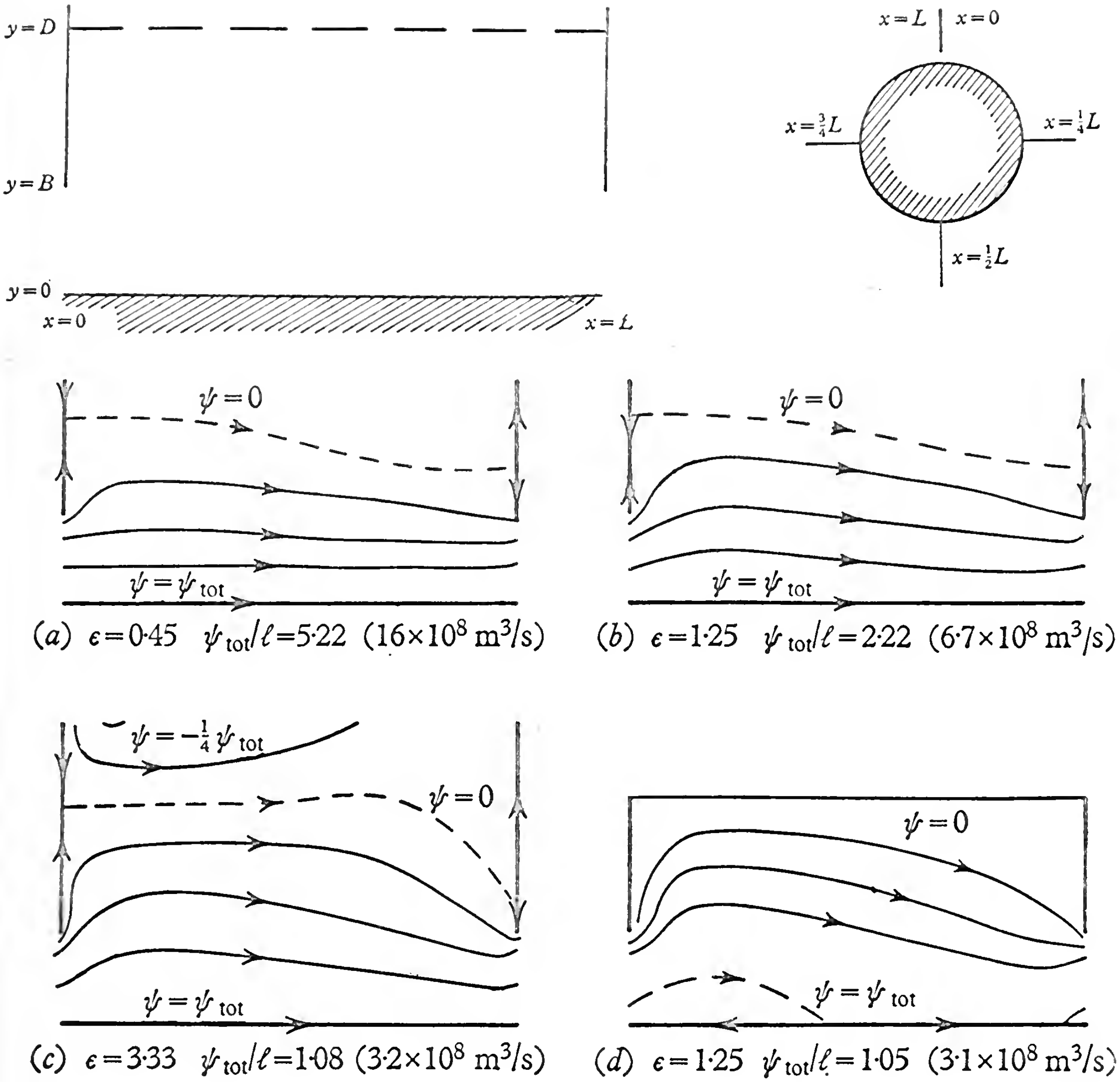
*By including friction as a retarding force Gill and Wyrtki deviate slightly from the Sverdrup solution. Physically, it is reasonable to expect that friction acts on the current at least near the Antarctic coast. Strictly speaking this is lateral rather than vertical friction, but the friction law used in these models is so general that it can be considered to represent either case.



(a) $\epsilon=0{\cdot}45$ $\psi_{tot}/\ell=5{\cdot}22$ $(16\times10^8\ m^3/s)$ (b) $\epsilon=1{\cdot}25$ $\psi_{tot}/\ell=2{\cdot}22$ $(6{\cdot}7\times10^8\ m^3/s)$

(c) $\epsilon=3{\cdot}33$ $\psi_{tot}/\ell=1{\cdot}08$ $(3{\cdot}2\times10^8\ m^3/s)$ (d) $\epsilon=1{\cdot}25$ $\psi_{tot}/\ell=1{\cdot}05$ $(3{\cdot}1\times10^8\ m^3/s)$

Figure 6. (Upper) The model geometry in Mercator and polar projection.

(Lower) Some numerical solutions showing the dependence on the friction parameter ϵ (= δ L/B), and on the wind stress distribution X(y). For cases (a), (b), and (c), X = $6/\pi$ sin ($\pi/4$ + $\pi y/6$; for case (d), X = $5/\pi \sin \pi y/5$. The maximum wind stress is further north in case (d). The contour interval for Ψ is $1/4\ \Psi_{tot}$. The equivalent dimensional mass transport is given in parentheses. (From Gill, 1968.)

where X, Y = components of dimensionless wind stress
δ = dimensionless friction parameter

and subscripts denote differentiation.

An analytical solution (valid for small δ) and a series of numerical solutions are given. Plots of numerical results for various combinations of δ and wind stress illustrate the effect of these parameters on the solution. Two wind stress functions are used. In both cases the meridional component Y is set equal to zero, and the zonal component is assumed to be a function of y only.

The theoretical transport lines, figure 6 (lower), resemble figure 2 in several respects. After passing through Drake Passage the model current swerves sharply to the north. In the western (Atlantic-Indian) portion of the basin the Current is broad and remains displaced to the north, but as it moves into the Pacific the transport lines converge and shift to the south. Within the Passage the transport lines are crowded in the northern part, where velocities are known to be highest. Total transport in the plots shown ranges from 3×10^{14} g sec^{-1} to 1.6×10^{15} g sec^{-1} depending on the combination of friction parameter and wind stress. However, the general shape of the Current is similar in every plot regardless of the choice of parameters. This would seem to indicate the overriding influence of the boundaries in determining the form of the flow.

Gill finds that the Current consists of two strongly coupled components: a zonal part which accounts for less than half the total transport and, to the north, an asymmetric part which makes up the rest. He shows that the asymmetry of the Current is due

more to the effect of the Passage* than to longitudinal variations in the wind field.

Discussion of Model Characteristics

Several analytical models of the Circumpolar Current have been presented above. In this section certain aspects of these models will be examined more closely to bring out similarities and differences in approach. The structures of the models will be compared with observed features of the current. Table 2 summarizes important characteristics of the models.

First, three assumptions which are shared by all the models (and are common in other studies of ocean currents) will be discussed: (i) unsteady and (ii) nonlinear terms are neglected in the dynamical equations, and (iii) depth is taken to be constant.

The lack of data with which to evaluate the relative importance of time-dependent terms forces a long-term average approach in the models. The neglect of unsteady terms in the equations of motion is consistent with this approach.

The relative importance of nonlinear accelerations in the equations of motion may be estimated with the Rossby number, $R_o = U/fL$, where U is a characteristic velocity, L is a characteristic length, and f is the Coriolis parameter. With $U = 20$ cm sec^{-1}, $L = 1000$ km $= 10^8$ cm, and $f = 10^{-4}$ sec^{-1}, it is found that $R_o < 10^{-2}$. Neglect of nonlinear terms is a

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* It is noteworthy that the bending of transport lines is achieved over a flat bottom. Stommel (1957) regards the barrier formed by the Scotia Ridge as essential, but Gill shows that the constricting effect of the Passage is enough to distort the flow. Perhaps the large-scale meanders in the Indian and Pacific sectors are related in a similar fashion to the influence of the African and Australian-New Zealand land masses.

Table 2

Summary of model characteristics

Model	Coordinate System	Homogeneous Water	Zonal Coriolis Force	Zonal Pressure Grad.	Wind Stress Function	Friction	Northern Boundary	Net Transport (g sec^{-1})
Munk & Palmen (1951)	Cylindrical	no	no	no	zonal, τ = Const	Lateral	Solid Wall at 45°S	10^{15} - 10^{16}
Hidaka & Tsuchiya (1953)	Rectangular	yes	yes	no	zonal, τ = Const	Lateral + Vertical	Solid Wall at 45°S	10^{15}
Stommel (1962)	Cylindrical	yes	yes	yes	Northward Ekman Drift Simulated by Distributed Sink at Surface	Vertical	Solid Wall at Unspecified Latitude	Not Calculated
Gill (1968)	Rectangular	no	yes	yes	zonal, $\tau = f(y)$	Vertical	Solid Wall at $y = D$	10^{14} - 10^{15}

All Models Assume:
(i) Steady State
(ii) Nonlinear Accelerations Negligible
(iii) Flat Bottom

reasonable approximation in these models of the large-scale flow.

It is more difficult to make a priori judgments as to the effect of bottom topography on the Circumpolar Current, for, with the exception of the area near the Scotia Ridge, it is not generally known whether the Current is deep enough to feel the bottom. If it isn't, bottom topography may be ignored. But if the flow is deep enough to feel the bottom, large-scale topographic features may be important. Sverdrup (1941) showed theoretically that the bending of streamlines over the Scotia Ridge may be caused by the effect of the Ridge on the Current. Similar distortions over the other submarine ridges are evident in figure 2. For the present the constant depth assumption must be regarded as being of questionable validity.

The models also display important differences in approach. These will be discussed under the headings Geometry and Boundary Conditions, and Driving and Retarding Mechanisms.

1. Geometry and Boundary Conditions

Defining model boundaries is the fundamental problem here. The southern limit is obvious--Antarctica--and the appropriate condition is that velocity vanishes at the boundary. However, the northern limit and boundary condition are not as clear-cut.

In the papers by Munk & Palmén and Hidaka & Tsuchiya the Current is constrained to flow between solid walls, i.e., in a channel of uniform cross section. The northern boundary condition is then $\underline{u} = 0$ along some latitude circle. It is evident from surface current charts (e.g., Sverdrup et al, 1942, Chart VII; Dietrich, 1963, Chart 5) that there is no zone of intense velocity shear surrounding Antarctica, as would be expected in the presence of a solid boundary. Such a boundary would prohibit

exchange between the Current and the oceans which border it to the north. In the ocean, however, a significant meridional exchange does occur. This exchange probably influences the dynamics of the zonal current, and it should not be arbitrarily eliminated.

Gill is more careful about choosing the northern boundary condition. He regards the northern limit of the Current as the position where it matches the Sverdrup-like circulation which develops in lower latitudes. In other words, the meridional extent of the Current is determined by the wind field. The boundary at $y = D$ may result in the formation of boundary currents which modify the Sverdrup regime. However, it is possible, through a judicious choice of parameters, to ensure that the dependence of the solution on D is weak.

Wyrtki also assumes that the northern boundary of the Current is determined by the anticyclonic gyre which exists at lower latitudes. He does not investigate the influence of different wind field distributions on this boundary as Gill does.

Stommel, Wyrtki, and Gill include a partial meridional barrier in their models. This has a profound effect. The most obvious result is that it eliminates the axial symmetry implicit in the channel configurations. It also has dynamical effects which are discussed below.

2. Driving and Retarding Mechanisms

Wind is the driving agent for the models studied in this paper. Munk & Palmén and Hidaka & Tsuchiya assume a constant eastward wind stress of 2 dyne cm^{-2} over the entire zone from 45°S to 70°S. Like the solid wall at 45°S, this is a highly artificial representation of natural conditions.

Gill and Stommel (1957) use zonal wind stress functions which vary in the north-south direction but not in the east-west direction. Over the region of the Circumpolar Current winds are westerly. Gill has a maximum stress of about 1.5 dyne cm^{-2}.

Wyrtki includes both longitudinal and latitudinal variations in the zonal wind stress. In figure 1 it is seen that the zone of maximum westerlies shifts toward the south in the Pacific. It is not clear whether this shift in the wind is an important factor in the asymmetry of the Current, since Gill obtains an asymmetric Current without allowing for longitudinal wind dependence.

Friction is included as a retarding mechanism in all models. Hidaka & Tsuchiya use vertical and lateral friction terms, but they indicate that lateral friction dominates. Munk & Palmén use only lateral friction. Wyrtki and Gill employ a simple form of bottom friction. Gill shows that the equations can also be written using lateral friction instead of vertical friction. The two models are similar in form and results.

The meridional barrier in Stommel's, Wyrtki's, and Gill's models introduces another retarding mechanism not found in the channel-flow models--a zonal pressure gradient. Its significance can be seen in the fact that the eddy coefficients required to give observed transport in Gill's model are $A_v = 10^3$ cm^2 sec^{-1} or $A_h = 10^8$ cm^2 sec^{-1}, while the coefficients required for the channel models are one to two orders of magnitude larger.

Concluding Remarks

In retrospect, Stommel's (1957) discussion of the Antarctic Circumpolar Current stands out as a turning point in the effort to explain theoretically the dynamics of the Current. He was the first to note the implications of boundary asymmetry with respect to dynamical structure. Polar asymmetry is a fundamental characteristic of the Current. Therefore, it is not surprising that inconsistencies appear in the simple channel-flow models of Munk & Palmén and Hidaka & Tsuchiya.

Recent papers by Wyrtki and Gill carry forward Stommel's suggestion. Polar symmetry is abandoned, and a partial meridional barrier is introduced. Boundary conditions and the wind stress distribution are more realistic. These models predict the transport and general form of the Current much better than the earlier zonal models. It is concluded that lateral boundary geometry, in particular the Drake Passage constriction, and latitudinal variations in wind stress are key elements in the dynamics of the Circumpolar Current.

Several of the large-scale features of the Circumpolar Current have been explained, but a number of interesting questions concerning its dynamics remain unanswered. Some of these are of general oceanographic interest and will not be discussed here. Four questions, however, are of particular significance to this subject:

1. Is there any reason to expect the Circumpolar Current to extend to much greater depths than do wind-driven currents in lower latitudes?

In several of the papers reviewed above reference

is made to the extreme depth of the Current. Observational support for this statement is rather scanty, consisting principally of geostrophic velocity sections such as figure 2. Nor has the matter received adequate theoretical attention.

Physically, three factors might be expected to cause an unusually deep flow: (i) the strength of the west winds, (ii) the great fetch over which they blow in the Southern Ocean, and (iii) the relatively homogeneous structure of Antarctic waters compared with tropical and subtropical waters.

2. If the Current is very deep, is it retarded significantly by bottom topography?

This question is aimed at investigating further the mountain effect discussed by Munk & Palmén. Although their analysis is too superficial to be conclusive, the possibility of zonal pressure gradients caused by submarine ridges should not be ignored. The retarding effect of underwater barriers could be studied in a laboratory model similar to the one proposed by Stommel (1962).

3. What is the nature of the meridional circulation, and what role does it play in the zonal flow?

Meridional flow must have both wind-driven (Ekman) and thermohaline components. The former has been studied theoretically by Sverdrup (1933) and Wyrtki (1960). Deacon (1937a) has discussed the thermohaline circulation. These papers are concerned primarily with the relationship between transverse water motions and the Antarctic convergence.

Meridional flow may play a larger part in the overall dynamics of the Circumpolar Current. The quasi-steady transverse circulation may act as a momentum-transfer agent in the manner proposed by Munk & Palmén. Barcilon (1966; 1967) suggests that the thermohaline meridional circulation driven by runoff from Antarctica sets up a westward countercurrent which opposes the Circumpolar Current.

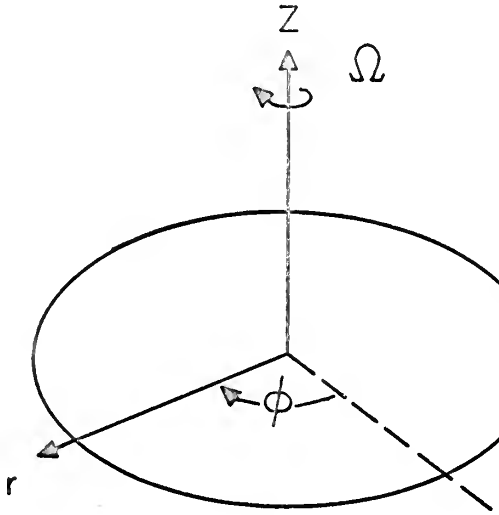
To properly evaluate the role of the meridional circulation it will be necessary to determine the size of the meridional transport more accurately than has been done in the past. Conventional geostrophic methods are of little help. A new approach will be required, perhaps use of the heat balance equation for the southern hemisphere.

4. What is the effect on the Current of the annual pack ice cycle?

It was noted earlier that during winter, pack ice covers a large portion of the Southern Ocean. The ice cover may enhance or reduce the transfer of energy from the wind to the water, depending on such factors as the roughness of the ice surface and internal friction within the pack. It is interesting to observe that Gordon (1967) finds a slight decrease in transport through Drake Passage in winter.

Appendix

Equations of motion in cylindrical coordinates for an incompressible fluid relative to a frame rotating with clockwise angular velocity Ω (adapted from Batchelor, 1967).



(tangential)

$$\frac{\partial u}{\partial t} + \underline{U}\cdot\nabla u + \frac{vu}{r} + 2\Omega v = -\frac{1}{\rho r}\frac{\partial p}{\partial \varphi}$$

$$+ A_h\left[\nabla_h^2 u - \frac{u}{r^2} + \frac{2}{r^2}\frac{\partial v}{\partial \varphi}\right] + A_v\frac{\partial^2 u}{\partial z^2}$$

(radial)

$$\frac{\partial v}{\partial t} + \underline{U}\cdot\nabla v - \frac{u^2}{r} - 2\Omega u = -\frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$+ A_h\left[\nabla_h^2 v - \frac{v}{r^2} - \frac{2}{r^2}\frac{\partial u}{\partial \varphi}\right] + A_v\frac{\partial^2 v}{\partial z^2}$$

Appendix (cont'd).

(vertical)

$$\frac{\partial w}{\partial t} + \underline{U}\cdot\nabla w = -\frac{1}{\rho}\frac{\partial p}{\partial z} + A_h \nabla_h^2 w + A_v \frac{\partial^2 w}{\partial z^2} - g$$

where $\underline{U} = u\hat{\xi} + v\hat{\sigma} + w\hat{\zeta}$; u, v, w = velocity components in φ, r, z - directions and $\hat{\xi}, \hat{\sigma}, \hat{\zeta}$ = unit vectors

Ω = magnitude of earth's rotation

p = pressure

ρ = density

g = gravity

A_h = kinematic coefficient of lateral eddy viscosity

A_v = kinematic coefficient of vertical eddy viscosity

∇_h^2 = horizontal Laplace operator

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}$$

and $$\nabla = \hat{\zeta}\frac{\partial}{\partial z} + \hat{\sigma}\frac{\partial}{\partial r} + \frac{\hat{\xi}}{r}\frac{\partial}{\partial \varphi} .$$

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Vita

Jeffrey Edwin Callahan was born in Cambridge, Massachusetts on 24 September 1943. He entered the United States Naval Academy in June 1961. Upon graduating in June 1965 he was awarded a Bachelor of Science degree and commissioned as an Ensign, United States Navy. His first duty was aboard the U. S. S. *Willis A. Lee* (DL-4) in the Atlantic Fleet. The author began his graduate studies at The Johns Hopkins University in June 1967.