U. S. DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY

TIDAL DATUM PLANES

H. A. MARMER

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PREFACE TO SECOND EDITION

In this revision of "Tidal Datum Planes," the first edition of which was issued in 1927, changes have been made to conform with the most recent practice of the Coast and Geodetic Survey. The section on lunitidal intervals has been eliminated since this matter is only of secondary interest in datum planes and is adequately covered in other Survey publications. The section on mean sea level has been extended to include the question of changing sea level, and consideration is given to datum planes of tides predominantly diurnal.

PREFACE TO FIRST EDITION

Of datum planes that may be used as planes of reference for elevations, those based on the rise and fall of the tide have the advantages of simplicity of definition, accuracy of determination, and certainty of recovery, even though all bench-mark connection be lost. Tidal datum planes are, therefore, the basic planes of reference used in the hydrographic and geodetic work of the Coast and Geodetic Survey.

In the present publication two objects have been kept in mind. It is aimed to provide a working manual for the determination of the more important tidal datum planes and at the same time to provide a sufficient discussion of the principles involved and accuracy attainable. Since no such discussion is elsewhere available, this phase of the subject has been treated in detail.

A considerable body of observational material formed the basis of the investigations here undertaken, long-continued observations being especially important. Of these latter, however, those at hand were limited almost without exception to the observations made by this bureau on the coasts of the United States. It is for this reason, and for the further reason that the publication is to serve as a manual in the Coast and Geodetic Survey, that the examples chosen are from observations on the coasts of the United States.

I. INTRODUCTORY

Definitions

A tidal datum plane is a plane of reference for elevations, determined from the rise and fall of the tides. Various tidal planes may be derived, and each is designated by a definite name, as, for example, the plane of mean high water, the plane of half-tide level, the plane of lower low water.

The tide is the name given to the alternate rising and falling of the level of the sea, which at most places occurs twice daily. The striking feature of the tide is its intimate relation to the movement of the moon. High water and low water at any given place follow the moon's meridian passage by a very nearly constant interval; and since the moon in its apparent movement around the earth crosses the meridian at any place 50 minutes later each day on the average, the tide at most places likewise comes later each day by 50 minutes on the average.

With respect to the tide, the "moon's meridian passage" has a special significance. It refers not only to the instant when the moon is directly above the meridian but also to the instant when the moon is directly below the meridian, or 180° distant in longitude. In this sense there are two meridian passages in a tidal day, and they are distinguished by being referred to as the upper and lower meridian passages or upper and lower transits.

The interval between the moon's meridian passage (upper or lower) and the following high water is known as the "high-water lunitidal interval." Likewise, the interval between the moon's meridian passage and the following low water is known as the "low-water lunitidal interval." For short they are called, respectively, high-water interval and low-water interval, and abbreviated as follows: HWI and LWI.

With respect to the rise and fall of the water due to the tide, high water and low water have precise meanings. They refer not so much to the height of the water as to the phase of the tide. High water is the maximum height reached by each rising tide and low water the minimum height reached by each falling tide.

It is important to note that it is not the absolute height of the water which is in question, for it is not at all infrequent at many places to have the low water of one day higher than the high water of another day. Whatever the height of the water, when the rise of the tide ceases and the fall is about to begin, the tide is at high water, and when the fall of the tide ceases and the rise is about to begin the tide is at low water. The abbreviations HW and LW are frequently used to designate high and low water, respectively.

The difference in height between a high water and a preceding or succeeding low water is known as the range of the tide or range. Since the heights of high and low water at any place vary from day to day, it follows that the range of tide likewise varies from day to day. It is, in fact, this variation which gives rise to the problems involved in the determination of tidal datum planes.

The Tide-Producing Forces

The tide arises as the result of the attractive forces of sun and moon on the rotating earth. The intensity with which a heavenly body attracts a particle of matter on the earth varies directly as its mass and inversely as the square of its distance. For the solid earth as a whole the distance is obviously to be measured from the center of the earth since that is the center of mass of the whole body. But the oceanic waters, which may be considered as lying on the surface of the earth, are on the one side of the earth nearer the heavenly body and on the other side farther away than the center of the earth. The attraction on the waters of the oceans is thus different in intensity from the attraction for the solid earth as a whole, and these differences of attraction give rise to forces that cause the ocean waters to move relative to the solid earth and bring about the tide. These forces are called the tide-producing forces.

The mathematical development of these forces brings out the fact that the tide-producing power of a heavenly body varies directly as its mass and inversely as the cube of its distance from the earth. Of the heavenly bodies, only the sun and moon need be taken into consideration insofar as the tide on our earth is concerned. For the other heavenly bodies are either too small or too far away to bring about any appreciable tides on the earth. The sun has a mass about 27,000,000 times as great as that of the moon but it is 389 times as far away from the earth. Hence the tide-producing power of the sun is to that of the moon as 27,000,000 is to $(389)^3$ or somewhat less than half. The moon is thus the principal tide-producing body.

When the relative motions of the earth, moon and sun are introduced into the equations, it is found that the tide-producing forces of sun and moon group themselves into three classes: (*a*) Those having a period of approximately half a day, which are therefore called the semidiurnal or semidaily forces; (*b*) those having a period of approximately one day, known as the diurnal or daily forces; (*c*) those having a period of half a month or more, known as the long-period forces.

The tide-producing forces are distributed in a regular manner over the earth, varying with latitude. But the response of the various oceans and seas to these forces differs, depending on the hydrographic features of the basins of the different oceans and seas. As a result, the tides as they actually occur differ markedly at different places but with no apparent relation to latitude.

The principal tide-producing forces are the semidiurnal forces. These forces go through two complete cycles in a tidal day, and it is because of the predominance of these semidaily forces that there are at most places two complete tidal cycles, and therefore two high and two low waters in a tidal day.

Tidal Currents

In its rise and fall the tide is accompanied by a horizontal forward and backward movement of the water called the tidal current. The two movements—the vertical rising and falling of the tide and the horizontal forward and backward movement of the tidal current—are intimately related, forming parts of the same phenomenon brought about by the tidal forces of sun and moon.

It is necessary, however, to distinguish clearly between tide and tidal current, for the relation between the two is not a simple one nor is it everywhere the same.

At one place a strong current may accompany a tide having a very moderate rise and fall, while at another place a like rise and fall may be accompanied by a very weak current. Furthermore, the time relation between current and tide varies widely from place to place. At some places the strength of the current coincides with high and low water, while at other places the slack of the current coincides with high and low water. Hence flood current is not everywhere synonymous with a rising tide nor is ebb current synonymous with a falling tide.

Unfortunately, there is no term in the English language by which to designate the whole phenomenon which includes both tides and tidal currents. Frequently "the tide" or "flood and ebb" is used in this general sense, and no confusion arises from this usage if the context clearly indicates that the term is intended in its general sense. For the sake of clearness, however, when the vertical movement of the water is meant, tide is to be used, and when the horizontal movement is meant, current is to be used.

Characteristics of the Tide

In its rise and fall the tide does not move at a uniform rate. From low water the tide begins rising very slowly at first, but at a constantly increasing rate for about three hours, when the rate of rise is at a maximum. The rise then continues at a constantly decreasing rate for the following three hours, when high water is reached and the rise ceases. The falling tide behaves in a similar manner, the rate of fall being least immediately after high water, but increasing constantly for about three hours, when it is at a maximum, and then decreasing for a period of three hours till low water is reached.

The rate of rise and fall and other characteristics of the tide may best be studied by representing the rise and fall of the tide graphically. This may be done by reading the height of the tide at regular intervals on a fixed vertical staff graduated to feet and tenths, plotting these heights to a suitable scale on cross-section paper and drawing a smooth curve through these points. A more convenient method is to make use of an automatic tide gage by means of which the rise and fall of the tide is recorded on a sheet of paper as a continuous curve drawn to a suitable scale. Figure 1 shows a tide curve for New York Harbor for the last two days of June 1934.

In Figure 1 the consecutive numbers from 0 to 24, increasing from left to right, represent the hours of the day beginning with midnight. Numbering the hours consecutively to 24 eliminates all uncertainty as to whether morning or afternoon is meant and has the further advantage of great convenience in computation. The numbers on the left increasing upward from 0 to 6 represent the height of the tide in feet as referred to a fixed vertical tide staff. The tide curve approximates the well known form of the sine or cosine curve.

The rise and fall of the tide at any place is characterized by numerous features which differ at different places. Of these features, three may be considered as constituting the principal features; namely, those relating to the time of tide, to the range of the tide, and to the type of the tide.

The time of tide has reference to the times of occurrence of high and low water with respect to the moon's meridian passage. That is, as a characteristic feature of the tide at a given place, the time of tide is specified by the high water and low water lunitidal intervals. These intervals are not constant, but vary periodically within

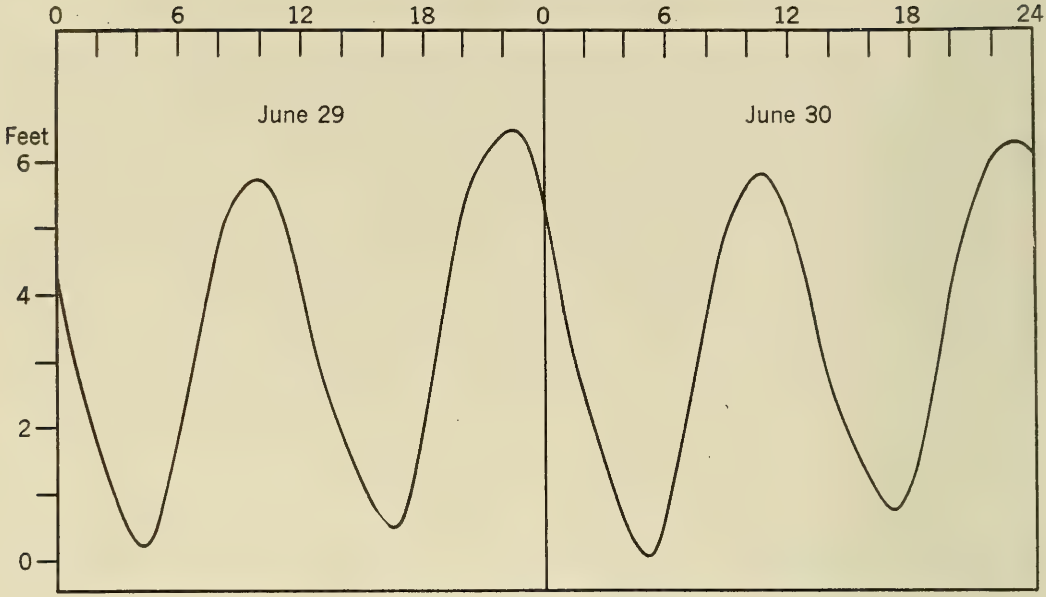

FIG. 1.—Tide curve, New York, June 29–30, 1934

relatively narrow limits as a whole. Since, however, the time of tide is only of minor importance in connection with the determination of tidal datum planes, this matter need not be pursued further here.

The range of tide has reference to the magnitude of rise and fall of the tide. This varies in different localities from less than a foot at many places to more than 40 feet in the Bay of Fundy. Moreover, the range of tide at any given place is not constant, but varies from day to day, the average rise and fall being called the mean range. In some localities the variation from the mean range is relatively small, but in others the variation may be considerable. This matter will be considered more fully in the section on variations in range.

The type of tide has reference to the characteristic form of the rise and fall of the tide as revealed by the tide curve. On investigation, it is found that quite apart from differences in time and in range, the tides at different places exhibit striking differences in regard to form of tide curve. For example, in New York Harbor, the tide exhibits two high and two low waters in the average tidal day of 24 hours and 50 minutes, each rise and fall occupying a period of approximately 6 hours and 12 minutes with succeeding tides resembling each other more or less closely. In San Francisco Bay, the tide likewise exhibits two high and two low waters in a day, but succeeding tides differ considerably. In Mobile Bay, on the other hand, the tide exhibits but one high and one low water in a day, each rise and fall occupying a period of approximately 12 hours.

Type of tide is an important matter in connection with tidal datum planes. As will be seen later, in the consideration of types of tides, there is a variety of forms of tide curves. Here it is sufficient merely to direct attention to the fact that the type of tide at any place is one of the important features of the tide, and together with the time and range constitute the principal features of the tide at that place.

Variations in Range

The range of the tide at any place is not constant, but varies from day to day; indeed, it is exceptional to find consecutive ranges equal. In part, this variation arises from the effects of wind and weather, but in much the larger part it is of a periodic nature, related to the positions of moon and sun relative to the earth. In the change in range from day to day, the tide reveals clearly the presence of three variations, each associated with a particular movement of the moon.

The most noticeable variation, as a rule, is that related to the moon's phase. During the phase cycle the tide rises higher and falls lower about the times of new and full moon, and rises least and falls least about the times of the moon's first and third quarters. The tides occurring about the times of new and full moon when the range is greatest are known as spring tides, while those occurring about the times of the moon's first and third quarters when the range is least are known as neap tides.

It is to be noted, however, that at most places there is a lag of a day or two between the occurrence of spring or neap tides and the corresponding phases of the moon; that is, spring tides do not occur on the days of full and new moon but a day or two later. Likewise, neap tides follow the moon's first and third quarters after an interval of a day or two. This lag in the response of the tide is known as the "age of phase inequality" or "phase age" and has different values in different localities. For example, in New York Harbor the phase age is 26 hours while in Boston Harbor it is 38 hours. That is, in New York Harbor spring and neap tides occur a day after the corresponding positions of the moon, while in Boston Harbor they occur one and a half days after these positions.

The second variation in range is that associated with the moon's varying distance from the earth. When the moon is nearest the earth or in perigee, high water rises higher and low water falls lower than usual, while when the moon is farthest from the earth or in apogee, the rise and fall is less than usual. The tides occurring at these times are known, respectively, as perigean and apogean tides.

In the response to the moon's changes in position from perigee to apogee, it is found that, like the response in the case of spring and neap tides, there is a lag in the occurrence of perigean and apogean tides. The greatest rise and fall does not come on the day when the moon is in perigee, but a day or two later. Likewise, the least rise and fall does not occur on the day of the moon's apogee, but a day or two later. This interval varies somewhat from place to place, and in some regions it may have a negative value. This lag is known as the "age of parallax inequality" or "parallax age." Taking Boston and New York again as examples, it is found that at the former place the parallax age is 58 hours, while at New York it is 31 hours.

The third periodic variation in the rise and fall of the tide is related to the moon's changing declination. When the moon is close to the equator the two high waters of a day, and likewise the two low waters, do not differ much; in other words, at such times morning and afternoon tides resemble each other. With the moon's increasing declination, differences between morning and afternoon tides appear, and at the times of the moon's maximum semimonthly declination these differences are most marked. The tides occurring when the moon is near the equator are known as equatorial tides, while those occurring when the moon is near its maximum semimonthly declination are known as tropic tides. Like the response to changes in the moon's phase and parallax, there is a lag in the response to the change in declination, this lag being known as the "age of diurnal inequality" or "diurnal age." Like the phase and parallax ages, the diurnal age varies from place to place, being generally about one day, but in some places it may have a negative value.

It is to be observed that although the three variations in the rise and fall of the tide described above are noted the world over, they are not everywhere exhibited in equal measure. In many regions the principal variation is that related to the moon's phase; in other regions it is that depending on the moon's distance or parallax; in still other regions the principal variation is that related to the moon's declination.

The month of the moon's phases (synodic month) is approximately 29½ days in length; the month of the moon's distance (anomalistic month) is approximately 27½ days in length; the month of the moon's declination (tropic month) is approximately 27⅓ days in length. It follows, therefore, that very considerable variation in the range of the tide occurs at any given place in consequence of the changing relations of the three variations to each other.

How great these variations may be is illustrated by the following examples. At Boston, the range of tide has an average value of 9.4 feet; yet within a fortnight the tide will vary in range from less than 6 feet to more than 12 feet. In Los Angeles Harbor, the mean range of tide is 3.8 feet; but within a week there may be individual ranges from less than a foot to more than 6 feet. At Seattle, the mean range is 7.6 feet; individual ranges within a single day, however, may vary from less than 5 feet to more than 15 feet.

The range of tide is also subject to other periodic variations. But the three discussed above are the principal variations.

Mean Values

Since the rise and fall of the tide varies from day to day, any tidal characteristics derived from a short series of observations may differ considerably from the average or mean values. In other words, to derive values that will represent averages, the results from short series of observations must be corrected to mean values.

The principal tidal variations are those connected with the moon's phase, parallax and declination, the periods of which are approximately 29½ days, and 27½ days, and 27⅓ days, respectively. It follows, therefore, that in a period of 29 days the phase variation will have almost completed a full cycle, while the other variations will have gone through a full cycle and but very little more. Hence, for tidal characteristics dependent primarily on the phase variation, tide observations covering 29 days or multiples, constitute a satisfactory period for determining approximate mean values of these characteristics. Such are the lunitidal intervals, mean range, mean high water and mean low water. For characteristics dependent primarily on the declination of the moon, as for example, higher high water or lower low water, observations covering 27 days or multiples, constitute the more satisfactory period.

As will be seen in the detailed discussion of the various tidal datum planes, the values determined from two different 29-day or 27-day periods may differ very considerably. This is due to the fact that these periods are not exact synodic periods for the different variations, and to the further fact that variations having periods greater than a month are not taken into account. Furthermore, meteorological conditions, which change from month to month, leave their impress on the tides. For accurate results, the direct determination of the tidal datum planes and other tidal characteristics should be based on a series of observations that covers a number of years. Values derived from shorter series must be corrected to mean values.

As a rule, it is impracticable to secure long series of tide observations at all places where tidal datum planes are desired. To correct to mean values the results derived from short series of observations, two different methods may be employed. One method makes use of tabular values, determined both from theory and from observations, for correcting for the different variations. The other method makes use of direct comparisons with simultaneous observations at some nearby place for which mean values have been determined from a series of considerable length. The latter method is the more satisfactory one in connection with the determination of tidal datum planes.

II. TYPES OF TIDE

The Three Primary Types

Of the principal features with regard to which tides differ, type of tide is the most fundamental. If the tides at two places are of the same type, but differ in time or in range, many of the other characteristics of the tide at both places will correspond, so that a knowledge of the tide at the one place permits the inferring of the characteristics of the tide at the other. But if the type of tide at the two places differs, then the fact that the range or time at the two places is the same does not prevent the occurrence of very profound differences in the other characteristics of the tide at the two places. In other words, differences in time and range of tide are merely differences in degree, but differences in type of tide are differences in kind.

Type of tide has reference to the characteristic form of the rise and fall of the tide as revealed by the tide curves. In general, it may be said that the tide curve for any particular place differs from the tide curve at any other place in some one or more respects. There is, therefore, great variety in tide curves. They may, however, be grouped into three large classes or types, namely, semidaily tides, daily tides, and mixed tides. Instead of the terms semidaily and daily, the terms semidiurnal and diurnal are sometimes used.

As the name suggests, the semidaily type of tide is one in which the full tidal cycle of high and low water is completed in half a day; in other words, in a day there are two high and two low waters in this type of tide. There is, however, the further implication that the two tidal cycles in each day resemble each other; that is, morning and afternoon tides do not differ much. In this connection, it is to be noted that a day in the tidal sense is a tidal day of 24 hours and 50 minutes and not the ordinary day of 24 hours.

The daily type of tide includes those tides in which but one high and one low water occur in a day. In this type of tide the rise and also the fall of the tide each occupies a period of approximately 12 hours against a period of 6 hours in the semidaily tide.

The mixed type of tide is one in which two high and two low waters occur in a day, but with marked differences between the two high waters or between the two low waters of the day. As will be seen later, the mixed type of tide arises as a mixture of semidaily and daily tides, and hence its name.

To exemplify the three types of tide there are shown in Figure 2 the tide curves for the last four days of May 1931 at Hampton Roads (Norfolk), Va., Pensacola, Fla., and San Francisco, Calif. The horizontal line associated with each tide curve represents the undisturbed or mean level of the sea, the rise and fall of the tide above and below mean sea level being measured by the scale to the left of each curve.

The upper curve, that for Hampton Roads, illustrates the semidaily type of tide. Two high and two low waters are seen to have occurred each day, with the morning and afternoon tides differing but relatively little. The middle curve, for Pensacola, illustrates the daily type of tide, one high and one low water occurring each day. The

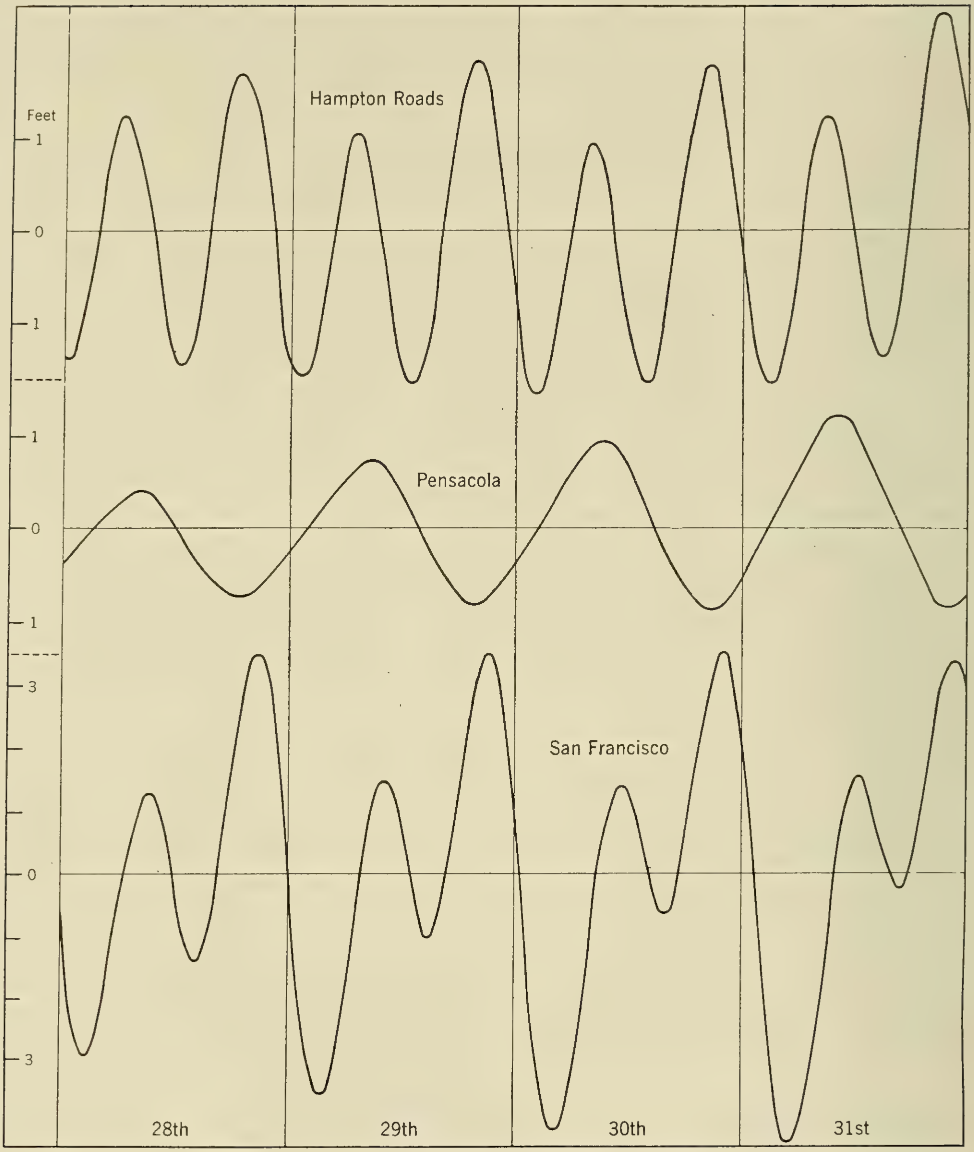

FIG. 2.—Tide curves, illustrating the three types of tide.

curve for San Francisco illustrates one form of the mixed type of tide. Two high and two low waters are seen to have occurred each day, but the forenoon tides differ considerably from the afternoon tides.

The distinction between the semidaily and mixed types of tide is based on the fact that in the former type morning and afternoon tides resemble each other, while in the latter type they exhibit differences. This difference between the two high or low waters of a day is known as diurnal inequality and is an important feature of the tide in connection with tidal datum planes. It will, therefore, be of advantage to consider this feature briefly before taking up the detailed discussion of the different types of tide.

Diurnal Inequality

Diurnal inequality, or difference between corresponding morning and afternoon tides arises primarily from the fact that the moon's orbit is inclined to the plane of the equator. This gives rise to daily and semidaily tide-producing forces, and as a result morning and afternoon tides differ in greater or lesser degree.

Diurnal inequality is thus a feature of all tides which have two high and two low waters a day. But the magnitude of the diurnal inequality is not the same at all places. In fact, the distinction between the semidaily and mixed types of tide is based on this difference in magnitude of diurnal inequality. In general, it may be said that in the semidaily type of tide the diurnal inequality is relatively small, while in the mixed type of tide it is relatively large. This statement is obviously only of a qualitative character and does not serve as a definite criterion for separating the mixed type from the semidaily type. In the detailed discussion of the different types of tide, however, it will be found that quantitative criteria can be formulated to separate these two types of tide.

In Figure 2 the tide curve for San Francisco was taken to illustrate the mixed type of tide. On glancing at that curve it will be seen that the morning low waters were lower than the afternoon low waters, and the morning high waters were lower than the afternoon high waters. In other words, the tide at San Francisco exhibits inequality in both high and low waters. It will be noted, however, that the inequality in the low waters is greater than in the high waters.

On investigation, it is found that the diurnal inequality in the tide at different places varies not only in magnitude, but also in the proportion in which it is exhibited by the high and the low waters. In some places the inequality is featured principally by the high waters; at other places the inequality is exhibited principally in the low waters; and at still other places the inequality appears in approximately equal degree in both high and low waters. This matter, however, it is more convenient to consider in connection with the detailed discussion of the different types of tide.

It is to be noted that the diurnal inequality in tides is featured not only in the heights but also in the times of the tide. Where there is considerable inequality in the heights of the tide, there will also be considerable inequality in the times, and this is evidenced by differences in the morning and afternoon lunitidal intervals and, therefore, by differences in the durations of rise and fall as between morning and afternoon tides. Thus, for the last day shown in Figure 2, the duration of rise of tide at Hampton Roads was 5.9 hours for the morning tide and 6.3 hours for the afternoon tide, or a difference of 0.4 hour. At San Francisco, for the same day, the durations were 7.4 hours and 5.9 hours, or a difference of 1.5 hours.

For any particular day, the difference between the heights of the two high waters or the two low waters would appear to be the natural measure of the magnitudes of the respective inequalities. For certain reasons, however, it is more convenient to use half these differences as the measure of the inequality. That is, the high water inequality is taken as half the difference between the two high waters of a day, and the low water inequality is taken as half the difference of the two low waters of a day. Technically, these are known as the diurnal high water inequality and diurnal low water inequality, and are abbreviated as DHQ and DLQ, respectively.

To distinguish the two tides of a day, definite names have been given to each of the two high and two low waters. Of the two high waters, the higher is called the "higher high water" and the lower the "lower high water." Likewise, of the two low waters of a day, the lower is called "lower low water" and the higher "higher low water."

The diurnal inequality in the tide depends primarily on the declination of the moon, which varies from zero to its maximum north or south declination in half a fortnight. Hence, the diurnal inequality in the tide likewise varies within a fortnight, being generally least when the moon is close to the equator and greatest when the moon is near its fortnightly maximum north or south declination.

The existence of diurnal inequality in tides and its variation within a fortnight find ready explanation in the existence of semidaily and daily constituents in the tide brought about, respectively, by the semidaily and daily tide-producing forces. This becomes evident from a consideration of the tide resulting from the combination of daily and semidaily constituent tides.

The Combination of Daily and Semidaily Constituent Tides

The daily and semidaily tide-producing forces of sun and moon bring about constituent tides of like periods in the waters of the sea. The relative ranges and times of these constituent tides at any particular place, however, depend not only on the relative magnitudes and phases of the corresponding tide-producing forces, but also on local hydrographic features. Hence, the relative ranges and times of the daily and semidaily constituents of the tide are different at different places.

Suppose that at a certain place the daily and semidaily constituents of the tide have equal ranges. The rise and fall of each of these constituent tides may be represented as in Figure 3, the semidaily constituent by the dotted curve and the daily constituent by the dashed curve. The height of the resultant tide at any moment is then clearly the sum of the heights of the two constituent tides at that moment. In Figure 3, the resultant tide is indicated by the heavy full-line curve.

Now, the times of the two constituent tides may have different relations to each other depending on local hydrographic features. In Figure 3, three different cases are considered. In the upper diagram the two constituent tides have such time relations that their low waters occur at the same instant; in the middle diagram the high waters of the constituent tides occur at the same time; and in the lower diagram the two constituents are at sea level at the same time. In each case the resultant tide exhibits considerable diurnal inequality, but there are profound differences with regard to the phase of the tide which exhibits the inequality.

When the time relations are as pictured in the upper diagram, the diurnal inequality in the height of the tide is exhibited wholly in the low waters. The middle diagram shows that with the ranges of the two constituents exactly the same as before, but with different time relations, the inequality in height is featured wholly in the high waters. Finally, the lower diagram shows that with the ranges of the two constituents still the same, but with time relations different than in the two preceding cases, the height inequality is featured in equal degree in both the high and low waters.

Without going into a detailed consideration of the matter, it is clear that the two upper diagrams of Figure 3 represent the limiting cases of the combination of daily and semidaily constituent tides of the same range. When the low waters of the two constituents occur at the same time, the inequality in height of the resultant tide is wholly in the low waters as represented by the upper diagram. As the times of the low waters of the constituent tides begin to differ, the inequality will begin to appear

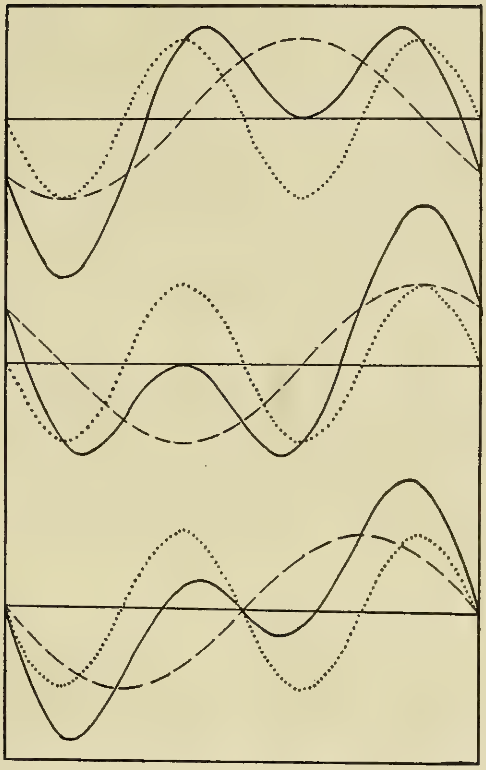

FIG. 3.—Combinations of semidaily and daily constituent tides.

also in the resultant high waters to some extent, the greater the difference in the times the greater the inequality in the high waters. When the time differences are such that the two constituents are at sea level at the same time, the inequality in the tide will appear in equal degree in both high and low waters, as pictured in the lower diagram. As the times of the constituent tides continue to differ still more, the inequality in the high waters will increase and the inequality in the low waters will decrease until finally as represented in the middle diagram when the high waters of the two constituents occur at the same time, the inequality will be exhibited wholly in the high waters.

With regard to the inequality in height of tide, therefore, tides may be grouped into three classes: (1) those in which the inequality is featured principally in the high waters; (2) those in which the inequality is featured principally in the low waters; (3) those exhibiting inequality in approximately equal degree in both the high and the low waters.

Thus far we have discussed only the combination of daily and semidaily constituents of equal ranges. In the tides as they actually occur, however, the ranges of the two constituents differ at different places. This gives rise to new features in the inequality, which a consideration of the lower diagram of Figure 3 will make clear. If the range of semidaily constituent remains as pictured in that diagram but the range of the daily constituent is greater, it is obvious that the lower high water will become lower and the higher low water will become higher. When the range of the daily constituent is taken twice that of the semidaily, it will be found that the lower high water and the higher low water have the same height, giving rise to what is known as the vanishing tide. As the range of the daily constituent is increased still further, the resultant tide shows but one high and one low water in a day.

In the tides as they actually occur at different places, not only do the times of the semidaily and daily constituents have different relations, but the ranges of the two constituents likewise differ. On investigation it is found that if the range of the daily constituent is less than twice that of the semidaily constituent, there will be two high and two low waters each tidal day; if the range of the daily constituent is between two and four times the range of the semidaily, there may be two high and two low waters or only one high and one low water a day; but if the range of the daily constituent is four or more times that of the semidaily, there will be only one high and one low water a day.

It is to be noted that both the daily and the semidaily tide-producing forces vary in intensity from day to day, the former being greatest when the moon is at its semimonthly maximum north or south declination, and the latter being greatest when the moon is over the equator. The tide at any given place, therefore, exhibits varying amounts of inequality within a fortnight.

With this brief discussion of diurnal inequality, we may now consider the three types of tide in greater detail.

The Semidaily Type of Tide

The semidaily type of tide, as implied by the name, is one in which the tidal cycle of high water and low water is completed in half a day; and there is the further implication that there is but little difference between the corresponding tides of successive half-day cycles. In other words, in the semidaily type of tide there are two high and two low waters in a day with but little diurnal inequality. Figure 4, which reproduces the tide curve for New York Harbor for the month of June 1934, illustrates the appearance of a tide curve of this type of tide for a period of a month. The horizontal line associated with each of the 10-day groups of tide curves represents the plane of mean sea level in New York Harbor. The heights of the tide are referred to this plane by the scales to the left of the diagram.

A period of 30 civil days of 24 hours each corresponds almost exactly to 29 tidal days of 24 hours and 50 minutes each. And Figure 4 shows that during the 30-day period of June 1934 there were 58 high waters and 58 low waters, or two high and two low waters in a tidal day. Furthermore, while there is some difference in height between the two high or two low waters of a day, this difference is seen to be relatively small as compared with the range of the tide. In other words, the diurnal inequality in the tide at New York is relatively small and hence the tide here is of the semidaily type.

In passing, it may be noted that Figure 4 shows clearly that the diurnal inequality of the tide at New York is greater for the high waters than for the low waters. That is, the little inequality that exists in the height of the tide in New York Harbor is exhibited principally by the high waters.

In the section on diurnal inequality it was noted that this feature of the tide varies within a period of a fortnight, depending on the declination of the moon. In Figure 4, it is seen that from the third to the fifth day of the month, the tide curve exhibits relatively little diurnal inequality, while from the 11th to the 13th the inequality is rela-

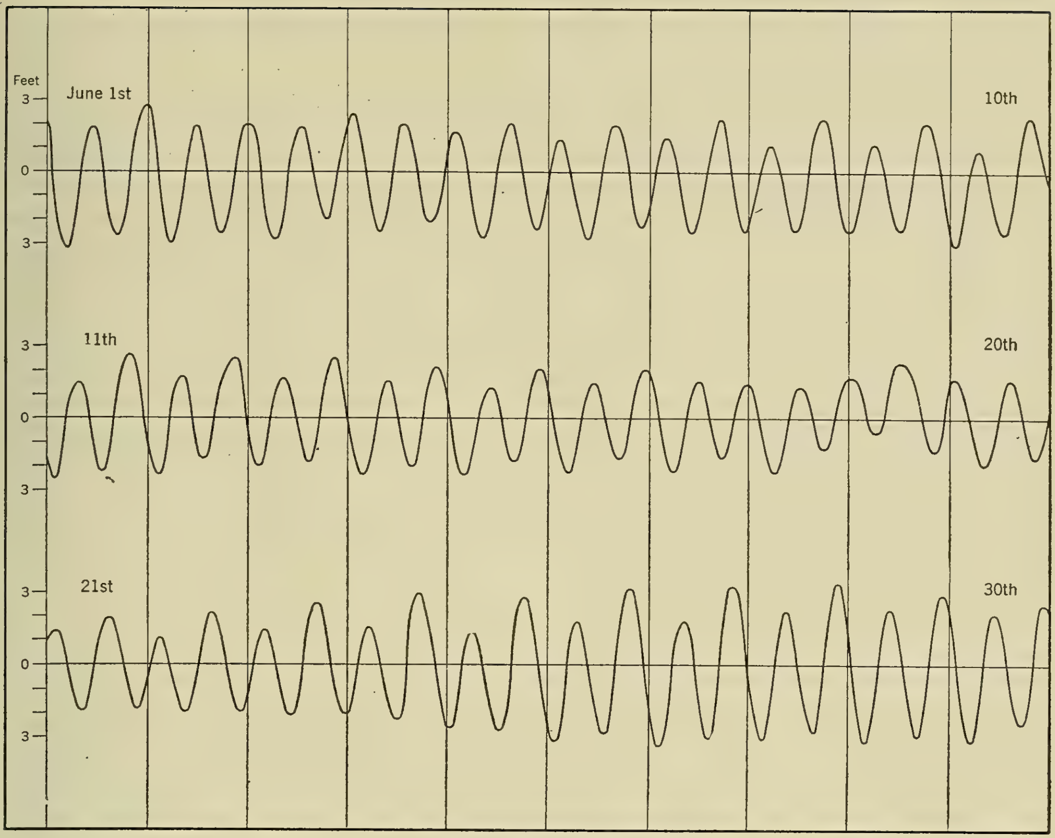

Fig. 4.—Tide curve, New York, June 1934.

tively large. Likewise, from the 18th to the 20th the inequality is small (except for the disturbing effect of a strong wind on the 19th), while from the 25th to the 27th it is again relatively large. If we examine a lunar table for June 1934, we find that on the 4th of the month and again on the 19th the moon was over the Equator, while on the 12th and 26th it was in the tropics.

The semidaily type of tide, therefore, exhibits some inequality, especially when the moon is at its maximum semimonthly north or south declination. The question therefore arises, how great must the inequality in the tide become before that tide ceases to to be classed as a semidaily tide? This question it will be more convenient to consider in connection with the mixed type of tide. For the present it will be sufficient to note that while the semidaily type of tide exhibits diurnal inequality, the magnitude of this inequality in relation to the range of the tide is small.

For the month shown in Figure 4, the tide in New York Harbor averaged 4.4 feet. The smallest range during the month occurred on the 19th, when the height difference between the morning high and low waters was 2.5 feet. The greatest range occurred on the 26th and again on the 28th when the difference between afternoon high water and the following low water was 6.6 feet. On a percentage basis, the smallest range was 43 percent less than the average range for the month, while the greatest range was 50 percent greater than the monthly average. In round numbers, therefore, it may be stated that the range during the month varied from 50 percent below its average value to 50 percent above.

The range of the tide at any place may be very greatly affected by wind and weather. But quite apart from the effects of disturbed meteorological conditions, the range varies in accordance with the astronomical positions of sun and moon relative to the earth, the principal variations being due to the phase, parallax, and declination of the moon, the tide at different places responding in different degree to each of these three causes.

In the semidaily type of tide, the variations in range during a month are largely those due to the phase and parallax of the moon, the variation due to the declination being but of minor importance. In response to the parallax variation, the tides rise higher and fall lower than the average when the moon is near its perigee, and rise and fall least near the times of apogee. In response to the phase variation, the tides rise higher and fall lower near the times of new and full moon, and rise and fall least near the times of the moon's quadratures. As a rough figure, 50 percent variation in range from the monthly average may be taken as characteristic of the semidaily type of tide.

As noted before, diurnal inequality is manifested both in the heights and in the times of the tide. But so far as tidal datum planes are concerned, only the height inequality is of importance, and hence diurnal inequality in times may here be left out of consideration.

The Daily Type of Tide

The daily type of tide may be defined as one in which one high and one low water occur in a tidal day. Figure 5, which pictures the rise and fall of the tide at Pensacola, Florida for the month of June 1934, may be taken as representative of this type of tide.

Figure 5 shows that the tide at Pensacola went through two periods of variation in range, with the minima occurring on the 5th and 20th and the maxima on the 12th and 28th. (The fluctuation from the latter part of the 19th to the early hours of the 20th, which was obviously due to disturbed weather conditions and not to tidal causes, is left out of consideration in this connection.) In June 1934, it will be recalled, the moon was over the Equator on the 5th and 19th, and in the tropics or at its semimonthly maximum declination on the 12th and the 26th. In the daily type of tide, therefore, the variation in range is principally in connection with the declination of the moon, while in the semidaily type of tide it is principally in connection with the phase and parallax of the moon.

For the month of June 1934, the range of tide at Pensacola averaged 1.44 feet. The least range occurred on the 20th when it was 0.5 foot, and the greatest range occurred on the 28th when it was 2.5 feet. On a percentage basis, the least range was 65 percent less than the average monthly value, while the greatest range was 74 percent above that value. In the daily type of tide, therefore, the variation in range of tide during the month is considerably greater than in the semidaily type.

It should be noted that places where the tide is at all times of the daily type are rather uncommon. At Pensacola, for example, there frequently occur two high and two low waters during the day at the times when the moon is over the Equator. In Figure 5 it will be noted that on the 5th of the month, towards the end of the day, there was a marked change in the slope of the tide curve which reveals the effects of the semidaily constituents of the tide. If the tide for the greater part of the month at any place is of the daily type, the tide at that place is designated as belonging to the daily type.

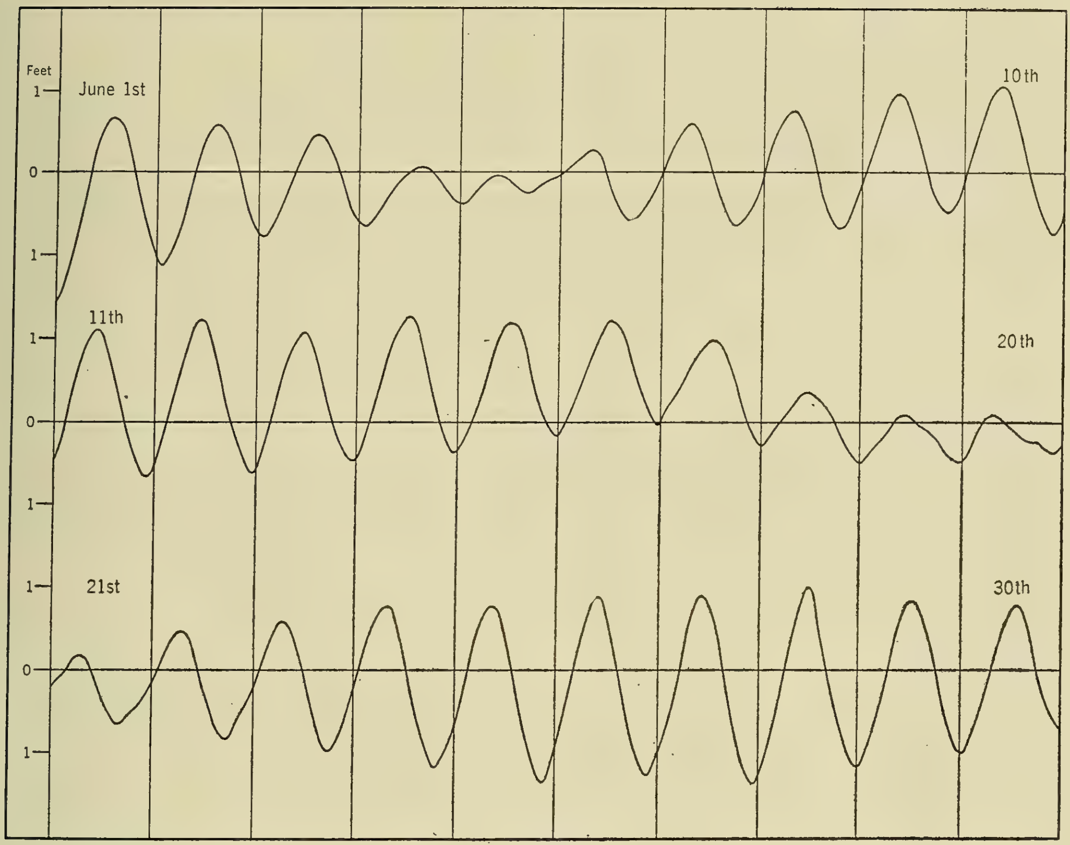

FIG. 5.—Tide curve, Pensacola, June 1934.

The Mixed Type of Tide

The mixed type of tide is defined as one in which two high waters and two low waters occur in a tidal day but with marked diurnal inequality. By its name this type of tide indicates that it arises as a mixture of daily and semidaily constituents of the tide. Strictly, all tides contain daily and semidaily constituents. In the semidaily type, however, the daily constituent is relatively so small that it is overshadowed by the semidaily constituent in the resultant tide. Similarly, in the daily type of tide, the semidaily constituent is relatively so small that the resultant tide exhibits primarily the features of the daily constituent. It is only when the two constituents do not differ greatly in magnitude that the resultant tide clearly reveals the presence of both constituents and the term "mixed tide" is then employed.

In the discussion of the combination of daily and semidaily constituent tides, it was found that the diurnal inequality in the height of the tide may be featured in three different ways: (1) principally in the high waters; (2) principally in the low waters; (3) in approximately equal degree by both high and low waters. The mixed type of tide, therefore, naturally divides itself into the above three classes. Figure 6, which reproduces the record of the rise and fall of the tide at Honolulu, Hawaii, for the month of June 1934, illustrates the mixed type of tide in which the inequality is featured principally by the high waters.

For the month represented in Figure 6, the high waters averaged 0.59 foot above sea level, and the low waters averaged 0.69 below sea level, giving an average range of 1.28 feet. If we take the higher high waters and lower high waters separately, we get an average height for the higher high waters for the month of 1.24 feet above sea level, and for the lower high waters 0.10 foot *below* sea level. Hence, the difference between the higher high and lower high waters for the month averaged 1.34 feet, which is 0.06 foot *greater* than the average difference between the high waters and low waters

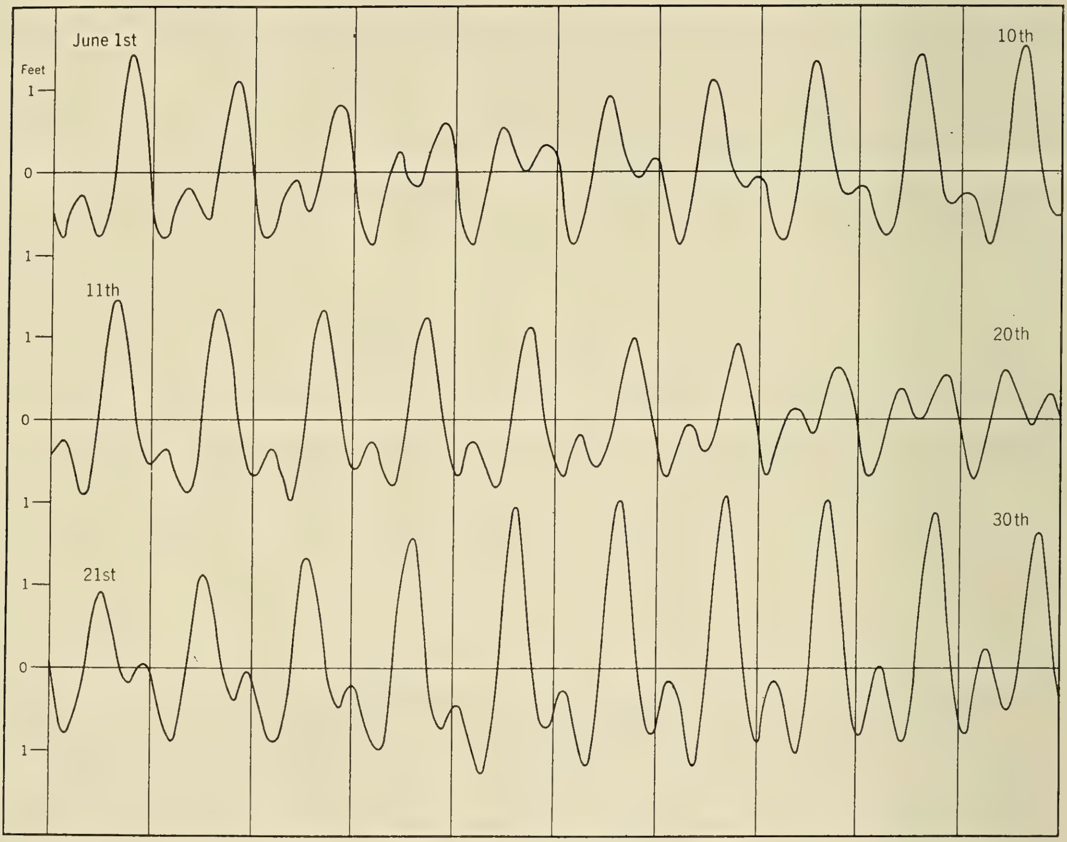

Fig. 6.—Tide curve, Honolulu, June 1934.

for the month. If we average the heights of the lower low and higher low waters for the month separately, we derive values of 0.89 foot and 0.47 below sea level, respectively. The average difference in the heights of lower low and higher low waters is thus 0.42 foot or less than one-third the difference between the higher high and lower high waters.

In Honolulu time, the moon was over the Equator on the 4th and 19th of the month represented in Figure 6, and at its maximum semimonthly declination on the 11th and 26th. The tide curve shows that about the times the moon was over the Equator, the difference between morning and afternoon tides was least, while about the times the moon was in the tropics, the difference was greatest. The greatest difference in height between the two high waters of a day during the month occurred on the

25th when higher high water rose 2.4 feet above the preceding lower high water. The least difference occurred on the 19th when the two high waters differed by but 0.1 foot.

It is of interest to note that on the 5th and again on the 19th, that is, when the moon was close to or over the Equator, there was considerably greater inequality in the low waters than in the high waters. If observations on those days only were available, it would be natural to class the tide here with that form of the mixed type in which the inequality is featured principally in the low waters. Because of the variations to which tides are subject from day to day, it is necessary to have at least a month of observations to determine the character of the tide at any place.

Turning now to a consideration of the second form of the mixed type of tide, in which the inequality is featured principally in the low waters, the tide at Seattle, Washington, may be taken for illustration. Figure 7 shows the tide curve at Seattle for the month of June 1934.

For the month shown in Figure 7, the high waters at Seattle averaged 3.88 feet above sea level and the low waters averaged 3.73 feet below sea level, giving an average range for the month of 7.61 feet. Deriving the heights of the higher and lower tides separately, we find higher high water to average 4.90 feet and lower high water 2.93 feet above sea level; lower low water 7.25 feet below sea level and higher low water 0.05

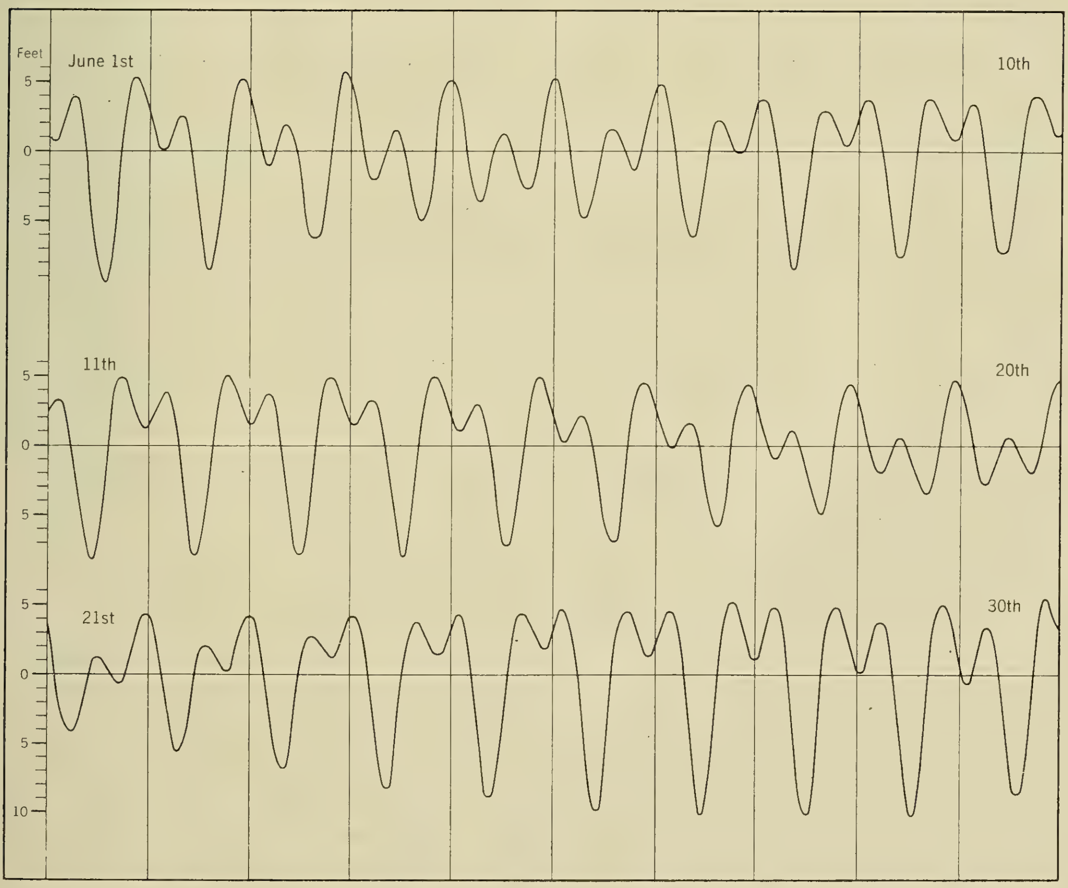

Fig. 7.—Tide curve, Seattle, June 1934.

above sea level. The average difference between the two low waters of the day, therefore, was 7.30 feet, or very nearly as much as the average difference between the high waters and low waters. For the high waters, the average difference between the higher high and lower high waters was 1.97 feet.

For the greater part of the month it is seen that the higher low water did not fall as low as mean sea level which, for each ten-day group of the observations, is represented by the corresponding horizontal line. The differences between the two low waters of a day are seen to be most marked near the times when the moon is in the tropics. In June 1934, the moon was at its maximum semimonthly declination on the 11th and on the 26th. On the 27th, the difference between morning and afternoon low waters was 11.5 feet.

Around the days when the moon was over the Equator, which occurred on the 4th and 19th of the month, it will be noted that the tide at Seattle exhibits the inequality principally in the high waters. This emphasizes the fact which was noted in connection with the Honolulu tides, that at least a month of observations is required to determine the character of the tide at any place.

The third form of the mixed type of tide, in which the inequality is exhibited in approximately equal degree by both high and low waters, is exemplified by the tide at San Pedro (Los Angeles Harbor), California. The tide curve for the month of June 1934 at that place is shown in Figure 8.

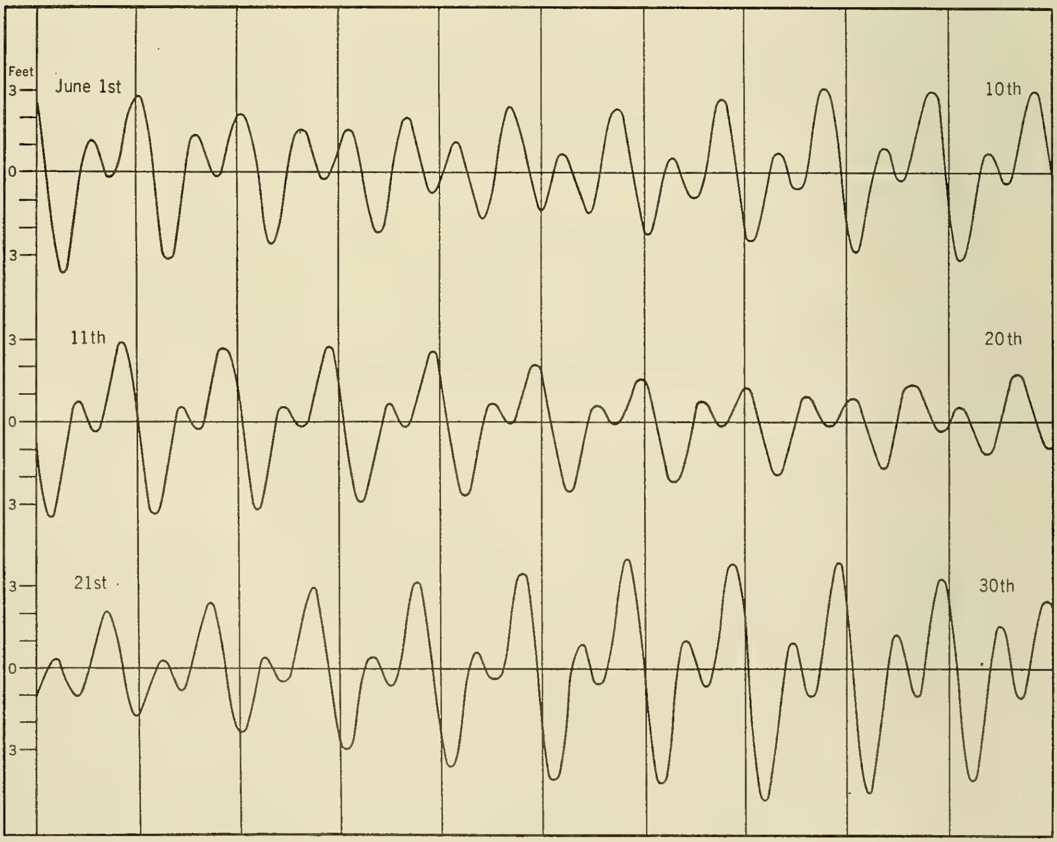

Fig. 8.—Tide curve, Los Angeles Harbor, June 1934.

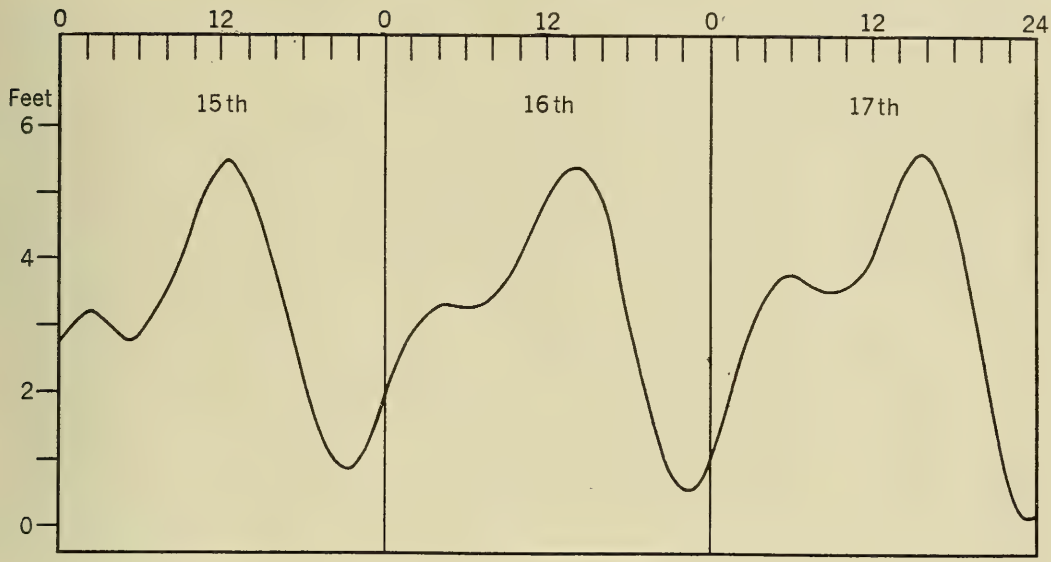

FIG. 9.—Tide curve illustrating the vanishing tide.

In June 1934, the range of tide at San Pedro averaged 3.71 feet, the difference between the higher high waters and lower high waters averaged 2.04 feet and the difference between the higher low waters and lower low waters averaged 2.48 feet. Corresponding to the moon's maximum semimonthly declination on the 11th and 26th the high-water differences were 2.5 feet on the former date and 3.4 feet on the latter date. For the low waters the corresponding differences were 3.3 feet and 3.8 feet, respectively. On the 4th and 19th the moon was over the Equator and the corresponding differences in the high waters and in the low waters were less than a foot.

It will be noted that on days when the inequality in the high waters and in the low waters is large, as for example on the 12th and 25th, the difference between the lower high water and higher low water is small. At certain times the difference becomes so small that these two tides merge, and thus but one high water and one low water occur in a day. This is illustrated by the tide at San Pedro for September 16, 1934, in Figure 9.

In Figure 9 the tides for the three-day period, September 15 to 17, are shown. On the 15th there were two high and two low waters, the difference between the morning high and low waters being 0.4 foot. On the 16th the morning high and low waters had the same height, merging to form what is known as a vanishing tide, resulting in a stand of the water for a period of about three hours. On the 17th, two high and two low waters again appear, the difference between the morning high and low waters being 0.3 foot.

Criteria for the Different Types of Tide

From the brief discussion of the three types of tide in the previous section, it is clear that there are no sharp dividing lines between the different types of tide. For general purposes it may be sufficient to define the semidaily type as one in which there are two high and two low waters a day with but little inequality, while the mixed type is defined as one with two high and two low waters a day which exhibit considerable inequality. But for technical purposes it is of obvious advantage that some definite criterion be available to separate the two types of tide.

Since diurnal inequality in the tide arises from the interaction of daily and semidaily constituents, the criterion employed for defining type of tide makes use of the magnitudes of these constituents. A formula frequently used, which is based on the harmonic constants of the tide, is the ratio of K_1+O_1 to M_2+S_2. In this formula, K_1 and O_1 represent the amplitudes of the principal daily constituents of the tide, and M_2 and S_2 represent the principal semidaily constituents. Where this ratio is less than 0.25, the tide is classed as semidaily; where it is between 0.25 and 1.50 the tide is classed with the mixed type; and where it is greater than 1.50 it is classed with the daily type.

To exemplify the use of this formula, we may employ it to determine the ratio of K_1+O_1 to M_2+S_2 for the tides used in the preceding pages to illustrate the different types. For New York this ratio is 0.18, for Pensacola 10.5, for Honolulu 1.04, for Seattle 0.97 and for San Pedro 0.77. Since the ratio for New York is less than 0.25 the tide there is classed as belonging to the semidaily type. The ratio for Pensacola is more than 1.50 and the tide there is therefore of the daily type. For Honolulu, Seattle and San Pedro the ratio is between 0.25 and 1.50 and the tide at those places is therefore of the mixed type.

When harmonic constants of the tide are not available, a rough approximation to the ratio of K_1+O_1 to M_2+S_2 can be derived from the mean values of the inequalities and range of tide. Approximately $(K_1+O_1)\div(M_2+S_2)$ can be taken as equal to $1.4\,(DHQ+DLQ)\div Mn$, where DHQ and DLQ are the mean values of the high water and low water inequalities and Mn is the mean range of the tide. The derivation of the mean values of these quantities will be considered later.

III. TIDE OBSERVATIONS

Location of Tide Station

In selecting the site for a tide station, a number of factors must be taken into consideration. Of these the more important are free communication for the tide, sufficient depth of water even at extreme low tide, shelter from storm waves, comparative freedom from freshets, and accessibility in all kinds of weather.

In passing over areas of shoal water, the tide is affected profoundly, both in time and in height; hence sites near the heads of tidal bays and rivers are not suitable for a tide station that is to be representative for any considerable area. In tidal rivers draining large areas, the effects of freshets or of seasonal variation in volume of drainage waters are most pronounced in the upper reaches, but become less pronounced farther seaward.

As will be seen later, the range of tide is sensitive to changes in the hydrographic features of a body of water. Hence small bays or bights connecting with the sea through narrow and shallow openings subject to change are, in general, not suitable for tide stations intended for furnishing fixed tidal datum planes.

Tide Staff

The simplest means for obtaining tidal observations consists in the use of a tide staff. This may be made from a board 5 to 6 inches wide and 1 inch thick, graduated to feet and tenths, with the numbers increasing upward. It should be of such length that the extreme fluctuation of the water in the locality in which it is to be used will be within its lowest and highest graduation, and it should be fastened securely in a vertical position to a pile or other suitable support.

Where the surface of the water is disturbed by considerable wave motion, it becomes difficult to read the height of the tide with any degree of accuracy. In such cases it is of advantage to fasten a glass tube to the face of the staff. Stock glass tubing about one-half inch in diameter, having a wall thickness of about one thirty-second of an inch and about 6 feet in length, has been found quite satisfactory. The tubing may be secured to the face of the staff by means of spring clips or cup hooks. The wave motion is reduced by partially closing the submerged end of the tube by a notched cork. A floating object introduced into the tube, such as a thin slice of cork having a diameter somewhat less than that of the tube, then permits the reading of the height of the water with ease to the nearest tenth or half tenth of a foot.

Where the tide staff is to be used for a considerable period of time, and therefore subject to weathering, it will be found of advantage to cut the graduations denoting the feet and tenths into the wood and to form the figures marking each foot from brass upholstery tacks.

To obviate the difficulties resulting from the defacement of the graduations on a wooden tide staff, which in polluted waters may become illegible in a comparatively short time, the Coast and Geodetic Survey makes use of vitrified scales. These are made by baking a vitrified coating on wrought iron strips. The strips are in 3-foot sections about 2½ inches wide, the sections being so graduated that when placed end to end they form a single continuous scale. They may be fastened to a suitable board or piece of timber to form the tide staff.

Readings on the staff should be recorded every half hour or hour, except near the times of high and low water, when the readings should be made every 15 minutes or even more frequently. Continuous observations covering both day and night are most satisfactory, but, where this is not feasible, daylight observations over a period of 13 consecutive hours every day should be made.

Bench Marks

The zero of the tide staff should be connected by spirit levels with at least three good bench marks. This will make possible the replacing of the tide staff at the same elevation during the progress of the observations, should it become destroyed or should its elevation be changed by accident. The bench marks will also serve the further purpose of preserving for future use the datum planes that are determined from the tidal observations. The bench marks should be placed at some distance from each other so that they are all not likely to be destroyed by a common cause.

It is the practice of the Coast and Geodetic Survey to establish and maintain at each tidal station not less than one standard disk bench mark for each year of observations up to 10 years, with a minimum of five such marks for a series of one year in length and a minimum of three for a series less than a year. Three of these bench marks are located within a short leveling distance of the tide staff while the remainder are more widely distributed to insure against loss from a common cause. Care is taken to avoid locating the bench marks on filled-in ground.

The qualities that distinguish a good bench mark are freedom from likelihood of change in elevation and ease of finding and identification. Disk bench marks fulfill these requirements well. The standard tidal bench mark of the Coast and Geodetic Survey consists of a brass disk about 3 inches in diameter, with a shank about 2½ inches long for insertion into a building or other substantial support, and carries the inscription shown in Figure 10.

Permanent and substantial buildings afford the best locations for setting the disk bench marks. The bench mark is countersunk, with its face flush with the surface of the part of the building into which it is set, and is securely cemented in, so that it will effectively resist extraction, rotation, or change of elevation. If the wall of a building is used, the bench mark should be set with its central line horizontal, for it is the elevation of this central line that is taken as the elevation of the bench mark. If a suitable location on a building is found which permits the disk bench mark to be set with its face horizontal, it is to be preferred, since this position is a more convenient one for placing a leveling rod.

A boulder or a ledge of rock makes a very satisfactory location for a bench mark. A good foundation for a bench mark is also furnished by a mass of concrete with its upper surface slightly above the level of the ground, about 2 feet square on the bottom and 1 foot square on top. The mass should extend not less than 3 feet below the surface of the ground; but in localities of severe winters the depth should be sufficient to with-

FIG. 10.—Standard tidal bench mark, United States Coast and Geodetic Survey.

stand frost action. A satisfactory mixture for the concrete consists of one part cement, three parts sand and five parts gravel or broken stone.

If standard disk bench marks are not available, a small cross cut on a rock, building, or other structure may serve the purpose of a bench mark. A copper bolt set into rock or into a cement block makes a satisfactory bench mark. Water hydrants, curbstones, and nails in growing trees, while frequently suitable for temporary use in leveling, do not make satisfactory permanent bench marks.

The bench marks and zero of the tide staff should be connected by a double line of levels, the lines being run in opposite directions. The instructions of the Coast and Geodetic Survey prescribe that when the forward and backward differences in the elevations of two tidal bench marks differ by more than $0.035\sqrt{K}$ feet (K being distance in statute miles leveled between the two bench marks), both the forward and backward leveling between these two bench marks are to be repeated until the difference falls within the required limit. It is important that the leveling record and the descriptions of the bench marks be made in such form that no ambiguity will arise in the use of such records at future times by other persons.

For convenience the following table of maximum closing error allowed in leveling between bench marks is given. It is based on the formula given above.

TABLE 1.—*Maximum allowable errors in leveling between bench marks*

Distance between B. Ms. (feet)	Maximum error allowed (feet)
500 or less	0. 011
1,000	. 015
2,000	. 021
3,000	. 027
4,000	. 030
5,000	. 034

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Box Gage

Where conditions make readings on a plain staff difficult, a box gage may be used. Essentially this consists of a float that rises and falls in a vertical box to which the tide has access. The box may be made of 1-inch boards 12 inches wide, the bottom being closed, except for a hole about 1 inch in diameter through which the tide has access and which reduces the wave motion in the float box very considerably. A convenient form of float for a box gage is a copper cylinder about 8 inches in diameter and 2 or 3 inches high with tapering top and bottom sections.

Various means may be used for determining the rise and fall of the float in a box gage. Where the range of the tide is moderate, a light wooden rod graduated to feet and tenths may be secured to the top of the float and at a convenient point above the top of the float box the rod made to pass through a metal ring secured in such wise that the axis of the rod is vertical. The metal ring serves the further purpose of furnishing a reference point for reading the height of the tide. It is to be noted that it is necessary to graduate the rod with the numbers increasing from top downward, in order that the heights of the tide as read on the rod may be direct and not inverted.

Where the range of the tide, or the distance from the top of the box gage to the surface of the water, is considerable, a graduated steel or phosphor-bronze tape is more convenient than a rod. In this case the lower end of the tape is attached to the float, and the upper end is made to pass over a fixed pulley. To keep the tape in tension, a weight is attached to its upper extremity, and for a reading point for measuring the rise and fall of the float, the tape may be made to pass a metal ring fixed at a convenient distance from the top of the float box, or a board may be fixed vertically near the tape and a reading line marked on the board.

The relation of the zero of a box gage to fixed bench marks on shore may be determined in two different ways. In the first method simultaneous readings of the box gage and a fixed tide staff are made and the relation of the zeros derived. The elevations of the bench marks above the zero of the fixed tide staff are then determined in the usual way, and the relation of the zero of the box gage to the bench marks is then determined through the difference of the zeros of the tide staff and box gage. In this method care must be taken to have the fixed tide staff near the box gage, so that at any instant the height of the tide is the same in the box gage as on the tide staff.

Another method of determining the relation of the zero of the box gage to fixed bench marks on shore consists in determining the elevations of the bench marks relative to the reading point and adding the length of the float rod or tape from the zero graduation to the line of flotation of the float. The first part of this operation is accomplished in the usual manner with the spirit level. The second part is accomplished by floating the float with rod or tape attached in a pan of water, care being taken to have the density of water in the pan the same as that in the float box, and measuring the distance between the zero of the rod or tape and the line of flotation of the float.

Automatic Tide Gages

Where the tide observations are to cover a period of several months, the automatic or self-recording tide gage is the more satisfactory. Various forms of automatic tide gages are on the market, some of these tracing a continuous curve and others printing the height of the tide at regular intervals. For certain purposes the printing gages are preferred, but for general purposes the curve-tracing gages have several advantages, among which may be mentioned the visualizing of any breaks in the record, whether due to stoppage or accidental change in adjustment. The curve-tracing gage furthermore permits the recording and studying of rapid changes in level and of any unusual features in the rise and fall of the tide.

The Coast and Geodetic Survey makes use of two types of curve-tracing tide gages, known respectively as the standard gage and the portable gage. The standard gage is generally used for tide observations except for short series or where local conditions are not suitable, in which cases the portable gage is used.

Standard Tide Gage

The essential parts of the standard tide gage consist of a clock that moves a roll of paper forward at a uniform rate and a float that is free to rise and fall with the tide and which is so connected with a pencil that the latter moves perpendicularly to the motion of the paper and proportional to the rise and fall of the tide. The combined motion of paper and pencil produces a continuous curve known as the tide curve, which shows the rise and fall of the tide to a reduced scale. From this tide curve the height of the tide at any given instant during the period of observations may be determined.

In addition to the clock which moves the paper, or motor clock, there is another clock, known as the time clock, the function of which is to make a short horizontal mark on the record at the beginning of each hour. The paper used on the gage is about 13 inches wide and is furnished in rolls about 70 feet long. Since the motor clock feeds the paper at the rate of 1 inch per hour, a roll is sufficient for 1 month of record. The approximate over-all dimensions of the standard gage are: length, 3 feet; width, 1½ feet; height, 1 foot. A view of the instrument is given in Figure 11.

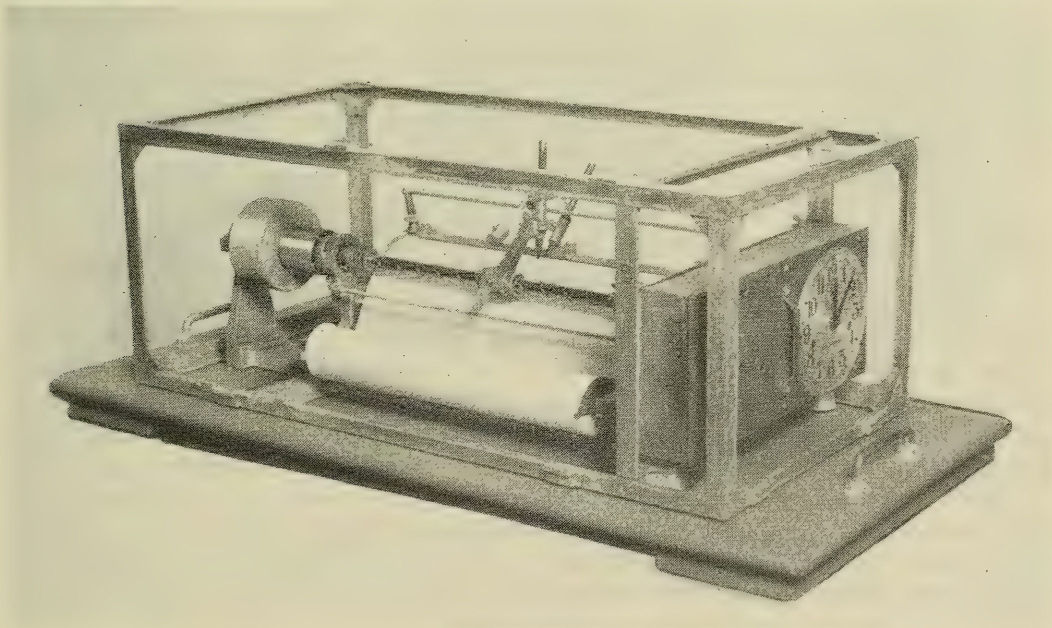

Fig. 11.—Standard tide gage.

In the operation of the standard tide gage, the observer visits the station once a day and makes the appropriate notations by which the record can be related to the fixed tide staff which is part of the installation. The gage also requires a float well and a suitable shelter. A detailed description of the standard tide gage, together with instructions for installation and operation, is given in Coast and Geodetic Survey Special Publication No. 196, entitled "Manual of Tide Observations."

Portable Tide Gage

For use by hydrographic parties in the field, the Coast and Geodetic Survey has developed a portable automatic tide gage shown in Figure 12. This gage is 10 inches square on its base and with its weatherproof metal cover in place is 10 inches high. It was designed to provide a gage which can be easily installed in remote localities where wharves and docks are not available.

The tide curve is made on cross section paper on a drum 7 inches long and 19.2 inches in circumference. This drum is geared to a clock movement within the drum so as to rotate once in 48 hours, giving a time coordinate of 0.4 inch to the hour. By the use of appropriate gear wheels, provision is made for five different height scales, allowing tides from less than 6 feet up to 25 feet to be recorded. In this gage, cross-section paper is used, so that the recording pencil is set to read the same as the tide staff. A detailed description of this gage, together with instructions for installation and operation, will be found in the above-mentioned Special Publication No. 196.

The Tide Record

When the tide record consists of visual readings made on a tide staff, it is of advantage in preparing it for tabulation, to plot these staff readings on cross-section paper to suitable time and height scales. Customarily the time is plotted along the horizontal axis and the height along the vertical axis. A smooth curve is then drawn through the plotted points, from which the height of the tide at any time, or the times and heights of the high and low waters can be scaled.

Plotting the staff readings on cross-section paper permits smoothing out accidental irregularities in the tide curve and the detection of errors. It also permits a more accurate determination of the times and heights of the high and low waters.

A convenient form for plotting staff readings consists in plotting a number of successive days under each other on the same sheet. Quite apart from the economy in cross-section paper, this method brings out any departures from normal conditions and aids in the interpolation of breaks in the record. Figure 13 shows on a reduced scale the plottings of the tide curves at Boston for the first 6 days of July 1944 derived by plotting the hourly heights of the tide. The height scale is shown from 8 to 12 feet for the first tide curve, but for the others only the 8-foot line is marked.

With automatic tide gages employing cross-section paper, as in the case of the portable gage, the recording pencil is set to give the height referred to a tide staff. Hence such a record is ready for tabulation as soon as taken off the gage. A comparison of the tide curve with the time and height notes made by the observer will indicate whether any time and height corrections are required. Generally no such corrections

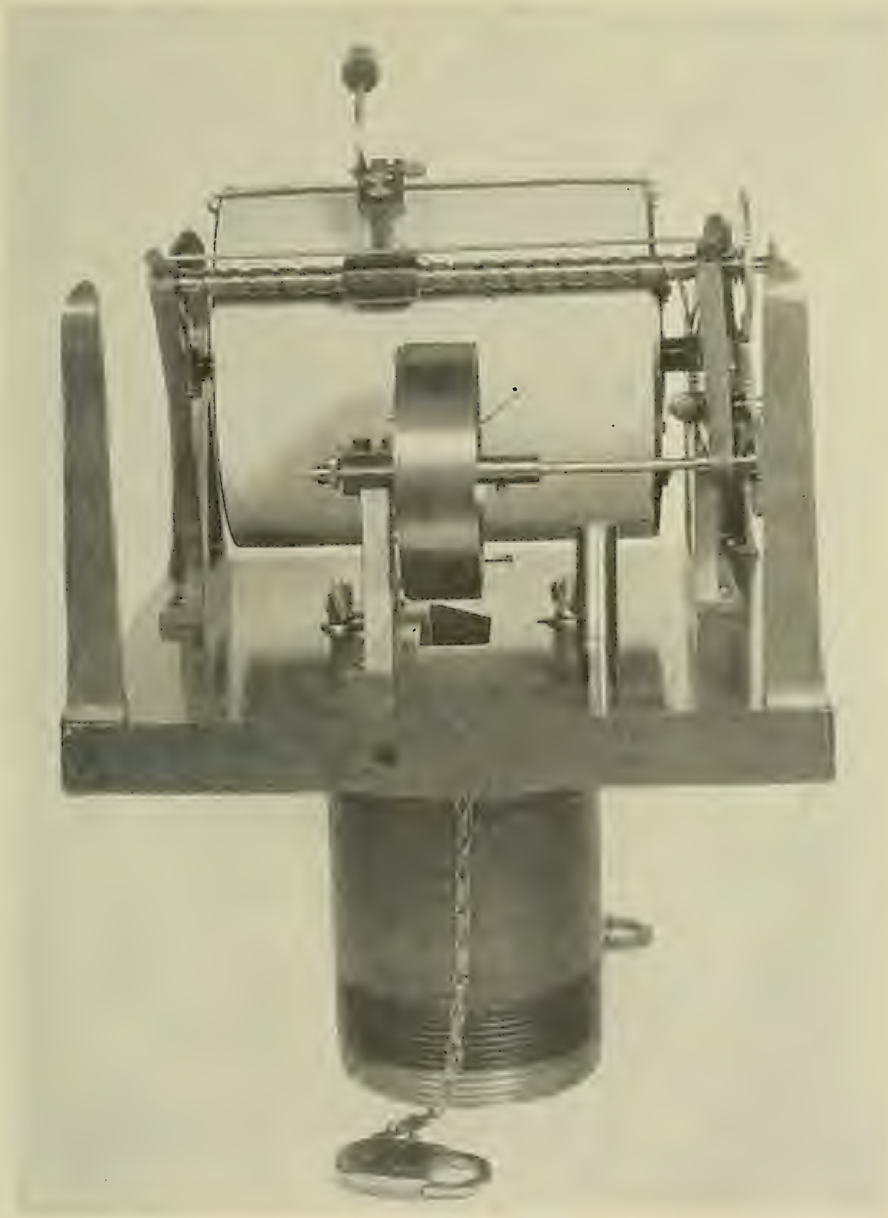

FIG. 12.—Portable tide gage.

are necessary since in tabulating tide records it is customary to tabulate times to the nearest tenth of an hour and heights to the nearest tenth of a foot, except in regions of little range of tide, in which case heights are tabulated to the nearest half tenth or even closer.

The standard gage does not employ cross-section paper. The record is made on a roll of plain paper on which the time scale is 1 inch to the hour, each roll generally containing the record for a calendar month. At the beginning of each hour the hour marking device on the gage causes the pencil to make a short horizontal stroke, thus

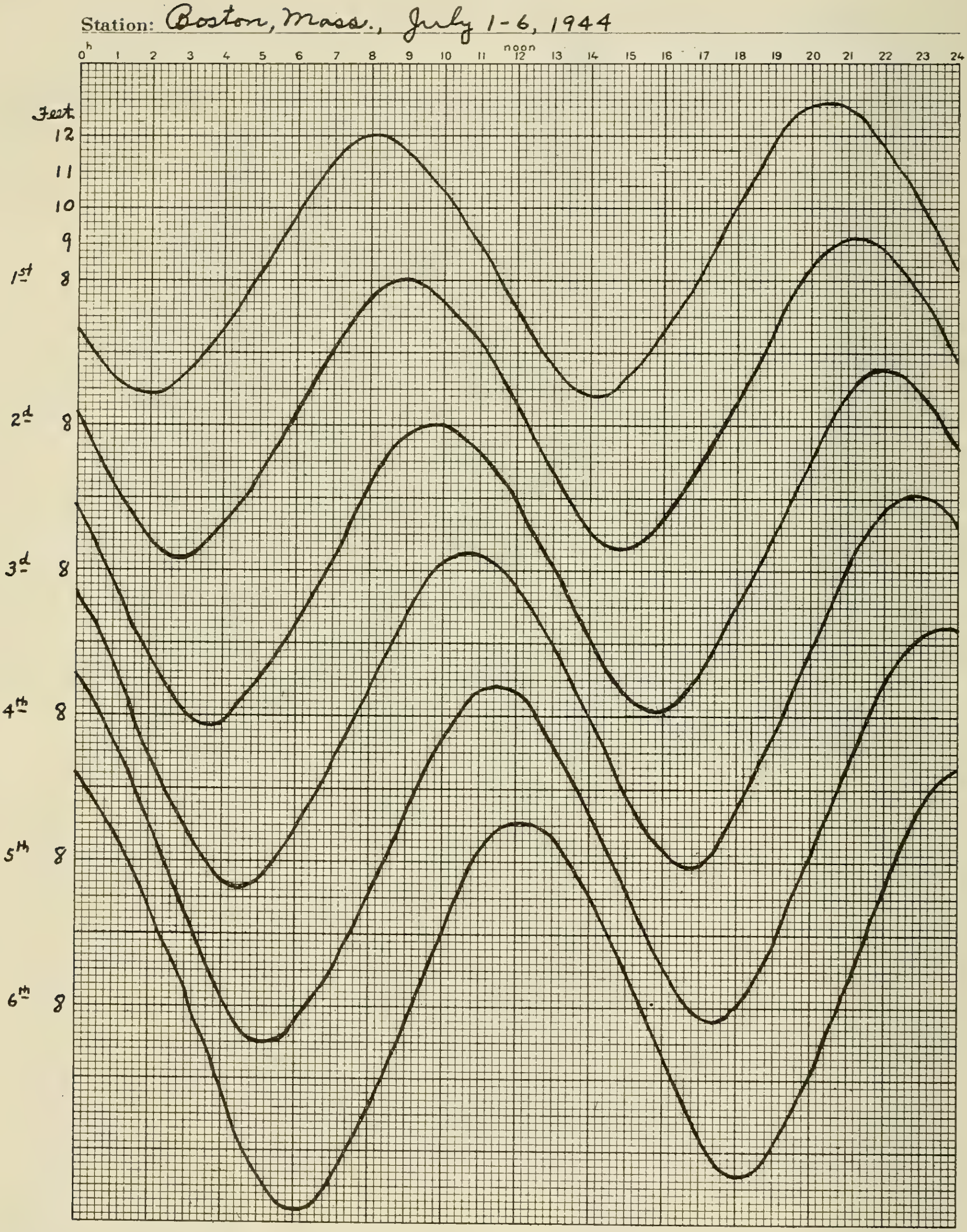

FIG. 13.—Tide curves, Boston, Mass., plotted from hourly heights of the tide.

indicating on the curve the beginning of the hour. Every day when the observer visits the tide station, he stamps and fills in the data of the note shown in Figure 14. This gives the necessary information for marking the successive hours of the day and for referring the heights on the curve to the tide staff. The observer reads the staff to the nearest half tenth of a foot if the water is free from waves, or to the nearest tenth giving the highest and lowest readings. The place on the curve to which the time and height note pertains is indicated by a vertical line.

On the first day of each month the tide roll is taken off the gage and a new roll put on. The observer notes on the roll the name of the station, the reduction-scale of the gage and the kind of time used. The roll is then ready for tabulation.

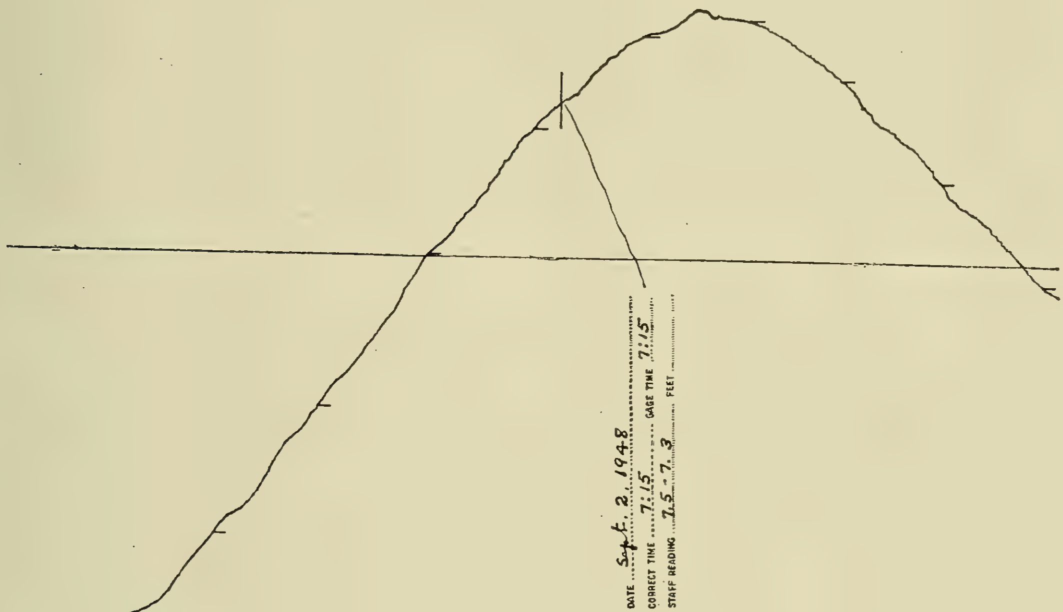

Fig. 14.—Tide curve from standard gage, and comparative note.

IV. TABULATION OF THE TIDE RECORD

Hourly Heights and High and Low Waters

A complete tabulation of a tide record comprises two sets of tabulations, the first giving the hourly heights of the tide and the second the times and heights of the high and low waters. In addition to constituting a full and convenient record of the tide, these two tabulations furnish the data requisite for the determination of all tidal datum planes and the characteristic features of the tide.

For convenience in tabulating and in filing, the Coast and Geodetic Survey makes use of printed forms 8 by 10½ inches for tabulating the hourly heights and the high and low waters. Both sides of the sheets are used, one sheet of high and low waters covering a month and one sheet of hourly heights covering two weeks. Specimen copies of these forms are shown on a reduced scale in Figures 15 and 16. The wide spacing of the days on the form for hourly heights is brought about by the fact that these tabulated forms are used for other purposes in connection with certain stencils which require that particular spacing.

The Record on Cross-Section Paper

If the tide record is on cross-section paper, whether made by an automatic tide gage or plotted from staff readings, the tabulation is a relatively simple matter. Generally no time corrections are necessary, since corrections up to three minutes are ignored; but should time corrections be necessary, the tabulator indicates them on the tide curves. The height of the tide pertaining to each hour of the day is then read from the tide curve and entered into its appropriate place on the hourly height form. The tabulation of the hourly heights for each sheet of curves is completed before taking up the tabulation of the high and low waters. A specimen page of hourly heights of the tide for the week beginning June 24, 1944, for Boston, Mass., is shown in Figure 15.

The horizontal and vertical sums shown to the right and bottom in Figure 15 are obtained later in connection with the determination of mean sea level. The figures in the horizontal column "Day of series" give the sequence of each day with reference to the beginning of the series. When a tide station is continued for a number of years it is most convenient to begin each series of observations on the 1st of January and continue the tabulation of the hourly heights consecutively throughout the year. Table 2 gives the day of series, the page, and the column corresponding to the 1st of every month and the last day of the year for a series beginning January 1. This table serves as a convenient check to insure against the omission or duplication of a day in the tabulation of the hourly heights of the tide.

On the sheet of hourly heights that has the tabulation for the last day of a month, the sum of the hourly heights is entered, and after dividing by the appropriate divisor

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Form 362
Ed. May, 1929
DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY

TIDES: HOURLY HEIGHTS

Station: Boston, Mass. Year: 1944

Observer: Lat. 42° 21.3' N Long. 71° 03.05' W

Time Meridian: 60°W Height datum is staff zero which is 21.45 ft. below B. M. 7

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Month and Day	mo. d. June 24	d. 25	d. 26	d. 27	d. 28	d. 29	d. 30	Horizontal Sum
Day of Series	176	177	178	179	180	181	182	
Hour	Feet	Feet	Feet	Feet	Feet	Feet	Feet	Feet
0	11.0	10.0	8.2	6.5	5.2	5.0	5.6	51.5
1	13.0	11.9	9.9	8.0	6.3	5.4	5.0	59.5
2	13.9	13.7	11.6	9.6	7.7	6.4	5.5	68.4
3	13.8	14.2	13.0	11.3	8.3	7.8	6.5	74.9
4	12.5	13.8	13.3	12.4	10.8	9.4	8.0	80.2
5	10.6	12.4	12.7	12.5	11.8	10.9	9.7	80.6
6	8.4	10.4	11.0	11.5	11.8	11.7	11.0	75.8
7	6.2	8.2	9.0	9.9	10.7	11.5	11.7	67.2
8	4.6	6.1	7.1	8.1	9.3	10.5	11.3	57.0
9	4.3	4.7	5.3	6.3	7.6	9.1	10.3	47.6
10	5.4	4.9	4.4	4.7	5.9	7.3	8.9	41.5
11	7.0	6.1	4.8	4.2	4.6	5.7	7.2	39.6
Noon	9.0	7.7	6.1	5.0	4.4	4.7	5.7	42.6
13	11.1	9.6	7.8	6.2	5.2	4.7	4.7	49.3
14	12.8	11.5	9.5	7.8	6.6	5.8	5.0	59.0
15	13.3	12.9	11.4	9.5	8.1	7.1	6.2	68.5
16	12.8	12.9	12.3	11.2	9.9	8.8	7.6	75.5
17	11.5	12.1	12.2	11.8	11.3	10.5	9.3	78.7
18	9.7	10.7	11.3	11.7	11.8	11.7	11.0	77.9
19	7.9	8.9	9.7	10.7	11.4	12.1	12.2	72.9
20	6.6	7.0	8.1	9.2	10.3	11.5	12.4	65.1
21	5.6	5.7	6.4	7.4	8.8	10.2	11.5	55.6
22	6.4	5.4	5.3	5.8	7.1	8.7	10.2	48.9
23	8.0	6.6	5.3	4.9	5.7	7.0	8.5	46.0
Sum	225.4	227.4	215.7		200.6	203.5	205.0	1483.8

Sum for June = 61807 Divisor=(28d) 672; (29d) 696; (30d) 720; (31d) 744. Mean for month = 8.58

Tabulated by M.A.W. Date 7-25-44 Summed by M.A.W. Date 7-25-44

FIG. 15.—Specimen sheet, tabulation of hourly height of tide.

TABLE 2.—*Day of series, page, and column for hourly height tabulations beginning January 1*

Common year				Leap year			
Month	Page	Column	Day of series	Month	Page	Column	Day of series
Jan. 1	1	1	1	Jan. 1	1	1	1
Feb. 1	5	4	32	Feb. 1	5	4	32
Mar. 1	9	4	60	Mar. 1	9	5	61
Apr. 1	13	7	91	Apr. 1	14	1	92
May 1	18	2	121	May 1	18	3	122
June 1	22	5	152	June 1	22	6	153
July 1	26	7	182	July 1	27	1	183
Aug. 1	31	3	213	Aug. 1	31	4	214
Sept. 1	35	6	244	Sept. 1	35	7	245
Oct. 1	40	1	274	Oct. 1	40	2	275
Nov. 1	44	4	305	Nov. 1	44	5	306
Dec. 1	48	6	335	Dec. 1	48	7	336
Dec. 31	53	1	365	Dec. 31	53	2	366

for the month the mean value of sea level for the month is entered, as shown on Figure 15.

After the tabulation of the hourly heights is completed, the high and low waters are tabulated. A specimen sheet of the tabulated high and low waters for the last half of June 1944 is shown in Figure 16. The first half of the month is tabulated on the other side of the tabulation form.

In tabulating the high and low waters, the tabulator notes in succession the highest and lowest points of the tide curve, tabulating the times to the nearest tenth of an hour and the heights to the nearest tenth of a foot read directly from the cross-section paper. However, with tides of small range, it is better to tabulate the heights to the nearest half-tenth or even to the nearest hundredth of a foot.

It is important to note that in determining the points of high and low water on the tide curve, which points give the times and heights to be tabulated, attention is to be centered on an arc of the curve that covers a time interval of about an hour each side of the high or low waters. The highest or lowest part of the smooth arc is chosen for the high or low water and not merely the highest or lowest point on the curve, which may be due to wave action or other disturbing factors. This matter will receive further consideration in connection with the discussion of irregularities in tide curves.

In tabulating times to the nearest tenth of an hour and heights to the nearest tenth of a foot, provision must be made for the tabulation of values which lie exactly half way between tenths. For example, 8.25 hours may be tabulated either as 8.2 or 8.3 hours and, likewise, 6.75 feet may be tabulated as 6.7 or 6.8 feet. Obviously, some definite rule is desirable for such cases. A rule sometimes used is to drop the last figure, but this introduces a systematic error. A much better rule in such cases is to make the first decimal place even; for example, 8.25 would be tabulated 8.2 while 8.35 would be tabulated 8.4.

When the tabulation of high and low waters for a calendar month has been completed, the heights of the high and low waters are summed and the average values for the month derived. As shown on Figure 16, there are spaces provided for deriving the

Form 138
DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Ed. Nov. 1929

TIDES: HIGH AND LOW WATERS

Station Boston, Mass.

Highest tide: Date 21, *Height* 15.2 *ft.* *Lowest tide: Date* 9, *Height* 1.9 *ft.*

$(K_1+O_1) \div M_2$ *or* $2(DHQ+DLQ) \div Mn=$ .... $F(Mn) \div$ .... $F_1=$ ....

DATE Year mo. d.	MOON'S TRANSITS (Greenwich mean civil) hr. dec.	TIME OF— High Water hr. dec.	TIME OF— Low Water hr. dec.	LUNITIDAL INTERVAL High Water hr. dec.	LUNITIDAL INTERVAL Low Water hr. dec.	HEIGHT OF— High Water feet	HEIGHT OF— Low Water feet	REMARKS
Brought forward						433.1	115.0	
18		10.2	3.9			13.1	2.7	
		22.4	16.4			14.1	3.3	
19		11.1	4.8			13.0	2.5	
		23.4	17.3			14.8	3.9	
20		11.7	5.6			13.3	3.0	
		—	17.8			—	4.2	
21		0.3	6.6			15.2	3.2	
		12.8	18.7			13.5	4.3	
22		1.0	7.2			14.6	3.2	
		13.4	19.5			13.2	4.3	
23		1.6	8.0			14.2	3.3	
		14.4	20.2			13.0	4.7	
24		2.5	8.5			14.0	4.2	
		14.9	21.1			13.4	5.6	
25		2.9	9.4			14.3	4.5	
		15.5	21.6			13.1	5.4	
26		3.9	10.1			13.3	4.4	
		16.4	22.4			12.5	5.1	
27		4.6	10.9			12.6	4.2	
		17.4	23.3			11.9	4.9	
28		5.6	11.7			12.0	4.3	
		18.1	—			11.8	—	
29		6.3	0.0			11.8	4.9	
		19.1	12.5			12.1	4.5	
30		7.2	0.7			11.8	5.0	
		19.6	13.3			12.5	4.7	
31								
						58	58	
Sums						762.2	219.3	
						13.14	3.78	
		Correction to intervals				3.78		
		Local intervals				9.36	*Mn*	
		Greenwich intervals				8.46	*MTL*	

	HHW	LLW	
			Sums
			Means

	Mn	DHQ	DLQ
Observed			
Factor			
Corrected			

Tabulated by *Date* *Checked by*

Reduced by *Date* *Checked by*

16—32185-1 U. S. GOVERNMENT PRINTING OFFICE

FIG. 16.—Specimen sheet, tabulation of high and low waters.

range of the tide for the month (Mn), the half-tide or mean tide level (MTL), higher high water, lower low water and the height inequalities.

The Record From the Standard Tide Gage

Before a tabulation of the hourly heights or of the high and low waters can be made from the tide record furnished by the standard tide gage, it is necessary to determine the relation of the curve to the zero of the tide staff. This is done by means of a tabulation of comparative readings of staff and curve, using a reading scale graduated in feet and tenths to the same scale as that to which the tide curve is drawn by the tide gage. A specimen sheet of the tabulation of the comparative readings for the tide record at Charleston, S. C., for the month of November 1948, is shown in Figure 17.

In the first three columns of the comparative readings tabulation the tabulator notes, respectively, the day, the time of staff reading, and the height of staff, which items are taken from the tide roll as recorded in the observer's notes. In the fourth column the tabulator notes the height of the curve by his reading scale at the time of the staff reading. This height obviously will depend on the height assumed for the datum line on the curve. It is most convenient to assume for the datum line a height which will be somewhat less than the staff reading for that point on the curve, so that the differences between staff and scale will be positive and lie between zero and 2 feet. The scale reading for the datum line in the specimen sheet of comparative readings shown in Figure 17 was taken as 5 feet. It will be noted that the staff height in the third column and the scale reading from the tide curve in the fourth column are taken to the nearest half-tenth of a foot.

In the fifth column of the comparative readings tabulation the difference between staff and scale is derived, and in the sixth column the phase of the tide at the time of staff reading is noted. The letters *F*, *R*, *H*, and *L* are used to designate, respectively, the falling tide, rising tide, high water, and low water.

For the period of the month shown in Figure 17, the mean difference between scale and staff is found to be 0.34 foot. With a preliminary setting of 5 feet the height of the datum line on the tide rolls is 5.34 feet. A constant of 0.01 foot is included to refer all the tabulations to a fixed datum, since in April 1948 a new tide staff was installed which leveling to bench marks showed was 0.01 foot higher than the old staff. Hence the correct height of the datum line is 5.35 feet. This height the tabulator marks on his reading scale which is then used for tabulating that month's record.

Any change in the adjustment of the gage during the month will change the relation between staff and scale. In such a case the two parts of the record are treated separately. The scale settings are computed separately for the two parts of the record and each part tabulated in accordance with its proper scale setting.

The differences between staff and scale in the fifth column of the comparative readings tabulation will vary somewhat from day to day, primarily because of the difficulty of reading the staff to the nearest half-tenth of a foot if any wave motion is present. Since the figures on the staff increase upward an error of a foot is occasionally made by the observer in reading the staff, and this error of 1 foot will appear in the column of differences. An error of this kind, however, is easily noted and should be corrected before the differences are summed for the derivation of the mean.

Form 455
DEPARTMENT OF COMMERCE
COAST AND GEODETIC SURVEY
Ed. Sept. 1929

TIDES: COMPARATIVE READINGS

Station: Charleston, S.C. Lat. 32° 46'.9 N
Party of Time meridian 75° W Long. 79° 55'.5 W
Obs. begin 1921 Obs. end Tabulated by M A W Date 12-13-48
Tide gage No. Scale 1:12 Preliminary scale setting of datum line 5.0 feet

DATE Year 1948 mo.	d.	TIME OF STAFF READING h.	m.	STAFF A feet	SCALE B feet	DIFFERENCE A—B feet	PHASE OF TIDE*
Nov.	1	13	34	2.38	2.10	0.28	F
	2	12	12	5.60	5.25	0.35	F
	4	13	07	6.20	5.90	0.30	F
	5	13	10	6.95	6.60	0.35	F
	6	10	39	7.65	7.30	0.35	R
	8	13	16	8.00	7.60	0.40	R
	9	13	13	7.70	7.30	0.40	R
	10	13	24	7.20	6.90	0.30	R
	11	12	50	5.55	5.15	0.40	R
	12	13	35	5.65	5.35	0.30	R
	13	11	41	3.20	2.90	0.30	L
	15	13	04	3.18	2.85	0.33	L
	16	13	07	3.25	2.95	0.30	F
	17	13	35	3.10	2.80	0.30	F
	18	13	21	4.50	4.20	0.30	F
	19	13	19	5.48	5.10	0.38	F
	20	10	59	7.80	7.45	0.35	F
	21	13	17	8.12	7.80	0.32	F
	22	13	07	8.50	8.15	0.35	H
	24	13	29	8.70	8.35	0.35	R
	25	11	18	5.00	4.60	0.40	R
	26	13	36	6.90	6.55	0.35	R
	27	11	04	2.45	2.10	0.35	F
	29	13	19	1.92	1.55	0.37	L
	30	13	41	2.65	2.35	0.30	F
Dec.	1	13	57	3.10	2.80	0.30	F

REMARKS

Scale setting for Nov. 1 to Dec. 1

Sum of differences	8.78
Mean difference (26)	0.34
Preliminary setting	5.00
Setting for reduction to tide staff	5.34
Constant for fixed datum	0.01
Setting for reduction to fixed datum	5.35

*In the column headed "Phases of Tide" write the appropriate one of the four following symbols: H, for high water; L, for low water; R, for rising tide; and F, for falling tide. Use Form 136 for tabulating high and low water.

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FIG. 17.—Specimen sheet, tabulation of comparative readings.

In general, during periods when no change is made in the adjustment of the gage, the differences between staff and scale will be approximately constant. Any difference which stands out strikingly from the others should be rejected from the computation of the mean difference.

It is to be observed that the differences between staff and scale will vary systematically if the inlet to the float well becomes clogged. In that case the difference will be greater than the average for the rising tide and less than the average for the falling tide. These differences thus furnish a check on the proper functioning of the float well.

With the determination of the corrected setting for the scale, the tabulation of the hourly heights of the tide and of the high and low waters is carried on as outlined in the preceding paragraphs for the tabulation of the record on cross-section paper.

In connection with the tabulation of the tide record it is assumed that throughout the period of observations the tabulations are referred to a staff the zero of which is maintained at a fixed level. If during the period of observations the staff is changed, the height relation between the two positions of the staff must be accurately determined. Whenever possible it is preferable to take account of this change in staff in connection with the tabulation, so that the whole series may be referred to the same staff. However, it frequently happens that the tabulations must be made prior to the determination of the exact relationship between the two staffs. In that case full explanation should be noted in the column of remarks of the hourly ordinates and high and low water tabulations; and, as soon as the correction necessary to reduce these readings to the zero of the previous staff is determined, this correction should be noted on the tabulated sheets.

Irregularities in Tide Curves

To secure a correct representation of the rise and fall of the tide, the inlet to the float well of an automatic tide gage is made sufficiently large to insure free communication with the water outside the float well. If the inlet is too small, the tide curve will show a smaller range of tide and a retardation in the times of high and low water. As a result of making it large enough to insure free communication, disturbed conditions of the sea will be reflected by irregularities in the tide curve. An example of such irregularities is shown in Figure 18, which is a representation on a reduced scale of the San Francisco tide curve for November 21, 1910.

Two kinds of irregularities in the tide curve are seen in Figure 18. The first consists of numerous small "saw teeth," which appear throughout the curve and which show up particularly well between the 11th and 12th hours of the day. These "saw teeth" represent the rise and fall of the larger waves and ocean swells which enter San Francisco Bay from the ocean. The second irregularity consists of larger and slower fluctuations, which appear suddenly about 4:45 a. m. and continue with diminishing amplitude until about 10 o'clock. These fluctuations are brought about by stationary-wave oscillations of the water within the bay and are known as seiches.

Seiches are brought about by various agencies. Heavy winds, sudden variations in barometric pressure, and seismic waves due to seaquakes—all these bring about seiche movements of the water which cause irregularities in the tide curve. The seiches shown in Figure 18 were caused by a rapid fall and rise of atmospheric pressure at San Francisco. In Figure 19 are shown seiches due to heavy winds. The curves of that figure reproduce the tide curves at Atlantic City, N. J., for the first 3 days of January

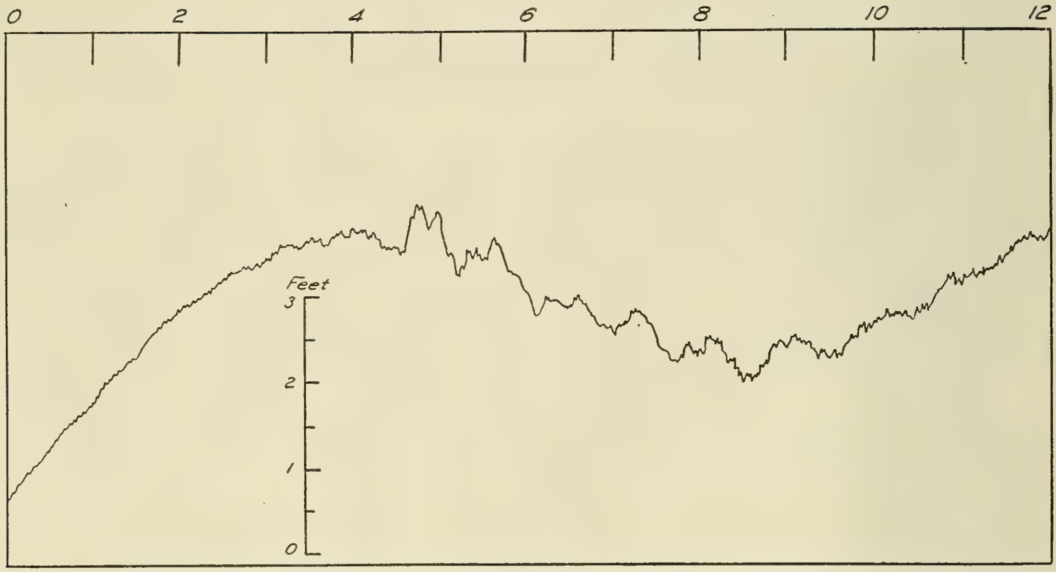

FIG. 18.—Tide curve, San Francisco, Calif., showing seiches due to changes in barometric pressure.

1925. Seiches are evident on the curve for January 1, but are especially marked on the 2d, between 11 a. m. and 12 p. m. On that day the wind blew from the northeast with velocities up to 78 miles per hour.

The Atlantic City tide gage is located about 1,500 feet from shore, on a pier that juts out into the open sea. The seiches must therefore represent oscillations of some part of a wide embayment of the coast.

Figure 20 is an example of seiches due to a seaquake. On April 1, 1946 at $12^h 29^m$ p. m., Greenwich civil time, a seaquake occurred in the North Pacific Ocean, about

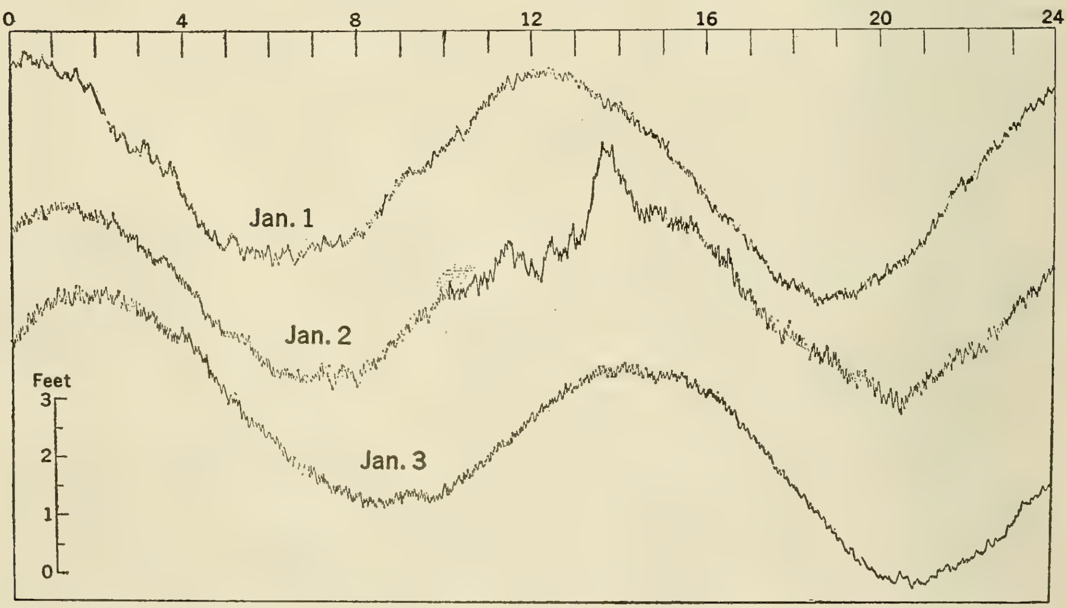

FIG. 19.—Tide curve, Atlantic City, N. J., showing seiches due to heavy winds.

60 miles south of Unimak Island, Alaska Peninsula. The curve of Figure 20 reproduces the tide curve at Valparaiso, Chile, for the first 10 hours of April 2, 1946. For the first six hours the curve is the normal tide curve for that place, but a little after 6, large seiches are recorded, these seiches resulting from the seaquake which occurred 8,000 miles to the northwest 18 hours previously.

At some places, seiche is an almost constant accompaniment of the tide. Figure 21 reproduces on a reduced scale the tide curve at Mormon Island inside of Los Angeles

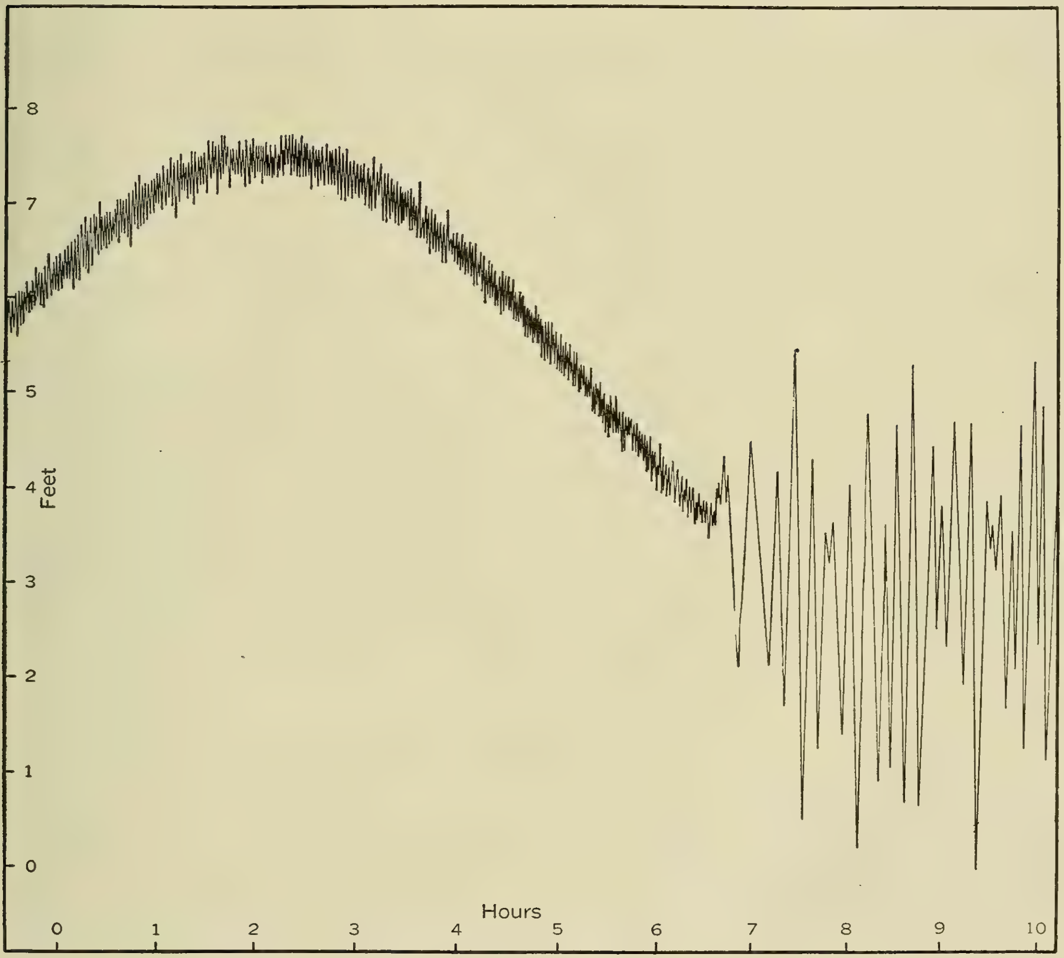

FIG. 20.—Tide curve, Valparaiso, Chile, showing seiches due to distant seaquake.

Harbor for the forenoon of January 10, 1951. The seiche here is practically a constant feature, although varying in amplitude throughout the year. For the forenoon shown, the range of tide was 4.5 feet and the seiche had a range of about half a foot, but at times the seiche here may have a range of 1½ feet.

In the tabulation of the tidal record "saw teeth" and seiches introduce difficulties. For use in the determination of tidal datum planes it is preferable to consider a smooth curve drawn through such irregularities and tabulate the hourly heights directly from this smooth curve. The times and heights of the high and low waters should also be

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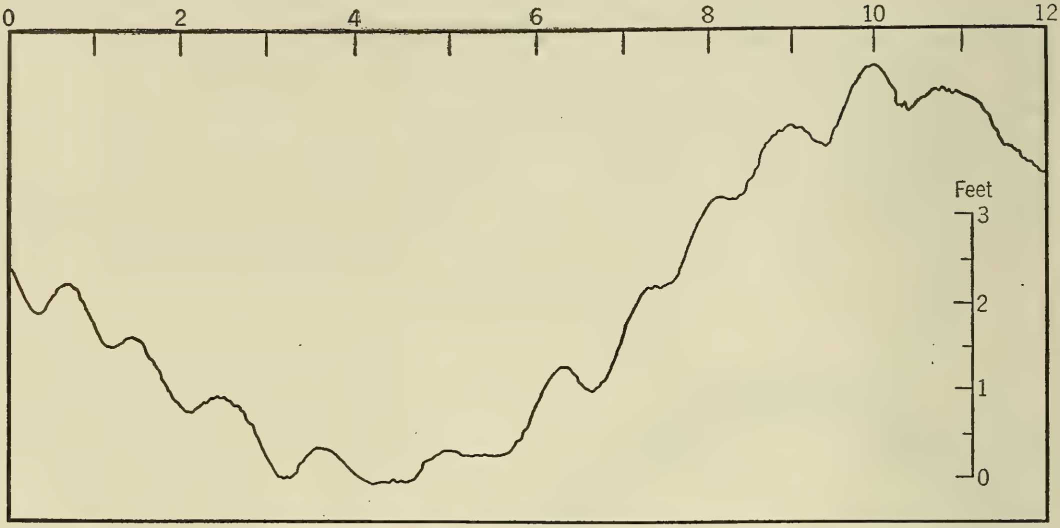

FIG. 21.—Tide curve, Los Angeles Harbor, illustrating seiches.

tabulated from the smooth curve, but note should be made in the column of remarks of the time and height of the highest (or lowest) point of the short-period oscillations.

Smoothing the tide curve must not be carried beyond the legitimate purpose of eliminating short-period oscillations. When the tide curve is disturbed in time and height by unusual weather conditions, the tabulator may be tempted to substitute for the actual tide curve a hypothetical tide curve which disregards the disturbances in time and height, on the mistaken notion that better mean values are derived through this substitution. A moment's reflection will make it evident that in such cases it is much better to tabulate the data directly from the actual tide curve and reject, if necessary, the disturbed values from the computation for mean values.

Tides of Small Range

For tides of small range—less than half a foot—the tabulation of the hourly heights presents no special problems, except that it may be desirable to tabulate these heights to the nearest half tenth or even to the nearest hundredth of a foot. But in the tabulation of the high and low waters of tides of small range, difficulties are encountered.

The tide curve is rarely free from the effects of wind and weather. With large ranges of tide these effects are generally only a fraction of the range and hence it is not difficult to pick out on the curve the high and low waters. But with tides of small range the weather effects are frequently of the same or even of greater magnitude than the purely tidal range, so that the highest and lowest points of the curve may depart widely from the times and heights of the undisturbed tide.

An even more troublesome feature is that the weather effects may produce more than the regular number of high and low waters in a day. It becomes difficult then to determine which of these to tabulate as the high and low waters of the day.

A similar difficulty is presented in tabulating tides of the mixed and daily types at times when the higher lows and lower highs tend to merge. At such times the range between the higher low and lower high is small, and even moderate weather effects so disturb the curve as to make it difficult to determine whether there are a true high water and a true low water intervening between the higher high water and lower low water.

In the latter case it is the practice to disregard the lower high waters and higher low waters when the difference between them is less than a tenth of a foot. While this practice furnishes a definite, even if an arbitrary criterion, in the tabulation of the tide curve, it does not altogether resolve the difficulty when it comes to the determination of the datums of mean high water and mean low water. Further consideration will be given this matter in the detailed discussion of these datums.

Interpolation of Breaks in the Record

Since the time and height of tide varies from day to day, it is desirable both for the purpose of determining mean values and for purposes of comparison to interpolate any breaks that may occur in the tide record at stations where the series of observations cover several months or more. Various methods may be used, depending on the location of the station and the duration of the break. In general the procedure is to tabulate the hourly ordinates and the high and low waters for such portions of the record as are complete, leaving the interpolations to be made later.

To distinguish interpolated values from those derived directly from the tide record, the interpolated values are tabulated in red ink or they are inclosed in parentheses. If the duration of the break is no more than a day or two, a convenient method is to interpolate linearly the times and heights of the high and low waters. These interpolated values are then used for constructing the tide curve on cross-section paper, from which the hourly ordinates are tabulated. An example will make this method clear.

Suppose that on June 26, 1944, the tide gage at Boston had failed to function. In tabulating the record for that month, the tabulator would leave that day blank in both the hourly heights and in the high and low water tabulations (figs. 15 and 16) and complete both tabulations before making the interpolations. The tabulation of the high and low waters illustrated in Figure 16 shows that on the day preceding the assumed break the morning high water came at 2.9 hours with a height of 14.3 feet, while on the day succeeding the break these values were, respectively, 4.6 hours and 12.6 feet. A direct mean of the above values gives for the time of the missing high water 3.8 hours and for the height 13.4 feet, as compared with 3.9 hours and 13.3 feet, the values actually observed.

In the same way the morning low water for the 26th would be determined as 10.2 hours and 4.4 feet, while the afternoon high and low waters interpolated from the corresponding tides the day previous and the day following are, respectively, 16.4 hours and 12.5 feet, and 22.4 hours and 5.2 feet.

To interpolate the hourly ordinates for the day in question, the values determined above for the times and heights of the high and low waters are plotted on cross-section paper and a curve drawn through these points as maxima and minima, the shape of the curve being made to conform to the curves of the days preceding and following. The hourly heights are then tabulated directly from this curve.

The linear method of interpolation obviously can be used only for relatively short breaks—rarely more than for 3 days. Breaks of greater duration may be interpolated by use of the observations at some other tide station, not too far away, which has a tide of the same type. The differences in the time and height of the tide at the two stations are determined from simultaneous observations and these differences applied to the observed times and heights of the high and low waters for the days in question, the hourly ordinates being interpolated as before.

Another method of interpolating a break of more than 3 days is to take a mean of the times and heights of the high and low waters 29 days before and after. This method is based on the fact that the three principal lunar cycles, the phase cycle, the parallax cycle, and the declinational cycle are, respectively, 29½ days, 27½ days, and 27⅓ days in length. As an example of this method, it may be used for interpolating the high and low waters at Boston for June 26, 1944, the day used to exemplify the method of linear interpolation.

Twenty-nine days prior to June 26th is May 28th, and 29 days after June 26th is July 25th. For May 28, 1944 the high waters at Boston occurred at 4.5 and 17.0 hours, the heights being 12.5 and 11.8 feet, respectively. For July 25th the corresponding values were 3.5 and 15.9 hours, and 12.8 and 12.4 feet. Hence the interpolated high waters for June 26th would be 4.0 and 16.4 hours, and 12.6 and 12.1 feet. The observed values were 3.9 and 16.4 hours, and 13.3 and 12.5 feet, so that the interpolated times are very close, but the interpolated heights differ by 0.7 and 0.4 foot, respectively. Means of the times and heights of the low waters on May 28 and July 25, 1944, are 10.2 and 22.4 hours, and 3.9 and 4.9 feet. The observed times and heights of the low waters on June 26th were 10.1 and 22.4 hours, and 4.4 and 5.1 feet. Again the times of the interpolated tides agree well with the observed, but the heights differ by 0.5 and 0.2 foot.

V. MEAN SEA LEVEL

Definition

Mean sea level at any point may be defined simply as the mean level of the sea at that point. It is the primary tidal datum plane, all the other tidal datum planes being generally derived with reference to mean sea level.

Strictly, mean sea level should be determined by integrating the tide curve. It is much more convenient, however, to derive mean sea level as the average of the tabulated hourly heights of the tide. For a very short period of observations the difference between the two determinations may be relatively large, but for a series covering a month or more the difference, if any, would be insignificant. The hourly heights of the tide are generally tabulated to the nearest tenth of a foot, and the mean sea level derived therefrom is taken to the nearest hundredth of a foot for series up to a year in length.

Mean sea level is generally assumed to constitute an equipotential surface; but as derived from tide observations at different places, mean sea level must be expected to deviate somewhat from a theoretical equipotential surface in consequence of the net or resultant effects of such agencies as winds or variations in barometric pressure. As a first approximation, however, mean sea level as derived from tide observations along open coasts may, for most purposes, be regarded as constituting an equipotential surface.

Within coastal bodies of water draining large areas subject to considerable fresh-water run-off the mean level of the sea obviously tends to stand somewhat higher than along an open coast. In tidal rivers in which variations in the fresh-water run-off cause relatively large fluctuations in level it is sometimes preferable to speak of mean river level rather than mean sea level, though this mean river level is determined in precisely the same manner as mean sea level, namely, by averaging the hourly heights of the tide.

It is convenient at times to use the expressions daily, weekly, monthly, and yearly sea level. These terms denote, respectively, the sea level derived by averaging the hourly heights of the tide for the period of a day, week, month, and year. With respect to weekly, monthly, or yearly sea level no ambiguity arises, but with respect to daily sea level it is necessary to define precisely how it is determined from the hourly heights of the day, for this determination is possible in three different ways.

If the hourly heights of the tide for any given day are denoted by $h_0, h_1, h_2, \ldots h_{23}, h_{24}$, in which h_0 is the height at midnight beginning the day and h_{24} the height at midnight ending the day, then, strictly, sea level for the day is given by $\frac{1}{24}(\frac{1}{2}h_0+h_1+h_2+h_3+\ldots+h_{22}+h_{23}+\frac{1}{2}h_{24})$. It is much simpler, however, to sum the hourly heights as tabulated; furthermore, no useful purpose is served by the refinement of the first and last terms of the formula. Hence daily sea level is frequently taken as $\frac{1}{25}(h_0+h_1+\ldots+h_{23}+h_{24})$. But, as shown in Figure 15, the hourly heights of the tide are tabulated with the 23d hour of the day as the last hour. It is therefore more convenient to derive daily sea level as $\frac{1}{24}(h_0+h_1+h_2+\ldots+h_{22}+h_{23})$. Throughout this publication, unless otherwise specifically stated, daily sea level will be derived in accordance with the last formula.

It is to be observed that except in regions of large range of tide, there is very little difference in the values of daily sea level determined by the three formulae. Thus taking the first day shown in Figure 15 (June 24, 1944), when the range of tide averaged a little over 9 feet, the first formula gives as the value of sea level on the tide staff at Boston as 9.37, the second 9.42 and the third 9.39. As will be seen later, sea level derived from one day of observations may differ from mean sea level by a foot or more. In such cases a difference of a few hundredths of a foot is negligible, and hence for most purposes the third formula is the most convenient one.

Half-Tide Level

Mean sea level must be carefully distinguished from half-tide level or, as it is frequently called, mean tide level. Half-tide level is the plane that lies exactly midway between the planes of mean high water and mean low water and is determined by averaging the heights of the high and low waters.

If the curve representing the rise and fall of the tide were that of a simple sine curve, the planes of mean sea level and of half-tide level would coincide. But the tide curve is not a simple sine curve; it is compounded of a number of simple sine curves, some of which have fixed phase relations with respect to each other. The average rise of high water above mean sea level is, therefore, generally not exactly the same as the average fall of low water below mean sea level, and hence mean sea level and half-tide level generally differ.

It will be more convenient to take up in detail the plane of half-tide level after the discussion of the plane of mean sea level. Here it will be sufficient to call attention to the fact that at any point on the open coast the planes of mean sea level and of half-tide level generally differ only by small quantities, and that over periods of a year or more the differences between these two planes are very nearly constant.

Variations in Sea Level

If the level of the sea were to fluctuate only in response to daily and semidaily tide-producing forces of unvarying periods, then mean sea level could be determined from one day of tidal observations. Averaging the hourly heights of the tide through one day would eliminate the effect of the tide, the resulting average height being the height of mean sea level. But the tide-producing forces to which the sea responds include, besides those of daily and semidaily periods, also those with periods of half a month or more. Daily sea level therefore varies from one day to another in consequence of these so-called long-period tides.

It can be shown that the variations in sea level from day to day resulting from the long-period tidal forces are relatively small. Far greater variations are brought about by the response of the waters to changes in wind and weather. It is a matter of common knowledge that a wind blowing toward the shore tends to raise the level of the sea along the shore, while a wind blowing from the shore tends to lower it.

Variations in barometric pressure likewise bring about fluctuations in sea level.

Indeed, as a first approximation, any arm of the sea may be regarded as constituting a huge inverted water barometer. When the barometric pressure over this arm of the sea rises, the level of the water will be lowered, while with a decrease in barometric pressure the level of the water will rise.

Daily Sea Level

Wind and weather vary from day to day; this, together with the variation due to the long-period tides, brings about variations in the height of sea level from day to day. Figure 22 shows in diagrammatic form the changes in sea level from day to day for 2 months of the year 1939 at Atlantic City, N. J., one a winter month, the other a summer month.

During January, as the upper curve of Figure 22 shows, sea level from one day to another varied from less than a tenth of a foot to more than 1½ feet. Furthermore, from the 19th to the 23d sea level fell 2¾ feet. Such large differences are obviously to be ascribed to the wide variations in wind and weather which characterize the month of January on the North Atlantic coast. An examination of the weather record for January 1939 shows that on the 18th easterly winds prevailed, with velocities up to 38 miles per hour. The next day the wind shifted to the northwest and continued from that direction with velocities up to 20 miles per hour until the 21st, when the wind shifted to the west, with velocities increasing up to 53 miles per hour on the 22d, and up to 40 miles per hour on the 23d.

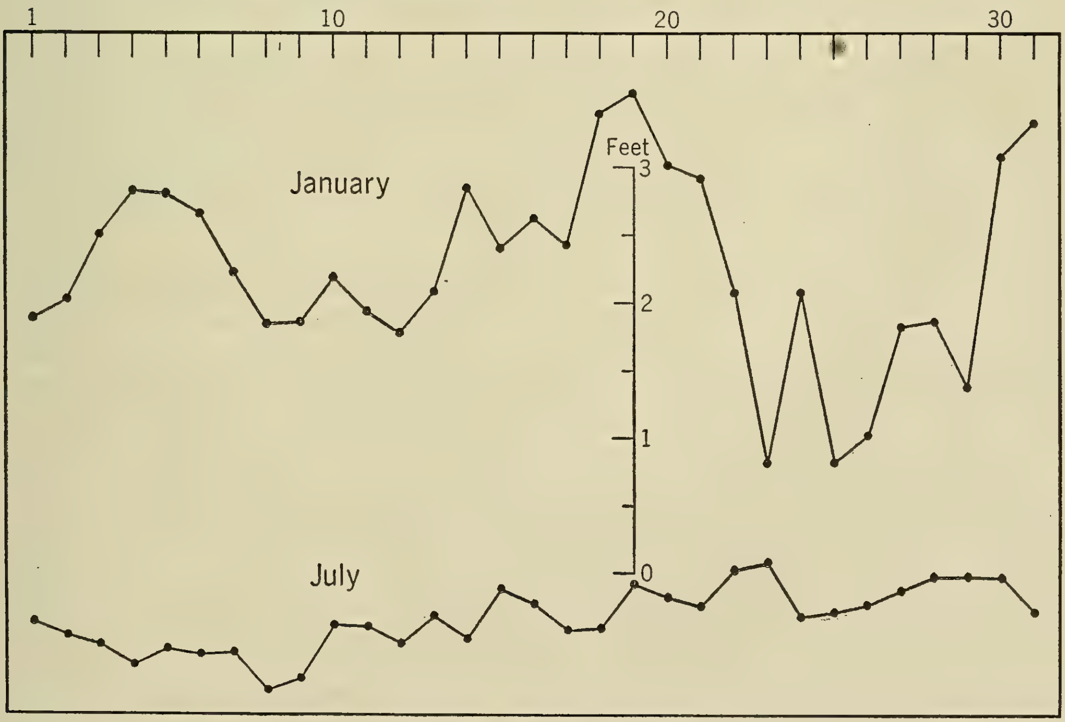

FIG. 22.—Daily sea level, Atlantic City, January and July 1939.

The lower diagram of Figure 22 shows that during July, when weather conditions were relatively more uniform, the changes in sea level from day to day were less than in January. Nevertheless, changes up to half a foot occur from day to day, and during the month sea level on the 23d was about a foot higher than on the 8th.

The change in sea level from day to day depends primarily on variations in meteorological conditions, hence such changes are not periodic; that is, from one day to the next, sea level may be either higher or lower, depending on the weather. But, as will be shown in the discussion of monthly sea level, there is a seasonal variation in sea level, or more precisely, an annual variation. Thus, at Atlantic City sea level is, on the average, lowest during the early months of the year and highest in the late summer or early fall months. Since within a single month daily sea level may differ by as much as 2¾ feet as shown in Figure 22, it follows that within a year the differences between two daily sea levels may be greater. During 1939 sea level at Atlantic City for August 29 was 1.9 feet above the average sea level for that year, and for December 8 it was 1.8 feet below. For these two days, therefore, sea level differed by 3.7 feet.

Regions subject to storms of great intensity, especially those fronting shallow bodies of water, exhibit much greater variation in daily sea level than found at Atlantic City. Likewise, in tidal streams subject to considerable fluctuation in drainage waters, there is greater variation in daily sea level than on the open coast, and this is especially marked in the upper reaches of the streams.

It is obvious that changes in sea level from day to day must, in general, be much the same at points near each other and which are subject to similar meteorological conditions. As will be seen later, advantage is taken of this fact in determining the plane of mean sea level from short series of observations by correcting the sea level derived from these observations to a mean value. Just how far two points may be separated and still exhibit similar sea level changes depends on a number of factors Within a long tidal river subject to considerable variation in fresh-water run-off the changes in daily river level may be quite different for points relatively near each other. But on the open coast and in tidal waters not subject to large variations in fresh-water discharge the changes in daily sea level resemble each other closely over areas of considerable extent.

In Figure 23 are plotted the heights of daily sea level for the month of October 1947 at six stations on the Atlantic coast from Portland, Maine, to Mayport, Fla. A glance shows that in regard to changes from day to day, Portland and Boston follow each other closely; likewise, New York and Atlantic City and to a somewhat lesser extent, Charleston and Mayport. There is some resemblance in the changes between the first pair and the second pair of stations, but none between the first pair and the third pair.

Examining the locations of these stations it is found that Portland and Boston are about 100 miles apart but lie in the same general embayment of the coast, and therefore the changes in sea level from day to day at the two stations are much the same. New York and Atlantic City likewise are about 100 miles apart and lie in the same embayment of the coast, which is different from the embayment on which Portland and Boston lie. Charleston is about 550 miles south of Atlantic City and in a different embayment of the coast, the distance and different exposure making the sea level changes different. Mayport and Charleston lie in the same embayment and although nearly 200 miles apart, show similar changes in sea level from day to day.

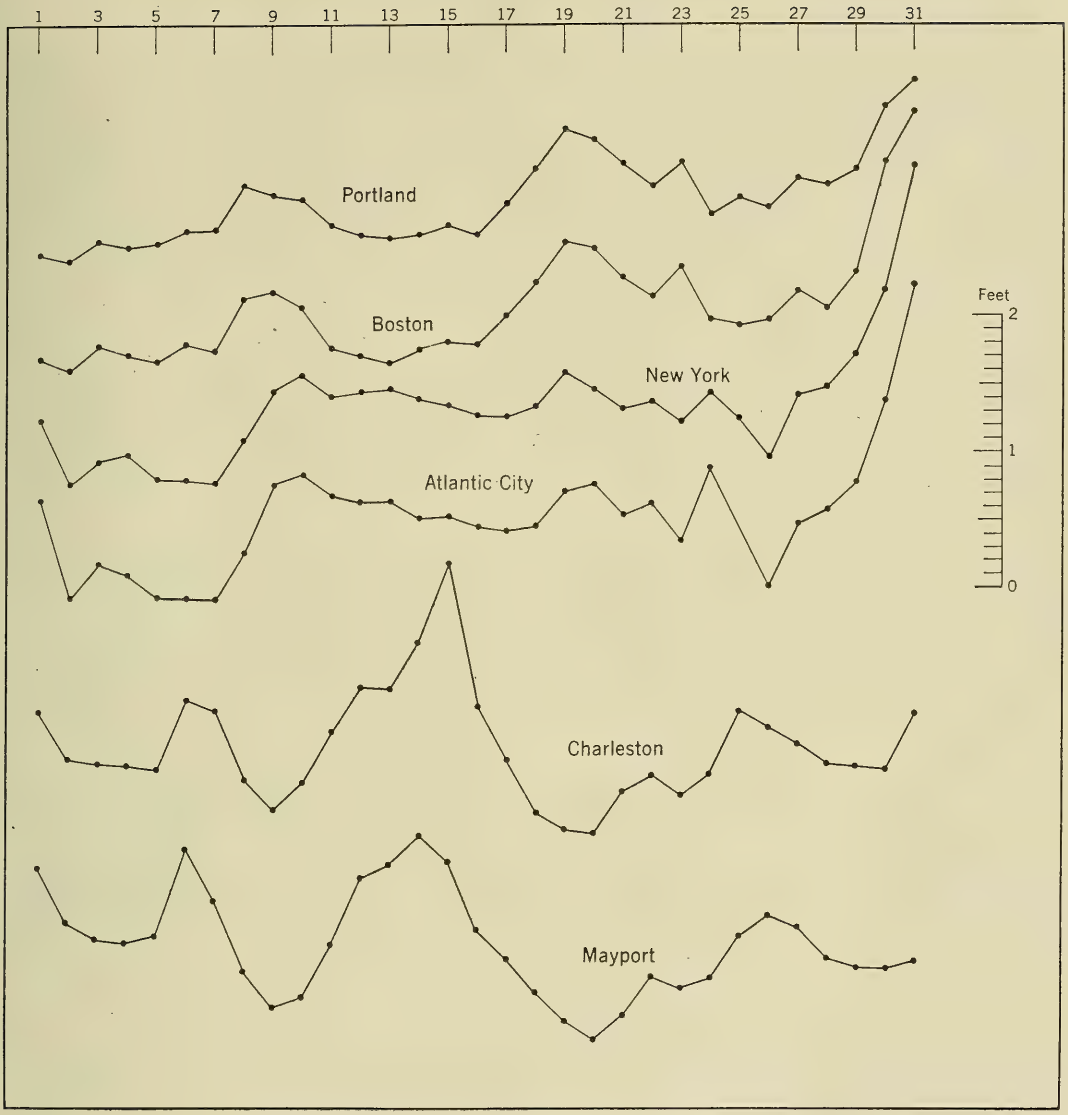

FIG. 23.—Daily sea level at six Atlantic coast stations, October 1947.

There appears to be an impression that sea-level changes are a function of the range of tide. That is, it is assumed that at stations where the range of tide is large, greater variations in sea level are to be expected than at stations having a small range of tide. That there is no basis for this impression, Figure 23 brings out as regards daily sea level. The average ranges of the tide at the six stations are as follows: Portland, 8.9 feet; Boston, 9.5 feet; New York, 4.4 feet; Atlantic City, 4.1 feet; Charleston, 5.1 feet; Mayport, 4.5 feet. At Boston, therefore, the range of tide is more than twice that at Atlantic City; nevertheless, the magnitude of the changes in daily sea level for the month shown is much the same. Indeed, for the last four days the changes at Atlantic City are somewhat larger.

Monthly Sea Level

It is obvious that sea level determined for periods of a week will show smaller variations than daily sea level. There is, however, no need of discussing such variations in detail, since they will lie between the daily variations and the monthly variations. Within a month the larger fluctuations exhibited by daily sea level will tend to balance out, so that monthly sea level shows much less variation than does daily sea level. For example, as noted in the discussion of daily sea level at Atlantic City, two daily sea levels in 1947, one in August and the other in December differed by 3.7 feet; during that same year the difference between the highest and lowest monthly values of sea level at Atlantic City was 0.9 foot.

In Figure 24 the monthly heights of sea level at the same six stations used in Figure 23 are shown for the 2 year period 1946–1947. The changes from month to month for each pair of stations—Portland and Boston, New York and Atlantic City, Charleston and Mayport—are closely similar, and there is a general resemblance for all six stations in that sea level is low in the later winter months and high in the autumn months.

In discussing the changes in sea level from day to day, attention was directed to the fact that such changes are in no way related to the range of tide. Figure 24 emphasizes this fact with regard to monthly sea level. New York with a mean range of 4.4 feet and Atlantic City with a range of 4.1 feet show much greater changes in monthly sea level than do Portland and Boston with ranges of 8.9 feet and 9.5 feet, respectively Still greater changes are shown by Charleston and Mayport, the ranges of which are, respectively 5.1 feet and 4.5 feet.

All six stations, but especially the four most southerly, give evidence of a seasonal or, more accurately, of an annual variation in sea level, reflecting the periodic seasonal changes in wind and weather. Since wind and weather do not repeat themselves exactly from year to year, the periodic annual variation in sea level in any one year may be somewhat masked; but if monthly heights of sea level for corresponding months are averaged over a number of years, the irregularities tend to balance out.

Annual Variation

The six curves of Figure 25 represent, as indicated, the annual variation in sea level at Atlantic City as derived from the monthly heights of sea level. The five upper curves give the monthly heights of sea level for each of the five consecutive years 1925–1929, while the lowest curve is the mean curve of annual variation derived by averaging the corresponding monthly heights of the five-year period. The horizontal line associated with each diagram represents the average value of sea level for the period in question.

For any one individual year shown in Figure 25 there are seen to be irregularities in the change of sea level from month to month. At the same time, however, there is a large element of periodicity in this change. The lowest curve shows that sea level is low in the winter and early spring months and high in the late summer and early fall months. And to a large extent this is seen to be the case for each of the 5 years.

The annual variation in sea level at any place is characteristic for a considerable area in its vicinity, but from this statement must be excluded the upper reaches of tidal streams subject to large fluctuations in fresh-water flow. Thus the annual variation

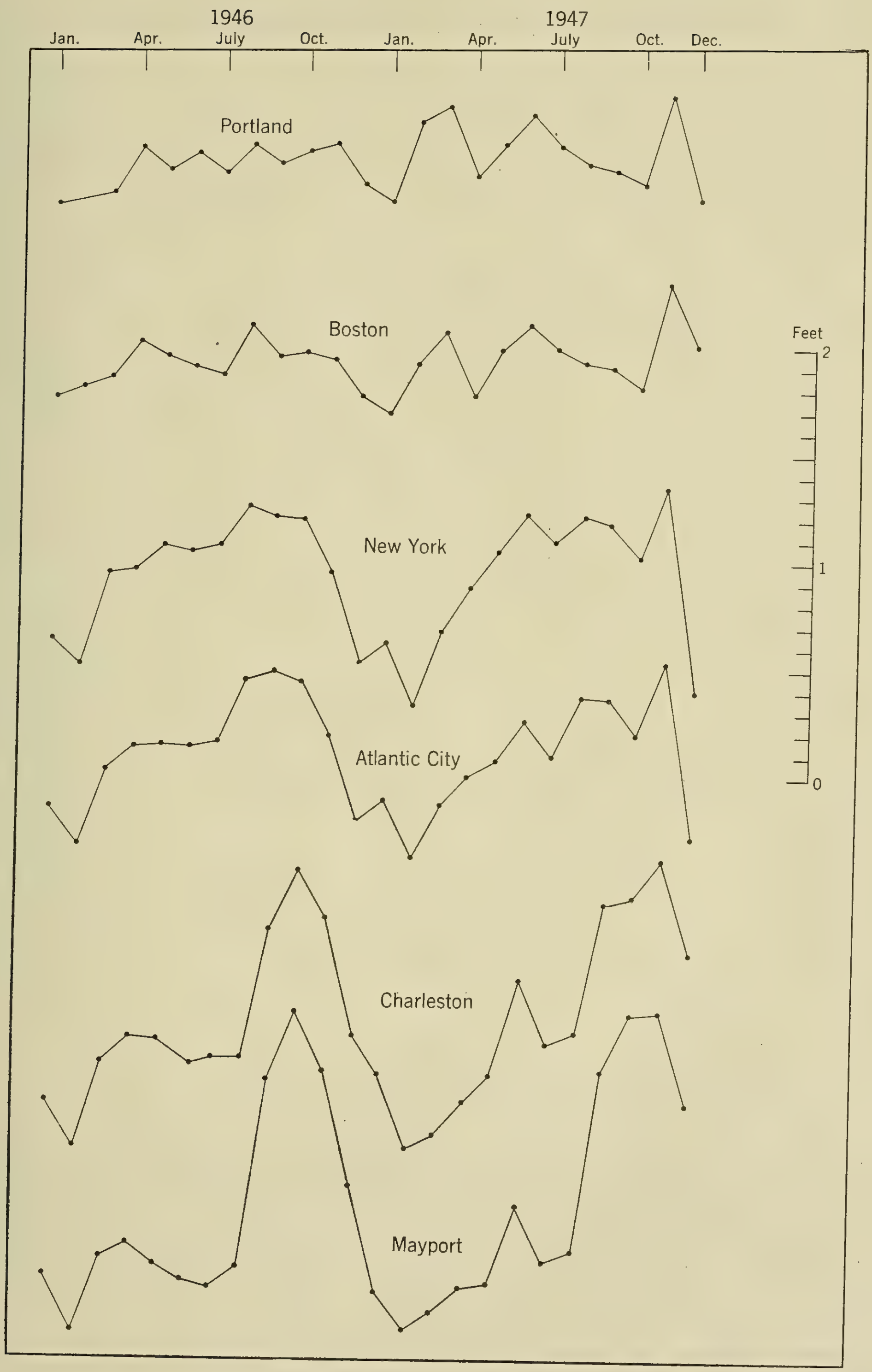

FIG. 24.—Monthly sea level at six Atlantic coast stations, 1946–47.

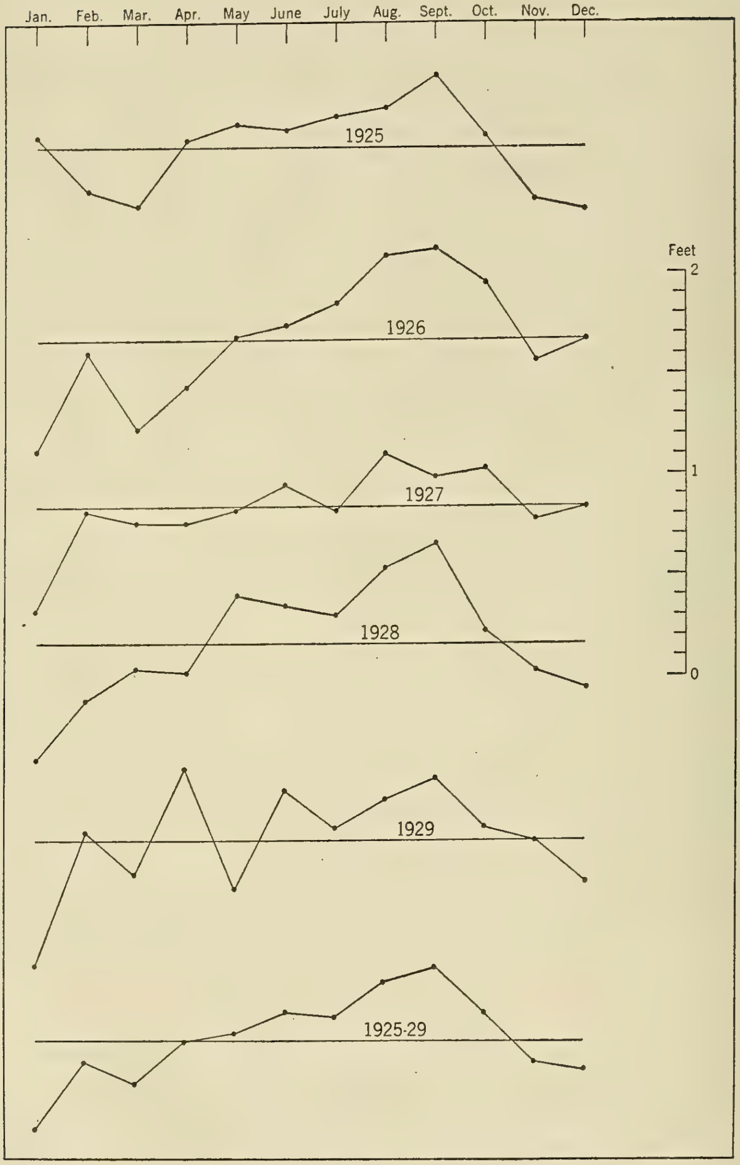

FIG. 25.—Annual variation in sea level, Atlantic City, 1925–29.

in sea level in New York Harbor at the mouth of the Hudson River is very much the same as at Atlantic City, on the open coast, although the two places are nearly 100 miles apart. But at Albany, near the upper end of the Hudson and but little farther from New York Harbor than is Atlantic City, the annual variation is much different. At Albany, freshets cause the highest river level to occur in April; while during the fall months, when sea level along the coast is highest, river level at Albany is several feet lower than in April.

Atlantic Coast.—The characteristics of the annual variation in sea level on the Atlantic coast of the United States at nine stations from Maine to Florida are shown in Figure 26. These curves are derived from the 19 year period of observations, 1930

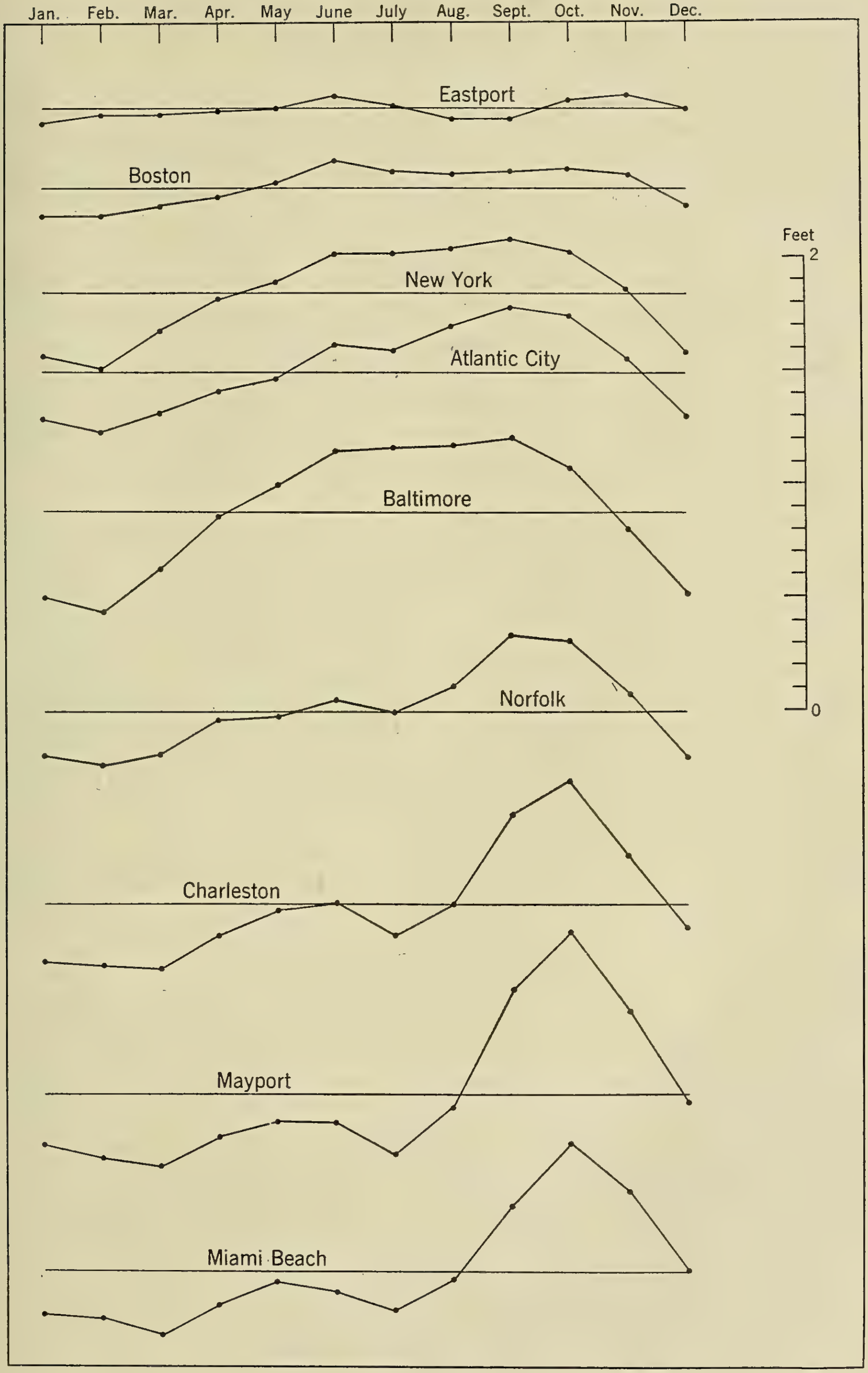

FIG. 26.—Annual variation in sea level, Atlantic coast.

through 1948, except for Miami, where the observations cover the 17 year period 1932–1948. The horizontal line in each diagram represents the average sea level for the years used at each station.

Figure 26 shows that with regard to detailed features, each station has a distinctive curve of annual variation in sea level. But in general this curve is much the same for relatively large stretches of the coast. Moreover, all along the Atlantic coast of the United States from Maine to Florida, sea level is lowest in the winter months and highest in the fall months.

The range of the annual variation increases more or less regularly from Maine to the mouth of Chesapeake Bay. At Eastport this range is 0.12 foot; New York, 0.58 foot; Atlantic City, 0.54 foot; Baltimore, 0.78 foot. From Chesapeake Bay to Florida the pattern of variation is much the same, but the range varies. At Norfolk it is 0.58 foot; Charleston, 0.83 foot; Mayport, 1.03 feet; Miami Beach, 0.85 foot.

South of Chesapeake Bay, there appears a well-developed secondary maximum and minimum, respectively, in May or June and in July. It is of interest to note that this secondary variation appears to be present in much diminished range in the northern stations also.

Gulf Coast.—In Figure 27 are shown the curves of annual variation at six stations on the United States coast of the Gulf of Mexico, from Key West to Port Isabel near the Mexican border. For Key West, Pensacola, and Galveston these curves are based on the 19 year period 1930–1948; for Cedar Keys on 10 years, 1939–1948 and for Port Isabel on 4 years, 1944–1948.

All along the Gulf coast, sea level is lowest in the winter months and highest in the fall months, the range of this variation being: Key West, 0.71 foot; Cedar Keys, 0.80 foot; Pensacola, 0.76 foot, Galveston, 0.81 foot; Port Isabel, 0.86 foot. Both Galveston and Port Isabel show a well-developed secondary maximum in May and a secondary minimum in July, while the other stations indicate the presence of this secondary variation, but in much less marked degree.

Pacific Coast.—For the Pacific coast of continental United States, Figure 28 gives the curves of annual variation at 7 stations from San Diego to Seattle. For all the stations with the exception of Crescent City, the curves are derived from 19 years of observations 1930–48. For the latter station the curve is derived for the 14-year period 1933–47.

The pattern of variation is much the same from San Diego to San Francisco, a stretch of some 450 miles. Sea level is lowest in April and highest in September, the range being: San Diego, 0.50 foot; La Jolla, 0.47 foot; Los Angeles, 0.50 foot; San Francisco, 0.34 foot. In this connection it should be noted that the San Diego tide station is located within San Diego Bay, La Jolla, and Los Angeles on the open coast, while the San Francisco tide station is located in San Francisco Bay which receives the drainage waters from the Sacramento and San Joaquin Rivers.

The Astoria tide station is located at Tongue Point, about 15 miles upstream from the mouth of the Columbia River. The curve of sea level variation at this station therefore reflects also the variation in discharge of the Columbia River. Lowest sea level at Astoria comes in August and highest in December, the range being 0.72 foot.

At Seattle the pattern of variation is quite different from that at the stations on the California coast. Highest sea level comes in December and lowest in August as at Astoria, and the range is 0.52 foot.

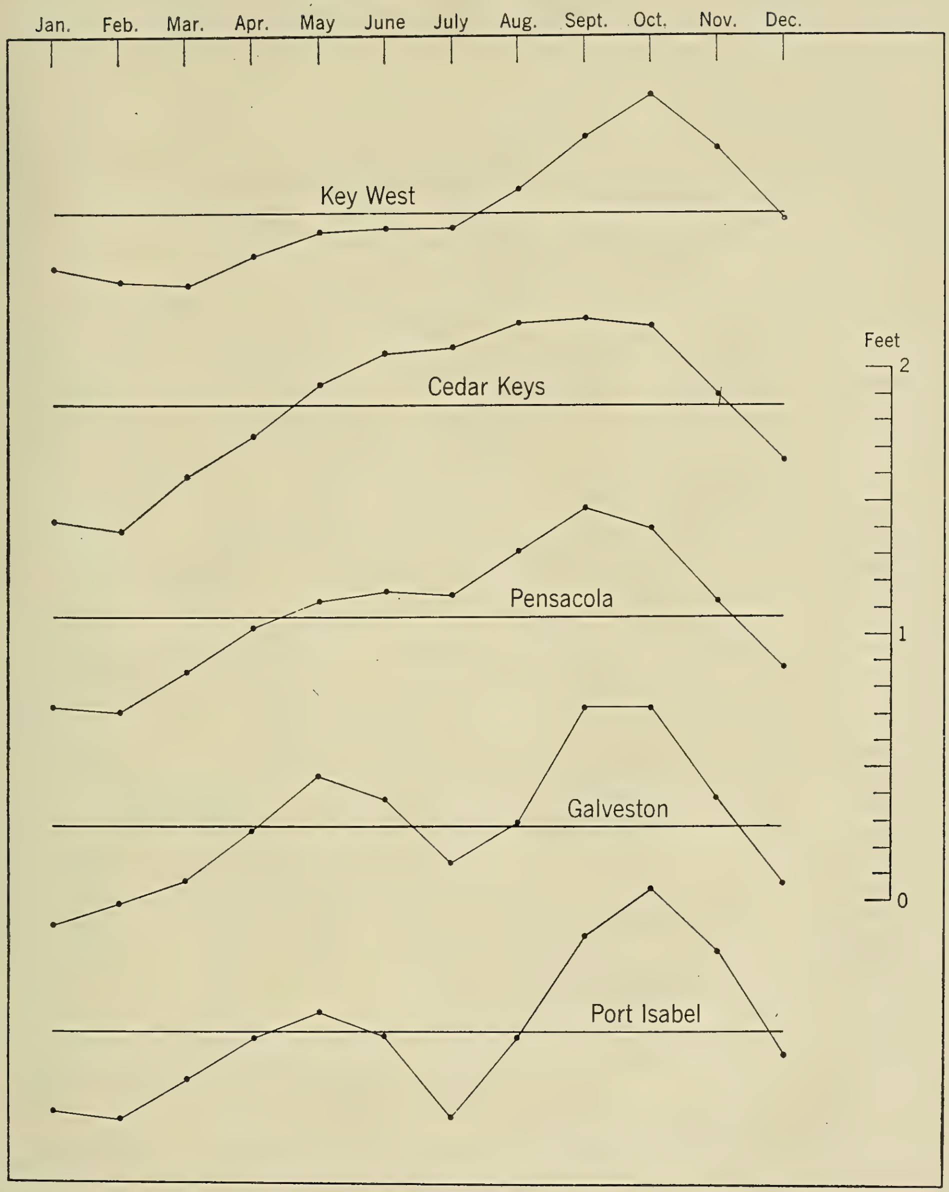

FIG. 27.—Annual variation in sea level, Gulf coast.

Alaska.—For the Pacific coast of Alaska, observations for four years or more are available at six stations for which the curves of annual variation in sea level may be derived, stretching from Ketchikan in southeast Alaska to Adak Island in the Aleutians. The curves for these stations are shown in Figure 29. The years of observations for each of these stations are indicated on the diagrams.

In general sea level all along this stretch is high in the late fall and early winter months and low in the summer months, except at Skagway which shows a secondary

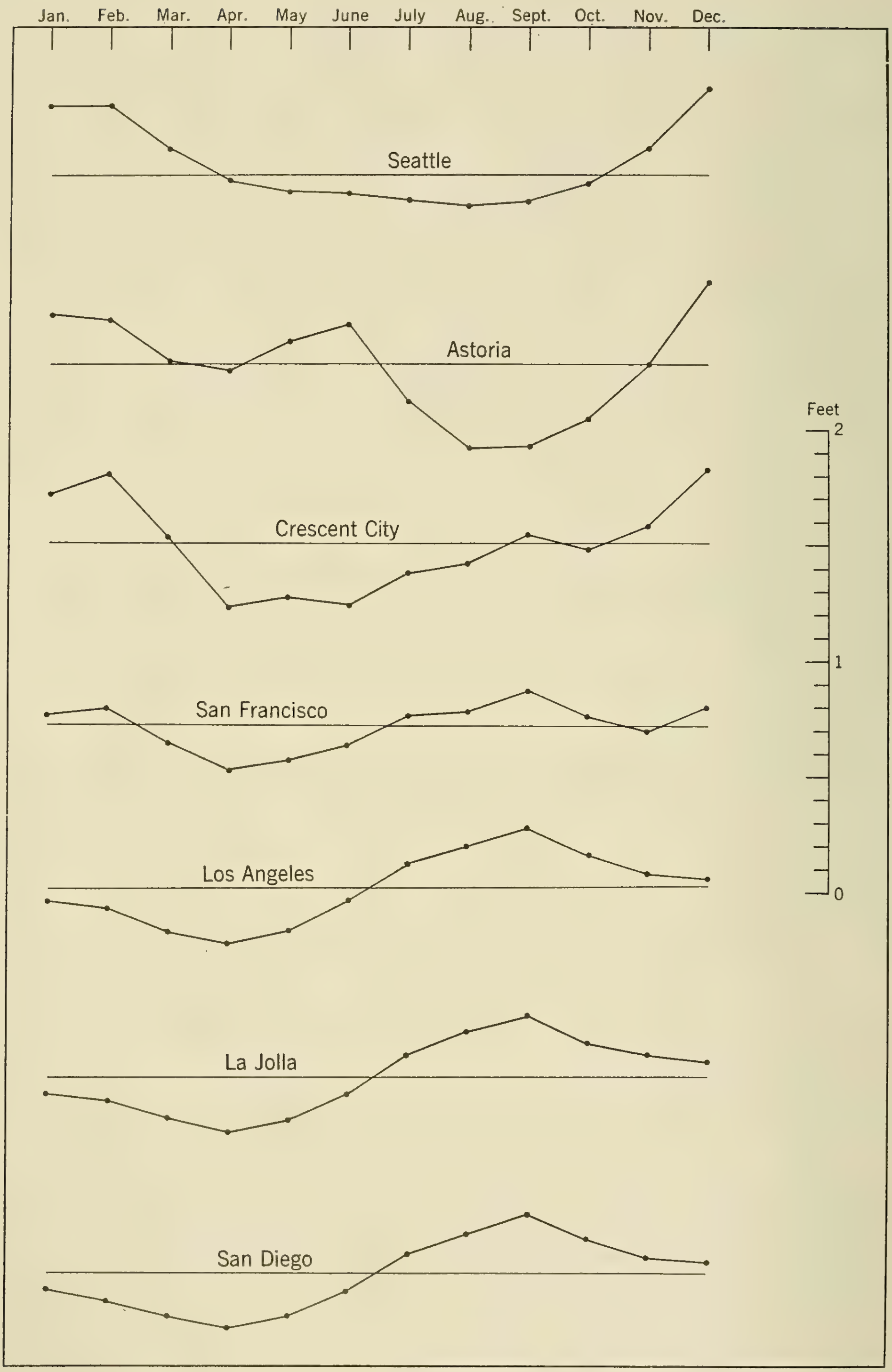

FIG. 28.—Annual variation in sea level, Pacific coast.

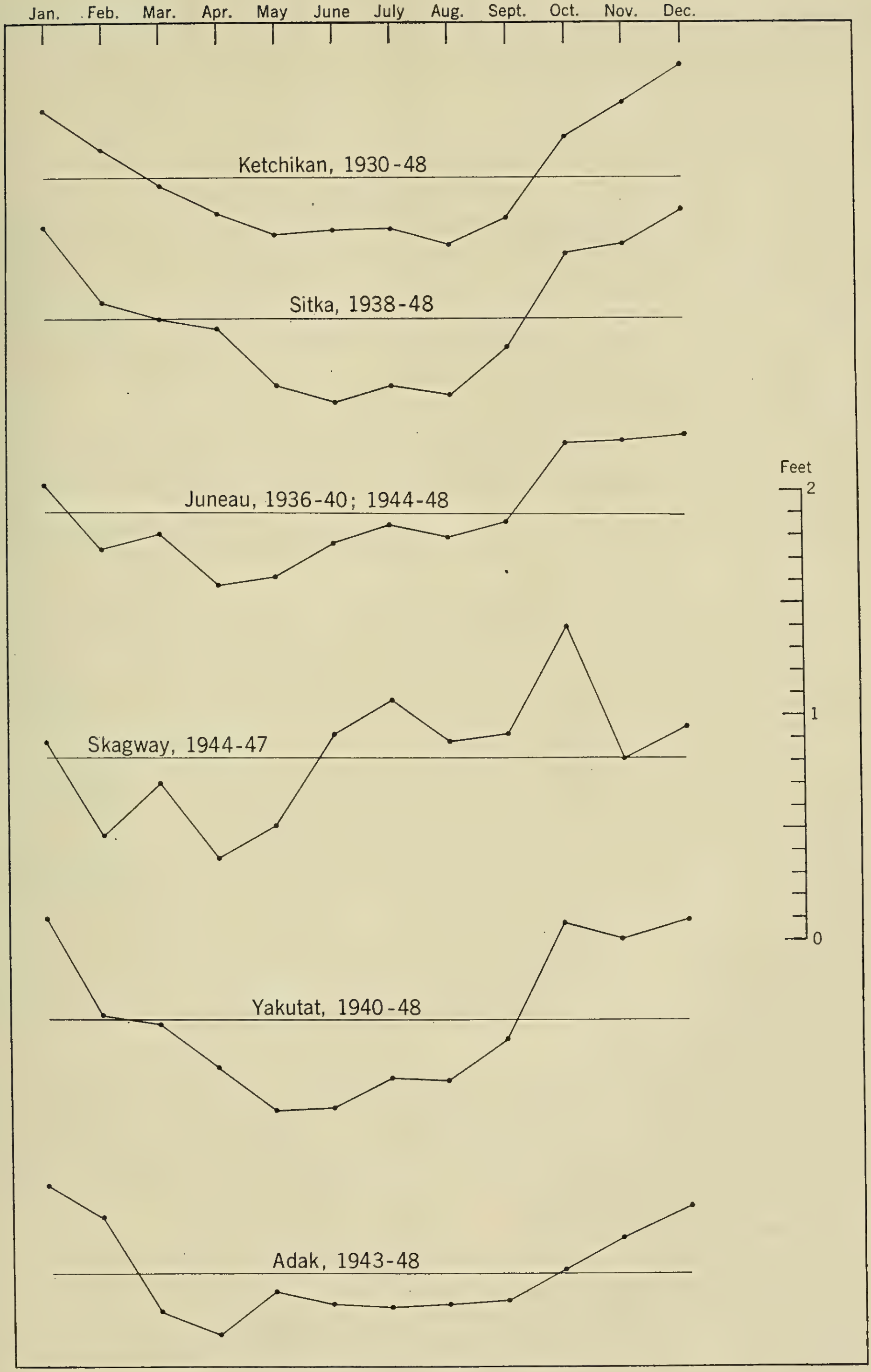

FIG. 29.—Annual variation in sea level, Alaska.

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maximum in July. The range of the variation varies somewhat at the different stations but approximates three-quarters of a foot, the lowest value being at Adak with 0.67 foot and the highest at Skagway with 1.03 feet.

Yearly Sea Level

In the previous section monthly sea level was found to be subject to an annual variation with a range up to a foot. This means that, due to this cause alone, mean sea level determined directly from one month of observations may be in error by half a foot. This error may be further augmented very considerably by the nonperiodic variation from month to month arising from variations in wind and weather. Within a year, however, the annual variation balances out, and it now remains to consider whether there are any variations in sea level from year to year.

Atlantic Coast.—In Figure 30 the yearly heights of sea level at eight stations along the Atlantic coast of the United States are shown for the period of observations available at each station. It appears at once that sea level does vary from year to year, though generally by relatively small amounts. In general the change in sea level from one year to the next is less than one-tenth of a foot, but at times it may be as much as 0.2 foot.

Two features stand out strikingly in Figure 30. The first is that the change in sea level from year to year is much the same for long stretches of the coast. When sea level at any station in Figure 30 during any year is high (or low), it is also high (or low) at stations several hundred miles distant. Thus, in the 500-mile stretch of the coast from Portland to Baltimore the year 1919 was one of high sea level at all five stations, while the years 1926 and 1930 were years of low sea level at these stations. In the same way the sea-level changes from year to year in the nearly 500-mile stretch from Charleston to Miami Beach parallel each other fairly closely.

The second striking feature is the steady progressive rise in sea level at all eight stations since about 1930. Prior to that year, both New York and Baltimore indicate a rise of sea level at the rate of less than 0.01 foot per year, but since 1930 the rise has been at the rate of about 0.02 foot per year.

Formerly it was thought that there were cycles in sea level of something like 4 years and 9 years, reflecting perhaps similar cycles in wind and weather. Apparently such cycles are accidental, and in any event are completely submerged in the progressive rise of sea level since 1930.

The up and down changes in sea level from year to year of a tenth of a foot or more must obviously be ascribed to the disturbing effects of wind and weather, which do not repeat themselves exactly from year to year. But the steady rise from 1930 must be due to more deep-seated causes, such as subsidence of the coast or actual rise in the level of the ocean waters.

Gulf Coast.—For the Gulf coast, Figure 31 gives the yearly sea levels at four stations from Key West to Galveston. At Cedar Keys there was a break in the observations between the years 1926 and 1939.

At Key West the change in sea level is much the same as on the Atlantic coast—little change from 1912 to about 1930 after which sea level has risen steadily, at the rate of approximately 0.02 foot per year. For Cedar Keys the break from 1926 to 1939 obscures the time of change but it is clear that from 1915 to 1925 there was

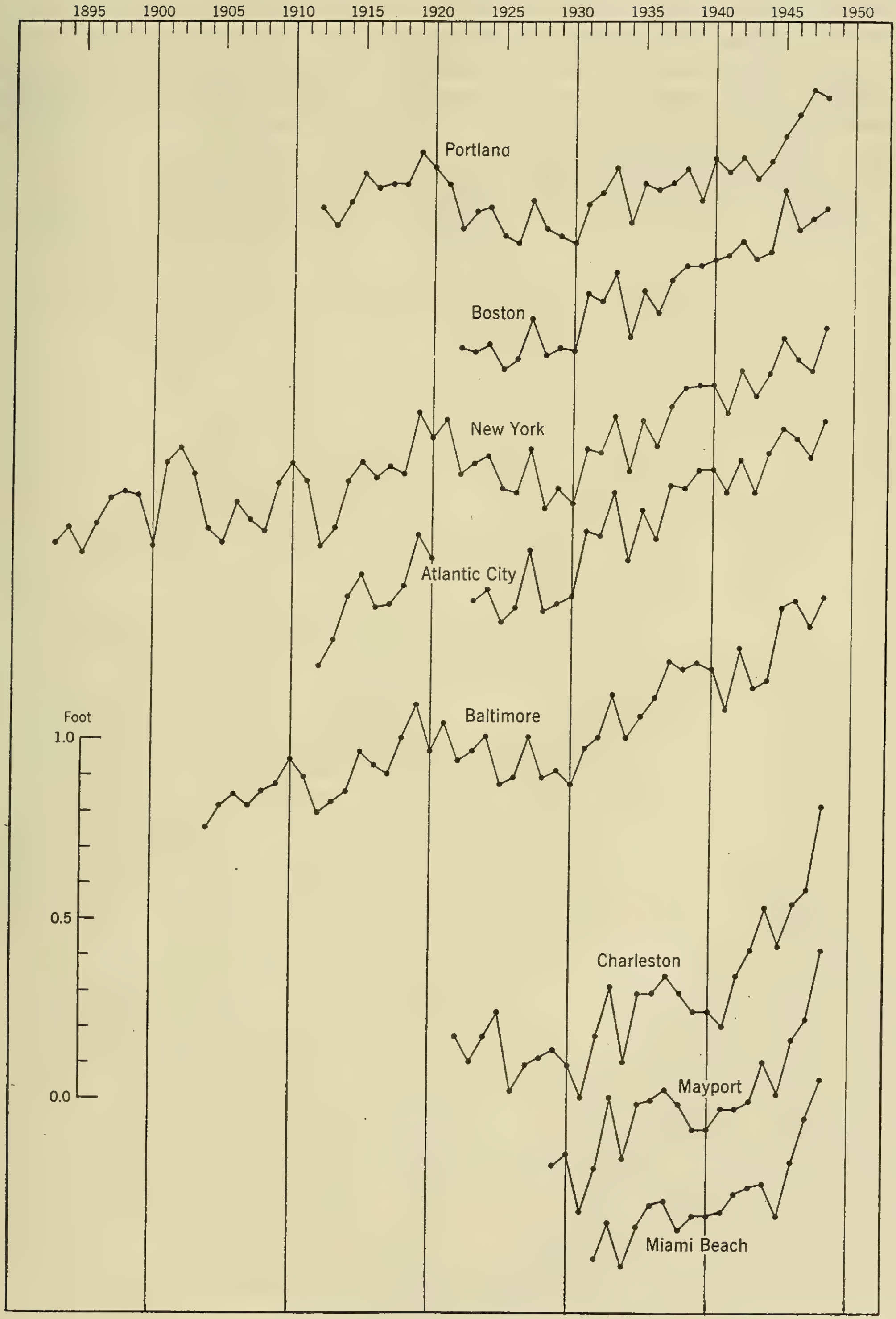

FIG. 30.—Yearly sea level, Atlantic coast.

little change in sea level. Since 1939 the rise has been steady, at the rate of about 0.03 foot per year.

At Pensacola the rise in sea level appears to have been in progress since the beginning of the observations in 1924. If we smooth the values in figure 31 by the method of moving means, we derive a rate of rise of about 0.015 foot per year up to 1942, after which the rise is about 0.04 foot per year.

For Galveston the observations extend over a period of 40 years, and during all this time sea level appears to have been rising. Between 1909 and 1940 the rise was at the rate of about 0.015 foot per year, and since that time at the rate of about 0.05 foot per year.

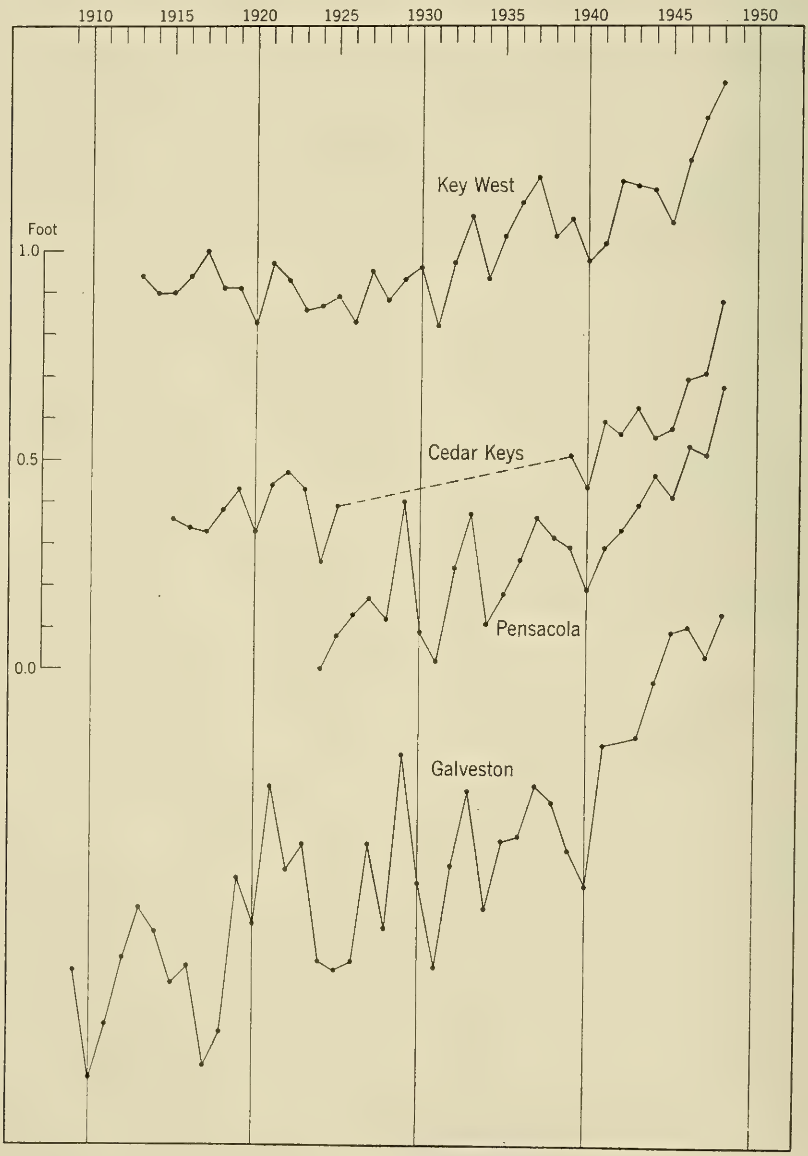

FIG. 31.—Yearly sea level, Gulf coast.

Since the rise along the Gulf coast is at different rates, the most plausible explanation is an assumption of local coastal movement. Apparently the coast in the vicinity of Galveston is subsiding at a more rapid rate than the coast to the eastward.

Pacific Coast.—Figure 32 gives the yearly values of sea level at four stations on the Pacific coast of continental United States. In general the changes from year to year at the stations are less than a tenth of a foot, although occasionally they may be as much as two- or three-tenths of a foot. In general, too, the variations are in the same direction at all four stations; when sea level is high (or low) at one station it is also high (or low) at the other stations.

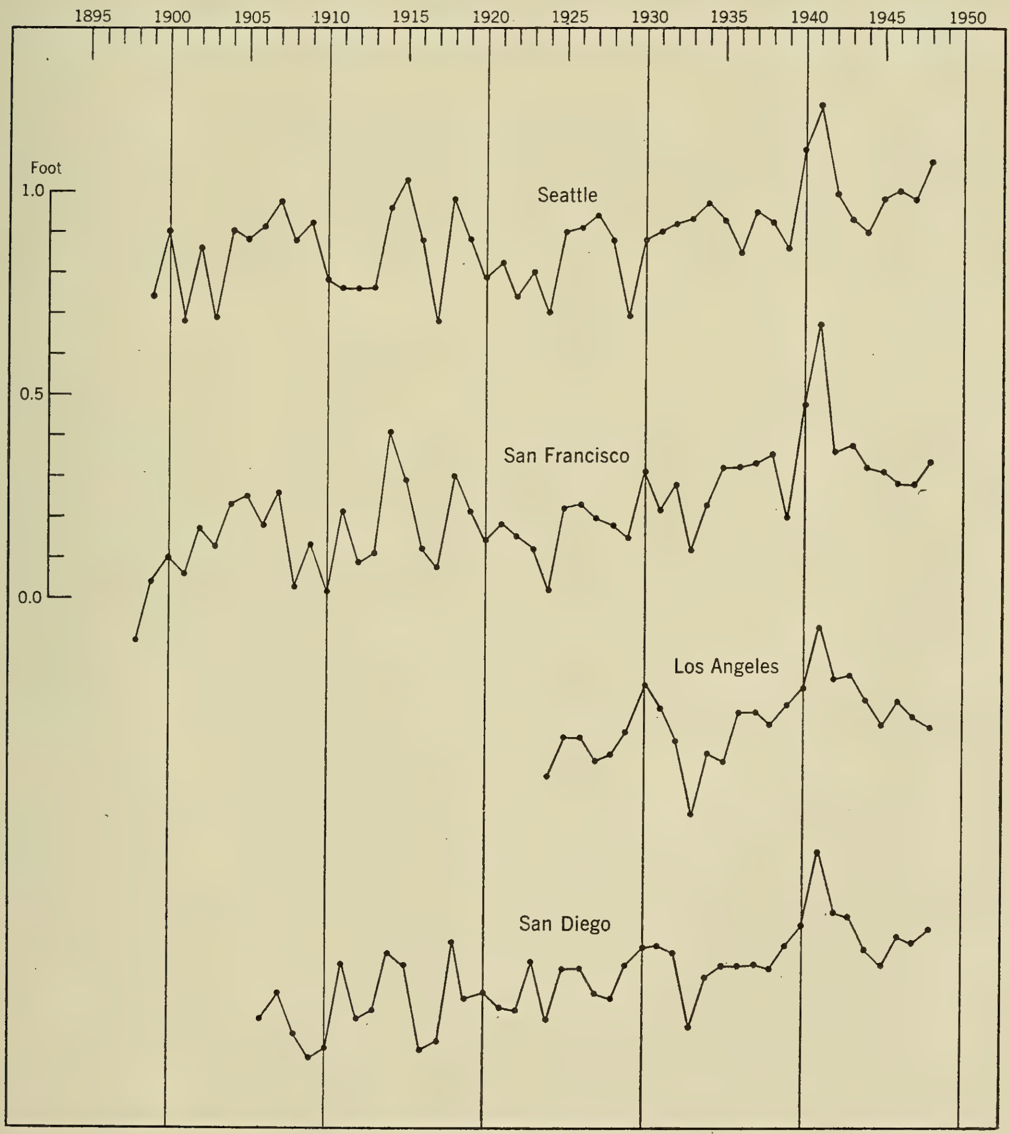

FIG. 32.—Yearly sea level, Pacific coast.

All four stations indicate a more or less progressive rise in sea level but at a much slower rate than on the Atlantic or Gulf coasts. At San Diego this rise was at the rate of about half a hundred of a foot per year to 1930 and at a slightly greater rate since then. Los Angeles indicates a rise of about 0.12 foot between 1924 and 1948, or at the rate of 0.005 foot per year. For San Francisco the rate is 0.005 foot per year from 1898 to 1930 and somewhat larger since then. For Seattle it was less than 0.005 foot per year up to 1930 and since then about 0.006 foot per year.

Alaska.—For Alaska there is only one station at which continuous observations for more than 20 years are available, namely, at Ketchikan. For Sitka, Juneau,

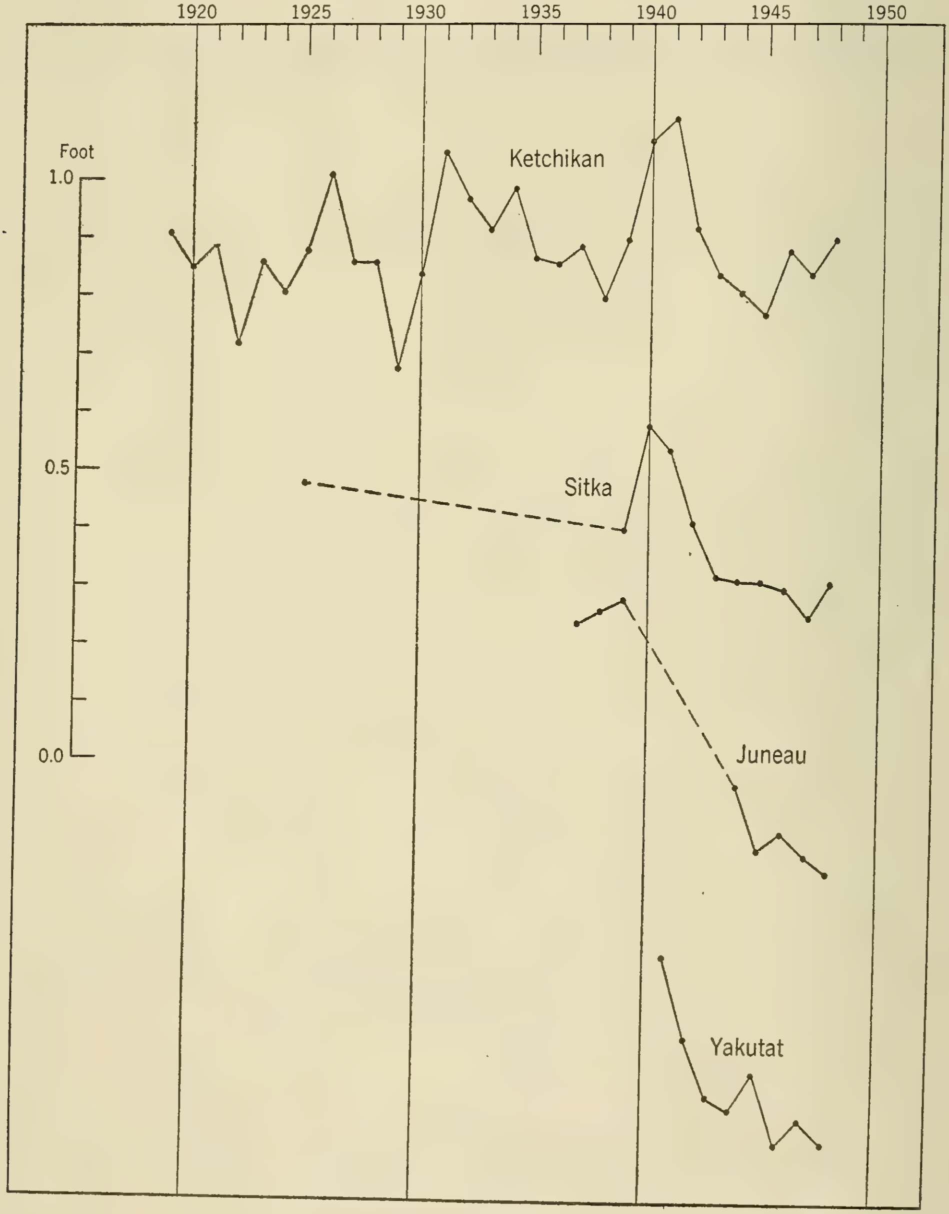

FIG. 33.—Yearly sea level, Alaska.

and Yakutat there are observations covering periods more than 5 years, and since along this coast sea level appears to be falling, a brief discussion of these will be of interest. In Figure 33 are plotted the yearly heights of sea level at the above-named four stations.

From the plotting for Ketchikan it appears that sea level at that station has been rising slowly from 1919 to about 1940, less than a tenth of a foot during this period. Since that time the indications are for a falling sea level, though it will require further observations to bring this out clearly.

For Sitka, Juneau and Yakutat the evidences for falling sea level are unmistakable. At Sitka there are available a year of tide observations from 1893 to 1894 and another year in 1925. In the latter year sea level was a quarter of a foot lower than in 1893–94. And in 1925 sea level was 0.08 foot higher than in 1939 at the beginning of the present continuous series.

At Juneau a year of observations is available in 1912 and this value of sea level is 1.27 feet higher than in 1937 when the present series began. It is to be noted that in this latter series there is a break in the observations between the years 1939 and 1944.

A rising sea level along the Pacific coast of continental United States and a falling sea level along the Pacific coast of Alaska must be interpreted as due to relative land movements in the two regions.

Primary Determination

The variations in sea level discussed in the preceding sections may be summarized as follows: At any point on the coast, sea level varies from day to day, from month to month, and from year to year. From one day to the next, sea level may vary by a foot or more, and within the same year two values of daily sea level may differ by 5 feet or more. Monthly sea level is subject to variations of both periodic and nonperiodic character, so that within a year sea level for two different months may differ by as much as a foot. Yearly values may differ by as much as one or two-tenths of a foot from one year to the next and in addition may be subject to a slow progressive rise or fall.

The determination of mean sea level therefore involves two problems. The first is, how long a series of tide observations is required to give an accurate determination of mean sea level? The second problem is, how can the sea level derived from a short series of observations be corrected to mean value?

A period of 19 years is generally considered as constituting a full tidal cycle, for during this period of time the more important of the tidal variations will have gone through complete cycles. It is therefore customary to regard results derived from 19 years of tide observations as constituting mean values. Hence sea level derived from 19 years of observations may be taken to constitute a primary determination and as giving accurately the datum of mean sea level.

If the mean level of the sea remained constant over long periods of time and if the coast were absolutely stable, we might expect sea level at any place determined from one 19-year series to be the same as that derived from another such series even if separated by a number of years. Apparently, however, this is not the case, and for precise purposes it is therefore necessary to specify the particular epoch used in the determination of mean sea level. The six long series of observations available for the

Atlantic, Gulf and Pacific coasts of the United States furnish illustrative examples of the necessity for specifying the epoch used.

For New York Harbor there are available 56 years of observations, from 1893 through 1948. This permits three 19-year series, 1893–1911, 1912–1930 and 1930–1948, the last two having the year 1930 in common. For the series 1912–30, sea level referred to a number of bench marks in the vicinity of the tide station was 0.09 foot higher than for the series of 1893–1911; for 1930–1948 it was 0.29 foot higher than for 1893–1911, and 0.20 foot higher than for 1912–1930.

At Baltimore the tide observations cover the 46 year period 1903–1948. Taking the first and last 19-year series, 1903–1921 and 1930–1948, we find the latter 0.26 foot higher. For comparison with New York, if we form two like 19 year series, 1912–1930 and 1930–1948, we find that sea level from the latter series was 0.23 foot higher as against a difference of 0.20 foot for New York.

For Galveston the 40 years of observations from 1909 permit two independent 19 year series, 1909–1927 and 1930–1948. Sea level derived from the latter series is 0.39 foot higher than from the former series. Figure 31 indicates an accelerated rise of sea level at Galveston since 1940, which if maintained, will bring about even greater differences for subsequent 19 year series.

For the three Pacific coast tide stations it will be sufficient to take comparable 19 year series at each of the stations, namely, 1906–1924 and 1930–1948. At San Diego the latter series gives sea level 0.14 foot higher; at San Francisco, 0.16 foot higher; at Seattle, 0.12 foot higher.

Specifying the epoch on which a given primary determination of mean sea level is based, permits correlation with mean sea level determinations made at other times, provided adequate bench marks are maintained.

Secondary Determination

Observations covering a period of 19 years for primary determinations of mean sea level are required at but few places on the coast. At all other places a satisfactory secondary determination of this datum plane can be made by means of observations covering much shorter periods if the results are corrected to a mean value by comparison with the primary determination at some suitably located tide station. The precision with which mean sea level can be derived by a secondary determination from various periods of tide observations can best be illustrated by examples.

Day.—Since weather conditions at widely separated places may be markedly different on the same day, it is obvious that, in deriving mean sea level at any point from one day of tide observations, comparison must be made with a near-by primary station at which the changes in sea level will be similar.

Figure 23 shows that at New York during the month of October 1947 sea level was lowest on the 2d and highest on the 31st, the values of sea level on the tide staff being respectively 4.94 and 7.28 feet, a difference of 2.34 feet. Suppose that from the observations for each of these two days it is desired to derive a value of mean sea level for New York, using Atlantic City, about 100 miles away, as primary tide station for comparison.

At Atlantic City for epoch 1930–1948, mean sea level on the tide staff reads 6.49 feet, while for October 2 and 31 the daily sea levels read, respectively, 6.09 and 8.41

feet. On the 2d, therefore, sea level was 0.40 foot below its mean value while on the 31st it was 1.92 feet above. Applying these corrections to the corresponding daily sea levels at New York, we derive a mean sea level value of $4.94+0.40=5.34$ feet for the 2d, and $7.28-1.92=5.36$ feet for the 31st.

From the New York observations for 1930–1948, the primary determination of mean sea level on staff reads 5.34 feet. Thus the two daily sea levels which differed from each other by 2.34 feet, give mean sea level values which differ from each other by 0.02 foot, and which in the one case agrees with the primary determination and in the other differs by 0.02 foot.

If we take each day of the month of October 1947 at New York and derive mean sea level by comparison with Atlantic City, it is found that on the average these daily mean sea level determinations differ from the primary determination by 0.07 foot, while the greatest difference is 0.29 foot.

If Boston and Portland, which likewise are about 100 miles apart, are used for the month of October 1947 shown in Figure 23, using Portland as the primary station it will be found that the mean sea level value for the 2d will differ by 0.06 foot from the primary determination, while for the 31st it will differ by 0.51 foot. But a glance at Figure 23 makes clear that on the 30th and 31st Boston sea level was responding to meteorological conditions prevailing southward which were reflected in much lesser degree at Portland. In fact, for the entire month of October 1947, mean sea level at Boston, when derived for each day by comparison with Portland, differs, on the average by 0.11 foot from the primary determination, the greatest difference being 0.51 foot for the 31st.

In general it may be taken that mean sea level determined from one day of observations when compared with simultaneous observations at some suitable primary tide station will give a value correct to within a quarter of a foot

Month.—As an example of the determination of mean sea level from a month of observations we may again take New York and Atlantic City. Figure 24 shows that for the two-year period 1946–1947, monthly sea level at New York was lowest in February 1947 and highest in November 1947, reading on the staff, respectively, 4.84 and 5.85 feet, a difference of 1.01 feet.

Determining mean sea level for each of these months by comparison with Atlantic City, we find that for February 1947 sea level at Atlantic City was 0.36 foot below the 1930–1948 value of mean sea level at that place while for November it was 0.55 foot above its mean value. Applying these corrections to the corresponding monthly values at New York we get mean sea level values of 5.20 and 5.30 feet respectively. At New York the 1930–1948 value of mean sea level on staff is 5.34 feet. Hence the values determined from each of the month's observations differ by 0.14 foot and 0.04 foot, respectively, from the primary determination.

If we derive mean sea level at New York for each month of the two years shown in Figure 24 by comparison with Atlantic City, it is found that the greatest difference from the nineteen year value is 0.17 foot, while the average difference is 0.08 foot. In general it may be taken that mean sea level determined from one month of observations, when compared with simultaneous observations at a suitable primary tide station, will give a value correct to within 0.1 foot.

Year.—To exemplify the secondary determination of mean sea level from one year of observations, it will be instructive to take stations farther apart than those used in connection with 1 month of observations. Boston and Baltimore are 360 miles apart by air line and considerably farther as measured along the coast line. Moreover, Baltimore lies on an arm of Chesapeake Bay 140 miles from the open sea, while Boston is less than 10 miles from the open sea.

From Figure 30 it is seen that for the 27 years of observations at Boston, sea level for the year 1925 was lowest, while for the year 1945 it was highest, the heights on the tide staff being, respectively 8.03 and 8.53 feet. At Baltimore mean sea level for the epoch 1924–1942 reads 4.25 on the staff, while for the years 1925 and 1945 the yearly sea levels read respectively, 4.08 and 4.57 foot. The corrections for the two years are therefore +0.17 and −0.32 foot, respectively. This makes mean sea level at Boston from the 1925 observations 8.03+0.17=8.20 and from the 1945 observations 8.53−0.32=8.21 feet.

From the continuous series of observations at Boston the direct 19 year mean value of sea level for the epoch 1924–1942 is 8.21 feet on the staff. The yearly sea levels for 1925 and 1945 which differed from each other by 0.5 foot thus give mean sea level values when corrected by comparison with simultaneous observations at Baltimore, which differ by 0.01 foot in the one case and agree exactly in the other case with the primary determination.

If we take each of the 27 yearly values of sea level at Boston from1922 through 1948 and determine mean sea level values by comparison with Baltimore, we find that on the average these values differ from the direct primary determination by 0.04 foot, the largest individual difference being 0.13 foot.

Had a station closer to Boston than Baltimore been taken, one more nearly subject to similar wind and weather conditions, closer approximation of the mean sea level values derived from a year of observations could be expected. Had New York been used, the mean sea level values from each year of the Boston observations would show an average difference from the primary determination of 0.03 foot, with the largest individual difference of 0.07 foot.

We may test the applicability of the method of correction by comparison to stations on the Pacific Coast. As an example we may take San Francisco and Los Angeles which are about 350 miles apart. From Figure 32 it is seen that at Los Angeles sea level was lowest in 1933 and highest in 1941, the heights on the tide staff being, respectively, 6.27 and 6.73 feet. Taking San Francisco as the primary station, mean sea level for epoch 1924–1942 reads 8.67, while for the years 1933 and 1941, the yearly sea levels were respectively, 8.52 and 9.07 feet.

The corrections to mean sea level for these years are therefore +0.15 and −0.40 foot. Applying these to the corresponding yearly sea levels at Los Angeles we derive mean sea level from the 1933 observations as 6.42 and from the 1941 observations as 6.33. From the observations at Los Angeles from 1924 through 1942 the direct primary determination is 6.48. So that the two yearly values which differed from the mean value by 0.21 and 0.25 foot respectively, after correction differ from the mean value, respectively, by 0.06 and 0.15 foot.

If each of the 25 years of observations at Los Angeles is corrected to a mean value by comparison with San Francisco, the average difference from the 1924–1942 mean is found to be 0.06 foot, the greatest difference being 0.15 foot for 1941.

Had San Diego, which is only 100 miles away from Los Angeles, been used as the primary station, closer approximations would have been derived. At San Diego the 1924–1942 value of mean sea level on staff is 6.31, the yearly sea levels for 1933 and 1941 being, respectively, 6.15 and 6.58 feet. The corrections for those years are therefore +0.16 and −0.27, which give mean sea level values at Los Angeles of 6.43 feet for 1933 and 6.46 feet for 1941, differing from the 1924–1942 value at Los Angeles by 0.05 and 0.02 foot, respectively.

Using San Diego as the primary station and deriving mean sea level for each of the 25 years of observations at Los Angeles, it is found that the average difference from the primary determination is 0.03 foot, while the greatest difference is 0.08 foot.

In general it may be taken that mean sea level determined from a year of observations, when compared with simultaneous observations at a suitable primary tide station will give a value correct to within 0.05 foot.

Three Years.—From the nature of the case it is clear that the longer the series the more precise is the determination of mean sea level. Where a continuous series of observations is being made, a preliminary determination is derived from a year of observations. This value is then maintained until a 3-year series is available, when a more precise determination is made if required.

The accuracy with which a 3-year series gives mean sea level, after correction by comparison, may be exemplified by taking Boston and Baltimore as in the example for 1 year. Forming 3-year running means of the 27 years of observations at Boston and correcting these to mean values by comparison with similar 3-year means at Baltimore, the average difference between these mean sea level values and the primary determination is 0.03 foot, while the greatest difference for any one 3-year group is 0.08 foot. These differences compare with the corresponding differences derived from 1 year of observations of these two stations of 0.04 foot and 0.13 foot.

Nine Years.—From the Boston and Baltimore observations for 1922–1948, 19 running 9-year means may be derived. Correcting these 9-year means to a 19-year value and comparing with a primary determination it is found that the average difference is 0.016 foot while the greatest difference for any one 9-year group is 0.03 foot.

Primary Tide Stations

The possibility of determining the plane of mean sea level from short series of observations is thus seen to depend on the existence of tide stations at which long series of observations are being made. Such tide stations are designated as primary tide stations. At the present time the Coast and Geodetic Survey is operating 30 such stations on the Atlantic coast, 8 on the Gulf coast, 15 on the Pacific coast, and 8 in Alaska.

The primary tide stations serve a number of purposes. They furnish primary determinations of mean sea level at these stations, which are then used as the starting and "tie-in" points of the precise level net which is being spread over the country. The data also permit the precise determination of other tidal datum planes at these stations and make possible the correction to mean values of the results of short series of tide observations in the vicinity. The records furthermore furnish data on the heights of the tide at any particular time, on the slow changes taking place in the relative elevations of land and sea and also the basic data for the study and advancement of the subject of tides.

Reference to Tide Staff and Bench Marks

From the tide observations at any point the plane of mean sea level is determined as corresponding to a certain height on the fixed tide staff used in the tide observations at that point. In other words, mean sea level at that point may be said to be so many feet and hundredths above the zero of a given tide staff. And if it were a simple matter to maintain that tide staff for many years without change in elevation, it would serve very well for preserving the determination of the plane of mean sea level.

But unfortunately it is only rarely that a tide staff can be maintained without change for a number of years. Deterioration of the material used, changes in wharves and piling, and accidents of one kind or another make it necessary to replace a tide staff at intervals more or less frequent. To make certain that the new tide staff will be replaced at the same elevation as the preceding one, so as to make succeeding observations comparable with those preceding, it is necessary to refer the zero of the tide staff to bench marks.

As soon as the elevation of the tide staff with reference to one or more bench marks is known it becomes possible to refer the determination of the plane of mean sea level to these bench marks. These bench marks, established in rock or concrete, or on some substantial structure, thus preserve the determination of mean sea level much better than would the tide staff; and generally this plane at any point is given as so many feet and hundredths below one or more bench marks.

VI. HALF-TIDE LEVEL

Definition

The plane of half-tide level, or mean tide level, as it is sometimes called, is defined as lying exactly halfway between the planes of mean high water and mean low water. It is thus a plane lying close to mean sea level, and frequently the two are taken as synonomous. As accurate datum planes, however, the two must be carefully distinguished.

Strictly, the plane should be designated as that of "mean half-tide level," rather than "half-tide level" in consonance with the distinction between sea level and mean sea level. No confusion, however, results from the dropping of "mean," since the context clearly indicates the sense in which the term is used, whether to designate the half-tide level for a short period of time, as a day, week, or month, or as a datum plane.

Prior to the invention of the automatic tide gauge the recording of the tide throughout the 24 hours of the day was a matter of considerable expense. It was therefore customary to observe the tide only near the times of high and low water. This permitted a tabulation of the high and low waters but not of the hourly heights. Half-tide level could be determined from such tabulations, but not mean sea level; and as a rule the earlier determinations were those of the plane of half-tide level.

While the tabulation of the hourly ordinates is necessary in the harmonic analysis of the tide, the only datum plane derived from such tabulation is the plane of mean sea level. From the high and low water tabulation, however, not only is the plane of half-tide level determined, but also the various high-water and low-water planes. Moreover, since mean high water and mean low water are symmetrical with respect to half-tide level, a determination of the one is also a determination of the other. It is therefore customary to derive these high-water and low-water datum planes with regard to half-tide level.

Variations in Half-tide Level

The tide oscillates about sea level, high water and low water being, respectively, the maximum and minimum of the oscillation. And, on the average, the rise of high water above sea level is approximately the same as the fall of low water below sea level. Since half-tide level lies halfway between high water and low water, it follows that it must vary in much the same way as sea level.

This conclusion is borne out by an examination of daily, monthly, and yearly values of half-tide level. The variations in sea level discussed in the previous section may be taken to represent also the corresponding variations in half-tide level. It is unnecessary, therefore, to go into a detailed discussion of the variations in half-tide level. It will be sufficient to note in summary that, like sea level, half-tide level at any point on the coast varies from day to day, from month to month, and from year to year. From one day to the next, half-tide level may vary by a foot or more, and within the same year two values of daily half-tide level may differ by 5 feet or more.

Monthly half-tide level is subject to variations of both periodic and nonperiodic character, so that within a year half-tide level for two different months may differ by as much as a foot. Yearly values of half-tide level may show differences of a quarter of a foot or even more.

Relation to Mean Sea Level

If the curve representing the rise and fall of the tide were that of a simple sine curve, the planes of mean sea level and of half-tide level would coincide. But the rise and fall of the tide does not take place in accordance with the ordinates of a simple sine curve. The movement of the tide is compounded of the movements of a number of simple sine curves, some of which have fixed phase relations with respect to each other. The rise of high water above sea level is therefore generally not exactly the same as the fall of low water below sea level, and hence mean sea level and half-tide level generally differ.

Obviously, any cause that tends to disturb the regularity of the tide-curve tends to change the relation between sea level and half-tide level. Decided changes in wind and weather may therefore change that relationship somewhat. In general, however, the relation is very nearly constant. Figure 34 shows in diagrammatic form for each day of the month of October 1947 the relation of sea level to half-tide level at Boston. Half-tide level for each day was derived as the average of the four high and low waters of the day. On days when but one high or one low water occurred, the other one occurring nearest to the day in question was used to make up the group of four high and low waters. Sea level for each day was, as heretofore, derived as the average of the 24 hourly heights of the tide.

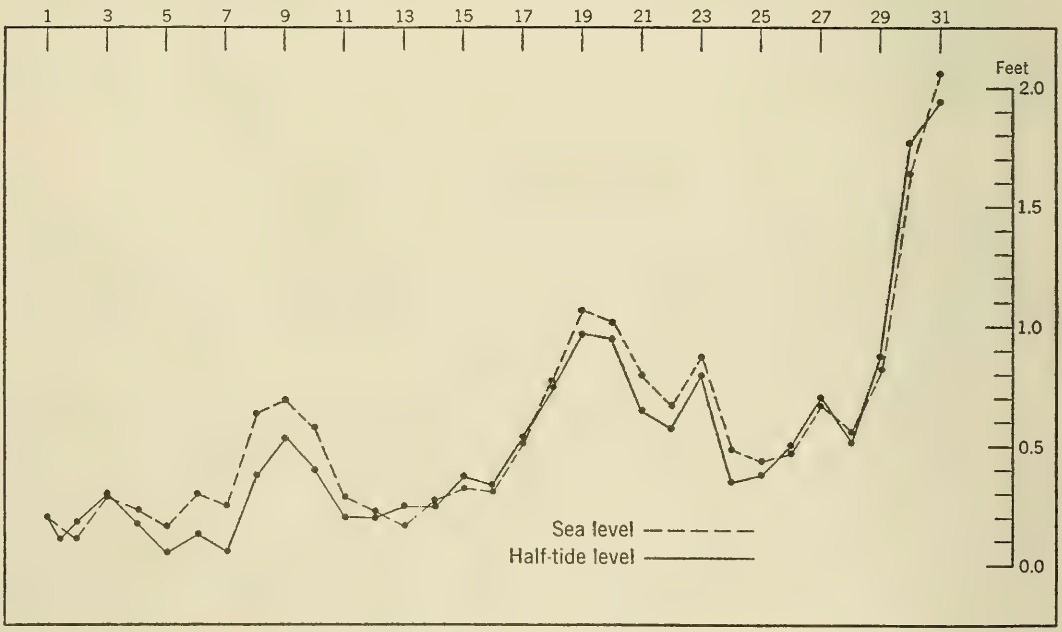

FIG. 34.—Daily sea level and half-tide level, Boston, October 1947.

From Figure 34 it is seen that despite the relatively large changes in sea level from day to day, the relation of sea level to half-tide level at Boston remains very nearly constant. To be sure, this relation changes somewhat from day to day, but these changes are relatively small. Sea level here is, almost without exception, above half-tide level, on the average by 0.12 foot.

It is not difficult to see why the relation of sea level to half-tide level is not constant from day to day. In the first place the fact that the tidal cycle has a period of very nearly 25 hours, and not 24 hours, introduces slight variations; and in the second place it is obvious that changes in wind and weather must vary that relationship. For example, suppose that at any given place we take two days during which the high waters, and likewise the low waters were exactly similar. Half-tide level for the two days would therefore be exactly the same. And if the weather conditions during the two days were similar, sea level likewise would be the same for the two days.

Suppose, however, that weather conditions on the second of the two days were the same as on the first day only until the occurrence of the last high or low water of the day (which, for the sake of illustration, we may assume to have occurred about 6 p. m.). Suppose that from that time to the end of the day the direction or velocity of the wind was different. Obviously, the half-tide level for that day would not be changed since the last high or low water used in deriving it has already occurred. But the hourly heights of the tide for the remainder of the day would differ from the corresponding heights on the first day, and hence, although half-tide level for the two days would still be the same, the sea levels would differ.

If monthly heights of sea level and half-tide level are compared, the relation between the two is found to be less variable than in the case of the daily levels. Figure 35 shows the monthly heights of sea level and half-tide level at Boston for the two years, 1947, and 1948. Without exception, monthly sea level is seen to be above half-tide level. For those two years sea level averaged higher than half-tide level by 0.10 foot, the least difference between monthly values being 0.05 foot and the greatest 0.15 foot.

A comparison of yearly heights of sea level and half-tide level shows a more nearly constant relation than between monthly values. For the 27 years of observations available at Boston from 1922 through 1948 the yearly values of sea level are above half-tide level, averaging 0.12 foot higher, the greatest yearly difference being 0.14 foot and the least 0.09 foot.

The relation between half-tide level and sea level at any place depends upon the amplitude and phase relations between the various constituents of the tide at that place. For tides of the semidaily and mixed types, the relation between half-tide level and sea level is given approximately, in the harmonic notation, by the formula

$$\mathrm{HTL}=\mathrm{SL}+\mathrm{M}_4 \cos\ (2\mathrm{M}_2^\circ-\mathrm{M}_4^\circ)-\frac{0.03(\mathrm{K}_1+\mathrm{O}_1)^2}{\mathrm{M}_2} \cos\ (\mathrm{M}_2^\circ-\mathrm{K}_1^\circ-\mathrm{O}_1^\circ),$$

in which HTL stands for half-tide level, SL for sea level, and the other terms have their usual significance in the harmonic notation.

Since the amplitudes of the various components vary somewhat from year to year, it follows that the relation between sea level and half tide level may differ from year to year. Furthermore, the cosine of $(2\mathrm{M}_2^\circ-\mathrm{M}_4^\circ)$ and also of $(\mathrm{M}_2^\circ-\mathrm{K}_1^\circ-\mathrm{O}_1^\circ)$ may be

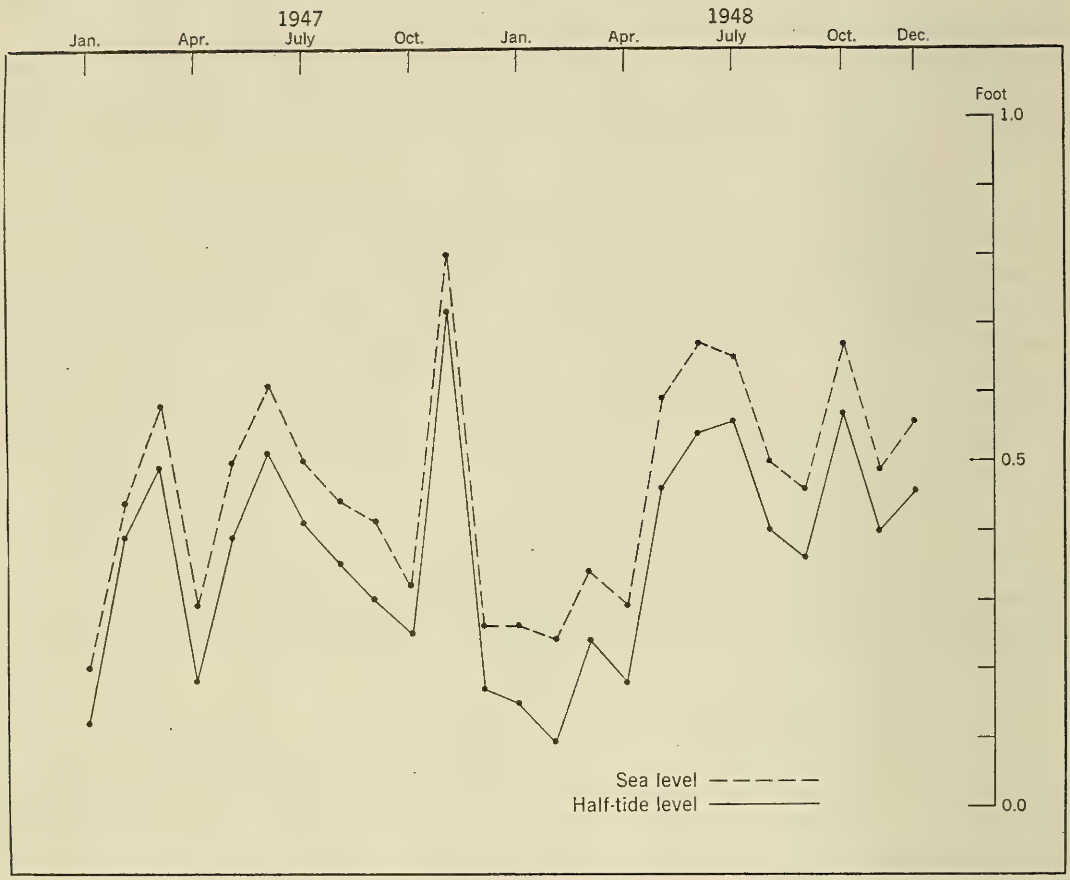

FIG. 35.—Monthly sea level and half-tide level, Boston, 1947–48.

either positive or negative. Hence sea level may be either above or below half-tide level, depending upon the phase and amplitude relations.

Along the Atlantic coast of the United States cos $(2M_2^\circ - M_4^\circ)$ is generally negative, while cos $(M_2^\circ - K_1^\circ - O_1^\circ)$ is generally positive. Hence along this coast half-tide level is below sea level, with but few exceptions. Along the Gulf coast both cosine terms in the formula are generally positive, so that here half-tide level may be either above or below sea level, depending upon which term has the greater value. On the Pacific coast the first cosine term is positive at some places, while at others it is negative; the second cosine term, however, is generally negative. Here, however, $\frac{(K_1+O_1)^2}{M_2}$ is, as a rule, more than 40 times as great as M_4, and therefore at most places along this coast half-tide level is above sea level.

The periodic variation in K_1 and O_1 from year to year is much greater than in M_4. Hence, where the ratio of $(K_1+O_1)^2$ to M_2 is large, appreciable variations in the relation of half-tide level to sea level may be expected from year to year. On the Atlantic coast this ratio is small, being at most places about 0.1; but on the Gulf and Pacific coasts it is relatively large, being at most places greater than unity. It is therefore to be expected that the relation of half-tide level to sea level will differ but little from year to year on the Atlantic coast, while on the Gulf and Pacific coasts larger differences will appear. This is brought out by the tide observations.

For tides of the daily type the above formula for the relation of half tide level to sea level does not apply, since the higher low waters and lower high waters which occur only infrequently are not taken into consideration. In determining half-tide level in the case of the daily type of tide, only the higher high waters and lower low waters are used. The relation of half-tide level to sea level at two nearby stations which differ in type of tide may therefore be quite different.

Galveston, Tex., furnishes a good example. Here the ratio of K_1+O_1, to M_2+S_2 is 1.8 and therefore according to the criterion on page 21 it should be classed with the daily tide. But it is only infrequently that but one high water and one low water occur in a day, and for some purposes it is useful to tabulate the tide at this station as if it were of the mixed type. This can be done easily by introducing a high and a low water having the same heights on any day when it is necessary to make the tidal day contain two high and two low waters.

When the tide observations at Galveston are tabulated as semidaily tides, half-tide level is found to be above sea level by 0.06 foot. If the tabulations are made in conformity with the practice for daily tides, that is, using only the higher high and

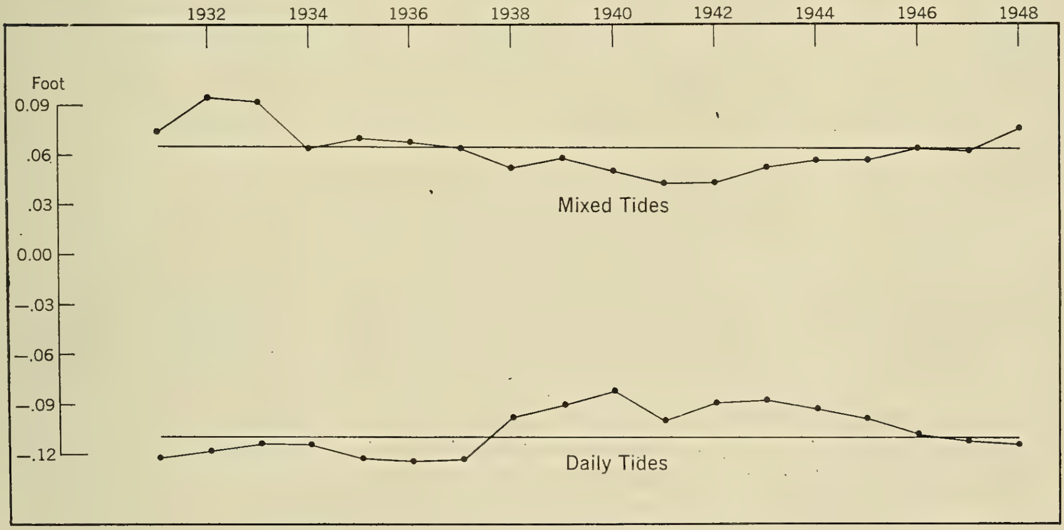

FIG. 36.—Relation of half-tide level to sea level, mixed and daily types.

lower low waters of each day, half-tide level is found to be below sea level by 0.11 foot. In other words, the two half-tide levels derived for the same station by treating them as different types of tide differ by 0.17 foot.

Figure 36 gives the yearly values of half-tide level minus sea level at Galveston for the 18-year period 1931 through 1948. The upper diagram gives the values determined by tabulating the tide record here as a mixed type, while the lower diagram gives the values derived by tabulating the tide record as a daily type. The horizontal line associated with each diagram represents the average value of HTL–SL for each of the respective series, in accordance with the scale at the left.

The average difference between the two tide level values is 0.17 foot, with a maximum difference of 0.21 foot in 1932 and a minimum difference of 0.13 foot in 1940 and 1942. In part these differences arise from the periodic variation in the relation of half-tide level to sea level which becomes appreciable where the ratio of $(K_1+O_1)^2$ to M_2 is large. For Galveston this ratio is 1.3. For Seattle, Washington, it is 5.1 and hence for this latter station the periodic variation in the relation of yearly half-tide level to sea level shows up much more clearly, as is illustrated in Figure 37. The ratio of K_1+O_1, to M_2+S_2 at Seattle is 0.97, and the tide here is definitely of the mixed type.

Figure 37 gives the yearly values of HTL–SL for the 38 years 1911 through 1948. The horizontal line shows the average value of this difference for the 38 years to be 0.02

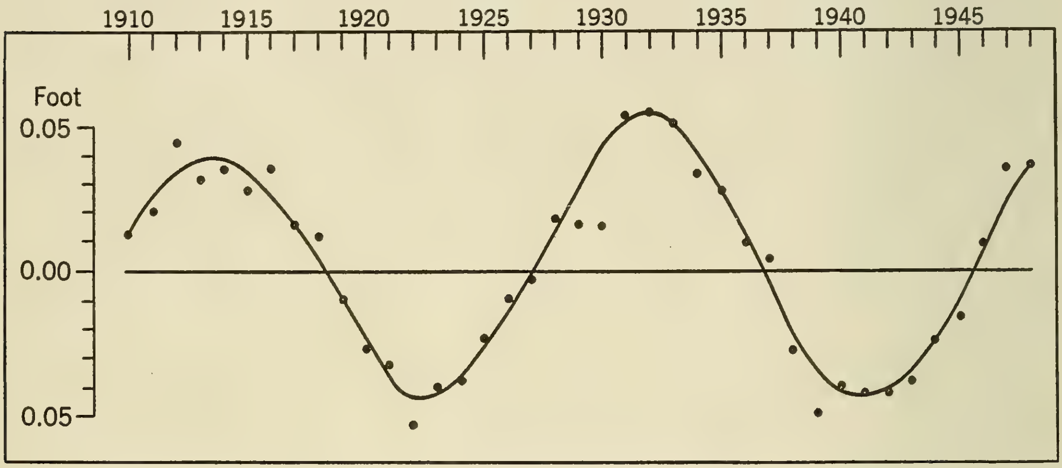

Fig. 37.—Relation of half-tide level to sea level, Seattle.

foot, and the yearly values range themselves very closely along the sine-like curve which has its maxima in the years of highest declination of the moon, 1913 and 1932, and its minima in the years of the moon's lowest declination, 1922 and 1941. The range of this variation, although well defined at Seattle, is relatively small, being only about 0.1 foot in a 19-year period.

Table 3 gives the relation of half-tide level to sea level as determined directly from the tide observations at a number of tide stations along the coasts of the United States. The table gives the values of half-tide level minus sea level. Negative values therefore indicate that sea level lies above half-tide level, while positive values indicate that sea level lies below half-tide level. Values in the table that are marked with an asterisk indicate that the half-tide level used was derived from the higher high and lower low waters.

In general it appears that along the Atlantic coast sea level is above half-tide level by about 0.1 foot, while along the Pacific coast sea level is below half-tide level by about 0.05 foot. The two sets of values for Eugene Island and Galveston show that half-tide level derived from the higher high and lower low waters is nearly 0.2 foot lower than that derived from all four tides, at those places.

TABLE 3.—*Half-tide level minus sea level*

	Foot
Atlantic Coast:	
Eastport, Maine	−0. 08
Portland, Maine	−0. 02
Boston, Mass	−0. 12
Woods Hole, Mass	0. 05
Providence, R. I	0. 13
Newport, R. I	0. 13
New London, Conn	−0. 06
New York, N. Y	−0. 08
Sandy Hook, N. J	−0. 01
Atlantic City, N. J	−0. 03
Philadelphia, Pa	−0. 23
Lewes, Del	−0. 02
Baltimore, Md	−0. 02
Annapolis, Md	−0. 02
Washington, D. C	−0. 01
Richmond, Va	−0. 14
Norfolk, Va	−0. 01
Wilmington, N. C	−0. 06
Southport, N. C	−0. 04
Charleston, S. C	−0. 12
Savannah, Ga	−0. 14
Mayport, Fla	−0. 05
Daytona Beach, Fla	0. 02
Miami, Fla	−0. 01
Gulf Coast:	
Key West, Fla	−0. 01
Cedar Keys, Fla	−0. 02
Pensacola, Fla	*0. 01
Bayou Riguad, La	*0. 00
Gulf Coast—Continued	
Eugene Island, La	0. 10 *−0. 10
Galveston, Tex	0. 06 *−0. 11
Port Isabel, Tex	*−0. 10
Pacific Coast:	
San Diego, Calif	0. 03
La Jolla, Calif	0. 01
Los Angeles Harbor, Calif	0. 03
Port Hueneme, Calif	0. 03
Port San Luis, Calif	0. 04
San Francisco, Calif	0. 05
Stockton, Calif	−0. 04
Crescent City, Calif	0. 04
Astoria, Oreg	0. 04
Neah Bay, Wash	0. 04
Seattle, Wash	0. 02
Friday Harbor, Wash	0. 22
Alaska:	
Ketchikan	−0. 01
Juneau	−0. 09
Sitka	0. 03
Skagway	−0. 11
Yakutat	0. 02
Sweeper Cove, Adak I	*−0. 30
Massacre Bay, Attu I	0. 12 *−0. 29
Hawaiian Islands:	
Honolulu	−0. 02
Hilo	−0. 02

Determination of Half-Tide Level

Since the variations in half-tide level are very nearly the same as those in sea level, the procedure employed in determining the plane of mean sea level becomes applicable also to the determination of the plane of half-tide level; and, as in the case of mean sea level, a determination based on 19 years of tide observations is taken as constituting a primary determination of the datum of half-tide level.

For deriving the datum of half-tide level from a short series of observations the direct determination is corrected by means of simultaneous observations at some nearby station for which a primary determination is at hand. The procedure is similar in all respects to that employed in correcting to a mean value the determinations of sea level from short series of observations. As an example, we may derive the datum of half-tide level at Annapolis, Md., from two different months of observations, using Baltimore, Md. as the primary station.

At Annapolis half-tide level for February 1947 read 4.04 feet on the staff and for June of the same year it read 5.24 feet. At Baltimore for these same months half-tide level was, respectively, 3.68 feet and 4.97 feet. For the 19-year period 1930–1948 half-tide level at Baltimore was 4.35 feet on the staff. Hence, for February 1947 a correction of $4.35-3.68=0.67$ foot is indicated and for June $4.35-4.97=-0.62$ foot. Applying these corrections to the respective half-tide levels at Annapolis the mean value for February is $4.04+0.67=4.71$ feet and for June, $5.24-0.62=4.62$ feet. Thus two monthly values which differed by 1.20 feet give mean values which differ by 0.09 foot.

For Annapolis there is available a direct determination of half-tide level for the 19-year period 1930–1948 from observations at that place. This value is 4.65 feet. Hence the mean values derived from one month of observations, after correction by comparison, differ by 0.03 foot and 0.06 foot, respectively, from the primary determination.

Annapolis and Baltimore are relatively near to each other, being about 20 miles apart. They are located, however, on different rivers. To exemplify the correction of a year's series we may take stations farther apart, Los Angeles and La Jolla, in California, which are nearly 100 miles apart, using Los Angeles as the primary station.

For the 19-year period 1928–1946, the lowest yearly half-tide level at La Jolla was for 1933 when it read 6.53 feet, and the highest was for 1941 when it read 6.90 feet. At Los Angeles the primary determination for the 19-year period 1928–1946 is 6.54 feet, while the yearly values for 1933 and 1941 are, respectively 6.30 and 6.75. Hence the corrections to these years are 0.24 foot and −0.21 foot, and therefore the mean value at La Jolla for 1933 is 6.77 feet and for 1941 is 6.69 feet. The direct primary determination of half-tide level at La Jolla is 6.71 feet. The two yearly values at La Jolla, which differed from each other by 0.37 foot, when corrected to mean values by comparison with Los Angeles differ from each other by only 0.08 foot, and from the primary determination by 0.06 foot and 0.02 foot.

To exemplify the determination of mean half-tide level for daily types of tide we may take Eugene Island, La., and Galveston, Tex., which are about 200 miles apart. This example will be instructive, furthermore, because at both stations, as Table 3 shows, two half-tide level datums can be derived, the one for the tide as mixed, and the other as daily.

For the 9 years of observations available at Eugene Island, 1940–1948, the lowest monthly value of half-tide level was for January 1940 when for the mixed tides it had a value of 1.42 feet and for the daily tide a value of 1.25 feet. The highest monthly value was for September 1948 with a value of 3.32 for the mixed tide and 3.06 for the daily tide. At Galveston the corresponding figures for January 1940 are 3.03 feet and 2.92 feet, and for September 1948, 4.97 feet and 4.80 feet.

The primary determination of half-tide level at Galveston for the 19-year period 1930–1948 is 4.00 feet for the mixed tides and 3.83 feet for the daily tide. Taking half-tide level for the mixed tides first, the corrections for January 1940 and September 1948 are respectively 0.97 foot and −0.97 foot which give for mean values at Eugene Island for these months 2.39 feet and 2.35 feet respectively. Thus two monthly values of half-tide level at Eugene Island which differed by 1.9 feet, when corrected to mean values by comparison with Galveston differ by but 0.04 foot.

Since only 9 years of observations are available at Eugene Island no direct primary determination of half-tide level can be derived. But it is clear that the value for the 9 years, after correction by comparison with Galveston will give a very close approximation to a primary determination. From these 9 years the value of half-tide level for the mixed tide at Eugene Island after correction by comparison with Galveston is 2.43 feet. The two derived mean values for the two months above therefore differ by 0.04 foot and 0.08 foot from a primary determination.

Taking up now the half-tide level for the daily tide, the primary determination of half-tide level at Galveston for the 19-year period 1930–1948 being 3.83 feet, the corrections for January 1940 and September 1948 are, respectively, 0.91 and —0.97, which give mean values for these months at Eugene Island of 2.16 feet and 2.09 feet respectively. The mean value of daily half-tide level at Eugene Island from the 9 years of observations when corrected by comparison with Galveston is 2.22 feet. Hence the derived mean values for the two months differ by 0.06 and 0.13 foot from a primary determination.

In general it may be said that the accuracy with which the datum of half-tide level can be derived from short series of observations is nearly the same as for sea level. By comparison with a suitable primary tide station, a day of observations will give mean half-tide level correct to within a quarter of a foot; a month of observations to within a tenth of a foot and a year of observations to within 0.05 foot.

Since half-tide level will be affected by slow changes in the relation of land to sea exactly as is sea level, it is necessary to specify the 19-year period which is used to derive mean values.

VII. MEAN HIGH WATER

Variations in Height of High Water

The height to which high water rises varies from day to day. Primarily these variations are related to the varying positions of the moon relative to earth and sun. These periodic variations were discussed briefly in Section II, Types of Tide. Superimposed on these periodic variations are non-periodic variations due to the effects of wind and weather. The resulting variations from day to day are exemplified in Figure 38 by the plottings of the heights of high water for the month of October 1947 at Atlantic City, Los Angeles, and Pensacola. These represent, respectively, the typical variations in the semidaily, mixed and daily types of tide.

The small circles in Figure 38 give the heights of each high water for the month, and to indicate clearly the succession, each high water is joined by a straight line to the preceding and succeeding high waters. At Atlantic City it is seen that there were two high waters each tidal day, succeeding high waters generally differing by several tenths of a foot and sometimes by as much as a foot or even more. For that month the lowest high water occurred on the morning of the 7th and the highest on the morning of the 31st, the difference between the two being 3.9 feet. The average range of tide at Atlantic City during that month, that is the average difference between the high and low waters was 4.0 feet, so that the difference between the highest and lowest of the high waters was very nearly the same as the average difference between the high and low waters.

At Los Angeles, there were likewise two high waters each tidal day, but the differences between succeeding high waters were generally greater than at Atlantic City. On the average the difference between the higher high waters and lower high waters for that month at Los Angeles was 1.1 feet, but several times this difference exceeded 2 feet. This difference in the behavior of the high waters at Los Angeles arises from the fact that at Atlantic City the tide is of the semidaily type while at Los Angeles it is of the mixed type.

For the month of October 1947, represented in Figure 38, the range of the tide at Los Angeles averaged 3.8 feet. The lowest high water occurred on the 6th and the highest on the 31st—almost exactly the same dates as for Atlantic City—the difference being 3.3 feet.

At Pensacola for the greater part of the month of October 1947 only one high water and one low water occurred. For the few days when two high waters occurred only the higher high water is plotted in Figure 38. The average range of the tide at Pensacola for that month, that is, the average difference in height between the higher high and lower low waters was 1.2 feet. The highest high water occurred on the 6th (with like heights on the 4th and 5th) and the lowest high water occurred on the 1st, the difference between the two being 1.4 feet. So that here during this month the difference in height between two high waters was greater than the average difference between the high waters and low waters of the month.

The fluctuations in the heights of the high waters pictured in Figure 38 are due both to periodic tidal variations and to the effects of wind and weather. The latter

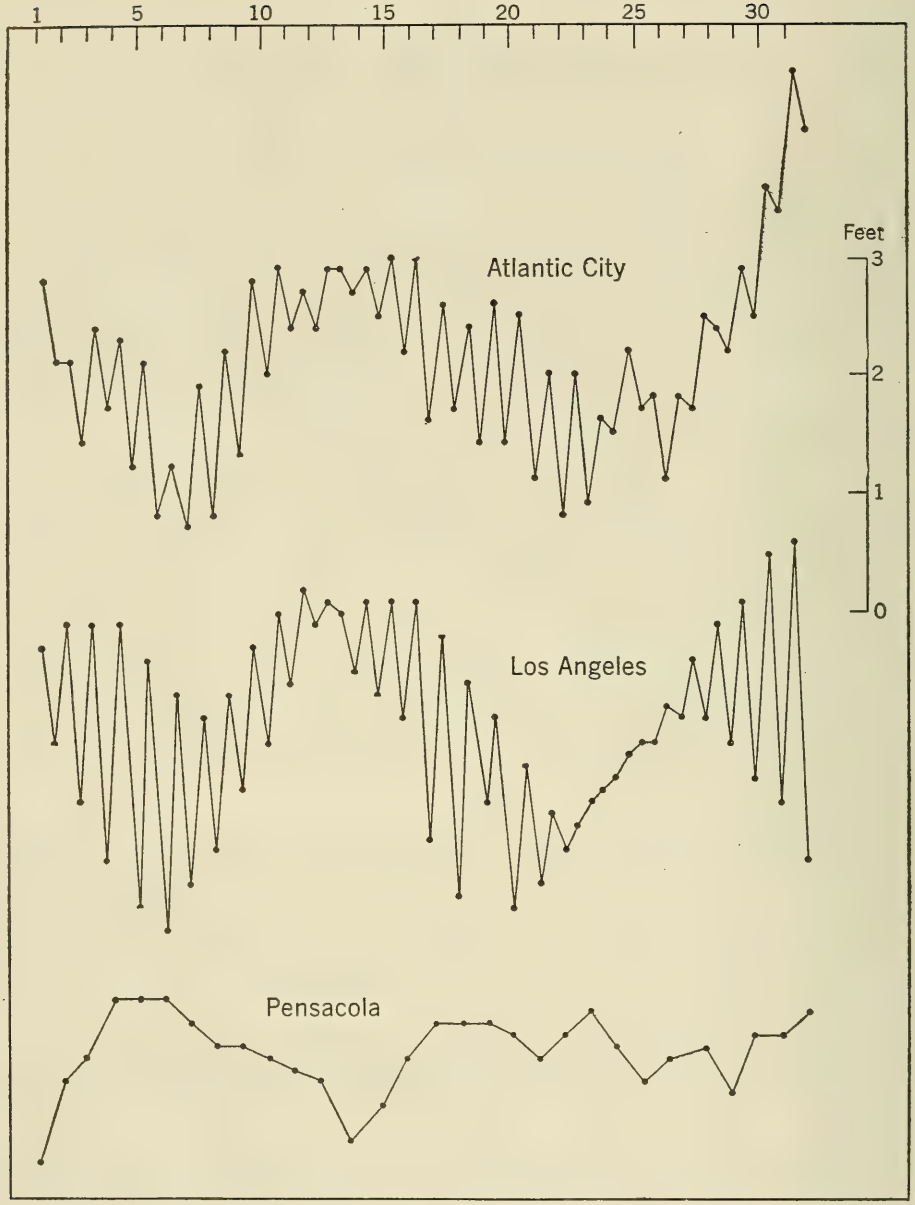

FIG. 38.—High waters, Atlantic City, Los Angeles, and Pensacola, October 1947.

effects can be practically eliminated by subtracting from the height of high water the instantaneous height of sea level. The resulting heights of the high waters at the three stations for the same month of October 1947 are shown in Figure 39. The instantaneous height of sea level for any high water is derived by averaging the 24 hourly heights of the tide which center about the time of that particular high water.

The times of the moon's phases, extreme declination and apogee and perigee are plotted at the top of Figure 39. The relation between the variations in the high waters and the moon's position now becomes clear. At Atlantic City the variation is primarily

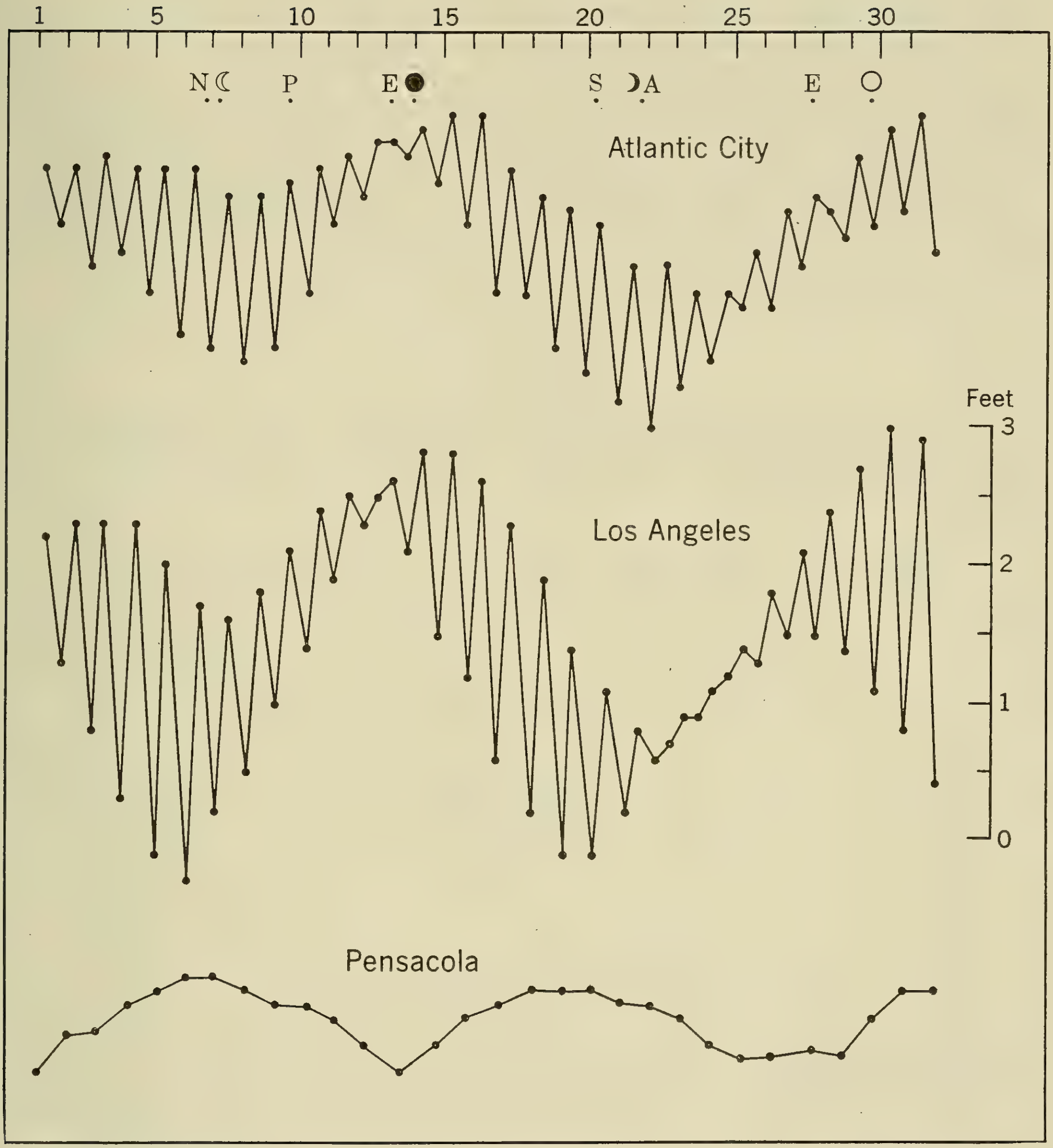

FIG. 39.—High waters, Atlantic City, Los Angeles, and Pensacola, October 1947, referred to instantaneous sea level.

with the variation in phase and in parallax, the highest high waters coming at the times of new and full moon, or when the moon is in perigee, while the lowest high waters come at the times of the moon's 1st and 3d quarters, or when the moon is in apogee. The effect of the moon's declination is also evident, giving greater inequalities in the high waters at the time of the moon's northing and southing, and least inequality, near the times when the moon is over the Equator. These are the typical variations in the semidaily type of tide.

At Los Angeles the variations are much the same as at Atlantic City, except that the inequality in the high waters is much greater. At Pensacola the variations in response to the changes in phase and parallax of the moon are very small, the primary variation depending on the moon's declination.

The daily height of high water is thus subject to relatively large variations both periodic and nonperiodic in character. The periodic variations depend primarily on the phase, distance, and declination of the moon, the periods of these being approximately 29½, 27½, and 27⅓ days, respectively. Such variations are therefore largely eliminated within a month. And within a month, too, the large variations in sea level due to wind and weather tend to balance out. It follows therefore that monthly high waters will show smaller variations than daily high waters.

Monthly High Water

In Figure 40 are shown the monthly heights of high water at the same three stations as in Figure 38 for the 2-year period 1946–1947. From one month to the next, high water is seen to vary from a few hundredths of a foot to as much as 0.8 foot.

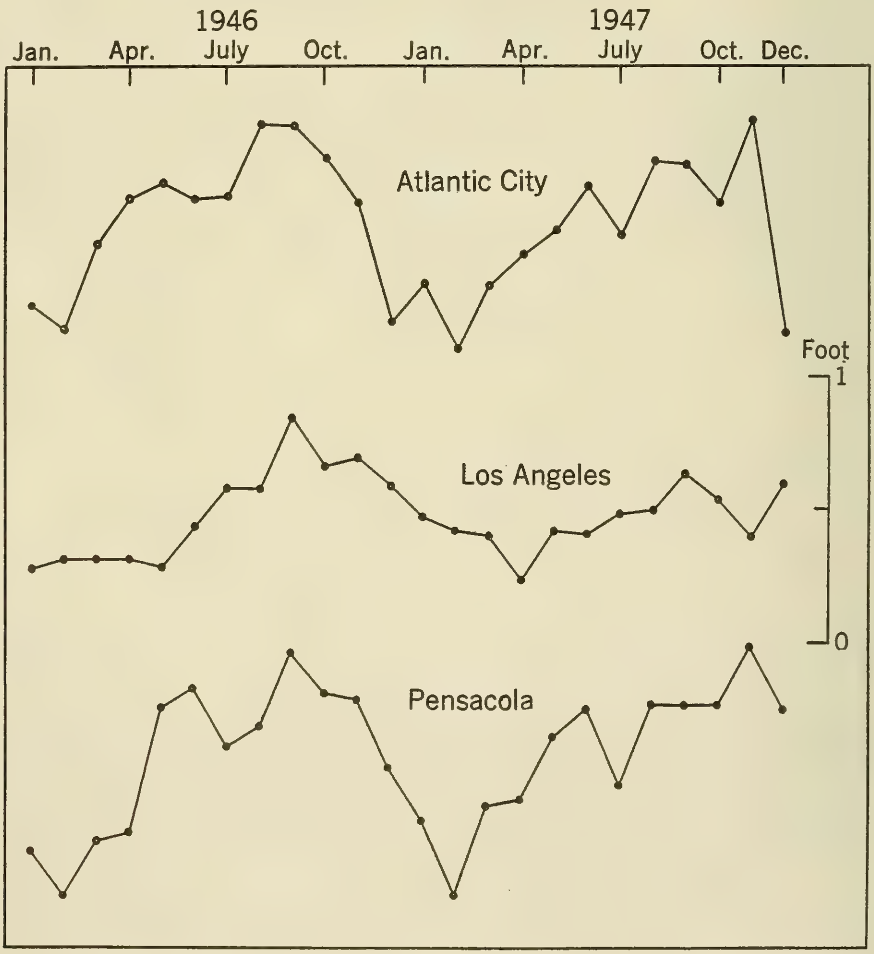

FIG. 40.—Monthly heights of high water, Atlantic City, Los Angeles, and Pensacola, 1946–47.

Obviously some of the variations must be due to the corresponding changes in sea level. On comparing the diagram for Atlantic City in Figure 40 with the diagram of monthly heights of sea level for Atlantic City in Figure 24 it is seen that there is a close parallelism between the two diagrams. It follows therefore that there must be an annual variation in high water similar to that in sea level.

Figure 41 gives the average heights of monthly high waters at the three stations as determined from the 19-year period 1930–1948. The three curves thus represent the annual variation in the height of high water at the respective stations. A comparison of each of the diagrams of Figure 41 with the corresponding diagrams in Figures 26, 27, and 28 shows very close parallelism. The horizontal line in each diagram represents the height of mean high water at that station.

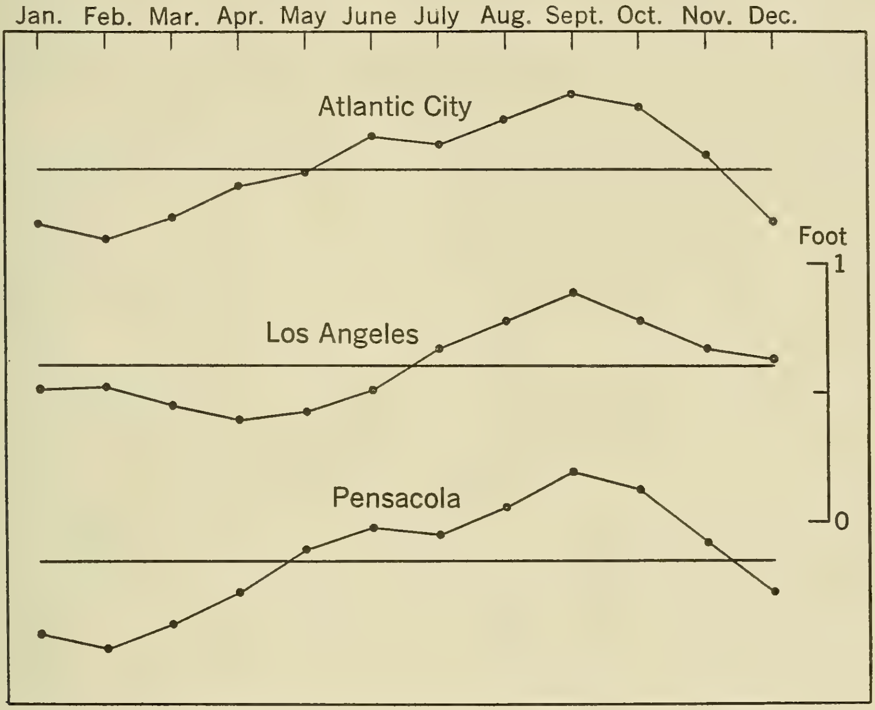

FIG. 41.—Annual variation in high water, Atlantic City, Los Angeles, and Pensacola.

Monthly high water at any point is thus subject to variations both periodic and nonperiodic in character; and in both of these it follows closely the like variations in sea level at that point.

Yearly High Water

Turning now to the variations in the height of yearly high water, it is found, as was to be expected, that these are much smaller than the variations from month to month. In Figure 42 are shown the yearly heights of high water for the 25-year period 1924–1948 at Boston, Los Angeles, and Pensacola. Generally, consecutive heights of yearly high water at any one of these places are seen to differ by not more than a few hundredths of a foot, but occasionally this difference may be as much as 0.2 foot.

Within the 25-year period the lowest and highest yearly values differed by 0.60 foot at Boston, 0.50 foot at Los Angeles and 0.76 foot at Pensacola.

A comparison of the curves of Figure 42 with the corresponding curves in Figures 30, 31, and 32 which give the yearly variations in sea level, shows that yearly high water varies in much the same way as yearly sea level. If the variation were exactly the

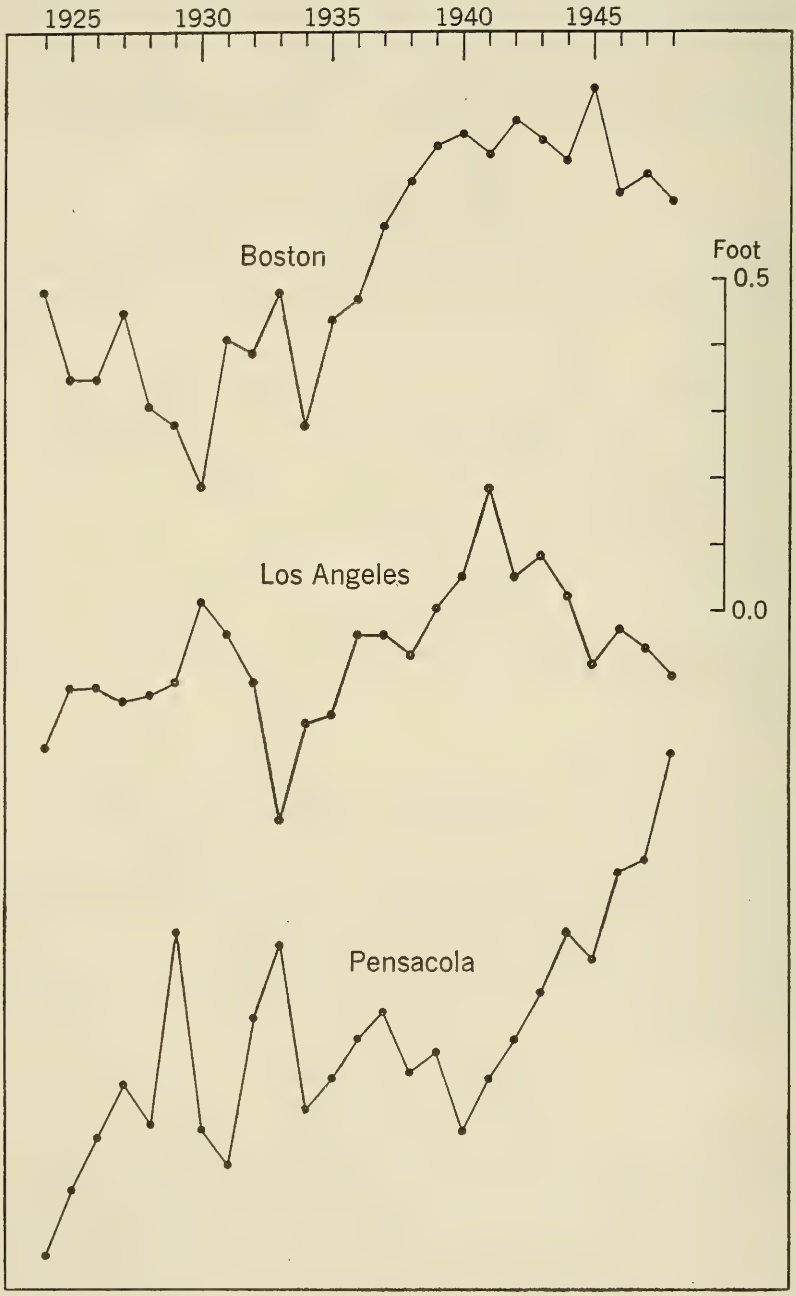

FIG. 42.—Yearly high water, Boston, Los Angeles, and Pensacola, 1924–48.

same, the rise of high water above sea level at any station would, from year to year, be constant. This, however, is found to be not the case, for investigation shows that the rise of high water above sea level varies from year to year.

In part such differences must be ascribed to difficulties inherent in the operation of a tide station over considerable periods of time, and also to the disturbing effects of wind and weather. Changes in hydrographic features, whether natural or artificial, likewise may change the relation of high water to sea level, for such changes generally do not affect sea level but do affect the range of tide.

The causes enumerated above for the variation in the relation of yearly high water to yearly sea level are not of a periodic character. If the height of high water above sea level from year to year is plotted for a number of years, a distinct periodic variation comes to light, the period of variation being approximately 19 years. In Figure 43

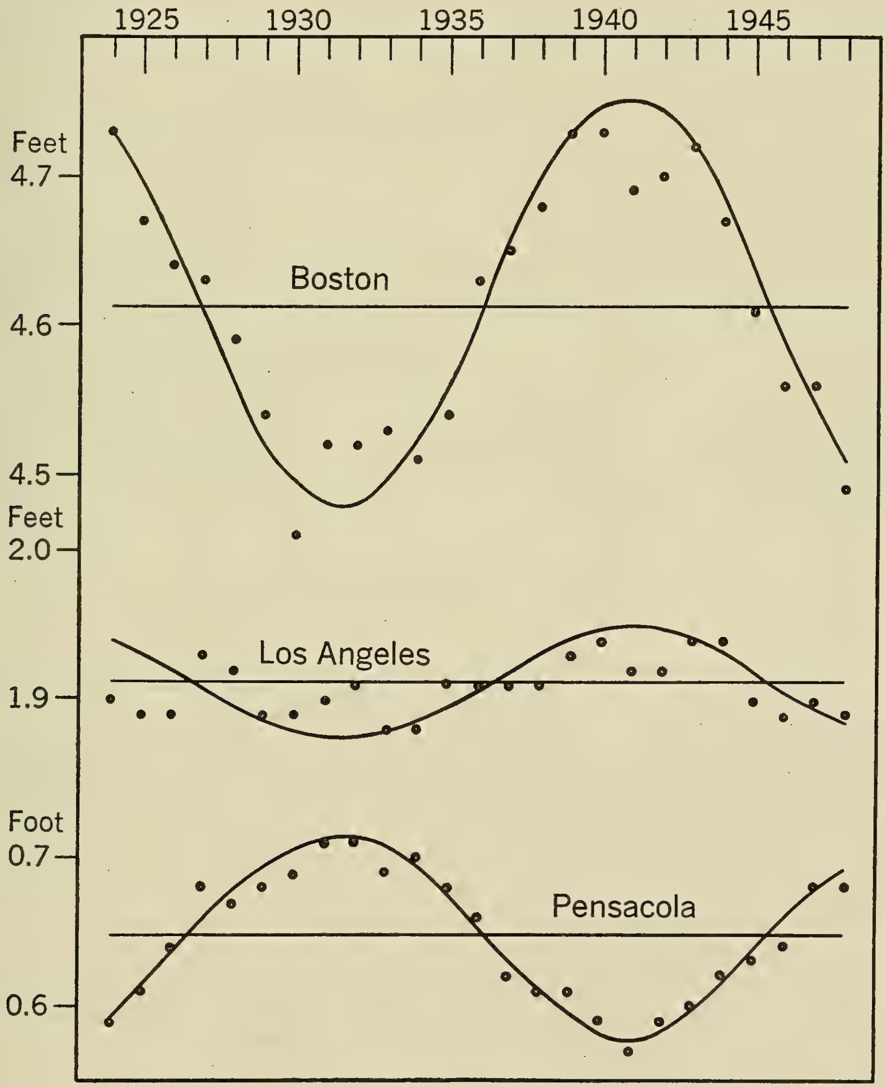

FIG. 43.—Variation of yearly high water in relation to yearly sea level.

is shown the relation of yearly high water to yearly sea level at the same three stations used in Figure 42.

The horizontal line associated with each of the diagrams of Figure 43 represents for each station the average height of high water above sea level for the 25 years represented, this height in feet being given by the figures at the left of the horizontal line. The yearly values of high water minus sea level are seen to range themselves more or less closely along the sine-like curves associated with each diagram.

From theoretical considerations of an astronomical character it can be shown that there should be a periodic variation in the range of the tide, and thus of the rise of high water above sea level, with a period of 18.6 years. This is brought about by the change in the longitude of the moon's node, which introduces a variation in the inclination of the lunar orbit to the earth's Equator. The effect of this, however, is different on the daily and semidaily constituents of the tide. The sine-like curves of Figure 43 were drawn in conformity with the above theoretical considerations.

In Figure 43 it is seen that the rise of high water at Boston and Los Angeles was least in 1931 and 1932, and greatest in 1941. The amplitude of this variation is different at the two places for two reasons: (1) this amplitude is a function of the range of the tide; (2) the variation depends on the relative amplitudes of the daily and semidaily tides.

At Pensacola it is seen the phase of the variation in rise of yearly high water above sea level is opposite to that at Boston and Los Angeles, the greatest rise coming in 1931 and 1932 and the least rise in 1941. At Pensacola it will be recalled that the tide is of the daily type while at Boston it is of the semidaily type and at Los Angeles it is of the mixed type. If we look into a nautical almanac we will find that in 1931 and 1932 the moon had its greatest declination while in 1941 it had its least declination.

Summarizing the variations of high water, it may be said that high water is subject to periodic variations from day to day, month to month, and year to year, in a period of approximately 19 years. In addition it is also subject to the nonperiodic variations found in sea level.

Definition of Mean High Water

In view of the variations to which the height of high water is subject, mean high water at any place may be defined simply as the average height of the high waters at that place over a period of 19 years.

In tides of the semidaily and mixed types no difficulties are encountered in the application of this simple definition. In applying this definition to tides which are predominantly of the daily type, however, the question whether or not to include secondary tides comes up. For where the tide is predominantly daily, there are periods every fortnight when two high and two low waters occur. The secondary tides in such cases are frequently difficult to detect on the tide record because of the disturbing effects of wind and weather. Moreover, on their first and last appearance each fortnight, they have small ranges and some arbitrary figure would have to be set for the range, below which the fluctuation on the record would not be considered as constituting a high or low water.

In the practical work of tide tabulation, it is customary to tabulate the heights of high and low waters to the nearest tenth of a foot, except in special cases of tides of very small range. Hence in tabulating tides of the daily type, secondary tides with ranges less than a tenth of a foot are disregarded for the purposes of high and low water tabulation.

In tides of the daily type, therefore, it is preferable to completely disregard the occasional secondary tides in determining mean high water, and use but one high water a day, the higher high water. Mean high water for the daily tide is thus the same as mean higher high water, and the simple definition of mean high water becomes applicable to all types of tide.

Primary Determination

A primary determination of mean high water is based directly on the average of the high waters over a 19-year period. And if there were no change in sea level from one 19 year period to another, we would expect two different 19 year determinations of mean high water at any place to agree, unless some change in tidal regime had taken place.

At Baltimore, continuous tide observations are available for the 46 year period 1903–1948. If we take the first 19 year series, 1903–1921, mean high water on the fixed staff reads 4.648 feet, while for the last 19 year series, 1930–1948, it reads 4.916 feet, a difference of 0.268 foot. Sea level for the first 19 year series reads 4.107 feet and for the last 19 years, 4.368 feet. With reference to the sea level for the respective 19 year series, therefore, mean high water at Baltimore for the period 1903–1921 was 0.541 foot above sea level and for the period 1930–1948, 0.548 foot above sea level. The difference of 0.007 foot between the two latter values is scarcely significant in view of the disturbing effects of wind and weather.

At Seattle, tide observations covering the 50 year period 1899–1948 are available. Three slightly overlapping 19 year series may be formed from these observations; 1899–1917; 1915–1933 and 1930–1948. The mean high water for each of these series on the staff is 7.881 feet, 7.897 feet and 8.011 feet, respectively. The difference between the first and second primary determinations is 0.016 foot and between the first and third is 0.130 foot. Referred to the respective 19 year sea levels, mean high water above sea level is, respectively, 3.837 feet, 3.842 feet and 3.849 feet, the difference between the first and the third being 0.012 foot. This again is so small as to be scarcely significant.

It appears therefore that primary determinations of mean high water above sea level are in practical agreement. But since sea level at many places appears to be subject to a slow change, for precise purposes the datum of mean high water must be specified with regard to the 19 year series used.

Secondary Determination

Primary determinations of mean high water are practicable at relatively few places. At other places this datum can be derived with sufficient precision for most purposes from observations covering much shorter periods than 19 years. Two methods are available: (1) comparison of simultaneous observations; (2) correction by tabular values.

Either method requires two separate corrections. Since high water varies periodically with respect to sea level, the first correction is to derive the value of mean high water above sea level for the period of observations. The second correction is to derive the value of mean sea level from the sea level of the period of observations.

In the tabulation of the high and low waters from the tide observations at any place, half-tide level is derived directly from this tabulation. The derivation of sea level requires the tabulation of the hourly heights of the tide. But as was found in the discussion of half-tide level, the variations in the latter follow closely those of sea level. Hence it is more convenient in determining mean high water to use half-tide level rather than sea level.

The method of comparison of simultaneous observations is generally the more satisfactory method and will be taken up first.

Comparison of Simultaneous Observations

To exemplify the determination of mean high water by this method, the procedure used and the accuracy attainable will be illustrated below for periods of various lengths.

Day.—Suppose that tide observations were made at Seaveys Island, Maine, near the mouth of the Piscataqua River, each fifth day of the month of May 1946 and that it was desired to determine mean high water from each of these days of observations using the primary tide station at Boston for comparison. In tabular form the data would appear as follows:

Mean high water from 1 day of observations

Date	Boston						Seaveys Island						
	Height on staff			HW-HTL	Corrections for—		Height on staff			HW-HTL	MHW above HTL	MHTL	MHW
	HW	LW	HTL		MHTL	Mean range	HW	LW	HTL				
1946	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Factor*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
May 1	14.10	1.75	7.92	6.18	0.28	0.764	11.35	0.85	6.10	5.25	4.01	6.38	10.39
May 6	13.55	3.15	8.35	5.20	−0.15	0.908	10.90	2.10	6.50	4.40	4.00	6.35	10.35
May 11	12.70	3.95	8.32	4.38	−0.12	1.079	10.20	2.80	6.50	3.70	3.99	6.38	10.37
May 16	12.95	3.75	8.35	4.60	−0.15	1.027	10.40	2.50	6.45	3.95	4.06	6.30	10.36
May 21	12.40	4.75	8.58	3.82	−0.38	1.237	10.05	3.40	6.72	3.33	4.12	6.34	10.46
May 26	12.85	4.25	8.55	4.30	−0.35	1.099	10.35	2.90	6.62	3.73	4.10	6.27	10.37
May 31	15.05	2.35	8.70	6.35	−0.50	0.744	12.15	1.60	6.88	5.27	3.92	6.38	10.30

From the high and low water tabulations the average height of the two high waters for each day is entered in the 2d column for Boston and in the 8th column for Seaveys Island. The 3d and 9th columns similarly give the average heights of the two low waters for each day. The 4th and 10th columns give the half-tide levels for each day, derived from the corresponding high and low waters, and the 5th and 11th columns give the heights of high water for each day above half-tide level, or the semi-range of the tide.

For the 19-year period 1930–1948, mean high water on the tide staff at Boston reads 12.920 feet, and mean half-tide level 8.196 feet. Hence the primary determination of mean high water above half-tide level, or the semi-range of the tide is 4.724 feet. In column 6 is entered the correction for the half-tide level to bring it to mean value, and in column 7, the factor by which the half range in column 5 is to be multiplied to give the mean value of the half range. The latter value obviously is derived by dividing 4.724 by the value in column 5.

The factors in column 7 are then applied to the corresponding values of column 11 to give the values of mean high water above half-tide level in column 12, and the corrections of column 6 are applied to the corresponding values of column 10 to give the values of mean half-tide level in column 13. By adding columns 12 and 13, the values of mean high water in column 14 are derived.

The observed average heights of daily high water in column 8 are seen to have varied from 10.05 feet to 12.15 feet, a difference of 2.10 feet. After correction to mean values the difference between the largest and smallest values in column 14 is seen to be 0.16 foot. From a 14-year series of tide observations at Seaveys Island, the value of mean high water corrected to the 19 years 1930–1948 is 10.38 feet. The values of mean high water derived from each of the one day of observations thus differ from the best determined value by less than 0.1 foot.

Boston and Seaveys Island are about 50 miles apart, have tides of the same type, and during the month of May 1946 the tide in the region was relatively free from large disturbances of wind and weather. Hence mean high water derived from one day of observations gave a value within 0.1 foot of the best determined value from a long series of observations. In general it may be taken that like sea level or half-tide level one day of observations will determine mean high water within a quarter of a foot, if a suitable primary tide station is available for comparison.

Month.—The tide at the station used in the previous section to derive mean high water from one day of observations is of the semidaily type. To exemplify the derivation of mean high water from a month of observations we may turn to the daily type. In this type of tide it will be recalled mean high water is the same as higher high water. At Mobile, Ala., and at Pensacola, Fla., the tide is principally of the daily type and we may use the tide observations at the former place for every other month of the year 1936 for deriving mean high water by comparison with Pensacola. The two places are about 50 miles apart but lie in different bays.

In the table below the data are given. The procedure is similar to that for deriving mean high water from one day of observations. An abridgement of the tabular form is made by omitting the column for low water since the half-tide level for the month can be taken directly from the high and low water tabulation.

Date	Pensacola					Mobile					
	Height on staff		HW–HTL	Corrections for		Height on staff		HW–HTL	MHW above HTL	MHTL	MHW
	HW	HTL		MHTL	Mean range	HW	HTL				
1936	*Feet*	*Feet*	*Feet*	*Feet*	*Factor*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
January	9.24	8.61	0.63	0.12	1.001	3.67	2.91	0.76	0.76	3.03	3.79
March	8.88	8.25	0.63	0.48	1.001	3.31	2.51	0.80	0.80	2.99	3.79
May	9.48	8.82	0.66	−0.09	0.956	3.87	3.08	0.79	0.76	2.99	3.75
July	9.42	8.75	0.67	−0.02	0.942	3.72	2.93	0.79	0.74	2.91	3.65
September	9.58	9.04	0.54	−0.31	1.169	3.84	3.17	0.67	0.78	2.86	3.64
November	9.09	8.42	0.67	0.31	0.942	3.21	2.43	0.78	0.73	2.74	3.47

For Pensacola the 19 years of observations 1930–1948 give mean high water as 9.363 feet on staff and mean half-tide level as 8.732 feet, so that the half range is 0.631 feet. From these values the corrections for each month are computed in columns 5 and 6, and used for deriving the values in columns 10 and 11, from which the last column is derived.

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At Mobile there are available four years of observations, from 1933 to 1937. From these 4 years, after correction by comparison with Pensacola to the 19 years 1930–1948, mean high water is 3.70 feet, mean tide level 2.96 feet and half the mean range 0.74 foot. These latter values, while not primary determinations may be considered as well-determined values. By comparing with the last column in the table above it is seen that mean high water determined from one month of observations is generally within 0.1 foot of a well-determined value. For November the difference is 0.23 foot, reflecting the effect of disturbed weather conditions.

Year.—To exemplify the determination of mean high water from a year of observations, it will be instructive to use the mixed type of tide. Los Angeles Harbor and La Jolla, both on the coast of California, are about 75 miles apart and both have the mixed type of tide. We may derive mean high water at La Jolla for every other year from 1936 to 1946 by comparison with Los Angeles Harbor. The procedure may be shortened from that used in the previous examples, by using half-tide level and range. The data are given in tabular form below.

Year	Los Angeles Harbor				La Jolla				
	Height on staff		Corrections for		HTL	Range	MHTL	Mean range	MHW
	HTL	Range	HTL	Range					
	Feet	*Feet*	*Feet*	*Factor*	*Feet*	*Feet*	*Feet*	*Feet*	*Feet*
1936	6.54	3.77	0.00	0.998	6.66	3.66	6.66	3.65	8.48
1938	6.51	3.78	0.03	0.995	6.63	3.62	6.66	3.60	8.46
1940	6.59	3.85	−0.05	0.978	6.77	3.72	6.72	3.64	8.54
1942	6.62	3.80	−0.08	0.990	6.75	3.66	6.67	3.62	8.48
1944	6.57	3.83	−0.03	0.982	6.72	3.67	6.69	3.60	8.49
1946	6.57	3.74	−0.03	1.006	6.78	3.62	6.75	3.64	8.57

In Los Angeles Harbor, the 19-year series 1928–46 gives half-tide level as 6.54 feet and the range as 3.762 feet. The corrections for the different years for half-tide level and range are derived from these values and entered in columns 4 and 5. These corrections are then applied to columns 6 and 7, deriving thereby the values in columns 8 and 9. The last column is then derived by adding one-half the mean range to the mean half-tide level.

The difference between the greatest and the least of the values in the last column is 0.11 foot and the average of the six values is 8.50 feet. From the 19-year series 1928–1946 available at La Jolla the primary determination of half-tide level is 6.71 feet and of the range 3.62 feet giving the primary determination of mean high water as 8.52 feet. The secondary determination for each of the years in the table above thus are correct to within about 0.05 foot as compared with the primary determination.

Correction by Tabular Values

The corrections to derive mean high water from a short series of observations are of two kinds: (1) correction of half-tide level to mean value; (2) correction of range to mean value. The former correction arises primarily from the effects of wind and weather, but the latter correction is primarily of a periodic character depending on the positions of sun and moon with respect to the earth. In the method of comparison of simultaneous observations, both corrections are determined by comparison. But it is also possible to derive the correction for range from theoretical considerations by the use of tabular values. When tides are observed at a place remote from a suitable primary tide station the range may be corrected by use of these tabular values.

In the discussion of yearly high water it was found that the variation of high water above sea level is a function of the range of the tide and depends also on the relative amplitudes of the daily and semidaily tides. For the purpose of deriving factors to correct the range to a mean value, the ratio of the daily to the semidaily constituents is taken from the harmonic constants as $(K_1+O_1)\div M_2$. Where harmonic constants are not available this ratio may be derived, approximately from the formula, $2(DHQ+DLQ)\div Mn$, in which DHQ is the mean high-water diurnal inequality, DLQ is the mean low-water diurnal inequality, and Mn is the mean range of the tide. The derivation of the diurnal inequalities will be taken up in connection with the higher high-water and lower low-water datums. In Table 4 the ratio of K_1+O_1 to M_2 at a number of points along the coasts of the United States is given.

TABLE 4.—*Ratio of K_1+O_1 to M_2*

Atlantic Coast:		Pacific Coast:	
Eastport, Me	0.10	San Diego, Calif	1.00
Portland, Me	0.19	La Jolla, Calif	1.10
Boston, Mass	0.19	Los Angeles Harbor, Calif	1.07
Woods Hole, Mass	0.56	Port Hueneme, Calif	1.15
Providence, R. I	0.19	San Francisco, Calif	1.10
Newport, R. I	0.22	Stockton, Calif	0.84
New London, Conn	0.34	Humboldt Bay, Calif	0.98
New York, N. Y	0.23	Crescent City, Calif	0.88
Albany, N. Y	0.31	Port Orford, Oreg	0.92
Sandy Hook, N. J	0.23	Coos Bay, Oreg	0.82
Atlantic City, N. J	0.30	Newport, Oreg	0.80
Philadelphia, Pa	0.22	Astoria, Oreg	0.66
Lewes, Del	0.31	Grays Harbor, Wash	0.63
Baltimore, Md	0.74	Neah Bay, Wash	0.97
Annapolis, Md	0.80	Port Townsend, Wash	1.82
Washington, D. C	0.18	Seattle, Wash	1.20
Richmond, Va	0.21	Olympia, Wash	0.93
Norfolk, Va	0.25	Friday Harbor, Wash	2.13
Wilmington, N. C	0.26	Alaska:	
Southport, N. C	0.25	Ketchikan	0.44
Charleston, S. C	0.24	Juneau	0.41
Savannah, Ga	0.18	Sitka	0.67
Mayport, Fla	0.22	Skagway	0.41
Daytona Beach, Fla	0.30	Yakutat	0.65
Miami, Fla	0.20	Cordova	0.56
Gulf Coast:		Seward	0.60
Key West, Fla	1.03	Kodiak	0.68
Cedar Keys, Fla	0.95	Dutch Harbor	2.13
Pensacola, Fla	12.29	Adak I	3.66
Mobile, Ala	18.60	Attu I	2.55
Eugene I., La	3.34	St. Michael	3.82
Galveston, Tex	2.42	Hawaiian Islands:	
Rockport, Tex	7.35	Honolulu	1.36
Port Isabel, Tex	4.12	Hilo	1.11

It will be noted in Table 4 that the ratio of K_1+O_1 to M_2 is approximately the same for large regions, so that at any desired point this ratio may be taken to be the same as at a station in the general vicinity. Thus for the Atlantic coast of the United States this ratio is about 0.2 while for the Pacific coast it is about 1.0. On the Gulf coast this ratio shows wide variations but even here the value at any given point may be taken as characteristic of the tide in the near vicinity.

The ratio K_1+O_1 to M_2 may also be used as a criterion of the type of tide. In the section discussing the criteria for the different types of tide the formula used was the ratio of K_1+O_1 to M_2+S_2. While the relation of M_2 to S_2 differs in different places, an approximate relation is $S_2=0.4M_2$. If this value of S_2 is substituted in the formulae defining the different types of tide, the semidaily tide has a ratio of K_1+O_1 to M_2 less than 0.35, the mixed tide a ratio between 0.35 and 2.10 and the daily type a ratio greater than 2.10. In round numbers therefore the ratio of K_1+O_1 to M_2 is less than 0.5 in the semidaily tide, between 0.5 and 2 in the mixed tide and greater than 2 in the daily tide.

With the ratio of K_1+O_1 to M_2 given, the periodic change in the range of the tide may be calculated and tabular values derived for each year. Table 5 gives these factors for each year of the 50-year period 1931 to 1980, calculated from Tables 6 and 14 of Harris' Manual of Tides. The tabular values may be calculated to yield factors which will give the range in the respective years, or the reciprocals of those values, which will then be the factors to correct the range observed in a given year to mean value. The latter factors are obviously more convenient for use in determining mean high water and it is these factors that are given in Table 5.

TABLE 5.—*Factors for correcting the range of tide to mean value.*

Year \ $\frac{K_1+O_1}{M_2}$	0.0 to 0.2	0.3 to 0.4	0.5 to 0.6	0.7 to 0.8	0.9 to 1.0	1.1 to 1.2	1.3 to 1.4	1.5 to 1.6	1.7 to 1.8	1.9 to 2.0
1931	1.029	1.028	1.026	1.023	1.020	1.017	1.013	1.008	1.001	0.994
1932	1.029	1.028	1.026	1.023	1.020	1.017	1.013	1.008	1.001	0.994
1933	1.026	1.026	1.024	1.021	1.018	1.015	1.011	1.006	1.001	0.995
1934	1.020	1.019	1.018	1.016	1.014	1.011	1.008	1.005	1.001	0.996
1935	1.011	1.011	1.010	1.010	1.008	1.007	1.005	1.003	1.001	0.998
1936	1.002	1.002	1.002	1.001	1.001	1.001	1.001	1.000	1.000	1.000
1937	0.991	0.991	0.992	0.992	0.994	0.995	0.997	0.998	1.000	1.001
1938	0.981	0.982	0.983	0.985	0.988	0.990	0.993	0.996	0.999	1.003
1939	0.975	0.976	0.977	0.980	0.983	0.986	0.989	0.993	0.999	1.005
1940	0.971	0.972	0.974	0.976	0.979	0.983	0.987	0.993	0.999	1.006
1941	0.971	0.972	0.974	0.976	0.979	0.983	0.987	0.992	0.999	1.006
1942	0.972	0.973	0.975	0.977	0.980	0.984	0.988	0.993	0.999	1.006
1943	0.977	0.978	0.979	0.981	0.984	0.987	0.991	0.994	0.999	1.005
1944	0.985	0.985	0.986	0.987	0.990	0.992	0.994	0.997	0.999	1.003
1945	0.995	0.995	0.995	0.996	0.996	0.997	0.998	0.999	1.000	1.001
1946	1.005	1.005	1.005	1.004	1.004	1.003	1.002	1.001	1.000	0.999
1947	1.014	1.014	1.013	1.012	1.010	1.008	1.006	1.004	1.001	0.997
1948	1.023	1.022	1.021	1.018	1.016	1.013	1.009	1.006	1.001	0.996
1949	1.028	1.027	1.025	1.022	1.019	1.016	1.012	1.007	1.001	0.995
1950	1.029	1.028	1.026	1.023	1.020	1.017	1.013	1.008	1.001	0.994
1951	1.028	1.027	1.025	1.023	1.020	1.017	1.013	1.008	1.001	0.995
1952	1.025	1.024	1.022	1.020	1.017	1.014	1.010	1.006	1.001	0.996
1953	1.017	1.016	1.015	1.014	1.012	1.010	1.007	1.004	1.001	0.997
1954	1.008	1.008	1.008	1.007	1.005	1.004	1.003	1.002	1.000	0.998
1955	0.998	0.998	0.998	0.998	0.998	0.999	0.999	1.000	1.000	1.000
1956	0.987	0.987	0.988	0.990	0.992	0.994	0.996	0.998	1.000	1.002
1957	0.979	0.980	0.981	0.983	0.985	0.988	0.991	0.995	0.999	1.004
1958	0.972	0.974	0.976	0.978	0.981	0.984	0.988	0.993	0.998	1.005
1959	0.971	0.972	0.974	0.976	0.979	0.983	0.987	0.992	0.999	1.006
1960	0.971	0.972	0.974	0.976	0.979	0.983	0.987	0.992	0.999	1.006
1961	0.974	0.975	0.977	0.979	0.982	0.985	0.988	0.993	0.998	1.005
1962	0.979	0.981	0.982	0.984	0.986	0.989	0.992	0.995	0.999	1.004
1963	0.989	0.989	0.989	0.990	0.992	0.994	0.996	0.998	1.000	1.002
1964	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000
1965	1.009	1.009	1.008	1.007	1.006	1.005	1.004	1.002	1.000	0.998
1966	1.018	1.017	1.016	1.014	1.012	1.010	1.007	1.005	1.001	0.996
1967	1.025	1.024	1.022	1.020	1.017	1.015	1.011	1.006	1.001	0.996
1968	1.029	1.028	1.026	1.023	1.020	1.017	1.013	1.008	1.001	0.995
1969	1.030	1.029	1.027	1.023	1.020	1.017	1.013	1.008	1.001	0.995
1970	1.027	1.026	1.024	1.022	1.019	1.016	1.012	1.007	1.001	0.996
1971	1.022	1.021	1.019	1.017	1.015	1.013	1.009	1.005	1.001	0.996
1972	1.013	1.013	1.012	1.011	1.009	1.008	1.006	1.003	1.001	0.998
1973	1.004	1.004	1.004	1.003	1.002	1.002	1.001	1.001	1.000	0.999
1974	0.994	0.993	0.993	0.994	0.996	0.997	0.998	0.999	1.000	1.001
1975	0.983	0.984	0.985	0.987	0.989	0.992	0.994	0.996	0.999	1.003
1976	0.976	0.978	0.979	0.981	0.983	0.986	0.989	0.994	0.999	1.004
1977	0.972	0.973	0.975	0.977	0.980	0.984	0.988	0.993	0.998	1.005
1978	0.970	0.971	0.973	0.975	0.978	0.983	0.987	0.992	0.999	1.006
1979	0.970	0.972	0.974	0.976	0.979	0.983	0.987	0.992	0.999	1.006
1980	0.976	0.977	0.978	0.980	0.983	0.986	0.989	0.994	0.999	1.004

An examination of Table 5 shows that the factor for correcting yearly high water to mean high water decreases with increasing values of the ratio K_1+O_1 to M_2. Thus, for a ratio of K_1+O_1 to M_2 of 0.0 to 0.2 it appears that the rise of high water determined from one year of observations may require a correction as great as 3 percent to derive a mean value, while, when the ratio is 1.5, this correction is, at most, barely 1 percent. In other words, this correction is relatively large for tides of the semidaily type and small for tides of the mixed type.

In the use of Table 5 for the determination of mean high water it is clear that only the range of tide is corrected to mean value by the use of that table. If no suitable primary tide station is available for correcting half-tide level to mean value, it will be sufficient to use the half-tide level determined from the particular series of observations as the best-determined value of half-tide level.

To exemplify the use of Table 5 it will be sufficient to employ it for correcting the range of tide to mean value. Ketchikan, Alaska, has a range of nearly 13 feet, and in the table below the yearly values of the range of tide for the 18-year period 1931–1948 are corrected to mean value.

Correction of yearly range of tide to mean value

KETCHIKAN, ALASKA

Year	Range	Factor	Mean range	Year	Range	Factor	Mean range
	Feet		*Feet*		*Feet*		*Feet*
1931	12. 61	1. 027	12. 95	1940	13. 37	0. 973	13. 01
1932	12. 61	1. 027	12. 95	1941	13. 20	0. 973	12. 84
1933	12. 55	1. 025	12. 86	1942	13. 21	0. 974	12. 87
1934	12. 65	1. 019	12. 89	1943	13. 25	0. 978	12. 96
1935	12. 83	1. 011	12. 97	1944	13. 07	0. 985	12. 87
1936	12. 96	1. 002	12. 99	1945	12. 89	0. 995	12. 83
1937	12. 95	0. 991	12. 83	1946	12. 75	1. 005	12. 81
1938	13. 11	0. 982	12. 87	1947	12. 68	1. 014	12. 86
1939	13. 32	0. 976	13. 00	1948	12. 59	1. 022	12. 87
Average	12. 843		12. 923		13. 001		12. 880

In column 2 of the table above, the range of the tide for the year in question is given. From Table 4 the ratio of K_1+O_1 to M_2 is 0.44. If the range of the tide were 5 feet or less it would be sufficient to take the factors from column 3 of Table 5. But since the range at Ketchikan is nearly 13 feet, it is better to take the factors by interpolating between columns 3 and 4, these factors being entered into the third column of the table above. The mean range is then determined by multiplying the values of column 2 by the factors in column 3.

From the continuous series of observations at Ketchikan for the 19-year period 1930–1948 the mean range of tide is 12.90 feet. The least yearly range for the 18-year period in the above table is 12.55 feet for 1933 and the greatest is 13.37 feet for 1940, a difference of 0.82 foot, and differing from the 19-year value by 0.35 foot and 0.47 foot, respectively. After correcting to mean values by the factors, the difference between them is 0.15 foot, and the differences from the 19-year mean are 0.04 foot and 0.11 foot respectively. The yearly values are thus corrected to within one percent of the mean range.

Table 5 may also be used for correcting the range from a month of observations to mean value. The values in that table are computed for July 1 of each year, so that values for each month can be determined by interpolation. As an example of the accuracy attainable from one month of observations by the use of the tabular values of Table 5, the observations for the years 1931 and 1940 at Portland, Maine, may be used. For purpose of illustration it will be sufficient to take every other month of each year.

Correction of monthly range of tide to mean value by tabular factors

PORTLAND, MAINE

Month	1931			1940		
	Range	Factor	Mean range	Range	Factor	Mean range
	Feet		*Feet*	*Feet*		*Feet*
January	8. 55	1. 029	8. 80	9. 28	0. 973	9. 03
March	8. 99	1. 029	9. 25	9. 20	0. 972	8. 94
May	9. 01	1. 029	9. 27	9. 11	0. 972	8. 85
July	8. 77	1. 029	9. 02	9. 03	0. 971	8. 77
September	8. 56	1. 029	8. 81	9. 38	0. 971	9. 11
November	8. 48	1. 029	8. 73	9. 16	0. 971	8. 89

The primary determination of mean range at Portland for the 19 year period 1930–1948 is 8.95 feet. For 1931 the computed mean ranges in the table above differ from the primary value by as much as 0.3 foot, and for the year 1940 by very nearly 0.2 foot. In this connection it must be noted since the months of the moon's phase, distance, and declination are respectively 29½, 27½, and 27⅓ days, the variations in the height of high water from day to day due to changes in the position of the moon will be more nearly eliminated in 29 days than in a month of 30 or 31 days. For that reason the use of a group of 29 days would give better results than a month of 30 or 31 days.

To illustrate the results derived for the same station by the method of comparison and that of tabular values it will be instructive to derive the mean range for the months of 1931 and 1940 at Portland by the method of comparison using Boston as the primary station. For the 19 year period 1930–1948, the mean range at Boston is 9.45 feet. The derivation is shown in tabular form below.

Correction of monthly range of tide to mean value by comparison

PORTLAND, MAINE

Month	1931				1940			
	Portland	Boston	Ratio	Mean range	Portland	Boston	Ratio	Mean range
	Feet	*Feet*		*Feet*	*Feet*	*Feet*		*Feet*
January	8. 55	9. 13	0. 937	8. 85	9. 28	9. 82	0. 945	8. 93
March	8. 99	9. 51	0. 945	8. 93	9. 20	9. 73	0. 946	8. 94
May	9. 01	9. 58	0. 941	8. 89	9. 11	9. 62	0. 947	8. 95
July	8. 77	9. 29	0. 944	8. 92	9. 03	9. 52	0. 949	8. 97
September	8. 56	9. 11	0. 940	8. 88	9. 38	9. 92	0. 946	8. 94
November	8. 48	9. 00	0. 942	8. 90	9. 16	9. 66	0. 948	8. 96

The results derived by the method of comparison are seen to be much better than those derived by means of the tabular values. The greatest difference from the primary value of 8.95 feet is by the former method only 0.1 foot while by the method of tabular values the greatest difference was 0.3 foot. In part the superiority of the comparison method lies in the fact that since the same number of days is used at both stations, the use of a 30 or 31 day month does not introduce the error that accompanies such use with tabular values that are better fitted for a 29 day month. In part, too, the comparison method tends to correct for disturbing effects of wind and weather.

Summary

In the preceding sections it was found that the height to which high water rises varies considerably from day to day and from month to month and, in lesser degree, from year to year. This variation is of two kinds—the first, in response to changes in the phase, distance, and declination of the moon; and the second, in response to the variation in sea level. With the exception of the variation due to change in longitude of the moon's node, the variations due to changes in the position of the moon balance out very largely within a month, so that in a sense the variation due to changes in sea level is the primary variation.

It was found, too, that the rise of high water above half-tide level, from observations covering periods of a month or more, may be corrected to a mean value either by factors derived from theoretical considerations or by comparison with simultaneous observations at some suitable primary station. But in this comparison, type of tide, and not nearness, determined suitability. In this regard the correction of high water to a mean value differs from the like correction of sea level, in which the suitability of a station for comparison purposes depends on the existence of like meteorological conditions. Furthermore, in correcting sea level to a mean value the changes in height at two nearby stations were taken as the same, whereas the changes in the rise of high water are taken as proportional.

In general it may be taken that, when corrected to a mean value, a year of tide observations will determine the rise of high water above half-tide level correct within 0.05 foot, a month within 0.1 foot, and a day within 0.5 foot. However, in regions of large range of tide, and of considerable variation in the rise of high water, a day of observations, especially when the factor for correction differs considerably from 1.00, may give a value differing by a foot or more from a primary value.

It should be noted that mean high water is determined with respect to half-tide level which is itself subject to variations. Hence the accuracy with which the plane of mean high water can be determined depends also on the accuracy with which the plane of half-tide level is determined. The degree of accuracy in deriving mean high water, noted above for observations covering various periods of time, refers only to the rise of high water above half-tide level.

To determine the plane of mean high water from any given series of observations the plane of half-tide level must first be determined; then the rise of high water above half-tide level is corrected to a mean value, and this gives the plane of mean high water above the plane of half-tide level.

VIII. MEAN LOW WATER

Variations in Fall of Low Water

The variations in the fall of low water resemble closely those in the rise of high water, especially in regard to those depending on the moon's position. Not only does the tide rise higher than usual at the times of full and new moon, but it also falls lower, while at the times of the moon's first and third quarters the less-than-average rise of high water is accompanied by a fall in low water also less than average. Similarly, when the moon is in perigee the fall, like the rise, is greater than usual, while at the time of the moon's apogee the rise and also the fall are less than usual.

The periodic semimonthly change in the declination of the moon brings about variations in the fall of low water, causing consecutive low waters to differ. This diurnal inequality in the low waters necessitates the distinction between higher low water and lower low water. When the moon is near its semimonthly maximum declination, the two low waters of a day show the greatest difference in fall, and when the declination of the moon is small—that is, when the moon is near the Equator—the difference between the two low waters is least.

In Figure 44 are plotted the heights of the successive low waters for the month of October 1947 at Atlantic City, Los Angeles, and Pensacola. These illustrate the typical variations from day to day in the semidaily, mixed, and daily types of tide, respectively.

At Atlantic City there were two low waters each tidal day and in general the heights of successive low waters do not differ much. On comparing this curve of low waters with the corresponding curve of high waters in Figure 38 it is seen that successive high waters at Atlantic City exhibit greater differences than do successive low waters. This is typical for the Atlantic coast of the United States, the diurnal inequality along that coast being greater in the high waters than in the low waters.

At Los Angeles there are seen to have occurred, likewise, two low waters each tidal day, but here the differences between successive low waters are considerably greater than at Atlantic City. On the 8th, for example, morning and afternoon low waters differed by 3.3 feet. On comparing this curve with the corresponding curve for the high waters at Los Angeles in Figure 38, it is seen that the inequality in the low waters is greater than in the high waters. For that particular month the difference between the two high waters of a day at Los Angeles averaged 1.1 feet while the difference between the low waters averaged 1.8 feet. This difference in inequality as between the high and low water is a characteristic feature of the tide at Los Angeles, but as we shall see later these inequalities are subject to cyclic variations.

It is of interest to note also that no high water at Los Angeles during the month of October 1947 fell below the average half-tide level for the month. In Figure 44 seven low waters rose above half-tide level. For that month the difference in height between the highest and lowest low waters—on the 21st and 31st, respectively—was 4.2 feet. The average difference between the high and low waters that month was 3.8 feet, so that the difference between two low waters that month exceeded the average difference between the high and low waters.

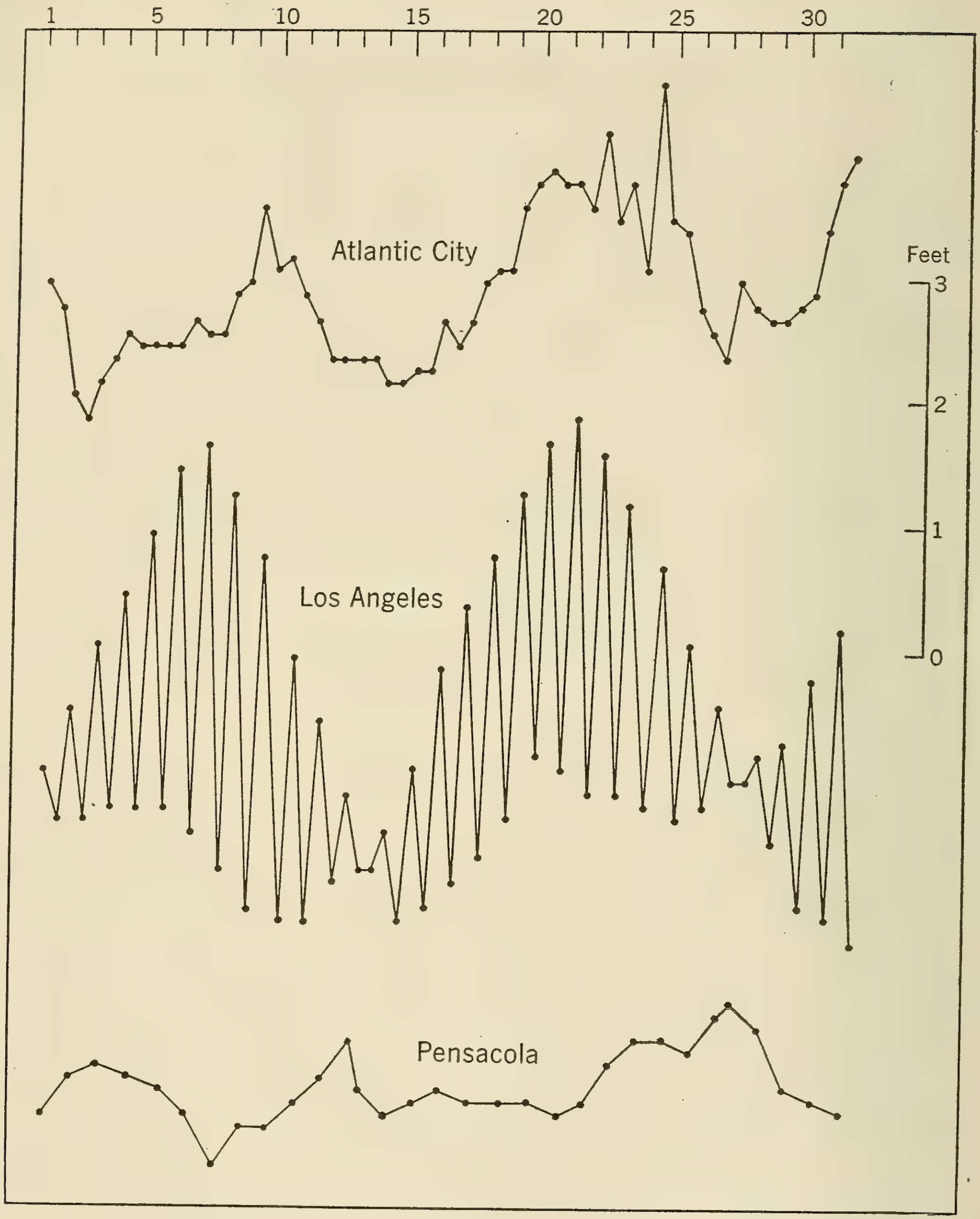

FIG. 44.—Low waters, Atlantic City, Los Angeles, and Pensacola, October 1947.

At Pensacola, the greater part of the month but one low water occurred each day. For the few days when two low waters occurred, only the lower low water is plotted in Figure 44. The average range of tide for the month was 1.2 feet, while the difference in height between the highest and lowest low water was 1.3 feet.

The fluctuations in the heights of the low waters shown in Figure 44 are due to both periodic tidal variations and to the disturbing effects of wind and weather. The latter effects can be very largely eliminated by subtracting the height of each low water from the instantaneous height of sea level. The resulting heights of the low waters at the three stations for the same month of October are shown in Figure 45. The instantaneous height of sea level for any low water is derived by averaging the 24 hourly heights of the tide which center about the time of that particular low water.

The times of the moon's phases, extreme semimonthly declination and of apogee and perigee are plotted at the top of Figure 45. The periodic fluctuations in the height of low water in relation to the moon's position now become clearer. For the semidaily and mixed types of tide Figure 45 shows that low water falls lower at the times of new

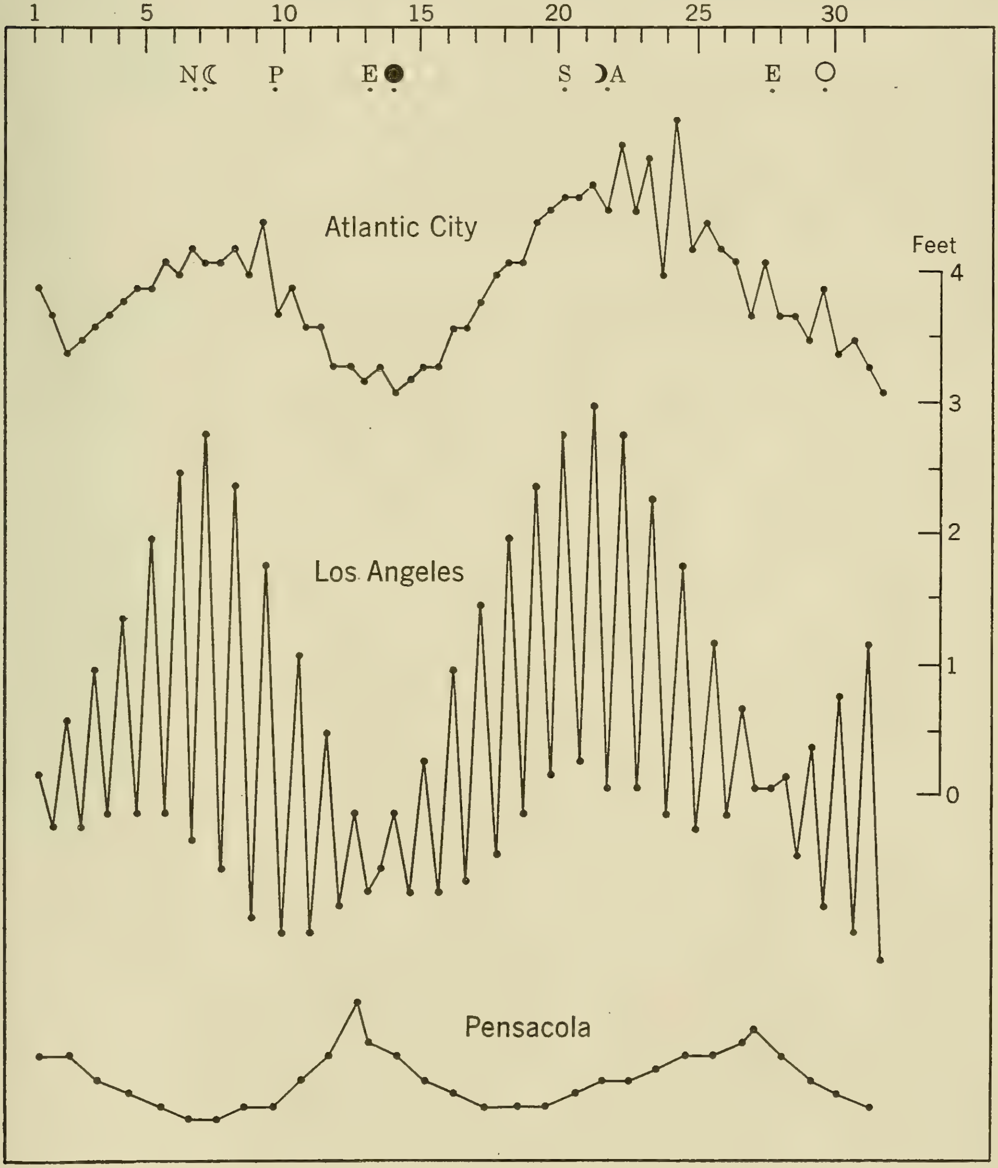

Fig. 45.—Low waters, Atlantic City, Los Angeles, and Pensacola, October 1947 referred to instantaneous sea level.

and full moon and less at the times of the moon's quadratures. In the mixed type of tide the inequality between the two low waters of a day is least at the time the moon is close to the Equator and greatest near the times of the moon's northing and southing.

At Pensacola, which exemplifies the daily type of tide, the low waters are seen to follow the moon's declinations rather than its phases. Low water falls lowest at the times of the moon's northing and southing, and least near the times the moon is close to the Equator.

On comparing the diagrams for Atlantic City in Figures 44 and 45 for the last 2 days of the month, it is seen that although the periodic variation in low water tended to make the fall of low water greater, the actual fall, represented in Figure 44, was less. In other words the effects of wind and weather more than counterbalanced the periodic variation and made the morning low water on the 31st higher by 0.9 foot than the morning low water on the 30th, whereas the periodic variation would have made it less by 0.2 foot.

The daily height of low water is thus subject to relatively large variations both periodic and nonperiodic. The periodic variations depend primarily on the phases,

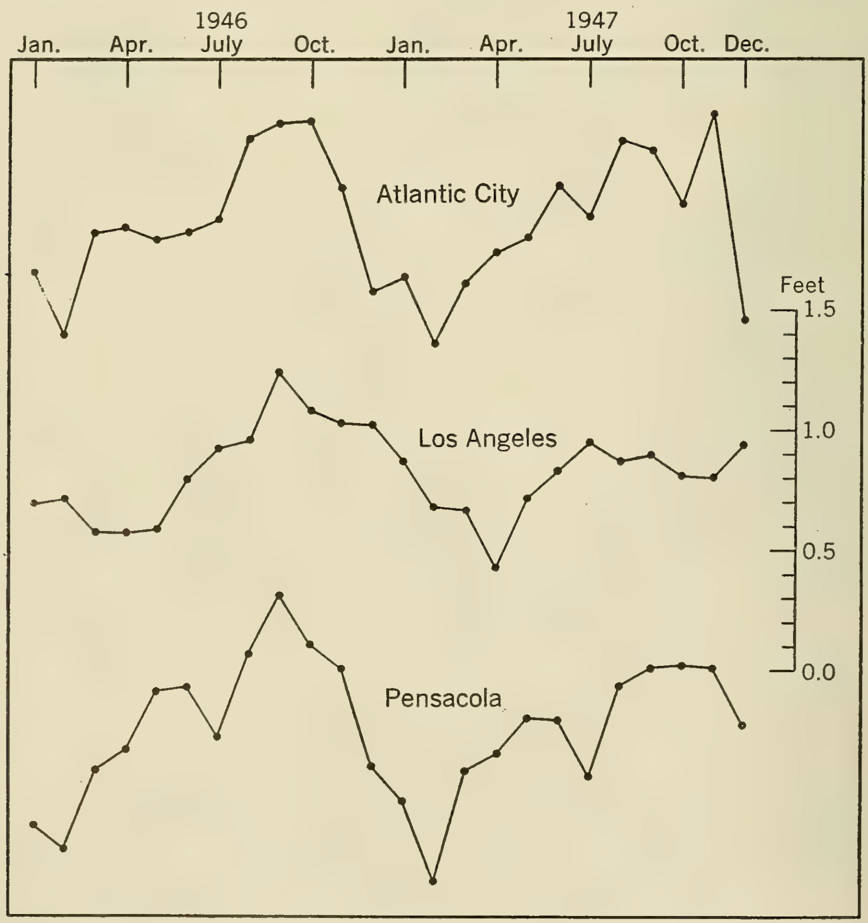

Fig. 46.—Monthly heights of low water, Atlantic City, Los Angeles, and Pensacola, 1946–47.

distance and declination of the moon, the periods of which are approximately 29½, 27½ and 27⅓ days respectively. Such variations are therefore largely eliminated in the period of a month. Within a month, too, the fluctuations in sea level from day to day due to changes in wind and weather tend to balance out. Hence it is to be expected that monthly low waters will show smaller variations than daily low waters.

Monthly Low Water

In Figure 46 are plotted the heights of monthly low water for each month of the two year period 1946–1947 at the three stations, Atlantic City, Los Angeles and Pensacola. From one month to the next, low water is seen to vary from a few hundredths of a foot to as much as half a foot or more. On comparing corresponding diagrams of monthly high and low waters in Figures 40 and 46 the variations from month to month are seen to be much the same. And since an annual variation in monthly high water was found, we may expect to find a similar annual variation in monthly low water.

Figure 47 shows the average heights of monthly low water at the three stations, derived from the 19 year period 1930–1948. Comparison with the corresponding diagrams of the annual variation in high water in Figure 41 shows close resemblance, both in amplitude and in phase. And as was noted in the discussion of the annual variation in high water, this annual variation depends on the annual variation in sea level.

Monthly low water at any point is thus subject to variations both periodic and nonperiodic in character, but in both of these it follows the like variations in sea level.

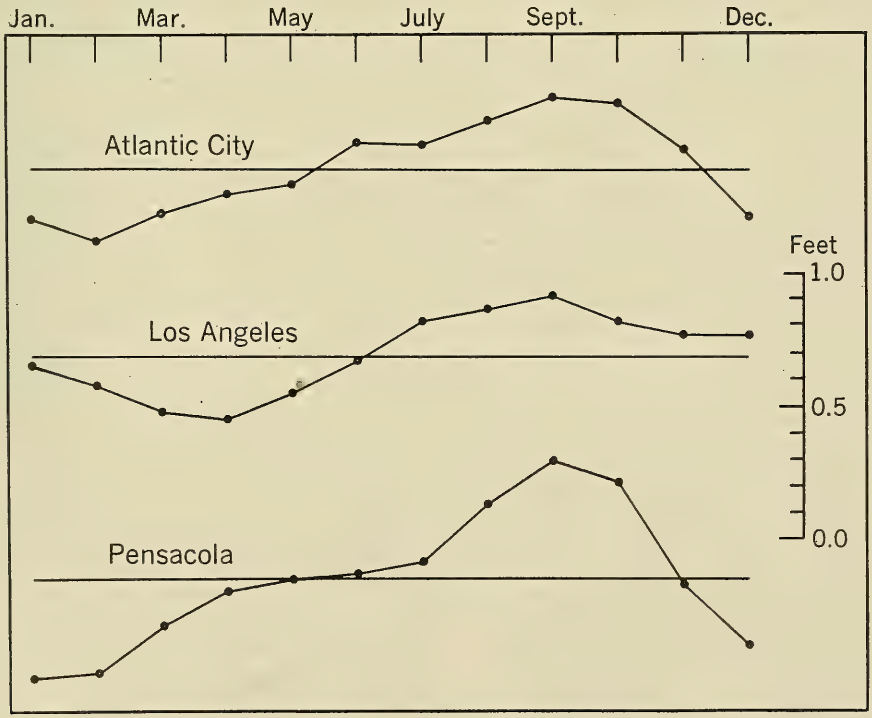

Fig. 47.—Annual variation in low water, Atlantic City, Los Angeles, and Pensacola.

And like the variations in high water and in sea level, the annual variation in low water shows definite regional characteristics.

Yearly Low Water

Within a year the annual variation in low water is eliminated, and hence yearly values of low water differ much less than monthly values. Figure 48 shows the yearly heights of low water for the 25 year period 1924–1948 at Boston, Los Angeles and Pensacola. Generally, consecutive heights of yearly low water at each of these stations are seen to differ by not more than a few hundredths of a foot, but occasionally this differ-

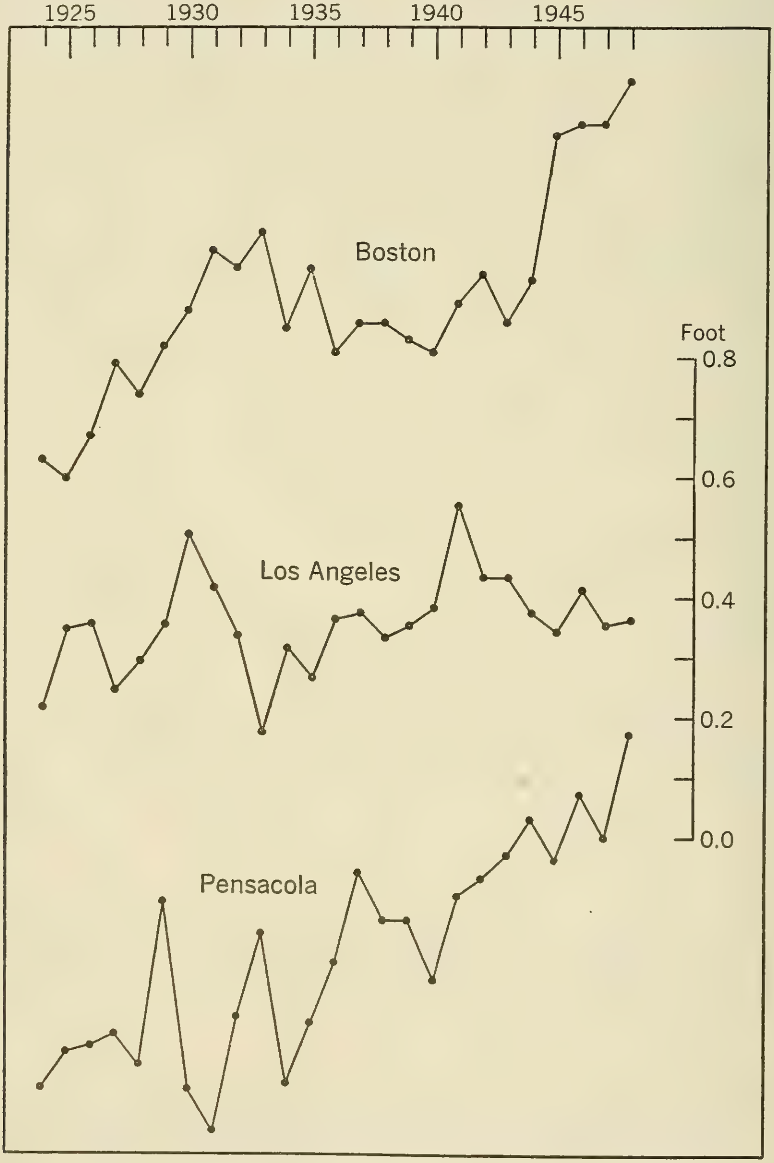

Fig. 48.—Yearly heights of low water, Boston, Los Angeles, and Pensacola, 1924–48.

ence may be as much as 0.2 or 0.3 foot. Within the 25 year period represented, the lowest and highest yearly values differed by 0.66 foot at Boston, 0.38 foot at Los Angeles and 0.66 foot at Pensacola.

Comparison of the diagrams of Figure 48 with the corresponding diagrams of yearly high water in Figure 42 brings out some resemblance but also some differences. And since low water like high water is affected by changes in sea level, we can eliminate the sea-level effects from yearly low water by subtracting the heights of these yearly low waters from the corresponding yearly values of sea level. Figure 49 shows the results in diagrammatic form.

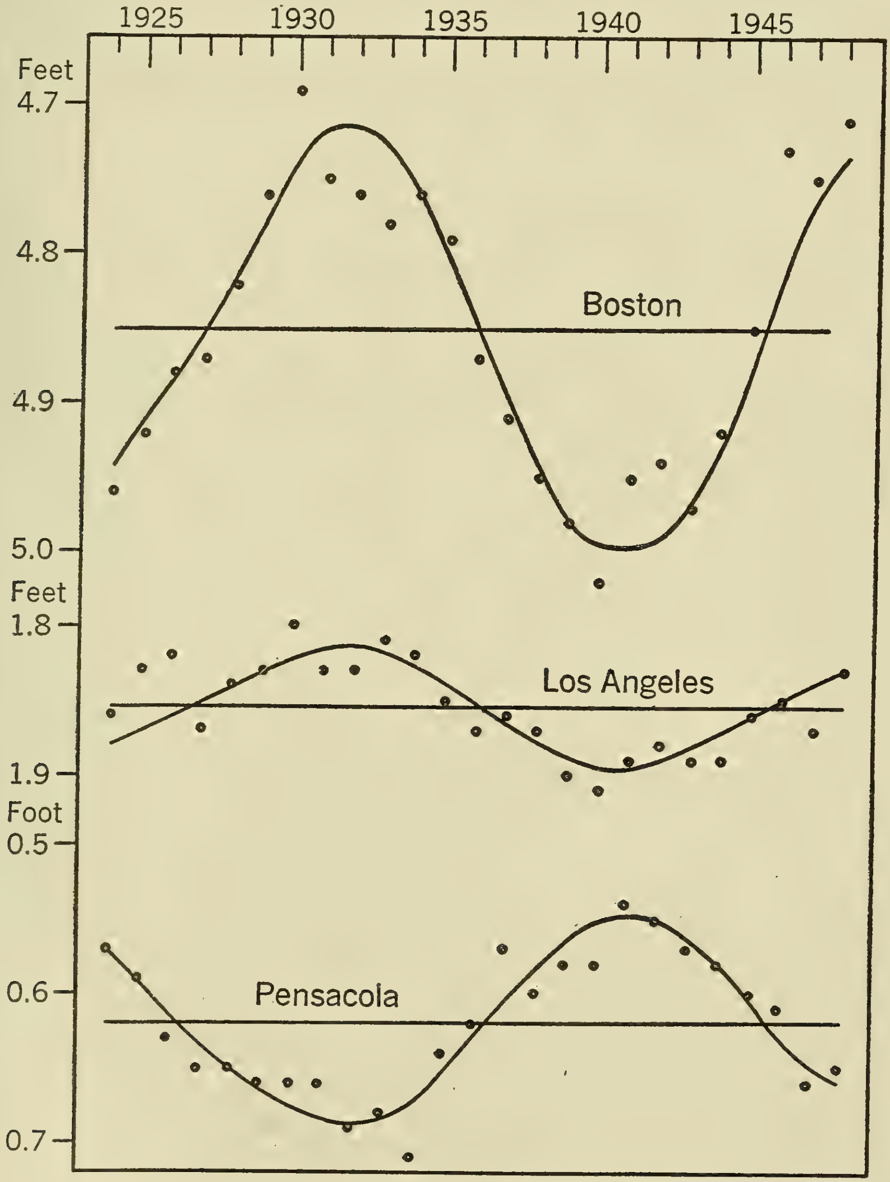

FIG. 49.—Variation of yearly low water in relation to yearly sea level.

The horizontal line associated with each of the diagrams of Figure 49 represents for each station the average height of low water below sea level for the 25 years represented, this height in feet being given by the figures at the left of the horizontal line. The yearly values are seen to range themselves more or less closely along the sine-like curves associated with each diagram.

Referring back to Figure 43 which represents the yearly variation in high water relative to sea level, it is seen that the two sets of curves in Figures 43 and 49 are complementary. In other words, when the rise of high water from sea level is above the average, the fall of low water below sea level will be greater than the average, and vice versa. This follows from the periodic variation in range of tide discussed in connection with Figure 43.

Low water therefore, like high water is subject to periodic variations from day to day, month to month and year to year. And like high water, too, low water is subject to the nonperiodic variations found in sea level.

Definition of Mean Low Water

In view of the variations to which the height of low water is subject, mean low water at any place may be defined simply as the average height of the low waters at that place over a period of 19 years.

For tides of the semidaily and mixed types of tide, this simple definition is immediately applicable. But for the daily type of tide the same difficulty comes up as in the case of mean high water, namely, whether or not to include the secondary tides that occur occasionally. Referring back to this matter (page 86), it is preferable in determining mean low water to disregard the secondary tides and use but one low water a day, the lower low water. Mean low water for the daily tide is thus the same as mean lower low water, and the simple definition of mean low water becomes applicable to all types of tide.

Primary Determination

A primary determination of mean low water is derived directly as the average of the low waters over a 19-year period. And if there were no change in sea level from one 19-year period to another, we would expect two different 19 year determinations of mean low water at any place to agree, unless some change in tidal regime had taken place.

From the 46 years of continuous tide observations at Baltimore we can get two 19 year series for the primary determination of mean low water. Taking the first 19 years, 1903–1921, mean low water averages 3.533 feet on the tide staff, while the last 19 years, 1930–1948 averages 3.786 feet, a difference of 0.253 foot. Sea level for 1903–1921 averaged 4.107 feet and for 1930–1948 it averaged 4.368 feet. Hence with reference to sea level for the respective 19-year series, mean low water at Baltimore for the period 1903–1921 was 0.574 foot below sea level and for the period 1930–1948 it was 0.582 foot below sea level. The difference of 0.008 foot between the two latter values is so small as to be of little significance, so that mean low water with reference to sea level is practically the same for the two 19-year periods.

From the 50 years of observations at Seattle, 1899–1948, three slightly overlapping 19 year series may be formed; 1899–1917, 1915–1933 and 1930–1948. The direct averages of low water on the tide staff for these 19 year periods are, in feet, respectively, 0.240, 0.259 and 0.359. The difference between the first and second of these primary determinations of mean low water is 0.019 foot and between the first and third, 0.119 foot. Referred to the respective 19 year sea levels, mean low water for each of the three series is, in feet, below sea level, respectively, 3.804, 3.796 and 3.803. The greatest difference between any two of these three values is 0.008, which is so small as to be scarcely significant.

It appears therefore that primary determinations of mean low water with respect to sea level are in practical agreement. Sea level at many places, however, is subject to a slow change. Hence, for precise purposes, the datum of mean low water must be specified with regard to the 19 year series used.

Secondary Determination

The procedures for deriving secondary determinations of mean low water are in all respects similar to the ones used for deriving mean high water, which have already been discussed. Both methods, comparison of simultaneous observations and correction by tabular values may be used, the former generally being the more satisfactory.

Either method of deriving a secondary determination of mean low water involves two separate corrections, the first correction being the derivation of mean low water below sea level for the period of observations, and the second correction being the derivation of mean sea level from the sea level of the period of observations. But as in the case of mean high water it is more convenient to use half-tide level instead of sea level.

Hence the examples used to illustrate the derivation of mean high water from a day, month or year of observations are also examples for the derivation of mean low water. For mean high water is derived as half the mean range above mean half-tide level and mean low water is derived as half the mean range below mean half-tide level. Thus after deriving the mean half-tide level and the mean range of tide, both mean high water and mean low water are determined.

The accuracy with which mean low water can be derived from a short series of observations is thus exactly the same as that with which mean high water can be derived. In general it may be said that when corrected to a mean value, a year of tide observations will determine the fall of low water below half-tide level correct within 0.05 foot, a month within 0.1 foot and a day within 0.5 foot.

It must be noted that mean low water is determined with respect to half-tide level which is itself subject to variations. Hence the accuracy with which the plane of mean low water can be determined depends also on the accuracy with which the plane of half-tide level is determined. The degree of accuracy in deriving mean low water noted above for observations covering various periods of time refers only to the fall of low water below half-tide level.

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IX. LOWER LOW WATER

Definitions

The apparent daily movements of sun and moon about the earth take place in planes inclined to that of the Equator, and this gives rise to two different constituents in the tide, one having a period of half a day and the other a period of a day. The actual tide, resulting from the interaction of the semidaily and daily constituents, is therefore characterized by differences as between morning and afternoon tides, or, more precisely, by diurnal inequality. In general, consecutive low waters differ in height, necessitating the distinction between lower low water and higher low water. Of the two low waters of a day the lower is designated as the lower low water and the higher as the higher low water.

Since the length of the tidal day is 24 hours and 50 minutes, there will be calendar days when but one low water occurs, the second one coming after midnight of that day and therefore on the next day. In such cases the question arises as to the designation to be applied to that single low water. Various rules may be formulated for this purpose, but for the practical purposes of datum plane determination a satisfactory rule is to give such a single low water the opposite name from the immediately preceding low water: that is, if the immediately preceding low water was the lower low water for the day, then the single low water in question will be designated as a higher low water, and vice versa. Thus, as shown in the column of low waters in the tabulation of Figure 16, only one low water occurred at Boston on June 28, 1944. The immediately preceding low water which occurred at 23.3 hours on the 27th was a higher low water; hence the single low water of the 28th would be designated as a lower low water.

At some places, however, only one low water may occur during a day because the tide is of the diurnal or daily type. In such cases the single low water of the day is designated as the lower low water. There is usually no difficulty in deciding whether the single low water of a day is due to the tide becoming diurnal or to the failure of the second low water to occur before midnight of that day. Diurnal tides occur only in regions having very considerable diurnal inequality, so that the characteristics of the tide for the day in question readily determine whether or not the single low water is due to the occurrence of a diurnal tide.

Variations

The depth to which lower low water falls varies from day to day. For Los Angeles for the month of October 1947 this variation is brought out graphically in the middle diagrams of Figures 44 and 45.

A detailed study of the lower low waters at any place brings out the fact that the variation from day to day is partly of a periodic nature, due to the change in position of the moon relative to earth and sun, and partly nonperiodic, due to changes in sea level. Referring to Figure 45, it is seen that with regard to the moon's declination, lower low water goes through a fortnightly cycle, being lowest about the time of maximum north or south declination and highest about the time when the moon is on the Equator. The effects of the phase and parallax cycles of the moon are also reflected in lower low water, but the declinational effect is the principal one.

The difference in the morning and afternoon tides of a day, which is known as diurnal inequality, arises from the existence of daily and semidaily constituents in the tide. The greater the daily constituents in relation to the semidaily, the greater the diurnal inequality until the tide becomes daily in type. Hence Table 4, which gives the ratio of K_1+O_1 to M_2 at a number of stations on the coasts of the United States, gives also a criterion for determining the existence of inequality at those stations.

It must be noted, however, that the magnitude of the diurnal inequality depends also on the magnitudes of the daily and semidaily tides, and these are not given in Table 4. Thus from that table the ratios of K_1+O_1 to M_2 for Los Angeles and Seattle do not differ much. But because the magnitudes of the daily and semidaily tides at Seattle are more than twice those at Los Angeles, the inequality at Seattle is more than twice that at Los Angeles.

In the consideration of diurnal inequality (page 11) it was found that it may be of three kinds. It may exist principally in the high waters, principally in the low waters, or equally in the high and low waters, depending on the phase relations of the daily and semidaily constituents. It happens that on the Atlantic coast the inequality is principally in the high waters. This, in conjunction with the small ratios of the diurnal to the semidiurnal constituents, makes the use of the plane of lower low water of little advantage on the Atlantic coast, and the plane of mean low water is used on that coast almost without exception. For this reason the examples given in connection with lower low water will be confined to the Pacific coast, where the diurnal inequality in the low waters is considerable and where the datum of lower low water is of practical importance.

At Los Angeles for the month shown in Figure 44 the difference between the highest and lowest lower low waters was 1.5 feet. At Seattle for the same month this difference was 4.0 feet while at Ketchikan, Alaska, for the same month the difference was 6.2 feet. In part the variation in lower low water from day to day arises from disturbing effects of changing meteorological conditions; but in regions of large range of tide, as for example Seattle and Ketchikan the variation is primarily of a periodic character, depending on the changes in the moon's declination, phase and parallax.

Monthly Lower Low Water

The declinational cycle of the moon has a period of 27⅓ days, the phase cycle has a period of 29½ days, and the parallax cycle a period of 27½ days. Hence in a period of a month the daily variations in lower low water should very largely be eliminated, and we may expect monthly values to show considerably less variation than daily values. In Figure 50 are shown the monthly lower low waters at Los Angeles, Seattle and Ketchikan for the two year period 1946–1947.

From one month to the next Figure 50 shows variations of as much as 0.4 foot at Los Angeles, 1.1 feet at Seattle and 1.0 foot at Ketchikan. Within the two year period

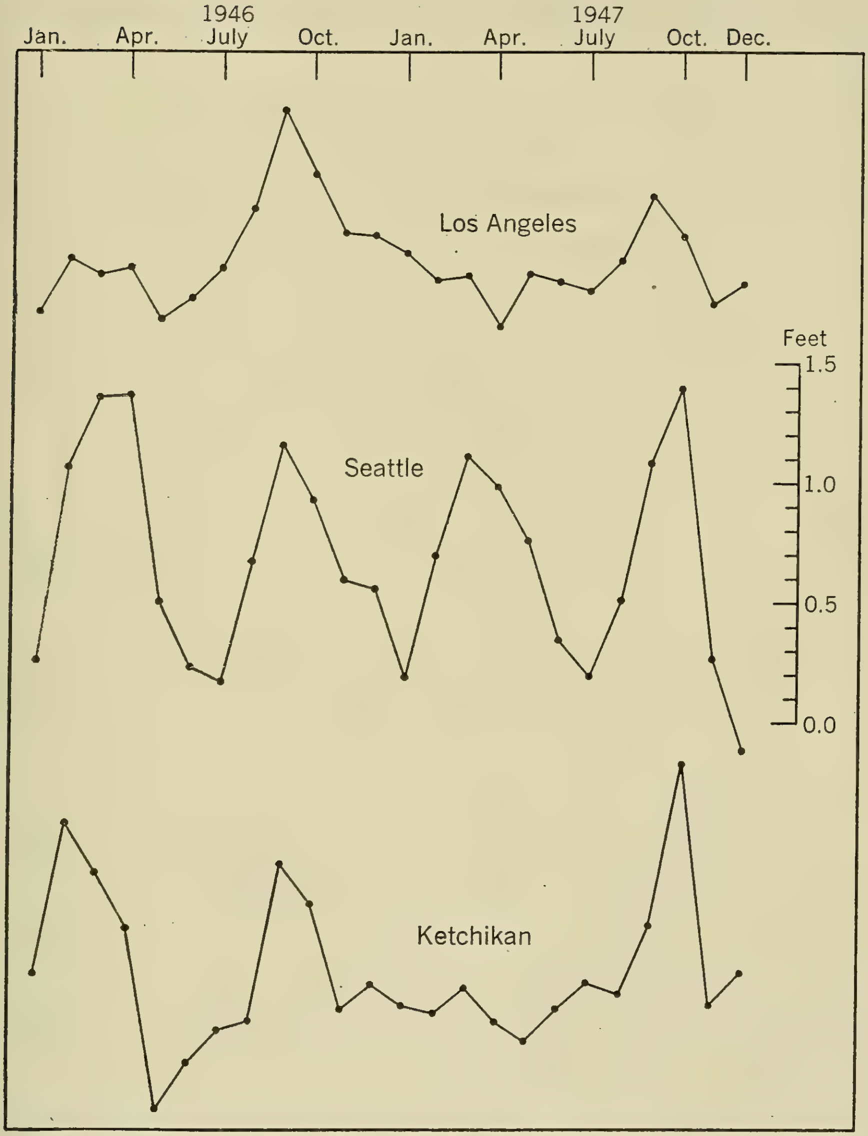

FIG. 50.—Monthly lower low water, Los Angeles, Seattle, and Ketchikan, 1946–47.

represented, the highest and lowest values of monthly lower low water differed by 0.9 foot at Los Angeles, 1.5 feet at Seattle and 1.4 feet at Ketchikan.

In the discussion of monthly sea level it was found that sea level at any place is subject to an annual variation which is characteristic for the region. In high water and in low water, likewise annual variations were found which closely resemble the annual variation in sea level at the place. It is therefore to be expected that lower low water should exhibit a similar annual variation. In Figure 51 are shown the monthly heights of lower low water at Los Angeles, Seattle and Ketchikan averaged from 19 years of observations, 1930–1948.

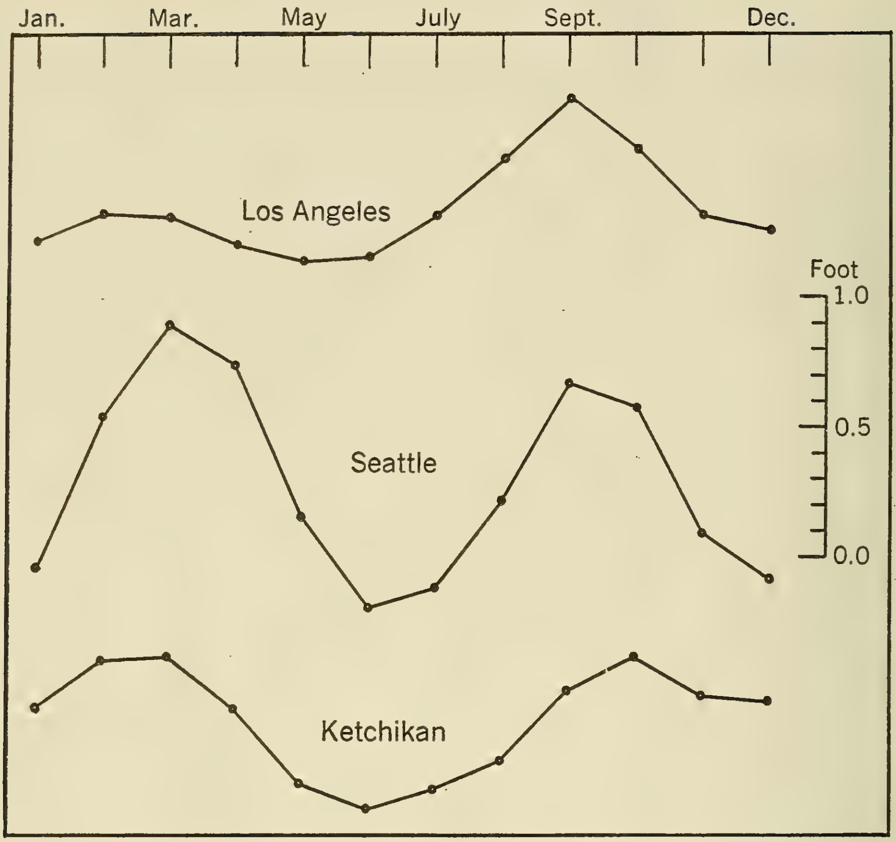

FIG. 51.—Annual variation in lower low water, Los Angeles, Seattle, and Ketchikan.

The annual variation in lower low water pictured in Figure 51 is seen to be quite different from the annual variation in sea level for the same stations shown in Figures 28 and 29. Furthermore, while the annual variation in sea level at the three stations has one maximum and one minimum, the annual variation in lower low water shows two distinct maxima and two distinct minima. The annual variation in sea level must certainly be reflected in the lower low waters, but obviously a more preponderant variation is present in the lower low waters.

We can eliminate the effect of the annual variation in sea level on the lower low waters by subtracting the montly values of lower low water from the monthly values of sea level. The results for three stations are shown in Figure 52, derived from 19 years of observations, 1930–1948. Comparing these diagrams with the corresponding diagrams of Figure 51 it is clear that the annual variation in lower low water is primarily periodic. The fact that the minima occur in June and December at all three stations points to the declination of the sun as the cause.

Two elements therefore enter into the annual variation of lower low water. There is, first, the variation due to changes in sea level; and this variation, as has been shown, depends not on the range of the tide but upon its location. Second, there is the variation depending on the sun's declination; and this variation, as will be seen later, varies with the diurnal inequality.

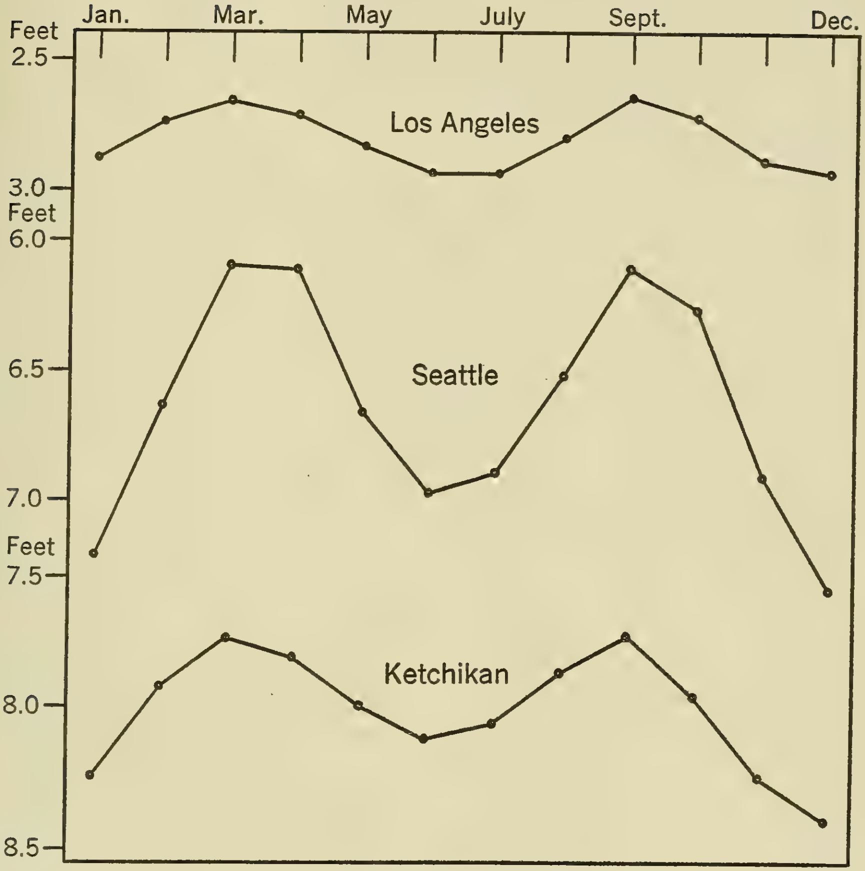

FIG. 52.—Monthly lower low water below monthly sea level.

Yearly Lower Low Water

The annual variation balances out within a year, and therefore yearly values of lower low water may be expected to show much smaller differences than monthly values. This is borne out by Figure 53 which is a plotting of the yearly heights of lower low water at three Pacific coast stations. Generally, lower low water from one year to the next differs by several hundredths of a foot, though at times it may be as much as 2 or 3 tenths of a foot. Within the 25-year period represented in Figure 53 the difference between the highest and lowest yearly values of lower low water was 0.6 foot at Los Angeles, 1.0 foot at Seattle and 0.5 foot at Ketchikan.

A comparison of the diagrams of Figure 53 with the corresponding diagrams of yearly sea level in Figures 32 and 33 shows some resemblance between them indicating that, in part, the variation of yearly lower low water is due to the variation in sea level. To determine the nature of any other elements in the variation of yearly lower low water, the variation due to change in sea level may be eliminated by subtracting each

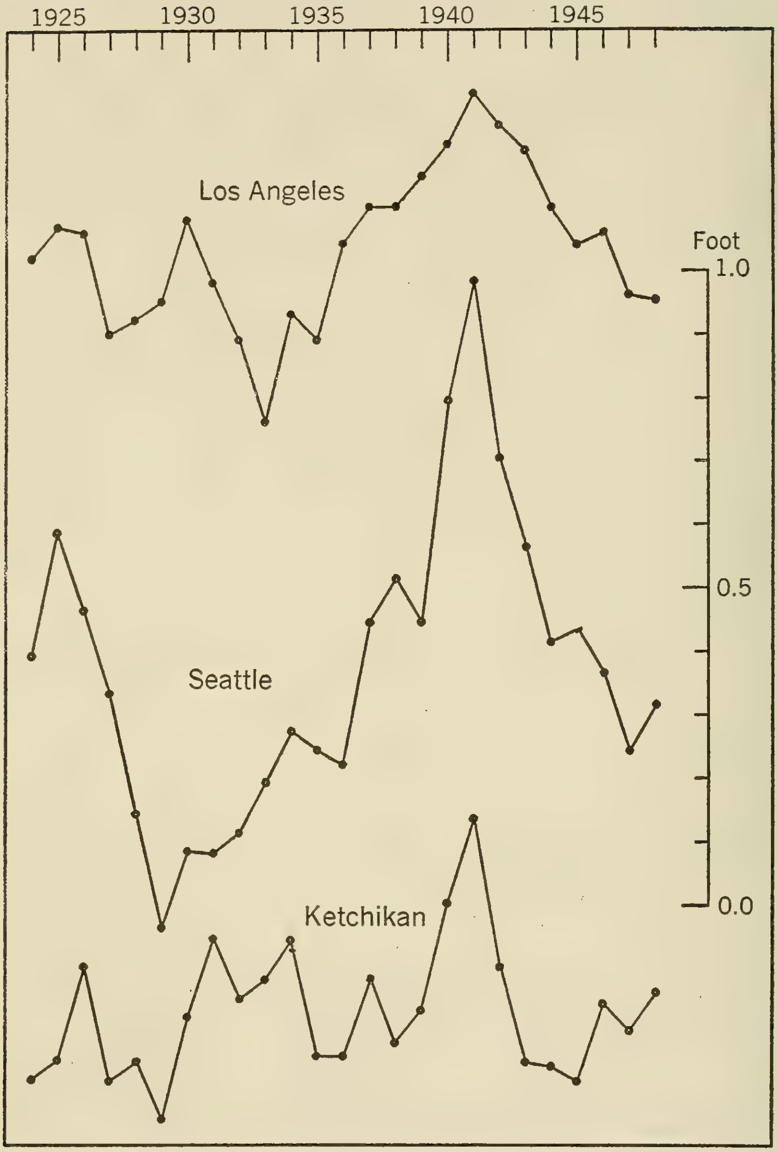

FIG. 53.—Yearly lower low water, Los Angeles, Seattle, and Ketchikan.

yearly height of lower low water from the corresponding yearly height of sea level. Figure 54 shows the result for the three Pacific coast stations used above.

The horizontal line of each of the diagrams of Figure 54 represents the average fall of lower low water below mean sea level, the figures to the left of each line giving this fall in feet. The yearly values are seen to range themselves more or less closely along the sine-like curves drawn for each diagram. The period of this periodic variation is 18.6 years and is associated with the period of the change in longitude of the moon's node. At Los Angeles the range of this variation in yearly lower low water is 0.2 foot; at Seattle, 0.6 foot and at Ketchikan 0.15 foot.

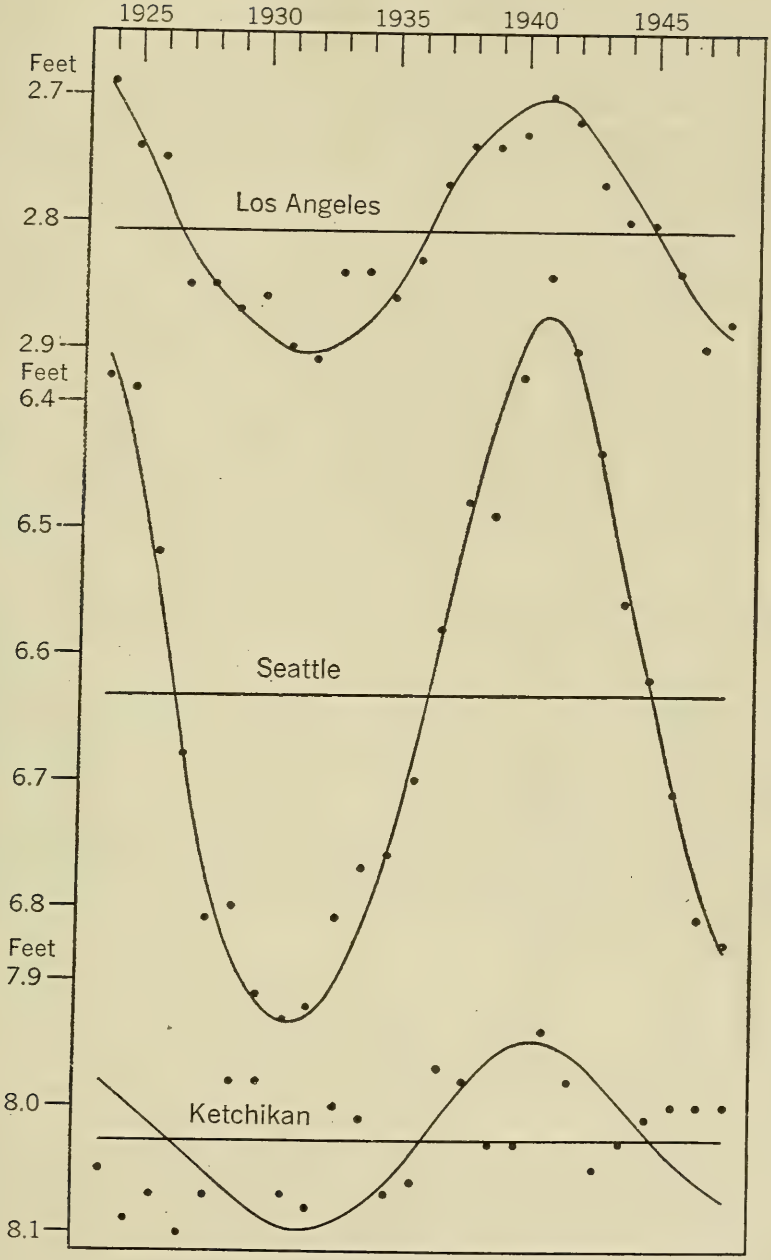

FIG. 54.—Yearly lower low water below yearly sea level.

Definition of Mean Lower Low Water

In view of the variations to which the height of lower low water is subject, mean lower low water at any place may be defined simply as the average height of the lower low waters at that place over a 19-year period.

Primary Determination

A primary determination of mean lower low water is based directly on the average of the lower low waters over a 19-year period. And if there were no change in sea level from one 19-year period to another, we would expect two different 19-year determinations of mean lower low water at any place to agree, unless some change in tidal regime had taken place.

At San Francisco, continuous tide observations are available for the 51-year period 1898–1948. Three slightly overlapping 19 year series may be formed from these observations; 1898–1916, 1914–1932 and 1930–1948. The values of mean lower low water on the staff for each of these series is 8.544 feet, 8.601 feet and 8.720 feet, respectively. Referred to the respective 19-year sea level, mean lower low water below sea level is respectively, 3.052 feet; 3.066 feet and 3.072 feet.

The greatest difference between the three primary determinations of mean lower low water below sea level is 0.02 foot. It must be noted that during the 51-year period the location of the tide station had been changed several times. It is possible too, that because of improvements in the harbor the range of tide has increased slightly. In any event, 0.02 foot is a small quantity, so that the three determinations are in practical agreement.

At Seattle, the 50 years of observations permit three slightly overlapping series; 1899–1917, 1915–1933 and 1930–1948. For these three series the value of mean lower low water on the staff is 14.044 feet, 14.055 feet and 14.162 feet, respectively. Referred to the respective 19 year sea level, mean lower low water below sea level is, respectively, 6.637 feet, 6.622 feet and 6.645 feet. The greatest difference between any two of these three primary determinations is again 0.02 foot. And as in the previous case this difference is so small as to be scarcely significant.

Primary determinations of mean lower low water below sea level are therefore in practical agreement. However, since sea level at many places appears to be subject to a slow change, for precise purposes the datum of mean lower low water must be specified with regard to the 19-year series used.

Relation to Low Water

It is necessary to distinguish not only between higher low water and lower low water, but also between the latter and low water. In general the term "low water" embraces both higher low water and lower low water; but, when used in contradistinction to lower low water, it refers to the average of the two low waters. For any day the difference between low water and lower low water is known as the diurnal low-water inequality, which in abbreviated form is written DLQ.

The diurnal low-water inequality varies from day to day throughout a fortnight, being greatest a little after the time of the moon's maximum north or south declination and least a little after the time when the moon is over the Equator. The mean value of the diurnal low-water inequality gives the difference between the planes of mean low water and lower low water. If, therefore, the plane of mean low water at any place is determined, the plane of lower low water becomes determined as soon as the mean value of the diurnal inequality is derived.

The heights of low water and lower low water vary in accordance with changes in sea level. Hence the low-water inequality, which is the difference between the heights of low water and lower low water, is independent of variations in sea level. This means that only the annual variation and the 19-year variation need be considered with regard to the diurnal inequality. These are of the same nature as the corresponding variations in lower low water, which were found to be due to changes in the relative positions of earth, moon, and sun. Factors for correcting monthly and yearly values of the diurnal inequality may therefore be derived from astronomical considerations, and in Table 6 such factors are given for each month and year of the 50-year period 1931–1980 as computed from Tables 6 and 32 of Harris' Manual of Tides.

TABLE 6.—*Factors for correcting diurnal inequality to mean value*

Year	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Mean
1931	0.79	0.91	1.03	0.97	0.82	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.879
1932	0.79	0.91	1.03	0.97	0.82	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.879
1933	0.79	0.91	1.04	0.98	0.83	0.76	0.79	0.92	1.05	0.98	0.83	0.77	0.888
1934	0.80	0.92	1.06	1.00	0.84	0.78	0.80	0.94	1.08	1.01	0.85	0.78	0.905
1935	0.82	0.95	1.10	1.03	0.87	0.80	0.83	0.97	1.13	1.05	0.88	0.81	0.937
1936	0.85	1.00	1.15	1.08	0.90	0.83	0.86	1.01	1.19	1.10	0.92	0.84	0.978
1937	0.88	1.04	1.22	1.14	0.94	0.86	0.90	1.07	1.26	1.16	0.96	0.88	1.026
1938	0.92	1.09	1.30	1.21	0.99	0.90	0.94	1.13	1.34	1.23	1.01	0.92	1.082
1939	0.97	1.15	1.38	1.28	1.04	0.94	0.98	1.18	1.42	1.29	1.05	0.95	1.136
1940	1.00	1.21	1.44	1.33	1.07	0.96	1.00	1.22	1.47	1.33	1.07	0.96	1.172
1941	1.01	1.22	1.46	1.34	1.07	0.96	1.00	1.22	1.47	1.33	1.07	0.96	1.176
1942	1.01	1.21	1.45	1.33	1.07	0.96	0.99	1.20	1.43	1.30	1.04	0.94	1.161
1943	0.98	1.18	1.39	1.28	1.03	0.93	0.96	1.15	1.36	1.24	1.00	0.91	1.118
1944	0.95	1.13	1.31	1.21	0.99	0.89	0.92	1.09	1.29	1.17	0.96	0.87	1.065
1945	0.90	1.06	1.23	1.14	0.94	0.85	0.88	1.04	1.20	1.10	0.91	0.83	1.007
1946	0.87	1.01	1.16	1.08	0.90	0.82	0.84	0.99	1.14	1.05	0.88	0.80	0.962
1947	0.83	0.97	1.11	1.03	0.87	0.79	0.82	0.95	1.09	1.01	0.85	0.78	0.925
1948	0.81	0.94	1.07	1.00	0.84	0.77	0.80	0.93	1.06	0.98	0.83	0.76	0.899
1949	0.79	0.91	1.04	0.98	0.83	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.882
1950	0.79	0.90	1.03	0.97	0.82	0.76	0.78	0.91	1.04	0.96	0.82	0.76	0.878
1951	0.78	0.91	1.04	0.97	0.82	0.76	0.78	0.91	1.06	0.97	0.82	0.76	0.882
1952	0.79	0.92	1.06	0.98	0.83	0.76	0.79	0.92	1.07	0.99	0.83	0.77	0.892
1953	0.80	0.94	1.08	1.01	0.85	0.78	0.81	0.95	1.11	1.02	0.86	0.79	0.917
1954	0.83	0.97	1.13	1.04	0.88	0.80	0.84	0.98	1.16	1.07	0.89	0.82	0.951
1955	0.86	1.01	1.19	1.09	0.91	0.84	0.87	1.03	1.23	1.12	0.93	0.85	0.994
1956	0.90	1.06	1.26	1.16	0.96	0.87	0.91	1.09	1.31	1.19	0.97	0.89	1.048
1957	0.94	1.12	1.34	1.23	1.00	0.91	0.95	1.15	1.39	1.26	1.02	0.92	1.102
1958	0.98	1.18	1.42	1.29	1.04	0.94	0.99	1.20	1.46	1.32	1.05	0.95	1.152
1959	1.01	1.22	1.48	1.33	1.07	0.96	1.01	1.22	1.50	1.34	1.06	0.96	1.180
1960	1.01	1.23	1.49	1.34	1.07	0.96	1.00	1.22	1.49	1.33	1.05	0.95	1.178
1961	1.00	1.21	1.45	1.31	1.05	0.94	0.98	1.18	1.43	1.28	1.02	0.92	1.148
1962	0.97	1.16	1.38	1.25	1.01	0.91	0.94	1.12	1.35	1.21	0.98	0.89	1.098
1963	0.93	1.10	1.30	1.18	0.96	0.87	0.90	1.07	1.27	1.14	0.93	0.85	1.042
1964	0.89	1.04	1.22	1.11	0.92	0.83	0.86	1.01	1.19	1.08	0.89	0.82	0.988
1965	0.85	0.99	1.15	1.06	0.88	0.80	0.83	0.97	1.13	1.03	0.86	0.79	0.945
1966	0.82	0.95	1.10	1.02	0.85	0.78	0.81	0.94	1.09	1.00	0.84	0.77	0.914
1967	0.80	0.93	1.07	0.99	0.83	0.76	0.79	0.91	1.06	0.98	0.82	0.76	0.892
1968	0.79	0.91	1.05	0.97	0.82	0.76	0.78	0.90	1.05	0.96	0.81	0.75	0.879
1969	0.78	0.90	1.04	0.96	0.82	0.75	0.78	0.90	1.04	0.97	0.81	0.75	0.875
1970	0.78	0.91	1.05	0.97	0.82	0.76	0.79	0.91	1.06	0.98	0.82	0.76	0.884
1971	0.79	0.92	1.06	0.99	0.84	0.77	0.80	0.93	1.09	1.00	0.84	0.78	0.901
1972	0.81	0.95	1.10	1.02	0.86	0.79	0.82	0.96	1.13	1.04	0.87	0.80	0.929
1973	0.84	0.98	1.15	1.06	0.89	0.82	0.85	1.00	1.19	1.09	0.90	0.83	0.967
1974	0.87	1.03	1.21	1.12	0.93	0.85	0.89	1.05	1.26	1.15	0.94	0.87	1.014
1975	0.91	1.09	1.29	1.19	0.98	0.89	0.93	1.11	1.34	1.22	0.99	0.90	1.070
1976	0.95	1.14	1.38	1.26	1.02	0.92	0.97	1.17	1.42	1.28	1.03	0.94	1.123
1977	0.99	1.20	1.45	1.31	1.06	0.95	1.00	1.21	1.48	1.33	1.06	0.96	1.167
1978	1.01	1.23	1.49	1.34	1.08	0.96	1.01	1.22	1.50	1.34	1.06	0.96	1.183
1979	1.01	1.22	1.48	1.33	1.07	0.96	1.00	1.20	1.47	1.31	1.04	0.94	1.169
1980	0.99	1.19	1.43	1.29	1.03	0.93	0.97	1.16	1.40	1.25	1.00	0.91	1.129

The factors for each month in Table 6 were computed for the middle of the month. In the last column the mean of the monthly factors for each year is given, and this may be taken as the factor for correcting the yearly inequality to a mean value.

Secondary Determination

Secondary determinations of mean lower low water, that is, determinations from series less than 19 years in length, are most conveniently derived by determining the mean value of the low water inequality and applying it to mean low water. The derivation of mean low water was discussed in Section VIII, and it therefore remains to discuss the derivation of the diurnal low water inequality.

Two methods are available for the determination of mean low water diurnal inequality: (1) comparison of simultaneous observations; (2) correction by tabular values. The method of comparison of simultaneous observations will be taken up first.

Comparison of Simultaneous Observations

To exemplify the determination of mean low water inequality by this method, the procedure used and the accuracy attainable will be illustrated below for periods of various lengths. The work will be carried through to the determination of mean lower low water.

In the tabulation of the high and low waters it is customary to derive the monthly averages of high water, low water, half-tide level, higher high water and lower low water. The difference between the monthly average low water and lower low water then gives immediately the monthly average of the low water diurnal inequality or DLQ. Yearly values of the various data are derived by averaging the corresponding monthly values.

Year.—To exemplify the derivation of mean lower low water from a year of observations we may take every other year from 1936 to 1946 at La Jolla, using Los Angeles as the primary station. The procedure is shown in tabular form below.

Year	Los Angeles		La Jolla		Factors for corrections		La Jolla		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	Mean DLQ	MLLW below HTL
	Feet	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
1936	1. 89	0. 96	1. 83	0. 93	0. 995	0. 990	1. 82	0. 92	2. 74
1938	1. 89	0. 87	1. 81	0. 84	0. 995	1. 092	1. 80	0. 92	2. 72
1940	1. 92	0. 82	1. 86	0. 80	0. 979	1. 159	1. 82	0. 93	2. 75
1942	1. 90	0. 84	1. 83	0. 82	0. 989	1. 131	1. 81	0. 93	2. 74
1944	1. 91	0. 91	1. 83	0. 88	0. 984	1. 044	1. 80	0. 92	2. 72
1946	1. 87	0. 99	1. 81	0. 96	1. 005	0. 960	1. 82	0. 92	2. 74

The epoch to which the data will be corrected will be taken as the 19 years 1928–1946. For this epoch low water below half-tide level at Los Angeles is 1.88 feet, and DLQ is 0.95 foot. In the sixth column the factors are derived by dividing 1.88 by the values in column 2, and in the seventh column they are derived by dividing 0.95 by the values in column 3. The values in columns 4 and 5 are then multiplied by their respective factors in column 6 and 7 to give columns 8 and 9, the addition of which gives column 10.

The primary determination of mean lower low water below mean half-tide level at La Jolla from the 19 year series 1928–1946 is 2.73 feet. The six values determined in the last column of the table above thus differ from the primary value by not more than 0.02 foot. Incidentally it may be observed that the primary determination of DLQ at La Jolla is 0.92 foot. So that from a year of observations the mean value of DLQ, as shown in the next to the last column, is determined within 0.01 foot.

Los Angeles and La Jolla are about 75 miles apart and their tides have ranges and inequalities much alike. It will therefore be instructive to take an example of tides having different ranges and inequalities and separated also by a greater distance. Neah Bay and Seattle, in the State of Washington are a little over 100 miles apart. At the former the mean range of tide is 5.62 feet and the low water inequality is 1.58 feet, while at Seattle they are respectively 7.65 feet and 2.84 feet. For the 19 years 1930–1948 mean low water below mean half-tide level at Seattle was 3.82 feet and mean DLQ 2.84 feet.

Year	Seattle		Neah Bay		Factors for corrections		Neah Bay		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	Mean DLQ	MLLW below HTL
	Feet	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
1938	3. 85	2. 62	2. 89	1. 44	0. 992	1. 084	2. 87	1. 56	4. 43
1940	3. 91	2. 46	2. 93	1. 35	0. 977	1. 114	2. 86	1. 50	4. 36
1942	3. 89	2. 46	2. 87	1. 40	0. 982	1. 114	2. 82	1. 56	4. 38
1944	3. 87	2. 69	2. 77	1. 50	0. 987	1. 056	2. 73	1. 58	4. 31
1946	3. 82	2. 92	2. 75	1. 65	1. 000	0. 972	2. 75	1. 60	4. 35
1948	3. 77	3. 10	2. 71	1. 75	1. 013	0. 916	2. 75	1. 60	4. 35

From the 14 years of continuous observations at Neah Bay, the mean value of low water below half-tide level corrected to the epoch 1930–1948 is 2.82 feet and the mean value of DLQ is 1.58 feet, so that the mean value of lower low water below half-tide level is 4.40 feet. The derived yearly values in column 8 thus are correct to within 0.07 foot of the mean value, in column 9 they are correct to within 0.08 foot, and in the last column to within 0.09 foot.

The fact that the comparison of La Jolla and Los Angeles gave more concordant values than the comparison between Neah Bay and Seattle is due in part to the lesser distance between the former pair of stations. In part too, it is due to the fact that La Jolla and Los Angeles have more nearly similar tides than Neah Bay and Seattle, evidenced by the values of the ratio of K_1+O_1 to M_2 for these stations in Table 4.

In general it may be taken that a year of observations when compared with a suitable primary station will give mean lower low water below HTL correct within about 0.05 foot.

Month.—To exemplify the determination of mean lower low water from a month of observations we may take again La Jolla and Los Angeles, and Neah Bay and Seattle taking every other month of the year 1946.

Date	Los Angeles		La Jolla		Factors for correction		La Jolla		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	Mean DLQ	MLLW below HTL
1946	*Feet*	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
January	1. 84	1. 11	1. 76	1. 08	1. 022	0. 856	1. 80	0. 92	2. 72
March	1. 91	0. 83	1. 85	0. 79	0. 984	1. 145	1. 82	0. 90	2. 72
May	1. 90	1. 03	1. 87	1. 00	0. 989	0. 922	1. 85	0. 92	2. 77
July	1. 88	1. 16	1. 79	1. 13	1. 000	0. 819	1. 79	0. 93	2. 72
September	1. 85	0. 81	1. 77	0. 76	1. 016	1. 173	1. 80	0. 89	2. 69
November	1. 88	1. 11	1. 83	1. 09	1. 000	0. 856	1. 83	0. 93	2. 76

The procedure is exactly similar to that used in deriving mean lower low water from a year of observations. In connection with that example it was found that for epoch 1928–1946 low water below half-tide level at Los Angeles is 1.88 feet and DLQ is 0.95 foot. The primary determination of mean lower low water below half-tide level at La Jolla was found to be 2.73 feet. The values derived for the six months in the last column of the table above thus differ from the primary value by not more than 0.04 foot. The example for Neah Bay and Seattle follows.

Date	Seattle		Neah Bay		Factors for correction		Neah Bay		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	DLQ	MLLW below HTL
1946	*Feet*	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
January	3. 81	3. 65	2. 74	1. 98	1. 003	0. 778	2. 75	1. 54	4. 29
March	3. 74	2. 46	2. 72	1. 35	1. 019	1. 150	2. 77	1. 55	4. 32
May	3. 91	2. 92	2. 84	1. 71	0. 977	0. 973	2. 77	1. 66	4. 43
July	3. 90	3. 33	2. 84	1. 82	0. 979	0. 853	2. 78	1. 55	4. 33
September	3. 79	2. 42	2. 72	1. 33	1. 008	1. 174	2. 74	1. 56	4. 30
November	3. 81	3. 04	2. 70	1. 84	1. 003	0. 934	2. 71	1. 72	4. 43

The best determined value for mean lower low water below half-tide level at Neah Bay for the epoch 1930–1948 is 4.40 feet. The six values determined in the last column of the table above, on the average differ by 0.07 foot from the best determined value, with 0.11 foot the greatest difference. It may therefore be taken that in general a month of observations will determine the value of mean lower low water below half-tide level within 0.1 foot when compared with a suitable primary tide station.

Day.—In determining mean lower low water from one day of observations, one observed value of lower low water at the secondary station is compared with one observed value at the primary station. Disturbances in regularity of rise and fall by meteorological conditions will therefore be less likely to balance out than in periods of several days. The accuracy attainable is illustrated below for every fifth day of the month of July 1946 for La Jolla using Los Angeles as primary station.

Date	Los Angeles		La Jolla		Factors for correction		La Jolla		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	Mean DLQ	MLLW below HTL
1946	*Feet*	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
July 1	2. 32	1. 70	2. 25	1. 75	0. 810	0. 559	1. 82	0. 98	2. 80
July 6	1. 32	0. 25	1. 22	0. 30	1. 424	3. 800	1. 74	1. 14	2. 88
July 11	1. 72	1. 20	1. 65	1. 20	1. 093	0. 792	1. 80	0. 95	2. 75
July 16	1. 98	1. 45	1. 88	1. 45	0. 949	0. 655	1. 78	0. 95	2. 73
July 21	1. 52	0. 20	1. 40	0. 20	1. 237	4. 750	1. 73	0. 95	2. 68
July 26	2. 45	1. 65	2. 35	1. 60	0. 767	0. 576	1. 80	0. 92	2. 72
July 31	2. 12	1. 20	2. 05	1. 15	0. 887	0. 792	1. 82	0. 91	2. 73

The primary values for Los Angeles used for determining the correction factors are 1.88 feet for mean low water below half-tide level and 0.95 for mean DLQ. The primary value for mean lower low water below half-tide level at La Jolla is 2.73 feet and for mean DLQ 0.92 foot. In the last column of the table above, the greatest difference between the primary value of mean lower low water below half-tide level and that determined from one day of observations is 0.15 foot.

If now we derive mean lower low water from one day of observations at Neah Bay, by comparison with Seattle, we find greater differences than in the preceding example. Every fifth day of July 1946 will again be used.

Date	Seattle		Neah Bay		Factors for correction		Neah Bay		
	LW below HTL	DLQ	LW below HTL	DLQ	LW	DLQ	MLW below HTL	Mean DLQ	MLLW below HTL
1946	*Feet*	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
July 1	4. 82	5. 25	3. 70	3. 05	0. 793	0. 541	2. 93	1. 65	4. 58
6	2. 95	0. 40	2. 02	0. 50	1. 295	7. 100	2. 62	3. 55	6. 17
11	3. 38	4. 10	2. 42	2. 00	1. 130	0. 693	2. 73	1. 39	4. 12
16	3. 95	4. 40	2. 95	2. 50	0. 967	0. 645	2. 85	1. 61	4. 46
21	3. 35	0. 05	2. 38	0. 55	1. 140	56. 800	2. 71	51. 24	53. 95
26	4. 45	5. 10	3. 35	2. 70	0. 858	0. 557	2. 87	1. 50	4. 37
31	4. 50	3. 30	3. 45	1. 90	0. 849	0. 861	2. 93	1. 64	4. 57

The best determined value for mean lower low water below half-tide level at Neah Bay for epoch 1930–1948 is 4.40 feet. Of the seven values in the last column of the table above, two differ by more than a foot from this best determined value, and one of these differs by more than 40 feet. It is seen that in both of these cases the difficulty comes from the determination of the mean DLQ, the best determined value of which at Neah Bay is 1.58 feet.

If for the day of observations the diurnal inequality departs widely from its mean value at the primary station, the derived value at the secondary station may vary considerably from its mean value, unless the type of tide at both stations is closely similar. On the 6th and 21st, DLQ at Seattle varied widely from its mean value, evidenced by the magnitude of the factors in 7th column for those days. In table 4, the ratio of K_1+O_1 to M_2 is 1.20 for Seattle and 0.97 for Neah Bay. The type of tide at these two stations is sufficiently different to make Seattle an unsuitable station for determining DLQ at Neah Bay from one day of observations. In the example for La Jolla and Los Angeles, the factors in the 7th column for the 6th and 21st likewise were relatively large. But the ratio of K_1+O_1 to M_2 is 1.10 for La Jolla and 1.07 for Los Angeles. The type of tide at these two stations is so nearly the same that even for these two days the DLQ derived did not differ much from the mean value and therefore gave concordant results.

Correction by Tabular Values

The correction of the results from a short series to mean value of lower low water involves two steps: (1) correction of low water to mean value; (2) correction of low water diurnal inequality to mean value. By the method of comparison of simultaneous observations, both these corrections are derived by comparing the observations with simultaneous observations at a primary station for which mean values are available.

The method of correction by tabular values derives the corrections directly from the observations by the use of tabular values that have been calculated from theoretical principles. The correction to mean low water makes use of the tabular values in Table 5 and the procedure has been exemplified in Section VII. The correction to mean DLQ makes use of the tabular values in Table 6 and will be exemplified below for a year and a month. For less than a month the method of tabular values is not applicable in the simplified form shown here.

Year.—We may take La Jolla for the same years as used in the example by comparison, namely, every other year from 1936 to 1948. Columns 2 and 4 give the yearly values of low water below half-tide level and DLQ respectively, from the observations at La Jolla. Columns 3 and 5 give the factors for each year taken respectively from Tables 5 and 6. Columns 6 and 7 give the respective products and column 8 the sums of the previous two columns.

Year	LW below HTL	Factor from Table 5	DLQ	Factor from Table 6	MLW below HTL	Mean DLQ	MLLW below HTL
	Feet		*Feet*		*Feet*	*Feet*	*Feet*
1936	1. 83	1. 001	0 93	0. 978	1. 83	0. 91	2. 74
1938	1. 81	0. 990	0. 84	1. 082	1. 79	0. 91	2. 70
1940	1. 86	0. 983	0. 80	1. 172	1. 83	0. 94	2. 77
1942	1. 83	0. 984	0. 82	1. 161	1. 80	0. 95	2. 75
1944	1. 83	0. 992	0. 88	1. 065	1. 82	0. 94	2. 76
1946	1. 81	1. 003	0. 96	0. 962	1. 82	0. 92	2. 74

The primary determination of mean lower low water below half tide at La Jolla is 2.73 feet, of mean low water below half-tide level 1.81 feet, and mean DLQ 0.92 foot. The yearly values derived in the last column are thus within 0.04 foot of the primary value. The mean DLQ was determined within 0.03 foot and mean low water below half-tide level within 0.02 foot. On comparing the values derived above with those derived by the method of comparison (page 116) it is seen that the latter method gave slightly more concordant results.

Taking now Neah Bay for every other year from 1938 to 1948, the data follow:

Year	LW below HTL	Factor from Table 5	DLQ	Factor from Table 6	MLW below HTL	Mean DLQ	MLLW below HTL
	Feet		*Feet*		*Feet*	*Feet*	*Feet*
1938	2. 89	0. 988	1. 44	1. 082	2. 86	1. 56	4. 42
1940	2. 93	0. 979	1. 35	1. 172	2. 87	1. 58	4. 45
1942	2. 87	0. 980	1. 40	1. 161	2. 81	1. 63	4. 44
1944	2. 77	0. 990	1. 50	1. 065	2. 74	1. 60	4. 34
1946	2. 75	1. 004	1. 65	0. 962	2. 76	1. 59	4. 35
1948	2. 71	1. 016	1. 75	0. 899	2. 75	1. 57	4. 32

For Neah Bay the best determined value of mean lower low water below half-tide level was found to be 4.40 feet. The yearly values derived in the last column table above are thus within 0.06 foot of this best determined value on the average, with 0.08 foot the greatest difference. The best determined value of DLQ at Neah Bay is 1.58 feet and from column 7 of the table above it is seen that the greatest difference from this best determined value is 0.05 foot. From a year of observations, therefore, mean lower low water below half-tide level can be determined correct to within 0.1 foot.

Month.—The determination of mean lower low water from a month of observations is carried out in exactly the same way as for a year. It is exemplified below for La Jolla and Neah Bay, taking every other month of the year 1946. The data for La Jolla follow:

Date	LW below HTL	Factor from Table 5	DLQ	Factor from Table 6	MLW below HTL	Mean DLQ	MLLW below HTL
1946	*Feet*		*Feet*		*Feet*	*Feet*	*Feet*
January	1. 76	1. 000	1. 08	0. 87	1. 76	0. 94	2. 70
March	1. 85	1. 001	0. 79	1. 16	1. 85	0. 92	2. 77
May	1. 87	1. 002	1. 00	0. 90	1. 87	0. 90	2. 77
July	1. 79	1. 003	1. 13	0. 84	1. 80	0. 95	2. 75
September	1. 77	1. 004	0. 76	1. 14	1. 78	0. 87	2. 65
November	1. 83	1. 005	1. 09	0. 88	1. 84	0. 96	2. 80

The primary value of MLLW below HTL for La Jolla is 2.73 feet. The greatest difference from this value of the derived monthly values in the last column of the table above is 0.08 foot. The primary value of DLQ at La Jolla is 0.92 foot and the greatest difference from this value of the derived monthly values in the 7th column is 0.05 foot.

Comparing the results in the last column derived by the use of tabular values with those derived by means of simultaneous observations (page 118), the values for individual months are seen to differ sometimes by as much as 0.1 foot. Comparing with the primary value of 2.73 feet, the results derived from tabular values give somewhat less concordant results.

The derivation of MLLW below HTL for Neah Bay from monthly observations is shown below.

Date	LW below HTL	Factor from Table 5	DLQ	Factor from Table 6	MLW below HTL	Mean DLQ	MLLW below HTL
1946	*Feet*		*Feet*		*Feet*	*Feet*	*Feet*
January	2. 74	1. 000	1. 98	0. 87	2. 74	1. 72	4. 46
March	2. 72	1. 001	1. 35	1. 16	2. 72	1. 57	4. 29
May	2. 84	1. 003	1. 71	0. 90	2. 85	1. 54	4. 39
July	2. 84	1. 004	1. 82	0. 84	2. 85	1. 53	4. 38
September	2. 72	1. 005	1. 33	1. 14	2. 73	1. 52	4. 25
November	2. 70	1. 006	1. 84	0. 88	2. 72	1. 62	4. 34

For Neah Bay the best deterimined value of DLQ is 1.58 feet and of MLLW below HTL, 4.40 feet. From column 7 it is seen that the correction of DLQ by tabular values may differ from the best determined value by as much as 0.14 foot. From column 8 the derived value of MLLW below HTL is seen to differ from the best determined value by as much as 0.15 foot. Comparing with the results derived by simultaneous observations (page 118) mean DLQ is about equally well determined by either method.

Summary

Lower low water is distinguished from higher low water and also from low water, the latter term in this connection referring to the average of the two low waters of the day. Lower low water, like low water, varies from day to day, from month to month, and from year to year, these variations being in part due to variations in sea level and in part to astronomic causes.

A direct primary determination of mean lower low water requires 19 years of observations. Shorter series may be corrected to mean value either by comparison with simultaneous observations at a primary station or by tabular values. In either case mean lower low water is derived with reference to half-tide level. In general it may be taken that from a year of observations mean lower low water can be determined, with reference to half-tide level, correct to within 0.1 foot; from a month, correct to within a quarter of a foot; and from a day's observations, correct to within about 1 foot. It is to be noted, however, that occasionally the value derived from one day of observations may be considerably in error, so that at least three days of observations should be used if it is desired to determine mean lower low water correct within a foot.

The secondary determination of mean lower low water involves the determination of half-tide level, mean low water, and mean diurnal low-water inequality; the distance of mean lower low water below half-tide level being the distance of mean low water below half-tide level plus mean low-water diurnal inequality.

X. HIGHER HIGH WATER

Definitions

The existence of daily and semidaily constituents in the tide gives rise to differences in consecutive high waters as well as to differences in consecutive low waters. As a rule, the two high waters of a day differ in height, the higher being designated the higher high water and the lower the lower high water.

On days when but one high water occurs, the rule for determining whether it should be designated as the higher high or lower high is framed in the same way as for the similar case of low water. The single high water is given the name opposite that of the preceding high water; that is, if the preceding high water was the higher high water of the day, then the single high water in question is designated as the lower high water, and vice versa. Thus, in the column of high waters in the tabulation of Figure 16, the single high water on the 20th will be designated as lower high water, since the immediately preceding high water is a higher high water.

Where the tide becomes diurnal—that is, where but one high and one low water occur in a day—the single high water obviously is a higher high water, for it is the merging of the lower high water and higher low water that gives rise to the diurnal tide.

Relation to Lower Low Water

Manifestly the relation of higher high water to the rise of the tide is of a similar nature to that which lower low water bears to the fall of the tide. Corresponding to low-water diurnal inequality is high-water diurnal inequality, which is the difference between high water and higher high water. As distinguished from higher high water, high water refers to the average high water, whether for the day, month, or year.

Diurnal inequality depends on the relative amplitudes of the daily and semidaily tidal constituents and also on their phase relations. With given amplitudes of the two constituents, the diurnal inequality may exist principally in the high waters, principally in the low waters, or equally in the high and low waters, depending on the phase relations of the daily and semidaily constituents. At most places the high-water and low-water diurnal inequalities differ. As mentioned in the section on lower low water, on the Atlantic coast of the United States the high-water inequality is the greater, while on the Pacific coast it is the low-water inequality that is the greater. However, the daily constituent of the tide has a small amplitude on the Atlantic coast, so that, notwithstanding the fact that on this coast the high-water inequality is the greater, it is relatively small.

Variations

Since the relation of higher high water to the rise of the tide is similar to that of lower low water to the fall, it follows that the variations in higher high water will be much the same as those in lower low water. These variations may be summarized as follows:

The height of higher high water varies from day to day, this variation being partly of a periodic nature, due to the change in position of the moon relative to earth and sun, and partly nonperiodic, due to secular variation in sea level. Referring to Figure 39, it is seen that with regard to the moon's declination higher high water goes through a fortnightly cycle, being highest about the time of maximum north or south declination and lowest about the time when the moon is on the Equator. The height of higher high water varies also with the phase and parallax of the moon, but the declinational effect is the principal one.

Monthly values of higher high water generally differ by several tenths of a foot from month to month, while within a year two such monthly values may differ by as much as a foot. The variation in monthly higher high water is partly in response to variations in sea level, but primarily in response to the change in declination of the sun, so that this latter variation has a period of a year.

From year to year higher high water varies about a tenth of a foot, although at times two consecutive yearly values of higher high water may differ by as much as three-tenths of a foot. The variation of yearly higher high water is in part due to secular variations in sea level and in part to a regular variation with a period of 19 years, depending on the longitude of the moon's node.

Definition of Mean Higher High Water

In view of the variations to which the height of higher high water is subject, mean higher high water at any place may be defined simply as the average height of the higher high waters at that place over a 19-year period.

Primary Determination

A primary determination of mean higher high water is based directly on the average of the higher high waters over a 19-year period. This implies that if there were no change in sea level from one 19-year period to another, and no change in tidal regime had taken place, two different 19-year determinations of mean higher high water at a given place would have the same value.

From the 51 years of observations, 1898–1948, at San Francisco three slightly overlapping 19-year series may be formed, 1898–1916, 1914–1932 and 1930–1948. The value of mean higher high water on the staff for each of these series is, respectively, in feet, 11.152, 11.214 and 11.354. The difference between the first two determinations is 0.062 foot and between the first and third 0.202 foot. These are relatively large differences, but these differences reflect changes that may have taken place in sea level. The mean sea level for each of the three 19-year periods on the staff is, respectively, 8.544, 8.601 and 8.720. Referred to the respective sea levels, therefore, the three 19-year series give mean higher high water above sea level as, 2.608, 2.613 and 2.634. The difference between the first and third determinations is 0.005 foot and between the first and third 0.026 foot. As noted in connection with the discussion of the primary determination of mean lower low water, the location of the San Francisco tide station had been changed several times. Furthermore it is possible too that improvements in the harbor may have changed the range of tide somewhat. In any event 0.026 foot is a small quantity and the three determinations are in practical agreement.

From the 50 years of observations at Seattle, 1899–1948, three slightly overlapping 19-year series may be formed, 1899–1917, 1915–1933, and 1930–1948. The value of mean higher high water on staff for each of these series is, respectively, 18.744, 18.754 and 18.867. The difference between the first and second determinations is 0.010 foot and between the first and third 0.123 foot. Referring these values to the respective mean sea levels for the three periods, the results are, respectively, 4.700, 4.699, 4.705. The greatest difference between any two of these three determinations is 0.006 foot.

Secondary Determination

In deriving mean higher high water from a short series of observations the procedure and accuracy attainable are exactly the same as in the like determination of lower low water. For a series varying in length from a month to several years two methods are available. One method consists in comparing with simultaneous observations at some suitable primary tide station, while the other method makes use of factors derived from theoretical considerations. The procedure is given in detail in the section on lower low water. Here it will be sufficient to exemplify the procedures briefly by an example of each method, using La Jolla as the secondary station and Los Angeles as the primary station, taking every other year from 1936 to 1946.

The epoch to which the data will be corrected is the 19 years 1928–1946. For this epoch high water above half-tide level at Los Angeles is 1.88 feet and mean DHQ is 0.75 foot. The factor for each year in column 6 is derived by dividing 1.88 by the corresponding value in column 2, and the factors in column 7 by dividing 0.75 by the corresponding value in column 3.

Mean higher high water from a year of observations, La Jolla, by comparison with Los Angeles

Year	Los Angeles		La Jolla		Factors for correction		La Jolla		
	HW above HTL	DHQ	HW above HTL	DHQ	HW	DHQ	MHW above HTL	Mean DHQ	MHHW above HTL
	Feet	*Feet*	*Feet*	*Feet*			*Feet*	*Feet*	*Feet*
1936	1. 89	0. 79	1. 83	0. 77	0. 995	0. 949	1. 82	0. 73	2. 55
1938	1. 89	0. 72	1. 81	0. 70	0. 995	1. 042	1. 80	0. 73	2. 53
1940	1. 93	0. 67	1. 86	0. 65	0. 974	1. 119	1. 81	0. 73	2. 54
1942	1. 90	0. 65	1. 83	0. 65	0. 989	1. 154	1. 81	0. 75	2. 56
1944	1. 92	0. 69	1. 84	0. 69	0. 979	1. 087	1. 80	0. 75	2. 55
1946	1. 87	0. 74	1. 81	0. 74	1. 005	1. 014	1. 82	0. 75	2. 57

From the 19-year series 1928–1946 at La Jolla the primary determinations are: MHW above HTL, 1.81 feet; mean DHQ, 0.74 foot; and MHHW above HTL, 2.55 feet. From the last three columns of the table above it is seen that the derived mean values differ from the primary mean values by not more than 0.02 foot.

To exemplify the method of correction by tabular factors, we may take La Jolla for every other month of the year 1946. The procedure is given in tabular form below.

Mean higher high water from a month of observations, La Jolla, by tabular factors

Date	HW above HTL	Factor from Table 5	DHQ	Factor from Table 6	MHW above HTL	Mean DHQ	MHHW above HTL
1946	*Feet*		*Feet*		*Feet*	*Feet*	*Feet*
January	1. 76	1. 000	0. 89	0. 87	1. 76	0. 77	2. 53
March	1. 86	1. 001	0. 57	1. 16	1. 86	0. 66	2. 52
May	1. 88	1. 002	0. 83	0. 90	1. 88	0. 75	2. 63
July	1. 79	1. 003	0. 95	0. 84	1. 80	0. 80	2. 60
September	1. 78	1. 004	0. 61	1. 14	1. 79	0. 70	2. 49
November	1. 83	1. 005	0. 64	0. 88	1. 84	0. 56	2. 40

The primary determination of MHHW above half-tide level at La Jolla is 2.55 feet The last column of the table above shows that the derived mean values differ from this primary value by less than 0.1 foot except for the last month when the difference is 0.15 foot. The larger part of this difference arises from the value of mean DHQ derived for that month which differs form the primary mean DHQ by 0.18 foot.

For a series of observations covering less than a month the most practicable method of deriving mean higher high water is by comparison with simultaneous observations at a suitable tide station or with the predicted tides at such a station, precisely as in the like case of determining mean lower low water, for which the detailed procedure was given.

In summary it may be stated that from a year of observations mean higher high water can be derived, with reference to half-tide level, correct within 0.1 foot; from a month, correct to within a quarter of a foot; and from a day's observations, correct to within about a foot. It must be noted, however, that occasionally the value derived from one day of observations may be considerably in error, so that at least three days of observations should be used if it is desired to determine mean higher high water correct within a foot.

XI. OTHER TIDAL DATUMS

Principal Datum Planes

The six datum planes discussed in the preceding pages, namely, mean sea level, half-tide level, mean high water, mean low water, lower low water, and higher high water, constitute the principal tidal datum planes. They are more easily determined than other tidal datum planes, and from a given series of tide observations they can be derived with a greater degree of precision.

Other tidal datum planes have at times been used. Thus, the planes of monthly lowest low water and spring low water and the Indian tide plane have been used in hydrographic surveying and in tide predictions. To determine accurately such datum planes directly from observations requires a much longer series of observations than is necessary for any of the principal planes, for spring tides or tropic tides occur but twice a month and monthly lowest low water but once a month. As a rule, however, approximate determinations of such planes are quite satisfactory, especially if their relation to mean sea level or half-tide level is stated.

When the use of some datum plane other than one of the principal datums is found of advantage, it is desirable that it be defined with reference to one of these principal datums. Thus, if a plane below mean low water or mean lower low water is to be used, it is best to define it by its distance below either of these datums or mean sea level rather than seek some secondary tidal datum which approximates it. Several datum planes have, however, been used heretofore, and it is proposed here to discuss them briefly.

Monthly Lowest Low Water

When a datum plane is desired which will be so low that most low waters will be above it, the plane of monthly lowest low water has sometimes been used. As its name signifies, it is the plane determined by the average height of the lowest low waters of each month over a considerable period of time.

This plane has sometimes been called the plane of extreme low water or of storm low water, but objections may be urged against both of those designations. Calling the lowest low water of each month an extreme low water is obviously arbitrary, while calling it a storm low water is even more arbitrary, for the lowest low water of a month is frequently not due to storms. The term monthly lowest low water is self-explanatory and definitely refers to the low water which, during the month in question, falls to the lowest level.

Within a year, the heights of monthly lowest low water may vary considerably. Yearly averages of these monthly lowest low waters, however, will vary by less than a foot, and a three-year average will not differ by more than a quarter of a foot from a mean based on a number of years.

Along the Atlantic coast of the United States, the plane of monthly lowest low water is below mean low water by the following amounts: from Maine to Rhode Island, about 2 feet; from New York to Georgia, about 1½ feet; Florida, about 1 foot.

Along the Gulf of Mexico coast this difference is from a foot to 1½ feet. Along the Pacific coast the difference is about 2½ feet along California and Oregon and from 4 to 5 feet along Washington. These differences are obviously only approximate, and along a given stretch of coast will vary with the hydrographic features of the coastal body of water.

Datum Planes From Harmonic Constants

The harmonic constants comprise the simple constitutent tides which are derived from the harmonic analysis of the tide observations. The basis of the harmonic analysis lies in the conception of the tide as the sum of a number of simple tides, each of which has a definite period that is determined by some motion of the moon or sun relative to the earth. The most complete list of harmonic constants for the world is the "List of Harmonic Constants" which is being published in loose-leaf form by the International Hydrographic Bureau, Monaco. In 1942 the Coast and Geodetic Survey issued Publications TH–1 and TH–2, listing the eight principal constants, TH–1 giving the constants for the Atlantic Ocean, including the Arctic and Antarctic regions, and TH–2 giving the constants for the Pacific and Indian Oceans.

Formulas have been developed by Harris,[1] by means of which the various datum planes may be derived through the harmonic constants. These formulas are somewhat involved if it is desired to derive the datum planes accurately, but for approximate determinations the formulas may be simplified considerably.

As examples, it may be noted that the plane of mean high water for tides of the mixed and semidaily types is given approximately by the formula $HTL+1.1M_2$, in which HTL is half-tide level and M_2 is the principal lunar semidiurnal constituent. In the same way mean low water is given approximately by $HTL-1.1M_2$. To test the degree of approximation of these formulas we may derive the plane of mean high water for Boston, Mass., and for Seattle, Wash.

The value of M_2 for Boston is 4.44 feet and for Seattle 3.50 feet. From the approximate formula above, mean high water at Boston is derived as 4.88 feet above half-tide level and at Seattle as 3.85 feet above half-tide level. These values compare with primary determinations of 4.72 feet at Boston and 3.83 feet at Seattle. The simple formula therefore gives mean high water above half-tide level correct within 0.1 or 0.2 foot.

An approximate value for the datum of lower low water on the Pacific coast of the United States is given by the formula $MLW—0.6(K_1+O_1)$, in which MLW is mean low water and K_1 and O_1, respectively, the principal lunisolar diurnal and principal lunar diurnal constituents. Since mean lower low water is given by subtracting the mean low-water diurnal inequality from mean low water, the formula amounts to taking MDLQ as equal to $0.6\ (K_1+O_1)$, which is obviously but a rough approximation. Thus if we derive the values of $0.6\ (K_1+O_1)$ for San Francisco and Seattle, we get 1.17 feet for San Francisco and 2.53 feet for Seattle which compare with primary values of 1.14 and 2.84 respectively.

The datum of higher high water on the Pacific coast is given approximately by $MHW+0.3(K_1+O_1)$, in which MHW is mean high water and K_1 and O_1 as above. It is to be emphasized, however, that the simple formulas given above for the planes

[1] R. A. Harris. Manual of Tides, Pt. III (Washington, D. C., 1895).

of mean high water, mean low water, lower low water, and higher high water are approximations, and where accurate determinations are desired these must be derived from the observations as outlined in the previous sections. For the plane of spring low water and for the Indian tide plane, however, the determination by means of the harmonic constants is to be preferred.

Spring Low Water

Spring low water has been used as a datum for hydrographic charts and for the prediction of tides. This datum may be defined as the average of the low waters that come at the time of spring tides. Spring tides are those that occur about the times of new and full moon, when the tide-producing forces of sun and moon conspire and bring about a greater rise and fall than usual. At most places there is a lag between full or new moon and the greatest rise and fall of the tide, this lag being known as the phase age of the tide. On the Atlantic and Pacific coasts of the United States the phase age of the tide is about one day; that is, spring tides come about one day after full and new moon.

It is obvious that there must be considerable variation in the height of spring low water from one fortnight to another. In the first place, it will vary in response to changes in sea level; and, in the second place, it will vary in response to changes in the positions of sun and moon as regards parallax and declination. It should be noted in passing that the two low waters coming nearest, one before and the other after, the time given by adding the phase age to the time of new and full moon are taken as constituting the spring low waters of any given new or full moon.

For all practical purposes it is sufficient to determine the plane of spring low water approximately, especially if its relation to mean sea level or half-tide level is given. This relation is given when spring low water is determined through harmonic constants, and this method therefore furnishes a convenient means for deriving the datum. As an approximate formula for the plane of spring low water, we may take it to be $MLW-S_2$ in which MLW is mean low water and S_2 the principal solar semidiurnal constituent.

When it is necessary to determine spring low water and harmonic constants are not at hand, it may be derived by comparison with the spring low water at some other place in the same general region. If R and SpLW represent, respectively, the mean range and spring low water at the comparison station and the same abbreviations, with subscript 1, the like quantities at the station for which spring low water is desired, then we have $SpLW_1 = \frac{R_1}{R}\ SpLW$. In both cases SpLW represents the distance of spring low water below half-tide level.

Indian Tide Plane

In predicting the heights of high and low water for tide tables it is obviously desirable to refer these heights to a plane such that no negative heights will be necessary; that is, the datum with regard to which the predictions are given should be so low that no low water will fall below it. But it is manifestly even more desirable that the plane used in the tide predictions for any given port should be the same as that used on the hydrographic charts of that port. This consideration limits the practical datums for such purposes to some low-water datum like mean low water, lower low water, or spring low water.

The Indian tide plane, or the harmonic tide plane, as it is sometimes called, has been used for a number of ports in India. It is defined as the datum plane that lies below mean sea level a distance given by adding the amplitudes of the principal lunar semidiurnal, principal solar semidiurnal, the principal lunisolar diurnal, and the principal lunar diurnal components. In the accepted notation it may be written as follows: $MSL-(M_2+S_2+K_1+O_1)$. The Indian tide plane is sometimes called also the plane of Indian spring low water. From its definition in terms of the harmonic constants it obviously corresponds approximately to tropic spring lower low water.

XII. CHANGES IN TIDAL DATUM PLANES

Implications in Assumption of Constancy of Tidal Datums

In the use of tidal datums as planes of reference for elevations it is implied that such datums at any given place remain constant over relatively long periods of time. Underlying this implied constancy are the tacit assumptions of coastal stability and constancy of hydrographic features. If changes take place in the relative elevation of land to sea or in the hydrogrpahic features of the body of water on which the given place is situated, changes will also take place in the tidal datum planes, which are fixed by reference to local bench marks.

With regard to periods of time measured in thousands of years, local changes in relative elevation of land to sea of considerable magnitude have been fully demonstrated. But for the lesser periods of time involved in everyday affairs any such changes, as a general rule, are so small that with respect to tidal datum planes they may be disregarded and coastal stability taken for granted for a number of years.

The changes in hydrographic features that bring in their train changes in datum planes are those that affect the local tidal régime. With regard to such changes in hydrographic features, distinction must be made between the open coast and inland bodies of tidal water. While the open coast is at all times under attack by wave and current and thus subject to change, such changes are relatively slight and only rarely bring about changes in the rise and fall of the tide, even over a period of a number of years. Hence, along the open coast, it may be assumed that tidal datum planes remain constant for periods covering many years.

But in inland bodies of tidal water changes in hydrographic features are as a rule followed by changes in the tidal régime, which are reflected by changes in the tidal datum planes. The changes that may be expected under different conditions will be discussed briefly in this section. But it will be of advantage to consider in this connection the changes in tidal datums that result from changes in relative elevation of land to sea.

Changes Due to Change in Relative Elevation of Land to Sea

If a coast is undergoing a slow gradual subsidence the first effect would obviously be an apparent elevation of all the tidal datum planes with respect to local bench marks by the same amount, this being the amount of the subsidence. That is, if after a number of years the subsidence of the coast in question amounted to one-tenth of a foot, mean sea level, half-tide and also the low-water and the high-water planes would, with respect to the local bench marks, stand one-tenth of a foot higher than at the beginning of the period.

But if the subsidence becomes sufficient to alter materially the hydrographic features of the coast, changes would ensue in the tidal régime along the coast, and as a consequence the different datums would change differently, as the following considerations will make clear.

Whatever the changes in the rise and fall of the tide along an open ocean coast due to gradual subsidence, it is obvious that the mean sea level, fixed with respect to local bench marks, will show an apparent change of the same magnitude as the subsidence, but in the opposite direction. That is, if S is the subsidence in feet, from the beginning of the period, the mean sea level would appear to have risen S feet. But, if because of the alteration of the hydrographic features consequent on the subsidence, the range of the tide is increased by A feet, mean high water will appear to have risen $S+\frac{1}{2}A$ feet, while for mean low water the apparent rise will be $S-\frac{1}{2}A$ feet. Thus the datums of mean high water, mean sea level, and mean low water, with respect to local bench marks will be changed by different amounts.

On a rising coast the changes that would take place in the tidal datums are similar but in the opposite direction to those just discussed. The first effect would be a lowering of all the tidal datum planes with respect to local bench marks by the amount of emergence. If the emergence becomes sufficient to alter materially the hydrographic features, and so bring about a change in the rise and fall of the tide, the changes ensuing from this latter cause would, as in the case of subsidence, be different for the different datums.

Changes Due to Alteration of Hydrographic Features

It is with regard to inland tidal waters that changes in datum planes due to alteration of hydrographic features become important. On the open coast it is reasonable to assume that only profound changes in the hydrographic features can bring about changes in the range of the tide. But in inland tidal waters, because of the relatively limited areas and depths involved, changes in the features of considerably lesser magnitude are sufficient to change the range of the tide and thus bring about changes in datums. While the quantitative relations subsisting between changes in the body of water and changes in datums are difficult to establish from general considerations, qualitatively we may determine the changes in the datums that will follow proposed changes in the hydrographic features.

Tidal rivers are good examples of such inland bodies of water. These rivers serve as highways to the sea for numerous ports, some of which are situated many miles from the coast. With the increased draft and size of modern vessels, changes in depth or other alterations are frequently found necessary; and such improvements, if of sufficient magnitude, result in changes in the local tidal datum planes.

The tides in rivers are due to the tides sweeping into them from the seas into which they open. Normally the tide travels upstream until stopped by falls or rapids. If the mouth of the river is widened or deepened, this makes for a freer entry of the tide from the open sea and thus for a greater rise and fall of the tide. As a first effect, therefore, of widening or deepening a tidal river at its mouth, we may expect a rise in the high-water datums and a fall in the low-water datums. This effect, it is reasonable to expect, will generally be greatest near the mouth of the river, becoming gradually less going upstream.

Tidal rivers serve, however, not only as highways for the tide, but also as channels for carrying to the sea the drainage waters from large territories. Normally the cross-sectional area of a river increases seaward, due to the seaward slope of the river bed and the increasing width between banks. As a consequence, the mean river level in a tidal river becomes higher in going upstream. Thus, precise leveling by the Coast and

Geodetic Survey shows that at Philadelphia, about 100 miles up the Delaware, the mean sea level (or mean river level) is about three-quarters of a foot higher than mean sea level on the coast, notwithstanding the fact that the range of the tide at Philadelphia is somewhat larger than at the mouth of the river.

Widening the mouth of a tidal river increases the cross-sectional area of the channel through which the drainage waters flow into the sea. As a consequence of the enlarged channel the drainage waters have freer outlet, which results in a lowering of the mean sea level some distance upstream. Deepening the mouth has a like effect; in addition, by reducing the friction per unit volume of water it brings about a further lowering of the mean level of the water.

If as a result of widening and deepening the mouth of a tidal stream the mean sea level at a given point of the river is lowered D feet and the range increased A feet, the changes in the different datums would be: Mean sea level, lowered D feet; mean high water, lowered $D-\frac{1}{2} A$ feet; mean low water, lowered $D+\frac{1}{2} A$ feet. The slope of mean sea level up a tidal stream is relatively slight, but the increase in range of tide consequent upon improvement is relatively large. For example, at Glasgow, on the Clyde, the range was increased 8 feet by river improvements. It follows, therefore, that as a rule in the above formulas D is less than $\frac{1}{2} A$. This means that $D-\frac{1}{2} A$ is negative and that high water, instead of being lowered, is raised somewhat. Low water, however, is lowered by the full amount of the increase in half range plus the depression of mean sea level.

These considerations are important in connection with the improvement of tidal rivers, since the depths in these are generally referred to mean low water. When improvements are contemplated the wording is generally to the effect that a certain depth at mean low water is to be attained. At first thought it would appear that if the desired depth below mean low water is a feet and the present depth is b feet the channel is to be deepened $a-b$ feet. But as the considerations outlined above show this is not the case, for the datum plane of mean low water is different in the two cases.

XIII. FORMS FOR TABULATIONS AND COMPUTATIONS

Standard Forms

In the tabulation of the tide record and in the computation of datum planes the work is facilitated by the adoption of standard forms. This permits of a uniform procedure, which, having been learned by the tabulator, reduces the time required for the tabulation and also lessens the chance of error.

Both in the tabulation and in the computation of the tidal data the Coast and Geodetic Survey makes use of printed forms of uniform size, 8 inches wide and 10½ inches long. This size is a very convenient one for handling, and it is also convenient for filing, since no folding is necessary.

Tabulation Forms

Comparative readings.—The form used for tabulating the comparative readings, necessary with the standard gage, was discussed in Section IV, "Tabulation of the tide record," and is illustrated, somewhat reduced, in Figure 17 (p. 38). The form is printed alike on both sides, one month being generally tabulated on each side.

Hourly heights.—The hourly heights of the tide are tabulated on a form printed on both sides, each side accommodating a week of observations. This form likewise was discussed in Section IV and is illustrated in reduced size in Figure 15. The sums at the bottoms of the vertical columns permit the daily sea level to be determined, while the horizontal sums are used in the harmonic analysis. A check on the correctness of the vertical and horizontal sums is given by the final sum in the lower right-hand corner, which must check from the 7 vertical sums and from the 24 horizontal sums.

The sum of the hourly heights for each month is entered on the last sheet of hourly heights of the month in the space provided for it at the bottom of the sheet, and the mean derived by dividing by the proper divisor for the month in question. The divisors for the various months are given on the sheet.

The wide spacing on the form between the consecutive days is due to the fact that the tabulated hourly heights are used for summing component hours for the harmonic analysis by means of stencils. When this is not the case the form can be ruled to accommodate half a month on one side and the other half month on the other side.

High and low waters.—The form used for tabulating the high and low waters is illustrated in Figure 16. The form is designed for one month of observations, the first 17 days being tabulated on one side of the sheet and the remaining days of the month on the other side. The lunitidal intervals and the heights of high and low water are summed and the means derived as indicated, from which follow the range of the tide (Mn) and half-tide or mean-tide level (MTL).

On the back of the sheet, illustrated in Figure 16, provision is made for the sums and means of higher high waters, lower low waters, and the inequalities of the month. A convenient method of summing the higher high and lower low waters directly from the sheet consists in checking each higher high and lower low with a small check mark.

These checked figures may then be readily summed directly from the sheet and the sums and means entered in their appropriate places. The DHQ and DLQ are then derived by subtracting the mean values of the high and low waters of the month from the corresponding values of higher high and lower low waters, respectively. Provision is also made on the back of the high and low water form for deriving mean values of the range of the tide and of the inequalities by use of the factors in Tables 5 and 6.

Computation Forms

Comparison of simultaneous observations.—In connection with a short series of observations the form illustrated in Figure 55 is useful, as all the results desired for datum planes are derived on a single sheet. It is designed to accommodate 7 days of observations, but it is to be noted that it is not necessary that these be consecutive days. In Figure 55 a week of observations at Falls Creek, Alaska, is compared with simultaneous observations at Ketchikan.

In the columns of heights and height difference the higher high and lower low waters are marked with a check, and from these the appropriate sums and means are derived. The back of the form carries instructions and explanations. For stations on the Pacific coast, where the datum on the charts is mean lower low water, the instructions call for filling in all the items shown for Falls Creek. For stations on the Atlantic coast, where the plane of reference on the charts is mean low water, the instructions call for the omission of items (4) to (9), (14) to (19), (25), (26), and the computation of DHQ and DLQ at the bottom of the page. In this case the heights of all the high waters are combined into a single sum and similarly the low waters, the headings being made to read HW and LW respectively.

The form is also adapted for the computation of datum planes where the tide is predominantly diurnal. In this case, it will be recalled, mean higher high water is the same as mean high water and mean lower low water the same as mean low water. The use of the form with predominantly diurnal tides is illustrated in Figure 56.

For completeness, the form also has columns for deriving mean values of the lunitidal intervals. For datum planes these are unnecessary, and have not been used in the illustrative examples of Figures 55 and 56. The columns of "Time difference," however, are used with the determination of datum planes to make sure that corresponding tides have been used at the subordinate and standard stations. If the difference for any particular tide in either of the columns of "Time difference" varies considerably from the other values in its column, it will call for examination, and if found due to disturbed conditions may necessitate the elimination of all the comparisons for that tide.

With a series of tide observations covering several or more months the form illustrated in Figure 57 is used. Any outstanding value in the column of differences or of ratios calls for examination of the corresponding values at both the standard and subordinate stations and helps in the detection of errors or elimination of observations disturbed by unusual conditions. In Figure 57, twelve months of observations at Avila are compared with simultaneous observations at Los Angeles. The corrected values for the various tidal data are then immediately available for determining desired tidal datum planes.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 248
Ed. May, 1928

TIDES: Comparison of Simultaneous Observations

(A) Subordinate station Falls Creek, Alaska Lat. 55°37′ N Long. 131° 20.6′ W
(B) Standard station Ketchikan, Alaska Lat. 55° 20′ N Long. 131° 37.6′ W.
Chief of party .... Time Meridian: (A) 120° W (B) 120° W

DATE.	(A) STATION. Time of—		(B) STATION. Time of—		(A)–(B) Time difference.		(A) STATION. Height of –		(B) STATION. Height of—		(A)–(B) Height difference.	
Year. 1949	HW.	LW.	HW.	LW.	HW.	LW.	HW.	LW.	HW.	LW.	HW.	LW.
Mo. D.	Hours.	Hours.	Hours.	Hours.	Hours.	Hours.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
Dec. 1	11.0	4.6	11.0	4.5	0.0	0.1	17.3	6.7	21.7	11.4	-4.4	-4.7
	23.6	17.4	23.7	17.4	-0.1	0.0	14.7	3.8	19.1	8.2	-4.4	-4.4
2	11.5	5.3	11.4	5.2	0.1	0.1	17.2	6.3	21.5	10.8	-4.3	-4.5
	—	18.0	—	18.1	—	-0.1	—	2.4	—	7.0	—	-4.6
3	0.4	6.0	0.4	5.9	0.0	0.1	14.6	5.9	19.0	10.3	-4.4	-4.4
	12.0	18.6	12.1	18.7	-0.1	-0.1	17.3	1.2	21.8	5.9	-4.5	-4.7
4	1.0	6.5	1.2	6.4	-0.2	0.1	15.4	6.6	19.9	11.2	-4.5	-4.6
	12.4	19.2	12.5	19.1	-0.1	0.1	18.6	1.4	23.1	6.2	-4.5	-4.8
5	1.5	7.1	1.7	7.1	-0.2	0.0	16.5	7.0	21.0	11.4	-4.5	-4.4
	13.0	20.0	13.0	19.8	0.0	0.2	18.7	0.3	23.0	4.9	-4.3	-4.6
6	2.3	7.7	2.2	7.6	0.1	0.1	15.6	6.2	20.0	10.6	-4.4	-4.4
	13.6	20.3	13.7	20.3	-0.1	0.0	18.0	-0.3	22.5	4.6	-4.5	-4.9
7	2.7	8.2	2.8	8.1	-0.1	0.1	15.6	6.4	20.1	10.9	-4.5	-4.5
	14.2	20.9	14.2	20.9	0.0	0.0	18.0	-0.1	22.6	5.0	-4.6	-5.1
Sums							7 HHW. 125.1	7 HLW. 45.1	7 HHW. 156.2	7 HLW. 76.6	7 HHW. -31.1	7 HLW. -31.5
Means							17.87	6.44	22.31	10.94	-4.44	-4.50
Sums							6 LHW. 92.4	7 LLW. 8.7	6 LHW. 119.1	7 LLW. 41.8	6 LHW. -26.7	7 LLW. -33.1
Means							15.40	1.24	19.85	5.97	-4.45	-4.73

HW.	LW.	
(1)=		=Mean difference in time of high and low water respectively.
(2)=		=Correction for difference in longitude. (Table on back of form.)
(3)=		=(1)+(2)=Mean difference in high and low water intervals, respectively.

Feet.

(4)= 17.87 =Mean HHW height at (A).
(6)= 15.40 =Mean LHW height at (A).
(8)= 2.47 =(4)–(6)=2DHQ at (A).
(10)= 16.64 =½[(4)+(6)]=Mean HW height at (A).
(12)= 12.80 =(10)–(11)=Mn at (A).

Feet.

(5) = 6.44 =Mean HLW height at (A).
(7) = 1.24 =Mean LLW height at (A).
(9) = 5.20 =(5)–(7)=2DLQ at (A).
(11)= 3.84 =½[(5)+(7)]=Mean LW height at (A).
(13)= 10.24 =½[(10)+(11)]=MTL at (A).

(14)= -4.44 =Mean HHW difference.
(16)= -4.45 =Mean LHW difference.
(18)= 0.01 =(14)–(16)=2DHQ difference.
(20)= -4.44 =½[(14)+(16)]=Mean HW difference.
(22)= 0.18 =(20)–(21)=Mn difference.

(15)= -4.50 =Mean HLW difference.
(17)= -4.73 =Mean LLW difference.
(19)= 0.23 =(15)–(17)=2DLQ difference.
(21)= -4.62 =½[(15)+(17)]=Mean LW difference.
(23)= -4.53 =½[(20)+(21)]=MTL difference.

(24)= 1.014 =(12)÷[(12)–(22)]=Mn ratio.

(25)= 1.004 =(8)÷[(8)–(18)]=DHQ ratio.
(26)= 1.046 =(9)÷[(9)–(19)]=DLQ ratio.

Results from comparison of Stations A and B.	HWI.	LWI.	MTL.	Mn.	DHQ.	DLQ.
Length of Series.	Hours.	Hours.	Feet.	Feet.	Feet.	Feet.
Accepted values for standard station, from 19 years, 1924 – 1942			14.30	12.94	0.89	1.55
Differences and ratios: (3), (23), (24), (25), (26)			-4.53	×1.014	×1.004	×1.046
Corrected values for subordinate station			9.77	13.12	0.89	1.62

Mean LW on staff at subordinate station=MTL–½Mn = 3.21 feet.
Mean LLW on staff at subordinate station=MTL–½Mn–DLQ= 1.59 feet.

Computed by J. M. S., Jan. 13, 1950 (Date.) Verified by E. C. M., Jan. 13, 1950 (Date.)

FIG. 55.—Form for comparison of simultaneous observations.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 248
Ed. May, 1928

TIDES : Comparison of Simultaneous Observations

(A) Subordinate station Bayou Rigaud, La. Lat. 29° 15'.5 N Long. 89° 58'.0 W
(B) Standard station Pensacola, Fla. Lat. 30° 24'.2 N Long. 87° 12'.8 W
Chief of party .... Time Meridian: (A) 90° W (B) 90° W

DATE.	(A) STATION.		(B) STATION.		(A)—(B)		(A) STATION.		(B) STATION.		(A)—(B)	
Year.	Time of—		Time of—		Time difference.		Height of—		Height of—		Height difference.	
1948	HW.	LW.	HW.	LW.	HW.	LW.	HW.	LW.	HW.	LW.	HW.	LW.
Mo. D.	Hours.	Hours.	Hours.	Hours.	Hours.	Hours.	Feet.	Feet.	Feet.	Feet.	Feet.	Feet.
July 1	6.8		6.5		0.3		5.5		9.5		-4.0	
		16.7		17.0		-0.3		4.7		8.5		-3.8
2	6.5		6.6		-0.1		5.6		9.6		-4.0	
		17.5		18.8		-1.3		4.5		8.2		-3.7
3	7.6		8.0		-0.4		5.6		9.7		-4.1	
		18.0		18.4		-0.4		4.4		8.3		-3.9
4	8.2		8.5		-0.3		6.0		10.0		-4.0	
		18.3		20.5		-2.2		4.5		8.1		-3.6
5	8.5		9.4		-0.9		6.0		10.1		-4.1	
		19.5		21.3		-1.8		4.4		8.2		-3.8
6	10.1		10.8		-0.7		6.1		10.3		-4.2	
		20.6		22.3		-1.7		4.5		8.2		-3.7
7	10.9		11.7		-0.8		6.3		10.4		-4.1	
		21.7		23.0		-1.3		4.7		8.4		-3.7
							7 HHW.	HLW.	7 HHW.	HLW.	7 HHW.	HLW.
Sums							41.1		69.6		-28.5	
Means							5.87		9.94		-4.07	
							LHW.	7 LLW.	LHW.	7 LLW.	LHW.	7 LLW.
Sums								31.7		57.9		-26.2
Means								4.53		8.27		-3.74

HW. LW.

(1)= .... | .... =Mean difference in time of high and low water respectively.
(2)= .... | .... =Correction for difference in longitude. (Table on back of form.)
(3)= .... | .... =(1)+(2)=Mean difference in high and low water intervals, respectively.

Feet. Feet.

(4)= .... =Mean HHW height at (A). (5) = ... =Mean HLW height at (A).
(6)= .... =Mean LHW height at (A). (7) = ... =Mean LLW height at (A).
(8)= .... =(4)−(6)=2DHQ at (A). (9) = ... =(5)−(7)=2DLQ at (A).
(10)= 5.87 =½[(4)+(6)]=Mean HW height at (A). (11)= 4.53 =½[(5)+(7)]=Mean LW height at (A).
(12)= 1.34 =(10)−(11)=Mn at (A). (13)= 5.20 =½[(10)+(11)]=MTL at (A).

(14)= .... =Mean HHW difference. (15)= .... =Mean HLW difference.
(16)= .... =Mean LHW difference. (17)= .... =Mean LLW difference.
(18)= .... =(14)−(16)=2DHQ difference. (19)= .... =(15)−(17)=2DLQ difference.
(20)= -4.07 =½[(14)+(16)]=Mean HW difference. (21)= -3.74 =½[(15)+(17)]=Mean LW difference.
(22)= -0.33 =(20)−(21)=Mn difference. (23)= -3.90 =½[(20)+(21)]=MTL difference.

(24)= 0.802 =(12)÷[(12)−(22)]=Mn ratio. (25)= .... =(8)÷[(8)−(18)]=DHQ ratio.
(26)= .... =(9)÷[(9)−(19)]=DLQ ratio.

Results from comparison of Stations A and B.	HWI.	LWI.	MTL.	Mn.	DHQ.	DLQ.
Length of Series.	Hours.	Hours.	Feet.	Feet.	Feet.	Feet.
Accepted values for standard station, from 19 years, 1924-1942			8.62	1.27		
Differences and ratios: (3), (23), (24), (25), (26)			-3.90	×0.802	×	×
Corrected values for subordinate station			4.72	1.02		

Mean LW on staff at subordinate station=MTL−½Mn = 4.21 feet.
Mean LLW on staff at subordinate station=MTL−½Mn−DLQ= .... feet.

Computed by .... , ... (Date.) Verified by .... , ... (Date.)

FIG. 56.—Comparison of simultaneous observations, diurnal tides.

DEPARTMENT OF COMMERCE
U. S. COAST AND GEODETIC SURVEY
Form 657
Ed. Dec. 1929

TIDES: Comparison of Monthly Means

(A) Subordinate station Avila (San Luis Obispo Bay) Calif. Lat. 35° 10!2 Long. 120° 44!4
(B) Standard station Los Angeles, Calif. Lat. 33° 43!2 Long. 118° 16!3

U. S. GOVERNMENT PRINTING OFFICE

MONTH	MTL			MSL			HWI			LWI		
	(A) Feet	(B) Feet	(A)−(B) Feet	(A) Feet	(B) Feet	(A)−(B) Feet	(A) Hours	(B) Hours	(A)−(B) Hours	(A) Hours	(B) Hours	(A)−(B) Hours
1948 July	5.86	6.64	−0.78	5.83	6.63	−0.80						
Aug.	5.87	6.64	−0.77	5.83	6.61	−0.78						
Sept.	6.08	6.76	−0.68	6.04	6.73	−0.69						
Oct.	5.84	6.52	−0.68	5.80	6.49	−0.69						
Nov.	5.74	6.54	−0.80	5.68	6.50	−0.82						
Dec.	5.64	6.52	−0.88	5.58	6.46	−0.88						
1949 Jan.	5.82	6.48	−0.66	5.80	6.45	−0.65						
Feb.	5.68	6.46	−0.78	5.65	6.43	−0.78						
Mar.	5.56	6.18	−0.62	5.53	6.14	−0.61						
Apr.	5.53	6.25	−0.72	5.50	6.22	−0.72						
May	5.67	6.34	−0.67	5.60	6.30	−0.70						
June	5.82	6.57	−0.75	5.77	6.53	−0.76						
Sums			−8.79			−8.88						
Means			−0.73			−0.74						
Accepted values for (B)			6.51	x x x x	x x x x	6.48	x x x x	x x x x		x x x x	x x x x	
Correct values for (A)			5.78	x x x x	x x x x	5.74	x x x x	x x x x		x x x x	x x x x	

Corrected value for MTL, MSL, HWI, and LWI, for subordinate station = accepted value for standard station + mean difference.

MONTH	Mn			DHQ			DLQ					
	(A) Feet	(B) Feet	(A)÷(B) Ratio	(A) Feet	(B) Feet	(A)÷(B) Ratio	(A) Feet	(B) Feet	(A)÷(B) Ratio			
1948 July	3.43	3.57	0.96	0.98	1.02	0.96	1.28	1.18	1.08			
Aug.	3.58	3.72	0.96	0.88	0.85	1.04	1.20	1.11	1.08			
Sept.	3.64	3.90	0.93	0.61	0.66	0.92	1.01	0.92	1.10			
Oct.	3.69	3.94	0.94	0.49	0.55	0.89	1.00	0.89	1.12			
Nov.	3.56	3.75	0.95	0.73	0.78	0.94	1.26	1.15	1.10			
Dec.	3.57	3.67	0.97	0.88	0.97	0.91	1.38	1.23	1.12			
1944 Jan.	3.55	3.67	0.97	0.89	0.93	0.96	1.27	1.18	1.08			
Feb.	3.55	3.73	0.95	0.79	0.77	1.03	1.05	0.97	1.08			
Mar.	3.51	3.80	0.92	0.56	0.58	0.97	0.86	0.80	1.08			
Apr.	3.52	3.74	0.94	0.61	0.66	0.92	1.04	0.94	1.11			
May	3.54	3.65	0.97	0.51	0.89	0.91	1.26	1.14	1.11			
June	3.55	3.64	0.98	0.95	0.99	0.96	1.32	1.21	1.09			
Sums			11.44			11.41			13.15			
Means			0.953			0.951			1.096			
Accepted values for (B)			3.76	x x x x	x x x x	0.74	x x x x	x x x x	0.95	x x x x	x x x x	
Corrected values for (A)			3.58	x x x x	x x x x	0.70	x x x x	x x x x	1.04	x x x x	x x x x	

Corrected value for Mn, DHQ, and DLQ for subordinate station = accepted value for standard station × mean ratio.

Remarks:

Computed by
Verified by

FIG. 57.—Comparison of monthly means.

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