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by

Cheng F. Lee

and

Kimio Morimune*

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*Associate Professor of Finance and Assistant Professor of Economics, University of Illinois at Urbana-Champaign, respectively. The second author is partly supported by NSF SOC 76-22232.

Abstract

Time aggregation technique is used to show that coefficient of determination, systematic risk and residual variance of the market model are generally not independent of the length of observed horizon used in the empirical study.

I. Introduction

In the last decade, the problems associated with the investment horizon in investment analysis have been studied in some detail. Jensen (1969) has shown that there exist some impacts of investment horizon on the systematic risk estimation; Levy (1972) has demonstrated that the Sharpe performance measure can be biased by the inappropriate investment horizon used in the empirical study; Cheng and Deets (1973) have raised some questions about Jensen's instantaneous systematic risk estimation method; Lee (1976A) has developed a method to test whether the investment horizon associated with individual security, portfolio and mutual is instantaneous or not; Lee (1976B) has derived the relationship between the estimated instantaneous systematic risk and the estimated finite systematic risk; Levhari and Levy (1977) have derived some mathematical formulas to show that there exist some relationships between the magnitude of estimated systematic risk and the length of investment horizon and to show that the estimated Treynor measure is biased unless a correct investment horizon is used in the empirical study; Schwartz and Whitcomb (1977) have derived some relationships to explain how the coefficient of determination (R^2) of the capital market model can be affected by the different investment horizons used in estimating the market model. In add-tion, Brenner (1977) has investigated the effect of model misspecification on the tests of the efficient market hypothesis.

The main purpose of this study is to use the time aggregation method proposed by Zellner and Montimarquette (1971), Tiao and Wei (1976) and others to show that estimated R^2, systematic risk and

residual variance of the market are generally not independent of the length of investment horizon used in the empirical study. It will be shown that the investment horizon problem can be treated either as a time aggregation problem or as a specification problem. In the second section the model used to investigate the effect of investment horizon on the magnitude of estimated parameters associated with market model are specified. In the third section the relationship between the estiamted R^2 of the market model and the length of the investment horizon developed by Schwartz and Whitcomb (1977) is reexamined and discussed. A generalized relationship based upon Zellner and Montimarquette's (1971) time aggregation technique is derived and interpreted. It is shown that the autocorrelation of the residual term of market model generally does not affect the estimated R^2. In the fourth section, the impact of the length of the investment horizon on the estimated systematic risk and the estimated residual variance is analyzed. It is shown that both the magnitude of estimated systematic risk and the results of testing the efficient market hypothesis are generally not necessarily independent of the length of the investment horizon used. Finally results of this paper are summarized, possible future research associated with time aggregation in capital asset pricing is also indicated.

II. The Model

Following Schwartz and Whitecomb (1977), the market model for any j^{th} firm or portfolio for a T year period is defined as

$$R_{Tij} = \alpha_{Ti} + \beta_{Ti} R_{TMj} + U_{Tij} \qquad (1)$$

Where $R_{TMj} = \log_e (I_{Tj}/I_{Tj-1})$, the "market" (log) rates of return per annum over the j^{th} period of length T.

$R_{Tij} = \log_e (P_{Tj} + D_{Tj}/P_{Tj-1})$, the log rate of return per annum over the j^{th} period of length T.

Then for any t^{th} short period of duration n years write the model (dropping the firm index i, and the observation index j for compactness) as:

$$r_{ti} = \alpha_n + \beta_n r_{tm} + U_t \qquad (2)$$

The relationship between R_{iT} and r_{it}, R_{mT} and r_{mt} is defined as:[1]

$$\begin{aligned} &(a) \quad R_{iT} = \sum_{t=1}^{T/n} r_{it} \\ &(b) \quad R_{mt} = \sum_{t=}^{T/n} r_{mt} \end{aligned} \qquad (3)$$

If r_{it} and r_{mt} represent monthly rates of returns and $T/n = 3$, then R_{iT} and R_{mT} will represent the quarterly rates of returns. To simplify the analysis, the market model deviation from the mean in terms of monthly rates of return is defined as:[2]

$$Y_t = \beta X_t + u_t \qquad t = 1, 2, \ldots, n \qquad (4)$$

Where $Y_t = r_{ti} - \bar{r}_{ti}$, $X_t = r_{tm} - \bar{r}_{tm}$, β is a scalar parameter, and U_t is a non-autocorrelated error term with $E(U_t) = 0$ and $E(U_t^2) = \sigma^2$ for all t.

Following Zellner and Montimarquette (1971) and the definitions defined in (3), the market model deviation from the mean in terms of quarterly rates of return is defined as:

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[1] These definitions are not exactly identical to Schwartz and Whitcomb (1977) definitions; however, they will not affect the results of this study.

[2] A model with autocorrelated residuals is developed in the Appendix.

$$q = AY = \beta AX + AU \qquad (5)$$

$$\text{Where } \underset{(QXn)}{A} = \begin{bmatrix} 1\ 1\ 1 & & & & \\ & 1\ 1\ 1 & & & \\ & & \ddots & & \\ & & & 1\ 1\ 1 & \\ & & & & 1\ 1\ 1 \end{bmatrix} ; \; Q = \frac{n}{3} ;$$

$q_t = A_t Y = \beta A_t X + A_t U$; and A_t is the t^{th} row of A.

In the following section, the relationship between the R^2 in terms of disaggregated rates of return and the R^2 in terms of aggregated rates of return is derived.

III. Impact of Investment Horizon on the Estimated R^2

Jacob (1971) has found that the R^2 estimated from monthly data is smaller than that from both quarterly and annual data; Altmon, Jacquillat and Levasseur (1974) have found that the R^2 estimated from quarterly data is smaller than that from both semiannual and annual data; McDonald (1974) has found that the R^2 obtained from monthly mutual fund data is smaller than that from both quarterly and annual mutual fund data. Schwartz and Whitcomb (SW) (1977) have tried to explain the above--mentioned findings by the time-variance relationship. Now, a new approach is used to explain the impact of time aggregation on the estimated R^2.

Given the assumptions made about the elements of U in connection with (4), we have E(AU) = 0, and the Q x Q covariance matrix for AU in (4) is:

$$E(AUU'A') = \sigma^2 \begin{bmatrix} 3 & & & & 0 \\ & 3 & & & \\ & & \ddots & & \\ & & & 3 & \\ & & & & \ddots & \\ 0 & & & & & 3 \end{bmatrix} \qquad (6)$$

The OLS estimate of β is defined as:

$$\tilde{\beta} = (X'A'AX)^{-1} \; X'A'AY \tag{7}$$

The minimum variance linear unbiased (MVLU) estimator for β is defined as

$$\begin{aligned} \beta^* &= [X'A'(A'A)^{-1}AX]^{-1} \; X'A'(A'A)^{-1} \; AY \\ &= [X'A'AX]^{-1} \; X'A'AY = \tilde{\beta} \end{aligned} \tag{8}$$

$\beta^* = \tilde{\beta}$ is essentially due to the fact that A'A is a diagonal matrix.

This result indicates that the ordinary least squares (OLS) estimator of systematic risk is equivalent to the generalized least squares (GLS) estimator. Hence the OLS estimator is a MVLU estimator.

To derive the relationship between the R^2 in terms of quarterly rates of return and the R^2 in terms of monthly rates of return, first the variance of q_t is defined as:

$$\mathrm{Var}(q_t) = \beta^2 \; (1 \;\; 1 \;\; 1) \; [\Phi] \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + (1 \;\; 1 \;\; 1) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \sigma^2 \tag{9}$$

Where:

$$\Phi = \begin{pmatrix} \mathrm{Var}(X_t) & \mathrm{Cov}(X_t, X_{t-1}) & \mathrm{Cov}(X_t, X_{t-2}) \\ \mathrm{Cov}(X_t, X_{t-1}) & \mathrm{Var}(X_{t-1}) & \mathrm{Cov}(X_{t-1}, X_{t-2}) \\ \mathrm{Cov}(X_t, X_{t-2}) & \mathrm{Cov}(X_{t-1}, X_{t-2}) & \mathrm{Var}(X_{t-2}) \end{pmatrix}$$

$$= \mathrm{Var}(X_t) \begin{pmatrix} 1 & \rho_1 & \rho_2 \\ \rho_1 & 1 & \rho_1 \\ \rho_2 & \rho_1 & 1 \end{pmatrix} \tag{10}$$

Equation (10) is derived by assuming that $\mathrm{Var}(X_t) = \mathrm{Var}(X_{t-1}) = \mathrm{Var}(X_{t-2})$ and $\mathrm{Cov}(X_t, X_{t-1}) = \mathrm{Cox}(X_{t-1}, X_{t-2})$.

In addition, the variance of Y_t is defined as:

$$\mathrm{Var}(Y_t) = \beta^2 \,\mathrm{Var} X_t + \sigma^2 \tag{11}$$

Equations (9) and (11) imply that there exists a relationship between the variance of monthly data and the variance of quarterly data.

Based upon the definition of R^2, the "quarterly" and "monthly" population goodness of fit measures, R_q^2 and R_m^2 associated with equations (3) and (4) can be defined as:

$$\begin{aligned} &(A) \quad R_m^2 = \beta^2 \,\mathrm{Var}(X_t)/(\beta^2 \,\mathrm{Var}\, X_t + \sigma^2) \\ &(B) \quad R_q^2 = \frac{\beta^2 (1\ 1\ 1)\ \Phi \begin{pmatrix}1\\1\\1\end{pmatrix}}{\beta^2 (1\ 1\ 1)\ \Phi \begin{pmatrix}1\\1\\1\end{pmatrix} + 3\sigma^2} \end{aligned} \tag{12}$$

Substituting (10) into (12B), we have:

$$R_q^2 = \frac{1}{1 + \dfrac{3}{\mathrm{Var}(X_t)(3 + 4\rho_1 + 2\rho_2)} \dfrac{\sigma^2}{\beta^2}} \tag{13}$$

from the equation (12A), we have:

$$\frac{\sigma^2}{\beta^2 \,\mathrm{Var}(X_t)} = \frac{1 - R_m^2}{R_m^2} \tag{14}$$

Substituting (14) into (13), we have:

$$R_q^2 = \frac{R_m^2}{R_m^2 + k(1 - R_m^2)} \tag{15}$$

Where $k = \frac{1}{(1 + \frac{4}{3}\rho_1 + \frac{2}{3}\rho_2)}$

ρ_1 and ρ_2 are first and second order autocorrelation coefficients of monthly market rates of return. Thus, if $k < 1$, $R_q^2 > R_m^2$. For example, if $R_m^2 = .3$ and $k = 1/4$, $R_q^2 = .63$. $k < 1$ implies that the monthly market rates of return have some positive autocorrelations. Working (1949) and Schwartz and Whitcomb (1977) have explained why the market rates of return generally have positive autocorrelation.

The relationship defined in (15) can be used to explain the findings by the previous empirical studies about the relationship between R^2 and the length of the investment horizon. Equation (15) is derived under the assumption that the residuals of the market model are not autocorrelated as defined in equation (4). But Schwartz and Whitcomb's (1977) equation (12) has regarded the existence of negative autocorrelation associated with the residual terms of the market model as essential in explaining the relationship between the length of the investment horizon and the estimated R^2. Previous empirical studies related to market model have shown that the autocorrelation of residual terms is generally trivial, hence, the approach used in this study is more realistic relative to that used by Schwartz and Whitcomb (1977).

Zellner and Montimarquette (1971) have pointed out that R_m^2 is not strictly comparable with R_q^2 since the dependent variables are different. Furthermore, they have regarded the difference between R_m^2 and R_q^2 as a pure "Mathematical Effect". However, previous empirical studies in capital asset pricing have used the estimated coefficient of

determination to determine whether aggregated data or disaggregated data should be used to do empirical tests.[3]

Note that the relationships developed in this section can also be used to investigate other type of aggregation, e.g., the weekly aggregation used by Cheng and Deets (1973), Schwartz and Whitcomb (1977) and Pogue and Solnik (1974). Pogue and Solnik (1974) have shown that both the estimated R^2 and the estimated systematic risk are not independent of either daily, weekly, biweekly or monthly data being used to fit the market model for American and several European common stock markets. If we want to explain the difference between the estimated R^2 associated with daily data and that associated with weekly data, then the transformation matrix, A, of equation(s) should be defined as

$$A = \begin{bmatrix} 1\;1\;1\;1\;1 & & & & \\ & 1\;1\;1\;1\;1 & & & \\ & & \ddots & & \\ & & & 1\;1\;1\;1\;1 & \\ & & & & 1\;1\;1\;1\;1 \end{bmatrix} \qquad (16)$$

Under this circumstance, the R^2 associated with weekly data (R_w^2) can be written in terms of the R^2 associated with daily data (R_d^2) as:

$$R_w^2 = \frac{R_d^2}{R_d^2 + (k'(1 - R_d^2)} \qquad (17)$$

$$\text{Where } k' = \frac{1}{(1 + \frac{8}{5}\rho_1' + \frac{6}{5}\rho_2' + \frac{4}{5}\rho_3' + \frac{2}{5}\rho_4')}$$

ρ_i's (i = 1, 2, 3, 4) represents the autocorrelation coefficients associated with daily market index.

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[3] McDonald (1974), for example, has used the magnitude of estimated R^2's associated with both monthly and quarterly mutual data to determine whether monthly or quarterly mutual fund data is more appropriate in testing the performance of mutual funds.

In the following section, the impact of time aggregation on the systematic risk and residual variances of the market model will be analyzed.

IV. Impact of Investment Horizon on the Systematic Risk and Residual Variance[4]

Equations (4) and (5) are simple regressions without intercepts. The estimated slope associated with quarterly data can be defined as:

$$\hat{\beta}_q^2 = \hat{r}^2 \frac{\mathrm{Var}(q_t)}{\mathrm{Var}(A_t X)} = R'^2_q \frac{\mathrm{Var}(q_t)}{\mathrm{Var}(A_t X)} \qquad (18)$$

Where $\hat{r}^2$ is estimated correlation coefficient between q_t and $A_t X$.

The second equality of equation (18) is due to the fact that the R^2 of a simple regression is equal to the square of the simple correlation coefficient between the dependent and independent variables. [See Theil (1971)].

Based upon the definition of (5), the variance of q can be defined as:

$$\mathrm{Var}(q) = (1\ 1\ 1) \begin{pmatrix} \mathrm{Var}(Y_t) & \mathrm{Cor}(Y_t, Y_{t-1}) & \mathrm{Cor}(Y_t, Y_{t-2}) \\ \mathrm{Cor}(Y_t, Y_{t-1}) & \mathrm{Var}(Y_{t-1}) & \mathrm{Cor}(Y_{t-1}, Y_{t-2}) \\ \mathrm{Cor}(Y_t, Y_{t-2}) & \mathrm{Cor}(Y_{t-1}, Y_{t-2}) & \mathrm{Var}(Y_{t-2}) \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= 3\,\mathrm{Var}(Y_t)h$$

Where $h = 1 + \frac{4}{3}\tau_1 + \frac{2}{3}\tau_2$; τ_1 and τ_2 are first and second order of autocorrelation for the dependent variable.

From equations (12) and (13), we also know that:

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[4] This section is derived in accordance with a capital market model with autocorrelated residual terms as specified in the appendix.

$$\mathrm{Var}(A_t X) = 3\ \mathrm{Var}(X_t)\ (1 + \frac{4}{3}\rho_1 + \frac{2}{3}\rho_2) \qquad (20)$$

Based upon the definition indicated in equation (18), the estimated slope associated with monthly data can be defined as:

$$\beta_m^2 = R_m'^2 \frac{\mathrm{Var}(Y_t)}{\mathrm{Var}\ X_t} \qquad (21)$$

From equations (18), (19), (20), (21) and definitions of $R_q'^2$, $R_m'^2$ and c defined in the Appendix, we obtain

$$\beta_q^2 = \frac{hc}{R_m^2 + c(1 - R_m'^2)} \beta_m^2 \qquad (22)$$

Equation (22) indicates that the estimated systematic risk obtained from quarterly data ($\hat{\beta}_q$) will not be equal to the estimated systematic risk obtained from monthly data ($\hat{\beta}_m$) unless the adjustment factor, $\frac{hc}{R_m^2 + c(1 - R_m^2)}$, is equal to unity. Now, the impact of the adjustment factor on the relationship between $\hat{\beta}_m$ and $\hat{\beta}_q$ is analyzed. $\hat{\beta}_q$ will be $\gtreqless \hat{\beta}_m$ when $R_m^2 \lesseqgtr \frac{hc - c}{1 - c}$. This implies that the estimated systematic risk from the disaggregated data can either be larger, equal to or smaller than the estimated systematic risk from the aggregated data. The magnitude of R^2 associated with disaggregated data and the magnitude and sign of autocorrelation associated with dependent and independent variables in terms of disaggregated data are important factors in determining the magnitude of the adjustment factor, $\frac{hc}{R_m^2 + c(1 - R_m^2)}$.

In sum, the relationship of (22) can be used to explain why the estimated systematic risk obtained by Cheng and Deets (1973), Pogue and Solnik (1974) and others are not independent of the length of investment horizon.

If there exists a true horizon for capital asset pricing as discussed by Levy (1972), Levhari and Levy (1977) and others. The the estimated systematic risk associated with inappropriate horizon will be biased. Furthermore, the estimated residuals will also be biased unless the appropriate horizon is used in the empirical work. In other words, the results of the cumulative residual technique suggested by Fama, Fisher, Jensen and Roll (1969) in testing the adjustment of stock price to new information may well not be independent of the length of investment horizon.[5] Griliches (1972) has pointed out that the aggregated dynamic model fails to assess the dynamic relationship accurately because the results obtained are in fact a mixture of model misspecification and temporal aggregation.[6] Brenner (1977) has shown that there exist some effects of model misspecification on tests of the efficient market hypothesis. Hence, the impact of time aggregation on testing efficient market hypothesis is still an open question to be investigated.

V. Summary and Concluding Remarks

Based upon the time aggregation technique, it is shown that the change of R^2 associated with the change of the degree of data aggregation is a pure "Mathematical Effect". In addition, the relationship

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[5] Tiao and Wei (1976) have investigated the effect of temporal aggregation a dynamic relationship they have shown that the effect of aggregation transforms the relationship into a feedback system unless the independent variable is not autocorrelated.

[6] It can be shown that the systematic risk obtained for the disaggregated data are generally more efficient than those obtained for the aggregated data. See Zellner (1971, p. 337) for detail.

between the change of the magnitude of estimated systematic risk and the change of the length of investment horizon is also derived in detail. Finally, the impacts and the implications of the change of the length of the investment horizon on testing the efficient market hypothesis are also discussed. In sum, the results derived in this study have demonstrated the importance of choosing an appropriate investment horizon for testing capital asset pricing.

Following the results associated with the impact of time aggregation on the coefficient of determination and systematic risk, a further research will investigate the effect of time aggregation on testing the efficient market hypothesis and the stability of systematic risk.

Appendix

If we assume the residual U_t following a first order autoregressive scheme

$$U_t = \gamma U_{t-1} + \varepsilon_t \tag{a}$$

where $|\gamma| < 1$ and ε_t satisfies the assumptions

$$E(\varepsilon_t) = 0$$

$$E(\varepsilon_t \varepsilon_{t+s}) = \begin{cases} \sigma_\varepsilon^2 & s = 0 \\ 0 & s \neq 0 \end{cases} \quad \text{for all } t$$

Following Theil (1971, 250-56), it can be shown that

$$\sigma_u^2 = \frac{\sigma_\varepsilon^2}{1 - \gamma^2} \tag{b}$$

and

$$\mathrm{Var}(AU) = \frac{\sigma_\varepsilon^2}{1 - \gamma^2} (1\ 1\ 1) \begin{pmatrix} 1 & \gamma & \gamma^2 \\ \gamma & 1 & \gamma \\ \gamma^2 & \gamma & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{c}$$

Based upon the definition of R^2 indicated in (12), we have

$$R'^2_m = \beta^2 \mathrm{Var}(X_t)/(\beta^2 \mathrm{Var}\ X_t + \sigma_u^2) \tag{d}$$

$$R'^2_q = \frac{\beta^2 (3 + 4 \rho_1 + 2 \rho_2)}{\beta^2 (3 + 4 \rho_1 + 2 \rho_2) + \sigma_u^2 (3 + 4 \gamma + 2 \gamma^2)}$$

$$= \frac{1}{1 + \frac{\sigma_u^2}{\beta^2 \mathrm{Var}(X_t)} \frac{3 + 4\gamma + 2\gamma^2}{3 + 4 \rho_1 + 2 \rho_2}} \tag{e}$$

From equations (d) and (e), we obtain

$$R_q'^2 = \frac{R_m'^2}{R_m'^2 + c\ (1 - R_m'^2)} \qquad \text{(f)}$$

where

$$c = \frac{1 + 4/3\ \gamma + 2/3\ \gamma^2}{1 + 4/3\ \rho_1 + 2/3\ \rho_2} \qquad \text{(g)}$$

It is clear that c will reduce to k if the first order autocorrelation coefficient, γ is equal to zero. If the residual terms of the market model are negatively autocorrelated, then c will be smaller than k and $(R_q'^2 - R_m'^2)$ will be larger than $(R_q^2 - R_m^2)$.

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