TRANSACTIONS

OF

THE ACADEMY OF SCIENCE OF ST. LOUIS.

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JANUARY 1902 TO DECEMBER 1902.

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CONTENTS.

CORRECTIONS.

P. 82, line 27. — For Jackson Country, read Shannon County.
Pl. VII. — For erythrosperma, read rhodosperma.

MEMBERS.

1. PATRONS.

Harrison, Edwin........................3747 Westminster pl.

2. ACTIVE MEMBERS.

Adkins, James..........................Park and Vandeventer avs.
Alleman, Gellert*.....................Swarthmore College, Swarthmore, Pa.
Alt, Adolf................................3036 Locust st.
Andrews, William Edward*........Taylorville, Ill.

Bain, Robert Edward Mather......900 Locust st.
Bailey, Liberty Hyde*...............Ithaca, N. Y.
Baker, Carl Fuller*...................Stanford University, Calif.
Ball, David C.*..........................27 William st., New York City.
Barck, Carl...............................2715 Locust st.
Bartlett, George M....................215 Pine st.
Bauduy, J. K............................2810 Olive st.
Baumgarten, Gustav..................5227 Washington av.
Bean, Tarleton Hoffman*...........Administration bldg., World's Fair.
Becktold, William B..................212 Pine st.
Bernays, A. C...........................3623 Laclede av.
Biebinger, Frederick W..............1421 S. 11th st.
Bixby, William Keeney..............13 Portland pl.
Boeckeler, William L.................4441 Laclede av.
Bolton, Benjamin Meade............4160 McPherson av.
Boogher, John H.......................4034 Delmar boul.
Brannon, Melvin A.*..................University, N. Dak.
Brennan, Martin S.....................1414 O'Fallon st.
Brimmer, George G....................6900 Michigan av.
Brookings, Robert S..................5125 Lindell av.
Brown, Daniel S........................2212 DeKalb st.
Brown, Willi.............................3526 Pine st.
Bryson, John P.........................209 N. Garrison av.
Budgett, Sidney Payne...............1806 Locust st.
Burg, William...........................1756 Missouri av.
Burnett, E. C............................University Club.

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* Non-resident.

Busch, Adolphus........................1 Busch pl.
Busch, Aug. A..........................Busch pl.
Bush, Benjamin Franklin*..........Courtney, Mo.

Calvert, Sidney*......................State University, Columbia, Mo.
Carpenter, George O..................Russell and Compton avs.
Carter, Howard*.......................Webster Groves, Mo.
Carver, George Washington*......Tuskegee, Ala.
Chaplin, Winfield S...................3636 West Pine boul.
Chappell, W. G.........................3810 Westminster pl.
Chase, E. C..............................Oriel bldg.
Chauvenet, Louis.......................5501 Chamberlain av.
Chessin, Alexander S.................Washington University.
Chouteau, Pierre.......................912 Security bldg.
Chouteau, Mrs. Pierre..............912 Security bldg.
Compton, P. C...........................4156 Washington boul.
Comstock, T. Griswold...............3401 Washington av.
Conklin, Harry R.*....................Joplin, Mo.
Cramer, Gustav.........................c/o G. Cramer Dry Plate Co.
Crandall, George C....................4287 Olive st.
Crunden, Frederick Morgan........Public Library.
Curtis, William S......................St. Louis Law School.
Cushman, Allerton S.*...............Bryn Mawr, Pa.

Dame, James E..........................2353 Albion pl.
Dameron, Edward Caswell*........Clarksville, Mo.
Davis, H. N..............................56 Vandeventer pl.
Davis, John D...........................421 Olive st.
Diehm, Ferdinand......................1834 Kennett pl.
Dodd, Samuel M.........................415 Locust st.
Douglas, Archer W.....................c/o Simmons Hardware Co.
Drake, George S........................64 Vandeventer pl.
Duenckel, Frederick William.......2912 Ellendale av.
Duggar, B. M.*.........................202 Hitt st., Columbia, Mo.

Eggert, Henry*..........................1001 Collinsville av.,
East St. Louis, Ill.
Eimbeck, August F.*..................New Haven, Mo.
Eliot, Edward C.........................5468 Maple av.
Eliot, Henry W..........................2635 Locust st.
Engler, Edmund Arthur*............11 Boynton st., Worcester, Mass.
Engman, Martin F......................2608 Locust st.
Erker, Adolph P.........................608 Olive st.

Espenschied, Charles................3500 Washington av.
Euston, Alexander....................3730 Lindell boul.
Evers, Edward........................1861 N. Market st.
Ewing, Arthur E......................6024 West Cabanne pl.

Favor, Ernest Howard *...............Box 842, Columbia, Mo.
Fernald, Robert Heywood..........Washington University.
Fischel, Washington E...............2647 Washington av.
Forbes, Stephen A.*..................Urbana, Ill.
Fordyce, John R.*.....................2223 Louisiana st.,
Little Rock, Ark.
Forster, Marquard......................2317 S. 13th st.
Francis, David R.......................4421 Maryland av.
French, George Hazen*.............Carbondale, Ill.
Frerichs, Frederick W................4608 S. Broadway.
Frick, John Henry*.....................Warrenton, Mo.
Fruth, Otto J..............................3066 Hawthorne boul.
Fry, Frank R..............................3133 Pine st.
Funkhouser, Robert Monroe.......3534 Olive st.

Gazzam, James Breading............514 Security bldg.
Gecks, Frank...............................3453 Magnolia av.
Gerling, H. J..............................4320 Cook av.
Glasgow, Frank A.......................3894 Washington boul.
Glasgow, William C...................2847 Washington av.
Goetz, Victor...............................129 Market st.
Goldstein, Max A........................3858 Westminster pl.
Goodman, Charles H.................. 3329 Washington av.
Graham, Benjamin B...................5145 Lindell boul.
Graves, William W......................1943 N. 11th st.
Graves, Willis Nelson..................2813 Lafayette av.
Gray, Melvin L............................3756 Lindell boul.
Grebe, E.......................................3839 Russell av.
Greeley, Arthur W......................Washington University.
Green, John.................................2670 Washington av.
Gregory, Elisha Hall...................3525 Lucas av.
Gregory, Elisha H., Jr.*.............Medical Dept., Univ. of Pa.,
Philadelphia, Pa.
Grindon, Joseph.........................4546 Laclede av.
Gundelach, Chas. H...................3900 Westminster pl.
Gurney, James.............................Tower Grove and Magnolia avs.
Guy, William Evans....................4380 Westminster pl.

Haarstick, Henry C ....Main and Walnut sts.
Habermaas, Albert ....3109 S. Jefferson av.
Hambach, Gustav † ....1319 Lami st.
Hardaway, W. A .... 2922 Locust st.
Hartmann, Rudolph ....2020 Victor st.
Held, George A ....International Bank.
Henske, A. A ....1504 St. Louis av.
Herzog, William ....3644 Botanical av.
Hirschberg, Francis D ....3818 Lindell boul.
Hitchcock, Albert Spear* ....U. S. Dept. Agriculture, Washington, D. C.
Hitchcock, George C ....3877 Washington av.
Holman, M. L ....3744 Finney av.
Holmes, Joseph A ....Mines Dept. La. Purch. Exposit'n.
Holzinger, John Michael* ....207 W. King st., Winona, Minn.
Homan, George ....323 Odd Fellows' bldg.
Hough, Warwick ....Circuit Court, Room 1.
Hughes, Charles Hamilton ....3860 West Pine boul.
Huiskamp, John E ....5554 Cabanne av.
Hume, H. Harold* ....Lake City, Fla.
Hunicke, Henry August ....3532 Victor st.
Hurter, Julius ....2346 S. 10th st.
Hyatt, Robert J ....U. S. Weather Bureau.

Ives, Halsey Cooley ....Museum of Fine Arts.

Johnson, J. B ....4244 Washington boul.
Johnson, Reno De O.* ....Desloge, Mo.
Jones, Breckinridge ....4010 Lindell boul.

Keiser, Edward H ....Washington University.
Keyes, Charles R.* ....State School of Mines, Socorro, New Mexico.
Kinealy, John H.* ....1108 Pemberton bldg., Boston, Mass.
King, Goodman ....78 Vandeventer pl.
Kirchner, Walter C. G ....1211 N. Grand av.
Kline, George R ....215 Pine st.
Kodis, Theodore* ....Schadow, Kowno, Russia.
Krall, George Warren ....Manual Training School.

† Elected a life-member January 3, 1882.

Lackland, Rufus J....................1623 Locust st.
Langsdorf, Alexander S.............Washington University.
Leavitt, Sherman......................Washington University.
Lefevre, George*......................State University, Columbia, Mo.
Leighton, George Bridge............803 Garrison av.
Letterman, George W.*.............Allenton, Mo.
Lichter, John J.........................5305 Virginia av.
Loeb, Hanau Wolf....................3559 Olive st.
Ludwig, Charles V. F................1509 Chouteau av.
Lumelius, J. George..................1225 St. Ange av.
Lyon, Hartwell Nelles................3910 Russell av.

Mack, Charles Jacob........ .. ...113 N. Broadway.
Mallinckrodt, Edward...............26 Vandeventer pl.
Markham, George Dickson.........4961 Berlin av.
Marx, Christian William*..........University of Cincinnati, Cincinnati, Ohio.
Maserang, Joseph, Jr................Washington and Leffingwell avs.
Mason, Silas C.*.......................Berea, Ky.
Matthews, Leonard.............. .....300 N. 4th st.
Meier, Theodore G........ ....3938 Washington boul.
Merrell, Albert.........................3814 Washington boul.
Meyer, John F...........................1739 Pennsylvania av.
Michel, Eugene H......................2721 S. King's Highway.
Miller, Charles F.......................1751 Missouri av.
Monell, Joseph T.*....................Flat River, Mo.
Monroe, Lee Ernest*.................Eureka, Mo.
Moore, Robert...........................61 Vandeventer pl.
Morton, Isaac W.......................c/o Simmons Hardware Co.
Mudd, Harvey G.......................2604 Locust st.
Muegge, Aug. H........................Grand av. and Hickory st.
Mueller, Ambrose*....................Webster Groves, Mo.

Nagel, Charles...........................3969 Washington boul.
Nasse, Aug........................ .....209 N. 2d st.
Nelson, Aven*...........................Laramie, Wyom.
Niedringhaus, George W............3745 Lindell boul.
Nipher, Francis E......................Washington University.
Norton, J. B. S.*.......................College Park, Md.

Oglevee, Christopher Stoner*......Lincoln, Ill.
Olshausen, Ernest P...................1115 Rutger st.
Olshausen, George R.*...............Armour Institute, Chicago, Ill.

O'Reilly, Andrew J.......... ....326 City Hall.
O'Reilly, Robert J......................3411 Pine st.
Outten, W. B............................Mo. Pacific Hospital.
Overstolz, Herman.....................100 N. Broadway.

Palmer, Ernest Jesse*...............321 S. Allen st., Webb City, Mo.
Pammel, Louis Hermann*.........Ames, Ia.
Pantaleoni, Guido......................415 Locust st.
Parker, George Ward.................417 Pine st.
Parsons, Charles........ ....2804 Pine st.
Pauls, Gustavus..........................St. Louis Altenheim.
Pettus, W. H. H.........................4373 Westminster pl.
Pfeiffer, Egmont.........................4247 Castleman av.
Pike, Sherman B.........................5881 Cates av.
Pitzman, Julius..........................1900 S. Compton av.
Poats, Thomas Grayson*............Clemson College, S. C.
Post, Martin Hayward...............5371 Waterman av.
Preetorius, Emil.........................c/o Westliche Post.
Prewitt, Theodore F...................4917 Berlin av.
Primm, Alexander T., Jr............c/o J. Kennard & Sons.
Pulsifer, William H.*.................The Grafton, Washington, D. C.

Quaintance, A. L.*....................Experiment, Ga.

Randall, John E.........................1910 Olive st.
Ravold, Amand...........................2806 Morgan st.
Reverchon, Julien*.....................Box 229, Dallas, Texas.
Richter, Phil. George.................2424 S. 18th st.
Rieloff, F. C..............................3837 W. Pine boul.
Robert, Edward Scott................1105 Union Trust bldg.
Robertson, Charles*...................Carlinville, Ill.
Roever, William Henry*.............Cambridge, Mass.
Rogers, Herbert F.......................c/o Provident Chemical Works.
Rolfs, Peter H.*.........................Tropical Laboratory, Miami, Fla.
Rosenwald, Lucian*...................Las Vegas, New Mex.
Ruf, Frank A..............................5863 Cabanne av.
Runge, Edward C.......................Supt. Insane Asylum.
Russell, Colton*......,.................325 S. Bunker Hill av.,
Los Angeles, Calif.
Ryan, Frank K... ....2725 Lawton av.

Sander, Enno.............................2807 Lawton av.
Sargent, Charles Sprague*..........Jamaica Plain, Mass.

Schmalz, Leopold......................2824 Shenandoah av.
Schneck, Jacob*.......................Mt. Carmel, Ill.
von Schrenk, Hermann..............Mo. Botanical Garden.
Schroers, John..........................1730 Missouri av.
Schrowang, Otto.......................Holland bldg.
Schwab, Sidney I......................2602 Locust st.
Schwarz, Frank.........................1520 Lafayette av.
Schwarz, Henry.........................1723 Chouteau av.
Schweitzer, Paul*.....................Columbia, Mo.
Scott, Henry C..........................64 Vandeventer pl.
See, Thomas Jefferson Jackson*...Naval Observatory, Washington, D. C.
Selby, Augustine Dawson*.........Wooster, O.
Senseney, E. M.........................2829 Washington av.
Sheldon, Walter L.....................4065 Delmar av.
Shepley, John F........................60 Vandeventer pl.
Shoemaker, William Alfred.........4383 Westminster pl.
Simmons, E. C..........................9th and Spruce sts.
Simmons, W. D..........................9th and Spruce sts.
Sluder, Greenfield.....................2647 Washington av.
Smith, Arthur George*..............422 N. Dubuque st., Iowa City, Ia.
Smith, D. S. H..........................3646 Washington boul.
Smith, Irwin Z..........................87 Vandeventer pl.
Smith, Jared G.*.......................Honolulu, Hawaiian Islands.
Soldan, F. Louis.......................3634 Flad av.
Spiegelhalter, Joseph................2166 Lafayette av.
Starr, John E.*.........................258 Broadway, New York City.
Staudinger, B...........................3556 Lindell boul.
Stedman, John Moore*..............State University, Columbia, Mo.
Stevens, Charles D....................1749 S. Grand av.
Stevens, Wyandotte James.........4043 Juniata st.
Stocker, George J.....................2833 S. King's Highway.
Stone, Charles H.......................5562 Clemens av.
Strauss, Julius C.......................3516 Franklin av.
Stuart, James Lyall....................5346 Maple av.
Sutter, Otto..............................3035 Bell av.

Taussig, Albert E......................2647 Washington av.
Taussig, William.......................3447 Lafayette av.
Teichmann, William C...............1141 Market st.
Terry, Robert James................. 2726 Washington av.
Thacher, Arthur........................4304 Washington boul.
Thiele, Albert...........................2746 Park av.

Thilly, Frank*..........................601 Hitt st., Columbia, Mo.
Thom, Charles......... ....239 Hazel st., Ithaca, N. Y.
Thomas, John R...................... ..420 N. 4th st.
Thomson, Wm. H., Jr...............3805 Lindell boul.
Thurman, John S.......... ....416 Lincoln Trust bldg.
Timmerman, Arthur H...............2633 Park av.
Tittmann, Harold H........... .....3726 Washington boul.
Trelease, William. ....Mo. Botanical Garden.
Tyler, Elza Edward*..................State University, Columbia, Mo.
Tyrrell, Warren Ayres..............3620a Folsom av.

Updegraff, Milton*....................12 Goldsborough Row, Annapolis, Md.

Vallé, Jules F..........................3303 Washington av.
VanOrnum, John Lane..............Washington University.
Vickroy, Wilhelm Rees..... ....2901 Rauschenbach av.
von Schrader, George F....... .....Wainwright bldg.
von Schrader, Otto U...............3749 Westminster pl.

Warren, William Homer ....1806 Locust st.
Watts, Millard F............... ....4362 Morgan st.
Weller, Stuart*...... ....University of Chicago, Chicago, Ill.
Westgate, John Minton*.............6023 Ellis av., Chicago, Ill.
Wheeler, H. A...........................3124 Locust st.
Whelpley, Henry Milton............2342 Albion pl.
Whitaker, Edwards....................300 N. 4th st.
Whitten, John Charles*..............Columbia, Mo.
Widmann, Otto.......... ....5105 Morgan st.
Wilson, Edward Allen...............3745 W. Pine st.
Winkelmeyer, Christopher...........3540 Lawton av.
Winslow, Arthur*......................104 W. 9th st., Kansas City, Mo.
Wislizenus, Frederick A.............3628 Cleveland av.
Witt, Thomas D.*.............. .....Rushville, Ill.
Wood, Obadiah M......................3016 Caroline st.
Woodward, Calvin Milton...........Washington University.

Zahorsky, John.........................1460 S. Grand av.

THE ACADEMY OF SCIENCE OF ST. LOUIS.

ORGANIZATION.

The Academy of Science of St. Louis was organized on the 10th of March, 1856, in the hall of the Board of Public Schools. Dr. George Engelmann was the first President.

CHARTER.

On the 17th of January following, a charter incorporating the Academy was signed and approved, and this was accepted by vote of the Academy on the 9th of February, 1857.

OBJECTS.

The act of incorporation declares the object of the Academy to be the advancement of science and the establishment in St. Louis of a museum and library for the illustration and study of its various branches, and provides that the members shall acquire no individual property in the real estate, cabinets, library, or other of its effects, their interests being unsufructuary merely.

The constitution as adopted at the organization meeting and amended at various times subsequently, provides for holding meetings for the consideration and discussion of scientific subjects; taking measures to procure original papers upon such subjects; the publication of transactions; the establishment and maintenance of a cabinet of objects illustrative of the several departments of science, and a library of works relating to the same; and the establishment of relations with other scientific institutions. To encourage and promote special investigation in any branch of science, the formation of special sections under the charter is provided for.

MEMBERSHIP.

Members are classified as active members, corresponding members, honorary members and patrons. Active membership is limited to persons interested in science, though they need not of necessity be engaged in scientific work, and they alone conduct the affairs of the Academy, under its constitution. Persons not living in the city or county of St. Louis who are disposed to further the objects of the Academy by original researches, contributions of specimens, or otherwise, are eligible as corresponding members. Persons not living in the city or county of St. Louis are eligible as honorary members by virtue of their attainments in science. Any person conveying to the Academy the sum of one thousand dollars or its equivalent becomes eligible as a patron.

Under the by-laws, resident active members pay an initiation fee of five dollars and annual dues of six dollars. Non-resident active members pay the same initiation fee, but annual dues of three dollars only. Patrons, and honorary and corresponding members, are exempt from the payment of dues. Each patron and active member not in arrears is entitled to one copy of each publication of the Academy issued after his election.

Since the organization of the Academy, 945 persons have been elected to active membership, of whom, at the present time, 293 are carried on the list. One patron, Mr. Edwin Harrison, has been elected. The list of corresponding members (Vol. X. p. xii) includes 205 names, among which are the names of 102 persons known to be deceased.

OFFICERS AND MANAGEMENT.

The officers, who are chosen from the active members, consist of a President, two Vice-Presidents, Recording and Corresponding Secretaries, Treasurer, Librarian, three Curators, and two Directors. The general business management of the Academy is vested in a Council composed of the President, the two Vice-Presidents, the Recording Secretary, the Treasurer, the Librarian and the two Directors.

The office of President has been filled by the following well-known citizens of St. Louis, nearly all of whom have been eminent in some line of scientific work: George Engelmann, Benjamin F. Shumard, Adolphus Wislizenus, Hiram A. Prout, John B. Johnson, James B. Eads, William T. Harris, Charles V. Riley, Francis E. Nipher, Henry S. Pritchett, John Green, Melvin L. Gray, Edmund A. Engler, Robert Moore, and Henry W. Eliot.

MEETINGS.

The regular meetings of the Academy are held at its rooms, 1600 Locust Street, at 8 o'clock, on the first and third Monday evenings of each month, a recess being taken between the meeting on the first Monday in June and the meeting on the third Monday in October. These meetings, to which interested persons are always welcome, are devoted in part to the reading of technical papers designed for publication in the Academy's Transactions, and in part to the presentation of more popular abstracts of recent investigation or progress. From time to time public lectures, calculated to interest a larger audience, are provided for in some suitable hall.

The following dates for regular meetings for the year 1903 have been fixed by the Council:—

Jan.	Feb.	Mar.	April.	May.	June.	Oct.	Nov.	Dec.
5	2	2	6	4	1		2	7
19	16	16	20	18		19	16	21

LIBRARY.

After its organization, the Academy met in Pope's Medical College, where a creditable beginning had been made toward the formation of a museum and library, until May, 1869, when the building and museum were destroyed by fire, the library being saved. The library now contains 14,491 books and 11,017 pamphlets, and is open during certain hours of the day for consultation by members and persons engaged in scientific work.

PUBLICATIONS AND EXCHANGES.

Twelve thick octavo volumes of Transactions have been published since the organization of the Academy, and widely distributed. Two quarto publications have also been issued, one from the Archaeological section, being a contribution to the archaeology of Missouri, and the other a report of the observations made by the Washington University Eclipse Party of 1889. The Academy now stands in exchange relations with 569 institutions or organizations of aims similar to its own.

MUSEUM.

Since the loss of its first museum, in 1869, the Academy has lacked adequate room for the arrangement of a public museum, and, although small museum accessions have been received and cared for, its main effort of necessity has been concentrated on the holding of meetings, the formation of a library, the publication of worthy scientific matter, and the maintenance of relations with other scientific bodies.

December 31, 1902.

RECORD.

From January 1, 1902, to December 31, 1902.

Meeting of January 6, 1902.

Vice-President Smith in the chair; about forty persons present.

The Council reported that the Academy had lost two members by the death of Judge George A. Madill and Mr. William McMillan, that the resignation of Messrs. C. T. Whittier, J. A. Conzelman and Ludwig Bremer had been accepted, and that the names of Messrs. A. I. Jacobs, E. W. Lazell, J. W. Lee and J. A. Seddon had been removed from the list of members.

The nominating committee reported that one hundred and twenty-nine ballots had been counted, and the following officers for 1902 were declared duly elected: —

President	Henry W. Eliot.
First Vice-President	D. S. H. Smith.
Second Vice-President	William E. Guy.
Recording Secretary	William Trelease.
Corresponding Secretary	Ernest P. Olshausen.
Treasurer	Enno Sander.
Librarian	G. Hambach.
Curators	G. Hambach,
	Julius Hurter,
	Hermann von Schrenk.
Directors	Amand Ravold,
	Adolf Alt.

Mr. Eliot, the President-elect, on taking the chair, spoke happily of his interest in the work of the Academy and his hope to further it to the extent of his power.

The Librarian submitted his annual report.

The Treasurer submitted his annual report* showing in-

* Transactions **11**: xl.

vested funds to the amount of \$6,500.00, and a balance of \$555.25 carried forward to the year 1902.*

A letter from Mrs. William Bouton was read, offering on behalf of herself and other contributors to a purchase fund, to present to the Academy a collection of 633 butterflies mounted on Denton tablets, and the containing cases, on condition that the collection be held as an educational exhibit for the benefit of the public, and in all feasible ways kept open to public inspection, and the following resolution was adopted: —

Resolved, That The Academy of Science of St. Louis gratefully accepts the gift as proposed by Mrs. Bouton, and agrees to hold the collection as an educational exhibit for the benefit of the public, and in all feasible ways to keep it open to public inspection.

The following papers were presented by title: —

K. K. Mackenzie and B. F. Bush, new species of plants from Missouri.

B. F. Bush, Revision of the North American species of Triodia.

Professor A. S. Chessin exhibited a gyroscope and explained how an accurately constructed and rapidly rotated gyroscope might be made to indicate the position of the meridian plane, the direction of the polar axis of the earth and the latitude of the place of observation, thus serving the purpose of the mariner's compass, but more accurately because of the fact that the compass indicates the magnetic and not the true pole. The following formulae pertaining to the subject were furnished: —

$$T = \pi \sqrt{\frac{A + C_1 + A_2}{C\,\omega\,\Omega \cos\lambda}}, \quad T_1 = \pi \sqrt{\frac{A + C_1 + A_2}{C\,\omega\,\Omega}}$$

where T and T_1 are the durations of a complete oscillation of the gyroscope when its axis is made to remain in the horizontal and the meridian planes respectively; ω and Ω the angular velocities of rotation of the earth and the gyroscope respectively; A, A_1, A_2 and C, C_1, C_2 the equatorial and the axial moments of inertia of the gyroscope and the two rings on which it is mounted. From these formulae the latitude (λ) of the place of observation is derived, namely: $\cos \lambda = \frac{T_1^2}{T^2}$.

Professor F. E. Nipher made a furthur statement concerning his results in the attempt to produce ether waves by the explosion of dynamite. He had obtained some results which seemed to show that magnetic effects could thus be produced. "There is apparently no doubt that great solar outbursts like the one which Professor C. A. Young saw at Sherman in 1872,* produce enormous distortions of the ether. Why should it not be possible to reproduce this result? It goes without saying that large sun-spots may be slowly formed, without such ether disturbance, and certainly we can hardly expect to reproduce solar velocities. But terrestrial explosions do yield tremors and sound vibrations, and these lead to great experimental difficulties. The nickel-silver coherer can be operated by the sound waves from a tuning fork. The coherer can be either opened or closed by sound waves, when the coherer is properly placed in a magnetic field. The same result may be produced by changes in the magnetic field, due to the slow approach of a horse-shoe magnet. After the coherer circuit has been closed by a spark, the slow approach of a horse-shoe magnet will often open the circuit, precisely as it does when the coherer has been closed by the magnet held in a position of reversed polarity. When the magnet fails to open the coherer circuit, the cause is either a too rapid approach, which causes the coherer to close by a reversal of magnetic polarity, or by a wrong presentation of the magnet which confirms the condition produced by the spark discharge. The conditions under which experiments are made as yet, with the jarring due to the street traffic and the explosions, and the changing magnetic field due to the electric cars, have proven to be a source of some perplexity. It throws some doubt upon the results reached. There, however, seems to be a residual effect which cannot thus be accounted for, and it may be due to an ether displacement."

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* The Sun, p. 156.

Meeting of January 20, 1902.

President Eliot in the chair, nineteen persons present.

Dr. George Richter addressed the Academy on the physical and chemical properties of gelatin, which he described as a spongy substance differing materially from other solids. The manner of manufacture of gelatin and its chemical and physical characters were described in detail, and considerable attention was given to the rate of absorption and evaporation of water by gelatin and the phenomenon of its apparent solution in water. A new hygrometer was exhibited and described, the action of which was based upon the water absorption of gelatin.

One person was proposed for active membership.

Meeting of February 3, 1902.

President Eliot in the chair, nineteen persons present.

Mr. Trelease presented, with the aid of lantern illustrations, some of the principal results of his recent studies of Yuccas and their allies.

One person was proposed for active membership.

Meeting of February 17, 1902.

President Eliot in the chair, twenty-nine persons present.

Dr. Gellert Alleman addressed the Academy on the chemical constitution and the manufacture of Portland cements. The growth of the cement industry was treated, the various steps of development being shown by lantern slides illustrating past and present types of machinery employed in its manufacture. Several slides were shown giving tabulated results of a number of analyses of different commercial Portland cements.

Mr. Charles Espenschied read a letter from Mr. Seymour Carter, of Hastings, Minnesota, in which was described a method of Professor Anderson of Columbia University, New

York, by which it was claimed that cereals could be directly transformed to food-stuffs. The process consists of inclosing the cereal to be treated in a hermetically sealed vessel and subjecting it to a temperature of about 450° F. for a certain time, and immediately thereafter opening the vessel, when it is found that the grains expand to six or eight times their normal size. The inventor claims that the process does not alter the composition of the cereal. Samples of several cereals treated in this manner were shown.

A written motion to amend Section 1 of Article V. of the Constitution by the addition of the words "and the Librarian" near the end of said article was submitted by Dr. Alt, seconded by Dr. Smith. Under the Constitution, action on this was deferred until the second following meeting of the Academy.

Messrs. Wm. L. Boeckeler and John F. Meyer, of St. Louis, were elected to active membership.

Two persons were proposed for active membership.

Meeting of March 3, 1902.

President Eliot in the chair, twenty-eight persons present.

The Council reported that the resignation of Professor H. F. Roberts had been accepted.

Mr. L. T. Genung gave a general discussion of the Lepidoptera, their structural characteristics, habits, and adaptations. He exhibited some of the more striking specimens of the Denton collection of butterflies, recently presented to The Academy of Science, and discussed the meaning of the various colors.

A paper by Mr. C. F. Baker, entitled A revision of the Elephantopeae, I., was presented by title.

Mr. Willi Brown and Dr. F. C. Rieloff were elected to active membership.

Two persons were proposed for active membership.

Meeting of March 17, 1902.

President Eliot in the chair, sixteen persons present.

A letter from Mr. W. W. Robinet, of Robinet, Tennessee, was read by the President, in which the writer presented to the Academy a small collection of fossils.

Dr. E. R. Buckley addressed the Academy on the work being done by the State Bureau of Geology and Mines, giving a brief review of the work done by the Bureau in the past, since its creation in 1839, and an outline of the plans for the future.

Meeting of April 7, 1902.

President Eliot in the chair, twenty-eight persons present.

Professor A. S. Langsdorf addressed the Academy on the subject of electric waves, illustrating his remarks by experiments, including some of the phenomena of self-induction, absorption, reflection, and resonance.

Dr. H. von Schrenk exhibited a sample of the impregnated wooden paving blocks used on some of the streets of London and Paris.

Mr. Frank Schwarz and Professor Frank Gecks were elected to active membership.

Meeting of April 21, 1902.

President Eliot in the chair, nine persons present.

The Council reported the resignation of Dr. C. M. Jackson and Dr. E. W. Oelfcken.

The Council reported that a ballot on the amendment of the Constitution had been canvassed, and that one hundred and one ballots had been cast; 97 for and 4 against the adoption of the amendment, so that the amendment had been carried, Article V, Section I, of the Constitution, as amended, now reading: —

Section 1. The President, the two Vice-Presidents, the Recording Secretary, the Treasurer, the Librarian, and the two Directors, shall constitute the Council of the Academy.

Mr. Arthur Thacher delivered an interesting address on the present and the probable future of the Missouri mining industry, with particular reference to lead.

One person was proposed for active membership.

Meeting of May 5, 1902.

President Eliot in the chair. There being no quorum present, the Academy adjourned.

Meeting of May 19, 1902.

President Eliot in the chair, fifteen persons present.

The Council reported that exchange relations had been established with the University of Montana, the University of Colorado and the State Historical Society of Columbia, Mo.

Professor C. M. Woodward gave, in abstract, the results reached by him in a study of the stresses in a rotating disk.

Mr. Egmont Pfeifer, of St. Louis, was elected to active membership.

Four persons were proposed for active membership.

Meeting of June 2, 1902.

Dr. John Green in the chair, seventeen persons present.

Professor A. S. Langsdorf described the factory tests that are made of electrical machinery, illustrating the subject by lantern diagrams, showing the circuits employed for the various tests, and by pictures of the machinery as set up for testing in the factory.

Mr. H. A. Wheeler spoke of the occurrence, near Hematite, Mo., some forty miles below St. Louis, of a number of granite boulders, some of them showing the polishing action of ice; and accounted for their occurrence at this point, or some fifty miles beyond the southern limit of the terminal moraine, by the theory that they had been carried there on cakes of ice during the Loess period.

Mr. Wheeler and Professor Nipher discussed a recent newspaper account of the alleged finding of a meteorite that was recently seen to fall in St. Louis, and agreed that the supposed meteorite, which both of them had examined, was merely a pyrite concretion from the coal measures, of the type called "sulphur-balls" or "nigger-heads," which had probably been raked out from the grate-bars of the adjoining factory and passed off on its discoverer as a meteorite.

Messrs. Tarleton H. Bean, Jos. A. Holmes, Geo. Richter, and Edward Allen Wilson of St. Louis, were elected to active membership.

The following biographical sketch of the late Dr. A. Litton, one of the first members of the Academy, by Dr. G. C. Broadhead, was presented by Dr. Hambach: —

ABRAM LITTON, M. D.

Dr. Abram Litton, the son of Joseph and Kate Warren Litton, was born in Dublin, Ireland, May 20, 1814, being next to the youngest of nine chil-n.

He died in St. Louis on September 22, 1901, at 2:45 P. M. aged 87 years, 4 months and 2 days, and in the house he had built 53 years before and in which he had lived ever since. His health had been very feeble for two years.

The father and mother of Abram Litton came to America about 1817. The family resided for awhile at Pittsburg, then came to Nashville, Tenn., where the older Litton and his wife resided during the remainder of their lives. Abram was educated in a private school in Nashville and for part of the time attended a school kept by a Mrs. Sterns. He then lived with his older sister, Mrs. Margaret Bostick at Franklin, Tenn., where he attended a school taught by Bishop Otey. In 1829 he entered the Junior Class of the University of Nashville, graduating in 1831, at the age of 17. He then studied another year with Dr. Lindsay. He then went to Paris, Tenn., and taught school. Then he taught two years at Jackson, Tenn.

In 1835, four years after graduation, he was offered the professorship of Mathematics and Natural Philosophy in Nashville University. This he accepted, remaining there three years. He then resigned his position in order to go to Europe and perfect himself in chemistry. He stayed a few weeks in London, then went to Paris, in July, 1840, and attended chemical lectures for about a year, but the laboratory not suiting, he decided to go to Germany.

He walked through Switzerland to Heidelberg but stayed there only a short time. He then went down the Rhine to Bonn, and six months after leaving Paris he settled down at Giessen, to work with Liebig, with whom he stayed six months. From Giessen he went to Berlin to work with Rosa in his Laboratory, Liebig having given him letters of recommendation. After six months he took a vacation in Switzerland. He then studied about a year with Wöhler in Göttingen.

He returned to America and married Julia Alice Manning in Nashville. He soon after came to St. Louis and in 1842 took the position as Professor of Chemistry in St. Louis Medical College. Here he procured his honorary degree of M. D. The first year he taught in the school with no salary, the

Respectfully
Yours &c
A. Litton

next year a small salary, then increased to $1,000. Here he continued to lecture for 49 years. One summer he lectured at Kemper College near St. Louis. He also gave a course of lectures at the Sacred Heart Convent. In 1849 he went to the State University at Columbia for a summer session, but being tendered a position as chemist to Belchers' Sugar Refinery, he returned to St. Louis. When Washington University was formed Dr. Litton accepted the Chair of Chemistry, without salary for the first year, then a small salary for part of his time so that he could continue his other work in the St. Louis Medical College.

During the first summer vacation he went East for the purpose of examining eastern laboratories so as the better to direct the building and fitting up of the University laboratories. To do this he borrowed the money, paying it back from his next year's salary.

He continued at Washington University for 35 years, resigning in 1891. During all this time he lectured both at Washington University and at the Medical school.

About 1850 he was for a while engaged in the Geological Survey of Wisconsin and Minnesota conducted by David Dale Owen.

In this connection, I would say that David Dale Owen had charge of the Survey and with him as principal assistant was Dr. Joseph G. Norwood. Other heads of sub-corps were J. Evans, B. F. Shumard, B. C. Macy, C. Whittlesey, A. Litton, and Richard Owen: other assistants were G. Warren, H. Pratten, F. B. Meek and J. Beal. Dr. Abram Litton was the last one left of these pioneer geologists.

In 1854 Dr. Litton, in the employ of the Missouri State Geological Survey, made an examination of the lead region of Southeast Missouri, which was published in the Geological Survey Report for 1855, occupying 94 pages of the volume. This was the first careful report made of that region, and was a very complete and carefully made report.

In his early life Dr. Litton's desire was to become a doctor and not a professor. He studied medicine while teaching in Paris, Tenn., and afterwards took a Doctor's degree. He practiced for six months in Potosi and then concluded that it was not his vocation, and gave it up.

In 1871 he went to Europe for instruments and self-improvement.

Dr. Litton was always interested in microscopy and physics and his amusement was to work with the spectroscope, the barometer, electric battery, etc. He accumulated a large and fine chemical and scientific library which, in 1899, he gave to the Missouri State University. This library included from 1,000 to 2,000 volumes, some of the volumes being very rare and expensive. They were in the English, French and German languages.

He gave a fine collection of rocks to the St. Louis High School, an institution which he was very much interested in, and he served one year as Superintendent of St. Louis Public Schools.

He had two sons, one a prominent lawyer, Joseph Norwood Litton, the other a good physician, Charles Manning Litton. Both died of consumption at the age of 33, and within three years of each other. He left one daughter, Alice M., his sole heir and executrix of his will and estate.

Dr. Litton was twice married, his second wife being related to the first. His second wife was cousin to the first, and James Manning, the first husband of the second Mrs. Litton, was the brother of the first Mrs. Litton.

Dr. Litton left some valuable scientific apparatus, worth probably several thousand dollars. His daughter has since donated this to the Missouri State University. The Faculty of the University passed appropriate resolutions on the death of Dr. Litton, referring to his scientific ability and his value as a teacher, and extending sympathy to his daughter in her bereavement.

At a meeting of the Alumni of Washington University, the St. Louis Medical School and other institutions of St. Louis, held in Memorial Hall, St. Louis, at Nineteenth and Locust streets, April 19th, 1895, special reference was made to Dr. Litton by Dr. Henry H. Mudd, from which I extract the following appropriate passages. This was six years before the death of Dr. Litton.

Dr. Mudd says: "Dr. Abram Litton is still with us — simple-minded as a child, but stern as ever in his unflinching demand for the truth. On May 15, 1843, he was appointed Professor of Chemistry and Pharmacy in the Medical Department of the St. Louis University. This was afterwards known as Pope's but now is the Medical Department of Washington University. His salary was $300, later increased to $600 and finally to $1,000. Besides this and work on the Missouri Geological Survey, he was also chemist to the Belchers' Sugar Refinery. He attended to this for a while, but concluding that they were paying him too much he resigned it. He was so modest that his merits were not known so well at home as abroad, and when Judge Treat asked Professor Horsford of Harvard for a chemist for Washington University, he said, 'Why not Litton of St. Louis?' In 1857 he was appointed to the Professorship in Washington University, which he held until 1892, for thirty-five years. Later, when he traveled in the East in the interest of the University, he refunded the money which had been advanced to him to pay the expenses incurred. Where is there another who would show such unselfishness." Dr. Mudd further says: "He infused into his teachings such a demand for accurate thinking and precise work as to call forth the best efforts of thoughtful students. A student of nature, he has sought nature's truths in the crucible, found the story of the sun in the spectroscope and wrung from the stars the mystery of the night. His whole life has been laboriously given to the accumulation of hard facts. Each one was stored away for future use, and became a part of the man. Fact upon fact until truth shines forth from every day in the mosaic of his life. There is not a black spot in the whole pyramid of truth which was thus erected from the daily labor of a long and industrious life. He was loyal to his friends, loyal to his own ideals, loyal to his trusts, loyal to every purpose which he consented to serve." Such are the deserved tributes offered by Dr. Mudd to the noble character of Dr. A. Litton.

Dr. Litton delivered many useful and instructive lectures, but the published list is small. The following I give: —

1. An Introductory Lecture to the course of Chemistry and Pharmacy in the St. Louis University — by A. Litton, Professor of Chemistry and Pharmacy, 1844.

2. Address to the graduates of the Medical Department of St. Louis University, 18 pages, St. Louis, 1851.

3. Belcher and Bro. Artesian Well, 7 pages and plate. Transactions of St. Louis Academy of Science, 1857.

4. Preliminary Report of some of the principal mines in Franklin, Jefferson, Washington, St. François and Madison Counties, Missouri, by A. Litton, chemist, 94 pages, included in Part II. of 2d Geological Report by G. C. Swallow, State Geologist, 1855.

Meeting of October 20, 1902.

President Eliot in the chair, seventeen persons present.

On behalf of A. S. Horwitz, J. J. Singer and G. L. Rosenberg, the President presented to the museum of the Academy a collection of fossil leaf prints from the Green River formation at Florissant, Colo., for which the thanks of the Academy were ordered extended.

A paper by Professor A. S. Chessin, On some relations between Bessel functions of the first and of the second kind, was presented by title and referred to the Council.

Professor Trelease exhibited photographs showing the variations in the ring or collar of *Lepiota naucinoides* and a series of lantern slides illustrating autumnal coloring of foliage.

Five persons were proposed for active membership.

Meeting of November 3, 1902.

President Eliot in the chair, eighteen persons present.

The Council reported that exchange relations had been established with the Institut Botanique, Bucarest, Musées Tcheque-Slavs, Caslav, and Botanisches Centralblatt, Leiden.

Mr. G. G. Hedgcock gave an illustrated account of the sugar beet industry in the United States and some of the difficulties attending it, tracing the development of the industry and reviewing some of the field and factory obstacles that it had been necessary to overcome, and speaking particularly of the fungous diseases of the crop.

Messrs. B. M. Duggar, of Columbia, Mo., August Eimbeck, of New Haven, Mo., and Charles H. Gundelach, A. A. Henske and Frank Ryan, of St. Louis, were elected to active membership.

One person was proposed for active membership.

Meeting of November 17, 1902.

President Eliot in the chair, eleven persons present.

Dr. M. A. Goldstein delivered an address on the uses of the tuning fork as a means of medical diagnosis.

Dr. George J. Engelmann gave a brief but interesting account of the history of the Western Academy of Sciences, organized by Dr. George Engelmann and Dr. Wislizenus in 1836, and tendered to the Academy the record book showing the proceedings of that organization from its beginning until it ceased to exist. He also tendered for such uses as the Academy might elect, a skeleton specimen which was prepared by Dr. Wislizenus and which formerly was the property of the Western Academy of Sciences.

Professor A. W. Greeley, of St. Louis, was elected to active membership.

Meeting of December 1, 1902.

President Eliot in the chair, twenty-five persons present.

The Council reported the resignation of Messrs. B. C. Adkins, G. F. Durant and A. Q. Kennett.

Dr. Adolf Alt delivered an address on the development of the eye, illustrated with colored drawings and stereopticon views made from microscopic sections prepared and photographed by him.

Two persons were proposed for active membership.

In accordance with the by-laws of the Academy, a committee — which consisted of Messrs. Evers, Hunicke and Langsdorf — was elected to nominate officers for the year 1903.

Meeting of December 15, 1902.

President Eliot in the chair, twenty-three persons present.

The nominating committee reported the following list of candidates for the year 1903: —

President	Henry W. Eliot.
First Vice-President	D. S. H. Smith.
Second Vice-President	Wm. K. Bixby.
Recording Secretary	William Trelease.
Corresponding Secretary	Ernest P. Olshausen.
Treasurer	Enno Sander.
Librarian	G. Hambach.
Curators	G. Hambach,
	Julius Hurter,
	A. H. Timmerman.
Directors	F. E. Nipher,
	Adolf Alt.

No nominations other than those by the committee were offered.

A paper by C. F. Baker, entitled A revision of American Siphonaptera, together with a complete list and bibliography of the whole group, was presented by title and referred to the Council.

Dr. C. B. Curtis delivered an illustrated address on color photography, outlining the theory of color vision and the various ways in which a given color sensation may be produced, and describing various processes by which the natural colors of objects can be reproduced by photographic means.

Messrs. Robert H. Fernald and Frank A. Ruf, of St. Louis, were elected to active membership.

Two persons were proposed for active membership.

Reports of Officers for the Year 1902. Submitted January 5, 1903.

The president, Mr. Henry W. Eliot, addressed the Academy as follows: —

It is customary at this, the annual meeting of members of the St. Louis Academy of Science, for the retiring officers to give an account of their stewardship. In accordance with this custom, I have to report that, while the Academy has not made as material progress as I had hoped, we still have reason to congratulate ourselves upon the season's work. Notwithstanding losses by death and resignations, the list of active members shows a net gain of six for the year. The average attendance at meetings has been good, though somewhat less than in 1901.

The number of publications has been above the average, and their character has been equal to any in the past.

The financial condition of the Academy is good. All the notes issued in payment of the Yandell collection have been met at maturity. There remains but one, to be paid in 1904. Collections have been prompt, and the number of uncollected dues has been decreased.

The Library has been thoroughly sorted and catalogued.

I regret to report the death of one member, Hon. Henry Hitchcock, for many years associated with this society, whose loss is a grievous one and much to be deplored.

The records of the Academy tell us that its history has been similar to that of other scientific institutions. The same difficulties have been encountered and overcome, and the same obstacles present themselves to bar active progress. The work of the Academy is not as thoroughly appreciated by our fellow-citizens as it should be. We have reached a period where it is necessary to go forward, or we will inevitably go backward. The time is soon coming, if not already at hand, when we shall be obliged to seek new quarters. The pressing need of the Academy is a home of its own. The advantages of quarters under our own control are so evident as to make discussion unnecessary. We have no place for our valuable and growing library. Its uses are therefore much curtailed. We have the nucleus of an excellent museum, but it is packed in boxes and stored in various parts of the city. The beautiful collection of butterflies, presented by Mrs. Bouton a year ago, is scarcely known to our members. All of our property is exposed to the danger of loss by fire.

We need a home of our own to give us a dignified standing in this community; to compel our people to recognize fully the invaluable service rendered so freely and unselfishly to the scientific world. It should no longer be said that the St. Louis Academy of Science is better known abroad than in its own city. I therefore call upon every member of this society to take this need seriously to himself. In due season Volume XII of Transactions will be mailed to you, and will serve as excellent material for missionary work.

The following table contains details of interest: —

Academy membership at date — January 1, 1903			293
Elections for year 1902		19	
Losses for the year		13	
By death	1		
Resignations	12		
Net gain for the year		6	

The active members are classified as follows: —

Patron	1
Resident	222
Non-resident	70

Number of meetings held in 1902	15	Average attendance	21
" " " " " 1901	16	" "	28
Total attendance, 1902	314		
" " 1901	448		

Number of papers published, 1902		10
By resident members	3	
By non-resident members	7	

The average for the preceding four years is 9.75.

The character of the publications has been as follows: —

		1902.	Average for four years (1898–1901.)*
Mathematical		3 or 30%	3.50 or 36%
Botanical		7 or 70%	3.25 " 33⅓%
Zoological and Anatomical		0	1.75 " 18%
Geological and Paleontological		0	1.52 " 12⅔%
Number of authors		5	8.25
Resident	1		3.75
Non-resident	4		4.50

The treasurer reported as follows: —

RECEIPTS.

Balance from 1901	$555 25	
Interest on invested money	496 60	
Membership dues	1,466 00	
Sales of publications	79 65	
Contributions, for Yandell collection	105 00	
		$2,702 50

EXPENDITURES.

Rent	$500 00	
Publication of Transactions	787 24	
Mailing Transactions and library expense	643 57	
Sundry expenses	133 60	
Payment, account Yandell collection	280 00	
Balance to 1903	358 09	
		$2,702 50

INVESTED FUND.

Invested on security	$6,500 00

The Librarian reported that during the year exchanges had been received to the number of 327 volumes and 667 pam-

* In which annual volumes have been issued.

phlets, an increase of 68 numbers as compared with 1901. It was reported that during the year the Transactions of the Academy had been distributed to 569 societies or institutions, chiefly by way of exchange — an addition of 8 as compared with the preceding year; and attention was called to the fact that the exchange list has now grown to such proportions that only 132 reserve copies remain after the regular distribution has been effected.

Transactions of The Academy of Science of St. Louis.

VOL. XII. No. 1.

ON THE TRUE POTENTIAL OF THE FORCE OF GRAVITY.

ALEXANDER S. CHESSIN.

Issued January 29, 1902.

ON THE TRUE POTENTIAL OF THE FORCE OF GRAVITY.*

ALEXANDER S. CHESSIN.

1. Assuming that the form of the earth is that of a solid spheroid covered by an ellipsoidal liquid mass of uniform density, that the density of the spheroid varies from center to surface but is constant in each one of the concentric layers, and that the liquid mass is in a state of equilibrium, Laplace arrives at the following expression for the potential of the force of attraction exercised by the earth on an external particle of unit mass: †

$$U = \frac{M}{R}\left\{1 + \frac{\left(\frac{1}{2}a\phi - ah\right)a^2}{R^2}\left(\mu^2 - \frac{1}{3}\right)\right\},$$

where M denotes the total mass of the earth, R the distance of the particle from the center of the earth, $a\phi$ the ratio of the centrifugal force to the force of attraction on the equator, ah the oblateness of the earth, a the mean value of its radius, and, finally, $\mu = \cos\theta$, θ being the angle of the radius vector of the particle with the axis of rotation of the earth. "According to the most probable hypotheses," says Laplace,‡ "the oblateness of the spheroid is less than $\frac{5}{4}a\phi$,.... These assumptions are the more probable because they become necessary in the case when the spheroid had originally

* Presented, and read by title, before The Academy of Science of St. Louis, December 2, 1901.

† Laplace, Oeuvres complètes. t. II. Livre III. p. 103.

‡ Ibid. p. 101.

been fluid." In the case of a homogeneous ellipsoid of revolution, ah would be equal to $\frac{5}{4}a\phi$ and the expression of the potential U would become

$$\frac{M}{R}\left\{1+\frac{3}{5}\frac{a^2}{R^2}ah\left(\frac{1}{3}-\mu^2\right)\right\}.$$

In general, $ah-\frac{1}{2}a\phi$ will differ but little from $\frac{3}{5}ah$. If, then, we put

$$(1) \qquad N=-\frac{1}{3}Ma^2\left(\frac{1}{2}a\phi-ah\right),$$

we shall have

$$(2) \qquad U=\frac{M}{R}+\frac{N}{R^3}(1-3\mu^2).$$

2. Let us take a rectangular system of axes, as follows: Z along the axis of rotation of the earth in the northward direction, X and Y through the center (C) of the earth in the plane of the equator and invariably fixed in it. Then μ will be equal to $\frac{z}{R}$ and

$$(3) \qquad \begin{cases} U=\dfrac{M}{R}+\dfrac{N}{R^3}-\dfrac{3Nz^2}{R^5}, \\ R=\sqrt{x^2+y^2+z^2}. \end{cases}$$

3. We will now transform our system of coördinate axes. The new origin (O) will be taken at an external point on or near the surface of the earth in the meridian plane ZCX; the axes $O\xi$ and $O\zeta$ in this plane, namely, $O\zeta$ in the direction of the *mean* * force of gravity at $O\zeta$, and $O\xi$ in the

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* that is, on the assumption that the forces of attraction by the sun and the moon are neglected.

southward direction perpendicular to $O\zeta$; finally, the axis $O\eta$ parallel to CY, eastward. Then, denoting by δ the distance of O from the center of the earth, λ the latitude of the point O, and θ the angle of the radius vector CO with the plane of the equator, the formulas of transformation will be:

$$(4)\qquad \begin{aligned} x &= \delta \cos \theta + \xi \sin \lambda - \zeta \cos \lambda, \\ y &= \eta, \\ z &= \delta \sin \theta - \xi \cos \lambda - \zeta \sin \lambda. \end{aligned}$$

4. The expressions of the partial derivations $\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}$, as obtained from (3), readily yield the derivatives of the same function with regard to ξ, η, ζ. Namely,

$$(5)_1\quad \frac{\partial U}{\partial \xi} = -\left[\frac{M}{R^3} + \frac{3N}{R^5} - \frac{15\,Nz^2}{R^7}\right](\delta \sin \epsilon + \xi)$$

$$+ \frac{6N}{R^5}\delta \sin \theta \cos \lambda - \frac{6N}{R^5}\xi \cos^2 \lambda - \frac{6N}{R^5}\zeta \sin \lambda \cos \lambda,$$

$$(5)_2\quad \frac{\partial U}{\partial \eta} = -\left[\frac{M}{R^3} + \frac{3N}{R^5} - \frac{15Nz^2}{R^7}\right]\eta,$$

$$(5)_3\quad \frac{\partial U}{\partial \zeta} = \left[\frac{M}{R^3} + \frac{3N}{R^5} - \frac{15\,Nz}{R^7}\right](\delta \cos \epsilon - \zeta)$$

$$+ \frac{6N}{R^5}\delta \sin \theta \sin \lambda - \frac{6N}{R^5}\xi \sin \lambda \cos \lambda - \frac{6N}{R^5}\zeta \sin^2\lambda,$$

where

$$(6)\quad \epsilon = \lambda - \theta,$$

$$(7)\quad R = \sqrt{\delta^2 + \rho^2 + 2\delta(\xi \sin \epsilon - \zeta \cos \epsilon)},$$

$$(8)\quad \rho = \sqrt{\xi^2 + \eta^2 + \zeta^2},$$

and z^2, in function of ξ, ζ, δ, θ, λ, is derived from the third one of the formulas (4).

5. The derivatives $\frac{\partial U}{\partial \xi}$, $\frac{\partial U}{\partial \eta}$, $\frac{\partial U}{\partial \zeta}$ will now be expressed in function of M, N, ϵ, λ, δ, ξ, η, ζ. All terms whose ratios to $\frac{M}{\delta^3}$ are small quantities of order higher than the first power of the oblateness of the earth will be neglected. It should be observed that ϵ and $\frac{N}{\delta^4}$ are small quantities of the order of the oblateness. In regard to ϵ this fact is well known. To be convinced of it in regard to $\frac{N}{\delta^4}$ one only needs to divide both sides of (1) by δ^4. Then

$$\frac{N}{\delta^4} = -\frac{1}{3}\frac{M}{\delta^2}\left(\frac{a}{\delta}\right)^2\left(\frac{1}{2}a\phi - ah\right)$$

and since $\frac{M}{\delta^2}$ represents the approximate value of the acceleration of the force of gravity at O, that is, $\frac{M}{\delta^2} = 9^m,78$ and $\frac{a}{\delta}$ differs little from unity, it is clear that $\frac{N}{\delta^4}$ is a small quantity of the order of $\frac{1}{2}a\phi - ah$, that is, of the oblateness of the earth. If, then, we put

$$(9)_1 \quad a = -\frac{M}{\delta^2}\sin\epsilon$$

$$+\frac{3N}{\delta^4}\left[\sin 2\lambda - 3(1+\sin^2\lambda)\sin\epsilon + 9\sin\lambda\cos\lambda\sin^2\epsilon\right],$$

$$(9)_2 \quad b = -\frac{3M}{\delta^3}\sin\epsilon + \frac{12N}{\delta^5}\sin 2\lambda,$$

$$(9)_3 \quad c = \frac{M}{\delta^2} \cos \epsilon$$

$$+ \frac{3N}{\delta^4} \left[1 - 3 \sin^2\lambda + 4 \sin 2\lambda \sin\epsilon + \frac{1}{2}(-11 + 23 \sin^2\lambda) \sin^2\epsilon\right],$$

$$(9)_4 \quad 2d = -\frac{M}{\delta^3} - \frac{9N}{\delta^5} + \frac{21N}{\delta^5} \sin^2\lambda,$$

$$(9)_5 \quad 2e = -\frac{M}{\delta^3} - \frac{3N}{\delta^5} + \frac{15N}{\delta^5} \sin^2\lambda,$$

$$(9)_6 \quad 2f = \frac{2M}{\delta^3} + \frac{12N}{\delta^5} - \frac{36N}{\delta^5} \sin^2\lambda,$$

we shall have

$$(10)_1 \quad \frac{\partial U}{\partial \xi} = a + 2d\xi + b\zeta,$$

$$(10)_2 \quad \frac{\partial U}{\partial \eta} = 2e\eta,$$

$$(10)_3 \quad \frac{\partial U}{\partial \zeta} = c + 2f\zeta + b\xi,$$

and, therefore,

$$(11) \quad U = a\xi + b\xi\zeta + c\zeta + d\xi^2 + e\eta^2 + f\zeta^2.$$

It may be well to note that the coefficients d, e, f satisfy the condition

$$(12) \qquad d + e + f = 0$$

in accordance with Laplace's equation $\Delta_2 U = 0$.

6. A particle moving on the surface of the earth is subject not only to the attraction by the terrestrial mass, but also to the attractions by the sun and the moon. This latter influence is exercised in part directly, in part indirectly through the medium of the tides, the liquid mass, which covers the

solid spheroid of the earth and which we have heretofore assumed to be in a state of equilibrium, being really in motion. The direct influence here referred to gives rise to a force Φ_1 which is the resultant of 1) the forces of attraction of the particle by the sun and the moon and 2) the forces equal but opposite in direction to the forces of attraction by the sun and the moon of a mass equal to that of the given particle and placed in the center of the earth. The indirect influence of the sun and the moon through the medium of the tides gives rise to another, also very small, force Φ_2.

Let Φ be the resultant of these forces Φ_1 and Φ_2. It will, of course, vary with the time, but for a comparatively small interval of time, as in the case of experiments and observations on the surface of the earth we may assume the force Φ, as well as its projections Φ_ξ, Φ_η, Φ_ζ on the coördinate axes, to be constant.* A rough computation will enable the reader to see that the forces Φ_ξ, Φ_η, Φ_ζ are small quantities of the order of $\frac{M}{\delta^3}$. To obtain the exact values of the components of the solar and lunar attraction it is necessary to resort to the rigorous methods of Celestial Mechanics. We will assume that these components have been actually computed for a given epoch t_0. Then these computed values, which we will denote by Φ_{ξ_0}, Φ_{η_0}, Φ_{ζ_0}, may be substituted for the forces Φ_ξ, Φ_η, Φ_ζ during the comparatively small interval of time under consideration.

7. The differential equations which express that a constrained particle is in a state of equilibrium relatively to the earth can now be written as follows:

---

* In this we follow the example of Puiseux. See his paper entitled "De l'équilibre et du mouvement des corps pésants en ayant égard aux variations de diréction et d'intensité de la pésanteur." Annales de l'École Normale. 2e Série. t. I. 1872. p. 23.

$$(13)_1 \quad 0 = \frac{\partial U}{\partial \xi} + \Phi_{\xi_0} + R_\xi + \omega^2 \delta \cos\theta \sin\lambda$$
$$+ \omega^2 \sin\lambda \, (\xi \sin\lambda - \xi \cos\lambda),$$

$$(13)_2 \quad 0 = \frac{\partial U}{\partial \eta} + \Phi_{\eta 0} + R_\eta + \omega^2 \eta,$$

$$(13)_3 \quad 0 = \frac{\partial U}{\partial \zeta} + \Phi_{\zeta_0} + R_\zeta - \omega^2 \delta \cos\theta \cos\lambda$$
$$+ \omega^2 \cos\lambda \, (\zeta \cos\lambda - \xi \sin\lambda),$$

R_ξ, R_η, R_ζ being the components of the reaction of the constraints to which the particle is subjected, and ω the angular velocity of rotation of the earth.

At the point O, where $\xi = \eta = \zeta = 0$, the equations (13) reduce to

$$0 = a + R_\xi + \omega^2 \delta \cos\theta \sin\lambda,$$
$$0 = R_\eta,$$
$$0 = c + R_\zeta - \omega^2 \delta \cos\theta \cos\lambda,$$

if we assume, at the same time, that $\Phi = 0$, that is, $\Phi_{\xi_0} = \Phi_{\eta 0} = \Phi_{\zeta_0} = o$. But, then, the axis $O\zeta$ would coincide with the direction of the *mean* * force of gravity at O. If, therefore, we denote by g the value of the *mean* acceleration of the force of gravity at that point we shall have

$$R_\xi = R_\eta = 0 \text{ and } R_\zeta = -g.$$

Hence,

$$(14) \qquad \begin{aligned} a &= -\omega^2 \delta \cos\theta \sin\lambda, \\ c &= g + \omega^2 \delta \cos\theta \cos\lambda. \end{aligned}$$

8. Introducing these expressions of a and c into equations (13) and putting for the sake of brevity

$$(15) \qquad \begin{aligned} B &= b - \omega^2 \sin\lambda \cos\lambda, \\ 2D &= 2d + \omega^2 \sin^2\lambda, \\ 2E &= 2e + \omega^2, \\ 2F &= 2f + \omega^2 \cos^2\lambda, \end{aligned}$$

---

* See foot-note, § 3.

we arrive at the following formulas:

$$- R_\xi = \Phi_{\xi_0} + 2D\xi + B\zeta,$$
$$- R_\eta = \Phi_{\eta_0} + 2E\eta,$$
$$- R_\zeta = \Phi_{\zeta_0} + 2F\zeta + B\xi + g.$$

Now, it is obvious that $-R_\xi, -R_\eta, -R_\zeta$ are exactly the components of the *true force of gravity* at the point (ξ, η, ζ). If, then, we denote by V the *true potential of the force of gravity*, we shall have

$$(16)\quad V = g\zeta + \Phi_{\xi_0}\xi + \Phi_{\eta_0}\eta + \Phi_{\zeta_0}\zeta + D\xi^2 + E\eta^2 + F\zeta^2 + B\xi\zeta.$$

9. In conclusion let us have a glance at the relative value of the several coefficients entering formula (16). As already remarked, the order of magnitude of $\Phi_{\xi_0}, \Phi_{\eta_0}, \Phi_{\zeta_0}$ is that of the *principal* terms in the expressions of the coefficients D, E, F, that is, of the order of $\frac{M}{\delta^3}$. The coefficient B and the *secondary* terms in the expressions of D, E, F are of the order of the product of $\frac{M}{\delta^3}$ by the oblateness of the earth.* Hence, if the components of the solar and lunar attraction are not taken into account when computing the forces of gravity at a point on the surface of the earth, it is needless to preserve any but the first term in the expression (16) of the potential, which, then, reduces to the familiar form

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* In the units here selected the approximate values of $\frac{M}{\delta^3}, \frac{N}{\delta^5}, \omega^2$ are:

$$\frac{M}{\delta^3} = 0.000\,001\,5\,,$$
$$\frac{N}{\delta^5} = 0.000\,000\,001\,5\,,$$
$$\omega^2 = 0.000\,000\,005\,3\,.$$

It may, also, be noted that, approximately,

$$\epsilon = 0.0034 \sin 2\lambda \quad \text{and} \quad \frac{M}{\delta^3} \sin \epsilon = 0.000\,000\,005 \sin 2\lambda.$$

$$V' = g\zeta. \tag{17}$$

This discussion, also, shows that it is absurd to retain terms of the order of ω^2 in treating the motion of a body relatively to the Earth if the force of gravity is assumed to be constant in magnitude and direction, *i. e.* if formula (17) is taken to represent the potential of this force. Yet the delusion that by retaining terms of the order of ω^2 they have reached more accurate results is still to be found among authors who discuss the motion of a body relatively to the earth, taking into account the latter's rotation about its axis.

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THE LESPEDEZAS OF MISSOURI.

K. K. MACKENZIE AND B. F. BUSH.

Issued March 19, 1902.

THE LESPEDEZAS OF MISSOURI.*

K. K. Mackenzie and B. F. Bush.

Various species of Lespedezas abound in the State of Missouri, especially in rocky woods in the Ozark Mountain region, in the southern and eastern portions. Observations in the field of the various forms which occur here have convinced the writers that there are several undescribed species in the State, and this has led to a more extended study of the specimens preserved in the herbarium of the Missouri Botanical Garden, and of those in the herbarium of Mr. K. K. Mackenzie. The result of this study is embodied in the following notes.

We beg to acknowledge our indebtedness to Professor Wm. Trelease for the loan of the entire Missouri collection of this genus in his care, and for other courtesies extended.

As an aid to the recognition of the species occurring in Missouri, the following key is offered: —

Annual; calyx-lobes broad. 1. *L. striata.*
Perennials; flowers yellow; calyx-lobes narrow.
  Peduncles exceeding the leaves. 2. *L. hirta.*
  Peduncles shorter than the leaves.
    Leaves glabrate above.
      Leaflets oblong. 3. *L. capitata.*
      Leaflets linear-oblong. 4. *L. capitata longifolia.*
    Leaves sericeous above. 5. *L. capitata sericea.*
Perennials; flowers purplish; calyx-lobes narrow.
  Peduncles exceeding the leaves.
    Plants prostrate and widely spreading.
      Glabrate or nearly so. 6. *L. repens.*
      Strongly pubescent. 7. *L. procumbens.*
    Plants strongly bushy-branched, from erect to slightly procumbent.
      Flowers strongly paniculate, on pedicels often 6 mm. long; larger leaflets 18 mm. long, or longer. 8. *L. violacea.*
      Flowers scarcely paniculate, on pedicels about 1.5 mm. long; leaflets much smaller, rarely 18 mm. long. 9. *L. violacea prairea.*

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* Presented and read by title before The Academy of Science of St. Louis, December 16, 1901.

Plants strictly erect or ascending, the few branches ascending.
Pods included or little exserted. 10. *L. Manniana.*
Pods much exserted.
Pods obtusish, strongly pubescent. 11. *L. Nuttallii.*
Pods long acute, slightly pubescent. 12. *L. acuticarpa.*
Peduncles shorter than the leaves.
Pods much exserted.
Leaflets linear or linear-oblong.
Glabrate or appressed-pubescent. 13. *L. Virginica.*
Strongly pubescent. 14. *L. neglecta.*
Leaflets oblong, oval or wider.
Glabrate or appressed-pubescent. 15. *L. frutescens.*
Downy-pubescent. 16. *L. Stuvei.*
Pods included or little exserted. 17. *L. simulata.*

1. Lespedeza striata (Thunb.) H. & A. Bot. Beechey. 262. 1841.

Abundantly introduced throughout the Ozark Mountain region in sterile fields and woods, and rarely extending as far north as Kansas City. Probably not of this genus, as the ovate calyx lobes and annual habit are clearly at variance with those characters in our other species.

Specimens examined. — Bush, Bismarck, Sept. 10, 1893. — Bush, *213*, Campbell, Aug. 3, 1895. — Bush, Dunklin Co., Oct. 21, 1892. — Bush, *215*, Sheffield, Sept. 5, 1895; *247*, Aug. 5, 1896. — Bush, Shannon Co., Sept. 12, 1888; July 12, 1891; July 23, 1891. — Mackenzie, *210*, Dodson, Sept. 18, 1895. — Trelease, *222*, Iron Mountain, Aug. 17, 1897. — Trelease, *321*, Pilot Knob, Aug. 18, 1897. — Russell, Pilot Knob, Sept., 1897. — Dewart, *33*, Poplar Bluff, Aug. 7, 1892; Aug. 14, 1892.

2. Lespedeza hirta (L.) Ell. Bot. S. C. & Ga. 2 : 207. 1824.

Abundant in dry rocky woods throughout the Ozark Mountain region, but not seen from other portions of the State.

Specimens examined. — Bush, Shannon Co., Oct. 21, 1893. — Bush, Bismarck, Sept. 10, 1893. — Bush, *35*, Eagle Rock, Sept. 28, 1896; *50*, Sept. 28, 1896. — Mackenzie, Eagle Rock, Sept. 28, 1896; Sept. 28, 1896, a slender decumbent form. — Eggert, St. Louis Co., Sept. 4, 1891. — Trelease, *227*, Carter Co., Sept. 27, 1897. — Russell, Piedmont, Sept., 1897. — Blankinship, Oregon Co., Aug. 18, 1888. — Engelmann, Pilot Knob, Sept. 5, 1859. — Engelmann, Meramec Hills, Nov., 1845.

3. Lespedeza capitata Michx. Fl. Bor. Am. 2 : 71. 1803.

Apparently common throughout the State in dry open woods and on dry prairies.

Specimens examined. — Bush, McDonald Co., July 24, 1893. — Bush, *230*, Raytown, Sept. 19, 1897.— Bush, Shannon Co., Oct. 21, 1893.— Bush, Newton Co., Aug. 29, 1893. — Mackenzie, Swope Park, Jackson Co., Aug. 23, 1896. — Mackenzie, Jones' Creek, Jackson Co., Sept. 19, 1897. — Weller, Springfield, Aug. 6, 1887.— Eggert, St. Louis Co., Sept. 21, 1877.

4. Lespedeza capitata longifolia (DC.) T. & G. Fl. N. A. 1 : 368. 1840.

A long narrow-leaved form of the last of very rare occurrence in the State.

Specimens examined. — Mackenzie, *123*, Dodson, Aug. 26, 1895.

5. Lespedeza capitata sericea H. & A. Comp. Bot. Mag. 1 : 23. 1835.

A silvery-leaved, usually quite distinct form of quite frequent occurrence on dry prairies and in open rocky woods.

Specimens examined. — Bush, Malden, Sept. 12, 1893. — Bush, Jackson Co., Sept. 22, 1893. — Trelease, *225*, Pilot Knob, Sept. 18, 1897. — Trelease, *226*, Clarksville, Oct. 10, 1897. — Mackenzie, *522*, Lone Jack, Sept. 24, 1901.

6. Lespedeza repens (L.) Bart. Prod. Fl. Phil. 2 : 77. 1818.

This species is only occasionally found in the Ozark Mountain region in the State, in dry rocky woods.

Specimens examined. — Bush, Malden, Aug. 12, 1893, in part. — Bush, Shannon Co., July 16, 1891; July 21, 1891.

7. Lespedeza procumbens Michx. Fl. Bor. Am. 2 : 70. 1803.

In similar situations to the last, but apparently much more common in the State. Probably intergrades with *L. repens*, but is usually quite distinct.

Specimens examined. — Bush, *40*, Eagle Rock, Sept. 28, 1896. — Bush, Campbell, Sept. 16, 1893. — Bush, Greene Co , Sept. 4, 1893. — Bush, Malden, Aug. 12, 1893, in part. — Bush, McDonald Co., Sept. 1, 1893. — Bush, Shannon Co., June 28, 1888. — Mackenzie, Eagle Rock, Sept. 28, 1896. — Russell, Piedmont, Sept., 1897. — Russell, Pilot Knob, Sept. 20, 1898.

8. Lespedeza violacea (L.) Pers. Syn. 2 : 318. 1807.

Probably not uncommon in rocky woods in the State, but not very often collected.

Specimens examined. — Bush, Jackson Co., Aug. 31, 1891, in part. — Mackenzie, *448*, Swope Park, Jackson Co., Sept. 13, 1901; *219*, Aug. 7, 1895. — Mackenzie, *208*, Westport, Oct. 5, 1895, an entirely apetalous form; Aug. 31, 1891, an entirely apetalous form. — Engelmann, St. Louis Co.

9. Lespedeza violacea prairea n. var.

Pl. I. f. 1, 2.

An ascending or somewhat spreading, much branched perennial, glabrous below, short appressed-hairy above; branches spreading, and much interwoven; leaflets obovate to obcordate, 6–18 mm. long, 4–9 mm. wide, mucronate, and obtuse or retuse at the apex, rounded below, smooth above, appressed-pubescent beneath; petiolules villous, 1 mm. long; rachis (2–6 mm. long) and petiole (2–12 mm. long) appressed-hairy; stipules linear-subulate, 4 mm. long; flowers subcapitate, not paniculate, 2–6 together, on peduncles less than 2 mm. long, but the inflorescence exceeding the leaves; often two peduncles arise from the same leaf axis; flowers on pedicels 1–2 mm. long, rarely a little longer; calyx 3 mm. long, appressed-hairy, the teeth lanceolate-subulate or narrower, about the length of the tube and much shorter than the corolla; corolla purplish, 6–8 mm. long, the keel noticeably longer than the wings and standard; non-petaliferous flowers in short-pedunculate or subsessile clusters; pods subsessile or very short-stiped in the calyx, orbicular-oval, 3–4 mm. long, acute, strongly reticulate-veined, short appressed-hairy or subglabrous, much exceeding the calyx.

Differs from the specific form in being smaller throughout, in its non-paniculate inflorescence, and in its much shorter pedicels. It is the common form in this State, and is intermediate between the species and *L. repens.* Its non-procumbent character at once distinguishes it from both *L. repens* and *L. procumbens.* Further study will probably prove that it is entitled to specific rank. It is very common on dry prairies, where *L. violacea* is never found, whence the name given it.

Type locality, Lee's Summit, Missouri; collected by Bush, No. 93, and Mackenzie, Sept. 2, 1895; type in herb. Missouri Botanical Garden, and in herb. K. K. Mackenzie.

Specimens examined. — Type specimens as cited under type locality. — Bush, Dunklin Co., Oct. 26, 1892. — Bush, Bismarck, Sept. 10, 1892. — Bush, Jackson Co., Aug. 31, 1891, in part. — Mackenzie, Lee's Summit, Sept. 9, 1901. — Thompson, Pacific, Sept. 19, 1898. — Fritchey, Bridgeton, Sept. 29, 1859. — Russell, Piedmont, Sept., 1899. — Dewart, Spring Park, Aug. 8, 1892. Somewhat approaching the specific form are the following: Bush, Dunklin Co., Oct. 21, 1892. — Blankinship, Greene Co., July 17, 1888. — Glatfelter, Pilot Knob, Aug. 20, 1895. A plant with strongly pubescent stems and foliage as in *L. procumbens*, but apparently ascending and bushy-branched as in *L. violacea prairea*, has been twice collected in the State, but the specimens are too few to enable us to definitely dispose of them. The specimens referred to are the following: Bush, McDonald Co., July 31, 1895. — Trelease, *223*, Carter Co., Sept. 9, 1897.

10. Lespedeza Manniana n. sp.

Pl. II. f. 1.

Perennial, 4–9 dm. tall, suberect, strongly pubescent above, glabrate below; branches erect-ascending; leaflets oblong, subcuneate, 1–4 cm. long, 4–12 mm. wide, obtuse and mucronate at the apex, cuneate or rounded at the base, somewhat appressed-pubescent above, especially near the mid-vein, densely appressed-pubescent beneath; petiolules densely hairy; petioles (2–20 mm. long) and rachis less so; petaliferous flowers in short racemes, on peduncles 12–24 mm. long, exceeding the leaves, the racemes often clustered near the summit of the stem, 6–12 flowered; flowers short-pedicelled; calyx strongly long-pubescent, the tube 2 mm. long, the long acuminate subulate teeth 4–6 mm. long, and often exceeding the corolla; corolla purplish, 5–6 mm. long, the keel usually not exceeding the wings and standard; non-petaliferous flowers in nearly sessile clusters; pods oval, acute, somewhat pubescent, from shorter than and included in the calyx, to slightly exserted.

Most closely related to *L. Nuttallii* Darl., and to *L. acuticarpa* (infra), but easily distinguished by its long sepals and nearly included pod. Named in honor of Rev. Cameron Mann, now Bishop of North Dakota, an ardent lover of nature and a skilled botanist, with whom the writers have spent many pleasant days in the field. It is very common in open post oak land and in limestone barrens south of Kansas City.

Type locality, Swope Park, Jackson County, Missouri; collected by K. K. Mackenzie, Aug. 23, 1896; type in herb. K. K. Mackenzie, duplicate in herb. Missouri Botanical Garden.

Specimens examined. — Type specimens as cited under type locality. — Mackenzie, same locality, Sept. 13, 1896. — Mackenzie, Red Bridge, Sept. 18, 1901. — Bush, *333*, Raytown, 1897. — Bush, Jones' Creek, Jackson Co., Aug. 31, 1891.

11. Lespedeza Nuttallii Darl. Fl. Cest. ed. 2, 420. 1837.

Apparently a rare species in Missouri, at least it is not often collected.

Specimens examined. — Bush, Wright Co., Sept. 18, 1885. — Engelmann, Pilot Knob, Sept. 8, 1859. — Trelease, *224*, Aurora Spring, Sept. 17, 1897.

12. Lespedeza acuticarpa n. sp.

Pl. III. f. 1, 2.

Erect or suberect perennial, 2.5–5 dm. tall, not bushy-branched, but growing in clumps; branches erect-ascending; stems in same plant varying from nearly glabrate to woolly-hairy; leaflets 8–28 mm. long, 4–8 mm. wide, oblong-elliptic, glabrate above, appressed-pubescent beneath; petiolules 1 mm. long, villous; leaf-rachis (2–6 mm. long) and petiole (2–20 mm. long) appressed-pubescent; stipules subulate; flowers in short few-flowered (6 or less) spikes, on peduncles 1–4 mm. long, the inflorescences much exceeding the leaves; flower pedicels 2–4 mm. long; flowers purplish, 6–8 mm. long, the keel somewhat exceeding the wings; calyx 2 mm. long, its long subulate teeth 3 mm. long, short appressed-pubescent; pod 6 mm. long, distinctly short-stiped, subglabrate or sparingly pubescent, ovate, twice the length of the sepals, strongly acute or almost acuminate, usually tipped by the long persistent style; non-petaliferous flowers sessile.

Not very common in open post oak woods and limestone barrens. Distinguished from *L. Nuttallii* Darl. by its narrower leaves, fewer flowers, and large acuminate, less hairy pod. *L. Manniana* differs in its very long hairy sepals, and nearly included scarcely pointed pod.

Type locality, Swope Park, Jackson County, Missouri; collected by K. K. Mackenzie, No. 449, Sept. 13, 1901; type specimens in herb. K. K. Mackenzie, duplicate in herb. Missouri Botanical Garden.

Specimens examined. — Type specimens as cited under type locality. — Bush, *67*, Eagle Rock, Sept. 28, 1896. — Mackenzie, Eagle Rock, Sept. 28, 1896.

13. Lespedeza Virginica (L.) Britton, Trans. N. Y. Acad. Sci. **12**: 64. 1893.

In dry rocky woods. Apparently the most common species of Lespedeza in the State, judging from the collections.

Specimens examined. — Bush, Jackson Co., Sept. 21, 1891; Sept. 29, 1891. — Bush, McDonald Co., Sept. 1, 1891. — Bush, Bismarck, Sept. 10, 1893; Bush, Jasper Co., Aug. 16, 1893. — Bush, Shannon Co., Sept. 12, 1893; July 24, 1891; Oct. 21, 1893. — Bush, Howell Co., Aug. 12, 1892. — Bush, Dunklin Co., Oct. 26, 1892. — Bush, Newton Co., Aug. 29, 1893. — Bush, *95*, Independence, Sept. 1, 1895. — Bush, Greene Co., Sept. 21, 1893. — Blankinship, Greene Co., Aug. 23, 1888; Aug. 27, 1888. — Mackenzie, *441*, Swope Park, Jackson Co., Sept. 13, 1901. — Mackenzie, Lee's Summit, Aug. 6, 1899. — Mackenzie, *948*, Dodson, Aug. 26, 1895. — Thompson, Pacific, Sept. 19, 1898. — Glatfelter, Pilot Knob, Aug. 20, 1895. — Russell, Pilot Knob, Sept., 1897. — Dewart, Meramec Highlands, July 4, 1892. — Trelease, *229*, Einstein Mine, Aug. 18, 1897. — Eggert, Forest Park, St. Louis, Oct., 1893.

14. Lespedeza neglecta (Britton).

Lespedeza Stuvei neglecta Britton, Mem. Torr. Club, **5**: 206. 1894.

What appears to be this species occurs in the southeastern part of the State. To us it seems more closely related to *L. Virginica* than to *L. Stuvei*, and appears to bear the same relation to that species that *L. Stuvei* bears to *L. frutescens*. It is well worthy of specific rank.

Specimens examined. — Engelmann, Pilot Knob, Sept. 8, 1859.

15. Lespedeza frutescens (L.) Britton, Mem. Torr. Club, **5**: 205. 1894.

Found in rocky woods throughout the Ozark Mountain region in Missouri, but usually does not occur in such abundance as some of the other species.

Specimens examined. — Bush, Shannon Co., Sept. 12, 1888. — Bush, McDonald Co., Sept. 1, 1893. — Bush, *53*, Eagle Rock, Sept. 18, 1896. — Mackenzie, Eagle Rock, Sept. 18, 1896. — Dewart, Poplar Bluff, Aug. 14, 1892. — Letterman, Poplar Bluff, Aug. 15, 1895. — Russell, Pilot Knob, Sept., 1897. — Trelease, *228*, Indian Hill, Dunklin Co., Sept. 28, 1897.

16. Lespedeza Stuvei Nutt. Gen. 2 : 107. 1818.

A common species in dry rocky woods throughout the Ozark Mountain region in the State.

Specimens examined. — Bush, McDonald Co., Sept. 1, 1893. — Bush, Campbell, Sept. 12, 1893. — Bush, Malden, Sept. 12, 1893. — Bush, Shannon Co., Oct. 21, 1893. — Bush, Newton Co., Aug. 29, 1893. — Bush, Jasper Co., Aug. 16, 1893. — Bush, *42*, Eagle Rock, Sept. 28, 1896. — Mackenzie, Eagle Rock, Sept. 28, 1896; Sept. 28, 1896, an entirely apetalous form.

17. Lespedeza simulata n. sp.

Pl. IV. f. 1, 2.

An erect perennial, 6–9 dm. tall, glabrate to strongly rather short-pubescent, the rather few branches appressed-ascending; leaflets oblong-linear to oblong-elliptic, rounded at both ends, but strongly mucronate at the apex, 12–30 mm. long, 4–10 mm. wide, appressed silvery-pubescent on both sides, especially beneath ; petioles glabrate or pubescent, less than 12 mm. long, the leaf rachis about the same length, or a little shorter; petiolules villous, 1 mm. long : stipules subulate, glabrate or hairy, 2–6 mm. long; petaliferous flowers in dense capitate spikes, the spikes sessile or on peduncles much shorter than the leaves; flowers on pedicels about 2 mm. long; flowers 6–8 mm. long, the purple carolla usually somewhat exceeding the long calyx-lobes, but sometimes almost included; calyx strongly hairy, 5–6 mm. long, the long acuminate sepals about twice the length of the tube; pod oval, acute or acutish, 4.5–5 mm. long, strongly pubescent, included or very slightly exserted ; non-petaliferous flowers in sessile axillary clusters.

This species is most closely related to *L. Manniana*, but is at once distinguished by having the flowering peduncles shorter than the leaves, while in *L. Manniana* they are much longer. The included pod and long sepals distinguish it from all the other purplish-flowered species, but make faded dried specimens greatly resemble some of the yellow-flowered species, whence the specific name. But here the presence of apetalous flowers at once furnishes a clue for its identification.

The species is found in southwestern Missouri and adjacent Indian Territory in the Ozark Mountain region, in dry rocky open woods, and on high rocky mounds and prairies.

Type locality, Newton County, Missouri; collected by B. F. Bush, Aug. 25, 1893; type in herb. Missouri Botanical Garden, duplicate in herb. K. K. Mackenzie.

SPECIMENS EXAMINED. — Missouri: Type specimens as cited under type locality. — Bush, *61*, Eagle Rock, Sept. 28, 1896. — Mackenzie, Eagle Rock, Sept. 28, 1896. Indian Territory: Bush, *1312*, Sapulpa, Creek Nation, Sept. 29, 1895.

EXPLANATION OF ILLUSTRATIONS.

PLATES I.–IV.

Plate I. — *Lespedeza violacea prairea.* 1, fruit, × 2. 2, flower, × 2.
Plate II. — *Lespedeza Manniana.* 1, fruit, × 2.
Plate III. — *Lespedeza acuticarpa.* 1, fruit, × 2. 2, flower, × 2.
Plate IV. — *Lespedeza simulata.* 1, fruit, × 2. 2, calyx-lobes, × 2.

Issued March 19, 1902.

LESPEDEZA VIOLACEA PRAIREA.

LESPEDEZA MANNIANA.

LESPEDEZA ACUTICARPA.

LESPEDEZA SIMULATA.

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Transactions of The Academy of Science of St. Louis.

VOL. XII. No. 3.

ON THE MOTION OF GYROSCOPES.

ALEXANDER S. CHESSIN.

A *Issued May 13, 1902.*

ON THE MOTION OF GYROSCOPES.*

Alexander S. Chessin.

The gyroscopes used to demonstrate the interesting phenomena connected with rapid rotary motion are, in many cases, so constructed that the subsidiary parts of the apparatus are apt to greatly influence the character of the motion. Yet, these subsidiary parts are never taken into account in mathematical investigations because of the difficulties involved. I have in mind, especially, the *gyroscope of Foucault* and the *polytrope of Sire.* In the theory of these instruments, it has been customary to neglect the mass of the subsidiary parts of the apparatus. It is the purpose of this paper to show how the character of the motion of a gyroscope may be determined without neglecting the mass of any part of the whole apparratus.

Sire's polytrope was invented to demonstrate the influence of the earth's rotation on spinning tops. It is, in principle, a Foucault gyroscope mounted on a metallic circle which represents a meridian of the earth and may be revolved about a diameter, thus producing an effect similar to that of the earth's rotation on the top. The polytrope of Sire, as demonstrating such effects, has a certain advantage over Foucault's instrument, because the metallic circle may be revolved with any velocity we please while, of course, experiments with a Foucault gyroscope are necessarily restricted by the actual angular velocity of the earth's rotation.

This is not the place to describe either of the above mentioned apparatus. A schematic figure will suffice to understand the derivation of our formulas.

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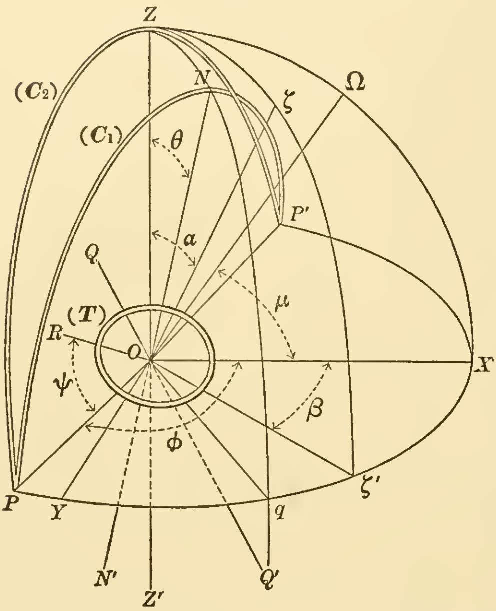
* Presented and read by title before The Academy of Science of St. Louis, April 21, 1902.

THE POLYTROPE OF SIRE.

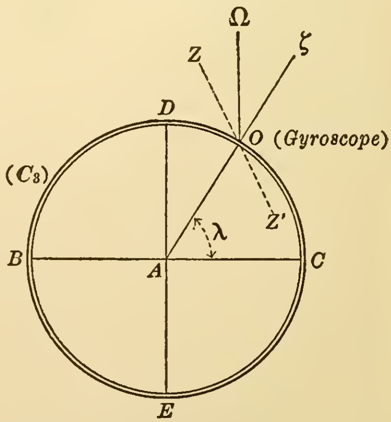
As already mentioned, the polytrope consists of two essential parts: (1) a large metallic circle (C_3) representing an earthly meridian and (2) a Foucault gyroscope mounted on this circle in such a manner that it may be fixed at any point of it, thus demonstrating the influence of the earth's rotation on the gyroscope at any latitude.

FIG. 1.



The Foucault gyroscope consists of the torus, or gyroscope proper (T) and two circular rings (C_1) and (C_2). The torus rotates freely about the diameter NN' of the circle (C_1), while the ring (C_1) itself revolves freely about the diameter PP' of the circle (C_2), the axes NN' and PP' being perpendicular to one another. The diameter ZZ' of the circle (C_2) is perpendicular to PP' and may be fixed in any direction we please relatively to the meridian circle (C_3).

FIG. 2.



The origin of the axes XYZ is taken in the center of gravity (O) of the gyroscope. This coördinate system is fixed with regard to the meridian circle (C_3) which revolves about its vertical diameter DE with a constant angular velocity ω. The direction $O\Omega$ is that of the *positive* axis of rotation of (C_3). The axis X is the projection of $O\Omega$ on a plane perpendicular to ZZ'; OR is a fixed line in the equatorial plane of the torus. The angles θ, ϕ, ψ are Euler's angles defining the relative position of the body with regard to the moving coördinate system XYZ. The axis QQ' is perpendicular to PP' and NN'. The angles which OP, OQ and ON form respectively with $O\Omega$ will be denoted by γ, γ_1 ard γ_2; the principal central moments of inertia of the torus (including its physical axis) about OP, OQ and ON by A, A and C respectively; A_1, A_1, C_1, and A_2, A_2, C_2 will indicate the principal central moments of inertia of the ring (C_1) about OP', ON, OQ and of the ring (C_2) about OP', OZ, Oq. If, then, T denote the kinetic energy of absolute rotary motion of the gyroscope about the point (O), we shall have

$$(1)\quad \begin{aligned} 2T = a\,(\theta' + \omega\cos\gamma)^2 &+ b\,(\phi'\sin\theta + \omega\cos\gamma_1)^2 \\ &+ C\,(\psi' + \phi'\cos\theta + \omega\cos\gamma_2)^2 + c\,\omega^2\cos^2\gamma \\ &+ d\,(\phi' + \omega\sin\mu)^2. \end{aligned}$$

where μ is the angle of $O\Omega$ with X, and

$$(2)\ldots\ldots \begin{cases} a = A + A_1, & b = A + C_1 - A_1, \\ c = A_2 - A_1 - C_2, & d = A_1 + A_2. \end{cases}$$

$$(3)\ldots\ldots \begin{cases} \cos\gamma = \cos\mu\cos\phi \\ \cos\gamma_1 = \sin\mu\sin\theta - \cos\mu\cos\theta\sin\phi \\ \cos\gamma_2 = \sin\mu\cos\theta + \cos\mu\sin\theta\sin\phi. \end{cases}$$

The differential equations of motion are given by the formula

$$(4)\qquad \frac{d}{dt}\left(\frac{\partial T}{\partial q'_k}\right) = \frac{\partial T}{\partial q_k}, \qquad q_k = \theta, \phi, \psi.$$

We obtain immediately two integrals, namely

$$(5)\qquad \psi' + \phi'\cos\theta + \omega\cos\gamma_2 = l_1,$$

$$(6)\quad a\,\theta'^2 + (d + b\sin^2\theta)\,\phi'^2 + C\,(\psi' + \phi'\cos\theta)^2 = 2\,h + \omega^2\,[(a + c)\cos^2\gamma + b\cos^2\gamma_1 + C\cos^2\gamma_2]$$

the latter being the integral of kinetic energy.

The relation between the angle μ and the latitude (λ) of the point (O) is expressed by the formula

$$(7)\qquad \sin\lambda = \cos a\sin\mu + \sin a\cos\mu\cos\beta.$$

a being the angle of the axis Z with the line $(O\zeta)$ passing through (O) and the center of the ring (C_3), and β the angle of the planes XZ and $ZO\zeta$.

Suppose, now, that the axis ZZ' be made parallel to $O\Omega$ and the mounting of the gyroscope then fixed in this position. In this case $\mu = \frac{\pi}{2}$ and, therefore,

$$\cos\gamma = 0, \qquad \cos\gamma_1 = \sin\theta, \qquad \cos\gamma_2 = \cos\theta,$$

$$(8)\quad 2T = a\,\theta'^2 + b\,(\phi' + \omega)^2\sin^2\theta + d\,(\phi' + \omega)^2 + C\,[\psi' + (\phi' + \omega)\cos\theta]^2.$$

We can, now, obtain a new integral, namely,

$$(9)\quad (d + b\sin^2\theta)(\phi' + \omega) + C\,[\psi' + (\phi' + \omega)\cos\theta]\cos\theta = l_2$$

while the integrals (5) and (6) take the form

$$(10)\qquad \psi' + (\phi' + \omega)\cos\theta = l_1$$

$$(11)\quad a\,\theta'^2 + (d + b\sin^2\theta)\,\phi'^2 + C\,(\psi' + \phi'\cos\theta)^2 = 2h + \omega^2\,(b\sin^2\theta + C\cos^2\theta)$$

With the help of equation (10) the integrals (9) and (11) may be presented thus:

$$(9)'\qquad (d + b\sin^2\theta)\,(\phi' + \omega) + Cl_1\cos\theta = l_2$$

$$(11)' \qquad a\,\theta'^2 = l_3 - \frac{(l_2 - Cl_1 \cos\theta)^2}{d + b\sin^2\theta}$$

where we have put

$$(12) \qquad l_3 = 2h - Cl_1^2 + 2\omega l_2 - d\omega^2$$

From $(11)'$ we find

$$(13)\ t = \pm\sqrt{a}\int_{\theta_0}^{\theta} \frac{\sqrt{d + b\sin^2\theta}\; d\,\theta}{\sqrt{l_3(d + b\sin^2\theta) - (l_2 - Cl_1\cos\theta)^2}}$$

Formula (13) shows that t is expressed in function of θ by a quadrature which involves, in general, hyperelliptic integrals. This is the reason why in the treatment of the present problem it has been customary to neglect the mass of the rings (C_1) and (C_2) of the gyroscope. In fact, if we put $A_1 = A_2 = C_1 = C_2 = 0$, we shall have $d = 0$,

$$t = \pm\sqrt{ab}\int_{\theta_0}^{\theta} \frac{\sin\theta d\theta}{\sqrt{bl_3\sin^2\theta - (l_2 - Cl_1\cos\theta)^2}},$$

and the integration may be performed with the aid of circular functions. But it is not at all necessary to perform the integration in order to get an idea of the character of the motion in the general case, as we will now proceed to show.

Let us put

$$(14)\left\{\begin{array}{ll} \Delta = C^2l_1^2 + bl_3, & \delta = \sqrt{l_3[(b+d)\Delta - bl_2^2]}, \\ m = \dfrac{Cl_1l_2 - \delta}{\Delta}, & n = \dfrac{Cl_1l_2 + \delta}{\Delta}. \end{array}\right.$$

Then, by formula (13)

$$(15) \qquad dt = \pm \sqrt{\frac{a}{\Delta}} \frac{\sqrt{d + b \sin^2 \theta} \;\; d\theta}{\sqrt{(\cos \theta - m)(n - \cos \theta)}},$$

from which we see at once that

$$m \leqq \cos \theta \leqq n \; ,$$

and, therefore, $m \leqq 1$. If $m = 1$ and θ_0 not $= 0$, the problem admits of no solution. In fact, the initial conditions are incompatible. If $m = 1$ and $\theta_0 = 0$, then the axis NN' of the torus will remain fixed relatively to the meridian circle (C_3). Thus, we are reduced to considering the case when $m < 1$.

The constants of integration l_1, l_2, l_3 are determined by the equations

$$(16) \left\{ \begin{aligned} l_1 &= \psi'_0 + (\phi'_0 + \omega) \cos \theta_0 \\ l_2 &= C l_1 \cos \theta_0 + (d + b \sin^2 \theta_0)(\phi'_0 + \omega) \\ l_3 &= a \, \theta'^2_0 + (d + b \sin^2 \theta_0)(\phi'_0 + \omega)^2 \end{aligned} \right.$$

Case I. $\qquad -1 < m < 1.$

We will put $m = \cos \eta$ and consider separately the cases of

$n >, =$ and < 1.

A. $n > 1$.

Equation (15) becomes

$$dt = \pm \sqrt{\frac{a}{\Delta}} \frac{\sqrt{d + b \sin^2 \theta} \;\; d\theta}{\sqrt{(\cos \theta - \cos \eta)(n - \cos \theta)}}$$

and shows that *the axis of the torus oscillates about ZZ', the amplitude of these oscillations being 2η.* If we put

$$(17) \qquad \tau = \sqrt{\frac{a}{\Delta}} \int_0^{\eta} \frac{\sqrt{d + b \sin^2\theta}\; d\theta}{\sqrt{(\cos\theta - \cos\eta)(n - \cos\theta)}}$$

and remark that

$$\int_0^{\eta} \frac{\sqrt{d + b \sin^2\theta}\; d\theta}{\sqrt{(\cos\theta - \cos\eta)(n - \cos\theta)}}$$

$$= \int_{-\eta}^{0} \frac{\sqrt{d + b \sin^2\theta}\; d\theta}{\sqrt{(\cos\theta - \cos\eta)(n - \cos\theta)}},$$

we find that *the oscillations of the axis of the torus are isochronic* and that *the period of a complete oscillation is* 4τ.

B. $n = 1$.

Equation (15) becomes

$$dt = \pm \sqrt{\frac{a}{2\Delta}} \frac{\sqrt{d + b \sin^2\theta}\; d\theta}{\sin\frac{\theta}{2}\sqrt{\cos\theta - \cos\eta}}$$

and shows that *the axis of the torus approaches assymptotically to the position* OZ. In fact, after the moment of maximum deviation (η) from ZZ', where the sign of θ' changes, this sign remains unchanged throughout the motion, and if τ_1 denote the time required for the passing of the axis of the torus from the position of maximum deviation (η) to the position OZ, we find that

$$\tau_1 = \sqrt{\frac{a}{2\Delta}} \int_0^{\eta} \frac{\sqrt{d + b \sin^2\theta}\; d\theta}{\sin\frac{\theta}{2}\sqrt{\cos\theta - \cos\eta}} > \sqrt{\frac{a\,d}{4\,\Delta \sin^2\frac{\eta}{2}}} \int_0^{\eta} \frac{d\theta}{\sin\frac{\theta}{2}}$$

i. e. $\tau_1 = \infty$.

C. $n < 1$.

We may put $n = \cos\eta_1$. Then

$$dt = \pm\sqrt{\frac{a}{\Delta}}\,\frac{\sqrt{d + b\sin^2\theta}\;d\theta}{\sqrt{(\cos\theta - \cos\eta)\,(\cos\eta_1 - \cos\theta)}}$$

and we see that *the axis of the torus oscillates between the positions $\theta = \eta$ (minimum deviation from ZZ') and $\theta = \eta_1$ (maximum deviation from ZZ'), never passing through ZZ'. The period of a complete oscillation is* $2\tau_2$ where

$$(18)\qquad \tau_2 = \sqrt{\frac{a}{\Delta}}\int_{\eta_1}^{\eta}\frac{\sqrt{d + b\sin^2\theta}\;d\theta}{\sqrt{(\cos\theta - \cos\eta)\,(\cos\eta_1 - \cos\theta)}}$$

Case II. $m = -1$.

Formula (15) now becomes

$$dt = \pm\sqrt{\frac{a}{2\Delta}}\,\frac{\sqrt{d + b\sin^2\theta}\;d\theta}{\cos\frac{\theta}{2}\sqrt{n - \cos\theta}}$$

where

$$(19)\qquad n = 1 + \frac{2Cl_1l_2}{\Delta}$$

Hence, two cases are possible: *A*) $n < 1$ and *B*) $n \geqq 1$. It should be observed that if $\theta_0' = 0$ the case *A*) alone is possible. In fact, if $\theta_0' = 0$, since $m = -1$, we have

$$\frac{d\,(l_2 - Cl_1\cos\theta_0)^2}{d + b\sin^2\theta_0} = (l_2 + Cl_1)^2,$$

from which it is clear that l_2 and l_1 must have different signs, and, therefore, $n < 1$.

A. If $n < 1$ *The axis of the torus oscillates about* ZZ'. *These oscillations are isochronic and their amplitude is* $2\eta_2$ *where* $\eta_2 = \cos^{-1} n$. *The period of a complete oscillation is* $4\tau_3$ and

$$(20) \qquad \tau_3 = \sqrt{\frac{a}{2\Delta}} \int_0^{\eta_2} \frac{\sqrt{d + b \sin^2 \theta}\, d\theta}{\cos \frac{\theta}{2} \sqrt{n - \cos \theta}}$$

B. If $n \leqq 1$, *the axis of the torus assymptotically approaches the position* OZ' (where $\theta = \pi$).

Case III. $m < -1$

Let us put $-m = s$, so that $s > 1$. Then

$$dt = \pm \sqrt{\frac{a}{\Delta}} \frac{\sqrt{d + b \sin^2 \theta}\; d\theta}{\sqrt{(s + \cos \theta)(n - \cos \theta)}}$$

A. If $n < 1$, *the axis of the torus performs isochronic oscillations about* ZZ'. *The amplitude of these oscillations is* $2\eta_3$ *where* $\eta_3 = \cos^{-1} n$, *and the period of a complete oscillation is* $4\tau_4$, having put

$$(21) \qquad \tau_4 = \sqrt{\frac{a}{\Delta}} \int_0^{\eta_3} \frac{\sqrt{d + b \sin^2 \theta}\, d\theta}{\sqrt{(s + \cos \theta)(n - \cos \theta)}}$$

B. If $n \geqq 1$, *the axis of the torus revolves about* PP' *without changing the direction of the revolution.* The *time required for a complete revolution is* $2\tau_5$ where

$$(22) \qquad \tau_5 = \sqrt{\frac{a}{\Delta}} \int_0^{\pi} \frac{\sqrt{d + b \sin^2 \theta}\;\; d\theta}{\sqrt{(s + \cos \theta)(n - \cos \theta)}}$$

It should, again, be observed that here, as in Case *II*, the condition $\theta'_0 = 0$ is compatible only with the condition $n < 1$.

Remark. The motion of the axis NN' of the torus includes the motion of the ring (C_1). In the cases *IA*, *IC*, *IIA*, *IIIA*, this ring oscillates about PP'; in the cases *IB* and *IIB* it assymptotically approaches the ring (C_2); finally, in the case *IIIB*, it revolves about PP' without a change in the direction of this revolution.

To complete the determination of the motion of the gyroscope it is necessary to evaluate the angles ϕ and ψ. The first angle defines the motion of the ring (C_2) about ZZ', the second determines the motion of the torus about its own axis NN'. These angles are given by the formulas

$$(23) \qquad \phi = \phi_0 - \omega t + \int_0^t \frac{l_2 - Cl_1 \cos\theta}{d + b\sin^2\theta}\, dt$$

$$(24) \qquad \psi = \psi_0 + l_1 t - \int_0^t \frac{(l_2 - Cl_1\cos\theta)\cos\theta}{d + b\sin^2\theta}\, dt.$$

In the following discussion the terms *precession* and *nutation* will be used to indicate the rates of change of the angles ϕ and θ respectively. The problem which we set ourselves to solve is to determine when the motion of the gyroscope proceeds so that the *nutation* is $= 0$.

The condition necessary and sufficient to obtain a motion without nutation is that

$$\sin\theta_0\,(l_2 - Cl_1\cos\theta_0)\,[(b + d)\,Cl_1 - bl_2\cos\theta_0] = 0$$

or, on account of the relations (16),

$$(25) \qquad \sin\theta_0\,(\omega + \phi'_0)\,[Cl_1 - b\cos\theta_0\,(\omega + \phi'_0)] = 0.$$

This equation may be satisfied in three ways.

1. By putting $\sin\theta_0 = 0$, *i. e.* at the start the axis of the torus is parallel to the axis $(O\,\Omega)$ of rotation of the meridian circle (C_3). Equation $(9)'$ shows that in this case *the precession has a constant value*, and equation (10), that *the angular velocity of rotation of the torus about its axis NN' is constant.*

2. By putting $\phi_0' = -\omega$. In this case *the precession has the constant value $-\omega$ and, besides, $\psi' = \psi_0'$.* This result is *a priori* obvious. In fact, the assumption of $\phi_0' = -\omega$, together with $\theta_0' = 0$, is equivalent to assuming that at the start the axis of the gyroscope was fixed in *absolute* space (*not* relatively to the meridian circle). But we know that, in this case, the axis retains its fixed position in absolute space.

3. By putting $Cl_1 = b\,(\omega + \phi_0')\cos\theta_0$, or, which means the same thing,

$$C\,\psi_0' + (C - b)\,(\phi_0' + \omega)\,\cos\theta_0 = 0.$$

Again, we find that both *the precession and the angular velocity of rotation of the torus about its axis, have constant values.*

We now pass to the gyroscope on the surface of the Earth.

THE GYROSCOPE OF FOUCAULT.

First of all, let it be remarked that we may neglect in our calculations all terms of the order of ω^2 and higher, ω being the angular velocity of rotation of the earth, if we assume that the force of gravity is constant in magnitude and direction for all positions of the gyroscope.* However, as we gain nothing by thus changing the appearance of our formulas, while we would be obliged, on the other hand, to retain the terms of the order of ω^2 if the point of observation were near one of the poles, it is advisable to leave the formulas as they are, with the explicit understanding, that *the results are correct only to terms of the order of ω.*

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* See a paper by the author in the Transactions of this Society, Vol. XII., No. 1, *On the true potential of the force of gravity.*

We may now immediately apply the formulas derived for the polytrope of Sire to the case of a gyroscope on the surface of the earth. The metallic ring (C_3) of the polytrope will, in the following discussion, be replaced by the meridian of the place of observation.

We will assume that ZZ' is parallel to the axis of the earth. Referring to the figures in the text it is clear that ω must be now replaced by $-\omega$, since the positive axis of rotation of the earth is directed southward.

We will consider the motion only under Foucault's conditions, *i. e.* when $\theta'_0 = \phi'_0 = 0$ and ψ'_0 is very great. Then only one among the cases discussed for the polytrope presents itself, namely the case IC, as can be readily seen. Therefore,

$$(26) \qquad dt = \pm\sqrt{\frac{a}{\Delta}}\; \frac{\sqrt{d + b\sin^2\theta}\, d\theta}{\sqrt{(\cos\theta - \cos\eta)(\cos\eta_1 - \cos\theta)}}.$$

If $\psi'_0 > 0$, *i. e.* the rotation of the torus appears from left to right to an observer standing along the axis ON with his feet at O, this axis forming an acute angle with the negative axis of the earth (*i. e.* its northward direction), then $\eta_1 = \theta_0 < \eta$. If, on the contrary, $\psi'_0 < 0$, *i. e.* the direction of rotation of the torus is opposite to the one just described, then $\eta = \theta_0 > \eta_1$. Hence, *the axis of the torus oscillates between the positions* $\theta = \theta_0$ *and* $\theta = \eta$ *or* η_1, *never passing through* ZZ' (which is parallel to the axis of the earth). *The period of a complete oscillation is* $2\tau_2$, where

$$(27) \qquad \tau_2 = \sqrt{\frac{a}{\Delta}} \int_{\eta_1}^{\eta} \frac{\sqrt{d + b\sin^2\theta}\quad d\theta}{\sqrt{(\cos\theta - \cos\eta)\,(\cos\eta_1 - \cos\theta)}}.$$

Of the three cases of motion *when the nutation* is $= 0$, only one is possible in Foucault's gyroscope, namely the one determined by the condition $\sin \theta_0 = 0$, *i. e.* $\theta = 0$ or π. Hence, *the axis of the torus can be in equilibrium only if it is parallel to the axis of the earth.*

Indeed, this fact can be readily ascertained by the ordinary rules of mechanics. Moreover, it is clear, that *one of the two positions $\theta_0 = 0$ or π, in which the positive axis of the torus is parallel to the positive axis of the earth, is the position of stable equilibrium, while the other position is that of unstable equilibrium;* i. e. if $\psi'_0 > 0$, then $\theta_0 = \pi$ is the position of stable equilibrium and $\theta_0 = 0$ the position of unstable equilibrium, while for $\psi'_0 < 0$ the position $\theta_0 = 0$ is that of stable equilibrium and $\theta_0 = \pi$ one of unstable equilibrium.

The motion is completely determined by the equation (26) and the following formulas giving the angles ϕ and ψ.

$$\phi = \phi_0 + \omega t + \int_0^t \frac{l_2 - Cl_1 \cos\theta}{d + b \sin^2\theta} dt,$$

$$\psi = \psi_0 + l_1 t - \int_0^t \frac{(l_2 - Cl_1 \cos\theta) \cos\theta}{d + b \sin^2\theta} dt.$$

It should be observed that

$$\cos\eta_1 - \cos\eta = \frac{2\delta}{\Delta} = \frac{2\omega}{\Delta} \sqrt{(d + b \sin^2\theta) [\Delta (b + d) - bl_3^2]}$$

i. e. the difference $\cos\eta_1 - \cos\eta$ is a quantity of the order of $\frac{\omega}{\psi_0'}$. The angular velocity of rotation of the earth (ω) being very small and ψ'_0 by hypothesis, very great, the oscillations of the axis of the torus (and of the ring (C_1) with it) will be exceedingly small. The same is true with regard to the period ($2\tau_2$) of these oscillations. It should also be observed that ϕ' is of the order of ω, i. e. the precessional motion is extremely slow. Finally, as ψ' differs from ψ'_0 by a quantity of the order of ω, the angular velocity of rotation of the tore about its axis will remain very nearly constant.

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Transactions of The Academy of Science of St. Louis.

VOL. XII. No. 4.

NOTES ON SOME PLANTS OF THE SOUTHWESTERN UNITED STATES.

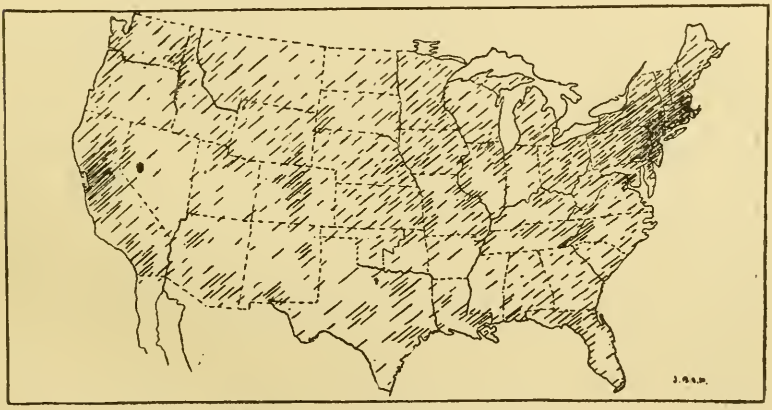
J. B. S. NORTON.

Issued May 14, 1902.

NOTES ON SOME PLANTS OF THE SOUTHWESTERN UNITED STATES.*

J. B. S. NORTON.

In connection with my work in the herbarium of the Missouri Botanical Garden during the past two years, a number of things which seemed worthy of publication came to my notice concerning the plants of the Southwest. The preparation for publication of the notes collected has been somewhat delayed by my removal to Maryland in August, 1901, and some of the species have already been published by



MAP OF THE UNITED STATES.
Showing extent of botanical exploration.

others. Mr. B. F. Bush and myself have worked up together the new species collected by him in Texas and southwestern Missouri. Some of these will be published by him in another paper.

The accompanying map shows the comparative thoroughness with which the United States has been worked for

* Presented by title to The Academy of Science of St. Louis, April 21, 1902.

botanical collections; the shading showing approximately where collections have been made, and the density of it indicating the number and extent of collections. The botanical centers of the country of course show up as spots of denser shading. The map is made up from lists of specimens found in our large herbaria of several large genera of wide distribution, species of which it would seem probable would be found in all parts of the United States. It will be seen that several regions of the west still need exploration. There is also a comparatively blank space along the eastern side of the Mississippi River. Portions of Arizona and Colorado have been well worked, and others are almost unknown. It may be that certain parts of the arid regions show blank on the map because of comparative dearth of species, northwestern Texas and part of Nevada for example. Most of the prairie region has been well explored, due to the energetic work in State botanical surveys.

There is a region extending from South Central Missouri through Arkansas and Indian Territory to northern Texas which is not perfectly covered by any of our manuals, which has recently furnished many new species and others will, with more exploration, no doubt be found. These are not mere transition forms between the Eastern and Texan or Western floras, but distinct species peculiar to this limited area which is somewhat characterized by the Ozark Mountains which may have isolated and preserved these peculiar forms.

The Middle Western States like Missouri and Arkansas would no doubt furnish a greater harvest for the "species makers" of recent years if they could be worked over as thoroughly as some of the Eastern States by persons of such tastes.

It is desirable as an aid to the study of geographical distribution, variation and the formation of species, that every distinguishable form, especially when they may also be geographically differentiated, should be published. However, in these studies, no effort has been made to discover new forms, although some have come to light simply in the determination of the plants in certain collections.

A study of the slightly variant forms of some widely distributed species in the desert regions of the Southwest seems to indicate that the isolation of mountain regions, affording favorable growth for certain species by intervening deserts where the same species cannot grow, offers a condition comparable with the differentiation of forms in insular floras. This seems to me to be especially true with some mountain species of Euphorbia, which I have examined, in which the means of dissemination is limited, most of the isolated mountains of Arizona and New Mexico having their peculiar variation not occurring elsewhere.

Cyperus longispicatus n. sp.

Perennial; 1 to 2 m. high; leaves several dm. long, 1 to 2 cm. broad; peduncle naked, surmounted by an involucre of several leaves, the larges as much as 1 m. long, 1 cm. or more wide, prominently veined, rough margined; umbel of 5 to 10 rays, some very short, others 15 cm. long, compound, with 2 to 5 spikes 1 to 3 cm. long; spikelets slightly flattened, 2 to 4 cm. long, 20- to 40-flowered, spreading, falling away from the axis and breaking up; scales concave or slightly keeled toward the apex, imbricated, ovate, short pointed, light yellowish-brown with a prominent green stripe on the back and two distinct nerves between this and the broad membranous margin, persistent on the swollen joints of the rachis, the membranous margins of which almost inclose the obpyramidal, 3-angled, light brown, pointed achenes; stamens 3; achenes 1 mm. long, the apex obtuse with a mucron which bears the 3-cleft style. — Plate V.

Type collected by B. F. Bush at San Antonio, Texas, 1900, no. 1248.

Common in water, conspicuous from its large size and the great length of the spikelets.

Dithyrea Wislizeni Engelm.

The range of this interesting crucifer is extended eastward by specimens collected in western Oklahoma by Mark White, in 1899.

ARGEMONE PINNATIFIDA n. sp.

Low, 3 dm. high, scarcely branched; leaves of the inflorescence small, thus giving a somewhat pedunculate appearance; spines few on stems and leaves, more abundant on the calyx; leaves deeply pinnatifid, the lobes deeply incised-dentate, the lobes and teeth spreading and not ascending as in related species; flowers rather small.

I have seen two specimens of this species, both from Texas. They are: Heller, Corpus Christi, no. 1378, 1894; Jermy, Gillespie Co., no. 218.

LINUM LEWISII PRATENSE n. var.

Less than 3 dm. high, wide spreading, diffusely branched with many sinuous branches, the main stem short; leaves crowded, narrowly linear, somewhat fleshy, 5 to 15 mm. long, 1 mm. wide; flowers small, whitish; capsule large, very obtuse. — Saline plains, Kansas to Texas. — Plate VI.

Type collected in Kearney Co., Kansas, Aug. 27, 1897, no. 1078, A. S. Hitchcock, in herb. M. B. G. Other specimens seen are: Hitchcock, Medicine Lodge, Kan., 1892, Sherman Co., Kan., 1892, Cloud Co., Kan., 1892; Carleton, Cheyenne Co., Kan., no. 192; Mark White, Grant Co., Okla., no. 196, 1899; P. J. White, Woods Co., Okla., no. 46, 1900; Bush, Catoosa, I. T., no. 1155; Hall, Dallas, Texas, no. 69, 1872. Transition forms occur from Texas to Arizona, for example: Toumey, Flagstaff, 1894; Wright, no. 69, W. Tex., 1849; Wislizenus, Rock Creek, N. M., no. 487, 1846; Fendler, N. M., no. 92, 1847.

The typical *L. Lewisii* Pursh, with tall, slightly branched stems, large blue flowers, larger, more lanceolate leaves and more pointed capsules, is found in the more mountainous parts of the Western States.

PLANTAGO RHODOSPERMA Decaisne, DC. Prodromus **13**: 722. 1852. (?). *P. Virginica longifolia* Gray, Synop. Flora **2**^{1}: 392. 1878.

The Texan plants referred to the above name were easily recognized by Mr. Bush as distinct from any recognized species of the United States, and they seem to be very easily placed with this Texan species of Decaisne. *P. purpurascens* Nutt. and *P. occidentalis*, Dec. may be much more easily referred to *P. Virginica* L. It is more likely that the plants referred to *P. Virginica longifolia* by Dr. Gray belong with *P. rhodosperma* than do the plants referred to it by him as synonyms. The following brief description is drawn from the plants mentioned below. The type is in DeCandolle's herbarium (see Prodromus l. c.).

Leaves entire to dentate with large blunt salient teeth; spikes several, 5 to 7 mm. thick, 12 to 20 cm. long; peduncles 5 to 7 cm. long, much shorter than the leaves; bracts as long as the calyx, rather rigid and projecting; flowers crowded, erect, the petals rather rigid, closed together, forming a point over the ripened fruit, not much longer than the tube; seeds flattish, with an indistinct border, red, 2 mm. long.—Plate VII.

The species may be readily distinguished by the large dense spike, pointed corolla and large red seeds, they being twice the length of the yellowish-brown seeds of *P. Virginica*.

Specimens have been examined as follows: Bush, Columbia, Texas, no. 107, 1900; Mearns, Ft. Clark, Texas, no. 1448, 1895; Ball, Alexandria, La., no. 552; Waugh, Oklahoma, no. 172; Palmer, Indian Territory, no. 213, 1868; Pringle, Tucson, Arizona, no. 15957; Bush, Columbia, Texas, no. 123.

Verbena polystachya HBK.

This species, reported in Gray's Synoptical Flora from California and Arizona as rare, is not at all uncommon from California to Florida. It has been confused with *V. urticaefolia* L. In the latter the sepals are but little longer than the ripened nutlets, giving the fruit a blunt appearance, while in *V. polystachya* they are enough longer than the fruit to make it appear pointed. This in addition to the thicker, smaller, less pointed, rougher leaves, and more dense spikes, easily distinguishes this tropical species. The following are representative specimens of *V. polystachya:* — Pollard, Scranton, Miss., no. 1191, 1896; Nash, Eustis, Fla., no. 1248, 1894; Curtiss, Jacksonville, Fla., no. 5111, 1894; Palmer, Biscayne Bay, Fla., no. 397, 1894; Hitchcock, Meyers, Fla., no.

269, 1900, with leaves more like *V. urticaefolia;* Tracy, South Pass, La., no. 22, 1900. Specimens are also common in West Indian collections. A specimen collected by Ball at Alexandria, La., no. 556, is a good example of *V. urticaefolia* from the Southern States.

Silphium integrifolium Michx.

Two rather distinct forms have been placed under this name in the Middle West, besides the many variations from Texas found in herbaria, most of which could probably be placed more properly with some of the Texan species now recognized. One form is a large plant with glabrous and glaucous stem or with a few hairs on the upper part, and large glabrous or scabrous leaves. This is well represented by many collections from Central Kansas. It is possible that this is the variety *laeve* of Torrey and Gray, and if so should stand as a good variety under that name if not as a species. The other form is smaller with smaller leaves, and has the stem and leaves covered with a short, dense, staring pubescence, which appears quite different from the longer scattering appressed hairs found on other forms. This occurs eastward and is well represented by the following specimens: Hitchcock, Olathe, Kan., 1892; Fink, Fayette Co., Iowa, no. 535, 1894; Wolf, Center, Ill., no. 28, 1881.

Brauneria paradoxa n. sp.

Stem glabrous or slightly scabrous or hispid, 3 to 8 dm. high; lower leaves narrowly lanceolate, 12 to 25 mm. wide, about 2 to 3 dm. long including the petiole which is sometimes 15 cm. long, glabrous or nearly so, entire; the 3 to 5 distinct nerves yellow, translucent; upper leaves smaller and wider in proportion, nearly sessile; peduncle 2 to 3 dm. long, comparatively slender, striated, somewhat pubescent at the apex; heads 2.5 cm. wide, 3 cm. high, dark brown; rays bright yellow, drooping or spreading, 4 to 6 cm. long. Common in prairie and barrens in Southwest Missouri. — Plate VIII.

The type is from Swan, Mo., collected by B. F. Bush, June 10, 1898, no. 155. Also collected by Mr. Bush at

Nichols Junction, Mo., 1898, no. 42, and Eagle Rock, Mo., 1897, no. 76, and in Texas by F. Lindheimer, Flora Texana exsiccata, 94, Fasc. 1, 1843.

HAPLOESTHES GREGGII Gray.

The range of this species is extended into Oklahoma from New Mexico by specimens collected by Mark White in Grant County, in 1899.

EXPLANATION OF ILLUSTRATIONS.

PLATES V–VIII.

Plate V. — *Cyperus longispicatus.* Habit, one-half natural size. 2, Portion of spikelet showing scales, with a joint of the rachis at base inclosing an achene, × 6.

Plate VI. — *Linum Lewisii pratense.* Habit, one-half natural size.

Plate VII. — *Plantago rhodosperma.* 1, Habit, one-half natural size. 2, Flower and seed, × 6. 3, Flower and seed of *P. Virginica*, × 6.

Plate VIII. — *Brauneria paradoxa.* Habit, one-half natural size.

Issued May 14, 1902.

CYPERUS LONGISPICATUS.

LINUM LEWISII PRATENSE.

PLANTAGO ERYTHROSPERMA.

BRAUNERIA PARADOXA.

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Transactions of The Academy of Science of St. Louis.

VOL. XII. – No. 5.

A REVISION OF THE ELEPHANTOPEAE.—I.

C. F. BAKER.

Plate 9

A Issued May 20, 1902.

A REVISION OF THE ELEPHANTOPEAE. — I.*

C. F. Baker.

Several years ago, while collecting in Alabama with Prof. Earle, the writer became convinced after numerous comparisons and observations in the field, that the three names *Carolinianus*, *tomentosus*, and *nudatus* with the very brief descriptions accompanying them in the manuals, did not at all represent the true status of the genus *Elephantopus* in the South. While at St. Louis, through the kindness of Dr. Trelease, an examination was made of the literature and of all the material at the Shaw Garden. Later, through the kindness of Dr. Robinson and Dr. Rose, the material of the Gray Herbarium and of the National Herbarium was also studied. Unfortunately the studies were not finished at that time, departure for the field preventing, but Dr. Trelease, with his usual hearty desire to place facilities in the hands of students, kindly forwarded the Shaw Garden material to Stanford University where this paper was prepared in Prof. Dudley's laboratory and various favors from the latter are here acknowledged.

These opportunities were wholly unexpected or arrangements would have been made for carrying out, in Alabama, a series of very important and necessary experiments, throwing light on many points which must remain obscure until such work is done. The range of variability in the flowers should be studied with a large quantity of fresh material at hand. No one thing of greater value could be done than the production and study of authentic hybrids between each two of the three species. Some most interesting surprises are likely to result from this. Seeds from a single parent plant (of which seeds the parentage is sure) should be raised under

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widely varying ecological conditions and the progeny carefully compared with each other and with the parent plants. It may be remarked here that such work in Experimental Ecology should have been gotten under way by our Gardens long ago. When this work shall have become coöperative it will yield results of wonderful import in systematic botany. A few years of it will give us more positive knowledge of plant forms than hundreds of years of herbarium study. Evidence drawn from the latter must remain circumstantial and opinionative. The experimental work would exercise a profound influence in the elucidation of such groups as *Viola*, *Aster*, *Antennaria*, *Apocynum*, *Senecio*, *Nemophila*, *Eschscholtzia*, *Sisyrinchium*, and the *Elephantopeae*.

The study of the North American *Elephantopae* has led into an examination of those of our tropics and finally into a study of the whole group. The notes presented here are distinctly preliminary and tentative. If they shall call marked attention to the group and the great necessity for the especial attention of field workers to it, the principal object of these first notes on the group (as a whole) to be published in English, will have been accomplished. The studies will be continued as more and better material accumulates.

Those plants grouped under the *Elephantopeae* differ from the *Vernonieae* and *Eupatorieae* in having the few-flowered (two to four) anthodia small, sessile, and gathered into compact terminal or lateral glomerulae.

In 1737 Linnaeus described the genus *Elephantopus*, of which *scaber* must be taken as the type. In 1792, Rohr followed with the genus *Pseudelephantopus*, using *spicatus* as its type. Later writers seem to have disregarded *Pseudelephantopus*, for Cassini founded *Distreptus*, and La Llave, *Matamoria*, on the same type. There seems to be no question as to the application of the unwieldly name *Pseudelephantopus* to our well-known plant. Lessing, in 1829, described *Elephantosis*, basing it on *quadriflora*, which afterwards he determined as synonymous with *angustifolius*. *Spirochaeta* Turczaninow, appeared in 1851 with *Funckii* as its type. This plant seems never to have been recognized except by its describer. The name *Euelephantopus* was first used as of sectional value by Endlicher in 1836. It is the typical section of the old genus and synonymous with *Elephantopus* Linnaeus (sens. strict.). In 1847, Schultz Bipontinus separated the section *Micropappus* for the very interesting species of the same name.

These generic and sectional names have met with varying treatment at the hands of botanical writers. In 1817 Cassini recognized *Elephantopus* and *Distreptus.* In 1829 Lessing reduced *Pseudelephantopus* to the later *Distreptus* and described *Elephantosis.* In 1836 De Candolle recognized *Elephantopus*, *Elephantosis* and *Distreptus.* In the same year Endlicher used *Euelephantopus*, *Elephantosis* and *Pseudelephantopus* as sections of the old genus, reducing *Distreptus* and *Matamoria* to synonyms of *Pseudelephantopus.* In 1830, Sprengel mentions *Matamoria* as a synonym of *Elephantopus.* In 1843 Walpers practically followed Endlicher. In 1847 Dietrich places all the species under *Elephantopus*, mentioning *Elephantopsis* and *Distreptus* as synonyms. In the same year Schultz Bipontinus rearranged the species, using the following names for sections of the old genus *Elephantopus: Euelephantopus*, *Micropappus*, *Elephantosis*, *Elephantopsis*, and *Pseudelephantopus* (= *Distreptus*). In 1873 Bentham and Hooker discussed *Matamoria* as a synonym of *Elephantopus*, *Elephantosis* as of sectional value, and *Spirochaeta* with *Pseudelephantopus* as equivalent to *Distreptus*, which is given sectional value. Finally, in the Index Kewensis, and in Engler and Prantl, all of these names are thrown together without remark under *Elephantopus.*

The groups given generic value in this paper, with their types and type localities, are as follows: —

1. *Elephantopus* L. — *scaber* L. — East Indies.
2. *Spirochaeta* Turcz. — *Funckii* Turcz. — Venezuela.
3. *Elephantosis* Less. — *angustifolius* Sw. — Jamaica.
4. *Pseudelephantopus* Rohr. — *spicatus* Sw. — Guiana.
5. *Elephantopsis* Sch. Bip. — *biflora* Less. — Brazil.
6. *Micropappus* (Sch. Bip.) — *micropappus* Less. — Brazil.

The synonymic list is as follows: —

1. *Elephantopus* L. 1737.
   = *Euelephantopus* Endl. 1836 (Sect.).
2. *Spirochaeta* Turcz. 1851.
3. *Elephantosis* Less. 1829.
4. *Pseudelephantopus* Rohr. 1792.
   = *Distreptus* Cass. 1817.
   = *Matamoria* La Llave 1824.
5. *Elephantopsis* Sch. Bip. 1847.
6. *Micropappus* (Sch. Bip.) 1847.

These names represent natural and well-marked groups in what would otherwise be a large, unwieldy, very heterogeneous assemblage of species. Even if the names are discarded we will still divide the group along the same lines. No attempt has ever been made to divide the group in any other way, and these names have been repeatedly used to designate the sections. These groups are as clearly differentiated as dozens of other so-called genera in the Compositae, as for instance *Liatris* and *Trilisa*, *Gutierrezia* and *Amphiachyris*, *Heterotheca* and *Chrysopsis*, *Brachychaeta* and *Solidago*, *Coreopsis* and *Bidens*, etc. Viewed in the light of a comparative study of the other genera, it is quite impossible, for instance, that the species *tomentosus*, *spicatus*, and *angustifolius* should be congeneric. Botanists have apparently thrown these genera together more often through a lack of knowledge, than because of any additional evidence. This seems especially evident in the case of *Spirochaeta*. With the other genera separated, *Elephantopus* L. (sens. strict.) remains a compact, homogeneous group of species, closely related and evidently congeneric. In addition, and finally, the retention of these names would be a matter of great convenience in many respects.

TABLE OF GENERA OF THE ELEPHANTOPEAE.

A. Heads four-flowered.
  B. Pappus in one series, with distinct chaffy paleae.
    C. Stem not stoloniferous; glomeruli subtended by specialized leafy bracts; inflorescence terminal or subracemose; pappus of 5 to 10 straight setae, the paleae rarely united. *Elephantopus*.
    CC. Stem stoloniferous; glomeruli subtended by simple reduced leaves; inflorescence spicate; pappus of about 4 to 6 spirally twisted setae. *Spirochaeta*.

BB. Pappus in two series, together of always more than 5 setae; inflorescence spicate; glomeruli subtended by simple reduced leaves.

C. Pappus homogeneous, of numerous straight setae having entire, scarcely distinct paleae.

D. Pappus several times length of achene. *Elephantosis.*

DD. Pappus only about a third the length of achene. *Micropappus.*

CC. Pappus heterogeneous; of comparatively few setae with strongly lacerate paleae; the two long sub opposite setae plicate near upper extremity. *Pseudelephantopus.*

AA. Heads two-flowered; pappus in outer series short, in inner longer, twisted, and deciduous. *Elephantopsis.*

Genus ELEPHANTOPUS Linn. Gen. ed. 1. 249. 1737.

In the study of the species of this genus the starting-point must be the exact determination of the type species — *scaber*. In Hort. Cliff., among the aggregate of forms there mentioned, the locality Jamaica is given. But in the Linnaean description of 1753, its habitat is given simply as "in Indiis," while Willdenow particularizes in "India orientali." Specimens occur in the American herbaria from India, the Philippines and Formosa. The plant in its native regions is low and roughly haired with usually very long narrow oblanceolate, rather obtuse, radical leaves, merely rough scabrous above, sparingly rough hairy below. The glomeruli resemble those of our *tomentosus*, the heads being about as long, but the achenes are smaller, the pappus shorter, and the paleae with much longer thicker pubescence. It seem probable that *scaber*, like *Pseudelephantopus spicatus*, has become widely disseminated in the tropics. Some specimens from the Isthmus of Panama may prove to be this, though nothing like it has been seen from elsewhere. The species *tomentosus* so far as the herbarium material examined goes, does not occur outside of the Eastern United States.

In the earlier days there seems to have been no question of the distinctness of *tomentosus*. In 1829 Lessing considered it distinct, but De Candolle in 1836, and Dietrich in 1847 do not even mention it. In 1847 Schultz Bipontinus properly reserves *scaber* for the old world form, but combines *all* the

American forms under *tomentosus*. Grisebach in Fl. British West Indies, considered *scaber* as represented in America, with *mollis* and *Carolinianus* as distinct from it. Baker in Fl. Bras. places both *mollis* and *tomentosus* under *scaber*. Hemsley in Biol. Cent. Amer. Bot. unites *tomentosus*, *Martii*, *mollis* and *Carolinianus* under *scaber*. This last must be considered as an extreme theoretical view. So far as the not inconsiderable material in American herbaria is concerned they are abundantly distinct with the possible exception of *Martii*.

Gray paved the way to a more critical separation of the forms by the description of *nudatus*. But with *nudatus* must be accepted several other forms in the United States and numerous others in the West Indies, Mexico, Central and South America, Africa and the Far East. The relationships of some of our common forms are provisionally indicated herein, but anything approaching this will be, for most of the forms, absolutely impossible at present. It seems to the author wholly immaterial at this juncture as to whether the names represent good "species" or not. They represent wonderfully distinct plant forms which must in any event be recognized, no matter to what grade of relationship they may eventually be assigned. It seems as if we would arrive at some clear and comprehensive view of the genus much sooner by means of clearly defined segregates than by shuffling off all responsibility in Hemsley's roomy aggregate. More and more attention is being given our tropical flora, so that many of the doubtful points will soon be cleared up.

SPECIES OF THE UNITED STATES.

These species, all of which occur east of the Rocky Mountains, may be listed as follows:—

1. *Carolinianus* Willd.
   (= flexuosus Raf.).
   Var. *violaceus* (Sch. Bip.).
2. *tomentosus*, L.
   (= Carolinianus var. simplex Nutt.)
   Var. *nudicaulis* (Poir.).
   (= nudicaulis Ell.).
3. *elatus* Bertol.
4. *nudatus* Gray.

They may be separated as follows: —

A. Stems leafy, lower cauline leaves like radical; paleae of pappus long triangular, gradually narrowed into the setae; vestiture of comparatively few loosely spreading hairs. *Carolinianus.*

AA. Stems scapiform, with a rosette of leaves at the base; cauline leaves differing widely from radical.

B. Paleae of pappus long triangular, narrowing gradually into setae; plant distinctly pubescent.

C. Pappus much longer than achene; glomeruli pubescent, but not densely canescent villose; heads long (10–12 mm.) with inner bracts of involucre acuminate; stems usually little branched above. *tomentosus.*

CC. Pappus usually about as long or shorter than achene; glomeruli densely canescent villose in typical forms; heads shorter (6–8 mm.) with inner bracts of involucre obtuse submucronate; stem usually much branched above. *elatus.*

BB. Paleae of pappus short and broad, suddenly narrowed into the setae; plant nearly naked. *nudatus.*

1. Elephantopus Carolinianus Willd.

Sp. Pl. 3: 2390. 1804.

This is the commonest and most easily recognized species in North America. The variety *violaceus* (Sch. Bip.) with purple pappus is not uncommon from Missouri to Louisiana and Alabama.

2. Elephantopus tomentosus Linn.

Sp. Pl. 814. 1753.

As the type of this species, is taken that form having short ovate or elliptical radical leaves. Canby's No. 62 (1898) from Alabama and Pollard's No. 1048 (1896) from Mississippi may be taken as very typical examples, though it is common from New Jersey to Texas. There is, however, a very great amount of variation in the vestiture and in the form of the floral and radical leaves. In some the radical leaves are very much narrowed proximally and it seems probable that the name *nudicaulis* Poir. applies to such varieties. Kearney's No. 738 (1897) from Tennessee and Heller's No. 77 (1890) from Pennsylvania well illustrate this form. The name *nudicaulis* Ell. apparently applies to the same thing. It was at first supposed these were the same as *elatus* Bertol., but for *nudicaulis* the involucral bracts are spoken of as longer than in *Carolinianus* while in *elatus* they are distinctly shorter.

3. Elephantopus elatus Bertol.

Misc. Bot. **11**: 21. 1851.

This very striking form from the Southern States has occasioned no little trouble. Dr. Robinson kindly furnished a transcription of the original description and a tracing of the plate. Here is referred the tall, heavily clothed, much branched form with radical leaves narrowed at base, and with short heads and pappus, as is beautifully illustrated by specimens of Chapman's, and by Rolf's No. 657, both from Florida. It apparently occurs throughout the Southern States east of the Mississippi and is somewhat variable. In the short heads it resembles *nudatus*. In its leaf forms it resembles var. *nudicaulis* of *tomentosus*. In its vestiture and length of pappus it is distinct from either. Some of the forms under consideration may be hybrids, though only actual experimentation can give us any direct evidence. There seems to be no reason why the name *elatus* is not available here.

4. Elephantopus nudatus Gray.

Proc. Am. Acad. **15**: 47. 1880.

Dr. Gray separated this species on the very short and broad paleae of the pappus, and the character of the vestiture. It occurs from Delaware to Florida and Louisiana. It is most typical in the Northeast, while towards the Southwest some very puzzling forms occur, though whether these are due to extreme variability within the species or to hybridization is impossible to say. With Prof. Earle, the author distributed some of these very peculiar forms from the Herbarium of the Ala. Biol. Survey.

The above species may be compared as to length of achene and pappus by the following table prepared by averaging in millimeters a number of measurements taken from fairly typical specimens.

ELEPHANTOPUS.	LENGTH OF ACHENE.	LENGTH OF PAPPUS.
Carolinianus	4.	5.
Tomentosus	4.3	6.8
Elatus	3.75	3.75
Nudatus	3.2	5.

The last three of these species seem usually to possess a hibernating winter rosette which the first does not have, though this calls for more extended observation.

SPECIES OF MEXICO.

In the literature at hand five species are credited to Mexico: *glaber* Sesse & Moç., *litoralis* Sesse & Moç., *cuneifolius* Fourn., *Colimensis* Sesse & Moç. and *mollis* HBK. The *glaber* of Sesse and Moçino seems to belong among some of the spicate forms. Neither *litoralis*, *Colimensis* or *cuneifolius* have been recognized among material at hand. The descriptions do not mention several most important characters. Forms of *mollis* apparently occur in this region, as illustrated by specimens of Dr. Palmer's collecting. More material and from type localities will readily clear up the whole matter. It may be here mentioned that a complete specimen suitable for study must include both flowers and fruit. Specimens in our herbaria usually lack good material of the former and too often of the latter. It seems likely that our *Carolinianus* extends to Mexico and may have been redescribed there, and similarly *tomentosus*, also.

SPECIES OF THE WEST INDIES.

Mollis has been reported from Porto Rico, Jamaica and other islands. Forms similar to our *Carolinianus* occur in Cuba and Jamaica, but the material examined is scarcely sufficient for definite determination. In Cuba occurs the very interesting and very distinct *pratensis* of Wright. It is a small plant as represented in our herbaria, low, the leaves cauline and lanceolate, the stems slender and usually numerous from a woody base, and the floral leaves glandular punctate and greatly produced at tips. This last character separates it from all other species of the tribe. The *sericeus* of Graham is described from "West Indies," though not noticed since its description.

SPECIES OF CENTRAL AND SOUTH AMERICA.

In the study of these species attention must first be centered on the exact determination of *mollis*. Dr. Gray followed Grisebach, who knew it well, in considering it a distinct and well-marked species, and our herbarium material amply supports this view. It seems to be entirely distinct from either *scaber* or *tomentosus*, which have the stem scapiform and the pappus paleae long and acuminate, gradually narrowed into the setae, while in *mollis* the stem is leafy without a rosette of leaves at base, and the pappus paleae are of the *nudatus* type — short and broad and suddenly narrowed into the setae. One of the most characteristic things about *mollis* is its vestiture. In general the plant is rather thickly clothed with rough hairs, but the lower surface of the leaves is soft tomentose or almost villose, its feeling to the touch being very characteristic. Bang's No. 497 of the Plantae Bolivianae seems to be a typical example, though it was distributed as *scaber*, which it is totally unlike. Rusby's No. 1105 from Bolivia is near *mollis* but apparently not that — certainly not *scaber* for which it was distributed. Morong's No. 258 from Paraguay is likewise neither *mollis* nor *scaber*, but apparently an interesting new thing. It seems likely that *Martii* represents a variety of *mollis*, as it is closer to it in many respects.

One of the most distinct of all the South American forms is *hirtiflorus*, which has a scapiform stem and most remarkable pappus, the limbs being flattened, not only at base but throughout their length, making it distinct in this regard from all other *Elephantopeae*. The whole plant is villose throughout — even to the corollas. The heads are of unusual length and the floral leaves are unmodified, which is unusual for species of this habitus. Likewise the coalesced and consequently coroniform paleae of *vaginatus* separates that species distinctly from all others.

The species, most of which are little known, or as to our herbaria, entirely unknown, may be listed as follows: —

1. *elongatus* Hook. — Brazil.
2. *hirtiflorus* DC. — Brazil.
3. *Martii* Grah. — Brazil.
4. *mollis* HBK. — Widely distributed.
5. *palustris* Gardn. — Brazil.
6. *paniculatus* Mart. — Brazil.
7. *racemosus* Gardn. — Brazil.
8. *Riedelii* Sch. Bip. — Brazil.
9. *riparius* Gardn. — Brazil.
10. *scaber* L. — Isthmus of Panama?
11. *vaginatus* Gardn. — Brazil.
12. *virgatus* Desv. — Guiana.

On the Isthmus of Panama also occur forms which are apparently undescribed, as for instance, Fendler's No. 163, as well as many others from other regions.

SPECIES OF AFRICA.

Only two species appear to be reported from Africa; *scaber* L. and *senegalensis* (Klatt.) O. & H., both representing distinct and well-marked forms. Herbarium material, however, indicates the existence of several other species in Africa.

SPECIES OF THE FAR EAST.

Here we find the original home of *scaber*. Besides India, it apparently occurs in Ceylon, Formosa, and many of the Malasian and Australasian islands. In the Flora Filip., Blanco describes *serratus* and *dubius*. We are likely to know more of these soon. Two species have also been described from Java — *sinuatus* Zoll. & Mor. and *ciliatus* Zoll. & Mor.

Genus Spirochaeta Turczaninow, Bull. Soc. Nat. Mosc. **24**[1]: 166. 1851.

This clearly separable genus, well described and founded on remarkably good characters, unique in the tribe, has either been wholly neglected or thrown in with *Elephantopus*, apparently because the single species did not have in the original description a separate characterization. But this cannot be held as a vital deficiency, when incorporated with the generic definition is a perfectly clear diagnosis of the species. This same thing has been done many times in other instances in both botany and zoology, and has been held as filling the requirements. The species was described from Colombia.

This plant has been collected a number of times since its description, and always labeled in our herbaria, "*Elephantopus spicatus*" with which it has scarcely anything in common excepting the spicate arrangement of the glomeruli. Indeed in the Bernhardi herbarium is a single sheet bearing a slip of *Spirochaeta* and another of *Pseudelephantopus* side by side under the same label. These specimens probably came from somewhere along the northeast coast of South America. Rusby's No. 1109 and Bang's No. 357, both from Bolivia, belong to this genus and apparently to this species, *S. Funckii*, though both were distributed as *E. spicatus*, without question or remark.

The freely rooting runners at base of stem are quite peculiar to this form, while the pappus clearly distinguishes it. This latter is in one series with the paleae minute and lacerate and the setae spirally twisted toward their tips. These setae are very brittle and easily broken off, so that an attempt to remove them entire should be made with care.

Genus Elephantosis Lessing, Linnaea. 4 : 322. 1829.

This genus, distinct by its pappus of numerous straight setae in two series, contains the common and variable West Indian *E. angustifolius* (Willd.) Less., and also several other species, but one of which, *quadriflora* Less., has ever been described. The latter was originally described from Brazil. Neither Bang's No. 344 from Bolivia nor Morong's No. 313, from Paraguay, appears to be *angustifolius*, though both were distributed as such. Far more material will be necessary to any satisfactory arrangement of the species. Lessing regarded the *nudiflorus* of Willdenow from Jamaica and San Domingo as distinct from *angustifolius*.

Genus Pseudelephantopus Rohr, Skrift. Nat. Selsk. Kobenh. 2 : 214. 1792.

All of the forms of this genus examined, present the same peculiar structure of pappus which marks them as widely distinct from any other portion of the group. Two of the setae are longer and stouter than the rest and plicate in the upper third. The chaffy paleae are very lacerate, the lacerations long, frequently becoming short setae. From the descriptions, *crispus* Cass., and *spiralis* Less., apparently do not belong in this genus. As a well recognized species, we have the very common tropical *spicatus*, first described from Jamaica, enormously variable and now introduced into the Far East. The miscellaneous series of specimens that have been picked up here and there, show this species to be composed of a number of very distinct and interesting forms. In no case in the Elephantopeae is there such need for the collection of large suites of specimens at every possible point. Kuntze in the Revis. Gen. Plant. **1**:335, recognizes two varieties, but this does not begin to enumerate the forms which exist, and hence which we must recognize. Perhaps the *glaber* of Sesse and Moçino is one of the many forms. Some specimens have the achene thickly haired, others thinly; in some the achene is about 6 mm. long, in others 7.5; in some the smaller setae of the pappus are but 5 or 6 in number, in others they are numerous; in some the floral leaves are shorter than the glomeruli, in others they are longer; and these characters occur in most confusing combination. Wide variations in vestiture and foliage commonly occur. Whether any of these be of specific value can only be ascertained by the examination of far more material.

Genus ELEPHANTOPSIS Sch. Bip. Linnaea. **20**:515. 1847.

This genus is represented only by the single species *biflora* Sch. Bip. from Brazil.

Genus MICROPAPPUS Sch. Bip. Linnaea. **20**: 515. 1847.

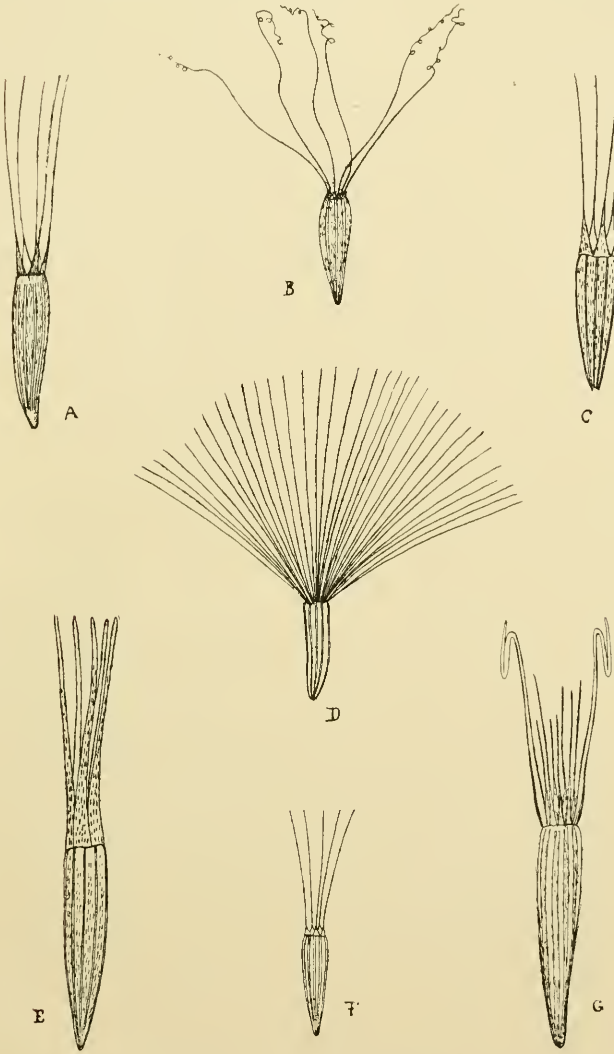
Represented only by the unique *M. micropappus* Sch. Bip. from Brazil.

EXPLANATION OF ILLUSTRATIONS.

PLATE IX.

Fig. A. — *Elephantopus Carolinianus.* — B. *Spirochaeta Funckii.* — C. *Elephantopus scaber.* — D. *Elephantosis angustifolius.* — E. *Elephantopus hirtiflorus.* — F. *Elephantopus mollis.* — G. *Pseudelephantopus spicatus.* — All drawn to same scale; see measurements of *E. Carolinianus.*

Issued May 20, 1902.



ACHENES OF ELEPHANTOPEAE.

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VOL. XII. No. 6.

THE NORTH AMERICAN SPECIES OF CHAEROPHYLLUM.

THE NORTH AMERICAN SPECIES OF TRIODIA.

B. F. BUSH.

Plates 10, 11

A *Issued June 9, 1902.*

THE NORTH AMERICAN SPECIES OF CHAERO-PHYLLUM.*

B. F. Bush.

Any one looking over a large collection of specimens of this genus, will at once be struck by the remarkable confusion of the species, such very different species as *C. Texanum*, *C. Tainturieri*, *C. procumbens*, and *C. dasycarpum*, being labeled *C. procumbens* or *C. Tainturieri* indiscriminately. All of this tends to show that the species of this genus have never been clearly understood, and it is my intention in the present paper to show that the species are readily distinguishable by good permanent characters. My attention was first drawn to this genus by some plants collected on my journeys to Texas in 1899 and 1900, which were so noticeably different from the supposedly same species of Missouri, that I began an investigation, which resulted in the conclusion that there was an unnamed species in Eastern Texas. Early in last year I had the pleasure of receiving Dr. Rose's splendid monograph of the North American Umbelliferae, in which I found he had described *Chaerophyllum Texanum*, the unnamed species that I had under consideration.

Having already done some work on the genus, I still continued to observe the species in the field during last year, and also studied the plants in the herbarium, and now present the results in this paper.

The conclusions I have come to are somewhat at variance with the disposition of the species by Dr. Rose in the monograph of the North American Umbelliferae, but Dr. Rose himself in this work differs very much from Dr. Gray, Dr. Chapman and Prof. Britton in the acceptation of species in this genus, giving specific rank to *C. Tainturieri* and establishing one more new variety, *C. Tainturieri Floridanum*,

* Presented by title to The Academy of Science of St. Louis, April 21, 1902.

besides recognizing *C. procumbens Shortii* and *C. Tainturieri dasycarpum*. Dr. Gray at first recognized only *C. procumbens* and *C. procumbens Shortii*. Dr. Chapman recognized *C. procumbens* and *C. Tainturieri*, the first being *C. Tainturieri* and the last being smooth *Tainturieri* and Dr. Rose's *C. Tainturieri Floridanum*. Dr. Britton in the Illustrated Flora recognized two species and one variety, *C. procumbens*, *C. procumbens Shortii*, and *C. Tainturieri*, the last largely made up of *C. Texanum*.

To my mind it is much better to consider these varieties as good species, especially as each exhibits individual characters that are very stable.

After several years of field observation and herbarium work, it is with no little confidence that I present the following:

ANALYSIS OF SPECIES.

Leaves coarsely divided. Northern species.	
Fruit beaked, smooth, ribs narrow.	1. *C. procumbens*.
Fruit beakless, pubescent, ribs narrow.	2. *C. Shortii*.
Leaves finely divided. Southern species.	
Fruit beakless, smooth, ribs broad.	3. *C. Texanum*.
Fruit beaked, pubescent, ribs broad.	4. *C. dasycarpum*.
Fruit beaked, smooth.	
Ribs thicker than the intervals.	5. *C. Tainturieri*.
Ribs narrower than the intervals.	
Base of fruit broad, obtuse.	6. *C. Floridanum*.
Base of fruit narrow, acute.	7. *C. reflexum*.

1. CHAEROPHYLLUM PROCUMBENS (L.) Crantz, Class. Umb. 77. 1767.

Scandix procumbens L. Sp. Pl. 1: 257. 1753.

From glabrous to very pubescent; stems weak, spreading or procumbent, or rarely erect, 2–5 dm. high; umbels sessile or peduncled, few-rayed; rays 1–5 cm. long; fruiting umbels open, of 2 to 4 fruits on weak and usually spreading pedicels 4–9 mm. long; fruit narrowly oblong, about 8 mm. long, glabrous, narrowed and commonly beaked at the apex; ribs filiform, much narrower than the broad intervals; styles very short; seed face deeply sulcate. Rich, shaded ground along streams, April and May.

Type locality: "Virginia;" collected by *Gronovius*, No. 147.

Carolinian and Louisianian areas. Alabama, Arkansas, District of Columbia, Delaware, Illinois, Indian Territory, Iowa, Kansas, Michigan, Mississippi, Missouri, New Jersey, New York, North Carolina, Ohio, Ontario, Pennsylvania, South Carolina, Tennessee and Virginia.

This species is very variable as to pubescence, some plants being nearly or quite smooth, and others pubescent to a marked degree. Fully 30 per cent. of all the specimens examined were very pubescent, the remainder being smooth or smoothish. This variability of pubescence has been the cause of much confusion, as very pubescent plants of this species have been placed with *C. Tainturieri*, and smooth or smoothish plants of *C. Tainturieri*, under this species. The species has a more northerly range than any other of the genus, reaching its southerly limit in Northern Alabama and Mississippi, the specimens upon which Dr. Chapman based his *C. procumbens* being clearly referable to *C. Tainturieri*.

SPECIMENS EXAMINED. — ILLINOIS: Proviso, *Agnes Chase*, May 17, 1900; American Bottom, *Engelmann*, 734, 1842; same locality, *Geyer*. — INDIAN TERRITORY: Limestone Gap, *Butler*, 1875. — IOWA: Moscow Lake, *Reppert*, 599, May 8, 1894; Decatur County, *Fitzpatrick*, May 8, 1897; Marshalltown, *Pammel*, 323, May 8, 1897. — KANSAS: Leavenworth County, *Hitchcock*, 699, 1896; Turner, *Mackenzie*, April 26, 1896. — MISSOURI: Greene County, *Blankinship*, May 5, 1888; same locality, *Weller*, April 20, 1889; same locality, *Shepard*, April 18, 1879; Independence, *Bush*, 149, May 23. 1894; Courtney, *Bush*, 78, April 19, 1896; Jackson County, *Bush*, April 30, 1888; Jefferson City, *Krause*, May 9, 1869; St. Louis, *Lüders;* same locality, *Engelmann*, 1833. — NEW JERSEY: Camden, *Parker*, 1866. — NEW YORK: Ithaca, collector not given, May 8, 1880. — OHIO: Cincinnati, *Lloyd*, April 26, 1890; same locality, *B. Frank*, April, 1835; Norwalk, *Stair*, May 3, 1894. — PENNSYLVANIA: Philadelphia, *M. C. Lea*, 1843; same locality, *Redfield*, 2540, May 24, 1872; same locality, *Beyrich*, in Bernhardi Herbarium. — TENNESSEE: No locality, *Buckley*.

2. CHAEROPHYLLUM SHORTII (T. & G.) B. F. Bush.

Chaerophyllum procumbens Shortii T. & G. Fl. 1: 637. 1840.

Stems 2–5 dm. tall, erect or decumbent, branched, pubescent or nearly smooth; fruit broadly oblong to ovate, about 6 mm. long, obtuse and blunt at the apex, not at all beaked, pubescent; ribs narrow, much narrower than the intervals. Moist rich ground in woods, April and May.

Type locality: "Kentucky;" collected by *Dr. Short*.

Carolinian and Louisianian areas. Arkansas, Kentucky, Louisiana, Maryland, Ohio, Pennsylvania and Virginia.

SPECIMENS EXAMINED. — OHIO: Cincinnati, *Lloyd*, 1888.

3. CHAEROPHYLLUM TEXANUM C. & R. Mon. N. A. Umb. 59. 1900.

An erect tall plant, 2–6 dm. high, usually quite smooth above the first fork, even the petioles smooth, or sometimes the lowest ones ciliolate; fruiting umbels more dense, of 8 to 15 fruits; fruit oblong, smaller and not beaked, tapering to a blunt point, 4–5 mm. long, glabrous; ribs very prominent, much wider than the intervals, nearly twice as wide and almost obliterating them; fruit with base about as wide as middle; involucres spreading. Dry, rocky barrens and prairies. May.

Type locality: near Houston, Texas; collected by *Dr. J. N. Rose*, No. 4173, May 6, 1899; type in U. S. Nat. Herbarium.

Lower Sonoran area. Kansas, Missouri and Texas.

SPECIMENS EXAMINED. — KANSAS: Cherokee County, *Hitchcock*, 1085, May 7, 1897. — MISSOURI: Glen Allen, *Russell*, May 20, 1898; Lee's Summit, *Mackenzie*, 57, May 22, 1898; same locality, *Mackenzie*, May 28, 1899. Sheffield, *Mackenzie*, May 16, 1897. — TEXAS: Columbia, *Bush*, 184, 1899.

4. CHAEROPHYLLUM DASYCARPUM Nutt. in T. & G. Fl. 1: 638. 1840.

Chaerophyllum procumbens dasycarpum (Nutt.) C. & R. Bot. Gaz. **12**: 160. 1887.

Chaerophyllum Tainturieri dasycarpum (Nutt.) Watson, Bibl. Index. 416. 1878.

Stems 2–6 dm. high, erect, much branched, very pubescent; fruit very pubescent, with a beak about one-fourth as long as the body; ribs broad, from one-third to a little over one-half as wide as the intervals. Low rich ground in fields and woods. Abundant in some places. March to September.

Type locality not given; collected by *Nuttall;* type in Gray Herbarium.

Lower Sonoran and Louisianian areas. Mississippi and Texas.

SPECIMENS EXAMINED.— MISSISSIPPI: Biloxi, *Tracy*, 4469, 1898.— AMERICA BOREALI: Bernhardi Herbarium. — TEXAS: Columbia, *Bush*, 6, 1899; same locality, *Bush*, 112, 328, 463, 1900; Hempstead, *Hall*, 260, 1872, in part. This has two specimens, the smaller whole plant is *C. dasycarpum*, the other is a branch of a nearly smooth larger plant of *C. Tainturieri;* Dallas, *Reverchon*, 2618, 1901; New Braunfels, *Lindheimer*, 616, 1848; Brazos Bottom, *Lindheimer*, 1839; Crab Apple, *Jermy;* Enchanted Rock, *Jermy;* Neuces Bay, *Heller*, 1521, 1894; no locality given, *Lindheimer*, 1847.

5. CHAEROPHYLLUM TAINTURIERI Hook. Comp. Bot. Mag. **1**: 47. 1835.

Chaerophyllum procumbens Tainturieri C. & R. Bot. Gaz. **12**: 160. 1887.

An erect tall plant, commonly pubescent, sometimes smooth or smoothish but with the foliage always more or less pubescent or puberulous, fruiting umbels more compact, of 6 to 8 fruits, either sessile or the outer on short stout erect or somewhat spreading pedicels sometimes reaching 6 mm. in length; fruit linear-oblong, 6–7 mm. long, with a distinct beak, glabrous, the beak about one-third the length of the body, the base obtuse and as wide as the widest part of the fruit; ribs very prominent, about as wide as the intervals or broader, but not obliterating them; styles longer than in *C. procumbens;* seed face less deeply sulcate; involucres spreading. Sandy soil at mostly low elevations. March to June.

Type locality: " N. Orl. " (New Orleans); collected by *M. Tainturier*.

Carolinian, Louisianian and Lower Sonoran areas. Alabama, Arkansas, Florida, Indian Territory, Louisiana, Mississippi, Missouri, Tennessee, Texas and Virginia.

SPECIMENS EXAMINED. — ALABAMA: Auburn, *Lee* and *Baker*, May 16, 1897. — ARKANSAS: Little Mamelle Bottom, *Engelmann*, 1837. — FLORIDA: Jacksonville, *Curtiss*, 1039, in part, April. This sheet contains two plants, the smaller pubescent plant is *C. Tainturieri* and the larger smooth plant is *C. Floridanum*. A sheet of the same No. 1039, in the Redfield Herbarium, has two small pubescent plants of *C. Tainturieri* and one large nearly smooth plant of *C. Floridanum;* Apalachicola, *Chapman*, three plants on one sheet, two evidently of same collection, and one of another later collection, in Chapman Herbarium; no locality, but presumably Apalachicola, *Chapman*, in Buckley Herbarium; no locality, but presumably Apalachicola, *Chapman*, in Parker Herbarium.— INDIAN TERRITORY: Sapulpa, *Bush*, 1020, May 29, 1895; same locality, *Bush*, 1083, May 13, 1895. — LOUISIANA: Lake Ponchartrain, *Lindheimer*, 1839; Baton Rouge, *Dodson*, 1896; New Orleans, *Joor*, April 1887, in Joor Herbarium. — MISSISSIPPI: Starkville, *Tracy*, 1350, April 9, 1892. —

MISSOURI: Dunklin County, *Eggert*, May 6, 1893. — TENNESSEE: Nashville, *Hubbard*, 1039, 1880; Dandridge, *Buckley*, April, 1842, in part. This sheet contains two plants, the larger is *C. Tainturieri*, the smaller is *Carum Carui*. — TEXAS: Dallas County, *Reverchon*, 2619, 1901; Mill Creek, *Lindheimer*, March, 1844, in Buckley Herbarium; no locality given, *Lindheimer*, 1844; Columbia, *Bush*, 80, 1900; Hempstead, *Hall*, 260, June 12, 1872, in part. This sheet contains two plants, the large smoothish plant is *C. Tainturieri*, the smaller one is *C. dasycarpum*; same locality, Hall, 261, June 8, 1872, in the Redfield Herbarium.

6. CHAEROPHYLLUM FLORIDANUM (C. & R.) B. F. Bush.

Chaerophyllum Tainturieri Floridanum C. & R. Mon. N. A. Umb. 60. 1900.

An erect nearly smooth or somewhat pubescent plant 2–6 dm. tall; fruit with a beak one-fourth or one-fifth the length of the body; ribs about one-half as wide as the intervals; fruit glabrous, oblong, the base about as broad as the middle; involucres spreading. Low, sandy ground. March and April.

Type locality: shell banks, Sister Islands, St. John's River, Florida; collected by *A. H. Curtiss*, No. 1040, March, 1880; type in the U. S. Nat. Herbarium, duplicate in the Missouri Botanical Garden Herbarium.

Louisianian area. Florida and South Carolina.

SPECIMENS EXAMINED: — FLORIDA: Shell banks, Sister Islands, St. John's River, *Curtiss*, 1040, March, 1880, in part. This sheet has a small pubescent plant, and a branch of a larger nearly smooth plant; the branch is *C. Floridanum*, and the whole plant is *C. Tainturieri*. A sheet of this same No. 1040 in the Redfield Herbarium has two small pubescent plants of *C. Tainturieri*, and one branch of a larger nearly smooth plant of *C. Floridanum;* Jacksonville, *Curtiss*, 1039, April, in part. This sheet also contains two plants, the larger smoother plant is *C. Floridanum*, and the smaller pubescent plant is *C. Tainturieri*. A sheet of this same No. 1039 in the Redfield Herbarium has one large nearly smooth plant of *C. Floridanum*, and two smaller pubescent plants of *C. Tainturieri;* Jacksonville, *Curtiss*, April. 1893, in part. This sheet also bears two different species, there being one large and one very small nearly glabrous plant of *C. Floridanum*, and one small pubescent plant of *C. Tainturieri*.

7. CHAEROPHYLLAM REFLEXUM n. sp.

An erect slender plant, 2–4 dm. high, commonly quite smooth above the first fork, the lower petioles sometimes ciliolate; fruit 6–8 mm. long, with a beak about one-third the length of the body, tapering to an acute base, broadest in the middle; ribs about one-half as wide as the intervals;

involucres strongly reflexed in fruit. Open rocky woods; uncommon. May and June.

Type locality: Eagle Rock, Barry County, Missouri; collected by *B. F. Bush*, No. 62, June 18, 1897; type in the Missouri Botanical Garden Herbarium, duplicates in U. S. Nat. Herbarium, and in Herbarium K. K. Mackenzie.

Lower Sonoran area. Missouri.

Specimens examined.— The type specimens are the only ones seen.

Issued June 9, 1902.

THE NORTH AMERICAN SPECIES OF TRIODIA.*

B. F. Bush.

While studying some specimens of Triodia collected in Texas, I was led to examine more closely the species of this genus, and through the kindness of Prof. Wm. Trelease was enabled to study the specimens preserved in the Herbarium of the Missouri Botanical Garden.

I now present the results of my observations, which are somewhat at variance with the commonly accepted interpretation of the species.

The species have been described under no less than five generic names, four of which must be considered as synonyms, one of which must stand for the genus, and these five generic names are: Sieglingia Bernh., Triodia R. Br., Tricuspis Beauv., Uralepis Nutt., and Windsoria Nutt., given in the order of their publication.

The first name was originally applied to some Old World forms, which are now generally conceded to be distinct from the North American species under consideration; the second name, Triodia, was given to the genus by Robert Brown in 1810, and appears to me to be the only tenable name for the genus, as the third name *Tricuspis*, given to the genus by P. Beauvois in 1812, is doubly barred by being a later name, and by being a homonym of the *Tricuspis* of Persoon, 1807, a genus in the Tiliaceae. Both of Nuttall's names are of a later date, and are therefore not tenable.

The following arrangement shows the result of my conclusions: —

TRIODIA R. Br. Prodr. Fl. Nov. Hall. **1**: 182. 1810.

Sieglingia O. Kuntze, Rev. Gen. Pl. **2**: 789. 1891, and authors, not Bernh. Syst. Verg. Pfl. Erf. 40. 1800.

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* Presented by title to The Academy of Science of St. Louis, January 6, 1902.

Tricuspis Beauv. Agrost. 77, 1812, not *Tricuspis* Pers. 1807, a genus in *Tiliaceae.*

Uralepis Nutt. Gen. **1**: 61. 1818.

Windsoria Nutt. Gen. **1**: 70. 1818. — Trans. Am. Phil. Soc. II. **5**: 147. 1837.

KEY TO THE SPECIES.

Spikelets subcompressed, oblong, in a rather dense spikelike panicle.
- Panicle elongated, spikelets about 5 mm. long.
  - Lower scales much longer than the lower flowers. 1. *T. stricta.*
  - Lower scales about as long as the lower flowers. 2. *T. albescens.*
- Panicle short, oblong, spikelets about 10 mm. long. 3. *T. congesta.*

Spikelets compressed, crowded in short subcapitate panicles on the simple nearly naked culms, bleaching white at maturity.
- Flowering scale entire or merely mucronate. 4. *T. pilosa.*
- Flowering scale deeply cleft at apex; awns from sinus exceeding the lobes.
  - Flowering scale lance-ovate; lobes subacute. 5. *T. grandiflora.*
  - Flowering scale linear-oblong; lobes obtuse. 6. *T. Nealleyi.*

Spikelets compressed, panicle verticillate, short, leafy.
- Flowering scales 4–6 mm. long. 7. *T. pulchella.*
- Flowering scales 2–3 mm. long. 8. *T. pulchella parviflora.*

Spikelets compressed, panicle open, branches spreading.
- Flowering scales entire or emarginate, sometimes mucronate.
  - Spikelets 6–12 mm. long; lower scales about one-half as long as the adjacent flowering scales. 9. *T. Texana.*
  - Spikelets 5–7 mm. long; lower scales nearly equaling or slightly exceeding the adjacent flowering scales.
    - Plant mostly smooth. 10. *T. eragrostoides.*
    - Plant scabrous throughout. 11. *T. eragrostoides scabra.*
- Flowering scales somewhat 3-toothed, the nerves all slightly excurrent.
- Panicle narrow, pedicels stout. 12. *T. Langloisii.*
- Panicle open, pedicels slender.
  - Sheaths pilose. 13. *T. Drummondii.*
  - Sheaths smooth.
    - Spikelets broad. 14. *T. Elliottii.*
    - Spikelets narrow.
      - Plant pale, panicle branches spreading with tufts of white hairs in axils of the panicle. 15. *T. Chapmani.*
      - Plant purplish, panicle branches upright. 16. *T. seslerioides.*

Spikelets terete, panicle narrowly linear.
- Lower scales both 1-nerved. 17. *T. mutica.*
- Second lower scale 3–5 nerved. 18. *T. elongata.*

1. Triodia stricta (Nutt.) Benth. ex Vasey, Gr. U. S. 35. 1883.

Windsoria stricta Nutt. Gen. 70. 1818.

Tricuspis stricta (Nutt.) Munro, A. Gray, Proc. Phila. Acad. Nat. Sci. **1862**: 335. 1863.

Tricuspis stricta Nutt. fide Heller, Cat. Pl. N. A. 28. 1900.

Tricuspis stricta (Nutt.) Thurb. Ms. name.
Sieglingia stricta (Nutt.) O. Kuntze, Rev. Gen. Pl. **2**: 789. 1891.

Culms 4.5–12 dm. tall, erect, a little compressed. Leaves 1.5–3 dm. long or more, flat, long-acuminate, smooth beneath, scabrous above; spike-like panicle 1.25–3 dm. in length, the branches appressed, the lower 2.5–5 dm. long; spikelets 4–10-flowered, 4–6 mm. long; lower scales usually about two-thirds as long as the spikelets, rarely extending beyond the flowering scales, acute, glabrous; flowering scale ovate, the nerves pilose for more than half their length, the middle and often the lateral excurrent as short tips. Low prairies and marshes; common. July to October. Perennial.

Carolinian and Louisianian areas. Alabama and Mississippi to Louisiana, Arkansas, Kansas, Indian Territory, Oklahoma, Texas and Arizona.

Type locality not ascertained.

SPECIMENS EXAMINED.— ALABAMA: Wilcox County, *S. B. Buckley*, 1878.— MISSISSIPPI: Starkville, *Kearney*, 81, 1896. — LOUISIANA: Lake Charles, *Tracy*, 3964, 1897; same locality, *K. K. Mackenzie*, 448, 1898; Feliciana, *Carpenter*. — ARKANSAS: Fayetteville, *Harvey*, 73. — INDIAN TERRITORY: Tulsa, Creek Nation, *Bush*, 795, 1894; Sapulpa, Creek Nation, *Bush*, 14[illegible]4, 1895. — KANSAS: Crawford County, *Hitchcock*, 917, 1896. — TEXAS: Texarkana, *Heller*, 4248, 1898; Dallas County, *Reverchon*, 12365; Columbia, Brazoria County, *Bush*, 1668, 1900; Houston, *Lindheimer*, 1840. — OKLAHOMA: Stillwater, *Waugh*, 60, 1893.

2. TRIODIA ALBESCENS (Munro) Benth. ex Vasey, Gr. U. S. 35. 1883.

Tricuspis albescens Munro, A. Gray, Proc. Phila. Acad. Nat. Sci. **1862**: 335, name only. 1863.
Sieglingia albescens (Munro) O. Kuntze, Rev. Gen. Pl. **2**: 789. 1891.
Rhombolytrum albescens (Munro) Nash in Britton, Man. Nor. States & Can. 129. 1901.

Culms 3–5 dm. tall, tufted, the sterile shoots one-half as long as the culm or more. Leaves smooth beneath, roughish above, acuminate, 6.25–27.5 cm. long, 2–4 mm. wide, panicle dense and contracted, white, 6.25–12.5 cm. long, 6–18 mm. broad, its branches erect or ascending, 2.5 cm. or less long; spikelets 7–11-flowered, 4–5 mm. long, the lower scales white, 1-nerved, about equal; flowering scales about 3 mm. long, 3-nerved, the lateral nerves vanishing below the apex, the midnerve excurrent in a short scabrous point. Dry prairies and barrens; not common. September and October. Perennial.

Louisianian and Lower Sonoran areas. Kansas to Texas, New Mexico and Arizona.

Type locality not ascertained.

SPECIMENS EXAMINED.— KANSAS: Kiowa County, *Hitchcock*, 916, 1896.— TEXAS: San Antonio, Bexar County, *Nealley*, 64, 1892; Crab Apple, Gillespie County, *Jermy*, 17; Fort Worth, Tarrant County, *Glatfelter*, 1898; San Antonio, Bexar County, *Bush*, 1250, 1900; Marion, Guadalupe County, *Hall*, 1872; Dallas County, *Bush*, 1171, 1900; no locality given, *Lindheimer*, 737, 1847.

3. TRIODIA CONGESTA (Dewey) B. F. Bush.

Sieglingia congesta Dewey, Contr. U. S. Nat. Herb. **2**: 538. 1894.
Tricuspis congesta (Dewey) Heller, Cat. Pl. N. A. 28. 1900.

Culms 4–8 dm. tall, from a perennial rootstock. Nodes and throats of striate sheaths usually purple; leaves 5–15 cm. long; panicle 5–8 cm. long; rays single, 1–3 cm. long, crowded with sessile spikelets; spikelets 10–15-flowered, 8–12 mm. long, turgid; lower scales slightly shorter than the adjacent flowers, 1-nerved; flowering scales 4–5 mm. long, ovate-oblong, obtuse, mucronate, the lateral nerves slightly or not at all excurrent, pubescent on the nerves near the base; palet one-fourth shorter than the flowering scale, the keels prominently arcuate near the base. Plains and rocky barrens; not common. September to October. Perennial.—Plate X.

Lower Sonoran area. Texas.

Type locality: "Corpus Christi, Texas;" collected by *Nealley*.

SPECIMENS EXAMINED.— Corsicana, Navarro County, *Reverchon*, 2558; Dallas County, *Reverchon*, 121, 1879; Corpus Christi, *Nealley*, 24, 1891, 1893; Guadalupe, *Wright*; Comanche Peak, Hood County, *Reverchon*, 3456, September. July.

4. TRIODIA PILOSA (Buckley) B. F. Bush.

Uralepis pilosa Buckley, Proc. Phila. Acad. Nat. Sci. **1862**: 94. 1863.
Tricuspis acuminata Munro, A. Gray, Proc. Phila. Acad. Nat. Sci. **1862**: 335, name only. 1863.
Triodia acuminata (Munro) Benth. ex Vasey, Gr. U. S. 35. 1883.
Sieglingia acuminata (Munro) O. Kuntze, Rev. Gen. Pl. **2**: 789. 1891.

Sieglingia pilosa (Buckley) Nash, Britt. & Brown, Ill. Fl. **3.** append. 504. 1898.

Tricuspis pilosa (Buckley) Heller, Cat. N. A. Pl. 28. 1900.

Culms 6.25–30 cm. tall, tufted, the sterile shoots 1 dm. tall or less. Sheaths smooth, a tuft of hairs on each side of the apex, much shorter than the internodes; leaves strict or curved, thick, linear, obtuse, 1-nerved, the margins white, serrulate, 3.75 cm. long or less, less than 2 mm. wide, folded, at least when dry, pubescent with long hairs, especially beneath; panicle almost raceme-like, long-exserted, 1.25–3.75 cm. long; spikelets 3–10, crowded, 8–12-flowered; lower scales acuminate, 1-nerved; flowering scales 6.–6.5 mm. long, acuminate, 3-nerved, the midnerve generally excurrent as a short tip, all the nerves pilose, the lateral at the top and bottom, the midnerve below the middle. Dry prairies and barrens; common southwestward. April to October. Perennial.

Upper and Lower Sonoran areas. Kansas and Colorado to Indian Territory, Oklahoma, Texas, New Mexico, Arizona and Mexico.

Type locality not ascertained.

Specimens examined. — Kansas: Gove County, *Hitchcock*, 914, 1896; Gray County, *Hitchcock*, 1897.— Colorado: Cañon City, *Shear*, 982, 1896.— Oklahoma: Gloss Mountains, *Mark White*, 165, 1899.— Texas: Dallas County, *Reverchon*, 1112, 1900; same locality, *Bush*, 631, 1900; Kerrville, Kerr County, *Miller*, 1637; Gillespie County, *Jermy*, 55; Santa Anna, Coleman County, and Chenate Mountains, Presidio County, *Nealley*, 817; San Antonio, *Bush*, 331, 1901. — New Mexico: Lincoln County, *Josephine Skehan*, 34, 1898; no locality given, *Fendler*, 738, 915, 1847. — Mexico: Monterey, *Trelease*, 1900; Diaz, *Pringle*, 8306, 1900.

5. Triodia grandiflora Vasey, Contr. U. S. Nat. Herb. 1 : 59. 1890.

Sieglingia avenacea grandiflora (Vasey) Dewey, Contr. U. S. Nat. Herb. **2** : 538. 1894.

Sieglingia grandiflora (Vasey) Beal, Gr. N. A. **2** : 471. 1886.

Tricuspis grandiflora (Vasey) Heller, Cat. Pl. N. A. 28, 1900.

Culms 2–3 dm. tall; leaves narrow, rigid, plane or conduplicate, 5–10 cm. long, lower with the sheaths softly pubescent; panicle oblong, dense, 3–5 cm. long, branches appressed; spikelets 6–8 flowered, 8–10 mm. long; lower scales unequal, lanceolate, the upper one 8 mm. long, 1-nerved, the lower one rather shorter, 3-nerved; flowering scales 7–8 mm. long, acute, apex 2-lobed, lobes acute, the fissure less than 2 mm. long, the awn about 2 mm. long, the lateral nerves densely ciliate the entire length, and the midrib below; palet narrow, a third as long as its scale, pubescent on the nerves, abruptly acute. Rare in high plains. August to October. Perennial.

Lower Sonoran area. Texas to Arizona, New Mexico and Mexico.

Type locality: "Chenate Mountains, Presidio County, Texas;" collected by *Nealley*.

Specimens examined.— Mexico: Chihuahua, *Pringle*, 406, 1885. — Cadenas, *Pringle*, 3930, 1891. — Texas: Chenate Mountains, Presidio County, *Nealley*, 823; Cibolo Cañon, *Nealley*, 154, 1892; Western Texas, *Wright*, 751.

6. Triodia Nealleyi Vasey, Bull. Torr. Club. **15**: 45. 1888.

Sieglingia Nealleyi (Vasey) Dewey, Contr. U. S. Nat. Herb. **2**: 538. 1894.

Tricuspis Nealleyi (Vasey) Heller, Cat. Pl. N. A. 28. 1900.

Culms 3–5 dm. tall, slender. Leaves short, blunt pointed, strongly keeled; panicle lance-oblong, 3–5 cm. long; spikelets 5–8 flowered, 6–8 mm. long; first empty scale nearly equaling and second slightly exceeding the lower flowers; flowering scales linear-oblong, deeply cleft, the lobes obtuse, prominently pubescent; palet about one-half as long as the flowering scale. Rare in mountain cañons. August to October. Perennial.—Plate XI.

Lower Sonoran area. Texas.

Specimens examined. — Texas: Chenate Mountains, Presidio County, *Nealley*, 825; Sierra Blanca, Presidio County, *Nealley*, 2305. 1893.

7. Triodia pulchella H. B. K. Nov. Gen. & Sp. **1**: 155. *t. 47*. 1815.

Tricuspis pulchella (H. B. K.) Torr. Pac. R. Rept. 156. 1857.

Uralepis pulchella (H. B. K.) Kunth, Rev. Gram. **1**: 108. 1829.

Sieglingia pulchella (H. B. K.) O. Kuntze, Rev. Gen. Pl. **2**: 789. 1891.

Culms 1 dm. tall, fasciculately branched from a stoloniferous base. Leaves 1–3 cm. long, fasciculate, involute; panicles short, leafy, verticillate; spikelets 5–10 flowered, 5–8 mm. long, white; flowering scales 4–6 mm. long, prominently pubescent, oblong, cleft to the middle. Dry sandy mesas; common southwestward. July to October. Perennial.

Lower Sonoran area. Texas to New Mexico, Utah, Nevada, Arizona, California, Mexico and Colorado.

Type locality: "In subfrigidis, siccis, apricis regni Mexicana inter Guanaxuato, Mina de Belgrado et Cubilente, alt. 1050 hexap."

SPECIMENS EXAMINED. — TEXAS: Chenate Mountains, Presidio County, *Nealley*, 826; Sierra Blanca, *Nealley*, 1893.— COLORADO: San Juan, *Brandegee*, 1203, 1875 — NEW MEXICO: Las Cruces, *Wooton*, 457, 1897; no locality given, *Wright*, 2059, *Parry*, 1867.— UTAH: No locality given, *Parry*, 260, 1874.— ARIZONA: Cienega, *Rothrock*, 575, 1874; Tucson, *Toumey*, 89, 1894; no locality given, *Pringle*, 540, 1882.— CALIFORNIA: Los Angeles, *Palmer*, 500, 652 and 1359.— MEXICO: No locality given, *Schaffner*; Durango, *Palmer*, 740, 1896.

8. TRIODIA PULCHELLA PARVIFLORA Vasey, Gr. U. S. 66. 1885.

Sieglingia pulchella parviflora (Vasey). Dewey, Contr. U. S. Nat. Herb. 2: 538. 1894.

Culms 1 dm. tall, slender, fasciculately branched from a stoloniferous base. Leaves .5–1.5 cm. long, fasciculate, involute; panicles very slender, leafy, verticillate; spikelets 3–4-flowered, 3–6 mm. long, white; flowering scales 2–3 mm. long, pubescent, oblong, cleft to the middle. Dry sandy mesas. August to October. Perennial.

Lower Sonoran area. Texas and Arizona to California.

Type locality not ascertained.

SPECIMENS EXAMINED.— CALIFORNIA: San Diego County, *Orcutt*, 1487, 1888.

9. TRIODIA TEXANA (Thurb.) Benth. ex Wats. Proc. Am. Acad. **18**: 180. 1883.

Tricuspis Texana Thurb. S. Wats. Proc. Am. Acad. **18**: 180. 1883.
Sieglingia Texana (Thurb.) O. Kuntze, Rev. Gen. Pl. **2**: 789. 1891.

Culms 3–5 dm. tall, slender, leafy. Leaves 2–3 dm. long, often exceeding the panicle; spikelets 6–10-flowered, 6–12 mm. long; lower scales unequal, about one-half as long as the adjacent flowers; flowering scales broadly ovate or rotund; palet broadly ovate or almost hastate lobed at the base, one-third shorter than the flowering scale. Mountains and hills; common. August to October. Perennial.

Lower Sonoran area. Louisiana and Texas to Arizona, New Mexico and Mexico.

Type locality not ascertained.

SPECIMENS EXAMINED — TEXAS: San Diego, *Nealley*, 65, 1892; Point Ysabel, Cameron County, *Nealley*, 829; Laredo, *Mackenzie*, 103, 1900; San Antonio, *Bush*, 810, 811, 1901. — MEXICO: Monterey, *Pringle*, 1970, 1888; Monclova, *Palmer*, 1371, 1880. — NEW MEXICO: No locality given, *Wright*, 776, 777 and 2055.

10. TRIODIA ERAGROSTOIDES V. & S. Contr. U. S. Nat. Herb. **1**: 58. 1890.

Sieglingia eragrostoides (V. & S.) Dewey, Contr. U. S. Nat. Herb. **2**: 539. 1894.

Tricuspis eragrostoides (V. & S.) Heller, Cat. Pl. N. A. 29. 1900.

Culms 6–9 dm. tall, leafy; sheaths longer than the internodes, roughish; ligule short, ciliate toothed; leaves 2–3 dm. long, flat, scabrous, acuminate; panicle large and spreading, 3 dm. long, the branches slender, rather distant, single or in twos, the lower ones 12.5–15 cm. long, lax-flowered; spikelets 5–9-flowered, 5 mm. long, short-pediceled, alternate, and mostly single; lower scales nearly equal, lanceolate-acuminate, 1-nerved; flowering scales 2–2.5 mm. long, 3-nerved, oblong, obtuse, emarginate, short-cuspidate, the lateral nerves and midrib pubescent below; palet one-fourth shorter than the flowering scale, obtuse and denticulate. Sandy soil; common. August to October. Perennial.

Louisianian area. Florida and Texas to New Mexico and Mexico.

Type locality: "Florida, *Blodgett*, Texas, *Buckley*, *Nealley*, *Reverchon*."

SPECIMENS EXAMINED.— MEXICO: Monterey, *Pringle*, 1972, 1888.—TEXAS: No locality given, *Nealley*, 822, *Reverchon*.— NEW MEXICO: No locality given, *Wright*, 426, 478 and 2054.

11. TRIODIA ERAGROSTOIDES SCABRA (Vasey) B. F. Bush.

Sieglingia eragrostoides scabra Vasey, Beal, Gr. N. A. **2**: 65. 1896.

Plant scabrous throughout; lower blades broader.

Louisianian area. Texas.

Type locality: "San Diego, Duval County, Texas."

SPECIMENS EXAMINED.— TEXAS: San Diego, Duval County, *Nealley*, 96. 1892.

12. TRIODIA LANGLOISII (Nash) B. F. Bush.

Tricuspis Langloisii Nash, N. Y. Bot. Gard. Bull. **1**: 293. 1899.

Culms 5–15 dm. tall, erect, tufted, round; basal leaves crowded, equitant and shining: culm leaves two or three; sheaths coarsely striate, much shorter than the intervals, often pilose at the apex; ligule a ring of copious silvery hairs about 1 mm. long; blades erect or ascending, flat, sometimes folded when dry, glaucous and minutely pubescent on the upper surface, long acuminate, the basal 2–4 dm. long and 2–5.5 mm. wide, the lower culm leaves 7–15 cm. long, and 2–3.5 mm. broad, the upper blade much smaller, 2 cm. or less long; panicle narrow, 1–2 dm. long, about 2 cm. broad, its triangular branches arranged singly, rarely in pairs, erect or nearly so, the larger 6–10 cm. long and usually subdivided; spikelets 6–7 mm. long, oval when mature, appressed to the branches, on stout pedicels about 1 mm. long; scales six to eight, the outer empty two acute, 1-nerved, or the second rarely 3-nerved, the lower half with ascending hairs about 5 mm. long, the callus pilose, the lower scales about 4 mm. long, and 2.25 mm. wide when spread out; palet as long as or a little shorter than the scale, 2-nerved, the nerves ciliate and about equally curved at the base and the apex, hence the internerve is elliptic or nearly so, about 1.6 mm. wide. Pine-woods and clearings. Perennial.

Louisianian area. Florida, Mississippi and Louisiana.

Type locality: "Slidell, Louisiana, in pine-woods clearings; collected by *A. B. Langlois;* type in Herbarium of Geo. V. Nash."

SPECIMENS EXAMINED.— FLORIDA: No locality given, Chapman, several collections. — MISSISSIPPI: Biloxi, *Tracy*, 1891; same locality, *Tracy*, 1897; same locality, *Kearney*, 208, 1896; Waynesboro, *Kearney*, 114, 1896; Bayou Porto, *Tracy*, 3869, 1897. — LOUISIANA: Slidell, *Langlois*, 1891.

13. TRIODIA DRUMMONDII (S. & K.) B. F. Bush.

Tricuspis Drummondii S. & K. Bull. U. S. Agrost. **4**: 37. 1897.

Culms 12–16 dm. tall, slender, erect, from strong, scaly rootstocks. Radical leaves 24–48 cm. long; nodes 2–4, dark purple; sheaths of the basal leaves crowded, somewhat compressed, closely imbricated, sparsely to densely pilose with long white hairs; upper leaf sheath shorter than the internodes, glabrous or pilose at the throat; ligule a dense fringe of very short white hairs; blades of the radical leaves about 5 mm. wide, alternate, acuminate, and involute toward the apex, shortly pilose below near the apex; uppermost cauline leaf 4 cm. long or less; panicle 18–24 cm. long, contracted, somewhat drooping, simple, the appressed rays solitary, the lowermost 2.5–5 cm. long, slightly glandular, but not villose, in the axils; spikelets commonly 3-flowered, 8–10 mm. long; lower scales ovate-acute, 1-nerved, whitish or purplish, except the prominent nerve, 4–5 mm. long, subequal; first flowering scale 5–6 mm. long, ovate-lanceolate, trifid, 3-nerved, the nerves extending into short, awn-like teeth, the central one equaling or a little exceeding the narrow obtuse lobes of the scale, nerves ciliate in the lower half with rather long, erect white hairs; palet slightly shorter or a little longer than the scale, oblanceolate, obtuse, minutely ciliate along the keels towards the apex. Dry soil in low pine barrens. July to October. Perennial.

Louisianian area. South Carolina and Georgia to Florida and Mississippi.

Type locality: "Jacksonville, Fla., *Drummond;* Aiken, S. Carolina, *Ravenel; Biloxi*, Miss., *Kearney*, 324. 1896. There is also a specimen in the National Herbarium from Georgia without locality."

Specimens examined. — Mississippi: Manuel, *Tracy*, 4560, 1898. — South Carolina: Aiken, *Ravenel*.

14. Triodia Elliottii B. F. Bush.

Poa ambigua Ell. Sk. 1: 165, 1817, not *Triodia ambigua*, R. Br. 1810.

Tricuspis ambigua (Ell.) Chapm. S. Fl. 559. 1860, not *Triodia ambigua* R. Br. 1810.

Triodia ambigua (Ell.) Benth. ex Vasey, Gr. U. S. 35. 1883, not *Triodia ambigua* R. Br. 1810.

Sieglingia ambigua (Ell.) O. Kuntze, Rev. Gen. Pl. 2: 789. 1891, not *Triodia ambigua* R. Br. 1810.

Culms 4–8 dm. tall, simple. Leaves attenuate, 1–3 dm. long; panicle 1–2 dm. long, open, branches spreading;

spikelets 5–8-flowered, 4–6 mm. long, short-oblong, broadly oval when mature, usually purple; lower scales slightly shorter than the lower flowers; flowering scales broadly oblong, pubescent below the middle; palet equaling the flowering scales, ciliolate on the nerves which are more curved at the base than at the rounded and apiculate apex. Low swampy pine barrens. July to October. Perennial.

Louisianian area. South Carolina and Georgia to Alabama, Florida, Texas and Arizona.

Type locality: "Found in the mountains of Carolina, by Dr. McBride, and in the lower country of Georgia, by Dr. Baldwin."

Specimens examined.— Florida: Apalachicola and Quincy, *Chapman;* Duval County, *Curtiss*, 10736; Jacksonville, *Curtiss*, 3455, 1877.— Texas: Point Ysabel, *Nealley*, 819.— Louisiana: De Quincy, *Bush*, 973, 1901.

15. Triodia Chapmani (Small) B. F. Bush.

Sieglingia Chapmani Small, Bull. Torr. Club. 22 : 365. 1895.
Tricuspis Chapmani (Small) Heller, Cat. Pl. N. A. 28. 1900.

Culms 9–15 dm. tall, mostly purple about the nodes, erect, wiry, glabrous, bright green. Lower leaves rather numerous, nearly erect, 4–6 dm. long, the upper few, divaricate, somewhat shorter, all firm, flat when young, soon involute and almost filiform, 7–11-ribbed, smooth and glabrous; lower sheaths about 1 dm. long, the upper ones often 2 dm. long, all 1/3 to 1/2 shorter than the internodes; ligule a short fringe of rigid villous hairs, above which on the surface of the leaf, is a tuft of longer villous hairs; panicle averaging about 2 dm. high, viscid above, broadly ovoid, its branches rigid, filiform, divaricate, the nodes tufted with bunches of silvery-villous more or less viscid hairs; spikelets very slender, pedicelled, rather few, 7–8 mm. long, tinged with purple, almost linear, about 5-flowered; empty scales lanceolate, 1-nerved, the lower one 3/4 longer than the upper; flowering scales oblong-elliptic, 5-nerved, 3-pointed by the excurrent nerves which are villous for one-half their length; palet 2-nerved, scabrous on the two nerves, slightly curved. Dry sandy pine barrens. August to October. Perennial.

Louisianian area. Georgia and Florida to Alabama and Texas.

Type locality: Slopes of Currahee Mountain, near Toccoa, Georgia, and along Yellow River in Gwinnett County, in the same State; collected by *J. K. Small* 1894; type in Herbarium of the New York Botanical Garden.

SPECIMENS EXAMINED. — FLORIDA: No locality given, *Chapman*, various collections; Duval County, *Curtiss*, 3454 — TEXAS: Paris, Lamar County, *Heller*, 4222; Texarkana, *Letterman*, 16, 1894; Columbia, Brazoria County, *Bush*, 952, 1901; no locality given, *Reverchon*; Camp 4, *Bigelow*.

16. TRIODIA SESLERIOIDES (Michx.) Benth. ex Vasey, Gr. U. S. 35. 1883.

Poa seslerioides Michx. Fl. Bor.-Am. 1: 68. 1803.
Triodia cuprea Jacq. Eclog. Gram. 2: 21. *pl. 26.* 1814.
Tricuspis seslerioides (Michx.) Torr. Fl. N. & Mid. U. S. 1: 118. 1824.
Sieglingia seslerioides (Michx.) Scribn. Mem. Torr. Bot. Club. 5: 48. 1894.
Sieglingia cuprea (Jacq.) Millsp. Fl. W. Va. 471.

Culms 6–15 dm. tall, erect, somewhat flattened, often viscid above. Sheaths sometimes villous at the summit, the lower short, overlapping and crowded, the upper longer, equaling or shorter than the internodes; leaves 1–3 dm. long or more, 6–12 mm. wide, flat, attenuate into a long tip, smooth beneath, scabrous above; panicle 1.5–4.5 dm. long, the branches finally ascending or spreading, the lower 1–2.5 dm. long, usually dividing above the middle; spikelets 4–8-flowered, 6–8 mm. long, purple; empty scales glabrous, obtuse, generally slightly 2-toothed; flowering scales oval, the nerves pilose, excurrent as short tips. Dry borders of woods and fields. July to October. Perennial.

Carolinian and Louisianian areas. Maine and New York to Minnesota, Iowa, Kansas, Missouri, Arkansas, Indian Territory, Illinois, Alabama, Florida, Tennessee, Texas, Louisiana, Oklahoma, Ohio, New Jersey, Mississippi and North Carolina.

Type locality: "Hab. in region Illinoensi et in montosis Carolinae."

SPECIMENS EXAMINED.— TEXAS: Dallas County, *Reverchon*, 1111, 1899; Marshall, *Bush*, 1015, 1901.— MISSOURI: Eagle Rock, Barry County, *Bush*, 1896; Clay County, *Mackenzie*, 646, 1901.— KANSAS; Wyandotte County, *Mackenzie*, 1896; Manhattan, *Kellerman*, 1888.— ARKANSAS: Fulton, Hempstead County, *Bush*, 1044, 1901.— INDIAN TERRITORY: Sapulpa, Creek

Nation, *Bush*, 1445, 1895.— OHIO: Cincinnati, *Lloyd*, 585, 1890.— NEW JERSEY: Camden, *Parker*, 9414, 1863.— OKLAHOMA: Stillwater, *Waugh*, 63, 1893.— NORTH CAROLINA: Cherokee, Swain County, *Beardslee & Kofoid*, 1891.— TENNESSEE: Point Rock, *Kearney*, 957, 1857. — ILLINOIS: Augusta, *Mead*, 1843. — MISSISSIPPI: Woodville, *Joor*, 1888. — PENNSYLVANIA: Manayunk, *Redfield*, 9415, 1870.— FLORIDA: Tallahassee, *Nash*, 2406, 1895.— NEW YORK: Harlem, *F. S. P.*, 1859.

17. TRIODIA MUTICA (Torr.) Benth. ex Wats. Proc. Am. Acad. **18** : 180. 1883.

Tricuspis mutica Torr. Pac. R. R. Rept. **4** : 156. 1857.
Sieglingia mutica (Torr.) O. Kuntze, Rev. Gen. Pl. **2** : 789. 1891.

Culms 3–5 dm. tall, slender, wiry. Leaves mostly involute, rather rigid; panicle 1–2 dm. long, branches appressed, spikelets 5–8-flowered, 8–10 mm. long; lower scales nearly equal, shorter than the lower flowers; flowering scales oblong, entire or emarginate, prominently pubescent near the base, nearly twice as long as the palet. Dry hills and barrens. June to October. Uncommon. Perennial.

Lower Sonoran area. Texas and Colorado to Arizona and Mexico.

Type locality not ascertained.

SPECIMENS EXAMINED. — TEXAS: Dallas County, *Reverchon*, 239. — ARIZONA: Tucson, *Pringle*, 13925, 1881; Fort Whipple, *Palmer*, 584, 1865. — MEXICO: Monterey, *Pringle*, 1980, 1888.

18. TRIODIA ELONGATA (Buckley) B. F. Bush.

Uralepsis elongata Buckley, Proc. Phila. Acad. Nat. Sci. **1862** : 189. 1863.
Tricuspis trinerviglumis Munro, A. Gray, Proc. Phila. Acad. Nat. Sci. **1862** : 333, name only. 1863.
Triodia trinerviglumis (Munro) Benth. ex Vasey, Gr. U. S. 35. 1883.
Sieglingia trinerviglumis (Munro) O. Kuntze, Rev. Gen. Pl. **2** : 789. 1891.
Sieglingia elongata (Buckley) Nash, Britt. & Brown, Ill. Fl. **3**. append. 504. 1898.
Tricuspis elongata (Buckley) Heller, Cat. Pl. N. A. 28. 1900.

Culms 3–9 dm. tall, tufted, erect, rough, the sterile shoots about one-half as long as the culms. Sheath rough, a ring of hairs at the apex; leaves rough, usually involute when dry, 7.5–25 cm. long, 2–4 mm. wide; panicle narrow, 1.25–2.5 dm. long, 1.25 cm. wide, its branches erect, 3.75 cm. long or less; spikelets 10–12-flowered, 9–12 mm. long, the empty scales scabrous, the first 1-nerved, the second 3-nerved; flowering scales about 6 mm. long, obtuse at the scabrous apex, 3-nerved, the lateral nerves vanishing at or below the apex, the mid-nerve usually excurrent as a short tip. Barrens and rocky prairies. Common southwestward. June to October. Perennial.

Upper Sonoran area. Missouri and Kansas to Arkansas, Indian Territory, Oklahoma, Texas, Colorado and Arizona.

Type locality not ascertained.

Specimens examined.— Missouri: Eagle Rock, Barry County, *Bush*, 639, 1896.— Texas: Dallas County, *Reverchon*, 445; same locality, *Bush*, 1134, 1900; same locality, *Reverchon*, 1109, 1900.— San Antonio, *Bush*, 842, 1901.— Kansas: Chautauqua County, *Hitchcock*, 919. 1896.— Colorado: Cañon City, *Brandegee*, 360, 1874.

EXPLANATION OF ILLUSTRATIONS.

Plates X.-XI.

Plate X.— *Triodia congesta,* habit, reduced one-half, and spikelet, × 2.
Plate XI.— *Triodia Nealleyi,* habit, reduced one-half, and spikelet, × 3.

Issued June 9, 1902.

TRIODIA CONGESTA.

TRIODIA NEALLEYI.

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VOL. XII. No. 7.

NEW PLANTS FROM MISSOURI.

K. K. MACKENZIE AND B. F. BUSH.

Issued June 9, 1902.

NEW PLANTS FROM MISSOURI.*

K. K. Mackenzie and B. F. Bush.

A more intimate acquaintance with the plants of Missouri has convinced the writers that there are quite a number of species that have never been described or have been mistakenly referred to species already described. A number of these have been under consideration for several years, but owing to lack of proper material for study, or for comparison, the results of our investigations have been held back until the present time. We now present the first paper upon the subject, hoping in a short time to be able to present the complete results of our observations. In the study of the following new species we have been very much assisted by Prof Wm. Trelease, who very kindly loaned us specimens for study of related species in the Herbarium of the Missouri Botanical Garden, and to whom we hereby acknowledge our indebtedness.

Muhlenbergia polystachya n. sp.

An erect or ascending much branched glabrous perennial, 6–9 dm. high, with long scaly rootstocks; nodes enlarged; sheaths equaling or somewhat exceeding the internodes; leaves 5–17.5 cm. long, 3–5 mm. wide at base, gradually attenuate, rough on both sides, especially so above, flat, or in drying slightly revolute; panicles 5–10 cm. long, long exserted, the branches all appressed and ascending, two to three together, the lower branches separated, the upper contiguous and usually much overlapping, the branches each bearing from three to about twenty-five sessile spikelets; spikelets 2 mm. or less long, the two outer glumes short cuspidate, about equal and from very slightly shorter to very slightly

* Presented by title before The Academy of Science of St. Louis, January 6, 1902.

longer than the equal and acuminate flowering glume and palet, lanceolate, scarious margined, strongly 1-nerved, the short cusps minutely barbed and the upper portion of the glumes minutely serrulate; basal hairs rather copious, half of the length of the flowering glume; flowering glume lanceolate, 3–5 nerved, the nerves scabrous, obtuse, but the middle nerves excurrent as a minute cusp. — Plate XII.

Type locality: near Sibley, Jackson County, Missouri; collected by *K. K. Mackenzie*, No. 637, October 14, 1901; type in Herb. K. K. Mackenzie, duplicate in Herb. Missouri Botanical Garden.

Rocky ground at the foot of low wooded bluffs along the Missouri River near Sibley, Jackson County, Missouri. This species is most closely related to *M. Mexicana* (L.) Trin. It is distinguished by its long exserted densely flowered inflorescence, its smaller spikelets and larger and more copious basal hairs.

The type specimens are the only ones known.

Iris foliosa n. sp.

Perennial from stout rootstocks; stems 20–38 cm. high, glabrous, usually very flexuous; leaves green, not glaucous, 12–28 mm. wide, strongly many nerved, the lower often over 6 dm. long and much exceeding the culm, the upper short, and the uppermost one or two sometimes but 7.5 cm. long; flowers axillary on pedicels 20–28 mm. long; bracts scarious, 3.75–7.5 cm. long, reaching beyond the perianth tube and in fruit loosely inclosing the capsule; perianth tube 14–22 mm. long; perianth segments about 3.75 cm. long, spreading, not crested, bluish; capsule oblong-cylindric, hexagonal, 3.75 cm. long or less, abruptly contracted at the apex and short beaked; seeds in two rows in each cell.

Type locality: Little Blue Tank, Jackson County, Missouri; collected by *K. K. Mackenzie*, June 6, 1897; type in Herb. K. K. Mackenzie, duplicate in Herb. Missouri Botanical Garden. Grows in dense masses in low open dry woods and prairies in Kentucky, Illinois, Missouri and Kansas. This species is distinguished from *Iris hexagona* Walt., a species of the Southern States, to which it has been referred by Watson and other American botanists, by its smaller pedicelled flowers.

Specimens examined.— Missouri: Type specimens as cited under type locality; Adams, Jackson County, *K. K. Mackenzie*, 159, June 12, 1898; Greenwood, Jackson County, *K. K. Mackenzie*, June 11, 1899; Glendale, Jackson County, *K. K. Mackenzie*, 158, June 12, 1898.— Kansas: Rosedale, Wyandotte County, *K. K. Mackenzie*, July 12, 1896.— Illinois: Carlinville, Macoupin County, *W. G. Andrews*, June 15, 1891, March 30, 1892, August 13, 1892.

Portulaca neglecta n. sp.

Annual, of robust growth, forming broad tufts, mostly 5–10 dm. wide; stems mostly ascending, large and thick, sometimes 12 mm. thick; leaves large, obovate-spatulate, 25–50 mm. long, and 12–25 mm. wide at widest part, mostly retuse or obtuse at apex, thickish, but equally thick across leaf; flowers beginning to appear about the first of August, opening in direct sunshine about 7:40 a. m., large for the section, 8–12 mm. wide; petals oblong, clear yellow, deeply cleft; sepals in bud large, ovate-oblong, acute or obtusish, carinate-winged at top; stamens numerous, from 12 to 18; style scarcely any; stigmas long, plumose, 5- or 6-parted; capsule large, 8–12 mm. long; seeds minutely, but under a lens distinctly, tuberculate, .7 mm. long, and .6 mm. wide, blackish when mature.

Type locality: Courtney, Jackson County, Missouri; collected by *B. F. Bush*, 1148, Aug. 24, 1900; type in Herb. Missouri Botanical Garden.

This species is abundant in bottoms along the Missouri River, and on rich prairies in Missouri and Kansas, and probably elsewhere. It takes the place in greater part of what has been passing for *P. oleracea* L. in Missouri, Kansas and Arkansas, and is very likely what has been taken for *P. retusa* Engelm., in Minnesota, Missouri and Kansas. It is easily distinguished from *P. oleracea* L., by its larger size, larger and broader, thinner leaves, more numerous stamens, different time of flowering, and its upright or ascending stems. Where the two grow together, *P. oleracea* L. opens its flowers about 9:30 a. m. From *P. retusa* Engelm., it is distinguished by its thicker leaves, larger flowers, and its smaller seeds.

SPECIMENS EXAMINED. — MISSOURI: Type specimens as cited under type locality.— KANSAS: Riley County, *J. B. Norton*, 1896.

DELPHINIUM NORTONIANUM n. sp.

An erect biennial or short-lived perennial from a woody branched root; 7.5–15 dm. high, simple or sometimes branched, the branches erect-ascending, 1.5–4.5 dm. long, finely pubescent or canescent, becoming smoothish below, the upper part, including the long pedicels, densely glandular-pilose and viscid; leaves rather variable, from 5–10 cm. wide, repeatedly divided into linear to linear-oblong divisions; racemes long and simple, often 4–6 dm. long; pedicels erect, 1–5 cm. long; bractlets narrowly linear, borne close under the calyx on the thickened end of the pedicel; sepals deep blue, with a tuft of yellow hairs on the back near the tip, canescent at the base and along the back to the tip, or sometimes all over without, glabrous within; spur stout, about twice as long as the petals, straight or slightly curved at the tip, of the same color as the sepals, strongly ascending, densely canescent; upper petals oblique at the summit, yellow below, tipped with blue; lateral petals with long white beard on upper portion, 2-cleft, the lobes not spreading, blue below, tipped with yellow; follicles cylindric, densely canescent when young, smoothish when old, nearly 2 cm. long, recticulate-veiny, cuspidate at tip; seeds 2–2.5 mm. long, brown, angular, strongly wing-margined, and strongly rugose-squamellate.— Plate XIII.

Type locality: Monteer, Jackson County, Missouri; collected by *B. F. Bush*, No. 377, May 24, 1900; type in Herb. Missouri Botanical Garden, duplicate in Herb K. K. Mackenzie.

Very common in rocky barrens in the Ozark Mountain region in Southern Missouri.

This species is most closely related to *D. Carolinianum* Walt., but differs from that species in the larger, more strongly rugose-squamellate seeds, the strongly ascending spurs, and the loose floccose pubescence. From *D. albescens* Rydb. it may be distinguished by the deep blue flowers, the strongly wing-margined seeds, and the presence of the bractlets under the calyx. Named in honor of Mr. J. B. S. Norton, with whom one of the writers has been associated in the study of some interesting species.

The type specimens are the only ones known.

PRUNUS LANATA (Sudw.) McK. & Bush.

P. Americana β mollis T. & G. Fl. N. A. 1 : 407. (1840), not *P. mollis* Torr. Fl. (1824).

P. Americana lanata Sudw. Bull. No. 14, Jan. 21, 1897.

A shrub or sometimes a large tree with a maximum height of about 9 meters and a diameter of about 4 dm.; branches very thorny; bark thick; leaves ovate to lanceolate, sharply and often doubly serrate; young twigs, pedicels, and both sides of the entire calyx-lobes densely short appressed-pubescent; leaves strongly pubescent below even at maturity, petioles biglandular near the blade; flowers white, 10–20 mm. broad, appearing in lateral sessile umbels before the leaves; pedicels 10–18 mm. long, densely appressed-pubescent; calyx-lobes entire, densely pubescent without and within; drupe globose, red or purple, 12–18 mm. in diameter, the skin tough, with a decided bloom, the stone somewhat flattened, its ventral edge acute, the dorsal grooved. Common along rivers and bottoms from Illinois and Iowa to Missouri, Texas and Mexico.

This species seems to us to differ very much from *P. Americana* Marsh., in its greater pubescence, its calyx-lobes pubescent on both sides, and in the appressed pubescent pedicels.

SPECIMENS EXAMINED. — MISSOURI: Fairmount Park, Jackson County, *Mackenzie*, 15, April 24, 1898; Dodson, Jackson County, *Mackenzie*, 23, May 1, 1898; same locality, *Mackenzie*, July 17, 1900; Independence, Jackson County, *Bush*, 274; June 7, 1895; St. Louis County, *Engelmann*, June, 1877. — IOWA: Muscatine, *Mackenzie*, 362, May 13, 1893; Iowa City, *Hitchcock*. — ILLINOIS: Dupage County, *Moffatt*, April 26, 1896. — ARKANSAS: Prescott, *Bush*, 184, April 25, 1901. — TEXAS: Marshall, *Bush*, 621, Aug. 8, 1901; San Antonio, *Bush*, 796, Sept. 16, 1901.

HYPERICUM PSEUDOMACULATUM, n. sp. B. F. Bush.

An herbaceous perennial from a woody base, erect, 4–9 cm. high, more or less black-dotted. Leaves sessile, or mostly clasping by a broad cordate base, oblong or oblong-lanceolate, obtuse, or the upper mostly acute, 25–40 mm. long, 7–15 mm. wide, copiously punctate with pellucid glands, and sparingly black-dotted; cymes terminal, many-flowered in rather loose clusters; flowers large, bright yellow, 15–30 mm. broad; sepals lanceolate, acute, very much shorter than the petals, copiously black-dotted; petals large, 12–15 mm. long, sparingly black-dotted and punctate with pellucid glands; stamens numerous, united in 3 or 5 sets; styles 3, variable in length; capsule not seen. — Plate XIV.

Type locality: Swan, Taney County, Missouri; collected by *B. F. Bush*, No. 106, June 6, 1899; type in Herb. Missouri Botanical Garden, duplicate in Herb. K. K. Mackenzie.

Rocky barrens in the Ozark Mountain region in southern Missouri, where it is abundant. This species has been taken for *H. maculatum* Walt., and *H. perforatum* L., from which two species it undoubtedly is distinct. From *H. perforatum* L. it differs in having larger leaves, black-dotted sepals, and punctate petals. It is readily distinguished from *H. maculatum* Walt., by its larger flowers which are both white- and black-dotted, and its copiously pellucid dotted leaves.

Specimens examined.— Missouri: Type specimens as cited under type locality; Eagle Rock, Barry County, *Bush*, 109, June 11, 1897.

Scutellaria cordifolia pilosissima n. var.

Differing from the specific form in being very strongly glandular pilose-hairy throughout, the corolla covered with short pubescence; corolla apparently not at all white.— Plate XV.

Type locality: Eagle Rock, Barry County, Missouri; collected by *B. F. Bush*, No. 190, June 24, 1897; type in Herb. Missouri Botanical Garden, duplicate in Herb. K. K. Mackenzie.

Rocky woods in the Ozark Mountain region in Southwestern Missouri, and probably through Arkansas to Texas.

Specimens examined.— Missouri: Type specimens as cited under type locality.— Texas: Dallas County, *Reverchon*, 769, 1900; same locality, *Reverchon*, 2531, May 2, 1901.

Physalis Missouriensis n. sp.

Annual, and from 2 dm. and little branched to 7 dm. tall, and 1 meter in diameter and much branched from the base;

stems, petioles, and peduncles strongly viscid and clammy pubescent, the hairs conspicuously unequal in length, leaves orbicular-ovate to ovate-oblong, 1.5–7 cm. long, 1.5–4.5 cm. wide, from rounded and a little, to strongly oblique at base, acute at apex, and from nearly entire to strongly undulate-dentate, more or less viscid-pubescent on both sides, especially on the veins and margin; petioles 8–60 mm. long, usually somewhat shorter than the leaves; flowers axillary on reflexed peduncles 2–8 mm. long; calyx strongly glandular, 4 mm. long, the lobes triangular-acute, the length of the tube; corolla twice the length of the calyx, 4–8 mm. broad, pure orange-yellow, without a dark eye, pubescent outside and slightly so within; stamens greenish-yellow, and shorter than the glabrous filaments; mature fruiting pedicels 12 mm. long; mature fruiting calyx 20–28 mm. long, not strongly angled, merely acute, reticulated, viscid-pubescent, especially on the veins, rounded or somewhat sunken at base; berry globular, very viscid, 8–10 mm. in diameter.

Type locality: Red Bridge, Jackson County, Missouri; collected by *K. K. Mackenzie*, 485, Sept. 18, 1901; type in Herb. K. K. Mackenzie.

This species is common in open rocky woods around Kansas City, and there can be scarcely any doubt that it is native. Prof. Gray and Dr. Engelmann called it a form of *P. pubescens* L. Prof. Watson named it *P. minima* Roxb. (*P. Lagascae* R. & S.), and this latter name is also taken up by Dr. Rydberg. However, that species lacks the viscid pubescence so strongly developed in the Missouri plant, which makes it so disagreeable to the touch when fresh. This characteristic, in addition to the pure yellow corolla, easily distinguishes it from all our other species.

Specimens examined. — Missouri: Type specimens as cited under type locality; Swope Park, Jackson County, *Mackenzie*, Aug. 26, 1896; Brush Creek, Jackson County, *Mackenzie*, Sept. 16, 1895; Kansas City, *Mackenzie*, 360, June 14, 1895; Jackson County, *Bush*, 1023, Aug. 6, 1888; Monteer, Shannon County, *Bush*, Oct. 21, 1893; Eagle Rock, Barry County, *Bush*, 162, June 28, 1897; Swan, Taney County, *Bush*, 173, June 12, 1898. — Kansas: Rosedale, Wyandotte County, *Mackenzie*, July 12, 1896; Manhattan, Riley County, *Kellerman*, Aug. 28, 1888.

PHYSALIS SUBGLABRATA n. sp.

An erect branching perennial, 3–9 dm. tall, from a stout rootstock; stem glabrous or very nearly so below, more or less pubescent with few and scattered appressed hairs above; leaves ovate or ovate-lanceolate, 2.5–7.5 cm. long, 1.25–5 cm. wide, entire or slightly undulate, obtuse or somewhat acute at apex, rounded or subcordate at base, and often slightly oblique, glabrous throughout, or occasionally with a few scattered appressed hairs; petioles shorter than the leaves, 6–36 dm. long, glabrous, or with a few scattered appressed hairs; pedicels strongly appressed hairy, 6–10 mm. long in flower, often twice as long in fruit; calyx 6–8 mm. long, appressed hairy, the lobes triangular and about the length of the tube; corolla yellow with a dark center, 10–14 mm. broad, and about as long; fruiting calyx angled and reticulated, depressed at base, 24–30 mm. long, 15–22 mm. wide, ovoid or oblong-ovoid, much inflated, the berry in the center; berry globular, 8–10 mm. broad, distinctly stiped.

Type locality: Sheffield, Jackson County, Missouri; collected by *K. K. Mackenzie*, June 14, 1896; type in Herb. K. K. Mackenzie.

This is the most common species of *Physalis* around Kansas City, occurring in great abundance along railroads, in fields, sandy woods, and river bottoms. It has been referred to *P. Philadelphica* Lam., but that species is annual, while the above proposed species is invariably perennial. It in fact appears to us much nearer *P. macrophysa* Rydb., and it differs from the published description of that species only in the more slender fruit, the nonglabrate flowering calyx, the noticeably appressed-pubescent pedicels, and the narrower entire leaves. It probably includes the greater part of the perennial *P. Philadelphica* of the Eastern United States.

SPECIMENS EXAMINED. — MISSOURI: Type specimens as cited under type locality; Brush Creek, Jackson County, *Mackenzie*, June 14, 1895; Lee's Summit, Jackson County, *Mackenzie*, July 6, 1900; Eagle Rock, Barry County, *Bush*, 165, June 27, 1897; Jackson County, *Bush*, 263, July 31, 1893; Swan, Taney County, *Bush*, 448, Sept. 24, 1899; Courtney, Jackson County, *Bush*, 286, July 8, 1896; Independence, *C. W. Tindall*, June 19, 1895; Clarksville, *Trelease*, 504, Oct. 10, 1897; Sac

River, Greene County, *Trelease*, 503, July 28, 1897; St. Louis County, *Glatfelter*, 1895. — ILLINOIS: Wady Petra, *V. H. Chase*, Sept. 24, 1897. — KANSAS: Quindaro, Wyandotte County, *Mackenzie*, July 11, 1897. — IOWA: Iowa City, *Hitchcock*, 1886.

SOLIDAGO LONGIPETIOLATA n. sp.

An erect slender tufted perennial, 3–6 dm. high; stems closely and finely short-canescent, but not whitened; leaves linear-oblanceolate, the lower and those on sterile shoots tapering into a long petiole, the upper sessile or nearly so; lower leaves (including petiole) 5–18 cm. long, the upper gradually becoming shorter, 3–11 mm. wide at the widest part, the lower usually short-crenate, the upper entire, green, but appressed-pubescent on both sides, 3-nerved, the mid-nerve prominent, the lateral often obscure; leaves often bearing small leaves in their axils simulating stipules; cyme 2.5–7.5 cm. long, rarely 15 cm. long, usually flat-topped and strongly one-sided, either bearing one long curving branch with smaller branches at base, or rarely bearing several curving branches; heads 5–7.75 mm. high on bracteolate pedicels; ray flowers 3–9, with rays 2 mm. long; disk flowers 8–12; involucral bracts in several rows, appressed, acutish and thickened at the end; achenes strongly pubescent.— Plate XVI.

Type locality: Lee's Summit, Jackson County, Missouri; collected by *K. K. Mackenzie*, No. 425, September 19, 1901; type in Herb. K. K. Mackenzie, duplicate in Herb. Missouri Botanical Garden.

This species is common in rocky woods and barrens from Michigan and Wisconsin through Iowa, Illinois, Missouri, Kansas and Indian Territory to Texas. Distinguished from *S. nemoralis* Ait. by the very long and narrow, more entire, linear-oblanceolate leaves, its larger heads and longer rays, more imbricated involucre, with sharper scales, lower height, more simple inflorescence and much more pubescent achenes.

SPECIMENS EXAMINED. — MISSOURI: Type specimens as cited under type locality; Lee's Summit, Jackson County, *Bush*, 1, Oct. 6, 1883 (marked "*radula ?*"); Monteer, Shannon County, *Bush*, 1134, Oct. 23, 1901; St. Louis, *Engelmann*, Aug., 1842 (marked *S. nemoralis ?*); Eagle Rock, Barry County, *Bush*, 74, Sept. 21, 1896; Martin City, Jackson County, *Mackenzie*, 477, Sept. 18, 1901. — ILLINOIS: American Bottom, Harber's, *Engelmann*, Aug., 1842; American Bottom, Harber's, *Engelmann*, Sept., 1843 (marked "var. fol. elongatis viridibus"), an unusual branched form. — WISCONSIN: St. Croix, *Hale*, 1861 (marked "var. of *nemoralis*"). — TEXAS: Spring Creek, Gillespie County, *G. Jermy*. — MICHIGAN: Manitou Island, Lake Michigan, *B. B. Brown*, Sept., 1892. — INDIAN TERRITORY: Sapulpa, *B. F. Bush*, 247, Sept. 25, 1894. — IOWA: Clinton, Lyon County, *Pammel*, 44, Sept. 4, 1896; Emmet County, *Cratty* and *Pammel*, 618, Aug. 28, 1897 (young plants); Ames, *Pammel*, 43, Aug. 24, 1896. — KANSAS: Manhattan, *Norton*, Aug. 7, 1892.

SENECIO PSEUDO-TOMENTOSUS, n. sp.

Perennial, 3–4.5 dm. high, densely and persistently woolly-canescent, tufted; basal leaves semi-orbicular to ovate or ovate-oblong, 24–36 mm. long, 7–24 mm. wide, narrowed, truncate, or semi-cordate at base, obtuse, sharply and often doubly serrate, varying to crenate-serrate, more or less persistently woolly even above; petioles often 5 cm. long, strongly and persistently woolly; stem leaves rather distant, short-petioled or sessile, few; the lower oblong in outline, 5–7 cm. long, strongly laciniate-pinnatifid, the terminal segment much the larger, 24–36 mm. long, 24 mm. wide; upper stem leaves lanceolate in outline; corymb 5–25-flowered; involucre 6–8 mm. high, 8–10 mm. broad, strongly tomentose, its linear-lanceolate segments acute, in a single row, with 1–3 lanceolate aristate-pointed bracts 2 mm. long at base; rays about 10, 4–6 mm. long; achenes hispidulous; pappus white. — Plate XVII.

Type locality: Monteer, Shannon County, Missouri; collected by *B. F. Bush*, No. 453, May 13, 1901; type in Herb. Missouri Botanical Garden, duplicate in Herb. K. K. Mackenzie.

This species is common in the rocky barrens in the Ozark Mountain region in Southern Missouri, which in spring are very wet and completely covered with herbaceous plants. The type specimens were collected in a rocky barren in Shannon County, associated with such plants as *Senecio Plattensis*, *Agave Virginica*, *Berlandiera Texana*, *Scutellaria Bushii*, *Delphinium Nortonianum*, and *Parthenium repens*.

This species belongs to the *aureus* group. Its persistently woolly tomentum distinguishes it from most of the Eastern United States species of the group; from *S. tomentosus* Michx., and *S. Plattensis* Nutt., with which it might be confounded, it is readily distinguished by its small, short and broad root-leaves. The type specimens are the only ones seen.

EXPLANATION OF ILLUSTRATIONS.

Plates XII.-XVII.

Plate XII. — *Muhlenbergia polystachya.* Habit, reduced one-half, and spikelet, × 6.

Plate XIII. — *Delphinium Nortonianum.* Habit, reduced one-half, flower, natural size, and seed, × 5.

Plate XIV. — *Hypericum pseudomaculatum.* Habit, reduced one-half.

Plate XIV. — *Scutellaria cordifolia pilosissima.* Habit, reduced one-half, and flower, × 2.

Plate XVI. — *Solidago longipetiolata.* Habit, reduced one-half, head, natural size, and ray flower, × 3.

Plate XVII. — *Senecio pseudo-tomentosus.* Habit, reduced one-half, head, natural size, and disk flower, × 3.

Issued June 9, 1902.

MUHLENBERGIA POLYSTACHYA.

DELPHINIUM NORTONIANUM.

HYPERICUM PSEUDOMACULATUM.

SCUTELLARIA CORDIFOLIA PILOSISSIMA.

SOLIDAGO LONGIPETIOLATA.

SENECIO PSEUDO-TOMENTOSUS.

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Transactions of The Academy of Science of St. Louis.

VOL. XII. No. 8.

SCLEROTINIA FRUCTIGENA.

J. B. S. NORTON.

Plates 18–21

A *Issued August 25, 1902.*

SCLEROTINIA FRUCTIGENA.

J. B. S. NORTON.

The fungus known as *Monilia fructigena* has been long known as the cause of a serious disease of fruits, especially in this country, and has been the subject of study by many investigators both from the economic and purely scientific standpoint. Among the many Experiment Station bulletins dealing with this fruit rot, may be mentioned those of Chester* and Quaintance.† Dr. Erwin F. Smith‡ has done the most valuable work on the fungus in this country. Humphrey§ has also contributed a good paper on the life history of *Monilia*. Among the host of European works, the excellent and beautifully illustrated paper by Woronin¶ is the most complete. Montemartini's ‖ paper in the Rivista di Patologia may be referred to for a more complete bibliography.

Although the ascospore stage of this fungus has not been reported by any writer, the species, from the morphology of its condia, the sclerotia formed and other peculiarities of life and structure, has been without question referred to the genus *Sclerotinia*, first by Schroeter, and later with more thorough work to uphold his views by Woronin, who after patient search for the apothecial stage published his paper on this subject with the view that either the apothecia had been lost from this species and also the similar *Sclerotinia cinerea*, or that the period of their development from the sclerotia was very long.

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* F. D. Chester, Delaware College Agricultural Experiment Station Bulletins 19. 1892; 29. 1895.

† A. L. Quaintance, Georgia Experiment Station Bulletin 50. 1900.

‡ Erwin F. Smith, Journal of Mycology **5**: 123–134. 1889; **7**: 26–38. *pl. 5–6.*

§ J. E. Humphrey, Botanical Gazette **18**: 85. 1893.

¶ M. Woronin, Mémoires de l'Académie impériale des sciences de St.-Pétersbourg. VIII. **10**[5]: 1–38. *pl. 1–6.* 1900.

‖ L. Montemartini, Rivista di Patologia Vegetale **8**: 210–218. 1900.

The latter proposition of Woronin seems to me the correct one. I found the apothecia developing abundantly this spring in Maryland peach and plum orchards; but in every case so far as I can judge, only from sclerotia in mummified fruits over one year old.

I first noticed the disk-like apothecia on the ground in an old peach orchard in Charles County, Md., April 10th of this year. Suspecting their origin, I was delighted on digging them up to find them attached to buried peaches. A short search revealed them in large numbers wherever the ground was moist and buried "mummy" peaches had been for some time undisturbed. Two days later I found them in the young peach orchard of the Maryland Experiment Station at College Park, and also on a few buried plums. They were also found in other orchards at College Park, and a week later one partially developed apothecium was found in northern Frederick County and on May 8 the dried up remains of many and a few fresh ones were found in Washington County, and one single apothecium was found after long search in the mountains of Garrett County on May 9, where peaches were just then in flower. In no case have I failed to find the cups where *Monilia* was abundant two years ago, but where it was abundant last year and not the year before, I have found none. This may offer some explanation of the appearance of blossom blight after a season when *Monilia* was not abundant: however the conidia alone appear able to carry the fungus over one year.

The apothecia are easily distinguished where perfectly developed, if on open ground, but they are usually found covered by trash. It hardly seems possible that they could appear every year, and not have been found by some of the excellent investigators who have looked closely for them. But it is well known that many fungi, including some species of *Sclerotinia* mentioned by Woronin, like some flowering plants do not produce certain stages in their life history for years, while on certain other years, these appear in great abundance.

I estimate from the condition in which I have found the fungus at different places that the first disks appeared at

College Park about April 1; the last ones I observed in perfect condition there were seen on April 27. The duration of the ascospore stage is then about that of the peach flowers. The single apothecia last only two or three weeks, after that drying up so as to be found with difficulty.

The apothecia arise from the familiar sclerotia in the tissues of the so-called mummy fruits beneath the soil or occasionally on the surface in moist places. Usually several arise from the under side of each fruit and appear in a ring around it at the surface of the ground, from one to twenty appearing above one fruit.

The sinuous stipe is from .5–3 cm. long, depending on the length it must grow to bring the spore-bearing surface above the ground. It is from .3–1.5 mm. thick. The lower part is covered with closely adherent particles of soil entangled in a mass of slender dark-colored septate rhizoids 1 mm. or less in length. These gradually disappear upward, the upper part of the stipe being smooth. The color is dark brown below running into the lighter brown of the disk above. The body of the stipe is made up of somewhat elongated cells in the center with shorter dark-colored cells on the outside, composing the cortex which continues around the outside of the disk and projects at the edges somewhat beyond the hymenium. The subhymenium is composed of elongated intertwined cells much like those in the center of the stipe.

The stipe enlarges into the at first campanulate disk, slightly broader below the top. The disk widens out until cup-shaped and finally flat. Older ones often have the edges torn and recurved. The disk becomes again campanulate in drying up and is then darker colored. The expanded disk is from 2–15 mm. wide, usually about 5–8 mm. In its later stages it is often whitish from a deposit of spores.

The line of demarcation seen in sections between the hymenium and subhymenium is composed of a dense mass of small hyphae from which the asci and paraphyses arise. These are of the usual form of the *Pezizaceae* and of the genus. The paraphyses are very slender and slightly enlarged at the apex. The asci are 45–60 μ long and 3–4 μ wide, with 8 spores in the apical half.

The spores are thrown off as the disk dries up. If fresh apothecia are kept for a time in a moist atmosphere and then exposed to a dryer or blown upon by currents of air or the breath, a distinct cloud of spores can be seen discharged which ascends several centimeters high and is wafted away by the slightest air current. When held in the sunlight the individual spores can apparently be seen with the naked eye. The discharge of spores can be several times repeated by additional drying, and after a rest still others may be given off. Not nearly all the spores are ejaculated, as old dry individuals contain many asci intact and others with a few spores.

The spores germinate in water in 6–10 hours sending out a small germ-tube or promycelium after swelling to twice their diameter. Usually this tube does not attain in water more than 30–40 μ in length. I have not observed the formation of sporidia such as are described by Woronin and Humphrey. The outer coat of the spore is probably thrown off in germination.

In bouillon or prune juice a much more vigorous growth takes place. A small branched mycelium is formed, which in drop cultures, which I have made, has rarely developed conidia. A few cells in some of the hyphae after long standing and slow growth took on a form approximating that of the *Monilia* spores and in some cases a few small sporidia(?) were developed (Plate XVIII, figs. 8, 9).

Beautiful and almost always pure, agar plate cultures were obtained by holding a sterile cover-glass with a drop of liquid for a moment over the discharging asci, dropping it into a tube of bouillon and then preparing the plates in the usual manner. This method was suggested to me by Dr. Erwin F. Smith, who has provided me with many facilities for making these studies. Several colonies developed in plates prepared in this way and in such large preponderance over others as to give certainty to their origin, although it must be remembered that *Monilia* conidia are apt to be plentiful in the air and contaminations of a very undesirable character result. The mycelium developed was very similar to that obtained in agar and bouillon tubes, a radiating dense layer of shining white, branched mycelium gradually growing wider to in some cases, with moist and warm air, 2 cm. or more from the original spore. After a few day's growth, most of these mycelia sent up branching hyphae into the air in a circle a short distance from the center. These bore in chains the characteristic yellowish-gray conidia of *Monilia fructigena.* After further growth, other rings of clusters of conidiophores were produced beyond the first, their production apparently depending on moisture conditions.

It remained to test inoculations of fruit and flowers with the ascospores or conidia developed from them. Some of the fresh apothecia were placed in contact with peach flowers moistened and inclosed in paper sacks and some on mutilated buds. Probably owing to dry weather no results were obtained. With cut twigs of blooming peach and plum at the same time placed under sterile jars indoors and similarly treated, in two or three days the peculiar browning of the petals seen in the *Monilia* blossom-blight appeared, followed by tufts of conidia. Although a great deal of blight appeared in the check cultures from *Monilia* spores already on the peach buds, several spots started in such places as to indicate undoubted infection from the ascospores. Finally, after a few weeks all the flowers blighted and became covered with dense masses of white hyphae often hanging down 2–3 cm. Although this may be some other mold, it appears to be connected with the *Sclerotinia.*

On May 12, peach petals collected the day before in Garrett Co., Md., were placed in a sterile Petri dish. On some were placed sterilized drops of water, others were touched by wet, fresh apotheca, and others with *Monilia* conidia from the peach flowers mentioned above. In 3 days those inoculated were blighted, turned brown, and later developed clusters of conidia. Those simply wet remained perfectly fresh and white, 3 weeks later.

I find a very good way to study the development from the ascospores is to cut very thin sections of the dry apothecia and place them in water or some nutrient liquid where the germination of the spores actually in the asci can be watched and thus the spores cannot be confused with those of other fungi that might be present. With such sections placed on a slide, the hyphae can be followed out until in nutrient solutions after 3 or 4 days the conidia are produced. The principal difficulty is in keeping them separate from the hyphae of other molds, which are difficult to keep out. By placing thin sections of fruit on the slide near the germinating spores, the course of the hyphae can be observed into and through the fruit tissues. I have used pieces of boiled prune and apple very well in this way, but the results with a few sections of green peaches and plums were negative.

On May 19 through the kindness of Mr. A. M. Ferguson of the University of Texas, I received some well developed peaches and partially ripe plums from Texas. I inoculated some of these by piercing them with a needle which had just been touched to conidia developed in plate cultures from ascospores or which held a section containing germinating ascospores. These were placed under a bell jar with check cultures of fruits pierced with a sterile needle. Most of those inoculated developed *Monilia* conidia in 3 to 5 days, preceded by the characteristic "brown rot." The checks remained fresh for two weeks. In the damp atmosphere of the bell jar the hyphae on some of the inoculated fruits developed a dense white mass over the surface of the fruit 3–8 mm. thick, much like that found on the flowers kept under jars mentioned previously. This does not have the usual appearance of *Monilia* on rotted fruits, but since it is preceded by the usual form of conidiophores in hemispherical clusters and the long white hyphae bear similar chains of spores, I do not believe that it is anything else.

My principal object has been to demonstrate the identity of *Monilia fructigena* and the ascus stage of the *Sclerotinia* which I found this spring. The cultures described and others not here referred to, I think have done this. *Monilia fructigena* Persoon must then become a synonym of *Sclerotinia fructigena* (Persoon) Schroeter.

SUMMARY AND CONCLUSIONS.

The apothecia of a *Sclerotinia* were found abundantly in April, 1902, developing from sclerotia in buried mummified peaches and plums in Maryland orchards. The ascospores developed coincident with the peach flowers. The ascospores were readily germinated in water, bouillon and prune juice and cultures made in agar and on sterilized dried apple and prune, from which conidia were developed not distinguishable from the *Monilia* associated with brown rot of fruits. Inoculations of peach and plum flowers and fruits from ascospores or from these conidia developed in 2–4 days brown rot and clusters of *Monilia* conidia. *Monilia fructigena*, Persoon, is then properly referred to *Sclerotinia fructigena* (Persoon) Schroeter.

EXPLANATION OF ILLUSTRATIONS.

PLATES XVIII-XXI.

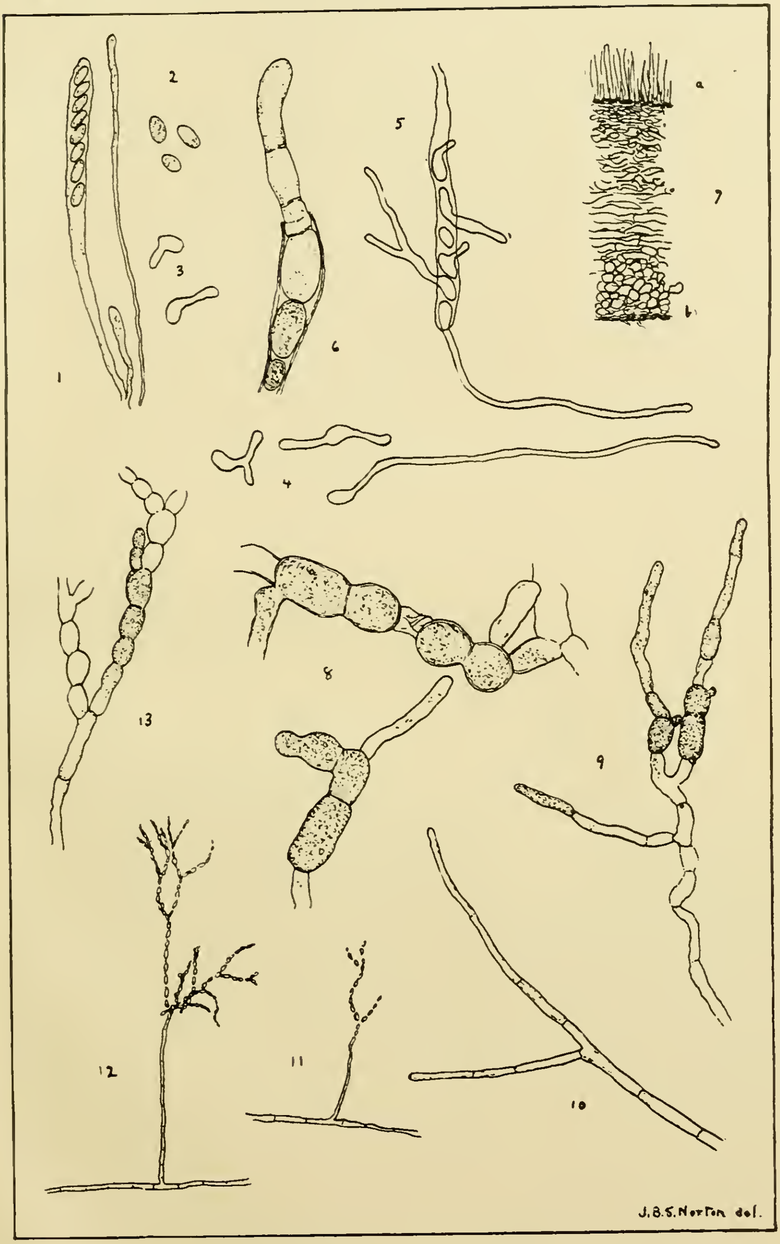
Plate XVIII. — *Sclerotinia fructigena.* 1, Ascus and paraphyses, × 700; 2, Spores, × 700; 3–5, Germinating spores, × 700; 6, Spores in end of ascus germinating, the lower spore natural size, the others swollen, × 1,000; 7, cross section of disk of apothecium, the base of the asci shown at *a*, *b* the cortex on the outside (somewhat diagrammatic), × 50; 8, 9, Mycelia from bouillon drop cultures with clamydospores(?) forming in the cells, — some sporidia(?) shown in 9, × 1,000; 10, End of hypha on plate culture, shown in plate XX, × 1,000; 11, 12, Conidiophores arising from branches of mycelium produced from ascospores. The chains of conidia were drawn from those that had fallen over and the conidia separated, × 50; 13, Chains of conidia attached, × 700.

Plate XIX. — *Sclerotinia fructigena.* Upper figure. Looking down upon a mummy peach surrounded by apothecia, natural size. Lower figure. Apothecia attached to mummied fruits; on the left appearing through the soil; in the center detached fragment showing the stipes, natural size.

Plate XX. — Mycelium of *Sclerotinia fructigena*, with clusters of conidia from agar plate culture developed from ascospore, × 6.

Plate XXI. — Fruits inoculated with ascospores of *Sclerotinia fructigena.* The plum in the upper left hand corner covered with a dense mass of hyphae bearing conidia in the grey spots at the left. To the right a peach and below an apple and plum, check cultures, simply pierced by sterile needle. The other two peaches on the right inoculated like the plum at the top. The middle peach shows at the right the usual form of clusters of conidia. All natural size.

Issued August 25, 1902.



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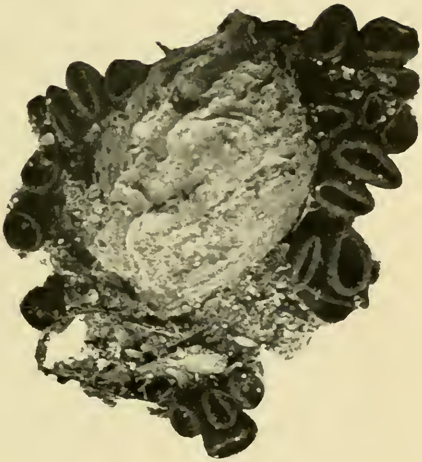

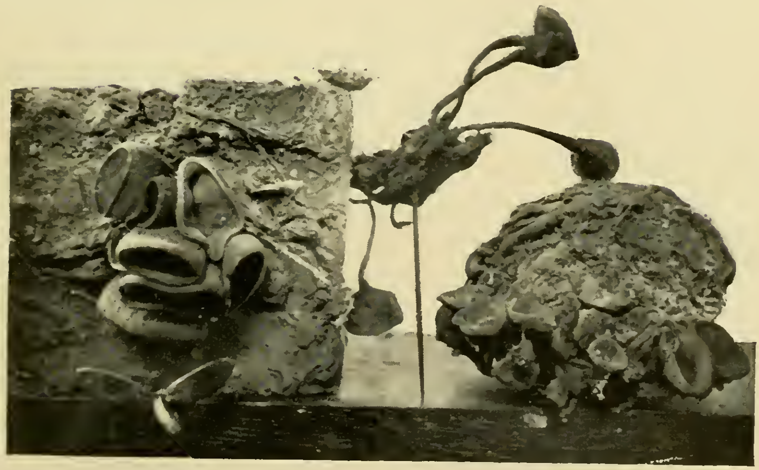

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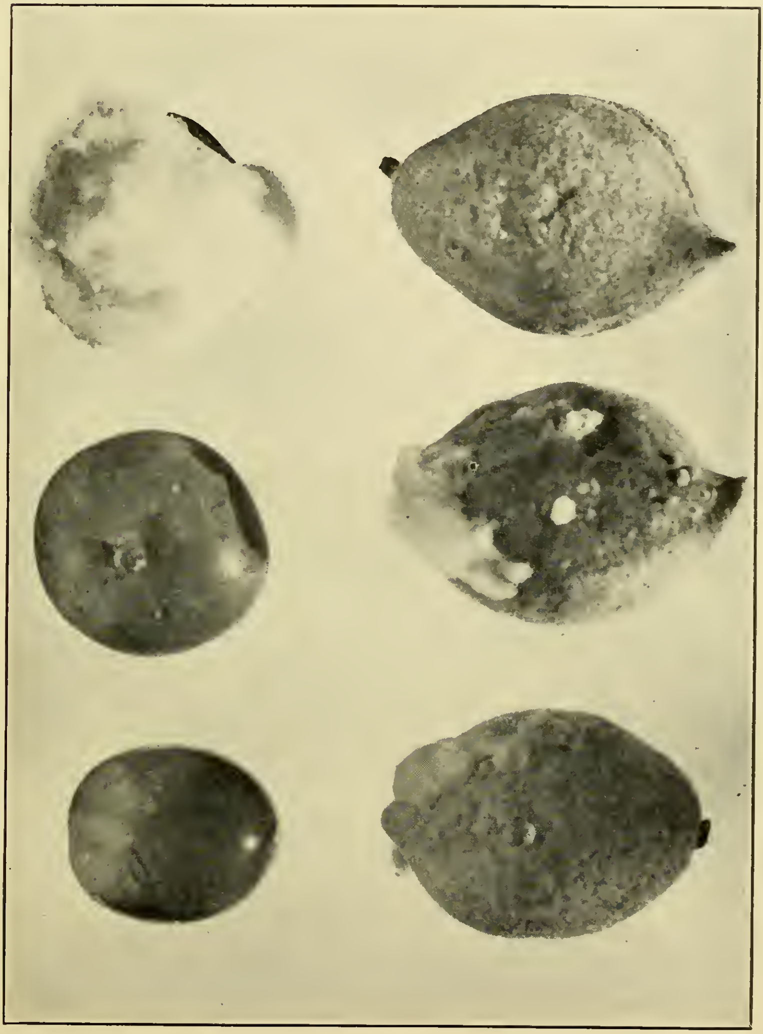

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VOL. XII. No. 9.

ON SOME RELATIONS BETWEEN BESSEL FUNCTIONS OF THE FIRST AND OF THE SECOND KIND.

ALEXANDER S. CHESSIN.

A *Issued November 24, 1902.*

ON SOME RELATIONS BETWEEN BESSEL FUNCTIONS OF THE FIRST AND OF THE SECOND KIND.*

ALEXANDER S. CHESSIN.

The general solution of Bessel's equation

$$(1) \ldots\ldots\ldots\ldots \frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right) y = 0$$

is of the form

$$(2) \ldots\ldots\ldots\ldots y = AJ_n(x) + BK_n(x),$$

or of the form

$$(2)' \ldots\ldots\ldots\ldots y = AJ_n(x) + BJ_{-n}(x),$$

according as n is, or is not, an integral number, A and B being arbitrary constants, while $J_n(x)$ and $K_n(x)$ denote Bessel functions of the first and of the second kind, the functions $J_n(x)$ and $J_{-n}(x)$ being distinct and independent when n is not an integral number.

The differential equation (1) may be presented in a different form, namely

$$(1)' \ldots\ldots\ldots\ldots \frac{d^2(y\sqrt{x})}{dx^2} + \left(1 - \frac{4n^2 - 1}{4x^2}\right) y\sqrt{x} = 0,$$

or, we may say that $u = y\sqrt{x}$ is the general solution of the differential equation

$$(3) \ldots\ldots\ldots\ldots \frac{d^2u}{dx^2} + uf(x) = 0,$$

* Presented by title to The Academy of Science of St. Louis, October 20, 1902.

where $f(x) \equiv 1 - \frac{4n^2 - 1}{4x^2}$, and y is given by the formulas (2) or (2)′ according as n is, or is not, an integral number.

Now, let v be a solution of the differential equation (3), and w a solution of the differential equation

$$(3)' \ldots\ldots\ldots\ldots \frac{d^2u}{dx^2} + u\phi(x) = 0,$$

$f(x)$ and $\phi(x)$ being arbitrary functions of x. Then

$$\frac{d^2v}{dx^2} + vf(x) \equiv 0,$$

$$\frac{d^2w}{dx^2} + w\phi(x) \equiv 0,$$

and, therefore,

$$v\frac{d^2w}{dx^2} - w\frac{d^2v}{dx^2} \equiv (f - \phi)vw.$$

But, on the other hand,

$$v\frac{d^2w}{dx^2} - w\frac{d^2v}{dx^2} \equiv \frac{d}{dx}\left(v\frac{dw}{dx} - w\frac{dv}{dx}\right);$$

hence,

$$(4) \ldots\ldots\ldots\ldots \frac{d}{dx}\left(v\frac{du}{dx} - w\frac{dv}{dx}\right) \equiv (f - \phi)vw$$

and

$$(4)' \ldots\ldots\ldots\ldots v\frac{dw}{dx} - w\frac{dv}{dx} = \int (f - \phi)\, vwdx + \text{Constant}.$$

We will proceed to make some applications of the last two formulas.

If we put

$$(5)_1 \ldots\ldots\ \ldots\ldots v \equiv \sqrt{x}J_n(ax), \quad w \equiv \sqrt{x}J_n(\beta x),$$

n being an integral number, then the function v will satisfy the equation (3) when $f(x) \equiv a^2 - \frac{4n^2 - 1}{4x^2}$, while the function w will be a solution of the differential equation (3)′ when $\phi(x) \equiv \beta^2 - \frac{4n^2-1}{4x^2}$. Therefore, by (4),

$$(6)_1 \quad \frac{d}{dx}\left\{ x\left(J_n(ax)\frac{dJ_n(\beta x)}{dx} - J_n(\beta x)\frac{dJ_n(ax)}{dx}\right)\right\} \equiv (a^2 - \beta^2) x J_n(ax) J_n(\beta x).$$

Likewise, n still being an integral number and the functions $f(x)$ and $\phi(x)$ being selected as above, if we take, in succession,

$$(5)_2 \qquad v \equiv \sqrt{x}J_n(ax), \qquad w \equiv \sqrt{x}K_n(\beta x);$$

$$(5)_3 \qquad v \equiv \sqrt{x}K_n(ax), \qquad w \equiv \sqrt{x}K_n(\beta x);$$

we obtain the formulas

$$(6)_2 \quad \frac{d}{dx}\left\{ x\left(J_n(ax)\frac{dK_n(\beta x)}{dx} - K_n(\beta x)\frac{dJ_n(ax)}{dx}\right)\right\} \equiv (a^2 - \beta^2) x J_n(ax) K_n(\beta x),$$

$$(6)_3 \quad \frac{d}{dx}\left\{ x\left(K_n(ax)\frac{dK_n(\beta x)}{dx} - K_n(\beta x)\frac{dK_n(ax)}{dx}\right)\right\} \equiv (a^2 - \beta^2) x K_n(ax) K_n(\beta x).$$

When n is not an integral number we take successively

$$(7)_1 \qquad v \equiv \sqrt{x}J_n(ax), \qquad w \equiv \sqrt{x}J_n(\beta x);$$

$$(7)_2 \qquad v \equiv \sqrt{x}J_n(ax), \qquad w \equiv \sqrt{x}J_{-n}(\beta x);$$

$$(7)_3 \qquad v \equiv \sqrt{x}J_{-n}(ax), \qquad w \equiv \sqrt{x}J_{-n}(\beta x);$$

and arrive at the identities

$$(8)_1 \quad \frac{d}{dx}\left\{ x\left(J_n(ax)\frac{dJ_n(\beta x)}{dx} - J_n(\beta x)\frac{dJ_n(ax)}{dx}\right)\right\} \equiv (a^2 - \beta^2) x J_n(ax) J_n(\beta x),$$

$$(8)_2 \quad \frac{d}{dx}\left\{x\left(J_n(ax)\frac{dJ_{-n}(\beta x)}{dx} - J_{-n}(\beta x)\frac{dJ_n(ax)}{dx}\right)\right\} \equiv (a^2-\beta^2)xJ_n(ax)J_{-n}(\beta x),$$

$$(8)_3 \quad \frac{d}{dx}\left\{x\left(J_{-n}(ax)\frac{dJ_{-n}(\beta x)}{dx} - J_{-n}(\beta x)\frac{dJ_{-n}(ax)}{dx}\right)\right\} \equiv (a^2-\beta^2)xJ_{-n}(ax)J_{-n}(\beta x),$$

of which the first one is identical with $(6)_1$ in form.

Now, we know that, whatever be n,

$$(9) \qquad \frac{dJ_n(ax)}{dx} \equiv \frac{n}{x}J_n(ax) - a\,J_{n+1}(ax),$$

$$(10) \qquad \frac{dJ_n(ax)}{dx} \equiv -\frac{n}{x}J_n(ax) + aJ_{n-1}(ax),$$

$$(11) \qquad \frac{dK_n(ax)}{dx} \equiv \frac{n}{x}K_n(ax) - aK_{n+1}(ax),$$

and that similar relations exist when a is replaced by β. Substituting into the identities (6) and (8) the above expressions for the derivatives of Bessel functions we readily obtain the formulas:

$$(12)_1 \quad \frac{d}{dx}\left\{x\left[aJ_n(\beta x)J_{n+1}(ax) - \beta J_n(ax)J_{n+1}(\beta x)\right]\right\} \equiv (a^2-\beta^2)xJ_n(ax)J_n(\beta x)$$

$$(12)_2 \quad \frac{d}{dx}\left\{x\left[aK_n(\beta x)J_{n+1}(ax) - \beta J_n(ax)K_{n+1}(\beta x)\right]\right\} \equiv (a^2-\beta^2)xJ_n(ax)K_n(\beta x)$$

$$(12)_3 \quad \frac{d}{dx}\left\{x\left[aK_n(\beta x)K_{n+1}(ax) - \beta K_n(ax)K_{n+1}(\beta x)\right]\right\} \equiv (a^2-\beta^2)xK_n(ax)K_n(\beta x)$$

when n is an integral number, and

$$(13)_1 \quad \frac{d}{dx}\left\{x\left[aJ_n(\beta x)J_{n+1}(ax) - \beta J_n(ax)J_{n+1}(\beta x)\right]\right\} \equiv (a^2-\beta^2)xJ_n(ax)J_n(\beta x),$$

$$(13)_2 \quad \frac{d}{dx}\left\{x\Big[aJ_{-n}(\beta x)J_{n-1}(ax)+\beta J_n(ax)J_{-n+1}(\beta x)\Big]\right\}$$
$$\equiv(\beta^2-a^2)xJ_n(ax)J_{-n}(\beta x),$$

$$(13)_3 \quad \frac{d}{dx}\left\{x\Big[aJ_{-n}(\beta x)J_{-n+1}(ax)-\beta J_{-n}(ax)J_{-n+1}(\beta x)\Big]\right\}$$
$$\equiv(a^2-\beta^2)xJ_{-n}(ax)J_{-n}(\beta x),$$

when n is not an integral number.

When $a=\beta$ the formulas just derived assume the form $0\equiv 0$. This is obvious in the case of $(12)_1$, $(12)_3$, $(13)_1$, and $(13)_3$. It is, however, quite as evident in the other cases if we take into consideration the relations

$$(14) \qquad J_{n+1}(x)K_n(x)-J_n(x)K_{n+1}(x)\equiv\frac{1}{x},$$

when n is an integral number, and

$$(15) \qquad J_n(x)J_{-n+1}(x)+J_{-n}(x)J_{n-1}(x)\equiv\frac{2\sin n\pi}{\pi x},$$

when n is not an integral number.* To derive the relations corresponding to (12) and (13) when $a=\beta$ we, therefore, differentiate both sides of the identities (12) and (13) with respect to a and put $\beta=a$ in the results. We then arrive at the formulas:

$$(16)_1 \quad \frac{d}{dx}\left\{x^2\Big[J_n^2(ax)-J_{n-1}(ax)J_{n+1}(ax)\Big]\right\}\equiv 2xJ_n^2(ax),$$

$$(16)_2 \quad \frac{d}{dx}\left\{x^2\Big[J_n(ax)K_n(ax)-J_{n-1}(ax)K_{n+1}(ax)\Big]\right\}$$
$$\equiv 2xJ_n(ax)K_n(ax),$$

$$(16)_3 \quad \frac{d}{dx}\left\{x^2\Big[K_n^2(ax)-K_{n-1}(ax)K_{n+1}(ax)\Big]\right\}$$
$$\equiv 2xK_n^2(ax),$$

when n is an integral number, and

* See the treatise on Bessel functions by Gray and Mathews, p. 16.

$$(17)_1 \quad \frac{d}{dx}\left\{x^2\left[J_n^2(ax) - J_{n-1}(ax)J_{n+1}(ax)\right]\right\} \equiv 2xJ_n^2(ax),$$

$$(17)_2 \quad \frac{d}{dx}\left\{x^2\left[J_n(ax)J_{-n}(ax) + J_{-n+1}(ax)J_{n+1}(ax)\right]\right\} = 2xJ_n(ax)J_{-n}(ax)$$

$$(17)_3 \quad \frac{d}{dx}\left\{x^2\left[J_{-n}^2(ax) - J_{-n+1}(ax)J_{-n-1}(ax)\right]\right\} \equiv 2xJ_{-n}^2(ax),$$

when n is not an integral number.

If, instead of applying formula (4), we had made use of formula (4)′; or, which amounts to the same thing, if we integrate both sides of the relations (12), (13), (16) and (17), we arrive at the formulas:

$$(18)_1 \quad x\left\{aJ_n(\beta x)J_{n+1}(ax) - \beta J_n(ax)J_{n+1}(\beta x)\right\} = (a^2 - \beta^2)\int xJ_n(ax)J_n(\beta x)dx + \text{Const.},$$

$$(18)_2 \quad x\left\{aK_n(\beta x)J_{n+1}(ax) - \beta J_n(ax)K_{n+1}(\beta x)\right\} = (a^2 - \beta^2)\int xJ_n(ax)K_n(\beta x)dx + \text{Const.},$$

$$(18)_3 \quad x\left\{aK_n(\beta x)K_{n+1}(ax) - \beta K_n(ax)K_{n+1}(\beta x)\right\} = (a^2 - \beta^2)\int xK_n(ax)K_n(\beta x)\,dx + \text{Const.},$$

when $a \neq \beta$, and

$$(19)_1 \quad x^2\left\{J_n^2(ax) - J_{n-1}(ax)J_{n+1}(ax)\right\} = 2\int xJ_n^2(ax)dx + \text{Const.},$$

$$(19)_2 \quad x^2 \left\{ J_n(ax)K_n(ax) - J_{n-1}(ax)K_{n+1}(ax) \right\}$$
$$= 2\int xJ_n(ax)K_n(ax)dx + \text{Const.},$$

$$(19)_3 \quad x^2 \left\{ K_n^2(ax) - K_{n-1}(ax)K_{n+1}(ax) \right\}$$
$$= 2\int xK_n^2(ax)dx + \text{Const.},$$

when $\beta = a$. Both sets of formulas (18) and (19) refer to the case of an integral n. When n is not an integral number these formulas should be replaced by the following ones:

$$(20)_1 \quad x\left\{ aJ_n(\beta x)J_{n+1}(ax) - \beta J_n(ax)J_{n+1}(\beta x) \right\}$$
$$= (a^2 - \beta^2)\int xJ_n(ax)J_n(\beta x)\,dx + \text{Const.},$$

$$(20)_2 \quad x\left\{ aJ_{-n}(\beta x)J_{n-1}(ax) + \beta J_n(ax)J_{-n+1}(\beta x) \right\}$$
$$= (\beta^2 - a^2)\int xJ_n(ax)J_{-n}(\beta x)dx + \text{Const.},$$

$$(20)_3 \quad x\left\{ aJ_{-n}(\beta x)J_{-n+1}(ax) - \beta J_{-n}(ax)J_{-n+1}(\beta x) \right\}$$
$$= (a^2 - \beta^2)\int xJ_{-n}(ax)J_{-n}(\beta x)dx + \text{Const.},$$

provided $a \neq \beta$. If $\beta = a$, the last relations assume the form

$$(21)_1 \quad x^2 \left\{ J_n^2(ax) - J_{n-1}(ax)J_{n+1}(ax) \right\}$$
$$= 2\int xJ_n^2(ax)dx + \text{Const.},$$

$$(21)_2 \quad x^2\left\{J_n(ax)J_{-n}(ax)+J_{-n+1}(ax)J_{n+1}(ax)\right\} = 2\int xJ_n(ax)J_{-n}(ax)dx + \text{Const.},$$

$$(21)_3 \quad x^2\left\{J^2_{-n}(ax)-J_{-n-1}(ax)J_{-n+1}(ax)\right\} = 2\int xJ^2_{-n}(ax)dx + \text{Const.}$$

With the exception of formulas $(18)_3$, $(19)_3$, $(20)_1$, $(20)_3$, $(21)_1$, and $(21)_3$, the integration may be assumed between the limits 0 and 1, the only restriction on the number n being that specified above. Thus, with $\beta \neq a$ and n being an integral number, positive or negative (zero, of course, included),

$$(22) \quad aJ_n(\beta)J_{n+1}(a) - \beta J_n(a)J_{n+1}(\beta) = (a^2-\beta^2)\int_0^1 xJ_n(ax)J_n(\beta x)dx,$$

$$(23) \quad aK_n(\beta)J_{n+1}(a) - \beta J_n(a)K_{n+1}(\beta) = (a^2-\beta^2)\int_0^1 xJ_n(ax)K_n(\beta x)dx,$$

$$(24) \quad J^2_n(a) - J_{n-1}(a)J_{n+1}(a) = 2\int_0^1 xJ^2_n(ax)dx,$$

$$(25) \quad J_n(a)K_n(a) - J_{n-1}(a)K_{n+1}(a) = 2\int_0^1 xJ_n(ax)K_n(ax)dx.$$

Likewise, with $\beta \neq a$, but n not an integral number,

$$(26) \quad aJ_{-n}(\beta)J_{n-1}(a) + \beta J_n(a)J_{-n+1}(\beta) = (\beta^2-a^2)\int_0^1 xJ_n(ax)J_{-n}(\beta x)dx,$$

$$(27)\quad J_n(a)J_{-n}(a)+J_{-n+1}(a)J_{n+1}(a)$$
$$=2\int_0^1 xJ_n(ax)J_{-n}(ax)dx.$$

Further, if n denote a positive, but not integral, number, formulas $(20)_1$ and $(21)_1$ yield the relations

$$(28)\quad aJ_n(\beta)J_{n+1}(a)-\beta J_n(a)J_{n+1}(\beta)$$
$$=(a^2-\beta^2)\int_0^1 xJ_n(ax)J_n(\beta x)dx,$$

$$(29)\quad J_n^2(a)-J_{n-1}(a)J_{n+1}(a)=2\int_0^1 xJ_n^2(ax)dx,$$

in form identical with (22) and (24); while n being a negative, but not integral, number, we derive from $(20)_3$ and $(21)_3$ the formulas

$$(30)\quad aJ_{-n}(\beta)J_{-n+1}(a)-\beta J_{-n}(a)J_{-n+1}(\beta)$$
$$=(a^2-\beta^2)\int_0^1 xJ_{-n}(ax)J_{-n}(\beta x)dx,$$

$$(31)\quad J_{-n}^2(a)-J_{-n-1}(a)J_{-n+1}(a)=2\int_0^1 xJ_{-n}^2(ax)dx,$$

which are, practically, identical with (28) and (29), since $-n$ in (30) and (31) is a positive, but not integral, number.

The integration between the limits 0 and 1 in formulas $(20)_1$ and $(21)_1$ when $n<0$, or in $(20)_3$ and (21_3) when $n>0$, is only possible if $|n|<1$. Hence, formulas (28) and (29) are still valid if $-1<n<0$; and formulas (30) and (31) hold good also when $0<n<1$.

Finally, in the relations $(18)_3$ and $(19)_3$ the integration may be taken between the limits 0 and 1 when $n=0$. We then obtain the formulas

$$(32)\quad aK_0(\beta)K_1(a) - \beta K_0(a)K_1(\beta) - J_0(a)J_0(\beta)\log\frac{a}{\beta} = (a^2 - \beta^2)\int_0^1 xK_0(ax)K_0(\beta x)dx$$

$$(33)\quad K_0^2(a) + K_1^2(a) - \frac{J_0^2(a)}{a^2} = 2\int_0^1 xK_0^2(ax)dx$$

which are readily verified with the help of well known expressions of Bessel functions of the second kind.

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