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ROYAL SOCIETY

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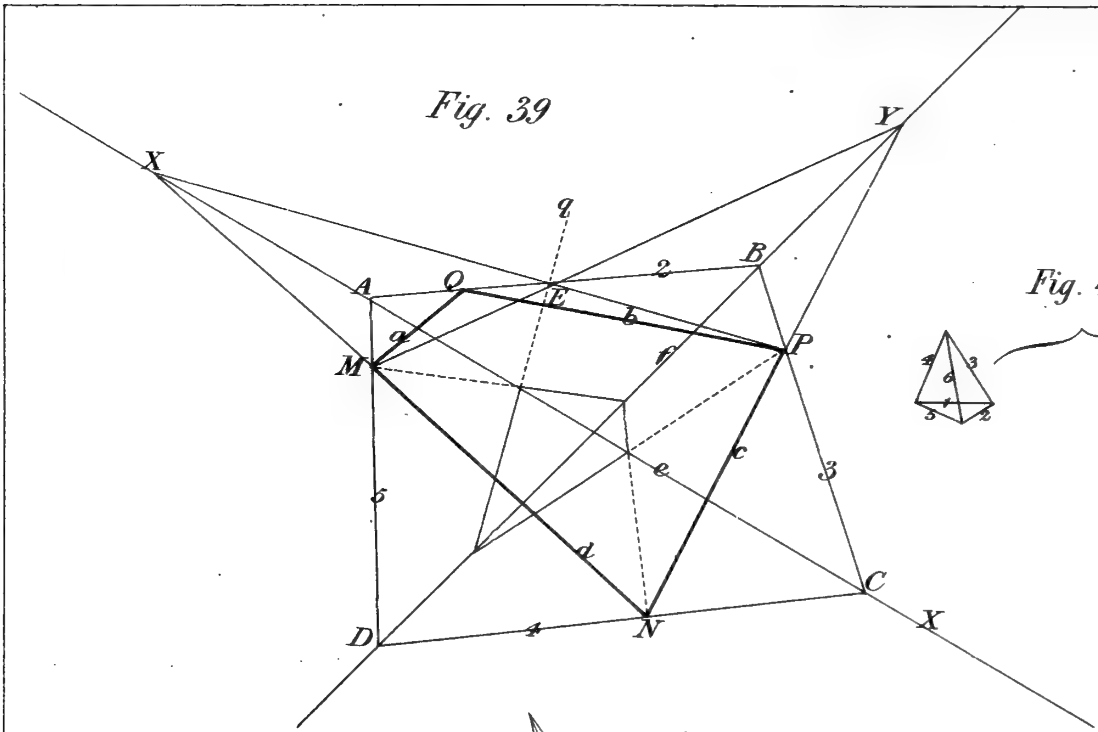

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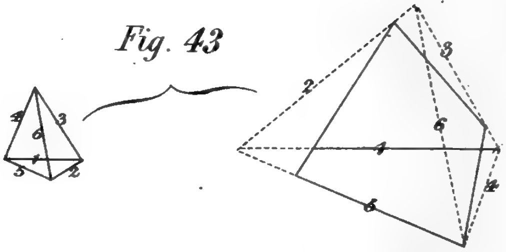

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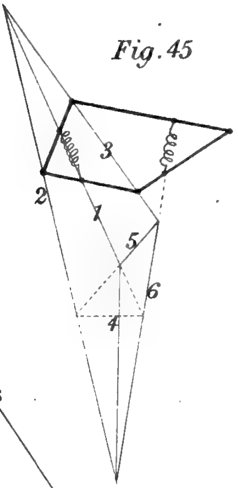

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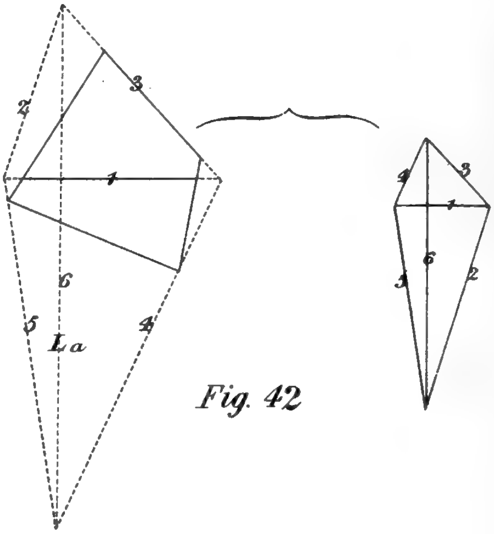

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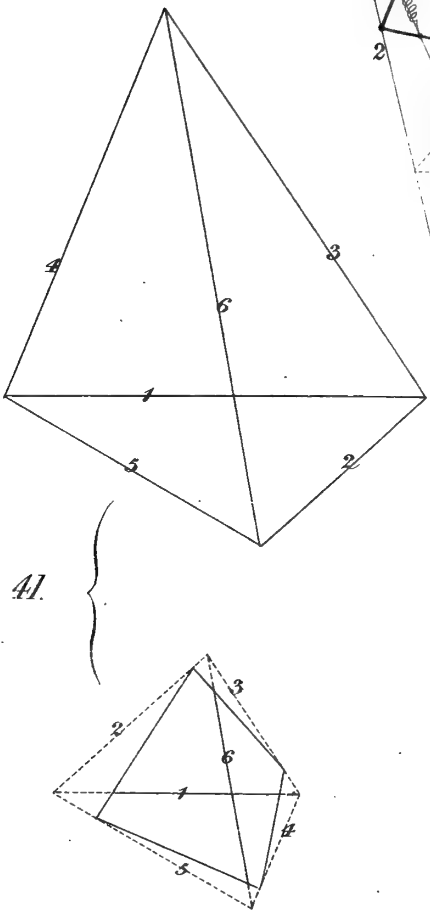

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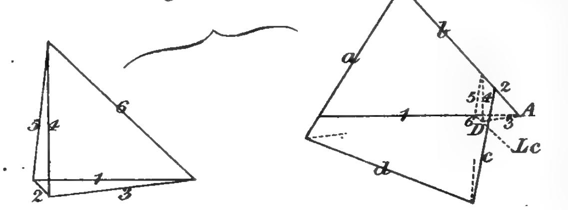
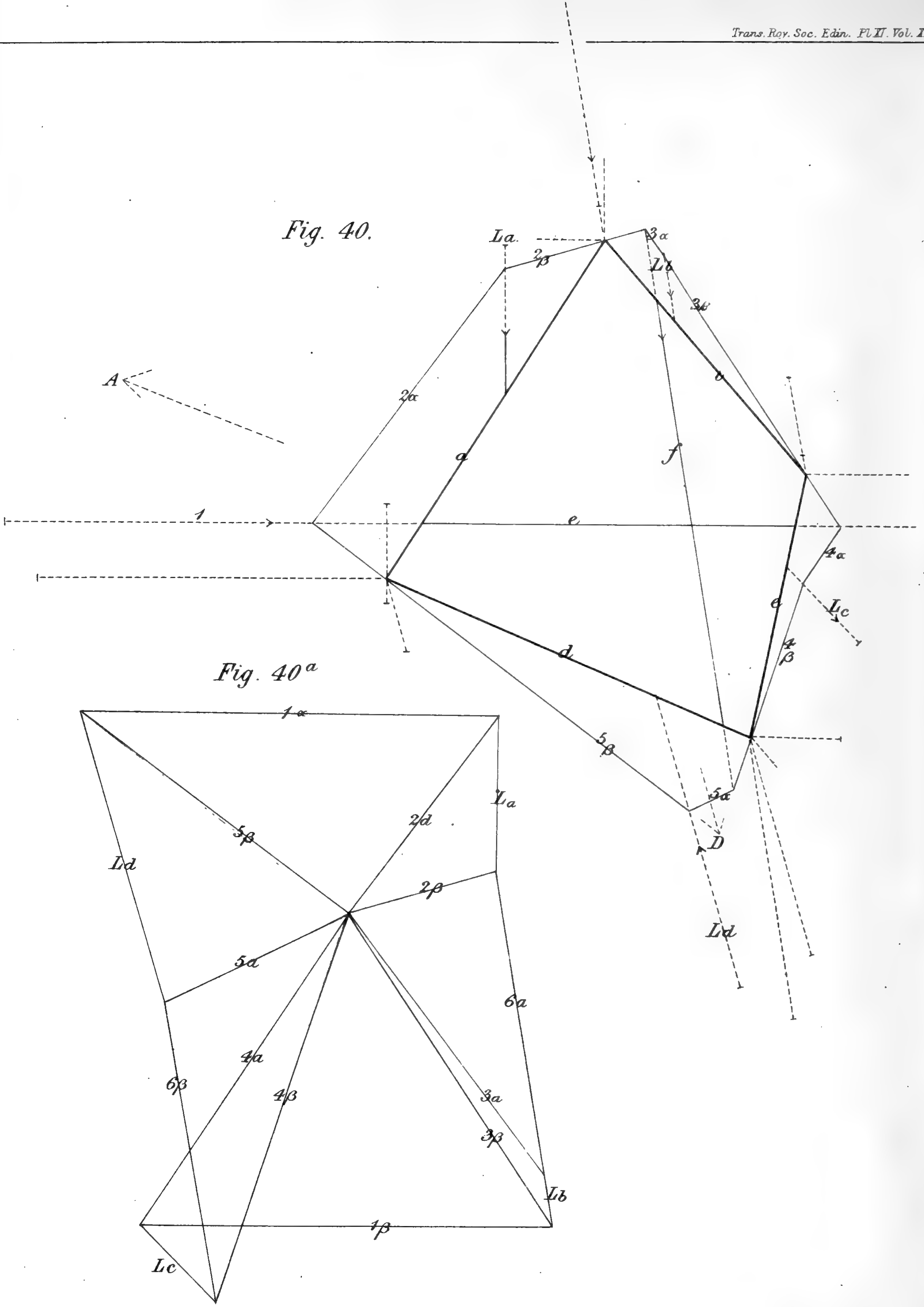


W. & A.K. Johnston, Edinburgh.

APPLICATION OF GRAPHIC METHODS TO THE DETERMINATION OF THE EFFICIENCY OF MACHINERY.

Fig. 40.

Fig. 40^a



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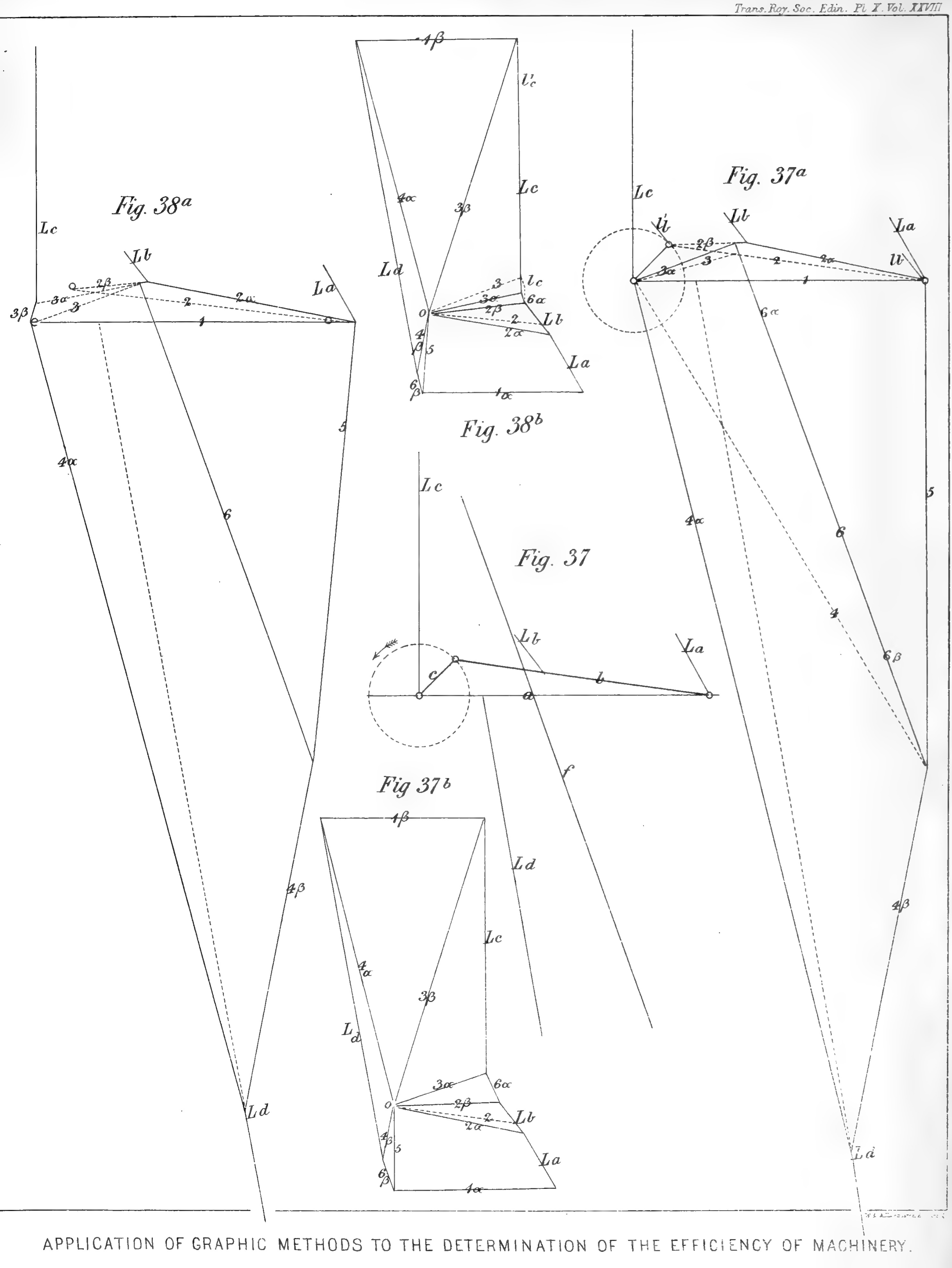

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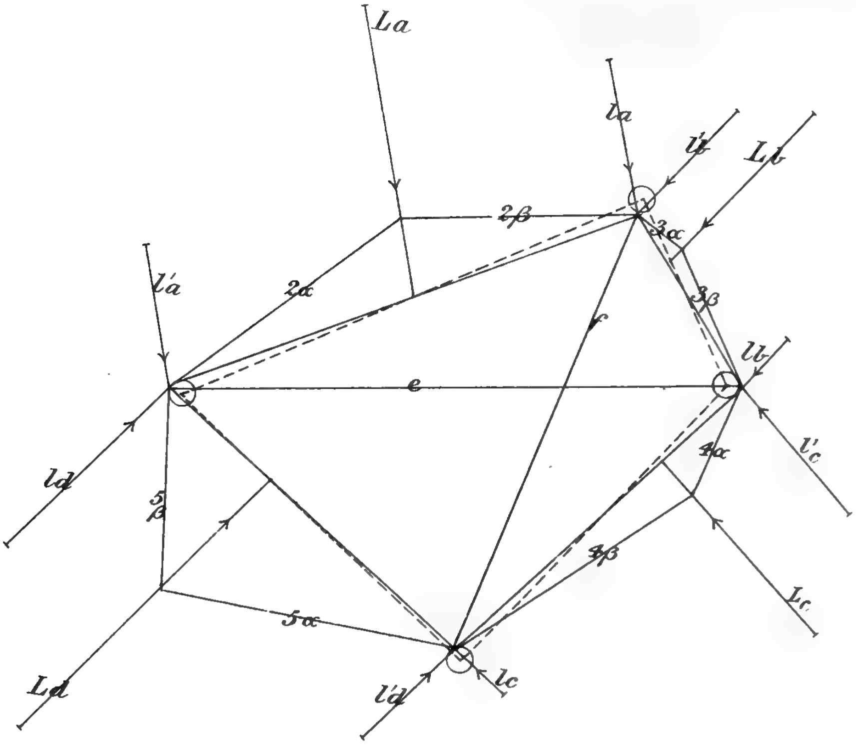

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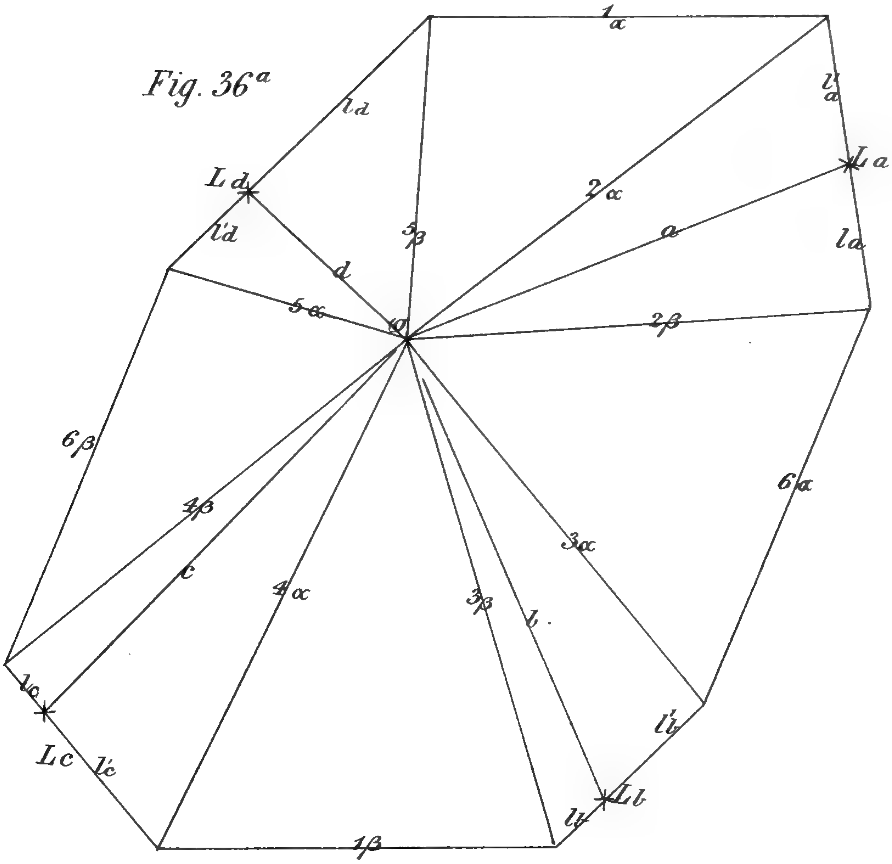

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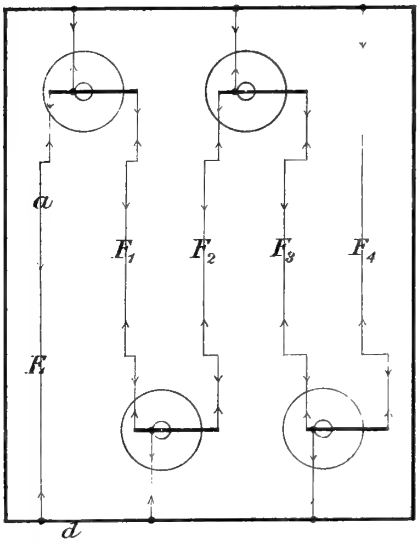

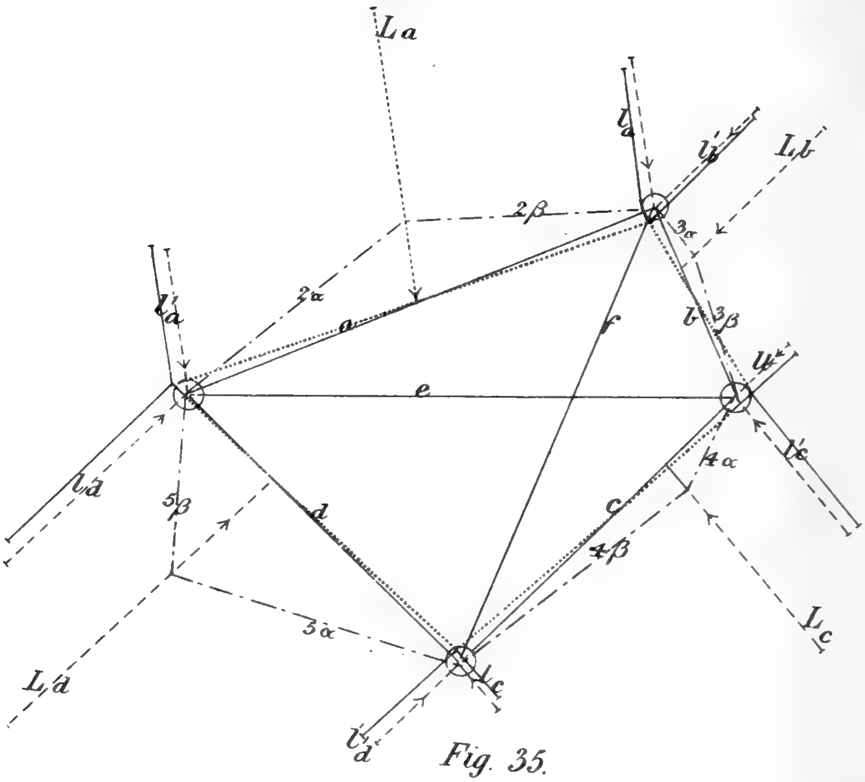

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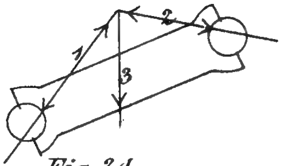

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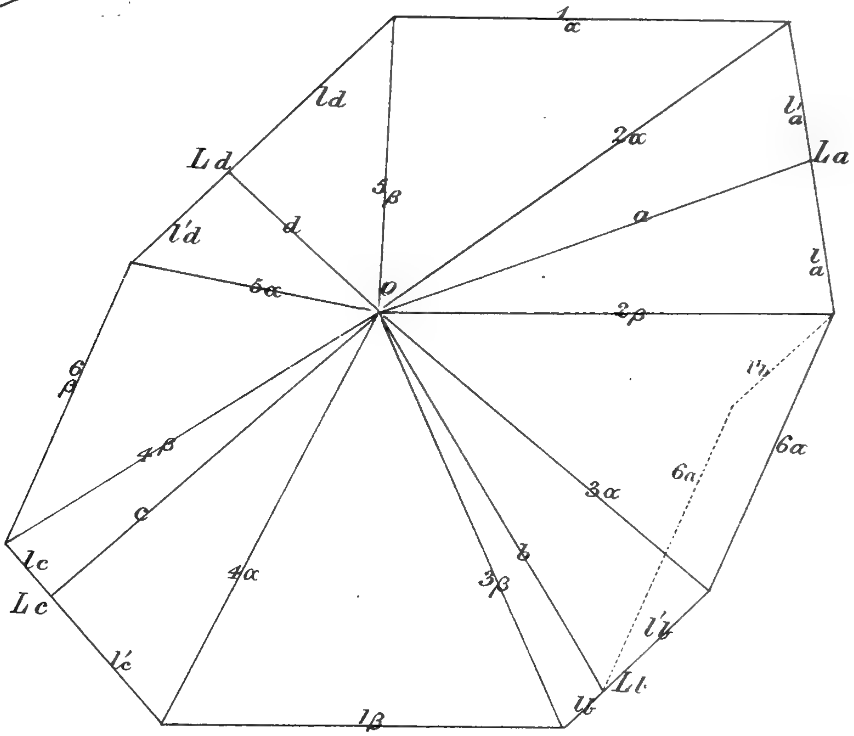

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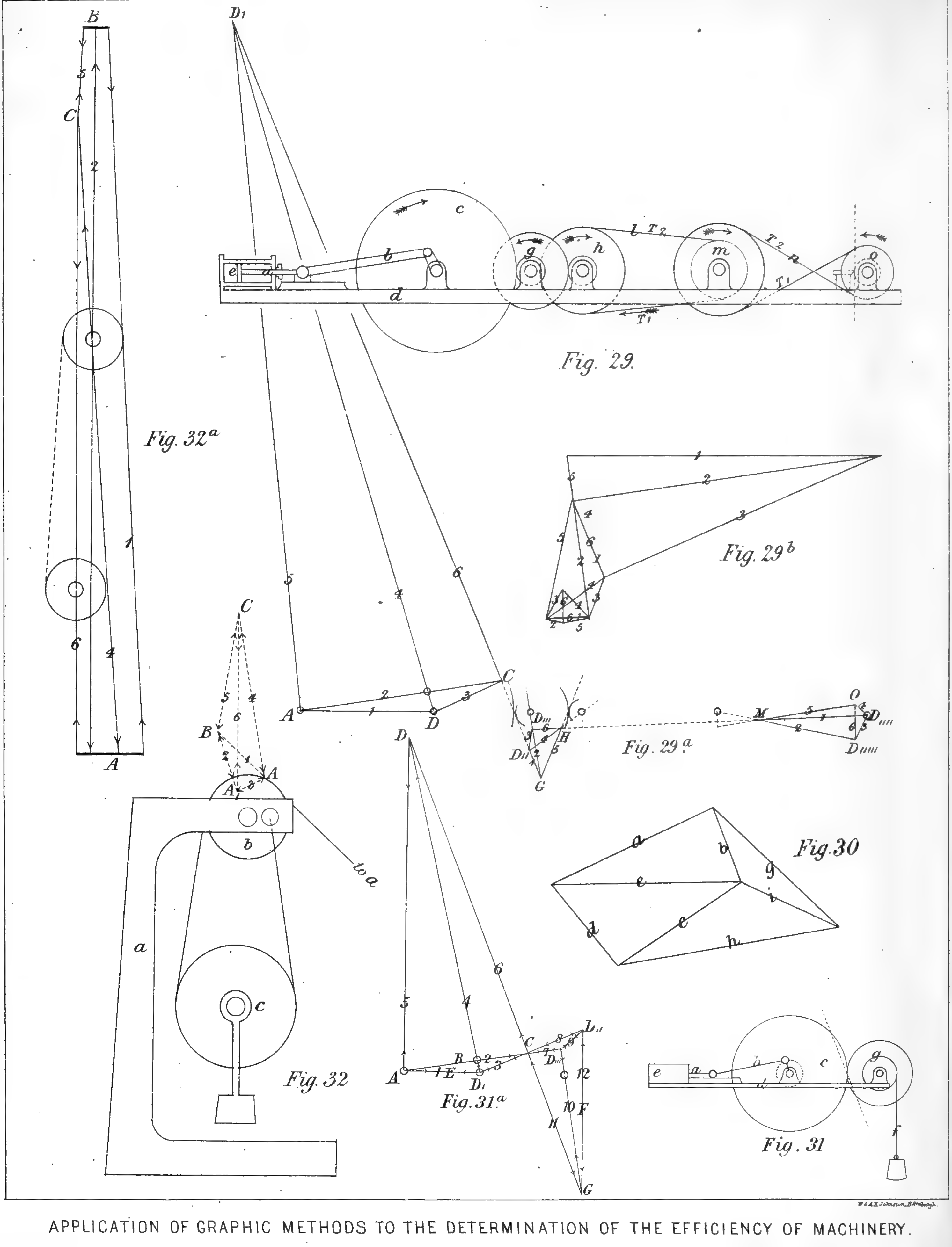

Fig. 29.

Fig. 29^b

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Fig. 32^a

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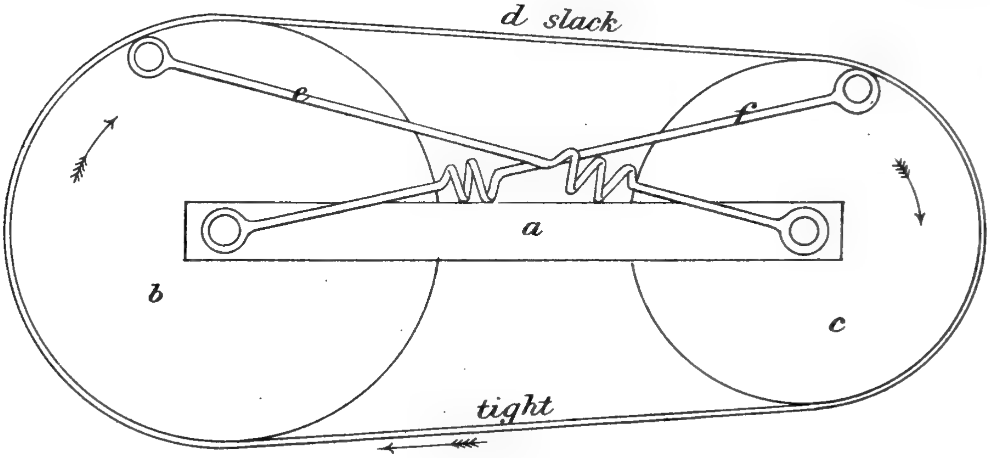

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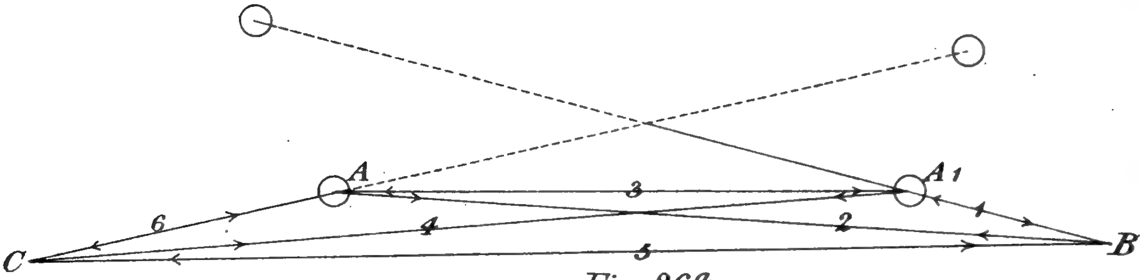

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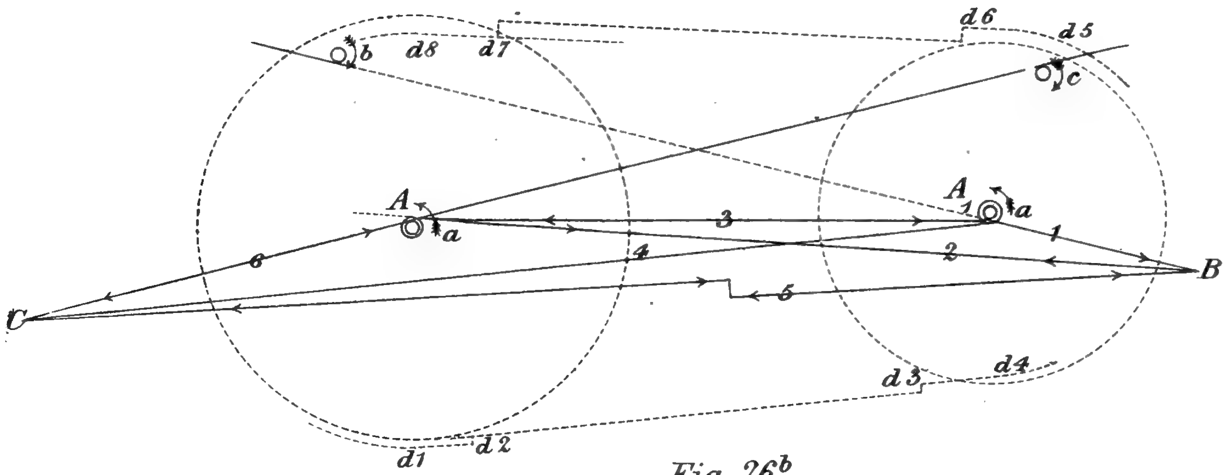

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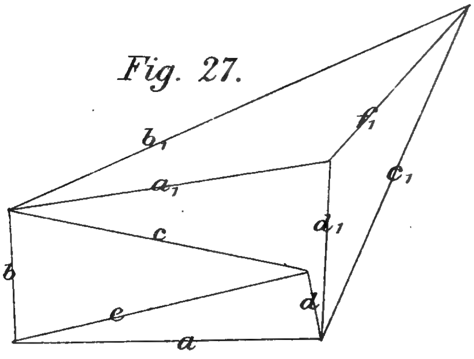

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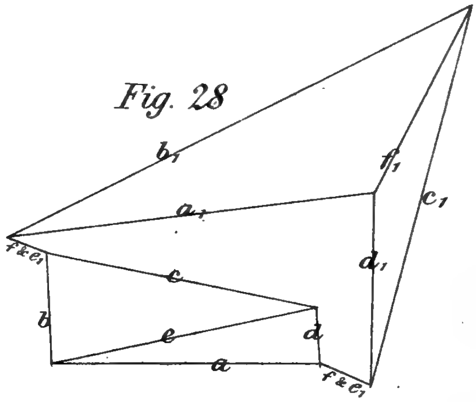

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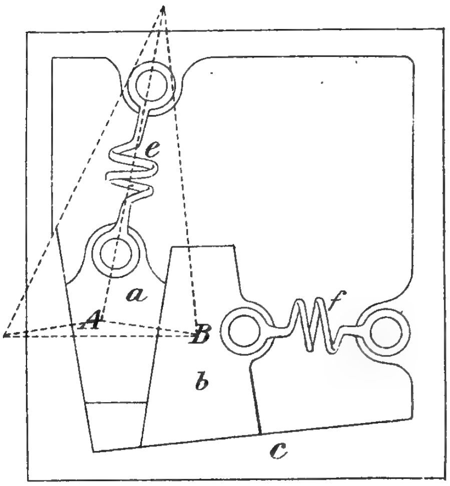

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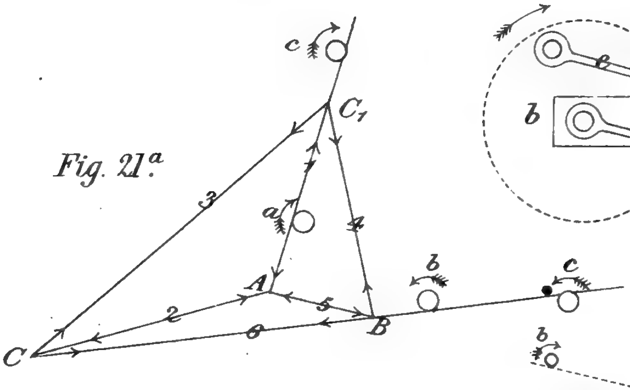

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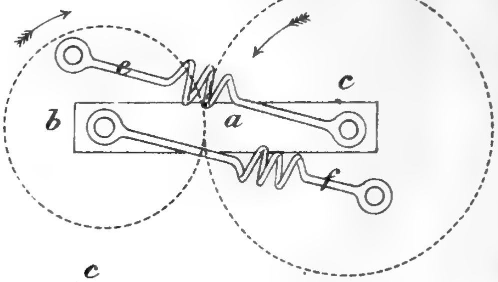

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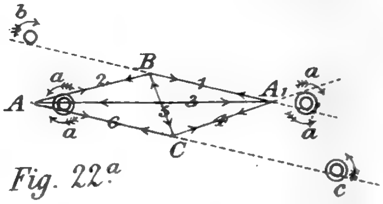

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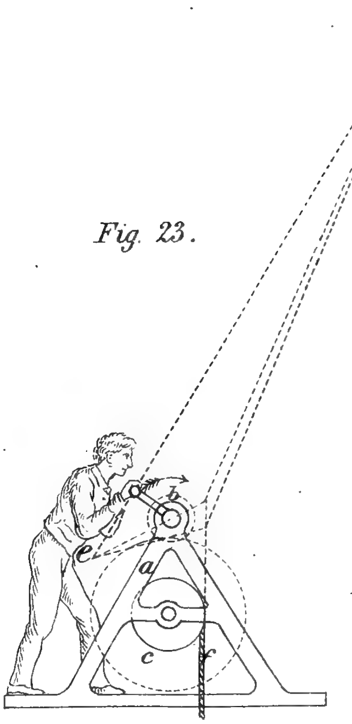

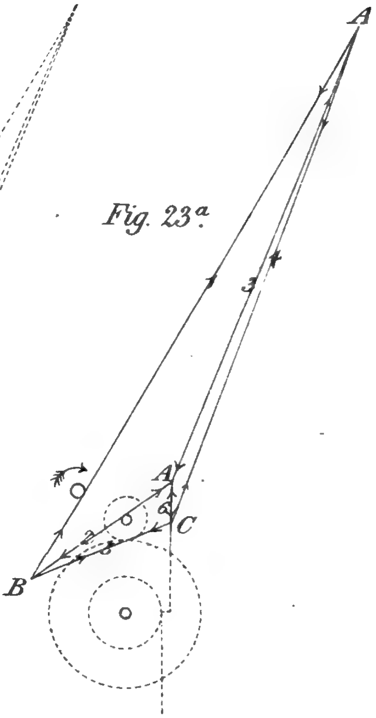

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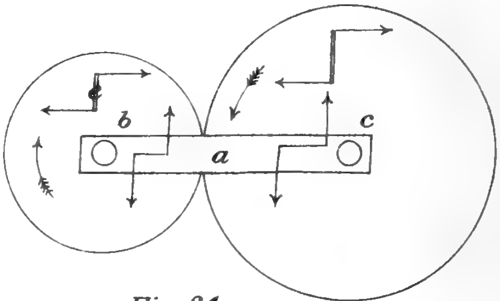

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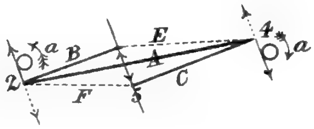

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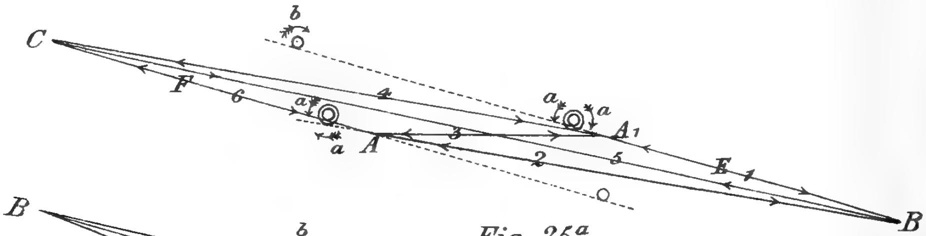

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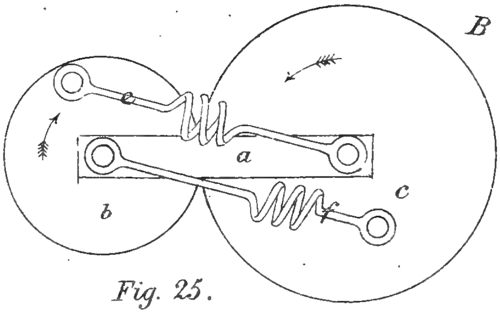

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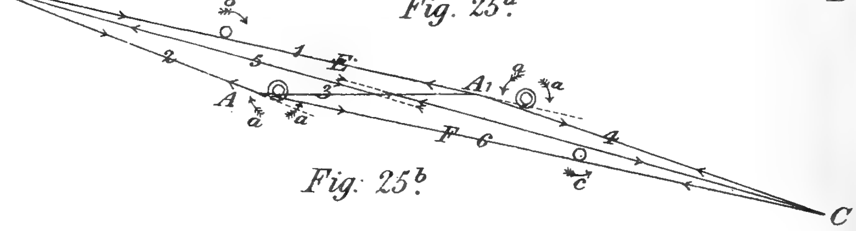

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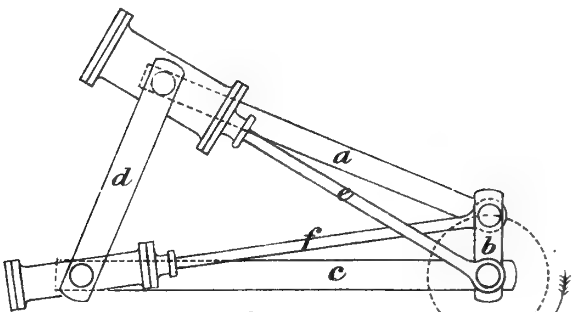

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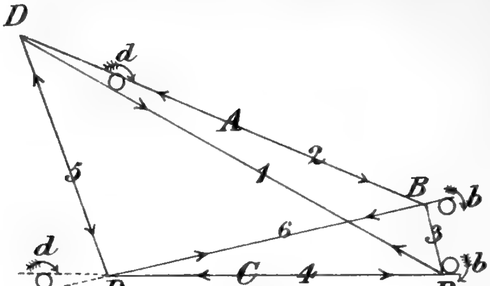

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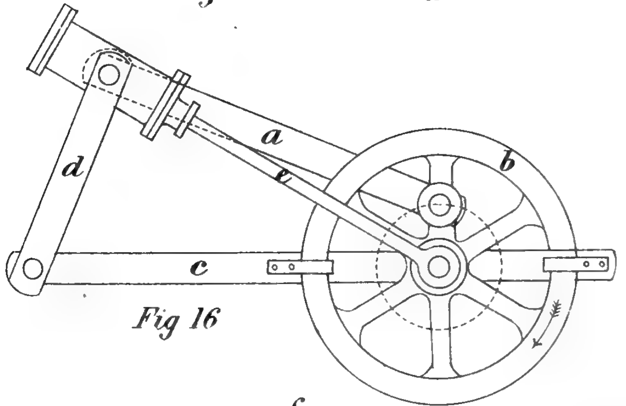

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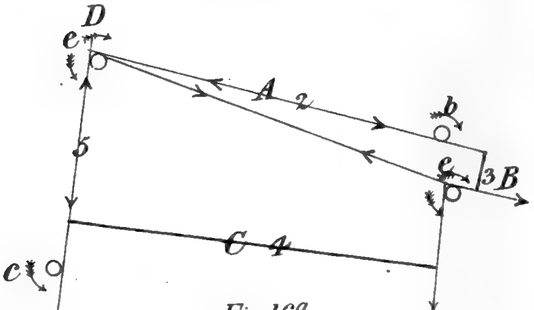

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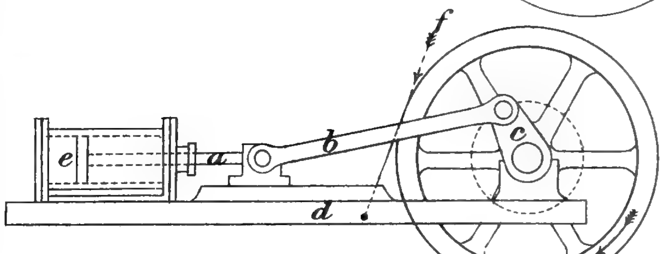

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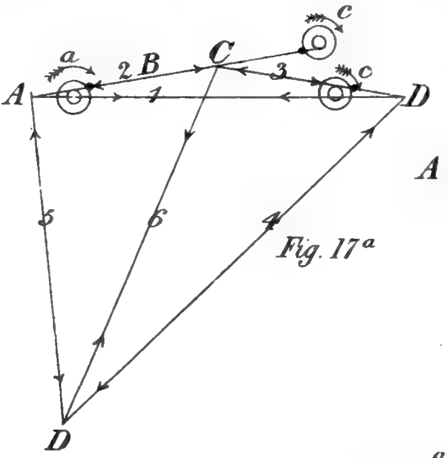

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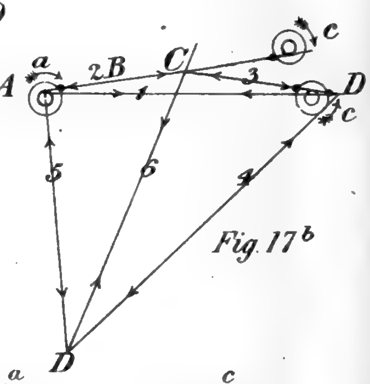

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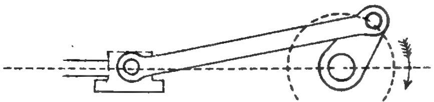

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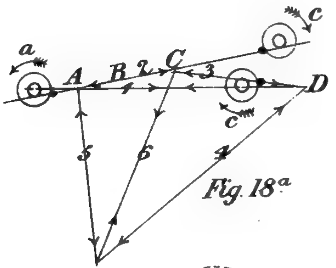

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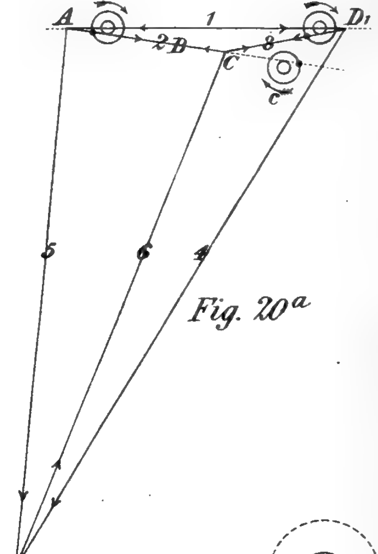

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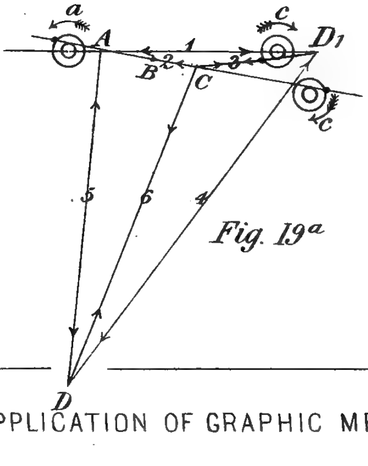

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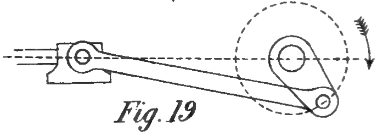

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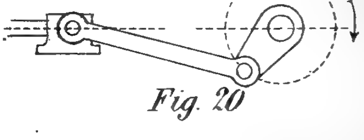

Fig. 20

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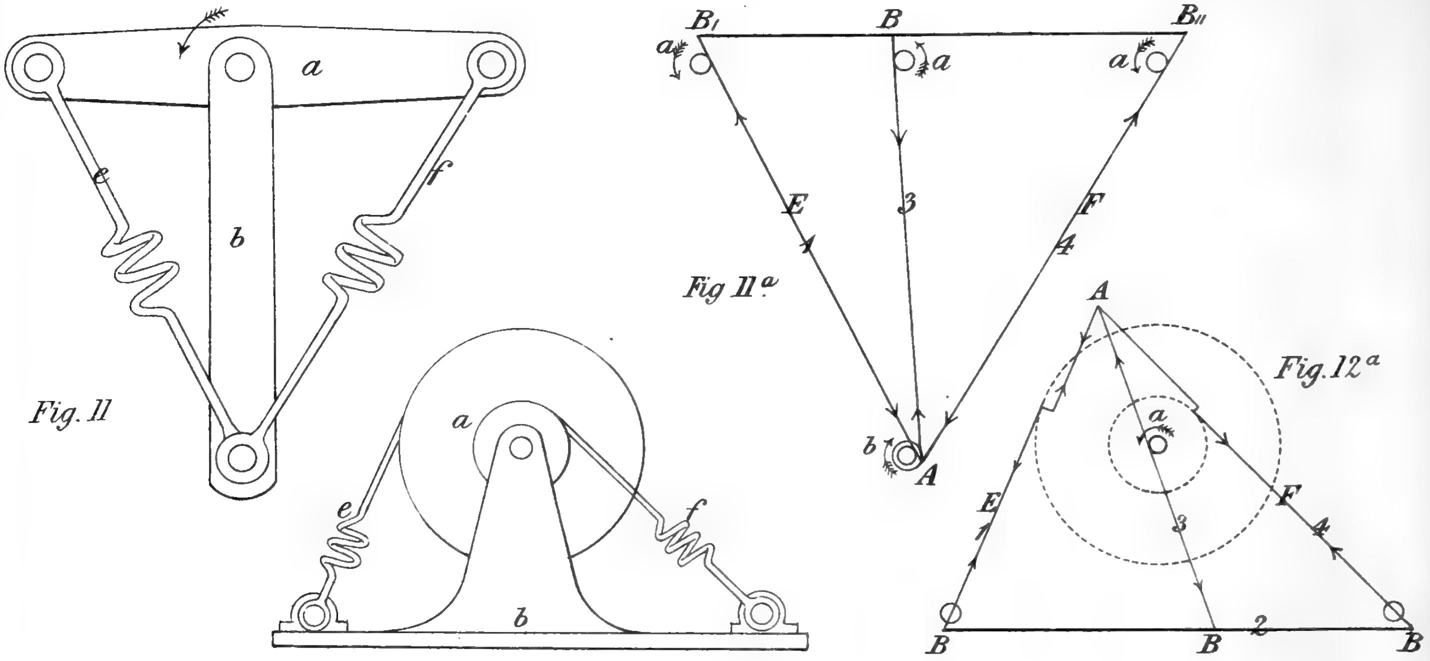
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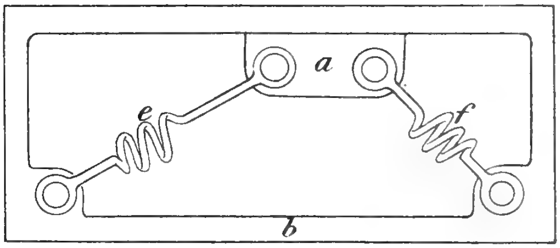
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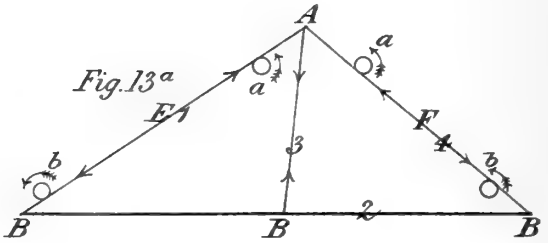
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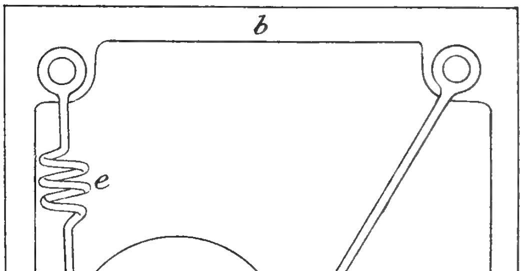
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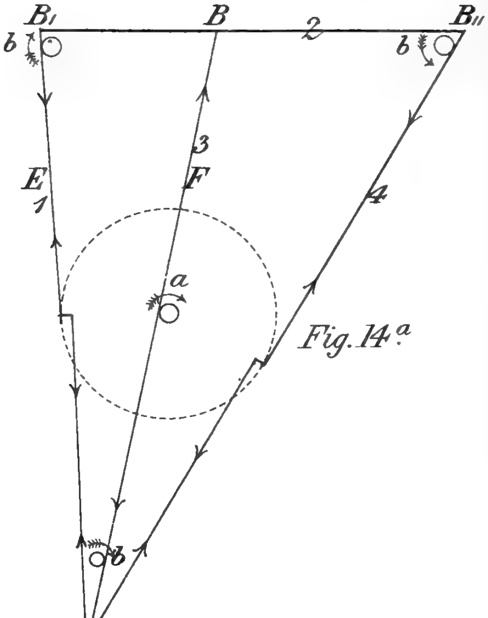
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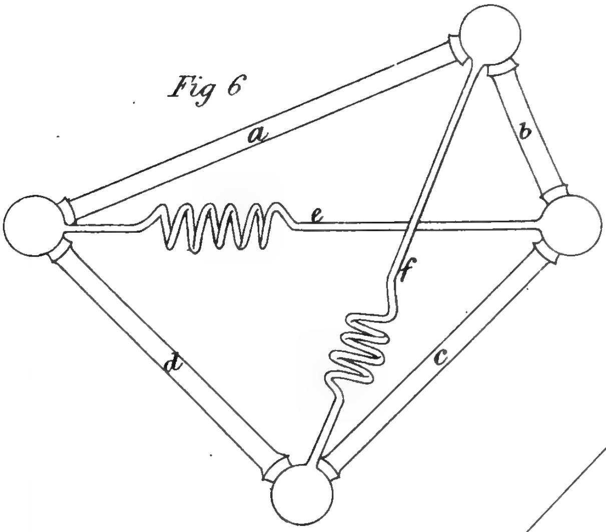
Fig 6



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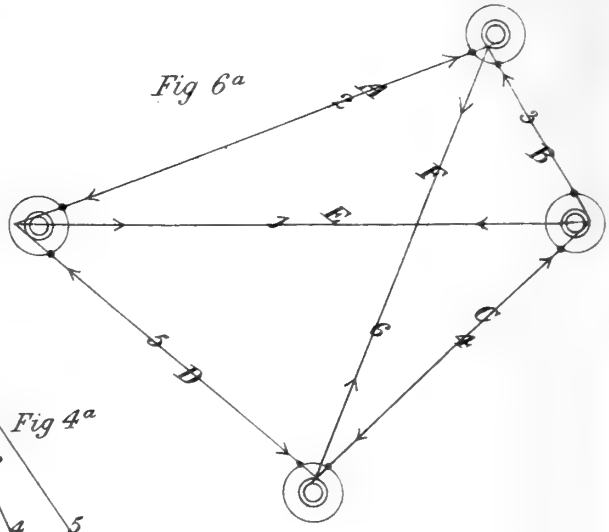

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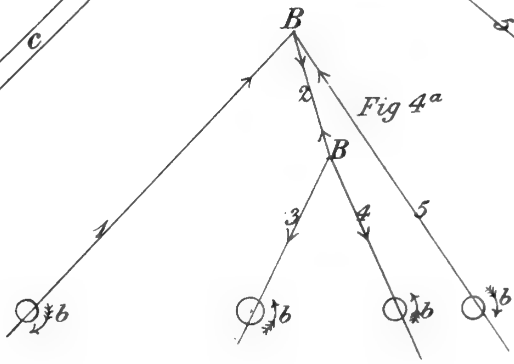

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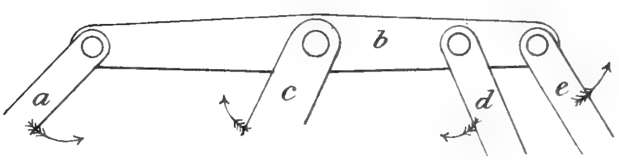

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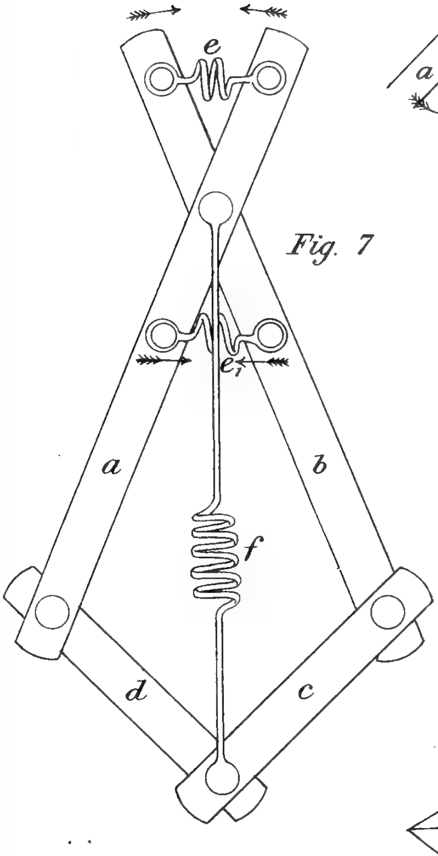

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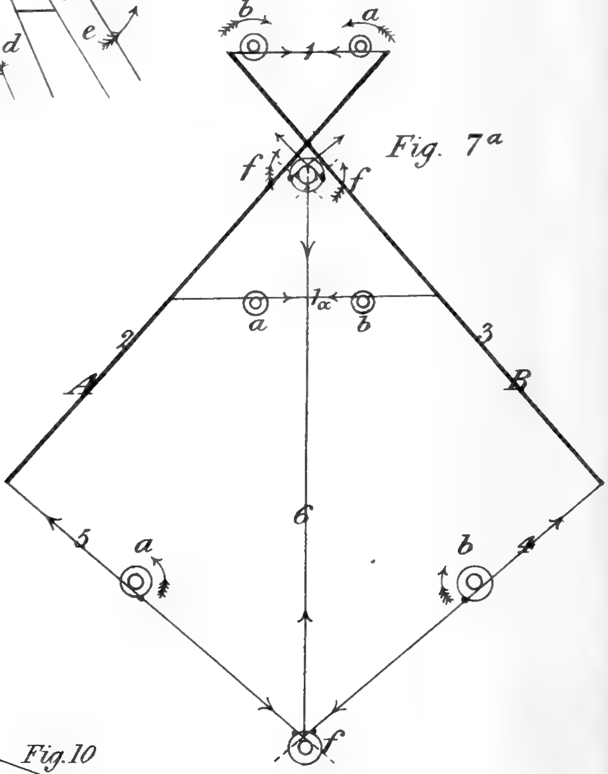


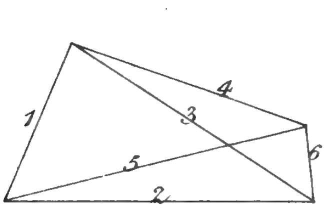
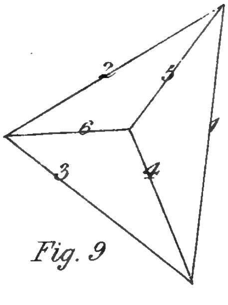


Fig. 8.

Fig. 9



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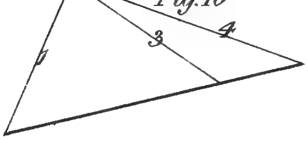

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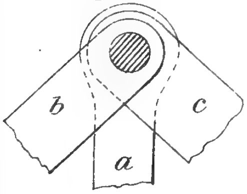
Fig 1



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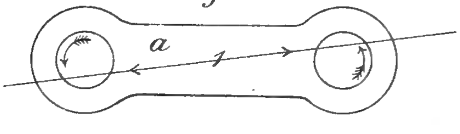

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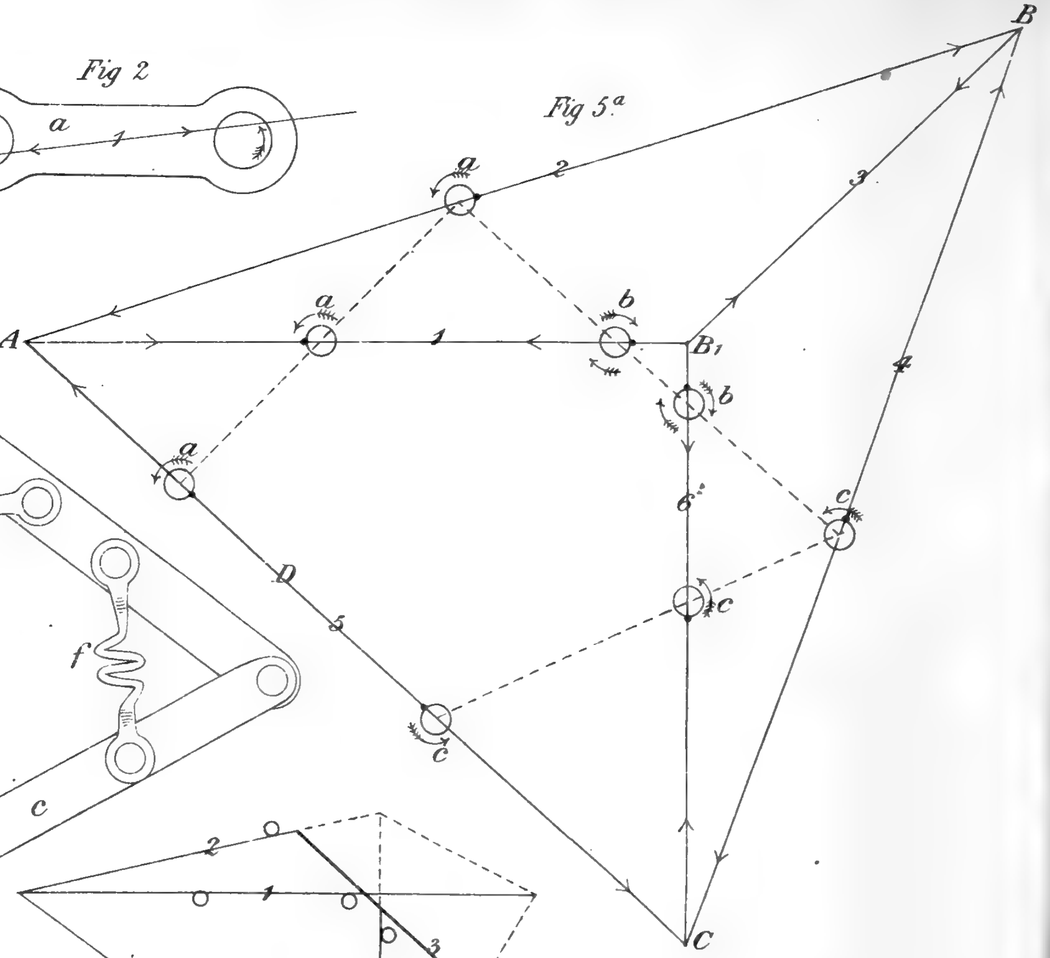

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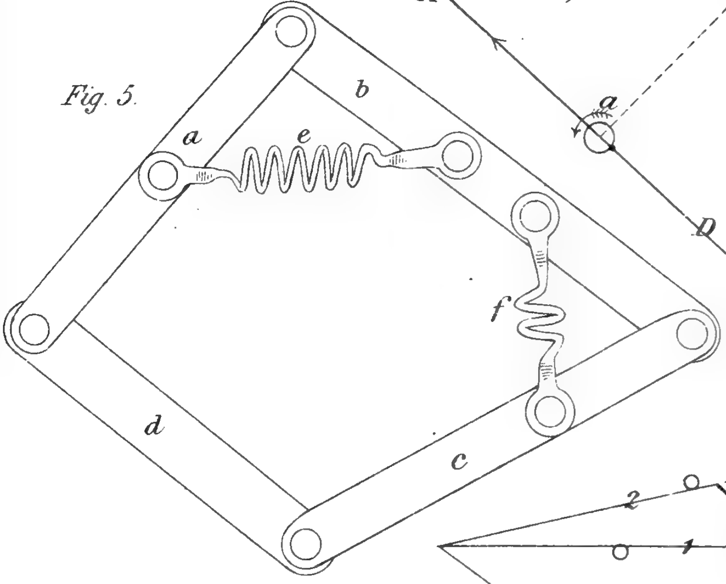

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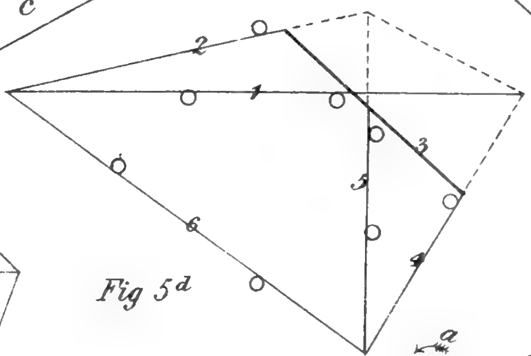

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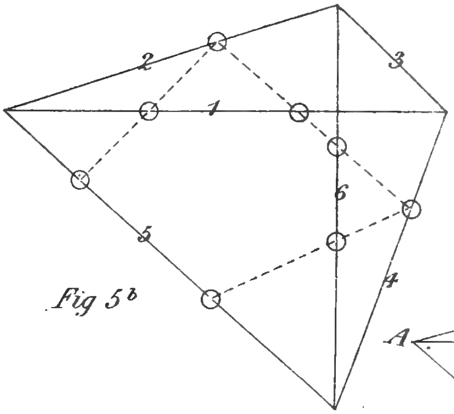

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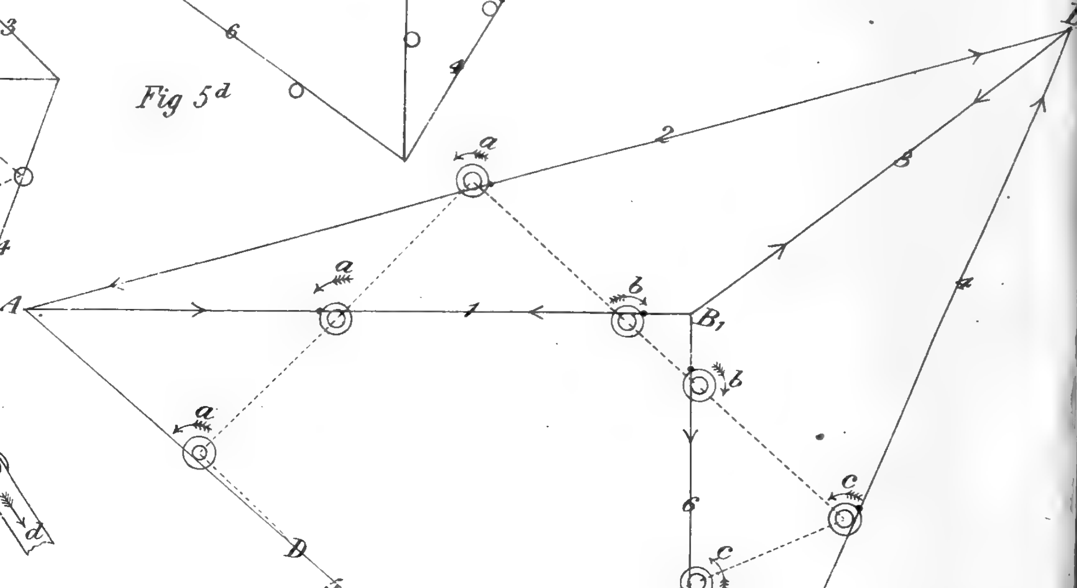

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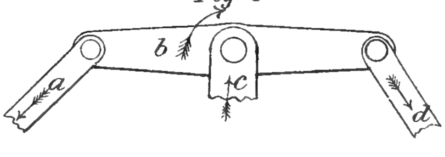

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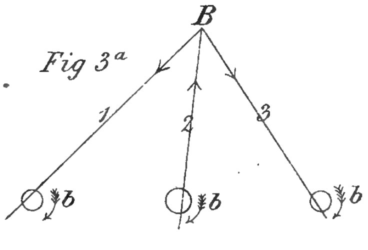

W & A K Johnston, Edinr

APPLICATION OF GRAPHIC METHODS TO THE DETERMINATION OF THE EFFICIENCY OF MACHINERY.

TRANSACTIONS.

I.—*On the Application of Graphic Methods to the Determination of the Efficiency of Machinery.* By Professor Fleeming Jenkin. (Plates I.–XII.)

(Read 2d April 1877.)

§ 1. The object of the present paper is to show how, by graphic methods, we may find the relation between the effort exerted at one part of a machine in motion, and the resistance overcome at another part; the solution found is rigorous for all motions in one plane, and takes count of the friction, weight, and inertia of the parts. It also takes into account the stiffness of ropes or belts.

The paper shows that we may represent any machine, at any given instant, by a frame of links, the stresses in which are identical with the pressures at the joints of the machine. This self-strained frame is called the *dynamic frame* of the machine, and may be so drawn as to represent the machine either rigorously, taking into account friction, weight, inertia, and rigidity, or approximately, omitting some of the conditions under which the machine works.

Moreover, it is shown that for all machines (in which the motions can be represented as in one plane), the dynamic frame is of one type, either simple or compounded. The dynamic analysis of machinery into parts represented by this simple frame is believed by the author to be novel. It is consistent with the kinematic analysis of Reuleaux.

The driving effort and the resistance are in the frame represented by stresses in links, and reciprocal figures afford an easy method of determining the relation between those stresses so soon as the frame has been drawn. Incidentally, the method also gives the resultant pressure at every joint in the machine, a result of considerable practical value. When the relation between the stresses due to effort and resistance has been found, it is easy to calculate the *efficiency* of a machine by taking into account the spaces through which these stresses act in equal times, so as to show any loss of energy which may arise from deformation, such as may be due to the stretching of ropes.

W. J. MACQUORN RANKINE introduced the word efficiency to denote the ratio of the *useful* work done by a machine to the *whole* work done. He showed that the efficiency of a train of mechanism is measured by the continued product of the efficiencies of all the successive pieces or combinations, and he gave methods by which the efficiency of certain elementary pieces and modes of connection could be ascertained. These methods assume that in each case the effort is known in magnitude, position, and direction; also that the position and direction of the resistance are known. The weight of the piece is taken into account, but no mention is made of the resistance due to inertia, which could, however, be treated in the same manner as the forces due to weight.

RANKINE did not carry his explanation of the subject so far as to show how to find for any actual complete machine the direction of the successive efforts and resistances; nor does he draw attention to the fact that they are interdependent. We cannot determine the efficiency of a whole machine by calculating the efficiency of each part separately without regard to its position in the machine, for it is this position which determines the directions of the driving and resisting efforts. These directions cannot be found by mere kinematic analysis, but depend not only on the form of the neighbouring parts, but also on their friction, weight, mass, and rigidity. In many cases the chief difficulty in determining the efficiency of a machine consists in determining the direction and point of application of the effort and resistance to the motion of each part. These directions are found by the *dynamic frame*. The writer has endeavoured to take up this subject where RANKINE left it, and to give a general method by which the efficiency of the great majority of actual machines can be practically calculated. In doing this, he has found the graphic method convenient.

Since the method adopted is based on a novel analysis of machines, it has been found necessary to begin by some elementary definitions.

§ 2. *Elements of Machines.*—All machines consist of parts so joined that any change in the force with which one part presses against another will produce some change in the force with which the other parts are pressed or held together. This condition obviously holds good when the parts are rigid, and it constitutes the test whether flexible ties or elastic fluids form part of a given machine.

A loose coil of rope cannot by this definition form part of a machine, but a rope is part of a machine, whenever a change in the tension of the rope changes the pressures or tensions between other parts of that machine. Similarly, steam or water forms part of a given machine whenever a change in the pressure of that steam or water changes the force exerted between other parts of that machine. This relation between the parts of a machine is dynamic, and is more general than the kinematic relation which exists between the rigid or inextensible parts of a machine. A definite position or motion of one part of a machine is not in every case accompanied by a definite motion or position of another part, irrespective of the forces in action, neither does the same relative position of all the parts of a machine necessarily imply the same forces between the parts, but a change in the force exerted between any two parts always implies a change in the forces between the other parts.

The word *element* will in this paper be used to designate the continuous parts of machines. Each *solid element* is a part which is continuous in respect that no portion can slide upon or break away from another portion in immediate contact with it. Any rigid bar or structure in a machine is an element, as, for instance, the connecting rod of an engine, the cylinder and bed-plate, the crank and fly-wheel. A belt between two pulleys, or a rope on an axle, when these form parts of machines, are separate elements. A slack belt or a slack rope forms no part of a machine, since it does not fulfil the first condition of altering the force at one end when the tension is altered at the other. A continuous fluid forming part of a machine will be called a *fluid element.* A portion of fluid is continuous when a change of pressure at one place is transmitted by the fluid so as to change the pressures throughout the element. Thus the steam in the cylinder of a steam engine is a fluid element, and, strictly speaking, this element comprises the steam in the pipes and in the boiler, as well as the water in the boiler. Generally, we need only consider the steam enclosed in the cylinder cut off and bounded by the slide valve. Wherever discontinuity of motion occurs, a solid element will be considered as terminated or bounded. Thus two rigid bars joined by a flexible tie will be treated as three distinct elements. The flexibility of the tie replaces the sliding motion at a joint which would otherwise be required. The discontinuity of motion here insisted upon as indicating the surface of separation between elements is the property by means of which we shall be enabled to determine the relative forces with which separate elements press on one another. Two elements are treated as separate when at any part they are discontinuous, and also in certain limiting cases when the relative motion at the surfaces of contact is infinitely small.

The elements of a machine as now defined correspond closely with Professor Reuleaux' kinematic links. All Professor Reuleaux' links are elements, but the definition now given of a dynamic element would embrace certain parts of machines which can hardly be called kinematic links, as, for instance, the steam and water mentioned above. Each element of a machine will in what follows be designated by a single small italic letter.

§ 3. *Joints.*—The name of joints will be given to the surfaces of separation between two elements. When the elements are rigid, joints can occur only where there is sliding or rolling contact between the elements. A surface of separation between a flexible tie and a rigid bar will also be treated as a joint, inasmuch as discontinuity of motion may occur at this surface similar to that which occurs at a surface of separation between rigid elements. A joint in the sense in which the word is here used is essentially a joint between *two* elements. There can be no common joint between three or more elements. In ordinary phraseology, we speak of three or more members of a frame as jointed at one place, and speak of the joint there as a single joint, meaning that they all abut against a single pin; but, in the present paper, each portion of the surface of such a pin as bears against a single element will be spoken of as a single joint. We shall always assume that a pin of this kind is fixed relatively to one of the elements, so as to reduce the elements of the machine to the smallest number. A joint will be designated by two italic letters, being the names of the elements between which it occurs. Thus in fig. 1 there are two joints, *ab* and *ac*, the pin being fixed on *a*. All actual joints are surfaces of greater or less extent. They may be divided into three classes:—1st, Joints between a solid and hollow sphere; this case includes that of a plane resting on a plane. 2d, A solid prismatic cylinder inside a hollow cylinder. When the cylinder has a circular cross section, relative turning and sliding are both possible. 3d, A solid screw inside a hollow screw. When the pitch is infinite we have one form of case 2d in which only translation is possible. When the pitch is zero we have a joint which only allows of rotation and is the joint or bearing most commonly met with in machinery. The third case is the most general, and may be considered as including the two others as special forms. The solid and hollow cylinder, which allow simple rotation of one element relatively to another, will be called the pin and eye of a joint.

A dynamic joint occurs wherever pairing occurs, the word pairing being used in the sense given it by Professor REULEAUX. The kinematic pair as defined by him differs from the dynamic joint in respect that complete pairing implies closure or complete restraint except in one direction. Moreover, Professor REULEAUX is able to classify pairs with reference to the mode of closure. A dynamic joint implies no restraint other than that given by the force exerted in the line of bearing pressure. All portions of the pair might be cut away, except a surface of sufficient strength round the bearing point; distinctions between force closure, pair closure, link closure, have no dynamic signification; and restraint, such as that afforded by collars on bearings, is of no importance dynamically when the machine is properly designed. A short distinction may be made between a dynamic joint and a kinematic pair, by saying that the latter might be self-strained by imperfect fitting, while the former cannot be self-strained.

In certain cases the surfaces in contact may approximately be represented by lines, as in what Professor REULEAUX calls the higher pairs, and we may even conceive a class of joint which requires only points of the elements to be in contact, as where a sphere is pressed against a plane. In connection with geometrical diagrams, the word joint will be applied to *points* round which intersecting lines may be said to turn relatively to one another. Geometrical joints of this kind will generally be designated by a single capital letter.

§ 4. *Definition of a Complete Machine.*—The object of machinery, considered dynamically, is the application of energy, or, in more popular language, power to the performance of useful work, and the name *complete machine* may be given to any combination of elements so joined that the energy developed in one element, or between two elements, is, by the relative motion of the elements, enabled to do useful work in overcoming a resistance exerted either by one element or between two elements. A complete machine is self-contained, and the internal action between its parts can change neither its momentum nor its angular momentum. Most actual machines have one portion fixed relatively to the earth, which then becomes part of one element of the machine.

§ 5. *Lines of Bearing Pressure.*—The elements of a complete machine are so held together at the joints by the forces which are in play when the machine is in action, that each element of the machine occupies a determinate position relatively to all the others, and presses against its neighbours at each joint with a force determinate in magnitude, direction, and position. A given position of the parts does not necessarily imply constant forces at the joints, but it does imply a determinate relation between the forces at the joints. A line indicating the direction and position of the equal and opposite resultant forces at a joint will be called a *line of bearing pressure.* In consequence of friction, a line of bearing pressure where it cuts the joint, must, when the machine is in motion, make an angle with the common normal to the surfaces equal to the angle of repose, or angle of which the tangent is equal to the coefficient of friction for the surfaces at the joint, and must be so inclined that the force exerted by the one element on the other has a tangential component directly opposed to the sliding of the second element relatively to the first. This condition will hereafter be frequently referred to, and will, for brevity, be spoken of as the condition that the bearing pressure shall make the *stated angle* with the joint.

§ 6. *Driving Element and Resisting Element—Driving Link and Resisting Link.*—The source of power in a machine is often contained in a single element, which is usually so combined with the others as to exert equal and opposite forces in two directions, and only in two directions. The steam in a cylinder of a steam engine affords one example of this kind of element, which will be called a *driving element.* If the directions of the equal and opposite forces exerted by the driving element lie in the same straight line, the power tends to lengthen or shorten the element in a definite direction. The two forces in this case tend to separate or draw together the other elements with which the first is jointed, as a straight elastic link would do if extended or compressed so that its line of action coincided with the direction of the forces exerted by the driving element. We may therefore conceive an actual driving element as replaced by an ideal elastic link producing the same force at the above named joints. Some machines have no material driving element, but are actuated by an attraction or repulsion between two of their elements, as, for example, when a machine is driven by a weight. In this case, as in the former case, we may conceive the machine as driven by an ideal elastic link between two elements. This link is as well suited to replace the equal and opposite forces due to gravitation as the equal and opposite forces due to steam. If the two opposite and equal forces exerted by a driving element do not lie in one straight line, these forces exert a couple between the elements with which the driving element is jointed; a similar couple may be exerted by the forces of attraction or repulsion between two elements. In either case, the machine may be considered as actuated by an ideal couple due to the elastic reactions of two links. What has been said of the driving power applies *mutatis mutandis* to the useful work done by a machine. In some cases the work is actually done in lengthening or shortening a separate resisting element, as when a bale of cotton is compressed. In other cases an attraction or repulsion is overcome between two elements, as when a weight is lifted. In both these cases we may conceive the resistance as represented by an ideal elastic link exerting definite, equal, and opposite forces in a definite position and direction. This ideal resisting link is equally suited to express the resistance by which cohesion, friction, or inertia opposes the relative motion of two elements. When the resisting element, or the resistance due to the united action of two elements, gives rise to a resisting couple, this resistance may be represented by two ideal links between two elements. Thus, we see that in every machine where there is only one source of power, we may represent that source by one ideal link (or two ideal links) connecting two elements, and acting on these elements as one member (or two members) of a strained frame acts on the rest of the structure. The name of *driving link* will be given to ideal links of this kind. The name of *resisting link* will similarly be given to ideal links used to represent the useful resistance in a machine. In all cases the cause of motion and the useful resistance must be considered as an action between two elements. We cannot properly speak of a driving point or resisting point, but only of driving or resisting links.

§ 7. *Links—Dynamic Frame.*—Any actual material element might at any instant be removed from a machine without altering the stresses on the other elements, if at each joint thus laid bare, a force could be applied corresponding in position, magnitude, and direction with the pressure supplied at that instant, and at that joint by the element removed. The series of forces supplied by any one element are necessarily such as would, when balanced by equal and opposite forces, leave the element in equilibrium. If, therefore, an element has only two bearing joints, and is without weight or inertia, the forces it supplies must lie in one straight line, and either push or pull, as the member of an actual frame does, when under simple tension or compression; the element, in fact, acts like one link of a frame, as shown in fig. 2, where the element *a* might be replaced* by the link 1, shown by the line on which arrow-heads are placed. The ideal link replacing the actual element does not necessarily or generally liein the geometrical axis of the element. When there are three bearing joints in an element having no weight or mass, the three lines of pressure must lie in one plane, and either be parallel or intersect in one point. Any one of the forces supplied may be looked upon as the equilibrant of the two others. The forces which this element supplies might therefore be replaced by three ideal half links, each coinciding in position and direction with the line of bearing pressure at the three joints, and all connected by an ideal geometrical joint without friction at the point of intersection (which, in the case of parallel forces, will be at an infinite distance). Thus element *b*, fig. 3, may be replaced by the half links 1, 2, 3 intersecting at the geometrical joint B, fig. 3*a*; links 1 and 3 would be half links in tension, link 2 a half link in compression. The direction of the arrows shows the direction of the stress in the links replacing *b*. The directions of the links 1, 2, and 3 do not coincide with those of the geometrical axes of *a*, *c*, and *d*; indeed, these elements may be stiff bars having many other joints; but if we know that the element *b* is moving relatively to *a*, *c*, and *d*, as shown by the arrow, then one condition determining the directions of links 1, 2, 3 is given us, for these directions must make the stated angle with the surfaces of the joints *ba*, *bc*, and *bd*. In fig. 3*a* the links 1, 2, and 3 are therefore shown, not passing through the centres of the circles at the joints, but passing on that side of the centre on which the force represented in the link would resist the motion of the pins supposed to be fast on *b*. The small arrows, fig. 3*a*, show the direction of rotation of these pins, and these arrows are lettered *b* to indicate that they represent the motion of element *b* relatively to *a*, *c*, and *d*. This plan of indicating the relative motion of the surfaces at joints will be followed in future diagrams. It will always be assumed that the pin is *fixed* in the element indicated by the letter at the arrow. When there are more than three joints, the forces supplied at each joint are such as would be given by a series of half links, one for each joint, corresponding with each line of bearing pressure, and themselves joined by other links, so as to form a frame which would be in

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* The writer has in this paper ventured to use the verb "to replace" as it is usually employed by writers on chemistry, namely as the translation of the French word "remplacer."

equilibrium under the action of external forces equal to those in the half links.* Thus any one of these forces may be looked upon as the equilibrant of the others, and as acting upon them through a series of links which are subject only to compression or tension. In fig. 4, plate II., if the member *b* is jointed with members *a, c, d,* and *e* by four parallel pins, we might replace *b* in any machine by the four half links, fig. 4*a*, 1, 3, 4 and 5, and a complete link 2, joining the intersection of 1 and 5 with that of 3 and 4: this last link will be in equilibrium under the action of the forces acting on *b*, as shown in the half links. In a more complicated example the link 2 might be replaced by a complete frame, or by a rigid plate. Since the forces acting on one element either intersect at one point or in a series of points which may be joined by other links, we may always designate the point or points in question by the same letter of the alphabet as is used to denote the element. The geometrical intersections will be marked by capital letters, whereas the elements are marked by italics. When there are several geometrical joints for one element, these points will be denoted by the same letter, but distinguished from each other by dots suffixed. The substitution of links or half links for an actual element may be effected even when forces are parallel, if we admit joints at an infinite distance. If now, all the elements of a machine in their relative positions, at any one instant be removed in succession, and replaced by their equivalent links or half links, we shall substitute for the original machine a self-strained frame of links such that the stress in each link passing through a joint, will be in all respects equal to that on the joint, while the stresses in the driving and resisting links will represent the effort and useful resistance in the machine. Each half link at a joint of one element is necessarily met and completed by the other half link in the same line, due to the reaction of the second element.

The self-strained frame, composed of links as described above, will be called the *dynamic frame* of the machine with its elements in the given relative position.

§ 8. *Example.*—An example will probably assist in showing what is meant by the dynamic frame. Let a machine be composed of the six elements, *a, b, c, d, e, f,* joined as in fig. 5. Let *e* be the driving element, and *f* the resisting element. We will suppose in this and in the following examples, that the resulting forces all lie in one plane, although the figures may not show the split joints necessary to ensure this result. For the present, the effect of weight and inertia will be neglected. The machine shown is a complete machine; the element *d* has two joints, the elements *a* and *c* have three joints, and the element *b* has four joints. The dynamic frame may be drawn assuming the friction at the joints to be

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* In this frame it might be necessary to include at least one stiff bar or frame to meet opposite and equal couples.

insensible, or it may be drawn taking friction into account. In the former case it will be represented as in fig. 5*a*, which is obtained as follows:—Link 1 may first be drawn through the axis of *e*, for we know that the half links at the joints *ae*, *eb* must lie normally to the surface of these joints (being frictionless), and must therefore lie in one straight line, passing through the centre of the pins at *ae* and *eb*. For similar reasons we draw 5 and 6 through the axes of elements *d* and *f*. The forces exerted by the links *d* and *e* on the element *a*, must be balanced by the force due to the third joint *ab*, therefore the direction of this balancing force must pass through the intersection of 1 and 5, and must be normal to the surface of the pin at *ab*. We therefore are now able to draw the link 2. For similar reasons we must draw the link 4 through the centre of the pin at *cb*, and through the intersection of the lines 5 and 6. The element *b* is in equilibrium under the action of the four forces acting at the four joints; in other words, the resultant of 2 and 4 must be equal and opposite to the resultant of 1 and 6, so that we may complete the dynamic frame by drawing the link 3 as shown; this last link, however, would be equally well placed as shown in fig. 5*b*, where it joins the intersection of 2 and 6 with that of 1 and 4. Both the frames shown in fig. 5*a* or fig. 5*b*, are kinematically equivalent to the actual machine shown in fig. 5 in the following sense:—A given small contraction of the element *e* would, supposing all the other elements to be rigid, produce a definite extension of the element *f* in the actual machine. A small contraction in link 1 equal to that in element *e* would, supposing all the other links of the frame inextensible, produce an extension in link 6 equal to that produced in *f* in the machine. We may calculate the relation between the stresses in *e* and *f* by the relative rates of their contraction and extension, that is to say by the principle of virtual velocities, or we may calculate the relative stresses between links 1 and 6 of the frame by the ordinary principles of statics, for instance, by a "reciprocal figure."* The ratio between the stresses in *e* and *f*, and that between the stresses in 1 and 6, would be identical whichever method were adopted. It need hardly be said that the method by virtual velocities would be much the simpler. It is not until we wish to take friction into account that the utility of the dynamic frame becomes apparent. In order to take friction into account we have to change the form of the frame only in this respect, that the links, instead of being normal to the surface of each joint, must be inclined so as to make the angle of repose with the normal to that joint, and must be so placed that the reaction due to the elasticity of the link—or in other words, the stress in the link—may oppose the motion of rotation of the pin in the eye of the link; in brief, the link must make the *stated angle* with the surface of the joint. In the present example,

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* *Vide* J. Clerk Maxwell on Reciprocal Figures, *Phil. Mag.* April 1864; and Fleeming Jenkin, *Trans. Roy. Soc. Ed.* vol. xxv. 1869.

where all the joints are made by circular pins and eyes, this is readily done by making the directions of the links tangent to, and on the proper side of, circles drawn with their centres at the centres of the pins, and having each a radius equal to $r \sin \phi$, where is r the radius of the pin in question, and ϕ the angle of which the tangent is equal to the coefficient of friction for the surfaces in question.

These circles will, in what follows, be called *friction circles.* The following mnemonic rules will be found useful in selecting the side of the circle to which any given link must be tangent. Case 1. When the link represents an element with only two joints, and therefore does not end in any geometrical joint named by the same letter as an element. Consider the link as terminating in an eye which rotates on a pin fastened to the other element. Mark by an arrow the direction of rotation of the pin inside the eye. Mark also by an arrow the direction of the force exerted by the link at the joint, then place the tangent so that the force indicated by the arrow in the link appears to oppose the motion of the pin. It must be remembered that the arrow indicating the direction of the force exerted by the link may, in the diagrams, frequently be found at the end of the link furthest from the joint. Case 2. When the link ends in a geometrical joint marked with the letter denoting the element in which any pin is fast, then the arrow on the half link, next that geometrical joint, must point as if opposing the motion of the pin relatively to the other element.

Fig. 5*c* shows the dynamic frame when the friction at the joints has been taken into account, or as it may be called the *dynamic frame with friction.* Eight friction circles are first drawn, arrows are placed on the links, as in fig. 5*a*, to indicate whether these are in tension or compression; the directions of stress are by hypothesis known in links 1 and 6, and we easily see what must be the direction of stress in the other links to keep links 1 and 6 in equilibrium. We next put arrows at each friction circle in fig. 5*c*, showing the motion of each pin relatively to its eye when *e* contracts, and *f* is lengthened; before doing this, we must choose in which element the pin is to be fixed. The links of the dynamic frame are then drawn tangent to the friction circles, so as to make the stated angle with the surface of each joint. The point where the pin bears against the eye, marked by a dot on the surface of each pin, will aid in showing the meaning of the diagram, as explained in § 7. The italic letters near the arrows denote the element in which the pin is fast, and the arrows show the direction of rotation of that element relatively to the other element of the joint. If *f* were made the driver the direction of these rotations would all be reversed, and consequently the links would all have to cross over to the other sides of the friction circles. This would materially alter the form of the diagram, and in many machines would result in an impracticable diagram, showing that the machine cannot be driven backwards.

§ 9. *Modification of the Dynamic Frame—Couples.*—When any element of a machine is in equilibrium under the forces applied at two joints the dynamic link will in direction and position correspond more or less closely with the direction in which the element of the machine lies between the joints. Thus, when the machine constitutes a material frame in one plane, as in fig. 6, the dynamic frame, fig. 6*a*, will have a general resemblance to the machine. The similarity between the actual frame composed of material links and the ideal dynamic frame in this case must not lead the reader to expect that he will always be able to identify a link in the dynamic frame as corresponding to an element in the actual machine. Half a dynamic link corresponds to each *joint* of the machine, and there may be dynamic links which, like link 3 of fig. 5, have no corresponding joint in the machine, but the dynamic link will only correspond with an actual element when the number of joints in the latter is limited to two. Since a two-jointed element has a link resembling it, we may sometimes name this link by the letter of the element; thus, all the links of 6*a* might be called indifferently by the letter of the element or the number assigned to the link; but in fig. 5*b* or 5*c*, we can name only the link 5 by the letter of an element. In fact, the capital letter always signifies an ideal joint, but in cases like that of link 5 this ideal joint is any point in a given straight line. When the dynamic joints are at an infinite distance owing to the parallelism of certain links, it is convenient to substitute ideal stiff frames, bars, or plates joining the parallel links and acting as the actual rigid elements or stiff frames would do, with the exception that the joints between the ideal bars and links are frictionless; this substitution may also be made when the links are nearly parallel. It is clear that, by the ordinary method of statics, we might calculate the relative stresses in all the links of the frame in fig. 5, if we were to imagine links 1, 2, 5, and 4 joined by a stiff bar or triangular frame to which they were jointed without friction; this gives a modified dynamic frame, as shown in fig. 5*d*. The position of this stiff bar is unimportant, but when used to connect parallel links, it is conveniently drawn perpendicular to these. When the links are not all in one plane, these bars become imaginary rigid plates or stiff frames. The rigid bars will be shown in the diagram by thicker lines than the links.

A couple in an actual machine can only be exerted between *two* elements which are acted upon in opposite directions. There is no such thing as "a solitary couple" in nature; we always find a pair of equal and opposite couples, as we find a pair of equal and opposite forces. Two equal and opposite couples require two rigid elements between which they are exerted, and these elements appear in the modified dynamic frame as two rigid bars, perpendicular to the forces producing the couples. Fig. 7 shows a simple machine in which the driving link of fig. 6 is replaced by a driving couple between the elements *a* and *b*. The driving couple is indicated by the two springs *e* and e_1, which it is assumed are producing exactly equal and opposite stresses between *a* and *b* in two parallel directions. Fig. 7*a* shows the dynamic frame for this machine with friction. First we draw the links 1 and 1*a* tangent to the friction circles for the joints *ea eb* e_1a e_1b The distance between the links 1 and 1*a* shows the arm of the driving couple as diminished by friction; next we draw links 4 and 5 by the rules already given, then, remembering that the force in link 5 at joint *ad* must produce an equal and parallel bearing pressure on the pin at the joint *af*, we draw this bearing pressure tangent to the friction circle, so as to make the stated angle with the surface of the joint; we also draw the bearing pressure due to element *b* on another part of the same pin, the intersection of these lines of pressure gives the dynamic joint through which the pull of link 6 must be exerted; this link is now drawn, and the diagram completed by the two bars drawn perpendicular to links 4 and 5 respectively. Let P_1 be the force of the original couple, and D the distance between link 1 and 1*a*. Let A be the distance between the lines of bearing pressure on element *a*; then P_5, the stress in link 5 is given by the expression $P_5 = \frac{P_1 D}{A}$. Similarly $P_4 = \frac{P_1 D}{B}$. The stresses in links 4 and 5 being known give the stress in link 6 by the simple composition of forces. The portion of fig. 7 referring to the material means of producing the two couples, does not necessarily belong to the diagram; the couple between *a* and *b* may in certain cases be produced by some other machine, being, in fact, the resisting couple of that other machine, and in that case the efficiency of the means of producing the couple must be determined by an examination of the first or driving machine. This case is, in fact, one case of compound machinery, and will be treated hereafter. In what follows, a driving couple may be occasionally described as existing between two elements, without reference to the mode in which it is applied; a resisting couple may be spoken of in the same manner.

§ 10. *Assumption that the Links of a Frame lie in one Plane.*—One object of our investigation is to find a means of ascertaining the efficiency of any mechanical arrangement—the word efficiency being understood in the sense given it by RANKINE—as the ratio of the useful work done in a machine to the whole work or energy expended. Now, RANKINE ("Millwork," § 371 A) has pointed out certain conditions which must be fulfilled to give the highest efficiency in any design, viz.:—First, that the useful resistance to the motion of any element, the effort to move it and the force due to the weight of the part must lie nearly in one plane, or else act in directions parallel to one another; and secondly, that the acting parts must not overhang the bearings. Injurious couples are introduced if these conditions are not fulfilled. RANKINE'S conditions are, however, generally fulfilled in all important designs of machinery, and when this is the case we may, without serious error, assume all the actions to take place in one plane parallel to the plane of action in the machine, and this hypothesis will be adopted in all that follows when the contrary is not stated.

§ 11. *Simple and Compound Dynamic Frames.*—The dynamic frame of a complete machine mustcontain a driving link or couple and a resisting link or couple, together with the links necessary to connect the driving and resisting links or couples in such a way as to form a self-strained frame. A complete machine may be either simple or compound. *Simple machines* are those having dynamic frames which cannot be decomposed into two or more self-strained frames, such that the resisting link or couple of the one becomes the driving link or couple of the other. *Compound machines* have dynamic frames so formed that they can be decomposed into the frames of simple machines so connected that the resisting link of one becomes the driving link of the next. If we exclude rigid bars as members, there is only one frame which can be self-strained, and which is yet incapable of analysis into two distinct self-strained frames. This frame has been already described, and consists of a quadrilateral, with two diagonals, as shown in figures 8 and 9. There is no essential difference between those two figures, which each consist of a quadrilateral figure 2, 3, 4, 5, having opposed angles joined by the two links 1 and 6. This simplest self-strained frame will be shown to be the dynamic frame of many elementary combinations in machinery. The driving and resisting links may in this frame be arranged in two ways. 1. They may, as in the examples hitherto given, be represented by two links, such as 1 and 6, 2 and 4, or 3 and 5, which are not jointed together; any one of these pairs may be considered as diagonals of a quadrilateral joined by the four remaining links, and each pair may be called conjugate links. For the convenience of description, when these links are placed as 2 and 4, or 1 and 6, fig. 8, they may be called opposite links. 2. The driving and resisting links may be adjacent, that is to say, they may, as in the case of 1 and 4, or 2 and 6, have a common intersection or joint. When the driving and resisting links are adjacent, as 1 and 4, those links which do not abut at the intersection of the two adjacent links* need not represent bearing pressures at working joints, as will be seen by considering an actual machine corresponding to the dynamic frame, as for instance that of fig. 11; the three elements corresponding in this case to links 2, 5, and 6 will then simply constitute a stiff system or frame, which might be replaced by a single rigid element. When this is the case the machine will belong to class 2, the dynamic frame of which is that of fig. 10, in which a bar is substituted for links 2, 5, and 6. When treating of compound machines,

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* In the frame of the machine shown in fig. 45, Plate XII., links 1 and 6 might be drawn so as to appear adjacent, by placing link 4 so as to join the intersection of 1 and 6 with that of 2 and 5. The links 2, 3, 5, do not, however, in this example, form a stiff frame, and the machine belongs to class 1. This is obvious when link 4 is placed so as to join the intersection of 1 and 5 with that of 2 and 6. Machines of class 2 have only 5 working joints.

we shall find reason to consider machines of class 2 rather as half machines than complete simple machines. When couples are admitted in place of driving and resisting links, the dynamic frame necessarily includes two stiff bars, between which the couple or couples act. We then have three cases :—1. The resisting and driving couples may act between the same pair of stiff bars. This gives a dynamic frame of merely two bars, with the two pairs of links by which the couples are exerted. 2. A driving or resisting couple between two bars may be combined with a resisting or driving link between the same bars. 3. A resisting couple or a driving couple may, in the quadrilateral of figs. 8 and 9, be substituted for any link, the couple being exerted between two bars replacing two links, which, together with the link removed, form a triangle. When we examine the usual combinations of elementary parts forming actual machines, we shall in all cases find that these combinations may be represented by a dynamic frame of one of the classes described.

§ 12. *Efficiency of Elements.*—The relation between the energy exerted and the useful work done in a machine is affected by a loss of energy in transmission through the elements, as well as by a loss in transmission past the joints. At each joint we may say that only a certain fraction of the energy received is transmitted, the remainder being wasted in overcoming useless friction; the fraction transmitted is the measure of the efficiency of the joint (RANKINE). Let this fraction be called J. Similarly, let the ratio between the energy received and that transmitted by each element be called e, then the efficiency of the whole machine consisting of a linear train of joints and elements will be the product $J_1 J_2 J_3 \ldots \times e_1 e_2 e_3 \ldots$ This formula is, however, of little practical use, because the values of $J_1 J_2$, &c., are materially influenced by the directions of the forces at each joint, and these cannot be assumed for one joint independently of the others. In other words, the values of J are not independent of one another. The value of the product $J_1 J_2 J_3, \ldots$ &c., can only be found by solving a large number of troublesome simultaneous equations, or by means of the dynamic frame. With respect to $e_1 e_2 e_3 \ldots$ we must distinguish between two cases. 1. Those in which the element is in equilibrium under the external forces independently of any progressive change in its own form. 2. Those in which the element is not in equilibrium under the forces applied at the joints. As an example of the first class, I may take a straight rope used to transmit power. Although the rope stretches, yet the whole pull at one end is transmitted to the other end, but there is a loss of work, because the distance traversed by the driving end is greater than that traversed by the following end. In all cases of this kind the values of e are not only independent of one another, but do not affect the values of J. They do not alter the relation between effect and resistance, and their aggregate effect is easily taken into account; e for each element is a constant fraction, which can be independently determined, and the fraction expressing the efficiency of the machine will be the product of two factors;—first, the efficiency as found by the dynamic frame, and secondly, a coefficient obtained by multiplying together all the values of $e_1 e_2 e_3$, &c., for the elements concerned. When the energy thus employed acts against a reciprocating resistance in the elements, as where it bends the beam of an engine, it is without influence on the whole efficiency for a complete cycle of operations, such as a whole revolution of the crank. It simply alters the relative efficiency at different periods of the stroke, and may, therefore, generally be neglected. Where, however, the lost work is done against a non-reciprocating force, as in stretching a rope, it cannot be safely neglected, and leads to sensible diminution of efficiency, as where power is transmitted by belts and pulleys. Coming to the second class of cases, it is clear that when a heavy element is being lifted, lowered, accelerated, or retarded, it is not in equilibrium under the action of the external forces at the joints calculated in the manner hitherto described; we shall, however, hereafter include the forces due to these causes in calculating the forces at the joints, and there remains only one mode in which a loss of energy occurs in the course of its transmission by an element, namely, its dissipation in overcoming an internal couple. This case finds an illustration in the case of a rope wound on to a pulley, or unwound from one; the pull on the rope is not transmitted in a direct line, as we have hitherto supposed, but in consequence of the couple required to bend or unbend the rope, the line of action is shifted sideways through a length $\frac{m}{\mathrm{F}}$, where m is the moment of the couple, and F the force transmitted. This translation of the force affects the values of J for all the subsequent joints. It can be represented in the dynamic frame by showing the line of action of the force shifted parallel to itself in a disadvantageous direction. If in the given problem we know the useful resistance, we must, in constructing the diagram, shift the force at the driving end; *vice versa*, if the driving effort is known, we must shift the force at the resisting end. The fraction expressing the loss of efficiency due to this cause is not, like that due to friction or stretching, independent of the magnitude of the forces involved, but will, on the contrary, always involve complete inefficiency when the force is very small, and implies a gradually increasing efficiency as the force transmitted increases. Thus, a small force exerted on a stiff rope passing over a pulley produces no effect on the further side, because it is insufficient to bend the rope. The loss by an internal couple always diminishes the resistance which a given driving effort can overcome, whereas the loss of internal work done in overcoming a single force has not this effect. The case of the transmission of power by fluids in pipes will be examined in a subsequent paper, after machines composed of solid parts have been analysed.

§ 13. *Simple Machines—Lever.*—Let *a*, fig. 11, be a lever to which a driving effort is applied by the spring *e*, and a resistance by the spring *f*. Let the lever have a fulcrum or bearing in the element *b* to which the elements *e* and *f* are jointed, making a complete or self-contained machine. This system is a self-strained frame, with one stiff bar, namely the lever. The bar in this, as in the other drawings, may be regarded as the symbol of a stiff frame, the form or design of which is unimportant in the given question.* The relation between the longitudinal stresses in the elements *e, f, b*, is given by the dynamic frame, fig. 11*a*, which takes the friction into account at all the joints. The friction circles are drawn for joints *ae, ab, af, be*, and *bf*; the circle for the joint *bf* is, for clearness in the diagram, supposed to be a little larger than that for *be*. Arrows are placed at each friction circle to denote the motion of one part relatively to the other at the joints; each arrow is marked with the letter of the element, the motion of which it denotes; thus, at the joint *ae* the arrow marked *a* shows that, when the driving element *e* moves the lever, the rotation of *a* is left-handed relatively to *e*. Similarly, the arrow marked *b* at the joints *be* and *bf* denotes that, relatively to *e* and *f*, the rotation of *b* is right-handed. The letter *b* also denotes that the pin is fixed in *b*; (it is not a matter of indifference in which element this pin is fixed). Links 1, 3, and 4 can now be drawn, each tangent to their two friction circles. We choose the side on which to draw them as follows:—The forces acting on A balance one another, and therefore meet in one point marked A (fig. 11*a*); the directions of the forces acting on *a* are marked by three arrow-heads near A, and the equal opposite forces by opposite arrow-heads near B. The links 1 and 4 appear as compression links in the dynamic frame, whereas they are tension links in the machine. The direction of the stress is also reversed in link 3. In explanation it must be remembered that the point A represents the lever, while the bar B_1BB_{11}, which may be drawn anywhere between the links, represents the element *b*. The links must be placed on that side of the circles where the arrow-heads of the forces acting on *a* (shown near A) oppose the motion of the arrows *a*, while the arrow-heads of the forces acting on *b* (shown near B) oppose the motion of the arrow *b*. The manner of drawing the figure for this example has been described in fuller detail than will in future be thought necessary. The relation between the driving effort in link 1 and the resistance in link 4 can be found from fig. 11*a* by the ordinary graphical or trigonometrical methods. The conception of a complete machine has not been recognised by any writer on mechanics as necessary for the statement of problems connected with the lever. If, however, these problems are to be practical, and not confined to abstractions, such as "forces applied to points," they do require the consideration of a complete

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* When the method of reciprocal figures is used to find the stresses in the links, it will be necessary in all cases to substitute a stiff frame of 3 links for the bars shown in the diagrams.

machine, as here drawn. The friction at *ab* is usually taken into account; that at *ae* and *af* is more generally neglected, and the friction at *be* and *bf* has perhaps never been thought of as an essential part of the problem. The reason of the neglect is clear. The forces represented by links 1 and 4 are in many problems due to attraction between some parts of element *a* and element *b*, as, for instance, when these forces are due to weights actually forming part of the element *a*, and attracted by the earth which supports, and is indeed part of, the element *b*. In this case the joints *ae*, *af*, *be*, and *bf* are frictionless, or may be said to disappear as joints. When the weights are hung by pins at *ae* and *af*, the friction at those pins must be taken into account, and whenever the forces represented by links 1 and 4 are due to another machine, to springs, or any other material element, the problem requires all the circumstances to be taken into account which are indicated in the dynamic frame as shown.

§ 14. *Wheel and Axle.*—The wheel and axle, with its driving element, resisting element, and bearing, forms a complete machine when the parts are connected, as shown in fig. 12. The wheel and axle constitute the element *a*, and the other elements have names given to them, corresponding to those for the lever. The dynamic frame is shown in fig. 12*a*. When the wheel and axle are circular, there is no friction at joints *eb* and *fb*; moreover the pins and eyes which form the joints at *ea* and *fa*, are replaced by the flexible rope. There is no friction at the joint *eb*, since *e* does not rotate relatively to *b*, and we may therefore assume that the force in the tie *e* is uniformly distributed relatively to its cross section: the resultant force, therefore, will pass through the centre of the pin at *eb*, and similarly the resultant of the resistance will pass through the centre of the pin at *fb*. If the ropes were perfectly flexible, we might, in fig. 12*a*, draw links 1 and 4 from the centres of the pins at *eb* and *fb*, tangent to the dotted circles drawn with the effective radii of the wheel and of the axle; from their intersection link 3 would be drawn tangent to the friction circle for the joint *ab*. The stiffness of the rope must, however, be taken into account, and this can be done by drawing links 1 and 4 as broken links, of which the lower halves are drawn as above described, while the upper halves represent the lines of action of the forces shifted sideways. The driving link is brought nearer the centre of *a*, and the resisting link removed further from this centre. The distances d and d_1, by which each half link is shifted, are given by the expressions $d=\frac{m}{\mathrm{P}}$, $d_1=\frac{m_1}{\mathrm{P}_1}$, where m or m_1 is the moment of the couple required to bend the given rope to the given radius, and P or P_1 is the stress in the link. It must be remembered that this stress is not the same in the driving and resisting links, and that if we are given the stress in *e* we must proceed, by trial and error or simultaneous equations, to find the stress in *f* before we can determine exactly the distance by which the link 4 is shifted. When the ropes are long, their efficiency must also be taken into account, when our object is to compare energy exerted with work done. When we simply wish to compare effort and resistance, the loss of energy due to the stretching of the rope may be neglected. Inasmuch as the axes of elements *e* and *f* are assumed to lie in parallel planes, perpendicular to the axis of *a*, the forces in the elements *e* and *f* (unless parallel) give rise to an injurious couple on the bearings, which, except when these are very far apart relatively to the distance between the planes of *e* and *f*, sensibly diminishes the efficiency of the machine.

§ 15. *Inclined Plane.*—The idea involved in problems on the "inclined plane" is that one element, sliding on another with a plane joint between them, shall be employed to maintain equilibrium between forces applied to the sliding element in a plane perpendicular to the joint. We may embody this idea in a simple complete machine, as shown in fig. 13, where *b* is a fixed element, *e* a driving element jointed to *b* and *a*, the sliding piece having a plane joint with *b* ; *f* the resisting element jointed with *b* and *a* : the axes of *e* and *f* are in a plane perpendicular to the joint *ab*. We have here a self-strained combination fulfilling all the required conditions. The dynamic frame with friction is shown in fig. 13*a*. Links 1 and 4 are drawn tangent to the friction circles, and link 3 is drawn from their intersection A, making the stated angle with the plane joint *ab*. The bar 2 may be drawn anywhere, but is conveniently shown parallel to the joint *ab*. When link 1 coincides with link 3 or makes a greater angle with the joint *ab* than link 3 does, the mechanism will not work.

§ 16. *The Hanging Pulley.*—The hanging pulley becomes a complete simple machine, when the driving element and resisting element are attached to a common element, as shown in fig. 14 ; *b* is the fixed element, *e* the rope by which the effort is exerted, *a* the pulley, *f* the element on which useful work is done. Links 1 and 4 are drawn for the dynamic frame in two parts as shown ; the moment of the couple dividing the parts being that required to bend and unbend the rope, and its force the force exerted on the rope. The pulley will take up a position in which the link 3 drawn tangent to its two friction circles cuts the intersection of 1 and 4 at A.

It is curious to observe that while Professor REULEAUX has very properly rejected the lever, inclined plane, hanging pulley, and wheel and axle, as elements of kinematic analysis, nevertheless these so-called mechanical powers do furnish the characteristic features of four simple machines of class 2 in which the number of elements is restricted to four. We shall find that the wedge is a characteristic feature in a simple machine with six dynamic links of class 1. These considerations show that dynamical and not kinematical reasoning guided the mechanicians who selected the so-called "powers."

§ 17. *Example of a complete Machine having a Dynamic Frame of Six Links.*—

A steam engine with an oscillating cylinder, steam-piston, and piston rod, to represent element *e*, and a pump to represent element *f*, as sketched in fig. 15, affords an example closely approximating to a simple machine of class 1 ; the typical dynamic frame of this engine has already been shown in fig. 6, and is here repeated with the slight variation that the pins are supposed to be fastened to *b* and *d* instead of *e* and *f*. It must be observed that the stress cannot be axial either in the driving or resisting elements ; indeed, these parts are not true elements, for there is a joint between two elements in each of them. They, in fact, with the links of the quadrilateral, constitute a machine working a machine such as will be discussed when treating of compound machines ; similarly, when we represent the driving and resisting element by springs, as in the foregoing cases, we might have observed that in an actual spring the stress would not be strictly axial. In most practical cases, however, the stress will be so nearly axial that for the present we may neglect these considerations, and assume that we know the stress in the driving or resisting link.* We then have the frame shown in fig. 15*a*.

If the same engine, fig. 16, were employed to overcome or transmit a couple substituted for link 6 or element *f*, the dynamic frame would become that shown in fig. 16*a*. The engine is shown with the piston rod in tension ; links 2 and 5 must be first drawn tangent to their friction circles, then the bearing pressures parallel respectively to 2 and 5, and tangent to the friction circles for *eb* and *ec*. The line of pull in the element *e* is given by the line joining the intersection of the bearing pressures with that of 2 and 5 ; the diagram is completed by the bar perpendicular to 5 and that perpendicular to 2 ; these bars are mere indications of the arms of the equal and opposite couples exerted on elements *c* and *b*. These elements must be stiff, and their rotation relatively to one another, is, by hypothesis, resisted by a couple such as would be produced by friction exerted on a wheel forming part of *b* revolving between clips or rubbers forming part of *c*. The diagram supposes that the frictional resistance thus obtained is so exactly equal at opposite ends of a diameter of the friction wheel as to constitute a couple which does not directly affect the pressures on the joints *eb*, *ec*.

§ 18. *Ordinary Direct acting Steam Engine.*—The ordinary direct acting steam engine with a single cylinder gives another example of a simple complete machine. Fig. 17 shows a sketch of an engine of this type with the resistance exerted as if by a link between the periphery of a flywheel and the fixed element or bed plate. We will assume that the position

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* The steam, piston, and cylinder constitute, with the resisting link, a simple inclined plane machine, *vide* § 15, and this machine drives the second machine, constituted by the piston rod, connecting rod, crank, bed plate, and resisting link ; the piston rod and bed plate are common to the two machines, *vide* § 24, 25.

and direction of this resistance is known, being that shown by the arrow on *f* in fig. 17. The elements are; *a,* the piston rod and block sliding on the guide bars; *b,* the connecting rod; *c,* the crank, axle, and fly wheel; *d,* the bed plate including the cylinder; *e,* the steam in the cylinder. This element is jointed with *a* and *d*; the position and direction of the force exerted are in this case determinate; for the cylinder does not oscillate and the piston with its rod are subject to no stress that is not axial; the element *a* is in equilibrium under the force due to *e,* and those due to the joints *ab* and *ad.* In fig. 17*a* link 1 is drawn coinciding with the axis of the cylinder, and represents the bearing pressure produced by *e* on its joints; the link 2 may next be drawn tangent to the friction circles for *ab* and *bc.* The third force under which *a* is in equilibrium is that due to the joint *ad*; the link 5 is drawn through the dynamic joint A, making the stated angle with the guide bars. This diagram corresponds to an engine in which the slide block is as usual fast on the piston rod.*

We next observe that element *c* has three joints—first at *cf,* secondly at *bc,* and lastly at *cd.* The resisting link must, in order that the machine may be complete, abut at its other end against the bed plate *d*; it may be due to actual friction, as when the fly wheel is, for experimental purposes, fastened between two friction blocks secured to *d.* We know, by hypothesis, the place and direction of its application, and therefore the position and direction of link 6. The intersection of 6 and 2 gives the dynamic joint C; the third force acting on *c* must pass through this joint, and make the stated angle at the joint *cd*; we therefore draw link 3 from C tangent to the friction circle for *cd.* Lastly, we observe that the element *d* is in equilibrium under the following forces:—1st that due to the joint *ed,* equal and opposite to that on *ae*; 2d, that due to the joint *ad*; 3d, that due to the resisting link *f*; 4th, that due to the joint *cd.* We have already on the fig. 17*a,* the position and direction of all these forces. They do not, however, all meet in one joint, and we must therefore, to complete the dynamic frame, add a link which shall receive the equal and opposite resultants of the forces compounded in two pairs; this we may do by joining the intersection between links 5 and 6 with that between 1 and 3, giving the complete diagram of fig. 17*a.* The diagram fig. 17*c* will help to explain the significance of the several links. The element *d,* which is the frame, is here shown by itself; it is in equilibrium under four external forces, indicated by arrows numbered as the links in the frame are numbered; and as this plate is itself in equilibrium, the resultant of 1 and 3 will be opposite and equal to the resultant of 5 and 6. The forces at the joints may therefore be repre-

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* If, however, there were a joint between these parts, such that the pressure from the guide bars must pass very near the centre of the pin at that joint, then links 5 and 2 would be first drawn, and link 1 drawn cutting their dynamic joint; this arrangement would cause the effort exerted by the piston to pass a little way from the axis of the cylinder, as shown in fig. 17*b*.

sented by the stresses in the four half links, 1, 5, 6, and 3, joined as in the diagram by a link 4, in which the stress will be that due to the resultant of 1 and 3, or 5 and 6. The frame, if drawn on the hypothesis of no friction, will be kinematically equivalent to the actual engine; that is to say, the infinitesimal compression of link 6, resulting from a given infinitesimal expansion of link 1, will be equal in length to the actual travel of the fly-wheel at its rim past the friction-block, when the piston advances by an amount equal to the given expansion of link 1. The diagram, independently of its value as a means of estimating the relation between effort and resistance, is of use in showing clearly the direction and magnitude of the stresses to which the bed plate is subject. As the revolution of the crank continues the dynamic frame changes. Figs. 17 to 20*a* show four positions of the engine with four dynamic frames; the bearing-points are marked in each case by dots on the circles representing sections of the pins. The changes in the direction of the stress in the connecting-rod, relatively to its axis, should be observed, as well as the sudden changes which take place in the points of bearing pressure as the crank shaft revolves. It is these sudden changes which give rise to "knocks" in the engine when the shafts or pins and bearings or eyes do not fit. At joint *ab* four sudden changes occur—two due to changes of relative motion between the pin and eye, and two to changes in the direction of the stress in the link; at joints *bc* and *cd* there are only two changes. It must be understood that the whole dynamic frame is modified by any change in the direction or position of the resisting link. It will be found easy to construct diagrams showing the modifications—resulting from a change of this kind or from the substitution of a couple—between *c* and *d* for the resisting link; this latter case corresponds to the arrangement of an engine employed to drive a long shaft with separate bearings co-axial with the crank shaft.

§ 19. *Wedge.*—When a wedge is employed to form a complete machine we find a dynamic frame of class 1, similar to that given by the direct-acting steam-engine. Fig. 21 shows a complete wedge machine. The letters on the parts indicate the elements, where *e* is the driving, and *f* the resisting element. Dotted lines on the same figure show the simple dynamic frame without friction. The dynamic frame with friction is given in fig. 21*a*; links 1 and 6 are determined by being made tangent to the friction circles for joints *ec*, *ea*, *fb*, and *fc*. Links 2 and 5 must intersect link 1 at the same point. If the wedge has plane joints, the position of this point is indeterminate, but with fair fitting it may be expected to lie at or near the centre of the joint. The direction of the links 2 and 5 is fixed by the condition that they shall make the stated angle with the joint, and their *position* is without influence on the relative proportions of the links of the frame. The intersection of links 6 and 5 gives the joint B; the intersection of link 6 with 2 determines the joint C. Link 4 is drawn making the stated angle with joint *bc*, and by its intersection with 1 gives joint C_1. The frame is completed by drawing the link 3 from C to C_1. The element *c* has four joints, the pressures on these joints are the stresses in the links 1, 4, and 2, 6. The equal and opposite resultants of these two pairs are met by the link 3, supplied in the original machine by the rigidity of *c*. The machine will cease to work when the joint C_1 falls inside the triangle CAB. The diagram suggests another arrangement of the wedge machine in which the wedge might be employed to open a pair of jaws corresponding to links 3 and 4, hinged at C_1. The analogy between the wedge machine and the direct-acting engine is curious. The connecting rod acts like a wedge, opening or closing the jaws, represented by the crank and bed-plate.

§ 20. *Spur Wheels.*—A simple complete machine can be made of two spur wheels *b* and *c*, with bearings in the same element *a*, and having a driving link *e* between *a* and *b*, and a resisting link *f* between *c* and *a*. The simplest type of this machine is shown in fig. 22, and its dynamic frame is given in fig. 22*a*; the frame is drawn as follows:—Links 1 and 6 are tangent to the friction circles for elements *e* and *f*. Link 5 passes through the pitch-point of the spur wheels, and makes the stated angle with the surface of the teeth; in other words, it makes an angle equal to ϕ with the normal, which is called by RANKINE the line of connection. ϕ here as elsewhere signifies the angle whose tangent is μ, the coefficient of friction. The intersection of 5 with 1 and 6 gives the joints B and C; from B and C links 2 and 4 are drawn tangent to the friction circles for *ab* and *ac*, and the frame is completed by joining AA_1. Each wheel is an element having three joints, and therefore gives three half links to the frame. These three half links for wheel *b* meet at B, and represent the pull of the spring *e*, the push from the joint where the teeth meet, and the reaction from the bearing *ab*. Wheel *c* gives the three corresponding half links at C; the frame is completed by link 3, so placed as to receive the equal and opposite resultants of the second halves of the links 1 and 4, 2 and 6. This link 3 lies in the direction of the stress on the element *a*. A practical example of this machine is afforded by a man *e*, fig. 23, turning a winch handle *b*, and lifting a weight *f* by the rope on an axle *c*, driven by a spur wheel gearing with a pinion on the shaft of the winch handle. The man stands on the element *a*, which also supports the bearings of the spur wheels. The dynamic frame, fig. 23*a*, for this example is drawn precisely as for the previous typical example. As before, we know the directions of the effort in link 1 and of the resistance in link 6 (the latter is shown shifted outward to allow for the stiffness of the rope). The effort of the man need not be perpendicular to the crank, but must be exerted between elements *a* and *b*; the resisting link is the attraction between the weight and the earth, that is to say, as before, it is a link between C and A. Links 5, 2, and 4 are drawn as before, and link 3, supplied by the rigidity of element *a*, completes the frame. The friction between the man's hand and the handle is important. A friction circle is accordingly shown at the joint *bc.* A handle which the man can grasp firmly and which turns easily round a well-oiled axis, makes the machine more efficient than when the man's hands must slip round the handle.

One of two spur wheels may be driven by a couple, and the other resisted by a couple; this is a case frequently arising in practice, when one wheel is driven by a shaft, and the other drives a shaft, both shafts having such additional bearings as prevent the ultimate driving effort or ultimate resistance from having any effect on the bearings of the simple machine. Fig. 24*a* shows the dynamic frame for this case. There are three bars, as we have two couples; the bars are lettered as elements. Two friction circles are drawn with their centres at the centres of the bearings *ab* and *bc.* The bent arrows show the direction of rotation of pins fixed in the element *a,* relatively to *b* and *c.* Link 5 is drawn as before through the pitch-point, and making the stated angle with the surface of the teeth; the directions of the equal and parallel pressures on the pins form part of links 2 and 4, which no longer cut line 6; the other halves of links 2 and 4 are the reactions from the bearings shown as dotted lines. The diagram is completed by drawing bars to represent *a, b,* and *c.* The position of these bars is really immaterial, but B and C may be conveniently shown perpendicular to the direction of link 5, and these letters will now be used to signify the perpendicular distances between these links. When the couple M_D is applied between *b* and *a,* it produces two forces equal to $\frac{M_D}{B}$, the one acting to compress link 5, and the other to force the bar B against the pin in the direction shown by the full arrow 2. The first force is resisted by the tooth and wheel, as if these formed part of a link under compression, the other force by the element *a* in the direction of the dotted arrow 2. The force $\frac{M_D}{B}$, is transmitted through the tooth to act on *c;* it produces an equal force at the distance C, forcing *c* down on its pin in *a,* as shown by the full link 4. This force is resisted by an equal and opposite force, as shown by dotted link 4. Thus the couple produced in *c* is $\frac{M_D C}{B}$, and this is the resisting couple M_R, which the driving couple M_D can overcome. The forces and couples are the same as would be produced if we could construct a material frame of the link 5 and the bars ABC of the dimensions shown, and having the driving couple between B and A, with the resisting couple between A and C.

§ 21. *Rolling Contact.*—The simple machine, made with two wheels which transmit power by a rolling contact between them, has a dynamic frame of the same character as that corresponding to spur wheels. This arrangement is shown in fig. 25, where the parts have the same names as were given in the case of spur wheels. The directions of links 1 and 6, fig. 25*a*, are known; the direction of link 5 is also known, if the machine is running with its maximum efficiency; that is to say, if there is no more tension on element *a* than is necessary. In that case, the line of pressure at the point of contact will make the stated angle with the surface of contact. Link 2 is drawn from the intersection of 5 and 1 to the friction circle at the centre of *b*. Link 4 is drawn from the intersection of 5 and 6 to the friction circle at the centre of *c*. The intersection of 4 and 1 is then joined by link 3 to the intersection of 6 and 2. The three lines meeting at B show the positions of the three forces under which *b* is in equilibrium, and the arrows show the directions of those forces. The three lines meeting at C show the forces under which the second wheel *c* is in equilibrium, and the arrows at C show the directions of those forces. The rules given in § 8 will enable the draughtsman to determine on which side of each friction circle the link is to be tangent. Fig. 25*b* shows the manner in which the diagram becomes modified when the directions of links 6 and 1 make smaller angles than link 5 makes with the normal to the joint *bc*. Fig. 25*b* also shows the effect of rolling friction at this joint, which, however, may generally be neglected. At the point of contact the material is continually being crushed, and the material is not perfectly elastic. We have, therefore, a resisting couple analogous to that met with in the case of ropes, and the effect is to shift in a disadvantageous direction one part of link 5 by a distance equal to the arm of this couple. If excessive tension is employed in *a*, the direction of link 5 will be found by compounding that tension with the force transmitted at the periphery. The diagram where one roller is driven by a couple and the other roller resisted by a couple, is easily deduced from that for spur wheels.

§ 22. *Belt and Pulley.*—The complete belt and pulley machine is shown in fig. 26. It is composed of the pulleys *b* and *c*; the element *a* in which their bearings run, the flexible belt *d*, the driving element *e*, and the resisting element *f*. The dynamic frame without friction is given in fig. 26*a*. Link 5 is the direction of the resultant of the tensions on the two bands, which may be considered as together forming one split link. When we assume that no more tension is used than is necessary, the ratio between the tensions on the tight and slack side of the bands is determined by the arcs round which the belts are in contact with the pulley, and by the coefficient of friction between the belt and the pulley. Consequently, the position of link 5 may be taken as known. The intersection of the driving link 1 with 5 gives joint B; the third force acting on *b* is the resultant of the two others, and must pass through the centre of the pulley *b*, this determines the direction of link 2. Similarly, C is given by the intersection of 5 and 6; link 4 is drawn from their intersection to the centre of the pulley *c*; the diagram is completed by drawing link 1. The forces which keep the pulley *b* in equilibrium are shown in position and direction at the joint B. Similarly, the forces which balance *c* are shown at C, but the driving and resisting links, as well as link 6, appear as if under compression. Before proceeding to draw the frame with friction, it may be well to show how the stiffness of the belt may be taken into account. In fig. 26 the arrows show the direction of the motion of the belt. The pulley *b* at the lower side has to exert a force P in the direction of the arrow on the belt, and also a right-handed couple *m* to bend the belt; the resultant of this force and couple is an equal force shifted to the left or downwards by a distance $\frac{m}{P}$; we might, therefore, represent the actual belt by a perfectly flexible belt placed as shown by the dotted line $d_1\,d_2$. The effect of the stiffness of the belt at the other places where it is bent and unbent, is also to shift the line of application of the force as shown by the dotted lines $d_3\,d_4$, $d_5\,d_6$, $d_7\,d_8$. Thus the resultants of the forces due to the tension of the belt on each pulley will not be opposite each other; the resultant will be shifted outwards on the driving pulley, and inwards on follower, or disadvantageously in both places. The actual amount of shifting cannot be ascertained until P and P_1, the tensions on the belt, are known; but the nature of the change is easily apprehended, and is therefore included in the dynamic frame, fig. 26*b*. This frame is drawn as for the case without friction.

§ 23. *Compound Machines.*—It is evident that the resisting link of one complete simple machine may be used as the driving link of another simple machine. This combination gives rise to what may be called a compound machine. If the two machines have no element in common, they must be connected by two joints in the manner of which fig. 27 gives a typical example. In this figure the lines may be considered to represent either the links of a dynamic frame, or the axes of a series of material elements, jointed without friction. The contraction of element *e* would cause an expansion of the line joining the joint *bc* with *da*. This line would indicate the position of the resisting link of the machine *abcde*, if this machine were a simple one. This same line lies between the joints $b_1\,a_1$ and $d_1\,c_1$ in the position required to enable the effort produced by the first machine *abcd* to drive the second $a_1\,b_1\,c_1\,d_1\,f_1$. In fact, the whole first machine may be considered as a somewhat complex driving element, relatively to the second machine; or the whole of the second machine may be looked on as a rather complex resisting element, relatively to the first machine. The two machines may obviously be treated as entirely separate. Let the first machine drive the second by contact between the elements *b* and b_1 and between *d* and d_1. Then it is necessary that at these joints the directions of the lines of bearing pressure shall be in one straight line. This line is the direction of the resisting link for one machine, and the driving link for the next. The joints bb_1, and ee_1, between two successive machines, will be called *transmitting joints.* They do not themselves belong to either machine. Distinct elements may be introduced between the two machines, as in the typical example, fig. 28. Here the second machine is tied to the first by two links, each lettered f and e_1. These elements, which must represent forces in one straight line, may be considered as equivalent to the driving element of the second machine, and the resisting element of the first. In the example given it would be necessary to make f a tie, so that the machine, as drawn, would only work when element e was expanding. In another case, one part of the link f might be omitted as between bc and a_1b_1, and replaced by a transmitting joint between b and b_1 as above. An element placed between two machines and serving to transmit the power, will be called a *transmitting element.* We see then that the communication of power can be made from one complete machine to another, either by two joints, by two elements, or by a joint and element. We may, with perfect propriety, give the name of complete machine to any one of a series, each of which drives its successor; for we may regard the driving system simply as a more or less complex driving element, and the driven system as a more or less complex resisting element. We are not concerned with the complex play of forces which produces the driving or resisting effort, but, so far as each complete machine is concerned, only with the fact that its elements are driven or resisted in a manner which may be represented by a single driving or resisting link.

§ 24. *Compound Machines with one common element.*—The compound machines described in the last paragraph have no elements common to two simple machines, but we may have compound machines in which either one or two elements are common to two successive machines. The examples most commonly met with in engineering practice are those in which there is one common element, namely, the framework or support which is continuous and common to a series of successive complete machines. The common element is necessarily in equilibrium under the whole series of stresses to which it is subject, but this equilibrium is not a matter of great interest. The driving link of the first machine usually abuts at one end against the common element. If the first simple machine stood alone, one end of its resisting link would abut against the same element; when it drives a series of machines, the resisting link of each must be so placed that in each case, if that particular machine were the last of the series, the resisting link would also abut against the common element. The common element takes the place of one transmitting joint or one transmitting link in the types given in the last paragraph. A practical example will serve to show the connection between successive complete machines, having one element in common. In fig. 29 a horizontal engine is shown driving a train of machinery. The engine consists of the elements *abcde*. The element d is a fixed frame; the element c comprises a fly-wheel and spur-wheel, which drives the pinion which is part of *g*; the spur-wheel of *g* drives a pinion which is part of *h*. A pulley, which is also part of *h*, drives a belt *l*, which, in its turn, drives a pulley *m*; a second pulley, also part of *m*, drives a second belt and pulley *n* and *o*. A piece of wood, forming part of *o*, is being turned by a tool which forms part of *d*. We have here three complete machines—1st, the engine; 2d, the machine *g h d*, driven by two transmitting joints and *gdcg*, and driving a transmitting link *l*; 3d, the machine *mnod*, driven by the transmitting link *l* and the joint *md*, and overcoming the useful resistance at the joint *od*. All these machines have the element *d* in common. The dynamic frame of the compound machine is shown in fig. 29*a*, and the reciprocal figure for that frame in fig. 29*b*. The driving element of the first machine is the steam which abuts against *d*, the bed-plate or support. The resisting element is the whole series of driven machines, in the last of which a resistance is overcome between an element of the machine and the bed-plate *d*. The resisting element is not complete unless we take into account the force it exerts at both ends; the one end of the resisting link of the first machine pushes up the periphery of the fly-wheel *c*, the other end pushes down the element *d*; this is precisely the action we should have, if the first machine had been completed by a single resisting link; the circuit must in either case be completed, so that the resisting link may abut against the common element *d*, against which the first driving link also abuts. The driving link of the second machine *ghd* is in the line of the resisting link of *abcde*. The direction of this link is for both machines determined by the transmitting joint *cg*, *i.e.*, by the form of the teeth of the wheels. The driving link of the machine *mnod*, is in the line of the resultant tension due to the band *l*, which is here a transmitting element. The driving or resisting link of each successive machine, if taken by itself, would abut against the common element *d*. The first driving and last transmitting element do abut against this common element. In any machine it will be found easy to analyse the series of parts so as to divide the whole structure into a series of complete machines, each joined to its neighbour by a transmitting joint or link. Each machine can then be treated as a separate whole. We see in the example given that the dynamic frame consists of three distinct quadrilaterals with diagonals. The connecting links may be considered as double links in each case. Thus we may consider the joint C, fig. 29*a*, as connected by one link with the joint G which it drives, while at the same time the joint D_1 is connected by a link with D_{11}. Thus, when the reciprocal figure for the whole frame is drawn, as in fig. 29*b*, it forms one connected whole. The stress corresponding to each connecting link is used twice as in all reciprocal figures. In fig. 29*b* each link is numbered from 1 to 6 for each successive machine. The length of the first line 1 corresponds to the driving effort. The length of the last small link 6 corresponds to the resistance which that effort can overcome at *od.* The diagram is drawn without taking friction or stiffness into account, so that the frame shown is kinematically equivalent to the machine. It would be easy in a drawing on a large scale, to show the complete effect of friction and stiffness, for we have already learnt to take these into account for each component simple machine. We see, therefore, that in any simple train of machinery, there can be little difficulty in estimating the true relation between effort and resistance (neglecting weight and mass); this difficulty never exceeds that met with in analysing a simple machine, and all simple machines are of one type.

§ 25. *Half Machines Compounded.*—In certain cases two successive machines have *two* elements in common, as well as the driving or resisting link. In this case we may consider the addition as in reality only half a machine. The typical example of this arrangement is given in fig. 30, where links *b* and *c* are common to two complete machines—1st, *abcde,* with its resisting link; and 2d, *bchgi,* with its driving link. It will be seen that the half machine *hig* has a certain analogy with the machines of class 2, only it is here the stiff bar which acts as a driving element by an alteration in its length; *h, i,* or *g* might represent the final resisting element. We are never driven to adopt this subdivision of a machine, except at one or other end of a train of machines which may not be divisible into a series of complete machines with joints or elements of transmission. Thus, if we (fig. 31) have a steam-engine with a spur-wheel on its crank shaft, driving a single pinion from the shaft of which a weight is hanging, we cannot divide the train into two distinct complete machines, each having a quadrilateral with diagonals as its dynamic frame. We may, however, draw two distinct dynamic frames, as shown in fig. 31*a,* where 7, 8, 9, 10, 11, and 12, are the links of a complete spur-wheel machine, similar to that of fig. 22*a,* driven by a link 7, but link 7 is in the same line as link 2 of the well-known engine frame. Links 8 and 3, 6 and 11, are also common to the two frames. The diagram shows that we might analyse the machine in two ways, calling, for instance, the steam engine a complete machine, and the extra spur-wheel with its weight, a half machine; or we might call the two spur-wheels a complete machine, and the piston, steam, and connecting-rod a half machine.

It is a matter of no consequence how we subdivide a train. The relative stresses in the first and last links will be the same, whether we use half machines with two common elements, or successive machines with one common element. This example shows us that the machines of class 2 may very properly be regarded as only half machines. This view is supported by the observation that when one machine of class 2 drives another of the same class, we get for the system a single complete dynamic frame, namely, the quadrilateral with two diagonals. As an example, we may take the hanging pulley and fixed pulley combined, as in fig. 32, on which is shown the dynamic frame of the combination. When the ropes are nearly parallel, as in fig. 32*a*, the bars shown by thick black lines may be considered as jointed to the six links, which would otherwise meet at the joints B, A, and A_1.

§ 26. *Reduplication of Cords.*—When a series of fixed and hanging pulleys are employed as in the ordinary blocks and tackle, it is found that, with the usual stiff ropes, no advantage can be obtained by using more than 5 or 6 plies of rope. The reason of this is shown clearly by the dynamic frame for a compound machine of this class, fig. 33. In this frame the successive pulleys and plies are arranged in one plane, so that the diagram may be better followed than could be the case if the pulleys were placed so as to be co-axial. Let the driving link act between the rope *a* and the fixed support *d*, and let the force applied by the driving link be called E, and the effective radius of each sheave R. The effect of the rope *a* is to produce a couple *m* diminishing R on the driving side by a length depending on the stiffness of the rope, and inversely proportional to the diameter of the pulley and to the tension on the rope. The effect of this is shown in the diagram by a shifting of the line of action of the force towards the centre of the pulley by a distance *s* equal to $\frac{m}{E}$. Let F_1 be the tension on the second rope. The couple required to unbend the rope has an effect which may be represented by shifting the line of action outwards to an amount s_1, equal to $\frac{m}{F_1}$; at each pulley a similar effect is produced, and as the value of the tension in the rope diminishes at each pulley, so the value of *s* increases at each pulley. The effect of friction at the axle is shown by shifting the joint in the bar representing each pulley towards the driving rope by a distance $r \sin \phi$, where *r* is the radius of the shaft. We then have the equation

$$E\left(R - r \sin \phi - \frac{m}{E}\right) = F_1\left(R + r \sin \phi + \frac{m}{F_1}\right)$$

$$\text{or } E(R - r \sin \phi) = F_1(R + r \sin \phi) + 2m.$$

From this equation we obtain F_1, and by a similar equation we could from this obtain F_2, &c. $\Sigma F = W$, the weight which can be raised. The result is after a few turns to reduce F_n to nil, after which no more plies can be of any service. The gradually diminishing efficiency of successive pulleys is very well shown by the diagram, fig. 33.

§ 27. *Loaded Dynamic Frame without Friction.*—Let us now consider the effect of the mass and weight of the elements of a machine. We may feel sure that the effect of weight and inertia may be shown by means of a dynamic frame, for this frame consists of lines indicating bearing pressures at the joints, and the direction and magnitude of these bearing pressures are always determinate. In the case of a material element supported by two joints, the lines of bearing pressure will no longer be directly opposite, or in the same straight line, but will intersect in a point on the line which indicates the resultant of the forces due to the weight and inertia of the element. Let the resultant of all the forces exerted on a given element other than those exerted at the joints be called the *load* on the element. This includes the equilibrant of the force producing acceleration. Then the action of an element with two joints, as in fig. 34, might be supplied by three forces represented by three half links 1, 2, and 3, fig. 34, showing in position and direction the bearing pressures at the joints, and the load on the element; this mode of representing a loaded element is commonly in use where the equilibrium of arches is discussed. The load 3 is here called a half link, for in the complete self-contained machine an equal and opposite load necessarily exists in some other element. This equal and opposite load is in general supplied by the reaction of the foundations, or more strictly by the reaction due to the mass of the earth.

Where an element has more than two joints, it will be found that the arrangement or form of the joints is generally, if not always, such as to render determinate the single joint or pair of joints by which it is supported. The effect of the load in modifying the direction of the bearing pressure can for these cases be as easily taken into account as in the simple case just cited.

Let us now consider the effect of four loads, L_a L_b L_c and L_d, on the four elements *abcd* of the elementary machine, fig. 35. We may, for the present, suppose the driving and resisting element to have no weight or inertia; the effect of these elements may then be treated as equivalent to the effect of four external forces, 1α 1β, and 6α 6β. The dynamic frame for this case will be a polygon of eight sides, fig. 35, 2α 2β, 3α 3β, 4α 4β, 5α 5β, having its angles on the lines of load, and so inclined as to be in equilibrium under these loads. The stresses in each link are the pressures at each joint. The reciprocal figure for this frame is shown in fig. 35*a*. The inclinations of the links are no longer independent of the magnitudes of the forces acting in the elements *e* and *f*, as was the case when we neglected weight and inertia. The effort in *e* or the resistance in *f* must therefore be given, as well as the loads, before the frame can be drawn. Let the effort in *e* or in link 1 be known, the frame and its reciprocal may then be drawn as follows, so as to solve the problem of finding the resistance which a given effort in *e* will overcome in *f*, neglecting the effect of friction, but taking into account the weight and inertia of the parts.

We know the position of four of the angles of the polygon, viz., the centres of the pins at the four corners of the machine. Let the loads be referred to these four points in the manner practised for the distributed loads on the actual rafters of a roof or members of a bridge; that is to say, let each load be replaced by two components acting at these four points. These components are lettered l_a l'_a, l_b l'_b, &c. The stress in element *f* will be the same as would be produced by the effort in *e* acting on the quadrilateral frame *abcd*, loaded at the joints in this manner. This stress in *f* is found by drawing the fig. 35*a*, beginning with the polygon d, l_d, 1α, l'_a, a. The directions of all these are known, and the magnitude of the stresses in all except *a* and *d*. The polygon serves to determine these stresses. To find the stress in *b* we require to draw the polygon a, l_a, 6α, l'_b, b, in which there are only two unknown stresses—those in 6α and *b*, the directions of which are however given. We cannot draw the polygon directly with the lines arranged in the manner shown in full lines, because 6α and *b* are not contiguous. If, however, the lines are drawn as dotted, we obtain a polygon which determines the stress both in 6α and in *b*, after which the lines may be rearranged in their natural order. By a similar process we find 6β and *c*, and can complete and check the drawing by adding the lines l'_c 1β and l_b. It is almost unnecessary to remark that 1α and 1β are equal, and that 6α and 6β are also equal. The sides of the polygon in fig. 35*a*, represent the loads on the elements of the machine in fig. 35, taken in their natural order. The lines *abcd* in 35*a* represent the stresses on the links *abcd*, or 1, 2, 3 and 4 of the machine in fig. 35. Let *o* be the point where *a*, *b*, *c*, and *d* intersect (fig. 35*a*), and let lines be drawn from *o* to the angles of the polygon; now, draw the lines 2α and 2β in fig. 35, parallel to the lines of the same name in fig. 35*a*, and so placed that they abut against the ends of link *a*, and intersect in the line L_a; draw 3α and 3β, 4α and 4β, 5α and 5β by a similar rule. The octagonal frame thus obtained and shown by a broken dotted line is a frame which will be in equilibrium under the four given loads and the two stresses in *e* and *f*. It is easy to see that this will be the case. The load L_a and the two stresses in 2α 2β of the frame form a polygon in the reciprocal figure. The same relation obtains between the other loads and the links supporting them. Moreover, the links 2α and 5β in the reciprocal figure, give a closed polygon with the line 1α, representing the stress in *e*. The lines abutting against the ends of *f* also give closed polygons with the stress 6, as shown in the reciprocal figure. The links 2α 2β, &c., of the octagonal frame do therefore represent the directions of bearing pressures at the joints on the hypothesis that there is no friction. The frame found by the method described will be called the *loaded dynamic frame without friction.*

The elements *e* and *f* have been represented as without mass; if their weight and inertia are to be taken into account, their load is to be referred to the joints in a manner similar to that indicated for the other elements. Links 1 and 6 would then be broken lines in the frame, with loads at their angles. It may be well to remind the reader that by hypothesis the machine is in equilibrium as a whole, and therefore in the reciprocal figure lines representing the loads necessarily form a closed polygon.

§ 28. *Loaded Dynamic Frame with Friction.*—The method by which we were enabled to draw the octagonal polygon described in the last paragraph depended on our knowledge of the four points which determined the position of four angles of the polygon, or one point on each of the eight lines. When we try to ascertain the actual lines of bearing pressure, taking into account the friction of the machine in motion, we find that the conditions determining their direction are more complex, since now we do not know any fixed point in any line. The conditions are, however, only changed to this extent, that the lines of bearing pressure must make the stated angle with the joints, instead of being normal to the surfaces of those joints. By trial, an octagon is easily found fulfilling this condition as well as the general condition of being in equilibrium under the forces applied at the joints. The manner of proceeding which seems most easy is to draw, first the polygon without friction, and then to sketch a modified polygon having its sides tangent to the friction circles, or making the stated angle with the joints. The sides of this trial polygon intersect at certain points which may be called *trial points*. When the friction circles are not very large, it is easy, by the exercise of a little judgment, to draw the trial polygon so as to make these *trial points* agree very closely with the true points of intersection, even at the first attempt. Then, referring the loads to the trial points, we draw a new polygon and reciprocal figure, figs. 36 and 36*a*, as for the frame without friction. If this second polygon has sides which make the stated angle with the joints, the problem is solved. Otherwise, a second selection of corrected trial points must be made, and a third trial polygon drawn: it will seldom if ever be necessary to make a third trial. We thus get a dynamic frame which truly represents the directions of the forces at every joint in the actual machine, and this frame will be called the *complete dynamic frame of the machine*, or the *loaded dynamic frame with friction*. The resistance which can be overcome by a given effort 1 in the driving link, is shown by the line 6 in the reciprocal figure 36*a*, which has been used as an auxiliary in drawing the loaded dynamic frame, and this resistance will be the actual resistance which could be overcome by the given effort in the given machine, under the given conditions as to speed, friction, mass, and weight.

§ 29. *Application of the Method to an ordinary Horizontal Single-Acting Steam Engine.*—In fig. 37, let the lines *b* and *c* represent the centre lines of the connecting rod and crank of an engine, while the line *a* represents the direction of the motion of the piston. Let the line *f* represent the direction and position of the resistance overcome; and let this resistance be represented as in previous examples by a stress between *d* and *c*. Let the lines L_a, L_b, L_c, and L_d represent the loads on the elements *a, b, c,* and *d* of this engine, where *d* is the frame, it being remembered that these loads must be such as would balance one another; in other words, L_d, the resultant reaction due to the foundation, must be the equilibrant of the three others; L_c is the weight of the balanced fly-wheel and crank-shaft, acting directly through the centre of the main bearing; L_b is the resultant of the weight of the connecting rod, compounded with the equilibrant of the force required to give the rod the acceleration (in respect of rotation and translation) which it actually has at the given instant. L_a is the resultant of the weight of the piston compounded with the resistance to acceleration. The loads shown in the figure have been calculated for an actual small direct acting engine, making 1 turn per second. To draw the dynamic frame without friction we proceed as follows:—L_b is represented by two components l_b and l'_b, acting at the joints *ab* and *bc*. L_a is wholly borne by the one joint, and the actual point of its application is a matter of indifference; L_c is wholly borne by the joint *cd*. The treatment of L_d will be explained hereafter. The simple dynamic frame without load or friction is shown by lines 1, 2, 3, 4, 5, 6 in fig. 37*a*, which also shows the loads referred to the proper points. Let it be remarked that l'_b is referred not to a joint in the simple frame, but to the point through which the actual bearing pressure 2β must pass. We may now begin the reciprocal figure 37*b* by drawing the effort 1α of the steam against the piston, the load L_a the load L_b, and the directions of the reaction 5, and the resistance 6α. The line L_b is then subdivided into its two components l_b and l'_b, and the line 2 is drawn from the point which subdivides L_b into its two components l_b and l'_b parallel to the line *b* in fig. 37 or 2 in fig. 37*a*. Let the point where line 2 intersects 5 be called *o*; then the polygon 1α, L_a, l_b, 2, 5 (fig. 37*b*) represents the forces in equilibrium at the joint *ab*; now, returning to fig. 37*a*, we are able to draw lines 5, 2α and 2β: the direction of 2α is given by the line of the same name in fig. 37*b*, drawn from *o* to the intersection of L_a with L_b; the line 2β is drawn in fig. 37*a* parallel to the line of the same name which in 37*b* joins *o* with the intersection of 6α and L_b; the line 2α and 2β abut against the joints at the end of link 2 (fig. 37*a*), and meet in the line L_b. The element *c* is in equilibrium under the action of the resistance 6α, the driving force 2β which we have just found, the weight L_c and the reaction of the main bearing. The load L_c is wholly borne by the joint *cd*, and therefore the direction of the remaining component of the reaction at the bearing is given by the full line 3α, fig. 37*a*, passing through the centre of the joint *cd*, and the intersection of 6 with 2β. From *o* draw 3α in 37*b*, parallel to 3α in 37*a*, and it will cut off a length 6α measuring the resistance which the effort is able to overcome; the polygon 2β 6α L_c and 3β represents the four forces under the action of which *c* is in equilibrium. 3β is the resultant pressure on the joint *cd*. We may complete the reciprocal figure by drawing the force 1β, the load L_d, and the forces 6β, 4α, and 4β. We may also complete the loaded frame in fig. 37*a* by drawing 4α and 4β parallel to the lines of the same name in 37*b*; a line from *o* parallel to link 4 of fig. 37*a* will subdivide L_d in the ratio in which L_d would be subdivided if referred to the two frame joints between links at the ends of link 4. The problem is rather simpler than that described in the last paragraph, inasmuch as link 5 of the dynamic frame is not loaded.

We will now consider how this figure is modified by the introduction of friction. Beginning with link 1*a* in fig. 38*b*, we can draw line 5 making the stated angle with the surface of the guide bars, and lines L_a L_b, which are identical with the lines of the same name in fig. 37*a*. We must, however, subdivide L_b in a new ratio, for it is clear that the line of bearing pressure 2α will pass over the friction circle, and so that the trial point where it intersects line 5 will be some distance to the right; 2β must pass under its friction circle, and the point which 2β must pass through on the left must obviously be near the bottom of its friction circle. Draw the line 2, joining two trial points, refer L_b to the two ends of link 2, or, in other words, subdivide L_b, fig. 38*b*, in the ratio in which L_b subdivides link 2; from the point of subdivision in fig. 38*b*, draw 2 parallel to 2 in fig. 38*a*. The intersection of 2 and 5 gives the point *o*. The piece *c* is held in equilibrium by four forces L_c, 6α, 2β, and the reaction from the bearing: the intersection of 2β with 6 (fig. 38*a*) gives one point through which the loaded link 3 must pass, and our trial point for the other end must obviously be a little to the left of the friction circle at the main bearing, where the tangent 3β cuts line 1, and this tangent 3β must be a little less steep than the line 3β in fig. 37*b*. The load L_c is now to be subdivided between the two ends of the loaded link 3, the two components being l_c and l'_c; the polygon of the forces in equilibrium at the upper right hand end of 3 can now be drawn in fig. 38*b*, these are 2β, l_c, 6α and 3; the two former are known and the directions of the two latter; the polygon can therefore be drawn with the lines arranged in the order named, and thus the magnitude of 6α can be determined. The problem is now solved, but if we wish to complete our drawing of the frame, we must rearrange the last drawn polygon, so that the forces in the reciprocal figure come in their natural order as shown by the full lines in fig. 38*b*, then finishing L_c we can, as in figures 37*a* and 37*b* complete the reciprocal figure and frame without difficulty. If we have chosen our trial points well, the lines of the frame will be tangent to the friction circles. If they cut these or do not touch them, we must correct our choice of the trial points until the desired figure is found. As the friction circles are generally very small, a single trial is generally sufficient for a draughtsman who has mastered the theory. The result is really remarkable. When the loads have been determined a reciprocal figure of nine lines enables us to ascertain the true relation between effort and resistance in a horizontal direct acting steam engine, taking into account the weight, inertia, and friction of every part of the simple train of joints and elements.

§ 30. *Loaded Dynamic Frame when neither* e *nor* f *are attached to the extremities of other members of the Machine.*—This case presents some geometrical peculiarities. Let the elements of the frame be the four lines *a, b, c, d,* shown as thick black lines in fig. 39, Plate XII., and let the elements *e* and *f* be joined to these at points intermediate between their extremities. Each element is then in equilibrium under the action of three forces, and the simple dynamic frame is the quadrilateral 2, 3, 4, 5, having its angles on the prolongations XX_1 and YY_1 of the directions of the stress in *e* and *f,* and having its sides so placed as to pass through the joints *ab, bc, cd, da,* denoted by the letters MNPQ. It is not quite obvious how this quadrilateral may be drawn. It may be proved that all quadrilaterals of which the angles lie in the lines XX_1 and YY_1 and of which three sides pass through M, N, and P, have their fourth sides so placed as to intersect at one point E; the point E can therefore be found by drawing two trial quadrilaterals, and this point can then be joined with Q and so give the direction of one side of the desired quadrilateral ABCD. Professor Tait, who pointed out this fact to the writer, also showed that the point E might be more simply found as follows:—Produce MN until it intersects *e* prolonged in X, join X with P; similarly, produce NP until it intersects *f* prolonged in Y, and join YM; the point E lies in the intersection of XP with YM; the line QE gives the direction and position of one side of the quadrilateral ABCD. A second quadrilateral has been drawn on the figure for the sake of illustrating the form which it assumes when the fourth point is *q,* chosen outside the angle XEY.

When, as in fig. 40, Plate XI., the four members *a, b, c* and *d* are all loaded, the problem becomes still more complex. The octagonal equilibrated polygon for the four loads and two stresses in *e* and *f,* otherwise named 1 and 6, are shown in fig. 40, with lettering analogous to that employed for the simpler cases. This polygon was formed in a somewhat indirect manner, and it is probable that a simpler geometric method may be found if the case should arise frequently in practice. (1.) The relation between a stress in *e* and one in *f* was found by a simple frame and reciprocal figure, fig. 41; (2.) The same process was repeated for a stress in *e,* and a stress L_a between the elements *a* and *d,* fig. 42; (3.) The process was repeated for a stress L_b between *b* and *d,* fig. 43; (4.) The process was repeated for a stress L_c between *c* and *d,* fig. 44; (5.) By addition the stress in *e* was found which was required to overcome the given stresses due to *f* and to the four loads; (6.) The several loads were referred to the joints; (7.) Polygons of force were drawn for each joint, and by these polygons the inclinations of 2β, 3β, 4β, and 5β, fig. 40, were found, the intersections of these lines with the loads (including *e* and *f*) gave the eight angles of the polygon; (8.) The reciprocal figure, fig. 40*a,* was drawn, by which the work was checked, and the inclinations of the sides of the polygon verified.

II.—*Additions to the paper "On the Establishment of the Elementary Principles of Quaternions, &c.," in the Transactions of the Royal Society of Edinburgh, Vol. XXVII.* By G. Plarr, Docteur ès-Sciences.

(Read 7th May 1877.)

Page 190, to the alinea (beginning with): "the sole condition to be satisfied, is,". &c. (ending with): ". . . the versor of the expression may then become *what it may,*" *add:*

provided this versor does not differ from the versor of the product

$$\rho\times[\varpi\times(\varpi\rho)]$$

made by successive multiplication (according to the definition of multiplication of more than two vectors), the quaternions $w+\omega$, $w-\omega$ being likened respectively to the products

$$\rho\varpi \quad \text{and} \quad \varpi\rho$$

by a proper choice of the vectors ρ and ϖ.

Page 191, to the fourth line from above, *add:*

It will be easily found that, in virtue of the value $\mathfrak{h}=-1$, the versor of the two products $\rho\times[\varpi\times(\varpi\rho)]$ and $(\rho\varpi)\times(\varpi\rho)$ become equal; *both* being equal to unity.

Let us designate the unit vectors of ρ, ϖ, σ, τ, respectively by ρ', ϖ', σ', τ'. Then for the calcul of the versor $[\varpi\times(\varpi\rho)]$ we admit, by an anticipation, not founded on $\mathfrak{h}=-1$, besides $\rho'^2=\mathfrak{h}$ and $\rho'\sigma'=\tau'$:

1°) $\mathfrak{g}=+1$, $\mathfrak{h}^2=+1$,
2°) $-\rho'\tau'=\sigma'$, $\sigma'\tau'=\rho'$;

and taking from pages 187, 182:

$$\begin{aligned}\varpi'\rho' &= \mathfrak{h}\cos u-\tau'\sin u\\ \varpi' &= \rho'\cos u+\sigma'\sin u,\end{aligned}$$

we have:

$$\rho'\times\varpi'\times\varpi'\rho'=\rho'(\rho'\cos u+\sigma'\sin u)(\mathfrak{h}\cos u-\tau'\sin u)$$

applying the distributive rule we get:

$$\rho'[\varpi' \times (\varpi'\rho')] = \cos^2 u - \mathfrak{h} \sin^2 u + \tau'(\mathfrak{h}+1) \cos u \sin u \,.$$

The square of the tensor of this product will be:

$$(\cos^2 u - \mathfrak{h} \sin^2 u)^2 + (\mathfrak{h}+1)^2 \cos^2 u \sin^2 u \,.$$

This expression being developed we see that the first power of $\mathfrak{h}$ disappears, and the result will be, by $\mathfrak{h}^2 = +1$:

$$T\rho'[\varpi'\chi(\varpi'\rho')]^2 = (\cos^2 u + \sin^2 u)^2 = +1 \,.$$

On the other hand, the product

$$(\rho'\varpi') \times (\varpi'\rho') = (\mathfrak{h} \cos u + \tau' \sin u)(\mathfrak{h} \cos u - \tau' \sin u)$$

being effected by the distributive rule, and having:

$$\mathfrak{h}^2 = +1 \,, \qquad \tau'^2 = \mathfrak{h} \,,$$

becomes, as we have seen (page 190):

$$= \cos^2 u - \mathfrak{h} \sin^2 u \,.$$

This being a scalar, the square of its tensor will be

$$\begin{aligned} &\cos^4 u + \sin^4 u - 2\mathfrak{h} \sin^2 u \cos^2 u \\ &= (\cos^2 u + \sin^2 u)^2 - 2(\mathfrak{h}+1) \sin^2 u \cos^2 u) \,; \end{aligned}$$

so that:

$$[T(\rho'\varpi') \times (\varpi'\rho')]^2 = 1 - 2\,(\mathfrak{h}+1) \sin^2 u \cos^2 u \,.$$

It appears therefore that the square of the tensor of $\rho \times [\varpi \times (\varpi\rho)]$ becomes equal to $T\rho^2 T\varpi^2$, whatever value we admit for $\mathfrak{h}$, provided $\mathfrak{h}^2 = +1$; whereas the square of the tensor of $(\rho\varpi) \times (\varpi\rho)$, in order to become $= T\rho^2 \times T\varpi^2$ *demands* the value $\mathfrak{h} = -1$, to the exclusion of the value $+1$.

Page 191, line twelve from below, *correct* vector into versor.

Page 196, to the eleventh line from below, *add:*

Provided that the versor of the product $(a+\alpha)\,(b+\beta)$ be the same as that of the product $\lambda \times [\mu \times (b+\beta)]$, when $a+\alpha$ has been assimilated to $\lambda\mu$.

Page 197, after the italics in the middle of page, *add:*

the verification as to the versor will result from the remark, at p. 200, on the equality $(\alpha\beta) \times (\gamma\delta) = \alpha \times [\beta \times (\gamma\delta)]$.

The distributive rule of multiplication of two quaternions by one another is thus legitimated in so far as the *tensor* of the product is concerned.

As to the *versor* of the product, and abstraction made of the tensor, as to the product itself, in so far as composed of scalar and vector, it will not be unnecessary to show: that the proposed expression of $c+\gamma$, as product of $(a+\alpha)(b+\beta)$, can be deduced as a consequence of the rule of multiplication of vector factors, adopted by definition, in the case of more than two factors, so that there will be only *one* rule of multiplication to be adopted by definition, and *not two* (*not* one for vectors and another for quaternions).

In such a case namely, Let us suppose that the product of several factors has been obtained. It will be a quaternion, which we may represent by $b+\beta$. Let us suppose that λ and μ represent the not yet employed factors. Then in order to form the product $\lambda\times[\mu\times(b+\beta)]$ we have to make the product $\mu b+\mu\beta$, and afterwards effect the multiplication by λ. Thus we will have:

$$\lambda\times[\mu(b+\beta)] = \lambda\mu b + \lambda\times\mu\beta.$$

But at page 199 it is shown (by vector multiplication) that the terms of each of the developments of the two products

$$\lambda\times\mu\beta \qquad \text{and} \qquad \lambda\mu\times\beta$$

are each to each the same, only under another form, and therefore we may consider the effected product under the form

$$(\lambda\mu)\times(b+\beta),$$

where $\lambda\mu$ is considered as a whole; only of course for the sake of effecting the multiplication of $b+\beta$ by $\lambda\mu$ the expression of $\lambda\mu$ will be represented by the several terms which constitute it.

But now we may suppose that λ, μ, have been chosen so as to represent $a+\alpha$ by their product, and then the product $(a+\alpha)(b+\beta)$, when effected, will have been effected by the rule of the multiplication of vectors only.

III.—*Note on the Bifilar Magnetometer.* By J. A. Broun, F.R.S.

(Read 7th May 1877).

In a paper on "The Bifilar Magnetometer, its Errors and Corrections," &c., which appeared in the Transactions of the Royal Society of Edinburgh, vol. xxii. p. 467, it was shown that, in turning the apparatus which carries the upper extremities of the two wires through an angle exceeding by v that through which the lower extremities and the magnet are moved, the upper end of each wire is also turned through an angle greater by v than the lower end. The whole force acting on the magnet, to turn it from the meridian, is compounded of the bifilar and the unifilar torsion—only the former of which had been considered in the method issued by the Committee on Physics of the Royal Society of London for the instruction of directors of observatories.

It was sought in the paper cited to show the change that the neglected forces would introduce into the expression for the unit coefficient, which was found to be

$$k = \cot (v + \beta) \, \Delta v ,$$

instead of

$$k = \cot v \Delta v ,$$

where Δv is the arc value of one scale division of the instrument, and β is the variation of v due to the unifilar torsion. There were errors of copying, including omissions in the investigation given. I also remarked, after the paper was printed, that the value of k found was *less* than when the unifilar torsions were neglected (since $\cot (v + \beta) < \cot v$); whereas all the determinations by other methods, in which the unifilar torsions were included, had shown a *greater* coefficient.

Having had some time ago to re-examine questions connected with this instrument, I found that there was an error in the investigation, which I now desire to correct. It was assumed that the right hand side of equation (3) of the paper (equation (1) of this note) might be represented by $G \sin v'$ (where $v' > v$), which cannot be allowed if v varies, especially as v is not small.*

Let us suppose as before, that, in turning the torsion circle carrying the upper ends of the two wires, the horizontal line in which they lie is carried through an angle of 90° + v, by which means the suspended magnet and the horizontal line passing through the lower ends of the wires are carried through an angle of 90° (that is, the magnet is then at right angles to the magnetic meridian);

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* The applications of the formula found (Edinburgh Royal Society Transactions, xxii. p. 468, arts. 5, 6, and 7) are not affected by this error.

the bifilar torsion (that is, the angle made by the upper and lower horizontal lines) will be v, and the torsion of each wire will also be v. If p be the torsion coefficient of a single wire for unity of arc, then the force exerted by the unifilar torsions will be equal to $2pv$; and as the force due to the bifilar torsion is $G \sin v$, the equation of equilibrium will be

$$mX = G \sin v + 2pv \qquad . \qquad . \qquad . \qquad (1)$$

where m is the magnetic moment of the magnet, X is the horizontal component of the earth's magnetism, and G is a constant depending on the mass suspended, and length and intervals of the wires.

If now we suppose the magnet to be in the magnetic meridian, and that *each* wire receives a torsion v, so that the magnet is moved from the meridian by an angle δ (the bifilar torsion being zero), then

$$2pv = mX \sin \delta \qquad . \qquad . \qquad . \qquad (2)$$

by (1) and (2)

$$mX = \frac{G \sin v}{1 - \sin \delta} \qquad . \qquad . \qquad . \qquad (3)$$

Differentiating (1), X and v only being variable, substituting the value of p from (2), and dividing by (3)

$$\frac{\Delta X}{X} = \left\{ \cot v + \sin \delta \left(\frac{1}{v} - \frac{1}{\tan v} \right) \right\} \Delta v \qquad . \qquad . \qquad (4)$$

Since $\frac{1}{v} > \frac{1}{\tan v}$, the coefficient of Δv is always greater than $\cot v$, as has been found in the experiments for this coefficient by deflections and by weights.

Experiments for the value of $\sin \delta$ have shown that equation (4) will give a near approximation to the coefficients by other methods; but the determination may always be vitiated through torsion introduced accidentally into the wires during adjustments. Thus, at Makerstoun, in two early experiments, the unit coefficient, with the same length and interval of wires, was found to be

$$k = 0{\cdot}0001185,^*$$

when the north end of the magnet was turned towards the east; and

$$k = 0{\cdot}0001522,$$

when the north end was turned towards the west: this difference was found due to torsion existing in the wires. As the methods by deflections, and by varying weights, include all the forces acting, it is much more satisfactory to employ one of these for the determination of the unit coefficient.

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* By a typographical error this was given 0·000185 in the Makerstoun Observations (Edinburgh Royal Society Transactions, xvii. part i. p. 37).

The quantity $2pv$ does not affect the unit coefficient only; it has a much more weighty influence on the temperature coefficient. This was proved experimentally in the paper cited, where, however, it was only shown that the *direction* of the temperature action was such as had been supposed by me previously; but as the quantities obtained prove also that the *amount* of the action was such as to explain the differences found by the two methods for the determination of the temperature coefficient, I shall examine the results here.

An unmagnetic weight was suspended by two silver wires; the weight was turned round a vertical axis by the horizontal pull of a small weight hanging at the end of a silk fibre, acting at right angles to the line joining the wires, and passing over a delicate friction wheel.* As the action of currents of air within the box containing the weight produced a regular diurnal movement, the pull was made in the first series of experiments to one side, and in the second series to the opposite side. In the first series, which (from causes indicated) was the least consistent of the two, it was found that an increase of the daily mean temperature within the box (derived from twenty-four hourly observations) corresponded to a movement of the weight in such a direction as to increase the bifilar torsion angle, so that the unifilar torsion force had diminished. In this case the mean reading changed by ± 4·0 scale divisions for ± 1° Fahr. When, as in the second series, the pull of the small weight was in the opposite direction, an increase of temperature was found as before to coincide with a diminution of the unifilar torsion force, the motion of the weight being also in the opposite direction, the daily mean reading changing by ∓ 2·5 scale divisions for ± 1° Fahr. Both results confirmed the hypothesis, that the action of an increase of temperature was to diminish the force due to the torsion of the separate wires. It should be remarked, that the total effect of temperature in expanding the wires, and the metal which separates them, is to *increase* the bifilar torsion force, but to *diminish* the bifilar torsion angle, which is just the opposite of the result shown by these experiments.

Taking the result of the second series of experiments as most free from error, we find that one scale division $= 0'{\cdot}327$, the angle $v = 65°$, whence employing the approximate coefficient from the bifilar torsion 65°,

$$\begin{aligned} q &= \cot 65° \,(\text{arc } 0'{\cdot}327)\ 2{\cdot}5 \\ &= 0{\cdot}00011\,. \end{aligned}$$

Let us apply this result to the case of the bifilar magnetometer. We know that the effect of increase of temperature on the magnet is to diminish the magnetic moment; the magnet then moves *from* the north; the result is as if the earth's magnetism had diminished: the action of increase of temperature on the unifilar torsion of the wires corresponds to a diminution of the force

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* See Edinburgh Royal Society Transactions, xxii. p. 472, art. 15.

which turns the north end of the magnet from the north; the magnet will therefore move towards the north. The one action is in just the opposite direction of the other, so that the conjoint action will be the *difference* of the two.

Experiments with hot water on bifilar magnets have shown that a change of 1° Fahr. corresponds to from $q = 0{\cdot}00022$ to $q = 0{\cdot}00038$. If we take the mean of the two, the conjoint coefficient, including the action of temperature on the wires, should be

$$q = 0{\cdot}00030 - 0{\cdot}00011 = 0{\cdot}00019;$$

that is, the true coefficient is only about two-thirds of the coefficient by hot water experiments. This is nearly the mean of the results deduced from the determinations of the temperature coefficient by my method of comparisons and by the hot water method (see art. 39 of paper on the Bifilar Magnetometer, previously cited).

I have endeavoured to make this explanation of the cause of the very different results obtained by the two methods as clear as possible, since two of the most eminent magneticians, while accepting the facts, have expressed their inability to offer any explanation of them, though the cause here shown was suggested in my earlier papers on the subject (see Edinburgh Royal Society Transactions, vol. xvi. p. 77, art. 20; Makerstoun Observations for 1843; Edinburgh Transactions, vol. xvii. part ii. p. 53, art. 70).

The conclusions arrived at for the unit and temperature coefficients have been confirmed in every instance in which the two methods for each coefficient have been tried; and it may safely be asserted, that if in any case this confirmation is not obtained, this will be due to some error in the determinations.

I believe that we have at present no more perfect magnetical instrument than the bifilar magnetometer, when proper precautions have been taken in its construction, and when the coefficients have been determined by the methods referred to in this note. It may be remarked, that the wires should be of gold ($\frac{9}{10}$ fine is probably the best), suspended by tubular pincers, and *not* wound on a roller. It is however, also, of the greatest importance that the true temperature of the magnet should be observed, and this can be got only when the thermometer bulb (resting on a thin metal plate) is within the *inner* box near the magnet. Every precaution also should be taken to make the changes of temperature as small as possible.

These precautions are essential if a scientific instrument is required. Bifilars with silk suspension threads are not scientific instruments; they are affected by humidity and by unceasing changes in the strain and disposition of the thread fibres, as well as by their frequent rupture, for all of which sources of error no correction is possible.

IV.—*On the Solutions of the Equation* $V\rho\phi\rho=0$, $\phi\rho$ *representing a Linear Vector-Function, generally not Self-Conjugate.* By GUSTAV PLARR, Docteur ès-sciences. Communicated by Prof. TAIT.

(Received July 26—Read December 18, 1876.)

INTRODUCTION.

Certain cinematical and physical questions lead to the problem: to determine the directions in which a given linear vector-function, $\phi\rho$, assumes a direction parallel to that of the vector, ρ, on which it depends.

The condition of parallelism is expressed by

$$V\rho\phi\rho=0\ ,$$

and it is translated into the equation

$$(\phi-g)\ \rho=0\ ,$$

where g represents a certain scalar, on whose determination the whole problem depends.

Following the method traced out by HAMILTON, we treat this equation successively by

$$S\alpha(\ \ )\ ,\ S\beta(\ \ )\ ,\ S\gamma(\ \ )\ ,$$

α, β, γ, being any system of vectors not coplanar between each other; but we will state at once that throughout the whole of this paper we shall assume α, β, γ, to form a system of *treble rectangular unit-vectors,* of which hypothesis the justification is evident.

Designating by ϕ' the conjugate of ϕ, defined according to

$$S\sigma\phi\rho=S\rho\phi'\sigma\ ,$$

we arrive at the known results (1st), that the vectors

$$(\phi'-g)\alpha\ ,\ (\phi'-g)\beta\ ,\ (\phi'-g)\gamma\ ,$$

are all three in one and the same plane, are coplanar, in one word; and (2d), that the sought-for direction of ρ, satisfying to $V\rho\phi\rho=0$, is perpendicular to that plane.

The first of these results gives us the scalar equation

$$(1)\qquad S(\phi'-g)\alpha(\phi'-g)\beta\ (\phi'-g)\gamma=0\ ,$$

which expresses the coplanarity of the three vectors forming the product; and the second of the results gives us the means of forming expressions which give the vector ρ, as for example, by an expression like

$$\rho C=\psi\gamma=V(\phi'-g)\alpha(\phi'-g)\beta,$$

C being a certain scalar, whose value will have to be determined as the case may be.

We now divide the subject into two parts.

In the first part (which will be the longer one), we shall consider the conditions which ϕ (and its scalar elements) will have to fulfil, in order that the three roots of the equation (1) be all three real; and in the second part, we shall examine the properties of the expressions giving ρ, for which the condition of parallelism with $\phi\rho$ is realised.

The results which we have tried to establish in the first part may be summarised briefly as follows:—

Supposing that the condition of reality of the three roots (1) be expressed by

$$\Gamma>0.$$

Γ being a known function of the scalar coefficients of equation (1), we transform the expression of Γ by introducing into it several auxiliary vectors, amongst which the vector commonly designated by ϵ, according to the definition

$(\phi-\phi')(\ \ )=2V\epsilon(\ \ )$, will play a prominent rôle;

and with the help of these vectors, we find for Γ an expression in function of the squares of the auxiliary vectors, and of scalars of some of their products.

We then reduce the utter indetermination of the question in treating the general case following:—We assume that the auxiliary vectors which constitute the self-conjugate part of ϕ remain invariable, and that ϵ, and all that depends on it, alone varies, and we construe the surface defined by

$$\Gamma=0, \tag{2}$$

in which the vector of the surface is to be the value and direction of ϵ, answering to the condition $\Gamma=0$.

We will be able to show that this surface, besides some particularities interesting in themselves, enjoys the property of embracing a closed and contiguous space round the origin of its vector, which is central.

The condition, then, $\Gamma>0$, will be realised for every value of ϵ which has its origin at the centre, and its extremity contained *within the inside* of the surface (2); and consequently for all these values of ϵ, the equation (1) will have three real and different roots g.

For the values of ϵ answering to the surface (2) itself, the equation (1) will have three real roots also, two of them, if not all three, being equal to one another. Finally, the value of Γ will be negative, and equation (1) will have only one real root when the extremity of ϵ lies *outside the surface* (2).

The case when $\epsilon=0$, and consequently when ϕ is self-conjugate, is included in the preceding. In this case, our expression of Γ becomes positive "*a priori*," being then composed of the sum of the squares of two tensors. This result of $\Gamma>0$, when $\epsilon=0$, has been established by other methods, but not under the compact form under which it is given by the quaternion method.

In the second part, we consider the connected problems of forming the expressions which satisfy respectively to

$$V\rho\phi\rho = 0\,, \qquad \text{and to} \quad V\rho'\phi'\rho' = 0\,.$$

It is known that those expressions, proportionate respectively to ρ, and to ρ', by a scalar factor, depend on an auxiliary vector, quite arbitrary in direction, in such a way that the unit vector of the expression remains invariable, whereas the tensor of the expression varies when the direction of the auxiliary vector varies.

The expressions which we establish for the tensors enable us to discern at once what the directions are of the arbitrary vector, to which correspond either the maximum of the tensor or its minimum (this latter being zero).

Likewise we find for the product $\rho\rho'$ an expression of striking simplicity, both as to the scalar and the vector of the product, from which it is easy to deduce some properties belonging to the planes determined by the three systems of vectors ρ, ρ', each system corresponding to one of the roots g, and some other properties relative to the angle between the vectors ρ, ρ', of each system.

Finally, a discussion of the cases in which the tensor of ψ vanishes, gives us an example of certain solutions ρ, ρ' (relating to a singular direction and value of ϵ) becoming indeterminate.

First Part.

§ 1. We designate, for any value of g, by fg the scalar :—

$$fg=S(\phi'-g)\alpha(\phi'-g)\beta(\phi'-g)\gamma\,, \tag{3}$$

and having between α, β, γ, the relations

$$\begin{array}{ll} \alpha\beta=-\beta\alpha=\gamma\,, & \alpha^2=\beta^2=\gamma^2=-1 \\ \gamma\alpha=-\alpha\gamma=\beta & \\ & \alpha\beta\gamma=-1 \\ \beta\gamma=-\gamma\beta=\alpha & \end{array}$$

we liken the function fg with

$$fg=g^3-m_2g^2+m_1g-m\,, \tag{4}$$

so that we have:

$$\left\{\begin{array}{l} -m = S\phi'\alpha\phi'\beta\phi'\gamma \\ -m_1 = \Sigma S\alpha\phi'\beta\phi'\gamma \\ -m_2 = \Sigma S\phi'\alpha.\beta\gamma = \Sigma S\alpha\phi'\alpha\,, \end{array}\right.$$

where Σ represents the summation of the three terms obtained by the permutation of α, β, γ, in circular order, according to α, β, γ, α, β, &c.

If we consider the function f as the symbol of an operator on a linear vector-function ϕ, we get

$$f(\phi) = \phi^3 - m_2\phi^2 + m_1\phi - m\,,$$

and we know by HAMILTON'S admirable theory that we have identically:

$$f[\phi(\rho)] = 0\,.$$

Like as in algebra we transform $f(g)$ by the introduction of another variable in the place of g, so we may also transform $f(\phi)$ by the introduction of another linear vector-function in the place of ϕ.

We will introduce successively the functions ϖ, ξ, defined by

$$\varpi = \frac{1}{2,3}\,\frac{d^2f(\phi)}{d\phi^2}\,, \qquad \xi = \frac{df(\phi)}{d\phi}\,,$$

the differentiations being of course only symbolical of the operations.

Thus we have for ϖ:

$$\varpi = \phi - \frac{m_2}{3}\,.$$

Once for all we put

(5) $$m_2 = 3m_3\,,$$

Then

(6) $$\varpi = \phi - m_3\,, \text{ or } \phi = m_3 + \varpi\,.$$

This gives:

$$f(m_3+\varpi) = fm_3 + \varpi f'm_3 + \varpi^2\frac{f''m_3}{2} + \varpi^3\,.$$

Owing to $f''g = 6g - 6m_3$, we have $f''(m_3) = 0$, and we put

(7) $$P = f'm_3\,, \qquad Q = fm_3\,.$$

Thus we transform $f(\phi) = 0$ into

(7 *bis*) $$f(m_3+\varpi) = \mathscr{F}(\varpi) = \varpi^3 + P\varpi + Q = 0\,.$$

We may now, besides the values of P, Q, given by definition, form other expressions for them, analogous to those of m, m_1, m_2, either by deducing them from the independent consideration of the function ϖ, or by the substitution of $m_3+\varpi$, in the place of ϕ, in the equation $f(\phi)=0$. By either method we shall arrive at the expressions following:

$$(8)\qquad \begin{cases} +Q=S\varpi\alpha\,.\,\varpi'\beta\,.\,\varpi'\gamma \\ -P=\Sigma S\alpha\varpi'\beta\,.\,\varpi'\gamma \\ \quad 0=\Sigma' S\alpha\varpi'\alpha=\Sigma' S\alpha\beta\varpi'\gamma\,, \end{cases}$$

ϖ' being the conjugate of ϖ.

Likewise as for any linear vector-function ϕ we have the relation

$$m\,\phi^{-1}V\theta\theta'=V\phi'\theta\phi'\theta'\,,$$

so also we have for the function ϖ:

$$-Q\varpi^{-1}V\theta\theta'=V\varpi'\theta\varpi'\theta'\,,$$

θ,θ', being two vectors whatever.

This last relation, combined with the equation (7 *bis*), $\mathscr{F}(\varpi)=0$, gives us other expressions for P, Q. Namely, writing $\mathscr{F}(\varpi)=0$ under the form:

$$-Q\varpi^{-1}=\varpi^2+P\,,$$

and forming the result $\Sigma S\alpha(\quad)\alpha$ we get:

$$-Q\Sigma S\alpha\varpi^{-1}\alpha=\Sigma S\alpha\varpi^2\alpha+P\Sigma\alpha^2\,.$$

But by the expression of P we have:

$$-P=\Sigma S\alpha V\varpi'\beta\,.\,\varpi'\gamma=\Sigma S\alpha(-Q)\varpi^{-1}\alpha,$$

Thus the preceding equation gives:

$$-P=\Sigma S\alpha\varpi^2\alpha+P\Sigma\alpha^2\,,$$

and as $\Sigma\alpha^2=-3$, we get the result

$$(9)\qquad 2P=\Sigma S\alpha\varpi^2\alpha\,.$$

The expression $\Sigma S\alpha\mathscr{F}[\varpi(\alpha)]=0$, namely

$$0=\Sigma S\alpha\varpi^3\alpha+P\Sigma S\alpha\varpi\alpha+Q\Sigma S\alpha^2\,,$$

in virtue of $\Sigma S\alpha\varpi\alpha=0$ (8), gives us:

$$(10)\qquad 3Q=\Sigma S\alpha\varpi^3\alpha\,.$$

We may also form $\Sigma S\alpha\varpi(\mathscr{F}\varpi\alpha)=0$, which gives us:

$$0=\Sigma S\alpha\varpi^4\alpha+P\Sigma\alpha\varpi^2\alpha\,,$$

in which Q disappears owing to its factor $\Sigma S\alpha\varpi\alpha=0$. Replacing the sum $\Sigma S\alpha\varpi^2\alpha$ by 2P we get:

$$(11)\qquad 2P^2=-\Sigma S\alpha\varpi^4\alpha\,.$$

These expressions (9) (10) (11) will be needed for further transformations.

§ 2. We introduce the six auxiliary vectors η, σ, τ, η_1, σ_1, τ_1 by the following definitions:

$$(12)\qquad \left\{\begin{array}{l} \eta=\Sigma\alpha S\alpha\varpi\alpha \\ \sigma=\Sigma\alpha S\beta\varpi\gamma \\ \tau=\Sigma\alpha S\gamma\varpi\beta \end{array}\right. \quad\Bigg|\quad \begin{array}{l} \eta_1=\Sigma\alpha S\alpha\varpi^2\alpha \\ \sigma_1=\Sigma\alpha S\beta\varpi^2\gamma \\ \tau_1=\Sigma\alpha S\gamma\varpi^2\beta \end{array}$$

The first three give:

$$(13)\qquad \left\{\begin{array}{lll} S\alpha\eta=-S\alpha\varpi\alpha\,, & S\beta\eta=-S\beta\varpi\beta\,, & S\gamma\eta=-S\gamma\varpi\gamma \\ S\alpha\sigma=-S\beta\varpi\gamma\,, & S\beta\sigma=-S\gamma\varpi\alpha\,, & S\gamma\sigma=-S\alpha\varpi\beta \\ S\alpha\tau=-S\gamma\varpi\beta\,, & S\beta\tau=-S\alpha\varpi\gamma\,, & S\gamma\tau=-S\beta\varpi\alpha\,, \end{array}\right.$$

and as we have generally,

$$\varpi(\omega)=-\Sigma\alpha S\alpha\varpi(\omega)\,,$$

so we get by the preceding table:

$$(14)\qquad \left\{\begin{array}{l} \varpi\alpha = \alpha S\alpha\eta + \beta S\gamma\tau + \gamma S\beta\sigma \\ \varpi\beta = \beta S\beta\eta + \gamma S\alpha\tau + \alpha S\gamma\sigma \\ \varpi\gamma = \gamma S\gamma\eta + \alpha S\beta\tau + \beta S\alpha\sigma\,, \end{array}\right.$$

these three expressions may also be deduced, each from the preceding, by the circular permutation of $\alpha\beta\gamma\alpha\beta$, &c., into $\beta\gamma\alpha\beta\gamma$, &c., and into $\gamma\alpha\beta\gamma\alpha$, &c.

With these expressions of $\varpi\alpha$, &c., we calculate $2P$, $3Q$, $2P^2$, &c.

By (9) and by (14) we have, namely:

$$2P=\Sigma S\alpha\varpi(\varpi\alpha) = \Sigma S\alpha[\varpi\alpha S\alpha\eta+\varpi\beta S\gamma\tau+\varpi\gamma S\beta\sigma]\,.$$

But by the table of values (13), this becomes

$$2P=\Sigma[-S^2\alpha\eta-S\gamma\sigma S\gamma\tau-S\beta\tau S\beta\sigma]\,.$$

Applying the general formula:

$$S\theta\theta'=-\Sigma S\alpha\theta S\alpha\theta'\,,$$

and remarking that the two last sums in the expression P are one and the same, as by circular permutation each is composed of the identical same terms, we get

$$(15)\qquad 2P=\eta^2+2S\sigma\tau\,.$$

Then by (10) and (14) we have:

$$3Q=\Sigma S\alpha\varpi^2(\varpi\alpha) = \Sigma S\alpha[\varpi^2\alpha S\alpha\eta+\varpi^2\beta S\gamma\tau+\varpi^2\gamma S\beta\sigma]\,,$$

and forming with the expressions of η_1, σ_1, τ_1, a table of values similar to (13), we get:

$$3Q=\Sigma[-S\alpha\eta_1 S\alpha\eta-S\gamma\sigma_1 S\gamma\tau-S\beta\tau_1 S\beta\sigma]$$

which gives

$$3\,Q=S\eta\eta_1+S\sigma_1\tau+S\tau_1\sigma\,. \tag{16}$$

Finally, (11) gives :

$$-2\,P^2=\Sigma S\alpha\varpi^2(\varpi^2\alpha)=-\Sigma S\alpha\varpi^2[\alpha S\alpha\varpi^2\alpha+\beta S\beta\varpi^2\alpha+\gamma S\gamma\varpi^2\alpha]$$

$$=-\Sigma[S\alpha\eta_1 S\alpha\eta_1+S\gamma\sigma_1 S\gamma\tau_1+S\beta\tau_1 S\beta\sigma_1]$$

$$-2P^2=\eta_1^2+2S\sigma_1\tau_1\,. \tag{17}$$

---

§ 3. We consider now the expression

$$\Gamma=(g_1-g_2)^2(g_3-g_1)^2(g_2-g_3)^2 \tag{18}$$

$g_1,\ g_2,\ g_3,$ being the three roots of the equation $f(g)=0$; and if we put

$$g_1=m_3+\mathfrak{h}_1,\ g_2=m_3+\mathfrak{h}_2,\ g_3=m_3+\mathfrak{h}_3,$$

$\mathfrak{h}_1,\ \mathfrak{h}_2,\ \mathfrak{h}_3,$ becoming thus the roots of the equation $\mathscr{F}(\mathfrak{h})=0$, we will have $g_1-g_2=\mathfrak{h}_1-\mathfrak{h}_2$, &c., and consequently we have also

$$\Gamma=(\mathfrak{h}_1-\mathfrak{h}_2)^2(\mathfrak{h}_3-\mathfrak{h}_1)^2(\mathfrak{h}_2-\mathfrak{h}_3)^2.$$

Now we borrow from algebra the knowledge that we have

$$\Gamma=-4\,P^3-27\,Q^2, \tag{19}$$

and we will now express Γ by the help of P, Q, and their transformations (15), (16), (17).

For this we put Γ under the form

$$\Gamma=(2P)\,(-2P^2)-3(3Q)^2, \tag{20}$$

and we get :

$$\Gamma=(\eta^2+2S\sigma\tau)\,(\eta_1^2+2S\sigma_1\tau_1)-3S^2(\eta\eta_1+\sigma_1\tau+\tau_1\sigma)\,.$$

Here we meet with a difficulty for further transformation, because the factor 3 is not in evidence in the first term of the second member.

We will not lengthen this already long deduction by indicating step by step the circuitous route which has led us to the discovery of a vector κ whose introduction in the stead of η_1, according to the relation :

$$\kappa=\eta_1-\tfrac{2}{3}P\delta, \tag{21}$$

where

$$\delta=\alpha+\beta+\gamma\,, \tag{22}$$

has enabled us to put in evidence the factor 3 in the first term.

[Suffice it to say, in the following short digression, that it was by transforming $f(\phi)=0$, in introducing the function

$$\xi=\frac{df(\phi)}{d\phi}=\frac{\mathscr{F}(\varpi)}{d\varpi}=3\varpi^2+\mathrm{P}\,,$$

and forming the operator

$$-27\,\mathscr{F}\{\varpi\,\mathscr{F}(-\varpi)\}=\mathfrak{f}\xi=0\,,$$

and finding

$$\mathfrak{f}\xi=\xi^3+3\mathrm{P}\xi^2-(4\mathrm{P}^3+27\,\mathrm{Q}^2)\,,$$

and then, from $\mathfrak{f}\xi=0$, deducing: $\Gamma\Sigma \mathrm{S}\alpha\xi^{-2}\alpha=6\mathrm{P}$, which gives

$$6\mathrm{P}\Gamma=\Sigma \mathrm{S}\Omega(\alpha)\Omega'(\alpha)\,,$$

where Ω represents $\xi^2+3\mathrm{P}\xi=\Gamma\xi^{-1}$, $\Omega'=$ &c., and defining

$$\eta_{11}=\Sigma\alpha \mathrm{S}\alpha\Omega\alpha\,,\quad \sigma_{11}=\Sigma\alpha \mathrm{S}\beta\Omega\gamma$$
$$\tau_{11}=\Sigma\alpha \mathrm{S}\gamma\Omega\alpha\,,$$

and finding by the help of $\mathscr{F}\varpi=0$:

$$\begin{aligned}\eta_{11}&=6\mathrm{P}\eta_1-9\mathrm{Q}\eta-4\mathrm{P}^2\delta\\ \sigma_{11}&=6\mathrm{P}\sigma_1-9\mathrm{Q}\sigma\\ \tau_{11}&=6\mathrm{P}\tau_1-9\mathrm{Q}\tau\,,\end{aligned}$$

I was led to introduce the vector κ, defined as above, in order to give to η_{11} the same form as that of σ_{11}, τ_{11}.

We will add the remark that the operator $\mathfrak{f}(\xi)=0$, when translated into an algebraic operation on $x=3\mathfrak{h}^2+\mathrm{P}$, $\mathfrak{h}$ being one of the roots of $\mathscr{F}(\mathfrak{h})=0$, provides us with a demonstration of the equality between the two expressions (18) and (19) of Γ, because the roots of $\mathfrak{f}(x)=0$ are: $x_1=\mathscr{F}'\mathfrak{h}_1=f'g_1$, $x_2=$, &c., namely:

$$-(g_1-g_2)(g_3-g_1),\ -(g_2-g_3)(g_1-g_2),\ -(g_3-g_1)(g_2-g_3)\,;$$

thus far our digression.]

Let us express η_1^2 and $\mathrm{S}\eta\eta_1$ in function of κ and η in the expressions (16), (17) of $-2\mathrm{P}^2$ and $3\mathrm{Q}$, before entering these latter quantities into (20).

The expression (21) gives us $\eta_1=\kappa+\frac{2}{3}\mathrm{P}\delta$,

squaring this we get

$$\eta_1^2=\kappa^2+\frac{4}{3}\mathrm{PS}\kappa\delta+\frac{4}{9}\mathrm{P}^2\delta^2\,.$$

As to $\mathrm{S}\kappa\delta$, we treat (21) by $\mathrm{S}\delta\,(\ )$; this gives:

$$S\kappa\delta=S\eta_1\delta-\tfrac{2}{3}P\delta^2 .$$

Now the expression (9) of P may be conceived under the form

$$2P=-\Sigma S\alpha\delta S\alpha\varpi^2\alpha ,$$

because we have by $\delta=\alpha+\beta+\gamma$:

$$S\alpha\delta=S\beta\delta=S\gamma\delta=-1 ,$$

and by the definition of η_1 we have

$$S\alpha\varpi^2\alpha=-S\alpha\eta_1 , \text{ \&c. Thus}$$

$$2P=+\Sigma S\alpha\delta S\alpha\eta_1=-S\eta_1\delta ,$$

and the first term of $S\kappa\delta$, namely

$$S\eta_1\delta=-2P .$$

Then as to δ^2, we have $\delta^2=\alpha^2+\beta^2+\gamma^2=-3$. This gives:

$$S\kappa\delta=-2P+2P=0 .$$

There remains for η_1^2:

$$\eta_1^2=\kappa^2-\tfrac{4}{3}P^2 .$$

Substituting this into the expression (17) we get:

$$-2P^2=\kappa^2-\tfrac{4}{3}P^2+2S\sigma_1\tau_1 ,$$

from which we draw,

$$(23)\qquad -2P^2=3(\kappa^2+2S\sigma_1\tau_1) .$$

Likewise, in treating (21) by $S\eta(\ )$, we get:

$$S\eta\eta_1=S\eta\kappa ,$$

because $S\eta\delta$ vanishes, being nothing but: $-\Sigma S\alpha\delta S\alpha\eta=+\Sigma S\alpha\varpi\alpha=\Sigma S\alpha\varpi'\alpha$, and this latter result is equal to zero by (8).

We have thus:

$$(25)\qquad 3Q=S(\eta\kappa+\sigma_1\tau+\tau_1\sigma) .$$

Substituting the expressions (15), (23), (25), into (20), divided by 3, we get:

$$(26)\qquad \tfrac{1}{3}\Gamma=(\eta^2+2S\sigma\tau)(\kappa^2+2S\sigma_1\tau_1)-S^2(\eta\kappa+\sigma_1\tau+\tau_1\sigma) .$$

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§ 4. If there was equality respectively between τ and σ, and between τ_1 and σ_1, the decomposition of Γ into a sum of squares would be possible at once. We therefore elicit the part of Γ responding to that hypothesis, in introducing into the place of σ, τ, σ_1, τ_1, the vectors defined by:

$$(27)\qquad \begin{cases} \sigma=\zeta+\epsilon , & \sigma_1=\zeta_1+\epsilon_1 \\ \tau=\zeta-\epsilon , & \tau_1=\zeta_1-\epsilon_1 . \end{cases}$$

The vector ϵ which we introduce here is the same as that which enters into the definition of $\phi-\phi'$.

$$(28)\qquad (\phi-\phi')(\ \ )=2V\epsilon(\ \ ),$$

because by (12) we have

$$\sigma-\tau=\Sigma\alpha S\beta(\varpi\gamma-\varpi'\gamma),$$

and as $\varpi=\phi-m_3$, $\varpi'=\phi'-m_3$ and $S\beta\gamma=0$, &c., we have by (28):

$$\sigma-\tau=\Sigma\alpha S\beta 2V\epsilon\gamma=-2\Sigma\alpha S\alpha\epsilon=+2\epsilon,$$

which is in accordance with (27).

We may at once also establish that $\epsilon_1=-\varpi\epsilon$, because, by (27) and (12) we have $2\epsilon_1=\sigma_1-\tau_1=\Sigma\alpha S(\beta\varpi^2\gamma-\gamma\varpi^2\beta)=\Sigma\alpha S(\varpi'\beta\varpi\gamma-\varpi'\gamma\varpi\beta)$, and replacing $\varpi'\beta$, $\varpi'\gamma$ respectively by $\varpi\beta-2V\epsilon\beta$, $\varpi\gamma-2V\epsilon\gamma$, we get:

$$2\epsilon_1=\Sigma\alpha S[(\varpi\beta-2V\epsilon\beta)\varpi\gamma-(\varpi\gamma-2V\epsilon\gamma)\varpi\beta]=-2\Sigma\alpha S\epsilon V(\beta\varpi\gamma-\gamma\varpi\beta).$$

We invoke here a general theorem, whose demonstration is easy, namely, that for any integer value of the number n, and for any linear vector function ϕ we have

$$(29)\qquad V(\theta\phi^n\theta'-\theta'\phi^n\theta)=-[\Sigma S\alpha\phi^n\alpha+\phi'^n]V\theta\theta^1.$$

In the case of ϖ, and for $n=1$, as by (8) we have $\Sigma S\alpha\varpi'\alpha=\Sigma S\alpha\varpi\alpha=0$, we get simply

$$V(\beta\varpi\gamma-\gamma\varpi\beta)=-\varpi'\alpha,$$

with two other similar results by permutation of α, β, γ, in circular order.

Thus we have, suppressing the factor 2:

$$(30)\qquad \epsilon_1=\Sigma\alpha S\epsilon\varpi'\alpha=\Sigma\alpha S\alpha\varpi\epsilon=-\varpi\epsilon.$$

We introduce now the vectors ζ, ϵ, ζ_1, ϵ_1, into the expressions (15), (23), (25), and because (27) gives us:

$$S\sigma\tau=\zeta^2-\epsilon^2$$
$$S\sigma_1\tau\zeta_1={}_1^2-\epsilon_1^2$$
$$S(\sigma_1\tau+\tau_1\sigma)=2S\zeta\zeta_1-2S\epsilon\epsilon_1$$

we have:

$$(31)\qquad \left\{\begin{aligned} 2P&=\eta^2+2\zeta^2-2\epsilon^2\\ -\frac{2}{3}P^2&=\kappa^2+2\zeta_1^2-2\epsilon_1^2\\ 3Q&=S(\eta\kappa+2\zeta\zeta_1-2\epsilon\epsilon_1). \end{aligned}\right.$$

§ 5. We might now substitute these values into the expression (20) of Γ, and develop. But, as the terms depending *explicitly* on ϵ, ϵ_1, in the development, would form a part the true sign of which could not be decided upon as a whole, we will put in evidence in all the terms those parts which depend on ϵ also *implicitly*, and then only make the development.

Let P' and Q' represent the values of P and Q which correspond to $\epsilon=0$, and let us put:

$$(32)\qquad \begin{cases} P = P' + P'' \\ Q = Q' + Q''. \end{cases}$$

First, as to η and ζ, they do not depend on ϵ. Because if we decompose ϖ and ϖ' into their self-conjugate part $\bar{\varpi}$ and a vector depending on ϵ, namely putting:

$$(33)\qquad \left\{\begin{matrix} \varpi = \bar{\varpi} + V\epsilon(\ \ ) \\ \varpi' = \bar{\varpi} - V\epsilon(\ \ ) \end{matrix}\right\} \qquad \text{with} \qquad S\theta\bar{\varpi}\theta' = S\theta'\bar{\varpi}\theta ,$$

we get by (12) and (27):

$$(34)\qquad \begin{cases} \eta = \Sigma\alpha S\alpha\bar{\varpi}\alpha \\ \zeta = \frac{1}{2}\Sigma\alpha S(\beta\bar{\varpi}\gamma + \gamma\bar{\varpi}\beta) = \Sigma\alpha S\beta\bar{\varpi}\gamma , \end{cases}$$

the terms in ϵ disappear, being

$$\Sigma\alpha S\alpha V\epsilon\alpha = 0 , \quad \Sigma\alpha S(\beta V\epsilon\gamma + \gamma V\epsilon\beta) = 0 .$$

For the partition of κ and ζ_1 we put

$$\eta_1 = \eta_1' + \eta_1'' ; \qquad \kappa = \kappa' + \kappa'' \qquad \text{and then} \qquad \begin{cases} \kappa' = \eta_1' - \frac{2}{3}P'\delta \\ \kappa'' = \eta_1'' - \frac{2}{3}P''\delta . \end{cases}$$

Now, by the expression (12) of η_1, and because:

$$\varpi^2 = (\bar{\varpi} + V\epsilon)^2 = \bar{\varpi}^2 + \bar{\varpi}V\epsilon + V\epsilon\bar{\varpi} + V\epsilon V\epsilon ,$$

we have

$$(35)\qquad \begin{aligned} \eta_1' &= \Sigma\alpha S\alpha\bar{\varpi}^2\alpha \\ \eta_1'' &= \Sigma\alpha S\alpha[\bar{\varpi}V\epsilon\alpha + V\epsilon\bar{\varpi}\alpha + V\epsilon V\epsilon\alpha]. \end{aligned}$$

But $S\alpha[\bar{\varpi}V\epsilon\alpha + V\epsilon\bar{\varpi}\alpha] = S\epsilon[V\alpha\bar{\varpi}\alpha + V\bar{\varpi}\alpha.\alpha]$, which is zero. Then

$$S\alpha V\epsilon V\epsilon\alpha = -V^2\epsilon\alpha = -\epsilon^2 - S^2\epsilon\alpha .$$

Thus:

$$\eta_1'' = \Sigma\alpha[-\epsilon^2 - S^2\epsilon\alpha] = -\epsilon^2\delta - \Sigma\alpha S^2\epsilon\alpha .$$

As to $P = \frac{1}{2}\eta^2 + \zeta^2 - \epsilon^2$, the partition is made. We have

$$P' = \frac{1}{2}\eta^2 + \zeta^2 , \qquad P'' = -\epsilon^2 .$$

Thus: (35 *bis*)

$$\begin{aligned} \kappa' &= \Sigma\alpha\, S\alpha\bar{\varpi}^2\alpha - \frac{2}{3}P'\delta \\ \kappa'' &= -\epsilon^2\delta - \Sigma\alpha S^2\epsilon\alpha + \frac{2}{3}\epsilon^2\delta , \end{aligned}$$

namely:

$$(36) \qquad \kappa''=-\frac{1}{3}\epsilon^2\delta-\Sigma\alpha S^2\alpha\epsilon.$$

Then putting $\zeta_1=\zeta_1'+\zeta_1''$, we have by (12) and by (34):

$$(37) \qquad \zeta_1'=\frac{1}{2}\Sigma\alpha S(\beta\bar{\varpi}^2\gamma+\gamma\bar{\varpi}^2\beta)=\Sigma\alpha S\beta\bar{\varpi}^2\gamma,$$

$$\zeta_1''=\frac{1}{2}\Sigma\alpha\left\{\begin{array}{c} S(\beta\bar{\varpi}V\epsilon\gamma+\gamma\bar{\varpi}V\epsilon\beta) \\ +S(\beta V\epsilon\bar{\varpi}\gamma+\gamma V\epsilon\bar{\varpi}\beta)\end{array}\right\}+\frac{1}{2}\Sigma\alpha S(\beta V\epsilon V\epsilon\gamma+\gamma V\epsilon V\epsilon\beta).$$

Now the first sum Σ of the 2d member is zero, because in another order the terms contain:

$$S\epsilon\left[V(\gamma\bar{\varpi}\beta+\bar{\varpi}\beta\,.\,\gamma)+V(\beta\bar{\varpi}\gamma+\bar{\varpi}\gamma\beta),\right.$$

which is identically zero.

The second sum Σ contains $S\beta V\epsilon V\epsilon\gamma=S\gamma V\epsilon V\epsilon\beta=-SV\epsilon\beta V\epsilon\gamma=-S\epsilon\beta S\epsilon\gamma$. Thus we get:

$$(38) \qquad \zeta_1''=-\Sigma\alpha S\beta\epsilon S\gamma\epsilon.$$

Now we have

$$3Q = 3Q'+3Q''=S\left[\eta(\kappa'+\kappa'')+2\zeta(\zeta_1'+\zeta_1'')-2\epsilon\epsilon_1\right]$$

thus:

$$\left\{\begin{array}{l} 3Q'=S(\eta\kappa'+2\zeta\zeta_1') \\ 3Q''=S(\eta\kappa''+2\zeta\zeta_1''-2\epsilon\epsilon_1).\end{array}\right.$$

By (36) we have

$$S\eta\kappa''=-\frac{1}{3}\epsilon^2 S\eta\delta-\Sigma S\eta\alpha S^2\alpha\epsilon.$$

We have already shown that $S\eta\delta$ is zero, being $=-\Sigma S\alpha\bar{\varpi}\alpha=0$, by (8). Thus, as $S\eta\alpha=S\alpha\varpi\alpha$, &c., we have:

$$S\eta\kappa''=+\Sigma S\alpha\bar{\varpi}\alpha S^2\alpha\epsilon.$$

Then, by (38): $2S\zeta\zeta_1''==2\Sigma S\alpha\zeta S\beta\epsilon S\gamma\epsilon$; and because by definition (27) and (12):

$$2S\alpha\zeta = S\alpha\sigma+S\alpha\tau=-S(\beta\bar{\varpi}\gamma+\gamma\bar{\varpi}\beta), \text{ we have}$$

$$2S\zeta\zeta_1''=+\Sigma S\beta\bar{\varpi}\gamma S\beta\epsilon S\gamma\epsilon+\Sigma S\gamma\bar{\varpi}\beta S\beta\epsilon S\gamma\epsilon.$$

Now, in the general terms of the two sums, we may operate a circular permutation of α, β, γ, in the first sum one step forward, in the second sum two steps forward (or one backward), so that

$$2S\zeta\zeta_1''=\Sigma S\gamma\bar{\varpi}\alpha S\gamma\epsilon S\alpha\epsilon+\Sigma S\beta\bar{\varpi}\alpha S\alpha\epsilon S\beta\epsilon.$$

Thus we get

$$\begin{aligned} S(\eta\kappa''+2\zeta\zeta_1'') &= \Sigma S[\alpha S\alpha\epsilon+\beta S\beta\epsilon+\gamma S\gamma\epsilon]\bar{\varpi}\alpha S\alpha\epsilon \\ &= -\Sigma S\epsilon\bar{\varpi}\alpha S\alpha\epsilon = -S\epsilon\bar{\varpi}\Sigma\alpha S\alpha\epsilon \\ &= -S\epsilon\bar{\varpi}(-\epsilon) = +S\epsilon\bar{\varpi}\epsilon\,. \end{aligned}$$

On the other hand, remembering $\epsilon_1=-\varpi\varepsilon$, we have

$$-2\,S\epsilon\epsilon_1=+2S\epsilon\varpi\epsilon,$$

and as $\varpi\epsilon$ and $\bar{\varpi}\epsilon$ are identical, because

$$\varpi\epsilon=\bar{\varpi}\epsilon+V\epsilon^2=\bar{\varpi}\epsilon\,,$$

we have finally owing to (30):

$$\begin{aligned} 3Q'' &= S\epsilon\bar{\varpi}\epsilon+2S\epsilon\varpi\epsilon=3S\epsilon\bar{\varpi}\epsilon \\ Q'' &= S\epsilon\bar{\varpi}\epsilon\,.^* \end{aligned}$$

Then also if we partition $-\frac{2}{3}P^2$ we have

$$-\frac{2}{3}P^2=-\left(\frac{3}{2}P'-\epsilon^2\right)^2.$$

The first term of the square will be $-\frac{2}{3}P'^2$, namely what $-\frac{2}{3}P^2$ becomes when we replace κ^2 by κ'^2, ζ_1^2 by $\zeta_1'^2$ in the expression (31) of $-\frac{2}{3}P^2$. Thus:

$$-\frac{2}{3}P'^2=\kappa'^2+2\zeta_1'^2,$$

and we put

$$-\frac{2}{3}P^2=-\frac{2}{3}P'^2+P''',$$

the explicit expression of P''' is not wanted in the following.

We have now, by recapitulation:

$$(39)\qquad \left\{\begin{array}{l} P=P'-\epsilon^2 \\ \\ Q=Q'+S\epsilon\bar{\varpi}\epsilon \end{array}\right. \quad \begin{array}{l} \left\{\begin{array}{l} 2P'=\eta^2+2\zeta^2 \\ -\frac{2}{3}P'^2=\kappa'^2+2\zeta_1' \end{array}\right. \\ \{\ 3Q'=S(\eta\kappa'+2\zeta\zeta_1')\,. \end{array}$$

§ 6. We now call Γ' what Γ becomes when we suppose in it $\epsilon=0$. Then, introducing this supposition into the expression (20) and dividing by 3 we get:

$$(40)\qquad \frac{1}{3}\Gamma'=(2P')\left(-\frac{2}{3}P'^2\right)-(3Q')^2$$

and substituting the preceding values we get:

$$(40\ bis)\qquad \frac{1}{3}\Gamma'=(\eta^2+2\zeta^2)(\kappa'^2+2\zeta_1'^2)-S^2(\eta\kappa'+2\zeta\zeta_1)$$

---

* A direct treatment of the expression (8) of Q will give $Q''=S\epsilon\bar{\varpi}\epsilon$ by a somewhat shorter process.

We might now be induced to consider the quaternion

$$\eta\kappa'+2\zeta\zeta_1',$$

but as it enters here only by its scalar we may also consider the other quaternion (not precisely a conjugate to the first), namely

$$q=\eta\kappa'+2\zeta_1'\zeta.$$

We have then the conjugate

$$\mathrm{K}q=\kappa'\eta+2\zeta\zeta_1'.$$

From this:

$$q\mathrm{K}q=(\eta\kappa'+2\zeta_1'\zeta)(\kappa'\eta+2\zeta\zeta_1')$$

$$\mathrm{S}^2q-\mathrm{V}^2q=\eta^2\kappa'^2+4\zeta^2\zeta_1'^2+2[\eta\kappa'\zeta\zeta_1'+\zeta_1'\zeta\kappa'\eta],$$

and as the vectors of the last two products are equal, but opposed in sign, we have

$$\eta\kappa'\zeta\zeta_1'+\zeta_1'\zeta\kappa'\eta=2\mathrm{S}\eta\kappa'\zeta\zeta_1'.$$

Then we draw from the preceding

$$\eta^2\kappa'^2+4\zeta^2\zeta_1'^2-\mathrm{S}^2q=-\mathrm{V}^2q-4\mathrm{S}\eta\kappa'\zeta\zeta_1'.$$

On the other hand, by developing $\frac{1}{3}\Gamma'$ we get

$$\frac{1}{3}\Gamma'=\eta^2\kappa'^2+4\zeta^2\zeta_1'-\mathrm{S}^2q+2[\zeta^2\kappa'^2+\eta^2\zeta_1'^2].$$

Substituting for the first three terms the prepared value we get

$$\frac{1}{3}\Gamma'=\mathrm{T}^2\mathrm{V}q+2[\zeta^2\kappa'^2+\eta^2\zeta_1'^2-2\mathrm{S}\eta\kappa'\zeta\zeta_1'].$$

Now

$$-2\mathrm{S}\eta\kappa'\zeta\zeta_1'=-2\mathrm{S}\eta[\kappa'\mathrm{S}\zeta\zeta_1'-\zeta\mathrm{S}\zeta_1'\kappa'+\zeta_1'\mathrm{S}\kappa'\zeta],$$

and the two first terms in the 2d member give

$$[-2\mathrm{S}\eta\kappa'\mathrm{S}\zeta\zeta_1'+2\mathrm{S}\eta\zeta\mathrm{S}\zeta_1'\kappa']=2\mathrm{S}\mathrm{V}\eta\zeta_1'\mathrm{V}\kappa'\zeta.$$

Replacing also $\zeta^2\kappa'^2+\eta^2\zeta_1'^2$ by

$$\mathrm{S}^2\kappa'\zeta-\mathrm{V}^2\kappa'\zeta+\mathrm{S}^2\eta\zeta_1'-\mathrm{V}^2\eta\zeta_1',$$

we see that the second term in the value of $\frac{1}{3}\Gamma'$ becomes:

$$2\left[\begin{array}{l}\mathrm{S}^2\kappa'\zeta-2\mathrm{S}\kappa'\zeta\mathrm{S}\eta\zeta_1'\quad+\mathrm{S}^2\eta\zeta_1'\\-\mathrm{V}^2\kappa'\zeta+2\mathrm{S}\mathrm{V}\kappa'\zeta\mathrm{V}\eta\zeta_1'-\mathrm{V}^2\eta\zeta_1'\end{array}\right],$$

which is

$$2[\mathrm{S}^2(\kappa'\zeta-\eta\zeta_1')-\mathrm{V}^2(\kappa'\zeta-\eta\zeta_1')=2\mathrm{T}^2(\kappa'\zeta-\eta\zeta_1')$$

Replacing now q, $=\eta\kappa'+2\zeta_1'\zeta$, and as under the sign V we have:

$$\mathrm{V}q=\mathrm{V}(\eta\kappa'-2\zeta\zeta_1'),$$

we get finally:

$$\frac{1}{3}\Gamma'=T^2V(\eta\kappa'-2\zeta\zeta_1')+2T^2(\kappa'\zeta-\eta\zeta_1'). \tag{41}$$

This result is *essentially positive.* Of course its use is only theoretical; there is no question of expanding it again by making use of the properties of the symbols T and V, &c.; that operation would lead to other expressions, and there can be no doubt that the expressions of $\frac{1}{3}\Gamma'$ which have been established by other methods (as for example, G. BAUER's, "Crelle," vol. lxxi.) might be established with the help of formula (41).

The value of Γ' being that of Γ for the case when $\epsilon=0$, we have therefore a demonstration, *by the quaternion method,* of the theorem, that when in the expression of the scalar

$$S(\phi-g)\alpha(\phi-g)\beta(\phi-g)\gamma = fg,$$

we replace ϕ by its self-conjugate part $\bar{\phi}$, then the equation

$$S(\bar{\phi}-g')\alpha(\bar{\phi}-g')\beta(\bar{\phi}-g')\gamma=f_1g'=0$$

is satisfied by *three real* roots g'.

---

§ 7. Let us now return to the originary question, the condition namely that Γ should be positive, and limit the question in the following manner:

Let us suppose that the elements of the self-conjugate part of ϕ, and consequently of ϖ, remain invariable, and given by the vectors η, ζ, κ', ζ_1' as defined in (34), and let us admit that the elements of ϵ, as defined by (27) or (33), namely by:

$$\epsilon=\frac{1}{2}\Sigma\alpha S\beta(\varpi\gamma-\varpi'\gamma)$$

are alone variable, $\varpi\epsilon$ varying in consequence.

Then under this supposition, let us determine the limits of the values of ϵ which satisfy to the condition

$$\Gamma>0.$$

The question will be solved when we can assign a surface limiting the space for the one side of which all points, having ϵ as vector, will give to ϵ, and consequently to Γ, a value satisfying the above inequality.

The equation of the surface will be

$$\Gamma=0, \qquad \text{or} \quad 4P^3+27Q^2=0, \tag{42}$$

wherein ϵ (drawn from an origin, of course common for all points), will represent *now* the vector of a point of the surface. When the surface has been constructed then we will be able to show that it is the *inside* of the surface which comprises the points for which the vector ϵ (as primitively defined) satisfies to

$$\Gamma > 0 \, .$$

By the expressions (39) of P, Q, the equation of the surface will become:

$$0 = 4(P' - \epsilon^2)^3 + 27(Q' + S\epsilon\varpi\varepsilon)^2. \tag{42 bis}$$

We call e the tensor of ϵ, which satisfies to the equation, and ϵ' its unit vector, putting

$$e = T\epsilon \, , \; \epsilon' = U\epsilon \, . \tag{43}$$

Then we have $S\epsilon\bar{\varpi}\epsilon = e^2 S\epsilon'\varpi\epsilon'$ and we put

$$E = -S\epsilon'\varpi\varepsilon' \, , \tag{44}$$

and generally we put

$$Fe^2 = -\frac{1}{4}\Gamma = P^3 + \frac{27}{4}Q^2 \, , \tag{45}$$

so that we have

$$F(e^2) = (e^2 + P')^3 + \frac{27}{4}(Ee^2 - Q')^2 \, . \tag{46}$$

Developing we get

$$F(e^2) = e^6 + (3P' + \frac{27}{4}E^2)e^4 + (3P'^2 - \frac{27}{2}Q'E')e^2 + (P'^3 + \frac{27}{4}Q'^2) \, .$$

The equation $F(e^2) = 0$, being of the 3d degree in e^2, assigns to e^2 at least one real value, and this value is positive because the last term is

$$P'^3 + \frac{27}{4}Q'^2 = -\frac{1}{4}\Gamma' ,$$

and we know by the preceding that $-\Gamma'$ is negative. Therefore there will be *at least one* real value for

$$T\epsilon = +\sqrt{e^2}$$

satisfying the equation, whatever be the value of $E = -S\epsilon'\bar{\varpi}\epsilon'$; and therefore the surface will have *at least one* point in every direction of ϵ', in fact in all possible directions.

Moreover, when $\epsilon = 0$ then Γ takes the value Γ' which is positive. It follows that the origin O of ϵ is one of the points for which the condition $\Gamma > 0$ is satisfied; and if we consider, for the present, the directions ϵ' for which the equation $F(e^2) = 0$ gives only *one positive root*, e^2, then we may conclude that all the points on those directions, from the origin O, up to the surface belong to the space for which $\Gamma > 0$ is satisfied.

The directions of ϵ' for which $Fe^2 = 0$ assigns *three positive values* to e^2 must now be determined in order to draw the corresponding conclusion in respect to the fulfilment of the condition, $\Gamma > 0$, in those directions.

§ 8. The question reduces itself to the determination of the limits between which E must be comprised, in order that $F(e^2)=0$ should have three positive roots.

For the solution of this question, we liken Fe^2 with

$$F(e^2)=e^6-3M_3e^4+M_1e^2-\frac{1}{4}\Gamma'$$

where
$$\begin{cases} M_3=-P'-\frac{9}{4}E^2 \\ M_1=3P'^2-\frac{27}{2}Q'E\,, \end{cases}$$

and calculating
$$X=F'(M_3)\;,\;\; Y=F(M_3)$$
$$\Delta=-4X^3-27Y^2,$$

we have to put the condition:

$$\Delta>0\,.$$

From (46) we deduce, by differentiation:

$$F'e^2=3(e^2+P')^2+\frac{27}{2}E(Ee^2-Q')\,,$$

This gives

$$X=\frac{3^5}{2^4}E^4-\frac{3^3}{2}E\left(\frac{}{2^2}E^3+EP'+Q'\right)$$

and by (46):

$$Y=-\frac{3^6}{2^6}E^6+\frac{3^3}{2^2}\left(\frac{3^2}{2^2}E^3+EP'+Q'\right)^2.$$

We introduce: $E^3=v\,,\; Q'+EP'=u\,,$ only for the moment.

Substituting and reducing, we get:

$$\begin{cases} X=-E\left[\frac{3^5}{2^4}v+\frac{3^3}{2}u\right] \\ Y=\frac{3^6}{2^5}v^2+\frac{3^5}{2^3}vu+\frac{3^3}{2^2}u^2 \end{cases}$$

Then as $E^3=v$, we deduce:

$$\Delta=2^2v\left[\frac{3^5}{2^4}v+\frac{3^3}{2}u\right]-3^3Y^2\,.$$

Developing and reducing, the terms depending on v^4, v^3u, v^2u^2, disappear, and there remains:

$$\Delta=-\frac{3^9}{2^4}u^3(v+u)\,.$$

But $v+u=E^3+P'E+Q'$; this is what the function $\mathscr{F}\mathfrak{h}=f(m^3+g)$, namely,

$$\mathscr{F}\mathfrak{h}=\mathfrak{h}^3+P\mathfrak{h}+Q\,,$$

becomes for $\epsilon=0$ and $\mathfrak{h}=E$. We put

(48) $$\mathscr{F}_1h=h^3+P'h+Q'\,:$$

Then we have:

$$(49)\qquad \Delta=-\frac{3^9}{2^4}(\mathrm{P}'\mathrm{E}+\mathrm{Q}')^3\cdot\mathscr{F}_1(\mathrm{E}).$$

The conditions that $\mathrm{F}(e^2)=0$ shall be satisfied by three *real* and *positive* roots e^2, and consequently that there should be three real values of e for a given value of E are thus:

$$(a)\qquad \mathrm{M}_3=-\mathrm{P}'-\frac{9}{4}\mathrm{E}^2>0$$

$$(b)\qquad \mathrm{M}_1=3\mathrm{P}'^2-\frac{27}{2}\mathrm{Q}'\mathrm{E}>0$$

$$(c)\qquad \frac{2^4}{3^9}\Delta=-(\mathrm{P}'\mathrm{E}+\mathrm{Q}')\mathscr{F}_1\mathrm{E}>0.$$

The discussion of these inequalities necessitates that we consider the roots of the equation

$$\mathscr{F}_1h=0.$$

We know that they are all three real by the very fact that $\Gamma'>0$.

Let h_1, h_2, h_3, be the three roots. The comparison of $\mathscr{F}_1(h)$ with its expression in function of the roots gives:

$$(50)\qquad \begin{cases}\Sigma h_1 = 0, & \text{namely,}\quad h_1+h_2+h_3=0\\ \Sigma h_1h_2=\mathrm{P}', & \text{,,}\quad h_1h_2+h_3h_1+h_2h_3=\mathrm{P}'\\ -h_1h_2h_3=\mathrm{Q}'. & \end{cases}$$

The first equality, $\Sigma h_1=0$, shows that one at least of the roots must be positive, and one at least must be negative. We fix the notation in accordance with the hypothesis:

$$h_1>h_2>h_3.$$

This comes to the admission $h_1>0$, $h_3<0$. As to the sign of h_2 it is the same as that of Q$'$, which is supposed a datum, because Q$'$ may be put under the form $\mathrm{Q}'=h_1(-h_3)\times h_2$ and $h_1(-h_3)$ is positive.

For the moment we put:

$$(50\ bis)\qquad h_2=h_1z,\quad\text{then}\quad h_3=-h_1(1+z).$$

Applying to these values the hypothesis $h_1>h_2>h_3$, we get for z the limits

$$1>z>-\frac{1}{2}.$$

As by squaring $\Sigma h_1=0$ we get

$$\Sigma h_1h_2=-\frac{1}{2}\Sigma h_1^2,\ \text{we have}$$

$$\mathrm{P}'=-\frac{1}{2}h_1[1+z^2+(1+z)^2],\ \text{namely}$$

$$(50\ ter)\qquad \left\{\begin{aligned} P' &= -h_1^2(1+z+z^2) \\ Q' &= h_1^3 z(1+z)\,. \end{aligned}\right.$$

Further we know that $\mathscr{F}_1 h=0$ is the resultant of the equation $(\varpi-h)\rho=0$ written for three directions λ, μ, ν, for which this equation is satisfied, or

$$V\lambda\varpi\lambda=0\,,\ V\mu\varpi\mu=0\,,\ V\nu\varpi\nu=0\,.$$

The unit vectors λ, μ, ν, being so defined, we know that they are treble rectangular, and we may suppose h_1, h_2, h_3 corresponding respectively to λ, μ, ν. Then we have:

$$\varpi\lambda=h_1\lambda\,,\ \varpi\mu=h_2\mu\,,\ \varpi\nu=h_3\nu\,;$$

and from this, supposing

$$\epsilon'=-\Sigma\lambda S\lambda\epsilon'\,,$$

we deduce

$$\varpi\epsilon'=-\Sigma h_1\lambda S\lambda\epsilon'\,.$$

And as $E=-S\epsilon'\varpi\epsilon'$, we get

$$(51)\qquad E=\Sigma h_1 S^2\lambda\epsilon'\,.$$

This expression, owing to $\Sigma S^2\lambda\epsilon'=1$, may be put under the form:

$$(51\ bis)\qquad \Sigma(h_1-E)S^2\lambda\epsilon'=0\,,$$

and it shows that the value of E must not be greater than h_1 nor smaller than h_3, lest all the three terms would be of the same sign.

Therefore we must always have

$$h_1 \overset{=}{>} E \overset{=}{>} h_3\,.$$

We have now prepared all that is required for the discussion of the three conditions (*a*), (*b*), (*c*). As to (*c*) we may write

$$\mathscr{F}_1 E=(h_1-E)(h_2-E)(E-h_3)\,,$$

and considering the limits of E just above we see that the product $(h_1-E)(E-h_3)$ will always be positive, and the sign of $\mathscr{F}_1 E$ will depend on that of (h_2-E). Thus (*c*) becomes

$$(c)\qquad (-P'E-Q')(h_2-E)>0\,.$$

Putting for $-P'$, Q' their expression in z, and suppressing the factor h_1^2, we get

$$[(1+z+z^2)E-zh_1(1+z)]\,[h_1 z-E]>0\,,$$

or as $1+z+z^2>0$:

$$(c)\qquad \left[E-h_1 z\left(\frac{1+z}{1+z+z^2}\right)\right][h_1 z-E]>0\,.$$

Now as: $-\frac{1}{2}<z<1$, we have $\frac{2}{3}<\frac{1+z}{1+z+z^2}<1$.

Therefore whatever be the sign of z, and consequently of h_2 and Q', the absolute values of E which satisfy (c) must be comprised between the absolute values of $h_1z=h_2$ and $h_1z\,\frac{1+z}{1+z+z^2}=\frac{Q'}{(-P')}$, and the former value will be the greater one. In fact, the limits are

$$(c\quad\begin{cases} 1^\circ)\quad \text{when}\quad h_2>0: & h_2>E>\frac{Q'}{-P'}>0\,,\\ 2^\circ)\quad \text{when}\quad h_2<0: & h_2<E<\frac{Q'}{-P'}<0\,.\end{cases}$$

The conditions (a), (b), have to be satisfied even when E^2 and $Q'E$, take their greatest value, namely h_2^2 and $h_2^2(-h_1h_3)$, both being positive. Namely, having in that case:

$$E^2=h_1^2z^2\,,\qquad Q'E=h_1^4z^2(1+z)\,,$$

the conditions (a), b), in virtue of (50 *ter*), after suppressing the factors h_1^2, h_1^4 respectively become

$$(a)\qquad 1+z+z^2-\frac{9}{4}z^2>0\,,$$

$$(b)\qquad (1+z+z^2)^2-\frac{9}{2}z^2(1+z)>0\,.$$

Introducing into (b) the quantity $w=\frac{1+z}{z^2}$, and decomposing $(w+1)^2-\frac{9}{4}w$ into the factors $=(w-2)(w-\frac{1}{2})$, and decomposing $w-2$, $w-\frac{1}{2}$ into factors depending on z, we get the conditions under the form:

$$(a)\qquad \frac{\sqrt{24}+2}{5}-z\Big)\left(\frac{\sqrt{24}-2}{5}+z\right)>0\,,$$

$$(b)\qquad (1-z)\left(\frac{1}{2}+z\right)(\sqrt{3}+1-z)(\sqrt{3}-1+z)>0\,,$$

and by approximate numbers into:

$$(a)\qquad (1{,}3798\ldots-z)\,(0{,}5798\ldots+z)>0\,,$$

$$(b)\qquad (1-z)\left(\frac{1}{2}+z\right)[2{,}732\ldots-z]\,[0{,}732\ldots+z]>0\,.$$

Now the limits of z give:

$$1-z\overset{=}{>}0\,,\quad \frac{1}{2}+z\overset{=}{>}0\,,$$

and the other factors likewise are positive in virtue of those limits. It follows that when condition (c) is satisfied by a certain value of E, the conditions (a), (b), will be fulfilled necessarily also, and the equation

$$F(e^2)=0$$

will give three real positive values for e^2 for those values of E comprised between the above limits of E, namely

$$\text{between} \quad h_2 \quad \text{and} \quad \frac{Q'}{-P'} = E_I,$$

which both are comprised between the general limits of E, namely, between h_1 and h_3.

---

§ 9. When we consider E as a parameter on which the values of e^2 depend by the equation $F(e^2)=0$, then the directions $\epsilon'=U\epsilon$, corresponding to a given value of E, will represent the generating lines of a cone of the 2d order, according to the formula (51 *bis*), in which E is supposed constant and ϵ' variable in direction.

For the extreme limits $E=h_1$ and $E=h_3$ the cone reduces itself to the straight lines λ and ν respectively.

For $E=h_2$, the cone reduces itself to two planes passing through μ, the angles n_0 of the normal to these planes with the axis ν, as for example, being

$$tg\ n_0 = \pm\frac{S\lambda\epsilon'}{S\nu\epsilon'} = \pm\sqrt{\frac{h_2-h_3}{h_1-h_2}}.$$

Let us designate these planes by R, R′.

When h_2 is positive, then the cone corresponding to $E = E_I$ will be comprised in the angles of R and R′ which contain the axes ν, and $-\nu$, because then we have

$$0 < E_I < h_2$$

and E beginning to grow from h_3 (which is negative) by degrees up to h_1, will reach E_I before it reaches h_2.

When h_2 is negative, then the cone corresponding to $E=E_I$ will be comprised in the angles of the above planes in which the axis λ and $-\lambda$ are situated, because then $h_2 < E_I < 0$.

For $E=E_I$ the equation of the corresponding cone takes the more simple form:

$$(51\ ter) \qquad 0 = \Sigma h_1^3 S^2\lambda\epsilon', \qquad \text{namely} \quad S\epsilon'\varpi^3\epsilon' = 0,$$

because we have

$$(h_1-E_I) = h_1\left(1 + \frac{Q'}{h_1P'}\right) = \frac{1}{P'}\,h_1(P'-h_2h_3).$$

and $\qquad P'-h_2h_3=h_1(h_2+h_3) = -h_1^2, \quad \therefore\ h_1-E_1=-h_1^3 : P', \ \&c.$

The equation $Fe^2=0$ assigns one or three real values to $e=T\epsilon$ for every value of E, that is, for every one of the cones just spoken of. It appears therefore that the surface $\Gamma=0$ is composed of a series of *spherical conics,* of which the radius of the sphere corresponding varies in function of E.

Between $E=h_2$, and $E=E_{\text{I}}$ the surface contains three curves on every cone, and therefore presents three sheets in the directions comprised between the cones corresponding to the adduced limits of E.

The curves corresponding to $E=h_2$ are circles, as they are situated in the planes R and R′ respectively.

The curve corresponding to $E=E_{\text{I}}$ and to the two equal roots e_{II} corresponding to it, presents a singular character, and we shall treat of it separately.

The calcul of the values of e for the five values of E following will give us the means of conceiving the outer shape of the surface.

Let us adopt the following designations, easily understood:

$E=h_3$	$E=0$	$E=E_{\text{I}}$	$E=h_2$	$E=h_1$
$e=e_\nu$	$e=e_0$	$e=e_{\text{I}}$	$e=e_\mu=e_1$	$e=e_\lambda$
		$=e_{\text{II}}$	$=e_2$	

The calcul of e_λ e_μ e_ν can be effected in one case, and the others be deduced by permutation. We have for $E=h_1$

$$0=(e_\lambda^2+P')^3+\frac{27}{4}(e_\lambda^2h_1-Q')^2.$$

Replacing P′, Q′ in function of the roots,

$$P'=\Sigma h_1h_2,\quad -Q'=h_1h_2h_3,$$

having

$$P=h_2h_3+h_1(h_2+h_3)=h_2h_3-h_1^2$$

we get

$$0=(e_\lambda^2+h_2h_3-h_1^2)^3+\frac{27}{4}h_1^2(e_\lambda^2+h_2h_3)^2,$$

we divide by $(e_\lambda^2+h_2h_3)^3$, and introduce

$$\frac{h_1^2}{e_\lambda^2+h_2h_3}=w.$$

This gives:

$$0=(1-w)^3+\frac{27}{4}w.$$

This equation has the root $w=4$ single, and $w=-\frac{1}{2}$ double, because the derivate $-3(1-w)^2+\frac{27}{4}$, is annulled by $w=-\frac{1}{2}$.

The single root gives:

$$e_\lambda^2 + h_2h_3 = \frac{1}{4}h_1^2 = \frac{1}{4}(h_2+h_3)^2,$$

therefore $$e_\lambda^2 = \frac{1}{4}(h_2-h_3)^2\,;$$

and always supposing

$$h_1 > h_2 > h_3\,,$$

we have the positive values:

$$e_\lambda = \frac{h_2-h_3}{2}\,,\quad e_\mu = \frac{h_1-h_3}{2}\,,\quad e_\nu = \frac{h_1-h_2}{2}\,.$$

The double root $w=-\frac{1}{2}$ cannot give us any real values for e corresponding to $E=h_1$, neither any for $E=h_3$, because we have seen *a priori* that there are no double values possible unless E be comprised between h_2 and E_1. Let us therefore take $E=h_2$ and $w=-\frac{1}{2}$. This gives:

$$\frac{h_2^2}{e_2^2+h_3h_1} = -\frac{1}{2}$$

$e_2^2+h_3h_1+2h_2^2=0$. Owing to $\Sigma h_1=0$, we have

$$\begin{aligned} h_3h_1+2h_2^2 &= h_3h_1+2(h_1^2+2h_1h_3+h_3^2)\,; \\ &= 2h_1^2+5h_1h_3+2h_3^2\,; \\ &= (2h_1+h_3)(h_1+2h_3) = -e_2^2\,. \end{aligned}$$

But $$2h_1+h_3=(h_1-h_2)\,,\quad h_1+2h_3=(h_3-h_2)\,.$$

Therefore

$$e_2 = \sqrt{(h_1-h_2)(h_2-h_3)}\,,\quad \text{which is real.}$$

We remark that owing to $h_1>h_2>h_3$ we have $e_2<e_1=\frac{1}{2}(h_1-h_3)$.

For $E=0$, the equation $Fe^2=0$ gives

$$0 = (e_0^2 + P')^3 + \frac{27}{4}Q'^2$$

$$e_0^2 = -P' - \left(\frac{27}{4}Q'^2\right)^{\frac{1}{3}}.$$

Now we have $4P'^3 + 27\,Q'^2 = -\Gamma'$, which gives $-P^3 = \frac{1}{4}\Gamma' + \frac{27}{4}Q'^2$, therefore

$$e_0 = \left[\left(\frac{1}{4}\Gamma' + \frac{27}{4}Q'^2\right)^{\frac{1}{3}} - \left(\frac{27}{4}Q'^2\right)^{\frac{1}{3}}\right]^{\frac{1}{2}},$$

which is real, the determinations of the fractionary powers are all supposed only numerical.

For $E = E_{\rm I} = \frac{Q'}{-P'}$, the equation gives:

$$0 = (e^2 + P')^3 + \frac{27}{4}\left(\frac{Q'}{-P'}e^2 - Q'\right)^2,$$

and the second term can be put under the form:

$$\frac{27}{4}\frac{Q'^2}{P'^2}(e^2 + P')^2.$$

Therefore we get

$$0 = (e^2 + P')^2\left[e^2 + P' + \frac{27\,Q'^2}{4\,P'^2}\right],$$

or

$$0 = (e^2 + P')^2\left[e^2 - \frac{\Gamma'}{4\,P'^2}\right].$$

We call $e_{\rm I}$ the single root, and $e_{\rm II}$ the double root for $E = E_{\rm I}$, and we have thus

$$e_{\rm I} = \frac{\sqrt{\Gamma'}}{-2\,P'}, \quad e_{\rm II} = \sqrt{-P'}.$$

It is clear that the double root corresponds to $P = 0$, $Q = 0$, because

$$P = e_{\rm II}^2 + P' = 0,$$

$$Q = -E_{\rm I}e_{\rm II}^2 + Q' = \frac{Q'}{P'}(-P') + Q' = 0.$$

We remark also that

$$e_{\rm II}^2 > e_{\rm I}^2, \text{ because } \Gamma' = -4\,P'^3 - 27\,Q'^2$$

gives

$$e_{\rm II}^2 = -P' = \frac{\Gamma'}{4\,P'^2} + \frac{27\,Q'^2}{4\,P'^2} > e_{\rm I}^2 = \frac{\Gamma'}{4\,P'^2}.$$

In fact $e_{\rm II}$ outpasses all other values of e given by $\Gamma = 0$.

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§ 10. We establish now the expression of the vector υ normal to the surface. For the differentiation we take up the originary expression (42) of Γ, and put it under the form:

$$\frac{\Gamma}{4 \times 27} = 0 = \left(\frac{P}{3}\right)^3 + \left(\frac{Q}{2}\right)^2$$

$$P = P' - \epsilon^2, \quad Q = Q' + S\epsilon\bar{\varpi}\epsilon.$$

Differentiating we get

$$0 = \left(\frac{P}{3}\right)^2 dP + \frac{Q}{2}\,dQ,$$

$$dP = -2\,S\epsilon d\epsilon, \quad dQ = 2\,S\bar{\varpi}\epsilon d\epsilon.$$

Therefore we have

$$0 = Sd\epsilon\left[-\epsilon\left(\frac{P}{3}\right)^2 + \varpi\epsilon\left(\frac{Q}{2}\right)\right].$$

By equation $\Gamma = 0$ there exists a relation between $\frac{P}{3}$ and $\frac{Q}{2}$. We will establish by definition

(52) $$\frac{Q}{2} = W^3, \quad \text{W having the sign of Q}.$$

Then $$\left(\frac{P}{3}\right)^3 + \left(\frac{Q}{2}\right)^2 = 0 \quad \text{gives}$$

(52 *bis*) $$-\left(\frac{P}{3}\right)^3 = W^6, \quad -\frac{P}{3} = W^2.$$

Thus the above scalar becomes:

$$0 = Sd\epsilon[-\epsilon W^4 + \varpi\epsilon W^3].$$

We now separate the factor W^3, reserving it in case it becomes = zero, and we have

$$0 = Sd\epsilon[-\epsilon W + \varpi\epsilon].$$

Thus the vector ν normal to the surface may be represented by

(53) $$\nu = (\varpi - W)\epsilon',$$

neglecting a scalar factor N, which we *might* suppose positive or negative when we wish ν to be directed always towards the *outside* of the surface. For the present we content ourselves with taking ν either towards the inside or towards the outside, as the formula will give it.

In looking upon e^2 as a function of the parameter E, we get the expression of $\frac{de}{dE}$ in differentiating, P and Q under the form:

$$P = P' + e^2, \quad Q = -e^2E + Q'$$

$$dP = \frac{de^2}{dE}\,dE, \quad dQ = -\left(\frac{de^2}{dE}\,E + e^2\right)dE,$$

and substituting into

$$0 = \left(\frac{P}{3}\right)^2 dP + \frac{Q}{2}\,dQ,$$

this gives us, reserving the factor W^3:

(54) $$\frac{de}{dE} = \frac{+\frac{1}{2}e}{W-E} = \frac{-\frac{1}{2}e}{E-W}.$$

If we designate by $\widehat{\epsilon' \upsilon}$ the angle between ϵ' and the normal, an angle which may vary from 0° to 180°, we have :

$$tg\,\widehat{\epsilon' \upsilon}=\frac{TV\epsilon'\upsilon}{-S\epsilon'\upsilon}=\frac{TV\epsilon'\bar{\varpi}\epsilon'}{E-W}.$$

So that

$$\frac{de}{aE}=\left(\frac{\frac{1}{2}e}{TV\epsilon'\varpi\epsilon'}\right)\left(-tg\,\widehat{\epsilon' \upsilon}\right).$$

We will calculate W, and E−W for the same values of E as before, and we easily get by

$$W=\left(\frac{Q}{2}\right)^{\frac{1}{3}}=\left(\frac{-Ee^2+Q'}{2}\right)^{\frac{1}{3}}=-\left(\frac{Ee^2+h_1h_2h_3}{2}\right)^{\frac{1}{3}}:$$

$$(55)\quad\left\{\begin{array}{l|l|l} \multicolumn{3}{l}{W_\lambda=-\frac{h_1}{2},\quad W_\mu=-\frac{h_2}{2},\quad W_\nu=-\frac{h_3}{2}} \\ W_{I}=\frac{3Q'}{-2P'} & W_1=W_\mu=-\frac{h_2}{2} & \\ & & W_0=\left(\frac{Q'}{2}\right)^{\frac{1}{3}} \\ W_{II}=0 & W_2=h & \end{array}\right.$$

This gives for υ, as $\varpi\lambda=h_1\lambda$, $\varpi\mu=h_2\mu$, $\varpi\nu=h_3\nu$

$$\left\{\begin{array}{l|l|l} \upsilon_\lambda=\frac{3}{2}h_1\lambda, & \upsilon_\mu=\frac{3}{2}h_2\mu, & \upsilon_\nu=\frac{3}{2}h_8\nu \\ \upsilon_{I}=\left(\bar{\varpi}+\frac{3Q'}{2P'}\right)\epsilon' & \upsilon_1=\left(\bar{\varpi}+\frac{h_2}{2}\right)\epsilon' & \\ & & \upsilon_0=\left[\bar{\varpi}-\left(\frac{Q'}{2}\right)^{\frac{1}{3}}\right]\epsilon \\ \upsilon_{II}=\bar{\varpi}\epsilon' & \upsilon_2=(\bar{\varpi}-h_2)\epsilon' & \end{array}\right.$$

Namely, these results are obtained as follows:

$$Q=Q'+S\epsilon\varpi\epsilon=-h_1h_2h_3-Ee^2$$

$$\text{For } E=h_1,\quad e^2=\left(\frac{h_2-h_3}{2}\right)^2$$

$$\left(Q\right)_\lambda=-h_1h_2h_3-h_1\left(\frac{h_2-h_3}{2}\right)^2$$

$$\left(\frac{Q}{2}\right)_\lambda=-\frac{h_1}{8}\left[4h_2h_3+(h_2^2-2h_2h_3+h_3^2)\right]$$

$$=-\frac{h_1}{8}\left(h_2+h_3\right)^2=-\frac{h_1^{\,3}}{8}$$

$$\left(\frac{Q}{2}\right)_\lambda = \left(-\frac{h_1}{2}\right)^3$$

$$W_\lambda = \left(\frac{Q}{2}\right)_\lambda^{\frac{1}{3}} = -\frac{h_1}{2}$$

Calul of W_2, by $W_2^3 = \left(\frac{Q}{2}\right)_2$, $E=h_2$

$$\begin{aligned} Q_2 &= -h_2 e_2^2 + Q' = -h_2(e_2^2 + h_3 h_1) \\ &= -h_2\left[(h_1-h_2)(h_2-h_3) + h_3 h_1\right] \\ &= -h_2\left[h_1 h_2 - h_2^2 - h_1 h_3 + h_2 h_3 + h_3 h_1\right] \\ &= -h_2^2\left[h_1 - h_2 + h_3\right] = -h_2^2(-2h_2) \end{aligned}$$

$\frac{Q_2}{2} = h_2^3$; therefore

$$W_2 = h_2$$

Also $W_{\rm I}^3 = \frac{Q}{2} = \frac{1}{2}\left(-Ee'_{\rm I} + Q'\right) = \frac{1}{2}\left[\frac{+Q'}{P'} \times \frac{\Gamma'}{4P'^2} + Q'\right]$

$$= \frac{Q'}{8P'^3}(\Gamma' + 4P'^3) = -\frac{27Q'^3}{8P'^3} = -\left(\frac{3Q'}{2P'}\right)^3$$

$$\left\{\begin{array}{l} (E-W)_\lambda = \frac{3}{2}h_1,\quad (E-W)_\mu = \frac{3}{2}h_2,\quad (E-W)_\nu = \frac{3}{2}h_3 \\ \left|\begin{array}{l|l} (E-W)_{\rm I} = +\left(\frac{Q'}{-2P'}\right) & (E-W)_1 = \frac{3}{2}h_2 \\ (E-W)_{\rm II} = \left(\frac{Q'}{-P'}\right) & (E-W)_2 = 0 \end{array}\right. \\ (E-W)_0 = -\left(\frac{Q'}{2}\right)^{\frac{1}{3}}. \end{array}\right.$$

§ 11. We will now, for preparing the construction of the surface $\Gamma=0$, ascertain for the five values of E, $= h_3$, 0, E_1, h_2, h_1, the corresponding values and direction of

$$e,\ v,\ de,\ dE,\ tg\widehat{\epsilon v}.$$

In the case $h_2>0$, the values of E are in the following order:

$$h_3 < 0 < E_1 < h_2 < h_1.$$

In the case of $h_2<0$, the order is:

$$h_3 < h_2 < E_1 < 0 < h_1,$$

but we will, for the present, consider the case $h_2>0$ exclusively.

We shall occupy ourselves only with the octant comprised between $+\lambda, +\mu, +\nu$, because for obvious reasons the surface will be in the other octants only the reproduction of what takes place in this first octant; namely, the surface in the first octant will be repeated in the other octants, either in the position of equality of superposition or in that of symmetry.

For $E=h_3$, the expression (51) gives

$$0=(h_1-h_3)S^2\lambda\epsilon'+(h_2-h_3)S^2\mu\epsilon'.$$

By $h_1>h_2>h_3$ the coefficients are both positive, and we must put

$$S\lambda\epsilon'=0,\ S\mu\epsilon'=0,$$

which gives $$\epsilon'=\nu.$$

The spherical conic reduces itself therefore to the point

$$\nu\frac{1}{2}(h_1-h_2).$$

The normal v becomes, as $\bar{\varpi}\nu=h_3\nu$,

$$v=\frac{3}{2}h_3\nu,\quad tg\,\widehat{\epsilon' v}=0,$$

and, as h_3 is negative, our expression gives v directed inversely to $\epsilon'=\nu$, that is towards the inside of the surface.

$$de=-\frac{1}{2}e\frac{dE}{E-W},\text{ becomes}$$

$$de=\frac{-\frac{1}{2}e_\nu dE}{h^{1}_{3}+\frac{1}{2}h_3}=e_\nu\left(\frac{-3}{h_3}\right)dE$$

the factor $-\frac{3e_\nu}{h_3}$ is positive, but for $\epsilon'=\nu$ we have $dE=0$, because

$$E=-S\epsilon'\varpi\epsilon' \text{ and } \epsilon'^2=-1 \text{ give}$$
$$dE=-2Sd\epsilon'\bar{\varpi}\epsilon',\text{ and } 0=S\epsilon' d\epsilon'$$
$$dE=-2\Sigma h_1 S\lambda d\epsilon' S\lambda\epsilon',\text{ and } 0=\Sigma S\lambda d\epsilon' S\lambda\epsilon'.$$

Now for $\epsilon'=\nu$, the last of these equations give

$$0=S\nu d\epsilon',\text{ and this gives } dE=0,\text{ so that } de=0,$$

namely, the tangent plane in $\epsilon=e_\nu.\nu$ is perpendicular to ν, and e undergoes a minimum.

A similar case is the one when $E=h_1$. Then the surface reduces itself to the point

$\epsilon=\lambda\,.\,\frac{1}{2}(h_2-h_3)$, with a tangent plane perpendicular to λ.

For $E=0$ the spherical conic is the intersection of the cone

$$0=\Sigma h_1 S^2\lambda\epsilon'$$

and the sphere

$$T\epsilon=e_0=\left[\left(\frac{1}{4}\Gamma'+\frac{27}{4}Q'^2\right)^{\frac{1}{3}}-\frac{27}{4}(Q'^2)^{\frac{1}{3}}\right]^{\frac{1}{2}}.$$

As $h_3<0$, the angle which the generating line ϵ' forms with ν is smaller when ϵ' is in the plane $\nu\lambda$, than when ϵ' is the plane $\nu\mu$, these angles being given by

$$tg^2 n_\lambda=\frac{-h_3}{h_1}<\frac{-h_3}{h_2}=tg^2 n_\mu,$$

owing to

$$h_1>h_2>0>h_3.$$

Also for

$$E=0,\ \left(\frac{de_0}{dE}\right)=\frac{-\frac{1}{2}e_0}{-W}=\tfrac{1}{2}e_0:\left(\frac{Q}{3}\right)_0^{\frac{1}{3}}.$$

Therefore e is increasing when E passes through zero.

For

$$E=E_{\mathrm{I}}=\frac{Q'}{-P'}>0\quad\text{when } h_2>0$$

the spherical conic on the first sheet is the intersection of the cone

$$0=\Sigma h_1^3 S^2\lambda\epsilon'$$

with the sphere

$$e_{\mathrm{I}}=T\epsilon_{\mathrm{I}}=\frac{\sqrt{\Gamma'}}{-2P'},=\frac{T(h_1-h_2)(h_3-h_1)(h_2-h_3)}{(h_1^2+h_1^2+h_3^2)}$$

The normal becomes

$$\nu_{\mathrm{I}}=\left(\varpi+\frac{3Q'}{2P'}\right)\epsilon',$$

$$\nu_{\mathrm{I}}=-\Sigma\lambda S\lambda\epsilon' h_1\left(1-\frac{3h_2h_3}{2\Sigma h_1h_3}\right),$$

$$\nu_{\mathrm{I}}=\left(\frac{-1}{2P'}\right)\Sigma\lambda S\lambda\epsilon' h_1(h_1-h_2)(h_3-h_1).$$

The generating line ϵ', of the cone $\Sigma h_1^3 S^2\lambda\epsilon'=0$, in the plane $\lambda\nu$ is given by

$$0=h_1^3 S^2\lambda\epsilon'+h_3^3 S^2\nu\epsilon',\ \text{and as } h_3<0,$$

this decomposes itself into :

$$0 = S\epsilon'[\lambda h_1^{\frac{3}{2}} - \nu(-h_3)^{\frac{3}{2}}] \times S\epsilon'[\lambda h_1^{\frac{3}{2}} + \nu(-h_3)^{\frac{3}{2}}].$$

The vector $\lambda h_1^{\frac{3}{2}} - \nu(-h_3)^{\frac{3}{2}}$ being perpendicular to the ϵ' which we want, we get

$$(60) \qquad M\epsilon' = \lambda(-h_3)^{\frac{3}{2}} + \nu(h_1)^{\frac{3}{2}},$$

and then the normal v_I for $S\mu\epsilon'=0$ becomes parallel to:

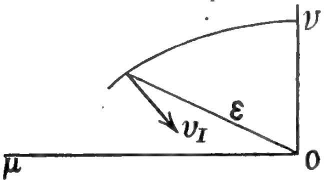
$$(M'v_I)_{S\mu\epsilon'=0} = [\lambda(-h_3)^{\frac{1}{2}}(h_1-h_2) - \nu h_1^{\frac{1}{2}}(h_2-h_3)].$$

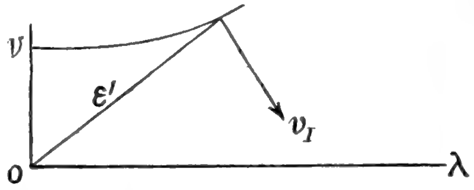
M′ being another scalar factor whose expression is unimportant. It is directed towards $-\nu$, and towards $+\lambda$.

Likewise, the normal v_I corresponding to the generating line ϵ' in the plane $\mu\nu$, is directed parallel to:

$$(M''v_I)_{(S\lambda\epsilon'=0)} = [-\mu(h_3)^{\frac{1}{2}}(h_1-h_2) - \nu h_2^{\frac{1}{2}}(h_1-h_3)],$$

that is, it is directed towards $-\mu$ and $-\nu$.





Then
$$\left(\frac{de}{dE}\right)_I = \frac{-\frac{1}{2}e_I}{(E-W)_I} = \frac{\frac{1}{2}\sqrt{\Gamma'} : 2P'}{Q' : 2P'},$$

$$\left(\frac{de}{dE}\right)_I = \frac{\frac{1}{2}\sqrt{\Gamma'}}{Q},$$

which is positive, so that e is increasing when E passes through E_I.

For $E = E_I$, also on the cone $0 = \Sigma h_1^3 S^2\lambda\epsilon'$, the equation $\Gamma = 0$ gives also:

$$e_{II} = \sqrt[2]{-P'}$$

as radius of the sphere of the spherical conic. And we have already remarked that this solution corresponds to

$$P = 0, \quad Q = 0,$$

annulling separately the two terms of Γ; for this reason, and for others too, this solution is of the kind termed a *singular solution.*

We have evidently

$$e_{II} > e_I, \text{ namely } -P' > \frac{\Gamma'}{4P'^2} \text{ which gives } -4P'^3 > \Gamma = -4P'^3 - 27Q'^2,$$

and so this curve belongs to another sheet of the surface than that which we have followed as yet.

But this new sheet is the beginning of a double sheet which does not extend to values of E smaller than E_{I}; namely:

Let us consider a point on the curve E_{I}, e_{II}, and let $e_{II}\epsilon'$ be its vector; and let us consider another point on the surface, whose vector we shall represent for the moment by

$$\epsilon = e_{II}\epsilon' + \rho'',$$

ρ'' being treated here as infinitesimal, of which we shall neglect the 2d order. Then we have

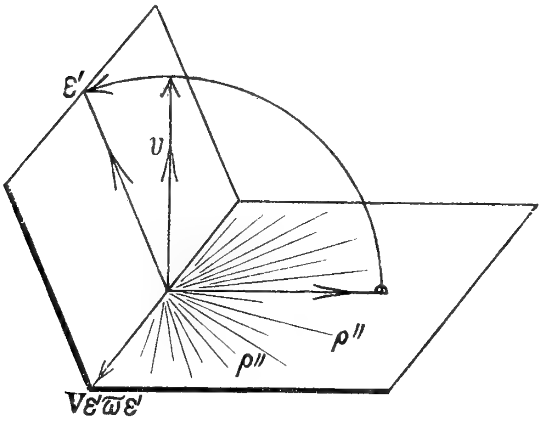
$$\begin{aligned}\epsilon^2 &= -e_{II}^2 + 2e_{II}S\epsilon'\rho'' + T_{\rho''}{}^2A' \\ S\epsilon\varpi\epsilon &= e_{II}^2S\epsilon'\varpi\epsilon' + 2e_{II}S\rho''\varpi\epsilon' + T_{\rho''}{}^2B'.\end{aligned}$$

Then, as P and Q are annulled as to their terms independent of ρ'', we get

$$\begin{aligned}P &= -2e_{II}S\epsilon'\rho'' + T_{\rho''}{}^2A'' \\ Q &= 2e_{II}S\rho''\varpi\epsilon' + T_{\rho''}{}^2B''.\end{aligned}$$

Then we must have:

$$0 = -8e_{II}^3S^3\epsilon'\rho'' + \frac{27}{4}4e_{II}^2S^2\rho''\varpi\epsilon' + T\rho''^4C'$$



the additional term $T\rho''^4C'$ being of at least the $(3+\frac{1}{2})^{\text{th}}$ order because we have to suppose

$$S^2\rho''\varpi\epsilon' \text{ to be of the order } S^3\epsilon'\rho'',$$

$$S\rho''\varpi\epsilon' \text{ namely of the } (\tfrac{3}{2})^{\text{d}} \text{ order.}$$ Thus we get

$$S\rho''\varpi\epsilon' = \pm\sqrt[2]{\frac{8}{27}}\,e_{II}(S\epsilon'\rho'')^{\frac{3}{2}}.$$

This shows that ρ'' can be real only when

$$S\epsilon'\rho''>0\,,\ \text{angle}\ \widehat{\epsilon'\rho''}>90°,$$

and consequently the surface $\Gamma=0$ extends not in all directions round the direction of ϵ', but only on one side of a certain plane passing through ϵ' and $V\epsilon'\varpi\epsilon'$, where $\widehat{\epsilon\rho''}>90°$.

As we have $W_{II}=0$, the normal is directed parallel to

$$(63)\qquad \nu_{II}=\overline{\varpi}\epsilon\,.$$

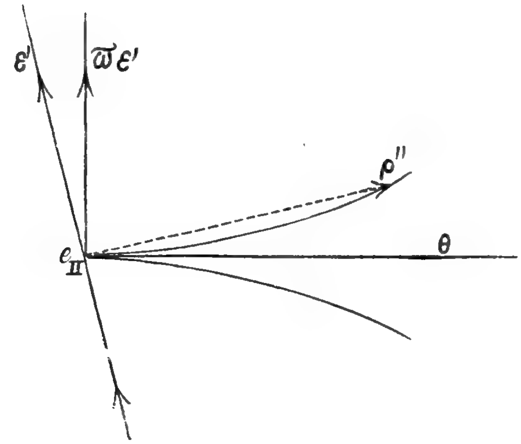
This accounts for $S\rho''\overline{\varpi}\epsilon'$ being of a smaller order than the order of ρ'', because ρ'' must be nearly directed in the tangent plane at the point $e_{II}\epsilon$.

We need not show that the directions of ρ'', which alone can take place, are those which render E greater than E_I (not smaller), because we have shown already that it is only when E is comprised between E_I and h_2 (supposing always here $h_2>0$) that there are three real roots for e given by $\Gamma=0$.

The double sign of the value of $S\rho''\varpi\epsilon'$ shows that there are two sheets beginning at $E=E_I$, $e=e_{II}$, the conic so determined forms therefore an edge-like termination of the surface in this region.

If we were to examine the section perpendicularly made to this edge, and call $\rho''=x\theta_{II}+y\varpi\epsilon'$ the vector of a point of the section beginning at $e_{II}\epsilon'$, θ_{II} being a unit-vector in the plane of ϵ' and $\varpi\epsilon'$, perpendicular to $\varpi\epsilon'$, we would find

$$yT^2\varpi\epsilon=\pm\sqrt{\frac{8}{27}e_{II}}\,.\,T^3(V\epsilon'\varpi\epsilon')x^{\frac{3}{2}}\,,$$



which gives the approximate expression for the beginning of the section of the surface at the point $e_{II}\epsilon'$.

We have for $\qquad W_{II}=0$

$$\left(\frac{de}{dE}\right)_{II}=\frac{-\frac{1}{2}e_{II}}{E_{II}}=-\frac{1}{2}\frac{(-P^1)^{\frac{2}{3}}}{Q^1}<0\,,$$

therefore e is decreasing when E increasing from E_{II} $(=E_I)$ towards h_2, for both branches.

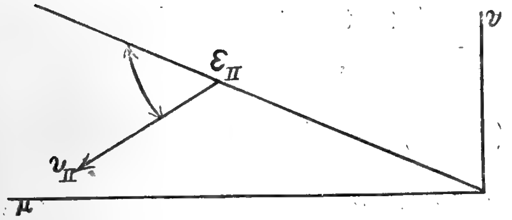
The normal v_{II} corresponding to the generating line ϵ' in the plane $\lambda\nu$ (for the expression (60) of ϵ', $M\epsilon^1=\lambda(-h_3)^{\frac{3}{2}}+\nu(h_1)^{\frac{3}{2}}$) will be parallel to

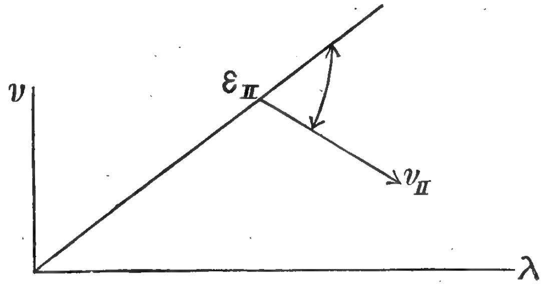
$$(Mv_{II})_{S\mu\epsilon'=0}=.\left[\lambda(-h_3)^{\frac{1}{2}}-\nu(h_1)^{\frac{1}{2}}\right],$$

that is directed downwards to $-\nu$.

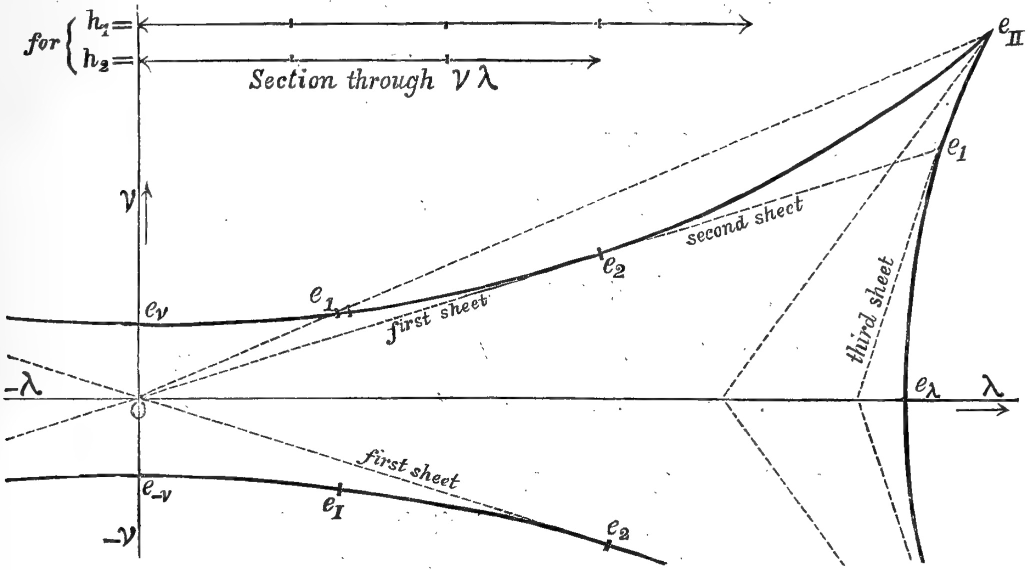
Likewise we find, in the plane $\mu\nu$:

$$(M'v_{II})_{S\lambda\epsilon'=0}=\left[\mu(-h_3)^{\frac{1}{2}}-\nu h_2^{\frac{1}{2}}\right]$$





We will call *first* sheet of the surface $\Gamma=0$, the one which we have followed from



$$\begin{cases}E=h_2\\ e=e_\nu\end{cases}\quad\text{through}\quad\begin{cases}E=0\\ e=e_0\end{cases}\quad\text{to}\quad\begin{cases}E=E_I\\ e=e_I\end{cases}$$

and which, as we shall see, extends as far as

$$\begin{cases} E=h_2 \\ e=e_2 \end{cases}.$$

We will call *second* sheet the one which begins at

$$\begin{cases} E=E_I \\ e=e_{II} \end{cases}, \text{ and which ends at } \begin{cases} E=h_2 \\ e=e_2 \end{cases};$$

and *third* sheet the other, beginning at

$$\begin{cases} E=E_I \\ e=e_{II} \end{cases} \text{ through } \begin{cases} E=h_2 \\ e=e_I \end{cases} \text{ ending at } \begin{cases} E=h_1 \\ e=e_\lambda \end{cases}.$$

For $E = h_2$ we have by $\Gamma = 0$, the two values of e, $e = e_2$, $e = e_1$; and the corresponding cyclical conic is transformed into two half circles, because the value of E gives the equation of the two planes R, R′:

$$0 = (h_1-h_2)S^2\lambda\epsilon' + (h_3-h_2)S^2\nu\epsilon'.$$

The plane R which cuts the first octant (both planes passing through the axis μ) is given by:

$$S\epsilon'(\sqrt{h_1-h_2}\,\lambda - \sqrt{h_2-h_3}\,\nu) = 0.$$

Replacing h_1-h_2, by $2e_\nu$, &c., the normal χ_2 to this plane R will be

$$\chi_2 = \lambda\sqrt{e_\nu} - \nu\sqrt{e_\lambda}. \tag{64}$$

The normal to the plane R′ will be

$$\chi_2' = \lambda\sqrt{e_\nu} + \nu\sqrt{e_\lambda}. \tag{65}$$

In representing by $a\lambda + b\mu + c\nu$ an ϵ', we get

$$\begin{aligned} M_2'\epsilon' &= \lambda\sqrt{e_\lambda} + \mu b N + \nu\sqrt{e_\nu} \text{ perpendicular to } \chi_2 \\ M_2''\epsilon' &= -\lambda\sqrt{e_\lambda} + \mu b N' + \nu\sqrt{e_\nu} \text{ perpendicular to } \chi_2'. \end{aligned} \tag{66}$$

for the ϵ' in the planes R, R′, respectively.

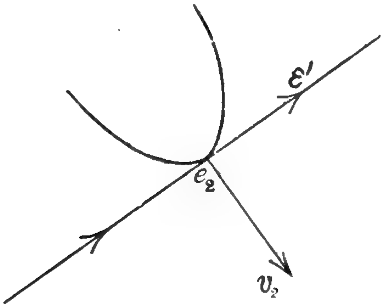
The circle of radius e_2 is comprised within the circle of radius $e_1 = e_\mu$, because the inequality $e_2^2 < e_1^2$ reduces itself to $0 < (3h_2)^2$.

Therefore the circle $\epsilon = e_2\epsilon'$ forms the junction curve between the first sheet and the second sheet. Along this circle the normal to the surface is perpendicular to the plane R of the circle; namely, we have, W being $= h_2$:

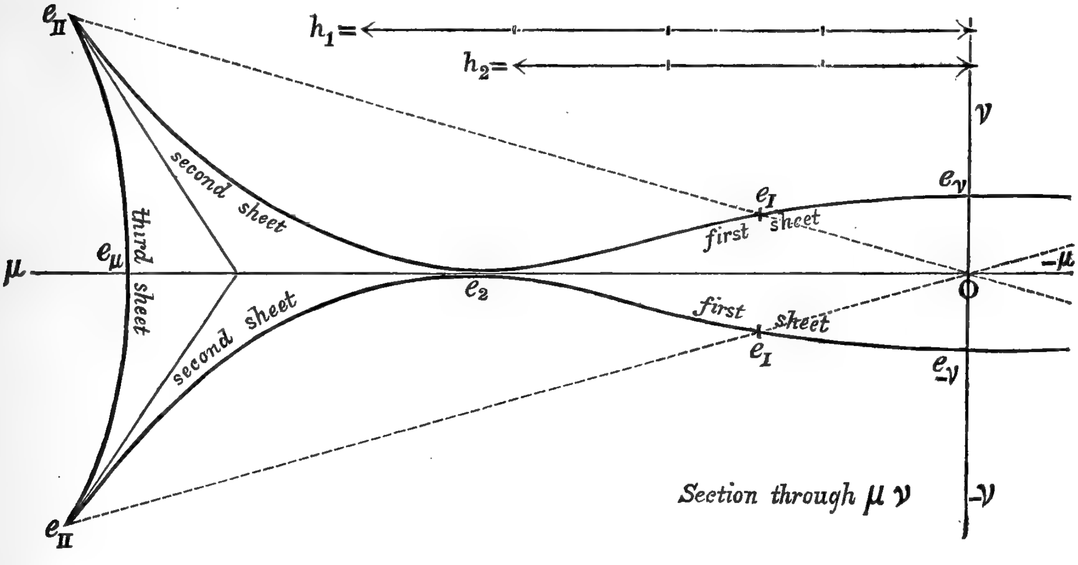
$$
\begin{aligned}
Mv_2 &= (\bar{\varpi}-h_2)\epsilon' \\
&= (\varpi-h_2)\left[\lambda\sqrt{e_\lambda}+\nu\sqrt{e_\nu}\,\right]
\end{aligned}
$$

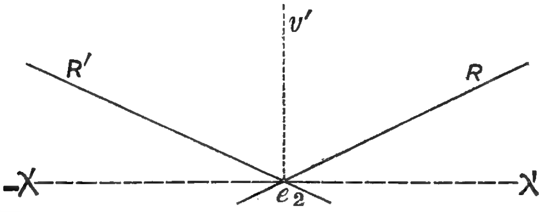
$$\text{parallel to } M'v_2 = \left[\lambda\sqrt{e_\nu}-\nu\sqrt{e_\lambda}\right] = \chi_2\,.$$

The expression of $\frac{de}{dE}$, owing to $E-W=0$, becomes infinite, a circumstance which explains the fact of the junction of the two sheets, the surface being tangent to the cone $E = h_2$ along the curve of contact.



The point $\epsilon = e_2\epsilon'$ corresponding to the direction μ is of course situated on





Ideal Section $\perp$ to μ at $\epsilon = e_2\mu$, for infinitely near that point only.

that axis, but here the tangent plane affects two definite positions, owing to the meeting of the two equal circles in the one point $\epsilon=e_2\mu$.

For $E = h_2$, $e = e_1 = e_\mu = \frac{h_1-h_3}{2}$, the spherical conic is a circle belonging to the third sheet, and whose plane is also in R.

The normal vector v, owing to $W_1 = -\frac{1}{2}h_2$, becomes

$$v_1 = \left(\varpi + \frac{1}{2}h_2\right)\epsilon',$$

and for the direction ϵ' situated in the plane of the circle, namely for (66)

$$M_2'\epsilon' = (\lambda\sqrt{e_\lambda} + \nu\sqrt{e_\nu}) + \mu N_1$$

we get

$$M_2'v_1 = \left[\left(\varpi + \frac{1}{2}h_2\right)\lambda\sqrt{e_\lambda} + \left(\varpi + \frac{1}{2}h_2\right)\nu\sqrt{e_\nu}\right]$$
$$+ \left(\varpi + \frac{1}{2}h_2\right)\mu N_1,$$

namely

$$M_2'v_1 = \left[\lambda\left(h_1 + \frac{1}{2}h_2\right)\sqrt{e_\lambda} + \nu\left(h_3 + \frac{1}{2}h_2\right)\sqrt{e_\nu}\right] + \mu\frac{3}{2}h_2N_1.$$

But we have

$$\left(h_1 + \frac{1}{2}h_2\right) + \left(\frac{1}{2}h_2 + h_3\right) = 0,$$

and

$$h_1 + \frac{1}{2}h_2 = \frac{1}{2}(h_1-h_3) = e_\mu, \quad \text{or } e_1.$$

Thus

$$M_3'v_1 = [\lambda\sqrt{e_\lambda} - \nu\sqrt{e_\nu}]e_\mu + \mu\frac{3}{2}h_2N_1.$$

If we compare this to v_2 (64)

$$M_2''v_2 = (\lambda\sqrt{e_\nu} - \nu\sqrt{e_\lambda}),$$

and calculate $\cos v_1v_2$ we get

$$\text{for } \mu\epsilon' = 90^\circ, \quad \cos\widehat{v_1v_2} = e_2 : e_1$$

$$\text{for } \mu\epsilon' = 0, \qquad \cos\widehat{v_1v_2} = 0, \qquad \widehat{v_1v_2} = 90^\circ.$$

---

§ 12. We must, at least for form's sake, not loose sight of the question which we put to ourselves at the end of § 7, and which is of easy solution now that we have sketched an outline of the principal features of the surface $\Gamma=0$, although we have not by far exhausted the subject, namely:

It will now be evident that the condition $\Gamma > 0$ will be satisfied by every value of ϵ whose extremity is comprised within the *inside* of the surface $\Gamma = 0$ taken in its whole; and in particular, that condition will be satisfied, for the proper values of e, when the direction of ϵ is such as to intersect the first, second, and third sheets at the same time; and this is due to the circumstance that Γ changes sign whenever, $U\epsilon$ remaining the same, the varying tensor e is such as to engender an intersection of ϵ by the surface.

If we now try to describe the general aspect of the surface $\Gamma = 0$, we may say (excepting the extreme cases of $\begin{cases} h_2 = h_1 \\ h_2 = h_3 \end{cases}$, or $h_2 = 0$) that when $h_2 > 0$, in looking at the surface from a point in the plane $\lambda\mu$, it would present a kind of continuous belt, formed by the third sheet, and whose axis (in a certain vague sense of axis) is the axis ν.

The belt would have its greatest breadth in the plane $\lambda\nu$, and its smallest in the plane $\mu\nu$, because the upper edge of it is formed by the spherical conic whose equations are

$$\text{the cone} \qquad 0 = \Sigma h_1^3 S^2 \lambda\epsilon'$$

$$\text{and the sphere} \quad T\epsilon_{II} = e_{II} = \sqrt{-P'},$$

so that the belt, as seen from the origin 0, would subtend the double angle whose tangent is

$$\text{in the plane } \lambda\nu, \quad tg\widehat{\lambda\epsilon_{II}} = \frac{S\nu\epsilon'}{S\lambda\epsilon'} = \left(\frac{h_1}{-h_3}\right)^{\frac{3}{2}},$$

$$\text{in the plane } \mu\nu, \quad tg\widehat{\mu\epsilon_{II}} = \frac{S\nu\epsilon'}{S\mu\epsilon'} = \left(\frac{h_2}{-h_3}\right)^{\frac{3}{2}},$$

both real because $h_3 < 0$, and the first greater than the second because of $h_1 > h_2$.

If we would trace or inscribe on the outer surface of the belt the spherical conics, we would find them belonging to two different sets, divided into two corresponding regions by the two circles of radius e_1 and whose planes, R, R′, passing through the axis μ, are symmetrical on both sides of the plane $\mu\nu$.

In the part of the surface comprised in the angle of the two planes R, R′, which contains the plane $\lambda\mu$ and the axis λ, the conics would in a certain sense be concentric about the central point $e_\lambda \lambda$, on the axis λ.

In the part of the surface of the belt comprised between the above two planes and in the angle which contains the plane $\mu\nu$, the conics belong to the set which has their quasi centre in the axis ν, in $e_\nu \nu$.

If now we would look at the surface from a point in the axis ν, we would see—1st, the *first sheet* spread out under the eye with its conics centred in the point $e_\nu \nu$, and extending as far as the obliquely seen circles of radius e_2, in the above two planes R, R′.

2d, Between these circles and the singular curve $E = E_{\text{I}}$, $e = e_{\text{II}}$, or upper edge of the belt, we would see the *second sheet* in continuation of the first.

3d, The *third sheet* will be shut out of view in this position of the eye, because the tangent plane along the curve forming the sharp edge of the belt is dipping downward and inward.

When $h_2 < 0$ the surface will present exactly the same general features, only the roles of the axis λ, ν, will be interverted.

Without entering into details about the special cases, when

$$h_2 = h_1 , \quad \text{or } h_2 = h_3 ,$$

we will only remark that in these cases the surface $\Gamma = 0$ becomes a surface of revolution round the axis ν, or λ, respectively.

When $h_2 = 0$ the surface looses the intermediate sheet, the one which we called the 2d sheet.

But in all cases the *sharp edge* corresponding to $P = 0$, $Q = 0$ is the feature most characteristic in the surface

$$\Gamma = 0 .$$

THE SECOND PART.

As we announced in our introduction, we will establish the directions and tensors of the expressions of ρ and ρ' satisfying to the conditions respectively of

$$V\rho\phi\rho = 0 , \quad V\rho'\phi'\rho' = 0 .$$

Both of these conditions lead us to establish the scalar equation $fg = 0$,

$$fg = g^3 - m_2 g^2 + m_1 g - m ;$$

where m, m_1, m_2, are scalars whose expressions we need not repeat, and which are the same when they are calculated either with the help of ϕ or with that of ϕ'.

Retaining the definition of α, β, γ, as a system of treble rectangular unit vectors, we put

$$\alpha_1 = (\phi'-g)\alpha\,,\quad \beta_1 = (\phi'-g)\beta\,,\quad \gamma_1 = (\phi'-g)\gamma\,,$$
$$\alpha_1' = (\phi-g)\alpha\,,\quad \beta_1' = (\phi-g)\beta\,,\quad \gamma_1' = (\phi-g)\gamma\,,$$

and we define ψ and ψ' by

$$\psi\alpha = V\beta_1\gamma_1\,,\quad \psi\beta = V\gamma_1\alpha_1\,,\quad \psi\gamma = V\alpha_1\beta_1\,,$$
$$\psi'\alpha = V\beta_1'\gamma_1'\,,\quad \psi'\beta = V\gamma_1'\alpha_1'\,,\quad \psi'\gamma = V\alpha_1'\beta_1\,,$$

the root g being supposed the same in both.

Representing by ω any vector, putting

$$\omega = \alpha u + \beta v + \gamma w\,,$$

we have generally

$$\rho D = \psi\omega = u\psi\alpha + v\psi\beta + w\psi\gamma$$
$$\rho' D' = \psi'\omega = u\psi'\alpha + v\psi'\beta + w\psi'\gamma\,,$$

D and D′ being scalars whose values are to be determined in the sequel. We know that the unit vectors of $\psi\omega$, $\psi'\omega$ are invariable; we will establish now that the tensors of these expressions depend on the direction of ω.

Let us suppose

$$\psi\alpha = A\rho\,,\quad \psi\beta = B\rho\,,\quad \psi\gamma = C\rho$$
$$\psi'\alpha = A'\rho'\,,\quad \psi'\beta = B'\rho'\,,\quad \psi'\gamma = C'\rho'\,.$$

A, B, &c., being particular values of D D′.

We find a relation between these scalars by the following way, namely:—

Any scalar of the product of three vectors, α_1, β_1, γ_1 suppose, satisfies to the identity:

$$3S\alpha_1\beta_1\gamma_1 = \alpha_1 V\beta_1\gamma_1 + \beta_1 V\gamma_1\alpha_1 + \gamma_1 V\alpha_1\beta_1\,,$$

and a similar one written for $\alpha_1'\beta_1'\gamma_1'$.

But $S\alpha_1\beta_1\gamma_1$ and $S\alpha_1'\beta_1'\gamma_1'$ are both equal to zero, as they represent fg, for the three roots of $fg = 0$.

Then we replace $V\beta_1\gamma_1$, &c., by their expressions $\psi\alpha$, namely $A\rho$, &c., and thus we get the two equations:

$$0 = (\alpha_1 A + \beta_1 B + \gamma_1 C)\rho$$
$$0 = (\alpha_1' A' + \beta_1' B' + \gamma_1' C')\rho'\,,$$

and as the tensors of ρ and ρ' are not generally vanishing, we have the equations:

$$0 = A\alpha_1 + B\beta_1 + C\gamma_1$$
$$0 = A'\alpha_1' + B'\beta_1' + C'\gamma'\,.$$

We treat these equations by $S\alpha(\ \ )$, $S\beta(\ \ )$, $S\gamma(\ \ )$, and we remark the relations:

$$S\alpha\alpha_1 = S\alpha(\phi'-g)\alpha = S\alpha(\phi-g)\alpha = S\alpha\alpha_1'$$
$$S\alpha\beta_1 = S\alpha(\phi'-g)\beta = S\beta(\phi-g)\alpha = S\beta\alpha_1', \text{ \&c.}$$

So that we have the table of values:

$$\begin{array}{lll} S\alpha\alpha_1=S\alpha\alpha_1', & S\alpha\beta_1=S\beta\alpha_1', & S\alpha\gamma_1=S\gamma\alpha_1' \\ S\beta\alpha_1=S\alpha\beta_1', & S\beta\beta_1=S\beta\beta_1', & S\beta\gamma_1=S\gamma\beta_1' \\ S\gamma\alpha_1=S\alpha\gamma_1', & S\gamma\beta_1=S\beta\gamma_1', & S\gamma\gamma_1=S\gamma\gamma_1' \end{array}$$

Applying this to the foregoing equation we get, by the first in A, B, C:

$$0=S\alpha_1'(A\alpha+B\beta+C\gamma)$$
$$0=S\beta_1'(A\alpha+B\beta+C\gamma)$$
$$0=S\gamma_1'(A\alpha+B\beta+C\gamma)$$

Thus the vector $A\alpha+B\beta+C\gamma$ must be perpendicular to the plane in which α_1', β_1', γ_1' are situated, the coplanarity of these latter being expressed already by $fg=S\alpha_1'\beta_1'\gamma_1'=0$.

But ρ, is also perpendicular to this plane. Therefore,

$$A\alpha+B\beta+C\gamma=t\rho'.$$

Likewise we conclude

$$A'\alpha+B'\beta+C'\gamma=t'\rho.$$

If we remember now the definitions of A, B, &c., namely

$$\psi\alpha=A\rho, \text{ \&c.,}$$

we deduce by multiplying *by* ρ the expression of $t\rho'$, and *by* ρ' the expression of $t'\rho$ we get:

$$\Sigma\,.\,\psi\alpha\,\alpha=t\rho\rho'$$
$$\Sigma\,.\,\psi'\alpha\,\alpha=t'\rho'\rho$$

Now the scalars of the first members are equal. For a demonstration, we will simply express ψ by the help of the coefficients of fg. We have:

$$\psi\alpha=V\beta_1\gamma_1=V(\phi'-g)\beta(\phi'-g)\gamma$$
$$=V\phi'\beta\phi'\gamma-gV(\beta\phi'\gamma-\gamma\phi'\beta)+g^2\beta\gamma.$$

This gives, by known properties:

$$\psi\alpha=(m\phi^{-1}-g\chi+g^2)\alpha;$$

Or developing and putting:

$$G=m_1-gm_2+g^2,\ H=m_2-g,$$

we get :

$$\psi=(\mathrm{G}-\mathrm{H}\phi+\phi^2)$$
$$\psi'=(\mathrm{G}-\mathrm{H}\phi'+\phi'^2)$$

and from these expressions there follows at once

$$\mathrm{S}\alpha\psi\alpha=\mathrm{S}\alpha\psi'\alpha\text{, \&c., because}$$

$$\mathrm{S}\alpha\phi'\alpha=\mathrm{S}\alpha\phi\alpha\,,\ \ \mathrm{S}\alpha\phi'^2\alpha=\mathrm{S}\alpha\phi^2\alpha\,,\ \ \text{\&c., \&c.}$$

Therefore we have

$$t'\mathrm{S}\rho\rho'=t\mathrm{S}\rho'\rho$$

and therefore the values of t and t' are one and the same.

We now have :

$$\mathrm{A}\,\alpha+\mathrm{B}\,\beta+\mathrm{C}\,\gamma=t\rho'$$
$$\mathrm{A}'\alpha+\mathrm{B}'\beta+\mathrm{C}'\gamma=t\rho\,.$$

Squaring we get :

$$-\Sigma\mathrm{A}^2=t^2\rho'^2\,,\ \ -\Sigma\mathrm{A}'^2=t^2\rho^2\,;$$

On the other hand, if we sum the squares of $\psi\alpha=\mathrm{A}\alpha$, &c., $\psi'\alpha=\mathrm{A}'\alpha$, &c., we form

$$\Sigma(\psi\alpha)^2=\rho^2\Sigma\mathrm{A}^2,\quad \Sigma(\psi'\alpha)^2=\rho'^2\Sigma\mathrm{A}'^2\,,$$

and replacing $\Sigma\mathrm{A}^2$, &c.,

$$\Sigma(\psi\alpha)^2=-t^2\rho^2\rho'^2=\Sigma(\psi'\alpha)^2.$$

For simplyfying we put :

$$t^2\rho^2\rho'^2=\mathfrak{T}=-\Sigma(\psi\alpha)^2$$
$$\sqrt{\mathfrak{T}}=t\mathrm{T}\rho\mathrm{T}\rho\,,$$

in taking for t and the radical a positive value.

Then our previous equations become

$$\Sigma(\psi\,\alpha\,.\,\alpha)=\sqrt{\mathfrak{T}}\mathrm{U}\rho\mathrm{U}\rho'$$
$$\Sigma(\psi'\alpha\,.\,\alpha)=\sqrt{\mathfrak{T}}\mathrm{U}\rho'\mathrm{U}\rho\,.$$

But now we will calculate scalar and vector of the first members by the explicit expressions of ψ, ψ'.

$$\Sigma\mathrm{S}\alpha\psi\alpha=\Sigma\mathrm{S}\alpha(\mathrm{G}\alpha-\mathrm{H}\phi\alpha+\phi^2\alpha)$$
$$=\mathrm{G}\Sigma\alpha^2-\mathrm{H}\Sigma\mathrm{S}\alpha\phi\alpha+\Sigma\mathrm{S}\alpha\phi^2\alpha$$
$$=-3\mathrm{G}-\mathrm{H}(-m_2)+(2m_1-m_2^2)\,.$$

Replacing G, H, and ordaining in respect to g, we get:

$$\Sigma S\alpha\psi\alpha=-m_1+2gm_2-3g^2,$$

which is nothing but

$$-f'g\text{, namely of }\frac{dfg}{dg}.\quad\text{Therefore}$$

$$\Sigma S\alpha\psi\alpha=\Sigma S\alpha\psi'\alpha=-f'g.$$

Then as to the vectors

$$\Sigma V.\psi\alpha\,\alpha=\Sigma[-HV.\phi\alpha\,\alpha+V.\phi^2\alpha\,\alpha],$$

as

$$-\Sigma V.\phi\alpha\,\alpha=\Sigma V\alpha\phi\alpha=2\epsilon$$

$$\Sigma V.\phi^2\alpha\alpha=-\Sigma V\alpha\phi^2\alpha=+2\phi\epsilon-2m_2\epsilon,$$

and as generally for any integer n:

$$\Sigma(V\alpha\phi^n\alpha+V\alpha\phi^{1n}\alpha)=0,$$

we get with

$$\Sigma V(\psi\alpha.\alpha)=+H.2\epsilon+2\phi\epsilon-2m_2\epsilon$$

namely:

$$\Sigma V(\psi\alpha.\alpha)=+2(\phi-g)\epsilon=-\Sigma V(\psi\alpha'.\alpha)$$

We have therefore;

$$\sqrt{\mathfrak{T}}SU\rho U\rho'=-f'g$$

$$\sqrt{\mathfrak{T}}VU\rho U\rho'=+2(\phi-g)\epsilon.$$

From this, as $T^2U\rho U\rho'=1$, we get:

$$\mathfrak{T}=(f'g)^2-4[(\phi-g)\epsilon]^2.$$

This shows, as for example, that when two of the roots g are equal, and consequently for them $(f'g)=0$, then the two directions ρ and ρ', corresponding to that root, will be at right angles to one another (always provided that ϵ be not zero).

Also, when we treat the vector of the product $\rho\rho'$ by $SV\epsilon\phi\epsilon(\ )$ we get

$$SV\epsilon\phi\epsilon V\rho\rho'=0,$$

and this whatever be the root g; so that it follows that the planes which ρ and ρ' determine, in the case of each of the three roots g (three planes), these three planes when drawn through the origin are all three cutting each other in the direction of the vector $V\epsilon\phi\epsilon$.

We have now the means of expressing the tensor of $\psi\omega$, by the help of

$$A\alpha+B\beta+C\gamma=t\rho',$$

from which we draw

$$-A = tS\alpha\rho', \; -B = tS\beta\rho', \; -C = tS\gamma\rho',$$

and as $\omega = -\alpha S\alpha\omega - \beta S\beta\omega - \gamma S\gamma\omega$,

$$\psi\omega = -\psi\alpha S\alpha\omega - \psi\beta S\beta\omega - \psi\gamma S\gamma\omega,$$

and remembering the definitions $\psi\alpha = A\rho$, &c., we get

$$\psi\omega = t\rho\Sigma S\alpha\rho' S\alpha\omega,$$

likewise

$$\psi'\omega = t\rho'\Sigma S\alpha\rho S\alpha\omega,$$

namely

$$\psi\omega = -t\rho S\omega\rho'$$
$$\psi'\omega = -t\rho' S\omega\rho.$$

Replacing t by $\sqrt{\mathfrak{T}} : T\rho T\rho'$, this gives

$$\begin{cases} \psi\omega = -U\rho\sqrt{\mathfrak{T}}\,S\omega U\rho' \\ \psi'\omega = -U\rho'\sqrt{\mathfrak{T}}\,S\omega U\rho. \end{cases}$$

This shows that the tensors $\psi\omega$ and $\psi'\omega$ are variable with the direction of ω, and that $\psi\omega$ has its *maximum* tensor when ω coincides, not with $U\rho = U\psi\omega$ as one might have surmised, but with $U\rho'$; and the tensor of $\psi\omega$ is *minimum*, namely zero, when ω lies in a plane perpendicular, not to ρ, but perpendicular to ρ'.

Similar remarks, *mutatis mutandis*, refer to the tensor of $\psi'\omega$.

We may now let ourselves be guided by the principle, drawn from observation, that when the expression of the tensor of a vector vanishes, the direction of the corresponding unit-vector will present a more or less marked degree of indetermination; and we institute a discussion of the cases in which $T\psi\omega$ vanishes.

We have

$$T\psi\omega = \sqrt{\mathfrak{T}} \,.\, TS\omega U\rho',$$

and we leave out of the question the second factor because its value depends on our own choice, and we discuss only the factor $\sqrt{\mathfrak{T}}$.

Originally we have defined $\mathfrak{T}$ by

$$\mathfrak{T} = t^2 T\rho^2 T\rho'^2,$$

but $T\rho$, $T\rho'$ are not susceptible to be annulled, because the original solutions of $(\phi-g)\rho=0$, $(\phi'-g)\rho'=0$, are dependent only on $U\rho$, $U\rho'$ respectively.

There remains the factor t, which however will be easier discussed when looked upon as implicitly contained in the expression of $\mathfrak{T}$, namely in

$$\mathfrak{T}=(f'g)^2+4\,T^2(\phi-g)\epsilon\,.$$

By the introduction of $\underset{\circ}{\mathfrak{h}}$, ϖ, by

$$g=m_3+\underset{\circ}{\mathfrak{h}}\,,\quad \phi=m_3+\varpi$$

we get also

$$\mathfrak{T}=(\mathscr{F}'\underset{\circ}{\mathfrak{h}})^2+4T^2(\varpi-\underset{\circ}{\mathfrak{h}})\epsilon\,,$$

where

$$\mathscr{F}\underset{\circ}{\mathfrak{h}}=\underset{\circ}{\mathfrak{h}}^3+P\underset{\circ}{\mathfrak{h}}+Q$$
$$\mathscr{F}'\underset{\circ}{\mathfrak{h}}=3\underset{\circ}{\mathfrak{h}}^2+P\,.$$

Incidentally we will state also that by the same new variable we have:

$$\psi\omega=(P+\underset{\circ}{\mathfrak{h}}^2+\underset{\circ}{\mathfrak{h}}\varpi+\varpi^2)\omega\,.$$

Now $\mathfrak{T}$ cannot vanish unless both of its terms vanish.

We omit, as being a *particular* case only, the case when $\epsilon=0$, and two of the roots h_1, h_2, h_3, are equal to one another (in which case $\mathfrak{T}$ may be annulled), because our hypothesis about the data in the present question is, that the elements of the self-conjugate part of ϕ are THE *data*, and ϵ alone is left free to be disposed of in tensor and direction.

So we suppose h_1, h_2, h_3 different from one another generally, and we suppose ϵ to take any value and direction at will.

Then we observe that none of the two terms of $\mathfrak{T}$, in the three values $\mathfrak{T}_1$, $\mathfrak{T}_2$, $\mathfrak{T}_3$, corresponding to the three roots $\underset{\circ}{\mathfrak{h}}_1$, $\underset{\circ}{\mathfrak{h}}_2$, $\underset{\circ}{\mathfrak{h}}_3$ (or at least those corresponding to the real values of $\underset{\circ}{\mathfrak{h}}$, roots of $\mathscr{F}\underset{\circ}{\mathfrak{h}}=0$), can be vanishing, unless it be for a value of ϵ corresponding to a point on the surface $\Gamma=0$.

In this case we have $\mathscr{F}'\underset{\circ}{\mathfrak{h}}=0$, because $\Gamma=0$ is the expression of the condition of the presence of equal roots of $\mathscr{F}\underset{\circ}{\mathfrak{h}}=0$.

Let $\underset{\circ}{\mathfrak{h}}_1$ $\underset{\circ}{\mathfrak{h}}_2$ be the two equal roots. We find by $\mathscr{F}\underset{\circ}{\mathfrak{h}}=0$ that

$$\underset{\circ}{\mathfrak{h}}_1=\underset{\circ}{\mathfrak{h}}_2=W\,,\quad \underset{\circ}{\mathfrak{h}}_3=-2W\,.$$

Thus we get

$$\begin{cases}\psi_1\omega=\psi_2\omega=(\varpi-W)(\varpi+2W)\omega\\ \psi_3\omega=(\varpi-W)^2\omega\end{cases}$$

$$\begin{cases}\mathfrak{T}_1=\mathfrak{T}_2=4T^2(\varpi-W)\epsilon\\ \mathfrak{T}_3=9W^2+4T^2(\varpi+2W)\epsilon\,.\end{cases}$$

These expressions of $\mathfrak{T}$ do not vanish for any point of the surface $\Gamma = 0$, save and except the two first, for the particular point

$$\epsilon = e_2\mu, \quad \text{where} \quad e_2^2 = (h_1 - h_2)(h_2 - h)_3,$$

because then we have $W = h_2$, and as generally

$$\varpi\epsilon = \bar{\varpi}\epsilon = -\Sigma h_1\lambda S\lambda\epsilon,$$

we get for the above point:

$$\varpi\epsilon = e_2 h_2\mu = h_2\epsilon,$$

So that $(\varpi - W)\epsilon = 0$, and $\mathfrak{T}_1$, $\mathfrak{T}_2$, vanish, and consequently $\psi_1\omega$ and $\psi_2\omega$ give the result zero for ω of any direction whatever.

But in this case the direct treatment of $V\rho\varpi\rho = 0$, with the hypothesis $\varpi\rho = \bar{\varpi}\rho + e_2 V\mu\rho$, gives for ρ the two following solutions:—

$$\rho = (-\lambda + \nu r), \quad \text{and} \quad \rho = (-\lambda r + \nu)z + \mu y,$$

where $r = \sqrt{\dfrac{h_2 - h_3}{h_1 - h_2}}$; z and y being independent from one another.

The first solution is *determinate*, and, according to (65) of § 11, it represents the normal to the plane R; the second is *indeterminate* in so far as it represents by (66) § 11 any direction parallel to the plane R′ (not R).

When on the curves characterised by $P = 0$, $Q = 0$, the value of W vanishes, and the cubic, $\mathscr{F}\varpi = 0$, reduces itself to $\varpi^3 = 0$, it does not follow that $\varpi = 0$, and neither will $T\varpi\epsilon$ vanish, nor $\mathfrak{T}_1$, $\mathfrak{T}_2$, will, nor $\mathfrak{T}_3$; the only remark to be made is: that the cubic, $\mathscr{F}\varpi = 0$, is founded on the supposition that Q does not vanish, and, when this circumstance takes place, the cubic is to be replaced by the equation from which it was originally deduced, namely, from $\psi = \varpi^2$, in this particular case of $P = 0$, $Q = 0$.

And yet, if the cubic $\mathscr{F}\varpi = 0$ fails in this instance, there exists at any rate a corresponding scalar equation in ϵ (see § 9, (51 ter.)), namely,

$$S\epsilon'\varpi^3\epsilon' = 0,$$

which is the resultant of $P = 0$, $Q = 0$, and in which any tensor may be reintroduced as a factor of the unit vector $\epsilon' = U\epsilon$.

ALPHABETICAL INDEX TO THE

Greek Letters.	Definitions and some Transformations.	Where found first.
α, β, γ	A treble rectangular system of unit vectors,	Introduction.
$\alpha_1', \beta_1', \gamma_1'$;	$\alpha_1, \beta_1, \gamma_1$; $\alpha_1' = (\phi - g)\alpha$, &c.; $\alpha_1 = (\phi' - g)\alpha$, &c.,	Second Part.
Γ	$= (g_1 - g_2)^2 (g_3 - g_1)^2 (g_2 - g_3)^2 = -4P^3 - 27Q^2 = -4F(e^2)$,	§ 3.
Γ^1	$=$ what Γ becomes for $\epsilon = 0$, $\Gamma' = 3[T^2 V(\eta\kappa' - 2\zeta\zeta_1') + 2T^2(\kappa'\zeta - \eta\zeta_1')]$,	§ 6.
δ	$= \alpha + \beta + \gamma$,	§ 3.
Δ	$= -4X^3 - 27Y^3 = -\frac{3^9}{2^4}(P'E + Q')\mathscr{F}_{(1)}(E)$,	§ 7.
ϵ	$= \frac{1}{2}\Sigma\alpha S(\beta\varpi\gamma - \gamma\varpi\beta) = \frac{1}{2}(\sigma - \tau)$,	§ 4.
ϵ'	$= U\epsilon$,	§ 7.
ϵ_1	$= \frac{1}{2}\Sigma\alpha S(\beta\varpi^2\gamma - \gamma\varpi^2\beta) = -\varpi\epsilon \vert,\vert \epsilon_1 = \frac{1}{2}(\sigma_1 - \tau_1)$,	§ 4.
$\zeta, \zeta_1, \zeta_1', \zeta_1''$	$\zeta = \frac{1}{2}(\sigma + \tau)$; $\zeta_1 = \frac{1}{2}(\sigma_1 + \tau_1) = \zeta_1' + \zeta_1''$, $\zeta_1' = \zeta_1$ for $\epsilon = 0$,	§ 4, § 5.
η, η_1, η_1'	$\eta = \Sigma\alpha S\alpha\varpi\alpha$, $\eta_1 = \Sigma\alpha S\alpha\varpi^2\alpha$, $\eta_1' = \eta_1$ for $\epsilon = 0$,	§ 2, § 4.
$\theta, \theta', \theta_{11}, \theta_2$	Auxiliary vectors in various places.	
ς, ς'	Tangents to the surface $\Gamma = 0$.	
$\kappa, \kappa', \kappa''$	$\kappa = \eta_1 - \frac{2}{3}P\delta = \kappa' + \kappa'' \begin{cases} \kappa' = \kappa \text{ for } \epsilon = 0, \\ \kappa'' = \text{the remainder}, \end{cases}$	§ 3.
λ, μ, ν	The three treble rectangular unit vectors, solutions of $V\rho\bar{\varpi}\rho = 0$, so that: $\bar{\varpi}\lambda = h_1\lambda$, &c.,	§ 8.
ξ		§ 3. Digression
$\varpi, \varpi', \bar{\varpi}$	$\varpi = \phi - m_3$, $\bar{\varpi}$ self-conjugate part of ϖ, ϖ',	§ 1.
ρ, ρ'	Solution of $V\rho\phi\rho = 0$, $V\rho'\phi'\rho' = 0$ respectively,	Introduction and Second Part.
ρ, ρ''	Sometimes auxiliary vectors,	§ 10, § 11.
$\sigma, \sigma_1, \sigma_{11}$	$\sigma = \Sigma\alpha S\beta\varpi\gamma$, $\sigma_1 = \Sigma\alpha S\beta\varpi^2\gamma \quad\vert\quad \sigma_{11} = \Sigma\alpha S\alpha\xi$, &c.,	§ 2, and
τ, τ_1, τ_{11}	$\tau = \Sigma\alpha S\gamma\varpi\beta$, $\tau_1 = \Sigma\alpha S\gamma\varpi^2\beta \quad\vert\quad \tau_{11} = \Sigma\alpha S\alpha\xi$, &c.,	§ 3.
Σ	Symbol of summation.	
$\upsilon, \upsilon_\lambda, \upsilon_\mu$, &c.	Normal vectors to surface $\Gamma = 0$, drawn towards the outside of it, $\upsilon = (\varpi - W)\epsilon' \times U(-W)$,	§ 10.
χ	Normal to the cone $E =$ const, drawn towards the outside of it, $\chi = (\varpi - E)\epsilon' U(h_2 - E)$,	
$\chi^{(n)}V\theta\theta'$	$= V[\theta\phi'^{(n)}\theta' - \theta'\phi'^{(n)}\theta] = -[\Sigma S\alpha\phi^n\alpha + \phi^n]V\theta\theta'$,	
$\phi, \phi', \bar{\phi}$	The 1st, *the* linear vector function; 2d, its conjugate; 3d, their self-conjugate part,.	§§
ψ, ψ^1	$\psi V\theta\theta' = V(\phi' - g)\theta(\phi' - g)\theta' \Vert \psi' V\theta\theta' = V(\phi - g)\theta(\phi - g)\theta'$,	Second Part.
ω	An auxiliary vector,	Second Part.

ATIONS IN THE PAPER ON $V\rho\phi\rho = 0$, &C.

Letters, &c.	Definitions and some Transformations.	Where found first.
, B , C , B′, C′	$\left.\right\}$ $A\rho = \psi a$, &c.; $A'\rho' = \psi' a$, &c.,	Second Part.
, (*b*), (*c*)	Designating three conditions concerning the roots of $\Gamma = 0$,	§ 7.
D , D′	$\psi\omega = \rho D$, $\psi'\omega = \rho' D'$,	Second Part.
e	$e_\lambda, e_\mu, e_\nu, e_{I}, e_{II}, e_1, e_2, e_0 \parallel e = T\epsilon$, and particular values,	§ 7.
$_I$, E_o, &c.	$E = -S\epsilon'\varpi\epsilon'$,* and particular values,	§ 7.
ee M_1 *also*)	$F(e^2) = -\frac{1}{4}\Gamma = (e^2 + P')^3 + \frac{27}{4}(Ee^2 - Q')^2$,	§ 7.
$\mathscr{F}$, $\mathscr{F}_{(1)}$	$fg = g^3 - m_2 g^2 +$, &c. $\parallel$ $\mathscr{F}\mathfrak{h} = \mathfrak{h}^3 + P\mathfrak{h} + Q$, $f' = \frac{df}{dg}$. $\parallel$ $\mathscr{F}_1 h = h^3 + P'h + Q'$,	§ 1. § 8.
$_1$, g_2, g_3	The variable of fg, and roots of $fg = 0$,	§ 1.
h_1, h_2, h_3	The variable of $\mathscr{F}_1(h)$, and roots of $\mathscr{F}_1 h = 0$,	§ 8.
G, H	$\psi\omega = (G - H\phi + \phi^2)\omega$,	Second Part.
$_1$, $\mathfrak{h}_2$, $\mathfrak{h}_3$	The variable $\mathfrak{h} = g - m_3$, and roots of $\mathscr{F}\mathfrak{h} = 0$,	Second Part.
$_1$, M_3	Coefficients in $Fe^2 = e^6 - 3M_3e^4 + M_1e^2 - \frac{1}{4}\Gamma^1$,	§ 8.
N, *n*, &c.	Auxiliary scalar factors,	§ 11, &c.
$_1$, m_2, m_3	$m_3 = \frac{1}{3}m_2$, in $fg = g^3 - m_2g^2 + m_1g - m$,	§ 1.
P	$= f'm_3$; $2P = \Sigma Sa\varpi^2a = \eta^2 + 2(\zeta^2 - \epsilon^2) = P' + P''$,	§ 1, § 5.
P′	$P' = (P)_{\epsilon=0}$, $-\frac{2}{3}P^2 = \Sigma Sa\varpi^4a = \kappa^2 + 2(\zeta_1^2 - \epsilon_1^2)$,	§ 4.
Q	$= fm_3$; $3Q = \Sigma Sa\varpi^3a = S(\eta\kappa + 2\zeta\zeta_1 - 2\epsilon\epsilon_1)$,	§ 1, § 5.
Q′ Q″	$\left.\begin{array}{l} 3Q' = 3(Q)_{\epsilon=0} = S(\eta\kappa' + 2\zeta\zeta_1') \\ 3Q'' = +3S\epsilon\varpi\epsilon = -3e^2E \end{array}\right\} Q = Q' + Q''$,	§ 5.
R, R¹	Auxiliaries, R R′, designating 2 planes, cutting $\Gamma = 0$ into circular sections,	§ 9.
s, *s*′	” defined by $h_2 = sh_1$, $h_2 = s'h_3$.	
T, U, V	The usual for scalar, tensor, &c.	
t, *t*′	Two scalar factors: $\left.\begin{array}{r} A a + B\beta + C\gamma = t\rho' \\ A'a + B'\beta + C'\gamma = t'\rho \end{array}\right\} t' = t$,	Second Part.
$\mathfrak{T}$	$= tT\rho'T\rho'$; $\mathfrak{T}^2 = (f'g)^2 - 4[(\phi - g)\epsilon]^2$,	Second Part.
, v, w	In $\omega = ua + v\beta + w\gamma$,	Second Part.
v, *w*, *z*	Scalar auxiliaries,	§ 8, &c.
X, Y	$X = F'(M_3)$, $Y = F(M_3)$, $\Delta = -4X^3 - 27Y^3$,	§ 7.
, *y*, *z*	Carthesian co-ordinates.	
W	Defined by $W^3 = \frac{1}{2}Q$; $W^2 = -\frac{1}{3}P$,	§ 10, and Second Part.

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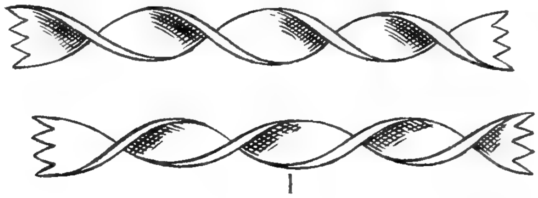
* $\varpi\epsilon' = -\Sigma\lambda h_1 S\lambda\epsilon'$, $E = +\Sigma h_1 S^2\lambda\epsilon'$.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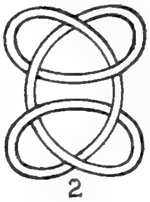
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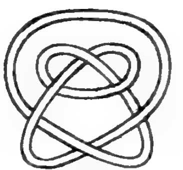
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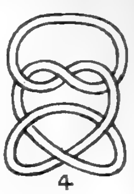
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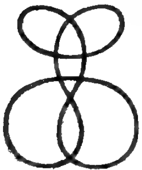
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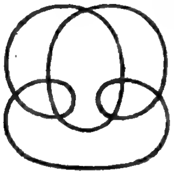
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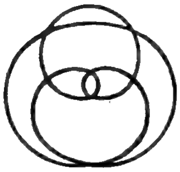
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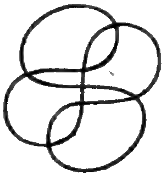
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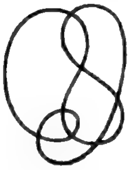
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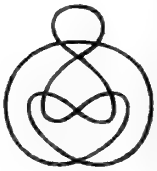
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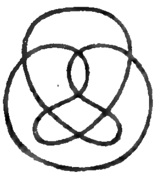
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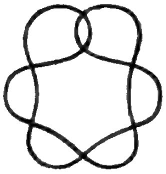
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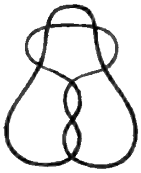
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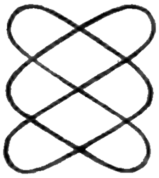
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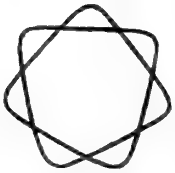
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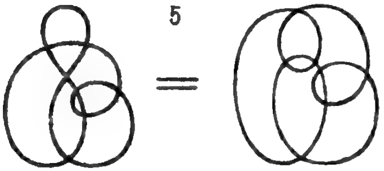
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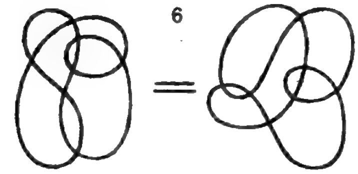
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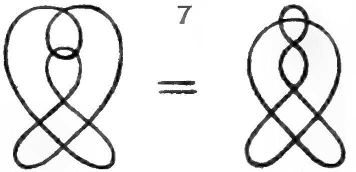
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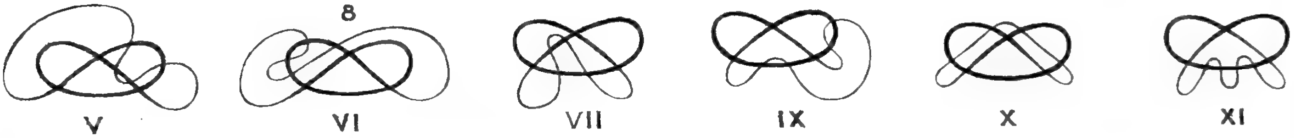
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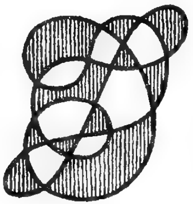
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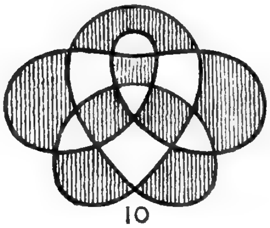
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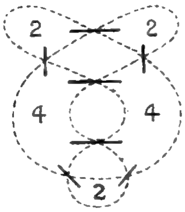

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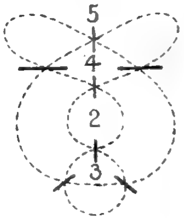

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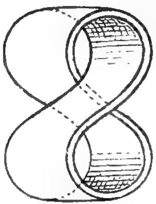
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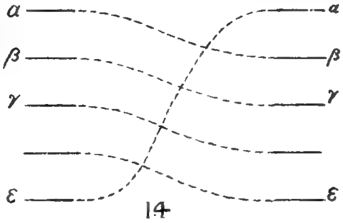

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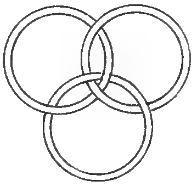
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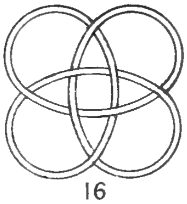
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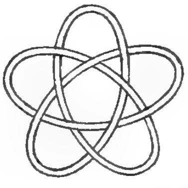
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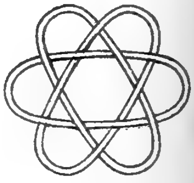
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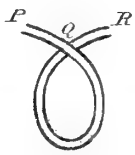

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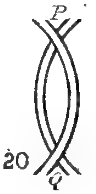

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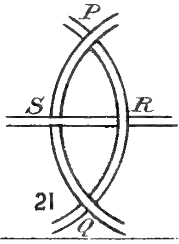

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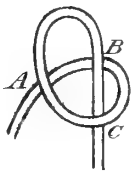

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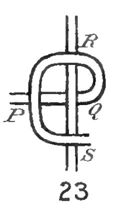

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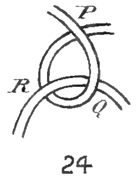

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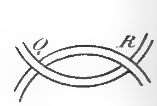

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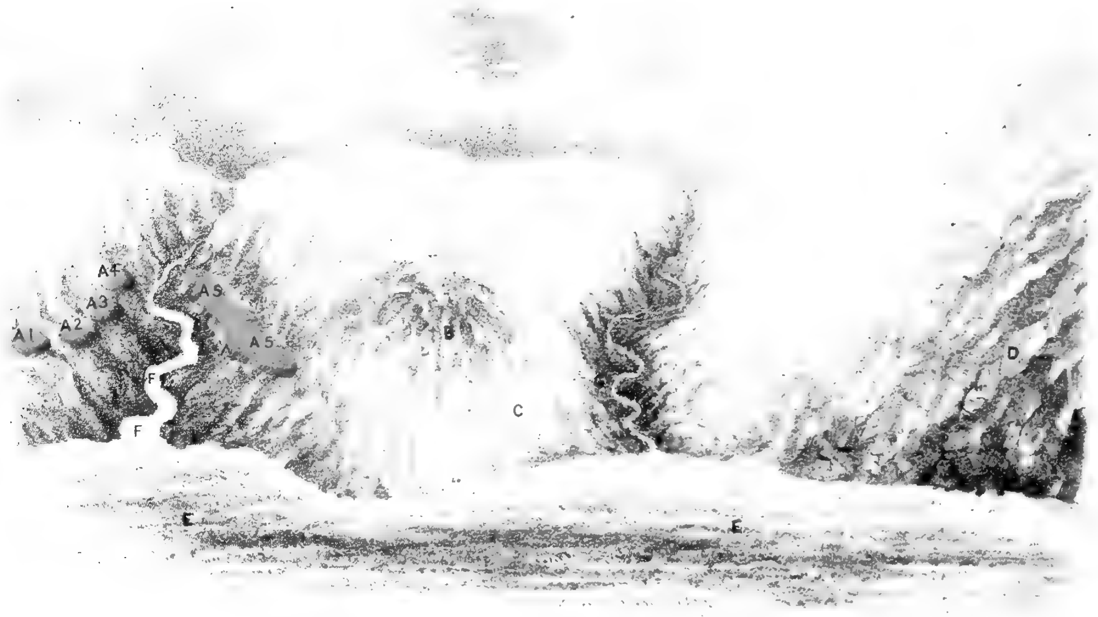
P. G. T.

J. Bartholomew, Edin^r

ROUGH SKETCH FROM MEMORY OF CORRY N' EOIN

(Referred to on Page 99 of Memoir)



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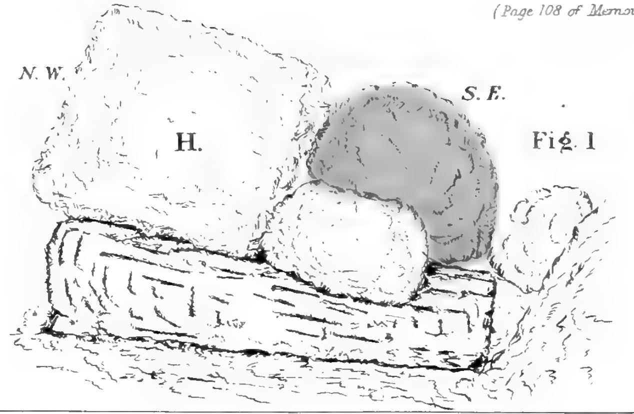

Fig. 1

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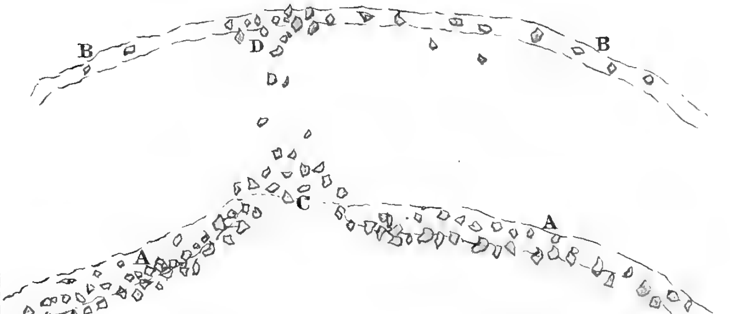
Fig. 2



GLEN SPEAN ESCARS

Covered with Boulders, AA are the highest. Their concave sides front lower part of Glen Spean - BB a smaller range - Groups of Boulders at C & D

GLEN SPEAN BOULDER

Resting on small Boulders, & a Bank facing N.W. by W. towards Inverlair - viz lower part of Glen Spean - Loch Treig bears W. by S.

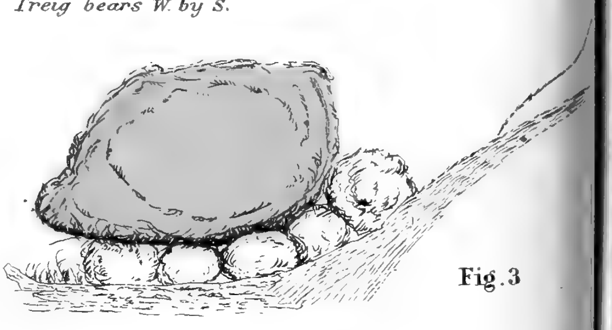


Fig. 3

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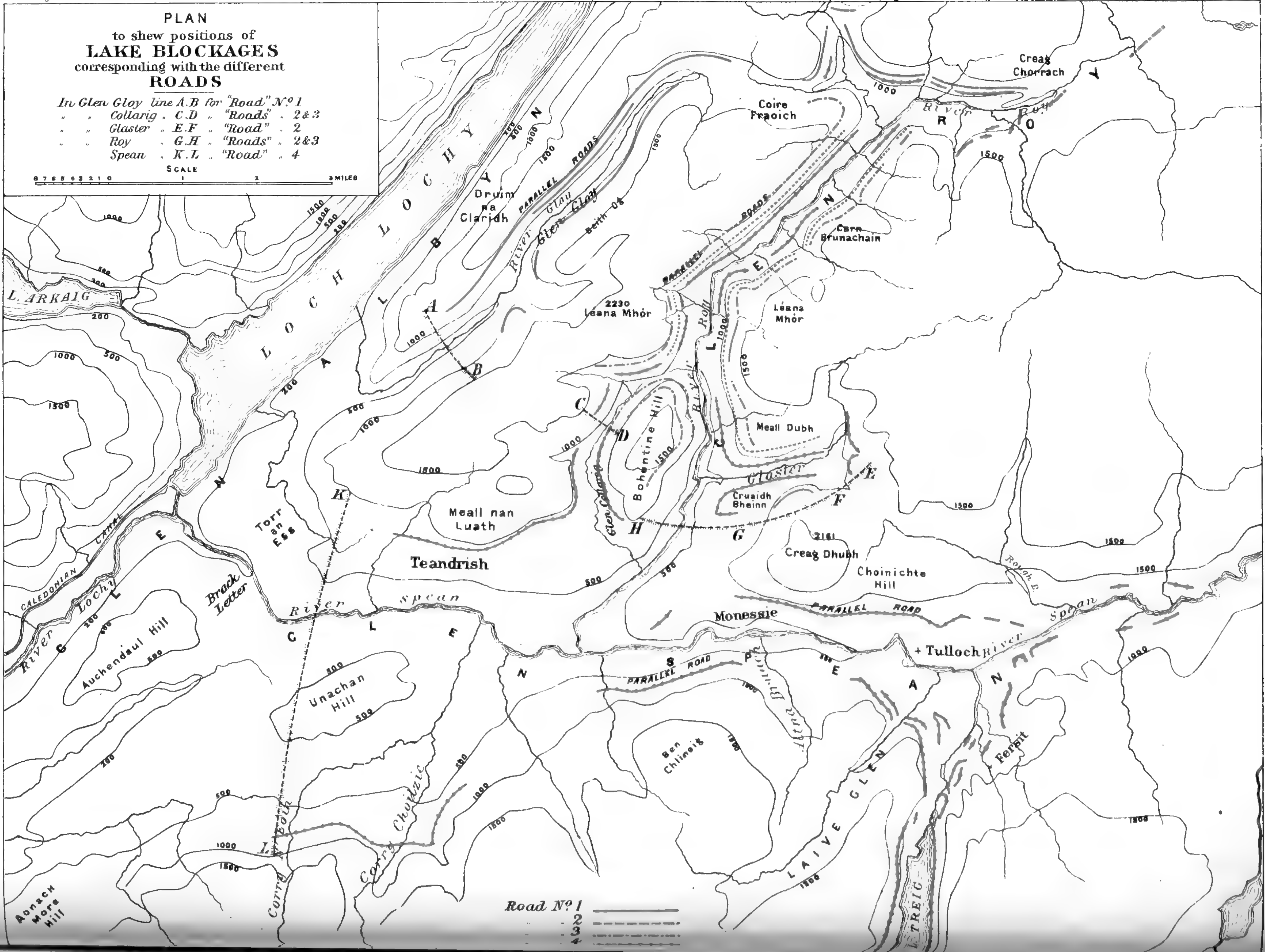
Lithographed by W & A K Johnston, [illegible]



PLAN
to shew positions of
LAKE BLOCKAGES
corresponding with the different
ROADS

In Glen Gloy line A.B for "Road" No. 1
" " Collarig " C.D " "Roads" " 2 & 3
" " Glaster " E.F " "Road" " 2
" " Roy " G.H " "Roads" " 2 & 3
Spean " K.L " "Road" " 4

SCALE

V.—*Additional Memoir on the Parallel Roads of Lochaber.* By DAVID MILNE HOME, LL.D. (Plates XIII., XIV.)

(Read 29th January 1877).

Towards the end of last winter session, a Memoir by me, on the "Parallel Roads of Lochaber," was read to this Society, and it has since been published in our Transactions. The subject was far from being exhausted. Nevertheless, I had no intention of continuing the inquiry, venturing to think, that enough had been adduced by me to support the conclusions at which I had arrived. But during the course of last summer, Dr TYNDALL of London visited Lochaber. He went for the special purpose of studying the "Roads," and of enabling him to give a public lecture regarding them in the Royal Institution, Albemarle Street, on 9th June.

In the course of his lecture, Dr TYNDALL alluded to my recent Memoir, and also to researches described in a previous Memoir. But he dissented from my solution of the problem, and told his hearers (I quote his words) that they might "with safety *dismiss* it (the detrital barrier theory) as incompetent to account for the phenomena. The theory which ascribes the Parallel Roads to lakes dammed by *barriers of ice,* has, in my opinion, an amount of probability on its side, which amounts to a practical demonstration of its truth."

These views having been rested on observations made in the district by Dr TYNDALL himself, I felt that it was only due to a person of his scientific reputation, to reconsider my own opinions, and to weigh well his reasons for coming to a different conclusion.

Accordingly, with Dr TYNDALL's printed lecture in my hand, I revisited Lochaber during last autumn, and I now propose to state the results of my farther researches.

I reserve for the close of this Memoir, a more special notice of Dr TYNDALL's lecture.

Before describing my most recent researches on the Glen Roy problem, let me very briefly notice the heads of the theory which I suggested as a solution of it in my last Memoir.

1*st,* I adduced cases of lakes in the Highlands, and some in the Lochaber district, now kept up in the valleys by blockages of detritus, and at levels above the sea, quite as high as the lakes which formerly filled the valleys of Gluoy, Roy, and Spean. In each of these cases, there were beach-marks on the sides of the valleys, indicating that the lakes had subsided from one level to another.

2d, I pointed out that in the Lochaber district, there is even yet an enormous accumulation of detritus, consisting of beds of clay, sand, and gravel, and that these beds occur at levels far higher than what had been the surface of the old lakes; so that ample materials for blockages at the requisite heights existed.

3d, I showed, by reference to the action of the rivers Roy, Spean, Spey, and other streams, that extensive masses of detritus had been cut through and removed, leaving scaurs or cliffs several hundreds of feet in height, so that it was reasonable to presume that, by similar agency, the blockages of the Glen Roy lakes might have been from time to time cut through and removed.

4th, I farther submitted, that the size or mass of the required blockages should not be estimated, by reference to the width and depth of the valleys at present; because, at the period when these lakes existed, the rivers now running in them must have occupied channels far above their existing channels.

These being the grounds on which I supported the detrital theory, I now proceed to mention the further observations recently made confirmatory of these grounds.

I. *Localities, where beds of Sand, Clay, and Gravel, at High Levels occur.*

1. In the district of *Stratherrick,* which is not far from Lochaber, on the east, I followed the course of the River Foyers up to the mountains, among which it takes its rise. This river runs into Loch Ness, on the south side.

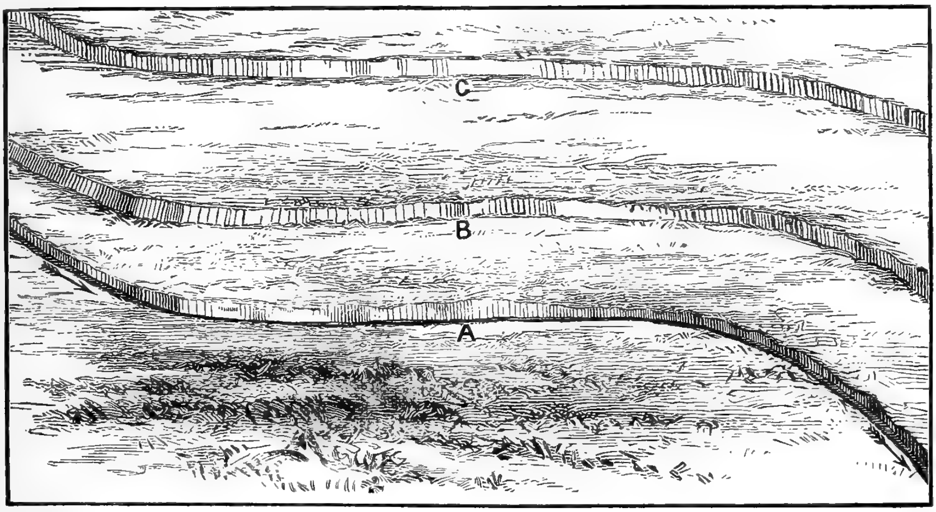
In various parts of its course, there are old haughs bounded by cliffs, showing the successive levels from which the river subsided, cutting through enormous deposits of sand and gravel.

About 3 miles above the upper Fall of Foyers, I took a rough sketch of two of these haughs. Both were bounded by steep cliffs, which had evidently been river banks, the one about 20 feet, and the other about 60 feet above the river.

The following diagram (page 95) exhibits these old river haughs and cliffs:—

In company with Captain Fraser of Balnain, who has a shooting lodge near the source of the river, I followed its course, till we reached a height above the sea of about 1774 feet. On each side of the river the hills are covered by great hummocks of sand and gravel, and occasionally clay containing pebbles and boulders. I did not ascend farther, but with the telescope I observed knolls and scaurs of detritus at least 300 feet higher; and learnt from Captain

Fraser that similar deposits exist to the very top of the ridge dividing Strath Errick from Strath Spey, at a height of about 2500 feet above the sea. In the adjoining glens of "Glen Markie," "Corry-an-Yack," and "Alt-our," I learnt that similar drift deposits exist, and at about the same heights above the sea.



A, present Channel of Foyers River.
B, line of old River, cliff about 20 feet above A.
C, line of older River, cliff about 60 feet above A.

In the upper parts of the Foyer River, there are numerous terraces formed on the drift, at heights of from 40 to 60 feet above the stream. Sloping as they do *down with* the stream, they must have been formed when its channel was at a higher level.

II. *Localities where Lakes exist, dammed by Detritus, and showing Subsidence or Disappearance.*

Loch Killin is traversed by the River Foyers. It is about half a mile long, and about 200 yards wide. At its west or lower end, there is a terrace on each side, from 40 to 50 feet above the present level of the lake.

Loch Duntelchak is situated the S.W. of Inverness, and about 8 miles distant.

There is an old detrital blockage at its lower or east end, through which the stream now issuing from the loch had evidently cut its present channel.

The mounds of drift, which formed the blockage, are about 40 feet above the lake, indicating that the lake had been that much higher.

In walking along the north bank of the lake, I found an old beach line about 40 feet above its present level.

At that time, there must have been a communication between *Loch Duntelchak* and *Loch Aschley,* situated about half a mile to the north, and the level of which is 16 feet above the level of Loch Duntelchak. The channel of communication between the two lakes is very manifest. It is now filled by a bed of peat from 10 to 15 feet in thickness. Below the peat, there is a bed of waterborne gravel, and below the gravel a bed of marl or clay.

Loch Farraline is in Stratherrick, and about 6 miles south of Loch Ness. It is about 620 feet above the sea.

It is now at least 50 feet below the level to which its waters once reached. Balnain House, belonging to Captain FRASER, on the south side of the loch, is on a flat which had been part of the bottom of the old lake. At Gorthleg, on the north side of the lake, there is asimilar flat, at the same height.

These flats with a bounding cliff are traceable distinctly along the south side of Stratherrick valley, for a distance of 6 or 8 miles. The present loch is about one mile in length.

But to allow of the lake standing 50 feet higher, and to extend so far beyond its present limits, a great blockage must have existed to the west of Boleskine, which blockage has entirely disappeared. This blockage probably consisted of the detritus, still existing in thick beds everywhere in Stratherrick, and which must have been removed by the Foyers and other rivers, now meandering through the valley.

Near the upper Fall of Foyers, at *Glenlia,* there has been a small lake, about a mile in length, through which the River Foyers had flowed eastward, to unite with the River Inverfarrigaig. The lake waters appear to have reached a rent or fissure in the rocks, by the wearing away of the detrital cover, and through which rent the River Foyers now flows more directly into Loch Ness. The effect of this change in the course of the river was to drain the Glenlia Lake.*

III. *Probable Position of the Blockages of the Lochaber Lakes.*

1. In my last paper, I pointed out exactly where the blockage in Glen Collarig occurred, its position being indicated by the termination of the several shelves, as shown on the Ordnance Map.

Before passing from that blockage, I may advert to the impossibility of accounting for the separation of the two sets of shelves, in any other way than by a detrital blockage, situated at a part of the glen intervening between the ends of the two sets of shelves.

The two uppermost shelves, 2 and 3 of Glen Roy, terminate in Glen Collarig at a point shown on the map. The lake which formed them

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* For many examples of ancient lakes, altogether or partially drained off in consequence of the wearing down of blockages of detritus, see a recent work on "The Jammoo and Kashmir Territories," by Frederic Drew, F.G S., 1875.

must therefore have stopped there. What hindered the lake extending farther down the Glen? Some blockage extending across the Glen must have existed.

It is also worthy of observation that, when the lake subsided from Shelf 2 to Shelf 3 (a fall of 81 feet), the lake was enabled to extend farther down Glen Collarig by about 40 yards. This is evident from the circumstance that Shelf 3 can be distinctly seen, and it is marked on the Ordnance Survey as terminating 40 yards beyond where Shelf 2 terminates.

This state of matters will be better understood by referring to fig. 10, on page 611, and to Plate XLII. of my former Memoir.

2. If it be established, as I venture to think it is, that the blockage in Glen Collarig, which kept in the lakes of Shelves 2 and 3, and separated these from the lower lake represented by Shelf 4, was detritus, and not ice, there should be the less hesitation in accepting a similar blockage for Glen Roy.

Sir HENRY JAMES, in his one-inch Ordnance Map, has indicated the position of two lake barriers in Glen Roy, calling them "*Ice-Barriers.*"

One of these barriers crosses Glen Roy a little above Cranachan, where the valley is about a mile wide, and the bottom of the valley about 800 feet below Shelf 2.

The other barrier Sir HENRY JAMES marks on his map as crossing Roy Valley between Cranachan and Boheenie. To reach Shelf 3 at its two extremities, this barrier must have been 1½ mile long. The bottom of the valley is here about 700 feet below Shelf 3.

In my last Memoir, I observed that the first of these barriers need not necessarily be at the place indicated by Sir HENRY JAMES. I suggested that it might have been at the head of Glen Glaster, where the col reaches a height of 1075; so that at this col, a blockage of only 81 feet in height instead of 800 feet, and a quarter of a mile wide instead of one mile, would be sufficient. This spot, therefore, is the more probable for the required blockage of Shelf 2.

I farther then stated, that the necessity of adopting this position instead of Cranachan would be established, were it ascertained that Shelf 2 extended into Glen Glaster. On my last visit, I discovered traces of Shelf 2 *on both sides of Glen Glaster*, so that there is now no room for farther question on this point.

With regard to the blockage for Shelf 3, between Cranachan and Boheenie, I admit that the difficulty of magnitude remains. But that difficulty any theory of barriers must encounter; for as to the fact of there having been a blockage here, of some kind, all are agreed.

The only question is—whether it was detritus or ice?

That there is, even yet, on the south side of the valley, an enormous accumulation of muddy detritus, must be perceived by any one who examines Shelves 3 and 4 in this locality. The facility with which this detritus is cut through and removed by streams is indicated in many places. It is owing to the same cause that Shelves 3 and 4, especially the former, are wider here than elsewhere, as the Ordnance Map (on the 6-inch scale) shows.

This point being of extreme importance towards the settlement of the question, I have given a map (see Plate XIII.) indicating the position of the different blockages. It will be observed that the blockages in Glen Glaster and Glen Roy (EF and GH) form one line. The most probable supposition is, that the blockages at both places were due to the same mass of detritus prevailing over the whole of this district.

A considerable stream, as the Ordnance map shows, now crosses the place where that blockage existed, so that it would be exposed to the risk of being cut through, and its materials removed by the operation of running water.

The succession of changes on the blockages, to allow of the subsidence and extension of the lakes, would be as follows:—

(1.) When the lake of Shelf 2 was flowing over the col at the head of Glen Roy, a lowering of the Glen Glaster blockage (EF) took place, first to the extent of 14 feet, next to the extent of 36 feet, and lastly 32 feet more, when the surface of the lake would reach the rocky col between Glen Glaster and flow out towards the Rough Burn.*

If the blockage there consisted of detritus, no long time would elapse between the successive erosions; and, accordingly, the "Roads" formed at these intermediate lines are only discernible at a few places.

(2.) The next great subsidence was from Shelf 3 to Shelf 4; the vertical distance between the two being 211 feet.

This was effected by a lowering of the Boheenie blockage (HG). But it was not all accomplished at once. A shelf intermediate between 3 and 4 was discovered by Mr Jolly and me in Glen Collarig, at a distance below Shelf 3 of 78 feet. This lower line was pointed out by us to the Ordnance Surveyors.

Moreover, above Shelf 4, between the mouth of Loch Treig and the Laire Burn, there is a Shelf about 30 feet above Shelf 4, indicating another intermediate subsidence.

The Boheenie blockage (GH), was therefore lowered from time to time, till it was totally removed, and thereafter the waters of the Glen Spean lake passed up into Glen Roy and Glen Glaster.

(3.) The only other blockage requiring notice is, that which kept in the lake of Shelf 4, viz., extending across the Unachan Moor, between Teandrish and Corry N'Eoin. It is marked by the thick line KL on the map (Plate XIII).

I agree with Sir Thomas Dick Lauder, in the position which he assigned to this blockage. At my last visit, I think I discovered a remnant of it where it had joined the steep bank to the north of the Corry.

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* These intermediate shelves are described by Dr Chambers and myself in our respective previous Memoirs. See also my last Memoir, p. 600.

On Plate XIV., there is a rough sketch (from memory) of the range of hills, looking at them from the eastward. The mouth of Corry N'Eoin is on the extreme left, a part of Aonach Mor (rocky hill) is D, on the extreme right. The red patches marked A^1 to A^5 indicate flats, which being on a level with Shelf 4, I consider to be remnants of it. F is the principal stream which flows out of the Corry upon the flat meadow land E. B is a projecting rock. C is a bank of detritus, cut through by the stream G, and forms a projecting bank.

On this detrital projection, there is no trace of Shelf 4. I therefore infer that the lake had not reached so far north. But, undoubtedly, there are traces of the lake in the mouth of the Corry, and on both sides of it, to the south-east of the above-mentioned detrital projection, as shown by the red patches.*

A line drawn from this projection across Unachan Moor to Teandrish (see line KL on Plate XIII.) indicates what would naturally be the line of blockage, being at right angles to the central axis of Glen Spean.

Unachan Moor reaches now to a height of 613 feet above the sea, which is only 243 feet below the level of Shelf 4; and on various parts of the moor there are unmistakable signs of great erosion.

The moor consists, as Dr CHAMBERS long ago stated, of an enormous mass of soft materials, chiefly gravel and sandy mud; so that denudation is quite intelligible.

There are powerful mountain torrents from the steep hills here, which afforded ample means of erosion at each end of the blockage. The stream now flowing through Corry N'Eoin seems at a former period to have flowed out upon the plain through a channel more to the west, in which case it would have had a greater effect in removing the blockage.

IV. *Supposition that Glaciers may have been formed in Corry N'Eoin and Loch Treig.*

1. In my previous Memoir, I pointed out that, even had there been glaciers in these glens, the levels of the country and the contours of the hills would not have admitted of their flowing to the places in Glen Roy and Glen Collarig, where the blockages were required.

The site of the Glen Spean blockage (between the north side of Corry N'Eoin and Teandrish) might have been occupied by a glacier from Corry N'Eoin, *if it were possible that a tongue of ice, five miles long,* could have protruded from that small Corry, and been pressed against the hills at Teandrish so tightly as to dam back Loch Spean.

2. But the fatal objection to the whole of this glacier theory is, that neither in Corry N'Eoin nor in Loch Treig, could there have been a glacier *at the time* when these lakes which formed the "Parallel Roads" existed; for, on an

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* See reference to this Corry at pages 631 and 632 of former Memoir.

examination of both glens, it turns out that at this "Parallel Roads" period, *these glens themselves were partially occupied by the Glen Spean Lake,* which formed Shelf 4.

(1.) With regard to Corry N'Eoin, a second visit to it last autumn enabled me to confirm my previous observation, that evidence of that lake having entered the corry is furnished by a series of flats on each side of its mouth, at exactly the level of Shelf 4, viz., 856 feet above the sea. (See sketch on Plate XIV.)

Sir THOMAS DICK LAUDER says that he also traced Shelf 4 into the mouth of Corry N'Eoin. He states (page 44) that this shelf, "though faint, is easily followed to a ravine called *Corr-a-Choilich,*[*] whence I thought I could even trace it, though with some little difficulty, *through an opening in a thin birch-wood,* on the side of Aonach Mor, nearly *as far as the projection of that mountain,* where all appearances of it are finally lost."

From this description, it is evident that Sir THOMAS LAUDER traced the shelf *beyond Corry Choilzie,* and through a thin birch wood, nearly as far as a projection from the hill called Aonach Mor. Now, there is a thin birch wood at the mouth of Corry N'Eoin. That is the place where the flats occur, and at a level exactly coincident with Shelf 4.

With Sir THOMAS LAUDER, I allow that the traces of the shelf here are faint. But even if there were no traces, it matters little, while there exists at the mouth of Corry N'Eoin a large accumulation of detritus; for such would undoubtedly have been swept clean away, had a great glacier flowed out of the Corry to form a huge ice-barrier stretching across to Teandrish.

(2.) With regard to Loch Treig, the only other place suggested for a glacier, I had likewise an opportunity of confirming my previous observations —that Shelf 4 certainly runs along both sides of its valley.

On this last occasion, I had the good fortune to obtain the use of a boat belonging to DONALD CAMERON, an intelligent shepherd, who, when I met him, was going from the foot of the loch to his dwelling at the head of the loch. From the boat I distinctly observed, as I passed along, two lines of beach, one about 40, the other about 90 feet above the water, the latter being about the level of Shelf 4.

At a distance of 2 miles from the foot of the lake, I landed on the north bank, at a sandy beach, where there was a large bank of detritus with a flat top, about 90 feet above the loch. One part of this bank being cut through by a stream from the hill, I saw that it consisted of detritus very similar to that prevailing in Glen Spean and Glen Roy. I found in the gravel some of the

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[*] This ravine, now known under the name of "Corry Choilzie," is situated a few hundred yards to the east of Corry N'Eoin. At Corry Choilzie Shelf No. 4 is quite distinct. Beyond Corry Choilzie, and towards Corry N'Eoin, the shelf exists only in patches.

pink-coloured Felspar pebbles which occur in Glen Spean, and there occasionally forming boulders, which are supposed by some of my friends to have been brought there by a glacier from Loch Treig. These pebbles I showed to Mr Cameron, and asked him if there were any rocks of the same kind in the hills adjoining Loch Treig. He replied, that he had never seen any in the Loch Treig hills, but that he had seen them about two miles to the west.

Finding that I had not time to go to the head of the loch, I drew Mr Cameron's attention to the mound of detritus on which we had been standing. I had also previously shown to him similar mounds at the foot of Loch Treig, and asked him whether mounds of the same kind existed at the head of the loch ? He said that there were such, a road having been cut through one of the mounds near his own cottage, which showed much sand and fine gravel in it.

On walking back to the foot of Loch Treig, I ascertained by aneroid, that the detritus in several places on its north bank reached to a height of fully 200 feet above the lake.

If ever glacier had been formed in Loch Treig and flowed out of it, it must have been at a period antecedent to the time when detritus had been laid down on its banks.

Not only is there detritus on both sides of Loch Treig, and bearing occasionally the impress of two water lines, but just below the foot of the loch where the river emerges from it, there are enormous masses of detritus, which, cut through by the River Treig and by its tributaries, exhibit vertical scaurs from 60 to 70 feet deep. On the top of this detritus, there are on the south side of the river, and close to the loch, two extensive flats evidently due to the action of a lake. The lowest seemed to correspond with the height of Shelf 4, visible on the opposite side of the river. The Ordnance Survey Map, however, makes it 10 feet higher.

But the important fact is undoubted, that here, as well as at Corry N'Eoin,—both inside and outside of these Glens, from which glaciers are imagined to have flowed into Glen Spean,—there is an enormous accumulation of detritus. It is upon this detritus, as all parties admit, that the "Parallel Roads" have been impressed; so that if glaciers ever existed in these glens and flowed out of them into the low country, it must have been at a period *before* the detritus was laid down, and the lake beaches formed.

V. *How the Detrital Blockages of Glen Gloy and Glen Spean were removed.*

In my last Memoir, I ascribed the removal of the blockages to one cause, viz., the agency of streams flowing through the main valleys, and also of streams rushing down upon the detritus, from the steep sides of the mountains adjoining.

I also (page 621) hinted at the possible action of the sea upon these blockages, when the sea stood at higher levels.

This last conjecture has now been strengthened, if not verified, by two things—*first,* evidence of the finding of sea shells on Unachan Moor at two places, at heights of from 200 to 400 feet above the sea; *second,* the recognition of flats or terraces, apparently marine, at heights from 350 to 450 feet above the sea.

In order not to interupt my argument, I put what I have to state on both of these points in an Appendix (see Notes A. B.)

Assuming, then, that when the Glen Gloy and Glen Spean lakes existed, at heights of 1150 and 856 feet above the sea respectively, the sea was at a height of say 500 feet above its present level, what would be the effect of this sea action on the detrital barrier?

The action would consist not merely of waves and tides, which on a cliff of soft materials would be considerable, but also of a current running through the Great Glen, now occupied by the Caledonian Canal, a current caused by the times of high water being different at the two ends of the kyle or strait.* It is also not improbable, that the sea might at this period have had masses of ice floating in it. The transport of the immense boulders which now lie high up on the mountains here and elsewhere in the Highlands, certainly indicates that when the sea stood at heights of 1200 and 2000 feet, it must have had in it huge masses of floating ice; and it is no unlikely supposition, that when the sea fell to 500 feet, it still had ice in it. I need not say how much greater, in that case, the effect of a sea current would be in undermining a cliff of soft materials composing the supposed lake blockages of Glens Gloy and Spean. (See Plate XIV. for the position of these blockages.)

It is also deserving of notice, that when the sea stood at the greater heights above mentioned, there probably was a strong ocean current from the W.N.W., because the direction of the parent rocks of the Lochaber boulders leads to that conclusion; and if this oceanic current continued when the sea had sunk to the level of 500 feet, the blockages of Glens Spean and Gloy would be exposed to the full force of that current.

VI. *Effect of the Removal of the Glen Spean Blockage.*

If this blockage was eroded and undermined by the united action of land streams and of sea, so as to allow of the escape of the waters of the lake, what else would happen?

The sea would then have free scope to flow up Glen Spean a certain distance, till stopped by the rising slope of the land.

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* According to Admiralty tide tables, when it is high water at Inverness, it is low water at Fort-William, with a difference of 12 feet between high and low water.

On the other hand, it does not follow that the whole of the old Glen Spean Lake would be drained off at once. There is evidence, indeed, that immediately after the rupture of the Teandrish barrier three smaller lakes, at lower levels, were formed. One of these still subsists, now Loch Laggan, and of the other two there are well-marked vestiges.

Loch Laggan forms a body of water, the level of which is about 40 feet below the original Glen Spean Lake, and now flows out, at its west end, instead of, as formerly, its east end. A trench through the detritus at its west end, of about 40 feet deep, allows its surplus water to escape down the valley of the Spean by the river Spean.

Formerly this river ran into a lake at a lower level, the western or lower end of which reached to near Inverlair. Its surface was about 640 feet above the sea. Its old beach-line is still visible, as is also the channel of the river by which its waters flowed down into the third lake. This third lake extended from Tulloch to near Monessie, a distance of about 3 miles, and stood at a level of 520 feet above the sea. Its beach-line, at that height, can be distinctly traced on both sides of the valley.

This lake must have existed for a long period, or down to a comparatively recent date, judging from the breadth of its old beach-lines.

I have in my previous Memoir explained, that this lower lake had been dammed by a blockage of detritus at its west end, and which had been cut through at one side, leaving the rest of the blockage still standing. (Page 609.)

The narrow passage in the rocks through which the river now rushes at Monessie, seems to have been an original fissure in the rocks, which probably had been at a former period so filled and choked with detritus, that the waters of the lake did not reach it.

Before this lower lake was drained off, the water from it would flow over the ridge which crosses the valley at Monessie, and form a stream reaching the sea somewhere near the Roman Catholic Chapel.

An old river course is visible, to the north of the present river channel, between the river and the turnpike road, at a height of about 420 feet above the sea, or about 80 feet above the present channel of the river at this place.

It seems not improbable that whilst this lower lake existed,—the sea reached up to near Monessie, in which case the fall from the lake to the sea may not have been more than 50 or 60 feet.*

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*It is a curious circumstance that the old Celtic names of several places in Glen Spean are in accordance with the conclusions of geological observation and reasoning. The Ordnance Map marks a spot at Inverlair as *Ceann-a-Mhuir*, which means the head of the sea, or lake. At Inverlair there is now neither lake nor sea. Was there a lake, reaching up to Inverlair, when this name was given? The Map likewise marks a spot lower down Glen Spean Valley as *Ceann-na-Mara*, which has exactly the same meaning as the above, though varying in form, in the same way as "Loch-end" and "End of the Loch." Can this refer to the west end of the supposed loch, or can it refer to the sea, When the sea began to retire, the river discharging from the lake would acquire more power of erosion, and would cut out for itself lower channels as the sea continued to subside.

VII. *The Glacial Markings in Lochaber, and their bearing on the Parallel Roads question.*

Having explained the grounds on which I consider that the blockages of the lakes were due to accumulation of detritus at the mouths of the glens, it is right that I should advert to the ground on which the ice theory rests.

There are undoubtedly marks of land ice in several of the glens. The upper part of Corry N'Eoin, at a height of about 1350 feet above the sea, is exceedingly narrow,—not more than a few hundred yards wide, with rocky sides, almost perpendicular. The floor of the valley is also rock; and in one part shows evidence of ice having moved down the valley, by long groovings and striations, in a direction parallel with the axis of the valley.

So also at and below the mouth of Loch Treig, there are rocks smoothed and striated, which seem to show, though not so unequivocally as in Corry N'Eoin, that ice has passed over these rocks—from Treig Valley.

But neither of these valleys is of sufficient size, as regards width, length, or depth, to have generated glaciers, even in the most favourable climate, of the dimensions required for the alleged ice barriers, and still less for reaching the sites of these barriers.

Independently, however, of this difficulty, it is important to observe at what period these glaciers existed. It was at a period in the world's history long antecedent to the formation of the Lochaber Lakes. It is quite evident that the detritus now in the district must have come at a period subsequent to the grinding and striation of the rocks;—because, in numerous places, these rocks are seen to be covered by the detritus.*

Now, it was not till after this detritus had been deposited, that the Parallel Roads were formed, because it is on the detritus that they were formed, as every geologist who has visited Lochaber, allows.

Moraines, it is alleged, occur in Glen Spean, and they are referred to as proving that large glaciers must have existed to produce these Moraines.

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which, when the name was given, came up to a point not far from this? "*Mur-laggan,*" situated on the north bank of the supposed lake, signifies "hollow by the sea or lake." "*Monessie,*" or "*Munessie,*" situated at the lower or west end of the supposed lake, signifies the plain by the waterfall. Was this the waterfall from the lake over the ridge or barrier of the lake? The word *Muir*, which makes its genitive in *Mara*, is evidently the same word as the Latin *Mare*, the English *Mere*, the French *Mer*, &c.

* See my last Memoir, p. 641, and JAMESON, *Geol. Soc. Proc.* vol. xix. p. 241.

Thus moraines are alleged to have been left by the Glen Treig glacier; and at Murlaggan there are large mounds, which have been so termed. I have carefully examined these mounds. They are composed entirely of beds of stratified sand, or sand and mud;—so that they cannot be moraines. They have been deposited by water—either the sea, or the waters of Lake Spean—for they are below the level of Shelf 4. So also, in the district between Craig Choinichte, Rough Burn and Fersit, there are huge lines of escar,—the materials composing which, consisting chiefly of coarse gravel, are at first sight, and when looked at from a distance, somewhat like moraines.

If there was a glacier from Corry N'Eoin, and of the size required to form a great ice barrier across Unachan, at least 4 miles long, that glacier should have left enormous moraines, both lateral and terminal, on Unachan Moor, and on the hill of Teandrich, against which the glacier must have pressed. But there are no such moraines. Some appearance of a moraine I observed in Corry N'Eoin itself, at a height of about 1100 feet above the sea. But if it be a moraine, its position within Corry N'Eoin shows that the glacier which formed it never reached so far as the mouth of the glen.

There have been some things ascribed to the action of glaciers which, as it strikes me, are due to a totally different cause.

(1). The smoothed and striated rocks, high up on the hills, are far above the reach of any imaginable glaciers. In my last Memoir, I pointed out various examples of such rocks on Craig Dhu at heights from 1400 to 1800 feet above the sea.

Mr Jameson takes special notice of rock smoothings at even greater heights. Thus, near the foot of Loch Treig, he mentions smoothings and scorings occurring up to 1280 feet; and he adds, "Not that I can affirm even this to be their upper limit; for on the mountain at the opposite side of the gorge I found the scoring fade away so gradually at these great heights, owing to the weathering of the rock, that I was unable to satisfy myself where it ended, *perched boulders,* and rounded surfaces occurring much higher; *and even up to the top,* which I made out to be about 3055 *feet bove the sea,* the gneiss, though it runs here in nearly vertical stratifications (dipping N.W. at an angle of about 70° or 80°), *is nevertheless so free of any loose fragments* on its surface, and the ends of the strata are often *so rounded in the outline, as to raise a suspicion that some denuding agent has flowed over it,* at a period geologically recent" (Geol. Soc. Proc. vol. xviii. p. 172).

This statement, alike of fact and of opinion, coming from a geologist so experienced as Mr Jameson, I consider of much importance. It is entirely in accordance with the view I have advocated, that perched boulders and smoothed rocks, on the sides and tops of mountains, at heights of from 2000 to 3000 feet, cannot possibly be ascribed to glaciers, but are due to ice floating in a sea, which overtopped the mountains.

The denuding agent of which Mr JAMESON speaks, flowing over the hills at a height of 3055 feet, leaving great boulders on the top, but sweeping off all smaller fragments, can scarcely be conceived to be anything else than a sea loaded with floating ice.

(2). With regard to the enormous mounds of gravel, and multitudes of boulders resting on them, situated in Glen Spean between the Rough Burn and Loch Treig, and which have been called the moraines of the Treig Glacier, I may observe, that any glacier which could ever have come from Loch Treig must have been far too insignificant to have produced effects on so large a scale. Moreover, several of the gravel mounds are at heights (viz., 1500 above the sea) which could never have been reached by any glacier flowing out of Glen Treig, where the waters of the lake are now only 740 feet above the sea.

In my last Memoir, I threw out a conjecture, that these so-called moraines were submarine banks, formed when the sea prevailed here, at a level of 2000 or 3000 feet above its present level.

This conjecture has been strengthened by a more special examination. At one end, viz., to the east of Loch Treig, the embankments run on lines nearly horizontal, and along the face of a hill, in an east and west direction, at a height of about 1500 feet above the sea. They then change their direction, and run first in a nearly north-east direction, and afterwards due north, forming two or more concentric crescents or curves, and about 200 yards apart, whose concave sides face down the valley of Glen Spean. They are even continued to the opposite side of the valley on the hill called Coinichte, situated to the north of Rough Burn. (See Map on Plate XLIII. of former Memoir.)

These embankments are, in respect of continuity and shape, more numerous and distinct when they are above the level of Shelf 4, which is 856 feet above the sea. Below that shelf, they were of course covered by the waters of the old Glen Spean Lake; and when that lake, or a large portion of it, rushed down Glen Spean, on the rupture of its blockage, the embankments in the central and lowest part of the valley would be to a great extent broken up; and cliffs or banks would be formed, approximately parallel with the axis of the valley. Such banks do occur along the course of the Spean, in the centre of the valley.

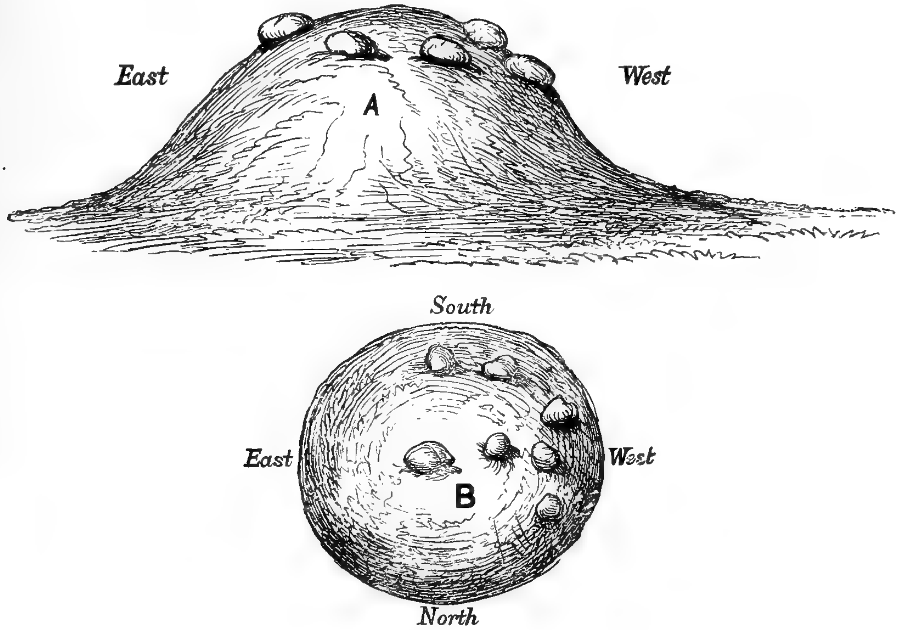
If the idea of submarine banks be adopted, the sea at this place may have been from 1000 to 2000 feet deep. In that view there would be a narrow passage at the west end, viz., between Ben Chlinaig and Craig Dhu, with a strong current running through it from the west,—and at the east end, viz., near Mukkul, a similar narrow passage; whilst in the district between Treig and Rough Burn, there would be a wide basin, with little current, where the gravel banks would be formed by eddies, many examples of such occur in sea charts. It appears also that in the Arctic regions such gravel banks and lines of boulders are occasionally due to the action of "pack ice."

In *one of the long circular banks* which stretch across this flat valley, there is a *singular breach* with an arrangement of boulders, which suggest the idea of injury by an iceberg, or more than one, coming across it from the west, and breaking through it, discharging cargoes of boulders at this place and beyond it. (See fig. 2 on Plate XIV.)

If these embankments are all due to one cause, I cannot conceive anything else to explain them, than sea currents bearing pack ice flowing in upon their concave sides. These currents, to produce the effects observed, must have flowed from the N.N.W. up Glen Spean.

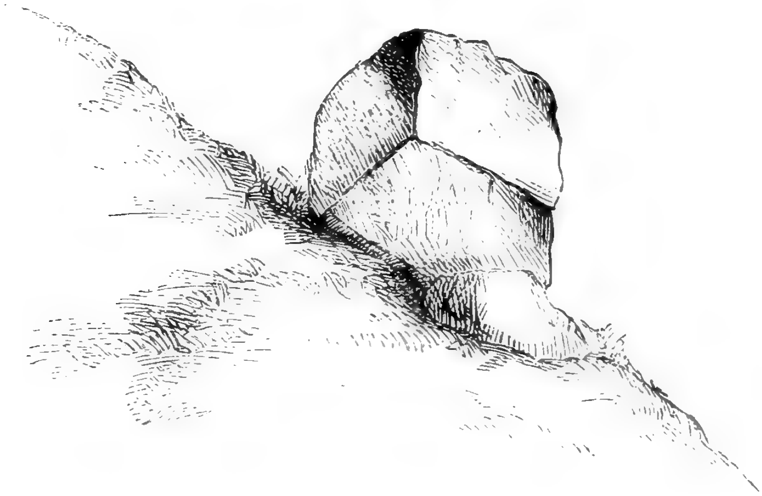
To the same conclusion I have been led by a study of the *boulders* on these banks. The vast majority of the boulders are on the concave slopes of the banks, and seem to have been dropped there by the ice on which they floated, being stopped by the banks in its farther progress eastward.

There are several knolls on this extensive flat district, which stand up from 50 to 60 feet above the general surface, whose tops are thickly crested with boulders. The tops of these knolls are generally rock, in which case the top, especially on its N.W. side, is smoother than any other. One of these knolls I found consisted entirely of detritus; five large boulders were on its top. It is represented in the following woodcut.



Knoll covered by boulders. A, Section; B, Ground Plan.

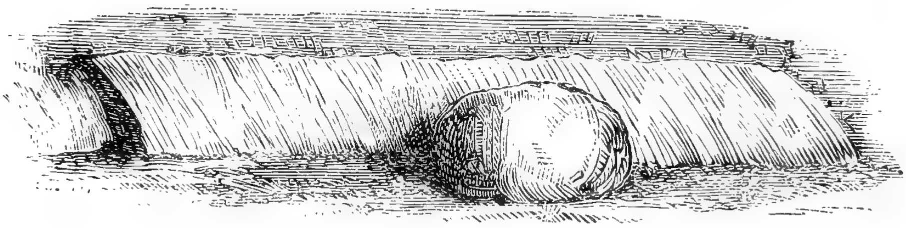
The following woodcut of an angular boulder about 8 feet square, resting on a steepish hill facing down Glen Spean, leads to the same conclusion. Former writers have assumed that all the boulders in this locality must have been transported from Loch Treig. If this boulder had been brought, whether by glacier or by floating ice, from Loch Treig, the hill on which it rests would not have intercepted it in its progress eastward.



Glen Spean.—Boulder resting on a hill which faces N.N.W., viz., towards lower part of the valley, indicating that it came up Glen Spean. If it had come from Loch Treig, which bears W.S.W., boulder would not have stuck where it is.

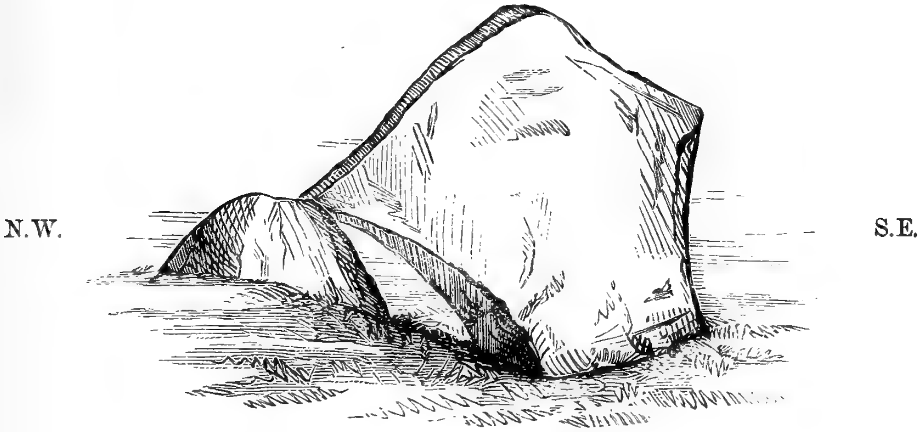
Figs. 1 and 3 on Plate XIV. show similar cases. The boulder marked H, on fig. 1, could not have obtained its position except by being brought there after the smaller boulders, against which it presses, had been deposited. This boulder H, must therefore have come from the N.W.—viz., up Glen Spean.

The following woodcut leads to exactly the same conclusion.



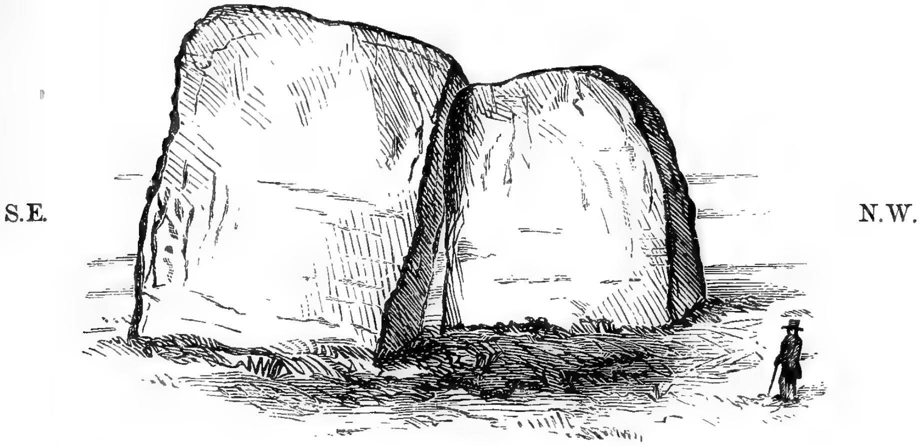
Glen Spean.—A granite rock, smoothed by some body passing over it from N.W. Length, about 20 feet; height, on an average, 4 feet. Smoothed face fronts N.W. Loch Treig bears from rock W.S.W.; centre of Glen Spean, N.W. by W. Smoothing agent therefore probably came up Glen Spean. The boulder, lying in front, has also probably come up Glen Spean—its further progress up Glen Spean having been intercepted by the rock.

The following woodcut is a case which very frequently occurs.



Glen Spean.—Large boulder, partly leaning on smaller boulder. The former apparently came from N.W. in order to obtain its position. Lower part of Glen Spean lies to N.W. If Boulder had come from Loch Treig (which lies to W.S.W.), it would have gone past smaller boulder, and not rested on it.

The following woodcut, is another case of the same kind.



Glen Spean.—Large boulder, 3 feet high and 5 feet wide, leaning on a smaller boulder. A line drawn through the point of contact and the centre of gravity of the large boulder runs N.W. by N., *i.e.*, down the centre of Glen Spean, indicating that the boulder came up Glen Spean. If it had come from Loch Treig, it would not have been in this position.

VIII. *Dr Tyndall's Lecture.*

Dr Tyndall in the outset states, that being an old student of glacial action, it was not inappropriate that he should take that side in the discussion.

Before explaining to his audience the grounds on which he supported the glacier theory, Dr TYNDALL endeavoured to combat the only theory opposed to it, viz., the theory of detrital blockages, first suggested by the late Sir THOMAS DICK LAUDER.

My own share in this question Dr TYNDALL was well aware of; for he refers to the two Memoirs which I read in this Society; but he gave no statement whatever of the facts and arguments advanced in these Memoirs. He went back to Sir THOMAS DICK LAUDER's paper, 59 years old, for an exposition of the grounds on which the detrital theory was maintained.

I am sorry to have to add that, in explaining to his hearers Sir THOMAS LAUDER's views, Dr TYNDALL inadvertently gave a version of these views not strictly correct.

The passage in Dr TYNDALL's lecture to which I refer, is as follows:—

"There are at the present moment vast masses of detritus in certain portions of Glen Spean. Out of such detritus, Sir THOMAS LAUDER imagined his barriers to have been formed. *By some unknown convulsion, the detritus had been heaped up.*"

Now, I affirm that Sir THOMAS LAUDER never supposed that the detrital barriers suggested by him had been *heaped up,* and still less that any *convulsion* effected that object. Nor has any supporter of the detrital theory entertained such an unlikely, not to say absurd, idea. The barriers which Sir THOMAS LAUDER supposed had kept in the lakes, consisted (to use his own words) of the "large depositions of alluvial clay, sand, rounded pebbles, and gravel, which present themselves everywhere, and more particularly towards the mouths of the different valleys."

Sir THOMAS considered that the "alluvial depositions," as he termed them, which overspread the hills and filled the valleys, supplied the required blockage. He did not imagine that to form a blockage, the detritus had been heaped up, either by a convulsion or by any special agency. The detritus existing naturally in the district, in his opinion, constituted the blockage.

The only difficulty which Sir THOMAS LAUDER had, was in regard to the *removal* of the barriers. He himself states, in regard to these, that it was "much easier to suppose the existence of them (the barriers), than to devise the means which operated in their removal" (page 51).

With regard to this last question, he referred to the Great Glen now occupied by the Caledonian Canal, as being probably an "immense rent produced by some extraordinary convulsion;" and he suggested that the convulsions attending the creation of this rent would so rive and shatter the country as to affect the lake barriers, and particularly those of Glen Gluoy and Glen Spean (page 56).

When Sir THOMAS LAUDER referred to a convulsion, it was not in explanation of how the lake barriers had been heaped up, but how they might have been ruptured and broken down.

Dr TYNDALL has unfortunately made another mistake in his representation of Sir THOMAS LAUDER's views. He says, that to explain the formation of the middle Parallel Road, "Sir THOMAS *invoked a new agency*," viz., "*a halt*" in the "breaking down or waste of his dam." I take leave to say that Sir THOMAS LAUDER invoked no agency whatever to explain the formation of the middle Parallel Road. The lake subsided by several steps from the uppermost to the middle Parallel Road, and stood there long enough to form that middle Parallel Road. The barrier held firm at that point long enough to allow of a strong beach line being formed. What new agency Dr TYNDALL can allude to, as having been invoked or invented by Sir THOMAS LAUDER, I cannot imagine.

The only other remark made by Dr TYNDALL by way of objecting to the detrital theory is, that "*no barriers of detritus could have existed, without leaving traces behind them.* But (he says) there is no trace left. The two highest Parallel Roads stop abruptly at different points near the mouth of Glen Roy. No remnant of the barrier against which they abutted is to be seen;" and he quotes an opinion, said to have been given and "insisted on by Professor GEIKIE, that *barriers of detritus would undoubtedly have been able to maintain themselves,* had they ever been there" (pp. 6 and 12.)

On this special ground, and, so far as I can discover, on this ground alone, Dr TYNDALL told his audience, that they "may with safety *dismiss the detrital barrier theory.*"

With regard to the opinion ascribed to Professor GEIKIE, I cannot help thinking, that the views of the learned Professor must have been misapprehended. No geologist has shown better than Professor GEIKIE himself, the enormous denudation of detritus effected by such agents as some of those suggested for the erosion and removal of the Lochaber blockages.

In Professor GEIKIE's popular work, entitled "Scenery and Geology of Scotland," the following passage occurs:—

"Let any one stand on the ice-worn barrier of rock between Loch Ness and Loch Oich. He will see there, that even on the supposition of an open fissure, the deep concavity of the (Great) Glen at this point *must be due to denudation.*" "The very arrangement of the rocks is enough to prove that the hollow has been worn out *by the agencies of nature. The glen at Fort Augustus* must be due mainly to *denudation*" (p. 178).

Farther, to show the effects of denudation on detritus, the Professor refers to the removal of lake barriers by the same agencies. He mentions that near Carstairs "the Kaimes stretch across the mouth of a broad valley, *where they must at one time have dammed back the drainage, so as to form a lake.* Since then they have been cut through by the Mouse water, and the lake has thus been drained. But its *site* is still visible" (p. 312).

Looking to these passages, and others, in Professor GEIKIE's writings, it is difficult to understand how he should have given it as his opinion, that had detrital barriers existed in Glen Roy to dam back the lakes, they "would undoubtedly have been able to maintain themselves, and be still extant."

There would, of course, be a greater probability of removal by rain and streams, if the detritus forming the blockage consisted of soft sandy mud. Now, as previously noticed, at and near the places where the Glen Roy and Glen Collarig blockages occurred, there is great abundance of such kind of detritus. To that circumstance is owing the great breadth of Shelf 3 on the N.E. shoulder of Craig Dhu, extending to 100 yards, as indicated even on the Ordnance 6-inch Map. To show how easily detritus of a soft muddy character may be eroded by rain and small burns, reference may be made to the circumstance that Shelf 4, which must have existed round the west shoulder of Craig Dhu and Meall Dherry, is for more than a mile not traceable. It, however, must have at one time existed there. As the lake reached to this part of the valley, a beach line must have been formed here as elsewhere, and the only explanation is, that the beach line at this place was washed away by the stream descending the sides of the hills.

The same remark applies to the discontinuance of Shelf 4 on the N.E. side of Ben Chlinaig. It will be seen from the map of Lochaber, that for more than a mile, the shelf is not traceable on this hill-side. But it must at one time have existed there also. The materials which composed it have been removed by streams and rain.

I have in the foregoing remarks assumed the correctness of Dr TYNDALL's statement, that not a trace is to be now seen of the detrital barrier by which the lake in Glen Roy was kept up at its two successive levels. But regarding the correctness of this assumption, some doubt exists. In my previous Memoir (p. 620) I mention that, at the places where Shelves 2 and 3 terminate in Glen Roy, there are banks of detritus in a direction transverse to the valley, which greatly resemble the remnants of a barrier.

I would only add, that I observe in Dr TYNDALL's lecture, with satisfaction, a full admission of the enormous extent of detritus abounding in the Lochaber district. He speaks of the "*vast masses of detritus* in certain portions of Glen Spean" (p. 5); and of "the friable drift *over-speading the mountains*" (p. 4). He, moreover, explains (p. 2) that "the Parallel Roads are terraces *formed in the yielding drift, which here covers the slopes of the mountains.*"

Having made these admissions, Dr TYNDALL must at all events allow that the materials were ample for forming the required blockages.

IX. *Dr Tyndall's Glacier Views.*

Dr TYNDALL's main position is that "*Glen Spean was at one time filled by a great glacier.* To the disciplined eye (he says) the aspect of the mountains is *perfectly conclusive* on this point" (p. 10).

How was Glen Spean so filled? He gives this answer,—"It is not difficult to restore in idea the process by which the *glaciers of Lochaber were produced, and the glens dammed by ice.* The great collecting ground of the glaciers which dammed the glens, and produced the 'Parallel Roads,' were the mountains south and west of Glen Spean. When the cold of the glacial epoch began to invade the Scottish hills, the sun at the same time acting with sufficient power upon the tropical ocean, the vapours raised and drifted on those northern mountains were more and more converted into snow. *This slid down the slopes, and from every valley, strath, and corry south of Glen Spean, glaciers were poured into that glen*" (p. 11).

Here we are presented with what must be admitted to be a very remarkable theory. The valleys, straths, and corries entering Glen Spean from the south were filled with *ice;* whilst the valleys on the north side of this same Glen were filled with *water.* Such a state of things implies an enormous difference of temperature in these respective valleys, though all are in one district of inconsiderable area. Yet Dr TYNDALL suggests nothing to show that such a difference of temperature between the two sides of Glen Spean must, or could have existed. The *ice* valleys are at about the same altititude above the sea as the *lake* valleys, and within a few miles of one another. That surely is a difficulty which deserved explanation. It is true that on the south side of the Glen, there is, as Dr TYNDALL observes, Ben Nevis, which is higher than any of the hills on the north side of the Glen. But how Ben Nevis, because higher, should have produced the wonderful effects of filling the valleys on one side with ice and those on the other side with water, and keeping them in that exact state for hundreds of years, so as to give time for the Parallel Roads to be formed, it is very difficult to understand.

And a more serious difficulty remains—Glen Spean is said to have been filled by a great glacier, which, crossing the mouth of Glen Roy, is supposed to have dammed the lakes in Glen Roy. But Glen Spean, at the very time that a lake filled Glen Roy, as previously shown, *was itself occupied by a lake.* One of the Parallel Roads is visible on both sides of Glen Spean,—as Dr TYNDALL himself allows, and represents on his map of the district. For this reason, the very possibility of a glacier in it, at the time when the Glen Roy Lake existed, is excluded.

And where did this supposed Glen Spean glacier come from? Dr TYNDALL says, "that glaciers were poured into it from every valley, strath, and corry, opening into that glen from the south." No one who has examined the locality, has ventured to point out any other valleys than two, viz., Loch Treig and Corry N'Eoin, from which glaciers might have come into Glen Spean.

But it now turns out, that at the very time that lakes filled Glen Roy and Glen Spean, a lake existed in Loch Treig also; nay, it was the same lake which existed in these three glens,—Shelf 4 being traceable in all of them. The existence of this shelf in Loch Treig is assumed by AGASSIZ and CHAMBERS. Sir THOMAS DICK LAUDER describes this shelf as surrounding Loch Treig. He so represents it in his map. I can myself vouch for having seen traces of Shelf 4, in the lower parts of Loch Treig,—the only parts examined by me: What is more, the map annexed to Dr TYNDALL's lecture represents Loch Treig as surrounded by the same "Parallel Road" as that in Glen Roy and Glen Spean!

With regard to *Corry N'Eoin,* any glacier from it, instead of flowing up towards Glen Roy, would, in consequence of the levels of the country, have flowed in a direction nearly opposite.

The barriers at Bohenie and in Glen Collarig admittedly necessary for keeping in the lakes which formed Shelves 2 and 3, are 7 or 8 miles distant from Corry N'Eoin. Before any glacier formed in that Corry could have pushed out a tongue of ice to form a barrier,—it had to cross a large extent of uneven surface of country, and must also have wheeled round several projecting hills, and have risen up at least 400 feet above its own original level!

But in this glen also, the existence of any glacier at the period when the Parallel Roads were formed, is more than doubtful. Having twice visited that Corry, I satisfied myself that the lowest shelf, or Parallel Road No. 4, exists at the Corry, near its south side; and that a mass of detritus exists at the mouth which would have been swept away had any glacier issued from that glen. (See Diagram on upper part of Plate XIV.)

In concluding and bidding farewell to the whole discussion, I offer the following programme of the various changes which appear to me to have taken place.

1*st.* Local glaciers occupied the valleys of the Highlands, so that the rocks occupying the floors of the valleys were smoothed and striated. Moraines were occasionally formed at the mouths of these valleys.

2*d.* A change then took place in the relative levels of sea and land. The land sunk, or the sea rose, so that the mountains of the country were submerged to the extent of 3000 feet or more. An oceanic current from the W.N.W. prevailed, bringing masses of ice loaded with boulders. The effect was to grind and round off the tops and sides of our mountains, and deposit on many of them, especially on their N.W. sides, boulders of all sizes.

3*d.* During this period of submergence, and as the sea retired or subsided, beds of clay, sand, and gravel were deposited, being the debris of rocks broken down and carried off by the sea and ice, from the submerged hills.

4th. When the sea subsided to such a level as to expose hill ranges, the force of the N.W. current would increase by being more confined between these hill ranges,—as in the Great Glen of Scotland, and the strath which runs through Lochaber into Strath Spey.

Then probably the detritus, previously forming beds more or less horizontal, would be formed into kaims or escars, whose direction would depend chiefly on the direction of the currents and tides.

5th, In reference to the curved kaims or escars in the part of Glen Spean formerly described, it will be remembered that as these are (at one end) 1500 feet above the sea; the sea must, when they were being formed, have been considerably above that height. On Ben Erin, one of the Glen Roy hills, there is a water line at a height of 1870 feet. At about the same height, there is a water line in Corry N'Eoin, on the rocky hill on the north side of the Glen. It was pointed out to me by Lord Abinger's gamekeeper.

A rapid current would at this time pass up Glen Spean, between Ben Chlinaig and Craig Dhu, both of which hills exceed 2000 feet in height, and this current would pass over into Strathspey. Glen Spean, whilst forming a narrow pass between Ben Chlinaig and Craig Dhu, opens out into the broad flat before described, occupied by the kaims and boulders. Just where the Glen so opens out, there stands a rocky hill called "Dun Dearg Mor," well bared on all sides, and particularly the west, rising to a height of about 800 feet. This rocky hill might cause a division of the current, as it flowed eastward, the larger portion flowing towards Treig, the smaller towards the Rough Burn. These streams, after curving past the adjoining hills, would unite farther east, and flow on through that part of the valley now occupied by Loch Laggan towards Strath Spey.

6th, Until the sea had subsided to a level below 1100 feet, none of the Lochaber lakes could have been formed. But in reference to materials for the blockage of these lakes, it is not unimportant to remember, that the extensive kaims just alluded to, consisting of detritus, exist at a level of 1500 feet, which is more than 300 feet above the highest of the required blockages, and that on the hills near the Rough Burn, there are beds of detritus 1700 feet above the sea.

In these circumstances, there is every reason to presume, that in the three valleys where blockages were required, viz. at 1170 and 856 feet, detritus must have *filled* the glens to the requisite heights, and that they were removed by natural agencies before explained.

7th, Lastly, I may observe, that whilst believing that the detrital theory is that which best explains how the lakes were dammed up, I can understand how other theories should have been suggested, and should have so long held their ground. The theory which ascribed the formation of the roads to the sea, a theory suggested by DARWIN, and supported by CHAMBERS and NICOL, had a certain amount of truth to rest on. So also the theory of glaciers;—(for the production of ice barriers,) was very naturally adopted, when rocks smoothed and striated were seen to occur in the district. Both of these theories were started, before all the facts necessary for a full representation of the question had been discovered. Confessedly, much has been ascertained since these theories were started. The facts so discovered suggest objections to the marine and glacier theories which, had these facts been known, probably would have prevented their adoption;—whilst other facts recently discovered seem (to me at least) to add greatly to the strength of the evidence on which the detrital theory rests.

APPENDIX.

Note A (p. 102.)

When at Lochalsh last September, I learnt that the innkeeper there, of Balmacara Hotel, Mr MACDONALD, had formerly been a residenter at Spean Bridge. In the course of conversation about the Parallel Roads, he expressed an opinion that they were sea-beaches. On asking his reason for thinking so, I was told by him that, when making drains, he had found in the land, under the peat, beds of sea-shells.

Not having time to take a note of this conversation, I requested the schoolmaster of the parish, Mr DUNCAN SINCLAIR, who was present, to make a written memorandum of it, and send it to me. The following letter was the result:—

"SCHOOLHOUSE, LOCHALSH, 20*th Sept.* 1876.

"DEAR SIR,—I have seen Mr MACDONALD, and his answers to your queries are—

"1. Year of finding shells?—About 35 years ago.

"2. What field found in:—'Acha-na-bo-ban' (White Cow field), about 1½ mile from Spean Hotel.

"3. Kind of shells?—Two or three kinds of wilks or periwinkles.

"He says that they were longer and more tapering than the ordinary edible sort, and of a bluish colour.

"4. He cannot give the name of any particular person who was along with him at the time of the shells being turned up. He says his companions of that period are mostly all dead, or abroad now."

After my conversation with Mr MACDONALD, but before receiving Mr SINCLAIR'S letter, I visited Lochaber, and saw the Rev. Mr CAMERON, minister of the parish. He stated that he knew Mr MACDONALD personally, and that he was an intelligent and trustworthy person.

I asked Mr CAMERON to make inquiry among the persons now residing at Spean Bridge, whether they had ever heard of sea-shells having been found in the neighbourhood.

After inquiry, Mr CAMERON reported to me that he had seen several persons who had heard a report to that effect, and that he had found one person, a respectable shopkeeper at Spean Bridge, who had seen the shells in a drain near the upper part of Unachan Moor, at a height of about 600 feet above the sea.

On my return home, I received Mr SINCLAIR'S letter. I transmitted it to Mr CAMERON, who returned it with the following answer:—

"BLAIR-OUR, KINGUSSIE, 13*th Oct.* 1876.

"DEAR MR MILNE HOME,—Ach-na-bo-ban is close on two miles from Spean Bridge, on the road to Fort-William. Peats have been cut all over the locality, and I should say the elevation of it is 20 feet higher than Spean Bridge, which is 211 feet above the sea.

"PETER M'FARLANE, the shopkeeper at Spean Bridge, declares that he himself had in his hands the shells seen on the top of Unachan hill, and was quite satisfied that they were sea-shells."

Note B (p. 102).

(1.) At *Brackletter*, on the left bank of the River Spean, about a mile from its junction with the Lochy, an extensive terrace of gravel occurs at a height of 430 feet above the sea.

(2.) On the right bank of the same river, and nearly opposite to Brackletter, several flats occur of detritus. One of these is a hill of detritus called "*Torr-an-Ess*," the top of which the Ordnance Survey makes 427 feet above the sea. Other flats to the N.W. from this hill, at the same level, are within sight of this hill.

(3.) On the same side of the river, about 2 miles higher up, near the turnpike road, there is a place called "*Blair-our*," with a shepherd's house, showing an extensive flat, bounded by a steepish cliff, at a height of 430 feet.

(4.) From this point, a good view can be obtained of the "*extended moor*" *of Unachan*, as Sir Thomas D. Lauder calls it, and on running the spirit-level along its slope, to the north, several terraces, at exactly the above level, are detected.

(5.) Having proceeded to those Unachan Terraces, I observed some flats on the side of the hill of Teandrish, both to the north and to the east of the manse occupied by the Rev. Mr Cameron.

(6.) In an old note book, I find the following entry, "There is an evident terrace on this (Unachan) hill, running towards Fort-William and approaching within 6 miles of it. It is by barometer 391 feet above the sea. Discovered that this same terrace runs far eastward even beyond High Bridge, places called Raw and Torinesh (Torr-an-Ess) being on it." *

The altitude of this flat is no doubt 40 feet lower than that of the places previously mentioned, but if an estuary prevailed, the sea-bottom would be lower towards the sea than near the head.

(7.) *Near the base of the Aonach More*, where limestone rock shows itself, covered with detritus, there are numerous "*pot-holes*" in the rock, besides detrital flats. The height is about 400 feet above the sea, but I cannot state it more precisely.

(8.) There are several places *in the upper parts of the River Spean*, above its junction with the River Roy, where very conspicuous terraces exist, at nearly the same height with those above mentioned at Brackletter and Torr-an-Ess. At the Roman Catholic Chapel I made the height 438 feet above the sea. Robert Chambers, in his "Sea Margins," notices these levels, and considered them to be at the same level.

(9.) In several parts of the district embraced by the foregoing observations, there are lower terraces, which appear horizontal. Thus at *Dalnabee* and *Inverroy*, there are extensive terraces about 349 feet above the sea. Near Liannachan (about 2 miles north of Corry N'Eoin) there is a terrace, occupied by boulders, 356 feet above the sea.

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* This terrace has a historical interest. My guide informed me that, in the year 1745, advantage was taken of it to form a rampart for cannon bearing on Fort-William; and he showed to me what he called the embrasures.

VI.—*Least Roots of Equations.*

By J. Douglas Hamilton Dickson, B.A., F.R.S.E., Fellow and Tutor of St Peter's College, Cambridge.

(Read 26th March 1877.)

(I.) Let $F(x) = 1_0 + b_1 x + \ldots + b_n x^n = 0$ be an equation whose roots are all real and different; then

$$-\frac{\phi(x)}{F(x)} = \ldots \Sigma\left(\frac{\phi(r)}{F'(r)\,.\,r^{p+1}}\right)x^p + \ldots \qquad . \qquad . \qquad . \qquad (1)$$

where $\phi(x) = 1 + a_1 x + \ldots + a_m x^m$, and Σ indicates summation for all the roots of $F(x) = 0$, of which r is a root.

Let also

$$\frac{\phi(x)}{F(x)} = A_0 + A_1 x + \ldots + A_p x^p + \ldots \qquad . \qquad . \qquad . \qquad (2)$$

then

$$(1 + a_1 x + a_2 x^2 + \ldots) = (1 + b_1 x + b_2 x^2 + \ldots)(A_0 + A_1 x + \ldots),$$

and equating coefficients of like power of x,

$$\begin{array}{rrrrll}
A_p + b_1 A_{p-1} + & b_2 A_{p-2} + & \ldots & + A_0 b_p & = a_p \\
A_{p-1} + & b_1 A_{p-2} + & \ldots & + A_0 b_{p-1} & = a_{p-1} \\
 & A_{p-2} + & \ldots & + A_0 b_{p-2} & = a_{p-2} \\
 & \ldots & \ldots & \ldots & \ldots \\
 & & & A_1 + A_0 b_1 & = a_1 \\
 & & & A_0 & = 1\,,
\end{array}$$

whence

$$A_p = (-)^p \begin{vmatrix} b_1, & b_2, & \ldots\ldots & b_p, & a_p \\ 1, & b_1, & \ldots\ldots & b_{p-1}, & a_{p-1} \\ 0, & 1, & \ldots\ldots & b_{p-2}, & a_{p-2} \\ . & . & . & . & . \\ & & \ldots\ldots & b_1, & a_1 \\ & & \ldots\ldots & 1, & 1 \end{vmatrix}_{p+1} \qquad . \qquad . \qquad . \qquad (3)$$

the suffix number indicating the order of the determinant.

Comparing the two expansions of $\frac{\phi(x)}{F(x)}$,

$$(-)^{p+1}\begin{vmatrix} b_1, & b_2, & \dots & b_p & , a_p \\ 1, & b_1, & \dots & b_{p-1}, & a_{p-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ & & \dots & b_1 & , a_1 \\ & & \dots & 1 & , 1 \end{vmatrix}_{p+1} = \Sigma\left(\frac{\phi(r)}{F'(r).r^{p+1}}\right).$$

If p be indefinitely increased, that term on the right hand side of the above equation will alone be comparable with the expression on the left hand side, which contains the smallest value of r, independent of its sign. In this case

$$\underset{p=\infty}{L}(-A_p) = \underset{p=\infty}{L}(-)^{p+1}\begin{vmatrix} b_1, & b_2, & \dots & b_p & , a_p \\ 1, & b_1, & \dots & b_{p-1}, & a_{p-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ & & \dots & b_1 & , a_1 \\ & & \dots & 1 & , 1 \end{vmatrix}_{p+1} = \frac{\phi(r)}{F'(r).r^{p+1}}$$

considering *now* r to be the smallest root of $F(x) = 0$.

From this

$$\underset{p=\infty}{L}\frac{A_p}{A_{p+1}} = r$$

or, if it be remembered, that $p = \infty$,

$$A_p - A_{p+1}r = 0. \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

As an example, consider the quadratic $x^2 - 5x + 6 = F(x) = 0$, and let $\phi(x) = F'(x) = 2x - 5$; then will

$$\frac{-5+2x}{6-5x+x^2} = -\frac{5}{6} - \frac{13}{6^2}x - \frac{35}{6^3}x^2 - \frac{97}{6^4}x^3 - \frac{275}{6^5}x^4 - \frac{793}{6^6}x^5 - \frac{2315}{6^7}x^6 \dots.$$

The series of values given by $\frac{A_p}{A_{p+1}}$ as p increases from 0 to 6 is as follows

$p =$	0	1	2	3	4	5	6	....
$r =$	2·31	2·23	2·17	2·12	2·08	2·06	2·04	....

in which the approximation of r to the smaller root 2 is evident.

II. Let $u_{2p+4} = A_{p+1}^2 - A_pA_{p+2}$

$$= \Sigma \left[\left(\frac{\phi(r)}{F'(r)}\right)^2 \frac{1}{r^{2p+4}} + 2\,\frac{\phi(r)\,.\,\phi(s)}{F'(r)\,.\,F'(s)}\,\frac{1}{(rs)^{p+2}} \right.$$

$$\left. - \left(\frac{\phi(r)}{F'(r)}\right)^2 \frac{1}{r^{2p+4}} - \frac{\phi(r)\,.\,\phi(s)}{F'(r)\,.\,F'(s)} \left(\frac{1}{r^{p+1}s^{p+3}} + \frac{1}{r^{p+3}s^{p+1}}\right)\right]$$

$$= \Sigma \left[\frac{\phi(r)\,.\,\phi(s)}{F'(r)\,.\,F'(s)}\,\frac{(r-s)(s-r)}{(rs)^{p+3}}\right] ;$$

when p increases indefinitely, supposing r and s to be the two smallest roots of $F(x) = 0$,

$$\underset{p=\infty}{L}\; u_{2p+4} = \frac{\phi(r)\,.\,\phi(s)}{F'(r)\,.\,F'(s)}\,\frac{(r-s)(s-r)}{(rs)^{p+3}} \,.$$

Again, let $-u_{2p+3} = A_{p-1}A_{p+2} - A_pA_{p+1}$

$$= \Sigma \left[\left(\frac{\phi(r)}{F'(r)}\right)^2 \frac{1}{r^{2p+3}} + \frac{\phi(r)\,.\,\phi(s)}{F'(r)\,.\,F'(s)} \left(\frac{1}{r^ps^{p+3}} + \frac{1}{r^{p+3}s^p}\right) \right.$$

$$\left. - \left(\frac{\phi(r)}{F'(r)}\right)^2 \frac{1}{r^{2p+3}} - \frac{\phi(r)\,.\,\phi(s)}{F'(r)\,.\,F'(s)} \left(\frac{1}{r^{p+1}s^{p+2}} + \frac{1}{r^{p+2}s^{p+1}}\right)\right]$$

$$= -\,\Sigma \left[\frac{\phi(r)\,.\,\phi(s)}{F'(r)\,.\,F'(s)}\,\frac{(r-s)(s-r)(s+r)}{(rs)^{p+3}}\right]$$

when p increases indefinitely, supposing r and s to be the two smallest roots of $F(x) = 0$.

$$\underset{p=\infty}{L}\; u_{2p+3} = \frac{\phi(r)\,.\,\phi(s)}{F'(r)\,.\,F'(s)}\,\frac{(r-s)(s-r)\,.\,(s+r)}{(rs)^{p+2}} \,.$$

Hence, remembering that p is to be increased indefinitely

$$\frac{u_{2p+3}}{u_{2p+4}} = r + s$$

$$\frac{u_{2p+2}}{u_{2p+4}} = rs \,,$$

therefore r and s are the roots of the quadratic equation

$$u_{2p+4}\,x^2 - u_{2p+3}\,x + u_{2p+2} = 0 \,,$$

that is, of

$$(A_{p+1}^2 - A_pA_{p+2})\,x^2 - (A_pA_{p+1} - A_{p-1}A_{p+2})\,x + (A_p^2 - A_{p-1}A_{p+1}) = 0 \,,$$

or, of

$$\begin{vmatrix} x^2 & , x & , 1 \\ A_p & , A_{p+1} , & A_{p+2} \\ A_{p-1} , & A_p & , A_{p+1} \end{vmatrix} = 0 \quad . \quad . \quad . \qquad (5)$$

Take the same quadratic as above for an example; the above determinant quadratic is the same as the given equation; for we have

$$\begin{array}{llll} x^2 & -5x & +6 & =0 \\ A_p & -5A_{p+1} & +6A_{p+2} & =0 \\ A_{p-1} & -5A_p & +6A_{p+1} & =0 \end{array}$$

whence the determinant quadratic in question.

This quadratic does not simplify, however, so easily in other cases.

Consider the expansion—quite general—of a fraction having

$$x^3+5x^2-2x-24\,, \quad \textit{i.e.}, \quad (x-2)(x+3)(x+4)$$

for denominator, and any other integral rational function of not more than three dimensions in the numerator. For example, expand the fraction

$$\frac{24+6x+3x^2}{24+2x-5x^2-x^3} = 1 + \ldots + B_n \frac{x^n}{(24)^{n-1}} + \ldots$$

The values of B_n beginning from $n = 1$, are

$$\begin{array}{rrr} \cdot 1\dot{6} & 1174.10^2 & 23701.10^6 \\ 7{\cdot}5 & 1016.10^3 & 29893.10^7 \\ 29\cdot & 1547.10^4 & 34699.10^8 \\ 938\cdot & 1587.10^5 & \\ 5924\cdot & 2125.10^6 & \end{array}$$

From these arises the series of quotients, $\frac{24.B_n}{B_{n+1}}$, which give the following values, beginning with $n = 3$,

$$\begin{array}{lllllllllll} n = 3 & , 4 & , 5 & , 6 & , 7 & , 8 & , 9 & , 10 & , 11 & , 12 & . \\ \frac{24B_n}{B_{n+1}} = & \cdot 74 , & 3{\cdot}8 , & 1{\cdot}21 , & 2{\cdot}77 , & 1{\cdot}57 , & 2{\cdot}34 , & 1{\cdot}79 , & 2{\cdot}15 , & 1{\cdot}90 , & 2{\cdot}07 . \end{array}$$

in which latter series the continued approximation to the number 2 is manifest.

It is noticeable that these coefficients are alternately greater and less than the root to which they approach. If the means are taken of the $(2r-1)$th and

$2r^{\text{th}}$ quotients, the root is approximated to much more quickly: thus beginning with $n = 5$ and 6, we get

$$1{\cdot}99\,,\ 1{\cdot}95\,,\ 1{\cdot}97\,,\ 1{\cdot}98\,,$$

the constancy of these means being very remarkable

The equation for the two smallest roots is

$$\begin{vmatrix} x^2 & , x & , 1 \\ A_p & , A_{p+1} & , A_{p+2} \\ A_{p-1} & , A_p & , A_{p+1} \end{vmatrix} = 0$$

which may be written in the form—using the above notation—

$$\begin{vmatrix} x^2 & , x, & 1 \\ \dfrac{24B_p}{B_{p+1}} & , 1, & \dfrac{1}{\dfrac{24B_{p+1}}{B_{p+2}}} \\ \dfrac{24B_{p-1}}{B_p} & , 1, & \dfrac{1}{\dfrac{24B_p}{B_{p+1}}} \end{vmatrix} = 0$$

(α) Putting $p = 7$, this is

$$\begin{vmatrix} x^2 & , x, & 1 \\ 1{\cdot}57 & , 1, & \dfrac{1}{2{\cdot}34} \\ 2{\cdot}77 & , 1, & \dfrac{1}{1{\cdot}57} \end{vmatrix} = 0, \text{ which reduces to } x^2 + {\cdot}877x - 5{\cdot}725 = 0$$

the roots of which are $\dfrac{-{\cdot}877 \pm 4{\cdot}865}{2}$, *i.e.*, $1{\cdot}999$, and $-2{\cdot}871$, nearly 2 and -3.

(β) Putting $p = 11$, this quadratic equation becomes,

$$\begin{vmatrix} x^2 & , x, & 1 \\ 1{\cdot}90 & , 1, & \dfrac{1}{2{\cdot}07} \\ 2{\cdot}15 & , 1, & \dfrac{1}{1{\cdot}90} \end{vmatrix} = 0, \text{ i.e., } x^2 + {\cdot}894x - 5{\cdot}784 = 0$$

the roots of which are $\dfrac{-{\cdot}894 \pm 4{\cdot}892}{2}$, *i.e.*, $1{\cdot}999$, and $-2{\cdot}893$, nearly 2 and -3.

It may be inferred from this example that the quadratic equation whose roots are the two smallest roots of the given equation, give the smallest root much sooner than the simple equation for the smallest root. For example, in the above $p=7$ gives, by the simple equation, 1·57 as the root, whereas the corresponding quadratic gives 1·999.

(III.) The quantities u_{2p}, u_{2p+1} may be reduced to determinants also. Thus $u_{2p+3}=A_p\,A_{p+1}-A_{p-1}\,A_{p+2}$

$$=\underset{p+1}{\begin{vmatrix} b_1 & b_2 & \dots & b_p & a_p \\ 1 & b_1 & \dots & b_{p-1} & a_{p-1} \\ \hdotsfor{5} \\ & & & b_1 & a_1 \\ & & & 1 & 1 \end{vmatrix}}\;\underset{p+2}{\begin{vmatrix} b_1 & b_2 & \dots & b_{p+1} & a_{p+1} \\ 1 & b_1 & \dots & b_p & a_p \\ \hdotsfor{5} \\ & & & b_1 & a_1 \\ & & & 1 & 1 \end{vmatrix}}-\underset{p}{\begin{vmatrix} b_1 & b_2 & \dots & b_{p-1} & a_{p-1} \\ 1 & b_1 & \dots & b_{p-2} & a_{p-2} \\ \hdotsfor{5} \\ & & & b_1 & a_1 \\ & & & 1 & 1 \end{vmatrix}}\;\underset{p+3}{\begin{vmatrix} b_1 & b_2 & \dots & b_{p+2} & a_{p+2} \\ 1 & b_1 & \dots & b_{p+1} & a_{p+1} \\ \hdotsfor{5} \\ & & & b_1 & a_1 \\ & & & 1 & 1 \end{vmatrix}}$$

(the underwritten letters indicating the order of the determinant above).

Let the p- and $(p+2)$- determinants be raised to the $(p+1)$ and $(p+3)$ orders respectively, and separate the determinant of the $(p+3)$ order into two, the constituents in whose first vertical lines are respectively (read downwards) b_1, 0, 0 . . . , and 0, 1, 0, 0 . . . . Thus

$$u_{2p+3}=\underset{p+1}{\begin{vmatrix} b_1 & b_2 & \dots & b_p & a_p \\ 1 & b_1 & \dots & b_{p-1} & a_{p-1} \\ \hdotsfor{5} \\ & & & b_1 & a_1 \\ & & & 1 & 1 \end{vmatrix}}\;\underset{p+3}{\begin{vmatrix} 1 & b_2 & b_3 & \dots & b_{p+2} & a_{p+2} \\ 0 & b_1 & b_2 & \dots & b_{p+1} & a_{p+1} \\ \hdotsfor{6} \\ & & & & b_1 & a_1 \\ & & & & 1 & 1 \end{vmatrix}}-\underset{p+1}{\begin{vmatrix} 1 & b_2 & b_3 & \dots & b_p & a_p \\ 0 & b & b_2 & \dots & b_{p-1} & a_{p-1} \\ \hdotsfor{6} \\ & & & & b_1 & a_1 \\ & & & & 1 & 1 \end{vmatrix}}\;\underset{p+3}{\begin{vmatrix} b_1 & b_2 & \dots & b_{p+2} & a_{p+2} \\ 0 & b_1 & \dots & b_{p+1} & a_{p+1} \\ \hdotsfor{5} \\ & & & b_1 & a_1 \\ & & & 1 & 1 \end{vmatrix}}$$

$$-\underset{p+1}{\begin{vmatrix} 1 & b_2 & b_3 & \dots & b_p & a_p \\ 0 & b_1 & b_2 & \dots & b_{p-1} & a_{p-1} \\ \hdotsfor{6} \\ & & & & b_1 & a_1 \\ & & & & 1 & 1 \end{vmatrix}}\;\underset{p+3}{\begin{vmatrix} 0 & b_2 & \dots & b_{p+2} & a_{p+2} \\ 1 & b_1 & \dots & b_{p+1} & a_{p+1} \\ . & 1 & \dots & b_p & a_p \\ \hdotsfor{5} \\ & & & b_1 & a_1 \\ & & & 1 & 1 \end{vmatrix}}$$

or adding the determinants in the first line, and putting

$$d_1=\begin{vmatrix} b_2 & b_3 & \dots & b_p & a_p \\ 1 & b_1 & \dots & b_{p-2} & a_{p-2} \\ . & 1 & \dots & b_{p-3} & a_{p-3} \\ \hdotsfor{5} \\ & & & 1 & 1 \end{vmatrix}\;,\quad d_2=\begin{vmatrix} b_1 & b_2 & \dots & b_{p-1} & a_{p-1} \\ 1 & b_1 & \dots & b_{p-2} & a_{p-2} \\ . & 1 & \dots & b_{p-3} & a_{p-3} \\ \hdotsfor{5} \\ & & & 1 & 1 \end{vmatrix}$$

$$\begin{array}{r|cccccccc|}
\multicolumn{1}{r}{} & & \mathrm{M}_1 & \mathrm{M}_2 & \cdot\,\cdot & \mathrm{M}_{p-1} & \mathrm{M}_p & & \multicolumn{1}{c}{} \\
u_{2p+3} = & d_1 & b_2 & b_3 & \ldots & b_p & b_{p+1} & b_{p+2} & a_{p+2} \\
& d_2 & b_1 & b_2 & \ldots & b_{p-1} & b_p & b_{p+1} & a_{p+1} \\
& . & 1 & b_1 & \ldots & b_{p-2} & b_{p-1} & b_p & a_p \\
& & . & . & . & . & . & . & . \\
& & & & \ldots\ldots & & 1 & b_1 & a_1 \\
& & & & \ldots\ldots & \ldots & \ldots & 1 & 1
\end{array}$$

The minors of the constituents of the first horizontal lines of d_1 and d_2 are the same; and d_1 and d_2 may be written

$$\begin{aligned}
d_1 &= b_2\mathrm{M}_1 + b_3\mathrm{M}_2 + \ldots + b_p\ \mathrm{M}_{p-1} + a_p\ \mathrm{M}_p \\
d_2 &= b_1\mathrm{M}_1 + b_2\mathrm{M}_2 + \ldots + b_{p-1}\mathrm{M}_{p-1} + a_{p-1}\mathrm{M}_p
\end{aligned}$$

where M_1, M_2, ... M_p, are the minors of the 1st, 2d, ... p^{th} constituents of the first horizontal row of d_1.

In u_{2p+3}, let there be added to the 1st vertical row, the next $p-1$ vertical rows, each multiplied by the minor written above it with its sign changed—then

$$u_{2p+3} = \underset{p\,+\,3}{\left|\begin{matrix}
a_p & b_2 & b_3 & \ldots & b_{p+1} & b_{p+2} & a_{p+2} \\
a_{p-1} & b_1 & b_2 & \ldots & b_p & b_{p+1} & a_{p+1} \\
a_{p-2} & 1 & b_1 & \ldots & b_{p-1} & b_p & a_p \\
 & . & . & . & . & . & . \\
1 & . & . & \ldots & b_1 & b_2 & a_2 \\
. & . & . & \ldots & 1 & b_1 & a_1 \\
. & . & . & \ldots & . & 1 & 1
\end{matrix}\right|}$$

i.e., u_{2p+3} is the same as $-\mathrm{A}_{p+2}$, with the constituents a_p, a_{p-1} ... 1, 0, 0 written down its first column instead of b_1, 1, ... 0.

In like manner it may be shown that u_{2p+4} is the same as $-\mathrm{A}_{p+2}$, with constituents a_{p+1}, a_p, a_{p-1} ... a_1, 1, 0, written down its first column instead of b_1, 1, ... 0

The quadratic equation

$$u_{2p+4}\,x^2 - u_{2p+3}\,x + u_{2p+2} = 0$$

may therefore be written in the following form:—

$$\underset{p+3}{\left|\begin{matrix}
a_{p+1} b_2 b_3 \ldots b_{p+2} a_{p+2} \\
a_p\ \ b_1 b_2 \ldots b_{p+1} a_{p+1} \\
a_{p-1} 1\, b_1 \ldots b_p\ \ a_p \\
.\ \ .\ \ .\ \ .\ \ . \\
1 \qquad \ldots b_1\ \ a_1 \\
0 \qquad \ldots 1\ \ 1
\end{matrix}\right|} x^2 - \underset{p+3}{\left|\begin{matrix}
a_p\ \ b_2 b_3 \ldots b_{p+2} a_{p+2} \\
a_{p-1} b_1 b_2 \ldots b_{p+1} a_{p+1} \\
a_{p-2} 1\, b_1 \ldots b_p\ \ a_p \\
.\ \ .\ \ .\ \ .\ \ . \\
0\ .\ .\ .\ b_1\ \ a_1 \\
0\ .\ .\ .\ 1\ \ 1
\end{matrix}\right|} x + \underset{p+2}{\left|\begin{matrix}
a_p\ \ b_2 b_3 \ldots b_{p+1} b_{p+1} \\
a_{p-1} b_1 b_2 \ldots b_p\ \ a_p \\
a_{p-2} 1\, b_1 \ldots b_{p-1} a_{p-1} \\
.\ \ .\ \ .\ \ .\ \ . \\
1\ .\ .\ .\ b_1\ \ a_1 \\
0\ .\ .\ .\ 1\ \ 1
\end{matrix}\right|} = 0$$

These determinant forms of the coefficients have already been made known.

(IV.) Let $F(x) = 0$ have imaginary roots, of which two conjugate ones are $\mu\xi$, $\mu\xi^{-1}$, where $\xi = \epsilon^{\theta.\iota}$ and $\iota = \sqrt{-1}$; and let their modulus, μ, be smaller than the modulus of any other pair of imaginary roots, and also smaller than any real root. Then, as in (1)

$$\frac{\phi(x)}{F(x)} = \ldots . - \Sigma\left(\frac{\phi(r)}{r^{p+1}. F'(r)}\right). x^p \ldots . = \ldots A_p x^p + \ldots .$$

therefore as p increases to infinity,

$$-A_{p+1} = \frac{\phi(\mu\xi)}{F'(\mu\xi)} \cdot \frac{1}{(\mu\xi)^p} + \frac{\phi(\mu\xi^{-1})}{F'(\mu\xi^{-1})} \frac{1}{(\mu\xi^{-1})^p} .$$

Let

$$2\frac{\phi(\mu\xi)}{F'(\mu\xi)} = P + Q\iota ,$$

then

$$-\mu^p A_{p-1} = \tfrac{1}{2}(P + Q\iota)(\cos p\theta - \iota \sin p\theta) + \tfrac{1}{2}(P - Q\iota)(\cos p\theta + \iota \sin p\theta)$$
$$= P \cos p\theta + Q \sin p\theta$$

$$\therefore \mu \frac{A_p}{A_{p-1}} = \frac{P \cos \overline{p+1}\theta + Q \sin \overline{p+1}\theta}{P \cos p\theta + Q \sin p\theta} = w_p \text{ (say)} ,$$

$$\therefore \frac{P}{Q} = -\frac{w_p \sin p\theta - \sin \overline{p+1}\theta}{w^p \cos p\theta - \cos \overline{p+1}\theta} ,$$

and similarly

$$= -\frac{w_{p+1} \sin \overline{p+1}\theta - \sin \overline{p+2}\theta}{w_{p+1} \cos \overline{p+1}\theta - \cos \overline{p+2}\theta} ,$$

whence reducing to a common denominator

$$\frac{(w_{p+1}\, w_p - 2w_p \cos\theta + 1) \sin\theta}{(w^p \cos p\theta - \cos \overline{p+1}\theta)(w_{p+1} \cos \overline{p+1}\theta - \cos \overline{p+2}\theta)} = 0 .$$

Let us suppose the solution of this equation given by

$$w_{p+1} w_p - 2w_p \cos\theta + 1 = 0 ,$$

or

$$2\cos\theta = w_{p+1} + \frac{1}{w_p} ,$$

that is,

$$= \mu\frac{A_{p+1}}{A_p} + \frac{1}{\mu}\frac{A_{p-1}}{A_p}$$

and similarly

$$= \mu\frac{A_{p+2}}{A_{p+1}} + \frac{1}{\mu}\frac{A_p}{A_{p+1}} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

therefore

$$\frac{\mu^2(A_{p+1}^2 - A_pA_{p+2}) - (A_p^2 - A_{p-1}A_{p+1})}{A_pA_{p+1}} = 0\,,$$

that is,

$$\mu^2 = \frac{A_p^2 - A_{p-1}A_{p+1}}{A_{p+1}^2 - A_pA_{p+2}} \quad . \quad . \quad . \quad . \qquad (7)$$

Again, equation (6) on simplification is

$$\mu^2A_{p+2} + A_p = 2\cos\theta\,.\,A_{p+1}\mu\,,$$

eliminating μ, this becomes

$$\frac{A_p^2 - A_{p-1}A_{p+1}}{A_{p+1}^2 - A_pA_{p+2}}A_{p+2} + A_p = 2\cos\theta\,.\,A_{p+1}\mu\,,$$

that is,

$$\frac{A_pA_{p+1} - A_{p-1}A_{p+2}}{A_{p+1}^2 - A_pA_{p+2}} = 2\cos\theta\,.\,\mu\,,$$

or

$$2\cos\theta = \frac{A_pA_{p+1} - A_{p-1}A_{p+2}}{\sqrt{(A_p^2 - A_{p-1}A_{p+1})(A_{p+1}^2 - A_pA_{p+2})}} \quad . \quad . \qquad (8)$$

These two results may be written thus

$$\mu = \sqrt{\frac{u_{2p+2}}{u_{2p+4}}} = \frac{u_{2p+2}}{\pm\sqrt{u_{2p+2}\,.\,u_{2p+4}}}$$

$$2\cos\theta = \frac{u_{2p+3}}{\pm\sqrt{u_{2p+2}\,.\,u_{2p+4}}}\,.$$

Note.—If

$$\frac{l + mx}{1 + ax + bx^2} = \,.\,.\,.\,.\, + A_px^p + \,.\,.\,.\,.$$

then

$$(p \overset{=}{>} 0) \qquad A_{p+2} + aA_{p+1} + bA_p = 0\,,$$

and

$$\frac{1}{\mu^2} = \frac{u_{2p+4}}{u_{2p+2}} = \frac{a^2A_p^2 + 2abA_pA_{p-1} + b^2A_{p-1}^2 - a^2A_p^2 - abA_pA_{p-1} + bA_p^2}{A_p^2 + aA_pA_{p-1} + bA_{p-1}^2} = b\,,$$

and

$$\mu\cos\theta = \frac{a^2A_{p+2} + A_p}{2A_{p+1}} = \frac{A_{p+2} + bA_p}{2bA_{p+1}} = -\frac{a}{2b}$$

therefore the roots are

$$-\frac{a}{2b} \pm \frac{\sqrt{a^2-4b}}{2b} \qquad \text{or} \qquad -\frac{a}{2b} \pm \iota\,\frac{\sqrt{4b-a^2}}{2b}$$

For example:

$$\frac{1}{4x^2+x+1} = 1-x-3x^2+7x^3+5x^4-53x^5$$
$$+13x^6+119x^7-171x^8-305x^9+989x^{10}$$
$$+231x^{11}-4187x^{12}+3263x^{13}+13485x^{14}-26537x^{15}$$
$$-27403x^{16}+133551x^{17}-23939x^{18}-510265x^{19}+606021x^{20}$$
$$+1435039x^{21}-\ldots$$

the coefficients being connected by the relation $A_{p+1}+A_p+4A_{p-1}=0$,

$$\therefore \frac{u_{2p+2}}{u_{2p+4}} = \frac{A_p^2+A_{p-1}A_p+4A_{p-1}^2}{(A_p^2+8A_{p-1}A_p+16A_{p-1}^2)-A_p(-3A_p+4A_{p-1})} = \tfrac{1}{4} = \mu^2$$

and

$$\mu \cos\theta = \frac{\mu^2 A_{p+2}+A_p}{2A_{p+1}} = -\frac{1}{8}$$

therefore the roots are

$$-\frac{1}{8} \pm 2\frac{\sqrt{15}}{8}$$

Again

$$\frac{7+4x}{6+2x+x^2} = \sum_{p=0}^{p=\infty} \frac{B_p}{6^{p+1}} x^p$$ where the values of B_0, B_1 . . . are as follows:—

7, 10, −62, 64, 244, −872, 280, 4672, −11024, −5984,
78112, −120320, −228032, 1177984, −987776,
−5092352, 16111360, −1668608, −93330944, 19667.10⁴,
16664.10⁴, −151330.10⁴, 202677.10⁴, 503626.10⁴, −2223314.10⁴,

i.e., the term in x^{24} is $-\dfrac{22233140000}{6^{25}}x^{24}$,

Here

$$6\frac{B_{p+2}}{6^{p+3}} + 2\frac{B_{p+1}}{6^{p+2}} + \frac{B_p}{6^{p+1}} = 0; \text{ or } B_{p+2}+2\,B_{p+1}+6\,B_p=0$$

$$\therefore \frac{u_{2p+2}}{u_{2p+4}} = 6^2\frac{B_p^2-B_{p-1}B_{p+1}}{B_{p+1}^2-B_pB_{p+2}} = 6^2\frac{B_p^2+2B_pB_{p-1}+6B_{p-1}^2}{4B_p^2+24B_pB_{p-1}+36B_{p-1}^2+2B_p^2-12B_pB_{p-1}} = 6 = \mu^2$$

and

$$\mu\cos\theta = \frac{\mu^2 A_{p+2}+A_p}{2A_{p+1}} = -1$$

therefore the roots are, $-1 \pm \iota\sqrt{5}$

(V.) To complete the discussion of the determinant quadratic for the two least roots of $F(x)=0$, it is necessary to show that it holds if the two least roots of $F(x)=0$ be equal.

Let $F(x) = (x - r)^2\psi(x)$ then we may write $\frac{\phi(x)}{\psi(x)} = f(x)$, and

$$\frac{\phi(x)}{F(x)} = \frac{f(r)}{(x-r)^2} + \frac{f'(r)}{x-r} + \Sigma\left(\frac{\chi(s)}{\psi'(s)}\frac{1}{x-s}\right)$$

$$= \ldots . + \left\{(p+1)\frac{f(r)}{r^{p+2}} - \frac{f'(r)}{r^{p+1}} - \Sigma\frac{\chi(s)}{\psi'(s).s^{p+1}}\right\}x^p + \ldots .$$

If r be the smallest root, then when $p = \infty$

$$\frac{A_{p+1}}{A^p} = \frac{p}{p+1}r = r \text{ we may say.}$$

Again

$$u_{2p+2} = A_p^2 - A_{p-1}A_{p+1}$$

$$= \left(\frac{(p+1)f(r)}{r^{p+2}}\right)^2 + \&c. - \frac{pf(r)}{r^{p+1}}\frac{(p+2)f(r)}{r^{p+3}} - \&c.,$$

the terms neglected ultimately vanishing, when p is infinite, compared with those retained: that is, ultimately

$$u_{2p+2} = \frac{(p^2 + 2p + 1 - p^2 - 2p)(f(r))^2}{r^{2p+4}}$$

$$= \frac{\overline{f(r)}|^2}{r^{2p+4}}$$

Also

$$u_{2p+3} = A_p A_{p+1} - A_{p+1}A_{p+2}.$$

ultimately

$$= \frac{\{p+1)(p+2) - p(p+3)\}.\overline{f(r)}|^2}{r^{2p+5}}$$

$$= \frac{2.\overline{f(r)}|^2}{r^{2p+5}},$$

therefore the determinant quadratic becomes

$$\frac{\overline{f(r)}|^2}{r^{2p+4}}x^2 - \frac{2.\overline{f(r)}|^2}{r^{2p+3}}x + \frac{\overline{f(r)}|^2}{r^{2p+5}} = 0.$$

or

$$x^2 - 2rx + r^2 = 0,$$

i.e., the determinant quadratic is true also for equal least roots.

(VI.) In the case in which $F(x) = 0$ has real and unequal roots, it has been proved that the equation

$$\begin{vmatrix} x & , & 1 \\ A_{p-1} & , & A_p \end{vmatrix} = 0$$

gives its least root; and that the equation

$$\begin{vmatrix} x^2 , & x , & 1 \\ A_{p-1} , & A_p , & A_{p+1} \\ A_{p-2} , & A_{p-1} , & A_p \end{vmatrix} = 0$$

gives the two least roots, when p is infinite. It will now be shown that the equation which gives the three least roots is

$$\begin{vmatrix} x^3 , & x^2 , & x , & 1 \\ A_{p-1} , & A_p , & A_{p+1} , & A_{p+2} \\ A_{p-2} , & A_{p-1} , & A_p , & A_{p+1} \\ A_{p-3} , & A_{p-2} , & A_{p-1} , & A_p \end{vmatrix} = 0$$

Let the coefficients of this equation be respectively u_{3p+3}, u_{3p+2}, u_{3p+1}, u_{3p}, as those of the quadratic are u_{2p+2}, u_{2p+1}, u_{2p}. Then evidently

$$\begin{aligned} u_{3p+3} &= A_p \; u_{2p+2} - A_{p-1} \, u_{2p+3} + A_{p-2} \, u_{2p+4} \\ u_{3p+2} &= A_{p-1} \, u_{2p+2} - A_{p-2} \, u_{2p+3} + A_{p-3} \, u_{2p+4} \\ u_{3p+1} &= A_{p+2} \, u_{2p-2} - A_{p+1} \, u_{2p-1} + A_p \; u_{2p} \\ u_{3p} &= A_{p+1} \, u_{2p-2} - A_p \; u_{2p-1} + A_{p-1} \, u_{2p} \end{aligned}$$

(It may be noticed that the values of these coefficients are not all symmetrical; the reason is, that in order to express the u's of the third order in terms of those already formed of the second order, the column of multipliers has to be so chosen as to bring in the u's of the second order; otherwise terms of the forms $A^2_p - A_{p-2}A_{p+2}$, $A_pA_{p+1} - A_{p-2}A_{p+3}$ would occur, which have not yet been considered, though each is homogeneous in terms of the least roots and of the orders $2p$, $2p+1$ respectively.)

The calculation of these coefficients may now be performed—in the light of what has already been done—by considering only the terms which contain the least roots. Let r, s, t be the three least roots—then

$$u_{3p} = \Sigma\left(\frac{\varphi(r)}{F'(r)} \frac{1}{r^p} \cdot \frac{\varphi(s).\varphi(t)}{F'(s).F'(t)} \frac{(s-t)(t-s)}{(st)^{p+1}}\right) - \Sigma\left(\frac{\varphi(r)}{F'(r)r^{p+1}} \cdot \frac{\varphi(s).\varphi(t)}{F'(s).F'(t)} \frac{(s-t)(t-s)(s+t)}{(st)^{p+1}}\right)$$

$$+ \Sigma\left(\frac{\varphi(r)}{F'(r)} \frac{1}{r^{p+2}} \cdot \frac{\varphi(s).\varphi(t)}{F'(s).F'(t)} \cdot \frac{(s-t)(t-s).st}{(st)^{p+1}}\right)$$

$$= \frac{\varphi(r).\varphi(s)\varphi(t)}{F'(r).F'(s)F'(t)} \frac{(s-t)(t-s)}{|r^p(st)^{p+1}} + \frac{\overline{\varphi(s)}|^2\varphi(t)}{F'(s)|^2F\ (t)} \frac{(s-t)(t-}{s^{2p+1}t^{p+1}} + \frac{\varphi(s).\overline{\varphi(t)}|^2}{F'(s).\overline{F'(t)}|^2} \frac{(s-t)(t-s)}{s^{p+1}t^{2p+1}} + \&c.$$

$$-\frac{\phi(r)\phi(s)\phi(t)}{\mathrm{F}'(r)\mathrm{F}'(s)\mathrm{F}'(t)}\frac{(s-t)(t-s)(s+\ )}{(rst)^{p+1}}-\frac{\overline{\phi(s)}|^{2}\phi(t)}{\overline{\mathrm{F}'(s)}|^{2}\mathrm{F}'(t)}\frac{(s-t)(t-s)(s+t)}{s^{2p+2}t^{p+1}}-\frac{\phi(s).\overline{\phi(t)}|^{2}}{\mathrm{F}'(s).\overline{\mathrm{F}'(t)}|^{2}}\frac{(s-t)(t-s)(s+t}{s^{p+1}t^{2p+2}}-\&\text{c.}$$

$$+\frac{(\phi(r)\phi(s)\phi(t)}{\mathrm{F}'(r)\mathrm{F}'(s)\mathrm{F}'(t)}\frac{(s-t)(t-s)st}{r^{p+2}(st)^{p+1}}+\frac{\overline{\phi(s)}|^{2}\phi(t)}{\overline{\mathrm{F}'(s)}|^{2}\mathrm{F}'(t)}\frac{(s-t)(t-s)st}{s^{2p+3}t^{p+1}}+\frac{\phi(s).\overline{\phi(t)}|^{2}}{\mathrm{F}'(s).\overline{\mathrm{F}'(t)}|^{2}}\frac{(s-t)(t-s)st}{s^{p+1}t^{2p+3}}+\&\text{c.}$$

+ Similar terms in each with s, t, r; t, r, s; written in turn for r, s, t, in the above expressions.

$$=\frac{\phi(r)\phi(s)\phi(t)}{\mathrm{F}'(r)\mathrm{F}'(s)\mathrm{F}'(t)}\frac{(s-t)(t-s)\{r^{2}st-(s+t)rst+(st)^{2}\}+(t-r)(r-t)\{\ \ \}+(r-s)(s-r)\{\ \ \}}{(rst)^{p+2}}+\&\text{c.}$$

$$=\frac{\phi(r)\phi(s)\phi(t)}{\mathrm{F}'(r)\mathrm{F}'(s)\mathrm{F}'(t)}\frac{(s-t)(t-s)(t-r)(r-t)(r-s)(s-r)}{(rst)^{p+2}}+\&\text{c.}\quad .\quad .\quad .\qquad (9')$$

This is the greatest term in the value of u_{3p}, provided the terms in

$$\frac{\overline{\phi(s)}|^{2}\phi(t)}{\overline{\mathrm{F}'(s)}|^{2}\mathrm{F}'(t)},\ \&\text{c.},$$

be not greater. The coefficient of the term in

$$\frac{\overline{\phi(s)}|^{2}\cdot\phi(t)}{\overline{\mathrm{F}'(s)}|^{2}\,.\,\mathrm{F}'(t)}$$

is

$$\frac{(s-t)(t-s)\{s^{2}t^{p+2}-(s+t)st^{p+2}+st^{p+3}\}}{s^{2p+3}t^{2p+3}}$$

which vanishes: similarly for the other terms of this form. Thus the only terms in u_{3p} which remain are of the form given in (9′): and when p becomes infinite

$$u_{3p}=\frac{\phi(r)\,.\,\phi(s)\,.\,\phi(t)}{\mathrm{F}'(r)\,.\,\mathrm{F}'(s)\,.\,\mathrm{F}'(t)}\frac{(s-t)(t-s)(t-r)(r-t)(r-s)(s-r)}{(rst)^{p+2}}\quad .\quad .\quad .\qquad (9)$$

The term u_{3p-1} may be calculated in the same way. For it is thus found that

$$u_{3p-1}=\frac{\phi(r)\phi(s)\phi(t)}{\mathrm{F}'(r)\mathrm{F}'(s)\mathrm{F}'(t)}\left(\frac{(s-t)(t-s)}{r^{p-1}(st)^{p+1}}-\frac{(s-t)(t-s)(s+t)}{r^{p-2}(st)^{p+2}}+\frac{(s-t)(t-s)}{r^{p-3}(st)^{p+2}}+\text{terms in } t, r, \text{and } r, s,\right)$$

$$+\text{terms in }\frac{\overline{\phi(s)}|^{2}\phi(t)}{\overline{\mathrm{F}'(s)}|^{2}\mathrm{F}'(t)}\ \&\text{c., whose coefficients vanish;}$$

therefore when r, s, t, are the three smallest roots, and p is infinite, this becomes

$$u_{3p-1}=\frac{\phi(r)\phi(s)\phi(t)}{\mathrm{F}'(r)\mathrm{F}'(s)\mathrm{F}'(t)}\frac{(s-t)(t-s)(t-r)(r-t)(r-s)(s-r)(r+s+t)}{(rst)^{p+2}}$$

Similarly it may be found that

$$u_{3p-2} = (\qquad\quad)\frac{(\qquad\quad)(rs+st+tr)}{(rst)^{p+2}}$$

$$u_{3p-3} = (\qquad\quad)\frac{(\qquad\quad)rst}{(rst)^{p+2}}$$

Thus the determinant cubic becomes

$$x^3 - (r + s + t)x^2 + (rs + st + tr)x - rst = 0$$

VII. Let now

$$c_m x^m + c_{m-1}x^{m-1} + \ldots + c_1 x + c_0 = 0 \quad . \quad . \quad . \quad (10)$$

be an equation whose roots are the m least roots of $\mathrm{F}(x) = 0$: and let r be the least root, then

$$c_m \frac{\phi(r)}{\mathrm{F}'(r)r^p} + c_{m-1}\frac{\phi(r)}{\mathrm{F}'(r)r^{p+1}} + \ldots + c_1 \frac{\phi(r)}{\mathrm{F}'(r)r^{p+m-1}} + c_0 \frac{\phi(r)}{\mathrm{F}'(r)r^{p+m}} = 0 .$$

Now $\frac{\phi(r)}{\mathrm{F}'(r)r^p}$ is what A_{p-1} becomes when p is infinite. The above equation may therefore be written in the form

$$\begin{aligned} &c_m\,\mathrm{A}_{p-1} + c_{m-1}\,\mathrm{A}_p + \ldots + c_1\,\mathrm{A}_{p+m-2} + c_0\,\mathrm{A}_{p+m-1} \\ &+ c_m\,a_{p-1} + c_{m-1}\,a_p + \ldots\ldots\ldots\ldots + c_0\,a_{p+m-1} = 0 \end{aligned}$$

where $a_{p-1} \ldots a_{p+m-1}$ ultimately vanish when p is infinite. The number of terms in the second line of this equation is finite, and if a' be the greatest of the quantities $a_{p-1} \ldots . a_{p+m-1}$, this equation cannot have a greater error than when

$$(c_m + c_{m-1} + \ldots . + c_0)a'$$

is written for its second line. But this, being the product of a finite quantity and a' which is ultimately zero, ultimately vanishes, and the quantities $c_m \ldots . c_0$ are connected by the equation

$$c_m\mathrm{A}_{p-1} + c_{m-1}\mathrm{A}_p + \ldots . + c_1\mathrm{A}_{p+m-2} + c_0\mathrm{A}_{p+m-1} = 0 .$$

Similarly, the following equations are true

$$c_m\mathrm{A}_{p-2} + c_{m-1}\mathrm{A}_{p-1} + \ldots . + c_1\mathrm{A}_{p+m-3} + c_0\mathrm{A}_{p+m-2} = 0 ,$$

$$c_m\mathrm{A}_{p-3} + c_{m-1}\mathrm{A}_{p-2} + \ldots . + c_1\mathrm{A}_{p+m-4} + c_0\mathrm{A}_{p+m-3} = 0 ,$$

&c.

Eliminating $c_m \ldots . c_0$ from m of these equations and equation (10) we have

$$\begin{vmatrix} x^m, & x^{m-1}, & \ldots . & x, & 1 \\ A_{p-1}, & A_p, & \ldots . & A_{p+m-2}, & A_{p+m-1} \\ A_{p-2}, & A_{p-1}, & \ldots . & A_{p+m-3}, & A_{p+m-2} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ A_{p-m}, & A_{p-m+1} & \ldots . & A_{p-1}, & A_p \end{vmatrix} = 0, \qquad . \quad (11)$$

an equation of the m^{th} degree which gives the least root.

But this equation possesses $m - 1$ roots in addition, and it will now be shown that these are the next $m - 1$ roots in order of ascending magnitude.

Let this determinant equation be written thus

$$\begin{vmatrix} \dfrac{\phi(x)}{F'(x),x^p} & , \dfrac{\phi(x)}{F'(x)x^{p+1}} & , \ldots . \\ \dfrac{\phi(r)}{F'(r)r^p} + \dfrac{\phi(s)}{F'(s)s^p} + \ldots ., & \dfrac{\phi(r)}{F'(r)r^{p+1}} + \dfrac{\phi(s)}{F'(s).s^{p+1}}, & \ldots . \end{vmatrix} = 0$$

Subtract the first horizontal line from the second, x times the first from the third, x^2 times the first from the fourth, and so on; and *after* having done so, write r for x. The determinant will now be of the form

$$\begin{vmatrix} \dfrac{\phi(r)}{F'(r)r^p} & , \dfrac{(\phi r)}{F'(r).r^{p+1}} & \ldots \\ \dfrac{\phi(s)}{F'(s).s} + \dfrac{\phi(t)}{F'(t).t^p} + \ldots, & \dfrac{\phi(s)}{F'(s).s^{p+1}} + \dfrac{\phi(t)}{F'(t).t^{p+1}} + \ldots, & \ldots \end{vmatrix} = 0$$

i.e., when expanded it will contain r *only* where x was in the previous equation; and as the form remains the same, this equation will vanish if s be written for r when p is infinite. But s is the next greater root of $F(x) = 0$. Thus the two least roots of $F(x) = 0$ are roots of the equation (11). In exactly the same manner, t and the succeeding higher roots of $F(x) = 0$ would be roots of equation (11), until m of them had been employed; or in other words, the m least roots of $F(x) = 0$ are given by the determinant equation (11) of the $\overline{m+1}^{th}$ order.

It is scarcely necessary to add that the process applied to the determinant quadratic when the two least roots were equal will also apply here. In fact the equation holds independently of the equality of the roots.

Thus, if the coefficients found by ordinary division in equation (2) be employed to construct equation (11), the solution of this equation will give the m least roots of $F(x) = 0$.

VII.—*On Eisenstein's Continued Fractions.*—By THOMAS MUIR, M.A.

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In CRELLE'S Journal for 1844, at the end of a paper on Cubic Forms,[*] EISENSTEIN gives the following results:—

$$1+\frac{x}{R}+\frac{x^2}{R^4}+\frac{x^3}{R^9}+\ldots\ldots+\frac{x^n}{R^{n^2}}+\text{in inf.} \qquad \ldots\ldots\ldots \text{(I.)}$$

$$=\cfrac{1}{1-\cfrac{x}{R-\cfrac{(1-R^2)x}{R^2-\cfrac{x}{R^3-\cfrac{(1-R^4)x}{R^4-\cfrac{x}{R^5-\cfrac{(1-R^6)x}{R^6-\text{\&c. in inf.}}}}}}}}$$

$$1+\rho x+\rho^4x^2+\rho^9x^3+\ldots\ldots+\rho^{(m-1)^2}\,x^{(m-1)} \qquad \ldots\ldots\ldots \text{(II.)}$$

$$=\cfrac{1-x^m}{1-\cfrac{x}{\rho^{m-1}-\cfrac{(1-\rho^{m-2})x}{\rho^{m-2}-\text{\&c.}\cdots\cdots\,\rho^2-\cfrac{(1-\rho)x}{\rho-\cfrac{x}{1-\cfrac{(1-\rho^{m-1})x}{\rho^{m-1}-\text{\&c.}\cdots\cdots\,\rho^3-\cfrac{(1-\rho)^2x}{\rho^2-\cfrac{x}{\rho}}}}}}}}$$

where m is any odd number and ρ a primitive root of the equation $z^m=1$;

$$\frac{2K}{\pi}=\left\{1+\cfrac{2}{p-\cfrac{1}{p^2-\cfrac{1-p^2}{p^3-\cfrac{1}{p^4-\cfrac{1-p^4}{p^5-\cfrac{1}{p^6-\text{in inf.}}}}}}}\right\}^2 \qquad \ldots \text{(III.)}$$

$$=1+\cfrac{4p}{1+p^2-\cfrac{p(1+p^2)^2}{1+p^4+\cfrac{p^3(1-p^2)^2}{1+p^6-\cfrac{p^3(1+p^4)^2}{1+p^8+\cfrac{p^5(1-p^4)^2}{1+p^{10}-\text{\&c.}}}}}} \qquad \ldots \text{(IV.)}$$

where

$$K=\int_0^{\frac{\pi}{2}}\frac{d\phi}{\sqrt{(1-k^2\sin^2\phi)}}\ ,\qquad K'=\int_0^{\frac{\pi}{2}}\frac{d\phi}{\sqrt{(1-k'^2\sin^2\phi)}}$$

$$k^2+k'^2=1 \qquad \text{and} \qquad p=e^{\pi\frac{K'}{K}}.$$

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[*] CRELLE xxvii. pp. 78, 79.

No demonstrations are given of these identities; but they are said by the author to be only particular cases of a very general equation, the complete theory connected with which he hoped to give on another occasion.

At p. 193 of the same volume of CRELLE he returns to the subject, not however for the purpose of giving the promised theory, but to add several other results similar to those before given. These are

$$\frac{z^{-1}-z^{-4}+z^{-9}-z^{-16}+\ldots}{1-z^{-1}+z^{-4}-z^{-9}+\ldots}=\cfrac{1}{z-1+\cfrac{z}{z^3-1+\cfrac{z^3}{z^5-1+\cfrac{z^5}{z^7-1+\cdots}}}} \quad \ldots\ldots \text{(V.)}$$

and one derived from this by changing the sign of z, and the signs of both sides;

$$\frac{z}{1-z}+\frac{z^2}{1-z^2}+\frac{z^3}{1-z^3}+\frac{z^4}{1-z^4}+\ldots=\cfrac{1}{t-1-\cfrac{(t-1)^2}{t^2-1-\cfrac{t(t-1)^2}{t^3-1-\cfrac{t(t^2-1)^2}{t^4-1-\cfrac{t^2(t^2-1)^2}{t^5-1-\cfrac{t^2(t^3-1)^2}{t^6-1-\cdots}}}}}} \quad \ldots\ldots \text{(VII.)}$$

where $t = z^{-1}$; and

$$1+\sigma x+\sigma^4x^2+\ldots+\sigma^{(2m-1)^2}x^{2m-1}=\cfrac{1-x^{2m}}{1-\cfrac{x}{\sigma^{2m-1}-\cfrac{(1-\sigma^{2m-2})x}{\sigma^{2m-2}-\cfrac{x}{\sigma^{2m-3}-\cfrac{(1-\sigma^{2m-4})x}{\ddots\,\sigma^3-\cfrac{(1-\sigma^2)x}{\sigma^2-\cfrac{x}{\sigma}}}}}}} \quad \ldots\ldots \text{(VIII.)}$$

where σ is a primitive root of the equation $z^{2m}=1$.

As has been said these additional results are also given without proof, and reference is again made to their author being in possession of a general method, for he adds: "Nous supprimons ici un grand nombre d'autres formules et de conséquences analogues."

In the next volume of CRELLE he takes up the subject again, giving another series of results connected with the theory of elliptic functions, viz:—

$$\sqrt{\frac{2\mathrm{K}}{\pi}}=1+\cfrac{2}{p-\cfrac{p}{p^3+1-\cfrac{p^3}{p^5+1-\cfrac{p^5}{p^7+1-\cdots}}}} \quad \ldots\ldots \text{(IX.)}$$

$$\sqrt{\frac{2k'\mathrm{K}}{\pi}}=1-\cfrac{2}{p+\cfrac{p}{p^3-1+\cfrac{p^3}{p^5-1+\cfrac{p^5}{p^7-1+\cdots}}}} \quad \ldots\ldots \text{(X.)}$$

where K, &c., have their usual signification as given above.

Putting, further,

$$\sqrt{-1} = i \quad \text{and} \quad z = e^{\frac{\pi x}{2\mathrm{K}} i},$$

he says

$$\sin \mathrm{am}x = \frac{2}{\sqrt[4]{p}\,\sqrt{k}} \sin\frac{\pi x}{2\mathrm{K}} (\mathrm{A}^2 + \mathrm{B}^2) \qquad \ldots\ldots \text{(XI.)}$$

where

$$\mathrm{A} + \mathrm{B}i = \frac{1}{1} - \frac{z^2}{1+p} - \frac{pz^2}{1+p^2} - \frac{p^2z^2}{1+p^3} - \cdots$$

and similar expressions are given in connection with

$$\cos \mathrm{am}x\,, \quad \frac{\sin \mathrm{am}x}{\Delta\, \mathrm{am}x}\,, \quad \frac{\cos \mathrm{am}x}{\Delta\, \mathrm{am}x}\,, \quad \Delta\, \mathrm{am}x\,, \quad \tan \mathrm{am}x\,;$$

after which he adds: "Il existe un grand nombre d'autres développements nouveaux des fonctions elliptiques, que je réserve pour un autre mémoire, où j'essaierai à les expliquer avec leur principes." In a paragraph separated from the preceding two other identities are given, viz.:—

$$1 + \omega^{-1} + \omega^{-3} + \ldots + \omega^{-\frac{1}{2}n(n-1)} = \frac{1}{1} - \frac{1}{1+\omega} - \frac{\omega}{1+\omega^2} - \cdots - \frac{\omega^{n-2}}{1+\omega^{n-1}} \qquad \ldots\ldots \text{(XVII.)}$$

$$(1-p^{-1}z)(1-p^{-2}z)(1-p^{-3}z)\ldots = 1 + \frac{z}{1-p} - \frac{z}{1+p} + \frac{p^2z}{1-p^3} - \frac{pz}{1+p^2} + \frac{p^4z}{1-p^5} - \frac{p^2z}{1+p^3} + \cdots \qquad \ldots\ldots \text{(XVIII.)}$$

the latter, like (V.) above, being introduced in connection with the subject of irrationality, as proved by means of Lambert's theorem. The closing statement bears, as formerly, on the generality of his method. "Toutes les fractions continues," he says, "que nous avons presentées ici, ne sont que des exemples particuliers. Les méthodes que nous avons employées pour y parvenir nous ont fournis des fractions continues d'une généralité telle, que j'ose assurer qu'elles renferment, outre une foule de résultats nouveaux et très rémarquable comme cas spéciaux, toutes les fractions continues trouvées jusqu'à présent, et surtout toutes celles de M. Gauss. C'est ce que nous expliquerons dans une autre occasion."

In the volume of Crelle for the following year (1845), Eisenstein recurs to the subject of these transformations, but only in a note exactly a page in length. Here he gives a second continued fraction for a series already dealt with, viz. :—

$$1+x+x^3+x^6+x^{10}+\ldots=\frac{1}{1}-\frac{x}{1}-\frac{x^2-x}{1}-\frac{x^3}{1}-\frac{x^4-x^2}{1}-\frac{x^5}{1}-\frac{x^6-x^3}{1}-\ldots\ldots \qquad \ldots\ldots \quad \text{(XIX.)}$$

and concludes as follows :—" Adjicere liceat formulam generaliorem—

$$\frac{(1-x)(1-px)(1-p^2x)\ldots}{(1-y)(1-py)(1-p^2y)\ldots}=\frac{1}{1}+\frac{x-y}{1-p}+\frac{py-x}{1+p}+\frac{p^2x-py}{1-p^3}+\frac{p^3y-px}{1+p^2}+\frac{p^4x-p^2y}{1-p^5}+\ldots\ldots \qquad \ldots\ldots \quad \text{(XX.)}$$

This, it would appear, was the last communication written by **Eisenstein** on the subject of continued fractions; and I am not aware that any one, with a single exception,[*] has attempted to solve the problems thus left, or made any reference to the subject, unless to record his inability to see how the results had been obtained. The object of the present paper is to supply this want.

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[*] In the year 1846 **Heine**, on the instigation of **Jacobi**, attempted to establish **Eisentein's** results by means of **Euler's** transformation. In this however he failed, except in the case of the identities numbered VII. and XVII. above, but his letter in **Crelle** (xxxii. pp. 205–209) closes with an indication of a method of considerable complexity, by which he says (I.) might be obtained. Immediately following this he has another paper on the series

$$1+\frac{(q^\alpha-1)(q^\beta-1)}{(q-1)(q^\gamma-1)}x+\frac{(q^\alpha-1)(q^{\alpha+1}-1)(q^\beta-1)(q^{\beta+1}-1)}{(q-1)(q^2-1)(q^\gamma-1)(g^{\gamma+1}-1)}x^2+\ldots$$

or say $\phi(a, \beta, \gamma, q, x)$, of which **Gauss'** hypergeometric series is a particular case, viz., when $q=1$. Treating this series step by step exactly after the manner of **Gauss**, he obtains the result

$$\frac{\phi(a, \beta+1, \gamma+1, q, x)}{\phi(a, \beta, \gamma, q, x)}=\frac{1}{1}-\frac{a_1x}{1}-\frac{a_2x}{1}-\frac{a_3x}{1}-\ldots\ldots$$

where $a_{2r}=\dfrac{(q^{\beta+r}-1)(q^{\gamma+r-a}-1)}{(q^{\gamma+2r-1}-1)(q^{\gamma+2r}-1)}q^{a+r-1}$ and $a_{2r+1}=\dfrac{(q^{a+r}-1)(q^{\gamma+r-\beta}-1)}{(q^{\gamma+2r}-1)(q^{\gamma+2r-1}-1)}q^{\beta+r}$;

and this, he says, includes those identities of **Eisenstein** which are given in **Crelle**, xxvii. and xxviii. In regard to the first of these papers it must be remarked that the author's failure was greater than it should have been, for there are at least *six* of **Eisenstein's** results obtainable by **Euler's** method, the additional ones being (V.) (VI.) (IX.) and (X.), which are derived from the transformation of the series

$$1+\frac{1}{p}+\frac{1}{p^4}+\frac{1}{p^9}+\ldots$$

The result given in the second paper is very valuable in itself, but as explanatory of **Eisenstein's** work it is of little importance. Instead of saying that the latter's results are contained in it, it would be more correct to say that they are *hidden* in it, if, indeed some of them, as, *e.g.*, those just referred to be there at all.

Of the twenty identities there are at most only five which are independent. These are (I.), (IV.), (V.), (VII.), and (XX.). From (I.) are deducible (II.), (III.), (VIII.), (XIX.). Thus, substituting ρ for $\frac{1}{R}$ in (I.), where ρ is one of the m^{th} roots of unity and m is odd, the left hand member becomes

$$\begin{array}{llll} & 1 + \rho x & + \dots\dots & + \rho^{(m-1)^2} x^{m-1} \\ + \rho^{m^2} x^m & + \rho^{(m+1)^2} x^{m+1} & + \dots\dots & + \rho^{(2m-1)^2} x^{2m-1} \\ + \rho^{(2m)^2} x^{2m} & + \rho^{(2m+2)^2} x^{2m+1} & + \dots\dots & + \rho^{(3m-1)^2} x^{3m-1} \\ + \cdots\cdots & & & \end{array}$$

or

$$\begin{array}{lll} & 1 + \rho x + \dots\dots & + \rho^{(m-1)^2} x^{m-1} \\ + x^m \{ & 1 + \rho x + \dots\dots & + \rho^{(m-1)^2} x^{m-1} \} \\ + x^{2m} \{ & 1 + \rho x + \dots\dots & + \rho^{(m-1)^2} x^{m-1} \} \\ + \cdots\cdots & & \end{array}$$

or

$$\frac{1 + \rho x + \rho^4 x^2 + \dots + \rho^{(m-1)^2} x^{m-1}}{1 - x^m} .$$

Making the same substitution in the right hand member, which thereby becomes terminate, and multiplying both members by $1 - x^m$, we obtain (II.). (VIII.) is derived from (I.) in a perfectly similar manner. Again, in (I.) writing p^{-2} for x and p for R, and dividing both members by p, the continued fraction becomes after some reduction

$$\cfrac{1}{p - \cfrac{1}{p^2 - \cfrac{1-p^2}{p^3 - \cfrac{1}{p^4 - \cfrac{1-p^4}{p^5 - \dots}}}}}$$

and the series becomes

$$\frac{1}{p} + \frac{1}{p^4} + \frac{1}{p^9} + \frac{1}{p^{16}} + \dots$$

Now, it is well known that

$$\sqrt{\frac{2K}{\pi}} = 1 + \frac{2}{p} + \frac{2}{p^4} + \frac{2}{p^9} + \frac{2}{p^{16}} + \dots \quad . \quad . \quad (a)$$

hence, the truth of (III.) is apparent. Lastly, putting x for $\frac{1}{R}$ in (I.), and making some simplifications, we obtain a result which is at once transformed into (XIX.) by writing x^2 for x.

Assuming now the truth of (V.) we may establish (IX.) and (X.). For, taking the reciprocals of both sides of (V.) and increasing them by 1 there results

$$\frac{1}{\frac{1}{z}-\frac{1}{z^4}+\frac{1}{z^9}-\frac{1}{z^{16}}+\dots} = z + \cfrac{z}{z^3-1+\cfrac{z^3}{z^5-1+\ddots}}$$

and writing $-p$ for z and taking reciprocals we have

$$\frac{1}{p}+\frac{1}{p^4}+\frac{1}{p^9}+\frac{1}{p^{16}}+\dots = \cfrac{1}{p-\cfrac{p}{p^3+1-\cfrac{p^3}{p^5+1-\ddots}}} .$$

whence, by means of (α), (IX.) is obtained; and (X.) is derived from it by changing the sign of p.

Again, assuming (XX.) to be established, we may deduce the results from (XI.) to (XVIII.) inclusive. Taking first the case of (XI.) we put $p^{-1}e^{\omega i}$ for y, p^{-2} for p, and $p^{-2}e^{\omega i}$ for x in (XX.), and on simplifying the resulting right-hand member, we find

$$\frac{(1-p^{-2}e^{\omega i})(1-p^{-4}e^{\omega i})(1-p^{-6}e^{\omega i})\dots}{(1-p^{-1}e^{\omega i})(1-p^{-3}e^{\omega i})(1-p^{-5}e^{\omega i})\dots} = \cfrac{1}{1-\cfrac{e^{\omega i}}{1+p-\cfrac{pe^{\omega i}}{1+p^2-\cfrac{p^2e^{\omega i}}{1+p^3-\ddots}}}}$$

and if we take ω to stand for $\frac{\pi x}{\mathrm{K}}$ and $\therefore e^{\omega i}$ for z^2 in EISENSTEIN's notation we shall have

$$\frac{(1-p^{-2}e^{\omega i})(1-p^{-4}e^{\omega i})(1-p^{-6}e^{\omega i})\dots}{(1-p^{-1}e^{\omega i})(1-p^{-3}e^{\omega i})(1-p^{-5}e^{\omega i})\dots} = \mathrm{A}+\mathrm{B}i .$$

$\therefore$ also

$$\frac{(1-p^{-2}e^{-\omega i})(1-p^{-4}e^{-\omega i})(1-p^{-6}e^{-\omega i})\dots}{(1-p^{-1}e^{-\omega i})(1-p^{-3}e^{-\omega i})(1-p^{-5}e^{-\omega i})\dots} = \mathrm{A}-\mathrm{B}i$$

and hence, by multiplication (the first factor of the first numerator with the first of the second, and so on)

$$\frac{(1-2p^{-2}\cos\omega+p^{-4})(1-2p^{-4}\cos\omega+p^{-8})(1-2p^{-6}\cos\omega+p^{-12})\dots}{(1-2p^{-1}\cos\omega+p^{-2})(1-2p^{-3}\cos\omega+p^{-6})(1-2p^{-5}\cos\omega+p^{-10})\dots} = \mathrm{A}^2+\mathrm{B}^2$$

Now the first number of this is known to be equal to

$$\left(\sin \operatorname{am} \frac{\mathrm{K}\omega}{\pi}\right) \div \frac{2\sin\frac{\omega}{2}}{p^{\frac{1}{4}}\,k^{\frac{1}{2}}} .$$

Hence,

$$\sin \mathrm{am} \frac{\mathrm{K}\omega}{\pi} = \frac{2 \sin \frac{\omega}{2}}{p^{\frac{1}{4}} k^{\frac{1}{2}}} (\mathrm{A}^2 + \mathrm{B}^2)$$

i.e.,

$$\sin \mathrm{am} x = \frac{2}{p^{\frac{1}{4}} k^{\frac{1}{2}}} \sin \frac{\pi x}{2\mathrm{K}} (\mathrm{A}^2 + \mathrm{B}^2)$$

as was to be proved. In a perfectly similar manner (XII.) . . . (XVI.) may be established. For the case of (XVII.) we put $x = q^2$, $y = q$, and $p = q^2$ in (XX.), and there results after simplification

$$\frac{(1-q^2)(1-q^4)(1-q^6)\ldots}{(1-q)(1-q^3)(1-q^5)\ldots} = \frac{1}{1-}\frac{q}{1+q-}\frac{q^2}{1+q^2-}\cdots$$

But the first member is known to be $= 1 + q + q^3 + q^6 + q^{10} + \ldots$, and therefore on writing ω^{-1} for q we have the identity required, in the case where n is infinite. Lastly, equating the reciprocals of the sides of (XX.), then putting $x = 0$, $p^{-1}z$ for y and p^{-1} for p, we obtain (XVIII.) almost at once.*

Having thus reduced the number of apparently independent results to *five*, it now remains to show how these may be established. Consider, first, the most important of the five, viz., (XX.). Making use of the observed fact that the continued product

$$(1-x)(1-px)(1-p^2x)\ldots .$$

is divided by $1-x$ by writing in it px for x, we can readily transform

$$\frac{(1-x)(1-px)(1-p^2x)\ldots .}{(1-y)(1-py)(1-p^2y)\ldots .}$$

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* Equating the reciprocals of the sides of (XX.), putting $y = 0$ and writing z for x, we have the same continued fraction equal to

$$\frac{1}{(1-z)(1-pz)(1-p^2z)\ldots}$$

Hence the identity

$$\left(1-\frac{z}{p}\right)\left(1-\frac{z}{p^2}\right)\left(1-\frac{z}{p^3}\right)\cdots = \frac{1}{(1-z)(1-pz)(1-p^2z)\ldots}$$

the truth of which may also be seen from the fact that were we to try to find, by the method of undetermined coefficients, the expansion of the two expressions in ascending powers of z, we should in each case make use of the observation that the change of z into pz is equivalent to multiplication by $1 - z$.

into

$$\frac{1+\dfrac{1}{p-1}x+\dfrac{p}{(p-1)(p^2-1)}x^2+\dfrac{p^3}{(p-1)(p^2-1)(p^3-1)}x^3+\dots}{1+\dfrac{1}{p-1}y+\dfrac{p}{(p-1)(p^2-1)}y^2+\dfrac{p^3}{(p-1)(p^2-1)(p^3-1)}y^3+\dots} \quad \dots (\beta)$$

Now it has been shown elsewhere* that

$$\frac{c_0+c_1x+c_2x^2+c_3x^3+\dots}{b_0+b_1x+b_2x^2+b_3x^3+\dots}=\cfrac{a_1}{1-\cfrac{a_2x}{1-\cfrac{a_3x}{1-\dots}}} \quad \dots (A)$$

$$\text{where } a_1=\frac{\Delta_1}{b_0},\quad a_2=\frac{\Delta_2}{b_0\Delta_1},\quad a_3=\frac{\Delta_3}{\Delta_1\Delta_2},\quad \dots\dots\quad a_n=\frac{\Delta_{n-3}\Delta_n}{\Delta_{n-2}\Delta_{n-1}}$$

$$\text{and } \Delta_n=\begin{vmatrix} b_0 & b_1 & b_2 & b_3 & \dots & b_{n-1} \\ 0 & b_0 & b_1 & b_2 & \dots & b_{n-2} \\ 0 & 0 & b_0 & b_1 & \dots & b_{n-3} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & c_0 & c_1 & \dots & c_{n-3} \\ 0 & c_0 & c_1 & c_2 & \dots & c_{n-2} \\ c_0 & c_1 & c_2 & c_3 & \dots & c_{n-1} \end{vmatrix}$$

the number of rows containing b in the determinant being the same as the number of rows containing c or one more according as n is even or odd. Putting $n=1$ in this, and substituting the terms of the numerator of (β) for c_0, c_1, c_2 . . . and the terms of the denominator for b_0, b_1, b_2 . . . in (A) we obtain

$$\frac{(1-x)(1-px)(1-p^2x)\dots}{(1-y)(1-py)(1-p_2y)\dots}=\cfrac{1}{1-\cfrac{\frac{x-y}{p-1}}{1-\cfrac{\frac{py-x}{p^2-1}}{1-\cfrac{\frac{p^2x-py}{(p^3-1)(p+1)}}{1-\dots}}}}$$

which, when the right-hand member is simplified, is identical with (XX.). On putting $b_0=1$, $b_1=b_2=b_3=\dots=0$ in (A) we have a special form (B), say, giving for a series in ascending powers of x an expression of the form

$$\cfrac{a_1}{1-\cfrac{a_2x}{1-\cfrac{a_3x}{1-\dots}}}$$

and on putting $c_0=1$, $c_1=c_2=c_3=\dots=0$, and taking reciprocals of both

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* Trans. R.S.E. 1875-6.

sides, there results another special form (C), say, which gives for a series of the same kind an expression of the form

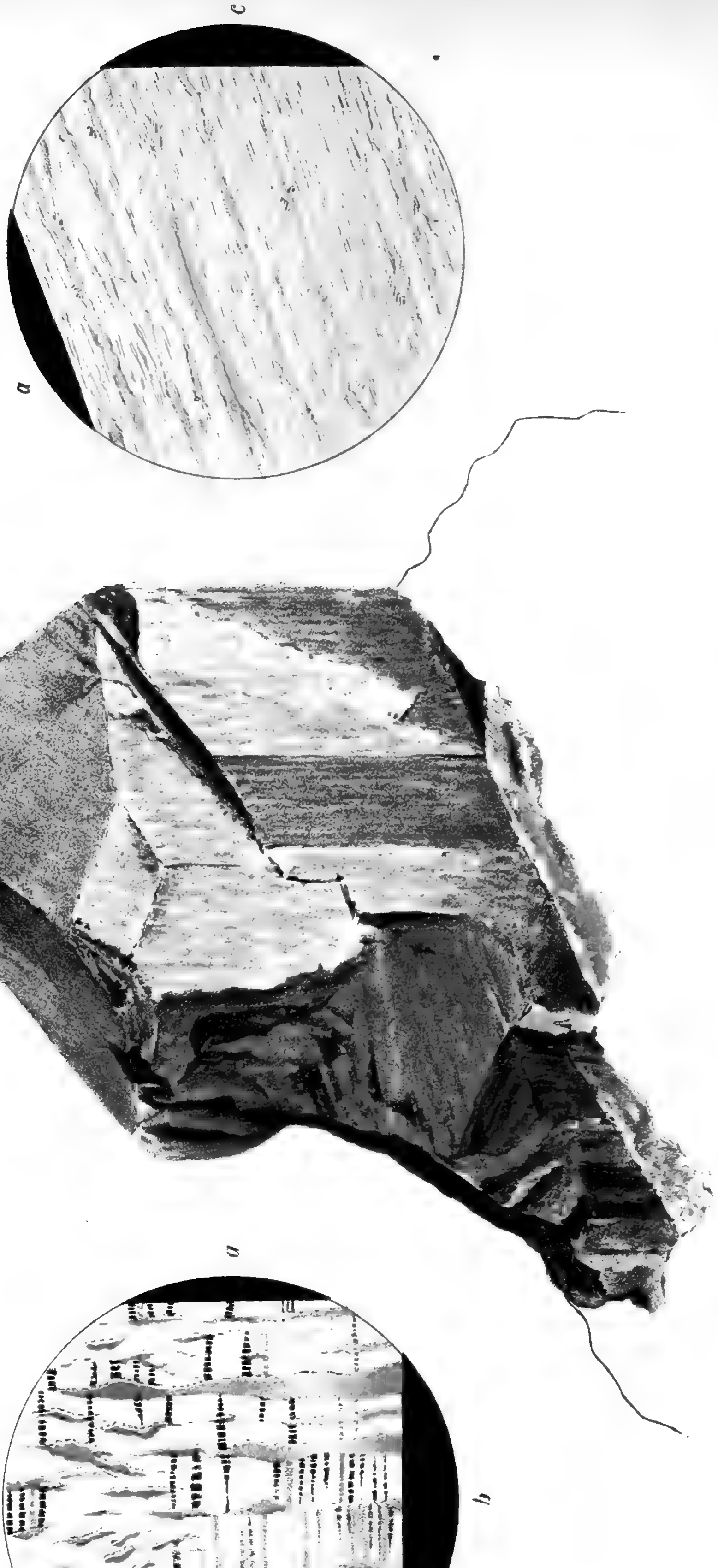
$$\beta_0 + \frac{\beta_1 x}{1} - \frac{\beta_2 x}{1} - \frac{\beta_3 x}{1} - \cdots\cdots$$

These results serve to establish (I.) (IV.) and (VII.). Writing 1, R^{-1}, R^{-4}, R^{-9} . . . for c_0, c_1, c_2, c_3 . . . in (B) we obtain (I.). In the case of (IV.) we replace $\frac{2K}{\pi}$ by its known equivalent

$$1 + \frac{4p}{p^2+1} + \frac{4p^2}{p^4+1} + \frac{4p^3}{p^6+1} + \ldots$$

and writing the terms of this in order for b_0, b_1, b_2, b_3 . . . in (C) and putting $x = 1$, the required result is obtained after some simplification. (VII.) is got from (B) in an exactly similar manner. The remaining identities (V.), (VI.), (IX.), (X.) have been already referred to in the footnote as being obtainable by EULER's method.

It is hard to derive from a study of EISENSTEIN's work any positive information regarding the method or methods which he employed. This is due not only to the fact that he confines himself in general to the statement merely of results, but also that occasionally he "darkens by elucidation," a certain air of mystery being sometimes induced by the few inevitable words which connect one result with another, and by the seeming capriciousness with which he selects his special cases, often preferring an intricate process of substitution for one which is more evident and equally effective. It seems, however, highly probable that the method was more akin to GAUSS' than EULER's, the majority of results being closely allied to those obtained by the former method. One thing is certain, EISENSTEIN knew nothing of EULER's transformation, for, otherwise, he would not have attached importance to those of his results which are obtainable by it, knowing, as he must then have done, that an infinitude of such results lay ready to the hands of any one, viz., one result, at least, in connection with every series in existence; and, besides, in one place he changes a series into a product, then the product into another series, and from this second series derives his continued fraction, whereas the result could have been got from the *first* series with the greatest ease by EULER's method. Although we could have wished more information, and an improved style in conveying it, it deserves to be said, however, that the method has given a number of interesting and useful results in analysis, and two, viz., (I.) and (XX.) which are decidedly notable.



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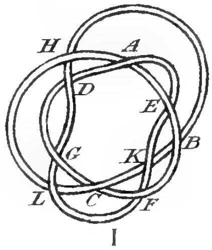

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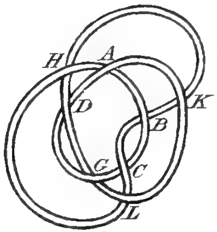

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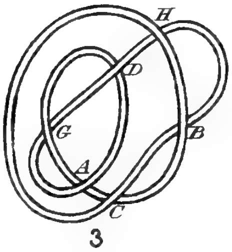

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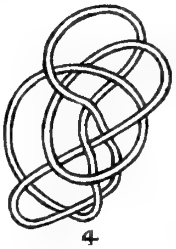

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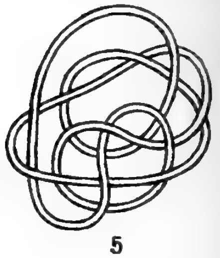

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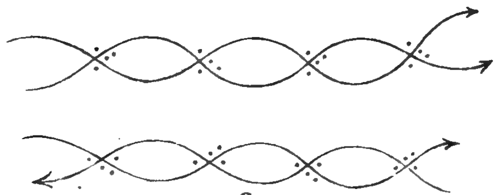

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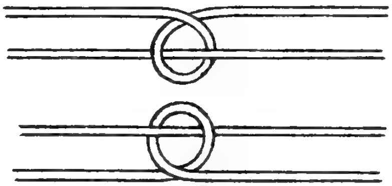

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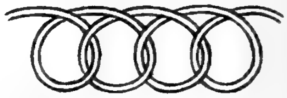

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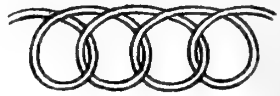

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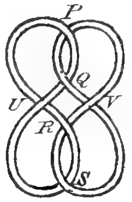

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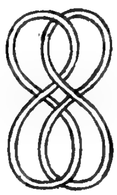

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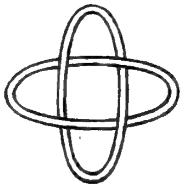

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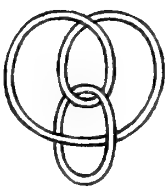

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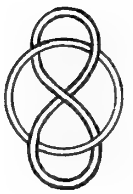

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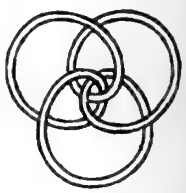

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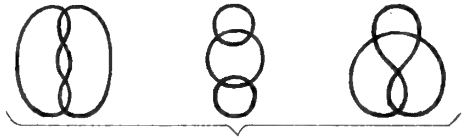

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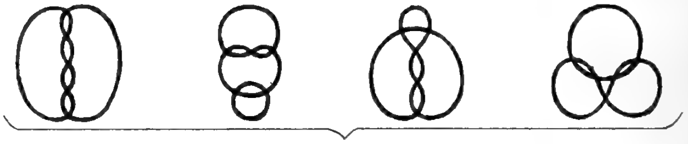

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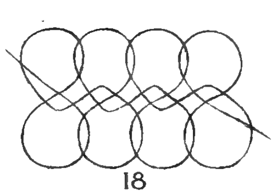

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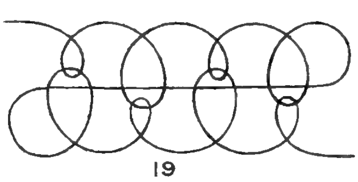

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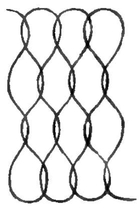

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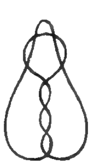

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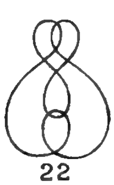

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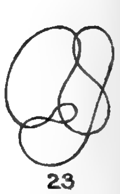

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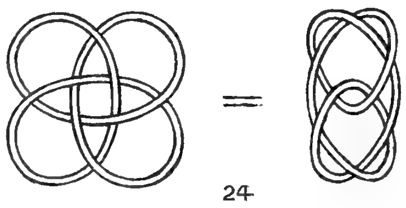

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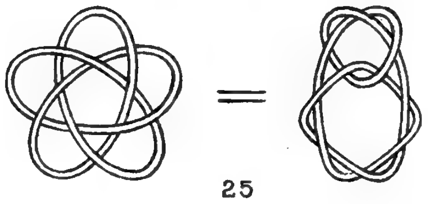

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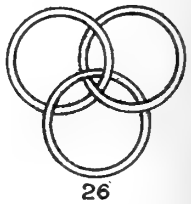

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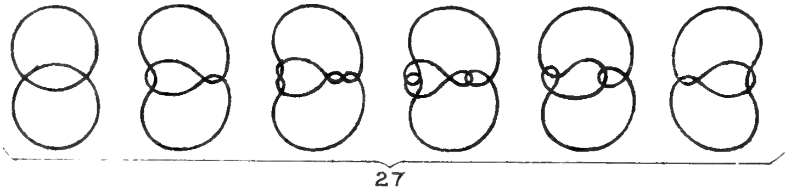

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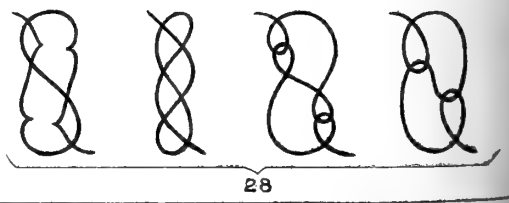

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P. G. T.   J. Bartholomew, Edinr.

VIII.—*On Knots.* By Professor TAIT. (Plates XV. and XVI.)

(Revised May 11, 1877.)

The following paper contains, in a compact form, the substance of several somewhat bulky communications laid before the Society during the present session. The gist of each of these separate papers will be easily seen from the abstracts given in the Proceedings. These contain, in fact, many things which I have not reproduced in this digest. Nothing of any importance has been added since the papers were read, but the contents have been very much simplified by the adoption of a different order of arrangement; and long passages of the earlier papers have been displaced in favour of short general statements from the later ones. With the exception of the portion which deals with the main question raised, this paper is fragmentary in the extreme. Want of leisure or press of other work may justly be pleaded as one cause; but there is more than that. The subject is a very much more difficult and intricate one than at first sight one is inclined to think, and I feel that I have not succeeded in catching the key-note. When that is found, the various results here given will no doubt appear in their real connection with one another, perhaps even as immediate consequences of a thoroughly adequate conception of the question.

I was led to the consideration of the forms of knots by Sir W. THOMSON'S Theory of Vortex Atoms, and consequently the point of view which, at least at first, I adopted was that of classifying knots by the number of their crossings; or, what comes to the same thing, *the investigation of the essentially different modes of joining points in a plane, so as to form single closed plane curves with a given number of double points.*

The enormous numbers of lines in the spectra of certain elementary substances show that, if THOMSON'S suggestion be correct, the form of the corresponding vortex atoms cannot be regarded as very simple. For though there is, of course, an infinite number of possible modes of vibration for every vortex, the number of modes whose period is within a few octaves of the fundamental mode is small unless the form of the atom be very complex. Hence the difficulty, which may be stated as follows (assuming, of course, that the visible rays emitted by a vortex atom belong to the graver periods):—"What has become of all the simpler vortex atoms?" or "Why have we not a much greater number of elements than those already known to us?" It will be allowed that, from the point of view of the vortex-atom theory, this is almost a vital question.

Two considerations help us to an answer. *First,* however many simpler forms may be geometrically possible, only a very few of these may be forms of kinetic stability, and thus to get the sixty or seventy permanent forms required for the known elements, we may have to go to a very high order of complexity. This leads to a physical question of excessive difficulty. THOMSON has briefly treated the subject in his recent paper on "Vortex Statics,"* but he cannot be said to have as yet even crossed the threshold. But *secondly*, stable or not, are there after all very many different forms of knots with any given small number of crossings? This is the main question treated in the following paper, and it seems, so far as as I can ascertain, to be an entirely novel one.

When I commenced my investigations I was altogether unaware that anything had been written (from a scientific point of view) about knots. No one in Section A at the British Association meeting of 1876, when I read a little paper on the subject, could give me any reference; and it was not till after I had sent my second paper to this Society that I obtained, in consequence of a hint from Professor CLERK-MAXWELL, a copy of the very remarkable Essay by LISTING, *Vorstudien zur Topologie*,† of which (so far as it bears upon my present subject) I have given a full abstract in the Proceedings of the Society for Feb. 3, 1877. Here, as was to be expected, I found many of my results anticipated, but I also obtained one or two hints which, though of the briefest, have since been very useful to me. LISTING does not enter upon the determination of the number of distinct forms of knots with a given number of intersections, in fact he gives only a very few forms as examples, and they are curiously enough confined to three, five, and seven crossings only; but he makes several very suggestive remarks about the representation of knots in general, and gives a special notation for the representation of a particular class of "reduced" knots. Though this has absolutely no resemblance to the notation employed by me for the purpose of finding the number of distinct forms of knots, I have found a slight modification of it to be very useful for various purposes of illustration and transformation. This work of LISTING'S, and an acute remark made by GAUSS (which, with some comments on it by CLERK-MAXWELL, will be referred to later), seem to be all of any consequence that has been as yet written on the subject. I have acknowledged in the text all the hints I have got from these writers; and the abstract of LISTING's work above referred to will show wherein he has anticipated me.

PART I.

The Scheme of a Knot, and the number of distinct Schemes for each degree of Knottiness.

§ 1. My investigations commenced with a recognition of the fact that in any knot or linkage whatever the crossings may be taken throughout alternately

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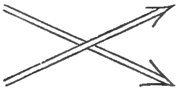
* Proc. R. S. E. 1875–6 (p. 59).

† Göttinger Studien, 1847.

over and under. It has been pointed out to me that this seems to have been long known, if we may judge from the ornaments on various Celtic sculptured stones, &c. It was probably suggested by the processes of weaving or plaiting. I am indebted to Mr DALLAS for a photograph of a remarkable engraving by DÜRER, exhibiting a very complex but symmetrical linkage, in which this alternation is maintained throughout. Formal proofs of the truth of this and some associated properties of knots will be found in the little paper already referred to.* They are direct consequences of the obvious fact that two closed curves in one plane necessarily intersect one another an *even* number of times. It follows as an immediate deduction from this that in going continuously round any closed plane curve whatever, an even number of intersections is always passed on the way from any one intersection to the same again. Hence, of course, if we agree to make a knot of it, and take the crossings (which now correspond to the intersections) over and under alternately, when we come back to any particular crossing we shall have to go *under* if we previously went *over*, and *vice versâ*. This is virtually the foundation of all that follows.

But it is essential to remark that we have thus two alternatives for the crossing with which we start. We may make the branch we begin with cross *under* instead of *over* the other at that crossing. This has the effect of changing any given knot into its own image in a plane mirror—what LISTING calls *Perversion*. Unless the form be an *Amphicheiral* one (a term which will be explained later), this perversion makes an essential difference in its character—makes it, in fact, a different knot, incapable of being deformed into its original shape.

LISTING speaks of crossings as *dexiotrop* or *laeotrop*. If we think of the edges of a flat tape or india-rubber band twisted about its mesial line, we recognise at once the difference between a right and a left handed crossing. (Plate XV. fig. 1.) Thus the acute angles in the following figure are left handed, the obtuse, right handed; and they retain these characters if the figure be turned over (*i.e.*, about an axis in the plane of the paper):—



but in its image in a plane mirror these characters are interchanged.

§ 2. Suppose now a knot of any form whatever to be projected as a shadow cast by a luminous point on a plane. The projection will always necessarily have double points,† and in general the number of these may be increased—

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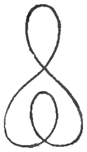
[*] "Messenger of Mathematics," January 1877.

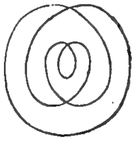
[†] Higher multiple points may, of course, occur, but an *infinitesimal* change of position of the luminous point, or of the relative dimensions of the coils of the knot, will remove these by splitting them into a number of double points, so that we need not consider them.

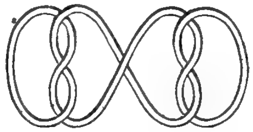
though not always diminished—by a change of position of the luminous point, or by a distortion of the wire or cord, which we may suppose to form the knot. This wire or cord must be supposed capable of being bent, extended, or contracted to any extent whatever, subject to the *sole* condition that no lap of it can be pulled through another, *i.e.*, that its continuity cannot be interrupted. There are, therefore, projections of every knot which give a *minimum* number of intersections, and it is to these that our attention must mainly be confined. Later we will consider the question how to determine this minimum number, which we will call *Knottiness*, for any particular knot; but for our present purpose it is sufficient to get rid of what are *necessarily* nugatory intersections, *i.e.*, intersections which no alteration of the mode of crossing can render permanent. These crossings are essentially such that if both branches of the string were cut across at one of them, and their ends reunited crosswise, so as to form two separate closed curves, these separate curves shall not be linked together, however they may individually be knotted, *i.e.*, that if they are knots they are separate from one another, so that one of them may be drawn tight so as to present only a roughness in the string. For in this case the nugatory crossing will thus be made to bound a mere *loop*.

[We may define a necessarily nugatory crossing as one through which a closed, or an infinitely extended, surface may pass without meeting the string anywhere but at the crossing. Or, as will be seen later (§ 20), we may recognise a necessarily nugatory crossing as a point *where a compartment meets itself*.]

In the first two of the sketches subjoined all the crossings are necessarily nugatory; in the third, only the middle one is so.







Now these diagrams, when lettered in the manner forthwith to be explained (see, for instance, Plate XVI. fig. 1), present respectively the following *schemes*:—

$\mathrm{A}A\mathrm{B}B \mid \mathrm{A}$
$\mathrm{A}C\mathrm{B}B\mathrm{C}A \mid \mathrm{A}$
$\mathrm{A}C\mathrm{B}D\mathrm{C}B\mathrm{D}A\mathrm{E}G\mathrm{F}E\mathrm{G}F \mid \mathrm{A}$

These and similar examples show that in a scheme a crossing is necessarily nugatory, if between the two appearances of the letter denoting that crossing there is a group consisting of any set of letters *each occurring twice.* The set may consist of any number whatever, including zero. For our present purpose it will be found sufficient to consider this last special case alone, *i.e.*, *the same letter twice in succession denotes a necessarily nugatory crossing.*

§ 3. If we affix letters to the various crossings, and, going continuously round the curve, write down the name of each crossing in the order in which we reach it, we have, as will be proved later, the means of drawing without ambiguity the projection of the knot. If, in addition, we are told whether we passed over or under on each occasion of reaching a crossing we can, again without any ambiguity, construct the knot in wire or cord. Passing over is, in what follows, indicated by a + subscribed to the letter denoting the crossing—passing under by a —. Any specification which includes these two pieces of information is necessarily *fully descriptive* of the knot; and when it is given in the particular form now to be explained we shall call it the *Scheme.*

If in accordance with § 1 we make the crossings alternately over and under, it is obvious that the odd places and even places of the scheme will each contain all the crossings. As the choice of letters is at our disposal, we may therefore call the crossings in the odd places A, B, C, &c., in alphabetical order, starting from any crossing we please, and going round the knotted wire in any of the four possible ways, *i.e.,* starting from any crossing by any of the four paths which lead from it, put the successive letters at the first, third, fifth, &c., crossings as we meet them. Then it is obvious that the essential character of the projected knot must depend only upon *the way in which the letters are arranged in the even places of the scheme.* Of course, the nature and reducibility (*i.e.,* capability of being simplified by the removal of nugatory crossings) of the knot itself depend also upon the subscribed signs. [In general there will be four different schemes for any one knot, but in the simpler cases these are often identical two and two, sometimes all four.]

§ 4. Here we may remark that it is obvious that when the crossings are alternately + and — no reduction is possible, unless there be essentially nugatory crossings, as explained in § 2. For the only way of getting rid of such alternations of + and — along the same cord is by *untwisting;* and this process, except in the essentially nugatory cases, gets rid of a crossing at one place only by introducing it at another. It will be seen later that this process may in certain cases be employed *to change the scheme* of a knot, and thus to show that in these cases there may be more than four different schemes representing the same knot; though, as we have already seen, a scheme is perfectly definite as to the knot it represents. Hence, in the first part of our work, we shall suppose that the crossings are taken alternately + and —, so that no reduction is possible. But it will afterwards be shown that, even when all essentially nugatory crossings are removed, it is not always necessary to have the regular alternation of + and — in order that the knot may not be farther reducible. It is easy to see a reason for this, if we think of a knot made up of different knots on the same string, whether separate from one another or linked together. For the irreducibility of each separate knot depends only upon the alternations of + and

— *in itself,* and the two knots may be put together, so that this condition is satisfied in the partial schemes, but not in the whole. As there cannot be a knot with fewer than three crossings, we do not meet with this difficulty till we come to knots with six crossings. And as there can be no linking without at least two crossings, we do not meet with linked knots on the same string till we come to eight crossings at least.

§ 5. We are now prepared to attack our main question.

Given the number of its double points, to find all the essentially different forms which a closed curve can assume.

Going round the curve continuously, call the first, third, &c., intersections A, B, C, &c. In this category we evidently exhaust all the intersections. The complete scheme is then to be formed by properly interpolating the same letters in the even places; and the form of the curve depends solely upon the way in which this is done.

It cannot, however, be done at random. For, *first,* neither A nor B can occur in the second place, B nor C in the fourth, and so on, else we should have necessarily nugatory intersections, as shown in § 2. Thus the number of possible arrangements of n letters (viz., $n.\,\overline{n-1}\;.\;.\;.\;.\;2.1$) is immensely greater than the number which need here be tried. But, *secondly,* even when this is attended to, the scheme may be an impossible one. Thus, the scheme

A *D* B *E* C *A* D *B* E *C* | A

is lawful, but

A *D* B *A* C *E* D *C* E *B* | A

is not.

The former, in fact, may be treated as the result of superposing two closed (and not self-intersecting) curves, both denoted by the letters A D B E C A, so as to make them cross one another at the points marked B, C, D, E, then cutting them open at A, and joining the free ends so as to make a continuous circuit with a crossing at A.

But in the latter scheme above we have to deal with the curves A D B A and C E C E, and in the last of these we cannot have junctions alternately + and — as required by our fundamental principle. In fact, the scheme would require the point C to lie simultaneously inside and outside the closed circuit A D B A.

Or we may treat A D B A and C E D C as closed curves intersecting one another and yet having only one point, D, in common.

Thus, to test any arrangement, we may strike out from the whole scheme all the letters of any one closed part as A——*A*, and the remaining letters must satisfy the fundamental principle, *i.e.*, that they can be taken with suffixes + and — alternately, or what comes to the same thing) that an even number of letters intervenes between the two appearances of each of the remaining letters.

Or we may strike out all the letters of any two sets which begin and end similarly, *e.g.*, A . . . *X*, X . . . *A*, the two together being treated as one closed curve, and the test must still apply.

More generally, we may take the sides of any closed polygon as A—*X*, X—*Y*, Y—*Z*, Z—*A*, and apply them in the same way. But in this, as in the simpler case just given, the sides must all be taken the same way round in the scheme itself.

A simple mode of applying these tests will be given later, when we are dealing with the question of *Beknottedness.*

It may be well to explain here how a change of the crossing selected as the initial one alters the scheme. Take the simple case of making B the first, and reckoning on from it. Then B becomes A, &c., and the scheme, which may be any whatever, suppose for example

A *F* B *L* C *E* D *H* . . . .

becomes (by writing for each letter that which alphabetically precedes it)

N *E* A *K* B *D* C *G* . . . .

or beginning with A,

A *K* B *D* C *G* . . . .

Hence the letters

F, *L*, *E*, *H*, ...

in the even places of a scheme are equivalent to

K, *D*, *G*, ... *E*,

i.e., we may change each to the preceding letter taken in the cyclical order of the alphabet and put the first to the end, or *vice versâ*, without altering the scheme. An arrangement of this kind is *unique* (reproducing itself) if the letters are in cyclical order; and if the number of letters be a prime, any arrangement is either unique or is reproduced after a number of operations of this kind equal to the number of letters. If it be not prime, arrangements may be found which will reproduce themselves after a number of operations equal to any one of its aliquot parts.

Another lawful change is this:—Begin from the A in the even places and letter as usual, *i.e.*, start from the same crossing as before, and in the same direction round the curve, but not by the same branch of the cord or wire. This will be evident from an example. Beginning at the second A, and lettering alphabetically every second crossing, we have the suffixed letters.

A *D* B *A* C *F* D *B* E *C* F *E* | A
F A B C D E

Now write the same equivalents for the same letters in the odd places, and the scheme in its new lettering is

A F C A D B F C E D B E | A

or the following are equivalents in the even places

D A F B C E
D F E B A C,

and each of these has, of course, five other equivalents found by the first of these two processes.

But we may also start from the same intersection A by either of these paths, but *in the reverse direction round the curve.* To effect this we have only to read the scheme backwards, beginning at either A, and changing the lettering throughout in accordance with our plan. Thus, taking the last example,

A *D* B *A* C *F* D *B* E *C* F *E* | A
F E D C B | A

we keep the terminal A unchanged, and write B, C, &c., for the 2d, 4th, &c., *preceding* letters. We have thus, as it were, the key for translating from the upper line to the lower. Apply this key to all the letters, and then write the result in the reverse order. Thus we get

A *C* B *E* C *F* D *B* E *A* F *D* | A

This new scheme has for its even places

C E F B A D

which is equivalent (in this particular case) to the *second* of the two direct schemes just given, viz.:—

D F E B A C.

Finally, if we read this reversed scheme from the A in the even places, its even letters become

E A F C B D

which (in this case) is the same as

D A F B C E

the even letters of the original scheme.

The notation we shall employ is this—*do, de, ro, re,* signifying the even places of the four cases

d o the *direct* scheme, read from A in the *odd* place
d e the *direct* scheme, read from A in the *even* place
r o the *reversed* scheme, read from A in the *odd* place
r e the *reversed* scheme, read from A in the *even* place

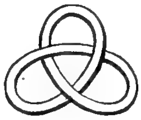
and we shall denote by an appended numeral the number of times the operation above has to be performed. Thus, in the example just given it will be found that

$$\begin{aligned} r\,o &= d\,e\,2 \\ r\,e &= d\,o\,2. \end{aligned}$$

§ 6. With one intersection or two only, a *knot* is thus impossible, for the crossings must necessarily be nugatory. Hence we commence with *three.* And here there is but one case, for by our rule we must write A, B, C in the odd places, and *we have no choice* as to what to interpolate in the even ones. Thus the only knot with three intersections has the scheme

A *C* B *A* C *B* | A

One of its two projections is the "trefoil" knot below.



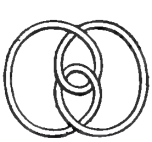
For *four* intersections our choice in the even places is restricted to C or D for the second, D or A for the fourth, &c., as expressed below,

$$\begin{array}{cccc} \mathrm{C} & \mathrm{D} & \mathrm{A} & \mathrm{B} \\ \mathrm{D} & \mathrm{A} & \mathrm{B} & \mathrm{C}. \end{array}$$

Now, if we take C to begin with, we obviously *must* take D next, else we shall not get it at all. Similarly A *must* come third. And if we begin with D, we *must* end with C, so that this case also is determinate. The only possible sets, therefore, are given by these two rows as they are written. But it is obvious that, as they are in cyclical order, the full schemes will be identical if one be read from the beginning, the other from the A in the even places. Thus they represent the same arrangement, and the sole knot with four intersections has the scheme

A *C* B *D* C *A* D *B* | A.

One of its two projections is given by the annexed figure :—



§ 7. When we have *five* intersections, our choice for the even places in order is limited to the following groups of three for each, viz. :—

C D E A B
D E A B C
E A B C D.

This gives the following thirteen arrangements :—

(1) C D E A B
(2) C E A B D
(3) C E B A D
(4) C A E B D
(5) D E A B C
(6) D E B A C
(7) D E A C B
(8) D A E B C
(9) D A E C B
(10) E D A B C
(11) E D A C B
(12) E D B A C
(13) E A B C D.

Now of these (1), (5), and (13) are unique; (6), (7), (8), and (10) can be obtained from (2) by cyclical alteration of the letters and bringing the last to be the beginning, and by the same process (4), (9), (11), (12) may be deduced from (3).

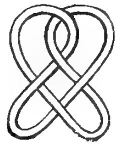
Hence the only possible forms are included in the following arrangements for the letters in the even places :—

C D E A B
C E A B D
C E B A D
D E A B C
E A B C D.

Of these the 1st, 3d, and 5th violate the conditions laid down in § 5 above. Hence there are but two schemes for five intersections, viz. :—

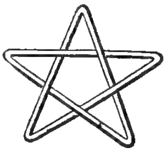
A *C* B *E* C *A* D *B* E *D* | A

of which this is one of the four forms



and A *D* B *E* C *A* D *B* E *C* | A

one of the two forms of which is the pentacle or Solomon's seal



§ 8. The case of six intersections gives the following choice :—

C	D	E	F	A	B
D	E	F	A	B	C
E	F	A	B	C	D
F	A	B	C	D	E.

I found, by trial, that there are 80 possible arrangements included in this form ; and that the following 20 alone are distinct. I have appended to each the number of apparently different forms in which it occurs among the 80 arrangements :—

1.	C D E F A B	Unique
2.	C D F B A E	Six forms
3.	C D F A B E	Six forms
4.	C D A F B E	Six forms
5.	C D B F A E	Three forms
6.	C E F B A D	Six forms
7.	C E F A B D	Six forms
8.	C E A F B D	Three forms
9.	D E F A B C	Unique
10.	C F E B A D	Two forms
11.	E F A B C D	Unique
12.	D F A B C E	Six forms
13.	C F A B D E	Six forms
14.	D F A C B E	Six forms
15.	D F B A C E	Three forms
16.	C F B A D E	Six forms
17.	C A F B D E	Six forms
18.	C A B F D E	Three forms
19.	D A F C B E	Two forms
20.	F A B C D E	Unique

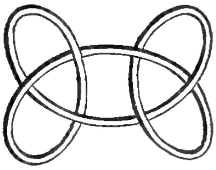
Of these, all but (5), (6), (7), (8), (12), (14), (15), (18), violate the conditions of § 5, and therefore do not correspond to real knots. Of those excepted the schemes agree in pairs when the branch first taken from the starting-point is changed.

Hence there are only *four* forms of 6-fold knottiness. These are as follows:—

(*a*). (5) and (18) agree in giving the scheme

A *C* B *A* C *B* D *F* E *D* F *E* | A

of which one form is the following :—

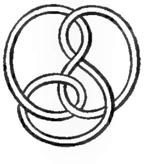


This form consists of two *separate* trefoil knots.

(β). (6) and (14) give the scheme

A *C* B *E* C *F* D *B* E *A* F *D* | A

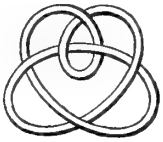
one form of which is as follows :—



(γ). From (7) and (12) we have

A *C* B *E* C *F* D *A* E *B* F *D* | A

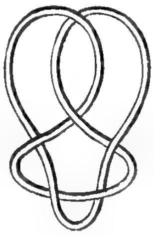
which has as one form



(δ). (8) and (15) give

A *C* B *E* C *A* D *F* E *B* F *D* | A

of which one form is



§ 9. The case of *seven* intersections is the only other to which I have found leisure to apply this method. As I did not see how otherwise to make certain that I had got all possible forms, I wrote out all the combinations of seven different letters, one from each column (in order) of the scheme—

C	D	E	F	G	A	B
D	E	F	G	A	B	C
E	F	G	A	B	C	D
F	G	A	B	C	D	E
G	A	B	C	D	E	F.

These I thus found to amount to 579. Then, by the help of an improvised arrangement of cardboard, somewhat resembling *Napier's Bones*, I rapidly struck off six of each equivalent set of 7. Thus 87 forms in all were left, viz., one form from each of 82 groups of seven, and 5 unique forms. Here they are—

1. C D E F G A B
2. C D E G A B F
3. C D E A G B F
4.* C D E B G A F
5. C D F B G A E
6. C D F G B A E
7. C D F A G B E
8. C D G F A B E
9. C D G F B A E
10. C D G A B E F
11. C D G B A E F
12. C D A G B E F
13.* C D A B G E F
14.* C D B A G E F
15.* C D B G A E F
16. C E F G A B D
17.* C E F G B A D
18. C E F G A D B
19. C E F A G B D
20.* C E G F B A D
21. C E G F D A B
22.* C E G A B D F
23. C E G A D B F
24.* C E G B A D F
25. C E G B D A F
26.* C E A G B D F
27. C E A G D B F
28. C E A B G D F
29. C E B G A D F
30. C E B G D A F
31. C E B A G D F
32. C F G A B D E
33.* C F G A D B E
34.* C F G A B E D
35. C F G B A E D
36. C F G B A D E
37. C F G B D A E
38.* C F A G B D E
39.* C F A G D B E
40.* C F A G B E D
41. C F A B G D E
42. C F A B G E D
43. C F B G A D E
44. C F B G D A E
45. C F B G A E D
46. C F B A G E D
47. C F B A G D E
48. C G E B A D F
49. C G E B D A F
50. C G F A B D E
51. C G F A B E D
52.* C G F A D B E
53. C G F B A D E
54. C G F B A E D
55. C G A F D B E
56. C G A B D E F
57. C G B A D E F
58. C A F G B D E
59. C A F G B E D
60. C A F G D B E
61. C A F B G E D
62. C A G B D E F
63.* C A B G D E F
64. D E F G A B C
65. D E G A B C F
66. D E G A C B F
67.* D E G B A C F
68. D E G C A B F
69. D E A G B C F
70. D E A G C B F
71.* D F G A B C E
72.* D F G A C B E
73. D F G B A C E
74. D F A G C B E
75. D G A B C E F
76. D G A C B E F
77. D G B A C E F
78. D G B C A E F
79. D A G B C E F
80. D A G C B E F
81.* E F G A B C D
82. E G A B C D F
83.* E G A B D C F
84. E G A C B D F
85.* E G B A D C F
86. F G A B C D E
87. G A B C D E F

On testing these by the rules of § 5, I found that 22 only, viz., those marked with an asterisk, correspond to real knots.

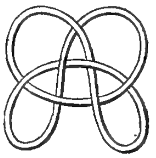
§ 10. When we study these groups by the method of § 5, we find that more than one of them correspond to different readings of the scheme of one and the same knot. Of course that knot will be the least symmetrical which has the greatest number of essentially different schemes. The following grouping has thus been arrived at (the notation is that of § 5 above):—

	do	*de*	*ro*	*re*
I.	(4)	1,(63)	(63)	6,(4)
	(13)	6,(15)	(15)	1,(13)
II.	(17)	3,(83)	5,(83)	2,(17)
III.	(20)	3,(85)	3,(85)	(20)
IV.	(22)	6,(33)	(22)	6,(33)
V.	(24)	(39)	(26)	5,(52)
VI.	(34)	(34)	6,(34)	6,(34)
VII.	(38)	(67)	(67)	(38)
VIII.	(40)	6,(40)	6,(40)	(40)
IX.	(71)	(71)	(71)	(71)
X.	(72)	(72)	5,(72)	5,(72)
XI.	(81)	(81)	(81)	(81)
	(14)	(14)	(14)	(14)

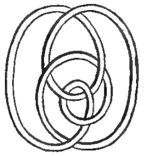
Thus it appears that the knot V., represented by any of the four schemes (24), (26), (39), and (52), is devoid of symmetry, while VI., VIII., IX., X., XI. have the highest symmetry. No number has been in this table affixed to (14), because it is only accidentally a 7-fold knot. It is represented by the third figure in § 2 above, and when the nugatory crossing is removed, it becomes (*a*) of the 6-fold type, § 8. Also it will be noticed that (4) and (63), although their common scheme differs from that of (13) and (15), are included with them under I. The reason is that the knot represented is a composite one, consisting of a 3-fold and a 4-fold knot, and that either may be slipped along the string or wire into any position whatever relative to the other. But even with this licence it appears that there are only 4 really distinct schemes.

In the second and third rows of figures of Plate XV. projections of each of these classes of 7-fold knottiness are given, with the number of the class attached.

§ 11. But the knots represented by these eleven forms are not all distinct. It will readily be seen that (by the process of inversion of § 15 below) II., when formed of wire, with crossings + and — alternately, may be brought into the form (whose *perversion* will be found in Sir W. THOMSON's paper on "Vortex-Motion," Trans. R. S. E., 1867–68, p. 244)



while IV. may be modified into



These are two of the three figures of 7-fold knots given as examples by LISTING; and he has stated, though without any explanation, that these two forms are equivalent, *i.e.*, convertible into one another. Hence II. and IV. form but one class of 7-fold knot.

How to effect this transformation has been already hinted in § 4. It is merely the passing of a crossing from one loop of the string to another (which intersects it twice) by a *twist* through two right angles. And the diagrams 5, 6, 7 of Plate XV. show the nature of this transformation, as well as of two others which I have since detected, viz., that of III. into V., and of VI. into VII. Hence there are in reality only *eight* distinct forms of 7-fold knottiness.

Thus, as the result of the last six sections, we have the following table:—

Knottiness,	3,	4,	5,	6,	7.
No. of Forms,	1,	1,	2,	4,	8.

§ 12. I have not attempted the application of the preceding method to forms with more than 7 intersections. Prof. CAYLEY and Mr MUIR kindly sent me general solutions of the problem, "*How many arrangements are there of n letters, when A cannot be in the first or second place, B not in the second or third, &c.*" Their papers, which will be found in the Proceedings R. S. E.,* of course give the numbers 13, 80, and 579, which I had found by actually writing out the combinations for 5, 6, and 7 letters. But they show that the number for 8 letters is 4738, and that for 9, 43,387; so that the labour of the above-described process for numbers higher than 7 rises at a fearful rate. I cannot spare time to attack the 8-fold knots, but I hope some one will soon do it. There is little chance of anything more than that, at least of an exhaustive character, being done about knots in this direction, until an analytical solution is given of the following problem:—

Form all the distinct arrangements of n *letters, when A cannot be first or second, B not second or third, &c.*

[Arrangements are said to be distinct when no one can be formed from another by cyclic alteration of the letters, at every step bringing the last to the head of the row, as in § 5.] This, I presume, will be found to be a much harder problem than that of merely *finding the number* of such arrangements,

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* 1877, p. 338, and p. 382.

which itself presents very grave difficulties, at least where n is a composite number. In fact it is probable that the solution of these and similar problems would be much easier to effect by means of special (not very complex) machinery than by direct analysis. This view of the case deserves careful attention.

In a later section it will be shown how, by a species of *partition*, the various forms of any order of knottiness may be investigated. But we can never be quite sure that we get *all* possible results by a semi-tentative process of this kind. And we have to try an immensely greater number of partitions than there are knots, as the great majority give links of greater or less complexity.

§ 13. But even supposing the processes indicated to have been fully carried out for 8, 9, and 10-fold knottiness, a new difficulty comes in which is not met with, except in a very mild form, in the lower orders. For when a knot is single, *i.e.*, not composite or made up of knots (whether interlinked or not) of lower orders, any deviation from the rule of alternate + and − at the crossings gives it, in general, nugatory crossings, in virtue of which it sinks to a lower order. But when it is composite, and the component knots are separately irreducible, the whole is so. Thus *there are more distinct forms of knots than there are of their plane projections.* For instance, the first species (α) of the 6-fold knots (§ 8) may be made of three essentially different forms, for the separate "trefoil" knots of which it is made may (when neither is nugatory) be both right-handed, both left-handed, or one right and the other left-handed. This species is thus, from the physical point of view, capable of furnishing *three* quite distinct forms of vortex-atom. And it will presently be shown that in each of these forms it is capable of having regular alternations of + and −, or a set of sequences at pleasure.

At least one knot of every even order is *amphicheiral, i.e.*, right or left-handed indifferently, but no knot of an odd order can be so. Hence, as there is but one 3-fold knot form, and one 4-fold, there are two possible 3-fold vortices, right and left-handed, but only one 4-fold. A combination of two trefoil knots gives, as we have seen, three distinct knots; that of two 4-fold knots would give an 8-fold, with only one form. When a 3-fold and a 4-fold are combined, as in Class I. of § 10, there are two distinct vortices, for the trefoil part may be right or left-handed. Thus it appears that though we have shown that there are very few distinct outlines of knots, at least up to the 7-fold order, and though probably only a very small percentage of these would be stable as vortices, yet the double forms of non-amphicheiral knots give more than one distinct knot for each projected form into which they enter as components.

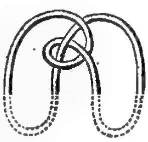
PART II.

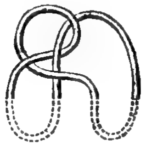
The number of Forms for each Scheme.

§ 14. A possible scheme being made according to the methods just described, with the requisite number of intersections, let it be constructed in cord, with the intersections alternately + and −. Then [since all schemes involving essentially nugatory crossings, like those mentioned in § 2, must be got rid of, as they do not really possess the requisite number of intersections] no deformation which the cord can suffer will reduce, though it may increase, the number of double points. If it *do* increase the number, the added terms will be of the nugatory character presently to be explained. If it do not increase that number, the scheme will in general still represent the altered figure. For, as we have seen, the scheme is a complete and definite statement of the nature of the knot. But, as already stated, in certain cases the knot can be distorted so as no longer to be represented by the same scheme.

All deformations of such a knotted cord or wire may be considered as being effected by bending at a time only a limited portion of the wire, the rest being held fixed. This corresponds to changing the point of view *finitely* with regard to the part altered, and yet *infinitesimally* with regard to all the rest. This, it is clear, can always be done, as the *relative* dimensions of the various coils may be altered to any extent without altering the character of the knot. In general such deformations may be obtained by altering the position of a luminous *point,* and the plane on which it casts a shadow of the knot. Any addition to the normal number of intersections which may be produced by this process is essentially nugatory. As is easily seen, it generally occurs in the form of the avoidable overlapping of two branches, giving *continuations of sign.*

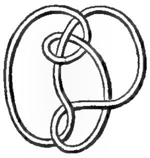
The process pointed out in § 11 gives a species of deformation which it is perhaps hardly fair to class with those just described, though by a slight extension of mathematical language such a classification may be made strictly accurate. It may be well to present, in passing, a somewhat different view of the application of this method. Thus, it is obvious at a glance that the two following figures are mere *distortions* of the second form of the 4-fold knot figured in § 17 below :—

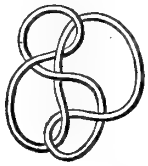




Also it will be seen that by twisting, the dotted parts being held fixed, either of these may be changed into the other, or changed to its own reverse (as from left to right).

We may now substitute what we please for the dotted parts. I give only the particular mode which reproduces the two forms stated by LISTING to be equivalent :—





Another mode of viewing the subject, really depending on the same principles, consists in fixing temporarily one or more of the crossings, and considering the impossibility of unlocking in any way what is now virtually two or more *separate* interlacing closed curves, or a single closed curve with full knotting, but with fewer intersections than the original one.

Another depends upon the study of cases of knots in which one or more crossings can be got rid of. Here, as will be seen in § 33 below, it is proved that *continuations* of sign are in general lost when an intersection is lost; so that, as our system has no continuations of sign, it can lose no intersections.

§ 15. Practical processes for producing graphically all such deformations as are represented by the same scheme are given at once by various simple mechanisms. Thus, taking O any fixed point whatever, let p, a point in the deformed curve, be found from its corresponding point, P, by joining PO and producing it according to any rule such as

$$\mathrm{PO} \cdot \mathrm{O}p = a^2,$$

or

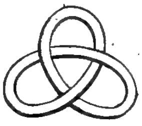
$$\mathrm{PO} + \mathrm{O}p = a, \text{ \&c., \&c.}$$

The essential thing is that points near O should have images distant from O, and *vice versâ*. And p must be taken in PO *produced*, else the distorted knot is altered from a right-handed to a left-handed one, and *vice versâ*, as will be seen at once by taking the image of the crossing figured in § 1 above.

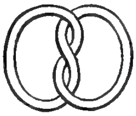
It is obvious, from the mode of formation, that these figures are all represented by the same scheme,—for the scheme tells the order in which the various crossings occur,—and it is easy to show that they give merely different views of the same knot. The simplest way of doing this is to suppose the knot projected on a sphere, and *there* constructed in cord, the eye being at the centre. Arrange so that one closed branch, *e.g.*, A——*A*, forms nearly a great circle. Looking towards the centre of the sphere from opposite sides of the plane of this great circle, the coil presents exactly the two appearances related to one another by the deformation processes given above. What was inside the closed branch from the one point of view is outside it from the other, and *vice versâ*. In fact, because the new figure is represented by the same scheme as the old, the numbers of sides of the various compartments are the same as before, and so also is the way in which they are joined by their corners. The deformation process is, in fact, simply one of *flyping*, an excellent word, very inadequately represented by the nearest equivalent English phrase "turning outside in."

Hence to draw a scheme, select in it any closed circuit, *e.g.*, A . . . . *A*—the more extensive the better, provided it do not include any less extensive one. Draw this, and build upon it the rest of the scheme; commencing always with the common point A, and passing each way from this to the *next occurring* of the junctions named in the closed circuit. [It is sometimes better to construct both parts of the rest of the scheme *inside*, and then invert one of them, as we thus avoid some puzzling ambiguities.] Inversions with respect to various origins will now give all possible forms of the scheme, though not necessarily of the knot.

§ 16. Applying these methods to the "trefoil" knot (§ 6)

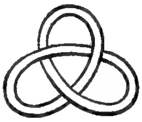


we easily see that if O be external, or be inside the inner *three*-sided compartment, we reproduce (generally with much *distortion*, but that is of no consequence, § 2) the same form; but if O be in any one of the *two*-sided compartments, we have the form



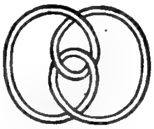
This again is reproduced from itself if O be external, or be within either of the *two*-sided compartments. But it gives the trefoil knot if O be placed inside either of the *three*-sided compartments.

Here notice that the angles of the two-sided compartments are left-handed, and those of the three-sided right-handed in each of the figures. The *perverted* or right-handed form is of course



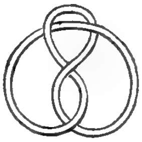
and its solitary deformation is the perversion of the other figure above.

§ 17. When we come to the deformations of the single 4-fold knot



we obtain a very singular result. If we place O external to the figure, we simply reproduce it; but if we put O inside the two-sided compartment in the middle we get the *perversion* of the same figure.

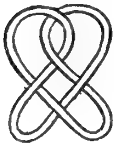
Again, if we place O in either of the *boundary* three-sided compartments we get

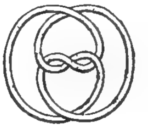


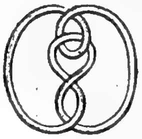
but if we place it in either of the *interior* three-sided spaces we get the *perversion* of this last figure.

Thus this 4-fold knot, in each of its forms, *can be deformed into its own perversion.* In what follows all knots possessing this property will be called *Amphicheiral.*

§ 18. The first of the two 5-fold knots (§ 7) has the following forms:—

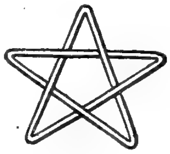


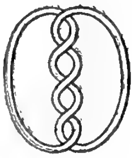




These I found were long ago given by Listing as reduced forms of a reducible 7-fold knot, and I have now substituted for my former drawing of the second form his more symmetrical one.

The second of the 5-fold knots has only two forms, viz.:—





§ 19. Plate XV. figs. 2, 3, 4, give various forms of the 6-fold knot distinguished as a in the classification in § 8. It will be seen that in the first of these the crossings are alternately over and under, but that it is not so in the others.

And in fig. 8 we have a collection (not complete) of forms of various species of the 7th order, drawn so as to show their relation to a lower form—the trefoil knot. It will be seen that in none of these is the connection merely *apparent*, the trefoil part having its signs alternately + and − if those of the complete knot have this alternation. But if, for instance, we had drawn the fine line horizontally through the trefoil, so as to divide each of the upper two-cornered compartments into two three-cornered ones, we should have got No. II. of the 7-fold forms, and the original trefoil would have been rendered only *apparent.*

§ 20. In my British Association paper, already referred to, I showed that any closed plane curve, or set of closed plane curves, provided there be nothing higher than double points, divides the plane into spaces which may be coloured black and white alternately, like the squares of a chess-board, or, to take a closer analogy, as the adjacent elevated and depressed regions of a vibrating plate, separated from one another by the nodal lines (Plate XV. figs. 9 and 10). I afterwards found that LISTING had employed in his notation for knots, in which the crossings are alternately over and under, a representation which comes practically to the same thing; depending as it does on the fact that in such a knot all the angles in each compartment are either right or left-handed, and that these right and left-handed compartments alternate as do my black and white ones.

I have since employed a method, based on the above proposition, as a mode of symbolising the form of the projections of a knot, altogether independent of its reducibility. I was led to this by finding that LISTING'S notation, though expressly confined to reduced knots, in which each compartment has all its angles of the same character, is ambiguous: in the sense that a *Type-Symbol,* as he calls it, may in certain cases not only stand for a linkage as well as a knot, but may even stand for two quite different reduced knots incapable of being transformed into one another.* The *scheme,* already described, has no such ambiguity, but it is much less easy to use in the classification of knots. Hence, following LISTING, I give the number of corners of each compartment, but, unlike him, only of those which are black or of those which are white. But I connect these in the diagram by lines which show how they fit into one another in the figure of the knot. An inspection of Plate XV. figs. 11 and 12 (species VII. of sevenfold knottiness) will show at once how diagrams are arrived at, either of which fully expresses the projection of the knot in question by means of the black or of the white spaces singly. The connecting lines in the diagrams evidently stand for the crossings in the projection, and thus, of course, either diagram can be formed by mere inspection of the other,† and the rule for

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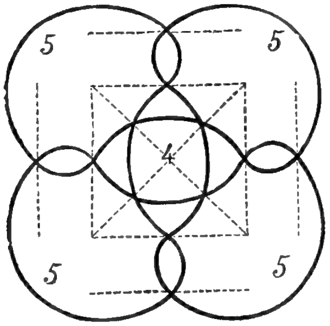
* Proc. R. S. E. 1877, p. 310 (footnote), and p. 325.

† Some further illustrations of this will be found in the abstract of my paper on "Links," Proc. R. S. E. 1877, p. 321.

drawing the curve when the diagram is given is obvious. Thus the annexed diagram shows the result of the process as applied to a symmetrical symbol.



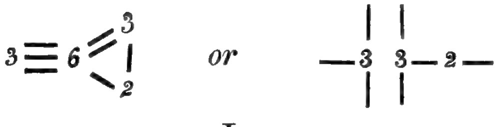
An inspection of one of these diagrams shows at once

(1.) The number of joining lines is the same as the number of crossings. Hence, as each line has two ends, the sum of the numbers representing the number of corners in either the black or the white spaces is twice the number of crossings.

(2.) Every additional crossing involves one additional compartment, for the abolition of a crossing runs two compartments into one. But where there is no crossing there are two compartments, the inside and outside (*Amplex*, in LISTING's phraseology), of what must then be merely a closed oval. Thus when there are n crossings there are $n+2$ compartments.

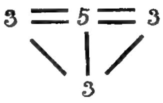
(3.) No compartment can have more than n corners. For, as the whole number of corners in the black or white compartments is only $2n$, if one have more than n, the rest must together have less, and thus some of the joining lines in the diagram must *unite the large number to itself, i.e.,* must give essentially nugatory intersections.

As an illustration, let us use this process in giving a second enumeration or delineation of the forms of 7-fold knottiness. The numbering of the various forms is the same as that already employed in §§ 10, 11 above.

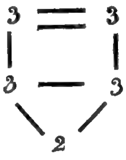


I.

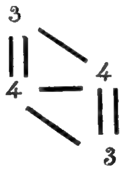
The second form of this symbol is particularly interesting as consisting of two parts. This accords with the composite nature of the knot.



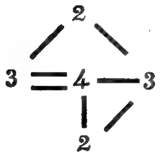
II.



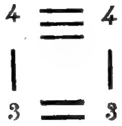
III.



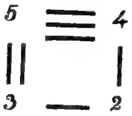
IV.



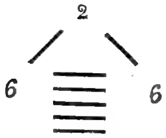
V.



VI.



VII.



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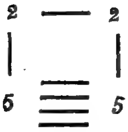
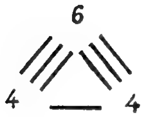


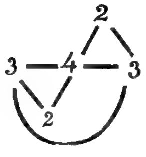
IX.



X.

XI.

The relations of equivalence in pairs among six of these forms, which were pointed out in § 11 and in Plate XV. figs. 5, 6, 7, are even more clearly seen as below :—



II.

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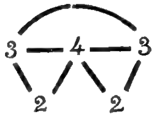

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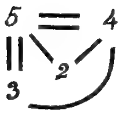

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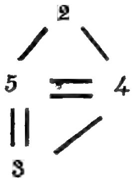

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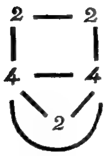

VI.

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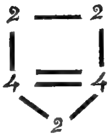

VII.

where the mode of passing from one form to the equivalent one is obvious.

§ 21. A tentative method of drawing all possible systems of closed curves with a given number (n) of double points is thus at once obvious.

Write all the partitions of $2n$, in which no one shall be greater than n and no one less than 2. Join each of these sets of numbers into a group, so that each number has as many lines terminating in it as it contains units. Then join the middle points of these lines (which must not intersect one another) by a continuous line which *intersects* itself at these middle points and there only. When this can be done we have the projection of a *knot.* When more continuous lines than one are required we have the projection of a *linkage.*

To give simple examples of this process, let us limit ourselves to 4 and 5 intersections.

The only partitions of 8, subject to the conditions above, are

(1) 4 4
(2) 4 2 2
(3) 3 3 2
(4) 2 2 2 2

Now the number of black and white compartments together must in this case be 4+2. Hence there are but four combinations to try, viz., (1) and (4), (2) and (2), (3) and (3), (2) and (3). Of these, the last is impossible; the others are as in Plate XVI. fig. 16. The third is the amphicheiral knot already spoken of, and the second may for the same reason be called an *amphicheiral link.*

The partitions of 10, subject to our rule, are

5 5
5 3 2
4 4 2
4 3 3
4 2 2 2
3 3 2 2
2 2 2 2 2

and the four figures (Plate XVI. fig. 17) give the only valid combinations of these. The third and the first are the knots already described (§ 18), the others are links.

§ 22. The spherical projection already mentioned (§ 15) will in general allow us to regard and exhibit any knot as a more or less perfect *plait.* It does so perfectly whenever the coil is *clear, i.e.,* when all the windings of the cord may be regarded as passing in the same direction round a common vertical axis thrust through the knot. When the coil is not clear some of the cords of the plait are doubled back on themselves. Thus by drawing the plait corresponding to a given scheme we can tell at once whether one of its forms is a clear coil or not.

Let us confine our attention for a moment to clear coils. It is easy to see that

If the number of windings is even the number of crossings is odd, and vice versâ.

Various proofs of this may be given, all depending on the fundamental theorem of § 1, but the following one is simple enough, and will be useful in some other applications.

First, in a clear coil of two turns there must be an odd number of intersections. For there must be one intersection, and the two loops thus formed must have their other intersections (if any) in pairs.

Now begin with any point in a clear coil, where the curve intersects itself for the first time. The loop so formed intersects the rest in an even number of points. Hence every turn we take off removes an odd number of intersections. Thus, as two turns give an odd number (or, more simply, as one turn gives none), the proposition is proved.

Thus, to form the symmetrical clear coil of two turns and of any (odd) number of intersections, make the wire into a helix, and bring one end through the axis in the same direction as the helix (not in the opposite direction, as in Ampère's *Solenoids*), then join the ends. [The solenoidal arrangement, regarded from any point of view, has only nugatory intersections.]

§ 23. A very curious illustration of the irreducible clear coils which have two turns only is given by the edges of a long narrow strip of paper. Bend it, without twisting, till the ends meet, and then paste them together. The two edges will form separate non-linked closed curves without crossings.

Give the slip *one half twist* (*i.e.*, through 180°) before pasting the ends together. The edges now form one continuous curve—a clear coil of two turns with *one* (nugatory) crossing.

Give *one full twist* before pasting. Each edge forms a closed curve, but there are two crossings. The curves are, in fact, once linked into one another. (See Plate XV. fig. 13.)

Give *three half twists* before joining. The edges now form one continuous clear coil with three intersections.

Two full twists give two separate closed curves with four crossings, *i.e.*, twice linked together. (See Plate XVI. fig. 12.)

Five half twists give the pentacle of § 7 above. And so on. In all these examples, from the very nature of the case, the crossings are alternately + and −.

§ 24. Now suppose that, in any of the above examples, after the pasting, we cut the slip of paper up the middle throughout its whole length.

The first, with no twist, splits of course into two separate simple circuits.

That which has half a twist, having originally only one edge, and that edge not being cut through in the process of splitting, remains a closed curve. It is, in fact, a clear coil of two turns, which, having only one intersection, may be opened out into a single turn. But in this form it has *two whole* twists, half a twist for each half of the original strip, and a whole twist additional, due to the bending into a closed circuit.

That with one whole twist splits, of course, into two interlinking single coils, each having one whole twist.

That with three half twists gives, when split, the trefoil knot, and when flattened out it has three whole twists.

From two whole twists we get two single coils twice linked, each with two whole twists. This result may be obviously obtained from a continuous strip, *with only half a twist.* One continued cut, which takes off a strip constantly equal to one quarter of the original breadth of the slip, gives a half twist ring of half breadth, intersecting *once* a double twist ring of quarter breadth. A second cut splits the wider ring into one similar to the narrow one, but there is now double linking.

§ 25. A good many of these relations may be exhibited by dipping a wire, forming a two-coil knot, into PLATEAU'S glycerine soap solution, and destroying the film which fills up the clear interior of the coil. Neglecting the surface curvature of the remaining film, it has twists similar to those of the paper strips above treated, and the integral amounts of twist show how far the wire-knot is, if at all, reducible.

This mode of regarding a clear coil of two turns, as, in certain cases, the continuous edge of a strip of paper whose ends are pasted together after any odd number of half twists, is one of many ways in which we are led to study *all clear coils* as specimens of more or less perfect *plaiting,* the number of threads plaited together being the same as the number of turns of the coil. Another mode in which we are led to the same way of regarding them is by supposing a cylinder to be passed through the middle of the (flattened) clear coil, and then to expand so as to draw all the turns tight. As there can be only a finite number of intersections, we have always an infinite choice of generating lines of the cylinder on which no intersection lies. Suppose the whole to be cut along such a line and rolled out flat. It would, of course, be a more or less perfect plait, but with a special characteristic, depending upon the fact that *it is formed from one continuous cord or wire.*

Call the several laps of the cut cord α, β, γ, &c. Then we may arrange the cut ends anyhow as follows :—α to γ, γ to ϵ, ϵ to β, β to δ, δ to α if there be but five; and similarly for any other number, *exhausting all before repeating any one oftener than once.* We may now, after having settled their order, *change their designations,* so as to name them, as they occur, in the natural order of the alphabet. Thus any such plait may be represented by a diagram as in Plate XV. fig. 14, where the dotted parts may cross and recross in any conceivable way, but must begin and end as above.

The number of ways in which such coils can be exhibited in plaits essentially distinct from one another is therefore, if n be the number of laps, $\overline{n-1}\,\overline{n-2}\,\ldots\,2.\,1.$ All the other possible arrangements, $n-1$ times the last written number, correspond to links or, at all events, to more than one continuous cord.

§ 26. From this point of view another notation for clear coils may be given in the form

$$\begin{array}{cccc}\alpha & \gamma & \beta & \alpha \\ \beta & \alpha & \gamma & \beta\end{array}\;\ldots\ldots$$

Here $\alpha, \beta, \gamma \ldots.$ are, as above, the several strings plaited, so that in the coil β is the prolongation of α, γ that of β, &c., and α that of the last of the series. The expression $\overset{\alpha}{\beta}$ means that α crosses *over* β. It is sometimes useful to indicate whether a crossing takes place to the right or left. This is done by putting + or − over the symbol. Thus the four crossings above may be more fully written as

$$\begin{array}{cccc}+ & - & + & - \\ \alpha & \gamma & \beta & \alpha \\ \beta & \alpha & \gamma & \beta\end{array}\;\ldots\ldots$$

The properties of this notation were examined in detail in my first paper; but as they are more curious than useful, I merely mention one or two.

Thus the combination just written cannot be simplified in itself; but

$$\begin{array}{cccc}+ & - & - & - \\ \alpha & \gamma & \gamma & \alpha \\ \beta & \alpha & \beta & \beta\end{array} = \begin{array}{cc}- & - \\ \gamma & \gamma \\ \beta & \alpha\end{array},\ \text{\&c.}$$

This notation requires care. For instance, the terms

$$\begin{array}{cc}\alpha & \alpha \\ \beta & \beta\end{array}$$

are simply nugatory, and may be cancelled. But, on the other hand, the terms

$$\begin{array}{cc}\alpha & \beta \\ \beta & \alpha\end{array}$$

usually add to the beknottedness of the whole scheme.

When the scheme is not compatible with a clear coil there occur terms of the form

$$\begin{array}{c}\alpha \\ \alpha\end{array},$$

and the application of this method becomes very troublesome.

§ 27. A question closely connected with plaited clear coils is that of the numbers of possible arrangements of given numbers of intersections in which the *cyclical* order of the letters in the 2d, 4th, 6th, &c., places of the scheme shall be the same as that in the 1st, 3d, 5th, &c., *i.e.*, the alphabetical. Instances of such have already been given above. In the first scheme of § 5, for example, the letters in the even places are

D E A B C.

Here the cyclical order of the alphabet is maintained, but A is postponed by two places. It is easy to see that the following statements are true.

Whatever be the number of intersections a postponement of *no* places leads to nugatory results.

A postponement of one place is possible for three and for four intersections only.

Postponement of two places is possible only for (*four*), five, and eight—three for seven and ten—four for nine and fourteen—five for (*eight*), eleven and sixteen,—six for (*ten*), thirteen, and twenty, &c. Generally there are in all cases n postponements for $2n+1$ intersections; and for $3n+2$, or $3n+1$ intersections, according as n is even or odd. The numbers which are italicised and put in brackets above, arise from the fact that a postponement of r places, when there are n intersections, gives the same result as a postponement of $n-r-1$ places. [It will be observed that this cyclical order of the letters in the even places is possible for *any* number of intersections which is not 6 or a multiple of 6.]

When there are n postponements with $2n+1$ intersections the curve is the symmetrical double coil, *i.e.*, the plait is a simple *twist.*

The case with $3n+2$ or $3n+1$ intersections is a clear coil of three turns, corresponding to a regular plait of three strands.

Figures 16, 17 of Plate XV. give the diagrams corresponding to the latter case for $n=2$, 3 respectively; *i.e.*, with 8 and 10 crossings. The diagrams 15 and 18, constructed according to the same plan for 6 and 12 intersections, show why there are no multiples of six in this form of coil. In fact, whenever the number of crossings in this three-ply plait is a multiple of 6, the strands are separate closed curves.

PART III.

Methods of Reduction.

§ 28. Before taking up the question of the complexity of a knot, a word or two must be said about the methods of reducing any given knot to its simplest form. I have not been able as yet to find any general method of doing this, nor have I even discovered, what would probably solve this difficulty, any perfectly general method of pronouncing at once from an inspection of its scheme or otherwise, whether a knot is reducible or not. It is easy to give multitudes of special conformations in which reduction can always be effected; but of these I shall give only a few, with the view of showing their general character.

One very simple case of such reduction has already been given, viz., *where a letter occurs twice in succession.*

For, if we have as part of a scheme, the letters

$$\ldots \mathrm{P\,Q\,Q\,R} \ldots.$$

the corresponding part of the coil must have the form shown in Plate XV. fig. 19. Whichever way the crossing at Q is effected, the loop can be at once got rid of, and it is thus nugatory, *because the scheme shows that it is not intersected by any other branch.*

If we put in the signs of the crossings, they must obviously be different for the two Q's; and thus in

$$\ldots \mathrm{P}\underset{+}{\mathrm{Q}}\underset{-}{\mathrm{Q}}\mathrm{R} \ldots.$$

we may treat them as + Q − Q = 0, and obliterate Q altogether.

An immediate consequence of this is, of course, that any group such as

$$\ldots \mathrm{P\,Q\,R\,R\,Q\,P} \ldots.$$

whatever be the number of letters arranged in this form, may be wholly struck out. Cases corresponding to this have been already figured in § 1.

§ 29. Another useful step in simplification occurs when we have a scheme containing the following terms:—

$$\ldots.\underset{+}{\mathrm{P}}\underset{+}{\mathrm{Q}}\ldots\ldots\underset{-}{\mathrm{P}}\underset{-}{\mathrm{Q}}\ldots.$$

for then both P and Q may be struck out.

[*N.B.*—The *order* of P and Q need not be the same at each occurrence, the essential thing is that they should *twice occur together, and with like signs.* This explanation shows that the process is not confined to clear coils.]

For the corresponding part of the diagram must evidently be of the form shown in Plate XV. fig. 20, since the scheme shows that there are no intersections between P and Q on either branch. Hence, as P and Q have the same sign for each branch, one branch may be slipped off from the other without otherwise altering the coil.

If a single turn of the coil pass across between P and Q, the only ways in which it can prevent the slipping off just described are that shown in Plate XV.

fig. 21, and the same looked at from the other side, *i.e.*, with all the signs changed.

Hence in the scheme

$$\ldots\underset{+}{P}R\underset{+}{Q}\ldots\underset{-}{P}S\underset{-}{Q}\ldots\ldots RS\ldots\ldots$$

(where the order is again indifferent in each of the groups) we can always leave out P and Q, unless R be negative and S positive, *i.e.*, unless this part of the scheme has in itself the greatest possible number of changes of sign.

But when we *can* thus strike out P and Q, it is necessary to observe that in R S or S R, which *must* occur at some other part of the scheme, the order is to be changed. Thus

$$\ldots\ldots\underset{+}{P}\;\underset{+}{R}\;\underset{+}{Q}\ldots\ldots\ldots\underset{-}{P}\;\underset{+}{S}\;\underset{-}{Q}\ldots\ldots\underset{-}{R}\,\underset{-}{S}\ldots\ldots$$

is simplified into

$$\ldots\ldots\underset{+}{R}\ldots\ldots\underset{+}{S}\ldots\ldots\ldots\ldots\underset{-}{S}\;\underset{-}{R}\ldots\ldots\ldots$$

§ 30. Such a portion as that figured in Plate XV. fig. 22 evidently goes out of itself, whatever be the character of B; *i.e.*, the whole of it

$$\ldots\underset{-}{A}B\underset{+}{C}\,\underset{+}{A}B\underset{-}{C}\ldots\ldots$$

may be struck out of any scheme. In fact, whichever sign be given to B, § 29 applies and removes two of the intersections. Then § 28 disposes of the remaining one.

This is merely a particular case of the general and obvious theorem, that any portion of a coil which may be treated as a separate coil, and which, if alone, could be reduced, may be reduced *in situ.*

A more general theorem, which includes the preceding, is that, if in

$$\ldots ABC\ldots\ldots GHA\ldots\ldots\ldots$$

the signs of B, C, . . . G, H, where they occur between the two A's, are all alike, all these intersections, including A, may be struck out. This is quite obvious, because it indicates a complete turn of the coil entirely above or below the rest. When one or more of B, C, G, H has a different sign from the others, a less amount of simplification is usually still possible.

Along with this we may take the case of fig. 23. Here we have

$$\ldots\ldots\underset{-}{P}\;\underset{-}{Q}\;\underset{+}{R}\;\underset{+}{P}\;\underset{+}{S}\ldots\ldots\ldots\ldots\underset{-}{R}\;\underset{+}{Q}\;\underset{-}{S}\ldots\ldots\ldots$$

If the sign of P were changed these parts of the scheme would contain alternately + and —. The scheme obviously loses three intersections, and becomes

$$\ldots\ldots\underset{-}{\mathrm{Q}}\ldots\ldots\underset{+}{\mathrm{Q}}\ldots\ldots$$

If the signs in the complete knot, with the exception of that of P, were all originally + and — alternately, there will generally be farther reductions possible.

§ 31. A glance shows that the first of the diagrams, 24, 25, Plate XV., can be reduced to the second. Hence in the scheme of a knot

$$\ldots\ldots\underset{+}{\mathrm{P}}\;\underset{+}{\mathrm{Q}}\;\underset{-}{\mathrm{R}}\;\underset{-}{\mathrm{P}}\ldots\ldots\underset{-}{\mathrm{Q}}\;\underset{+}{\mathrm{R}}\ldots\ldots$$

may be simplified into

$$\ldots\ldots\underset{+}{\mathrm{Q}}\;\underset{-}{\mathrm{R}}\ldots\ldots\ldots\underset{+}{\mathrm{R}}\;\underset{-}{\mathrm{Q}}$$

[*N.B.*—The essential point is that P and Q should have the *same* sign, and R the opposite. If Q and R had the same sign they might both be struck out § 29. But if P and Q have different signs, as also Q and R, no simplification can be effected, though, as has been shown in § 11, a change of scheme is practicable.]

§ 32. The scheme

$$\ldots\underset{+}{\mathrm{A}}\;\underset{+}{\mathrm{B}}\;\underset{+}{\mathrm{C}}\ldots\underset{+}{\mathrm{E}}\;\underset{+}{\mathrm{F}}\;\underset{+}{\mathrm{G}}\ldots\ldots\underset{-}{\mathrm{A}}\;\mathrm{M}\;\mathrm{N}\ldots\mathrm{P}\;\mathrm{Q}\;\underset{-}{\mathrm{G}}\ldots$$

always admits of striking out A and G. But special consideration is necessary as to what is to take the place of B, C, . . . E, F. Their substitutes will all be positive, and may be called *m*, *n*, . . . *p*, *q*, since they are in number the same as M, N, ... P, Q—irrespective altogether of the number of B, C, ... E, F. In fact, M and *m*, N and *n*, ... &c., lie (as near one another, in pairs, as we please) on the several turns of the coil which intersect the arc A M . . . . Q G. And *m*, *n*, ... &c., are on the *opposite* side of that arc from B, C, ... F.

§ 33. There are numberless other special rules, but those just given are among the simplest, and they are in general sufficient for coils with only a moderate number of intersections. With the present notation it is not easy to classify them, or to show how they may be exhibited as particular cases of more general rules. We will therefore, for the present, employ them only for the simplification (where possible) of a few diagrams of knots. But it must be particularly noticed that the simplifications above are mainly such as *tend to remove continuations of sign from a scheme*, none of them but the first being applicable to a scheme whose signs present no continuations.

§ 34. *Examples.*

$$\begin{array}{lcccccccccccccccccccccc} \text{I.} & A & E & B & F & C & G & D & A & E & K & F & L & G & D & H & B & K & C & L & H & | & A \\ & - & + & + & + & - & + & - & + & - & + & - & + & - & + & - & - & - & + & - & + & & \end{array}$$

This is, of course, rendered irreducible by changing the sign of B. It is figured Plate XVI. fig. 1.

[If we were to change the signs of F, L, H, the knot would acquire a great increase of beknottedness, and would consist, in its simplest form, of a pentacle and a trefoil knot linked together, as in Plate XVI. fig. 25.]

(*a*) Now

$$\begin{array}{ccccccccccccc} \ldots & E & B & F & \ldots & E & K & F & \ldots & B & K & \ldots \\ & + & + & + & & - & + & - & & - & - & \end{array}$$

become

$$\begin{array}{cccccccc} \ldots & B & \ldots & K & \ldots & K & B & \ldots \\ & + & & + & & - & - & \end{array}$$

(*b*) Two intersections being thus lost, the knot has now the form, Plate XVI. fig. 2, with the scheme

$$\begin{array}{cccccccccccccccccc} A & B & C & G & D & A & K & L & G & D & H & K & B & C & L & H & | & A \\ - & + & - & + & - & + & + & + & - & + & - & - & - & + & - & + & & \end{array}$$

Now in

$$\begin{array}{ccccccc} \ldots & D & A & K & L & G & \ldots \\ & & + & + & + & & \end{array}$$

with G before or D after, we can at once get rid of K, L, if A be put close to G.

(*c*) Hence the scheme becomes

$$\begin{array}{cccccccccccccc} B & C & A & G & D & A & G & D & H & B & C & H & | & B \\ + & - & - & + & - & + & - & + & - & - & + & + & & \end{array}$$

and the knot is as in the figure 3, Plate XVI.

Now

$$\begin{array}{ccccccc} H & B & \ldots & H & B & \ldots & \text{go out (§ 29).} \\ - & - & & + & + & & \end{array}$$

(*d*) The scheme is now

$$\begin{array}{cccccccccc} C & A & G & D & A & G & D & C & | & C \\ - & - & + & - & + & - & + & + & & \end{array}$$

so that C goes out by § 28, and we have finally

$$\begin{array}{cccccccc} A & G & D & A & G & D & | & A \\ - & + & - & + & - & + & & \end{array}$$

the trefoil knot.

II. The knot figured in Plate XVI. fig. 4 has no beknottedness.

III. That in fig. 5 is reducible to the trefoil.

These are left as exercises to the reader.

Part IV.

Beknottedness.

§ 35. Recurring to the two species of five-crossing knots discussed in § 18, we easily see that there is less entanglement or complication in the first species than in the second. For if the sign of *either* of the two crossings towards the top of the first figure be changed, it is obvious that it will no longer possess any but nugatory crossings. But if we change the sign of any one crossing in the pentacle, that crossing, and *one* only of the adjacent ones, become nugatory, so that the knot becomes the trefoil with alternating + and −. This, in turn, has all its intersections made nugatory by the change of sign of any one of them. Thus one change of sign removes all the knotting from the first of these knots, but two changes are required for the second.

In what follows the term *Beknottedness* will be used to signify the peculiar property in which knots, even when of the same order of knottiness, may thus differ: and we may define it, at least provisionally, as *the smallest number of changes of sign which will render all the crossings in a given scheme nugatory.* This question is, as we shall soon see, a delicate and difficult one. It is probable that it will not be thoroughly treated until one considers along with it another property, which may be called *Knotfulness*—to indicate the number of knots of lower orders (whether interlinked or not) of which a given knot is in many cases built up. But this term will not be introduced in the present paper.

§ 36. It may be well to premise a few lemmas which will be found useful in examining for our present purpose the plane projection of a knot.

(*a*). Regarding the projection as a wall dividing the plane into a number of fields, if we walk along the wall and drop a coin into each field as we *reach* it, each field will get as many coins as it has corners, but those fields only will have a coin in each corner whose sides are all described in the same direction round. For we enter by one end of each side and leave by the other. The number of coins is four times the number of intersections; and two coins are in each corner bounded by sides by each of which we enter, none in those bounded by sides by each of which we leave. Hence a mesh, or compartment, which has a coin in each corner has all its sides taken in the same direction round; and we see by fig. 6, Plate XVI., that this is the case with twists in which the laps of the cord run opposite ways, not if they run the same way. Compare this with the remarks of § 35, as to the two species of 5-fold knottiness.

(β). To make this process give the distinction between crossing *over* and crossing *under*, we may suppose the two coins to be of different kinds,—silver and copper for instance. Let the rule be:—silver to the right when crossing over, to the left when crossing under. Then, however the path be arranged, of the four angles at each crossing, one will have no coins, the vertical or opposite corner will have *two* silver or *two* copper coins, the others *one* copper or *one* silver coin each.

It is easily seen that a reversal of the direction of going round leaves the single coins as they were, but shifts the pair of coins into the angle formerly vacant: also that in all deformed figures the circumstances are exactly the same as in the original. Hence we may divide the crossings into silver and copper ones, according as two silver or two copper coins come together. And the excess of the silver over the copper crossings, or *vice versâ*, furnishes an exceedingly simple and readily applied test (not, however, as will soon be seen, in itself absolutely conclusive of identity, though absolutely conclusive against it), which is of great value in arranging in family groups (those of each family having the same number of silver crossings), the various knots having a given number of intersections.

(γ). Or, still more simply, we may dispense altogether with the copper coins, so that, going round, we pitch a coin into the field to the *right* at each crossing *over*, to the *left* at each crossing *under*. When the coins are in the same angle the crossing is a silver one, when in two vertical angles it is copper. Each of these three processes has its special uses.

§ 37. This process, thus limited, is obviously intimately connected with that required for the estimation of the work necessary to carry a magnetic pole along the curve, the curve being supposed to be traversed by an electric current. Hence it occurred to me that we might possibly obtain a definite measurement of beknottedness in terms of such a physical quantity: as it obviously must be always the same for the same knot, and must vanish when there is no beknottedness. To make the measure complete, we must record the numbers of non-nugatory silver and copper crossings separately, with the number to be deducted as due merely to the *coiling* of the figure. This last is a very important matter, and will be dealt with later.

§ 38. When unit current circulates in a simple circuit, it is known that the work required to carry unit magnetic pole once round any closed curve once linked with the circuit is $\pm 4\pi$. Instead of the current we may substitute a uniformly and normally magnetized surface bounded by the circuit. The potential energy of the pole in any position is measured by the spherical aperture subtended at the pole by the circuit; but its sign depends upon whether the north or south polar side is turned to the pole. Hence the pole has no potential energy when it is situated in the plane of the circuit but external to it, and the potential energy is $\pm 2\pi$ when the pole just reaches the plane of the circuit internally.

In fact the electro-magnetic force exerted by an element $d\alpha$ of a unit current, on a unit north pole placed at the origin of α, is

$$\frac{\text{V}\alpha d\alpha}{\text{T}\alpha^3}$$

or, as we may write it,

$$\text{V}.d\alpha\nabla\frac{1}{\text{T}\alpha}.$$

This is identical in form with the expression for the differential whose integral, taken round a closed circuit, is AMPÈRE'S *Directrice.*[*]

Hence the element of work done by the closed circuit while the pole describes a vector $\delta\alpha$, is

$$\delta\text{W}=-\text{S}.\delta\alpha\int\frac{\text{V}\alpha d\alpha}{\text{T}\alpha^3}=-\text{S}.\delta\alpha\int d\alpha\nabla\frac{1}{\text{T}\alpha}.$$

But, if $d\Omega$ be the spherical angle subtended at α by a little plane area ds, whose unit normal vector (drawn *towards* the origin of α) is $\text{U}\nu$, obviously

$$d\Omega=\frac{\text{S.U}\nu\text{U}\alpha}{\text{T}\alpha^2}ds=-\text{S.U}\nu\nabla\frac{1}{\text{T}\alpha}ds.$$

Now, in the general formula (Trans. R. S. E. 1870, p. 76)

$$\int\text{V}\sigma d\alpha=\iint ds\,\text{V}.(\text{VU}\nu\nabla)\sigma,$$

put

$$\sigma=\nabla\frac{1}{\text{T}\alpha}$$

and we have

$$-\int\frac{\text{V}\alpha d\alpha}{\text{T}\alpha^3}=\iint ds\left(\text{U}\nu\nabla^2\frac{1}{\text{T}\alpha}-\nabla\text{SU}\nu\nabla\frac{1}{\text{T}\alpha}\right)$$

$$=\iint ds\,\text{U}\nu\nabla^2\frac{1}{\text{T}\alpha}+\nabla\Omega.$$

Now the double integral always vanishes while $\text{T}\alpha$ is finite, and we have therefore

$$\delta\text{W}=-\int\frac{\text{S}.\alpha\delta\alpha d\alpha}{\text{T}\alpha^3}=-\text{S}.\delta\alpha\nabla\Omega=\delta\Omega.$$

That is, the work done during any infinitesimal displacement of the pole is numerically equal to the change in the value of the spherical angle subtended by the circuit. The angle is, of course, a discontinuous function, its values differing by 4π at points indefinitely near to one another, but lying on opposite

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[*] *Electrodynamics and Magnetism*, §§ 5–8, Quarterly Math. Journal, 1859.

sides of the uniformly and normally magnetized surface whose edge is the circuit. There is, however, no discontinuity in the value of the work, for the element of the double integral is finite, and equal to 4π, when $T\alpha=0$.

Gauss * says (with date January 22, 1833) :—“ Eine Hauptaufgabe aus dem *Grenzgebiet* der *Geometria Situs* und der *Geometria Magnitudinis* wird die sein, die Umschlingungen zweier geschlossener oder unendlicher Linien zu zählen.” And he adds that the integral

$$\iint\frac{(x'-x)(dydz'-dzdx')+(y'-y)(dzdx'-dxdz')+(z-z)(dxdy'-dydx')}{((x'-x)^2+(y'-y)^2+(z'-z)^2)^{\frac{3}{2}}},$$

extended over both curves, has the value

$$4m\pi,$$

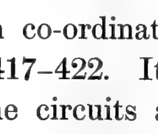
where m is the number of linkings (Umschlingungen). This is obviously the same as the integral of δW above, viz. :—

$$\iint\frac{S.\,\alpha d\alpha\delta\alpha}{T\alpha^3},$$

extended round each of two closed curves, of which $d\alpha$ and $\delta\alpha$ are elements.

§ 39. A very excellent investigation, by means of Cartesian co-ordinates, will be found in Clerk-Maxwell’s *Electricity and Magnetism* §§ 417–422. It is there shown that the above integral may vanish, even when the circuits are inseparably linked together. In fact m may vanish either because there is no real linking at all, or because the number of linkings for which the electro-magnetic work is negative is the same as that for which it is positive. For our present application this is of very great consequence, because it shows that the electro-magnetic work, under the circumstances with which we are dealing, cannot in all cases measure the amount of beknottedness. In fact the processes, soon to be described, enable us, without trouble for any given *linkage*, to find the value of m in Gauss’ formula; but there are special ambiguities when we try to apply the process to knots.

§ 40. To construct the magnetized surface which shall exert the same action on a pole as a current in any given closed circuit does, we may either suppose a surface extending to infinity in one direction (say for definiteness, upwards from the plane of the paper), and having the circuit for its edge; or we may form, as in the figure, a finite autotomic surface of one sheet, having the circuit for its edge. In dealing with the *two* curves of Gauss’ proposition, our procedure is perfectly definite; but when one and the same curve is to be the current and also the path of the pole, there is an ambiguity in esti-



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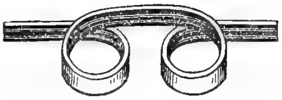
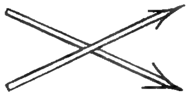
* *Werke*, Göttingen, 1867, v. p. 605.

mating the electro-magnetic work. To clear this away we require a definite statement of how the pole moves along the curve itself. For if its path screw round the curve $\pm 4\pi$ must be added to the work for each complete turn. As an illustration, suppose we bend, as in the figure, an india-rubber band coloured black on one side, so that the black is always the concave surface, and so that one loop is the perversion of the other, we find on pulling it out straight that it has no twist. If both loops be made by *over*laying, when pulled out it becomes twisted through two whole turns. This illustrates the kinematical principle that spiral springs act by torsion. An excellent instance of its connection with knots is to be seen in the process employed in § 11. For if we have portions of a cord, as in the diagram (Plate XVI. fig. 7), the pulling out of the loop in the upper cord changes the arrangement, as shown in the second figure.



A practical rule, which completely meets the GAUSSIAN problem, may easily be given from the consideration of the cylindrical magnetized surface above mentioned. Go round the curve, marking an arrow-head after each crossing to show the direction in which you passed it. Then a junction like the following gives $+4\pi$ for the upper branch, and nothing for the lower (which, on this supposition, does not pass through the magnetic sheet). Change the crossing from *over* to *under*, and this quantity changes sign. The junction figured above would, in our first illustration, be a silver one. But a still simpler process is to go round, as in § 36 (γ), putting a dot to the *right* after each crossing *over*, and *vice versâ*.



§ 41. Now, in order that our rule when applied to *knots* may give no work where there is no beknottedness, we must make the required expression such as to vanish whenever all the intersections are nugatory. Those which are nugatory only in consequence of their signs are in pairs, silver and copper, and will take care of themselves, as we see by special examples like the following. Hence the only part to correct for is that depending on the number of whole turns, and the sketch of the india-rubber band above shows that the work at the vertex of each such partial closed circuit is simply not to be counted, *i.e.*, that the 4π, which would be reckoned for each such crossing by our rule (positively or negatively as the case may be), is to be considered as made up for by the corresponding screwing of the pole round the curve.

§ 42. There must be some very simple method of determining the amount of beknottedness for any given knot; but I have not hit upon it. I shall therefore content myself with a few remarks on the subject, some of which are general, others applicable only to certain classes of forms. There seems to be little doubt that the difficulty will be solved with ease when the true method of attacking amphicheiral forms is found.

1. To form from a given projection the knot with the greatest amount of beknottedness, it is clear that we must in general so arrange the crossings over and under as to make *all* the crossings simultaneously silver or copper ones. And when this is done, a projection will give greater beknottedness for the same number of crossings the smaller is the number of crossings which have to be left out of account. Thus the simple *twists* (or clear coils with two turns) are the forms which, with a given amount of knottiness, can have the greatest beknottedness. For in them (see § 41) only one crossing has to be left out of the reckoning. Even a regular plait if of more than two strands cannot have so much beknottedness as it would acquire with the same amount of knottiness if two of its strands were first twisted together, then a third round these, and so on. And thus also entirely nugatory forms like the two first cuts in § 1 can have no beknottedness, for *all* their crossings have to be left out of the reckoning.

As an illustration, take the figure (Plate XVI. fig. 8) where the supposed number of loops may be any whatever. The free ends must, of course, be joined externally.

If we make the crossings alternately + and − it will be seen at a glance that a change of *one* sign (*i.e.*, that of the extreme crossing at either end) removes the whole knotting; so that there is but one degree of beknottedness. The crossings in this figure are in three rows. Those in the upper row are all copper (the last, of course, becomes silver when its sign is changed), and their number is n the number of loops. Each of the other rows has $n-1$, and all of them are silver. Thus when the one sign is changed there are $n-1$ copper crossings, and $2n-1$ silver. By pulling out the right hand loop we change n to $n-1$, so that one copper and two silver crossings are lost. After $n-1$ operations like this there remains only one (silver) crossing. It is easy to see from this that the crossings to be omitted in the reckoning of beknottedness (as in § 41) must be the lower row. To prove that it is so, study the beknottedness when the crossings are made so that the upper row are copper, silver, copper, &c., alternately, and those of the two other rows, silver, copper, silver, &c., alternately. It will be easily seen that with five loops there are two degrees of beknottedness, &c., and thus that our rule is correct. It is a curious problem to investigate the torsional and flexural rigidities of a wire bent in this form.

To give the greatest beknottedness to a knot with the same projection, it is obvious that all we have to do is to make the copper crossings into silver ones, *i.e.*, change the sign of each of the upper row of crossings. This gives fig. 9. With five loops it has four degrees of beknottedness.

Another excellent illustration is given by the coils of the class figured in

Plate XV. figs. 16 and 17, which have been already described (§ 27). A full investigation of the higher knottinesses of this class (especially when fully beknotted) would well repay the trouble it would involve.

As they are all amphicheiral, and in each case the crossings are divisible into two sets, those of each set being in all respects alike, while those of different sets differ only as to silver or copper, it is no matter (so far as testing beknottedness is concerned) which crossing we suppose to have its sign changed.

In the 8-fold amphicheiral of fig. 16 the change of any one sign reduces the whole to the irreducible trefoil knot (§ 16), right or left-handed according as we have changed one of the four outer, or of the four inner, crossings in the figure. Hence it has *two* degrees of beknottedness. But if we change the signs of one set of crossings (Plate XVI. fig. 24) so as to make all the crossings alike silver (or copper), we find the knot irreducible, though with continuations of sign; but with *three* degrees of beknottedness. And it is easy to see that it can now be analysed into two right-handed trefoil knots linked together as shown in the other part of the figure. But the linking is *left-handed.* Had it been right-handed we should have had + and − alternately, and thus we could not have transformed back to the form with continuations of sign (§ 4).

Similar remarks apply to the 10-fold amphicheiral plait (Plate XV. fig. 17). Change of any one sign reduces it to the third form of 6-fold knottiness (γ, § 8), which has only one degree of beknottedness. Hence the 10-fold plait has but *two* degrees of beknottedness when its signs are alternate. If we make all its crossings silver (or copper), as in Plate XVI. fig. 25, it has *four* degrees of beknottedness; and the reason is obvious from the other half of the figure, where it is seen to be made up of a pair of irreducibles—a pentacle and a trefoil, once linked together. There is one degree of beknottedness for the trefoil, one for the link, and two for the pentacle. The trefoil and pentacle are right-handed, the link left-handed, else we should not have had the continuations of sign which the figure must show.

A very curious illustration of this is to be found in the excepted cases, where the number of crossings is a multiple of six. From the two figured (Plate XV. figs. 15, 18) it is obvious that all of these are formed by three unknotted closed curves, *no two of which are linked together*, yet the whole is irreducible, having alternate signs. Hence we require a *third* term to complete our descriptions—knotting, linking, locking (?).

To give the greatest amount of belinkedness to these figures, let us suppose the ovals taken all the same way round, and arrange so that all the crossings shall be silver. Then we have continuations of sign (Plate XVI. fig. 26) as in the knots of the same series. But whereas Plate XV. fig. 15, if made of wire, is particularly stiff, the new figure is eminently flexible. This seems to have been practically known to the makers of chain armour.

The 9-fold knot of Plate XVI. fig. 15 has its crossings so drawn as to be all copper. Three must be left out of reckoning for the coiling, so it has *three* degrees of beknottedness.

But if we made the crossings alternately + and — we should find zero for the corrected electro-magnetic work—three copper and three silver crossings remaining. Change, then, the sign of any one of the three outer or inner crossings, and the whole reduces to the 4-fold knot. Hence it has *two* degrees of beknottedness.

If the crossing whose sign is changed be neither an outer nor an inner one, the result is a very singular 8-fold knot (irreducible, though having continuations of sign), differing from that of fig. 24, Plate XVI., in the fact that its component trefoil knots are *unsymmetrically* linked together. And it has but *one* degree of beknottedness, while that of fig 24 has *three.*

I have called attention to this example because of its bearings on the question of *the numbers of different irreducible knots having the same projection,* which we meet with as soon as we reach 8-fold knottiness.

2. To remove all beknottedness from a projection it is only necessary to make every crossing in its scheme + (or —) when it is first met with, reading from any point whatever. For then the several laps of the coil are, as it were, paid out in succession one over the other. When the beknottedness of a scheme so marked is calculated (as in § 41), it will be found that there is always at least one choice of a set of crossings such that, when these are omitted from the count, the electro-magnetic work is zero.

As an illustration take the very simplest form, the trefoil knot, with the suffixed signs determined by this rule. The scheme is

$$\begin{array}{cccccc|c} - & + & - & - & + & - & \\ A & C & B & A & C & B & A\,. \\ + & + & + & - & - & - & + \end{array}$$

Now, by § 41 we are entitled to leave out of count either A, B, or C. Leaving out either A or B gives zero for the electro-magnetic work, as it ought to be; but leaving out C gives -8π.

3. The only way in which we can have the intersections + and — alternately while every letter is + on its first appearance, *i.e.*, when there is no beknottedness, obviously the wholly nugatory scheme

$$\begin{array}{cccc} A & A & B & B, \&c. \\ + & - & + & - \end{array}$$

§ 43. To illustrate these methods let us take again the 5-fold knots (as in § 18) whose schemes are

$$\begin{array}{cccccccccc|c}
+ & + & + & + & + & + & + & + & + & + & \\
\mathrm{A} & \mathrm{D} & \mathrm{B} & \mathrm{E} & \mathrm{C} & \mathrm{A} & \mathrm{D} & \mathrm{C} & \mathrm{E} & \mathrm{B} & \mathrm{A}, \\
- & + & - & + & - & + & - & + & - & + &
\end{array}$$

$$\begin{array}{cccccccccc|c}
- & - & - & - & - & - & - & - & - & - & \\
\mathrm{A} & \mathrm{D} & \mathrm{B} & \mathrm{E} & \mathrm{C} & \mathrm{A} & \mathrm{D} & \mathrm{B} & \mathrm{E} & \mathrm{C} & \mathrm{A}. \\
- & + & - & + & - & + & - & + & - & + &
\end{array}$$

The lower signs refer to over or under, the upper to the electro-magnetic work, or to the silver-copper distinction.

Hence to determine the electro-magnetic work we must divide each scheme into independent circuits, no one of which includes a less extensive one; and omit from the reckoning the work for the terminal of each such circuit, and for each of the intersections which is not included in any one of the separate circuits. There are usually more ways than one of doing this. Sometimes these agree in their results; but the rule for choosing which to omit is to take them such that *with their proper signs,* and the rest with any signs whatever, they may be capable of making each letter positive on its first appearance. But there are cases even when the knot is not amphicheiral in which this process cannot be carried out. These occur specially when a part of the knot forms a lower knot with which the string is again linked.

In the first of the two schemes above there is but one independent non-autotomic circuit, which may be taken as

A D B E C A.

In this all the intersections are included, so that the whole work is to be found by leaving out that for A only; *i.e.,* it is $-16\,\pi$.

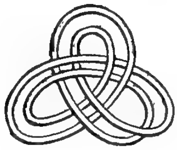
But in the second scheme we may take the two circuits

B A D B and C A D C,

and E is not included in either. Hence we must leave out of count the work for B, C, and E. This is found to satisfy our test, and thus the whole work is only $-8\,\pi$.

This is an instance in which the estimate by the electro-magnetic process exactly agrees with the result of simpler considerations, as given in § 35 above.

§ 44. It will be found that the alteration of five signs is sufficient to remove the knotting from the annexed figure, and the stages of operation of the various modes of reduction show that this form can be regarded as made up of simpler knots intersecting one another on the same string. These separate knots are virtually independent, and to change *all* the signs in any one of them does not in cases like this necessarily simplify the knot. Uncorrected the work is $-13 \times 4\pi$. Corrected it is $-10 \times 4\pi$, which agrees with the removal of the beknottedness by change of *five* signs only.



If the sign of the one unsymmetrical crossing be altered, four changes of sign will suffice; for the uncorrected work is $-11 \times 4\pi$; corrected it is $-8 \times 4\pi$, corresponding to four changes of sign.

§ 45. It is clear from what precedes that the GAUSSIAN integral does not, except in certain classes of cases, express the measure of what may be called, by analogy with § 35, *Belinkedness.* It may be well to examine a simple form of link with all its possible arrangements of sign to see what the integral really gives in each of these. Let us choose for this purpose two lemniscates having four mutual crossings, as in the edges of the band shown in fig. 13, Plate XV.

If we suppose the signs to be made alternately + and −, as in Plate XVI. fig. 10, the form is a six crossing one, and irreducible. The silver or copper character of the *self* crossings does not depend upon the directions in which we suppose the lemniscates to be described, that of the *mutual* crossings does. We thus have, from another point of view than that of § 41, a proof that these are to be left out of account in the reckoning.

The four crossings of the *two* curves are copper, if these curves are supposed to be described in the same way round; those of the separate curves (which do not count) are silver. Hence the work is -16π, or two degrees of belinkedness.

Change the sign of any *one* of P, Q, R, S, that and the adjacent one slip off, U and V become nugatory. The linkage is the simplest possible, and the integral is 8π.

Change the sign of either or both of U and V. In either of these three cases both become nugatory, and the whole takes the form of two doubly-linked ovals, with the integral $= -16\pi$. (Plate XVI. figs. 12, 13.)

If the signs of both R and S be changed the value of the integral is obviously $4(2-2)\pi$, for R and S have become silver, while P and Q remain copper.

If in addition the signs of U and V be both, or neither, changed, only one crossing is got rid of, and the link may be put in the form (Plate XVI. fig. 14). It cannot be farther reduced, because the crossings are alternately over and under.

But if the sign of one only of U, V be changed, it will be seen that there is no linking (Plate XVI. fig. 11). Here the integral vanishes because there is really *no work,* not as in the last case, where there are *equal amounts of positive and negative work.*

§ 46. This gives a hint as to the reckoning of beknottedness from the silver and copper crossings in the cases where we have found a difficulty. After omitting from the reckoning the crossings which belong merely to the *outline* of the figure, there must remain an *even* number of crossings (§ 22). Hence, whatever numbers be silver and copper respectively, the excess of the one of these over the other must be an even number (zero included). In general, *half this number is the beknottedness.* But when the knot, or even part of it, is amphicheiral there is usually more beknottedness than this rule would give. And, in particular, there may be beknottedness when the number is zero. In this case the number of silver (and of copper) crossings is even, and is double the degree of beknottedness.

As I have already stated, I have not fully investigated this point, and therefore for the present I content myself with giving two instructive examples from the six-fold knots. The observations which will be made on these contain at least the germ of the complete solution.

The form γ (of § 8) is not amphicheiral. As there drawn, it has four copper and two silver crossings, the latter being the intersections of the loop with the trefoil; but the scheme shows that two copper crossings must be omitted from the reckoning, one of these being necessarily that which is uppermost in the figure. If the sign of this last be changed, the knot opens out, so that it has but one degree of beknottedness. Hence, in this case, the two copper and two silver crossings correspond to one degree of beknottedness only. But if we change the sign of *any one* of the other three copper junctions the knot sinks to the 4-fold amphicheiral, retaining its one degree of beknottedness.

In the amphicheiral form β (of § 8) there are three silver and three copper crossings. As the figure is drawn, these are to the right and left of the figure respectively; and either crossing at the end of the lower coil may be left out, along with any one of the three on the other side. Thus there remain, as in the former case, two silver and two copper ones. This corresponds to one degree of beknottedness, as in the last case, for the change of sign of *either* crossing at the end of the lower coil unlooses the knot. But if any one of the other four crossings (alone) have its sign changed, the whole becomes a right or left-handed trefoil knot, retaining, as in the former example, its one degree of beknottedness.

To give the greatest beknottedness to these forms, we must alter two signs in (γ) and three in (β). In each case one crossing is lost, and the form becomes the pentacle (§ 7) with its two degrees of beknottedness.

Part V.

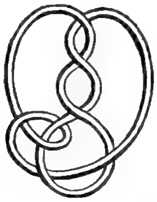
Amphicheiral Forms.

§ 47. These have been defined in § 17, and several examples have been given, not only of knots, but of links, which possess the peculiar property of being transformable into their own perversions.

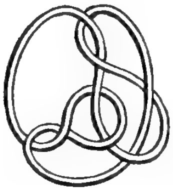
The partition method (§ 21) suggests the following mode of getting amphicheirals:—Since the right-handed and left-handed compartments must agree one by one, and since (§ 20) the whole number of compartments is greater by 2 than the number of crossings, the number of crossings must be even. Let it be $2n$, and let $p_1, p_2, \ldots p_{n+1}$ be the partitions. Then our selection must be made from the numbers which satisfy

$$p_1 + p_2 + \ldots + p_{n+1} = 4n,$$

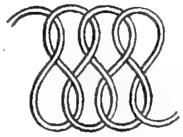
no one being greater than the sum of the others. If a set of such can be grouped as in § 20 so that the other set for the complete scheme shall be the same numbers *with the same grouping*, we have an amphicheiral form. The words in italics are necessary, as the following example shows; for here the black and white compartments have the same set of partitions but not the same grouping, and the knot is not amphicheiral:—



But a different grouping of the *same* set of partitions gives the amphicheiral form below

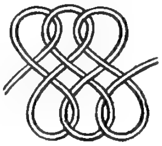


But an easier mode of procedure, though even more purely tentative, is the following:—If a cord be knotted, any number of times, according to the pattern below,

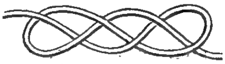


it is obviously *perverted* by simple *inversion*. Hence, when the free ends are joined it is an amphicheiral knot. Its simplest form is that of 4-fold knottiness. All its forms have knottiness expressible as $4n$.

The following pattern gives amphicheiral knottiness $2+6n$:—



And a little consideration shows that on the following pattern may be formed amphicheiral knots of all the orders included in $6n$ and $4+6n$:—



Among them these forms contain all the even numbers, so that *there is least one amphicheiral form of every even order.*

Many more complex forms may easily be given. See, for instance, Plate XVI. figs. 18, 19, 20. Some are closely connected with knitting, &c.

An excessively simple mode of obtaining such to any desired extent is to start with an amphicheiral, whether knot or link, and insert additional crossings. These must, of course, be inserted symmetrically in pairs, each in the original figure being accompanied by another which will take its place in the perversion or image.

Thus, taking the simplest of all amphicheirals, the single link (Plate XVI. first of figures 27), we may operate on it by successive steps as in the succeeding figures.

The second, third, and fourth are formed from the first by adding, the fifth and sixth from the fourth by removing, pairs of crossings. The third, like the first, is a link; the others are knots.

Figures 28, Plate XVI., give another series, of which the genesis is obvious. The protuberances put in the first figure, for instance, show how it becomes the second. The fifth of fig. 27, and the second and fourth of fig. 28, all alike represent the amphicheiral form (β) of § 8. But we need not pursue this subject.

§ 48. It will be seen at a glance that the first pattern in last section gives for two loops (*i.e.*, four crossings) the knot of § 6; while the third pattern as drawn is simply β of § 8. In this form of the knot, the two dominant crossings (§ 46) are those in the middle, and mere inspection of the figures shows that the whole knotting becomes nugatory if the sign of either of these be changed.

It might appear at first sight that amphicheirals of the same knottiness, formed on such apparently different patterns as the two first of last section, would be necessarily different. But the very simplest case serves to refute this notion. For the lowest integers which make

$$4n=2+6n'$$

give 8 as the value of either side. Figs. 22, 23, Plate XVI., represent the corresponding amphicheirals, apparently very different, but really transformable into one another by the processes of § 11. Fig. 21, Plate XVI., represents another 8-fold amphicheiral form, suggested by a somewhat similar pattern. I hope to return to the consideration of this very curious part of the subject, and at the same time to develop a method of treating knots altogether different from anything here given, which I recently described to the Society.*

After the papers, of which the foregoing is a digest, had been read, I obtained from Professors LISTING and KLEIN a few references to the literature of the subject of knots. It is very scanty, and has scarcely any bearing upon the main question which I have treated above. Considering that LISTING's Essay was published thirty years ago, and that it seems to be pretty well known in Germany, this is a curious fact. From LISTING's letter (Proc. R. S. E. 1877, p. 316), it is clear that he has published only a small part of the results of his investigations. KLEIN† himself has made the very singular discovery that *in space of four dimensions there cannot be knots.*

The value of GAUSS' integral has been discussed at considerable length by BOEDDICKER (by the help of the usual co-ordinates for potentials) in an Inaugural Dissertation, with the title, *Beitrag zur Theorie des Winkels,* Göttingen, 1876.‡

An Inaugural Dissertation by WEITH, *Topologische Untersuchung der Kurven-Verschlingung,* Zürich 1876, is professedly based on LISTING's Essay. It contains a proof that there is an infinite number of different forms of knots! The author points out what he (erroneously) supposes to be mistakes in LISTING's Essay; and, in consequence, gives as something quite new an illustration of the obvious fact that there can be irreducible knots in which the crossings are not alternately over and under. The rest of this paper is devoted to the relations of knots to RIEMANN's surfaces.

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* Proceedings, R. S. E., May 7th, 1877.

† *Mathematische Annalen,* IX. 478.

‡ Professor FISCHER has just shown me an enlarged copy of BOEDDICKER's pamphlet above mentioned. Twenty pages are now added, mainly referring to the connection of knots with RIEMANN's surfaces, and the title is altered to *Erweiterung der Gauss'schen Theorie der Verschlingungen.* Stuttgart, 1876.

ERRATA.

Page 167. In the lower group of figures the numbers II. and IV. are to be interchanged. Also the cut marked III. is merely a form of V. The correct symbol for III. differs from these by having the 2 *between* the two lines joining 5 and 4.

,, 185. Interchange the words "first" and "second" in lines 20 and 25 respectively.

IX.—*On the Toothing of Un-round Discs which are intended to Roll upon each other.* By Edward Sang, Esq.

(Read April 16, 1877.)

The construction of the toothed wheels used in machinery gives rise to some very interesting investigations in the geometry of motion. The general problem is so to shape the contours as that they shall remain in contact while the wheels turn on their centres with uniform angular velocities.

The inquiry becomes more extensive when the velocities of the wheels are to be variable; as, for example, when we seek to imitate the revolutions of the planets round the sun, and for that purpose introduce the equation of the centre.

In these cases the wheels are supposed to turn on fixed centres; but we may still farther extend the scope of our researches by removing the centres and subjecting the discs to the single condition that they roll upon each other.

If two discs A and B touch at the point S, and if they so move as that the point of contact shifts equally along the two boundaries, they are said to roll on each other; that is to say, if we measure equal distances S T, S V along the two boundaries, the points T and V will come together in the course of the movements.

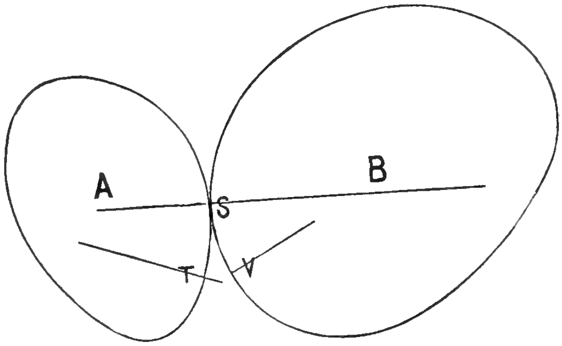


Fig. 1.

This rolling may be effected in various ways. One of the discs, as A, may be held fast while the other is rolled round about it. In this case the motion of B is regulated by its own form as well as by the form of A. In order that the motion of each disc may depend on its own shape alone, we may imagine the point of contact S to remain motionless, while the contours of the two discs slide past it in such a manner as to be continually perpendicular to a fixed straight line. In this way when

T comes to the *pitch-point* as it is called, the normal to the curve at T must lie along the straight line S A, while the corresponding normal at V must be along S B; and the motion of each disc becomes an instantaneous rotation round the centre of curvature.

Were such discs employed for the transmission of pressure, the limit of their action would be determined by the co-efficient of friction between the two substances, and would, at all times, be precarious. For the purpose of preventing any slipping of the one boundary past the other, we propose to notch and tooth them. The object of the present paper is to investigate the principles according to which this toothing must be accomplished.

In the first place we oqserve that if the discs be to turn round and round upon each other, their peripheries must be commensurable, and the distance from tooth to tooth must be a common measure of them.

We may next observe that when both discs are convex all round, they may roll upon each other; but that if one of them have a sinuosity as in figure 2, no part of the other's contour can apply to the hollow unless it be convex and have its radius of curvature less than the osculating radius of the concavity. This, and similar limitations, caused by the impenetrability of the solids, may be set aside when we are considering only the geometry of the subject.

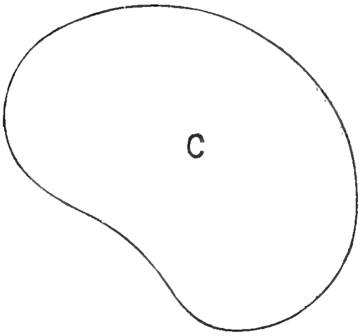


Fig. 2.

The contour of the toothed disc must undulate on either side of the pitch-line; our first business is to inquire into the general law of its formation.

Let *a* and *b*, figure 3, be the centres of instantaneous revolution of two discs; let P be the point at which their teeth touch, and let S P T be the direction of their common tangent.

Imagine now that the discs move forward by an indefinitely small quantity, and that the common tangent takes up the new position *s t*, sensibly parallel to S T; then making *a* P *s* and *b* P *t* each a right angle, *s* and *t* will indicate the new positions taken up by the points P of the discs, and *s t* will represent the distance through which the peripheries slide upon each other. Draw P *r* perpendicular to *a b*, and P Q perpendicular to S T.

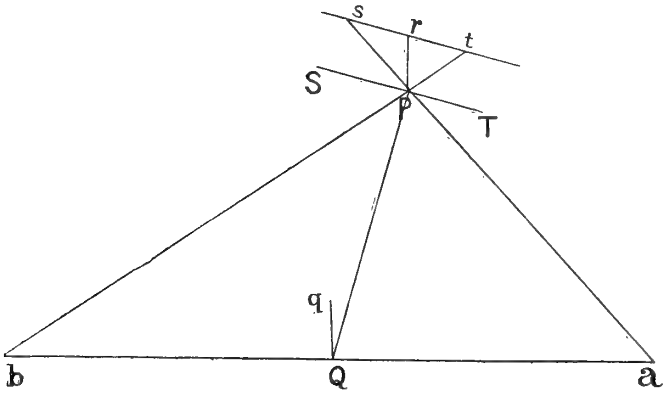


Fig. 3.

The sides of the trigon $s\,r\,P$ are perpendicular, respectively to those of $P\,Q\,a$, wherefore

$$AP : PS :: AQ : Pr.$$

Now $P\,a\,s$ is the angular displacement of the disc A, wherefore, if we make $Q\,q$ parallel and equal to $P\,r$, $Q\,a\,q$ must also represent the same angular displacement. Also for the same reason $P\,b\,t$, and its like $Q\,b\,q$, represents the angular displacement of the disc B; so that the angular movements are in inverse proportion to the radii $a\,Q$ and $b\,Q$; in other words Q is the pitch-point and $Q\,q$ or $P\,r$ the motion of the pitch-lines.

During this minute change of position, the point of contact has moved from P to some point in the line $s\,t$; the direction of its motion being undetermined.

This investigation shows us, in the first place, that the line joining the pitch-point with the point of contact is always normal to the surfaces of the teeth. Secondly, since the augmentation of Q P is the distance between the two parallels S T, $s\,t$, it follows that the velocity of the pitch-lines is to the rate of increase of Q P as radius is to the cosine of the inclination $q\,Q\,P$. And thirdly, the motion of the contact-point is independent of the radii of curvature $a\,Q$, $b\,Q$, of the pitch-lines.

Hence we have this noteworthy porism: If the point P move along some line, which we may call the contact-path, in such a way as that the rate of increase of Q P shall be to the rate of motion of the pitch-lines, as the cosine of the angle R Q P is to the radius, the motion of P combined with the appropriate motion of any of the discs A, B, C, will generate, on the planes of those discs, the outlines of the teeth. In this way the peculiar form of the pitch-line of the disc is eliminated, as it were, from our investigation, and we have to consider chiefly the form of the contact-path.

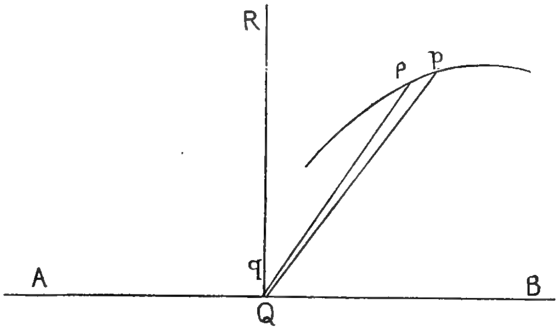


Fig. 4.

Since the same action has to be repeated, the entire contact-path must be retraced during the passage of each successive tooth; and therefore, if there be only one point of contact, the path must return into itself. Also, since the tooth outline must cross the pitch-line of the disc in all actual cases, the contact-path must pass through the point Q, and must lie partly on either side of the line $Q\,q$; it must also lie above and below A Q B, and so must be distributed among the four quadrants round Q, while it is obvious that for all business purposes, the four parts should be symmetrically placed.

From the general law of its motion it follows that the point P cannot cross A Q B obliquely except at Q, and that it cannot cross R Q S otherwise than obliquely, wherefore the general character of its path must be as represented in figure 5; and, when there is only one contact-point, that point must trace the curve Q G K F Q I L H Q continuously during the passage of each tooth. In order that this be possible, the distance Q P must increase while P moves along the part Q G K, and must attain its maximum at K. Whenever P passes into the part K F Q, Q P must decrease because the cosine of R Q P is then subtractive; and similarly for the remaining half of the path.

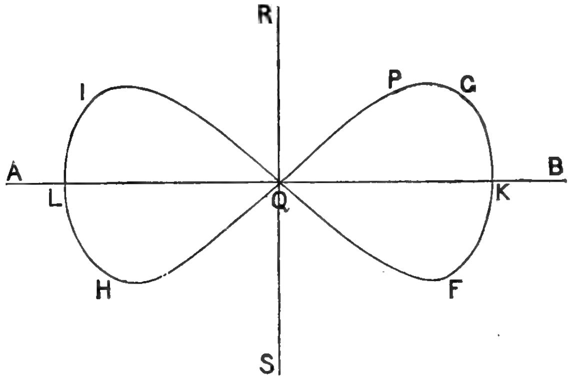


Fig. 5.

Teeth formed by help of such a path would be useless in determining mechanically the relative positions of two discs; the single contact might prevent A from moving forward without taking B along with it, but then it would not hinder A from being turned backwards; wherefore it is requisite to have more contacts than one.

If the contact-path pass outside of the circle described from Q with the radius Q K, as in figure 6, the motions at K and L must be upwards, or in the same direction with that of the pitch-lines. Let F, G, H, I, be the four points at the maximum distance from Q, then if the contact begin at F, it must separate there into two, one contact proceeding along F Q I, the other along F K G. The former of these will reach I just as a third contact proceeding along the line H L I arrives there, and the two coalescing will cease. In the same way that contact which has passed along F K G will coalesce at G with another that has passed along H Q G; and the motions will be timed so as that the arrivals at K, Q, L, Q, divide the entire passage of a tooth into four equal parts.

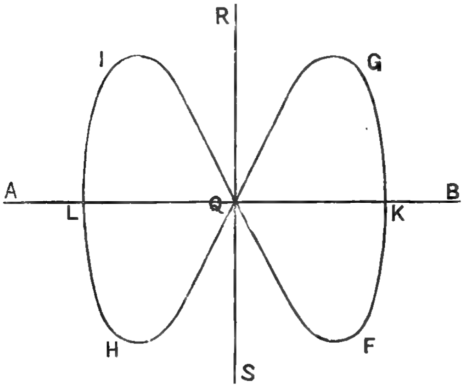


Fig. 6.

Since the contacts appear and disappear in pairs, their number must be odd, excepting just at the instant of beginning or ending. By augmenting the ordinates parallel to R S, thereby making the curve flatter at K and L, we can increase the number of contacts; and it is possible so to determine this augmentation as that a new contact may appear at F or H just as a disappearance occurs at G or I. When this has been done, the number of points in contact remains constant, and these are distributed so that the total number of them on the two parts F Q I, H Q G, exceeds the total number on F K G, H L I, by one.

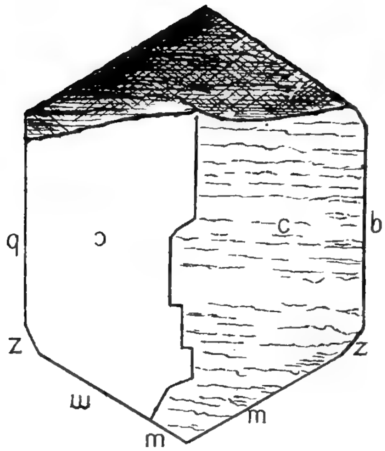
The simplest arrangement admissible in practice is that with *three* contacts. Of these one, occuring on H Q G, is on the front of a tooth, a second occuring on F Q I is on the back, while the third happening alternately on F K G and H L I is on the top or bottom of the tooth. The first and second of the contacts serve to determine the relative positions of the discs in the direction of their pitch-lines, and to communicate the motion of rotation from the one to the other. The third serves to resist the pressure exerted to keep the discs in contact.

When the discs turn on fixed centres the resistance perpendicularly to their pitch-lines is exerted on the centre pins or axes; but, in the case of un-round discs generally, the action at the tops and hollows is indispensable.

If such a contact-path graduated for equidifferent positions of the pitch-lines, be carried round any of the discs A, B, C, in such a way as that the point Q is placed at the successive points of division while the line A Q B lies along the normals at these points, marks made through its appropriate divisions upon the plane of the disc, indicate the forms of the teeth; and the discs so toothed work truly with each other. This mode of delineating the teeth of irregular discs is a generalisation of that explained in my "New General Theory of Wheel Teeth" as applicable to ordinary wheels.

The teeth placed round any one of these discs necessarily vary in shape according to the radius of curvature of the pitch-line; their mechanical possibility is limited by certain considerations. If, while a point moves along the part Q G, a normal to the curve accompany it, the point at which that normal crosses Q B, will move away from Q, will reach a maximum distance and thence return to Q. Whenever the centre of curvature of the pitch-line lies within this extreme distance, the outline of the side of the tooth becomes folded in; also whenever it lies within the extreme limit of the normal applied to the curve on the part L I, the outline is replicated at the top of the tooth; and the greater of these two limits indicates the impossibility of the mechanical action. This matter is thoroughly discussed in a paper entitled "Search for the Optimum System of Wheel-Teeth."

Sutherland Amazonstones.



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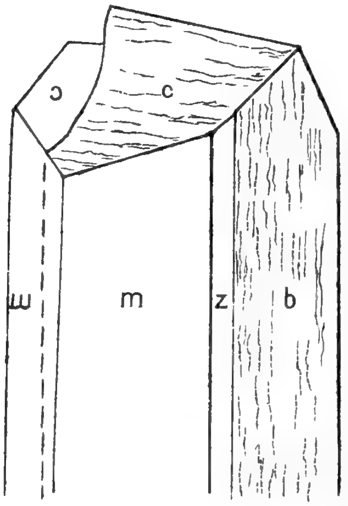

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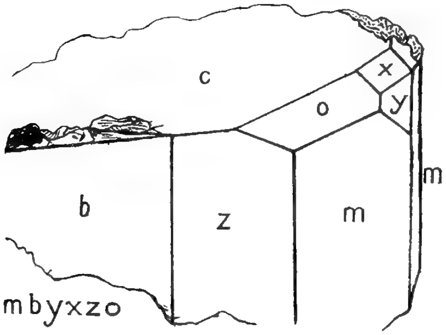

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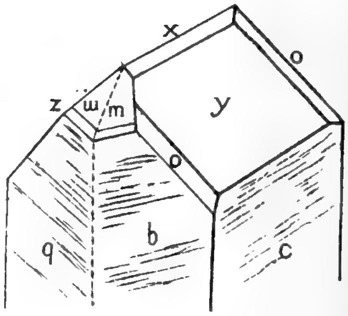

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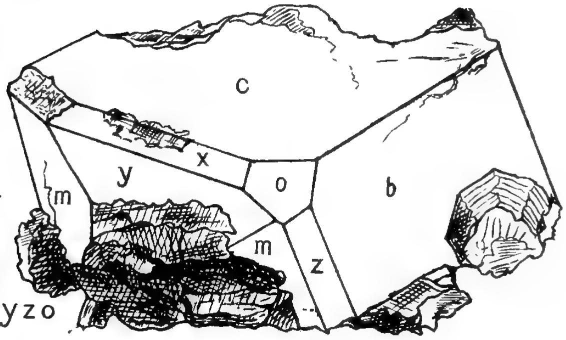

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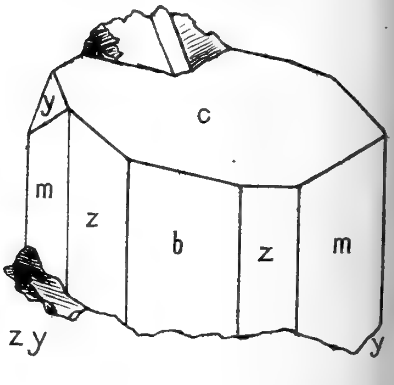

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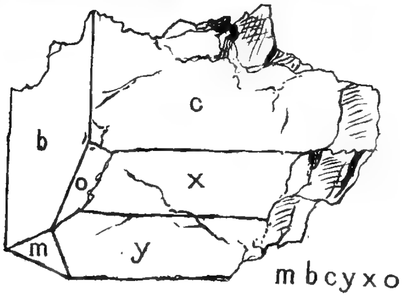

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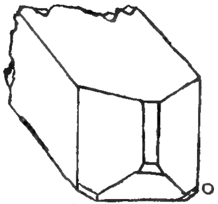

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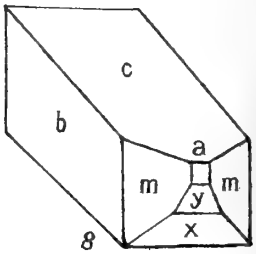

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X.—*Chapters on the Mineralogy of Scotland. Chapter Second.—The Felspars. Part I.* By Professor HEDDLE. (Plates XVII., XVIII.)

(Read 15th April 1877.)

Before submitting the results of my examination of the silicates, it is incumbent upon me to give an outline of the processes adopted for their analysis, seeing that the amount of confidence which is to be placed in the results must rest very immediately thereupon.

1. *Determination of the Water.*—In all silicates which were found, after thorough drying, to give out moisture when heated in the closed tube, the amount of loss upon ignition was calculated as water. About 2 grammes, crushed in the diamond mortar, were first merely heated throughout in the air-chamber of a water-bath, the contact temperature of which was 212° Fahr., the temperature of the air-space being never below 199°. The weight after this heating was taken; and, except in the case of a mineral functioning in any peculiar way, the loss in weight was estimated as *hygroscopic moisture.* The mineral was now continuously heated in the bath until the weight was found to be constant. It was then exposed for the space of one hour to a full red heat, in the flame of a BUNSEN burner, after which it was re-weighed. Finally, it was exposed for ten minutes to a heat approaching whiteness, in a three-jet GRIFFIN blast-furnace,* again re-weighed, and again heated in this furnace, until the weight was constant.

It was found that minerals which contained any considerable quantity of water, lost markedly different proportions thereof during the three latter stages of this treatment.

2. *Determination of the "Silica" and the Bases.*—The minerals were decomposed in the ordinary way by fusion with 4½ times their weight of FREZENIUS' flux. The fritting and fusion occupied one hour in a BUNSEN flame; the crucible was then heated for ten minutes in the GRIFFIN furnace.

Although the exposure of the platinum crucibles to these naked flames was found destructive to a very considerable extent, still it was not by any means so much so as the necessary exposure in a GRIFFIN draught-furnace proved to be. FLETCHER's furnace, with plicated copper burner, was discarded after two

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* I cannot too highly recommend this simple and cheap little furnace; it commands every temperature, and, for mineral analysis, is superior to all others which require a blowing apparatus.

trials; the fusions, though in an unusually close crucible, having been found to be impregnated throughout with copper.

The acidified solution of the "glass" was evaporated to *absolute* dryness, with granulation, in a water-bath, which kept its beakers at the average temperature of 186° Fahr. In the case either of an unusually large quantity of material, or an unusually large proportion of alkalies, the saline mass from the first granulation was redissolved in water and regranulated.

After the separation of the main silica, the iron was peroxidised by nitric acid, it and the alumina thrown down as basic acetates, filtered off hot, and washed with sodium acetate. The precipitate was redissolved in hydrochloric acid, and evaporated to dryness with granulation, to separate the hitherto soluble silica.

The acid solution of the iron and alumina, now free from silica, was made up to the bulk of 250 c.c.; 100 of these were employed for determining the total ferric oxide and alumina combined; measured quantities of the remainder being, after reduction with zinc, used for the ascertaining the total iron, by permanganate of potassium. This calculated to ferric oxide, and as such deducted from the total ferric oxide and aluminia, gives finally the amount of alumina. In the filtrate from the basic acetates, the manganese was thrown down by bromine water. The lime and magnesia were determined as usual; the ammonium oxalate precipitate being invariably redissolved, and the lime reprecipitated; in the case of substances with large amounts of magnesia, it was found necessary to repeat the resolution and precipitation.

3. *Estimation of the Ferrous Oxide.*—About ·3 grammes of the finely powdered mineral were intimately mixed with a considerably larger amount of colourless and pure fluorspar, in a platinum crucible, the lid of which has a small central orifice,—through this orifice sulphuric acid is added drop by drop. The crucible fits into and fills up an opening in the bottom of a copper chamber, which stands over one of the holes of the water-bath; over the crucible, but within the chamber, there stands a three-necked WOULFF's bottle, from which the bottom has been removed by grinding. Through a recurved funnel tube in the central neck more acid may be added if necessary; through one of the lateral necks a continuous current of carbonic acid is conveyed into the chamber;—the tube in the third neck gives exit to this current.

With sulphuric acid, the decomposition of a silicate is complete in about three minutes, and the whole fluosilicic and hydrofluoric acid driven off in less than twenty. With hydrochloric acid, the action is neither so speedy nor so certain.

After the dissipation of the fluorine acids, the contents of the crucible are washed into about five cubic inches of recently boiled water, cooled in vacuo; and the iron, in the state of ferrous oxide, titrated by permanganate. The whole iron present is now reduced by zinc, and again titrated by permanganate—giving a second determination of the *total* iron, as a check upon that already made on the mixed solution of iron and alumina.

We have now ascertained the absolute amount of ferrous oxide, and this calculated to ferric oxide, and as such deducted from the total ferric oxide, as ascertained after the original peroxidation, gives the absolute amount of ferric oxide existing as such in the mineral.

A blank result from numerous blank experiments, and a closely accordant result in numerous experiments on the same mineral, repeatedly instituted, showed that this process is to be relied on.

As regards silicates, at least, its expedition leaves nothing to be desired.

I have been constrained to prefer it to EARLY'S very similar process, because I have repeatedly failed in procuring hydrofluoric acid free from iron; and I find that the substitution of the ammonium fluoride for fluorspar is not satisfactory. With the former salt, the reaction is so energetic, that there is a risk of loss of substance by projection from the crucible, while the duration of the action is not sufficient for the complete decomposition of the substance operated on. Ammonium fluoride, moreover, is frequently contaminated with iron; could it be obtained pure, its employment in the above process would enable us to determine alkalies by a method superior to any in use; and were we in possession of the permanganate of ammonium we might also in the same operation—*i.e.*, in the same quantity of material—determine likewise the amount of both the oxides of iron.

4. *Determination of Alkalies.*—In all my more recent analyses I have employed LAURENCE SMITH'S process; of this I think very highly, perhaps as highly as does its author, though I am not prepared to admit the extreme expeditiousness which he assigns to it. From the frit or fusion in by far the greater number of cases forming a solid mass which dissolves extremely slowly if at all in water, and from the consumpt of time in the getting down so much lime, I now invariably dissolve the fusion in acid. When this is done, I find that a determination may be made in about a day, if little attention is paid to anything else. I also hold that it is absolutely necessary *finely* to pound the mineral—more so, indeed, than for a fusion with an alkaline carbonate. I use baryta-water whenever there is much magnesia present; and I find that it requires watchfulness and unusual care to prevent a slight loss of sodium chloride, in getting rid of so large a quantity of ammonium salts.

5. *The "Silica" of these Analyses. What is it? and in what state did it exist in the mineral?*—Probably no one who has made many analyses of silicates, and examined the "silica," will confidently say that it is silica. But to begin at the beginning, what is silica? We actually know eight silicas; and we speculate upon several "condensed silicas." The silicas

that we do know are very different things indeed, both physically and chemically. Without knowing whether they are present or not, or how much of them are present, we jumble them all together, and set them down as *one* thing, in formulating minerals. Is it rational to set down as one thing that which we know to contain two or more perfectly different things, and call our elaboration a rational formula?

In the analyses which are to follow, the "silicas" which have come out of different minerals,—nay, the "silicas" which have come out of the same mineral from different localities,—nay, the "silicas" which have come out of more than one portion of the same specimen, analysed by treatment purposely identical, have proved so different as to be perplexing beyond measure.

Tabulating the results of the researches of RAMMELSBERG, JENZSCH, VOM RATH, and GUSTAV ROSE, we have—

Crystalline Silicas.		Spec. Grav.
Hexagonal, hemihedral,	1. Silica, insoluble in hot potash solution,	2·65
	2. Silica, soluble " " "	2·65
Hexagonal, holohedral,	3. Tridymite, insoluble in hot potash solution,	2·25
Prismatic,	4. Asmanite, (?)	2·245
Pyramidal,	5. In Zircon,	(?)
Amorphous Silicas.		
	6. Opal Silica, insoluble in hot potash solution,	2·185
	7. Opal Silica, soluble " " "	2·1
	8. Jenzschite, soluble in hot potash solution,	2·64

2 and 8 possibly are the same,—the interpenetration of a moderate amount of a colloid within a much greater amount of a crystalloid might not materially interfere with the optical properties of the latter; and RAMMELSBERG found but a small quantity of 2 in the general mass of the crystals of 1.

That some of these modifications, if the word may be used, are mutually convertible into each other is unquestionable; and that the processes employed in our analyses must and do so convert them is also unquestionable; but this only renders it the more difficult for us to ascertain in what state or states the silica was originally present in the mineral;—or, in other words, which one, or which two, or how many of these modifications, and how much of each was present, before our operations induced changes upon them.

Seeing that under the adoption of a uniform process we obtain the silica from certain minerals only in one state—from certain other minerals in two states—and from others in three, is it irrational to suppose that in these latter cases it had existed in the mineral in more states than one?

If this be granted, is it, to go a step further, irrational to suppose that one of the modifications was in union with one ingredient, or one isomorphous set of ingredients—another modification with another set—the third with a third;

—that one silica saturated the sesquioxides, another the protoxides, while an opal silica took up any non-basic water? Should such a conclusion prove to be correct, we may hope to explain how a certain quantity of silica, and no more, may be replaced, as in hornblende, by alumina.

Even, however, as regards the question of the convertibility of one modification into another, we do not find a conformity of opinion.

Of our methods of examining into the condition and the purity of our silicas, two only may be said to be in general use—evaporation with hydrofluoric acid, and boiling in a saturated solution of sodium carbonate: the first, from the difficulty of procuring a sufficiently pure acid, and from the destructive nature of its fumes, being little employed.

Rammelsberg, making use of the latter test of purity, holds that pure silica is only soluble in boiling solution of sodium carbonate, if it has been *gently* heated; and that the same silica is rendered more or less insoluble in the same solution, if continuously *highly* heated; as it is thereby converted into tridymite. The application of this view would be that the very great difference in the solubilities of the silicas got from various minerals by different analysts is due to different degrees, and undue continuance of heating.

The late Mr Wallace Young, on the other hand, remarks—"In testing the residual silicic acid by boiling with sodium carbonate solution, a great deal of trouble was experienced frequently by its sparing solubility in that reagent. I have not as yet been able to find out why the silicic acid should be quite soluble in some cases and not so in others, when subsequent examination showed it to be equally pure in both cases. For instance, in one specimen of analcime, which had been decomposed with hydrochloric acid, the silicic acid was almost insoluble in sodium carbonate solution; prolonged boiling and using different proportions of water, &c., did not seem to have any effect. On being fused with alkaline carbonate and separating as usual, it was found to be pure, and was now quite soluble in the sodium carbonate solution. Another specimen of analcime decomposed by acid gave 55·56 per cent. of silicic acid very sparingly soluble; a fresh portion of the mineral fused with alkaline carbonate gave 55·52 per cent. of silicic acid which dissolved in sodium carbonate, and gave a clear solution. *The difference of time and intensity of heat employed in the ignition of the silicic acid did not seem to affect its solubility.*"

And Mr Young makes the following remark—most pertinent to the analysis of rocks:—"The occasional sparing solubility of silicic acid comes to be of great importance in an analysis of mixed silicates only partially soluble in acids, when, after the acid treatment, the residue is boiled in sodium carbonate solution to extract the separated silicic acid, washed and dried, and the insoluble silicates fused with alkaline carbonate as usual. The silicic acid would be apt to come out too low for the soluble silicates, and too high for the insoluble ones."

In consequence of this conflict of opinion, and being as much puzzled as Mr Young, I made the following series of experiments :—

1. *Is soluble Silica converted by a high and long-continued heat into insoluble Silica?*

1st. A quantity of impalpable silica was prepared by the fluosilicic acid process. This silica, after thorough air drying, was found to lose 9·854 per cent. of water when heated to 212° Fahr. It was found to be then almost totally soluble in hot sodium carbonate solution. It was now cautiously heated and kept at a barely visible red heat for five minutes. After this treatment 9·9 per cent. were found to be insoluble. Another portion of the same make was heated to the highest heat of a fine Bunsen burner for seventy-five minutes, of this 75·19 per cent. were insoluble.

The specific gravity of this powder was not taken, but if it be tridymite the above silica is the best variety to employ in the manufacture of that modification.

2d. A piece of perfectly limpid and colourless rock crystal was powdered, fused with flux, and obtained as recognised silica—entirely in the granular form. After drying in the water bath it was soluble all but ·397 per cent.

By the same treatment as first applied to the impalpable silica, it yielded 4·492 per cent. of insoluble ; while by the same treatment as secondly applied to the impalpable, it yielded 6·963 per cent. of insoluble.

A continuous high heat, therefore, converts soluble into insoluble silica ;—and the finer the state of division of the silica, the greater the proportional amount so converted.

2. *What amount of influence upon the solubility of the "Silica" is affected by over-heating during the granulation of the saline mass? And what is the cause of this influence?*

The pseudo hypersthene (augite) of Coruisk was found when granulated in the water bath at a temperature of a 186° to give a "silica" of which 23·4 per cent. was insoluble. The same mineral was granulated in the hot air bath at a temperature of 320°, when 53·98 per cent. were found to be insoluble.

Anthophyllite from Hillswick, Shetland, was found when granulated over the water bath to give a silica of which 2·87 was insoluble ; the same mineral, treated and granulated at 270° in the hot air bath, gave a silica, 7·25 per cent. of which was insoluble.

Andesine, from Portsoy, at the low heat granulation, gave a silica with 8·9 per cent. insoluble ; when granulated at 380°, 71·25 per cent. were insoluble.

The low heat insoluble silica from the augite was found to contain only traces of alumina and lime; the high heat insoluble silica contained 1·27 per cent. of ferric oxide, ·83 of lime, ·83 of magnesia, and traces of alumina.

The low heat insoluble silica from the anthophyllite contained only traces of alumina; the high heat insoluble "silica" contained 21·76 per cent. of alumina, 19 per cent. of lime, and ·44 per cent. of magnesia.

The low heat insoluble silica from the andesine contained no impurity; that from the high heat contained 1·34 per cent. of alumina, 1·17 per cent. of lime, and ·18 per cent. of magnesia.

Over-heating during granulation, therefore, largely increases the amount of insoluble silica; and this is due to the chlorides being decomposed, and the silica re-entering into combination with the bases present in the mineral.

3. *What are the Conditions or Modifications in which "Silica" is separated during an Analysis?*

There are—1st, The flocculent or semi-gelatinous; 2d, the hyaline granular; 3d, the white opaque granular; as seen from minerals which have been fused with alkaline carbonates. Minerals which have been decomposed by acid show only the first two.

Whatever be the reason, the "silica" obtained from minerals decomposed by acid is, after dehydration by heat, as a rule, much less soluble than that obtained from those which have been fluxed; the acid would appear to decompose by soaking out the bases while it combined with them, leaving the silica behind in the crystalloidal state; the fusion with the alkaline carbonate converts the crystalloidal silica into its colloidal and more soluble form; though an after-heating may reconvert a certain portion.

The silica obtained in an insoluble state, or rendered insoluble by unduly continued dehydration of the first and second varieties, is of about equal purity. It cannot be set down as pure silica, for I find that it generally contains traces of alumina, sometimes as much as 5·9 per cent., sometimes also traces of lime. Should it be even slightly opaque or dull white, its purity is doubtful.

Singularly enough, the most impure insoluble "silica" obtained by a proper method of granulation which I have examined* was considered suspicious from its being unusually light and slimy. It was got from a margarodite, the silica of which yielded 5·68 per cent. of the suspicious powder. This powder gave in 100 parts—silica, not yet absolutely soluble, 69·27 per cent., alumina 27·11, lime 2·01, magnesia 1·61.

Of the above noted "silicas," the first is impalpable to the touch of a glass rod; the second is crystalline gritty; the third is hard, crunching only on the application of an amount of force which might fracture the containing beaker.

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* I except, of course the cases where titanic acid is present.

This last, generally in grains nearly the size of those of sand, is usually regarded as undecomposed mineral; this it cannot possibly be, if the powdered mineral has been dusted through fine linen, and so thoroughly mixed with the flux that reagglutination has not taken place; it may, however, be some new compound formed during the fusion, especially if the latter has been overheated.

Although white and hard grains are to be regarded with suspicion, crushed, and treated longer with acid, yet they frequently prove to be little else than silica. I have separated and examined by refusion and boiling in solution of sodium carbonate this variety several times, generally finding mere traces of alumina and lime, never having got over 3·7 per cent. of alumina.

The desiccation and granulation of the mixed silica and saline mass, generally converts the flocculent silica into the granular; and the hard gritty, which should always be crushed up, in the fear of its not being pure, also into the same. Should there, after granulation, be any new appearance of a hard, white, gritty powder, there has been overheating; and recombination between the silica, alumina, and lime; a white powder, so appearing, contains much larger quantities of bases than the before-mentioned white "silica."

4. *Is the Amount of Contamination incident upon the employment of Vessels of Glass of signal importance?*

Weighings of three German glass beakers used in the one operation, namely, the dissolving up and granulating the acid solution, showed losses in weight of ·01, ·078, and ·189 grains.

If the solutions be allowed to remain alkaline for any length of time, the amount of material dissolved from the glass in common use is very material.

5·646 grains of silica were, after fusion with six times their weight of Frescenius' flux, dissolved in water, *without being acidified*, and dried twice in a glass beaker, sold as Bohemian; on re-solution there was got 6·361 grains of insoluble matter—a gain of 12·68 per cent. on the small quantity. In another similar operation, in which 4·88 grains of silica were used, the beaker lost ·246 of a grain. Two others gave respectively a loss of ·64 and ·111 grains.

Though such circumstances, or modes of operating in glass vessels, do not occur in an analysis properly conducted, yet so material is the corrosion of glass vessels during many parts of the course, that I look upon an excess of from ·2 to ·7 per cent., when about 20 grains are employed, as indicating a more correctly executed analysis than anything more nearly approaching the 100. It being, therefore, recognised that during any overheating in the desiccation of the acid solution of the fluxed mineral, the silica re-enters into union with small quantities of certain of the bases; and it being also recognised that during the long continued heating in the beakers, a portion of their substance is dissolved, mingling with the contents, it is evident that a silica originally separated pure, may, by the faults of the process adopted, be rendered impure.

We cannot hope to execute correct analyses of silicates until we employ vessels of a material other than glass.

The above results support the finding of Rammelsberg, that a high heat converts silica soluble in the test solution into an insoluble variety; but they go to show more than this.

First, That different varieties or states of silica are so converted in very different amounts; *second,* That so far as experiments go, it is the more finely-divided silica which suffers the greatest amount of change; and *thirdly,* That the insoluble powder left after boiling in the test-solution should never be confidently set down as being pure silica, or tridymite; seeing that it may contain as much as 6 per cent. of its weight of alumina: so that in every case where the analysis is intended to be founded on for the deduction of a formula, this so-called tridymite should be fluxed, and itself analysed.

But, though thus finding, with Rammelsberg, that the continuance of a high heat converts a soluble silica into an insoluble, I am unable to accept this as the explanation of the very varying amounts of insoluble silica obtained from minerals by different analysts;—an explanation altogether different is required.

It is unquestionably the fact that the more carefully every part of the process connected with the separation of the silica is performed, the greater will be the proportion of it soluble; but I find that even the same mineral, when treated more than once *in identically the same way,* may yield very different quantities of insoluble silica; the silica, therefore, must come out of the mineral in different states, from modifications of the treatment, so slight as to be unavoidable in ordinary manipulation.

The treatment to which the silicas obtained in my analyses were subjected was the following:—

After thorough drying in a filter-drying bath, the silica, removed from the filter and placed in a platinum crucible, was very gradually brought down into the flame of a Bunsen burner during 20 minutes, and was then heated in that flame for 40 minutes longer. After weighing, it was boiled for four hours in 6 cubic inches of a saturated solution of sodium sesquicarbonate; thrown on a filter, thoroughly washed, re-heated, and the insoluble portion weighed. Should there have been anything the least suspicious in its appearance or colour, either when hot or cold, it was re-fused and examined.

Of the "silicas" of 181 minerals so examined, the average percentage of the silica insoluble was 4·054. The highest quality was that of steatite, from Scalpa, in Harris, 28·48 per cent.; the lowest that from the silica of orthoclase, from Froster-Hill, in Aberdeenshire, ·027 per cent. The steatite was a very pure variety—the finest in Scotland—from a vein in serpentine. The orthoclase was from syenite,—it was neither fine nor characteristic in appearance,—rather resembling oligoclase.

In the 181 minerals, every kind of proportion of insoluble silica appears, up to 14 per cent.—very few above that.

So far for different minerals; but very different amounts appear for the same minerals from different localities.

I instance

Orthoclase—

Froster-Hill, Aberdeen; vein in syenite,	·027	per cent.	insoluble.
Glen Beg, Glenelg; from limestone,	·422	„	„
Glen Fernate, Perth; vein in mica schist,	·651	„	„
Lairg, Sutherland; syenitic granite,	1·305	„	„
Rubislaw, Aberdeen; vein in granite,	1·33	„	„
Struay, Ross; pink, layer of gneiss,	1·553	„	„
Canisp, Sutherland; from porphyry,	1·573	„	„
Struay, Ross; blue,	1·656	„	„
Tongue, Sutherland; vein in "syenitic granite,"	1·66	„	„
Loch of Leys, Aberdeen; layer of gneiss,	1·82	„	„
Cowhythe Head, Banff; vein in talc slate,	1·84	„	„
Corrybeg, Arran; from pitchstone,	3·923	„	„
Ben Capval, Harris; vein in hornblendic gneiss,	11·52	„	„
Kinkell, Fife; crystals in trap tuff,	14·557	„	„
Cowhythe Head, Banff; vein in talc slate,	18·62	„	„

being a difference almost as great as that of 1 to 690.

Saponite—

Gapol, Kincardine; green,	·061	„	„
Quirang, Skye; white,	7·83	„	„
Do., do.; yellow,	24·98	„	„

or a difference equal to that of 1 to 410.

Precious Serpentine—

Portsoy, Banff; olive-green veins,	·485	„	„
Do., do.; chocolate-brown veins,	4·16	„	„

the composition of the two being almost identical.

The two instances last cited show that different amounts are obtained from the same mineral from one and the same locality.

While, *lastly*, in examining the "silicas" of the same mineral, the analysis of which was repeated, I have on some occasions obtained differences as great as 2 to 1.

Seeing, then, that the treatment, from the first dusting of the powdered mineral through the finest linen, to the final weighing of the insoluble residue, was throughout, as far as possible in the course of continuous laboratory work, *identical,* I can only say with Mr WALLACE YOUNG that I "have not as yet been able to find out why the silicic acid should be so soluble in some cases and not so in others, when subsequent examination showed it to be equally," or nearly equally, pure in all.

A perfectly new field of research lies open here.

The Selection and Purification of Specimens for Analyses.—This is not only of the first importance, but it is all-important. It is evident that it is better to refrain altogether from analysing than to operate on material which has not been purified as far as it is possible for our appliances and our patience to effect. It is not altogether unusual for those petrologists, who within the last few years have been availing themselves of the aid lent by microscope in the examinations of sections of rocks, to disparage the labours of the analyst by the assertion that he works upon material which is visibly impure, and that his results should therefore be disregarded. Very fittingly might the chemist reply, that the knowledge of *what* is seen in the microscope is primarily the result of his labours. It is possible that the eye of the chemist may be as capable of microscopic culture as that of the geologist, and therefore he himself may be fully qualified to put the proper value on his work; while it is very probable indeed that the eye of the mineralogist has even the advantage, in the recognition of the several components of a minutely crystalline mass. Let us inquire how far the histological geologist can advance in independent isolation. The numerical range of his power of recognition of mineral constituents is very limited indeed, and uncertain at the best. Hornblende can be distinguished from augite,—not always; orthoclase from the plagioclastic felspars always,—but what then? Then the geologist has arrived but at the commencement of the examination, which it behoves him to prosecute; and the microscope is, in the present state of our knowledge, powerless to enable him to discriminate between the felspars further.

His primary objection, moreover, is a hypercriticism formed on quite mistaken grounds. The geologist examines cryptocrystalline rocks, the ingredients of which were paragenetic, or nearly so, in time. The chemist seeks out highly perfect giant crystals, which have crystallised out of these rocks through exfiltration; and which are crystals of great size and perfect form, just in virtue of their purity. These crystals are associated, it is true, with those of other substances; but they are not to any great extent incorporated with one another; just because, though paragenetic in space, they were not so in time. The geologist is dealing with a congeries of interpenetrating crystals.—the result of a rapid and nearly simultaneous crystallisation; the mineralogist and chemist with one or other of a series of superimposed epallelogenetic crystallisations; the sequential products of a long-protracted deposition and growth of matter. In the change of state assumed by the fluent or plastic rock, its atoms have rushed towards solidity. In the formation of what the Abbé HAUY called "the gems of crystallisation," the atoms, under the guidance of the crystallipolar force, have, with slow and stately march, assumed their destined positions, jostling aside only the foreign substances which all great crystals loathe.

The geologist of the present day, moreover, from the limited range of his familiarity with minerals, is incapable of estimating the quality of the chemist's work; and it so happens that it is just the few minerals with which the former interests himself which are most commonly and most largely contaminated with foreign and included matter. Felspar, with its spangles; micas, with their crystals; quartz, with its fluid cavities; garnets, with its granules; and pitchstones, with what the geologists call "hairs," which, being crystals of actynolite, are just as far removed from hairs, as an inorganic substance is from an organic.

But while it is easy to repel these criticisms, it must be admitted that this part of the chemist's work calls for an amount of patience and mineralogical discrimination not always, it is to be feared, exercised. The writer, who invariably reserves the final examination of a picked mineral as his own work, was never yet able to educate an assistant's eye so as to thoroughly accomplish the separation of quartz from oligoclase; but he trusts the perusal of the subjoined letter from a former assistant will satisfy all geologists that conscientious care had been carried to the very limits of human patience:—

EDINBURGH, *June* 31, 1876.

DEAR DR HEDDLE,—In reply to your note asking me to say how long I was engaged at the picking out of the Withamite, I have to say that it was my regular employment, when not called away for a temporary matter, from March to July, both inclusive, of Session 1873–74, and for some weeks of Session 1874-75.—I am, yours truly,

R. M. MURRAY.

In order, however, to meet as far as possible all criticism on this score, the associated minerals—the contact minerals—and the conjectured *possible* impurity, will, under each analyses, be stated. It has also to be mentioned that, wherever practicable, portions of the material selected were, in thin slices, examined under the microscope by ordinary and by polarised light.

ORTHOCLASE.

Orthoclase from intrusion Veins,—Dykes.

Dykes in Hornblendic Gneiss.—1. From the great dyke of Ben Capval (K-haipeval), Harris.

Quite a number of granite dykes are to be seen in the south end of Harris: towards the south-west these run nearly parallel to one another, cutting the strata at about right angles to their strike, and protruding from the inclosing rock like huge walls. Those of the south-east, or about Roneval, do not exhibit this parallelism, or general N.N.E. and S.S.W. direction.

The dyke of Ben Capval is markedly the largest of the series, and is the grandest granite dyke in Scotland. Nearly two miles long, of about thirty feet in thickness, and protruding with a smooth and inaccessible front in some places to nearly an equal height above the surface, it forms quite a feature in the scenery, even when viewed from a distance.

To the mineralogist this vein is interesting, as affording the largest, though perhaps hardly the finest, specimens of graphic granite to be obtained in Scotland; and also the most delicately-coloured specimens of rose quartz. Pale blue quartz, olive green muscovite, Haughtonite, magnetite, garnet, and crystalline yellowish green talc are also sparingly to be met with; and white opaque beryll is said to have been found; but this was searched for unavailingly both by Mr DUDGEON and myself for the greater part of a day. In but one spot, where the rock begins to slope to the south, a blue felspar of unusual depth of colour was very sparingly found. Its unusual colour led to its being analysed.

Colour, lavender blue. Specific gravity, 2·565; cleavage angle, 89° 50′; exhibits a structure to be afterwards noticed.

Contact mineral, quartz; visible impurity, none.

1·7105 grammes yielded—

Silica,	1·0855		
From the alumina,	·024		
	1·1095	=	64·86
Alumina,			18·469
Ferric Oxide,			·671
Magnesia,			·71
Potash,			12·98
Soda,			1·895
Water,			·501
			100·086

11 · 52 per cent. of the silica were insoluble in a boiling solution of sodium carbonate.

Possible impurity, quartz.

2. From a great dyke which protrudes from the steep shore slope facing the rock of Stromay, in the Sound of Harris.

This dyke, though of much greater thickness than the last, is not nearly so rich in minerals; it is mentioned in the 3d vol. of the "Edinburgh Philosophical Journal," as containing "moonstone"; besides this, it contains Haughtonite, and fine specimens of graphic granite, of which the felspar is, in Macculloch's words, "white, translucent, and nacreous,—acquiring, after exposure, an argentine brilliancy." It likewise contains imbedded crystalline masses of orthoclase of wondrous purity of appearance, somewhat foliated in structure.

Pellucid, almost transparent; but with fibrous-looking portions which are opaque or brilliantly lustrous, according to the direction in which they are viewed; this structure will be particularly noticed below.

Colour, gray. Lustre, somewhat vitreous, but brilliant. S. G., 2 · 574. Cleavage angle, 89° 55′.

Contact mineral, the graphic felspar. Visible impurity, none.

1 · 633 grammes yielded—

Silica,	1 · 0568		
From Alumina,	· 0103		
	1 · 0671	=	65 · 346
Alumina,			17 · 68
Ferric Oxide,			· 92
Lime,			· 68
Magnesia,			· 253
Potash,			13 · 13
Soda,			2 · 505
Water,			· 18
			100 · 694

Possible impurity, unknown.

From Dykes in Mica Slate.

3. At a turn of the road, which leads up Glen Fernate, in Perthshire, on the east side of the stream, just where a bend is made to the west about two

and a half miles from the foot of the glen, a small vein of granite cuts the strata, which consist of a somewhat granular mica schist; this schist a little further on, on the other side of the stream, becoming more dense and coherent, has been dignified by Macculloch by the name of avanturine.

The felspar of this vein is of so high a pink colour that, under the impression that it could not but be due to manganese, it was analysed.

Colour, bright pink. S.G., 2·525. Angle, 90°. Associated minerals, little quartz and mica.

Contact mineral, mica. Visible impurity, none.

1·5 grammes gave—

Silica,	·936		
From Alumina,	·023		
	·959	=	63·993
Alumina,			17·058
Ferric Oxide,			2·475
Lime,			·522
Magnesia,			·066
Potash,			14·851
Soda,			·53
Water,			·651
			100·146

·651 of the silica insoluble.

Possible impurities, quartz and mica.

There was not a trace of manganese.

From Dykes in Talcose Slate.

4. The promontory called Cowhythe Head, which lies about a mile to the east of Portsoy, in Banffshire, consists of highly-tilted beds of a talcose and micacious schistus, with occasional bands of limestone, serpentine, and "primitive greenstone,"—a compound of augite and labradorite. Dykes of granite, seven at least in number, stand among these tilted beds; they simulate beds themselves, as it is only occasionally that they are to be seen disturbing or cutting the rocks with which they are in contact.

These granitic dykes are very interesting, from the fact that, though following close upon one another, they are yet of very different constitution.

That which is first met with on the west is a pale granite, containing much white crystallised quartz,—which is itself unusual,—a nearly colourless felspar, rarely tourmaline and *graphic garnet*, and a mica so colourless and lustrous that this locally receives in the district the name of the "sheepy silver vein." Within a few yards of this, there follows a vein with a felspar seldom surpassed in the fiery redness of its colour in Scotland; this vein has a rough scorified appearance. There is in it little quartz, and almost no mica.

Just at the turn of the promontory two closely-contiguous veins show themselves; and these interfere somewhat more markedly with the bedding of the strata. The felspar of both of these is somewhat similar in appearance; but the association of minerals is so very different that the felspar of both was analysed.

At no great distance to the east of these, the well-known graphic granite—correctly *pegmatite*—vein is to be seen protruding in a line of bosses from the turf, and finally thrusting itself in a terminal bluff out of the grass-clad bank. The structure of this vein is so peculiar, and the development of the graphic lettering so well marked as to call for a more extended notice than can be here given.

In the uncovered rocky beach immediately to the east of the graphic vein, another, formed like a tesselated pavement, succeeds. The whole mass of this vein consists of numberless brick-shaped blocks of felspar, disposed upon their narrow sides. These blocks are so arranged as to form sockets for one another, from which they can easily be removed; they are not rude crystals, their fracture exhibiting quite an unique appearance. A kind of rough twinning of crystals, in which there is a parallelism in their axes, shows, by the oscillation of the reflected light, like a dovetailing in carpentery, or still liker to the interlacing of the fingers in clasped hands. Some twenty yards still further west the last of the veins is to be seen, running, however, as it skirts the sea, in quite a different direction from the others. This vein is of massive felspar, which is everywhere studded with fine plumose crystallisations of muscovite.

Probably at no other spot in Scotland can so many varieties of granitic veins be seen.

From only two of these, however, could felspar pure enough for analysis be got—namely, the third and fourth,—those protruding from the rocks immediately at the point.

The third vein is quite a mineral casket, the felspar being plentifully studded throughout with finely radiated crystals of black tourmaline,—lustrous muscovite,—occasionally crystallised green apatite,—large crystals of pale red garnet with as perfect a graphic structure as that seen in granite,—and either epidote, or green tourmaline in minute threads.

The orthoclase of this vein is lustrous, of a rich flesh red, and a specific gravity of 2·561. Contact mineral, mica. Visible impurity, none.

25 grains yielded—

Silica,	64·745
Alumina,	18·3
Ferric Oxide,	1·99
Magnesia,	·038
Lime,	·975
Potash,	9·873
Soda,	3·336
Water,	·173
Fluorine,	tr.
	99·43

Insoluble silica, 1·842 per cent. Possible impurity, none.

5. The fourth vein consists almost solely of felspar; a few imbedded garnets, and an occasional imbedded crystal of a bright red felspar (?) being alone visibly associated with it.

The felspar is of very pure appearance, of pale flesh colour and watery lustre. S. G., 2·559. Cleavage angle, 89° 40′.

25·57 grains yielded—

Silica,	16·57		
From Alumina,	·307		
	16·877	=	66·003
Alumina,			18·302
Ferric Oxide,			2·035
Lime,			·997
Potash,			10·018
Soda,			3·191
Water,			·165
			100·713

18·62 per cent. of the silica insoluble; possible impurity, the bright red felspar.

This analysis of perhaps the purest orthoclase I have examined, so far as mere appearance goes, departs more from the formulaic requirements, as regards the amount of silica and soda, than any other. An admixture with quartz would have partly explained this departure, but there was present no visible quartz. It may be microline.

I cannot even conjecture what the imbedded bright red crystals were—they had apparently the lustrous cleavages of a felspar.

Excepting this difference of one and a quarter per cent. in the silica, these two felspars are of nearly identical composition; the one vein carried little else than felspathic matter; the felspar of the other crystallised out of a melange of the constituents of tourmaline, apatite, and mica; or to put itmore correctly, as these minerals were imbedded in the felspar, it was the residue of their separation in the solid form.

Orthoclase of Porphyritic Dykes.

6. A great dyke of red porphyry crosses the Don, in Aberdeenshire, about half a mile north of Monnymusk, also the railway cutting to the west of Tillyfourie, shows itself in the pass above Greenfolds, protrudes as a huge rampart for nearly two miles in the slack between the Greenhill and Corrennyhill, curves round at Upper Broomhill and Nether Dalgie so as to cross the Deeside railway near Balnacraig, and the Dee at Potarch Bridge,—shows itself here and again, according to information furnished me by Nicol, near the Muckle Ord road,—stands boldly up like a wall in the slack of Glen Dye, forms the summit of Mount Shade, and crosses the Birnie-slack burn, not as a single great band, but as several minor parallel branches.

Throughout the whole of this extended reach, the granular felspathic base contains large imbedded crystals of orthoclase of dull lustre and a light fawn colour. So far may this great dyke be traced almost continuously; speculation connects it northward and eastward at Buchanness to the Boddam black porphyry, and southward with the dyke on Herscha Hill, near Auchenblae; while that, in turn, may be supposed to be continued in the dyke on the south side of the Bervie Water, between Arbuthnot and its mouth.

The crystal chosen for analysis was one taken from the vein near the limestone quarry at Clattering Briggs; it was about two inches by one-half inch in dimensions; no pure portion sufficiently large for ascertaining the specific gravity could be got.

·809 grammes gave—

Silica,	·516	
From Alumina,	·002	
	·518	= 64·029
Alumina,		19·167
Ferric Oxide,		·301
Manganese Protoxide,		·223
Magnesia,		·938
Lime,		1·384
Potash,		11·839
Soda,		1·373
Water,		·574
		99·828

Probable impurity, some of the less pure felspathic base; here doubtless somewhat more calcareous than normal; possibly also quartz.

From exfiltration Veins in Granite.

7. From the great vein in Rubislaw quarry, near Aberdeen. This vein being now quite covered, and my attention not being directed to the subject when I last saw it uncovered—now many years ago—I cannot affirm that it is an exfiltration vein; it certainly was not like those of that class at present to be seen in the quarry.

Twins of orthoclase of large dimensions are now and again to be seen at Rubislaw; several years ago I measured one eight and a half inches in length, from the above vein; and in the summer of 1876 one was measured by Professor Nicol and myself which was eight inches in width.

These crystals are lustrous on their cleavage faces, of a reddish flesh colour, and in parts translucent. They exhibit at times the peculiarity of structure already mentioned, which here at first sight somewhat resembles the striation of repeated twinning.

In addition to these giant crystals of felspar, the old vein of Rubislaw afforded as associated minerals, fine specimens of tourmaline, garnet, beryll (Davidsonite), apatite, and muscovite,—all imbedded in a paste of quartz.

There are also present large crystals of a black, slightly biaxial mica, which differs from lepidomelane in having almost all the iron present in the state of *protoxide,*—as in the magnesian Biotite. The description and analysis of this new mica will be afterwards submitted, under the name of *Haughtonite.*

A pure fragment of the orthoclase, obtained and analysed in 1856, with cleavage angle 89° 58′, and S. G. 2·554, gave—

Silica,	64·54
Alumina,	18·36
Ferric Oxide,	·32
Magnesia,	·09
Lime,	·36
Potash,	13·05
Soda,	2·58
Water,	·086
	99·386

1·33 per cent. of the silica insoluble. Possible impurity, quartz or muscovite.

From exfiltration Veins in "Syenitic Granite."

8. From a vein in the wood on Cnoc dubh, about a mile east of Lairg, Sutherland.

This granite consists of little quartz, much white felspar (? oligoclase), much hexagonal black mica (Biotite ? lepidomelane), no muscovite, rarely sphene, and very rarely any hornblende. The propriety of the term *syenitic* is therefore very questionable.

The vein from which the orthoclase was taken contains also little quartz, crystals of orthoclase and oligoclase in about equal quantity,—the crystals of each being about two inches in length and breadth,—large crystals of green Haughtonite, the largest crystals of Allanite yet got in Scotland, and crystals of sphene, of sometimes half an inch in size, which much resemble Keilhauite in colour and lustre.

There was also in cracks a minute quantity of a purple substance, much resembling yttrocerite, but which more probably was fluor: it did not lose its colour when heated.

The orthoclase was of a pale pinkish tint; its cleavage angle, 89° 59′; its S. G. 2·555.

25 grains yielded—

Silica,	14·898		
From Alumina,	·756		
	15·654	=	62·616
Alumina,			19·634
Ferric Oxide,			·064
Magnesia,			·636
Lime,			·604
Potash,			13·72
Soda,			2·92
Water,			·13
			100·324

1·305 per cent. of the silica insoluble. Possible impurity, oligoclase.

9. Amazonstone from a vein in the syenitic granite of the north of Sutherland. This white granite is very similar to that at Lairg; as it contains very few sphenes, more quartz, and little or no hornblende, it has still less claim to be called syenitic.

I am indebted to Dr Joass of Golspie and Professor Nicol for the fragments of this very rare variety of orthoclase which were analysed. Fragments and a few crystals of it were sent, as "a pretty green stone from a boulder," to Dr Joass, and deposited by him in the museum of the Duke of Sutherland at Dunrobin. To the courtesy of that nobleman the author is indebted for the fine crystal, a plate of which accompanies this paper.

This boulder, doubtless derived from the adjacent mass of Ben Laoghal, lay upon the west slope of Ben Bhreck, Tongue. Having been broken up by the author for the Duke of Sutherland in the interests of science, it proved to be quite a mineral casket. Specimens of amazonstone of unparalleled magnificence were profusely distributed among an assemblage of minerals occurring, within the same space, certainly nowhere else in Scotland.

The boulder may have weighed somewhere about 100 tons: the vein was about two feet in width. The accompanying minerals, arranged as far as possible in the order of their occurrence from without inwards in the vein, were:—Babbingtonite—which is found in one other locality only in Scotland,—purple fluor, sphene, Allanite, zircon, orangite, passing into thorite—found nowhere else in Scotland,—magnetite, lepidomelane, radiated Cleavlandite, ilmenite, glauconite (?),* quartz, specular iron, and crystals of a new radiated hydro-carbonate of lime. The amazonstone was inferior to the four latter.

Of the numerous forms in which the amazonstone crystallised, certain are depicted in Plate XVIII.;—two crystals, which were unterminated prisms, and which were unavoidably broken in the extraction, had the following very unusual dimensions:—

	Length along *b* and *c*.	Breadth over *b*.	Breadth over *c*.
1st Crystal, . .	15½ inches,	10 inches,	8 inches.
2d „ . .	12½ „	8 „	6 „

A magnificent museum specimen, now in the possession of the Duke of SUTHERLAND, shows, on a surface of some three square feet, eight perfect and some half a dozen imperfect crystals of amazonstone, of the size of the fist.

This amazonstone, frequently in hemitrope crystals, is pervaded throughout with a white material forming a corded structure, which will be further specially noticed.

The finest of these amazonstones are of a beryl green colour, and translucent. They have a cleavage angle of 89° 43′, and a specific gravity of 2·569. The contact minerals were quartz and lepidomelane; but the specimen picked seemed quite pure from all but the whiter layers.

1·313 grammes yielded—

Silica, ...	·81	
From Alumina, . .	·033	
	·843	= 64·204
Alumina, . . .		18·395
Ferric Oxide, . .		·455
Protoxide of Manganese,		·152
Lime, ...		·725
Magnesia, . . .		·076
Potash, . . .		12·752
Soda, ...		2·952
Water, ...		·512
		100·223

Insoluble silica, 1·66 per cent.

Copper, chromium, and nickel were specially sought for, but no trace found. Nor was there a trace of Ferrous Oxide.

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* Or Saponite or Strigovite;—not yet analysed.

From Syenite.

10. At the locality—about half a mile south of Froster Hill, near Old Meldrum,—whence this orthoclase was taken, the rock is as true and as simple a syenite as any I have seen.

It consists sometimes solely of a pure, opaque, almost lustreless orthoclase, acting as a paste to imbedded crystals of hornblende, of about half an inch in size, and of a deep-green colour. Occasionally a lighter green somewhat fibrous hornblende is added, very rarely a plate of a bronzy mica (? Biotite), a few brilliant minute brown sphenes, and small flat plates of blue-black titanic iron. Being in plates, they are probably not magnetite. There is no visible quartz.

The bit of orthoclase analysed was of unusual size. It was quite crystalline, in broad cleavages, with a somewhat fatty lustre; it was thought to be oligoclase.

Cream-coloured, tough, cleavage angle 89° 58′; specific gravity, 2·548.

1·499 grammes gave—

Silica,	·939	
From Alumina	·020	
	·959	= 63·308
Alumina,		18·166
Ferric Oxide,		·846
Lime,		1·046
Potash,		13·272
Soda,		2·060
Water,		·808
		99·506

Insoluble silica, ·027 per cent.; possible impurity, hornblende.

From Micaceous Gneiss.

11. The specimens I have analysed from the above rock were taken from what are regarded by me as being the largely amplified felspathic layers of the gneiss; they are so regarded because I have observed a gradual augmentation of these layers, from quite a thin and crypto-crystalline band lodged between the dark mica layers, to bands, or what might even be called beds of several feet in thickness;—their thickness, however, is ever varying; they never cut the micaceous layers, and the contortions and foldings of the rock are common to both.

They are to be regarded, I believe, as the results—the incipient results—of an exalted metamorphism;—a metamorphism which has accomplished the segregation, and probably the fusion of the felspathic material; but fallen short of affecting the more intractible mica.

When the fluent felspar, sapping into the darker layers, loosens the continuity of the granular quartz and flakey mica,—incorporating them in its mass,—the magma, upon resolidification, would assume the form of a granitic breccia, or a granite, according to whether the felspar, in its action as a solvent, failed or succeeded in accomplishing the solution of the other two minerals.

A friend, however, of far greater petrological experience than myself, and much more extended knowledge of the district in which they occur,—one, moreover, to whose opinion I pay so much deference, that I am almost afraid to differ from him, holds these layers to be granitic veins. I have indeed been shown localities where hand specimens of these layers could not be called anything but granite; but where, in my opinion, the rock as a whole was as unmistakably gneiss. These localities are however, I believe, all in the neighbourhood of the granite; and, though it may not be possible to show it everywhere, there are localities where,—the metamorphism affecting the rock as a whole,—the gneissose structure may be seen gradually to fade away into the gray granite of Aberdeenshire; so that no two authorities would agree where the one rock ceases and the other commences.

And if the nature of the rock here is puzzling, so are the specimens of the felspar difficult to recognise.

The first specimen analysed was handed to me by Professor Nicol as oligoclase, from Banchory; and this it was considered by myself also to be, till the cleavage angle was measured. I obtained specimens of it myself, about a mile north of Blirydrine, in Kincardineshire, where it is associated with pale green apatite and brown Haughtonite, which here weathers green. The orthoclase itself in this district is so largely graphically charged with quartz, that it is but seldom that specimens can be sufficiently separated from it for analysis. Throughout the whole of Deeside, especially in localities adjacent to limestone, the felspar is of the same type.

Colour, that of skim milk; weathers buff; lustre, somewhat fatty; cleavage, 89° 41′; specific gravity, 2·551.

25 grains gave—

Silica,	63·59
Alumina,	19·582
Ferric Oxide,	1·088
Lime,	·682
Magnesia,	·08
Potash,	12·53
Soda,	2·76
Water,	·421
	100·733

Possible impurity, quartz.

12. From large masses,—probably from Brathans,—which were found loose in a mole which crosses the Loch of Leys, near Banchory. In large crystals of a white colour, quite fresh, but very opaque and lustreless, associated with Haughtonite; specific gravity, 2·542.

On 25 grains—

Silica,	15·245	
From Alumina,	·5325	
	15·7775	= 63·11
Alumina,		18·98
Ferric Oxide,		·982
Lime,		·88
Magnesia,		·569
Potash,		13·06
Soda,		2·34
Water,		·34
		100·261

Insoluble silica, 1·82 per cent. Possible impurity, quartz.

13 and 14. From a tilted granitic band in gneiss, exposed in a quarry about half a mile south of Struay Bridge Inn, Ross-shire. This band much simulates a dyke, but it follows obediently the contortions of the gneissose layers. The associated minerals are a brilliant pale olive-green muscovite, tourmaline, and garnet of a lively red tint, in a paste of hyaline quartz.

The orthoclase occurs of two very different appearances.

That which arrested attention, and led to the analysis of both, is of a saccharoid or large granular structure. The crystalline facetts, lying in every direction, with a high lustre, give it a spangling appearance, like statuary marble; and the colour being precisely that of the pink variety of petalite, it was taken for that mineral. It was associated with, sometimes imbedded in, a lavender-blue, cleavable, somewhat dull variety, which occurred in masses of considerable size.

As the analysis of the pink variety showed it to be orthoclase, the blue was likewise examined,—it being hardly conceivable that one and the same substance could assume, in juxtaposition, two such diverse structures and colours.

The Pink.		The Blue.	
S. G.	2·543	S. G.	2·545
On 1·3 grammes—		On 1·304 grammes—	
Silica,	·841	Silica,	·84
From Alumina,	·004	From Alumina,	·007
	·845		·847

Silica,	65·	64·187
Alumina,	17·033	17·393
Ferric Oxide,	1·428	1·203
Manganous Oxide,	·692	·461
Lime,	·732	·687
Potash,	13·823	13·311
Soda,	1·003	1·96
Lithia,	tr.	...
Water,	·501	·557
	100·211	99·750
Loss in Bath,	·167 p.c.	·158 p.c.
Insoluble Silica,	1·656 p.c.	1·553 p.c.

Possibly the manganese is the cause of the colour of the pink: candour compels the confession that the chemistry of the present day fails in the detection of the colouring ingredients of the felspars. In the greater number of my analyses of orthoclase the state of the oxidation of the iron was determined, and, as it invariably proved to exist as ferric oxide, it was also set down as such in the few in which no special examination was made.

From primitive Limestone,—"Necronite."

15. The specimen examined was from a mass which lay between granular limestone and its associated augitic belt, on the hill slope above the farm-town of Balvraid, Glen Beg, Glenelg.

Besides limestone, the mass contained Biotite, a hydrated labradorite of a fibrous structure, and a new mineral species—to be called *Balvraidite.*

The orthoclase was very peculiar—it was considered oligoclase. Its colour was dove-blue, its lustre somewhat pearly; it was in cleavable masses; its S. G. 2·558.

1·499 grammes gave—

Silica,	·929		
From Alumina,	·016		
	·945	=	63·042
Alumina,			19·312
Lime,			·971
Magnesia,			·213
Potash,			14·63
Soda,			1·016
Water,			·56
			99·744

Insoluble silica, ·422 per cent.; possible impurity, labradorite.

In my experience such an occurrence and association of orthoclase is unique in Scotland; the specimens were somewhat lighter in colour and higher in lustre than a specimen of the original necronite which I possess from Baltimore.

From Pitchstone Porphyry.

16. The "glassy felspar," from Corriegills shore, Arran: colourless to slightly white translucent crystals of about a third of an inch in length, imbedded promiscuously throughout a paste of blue-black glassy pitchstone. Colourless,—seldom light amber-brown. These crystals were so flawed and brittle that neither cleavage angle nor specific gravity could be taken.

1·792 grammes yielded—

Silica,	1·182		
From Alumina,	·016		
	1·198	=	66·852
Aluminia,			17·241
Ferric Oxide,			·42
Lime,			1·218
Magnesia,			·055
Potash,			9·203
Soda,			4·316
Water,			·864
			100·169

Insoluble silica. 3·903 per cent.; possible impurity, pitchstone.

From interbedded Porphyry.

17. The quartzite which caps the peaked summit of Canisp, in Sutherland, contains great beds of porphyry, highly characteristic from the rapidity of their decomposition, and the spongy clay which is the result of that decomposition.

Banks of this clay spot the south-eastern slope, just below the summit; and from out of this clay, brick-red, well-formed crystals of orthoclase may be picked in numbers, along with less frequent and smaller crystals of ochre-yellow to cream-coloured oligoclase. The crystals of orthoclase are of all sizes, from that of a pea to one inch. Their specific gravity is 2·245.

1·3 grammes yielded—

Silica,	·825		
From Alumina,	·001		
	·826	=	63·538
Alumina,			17·363
Ferric Oxide,			1·867
Manganous Oxide,			·384
Lime,			1·335
Potash,			12·932
Soda,			1·695
Water,			1·123
			100·237

Loss in bath, ·395 p. c. Insoluble silica, 1·573 p. c.; possible impurity, oligoclase.

From Trap Tufa.

18. Large twin crystals of glassy felspar occur somewhat rarely imbedded in the tuff of Kinkell, near St Andrews. These crystals are sometimes about two inches in size, generally about one inch. They are invariably twins, and also invariably much worn, rounded, and smoothed on the angles, preserving only rudely the form of crystals. They lie in the loose friable tuff in no special direction, and in no way in connection with either drusy cavity, exfiltration vein, or with any other mineral. The wearing is neither like that of water or sand friction; they have more the appearance of a portion of their substance having been dissolved away. They are quite unaltered, fresh-looking, of a brilliant lustre, transparent, but sometimes much fissured in the interior.

To the writer the position of these crystals is an enigma. Colour, dull-yellow brown. Cleavage angle, 89° 50′; specific gravity, 2·609.

1 · 503 grammes yielded—

Silica, ...	63 · 073
Alumina, ...	18 · 686
Ferric Oxide, . . .	2 · 47
Oxide of Manganese, .	· 059
Lime, ...	2 · 198
Potash, ...	6 · 619
Soda, ...	5 · 503
Water, ...	1 · 388
	99 · 996

14 · 557 per cent. of the silica insoluble; possible impurity, the matrix.

The structure of some of these crystals is of a very extraordinary nature, and will, in a future chapter, be described.

In the somewhat less friable tuff of the Kincraig, near Elie, and of two dykes to the east of that town, a felspar identical in every way in appearance is found.

Both 16 and 18 are sanidine.

ORTHOCLASE.

	Colour.	Cleavage Angle.	Specific Gravity.	$\ddot{Si}$.	$Al_2\dddot{O}$.	$Fe_2\dddot{O}$.	$\dot{Mn}$.	$\dot{Mg}$.	$\dot{Ca}$.	$\dot{K}_2$.	$\dot{Na}_2$.	$\dot{H}_2$.	Total.
Ben Capval, . .	Blue	89° 50′	2·565	64·86	18·47	·67	...	·71	...	12·98	1·89	·5	100·09
Stromay, Harris, .	Grey	89 55	2·574	65·35	17·68	·92	...	·25	·68	13·13	2·5	·18	100·69
Glen Fernate, .	Pink	90	2·525	63·99	17·06	2·47	...	·07	·52	14·85	·53	·65	100·17
Cowhythe, 3d vein,	Flesh	...	2·561	64·74	18·3	1·99	...	·04	·97	9·87	3·34	·17	99·43
Do. 4th vein,	Do.	89 40	2·559	66·0	18·3	2·03	...	...	1·	10·02	3·19	·16	100·71
Clattering Briggs, .	Fawn	...	...	64·03	19·17	·3	·22	·94	1·4	11·84	1·37	·57	99·83
Rubislaw, . .	Flesh	89 58	2·554	64·54	18·36	·32	...	·09	.36	13·05	2·58	·09	99·39
Lairg, . . .	Buff	89 59	2·555	62·62	19·63	·06	...	·64	·6	13·72	2·92	·13	100·32
Tongue, . .	Green	89 43	2·569	64·2	18·39	·45	·15	·07	·72	12·75	2·95	·51	100·22
Froster Hill, . .	White	89 58	2·548	63·31	18·17	·87	...	...	1·07	13·27	2·06	·81	99·51
Blirydrine, . .	White	89 41	2·551	63·59	19·58	1·09	...	·08	·68	12·53	2·76	·42	100·73
Banchory, . .	White	90	2·554	63·11	18·98	·98	...	·57	·88	13·06	2·34	·34	100·26
Balvraid, . .	Blue	89 58	2·558	63·04	19·31	...	...	·21	·97	14·63	1·02	·56	99·74
Struay, Ross, .	Pink	...	2·543	65·	17·03	1·43	·69	...	·73	13·82	1·	·50	100·20
Do. Do. .	Blue	89 50	2·545	64·19	17·39	1·2	·46	...	·69	13·31	1·96	·56	99·76
Canisp, Sutherland,	Brick	...	2·545	63·54	17·36	1·87	·38	...	1·33	12·93	1·69	1·12	100·24
Ratio of Soda to Potash ⅔ to 1; to 2 to 1, Sanidines.													
Corriegills, .	Colourless	...	...	66·85	17·24	·42	...	·06	1·22	9·2	4·32	·86	100·17
Kinkell, .	Yellow	89 50	2·609	63·07	18·69	2·47	·06	...	2·2	6·62	5·5	1·39	99·99

The peculiarity of structure, more than once noticed above, I shall now describe. I first observed it, many years ago, in the Rubislaw crystals; and now find it to be very generally, if not always, to be seen in the felspar of veins or dykes, especially of exfiltration veins.

When viewed only on the surface of an opaque crystal, the structure resembles, in its somewhat interrupted markings, a graphic delineation of the distant waves of the sea; or the light reflected from a polished piece of fine-grained ivory, cut parallel to the length of the tusk. When viewed in a transparent crystal through a lens, it resembles the fibres, great and small, of a partially untwisted cord, spread loosely and more or less uniformly across the crystal—the direction being invariably parallel to the face *a* of BROOKE and MILLER (ii. of DANA).

Professor NICOL of Aberdeen, who quite independently observed the same thing in the crystals of amazonstone lately obtained in Sutherland, writes me (12th Nov. 1875) as follows:—"The most curious felspar I have seen is one from a boulder near Ribigill. It is a macle like those in p. iii. of my 'Elements,' but broken at the ends; it is about 3 inches long by 2 broad. It consists of red and green layers running across the cleavage face, and down the side of M, approximately parallel to its edge, and at right angles to the plane of cleavage, almost like fibres of wood or muscle. The specimen, I understood, was from a druse in a boulder broken up when clearing some fields. The brown and green are not in twins, but mixed in irregular plates—the cleavage runs right across both. The layers look like fat and flesh fibres in a piece of well-mixed beef. These fibres are not quite continuous, but more or less interrupted or broken. They are thicker near the outside.

"You call it amazonstone, but I think it is rather orthoclase, *mixed* with the green mineral. This is more gem-like and less altered than the red. I am inclined to think also harder."

Up to the time of receiving Professor NICOL's letter I was uncertain whether these markings were to be regarded as due to a peculiarity of structure, or to an actual difference in material; though I inclined to the latter belief from having observed the fact, noted above by Professor NICOL, that the fibrous-like substance invariably weathered faster than the general mass of the crystal. The opinion of so acute an observer as Professor NICOL strengthened me in this belief; but it was not until I had analysed many such felspars, and more minutely examined the structure, that I definitely came to the conclusion that NICOL's view as to their being *actually two chemically different substances conjoined in one crystal* was the correct one.

The observations I have made, so far as yet carried out, are the following:—

1. *The Structural Appearances.*

These fibres and fibrillæ invariably differ in colour, lustre, and transparency from the general substance of the crystal through which they are spread. When viewed perpendicularly to the face *c*,—that of most perfect cleavage,—they frequently appear opaque, white, or brownish red,—always lighter in colour than the general mass of the crystal; but this opacity is, except in the case of weathered crystals, the result of the manner in which the light is reflected from them.

Placing the crystal as usual, with the macrodiagonal transverse to the observer, and reflecting light from the general surface of the face *c*, the striæ pass as dull or opaque white interrupted lines from the observer, the face itself frequently exhibiting a faint lineation between the fibres, parallel to the macrodiagonal; but on revolving the crystal upon the diagonal about 4° either to or from the observer, the corded structure starts out with a nacreous glimmer, somewhat similar to that in sonnenstein; the edge between the two reflecting faces lies parallel to the macrodiagonal of the large crystal, and the light appears to the lens to be reflected from innumerable interrupted cleavages.

When the somewhat imperfect *a* cleavage is obtained, a less interrupted and somewhat more brilliant flash is thrown from spots upon its surface;—careful observation being required before the impression is done away with that the light is reflected from thin plates of talc.

In the more brilliantly green-tinted and transparent amazonstones, the colour of these so-called fibres is usually of an opaque white; in some duller crystals they are of exactly the tint of dead muscle.

2. *Is the Material of the duller portions different from that of the general Mass?*

In no specimen that I have seen is it within the bounds of possibility to separate the one from the other, so as to determine this by actual analysis; but it may reasonably be concluded that they are different from (1st) the higher lustre of the filamentous portions when unweathered; (2nd) the greater readinesss with which these portions do weather; (3rd) their greater opacity when weathered; (4th) their inferior hardness,—a soft knife drawn across both does not scratch the general mass, but only the fibrous portions.

3. *What proportion of the Crystal does this intruded or extruded material bear to the ordinary Orthoclastic substance?*

The habit of taking the specific gravity of all minerals examined led to my being able to form a fairly approximate estimate of this.

Many determinations of specific gravities of orthoclase from diverse localities give an average of 2·555.

A crystal of the variety called Murchisonite, obtained at Loch Ransa, in

Arran, gave the exceptionally low gravity of 2·3;—the weight of the crystal was 163·3 grains in air; 92·3 in water:

$$163\cdot3 - 92\cdot3 = 71; \frac{71}{163\cdot3} = 2\cdot3.$$

It was boiled, when it gave out much air, in lines transverse to the face *c*; it was now cooled under water, dried with blotting-paper, and re-weighed in air, and in water. It now weighed in air 169·5 grains, and in water 98·3 grains,—the buoyant effect of the air previously in the pores being done away with.

$163\cdot3 - 98\cdot3 = 65: \frac{65}{163\cdot3} = 2\cdot512$ is therefore the true specific gravity of the solid matter of this crystal.

But the weight of the crystal in air with its pores filled with air, was 163·3 grains; and with its pores filled with water, was 169·5 grains; the weight of a bulk of water = its pores is therefore 6·2 grains; and the substance having a specific gravity of 2·512, the bulk of its pores in its own material would weigh $6\cdot2 \times 2\cdot512 = 15\cdot57$ grains. This, added to its original weight, so as to get the weight which the crystal would have been if solid, gives $163\cdot3 + 15\cdot57 = 178\cdot87$ grains;—the weight of the bulk of the pores was, 15·57,—and $178\cdot87 \div 15\cdot57 = 114\cdot78$.

So that these pores are about one-eleventh and a half, $\frac{1}{11\cdot5}$, of the total bulk of the solid.

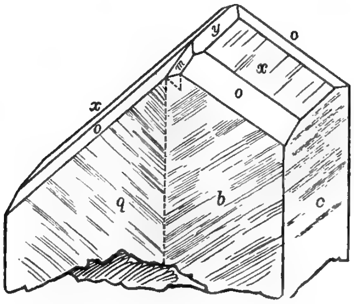
The relation between these pores in Murchisonite and the subject under consideration is now to be shown.

Murchisonite is chemically an orthoclase, but it is characterised by an extra cleavage—not seen, so far as I know, in Scotch specimens,—and by a peculiar pearly glimmer, when viewed in certain positions. An examination into the cause of this optical peculiarity seemed to show that it was due to a structure identical with that above described as occurring in the amazonstone,—only that vacuities took the place of the material constituting the fibrous net-work.

The crystal I was examining, however, being a complex twin of eight individuals, was ill adapted for the display of internal structure; and I was fortunate in having in my hands a simple and well-developed crystal which Mr Dudgeon had sent me to figure.

I was not, of course, able to get a section of it, but an examination by the lens at once and unmistakably showed that the characteristic lustre was due to internal reflection from a multitude of flattened microscopic pores, arranged accordant with the axis of the crystal, parallel to the face *a*.

So far as the power of the lens went, the pores seemed similar to those of hypersthene (Paulite).



Hill of Fare—hemitrope.

The structure, therefore, is identical with that under consideration, only that in Murchisonite it is porous, or empty.

A crystal lent me by Professor NICOL actually showed this identity. This crystal was a hemitrope of the form depicted; it was got in a quarry at Upper Craigton, Hill of Fare, Aberdeenshire.

The general colour of this hemitrope is a reddish brown; the faces *b*, which are smooth and somewhat transparent, show the fibrous structure shining up from the depths, on account of the relative lightness of its colour. The face *c* again shows a lineation through deficiency of substance; consisting of a transverse series of pitted markings, which are the openings of the pores.

A point of considerable interest in this crystal is that the portion covered by the face *m* is either of a purer, or of a different nature from that of the general mass. It forms a small wedge of nearly colourless and transparent material *let in* to the general substance.

The weight of this crystal in air was 48·28 grains; in water, 27·12.

$$48{\cdot}28-27{\cdot}12=21{\cdot}16,$$
$$48{\cdot}28\div21{\cdot}16=\ 2{\cdot}282,$$

being the specific gravity with pores *air-filled.*

Boiled in water, air escaped from the pitted markings; after cooling in water and drying it weighed 50·13 grains in air, and 29·13 in water.

$$48{\cdot}28-29{\cdot}13=19{\cdot}15,$$
$$48{\cdot}28\div19{\cdot}15=\ 2{\cdot}521,$$

which is thus the true specific gravity.

But the water weight of the pores is equal to the difference between 48·28,—its weight with these pores air-filled, and 50·13,—its weight with pores water-filled = 1·85.

And $1{\cdot}85\times2{\cdot}521=4{\cdot}66$,—the weight of the bulk of the pores if they were solid.

This 4·66 added to the original weight of the crystal, 48·28, gives 52·94 as what would have been the weight of the crystal if it had been solid; and $52{\cdot}94\div4{\cdot}66$ gives 11·36.

So that in this crystal the amount of the foreign matter now represented by pores is about one-eleventh and a third, $\frac{1}{11 \cdot 3}$, of the whole crystal. A remarkable coincidence with the proportions in the Arran crystal.

I find that many crystals of orthoclase exhibit these vacuities in their substance, exactly corresponding in position to that of the structure which has been described. Crystals of adularia from St Gothard are frequently hatched, and cross-hatched in twins, through the lineation produced by this deficiency of substance. I possess a crystal which is little better than a skeleton from this cause.

4. *What is the Material of which this knitted Structure is composed?*

It would not, as before mentioned, be practicable to separate the material from the orthoclastic mass: by a consideration, however, of the composition of those felspars which exhibit the structure, a conclusion, which it would not be unreasonable to consider as closely approximative to the truth, may be drawn.

These analyses show so slight a departure from the average analyses of orthoclase, that it is evident that it cannot be a substance very far removed in composition. The only decidedly noticeable differences are an excess of soda over that normal to orthoclase, and the introduction of some lime.

The first point here to be noted is that as the amount of silica is quite normal, and as the lineation is the softer part of the structure of the crystal; it cannot consist of quartz: had it done so its occurrence in the position occupied would not have been altogether inexplicable, for then it would have consisted of the excess of silica over and above that necessary to saturate the bases in the formation of felspar; which excess indeed the quartz in granite veins must probably all be regarded as being. Such an excess of silica, *and so separating*, forms in the graphic felspar of Ben Capval and Stromay, a structure wondrously similar to that under consideration.

The sonnenstein appearance of the layers gives rise at once to a suspicion of their being composed of some species of felspar; but how a felspathic material should, in thus being extruded from another of similar nature, form so strange a structure, instead of regular crystals, is the problem to be solved.

Now, seeing that other felspars are paragenetic with orthoclase in veins, that these are softer than it, and, all but albite, more readily decomposed, it is very probable that the material consists of one or other of these.

There are, however, for lithological relationships afterwards to be pointed out, only two that it can well be—albite and oligoclase—and the evidence appears to go entirely in favour of its being the latter.*

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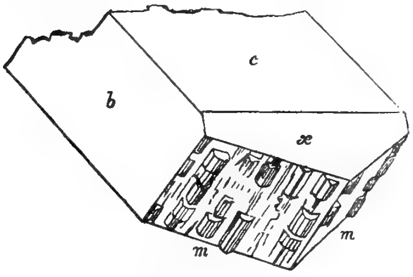
* Were it not so, we would have to regard these as being all *microlines;* but DESCLOISEAUX, (*Comptes rendus*, April 17, 1876,) holds these to consist of a mixture of orthoclase, microlin, and

An admixture of it with the orthoclase would introduce both soda and lime,—albite would only bring in the former, and then only at the cost of simultaneously increasing the quantity of the silica,—which oligoclase would not do.



Murdoch's Cairn.

Again, albite is exceedingly rare in Scotland in granite. I only know it at Stirling Hill and Murdoch's Cairn quarries near Peterhead, and in smaller quantity in the quarry of Craigton, Hill of Fare; and at these localities it does not occur in veins, but in druses in the granitic mass.

Albite, moreover, it is well known, weathers less rapidly than orthoclase; but the material of these laminæ, or whatever they be termed, weathers decidedly more rapidly.

And lastly, albite is not paragenetic in time with orthoclase—the latter is always proterogenetic to albite, the crystals of which are generally disposed on the top of the former;—indeed in Murdoch's Cairn the albite is evidently a product of a change in the orthoclase; being always disposed on apparently corroded *m* faces of the latter, with their axes accordant. (See figure.)

Oligoclase, on the other hand, is in all respects paragenetic with orthoclase,—at Lairg they are imbedded side by side in quartz; at Sclatty, Rubislaw, Craigiebuckler, &c., they are mutually assertive, and mutually interpenetrating.

At the very locality in question, however, though ordinary albite does not occur, yet the sheafy Cleavlandite variety is somewhat rarely found; but, singularly enough, it is here markedly proterogenetic to the amazonstone,—forming almost a "basement mineral" to the crystals of the latter.

It has to be stated, moreover,—whether it be an argument for or against the above view I know not—that at certain of the localities oligoclase does not occur otherwise—that is in separate crystals,—indeed that is the case in those localities where the structure is best seen. The same thing, however, applies to albite, with the single exception of that at the Stromay locality.

I have here to note that in the summer of 1876 a single mass of apparently a dark green felspar was broken out of a vein in Rubislaw Quarry by Professor Nicol and myself; this upon examination was found to exhibit the structure very plainly; but here the included and colouring material—so far as the small quantity obtained has as yet enabled me to determine — was the mineral strigovite. Some of this substance in a minute scaley form, and also in hexagonal crystals of the size of peas, filled small druses.

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albite. In *perthite*,—where there is a somewhat parallel banding of orthoclase and albite,—the two were probably paragenetic in time; but the microscopic structure of perthite—the only mineral in which there has been a chemical determination of the nature of the layers—is so different from *this*, as to form the very strongest argument against the view that the intruded material is here *albite*.

5. *Has this included Foreign Matter any effect, and what, upon the enclosing Crsytals?*

The cleavage angle of every one of the felspars showing this structure departed more or less from 90°; and *the departure from a right angle is greatest in the felspars which show the structure in its strongest development.*

Lairg, 89° 39′; Rubislaw, 89° 58′; Stromay and Ben Capval, 89° 50′; Tongue, 89° 43′; Cowhythe Head, 89° 40′; Blirydrine, 89° 40′; Eslie, 89° 40′; Anguston, 89° 49′; Yestnaby, 89° 56′, are a few quotations,—these being, out of measurements of several fragments, those which were the most common; though occasionally measurements nearer to 90° were obtained, none were absolutely at the right angle.

If we suppose that the foreign substance is oligoclase, and that the polar axes lie accordant with one another,—which it will be shown that they do,—the obliquity of the cleavage angle in that substance should distort more or less the orthoclastic cleavage.

The angle of oligoclase differs 230′ from that of orthoclase; the amount of oligoclase present seems about $\frac{1}{11}$; the eleventh part of 280′ is 21′.

The case of those crystals which, like the Murchisonite, are systematically porous, here calls for consideration. The crystal from Arran, being a complex twin, was unfitted for the determination of the angle.

If the open structure and the filled-up structure be in reality the same, and if the crystals now vacuous be also more or less triclinic, we must conclude that the oligoclastic material segregated out of the orthoclastic when both were solidifying, nearly contemporaneously; and that in the hollow crystals the more decomposable mineral has been weathered or dissolved out. All Murchisonites I have seen were either loose stream-rolled crystals, or they were weathered; but it was not so with the Hill of Fare specimen; it was taken from a freshly-opened cavity.

It cannot be supposed that the crystalline molecules of the orthoclase arranged themselves preconcertedly, so as to distort the crystal; and also so as to leave vacuities for the reception of *a pre-selected* substance. The action of the crystallipolar force, in its wondrous production of hemitrope, twin, hemihedral, and hemimorphic crystals, must be said to be *determinative*, but we cannot assign to it the function of being *deliberative*. And though it is quite conceivable that pre-existent vacuities might be plugged up by the accretion of a subsequently solidifying felspathic material, or, in its absence, of the siliceous paste, still it is inconceivable that vacuities—the absence of material—could distort the angle of a crystal. "Nothing can come of nothing,"—nothing can do nothing.

An insufficiency of accreting molecules produces modifications—new faces on a crystal, but is powerless to alter its angles; and we are thus forced to conclude, as above, that the contemporaneous, or nearly contemporaneous, solidification of the triclinic substance within the orthoclastic distorted it latterally; and that the open structure in all felspars in which the angle departs from 90° is in a double sense an *outcome*—due to the removal of a material soluble in a solvent incapable of attacking the orthoclase.

I have somewhat particularly described this structure, as I have known it to be mistaken for that striation which is founded on by some geologists as a reliable criterion in the discrimination between orthoclase and the plagioclastic felspars;—the following distinctions may, however, be pointed out:—

The structure differs from the striation of twinning in that its markings are not rectilinearly parallel, but in undulating lines,—more undulating in *c* than in *b*; they *cross* the face *c*,—being at right angles to the macrodiagonal, while the lineation of repeated twinning is parallel to the macrodiagonal; they are not alternately lustrous with the other parts of the crystal on revolution on an axis parallel to their own strike; they are coalescent; and they are lighter in colour than the rest of the crystal.

I might add that, as these markings are as frequently seen in these vein orthoclases, as striation is in the plagioclastic felspars, they are as good a mode of discrimination between the two as the latter; and as they have never been seen in a plagioclastic felspar, while striation may occasionally be seen in *every* plagioclastic felspar, where seen they give some absolutely *specific* information, which striation can never do.

As the hemitropism of the crystals of amazonstone exhibits this singular structure in a modified development, it calls for notice.

This amazonstone, as before noticed, is frequently in hemitrope crystals which are pervaded throughout with the corded structure, and to such an extent that the white opaque filamentous matter acts as tenons to the two halves of the hemitrope, binding them together, until separated by force; when the white substance is seen with fractured edges like ruptured tenons, or the broken dovetails of the separated sutures of the disconnected bones of a skull.

The faces of the separated halves of the hemitrope are found to be highly polished and lustrous; the white ruptured lines passing across them in relief or depression, according to where the fracture operated,—like the rugosities of a single-struck file.

On pressing the two halves together, a certain amount of cohesion is re-established, as in the forcible reinsinuation of the sutures of the bones of the skull.

Difficult as it is to account for the arrangement of the molecules during the thrawn polarity which is operating whilst a twin or hemitrope crystal is being built up, here is something more difficult still: for, across the path of the molecules of the green matter, while moving towards their appointed positions, there must simultaneously have been moving those other white molecules engaged in their work of building up an inosculatory net-work. A system of net-works rather, extending not like the float of the fisherman in single line, but in innumerable parallel dispositions, stretched with perfect verticality* across the crystal, as the net of the fisherman is stretched by its leads with perfect verticality in a tranquil sea. But a consideration of what must be the relative motions of the molecules which are going to form a hemitrope crystal, would necessitate our comparing their influence upon the white layers in such crystals, to that of a flood-tide impelling the buoy-rope and also the lead-line of a net the one way, while a medial current of ebb swept its corded structure in the opposite. In accordance with this, at the face of revolution of the two halves of the hemitrope, the line of direction of this multitudinous net-work abruptly bends, and in its altered course maintains, as before, a perfect parallelism to the face *a* of the revolved segment of the crystal. (See figure 3, also Fare crystal.)

But this white matter, if oligoclase, must not be regarded as merely extruded from the green, in the manner that amorphous impurities would be.

As a definite chemical compound, it is itself subject to the operation of the crystallising force, in its double function as a physical separater and a symmetrical arranger. It is so arranged here, and the law of the arrangement is a singular one. It has been stated above, that the white reticulated structure is generally parallel to the face *a*, and gives a glimmering cleavage reflection when revolved about 4° either to or from the position of ordinary reflection of the face of the crystal as a whole. This glimmering reflection is unmistakeably from repeated cleavage faces, which give between each other an angle of about 9°. Such a cleavage angle is unknown in felspar; but the angle is as near that of the salient between the faces *p* of hemitropes of oligoclase, as reflections from so small surfaces could be expected to afford. Such hemitropes, however, are formed by the revolution of one-half of the crystal round an axis at right angles to the face *m*—that is, round the brachydiagonal—which, of course, is at right angles to the axis of revolution of the main crystal.

In fine, we have here a squat crystal of orthoclase—squat as regards the length of the vertical axis—which crystal is a hemitrope accordant with face *c*, laced throughout by thread-like hemitropes of oligoclase (?), the main axes of the two being accordant, but the axes of the hemitropism being at right angles to each other. We have molecular repulsion coexistent with a peaceful crystalline consorting in one and the same fabric—a consorting which, perhaps, is all the more firmly interwoven, that, as regards hemitropic arrangements, the substances uniting have agreed to differ. Perhaps only those who have studied the motions of the molecules which, in a fluent menstruum, are engaged in the building up of regular solids, can in any measure appreciate the intricate evolutions of the molecules of these two substances, as they jinkingly evaded one another in finding each its allotted place.

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* Supposing the crystal to be positioned as usual with the faces *m* vertical.

I would propose the term *corded structure* for what has been described; and I adopt to the full NICOL's view, that *two substances of a nature differing both in composition and molecular segregation,—differing also unquestionably in the system of crystallisation to which they belong, have here harmoniously united in one structure, in which the rectilinear rigidity of crystalloid arrangement is pervaded but not infringed by the flowing curvatures and wavy lines of a structure as involved as the anastomosis of plant or animal life.*

I must, however, refrain from proposing a name for the crystals which exhibit this structure, until their optical properties disclose whether the included substance is albite, or, as I suppose, oligoclase; should it prove to be albite, these are all, of course, *microlines;* should it be oligoclase, the crystals have the same claim as that mineral to a specific name.

Of the two appended columns, the first enumerates some of the felspars in which the structure is clearly to be seen; the second, those in which it is clearly absent. The latter are, in fact, few in number.

From	Corded.	Plain.
Intrusion veins in Hornblendic Gneiss.	Ben Capval, Harris.	Rispond, Sutherland.
	Stromay, do.	Hillswick, Shetland (?)
	Scaire Ruidh, do.	
Exfiltration veins in do.	West Stocklet, do.	
	Shermaig, Sutherland.	
Felspathic layers of do.	Erribol, do.	
	Cape Wrath, do.	
Intrusion veins in Micaceous Gneiss.	Blackwater, Ross.	? Glen Urquhart, Inverness.
	Pitfechie, Aberdeen.	Cowhythe, 6th vein, Banff.
	Blirydrine, Kincardine.	
Felspathic layers of do.	Midstrath, Aberdeen.	
	Brathans, Kincardine.	
	Struay, Ross.	
	Ben Resipol, Argyll.	
Intrusion veins in Granite.	Rubislaw, Aberdeen.	
	Cove, Kincardine.	
Exfiltration veins in do.	Rubislaw, Aberdeen.	Amazonstone, Ross.*
	Yestnaby, Orkney.	Stromness, Orkney.
	Tongue, Sutherland.	
	Anguston, Aberdeen.	
	Cairngorm, Banff.	
Drusy Cavities in do.	Upper Craigton, Aberdeen.	
	Arran ?	
Syenite,	. . . . . .	Froster Hill, Aberdeen.
Granular Limestone,	Eslie, Aberdeen,	Glen Beg, Inverness.
Felsite,	. . . . . .	Hausemann, Kincardine.
Porphyry,	. . . . . .	Skaw, Unst, Shetland. Canisp, Sutherland.
Tufa,	. . . . . .	*Sanidine,* Fife.

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* This consisted of a single crystal of nearly an inch in size, imbedded in granite—or part of a granitic vein,—it was found and given to me by the late T. Bell of Ballygrogan. The precise locality has escaped my memory. It has a pale but fine green colour.

Most of those in the first column are vein felspars; of those in the second column only Rispond, Urquhart, and Cowhythe are vein felspars, and these are all graphic with quartz. I have never seen one of the corded felspars graphic with quartz.

The structure which is visible to the eye, aided by the power of a lens, has, thus far, alone been considered; but when thin slices of the amazonstone are examined in the microscope, especially with the aid of polarised light, a still more intricate and wonderfully beautiful arrangement of the parts unfolds itself.

The lineation on *c*, which is parallel to its edge, but transverse to the corded structure, is seen to be the result of the intersections of acutely wedge-shaped crystals, two sets of which interlace with each other, like the teeth of two combs placed in opposition with forcible mutual insinuation.

With parallel Nicols, one set of these crystals—the teeth, as it were, of one of the combs—is highly coloured, the other being colourless; but each individual crystal—each tooth of the comb—is transversely banded by stripes of complimentary colour; as if itself built up of myriad crystals,—a pile of numerous twins, the length of the individuals whereof diminishes insensibly to the summit.

With crossed Nicols all this disappears from the lately coloured set,—that previously colourless assuming an identical appearance.

The twinning thus developed,—being at right angles to the twin face of the large crystal, and also at right angles to the supposed twin face of the myriad individuals which build up the corded layers,—produces therewith a structure of wonderful intricacy, and one which is altogether inexplicable.

The appearances, when examined by polarised light, of slices cut parallel to *c* and to *b*, are shown in the plate.

No optical phenomenon whatever is developed in slices parallel to *a*; this goes to negative the supposition of there being a twinning of crystals in the inclosed oligoclase.

ALBITE.

From Granitic Dykes in Hornblendic Gneiss.

1. From the vein on the south shore of Harris, facing the rocky islet of Stromay. This locality is noted in an old work, the entry being "moonstone from granite, opposite the Rock of Stromay." The exact spot, which is near the water edge, on the terminal face of the vein, was refound by Dudgeon. The mineral much resembles the moonstone of Norway, being possessed of a

delicate lavender reflection of considerable brilliancy and great beauty. The associated minerals are the grey orthoclase, the analysis of which has been given, and quartz. But few pieces of the moonstone could be got.

Colour, greyish white, translucent in patches. Cleavage angle, 86° 21′; specific gravity, 2·627.

1·401 grammes yielded—

Silica,	·9172		
From Alumina,	·021		
	·9382	=	66·966
Alumina,			19·46
Ferrous Oxide,			·601
Magnesia,			·214
Lime,			2·038
Potash,			1·228
Soda,			9·545
Water,			·314
			100·366

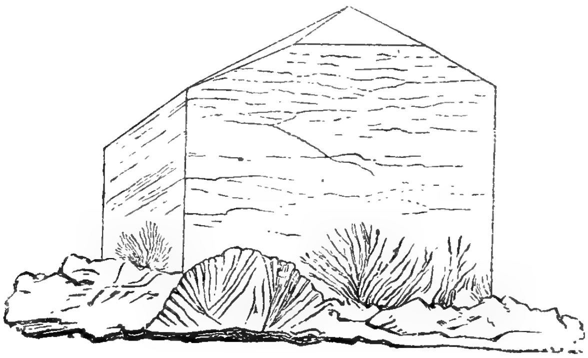
Insoluble silica, 4·584 per cent.; possible impurity, quartz.

From exfiltration Veins in Syenitic Granite.

2. From the exfiltration vein in the boulder of syenitic granite on the slope of Ben Bhreck, Tongue.

The associated minerals have been already mentioned. The mineral occurs in the rare form of radiated Cleavlandite. It has been proterogenetic to amazonstone, quartz, specular iron, and the radiated hydrated carbonate of lime. The other minerals were of anterior solidification.

Occurring only towards the centre of the vein, it forms mamillated bands of about one inch in thickness, with a radiated sheafy structure; sometimes it occurs in leaves. The sheafs are loose, and divergent at their circumferences; and are there imbedded either in quartz or amazonstone, — more rarely in the general mass of the vein. The specimens were very pure. The colour is cream-white. The lustre pearly on the flat faces, vitreous on the edges. The S. G. 2·622.



Radiated Cleavlandite in Amazonstone.

1·3 grammes yielded—

Silica,	·824	
From Alumina,	·047	
	·881 =	67·789
Alumina,		18·764
Ferric Oxide,		1·428
Manganous Oxide,		·076
Lime,		·516
Potash,		·757
Soda,		10·492
Water,		·159
		99·981

Insoluble silica, 1·362 p.c. Possible impurity, magnetite.

From Serpentinous and Talcose Rocks.

3. On the south shore of Colafirth Voe, in the mainland of Shetland, there is an assemblage of such rocks lying between the red aplite of Roeness and Colafirth hills and the mica slate of the east shore. Several varieties of serpentine occur in close proximity with the aplite. These are all bedded, but thrown up at a high, almost a vertical angle. Three well-marked varieties,—a dark green,—"potstone" like,—granular serpentine; a light green, with blue dendritic markings; and a verdigris green,—cannot be distinguished from three similar varieties which occur at Portsoy: there also they occur as tilted beds; the strike is the same, and the order of succession from west to east identical. Dr HIBBERT states that he found the breadth of two of the masses of serpentine to be 90 and 240 feet—the writer found the two principal masses at Portsoy to measure 80 and 240.

HIBBERT, in describing this spot, writes—"On the south of Colafirth Voe, the particles of the gneiss are disposed into distinct striæ of hornblende, quartz, felspar, or mica. At the same place, where there is one of the finest sections that is to be seen in Shetland, a beautiful rock succeeds, that is composed of nothing more than striæ of quartz and hornblende." Further on he adds—"In addition to these substances, beds of quartz from 3 to 7 feet broad occur, with some trifling quantity of a pure white limestone."

HIBBERT's use of the words *particles* and *striæ* here is peculiar. By particles I understand him to refer to the *several constituents;* by striæ, as used by him the first time, I understand beds; as used the second time, I understand layers or bands.

There is at least no mistaking his "beautiful rock," so well does it deserve the name; even in hand specimens nothing can be more striking than the contrast presented by the succession, in varying thicknesses, of the different layers of its two mineral constituents. But I am unable to agree with him as to their nature, as I am unable to admit that the beds of quartz mentioned should be called quartz.

The beautiful rock lies immediately to the west of the dark serpentine, near the south-west corner of the Voe. After a hasty examination it would be pronounced hornblendic gneiss; closer inspection inclines one to believe the green mineral to be more probably augite than hornblende, while the white layers should hardly have been taken for a granular quartz. The use of the knife would have determined the absence of any quantity of quartz.

It is a portion of the largest and purest band of the white mineral of which the analysis is now given. Structure,—a granular mass of crystals lying in every direction—the crystals are not striated; (the augite (funkite) crystals are all platey, or parallel to the bands of the felspar.)

Colour, white; specific gravity, 2·622.

1·544 grammes gave—

Silica, . . .	66·80
Alumina, . . .	17·832
Ferrous Oxide, . .	1·128
Magnesia, ...	·138
Lime, . . .	1·504
Potash, . . .	·919
Soda, ...	11·517
Water, . . .	·484
	100·322

Of the silica 1·591 per cent. were insoluble; possible impurity, quartz and funkite.

4. Hibbert's so-called "quartz vein" proved, on examination, to be a bed of a splendent white felspar, plentifully interspersed with quartz graphically arranged. The bed lies on the other or east side of the serpentine, and closely adjacent to a bed of dense granular white limestone. This is the finest massive albite in Scotland; though there are no free crystals, its cleavage faces are of large size, great purity of appearance, and brilliancy of lustre.

It very rarely exhibits striation. It is separated from quartz with great difficulty.

Its cleavage angle is 86° 45′; its specific gravity 2·61.

1·54 grammes gave—

Silica,	66·838
Alumina,	16·733
Ferrous Oxide,	2·42
Magnesia,	·372
Lime,	·942
Potash,	·733
Soda,	10·76
Water,	·894
	99·692

5·03 per cent. of the silica insoluble; possible impurity, quartz.

From Hornblendic and Chloritic Rocks.

5. At Hillswickness, on the same island of the Shetland group, strata of much the same general nature as those of Colafirth are to be seen; the serpentine, however, is found in but small quantities, and only of the precious variety; larger beds of serpentine possibly lie in the bay to the east; all the rocks in the immediate vicinity are highly contorted and fractured; large masses of porphyry being intruded into them in all directions, on the west side of the ness.

At the spot called the "banks of Nudista" (Nithista of Greg), large quantities of hornblende of several varieties occur, sometimes in crystals imbedded in snow-white albite; and these minerals were found immediately associated with imbedded crystals of a pink lustrous felspar, of which the following is the analysis.

Specific gravity, 2·615.

1·499 grammes yielded—

Silica,	·992	
From Alumina,	·008	
	1· =	66·711
Alumina,		19·813
Ferrous Oxide,		·9
Magnesia,		·093
Lime,		1·382
Potash,		1·264
Soda,		9·229
Water,		·542
		99·934

·7 per cent. of the silica were insoluble; possible impurity, unknown.

The white albite does not exhibit striation; the pink does, but only under a high power.

Albite in small but highly-modified pellucid crystals occurs disposed on the surface of the larger crystals of orthoclase in the syenitic granite of the Peterhead district. It has specially been found in the Stirling Hill and Murdoch's Cairn quarries. HAUGHTON having published an analysis of the crystals from the former quarry, I have not thought it necessary to analyse those I found in somewhat greater abundance in the latter; but I have (page 230) figured an interesting specimen from that locality, in which the albitic crystals stood in parallel disposition on the faces *m* only of the orthoclase; and, as these faces, and these faces only, are pitted, and of a corroded appearance, the albite would seem to have been formed from the decomposition of the orthoclase, through the substitution of soda for potash.

ALBITE.

	Colour.	Cleavage Angle.	Specific Gravity.	$\ddot{Si}$.	$Al_2\dddot{O}$.	$\dot{Fe}$.	$\dot{Mg}$	$\dot{Ca}$.	$\dot{K}_2$.	$\dot{Na}_2$.	$\dot{H}_2$.	Total.
Stromay, Harris,	Grey	86° 21′	2·627	66·97	19·46	·6	·21	2·04	1·23	9·54	·31	100·36
Colafirth, Shetland—"beautiful rock,".......	White	...	2·622	66·8	17·83	1·13	·14	1·5	·92	11·52	·48	100·32
Colafirth, ...	White	86 45	2·61	66·84	16·73	2·42	·37	·94	·73	10·76	·89	99·69
Hillswick, ...	Pink	...	2·615	66·71	19·81	·9	·09	1·38	1·26	9·23	·54	99·93
.Cleavlandite,.....	White	...	2·622	67·79	18·76	1·43*	...	·52	·76	10·49	·16	99·96

OLIGOCLASE.

From intrusive (?) Veins in Hornblendic Gneiss.

1. On the north side of the little harbour of Rispond, in Sutherland, a great lump rather than a vein of graphic granite lies in the gneiss in a tortuous rent, cutting the strata, here nearly vertical, at right angles.

The felspar of the granite is highly lustrous, flesh-coloured, and graphic with quartz throughout; it contains sparsely diffused imbedded crystals of oligoclase, Haughtonite, and magnetite in about equal amount.

The magnetite is of the blue-black, hackly fractured variety, which weathers with a brown strain, and possibly is titaniferous. The oligoclase is, as regards the size of the crystalline masses and its lustre, the finest in Scotland. It is pure white, in colour translucent, with striation highly developed and large in character.

---

* $Fe_2\dddot{O}$, also ·08 $\dot{Mn}$.

Its cleavage angle is 86° 14′; its specific gravity, 2·636.

1·017 grammes yielded—

Silica,	·625		
From Alumina,	·004		
	·629	=	61·848
Alumina,			21·703
Ferric Oxide,			3·37
Oxide of Manganese,			·196
Magnesia,			·09
Lime,			4·129
Potash,			1·63
Soda,			6·952
Water,			·375
			100·293

Insoluble silica, 1·113 per cent.; possible impurity, quartz.

From Hornblendic Slate.

2. The north-west eminence of the serpentinous hills of the Coyle, in Aberdeenshire, consists of hornblende rock; very flaggy on its eastern side. A quarry has been here opened for slabs, at the foot of the last heave of the hill; in this, and a little to the east of it, in actynolite slate, thin tortuous veins of cream-coloured oligoclase occur; these veins cut the beds.

The oligoclase is very opaque, with a somewhat greasy lustre, affording cleavages somewhat curved, but giving the angle 86° 32′. It is not striated. Its specific gravity is 2·627.

1·3 grammes yielded—

Silica,	·813		
From Alumina,	·013		
	·826	=	63·538
Alumina,			21·45
Ferric Oxide,			1·857
Magnesia,			·23
Lime,			3·876
Potash,			1·073
Soda,			7·638
Water,			·438
			100·1

Insoluble silica, 3·648; possible impurity, the matrix.

From a Vein in vicinity of Serpentine.

3. Barra hill, in the vicinity of Old Meldrum, in Aberdeenshire, is composed of an almost black serpentine which contains, porphyritically inbedded, patches or spots of apparently a highly altered felspar; giving much the appearance of an amygdaloid.

This serpentine indeed simulates an igneous rock more than any in the north of Scotland, with the one exception of that occurring near the railway station at Rothiemay.

A low spur or ridge is thrown out by the hill on its south-west flank pointing in the direction of Premnay, which is the nearest spot at which serpentine occurs to the west. About half-way up the ridge a quarry has been opened for dyking purposes—this quarry contains veins of granular labradorite, which in their fissures show radiating crystals of Wollastonite.

A dyke built out of this quarry and flanking its side, contained a mass which was apparently a portion of a vein, which might have lain either between the serpentine and the adjacent gneiss, or have cut the gneiss itself.

This mass consisted of a pale blue vitreous quartz, a little muscovite, and the oligoclase analysed; this was much run through with quartz, from which it was with difficulty separated.

Colour, milk white; no striæ. Cleavage angle from 86° 8′ to 86° 18′; specific gravity, 2·834. This high gravity led to its being considered labradorite, notwithstanding its association with quartz and muscovite.

1·302 grammes yielded—

Silica,	·834		
From Alumina,	·008		
	·842	=	64·67
Alumina,			22·18
Ferric Oxide,			1·437
Magnesia,			·015
Lime,			1·893
Potash,			1·542
Soda,			7·642
Water,			·15
			99·529

Insoluble silica, 5·76 per cent.; possible, perhaps probable impurity, a trace of quartz.

From Exfiltration Veins in Grey Granite.

4. A large vein, almost solely quartz, occurs in the granite quarry a mile west of Dyce, in Aberdeen. Rarely in the massive quartz there are imbedded crystals of silvery muscovite associated with finely formed crystals of nearly an inch in size of oligoclase. These crystals are, however, perfectly opaque, white in colour, and lustreless,—they are not striated. There is no orthoclase in the vein.

The cleavage angle is 86° 15′; the specimens were too porous to determine the specific gravity upon.

25 grains yielded—

Silica,	15·8635		
From Alumina,	·348		
	16·2115	=	64·846
Alumina,			23·2
Magnesia,			·202
Lime,			·964
Potash,			3·774
Soda,			8·125
Water,			·008
			101·119

Insoluble silica, 18·23 per cent.; possible impurity, quartz. This being an almost non-calcareous oligoclase.

5. From veins in Sclattey Quarry, near Buxburn, Aberdeenshire. The oligoclase here is associated with delicately flesh-coloured orthoclase, and Haughtonite. It is white in colour, and obscurely and very minutely striated.

1·264 grammes yielded—

Silica,	·7225		
From Alumina,	·03		
	·7525	=	59·533
Alumina,			21·05
Ferric Oxide,			1·81
Magnesia,			·88
Lime,			3·632
Potash,			4·73
Soda,			7·23
Water,			1·881
			100·746

Insoluble silica, 4·12 per cent.; possible impurity, orthoclase.

6. From Rubislaw Quarry. No better illustration of exfiltration veins could be seen than those from which the fine oligoclase now noticed was taken. Two great tortuous tears are to be seen in the depth of the quarry crossing the jointings more than once, but fading away at both ends into the unruptured rock. These tears are filled up with a mass of crystals of grey-pink orthoclase, milk-white oligoclase in large twinned crystals, crystallised muscovite, Haughtonite, apatite, schorl, and rarely garnet, in a paste of quartz.

At the centre of the wider portions of the rents these crystals are of considerable size; but that diminishes towards the sides until they graduate down by insensible transition to the dimensions of the crystals of the rock itself. There is no line of demarcation, no slickenside markings, and no interstitial mineral, or skin,—as in the case of intrusion veins.

The crystals of oligoclase are so minutely striated as to require the aid of a powerful lens, for the detection of the lines.

Their cleavage angle is 86° 14′; their specific gravity, 2·637.

1·476 grammes yielded—

Silica,	·909		
From Alumina,	·014		
	·923	=	62·533
Alumina,			23·518
Ferric Oxide,			1·277
Magnesia,			·365
Lime,			4·97
Potash,			1·324
Soda,			6·194
Water,			·6
			100·777

Insoluble silica, 2·845; possible impurity, orthoclase or quartz.

7. From the Quarry of Cragie Buckler, near Aberdeen. The specimen was sent me by Professor Nicol.

It was very similar in appearance to that described from Sclattey; the associated minerals were the same; the only difference being that the imbedded crystals of oligoclase were better defined, larger, more distinctly striated, and the striæ much coarser than those from the former locality.

Cleavage angle, 86° 14′; specific gravity, 2·622.

On 25 grains—

Silica,	61·58
Alumina,	22·
Ferric Oxide,	1·242
Magnesia,	·32
Lime,	4·192
Potash,	1·522
Soda,	8·27
Water,	·54
	99·666

Insoluble silica, ·922 per cent.; possible impurity, orthoclase.

From the Vein in the Syenitic Granite near Lairg, Sutherland.

8. Occurs along with the orthoclase,—the analysis of which has been given,—and the associated minerals already mentioned.

The oligoclase is in crystals of about two inches in size, imbedded in the paste of quartz. These crystals are yellowish cream colour, to white; opaque; dull throughout; and not striated, but cleaving so readily at the twin face as to appear somewhat foliated.

In breaking up a large crystal, a portion was obtained from the centre which was colourless, translucent, and distinctly striated. The non-striated crystal gave the angle 86° 10′; the striated portion, the angle 86° 15′. The specific gravity of that portion was 2·618.

	Cream-coloured.	Colourless.
Silica,	62·052	62·813
Alumina,	22·444	22·919
Ferric Oxide,	·352	·156
Magnesia,	·14	·08
Lime,	4·2	4·25
Potash,	...	·84
Soda,	...	8·53
Water,	·36	·29
		99·878

Possible impurity of first, orthoclase.

The striking fact here is, that what may be regarded as incipient weathering obliterates absolutely the reflection which developes striation.

From bedded Porphyry.

9. The mode of occurrence of the oligoclase found in the porphyry of Canisp has been noted before. Occasionally a small crystal of the red ortho-

clase is centrally imbedded in a larger one of the substance analysed, indicating that the latter was of later formation. Very rarely the red orthoclase is penetrated by crystals of the oligoclase.

The analysis, though pointing most clearly to oligoclase, is not altogether inconsistent with the mineral being albite,—unfortunately the crystals, though fairly well formed, had faces too rough for measurement. They decompose more rapidly than the orthoclase; this points to oligoclase, and not albite. There was no visible striation.

1·302 grammes yielded—

Silica,	·831	
From Alumina	·008	
	·839 =	64·439
Alumina,		20·436
Ferric Oxide,		·877
Manganous Oxide,		·384
Lime,		1·333
Potash,		1·135
Soda,		9·962
Water,		1·463
		100·029

Loss in bath, ·47 p.c. Insoluble silica, 1·668 p.c.

OLIGOCLASE.

	Colour.	Cleavage Angle.	Specific Gravity.	$\ddot{Si}$.	$Al_2\dddot{O}$	$F_2\ddot{O}$	$\dot{Mn}$.	$\dot{Mg}$.	$\dot{Ca}$.	$\dot{K}_2$.	$\dot{Na}_2$.	$\dot{H}_2$.	Total.
Rispond,	White	86° 14′	2·636	61·85	21·7	3·37	·2	·09	4·13	1·63	6·95	·37	100·29
Coyle,	Cream	86 32	2·627	63·54	21·45	1·86	...	·23	3·88	1·07	7·64	·44	100·1
Barra Hill,	Milk	86 12	2·834	64·67	22·18	1·44	...	·01	1·89	1·54	7·62	·15	99·53
Dyce,	White	86 15	...	64·85	23·2	...	...	·2	·96	3·77	8·12	·01	101·12
Sclatty,	White	...	...	59·53	21·05	1·81	...	·88	3·63	4·73	7·23	1·88	100·75
Rubislaw,	White	...	2·637	62·53	23·52	1·28	...	·36	4·97	1·32	6·19	·6	100·78
Craigiebuckler,	White	86 14	2·622	61·58	22·	1·28	...	·32	4·19	1·52	8·27	·54	99·67
Lairg,	Colourless	86 15	2·618	62·81	22·92	·16	..	·08	4·25	·84	8·53	·29	99·88
Canisp	Cream	...	...	64·44	20·47	·88	·38	...	1·3	1·13	9·96	1·46	100·02

ANDESINE.

From Granular Limestone in Micaceous Gneiss.

1. From Glen Urquhart.

The serpentine of Polmally is on the north-east, wrapped round with an unusually plicated bed of granular limestone, so convoluted and fractured, and cut up by dykes, that it is difficult to determine whether there be not a greater number of beds than one.

The lime is best seen about a mile to the north of Milltown. It has been quarried here and there, where anticlines brought it to the surface. In one of these openings, a large mass of a substance simulating tremolite in form, but quite distinct in composition, was found; and among the crystals of this substance a single crystal of andesine was also found.

This crystal, which was from the centre of the mass, was perfectly fresh and unaltered. It was over an inch in every direction, had distinct striæ, was moderately lustrous and somewhat translucent, of a pale-blue colour, a cleavage angle about 86° 28′ or 30′, and specific gravity 2·672.

The only substances immediately associated with it were the new mineral, calcite, and minute tufts of light green tremolite. In granitic veins in the vicinity there was an orthoclase of a dull-white colour, very similar in appearance to oligoclase, and Biotite.

No impurity was visible in the portion analysed.

1·3 grammes yielded—

Silica,	·737		
From Alumina,	·022		
	·759	=	58·384
Alumina,			22·496
Ferric Oxide,			2·119
Manganous Oxide,			·153
Magnesia,			tr.
Lime,			5·341
Potash,			3·197
Soda,			5·209
Water,			3·409
			100·308

Insoluble silica, ·527; possible impurity, tremolite.

2 and 3. From the Limestone Quarry at Delnabo, Glen Gairn, Aberdeenshire.

The rock in the neighbourhood of this granular limestone is the ordinary Haughtonite gneiss of Aberdeenshire; where it approaches the lime it becomes very highly metamorphosed; where in near contact with it, it is a nondescript mixture of a flakey biotitic gneiss, with a granular paste of coccolite. Imbedded partly in this singular compound, and partly in the lime, are numerous minerals, some more affecting the one, some the other; the less siliceous and more highly calcareous lodge in the lime; the more siliceous and less calcareous in the including rock.

There can be no room for doubt that these minerals are the results of an exalted metamorphism, which has resulted in a direct combination between the constituents of the gneiss and the limestone.

One of the many species which are to be found lodging in the intermediate zone of rock is andesine, and it is to be found of three appearances; it might almost be said in three conditions.

In all these it presents itself in large crystals, sometimes over two inches in length and breadth, by one-third of an inch in thickness. Of these crystals some are readily cleavable, clear, translucent, lustrous, and of a blue-white colour; they are simple twins, but not striated; others of the crystals are duller, opaque, and cream-coloured; they are, however, distinctly cleavable; while, lastly, some have a slightly greenish-white colour, a minutely granular structure, no cleavage, a glimmering lustre, and they present vacuities studded with minute crystals, which pass into a granular structure; these last are, in fact, pseudomorphic, and the material is Prehnite. In one and the same crystal a distinct transition may be seen from the second to the third.

When Prehnite replaces andesine, there must—according to the law of pseudomorphic interchange, as laid down at page 506 of vol. xxvii. of these Transactions, and calculated out as regards this particular case at page 510—be a vacuity equal to the difference between the numbers 686 and 795—about one-eighth; hence the vacuities in the pseudomorphosed crystal.

The blue lustrous crystals gave a cleavage angle of 86° 21′; the white somewhat massive variety, was curved on the cleavage faces; the specific gravity of the first was 2·705, of the second 2·689. For comparison, the analyses are placed in juxtaposition. There would appear to be in the second an incipient transition into the Prehnite, the analysis of which is appended to exhibit this.

The crystals were not striated.

	The Blue, on 1·651 grm.	The White, on 1·623 grm.	The Prehnite, on 1·51 grm.
Silica,	57·177	56·961	44·105
Alumina,	24·042	23·81	22·568
Ferric Oxide,	1·124	·938	2·894
Magnesia,	·121	·086	·529
Lime,	6·105	7·984	25·478
Potash,	2·832	2·565	...
Soda,	7·132	6·853	tr.
Water,	1·596	1·621	4·604
	100·129	100·814	100·178

Insoluble silica, 9·11 per cent.

All three possibly contained traces of coccolite. The Prehnite pseudomorphs were set round with a sheath of minute crystals of that substance. No trace was visible to the lens, however.

4. From Crathie.

The stratum of lime which has been worked in the hill face opposite to Crathie is doubtless the same as that at Delnabo; it may be traced from Crathie for a couple of miles in a northward direction. The crystals of andesine are here neither so common nor so fine; nor have we here the transition into Prehnite, though Prehnite does occur in the quarry: there is here also altogether a feebler development of mineral formation, and of exceptional metamorphism.

A singular rock is, however, the cap of the limestone; at a small distance it has somewhat the appearance of a black porphyry, and it is porphyritic in structure.

When examined with the lens it shows a basis composed solely of minute brilliant crystals of brown augite, somewhat translucent—("pseudo-hypersthene.") Among these there lie porphyritically imbedded pale-blue hyaline crystals, which I believe to be andesine, but which may be labradorite; very rarely a spangle of Biotite may be seen, but nought else.

The andesine was here immediately associated with Wollastonite and coccolite; it was not striated; its cleavage angle was 86° 24′: its specific gravity, 2·677.

The first analysis was incomplete through an accident. The second was on 1·299 grammes, which yielded—silica ·712; from the alumina, ·015 = ·727.

			Average.
Silica, . . .	56·64	55·996	56·303
Alumina, . . .	...	25·706	25·706
Ferric Oxide, . . .	...	·967	·967
Protoxide of Manganese,	...	tr.	tr.
Lime,	...	9·354	9·354
Potash, . . .	1·462	1·514	1·488
Soda, . . .	4·89	4·557	4·724
Water, . . .	1·611	2·024	1·817
		100·088	100·359

The insoluble silica in the second was 2·063 per cent.; possible impurity, Wollastonite or quartz.

From Diabase, near Limestone.

5. The rock immediately on the east side of the harbour of Portsoy, in Banffshire, first got the name of "primitive greenstone," and then "syenitic

greenstone:"—in the district, from its toughness and untractableness, it is denominated "heathen."

Much of the "heathen," however, spread over the country eastward and southward is from the Morven range; and that consists of true syenite.

The rock here comes nearest to diabase in constitutents, though from its grey-brown colour it might not at first be recognised as such; indeed, from the rapidity with which the structure varies, in a small space, from large to small-grained, it becomes difficult, on first inspection, to believe all to be the same rock.

To the west of Portsoy, immediately beyond the battery, the rock exhibits a coarser structure than at any other spot in the district: no ingredients are visible in quantity except a somewhat greasy-looking bluish-grey labradorite, and brownish-grey augite; which augite, when cut into small pieces, and looked at by transmitted light, is found to be purple-red, like garnet—so changed is the appearance of the small partcles that there is difficulty in coming to a belief that the substance is the same.

Say fifty yards across the section westward of this point, a mass of aphanite appears, difficultly distinguishable from diorite; two veins of greasy quartz alone visibly intervening, though probably several beds of limestone have been here washed out. At an equal distance to the east of the first-mentioned spot the rock has become fine grained; the augite being greener, the labradorite waxy and diaphanous. Passing further east, to the other side of the harbour, the labradorite almost disappears; while the augite has undergone incipient decomposition, and its cleavage faces have assumed a pseudo-hypersthenic glimmer. Specimens from this spot were indeed sold by ABRAHAM CLARK, a former mineral dealer in the place, as "Norwegian hornblende;" but true hypersthene is, at this locality, to be met with only in very minute crystals;—so far as I know, this is the only locality in Scotland where this mineral is to be found *in situ.*

Passing still further eastward, beds of serpentine and limestone are met with, and immediately over the latter, on the east shore of the bay of the Ard, masses of the igneous rock again appear, bearing now no small resemblance to the peculiar variety noted at Crathie. The felspar, however, is not so markedly porphyritic, but its singular appearance, like half-cold suet, is strikingly similar to the Crathie variety; and through this, again fine-grained rock, there occurs a small tortuous vein of andesine, differing in appearance altogether from that met with elsewhere in Scotland. It has much the appearance of a massive labradorite, being fine granular, passing into obscurely fibrous or plumose; a white colour, dirtied with a dash of grey-green; and a lustre between vitreous and greasy. No striation could anywhere be seen.

The immediately associated, indeed imbedded, minerals are buff-coloured sphenes, and one minute crystal of Babbingtonite was also found.

In the neighbourhood true hypersthene, in foliated patches the size of peas, is very rarely found;—but so far as I know only in loose blocks.*

The cleavages of this andesine were too curved for measurement; the specific gravity was 2·692.

1·62 grammes gave—

Silica,	58·36
Alumina,	23·344
Ferric Oxide,	·24
Magnesia,	·5
Lime,	8·24
Potash,	1·151
Soda,	7·844
Water,	·53
	100·209

ANDESINE.

	Colour,	Cleavage Angle.	Specific Gravity.	$\ddot{\mathrm{Si}}$.	$\mathrm{Al_2\dddot{O}}$.	$\mathrm{Fe_2\dddot{O}}$.	$\dot{\mathrm{Mu}}$.	$\dot{\mathrm{Mg}}$.	$\dot{\mathrm{Ca}}$.	$\dot{\mathrm{K}}_2$.	$\dot{\mathrm{Na}}_2$.	$\dot{\mathrm{H}}_2$.	Total.
Glen Urquhart,......	White	86° 28′	2·672	58·38	22·5	2·12	·15	tr.	5·34	3·2	5·21	3·41	100·31
Glen Gairn, ...	Blue	86·21	2·705	57·18	24·04	1·12	...	·12	6·11	2·83	7·13	1·6	100·13
Do.	White	...	2·689	56·96	23·81	·94	...	·09	7·98	2·56	6·85	1·62	100·81
Crathie, ...	White	86·24	2·677	56·3	25·71	·97	tr.	...	9·35	1·49	4·72	1·82	100·36
Portsoy, ...	White	...	2·692	58·36	23·34	·24	...	·5	8·24	1·15	7·84	·53	100·21

LABRADORITE.

From Gneissose and Serpentinous Rocks.

1. Immediately to the west of the aphanite mentioned at page 250, as occurring near Portsoy, there is a tilted bed of euphotide rock, entirely converted in those portions which are above high water line into a mottled serpentine; after passing over a quantity of debris from this rock, lying probably in the void of a washed-out lime stratum, a bed almost fifteen feet in thickness, of white or pale greenish-grey labradorite is seen; the workings of an old mine, said to have been *lead,* runs along its west face; a stratum of talcaceous quartzite formed the west wall of the mine.

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* Somewhere near this spot, Mr Grieve of Burntisland found a loose piece of hypersthene, showing a cleavage face over an inch in extent.

As *black lead* has been wrought at Rothiemay, Huntly, and other spots along the strike of these rocks, the "*lead*" here was doubtless also graphite.

The bed of labradorite is the largest mass of that substance that I know of; it has generally been considered Saussurite.

It contains imbedded many minute crystals of olive-green talc, from which it is freed with much difficulty; it also contains minute sphenes, small crystals of pyrite, and specks of a lustrous black mineral too minute to be identified (ilmenite?). Rarely in its mass twin crystals, apparently of the mineral itself, lie imbedded; these are somewhat higher in lustre than the general mass, probably from the reflection being from a more continuous surface than the general somewhat saccharoid massive mineral; they were not striated. JAMESON, who paid only a hurried visit to Portsoy, describes this as a bed of marble.

This labradorite is hyaline vitreous in lustre, and of specific gravity 2·672.

25 grains gave—

Silica,	13·01			
From Alumina,	·248			
	13·258		=	53·032
Alumina,				29·852
Ferric Oxide,				·128
Magnesia,				·612
Lime,				11·436
Potash,	·62	·665	av.	·642
Soda,	3·811	4·617	„	4·214
Water,				·42
				100·336

Indissoluble silica, 2·881 per cent.; probable impurity, a trace of talc.

From hyperitic Diabase.

2. From the Cuchullin Range, Skye.

The specimens were given me by the late Principal FORBES, as from Hart-o-Corry. The bulk of these consists of imbedded crystalline masses of a fine lustrous green augite, which weathers of a bronzy lustre, and hence has been considered hypersthene. The "felspar" is for the most part in such small ill-defined crystals that it might be called granular; it was thought commonly to be Saussurite. The other constituents of the rock are magnetite,—pervading it, and crystallised in octohedra in the rifts; Biotite in small quantity, a rare trace of chlorite(?), and according to JAMESON "glassy actynolite."

The labradorite of this locality does not seem, as is generally the case in these plutonic rocks, to have separated out first, but it is proportionally in such small amount as to make it difficult to determine this. Among the granules there are rarely elongated crystals; these are not striated, though sometimes twins. It was colourless, or with a faint tinge of green, and had a vitreous or greasy lustre. Neither cleavage angle nor specific gravity could be taken.

	On 1·315 grm.	On 1·621 grm.	Average.
Silica,	49·387	48·922	49·155
Alumina,	29·616	29·624	29·62
Ferric Oxide,	1·154	1·151	1·152
Magnesia,	·89	·932	·911
Lime,	lost	15·309	15·309
Potash,	...	·695	·695
Soda,	...	2·914	2·914
Water,	·719	·741	·73
		100·288	100·386

Insoluble silica, 2·11 per cent.; probable impurity, a trace of augite. In the lime and alkalies this approaches anorthite, of which there was possibly an admixture.

3. From near the Head of Loch Scavaig.

This specimen, given me by Mr Grieve, was very unlike the previous one. It consisted of a large imbedded crystal, an inch and a half in length; it was associated with large greenish-brown, slightly bronzy-lustred crystals of augite, these being penetrated by small crystals of magnetite. The crystal of labradorite was of a bluish-grey colour, feebly lustrous, and indistinctly but finely striated.

The cleavage angle was 86° 42′; the specific gravity, 2·715.

1·64 grammes gave—

Silica,	50·811
Alumina,	29·48
Ferric Oxide,	·252
Magnesia,	·124
Lime,	12·69
Potash,	·552
Soda,	3·922
Water,	2·481
	100·292

Insoluble silica 12·58 per cent.; possible impurity, augite.

Mr DUDGEON has a specimen of the Cuchullin diabase from the east slope of Scuir-na-Gillean, in which the labradorite is of the colour of the last analysed, but which has iridescent reflections which are fairly brilliant in colour.

From Diorite.

4. On the north side of Glen Bucket a dyke of diorite (?), with gigantic crystals of jet black hornblende imbedded in snow-while labradorite, appears in the low flanks of Craig-an-Innean, striking in the line of Tullocharroch.

The associated minerals are ilmenite, Biotite, apatite, sphene, ripidolite or chlorite, and very rarely a rose-coloured felspar in crystals—the last five being rare.

The labradorite is in confused crystalline granules; instead of being tough, as this mineral usually is, it is brittle; its specific gravity is 2·674.

11·53 grammes yielded—

Silica,	·7665		
From Alumina,	·008		
	·7745	=	50·588
Alumina,			28·334
Ferric Oxide,			3·05
Magnesia,			·588
Lime,			11·174
Potash,			2·183
Soda,			2·558
Water,			1·417
			99·892

10·25 per cent. of this silica insoluble; possible impurity unknown.

The associated mineral is named hornblende, both from cleavage angle and analysis; according to ROTH, labradorite and hornblende exclude each other; it is here not so. The rock is a diorite, having labradorite as the felspar.

From Gabbro.

5. The great mass of this rock in Unst, in Shetland, terminates about the north of the island of Balta. On the south side of Brough Geo, in that island, two pseudo veins are to be seen protruding through the turf, and striking over the edge of the cliff. The one consists of a paste of granular labradorite, carry-

ing giant crystals of hornblende; the other, some dozen feet apart, has crystals of augite of large size replacing the hornblende.

In both cases the labradorite is massive or impalpable granular—a crypto-crystalline or felsitic variety; where associated with hornblende, it is, in the interior, of a pale lavender-blue colour, shading off to white on the exterior,—perhaps from weathering. The variety associated with the augite is always white. The blue had a specific gravity of 2·95.

25 grains yielded—

Silica,	52·212
Alumina,	29·64
Ferric Oxide,	·48
Magnesia,	·263
Lime,	12·428
Potash,	·443
Soda,	3·998
Water,	·111
	99·575

3·86 per cent. of the silica were insoluble; possible impurity unknown. Roth's law, above referred to, is here again not borne out.

6. That with the augite had a specific gravity of 2·954.

25 grains yielded—

Silica,	53·136
Alumina,	29·992
Ferric Oxide,	·248
Magnesia,	·208
Lime,	12·296
Potash,	·472
Soda,	3·86
Water,	·21
	100·422

Indissoluble silica, 1·64 per cent.; possible impurity unknown.

7. The rock which is found in the diabase of the cliffs immediately to the west of the battery at Portsoy, consists almost solely of crystals of labradorite imbedded in a confusedly crystalline mass of augite; Biotite, and iserine in specks rarely occur. In cavities—which are very rare in such a rock—the labradorite is occasionally met with regularly crystallised; and, throughout, it impresses its form upon the augite; but by the action of the sea the labra-

dorite is distegrated and dissolved at a faster rate than the augite, so that the latter is left protruding from the surface of the surf-beat cliffs, with a false appearance of being the better crystallised mineral.

The labradorite here occurs in grey crystals, which are minutely striated; they are possessed of little lustre. Its specific gravity is 2·831.

1·567 grammes yielded—

Silica,	·804		
From Alumina,	·022		
	·826	=	52·411
Alumina,			28·959
Ferric Oxide,			·149
Protoxide of Manganese,			·913
Magnesia,			·54
Lime,			10·85
Potash,			1·61
Soda,			3·485
Water,			·927
			99·844

7·73 per cent. of the silica were insoluble; possible impurity, augite.

Immediately to the back of a store, south of the battery, there is a thin vein of granular grey labradorite, with imbedded lustrous white crystals; this is here the matrix of splendent crystals, either of the pseudo-hypersthene variety of augite, or more probably of diaclasite: the habit of the labradorite has, therefore, within a distance of about fifty yards, in every way altered. A fine polished slice from this vein is in the Industrial Museum of Edinbugh.

From Micaceous Gneiss

Scattered sparsely over the hill slopes, built into the dykes, or gathered into the road-metal heaps, there were at one time found, in the parish of Kildrummy, in Aberdeenshire, masses of a seam stone, consisting of an extraordinary tough matted mixture of crystals of red andalusite, white fibrolite, lepidomelane or Biotite, margarodite, and cream-coloured labradorite.*

A plicated double skin of mica on all the specimens showed that the gneiss had been the matrix, and that they had not been borne far from their original site.

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* The late Rev. Mr Morgan of Stonehaven had so great an admiration for these specimens, that he was wont to spend months in wandering over the Kildrummy and Clova Hills in search of them; and so thoroughly had he scoured the district, that the present writer, who has made repeated journeyings with the same object, was never successful in finding either the original locality, or a single specimen.

This labradorite is in somewhat large crystals, of a cream colour, striated, brittle, and with a specific gravity of 2·674. It yielded—

Silica,	51·312
Alumina,	26.756
Ferric Oxide,	1·818
Manganous Oxide,	·76
Magnesia,	·41
Lime,	10·137
Potash,	2·11
Soda,	6·43
Water,	·684
	100·417

From Porphyrite.

9. One of the felsitic intrusive traps, which occur so persistently among the old red conglomerates of Kincardineshire, shows a cliffy escarpment at a turn of the road from Bervie to Catterline, just where the branch road to the church of Kinneff diverges. This spot is interesting from the trap being at one and the same place highly prophyritic in structure, as well as markedly amygdaloidal.

The imbedded crystals are large flat twins of labradorite. The drusy cavities, which are of considerable size, though not numerous, contain large sheafs of red stilbite, and finely developed crystals of Heulandite, both of a vermilion tint,—radiated quartz,—pale green fibrous, and massive chocolate-coloured saponite. The labradorite is colourless to brownish-grey, translucent, and vitreous, but weathers dull and white; much flawed, and not striated.

1·301 grammes yielded—

Silica,	·672		
From Alumina,	·02		
	·692	=	53·189
Alumina,			26·431
Ferric oxide,			2·854
Manganous oxide,			tr.
Magnesia,			·922
Lime,			9·684
Potash,			1·511
Soda,			4·594
H_2O,			·726
			99·911

Insoluble silica, 2·167 per cent.; impurity doubtless a trace of the matrix.

10. From Glen Beg, Glenelg.

The exact relationships of the specimens, now to be described, it is not easy to determine, as the block from which they were taken was a loose, though not a transported one.

It lay, along with a number of masses of primary serpentinous limestone, at the foot of a cliff, in the bed of lime which lies about five hundred yards above the hamlet of Balvraid.

Over the limestone the ground is, in the immediate neighbourhood, so covered that its junctions cannot be seen; backward, in the hill, gneiss is first come upon.

On the side of the stream opposite to Balvraid, and further down the glen, where the lime curves round the foot of Bieneghapple, an augite rock lies under it; while it is covered by a rock composed of granular augite, dark red garnet, and pyrite;—a compound of wonderful toughness.

I incline to think that this rock may accompany the lime in its northern fold, and that a labradorite belt intervenes, as in so many other localities.

The chief portion of the mass of rock mentioned, consisted of a brown, granular mineral, which is new—and will be after described under the name of *balvraidite*,—the labradorite under consideration, the blue lustrous orthoclase, the analysis of which has been given,—and smaller quantities of Biotite: there was also some granular calcite.

The labradorite occurs in two forms, both of which are peculiar. The first, of which there was but little, may be called saccharoid, from its resemblance to loaf-sugar; the second much resembled the Norwegian radiated Cleavelandite, in all but the want of divergence in the pseudofibrous structure, that being of a parallel disposition in the Balvraid mineral. In both varieties the colour and lustre was that of bleached wax, and both fuse readily, with much frothing, into a blebby glass.

The specific gravity of the granular variety was 2·705; 1·703 grammes yielded—

Silica,	·788		
From Alumina,	·022		
	·81	=	47·437
Alumina,			28·023
Ferric Oxide,			·343
Magnesia,			·41
Lime,			11·033
Potash,			3·515
Soda,			4·613
Water,			5·202
			100·577

11·885 per cent. of the silica insoluble; possible impurity, orthoclase or balvraidite.

11. The fibrous, or rather flat fibrous—for the structure is not so perfect in one direction—resembled a pale Wernerite,—it nowhere showed striation; it had a specific gravity=2·708; 1·494 grammes yielded—

Silica,	·722	
From Alumina,	·015	
	·737 =	49·33
Alumina,		26·698
Ferric Oxide,		·25
Magnesia,		·072
Lime,		11·02
Potash,		2·59
Soda,		5.254
Water,		4·845
		100·059

4·85 per cent. of the silica were insoluble; possible impurity, balvraidite.

In both of the above, the water is in such quantity as to constitute a hydrated labradorite; its presence indeed confers upon the mineral its most decided blow-pipe reaction,—the frothing being as well marked as that of most zeolites. But hydrated labradorites are by some regarded as alteration products. Now, this is certainly not the case here: the mineral is in no way weathered or decomposed; it is perfectly fresh and lustrous; in fact, it is, with the single exception of the Glen Bucket specimens, markedly the freshest labradorite I have seen;—while the Balta specimens, which contain only traces of water, are as unquestionably the most weathered in appearance.

LABRADORITE.

	Colour.	Cleavage Angle.	Specific Gravity.	$\ddot{\text{Si}}$.	$\overset{\cdots}{\text{Al}_2\text{O}}$.	$\overset{\cdots}{\text{Fe}_2\text{O}}$.	$\dot{\text{Mn}}$	$\dot{\text{Mg}}$	$\dot{\text{Ca}}$	$\dot{\text{K}}_2$	$\dot{\text{Na}}_2$	$\dot{\text{H}}_2$	Total.
Portsoy, massive, .	White	...	2·672	53·03	29·85	·13	...	·61	11·44	·64	4·21	·42	100·34
Hart-o-Corry, . .	White	...	...	49·15	29·62	1·15	...	·91	15·31	·69	2·91	·73	100·39
Loch Scavaig, . .	Grey	86°42′	2·715	50·81	29·48	·25	...	·12	12·69	·55	3·92	2·48	100·29
Glen Bucket, . .	White	...	2·674	50·59	28·33	3·05	...	·59	11·17	2·18	2·56	1·42	99·89
Balta, ...	Lavender	...	2·95	52·21	29·64	·48	...	·26	12·43	·44	4·	·11	99·57
Do.	White	...	2·954	53·14	29·99	·25	...	·21	12·3	·47	3·86	·21	100·42
Portsoy crystals, .	Grey	86 42	2·83	52·41	28·96	·15	·91	·54	10·85	1·61	3·48	·93	99·84
Kildrummy, . .	Cream	...	2·674	51·31	26·76	1·82	·76	·41	10·14	2·11	6·43	·68	100·42
Kinneff, ...	Colourless	86 40	...	53·19	26·43	2·85	tr.	·92	9·68	1·51	4·59	·73	99·91
Balvraid, granular,	Wax	...	2·708	47·44	28·02	·34	...	·41	11·03	3·51	4·61	5·2	100·58
Do. fibrous, .	Wax	86 40	2·705	49·33	26·7	·25	...	·07	11·02	2·59	5·25	4·84	100·06

ANORTHITE.

From Anorthic Diorite.

1. The rocks at the south-east corner of the bay of Trista in Fetlar, in Shetland, exhibit a good section.

A graphitic slate is succeeded on the east by an ordinary clay state, that by diorite, and that in turn by serpentine.

The association of mineral constituents in the diorite being unusual, the propriety of the name assigned to it may be questioned.

It is best seen in a gentle height about due east of the bay. Its appearance is quite startling; the diorite of Glenbucket may be more striking when considered in detail, from the great size of its crystals of hornblende; but regarded as a rock mass, I know of nothing so striking as this in all Scotland.

When broken into, it is found to consist of an almost compact cryptocrystalline cream-coloured felspar, containing imbedded crystals of dark green, somewhat dull hornblende, several inches in size. In the slightly weathered surface the felspar is quite white, while the hornblende is jet black and highly lustrous. When seen in sunshine, with the light flashing from the crystals of hornblende, the effect is so extraordinary, and the appearance altogether so beautiful, that one hesitates even to walk over it, in the fear of damaging so splendid an object.

The felspar, which proved to be anorthite, was both hard and tough; its specific gravity was 3·099.

1·511 grammes gave—

Silica,	·696		
From Alumina,	·013		
	·709	=	46·922
Alumina,			30·775
Manganous Oxide,			tr.
Magnesia,			·094
Lime,			16·344
Potash,			1·498
Soda,			3·066
Water,			1·583
			100·237

Insoluble silica, 10·24 per cent.; there was absolutely not a trace of iron, so that there could have been no contamination from hornblende, the only contact mineral. There is an approach to Saussurite in this composition, and in gravity.

From Gabbro.

2. This specimen was sent me by Mr Grieve of Kirkbank, Burntisland; it was broken out of the diallagic rock at Lendalfoot, Ayrshire.

It consisted of imbedded crystals about half an inch in size, which were of a greyish-white colour, distinctly and coarsely striated, and of specific gravity 2·761.

1·205 grammes yielded—

Silica,				44·224
Alumina,				31·442
Ferric Oxide,				1·955
Magnesia,				1·
Lime,				14·18
Potash,				1·48
Soda,				1·625
Water,	4·023	3·348	av.	3·686
				99·592

4·023 per cent. of the silica were insoluble; possible impurity, diallage.

Mr Grieve remarked to me on the fact of the diallage of this rock weathering so much more rapidly than its felspar, as to leave the latter protruding from the surface; while in the compound which occurs a mile north, at Pinbain, the diallage is the mineral which endured, and is left protuberant. I find the mineral which is associated with the diallage at Pinbain to be a hydrated Saussurite. The weathering at Lendalfoot is due to subaerial,—that at Pinbain to subaqueous action. The general rule is that in gabbros, the felspar weathers much the faster, but elsewhere in Scotland the felspar is labradorite.

From Granular Limestone.

3. I doubt if the next substance should be classed here; it was found at the top of the limestone quarry at Delnabo, Glengairn, at the contact of the lime with the gneiss.

It consisted of a band about two inches thick, of a granular translucent pale-green substance, somewhat resembling prase,—except in its granular structure, and its hardness, which was only about 6. It was associated with Latrobite and coccolite; and contained, diffused throughout it, a probably small quantity of the latter, as it was unequal in colour. It had a glimmering lustre, and a specific gravity of 2·958.

1·301 grammes yielded—

Silica,	·589		
From Alumina,	·015		
	·604	=	46·425
Alumina,			21·864
Ferrous Oxide,			5·918
Manganous Oxide,			·691
Magnesia,			2·92
Lime,			18·38
Potash,			1·262
Soda,			1·695
Water,			1·078
			100·233

Insoluble silica, 2·814.

I have lately seen a thin band of the same substance in the limestone quarry of Boulchach—S.W. of Abergeldie.

LATROBITE.

Most authorities regard Latrobite as being a species distinct from anorthite; DANA places them together. This is certainly ill-advised. Anorthite is a lime felspar, with but traces of potash; Latrobite is a potash lime felspar.

The classification of the felspars is essentially an *alcaline* one—every consideration should make it so. DANA unites this with anorthite, from the quantity of its *acid;* but it is doubtful if that quantity is the same as in anorthite.

Latrobite has different angles, and much inferior hardness.

The specimens I have found were associated with the last mentioned felspar. There was very little of it; from the appearance and hardness being nearly the same, it was taken for rhodonite, until the specific gravity was determined.

The colour was pale rose red, the structure fine granular, the lustre feeble, the hardness 5, the specific gravity 2·749.

Two specimens from different parts of the quarry were analysed; the first was pure,—the second might not have been totally separated from coccolite.

	1·5 grm. yielded—	1·3 grm. yielded—
Silica,	·654	·596
From Alumina,	·024	·013
	·678	·609

Silica,	45·2	46·846
Alumina,	31·038	29·313
Ferric Oxide,	3·428	2·306
Ferrous Oxide,	tr.	·111
Manganous Oxide,	·68	1·153
Magnesia,	1·2	1·384
Lime,	5·21	6·461
Potash,	7·116	7·314
Soda,	·491	·83
Water,	5·697	4·491
	100·06	100·209

The insoluble silica of the first was 2·802 per cent.; of the second, 3·12.

One other locality only of this mineral is known. GMELIN'S analysis of two pecimens from that locality accord fairly with the above.

ANORTHITE.

	Colour.	Cleavage Angle.	Specific Gravity.	$\ddot{\mathrm{Si}}$.	$\mathrm{Al}_2\dddot{\mathrm{O}}$.	$\mathrm{Fe}_2\dddot{\mathrm{O}}$.	$\dot{\mathrm{Fe}}$.	$\dot{\mathrm{Mn}}$.	$\dot{\mathrm{Mg}}$.	$\dot{\mathrm{Ca}}$.	$\dot{\mathrm{K}}_2$.	$\dot{\mathrm{Na}}_2$.	$\dot{\mathrm{H}}_2$.	Total.
Fetlar, Shetland	Cream	...	3·099	46·92	30·77	...	...	tr.	·09	16·34	1·5	3·07	1·54	100·24
Lendalfoot, Ayr	Greyish	86·42	2·761	44·22	31·44	1·95	...	...	1·	14·18	1·48	1·63	3·69	99·59
Glengairn . .	Green	...	2.958	46·42	21·86	...	5·92	·69	2·92	18·38	1·26	1·69	1·08	100·23

LATROBITE.

	Colour.	Cleavage Angle.	Specific Gravity.	$\ddot{\mathrm{Si}}$.	$\mathrm{Al}_2\dddot{\mathrm{O}}$.	$\mathrm{Fe}_2\dddot{\mathrm{O}}$.	$\dot{\mathrm{Fe}}$.	$\dot{\mathrm{Mn}}$.	$\dot{\mathrm{Mg}}$.	$\dot{\mathrm{Ca}}$.	$\dot{\mathrm{K}}_2$.	$\dot{\mathrm{Na}}_2$	$\dot{\mathrm{H}}_2$.	Total.
Glengairn . .	Rose	...	2·749	45·2	31·04	3·43	tr.	·68	1·2	5·21	7·12	·49	5·7	100·06
Do. . .	Do.	...	...	46·87	29·31	2·31	·11	1·15	1·38	6·46	7·31	·83	4·49	100·21

In reviewing the results of this extensive series of analyses, the first point calling for consideration is the light (if any) which is thrown by it on the ever-recurring question as to the specific distinction between the felspars,—the number of species into which they are to be divided.

In speaking of the light that may be thrown upon the question, I would not have it thought that I consider it now as an open one; I conceive that the masterly paper by DESCLOISEAUX in the *Comptes rendus*[*] has definitively settled the matter, and must have satisfied TSCHERMAK himself. Nor would I reduce it to a mere question of personal opinion or experience, by enumerating the grounds which have induced me long to take the view advocated by DESCLOISEAUX; but,—postponing, to the second part of this paper, the chemical consideration of the subject,—I would only endeavour meantime to strengthen his position by drawing attention to the evidence of the rocks themselves, as clearly declared by the stratigraphical position of the minerals.

All mineralogists admit the three species, orthoclase, albite, anorthite;—some also labradorite,—but do not admit oligoclase or andesine.

It appears to me that, in Scotland, the lithological relations of the three last bear out their specific individuality quite as thoroughly as that of the universally acknowledged species is borne out.

Reviewing each in turn,—and admitting only the evidence of such specimens as have either been actually analysed, or otherwise determined by characteristics sufficiently cumulative to leave no room for doubt,—we find true or ordinary *orthoclase* occurring in crystalline schists—as in gneiss at Glen Urquhart, Dee side, and central Sutherland; the associated minerals being hyaline quartz paste, carrying either Haughtonite or lipidomelane, and rarely apatite. Rarely in cavities of chlorite schist, as in Strath Alnack; the associates being chlorite and rutile. Exceptionally, otherwise than crypto-crystalline, in porphyry, with granular quartz and pinite, or black hexagonal mica (? Biotite) as associates. Exceptionally in syenite, as on Morven and Froster hills, and near New Leslie, in Aberdeenshire; the associates there being hornblende, menaccanite, and sphene. And very exceptionally (*necronite*) in granular limestone, the immediate associates being Biotite, balvraidite, and hydrous labradorite.

The more sodaic *sanidines* occur in tufa or in pitchstone.

The *corded orthoclases* alone occur in granitic veins or bands; the ordinary associates being oligoclase and Haughtonite in a quartz paste; the more occasional associates, arranged in the order of the frequency of their occurrence, being either muscovite or lepidomelane, tourmaline, apatite, garnet, beryl, magnetite, rarely ilmenite and pinite; and—where the veins cut syenite,—sphene, menaccanite, Allanite, Babbingtonite, zircon, and thorite.

Though a constituent of a greater number of rocks than any other of the

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[*] 8 Fevrier 1875.

felspars, there are rocks in which orthoclase finds no place:—its lithological habitudes are therefore clearly defined.

Albite in Scotland is confined to the somewhat hornblendic and epidotic red granite of Peterhead, the rare associates being agalmatolite and fluor.[*]

A rock *sui generis,* which is associated with serpentine, and which stretches in a belt from Fetheland Point to Hillswick Ness, contains it in association with hornblende or with augite; a graphic vein near serpentine, at Bigsetter Voe, may be an offset from the above.

An exceptional occurrence is that of the moonstone of Stromay, where it would appear to replace oligoclase. The lithological relationships of albite in Scotland are therefore ill-developed, if not also ill-defined.

Anorthite in Scotland seems to replace oligioclase in the diorite of Fetlar, and in portions of the gabbro of Ayrshire; as Latrobite it very rarely is found in granular limestone in Glen Gairn. Its relationships therefore are also ill-defined.

The habitudes of the other—the doubted—species are much more definite and clear.

Labradorite finds its place, admitting no other felspar to play its part, or even associate itself with it, in all the varieties of diabase,—gabbros, primitive greenstones, diallage rock, "heathens"—or whatever name their varying features may procure them—from Balta to Ballantrae. Showing a slight tendency to pass over to anorthite very rarely; only in one case (Fetlar) yielding to it. Even when in Glen Bucket the rock actually passes into diorite, the felspar is unchanged.

Nor has it stepped far afield when it also asserts itself as the felspar special to the porphyritic amygdaloids of the south-west and north-east. Le Hunt showed it to be the felspar of the traps on both sides of the Clyde; and we now find it to be that of Bervie, Kinneff, Thornyhythe, Tremuda, and all that coast.

Nothing could be stronger than the evidence of the rocks as to the specific individuality of this mineral.

Oligoclase again presents itself as the very frequent associate of the corded felspar in granitic veins[†]—only in one locality (Coyle) is it found in another association; the evidence as regards it is also perfectly definite. No one who has become familiar with the ever-recurring exfiltration veins—called *crocus* by the quarrymen—which lace the grey granite of Aberdeenshire, will hesitate in

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[*] The "felspar" of the rock of Corstorphine Hill is usually called albite,—it has not been analysed.

[†] It also constitutes the bulk of the grey granite of Aberdeen; this I find to consist of a great deal of oligoclase, little orthoclase, little quartz, very small quantities of muscovite, and a good deal of Haughtonite, such a compound as G. Rose calls *granitite.* The hornblendic gneiss of the Cape Wrath district frequently consists almost solely of a granular mixture of oligoclase and hornblende.

considering the well-defined crystals of the *white* felspar of these veins to be as thoroughly good a species as the accompanying flesh-coloured orthoclase.

A similar association with orthoclase is also to be seen in the veins—whether intrusive or exfiltration—which occur in hornblendic gneiss,—as at Rispond and Geo-na-Shermaig.

And I consider that it is even more singularly clear as to the last of the felspars—that which is generally held to be the most doubtful.

Andesine is by many held to be merely an altered oligoclase; but in Scotland oligoclase is a *granitic* felspar, andesine is a *gneissic* one; and it occurs only in, or in near proximity to primary limestone. One would have expected that the felspar specially pertaining to limestone would be the most calcareous of all; but it is not so; with the exception of the limestone locality at Glen Beg, andesine is the only felspar I have found in limestone—and it occurs only with limestone.

In Scotland it cannot, for two reasons, be altered oligoclase:—first, there is no oligoclase with it, or near it; and second, it is perfectly fresh, lustrous, undecomposed,—and in larger crystals than the oligoclase which occurs elsewhere.

In Scotland, therefore, the felspars belong so specially each to its own rock, that if we can tell the felspar, we may say that we are well on in the determination of the rock.

But can we tell the felspars? That is—can we discriminate between them as they ordinarily occur in rock masses?

That they can* be distinguished by the employment of the polariscope, according to the method established by DESCLOISEAUX, we know; but the bulky polariscope could not be employed in the field, even supposing that the obtaining plates sufficiently delicate for that mode of investigation were an effort of ordinary skill, which is very far from being the case.

We have the old physical distinction of *striation*,—laid down as infallible in separating the ortho from plagio-clastics; let us see if this be an infallible test, or what value is to be assigned to it.

No twinning of orthoclase has as yet been shown to be so repeated as to exhibit the phenomenon.

Does the absence of striation then entitle us to pronounce the felspar to be orthoclase?

Reverting to the physical descriptions of the felspars analysed, we find that to the eye, or to the lens there exhibited striation—of four albites, *one*—rarely; of ten oligoclases, *four*; of four andesines, *one*; of ten labradorites, *three*; of two anorthites, *one*; of one latrobite, *none*; or, altogether in the thirty-one plagioclastic felspars, striation was ten times seen, twenty-one times not seen.

The oligoclase near Lairg,—the crystals of which are perhaps larger than at

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* Except as regards the discrimination of oligoclase and andesine.

any other locality,—is cream-coloured, and possibly somewhat altered by incipient weathering; it does not show striation. From the centre of one of these crystals, however, a clear colourless translucent portion was obtained, which was distinctly striated: here then, one and the same crystal had a striated and an unstriated portion, the striated being inside.

We are hardly therefore justified in going even the length which COTTA does when he says—"This striping, when observable, is a very characteristic sign; but its absence is not equally so."

But it can only profess to be characteristic as between the ortho- and the plagio; for its being seen indiscriminately, sometimes in coarse pattern, sometimes of microscopic minuteness, among all the plagioclastics, renders it inapplicable as a mode of discriminating among *them;* and there is reason to believe that the time even now is, when it is more imperative that the field geologist should be able to determine between the various plagioclastic felspars, than even to recognise "common felspar."

As regards striation, then, its presence enables us to say very little,—its absence confers upon us no absolute knowledge whatever.

Let us now see what aid we derive from specific gravities.

The specific gravities of Scottish Orthoclases, numerically arranged, are the following,—the numbers being in some cases the mean of several determinations:—

Pitfechie Monnymusk, Aberdeen,	2·503
Glen Fernate, Perthshire,	2·525
Ben-na-chie, Aberdeen,	2·534
Ben Resipol, Argyle,	2·538
Blackwater, Ross-shire,	2·54
Loch of Leys, Aberdeen,	2·542
Struay, Ross,	2·544
Sclattey, Aberdeen,	2·545
Canisp, Sutherland,	2·545
Froster Hill, Aberdeen,	2·548
Yestnaby, Orkney,	2·549
Rinashat, Aberdeen,	2·55
Blirydrine, Kincardine,	2·551
West Stocklet, Harris,	2·552
Rubislaw, Aberdeen,	2·554
Lua yayi, Erribol, Sutherland,	2·554
Torry, Kincardine,	2·554
Lairg, Sutherland,	2·555
Scaire Ruidh, Harris,	2·555
Anguston, Aberdeen,	2·555
Scoltie Hill, Aberdeen,	2·555
Midstrath, Aberdeen,	2·555
Glen Beg, Glenelg,	2·558
Rubislaw, pinkish,	2·559
Shinness, Sutherland,	2·56
Cowhythe, Banffshire,	2·56
Spyhill, Kincardine,	2·562
Rubislaw, grey,	2·562
Geo-na-Shermaig, Cape Wrath,	2·563
Ben Capval, Harris,	2·565
Tongue, Sutherland,	2·569
Stromay, Harris,	2·574
Pitfodles, Aberdeen,	2·579
Cowhythe Head, Banff,	2·58

The average of these thirty-four is 2·553: the number 2·555 (occurring five times) may be more easily retained in the memory. The range ·077 is so small that the gravity of orthoclase must be regarded as a highly characteristic physical feature.

The specific gravities of the Albites determined are—

Colafirth Voe, Shetland (av. of 4),	2·607
Nudista, Hillswick (av. of 2),	2·615
Beautiful Rock, Colafirth,	2·622
Cleavlandite, Sutherland,	2·622
Moonstone, Harris,	2·627

Average, 2·618; the range, ·02.

Of Oligoclase:—

Anguston,	2·611
Lairg, Sutherland,	2·618
Craigiebuckler, Aberdeen,	2·622
Ben Dubh, Coyle, Aberdeen,	2·627
Rispond, Sutherland,	2·636
Rubislaw, Aberdeen,	2·637
Geo-na-Shermaig, Cape Wrath,	2·654
Barrahill, Aberdeen,	2·834

Average, 2·655; the exceptionality of the Barra Hill gives here the large range ·223. Excluding the anomalous Barra Hill specimen, the average is 2·629, and the range only ·043.

Of Andesine—

Glen Urquhart, Inverness-shire,	2·672
Crathie, Aberdeenshire,	2·677
Glen Gairn, white, Aberdeen,	2·689
Cowhythe, Banffshire,	2·692
Glen Gairn, blue,	2·705

Average, 2·687; range, ·033.

Of Labradorite—

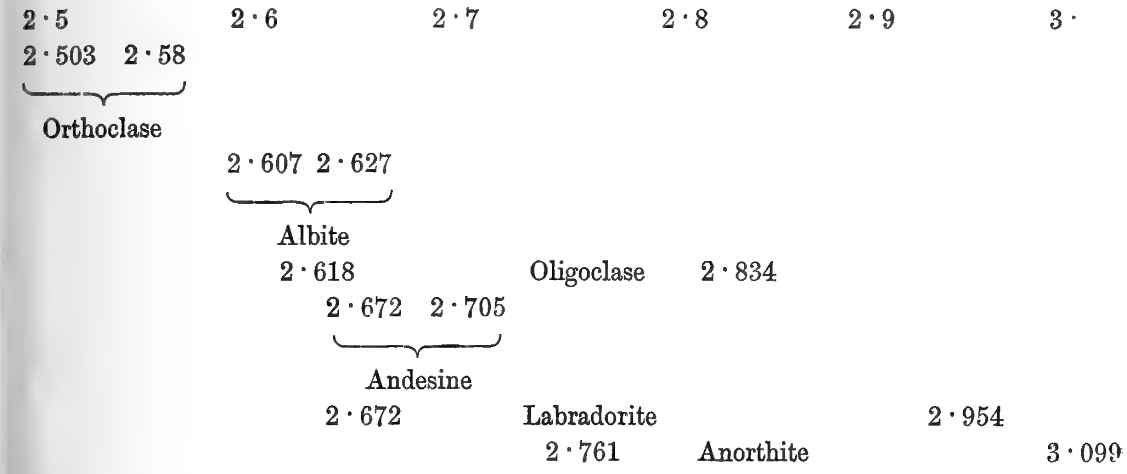
Portsoy, massive, Banffshire,	2·672
Glen Bucket, Aberdeen,	2·674
Kildrummy, Aberdeen,	2·674
Badnagoach, Aberdeen,	2·681
Balvraid, Glenelg, granular,	2·705
Balvraid, Glenelg, fibrous,	2·708
Loch Scavig, Skye,	2·715
Portsoy, crystallised,	2·813
Balta, blue, Shetland,	2·95
Balta, white,	2·954

Average, 2·755; range, ·282.

Combining Anorthite and Latrobite, we have—

Latrobite, Glen Gairn,	2·749
Lendalfoot, Ayrshire,	2·761
Fetlar, Shetland,	3·099

Average, 2·87; range, ·35.



A consideration of the above diagram shows that, as there exists a well-defined gap between the specific gravities of orthoclase and the other felspars, the former might, failing other distinctions, be recognised by its low gravity: secondly, that as the gravities of each of the plagioclastic felspars *overlaps* from some localities those of the others, no one of these species can be individualised by the determination of its gravity.

It is interesting, however, to observe the gradual increase of gravity, as the percentage of silica is diminished, and that of alumina increased, in the different members of this family.

Gravity having thus failed to lend much aid, we turn to the cleavage angle. The use of the reflecting goniometer, in far shorter time than any other process of determination, and with unerring certainty, enables us to determine the orthoclases. But when we measure such rough cleavages as are to be obtained by the splitting up of imbedded crystals of the other felspars, we do not obtain all the aid that we might hope for.

Of Albites,—true angle 86° 24′,—there were got—

Stromay,	86° 21′
Nudista, curved,	86 32
Colafirth,	86 45

Of Oligoclase,—true angle 86° 10′,—there were got—

Anguston, . .	86° 10′
Lairg, colourless, .	86 10
Rispond, . .	86 14
Craigiebuckler, . .	86 14
Dyce, . . .	86 15
Lairg, cream, . .	86 15
Rubislaw, . .	86 15
Coyle, . . .	86 32

Of Andesine,—angle according to DESCLOISEAUX 87° to 88°,—there were got—

Glen Gairn, . .	86° 21′
Crathie, . .	86 24
Glen Urquhart, .	86 28

Of Labradorite,—true angle 86° 40′,—there were got—

Kildrummy, .	86° 28′ to 86° 40′
Kinneff (curved), about	86 40
Balvraid, about .	86 40
Scavig, . . .	86 42
Portsoy, . .	86 42
Badnagoach, . .	86 45

Of Anorthite,—true angle 85° 50′,—there was got—

Lendalfoot, *hydrous*, .	86° 42′

This is not altogether satisfactory; we have much the same overlapping that we had in the gravities: an angle of 86° 15′, or under that, would pretty certainly indicate oligoclase;—of 86° 40′ or over it, would indicate labradorite. The low angle with a low gravity, and the high angle with a high gravity, would make assurance more than doubly sure:—this is about all that we can say; and it takes no account of albite, andesine, and anorthite, as regards at least their discrimination from one another.

Albite usually occurs in free crystals superimposed upon orthoclase;—this is, however, the chronicling of a habit, not the observation of an inherent property.

Truly we are sadly in want of a rapid and trustworthy test.

As to whether the eye, with the simple aid of the lens, can be educated to the recognition of all of these felspars, as they ordinarily occur in rocks, I can only say that,—after having selected from the rocks the specimens to be analysed, after having manipulated them by grinding on the lapidary's wheel for testing their gravity, after having examined with the lens their myriad chips in purifying them for analysis, and after having analysed them,—that is, after having obtained that absolute knowledge which enabled me to correct my own errors,—I should not even now like to have to pronounce upon certain obscure varieties;—but, that if I did see such an amount of recognisable features as to warrant the expression of an opinion, I should be inclined to maintain it against that of any one whose eye had not been educated to a similar extent.

The microscopic structure and optical properties of the felspars, with the aid to their recognition thereby afforded, will be considered in Part II. of this paper.

Andesine, anorthite, and Latrobite are now first introduced as British minerals.

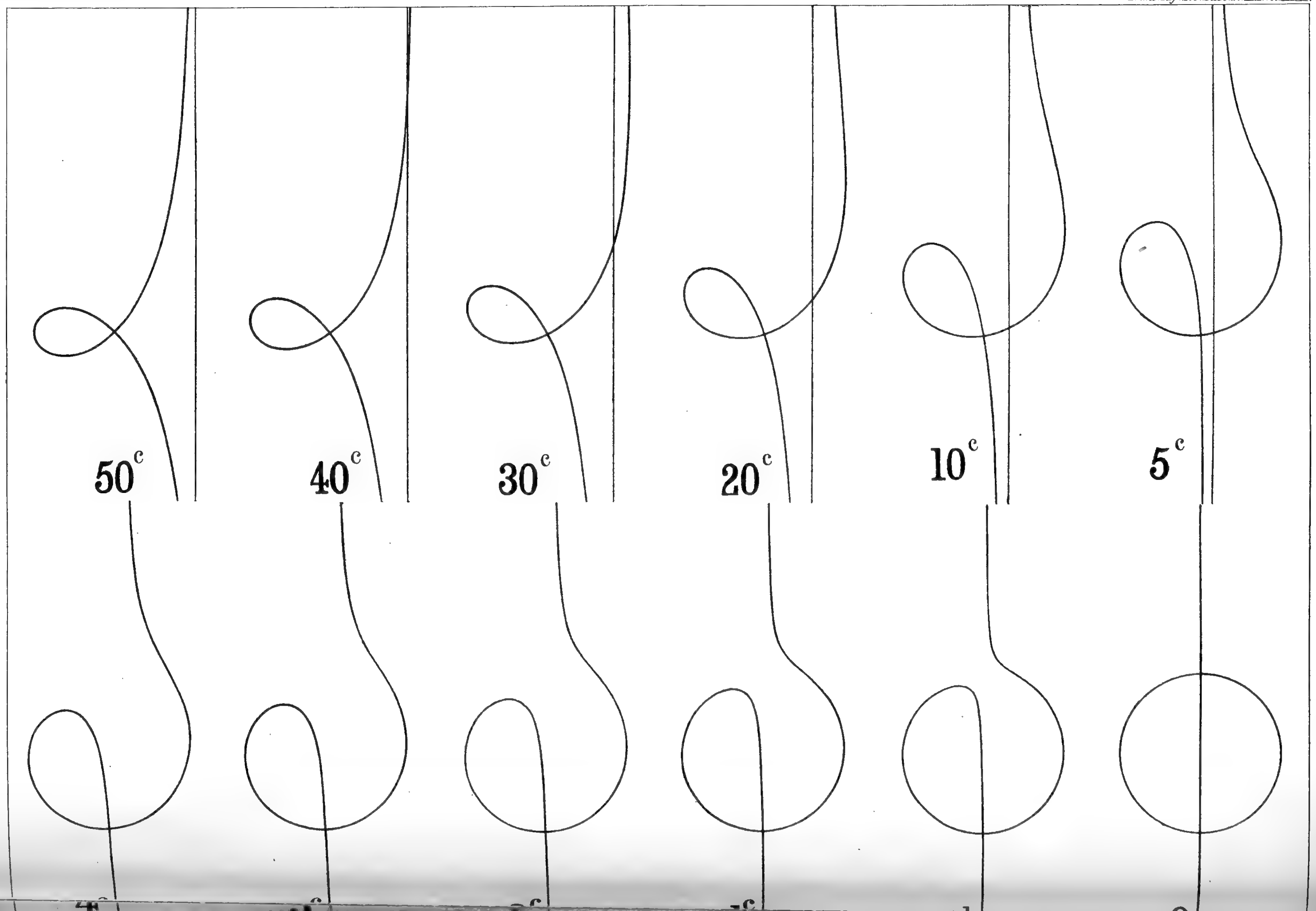
EXPLANATION OF PLATES XVII. AND XVIII.

Plate XVII.—In the centre is figured to size one of the crystals of amazonstone from Tongue. The figure on the left side thereof shows the appearance, with polarised light, of a slice of the same, cut parallel to *c*; magnified about 25 diameters: the undulating "corded structure" runs parallel to *a*, and exhibits gradations of colour on account of a difference in the thickness of the slice at its two edges: the twinned and dovetailed lineated structure, which is at right angles to the other, is parallel to *b*. The figure on the right shows the appearance of the "corded" structure in a slice cut parallel to *b*.

Plate XVIII.—Some of the forms of the Tongue crystals. No. 1 is in the possession of Professor Nicol of Aberdeen. Nos. 6 and 7 (Murchisonites from Arran) belong to Mr Dudgeon of Cargen. The others are in the collections of Mr Dudgeon and the Author.

REFLECTION FROM A REVOLVING WIRE.

Trans. Roy. Soc. Edin. Pl. XIX. Vol. XXVIII.



XI.—*On the Curves produced by Reflection from a Polished Revolving Straight Wire.* By EDWARD SANG, Esq. (Plate XIX.)

(Read 5th February 1877.)

If light, emanating from a fixed source, be reflected to the eye, also fixed, from the surface of a polished cylinder, which cylinder changes its position in some definite manner, the point of reflection moves in some curve or locus whose nature may be made the subject of investigation.

In the present paper I propose to examine that case in which an indefinitely thin cylinder is restricted to pass through a fixed point. The locus of the point of reflection is then a curved surface. For the present I shall still farther restrict the polished line to the plane passing through the vertex, the source of light, and the eye; the locus being then a plane curve.

Let then a fine polished straight line, extended indefinitely both ways, turn on the vertex O, while light emanating from the source A is reflected to the eye at B; it is required to investigate the locus of the point C at which the reflection takes place.

The reflection is only possible while the polished line passes outside of the angle AOB. When the wire lies along AO, the point of reflection is at A; when along OB, at B; and when it is equally inclined, outwards, to OA and OB, the reflection occurs at O, wherefore the curve must pass through the three points O, A, and B.

When the direction of the wire is between OA and OB, the optical genesis of the curve falls to be supplemented by the geometrical one, that the two lines AP, BP drawn to a point in it must make equal angles OPA, OPB on opposite sides of the wire OP.

The shape of the curve would seem to depend on two arguments, namely, the angle AOB and the ratio of OA to OB. However, if we take two positions OC and OD of the wire, making equal angles AOC, BOD outwards, and find the points C and D belonging to these positions, we find that the angle CAO is equal to DAO and CBO to DBO. So that if C were taken as the source of light and D as the position of the eye, A and B are points in the curve belonging to them; and hence we conclude that the curve belonging to C and D is identic with that belonging to A and B.

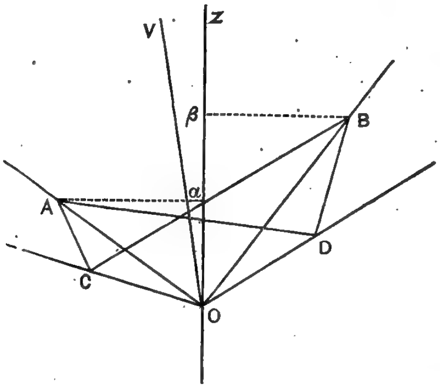


Fig. 1.

If OZ be drawn to bisect the straight line AB, the perpendiculars Aα and Bβ drawn to it are of equal length and the tracing point is at an infinite distance along it in either direction. Now it will be shown that the perpendiculars from C and D upon the same line are also alike, or that OZ bisects the straight line CD, while OV drawn to bisect the angle AOB, also bisects COD; wherefore the angle VOZ formed by OZ and OV is the same whether derived from AOB or from COD; it is thus the characterising angle of the curve.

If, for shortness, we put a, b, c, d, for the lengths of the four lines OA, OB, OC, OD, we easily obtain

$$c=\frac{ab\,.\sin \mathrm{COD}}{a\sin \mathrm{BOD}+b\sin \mathrm{AOD}}\ ;\qquad d=\frac{ab\,.\sin \mathrm{COD}}{b\,.\sin \mathrm{BOD}+a\sin \mathrm{AOD}}\,.$$

Observing that $\sin \mathrm{AOD}^2-\sin \mathrm{BOD}^2=\sin \mathrm{AOB}\,.\sin \mathrm{COD}$, we convert these into

$$a=\frac{cd\,.\sin \mathrm{AOB}}{-c\,.\sin \mathrm{BOD}+d\,.\sin \mathrm{AOD}}\ ;\quad b=\frac{cd\,.\sin \mathrm{AOB}}{-d\,.\sin \mathrm{BOD}+c\,.\sin \mathrm{AOD}}\ ;$$

which are just what would have been obtained on supposing c to be the source of light and D to be the position of the eye.

The angle VOZ as found from AOB, has for its tangent $\frac{b-a}{b+a}\tan \mathrm{AOV}$; as found from COD the expression becomes $\frac{d-c}{d+c}\tan \mathrm{COV}$; and on substituting for c and d their values in terms of a and b, we find these expressions to be identic; wherefore OZ bisects the straight line CD. From this it is obvious that the character of the locus depends on the angle VOZ.

Taking OZ for the axis Z of rectangular continates, let us put $Z_A=O\alpha=\alpha$; $Z_B=O\beta=\beta$, and $A\alpha=-X_A=B\beta=+X_B=\mu$, then

$$\tan \mathrm{AOZ}=\frac{\mu}{\alpha}\ ,\ \tan \mathrm{BOZ}=\frac{\mu}{\beta}.$$

whence $$\tan(\mathrm{AOZ}-\mathrm{BOZ})=\tan 2\mathrm{VOZ}=\frac{(\beta-\alpha)\mu}{\alpha\beta+\mu^2}\,.$$

Also, writing θ for the varying angle ZOD,

$$a\,.\sin \mathrm{AOD}=\alpha\,.\sin\theta+\mu\,.\cos\theta\ ;\ a\,.\cos \mathrm{AOD}=\alpha\,.\cos\theta-\mu\,.\sin\theta$$
$$b\,.\sin \mathrm{BOD}=\beta\,.\sin\theta-\mu\,.\cos\theta\ ;\ b\,.\cos \mathrm{BOD}=\beta\,.\cos\theta+\mu\,.\sin\theta\,,$$

wherefore the value of OD becomes

$$\mathrm{OD}=\frac{(\alpha\beta+\mu^2)\sin 2\theta+(\beta-\alpha)\mu\,.\cos 2\theta}{(\alpha+\beta)\sin\theta},$$

now
$$\alpha\beta+\mu^2=\surd(\alpha^2+\mu^2)\,.\,\surd(\beta^2+\mu^2)\cos 2\mathrm{VOZ}$$
$$(\beta-\alpha)\mu=\surd(\alpha^2+\mu^2)\,.\,\surd(\beta^2+\mu^2)\sin 2\mathrm{VOS},$$

so that
$$\mathrm{OD}=\frac{ab}{\alpha+\beta}\;\frac{\sin 2\mathrm{VOD}}{\sin \mathrm{ZOD}}.$$

The fourth proportional to $\alpha+\beta$, a and b determines the size of the curve, and may be called its *parameter*, p; and we may write V for the characterising angle VOZ, and so have, more concisely,

$$\mathrm{OD}=p\,.\,\frac{\sin(2\mathrm{V}+2\theta)}{\sin\theta};$$

here we may observe that $\mathrm{OD}\,.\sin\theta=x_{\mathrm{D}}$, and thus determine the curve from the equations

$$x=p\,.\sin 2(\mathrm{V}+\theta)\,;\;\; z=x\,.\cot\theta\,.$$

On eliminating θ from these equations we get

$$z^2x+x^3=p\,.\sin 2\mathrm{V}\,(z^2-x^2)+p\,.\cos 2\mathrm{V}\,.\,2xz,$$

which is of the third order in regard to x, of the second order in regard to z.

From these equations it is seen that the whole curve is included between the two indefinitely extended parallel lines $x=+p$ and $x=-p$, and that z is infinite when $\theta - 0$, and consequently when $a=p\,.\,\sin 2\mathrm{V}$; wherefore the line $x=p\sin 2\mathrm{V}$ is an asymptote to the curve. The intersections of the curve with a line drawn parallel to the asymptote may be found by a very simple construction. Describe a circle round O with the parameter p for its radius, cutting the asymptote in E and intersecting the proposed parallel to it in F and f, join EF, Ef and draw parallel to those OP, Op; P and p are points in the curve.

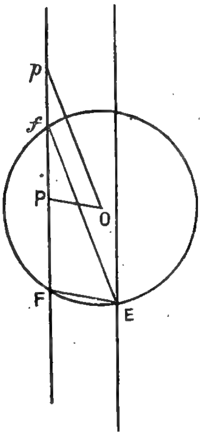


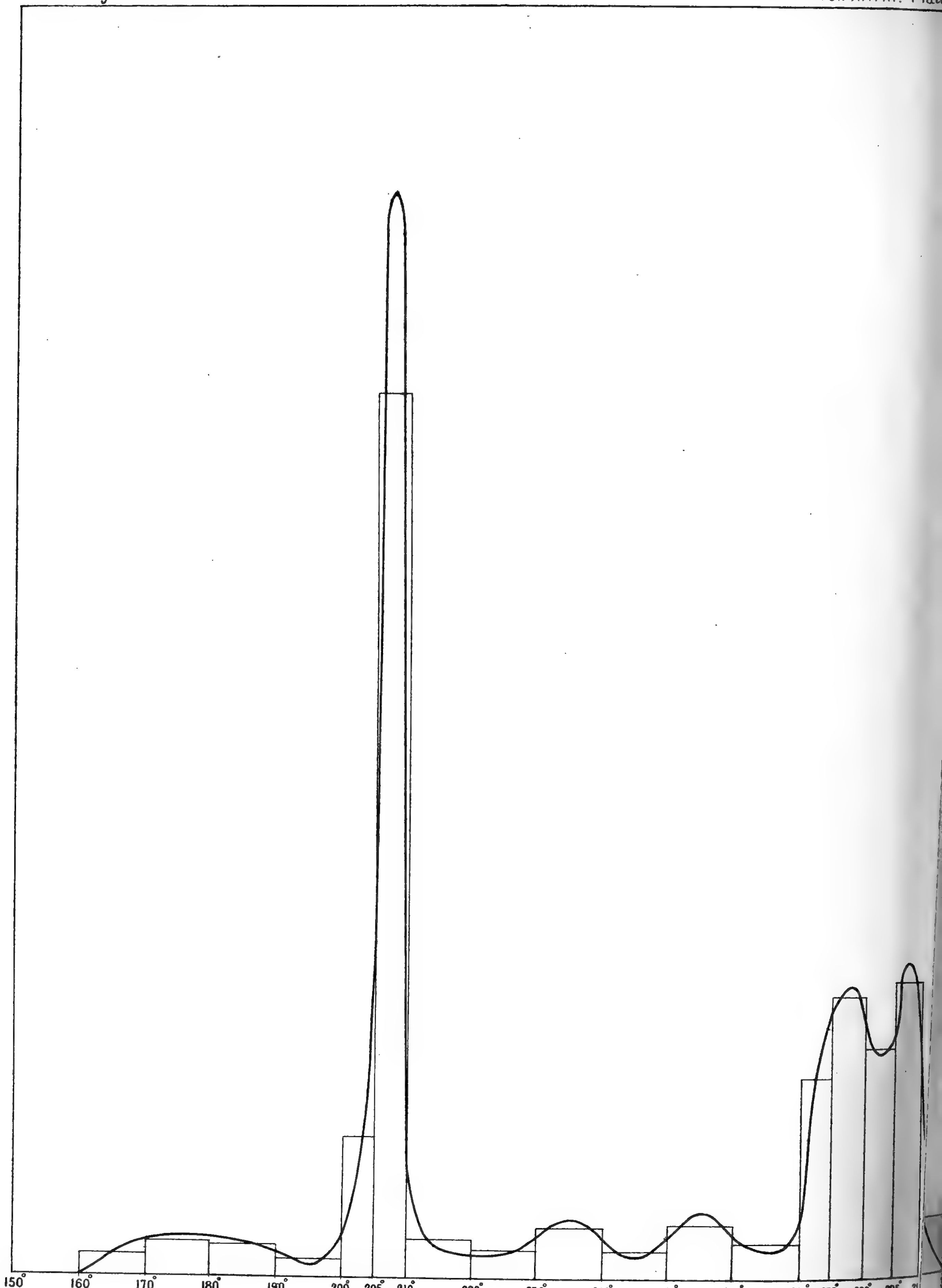
Fig. 2.

The aspect of this curve changes remarkably for different values of the angle V; those changes may be better shown by diagrams than explained by words; for this the accompanying twelve accurately-drawn varieties have been prepared. The first of these, corresponding to $\mathrm{V}=50^{\mathrm{c}}$, shows a symmetric curve lying all on one side of its asymptote and without any reflexure. The last of them, corresponding to $\mathrm{V}=0$, shows a circle transfixed by a diameter extended indefinitely both ways. It is interesting to study the transition from the one to the other of these incongruous forms. I have divided the half right angle into five equal parts and constructed the curves for

$V=40^c$, 30^c, 20^c, and 10^c, which should form, as it were, equal steps between the two extremes. The asymptote approaches to O, and the upper part of the curve passes across that asymptote, wherefore, after having reached a maximum distance beyond, it must again bend inwards, and so must have a point of reflexure; as is seen in the figure for $V=10^c$.

The difference in character between the figures for 10^c and for 0 being very marked, another set was interpolated, namely, the forms for 5^c, 4^c, 3^c, 2^c, and 1 centesimal degree. In the last of these we see well the progress of the upper part of the loop towards infinite curvature, as also that of the curve at the point of reflexure; and in order that this may be still better seen, the form for $V=$ one-half of a centesimal degree has been interjected.

This sudden change in the character of the curve is an interesting case of a general class of such transformations.


150°
160°
170°
180°
190°
200°
205°
210°
220°
230°
240°
250°
260°
270°
275°
280°
285°

XII.—*On the Solid Fatty Acids of Coco-Nut Oil.* By G. CARR ROBINSON, F.R.S.E., *Demonstrator of Chemistry, Public Health Laboratory, University of Edinburgh.* (Plate XX.)

(Read 21st January 1878.)

Most common oils and fats are mixtures of ethereal salts, termed glycerides or glyceric ethers, formed from glycerine and acids of the fatty, the oleic, and other allied series.

Of the methods employed for the decomposition of oils and fats, the following are the most usual:—

1. Distillation of the oil or fat; those which yield *volatile* acids, such as the acetins, butyrins, &c., may be more or less distilled without decomposition; but those which yield *fixed* acids, *e.g.*, palmitin, olein, &c., are almost wholly decomposed by a heat of 300° C., yielding acrolein, numerous hydrocarbons, &c.

2. Distillation with dilute sulphuric acid, by which the volatile acids are separated from the non-volatile.

3. Distillation in a current of superheated steam, when the glycerides are decomposed, the fatty acids being liberated.

4. Saponification; oils and fats are decomposed by caustic alkalies into glycerine and a soap, the soap, when boiled with dilute sulphuric or hydrochloric acid, being decomposed into a sulphate or chloride of the alkali employed and the fatty acid set free.

To separate a mixture of fatty acids various methods are employed, of these may be mentioned—the separation of the *volatile* from the *fixed* acids by distillation; fractional distillation of the volatile acids; Liebig's method of partial saturation; whilst the *fixed* acids are separated by dissolving in alcohol and fractional precipitation with acetate of lead. Another method is that usually adopted in the preparation of ethereal salts, and consists in saturating with gaseous hydrochloric acid the alcoholic solution of the mixed acids, separating the ethers so produced by fractional distillation, saponifying the pure ethers, decomposing the soap, and examining the fatty acid by analysis, melting point, &c., or by converting it into baryta, silver, or lead salt, and examining these.

It was determined, at the suggestion of Dr LETTS and Dr CRUM BROWN, that an examination should be made in this manner of coco-nut oil.

Coco-nut oil or coco butter is obtained by pressure from the fruit of certain coco-palms. It is whitish, of unctuous consistence, with a peculiar fatty smell. It is a mixture of several glycerides, containing also free acids.

Though the frequent subject of investigation, this research into coco-nut oil was undertaken with the view of ascertaining if the process of converting the mixed acids into ethers, and separating these by fractional distillation, would give results agreeing with those of other observers.

One of the most exhaustive memoirs on the constitution of coco butter is that by OUDEMANS (*Journal für Praktische Chemie,* lxxxi.). He saponified the coco butter with caustic potash, and from the potash soap obtained a lime salt by precipitating with ammonia and chloride of calcium; after treating the lime salt with ether, to remove any unsaponified fat, it is decomposed by hydrochloric acid, and the fatty acid, dissolved in alcohol, is subjected to fractional precipitation with acetate of baryta. In this way OUDEMANS obtained nine baryta salts; these were in turn decomposed by hydrochloric acid for the liberation of the fatty acids, the melting points of which were taken, and their composition arrived at by analysis. By this method OUDEMANS found in coco butter capric, caproic, caprylic, lauric, palmitic, and margaric acids, and denied the existence of cocinic acid ($C_{13}H_{26}O_2$).

Of other investigations of coco-nut oil may be mentioned those of BROMEIS (*Ann. Chem. Pharm.* xxxv.) and ST EVRÈ (*Ann. Chem. Phys.* xx.).

BROMEIS separated the acids by repeated crystallisation from alcohol; ST EVRÈ, in addition to this, prepared lead salts, extracted these with ether, and from them separated a body which was subjected afresh to crystallisation.

FEHLING (*Ann. Chem. Pharm.* liii.) obtained the volatile acids by distilling the coco butter with dilute sulphuric acid, saturating the distillate with baryta, and separating the baryta salts by their different solubility in water.

The following is the process employed in this research:—A quantity of coco butter, 5 kilograms, is melted with hot water, and boiled with a very weak soda lye for a period of eight hours; on cooling, the hard white soap is broken up, steam passed into it, and the soap solution boiled for another period of eight hours. After cooling the soap is well washed with cold water, then melted, hot water added, and the soap decomposed by the gradual addition of sulphuric acid. The fatty acids are separated from the sulphate of soda, glycerine, and excess of sulphuric acid, washed several times with boiling water, and again saponified, using at first a very weak lye. On cooling, the cake of hard soap is washed and decomposed with sulphuric acid as before.

The fatty acids so obtained, weighing about 3½ kilograms, are thoroughly washed with hot water, dried, and dissolved in 1·25 kilograms of absolute alcohol, and the alcoholic solution saturated with gaseous hydrochloric acid; the ethers, separated from the water, dissolved again in alcohol, and a second time saturated with gaseous hydrochloric acid, washed from all trace of hydrochloric acid, dried first with chloride of calcium, and finally with phosphoric anhydride.

The mixture of ethers, dried in this manner, was then submitted to fractional distillation; the first distillation showed no trace of acrolein, whilst some previous experiments on the fractional distillation of fatty acids had to be abandoned owing to the continual production of this substance. Water, however, clung to the ether fractions with great obstinacy, and was with difficulty removed even by frequent treatment of the ethers with phosphoric anhydride.

This frequent treatment with phosphoric anhydride, along with the necessary drying of the distilling apparatus between each fractionation, entailed very considerable loss in ethers, so that the subjoined table of ether fractions, giving the weights of the various fractions after the ethers had been separated as much as was possible by distillation, must be accepted as representing only approximately the acids in coco-nut oil.

The quantities and boiling points of the seventeen ether fractions so obtained are represented on the diagram, the area of each blue rectangle representing the mass of the distillate between the temperature corresponding to the two ordinates. (Plate XX.)

Ether Fractions.

B	Ether boiling point	between	160° and	170° C. =	6	grammes.
C	"	"	170	180	10	"
E	"	"	180	190	9	"
	"	"	190	200	5	"
D	"	"	200	205	21	"
A	"	"	205	210	133	"
F	"	"	210	220	11	"
	"	"	220	230	8	"
H	"	"	230	240	15	"
I	"	"	240	250	8	"
J	"	"	250	260	16	"
	"	"	260	270	10	"
L	"	"	270	275	30	"
M	"	"	275	280	42	"
N	"	"	280	285	34	"
O	"	"	285	290	44	"
P	"	"	290	300	17	"

The ether marked B, boiling between 160° and 170° C., was now saponified by boiling with caustic soda, the solution of soap decomposed by sulphuric acid, the fatty acid separated, and washed to free it from sulphate of soda and excess of sulphuric acid used, then boiled with water and carbonate of baryta, filtered, the baryta salt crystallised out and purified by fractional crystallisation.

The baryta salts from ethers, ranging from B to L, were all obtained in this way; the remaining four, M to P, were prepared by saponifying the ether, and after separating the fatty acid as before, dissolving this in alcohol, and boiling it with an alcoholic solution of acetate of baryta, and, when possible, crystallising the new baryta salt from alcohol.

The following are the analyses of these baryta salts :—

The barium was estimated in the usual way, by igniting the salt with sulphuric acid, and weighing as $BaSO_4$.

Analysis of baryta salt, marked A, from ether boiling between 205° and 210° C.

Five estimations of barium gave a mean of 32·50 per cent. barium.

Estimation of carbon and hydrogen ; combustion made with chromate of lead.

0·21625 grammes baryta salt gave—

0·337	grammes CO_2 =	42·54	per cent.	carbon.
0·13075	„ H_2O =	6·70	„	hydrogen.

Allowing for the carbon retained by the baryta as $BaCO_3$, and adding this to the carbon found, gives 45·31 per cent. carbon in salt,* thus—

100 grammes baryta salt yields, on analysis, 32·5 grammes Ba.
0·21625 grammes of salt (quantity used in combustion) = 0·07028 grammes Ba. And
137 grammes Ba require 44 grammes CO_2 to form $BaCO_3$.
∴ 0·07028 grammes Ba require 0·0225 grammes CO_2.
0·0225 + 0·337 (CO_2 found) = 0·3595 grammes CO_2.
= 45·31 per cent. carbon.

Analysis of baryta salt, marked A—

Carbon,	45·31
Hydrogen,	6·70
Barium,	32·50
Oxygen (by difference),	15·49
	100·00

Agreeing with barium caprylate $(C_8H_{15}O_2)_2Ba$, which requires—

Carbon,	45·39
Hydrogen,	7·09
Barium,	32·38
Oxygen,	15·14
	100·00

A weighed quantity of this salt, A, on being exposed for three hours to 100° C., did not lose weight; it is therefore anhydrous.

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* Explanation of this will be found at end of analyses of these salts.

From the acid obtained from ether, marked H, boiling point 230° to 240°, both a baryta and a lead salt were prepared.

Baryta salt, marked H, gave 28·53 per cent. Ba.

Analysis of lead salt, marked H.

0·3805 grammes lead salt gave—

0·60625 grammes CO_2 = 43·45 per cent. carbon.
0·252 „ H_2O = 7·35 „ hydrogen.

0·20225 grammes lead salt gave, after ignition with sulphuric acid,—

0·10775 grammes $PbSO_4$ = 38·03 per cent. lead.

Analysis of lead salt, marked H—

Carbon,	43·45
Hydrogen,	7·35
Lead,	38·03
Oxygen (by difference),	11·17
	100·00

Agreeing with lead caprate $(C_{10}H_{19}O_2)_2Pb$, which requires—

Carbon,	43·71
Hydrogen,	6·90
Lead,	37·70
Oxygen,	11·69
	100·00

Barium caprate $(C_{10}H_{19}O_2)_2Ba$ requires—

28·60 per cent. barium.

Analysis of baryta salt, marked N, from ether boiling point, 280° to 285° C.

This salt is anhydrous. 0·233 grammes salt, exposed for two hours to 100° C., did not lose in weight.

Three estimations of barium gave a mean of 25·40 per cent. Ba.

Estimation of carbon and hydrogen; combustion made with oxide of copper.

I. 0·05025 grammes baryta salt gave—

0·093 grammes CO_2 = 51·36 per cent. carbon.
0·04325 „ H_2O = 9·56 „ hydrogen.

II. 0·323 grammes baryta salt gave—

0·6045 grammes CO_2 = 51·04 per cent. carbon.
0·25125 „ H_2O = 8·66 „ hydrogen.

III. 0·1905 grammes baryta salt gave—

0·35775 grammes CO_2 = 51·21 per cent. carbon.
0·1545 „ H_2O = 9·00 „ hydrogen.

Allowing for carbon retained, as $BaCO_3$ (as in A), these show respectively—

I. Carbon,	53·28
II. „	53·67
III. „	53·45

	I.	II.	III.
Carbon,	53·28	53·67	53·45
Hydrogen,	9·56	8·66	9·00
Barium,	25·35	25·57	25·34

Agreeing with barium laurate $(C_{12}H_{23}O_2)_2Ba$, which requires—

Carbon,	53·82
Hydrogen,	8·59
Barium,	25·60
Oxygen,	11·99
	100·00

Analysis of baryta salt, marked O, from ether, boiling point 285° to 290° C. Two estimations of barium gave—

25·09 and 25·26 per cent. barium.

Estimation of carbon and hydrogen; combustion made with chromate of lead.

I. 0·13175 grammes baryta salt gave—

0·25625 grammes CO_2 = 53·39 per cent. carbon.
0·10975 „ H_2O = 9·25 „ hydrogen.

II. 0.17675 grammes baryta salt gave—

0·34525 grammes CO_2 = 53·27 per cent. carbon.
0·141 „ H_2O = 8·86 „ hydrogen.

Allowing for carbon retained as $BaCO_3$ (as in A and N), these show—

I. 55·11, and II. 55·46 per cent. carbon.

	I.	II.
Carbon,	55·11	55·46
Hydrogen,	9·25	8·86
Barium,	25·09	25·26

Agreeing with barium cocinate $(C_{13}H_{25}O_2)_2Ba$, which requires—

Carbon,	55·41
Hydrogen,	8·88
Barium,	24·33

The last analyses to be mentioned are those of baryta salt, marked P, from ether, boiling point 290° to 300°.

Analysis shows this salt to be the same as O.

Two estimations of barium gave—

25·10 and 24·63 per cent. barium.

Estimation of carbon and hydrogen; combustion made with chromate of lead.

I. 0·2095 grammes baryta salt gave—

0·407 grammes CO_2 = 52·98 per cent. carbon.
0·1685 „ H_2O = 8·93 „ hydrogen.

II. 0·1405 grammes baryta salt gave—

0·2725 grammes CO_2 = 52·89 per cent. carbon.
0·1175 „ H_2O = 9·28 „ hydrogen.

III. 0·551 grammes baryta salt gave—

1·0765 grammes CO_2 = 53·26 per cent. carbon.
0·412 „ H_2O = 8·29 „ hydrogen.

Allowing for carbon retained as $BaCO_3$ (as in A, N, and O), these show—

	I.	II.	III.
Carbon,	55·10	55·06	55·40

Analysis—

	I.	II.	III.
Carbon,	55·10	55·06	55·40
Hydrogen,	8·93	9·28	8·29
Barium,	25·10	24·63	

To prove that the corrections made in the carbon determinations of these analyses were not only justifiable but necessary, two analyses were made of the same baryta salt P with chromate of lead mixed with one-tenth of its weight of fused bichromate of potash, with the following results:—

I. Carbon,	55·40 per cent.
II. Carbon,	55·39 „

These agree well with carbon percentage demanded by formula $(C_{13}H_{25}O_2)_2$ Ba. Theory, 55·41; found 55·40 and 55·39.

These results show conclusively that when the combustion of baryta salts of the higher fatty acids is made with chromate of lead, that not only is the "carbon found" *always* less than that demanded by theory, but also that the deficiency in carbon is exactly accounted for by *all* the barium being left as carbonate, hence it is absolutely necessary to employ, along with chromate of lead, some other substance that readily gives up oxygen, such as bichromate of potash.

A mixture of fused borax and oxide of copper has been recommended in the analyses of salts such as these. Experiments made with this mixture were very unsatisfactory, the "carbon found" being the same as when oxide of copper alone is used.

The analysis of these baryta salts, prepared from the ethers, shows the presence of the same fatty acids as those found by other observers and by other methods.

OUDEMANS denies the presence of the acid $C_{13}H_{26}O_2$, cocinic acid, in coco butter, and doubts the existence of such an acid; whilst BROMEIS and ST EVRÈ both found an acid, melting point 35° C., and having the formula $C_{13}H_{26}O_2$.

FEHLING found in coco-nut oil an acid resembling, in appearance and melting point, that prepared by BROMEIS.

HEINTZ considers this acid a mixture on account of its low melting point, which is lower than that of lauric acid, $C_{12}H_{24}O_2$, whereas it should be intermediate between that of lauric acid, melting point 44° C., and myristic acid, melting point 54° C.; he finds, moreover, that a mixture of these two acids in the proportion of fourteen parts of lauric to two parts of myristic does melt at about 35° C.

On the other hand, the existence of this acid, $C_{13}H_{26}O_2$, in coco butter is confirmed by the analysis of the baryta salt marked P; the formula $(C_{13}H_{25}O_2)_2$ Ba, as already shown, requiring

Carbon,	55·41
Hydrogen,	8·88
Barium,	24·33

whilst these analyses, made with a mixture of chromate of lead and bichromate of potash, show—

	I.	II.
Carbon,	55·40	55·39

also the acid, from which this baryta salt was prepared, had a melting point of 45° C., and solidifies at very nearly the same temperature.

Having thus shown in this research the existence in coco butter of acids proved by others to be present, the process of converting the mixture of fatty acids into ethers, and separating these by fractional distillation, may be considered generally applicable for the examination of ordinary oils and fats.

In this investigation the chief difficulties met with were, first, in saponifying the coco-nut oil, when it was found that only by prolonged boiling with a weak lye, decomposing the soap, and boiling for a second time with caustic soda, was complete saponification effected; secondly, there was great difficulty experienced in thoroughly drying the ethers, even phosphoric anhydride, as already mentioned, only removing the water after prolonged contact with them. Chloride of calcium was of no use for drying them, and baryta could not be employed, as at ordinary temperature it slowly decomposes the ethers.

As before remarked, the accompanying diagram (Plate XX.) shows the ether-fractions after distillation had been carried on, until no longer any appreciable separation was effected. The rectangular areas represent the quantity in grms. of distillate between given intervals of temperature, whilst the curve represents what may be supposed to be the limit when the intervals of temperature are indefinitely diminished.

Such a curve is obviously of a highly characteristic appearance, depending upon the nature of the simple fats and the proportion in which they occur in the original mixture; and it is possible that curves such as this may be of use in identifying particular mixtures of volatile substances.

XIII.—*On the Tabulation of all Fractions having their Values between Two Prescribed Limits.* By EDWARD SANG, Esq.

(Read 21st January 1878.)

The object sought to be gained by this tabulation may be best seen by considering a specific case. When we wish to produce, by help of toothed wheels, a certain ratio between the rotations of two axes, we must have that ratio represented by integer numbers, and these numbers must contain no prime factors exceeding the limit of the number of teeth which can conveniently be cut in any wheel. If, for example, we wish to represent the mean motions of the planets, we have to express the ratios of their periodic times by decomposable numbers; and as this, in general, can only be done approximately, we have to prescribe some limit of error on either side, and have to seek for all fractions between those limits, and then among these to search for those that are suitable.

In my treatise on wheel-teeth a solution of this problem is given, complete so far as the discovery of the fractions is concerned, but imperfect in this, that it does not place those fractions in the order of their magnitudes. I propose now to supply this deficiency by explaining an exceedingly simple process, which enables us to make a complete list, arranged in the order of their values, of all irreducible fractions whose denominators shall not exceed some specified number; and also to take up and interpolate any portion of the list. This process is founded on a general theorem flowing directly from the doctrine of continued fractions, but of which the following simple demonstration may be given.

If there be two fractions, $\frac{A}{\alpha}$, $\frac{C}{\gamma}$, the cross products of whose members differ by unit, any fraction, as $\frac{B}{\beta}$, intermediate in value between them must be of the form—

$$\frac{B}{\beta}=\frac{pA+qC}{p\alpha+q\gamma}$$

where p and q are integer numbers, and this fraction is irreducible if p and q be prime to each other.

If we suppose $\frac{C}{\gamma}$ to be the greater of the two, so that $\alpha C-\gamma A=1$, the differences,

$$\frac{B}{\beta}-\frac{A}{\alpha}=\frac{\alpha B-\beta A}{\alpha\beta} \text{ and } \frac{C}{\gamma}-\frac{B}{\beta}=\frac{\beta C-\gamma B}{\beta\gamma},$$

must both be positive, and we may write $\alpha\mathrm{B}-\beta\mathrm{A}=q$, $\beta\mathrm{C}-\gamma\mathrm{B}=p$, p and q being necessarily integer and positive. On eliminating first β and then B from these, we obtain—

$$(\alpha\mathrm{C}-\gamma\mathrm{A})\mathrm{B}=p\mathrm{A}+q\mathrm{C}\,;\ (\alpha\mathrm{C}-\gamma\mathrm{A})\beta=p\alpha+q\gamma,$$

that is—

$$\mathrm{B}=p\mathrm{A}+q\mathrm{C}\,;\ \beta=p\alpha+q\gamma\,.$$

Hence, if we have two fractions, as $\frac{5}{7}$ and $\frac{8}{11}$, we may obtain others intermediate between them by inserting in the formula

$$\frac{5p+8q}{7p+11q}$$

any positive values for p and q, observing, however, that if p and q have a common divisor, the resulting fraction is reducible.

The fraction $\frac{\mathrm{B}}{\beta}$, however, so inserted between $\frac{\mathrm{A}}{\alpha}$ and $\frac{\mathrm{C}}{\gamma}$ does not necessarily give, with either of these, cross products differing by unit, for we have—

$$\alpha\mathrm{B}-\beta\mathrm{A}=q(\alpha\mathrm{C}-\gamma\mathrm{A})=q$$

and

$$\beta\mathrm{C}-\gamma\mathrm{B}=p(\alpha\mathrm{C}-\gamma\mathrm{A})=p,$$

and thus the same law of interpolation does not hold good; but if p and q be each taken as unit, the cross products on each side differ by unit.

From this we see that the lowest fraction intermediate in value between $\frac{5}{7}$ and $\frac{8}{11}$ is $\frac{13}{18}$, and that each interval of the progression, $\frac{5}{7}, \frac{13}{18}, \frac{8}{11}$, may be filled up in the same way, so as to give $\frac{5}{7}, \frac{18}{25}, \frac{13}{18}, \frac{21}{29}, \frac{8}{11}$.

If, then, we wish to insert between the limits $\frac{5}{7}$ and $\frac{8}{11}$ all fractions whose denominators shall not exceed, say 50, we have only to continue this process until the sum of the contiguous numbers do not exceed 50. Our list is then—

$$\frac{5}{7}, \frac{33}{46}, \frac{28}{39}, \frac{23}{32}, \frac{18}{25}, \frac{34}{47}, \frac{21}{29}, \frac{29}{40}, \frac{8}{11}.$$

As an interesting application of this method, I shall propose to compute those trains of wheels which shall convert mean solar into mean sidereal time so accurately as not to err by one second in the year.

For this we observe that, according to the computations by Bessel, the equinoctial year consists of 36 524 221·7 solar, or of 36 624 221·7 sidereal seconds, and that, therefore, these numbers represent the ratio to be produced by the wheel-work. Also, the coincidence of the beats of the solar and sidereal clock occurs at intervals of 365 solar or 366 sidereal seconds, wherefore an error of that much in the estimated length of the year would produce a discrepancy of one second between the two clocks. The limits of the ratio become, then, on the one hand $\frac{36\,524\,586\cdot 9}{36\,624\,586\cdot 9}$, and on the other $\frac{36\,523\,856\cdot 5}{36\,623\,856\cdot 5}$, and our business is to make a list of all those fractions whose values are between these limits. Moreover, since the members of these fractions have to be resolved into their factors, and since BURCKHARDT's table is the only one available for this resolution, we have the further restriction that the members of the fractions shall not exceed three million.

On converting these into continual fractions, we find the quotients 1, 365, 4, 14, &c., and 1, 365, 4, 5, &c., respectively, giving the successive approximations

$$\frac{365}{366},\ \frac{1461}{1465},\ \frac{20819}{20876},\ \text{and}\ \frac{365}{366},\ \frac{1461}{1465},\ \frac{7670}{7691};$$

wherefore every fraction between the prescribed limits must be of the form

$$\frac{365p+1461q}{366p+1465q},$$

in which q exceeds $5p$, and is less than $14p$; hence, assuming $p=1$, and making q successively 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, we obtain the series $\frac{7670}{7691}$, $\frac{9131}{9156}$, $\frac{10592}{10621}$, $\frac{12053}{12086}$, $\frac{13514}{13551}$, $\frac{14975}{15016}$, $\frac{16436}{16481}$, $\frac{17897}{17946}$, $\frac{19358}{19411}$, $\frac{20819}{20876}$, in which the cross products of every adjoining pair differ by unit; so that no fraction in lower terms can have its value intermediate between any two.

If we agree to use only denominators under 21,000, we may insert $\frac{16801}{16847}$ between the first and second, $\frac{19723}{19777}$ between the second and third.

On examination, by help of BURCKHARDT's table of divisors, we find that not one of these twelve ratios can be resolved into others admissible in clock-work. For example, the fraction $\frac{10592}{10621}$ may be written $\frac{2.2.2.2.2.331}{13.19.43}$; but 331 is an inconveniently large number for the teeth of a clock-wheel. We must, therefore, go to fractions with higher denominators.

The most convenient way of arranging the work is to write each fraction on a separate card, placing the cards in their proper order: to interpolate between two, we write the sums of the two members on a new card, which is then placed between the others, and so is ready for other interpolations.

On computing in this way all fractions with members of five places, and whose values are between the prescribed limits, we obtain 173 cases, of which, however, only eight can be resolved into products of primes less than 240, which may be taken as the limit of the number of teeth for a clock-wheel. These cases are shown in the following list:—

Fraction.	Year.	Annual Error.
$\frac{87292}{87531} = \frac{4.139.157}{3.163.179}$	·238 494	+1·02
$\frac{41272}{41385} = \frac{8.7.11.67}{3.5.31.89}$	·238 938	+ ·90
$\frac{99345}{99617} = \frac{3.5.37.179}{7.7.19.107}$	·238 970	+ ·89
$\frac{59534}{59697} = \frac{2.17.17.103}{9.9.11.67}$	·239 264	+ ·81
$\frac{88023}{88264} = \frac{3.13.37.61}{8.11.17.59}$	·240 664	+ ·42
$\frac{50038}{50175} = \frac{2.127.197}{9.25.223}$	·240 876	+ ·37
$\frac{91676}{91927} = \frac{4.13.41.43}{11.61.137}$	·243 028	− ·22
$\frac{34333}{34427} = \frac{13.19.139}{173.199}$	·244 681	− ·67

The fifth case in this list gives a train of low-numbered wheels, viz.,

$$\frac{37}{34} \times \frac{39}{44} \times \frac{61}{59},$$

which gives an annual gain by the sidereal index of only ·42 of a decimal second; that is, of ·36 of a common second; that is to say, an error of about 1 second in three years.

By inserting fractions in the intervals of these 173, we obtain others with larger numbers, and among these we may find new cases available for our purpose. But we have already obtained a low-numbered train with an error in defect of only 155·3 in the length of the year; wherefore it would be a waste of labour to compute the fractions outside of that degree of accuracy; wherefore we may now restrict our inquiries to ratios between the limits

$$365·240\,664 : 366·240\,664 \text{ and}$$
$$365·243\,770 : 366·243\,770;$$

or, what is the same thing, we have to insert fractions in the intervals of the series from $\frac{88023}{88264}$ to $\frac{73414}{73615}$ already found, there being 71 of these intervals. Of fractions whose denominators are below 200 000, we find 284; of which, therefore, 212 are new, or between 100 000 and 200 000; and among these we find only three with prime factors less than 240. These are—

Fraction.	Year.	Annual Error.
$\frac{119\,799}{120\,127} = \frac{3.9.9.17.29}{7.131.131}$	·240 854	+ ·35
$\frac{177\,873}{178\,360} = \frac{3.211.281}{8.5.7.7.7.13}$	·242 300	— ·02
$\frac{183\,717}{184\,220} = \frac{3.3.137\ 149}{4.5.61.151}$	·242 545	— ·09
$\frac{133\,679}{134\,045} = \frac{7.13.13.113}{5.17.19.83}$	·243 169	— ·26

among which is written one fraction having the prime factor 281, but noticeable on account of the very close approximation.

Restricting now our inquiries to fractions representing the year as between 365·241 890 and 365·242 545, that is, to a limit of the error of the clock of one second in eleven years; and, at the same time extending the range of the denominators to 400 000, we obtain a new series of approximations.

We have already 60 fractions within these limits; on interpolating we get 177 new ones, among which three only have their members decomposable into prime factors sufficiently small; these are—

Fraction.	Year.	Annual Error.
$\frac{266\,992}{267\,723} = \frac{16.11.37.41}{9.151.197}$	·242 134	+ ·023
$\frac{271\,375}{272\,118} = \frac{125.13.167}{2.3.7.11.19.31}$	·242 261	— ·012
$\frac{256\,035}{256\,736} = \frac{3.5.13.13.101}{32.71.113}$	·242 510	— ·080

one of which errs by only the 80th part of a decimal second in the year. Hence in our further search we may narrow the limits of the error to between 365·242 171 and 365·242 261 for the length of the year, that is, to between the fractions

$$\frac{174\,951}{175\,430} \text{ and } \frac{271\,375}{272\,118}.$$

On continuing this interpolation to all denominators less than one million, we get 169 new fractions between these limits, not one of which is decomposable into prime factors below 240; we get, however—

Fraction.	Year.	Annual Error.
$\frac{618\,355}{620\,048} = \frac{5.19.23.283}{16.11.13.271}$	·242 175	+ ·011
$\frac{879\,138}{881\,545} = \frac{2.3.3.13.13.17.17}{5.7.89.283}$	·242 210	+ ·002

The latter of these has the inconveniently large prime factor 283, but gives an approximation perhaps within the limits of error in the determination in the length of the equinoctial year.

This mode of interpolation enables us to suit the order of proceeding to the circumstances of the case. We might, for example, have at first taken very narrow limits of error, pushed the interpolation therewithin to the utmost extent of our table of divisions, and only enlarged the limits of error when compelled by the non-discovery of useful results.

On representing the ratio 365·242 217 : 366·242 217 by a chain-fraction we get the quotients 1, 365, 4, 7, 1, 3, 1, 1, 3, 7, which give the approximations—

$$\frac{10\,592}{10\,621},\ \frac{12\,053}{12\,086},\ \frac{46\,751}{46\,879},\ \frac{58\,804}{58\,965},\ \frac{105\,555}{105\,844},\ \frac{375\,469}{376\,497},\ \frac{2\,733\,838}{2\,741\,323},\ \&c.,$$

alternately on the one and on the other side of the true value. Arranging these in the order of their magnitude, and writing the corresponding length of the year opposite each, we get

$$\frac{10\,592}{10\,621}\ ·241\,380;\ \frac{46\,751}{46\,879}\ ·242\,187;\ \frac{105\,555}{105\,884}\ ·242\,214;\ \frac{2\,733\,838}{2\,741\,323}\ ·242\,217;$$
$$\frac{375\,469}{376\,497}\ ·242\,218;\ \frac{58\,804}{58\,965}\ ·242\,236;\ \frac{12\,053}{12\,086}\ ·242\,424.$$

Now here we observe that the values ·242 214 and ·242 218 are within the range of the error of astronomical determination; wherefore any fraction between the limits $\frac{105\,555}{105\,884}$ and $\frac{375\,469}{376\,497}$ may be regarded as expressing the true ratio. Between these limits we have already inserted and examined all those fractions whose denominators are below one million, and have now to treat those expressed by numbers between 1 000 000 and 3 036 000, the limit of Burckhardt's table.

In this way, by a process exceedingly simple in its principles and mode of application, but very laborious in the actual work, we can discover all those combinations of prime factors which express, with sufficient precision, any proposed ratio. The great labour is in seeking the divisions of large numbers; and here it may be permitted to me to make a few remarks on the usefulness of BURCKHARDT's table.

To the great majority of computers the knowledge of the divisors of large numbers is of no moment, and the author of a table of such divisors up to several millions may be regarded as a mere enthusiast. Not one among a hundred practised calculators may ever have occasion to consult such a work. Yet, for all that, there is no arithmetical table which, in proportion to the labour bestowed upon it, effects such a saving to those investigators who require its aid. For example, in the preceding search for a train of wheels to connect the indicators of solar and sidereal time, we have had to decompose many very large numbers. The examination of one of these, particularly when it turns out to be a prime, might have cost us a whole day. So irksome, too, is this operation, that had such an inquiry been imperative, we might even have preferred to prepare for it by first constructing the table of divisors.

The method of differences, of so much use in the formation of many other tables, is here inapplicable; each number has to be examined by itself, the decomposition of one being no guide to the factors of the adjoining numbers. The user of the table has to rely implicitly on its accuracy. If the tabular statement be that a proposed number is divisible we can easily verify it by actual division; were we to find it indivisible we should remain as ignorant of the factors of the proposed number as if we had had no table. And when the assertion is that the number is prime, we have no means of verification other than the tedious one of actual trial.

Having had occasion to verify the divisibility of many hundred numbers, as shown in BURCKHARDT's table, I am desirous to bear testimony to its wondrous freedom from error. Only in one case have I found a mistake. In my copy of the work the number 854647 is marked as divisible by 7, which it is not, and the succeeding number 854651 is marked as prime, while it is divisible by 7. I have ascertained that 854647 is in reality prime. The work has been printed from movable, not from solid type, and, in all probability, the error has been caused by the withdrawal and misplacement of a type while at press. There is also another (unimportant) fault; in place of the title 6400, 6300 is printed.

The processes above explained enable us to make a progressive list of all ratios expressible by numbers below a prescribed limit, and to select that portion of the progression which may be applicable to our purpose. By help of BURCKHARDT's table of divisors we are then able to decompose those numbers into their factors, and so to discover what of those ratios may be produced by the combination of wheels.

Working from the exact ratio of solar to sidereal time, and extending the list on either side for numbers below 3 036 000, until we find a decomposable ratio, we come, on the one hand, to $\frac{2\,200\,219}{2\,206\,243} = \frac{7.19.71.233}{13.17.67.149}$, corresponding to 365·242 199 for the length of the year; and, on the other hand, to $\frac{1\,871\,136}{1\,876\,259} = \frac{32.9.73.89}{7.7.11.59.59}$ corresponding to 365·242 241.

The latter of these gives a train of very low-numbered wheels, with a retardation of the sidereal index by one decimal second in 152 years, or one common second in 175 years.

As clocks showing both solar and sidereal time are coming into use, I subjoin a note of the approximations found in the course of this inquiry :—

$$\frac{87\,292}{87\,531} = \frac{4.139.157}{3.163.179} \qquad \cdot 238\,494$$

$$\frac{41\,272}{41\,385} = \frac{8.7.11.67}{3.5.31.89} \qquad \cdot 238\,938$$

$$\frac{99\,345}{99\,617} = \frac{3.5.37.179}{7.7.19.107} \qquad \cdot 238\,970$$

$$\frac{59\,534}{59\,697} = \frac{2.17.17.103}{9.9.11.67} \qquad \cdot 239\,264$$

$$\frac{88\,023}{88\,264} = \frac{3.13.37.61}{8.11.17.59} \qquad \cdot 240\,664$$

$$\frac{119\,799}{120\,127} = \frac{4.8.17.29}{7.131.131} \qquad \cdot 240\,854$$

$$\frac{50\,038}{50\,175} = \frac{2.127.197}{9.25.223} \qquad \cdot 240\,876$$

$$\frac{255\,304}{256\,003} = \frac{8.7.47.97}{11.17.37.37} \qquad \cdot 241\,774$$

$$\frac{368\,529}{369\,538} = \frac{3.7.7.23.109}{2.13.61.233} \qquad \cdot 241\,823$$

$$\frac{741\,076}{743\,105} = \frac{4.7.7.19.199}{5.11.59.229} \qquad \cdot 241\,991$$

$$\frac{266\,992}{267\,723} = \frac{16.11.37.41}{3.3.151.197} \qquad \cdot 242\,134$$

$$\frac{2\,200\,219}{2\,206\,243} = \frac{7.19.71.233}{13.17.67.149} \qquad \cdot 242\,199$$

$$\frac{1\,871\,136}{1\,876\,259} = \frac{32.9.73.89}{7.7.11.59.59} \qquad \cdot 242\,241$$

$$\frac{1\,471\,561}{1\,475\,590} = \frac{7.13.103.157}{10.41.59.61} \qquad \cdot 242\,244$$

$$\frac{1\,100\,475}{1\,103\,488} = \frac{9.25.67.73}{8.16.37.233} \qquad \cdot 242\,283$$

$$\frac{450\,709}{451\,943} = \frac{7.31.31.67}{41.73.151} \qquad \cdot 242\,301$$

$$\frac{761\,165}{763\,249} = \frac{5.41.47.79}{17.17.19.139} \qquad \cdot 242\,322$$

$$\frac{679\,716}{681\,577} = \frac{4.9.79.239}{13.13.37.109} \qquad {\cdot}242\,343$$

$$\frac{550\,055}{551\,561} = \frac{5.11.73.137}{43.101.127} \qquad {\cdot}242\,364$$

$$\frac{420\,394}{421\,545} = \frac{2.13.19.23.37}{3.5.157.179} \qquad {\cdot}242\,398$$

$$\frac{256\,035}{256\,736} = \frac{3.5.13.13.101}{4.8.71.113} \qquad {\cdot}242\,510$$

$$\frac{183\,717}{184\,220} = \frac{3.3.137.149}{4.5.61.151} \qquad {\cdot}242\,544$$

$$\frac{91\,676}{91\,927} = \frac{4.13.41.43}{11.61.137} \qquad {\cdot}243\,028$$

$$\frac{133\,679}{134\,045} = \frac{7.13.13.113}{5.17.19.83} \qquad {\cdot}243\,169$$

$$\frac{34\,333}{34\,427} = \frac{13.19.139}{173.199} \qquad {\cdot}244\,681$$

In this way we are able to compute the train of wheels giving a close approximation to any proposed ratio, and so could construct a machine to indicate say the mean longitudes and anomalies of the planets, with the arguments for their perturbations, with a precision commensurate with our knowledge of the elements of their motions.

The series of fractions thus inserted between two prescribed limits may be regarded as a small portion of a list extending from unit to $\frac{0}{1}$ on the one side, and to $\frac{1}{0}$ on the other. The filling in of each interval between two fractions whose cross-products differ by unit is, in truth, an epitome of the complete series from zero to infinity. The fraction $\frac{\mathrm{B}}{\beta}$ inserted between $\frac{\mathrm{A}}{a}$ and $\frac{\mathrm{C}}{\gamma}$ is near to the former when $\frac{p}{q}$ is greater than unit, and is near to the latter when $\frac{p}{q}$ is almost zero; that is to say, the complete insertion of all fractions between $\frac{\mathrm{A}}{a}$ and $\frac{\mathrm{C}}{\gamma}$ requires that we assign to $\frac{p}{q}$ all values from $\frac{1}{0}$ to $\frac{0}{1}$, represented by integer numbers prime to each other. In the actual tabulation we are tied down to some limit for the magnitude of the numbers; thus, in the example which has been given, B and β must be within the limits of our table of divisors, and the values of p and q are restricted by the condition $p\mathrm{A}+q\mathrm{B}<3\,036\,000$.

If, after having made a list of fractions represented by numbers under n, we wish to interpolate those represented by numbers up to some much larger number N, we may use for p and q the values contained in a more limited list from $\frac{1}{0}$ to $\frac{0}{1}$. For the purpose of facilitating such computations, I have constructed the subjoined table of all ratios represented by numbers up to 20, with their values in decimals to six places. In order to study the effects of the use of such a minor list, let us insert between the fractions $\frac{\mathrm{A}}{\alpha}$, $\frac{\mathrm{C}}{\gamma}$ first one intermediate by help of the multipliers p, q; and then another intermediate by help of r and s; these intermediate fractions are—

$$\frac{p\mathrm{A}+q\mathrm{C}}{p\alpha+q\gamma} \text{ and } \frac{r\mathrm{A}+s\mathrm{C}}{r\alpha-s\gamma}\,.$$

The cross products of the members of these fractions are—

$$pr\mathrm{A}\alpha+ps\mathrm{C}\alpha+qr\mathrm{A}\gamma+qs\mathrm{C}\gamma$$
$$\text{and } pr\mathrm{A}\alpha+ps\mathrm{A}\gamma+qr\mathrm{C}\alpha+qs\mathrm{C}\gamma,$$

which differ by $(ps-qr)$ $\mathrm{C}\alpha-\mathrm{A}\gamma)$, that is to say, by the product of the corresponding differences for the pairs of fractions $\frac{\mathrm{A}}{\alpha}$, $\frac{\mathrm{C}}{\gamma}$ and $\frac{r}{s}$, $\frac{p}{q}$. Hence if each of these pairs be contiguous in the lists, the interpolated fractions are also contiguous; that is to say, no fraction of intermediate value can be in lower terms.

By using for p and q the successive pairs of values taken from such a table as the subjoined, we are almost enabled to dispense with the use of movable cards.

If the two limiting fractions $\frac{\mathrm{A}}{\alpha}$, $\frac{\mathrm{C}}{\gamma}$ be not contiguous, that is, if $\alpha\mathrm{C}-\mathrm{A}\gamma$ be other than unit, the interpolated fraction $\frac{\mathrm{B}}{\beta}$ may not be in its lowest terms. Let us suppose that $p\mathrm{A}+q\mathrm{C}$ and $p\alpha+q\gamma$ have a common divisor n, or that

$$p\mathrm{A}+q\mathrm{C}=n\mathrm{B}\ ,\ p\alpha+q\gamma=n\beta.$$

we find, on eliminating p,

$$n(\mathrm{B}\alpha-\mathrm{A}p)=q(\mathrm{C}\alpha-\mathrm{A}\gamma),$$

and on eliminating q,

$$n(\mathrm{C}\beta-\mathrm{B}\gamma)=p(\mathrm{C}\alpha-\mathrm{A}\gamma),$$

wherefore, n must be a divisor of q $(\mathrm{C}\alpha-\mathrm{A}\gamma)$ and also of $p(\mathrm{C}\alpha-\mathrm{A}\gamma)$; now, p and q are always supposed to be prime to each other, wherefore n must be a divisor of $\mathrm{C}\alpha$—$\mathrm{A}\gamma$. That is to say, a fraction interpolated between $\frac{\mathrm{A}}{\alpha}$ and $\frac{\mathrm{C}}{\gamma}$ by means of two multipliers p and q prime to each other, can be reduced to lower terms by a common divisor, either $\mathrm{C}\alpha-\mathrm{A}\gamma$ itself, or some of its factors.

By taking note of this useful theorem, we are enabled to begin our operations with any two fractions whatever.

As an example, we may propose to compute those fractions which are intermediate in value between $\frac{9}{17}$ and $\frac{20}{23}$, where the difference between the cross products is $133=7\cdot19$. Proceeding by the addition of the members of the adjacent fractions, we have—

$$\frac{9}{17},\ \frac{29}{40},\ \frac{20}{23},$$

where the difference 133 still subsists. Another step gives us—

$$\frac{9}{17},\ \frac{38}{57},\ \frac{29}{40},\ \frac{49}{63},\ \frac{20}{23},$$

where we observe that the second fraction may be simplified by the divisor 19, the fourth fraction by the divisor 7, thus giving—

$$\frac{9}{17},\ \frac{2}{3},\ \frac{29}{40},\ \frac{7}{9},\ \frac{20}{23}.$$

In the first and second intervals the difference is 7, in the third and fourth 19. The next operation gives—

$$\frac{9}{17},\ \frac{11}{20},\ \frac{2}{3},\ \frac{31}{43},\ \frac{29}{40},\ \frac{36}{49},\ \frac{7}{9},\ \frac{27}{32},\ \frac{20}{23},$$

which are all in their lowest terms; the differences of the cross products remaining 7 and 19 as before. In the same way we may continue until all the differences become unit.

Instead of proceeding by simple addition, that is, by using $p=1$, $q=1$, we may determine p and q directly, so as to give a common divisor 7 or 19. Thus, taking the first pair of the above series, we may resolve the indeterminate equation $9p+11q=7n$, or $1p+2q=7n$, giving $p=1$, $q=3$, whence the intermediate fraction $\frac{42}{77}=\frac{6}{11}$. The list then is $\frac{9}{17}$, $\frac{6}{11}$, $\frac{20}{37}$, &c.

Though this method appear to be more direct, it is in reality much more laborious than that which we used at first.

Table of all Ratios represented by Numbers up to 20.

·050 000	1 : 20	20·000 000	·150 000	3 : 20	6·666 667
·052 632	1 : 19	19·000 000	·153 846	2 : 13	6·500 000
·055 556	1 : 18	18·000 000	·157 895	3 : 19	6·333 333
·058 824	1 : 17	17·000 000	·166 667	1 : 6	6·000 000
·062 500	1 : 16	16·000 000	·176 471	3 : 17	5·666 667
·066 667	1 : 15	15·000 000	·181 818	2 : 11	5·500 000
·071 429	1 : 14	14·000 000	·187 500	3 : 16	5·333 333
·076 923	1 : 13	13·000 000	·200 000	1 : 5	5·000 000
·083 333	1 : 12	12·000 000	·210 526	4 : 19	4·750 000
·090 909	1 : 11	11·000 000	·214 286	3 : 14	4·666 667
·100 000	1 : 10	10·000 000	·222 222	2 : 9	4·500 000
·105 263	2 : 19	9·500 000	·230 769	3 : 13	4·333 333
·111 111	1 : 9	9·000 000	·235 294	4 : 17	4·250 000
·117 647	2 : 17	8·500 000	·250 000	1 : 4	4·000 000
·125 000	1 : 8	8·000 000	·263 158	5 : 19	3·800 000
·133 333	2 : 15	7·500 000	·266 667	4 : 15	3·750 000
·142 857	1 : 7	7·000 000	·272 727	3 : 11	3·666 667

Table of all Ratios represented by Numbers up to 20—continued.

·277 778	5 : 18	3·600 000	·636 364	7 : 11	1·571 429
·285 714	2 : 7	3·500 000	·642 857	9 : 14	1·555 556
·294 118	5 : 17	3·400 000	·647 059	11 : 17	1·545 455
·300 000	3 : 10	3·333 333	·650 000	13 : 20	1·538 462
·307 692	4 : 13	3·250 000	·666 667	2 : 3	1·500 000
·312 500	5 : 16	3·200 000	·684 211	13 : 19	1·461 538
·315 789	6 : 19	3·166 667	·687 500	11 : 16	1·454 545
·333 333	1 : 3	3·000 000	·692 308	9 : 13	1·444 444
·350 000	7 : 20	2·857 143	·700 000	7 : 10	1·428 571
·352 941	6 : 17	2·833 333	·705 882	12 : 17	1·416 667
·357 143	5 : 14	2·800 000	·714 286	5 : 7	1·400 000
·363 636	4 : 11	2·750 000	·722 222	13 : 18	1·384 615
·368 421	7 : 19	2·714 286	·727 273	8 : 11	1·375 000
·375 000	3 : 8	2·666 667	·733 333	11 : 15	1·363 636
·384 615	5 : 13	2·600 000	·736 842	14 : 19	1·357 143
·388 889	7 : 18	2·571 429	·750 000	3 : 4	1·333 333
·400 000	2 : 5	2·500 000	·764 706	13 : 17	1·307 692
·411 765	7 : 17	2·428 571	·769 231	10 : 13	1·300 000
·416 667	5 : 12	2·400 000	·777 778	7 : 9	1·285 714
·421 053	8 : 19	2·375 000	·785 714	11 : 14	1·272 727
·428 571	3 : 7	2·333 333	·789 474	15 : 19	1·266 667
·437 500	7 : 16	2·285 714	·800 000	4 : 5	1·250 000
·444 444	4 : 9	2·250 000	·812 500	13 : 16	1·230 769
·450 000	9 : 20	2·222 222	·818 182	9 : 11	1·222 222
·454 545	5 : 11	2·200 000	·823 529	14 : 17	1·214 286
·461 538	6 : 13	2·166 667	·833 333	5 : 6	1·200 000
·466 667	7 : 15	2·142 857	·842 105	16 : 19	1·187 500
·470 588	8 : 17	2·125 000	·846 154	11 : 13	1·181 818
·473 684	9 : 19	2·111 111	·850 000	17 : 20	1·176 471
·500 000	1 : 2	2·000 000	·857 143	6 : 7	1·166 667
·526 316	10 : 19	1·900 000	·866 667	13 : 15	1·153 846
·529 412	9 : 17	1·888 889	·875 000	7 : 8	1·142 857
·533 333	8 : 15	1·875 000	·882 353	15 : 17	1·133 333
·538 462	7 : 13	1·857 143	·888 889	8 : 9	1·125 000
·545 455	6 : 11	1·833 333	·894 737	17 : 19	1·117 648
·550 000	11 : 20	1·818 182	·900 000	9 : 10	1·111 111
·555 556	5 : 9	1·800 000	·909 091	10 : 11	1·100 000
·562 500	9 : 16	1·777 778	·916 667	11 : 12	1·090 909
·571 429	4 : 7	1·750 000	·923 077	12 : 13	1·083 333
·578 947	11 : 19	1·727 273	·928 571	13 : 14	1·076 923
·583 333	7 : 12	1·714 286	·933 333	14 : 15	1·071 429
·588 235	10 : 17	1·700 000	·937 500	15 : 16	1·066 667
·600 000	3 : 5	1·666 667	·941 176	16 : 17	1·062 500
·611 111	11 : 18	1·636 364	·944 444	17 : 18	1·058 824
·615 385	8 : 13	1·625 000	·947 368	18 : 19	1·055 556
·625 000	5 : 8	1·600 000	·950 000	19 : 20	1·052 632
·631 579	12 : 19	1·583 333	1·000 000	1 : 1	1·000 000

XIV.—*Chapters on the Mineralogy of Scotland. Chapter Third.—The Garnets.*

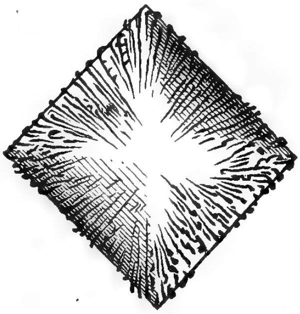
By Professor HEDDLE.

(Read 7th January 1878.)

Abundant as are the localities in which garnet is found in Scotland, there are but few which yield specimens such as can be analysed.

This is on account of an intermixture of quartz—for the most part in a granular form—the granules being promiscuously scattered throughout the mass of the crystals. In three localities the intermixture is not promiscuous, but has been governed by some intermittent crystalline action. These localities are Glen Skiag in Ross, where, around a central nucleus of leucitoidal crystals of garnet, translucent quartz is arranged in layers which alternate with those of the garnet, conformably to the figure of its crystal.

The two other localities are the first and third granitic veins to the east of Portsoy; in both of which garnet of a pale brown colour is laced with quartz, which is arranged in a *graphic* manner, as in the felspar from an adjacent vein. At no other locality do I know of such an occurrence. The appearance of a section of a rhombic dodecahedron is somewhat like the figure.



Lime and Alumina Garnet.

$$(\dot{\mathrm{C}}\mathrm{a}^3)^2\ \ddot{\mathrm{S}}\mathrm{i}^3 + \dddot{\mathrm{A}}\mathrm{l}_2{}^2\ \ddot{\mathrm{S}}\mathrm{i}^3 .$$

Colourless Garnet; Water Garnet.

This very rare garnet I found in small quantity in one of the old limestone quarries in the wood on Craig Mohr, opposite to Balmoral.

It was in small, perfectly colourless, dodecahedral crystals, which were associated with grossular.

Qualitative trials showed the absence of iron in either state of oxidation.

Lime—Alumina, Iron Garnet.

$$(\dot{\mathrm{C}}\mathrm{a}^3)^2\ \ddot{\mathrm{S}}\mathrm{i}^3 + (\dddot{\mathrm{A}}\mathrm{l}_2,\ \dddot{\mathrm{F}}\mathrm{e}_2)^2\ddot{\mathrm{S}}\mathrm{i}^3 .$$

Grossular.

1. This also very rare garnet was associated with the former; it was imbedded in limestone along with idocrase and rarely epidote.

It was found generally in crusts of pale pea-green crystals, in the form of the rhombic dodecahedron. These crusts of crystals passed frequently internally into a massive granular form, generally of the same colour—rarely becoming somewhat red-brown.

The crystals are sometimes isolated, of the size of peas, the colour of which they more resemble than that of the gooseberry.

The tint is finer and much more uniform than that of the Wilui crystals;—they are, however, more opaque and muddy.

Their specific gravity is 3·545.

1·313 grammes yielded—

Silica,	·494		
From Alumina,	·029		
	·523	=	39·832
Alumina,			9·737
Ferric Oxide,			15·065
Ferrous Oxide,			·108
Manganous Oxide,			·353
Lime,			33·565
Magnesia,			1·012
Water,			·045
			99·717

Insoluble silica, 17·208 per cent.; possible impurity unknown; it has to be remarked, however, that in all analyses of garnet there must be a small admixture of quartz, from abrasion of the agate mortar.

These crystals are to be very rarely obtained in two or three of the small quarries on the south face of the hill; in one of these I obtained a specimen of *aplome*, of a brown colour—the striation parallel to the shorter diagonal of the rhombic face was exceedingly well marked.

Grossular also occurs very rarely in the limestone quarry at Crathie, Deeside.

Lime, Iron—Alumina Garnet.

$$(\dot{\mathrm{Ca}}^2, \dot{\mathrm{Fe}})^2\, \dddot{\mathrm{Si}}^3 + \dddot{\mathrm{Al}}_2^2\, \dddot{\mathrm{Si}}^3$$

Cinnamonstone.

2. This variety occurs in great abundance in the limestone quarry at Dalnabo, Glen Gairn. Here it occurs of an exceedingly fine colour, and also of a brownish tint. Of this latter colour it is also found in the quarryings on Leach Ghorm, at Crathie, at Boultshoch, and elsewhere in the district.

At Dalnabo it seems occasionally to pass into idocrase, which, with many other species, is abundant here; the garnets occur towards the more central and purer portions of the limestone bed.

1·5 grammes yielded—

Silica,	·571		
From Alumina,	·018		
	·589	=	39·266
Alumina,			21·976
Ferric Oxide,			1·487
Ferrous Oxide,			3·933
Manganous Oxide,			·333
Lime,			31·882
Magnesia,			·6
Soda,			tr.
Water,			·184
			99·661

6·281 per cent. of the silica was insoluble; possible impurity unknown.

As far as colour is concerned, this cinnamonstone is superior to the mineral obtained from Ceylon; it is, however, so much flawed that it is useless for purposes of jewellery.

In connection with the very common occurrence of garnets in limestone the question arises—can any explanation thereof be suggested?

In venturing upon an explanation, there are three facts upon which I have to found.

The first: That the localities in which garnets are found to occur in limestone are—as regards the occurrence of *silicates generally*—the most richly mineral-bearing of any in the country.

The second: That close observation of limestone strata among the metamorphic rocks of the Highlands of Scotland shows that these limestones function in a manner somewhat similar to that of igneous rocks—I mean as regards the changes they induce, or *seem* to induce in the including rocks.

The third: That this special mode of functioning—at first sight very anomalous as regards a substance and structure of unquestionable organic origin—is only to be observed in certain circumstances.

If the limestone bed is thin, whether disturbed or undisturbed—convoluted or not so—there is no change. The stratum itself is still *limestone*, a more or less impalpable paste, amorphous in its nature, structureless; or if with recognisable structure, then it is that of an imbedded and badly developed organism.

Here the including rock shows no greater amount of metamorphism in the vicinity of the lime than elsewhere; while in neither lime nor including rock are there to be found any minerals, other than the ordinary ingredients of their general mass. And all this also holds with equal force in the circumstances of the lime stratum being of considerable extent, so long as it is untroubled—unconvoluted.

But it is not so if the stratum be at one and the same time large in mass, and either itself highly contorted, or, in those cases in which it is not sufficiently exposed for this to be determined, if it be seen to be lying in the midst of highly contorted rocks. Then the calcareous mass no longer is amorphous in its substance, and organic in its structure; it presents itself as a *granular limestone,* often as a true marble; that is, it is *calcite,* and is crystalline in structure, and in all its inherent properties. Again, the including rock is, in the immediate vicinity, much more highly metamorphosed than it is throughout its general mass. While lastly, imbedded in the limestone, and to a smaller extent in the altered rock, but in both cases near their point of contact, there are found numerous minerals, *all* of which are such as can be formed by the union of the constituents of the inclosed limestone with those of the inclosing gneiss.

The assigning of the above local changes to the mere presence of limestone, in a rock which is undergoing ordinary metamorphic change, will not suffice as an explanation of what is found in the above case.

Ordinary metamorphic change is, in our almost total ignorance of the subject, usually assigned to a hydro-thermal action, which has taken place at great depths, *i.e.,* under enormous pressure. Such a thermal change should affect a plicated, and a nonplicated included rock alike; and should certainly affect a thin bed of lime more than a thick. As the result of any change thus passing from *without inwards,*—that is, from the gneiss to the limestone,—we should expect to find the thinner parts of the stratum of the latter wholly converted into large granular marble, and very fully pervaded with mineral species; while the thicker parts of the lime should be much less altered in both respects: but this is the opposite of what obtains; and so any explanation which requires that the agent of change should act from *without inwards* does not suffice.

The whole facts of the case seem to point to an action taking place from *within outwards.* So at least is it to be seen how there should be greater change where the stratum is thickest;—the greater the mass, the greater must be the amount of action, if that action proceed from within.

But what action, physical or chemical, can be conceived to take place within or emanate from a sedimented, amorphous, organic limestone, which shall credit it with being at one and the same time the agent of its own metamorphosis into its crystalline allomorph,—of augmenting the metamorphosis of a rock in its immediate neighbourhood,—and of stimulating to the chemical union of the material of its own mass with the constituents of that rock?

Among the physical agencies which induce a change from the amorphous or colloidal to the structural or crystalloidal condition, we find *plication* and *heat*. Sir JAMES HALL'S experiments have shown that direct heat under pressure will change limestone into marble or calcite; and *plication*, in the very act transforming a colloid into a crystalloid, does so at the cost, so to speak, of an elimination of heat from the substance so plicated. Melt lead, pour it out on a flat stone,—in cooling it assumes the structureless, amorphous form, with rounded colloidal outline; if, after cooling, it is bent in the hand, it gives out heat, becomes crystalline in structure, gradually loses its pliability, and assumes the brittleness inseparable from the crystalloidal state. The same holds for iron and all metals capable of assuming the colloidal state; bend or beat them, they give out heat, becoming crystalline and brittle in the so doing.

The heat here eliminated is doubtless partially the representative of arrested motion; but it is probably *chiefly* an outcome of the change from the colloidal to the crystalloidal condition; the agent of the elimination was the plication to which the substance was subjected, which plication was at the same time the immediate agent of the physical transformation.

I would venture to predict that it will come to be found that *the specific heat of substances in their colloidal state is always greater than that of their crystalloidal.*

I am not aware that any researches have been made specially to determine this point; or even that the attention of physicists has been directed to it; but the following table, which embodies all that I have been able to find, seems clearly to bear out such a view:—

COLLOID.		CRYSTALLOID.	
Water,	1·	Ice,	·72
Lamp Black,	·26	Graphite,	·201
		Diamond,	·174
Limestone and Chalk,	·264	Calcite and Marble,	·201
Chalcedony,	·195	Quartz,	·179
Titanic acid (artificial),	·172	Rutile,	·163
Peroxide of Iron (artificial).	·176	Hematite,	·166

Should it prove to be the case that the amorphous form has a specific heat always exceeding that of the crystalline, a step probably will be gained in the explanation of metamorphism generally; it may at least be held that the high specific heat of limestone, and the much lower heat of calcite or granular marble, explains the local metamorphism which we are now considering; as the plication, crushing and folding of the strata *expressed, or extricated as heat* *of active energy* the difference in the amount of the specific heats of carbonate of lime in its two states,—the excess which is special to it in the colloidal. This expressed heat left only a residue, so to speak, sufficient for granular limestone; while it became the active agent in stimulating the chemical affinities existing between lime and the silica, alumina, and alkalies of the gneissose matrix. It thus led directly to the formation of minerals, while it at the same time expedited or perfected a more thorough metamorphosis, *i.e.*, the assumption of a more perfectly developed crystalline structure, in the previously only partially metamorphosed rock.

The assumption of a definite crystalline structure,—a character or property directly attached to a definite chemical composition,—must of necessity extrude from the resultant calcite the phosphate of lime and fluoride of calcium which limestones contain uniformly distributed throughout their mass. *Apatite* and *Fluorspar* are accordingly among the crystallised minerals found imbedded in the saccharoid "primary" limestones.

Whether the extruded heat can actually fuse the residual calcite may possibly yet be ascertained by direct experiment; but it would seem to be almost a necessary deduction, that no mineral could be formed by such an action possessed of or requiring a specific heat greater than that of the original source of the expressed heat.

It must be borne in mind that chemical elements do not alone go to the formation of any substance;—a due amount of certain physical agencies is the special portion of each, lodging as it were in their pores as resting places. Of the chemical ingredients, one alone may suffice; of the physical, among which, heat, phosphorescence, and magnetism may be said to be those most germain to minerals, heat is the only one which is never absent. Chemical affinity then can only *predispose* to the union of the constituents which go to form the substance; for something more is requisite before the formation can be accomplished,—before the substance can, so to say, assume an independent existence,—namely, the supply of the heat special to it,—to the perfect putting together of the whole as a mineral species.

It has, however, to be admitted that we are unable positively to affirm that the expressed heat may not be so concentrated in the spots where the chemical action is operating as to afford any specific heat required.

Two arguments against such a view may, however, be adduced.

The first—That the heat extricated throughout the general mass would not readily be localised, or carried specially to any point in so badly conducting a substance as granular limestone or marble; more probably would it accumulate within the limestone itself, even to the point of its fusion;—and indeed the occurrence of porphyritically disposed crystals of quartz with rounded angles throughout the general mass of some marbles (LEDBEG, SUTHERLAND, *e.g.*) would seem to indicate actual fusion.

The second—That it has not yet been shown that there is the necessity for such concentration.

The high specific heat of the matrix is, in the present case at least, amply sufficient; no one of the minerals found in the above localities possessing so large an amount of heat as ·264 — that of limestone.

That of the following is known—

Fluor,	·19
Apatite,	·17
Tetrahedrite,	·192
Actynolite,	·204
Augite,	·194
Diopside,	·19
Pyrrhotite,	·16
Molybdenite,	·102

It is a somewhat significant fact that *limestone has a specific heat superior to that of any other rock mass;* we speak of mollusks and crustaceans as "cold blooded,"—though it is believed that they have a temperature somewhat higher than that of the medium in which they live;—but the calcareous matter which they have secreted from solution in that medium, for the defence of their soft parts, is deposited and arranged with an organic and non-crystalline structure, and consequently has had conferred upon it the high specific heat of the amorphous state; and so the long buried skeletons of these organisms are abiding store-houses of heat, until called upon to yield up their surplus store, and thus become the active agents of future change.

In illustration of the foregoing speculation, a brief sketch may now be appended of the localities in Scotland where limestones of the age above referred to occur.

Although there are many localities where metamorphic rocks contain calcareous beds, yet their continuity and connections can nowhere so easily be made out as in the counties of Banff, Aberdeen, and the north of Perth.

In this district the general trend of the outcrop is from N.N.E. to S.S.W., the dip, usually at a high angle, being to the E.S.E. The lowest member of the series which includes calcareous beds is an argillaceous mica-schist; this is succeeded, to the east, by a highly metamorphosed quartzite of a pinkish tint, the dip of which, in its northern reaches at least, is somewhat more southerly than that normal to the series. Eastward of the quartzite the characteristic "gnarled gneiss" of Scotland supervenes,—rolling over the country in repeated folds, convoluted and contorted in the extreme. This gneiss is composed of orthoclase of a somewhat bluish tint, passing occasionally to cream colour, and containing over half a per cent. of lime, a black or bronzy mica, probably lepidomelane, and pale blue quartz. This rock, which is highly characteristic from the manner in which its dark micaceous layers display the marvellous plications into which it has been crumpled, dominates throughout the county of Aberdeen to a much greater extent than the granite-blotched geological maps admit.

In no place perhaps can the gneiss be seen actually abutting upon the quartzite, for there intervenes a bed, or a series of beds of limestone, associated for the most part with serpentine and with diorite.

It has been stated above that the dip of the quartzite was not absolutely conformable with that of the mica state—the relations of this quartzite have yet to be thoroughly investigated; possibly it may prove to be the lowest member of the series; it is in the north somewhat talcose, and micaceous in its southern reaches.

Such being the general relationships of the rocks—so far as regards the included limestones—their geographical position has now to be pointed out.

The argillaceous mica schist is first seen in the north a few miles south of Cullen, in Banffshire: stretching thence to the S.W. it occupies the whole of the low ground of the vale of Deskford, Glen Isla, Glen Rinnes, Glen Livat, the Braes of Abernethy, and the upper reaches of Strath Alnack. The lower part of these hollows is occupied by a frequently interrupted, but on the whole very persistent bed of limestone. The general character of both schist and limestone is an absence of disturbance, trouble, folding, or dislocation of their beds.

Of the quartzite little need be said: a considerable though detached mass of it appears in the neighbourhood of Cullen; the portion usually supposed to overlie the argillaceous schist of the western valley commences in the Durn Hill, south of Portsoy, and stretches in two parallel ranges of highly elevated land, protruding as it were through the sea of micaceous mud which lies at their feet, with but few breaks in the continuity of their course, till, converging after forming the buttressed walls of Glen Clunie, and sweeping westward along the ridge of Cairn Geodeh (Gey), they constitute the great masses of Ben Uran and Ben-y-Gloe. This quartzite reappears in small amount here and again, but is finally denuded off after forming the terminal 50 feet of the sharp summit of Aonach Beg, near Loch Ousan.

Immediately above the quartzite there comes in a series of beds of serpentine, of at least three varieties, which alternate with limestones. These limestones are of so trifling and irregular a character as to lead to the suspicion that they are not sedimentary limestones, but are a product of the metamorphism of the overlying diallage and diorite. These rocks, there is every reason to believe, have, by the action of carbonated waters, been transmuted into beds of serpentine and calcite. The beds of serpentine all show themselves in the north in the neighbourhood of Portsoy, in Banffshire; they hold a parallel and generally a closely united course as far as the head of the Blackwater in the same county,* there they appear to separate or trifurcate. One, which continues to hug the quartzite, is, according to MACCULLOCH, seen in the Ey Forrest—this probably loses itself in the serpentinous marble of Glen Tilt. Another, seen on the Kindy, the north-east shoulder of Culbleen, the Coyle, and little Kilrannock, is lost among the cliffs of the Canlochan of Glen Isla. While the third, associated almost throughout with diorite and true syenite, crosses the Alt Dovern at Redford in the Cabrach, skirts the north foot of the Buck, forms the hill of Tombreck or Towanrieff, reappears at Knockespock, Chapeltown, Premnay, Barra Hill, Beauty, and Belhelvie,—passing five miles north of Aberdeen into the German Ocean as the Schiller-spar of the Black Dog Rock.

Throughout the whole of this extended reach the serpentine is so commonly associated with limestone that, where they do not appear in company, they may be said mutually to vouch for the near presence of the other; both, more or less included in the gneissose rocks, share the vicissitudes of dip, strike, and plication to which the including rock has been subjected; and these limestones offer a marked contrast in almost every respect to the undisturbed and comparatively barren bed first mentioned as occurring in the argillaceous schist.

It is in those portions of the limestone beds which are not associated with serpentine, but which lie among the most highly plicated, disturbed, and fractured strata, that the largest number and the largest amount of mineral bodies are found. Doubtless the fact of one stratum having an argillaceous and the other a gneissose matrix cannot be without influence; but the absence of minerals in the gneissose beds, when these are either small in quantity or unplicated, shows the amount of that influence to be but small.

The following contrasted columns present these differences to the eye:—

---

* The localities where it may be seen are Dambead, with steatite, on the east side of Durnhill; Badenochs, north of Knockhill; Limehillock, over lime, north of Grange; Rothiemay Station; Drumhead, near Ruthven; the hill of Sockach, south of Glass; Craig Carnie, near Baldornie; southward of this it forms a serrated ridge to the west of Boghead and Greenloan, till it reaches the larger mass of Craig Lui; it appears along with diorite in the east and west bends of the Blackwater; again assumes the form of a ridge till it reaches the Blackwater Lodge; and lastly, forms finely buttressed cliffs and castellated pinnacles which overhang the Scores-burn,—the source of the Blackwater.

LOWER LIMESTONE.

Generally in argillaceous schist, and usually unplicated.

REDHYTHE, BANFFSHIRE.
Matrix—gneiss, stratum thin, very highly plicated, moderate dip.
Talc, in small quantity.
Pyrrhotite, in small quantity.
Biotite, very rare.
Rutile, very rare.

FORDYCE.
Matrix—gneiss, stratum thick, not plicated, horizontal.
Talc, rare.

MAISLEY.
Matrix—argill. slate, stratum thick, not plicated, low dip.
Antimonite.
Fluor.

BOHARM.
Matrix—argill. slate, stratum thin, not plicated, high dip.
Fluor.
Hornblende,
Kyanite,
Staurolite,
Garnet,
in matrix in vicinity.

BOTRIPHNIE.
Matrix—argill. slate, stratum thick, not plicated, low dip.
Kyanite,
Margarodite,
in quartz seams.

DUFFTOWN.
Matrix—argill. slate, stratum thick, not plicated.
Quartz, rare.

GLEN RINNIES.
Matrix—argill. slate, stratum thick, not plicated, low dip.
No minerals.

CANDELMORE.
Matrix—argill. slate, stratum thick, not plicated, low dip.
Chlorite, rare.

UPPER LIMESTONE.

In gneissose rocks, and usually highly plicated.

BOYNDIE BAY, BANFFSHIRE.
Matrix—gneiss, stratum thick, not plicated, moderate dip.
Mountain leather, rare.
Mountain cork, rare.

ARDONALD.
Matrix—gneiss, with cover of clay slate stratum thick, not plicated, high dip.
Margarodite, rare.
Kyanite, in seams of, and
Grenatite, in the slate.

LIMEHILLOCK, GRANGE.
Matrix—gneiss, stratum thick, not plicated.
No minerals.

GLEN BUCKET.
Matrix—gneiss, stratum thick, not plicated, low dip.
Margarodite.
Pyrite.
Actynolite.

GREEN HILL OF STRATHDON.
Matrix—mica slate, stratum thick, not plicated.
No minerals.

STRATH EARNAN.
Matrix—gneiss, stratum thin, not plicated, low dip.
No minerals.

DALNEIN.
Matrix—gneiss, stratum thick, not plicated.
Serpentine.

GLEN GAIRN.
Matrix—gneiss, stratum thick, much plicated and fractured, high dip, inverted, central bed of rock.
Cinnamonstone, abundant.
Idocrase, abundant.
Pyrrhotite, abundant.
Andesine, common.
Diallage, very rare.

LOWER LIMESTONE—*Continued.*

ALT NA CORLEACHAN.

Matrix—argill. slate, stratum thick, not plicated.

No minerals.

CARN ELLICK.

Matrix—argill. slate, stratum thin, not plicated.

No minerals.

GAULRIG.

Matrix—argill. slate, stratum thin, not plicated, low dip.

Fluor, common.
Steatite (?), scarce.
Sphene, rare.
Ripidolite, rare.

WEST AND EAST BED.

Always in gneiss, sometimes with high dip, sometimes horizontal.

LEACH GHORM.

In gneiss, but capped by granite! stratum pervaded by layers of granular malacolite (?) much fractured and troubled, granitic belt in the lime.

Idocrase.
Garnet.
Malacolite.
Pyrrhotite.

In the granitic belt.

Biotite.
Orthoclase.
Andesine.

CREAG MOHR.

Matrix—gneiss, stratum thin, not plicated, low dip.

Idocrase, common.
Garnet, common.
Grossular, very rare.
Epidote, rare.

CRATHIE.

Matrix—gneiss, stratum thick, contorted, central bed of rock, high dip.

Hornblende and Actynolite.
Sahlite and Coccolite.

UPPER LIMESTONE—*Continued.*

Funkite, scarce.
Coccolite, common.
Prehnite, scarce.
Epidote, common.
Wollastonite, scarce.
Actynolite, scarce.
Lamellar quartz, common.
Pseudo-prehnite, common.
Greenovite, rare.
Rhodonite, pseudo after sphene, very rare.
Latrobite, very rare.
Anorthite ? very rare
Biotite, rare.
Molybdenite, very rare.
Apatite, scarce.
Calcite crystallised, scarce.
Pyrite, very rare.
Lepidomelane ? in matrix.

FROSTER HILL.

Matrix—gneiss, stratum thick, slightly plicated, high dip.

Augite,
Actynolite,
Pyrrhotite,
Pyrite,
Andesine, } all in small quantity.
Biotite,
Talc,
Sphene,
Chlorite,

SHINNESS, SUTHERLAND.

Matrix—gneiss, hornblendic cap, stratum thick, medial belt of rock, high dip, moderately plicated.

Malacolite, abundant.
Sahlite, abundant.
Funkite, rare.
Actynolite, scarce.
Biotite.
Sphene, scarce.
Molybdenite, very rare.
Apatite, rare.
Pyrrhotite, rare.
Tremolite, rare.

WEST AND EAST BED—*Continued.*

Wollastonite, rare.
Prehnite, very rare.
Grossular, very rare.
Garnet, abundant.
Greenovite, very rare.
Idocrase, abundant.
Pyrrhotite, abundant.
Pyrite.
Andesine, rare.
Fatty quartz.
Biotite.
Fluor, rare.
Apatite.

BOULTSHOCH.

Matrix—gneiss, stratum thin, high dip, granitic belt, central bed of rock.

Idocrase.
Coccolite.
Garnet.

In the granitic belt.

Biotite.
Orthoclase.
Andesine ?

CORN TULLICH.

Matrix—gneiss, stratum thin, horizontal, not fractured.

Malacolite, granular.
Pyrrhotite.
Graphite.
Wollastonite, constituting a bed 24 inches thick overlying the lime, impregnated with graphite and sphene.

MUIR AND MIDSTRATH.

Matrix—gneiss, stratum thick, horizontal, not plicated.

Malacolite, granular.
Specular iron.
Graphite ?
Pyrrhotite, common.
Sphene, scarce.
Fluor, scarce.

UPPER LIMESTONE—*Continued.*

Asbestus, rare.
Talc, very rare.
Steatite, rare.
Pyrite, rare.
Chlorite, rare.
Lepidomelane, } contact minerals.
Orthoclase, }

MILLTOWN, GLEN URQUHART.

Matrix—gneiss, highly contorted, fractured, and with high dip, with granitic belt.

Tremolite, common.
Andesine, rare.
Urquhartite.
Edenite, common.
Pyrrhotite, common.
Sphene, very rare.
Apatite, rare.

In the granitic belt.

Biotite.
Orthoclase ?
Andesine.

WEST AND EAST BED—*Continued.*

MUIR AND MIDSTRATH.

In vertical vein.

Quartz, with
Andesine ? (granular).

In the matrix.

Orthoclase.
Haughtonite ?
Apatite.

ESLIE.

Matrix—gneiss, stratum thin, much plicated, high dip.

Sahlite.
Pyrrhotite.
Actynolite, rare.
Orthoclase !
Specular Iron.
Graphite ?
Sphene, scarce.
Talc, rare.
Apatite.

Magnesia, Iron—Alumina Garnet.

$$(\dot{Mg}, \dot{Fe}, \dot{Ca})^2\ddot{Si}^3+\dddot{Al}_2{}^2\ddot{Si}^3.$$

Pyrope.

3. The "Elie rubies" occur imbedded in two dykes of basalt, which cut tufa about a mile to the east of Elie, in Fife. Here they are associated with nigrine, saponite, and sanidine.

They also occur in imbedded fragments in the tuff of Elie Ness. In Sowerby's "British Minerals" there is a plate of a portion of a vein of greasy quartz from this spot, carrying rhombic dodecahedra of the pyrope.

Lastly, they are to be found, also in imbedded fragments, along with nigrine and large rough crystals of fissured olivine in columnar basalt, at Ruddock Point, about one mile west of the Kincraig.

The chips analysed were from Elie Ness. Their colour was deep port wine; specific gravity, 4·124.

1·3 grammes yielded—

Silica,	·515		
From Alumina	·017		
	·532	=	40·923
Alumina,			22·452
Ferric Oxide,			5·463
Ferrous Oxide,			8·107
Manganous Oxide,			·461
Lime,			5·04
Magnesia,			17·846
Water,			·098
			100·39

Insoluble silica, 1·127 per cent.

Mr Connell believed this pyrope to contain a little chromium. I did not find a trace.

Though somewhat wanting in transparency,—and although it is surpassed in brilliancy of appearance by the blue topaz, the beryl, and the citrine cairngorum of the Braemar district,—the Elie pyrope certainly is, weight for weight, the most valuable gem obtained in Scotland.

Iron—Alumina Garnet.

$$(\dot{\mathrm{Fe}}^{3})^{2}\ddot{\mathrm{Si}}^{3}+\dddot{\mathrm{Al}}_{2}{}^{2}\ddot{\mathrm{Si}}^{3}.$$

Common Garnet.

From Gneiss.

4. Occurs imbedded abundantly in micaceous gneiss about two miles north-west of Burra Voe, Yell, Shetland, near the junction of said gneiss with an epidotic rock (? the epidotic syenite of HIBBERT). At this spot the epidotic rock overlies micaceous gneiss, and is itself overlaid by the hornblendic. The dip is to the west of north, at an angle of about 15°.

The garnets here are fine in colour, being of a lively pinkish red; they are much flawed. Specific gravity, 3·997.

1·303 grammes yielded—

Silica,	·468	
From Alumina,	·018	
	·486	= 37·298
Alumina,		21·095
Ferric Oxide,		7·47
Ferrous Oxide,		24·023
Manganous Oxide,		2·141
Lime,		4·426
Magnesia,		3·53
		99·983

Insoluble silica, 2·056 per cent.; possible impurity, quartz.

From Mica Slate.

5. From Ben Bhrackie and Killiecrankie, in Perthshire. On the south-west slopes of Ben Bhrackie the garnets are of small size; they occur along with lanceolate crystals of hornblende, imbedded in a granular paste of felspar and quartz of a cream colour; forming a rock which, when slit and polished, is possessed of considerable beauty.

At Killiecrankie the garnets are of the size of bullets, and are imbedded in a dense fine-grained laminated paste of mica. Their colour is a brownish red; their structure is fine granular, with minute granules of quartz very frequently intermixed. Specific gravity, 3·688.

On 25·01 grains—

Silica,	37·59
Alumina,	13·664
Ferric Oxide,	3·658
Ferrous Oxide,	32·311
Manganous Oxide,	4·468
Lime,	4·116
Magnesia,	3·464
Water,	·32
	99·591

Insoluble silica, 11·602 per cent.; probable impurity, quartz,—from which it was separated with extreme difficulty.

6. From the summit of Meall Luaidh, north-west of Ben Lawers, Perthshire. Garnets of the size of peas lie loose on the summit of this hill in quantities, especially in the neighbourhood of a bed of yellow quartz.

Their colour is a dull red brown.

1·179 grammes afforded—

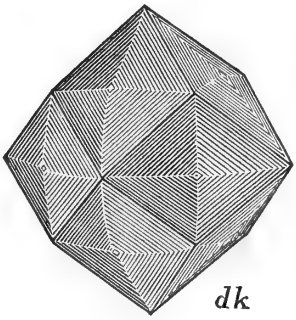
Silica,	·419	
From Alumina,	.025	
	·444 =	37·659
Alumina,		14·803
Ferric Oxide,		4·56
Ferrous Oxide,		32·973
Manganous Oxide,		2·374
Lime,		5·889
Magnesia,		1·806
		100·064

Insoluble silica, 11·871 per cent.; possible impurity, quartz.

From Diorite.

7. The rock from which the garnets now noticed are obtained I have never seen *in situ;* it is said to occur at the foot and to the north side of Knock Hill.

Large loose masses of it I have observed on the summit of the cliffs on the east side of the Bay of the Durn,—to the east of Cowhythe Head,—and near Retannach, in Banffshire. The rock, which is of a very fine grain, seems to be a labradoric diorite, in which the hornblende is replaced almost entirely by a dark mica. Near Retannach it contains patches of snow-white granular labradorite; elsewhere the whole rock is crypto-crystal-



line. The crystals, said to be from Knock, are of large size, and exhibit the interesting oscillation of faces shown in figure. They are of a port-wine colour, and their specific gravity is 4·116.

1·237 grammes yielded—

Silica,	·44		
From Alumina,	·019		
	·459	=	37·105
Alumina,			14·899
Ferric Oxide,			10·125
Ferrous Oxide,			32·409
Manganous Oxide,			1·212
Lime,			2·17
Magnesia,			2·934
			100·854

Insoluble silica, 6·57 per cent.; no visible impurity.

Iron—Alumina, Iron Garnet.

$$(\dot{F}e^3)^2\ddot{S}i^3 + (\dddot{A}l_2\dddot{F}e_2)^2\ddot{S}i^3 .$$

Almandite.

From Micaceous Gneiss.

8. Is found, somewhat rarely, in the quartzose belts of the rocky bluff named Clach an Eòin, between the mouths of the Navir and the Borgie, in Sutherland.

The colour is brown red; the associated minerals are Haughtonite, ilmenite, chlorite, and rutile.

1·3 grammes yielded—

Silica,	·502		
From Alumina,	·016		
	·518	=	39·923
Alumina,			19·808
Ferric Oxide,			13·69
Ferrous Oxide,			13·294
Manganous Oxide,			1·
Lime,			9·132
Magnesia,			3·307
			100·154

Insoluble silica, 2·509; possible impurity, quartz.

The large amount of lime in this garnet is peculiar, when we consider the nature of its matrix.

Iron, Manganese—Alumina, Iron Garnet.

$$(\dot{Fe}^2Mn)^2\dddot{Si}^3 + (\dddot{Al}_2\dddot{Fe}_2)^2\dddot{Si}^3$$

Precious Garnet—Manganesious Garnet.

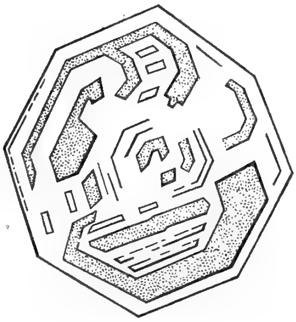
From Granitic Belts in Micaceous Gneiss.

9. From the railway cutting at Glen Skiag, west of the Raven's Rock, Strathpeffer, Ross-shire.

In carrying the railway through one of the ridges which run north and south in this small glen, a number of huge egg or lens-shaped masses were dislodged from the rock. These masses showed on their exterior a platey covering of glistening greenish mica. Their bulk necessitated their being broken up for removal; their interior was then seen to consist chiefly of quartz, plentifully studded in all directions with large and brilliant plates of mica, and garnets of very unusual beauty of colour.

In addition, there was found, though in much smaller quantity, black and *green* tourmaline, rarely zircon, and still more rarely apatite.

The lens-shaped masses were connected with one another through the continuity of their micaceous coating, forming what may be called a nodose vein; the mica alone, however, truly constituted the vein—the quartz, with its included minerals, assuming the nodose arrangement. Garnets of two colours occur in this rock—the more brilliantly coloured is probably the finest in Scotland; it is of a light red-currant tint. The crystals of this variety are never much over an inch in size; the others are larger, of duller lustre, almost granular in structure, and of a brownish-red tint. Those of a bright currant red are very transparent, but much flawed; and they sometimes contain layers of white quartz arranged in a concentric manner, parallel to the sides of the leucitoidal crystals.



Of the bright red garnet the specific gravity was 4·125.

1·528 grammes yielded—

Silica,	·536		
From Alumina,	·014		
	·55	=	35·99
Alumina,			16·221
Ferric Oxide,			8·638
Ferrous Oxide,			23·27
Manganous Oxide,			15·24
Lime,			·403
Magnesia,			·471
Water,			·249
			100·482

Insoluble silica, 7·211; possible impurity, quartz.

10. Of the brownish tinted larger variety one crystal was seen over five inches in diameter.

On 1·3 grammes—

Silica,	·459		
From Alumina,	·01		
	·469	=	36·076
Alumina,			18·957
Ferric Oxide,			7·033
Ferrous Oxide,			21·56
Manganous Oxide,			13·615
Lime,			·904
Magnesia,			1·769
Water,			·325
			100·239

11. From the Blackwater, Loch Garve.

Immediately to the south of the outflow of Loch Garve into the Blackwater, two large granitic veins occur; these carry well-crystallized plates of greenish mica, and large red-brown garnets, in rhombic dodecahedral crystals.

These yielded in 1·3 grammes—

Silica,	·463		
From Alumina,	·007		
	·47	=	36·153
Alumina,			21·935
Ferric Oxide,			15·15
Ferrous Oxide,			15·085
Manganous Oxide,			7·846
Lime,			2·067
Magnesia,			1·615
Water,			·31
			100·161

Insoluble silica, 4·68 per cent.; possible impurity, mica. This garnet contains but half the quantity of the manganese present in the others.

12. From the tilted granitic belt already noticed, in my chapter on the Felspars, as occurring near Struay Bridge, Ross-shire.

The crystals of garnet are in the leucitic form, of about an inch in size, and of a somewhat fine red; they are deficient in transparency, and much flawed. The associates have been before noted, with the exception of zircon, which I have lately found imbedded in the pink felspar.

1·304 grammes yielded—

Silica,	·465		
From Alumina,	—		
	·465	=	35·695
Alumina,			15·804
Ferric Oxide,			21·084
Ferrous Oxide,			14·941
Manganous Oxide,			11·426
Lime,			1·116
Water,			·06
			100·09

Possible impurity, quartz.

13. From granitic belts on the north-east side of Ben Resipol in Argyllshire, near the summit. The associates are cream-coloured opaque orthoclase, and muscovite.

The garnets are of a fine lively red, transparent, and less fissured than usual in Scotland.

On 1·3 grammes—

Silica,	·471		
From Alumina,	·008		
	·479	=	36·846
Alumina,			21·237
Ferric Oxide,			7·381
Ferrous Oxide,			18·378
Manganous Oxide,			14·461
Lime,			·775
Magnesia,			·846
			99·924

The average composition of these garnets, excluding the Garve specimen, is as follows:—

		Oxygen.		
Silica, . . .	36·14	19·27		19·27
Alumina, . .	18·05	8·41	11·12	19·54
Ferric Oxide, .	9·04	2·71		
Ferrous Oxide, .	21·35	4·74	8·42	
Manganous Oxide,	13·69	3·09		
Lime, . . .	·77	·22		
Magnesia, . .	·55	·22		
Water, . .	·16	·15		

The alumina is to the ferric oxide as 3 to 1; the ferrous oxide to the manganous oxide as 3 to 2. Though the protoxides do not here balance the sesqueoxides well,* the formula may be stated generally thus—

$$(\dot{\mathrm{Fe}}^3\dot{\mathrm{Mn}}^2)^2\ddot{\mathrm{Si}}^3 + (\dddot{\mathrm{Al}}_2{}^3\dddot{\mathrm{Fe}}_2)^2\ddot{\mathrm{Si}}_2\,,$$

and no garnet yielding even approximatively such a proportion of manganese has before been noticed. The manganese is doubtless the cause of the lively colour.

This may be called a *manganesious* garnet, in contradistinction to the true manganesian garnet, found in America, Miask, and elsewhere, and of which the formula is—$(\dot{\mathrm{Fe}}, \dot{\mathrm{Mn}}^2)^2\ddot{\mathrm{Si}}^3 + \dddot{\mathrm{Al}}_2{}^2\ddot{\mathrm{Si}}^3$.

	S. G.	$\ddot{\mathrm{Si}}$.	$\dddot{\mathrm{Al}}_2$.	$\dddot{\mathrm{Fe}}_2$.	$\dot{\mathrm{Fe}}$.	$\dot{\mathrm{Mn}}$.	$\dot{\mathrm{Ca}}$.	$\dot{\mathrm{Mg}}$.	$\dot{\mathrm{H}}_2\mathrm{O}$.	Total.
Grossular—										
Creag Mohr, . .	3·545	39·83	9·74	15·07	·11	·35	33·57	1·01	·05	99·73
Cinnamonstone—										
Glen Gairn, . .	...	39·27	21·98	1·49	3·93	·33	31·88	·6	·18	99·66
Pyrope—										
Elie, ...	4·124	40·92	22·45	5·46	8·11	·46	5·04	17·85	·1	100·39
Common Garnet—										
Burra Voe, Yell, . .	3·997	37·3	21·1	7·47	24·02	2·14	4·43	3·53	...	99·98
Killiecrankie, . .	3·688	37·59	13·66	3·66	32·31	4·47	4·12	3·46	·32	99·59
Meall Luidh, . .	...	37·7	14·8	4·56	32·97	2·37	5·89	1·81	...	100·05
Knock Hill, . .	4·166	37·11	14·9	10·13	32·41	1·21	2·17	2·93	...	100·85
Almandite—										
Clach an Eòin, . .	...	39·93	19·81	13·69	13·29	1·	9·13	3·31	...	100·15
Precious Garnet—										
Glen Skiag (red), .	4·125	35·99	16·22	8·64	23·27	15·24	·4	·47	·25	100·48
Do., do. (brown), .	...	36·08	18·96	7·03	21·56	13·62	1·77	·9	·33	100·29
Loch Garve, . .	4·122	36·15	21·94	15·15	15·09	7·85	2·07	1·62	·31	100·16
Struay Bridge, . .	...	35·66	15·8	13·12	22·21	11·43	1·12	...	·06	99·39
Ben Resipol, . .	..	36·85	21·24	7·38	18·38	14·46	·78	·85	...	99·92

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* This possibly may, in one or two of the specimens, be from the decomposition with the fluor spar and sulphuric acid not having been absolutely perfect. My assistant conceives that a mixture of potassium fluoride with fluor spar is, from the greater energy of the reaction and the smaller quantity of resultant calcium sulphate, to be preferred. The greater energy of the reaction, however, entails a certain amount of risk of projection from the crucible. It is much to be desired that pure ammonium fluoride could be procured.

These analyses show the lime garnets to occur in primitive limestone; the magnesian garnet in igneous rock; and common garnet in micaceous gneiss, and at one locality in diorite.

The new manganesious variety is found in granitic veins, which lie in micaceous gneiss.

The vein which carries garnet at Glen Skiag is probably the same which is seen at Struay Bridge; the similarity in appearance and composition of the garnet which occurs in these veins may aid in identifying them.

At one time I indulged the expectation that the occurrence of garnet in the limestones which are found towards the upper reaches of the Dee would enable us to follow out the bed or beds throughout the numerous and far-reaching windings to which they have been subjected. The two great beds of lime which course up the country from the west and east of Portsoy,—the westerly beneath, the easterly above the quartzite,—may without difficulty be traced until they approach the upper portions of the Don—into the vicinity in fact of the granite. Here, from a direct, they are suddenly thrown into a much winding course; and here they come into near proximity with an altogether different bed, which, with a general west and east trend, undulates down the valley of the Dee.

To follow out their relationships, and even to determine their identity in this highly disturbed district, is a problem of much difficulty.

If it could be proved that the garnet was confined to one bed—that in which it first appears to the westward on the slopes of Leach Ghorm, and which can be clearly traced to Creag Mohr,—then it must be held that this bed takes a sudden sweep to the northward, and is that which, with a westerly dip, is seen in Glen Gairn. It must also be held that it is this same bed which, having formed an anticline—now denuded off—reappears with an easterly dip at Crathie, and, crossing the river to Boultshoch, is again seen at Corn Tullich,—henceforth holding an almost rectilinear course as far at least as Eslie.*

But it is probably as correct a view to hold that the limestones become garnetiferous when they are approached by the granite, or became partially at least involved in that process, which in their near neighbourhood resulted in granitic formation. If so, the Glen Gairn, if not also the Crathie lime, must be regarded as the continuation of the great north and south bed last seen in the neighbourhood of Tornahaish.

It is to be looked for that the occurrence of the garnet may aid in the determination of the true position of the limestones in this troubled district.

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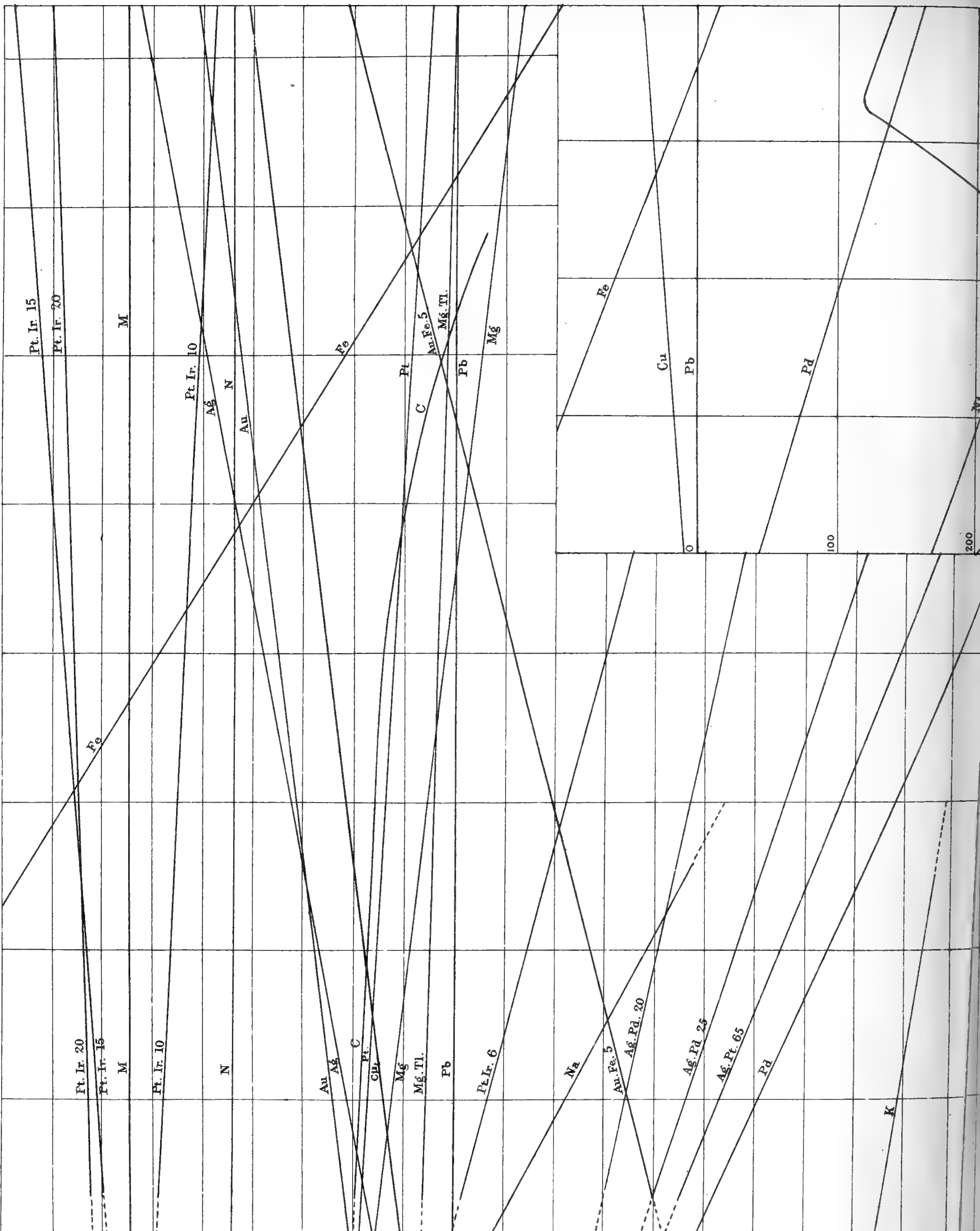
* The localities at which this *west and east* bed is to be seen are—Leach Ghorm; near Carn na Cuimhne; on the south of the Dee in the Balmoral Park; on Creag Mohr; at Boultshoch; at Corn Tullich; in Knappy Park, at Aboyne; at Craigs, Muir, Midstrath, and Wood Cottage, in Birse; west of Arbeadie, Banchory; on the Aberdeen road, near Banchory; east of Feugh Bridge; and in three or four spots on the Hill of Tilquihillie. That only one bed appears at all these spots may however be doubted.

SUPPLEMENTARY THERMOELECTRIC DIAGRAM.



XV.—*On the Thermo-Electric Properties of Charcoal and certain Alloys, with a Supplementary Thermo-Electric Diagram.* By C. G. Knott, B.Sc., and J. G. MacGregor, D.Sc. (Plate XXI.)

(Received 24th January 1878. Read 4th March 1878.)

The determination by experiment of the thermo-electric relations of any one substance belonging to the electromotive series to all other such substances is sufficient to fix all mutual thermo-electric relations among these. The first endeavour of the experimenter is then to obtain as convenient a substance for this purpose as possible. In investigating charcoal and certain alloys, we have in almost all cases employed one or other of two alloys of platinum and iridium, which have been already used by Professor Tait for a like purpose. The wires we used were the same which he discusses in his "First Approximation to a Thermo-Electric Diagram,"* under the names of M and N. Their complete freedom from oxidation, their elasticity, and the high temperatures of their fusing points, rendered them peculiarly suitable for thermo-electric investigations through long ranges of temperature.

Generally both the M and N wires were firmly bound, each by its one extremity to the end or ends of the wire or wires respectively which were under investigation, in a multiple junction. This triple or, as it was in some cases, quadruple junction constituted the "hot junction." The free extremities of the wires thus united were each bound to a moderately thin copper wire by very thin wire of the same metal, and the copper wires were led from these "cold junctions" to a commutator, which was in connection with a galvanometer. The commutator consisted of an arrangement of small mercury pools, into which the galvanometer and circuit wires, carefully amalgamated to ensure contact, dipped. All the junctions were formed in the manner indicated above, namely, by tightly binding the extremities of the wires together by thin copper wire.

The hot junction was made to vary in temperature according to the method referred to by Professor Tait on page 135 of the paper cited above. A wrought-iron tube, four inches long, two inches in diameter, with a bore of one inch diameter, which opened only below, was heated to the required temperature, and suspended by a hook fixed in its upper and closed end over the multiple

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* "Trans. R.S.E." 1872–3, p. 125.

junction. It was then lowered until the junction occupied a position about the centre of the tube, and about an inch above the open end. The junction speedily acquired the temperature of the region inside the tube or hollow cylinder, and gradually cooled with it. The cold junctions were immersed in distilled water contained in small beakers, which were all set in one large vessel, through which a constant stream of cold water flowed from a cistern. In this way, notwithstanding the proximity of the heated cylinder, the temperature of the cold junctions, as determined by a mercury thermometer, was kept nearly constant during a whole series of observations. In many cases no change was perceptible, and the variation never exceeded one-fifth of a degree centigrade. To guard against the possibility of galvanic or of electrolytic action, the portions of the wires of the cold junctions which were in the water were carefully covered with a thin coating of a non-conducting varnish.

To determine the temperature of the hot junction, the thermo-electric pair composed of the alloys M and N was used. It was for this reason solely that *both* of these wires were bound to the wire under investigation. They were peculiarly fitted for measuring temperature, since the variation with temperature of the electromotive force of a thermo-electric circuit formed of wires of these alloys had been thoroughly investigated by Professor TAIT and his students. The special peculiarity of this variation established in Professor TAIT's paper above cited, and fully corroborated by our own results given below, is that, through considerable ranges of temperature, the electromotive force of the M-N circuit may be taken as directly proportional to the difference of temperature of the hot and cold junctions. The current intensities were measured by the deflections produced on one of THOMSON's "dead beat" mirror galvanometers. The deflections were so small that they might be regarded as proportional to the currents producing them. The galvanometer terminals and the free extremities of the wires, which formed the multiple junction, were so connected to the mercury pool commutator, that any one of the possible thermo-electric junctions could be thrown into circuit with the galvanometer at will. It was not necessary to take measurements of the electro-motive force of *all* the possible pairs. Ordinarily the pair composed of either M or N and the wire under investigation, and that formed of M and N, were joined in circuit with the galvanometer alternately and in rapid succession at equal intervals of time. The circuit containing the latter pair may be distinguished as the thermometric circuit; and the mean of any two consecutive readings of the thermometric circuit, made in this way during the cooling of the hot junction, gave the measure of the electromotive force which corresponded to the temperature of the hot junction at the instant at which the reading for the other circuit was taken. In one or two cases the wire to be investigated was taken in thermo-electric connection with a wire of known thermo-electric relations other than either M or N. The hot junction was then quadruple, since in all cases in which the heated hollow cylinder was used, the M-N circuit was employed as the thermometer. After the completion of each experiment, which lasted until the temperature of the hot cylinder fell to very little above the temperature of the room, the junction was heated in olive-oil, and a series of measurements of temperature and corresponding M-N deflections was made. The temperature measurements were made by means of a mercury thermometer corrected by comparison with a Kew standard centigrade thermometer. We thus calibrated our galvanometer, and could at once substitute for the M-N deflections their equivalents in degrees centigrade, and thus obtain the law of variation of the electromotive force of the given thermo-electric pair with temperature. The observations of temperature of the cold junctions were also corrected by a similar comparison. It was found convenient to make the resistances of the circuits which were to be compared equal.

The results given below are subject to the following sources of slight inaccuracy:—(1.) The variation of temperature of the cold junctions, which, however, never exceeded ·2° C., and which in almost all cases was very small compared to the difference of temperatures of the hot and cold junctions. The assumption of a constant temperature of cold junction could not appreciably affect the interpretation of results. (2.) A necessary error in the estimation of the higher temperatures—due to the merely approximate parallelism of the M-N diagram lines. For the lower and really important temperatures, however, this error was so small as to be inappreciable. (3.) The variation in resistance of the circuits due to heating. The thickness of the M-N wires, and the small portions of them which were in the heated cylinder, and which were perhaps $\frac{1}{5000}$ of the whole resistance in the circuit, warrant us in believing that this variation in the M-N circuit was within the unavoidable errors of observation. Most of the wires we examined, however, were very thin, and had a great resistance per unit length, so that we could not put very great lengths in circuit; consequently a comparatively great proportion of the whole resistance was subject to increase from rise of temperature. For high temperatures then we do not consider ourselves warranted in placing more than a guarded confidence in the determinations, and consequently we give below no measurement made above 400° C. Further, for these high temperatures, the thinness of the wires rendered them peculiarly sensitive to currents of air, which could not be wholly screened off.

I. *Charcoal.*

The charcoal which we examined was an ordinary piece of gas-coke, such as is used in the Bunsen cell. It was cut in the form of a cylinder about 15 cm. in length and 1·5 cm. in thickness, and was arranged exactly as above described.

In Table I. we do not give the observed deflections of the thermometric circuit, but the averages of contiguous observed deflections, which are therefore the deflections corresponding in time to those of the thermo-electric circuit. The former are contained in the column headed M-N Deflections, the latter in that headed N-C Deflections (observed). The order of the letters in these headings indicates the arrangement of the wires, &c., in circuit. In the thermometric circuit this order was: M wire, N wire, galvanometer; in the other: N wire, charcoal, galvanometer; the charcoal being joined to the same galvanometer terminal in the one circuit as the N wire in the other. The column headed Temperature gives the temperatures of the heated junction as calculated from the deflections of the M-N circuit, by means of the observations contained in Table II. The other junctions had throughout the temperature 13·6° C. The column headed N-C (calculated) will be explained below.

Table I.

M-N Deflections.	Temperature.	N-C Deflections.	
		Observed.	Calculated.
+184·3	364·3 C.	+301	263·0
179·5	355·5	285	255·3
165·7	329	251	232·3
156	310·3	230	216·4
149	297	215	205·2
134·5	269·6	187	182·7
127	255·5	174·5	171·3
120·7	243·7	164	161·9
113·5	232·2	152	152·8
107·3	218·3	143	142
102	208·3	135	134·3
91	187·6	118·5	118·6
84·7	176	110	110
79	165·2	101·5	102
64	136·8	81·5	81·3
59·7	128.8	76·5	75·7
48	106·6	60·5	60·5
40·3	92	50·5	50·6
36	83·7	45·5	45
28	68·8	34·5	35·1
22·5	58·5	27·5	28·4
17	48	20	21·6
13	40·3	15	16·7

Table II. gives the experiments which show that the temperatures given above are those indicated by the M-N deflections against which they stand.

TABLE II.

Temperature of Oil round the M-N junction.	M-N Deflection.	Temperature of Oil round the M-N junction.	M-N Deflection.
288·1° C.	143	173·2° C.	82·7
280·8	140·4	150·2	70·5
273·6	136·7	141·2	65·7
266·4	133	132	61·4
257·5	128	123·9	57·5
248·6	123·7	107·8	48·5
239·7	119	92·5	39·7
229·1	113·5	79·2	32·7
211·2	102·5	66·2	26·2
201·1	97·7	60·3	23·7

When the numbers of Table II. are plotted as abscissae and ordinates, they give a straight line from which the temperatures of Table I. are obtained graphically. Up to 350° C. the deviation from a straight line has been shown, both by Professor TAIT's experiments and our own, to be inconsiderable. For higher temperatures, however, the deviation is more appreciable, indicating a neutral point for M and N at about a white heat. Accordingly, we do not feel warranted in giving the results for the higher temperatures which this experimental method supplies; and in the tables which follow we give those deflections only which correspond to temperatures below 400° C. Up to this temperature other errors pointed out above are also of extremely small magnitude. As it would require too much space to give full tables corresponding to Table II. for all substances which we have investigated, we give for the rest simply sufficient data to determine the relation between the deflection and the temperature of the heated junction of the M-N circuit.

In deducing from our observations formulæ showing the relation between the temperatures and the corresponding deflections, we have based upon Professor TAIT's deduction from theory that the electromotive force of a thermo-electric circuit is a parabolic function of the temperature,[*] a deduction which is supported by a long series of experimental facts. We have therefore thought it unnecessary to seek for any other expression for the relation between deflection (proportional, of course, to electromotive force) and temperature than is given by the expression of the former in terms of the first

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[*] "Trans. Roy. Soc. Edin." 1872–3, p. 127.

and second powers of the latter. In general the agreement between our observations and the results of the substitution of the corresponding values of the temperature in the formulæ will show that this assumption was not unwarranted.

In the case of charcoal, however, we find that we cannot express the deflection in terms of the first and second powers of the temperature. For a small range of temperature up to about 230° C. it is possible. From 30° or 40° up to that temperature the following formula (in which δ stands for deflection, and t for temperature) holds:—

$$\delta = -8{\cdot}29 + {\cdot}604t + {\cdot}000385t^2$$

The fourth column of Table I. gives the values of δ as calculated from this formula. Up to 230° they agree well with the observed results. Above that, however, they do not, and, perhaps because of chemical changes produced by heat, charcoal seems to be an exception to the general law. When it forms part of a thermo-electric circuit the electromotive force is not capable of being represented as a parabolic function of the temperature, except for comparatively low temperatures.

The above formula and a graphic treatment of the observations at higher temperatures enable us to determine the position of the charcoal line on the thermo-electric diagram.*

Differentiating the equation given above, we find

$$\frac{d\delta}{dt} = {\cdot}604 + {\cdot}00077t\,,$$

whence, if $t = 0° \text{ C.}, \frac{d\delta}{dt} = {\cdot}604\,,$

and if $t = 200° \text{ C.}, \frac{d\delta}{dt} = {\cdot}758\,.$

Now the results of Table II. are capable of representation by the formula

$$\delta_1 = \text{const.} + {\cdot}5307t,$$

where δ_1 stands for the deflection of the M-N circuit when the junction was immersed in oil, t for the corresponding temperature.

Hence $\frac{d\delta_1}{dt} = {\cdot}5307$

for all values of t.

Since the circuits had equal resistances we find, by dividing $\frac{d\delta}{dt}$ by $\frac{d\delta_1}{dt}$, the ratio of the "thermo-electric powers" of the N-C and of the M-N circuits at 0° and 200° C. This ratio may be written $\frac{d\delta}{d\delta_1}$.

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* "Trans. Roy. Soc. Edin." 1872–3, p. 131.

We thus know the ratio of the distance from the N line of the point corresponding to 200° C. on the charcoal line, to the distance between the M and N lines; and it only remains to determine their general relative position on the diagram. This is indicated by the signs of the deflections given in Table I. These show that the arrangement M, N, galvanometer, gives a current in the same direction as N, C, galvanometer, *i.e.*, if a current flows across the heated M-N junction from M to N, it flows across the heated N-C junction from N to C, and thus on the diagram the C line must be on the same side of the N line as the N line is of the M line.

The above data enable us to draw the C line from 30° or 40° up to 230°. To fix the line for higher temperatures, we have determined $\frac{d\delta}{dt}$ graphically as follows:—

$$\text{for } t = 290^\circ \text{ C.}, \frac{d\delta}{dt} = 1{\cdot}01$$

$$t = 340^\circ \text{ C.}, \frac{d\delta}{dt} = 1{\cdot}29 .$$

The line thus determined will be found lettered C in the supplementary thermo-electrical diagram which we give in Plate XXI. It will be noticed that we have found the value of $\frac{d\delta}{dt}$ for $t = 0^\circ$ C., although 0° C. is beyond the limits of our experiments. Consequently the line at and somewhat above 0° C. is dotted. The diagram which we have prepared is simply an extension of that of Professor TAIT. It is drawn to exactly the same scale. In the equations to lines on this diagram, the point at which the 0° C. line cuts the lead line is taken as origin, and the lead line as the axis of temperature. The equation to that part of the carbon line which is straight is

$$y = -{\cdot}029t + 20{\cdot}54,$$

where the ordinate y is expressed in terms of a unit of electromotive force, the number of which corresponding to different lengths of ordinates is given on the margin of the diagram, and t in degrees centigrade.

For the value of the above equation in determining the electromotive force in a circuit containing carbon, see the conclusion of this paper.

The position which is thus assigned to the charcoal line agrees generally with the results which E. BECQUEREL and MATTHIESSEN have obtained for the same substance. The position in thermo-electric series which the former finds to hold for the low temperatures is indicated as follows[*]:—(+) cadmium, silver, platinum, charcoal, tin, lead (—). Taking the lines for these metals as given in Professor TAIT's diagram, our order for low temperatures is:—(+) cadmium,

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[*] "Ann. de Chim. et de Phys." (4) T. viii. p. 415.

silver, charcoal, platinum, tin, lead(—). The position of platinum is known to vary very much according to the state of the wire which is used in the determinations. With regard to numerical values of the electromotive force of circuits containing charcoal, BECQUEREL'S results and ours do not agree so well. But considering the very variable constitution of gas-coke, more than a general agreement is not to be expected. With MATTHIESSEN'S results,* ours show also a general agreement. He gives charcoal the following position :—(+) cadmium, zinc, charcoal, silver, gold, platinum, lead (—). Ours for the same temperature is :—(+) cadmium, gold, zinc, charcoal, platinum, silver, lead (—).

II. *Alloys.*

The alloys which we have investigated were all (except that of magnesium and thallium) prepared for Professor TAIT by Messrs JOHNSON & MATTHEY, London. They were all in the form of very thin wires, and were prepared from pure metals. Their constitution is given in percentages of mass.

Silver-Palladium Alloys.

We have investigated two, one of which contained 20 per cent. the other 25 per cent. of palladium. As both wires were thin, and a junction of four wires could be made so small as to be certainly at the same temperature throughout, we examined both wires at the same time. The following modifications in the method described above were thus rendered necessary :—The junction in the tube was a junction of four wires. The connections were so made that there were three circuits, viz., each of the alloys with N, and M-N. The commutator was so arranged that, by being placed in various positions, contact was made in each of these circuits. Readings were taken at short and nearly equal intervals of time in the following order :—M-N, first alloy circuit, second alloy circuit, first alloy circuit, M-N ; and the averages of the deflections of the M-N and of the first alloy circuit gave deflections corresponding in time, and therefore in temperature, with those of the second alloy circuit. The time occupied in taking the five observations was so short as to warrant the taking of averages.

We give in one table the results of our experiments with these alloys. The end of N was attached to the same terminal of the galvanometer in all three circuits. The direction of the deflection of the M-N circuit is considered positive. The temperature of the beakers was almost constantly 14·1° C., rising for a short time to 14·2°, and falling also for a very short time to 14°.

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* "Pogg. Ann." Bd. ciii. p. 412.

TABLE III.

M-N Deflections.	Temperature.	$AgPd_{20} - N$ Deflections.		$AgPd_{25} - N$ Deflections.	
		Observed.	Calculated.	Observed.	Calculated.
+58	333·9° C.	–265·8	266	–319	321·8
53·8	310·4	243·5	242·5	293·5	292·2
51·6	298·3	228	230·7	271·2	277·4
47·9	278·6	212·1	211·8	252·6	253·8
44·5	259·5	191·5	193·9	227·5	231·5
40·3	236	175	172·4	205	204·9
36·8	216	154·3	154·6	182·5	183
33·5	197·5	138	138·5	162·8	163·4
30	178·8	124·5	122·7	146·5	144·1
26	156·9	105	104·6	123	122·3
21·8	133	85·3	85·5	100	99·4
16·7	105	64·1	64	73·8	73·8
13·3	86·2	49·3	50	57·2	57·4
9·75	67	37	36·1	42·5	41·3
9	62·8	33·5	33·1	38·5	37·8
8·25	58·5	29·5	30·1	34·5	34·3
6·25	47·5	24	22·5	27·5	25·5
5	40·8	18·7	17·9	20·7	20·3
4·25	36·2	14·7	14·7	17	16·7

The following table gives various temperatures of the heated junction of the M-N circuit, with the corresponding deflections.

TABLE IV.

Temperature.	M-N Deflection.
14·1° C.	0
54	7·2
93	14·5
141·2	23
187·2	32

(1.) *Alloy of which* 20 *per cent. is Palladium.*

The signs of the deflections show that if the current flows from M to N, across the heated M-N junction, it flows from N to $AgPd_{20}$ across the heated junction of these substances. Hence the diagram lines of M and $AgPd_{20}$ are on opposite sides of the N line. From the above observations we deduce the formula

$$\delta = -9{\cdot}015 + {\cdot}636t + {\cdot}000562t^2.$$

The close agreement between the observed and calculated values of δ speaks for the accuracy of the formula.

Hence

$$\frac{d\delta}{dt} = \cdot 636 + \cdot 001124t\,;$$

If, therefore,

$$t = \quad 0^\circ \text{ C.}, \frac{d\delta}{dt} = \cdot 636\,,$$

and if

$$t = 300^\circ \text{ C.}, \frac{d\delta}{dt} = \cdot 9732\,.$$

From the observations of Table IV. may be deduced the equation

$$\delta_1 = \text{const.} + \cdot 18137t,$$

whence

$$\frac{d\delta_1}{dt} = \cdot 18137.$$

We are now able to find $\frac{d\delta}{d\delta_1}$ for $t=0^\circ$ and $t=300^\circ$ C., and thus the position of the line on the diagram is determined. It will be found lettered $AgPd_{20}$ on Plate XXI. This mode of writing is adopted to show readily the composition of the alloy. The number expresses in all cases the percentage of the substance whose symbol is written second. Thus $AgPd_{20}$ means the alloy, 20 per cent. of which is palladium.

The equation to this line is readily found to be

$$y = -\cdot 124t - 26\cdot 83.$$

(2.) *Alloy of which 25 per cent. is Palladium.*

The signs of deflections being the same as in the case of the former alloy, the lines of M and $AgPd_{25}$ are also on opposite sides of the N line.

The equation deduced from the above observations is as follows:—

$$\delta = -10\cdot 206 + \cdot 71189t + \cdot 0008457t^2.$$

The column of calculated deflections shows in some cases somewhat large differences, but in number the differences are about equally divided between those which are greater and those which are less than the observed results.

Hence

$$\frac{d\delta}{dt} = \cdot 71189 + \cdot 0016914t\,;$$

if, therefore,

$$t = \quad 0^\circ \text{ C.}, \frac{d\delta}{dt} = \cdot 71189,$$

and if

$$t = 300^\circ \text{ C.}, \frac{d\delta}{dt} = 1\cdot 21931.$$

As before,

$$\frac{d\delta_1}{dt} = \cdot 18137,$$

and two points being determined of the $AgPd_{25}$ line, its line is determined. It will be found in Plate XXI. Its equation is found to be

$$y = -{\cdot}1865t - 35{\cdot}2$$

Along with the lines for these alloys we give in the diagram those of silver and palladium (after TAIT). We may notice that the alloy lines lie between those of the metals. But they resemble in position and in general relation to others much more that of palladium than that of silver, although they contain a much greater proportion of the latter than of the former metal. The specific heat of electricity of both alloys is negative, that of palladium being negative, that of silver positive.

Platinum-Iridium Alloys.

We have investigated four, in which the percentages of iridium were 6, 10, 15, 20 respectively, and, as above, two at a time. We give in Table V. the results of our observations on the first two, viz., the alloys $PtIr_6$ and $PtIr_{10}$. The N wire of each alloy circuit was connected with the same terminal of the galvanometer as the N wire of the M-N circuit. The temperature of the beakers was constantly 13·4° C.

TABLE V.

M-N Deflections.	Temperature.	$PtIr_6$-N Deflections.		$PtIr_{10}$-N Deflections.	
		Observed.	Calculated.	Observed.	Calculated.
+71·3	386·3° C.	−257·2	255·2	+32·5	33·4
66·8	362·7	232·5	233	32·3	32·3
62·8	341·8	213·5	213·9	31	31·2
58·5	319·7	195	194·4	30	29·9
55·8	305·3	182·8	182·1	29	29·1
52·8	289·9	168·8	169·2	28	28·1
47·6	262·7	147·3	147·5	26·5	26·1
43·3	240	130·5	130·1	25	24·4
40·5	226	118·7	119.9	23·6	23·2
36·3	204·2	105·8	104·3	21·5	21·3
32·5	184·2	91·5	90·7	20	19·5
29·5	169·4	80	81	18	18·1
24·3	142·2	63·5	64·1	15	15·3
20	119·8	51·5	50·9	12·7	12·9
16·5	101·3	41·3	40·6	10·5	10·8
13·8	86·6	33·2	32·8	9	8·8
10·5	69·5	26·5	24·1	5·7	6·9
8·8	60	21·5	19·4	5·7	5·8
6	45·9	12	12·8	4	4

Table VI. gives observations which enable us to find the temperatures given in the 2d column of Table V., and to find the rate of variation of the deflections of the M-N circuit with the temperature of the heated junction :—

TABLE VI.

Temperature.	M-N Deflection.
13·4° C.	0
87·7	13·9
111·8	18·5
118	19·7
123·8	20·7

(1.) *Alloy,* 6 *per cent. of which is Iridium.*

The deflections of the circuit containing this alloy being negative, its line is below that of N in the diagram. The relation between the deflections and the temperature of the heated junction of the circuit containing this alloy is given by the formula

$$\delta = -6{\cdot}8675 + {\cdot}39398t + {\cdot}0007367t^2.$$

The 4th column of Table V. shows that the deflections calculated from this formula agree with those observed up to about 390° C. Hence if

$$t = \quad 0^\circ \text{ C.}, \frac{d\delta}{dt} = {\cdot}39398$$

and if

$$t = 300^\circ \text{ C.}, \frac{d\delta}{dt} = {\cdot}836$$

From Table VI. we find that

$$\delta_1 = \text{const.} + {\cdot}1914t\,.$$

Hence

$$\frac{d\delta_1}{dt} = {\cdot}1914\,.$$

The line thus determined will be found in Plate XXI. Its equation is

$$y = -{\cdot}154t + 2{\cdot}13.$$

(2.) *Alloy containing* 10 *per cent. of Iridium.*

The deflections of column 5, Table V., being positive, the line of this alloy lies above that of N in the diagram. The formula showing the relation between deflection and temperature is as follows :—

$$\delta = -2{\cdot}259 + {\cdot}14172t - {\cdot}00012822t^2.$$

The 6th column of Table V. shows the accuracy of this formula up to about 390° C. Hence if

$$t = \quad 0^\circ \text{ C.}, \frac{d\delta}{dt} = {\cdot}14172$$

and if

$$t = 300^\circ \text{ C.}, \frac{d\delta}{dt} = {\cdot}06479\,.$$

As above $\frac{d\delta^1}{dt}=\cdot 1914$; and thus two points on the line of this alloy are fixed. The line will be found on Plate XXI. Its equation is

$$y=-\cdot 0268t+58\cdot 11.$$

The other two platinum-iridium alloys we examined also together, the general arrangement being such as was described in the experiments with the silver-palladium alloys. In this case the N wires of the alloy circuits were connected with the other terminal of the galvanometer than that with which the N wire of the M-N circuit was connected. The temperature of the beakers was almost uniformly 13·3° C. It rose for a short time to 13·4°. The results of our experiments with these wires are given in Table VII.

Table VII.

M-N Deflections.	Temperature.	N-PtIr$_{15}$ Deflections.		N-PtIr$_{20}$ Deflections.	
		Observed.	Calculated.	Observed.	Calculated.
+95	384·3° C.	−163	158·7	−148	146·5
87·3	354	144	142·8	134	136·8
81	329·6	132	130·4	124	123·7
72·3	295·4	114·8	113·6	111·3	108·3
64	263·1	97·5	98·4	93·5	94·9
54·3	225·1	81	81·2	78·8	79·5
45	189	66	65·6	65·3	65·1
39	165·3	56·8	55·8	55·8	55·9
34·5	147·5	50	48·6	49·5	49·1
26·9	118·2	36·6	37·2	37·6	38
23·3	103·8	31·8	31·8	32·6	32·6
19·8	89·6	26·5	26·5	28	27·4
17·3	80	23	23	24	24·9
14·3	68·3	19	18·9	19·9	19·6
12·5	61·5	15	16·5	15·3	17·2

Table VIII. gives the observations of various deflections of the M-N circuit and the corresponding temperatures of the heated junction.

Table VIII.

Temperature.	M-N Deflection.
12·9° C.	0
103·8	23·7
142·2	33
186·1	44·3

(3.) *Alloy containing* 15 *per cent. of Iridium.*

The deflections of the circuit containing this alloy being negative, it follows (from the arrangement of the circuits in these experiments) that the thermo-electric line of $PtIr_{15}$ is above that of N on the diagram. From the observations of the 3d column of Table VII. the following formula has been found :—

$$\delta = -3{\cdot}8992 + {\cdot}31411t + {\cdot}00028348t^2 .$$

The 4th column of the same table contains the deflections calculated from this formula for various temperatures. They show the accuracy of the formula up to about 380° C. Hence

$$\frac{d\delta}{dt} = {\cdot}31411 + {\cdot}00056696t .$$

If, therefore, $t = \ 0° \text{ C.}, \ \frac{d\delta}{dt} = {\cdot}31411 ,$

and if $t = 300° \text{ C.}, \ \frac{d\delta}{dt} = {\cdot}4842 .$

The results of Table VIII. show that the relation between deflection and temperature of the M-N circuit is given by the formula

$$\delta_1 = \text{const.} + {\cdot}25577t .$$

Hence

$$\frac{d\delta_1}{dt} = {\cdot}25577 .$$

All three circuits having exactly the same resistance, we have, therefore, values of $\frac{d\delta}{d\delta_1}$ for two temperatures which determine the position on the diagram of the $PtIr_{15}$ line. (See Plate XXI.) Its equation is found to be

$$y = + {\cdot}0443t + 67{\cdot}86 .$$

(4.) *Alloy containing* 20 *per cent. of Iridium.*

As in the last case, and for the same reasons, the thermo-electric line of $PtIr_{20}$ on the diagram is above that of the N line. The observations recorded in the 5th column of Table VII. give the formula

$$\delta = -4{\cdot}4606 + {\cdot}34414t + {\cdot}00012721t^2 ;$$

which is tested by the calculations contained in the 6th column, and found to be accurate up to nearly 380° C. Hence

$$\frac{d\delta}{dt} = {\cdot}34414 + {\cdot}00025442t .$$

If, therefore, $t = \ 0° \text{ C.}, \ \frac{d\delta}{dt} = {\cdot}34414 ,$

and if $t = 300° \text{ C.}, \ \frac{d\delta}{dt} = {\cdot}42047 .$

As before

$$\frac{d\delta_1}{dt} = \cdot 25577\,,$$

and thus the $PtIr_{20}$ line is determined. It will be found on Plate XXI. Its equation is

$$y = +\cdot 0199t + 70\cdot 21\,.$$

The platinum line (according to TAIT) is laid down on the diagram: the iridium line is not yet determined, but at low temperatures it is, according to SEEBECK* and MATTHIESSEN† not far from the platinum line. Thus, the lines of none of these alloys are between those of their constituent metals. So far as the above four are concerned, at low temperatures the greater the percentage of iridium the higher is the line on the diagram. For temperatures above about 100° C., however, the order in the thermo-electromotive series is $PtIr_{15}$ $PtIr_{20}$, $PtIr_{10}$, Pt,‡ $PtIr_{6}$. When we consider the position of the lines of alloys of platinum and iridium, which have been determined by Professor TAIT, the above simple relation between the constitution of the alloy and position on the diagram does not seem to hold even for low temperatures. Comparison of our diagram with his shows the order in the thermo-electromotive series at about 10° C. to be: (+) $PtIr_{20}$, $PtIr_{15}$, $PtIr_{10}$, $PtIr_{15}$,‡ $PtIr_{5}$‡ $PtIr_{10}$,‡ $PtIr_{15}$,‡ PtIr‡, $PtIr_{6}$. At 300° C the order is: (+) $PtIr_{15}$, $PtIr_{20}$, $PtIr_{10}$, $PtIr_{15}$,‡ $PtIr_{15}$,‡ $PtIr_{5}$,‡ $PtIr_{10}$,‡ Pt, $PtIr_{6}$ (−). The difference in the position of lines of alloys of the same composition can be due only to differences of molecular state, or to slight impurities. These results do not warrant the conclusion to the relation between the constitution of the alloys and their order in the series which ROLLMANN and others have found to hold in the case of tin-bismuth and other alloys.§ Possibly, however, high temperatures would so change the series as to give this relationship only a narrow validity. It is interesting to note that the coefficient of proportionality of the specific heat of electricity to absolute temperature for platinum-iridium alloys is in some cases zero, in some greater, in others less than zero.

Iron-Gold Alloy, containing 5 *per cent. of Iron.*

This alloy was examined by itself. The N wire in both circuits was attached to the same end of the galvanometer wire. The temperature of the beakers was 14·6° C. For a short time it sank to 14·4°. The observations are given in Table IX.

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* "Pogg. Ann." Bd. vi. pp. 133, 253.

† "Pogg. Ann." Bd. ciii. p. 112.

‡ See Professor TAIT's paper cited above, pp. 138, 139.

§ WIEDEMANN's "Galvanismus," Bd. i. § 594, p. 814.

TABLE IX.

M-N Deflections.	Temperature.	AuFe$_5$-N Deflections.	
		Observed.	Calculated.
+97	388·3° C.	−263·5	263·2
90·3	362·3	256·5	253·6
83·3	335·3	244·5	242·2
75·5	305·3	226·8	227·9
71	287·9	218·3	218·8
66	268·9	208	208·8
62·5	255·4	197·5	200·2
58	237·7	189	189·3
54·8	225·5	181	180·4
50	207·2	169	169
47	196	160	161·1
43	180·3	151·5	149·5
42	176·4	144·5	146·6
38	161·2	138	135
37	157·1	130	131·7
35·1	150	124·8	126·1
33·3	143	118	120·4
31·3	135	111·5	113·6
29	127·1	103·5	107·1
25	111·6	93	93·7
23	104	90·5	87
20	92·4	78	76·5
19·5	90·5	72·8	74·8
16·5	79	63	64
14·8	72	56·5	57·4
13·3	66·2	50·5	52·2
10·3	54	39·5	40
8·3	46·5	34·25	32·4
6·7	40·2	27	26
6	38	24·5	23·8
5·4	35·3	21	21
5	34	19	20·3
3·5	27·6	12·5	13·1
1·1	18·2	5	3·2

The deflections of the alloy circuit being negative, the line of AuFe$_5$ is below that of N on the diagram. The third column of the above table gives the formula

$$\delta = -16{\cdot}4 + 1{\cdot}09445t - {\cdot}0009641t^2$$

whose accuracy up to nearly 390° the fourth column attests. Hence

$$\frac{d\delta}{dt} = 1{\cdot}09445 - {\cdot}0019282t.$$

If, therefore, $t = 0° \text{ C.}, \frac{d\delta}{dt} = 1{\cdot}09445,$

and if $t = 300° \text{ C.}, \frac{d\delta}{dt} = {\cdot}51599.$

In Table X. will be found deflections of the M-N circuit with the corresponding temperatures of the heated junction.

TABLE X.

Temperature.	M-N Deflections.
14·6° C.	0
142·2	32·9
147·9	34·4
168·2	40
181·2	43·2

From the observations of Table X. we deduce the formula

$$\delta_1 = \text{const.} + \cdot 25947t\,;$$

and therefore,

$$\frac{d\delta_1}{dt} = \cdot 25947\,.$$

The $AuFe_5$ line thus determined is given on Plate XXI. Its equation is

$$y = + \cdot 149t - 41\cdot 06\,.$$

The iron and gold lines (after TAIT) are laid down on the diagram. The line of the alloy resembles that of gold more than that of iron, as might be expected from the small quantity of iron present.

Platinum-Silver Alloy, containing 35 *per cent. of Silver.*

To make observations with a wire of this alloy we found it convenient to join it in a circuit with a palladium wire (the same which Professor TAIT used in determining the palladium line on his diagram), instead of the usual N wire. Otherwise the arrangement and method were the same as before. The N wire of the thermometric circuit, and the platinum-silver wire of the thermo-electric circuit, were joined to the same terminal of the galvanometer. The temperature of the beakers was almost constant at 13·4° C. For a short time it was 13·45°, and again for a short time 13·5°. The observations are given in Table XI.

TABLE XI.

M-N Deflections.	Temperature.	$Pd\text{-}PtAg_{35}$ Deflections.	
		Observed.	Calculated.
+85	377·5° C.	+51	51·7
80·5	358·3	49	47·7
74·3	331·7	42	42·2
70·5	315·7	39	39·2
68	305	36	37·2
65·3	293·3	35	35
62·7	282·7	33	33·1
57·8	261·4	29	29·5
54·5	247·5	28	27·2
51	232·6	26	24·8
44·5	205·2	18·5	20·6
41	190	19	18·5
38	177·5	17	16·8
34·5	162	14·5	14·7
30·9	147·3	12·5	12·9
26·3	127·2	10	10·5
23·3	114·3	8·75	9
21	104·9	7·5	8
19	96·5	7	7·1
16·7	87	8	6·2
16·5	86	6	6·1
15·3	80·3	5·5	5·5
13·4	72·5	5	4·8
12·6	69	4·5	4·5
12·2	68	5	4·4
11·2	63·5	4	4
10·2	59	3·5	3·6
8·7	54	3·75	3·1
6·5	43·3	2·5	2·2
5·7	40	2	1·9
5·2	38	1	1·8
5	37	1·25	1·7
3·8	32	·5	1·3

The deflections of both circuits being of the same sign, the line of this alloy must be above that of palladium on the diagram. The following is the formula deduced from the third column of the above table :—

$$\delta = -\cdot 91017 + \cdot 06422t + \cdot 0001988t^2.$$

The fourth column shows the accuracy of this formula up to nearly 380° C. Hence

$$\frac{d\delta}{dt} = \cdot 06422 + \cdot 0003976t.$$

If, therefore, $t = \ \ 0^\circ \text{ C.}, \frac{d\delta}{dt} = \cdot 06422,$

and if $t = 300^\circ \text{ C.}, \frac{d\delta}{dt} = \cdot 1835.$

Table XII. contains deflections of the M-N circuit and corresponding temperatures of the heated junction of that circuit.

TABLE XII.

Temperatures.	M-N Deflection.
13·4	0
88·7	17
180·5	38·6
197·9	42·75
202·2	44

Thus $$\delta_1 = \text{const.} + \cdot 23496t,$$

and $$\frac{d\delta_1}{dt} = \cdot 23496\,;$$

and therefore $\frac{d\delta}{d\delta_1}$ is determined for two temperatures. $\frac{d\delta}{d\delta_1}$ is here the ratio of the length of the particular line of temperature intercepted by the Pd and alloy lines to that intercepted by the M and N lines. The line of $PtAg_{35}$ will be found on Plate XXI. It will be noticed that the accuracy of this line depends on the accuracy of the palladium line. If the latter should be found not quite accurate, and be changed in position, the $PtAg_{35}$ line must be drawn from it in its new position. The equation to the line of this alloy is found to be $y = -\cdot 241t - 40\cdot 93$. It lies much lower on the diagram than the lines of its constituent metals.

Magnesium-Thallium Alloy, containing 25 to 50 per cent. of Thallium.

This alloy was furnished to Professor TAIT by Dr GORE. We joined a wire of it in circuit with the N wire. The ends of the N wire in both circuits were joined to the same terminal of the galvanometer. The temperature of the beakers varied from 13·4° to 13·5° C. The observations are contained in Table XIII.

TABLE XIII.

M-N Deflections.	Temperature.	MgTl-N Deflections.	
		Observed.	Calculated.
+ 180	394·1° C.	− 360	356·3
169·7	372·4	336	334·6
157·7	347·3	309	309·8
148·5	327·5	290·5	290·3
138·7	306·8	269	270·1
129·7	287·8	248	251·8
121	269·3	236	234

TABLE XIII. (*continued*).

M-N Deflections.	Temperature.	MgTl-N Deflections.	
		Observed.	Calculated.
114·7	256·1 C.	224	221·4
107	240	206	206·1
99·5	224	191·5	191
90	204·2	172	173·4
79·7	182·4	152	152
69·5	160·5	132	132
59·7	140	115·5	113·25
57	134	106	107·8
51	121·2	97	96·2
47	112·7	87·5	88·5
44·2	107	81	83·4
39·5	96·7	74	74·2
35·2	87·7	66·5	66·1
30·2	77·4	57	57
27·7	70	49	50·4
23·7	63·7	45	44·8
22·2	60·5	42	42
21	58	38·5	39·8
19	53·8	34·5	36·1
16·8	49	31·2	31·9
14	43·1	27	26·7
13	41	25	24·9
11·5	37·5	22	21·8

The deflection of the alloy circuit being negative, the alloy line is below that of N. From the third column we deduce the formula

$$\delta = -10{\cdot}6516 + {\cdot}85965\,t + {\cdot}0001812\,t^2.$$

The fourth column shows its accuracy up to about 400° C. Hence

$$\frac{d\delta}{dt} = {\cdot}85965 + {\cdot}0003624\,t.$$

If, therefore, $t = \quad 0^\circ \text{ C.}, \frac{d\delta}{dt} = {\cdot}85965,$

and if $t = 300^\circ \text{ C.}, \frac{d\delta}{dt} = {\cdot}96837.$

The results of experiments with the M-N junction of variable temperature in oil are contained in Table XIV.

TABLE XIV.

Temperature.	M-N Deflection.
13·5° C.	0
32·8	9
155·9	67
201·1	88·5
216·4	96

These observations give us the equation

$$\delta_1 = \text{const.} + \cdot 47284t\,.$$

Hence

$$\frac{d\delta_1}{dt} = \cdot 47284\,.$$

The line is laid down on Plate XXI. Its equation is

$$y = - \cdot 0153\,t + 6\cdot 64\,.$$

The magnesium line (after TAIT) will be found on our diagram. The line of the alloy is near it, but its inclination to the lead line is different. The thallium line is not yet determined.

The unit of electromotive force which we employed in the above was an arbitrary unit. It was necessary then to find its value. For this purpose we used a DANIELL's cell, consisting of a plate of copper dipping in a solution of pure copper sulphate whose density was 1·125, and a plate of amalgamated zinc dipping in a solution of zinc sulphate (free from iron), whose density was 1·098. The temperature of the solutions was about 11° C. We compared the deflection produced in the same galvanometer, when the current from the shunted cell was sent through it, with that caused by the current of the M-N thermo-electric circuit with the junctions at definite temperatures. The following are the measurements made :—

(1.) *Circuit containing Daniell's Cell.*

Resistance of galvanometer, . . . = 22·588 ohms.
Resistance of wire in circuit, . . . = 20,000·08 „

Resistance of Shunt.	Average of several Deflections.
1 ohm.	134·2
2 „	222
5 „	365·8

(2.) *Thermo-Electric Circuit containing* M *and* N.

Resistance of galvanometer, as before, . . =22·588 ohms.
Resistance of M, N, and other wires in circuit, . = ·841 „

Temperature of M-N Junction.	Temperature of other Junctions.	Deflection.
51·5° C.	11·2° C.	46·5
149·5	11·3	160

From these data it is found that the arbitrary unit which we have used in drawing our diagram has the value 105×10^{-9} of a DANIELL's cell of the above construction.

Professor TAIT has given the following formula for the electromotive force of a circuit consisting of two substances a and b, viz. :—

$$E = (k_a - k_b)(t - t_1)\left(T_{ab} - \frac{t+t_1}{2}\right),$$

where t and t_1 are the temperatures of the junctions, T_{ab} the "neutral point" of the substances, and k_a, k_b constants whose magnitude depends on the peculiarity of the substance and on the units employed in the measurement of electromotive force and temperature. It may be easily shown that k for any metal is numerically equal to the tangent of inclination to the t axis of its line on the diagram. Hence we obtain the electromotive force of any pair in terms of our arbitrary unit by substituting in the above formula the various temperatures in °C., and writing for k_a, k_b the coefficients of t in the equations to the diagram lines of the metals a, b. The neutral point of any two substances is found from the equations to their diagram lines as given above by the elimination of y. To obtain the electromotive force in terms of any other unit is a mere matter of arithmetic. For convenience we give in Table XV. the values of k, which, when substituted in the above formula, give the electromotive force in terms of 10^{-6} of a DANIELL's cell. Columns (1) and (2) contain the values of the constants $\frac{dy}{dt}$ and A in the equations to the diagram lines of the various substances (the equations being of the form $y = \frac{dy}{dt} t + A$). The fourth column contains the values of k (after TAIT*) expressed in terms of a unit equal to "nearly 10^{-5} of a GROVE's cell."

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* "Trans. R. S. E." vol. xxvii. 1872-73, pp. 126 and 139.

TABLE XV.

Substance.	$\frac{dy}{dt}$ (diagram unit).	A (diagram unit).	k in terms of 10^{-6} of a DANIELL'S cell.	k (after TAIT) in terms of nearly 10^{-5} of a GROVE'S cell.
Charcoal .	− ·0290	+ 20·54	− ·00304	− ·000193
$AgPd_{20}$.	− ·1240	− 26·83	− ·01302	− ·000830
$AgPd_{25}$.	− ·1865	− 35·20	− ·01958	− ·001243
$PtIr_{6}$ . .	− ·1540	+ 2·13	− ·01617	− ·001026
$PtIr_{10}$ . .	− ·0268	+ 58·11	− ·00281	− ·000179
$PtIr_{15}$ . .	+ ·0443	+ 67·86	+ ·04651	+ ·000296
$PtIr_{20}$ . .	+ ·0199	+ 70·21	+ ·00209	+ ·000133
$AuFe_{5}$ . .	+ ·1490	− 41·06	+ ·01564	+ ·000991
$PtAg_{35}$ . .	− ·2410	− 40·93	− ·02530	− ·001607
MgTl .	− ·0153	+ 6·64	− ·00161	− ·000102

The above experiments were conducted during the summer months of 1877 in the physical laboratory of the Edinburgh University. The alloys were kindly supplied us by Professor TAIT, at whose desire we undertook the investigations, and to whom we are indebted, not only for the use of his laboratory and apparatus, but also for his invaluable advice.

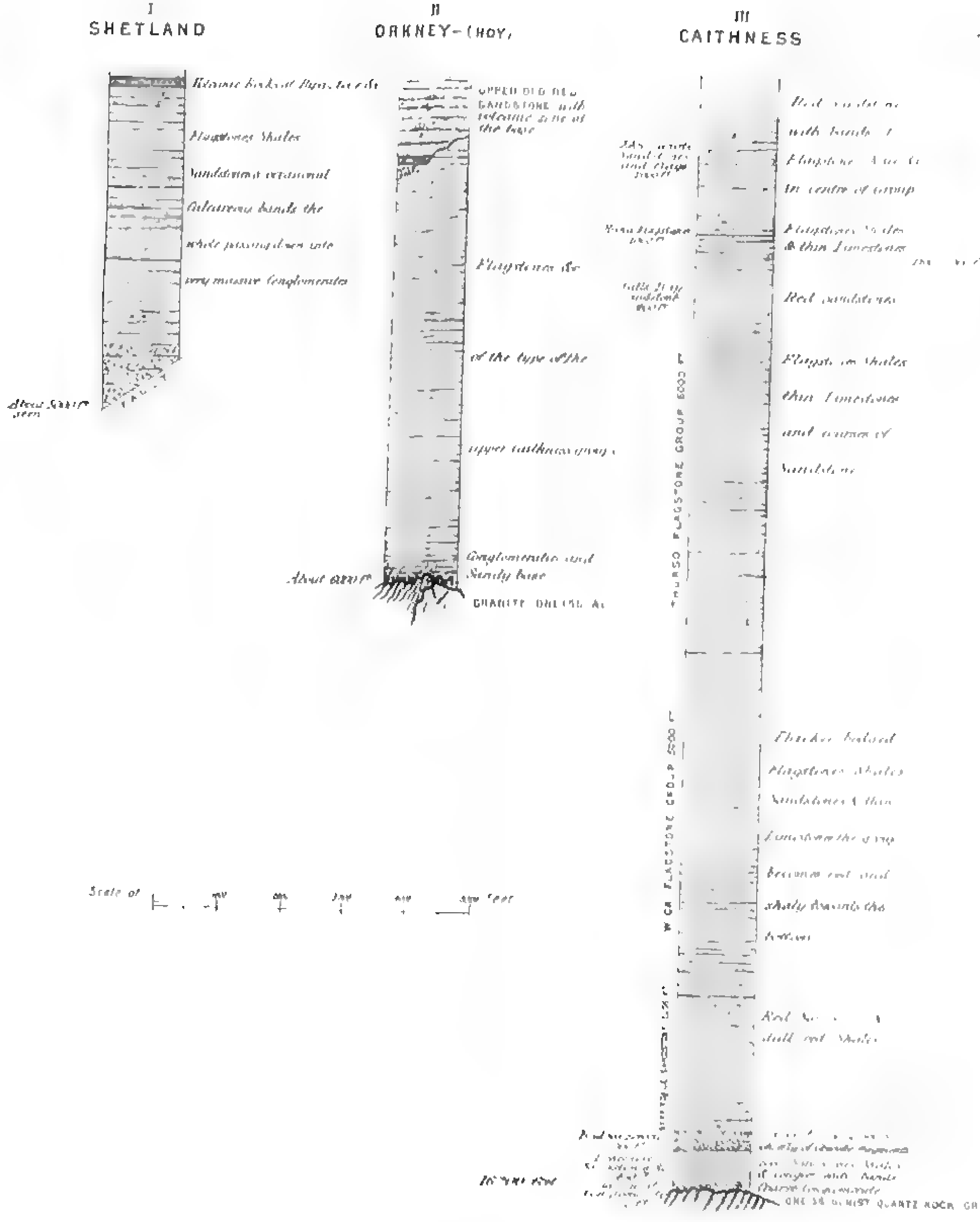
In addition to the lines of the substances which form the subject of this paper, we give on the diagram the lines of the metals potassium, sodium, and cobalt. These were investigated experimentally in Professor TAIT'S laboratory after the publication of the "First Approximation to a Thermo-Electric Diagram," so that, though their positions were indicated in notes read before the Society in the years 1874 and 1876, they had not been presented graphically on the diagram. They all lie below the lead line, and are inclined like iron and palladium; but the cobalt is so far down that its position relatively to palladium cannot be represented except on a diminished scale. Accordingly, in the lower right-hand corner of the diagram, a smaller diagram is constructed which gives the relative positions of the iron, lead, palladium, nickel, and cobalt lines. The numerical constants are as follows:—

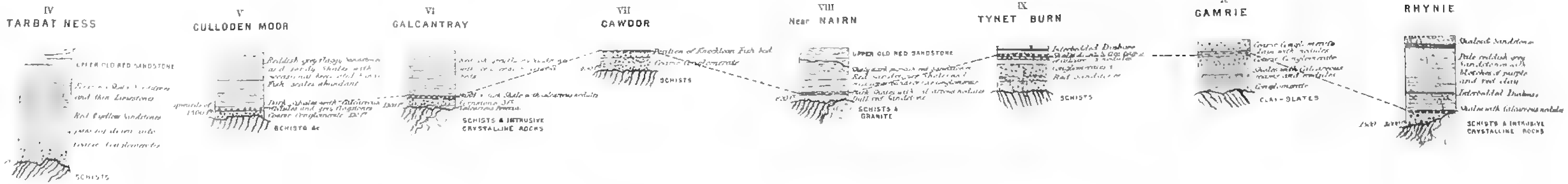
Substance.	$\frac{dy}{dt}$	A.	k in terms of 10^{-6} of a DANIELL'S cell.	k (after TAIT).
Na	− ·32	− 6	− ·0336	− ·00213
Pd	− ·27	− 47	− ·0284	− ·00182
K	− ·10	− 84	− .0105	− ·00067
Co	− ·88	− 200	− ·0924	− ·00585

To accompany Memoir by Prof Geikie on Old Red Sandstone of Western Europe

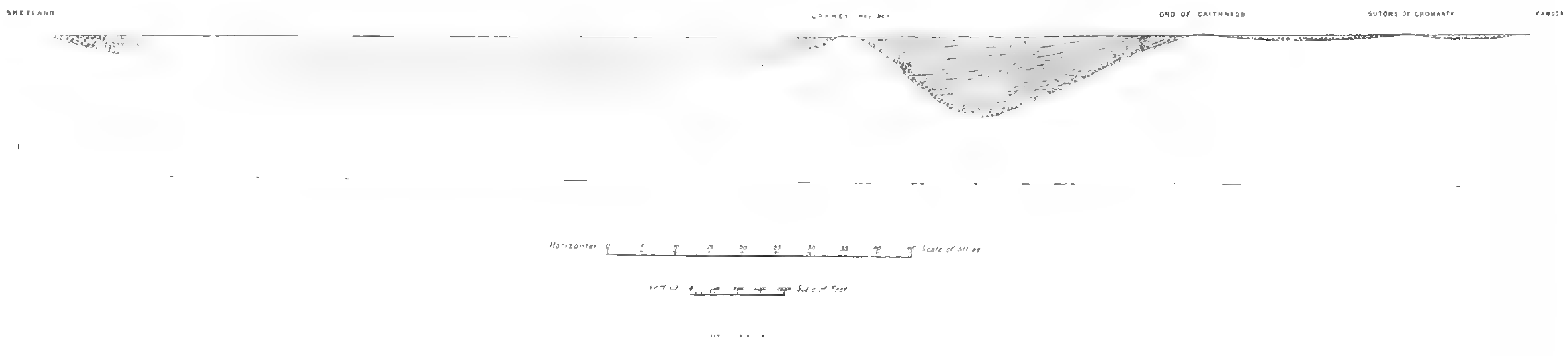
VERTICAL SECTIONS OF THE OLD RED SANDSTONE OF THE NORTH OF SCOTLAND

(The Sections are all drawn on the same Scale The Figures at the bottom of each Section do not include the thickness of the Upper Old Red Sandstone)





HORIZONTAL DIAGRAM—SECTION TO EXPLAIN THE DEPOSITION OF THE OLD RED SANDSTONE IN THE NORTH OF SCOTLAND



XVI.—*On the Old Red Sandstone of Western Europe.* By Professor GEIKIE, LL.D., F.R.S. (Plate XXII.)

(Read 1st April 1878.)

PART I.

HISTORICAL INTRODUCTION.

In the early part of the present century, when stratigraphical geology, starting from the clear succession of Secondary rocks of England, was groping its way among the older formations in this country and abroad, the Old Red Sandstone occupied a somewhat indeterminate position. The series of deposits comprised under that name had been recognised chiefly in the British Islands, hardly at all on the opposite mainland of Europe. By most geologists they were classed as a subordinate and inconstant portion of the Carboniferous system, while by some they were placed rather at the top of the yet unexplored "Transition" or "Greywacke" series. MURCHISON first claimed for them the dignity and importance of a distinct system.* On the whole, they had yielded comparatively few organic remains; they consequently seemed to lie as a thick red barren zone between the richly fossiliferous Silurian deposits below them and the equally fossiliferous Carboniferous limestone above. By degrees, however, as they brought forth a rich harvest of new and strange ichthyolites, they indicated their own right to recognition, and when they were found covering a vast space in Russia with many of the same types of fish as they had yielded in Britain, their claim to rank as a distinct and independent system was no longer contested.

In the year 1839 SEDGWICK and MURCHISON, adopting a suggestion of LONSDALE'S, established the Devonian system, and showed that it extended over a considerable area in Central Europe, with everywhere the same characteristic marine fauna. The validity of this step in the classification of the geological record was ere long universally admitted. The new term "Devonian," as a convenient euphonious adjective, and one readily transferable into other languages, passed at once into general use; while the earlier name of "Old Red Sandstone," never much in vogue out of this country, fell somewhat into disuse. Even the Old Red Sandstone of the original and typical Welsh and Scottish areas, from which not a single shell or trilobite like those of Devonshire had been obtained, was now often spoken of as "Devonian,"—that term being

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* In his "Silurian System" (1839).

employed to embrace all the deposits which were laid down between the close of the Silurian and the base of the Carboniferous system. In Canada and the north-eastern regions of the United States, a vast area, extending east and west for nearly 700 miles, and from the north of Michigan far into the middle states, was found to be occupied by a great thickness of strata (18,000 feet in some places), intermediate between the Silurian and Carboniferous formations. These have been identified with the European Devonian rocks, but have been claimed, from their thickness, their extent, and their varied organic contents, as a more ample, clear and typical development of the Devonian system than can be found in Europe.

The first attempt to point out the distinction between the typical Old Red Sandstone areas and those where rocks of the Devonshire type occurred was made by Mr GODWIN AUSTEN, in his very suggestive memoir "On the possible Extension of the Coal-Measures beneath the South-Eastern part of England." This paper appeared in 1855, and opened up a new era in the investigation of the history of the Old Red Sandstone.* The author boldly claimed for these red rocks the lacustrine origin which had been many years previously suggested by Dr JOHN FLEMING; and he endeavoured to sketch out what seemed to have been the broad features of the physical geography of Western Europe during the Palæozoic periods. The influence of this paper may be traced in all the subsequent literature of the subject. The lacustrine character of the Old Red Sandstone, as distinguished from the marine strata known as "Devonian," being enforced from fossil evidence by Professor RUPERT JONES,† and from broad lithological considerations by Professor A. C. RAMSAY, ‡ has been very generally admitted by British geologists. The two terms have thus acquired a distinctive meaning, though both applied to rocks which the majority of geologists probably still regard as geologically contemporaneous: Old Red Sandstone has in Britain been restricted to those deposits in which few or no unequivocally marine remains occur, but which by their lithological characters, and often by their organic remains indicate that they were laid down in inland areas of deposit; while Devonian has been applied to the supposed marine equivalents of these lacustrine deposits. Although the alleged contemporaneity of these two groups of strata had, in England at least, been assumed rather than proved, it was received with such acceptance as to find a place in text-books and manuals as one of the recognised facts of the science. My lamented friend and colleague, the late Mr J. B. JUKES, vigorously opposed the general assumption on this point. He contended that the "Devonian" rocks were younger than the Old Red Sandstone, and really formed the lowest division of the Carboniferous

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* "Quart. Journ. Geol. Soc." vol. xii. p. 38.

† "Monograph on Fossil Estheriæ" (Palæontographical Society), p. 22.

‡ "Quart. Journ. Geol. Soc." vol. xxvii. (1871), p. 241.

system; while between their base and the top of the Upper Silurian formations lay the vast masses of lacustrine sandstones and conglomerates of the Old Red series, marking the lapse of a prodigious interval of time. I do not propose in the present memoir to enter into this disputed question. I believe, however, that some progress may be made towards a solution of the difficulty by a more thorough examination into the distribution and history of the deposits which admittedly belong to the Old Red Sandstone.

At the present time these deposits, as they occur in Europe, are divided into three groups, Lower, Middle, and Upper, in accordance with the classification proposed by MURCHISON. To the Lower series are assigned the red sandstones, shales, and conglomerates which graduate downward into the Upper Silurian system, and which are characterised by cephalaspid and pteraspid fishes, and by large eurypterid crustaceans. In the Middle group are placed the flagstones and nodular clays of the north of Scotland, containing numerous dipterine and acanthodean fishes. To the Upper division are relegated the red and yellow sandstones, lying conformably below the Carboniferous system, and, when fossiliferous, containing such characteristic fishes as *Holoptychius*, *Bothriolepis*, and *Phaneropleuron*. Never having been able to find any stratigraphical support for this classification, I can hardly resist the suspicion that, plausible though the argument from fossil evidence appears, the threefold subdivision was unconsciously suggested by the seemingly well established threefold arrangement of the true Devonian rocks, and by the natural desire to establish a closer analogy between these rocks and the Old Red Sandstone. MURCHISON asserted that in the north of Scotland a clear ascending series could be traced through the three groups of the Old Red Sandstone, though it neither went down so far as the top of the Upper Silurian series, nor reached up as high as the base of the Carboniferous.* My own work in the centre and south of Scotland had proved the Old Red Sandstone to consist of two great divisions,—a lower passing down conformably into the Upper Silurian shales, and an upper graduating upward into the Lower Carboniferous sandstones, with a complete discordance between the two series.† Mr JUKES and Mr DU NOYER had made out a similar arrangement in the south-west of Ireland; but no fossils had been found in the lower subdivision there, so that its title to rank as part of the Old Red Sandstone had at least no palæontological support.‡ In the southern half of Scotland, however, the evidence both from fossils and from stratigraphical succes-

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* "Quart. Journ. Geol. Soc." vol. xv. p. 493 *et seq.*; and "Siluria," 4th edit. p. 250.

† "Quart. Journ. Geol. Soc." vol. xvi. (1860), p. 312.

‡ See "Explanation to Sheets 160, 161, 171, and 172 of the Geological Survey of Ireland" (1863). It is much to be wished that some fossil evidence could be obtained to fix the limit of the Upper Silurian series in the south-west of Ireland. The lithological argument seems to favour the classification adopted by Mr JUKES, for a great part of his Dingle beds would answer well for much of the Lower Old Red Sandstone.

sion was complete. Murchison ingeniously contended that the great hiatus shown by me to exist between Lower and Upper Old Red Sandstone could be bridged over by his Middle or Caithness Flag group of the north; and he cited the Devonian rocks of Russia, where a number of the fishes of the Old Red Sandstone of the north of Scotland are associated with marine shells belonging to Middle Devonian types. The introduction of a middle Old Red Sandstone into the series was described as a "masterly suggestion," and as "the greatest advance made of late years in the classification of the British Devonian rocks."*

I shall venture in a later part of this essay to show grounds for doubting whether such a middle Old Red Sandstone really has any existence. The three groups not only do not occur in any one continuous section, they are not even met with together in the same region. I shall show that even as far north as Orkney the same twofold grouping with intervening unconformability can be seen, which is so persistent in other parts of the island.

But, apart from questions of classification, there are features of such peculiar interest connected with the Old Red Sandstone as to give that series of rocks a claim for much more thorough investigation than it has yet received. From these venerable deposits we obtain some of the earliest traces of land on the surface of the globe. They bring before us, dimly it is true but still certainly, portions of the Palæozoic continent which preceded our modern Europe. We can make out from them the positions of a few of the great lakes of that time, and can trace the sites of some of the larger rivers. We have fragments of the vegetation which covered the land, and can in some measure realise the nature of the life which teemed in some of the sheets of water, or found only a precarious subsistence in others.

No attempt has yet been made, by working out in detail the stratigraphical order of the various Old Red Sandstone tracts of the British Islands, to present a connected view of their relations to each other, and of the history which they record. Occurring in detached areas, the Old Red Sandstone of this country has been very commonly looked upon as a very fragmentary formation. Its enormous depth, and the remarkable vicissitudes of physical geography which it has chronicled, are still but vaguely appreciated, even by British geologists. There need be no wonder, therefore, that its importance has not been recognised by our fellow-workers in other countries, or that we should find an able writer on the other side of the Atlantic cautioning his readers "that they should not measure the Erian (Devonian) formations of America or the fossils which they contain by the comparatively depauperated representation of this portion of the geological scale in Europe."† I venture to affirm that a better comparison of

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* Salter, "Quart. Journ. Geol. Soc." vol. xix. p. 493.

† J. W. Dawson on "Fossil Plants of the Devonian and Upper Silurian Formations of Canada." —"Geol. Survey of Canada Memoirs," 1871, p. 10.

the development of the formation on either side of the Atlantic will show that neither in depth of strata nor in palæontological interest is the European Old Red Sandstone so depauperated as has been supposed, and I hope that the present memoir may be so far useful in affording materials for such a comparison.

An essentially lacustrine series of deposits, besides being originally of limited extent, necessarily runs great risk of being largely removed by denudation, or of being buried and concealed under later, and especially marine, accumulations. The Old Red Sandstone, even at the first developed only locally, and subsequently overspread successively by younger geological formations, now occupies a comparatively small area in Western and Northern Europe. From the south-west of Ireland it may be followed through the rest of Britain as far as the distant Shetland Islands, and across the intervening tract of sea into the south-west of and south of Scandinavia; while far to the east in Russia it reappears in almost horizontal strata, which cover a space considerably larger than the area of the British Islands.

In no part of its European distribution does the Old Red Sandstone attain the thickness and variety which it presents in Scotland. In that country its most characteristic features—sandstones, flagstones, conglomerates, lavas, tuffs, fishes, crustaceans, and plants—are so admirably preserved, that the Scottish Old Red Sandstone may with reason be described as a type for the system. Numerous sections on the sea-coasts, in inland ravines, and river courses, as well as on bare hillsides, allow of the thorough investigation of almost every stratigraphical detail. For many years its abundant fossil treasures have been sedulously gathered by many enthusiastic collectors. But notwithstanding the numerous memoirs which have from time to time appeared, much remains to be done before our knowledge of the history of the Old Red Sandstone in Scotland and the surrounding regions is brought up to the same fulness as our acquaintance with that of the Carboniferous system which followed it. Over large tracts of country the stratigraphical relations of the various portions of the system have never been determined. The order of succession in the Old Red Sandstone on the north side of the Grampian mountains, for example, has never yet been adequately worked out. Yet this part of the subject is of paramount consequence in any attempt to unravel the chronology of events of which the strata of the Old Red Sandstone are the records.

I propose in the present memoir to attempt to trace a series of changes in the physical geography of Western Europe, which took place between the close of the Upper Silurian and the commencement of the Carboniferous period. As this history must be based upon a rigorous examination and comparison of sections in different districts, I shall be under the necessity of presenting hitherto unpublished observations in greater detail than I could have wished. After many years devoted specially to the investigation of the Old Red sion was complete. MURCHISON ingeniously contended that the great hiatus shown by me to exist between Lower and Upper Old Red Sandstone could be bridged over by his Middle or Caithness Flag group of the north; and he cited the Devonian rocks of Russia, where a number of the fishes of the Old Red Sandstone of the north of Scotland are associated with marine shells belonging to Middle Devonian types. The introduction of a middle Old Red Sandstone into the series was described as a "masterly suggestion," and as "the greatest advance made of late years in the classification of the British Devonian rocks."*

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Thus, in spite of the prevalent and long-continued subsidence, the succession of Silurian deposits over the British area was frequently interrupted. Professor RAMSAY many years ago drew attention to these local breaks in the succession of the strata, and connected them with corresponding local interruptions of the continuity of organic forms.* Reflecting upon this curious and varied history, we may believe that apart from any upheaval of the sea-floor, a mere cessation of the downward movement could not but produce a great change in the geography of the region, and, consequently, upon the distribution of life. Let us suppose that such a pause in the subsidence took place at the close of the Silurian period. The wide but shallow Silurian sea would come to be silted up over considerable tracts. Sand-bars and mud-flats would gradually rise above the level even of the highest tides. Portions of the sea would thus be isolated from the main body of the water, and under the influence of evaporation would be converted into *salinas*, too bitter for the support at least of an abundant fauna. Yet, on the breaking down of any of these barriers by the waves, an irruption of the sea into the lagoons might temporarily reintroduce some of the forms of life which had previously been killed by the concentration of the water. If, now, we suppose that in addition to the shallowing and narrowing of the sea by means of the constant deposit of sediment, portions of the bottom were from time to time ridged up above the sea-level, so as still further to complete the isolation of different water-basins, and even to permit these to become slowly freshened into lakes, we can realise how completely the aspect of the north-western European area would thus be changed, and how the Silurian fauna might be permanently driven from that region.

It was by some such physical changes as these that the era of the Old Red Sandstone was ushered in. In spite of many subsequent dislocations, of enormous denudations, and of the overspread of later sedimentary formations, it is still possible to trace some of the more salient features in the geography of Britain at that ancient geological epoch, and to follow the more marked vicissitudes through which the region passed during the vast interval which separated the Silurian from the Carboniferous period.

To do this with completeness, as far as the evidence permits, will require a much more thorough analysis and comparison of the infra-Carboniferous sections, both in this country and on the Continent, than has yet been attempted. I believe, however, that a much larger and more comprehensive fragment of the record of the Old Red Sandstone remains in the northern half of the

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* "Quart. Journ. Geol. Soc.," vol. xix. (1863) p. xxxvi.

British Islands than exists elsewhere, and that the unravelling of its history will probably throw light upon most of the leading changes by which the west of Europe was affected during that interval of geological time. I shall in the present memoir restrict myself mainly to the Scottish tracts.

For the sake of clearness in description, it will be of advantage to anticipate some of the conclusions which form the subject of discussion in the following pages. Thus, agreeing in the now very generally accepted view of the lacustrine origin of the Old Red Sandstone, I shall speak of the separate basins of deposit as *lakes*, to which, for ease of reference, different names will be given. As the history of the deposits in these basins often varies greatly, I shall adopt the geographical mode of treatment and describe each lake separately, with its varied accumulations, and the changes which they indicate. Viewed in a large way, the Old Red Sandstone of Great Britain naturally groups itself, stratigraphically, into two divisions, which as a rule are strongly marked off from each other, both by physical structure and by difference of organic contents. These may be called Lower and Upper. I shall first trace the history of the various lakes of the earlier period. To some extent these basins still remained during the later period, but the geography of the whole region had greatly altered in the interval. A sketch of these subsequent changes will show how the way was prepared for the Carboniferous system.

THE LOWER OLD RED SANDSTONE.

The Basins of Deposit in the British Area.

Among the earlier Palæozoic formations, there is often such a marked persistence of lithological characters that the same group of rocks can be recognised with confidence in widely separated areas, even where the fossil evidence is scanty. A thin zone of black shale, found among the Llandeilo rocks of the south of Scotland, has been traced for more than a hundred miles, retaining its distinctive features all the way. The Wenlock and Ludlow shales, and mudstones of the typical Silurian country, reappear with their familiar aspect in Westmoreland, Liddesdale, Kirkcudbright, Ayrshire, and Midlothian. When we leave the marine formations, however, and pass from the top of the Upper Silurian groups into the Old Red Sandstone, no such general uniformity of stratification presents itself. On the contrary, with the accumulation of the deposits in limited basins, come local and often peculiar features, whereby even contiguous tracts are distinguished from each other. It is still possible roughly to make out with more or less clearness the limits of these basins, which seem sometimes to have been connected by narrow or shallow, and doubtless occasionally closed, water-channels; in other cases to have been completely isolated.

As the ancient basins only very partially correspond with any modern geographical subdivisions, and as the use of cumbrous descriptive designations should be avoided, I venture to propose short reference names which, when the sense in which they are employed has been once explained, will save the inconvenience of more lengthy epithets. In the reconstruction of such ancient aspects of geography, it is as if we were groping our way into the interior of an unexplored continent. Collecting all obtainable information, we throw it into the form of a map, whereon names are given to the more prominent features for convenience of reference. No harm can be done even though two names may be applied to different parts of what may eventually prove to be the same river or lake, so long as we distinguish between what is inferred and what can be actually proved.

The subjoined table shows the geographical areas into which I propose to divide the Old Red Sandstone of the British region. In the first column are given the position and general limits of the different basins of deposit; on the opposite side, in the second column, are placed the short reference names which I shall use to save the repetition of description.

Basins of Deposit of the Lower Old Red Sandstone in Britain.

Area of the Basins.	Short Reference Names proposed to designate them.
1. A wide region, embracing all the Old Red Sandstone to the north of the Grampian range. Its southern and western border skirts the northern base of the Highland mountains, running up here and there in long fjords or bays. It includes the whole of the Orkney Islands, while its northern margin may be traced in the Shetland group.	Lake Orcadie.
2. The central valley of Scotland, between the range of the Highland mountains on the north, and that of the Silurian pastoral uplands of the southern counties. This basin was probably prolonged across the Firth of Clyde into the north of Ireland.	Lake Caledonia, or the Mid-Scottish Basin.
3. A portion of the south-east of Scotland and the north of England, extending from near St Abb's Head south-westward, along the base of the Silurian hills to the head of Liddesdale, and including the area of the Cheviot Hills.	Lake Cheviot.
4. The Old Red Sandstone region of Wales, bounded on the north and west by the Cambrian and Silurian rising grounds, but its eastern and southern extension obscured by later formations.	The Welsh Lake.
5. A district in the north of Argyllshire, extending from the south-east of Mull to Loch Awe, and perhaps northward up the line of the Great Glen.	Lake of Lorne.

The most northerly of these basins, Lake Orcadie, presents in its deposits and their fossil contents such marked peculiarities as to show that it remained for the most part completely separated from the more southern areas, though a communication may have been occasionally open between them, perhaps by the very ancient hollow of the Great Glen. This extensive sheet of water in a late part of the Lower Old Red Sandstone period, or even in the time of the Upper Old Red Sandstone, may have stretched eastwards by the southern part of Scandinavia, so as to be connected in such a way with the great Russian lake as to allow some species of fishes to become common to both. Other British basins are marked by sandy and gravelly accumulations, which do not differ sufficiently to offer a satisfactory means of discriminating between the respective areas of deposit. In some cases the margins of the basins can be laid down for many miles with accuracy. This is particularly true of Lake Caledonia. In other cases the borders are more or less completely buried under later formations.

I. LAKE ORCADIE.

1. Area of the Region.

In accordance with the plan proposed in this memoir, I shall, for the sake of convenience of reference, unite under one geographical designation all the Old Red Sandstone lying to the north of the Scottish Highlands, using for this purpose the name of Lake Orcadie. That this great basin was an area of deposit, distinct from that of central Scotland, may be shown both from petrographical and palæontological evidence. Of course little more than its southern and western margin can be seen on the mainland of Scotland. But taking that part in connection with the islands to the north, we can form some notion of the area of this lake, and of the changes in physical geography which it suffered. Fortunately, large boundary faults do not here form the usual line of demarcation between the Old Red Sandstone and the older rocks, as is the case on both sides of the great Old Red Sandstone basin of central Scotland. For the most part, the marginal belt of the ancient lacustrine area is marked by thick conglomerates, which lie directly and unconformably upon the worn edges of the metamorphic rocks. In these marginal conglomerates, therefore, the shore lines of the old lake may still be traced.

The present southern coast-line of the Moray Firth seems not to deviate very widely from that of this Old Red Sandstone lake; but is much less sinuous and picturesque than the ancient shore must have been. Long, narrow, fjord-like inlets allowed the waters of the lake to run far inland, up even to the very roots of the mountains. The line of the Great Glen was even then marked by a long straight valley in which the lake stretched far to the south-west, if indeed there was not a communication, more or less interrupted, between this northern basin and the Lake of Lorne. The mountainous country of Ross and Sutherland formed a bold border-land on the west side. But the coast-line appears to have turned westwards across the north-eastern part of Sutherlandshire, at least as far as the base of the noble granite peaks of Ben Laoghall and the Kyle of Tongue. Towards the north, Lake Orcadie stretched over the site of Caithness and the whole of the Orkney Islands, but save one or two fragments of its ancient islets, no trace of its shores is to be seen until we reach the southern end of the Shetland group, and meet there with some of its littoral conglomerates.

On the west coast of Norway, about the mouths of the Sognefjord and Dalsfjord, and on the neighbouring islands, huge cliffs of a massive conglomerate occur, the resemblance of which to those of the north of Scotland was pointed out many years ago by NAUMANN.* Red conglomerates and sandstones likewise occur round Christiana, which have been referred to the Old Red Sandstone. Of course these identifications in the absence of fossil evidence must be regarded as only provisional; and even if they are eventually sustained, they would not prove that Lake Orcadie extended continuously across what is now the trough of the North Sea to the margin of the Scandinavian highlands. At the same time this former north-easterly prolongation of the basin seems to me extremely probable. The occurrence of a few species of fishes common to the Old Red Sandstone of the north of Scotland and of Russia goes far to show a connection, more or less restricted, no doubt, between these distant areas, or at least suffices to indicate that any watershed which separated them was not a wholly insuperable barrier to their respective faunas.

Even if we restrict the area of Lake Orcadie merely to the space over which its deposits can be traced within the Scottish islands, the basin will be found to have had no inconsiderable dimensions. Like the majority of the larger features of the country, it probably had a north-easterly trend. From its southern margin at Loch Ness to its northern limit in Shetland is a distance of 250 miles. Were we to include the Norwegian conglomerates of the Bergenhuus as marking the shore-line in that direction, the length of the basin would be increased to somewhere about 400 miles. Of its breadth little can be said from the fragmentary portions which alone remain; but a line drawn from the Kyle of Tongue to Aberdour in Aberdeenshire, would give a breadth of about 120 miles. I shall have occasion to show, however, that there is good reason

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* "Beiträge zur Kenntniss Norwegens," vol. ii. p. 118, *et seq.*

to believe the present southern margin of the Old Red Sandstone area to mark the coast-line, not of the earlier but of the later part of the history of Lake Orcadie; in other words, that for a long time the area of the Moray Firth was land, and that only towards the close of the existence of that lake, by a gradual submergence of the area, did the water creep southward, and accumulate the present marginal deposits.

2. Work of Previous Observers.

Before entering upon the history of this great northern basin of Old Red Sandstone, which in many respects, and notably in regard to its organic remains, is one of the most interesting areas of that geological system which has yet been noticed, let me refer briefly to the labours of previous observers, and to the present state of our knowledge on this part of my subject.

While the general area covered by the Old Red Sandstones and conglomerates in the north of Scotland had already been so well traced as to find tolerably accurate expression in the geological map, published by Boué, as far back as 1820; there does not appear to have been any attempt to work out the structure and subdivisions of the system in that region until the year 1827, when Sedgwick and Murchison examined the wide belt of country extending from the west of Ross-shire, northward through Sutherland, eastward across Caithness, and then southward by the shores of the Dornoch, Cromarty, Beauly, and Moray Firths.* In the important Memoir which gave the results of their labours, these authors not only traced more accurately the area covered by the Old Red Sandstone, but ascertained the general order of succession of its component groups of strata, noted some of their fossil contents, and showed the probable connection of the fossiliferous deposits even at the opposite extremities of the district. As the larger part of their Memoir deals more particularly with the formations as these are exhibited in Caithness, this portion of it will be again more specially referred to when the Caithness development of the system is treated of. Sedgwick and Murchison considered the Old Red Sandstone of the north of Scotland to consist essentially of three groups,—1*st*, A mass of red conglomerate and sandstone forming the base, and lying unconformably upon the primary rocks, from the waste of which it had been formed; 2*d*, A middle series consisting, in Caithness, of a thick pile of calcareo-bituminous schists with fossil fishes, and along the Moray Firth of red and grey sandstone, marls, and calcareous nodules, and cornstone; 3*d*, An upper set of light yellow and reddish sandstones. They remark that while the con

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* "Trans. Geol. Soc." 2d ser. vol. ii. p. 125.

glomerates, so conspicuous to the west and north, have thinned off and nearly disappeared on the southern side of the Moray Firth, the true order of succession is much concealed by superficial detritus, and the three groups of the system are there very ill defined. They do not therefore attempt to enter into any details regarding the Old Red Sandstone of that region, contenting themselves with a notice of the cornstone so copiously developed there, and of the strata with which it is associated.

In the year 1836, Mr MARTIN of Elgin, who had a year previously obtained a prize from the Highland and Agricultural Society for an "Essay on the Geology of Morayshire,"* discovered fossils in the conglomerate of Scat Craig near Elgin. These were eventually recognised as similar to forms which had already been obtained from undoubted Old Red Sandstone elsewhere, so that the reference of the Morayshire conglomerates and sandstones to that formation was confirmed by palæontological evidence. Two years afterwards, viz., in the autumn of 1838, Dr JOHN MALCOLMSON began an extensive and laborious investigation of the region, specially with the object of unravelling the history of its Old Red Sandstone, and of ascertaining whether or not it could not be proved to be much more fossiliferous than the solitary locality near Elgin seemed to denote. He carried on his observations along the whole of the northern sea-board, from the cliffs of Aberdeenshire to the shores of the Cromarty Firth, discovering in many localities remains of fossil fishes, some of which he could identify with forms found at Cromarty, in Caithness, and in Orkney, while others appeared to be new. In a paper read before the Geological Society (June 1839), he announced the important conclusions to which he had been led, and which may be briefly summarised as follows:—The Old Red Sandstone system of the north of Scotland may be arranged in three divisions. Of these, the lowest member (I.), consists of three formations—(*a*) The great conglomerate; (*b*) Red sandstones, shales with calcareous nodules and limestones abounding in remains of fishes and plants; (*c*) Argillo-calcareous red sandstones and conglomerates. The central member, or cornstone, group (II.), is composed of sandstones, calciferous conglomerates, marls, and cornstones, and abounds in remains of fishes all different from those of the middle zone of the lower beds. The highest member (III.) consists of fine white, grey, and yellow siliceous sandstones, conglomerates, and cornstones. He places the middle zone of the lowest division on the same parallel with the flagstones of Orkney and Caithness, and the tile-stones of England; while he brackets together, as equivalent strata, his central division with the sandstones of Clashbennie in Perthshire (*Holoptychius Nobilissimus* beds), and the Herefordshire cornstones. It is impossible to traverse the scattered sections of Moray and Nairn without

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* "Trans. Highland Soc." new series, vol. v. p. 417.

rendering a tribute of admiration to the skill with which they were pieced together by this pioneer in the geology of the north of Scotland. Had he lived he would doubtless have modified some of his inferences and withdrawn others; but even with these emendations, his paper will always remain a landmark in the literature of the subject, and raise a regret in its readers that he should have been cut off in the midst of such promise of ample and admirable geological work.

Meanwhile other observers had been quietly gathering the strange and abundant fossil fishes of the Old Red Sandstone of the north of Scotland. Dr TRAILL and Mr H. E. STRICKLAND had made collections of them from the Orkney Islands. HUGH MILLER had discovered them at Cromarty; Lady GORDON CUMMING, Mr PATRICK DUFF, the Rev. Dr GORDON, Mr ALEXANDER ROBERTSON of Elgin, and other zealous collectors found them in new localities along the southern shores of the Moray Firth. Lord ENNISKILLEN and Sir PHILIP EGERTON had acquired some fine specimens from these regions, and had brought them to the notice of geologists and palæontologists. Much confusion still existed, however, as to the zoological grade of the organisms. That most of them were fishes, even though of very remarkable types, was generally admitted; though some of the larger teeth had been popularly described as parts of reptiles; the broad head plates of *Coccosteus* had been referred to as those of an ancient tortoise; while the strange winged form of the *Pterichthys* had been gravely figured and described as that of a primeval beetle. It was not until the time when AGASSIZ, in the course of his researches into the natural history of fossil fishes, made acquaintance with the undescribed forms which had been obtained from the Old Red Sandstone of Scotland, that firm data could be used in comparing the palæontological characters of the system in different parts of the country. He first visited Scotland in the year 1834, and again in 1840, when he had opportunities of personally inspecting the collections of ichthyolites from the northern counties, and collecting materials for his great work on fossil fishes. So large, however, did he eventually find these materials to be, that he devoted to their illustration a special volume—his well-known *Poissons Fossiles du Vieux Grès Rouge.* It is hardly possible to overestimate the importance of this work in the history of inquiry into the palæontology of the Old Red Sandstone. To the labours of AGASSIZ we are indebted for most of the knowledge which we possess regarding the curious fishes which he was the first to recognise and describe. Working, as he did, often with imperfect materials, and unfamiliar as he was with the singular differences in the state of preservation, and consequent aspect of the organisms according to the varying nature of the matrix in which they are enclosed, he, no doubt, must have been occasionally puzzled and deceived by the specimens before him, and may have been led to multiply species which a succeeding naturalist will be able to reduce to smaller numbers. But none the less will the name of AGASSIZ stand in the fore front of those by whom the palæontology of the Old Red Sandstone has been worked out.

In his classic "Old Red Sandstone," which appeared early in the summer of 1841, HUGH MILLER gave a section of the formation as displayed at Cromarty, and connected it with the much greater development of the corresponding beds in Caithness. He regarded these *Dipterus*-bearing rocks as forming the true Lower Old Red Sandstone; while the *Cephalaspis* flagstones of Arbroath he considered to be the middle part of the system; the *Holoptychius* sandstones forming the upper division. This continued to be the prevalent belief for somewhere about fifteen or twenty years. MILLER'S graphic descriptions of the little known fishes, which he had been among the first to disinter from their ancient burial-places, roused the attention not only of professed geologists but of general readers all over this country and America, and the name "Old Red Sandstone" became at once a household word. He claimed for his favourite group of rocks an importance in geological history which had never been admitted, and he lived to see his claim fully recognised. The popularity of his work incited other collectors to explore the Old Red Sandstone of the north of Scotland. Many new species and many magnificent specimens of known ones were in this way brought to light. Not a few of these were sent to HUGH MILLER. His collection was thus enriched by the contributions of other fellow-labourers. Conspicuous among those who assisted him in this way was the late ROBERT DICK of Thurso—an enthusiast in the study of the Old Red Sandstone of his native county, to whom science is indebted for much that is now common knowledge regarding the fishes of that formation. Of still greater service have been the sedulous and unobtrusive but sagacious labours of my friend Mr C. W. PEACH. Probably no one has done so much as he towards working out the palæontology of the Caithness flagstones. He has not merely discovered several new species of fishes, but has collected diligently from every horizon in that series of deposits, and has amassed a vast amount of information regarding the natural history of the fauna of Lake Orcadie. To him belongs the merit of having early perceived the terrestrial origin of the plants of the Caithness flagstones. He has generously placed his knowledge at my service, and I shall have occasion in a subsequent part of this memoir to avail myself fully of it.

Hitherto no attempt beyond vague generalisations had been made towards a correlation of the Old Red Sandstone of the north of Scotland with that of other regions. The general belief was that of HUGH MILLER, that these northern deposits belonged to the lower subdivision of the system. It was not until the year 1858 that Sir RODERICK MURCHISON took up the subject anew, and endeavoured to show that the Caithness flagstones could not possibly belong to the Lower Old Red Sandstone, but must be younger than the Forfarshire flagstones, in which such old forms as *Cephalaspis* and *Pterygotus* occur.[*] Accordingly he grouped them as a "middle" division, in conformity with the triple classification so much in vogue. He sought to establish an analogy between the three groups of the Old Red Sandstone and the three groups of the Devonian system, maintaining that they are essentially the equivalents one of the other. So far as I have been able to gather, the following reasons appear to have guided him to these conclusions. He believed and affirmed—1*st*, That while in the true Lower Old Red Sandstone several genera of fishes and crustaceans occur (*Pteraspis, Pterygotus,* &c.), which descend into the Upper Silurian rocks, not one of these occurs in the flagstone series of the north of Scotland. 2*d*, That the ichthyic fauna of the latter series, differing so greatly as it does from that of the true Lower Old Red Sandstone, cannot be of the same age, and from the absence of such early forms as *Pteraspis, Cephalaspis,* &c., must be presumed to be of younger date. 3*d*, That at the base of the flagstones of Caithness, red sandstones and conglomerates, in which Mr C. W. Peach found *Pterygotus,* may be recognised as equivalents of the Lower Old Red Sandstone, occupying their proper place below the great "middle" formation with its abundant and peculiar fishes. 4*th*, That the upper portion of this middle formation passes upwards in the north of Scotland into the Upper Old Red Sandstone, which it could hardly have been expected to do had it really been of so ancient a date as the flagstones of Forfarshire. 5*th*, That the fishes of the "middle" Old Red Sandstone of Scotland occur in the middle Devonian rocks of Russia, thus showing so remarkable a correspondence between the Old Red Sandstone and the Devonian groups in the west and east of Europe as to justify the conclusion that they are mutually equivalent.

Now it is impossible to deny the ingenuity and apparent cogency of this reasoning. Mr Salter, in the passage already cited, declared it to be "a masterly suggestion," and "the greatest advance made of late years in the classification of the British Devonian rocks."[†] I venture to think, however, that the argument is based on such evidence as to render it more than inconclusive. Some of the supposed facts on which it rests may be shown to be erroneous; while another, and, as it seems to me, quite as probable an interpretation, may be put upon those which remain unchallenged. After long research in the field, and much anxious consideration of the whole subject, I am unable to find any valid reason for the erection of the so-called "Caithness flagstones" into a "middle" division of the Old Red Sandstone. Nowhere in the British area, so

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[*] "Quart. Journ. Geol. Soc." xv. p. 400; and "Siluria," 3d edit 1859, p. 284.

[†] "Quart. Journ. Geol. Soc." xix. 493.

far as I know, does any group of rocks exist which requires to be ranked as Middle Old Red Sandstone. I can discover only two great well-marked series of strata, which build up the Lower and Upper divisions of this Palæozoic system of deposits. The Old Red Sandstone of Lake Orcadie I would place in the Lower series, and would explain its peculiarities by the geographical circumstances under which it was laid down. This conclusion has been forced upon me by the evidence which I shall now proceed to adduce.

1. Taking the British area, we have nowhere any strata which could possibly be claimed as forming a distinct "middle" series save those of the north of Scotland. Everywhere else the twofold classification into Lower and Upper is manifest. Yet to the south of the Grampian range these two subdivisions attain the vast development of more than 20,000 feet. In South Wales, and again in Ireland, they attain an enormous thickness. On the view which I am combating we must suppose that the break between these two series is everywhere so great as to be chronologically equivalent to the whole of the vast depth of the "Caithness and Orkney flagstones." Though the Lower and Upper Old Red Sandstones are certainly strongly unconformable in many places, yet in wide tracts, where derangements of strata are otherwise plentiful, they lie with so little discordance that it is difficult to believe the interval between their deposition to have been so vast.

Though arguments founded on the thickness of strata are notoriously unsafe, I may be allowed to point out that, on the supposition that the Old Red Sandstone of the north of Scotland forms a division of the system elsewhere unrepresented in Britain, the total thickness of the Old Red Sandstone would be increased to a united depth of between 30,000 and 40,000 feet. I prefer to reduce this thickness by making the Old Red Sandstones on the north side of the Scottish Highlands the general equivalents of those on the south.

2. While in none of the tracts where Lower and Upper Old Red Sandstone occur is there any true "middle" group, in the north of Scotland, where the so-called "middle" group attains so great a development, there is no representative of any lower series. Sir R. MURCHISON affirmed, indeed, that the Caithness flagstones may be seen in various places to graduate downwards into equivalents of his Lower red sandstones, and upward into the Upper light red and yellow sandstones. I shall be able to show in this memoir that what he considered to be equivalents of the Arbroath flagstones or Lower Old Red Sandstone, form merely the variable and sandy or pebbly base of the Caithness series, and occur on many different horizons throughout that series. They cannot be regarded as in any sense equivalents of the whole vast succession of sandstones and conglomerates in central Scotland. With regard, also, to the

upper limit of the Caithness series, I shall prove from clear natural sections that it is marked by a strong unconformability, with which the overlying Upper yellow and red sandstones make their appearance. In short, there are in the north of Scotland only two great divisions of the Old Red Sandstone, and it seems at least probable that they represent the two divisions which occur everywhere else in Britain.

3. There can be no doubt that lithologically the Old Red Sandstone of the north of Scotland differs in a marked way from that of any other district within these islands. So striking, indeed, is this divergence, that even a geologist whose eye has long been familiar with the Old Red Sandstone in more southern regions, finds at first some difficulty in believing that the dark shaly flagstones and limestones of Caithness and the Orkney Islands can form part of the Old Red Sandstone within the same geographical area in which red and purple sandstones and conglomerates are elsewhere so prevalent. In other tracts he is familiar with the general barrenness of the red sandy strata in regard to organic remains; but in these northern rocks he meets with layers which are crowded with well-preserved teeth, scales, and bones of fishes. Mere difference of lithological character does not necessarily point to difference of age, and certainly cannot be cited as evidence in favour of a threefold classification of the Old Red Sandstone. But this striking contrast in the nature of the strata does point to markedly dissimilar conditions of deposit. The Old Red Sandstone of the north of Scotland must have accumulated in a distinct geographical basin, and under circumstances to which as yet no parallel has been found in the Old Red Sandstone of other parts of Britain. So singularly characteristic is this lithological discrepancy, that the observer who comes upon it for the first time naturally expects to find it accompanied by a corresponding palæontological divergence. He feels that the Caithness and Orkney flagstones are themselves so peculiar that the waters in which they were laid down might reasonably be expected to have been tenanted by other forms of life than those which have been preserved among the sandstones and shales of Forfarshire.

4. It was on the fossil evidence that Sir R. MURCHISON chiefly relied when he proposed his "middle" Old Red Sandstone group. His position seems to be strong and well chosen. Among the numerous ichthyolites of the Caithness flagstones, none of the cephalaspids so characteristic of the Lower Old Red Sandstone of Forfarshire and Herefordshire occur. The Caithness fishes, on the other hand, as he contended, are not represented in the Lower Old Red Sandstone. Deposits differing so much palæontologically cannot therefore be contemporaneous. The Caithness flagstones must thus be younger than the cephalaspid sandstones which are found passing down into the Upper Silurian series. On the other hand, they are certainly older than the *Holoptychius*-

bearing Upper Old Red Sandstone. Consequently they must form a middle group by themselves.

I propose to enter in some detail into the examination of the evidence on which this reasoning is based. In the meantime, I may remark that the palæontological discrepancy between the Old Red Sandstones of the north of Scotland and that of the rest of the country, seems to be capable of reasonable explanation by isolation and differences in the conditions of deposit, and that it is really not so complete as is commonly supposed.

In the first place, several genera, and perhaps species, are common to the Old Red Sandstone on both sides of the Highlands. The acanthodean fishes are eminently characteristic of the flagstones of Caithness and of Forfarshire. The genera *Acanthodes* and *Diplacanthus* abound; and though the species found in the southern area are pronounced to be different from their congeners in the northern, they admittedly stand in close relation to each other. *Parexus incurvas* is found in both tracts. Even the crustacean genus *Pterygotus*, which has been regarded as so characteristic of Lower Old Red Sandstone and Upper Silurian strata, occurs on two widely separated horizons among the Caithness and Orkney flagstones. So far as any comparison is possible at present between the fossil plants, there appears to be a close similarity in the northern and more southerly regions. The lycopodian genus *Psilophyton* is common to both, besides lepidodendroid stems, and probably other still undetermined forms.

In the second place, there does not seem to be any valid reason why the ichthyic fauna of two adjacent but completely disconnected water-basins should not have differed considerably in Old Red Sandstone times, as they do at the present day. Even in the same river-system it is well known that the fishes of the higher portions of the basin are sometimes far from corresponding with those in the maritime parts of the area. Neighbouring drainage-basins, divided by a comparatively unimportant watershed, sometimes show a remarkable contrast in their fishes. This has been well pointed out by Professor E. D. COPE, in a suggestive paper "On the Distribution of Fresh-water Fishes in the Alleghany Region of South-Western Virginia."* The James and Roanoke rivers descend the eastern slope of the continent and discharge into the Atlantic. In their upper waters they have only four species of fish in common. In the upper waters of the rivers Holston and Kanawha, which flow south-westwards into the Mississippi basin, there are only two species alike. Between those eastern and western pairs of rivers runs the more marked water-parting of the Alleghany chain. Out of fifty-six species of fish obtained from the head waters of the four rivers, five were found by Mr COPE on both sides of the watershed. There is likewise considerable disparity in the genera represented in the

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* "Journ. Acad. Nat. Sci. Philadelphia," vi. 2d series (1860–69), p. 207.

different rivers. The still more important barrier of the Rocky Mountains separates ichthyological areas yet more sharply marked off from each other. Such isolated basins as Lake Baikal, Lake Titicaca, and the Caspian Sea show by their peculiar assemblages of fishes how much ichthyic types may be modified by prolonged isolation. The differences, therefore, between the fauna of Lake Orcadie and Lake Caledonia during the Old Red Sandstone, as I venture to hold, are not incompatible with the idea that the two lakes were in a general and geological sense contemporaneous, though separated from each other by the barrier of the Grampian Mountains, which formed an effectual boundary between two ichthyic faunas.

In the third place, it can be shown that even among the tracts covered by undoubted Lower Old Red Sandstone considerable divergences in the fishes and crustaceans exist, pointing to separate areas of deposition. It is doubtful if a single species of fish or crustacean found in the Lower Old Red Sandstone of England and Wales can be identified with one found in Scotland. Most of the genera, however, are the same. This certainly indicates the influence of geographical distribution, though it shows that the barrier between the English and Scottish basins was not so complete, or had not existed for so long time, as that which interposed between Lake Caledonia and Lake Orcadie.

5. So far, therefore, as the decision of the question rests upon evidence obtainable within the area of the British Islands, I submit that the presumption is in favour of the Old Red Sandstone on the north side of the Scottish Highlands being the same as that on the south side, and that consequently the so-called middle division of the system does not really exist there. So strong does this presumption seem to me that it cannot, I think, be set aside, save by much more convincing reasoning than that by which the Middle Old Red Sandstone of this country was considered to be established.

But Sir R. MURCHISON was probably quite aware of the weakness of his argument, so far as the case of the British rocks are concerned. He endeavoured to strengthen it by an appeal to the Old Red Sandstone of Russia, where, he asserted, the fishes of his Middle Old Red Sandstone group of Caithness and the Moray Firth occur in the very same beds with true Middle Devonian shells. He laid great stress on this argument, regarding it indeed as unanswerable in itself and as affording the most cogent possible proof of the soundness of his threefold classification of the Devonian or Old Red Sandstone system. And certainly it appears at first to deserve the confidence which he reposed in it. But an inquiry into the facts on which he depended, shows that to some extent at least he was misled.

In the first place, in making the comparison between the Scottish and Russian ichthyolites MURCHISON availed himself of fossil lists wherein the two very distinct Old Red Sandstone groups of the Moray Firth were not sufficiently recognised. As will be explained in the sequel of this memoir, there exist in that region representatives of the Caithness and Orkney beds covered probably unconformably by the true Upper Old Red Sandstone. The structure of the country, owing to paucity of sections, had not been satisfactorily determined; consequently some confusion could not fail to arise as to the true horizons of certain fossils. In particular, genera which elsewhere in this country are characteristic of Upper Old Red Sandstone were given as occurring in the same strata with characteristic forms of the true Caithness series. I shall again advert to this source of confusion. Prior to any comparison between the Old Red Sandstone of the north of Scotland and of Russia it is necessary to know precisely between what subdivisions of the system the comparison is to be made.

In the second place, in the comparisons made by MURCHISON between the fauna of the so-called Middle Old Red Sandstone of Scotland and that of the Middle Devonian rocks of the Continent, there was an obvious vagueness which, no doubt, led to its being "slightingly spoken of," as he himself remarked.* He nowhere, so far as I am aware, gave any list of the species of ichthyolites and of molluscs found together, but contented himself with citing the names of a few genera of fish which had been obtained in Russia associated with true Devonian shells. In the palæontological volume of the great work on "Russia and the Ural Mountains," AGASSIZ inserted a list of the various ichthyolites which up to that time (1845) had been obtained from the Old Red Sandstone of Europe. Eighteen species are there marked as occurring both in Scotland and in Russia. Of these, thirteen are forms belonging to the Upper Old Red Sandstone of Scotland. The remaining five (*Osteolepis major*, *Diplopterus macrocephalus*, *Glyptolepis leptopterus*, *Asterolepis Asmusii*, and *A. minor*) occur among the shales and nodules of the Moray Firth and the flagstones of Caithness. It is impossible from AGASSIZ' list or MURCHISON's descriptions to be certain whether or not these five species occur in the very same strata with the others. The Russian development of the Old Red Sandstone, however, though it covers so wide an area, consists evidently of very flat and little disturbed beds, and, if one may judge from the sections in "Russia and the Ural Mountains," does not attain a great thickness. It would appear to represent merely the upper part of the Old Red Sandstone of Britain. The occurrence in it of a few other types of fishes like those just cited, cannot be regarded as evidence sufficient to establish a "middle" Old Red Sandstone series in Britain. It may merely show that in the basin of the east of Europe certain species of fishes survived longer than they seem to have done in Lake Orcadie.

Since the appearance of MURCHISON's paper in 1858, several interesting contributions have been made by other writers to the literature of the subject and

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* "Siluria," 4th edit. p. 362.

to our knowledge of the distribution of the ichthyolites in the Old Red Sandstone north of the Grampians. To some of the more important of these I shall make reference in subsequent pages. But as regards the state of opinion on the classification of the Old Red Sandstone, it remains very much as MURCHISON left it. The threefold grouping is received without question, and the Caithness flagstones are acknowledged to form a great middle series.

My own labours in the north began with the endeavour to ascertain the relation between these so-called "middle" beds and the upper division with *Holoptychius*, &c. In working out this point I have been gradually led to extend my observations over most of the area of what I have termed Lake Orcadie. In particular I found it necessary to make out in some detail, with the co-operation of my friend and colleague in the Geological Survey, Mr B. N. PEACH, the order of succession among the flagstones of the typical Caithness area, and to ascertain, as far as the available evidence would permit, the extent to which palæontological subdivision could be introduced into them. This task had never been attempted, and yet, until it had been in some measure at least accomplished, it was obviously hopeless to undertake any correlation of the fragmentary portions of the Old Red Sandstone of the north of Scotland, and still less any comparison between them and the area south of the Grampians. Over a large part of Caithness I have been accompanied and assisted by Mr B. N. PEACH, who likewise took part with me in traverses of Orkney and Shetland, and to whose skill and energy as an observer, always cordially given, I am under many obligations. Another Survey colleague, Mr JOHN HORNE, accompanied and assisted me in a traverse of the coast of Caithness, between Thurso and Duncansbay Head, as well as in the southern and western parts of the county. To the unravelling of the history of Lake Caledonia, on which I had been engaged for many years previously, a singular fascination had attached. I soon felt that a similar impulse was given by the study of the northern tracts. It led me to the shores of the Moray Firth and of Caithness summer after summer, and through the Orkney Islands to the remote Shetland. In now presenting the result of these journeys I am well aware that it cannot be regarded as more than a first sketch of the subject, that some parts are much less fully worked out than others, and that many modifications and corrections may eventually be needed throughout. I have myself, however, so long desired to possess such a first general outline for my own guidance that I venture to hope it will not be without interest and usefulness to geologists who are studying the history of the palæozoic rocks of this country.

The area which I have included under the general name of Lake Orcadie may be conveniently divided into four geographical districts,—1st, Caithness and Sutherland; 2d, the Orkney Islands; 3d, the Shetland Islands; and 4th, the Basin of the Northern Firths. In the north of Caithness and throughout the

Orkney Islands are presented to us the deposits of the deeper or at least more open waters of the lake. In the other districts we meet with the littoral accumulations of that great water-basin.

3. Description of Different Districts.

A. Caithness and Sutherland.

1. *Structure and Sections.*

The county of Caithness, in its geological structure as well as in its scenery, differs essentially from any other county in Scotland. It may be appropriately described as a wide table-land of triangular form, sloping up into the Sutherlandshire mountains on the south-west side, and truncated by the sea on the northern and eastern margins. When seen from any elevation beyond its borders, such as Ben Griam or Morven, this table-land, of which the average elevation may be taken at probably less than 350 feet, spreads out as a wide flat expanse of black peat rising here and there into gentle swells or ridges and sinking into numerous tarns and lochs. The geologist who tries to penetrate this interior soon finds himself beyond the limits of cultivation; roads, quarries, and every ordinary artificial opening into the rocks disappear; over broad spaces there are no streams save the dark brown runnels which trickle from the peat mosses, and carry the discoloured drainage of these barren wastes to the sea. Fortunately, however, the deep and widespread pall of peat gives way along the coast-line to some of the most magnificent ranges of mural precipices to be seen anywhere in Britain. There the obscurity of the interior is amply compensated by the full display of almost every bed and layer in the formation from base to summit. The angles of inclination are usually low, so low indeed that any one accustomed to the Old Red Sandstone in the central and southern counties of Scotland is rather inclined to look upon the Caithness strata as almost horizontal. Gentle anticlinal and synclinal folds repeat the same beds again and again, while a further reduplication is caused by small faults. But on the whole one cannot fail to be impressed by the general absence of disturbance in the Old Red Sandstone, not in Caithness only, but everywhere to the north of the Grampian range—a character which does not extend to the same system of rocks on the south side of that ridge. Were it not indeed for the occurrence of these faults and undulations, which either throw out or repeat portions of the strata, and for the absence of any very readily recognizable bands, which might serve as horizons and thereby allow the actual extent of the dislocations to be measured, there would be no difficulty in constructing a detailed section showing every minute variety in the stratification of many thousand feet of rock.

The eastern coast cuts the Old Red Sandstone of Caithness from its base to some of the highest parts of the series, and thus affords the most continuous and complete section, particularly of the lower parts of the formation. The northern coast exhibits in long lines of cliff and of shore-reef many admirable sections of the higher portions. No one section goes continuously from bottom to top of the whole succession of the Old Red Sandstone in this district. A traverse of the district from about the sources of the Isauld Burn in a northerly direction to Holburn Head probably crosses the most continuous and least undulated succession of beds in the county; but unfortunately it is only at occasional intervals that the rocks along that line can be seen. Yet by comparing the order of the beds displayed on the coast-line with such artificial or natural exposures as are available inland, a tolerably complete section of the whole series may be constructed.

The Old Red Sandstone of Caithness rests unconformably upon a portion of the altered Lower Silurian rocks of the northern Highlands. These masses are seen to rise from beneath it on the south-eastern coast at the Ousedale Burn, whence, striking inland, they form the range of the Scarabin Hills and the low mossy moors about the centre of the watershed between Sutherland and Caithness. Northward beyond this central moorland they rise again into more hilly ground as they approach the sea, while to the west they sweep onward into the rough mountainous country which extends to the Atlantic sea-board. No geologist can trace the relation of this platform of old crystalline rock to the unconformable and usually little disturbed strata which cover it without being struck by the singular unevenness of its surface, and by the evidence that this rugged character existed at the time when the Old Red Sandstone was laid down. At the eastern end of the Scarabin Hills, the Berriedale Water has cut a deep transverse section across this uneven platform, and shown how its inequalities are wrapped round by and buried under the conglomerates of the later formation. Yet from this marginal belt—doubtless a shingly beach at the beginning of the Old Red Sandstone deposits—the Scarabin range towers steeply to a height of 1600 feet. Knobs of the same ancient surface rise up among the sandstones and flagstones to the north and west, as at Dirlet Castle, Isauld Mill, Port Skerra, and Coalbackie, near Tongue. The platform upon which the Caithness flagstones and conglomerates were deposited must have been not unlike portions of the country lying to the westward from which these overlying strata, once far more widely extended in that direction than now, have been removed. Another feature, even more readily recognised by the observer, is the constant relation between the nature of the rock constituting the platform and the character of the overlying conglomerate. Towards the southern corner of Caithness the underlying crystalline rock is chiefly a rather fine-grained pink granite. Further north this gives way to flaggy micaceous gneiss, dark greywacke, and quartz-rock, which in turn, dipping gently towards north-west, are overlaid by the thick but lenticular band of white quartz-rock of the Scarabin Hills. Higher still in the series, but forming a band of lower ground on the north side of these hills, comes a remarkably beautiful gneiss, in which the most distinguishing feature is the occurrence of abundant wavy ribbons and irregular kernels of pink orthoclase. Other bands of white quartz-rock and dark schist occur further to the north. Veins of pink granite and pink quartz-porphyry or elvanite abundantly traverse all the crystalline rocks. In tracing, therefore, the limits of the conglomerates and breccias at the base of the Old Red Sandstone, we find them to indicate with considerable distinctness the character of the more ancient rock beneath them. Near the granite they are made up in great measure of granitic *debris*. Round the quartz-rock they are largely composed of that material. The existence of the well-veined orthoclase gneiss is indicated some distance before the underlying rock is actually seen by the abundant fragments of beautifully fresh cleavable pink felspar in the conglomerates. Hence, even when faults occur between the conglomerates and the rocks of the older platform, they do not materially obscure the section, as the connection of the fragmental with the contiguous crystalline masses remains sufficiently clear. As on the southern margin of the Highlands, so here, the crystalline or metamorphic character of the older rocks was as distinct as now before the formation of the conglomerates. The metamorphism of the gneiss quartz-rock and other associated bands, and the extravasation of the multitude of granite and quartz-porphyry veins, had been completed long before any portion of the Caithness Old Red Sandstone began to be laid down. And, as elsewhere throughout the Highlands, the denudation of these metamorphosed rocks must already have been enormous before any of the now visible conglomerates were formed.

The eastern coast-line of Caithness shows at its southern extremity a portion of the granite of the crystalline platform, forming the high headland of the Ord. A small fault intervenes at the Ousedale Burn between the granite and the Old Red Sandstone, its effect being to cut out some of the basement conglomerate. From this point, however, the strata can be followed in ascending order as far as Berriedale village, where, a little to the north of the mouth of the river, a considerable fault cuts the cliff, and extends inland along the southern flank of the Scarabin Hills. By means of this dislocation, and a reversal of the dip, a low portion of the series is again brought up, including part of one of the conglomerate masses. A steady dip towards north-east and east now sets in as far as Latheron, so that, from the axis of the conglomerate near Berriedale to that place, there must be included a thickness of at least 8000 feet of strata. Beyond Latheron the beds rise again with a south-westerly and then a westerly dip as far as Sarclet, where the Berriedale conglomerate once more rises to the surface. From the centre of the trough at Latheron to

Sarclet a protraction of the angles of dip gives a total thickness of strata amounting to about 9000 feet; so that either the series is rather thicker on the north side of the trough, or the effect of some small faults is to repeat portions of the strata so as to cause them to be reckoned twice in the estimate. At Sarclet the rocks arch over, and then dip to the north-west. The lower or red sandy portion of the Caithness development of the Old Red Sandstone is then repeated on the northern side of the anticlinal axis to beyond Ires Goe, fully 2500 feet of strata being exposed along the cliffs before the more flaggy series begins. To the east of Hempriggs House, however, another dislocation occurs, whereby a portion of the lower red beds is once more carried up to the surface. The dip thereafter remains more steadily towards north-west, bending towards north-east at Staxigoe, flattening and undulating towards Noss Head, and then turning round at Castle Sinclair towards north-west as far as Ackergill, where the centre of another synclinal trough is reached. It is hardly possible at present to say definitely, or within other than tolerably wide limits, what may be the cumulative effects of the obviously small faults which occur in the strip of coast-line between Sarclet and Ackergill. Probably somewhere about 3500 feet of flagstones are exposed between the Old Man of Wick and Ackergill. This, added to the 2500 feet of red strata extending southwards to the Sarclet axis, would give a total depth of 6000 feet. The coast-line northwards from Ackergill runs very nearly along the line of strike on the west side of the Sinclair's Bay syncline. After passing Brough Head we begin to descend slowly in the flagstone series; but before any considerable thickness of strata is passed over we encounter a set of yellow and red false-bedded sandstones, which perhaps lie unconformably on the flagstones, and occupy the north-western side of Freswick Bay. At Skirsa Head the flagstones reappear, and continue to dip at low angles in a general southerly direction as far as Fast Goe, where they are faulted against another mass of red and yellow sandstone, beyond which they reappear on the further side of a second fault, and form the bold bluff promontory of Duncansbay Head. It is evident, therefore, that along the eastern coast the first portion of the section, or that which runs from the granite of Ousedale to the centre of the synclinal fold at Latheron, affords the best evidence of the succession of the strata, and can best be compared with, and supplemented from, the other exposures of the same beds, as they are repeated by successive anticlinal and synclinal folds.

Comparing the rocks in that part of the section with those exposed along the northern coast-line, we soon perceive that the latter, on the whole, differ considerably in lithological character from those on the east coast. Taking this fact in connection with the general seaward inclination of the beds in the interior, from the Sutherlandshire moors northwards to Holburn Head, we cannot hesitate to place the northern series on a higher general horizon than the strata of the east side. From the fault on the west side of Duncansbay Head, an admirable section can be traced, partly in shore-reefs and partly in low but highly picturesque cliffs, westward to the fault at Brough, where the flagstones are thrown out by a fault which brings down the upper yellow and red sandstones against them. To the west of this interruption, however, they resume their position at Castletown, whence they continue without intermission to form the coast-line westwards for about twenty-five miles, rising here and there, as at Holburn Head, Brims Hill, Sandside Head, and Bighouse, into magnificent mural precipices between 200 and 300 feet in height. In no part of the northern coast does the lower or red portion of the Caithness Old Red Sandstone appear, nor do we apparently ever reach there even so low a position as the massive sandstones and flagstones of Wick. The highest visible members of the flagstone series occur about John o' Groat's House. A steady easterly dip is traceable along that part of the coast-line with one or two trifling undulations for seven miles. But the angles of inclination are low, seldom rising so high as 25°, and usually less than 15°. At the eastern or upper end of the section occur some red sandstones with flagstones and shales; these are underlaid by flagstones at Huna, below which lies a second zone of red sandstone, forming the west side of Gills Bay. Flagstones and shales succeed, rising at first with a steady easterly dip, and then bending round to north-east and north. At Harrow the north-easterly inclination becomes predominant, and it continues thence by Scarfskerry to Ham; while the angle of dip rises to from 30° to 50°. These are by far the most extensively disturbed beds on any part of the Caithness coast. Some sharp folds and crumplings occur, particularly between Scarfskerry and Sir John's Castle. At Ham the strata arch over towards the west, and the series is repeated as far up as the portion which lies between Scarfskerry and Harrow. The fault at Brough brings the section to an abrupt close. The total thickness of strata between Duncansbay Head and Brough must exceed 8000 feet. Probably the lower portions represent the higher parts of the Latheron synclinal beds. If we suppose 1000 feet of them to be thus representative, we obtain a total thickness for the Caithness flagstones of at least 16,000 feet.

Passing to the coast sections west from Dunnet Head, we find a succession of flagstones of which the general dip is towards north-west, at angles varying from 15° to 20°, the average being apparently from 8° to 10°. In this portion of the traverse a depth of probably not far short of 4000 feet of flagstones may be seen. But beyond Holburn Head, owing to the want of correspondence between the trend of the coast and the strike of the rocks, a gradual ascent is made through at least 800 or 900 feet of additional flagstones. Hence to the west of Dunnet Head a thickness of about 4800 feet of flagstones may be seen until their top dips under the waters of the Atlantic. There can be little doubt that this mass of strata is, on the whole, a repetition of those found to the east of Brough, but without reaching so high as the zone of red sandstone seen to the east of Mey. They do not therefore require to be added to the total thickness above given for the Caithness flagstones.

If now, by way of comparison, we trace a section from the base of the Old Red Sandstone about Loch Shurrery across the prevalent strike of the strata to Holburn Head,—a distance of 10 miles; and if we assume the average general angle of dip to be 10°, which is probably very near the truth, we obtain a thickness of strata along that line amounting to nearly 9000 feet. To that thickness would require, of course, to be added the 800 or 900 feet of flagstones overlying the Holburn Head beds to the west, and likewise the still higher portions of the Thurso flagstone series, as well as the depth of the red sandstone and flagstone series of Mey, Huna, and John o' Groat's, amounting to about 3400 feet. These sums would make a total probably not very widely different from the measurement already given from actual observations on the coast-line. We may regard the Old Red Sandstone in Caithness, therefore, as probably attaining a thickness of more than 15,000 feet.

Before quitting this part of the subject, a remarkable feature may be noticed here, to which fuller reference will afterwards be made. On glancing at the arrangement of the rocks, as shown by the dip arrows on the map (fig. 4), we observe that in the north-western part of the county the flagstones for some miles strike persistently at the crystalline rocks of the underlying platform. It might be supposed that this structure is due here, as it is commonly, to the influence of a fault, by which the older and younger rocks have been brought abruptly against each other along a sharp and rectilinear boundary-line. Such, however, is not the case. From a point on the coast about a mile to the west of Sandside Head, the gneiss, granite, and other crystalline masses may be followed south-eastwards for five or six miles, with the conglomeratic and sandy ends of the flagstone strata resting upon them; but, instead of dipping away from them, retaining still their prevalent north-westerly inclination. Hence the strata which eastwards from Sandside Bay occur as ordinary calcareous fissile flagstones, pass along their line of strike into sandstones and conglomerates as they abut upon the fundamental platform. A conglomeratic base at any point does not therefore necessarily mark the actual bottom of the whole formation. On the contrary, the conglomerates of Reay belong to a high zone in the flagstone series. But they are of course merely local, and disappear as soon as the strata begin to recede from the older rocks. We perceive also how importantly this structure bears upon the history of the Caithness flagstone series, since it proves the gradual depression of their area of deposit, and enables us in some measure to trace the outline of that area and the direction of its greatest depression. Similar evidence is furnished in Orkney as well as along the basin of the Moray Firth.

2. *Order of Succession among the Strata.*

It is now necessary to show the order and the lithological and palæontological characters of the Caithness flagstones as obtainable from a comparison of the various natural sections which have just been described. For the sake of clearness the following general table may be prefixed to this section of the memoir, as it shows the subdivisions which I have been able to make out, their thicknesses, and the localities where they are best seen, and where the characters to be afterwards described were observed in the field.

TABLE showing the Order, Thickness, and Typical Localities of the leading Subdivisions of the Old Red Sandstone of Caithness in descending order.*

Subdivisions.	Strata.	Thickness in Feet.	Localities where the Rocks occur.
John o' Groat's Sandstone and Flagstone Group.	9. Red sandstones, with occasional bands of flagstone, thin impure limestone, and shale.	2000	John o' Groat's House, on the shore.
Huna Flagstone Group.	8. Flagstones, shales, and thin impure limestones.	1000	Shore at Huna.
Gill's Bay Sandstones.	7. False-bedded red sandstones.	400	Gill's Bay, on Pentland Firth.
Thurso or Northern Flagstone Group.	6. Dark-grey and cream-coloured flagstones, grey and blue shales, and thin limestones; some beds strongly bituminous. This group more fissile, shaly, and calcareous than No. 5.	5000	Coast-line of Caithness from Reay to Dunnet Bay, and from Brough to Gill's Bay. Stroma and the Orkney Islands.
Wick or Eastern Flagstone Group.	5. Dark-grey flagstones, often thick-bedded, thin shales and limestone bands passing down into red shales and sandstones.	5000	Coast on either side of Wick, and inland to Banniskirk.
Lower Red Sandy and Conglomeratic Groups.	4. Dull red sandstones and occasional red shales and bands of fine conglomerate passing inland into conglomeratic sandstone.	2000	Coast south and north of Berriedale Water, Sarclet, Braemore, Morven, &c.
Do.	3. Brecciated conglomerate.	300	Coast at Badbea.
Do.	2. Dull chocolate-red sandstones and sandy shales or clays.	450	Berriedale Water around Braemore.
Do.	1. Coarse basement conglomerate.	50	Berriedale Water below Braemore, and mouth of Ousedale Burn.
		16,200	

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* In the field observations from which I have plotted this table, I was materially aided by Mr B. N. PEACH, whose active and helpful co-operation I would again heartily acknowledge.

These subdivisions will now be described in order, beginning with the lowest.

1. *Basement Conglomerate.*—This rock is partially seen near the mouth of Ousedale Water, where it was observed by SEDGWICK and MURCHISON. It does not rest there directly upon the granite, but is brought against it by a small fault which runs up the Ousedale valley, and strikes into the hills above Ousedale farm. As described by these writers, it there presents a remarkable granitoid aspect, insomuch that where the lines of stratification fail, and the texture of the rock is at its coarsest, neither in hand specimens nor in the sections of the cliff is it easy to determine where the conglomerate ends and the granite begins. A similar character is shown by the brecciated conglomerate of Badbea. It is on the Berriedale Water, however, that the relations of this lower conglomerate to the metamorphic platform are best seen. That stream, as already mentioned, has cut through the eastern end of the Scarabin range, and in so doing has laid open a fine section of the conglomerate for nearly a mile along its junction with the older rocks not far below Braemore. Wrapping round the ends of the beds of coarsely foliated orthoclase gneiss above referred to, it dips gently away from the hills, and passes under the sandstones and shales of Group No. 2. Laid bare both in the bed of the stream and in the cliffs on either side, its position on an uneven surface of the older rock is made very clear. Its component blocks vary in size up to as much as a yard, or even more, in length, and consist of gneiss, pink granite, quartz-porphyry, quartz-rock, mica-schist, and other crystalline rocks, with abundance of pink cleavable orthoclase derived from the underlying gneiss. They are for the most part tolerably well rounded, and in this respect, as well as in their larger size, they serve to mark this basement rock from the brecciated mass of Badbea. There is hardly any further trace of bedding than is sufficient to indicate that the rock inclines at a gentle angle against the gneiss, and dips under the sandstones. It is a coarse tumultuous shingle, such as might have been heaped up against a steep rocky shore, exposed to waves, driven by prevalent winds across a broad surface of water. In Old Red Sandstone times, the Scarabin ridge probably rose steeply from the southern margin of Lake Orcadie, which stretched thence northwards as far as the hills of Shetland—a distance of 140 miles. If in that ancient period the north-east wind blew with any approach to the fury with which it rushes along the cliffs at the present day, a sweep of 140 miles would be amply sufficient to raise waves capable of moving and rounding as coarse shingle as that which forms the basement conglomerate.*

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* When walking along the summits of the cliffs in Caithness and Orkney, I have often been struck by the evidence of the enormous force of the wind, which, caught in the clefts of these precipices, and converging upwards as in so many funnels, sweeps across the bald and barren ground at the top.

The anticlinal fold at Sarclet has been the means of bringing up on the coast-line a coarse conglomerate, which greatly resembles the mass as seen on the Berriedale Water, with such differences as might have resulted from a somewhat greater distance from the source of supply of its materials. The base of the rock is concealed by the sea, but a thickness of about 250 to 300 feet is visible. The matrix, red in colour, and less strongly felspathic than towards the south, contains large and usually rather well water-worn fragments of quartz-rock, granite, felspar, porphyry, and red sandstone. It may be matter for doubt whether this band should be placed on the same parallel with the basement conglomerate or with the brecciated beds of Badbea. On the whole, it seems rather to belong to the former, while a probable representation of the Badbea breccia occurs higher up in the series at Ulbster. The Sarclet conglomerate is distinctly bedded, and in its upper part has intercalated seams and bands of red sandstone. The whole of the strata there are considerably contorted and broken, though the main inclination and the order of succession remain perfectly clear.

From what has been stated above regarding the conglomerates around Reay, it is evident that we cannot be certain that the conglomerate here spoken of as the basement subdivision really forms the true base of the whole Old Red Sandstone series of this region. If, indeed, we may place the Sarclet rock on the same horizon with that of the Berriedale Water, the fragments of red sandstone which it contains serve to indicate the existence somewhere in the neighbourhood of other and older strata, which may have formed an earlier part of the system. We may, however, at least affirm that underlying all the visible portions of that system in Caithness it is the oldest known member. Whether

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This evidence was more particularly striking in an examination (made in company with my colleague, Mr B. N. Peach) of the summits of the magnificent precipices which on the west side of Hoy rise vertically from the Atlantic breakers to a height of more than 1300 feet. Back from the broken and ruinous summit the ground is covered with a coating of peat, heather, and coarse grass. Over the growing vegetation abundant fragments of sandstone are scattered for a distance of many yards from the edge of the cliff. The larger pieces are chiefly flat, and may weigh a pound or more. On lifting them the vegetation underneath was found to be quite green, indicating that they had been only recently deposited. They are evidently torn by the wind, partly from the crumbling sandstone strata of the cliff, partly from holes which have been worn through the peaty and heathy soil, and they are moved up the slope by successive powerful gusts. Further proof of the force of the wind is furnished by the number of little pools, ponds, and miniature tarns scattered over the ground above the edge of the cliff. The wind, taking advantage of hollows and little gullies or holes worn in the peaty covering by runnels formed after heavy rain, tears them wide open. When in dry weather the surface of peat becomes loose and powdery, the dust and loose fibres are blown away. This, of course, takes place more especially on the sides and bottoms of the hollows, which are thus further widened and deepened. The return of heavy rain serves to fill these hollows with black or brown peaty water. But the denuding influence of the wind does not cease, for the water is thrown into ripples and waves, which, beating against the black peaty sides of the pools, loosens them and removes the peat, partly in solution and partly in suspension, so as to allow of its being carried away in the outflow. In this manner the ground comes to be covered with shallow ponds and sheets of black water, which remain until they are either filled up with decayed peat *débris*, or emptied by the lowering of their margin at the point of exit.

or not there may be still older conglomerates and sandstones bearing to this mass a relation somewhat like that which it bears itself to the conglomerates of Reay, but lying concealed under the sea, must always be matter of conjecture. In any case an enormous interval of time must have elapsed between the formation of the underlying crystalline rocks and that of this basement conglomerate, though no strata belonging to this interval may have been laid down or preserved within the district.

2. *Braemore and Ousedale Sandstones.*—At the mouth of Ousedale Water the conglomerate just described passes under a series of red sandy and shaly beds. Thick bands of dull chocolate-red and reddish-grey sandstone, with courses of dull red sandy shale, are found cropping out along the slope of the valley, and running out to sea in successive shore-reefs. Several thick zones of the same dull red sandy shale occur in the upper part of the group, while in the lower two-thirds the beds are chiefly of sandstone, but with occasional courses of shale. It is noteworthy how abundantly pink orthoclase occurs in the matrix of many of the sandstones. In one particular bed, exposed at a little fishing creek below Badbea, the composition is so felspathic, and the felspar so fresh and distinctly cleavable, that the rock might readily be mistaken for a porphyry vein. The total thickness of this group in the Ousedale locality may be between 400 and 500 feet. No fossils were observed in it.

The same group of red sandstones and shales, faulted on the west side against the granite, runs up the Ousedale valley into the Langwell Water. On the further side of the Scarabia Hills it reappears, and covers several square miles of surface, filling the valley of Braemore, and stretching westwards under the Maiden Pap, Smean, and Morven. Exposed in the Berriedale Water, and in the gullies descending from the hills on either side, it presents everywhere the same dull chocolate-red hue as at Ousedale. The sandy shales strongly remind one of some of the red shales at the base of the Lower Old Red Sandstone in Lanarkshire and elsewhere.

Above the coarse conglomerate of Sarclet comes a series of red sandstones, and red, green, and grey shales, and calcareous flagstones. Between Ellen's Goe and the Stack of Ulbster this series of strata can hardly be less than 800 feet thick. It may perhaps be placed on the same parallel as the red shales and sandstones of Braemore, like which it rests on coarse conglomerate, and is covered by red brecciated conglomerate; but it contains a considerable admixture of greenish and grey calcareous flagstone, showing that the peculiar type of the Caithness Old Red Sandstone had already appeared. Traced northwards, the group appears to assume still more of that type, and it becomes impossible to draw any line between it and the group of sandstones No. 4.

The red shales and sandstones of Braemore are sometimes pitted as if by rain-prints, but have as yet yielded no fossils. In a sandstone at Ulbster, Mr C. W. Peach and Mr Shearer found plates of *Pterygotus.* As this crustacean genus has been supposed to be characteristic of Lower Old Red Sandstone and Upper Silurian strata, Sir Roderick Murchison naturally regarded its occurrence here as proof that the red sandstones of Sarclet belong to the Lower Old Red Sandstone, the overlying flagstones, devoid of *Pterygotus,* being placed in the middle of the system.* It ought to be noted, however, that distinct grey calcareous flagstones of the true Caithness type occur below the horizon from which the specimens were obtained at Ulbster; so that, at all events, the remarkable physical conditions to which the Caithness deposits were due had begun before the formation of the *Pterygotus* sandstone. Not only so, but this genus appears to have survived far into the time of the true Caithness flags. Mr Peach found some pieces, probably of *Pterygotus,* in one of the calcareous flagstones at Kilmster, near Wick, which is well up in the lower flagstone group. Again, in Orkney, from a bed high in the upper flagstones, a well-sculptured fragment of *Pterygotus* has been obtained. If, then, this genus be assumed to mark a zoological platform not higher than the inferior parts of the Lower Old Red Sandstone, we must assign a considerable part of the Caithness flagstones to that position.

With regard to the extension of the Braemore sandstones and shales, it may be remarked that though, owing to the overlap of higher strata, that group is concealed towards the north, except at the Sarclet anticline, red sandstones of similar aspect are seen below the conglomerate which far to the west caps the hills above Tongue. These westward prolongations of the lower parts of the Caithness system will be referred to in a subsequent paragraph.

3. *Badbea Breccia and Conglomerate.*—By far the most conspicuous member of the Old Red Sandstone series in the south of Caithness is a remarkable breccia or brecciated conglomerate which crowns the heights to the east of Ousedale and above Badbea, and projects in a line of rough broken escarpment. For nearly a mile and a half its edge runs approximately parallel with the coast-line, until by a rapid change of dip and angle it turns round, and, dipping towards the north-east, descends to the sea and passes under higher strata. It occurs in thick beds, wherein little or no trace of stratification may be found. The stones, considerably smaller than in the basement conglomerate of Braemore, seldom exceed five or six inches in length. They consist mainly of pink cleavable orthoclase, pink granite, grey quartz-rock, white vein-quartz, and occasional pieces of red sand-

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* "Siluria," 4th edit. p. 256.

stone. The felspar is the predominant ingredient, and likewise enters largely, in a comminuted state, into the composition of the paste. The rock has thus a strongly felspathic character, and has evidently been in great part derived from the detritus of such a gneiss as that already referred to as exposed on the north side of the Scarabin range. Owing, indeed, to the remarkably fresh and crystalline nature of the felspar, many portions of the fine-grained breccia would at first be most probably set down as granite, or some variety of the intrusive porphyries of the old metamorphic rocks. As seen at Badbea, this brecciated mass has a thickness of at least 300 feet. It passes upward into pebbly sandstones.

Owing to the fault already mentioned as flanking the Scarabin Hills on the south and emerging on the coast-cliffs at Berriedale, the Badbea breccia is brought up again along with the sandstones which cover it, forming together the projecting headland Ceann Leathad nam Bo. It retains here the same characters as at Badbea. By a rapid reversal of dip, the breccia or brecciated conglomerate arches over to the north-east under its overlying sandstones. The section then continues an ascending one as far as the Latheron synclinal axis. Eastward from that part of the coast, however, the whole series rises again to the surface, until, as already stated, the lower strata are once more brought up along the Sarclet anticline. Although the rocks round Sarclet are not exactly similar to those exposed about Berriedale and southwards, we are probably not far from the truth in placing the conglomerate seen at Ulbster on the same horizon with that of Badbea. Two bands of conglomerate occur on the coast of Ulbster,—the upper one measures only a few feet in thickness; the lower is about fifty feet thick, and contains angular fragments of quartz-rock, sandstone, &c. This conglomerate zone cannot be followed far inland. It seems to die out before reaching the shore again, on the other side of the Sarclet axis. This northward attenuation of the thick conglomerates and breccias of Badbea affords further evidence of the position of the ancient shore, and that the more open water of Lake Orcadie lay to the north.

Turning inland from the Berriedale shore, we find the conglomerate running along the hill-tops on the left side of the Berriedale Water, with the red shales and sandstones emerging from beneath it, and cropping out along the lower slopes. On the right bank of the stream, outlying portions rise into the Maiden Pap, Smean, and Morven, but are so blended with the next succeeding sandstones as to form with these one continuous group.

4. *Langwell and Morven Sandstones and Conglomerates.*—Between Badbea and the Berriedale Water, the breccias now described pass up into a thick series of dull chocolate-red, grey, and yellow sandstones, with layers of dull-red and olive-coloured shales and of fine conglomerate. They form the

high headland of Cnoc na Croiche, and a range of sea-precipices nearly two miles in length. In their lower part they are highly felspathic, with a rather coarse texture, and many scattered pebbles, as well as nests and bands of conglomerate. Higher up they are more flaggy, and they finally pass upward imperceptibly into the dull-red and grey flagstones of Berriedale. Nothing but an arbitrary line can be drawn for their upper limit; but if we place that line a little to the south of the mouth of the Berriedale Water, the thickness of this red sandstone group is found to be about 2000 feet. The group has as yet yielded no fossils.

Before tracing these strata inland and westward, we may look at their prolongation northward along the shore. On both sides of the Berriedale anticline they are found overlying and dipping away from the brecciated conglomerate. On the north side they are particularly well seen. They have there already lost the coarse pebbly and conglomeratic features so conspicuous on Cnoc na Croiche. They become fine flaggy sandstones, with sandy shales, the whole having a prevailing dull chocolate-red tint, through which seams of greenish-grey shales and flags occur. At last, at the headland of An Dun, these strata become very thin-bedded and flaggy, and assume most of the ordinary characters of the Caithness flags, though still retaining their prevailing red colour. At the Sarclet anticline, further interesting evidence is supplied of the gradual diminution of coarse sandy sediment towards the north. The red sandstones are thin-bedded, and alternate with red and grey shales, and greenish-grey calcareous flagstones, until they pass insensibly into the ordinary flagstone series. As was observed by SEDGWICK and MURCHISON, the red tint is often local and even superficial, the same stratum being red or green at different parts of its course. Beyond the Sarclet anticlinal axis, the red sandstones and shales are prolonged, partly by the agency of faults, as far as the Brough, near the Castle of Old Wick, where a series of dull-red flaggy sandstones and shales, without thick beds, forms a large isolated stack. These thin-bedded rocks doubtless represent the An Dun beds just referred to.

While, then, the red strata become more and more mixed with fine muddy sediment as they advance northwards, they present very different characters as we trace them inland to the west. In the Langwell Water they are admirably displayed with the same general type as on the coast. But on the north side of the Scarabin ridge they so increase in the quantity of coarse detritus scattered through them, that for hundreds of feet in vertical depth they may either be called highly conglomeratic sandstones or sandy conglomerates. It is this coarse mass of sediment which, rising above the red sandstones and shales of Braemore, forms the remarkably picturesque cone of the Maiden Pap, the rough-topped ridge of Smean, and the huge pyramid of Morven. It must once have spread westward, with nearly the same characters, for at least thirty-five miles, since it occurs in detached outliers as far as the Kyle of Tongue, where it passes under the Atlantic. The accompanying section (fig. 1) explains the structure of the ground between Braemore and Morven, and shows the great denudation which the rocks have there undergone. Morven, the highest and most conspicuous hill in Caithness, reaching a height of 2313 feet above the sea, is an enormous outlier of gently inclined sandstones and conglomerates, of which the truncated edges, seen all round the flanks of the mountain, look over the wide plain of Caithness and the undulating uplands of Sutherlandshire.* The lower two-thirds or thereabouts of the mountain consist of well-bedded coarse reddish-grey pebbly sandstone, often passing into brecciated conglomerate, and sometimes with intercalated courses of dull red shale. The upper portion is a coarse brecciated conglomerate, in massive irregular beds, much jointed, and standing out in broken crags. The paste of this rock abounds in the same pink orthoclase already noticed, while fragments of the same mineral are specially conspicuous among the pebbles. Other included fragments consist of quartz-rock, pink granite, pink porphyry, vein-quartz, gneiss, schist, and a dull-red sandstone. They vary in size up to five or six inches in length. Though sometimes tolerably well rounded, they are mostly subangular, in some cases sharply angular. I did not observe the coarse basement conglomerate at the foot of Morven, though immediately to the west of the mountain the snowy-white quartz-rock appears in a remarkably flowing ice-worn ridge. Smean resembles Morven in structure, but is less lofty and conical in form, though its otherwise tame outline is relieved by the splintered crags of conglomerate along its crest, especially when seen from the east. The Braemore sandstones and shales are exposed along its flanks, whence they sweep eastward in a broad smooth ridge, which slopes down into Braemore. On the highest part of this ridge stands the

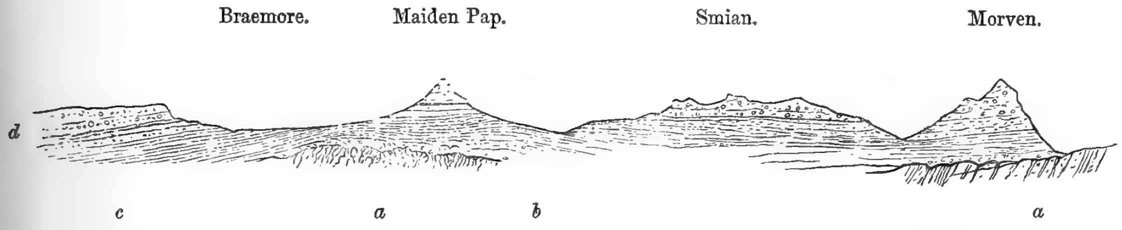

Fig. 1.—Section through the bottom beds of the Old Red Sandstone at Morven and Braemore, Caithness. *a*, Schists, Quartzites, &c.; *b*, Bottom Conglomerate; *c*, Sandstones, Shales, and Conglomerates; *d*, Conglomerate.

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* Some remarkably steep angles of repose for the detritus of the mountain are to be seen on the south and east slopes, the angle seldom falling below 27° and rising sometimes to 35°, as measured by my colleague, Mr JOHN HORNE, who accompanied me in the ascent. The north and west sides are precipitous.

conical outlier of conglomerate and sandstone forming the Maiden Pap, with its knob-roughened sides presenting a sharp contrast with the featureless surface formed by the softer red strata on which it rests.

Westward from this district, it is no longer possible to identify subordinate groups of beds in the lower sandy and conglomeratic portion of the Old Red Sandstone of this part of Scotland. Even in the Morven area, we see the gradual blending of some of the zones which are so distinct on the eastern shore. Beyond the water-shed of Caithness and Sutherland, two massive outliers of the same character as Morven form the two isolated conical mountains of Ben Griam Mohr and Ben Griam Bheag. Rising steeply from the brown waste of peat-mosses between the headwaters of the Halladale and Helmsdale rivers, these two eminences afford a most impressive lesson as to the denudation of this region since Old Red Sandstone times. They form conspicuous landmarks from all the low country to the north and north-west. Their strata, inclined at low angles, run in terraced bars along the sides of the mountains like lines of masonry, while from their base the platform of crystalline rocks undulates in all directions. On the east side of Ben Griam Bheag, well ice-worn bosses of grey quartz-rock and gneiss, with veinings of granite and pink porphyry, ascend for about a third of the height of the mountain (fig. 2). They are unconformably overlaid by a very coarse conglomerate not less than 150 feet thick, capping an eastern spur of the cone. In this rock the stones are tolerably well rounded, and sometimes two feet long. They include pieces of pink granite, greywacke, gneiss, and other crystalline rocks. Above this mass lie thick beds of pale reddish and reddish-brown gritty and pebbly sandstone, sometimes passing into conglomerate. In general composition, texture, and stratification, these rocks are quite like those of Morven. In successive ledges and terraces, they may be followed for a thickness of probably about 1000 feet to the summit of the mountain, which is formed of a hard conglomeratic bed, dipping, like the beds below, towards south-west at a very gentle angle.

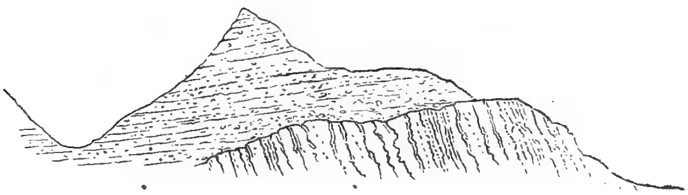

Fig. 2.—Section of Ben Griam Bheag.

When the wild tracts included within the basin of the Naver are examined in detail, other outliers of similar sandstones and conglomerates may be found.

I was informed, indeed, by Mr Crawford, Tongue House, that on Beiun Armuinn, a mountain, rising to a height of 2338 feet, in the heart of eastern Sutherlandshire, conglomerate occurs.

The extreme north-western limit to which the Old Red Sandstone can now be traced may be drawn at the Kyle of Tongue. Detached ridges and knolls referred to this formation were observed there by Sedgwick and Murchison in 1827.* They were described in further detail, and inserted on his map by Hay Cunningham, in his "Geognostical Account of Sutherlandshire." † But by much the most extensive and instructive series of sections of these deposits occurs, not on the mainland, but on the Roan Islands, at the mouth of the Kyle of Tongue, though there they do not show their junction with the crystalline rocks, as they are surrounded by the sea. These islands consist entirely of conglomerate, a coarse well-bedded rock, with nests and partings of dull red sandstone. The strata vary from less than a foot to several yards in thickness, and are marked off chiefly by these intercalated ribs of sandstone, for in the mass of the conglomerate itself the pebbles very commonly show no trace of stratified arrangement. The bedded character of the rock, however, comes out very clearly even from a distance, the successive ledges shelving down into the water, and dipping out to sea at angles of 10° to 20°. By means of the joints, slices have been cut away from the cliffs, leaving vertical walls against which the Atlantic surge is always heaving. These precipices are deeply fissured by the opening of the joints, whereby the waves have been able to perforate the rock in many gullies and caves. On the north side, where the beds slope like a long breakwater into the sea, huge masses of the conglomerate have been loosened and moved down the inclined surface of the beds below them. But the most ruinous scene is presented by the south-western or inland face of the larger island. Owing to the abundant joints and the more decomposing nature of the rock at that place, the ground seems caverned and honeycombed in all directions.‡ At the north end of the smaller island, in Meal Chalam and the neighbouring headlands, the rock assumes its noblest forms, rising into massive buttressed sea-cliffs, a fitting termination for the Old Red Sandstone in the north-west of Scotland.

The component pebbles of the conglomerate vary in size up to one foot or rather more in length. They are mostly rounded in shape, but include subangular stones, particularly where the material happens to be of a jointed

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* "Trans. Geol. Soc.," 2d series, vol. iii. p. 127.

† "Trans. Highland Society," vol. vii.

‡ The tradition, so commonly told in such districts, is repeated here, of a dog having fallen into one of the subterranean passages, and having worked its way underground until, after some days, it succeeded in crawling out at an opening on the opposite side, but devoid of its hair, which had been scraped or shaved off by the rough walls of the narrow passages through which it had to force its way!

shivery kind. They consist entirely of the surrounding metamorphic rocks, quartz-rock and flaggy gneiss being particularly abundant. The matrix has a red sandy character.

On the mainland, conglomerate of a similar kind begins at Coalbackie, not far from the mouth of the Kyle of Tongue, and stretches southward in a conspicuous ridge and detached craggy eminences as far as Ben Stomino, on the east side of Loch Laoghal. Its unconformable relation to the underlying crystalline rocks on the shore was clearly pointed out by CUNNINGHAM.* Another feature to be noticed at that locality is the remarkable unevenness of the platform on which the conglomerate was deposited. The accompanying diagram (fig. 3) will illustrate this structure. On the shore we find the upturned edges of the flaggy micaceous quartz-rock wrapped round with coarse red conglomerate, and forming a small isolated stack within tide-marks. A large mass of the conglomerate runs inland for a few yards, and presents a vertical face on one side. Immediately beyond it the underlying quartz-rock reappears, while at the base of the long slope which rises up to Coalbackie the conglomerate again occurs. A small fault may form the upper limit of this patch. At the summit of the slope lies the huge conglomerate cliff of Cnoc Vreckan, the final north-westerly escarpment of the Old Red Sandstone on the mainland.

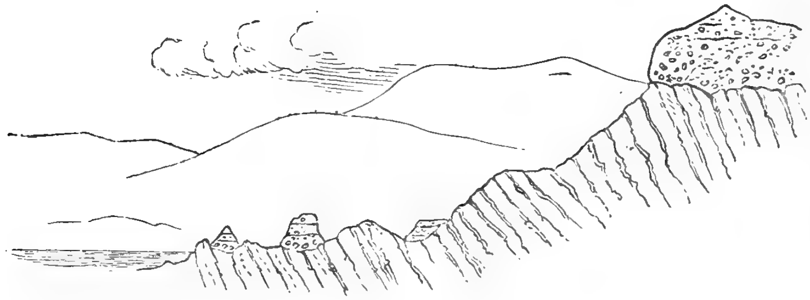

Fig. 3.—Sketch-Section of the relations of the Old Red Conglomerate to the underlying Quartz-Rock in the Kyle of Tongue.

From the steep truncated end of the ridge above Coalbackie, the conglomerate (with sometimes a basement of red sandstone, seen particularly below the east slope of Cnoc Vreckan) runs southward for several miles. It forms the detached outlier of Cnoc Craggie, and rises high upon the sides of the granite mass of Ben Stomino. Its bottom there must be somewhere about

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* *Op. cit.*

1500 feet higher above the sea than on the shore of the Kyle of Tongue. Yet the distance between the two points, in a straight line, is not more than about seven miles.

One further fragment of the Old Red Sandstone in the north of Sutherlandshire occurs in the little bay of Kirktomy, where it was observed by CUNNINGHAM. It consists of dull red and reddish-grey sandstone, sometimes mottled and marked with nodular bands and intercalations of soft dull red sandy clay or "marl." These strata rest upon the gnarled quartzose schists which run out into the promontory of Kirktomy Point. They dip gently towards the north-west, and are probably cut off by a fault on the west side. There can be little doubt that these beds occupy a considerably higher zone of the Old Red Sandstone than the Morven conglomerates and sandstones. They lie well to the north of what must have been the old shore-line. Possibly they may belong to the same horizon as the sandstones and conglomerates of Strathy, to be afterwards described.

Looking at the distribution of the Sutherlandshire outliers of Old Red Sandstone towards the north-west, we perceive that they must represent merely the last fragments of a once much more extensive deposit. And this conclusion is strengthened when we observe on the ground how very gentle is the inclination of the strata in these outliers, and how, accordingly, the truncated edges of the beds cut the sky-line from every point of view. At the same time, if, in endeavouring to restore in imagination what has been removed, we fill up the whole of the intervening space with conglomerate, and thus cover most of the east of Sutherlandshire with a deposit at least as thick as what remains in the outliers, that is, about 1000 or 1500 feet, we may err by somewhat exaggerating the quantity of material denuded. Conglomerates and coarse conglomeratic sandstones are notoriously local formations, suddenly swelling out into great masses, and as rapidly dwindling down again or disappearing altogether. The conglomerate may indeed have once extended from Morven to the Kyle of Tongue in a continuous belt, as it still does for many miles down the east side of Sutherlandshire and Ross-shire. On the other hand, though continuous along the ancient shingly coast-line of the Old Red Sandstone period, it probably varied greatly in thickness, even at the time of deposit. Some parts of the shore, owing to the nature of their rocks, to the set of the prevalent winds, or to the quantities of shingle brought down by streams from the interior, may have received and accumulated enormous piles of conglomerate, while intermediate tracts, from failure of supply, witnessed the formation of none at all, or, at most, of trifling beds of sand and gravel. On the subsequent consolidation and denudation of the formation it might well happen that the local piles of shingle, compacted into conglomerate, should remain still conspicuous masses, notwithstanding the general degradation of the surface during which the thinner connecting sheets of sandy detritus disappeared.

The four groups of strata, above described, may be regarded as forming together a red sandy and conglomeratic base, of very variable thickness, on which lies the great flagstone series of Caithness now to be discussed. This latter remarkable and most characteristic development of the Old Red Sandstone in the north of Scotland, was first examined and delineated with much skill in the Memoir of SEDGWICK and MURCHISON. These authors comprised in their narrative the description of the two coast sections on the northern and eastern margins of Caithness. They regarded the traverse from Strathy to Duncansbay Head as an ascending section, and that from Duncansbay Head to the Ord as a descending one, though they admitted that each presented peculiarities not found in the other. From what has already been said regarding the geological structure of the region, it will be apparent that the two coast sections cannot properly be put in comparison. For in the first place, the supposed basement conglomerates at the west end of the northern section are but of trifling extent, and, instead of representing the thick masses already described as occurring towards the southern end of the eastern section, belong really to a higher horizon than is anywhere reached among the flagstones on the east side; in the second place, owing to anticlinal and synclinal foldings, as well as to faults, the same beds are repeated again and again, so that it is in some cases only from a protraction of the measured angles of dip that we can be sure, after several miles of coast section, whether we have reached a higher or a lower part of the series. In no one natural section do we meet with a continuous succession of the whole flagstone series. Fortunately, however, owing to the many arches and troughs into which the strata have been folded, most parts of the series are exposed in more than one coast section. Their varying lithological characters and thicknesses can thus be compared, and probably a more generally accurate table of the whole mass can be compiled than if the data were obtainable only from a single exposure.

In endeavouring to construct such a table, much difficulty is experienced owing to the remarkable sameness of lithological characters throughout the flagstones, the want of strongly defined bands or zones which could be recognised at each successive reappearance at the surface, and the uniformity of the palæontological contents of this whole series of strata. No doubt when the county of Caithness comes to be geologically surveyed in detail, many traceable bands may be detected, and will then be used in unravelling the minute structure of the county, and in forming a more accurate and detailed tabular arrangement of the flagstones than can at present be attempted. As a provisional grouping, it may be convenient to arrange the principal mass of the flagstones into two great groups, which seem to be tolerably well defined both lithologically and palæontologically. The lower of these, well exposed along the east coast, may be called the Wick and Lybster or eastern group; the upper, copiously developed along the north coast from Reay by Thurso to the Pentland Frith, may be termed the Thurso and Reay or northern group.

5. *Wick and Lybster Flagstones, Eastern or Lower Group.*—Returning now to the east coast section we observe, as already remarked, that in the first ascending section from the basement beds at Ousedale, the red sandy and conglomeratic groups pass upward into sandy red and blue flagstones, which crop out in the lower reaches of the Langwell and Berriedale Waters. Some calcareous beds are associated with them, in one of which, under the ruined castle at the mouth of the last-named river, *Dipterus* was obtained by Mr C. W. Peach. The Berriedale fault, however, throws this group out, so that we need to pass over a space of rather more than two miles until we reach the equivalent strata about An Dun. From this latter part of the coast a continuous section can be followed as far as the centre of the synclinal trough at Latheron, interrupted by some dislocations, of which the only one of importance appears to be that which crosses the coast-line about a mile to the south of Janetstown harbour. On the north side of that fault a repetition of part of the flagstones probably takes place, accompanied by some crumpling of the beds. But as this occurs near the top of the basin it does not seriously interfere with the construction of the vertical table. For rather more than two miles beyond the fault the trend of the coast-line nearly coincides with the strike of the strata; but from Port na Muic to the red sandy group of Ulbster there is a continuous descending section, only interrupted by what appear to be unimportant faults. On either side of the trough, therefore, the same series of strata is to be found. On the south side the thickness of rock exposed above the red sandstones near An Dun is about 5500 feet; on the north side a protraction of the angles taken on the shore gives a thickness of 5000 feet. The comparatively slight discrepancy between these two measurements may be due partly to the thinning out of the strata, but more probably in the main to the faulting just referred to. If then we place the top of the lower group of flags somewhere about the centre of the Lybster syncline, we shall have a mass of strata composing this group to a depth of about 5000 feet.

The base of the flagstones passes down, by imperceptible gradations, into the red sandy strata of the Langwell group. The thin-bedded red and grey sandy shales and flags of An Dun and the Brough, near Wick, may be regarded as the passage beds between the two sets of strata. Above this intermediate zone, the flagstones become thick-bedded, with intercalated sandstone bands, and occasional seams of shale and argillaceous limestone. The prevailing colour is bluish-grey, though the red hue of the group below still appears both as a local superficial stain and as the tint of separate layers. To the south of Wick, thick calcareous blue flagstones and hard grey well-bedded sandstones have long been extensively quarried for building purposes. In lithological character this group of strata is distinguished from that which overlies it by the greater massiveness of its flagstones, and by their less calcareous composition and less fissile or shaly texture. But, in general aspect, the two groups so agree that they must be regarded as essentially one great flagstone series. Their more minute characters will therefore be detailed after the position and thickness of the second group have been given, and these details will, for the most part, apply to all the flagstone groups.

Palæontological Features.—The palæontological distinctions of the group are in like manner rather vague. I may remark, in reference to this part of my subject, that I am mainly indebted for my information regarding the distribution of organic remains in Caithness, to the assistance so freely rendered to me by my friend Mr C. W. PEACH. Long years of patient collecting in that county made him familiar with the best localities for fossils, and he noted what species had been met with at each place. Having myself worked out on the ground the order of succession of the strata, I was enabled, by means of Mr PEACH's list of localities, to construct the table appended to this memoir, showing the vertical range of every species known to occur in Caithness. Many of the same forms are also met with in Orkney; but, as will be afterwards pointed out, their precise localities there have not always been noted, so that, in the meantime, no corresponding table of ranges can be drawn up for the Orkney region. I have placed, however, in Table II., p. 451, the names of all the species hitherto met with in Orkney; so that a simple inspection of this table will afford materials for a comparison between the ancient ichthyic fauna of Lake Orcadie, as preserved over the areas of Caithness and Orkney respectively.

In various horizons throughout this group remains of terrestrial plants have been found by Mr PEACH, notably at the South Head of Wick, at Kilminster (or Kilmster), at Ackergill, at Kiess, and northward in the pale sandstones of Freswick, which probably belong to one of the upper groups of Caithness. The Kilminster bed, which particularly abounds in these remains, is a grey shale, or "calmstone," lying between flagstones, and having its surface covered with carbonised vegetation. Among the forms DAWSON's genus *Psilophyton* predominates, both here and throughout the plant-bearing beds of Caithness. Mr PEACH thinks there are probably two species. He has obtained some specimens showing fructification at Kilmster and at Ackergill. Large sheets of these plants, matted together, appear to have been entombed in the mud of Lake

Orcadie, both over the area of Caithness and Orkney. Of *Caulopteris Peachii,* believed by SALTER to be a tree-fern, large stems have been obtained by Mr PEACH from South Head, Wick, and from the upper flagstones near Thurso. *Pinnularia* occurs at South Head. The most highly organised vegetation yet obtained from these strata are remains of coniferous wood assigned to the genus *Araucarioxylon.* The true coniferous internal structure was originally detected by QUECKETT, and his observations have been confirmed by Mr PEACH. No true marine plants have yet been found in any part of the Caithness flagstones.

Fragments, probably of *Pterygotus,* certainly of some large crustacean, occur in the plant-bearing calmstone of Kilmster. Annelide burrows, in pairs, were figured by Mr SALTER as occurring at the same locality.* The small phyllopod crustacean, *Estheria membranacea,* occurs both at Reiss and at Wick. It will be observed from the table, p. 451, that several of the species of fish range through almost the entire series of the Caithness flagstones. *Dipterus macrolepidotus,* in particular, has the widest range of any of the Caithness fishes, for it occurs far down in the eastern group at Berriedale, and likewise in the beds of Huna, near the top of the northern or upper groups. *Diplopterus borealis,* and *Osteolepis macrolepidotus,* have likewise a wide vertical range. At the same time, the table enables us to recognise certain positive and negative characters as distinguishing the two flagstone groups. *Dipterus Valenciennesii,* and *Osteolepis arenatus,* which occur in great numbers at South Head, Wick, seem to be peculiar to the lower or Wick group ; and there may be other forms or varieties which do not pass upward from these strata. But according to present information, it is rather by the absence of many genera and species found in the higher groups, than by the presence of forms peculiar to itself, that the great group of the lower flagstones is distinguished. Thus the genera *Acanthodes, Diplacanthus,* and *Parexus,* found in the Forfarshire flagstones, have only been noticed in Caithness from the Thurso group. The *Osteolepis microlepidotus,* so characteristic of that group, has not been observed in the Wick flagstones. The range of *Coccosteus pusillus* is equally restricted. The land plants, also, in the lower group are neither individually nor generically so numerous as they become higher in the series. How far these palæontological distinctions are radical, depending upon the chronological sequence of the organic forms, or accidental, arising from original conditions of hydrography and deposit, it would be premature at present to decide. I believe, however, and will be able to show in a later part of this memoir, that they probably do point to chronological sequence, and may therefore be used to some extent as a means of making out the stratigraphical relations of the Old Red Sandstone of the north of Scotland.

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* "Quart. Journ. Geol. Soc." vol. xiv. p. 72, and plate v. fig. 6.

6. *Thurso and Reay Flagstones.*—The lower group of flagstones, after its numerous undulations and fractures to the south of Wick, dips towards the north-west at that town, but from Staxi Goe bends round to the north-east with low angles as far as the Noss Head, where the beds once more resume their north-westerly inclination. After some crumpling and dislocation between Castle Girnigo and Ackergill Castle, a steady south-easterly dip sets in, which may be traced from Reiss up to Freswick Bay. There is thus a syncline in Sinclair's Bay, of which the axis runs parallel with the trend of the coast between Keiss and Freswick: consequently no higher beds are anywhere seen here than in the centre of the trough at Ackergill, where they probably do not reach to the top of the lower group. Beyond this syncline to the north, owing partly to faults and partly to the spread of certain upper red sandstones, the passage of the two groups into each other is not exposed on the coast. From about Freswick Bay the strata arch over again and dip towards north-west and north. They must continue to do so steadily, for in a space of four miles the whole of the higher flagstone group and the upper portion of the lower must be comprised, yet the greater part of this space is occupied at the surface by red and yellow sandstones, belonging, probably, to the Upper Old Red Sandstone.

It is unfortunate that no continuous section can be traced through the whole flagstone series. Each of the two groups is admirably exposed on the coast sections, but their passage into each other cannot at present be satisfactorily recognised. Possibly some of the lower parts of the higher group may occur in the centre of the Latheron syncline; and, on the other hand, a portion of the lower group may rise to the surface from below the upper on the north coast near Brough. But until a detailed survey of the ground has been made this point must remain undecided.

The interior of the county being for the most part obscured by peat and drift, no continuous section can be followed there, though the rocks are exposed in a sufficient number of places to allow of the respective limits of the two flagstone groups being traced. A line drawn from the Wart Hill of Duncansbay, south-westwards to somewhere about Halkirk, and then westwards by Loch Calder to the edge of the crystalline rocks, would approximately mark the position of the passage beds between the two groups.

To the north of that line the upper group may be traced in many isolated inland exposures and in continuous coast sections. A traverse from Broubster, near Loch Calder, to Holburn Head, crosses all the lower and central portions of the group, but does not reach the top. The coast west from Holburn Head trends almost along the strike of the strata, so that no great thickness is there to be seen. Eastwards from Holburn Head the shores of the Thurso and Dunnet Bays show a descending series for several thousand feet without reaching the base. It is on the coast from Brough to John o' Groat's that the most complete sections of the group occur.

In comparing these sections, the first feature to arrest attention is the remarkable overlap of the beds upon the old crystalline platform to which reference has already been made on p. 373. The flagstones which, towards the east, retain the usual normal characters of fissile calcareous strata, pass into sandstones and conglomerates as they approach and rest upon the granite and gneiss (A in fig. 4). Hence, from Reay south-eastwards, these latter rocks are fringed with sandy and pebbly strata, not, however, as a continuous mantle sloping away from them, but in successive beds, abutting against the eastern edge of the crystalline era, and dipping towards the north, north-west, or north-east. Such a structure, as I have before remarked, might at first be attributed to the effect of a fault between the two masses of rock, but many natural sections may be examined where the pebbly strata rest directly upon and are formed out of the crystalline rocks beneath them. The subjoined map will explain this interesting and somewhat uncommon structure.

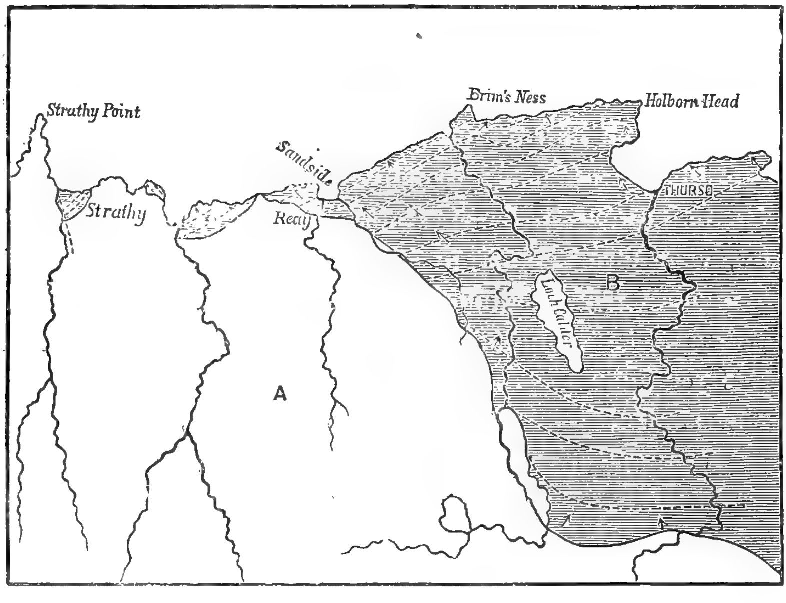

Fig 4.—Map of Part of the North-West of Caithness, showing the overlap of the Thurso Flagstone Group upon the Crystalline Rocks. A. Schists and Crystalline Rocks. B. Old Red Sandstone. The dotted lines mark the strike, and the arrows the dip of the strata.

For the sake of clearness in description it will be most convenient to trace first the section of the strata exposed to the west of Dunnet Bay, and then that on the east side between the two headlands of Dunnet and Duncansbay.

Beginning on the shore at Castletown, we meet with thin-bedded and shaly flagstones dipping gently to the north-west. These strata have long been extensively quarried here for the well-known Caithness flags of commerce. They are also exposed further inland in the large quarries of Stonegun and

Weydale. They are dark-grey, hard, fissile, calcareous, and partly bituminous strata, capable of being split into large, clean, smooth flags, an inch or less in thickness and of uncommon toughness. The zone which emerges on the coast below Castletown is probably the lowest, from which the best, that is, toughest and thinnest flagstones can be procured. An analysis of four specimens from the Castlehill quarries by Dr Hofmann, to whom they were submitted by Sir R. Murchison, gave the results quoted below.*

From Castletown to Thurso the succeeding strata are admirably laid bare, both in vertical sections along the low and easily scaled cliffs, and in horizontal ground-plan on the beach, where broad sheets of the rock slope gently into the sea. The prevalent dip continues north-westerly, with one or two gentle rolls, and the angle of inclination seldom exceeds 8°. By an observer coming fresh from the more massive flagstones of the lower group the first features to be noticed are the thin-bedded, almost shaly character of the strata here, their pale-yellow and green colours on weathered surfaces, and their evidently calcareous composition. On closer examination they are found to consist of fissile, calcareous, grey, hard flagstones, green, grey, and brown calcareous (and frequently bituminous) shales, with thin bands of calcareous gritty sandstone and argillaceous limestone ("calmy limestone"), seldom more than a few inches in thickness. The influence of weathering generally produces a yellow or greenish-grey tint on the surface of the rocks, while at the same time it reveals the exceedingly fine lines of deposit on many of the flagstones. Even when split into smooth sheets an inch or less in thickness, these hard, tough layers show on their yellow, weathered edges successive paper-like but mutually adherent laminæ. The general thin-bedded, shaly, and calcareous characters of these strata recall the aspect of some of the lower parts of the Carboniferous system in the south of Scotland.

The next feature to engage the attention of the observer is probably the extraordinary abundance of ripple-marked surfaces and sun-cracks. Though these markings abound also in the lower flagstone group, it is here that they attain their greatest development. Surfaces of flagstone or shale, many square

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* "Quart. Journ. Geol. Soc." vol. xv. p. 402.

	Silica and Silicates Insoluble in HCl.	Oxide of Iron and Alumina.	Carbonate of Lime.	Organic Matter.	Water, Loss at 100° C.	Salts of Magnesia, the Alkalies, &c.	Total.
No. 16, Top Flag, .	68·40	10·21	10·93	3·88	0·42	6·16	100·00
No. 7, Middle Flag, .	69·45	11·50	10·66	5·79	0·40	2·20	100·00
Bituminous Shale, .	69·96	8·15	7·72	10·73	0·53	2·91	100·00
No. 1, Bottom Flag, .	61·39	4·87	21·91	3·40	0·20	8·23	100·00

yards in extent, are profusely covered with fine rippled lines as sharply preserved as if only to-day imprinted on the soft sediment. In many places every successive stratum or leaf of rock is thus marked, so that several distinct rippled surfaces may be counted in the thickness of a few inches of rock. It is likewise observable that the rippling is generally close-set, sometimes not exceeding an inch in breadth from crest to crest of the ridges.

More abundant and admirable illustrations of sun-cracks could hardly be found than occur along this coast. Broad gently-inclined sheets of rock again and again present themselves to view so covered with reticulations as to look like tesselated pavements. It may be noticed that the cracks not infrequently descend through many of the fine laminæ of deposit for a depth of five or six inches with occasionally a breadth of three or four inches. The material filling up the interstices abounds with small, occasionally curved pieces of shale. These may, no doubt, be regarded as portions of the upper muddy layer which cracked off and curled up during desiccation, as may often be observed on dried-up pools at the present time. Some pittings, occasionally seen on the sun-cracked surfaces, may perhaps represent rain-drops. Altogether, no evidence could more conclusively indicate a long-continued, tranquil deposit of fine sediment in shallow water, which frequently retired and left wide tracts of muddy shore to be dried and cracked by exposure to the sun.

Reserving the consideration of the palæontology of the upper flagstone group for a subsequent section, it may be stated here that organic remains abound in the strata exposed on the shore between Dunnet Bay and Reay. Fragments of fish and coprolites are scattered abundantly through most of the flagstones. Some of the calcareous shales are full of *Estheria,* while traces of plants occur in great numbers, though generally in a somewhat macerated condition.

In tracing the succession of beds westwards beyond Thurso, we encounter a thick series of strata in which the characters now described are still fully developed. They rise, however at Holburn Head into noble vertical walls from 100 to 250 feet in height, against the base of which the rapid tides of the Pentland Firth are ever chafing. In these precipices the thinly bedded nature of the flagstones, their clean-cut joints, and the tardy but not ineffectual influence of the weather upon the edges of the gently sloping strata, can be traced on headland and "goe" for many miles. The dip continues very steadily towards N.N.W. at angles ranging from 3° to 15° or even 20°, but probably averaging not more than 8° or 10°. A marked but strictly local curvature occurs at the Port of Brims. With this exception, there is hardly any interruption of the prevalent dip even so far as the extreme western end of the flagstone section. Though the trend of the coast nearly coincides with the strike of the beds, nevertheless, as already stated (p. 373), a sufficient divergence exists between them gradually to bring in higher parts of the group as we proceed westwards. Thus the strata of Sandside Head lie on a platform several hundred feet higher in the flagstones than those of Holburn Head. This is a point of considerable importance with reference to the overlap already spoken of. On the east side of Sandside Bay an admirable section exposing every bed may be traced from the outer reefs to the line of sandy beach at the mouth of the Isauld Burn.* Dark-grey and blue shaly flagstones and "calmy" or calcareous shales dip out to sea (N.N.W.) at 12°–15°. Harder bands, varying from less than an inch to a foot or more in thickness, are intercalated with them and project as ribs from their weathered surfaces. These are underlaid by about 70 feet of white and yellow sandstone, below which lie some calcareous shales, flagstones, and an impure limestone band. After a short space obscured by blown sand and other superficial accumulations, the section is resumed in the channel of the Isauld Burn by flaggy yellow pebbly sandstones, becoming more conglomeratic till they rest upon an uneven surface of dark granite full of black mica. Similar sandy conglomeratic strata with occasional bands of limestone continue to emerge from under the Isauld beds, with a dip to N.N.W.; but eastwards on the line of strike they pass into the flagstones, as already explained. (See map, fig. 4.) The thick yellow sandstone, with its underlying limestone band and shales seen in Sandside Bay, strikes westward and comes against the granite beyond Sandside house. This sandstone is quarried for building purposes, and, with its well-defined beds and occasional false-bedding, could hardly be distinguished from one of the ordinary Carboniferous sandstones. It is well exposed in the quarry and in the water-course to the north of Sandside house, where a deep goe also affords an excellent section of the overlying flagstones and shales. At the mouth of this inlet the shaly calcareous flagstones, with their nests of sand, concretions of bitumen, rippled and sun-cracked surfaces, present all the typical characters of the group; yet only a quarter of a mile inland they pass down into the sandy beds which rest upon the crystalline rocks.

Three quarters of a mile westwards from the Sandside Goe, the prevalent dip to N.N.W. having continued undisturbed through that interval on a bold rugged projection of the cliff-line, a portion of the uneven platform of crystalline rocks rises up through the later deposits. Bosses of gnarled gneiss and dark mica-schist, traversed with pink granite veins and with an easterly dip at high angles, appear on the shore-reefs and rise up to the summit of the cliff. The singularly unequal surface presented by these ancient rocks when the Old Red Sandstone began to be laid down on them is well shown by this part of the coast. A pink granitic breccia with intercalated seams of green sand-

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* Murchison, "Quart. Journ. Geol. Soc." vol. xv. p. 403.

stone and flagstone occurs at the base of the overlying strata, filling up hollows of the gneiss and allowing projecting parts of that rock to be wrapped round by the succeeding sandstones. This breccia is so hard and crystalline as to be in part scarcely distinguishable from a jointed granite. Some calcareous bands are here as usual associated with the base of the flagstones. A few small faults have the effect of here and there bringing the sandstones and the gneiss together along a vertical junction line.

Westwards from this granitic interruption a noble range of mural precipices, culminating in the Cnoc Geodh Stoir (314 feet high), displays a characteristic section of beds in the upper flagstone group. As the dip continues to be N.N.W. to N.W., while the coast trends nearly east and west, a gradually ascending succession of strata is passed over until, at the mouth of Bighouse Bay, the dip bends round to N.N.E. Since, however, the angle of inclination remains the same (15°), and there is not room for the reappearance of more than merely the upper part of the series between this point and the crystalline rocks referred to, it is evident that the beds at Bighouse Bay must be several hundred feet higher in the flagstone group than those resting on the gneiss to the west of Sandside, and therefore higher than the typical calcareous and bituminous fish-bearing shales and flagstones of Sandside Head; the total depth of flagstones exposed between Holburn Head and Bighouse being perhaps not much under 1000 feet. Yet at the head of Bighouse Bay green and grey sandstones and sandy flags, with a band of striped limestone, occur as if again the underlying crystalline rocks existed immediately below. The strike of these strata carries them obliquely across the bay, and on the western side we actually again encounter the granitic platform, covered by breccias and sandstones. It is on the shore to the north of Portskerry that these strata and their relations to the old rocks, so well described by SEDGWICK and MURCHISON [*] and by HAY CUNNINGHAM [†] are best examined. On the west side of the Portskerry harbour knobs of pink granite and gneiss are wrapped round and covered over by the sandstones, the unconformability being admirably displayed both on the low cliff and on ground plan upon the beach. The lowest strata, consisting of breccia, perhaps 20 to 30 feet thick but inconstant and sometimes absent, are red or pink in colour, owing mainly to the abundance of the included fragments of fine red granite. The angular detritus of this deposit has been obtained from the waste of the underlying and surrounding crystalline rocks. The breccia passes up into pale-yellow and greenish sandstones, through which scattered pieces of granite or other crystalline rock may be observed, especially towards the base and where the breccia has thinned away. In one of these pale sandstones Mr PEACH found

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[*] "Trans. Geol. Soc.," 2d ser. vol. iii. See also MURCHISON, "Quart. Journ. Geol. Soc." vol. xv. p. 403.

[†] "Trans. Highland Society," vol. vii.

an ichthyolite, which was regarded by Professor Huxley as resembling the *Glyptolœmus* of the Upper Old Red Sandstone of Dura Den.

There is, of course, no absolute proof that the Portskerry beds form a continuous portion of the flagstone group. It might be contended, especially in view of the resemblance of the fish to an Upper Old Red Sandstone form, that the pale sandstones last described belong really to the same part of the system as those displayed at Dunnet Head and Hoy. But, on the other hand, it should be observed that the sandstones perfectly resemble some of the yellow sandstones of Reay and Sandside, which have already been described as clearly interposed among the flagstones; that from the dip of the flagstones on the east side of Bighouse Bay they must, unless faulted, overlie the beds at Portskerry; and that their increasing sandy character towards their base plainly points to an approach to the crystalline platform. The sandy and brecciated strata at Portskerry are exactly what we should expect to meet with in the westward prolongation of the flagstones. It would never indeed occur to any observer examining the coast-line to separate them from the rest of the Old Red Sandstone around them.

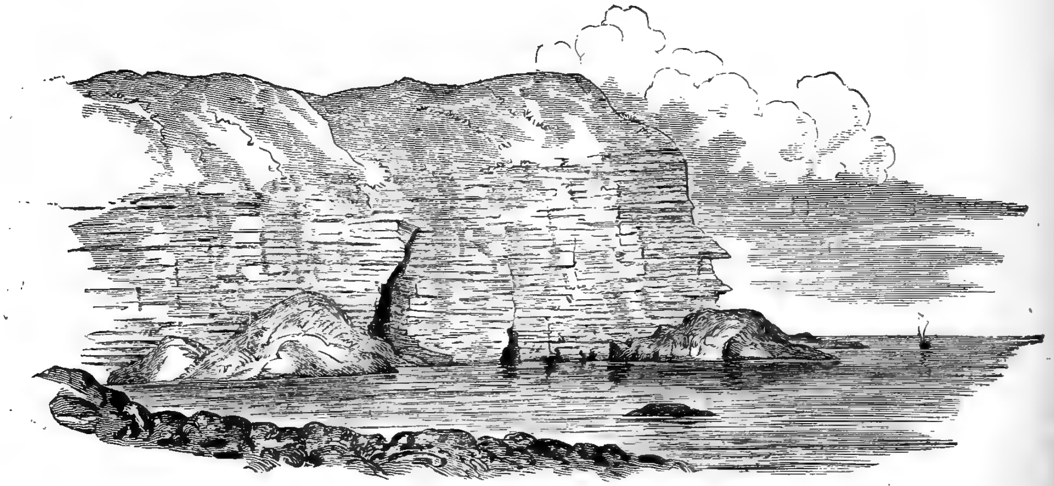

Fig. 5.—Unconformable Junction of Old Red Sandstone on Crystalline Rocks, Portskerry.

To the west of Portskerry, for an interval of more than a mile, the cliffs consist of granite and a gnarled granitic gneiss, deeply trenched by narrow sea, inlets or goes, and fringed here and there with skerries and stacks. These rocks run out into the promontory of Rudha na Cloiche. But nearly on the furthest point of that headland a patch of grey sandstone, only a few square yards in extent, lies perched, with a gentle seaward dip, to indicate the westward extension of the deposits we are now tracing, and to connect the area, which has been already described, with the last outlier of the flagstones. Half a mile to the west of the promontory the fine-grained pink granite passes under grey, green, and yellow sandstones, portions of which may be seen adhering to the sea-face of the underlying rock. There may be in all from 80 to 100 feet of these sandy strata at this locality. They are, however, banded with well-marked flagstones and calcareous shales, containing abundant and excellently preserved remains of *Dipterus*, *Osteolepis*, and *Coccosteus*. One of the most remarkable of these associated beds occurs near the base. It is a striped limestone, like that referred to as lying among the sandstones on the east side of Bighouse Bay. The occurrence of workable limestone in the Old Red Sandstone of the district was noted in the early memoir of SEDGWICK and MURCHISON.* The seam, three or four feet in thickness, lies imbedded among calcareous shales. It is dull blue to grey, or pale brown, on the fresh fractures, but weathers with a yellowish crust. It is finely striped by its laminæ of deposit, along many of which lie numerous minute oval calcareous grains, and some larger oval cavities filled with crystalline calcite. I observed no organic remains in it. But the white calcareous bodies often suggested to me the cyprid cases of other Palæozoic limestones, particularly that of Burdiehouse, to which the rock now referred to bears a further striking resemblance in its abundant fine pale and dark laminæ. Some bands of the limestone resemble cornstone, and have a strongly fœtid odour when broken.

The sandstone strata, with their interbedded calcareous flagstones, shales, and limestones, are well displayed on the cliffs at the mouth of the Baligill Burn, whence they strike inland, passing under the hamlet of Baligill, and ranging south-westwards by the village of Strathy. As shown by the shore cliffs, they pass up into flagstones of the usual type. These overlying strata form a noble range of precipices nearly 300 feet in height as far as Strathy Bay, where they seem to be thrown against the crystalline rocks by a fault flanking the east side of Strathy Point. In these cliffs the Caithness flagstones finally disappear in a westerly direction. Yet they retain unchanged to the last their normal and distinctive features—vertical and overhanging walls, defined by clean-cut joints, striped by the rapid alteration of harder and softer layers, projecting in huge quadrangular buttresses, trenched by dark perpendicular goes, and fretted into stack, skerry, and cave by the restless waves at their base.

Coast Section from Dunnet Head to Duncansbay Head.—The section now to be described, though inferior in the impressiveness and variety of its cliff-scenery, is unquestionably the most varied and important in Caithness as

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* *Op. cit.* p. 131.

regards the structure and history of the Old Red Sandstone. Unfortunately, owing partly to the obscurity arising from extensive accumulations of blown sand and of peat, partly to the existence of faults and frequent foldings of the rocks, it is not possible in the meantime positively to identify any of the beds now to be traced with those which have already been described. Judging from the evidence at present available, we may with some probability place the flagstones of Brough, which lie at the western end of the section, not far from the position of those of Castletown. As the dip is mostly towards E.N.E., there must be a repetition of the strata with a gradually ascending section towards the east. But in this case the line of coast cuts sharply across the strike of the beds, with the result of bringing us to much higher portions of the flagstones than can be seen to the west of Dunnet Head.

Between Dunnet Bay and Brough the great promontory of Dunnet Head projects into the Pentland Firth, with its long lines of precipice barred by the gently inclined stratification of their yellow and red sandstones. These strata, so different from everything around them, have no doubt been correctly assigned by MURCHISON to the Upper Old Red Sandstone. They will be again referred to in a subsequent part of this memoir. On the west side of the promontory no junction can be seen between these flat or very slightly sloping strata and the more highly inclined flagstones of Dunnet Bay. On the east side, however, the contact of the two series of rocks can be examined to great advantage in the Bay of Brough. Through the centre of that bay there runs a fault, by which the yellow sandstones on the west are thrown on end and contorted against the flagstones on the east side. (See fig. 6.)* Sir RODERICK MURCHISON believed that the Caithness flags pass up conformably into these yellow sandstones, and referred to the shore at Brough as one of the places where this passage could be made out. Apart, however, from the striking dislocation between the two formations, by which of course any chance of tracing a gradation between them must be

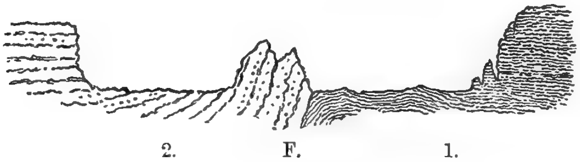

Fig. 6.—Section in Brough Bay. 1. Caithness Flagstones; 2. Upper Yellow Sandstones; F. Fault.

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* The existence and effect of this fault were noted by SEDGWICK and MURCHISON, *op. cit.* p. 133. In his later memoir, on the "Succession of the Older Rocks in the Northern Highlands," MURCHISON, as stated above, referred to the locality as one where the flagstones pass under the overlying yellow sandstones, and he actually gives a section showing the conformable superposition of the latter strata upon the former. ("Quart. Journ. Geol. Soc." xv. 409.)

destroyed, it will be clear upon reflection that the flagstones of Brough cannot possibly graduate into the yellow sandstone. Instead of lying at the top of the Thurso flagstone group, they must be somewhere near its bottom; separated, therefore, by more than 8000 feet of strata from the yellow sandstones, even if these reposed conformably upon the Caithness flagstone series. We have no reason to suppose the throw of the Brough fault to be very large; but even should it be considerable, it could not account for the position of the flat sandstones of Dunnet, against which the more highly inclined flagstones, on the south-west side of Dunnet Bay, are striking, and under which they no doubt pass. Though an unconformability cannot be proved here by any actual section, that is the only relation of the two formations which will explain the geological structure of the ground. It will be afterwards shown how the unconformability is actually demonstrated by clear sections on the opposite island of Hoy.

Starting, then, from the fault in the middle of Brough Bay, let us trace the succession of beds eastwards. At first the flagstones undulate slightly; they then assume an inclination to N.N.W., which soon veers round to W.N.W., the angle of dip rising rapidly from 15° to 40°. These strata exactly resemble parts of the section in Dunnet Bay. They continue in descending section as far as Ham, where an anticlinal axis occurs, beyond which the dip, with only occasional and trifling exceptions, remains easterly, or tending towards north-east. It is on this axis that the lowest strata along the south side of the Pentland Firth are brought up. From that point on the coast an ascending series can be traced through the upper flagstones into higher strata than are elsewhere in Caithness seen in conformable sequence. Much greater facilities exist for the detailed examination of the rocks along this northern shore than in other parts of the coast-line of the county; for, except at the cliffs of Brough, where the flagstones rise into mural precipices 100 feet in height, these strata form a sinuous line of low indented cliffs, easily accessible in many places, and fringed by shore ledges and reefs, where wide sheets of the strata can be walked upon.

At Ham grey flagstones of the usual type form the crown of the anticlinal arch, dipping to north-west and north-east in broad sheets, which slope into the water at angles of from 8° to 15°. Strata of the same character succeed in ascending order towards the east, the dip continuing north-easterly at angles ranging between 8° and 25°. Here and there small sharp folds occur, sometimes accompanied by faults. In one case the line of an arch of the flagstones coincides with the trend of a picturesque goe; in another, at the haven of Skarfskerry, the strata, bent into a sharp fold, with crumpling of the shales, run out as a reef into the sea. In the western promontory of Skarfskerry thicker bedded flagstones appear, and among these there occurs a band of lumpy dull-red shale, not unlike some of the shales or "marls" of the lower red parts of the Caithness Old Red Sandstone. A similar, perhaps the same, band occurs on the west side of the Ham anticline at Kerry Goe. Thin bedded flagstones and shales succeed and continue to occupy the shore as far as St John's Point. In no part of Caithness can the characteristic features of the upper flagstones be more conveniently studied than on this shore. The thinness of the layers, and their alternations of harder and softer texture, give rise to numerous parallel shore-reefs and ledges. A prevailing bright or cream-yellow crust, graduating into a greenish or even pinkish tint, is superinduced upon the strata by weathering, but when broken they are found to show the usual smoke-grey to brown tint. They consist of well-bedded fissile flagstones, often very shaly, and with partings of green shale. Many of them are so calcareous that, if found in the Scottish coal-fields, they would receive the name of "calmy" (pale argillaceous) limestones. The resemblance which, as already mentioned, they bear to some of the so-called fresh-water limestones and cement-stones at the base of the carboniferous system of the south of Scotland, cannot but strike any one who is familiar with the latter strata. This likeness includes not only the composition, colour, and mode of weathering, but even the minute wavy lamination indicative of intermittent but tranquil deposit. Other shaly layers are strongly pyritous. The bituminous impregnation already referred to is likewise well shown on this part of the coast. A shale, for example, near Barrogill Castle was found by Dr HOFMANN to contain 30 per cent. of volatile matter, and to yield on an average 7500 cubic centimetres of a very luminous gas from 100 grammes of the rock.* So large is the quantity of bituminous matter diffused through some portions of the group that it may be perceived, not only in specks and kernels, but even in black, soft, tar-like exudations from the joints.† Another characteristic feature among these flagstones is well displayed along the shores of the Pentland Firth—the occurrence of abundant sandy and calcareous concretions, varying in size from mere pea-like spherules up to masses five or six inches in diameter. These often roughen the sheets of flagstone with their projecting surfaces, and, where large in size, give a kind of rudely honeycombed aspect to the rock in which they occur. In some of the Orkney flagstones similar concretions occur. In Rowsay, for example, they weather out, leaving casts so exactly resembling footprints that specimens have actually been sent to me as the "footprints of men, women, children, and animals."

Remains of plants occur more or less abundantly diffused through the flagstones on this coast. They are particularly observable at Mey, where the thin flagstones have long been quarried. Scales, teeth, plates, and other fragments

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* "Quart. Journ. Geol. Soc." xv. 402.

† This was noticed by SEDGWICK and MURCHISON, "Geol. Trans." 2d ser. vol. iii. p. 134.

of fossil fishes are so constantly present as to form a striking feature of the strata.

At St John's Point layers of red sandstone and shale begin to appear in the flagstones, which likewise assume a reddish-grey colour and more sandy texture, until at a fault which runs through the creek called Scotland's Haven, they are succeeded by a higher set of red strata, forming the zone of the Gill's Bay sandstones.

Palæontological Distinctions of the Thurso Flagstone Group.—A considerably larger suite of fossils has been collected from this than from the Wick group, though the general facies remains so similar as to show that the whole vast pile of flagstones forms but one natural and continuous series. By far the most characteristic palæontological distinction of the Thurso group is the occurrence of fishes belonging to the ganoid sub-order *Acanthodidæ*. None of these have yet been found in the lower group. Three species of *Acanthodes* have been obtained by Mr PEACH from the flagstones which extend from Mey Hill to Strathy. One of these (*A. Peachii*), named after its discoverer by Sir PHILIP EGERTON, has a considerable range, since it occurs not only towards the base of the group among the Weydale and Stonegun flags, but at intervals up to the top, and even ascends into the highest set of beds in the flagstone series of Caithness. But it has not been obtained from lower horizons. Three species of *Cheiracanthus* occur at Holburn Head and among the flagstone quarries to the south-east of Thurso. *C. grandispinus* is rare, and only appears in the form of detached spines. *C. pulverulentus* is more frequent. Three species of *Diplacanthus* have been rarely found at the Hill of Forss, but none of them in any other Caithness locality. Of *D. crassispinus* and *D. longispinus* only spines occur. Again, fragments of the large plates of *Asterolepis* have long been known from the labours of ROBERT DICK, HUGH MILLER, JOHN MILLER, and C. W. PEACH to be eminently characteristic of the flagstones round Thurso. Mr PEACH has only found one piece of this fish from the lower or Wick flagstones at Noss Head. *Coccosteus cuspidatus* and *C. decipiens* have a wide range, since they occur as low down as the thick beds of Noss Head, and high in the thin shaly flags of Mey Hill; but *C. pusillus* occurs only in the Thurso group, wherein Mr PEACH first observed it in abundance on the shores of Thurso Bay. *Osteolepis macrolepidotus* is another fish with a great vertical range; it occurs in the thick flags to the south of Wick, and on many different platforms up to the Huna flagstones. It is specially abundant and beautifully preserved at Reay. *O. microlepidotus*, on the other hand, does not appear ever to descend into the Wick group. But in the Thurso group, and particularly in its lower members, this last-named fish is remarkably abundant. As many as a hundred individuals may be counted within a space of three or four square feet, mingled with remains of *Acanthodes* and *Cheiracanthus*. It is emphatically the fish of the flagstones of commerce, and Mr Peach informs me that none of the best flagstones are found below where it ceases to appear. *Glyptolepis elegans* occurs plentifully along the shore of the Pentland Firth. Mr Peach has never found this species in the Wick flagstone group except at Noss Head, and even there only a few scales have been observed. Of *Parexas incurvus*—a species first described by Agassiz from the Lower Old Red Sandstone of Balruddery*—spines have been detected by Mr Peach in two localities near Thurso, but they are extremely rare. They are imbedded in a light bluish-grey flagstone, each spine being surrounded with a dirty mass of organic *débris,* in which other fish fragments may be recognised, as if they had been vomited by, or had passed undigested through, some larger predaceous fish. *Estheria membranacea* (Pacht) crowds the surfaces of the shales and flagstones to the east of Thurso, its valves, sometimes still united, appearing as small black glossy membranous shell-like markings. Some of these Estheria shales remind one of the leperditia beds in the Calciferous Sandstone group near Edinburgh, or of the cyprid shales in parts of the Weald, and in the Isle of Wight Tertiary series.

Plant remains abound in many parts of the Thurso flagstone group. So far as yet known not a single true marine plant occurs there; all the traces hitherto noticed point to terrestrial vegetation. In general facies this flora recalls that described from the Old Red Sandstone of Gaspé by Dr Dawson, who has himself dwelt upon the similarity. Particularly frequent are flat stems completely carbonised, varying up to four or five inches in breadth, and sometimes as many feet long. As a rule they show no structure, though traces of a coniferous organisation have been detected among them. On the better preserved specimens may sometimes be observed a regular calamite-like fluting, but without the transverse joints of ordinary calamites. These resemble the "corduroy" stems so common at Lerwick in Shetland. Sigillaroid stems likewise occur in the plant-bearing flagstones of Mey. Several lepidodendroid forms may be seen in these strata, as well as at the Hill of Forss, and, indeed, generally throughout the northern flagstones up to the top of the whole series at John o' Groat's. *Lycopodites Milleri* (Salter) and *Lepidodendron nothum* (Unger), both of frequent occurrence, were described by Mr Salter from these strata. Dr Dawson recognised in Mr Peach's collection what appeared to be closely allied to, if not identical with, his Canadian species *Psilophyton princeps*. *Caulopteris Peachii* occurs in a stratum charged with plant remains below the flagstones at Stonegun and elsewhere.

7. *Gill's Bay Red Sandstone.*—The fault which occurs at Scotland Haven probably does not displace the rocks to any considerable extent, for, as already

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* "Vieux Grès Rouge," p. 120. His specific name was *recurvus*.

pointed out, intercalations of red strata, the precursors of the thick mass of Gill's Bay, are found on the west side of the dislocation regularly interstratified in the upper part of the flagstone group. We have here, therefore, in ascending succession a higher conformable series of red sandstones. In the memoir by SEDGWICK and MURCHISON the red sandstones of the north of Caithness are spoken of as one series, sometimes as "the newer sandstone," sometimes as "the upper red sandstone." Under these terms the sandstones of Dunnet Head, Gill's Bay, and John o' Groat's were included as portions of one great set of deposits resting conformably upon, and passing down into, the flagstones. It will now be shown, however, that there are in this part of Caithness three distinct zones of red sandstone. Two of these belong to the higher part of the flagstone series; while the third or Dunnet Head zone, which has with probability been assigned to the Upper Old Red Sandstone, as indicated in an earlier part of this essay, must lie unconformably upon the flagstones.

On the west side of Gill's Bay the shore exposes a series of ledges of red false-bedded sandstone, dipping gently eastward. Many of these strata greatly resemble parts of the Dunnet Head or Upper Old Red Sandstone series, so that it is not in the least surprising that they should have been classed together. My first impression was that they were identical. Further examination, however, showed not only that the base of the sandstones was interstratified with the flagstones, but that their top in like manner was interleaved with seams of flagstone, and passed under a thick overlying and conformable flagstone group. The latter relation can be satisfactorily established along the southern shore of Gill's Bay. Alternations of friable red sandstones with red and grey flagstones, after dipping eastward like the thick zone of sandstone underneath them, begin to undulate and then turn round so as to dip towards N.N.W. This change so nearly coincides with the trend of the coast-line that the reefs of hard red sandstone and flagstone (one of which is particularly prominent) run almost parallel with the beach as far as the Ness of Quoys, where a portion of the Gill's Bay sandstone is brought up along a sharp anticline. To the east of that promontory there is a steadily ascending section for nearly three miles, the strata being always visible in shore reefs, and also for much of the distance in a line of low sea-cliff. The same alternations of red sandstones and grey flags occur on the east side of the Ness of Quoys, and these are soon succeeded by a thick zone of grey flagstone, which extends along the shore below Huna.

Making allowance for the undulations of dip, and taking the limits of the Gill's Bay sandstone zone to be defined by the cessation of the red sandstone bands in the flagstones below and above, we may allow 400 feet for the thickness of this group. No fossils have yet been found in these sandstones, so far as I have been able to learn. The group is seen nowhere else in Caithness;

but it may be the same as some of the red freestone associated with the flagstone series in Orkney.

8. *Huna Flagstones.*—To the alternations of red sandstone and flagstone on the east side of the Ness of Quoys a group of flagstones succeeds, having characters similar to those already described in the Thurso Bay and Pentland Firth sections. The flagstones are thin-bedded, blue, grey, and yellowish, flaggy and shaly strata, often strongly calcareous. They dip a little to the north of east at from 8° to 15°, and their thickness may be estimated at 1000 feet. Some of the strata contain well preserved remains of fishes. Among the fossils we still meet with *Dipterus macrolepidotus* and *Osteolepis macrolepidotus.* The little *Coccosteus* (*C. pussillus*) and the Acanthodians, so characteristic of the Thurso flagstone group, occur likewise here. From the strike of the beds, and from similarity of dip and lithological features, this group appears to pass across the Pentland Firth to form Stroma, the most southerly of the Orkney islands, whence it no doubt extends northwards into other members of that archipelago.

9. *John o' Groat's Red Sandstones and Flagstones.*—The Huna flagstones occupy the shore as far as the east side of the Ness of Huna, where they are succeeded by another mass of false-bedded red sandstones, with intercalations of blue flagstones. The line of junction between the two groups is here, again, along a line of fault. But as the strata are so little disturbed by the dislocation, and the prevailing gentle dip towards E. 10° N. continues on both sides of it, the amount of displacement is probably trifling. The red sandstones which now appear much resemble those of Gill's Bay. They occur in successive thicker zones, between which lie many alternations of red sandstone, red and blue flagstones, grey shale, and impure limestone. These latter strata are quite undistinguishable from portions of the older flagstone groups. The highest part of the group consists of a thick mass of false-bedded red sandstone, without flagstone or shale. Fossils occur in some of the blue flagstones and impure limestones on the beach to the east of John o' Groat's House. Among these may be noticed *Acanthodes Peachii* (Eg.), *Dipterus sp. nov.*, *Pterichthys Dicki* (Peach)—the only known *Pterichthys* from the Old Red Sandstone of Caithness; *Tristichopterus alatus* (Eg.), another peculiar form; and *Holoptychius Sedgwickii* (M'Coy).* These highest members of the series are thus distinguishable by their ichthyic remains. Their plants, also, are perhaps more varied in kind than those of any other member of the Caithness flagstones. Mr Peach has obtained from them the same *Lepidodendron* (allied to

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* I cannot distinguish between this species and a *Glyptolepis.* On mentioning this to my friend Dr Traquair, he fully corroborated my suspicions, and said that he believed it to be only *G. leptopterus.*

L. Gaspianum) which occurs so frequently in the northern flagstones from Huna to Reay. Another is, according to DAWSON, "obviously of the same type as his *Cyclostigma densifolium;*" a third is "a *Cyclopteris* of the type of *C. Brownii,*" while a fourth is "a *Calamite* resembling *C. transitionis.*"* Other less determinable forms, including also a *Stigmaria,* occur. The general resemblance of this vegetation to that of pre-carboniferous lands in Canada is interesting in its bearings upon palæozoic geography.

The total thickness of the John o' Groat's group may be set down at about 2000 feet. Its component strata are seen in a continuous ascending section as far as the Ness of Duncansbay, where the thick zone of red sandstone forming the highest visible portion of the group bends into a trough, and rises again with a north-westerly dip, which increases in steepness until a fault on the south side of Duncansbay Head brings up a mass of grey flagstones to form that grim, steep-walled and deeply-cleft headland. To the south of Duncansbay Head a range of red sandstone cliffs, separated from the flagstones of the promontory by a well-exposed fault, range southwards for more than a mile and a half, until by another dislocation the grey flagstones are again brought up at Fast Goe. These Duncansbay red sandstones have much in common with those of Dunnet Head and Hoy, with which, in the absence of fossil evidence, I would provisionally place them. They will therefore be again referred to in connection with the development of the Upper Old Red Sandstone.

In the centre of the synclinal trough at the Ness of Duncansbay lie the only igneous rocks which (with the exception of occasional basalt dykes, presumably of Tertiary date) I have met with in the Old Red Sandstone of Caithness. Their interest is heightened by the fact that they include both crystalline and fragmental materials, and that they point distinctly to volcanic activity in this part of Scotland at some time subsequent to the consolidation of the John o' Groat's sandstones. The above rough eye-sketch shows the general

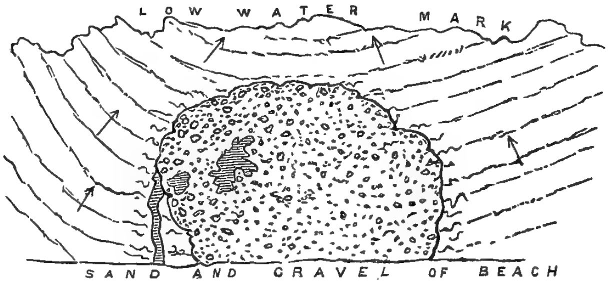

Fig. 7.—Plan of Volcanic Neck in Red Sandstones on Beach near John o' Groat's. The arrows show the dip of the sandstones. The dark vein and two patches in the neck are of diabase.

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* DAWSON on Fossil Plants of Devonian Rocks of Canada, "Memoirs of Geol. Surv." Canada, 1871.

arrangement of these rocks. It will be seen that they are exposed between tide-marks, and that they lie near the centre of the trough of sandstones. A dyke of diabase cuts through the sandstones, while immediately to the east lies a well-marked neck of volcanic agglomerate, about 300 feet in diameter. This rock consists of a dull greenish diabase paste, stuck full of blocks of diabase, red sandstone, flagstone, limestone, black cleavable augite,* gneiss, &c. Some diabase blocks measure three feet or more across. On the west side of the neck some irregular veins, or perhaps huge included amorphous masses of diabase and hardened agglomerate occur. The sandstone round the margin of the neck is perceptibly hardened and jointed.

It is impossible to regard without some interest this little volcanic pipe placed alone almost on the extreme north-eastern point of Scotland. When it was discovered by Mr B. N. Peach and myself, we knew of no other volcanic mass associated with the Old Red Sandstone on the north side of the Grampian mountains. Not a single dyke or bed had up to that time appeared to us among the wonderfully continuous coast sections of Caithness. We could not but suspect, however, that this ancient volcano of John o' Groat's might be one of a series, which, as we had failed to find any other of its members to the south, might hopefully be sought for among the Orkney Islands. Its precise geological date could not of course be fixed from the evidence of the neck itself. In lithological character, it quite resembles some of the agglomerates associated with the Old Red Sandstones and Lower Carboniferous rocks of central Scotland, and we were inclined to regard it as probably an outlier of some volcanic series associated with the Upper Old Red Sandstones of the north of Scotland. This conjecture was soon after verified by the discovery of a much more abundant and complete volcanic series at the base of the Upper Old Red Sandstone of the island of Hoy.

B. The Orkney Islands.

From the northern headland of Caithness the eye looks across the Pentland Firth to the long, low, featureless outline of Orkney, which seems to be rather a single detached area of the tableland of Caithness than a series of independent islands. To the westward, indeed, the low land rises into the rounded hills of Hoy, which descend in lofty lines of yellow cliff into the sea; but with this exception, neither as seen from a distance, nor, when actually visited, is there any rough or hilly ground to be met with in Orkney. The scenery of one island is almost entirely the counterpart of that of its neighbours. Each forms a little table-land, sometimes entirely girded by cliffs, but usually sloping

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* I supplied Dr Heddle with a piece of this remarkable augite for analysis. His determination of its composition and my description of its occurrence will be found in his paper published in this volume.

on one or more sides to a sandy beach. The cliffs are repetitions of those of Caithness, down even to the minutest details of form and colour. It is only on these sea-walls that any angular outlines, or, indeed, any considerable exposed faces of rock can be seen. Back from the edge of the precipice, the ground spreads out into a great plain, or a gently undulating surface, only varied now and then by some higher swell, or by the conical outline of a "Wart Hill." This monotony of external configuration affords a good indication of the sameness of the geology. Orkney is in fact simply a prolongation of the north of Caithness.

These northern islands have long been of geological interest, from the number and variety of ichthyolites which they have yielded. Probably the earliest geological examination of them was that of JAMESON (1800), who, with characteristic method and perseverance, traversed their rocks, but grew weary of their monotony. "It required," he says, "nearly six weeks to traverse the Orkney Islands, and that journey proved the most uninteresting I had ever made."* He makes no mention of organic remains, and does not attempt to connect the rocks with the formations of other regions. Professor TRAILL, of Edinburgh University, made a large collection of the fossil fishes from the flagstones. He supplied some of the species named and described by AGASSIZ, whose works called more special attention to this tract. In later years (1850) the Stromness flagstones became widely known from HUGH MILLER's chapters on "The Footprints of the Creator; or the Asterolepis of Stromness." Other collectors added to the number of species, some of which were described by Professor M'COY.† From the general similarity of organic remains, it had long been known that the Orkney Old Red Sandstone belonged to the same division of the system as the Caithness flagstones. The first attempt, however, to ascertain the structure of the islands, and to connect them stratigraphically with the mainland, appears to have been that of Sir R. MURCHISON, who visited Orkney in 1858, and passed rapidly through the islands.‡ His observations went to show that two divisions of the Old Red Sandstone occur in Orkney—the flagstones, and certain overlying yellow sandstones, which he assigned to the upper member of the system.§ He believed that the base of the whole flagstone series was to be seen at Stromness, where certain conglomerates rest upon a ridge of crystalline rocks, and that this series passes upwards, conformably into the yellow sandstones of Hoy. In both of these inferences he was mistaken, as will be pointed out further on.

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* "Mineralogy of the Scottish Isles," vol. ii. p. 252. See also P. NEILL's "Tour in Orkney and Shetland," 1806.

† "Synopsis of Classification of British Palæozoic Rocks," &c., 1855, p. 579, *et seq.*

‡ "Quart. Journ. Geol. Soc." xv. 410.

§ These subdivisions had been noticed before by Dr MALCOLMSON (1839, "Quart. Journ. Geol. Soc." vol. xv. p. 336) and by HUGH MILLER ("Footprints of the Creator," p. 2).

I have myself twice visited Orkney, accompanied on each occasion by my colleague in the Geological Survey, Mr B. N. PEACH, with whose co-operation all my observations were made. My object on the first visit was to ascertain whether the yellow sandstones of Hoy really passed down into the flagstones as had been repeatedly stated by Sir R. MURCHISON. For this purpose we surveyed with some care the northern end of the island. On the second visit we re-examined that locality and mapped the rest of the island. Having established an unconformable relation between the two formations, and having examined the various sections of a remarkable volcanic zone at the base of the yellow sandstones, we extended our explorations through the archipelago, crossing the Mainland in several directions, and then passing by steamer among the central and northern islands. We had not at our disposal the "six weeks," which JAMESON found necessary for his traverse of Orkney, and we did not attempt to work out in detail the structure and succession of the strata. Fortunately, however, the coast sections are so continuous and clear, that one who has familiarised his eye with the characters of the flagstones in Caithness has no difficulty, even at a distance, in recognising the stratification and other features of that remarkable series of strata. We saw enough to enable us to follow the general arrangement of the rocks in the islands. The following narrative is drawn up from our observations :—

1. *Geological Structure and Sections.*—Almost the whole of the Orkney islands consist of flagstones and sandstones of the Caithness flag series. The only other formations present appear in the south-western part of this group. A small ridge of the underlying crystalline rocks rises to the surface at Stromness. In the island of Hoy a group of conical mountains with their underlying volcanic platform represents the upper division of the Old Red Sandstone. Here and there a few basalt dykes—far outlying portions, no doubt, of the great Tertiary series of the west of Scotland—cut through the flagstones with a prevalent direction towards west or north-west.

Each island may be looked upon as a low undulating plateau, truncated on one or more sides by a line of sea-cliff ranging in height up to 200 feet or upwards. In many places these cliffs rise directly out of the water, so that no tidal margin of *débris* occurs by which their base may be examined on foot. Like those of Caithness they are split by innumerable vertical or highly inclined joints, and have been eroded into long, deep, and narrow gullies or "goes," often with great, square, detached columns of rock in front, and dim, resounding caves at the upper end. But where the coast shelves gently into the sea, the flagstones commonly appear upon the beach forming shore skerries, where each successive stratum may be followed. Hence there can hardly be any difficulty in making a tabular arrangement of the rocks in almost any one of the islands.

The flagstones retain the same features so well-marked in Caithness. Sometimes, as at Skaill in Pomona, they are exceedingly hard, fissile, bituminous, and crowded with fossil fish. In other places, as near Kirkwall, they form thicker beds, and can be quarried for building materials, like those which are similarly used at Thurso. Bands of dull red and even yellowish sandstone occur interstratified with the flagstones, as in Scapa Bay, Meal near Kirkwall, Eday, and other places. Where the flagstones rest on the old crystalline rocks they become for a short space conglomeratic at the base. A conglomerate, as I am informed by my friend Professor HEDDLE of St Andrews, occurs at Heglabir, on the west side of Sanday, upon a band of red sandstone which passes down into the flagstone series. Sir R. MURCHISON thought it probable that the light yellow and whitish sandstones of Eda and Shapinsha belong to the Upper Old Red Sandstone.* I did not land on these islands, but passing close to their shores, I saw nothing which did not seem to me a conformable part of the flagstone series. Dr HEDDLE, at my request, examined this question on the ground, and confirms my inference. Dark flagstones always rise from under these sandstone bands, which therefore cannot be identified with the great lower red sandstone masses of Caithness. Indeed there does not appear to be in Orkney any trace of these lower parts of the system. (See Plate XXII., column ii.)

2. *Position and Order of Succession among the Strata.*—As the flagstones of the Orkney Islands have supplied a considerable proportion of the fossil fishes of the Scottish Old Red Sandstone, it becomes of importance to define as nearly as possible their place in the system. Fortunately, from their position with reference to the clear sections in the adjacent Caithness tract, this can be satisfactorily done.

It has been already mentioned that the upper parts of the great Caithness series stretch across the Pentland Firth and reappear in the southern of the Orkney Islands. Stroma is formed merely of a prolongation of the Huna group of flagstones. In South Walls, Fara, Flota, and South Ronaldsha similar flagstones extend mile after mile along the picturesque cliffs of these flat-topped and otherwise featureless islands. Here and there, as in the north-west of South Ronaldsha, occur the zones of red sandstone above referred to, which may be parallelled with the John o' Groat's and Gill's Bay groups of Caithness. As we advance northwards among the islands the same petrographical characters continue. The dip of the strata

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* *Op. cit.* p. 410.

is perhaps on the whole towards W. or N.W. But it undulates usually in broad folds, though sometimes, as at the southern end of Eda, in sharper bends. But neither by the prevalence of a uniform dip, nor by any marked change in the lithology, is the traveller led to notice any great ascending series of strata. He seems to be for ever meeting with repetitions of the same rocks. No doubt when these islands come to be mapped in detail, the real thickness of flagstones will be found to be more considerable than might at first have been surmised.

Although no equivalents are met with of the massive red sandstone, shales, and conglomerate groups at the base of the Caithness series, yet on the south-west side of the Mainland, the axis of old crystalline rocks above mentioned rises through the flagstone. It begins on the coast at Inganess, runs S.S.E. to Stromness, and reappears in the middle of the island of Gremsa. In this axis granite, and a coarse gneiss passing into a foliated granite, and crossed by many veins and threads of pink felsite, come to the surface. Its north end is truncated by a small fault. But both on the east and west sides the overlying flagstone series may be seen resting unconformably upon and partly derived from these ancient rocks. A few yards of brecciated conglomerate and sandstone form the base of the series. There is but little here to recall the base of the Old Red Sandstone of Caithness. Indeed, in walking over the sections the observer is constantly led to realise that he has here but a mere local interruption of the flagstone series, due to the rise of an old ridge of rock from the surface of the sheet of water in which these strata were accumulated, and that the conglomerates are not the actual base of the formation. Passing southwards into Hoy and the islands lying to the south-east he finds that as the general average dip there is towards the north or the north-west, the strata overlying the crystalline ridge must really lie some considerable way up in the Orkney series. Here therefore is another evidence, in addition to those cited from Caithness, of the gradual subsidence of the very uneven surface of the Old Red Sandstone land beneath the waters of Lake Orcadie.*

In connection with the position of the Stromness and Gremsa conglomerate, it should be noticed that the Upper Old Red Sandstone of Hoy passes unconformably over a portion of the flagstone series not far above that conglomerate. A section drawn from the centre of Gremsa to the opposite hills of Hoy gives a thickness of probably not less than 2000 feet of flagstone between the con-

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* Sir R. MURCHISON ("Quart. Journ. Geol. Soc." vol. xv. p. 410), in describing the conglomerate of Stromness, refers it without question to the same basement position as the lowest conglomerates of Caithness. He likewise speaks of the red sandstone of Kirkwall as occupying a similar low horizon. Further examination, however, would have shown that these sandstones not merely underlie, but are interstratified with and overlie flagstones. The coarse conglomerate of well-rounded sandstone blocks at Heglabir on the west side of Sanday, which has been long known (see BARRY's "Orkney," p. 56, and NEILL's "Tour"), seems to occur at a greater distance from the local base, for it is said to overlie sandstones and flagstones. (See *ante*, p. 409.)

glomerate and the nearest part of the base of the Upper Old Red Sandstone (see fig. 8). This approach of the latter formation affords good ground for believing that we must here be very far removed from the true base of the flagstones. The deposits of Lake Orcadie have on the whole suffered little disturbance. At the time of the Upper Old Red Sandstone they must have been considerably less deranged than they are now. That formation therefore could hardly have been deposited on or near to the upturned and denuded base of the flagstones. The importance of fixing as definitely as possible the position of these strata in the south-west of the Mainland arises from the fact that this particular district is one of the chief localities for the fossils of the Orkney Islands.

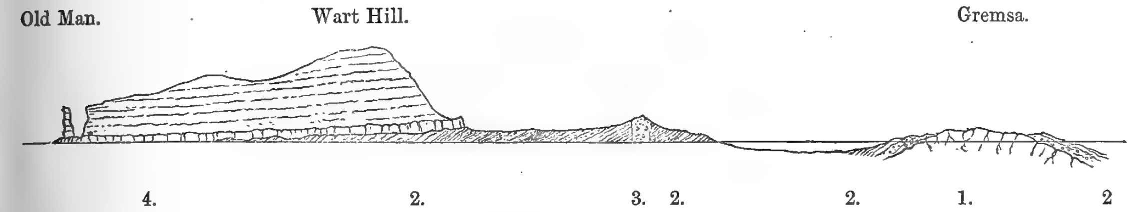

Fig. 8.—Section from Gremsa to Old Man of Hoy, Orkney. 1. Granitic and schistose rocks. 2. Flagstones with conglomerate base. 3. Volcanic "neck." 4. Lavas and tuffs forming base of Upper Old Red Sandstone, and lying unconformably on flagstones. 5. Yellow and red Upper Old Red Sandstones.

3. *Fossils.*—Many of the flagstones of Orkney are charged with organic remains. Especially is this the case with some of the dark, hard, fissile, bituminous bands. On the surfaces of these strata remains of the characteristic ichthyolites are crowded thickly together, and usually in such a tolerable state of preservation as to show that the fishes must have died where their remains are found, or at least that they could not have been subjected to any prolonged exposure and transport before they were buried under the accumulating sediment. As a rule, the fossils have been converted into a brittle jet-like substance, which is so liable to crack and scale off, that unless great precautions are taken, an organism, which at first showed external sculpture in great beauty, becomes eventually a mere black bituminous patch, retaining only the outline of the original specimen. It is not difficult, in most cases, to distinguish an Orkney from a Caithness ichthyolite.

The fossil plants of Orkney include most of the forms found in the Thurso and upper Caithness groups.

Crustacea are represented in the Orkney flagstones by two forms, according to present knowledge. The little *Estheria membranacea*, so abundant at Thurso, was first observed by Sir R. MURCHISON and Mr C. W. PEACH to be equally plentiful in some of the flagstones at Kirkwall,* thus corroborating by

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* "Quart. Journ. Geol. Soc." xv. p. 404.

fossil evidence the conclusion, otherwise reached, that the Orkney beds belong to the upper part of the Caithness series. Of the other crustacean, so far as I have been able to learn, only one specimen has been recognised. This is in the British Museum. With the view of obtaining its history and zoological relations, I applied to my friend Mr HENRY WOODWARD, who kindly supplied me with the following notes:—"The slab came from near Stromness [it exactly agrees with the well-known Skail flagstone so crowded with fossil fishes], and was purchased some years since of Mr J. R. GREGORY, together with specimens of *Pterichthys cancriformis, Cheirolepis Traillii, Cheiracanthus minor, C. pulverulentus, Diplacanthus crassispinus, Osteolepis brevis, Gyroptychius angustus,* &c. All these fishes are in the same dark, highly bituminous shale, and they are preserved as bright jet-black enamelled objects. I have never doubted the fossil on this slab being *Pterygotus;* but it is at the same time *very obscure* and fragmentary. It is part of a basal joint of one of the large ectognaths or swimming jaw-feet (maxillipeds), and has traces of the characteristic *squamæ* along the lower border. The fossil is much distorted in form."

The genus *Pterygotus* has always been regarded as specially characteristic of the higher part of the Upper Silurian and lower part of the Old Red Sandstone system. Its occurrence high up in the flagstone series of Orkney is significant of the true position of that series. Taken in conjunction with the other evidence already cited, it helps to indicate that the Old Red Sandstone of the north of Scotland was not so entirely posterior to that of Forfarshire as to deserve the name of a "middle" series, but rather that both may have been deposited during the life of the same crustacean forms.

The Orkney ichthyolites have been described chiefly by AGASSIZ, partly by M'COY. But they have never had the advantage of being collected by a resident, keen-eyed, enthusiastic, and able naturalist like Mr PEACH. No precise record appears to have been kept of the localities from which they have been obtained. In the fossil lists the word "Orkney" is usually the only indication of the source of the specimens. It is as yet impossible, therefore, to construct any table of ranges for the Orkney fishes. If, however, the Orkney flagstones are the northern extension of the upper rather than the lower groups of Caithness, we may expect their ichthyolites to bear out this correlation. The list of species given in Table II. shows it to be thus sustained. It may be surmised that the species at present peculiar to Orkney will probably in great measure prove to be identical with forms from Caithness and the Moray Firth. Very considerable differences of aspect arise from the peculiar conditions of fossilisation. An Osteolepis, for example, from the Orkney flagstones, with its pitch-black glossy surface, and its frequently indistinct preservation of scales and fins, presents a very distinct appearance from the dull black-grey hue and admirably defined form and sculpture of a Caithness specimen. Both of these, again, contrast singularly with a white and reddish enamelled ichthyolite, often lying on its back or jumbled asunder in the heart of a nodule from Lethen Bar, or with a grey bone-coloured specimen from Gamrie, showing, perhaps, only a few plates or bones, but all in an admirable state of keeping. It is only after a great many specimens exhibiting all these varieties of preservation has been examined that a palæontologist can secure himself against the risk of multiplying species. I quite anticipate, that as the able naturalists who named the ichthyolites from the north of Scotland had not the advantage of this extended experience, there will be considerable pruning of the fossil lists when they are revised by a competent authority.

It will be seen from Table II. (p. 452) that the great preponderance of the Orkney ichthyolites belong to genera, and in many cases even to species, characteristic of the higher groups of Caithness, and found also along the western and southern shores of the Moray Firth. The acanthodean fishes are specially noticeable, since they have not been observed among the Wick flagstones. *Cheiracanthus* and *Diplacanthus* are well represented. *Cheirolepis*, a Moray Firth form, likewise occurs. The wide-ranged *Coccosteus* is common in Orkney, likewise *Asterolepis Asmusii*, *Diplopterus* (three species), *Osteolepis* (the two abundant Caithness forms and *O. brevis* of M'Coy), and the long-enduring species of *Dipterus* (*D. macrolepidotus*). The forms of *Glyptolepis* so common on the south side of the Moray Firth, as well as in the higher Caithness groups, including also the so-called *Holoptychius Sedgwickii*, are found in Orkney. A further point of connection between the flagstones of these islands and the fish-bearing strata of the Moray Firth, is furnished by the occurrence in both districts of two species of *Pterichthys* (*P. cancriformis* and *P. Milleri*).

4. *Origin of some Features of Orkney Scenery.*—In quitting Orkney I would refer to the many admirable sections exposed along the mural sea-cliffs of that storm-swept group of islands, and to the endless instructive lessons furnished by them on stratification, jointing, and other elementary questions in physical geology, as well as on the progress of weathering, its relation to rock-structure, and the proportional share taken in it by the sea and the atmosphere. As a rule the flagstones crumble slowly; they form, indeed, a most durable material, and it is well for the Orcadians that their cliffs stand up so stoutly against the dash of rain and sea-spray. But like those of Caithness, they are traversed by many joints, and it is through these diagonal lines that they are mainly broken up. Frost is there a comparatively feeble agent. Nevertheless, with the co-operation of wind, rain, and waves, the lines of joint are slowly opened into narrow vertical rifts, extending from bottom to top of a sea-wall 150 or 200 feet high. These might not unnaturally be regarded as rents due to earthquake shocks. By the widening of these

fissures huge quadrangular buttresses, wedged off from the main cliff, rise in perilous independence above the surges of the northern sea. Where the dip of the rocks inclines seawards, and where, therefore, the strike-joints slope at high angles inland, the resultant cliff is often found to be a beetling, overhanging precipice. These features are well seen on the western headlands of the Mainland, particularly on the Brough of Birsa. On the east side of Stronsa also they reappear in the noble precipice of Odin Ness. In these and the examples cited from Caithness, we see how overhanging walls of rock owe their singular form, not to the undermining of their base by the waves, but to the progress of weathering along their joints. Even, therefore, in so exposed a coast as that of the Orkney Islands, it is not so much the breakers (though their force is enormous), as the less conspicuous action of atmospheric agents, which cuts slice after slice from the edge of the land.

C. The Shetland Islands.

Beyond the furthest extremity of Orkney, at a distance of about 45 miles, rises the bold promontory of Sumburgh Head, the most southern point of Shetland. Here again we encounter cliffs and skerries of the Old Red Sandstone, which so closely resemble those of Orkney and Caithness as at once to suggest that the whole form parts of one continuous area. By one who has spent some time among the latter localities, and has become familiar with the forms of cliff and goe, stack and cave, by which the flagstones are pierced, the outlines of Sumburgh Head are readily recognised as belonging to the rocks of the northern type of Old Red Sandstone; and he naturally would anticipate another succession of flat cliff-girt islets like that which he has left behind him in Orkney. But he soon discovers that here at last he seems to reach the limit of the formation. He finds but a narrow margin of it on the south-east, and a still more limited tract on the south-west side of the Shetland group; while the main mass of that group consists of crystalline rocks, which stretch northwards as if to connect themselves with those of Norway.

The Old Red Sandstone of Shetland has been described by different observers; but its geological position and its relation to the rest of the system in the north of Scotland have not been determined. Most of the accounts which have been given of it have been mineralogical.* In the beginning of 1853 some fossil plants from the sandstones of Lerwick in Shetland, sent to the

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* See Jameson, "Mineralogy of the Scottish Isles" (1800), vol. ii. p. 186; Traill in "Neill's Tour in Orkney and Shetland," 1806; Fleming, "Memoirs of Wernerian Society," vol. i. (1808), p. 162; and in Shireff's "Agriculture of the Shetland Islands" (1814), p. 120; Hibbert, "Description of the Shetland Islands" (1822), p. 157.

Geological Society of London by Mr TUFNELL, were described by Dr HOOKER as apparently belonging to two species of *Calamites*. At the same time Sir R. MURCHISON, while quoting the opinions of Dr J. FLEMING and Dr TRAILL, stated that the sandstones from which these vegetable remains came, should, in his opinion, be regarded as belonging to the Upper Old Red Sandstone, and as overlying the Caithness flagstones.* A few years later (1858), having meanwhile had an opportunity of personally visiting Shetland, he announced that the ichthyolites of the Caithness flags had not been found in Shetland, yet that strata of that age could now be shown to exist by the discovery at Lerwick of the characteristic *Estheria* of Thurso and Kirkwall; but he immediately adds that there can be little doubt that the plant-beds of Lerwick and the sandstone of Bressay Lighthouse pertain to the younger portion of the Old Red Sandstone.

1. *Geological Structure of Old Red Sandstone Area.*—In the summer of 1876 I visited Shetland with Mr B. N. PEACH, to determine if possible the connection between the Old Red Sandstone of that area and of Orkney and Caithness, and to ascertain whether there existed any evidence of contemporaneous volcanic action in the system as developed at the northern extremity of the British Islands. We found the south-eastern strip of sandstone to have been laid down by HIBBERT with approximate correctness, but to occupy on his map a larger space and to be more continuous than it is in reality. A few preliminary traverses showed us that a fundamental and hitherto unobserved feature in the structure of the Old Red Sandstone tract of the south-east of Shetland consisted in a fault, which by bringing down that formation against the older or crystalline rocks, prevents the actual base of the sandstone from being reached. This fault (or perhaps a succession of faults, having, however, the same general effect) begins at Rovey Head, a little to the north of Lerwick, and, after sweeping inland along the flanks of the schistose hills, turns shorewards again and reaches the coast on the south side of Gulber Wick. It must skirt the eastern shore a little way out to sea, for where the land next projects well beyond the main coast-line at Haly Ness it cuts across the head of the peninsula to Aith Voe. In the same way it traverses the Sand Lodge promontory to the bay at Hoswick. As it passes across the Lambhoga and Sumburgh Head region from Levenwick to Quendale Bay it dies out, and a lower series of flagstones and conglomerates is there seen to rest unconformably upon the metamorphic schists. All the islands lying to the east of this line—Bressay, Noss, and Mousa—consist of Old Red Sandstone.

The largest mass of the formation, and the best sections to show the general succession of the beds, are to be found in the northern portion of the area.

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* "Quart. Journ. Geol. Soc." vol. ix. p. 49.

From Gulber Wick to Lerwick a series of promontories and inlets with rocky sides reveals the characteristic features of the Old Red Sandstone of Shetland. But if we take the exposures at the north end of Bressay Sound and connect them with those from the north-western extremity of Bressay to the precipice of the Noss Head, we obtain a nearly continuous section across the strike of the strata, and see the thickest succession of beds in this region. A dislocation of the strata occurs near the north-west end of Bressay, by which perhaps some of the beds may be repeated. The total length of the line from Rovey Head to Noss Head is 6½ miles. On protracting a section from the measured angles of dip, taken while boating along the base of the cliffs, I find that if we make a liberal allowance for the effect of crumpling and dislocation, we shall still probably be within the truth in assigning a depth of about 5000 feet to what remains of the Old Red Sandstone of Shetland.

2. *Petrographical Characters and Order of Succession among the Strata.*—The lowest visible strata occur on the mainland at Rovey Head, and consist of coarse conglomerate beds placed on end against the slates and limestones by the fault. So large are the blocks in these beds that we may reasonably infer that in spite of the fault they cannot lie far from the base of the Old Red Sandstone of this district. The dislocation is probably not an extensive one, and at this part of its course seems to run nearly coincident with what was the actual margin of the conglomerate area. With the conglomerates are interstratified bands of dark flagstone and grey sandstone. The north-easterly strike of these lower beds carries them across the northern entrance of the Sound into the island of Bressay, the north-western promontories of which consist of reefs of very coarse conglomerate. As the strata ascend in the section they become less conglomeratic, though pebbles still occur in them, either scattered irregularly or gathered into layers and nests. On the Score Head the grey sandstone besides its pebble-bands contains seams of dull red shale. At first the sandstones are somewhat thick bedded, but after passing Cullensbro' Wick they assume the flaggy character which they retain through the rest of the section. The resemblance to the flags of the typical region of Caithness and Orkney increases as we round the precipices of Noss. When at last we reach the stupendous Head, on the eastern front of that island, rising as a sheer wall 577 feet above the surface of the deep water which frets its base,

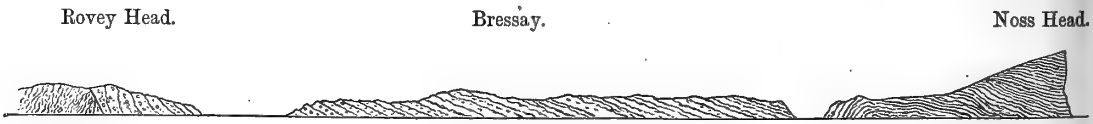

Fig. 9.—Section across the Old Red Sandstone of Shetland, from Rovey Head to Noss Head.

we cannot fail to be struck by the close lithological resemblance of the rocks to those of some of the Caithness cliffs. Grey and reddish flaggy sandstone, with bands of red shale and dark grey flagstone in rapid alternation, give the same striped aspect to the mural faces of rock. Many of the beds also are strongly calcareous, and weather with the same pale surface and worn edges. A further resemblance to parts of the Caithness cliffs is found in the occasional sudden rise in the angle of inclination, owing to a local plication or a fault, as at the north-west promontory of Noss.

The beds which form the southern part of Noss seem to be the highest parts of the Shetland Old Red Sandstone. They dip gently southwards; but must rise again with a contrary inclination under the sea, for on the opposite side of the Noss Sound they form a range of cliffs on the east wall of Bressay, where the north-easterly dip of the beds is well seen. At the south end of that island another noble range of precipices affords an admirable section of the middle part of the series. Grey, thick-bedded, and flaggy sandstones, with here and there a prominent red band, rise into the Bard of Bressay and the Ord Head. Huge quadrangular buttresses, isolated as the rock splits off along its joints, stand out from the face of the cliff, and are here and there detached from it or united to it at the top, so as to form such striking features as the Giant's Leg. Several sea-caves have been tunnelled into the cliffs. Of these the largest, known as the Orkneyman's Cave, shows how strongly calcareous the strata must be, since its walls and roof are crusted and ribbed with stalagmite.

At Sand Lodge grey flaggy sandstones, with bands of darker shaly flagstones, are brought down against the schist without the intervention of the basement conglomerates. Some of these strata, particularly the dark shaly layers, abound in plant remains. The opposite island of Mousa consists of similar strata with the same fossils. Some of its beds are so strongly calcareous as to pass into an impure argillaceous limestone. At the extreme southern point of Shetland the cliffs of Sumburgh Head show fine grey micaceous sandstones and flagstones, with bands of conglomerate, which pass down westwards into a coarse conglomerate like that which flanks the eastern side of the hills north to Rovey Head. On the whole, the strata of the Shetland Old Red Sandstone are more arenaceous than those of the typical Caithness flags, but not more so than some portions of that series, particularly the older half of the Wick flagstones, and the underlying sandy groups.

3. *Fossils.*—As in Caithness, so in Shetland organic remains are not uniformly distributed. They occur least frequently in the sandy strata, most commonly in those which are shaly and calcareous, consequently the Shetland beds will probably never prove markedly fossiliferous. Some of the strata are so exactly like those which would have yielded fossils in Orkney and Caithness that they

may be searched with considerable hope of success. I could not find, indeed, any trace of fish remains among them. Dr HEDDLE, however, informs me that he was shown some ichthyolites (*Coccosteus*, &c.) in Bressay, which he was assured had been found among the flagstones of that island. But in some of the flaggy beds near Lerwick the little *Estheria*, so common near Thurso and in different parts of Orkney, has been met with.* In the sandstone quarry at the south end of Lerwick many plants occur in the form of casts.† They consist of stems and roots. The stems, sometimes in fragments five or six feet long, are fluted longitudinally, like *Calamites*, to which genus they were referred by Dr HOOKER. As he pointed out, however, they have no articulations, but, on the contrary, show traces of projecting knobs, perhaps spirally arranged. I found the stems to be sometimes dichotomous, and to be furnished with massive divergent roots, like the knarled terminations of old dwarf Scotch firs. They suggested *Sigillaria* rather than *Calamites* as their probable analogue. They are known as the "Corduroy" plants of Shetland.

In the dark shaly beds at Sand Lodge many small forms of vegetation occur. They have evidently been much macerated previous to deposition. I procured some which looked like the ill-preserved raches of ferns, but too indefinite for identification. Further search, however, at that locality is desirable.

4. *Volcanic Rocks.*—On the west side of the Mainland of Shetland Dr FLEMING observed that red sandstone occurs on either side of Papa Sound, at the south end of St Magnus Bay, and he suggested that it belonged to the so-called "independent coal-formation" of the Wernerian school.‡ He showed that it was associated with certain amygdaloids and claystones. Since the appearance of his paper about seventy years ago, no one, so far as I know, has published any further account of these rocks. In Dr FLEMING'S description, which is written in the true Wernerian style, no reference is of course made to volcanic action, or to the intrusive character of any of the rocks. Yet he was so accurate and minute in his observations, that knowing the volcanic history of the Old Red Sandstone on the south side of the Grampians, I had long believed from his narrative that a somewhat similar history must await deciphering in the far north among the western islands and voes of Shetland. It was with considerable interest and pleasure that I visited that region in company with Mr B. N. PEACH, and found a fine series of natural sections, in which the existence of volcanic activity at the northern extremity of the British area during the Old Red Sandstone was admirably demonstrated.

The sandstone which occurs on the Mainland at Melby, between Norbie

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* MURCHISON, "Quart. Journ. Geol. Soc." xv. p. 413.

† See HOOKER, *Ibid.* ix. p. 49.

‡ "Mem. Wernerian Soc." vol. i.

Noup and Sand Ness, cut off from the red quartzite and altered sandstones to the south by a fault, resembles part of the Bressay series, but is redder in colour, and is invaded here and there by a pink or salmon-coloured porphyry. It is on the opposite island of Papa Stour, however, that the most instructive sections are to be seen. Exposed to the full sweep of the Atlantic storms, the coast-line of that island presents a picture of utter ruin,—of solid rocks stubbornly standing their ground, yet trenched, and splintered, and crumbling under the assaults of the ocean-battery; cut into caverns, and arches, and into isolated stacks and skerries, which, rising in advance of the present cliff, serve to mark how much it has receded. The fundamental rocks of the island consist of purple sandstones and flags, with bands of quartzose and felspathic conglomerate. These strata are seen on either side of the chief bay on the east side of the island. They have a south-westerly dip, but cannot be followed westwards in ascending section, owing to a large mass of pink porphyry which occupies the higher ground, and appears to have broken through and to overlie them. At the southern end of the island, however, similar flaggy sandstones, having a westerly dip at 20°, and therefore, it may be presumed, stratigraphically higher than those on the east side, appear on the shore. They are interstratified with several successive beds of a dull, dirty green, sometimes very amygdaloidal and slaggy diabase-porphyrite. These igneous rocks closely resemble some of the typical diabase-porphyrites of the Sidlaw, Ochil, and Pentland Hills, and may without hesitation be regarded as contemporaneous lava-streams in the Old Red Sandstone of this part of Scotland. Fragments of similar rocks abound in the conglomerates and sandstones below and above them, so that their truly coeval eruption is put

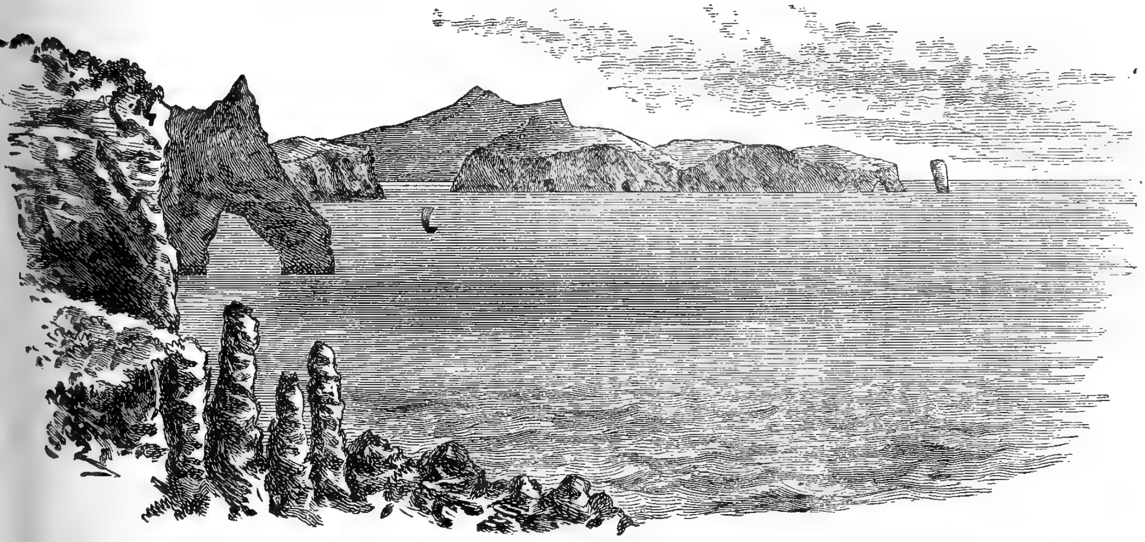

Fig. 10.—View on the West Side of Papa Stour. (Island of Foula in the distance.)

beyond doubt. Owing to the protrusion of a remarkable pink porphyritic mass, the bedded rocks can only be seen at intervals along the shore. On the north side of Hamna Voe, but still more strikingly on the singularly indented cliffs towards the north-western headland, the diabase-porphyrite appears, with an overlying mass of red and grey flaggy sandstones, the whole being cut through by the pink porphyry, and traversed by many small faults. Again, at the north end of the island, on the promontory to the east of the Bardie, a dark, dirty green slaggy and amygdaloidal diabase-porphyrite forms the base of the cliff, and passes under a series of sandstones, tuffs, and fine felspathic conglomerates. The detritus of which some of these strata have been formed, consists mainly of a pink felsitic rock, like that which covers so much of the island.*

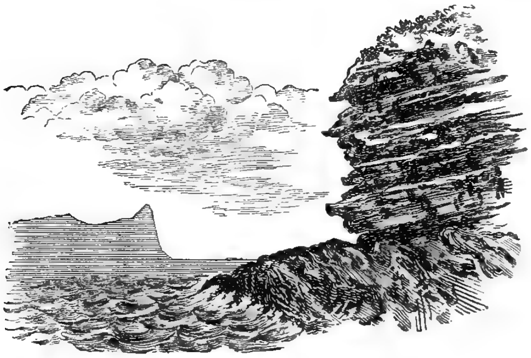

Fig. 11.—View on west side of Papa Stour. Sheets of diabasic lavas overlaid by sandstones. (Foula in the distance.)

Above the stratified beds and associated lava-streams lies an enormous mass of a very different character. This apparently intrusive rock consists of a compact pink or pale salmon-coloured "porphyry," or, in the nomenclature of Renard and De la Valleé Poussin, "porphyröid," generally decomposing with a dull, meagre surface, and then assuming the character of what was termed in the Wernerian nomenclature, a claystone. Here and there, as on the slope to the east of the Bardie, it assumes a curious concretionary structure. In several sections which I have examined under the microscope, the rock presents a remarkable absence of any crystalline structure, but on the contrary, shows many wavy lines and streaks, formed by layers of finer ferruginous felsitic matter,—an arrangement strongly resembling that possessed by many true tuffs. In some of the sections a spherulitic character appears, the concretions or spherules having an internal fibrous divergent structure. Many cavities occur more or less completely filled with secondary quartz. Occasional crystals of orthoclase and octohedra of magnetite may be detected; but as a rule the rock has been very much altered. It resembles some of the materials which, in the Pentland Hills and elsewhere, fill up volcanic orifices of the age of the Lower Old Red Sandstone; and in connection with these rocks will be referred to in a succeeding portion of this memoir. If it has had a similar origin we may be able to reconcile its tuff-like petrographical aspect with the fact that

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* Jameson ("Mineralogy of Scottish Isles," ii. 207) speaks of the "wacken" (felstone) lying upon a kind of breccia at the north end of the island, and traversed by veins of greenstone and basalt.

it presents in many sections on the west side of the island truly intrusive characters. Sometimes the mass is seen overlying the sandstone, then it may be observed creeping across the edges of the strata and cutting out portions of them until it comes to rest directly upon the underlying diabase-porphyrite.

At one point, a northern promontory called the Horn of Papa, an arched mass of this porphyry, projects into the sea, and is covered by nearly flat felspathic sandstones. I could not satisfy myself whether these strata were later than the igneous rock, and deposited upon its denuded surface, or belonged to the same series as the strata below, but now disjoined and borne up from them by the intrusive mass.

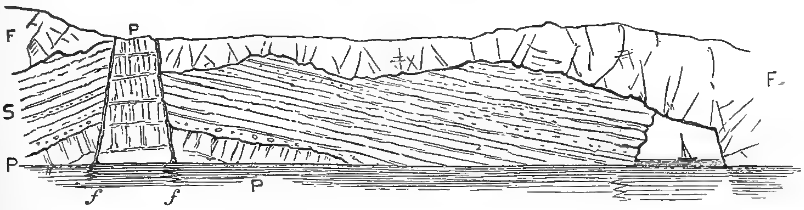

Fig. 12.—Section on north-west side of Papa Stour. P. Diabase sheets (=contemporaneous lavas). S. Sandstones and conglomerates. F. Pink and yellow porphyry or "porphyroïd." *f. f.* faults.

In Papa Stour, therefore, we have clear evidence of the eruption of slaggy diabasic lavas, the formation of sandstones and conglomerates, partly from the eroded surfaces of these lavas, and partly, perhaps, from fragmentary volcanic ejections, and the subsequent invasion of all these rocks by a curious tuff-like porphyry. That these rocks belong to the Old Red Sandstone system hardly admits of question. The sandstones and flagstones, where free from volcanic *débris*, are so closely repetitions of the Bressay beds on the east side of the Mainland, that we may regard them as belonging, not only to the Old Red Sandstone, but to the same part of the system as some portion of the sandstones and flagstones on the eastern coast.

The volcanic activity of that ancient period was not limited, however, to the eruptions at Papa Stour. On the northern side of St Magnus Bay, according to Dr Hibbert, masses of "claystone," "amygdaloid," sandstone, and conglomerate occur. From his description, I have little doubt that these rocks are a repetition of those of which I have just given an account. But I had not an opportunity of examining them. Dr Hibbert's narrative is strictly geognostical, and affords no clue to the geological structure of the ground. Dr Heddle informs me that some of the rocks are "trappean breccias or conglomerates." A specimen which he kindly sent to me shows under the microscope the same streaky character as occurs in the mass of Papa Stour. It contains scattered fragments, and is no doubt a tuff.*

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* After the visit to Shetland, of which the results are given above, my former pupil, Dr George

D. Basin of the Northern Firths.

Within the area included under this name, I place all the Old Red Sandstone lying on the mainland of Scotland to the north of the Grampian chain, except Caithness and the north of Sutherlandshire. It appears to have formed originally one great basin of deposit, or at least a connected series of minor basins. It presents strongly marked lithological distinctions from the Caithness and Orkney region, which may have been either a portion of the same basin, but so far from shore as to possess a very different kind of bottom, and to receive a strikingly distinct set of deposits, or an adjoining but more or less completely isolated area.

It will be seen from the map, that while from the coast of Banffshire at Buckie a continuous belt of Old Red Sandstone extends along the southern margin of the Moray Firth westwards to Inverness, this does not by any means represent the original limit of that system here, or include all the remaining tracts in which portions of the system still survive. Small outliers occur on the coast to the east of Buckie, while larger areas extend inland, and even penetrate far upward into the heart of the Highland mountains. That the description of the main mass may not be interrupted, but may be followed in order round the whole of the basin, it will be convenient to take the outliers first.

1. *The Outliers.*—Passing, then, across the great ridge of the Scottish Highlands, we first encounter at the mouths of the Don and Dee deposits which are no doubt referable to the Old Red Sandstone. They consist of a soft red and grey coarse conglomerate and red sandstone. The conglomerate, formed chiefly of rounded blocks of grey granite, with angular and sub-angular pieces of gneiss, schist, quartz, and other metamorphic rocks, imbedded in a loose ferruginous granite sand, is seen on the Don below the old bridge of Balgownie, dipping gently to E.S.E., and resting on the upturned edges of the gneissose rocks. Similar deposits have been met with in sinking wells and in other subterranean operations at Aberdeen.* As the outlier is bounded both towards the sea and inland by the crystalline masses, it seems to lie in a narrow strip, the mere end, perhaps, of a larger mass now concealed under the sea. It will be seen how

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A. Gibson, at my suggestion, undertook a further examination of the Old Red Sandstone of these islands, and prepared an essay on the subject, which he submitted to the Senate of the University of Edinburgh as his thesis previous to presenting himself for examination for the degree of Doctor in Science. Since the foregoing pages were written, the essay has been published, and I am glad of this opportunity of referring to it. He has traced the faults with care, and has extended his observations to Foula, where he finds red and grey sandstones similar to those of Shetland rising into the vast sea-precipice for which this island has long been famous. See his "Old Red Sandstone of Shetland." Edinburgh, 1877. [See note appended to the present Memoir.]

* See Hugh Miller's "Old Red Sandstone," 4th edit. p. 54, note.

frequently the conglomerates of the region now being described run inland in narrow valleys.

Far up the valley of the Don a much more important outlier extends northwards from this river through the parishes of Kildrummy, Auchindoir, and Rhynie, as a strip about nine miles long and a mile and a half broad. This tract was laid down upon MACCULLOCH'S map; fossil fish like those from Lethen, to be afterwards noticed, were said to have been obtained from it as far back as 1839, by the Rev. Dr Gordon of Birnie,* and in later years (1854) obscure remains of fluted but jointless plants were found there by the Rev. A. MACKAY.† With the co-operation of Mr B. N. PEACH, I traced the boundaries of this area in the summer of 1876, and found it to cast important light upon the physical geography of the north of Scotland at the time when the Old Red Sandstone was laid down. It extends as a narrow belt from the valley of the Bogie below the Muir of Rhynie southwards to a little beyond Kildrummie Castle, and thus crosses the watershed of Aberdeenshire. It is bounded on the west side by a fault, which, running along its whole extent, brings its highest strata against the granite, gneiss, serpentine, and other crystalline rocks of this part of the Highlands. The east side presents a more sinuous boundary, and though the actual junction between the sandstones and the underlying metamorphic masses is much obscured by drift, it can be distinctly seen at several places between Cottoun and Westhills, particularly on the hill between Cottoun and Druminnor House, where flaggy sandstones and conglomerates lie upon a denuded surface of a decomposing hornblendic rock of the metamorphic series. From the evidence presented by the ground to the south of Druminnor, it appears probable that the sandstone series was laid down against a somewhat steep and irregular surface, and sometimes at a considerable angle. The best sections are those in the water-courses on both sides of the valley between the Rhynie Quarries and Lumsden. From a comparison of these the following table has been constructed, representing the order of the strata and their approximate thicknesses. The total depth of Old Red Sandstone preserved at this locality probably measures between 1500 and 2000 feet.

6. Greenish grey shales, with beds of flaggy sandstone. Dryden.
5. Thick group of hard pale grey and reddish or purplish sandstones, with occasional pebble beds, and numerous pipes, "galls," and irregular veinings of red clay. Rhynie quarries, Burn of Craig, about 1000 feet.
4. Band of diabase-porphyrite, seen between Contlach and Auchindoir Manse.

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* MALCOLMSON, "Quart. Journ. Geol. Soc." xv. p. 350, footnote, where a brief reference to the order of succession of the strata is given.

† *Op. cit.* p. 432.

3. Very soft crumbling grey and red pebbly sandstones, and conglomerates of well-rounded pebbles, with bands of red shale, 300 or 400 feet, seen below Glenbogie, where the valley is cut out of this soft series.
2. Red shales, with calcareous red nodules, 40 or 50 feet; seen in small ravine to east of Glenbogie.
1. Band of red and yellow conglomerate and breccia, sometimes with calcareous cement. This lowest deposit immediately underlies the shales at the last-named locality, and rests on the crystalline rocks.

It will be afterwards seen that in several respects the Rhynie section affords points of comparison with those along the borders of the Moray Firth. The basement conglomerate (1) is a counterpart of that which occurs on the shore at Buckie, in the markedly calcareous character of parts of its matrix, and in the occurrence in it of cornstone. It is probably only a few feet thick to the east of Glenbogie, though it may swell out southward. The shale zone (2) closely resembles the shales of Fochabers, Tynet, and Gamrie, but is redder in colour. The calcareous nodules are flattened oval or circular concretions of hard, finely crystalline, grey limestone. So exactly do these present the characters of those so long known from the localities just mentioned, that they may be expected to yield an adequate number of the typical fossils.*

Immediately above the shales comes the series marked No. 3, which consists mainly of soft sandstones, with many well-rounded pebbles partly scattered irregularly through the strata, but more usually grouped in lenticular patches, layers, and nests. The curious interrupted arrangement of these layers, and of the sandy strata, with a dominant inclination in one direction, seems to point to deposit against a steep bank or shore. Bands of conglomerate, formed by the aggregation of similar well-rounded pebbles, occur in the group, one particularly striking bed lying nearly at the base, and close to the top of the shales. While the prevailing colour of the group is pale reddish-grey, every gradation of tint may be traced to deep blood-red. A considerable interval of obscured ground separates the highest strata of the third group, near Glenbogie, from the sandstones of Rhynie (5). But a little to the south-west, beyond the Burn of Craig, there occurs in that part of the series a remarkable bed of black, compact, finely-vesicular diabase, or diabase-porphyrite. It has been partially quarried for road material in a knoll which rises out of the surrounding drift. It very closely agrees in petrographical characters with many of the dark basic volcanic rocks associated with the Old Red Sandstone in the central valley of Scotland. Though I did not observe its immediate relations to its neighbouring strata, I had little doubt that it is a truly contemporaneous volcanic product,

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* It was from these nodules that Dr GORDON obtained the fossils already referred to. He informs me, however, that no record seems to have been kept of them, and that he cannot say what were the species.

erupted as a lava during the formation of the Old Red Sandstone of this valley. A peculiar interest attaches to it therefore; for if this be truly its history, it is the only interbedded volcanic mass yet discovered on the mainland of Scotland, to the north of the Grampian barrier.*

The Rhynie or Quarry Hill Sandstones have long been worked, and are well exposed in the quarries to the south of Muir of Rhynie. A good section has likewise been cut through the lower portion by the Burn of Craig. One of their most obvious features is the great number and variety of the clay-patches above mentioned. Many of them so closely resemble worm-burrows that, if found alone, they could hardly fail to be regarded as such. But they are associated with, and indeed pass into, others of the most irregular branching shapes, sometimes clustered in bunches, or dwindling into flattened leaf-like ribbons, which rather suggest a vegetable origin. The red clay of these concretions is sharply defined from the surrounding pale grey sandstone, and as they are singularly abundant in some of the beds, they give a very striking character to the rock. The general colour of the sandstones is pale grey, inclining now to a dirty green, now to a pale purple, with many dark-red and purple blotches, besides the pellets, pipes, and branching threads and stems of red clay. Some of the beds are well ripple-marked. Neither my companion nor I could observe any fossils, though the Rev. A. MACKAY found there in 1854 some organic remains, among which one was stated by Sir R. MURCHISON to be "unquestionably a fragment of a large stem of a plant, which measures 4 feet in length by 5 inches in breadth. It is nearly cylindrical, and is fluted irregularly near the pointed tip. No joints or nodes are visible, as in Calamites; but the surface is coarsely striated. The striæ or ribs are too obscure to warrant us in placing this fossil plant in the genus *Columnaria* of STERNBERG, which it most resembles." His description seems to point to the wide-spread "corduroy" stems of Shetland. Some of the other markings on these Rhynie sandstones were regarded by HUGH MILLER as the tracks of crustaceans. "An inspection of these very imperfect impressions conveyed to Mr SALTER the idea that they might have been made by the pectoral fins of fishes swimming in shallow water."†

The highest visible strata of the district occur on the west side of the Quarry Hill, and consist of green and grey sandy and calcareous shales. No fossils were observed in them, but they might be searched with some prospect of yielding plants and fish remains. They resemble some of the strata seen in the valley of Nairn, about Clava, like which they remind one of the so-called "calmy" shales in the Cement-Stone series of the Lower Carboniferous rocks of central Scotland.

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* Since this was written another volcanic locality has been found. See p. 435, note.

† MURCHISON, "Quart. Journ. Geol. Soc." xv. 432.

There need be little hesitation in placing the sandstones, shales, and conglomerates of Rhynie on the same horizon with those of Gamrie, Turriff, and the Spey. Their position in Strathbogie, so far south from the main mass of Old Red Sandstone, shows over how much wider an area that system of deposits once extended than it now covers, stretching, as in this case, far up one of the valleys among the uplands of Aberdeenshire.

But a still more remarkable proof of the prolongation of the Old Red Sandstone into ancient valleys or hollows, even in the heart of the Highlands, is furnished by another outlier which, as was shown many years ago by HAY CUNNINGHAM,* occupies a part of the valley of the Avon, near Tomintoul. Ascending Strathdon from the sandstone tract of Rhynie and Kildrummie, we find ourselves at last on the spurs of the Cairngorm Mountains, and, crossing the watershed, we look down upon the wild valley of the Avon, which rises at the base of the loftiest summits in the Grampian range. Granite, quartz-rock, gneiss, mica-schist, clay-slate, and other metamorphic rocks form these high grounds. It is not without surprise that following the Avon down its course, we suddenly come upon some deep ravines near Tomintoul, which the river and its tributaries have excavated in a coarse conglomerate. In this case, as at Rhynie, the deposit occurs as a long narrow strip, doubtless still representing approximately the position and trend of the ancient valley in which it was laid down. That former valley, perhaps occupied by the palæozoic predecessor of the modern Avon, ran in a north-north-easterly direction from the Avon to Glen Livet, a distance of rather more than seven miles, across the lines now trenched by the Conglass and Chabet Waters. The ground on either side still slopes up to heights of from 250 to nearly 800 feet above the platform of conglomerate. At the upper end, the conglomerate reaches a height of 1300 feet above the sea, but its northern or lower extremity is not more than about 800 feet.

At several places the unconformable junction of the Old Red Sandstone of this outlier upon the crystalline rocks of the Highlands can be seen. Thus on the Livet Water above Tomnavoulin a coarse conglomerate, with well-rounded blocks of metamorphic rocks, rests upon the reddened edges of the schists and quartz-rocks. But the best sections in the whole basin are those of the Avon and its tributary, the Water of Ailnack. Behind the village of Tomintoul the valley of the Avon contracts, and the river and its tributaries flow in magnificent gorges sometimes more than 200 feet deep. These picturesque features have been excavated out of a crumbling red, green and grey conglomerate of exceeding coarseness. There is perhaps no other conglomerate in the north of Scotland so coarse and so tumultuous in its arrangement. Blocks six feet long are not uncommon. The mass, however, is a conglomerate and not a breccia, for

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* "Trans. High. Soc." vol. vii.

the stones are mostly well water-worn. But with so little trace of stratification have these coarse materials been accumulated, that it is often only when we retire from the front of a cliff, and look at it as a whole, that we can make out a general gentle inclination down the valley. Large oblong stones may often indeed be found stuck on end in the conglomerate, or tilted up at a high angle. A marked structure in some parts of the conglomerate, particularly observable in the gorge of the small brook which joins the left bank of the Avon near its union with the Ailnack, is a kind of false-bedding of the stones. Between the gently inclined lines which mark the true dip, the stones of each bed of conglomerates are arranged in a rude stratification obliquely across the line of the valley at angles of 17° to 20°. They thus appear to dip towards the hills, while the true inclination of the conglomerate beds is away from them. The stones are all derived from the waste of the metamorphic rocks, quartz-rock being particularly abundant. At several points the conglomerate may be seen resting upon and wrapping round knobs and stacks of the reddened schists and slates, indicating on what a very uneven rugged surface it was deposited.

There can hardly be any doubt that this outlying portion of the Old Red Sandstone was formed either in a long valley connected with the main area of deposit to the north, or in a separate and isolated hollow. In neither case does it necessarily imply the denudation of the Old Red Sandstone from a wide extent of country. Undoubtedly it has suffered greatly in the general waste of land since palæozoic times, so that we can but dimly perceive what must have been the original physical geography of the district. Nevertheless, it is not an outlier like Morven or the Ben Griam hills, where mountains are capped with conglomerate, which must assuredly have had formerly a far wider extension. The worn and trenched slopes which rise on either side of the Tomintoul patch of conglomerate doubtless represent still the slopes of the hollow or valley, whether of lake or river, in which the coarse shingle of that deposit was accumulated.

I am not aware that any other outliers of the Old Red Sandstone occur in the interior, though a few patches of small size may yet be found in their original hollows along the northern slopes of the Grampian range. Turning therefore to the coast-line, we find that the continuous belt of this system, which skirts the Moray Firth from Loch Ness to Buckie beyond the mouth of the Spey, is prolonged eastward by a number of detached patches—the broken fringe of deposits which probably exist in a continuous sheet underneath the sea to the north.

Of these outliers the most easterly, and, at the same time, by far the most important, is that which stretches along the coast from Gamrie to Aberdour, and extends inland for upwards of 15 miles. This patch though long known to exist and to belong to the same series with the red sandstones and conglomerates of the north,* was first brought prominently into notice by MURCHISON, who announced that fossil fishes had been found in it.† Subsequently Mr PRESTWICH examined its structure as shown in the coast sections. He believed that at Gamrie he could trace a red sandstone passing down into the schistose rocks, and assuming it to be the equivalent of the Old Red Sandstone of England, he found it covered unconformably by the conglomerates and clays among which the fish-bearing nodules occur. He therefore concluded that the upper fossiliferous deposits "belonged to the Carboniferous series, and most probably to be the representative of the Millstone Grit or Mountain Limestone." ‡ Dr MALCOLMSON, in the course of the explorations already referred to, could find no passage of the red sandstone into the schistose rocks, nor any evidence of an unconformability in the Red Sandstone series. He recognised the Gamrie strata as both the lithological and palæontological equivalents of those which he had explored on the Spey and in Nairnshire, and he referred the whole without hesitation to the same Old Red Sandstone series, which included also the fish beds of Cromarty, Caithness, and Orkney.§

There is probably no section on any part of the Scottish coast-line where the relations of the Old Red Sandstone to the older rocks, the rapid increase and diminution of the conglomerates, the position of calcareous and shaly zones, and the abundant dislocations which the whole system has undergone, can be more clearly understood than in the six miles between Aberdour and the More Head of Gamrie. Mr PRESTWICH's clear descriptions refer only to the western end of this line of section, of which as a whole, so far as I am aware, no detailed account has yet been given.

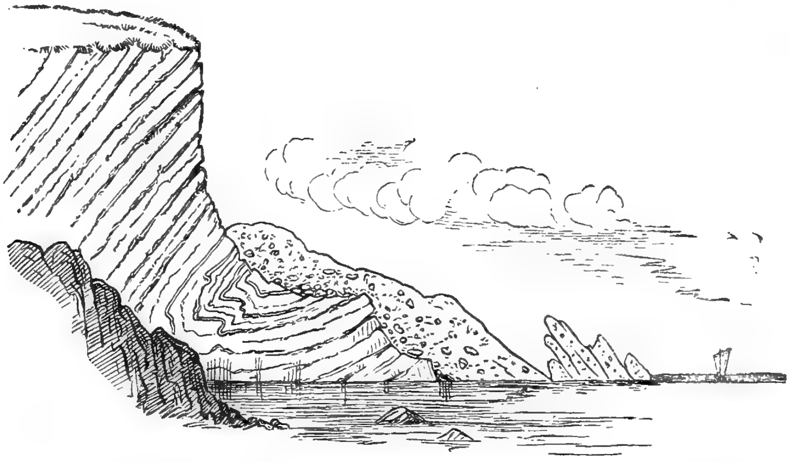

Fig. 13.—Junction of Old Red Sandstone and Metamorphic Rocks near Aberdour.

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* See Boué, "Essai," p. 101.

† "Trans. Geol. Soc." 2d series, vol. ii. p. 363.

‡ "Trans. Geol. Soc." (1835), 2d series, vol. v. 145.

§ "Quart. Journal Geol. Soc." xv. 349. Read in 1839.

At the eastern extremity the junction of the Old Red Sandstone with the crystalline rocks is seen to great advantage on the shore of Aberdour Bay beyond Dundarg Castle. The metamorphosed slates and greywackes, here and there twisted into sharp folds, form a steep rocky bank, against which the later red deposits have been formed (fig. 13). At the northern end of the junction a coarse brecciated conglomerate a few yards thick, and formed mainly from the underlying rock, constitutes the lowest bed, and wraps round projecting knobs of slate. A little to the south this basement conglomerate thins away, and the uneven denuded edges of the older rocks are then covered directly by red sandstones, consisting of a granitic sand with many rounded pebbles of granite, and abundant angular fragments of the surrounding slates. Sandstone of this character, but becoming finer in texture, continues along the bay to the west side of the promontory on which Dundarg Castle stands, where it passes under a series of dirty red and mottled shales and clays containing abundant calcareous nodules and lenticular beds of red, and less commonly yellow, cornstone. By means of a fault this shale zone is soon thrown out, and red sandstone appears on the beach, extending to the Dour Burn, which falls into the middle of the bay. This sandstone much resembles that which underlies the shale. In great part it is often rather a conglomerate than a sandstone, having a coarse granitic paste through which pebbles and especially angular chips of slate are scattered or crowded into nests and lenticular bands. As they so closely correspond to the conglomeratic sandstone below, and pass beneath a similar shale zone, it is possible that the two sandstones are the same, though a considerably greater thickness of the deposit occurs to the west of Dundarg than intervenes between the shale and the slates to the east. At the mouth of the Dour Burn the sandstone passes under a series of shales and shaly flagstones of dark brown, red, and purple colours, with abundant calcareous nodules in some bands. These beds, though soon thrown out by two faults, which bring up the old slates, agree so closely with the shale zone at Dundarg that they may with probability be regarded as the same. The coast-line for three-quarters of a mile is now occupied by slates, schists, and greywackes, until on the west side of Strathangles Point another fault occurs, by which the conglomerate is again brought in on the face of a tall cliff along a nearly vertical line against the slates. This rock, which, there can be little doubt, is the prolongation of the conglomeratic sandstone and conglomerate just mentioned, rises between that promontory and the fishing village of Pennan, into the noblest sea-cliffs in the north of the mainland of Scotland. Vertical walls and huge quadrangular buttresses tower above the waves to a height of from 400 to 500 feet. The horizontal or gently undulating strata, admirably exposed along these magnificent natural sections, are seen to consist of the same coarse angular detritus, with seams of finer sandstone. The mural character of the precipices is doubtless due, like that of those in Caithness and Orkney, to the joints along which the rock weathers, and by means of which huge slices are from time to time cut sharply away from their face. Apart from the joints, the conglomerate, as may be seen at the highest point of Pennan Head, is apt to weather into little *aiguilles*, like those so often assumed by crumbling cliffs of boulder clay. In descending upon the picturesquely placed houses of Pennan, we find that the conglomerate passes into a greenish breccia of slate fragments and dips eastward. The beds gradually incline to north-east and then to north-west, but continue to show a brecciated character, and to consist mainly of angular slaty *débris*, with here and there large rounded blocks of granite, felsite-porphyry, and other crystalline rocks. The Lion's Head is a fine projecting bluff of greenish brecciated conglomerate, pierced by a cave which enters from the west side, and issues on the middle of the slope on the east side as a huge chasm, called "Hell's Lum." The north-westerly gales which send the breakers against the western cliff carry the spray through the passage, and drive it in successive clouds out of the opening at the "Lum." The conglomerate ends off abruptly against a fault which cuts the coast due north from Troup House. Preserving to the last its impressive mural escarpment, it rises into a cliff 200 feet high, which, owing to the inward slope of the joints, even in some places overhangs its base.

By means of the last-named fault the Old Red Sandstone is once more thrown out, and the coast projects into the rugged Troup Head, consisting of the old slates and schists. On the west side of this interruption at the fishing hamlet of Crovie, the same fault again strikes the shore, and the Old Red Sandstone reappears. From this point to the next headland—the More Head of Gamrie—the bay is entirely occupied by that formation, which comes out in shore ledges and rises into red cliffs and crumbling slopes. The general dip of the beds inclines towards W.S.W., but a large fault, as was shown by Mr Prestwich, runs in a south-westerly direction behind Gardenstoun, and has the effect of bringing up again on the west side a considerable portion of the lower sandstones. Arranged in tabular form the Old Red Sandstone of Gamrie Bay appears to consist of the following members:—

7. Coarse brecciated conglomerate.
6. Grey and red clays and shales with calcareous nodules containing ichthyolites 20 to 25 feet.
5. Thick coarse red brecciated conglomerate, with occasional bands of red sandstone above Gardenstoun.
4. Bright red sandstone, seen to the west of Gardenstoun and to the south-west of Crovie.
3. Dull red and grey shaly flagstones, with lenticular grey calcareous seams and calcareous nodules.

2. Dull red pebbly sandstone.
1. Brecciated conglomerate.
   (Fault).
   Clay slates.

Owing to the large fault at Gardenstoun, and to one or two of minor extent on the shore, it is not possible to estimate satisfactorily the thickness of these various beds until the ground has been surveyed in detail. Their total depth probably does not fall much short of 1000 feet. Nos. 1 to 4 may be parallelled with the basement beds on the shore in Aberdour Bay. There can be little doubt that the coarse brecciated conglomerate (5 and 7) containing the ichthyolite-bearing clays (No. 6) is the same as that which forms so conspicuous a feature of the coast between Gamrie and Aberdour. I did not observe the ichthyolite beds along that coast, but Dr MALCOLMSON mentions the occurrence of shales with nodules like those of Gamrie, a little below the Manse (Mains) on the estate of Auchmedden.

The argillaceous zone containing the ichthyolites is exposed in two ravines on either side of the farm of Findon. It occurs likewise at several points further inland where water-courses have cut down through the overlying conglomerate, as on the road below Cushnie. In one of the upper bands of bluish-grey shaly clay, about three or four feet thick, nodules of grey impure fetid limestone occur. These nodules are flattened disks, averaging perhaps six or eight inches in diameter. The limestone of which they consist is distinctly stratified, as may be best seen upon weathered surfaces, and it breaks open along the lines of deposit, but more particularly along the middle, which is usually occupied by a flattened ichthyolite. Most of the nodules are crusted over with an outer layer of fibrous limestone, about an inch in thickness, the fibres being directed inwards towards the centre of the stone. This peculiar feature is one of the characteristics of the fish-bearing nodules along the whole of the southern borders of the Moray Firth. It reappears likewise at Cromarty. Though usually in detached pieces the fish are admirably preserved in these nodules. Their colour is a dull yellowish or brownish horn-like grey; none of them have the deep black of the Cromarty beds, nor the rich colours of those of Altyre and Tynet Burn.

The true horizon of these Gamrie beds is well brought out by a list of their ichthyolites. The following species were named by Agassiz as coming from this locality:—*Cheiracanthus Murchisoni, Diplacanthus longispinus, Cheirolepis ouragus, Diplopterus affinis, Glyptolepis elegans* (common), *Osteolepis major, O. arenatus, Pterichthys Milleri, P. oblongus.*

Though the Gamrie outlier extends so far inland, few good sections are to be seen after we leave the coast, and the ravines leading down to it. About two miles and a half to the south-east of Turriff, some quarries have been opened in a dull reddish or purplish-grey sandstone, which is not only full of pebbles, but contains layers and beds of coarse red conglomerate. The included stones consist of well-rounded pieces of granite, quartz-rock, and other crystalline rocks with angular flakes of schist. No fossils were observed there. Similar pebbly sandstone and conglomerate have been laid open in several other parts of the outlier, as near Dalgaty Castle, and on the road near Slap Farm.

To the west of Gamrie the coast-line for fourteen miles exposes sections of the slates, quartz-rocks, greywackes, serpentines, limestones, and intrusive rocks of the metamorphosed Highland series. On reaching Sandend Bay we again come upon traces of the Old Red Sandstone, meagre and fragmentary, but enough to indicate roughly the margin of the waters in which that formation was here laid down. On the east side of the bay below Redhaven a large detached stack of red brecciated conglomerate, very like the Gamrie deposit, stands on the upper edge of the beach, while ledges of the same rock extend along the shore to about the middle of the bay, and ascend for a short way up the Burn of Fordyce at Craig Mills. Those red strata rest upon and wrap round the truncated ends of the underlying quartz-rocks and limestones which are reddened at the junction. The breccia has a dull deep red colour, and its angular detritus consists of the mere shivers of the surrounding metamorphic masses, with so little trace of bedding that a face of the breccia might at first be taken for a section of boulder-clay. No trace of any organic remains has yet been met with in this outlier.

In the next large indentation of the coast-line about four miles further west, red breccia is again met with, forming the picturesque stacks in Cullen Bay, known as the Kings of Cullen. They rise from the platform of the lowest raised beach, and their bases are wrapped round with folds of blown sand. The same rock forms a cliff or bank behind them, and extends westward, passing under bright brick red sandstone with breccia bands, through which the underlying white quartz-rock rises to the surface. The sandstone and breccia may be traced as far as Portknockie, where they fill a hollow in the quartz-rock; but beyond this point the underlying metamorphic rocks occupy the shore. This outlier like that of Sandend Bay, is interesting in the evidence it furnishes as to the position of the shore-line during a part of the Old Red Sandstone period, and the nature of the *dêbris* which gathered there. We have, as before, a course breccia derived from the decomposition of the rocks immediately below and around. The angular quartz-rock fragments are imbedded in a dull red sandy ferruginous paste, and the bedding of the mass is indicated by the included lenticular seams and nests of red sandstone, which dip north-westwards at 15° to 20°. This inclination carries the breccia below the sandstone just referred to. Yet so uneven must have been the bottom on which these deposits accumulated, that the underlying quartz-rock rises up into the red sandstone, and has the ends of its highly inclined beds wrapped round by it. No bands of shale and no trace of calcareous nodules was observed at this locality, nor have any fossils yet been obtained here.

2. *The South Coast of Lake Orcadie from Buckie to the Spey.*—These Cullen patches form the last outlying portion of the Old Red Sandstone of the Moray Firth. About five miles of a rough rocky coast, where the metamorphic rocks run out in sharp ledges into the sea, now intervene, and on the west side of this interval we finally enter upon the main area of the formation. Evidence of the great waste of the conglomerates and sandstones of this region is afforded, as we approach that main area, by the red boulder-clay scars, which are full of fragments from these deposits. On the east side of Buckie harbour the red rocks occur *in situ.* Thence they may be traced, as they were originally by Dr MALCOLMSON, up the valley of the Spey and through the vale of Rothes; westwards by the valley of the Lossie, round the base of the Pluscardine and Forres hills into the gorges of the Findhorn as far as Sluie, and by Lethen Bar and the base of the hill of Rait into the valley of the Nairn, up which they extend along the northern declivities of the Highlands, into the hollow of the Great Glen. Throughout this wide region the rocks were shown by Dr MALCOLMSON to preserve the distinctive lithological characters of their subdivisions, and to maintain likewise a palæontological uniformity. By means of his only too brief researches, it was made for the first time possible to compare the development of the Old Red Sandstone on the opposite shores of the Moray Firth. His descriptions of localities are so minute and accurate that reference may be made to them for details, which thus need not be inserted in the present memoir.

Dr MALCOLMSON, as already stated, arranged the Old Red Sandstone of the borders of the Moray Firth in three divisions. His lower division, consisting of—(*a*) conglomerates; (*b*) red sandstone and shales, and (*c*) argillo-calcareous sandstones and conglomerates, and containing an ichthyic fauna of the Caithness type, is that with which we have at present to deal. He believed that these strata were surmounted conformably by what he called the central or cornstone division, containing *Bothriolepis, Holoptychius,* &c. There can be now no doubt that the central division in which these fossils occur, and which he rightly identified with the deposit of Clashbennie, represents the Upper Old Red Sandstone. I shall adduce evidence to show that, in spite of the want of good sections, that division must really lie here, as elsewhere, unconformably on older parts of the Old Red system. It occurs as a strip, with detached outliers, and extends from the flanks of Finlay Seat, past Elgin and Forres, ascending the Findhorn to Sluie, and then curving round the northern slopes of the Lethen hills, until it crosses the lower reaches of the Nairn, beyond which it skirts the shore for a few miles, until lost under raised beaches and blown sand. Thus the area of Upper Old Red Sandstone, by coming directly against the crystalline rocks to the south of Elgin and in the Findhorn, separates the lower or Caithness flag division of the system into at least four tracts. Of these one extends from Buckie up the valley of the Spey, between the slopes of the Rafford hills and the tongue of Upper Old Red Sandstone, which protrudes up the valley of the Findhorn. A second smaller tract of the lower division appears at Altyre; while on the west side of that tongue a third area rises into the heights of Lethen Bar and Clune, and extends up the valley of the Muckle Burn. To the west of the hills of Rait and Urchany the lower division is found interposed between the upper sandstones and the crystalline rocks, and gradually swelling out westwards, until on the Drummossie Muir it fills the whole space between the base of the hills and the sea.

It is only when this structure of the ground is made out, that we can understand the differences in the character of the strata which lie next to the gneiss and granite, even in sections at short distances from each other. We must also bear in mind the irregular indented character of the ancient coast, as well as the uneven and, indeed, rugged slopes on which the conglomerates and sandstones were deposited. Thick masses of conglomerate are apt to die out rapidly, and to appear on different horizons, so that we cannot always safely identify two conglomerate bands, even though they both rest upon a common platform of the crystalline rocks, and may not be more than two or three miles apart.

Beginning, then, at the eastern extremity of the Old Red Sandstone belt of Moray and Banff, we find at Buckie an admirable shore section, showing the unconformable position of this system on the crystalline rocks below. The quartz-rocks and quartzose flags, with partings of garnetiferous mica-schist, which occupy the shore between this point and where we left the conglomerate at Portknockie, here once more pass under a coarse red breccia, with a calcareous matrix. But unlike the massive deposit of Gamrie and Aberdour, the breccia is only 20 or 30 feet thick, and is immediately succeeded on the west side of the harbour by thin purplish-red and grey flagstones, with bands of coarse, somewhat brecciated conglomerate. In these strata Dr MALCOLMSON found fish plates and scales, as at the Tynet section, to be immediately noticed. At the west end of the village of Buckie additional proof is afforded of the uneven denuded surface on which the Old Red Sandstone was laid down. A small knob of the underlying strata projects from among the sandstones, of which the lowest bed is a very calcareous breccia, in part a cornstone. Owing to the low angle of dip, and the coincidence between the strike of the strata and the trend of the coast-line, only a very limited thickness of beds can be seen between Buckie and the mouth of the Tynet Burn. The coast, in fact, is fringed with a mere strip of nearly flat Old Red Sandstone, only about a quarter of a mile broad. At Portgordon the thin red flagstones pass under a sandy beach, which, with two wonderfully well-preserved raised beaches, runs westwards along the shore towards the Spey. In the Burn of Tynet, however, a good and tolerably continuous section is afforded, one portion of which has long been known from the beautiful ichthyolites originally obtained from it by Dr MALCOLMSON.

The total thickness of strata exposed in that section probably falls short of 400 feet. The upper portion, perhaps 150 feet thick, consists of soft red sandstones, with occasional bands of conglomerate, and of greyish-purple clay with calcareous nodules. These beds are underlaid by a conglomerate, which, though finer in its upper part, passes downward into a coarse mass, with well-rounded pebbles. As several faults traverse this part of the section, the true thickness of this conglomerate can only be inferred. Probably it is not less than 130 feet. On the south side of one of these faults the conglomerate is replaced by purplish-grey, red, and greenish shaly sandstone, and sandy shale with calcareous nodules, which occur chiefly in the grey shales or clays, but also in the red bands. The lower or more shaly part of this zone is about 50 or 60 feet thick. It is here that the fish-bearing nodules chiefly occur. Beneath the shales fine conglomerate crops out, and continues up the stream, until the section is obscured towards Tynet Bridge by overlying drifts. But the crystalline rocks appear a short way further up.*

It is evident from the facts now stated, that here again, as at Gamrie, the ichthyolite beds occur in the heart of a thick mass of conglomerate. The

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* Since my description of the Old Red Sandstone of these northern regions was written, the work of the Geological Survey has commenced in that area. Mr J. G. WILSON, who has been intrusted with the mapping of the district above referred to, has made the interesting discovery of a bed of diabase-porphyrite, interstratified in the lower part of the section of sandstone and conglomerate in the Gollochy Burn, near Buckie. This is a true lava-flow; he has observed pebbles of the rock in some of the overlying strata. With the exception of the Rhynie diabase already referred to, it is the only example yet noticed of the occurrence of contemporaneous volcanic rocks in the Lower Old Red Sandstone on the north side of the Grampian mountains, until we reach the far distant Shetland Islands. I have examined it microscopically, and find it to be identical in character with some of the lavas of the Lower Old Red Sandstone of central Scotland. It has a characteristic porphyry ground-mass through which are scattered decayed plagioclase crystals and numerous opaque ferruginous pseudomorphs, many of which appear to represent former augite. The characters of the volcanic rocks of the Old Red Sandstone will be described in a subsequent portion of this Memoir. [Since this Memoir was read I have had an opportunity of examining the locality where this volcanic sheet occurs, and of confirming the view taken of its relations by Mr WILSON. If we may judge from the different petrographical aspects of the mass, it would seem to consist of more than one flow, but with no intercalated tuff or other strata.]

nodules in which the fossils chiefly occur are oval or oblong in shape, according to the form of the organism which they enclose. They consist of a dull grey compact, somewhat argillaceous limestone, and show lines of deposit corresponding in plane to the stratification of the shales among which they lie. They are sometimes crusted with the same fibrous divergent calcite layer, as at Gamrie. The fishes are admirably preserved; indeed, they here surpass those from any other nodule-bearing locality in the region of the Moray Firth. They are not strictly confined to the nodules, though far more abundant and perfect there; they may be found occasionally even in some of the shaly and sandy layers in the overlying conglomerate.*

Three miles to the south-west of the Tynet Burn a section has been cut by the Spey on its left bank between Fochabers Bridge and Dipple, exposing strata similar to those last described. Soft dull red pebbly, sometimes rain-pitted, sandstones, shales, and clays, with bands of fine conglomerate, dip gently towards N.N.W., resting upon a very coarse conglomerate seen at the bottom of the bank, and passing under a higher conglomerate and red pebbly sandstones.† The red shaly bands contain red calcareous nodules in which many ichthyolites have been obtained. Hence we observe that the same intercalation of the ichthyolitic nodules in a conglomeratic deposit continues to be the rule.

The species of fossil-fishes obtained from Tynet and Dipple include the following:—*Acanthodes pusillus* (Ag.); *Cheiracanthus microlepidotus* (Ag.); *C. Murchisoni; Diplacanthus striatus* (Ag.); *D. striatulus; Glyptolepis leptopterus* (Ag.); *Osteolepis major* (Ag.).

From underneath these shaly strata there rises along the right bank of the Spey a great mass of conglomerate, presenting many points of resemblance to that which underlies the fish-beds of Gamrie. It is a red brecciated rock, so loosely compacted, sub-angular, and rudely stratified, that it can hardly be distinguished from the red boulder-clay which caps it. A further resemblance to the drift deposit is afforded by the peculiar style of weathering. Every little runnel has cut for itself out of the conglomerate a deep ravine, from the sides of which project picturesque groups of buttresses and pillars, each with its capping of boulder-clay. Those singular features remind one of the well-known earth pillars in some of the valleys of the Tyrol. The conglomerate extends up the valley as far as Boat of Bridge, beyond which it is succeeded by the metamorphic rocks. Though its angle of inclination is low (1° to 5°), it probably reaches a depth of at least 500 or 600 feet. This thick mass, however, must be of very local occurrence. It disappears northward, or dwindles down into a

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* MALCOLMSON, *op. cit.* p. 346.

† See MALCOLMSON, *op. cit.* p. 345.

thickness of a few feet. It seems to have been accumulated in a great bay, like the conglomerate of Cawdor to be immediately referred to.

3. *From the Spey to the Nairn. The two divisions of Old Red Sandstone in Morayshire.*—Between the conglomeratic series which I have described as stretching in an interrupted course from Gamrie to the Spey, and the conglomerates which lie immediately to the west, and are interstratified with bright red and yellow or grey Holoptychius-bearing sandstones, there is often too little lithological difference to allow of their being at once and by the eye distinguished from each other. When we consider that they have both been formed from similar materials, apparently under very similar conditions, and that the later conglomerates indeed have probably been more or less derived from the waste of the older masses, we need not be surprised that there should be difficulty in drawing a line of boundary between them. Such a line cannot be altogether satisfactorily traced until the ground is carefully mapped out in the minutest detail.

In the meantime, however, we shall probably not err in classing with the great brecciated conglomerate of the Spey, as a part of the Caithness flag series, the similar rock which partially fills the ancient hollow of the vale of Rothes. It is a loose, incoherent, rudely stratified deposit like that of the Spey ravines, but with many well-rounded stones scattered through the sub-angular and angular detritus of quartz-rock, gneiss, schist, and other metamorphic rocks. These underlying masses appear here and there on both sides of the valley, which is thus of as early a date as the time of the Old Red Sandstone. After forming a marked feature on the east side of the valley, the conglomerate is succeeded below Coleburn's Mill by the pebbly red, green, and grey sandstones, and green and red conglomerates of the well-known Scat Craig.

In his memoir on the sandstones of Morayshire* (1858), Sir R. MURCHISON, while describing the sections to the south of Elgin, which had been so well worked out by the Rev. Dr GORDON, of Birnie, speaks of the Scat Craig conglomerate as affording geological evidence of a transition from the Caithness flags into the Upper Old Red Sandstone, there being here, he affirms, "a union in this one mass of conglomerate of genera which in other places mark the central and upper members of the series." He remarks that "the interest which specially attaches to the fossil fishes found at Scat Craig, is that whilst the *Pterichthys major* and other species of that genus, as well as the *Asterolepis*, are common to Caithness and the west of Moray (Lethen Bar, Clune, Altyre, &c.), there are other forms such as the *Dendrodus latus* and *D. strigatus*, *Lamnodus*, and *Cricodus*, which as well as the *Asterolepis Asmusii*, are common in the Old

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* "Quart. Journ. Geol. Soc." xv. p. 425.

Red of Russia, where I have myself detected them."* If the fossils from Scat Craig really proved what MURCHISON here contends for, they would invest that locality with no ordinary importance; for from no other place in Scotland can palæontological evidence be adduced to show a passage from the fauna of the Caithness flagstones into that of the Upper Old Red Sandstone. But a critical examination of the supposed proofs of a union of the two ichthyic types, leads to a conclusion directly opposite to that of my old chief. So far as I have been able to discover, not a single true Caithness flag fish has ever been found at Scat Craig. The *Pterichthys major* is common in the Upper Old Red Sandstone of Nairn and the Findhorn, but does not occur in Caithness. The teeth so common at Scat Craig (*Dendrodus lamnodus*, &c.) are likewise absent in Caithness. *Holoptychius* and *Bothriolepis* are typical Upper Old Red Sandstone forms. The *Asterolepis*, however, is a characteristic genus of the Caithness flags; if it could be established as a Scat Craig fossil also, a connection between the conglomerate of that locality and the true Caithness flags might be established. But I cannot learn that it has ever been found there. The *A. Asmusii*, *A. minor*, and *A. Malcolmsoni*, are given by AGASSIZ as from the "neighbourhood of Elgin"; but though he was acquainted with the remains from Scat Craig, he does not give that locality as one of the sources of *Asterolepis*. The "neighbourhood of Elgin" is rather a vague description, and may include the Altyre beds on the one side and those of the Spey on the other. There can be no doubt, also, that some of the early identifications of the species, and even genera of fossil fishes from that region, were erroneous. *Cephalaspis*, for instance, was named as one of the fossils of the upper sandstones of Elgin.† I am disposed to believe that no true *Asterolepis* has ever been found either in the Scat Craig beds or in any of the upper conglomerates and sandstones of Elgin.

Before returning from this digression, I may remark that the ichthyolites at Scat Craig are imbedded in a ferruginous conglomerate, and are usually more or less water-worn. Though there can be no doubt that these fishes, as a whole, really lived at or immediately before the time when the gravel was deposited in which their rolled bones, spines, scales, and teeth, have been preserved, it

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* *Op. cit.* p. 425.

† MALCOLMSON, "Quart. Journ. Geol. Soc." xv. p. 344. [Since this paper was read I have had an opportunity of examining the collection of Old Red Sandstone fishes in the Museum of Practical Geology, Jermyn Street. In the "Catalogue of Fossils," published in 1865, three specimens of *Asterolepis* are marked as occurring, two of them in the Upper Old Red Sandstone of the Moray Firth, and one in that of the Heads of Ayr. Being convinced that no *Asterolepis* was likely to have been obtained from those localities, I was gratified to find on inspection that one of the specimens was a fine plate of *Pterichthys major*, and that the others were *Holoptychius* scales—fossils eminently characteristic of the Upper Old Red Sandstone. My colleague Mr Etheridge at once acknowledged that the fossils had been erroneously entered in the catalogue. No doubt much of this confusion may be traced to the fault of the original synonymy. The *Asterolepis* of Eichwald and Pander is the *Pterichthys* of Agassiz; the *Asterolepis* of the latter naturalist is equivalent to the *Homosteus* of Asmus and Pander.]

seems not impossible that fragments of some of the larger fishes long previously entombed in the nodules or flagstones of the older part of the system might have been reached during the denudation of these strata, and brought into the gravel banks of Scat Craig.

But as I have already indicated, no passage from the Scat Craig beds downwards into strata, containing unquestionable Caithness flag fishes, has ever been made out, while on the contrary these beds pass upwards into undoubted Upper Old Red Sandstone, from which neither palæontologically nor stratigraphically does there appear to be the slightest reason to separate them. Let me now, however, endeavour to show that, as I have mentioned in connection with the areas respectively occupied by the upper conglomerates and sandstones and the Caithness flag series in Morayshire, these two members of the Old Red Sandstone lie unconformably upon each other.

To the south of Elgin, as the observations of the Rev. Dr GORDON* and the section of Sir R. MURCHISON† clearly show, the crystalline schists are immediately overlaid by fossiliferous conglomerates (Scat Craig, &c.), which pass up into the yellow sandstones, conglomerates, and cornstones, of Elgin, containing *Holoptychius* and other Upper Old Red Sandstone fishes. There is no trace of the older ichthyolitic beds of the Spey, though to judge from the dip which these had when last seen at Dipple, they might have been expected to spread over the plain of Moray, and to flank the base of the hills. They are entirely overlapped by the younger series which runs up the valley of the Lossie, and curving round the flanks of the Monaughty hill, where the Highland rocks project into the plain, ranges westwards into the valley of the Findhorn. If we may judge indeed from scattered patches of conglomerate and sandstone, these rocks must at one time have extended considerably further up some of the hollows than they do now. In the ravines of the Findhorn, a magnificent succession of cliffs has been cut through the upper yellow and red sandstones and conglomerates down to the gneissose rocks on which they directly lie. Here again we find no trace of the presence of any intermediate zone between these two formations. Immediately to the east, however, in the Burn of Altyre, flaggy sandstones occur from which Lady GORDON CUMMING obtained characteristic Caithness flag fishes.‡ It would appear, therefore, that on the east side of the old bay in which the Findhorn sandstones were deposited, a portion of the underlying series intervenes between these strata and the gneiss.

Passing to the west of the Findhorn we obtain still further evidence to show an unconformability. Though on that river at Sluie the upper series lies directly upon the gneissose rocks, immediately to the west a great thickness of strata

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* "Edin. New. Phil. Journ." new series x. p. 29, *et seq.*

† "Quart. Journ. Geol. Soc." xv. p. 424.

‡ See MURCHISON, "Quart. Journ. Geol. Soc." xv. p. 423.

comes in between them. These interposed beds form the high grounds of Lethen Bar and Broadshaw, and are well exposed along the course of the Muckle or Lethen Burn. They were carefully observed by Dr MALCOLMSON, who found them to consist of a thick loose conglomerate, passing up into a series of shaly sandstones, shales, and clays, containing calcareous ichthyolitic nodules and remains of plants. Lithologically and palæontologically, these strata present a great contrast to those of the neighbouring Findhorn. We recognise in their dark grey colour, their flaggy and shaly character, their enclosed nodules with the familiar fibrous crust, the same type of deposits as those of Dipple, Tynet, and Gamrie. The fossil evidence entirely bears out this identification. From Clune and Lethen Bar many fine specimens were originally obtained by Lady GORDON CUMMING and Dr MALCOLMSON. The following species have been noted:—

Pterichthys latus (Ag.); *P. Milleri* (Ag.); *P. productus* (Ag.); *P. cornutus* (Ag.); *P. oblongus* (Ag.); *Coccosteus oblongus* (Ag.); *C. maximus* (Ag.); *Cheiracanthus microlepidotus* (Ag.); *Diplacanthus striatulus* (Ag.); *D. longispinus* (Ag.); *Cheirolepis Cummingiæ* (Ag.); *C. curtus* (M'Coy); *Osteolepis major* (Ag.); *Diplopterus macrocephalus* (Ag.); *Glyptolepis leptopterus* (Ag.); *G. microlepidotus* (Ag.).

It will be observed that no *Holoptychius, Bothriolepis,* or other species found in the Findhorn section occurs in this list, while, on the other hand, not one of the above fossils is to be met with on the Findhorn. But the Findhorn strata extend westward into the lower reaches of the Muckle Burn, where with their northerly dip, they overlie the Lethen sandstones and nodule-bearing clays, as Dr MALCOLMSON first pointed out. They have not entirely filled up the great bay in the Highland hills between Forres and Nairn, or at least have been considerably denuded there. Hence the mass of the lower or Caithness series, several hundred feet thick, which rises into the Lethen hills between Rait and the Findhorn.

Owing, however, to the long promontory of gneiss and schist, which runs through the hill of Urchany to the granite of Newton of Park, near Auldearn, the lower division of the Old Red Sandstone is again reduced to narrow limits, or disappears altogether as the upper series sweeps westwards by Nairn. But it reappears immediately on the west side of that ridge, and continues to flank the hills by Cawdor and Kilravock, until it swells out in the Drummossie Muir to the dimensions already referred to.

From these facts it will be seen that two distinct margins or coast-lines are presented to us by the Old Red Sandstone of the south side of the Moray Firth. There is first the coast-line of Lake Orcadie, marked by the unconformable junction of the older or Caithness flag division upon the uneven surface and sinuous border of the crystalline rocks of the Highlands. Next comes the edge of the upper division which winds to and fro with little regard to the earlier shore-line, sometimes leaning directly against the Highland slopes, sometimes running up into bays worn out of them, sometimes retiring and exposing a considerable breadth of the older sandstones and conglomerates with their earlier shore. Some of these phenomena did not escape the observant eye of Dr MALCOLMSON, who believed that "great denudations were in progress during the whole period of the deposition of the Old Red Sandstone, by which different superior members of the system were placed in contact with the inferior rocks,"* and who specially points out the rapid disappearance or attenuation of the massive conglomerate as inconsistent with the notion that this formation merely thins off or appears in a degraded form. I have not found, however, any evidence of such extensive denudation of the Old Red Sandstone of Morayshire, except at the great interval between the lower and upper divisions of the system. The rapid diminution of the great conglomerate of Cawdor, which Dr MALCOLMSON cites as proof of these changes, is almost certainly due to the circumstances under which the deposit was accumulated and not to subsequent denudation, as will be pointed out further on.†

4. *From the Nairn to Inverness.*—For the present, therefore, I shall omit further description of the upper or *Holoptychius* sandstones of Moray and Nairn, seeing that neither stratigraphically nor palæontologically can they be classed with the true Caithness flags. This latter series runs, as I have said, in a gradually widening strip along the base of the Highland hills to the south-west of Nairn. In that part of its course it presents many of the lithological peculiarities which mark its occurrence in the country to the eastward. A band of grey clay and shale, full of calcareous nodules containing plates and scales of *Coccosteus* and other characteristic ichthyolites, forms a recognisable horizon, and was traced for many miles up the Nairn valley by MALCOLMSON. The remarkable changes which occur especially in the thickness of the basement conglomerates cannot be better displayed than in the four vertical sections which my colleague in the Geological Survey, Mr JOHN HORNE, has been good enough to prepare for me. (See Plate XXII., columns v., vi., vii. and viii.). They are arranged to show the variations along the strike of the beds as we proceed from south-west to north-east. In section viii. the strata consist almost entirely of sandstones and sandy shales. About 180 feet above the base lies the thin zone of grey shales and clays, with calcareous nodules and seams, which occurs at Knockloan and is well-known for its fossil fishes. Even at a glance one can see the close similarity of these concretions to those of the Banffshire sections; and on closer examination this similarity is

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* *Op. cit.* p. 338.

† [Since this was in type Mr HORNE has extended the work of the Geological Survey into the Findhorn district, and has traced the unconformable overlap of the Upper Old Red Sandstone.]

found to include even the thin fibrous crust of calcite so characteristic at Gamrie and elsewhere. Though the actual junction of the Old Red Sandstone with the Highland rocks is here obscured, the two formations approach so nearly as to leave room for no more than a very thin band of conglomerate. Only two miles to the west, however, as appears by section vii., a mass of coarse conglomerate, about 400 feet thick intervenes between the position of the Knockloan fish-bed and the base of the series. Spreading out over a space about two miles in breadth, this conglomerate forms the material out of which the gorge above Cawdor Castle has been excavated—a narrow picturesque gully often more than 130 feet deep, with tortuous convex and concave sides, which here and there approach within a few yards of each other at the top. Despite its rent-like aspect, it is a true case of brook-erosion, as is shown by the numerous segments of old pot-holes at many levels far above the present reach of the highest flood. The stones of the conglomerate are mostly water-worn, though more angular shapes appear among them. They consist of fragments of the gneiss, quartz-rock, granite, and other crystalline rocks of the district, the larger blocks, one foot or more in length, being often well-rounded. The stratification is shown partly by the position of the flatter stones and partly by the intercalation of occasional thin lenticular bands of dull purplish-red sandstone, the dip being so gentle towards the valley of the Nairn that it nearly coincides with the slope of the ground. Mr HORNE infers, I think with much probability, that this thick mass of conglomerate accumulated in a bay of the old coast-line of the lake. He finds that while it disappears so rapidly towards the north-east, it quickly diminishes also in thickness in a south-westerly direction. About three miles to the south-west it is 250 feet thick. Near its base at Galcantray, a thin lenticular band of cornstone occurs, in which fish-scales and bones may be detected. The lowest strata consist of a breccia sometimes highly calcareous, like those of Banffshire, sometimes made up entirely of broken *débris* of a pink quartz-porphyry, on which it rests. Grey shales and clays about 30 feet thick, with calcareous ichthyolitic nodules, no doubt the same band as that of Knockloan, crop out near Cantray, about 250 feet above this lower cornstone band. Westward at Clava, apparently on the same horizon as these nodule-bearing clays, there occurs in the channel of the river Nairn a series of hard grey-blue and reddish calcareous flagstones, shales, and impure limestone, which, since Dr MALCOLMSON's early observations, have been known to yield fish remains. I was struck by the resemblance of these strata to some parts of the true flagstone groups in Caithness. From the Nairn valley at this locality, northward across the Culloden or Drummossie Muir, to the low country skirting the Moray Firth, occasional exposures of the strata are to be seen. Putting the available evidence together Mr HORNE has compiled the vertical column v. This is only a provisional computation. It shows, however, what cannot be observed to the east, that the fish-bearing nodules, shales, and clays are overlaid by a thick series of reddish and grey flagstones and shales. These beds are well exposed in the stream above the farm of Morayston. They at once reminded me of portions of the Caithness flagstones about Berriedale. In some of the strata fish remains occur abundantly; Mr HORNE and another Geological Survey colleague, Mr LINN, have recently obtained some fine large bucklers of *Asterolepis Asmusii*—a form which, so far as I am aware, has not been previously observed in this region, if indeed it has ever been actually found on the south side of the Moray Firth.

Much higher up the valley of the Nairn the nodular fish-bed has recently been detected near Nairnside House, by Mr WALLACE, who has obtained numerous fragmentary specimens of the large Thurso *Dipterus* (*D. macrolepidotus*)—the first time, so far as I am aware, that this fish has been met with on the southern shores of the Moray Firth. As far back as 1827 the occurrence of ichthyolitic nodules at Inches, 3 miles to the south-east of Inverness, was brought to the notice of SEDGWICK and MURCHISON.* At present the work of the Geological Survey is being prosecuted in this region by Mr JOHN HORNE, who has brought to my notice the very fine section of the lower portion of the Old Red Sandstone in the ravines of the Nairn, near Daviot, where the flaggy characters of many of the strata at once recall the true flagstone groups of Caithness.

5. *Loch Ness to Sutherlandshire.*—Beyond these limits no further palæontological evidence has yet been found to guide us in unravelling the broken series of conglomerates and sandstones which stretch up Loch Ness. That region has still to be worked out. On the west side of Loch Ness good sections are to be seen of the dull red sandstones and conglomerates, which rise into the huge dome-shaped mass of Mealfour Vounie (2284 feet). They are much broken and hardened—a result, doubtless, of the movements which from a very early period have taken place along the line of the Great Glen. On both sides of Glen Urquhart the unconformable position of the conglomerate upon the schistose rocks is well exposed. At the same time, the immense interval which must have separated the extrusion of the north Highland granites from the date of these conglomerates is made strikingly apparent. The conglomerates (or in many cases breccias, for the stones are often more angular than rounded) consist largely of fragments of granite, quartz-porphyry, and other intrusive rocks of the Highland series, mixed with pieces of the various gneisses and schists. The granitic materials are particularly abundant in the conglomerates of the rounded hills which guard the mouth of Glen Urquhart. Hardly any trace of bedding can be made out, and though

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* See their Memoir, "Trans. Geol. Soc." 2d ser. iii. 147.

the rock, by virtue of its joints, splits up into large angular blocks, it is exceedingly durable. Hence it has been well adapted for the formation of boulders, and, no doubt, from these hills many of the blocks of granitic conglomerate travelled, which are found scattered over the lowlands to the north and east of the Great Glen. Similar conglomerate reappears on the north side of the Beauly Firth to the north of Kessock Ferry.

The geological structure and order of the strata in the tract of country between the Dornoch Firth and Inverness was well sketched in the early memoir by SEDGWICK and MURCHISON. They showed that the margin of the Old Red Sandstone area is occupied by a very massive, coarse conglomerate, derived from the waste of the underlying and surrounding metamorphic rocks. This rock rises into conspicuous rounded hills, which run northwards almost to the borders of Caithness. The angle of dip appears to be generally very low, as we have seen to be the case in Caithness on the one hand, and along the south side of the Moray Firth on the other. Overlying the conglomerate, and passing down by intercalations into it, are dull red sandstones and flagstones, with reddish-grey bituminous and calcareous flagstones and shales. There can be little hesitation in identifying these strata with these which occupy a similar position at Clava and Culloden. They are seen on the Alness river "mixed with red and greenish-red marls;" on the Ault Graat, above the well-known gorge; in the hills about Tulloch Castle, near Coul, and for a considerable way down Strathpeffer.* They appear to run northwards also for a long way, flanking the great marginal band of conglomerate. They have been detected by the Rev. Dr JOASS of Golspie, at Edderton on the Dornoch Firth, and he has seen a tuberculated scale from the Brora district, which he is disposed to refer to *Coccosteus.* With regard to the Edderton locality, he has been kind enough to furnish me with the following particulars:—"The fossils there occur in calcareous nodules imbedded in two bands of red clay, interstratified with dark red sandstone, on the right bank of the Craig-roy Burn, about a mile above the bridge on the coast-road, which is half a mile from the Edderton Station. In the order of their abundance they were—*Coccosteus, Osteolepis, Diplopterus, Cheiracanthus, Cheirolepis, Glyptolepis* (? *Holoptychius Sedgwickii*), *Pterichthys,* and *Fucoids* (?)."

In these flaggy and ichthyolitic strata SEDGWICK and MURCHISON recognised the equivalents of the massive Caithness flagstone series. They are succeeded by an upper conglomerate band perhaps exceeding 300 feet in thickness. It is this rock through which the remarkable fissure-like ravine of the Ault Graat has been excavated. It passes up into red or reddish-grey sandstones, with occasional partings of grey shale. These overlying sandstones are seen in

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* *Op. cit.* p. 146.

many quarries to the south and north of Dingwall. They seem to spread over most of the basin or valley in which the Cromarty Firth lies. Down that hollow, as SEDGWICK and MURCHISON pointed out, a synclinal fold of the Old Red Sandstone runs, the strata being sharply bent up on the east side against the ridge of metamorphic rocks on which stand the Sutors of Cromarty. On that side of the trough the fish-bearing clays and nodules rise to the surface from under their overlying sandstones and conglomerates. They were first made known by HUGH MILLER as occurring at Cromarty on both sides of the ridge of the older rocks. His original section* shows that the fish-bed, which he has made world-famous in the history of geology, lies upon a band of conglomerate which rests directly upon the gneiss of the Sutor, and that it is overlaid by yellow sandstone. It is a band of the usual grey clay, full of calcareous nodules like those on the south side of the Moray Firth. The most characteristic organisms in these nodules are—*Coccosteus cuspidatus, C. decipiens, Diplacanthus longispinus, D. striatus, Glyptolepis elegans, G. leptopterus, Pterichthys Milleri, P. oblongus.*

The axis of the Black Isle is prolonged across the mouth of the Cromarty Firth into the northern Sutor. On the east side of that ridge the Lower Old Red Sandstone again appears in conglomeratic masses, which dip away from the nucleus of crystalline rocks. North of Shandwick these lower strata, which appear to be equivalents in time and position of the lower conglomerates of the Moray Firth, pass upward into red flaggy sandstones, grey flagstones, blue shales, and thin limestones, which, like the strata of Culloden Muir, have many of the peculiar lithological characters of the true Caithness flags. Among these strata SEDGWICK and MURCHISON found fragments of fossil fishes.† More recently the Rev. Dr GORDON and Dr JOASS have re-examined the locality and have obtained from it *Coccosteus* and other undoubted Caithness ichthyolites.‡ Professor HARKNESS has described the coast section.§ He estimates the lower sandstones and conglomerates to be 1500 feet thick, succeeded by 350 feet of the flaggy and shaly beds in which fish remains occur. My estimate would considerably increase the depth of the latter series. It is evident, however, that even on the largest allowance we have here but a feeble representation of the vast masses of the Caithness Old Red Sandstone. Nevertheless, we see the same agreement alike in lithological characters and in fossil contents which is presented also by the rocks on the south side of the Moray Firth.

A little beyond Geanies the flaggy and shaly ichthyolitic strata, which have a prevalent dip towards N.N.W., are abruptly succeeded by a series of yellow

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* "Old Red Sandstone." Plate of Sections, figs. 4 and 5.

† *Op. cit.* p. 150.

‡ "Quart. Journ. Geol. Soc." xix. p. 507.

§ *Op. cit.* xx. 437.

sandstones, which extend to Tarbat Ness, and have been referred to the Upper Old Red Sandstone. In examining this coast section I could not satisfy myself of the existence of any unconformability between the two sets of rocks. But of course it might very well exist without being traceable in a few yards of beach. Some faulting occurs at the junction, which rather obscures the relations of the rocks.

Horizon of the Lower Old Red Sandstone in the Basin of the Northern Firths.

We have now traced the deposits along the remaining coast-line of the ancient Lake Orcadie from Banffshire round to Sutherlandshire. We have found them throughout that extended tract mainly conglomeratic and arenaceous —the conglomerates in thick masses coming in again and again on successive platforms, while interstratified with them lie bands of grey clay and shale full of calcareous nodules containing fish remains. These fossiliferous bands retain their distinctive characters, lithological and palæontographical, throughout the whole district. In no part of this Old Red Sandstone belt does any great thickness of strata appear, the belt being itself narrow and the usual angles of inclination low. There can be little hesitation in regarding the whole of this tract as part of one continuous area of deposit, and in recognising in its strata the deposits of an old shore and of the shallow water near land.

When we trace these deposits almost up to the verge of Caithness, and, crossing into that country, come upon the vast continuous flagstone series, we cannot but be struck by the remarkably rapid change in the character of the strata. The hills of Sutherlandshire, ending in the granitic projection of the Ord of Caithness, separate two very distinct groups of rocks. In trying to parallel these groups, we naturally begin by assuming that the basement conglomerates on the one side represent in stratigraphical position, and generally in time the corresponding strata on the other side. The reddish-grey sandstones, blue-grey nodule-bearing clays, and rapid alternations of grey and dark bituminous flagstone, shale, and thin limestone, which succeed the conglomerates in Easter Ross, do not, however, at all accord with the flaggy sandstones and enormously thick flagstones of Caithness. It is hardly possible that the comparatively trifling thickness of strata in the Moray Firth can really be the true equivalent of the vast Caithness series. The attenuation of so massive a succession of deposits would be too rapid. I believe the true explanation to be that the Old Red Sandstone south of the barrier of the Ord of Caithness represents only the upper part of the Caithness flagstone series. We have already seen on what an uneven surface that series was laid down, and likewise how unequal was the movement of depression during which the deposition took place. The Thurso and Forss flagstones pass westwards into sandstones and conglomerates as they approach the old shore-line at Reay (*ante*, p. 373). In like manner, as it seems to me, the flagstones of the upper Caithness groups pass southwards into the conglomerates and alternating sandstones and clays of the great recess in Lake Orcadie, which is now chiefly occupied by the different northern firths. That the strata south of the Ord of Caithness are equivalents of some of the higher rather than of the lower parts of the Caithness flagstones is, I think, probable from the following considerations:—

1. On the north side of the Ord of Caithness there is abundant evidence of an ancient shore-line; but though conglomerates and sandstones occur along that line, they can be distinguished from those of the Moray Firth district in particular by the absence of those nodule-bearing clays, dark shales, and limestones which form so characteristic a feature throughout the latter district. Had the coast-line of Lake Orcadie at the beginning of the flagstone series run continuously from Aberdeenshire round the present margin of the Old Red Sandstone up to the Pentland Firth, we should have expected that the series of clays and nodules, which, with apparently every variety of shore,—bays, cliffs, inlets, gravel, sand, and mud,—yet continued to be formed along the whole of that extensive coast-line, would have stretched into Caithness also; at least we could hardly be prepared for their sudden cessation.

2. By regarding the conglomerates, sandstones, and clays south of the Ord as parts of the higher division of the Caithness flagstones, we overcome the difficulty which would otherwise arise, to account for the position of the Upper Old Red Sandstone along the southern shores of the Moray Firth. On this supposition the latter formation occupies the same relative place there as it does in Caithness and Orkney; that is, it rests unconformably on various members of the higher groups of the flagstone series. But if we make the underlying conglomerates and sandstones equivalents of the lower part of the Caithness series, we have to account for the want of any of the higher groups, and for the enormous denudation which must have intervened in that case previous to the time of the Upper Old Red Sandstone. There was undoubtedly some disturbance as well as denudation during the interval between the two formations, but the still gentle inclination of the strata all round the Moray Firth seems to indicate no very serious displacement of the older rocks.

3. The strata south of the Ord agree more in petrographical characters with the higher than with the lower parts of the Caithness series. They much resemble some portions of the section about Huna and John o' Groat's, where alternations of red sandstone, grey shales, and clays, and dark limestones occur. The peculiar fish-bearing nodules of the Moray Firth I have not detected anywhere in Caithness, but nodular shales occur at intervals, even as far west as Balligil, in Sutherlandshire.

4. But perhaps the argument which will have most weight in support of the position for which I am contending is based upon the fact that the fossils of the area south of the Ord are unequivocally those of the upper flagstones of Caithness and Orkney. The following species, common to both areas and not known or very rare in the lower flagstones, sufficiently establish this assertion:—*Acanthodes pusillus, Asterolepis Asmusii, Diplacanthus longispinus, D. striatus, Glyptolepis leptopterus, Pterichthys Milleri.* Besides these, each area contains what are said to be distinct species of *Cheiracanthus* and *Cheirolepis*, two genera not yet found among the lower flagstones. I have little doubt, when the fauna of Lake Orcadie comes to be revised, that a still greater community of species will be shown to exist.

If, then, the statement be accepted that the Lower Old Red Sandstone south of the Ord represents the upper portion only of the great Caithness flagstone series, an interesting light is thrown upon part of the history of Lake Orcadie. We see that during the greater part of that history the southern margin of the lake did not extend beyond the Ord. All south of that granitic ridge was land. The mainland of Scotland, consequently, had at that time a considerably greater extension northward than it can boast of now. The northern slopes of the Highlands extended over the area of the Moray Firth. The depression which went on to the north and carried down the bottom of the lake did not begin seriously to affect the ground south of the Ord until late in the history of the flagstones. But at last the land bordering the lake on its southern margin began to go down. The waters crept southward, until in the end they filled the hollows and glens leading up into the Grampians; and as we have seen, perhaps even penetrated into the very heart of these high grounds. During this subsidence, as the waters encroached on the land, banks of gravel would be formed along the shore at different successive levels, especially in recesses of the coast-line like that of Cawdor. Some of the same fishes which lived in the more open water towards the north would find their way southward with the gradual encroachment of the lake. But in so doing, they passed into tracts where the conditions of the water and of the bottom were somewhat different, and where at intervals, now marked by the nodule-bearing clays, their bodies, when they sank upon the silt, served as centres round which calcareous matter was segregated. Such intervals were, however, comparatively infrequent during the whole of the period represented by the deposits of the Moray Firth. As a rule, sand and gravel were the materials which there gathered on the lake bottom, and in these the traces of the fauna of the time are scarcely ever to be found.

NOTE ON THE VOLCANIC ROCKS OF SHETLAND.

(*Added August* 12, 1878.)

During the summer of the present year, my colleagues, Messrs B. N. PEACH and JOHN HORNE, have spent some weeks together in Shetland, chiefly studying the glacial geology of these northern islands. In the course of their investigations they made some interesting observations on the volcanic rocks associated with the Old Red Sandstone in parts of Shetland which I had not visited. I am glad to be able to supplement my Memoir with the following notes from Mr HORNE's verbal communications and Mr PEACH's written memoranda:—

East side of Shetland.—Necks of volcanic agglomerate pierce the sandstones on each side of Noss Sound. One on the Bressay side is irregular in form, about 230 yards in length by 50 in extreme breadth, sending fingers into the surrounding sandstones, which are shattered, discoloured, and baked to an extraordinary degree. The neck itself is occupied almost entirely with angular and sub-angular fragments of the altered sandstone (including blocks of great size, some of them several yards in diameter), set in a matrix of comminuted sandstone. The only lavaform material is that which forms a thin vesicular dyke, seen on south-west margin, and which in many places coats the walls of this neck with a crust a few inches in thickness. The neck on Noss presents the same general features. That these volcanic orifices belong to the period of the sandstones among which they occur, is shown by the occurrence of a bed of fine-grained volcanic tuff, intercalated among the sandstones on the north-east of Bressay. It is not more than 3 or 4 feet thick.

West side of Shetland.—The conjecture offered on p. 421 of the foregoing Memoir has been amply confirmed by my colleagues. They have found that the district stretching from the mouth of Ronas Voe to the head of Braewick consists of a splendid series of lavas and tuffs, with a few intercalated beds of chocolate-coloured and grey flaggy sandstones. The lavas are of the same dark purplish-red and greenish tints, crystalline to compact textures, and prevailing amygdaloidal and slaggy characters which distinguish the lavas of the Old Red Sandstone of central Scotland. I shall describe their petrographical characters in a succeeding part of this Memoir. Thick bands of coarse volcanic conglomerate or tuff sometimes separate the sheets of lava, which in other instances lie directly upon each other without any fragmental intercalation. The cliffs at Ockren Head and the Grind of the Navir present magnificent sections of these interbedded lavas and conglomerates.

The pink quartz-felsite ("pink granite" of HIBBERT), which stretches over a considerable area on either side of Ronas Voe, seems to have been erupted during the volcanic period of the Old Red Sandstone, and not to belong to the older metamorphism of the surrounding and underlying schists, gneisses, &c.

Unfortunately, its actual relations to the Old Red Sandstone rocks are everywhere obscured. But as it forms a great flat cake, with vertical joints, spreading over the edge of the upturned metamorphic beds, and must almost certainly have been intruded between the crystalline platform and an overlying cover of rocks, now removed by denudation, the reference of it to the Old Red Sandstone period may be accepted as highly probable. Dykes of a material very like or identical with the lavas of Hillswick traverse this pink rock on Ronas Voe and Muckle Rooe; so that it would appear to be at least older than some of the igneous rocks of the district. Though the general aspect of the coast cliff of the Ronas Voe rock recalls that of the remarkable pink "porphyry" of Papa Stour described at p. 420, the two rocks differ very much in their external petrographical and internal microscopical characters.

An interesting and important discovery made by Messrs PEACH and HORNE is the occurrence of numerous plants, evidently similar to those of Lerwick, in the altered sandstones or quartzites south of Melby. The whole of that district would thus appear to consist of Old Red Sandstone which has been much metamorphosed and invaded by pink porphyry.

With regard to the fossil lists, I have to add that Dr TRAQUAIR has kindly looked over the Table II. on p. 452, and given me a few additional references. He informs me that he has made some progress with the revision of the ichthyology of the Old Red Sandstone, and that considerable changes will require to be made on all published lists.

APPENDIX.

TABLE I.—Showing the Vertical Range of the Known Fossils of the Old Red Sandstone of Caithness.

(Compiled chiefly from data furnished by Mr C. W. PEACH.)

		Caulopteris Peachii.	Cyclopteris (allied to C. Hibernicus).	Cyclopteris (allied to C. Brownii).	Pinnularia.	Anarthrocanna.	Lycopodites Milleri.	Lepidodendron Nothum.	Lepidodendron sp.	Psilophyton 2 sp.	Calamites.	Stigmaria.	Sigillaria.	Araucarioxylon.	Palæopitys.	Annelide markings ?	Estheria membranacea.	Pterygotus.	Acanthodes Peachii.	Acanthodes coriaceus.	Acanthodes pusillus.	Asterolepis Asmusii.	Cheiracanthus grandispinus.	Cheiracanthus pulverulentus.	Cheiracanthus sp.	Coccosteus cuspidatus.	Coccosteus decipiens.	Coccosteus pusillus.	Diplacanthus crassispinus.	Diplacanthus striatus.	Diplacanthus longispinus.	Diplopterus borealis.	Dipterus macrolepidotus.	Dipterus Valenciennesi.	Dipterus sp. nov. ?	Glyptolepis elegans.	Glyptolæmus ?	Gyroptychius angustus (? sometimes weathered scales of Osteolepis, &c.).	Gyroptychius dipteroides (? weathered scales of Dipterus).	Holoptychius Sedgwickii (? Glyptolepis).	Osteolepis arenatus.	Osteolepis macrolepidotus.	Osteolepis microlepidotus.	Parexus incurvus.	Pterichthys Dicki.	Tristichopterus alatus.
Upper Flagstone Group.	John o' Groats' Beds,	...	...	×	...	...	...	...	×	×	×	...	×	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	...	...	...	×	...	...	...	...	×	×
	Freswick,	...	...	...	×	...	...	...	...	×	...	×	...	×	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Huna Flagstones,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	×	...	...	×	...	...	...	...	...	×	...	...	...	...
	Gill's Bay Sandstones,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Mey Hill,	...	...	...	...	...	×	×	...	×	×	×	...	...	...	...	...	...	×	×	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	×	...	×	×	...	...	...	×	...	...	...
	Baligill, Strathy,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	×	×	...	...	...	...	×	×	...	...	...	×	...	...	...	...	×	...	...	...	...
	Bighouse,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Reay,	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	×	...	...	...	...	...	×	×	...	...	× × ×	×	...	...	...
	Holburn Head,	...	...	...	...	...	×	×	...	×	...	...	...	...	...	...	...	...	×	...	×	×	×	×	×	...	×	...	...	...	...	...	...	...	...	×	...	×	×	...	...	×	× × ×	...	...	...
	Scrabster,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...
	Hill of Forss,	...	...	...	...	...	×	×	...	×	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	×	×	×	×	×	×	...	...	×	...	...	...	...	...	×	×	...	...	...
	Thurso, Gallowshill,	...	...	...	...	...	×	×	...	×	...	...	...	...	...	...	...	...	×	×	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	...	...	...
	Thurso Castle,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	...	...	×	×	×	...	...
	Murkle Bay,	×	...	...	...	...	×	×	...	×	...	...	...	...	...	...	×	...	...	...	...	×	...	...	...	...	...	×	...	...	...	...	...	...	...	×	...	×	×	...	...	×	×	...	...	...
	Weydale, Stonegun, and Castletown,	×	...	...	...	...	×	×	...	×	×	...	...	...	...	...	×	...	×	...	...	×	×	×	×	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	×	× × ×	×	...	...
	Olrig,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Wick, or Lower Flagstone Group.	Banniskirk,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	...	...	...	...	×	×	×	...	...	...	...	...	...	...	×	...	...	...	...
	Latheron,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...
	Halkirk,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	...	...	...	...	...	×	...	...	...	...	×	×	...	...	×	...	...	...	...
	Lybster,	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	...	...	...	...	×	×	...	×	...	...	×	×	...	...	×	...	...	...	...
	Thrumster,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	...	...	...	...	...	...	...
	Ackergill,	...	...	...	...	...	...	...	...	×	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	×	...	...	...	...
	Reiss,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	...	...	...	...	×	...	...	...	...	...	...	...	...	...	×	...	...	...	...
	Keiss,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...
	Noss Head,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	×	×	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...
	Loch Watten, Strath,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	×	...	...	...	...	...	...	...
	Kilmster, Wick,	...	Harland nr. Wick.	...	×	...	...	...	...	×	...	...	...	×	...	×	...	×?	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...
	South Head, Wick,	×		...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	×	...	...	...	×	×	...	×	×	...	...	...	...
	Spital Quarries,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	×	×	...	...	×	...	...	...	...
	Hempriggs,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Berriedale Flags,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...
	Ulbster,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	×	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
	Lower Red Sandstones and Conglomerates,	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...

Note.—It must be borne in mind that any list of the fossil fishes of the Old Red Sandstone can be regarded in the present state of science as merely provisional. The whole subject requires revision. No doubt several species may have been confused under one name, and distinct names have doubtless been affixed to specimens of the same species. Dr Traquair, of the Museum of Science and Art, Edinburgh, has undertaken the investigation of these interesting fossils, and in his competent hands it will be exhaustively carried on.

TABLE II.—List of the Fossil Fishes of the Lower Old Red Sandstone of the North of Scotland.

	Orkney.	Caithness.		Cromarty.	South side of Moray Firth.
		Lower Group.	Upper Group.		
Acanthodes Peachi, *Eg.*	...	...	×	...	...
coriaceus, *Eg.*	×	...	×	...	...
pusillus, *Ag.*	...	...	×	...	×
Asterolepis Asmusii, *Eichw.*	×	×	×	...	...
Cheiracanthus grandispinus, *M'Coy*	×	...	×	...	...
latus, *Eg.*	...	...	...	×	×
microlepidotus, *Ag.*	...	...	...	...	×
minor, *Ag.*	×	...	...	...	...
Murchisoni, *Ag.*	...	...	...	...	×
pulverulentus, *M'Coy*	×	...	×	...	...
sp.	...	...	×	...	...
Cheirolepis Cummingiæ, *Ag.*	...	...	...	...	×
curtus, *Ag.*	...	...	...	...	×
macrocephalus, *M'Coy*	×	...	...	...	...
Trailli, *Ag.*	×	...	...	...	...
ouragus, *Ag.*	...	...	...	...	×
velox, *M'Coy*	×	...	...	...	...
Coccosteus cuspidatus, *Ag.*	×	×	×	×	...
decipiens, *Ag.*	×	×	×	×	...
microspondylus, *M'Coy*	×	...	...	...	...
oblongus, *Ag.*	...	...	...	...	×
pusillus, *M'Coy*	×	...	×	...	...
trigonaspis, *M'Coy*	×	...	...	...	...
Diplacanthus crassispinus, *Ag.*	×	...	×	...	...
longispinus, *Ag.*	×	...	×	×	×
gibbus, *M'Coy*	×	...	...	...	...
perarmatus, *M'Coy*	×	...	...	...	...
striatus, *Ag.*	×	...	×	×	×
striatulus, *Ag.*	...	...	...	...	×
Diplopterus affinis, *Ag.*	...	...	...	...	×
borealis, *Ag.* (Agassizi, *Traill*)	×	×	×	...	...
gracilis, *M'Coy*	×	...	...	...	...
macrocephalus, *Ag.*	...	...	...	...	×
Dipterus macrolepidotus, *Val and Pentl*	×	×	×	...	×
Valenciennesi	×	×	...	...	...
sp. nov.	...	×	×	...	...
Glyptolepis elegans, *Ag.*	×	×	×	×	×
leptopterus, *Ag.*	×	...	...	×	×
microlepidotus, *Ag.*	...	...	...	...	×
sp.	...	...	...	...	×
Glyptolæmus (?)	...	...	×	...	...
Gyroptychius angustus, *M'Coy*	×	×	×	...	...
diplopteroides, *M'Coy* (? Diplopterus)	×	×	×	...	...
Holoptychius Sedgwicki, *M'Coy* (? Glyptolepis)	×	...	×	...	...
Osteolepis arenatus, *Ag.*	×	×	...	...	×
brevis, *M'Coy*	×	...	...	...	...
macrolepidotus, *Ag.*	×	×	×	×	...
major, *Ag.*	...	...	...	...	×
microlepidotus, *Ag.*	×	...	×	(?)	...
Parexus incurvus, *Ag.*	...	...	×	...	...
Platygnathus paucidens, *Ag.*	×	(?)	(?)	...	...
Pterichthys cancriformis, *Ag.*	×	...	...	...	...
cornutus, *Ag.*	...	...	...	...	×
Dicki, *Peach*	...	...	×	...	...
latus, *Ag.*	...	...	...	...	×
Milleri, *Ag.*	×	...	...	×	×
oblongus, *Ag.*	...	...	...	×	×
productus, *Ag.*	...	...	...	...	×
quadratus, *Eg.*	...	...	...	...	×
testudinarius, *Ag.*	...	...	...	×	...
Tripterus Pollexfeni, *M'Coy*	×	...	...	...	...
Tristichopterus alatus, *Eg.*	...	...	×	...	...

XVII.—*Chapters on the Mineralogy of Scotland. Chapter Fourth.—Augite, Hornblende, and Serpentinous Change.* By Professor HEDDLE.

(Read 1st April 1878.)

AUGITE.

Malacolite.

$$(\dot{\mathrm{C}}\mathrm{a}\ \dot{\mathrm{M}}\mathrm{g})\ \ddot{\mathrm{S}}\mathrm{i}.$$

Malacolite was first observed in Scotland by Dr MACCULLOCH, who included it with the second sub-species, under the name of Sahlite.

He, in his different works, noted its occurrence—"near the church of Rodel in Harris,"–"in Tiree,"–"Glen Elg,"–"Glen Tilt,"–and "in Rannoch." With his usual power he describes the occurrence, varieties, and even the crystalline form of the mineral,—the clearness and precision of his descriptions being only equalled by the utter want of precision and frequent mystifications as to their localities.

The above sentence conveys all the *direct* information which Dr MACCULLOCH vouchsafes as to the five localities *indicated* by him, though in a vague manner he may occasionally lead one to conjecture that certain particular spots are meant.

By the time that the mineralogist, led by Dr MACCULLOCH to Glen Elg, Tiree, or Rannoch, has walked some twelve or twenty miles in an unsuccessful search, he begins to think that if he be successful, he has some claim to be himself regarded as an independent discoverer.

From Granular Limestone.

1. Found at Shinness, Sutherland, sometimes imbedded in distinct layers of the lime, and occasionally at its point of contact with the inclosing gneiss. The mineral here is both in larger quantity and in larger crystals than elsewhere in Scotland. Frequently it is found in widely radiating crystals, which are, however, imbedded in the lime. Crystals from a foot to sixteen inches in length, by an inch or two in breadth, occur; these are frequently fissured transversely—delicate amianthiform fibres stretching over the gaps to connect the severed portions.

It is here associated with the following large assemblage of minerals:—

Sahlite, abundant.
Biotite, common.
Molybdenite, very rare.
Pyrrhotite, rare.
Actynolite, scarce.
Sphene, scarce.
Apatite, rare.
Augitic tremolite, rare.

Amianthus.
Pyrite, rare.
Talc, very scarce.
Steatite, rare.
Chlorite, rare.
Orthoclase, rare } in covering rock.
Haughtonite ? } in covering rock.

The sahlite is found of two very different colours: the more common, a pale leek-green; the rarer, a very dark green. These three varieties of the same mineral—the malacolite, the sahlite, and the augite—in no way pass into one another; they are perfectly different in appearance, and the malacolite sometimes occurs in cavities in distinct crystals, which are super-imposed upon the sahlite. An augitic asbestus rarely occurs also. Another point of note here is the paragenetic occurrence of the hornblendic type of mineral, which also presents itself in three varieties, which do not shade into one another—amianthus, tremolite, actynolite.

The lime carrying these minerals is highly granular, but for the most part dense; it shows rarely a belt or belts of a brilliant red. Two beds, separated only some half dozen feet, occur; they dip at an angle of about 20° to the north; they are immediately overlaid by ordinary gneiss, but this in turn underlies a bed of hornblende rock; and the gneissic beds which overlie this are for some distance persistently hornblendic.

Somewhat south of the line of strike, the same beds of lime, with the same dip, appear at Arskaig, on the south side of Loch Shin, but only for a short distance; they lie between two cross faults.

The malacolite is of a white colour, a cleavage angle of 87·5, and specific gravity, 3·149. 1·306 grammes yielded—

Silica,	·688	
From Alumina,	·005	
	·693 =	53·062
Alumina,		·193
Ferric Oxide,		1·773
Ferrous Oxide,		·47
Manganous Oxide,		·153
Lime,		23·626
Magnesia,		19·295
Water,		1·546
		100·118

Insoluble silica, 2·741 per cent.; the specimen was absolutely pure, the only contact mineral being lime.

The *c* cleavages of the malacolite and sahlite of Shinness, and of the malacolite of Scotland generally, is so eminent that the crystals fall asunder along it frequently in being extracted; yet this cleavage is not even mentioned in BROOKE and MILLER'S "Mineralogy."

These *c* cleavages are evident to the sight by a parallel lineation of the mineral: in the more transparent of the Shinness specimens this lineation appears as narrow, opaque white bands; in the sahlites generally, as lines of a depth of colour paler than that of the general mass. This lineation constitutes the most characteristic microscopic feature of the stone.

2. From granular limestone, about two hundred yards to the south-west of Totaig, on Loch Ailsh.

The malacolite here was rare; it occurred in imbedded crystalline lumps of a somewhat platey structure, and a pale dove-blue colour. The associated minerals were, nodular imbedded serpentine of green and blue-black tints, and a semi-pseudomorphic substance, afterwards to be noticed under the name of *Totaigite.* Specific gravity, 3·2.

On 1·31 grammes—

Silica,	·66		
From Alumina,	·004		
	·664	=	50·687
Alumina,			·029
Ferric Oxide,			·935
Manganous Oxide,			·068
Lime,			25·777
Magnesia,			18·091
Potash,			·503
Soda,			1·426
Water,			2·616
			100·122

Insoluble silica, 2·56 per cent.; possible impurity, lime.

3. The limestone of Totaig, lying in micaceous gneiss, with a gentle easterly dip, is first seen immediately to the east of the pier, thrusting itself into the water in a rounded bluff. This much corroded promontory is studded with projecting nodules of large size. These consist of a matted agglomeration of crystallised malacolite, with crystals sometimes so free as to exhibit fairly well-developed forms. The only other substance of interest here is dark augite passing into serpentine, the transition being very clearly exhibited. In the neighbourhood, a little to the west, an augitic tremolite and an augitic amianthus, coat the exposed layers of the limestone. From the shore of Loch Ailsh the limestone strikes across the country in a southerly direction. Passing under the western slopes of the hill of Ben Chourn in cliffy escarpments, it sinks with the falling ground into the greater glen of Glen Elg towards its lower third. After rising from its concealment in the detritus of the valley, it

reappears among its southern slopes, curving far inland and upward among the hill tarns to the eastward, again to turn back with a westerly sweep as the land sinks into the hollow of Glen Beg. It now passes behind the hamlet of Balvraid, crosses the stream which waters this glen, and skirts its southern banks as it undulates with a westerly strike among the low heights which form the base of Beineghapple. The limestone thus in its escarpment pursues a course much resembling in outline the figure 5.

Here and again augitic minerals are to be found in it, most largely under Ben Chourn and Beineghapple.

At the former hill, very fine masses of milk-white malacolite, in association with a sprinkling of small particles of yellow serpentine, occur imbedded in the lime; sahlite in radiated crystals being also rarely found. Under the slopes of Beineghapple, nodular masses, somewhat resembling solid amygdaloids, of pale-green to white radiating sahlite, abundantly occur. Possibly these may be the "zoisite" of JAMESON.

Beneath the lime there is here found massive, cleavable, dark smaragdite-green augite; while, over the lime, there lies a singular rock composed of hornblende, garnet, and pyrite, forming a compound of most unusual hardness and toughness.

The malacolite, the analysis of which is now given, was got from the north side of Ben Chourn, about half way between Totaig and Glen Elg. It occurred in large foliated crystals; was semi-transparent and lustrous.

Cleavage angles, *c* on *m* 100° 52′, to 100° 45′; *c* on *a* 105° 50′; *a* on *m* 133° 32′; specific gravity, 3·155.

1·71 grammes yielded—

Silica,	·874		
From Alumina,	·008		
	·882	=	51·578
Alumina,			·11
Ferric Oxide,			·328
Lime,			22·007
Magnesia,			19·59
Potash,			·491
Soda,			1·008
Water,			4·643
			99·755

Insoluble silica, 10·09 per cent.; possible impurity, lime.

4. From the limestone of Glen Tilt.

This limestone, probably from its proximity to an intrusive granite, has assumed the characters of a statuary marble, though it is much too close-grained and dense to be regarded as possessed of beauty. It also, like the Skye marble, either has naturally, or rapidly assumes, a greasy opacity, which quite unfits it for the purposes of the sculptor, or even for domestic architecture. Of the yellow serpentinous marble which is associated with it, quite a different account must be given. It unquestionably is the most beautiful true marble in Scotland. It may be doubted if any more striking ornamental stone can be seen than that of which the mantel-pieces in the hotel at Blair-Athol are constructed.

The white marble occurs at various points in the glen, being well exposed in the bed of the stream.

At a few hundred yards above the Forest Lodge a bed crosses the stream. Formerly it here formed a fine natural arch;—a quarry having been opened at the spot, this has long since given way. Here the lime overlies a stratum of a dark, probably Biotitic slate. This has been regarded by some geologists as an igneous rock.

The marble of this quarry, during the time when it was wrought, yielded specimens of tremolite of very remarkable beauty; but the malacolite, of more subdued appearance, was hardly noted. It is found in a belt or band, being very similar in its mode of occurrence to that at Shinness, though it has more of a foliaceous massive, and less of a crystalline character. Milk-white, and a very pale green are the tints it here assumes. Its associates are yellow serpentine in patches, and radiated greenish-grey tremolite.

Margarodite in small crystals, simulating talc, disposed with a parallel arrangement in the marble, is also to be occasionally found. The rock then assumes more or less of a schistose structure.

The white malacolite has a specific gravity of 3·124; the greenish, of 3·1625. That with the faint green tint was analysed. 1·495 grammes yielded—

Silica,	53·244
Ferrous Oxide,	2·709
Manganous Oxide,	·133
Lime,	22·774
Magnesia,	18·862
Water,	2·167
	99·889

·4 per cent. of the silica insoluble; possible impurity unknown.

Dr Macculloch mentions the occurrence in this quarry of a quartz with an unusually high gravity. A thin layer of a fine granular quartz, very dense in structure, overlies the malacolite, but its specific gravity was found to be 2·645.

From the Vicinity of Serpentine.

5. On the north-east side of the serpentinous hills of the Coyle, Aberdeenshire, a pathway leads through fields past the house of Alltcailleach. On the south side of this pathway, about half-a-mile west of the house, two large, apparently loose blocks of malacolite, with a somewhat radiating crystalline structure, and a pigeon-blue colour, protrude from the soil. The only associated mineral is Biotite in crystals.

Lime is said to occur near the stream which flows northward from the horse-shoe group of hills.

The specific gravity of this malacolite is 3·183. On 1·3 gramme—

Silica,	·663		
From Alumina,	·002		
	·665	=	51·
Ferric Oxide,			1·372
Ferrous Oxide,			1·595
Manganous Oxide,			·384
Lime,			26·363
Magnesia,			17·076
Potash,			·63
Soda,			1·112
Water,			·261
			99·893

Possible impurity, Biotite.

Besides the above, malacolite occurs in most of the granular marbles and highly altered limestones of Scotland,—as at Rodil in Harris, Balliphætrich in Tiree, Glen Mark, Morenish on Loch Tay, Strath Dee, &c.

The finest crystals of malacolite which I have seen in Scotland were found by Dr Lauder Lindsay, "somewhere in Glen Callater;" they were associated with actynolite.

Sahlite.

$$(\dot{\mathrm{Ca}}\ \dot{\mathrm{Mg}}\ \dot{\mathrm{Fe}})\ \ddot{\mathrm{Si}}.$$

From Granular Limestone.

Of the two varieties of sahlite which are associated with malacolite in the limestone of Shinness, all that need be noted, in supplement of the observations already made, is that the lighter—the sap green variety—does not occur in crystals much over half the dimensions of the white variety; while the dark green —almost augitic variety—is in crystals of only about an inch in size. However closely associated are the three varieties, they never pass into one another.

6. From Ben Chourn, Glen Elg.

The sahlite, mentioned as occurring in the lime on the north slopes of Ben Chourn, was found in only a few rounded lumps, which had a radiating crystalline structure, the crystals being about two inches in length. The mineral was of a pale leek-green colour, and was coated with an ochrey rust.

1·507 grammes yielded—

Silica,	54·479
Ferrous Oxide,	3·133
Manganous Oxide,	·245
Lime,	22·816
Magnesia,	17·584
Potash,	·438
Soda,	·79
Water,	·424
	99·909

9·025 per cent. of the silica were insoluble; possible impurity, unknown.

7. From the well-known Tiree marble.

This now almost exhausted marble occurs at Balliphætrich, on the west side of Tiree. It has been admirably described by Macculloch in his "Western Islands." It owes its peculiarity to numerous imbedded minute crystals of dark green sahlite which are promiscuously sprinkled through a granular calcareous paste, which is tinged of a peculiar red in blotches. This coloration is due to minute tufts of crystals of a substance afterwards to be noticed. Dr Macculloch has remarked upon the rounded and polished outlines of the imbedded crystals of sahlite; and in Sowerby's "British Minerals," a peculiar pale cross banding (? *c* cleavages) which they present is admirably depicted. They are commonly of the size of shot, rarely of large size—an inch or so,—sometimes in cleavable masses, and are occasionally associated with a pale-blue, watery variety,* and with minute highly lustrous crystals of sphene, which latter also exhibit a rounded contour in their faces.

Of more marked interest in this marble is the occurrence of portions of sahlite, in which an incipient as well as a perfected passage into serpentine is beautifully seen.

At this locality unaltered sahlite, however, prevails:—while, in contradistinction to this, in the Glen Elg limestone there is a very similar sprinkling of granules of yellow serpentine, the unquestionable representatives of former crystals of sahlite,—the unaltered sahlite being there the exceptional occurrence.

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* In Jameson's "Mineralogical Travels," vol. ii. p. 33, we see that this pale variety was considered corundum by the Hon. Mr Greville. Jameson considered the sphenes to be garnets.

Specific gravity, 3·142.

1·486 grammes of the Tiree sahlite yielded—

Silica,	·74		
From Alumina,	·011		
	·751	=	50·538
Alumina,			4·688
Ferric Oxide,			4·144
Ferrous Oxide,			·037
Manganous Oxide,			·686
Lime,			23·59
Magnesia,			14·401
Potash,			·314
Soda,			·633
Water,			1·476
			100·507

2·961 per cent. of the silica insoluble.

The crystals having been dissolved out of the lime by weak acid, must have been pure from all contamination, except, perhaps, a minute crystal of sphene, of which there is no evidence.*

8. From Eslie, near Banchory, Kincardineshire.

The bed of lime which courses down the south side of the Dee, and which is thrown in wrinkles to the surface every four or five miles, between Inver Inn and Banchory, nowhere carries augitic minerals, except in ill-developed specks, until it shows itself in the hill above Eslie, three miles south of Banchory.

In the quarry on the south of the top of the hill, crystals of sahlite an inch or two in size were found by Nicol. These are associated with pyrrhotite, sphene, talc, rarely oligoclase (?) and pale blue orthoclase; which last is actually imbedded in the lime,—a most unusual occurrence.

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* In the *Chemical News*, Feb. 17, 1871, Mr E. C. C. Stanford gives the following analysis of the dark green crystals imbedded in the pink matrix of the Tiree marble :—

Silica,	60·
Alumina,	22·18
Ferric Oxide,	3·42
Manganous Oxide,	tr.
Lime,	11·64
Magnesia,	3·32
	100·56

which analysis is said to correspond with some already published of aluminous hornblende, and to prove these crystals to be *hornblende*.

It has first to be remarked that Jameson, Macculloch, and all mineralogists who have examined these crystals recognise them as, without doubt, *augite*. Second, that this analysis of the same substance as that noted above, presents an extraordinarily discordant result therewith. And lastly, that in no particular whatever, except in the quantity of lime, does it agree with any one of the 52 analyses of aluminous hornblendes published by Dana. In hornblende, alumina, when present, replaces silica; but, without even deducting the 22 per cent. of alumina, the silica here is in excessive amount.

The crystals of sahlite appear to be undergoing an incipient change into serpentine, being duller and softer than usual. Such a change is, however, not borne out by the analysis. They also rarely contained imbedded bunches of radiating crystals of dark green lustrous actynolite, which crystals cut across the cleavage plains of the sahlite.

The colour of this sahlite is pale grass-green.

1· gramme yielded—

Silica,	·487		
From Alumina,	·008		
	·495	=	49·5
Alumina,			1·965
Ferrous Oxide,			11·057
Manganous Oxide,			·4
Lime,			24·08
Magnesia,			10·81
Potash,			·567
Soda,			·795
Water,			·688
			99·862

The quantity of iron is, when the colour is considered, strikingly large; and as the mineral contains actually less magnesia than usual, no serpentinous change has here taken place. In fact the whole of this stratum is markedly deficient in serpentinous matter; though in some places, as at Muir and Midstrath, the lime is little more than a granular malacolite, with but little lime lodging between the crystals. As the quarries at the two above-mentioned localities are somewhat extensively wrought, and the rock bears in the district the character of being an excellent binding lime, the writer proceeded to analyse it. He was much surprised to find that a fragment placed in acid effervesced for an exceedingly brief period, and was, after this treatment, absolutely unchanged in form, and only slightly vesicular—showing a granular congeries of minute crystals of malacolite, interspersed with still smaller ones of specular iron.

There being a possibility that calcined malacolite might combine with water and "set," so as to form a mortar, a quantity was treated and tested in such a manner, with a totally negative result. The excellence of this lime must therefore be in all respects open to question.

Coccolite.

From Gneissose Rocks.

9. In association with limestone.

Dr Joass sent me from the Gruagach Cliff, near Loch Ailsh in Ross, a lump of coccolite passing into cleavable augite.

The colour was deep, rich green. The portion upon which the specific gravity was taken was probably somewhat porous; it gave 3·048.

1·303 grammes yielded—

Silica,	·627		
From Alumina,	·012		
	·639	=	49·04
Alumina,			6·086
Ferric Oxide,			1·393
Ferrous Oxide,			2·939
Manganous Oxide,			·46
Lime,			23·336
Magnesia,			15·118
Potash,			·816
Soda,			·791
Water,			·174
			100·153

Of the silica, 4·381 per cent. were insoluble; possible impurity, margarodite. A more typical coccolite or granular Funkite occurs with cinnamonstone in the limestone of Delnabo, Glen Gairn. A sufficiency for analysis could not be got.

Diallage—"Thin foliated Augite."

$$(\dot{\mathrm{Ca}}, \dot{\mathrm{Mg}}, \dot{\mathrm{Fe}}), (\ddot{\mathrm{Si}}, \dddot{\mathrm{Al}}_2).$$

From Metamorphic Rocks.

10. From the diallage rock of Unst, Shetland.

Here the rock is palpably metamorphic. Regularly interstratified among bedded rocks a little north of Loch Trista in Fetlar, it sweeps northward conformably along with them, with a strike gradually curving to the east, till they leave the shores on the north of the island. Reappearing in due course to cross the island of Uya, they again appear on the southern shore of Unst, trending more easterly still, until they shade off into the serpentine of the north of Balta Sound,—the greatest mass of that rock in Scotland.

For the most part this diallage consists merely of a fine-grained melange of crystals of the mineral itself with minutely granular labradorite; but it here and again gives evidence of pre-existent stratification in bands or belts of coarse-grained structure which hold a uniform and parallel course throughout it. These are most highly developed and most easily seen among the winding cliffs of Balta and Hunie.

Here the segregation of matter into giant crystals repeatedly produces the appearance of pseudo veins;—the crystalline masses of diallage which stud them being occasionally of the size of the palm of the hand; these lie bedded in still larger masses of fine granular labradorite. Even in these localities of higher crystalline development, no accessory minerals, with the sole exceptions of veins of quartz, carrying epidote, are to be found; the rock, throughout its whole stretch, being of singularly simple constitution.

Of the pseudo veins, the two finest appear on the south side of a little "geo" in the island of Balta, which, from its lying to the immediate south of a small brough, I have named Brough Geo. Of these veins, which are closely adjacent, one carries diallage, the other hornblende.

It is of this diallage that the analysis is given. In colour it is of a somewhat dull grass-green, the structure is platey, tending to foliated, the lustre is not so pseudo-metallic or splendent as is sometimes seen in this variety of augite; the specific gravity is 2·965.

1·501 grammes yielded—

Silica,	·742		
From Alumina,	·012		
	·754	=	50·233
Alumina,			5·845
Ferrous Oxide,			5·223
Lime,			11·23
Magnesia,			21·586
Potash,			1·199
Soda,			·582
Water,			4·167
			100·074

The quantity of water was sometimes as high as 5·218.

Insoluble silica, 2·652 per cent.; possible impurity, labradorite.

11. From the diallage rock of Pinbain, near Lendalfoot, Ayrshire.

The relationships of this rock have been so clearly made known by Dr James

Geikie, in the explanatory chapter attached to the Ayrshire Survey, that there is no need of enlarging on them.

The specimens examined were from a low spit, which protrudes far seaward about mid-tide level. This consists for the most part of the diallage, in large platey and intermatted crystals; these lie imbedded in a granular massive hydrated Saussurite; the rock here, also, being of singularly simple constitution, with no accessories.

The diallage here is an excellent type of this variety of augite: broad cleavage foliations, and even crystal faces, flash with a splendent lustre, but with a uniformity which is frequently broken by a singular reticulated or arborescent appearance; the interrupting duller structure having the ordinary non-lustrous appearance of the stone.

The pseudo-metallic semi-nacreous flash, brought out in certain positions by reflected light, appears to arise from an internal reflection from flat fissures or broad cleavages.

Rough crystals, of half the size of the palm of the hand, may here be obtained. The colour is olive-green; the specific gravity, 3·251.

1·3 grammes yielded—

Silica,	·66	
From Alumina,	·013	
	·673	= 51·769
Alumina,		2·1
Ferrous Oxide,		2·955
Manganous Oxide,		·307
Lime,		22·098
Magnesia,		18·461
Potash,		·628
Soda,		·579
Water		1·085
		99·982

Insoluble silica, 3·863. Impurity unknown.

Smaragditic Augite.

12. True smaragdite, according to Descloiseaux, has the angles and cleavages of hornblende. Haidinger found that from Bacher to consist of alternate laminæ of hornblende and augite, in parallel disposition.

The mineral which I have designated as above cannot be of such a nature, as it has needles of actynolite here and there, in confused arrangement, matted into a generally foliated mass, and penetrating the augite crystals. Minute transparent gem-like crystals of augite are also to be seen.

The mineral was got in the southern—the *Beg* glen of Glen Elg—on its south side, about three miles from its embrochure, a little above where the limestone disappears, and at a point where a small stream crosses a pathway in passing out of a wood to join the river.

It occurred in somewhat large masses of interpenetrating crystals, of an inch or more in size; with eminent cleavages, a somewhat fibrous structure, and a lively green colour. Specific gravity, 3·242.

1·505 grammes yielded—

Silica,	54·219
Alumina,	·171
Ferrous Oxide,	6·723
Manganous Oxide,	·398
Lime,	19·572
Magnesia,	16·97
Potash,	·504
Soda,	·449
Water,	·956
	99·962

4·289 per cent. of the silica insoluble. Possible impurity, a trace of actynolite. The small quantity of alumina entitles us to consider this merely a dark sahlite.

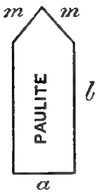
A mineral very similar in structure—a readily cleavable augite with penetrating crystals of actynolite—was found by the writer in micaceous gneiss at the col—3200 feet high—between Aonach Beg and Ben Aiblen in Inverness-shire; also, near serpentine, at Greenloan, on the Blackwater; and specimens, dentical in appearance with the last, were sent to him by the Rev. Mr PEYTON of Portsoy, from a crook of the Deveron, a mile west of Kinnairdy Castle.

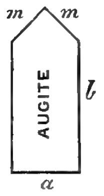
Augite.

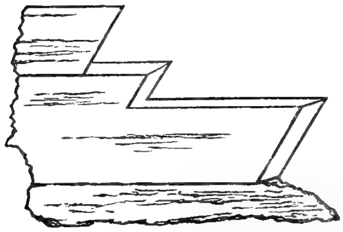
From Diorite. (?)

13. This variety presents an appearance so unusual to augite, that, after considerable doubt, I inclined to believe it to be either diaclasite, or one of the non-fibrous and non-bronzy varieties of enstatite. I cannot say that I was able to satisfy myself that the masses of rock from which I obtained it were *in situ*. They lay, in somewhat linear disposition, after the manner of a loosened or disjointed outcrop. Their site was the west side of the height called Craig Buiroch, in Banffshire. They stretched southward along the elevated ridge in the direction of Retannach. The ordinary rock of the hill was a some-

what laminated variety of apparently the same mineral, and labradorite, in a crypto-crystalline form. The larger masses of the mineral in question occurred either in imbedded crystals, in coarse-grained patches of the rock, or in exfiltration veins. In the latter they were associated with Paulite, menaccanite, labradorite, pyrite, and Biotite. (?)







The rock, as a whole, may be regarded as intermediate between diorite and hyperite.

This augite is occasionally in rough imbedded crystals of the form shown. More generally it presents flat cleavable surfaces like the lower figure. The colour is brownish or yellowish brown; it is semi-transparent.

The cleavage is flat and decided, parallel to *a*, of which the lustre is vitreous. There is no face of bronzy or metallic lustre. There is a slight appearance of decomposition; and in specimens, evidently of the same mineral, got from the shore at Cowhythe, the colour is greenish, with, on weathering, a pinchbeck lustre; and, here, polished faces seem to show *an absolute passage of the unaltered greenish mineral into true unweathered violaceous Paulite.*

The specific gravity is 3·28.

1·3 grammes yielded—

Silica,	50·307
Alumina,	4·479
Ferric Oxide,	3·924
Ferrous Oxide,	5·763
Manganous Oxide,	·307
Lime,	17·575
Magnesia,	16·615
Potash,	·188
Soda,	·899
Water,	·378
	100·435

This, apparently, is the mineral which forms the greater part of the bulk of the beds of diorite—if they are to be so named—which occur in the bay of Durn near Portsoy. The rock there would appear to contain this mineral (almost entirely replacing hornblende), Paulite in varying amount, labradorite, andesine rarely—either in small tortuous veins, or in stellate crystallisations—Biotite, menaccanite, pyrite, and pyrrhotine. There are at least five alternations of this rock with serpentine in the space of some 50 yards, and some of the beds are evidently being changed into serpentine,—even the unchanged beds being crossed in all directions by thin veins of serpentine.

The rock at Craig Buiroch probably is an extension of one or other of these beds.*

From Hyperite.

14. It is with no little hesitation that I call the rock I have at present to notice by the above name. As it differs markedly in outward aspect and composition from typical diabase and gabbro alike, I adopt the name applied by MACCULLOCH to the rock which it most resembles.

This rock forms, if not the chief mass, at least that which confers their distinctive features upon the three great hills of Halival, Haiskeval, and Tralival in Rum. It also forms many others of the unapproachably rough uplands of the same island; and it, moreover, passes, if not always by insensible gradation, at least in an unmistakable manner, into many of the other bedded igneous rocks of the island.

The name to be assigned to this rock may be of little moment; its precise lithological character may not be of much more; but the fixing its true position as regards the strata connected with it, and the proper correlation of it with those of the adjoining island of Skye, are of very prominent importance.

As two geologists of repute have referred to it—JAMESON and MACCULLOCH—their opinion as to its nature, and evidence as to its relationships, may be cited.

JAMESON, writing in 1813, says that, on ascending the hill of Halival, he found "greenstone, associated with basalt, lying upon the top of sandstone," and that all the higher hills which he saw "were apparently composed of trap-formation rocks." "I observed several pieces of a rock resembling Lydian stone which seemed to traverse the greenstone in the form of veins."

MACCULLOCH, writing in 1819, says: "Immediately incumbent on the sandstone is observed a great body of augite rock, which is nowhere else so conspicuous and abundant in Scotland. It varies considerably in aspect, being sometimes of a small granular texture, and scarcely to be distinguished from common greenstone."

From an admirable description of the relations of this rock, extended through twelve pages of his book, the following statements are gathered:—"It is in a perpetual state of transition into other members of the trap family." "There is an evident transition into basalt." "It passes into the same syenite

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* I have in the figure, alongside of the rough crystals of this augite, placed for comparison a figure of the equally rough cleavage crystals of the Paulite (true hypersthene), with which they are sometimes associated; in these the *m* angle is slightly more accute; *b* is the face of facile cleavage, its lustre being bronzy and somewhat purple. There is, at right angles to this, more a fracture than a cleavage, *a*, the colour of which is black.

as that described as constituting a large portion of Skye, as well as of Mull and St Kilda." "This syenite also reposes immediately on the sandstone." "This syenite ought to be distinguished by an appropriate name, since the ingredient united to the felspar is augite and not hornblende." "In the neighbourhood of Harris I obtained hypersthene, which in mineral character was not to be distinguished from the specimens described as existing in Skye; it occurs in compound veins traversing the augite rock. One variety of the rock has everywhere the aspect of rusty iron. It gives a singularly barren and desolate look to the part of Rum which it occupies, and in this respect and its russet hue strongly reminds the spectator of the naked and sterile appearance of Loch Scavig. This rock is remarkably sonorous when struck, rendering a sound precisely like that of iron." "Under Scuirmore the sandstone beds are traversed in one place by a vein of the augite rock which forms a considerable portion of the island."

From Dr MACCULLOCH's description of his *hypersthene rock* of Skye, the following quotations are made:—"On the shore at the foot of Garsven the overlying position of the hypersthene to the red sandstone is distinctly visible. The strata of the latter are traversed by numerous veins of the former." "A question will naturally arise here, namely, whether there is any difference between the several rocks of the unstratified division; whether, for example, the syenite is of prior or posterior origin to the hypersthene rocks of the Cuchullins. If that which is just related respecting the interchanges of the two be correct, there is no reason to imagine any such differences to exist. Still, however, certain portions may be seen which appear posterior to the syenite." "In some places large veins are seen composed of a very compact hard substance, as sonorous as cast iron; there occur, also, thin veins of a black substance rather resembling Lydian stone than any form of basalt." "The large concretions of hypersthene are found in veins." "In the simplest small-grained variety of the rock it cannot easily be distinguished from a common greenstone."

The inference to be drawn from the above must be, that, if not one and the same, MACCULLOCH's "hypersthene rock" of the Cuchullins and his "augite rock" of Rum are but slightly varying modifications of one and the same, and that the rock is igneous.

Both rest upon his "syenite," which in both islands rests upon red sandstone. According to him, both alike pass by insensible gradations into that syenite, as they also do into a great many other varieties of igneous rock. When fine-grained, neither can be distinguished from greenstone. Both carry veins resembling "Lydian stone," and of a coarse-grained variety which contains MACCULLOCH's "hypersthene."

It is necessary here to make another quotation from MACCULLOCH. He writes:—"I am under the necessity of describing the next of the overlying rocks. . . . . On the sides of Halival and Haiskeval* it is found intermingled with the augite rock in large masses, nor could I after much research determine the precise nature of the relation between them. The character of this trap varies in different places. In one place there occurs a mass, of which the weathered surface displays imbedded fragments of the same rock. In another place there is a still more remarkable mass, consisting of an ordinary black basalt, with fragments of the red sandstone scattered at considerable distances through it. It is by no means common to find trap veins entangling fragments of the rock which they traverse. . . . . Notwithstanding the difficulty of determining the exact nature and connection of these masses, it is probable they are portions of large veins, which the ruined and encumbered nature of the ground does not permit to be traced. This trap, therefore, is posterior to the augite rock and syenite, although it cannot properly be called superincumbent."

Though, as I have remarked, the inference to be drawn from MACCULLOCH's observations must be that the hypersthene rock of Skye and the augite rock of Rum are mere modifications of the same rock, and that an *igneous* one, such is not the conclusion which has generally been arrived at. "Metamorphosed Laurentian gneiss" was the view at one time held. Dr STERRY HUNT, who appears to be the last writer who notices these rocks, in summing up the evidence regarding them (pages 279 and 281 of "Chemical and Geological Essays"), concludes that they are "identical with the North American norites, whose stratified character is undoubted." He quotes EMMONS in support of this view; and HAUGHTON,—who regards them as evidently a "bedded metamorphic rock."

"Stratified character" is not precise; neither is "bedded metamorphic rock" altogether so. An igneous rock may have a stratified character; and a bedded igneous rock may have suffered metamorphosis.

These expressions, however, though by no means clear in themselves, are generally employed as contradistinctive of intrusive or eruptive; and the context of the two pages mentioned, clearly shows that they were so used.

STERRY HUNT's view, however, is evidenced by the following quotations:—"I propose to designate as the Norian series the great formation of crystalline stratified rocks, of which the norites make up so large a part."

"I accept in its widest sense the view that all the crystalline stratified rocks have been produced by the alteration of mechanical and chemical sediments."

"In the second class of sediments we have" a series of specified chemical changes and crystalliferous admixtures, "which give rise to diorite, diabase, euphotide, eklogite, and similar compound rocks."

This "second class of sediments," or "series," is the Norites, which are placed below the Huronian, in which latter he includes the Cambrian, among its upper members.

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* Aisgobhall of JAMESON.

It would, therefore, appear that, according to the views of Drs Sterry Hunt and Emmons, "the hypersthene rocks of the Western Islands of Scotland" are metamorphosed sediments, which underlie, at an enormous depth, the Cambrian rocks.

From such a conclusion I must entirely dissent.

First, with regard to the rocks as seen in Skye;—it is to be doubted if any one who examined the pillared line of cliff which the so-called "syenite" forms at Ard Bhornis, opposite to Raasay, and the manner in which a sandstone bed is involved in it, in Glamich, could come to any conclusion other than that the syenite is igneous.

Its relation to the hypersthene of the Coolins may be studied about the middle of Glen Sligican; where, in the lower slopes of Blaaven, it is seen to be continued from Marscow and overlaid by the dark rock, and that in a manner very suggestive of denudation of the syenite previous to its envelopment. In the steep slopes of the south side of Hart o' Corry it at first appears to be interbedded with the darker rocks; but a closer inspection leads to the conclusion that the small portions of syenite here seen had been connected with the large masses on the east side of the glen,—that intruded processes of the hypersthene had been thrust into the mass of the older rock, and that the cutting out of the strath had severed the original connection.

As regards its connection with stratified rocks, Macculloch has pointed out that "it is found for a considerable space distinctly incumbent on the limestone and shale of the lias on the shore near Broadford"; that it alters these rocks;—and it can be seen on the north slopes of Glamich to have broken through, and tilted to the north at a high angle, limestones and other rocks of the lias.*

Passing to the relationship of the very similar rock of Rum, we find the north of that island to consist of the same red sandstone as that seen to underlie the hypersthene of Garsven; but that red sandstone is in turn, on the east coast of the island, underlaid by a series of grits, schists, and cherty or hornstone-like beds, of prevailing blue, green, and almost black colours.

In making two parallel sections of the island, nearly from north to south, —namely, from Camusplesaig, past the head of Loch Scresort, up the Allt an Eassain, over Scuir na Gillean, and down to the cliffs of Riesval; the second from the shore of Scresort, over Halival, Haiskeval, Ben More, and down to Pappadill, we find as follows:—

Between the north shore and the head of Scresort we walk for a couple of miles over strata, finer, closer, and more sandy than those generally assigned to the Cambrian in the north of Scotland; these dip at angles 7° to 10° to the north-

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* Dudgeon and the Author found on the banks of a stream which flows from the north side of Glamich loose masses of a volcanic breccia—somewhat similar to that of Rum—but which, among fragments of schists and conglomerates, contained many of blue-grey "indurated" limestone.

west. The same rock continues, with slightly increasing angles, up the channel of the Allt for half a mile, when we come upon the junction with the schistose beds; these conformably underlie the red beds at this point. Following up the burn, the dip of the older beds rapidly increases to nearly the vertical, when, after having passed over a considerable thickness of denuded strata, there is a sudden anticline, with an equally high dip in the opposite direction.

There is now evident alteration, a good deal of fracturing,—occasioning many falls in the burn, and the heather-covered holes into which Professor JAMESON tells us he had many falls. In the col between Arstival and Halival we come upon the augitic rock, lying in flat sheets upon the upturned and most irregular edges of the strata; the contact is immediate, but the surface of junction is usually a very rough one. Passing under the toppling cliffs of Haiskeval, we observe many varieties and gradations of the rock. When first seen, it was a rough-grained mixture of greenish-brown augite, and pale, greenish-white, glassy labradorite, in which the crystals, less than half an inch in size, were mutually assertive;—the decomposition of the felspar leaving runs of gravelly-brown sand.

The masses, fallen from the cliffs of the central hill, show gradations in both directions,—both as regards augmentation or diminution of the relative proportion of either constituent, and also as regards augmentation or diminution in the size of the component crystals.

A compound which, from an unusual brilliancy of lustre and brightness of colour, "could scarcely be distinguished from greenstone," is common. Aphanitic masses are equally so; and in the clean-cut face of the towering cliff, the bedded arrangement is markedly developed by the suddenness of the alternations.

In the col (1750 feet) between Tralival and Scuir na Gillean a breccia is seen; the sharp-angled fragments consist solely of varieties of the augitic rock, —not a particle of either sandstone, or schist, or grit being visible; the paste is a green-blue "claystone." This doubtless is the *first* of the breccias noted by MACCULLOCH.

This breccia is sticking on to the sides of Tralival; whether it passes into its mass cannot here be seen. The hill towers for 550 feet above,—a mass of little else than crystalline brown augite, presenting a surface of most repellant roughness.

The bulk of Scuir na Gillean, which overlies the volcanic breccia, is composed of slabby felstone, of a slate-blue colour, and generally of a porphyritic structure,—the imbedded crystals being a white or pale-blue felspar.

The breccia and overlying rock do not seem to rest conformably upon the augitic beds, but to dip to the south-east. Upon the opposite side of Glen Dibadale a patch of the same rock is adherent to the south slopes of an outlier of Haiskeval; and it seems to descend the southern slopes of the Scuir to a lower altitude than on the northern. Be this as it may, it is, on the southern slopes and towards Riesval, seen distinctly to overlie the augitic rock. In the north centre of Arstival a small patch of it is seen in the same position, but in no other part of the island is this bed, or the highly elevated layer of breccia which underlies it, to be seen.

Ascending the north slopes of Halival,—on the second parallel,—we cross the tilted beds of the flaggy, somewhat gneissic rock, which here rises into a shoulder some 1200 feet in height. A slight depression or transverse valley is immediately in front, before us there towers a grandly-terraced conical hill, hardly surpassed in characteristic outline by any spot in Faröe. Of this the boldly pronounced *trappean* beds dip very slightly to the west; while, turning to look northward, we see the near bulk of the Cuchullins distinctly lined, from Garsven to Scuir na Gabhar, with three great belts of rock, likewise dipping slightly to the west.

Descending into the transverse depression, we suddenly come upon the denuded bank of a bed of breccia, lying directly on the edges of the strata of the sedimentary rock.

This is what Macculloch refers to as his *second* breccia, considering it as probably a portion of a vein, posterior to the overlying rock.

This cannot be conceded: the imbedded fragments consist solely of the underlying rocks,—grits, cherty lumps, and sandstones; not a particle of the augitic compound is to be seen; the paste, relatively small in amount, is felspathic and scoriaceous; and it is *a bed* which is overlaid by, and altered at its point of contact with the lowest of the augitic bands.

Moreover, it is laced in all directions by tortuous veins of aphanite, resembling Lydian stone. These do not enter the overlying rock, and therefore in all probability have been an exfiltration from the breccia itself, antecedent to its envelopment.

Neither could it have been an *intrusive* sheet, thrust between the augite rock and the tilted strata;—the sharp casting which the augite has taken of the rough floor on which it lies, amounting in places to an interlocking of the two, forbids such a view. Before it could be tenable that there had been a forcible insinuation of so extended a sheet, the relative quantity of paste to fragmentary matter in the intruding mass should be very great;—that fragmentary matter should also indiscriminately contain augite and schist alike; a mass so intruded could not possibly contain the *red* sandstone; and, lastly, on such a view, the brecciated bed should have affected the augite rock,—but in it no change is visible.

Gravel beds of loose augite crystals lie glancing in the sun, with splendent but never *bronzy* lustre; and during the scaling of the cliff-capped terraces, abundance of loose rough crystals of glassy striated labradorite may be got. One bed of pure labradorite also occurs, it is a mass of small crystals in sugar-loaf arrangement.

The ascent of Haiskeval from the north is, from the toppling condition of the huge blocks into which its aphanite beds have been rent, attended with very considerable risk, and is, for a single individual, quite impracticable.

Its beds show many transitions from coarse to fine grain, from loose to close structure; from brown opaque to green translucent augite. The rock, besides the two ingredients mentioned, seems generally to contain yellow olivine; bronzy Biotite is sometimes present; martite in thin bands is an accessory; and the occurrence of a radiated zeolite lends its aid to the proof of the matrix being of igneous origin.

The continuation of this line of section leads to the claystone porphyry mass of Ben More, which is altogether similar to that of Scuir na Gillean. At the shore at Pappadill the sandstone is cut off, bent upon itself, and enveloped in trap, which takes the form of a coalescent bundle of vertical dykes.

This is a fine-grained augitic rock, similar to the central mass; but as no connection can be shown to exist between the two, no argument can be drawn from it, or probably from the existence of a lake which obscures some faulting.

A section drawn from the east shore a little south of Strone, westward to the point of Bridianach, or to Scuir More, shows first, the deep-seated green flags, here dipping slightly to the north-west: they are frequently coated with rock milk, and the cement of the gritty varieties is calcareous. Following these into the higher ground, we find, as we approach the craggy central rocks, that the dip is inverted; and finally we have the augite rock showing itself on the crest of their ruptured and upturned edges. Passing over the central group of augitic hills, the land falls into a valley, the west side of which is entirely formed by the extended bulk of Oreval,—a felspathic or granitic hill ("syenitic granite"). The junction cannot here be seen on account of surface covering. At Harris, on the south shore, MACCULLOCH states that the one rock insensibly passes into the other. Seen from a boat, it seemed as if the augitic rock here exhibited intrusive features, and ramified for some little distance through the other, as though a restraining and pre-existent mass.

Westward of the syenite, red sandstone, in enormous and high tilted beds, falls away to the west, forming sheeted and terrific slopes, at an angle of about 78°, and probably of 800 feet in height.

At the north end of Oreval, at Scuir More, and at a hill north of Oreval, amygdaloidal and basaltic rocks hang on to the skirts, or overlap the tops of the syenitic hills,—overlap unconformably the sandstone,—and overlie a denuded spur of the augite rock; these three connections being seen within a narrow compass. The many-bedded strata of these "trap rocks" dip slightly to the west.

The conclusions to be drawn from these facts are, as regards Skye, small in amount, but they are irresistible.

An igneous rock, of a granitic or felspathic nature, burst through, overlapped, and altered pre-existent liassic rocks; to be in turn invaded and overlapped by a series of rocks, up to this day termed hypersthenic.

In Rum the inferences are more extended. Red sandstone rocks, assigned to the Cambrian epoch, and underlying beds conformable thereto, on three sides fell away from a central mass at high angles; on two of these sides these rocks assume low angles, and a uniform strike and dip, at a short distance from that central mass.

When in close contact with that mass they are fractured and altered; and they are overlaid by that mass. That mass has the composition and the structural arrangement of recognised igneous rocks; and it contains zeolites. These masses, therefore, are *igneous* rocks,—in the first island manifestly more recent than the Lias; in the second as manifestly more recent than the Cambrian.

But, inasmuch as the earliest and the latest members of these rocks—the syenite, and the amygdaloids with basalts—are common to the two islands, while the intermediate members show but trifling lithological differences, if any, the conclusion may safely be drawn, that the two rock masses were, as a whole, contemporaneous;—that, instead of being metamorphosed sediments of very great antiquity, they are *eruptive rocks of comparatively very recent times.*

In adducing *chemical evidence* to show that the rock in Rum, termed augitic by MACCULLOCH, is but a modification of that which occurs in Skye,—and to which, from its containing a mineral of supposed specific distinctiveness, he gave the name of hypersthene rock, since contracted into hyperite,—I have in the first place to refer to my paper on the Felspars. In this I have shown, by the analysis of specimens purposely received from the hands of others, that the felspar of the rock is labradorite. Though I have not yet analysed the felspar from the rock of Rum, I have no hesitation in pronouncing it to be the same. Crystals of the mineral may, at the north side of the summit of Haiskeval, be obtained over an inch in size in every direction. These are finely striated, very glassy and pellucid; if they differ from the mineral elsewhere found in Scotland, it is only in their being finer in all respects.

As regards the occurrence of the substance called hypersthene by MACCULLOCH, and faultily analysed by MUIR, it is to be observed that it is, in its lustrous slightly-bronzy form, by no means of common occurrence in the rocks of the Cuchullins. MACCULLOCH himself says that the large characteristic specimens only occur in veins; in which he himself likewise declares that it is to be found in Rum. If we except the known localities, these veins are so rare in the Cuchullins that I have never myself found one of them; or even a single bit of the characteristic mineral, except in loose blocks,—evidently portions of veins.

MACCULLOCH, it is true, maintains that it is the same mineral which, in minute crystals, forms the general mass of the rock; this, it will be seen, is true in a certain sense; but it is certainly not the case that a mineral with a bronzy true-hypersthene lustre forms the general mass of the rock. The mineral which does so is brown and rusty externally, of vitreous lustre, and dark green internally when fresh. It is, in fact, perfectly similar to the variety to be seen in the ordinary rock of Rum; and specially in the more central, somewhat lower range of Baikeval, Arstival, and Tralival. Every feature of the two rocks is the same, only that they are somewhat heightened as regards roughness, sonorous ring, and sterility, in this portion of the latter island. Bare and rugged as the Cuchullins are, there is no spot in them which even approaches in these respects to the ridge of Tralival;—JAMESON speaks of this part of the island as "terrible, bare, rugged;"—MACCULLOCH says "Rum is the wildest and most repulsive of all the islands."

But it can be shown that, accurate and cautious as an observer as Dr MACCULLOCH was in the mineralogical department of his science, he was hasty in founding a species upon such characters as he observed in this mineral; and still more hasty in naming a great mountain mass therefrom.

The marked feature insisted on in the mineral is its bronzy lustre; it conferred upon the rock which it goes to form its own name, from a supposed and assigned unusual resistance to the blows of the hammer.

Now it has to be maintained that it is not consistent, ordinarily practicable, or proper to characterise or confer upon a new creation a name which shall be expressive of the features which it displays during the period of its dissolution and its decay. Yet this is just what Dr MACCULLOCH has in this case done; and moreover that which, in his acting designedly and *expressedly* in the same way as regards the mineral found in the Scuir More of Rum, he laid down as the principle of his nomenclature.

I have to maintain in the first place that the bronze-lustre which has been assigned as the characteristic appearance of this mineral is only to be seen in it *during its decay;* and in the second, that the toughness assigned to it in its name is neither characteristic of it, or a feature of it at all,—but only of the rock mass which it goes to form. If in addition I can show that its composition and properties are identical with those of a well-known mineral, it can have no claim whatever to a title which expresses specific individuality.

The bronzy or semi-metallic lustre is only to be seen *in weathered specimens,* and it only extends in them to a depth commensurate with the amount of their weathering.

The specimens which most markedly of any that I have seen exhibited the somewhat bronzy lustre—and in these there was even a slight amount of the violaceous hue typical of true Paulite—were given me by Principal FORBES. The analysis of these is given; they were from Hart o' Corry. The bronzy portion occupied a layer certainly not the twentieth of an inch in thickness;

the unaltered material beneath having the finest *green* colour, and the clearest *vitreous* lustre of any specimens I have seen from the Cuchullins.

The specimen which showed the semi-metallic lustre to the greatest depth in splitting it up was a loose crystal which I got from GRIEVE;* this was from Drum na Raave, and its analysis is given. It was about an inch in thickness; the small amount of metallic lustre it possessed gradually, in the inner parts of the stone, merged into a dull brown, and finally into a dirty green.

As regards toughness, this is a property which it does not, when separated from the rock, possess in any marked degree; it may be readily split along its cleavages, bruised by a hammer, or powdered in a mortar.

The rock as a whole is somewhat tough in its exterior layers, but not more so than the rock in Rum, and very much less so than many rocks of a somewhat similar type.

The external toughness which the rock presents is doubtless due to surface peroxidation, which cements the granules of the rock; and, acting like a "pan," or like bog iron ore, excludes in greater part the falling water, and so, preserving it from decay, enables it to a great extent to present that appearance of indestructibility which may have influenced Dr MACCULLOCH when assigning to it a name.

As my analyses show that the mineral is merely augite of a composition very similar to that of Rum, the question now is, what are we to call the rock which contains it?

The mineral it was, which was supposed by Dr MACCULLOCH to confer upon the rock its toughness; and so, from the mineral, the rock was called hypersthenic. This has now been changed for the somewhat meaningless abbreviation of hyperite.

There does not seem to be much judiciousness in attempting to change the name. Similar rocks which occur elsewhere are known by it; and although it may prove to be the case that they also contain no true Paulite, yet there is no recognised name which can be altogether fittingly applied to it.

Were we to be guided only by mineralogical characters, as it consists almost solely of labradorite and augite, we would be constrained to call it dolerite; but as no dolerite presents the same lithological features, and as from its position and physical structure it has much the appearance of having been the crystalline slag which plugged the void from whence active volcanic forces had once operated,—filling up at the same time the rents resulting from their operations,—it must rather be regarded as plutonic than a volcanic rock.

Of such rocks it can only fall under either diabase or hyperite. Its freedom

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* My friend, who most generously made over to me the cherished gleanings of his many and even midnight wanderings among the Coolins,—repelling my slanders concerning the beautiful product of the hills he loved so well,—put the most bronzy specimens remaining into the hands of a lapidary to be cut into brooches. He was horrified when the stone was returned to him in fragments, with the explanation that they were "all rotted."

from chlorite, however, must be held to exclude it from the former; and we are thus brought back to reassign it to the latter—a hyperite in which augite takes the place of hypersthene, and so one under the category of which the Rum rock also immediately falls.*

The augite of Rum is of at least two well-marked varieties. The first and most common may be picked up in loose rough crystals, which strew the gravelly banks under the pyramidal second heave of Halival; they lie glancing in the sun in quantities. This variety is dull green when fresh, but always opaque; they weather brown, but never bronzy (iron-brown, JAMESON); they resemble the ordinary variety of the Cuchullins.

The second variety is transparent or translucent. It was first noticed by JAMESON, who considered it to be pitchstone. He thus describes it:—"Crystallised pitchstone, top of Halival, Rume. It is of an olive or dark leek-green colour, and has the usual lustre, transparency, and hardness. The crystals are immersed in a rock formed of felspar, with a few scales of tombac-brown coloured mica. They are from the tenth to the half of an inch long. I could not discover the form of the crystals, on account of their being much broken. It is well deserving the attention of those who visit Rume."

Crystals of considerably larger dimensions than here noted are to be found. Bright yellow olivine is another associate, and the labradorite crystals are finely striated, and the most pellucid in Scotland. This transparency is also the marked feature of the augite; and, taken in conjunction with its vitreous lustre, and the fact that it does not cleave, but yields conchoidal fractures, quite accounts for JAMESON'S having taken it for a variety of pitchstone. It might also quite excusably be taken for olivine, to which it bears a much greater resemblance than does the associated yellow olivine itself.

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* While correcting the proofs of this paper, I have, for the first time, had the pleasure of perusing Professor JUDD'S classical research on the rocks of Mull. In this I find that he, throughout, unhesitatingly calls the Rum rock *gabbro.* Now—whatever rock Von Buch originally applied the name to,—the term gabbro is now, I believe, universally attached to a combination of labradorite with *diallage.* I went round the whole shore of Rum, traversed the island in three directions, ascended seven of its highest hills, skirted the flanks of these, and yet, not only did I never see a particle of diallage in the island, but I never saw a bit of the augite which even tended in features to that modification. I need hardly, therefore, say that I saw no gabbro. It may be held that this is mere fighting for a word; but it is not so, it is fighting for precision in language. If *gabbro* is to be extended or loosened in its application so as to include an ordinary augitic rock, I have nothing to say, further than to show that it would be most unadvisable to do so. But till a consensus of geologists resolves to do so, it must tend to endless confusion, that the author of so masterly a research as that referred to,—speaking almost *ex cathedra,*—applies the term to a rock so absolutely different in appearance as the present is from any typical gabbro. I am the more sensitive as regards *writing by the book* in the present case, because, so far as my experience goes, the gabbros of Scotland—that is, the rocks carrying *thin, foliated lustrous augite,* and they are very typical—are all metamorphic. Be this as it may, it is not advisable that the name to be adopted for a *class* of rocks should be one which in the past has designated a variety, the marked feature of which is, that it contains a peculiar modification of that substance which is the chief constituent of all the members of that class. We might, it is true, designate that variety by the term diallagite; but let us understand each other, as a necessary preliminary to our entertaining the hope that others will understand us.

Altogether this Rum augite must be called gem-like. Its specific gravity is 3·481, being the heaviest augite I know.

1·302 grammes yielded—

Silica,	50·537
Alumina,	3·348
Ferric Oxide,	1·338
Ferrous Oxide,	4·423
Manganous Oxide,	·23
Lime,	21·419
Magnesia,	17·05
Potash,	·252
Soda,	·53
Water,	·706
	99·833

Pseudo-Hypersthene.

From Hyperite.

15. The specimens I have now to describe have been almost uniformly and very persistently set down as true hypersthene (*Paulite*). Macculloch, Thomson, Forbes, Traill, Nicol, and Greg have all called it so; Jameson termed it "glassy actynolite," which is certainly not further from the truth. Haughton and V. Rath analysed it, giving it different names, but without pointing out the previous error. The writer has always disputed its identity with the Paulite of Labrador.

To be assured that the analyses were conducted on specimens maintained to be true Paulite, and conclusively to settle the point as to that mineral occurring in Skye, the writer obtained his specimens from individuals who presented them as *hypersthene;* and he analysed examples from different spots in the Cuchullin range.

From Corry na Creech. Specimens from Principal Forbes. In large, lustrous, greyish-green crystals, associated with greenish-white labradorite.

Cleavages and angles:—

Cleavage *a.* (*i i*) perfect, flat, splendent.
b. (*i i*) common, interrupted, somewhat dull.
M. (I) very rare, but nearly perfect, vitreous.

Angles *a* on *b*=90°,
M on *a*=133° 38′ to 134° 08′;—133° 58′ common,
M on M=87° 56′,

which are augitic angles.

Specific gravity, 3·3293.

25 grains yielded—

Silica,	53·046
Alumina,	4·816
Ferrous Oxide,	11·389
Manganous Oxide,	·078
Lime,	19·808
Magnesia,	11·576
Water,	·626
	101·339

Seemed pure.

16. From Hart o' Corry. From Principal FORBES. In large green crystals, which weather pale, and assume on an outer film a lustre which is metallic and somewhat bronzy. Associated with magnetite, pale-green labradorite, anorthite (?), very little olivine, and traces of earthy chlorite, or a substance resembling it.

Specific gravity, 3·329.

1·505 grammes yielded—

Silica,	51·362
Alumina,	1·662
Ferrous Oxide,	8·968
Manganous Oxide,	·332
Lime,	20·837
Magnesia,	16·471
Water,	·54
	100·172

9·185 per cent. of the silica insoluble; possible impurity, labradorite (?).

17. From Drum na Raave (Rabm). Specimens from Mr GRIEVE. Slightly weathered, with a brownish-green colour, and a slightly bronzy lustre; no impurity visible to lens, but there were barely visible cavities which had held magnetite; which therefore may be present in minute quantity in the centre of the mass. Was associated with pale-green translucent labradorite; this, when in contact with the somewhat bronzy hypersthene, is usually of a grey colour.

Specific gravity, taken on a very fine piece of 641·15 grains, = 3·335.

The magnet extracted nothing from the crushed mineral.

Cleavage *a*. perfect, splendent, and bronzy.
b. flat and lustrous.
M. interrupted and somewhat dull.

Angles *a* on *b* 90°.
M on M 87° 22′ to 87° 05′;—87° 15′ common.

25 grains yielded—

Silica,	51·936
Alumina,	1·322
Ferrous Oxide,	13·9
Manganous Oxide,	·25
Lime,	19·363
Magnesia,	13·85
Titanic Acid,	·38
Water,	·2
	101·252

Possible impurity, magnetite.

18. From near the shores of Loch Scavaig. Specimen from Mr Grieve. Large, dark-green, cleavable masses, associated with large (one and a half inch) crystals of grey striated labradorite, and, more rarely, imbedded magnetite. Specific gravity, 3·321.

1·573 grammes yielded—

Silica,	49·268
Alumina,	·222
Ferric Oxide,	2·17
Ferrous Oxide,	12·15
Manganous Oxide,	·381
Lime,	20·256
Magnesia,	14·812
Water,	·719
	99·978

6·193 per cent. of the silica were insoluble. Possible impurity, magnetite.

The above are all varieties of the pseudo-hypersthene form of augite, but furnished to me as typical specimens of true hypersthene; and they were the finest and most typical specimens of the peculiar mineral of the locality which I have seen. They are not Paulite,—I have never seen, nor do I believe in the existence of that mineral here at all. The likest thing to it I have seen from the Cuchullins, is a polished specimen in Dudgeon's Cabinet, found not far from Sligican Inn, and which I believe to be Biotite lying in a peculiar position. The *structure* of Paulite under the microscope is perfectly characteristic and unmistakable; the stone embraces a multitude of parallel flattened pores, of such dimensions as to reflect light of a mingled purple and dark-brown colour.

I have, in the table, appended to my own, three other analyses of the Skye mineral; that by Muir is imperfect as regards the separation of certain of the bases; but it is palpable that all were made on specimens from the same hill range, and that all are augite.

Allomorphs of Augite.

Vitrified Augite—Augitic Glass.

From Volcanic Rocks in the Coal Formation.

19. So far as I know, this peculiar variety of the mineral has not before been described.

I first obtained it in a small dyke in the tufa, a little to the west of the old summer-house on Elie-ness. It occurred totally filling small druses, or else in rounded lumps of the size of pigeon's eggs; these lumps were fissured in every direction to such an extent that the mineral fell to pieces on attempting to extract it. The fragments showed conchoidal fractures on all their sides: a high vitreous lustre, a deep greenish-brown colour, and they were slightly translucent on the edges. They were set down without hesitation as a highly ferruginous and somewhat dull-lustred olivine. No cleavage could be obtained; the fragments, on the smallest application of force, falling into minute portions, with a facility which seemed to point to unequal tension in the particles—as in the case of the Prince Rupert's drop.

The only associated mineral was pyrope in imbedded fragments.

Similar specimens were also obtained from an intrusive dyke, which cuts tufa about half a mile to the east of the above-mentioned locality. There were here no closely adjacent minerals, the nearest being Delessite, sanidine, glauconite (?), cleavable and brilliant black hornblende, and nigrine.

The mineral was very dense, reaching the high gravity of 3 · 327.

1 · 303 grammes yielded—

Silica,	· 625	
From Alumina,	· 014	
	· 639 =	49 · 04
Alumina,		9 · 71
Ferric Oxide,		1 · 25
Ferrous Oxide,		5 · 156
Manganous Oxide,		· 306
Lime,		16 · 254
Magnesia,		16 · 884
Potash,		· 313
Soda,		· 79
Water,		· 305
		100 · 008

Insoluble silica, 3·286 per cent.; possible impurity, Delessite—the rifts being sometimes lined therewith.

I have also obtained the same substance at the Giant's Causeway in Ireland, both in the basalt and in a bed of red clay; (this is the "plynthite" of THOMSON, which I find to be bole, and possibly was a bed of overflowed and calcined mould). More lately I have obtained it in the basalt at Kinkell, along with black lustrous hornblende, sanidine, and Delessite.

From Volcanic Rocks in Old Red Sandstone.

20. This specimen was forwarded to me by Professor GEIKIE. It was of a dark bottle green colour, and high vitreous lustre; the specific gravity was 3·36.

1·3 grammes yielded—

Silica,	·591		
From Alumina,	·008		
	·599	=	46·076
Alumina,			11·391
Ferrous Oxide,			7·921
Manganous Oxide,			·461
Lime,			16·067
Magnesia,			15·653
Potash,			·818
Soda,			1·058
Water,			·38
			99·825

Lost in Bath, ·142 per cent. of moisture; probable impurity, a trace of sanidine, or even a more sodaic felspar.

The large quantity of alumina in both of these augitic glasses is remarkable.

In forwarding this augite, November 2d, 1877, GEIKIE writes:—

"You should receive to-morrow a parcel containing the pieces of the John o' Groat's augite.

"I have not time to send you full particulars at present, but will do so in time for insertion in your paper. It occurs in rough, rounded, and broken lumps, imbedded in the coarse agglomerate of a volcanic rock, belonging to the Upper Old Red Sandstone times.

"I have had it sliced, and have examined it microscopically. It is not in the least dichroic, and has none of the other features of hornblende. I have no doubt it is augite. Some of the rough granular pieces suggested, at first, olivine to me. But I found the granular character to be exceptional, and the mineral is fusible, and with hardness equal to 5·5.

"It is by far the most interesting volcanic augite I am acquainted with."

On the 4th December he writes :—

"The specimen I send you is one of a number of pieces of what seems to me to be a form of augite, which I have found in the most northerly volcanic rock on the mainland of Scotland.

"This occurs on the shore, half a mile to the east of John o' Groat's House.

"While the whole of that part of Caithness lies upon the so-called 'Caithness flagstones' of the Old Red Sandstone, the coast at the locality in question affords an admirable section of the red sandstones, with blue and gray flags and shales (sometimes with well preserved ichthyolites) which form the highest visible portion of the Caithness flagstones.

"No igneous rocks of any kind are interbedded among these strata, or in any other part of Caithness.

"It was, therefore, with the utmost surprise that, when examining the coast-line in the year 1874, in company with my colleague in the Geological Survey, Mr R. N. Peach, I came upon a well-marked volcanic 'neck' or pipe, which had been drilled through the John o' Groat's sandstones.

"I was not at that time aware of any truly contemporaneous igneous rocks associated with any part of the Old Red Sandstones in this northern region, though I knew of the existence of what are called 'trap dykes' both in Caithness and among the other Orkney Islands.

"But in prolonging my observations into the opposite island of Hoy, I saw abundant evidence of the intercalation of true lavas and tuff, at the base of the Upper Old Red Sandstones.

"These sheets of volcanic material were found to lie in complete discordance upon the Caithness flagstones. I found among them some material exactly similar to that which fills the John o' Groat's neck. I have no doubt, therefore, that this neck marks the chimney of one of these Upper Old Red Sandstone volcanoes.

"I again visited Caithness last summer, accompanied by another Survey colleague, Mr John Horne, and made further notes about this interesting locality.

"The neck is irregularly oval in shape; the diameter, on this edge truncated by the sandy beach, being about 300 feet. The sandstone round it is somewhat hardened and jointed, as may usually be observed in similar cases. The whole space of the neck is filled up with a coarse tumultuous agglomerate of a dirty green colour, which makes it stand out in strong contrast to the surrounding red sandstones. The paste of this rock is a granular tuff, formed evidently of comminuted diabase.

"The blocks imbedded in it are angular, sub-angular, and rounded, of all sizes up to masses of a yard or more in diameter.

"I observed among them pieces of diabase (these form the great majority), red sandstone, grey flagstone, gneiss, and abundant fragments of the augite, which I send you.

"Some of the diabase blocks were of great size, but as veins of a similar rock traverse the neck, it was not always possible to tell which were parts of veins and which were really detached blocks. The augite fragments are all rounded, as if they had undergone considerable trituration before they came to rest. Here and there they show a rough cleavage-face, which may have been produced by their striking against another ejected block. They vary in size from mere seed-like grains up to blocks at least 8 or 9 inches in diameter.

"No sign of fusion, or even of good baking, was to be seen in the bits of imbedded sandstone in the agglomerate. I should add that I did not observe any minerals associated with the augite, except such as had resulted from the alteration of the tuff.

"There cannot be the least doubt that the fragments of augite are true ejected blocks, and did not originate in the matrix where we found them.

"They are by no means the only examples to be met with among the Scottish tuffs, though they are very much larger than any I have before observed. Next to these in size, were some which I found in a neck on the shore to the south of Fairlie in Ayrshire, belonging to the Lower Carboniferous volcanoes."

These masses proved on examination to be an augitic glass, somewhat similar to, but neither so much vitrified or flawed as the Elie or Giant's Causeway specimens.

It will be observed that GEIKIE remarks on the granular pieces suggesting olivine to him; which mineral the Elie specimens were considered by the writer to be, until their analysis established the occurrence of augite in this peculiar allomorphic form.

The resemblance to olivine is, in the John o' Groat's specimens, by no means—even in the most granular portions—so striking as in those already mentioned; while some portions of the specimens show, unchanged, the features of ordinary augite.

This fact of the specimens presenting a changed and an unchanged portion—having been caught or arrested in the middle of the process of change—renders them certainly far more interesting, from the amount of information they convey, than those in which the glassy condition is perfect.

"Rough, rounded, and broken lumps." The words well describe them as hand specimens. We will consider these features in a different order, that we may see what they teach. "Broken"; "rough" because broken; and "rounded"—that is, partially rounded, or they would not still be rough,—rounded on account of some attriting action which operated after they had been broken.

Evidence of their having been broken, i.e., that they are the fragments of a larger mass. Mere roughness could not prove this; the steam holes of igneous rocks, though usually more or less spherical, and presenting rounded or curved outlines, are not invariably so; so that a mineral which ultimately plugged these, through endosmose, would, in the taking a cast of a rough cavity, assume a rough outline; while again, if the augite, after thorough fusion, was on the abstraction of the heat to assume the solid state, while the rock matrix was still in a plastic condition, it might form a confusedly crystalline mass with rough surfaces.

That the roughness of outline is not here to be so explained, but is really due to irregularity of fracture, is shown, *first,* by large and fairly brilliant cleavages of the augite abutting against what have been the sides of the masses, as shown by a thin investing layer of calcite and skin of saponite; and abutting at an angle to these sides, which could be formed neither by a crystalline arrangement which radiated from the sides of a cavity, nor by one radiating from the centre of the mass. The above cleavages, *secondly,* are of such a size as show that they must have originally belonged to masses of much greater dimensions than those in which they are now found. And *thirdly,* the rough fractured sides occasionally exhibit—as also do internal fractures—portions of crystals of a size much too great to have been formed during the solidification of masses so small as these.

As regards the rounding, it amounts in general to little more than the abrasion of the protruding cleavage angles, not to the giving a rounded outline to the general mass. As the surfaces are now covered with a thin coating of calcite, the nature of the abrasions cannot be seen; it certainly has not been of the continuous nature of wave battering, or river grinding, or sand blowing.

As regards internal structure, there is here the same confused and hackly fissuring described in the Elie mineral, the fissures being however narrower, and the whole mass more coherent. These fissures are, for the most part, filled with saponite (?)—the probable result of incipient change,—and rarely with calcite.

The recent fractures are, like those of the Elie, conchoidal; though here and there true cleavages in broad sheets cross the conchoidal fractures, and even lie in directions non-accordant with the cleavages of the less altered portions of the mass. These latter have been, in fact, posterior to the vitrifaction. The conchoidal fractures exhibit a perfect bottle-glass appearance, very similar to that of augite melted in a crucible. Close inspection with a lens, however, shows on the crystalline surfaces a cupped appearance, quite similar, though not so minute, as that which in decomposed glass was shown by Sir DAVID BREWSTER to be the cause of the iridescent colouring. Closer inspection still, demonstrates that these cup-shaped hollows had been the receptacles of little spheroids of felspar, which may be seen arranged in layers in portions of the mass of the stone. I call the spheroids *felspar* merely from the lustre of their cleavages, and from the presence of alkalies,—as shown by analysis.

Two questions at once connect themselves with these "rough, rounded, and broken lumps," which helped to choke up a former volcanic orifice.

The first—*Whence came they, or what were they broken from?*

The second—*Do they, in themselves, give us any information as to the amount of heat to which they had been subjected in the process of their vitrification?*

These two questions may resolve themselves into the more general one—*Is there any liklihood that we may be able*—by their recognition as components of a well-known stratum, or by the amount of alteration which has been effected upon them by heat—*in any measure to arrive at an estimate of the depth at which the volcanic turmoil originated?*

Such a question is too wide a one to be entered upon in its entirety here; but although a consideration of it, in the two directions indicated above, may lead us but a small extent to any definite answer, yet to that small extent it unquestionably must lead us.

All fragments must have been brought by an outflow from beneath the parent rock from which the fragments were torn: the amount of change, if incomplete, is the register of *the highest degree of the heat,* which was the active agent of the change.

Asking ourselves in this instance: Whence came these lumps of augite? we are able to reply unhesitatingly,—from no part of the Old Red;—no formation, not even the chalk, could have less claim *in itself* to an augitic mineral. Inferior to the Old Red, we have in this part of Scotland gneisses and flaggy schists, thrown into wavy folds, where they cross the breadth of Sutherland. These are probably similarly crumpled, and are at no very great depth where they underlie the sandstones here. Beneath these fractured and folded rocks, again, we come upon the gently inclined beds of a highly siliceous conglomerate; which beds vary, not at all in the nature of their constituents, but only in the amount of attrition to which they have been subjected; they are altogether destitute throughout of vein, or seam, or cavity, within which a crystalline or cleavable mineral had space or time to arrange its atoms.

Going lower still, we may walk for tens of miles across the upturned edges of a rock, the beds of which strike vertically downwards with such an unvarying determination of purpose that we are impressed with the whimsical fancy that if we ever again meet with them it must be when they come out at the other side.

After many traverses of the formations mentioned, my experience does not enable me to indicate any one spot or bed which could afford an augite which is the representative of this.

The Shinness limestone locality shows, throughout its great abundance, no trace of such a variety. I know of but a single doubtful occurrence of augite in the midst of the vast abundance of hornblende in the lower gneiss.

The well-known conversion of hornblende into augite by heat may possibly account for its occurrence; but in this case a double action is required. The first—that by means of which the transmutation was effected—resulting in the production of the broadly cleavable masses. The second—that which produced the final vitrification of these masses. Of any such double action, we have here evidence only of the feebler—the vitrifying.

The second part of our query, namely, the topmost limit of the heat to which the augitic lumps had been subjected, is more easily answered; seeing that a near approximation may be arrived at by direct experiment.

Many years ago I had an opportunity of determining the temperature to which fragments of different of the members of the lower coal measures had been subjected, when caught up among the ashes which form the small volcanic cones and necks around the shores of Fife.

In the shore sections of the wave-worn bosses of tufa which stud the coast near Kinkell, much fragmentary matter of various description is to be seen promiscuously imbedded, and characteristically and instructively altered.

The bituminous shales have lost all their illuminants; and, of organic matter, retain only the blackening stain of sparsely distributed carbonaceous particles.

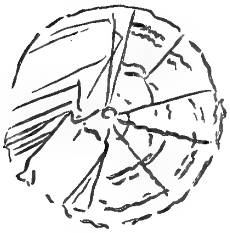
The encrinal limestone has become granular and crystalline.

The included freestone masses present themselves as a quartzite,—with firmly agglutinated grains, splintery fracture, elastic resiliance, and sonorous ring.

Shivery masses of carbonaceous or "black chalk" clay show every stage of a passage into Lydian stone; while volcanic bombs of the latter—perfect in the transmutation—lie impacted in the encircling volcanic mud, with surfaces which exhibit rounding and abrasion.

We have here abundance of material to work upon, in estimating the quantum of the energy expended in the production of these changes.

Close observation of the altered calcareous fragments shows most clearly that the period of time during which they had been subjected to the heat had been of but short continuance; for while it will be found that the smaller of these fragments are, to their very centres, converted into a crystalline mass, reflecting light from innumerable facettes, the larger exhibit the change only to a certain depth, the interior presenting itself as an impalpable calcareous basis, studded occasionally with joints of the encrinile stems.



More than once I have found a larger than ordinary joint, in part converted into cleavable calcite, which flashed back reflected light; while the greater part showed the structure of the fossil in no degree effaced.

In the estimation of the temperature to which these erupted fragments had been heated, I operated upon the ordinary bituminous shales of the district;—the imbedded shaly coke which is found in the tuff;—and on the bombs of Lydian stone imbedded therein.

First, as regards the shales, that from the Kenly burn, the nearest point where it crops out, yielded—

S. G.	Water.	Gas.	Carbon.	Ash.
1 · 79	4 · 54	25 · 29	9 · 27	60 · 09

That from Kinkell imbedded in the tufa—

2 · 57	2 · 64	6 ·	· 44	90 · 92

The gas from the latter is non-combustible, or almost so. The shale from the tuff is grey-black,—very dark when the small amount of carbon is considered.

From the Kinkell shale, therefore, the greatest amount of the gas and water has been distilled off, and the greatest amount of the carbon burnt away.

It was next ascertained that the shales of the district parted with their gas below a red heat; for antimony did not melt when inclosed along with them in the retort in which the distillation was conducted.

In working downward, as regards temperature, in order to ascertain the lowest point at which the change could be effected, the following process was adopted—

A flat block of iron, of some pounds in weight, had a dome-shaped cavity turned out of one of its surfaces. On the surface of a large quantity of lead, which had been melted and cooled in an iron pot, a number of fragments of the bituminous shales were placed, and covered with the iron block, so as to be inclosed in and under the dome. The lead was melted in a dark room in a Bunsen furnace, which fitted closely round the sides of the pot, so that only the faintest glimmer of light escaped from between the surfaces of contact.

Upon the melting of the lead, the floating iron sank slightly into it, effectually inclosing the shale fragments. Very shortly after the lead had liquefied, the iron was heard bumping against the sides of the pot; the sound of bursting bubbles was likewise audible; and wreaths of a flickering lambent vapour, which reflected apparently more light than that which escaped from the crevice in the furnace, were seen to ascend for a height of a foot or more from around the sides of the floating iron. The form of the iron was clearly delineated in black shadow in the midst of these luminous fumes, while the surface of the lead emitted no light whatever. Whether these gaseous exhalations be phosphorescent or not, the experiment was not arranged to determine,—they so appeared. The result sufficed to show that the organic matter could be and was driven out of the shale at a heat below redness.

A quantity of the shale was next inclosed in a capacious iron retort, along with a large quantity of mercury; the retort placed in a furnace, and the orifice of its tube conducted beneath jars filled with water, and standing in the pneumatic trough.

There distilled over—*first,* water; *second,* carbonic acid, mixed with white vapours of a hydro-carbon; *third,* a combustible faintly-illuminating gas; *fourth,* mercury, along with vapours of paraffin, a highly-illuminating gas, and a quantity of one of the paraffin oils; *fifth,* mercury abundantly, much heavy oil, and little feebly-illuminating gas; and finally, after anything but mercury had ceased to distil, the retort was opened; when there was found much mercury remaining, with the fragments of shale, unaltered in shape and bulk, darker in colour, and somewhat greasy in appearance.

On being analysed they yielded almost no volatile ingredients, and about 5 per cent. carbon, which is less than when decomposed by a red heat.

These shales, therefore, are decomposed, partially at a temperature somewhat below that at which mercury boils, and totally, as regards illuminants, at that temperature.

Upon the examination of the Lydian stone bombs, it was found that they contained 6·9 per cent. of water, of which 1·01 was hygroscopic; now this is about the normal quantity of water for that mineral wherever obtained; the heat, therefore, had not reached the point at which Lydian stone is dehydrated.

In operating to discover the temperature at which ordinary Lydian stone is dehydrated, it was found to lose, when continuously heated just below visible redness, 1·963 per cent.; at a feeble red heat, a total of 4·272 per cent.; at a full red, 4·503 per cent.; and when heated to whiteness in the furnace, 5·889 per cent. At the full red heat the powdered mineral slightly agglutinated; while at the white heat it fused to a blebby scoriaceous slag, of a fawn colour, very similar in appearance to some vesicular lavas.

The data presented to us by the first experiments were that the shales had lost all their combustible ingredients, while the Lydian stone had not been deprived of any of its water. We have ascertained that the shales are totally decomposed at the temperature of boiling mercury, while the Lydian stone is partially decomposed at a heat below redness; we are therefore in a position to say that—though the restraining effect of pressure may have counteracted the decomposing power of a higher temperature at great depths,—the temperature at which the ashes were *finally ejected* from the volcanic vents, which ruptured the Lower Coal strata, probably lay between 660° and 900° Fahr.

Let us now see what data we have to work upon, in attempting to estimate the temperatures at which ashes, cinders, mud, and fragments of penetrated rocks were ejected in Old Red Sandstone days.

As regards the augitic masses described, we have two circumstances to found upon: the first, that the included felspar—clearly one or other of the soda felspars, probably labradorite—had been so perfectly liquefied that it had assumed spherical forms; a fact which also necessitates at least a viscous condition in those portions of the augite which included the felspathic spheres; the second, that we have, in the old cleavages and unaltered portions of the augitic masses, evidence either that the heat had not been high enough to vitrify them throughout, or that the period of time during which they had been subjected to that heat had not been sufficiently extended to enable them to be uniformly altered. In other words, the heat had been such as thoroughly to liquefy the felspar, but not such as thoroughly to liquefy *masses* of the augite.

What heat then had sufficed?

The process employed in the determination of the water in labradorites had shown that mineral, and the plagioclastic felspars generally, to be liquefied at the temperature of a full, but hardly a bright yellow heat.*

The point of liquefaction of the augites varies largely,—from readily fusible varieties, to the almost infusible diallage.

Direct experiments on the mineral in question were therefore necessary.

Preliminary trials in the blowpipe showed that ordinary-sized chips of the mineral were only rounded on the edges, or fused with difficulty in the best flame that could be obtained; but when subjected to FLETCHER'S blowpipe, in which the air and gas are both passed through red-hot tubes before combustion, a perfect glass, of a colour somewhat paler than the mineral, was readily obtained.

A quantity, in chips and powder, was heated at gradually increasing temperatures in a platinum crucible; it was found that the highest heat of a fine

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* Orthoclase requires nearly a white heat.

Bunsen burner had no effect in causing agglutination, even when the crucible was inclosed in a jacket.

The crucible and its contents were now subjected to gradually increased temperatures in Griffin's blast furnace. At a bright-yellow heat there was no change, and only when the crucible was at the point of bright ignition, approaching to a white heat, did the powder coalesce and liquefy.

According to Pouillet, this accords with a temperature of 2200° or 2250° Fahr.

Tending to the conclusion that the depth at which the volcanic force operated at the epoch of the Upper Old Red Sandstone was much greater than at the time when the Lower Coal was disrupted.

AUGITE.

	S. G.	$\ddot{\text{Si}}$.	$\dddot{\text{Al}}_2$.	$\dddot{\text{Fe}}_2$.	$\dot{\text{Fe}}$.	$\dot{\text{Mn}}$.	$\dot{\text{Ca}}$.	$\dot{\text{Mg}}$.	$\dot{\text{K}}_2$.	$\dot{\text{Na}}_2$.	$\dot{\text{H}}_2$.	Total.
Malacolite—												
Shinness, white, . .	3·15	53·06	·19	1·77	·47	·15	23·63	19·29	...	...	1·546	100·12
Totaig, blue, . .	3·2	50·69	·03	·93	...	·07	25·78	18·09	·5	1·43	2·62	100·12
Ben Chourn, white, .	3·16	51·59	·11	·33	...	...	22·01	19·59	·49	1·01	4·64	99·75
Glen Tilt, white, . .	3·16	53·24	...	2·71	...	·13	22·77	18·86	...	...	2·18	99·89
Glen Muick, blue, .	3·18	51·	...	1·37	1·59	·38	26·36	17·08	·63	1·12	·26	99·89
Sahlite—												
Ben Chourn, . .	...	54·48	...	...	3·13	·24	22·82	17·58	·44	·79	·42	99·91
Tiree, ...	...	50·54	4·69	4·14	·08	·69	23·59	14·4	·31	·63	1·48	100·51
Eslie, ...	...	49·5	1·96	...	11·06	·4	24·08	10·81	·57	·8	·69	99·86
Coccolite—												
Gruagach, . . .	3·05	49·04	6·09	1·39	2·94	·46	23·34	15·12	·82	·79	·17	100·15
Diallage—												
Balta, ...	2·96	50·23	5·84	...	5·22	...	11·23	21·59	1·2	·58	4·17	100·07
Pinbain, . . .	3·25	51·77	2·1	...	2·96	·31	22·1	18·46	·63	·58	1·08	99·98
Augite—												
Glen Beg, . . .	3·24	54·22	·17	...	6·7	·4	19·57	16·97	·5	·45	·96	99·96
Halival, Rum, . .	3·48	50·54	3·35	1·34	4·42	·23	21·42	17·05	·25	·53	·71	99·83
Craig Buiroch, . .	3·28	50·31	4·48	3·92	5·76	·31	17·57	16·62	·19	·89	·38	100·45
Pseudo-Hypersthene—												
Corry na Creech, green, .	3·33	53·05	4·82	...	11·39	·08	19·81	11·58	...	...	·63	101·34
Hart o' Corry, bronzy, .	3·33	51·36	1·66	...	8·97	·33	20·84	16·47	...	...	·54	100·17
Drum-na-Rabm, . .	3·34	51·94	1·32	...	13·9	·25	19·36	13·85	...	$\ddot{\text{Ti}}$ ·38	·2	101·25
Loch Scavig, . .	3·32	49·27	·22	2·17	12·15	·38	20·26	14·81	...	...	·72	99·98
Cuchullins (Muir), .	3·34	51·35	...	...	33·92	...	1·84	11·09	...	...	·5	98·70
Skye (Haughton), .	...	50·8	3·	...	9·61	1·08	19·35	15·06	...	·66	·6	100·16
Cuchullins (Rath), .	3·34	51·30	·76	...	13·92	·25	20·15	14·85	...	...	·21	101·44
Augitic Glass—												
Elie, ...	3·33	49·04	9·71	1·25	5·16	·31	16·25	16·88	·31	·79	·3	100·01
John o' Groats, . .	3·36	46·08	11·39	...	7·92	·46	16·07	15·65	·82	1·06	·38	99·82

Alteration Products of Augite.

Conversion into Serpentine.

All the serpentines of Scotland which I have had opportunities of properly studying are metamorphic rocks, formed for the most part by a change of augitic and hornblendic rocks—as diallage, euphotide, and diorite.

The serpentines of Unst in Shetland are derived from diallage. Of the two beds to the west of Portsoy, the first from gabbro, the larger apparently from euphotide; the beds to the east, from a rock chiefly augitic.

The peculiar structure of the serpentine of the hill of Towanrieff would lead to the conclusion that gneiss was the original; but the nearest rock is a laminated diorite, composed of labradorite and black mica.

The Green Hill of Strathdon, from diorite rich in augite; that of Culbleen, Coyle, and some of the Shetlands, is obscure. A bed in the Farrid Head, Sutherland, unquestionably from gneissic schists.

Though these conclusions are chiefly the result of geognostic observations of the district, there are many localities where the transition may be traced through a gradual change in the *minerals* composing the rock. Such comparatively molecular transformation may be well studied on the north shore of Swinaness, in Unst, in several places in the neighbourhood of Portsoy,* on the north side of the hill of Towanrieff, and on the northern slopes of the Green Hill of Strathdon. At the last-mentioned locality there may be obtained unaltered, or apparently unaltered, diorite;—the same with the hornblende duller in lustre and softer than normal, and the felspar dull, semi-opaque, and of a greasy lustre;—and lastly, almost perfectly formed serpentine, in which, however, the granular structure of the altered rock is plainly visible. These three occur within the space of a few feet of each other. It is not, however, easy to select for analysis, from rocks—the several crypto-crystalline ingredients of which give way to the transmuting agent at different periods of time—specimens at once typical and sufficiently pure. I have met with more success in this direction in working among the serpentinous marbles—those which contain imbedded granules or patches of serpentine—than I have among the larger masses of the serpentine rock itself.

One fact I would direct attention to, seeing that it has perhaps not been clearly enough considered, namely, that great beds of serpentine must have been formed by the metamorphism of pre-existent rocks *as a whole;* that although the change took place step by step, one ingredient giving way before another, still, ultimately, all participated more or less thoroughly in the change. The molecular or crypto-crystalline transformation had thus as its result a lithological transmutation. To be more precise, where a great bed of diallage rock has been converted into serpentine, the felspar as well as the augite has gone to form the latter. This magnesian metamorphosis of labradorite does not seem to have been sufficiently recognised; but though the general rule is that the augitic mineral is the first which suffers alteration, there are localities in which the felspar would seem to have been first affected. It is true, that in many cases the felspar may not have been converted into true serpentine, but merely into an impure kaolin, which, disseminated throughout a serpentinous basis,

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* Specially at the Bay of Durn, north of the Battery, and the first bed to the west thereof.

may defy individual recognition, from the similitude of kaolin to serpentine itself. Such an intermixture may account for the large quantity of alumina in some serpentinous rocks; indeed, any serpentine rock which contains much alumina may be held to have originated from a *primary* rock of which one or other of the felspars was an ingredient.

Change by Hydration; removal of Lime; of Silica; and partial peroxidation of the Iron.

Pseudo-Augite.

21. Portions of the larger bed of serpentine to the west of Portsoy form, when polished, a highly ornamental stone, there being a mixture of blotches of a bright-red colour throughout a groundwork of green and white. These red blotches here and again assume regular forms—those namely of augite; they are simply pseudomorphs thereof, set in a matrix composed of green steatite, a pale-green hydrated asbestus, and ordinary serpentine. The rough crystalline forms are one or two inches in length; they are sometimes isolated, sometimes radiate from each other, or form a layer of a basement-like structure.

They are seamed with a reticulation of a somewhat darker and slightly harder substance; but are otherwise homogeneous, and of a minutely granular structure.

On 1·302 grammes—

Silica,	·475		
From Alumina,	·011		
	·486	=	37·327
Alumina,			1·13
Ferric Oxide,			4·357
Ferrous Oxide,			4·047
Manganous Oxide,			·384
Lime,			1·204
Magnesia,			36·712
Potash,			·875
Soda,			·734
Water,			13·374
			100·144

Insoluble silica, 1·646 per cent.

As regards the proportions of silica, alumina, and water, this is a true serpentine; the abstraction of iron is, however, only partial, and the peroxidation of part thereof gives the red colour.

22. An increase in the extent of the peroxidation is seen in the following case:—

It is that of a mineral which is found imbedded in the granular massive dark-green serpentine of the Balhammie Hill, near Colmonell, in Ayrshire.

Dr James Geikie, who surveyed this district, calls it *Bronzite.* A very similar, if not identical substance, is found at Knockdow Hill, Lendalfoot, and Byne Hill, Girvan.

The mineral has much resemblance to the enstatitic, or pale variety of bronzite; it is of a pale, somewhat greyish olive-green colour, a greasy sub-vitreous lustre, and is very soft.

Through the kindness of Professor Geikie, a sufficiency of fragments from specimens collected by the officers of the Survey was put into my hands. The imbedded crystals were about one-fourth of an inch in size; their form could not be made out. They had somewhat of a fibrous structure, were semi-transparent, and had an appearance between that of schiller-spar and diallage.

1 · 313 grammes yielded—

Silica,	·491		
From Alumina,	·005		
	·496	=	37 · 776
Alumina,			2 · 123
Ferric Oxide,			5 · 069
Ferrous Oxide,			2 · 095
Manganous Oxide,			· 076
Magnesia,			37 · 014
Potash and Soda,			traces.
Water,			16 · 07
			100 · 223

Loses in bath, 3 · 957 per cent. of water, beyond the above. Was probably impure from non-separable serpentine, to the extent of about 1 per cent.

The total absence of lime seems to indicate that this is an altered enstatite. Be this as it may, the change is the same as that of augite. Enstatite, however, changes less rapidly, and seems to retain, after alteration, much of its original lustre.

Change by Hydration; removal of Lime; of Silica; and total peroxidation, but no removal of the Iron.

23. The smaller—the more easterly of the two beds of serpentine to the west of Portsoy—has an internal structure and appearance which is quite unique.

It protrudes out of the grass-clad bank and extends northward for some hundred feet, terminating in the sea. As it dips at a high, almost a vertical angle, it much resembles an intrusive dyke, for which it has indeed been taken.

The portion which stands in the water has by wave action been separated from the main body, and consists of an altogether unaltered gabbro, with bright green diallage crystals of the size of peas, imbedded in a paste of ladradorite.

The terminal end of the chief mass—that which immediately faces the sea-stack of gabbro—is in a much altered, transition state. The augite is grey-green in colour, dull in lustre, and much softened; the felspar has darkened in colour, being greyish brown; it is somewhat translucent and of a waxy appearance. Unaltered asbestus is associated with it.

A little further southward the change is completed. What had been the felspar is now of a pale-green colour, soft and greasy;—in fact, it is true serpentine. That which now represents the augite has a peculiar appearance. The colour is blue-black, the lustre is feeble and glimmering, being reflected from apparently a former cleavage plain, there being now little or no fixedness in the direction of its fracture. Its specific gravity is 2·618.

The appearance is so similar to the hydrophite of Taberg, that it was believed before analysis to be that mineral.

The peculiar appearance of the fractured rock results from the isolated and promiscuous way in which these almost black crystals are sprinkled throughout a paste of yellow-green serpentine, giving an appearance somewhat similar to that of a leopard's skin.

1·3 grammes gave—

Silica,	·445		
From Alumina,	·004		
	·449	=	34·538
Alumina,			1·159
Ferric Oxide,			15·201
Ferrous Oxide,			·333
Manganous Oxide,			·284
Magnesia,			36·384
Water,			12·197
			100·096

Here the relative proportions of silica, magnesia, and water are those normal to serpentine; the lime has been totally removed; none of the iron has apparently been abstracted, but it has been entirely converted into the higher oxide.

It is a point deserving of much consideration that the portion of this stratum, which is now under the influence of wave action, has suffered no change whatever.

24. Though not immediately connected with *augitic* change, yet as bearing on the origin of serpentine as a whole, I here insert the analysis of the yellow-green matrix in which these black pseudomorphs of augite were imbedded,—the matrix which here represents the original labradorite-paste of the gabbro. Specific gravity, 2·616.

Silica, ...	38·834
Ferric Oxide, ...	2·021
Ferrous Oxide, ...	2·028
Manganous Oxide, . .	·767
Lime, ...	1·074
Magnesia, ...	38·756
Water, ...	16·58
	100·06

So that the felspar here has been converted into a fully more typical serpentine than that which resulted from the change of the augite. This cannot be regarded as a common occurrence; the felspathic constituent of the altered rock usually forming a substance more or less allied to kaolin.

It is, moreover, anomalous in another respect. As will be afterwards shown, the serpentinous change induced in the augitic mineral is generally effected not by the direct introduction of magnesia, but by the abstraction of portions of the other ingredients, and the resultant proportional increase of the non-abstracted magnesia. But here a direct insertion is requisite for the transmutation of the non-magnesian felspar, or at least an interchange between magnesia and certain constituents of the felspar. This will be after considered.

25. My next illustration of the same mode of change was got by Professor Nicol, Mr Dudgeon, and myself on the north slope of the Green Hill of Strathdon, in Aberdeenshire. It occurred in one of the bare spots on that grass-clad hill, near the summit, in pieces about half the size of the hand; they were somewhat loosely bedded. These pieces were of a dull olive-green colour passing to brown; they were of a laminated or flatly cleavable structure, much resembling that of malacolite or sahlite. Between the cleavages there are reticulating crystals of extreme tenuity, apparently of talc, possibly, however, of Brucite. This gives the stone a false glimmering lustre resembling that of enstatite, while in reality it is lustreless, somewhat porous, and resembles a close-grained gingerbread.

It is soft and easily crushed.

The specific gravity is 1·753 when dry,—2·158 after soaking for five hours in water.

On 1·305 grammes,—

			Dry (calculated).
Silica,	·461		
From Alumina,	·005		
	·466	= 35·708	37·412
Ferric Oxide,		12·925	13·542
Ferrous Oxide,		·054	·056
Manganous Oxide,		·229	·24
Lime,		·171	·179
Magnesia,		33·18	34·764
Water,		17·52	13·594
		99·787	99·787

Loses 4·545 per cent. of water in the bath; contained traces of potash and soda.

Change by Hydration; removal of Silica; of the Iron; of most of the Lime; with abnormal and possibly direct augmentation of the Magnesia.

Totaigite.

26. Serpentine is generally said to result from a change induced in augitic minerals, through the operation of what is called "the magnesian process;" *i.e.*, the direct insertion of magnesia by magnesian waters, and the consequent increment of that material in the product.

It is hardly necessary to show that such an operation, acting by itself, could never accomplish the necessary transformation. Pre-existent constituents have to be abstracted, and the mere abstraction of these will, of itself, by the consequent *proportional incrementation* of the non-abstracted magnesia, suffice to determine the required change. A simultaneous intrusion of magnesia is thus in no way required.

In the substance now to be described, however, there is, as regards the magnesia, an undue amount of incrementation,—under the supposition that serpentine was the substance aimed at, so to speak, in the change. Very unwillingly have I attached to a substance, palpably a product of pseudomorphic conversion, a specific name. I do so to direct attention thereto, as a material which will have to be considered in the investigation of serpentinous formation.

About 200 yards to the south-west of the ferryhouse at Totaig, in Ross-shire, there occurs a substance in association with the blue malacolite already noticed, which Mr Dudgeon and I at first imagined might be one of the substances which had been considered chondrodite; but which, answering neither to the description of that mineral nor of any other, was examined and analysed.

It occurred, so far as our observations went, only at the spot mentioned; and, of another colour, at a second spot afterwards to be noticed. Imbedded in the granular limestone surrounding the malacolite, were granules up to the size of peas, of a substance which bore a certain amount of resemblance to chondrodite, but which was so identical in appearance to danburite as to be almost undistinguishable. The quantity of material unfortunately was so small that it had to be picked for analysis under the microscope; and even then, adherent, or rather imbedded malacolite, could not be *absolutely* removed. Its colour was pale fawn; its lustre weak and glimmering; it was slightly softer than danburite; it had distinct cleavages, but also a conchoidal fracture; it was surrounded in the limestone by imbedded granules of yellow, green, and dark-grey serpentine.

Its occasionally investing granules of malacolite leads to the impression that it may be a product of their change. As the surrounding granules of serpentine were frequently in the form of augite, there could be no question as to them; the associated blue malacolite, however, was perfectly unaltered, as were the specks of colourless malacolite imbedded in the mineral. The mineral in itself also did not resemble an alteration product.

1·303 grammes yielded—

Silica,	.48		
From Alumina,	·005		
	·485	=	37·221
Alumina,			·757
Ferrous Oxide,			1·045
Manganous Oxide,			·23
Lime,			5·243
Magnesia,			44·973
Water,			10·643
			100·112

Insoluble silica, 1·422 per cent.

27. Immediately to the south of the pier at Totaig, the limestone thrusts itself as a rounded bluff into the sea; projecting from its corroded surface are numerous confusedly crystalline segregations of malacolite, and very rarely among these, the same mineral as that already described, differing in no respect from it, except in its having a blue-black colour, and a somewhat higher lustre. It was here found in masses of some size; it occasionally invested the crystallised malacolite, the structure and lustre of both being similar.

Its surface, where exposed, was ochre yellow, softer, and serpentinous looking. It occasionally seemed to pass into ordinary dark sahlite, and was sometimes associated with massive green serpentine carrying crystals of talc;—this serpentine was well crystallised in the form of augite.

When steeped in weak acid this totaigite became white. Its specific gravity lay between 2·84 and 2·893.

1·76 grammes yielded—

Silica,	·631		
From Alumina,	·004		
	·637	=	36·193
Alumina,			·264
Ferric Oxide,			·286
Ferrous Oxide,			2·958
Manganous Oxide,			·454
Lime,			3·272
Magnesia,			45·57
Potash,			·252
Soda,			·424
Water,			10·2
			99·973

Insoluble silica, 2·04 per cent.; loss in bath, ·568 per cent.

These two are unquestionably to be ranked as the same substance; the second had certainly the appearance of being an intermediate stage in the conversion of sahlite into serpentine; the difficulty in coming to the conclusion that it was so, lies in the immediate association of perfect serpentine, and also in the supposed intermediate substance containing *more* magnesia than that requisite for the perfected change.

Schiller Spar.

28. This variety—new to Britain—I first found in two serpentinous masses of rock which lie imbedded in the sand, below high-water mark, a few yards to the north of the Black Dog rock, north of Aberdeen. It has, more lately, been found by Professor Nicol and myself in two quarries to the east of Craigie, near Beauty Hill, some eight miles north of Aberdeen.

It is in every way identical in appearance with the Basta specimens, being perhaps somewhat larger in the glimmering foliations. These foliated portions were, in the Black Dog specimens, of a fine leek-green colour; in those from

Craigie the colour was somewhat bronzy—indeed, the finders were guided to the quarry at the latter locality by flashes of light reflected from what looked like bronzy buttons, on a road which had been metalled from the quarry. At both localities the lustrous foliations were stippled with dark green duller portions, consisting of ordinary serpentine. These portions apparently presented the section of former crystals of augite, which had pierced at right angles the less-changed, lustrous and foliated mineral.

If these truly serpentinous pseudomorphs be after augite, it is probable that the still lustrous portions represent diaclasite,* to which they bear a strong resemblance. So closely packed were the dull pseudomorphs in the general mass, that it was vain to attempt absolutely to separate them, but the green glimmering portions were as far as possible selected.

The specific gravity of the Black Dog mineral was 2·649.

On 1·092 grammes—

Silica,	·407		
From Alumina,	·01		
	·417	=	38·186
Alumina,			2·178
Sesquioxide of Chromium,			·276
Ferric Oxide,			·028
Ferrous Oxide,			8·479
Manganous Oxide,			·513
Lime,			2·912
Magnesia,			32·418
Potash,			1·401
Soda,			·065
Water,			14·03
			100·486

Insoluble silica, 5·914 per cent.

In one—the more southerly—of the quarries where this mineral occurs, at Craigie, it is associated with arborescent filaments of pyrite, of a gold-yellow colour. Gold is said to have been found here !

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* Streng considers Schiller spar as altered bronzite (enstatite) ; Descloizeaux, however, finds that it has a negative bisectrix, which makes it altered diaclasite. In the lustrous portion there is really no *apparent* alteration ; and the hardness, gravity, and colour are all nearer diaclasite.

ALTERATION PRODUCTS OF AUGITE.

Conversion into Serpentine.

	S. G.	$\ddot{Si}$.	$\dddot{Al}_2$.	$\dddot{Fe}_2$.	$\dot{Fe}$.	$\dot{Mn}$.	$\dot{Ca}$.	$\dot{Mg}$.	$\dot{K}_2$.	$\dot{Na}_2$.	$\dot{H}_2$.	Total.
Typical Sahlite, ...	...	53·7	...	...	8·	...	24·9	13·4	.	...	...	...
Hydration,—Loss of $\dot{Ca}$, and $\ddot{Si}$,—peroxidation partial—												
Pseudo-Augite, Portsoy, . .	...	37·33	1·13	4·36	4·05	·38	1·2	36·71	·88	·73	13·37	100·14
Pseudo-Enstatite (?), Greenhill, .	2·16	37·78	2·12	5·07	2·1	·76	...	37·01	tr.	tr.	16·07	100·22
Hydration,—Loss of $\dot{Ca}$, and $\ddot{Si}$,—total peroxidation—												
Pseudo-Diallage, Portsoy, . .	2·62	34·54	1·16	15·2	·33	·28	...	36·38	...	...	12·2	100·09
Pseudo-Enstatite (?), Balhammie,	. .	37·41	...	13·54	·06	·24	·2	34·76	...	...	13·59	99·78
Totaigite, Yellow, Totaig, . .	...	37·22	·76	...	1·05	·23	5·24	44·97	...	...	10·64	100·11
,, Black ,, . .	2·87	36·19	·26	·29	2·96	·45	3·27	45·57	·25	·42	10·2	99·97
The same,—with no peroxidation—												
Schiller Spar, Black Dog Rock, .	2·65	38·19	2·18	·07*	8·48	·51	2·91	32·42	1·4	·07	14·03	100·49
Typical Serpentine, ...	...	44·14	...	...	...	...	...	42·97	...	...	12·89	...

* $\dddot{Cr}_2$, ·28.

HORNBLENDE OR AMPHIBOLE.

Lime—Magnesia Amphibole.

$(\dot{Ca}, \dot{Mg})\, \ddot{Si}$, or rather $(\dot{Ca}\, \dot{Mg}^2)\, \ddot{Si}$.

Amianthus, Asbestus, Tremolite, Nephrite.

Amianthus—Flexible Asbestus.

1. The purest and most flexible amianthus in Scotland—probably in the world—is to be found in thin rifts in the diallage rock of Balta in Shetland. It occurs at Doo's Geo, and Muckle Head Geo; chiefly, in both cases, over the edge of the cliffs. It is here associated with a peculiar antigoritic allomorph, and a deep green hydrated asbestus. The fibres are not generally over three or four inches in length, but they are finer and softer than those of any other locality. They have very slight tenacity;—when rolled between the fingers, they work first into a delicate felt, and ultimately into the smoothest and softest conceivable powder. This is absolutely impalpable.

This variety is not at all suited for the manufacture of incombustible paper; but as a lubricant it would be invaluable; stuffing-boxes packed with it would be as devoid of friction as if the most highly-purified graphite were employed; no talc that I have seen is softer, no French chalk nearly so soft. The colour is a very pale watery grey-green. The specific gravity is 2·949.

1·3 grammes yielded—

Silica,	·725		
From Alumina,	·005		
	·730	=	56·153
Alumina,			1·539
Ferric Oxide,			·388
Ferrous Oxide,			3·111
Manganous Oxide,			·769
Lime,			11·716
Magnesia,			22·461
Potash,			·188
Soda,			·692
Water,			2·5
			99·517

Insoluble silica, 3·382 per cent.; was pure.

Asbestus.

From Granular Limestone.

2. This was sent me from Shinness in Sutherland, by Dr Joass of Golspie. Asbestus was found in such quantities in Corsica that Dolomieu packed his minerals with it. I suppose that, at one time, some such abundance must have prevailed at Shinness, for in a box of specimens sent to me by Dr Joass a quantity of this substance lay loose at the bottom, and also separated the specimens from each other. I likewise obtained from the same gentleman long hatchet-shaped masses of closely packed parallel fibres of a pale-green colour, with occasional imbedded crystals of sahlite (?).

The asbestus was hard, rigid, and rough, though very similar in appearance to the amianthus from Balta.

1·3 grammes yielded—

Silica,	·733		
From Alumina,	·006		
	·739	=	56·864
Alumina,			·232
Ferric Oxide,			·484
Ferrous Oxide,			2·124
Manganous Oxide,			·23
Lime,			12·535
Magnesia,			23·923
Potash,			·439
Soda,			·538
Water,			2·525
			99·866

Insoluble silica, 3·924 per cent.; was pure.

From Serpentine.

3. From the serpentine of Portsoy.

Much asbestus is said, or is imagined to have been obtained hereabout; and a couple of small quarry-holes have been pointed out to me as the spots whence it was obtained. The only place where I have myself procured it was at the west side of the foot of the terminal bluff or cliff of the most easterly of the two beds of serpentine. Here it occurs in veins of an inch in width, in a dense grey rock, which appears to be gabbro passing into serpentine. The fibres of the asbestus here lie transversely to the course of the veins in which they occur; this is very unusual as regards asbestus. The colour was a pale greenish-grey, the lustre somewhat silky; it was unctuous to the touch, tending to amianthus. Specific gravity, 2·986.

1 · 3 grammes yielded—

Silica,	· 725	
From Alumina,	· 007	
	· 732 =	56 · 307
Alumina,		· 77
Ferric Oxide,		· 527
Ferrous Oxide,		2 · 323
Manganous Oxide,		· 153
Lime,		12 · 578
Magnesia,		23 · 307
Potash,		· 439
Soda,		· 633
Water,		2 · 941
		99 · 978

Insoluble silica, 3 · 005 per cent. Loss in bath, · 32 per cent.; was pure.

Antigoritic Allomorph—Nephrite.

4. I place this very remarkable variety here, on account of its paragenesis with the amianthus of Balta. It occurs as a layer, immediately in contact with the vein of the former at Doo's Geo. This layer is about a couple of inches in thickness. It presents itself as a fissile schist, which may readily be split up into laminæ of extreme thinness. Its colour is a very pale pea-green; it is translucent, and altogether so similar in appearance to antigorite that it was without hesitation assigned to that mineral.

Being of extreme toughness, and naturally splitting into axe-shaped fragments, it would by Dana be classed as nephrite; and in composition it would stand immediately central in his list of analyses of that substance.

It is altogether dissimilar to the amianthus, with which it is associated; but its most remarkable peculiarity is, that where it is exposed to the air it passes into it,—the amianthus appearing to grow out of the solid and fissile stone.

Moreover, although this stone may be scraped down into powder by the knife, like steatite or slate-pencil, yet if it be crushed in a mortar or beaten with a hammer, it is immediately matted into a felt of amianthoid fibres.

The laminæ of this stone have a rough cleavage in two other directions, forming angles of about 122° with each other. This angle is sufficiently near the normal to lead to supposition that it may be due to a feeble development of crystalline arrangement of parts. Its powder and cut surface are both very smooth. Its specific gravity is 2 · 957.

1·421 grammes yielded—

Silica,	·785		
From Alumina,	·007		
	·792	=	55·734
Alumina,			·045
Ferrous Oxide,			5·203
Manganous Oxide,			·008
Lime,			13·241
Magnesia,			22·696
Potash,			·138
Soda,			1·12
Water,		2·305	2·438
			100·625

Was perfectly pure.

Schistose Allomorph.

5. This still more remarkable variety is similar to the last; but it is very much softer, looser in structure, and more schistose.

It occurs in the sea-banks at Leegarth, on the east side of Trista Voe, in Fetlar. It was apparently considered to be talc-slate by Hibbert, and its extreme unctuosity gave some countenance to such a view.

It is white, with a slight tinge of green; it readily breaks into fissile slates, the cross fracture of which shows a loose luminated arrangement of particles, which, under the lens, appear to be minute granules, with occasionally an obscure fibrous structure. It is so soft as not only to be abraded readily by the nail, but also to leave, when handled, a powder of extreme unctuosity upon the skin. It can with great ease be crushed into powder. It is associated with strata of a darker variety of the same substance, which variety carries minute crystals of magnetite, and is slightly actynolitic. Fasciculitic asbestus also occurs at this spot.

The specific gravity of the schistose mineral is 2·955.

It yielded—

Silica,	·737		
From Alumina,	·003		
	·74	=	56·923
Alumina,			·22
Ferrous Oxide,			4·647
Manganous Oxide,			·076
Lime,			12·32
Magnesia,			22·076
Alkalies,			tr.
Water,			3·4
			99·662

No loss in bath; was perfectly pure. This is a simple mineral, functioning as a rock mass. Both it and the previously described variety might, in the state of powder, be used as lubricants, and possibly for polishing purposes.

Tremolite.

6. I am indebted to Dr Joass for sending me from the lime quarries of Shinness, in Sutherland, a specimen of this variety; this surpasses in beauty anything of the kind I have ever seen from abroad. Nearly a foot in size in all directions, the mass is covered with parallel bundles of glancing fibres of the most delicate tenuity; these flash with a brilliant but chastened light, of more than silvery whiteness.

No natural object, devoid of colour, could possibly be more beautiful.

The specific gravity is 2·964.

1·3 grammes yielded—

Silica,	·725		
From Alumina,	·005		
	·730	=	56·153
Alumina,			·857
Ferric Oxide,			1·617
Ferrous Oxide,			·716
Manganous Oxide,			·069
Lime,			13·31
Magnesia,			24·138
Potash,			·441
Soda,			·211
Water,			2·5
			100·012

Insoluble silica, 1·095; there was no fluorine. It is most strange that a substance which seems the very type of purity and simplicity of constitution, so far as appearance goes, should have so complex a composition.

I strongly incline to the opinion that this is an *augitic* tremolite, but am unwilling further to diminish the specimen for the determination of the point.

7. From the most north-easterly of the limestone quarries of Milltown, Glen Urquhart.

This variety, being highly aluminous, probably should not be placed here; but where better to place it I know not, for in form it is distinctly tremolite. It occurs in crystals of one-fourth of an inch in size; these are of a somewhat bright yellow colour; they are imbedded promiscuously in the granular limestone, and are so closely associated with talc that their separation therefrom was very tedious and difficult. The colour was not due to weathering; as crystals dissolved out of the mass of the lime had as high a colour as those lying upon the exposed surfaces; and the centres of the crystals also were as highly coloured as were the surfaces.

They were analysed in a hitherto vain attempt to find out what substance it is which has been called "chondrodite from Loch Ness."* The single specimen to which that name was applied is now in the British Museum (I believe it to be yellow serpentine): it is imbedded in a limestone which in no way resembles the Urquhart lime. Were it not that arsenical pyrites is to be seen in this specimen, I should consider it as coming from Glen Elg.

Of this tremolite, 1·3 grammes yielded—

Silica,	·732		
From Alumina,	·013		
	·745	=	57·307
Alumina,			6·676
Ferric Oxide,			1·082
Ferrous Oxide,			3·229
Manganous Oxide,			·307
Lime,			12·361
Magnesia,			16·615
Water, with Fluorine,			2·5
			100·077

Insoluble silica, 2·147 per cent. It was found impossible absolutely to separate the talc; the amount present could not, however, have amounted to more than the hundredth part.

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* See GREG and LETTSOM.

Actynolite.

Magnesia, Lime, Iron—Amphibole.

$(\dot{Ca}\,\dot{Mg}\,\dot{Fe})\,\ddot{Si}$.

From Rocks of Hornblendic Gneiss.

8. Found at the well-known locality of the banks of Nidister, Hillswick, Shetland. The exact spot is just where the low banks, trending nearly south, are suddenly met by a cliff of several times their height, which lies at right angles to their course. There is, if not a junction here, much vein confusion; and among these veins one of about two feet in thickness, composed entirely of long tortuous crystals of glassy actynolite, lying in silvery white talc and soft chlorite, strikes with a north-easterly course out of the bank. This vein has one of serpentine on its north side, and another of anthophyllite on its south. It seems to pass into chlorite and Biotite, where it is covered by the shingle of the beach. The specimens of this actynolite, though exceedingly fragile from the softness of their matrix, and from their own brittleness, are the finest in Scotland. The crystals are well formed, though not terminated; they are semi-transparent, highly lustrous, and bright green; their angle is 124° 29′; their specific gravity, 2·993.

On 1·3 grammes—

Silica,	·711		
From Alumina,	·004		
	·715	=	55.
Alumina,			1·512
Ferric Oxide,			·994
Ferrous Oxide,			3·456
Manganous Oxide,			·307
Lime,			10·381
Magnesia,			23·307
Potash,			1·12
Soda,			1·097
Water,			2·898
			100·072

Insoluble silica, 1·958; loss in bath, ·266 per cent.; possible impurities, chlorite and talc.

ALUMINOUS AMPHIBOLES.

Magnesia Lime Amphibole.

Edenite.

$$(\dot{Ca}\,\dot{Mg})\ (\ddot{Si}\,\dddot{Al}_2).$$

From Granular Limestone.

9. In several of the small quarry-holes above Milltown, Glen Urquhart, edenite occurs in sheaf-like, or *fasciculitic* tufts, as they have been called by HITCHCOCK.

The tufts are about half an inch in length, their fibres being very delicate, but perfectly rigid. The plumose tufts are arranged in every conceivable position to each other, branching out from centres which are frequently specks of pyrrhotite. They are invariably imbedded in the lime.

Specimens possessed of this divergent fibrous structure are seen of two colours—pale green and slate blue; and the composition of these varieties is different.

The immediately associated minerals are pyrrhotite and calcite. The mineral has somewhat of a soapy feeling when rubbed. 1·3 grammes of the green variety afforded—

Silica,	·653		
From Alumina,	·001		
	·654	=	50·307
Alumina,			8·538
Ferric Oxide,			·118
Ferrous Oxide,			2·76
Manganous Oxide,			·076
Lime,			11·63
Magnesia,			20·769
Potash,			·5
Soda,			1·156
Water,			4·134
			99·988

Insoluble silica, 1·375 per cent.; there was no fluorine; possible impurity, calcite.

10. The blue-black coloured variety afforded—

Silica,	51·306
Alumina,	2·215
Ferric Oxide,	·155
Ferrous Oxide,	7·663
Manganous Oxide,	·487
Lime,	11·17
Magnesia,	21·872
Potash,	2·196
Soda,	·463
Water,	2·125
	99·652

Possible impurity, calcite.

Magnesia, Lime, Iron,—Aluminous Amphiboles.

Pargasite.

$$(\dot{Ca}\ \dot{Mg}\ \dot{Fe})\ (\ddot{Si}\ \dddot{Al}_2).$$

11. This variety was found among the heaps thrown out in some searchings after copper on Mount Errins; the pits are some three miles west of Urin or Errins, north of Tarbet in Kantyre.

The mineral was of characteristic appearance. It formed a bristly or hackly mass of parallel acicular crystals, of half an inch in length; these crystals stood erectly transverse to the surfaces of small veins in the rock. This rock, which lies in the district of chlorite schist, is here chiefly a vitrified-looking gneiss, not unlike a Cornish elvan; but it is not unfrequently a foliated and beautifully plicated hornblende rock, with a subfibrous structure.

The traces of copper consisted of chalcopyrite, in pyrite; there being also rarely crystals of quartz, felspar, and calcite. A pale-green mineral, which occurred in mammillated crusts, and much resembled pennine or grastite, was also not infrequent. Magnetite, pseudo after cubical pyrite, was rarely seen. Dolomite is also rare.

This variety of amphibole, which, solely from chemical resemblance, I have classed under pargasite, was in translucent, hard, and brittle acicular crystals; these were of a fine rich green colour, and sometimes coated with the mammillated green mineral.

1 · 3 grammes yielded—

Silica,	·679		
From Alumina,	·006		
	·685	=	52 · 692
Alumina,			2 · 556
Ferric Oxide,			4 · 086
Ferrous Oxide,			9.772
Manganous Oxide,			· 23
Lime,			11 · 419
Magnesia,			15 · 769
Potash,			· 57
Soda,			· 693
Water,			2 · 125
			99 · 912

Insoluble silica, 1 · 897 per cent. ; loss in bath, · 242 per cent. ; there was a suspicion of coloration from copper, but none was found.

Octohedrite (*Anatase*) and chabasite have been said to occur here. The substance mistaken for the first consisted of minute *octohedral* crystals of magnetite ; that taken for the second was rhombs of pale-brown dolomite ! The crystals of magnetite are imbedded in the hornblendic schist.

Magnesia, Iron, Lime Amphibole.

Hornblende Proper.

From Diallage Rocks.

12. The diallage of Balta has been stated,—both in my chapter on the Felspars and in speaking of the Augites,—to contain, opposite the little Brough, two pseudo veins, of which the matrix is massive labradorite, but of which the one carries imbedded crystals of diallage, and the other of hornblende. It is this hornblende which is here described. It is in large dark-green, dull, somewhat foliated crystals ; these show thin rifts filled with calcite, possibly the result of incipient alteration. Specific gravity, 3 · 112 ; cleavage angle, 124° 27′.

On 1·5 grammes—

Silica,	·671		
From Alumina,	·017		
	·688	=	45·866
Alumina,			8·779
Ferrous Oxide,			14·151
Manganous Oxide,			·133
Lime,			9·818
Magnesia,			14·4
Potash,			·821
Soda,			1·43
Water,			2·301
Carbonic Acid,			not dct.
			97·699

Impurity, a trace of calcite.

From Diorite.

13. From the north-east side of the entrance of Trista Voe, Fetlar, Shetland. The mode of occurrence has been noted under anorthite, in the chapter on Felspars.

The hornblende is somewhat fibrous-looking, and green on the fresh fracture; jet black and highly lustrous on the exposed surfaces. A portion from the unaltered green interior was taken. Specific gravity, 3·09.

1·3025 grammes yielded—

Silica,	41·628
Alumina,	11·631
Ferric Oxide,	1·85
Ferrous Oxide,	8·949
Manganous Oxide,	·307
Lime,	9·247
Magnesia,	18·509
Potash,	·63
Soda,	1·218
Water,	5·399
	99·368

Loss in bath, ·423 per cent.

The large quantity of water led to the suspicion of alteration, but the powdered mineral showed no effervescence with acid. The quantity of alkalies, in presence of the fact that a lime felspar is the associate, is also singular.

14. From the diorite of Portsoy.

The rock from which the mineral under consideration was taken has, through error on my part as to the nature of the hornblendic mineral, been called in my paper on the Felspars, *diabase.* The subjoined analysis of the mineral, taken in conjunction with the determination of its cleavage angle, and with the ascertaining that it is dichroic, shows it to be hornblende, and the rock hence *diorite.*

This diorite first shows itself a little to the west of the Battery; it is here of a coarser grain than elsewhere to the east, being formed of labradorite and of the mineral in question, porphyritically arranged; the crystalline masses are of an inch in size.

Of the associated labradorite, the analysis has already been given at page 256 of these "Transactions." These minerals form the great mass of the rock, which here also contains in small quantity, menaccanite, sphene, a dark brown mica, and specks of pyrrhotite.

The hornblende in mass appears dark grey in colour; when closely viewed by transmitted light—for it is transparent in small fragments—it is reddish-purple, being very similar to some garnets.

Its cleavage angle is 124° 33′; its specific gravity, 3·252; it also affords the rectangular cleavage.

1·402 grammes afforded—

Silica,	·688		
From Alumina,	·042		
	·73	=	52·068
Alumina,			2·568
Ferrous Oxide,			9·725
Manganous Oxide,			tr.
Lime,			19·052
Magnesia,			14·407
Potash,			·746
Soda,			·569
Water,			·852
			99·987

Possible impurity, labradorite.

The determination of the nature of this rock, I regard as one of considerable importance, as it will be found to have a very immediate bearing upon the question whether rocks of this class are, as regarded by Cotta, wholly or solely igneous,—or, as regarded by Dana, metamorphic.

Concerning the *lithological* nature of the rock itself, there has, in the past, been considerable difference of opinion.

By most writers it has been described under the vague name of "primitive greenstone."

It found its way into many collections, from the stores of a local mineralogist who termed it "Norwegian hornblende." HAY CUNNINGHAME[*] describes it particularly under the name of syenite; and, in his geological map of Banffshire, lays it down as occurring of a width of about seven miles in the north of that county; and as stretching, with an average width of three miles, for a distance of about fourteen, southward,—until in fact it reaches the Isla, the boundary of the county in that district.

Though, from the covered nature of the ground, it is impossible by actual determination to prove so great or so continuous an extent, still I am prepared to admit these limits in a general way; though I do not regard it as one single mass.

The first point I would direct attention to, as regards the rock occurring within the limits assigned to it by CUNNINGHAME, is that it occurs therein presenting such differences in type, that I do not think that any lithologist who had not studied it by the convincing process of slow pedestrian exploration, could be brought to believe that its very characteristic varieties could possibly belong to one and the same substance.

Having so done, I agree with CUNNINGHAME that all that is visible within the boundaries above assigned, is to be referred to one and the same general species of rock; and having, after doubts and difficulties, had to admit so much, I have to maintain that rocks, extending far beyond the limits of CUNNINGHAME'S survey, must be referred to the same mass also. Such an extension is almost demonstrable as far as Colquhanny in Strathdon.

The conclusions and admissions, above referred to, have been arrived at chiefly through having been able to trace the gradual passage of the one variety into the other; and also through having to maintain that, although the *extreme* varieties depart to so signal an extent from the typical as to leave some room for doubt, there is yet no other known rock to which they can, with so great a degree of consistency, be referred.

The constitution of the rock at its extreme north-westerly limit has been given; in appearance it may be said to be a very dark brown, hackly-fractured rock.

COTTA would define it, probably, as porphyritic diorite. CUNNINGHAME terms it "large granular." The hornblende is in great excess; the imbedded crystals of labradorite have little effect in lightening their colour, through the deadening effect of their dark setting. The rock as a whole in fact is almost black.

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[*] "Trans. Highland Soc." vol. xiv.

Some fifty yards to the east, it is almost as nearly white. Here the labradorite greatly predominates. The whole character of the rock has at the same time altered.

The labradorite is now the matrix, the hornblende the porphyritically disposed substance.

The labradorite, instead of occurring in large striated translucent crystals, is now granular to structureless. The hornblende occurs to about the seventh part only of the bulk of the labradorite; it is now light green and somewhat platey,—it may be *uralite.* The appearance is similar to what a quantity of chopped leeks diffused through cold suet would present.

This, though so contiguous in position to the first described, is the furthest removed in mineral constitution in *one* direction—that, namely, in which the felspar is in excess. The change, however, is elsewhere much more marked in other respects.

Immediately to the east of the harbour of Portsoy, the structure of the rock is distinctly laminated, when considered in hand specimens; and clearly bedded,—the beds frequently exhibiting well-marked contortions. The felspar and hornblende are here separated into belts; the structure altogether is gneissose.

A third set of beds, which, though not altogether the same in composition (for they contain small crystals of Paulite and are cryptocrystalline), must be assigned to the same general type, occurs higher up in the series, on the west shore of the bay of the Durn. Within a space of some sixty yards, this rock alternates—at least five times,—with an equal number of beds of serpentine. The augitic type of mineral, also, here prevails, if it does not absolutely exclude the hornblendic; Paulite and Biotite in small quantity find a place; the felspar is the same.

The variety first described was stated to present occasionally a flake of brown mica (Biotite ?). This mica in the upper beds takes the place—in increasing quantity as we pass eastward—of the augite, and of what hornblende may be present; and it is this increased and ultimately total replacement of the one for the other, which forms a rock which can be referred to diorite with so much difficulty. Varieties which show the augite and Biotite in nearly equal amount are to be seen, in large pattern, in the heights above Cowhythe Head; while the rock which forms the cap of—or at least occurs immediately eastward of the granite of—Barry Quarry, has a minute, almost granular structure, with apparently no hornblende.

The *total* replacement of augite by Biotite gives rise to the variety which departs to the most marked extent from the typical. This is to be seen in beds on the shore near to East Head and a little to the west of Cowhythe Head, where the rock consists solely of white labradorite and bronzy Biotite;

the mixture may be said to be almost foliated; the felspar, however, is somewhat granular. The rock is sometimes of so minute a structure that its grains are little larger than mustard seed. A hand specimen would be pronounced a fine-grained gneiss. CUNNINGHAME speaks of this as a slaty variety, but does not seem to have observed the replacement of hornblende by mica. A rock having the same components, but of much coarser structure, is to be seen on the south side of Craig Burn, in Auchindoir; and, in a dyke at New Merdrum near Rhynie, there are two large masses of the same rock, in which the crystals of both the bronzy mica and of the labradorite—here grey—are one to two inches in size.

The rock from these last two localities contains nothing but labradorite and Biotite, if we except a very occasional speck of iserine or magnetite: it seems a straining of nomenclature to apply the term *diorite* to such a compound, but it must be remembered that the gradual interchange of augite for hornblende, and then of Biotite for augite, can be traced in an altogether unmistakable manner. It has to be stated that, except at Cowhythe Head, the rock is not in the slightest degree fissile.

CUNNINGHAME clearly defines what is indubitably a variety intermediate between the two last varieties. In this, true hypersthene (*i.e.*, Paulite), augite, and bronzy mica take the place of hornblende. This rock, of which a fine-grained variety may be seen *in situ* on the west side of the Bay of Durn, would appear to extend up the country, to re-appear on the west side of Craig Buirroch, and near Retannoch. In these localities the constituents of the rock are best seen in exfiltration veins. These veins carry crystals of a yellowish-brown, translucent augite (analysis, page 466), bronzy Paulite, menaccanite, pyrite, pyrrhotite, and rarely a bronzy mica.

The rock in which these veins occur is a fine-grained mixture of apparently identical ingredients; and also with no hornblende. As one—the platey cleavage of the augite—is dominant, this rock may be called a hyperitic gabbro. But, if not *diorite,* what name should be given to the binary compound of labradorite and Biotite, I know not.

Two questions at once suggest themselves in connection with these varieties.

The first is—*Are these not indications of the passage of the rock into other rocks?*

Let us see what CUNNINGHAME says on this point. At page 488 of the "Transactions" he says—"At Portsoy the syenite is connected with hornblende rock and hornblende slate, into which it passes in the most gradual and insensible manner:" at page 489—"We assert that there is a gradation from the one into the other as perfect as may frequently be found to take place between gneiss and granite, and to which it seems to be analogous."

In the same page he also describes a locality near Boyndie, where there is a transition of greywacke into his syenite, the rock "exhibiting a great quantity of micaceous scales,"—remarking: "We can only recognise it as a peculiar variety connecting it with more crystalline strata, and ably supporting the propriety of retaining the term 'Transition Series' as expressing a group of which greywacke is the characteristic type."

At page 493 he considers its relation to serpentine, remarking: "From the district round Portsoy the examiner may soon assure himself that the serpentine presents characters which, when conjoined with those of the syenite, indicate that both form parts of a mass which has been the product of one epoch, and the elaboration of one great system of causes. Whenever the two rocks meet each other, and there are no instances in which they are far distant, there is an intermixture so gradual, that whatever their mode of formation has been, we can only consider the one as a variety of the other." DE LA BECHE is quoted as saying of the rocks towards Roscreage Beacon, Cornwall: "Well characterised hornblende slate is found in many places near the serpentine, and also so mixed with ordinary greenstone rocks that to attempt a separation would appear a violation of natural union."*

Having fortified myself with the above very emphatic evidence, I shall only say that the extreme micaceous variety which I have described from Cowhythe and Craig Burn would, had I not traced as far as possible its connections, have been taken by me for a peculiarly granular variety of the ordinary lepidomelane gneiss of the district, or for a highly metamorphosed greywacke. There is a rock to be seen a little above the bridge at Huntley which seems to be the same, with merely the addition of grains of quartz and an increased amount of the mica: its appearance is quite that of a highly micaceous and laminated gneiss, and it seems to shade off into the ordnary gneiss of the district.

I have not seen the transition into greywake; but, with a certain qualification of CUNNINGHAME's expression, "the product of one epoch," I maintain to the full the most intimate connection with serpentine. I go a step further, however, and say, another rock must come immediately, and, as regards Portsoy, sometimes *interstitially* into the same category, namely, diallage. Diallage is here a quite subsidiary variety of diorite, and it is this variety which usually passes into serpentine. CUNNINGHAME must concede this, for he says: "Serpentine is geognostically the frequent attendant of diallage, passing into it by the most gradual transitions."

Here, then, we have the independent evidence of three observers that, *as matters of fact*, diorite passes by "insensible gradations" into greywacke, or

---

* Would it not, then, be a "violation of natural union" to "attempt a separation" of the ten beds which, on the west shore of the Bay of Durn, alternate with, without once intersecting, each other?

metamorphosed siliceous schists, if not into lepidomelane gneiss, on the one hand; and into hornblende schist and serpentine on the other.

That a rock, usually regarded as belonging to the basic igneous division,—passes into acidic sedimentary rocks,—and into a basic sedimentary rock.

Question the second then is—*is it an igneous rock?* I hesitate not to say that in the district of which I write, the great mass of the rock, namely, that passing from the harbour of Portsoy up the country to the south-west, following the quartzite throughout, implicitly yielding to its every sinuosity, and preserving an unvarying position to the contiguous strata, never cutting through them or displacing them in the smallest degree, is just as much a metamorphic rock as the quartzite beneath it or the schists above it. It is not quite so much a metamorphic rock, however, as the underlying serpentine, which will, in the sequel, be shown to be a product of a secondary metamorphism or transmutation of the diorite itself, and therefore not strictly a "product of the same epoch."

Such a deduction has never been entertained by CUNNINGHAME, who, throughout, regards the diorite (his syenite) as an igneous mass; he places it among the unstratified rocks;—distinctly says, that he is to be understood as not endeavouring to call in question its igneous origin; and commits himself absolutely when he says: "It forms the base of the hills of Knock and Durn; and in the former we found masses high up the acclivity, derived not improbably from veins in the quartz that caps it." He also figures it in his map as extending for about two miles up the *western* side of Durn Hill; and this as an extension of that "principal mass" which he states appears as great beds included in, though sometimes crossing, the strata at Denbrae, now Whyntie Head. As Denbrae Head is some thousands of feet higher up in the series than Durn and Knock hills, CUNNINGHAME has placed himself in the dilemma of either making these hills cap strata much more recent than themselves; or he must set down a hill of 1400 feet in height as being a mere fragment uplifted by the igneous rock, without alteration of its dip, or visible severance from those portions of the same stratum which are in no way associated with diorite.

The above random speculation is, in fact, perhaps the only blot in a paper of extreme merit.

The fact is, that there is no evidence that the dykes and beds at Denbrae connected with any principal mass of diorite; and if they are, there is strong evidence which tends to disconnect such a "principal mass" from *the* principal mass which is associated with the quarzite; and which is clearly associated with it in this regard, that it lies perfectly conformably above it.

No one who takes his stand on the diorite immediately to the west of the battery at Portsoy, and regards even the "coarse granular" rock as a whole, can fail in good light to see the gneissose structure, still apparent when viewed at a distance of a dozen of feet or so; and no one who follows up the regular bedding of the diorite, diallage, and limestones, with associated thin rifts of serpentine, quartzite, and gneiss, throughout the forty or fifty miles which they extend, with flexures ever accommodating themselves to those of the inclosing rock, can but regard them as regularly bedded and non-intrusive.

That it may not *primarily* have been an interbedded igneous rock, it might be rash absolutely to deny; but the convoluted schistose structure occasionally evident in the rock, on the one hand, and its apparent transitions into laminated sedimentary rocks on the other, go far to preclude such a view. It must be admitted, however, that one belt of rock on the west side of the Bay of Durn has very much the appearance of an igneous origin.

There is another relationship of this rock, however, or of a very similar one, which, having been somewhat forcibly insisted on, calls for some notice.

MACCULLOCH in his "Classification of Rocks" has, under binary granites, the following:—

> D. FELSPAR AND HORNBLENDE.
>
> *a.* Large-grained, or the hornblende crystallised.
>
> *b.* An uniform granular mixture; the respective ingredients varying materially in their sizes and proportions, so as to produce a great variety of aspect.
>
> *c.* Intimately mixed so as to be nearly undistinguishable.
>
> Var. *b* often resembles the greenstones of the trap family; and is in fact only distinguished by its geological connection with granite. Var. *c* is often similarly undistinguishable from basalt; occasionally from the non-fissile hornblende schists; but, like variety *b*, it is connected with, and passes into, granite of the common character.
>
> These varieties occur in Aberdeenshire, where they are connected with the most ordinary granite, subjacent to gneiss, both by transition and alteration, in a manner so distinct as to leave no doubt respecting their true place in a geological classification like the present.

In NICOL'S "Guide to the Geology of Scotland," page 185, we read:—

> In Bennachie the granite is a reddish-brown binary compound of quartz and felspar, both in regular crystals, whilst on the northern face it approaches to greenstone or diorite. Some rare varieties also occur between Old Rayne and Meldrum, the common granite passing by imperceptible transitions to a dark dioritic greenstone, and this to a uniform mixture not distinguishable from basalt. But the identity does not cease even here, since in many places it passes in the same uninterrupted manner into a soft claystone, with a schistose tendency on exposure, in no respect differing from those of the trap islands of the western coast. In these varieties felspar and hornblende in various proportions compose the entire rock. The former mineral is sometimes white, inclosing hornblende crystals, when the rock has a very beautiful appearance. In the coarser mixtures the felspar is commonly white, but in the finer it becomes greenish; whilst the basalts seem mere hornblende invariably black. In a word, quartz, felspar, mica, and hornblende may be found in almost every conceivable variety of composition and

relative degree of abundance. There is a complete transition from the oldest and most regular granite to rocks differing in nothing from recent trap unless in possessing hornblende instead of augite.

The observations that have to be made upon the above very precise statements, of two very precise observers, are,—that the interchanges and transitions referred to are certainly to be seen in the rocks of the trough which lies between Bennachie and the Froster and Barra Hill ridge;—that our knowledge regarding the different felspars now enables us to follow out to a certain extent the precise nature of the interchanges;—that the connection of the rocks of this trough with those of the north and south line of strata to which I have been directing attention has not only never been made out,—but that, in that north and south system of rocks, any transition into the granites there occurring is absolutely denied by CUNNINGHAME, and can at Barry quarry be clearly seen not to exist;—and, lastly, that if such a connection be proved, then *so much the worse for the granite* of the eastern districts of Aberdeen,—so far as regards the maintaining that it is of an igneous and not a metamorphic nature.

15. From the diorite of Glenbucket.

The mode of occurrence and associates have been already noticed in the chapter on Felspars. This hornblende, though occurring in comparatively small quantity, surpasses everything of a similar kind in the startling magnitude of its crystals, and the striking contrast which their brilliant blackness offers to the snow-white labradorite in which they are imbedded.

Crystals over 21 inches in length by 2 in breadth have occurred here; through the breaking up and removal of the larger masses for rockeries, the finest have now been defaced.

This hornblende bears a very strong resemblance to Arfwedsonite. It fuses readily; its cleavage angle is 124° 24′, and its specific gravity is 3·218.

Silica,	·638		
From Alumina,	·037		
	·675	=	45·
Alumina,			9·412
Ferric Oxide,			1·551
Ferrous Oxide,			16·758
Manganous Oxide,			·333
Lime,			11·237
Magnesia,			11·193
Potash,			1·363
Soda,			1·663
Water,			1·345
			99·855

This diorite is probably an extension of the Portsoy beds. It is well seen in association with serpentine at Shenwall, on the Blackwater; and in similar association in a line of rugged cliffs which, at a high altitude, divide the upper waters of that stream. Though there is no serpentine visible in Glenbucket, yet it reappears in the next glen; the diorite of Colquhanny—unquestionably the continuation of the present bed—occupying the normal position to the east of, or above the serpentine. . . .

. .

. .

From Hornblendic Gneiss.

16. The mass of rock from which I took the specimen now noticed was loose. It lay on the shore of the Kyle of Durness.

It consisted chiefly of a felspar much resembling in appearance a pinkish granular marble. Though very unlike ordinary orthoclase, analysis proved this to be that substance. Imbedded in this felspar were crystals of half an inch in size, of an appearance intermediate between that of hornblende and sahlite. After an examination of these crystals, I would not be prepared to say to which mineral they belong; for although there are appearances of the hornblendic cleavage, still the *c* cleavage of augite is quite distinct; so that we may here have a physical compound like the *Uralite*. Sphene of a light brown colour is associated in crystals.

During a late visit to Durness, I have, however, been able to find, if not the original site of the transported block, at least a locality affording precisely similar substances in similar association.

This is the north-west slope of Ben Spinna, at a height of about 600 feet.

Several streams cut deeply into the rounded shoulder of the hill, and admirably display the strata. These here dip to the north-east at an angle of from 20° to 32°.

Although the rock is clearly a hornblendic gneiss, I am far from satisfied that it is the same as the neighbouring rock of Kean na Bin, which has the usual W.S.W. dip: I incline to regard this as a fragment overlying the Durness lime—a fragment of a rock to be seen to the east of Whitten Head, let down by a great fault running N.E. and S.W. a little higher up the hill. The green mineral here occurs both of the same appearance as the Durness specimen analysed, and also of one more resembling ordinary hornblende.

The Durness specimen was of a rich green colour, and was somewhat harder than usual.

1·3 grammes yielded—

Silica,	·658		
From Alumina,	·011		
	·669	=	51·461
Alumina,			2·968
Ferric Oxide,			2·451
Ferrous Oxide,			9·661
Manganous Oxide,			1·076
Lime,			20·073
Magnesia,			10·461
Potash,			·683
Soda,			1·305
Water,			·683
			100·822

Probable impurity, a trace of the felspar.

From Igneous Rocks.

17. In speaking under Augite of its vitreous allomorph, which is to be found at Elie, Kinkell, &c., the associated occurrence of glossy black hornblende at both of these localities was noticed. This hornblende is found at Elie, rarely in the tuff, and more commonly in the injected dykes which cut it, to the east of the town.

The appearances which it presents are well marked and peculiar. It occurs in cleavable masses of the size of beans to that of small eggs; the cleavages give the angle 124° 19′. The colour is black, with a shade of green; it is highly lustrous.

It is not easy to form an opinion as to whether these masses are imbedded, worn fragments; or crystalline infiltrations filling pre-existent cavities which had a rounded outline.

The argument for the former view is that crystals of sanidine, which might be held to be water or sand worn, are found imbedded in the same rock, at no great distance. An argument which points in the opposite direction is, that the cleavages of a newly detached mass invariably pass over its whole surface uninterruptedly, abutting against the containing rock, without exhibiting, near the surfaces of the fragment, any internal fractures, bruise, or perceptible paling of the colour; all of which we should expect to find in a rolled and fractured fragment.

On the whole, the appearances are most favourable to *formation in situ*; though it must be admitted that the single broad cleavage is altogether anomalous; infiltrated minerals, as an almost invariable rule, presenting themselves with a structure which exhibits bundles of fibro-crystalline radiations, which diverge or converge from the sinuosities of the drusy surface, according as i presents a convex or concave outline.

The specific gravity of this hornblende is 3·375.

1·3 grammes yielded—

Silica,	·502		
From Alumina,	·23		
	·525	=	40·384
Alumina,			19·012
Ferric Oxide,			2·124
Ferous Oxide,			7·284
Manganous Oxide,			·461
Lime,			11·544
Magnesia,			17·5
Water,			1·173
			99·482

Insoluble silica, 1·904 per cent.; no visible impurity; possible, unknown.

Although I have no analysis to offer of it, there is still another variety to which I would direct attention. It is a mineral which occurs in a dyke at Crawford-John. Specimens of the rock were sent to me by Geikie as containing augite, to see if it were possible to extract therefrom a sufficiency for examination.

Upon writing to Professor Geikie that the mineral appeared to me to be hornblende, he stated that he had at first regarded the included dark mineral as being hornblende, but he now regarded it as augite, seeing that under the microscope it possessed all the characters of that substance.

In separating chips for the measurement of the cleavage angle, I found it extremely difficult to obtain anything like a cleavage at all, from the presence of projecting points or rivets which interrupted the cleavage. On attempting to measure the angles of such imperfect cleavages as I did obtain, I was much puzzled at obtaining—in reflecting from cleavages in one zone—reflections at about the angles of 133°, 124°, and 87°; these are near the angles of augite in the first and last place, and of hornblende in the second.

The fragments operated upon were not of a sufficiently satisfactory character to speak positively on the point; but, if I be right regarding them, we have here an illustration of what has been previously noticed both by G. Rose and Haidenger.

The former found unions (verwacksungen) of augite and hornblende crystals, each with its own cleavage. HAIDENGER'S observation was one still closer, if not identical in character with the present; the grass-green smaragdite from the Bacher was found to consist of augite and hornblende in alternate layers; one of the cleavages of the hornblende being parallel to the face *a* of the augite; and the intersection of the cleavages parallel to the edge *a b*,—which is precisely what obtains here.

HORNBLENDE.

	S. G.	$\ddot{Si}$.	$\dddot{Al}_2$.	$\dddot{Fe}_2$.	$\dot{Fe}$.	$\dot{Mn}$.	$\dot{Ca}$.	$\dot{Mg}$.	$\dot{K}_2$.	$\dot{Na}_2$.	Fl.	$\dot{H}_2$.	Total.
Amianthus—													
Balta,	2·95	56·15	1·54	·39	3·11	·77	11·72	22·46	·19	·69	...	...	99·52
Asbestus—													
Shinness,	...	56·85	·23	·48	2·12	·23	12·54	23·92	·44	·54	tr.	2·53	99·87
Portsoy,	2·99	56·31	·77	·53	2·32	·15	12·58	23·31	·44	·63	tr.	2·94	99·98
Nephrite—													
Balta,	2·96	55·73	·05	...	5·2	·01	13·24	22·7	·14	1·12	...	...	100·62
Schistose—													
Leegarth Fetlar,	2·95	56·92	·22	...	4·65	·08	12·32	22·08	tr.	tr.	...	3·4	99·66
Tremolite—													
Shinness,	2·96	56·15	·86	1·62	·72	·07	13·31	24·14	·44	·21	tr.	2·5	100·01
Glen Urquhart,	...	57·31	6·68	1·08	3·23	·31	12·36	16·62	...	...	tr.	2·5	100·08
Actynolite—													
Hillswick,	2·99	55·	1·51	·99	3·46	·31	10·38	23·31	1·12	1·1	...	2·9	100·07
Edenite—													
Urquhart, green,	...	50·31	8·54	·12	2·76	·08	11·63	20·77	·5	1·16	none.	4·13	99·99
,, black,	...	51·31	2·21	·16	7·66	·49	11·17	20·87	2·2	·46	...	2·12	99·65
Pargasite—													
Errins,	...	52·69	2·56	4·09	9·77	·23	11·42	15·77	·57	·69	...	2·13	99·91
Hornblende—													
Balta,	3·11	45·87	8·78	...	14·15	·13	9·82	14·4	·82	1·43	...	2·3	97·7
Fetlar,	3·09	41·63	11·63	1·85	8·95	·31	9·25	18·51	·63	1·22	...	...	99·37
Portsoy,	3·25	52·07	2·57	...	9·72	tr.	19·05	14·41	·75	·57	...	·85	99·99
Glenbucket,	3·22	45·	9·41	1·55	16·76	·33	11·24	11·19	1·36	1·66	...	1·34	99·85
Durness,	...	51·46	2·97	2·45	9·66	1·08	20·07	10·46	·68	1·31	...	·68	100·83
Elie,	3·37	40·38	19·01	2·12	7·28	·46	11·54	17·5	...	...	...	1·17	99·48

Alteration Products of Hornblende.

Incipient Incrementation of Water; Peroxidation of the Iron; Decrement of Lime.

18. The serpentinous change of augite on the Green Hill of Strathdon has already been noticed. A little further up the slope of the north side of the hill

than the spot whence the altered augite was taken, specimens very similar in appearance to bronzite were found lying loose; these were traced to a bed of what appeared to be an unaltered or little altered hornblende. This was devoid of any bronzy lustre, which was now seen to be clearly the result of alteration. The mineral was in broad cleavable masses, of a dark green colour, a structure between foliated and fibrous, a slightly greasy lustre, and a hardness a little less than the normal. The specific gravity was 3·01.

1·302 grammes yielded—

Silica,	·661		
From Alumina,	· 2		
	·663	=	50·921
Alumina,			1·893
Ferric Oxide,			9·427
Ferrous Oxide,			2·085
Manganous Oxide,			·307
Lime,			8·645
Magnesia,			21·582
Potash,			·343
Soda,			·433
Water,			4·536
			100·172

Loses in bath ·454 per cent. of moisture.

Comparing this with the analysis of the altered augite which was found about a hundred yards from this locality, the greater durability of the hornblendic type of mineral is well shown; so slight, indeed, was the *apparent* change, that the eye alone could not have detected it, nor could so extensive an amount of peroxidation of the iron have been supposed compatible with the retention of a bright green colour and a brilliant lustre.

This is the only hornblende which I have met with in which the broad foliated massive variety showed any such change. It may be theorising overmuch to suggest that the state of change in the more readily decomposable surrounding mineral had induced a similar state in this. Doubtless the bronzy-looking specimens had suffered a still greater amount of change; the bronzy tint being merely superficial, they were not analysed.

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[*] The occurrence of this variety is another proof of the undesirableness of naming minerals from a single external character. There are numerous *bronzy* minerals in Scotland; I do not *know* that true *bronzite* occurs. The name is nearly as unsatisfactory as is that of *chlorite.*

Through Increment of Water; Hydrated Amphiboles.

In small amount.—" Hydrous Anthophyllite;" Hydrated fasciculitic Edenite.

19. From the north-west point of the bay of Scoorie, Sutherland, opposite the island of Handa.

Here is to be seen a wonderfully tortuous convolution of the gneiss. The domed summit of a plicated protrusion fold having been denuded off, or scalped by marine breaching, the dark hornblendic layers at one spot exhibit in section a singularly close resemblance to a huge soup-ladle.

This locality was discovered by DUDGEON. It is altogether a most extraordinary and extreme illustration of the pliability and plasticity of solid or semi-solid matter under the exercise of enormous pressure. At the periphery of the bowl of the ladle the darker layers of rock are folded upon and within themselves, to an extent that is not surpassed by the convolutions of the brain, or the intricate structure of the tooth of the labyrinthodon.

In chiselling off portions of these hornblendic layers,—here composed of actynolytic crystals,—the tool sank into a quantity of underlying pulpy yellow-looking mud or clay. This was perfectly plastic, soiling the hands and clothes, but was found to consist of, or contain speculæ of a crystalline matter. Portions of this wrapped in paper speedily toughened; they could the next day be broken, though with difficulty through retaining somewhat of their plasticity. In the course of some months they constituted an easily crumbled stone. They afterwards hardened to the ordinary consistence of rock, having all the appearance of hydrous anthophyllite.

This remarkable change has ofttimes been before noticed; it is said to occur even in the beryl.

The structure of the rigid mineral is somewhat similar to the fasciculitic edenite of Urquhart; the lustre is, however, somewhat pearly or greasy. The colour, which in the plastic condition was fawn, is now pale olive-green. Some specimens contain small crystals of talc or ripidolite of the same colour. When reduced to powder the colour becomes rapidly that of chocolate.

The specific gravity is 2·917.

1·22 grammes yielded—

Silica,	·538			
From Alumina,	·017			
	·555	=		45·508
Alumina,			6·376	6·388
Ferrous Oxide,			14·331	14·285
Lime,			...	4·437
Magnesia,			...	22·14
Alkalies,			...	traces
Water,			...	6·721
				99·479

It is possible that the so-called actynolitic crystals are the same substance, darkened in colour by exposure—they are dark green and brittle.

In large amount.—Hydrated Amianthus (?) *; Mountain Cork; Mountain Leather.*

From Granular Limestone.

20. From the limestone which occurs in the small promontory immediately to the west of the mouth of the Burn of the Boyne, in Banffshire.

The mineral here is of a buff or wash leather colour. It is found in thin flexible sheets, and also in rigid cork-like masses. Rarely it is colourless. No structure is, to the lens, visible in the rigid masses; the thinner sheets seem composed of felted fibres. A portion of the white mountain cork was analysed.

·77 grammes yielded—

Silica,	·393		
From Alumina,	·003		
	·396	=	51·428
Alumina,			7·515
Ferric Oxide,			2·06
Ferrous Oxide,			2·486
Manganous Oxide,			1·298
Lime,			·581
Magnesia,			9·35
Water,			25·043
			99·761

Loses in water bath 10·88 per cent. of water; insoluble silica, 1·428 per cent.

From Silurian Slates.

21. Specimen from Jameson Torry's collection ticketed "Lead hills." This was nearly white, tough, but flexible; seemed formed of matted fibres.

On 19·8 grains—

Silica,	51·45
Alumina,	7·981
Ferric Oxide,	·973
Ferrous Oxide,	3·286
Manganous Oxide,	1·487
Lime,	1·97
Magnesia,	10·15
Water,	21·7
	98·997

Loses 5·96 per cent. of water in bath; contained some calcite.

From Old Red Sandstone.

22. The conglomerate of the "Old Red" at the Tod Head, in Kincardineshire, contains a singular vertical vein of about one foot in width, which consists for the most part of calcite. This vein runs about S.E. and N.W. Towards its centre there is an almost continuous sheet of mountain leather. This sheet on the one side has always cockscomb crystals of baryte penetrating it; on the other there is calcite alone. Here and there fragments of the conglomerate are matted into the mass of the leather.

The appearances convey the idea that the mountain leather was the last in the formation. A cross vein in the neighbourhood, formed of lustrous calcite, carried imbedded crystals of Laumontite.

Colour white, more or less stained with iron oxide. Tough, and difficultly flexible.

On 17·3 grains—

		Strontian.
Silica,	52·483	51·65
Alumina,	6·326	9·505
Ferric Oxide,	·6	...
Ferrous Oxide,	2·111	5·805
Manganous Oxide,	2·878	some.
Lime,	1·342	10·005
Magnesia,	11·954	2·065
Water,	21·702	21·7
	99·396	100·73

Loses in bath 5·995 per cent. of water; insoluble silica, 1·618 per cent.

If mountain leather be an alteration product, and not an independent mineral, amianthus has probably been the original substance.

Attention may here be directed to the fact that Dr THOMSON published an analysis of mountain leather from Strontian, wherein he gave the quantity of water at 21·7 per cent.; this appeared so excessive that the analysis was not admitted into most works. The above three analyses give much the same quantity, and I therefore insert Dr THOMSON's analysis along with the above, as affording another illustration. The mineral from Tod Head is so impregnated with minutely granular quartz that it had to be three times picked and analysed before the results accorded with those from the other localities.

From Amygdaloid.

23. It has been in accordance with the view generally held,—namely, that mountain leather, cork, &c., are products of the alteration of a hornblendic mineral,—that I have inserted them above, but I have lately found in the railway cutting in the immediate vicinity of Partan Craig, Fife,* a variety of the same, occurring in an association which leaves little doubt of their being, sometimes at least, an unaltered and quite distinct substance.

The highly porphyritic amygdaloid here contains steam cavities filled with saponite, agates, and rarely celedonite; it also has its rents filled with veins, about an inch in thickness, of calcite, rarely crystallised.

The sides of these veins are coated with a fibrous mineral, which also rarely is imbedded in the calcite. The fibres are of extreme tenuity, and rarely felted; they are highly lustrous, and pure white, rarely passing into saponite green, or azure blue. A fitting trivial name would be mountain silk. The fibres are tough, and reduced to powder with difficulty. The specific gravity is 2·108.

1·201 grammes yielded—

Silica,	·649		
From Alumina,	·004		
	·653	=	54·371
Alumina,			11·27
Ferric Oxide,			·212
Ferrous Oxide,			1·094
Manganous Oxide,			·333
Lime,			·979
Magnesia,			9·492
Water,			22·407
			100·158

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* Lately corrupted to Ferry Port on Craig.

Loses in bath 9 · 259 per cent. of water.

The occurrence in unaltered calcite leads to the conclusion that this must be an unaltered mineral.

Increment of Water; decrement of Lime and partially of other Bases, but no decrement of Silica; hence an apparent increment of Magnesia.

Incipient passage into Serpentine.

Hydrous Asbestus.—Picrolite.

24. 25. Alteration products showing this change are to be found in several places in the Shetlands.

The lime and part of the iron first go, possibly by the direct action of carbonic acid.

At Doo's Geo in Balta veins of a picrolitic substance are to be found; there is not in these in themselves any appearance of change; they are quite unweathered-looking, quite fresh, and to all appearance undergoing no alteration except in this, that the comparison of different specimens shows a gradual fading of the characteristic appearances of the one mineral, and a gradual assumption of the features of the other; and secondly, that in the specimens in which the latter is well marked, there is generally more or less fibrous calcite lodging between the filaments of the mineral.

As a general description of these, they may be said to occur in dark green, finely fibrous masses of a greasy lustre and unctuous feeling; but it would be difficult to pronounce whether they were asbestus or picrolite.

A specimen, presenting the extremes of each appearance, was analysed. The second specimen was more lustrous, more serpentinous, and somewhat less fibrous. Specific gravity of first, 2 · 693; of second, 2 · 634.

	First. On 1 · 3 grm.	Second. On 1 · 302 grm.
Silica,	50 · 193	50 · 076
Alumina,	2 · 099	1 · 876
Ferrous Oxide,	4 · 393	6 · 087
Manganous Oxide,	· 007	· 23
Lime,	5 · 067	· 86
Magnesia,	29 · 226	31 · 566
Soda,	· 743	· 341
Water,	8 · 5	9 · 302
	100 · 228	100 · 338

Insoluble silica of first, 4 · 636 per cent.

With Gradual Removal of the Silica.

26. That mineral which was noticed by JAMESON as occurring in Glen Urquhart, and called by him anthophyllite, is an excellent illustration of the commencement of this,—the next stage in the process.

The mineral is that termed hydrous anthophyllite, and was probably originally an asbestiform tremolite. It occurs in very beautiful fibrous specimens, which constitute veins in a very singular if not unique rock.

It is an excellent illustration of the distinction which should be drawn between an altered rock, or *alteration product,* and a decomposed rock, or *weathered mineral.* There is certainly here no decomposition, no *degradation.* The substance is perfectly fresh, unrotted, and probably more beautiful in its silky lustre and specific integrity than was the asbestus from which it probably originated.

This rock is composed almost solely of large crystals of green zoizite and plates of Biotite; very rarely a plate of chlorite is seen; still more rarely nodules of chondrodite occur. Its veins carry Wollastonite of a skim-milk colour and fibrous structure.

The fibres of both this and the hydrous anthophyllite run transverse to the vein.

The fibres of the anthophyllite are 4 to 5 inches in length, of a greenish-brown colour, a silky lustre, and great toughness.

The locality is east of the Free Church of Milltown, near the serpentine.

Specific gravity, 2·811.

1·251 grammes afforded—

Silica,	·586		
From Alumina,	·011		
	·597	=	47·721
Alumina,		3·926	3·837
Ferric Oxide,		...	·176
Ferrous Oxide,		5·819	5·741
Manganous Oxide,		...	·159
Lime,		...	5·64
Magnesia,		...	28·745
Potash,		...	·186
Soda,		...	·264
Water,		...	7·648
			100·117

Insoluble silica, 4·187 per cent.; was quite pure.

27. From the great bed of serpentine at Portsoy.

In speaking of the change of augite into serpentine at this spot, it was mentioned that the augitic pseudomorphs which were analysed lay imbedded in what appears to be a hydrated asbestus. This substance is to be here seen of different appearances, which are all more or less perfected passages into serpentine.

The mineral bears in general some resemblance to a soft nephrite. It is of a pale green colour, occasionally of a matted fibrous structure, and of great toughness; its specific gravity is 2·388.

Three specimens, illustrating the progressive steps of the conversion, were selected; the two first were only partially,—that in which the change seemed perfected was completely examined. The first was tough, close in structure, and harder than the others, being altogether less serpentinous-looking.

	First. On 1·grm.	Second. On 1·2 grm.	Third. On 1·3 grm.	
Silica,	...	46·716	46·923	
Alumina,	...	·625	·633	
Ferric Oxide,	...	·007	·007	
Ferrous Oxide,	...	1·712	1·674	
Manganous Oxide,	...	·71	·769	
Lime,	...	not det.	9·907	
Magnesia,	...	25·758	25·846	
Potash,	·567	...	·567	
Soda,	·57	...	·582	
Water,	9·9	...	12·84	
			99·748	
Insoluble Silica,	...	2·32	2·295	per cent.
Loss in bath,	...	3·2	3·2	„

In this case the alkalies are retained during the change.

Though I have said that the above substance appears to be a hydrated asbestus, it is by no means improbable that it represents what was originally the felspathic portion of the rock. As a general rule, the fibrous structure is obscure, and two questions in this connection call for answer.

The first,—if the transmuted rock was primarily a gabbro, what now represents the felspar?

The second,—if this pale green mineral was originally asbestus, then the rock must have consisted of a great mass of matted asbestus, holding augitic crystals in their meshes, and of little else. Such a compound is unknown.

Now, one subsidiary bed of the serpentine here—that of which the portions we are considering are merely extremely well characterised illustrations—consists solely of the previously described red pseudomorphs of augite, set in or studding a kind of pale-green paste, much resembling serpentine; and this paste would seem to pass by insensible gradations into the subfibrous mineral, of which the above is the analysis.

So that it may be the correct view to regard this as being a transmutation from labradorite in the first place.

Here, then, as in the more easterly bed, we have to call in the actual transmutation by magnesian waters; and as portions of this transmuted material are distinctly fibrous, we have to ask ourselves if, as a preliminary stage in the process, the labradorite had been, through the insertion of magnesia, converted into a fibrous hornblende. Such a change Bischof considers possible; but it has to be remarked, that while the possibility of the increment of magnesia must be granted, it is not easy to account for the simultaneous removal of the alumina.

28. For an illustration of the increased abstraction of the silica, we will go to Pundy Geo, near Fethaland, in the mainland of Shetland.

Here a picrolitic mineral is to be found in large, coarse, fibrous masses with a columnar appearance. The form is that of actynolite; the rough crystals are several inches in length, and of a third of an inch in breadth. The colour is dark green. The specific gravity 2·65.

On 1·314 grammes—

Silica,	·563		
From Alumina,	·008		
	·571	=	42·932
Alumina,			1·847
Ferric Oxide,			5·104
Manganous Oxide,			·419
Lime,			·8
Magnesia,			36·195
Potash,			·81
Soda,			·366
Water,			11·5
			99·973

Absorbs 1·217 per cent. of water; insoluble silica, 4·378 per cent. Here the change into serpentine—a ferruginous serpentine—is well-nigh perfect. There can be no doubt that the mineral changed is actynolite. Associated minerals here are chlorite, with imbedded simple and twin crystals of blue-black magnetite, the finest in Scotland.

29. There is a mineral obtained in Ayrshire (the locality I cannot give—my specimens were purchased from the dealer, Patrick Doran) which has passed

under the name "hydrous anthophyllite." Doubtless this occurs somewhere about Pinbain, in the serpentinous district.

The mineral bears considerable resemblance to true hydrous anthophyllite, in colour, lustre, and in structure; it also, however, much resembles the fibrous talc from Cairney in Aberdeenshire.

The fibres radiate from separate centres of crystallisation, interlacing at their terminations, forming more or less circular masses. The lustre is subpearly, the colour grey, with here and there a brownish or a greenish tint. Specific gravity, 2·806.

In this mineral the change is not altogether what is usual; the silica has been reduced to the amount contained in serpentine, before the whole of the lime or of the peroxide of iron has been removed.

1·303 grammes yielded—

Silica,	·516		
From Alumina,	· 2		
	·518	=	39·754
Alumina,			·493
Ferric Oxide,			5·296
Ferrous Oxide,			4·114
Manganous Oxide,			·23
Lime,			6·274
Magnesia,			26·247
Potash,			·757
Soda,			·105
Water,			16·832
			100·102

Loss in bath, ·822.

30. A still more perfect change is to be seen in the so-called Baltimorite of Corrycharmaig, near Killin, in Perthshire.

This mineral is, from its structure, evidently the result of a change induced on an asbestiform variety either of augite or hornblende; its association with chromite and ripidolite leads to the assigning it rather to the augitic type.

It is of a fine dark sap-green colour, a fibrous slickenside-like structure and lustre;* it is somewhat brittle, and cuts like slate-pencil. Its specific gravity is 2·628.

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* It may appear strange to speak of a *slickenside lustre*, but, except in the case of metallic minerals, the lustre of all slickenside surfaces is much the same, and quite peculiar;—it may be said to be the lustre of reflected light.

1·131 grammes yielded—

Silica,	·466		
From Alumina,	·003		
	·469	=	41·467
Ferric Oxide,			4·005
Ferrous Oxide,			4·83
Manganous Oxide,			·265
Magnesia,			37·135
Water,			12·5
			100·202

Loses in bath 1·557 per cent.

Here, in the total abstraction of lime and of alkalies, we have a close approach to the simplicity of composition of serpentine. The relative amounts of silica, magnesia, and water are almost those normal to that mineral; there is still, however, a retention of iron.

31. If serpentine be a mineral incapable of assuming a crystalline form, or of having its particles grouped in an approach to a crystalline arrangement, then must the fibrous varieties, called chrysotile—which appear like acicular crystallisations—be pseudomorphic.

Even in these, however, the abstraction of the iron is not complete.

Chrysotile of wondrous beauty was found by Dudgeon near Hesta Ness, which terminates the south side of the bay of Gruting, in Fetlar, Shetland.

It consisted of veins of about one-fourth of an inch in thickness, which veins—sometimes in double rank—traversed a peculiar granular magnetite. The fibres of the chrysolite, as is usual with this mineral, lay transverse to the vein. These fibres were of a golden yellow, tinged with green; they were of extreme delicacy and had a brilliant silky lustre. Altogether they constitute one of the most beautiful of Scotch minerals.

1·495 grammes yielded—

Silica,	·578		
From Alumina,	·016		
	·594	=	39·732
Alumina,			·096
Ferrous Oxide,			2·923
Magnesia,			41·605
Water,			15·659
			100·015

As the magnetite here is itself saturated with a serpentinous basis, there is a possibility that the fibrous structure of the chrysolite may be the result of its protrusion, by an exfiltration process, through the interstices of the granular magnetite, which interstices acted upon the mineral in a manner similar to the holes in a draw-plate. Feather-alum and similar salts have their peculiar form impressed upon them in some such way.

ALTERATION PRODUCTS OF HORNBLENDE.

	S. G.	$\ddot{Si}$.	$\dddot{Al}_2$.	$\dddot{Fe}_2$.	$\dot{Fe}$.	$\dot{Mn}$.	$\dot{Ca}$.	$\dot{Mg}$.	$\dot{K}_2$.	$\dot{Na}_2$.	$\dot{H}_2$.	Total.
Hydration—Loss of $\dot{C}a$—Peroxidation—												
Hydrated Hornblende, Greenhill,	3·01	50·92	1·89	9·43	2·09	·31	8·65	21·58	·34	·43	4·54	100·17
Hydrous Anthophyllite, Scoorie, .	2·92	45·51	6·39	...	14·29	...	4·44	22·14	tr.	tr.	6·72	99·44
Mountain Cork, Portsoy, . .	...	51·43	7·52	2·06	2·49	1·3	·58	9·35	...	...	25·04	99·76
,, ,, Leadhills, . .	...	51·45	7·98	·97	3·29	1·49	1·97	10·15	...	...	21·7	99·
Mountain Leather, Tod Head, .	...	52·48	6·33	·6	2·11	2·88	1·34	11·95	...	...	21·7	99·4
Mountain Silk, Partan Craig, .	2·11	54·37	11·27	·21	1·09	·33	·98	9·49	...	...	22·41	100·16
Hydration—Loss of $\dot{C}a$ and ases—Passage into Serpentine—												
Picrolite, Balta, ...	2·69	50·19	2·1	...	4·39	·01	5·07	29·23	...	·74	8·5	100·23
,, serpentinous, Balta, .	2·63	50·08	1·88	...	6·09	·23	·86	31·57	...	·34	9·3	100·34
Above,—with Removal of Silica—												
Hydrous Anthophyllite, Urquhart,	2·81	47·72	3·84	·18	5·74	·16	5·64	28·75	·19	·26	7·65	100·12
Hydrous Asbestus, Portsoy, .	2·39	46·92	·63	·01	1·67	·77	9·91	25·85	·57	·58	12·84	99·75
Picrolite, Fethaland, . . .	2·65	42·93	1·85	5·1	...	·42	·8	36·19	·81	·37	11·5	99·97
Hydrous Anthophyllite, Ayrshire,	2·81	39·75	·49	5·3	4·11	·23	6·27	26·25	·76	·11	16·83	100·1
Baltimorite, Killin, . . .	2·63	41·47	...	4·01	4·83	·26	...	37·14	...	...	12·5	100·2
Chrysotile, Fetlar, . . .	...	39·73	·1	...	2·92	...	...	41·61	...	...	15·66	100·02

In taking a periscopic review of the foregoing analyses, we will direct our attention first to the information accorded to us by an examination of the alteration products of these closely allied species.

It has to be remarked, in the first place, that as the above analyses of these alteration products were made in the course of a progressive examination of all uncertain minerals, and in no way with the object of determining the nature of what has been unfortunately designated as "the magnesian process," these analyses come forward as altogether unbiassed witnesses as to what that process has been. They were not selected from the rock as specimens suitable or likely to prove any theory whatever, but were simply chosen as the best obtainable illustrations of things, the nature of which it was desirable to determine.

The first clearly notable point is that the so-called "serpentinous change" affects all the allomorphic forms of augite, the more solid and massive equally with the fibrous and foliaceous; while it seems for the most part inert as regards the denser varieties of hornblende, it being merely the fibrous and more delicate forms of that mineral which are affected.

As regards the transformation itself, it has, in considering certain of the analyses, been pointed out, that though serpentine is generally said to result from a change induced in augitic minerals through *the direct insertion* of magnesia, by magnesian waters, and the consequent increment of that material in the product, such an operation acting by itself could never accomplish the necessary transformation. Pre-existent constituents have to be abstracted, and the mere abstraction of these, without any direct insertion of magnesia, will, of itself, by the consequent proportional incrementation of the non-abstracted magnesia, suffice to determine the required change.

While it is much to be desired that a still more extended series of analyses be undertaken to throw more light on this matter, a glance at the tabulated results of the analyses of these alteration products will show that no inconsiderable amount of information is afforded to us thereby.

These results demonstrate that the process of change consisted, in progressive order—

First, in a direct increment of the water.

Second, in a decrement of the lime.

Third, in a gradually increased decrement of iron.

Fourth, in a decrement of silica.

In some situations, however, there is no decrement in the iron, but a peroxidation thereof.

What the rationale of the change under the *first* of these heads may be, I could not, in the circumstances of so small an amount of matter to found on, attempt to show.

What were the circumstances under which minerals, usually inert as regards any tendency to become hydrated, assumed to themselves, *often without any appearance of alteration,* so large a quantity of water, we do not in any measure know.

The circumstances under which some at least of the other changes may be effected we do know; and our knowledge thereof may be grouped under certain heads.

And here it may be remarked, that *unless we are able to show, upon recognised chemical principles, the mode in which one rock mass may be transmuted into another, we are still within the region of mere speculation.*

The leading summary of our knowledge may be stated thus:—

The primary agent of change is meteoric water, holding carbonic acid and oxygen in solution.

The secondary agent is spring water, holding less oxygen, more carbonic acid, and certain salts in solution.

The third agent is these same waters, sinking downward or rising upward, but now holding more complex salts—the products of the first operation of the waters themselves,—these salts being the agents of a second set, perhaps an endless cycle of changes, generally more potent than the originals.

As regards the substances operated on, we know that those most easily attacked are carbonates and silicates of the alkalies, the waters thus becoming charged with most potent graving tools.

Next we have silicates which contain lime, protoxides of iron, and of manganese.

Lastly, we know that silicates of alumina and magnesia are the most stable of all; for carbonated water has no action upon silicate of alumina, and but a slight one on silicate of magnesia.

In virtue of the above,—from compound silicates carbonated waters will abstract the silicates of lime, iron, and manganese; leaving the silicates of magnesia and alumina as residues.

In virtue of the above,—the rock masses which we find in nature to be least prone to decomposition, are either immediately silicates of alumina and magnesia, or they are such as have originated from the alteration of the less stable silicates.

Such are—steatite, talc, silicate of alumina, clay, kaolin, and sand itself—among simple silicates; and mica, chlorite, serpentine, asbestus, and mountain leather—among compound ones.

These, however subject they may be to more complex changes induced by saline or alkaline waters, are no longer liable to further alteration through the operation of atmospheric agents—such as oxygen, carbonic acid, and water.

Thus it is, then, that serpentine *asserts itself* wherever occurring; protruding as lines of rugged eminences,—fitter types of the attribute assigned to the "everlasting hills" than the lordlier granitic masses around it; thus it is that the mica crystal, which, torn from that granite, and mechanically comminuted but intrinsically unchanged, had served its purpose of giving continuity and sparkle to sandstone of newer and still newer epoch, glitters yet untarnished mid the sands of the sea-shore; and thus it is that these sands themselves, buffetted by the waves of Cambrian, and Old Red, and Coal Measure, and Permian, and it may be still more recent epochs, amid many surrounding changes have known none, but, atomies though they be, seem quite large enough and hard enough again to complete a like extensive cycle.

Thus it is that the clay which, as impure kaolin, the rain drop has gouged out of the felspar of that granite,—which, soft as mud, gives way to everything, but can be changed by nothing,—is seized upon by man to be fashioned into a structure, harder, less compressible, more durable than stone itself.

Thus, then, the mere passage of a current of carbonated water over minerals containing, or rock containing lime, iron, and silica, is sufficient to sweep these substances in solution out of the rock, and to do so, moreover, with great rapidity.

Some years ago I had an opportunity of noting this. In an investigation into the relative excellence and the durability of different paving stones, I first ascertained the quantity of water which perfectly fresh unaltered pieces thereof would absorb in 24 hours; and secondly the alteration effected upon them by the action of moist carbonic acid.

It was found that—

	Specific Gravity.	Contained of Hygroscopic Water, per cent.	Absorbed in 24 Hours, per Cent.
Ardshiel granite,	2·774	·04	·139
Furness granite,	2·603	·626	·282
Bunawe granite,	2·662	·241	·18
Ratho "greenstone,"	2·882	1·62	1·52
Marchburn "greenstone,"	2·996	·059	·031
Knockdow "greenstone,"	2·997	·062	·029

Here there is the power of taking into their pores the instrument of change.

In testing the power of that instrument, the following process was adopted:—One ounce of coarsely pulverised fragments of the rock were suspended in four ounces of water by the agitation of a current of carbonic acid, which was passed through them for two and a half hours.

This treatment was found to dissolve of the Knockdow greenstone 1·1 per cent., of which ·0475 was silica; = 4·32 per cent. of what was dissolved.

Of the Ratho stone there were dissolved 0·895 per cent., of which ·04 was silica; = 4·47 per cent.

As it is a recognised fact that silica is less soluble in carbonated than in ordinary water, this rapid solubility in the acidified water shows how great must be the solubility in ordinary waters.

The substances which were found to make up the remainder of the portion dissolved were the alkalies, lime, and iron; the latter becoming peroxidised rapidly and totally during the evaporation of the solution.

The point of chief moment was that only traces of magnesia appeared in the solution.

It was Bischof who first clearly pointed out the potency of carbonated waters in effecting decomposition of rocks containing the substances above noted as soluble therein; and its absolute want of power to remove magnesia from them on account of the insolubility of silicate of magnesia in carbonated waters or even in carbonated alkalies, supposing these to be formed as a first step in such a process.

Hence the direct and unfailing action of such waters upon augitic rocks must be their conversion into serpentine.

It has been shown that inasmuch as carbonic acid does not combine with alumina it can have no power to remove that alumina; and so a serpentine formed by the alteration of an aluminous mineral must be a more or less aluminous serpentine. And inasmuch as it is in hornblende and not in augite that alumina replaces silica, we may, if we find on analysis that a serpentine is notably aluminous, be able to say that it was formed by the transmutation of diorite or other hornblendic rock.

Again, inasmuch as carbonic acid does not combine with the peroxide of iron, if that substance is either present, or once formed in a mineral, it cannot be removed by such a process; and so it is that we have the iron in these serpentinous products unremoved if it be thrown into the state of peroxide.

Its retention as such may even aid us in determining the depth at which the transmutation took place.

If the change was effected near the surface, we know that the transmuting water was aerial,—"meteoric water," as it has been called. Each gallon of such water holds in solution 2 cubic inches of oxygen and 1 of carbonic acid. This water, holding so small a charge of carbonic acid, could effect the transformation with extreme slowness. We have seen that the lime was removed in the first place: if the quantity of acid did not suffice to remove both lime and iron, during the time that that acid was engaged in taking up the lime, the oxygen in solution would simultaneously be engaged in peroxidising the iron.

So we would be entitled to hold that serpentines with red or brown colours, and such as retained iron as peroxide, had been formed near the surface. So can we explain, also, the ferruginous crust which is so characteristic of most serpentines at their outcrop.

Spring waters, again, much more highly charged with carbonic acid, but not carrying so large a supply of oxygen, would effect the change more rapidly, and sweep away, more or less, perhaps, all of the iron as protocarbonate, leaving only residual traces of protosilicate, which impart the green coloration.

We have at Portsoy the most direct evidence conceivable of the conversion of a diallagic rock into serpentine, in the fact that one end of the stratum still remains as gabbro; and in immediate contact with it we have limestone, here very siliceous. Now, the frequent association of thin beds of limestones with serpentine supplies very direct evidence of the conversion of hornblendic and augitic rocks into serpentine. In that fact we have a ready answer to the question, "What becomes of the carbonate of lime necessarily formed during such an alterative process as the above?"

I will not say that limestone is always to be found in such association; we do not always find limestones even where we have indubitable evidence that they once existed; for here the very thing that makes can unmake, or sweep away. The carbonate of lime thus fashioned out of the rock forms a belt beneath the residual serpentine, thicker or thinner in accordance with the original thickness of the stratum of transformed rock; also thicker or thinner according to whether that rock was augitic or hornblendic; for the former can supply considerably more lime than the latter. This calcareous belt must lie beneath the parent rock, sealed against any great amount of further change, unless or until upheaval or denudation expose it to meteoric influences. *Then* water, flowing either downward or upward, may—nay, in time must—sweep it away in solution, leaving lime-sink, or collapsed-void to evidence its former existence. But if the limestones, so frequently associated with serpentines, are thus to be assigned to the decomposition of the rock which yielded these serpentines, we have a crucial test of the soundness of the theory of the change, in the inquiry as to whether *unchanged* gabbro, or other such rocks occur in contact with lime.

That it never does, I will not say; but, in glancing at a sketch geological map which I have constructed of the district where these rocks occur, I find, as regards the great belt of diorite and diallagic rock which sweeps up central Scotland, that where either the limestone appears in contact with it, or a "wash-out" discloses its former existence, there the rock is serpentine; where it appears as unaltered rock there is no lime.

I find, moreover, that wherever the association can be observed, the lime invariably is beneath the serpentine. So it is with the loch of Cliff lime and the serpentine of Unst; both of the lime and serpentine beds at Polmally; both of the lime and serpentine beds at Portsoy; at Limehillock; Tombreck; the Green Hill of Strathdon; and Beauty Hill; and in enumerating these I have named all the most important masses in the country.

The evidence of our transmuted minerals thus goes a long way to prove serpentine to be a metamorphic, and not an igneous, rock, whether the chemical process proposed in explanation of the *modus operandi* of the change be or be not considered satisfactory.

But there are occurrences of serpentinous matter differently circumstanced, where the above explanation can by no means suffice.

By the dissolving out of the lime from a stratum either of gabbro or of labradoric diorite, we would obtain a great mass of limestone, it is true; but that mass, relatively to the simultaneously formed serpentine, should be comparatively small.

The hornblende of Portsoy would yield equal to about 34 per cent. of carbonate of lime—its labradorite would yield about 20 per cent; and, as in the diorite rock, the hornblende is to the labradorite in about the proportion of 2 to 1, this rock as a whole would yield about 29 per cent. of its original bulk; of which, however, there remains still 84 per cent., for 13 of the 29 consist of direct addition of carbonic acid. Here, then, we have a rock yielding by its metamorphoses about one-third of its original bulk of carbonate of lime.

But, in what are called the serpentinous marbles, quite a different state of matters exists. These marbles I know from one part of the bed in Tiree; from Loch Bhalumais, in Lewis; Rodal, in Harris; Dalnein, in Strathdon; Glen Elg; Glen Tilt, and some neighbouring localities. It is of these only I am entitled to speak.

In all, the appearance is the same,—granular imbedded particles of serpentine, from the size of a shot to that of a bean, sparsely sprinkled throughout a great mass of lime, in an amount which is altogether quite trifling.

Unhesitatingly I say that these granulars are, one and all, pseudomorphs of pre-existent crystals of augite. That mineral may be seen unchanged and changing in the Tiree marble. In Glen Elg and other localities the pseudomorphic forms are so perfect that the crystalline form of the augite is indubitable.

These trifling specks could never have been the origin of a lime stratum tens of feet in thickness.

I would suggest the following explanation of the serpentinous formation of these pseudomorphs,—which had palpably pre-existed as augitic crystals imbedded in lime.

It is well known that when silica or silicates are heated along with carbonate of lime, there is, in the first instance, a disengagement of carbonic acid and a formation of silicate of lime; or, in the presence of other bases, of more complex silicates. Bischof found this decomposition to take place unfailingly even at the temperature of boiling water. No great depth in the earth's crust would suffice for the attainment of such a temperature. Thus at the point of contact of the lime with the including rock, and also with inclosed silicous minerals, would there be disengaged the very acid which we have already seen to be the active agent in serpentinous change; and thus also would there be formed new silicates, such as Wollastonite, silicate of lime, and tremolite, which, in fact, are found associated with the pseudomorphs of the serpentinous marbles.

In the outset I directed attention to the fact that serpentines were occasionally formed by the transmutation of such rocks as diallage and diorite *as a whole,—i.e.*, that the labradoric as well as the augitic ingredient had suffered conversion. The analyses afford one if not two instances of such conversion, and I now instance another which may be a case in point.

32. Beauty Hill, to the north of Aberdeen, is composed of serpentine, and in small quarrys on its north-east side it will be seen to be as distinctly bedded a rock as any recognised sedimentary deposit.

On its eastern slope some masses of gabbro protrude through the sward of a field. This gabbro is, for the most part, unaltered. It consists of dark somewhat granular crystals, apparently of augite, of the size of large shot, which are imbedded, singly or separately, in a waxy-looking massive labradorite.

In this rock a vein of a pale sap-green mineral was found by Professor Nicol and myself. This was set down by my friend as precious serpentine; to me it appeared somewhat too hard for this, as it cut with less ease than slate-pencil. It was translucent, tough, and had a specific gravity of 2·59.

On analysis it afforded—

Silica,	34·731
Alumina,	12·444
Ferrous Oxide,	2·684
Manganous Oxide,	1·17
Lime,	1·595
Magnesia,	34·098
Water,	13·1
	99·822

Now this is precisely the composition of the massive variety of penninite which, from its resemblance to serpentine, has received the name of pseudophite. But it may also be regarded as a highly aluminous serpentine; for if that earth be abstracted the residue is just serpentine. The point of interest which attaches to this substance is, that although it formed a true exfiltration vein in the rock, if the rock in contact with the vein be examined, *the waxy labradorite is also seen to pass, within a space of about an inch, into this mineral by insensible gradation.* So it matters not whether it, in a systematic arrangement, be consistently classed with penninite or not, it is here unquestionably a transmutation product, the result of a serpentin*ous* change of the labradorite.

The *modus operandi* of the change of felspar into serpentine is much more subtle than that of the conversion of augite, and also, it must be said, much less certain. As labradorite contains much alumina, and no magnesia, we have, in *its* serpentinous change, to account for the removal of alumina and the direct insertion of magnesia. The term "magnesian process" may with perfect fitness be applied to such a change.

Such an action as that above shown to suffice for the transmutation of augites could not in any degree effect a similar change in labradorite; carbonated waters do not affect silicate of alumina, and carbonated waters cannot directly purvey magnesia.

It is true that immediately over the bed of serpentine at Portsoy which evidenced such a change, there is a washed out bed of *something,* and it might be argued that that something had been dolomitic limestone; which would yield bicarbonate of magnesia to waters passing through it;—which bicarbonate of magnesia, by a recognised interchange with silicate of lime, would yield silicate of magnesia. To such a view it has to be replied, first, that there is no evidence that the removed stratum was limestone,—the rock immediately beyond the void is a perfectly unaltered diorite, which therefore could not have supplied any lime;—secondly, that, in Scotland at least, the limestones associated with serpentine are not dolomitic; and this because, according to the opposite view, the magnesia is retained in the resultant serpentine;—and lastly, that there is no provision, according to such a view, for the removal of the alumina.

For aid in the explanation, we must ascertain what are the circumstances in which silicate of alumina can either be decomposed or dissolved; and what those in which silicate of magnesia can in any way be introduced:—we will then be in a position to say whether any of the said circumstances, in the case in question obtain.

Silicate of alumina can be decomposed by chloride of magnesium or sulphate of magnesium in solution,—silicate of magnesia being formed.

Silicate of magnesia, again, is also formed by the decomposition of bicarbonate of magnesia, by silicate of lime, or by silicates of the alkalies.

Now the two first of these salts are present in river water, more largely in spring water, and most largely in sea water; while bicarbonate of magnesia is very frequently present in spring water. Though it is unnecessary to call in the operation of sea water, in presence of the fact that the waters which permeate all rocks are themselves charged with these potent agents of perpetual change, still, seeing that the altered rocks in question have been beneath the sea, and that their inmost pores would then be more urgently saturated by hydrostatic pressure, it may be very fittingly argued that diallagic rocks which have their felspar as well as their augite converted into serpentine, suffered the alteration during an epoch of marine submergence.

Bischof writes: "It would be very illogical to suppose that the calcareous and magnesian salts dissolved in sea water do not take part in chemical alteration, when it is so manifest that alteration is effected by these salts dissolved in much smaller quantity in the water percolating through rocks. It would be inconsistent with the relation of mutual compensation perceivable in all natural phenomena to suppose that saline substances were continually carried into the sea, without being consumed in the formation of new substances." To apply this to the present case, there would seem to be no readier way of accounting for the abstraction of the vast quantities of magnesian salt present in the sea, and constantly being added to it, than to use it up, so to speak, in the formation of serpentine. How else abstract so soluble a salt as chloride of magnesium? Its extreme solubility and deliquescence would negative its being abstracted through the direct formation of minerals; but, in virtue of the above interchange, lime salts replace it in the ocean, to be continuously removed in turn by crustaceans, mollusks, and coral insects.

Serpentines, as a rule, are denser, more solid,—give fewer and feebler evidences of being the result of change effected upon pre-existent rocks, when examined *in their depths* than on the exterior. Without entering upon the many and complex reactions which take place as secondary results of the interchanges already noticed, it may be sufficient to show that the products of surface change, as they percolate through the deeper portions of a rock, may effect a somewhat dissimilar change therein, and may also, in their passage through these rocks, plug up their pores.

Silicate of lime, formed by the action of meteoric water in the superficial portions of a rock, and meeting with magnesian salts within, would by interchange supply serpentinous matter in the solid form to the more porous portions of the deeper-seated beds, to render the whole mass more uniform in structure, while it might also thereby be diversified in colour.

When we remember that the decomposition of both augite and labradorite is effected through the operation of what may be called the ordinary agents of exposure—carbonic acid, oxygen, and water—but that, in virtue of their difference in composition, the nature and rate of the decomposition varies, it is easy to explain how it is that the crystals of labradorite are protuberant from the general mass of the augite at Lendalfoot, while at Pinbain the augitic crystals stand in high relief above the felspar in that wave-washed situation.

Augite, containing iron in the state of protoxide, most prone to higher oxidation, rapidly and readily gives way when subjected to aerial exposure, where oxygen, carbonic acid, and water are alike free to operate upon it. Labradorite, containing little or no protoxide, is subject to the operation of the two last only, and so, in the air, is the more enduring. When plunged beneath water, however, the oxygen is in great part shut off from the augite, which is thus protected; while here, the labradorite is subjected to the attack of water upon its alkaline silicates, suffering thereby rapid degradation.

Varying circumstances, therefore, must ever vary the mode of degradation of a rock.

From this consideration of the rationale of the process, elaborated at the desk, I turn to an illustration of its working, as shown in nature.

That portion of the Long Island which is called Harris is almost entirely composed of a hornblendic gneiss, which has been assigned to Laurentian age. It is certainly the oldest of all known rocks in Scotland.

In the northern portions of the district the rock dip is that normal to it on the mainland of Scotland,—namely, to the S.S.W.

This is well seen in the striking hills of Totam and Cleesham, which present great precipices to the north, and accessible slopes southward.

In the south of the island, towards the Sound of Harris, the dip seems to be the other way, but it is somewhat obscure. This obscurity is in part due to the occurrence of an almost non-laminated massive rock, which my *confrère*, Mr Dudgeon, and I claim to have discovered here, as no one of the writers who have described the country has noticed it.

This rock consists in greatest part of garnet, in less of smaragdite, and in still less of kyanite—it is in fact eklogite.

The rock forms a ridge extending from Ben Capval on the west, to Roneval on the east; it is somewhere about eight miles in length, by two in breadth, with an altitude of 1000 feet on the west and 1500 on the east.

On rounding to the northward the eastern extremity of this ridge, the hornblendic rock is again met with, its layers laced and bound together by huge granitic dykes.

This extends to the northward as a gently undulating country,—if such a word is here applicable,—for a distance of some seven miles, with a breadth of about four.

In no other part of Scotland is such an expanse of utterly barren waste to be met with. There is copse on Cruchan Ben, heath bells in deep Glen Coe; there are at least waterfalls which represent that motion which is akin to life at Coruisk, and there is plenty of heather, along with the grey boulder stones on the dreary moor of Rannock. But here there is *nothing*. And had those writers, who in their eloquence have expended all the synonyms of desolation in their descriptions of the above localities, but visited this, they would have found themselves with an exhausted vocabulary in presence of a scene which, for dreary barrenness, transcends them all.

A great flat—and yet not a flat; roll after roll of bared and bleached rock,—like the swellings of the ocean petrified after storm; no covering whatever,

> "Worn and wasted to the bones;"

nothing which speaks of life; nothing which, at first sight, even speaks of motion past. The only objects which speak at all, speak of death,—every loose stone has been sedulously collected to form those cairns which mark the resting-places of the funeral corteges which for generations have carried their burdens from sea to sea.

In single line these cairns stretch from east to west, and, some little way north of them, there also stretches a line of towering eminences, connected ridge-wise into an elevated Scuir.

Crossing transversely the scalped and wasted outcrops of one of the hardest of known rocks, curiosity is strained to the utmost as one speculates upon the nature of this enduring ridge.

If this primal rib of old mother earth upon which we walk has been so cut in upon, surely the rock which has withstood the operation of the agent of decay must be of adamant itself.

But when, with a dull thud, the hammer almost sinks into its substance, when the knife cuts into it with ease, when the nail shows the hardness of the mass to be little superior to its own, curiosity gives place to wonder.

The ridge is serpentine—one of the softest of rocks; and yet, in lofty eminences, it overtops the worn and troughed-out gneiss.

Truly here is a riddle difficult to be read. Of what strange temper was the graving tool which, while it gouged out the gneiss, was deflected by the serpentine.

Or did it escape the operations of the tool? Did it, aiguille-like, tower above the zone of its action?—for no one who has learnt ice-sign can fail to recognise it here; in all scalped Scotland there is no such illustration of its power. True that there is no direct evidence that the low-lying flat resulted from *its* work alone; but the top-dressing certainly did;—the rounding of the outlines, the cutting out of the tarns, the smoothing of the surface, were the work of ice alone.

And the serpentine did not escape,—was not above the scope of its operations. Terrific gougings 30 and 40 feet in length, and large enough to hold a limb, if not a body; trenches, along the weaknesses of the jointing, in which a herring boat might lie concealed; and rounded haunches, after the similitude of the hind-quarters of an elephant,—these showed that the ice had not spared the serpentine.

Truly the riddle of the Scair Ruidh, the Red Scur, is hard to read; but, thinking of another Scottish Scoor, I resolved to master it. The riddle of that other Scottish Scoor was read by a talented member of our body; and though he may in the future place many feathers in his cap, that which he placed there by the reading of that riddle may, perchance, remain the loftiest.

But when Geikie read the riddle of the Scur of Eig, he had something more to do than what is before us here; he was in the position of Daniel before Nebuchadnezzar—he had not only to expound the dream, but he had to *declare it.* Geologists of great repute had speculated on Eig before; they expounded capitally; only, unfortunately, they expounded the wrong dream.

But here no such difficulty is before us; the dream is declared; it is the exposition only that is called for.

Given, a high ridge of about the softest known rock, dominating over an expanse of one of the hardest, both exhibit unmistakable evidences of ice graving, but the soft rock to much the greater extent. How comes it that the soft rock has been left protruding?

The solution is, to a certain extent, easy: it is very evident that it must have been highly protuberent above the gneiss when the ice began its work, and that the duration of that work was not sufficient to bring about what may be termed the normal result.

But how came it that the soft rock was thus protuberent? And this is what serpentinous change, considered along with other simultaneously acting changes, alone can show.

"Who scalped the brows of old Cairngorm?" "'Twas I,—the Spirit of the Storm." Now, this Spirit, in making free with those poetic fancies called "the everlasting hills," makes use of several kinds of scalping knives.

Certain of these are chemical, some physical. Among the chemical we have the gnawing tooth of carbonic acid, and the rusting fang of oxygen. Among the physical there is the solvent soak of water, the expanding heave of that water when it becomes ice, the chiselling chip of the sand blast, and, more locally, the rasping and the bruising of marine breaching.

Let us see how they have been operating here, for operate they did before the era of the ice,—there could have been no serpentine else. The ribs of mother earth, when swathed in ice, must have been as effectually preserved from atmospheric change, as are those other ribs which are brought over the Atlantic in ice-sheathed holds.

The result of their operation—of the operation of all yet noticed—is in one respect the same. This has been emphatically pointed out by the gentleman already mentioned; *they* all *roughen*, one alone *smoothes.*[*] It is the function of ice alone to "make the rough places plain."

And the planed surfaces present a fair field for the study of the mode of action of the atmospheric agencies, when the re-exposure of the surfaces left them again subject to such action.

That action has been but trifling in amount after all. The hieroglyphics on those polished tablets of stone—these glacial Runes "wrought by the hand which works unseen"—we know were not of yesterday; but, so sharply tangible to feeling as well as to sight are they, that we might almost conclude that they were. The thousand years has here been as one day.

At Canisp and other parts of Sutherland the tracings may be seen to run to the brink of what are but greater scratchings after all;—a trough which lies between it and Coul More on the south is 6 miles wide and 1500 feet of clear depth;—another, between the same hill and Quinaig on the north, is 5 miles in width and 2600 feet in depth.

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[*] Water in certain situations *appears* to polish—as in the rock-runnels of streams, the pot-holes of rivers, the back-tow crevasses of the seashore, and some undercut beach rocks or cliffs. In all these cases, sand or stones, held in the grasp of the water, have done the work, which was of the nature of grinding or chiselling. Most interesting illustrations of simple wave-action, and of this compound pummelling and rasping process, are to be seen at low water beneath the Kincraig near Elie.

And these were cut by the operation of the very agents which—acting during an epoch that may to a certain extent be measured by that of the human tenancy of the globe—have failed even to smooth out the minute tracery of the ice.

Trifling as the amount of the action is here, and vast as it has been there, that action, and the chief agent in that action, will be found to be the same in both cases. Water—water everywhere.

Arenaceous rocks are disintegrated through the solution of the trifling amount of the calcareous and magnesian cement which unites their grains; and which, once soaked out from between these grains, leaves them to the brushing of winds or the scouring of waters. Rocks which contain felspar assume the appearance of a rasp, from the decay of that felspar, the alkalies of which are called for elsewhere, and are borne away on the chariot wheels of the raindrop;—"the highest parts of the dust of the world" may have had nobler uses assigned to it than the mere nurturing of lichens.

Carrying with us to Harris the lesson learned in the schoolroom of Sutherland, and holding our course along the ridge of the Red Scuir, we note the geognostic facts that its eminences gradually diminish in height and in width as we proceed westward; and that they are accompanied, for about the first half of the six or seven miles of their length, by a series of lakes, also gradually diminishing in size. As these lakes maintain a constant distance from the serpentinous ridge, and underlie it in position, they have the greatest possible resemblance to a washed-out lime stratum.

As a lithological fact, again, it is to be noted that while at the more easterly, the loftier end of the red ridge, the whole rock is a massive perfectly-formed serpentine; as we approach the west, foliaceous flakes of enstatite, or some one of the minerals which cluster round augite, become visible. Further west still, where the ridge begins to diminish in height, and the lakes are represented merely by swampy ground, the hornblendic type of mineral appears as a matted asbestiform actynolite;* while at its termination, in the comparatively low Dun of Borve, true unaltered hornblende of the smaragdite type is alone seen,—mixed, however, with a concreting felspar.

Southward of, that is beneath this, there is no appearance of swamp, or any evidence direct or indirect of the presence of lime.

The condition of things when the outcrops of the system of rocks was first exposed to the operation of the agents of change would therefore appear to have been this;—there were to the north, beds of the ordinary hornblendic gneiss, succeeded, in descending order, by a bed of rock which was hornblendic to the

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* JAMESON says that he found enough of this in the island to load an Indiaman,—and this is the spot where it occurs.

west and augitic to the east; this, again, by other strata of the gneiss; and these, lastly, by the garnetiferous eklogite.

The action of the graving tools on each variety of rock is, so far, certain.

The gneissic rock, physically tough, is inherently most prone to degradation; highly felspathic, it yields the alkalies of the felspar to carbonated waters; the residual kaolinic mud is swept away by every raindrop and every stream; and the loosened quartz grains are removed from its surface by every gust of wind. Vast tracts of barren sands along the shores of these islands speak unmistakably to the great waste which the rock has suffered.

Of the succeeding stratum, one extremity—the augitic—is more subject to change perhaps than even the gneiss; but it is a change which does not result in its disintegration and its waste. Considered as an augitic rock, it may be said to be most prone to *rotting;* but it is a process of rotting which enables it to endure. As a rock mass, it is a transmutation, not a decay.

The outcome of its primal weakness is its abiding strength. After the transmutation, it is no longer subject to the further operation of the agents which transmuted it. Into its dense structure the carbonic acid cannot force its way; nor is there aught now there for that carbonic acid to combine with; while off its oily surfaces the raindrop flows unabsorbed. It might, not altogether inaptly, be termed the *adipocere* of rocks.

During all the period of the transmutation of the augitic extremity of the ridge, the western, formed of the less alterable hornblendic mineral, is falling to pieces from the disintegration of its felspar, and is being swept away as a hornblendic sand.

The last rock of the district, consisting chiefly of garnet—rich in alumina, which forms no natural carbonate—almost defies decay; garnetiferous sands being residues almost as enduring as siliceous ones.

So that, when, after long subjection to the operations of the Spirit of the Storm, he thought to put some sort of a polish upon his work, and the ice settled down upon the land, it found in the southward a lofty elevated ridge of enduring garnet, succeeded to the north by a low and wasted plain, in the midst of which there projected, high in air, a rugged ridge with a russet coat.

Doubtless the gouging ice would soon have made an end of the almost pulpy protuberances,—those terrific gashes showed that the scalping-knife was making quick work; but the great ice age came to an end first; the fiat, "thus far," or "thus long," had gone forth; and the Red Rock henceforth shall stand erect in the grim wilderness, a type of those things which "out of weakness are made strong."

Turning our attention next to the modes of occurrence and lithological associations of the unaltered varieties of augite and hornblende, the first direction our inquiry must take is as regards any evidence the analyses may afford, bearing upon the union of the two species into one.

It may be at once conceded that this question cannot be decided by any merely chemical evidence; the wide range of composition, hardly referable to anything closer than a general formula in either mineral, precludes such a hope. The little information that we do obtain tends in the direction of the separation of the two; it consists in this, that in what may be designated as "recognised individuals" of the hornblendic type we have here and again a distinct replacement of silica by alumina; and this we have not in recognised individuals of the augitic type.

Passing from this to the lithological information, we find that a consideration of the records above made by no means tends to strengthen certain of the lines of demarcation which have been drawn between them.

Put in a general form, two of these lines are thus defined: "These two minerals occur in distinct geognostic positions. Hornblende in rocks containing quartz or free silica, and mostly with minerals that are neutral compounds of silica, as orthoclase and albite. Augite in rocks that do not contain free silica, and mostly with minerals that are not neutral silicates, as labradorite, olivine, and leucite.

"Hence, there are two distinct series of igneous rocks: the hornblende series, including granite, syenite, diorite, &c.; and the augite series, or hypersthene rock, gabbro, dolerite, &c.

"In some rare instances these two minerals have been found together, either regularly conjoined or in distinct conditions."*

Here we have it clearly laid down that two "distinct series of igneous rocks" are established on the specific distinction between the two minerals; and secondly, that one argument for these minerals being themselves distinct is to be drawn from the fact that they are associated in but "rare instances."

It would appear at first sight to be an easy matter to enter at least upon the consideration of questions couched in language so precise as the above; but it proves to be not altogether so.

We have it *first* laid down here that hornblende occurs in rocks with free silica, and along with orthoclase and albite.

Secondly, that augite occurs in rocks devoid of free silica, and,—in contradistinction to hornblende,—along with labradorite, olivine, &c.

Thirdly, that the hornblendic series of rocks,—granite, syenite, diorite,—are "distinct" from the augitic series,—hypersthene rock, gabbro, dolerite, &c.

Fourtly, that the two minerals, on the distinctiveness of which the above separations have been made, occur together in rare instances.

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* NICOL's "Manual of Mineralogy," p. 208. An epitomy of the views of GUSTAV ROSE.

Let us take evidence from the specimens analysed as to these points.

Setting aside, meanwhile, the evidence of all those fibrous hornblendes which occur in serpentinous rocks, which, it has been shown, may and do result from the metamorphosis of rocks of both a hornblendic and augitic nature,—and waving all present consideration of the question whether several of the rocks mentioned are necessarily of an igneous origin, we come first to the hornblende of Balta.

This occurs imbedded in a diallage rock, one of the augitic series, a rock in which there is no free silica, no orthoclase or albite; the immediate matrix is labradorite,—a bed of which, some dozen feet apart, also carries augite. So thus our first appearance of hornblende, as above noted, contradicts at once the first, third, and fourth of the principles laid down above.

Next we have the hornblende of Fetlar, which with anorthite forms a well-marked rock. DELESSE having found anorthite in certain rocks denominated diorite, we grant the applicability of the name. No augitic mineral is here present and no quartz; but if the Balta specimens did not, in having labradorite as their associate, conform to the requirements which demanded that the hornblende should be accompanied by a *neutral* silicate, we have a further departure here in the presence of the *most highly basic* of all the felspars.

The huge crystals of Glenbucket stand the next in order. The rock they go to form has labradorite as its felspar, with small quantities of other things; but quartz and the neutral silicates again find no place. The requirements of the first postulate are not here met in either particular, and it will afterwards be shown that this occurrence goes far to nullify the third.

The same remarks, in their totality, apply to the hornblendic occurrence in the labradoric diorite of Portsoy, our fourth example, and these are all that in any way do apply. As regards postulate number one, we have had contradiction throughout. Three of the four cases brought forward were those of the finest and best developed of the occurrences of the mineral in the country. That which stands next in this respect, namely, the hornblende of the Deskery, bears evidence which is of the same character. I do not mean to insist that in Scotland there is never a rock association between hornblende and the more acidic felspars; but the syenite of Froster Hill, consisting of hornblende, orthoclase, and some menaccanite,—the "beautiful rock" of Colafirth, consisting of hornblende (?) and albite,—the rock of Hillswick, of actynolitic hornblende and albite,—and hornblendic gneiss, are the only good illustrations which I remember where the above requirements are complied with.*

As regards postulate the second, it will be seen that the evidence of the section of the chapter which bears upon this is entirely in its favour.

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* The felspar of the Morven syenite I have not yet determined. There is also a peculiar rock found north of Shinness in which the felspar *may* be orthoclase.

In approaching postulate the third, no small difficulty presents itself at the outset, on account of the different views held by different authorities regarding the constitution of certain of the rocks named.

In the foregoing general classification we have, on the one hand, diorite set down among hornblendic rock, *i.e.*, those in which hornblende is associated with orthoclase and albite; and, in the augitic series, we have hypersthene, gabbro, and dolerite.

From COTTA we learn that diorite is a compound, not of hornblende and albite, but of hornblende with oligoclase. The same authority gives us diabase as an augitic compound, with hyperite as its synonym; while elsewhere, in the same work, hyperite is defined as a variety of gabbro.

DANA distinctly states * that diorite is a compound of albite and hornblende, diabase † one of hornblende and labradorite; and he seems to indicate that this last rock passes into hyperite.

Professor GREEN tells us that diorite is composed of oligoclase and hornblende; that hyperite is an augitic rock resulting from the substitution of hypersthene for the augite of gabbro or dolerite; and he gives us anorthite as a component only of Corsite.

BISCHOF so strongly disallows the presence of either labradorite or anorthite in diorite, that he, in his third volume, fences with every one of the analyses of DELESSE, which clearly demonstrate their occurrence; while, at page 206 of his second volume, he calls diorite an augitic rock which has labradorite as its felspar.

From such a confusion as the above—which demands a congress of lithologists—we turn to the evidence of the rocks. The felspathic, as well as the other compound of these rocks, having been analysed in the instances which I shall cite, the correctness of their evidence cannot be called in question.

Calling the hornblendic rock diorite, with what do we in Scotland find that hornblende associated?

With labradorite, in the exceptional occurrences of hornblende in the diallage of Balta; with anorthite in the diorite of Fetlar; and with labradorite in the diorite of Portsoy, Glenbucket, Colquhanny, Deskery, and wherever it appears, in the long reach from the sea to the vicinity of the Dee.

In this rock neither albite and orthoclase, on the one hand, or oligoclase, on the other, ever find a place.

This association of hornblende with labradorite in diorite is exceptional, though not altogether unknown, DELESSE having recognised both labradorite and anorthite as the felspar of diorite.

BISCHOF, again, unable to get a sufficiency of lime from oligoclase to explain the composition of some diorites, first endeavoured to supply it by supposing the presence of a calcareous augite; but finally, dissatisfied with this supposition,

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* "Mineralogy," p. 240.

† *Ibid.* p. 343.

concludes by saying, "there is no doubt a class of hornblendic rocks which contain a highly calcareous felspar." This, in their having labradorite as the felspar, is precisely what all the diorites of Scotland which I have examined do contain.

Next, as to the nature of hyperite,—that is of true hyperite; the Skye and Rum rock need not be considered, as the name can only be applied to it in virtue of a certain lithological similitude.

In the diorite of Craig Burroch veins of hyperite, consisting of labradorite and hypersthene (Paulite), appear; and the rock itself shades off into hyperite, both there, at Retannach, and at the Bin of Huntly.

The *mineral* Paulite stands nearer to augite than to amphibole, and in the only localities where, so far as I know, true hyperite rock appears, it is a mere variety—out of many others—of a rock which has diorite as its simplest type—but in which a transition has been accomplished through gradual replacement by augite—the intermediate mineral acting here as an intermediate in the lithological change.

Here, then, the requirements of postulate third are not conformed to. My own opinion is, that through insensible gradations—intermediate varieties—augitic and hornblendic rocks pass into one another more frequently than is imagined. The two minerals also, in certain circumstances, are so very similar that I do not believe that any mode of discrimination which can be applied in the field, or any more expeditious than the reflective goneometer, can serve to determine them; so that defective nomenclature may have been the cause of the drawing of too sharp a line of demarcation between the rocks which contain them.

I have little doubt that the diorite of Portsoy, last seen in the neighbourhood of Colquhanny,* will be found to shade off into the "syenite" of Morven; that the hornblendic type of rock, the augitic type of rock, the hyperitic type of rock, and the Biotitic type of rock occurring in the neighbourhood of Portsoy will come to be regarded as mere mineralogical varieties; and that a somewhat similar rock, occurring first about half a mile to the east of the mouth of the Durn of the Boyne in Banff, composed there of a foliaceous mineral—whether augite or hornblende the eye could not determine—and which runs up the country, taking a felspar as its associate, will, through several such changes, be found to be the northern extension of the syenite of Froster Hill. While it may prove to be this same belt which reappears in association with the serpentine of Barra Hill,—its hornblende having given way to a mixture of brown augite and Paulite (or bronzite), and its orthoclase to labradorite.

A somewhat similar rock again appears at Beauty Hill, once more in association with serpentine; in this there is no Paulite, and the augite is green.

As to postulate fourth—the question whether the distinctiveness of the two

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* The belt on the Deskery may be the same.

minerals acts as a bar to their paragenetic occurrence. The subjoined columns exhibit the information which is to be derived from the specimens that have been analysed.

In Juxtaposition.	
Shinness,	Malacolite and Actynolite.
Glen Tilt,	Malacolite and Tremolite.
Tiree,	Sahlite and Actynolite.
Eslie,	Sahlite and Actynolite.
Balta,	Augite and Hornblende.
Elie,	Augite and Hornblende.
Portsoy,	Augite and Asbestus.
Glen Gairn,	Sahlite and Actynolite.
Crathie,	Sahlite and Actynolite.
Colafirth,	Sahlite and Actynolite.
Glen Beg,	Augite and Actynolite.
Craig Lui,	Augite and Actynolite.

Apart.	
Totaig,	Malacolite.
Beinnegapple,	Augite.
Alltcailleach,	Malacolite.
Ben Chourn,	Sahlite.
Pinbain,	Diallage.
Tarfside,	Sahlite.
Loch Tay,	Sahlite.
Cuchullins,	Augite.
Rum,	Augite.
John O' Groat,	Augite.
Coyle,	Actynolite.
Fetlar,	Hornblende.
Nudista,	Actynolite.
Urquhart,	Actynolite.
Errins,	Actynolite.
Portsoy,	Hornblende.
Glenbucket,	Hornblende.
Enesay Island,	Actynolite.

This tabulation does not point to any marked non-consorting of the one mineral with the other. The very frequent apparently solitary occurrence of asbestus and other fibrous varieties of hornblende in serpentine rocks proves nothing, as here these rocks may have been formed from augite, the less alterable hornblendic mineral being left unchanged.

One other point calls for brief notice,—the question as to augite being a volcanic or fusion-form of hornblende. It will be seen above that both of the minerals occur in granular limestones; these afford very distinct evidence of the operation of great heat.

Again, it will be observed that both occur together in the basalts and dykes of Elie and Kinkell; in these the one mineral—the augite—has undergone fusion; while the other—*the more fusible*—appears unchanged. This circumstance becomes a strong argument in favour of the latter having been formed *in situ*,—*i.e.*, after the eruption and cooling of the containing rock,—an exfiltration product in fact.

Here it may be the case that the fused augite has at one time been formed from hornblende, but the two could not here have been erupted together; so that no direct evidence is to be gleaned from this apparent paragenesis in space, which palpably was not a paragenesis in time.

XVIII.—*An Account of some Experiments on the Telephone and Microphone.*

By James Blyth, M.A.

(Read 3d June 1878.)

The first experiment which I have to describe relates to an attempt to employ the telephone as a means of measuring the hearing power of the ear in different individuals, or in the same individual at different times. For this purpose a constant source of sound, driven by clockwork, is made to act upon the disc of a telephone in a distant room. In the circuit of this telephone are included two others, so placed on a stand, that they can be brought close to both ears of the person to be experimented on. Soft india-rubber rings are attached to these telephones so as to enclose both ears, and serve the double purpose of avoiding any disagreeable pressure on them, and preventing, as far as possible, all extraneous sounds from entering. In the circuit of the telephones there is also included a column of slightly acidulated water whose length can be varied at pleasure, so as to introduce a greater or less resistance to the electric current. This is managed in the following way:—A pretty wide glass tube, about 24 centimetres long, closed at the lower end, and graduated to millimetres, is mounted vertically on a stand. Projecting through the lower end of the tube is a platinum wire melted into the glass and having its point inside opposite the zero of the scale. By means of a rack motion another platinum wire is made to move up and down the axis of the tube through its entire length. This tube is filled with water, containing a few drops of sulphuric acid, and is placed in the circuit of the telephone by means of the platinum wires. When the instrument is used the platinum wires are first placed in contact, so that the clockwork sound is clearly heard in the telephones. The platinum wires are then gradually separated, introducing more and more of a liquid resistance into the circuit, the sound meanwhile getting fainter and fainter, until a point is reached when no sound at all is heard. At that point the distance between the points of the platinum wires as read on the millimetre scale, gives the measure of the hearing power of the ear for the particular individual examined. Tested in this way the hearing power of no two persons will be found to be identical, while even in the same person it varies remarkably at different times. It should be noted, however, that the reading will be different for the same individual on a first trial from what it will be after he has got a sort of training in the hearing of telephonic sounds.

The next experiment which I have to mention was suggested by a description of the microphone lately invented by Professor HUGHES. Instead of the pointed piece of retort carbon supported between pieces of the same material, as used by him, it struck me that gas-coke or cinders were likely to answer the purpose tolerably well. To test this I included in the circuit of an ordinary telephone a single Leclanché cell and a jelly jar half filled with cinders broken into coarse fragments. The connections were made by slipping down at opposite sides, between the cinders and the sides of the jar, two strips of tin to which the circuit wires were attached. When this was used as a transmitter articulate sounds were heard very loud and distinct in the distant telephone, though occasionally marred by what appeared to be the rattling of the cinders in the jar. With this transmitter the words were also quite audible, even when the speaker stood several yards away from it. I next took a shallow box made of thin wood about 15 in. by 9 in., and filled it with cinders, taking care in the first place to nail to the inside of its ends two pieces of tin to which wires could be attached. Having nailed down the thin lid of the box, and included it in the circuit of the telephone along with one Leclanché cell, I found that it made both a very sensitive microphone and also a most excellent transmitter for the ordinary telephone. With three of these boxes hung up like pictures on the walls of a moderately sized room, and connected in circuit, almost any kind of noise made in any part of the room was distinctly revealed in the telephone. Speaking was distinctly heard, and a part-song sung by two voices in the middle of the floor was rendered with surprising clearness and accuracy.

In my next experiment, [still using the same cell in the circuit, I tried as transmitter a single elongated cinder with the wires wound tightly round each end. Sounds uttered close to this cinder were quite audible in the telephone, but I failed to hear them when there was substituted for the cinder a carbon from a Bunsen battery with brass clamps at each end, into which the circuit wires were tightly screwed. Possibly, either the more porous and friable nature of the cinder, or the comparative looseness of the wire connections with the cinder may have had something to do with this difference of effect.

I next removed the Leclanché cell entirely from the circuit, and used as transmitter a jelly jar containing dry cinders. With this I sometimes fancied that I heard sounds, but they became distinctly, though faintly, audible as the cinders became somewhat moistened by the breath of the speaker. I then poured water into the jar so as almost to cover the cinders, and then the sounds in the telephone were almost as distinct as when the Leclanché cell was in the circuit. I did not, however, hear any sound with the cinders removed and water only in the jar, not even when the resistance of the water was diminished by being slightly acidulated.

In my next experiment I tried if the jar with the cinders would act as receiver as well as transmitter, and was not a little surprised to find that it did so. For this purpose I used similar jars of cinders, both for transmitter and receiver, and included a battery of two Grove's cells in the circuit. Articulate sounds uttered in the one cinder jar were distinctly heard in the other, and even voices could be distinguished. The results, however, were not so good as I have no doubt they will yet be when better forms of transmitter and receiver are used. I also tried successfully an ordinary telephone as transmitter and a jar of cinders as receiver. In this case, however, the sounds were somewhat fainter, and not so easily distinguished. I remarked that when an intermittent current was sent through the jar of cinders, a very distinct rattling noise issued from it. In order to find out if the cinders in the receiving jar were at all jostled about while sounds were being transmitted to it from a similar jar, the following plan was adopted:—A strong battery was included in the circuit and a clear glass vessel containing the cinders taken as receiving jar. When this was taken into a dark room, on looking through the cinders, small flashes of light were here and there seen among the cinders when sounds were being transmitted.

XIX.—*On Some New Bases of the Leucoline Series.* By G. Carr Robinson, F.R.S.E., Demonstrator of Chemistry, Public Health Laboratory, University of Edinburgh.

(Read 1st July 1878.)

Of the bases of the $C_nH_{2n-11}N$ series, obtained from coal-tar and from the "acid-tar" from the distillation of shale, only three are known, viz., Leucoline, C_9H_7N; Iridoline, $C_{10}H_9N$; and Cryptidine, $C_{11}H_{11}N$. These are isomeric with the bases obtained by the distillation of cinchonine with caustic potash, and are described by their discoverer, Greville Williams, in "Trans. Royal Society, Edinburgh," vol. xxi. The leucoline series of bases, at first thought to be identical with the cinchonine series, Williams has shown to be isomeric, but otherwise having important differences, these differences chiefly being apparent in their boiling points, their slight tendency to form crystallisable salts, and their reaction with iodide of amyl.

The source of the leucoline bases of Greville Williams was "coal-oil of a very high boiling point, and a density greater than that of water;" his process of separation was to treat the oil with sulphuric acid, and distil the acid liquid with lime; that portion of the distillate which sinks in water is treated with nitrite of potash and hydrochloric acid to destroy phenols; the acid liquid is placed in an iron retort and a current of steam passed through it; the residue remaining in the retort, after being treated with caustic potash, is distilled, and the distillate, consisting of the mixed bases, purified by fractional distillation; the bases were then identified by converting the fractions into platinum and other salts. Proceeding in this manner, Greville Williams separated the three bases—leucoline, iridoline, and cryptidine; the last, cryptidine, $C_{11}H_{11}N$, being found in his highest fraction, that boiling 270°–275° C., and from what I can learn from other authorities, the isolation of the higher members of this series has not been attempted.

For the material on which I have worked, the results of which investigation I now lay before the Society, I am indebted to the kindness of Messrs Gellatly and J. S. Thomson, chemists to Young's Paraffin Oil Company, West Calder, who have also kindly supplied me with the following account of the process of purification the bases went through before being sent to me:—

"*Extraction and Purification of Bases from Acid Tar from Shale Oil.*

"In order to set free the bases and remove the colouring matter and foreign substances, the tar was subjected to the following treatment, viz.:—It was neutralised with caustic soda, and the crude bases so separated distilled; the distillate was then dissolved in dilute sulphuric acid, allowed to stand some time to separate tarry matter, and the aqueous solution once more neutralised with caustic soda, when the bases so liberated were distilled off from an excess of caustic soda and potash. The semi-purified bases were now dissolved in the naphtha known as 'colourless oil,' s.g. 760, in the proportion of four parts naphtha to one part bases, and after standing some hours, the bases were dissolved out with dilute sulphuric acid, and the acid solution once more neutralised with caustic soda. The separated bases were then digested in an iron still connected with an inverted condenser, with an excess of caustic soda, for twelve hours, and then distilled off from the alkali; this last process of digestion was repeated a second time, when the bases were considered pure."

In addition to the above purification process, the bases had undergone twelve complete fractional distillations, in order to obtain specimens of leucoline, iridoline, and cryptidine; and only that portion of the distillate boiling above 315° C. was forwarded to me, and to which my examination was confined.

The mixed bases, about 1 kilogram, said to boil above 315° C., were fractionated three times, the distillate being received in two portions,—that boiling between 305° and 320° C., marked A; and that boiling above 320° C., marked B.

The high boiling bases, B, were dissolved in dilute hydrochloric acid, the solution filtered, then saturated with caustic soda, the separated bases washed with water, dried with sticks of caustic potash, and distilled off.

Attempts were then made to get crystallisable salts from the mixed bases; the double chlorides of platinum, gold, cadmium, mercury, lead, and zinc were tried, but without success, only resinous sticky masses being obtained; the same failure in getting crystallisable salts was experienced when the bases were treated with sulphuric, hydrochloric, nitric, and oxalic acids. It was thought that separation of the bases might be effected by converting them into methyl- or ethyl- iodide compounds, as GREVILLE WILLIAMS, "Trans. R.S.E.," vol. xxi., obtained well-defined crystalline salts in this way. This method was tried, reference to which will be made in another part of this paper, but it was not entirely satisfactory; for though the methyl-iodide compounds are crystalline, they decompose somewhat readily, and this method of separation was abandoned; and fractional distillation, though such a tedious and wasteful process, was resorted to.

Fractional Distillation of the Mixed Bases.

The portion of the first distillate from the mixed bases, marked A, boiling between 305° and 320° C., was fractionated, and after twenty-five complete fractionations, the fractions being fairly constant, distillation was stopped. About one-third of original quantity now appeared below 270° C., one-third between 270° and 280°, and the remainder in very small fractions ranging from 280° to 315°.

The high boiling portion from the first distillate from mixed bases, boiling above 315°, marked B, was next fractionated, and the fractions which then came over below 315° added to the corresponding fractions of A.

Examination of Fractions.

Cryptidine, $C_{11}H_{11}N$, boiling 270°–275° (WILLIAMS), and the members of this series differing in their boiling points by 18° for each addition of CH_2, the next base, $C_{12}H_{13}N$, would be expected to be found in fraction 290°–295°; so a portion of this fraction was dissolved in dilute hydrochloric acid, and the solution boiled with a few drops of nitric acid, to separate tarry matter; the filtered solution diluted further, cooled down in a freezing mixture, and platinum chloride added; the precipitated chloro-platinate filtered off, washed with ice-cold water, then with alcohol and ether, and dried over sulphuric acid and finally at 100° C.

Analysis of Chloro-Platinate from Fraction 290°–295°.

0·2585 grms. gave
0·0675 „ platinum = 26·11 per cent. platinum.

This agrees with the percentage of platinum demanded by the chloro-platinate of the new base $C_{12}H_{13}N$, the formula $2C_{12}H_{13}NHCl, PtCl_4$ requiring 26·12 per cent. platinum.

Another portion of this fraction, 290°–295°, was mixed with methyl-iodide and heated in a sealed tube in a water bath, and a small quantity of the methyl-base obtained, but this, whilst under process of conversion into platinum salt, was lost by an accident.

The remainder of the same fraction, reduced now to a few drops, was dissolved in dilute hydrochloric acid, and a chloro-platinate prepared as in the first case; the chloro-platinate was dried over sulphuric acid and finally at 100° C.

Analysis of Chloro-Platinate, No. 2, from Fraction 290°–295°.

0·54 grms. gave
0·141 „ platinum = 26·11 per cent. platinum,

agreeing with first analysis.

	I.	II.	Theory.
Platinum, . . .	26·11	26·11	26·12

These two platinum determinations show that the fraction 290°–295°, obtained after twenty-five complete fractionations, consisted of the new base $C_{12}H_{13}N$; but the whole of the chloro-platinate being used in these analyses, I am unable to give the carbon and hydrogen proportions.

Treatment of Mixed Bases with Methyl-Iodide.

As previously mentioned, before commencing the fractional distillation of the bases, some experiments were made with a view to separating them in the form of methyl-iodide compounds. For this purpose 100 grms. of the mixed bases were dissolved in 200 grms. of methyl-iodide; the solution was then digested on a water bath for 2 hours, in a flask of 2 litres capacity fitted with an inverted condenser; the excess of methyl-iodide distilled off and the semi-solid crystalline mass in the flask treated with absolute alcohol, to dissolve out unacted-on bases, and filtered; the crystalline mass washed with cold water, dissolved in warm water, and the solution set aside to crystallise, from which three crops of crystals were obtained, those marked a^1, a^2, a^3.

The first crop of crystals, a^1, was washed, dissolved in water, and the solution treated alternately with silver nitrate, dilute hydrochloric acid, and platinum chloride; the precipitated chloro-platinate of methyl-base, washed, and dried at 100°, gave on analysis—

0·543 grms. gave
0·1255 „ platinum = 23·11 per cent. platinum.

From the second crop of crystals, a^2, the chloro-platinate of methyl-base was prepared in same manner as a^1. This gave on analysis—

0·29325 grms. gave
0·07225 „ platinum = 24·63 per cent. platinum.

The third crop of crystals, a^3, had decomposed before it could be dried, so its examination was not proceeded with.

From these two analyses it would seem that the base of highest molecular weight is precipitated first as methyl-iodide compound; for a^1, being the chloro-platinate of methyl-base from first crop of crystals of the iodide, gave 23·11 per cent. platinum, which approaches the percentage of platinum from the methyl chloro-platinate of base $C_{14}H_{17}N$, the formula of which, $2C_{14}H_{17}NCH_3Cl$, $PtCl_4$, requires 23·5 per cent. platinum; whilst a^2, giving 24·63 per cent. platinum, approaches the percentage of platinum from the methyl chloro-platinate of base $C_{13}H_{15}N$, the formula of which, $2C_{13}H_{15}NCH_3Cl$, $PtCl_4$, requires 24·32 per cent. of platinum.

These results with methyl-iodide can of course be accepted as only giving a mere indication of the presence of these bases; but I hope to be able to extend this investigation when I can get a larger quantity of material to work on.

The remainder of the fractions of the bases being so small as to render any satisfactory examination of them quite impossible, Messrs GELLATLY and THOMSON again kindly supplied me with a further quantity of the mixed bases, which had been purified in the same manner as the first quantity. This second lot of bases was first distilled without a thermometer, then submitted to a rigorous fractional distillation: after twelve complete fractionations the fractions had become very fairly constant in their boiling points, and ranged from 260°–265° to 325°–330°.

It being expected that the base $C_{12}H_{13}N$, the first above cryptidine, would be found about the fraction 290°–295°, the fractions ranging from 285°–305° were submitted to four more complete fractionations, and the examination of these fractions was commenced.

Examination of Fraction 290°–295°.

About 5 grms. of this fraction was dissolved in ordinary nitric acid, and the solution evaporated on the water-bath; the resinous mass, on digestion with cold water, yielded a yellow solution, whilst the tarry matter was insoluble, this was filtered off, the solution cooled down in freezing mixture, and platinum chloride added, when the chloro-platinate was precipitated in a fine granular condition, this was collected on a filter, washed with ice-cold water, then with alcohol and ether, dried over sulphuric acid, and finally dried at 100° C.

GREVILLE WILLIAMS, "Trans. R.S.E." vol. xxi., in preparing the chloro-platinate of cryptidine, observed that on the addition of platinum chloride to the solution of the base, an adhesive precipitate came down, which crystallised, from its solution in boiling water, in yellow needles. As already stated, the precipitate from the solution of the base, cooled in a freezing mixture, was granular, and, examined by the microscope, had the appearance of fine tufts of silky crystals; but I completely failed to get crystals from the solution of the precipitate in boiling water, for as the solution cooled the chloro-platinate was deposited on the sides of the vessel in sticky globules.

Analysis of Chloro-Platinate from Fraction 290°–295°, *dried at* 100° C.

I. 0·53825 grms. salt gave
0·135 „ platinum = 25·08 per cent. platinum.

II. 0·537 „ salt gave
0·1355 „ platinum = 25·23 „ „

III. 0·50475 „ salt gave
0·732 „ CO_2 = 39·52 „ carbon.
0·2085 „ H_2O = 4·57 „ hydrogen.

IV. 0·291 „ salt gave
0·4255 „ CO_2 = 39·87 „ carbon.
0·123 „ H_2O = 4·46 „ hydrogen.

	I.	II.	III.	IV.
Carbon, ...	39·52	39·87	...	...
Hydrogen, . . .	4·57	4·46	...	. .
Platinum, . . .	...	...	25·08	25·23

These analyses show that the fraction 290°–295° consists of the new base $C_{13}H_{15}N$, the chloro-platinate of which, $2C_{13}H_{15}NHCl, PtCl_4$, requires—

		Found.	
Carbon, ...	39·89	39·52	39·87
Hydrogen, ...	4·09	4·57	4·46
Platinum, ...	25·19	25·08	25·23

The base, too, is as pure as can well be obtained by fractional distillation; and analysis of the fraction itself would not throw any light on it, as in this series of homologous bases, differing by CH_2, there is a difference of only 0·2 in the carbon percentage for each carbon atom.

The presence of the base $C_{13}H_{15}N$ was entirely unexpected in this fraction; for cryptidine, $C_{11}H_{11}N$, boiling at 274° C., the next member of the series, $C_{12}H_{13}N$, should be in fraction 290°–295°; and the base $C_{13}H_{15}N$ would then come in at fraction 310°–315°; *i.e.*, taking 18° as the rise in boiling point for each addition of CH_2. Thinking the thermometer employed in the fractional distillation might be at fault, the fraction 270°–275° was again distilled with another thermometer, but no error was here detected, the fraction still being very constant, only a few drops coming over either below or above its original boiling point 270°–275°; that portion of the fraction distilling between 270° and 275° was collected separately.

Examination of Fraction 270°–275°.

About 5 grms. of this fraction was dissolved in nitric acid, and a chloro-platinate prepared in exactly the same manner as in the preparation of chloro-platinate from fraction 290°–295°—viz., by adding platinum chloride to the solution of the base cooled down in freezing mixture, collecting the precipitate on a filter, washing with water, then with alcohol and ether, and drying at 100° C.

Analysis of Chloro-Platinate from Fraction 270°–275°.

I. 0·2493 grms. salt gave
0·0652 " platinum = 26·19 per cent. platinum.

II. 0·2188 " salt gave
0·0571 " platinum = 26·09 " "

III. 0·1245 " salt gave
0·3265 " platinum = 26·22 " "

IV. 0·2695 " salt gave
0·3755 " CO_2 = 37·99 " carbon.
0·1175 " H_2O = 4·32 " hydrogen.

V. 0·176 " salt gave
0·244 " CO_2 = 37·78 " carbon.
0·074 " H_2O = 4·54 " hydrogen.

These results confirmed the belief, that as the base $C_{13}H_{15}N$ was found in fraction 290°–295°, so would the base $C_{12}H_{13}N$ be found in this fraction, and not cryptidine; the chloro-platinate of cryptidine, $2C_{11}H_{11}NHCl, PtCl_4$, has the composition—

Carbon, . . .	36·36
Hydrogen, . . .	3·30
Platinum, . . .	27·12
Chlorine, . . .	29·31
Nitrogen, . . .	3·85

whilst $2C_{12}H_{13}NHCl, PtCl_4$, the chloro-platinate of the base $C_{12}H_{13}N$, has the composition—

Carbon, . . .	38·19
Hydrogen, . . .	3·72
Platinum, . . .	26·12
Chlorine, . . .	28·24
Nitrogen, . . .	3·73

with which the above analyses agree.

	I.	II.	III.	IV.	V.	Calculated.
Carbon, . .	...	...	...	37·99	37·78	38·19
Hydrogen, . .	...	...	...	4·32	4·54	3·72
Platinum, . .	26·19	26·09	26·22	...	...	26·12

In accordance with these views, the next base would be found in fractions 305°–310° or 310°–315°, therefore this latter was the next one examined.

Examination of Fraction 310°–315°.

About 5 grms. of fraction 310°–315° was dissolved in nitric acid, and the chloro-platinate prepared just as in case of fractions 270°–275° and 290°–295°.

Analysis of Chloro-Platinate from Fraction 310°–315°.

I. 0·7385 grms. salt gave
0·17475 „ platinum = 23·66 per cent. platinum.

II. 0·528 „ salt gave
0·1275 „ platinum = 24·14 „ „

III. 0·2525 „ salt gave
0·38025 „ CO_2 = 41·07 „ carbon.
0·121 „ H_2O = 5·18 „ hydrogen.

IV. 0·2575 „ salt gave
0·388 „ CO = 41·08 „ carbon.
0·121 „ H_2O = 5·04 „ hydrogen.

These results are in accordance with the opinion stated above, that this fraction, 310°–315°, consisted of the third new base $C_{14}H_{17}N$, the chloro-platinate of which, $2C_{14}H_{17}NHCl, PtCl_4$, has the composition—

Carbon, ...	41·48
Hydrogen, ...	4·44
Platinum, ...	24·32
Chlorine, ...	26·29
Nitrogen, ...	3·45

whilst analysis gave—

	I.	II.	III.	IV.	Calculated.
Carbon, . .	...	...	41·07	41·08	41·48
Hydrogen, . .	...	...	5·18	5·04	4·44
Platinum, . .	23·66	24·14	...	...	24·31

Having now shown the presence of the three new bases—

$C_{12}H_{13}N$,
$C_{13}H_{15}N$,
$C_{14}H_{17}N$,

in the fractions 270°–275°, 290°–295°, and 310°–315° respectively, it remains to be seen how their existence in these particular fractions is to be reconciled with the accepted boiling points of the three first members of the series—

Leucoline, . . .	C_9H_7N .	B.Pt.	238° C.
Iridoline, . . .	$C_{10}H_9N$	„	256° C.
Cryptidine, . . .	$C_{11}H_{11}N$	„	274° C.

for it is evident that in such a series of homologous bodies, it would be an anomaly to have two members of the series with the same boiling point, as cryptidine, $C_{11}H_{11}N$, bg. pt. 274° C., and the first new base, $C_{12}H_{13}N$, from fraction 270°–275°, apparently are, whilst the remainder conform with the recognised difference of 18° C. for each addition of CH_2.

The only explanation I can in the meantime offer is this : the first quantity of mixed bases worked upon was submitted to *twenty-five* complete fractionations, the fractions ranging from 270°–315°—this represents about 225 distillations; whilst the second quantity of mixed bases underwent sixteen complete fractions. The difficulty of separating substances by the method of fractional distillation is well known, and this difficulty is increased by several important factors, *e.g.*, high molecular weights, and consequently high boiling points; and also by the constitution of the substances under examination, as, in this case, a series of homologous bases.

I think, therefore, that had it been possible to continue the fractional distillation of the second quantity of bases to as many as 25 or 30 fractionations, the three new bases would have been found in their hypothetical fractions 290°–295°, 310°–215°, and 325°–330°, instead of in the fractions 270°–275°, 290°–295°, and 310°-315° ; and this opinion is confirmed by the fact, that from the first quantity of bases where distillation had been pushed to *twenty-five* fractionations, the fraction 290°–295° *did* yield a chloro-platinate, the percentage of platinum in which corresponds with the first new base, $C_{12}H_{13}N$, and though for want of chloro-platinate the carbon and hydrogen proportions could not be ascertained, still the two platinum results stated were obtained by the analysis of *two* chloro-platinates, prepared in the same manner, but independently of each other.

Again, it is to be noted that the actual boiling points of the three new bases have not yet been attempted, but that merely in these three particular fractions such bases have been found. GREVILLE WILLIAMS, also ANDERSON, "Trans. R.S.E.," vol. xx. part 2 page 247 ; and Dewar, "Proc. Royal Society," No. 179, 1877, remark that a wide range must be allowed in the fractional distillation of these and the chinoline series, or that the same base is found extended through several fractions.

I hope to be able to continue the investigation on a considerably larger quantity of material, when it will be possible to continue fractional distillation at least 25 or 30 times, and also to complete other points in the examination of the chemical and physical properties of these bases, which, in this paper, time has not allowed of.

These bases, when recently distilled, are nearly colourless, and rapidly darken when exposed to the air ; they have the same sooty smell as the first three of the series, though not nearly to the same degree ; they give no blue colour with amyl-iodide and caustic potash, proving them to be members of the leucoline, and not of the chinoline, series.

I have much pleasure in acknowledging the valuable assistance rendered by Mr L. Johnstone and Mr W. Goodwin in the fractional distillation of these bases.

XX.—*On Dimethyl-Thetine and its Derivatives.* By Professor Crum Brown and Dr E. A. Letts.

(Read 24th November 1873 and 4th May 1874.)
(Sent for Publication 19th August 1878.)

The analogies existing between elements belonging to one "family," such, for instance, as the nitrogen family or the sulphur family, have long been recognised, and are pointed out and insisted upon even in elementary text-books; but the very important analogies existing between substances of different quantivalence are apt to be forgotten or overlooked. For illustrations of such analogies we may point to boron and silicon, elements closely resembling one another in themselves and also in their compounds,—differing, indeed, in little else but that the one is triad and the other tetrad. A similar relation exists between gold and platinum.

The elementary substances, sulphur and phosphorus, have many points of similarity: both fuse at a comparatively low temperature, both are transformed by heat into amorphous insoluble modifications, and both have anomalous vapour densities.

If we turn to their chemical relations and examine the compounds which they form, we find similarities of a very striking kind,—similarities as close as the difference in quantivalence of the two elements will allow.

To illustrate this we have arranged some typical compounds of sulphur and of phosphorus in parallel columns—

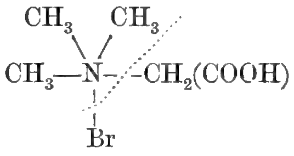
P^{III}.	P^{V}.	S^{II}.	S^{IV}.	S^{VI}.
PH_3		SH_2		
PR_3		SR_2		
	PR_4I		SR_3I	
	$PR_4(OH)$		$SR_3(OH)$	
P_2O_3			SO_2	
$P(OH)_3$ or	$POH(OH)_2$		$SO(OH)_2$ or	$SO_2H(OH)$
	PR_3O		SR_2O and	SR_2O_2
	$PR_2O(OH)$ $PRO(OH)_2$ }		$SRO(OH)$ and	$SRO_2(OH)$
	P_2O_5			SO_3
	$PO(OH)_3$			$SO_2(OH)_2$
	$(PO)_2O(OH)_4$			$(SO_2)_2O(OH)_2$

This analogy is not confined to the formulæ of the substances; the substances themselves have close resemblances, as any one can see by running the eye over the list given above. It will further be noticed, that in compounds with hydrogen and electro-positive elements or radicals, triad and pentad phosphorus correspond to dyad and tetrad sulphur respectively, while in compounds with oxygen and electro-negative elements or radicals, triad and pentad phosphorus correspond to tetrad and hexad sulphur respectively.

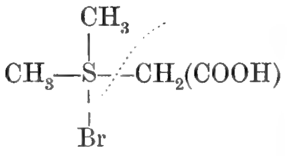
The following investigation was undertaken with the view of extending the number of comparable compounds by preparing a sulphur compound corresponding to betaine in the nitrogen series, and to the analogous phosphorus compound prepared by HOFMANN.* In this case the term in the nitrogen series exists, and is better known than that in the phosphorus series, so that we may compare our new substance to betaine, although probably it more closely resembles HOFMANN's phosphorus base.

We have given the substance the name *thetine* to recall its relation to betaine, and the fact that it contains sulphur. The salts of thetine may be regarded as compounds of sulphide of methyl, with substitution products of acetic acid; or as salts of trimethyl-sulphine, in which one atom of hydrogen in one methyl is replaced by carboxyl. Similarly, the salts of betaine may be regarded as compounds of nitride of methyl (trimethylamine), with substitution products of acetic acid, or as salts of tetramethyl-ammonium, in which one atom of hydrogen in one methyl is replaced by carboxyl. If other compounds of this kind are prepared, the nomenclature can easily be adapted to them; thus our base may be called *dimethyl-aceto-thetine,* just as betaine may be called *trimethyl-aceto-betaine.*

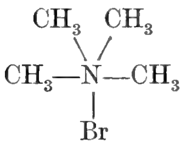
These relations are best shown by means of graphic formulæ. Thus—



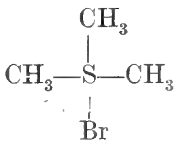
Hydrobromate of Betaine.



Hydrobromate of Thetine.



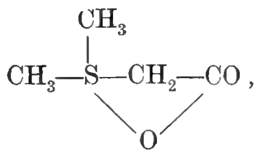
Bromide of Tetramethyl Ammonium.



$$\begin{array}{c} CH_3 \\ | \\ CH_3—S—CH_3 \\ | \\ Br \end{array}$$

Bromide of Trimethyl Sulphine.

Free thetine may be regarded either as—



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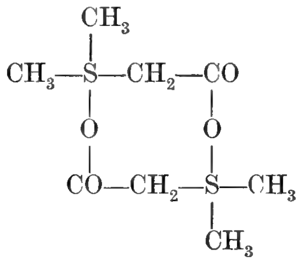
* HOFMANN, "Proceedings of the Royal Society of London," xi. 525.

or as—



exactly as similar alternative formulæ have been proposed for free betaine.

Hydrobromate of Dimethyl-Thetine.—This substance, from which all the derivatives of dimethyl-thetine are obtained, is produced by the action of sulphide of methyl on bromacetic acid at ordinary temperatures.

For its preparation, sulphide of methyl and bromacetic acid are mixed in a flask connected with a vertical condenser. The flask with condenser attached is then placed in a tub of water, and so arranged that from time to time it may be removed and agitated. Conveniently the tub of water is placed in the corner of a room and the condenser supported by leaning against the angle.

The bromacetic acid rapidly dissolves, but to ensure complete solution the flask should be well shaken, otherwise the thetine compound begins to separate out before the whole of the acid dissolves.

The bromacetic acid is so soluble in sulphide of methyl that the temperature of the mixture sinks below zero, and in one experiment a minimum of − 5° C. was observed.

As soon as the whole of the bromacetic acid has dissolved, the mixture begins to grow cloudy, and almost immediately yellowish oily drops begin to separate out; these rapidly increase in quantity, and soon unite to a layer, which eventually forms the greater part of the product. As the oily liquid increases in quantity the temperature of the mixture rises; the action, indeed, is so energetic that unless it be checked by cooling the flask in water, the sulphide of methyl boils off with such violence that even with a long condenser the greater part may be lost. On the termination of the action (which occurs in an hour or two) the product consists of two distinct layers. The lower of these is almost pure dimethyl-thetine hydrobromate; the upper a solution of the thetine compound in sulphide of methyl. The condenser is now removed, the flask loosely corked * and left to itself for a night. Next day the lower oily layer is found to have completely solidified, whilst the upper

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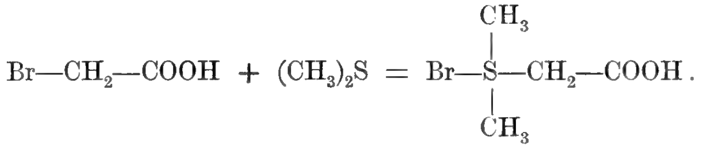
* If tightly corked, the flask, unless strong, may burst inwards owing to the absorption which occurs.

layer is almost solid from a network of snow-white needles. The reaction which occurs is represented by the equation—



$$\mathrm{Br{-}CH_2{-}COOH} + \mathrm{(CH_3)_2S} = \mathrm{Br{-}\overset{\displaystyle CH_3}{\underset{\displaystyle CH_3}{S}}{-}CH_2{-}COOH}\,.$$

It should be remarked that, although combination of by far the greater part of the two substances occurs, a certain quantity of each always remains uncombined.*

For the preparation of most of the derivatives of dimethyl-thetine, the crude hydrobromate obtained by the above method is sufficiently pure. But if it be desired to obtain the pure substance, the crude product is best purified by pounding it in a mortar with ether, allowing the mixture to subside, pouring off the supernatant liquid (which contains sulphide of methyl and bromacetic acid), and repeating this process two or three times. The resulting mass of finely powdered thetine compound (which should be quite white) is then rapidly dried between filter paper, and the desiccation completed by placing it over sulphuric acid; or it may be recrystallised from hot alcohol.†

The results of its analysis are as follows:—

	Calculated for $C_4H_9SO_2Br$.	Obtained.	
Carbon,	23·9	23·7	23·1
Hydrogen,	4·5	4·5	4·8
Oxygen,	51·9	...	...
Sulphur (*a*),	15·9	...	16·1
Bromine (*b*),	39·8	40·0	39·7
	100·0		

(*a*) *Determined as sulphate of baryta after fusion with chlorate and carbonate of potash.*

(*b*) *Determined as bromide of silver by precipitating the solution of the hydrobromate direct with nitrate of silver.*

Hydrobromate of dimethyl-thetine is a very deliquescent body. It dissolves with ease in hot alcohol, and, if the solution be sufficiently concentrated, crystallises on cooling in large transparent plates, which are apparently rectangular. Large and perfect crystals more than half an inch across and more than an eighth of an inch in thickness are easily grown. It is also soluble,

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* See pp. 607–611.

† It should not be boiled too long with alcohol, otherwise it decomposes slowly.

though to a less extent, in sulphide of methyl. It is insoluble in ether. After drying on filter paper the crystals often become brown on the surface; this also occurs with the ethyl compound, and has been noticed with bromacetic acid itself.

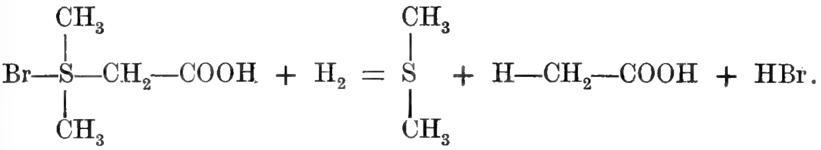
Hydrobromate of dimethyl-thetine yields magnificent orange-coloured crystals with chloride of platinum; these are probably isomorphous mixtures of the chloro-platinate and bromo-platinate.

Bromaurate of Dimethyl-Thetine.—This salt was obtained in beautiful, glittering, dark red scales, on mixing alcoholic solutions of bromide of gold and hydrobromate of dimethyl-thetine. It decomposes when heated to 100° C., and also when boiled with alcohol. It was not completely analysed.

Bromo-Platinate of Dimethyl-Thetine was obtained in magnificent dark red crystals, resembling ferricyanide of potassium in appearance, by allowing a solution of hydrobromate of dimethyl-thetine and bromide of platinum to evaporate in a desiccator.

The formula $2(C_4H_9SO_2Br), PtBr_4$ requires 21·3 per cent. of platinum, whereas 21·5 was obtained in two determinations.

Hydrobromate of dimethyl-thetine is completely decomposed by nascent hydrogen, and, as might be expected, yields sulphide of methyl and hydrobromic and acetic acids—



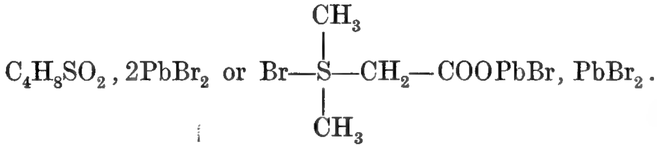
$$\mathrm{Br{-}}\underset{\underset{\mathrm{CH_3}}{|}}{\overset{\overset{\mathrm{CH_3}}{|}}{\mathrm{S}}}\mathrm{{-}CH_2{-}COOH} + \mathrm{H_2} = \underset{\underset{\mathrm{CH_3}}{|}}{\overset{\overset{\mathrm{CH_3}}{|}}{\mathrm{S}}} + \mathrm{H{-}CH_2{-}COOH} + \mathrm{HBr}.$$

This decomposition occurs when the hydrobromate is treated with zinc and a dilute acid, and also when its aqueous solution is mixed with zinc dust alone—the mixture growing very hot, and sulphide of methyl escaping in abundance.

Hydrobromate of dimethyl-thetine has a pleasant sour taste, and its solution reddens litmus. It acts readily on metallic carbonates, hydrates, and oxides, but the resulting compounds are not necessarily salts simply derived from the hydrobromate by the replacement of the hydrogen of its carboxyl group by metals; but are, in some cases at least, double salts, which may be conveniently represented as compounds of the base dimethyl-thetine with metallic bromides. Of these the most readily prepared is the—

Double Salt of Dimethyl-Thetine and Bromide of Lead.—This salt is produced when an aqueous solution of the hydrobromate is boiled with litharge, carbonate or hydrate of lead, but is most conveniently prepared by means of the latter. The hydrobromate is dissolved in a considerable quantity of water, and recently precipitated hydrate of lead is added. This at once becomes curdy, and, on warming the solution, gradually dissolves. The addition of the hydrate is continued until the boiling liquid ceases to dissolve it; the solution is then rapidly filtered; as it cools the double salt separates in beautiful silvery scales, which may be purified from mother liquor by washing with a little cold water.

The analysis of this salt shows that it is a compound of 2 molecules of bromide of lead with 1 molecule of dimethyl-thetine—



$$C_4H_8SO_2\,,\ 2PbBr_2 \text{ or } Br\text{—}\overset{\displaystyle CH_3}{\underset{\displaystyle CH_3}{S}}\text{—}CH_2\text{—}COOPbBr,\ PbBr_2\,.$$

	Obtained.		Calculated for $C_4H_8SO_2$, $2PbBr_2$.
Carbon, ...	5·7	5·7	5·6
Hydrogen, ...	0·9	0·9	0·9
Bromine, ...	37·0	37·5	37·0
Lead, ...	48·4	48·5	48·4

Its formation may be represented by the equation—

$$4(C_4H_9BrSO_2) + 2Pb(OH)_2 = C_4H_8SO_2, 2PbBr_2 + 3(C_4H_{10}SO_3) + H_2O\,.$$

That hydrated dimethyl-thetine is produced along with the lead salt was proved by evaporating the mother liquors on a water bath, extracting the residue with alcohol, and adding to the alcoholic solution hydrochloric acid and chloride of platinum, which caused the precipitation of the orange-coloured chloro-platinate of dimethyl-thetine, the composition of which was verified by a platinum determination. The lead salt is very sparingly soluble in cold water, but dissolves to a considerable extent in boiling water, from which it may be recrystallised.

Action of Hydrobromate of Dimethyl-Thetine on Ethylate of Sodium, Oxide of Copper, Oxide of Mercury, and Ammonia.—The action of the hydrobromate on ethylate of sodium appears to vary with the conditions, and has not been thoroughly investigated. In one experiment, on boiling the hydrobromate for some time with ethylate of sodium dissolved in alcohol, the solution almost solidified to a crystalline mass. These crystals were with difficulty soluble in boiling alcohol. An estimation of the sodium which they contained gave 14·6 per cent., whereas the compound $C_4H_8SO_2$, 2NaBr requires 14·4 per cent. A bromine determination gave, however, too small a quantity for the above formula, viz., 36·8 per cent. instead of 49·0. In another experiment, 5 grms. of hydrobromate and 6 grms. of sodium* were separately dissolved in absolute alcohol, and the solutions mixed together, warmed, and filtered. The filtered

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* These quantities represent 1 molecule of the hydrobromate and 1 atom of sodium.

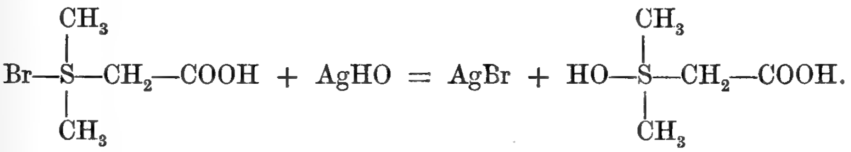
liquid grew cloudy and gradually deposited oily drops, which did not crystallise. On evaporating the solution over a water bath a gummy mass was left, which did not invite a closer examination.

Hydrobromate of dimethyl-thetine dissolved in water, and boiled with oxide of copper, yields a beautiful dark blue solution, which becomes of a magnificent purple colour when almost dry. If alcohol be added, purple flakes are precipitated, and alcoholic ammonia produces a magnificent dark blue crystalline precipitate. The blue solution obtained in the first instance when evaporated in vacuo over sulphuric acid gave eventually a purple syrup which would not crystallise.

Alcoholic ammonia digested for a couple of days with the hydrobromate in a sealed tube at 100° C. gave crystals of bromide of ammonium. The solution filtered from these gave a syrupy liquid, which, on evaporation, deposited crystalline matter. This, however, was not examined more closely.

Oxide of mercury, when boiled with the hydrobromate in aqueous solution, is gradually dissolved ; oily drops separate, which gradually become crystalline. They consist, in all probability, of a double salt.

That the hydrobromate of dimethyl-thetine behaves as the hydrobromate of a base, and no longer possesses the properties of bromacetic acid, is proved, amongst other of its properties, by the reaction which occurs when its aqueous solution is treated with oxide of silver. The mixture becomes warm, bromide of silver is precipitated, and hydrated dimethyl-thetine remains in solution—



$$\mathrm{Br{-}\underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{S}}{-}CH_2{-}COOH + AgHO = AgBr + HO{-}\underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{S}}{-}CH_2{-}COOH.}$$

In the case of the action on hydrobromate of dimethyl-thetine of hydrates or oxides of metals, the bromides of which are more or less soluble in water, the bromide first formed combines with the base thetine to form a more or less soluble double salt. In the case, however, of the silver oxide, no such double salt is produced.

Dimethyl-Thetine.—This substance may be obtained either by the action of oxide of silver on the hydrobromate, or by action of carbonate or hydrate of barium on the sulphate of dimethyl-thetine. To prepare it with oxide of silver, a weighed quantity of the hydrobromate is dissolved in water and mixed with rather more than the theoretical quantity of oxide of silver.* The oxide at once becomes yellow, and the mixture grows very hot. It is digested for some

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* Conveniently rather more than the equivalent weight of nitrate of silver is converted into oxide by addition of potash solution, and the oxide employed without further weighing. (Equal weights of hydrobromate and nitrate of silver were usually employed.)

time in a water bath, and from time to time tested with freshly precipitated oxide of silver. When this ceases to be affected, the mixture is filtered, and the small quantity of silver contained in solution * removed by the cautious addition of hydrochloric acid. The solution is again filtered and concentrated on a water bath till it is syrupy, and then placed in a desiccator. After some time (usually a day or two) large crystals of the base separate. These may be purified by re-crystallisation from boiling alcohol.

Hydrated dimethyl-thetine is a colourless substance which may be obtained in large crystals. It is very hygroscopic, and hence crystallises with difficulty from an aqueous solution; but although soluble to a considerable extent in alcohol, it is much less so in that liquid than in water, and readily crystallises on cooling its saturated alcoholic solution.

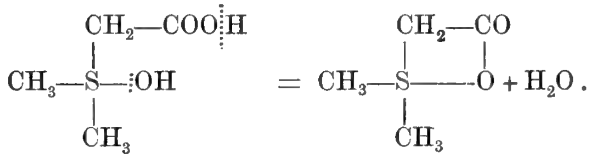
Its composition was determined by combustion with oxide of copper. The numbers obtained are as follows:—

	Calculated for $C_4H_{10}SO_3$	Obtained.	
Carbon, . . .	34·7	34·0	33·9
Hydrogen, . . .	7·2	7·3	7·3

It has a taste which is at first slightly burning—afterwards saline. It is neutral to litmus.

As might be expected, it is but a weak base, and does not combine with carbonic acid—indeed, it may be obtained by the action of carbonate of barium on its sulphate, carbonic acid escaping. Attempts made to prepare its cyanide were also unsuccessful, hydrocyanic acid being evolved when solution of the hydrobromate was mixed with cyanide of silver, and the base remaining on concentrating the solution over sulphuric acid.

The hydrated base does not lose water when placed in a desiccator over sulphuric acid. Heated to 100° C. it decomposes.† But if it be placed in vacuo over sulphuric acid it swells up, becomes opaque, and loses a molecule of water, becoming transformed into the anhydrous base—



$$\begin{array}{lcl} & CH_2\text{—}COO\vdots H & \\ & | & \\ CH_3\text{—} & S\text{—}\vdots OH & \\ & | & \\ & CH_3 & \end{array} = \begin{array}{llll} & CH_2\text{—} & CO & \\ & | & | & \\ CH_3\text{—} & S\text{———} & O & + H_2O\,. \\ & | & & \\ & CH_3 & & \end{array}$$

It is quite possible, however, that the formula of the anhydrous base is double that of the above, as in the case of glycocol, &c., viz.—

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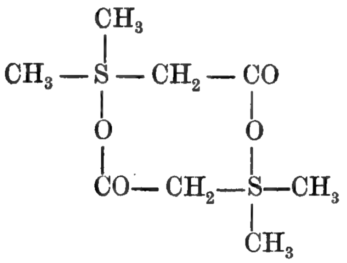
* This silver was probably dissolved by the free bromacetic acid contained in the crude hydrobromate employed for the preparation of the base.

† Its products of decomposition are described in a separate paper. See page 598.



The existence of the hemi-hydriodate described further on supports such a view. The dehydration requires eight or nine days for completion.

The loss of water was determined quantitatively, and amounted to 13·1, theory requiring 13·0.

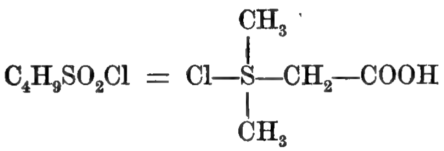
Dimethyl-thetine combines with strong acids such as sulphuric and hydrochloric to form the corresponding sulphate and hydrochlorate; it also combines with hydriodic acid. Most of its salts are, however, more conveniently prepared either from the hydrobromate or sulphate by double decomposition with the corresponding silver or barium salts.

Hydrochlorate of Dimethyl-Thetine.—This salt may be prepared by the action of hydrochloric acid on the base, but more conveniently by the action of chloride of barium on its sulphate.

A weighed quantity of the hydrobromate was dissolved in water and treated with rather more than the equivalent quantity of sulphate of silver. As soon as this ceased to act, the solution was filtered from the bromide of silver, mixed with the equivalent quantity of chloride of barium dissolved in water, the mixture boiled, allowed to settle, and chloride of barium cautiously added till the whole of the sulphuric acid was precipitated. The clear solution was then filtered off and concentrated in vacuo over sulphuric acid, when, after some time, it solidified to a colourless crystalline mass.

Hydrochlorate of dimethyl-thetine is a colourless crystalline substance, having a pleasant sour taste and acid reaction. It is very soluble in water, and is deliquescent. In alcohol it is much less soluble, and may in fact be precipitated by that liquid from its concentrated aqueous solution.

The formula—



$$C_4H_9SO_2Cl = Cl-\overset{CH_3}{\underset{CH_3}{S}}-CH_2-COOH$$

was verified by a chlorine determination—

	Calculated in 100.	Obtained.
Chlorine, . . .	22·7	23·3 *

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* The excess of chlorine was probably due to a small quantity of hydrochloric acid enclosed within the crystals, as the specimen analysed was obtained from the base and hydrochloric acid.

A further confirmation of the above formula was afforded by the composition of the double salt which the hydrochlorate forms with chloride of platinum.

Chloro-Platinate of Dimethyl-Thetine.—This salt crystallises out in beautiful orange-yellow needles when tolerably concentrated aqueous solutions of the hydrochlorate and chloride of platinum are mixed. It is not very soluble in cold water, but is much more so in boiling water, from which it may be recrystallised. It is insoluble in alcohol, and when heated with that liquid fuses below its surface, but becomes solid again on cooling.

The salt obtained from aqueous solutions contains 2 molecules of water of crystallisation, and has the formula—

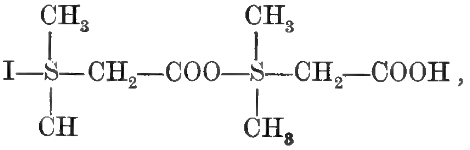
$$2(C_4H_9SO_2Cl) , PtCl_4 , 2H_2O .$$

The salt was analysed by determination of water and platinum—

	Calculated in 100.	Obtained.	
Platinum, . . .	28·6	28·5	28·4
Water, . . .	5·2	5·4	...

Hydriodate of Dimethyl-Thetine.—Up to the present time no salt having the composition of the normal hydriodate has been obtained. A solution of the base when warmed with hydriodic acid gradually becomes brown from the separation of free iodine. The same solution, not warmed but allowed to evaporate in vacuo over sulphuric acid, also became brown, and eventually deposited splendid lustrous crystals resembling permanganate of potash in appearance. Attempts to prepare the hydriodate from the sulphate of dimethyl-thetine and iodide of barium were equally unsuccessful, the solution of the hydriodate thus obtained decomposing during evaporation and yielding free iodine.

In another experiment a very concentrated aqueous solution of the base was mixed with exactly the equivalent quantity of recently distilled hydriodic acid of constant boiling point. The mixture was then placed in a desiccator, and after a day or so yielded comparatively colourless crystalline crusts. These were found to contain 34·1 per cent. of iodine. They were recrystallised from a little hot water, and were then found to contain 33·4 per cent. of iodine. A hemi-hydriodate, having the composition expressed by the formula $2(C_4H_8SO_2),HI$—*i.e,* consisting of a compound of 2 molecules of the anhydrous base and 1 molecule of hydriodic acid, requires 34·5 per cent. of iodine—a number closely agreeing with that obtained with the product which had not been recrystallised. Such a salt might be capable of existence thus—



$$I—\underset{\underset{CH}{|}}{\overset{\overset{CH_3}{|}}{S}}—CH_2—COO—\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{S}}—CH_2—COOH ,$$

and might decompose when treated with water into hydriodic acid and the hydrated base. The presence of a small quantity of the latter in the recrystallised product would account for the slight deficiency of iodine.

Polyiodide of Dimethyl-Thetine.—The beautiful crystalline salt resembling permanganate of potash in appearance,—which was obtained by the spontaneous decomposition of the hydriodate, or by allowing dilute hydriodic acid to remain for a long time in contact with the base,—is a polyiodide of dimethyl-thetine. It forms well-defined crystals, which are insoluble in water, but soluble in alcohol and ether. Warmed with water they fuse to oily drops, which on cooling do not readily solidify.

The analysis of the salt was effected by dissolving it in alcohol and adding standard hyposulphite solution; as soon as it was decolorised, the total iodine was determined by precipitation as iodide of silver.

The numbers obtained agree with the formula—

$$(C_4H_8SO_2), HI, I_2.$$

	Calculated in 100.	Obtained.		
Free Iodine, . . .	50·6	50·7	50·5	50·4
Total Iodine, . . .	75·9	75·4	...	...

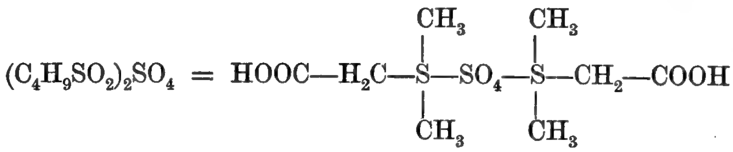
Other polyiodides were obtained by adding iodine to a solution of the base in hydriodic acid. These were beautiful salts with a greenish metallic lustre. As they were probably mixtures, they were not examined more closely.

Sulphate of Dimethyl-Thetine.—This salt is obtained by the action of sulphate of silver on the hydrobromate.

The hydrobromate is dissolved in water, rather more than the equivalent quantity of sulphate of silver added, and the mixture warmed in a water bath and well stirred. From time to time a little of the solution is removed and tested with sulphate of silver, and when this ceases to be affected, the whole is thrown on a filter, and the solution, after removing the small quantity of dissolved sulphate of silver by addition of hydrochloric acid, concentrated either over a water bath or in vacuo over sulphuric acid. When syrupy the solution solidifies to a crystalline mass.

Sulphate of dimethyl-thetine is a colourless crystalline body, and is perhaps the least soluble of all the dimethyl-thetine salts, and is not deliquescent. Nevertheless, its aqueous solution may be concentrated till of syrupy consistence, and frequently remains thus without crystallising. The salt is very slightly soluble in alcohol, and is readily precipitated by that reagent from its concentrated aqueous solution. It has a pleasant sour taste, and its solution reddens litmus paper.

The formula—



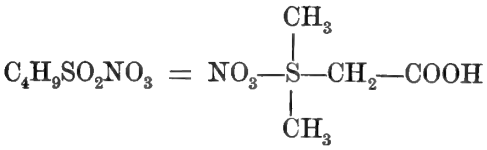
$$(C_4H_9SO_2)_2SO_4 = HOOC{-}H_2C{-}\overset{CH_3}{\underset{CH_3}{S}}{-}SO_4{-}\overset{CH_3}{\underset{CH_3}{S}}{-}CH_2{-}COOH$$

was verified by a determination of sulphuric acid—

	Obtained in 100.		Calculated.
SO_4	28·1	28·2	28·4

Nitrate of Dimethyl-Thetine.—This salt was prepared by the action of nitrate of silver on hydrobromate of dimethyl-thetine. A weighed quantity of the latter was dissolved in water, and a solution containing the equivalent quantity of nitrate of silver added. The mixture was then filtered from the precipitated bromide of silver, and concentrated in vacuo over sulphuric acid. The nitrate separated in large colourless crystals when the solution was of small volume. It has a sour taste, and its solution reddens litmus. It is readily decomposed even when heated at 100° C., and its aqueous solution when boiled also suffers decomposition, red fumes being given off.

The formula—



$$C_4H_9SO_2NO_3 = NO_3{-}\overset{CH_3}{\underset{CH_3}{S}}{-}CH_2{-}COOH$$

was verified by a determination of carbon and hydrogen—

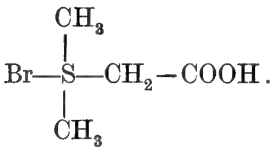
	Calculated in 100.	Obtained
Carbon, ...	26·2	26·4
Hydrogen, ...	4·9	4·9

XXI.—*On the Compounds of Ethyl-, Propyl-, Butyl-, and Amyl- Thetines.*

By Dr E. A. Letts.

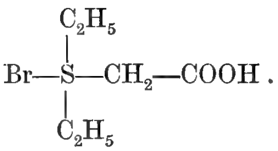
(Sent for Publication 19th August 1878.)

Compounds of Ethyl-Thetine.—In the course of the investigations, the results of which are recorded in the preceding paper, Professor Crum Brown and I observed that sulphide of ethyl, like sulphide of methyl, forms a crystalline compound with bromacetic acid.[*] This we called *hydrobromate of diethyl-thetine,* in conformity with the system of nomenclature we had proposed for such bodies—



$$\mathrm{Br{-}\overset{\displaystyle CH_3}{\underset{\displaystyle CH_3}{S}}{-}CH_2{-}COOH}.$$

Hydrobromate of Dimethyl-Thetine.



$$\mathrm{Br{-}\overset{\displaystyle C_2H_5}{\underset{\displaystyle C_2H_5}{S}}{-}CH_2{-}COOH}.$$

Hydrobromate of Diethyl-Thetine.

Owing, however, to the extremely deliquescent nature of the substance, we did not at the time subject it to a closer examination, but devoted our attention to the methyl compound and its derivatives. The following are the results of an examination which I have since made of the hydrobromate of diethyl-thetine and its derivatives:—

Hydrobromate of Diethyl-Thetine.—50 grms. of sulphide of ethyl and 70 grms. of bromacetic acid[†] were shaken together. The acid slowly dissolved, occasioning a fall of temperature in so doing from 20–14°. After about half an hour's standing, the mixture grew warm, and deposited a slightly brown, oily liquid. The reaction was very much less intense than in the case of the methyl compound, when the sulphide of methyl actually boils and the mixture has to be cooled to prevent loss of product.

Next day a greater part of the oily liquid had crystallised, but the formation of crystals continued for about three days, and even then a considerable quantity of sulphide of ethyl remained unacted on.

The whole of the solid crystalline matter that had formed was pounded in a mortar with ether, and this treatment was repeated twice, to remove excess of sulphide of ethyl and bromacetic acid. Most of the product was then dried in a desiccator and reserved for the production of other salts, but a considerable quantity was dissolved in hot alcohol, the solution allowed to cool, and

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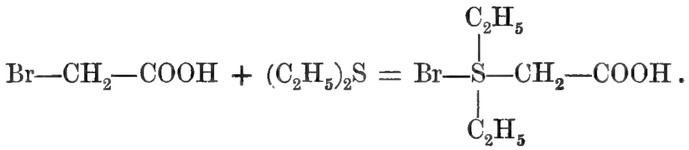
[*] "Proceedings," viii. 220, 385.

[†] Equi-molecular quantities of the two substances require, for 50 of sulphide of ethyl, 77 of bromacetic acid; the sulphide of ethyl was therefore slightly in excess.

then placed in a desiccator in order that the alcohol might evaporate. After some time magnificent colourless crystals separated from the solution, which are either oblique rhombic or quadratic prisms—which, could not be determined by simple inspection. Several of them grew till they were fully a quarter of an inch in across, and could no doubt be grown to almost any size if allowed to remain for a sufficient length of time in the mother liquor.

The crystals, there can be no doubt, consist of hydrobromate of diethyl-thetine, formed according to the equation—



$$Br—CH_2—COOH + (C_2H_5)_2S = Br—\overset{\overset{\displaystyle C_2H_5}{|}}{\underset{\underset{\displaystyle C_2H_5}{|}}{S}}—CH_2—COOH .$$

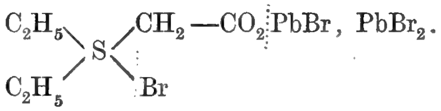
The analysis of the lead and platinum salts derived from it, as well as its analogies with the methyl compound, leave no doubt as to its composition, but owing to its extreme deliquescence no analysis has been made.

Hydrobromate of diethyl-thetine, like the corresponding methyl compound, is readily stained brown by contact with filter paper. It is soluble in water and alcohol with ease, less so in sulphide of ethyl, but still considerably, and, as far as could be judged, insoluble in ether. Well-developed crystals could not be distinguished from the methyl compound. Both apparently crystallise in the same form, but this point requires investigation.

Ethyl-Thetine Lead Salt.—A quantity of the hydrobromate was dissolved in water, and carbonate of lead added. Effervescence occurred immediately, and on boiling the mixture, the carbonate of lead gradually dissolved. When this no longer occurred, the solution was filtered, and as it cooled deposited very beautiful tufts of small crystals, which consist either of needles or of narrow plates. This salt is very sparingly soluble in cold water, but comparatively easily soluble in boiling water. Its analysis gave the following numbers :—

	Obtained.		Calculated for $C_6H_{12}SO_2$, $2PbBr_2$.
	I.	II.	
Carbon, ...	8·1	...	8·16
Hydrogen, ...	1·6	...	1·36
Bromine, ...	35·6	35·0	36·28
Lead, ...	46·6	46·3	46·94

It is analogous in composition and properties to the corresponding methyl compound, and may be formulated thus—



Its formation may be represented by the equation—

$$\underbrace{4(C_6H_{13}SO_2Br)}_{\text{Hydrobromate of Diethyl-Thetine.}} + 2PbCO_3 + H_2O = \underbrace{C_6H_{12}SO_2, 2PbBr_2}_{\text{Lead Salt.}} + \underbrace{3(C_6H_{14}SO_3)}_{\text{Ethyl-Thetine.}} + 2CO_2.$$

Hydrochlorate of Diethyl-Thetine.—About 12 grms. of the hydrobromate were converted into sulphate by treating the solution with sulphate of silver. To the resulting solution, filtered from the bromide of silver, the calculated quantity of chloride of barium dissolved in water was added. The solution of hydrochlorate of diethyl-thetine thus obtained was filtered from the precipitated sulphate of barium, and concentrated at ordinary temperatures in a desiccator. The solution left a syrupy liquid, which did not crystallise even after weeks' standing. It can scarcely be doubted that the hydrochlorate is a solid substance; but up to the present time it has not been obtained in the crystalline condition, and I have therefore not attempted its analysis. As to its composition, but little doubt can exist on account of the splendid salt which it yields with chloride of platinum.

Chloro-Platinate of Diethyl-Thetine.—Some of the solution of the hydrochlorate was mixed with chloride of platinum, and the mixture evaporated somewhat in a water bath and then left to itself. Beautiful dark orange crystals of considerable size soon separated. This salt, unlike the corresponding methyl compound, contains no water of crystallisation; otherwise the two bodies have a similar composition,* as shown by a determination of the platinum—

·1848 gave ·0515 platinum = 27·9 per cent.
$(C_6H_{13}SO_2Cl)_2PtCl_4$ requires 27·9 „

Sulphate of Diethyl-Thetine.—About 30 grms. of the hydrobromate were dissolved in water, a slight excess of sulphate of silver added, and the mixture digested in a water bath. From time to time some of the liquid was removed and tested with fresh sulphate of silver, to see that the whole of the hydrobromate had been decomposed. When this was found to be the case, the solution was separated from the bromide of silver by filtration, a few drops of hydrochloric acid added to precipitate sulphate of silver which had dissolved, the solution again filtered and set to evaporate in a desiccator. After some weeks a syrupy liquid remained, which refused to crystallise even after months'

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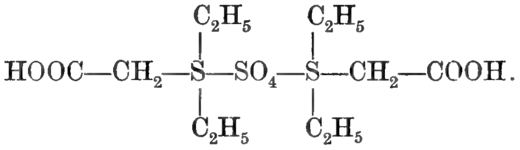
* Chloro-platinate of dimethyl-thetine crystallises with 2 molecules of water and has the composition expressed by the formula $2(C_4H_9SO_2Cl), PtCl_4, 2H_2O$.

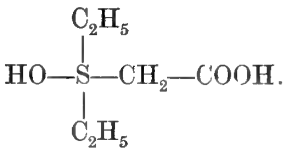
standing. Although there can be no doubt that the syrupy liquid consists essentially of sulphate of diethyl-thetine—



$$\mathrm{HOOC{-}CH_2{-}\underset{C_2H_5}{\overset{C_2H_5}{S}}{-}SO_4{-}\underset{C_2H_5}{\overset{C_2H_5}{S}}{-}CH_2{-}COOH.}$$

owing to the impossibility of obtaining the salt in the crystalline state, it was not further examined.

Diethyl-Thetine Base.—Some of the hydrobromate was dissolved in water, and oxide of silver added to the solution until it ceased to be converted into bromide. The solution was then filtered, a few drops of hydrochloric acid added to precipitate any silver that remained in the solution, and the latter again filtered. It was then concentrated in a desiccator, and finally remained for some weeks over phosphoric anhydride. It, however, simply dried up to a very thick syrupy liquid and refused to crystallise. The base, no doubt, has the composition expressed by the formula—



$$\mathrm{HO{-}\underset{C_2H_5}{\overset{C_2H_5}{S}}{-}CH_2{-}COOH.}$$

but the syrup was not thought sufficiently pure for analysis.

Hydrobromate of Dipropyl-Thetine.—Sulphide of propyl was prepared by digesting for some time in a flask provided with an upright condenser—chloride of propyl[*] and alcoholic solution of sulphide of potassium. The mixture was heated in a water-bath for three or four hours, then left for a night, and distilled. The sulphide of propyl contained in the distillate was separated with water, filtered, and employed without further purification; 6·5 grms. of the sulphide thus prepared, and 6·5 grms. of bromacetic acid[†] (carefully dried between filter paper), were mixed and shaken. The acid dissolved with fall of temperature, and after a few minutes the mixture grew cloudy, and soon a dense syrupy layer separated, which gradually increased in quantity. This did not, however, show any signs of crystallising even after several days' standing, nor was any change effected by cooling it in a freezing mixture.

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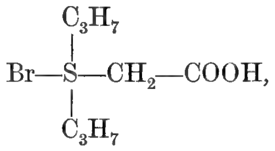
[*] The specimen employed was from Kahlbaum of Berlin.

[†] Equimolecular quantities require for 6·5 of bromacetic acid 5·6 of propyl sulphide.

Up to the present time hydrobromate of dipropyl-thetine—



$$Br{-}\overset{\displaystyle C_3H_7}{\underset{\displaystyle C_3H_7}{S}}{-}CH_2{-}COOH,$$

has not been obtained in the solid state, nor was any analysis of the syrupy liquid attempted; but there can be little doubt that the latter consists of the thetine compound—the phenomena attending its formation as well as its properties indicating that such is the case.

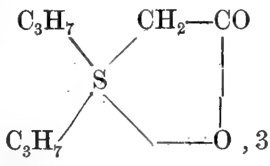
The product of the action of bromacetic acid on sulphide of propyl was mixed with water, which dissolved the greater part, but left about 4 grms. of sulphide of propyl, showing that only about half the theoretical quantity had been converted into the thetine compound.

The aqueous solution was employed for the preparation of the lead and platinum salts.

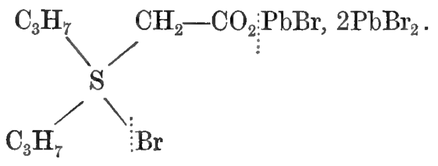
Action of Hydrobromate of Dipropyl-Thetine on Carbonate and Hydrate of Lead.—Some of the solution was boiled with carbonate of lead; effervescence occurred, and when this ceased the solution was filtered. On cooling, it deposited minute colourless needles or narrow plates (A). On filtering the solution from these, and allowing it to remain undisturbed for some time, minute tufts of needles were deposited (B). In a second experiment with a larger quantity of the solution, recently precipitated hydrate of lead was employed instead of the carbonate. The solution as it cooled deposited beautiful glistening thin plates (C). The mother liquor from them was again saturated with hydrate of lead, and yielded a second crop of crystals resembling the former in appearance (D). Determinations of lead in these four salts gave the following numbers:—

	Lead in 100.
(A)	49·0
(B)	45·4
(C)	48·8
(D)	49·3

A, C, and D have apparently the same composition, and the lead which they contain agrees with the quantity calculated for a compound of one molecule of the base dipropyl-thetine and three molecules of bromide of lead—



, $3PbBr_2$, or



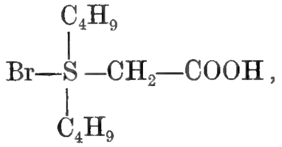
B contains a quantity of lead which agrees with that required for a compound of one molecule of the base and two molecules of bromide of lead, and therefore similar in composition to the lead salts obtained from the hydrobromate of dimethyl-thetine and of diethyl-thetine.

$(C_8H_{16}SO_2), 2PbBr_2$ requires 45·5 per cent. lead.
$(C_8H_{16}SO_2), 3PbBr_2$ „ 48·7 „

Up to the present time no other crystallised derivatives of dipropyl-thetine have been obtained. The aqueous solution of the hydrobromate is readily acted on by sulphate of silver to form the sulphate of dipropyl-thetine, and from this the hydrochlorate is obtained by the action of chloride of barium; but both of these compounds could only be obtained as syrupy liquids, and even the chloro-platinate refuses to crystallise, and dries up to an orange-coloured syrup.

Hydrobromate of Di-isobutyl-Thetine.—7 grms. of bromacetic acid and 7 grms. of isobutyl sulphide* were mixed and shaken. The acid dissolved with a considerable fall of temperature. The mixture was then heated in a water-bath and rapidly separated into two layers, the lower of which was very syrupy. The whole was left to itself for a night, and next morning the excess of sulphide of isobutyl poured off; this amounted to 3·3 grms., and thus only about half of the sulphide was acted on.

The syrup consists in all probability of hydrobromate of di-isobutyl-thetine—



$$\mathrm{Br{-}}\overset{\displaystyle C_4H_9}{\underset{\displaystyle C_4H_9}{\overset{|}{\underset{|}{S}}}}\mathrm{{-}CH_2{-}COOH},$$

but like the propyl compound it has not been obtained in the solid state, and therefore in a fit condition for analysis, so that its composition can only be inferred from the phenomena attending its formation and from its properties.

The only crystallised derivatives that could be obtained from it were lead salts, for the preparation of which most of the aqueous solution of the crude thetine was employed.

The solution was diluted, boiled with hydrate of lead till the latter ceased to be dissolved, then filtered and allowed to cool when an oily liquid precipitated.

The supernatant liquor (1) was poured off and the oily liquid warmed with water, when it became crystalline almost immediately; the crystals

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* These represent equimolecular quantities.

dissolved on boiling the mixture, and separated on cooling, in beautiful silvery plates (A) resembling the propyl-thetine lead salt.

The liquor (1) was again saturated with hydrate of lead, and yielded another crop of similar crystals. These were recrystallised (B). The mother liquors from both it and (A) were concentrated, and yielded another crop of crystals (C). Determinations of lead in these three salts gave the following numbers :—

	Lead in 100.
(A)	57·4
(B)	50·8
(C)	47·7

which compare thus, with those calculated for bromide of lead and compounds of it with hydrobromate of di-isobutyl-thetine—

	Lead in 100.
$PbBr_2$ requires	56·2
$(C_{10}H_{20}SO_2), 5PbBr_2$	50·7
$(C_{10}H_{20}SO_2), 3PbBr_2$	47·6

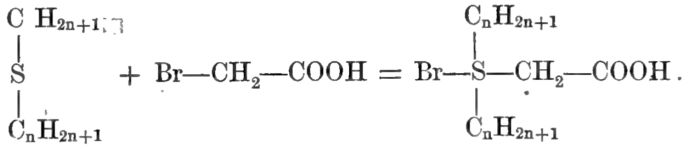
From which it appears probable that (A) consists, if not entirely, at least for the greater part, of bromide of lead, and (B) and (C) of compounds of a molecule of di-isobutyl-thetine, with 5 and 3 molecules respectively of bromide of lead.

The solution of the hydrobromate of di-isobutyl-thetine was readily acted on by sulphate of silver, but the resulting sulphate could not be obtained in the crystalline condition, as was also the case with the hydrochlorate obtained from the sulphate by treating it with chloride of barium.

Hydrobromate of Diamyl-Thetine.—The phenomena attending the action of sulphide of amyl on bromacetic acid are exactly similar to those observed with the isobutyl sulphide, only the formation of an oily layer takes place very slowly and even more of the sulphide remains unacted on. No crystalline compounds could be obtained—with the exception of lead salts which were not specially examined. The hydrobromate and the other salts of diamyl-thetine consist apparently of uncrystallisable syrups.

In concluding this paper it may not be superfluous to make a few remarks on the thetines as a class.

A sulphide of the series $(C_nH_{2n+1})_2S$, when treated *in the cold* with bromacetic acid, combines with it to form the hydrobromate of a thetine—



$$\overset{C_nH_{2n+1}}{\underset{C_nH_{2n+1}}{S}} + Br—CH_2—COOH = Br—\overset{C_nH_{2n+1}}{\underset{C_nH_{2n+1}}{S}}—CH_2—COOH.$$

The intensity of the reaction and the quantity of hydrocarbon sulphide combining with the bromacetic acid decrease as the series is ascended. In the case of the first member of the series—sulphide of methyl—nearly the whole of it combines with the bromacetic acid, and the action is so energetic that unless checked by cooling, the sulphide of methyl boils off; whereas in the case of sulphide of amyl the mixture has to be warmed before a reaction occurs, and even then less than half the sulphide enters into combination.

The production of a thetine hydrobromate is almost invariably attended with the following phenomena:—The bromacetic acid dissolves in the sulphide with fall of temperature—the solubility of the acid and therefore the extent of this fall decreasing as the series of sulphides is ascended. An oily liquid gradually separates from the solution as soon as the bromacetic acid is dissolved, and this liquid consists for the greater part of the thetine hydrobromate. The latter is more or less soluble in excess of the hydrocarbon sulphide. The separation of oily liquid appears to be characteristic of the formation of a thetine hydrobromate.

Considerable differences are observed in the properties of the thetine compounds. All the derivatives of dimethyl-thetine crystallise with remarkable ease. In the ethyl series only the hydrobromate, lead salt, and chloro-platinate have been obtained crystallised; and in the higher series only the lead salts (which are perhaps not of definite composition) are crystalline.

The action of heat on the thetine compounds is described in a separate paper, p. 591, as also the action of hydrocarbon sulphides on bromacetic acid, p. 612. It appears probable that only sulphides of the series $(C_nH_{2n+1})_2S$ are capable of combining with bromacetic acid, and that in other series the action is similar to that which occurs when a sulphide of the $(C_nH_{2n+1})_2S$ series is *heated* with bromacetic acid.

The action of oxidising agents on the compounds of dimethyl-thetine, p. 601, shows that in the latter the hydrocarbon sulphide retains to a certain extent its original properties.

Action of Heat on Compounds of Dimethyl-Thetine.

By Dr E. A. Letts.

(Sent for Publication August 19, 1878.)

I was induced to study the action of heat on the compounds of dimethyl-thetine, from having observed that they very readily suffered change when heated alone, or even, indeed, in certain cases when their solutions are boiled. I noticed that this change is attended with the disengagement of sulphide of methyl in all the salts except the nitrate, and this led me to think,—bearing in mind the ease with which the methyl-sulphine compounds are decomposed into sulphide of methyl and an ether of methyl,*—that in the case of the thetine compounds, dissociation also occurred. Thus I believed that the hydrobromate of dimethyl-thetine is decomposed by heating into bromacetic acid and sulphide of methyl,—the substances from which it is produced by direct addition.

In order to test the truth of this supposition, weighed quantities of the hydrobromate (carefully dried over sulphuric acid till they ceased to lose weight) were heated on watch-glasses and in beakers in a water oven. The results of these experiments confirmed the fact of the decomposition of the salt, though at this temperature it took place very gradually, loss in weight continuing for more than a week.

But it soon became clear that the loss in weight which the hydrobromate suffered when thus heated was greater than could be accounted for on the assumption that simple dissociation occurred. Moreover, the residue was not deliquescent, and had neither the appearance nor properties of bromacetic acid.

In order to ascertain the nature of the products of decomposition, a quantity of the hydrobromate was sealed up in a tube constructed according to the sketch. The thetine salt was placed in the limb A, and the limb B immersed in cold water; A was then heated in oil or water. After five hours' heating in a water-bath the hydrobromate had fused, and B contained a few drops of a colourless liquid. A was then heated in an oil-bath. At

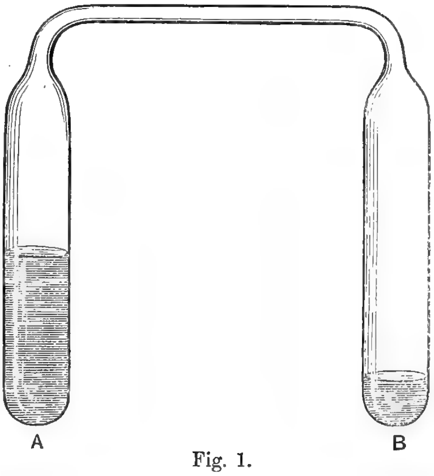


Fig. 1.

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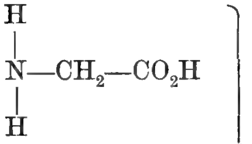
* Crum Brown and Blaikie, "Proceedings," vol. ix. pp. 565, 712.

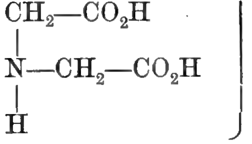
120° C., the thetine salt began to bubble, and the decomposition increased in rapidity with rise of temperature. The heating was continued for some hours, the final temperature being from 150° to 160° C. The contents of the limb B—which at the close of the experiment consisted of a clear liquid—solidified during the night to a colourless crystalline mass.

B was cut* off and placed for a day or two in a desiccator, in order that any sulphide of methyl might escape. The crystals contained bromine, and on determining its amount it was found to agree with that required for the bromide of trimethyl-sulphine. Assuming the decomposition of the thetine compound to be such that this body is produced, it appeared possible that its formation might be attended with that of substances analogous to HEINTZ's diglycol- and triglycol- amidic acids, which are formed along with glycocoll by the action of ammonia on bromacetic acid†—



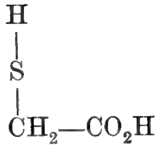
$$\begin{array}{l} H \\ | \\ N—CH_2—CO_2H \\ | \\ H \end{array}$$

Glycocoll.



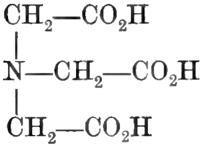
$$\begin{array}{l} CH_2—CO_2H \\ | \\ N—CH_2—CO_2H \\ | \\ H \end{array}$$

Diglycol-amidic Acid.



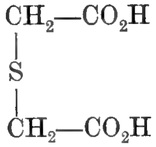
$$\begin{array}{l} H \\ | \\ S \\ | \\ CH_2—CO_2H \end{array}$$

Thioglycollic Acid.‡



$$\begin{array}{l} CH_2—CO_2H \\ | \\ N—CH_2—CO_2H \\ | \\ CH_2—CO_2H \end{array}$$

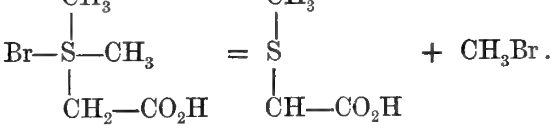
Triglycol-amidic Acid.



$$\begin{array}{l} CH_2—CO_2H \\ | \\ S \\ | \\ CH_2—CO_2H \end{array}$$

Thiodiglycollic Acid.

The formation of thioglycollic acid could not be expected, but methyl-thioglycollic acid might result from the hydrobromate of dimethyl-thetine by loss of bromide of methyl—



$$\begin{array}{l} \quad\quad CH_3 \\ \quad\quad | \\ Br—S—CH_3 \\ \quad\quad | \\ \quad\quad CH_2—CO_2H \end{array} = \begin{array}{l} CH_3 \\ | \\ S \\ | \\ CH—CO_2H \end{array} + CH_3Br.$$

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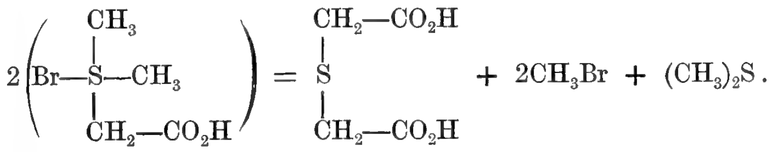
* Considerable pressure was observed on opening the tube.

† HEINTZ, "Ann. der Chem. u. Pharm." cxxxvi. 214, and cxlv. 49.

‡ Thioglycollic acid may be compared both with glycocoll and diglycol-amidic acid, as in each of these substances the nitrogen is partly saturated with hydrogen and partly with glycolyl. The properties of thioglycollic acid, however, point to a stronger resemblance to diglycol-amidic acid than to glycocoll.

And the formation of thiodiglycollic acid from the hydrobromate of dimethyl-thetine would be represented by the equation—



$$2\left(\begin{array}{l}\quad\;\; CH_3 \\ \quad\;\;\; | \\ Br—S—CH_3 \\ \quad\;\;\; | \\ \quad\;\; CH_2—CO_2H\end{array}\right) = \begin{array}{l} CH_2—CO_2H \\ | \\ S \\ | \\ CH_2—CO_2H \end{array} + 2CH_3Br + (CH_3)_2S\,.$$

The sulphide and part of the bromide of methyl would unite to form bromide of trimethyl-sulphine.

Before proceeding to examine the non-volatile products of the action of heat on hydrobromate of dimethyl-thetine, in order to ascertain the presence or absence of these acids, it was judged expedient to repeat the experiment with somewhat larger quantities, and in a somewhat different manner,—first, to have a sufficient quantity of material to examine; and next, to ascertain the nature of the gaseous or very volatile substances which caused the pressure observed in the experiment with the sealed tube. Three separate quantities of the hydrobromate were heated in a strong reagent bottle connected with a Liebig's condenser, which, in its turn, was connected with a receiver placed in a freezing mixture, and the latter attached to a mercury manometer. Thus the volatile products of the decomposition would be subjected to a pressure of at least 30 inches of mercury. (See sketch.)

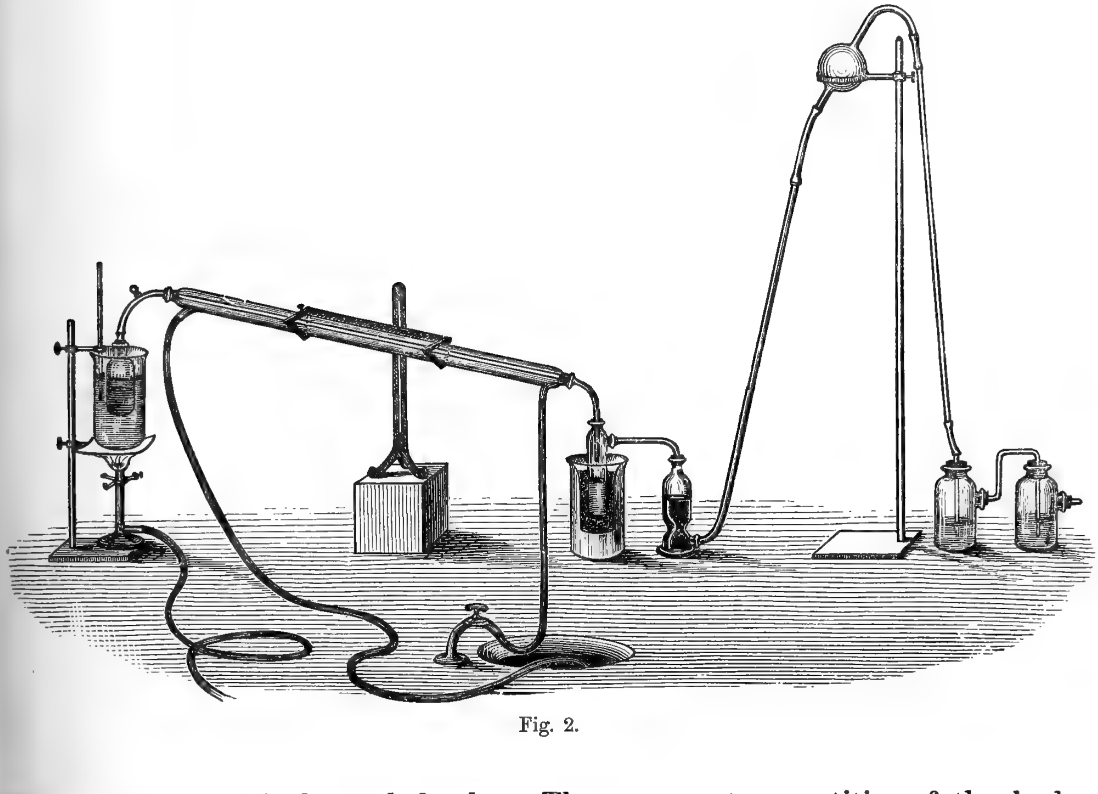


Fig. 2.

In all three experiments the phenomena observed were much the same. The thetine salt fused and frothed at a temperature a little above 100° C., and the frothing continued to about 170° C. A volatile liquid passed over, which partially solidified in the receiver (after standing some hours); at the same time some permanently gaseous products were formed, for at the end of the experiment the mercury stood at about 30 inches, and after some hours had diminished only to about 15 inches. No doubt part of these gaseous products consisted of hydrobromic acid, as the corks used for connecting the different parts of the apparatus were charred, and acid clouds were formed when the apparatus was disconnected. In one of the experiments in which 50 grms. of the hydrobromate had been taken, there remained 20 grms. of residue, and thus 30 grms. of volatile products had been given off.

The partly solidified volatile products were distilled from a water-bath, under pressure.

A few cubic centimetres of a colourless liquid passed over, which smelt more like bromide than sulphide of methyl. During the distillation the pressure rose to nearly 30 inches.

The solid crystalline residue which remained, fumed, and the corks connected with the apparatus in which it was heated were much charred. It was extracted with dry ether several times, then dissolved in boiling alcohol, and the solution filtered. On cooling, colourless plates separated, which were washed with cold alcohol, dried in a desiccator, and analysed—

·4665 grms. gave ·559 bromide of silver = ·238 bromine = 51·0 per cent.
·3560 „ ·438 „ = ·187 „ = 50·9 „

Calculated for $(CH_3)_3SBr$ . . . . . . = 50·9 „

The identity of the substance with the bromide of trimethyl-sulphine was further proved by its properties.

The non-volatile products when first formed were dark-coloured and syrupy, but after standing for some hours they solidified to a buttery, crystalline mass. They had an acid taste, dissolved with great ease in water, effervesced with carbonates, and yielded insoluble precipitates with solutions of lead and silver salts.

They were extracted seven or eight times with warm and perfectly dry ether (the ether was boiled with them each time, well agitated, and poured off). After each extraction they seemed to grow more pasty and solid. The residue I shall call for convenience A. The ethereal solution deposited small crystals as it cooled,—apparently rhombohedra. The cold ether was poured away from these, and partly distilled off, when another crop of crystals separated. The mother liquor was again poured off, and this time distilled to dryness, when it left a crystalline crust, impregnated with an oily substance which had a pungent and disagreeable smell. The first and second crops of crystals, and the crystalline crust, I shall call for convenience B1, B2, and B3 respectively.

B3 could not be readily crystallised from alcohol or water, as it was exceedingly soluble in these.

Its aqueous solution yielded with—

(1.) Nitrate of silver solution, a yellowish precipitate which dissolved on stirring. The silver salt added in excess caused a permanent yellow precipitate which was not crystalline.

(2.) Solution of acetate of lead, a white, curdy, precipitate, which rapidly became crystalline.

(3.) Chloride of zinc solution (after neutralisation with ammonia), a crystalline precipitate.

B2 dissolved in water yielded with—

(1.) Nitrate of silver solution, a white, curdy, precipitate, which rapidly became crystalline.

(2.) With acetate of lead, a precipitate behaving in a similar manner.*

B1 behaved as B2.

A quantity of the lead and silver salts of B2 were prepared and analysed. A combustion of B2 was also made.

From the results of these analyses there can be little doubt that B2 consists of thiodiglycollic acid $S(CH_2COOH)_2$, first prepared by SCHULZE,† and from subsequent observations B1 and B3 were found to consist of the same substance—B1 in a somewhat purer, B3 in a somewhat less pure condition.

The results of the analyses are as follows:—

B2.

·3635 grms. gave ·1478 grms. water . . . = 4·5 per cent. hydrogen.
·3635 „ „ ·435 „ carbonic anhydride = 32·6 „ carbon.

$S(CH_2COOH)_2$ requires . . . . . . $\begin{cases} 4·0 \text{ „ hydrogen.} \\ 32·0 \text{ „ carbon.} \end{cases}$

Lead Salt of B2.

·3283 grms. gave ·2765 sulphate of lead . . = 57·5 per cent. lead.
·2737 „ „ ·1587 „ . . = 57·8 „ „
1·0208 „ „ ·5970 sulphate of barium . . = 8·7 „ sulphur.
·9353 „ „ ·5943 „ . . = 8·0 „ „
1·2245 „ „ ·6047 carbonic anhydride . . = 13·5 „ carbon.
1·2245 „ „ ·1398 water . . . . . = 1·2 „ hydrogen.

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* This change, when observed under the microscope, was very curious. The acetate of lead first occasioned the precipitation of warty masses, which dissolved of their own accord, and from the solution groups of needles shot out.

† SCHULZE, "Jenaische Zeitschr." i. p. 470 (1864); "Bull. Soc. Chim." v. p. 130 (1866).

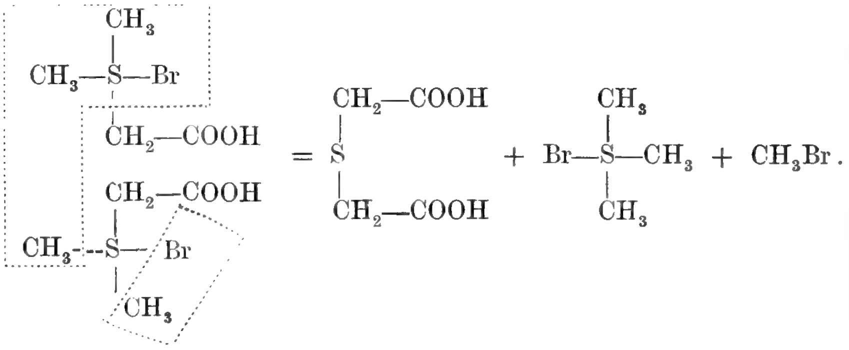
	I.	II.	Calculated for $S(CH_2COO)_2Pb$.
Lead, ...	57·5	57·8	58·3
Sulphur, ...	8·7	8·7	9·0
Carbon, ...	13·5	...	13·5
Hydrogen, ...	1·2	...	1·1
Oxygen, ...	19·1	...	19·1
	100·0		100·0

Silver Salt of B2.

·3065 gave ·179 silver = 58·4 per cent.
·304 „ ·178 „ = 58·5 „

$S(CH_2—COOAg)_2$ requires 59·3 „

These results clearly indicate the nature of the decomposition which the thetine salt suffers when heated. Two molecules of the hydrobromate are resolved into a molecule of thiodiglycollic acid, a molecule of bromide of methyl, and a molecule of the sulphine compound.



$$\begin{matrix} CH_3 \\ | \\ CH_3—S—Br \\ | \\ CH_2—COOH \\ CH_2—COOH \\ | \\ CH_3—S—Br \\ | \\ CH_3 \end{matrix} = \begin{matrix} CH_2—COOH \\ | \\ S \\ | \\ CH_2—COOH \end{matrix} + \begin{matrix} CH_3 \\ | \\ Br—S—CH_3 \\ | \\ CH_3 \end{matrix} + CH_3Br .$$

Thiodiglycollic acid is the true sulphur analogue of triglycol-amidic acid, and the action of heat on the hydrobromate of dimethyl-thetine (which we may regard as the action of heat on sulphide of methyl and bromacetic acid) may be strictly compared with the action of heat on ammonia and bromacetic acid. In both cases the final product is a body resulting from the replacement of the radicals bound to the sulphur and nitrogen respectively (methyl and hydrogen) by the radical glycolyl.*

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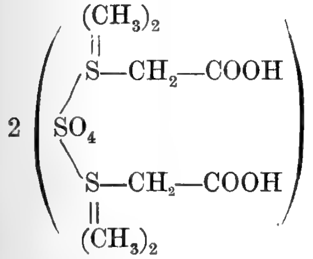
* The above experiments were made before those described on pp. 612–617 on the action of hydrocarbon sulphides on bromacetic acid, which go still further to establish an analogy between the action of ammonia on bromacetic acid and the action of hydrocarbon sulphides generally, on the same substance, and also before the experiments on the action of sulphide of methyl on iodacetic ethyl ether, which led to the discovery of methyl-thioglycollic acid. It is quite possible that the latter substance is formed along with thiodiglycollic acid when hydrobromate of dimethyl-thetine is heated, and that in the above experiments it remained in the residue of the non-volatile products after the latter had been extracted with ether (product A). At the time the experiments were made however, this

Action of Heat on Sulphate of Dimethyl-Thetine.—Having ascertained the nature of the decomposition which occurs when the hydrobromate of dimethyl-thetine is heated, I entertained but little doubt that the sulphate would suffer a similar change—that is to say, that it would split into sulphate of trimethyl-sulphine, sulphate of methyl, and thiodiglycollic acid, according to the equation—



$$= 2\,S(CH_2{-}COOH)_2 + \left\{(CH_3)_3\right\}_2SO_4 + (CH_3)_2SO_4\,.$$

The experiment was made with the same apparatus as that employed for the hydrobromate, the manometer being retained in order that any sulphide of methyl produced by the dissociation of the sulphine compound might be the more readily condensed.

The sulphate fused at about 140° C., and abundance of volatile products were disengaged. Some of these condensed in the receiver, but a large portion consisted of permanently gaseous substances, which escaped after the mercury in the manometer had risen above the full height of the tube.

When this escape of gas had occurred once or twice it was thought worth while to collect it for examination. It was accordingly made to pass into wash bottles arranged as in the sketch.*

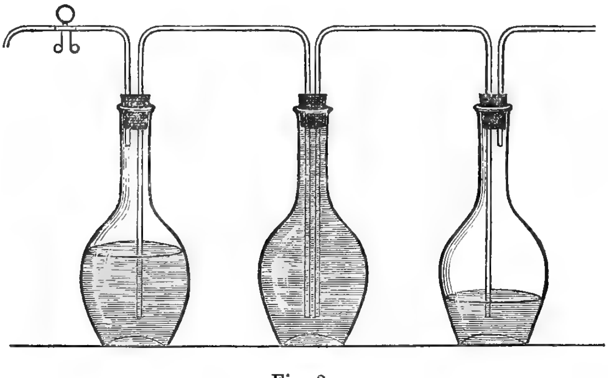


Fig. 3.

The gas thus collected was in great measure absorbed after a few hours'

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residue was taken to be impure thiodiglycollic acid, and was only roughly examined to ascertain the presence of that substance. It is also possible that it was dissolved with the thiodiglycollic acid by the ether, and formed the oily liquid which saturated B3 after the ether had been evaporated off. The experiment, therefore, needs repeating.

[*] I have found this form of apparatus exceedingly convenient, not only for storing up gases, but also for absorbing ammonia, hydrobromic acid, &c. It is especially useful for this purpose in determining ammonia.

standing. That which remained, when tested with baryta water, was found to contain considerable quantities of carbonic anhydride.

After the heating of the thetine salt had continued for a sufficient length of time—*i.e.*, until it ceased to froth, the experiment was stopped, and the dark-brown viscous residue allowed to remain a night, but it did not solidify. It was then treated with water (in which it readily dissolved for the greater part) and the mixture distilled. A few oily drops passed over, which were not sufficient in quantity for examination, but were probably sulphate of methyl. The residue was still further diluted, and mixed with baryta water. An abundant white precipitate of sulphate of barium occurred.

The solution filtered from this was evaporated over a water-bath and left a syrupy liquid which did not crystallise on cooling, nor did it yield a fixed residue when calcined, showing that no barium salts had been formed. It gave an abundant yellow precipitate with chloride of platinum, which dissolved in boiling water, and as the solution cooled orange-red crystals separated. These on analysis were found to consist of chloro-platinate of trimethyl-sulphine—$2(CH_3)_3SCl, PtCl_4$—

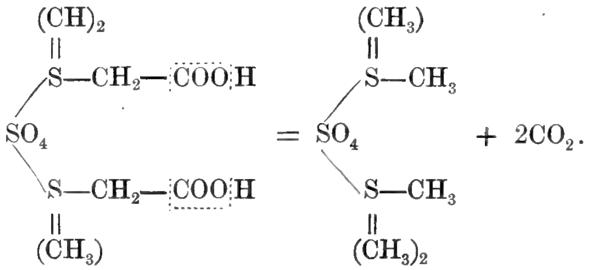
	I.	II.	III.	Theory.
Platinum, . . .	34·4	34·49	34·22	34·9
Carbon, . . .	12·82	12·68	12·99	12·7
Hydrogen, . . .	...	3·46	3·46	3·2

The volatile products of the reaction did not solidify on remaining at rest, and appeared to consist wholly of sulphide of methyl.

Not a trace of thiodiglycollic acid was present in the non-volatile products, a fact which was proved by the absence of a solid residue when these products were neutralised with baryta and calcined.

It is quite obvious that the sulphate of dimethyl-thetine suffers, when heated, a totally different decomposition from the hydrobromate.

The substances produced from the former body indicate that its decomposition when heated is expressed by the following equation—



the sulphide of methyl which escaped being no doubt due to the dissociation of the sulphine sulphate.

Action of Heat on Hydrate of Dimethyl-Thetine.—Having found that the hydrobromate is decomposed in quite a different manner from the sulphate, it appeared to be of interest to extend the experiment to the hydrate, in order to ascertain which of the two kinds of decomposition it would suffer when heated. If it behaved as the hydrobromate, then hydrate of trimethyl-sulphine (or methyl alcohol and sulphide of methyl) would be produced together with thiodiglycollic acid; whereas if it behaved as the sulphate, the products would be carbonate of trimethyl-sulphine and carbonic anhydride.

A quantity of the hydrate (well dried in a desiccator) was heated in a distilling flask placed in an oil-bath. It fused easily, and gaseous products were at once evolved. That these contained considerable quantities of sulphide of methyl was proved by passing them through alcoholic solution of corrosive sublimate, when the characteristic white crystalline precipitate was produced. That they also contained large quantities of carbonic anhydride was shown by the abundant white precipitate which occurred when they were allowed to bubble through baryta water.

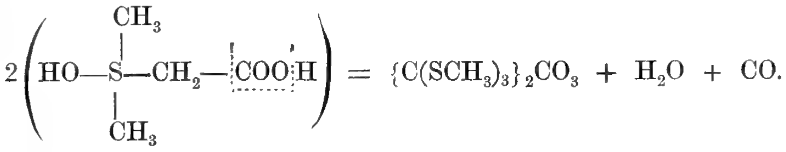
As soon as no further disengagement of volatile products occurred, the experiment was stopped, and the syrupy liquid which remained examined.

It effervesced violently when mixed with hydrochloric acid, the gas given off consisting of carbonic anhydride. The resulting solution gave a sparingly soluble crystalline salt with chloride of platinum, which was proved by a platinum determination to consist of chloro-platinate of trimethyl-sulphine—

	I.	II.	Theory.
Platinum . . .	34·5	34·8	34·9.

No thiodiglycollic acid could be detected.

The reaction which occurs when the hydrate of dimethyl-thetine is heated is therefore similar to that which the sulphate suffers, and is expressed by the following equation :—



$$2\left(HO-\overset{CH_3}{\underset{CH_3}{S}}-CH_2-\boxed{COO}H\right) = \{C(SCH_3)_3\}_2CO_3 + H_2O + CO.$$

The sulphide of methyl no doubt arises (as in the case of the sulphate of dimethyl-thetine) from the dissociation of the sulphine compound.

The preceding experiments show that the salts of dimethyl-thetine suffer one of two distinct decompositions when heated.

Either they are resolved into thiodiglycollic acid and a sulphine compound—as in the case of the hydrobromate; or they split into carbonic anhydride and a sulphine compound—as in the cases of the sulphate and hydrate. When this latter decomposition occurs, the carbonic anhydride separates from the glycolyl group, leaving a complete methyl group, which remains attached to the sulphur—

$$—CH_2—COOH = —CH_3 + CO_2.$$

No doubt the other salts of dimethyl- and other thetines would suffer similar changes,—the hydracid salts probably the first, the oxyacid salts the second. The decomposition of the nitrate would probably not be so simple and would give rise to oxidation products. Whether both kinds of decomposition can occur with the same salt has not as yet been ascertained in the case of the hydrobromate,* but certainly does not occur to any appreciable extent with the other two salts experimented on.

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* The disengagement of hydrobromic acid in considerable quantity, before alluded to as occurring when this salt is heated, points to a different decomposition from either of these, the nature of which I have not as yet ascertained. A disengagement of hydrobromic acid also occurs when bromacetic acid is heated with sulphide of ethylene. Perhaps the reaction is similar in both cases.

On the Action of Oxidising Agents on Compounds of Dimethyl-Thetine.

By Dr E. A. Letts.

(Sent for Publication August 19, 1878.)

In a former paper by Professor Crum Brown and myself,* some preliminary experiments were described on the products of the oxidation of dimethyl-thetine and its salts. We stated that when nitrate of dimethyl-thetine is acted on by nitric acid, two distinct products of oxidation are formed,—the one a neutral substance, readily crystallising from alcohol in long colourless needles, the other apparently an uncrystallisable body possessing acid properties, and readily forming a crystalline barium salt. We found that the base dimethyl-thetine is readily acted on by permanganate of potash in acid or alkaline solution, and that apparently only the crystallisable product of oxidation results.

We also found that fuming nitric acid acts readily on solid hydrobromate of dimethyl-thetine; bromine is separated, and on driving off the excess of nitric acid a syrup remains which fumes like hot sulphuric acid. This syrup also possesses acid properties and forms a well-marked barium salt.

The last oxidising agent with which we experimented was chromic acid. That substance, strange to say, exercises no oxidising action at all on the base dimethyl-thetine, but simply combines with it to form a chromate, which on drying solidifies to a yellow gummy mass, and cannot be obtained in crystals.

The same compound was obtained by the action of chromate of silver on the hydrobromate of dimethyl-thetine.

These results appeared sufficiently interesting to warrant a somewhat closer study of the action of oxidising agents on dimethyl-thetine and its compounds, and accordingly I undertook the investigation, the results of which follow.

Action of Nitric Acid on Nitrate of Dimethyl-Thetine.—A solution of the nitrate was prepared by acting on a solution of the hydrobromate of dimethyl-thetine with nitrate of silver, filtering from the precipitated bromide of silver, and removing the slight excess of nitrate of silver by careful addition of hydrochloric acid. The solution of the nitrate thus prepared was mixed with an equal volume of the ordinary strong nitric acid of the laboratory. The mixture was then heated in a water-bath, when copious red fumes were disengaged. When these ceased to be evolved the greater part of the nitric acid was distilled off, and the remaining liquid heated on a water-bath until it ceased to give the nitric acid reaction. On then allowing it to cool, it solidified

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* "Proceedings," vol. viii. p. 508.

to a crystalline mass. This was broken up, drained in a funnel, and washed with methylated spirit. The strongly acid solution which drained from the crystals, together with the methylated spirit washings, was neutralised with baryta water, when a slight precipitate of sulphate of barium resulted. The excess of baryta was removed by a current of carbonic acid, and the filtered solution evaporated to dryness on a water-bath. The solid mass which remained was extracted with boiling alcohol till it ceased to take up any of the crystalline substance. The remaining barium salt was dissolved in water, the solution evaporated to small bulk and allowed to cool, when hexagonal tables separated out. These were washed with alcohol to remove colouring matter, redissolved in water, and the solution concentrated. Unfortunately it was allowed to evaporate to dryness, and a portion of the salt was charred; the rest, however, was recrystallised from a very little boiling water, and presented the appearance of small white spangles. Previous to analysis it was dried on blotting paper in air.

The results of its analysis are as follows:—

	I.	II.	III.
Water, . . .	7·55(*a*)	...	...
Barium, . . .	38·5(*b*)	38·8(*c*)	38·8(*c*)
Sulphur . . .	...	18·8(*c*)	18·0(*c*)

(*a*) *Dried at* 100.

(*b*) *Same quantity as* (*a*) *treated with sulphuric acid after ignition.*

(*c*) *Oxidised by fusing with a mixture of caustic potash and nitrate of potassium. The mass was then treated with water, and the barium and half the sulphur determined by filtering off and weighing the sulphate of barium. The other half of the sulphur was determined in the filtrate by precipitation with a barium salt.*

The numbers obtained agree with those calculated for methyl-sulphite of barium, containing three molecules of water of crystallisation to two molecules of the anhydrous salt. Thus—

	Calculated for $2(CH_3SO_3)_2Ba, 3H_2O$.
Water, . . .	7·6
Barium, . . .	38·7
Sulphur, . . .	17·6

I am not aware, however, that a salt having that composition has ever been obtained, the only one that I can find mention* of containing one molecule of water of crystallisation to one of the anhydrous salt. It is quite possible that the salt suffered some change by the accidental heating to which it was subjected, but on the other hand a barium salt obtained by the oxidation

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* Muspratt, "Ann. d. Chem. u. Pharm." lxv. 260.

of hydrobromate of dimethyl-thetine by fuming nitric acid gave similar numbers.

The crystalline substance which separated from the product of the action of nitric acid on the thetine salt, after the excess of the former had been driven off by heat, was characterised by its readiness to crystallise from a hot alcoholic solution, in long colourless needles. It did not taste acid, but after recrystallisation from alcohol gave the acid reaction when placed on litmus paper and moistened with a drop of water. It did not combine with bases nor manifest any other acid properties. Heated in a tube, it sublimed in very beautiful crystals, even (though slowly) at the temperature of 100° C. Heated on platinum, it burnt away completely with a blue flame, giving off sulphurous acid.

Its fusing (108·5°) and solidifying point (96°) agree with those given by SAYZEFF * for dimethyl-sulphone, $(CH_3)_2SO_2$—*i.e.*, 109° C. and 99° C.

Its identity with that substance was established beyond doubt by the analysis of the product of the oxidation of dimethyl-thetine by permanganate of potash.

Action of Fuming Nitric Acid on Hydrobromate of Dimethyl-Thetine.—Solid hydrobromate of dimethyl-thetine, added to fuming nitric acid, dissolved to a deep red liquid smelling strongly of bromine. No violent action took place, but the solution grew slightly warm, and a few bubbles of gas came off. On warming, however, a violent action ensued, torrents of red fumes escaping; but it was not found necessary to check the action. When no further evolution of gases occurred, the nitric acid was evaporated off on a water-bath, and a syrupy liquid of disgusting odour remained. This fumed like hot sulphuric acid, even when heated to 100° C. On cooling, no crystalline substance separated out. It was mixed with baryta water, the excess of baryta separated by carbonic acid, and the filtered solution evaporated to small bulk. The crystalline mass, which separated when the solution had cooled, was pressed between filter paper, dissolved in water, and alcohol added. After some time a deposition of crystals occurred; these were filtered off (No. 1 barium salt), and more alcohol added to the mother liquor, when a second crop of crystals separated (No. 2 barium salt).

The results of the analyses of these two salts are as follows:—

No. 1 *Salt.*

	I.	II.	
Barium, . . .	39·3	40·01	...
Water, . . .	5·97	6·14	6·00
Carbon, . . .	6·73	...	...
Hydrogen, . .	2·47	...	...

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* SAYZEFF, "Ann. der Chem. u. Pharm." cxliv. 152.

No. 2 Salt.

	I.	II.
Barium, . . .	38·8	38·7
Water, . . .	7·9	7·6

	Calculated for $(CH_3SO_3)_2Ba, H_2O$.
Barium, . . .	39·7
Water, . . .	5·2
Carbon, . . .	6·9
Hydrogen, . .	2·3

From these numbers it is evident that No. 1 is methyl-sulphite of barium. The analysis of No. 2 agrees with that of the previously-described barium salt obtained by the action of dilute nitric acid on nitrate of dimethyl-thetine. If this salt be a second hydrate of methyl-sulphite of barium, it is rather remarkable that it should be formed in presence of such a strong dehydrating agent as alcohol.

An experiment was also made with hydrobromate of diethyl-thetine. It was treated with strong nitric acid, and a violent action occurred. When this was over, the excess of nitric acid was removed by heating the solution on a water-bath, and the strongly acid syrup which remained was diluted with water and neutralised with carbonate of barium. The solution of the barium salt thus obtained was mixed with alcohol, when a beautiful white crystalline powder separated. This, when analysed, gave the following numbers :—

	Obtained.	Calculated for $(C_2H_5SO_3)_2Ba, H_2O$. *
Barium, . . .	37·0	36·8
Water, . . .	4·9	4·8
Carbon, . . .	13·7	12·9
Hydrogen, . . .	3·2	3·0

Action of Dilute Nitric Acid on Hydrobromate of Dimethyl-Thetine.—The hydrobromate dissolved without any apparent action in dilute nitric acid,† but on very slightly warming the mixture, apparently the whole of the bromine contained in the salt separated out. On heating the mixture a steady action set in, and when this had ceased the nitric acid was evaporated off. A fuming acid syrup remained from which no crystalline substance separated. It did not smell like the product obtained by the action of the fuming acid on the thetine compound. Neutralisation with baryta water showed that a considerable quantity of sulphate of barium had been formed. The barium salt was not obtained in quantity sufficient for analysis.

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* MUSPRATT obtained this salt from ethyl-sulphurous acid, which he prepared by acting on sulphocyanate of ethyl with concentrated nitric acid. (MUSPRATT, "Annalen. d. Chem. u. Pharm." lxv. 253, 254.)

† Equal vols. of strong nitric acid of the laboratory and water.

Action of Permanganate of Potash on Dimethyl-Thetine (*base*).—Several experiments were tried on the oxidising action of permanganate of potash on dimethyl-thetine, both with and without the addition of sulphuric acid. The results appear to be the same in either case, and may be summed up as follows: —The solution of permanganate of potash is soon decolorised when added to the thetine solution, and whether sulphuric acid has previously been added or not to the mixture, a brown hydrate of manganese (hydrate of peroxide ?) precipitates. This gradually disappears when the mixture is boiled, if sulphuric acid has been previously added, forming sulphate of manganese, but remains undissolved of course when the mixture has not been acidulated.

In the case of the mixture of permanganate, thetine, and sulphuric acid, much carbonic acid is evolved, in fact in several experiments the solution actually effervesced from the escape of the gas; sulphide of methyl, or some substance closely resembling it in odour, also escapes, though in small quantity. The solution, after the reaction is completed, appears to contain but a single product of oxidation, which, by its analysis and properties, was proved to be dimethyl-sulphone. The presence of methyl sulphurous acid, which appears to be always formed along with the sulphone when nitric acid is employed as the oxidising agent, could not be ascertained.

The following is an outline of the methods employed in the experiments on the action of permanganate on the thetine. When a mixture of sulphuric acid and permanganate was used, excess of the latter remaining after the reaction was completed was removed by sulphurous acid, the solution was then filtered, if necessary, from hydrate of manganese, and evaporated to dryness. The residue, consisting of sulphates of potash and manganese, sulphone, and any other body that might have been formed, was treated several times with boiling alcohol to remove the sulphone; and the alcoholic solution, after evaporation to dryness, tested for organic salts by heating on platinum. The residue which remained after extracting with alcohol was also tested in a similar manner. When the permanganate solution was employed without the addition of sulphuric acid, the products of the reaction were filtered, by which means hydrate of manganese was removed; the solution was then saturated with carbonic acid, evaporated to dryness, and treated with alcohol as just described. The residue was also tested for organic salts by heating a portion of it on platinum.

The results of the analysis of the sulphone are as follows:—

	I.	II.	Calculated for $(CH_3)_2SO_2$.
Carbon, . . .	25·78	25·31	25·5
Hydrogen, . . .	6·76	6·49	6·4

Finally, I may mention the action of permanganate of potash on hydrobromate of dimethyl-thetine. This is similar to the action of the same oxidising agent on the base, except that bromine is liberated. A very small quantity of a volatile crystalline compound, smelling like bromoform or bromide of carbon, is also produced, together with a very small quantity of acetic acid.

The experiments just described show that the action of oxidising agents on the compounds of dimethyl-thetine is the same as that of oxidising agents on their constituents, *i.e.*, sulphide of methyl and a substitution derivative of acetic acid. In fact, the action in all cases may be regarded as consisting, in the first instance, of the dissociation of the thetine compound into sulphide of methyl,—(OH), (Br), (NO_3), &c., according to the thetine compound employed,—and glycolyl CH_2—COOH. The sulphide of methyl is then oxidised either to methyl sulphurous acid or dimethyl sulphone, or to both, the glycolyl to carbonic anhydride.

In conclusion, I have to express my thanks to my assistant, Mr A. RICHARDSON, for his assistance in this investigation.

Action of Alcohol on Hydrobromate of Dimethyl-Thetine.

By Dr E. A. Letts.

(Sent for Publication August 19, 1878.)

In order to purify the hydrobromate of dimethyl-thetine in some of the earlier experiments with the thetine compounds, crystallisation from alcohol was resorted to.

About 300 grms. of the crude product obtained by the action of equimolecular quantities of bromacetic acid and sulphide of methyl were boiled with methylated spirit. During the boiling an exceedingly pungent substance, which irritated the eyes and nostrils most powerfully, volatilised with the alcohol. On allowing the solution to cool, abundance of the thetine hydrobromate crystallised out. In order to obtain more of the crystals, the mother liquors were boiled, but the continuance of the pungent odour indicated that volatile products were escaping.

As it seemed probable that these consisted of some substance or substances produced by the action of the hydrobromate on the alcohol, which it might be worth while to examine, the digestion was stopped and the solution mixed with water. A dense oily liquid of very pungent odour separated, which was washed with more water and reserved for examination. It was soon found that this oily liquid yielded a crystalline compound when treated with ammonia. On mixing some of it with alcoholic ammonia, and allowing the whole to evaporate spontaneously for some weeks, beautiful needles of considerable size separated. These were quite white, and yielded on analysis—

	I.	II.
Carbon, . .	13·1	12·99
Hydrogen, . . .	2·04	1·87
Nitrogen, . . .	5·88	...
Sulphur, . . .	1·17	...
Bromine, . . .	...	...

The liquid was also acted upon with great ease by aqueous ammonia. A quantity shaken with liquor ammoniæ gradually dissolved, and the solution solidified to a mass of crystals. These were recrystallised from boiling water, in which they were readily soluble, whereas they were sparingly soluble in cold water. They consisted of blunt needles of considerable length, some of them at least half an inch. They evolved ammonia when boiled with caustic potash. Their solution was not precipitated by nitrate of silver, but after they had been fused with caustic potash and the mass dissolved and acidulated, bromide of silver was precipitated in large quantity on adding the nitrate. Unfortunately the amount of oily liquid at my disposal was small, as it was not produced in large quantity, and this is the more to be regretted because all the attempts I have subsequently made to obtain the same body have been unsuccessful.

I have repeated the experiment of boiling the crude thetine hydrobromate with methylated spirit several times, also with absolute alcohol, and each time after the greater part of the thetine had crystallised out and water was added to the mother liquors heavy oily liquids were obtained which possessed a pungent and irritating odour. Each time, however, they appeared to have a different composition. They were all dissolved by ammonia more or less rapidly, but the resulting products differed on each occasion; sometimes they were only moderately soluble in water, sometimes they were so deliquescent that they could not be recrystallised. Once bromide of ammonium was obtained. Owing to the small quantities of the oily liquids obtained, and consequently also of the products of the action of ammonia on the latter, I have been able to make only a few analyses of the latter. Once, however, a sufficiently pure product was obtained, which after recrystallisation from water yielded the following numbers:—

	I.	II.	III.
Carbon, . . .	32·97	33·72	34·26
Hydrogen, . . .	5·77	6·14	6·11

The mother liquors on concentration yielded more of the crystalline substance, which, when recrystallised, gave—

Carbon, . . .	34·81	35·22
Hydrogen, . . .	6·9	6·9

A comparison of these results indicate that the substance was not pure.

I should remark that attempts made to fractionate the oily liquids gave no satisfactory result, the temperature gradually rising.

The fact that the oily liquids possessed a pungent irritating odour, and that they did not occur in any considerable quantity when the thetine compound was boiled for a *short* time with alcohol, led me to think that they might consist of mixtures of monobromacetic and dibromacetic ether. The former would result from the action of the alcohol on monobromacetic acid, which had not combined with sulphide of methyl (and, as a fact, I have observed that in the preparation of all the thetine hydrobromates, some of the hydrocarbon sulphide remains unacted on,—the quantity so remaining increasing with each higher homologue). The latter, the dibromacetic ether, would be formed by the action of alcohol on the dibromacetic acid contained in the crude bromacetic acid employed, which, as a rule, was not specially purified, but boiled between 200° to 215° C., the pure acid boiling at 208° C. In order to ascertain whether the oily liquids owed their production to the presence of these acids, the following experiments were made :—

143 grms. of bromacetic acid (boiling point 200°–215°) were treated in the usual way, for the preparation of hydrobromate of dimethyl-thetine, with 80 grms. of sulphide of methyl (about 16 grms. in excess of the theoretical quantity). Next morning, when the action was complete, the crude thetine compound was purified by washing it repeatedly with ether in which bromacetic and dibromacetic acids readily dissolve. The residue, after this washing, consisted of a pure white powder, from which the last quantities of ether were removed by heating it in a water-bath. It was then boiled for about 10 minutes with 150 c.c. of absolute alcohol. The resulting solution had a very feebly irritating odour, and was not precipitated by water, even on the addition of common salt. It was then left a night, and yielded magnificent colourless crystals of the thetine compound. The residual solution was poured off from these and heated in a flask, to which was fitted a vertical condenser, the inner tube of which was connected by means of a glass tube with a vessel containing alcohol to absorb volatile products, especially sulphide of methyl. The alcoholic solution of the thetine was boiled for several days. From time to time it was tested with water to see whether oily drops precipitated, and this was found to occur after some time, the quantity increasing day by day. When the quantity of oily liquid thus precipitated appeared to cease increasing, the mixture was distilled from a water bath. The residue was then mixed with water, and the liquid which separated washed several times with water. It weighed about 44 grms. The aqueous washings and the water employed to precipitate the oily liquid were evaporated over a water-bath, and left a residue which solidified to a crystalline mass.

The aqueous solution of this gave—

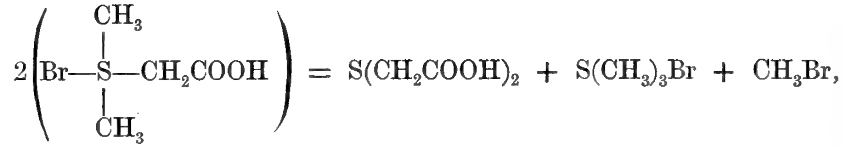
With acetate of lead in the cold, a white amorphous precipitate, which dissolved on warming it with the solution, and was almost immediately replaced by a white crystalline salt.

When neutralised with ammonia, and mixed with a concentrated solution of sulphate of copper—a greenish-white crystalline precipitate.

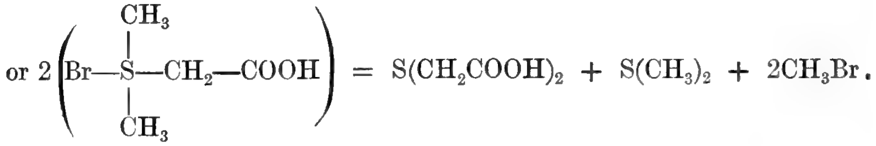
When neutralised with solution of caustic baryta, and warmed—white crusts.

When neutralised with hot concentrated baryta solution, and the mixture allowed to cool—tufts of colourless blunt needles.

These reactions are characteristic of thiodiglycollic acid. The production of thiodiglycollic acid is simply due to the decomposition of the hydrobromate of dimethyl-thetine by heat, which I have already shown occurs according to the equation—



$$2\left(\mathrm{Br}-\overset{\mathrm{CH_3}}{\underset{\mathrm{CH_3}}{\mathrm{S}}}-\mathrm{CH_2COOH}\right) = \mathrm{S(CH_2COOH)_2} + \mathrm{S(CH_3)_3Br} + \mathrm{CH_3Br},$$



$$\text{or } 2\left(\mathrm{Br}-\overset{\mathrm{CH_3}}{\underset{\mathrm{CH_3}}{\mathrm{S}}}-\mathrm{CH_2}-\mathrm{COOH}\right) = \mathrm{S(CH_2COOH)_2} + \mathrm{S(CH_3)_2} + 2\mathrm{CH_3Br}.$$

During the whole of the experiment sulphide of methyl escaped through the vertical condenser, and was caught in the alcohol. Its presence was proved by the odour, and production of the characteristic compound with corrosive sublimate.

The oily liquid precipitated by water was next examined. It appeared probable that it consisted of thiodiglycollic ether, formed by the action of alcohol on the thiodiglycollic acid. It had much the same odour as thiodiglycollic ether*—by no means pungent, and recalling faintly the smell of peppermint.

Some of it was saponified with caustic potash, the resulting solution neutralised with acetic acid, and acetate of lead added ; this occasioned a white crystalline precipitate, which was collected and dried on blotting paper. Determinations of lead gave the following numbers :—

·2285 gave ·1948 sulphate of lead, = 58·2 per cent. lead.
·2795 „ ·2375 „ „ = 58·0 „
Calculated for $\mathrm{S(CH_2CO_2)_2Pb}$, 58·3 „

Another quantity was saponified with caustic baryta, and the solution on standing some time yielded the characteristic barium salt of thiodiglycollic acid, crystallising in blunt needles.

These experiments show that pure hydrobromate of dimethyl-thetine, when boiled with alcohol, is resolved, even at the comparatively low temperature, into thiodiglycollic acid, and bromide, and sulphide of methyl. The pungent

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[*] Thiodiglycollate of ethyl has been studied by E. SCHULZE ("Jenaische Zeitschr." i. 472, 477, 1864), who states that it may be prepared by the action of hydrochloric acid on an alcoholic solution of thiodiglycollic acid, and probably also by treating monochloracetic ether with sulphide of ammonium. According to SCHULZE, it is a colourless liquid, boiling with slight decomposition at 240–250° C., and has a feeble ethereal odour. Wishing to prepare some of the ether for purposes of comparison, I treated chloracetic ether with sulphide of potassium, and obtained a liquid which boiled constantly at 161–163° C., and which did not appear to suffer any decomposition when distilled. It possessed, however, all the properties of thiodiglycollic ether, and yielded thiodiglycollic acid by saponification with potash—precipitation with acetate of lead, and decomposition of the resulting lead salt by sulphuretted hydrogen.

liquids obtained by boiling the crude thetine compound with alcohol are most probably mixtures of bromacetic and dibromacetic ether; their odour supports this view, as does also the fact that they are readily acted on by ammonia.

The mean results of the determination of carbon and hydrogen in the compounds obtained by treating the oily liquids with ammonia, compare as follows with the numbers calculated for monobromacet-, dibromacet-, and thiodiglycol-amides :—

	I.	II.
Carbon, . . .	13·1	33·6
Hydrogen, . . .	1·9	6·0

	Calculated for $CH_2BrCONH_2$.	Calculated for $CHBr_2CONH_2$.	Calculated for $S(CH_2CONH_2)_2$.
Carbon, . . .	17·4	11·1	32·4
Hydrogen, . . .	2·9	1·4	5·4

I am, therefore, inclined to regard those liquids as consisting—the first of a mixture of monobromacet- and dibromacet- amides; the second of thiodiglycol-amide.

In conclusion, I may mention that I have endeavoured, without success, to obtain ethers of hydrobromate of dimethyl-thetine, or rather methyl or ethyl—bromates of dimethyl-thetine, by the action of methyl or ethyl alcohol on hydrobromate of dimethyl-thetine. An attempt to prepare the methyl-bromate by passing hydrochloric acid gas through a solution of the hydrobromate of dimethyl-thetine in pure methyl alcohol, and allowing the mixture to evaporate to dryness, led to the somewhat curious and unexpected result, that the bromine of the former was replaced by chlorine, and, therefore, that hydrochlorate of dimethyl-thetine remained.

An ethyl bromate of dimethyl-thetine has, however, since been obtained by the direct addition of bromacetic ether to sulphide of methyl.

In conclusion, I have to express my thanks to my assistant, Mr W. W. J. Nicol, and to my pupil, Mr J. E. Baker, for their assistance in the above investigation.

Action of Hydrocarbon Sulphides on Bromacetic Acid.

By Dr E. A. Letts.

(Sent for Publication August 19, 1878.)

Action of Sulphide of Benzyl on Bromacetic Acid.—The starting-point of these experiments was an observation made some time ago, that if benzyl sulphide and bromacetic acid be warmed together, and the resulting mixture allowed to cool, it remains liquid for weeks; whereas, if benzyl-thetine hydrobromate were formed it would certainly be a solid substance, and would crystallise out; while, on the other hand, if no action occurred, the mixture ought to solidify on cooling, as both its ingredients are solid substances at ordinary temperatures. The fact, then, that the mixture remained liquid indicated that a reaction had occurred, not analogous to that which takes place when a sulphide of the $(C_nH_{2n+1})_2S$ series acts on bromacetic acid.

That a reaction between sulphide of benzyl and bromacetic acid does occur, and not that the mixture of the two substances remains liquid simply from physical causes, soon became evident after another experiment had been tried with the two substances. This time a mixture of them (both specially purified and dried) was very gently warmed and agitated till a homogeneous liquid resulted; the heat required for this was very slight, the temperature not exceeding 80–60° C. On cooling, the liquid solidified to a mass of crystals, which no doubt consisted of a mixture of bromacetic acid and sulphide of benzyl. This solid mixture, however, in course of time gradually liquefied, and at the end of six or seven days was perfectly fluid; moreover, it no longer smelt of sulphide of benzyl, but possessed a powerful odour exceedingly irritating to the eyes and nostrils. The mixture was allowed to remain undisturbed for several weeks, and eventually warty crystals were deposited in it, which increased in quantity as the time went on.

In order to ascertain whether the reaction would occur more rapidly at a higher temperature, the two substances were boiled together for a few minutes and then allowed to cool; after a day or two's standing, crystals separated, and by continued boiling more and more of these were produced.

In order to ascertain whether they consisted of hydrobromate of dibenzyl-thetine, some of them were treated with water (in which they dissolved with great ease), and the solution was tested with nitrate of silver; a *white* precipitate resulted, apparently crystalline, and not at all resembling bromide of silver. This showed that they did not consist of a thetine compound.

It occurred to me that even if no thetine compound resulted, the mixture of sulphide of benzyl and bromacetic acid might behave as such when heated, or at least yield similar products, viz., bromide of benzyl and thiodiglycollic acid, as I have shown that the hydrobromate of dimethyl-thetine very readily splits under the action of heat into bromide of methyl, sulphide of methyl, and thiodiglycollic acid.

In order to ascertain whether this reaction really occurred, the mixture of solid and liquid products obtained by heating benzyl sulphide and bromacetic acid for some time, was repeatedly extracted with water. The residual oil had a pungent and irritating odour, which acted on the nostrils and eyes powerfully. It was distilled in a current of steam, separated from the water which accompanied it, and saponified with alcoholic potash solution. The mixture was then acidulated with nitric acid and yielded a copious precipitate with nitrate of silver, which was yellow, coagulated when shaken, and possessed the other properties of bromide of silver.

The aqueous washings employed to extract the products of the action of heat on the sulphide of benzyl and bromacetic acid were neutralised with caustic baryta solution, excess of baryta removed by a stream of carbonic anhydride, and the filtered solution evaporated and allowed to cool, when a barium salt separated out in characteristic radiating blunt needles. A determination of barium and water in this salt showed that it consisted of thiodiglycollate of barium—

	I.	II.	Calculated for $S(CH_2COO)_2Ba, 5H_2O$.
Water of Crystallisation, . .	24·00	...	24·0
Barium, ...	36·9	36·5	36·5

The crystals which separated after some weeks from the mixture of benzyl sulphide and bromacetic acid, which was only just sufficiently heated to fuse the two substances, also behaved as thiodiglycollic acid, yielding the very characteristic barium salt and the equally characteristic lead salt. This lead salt is precipitated in an amorphous flocculent condition by the addition of acetate of lead to solution of thiodiglycollic acid. On warming the mixture it dissolves, but is almost immediately replaced by a beautiful salt crystallising in glittering spangles.

These results show that when sulphide of benzyl and bromacetic acid are mixed together they yield thiodiglycollic acid and bromide of benzyl—in other words, that double decomposition occurs between them, the bromine of bromacetic acid becoming replaced by sulphur, and the sulphur of sulphide of benzyl by bromine. Thus—

$$2(CH_2—CO_2H)Br + (C_7H_7)_2S = 2C_7H_7Br + S(CH_2CO_2H)_2.$$

This reaction occurs rapidly when the two substances are heated together—slowly when they remain in contact at ordinary temperatures.

Action of Sulphide of Allyl on Bromacetic Acid.—The next experiments tried were with allyl-sulphide—the oil of garlic.

In a preliminary experiment, a little sulphide of allyl was warmed with bromacetic acid. The latter dissolved, but no separation of oily liquid occurred (characteristic of the production of a thetine compound), nor was any change apparent. After about a week, however (during which the mixture remained at ordinary temperatures), it became very syrupy, and warty crystals were deposited, which increased in quantity when the liquid was shaken. In a second experiment 6 grms. of allyl-sulphide (mol. wt. 114), and 7 grms. of bromacetic acid (mol. wt. 139), carefully dried between blotting paper, were placed in a test tube, and the mixture simply shaken, and not warmed. Most of the bromacetic acid dissolved, but some remained undissolved even after a day's contact with the sulphide. On the second day all had dissolved, and there remained two layers of liquid—the lower oily and slightly brown; the upper much more mobile and lighter in colour. In two or three more days the upper layer had nearly disappeared, being absorbed into the lower. After a week or two, crystalline nodules appeared in the liquid, and gradually increased in quantity. From time to time the liquid was poured off from these, and allowed to remain at rest, when more and more of the crystals separated.

After about five weeks it was judged that the reaction was complete.

A portion of the crystals which had separated out was washed with ether (to remove adhering sulphide of allyl and bromacetic acid), then dissolved in water, the solution neutralised with caustic baryta, and evaporated somewhat. On cooling, the characteristic barium salt separated out, which was identified as thiodiglycollate by a determination of barium—

·1213 grms. gave ·0748 sulphate of baryta = 36·3 per cent. barium.
$S(CH_2CO_2)_2Ba, 5H_2O$ requires . . . 36·5 „

Another portion of the crystals was dissolved in water, and precipitated with acetate of lead. On warming the mixture, the amorphous flocculent salt at first precipitated dissolved, but was rapidly replaced by crystalline spangles. This reaction, as before mentioned, is characteristic of thiodiglycollic acid. The lead was determined in the spangles by heating them with sulphuric acid—

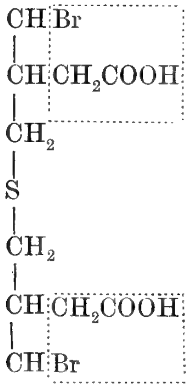
·135 grms. gave ·114 sulphate of lead = 57·6 per cent. lead
$S(CH_2CO_2)_2Pb$ requires . . . 58·2 „

The liquid products of the action of sulphide of allyl on bromacetic acid were distilled in a current of steam, and yielded at first an oily liquid heavier than water (bromide of allyl, sp. gr. 1·4)—later an oily liquid lighter than water (sulphide of allyl is lighter than water). These two were boiled for some time with solution of caustic baryta; excess of nitric acid was then added, and nitrate of silver solution, when an abundant yellow precipitate of bromide of silver was produced.

The reaction which occurs between sulphide of allyl and bromacetic acid is then of the same nature as that between sulphide of benzyl and bromacetic acid—viz., production of thiodiglycollic acid and bromide of the hydrocarbon radical—

$$(C_3H_5)_2S + 2(CH_2CO_2H)Br = 2C_3H_5Br + S(CHCO_2H)_2 .$$

Whether this reaction is preceded by the formation of a thetine compound has not as yet been ascertained. It might have been thought probable, considering the ease with which bromine quits bromacetic acid, that an addition product containing the triad radical glyceryl might be formed, namely—



but the experiments just described show that such a substance is not produced.

Action of Bromacetic Acid on Sulphide of Ethylene.—It occurred to me that it would be of interest to study the action of these two substances on each other, to ascertain whether a thetine compound, containing a diatomic hydrocarbon radical, was capable of existence, or at least of ready formation, and if not, to see what course the reaction would take.

Preliminary experiments showed that when the amorphous sulphide of ethylene is warmed with bromacetic acid, a brownish liquid results, which shows no tendency to crystallise on cooling, even though left to itself for some days.

On heating the mixture, a volatile liquid passes off, which is accompanied later by hydrobromic acid, and when the reaction appears to have terminated a brown syrupy residue remains.

These results induced me to study the action more carefully.

4·5 grms. of carefully washed and purified sulphide of ethylene, and 15 grms. of bromacetic acid carefully dried on blotting paper, were placed in a small distilling flask heated in an oil-bath and connected with a LIEBIG'S condenser. The oil-bath was heated to 150–170°.

The mixture soon fused, grew quite brown, then black; a heavy liquid passed over very gradually, and hydrobromic acid was disengaged in large quantity.

The experiment was stopped when the action appeared to have ceased, which only occurred after two or three hours. The volatile product which passed over was not quite homogeneous, but consisted of two distinct liquids,—one of which was present in large quantity, and was very heavy; the other floated as a distinct layer, and amounted only to a very small quantity. Both together weighed 6 grms.

The liquid present in largest quantity was bromide of ethylene. This was proved by its odour, high specific gravity, and by the fact that when mixed with alcoholic sulphide of potassium the mixture grew warm and the characteristic amorphous sulphide of ethylene was precipitated,—the aqueous washings from which yielded a copious yellow precipitate when treated with nitrate of silver, which was easily identified as bromide of silver.

The non-volatile products of the reaction solidified on cooling to a crystalline mass. This was treated with water, and yielded a claret-coloured solution, together with a considerable quantity of black unsoluble matter, which appeared to consist of charred products.

The colour of the solution was much altered by an alkali—becoming brown; but on the addition of acid to the alkaline solution the original colour was restored.

The whole of the solution was neutralised with baryta and evaporated down. During the evaporation brownish flakes were precipitated, and the colour of the solution became much lighter. As soon as the concentration was judged to be sufficient, it was allowed to cool and remain at rest for some time, when abundance of crystals having the appearance of thiodiglycollate of barium separated out. That these consisted of the thiodiglycollate was proved by a barium determination (36·6 per cent. of barium was found instead of 36·5 per cent., the theoretical quantity), and also by the production of the characteristic thiodiglycollate of lead, and a determination of lead in it (58·1 per cent. of lead was found instead of 58·2).

These results are sufficient to show that the action of sulphide of ethylene on bromacetic acid is of the same kind as the action of the sulphides of benzyl and allyl on bromacetic acid, and is, in fact, a case of double decomposition—

$$C_2H_4S + 2(CH_2CO_2H)Br = C_2H_4Br_2 + S(CH_2CO_2H)_2.$$

That an altogether different action, however, occurs at the same time was shown by the disengagement of hydrobromic acid in abundance (1 grm. escaped in the experiment described), by the colour of the solution of the non-volatile products, and by the fact that, along with the thiodiglycollate of barium, another and far more soluble salt was produced which crystallised from the solution after the thiodiglycollate had separated out.

Action of Iodacetic and of Bromacetic Ethyl Ether on Sulphide of Methyl.

By Dr E. A. Letts.

(Sent for Publication August 19, 1878.)

Among other experiments which were tried before the compounds of dimethyl-thetine had been investigated, was the action of iodacetic ethyl ether on sulphide of methyl, as it was thought that these two substances would yield an addition product more readily than bromacetic ethyl ether and sulphide of methyl. It was found, however, that the reaction took quite a different course—iodide of trimethyl-sulphine being produced in abundance.*

It seemed highly interesting to submit this reaction to a closer investigation, especially after the experiments on the action of bromacetic acid on hydrocarbon sulphides had shown that the bromine of the one and the sulphur of the other simply change places. It appeared to me to be of importance to ascertain whether the reaction in question was of a similar nature.

The iodacetic ethyl ether employed in these experiments was prepared from chloracetic ethyl ether by acting on it with iodide of potassium. Rather more than the equivalent quantity of the latter was dissolved in as small a quantity of water as possible, the solution largely diluted with alcohol, then mixed with the chloracetic ether, and the whole distilled from a water bath till no more liquid passed over. The residue in the retort was then mixed with water, and the crude iodacetic ether (which was always brown from the presence of free iodine) first decolorised with hyposulphite of soda solution and then washed with water.

A great many experiments were made with the iodacetic ethyl ether thus prepared and sulphide of methyl. The following is a summary of the method of investigation pursued, and of the results obtained:—

When iodacetic ethyl ether and sulphide of methyl are mixed, a reaction occurs almost immediately,—free iodine separates, colouring the solution brown, and soon dense oily drops collect. The reaction is so energetic, that, unless checked by immersing the vessel in which the experiment is made in cold water, the temperature may rise sufficiently high to cause the sulphide of methyl to boil off. The oily drops gradually accumulate to a layer of liquid which sinks to the bottom of the vessel. On leaving the whole to itself for a night, this oily layer is found to have solidified to a mass of crystals—in fact, the whole product is almost solid.

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* "Proceedings," 1873–1874, p. 220.

It was soon found that these crystals consisted of iodide of trimethyl-sulphine. This was proved by a determination of iodine in them after they had been purified, and by the characteristic manner in which they crystallised from alcohol.

In order to ascertain the nature of the other products of the reaction, the crude, semi-solid product was distilled from a water-bath, and yielded a liquid distillate in small quantity, which rapidly solidified to a mass of crystals of iodide of trimethyl-sulphine.

The residue was extracted with perfectly dry ether (in which iodide of trimethyl-sulphine is practically insoluble) until it was perfectly colourless, and the ethereal extract after filtration was distilled from a water-bath. This left a brown oily liquid, which was treated with hyposulphite of soda solution to remove free iodine, and then washed with water. Thus purified, it was still slightly brown.

When boiled with an aqueous solution of caustic baryta, the liquid product soon disappeared, showing that it consisted of an ether or of a mixture of ethers.

A considerable quantity of the product was thus saponified with caustic baryta, the excess of the latter removed by a stream of carbonic anhydride, and the solution filtered off. On evaporating a portion of it to small bulk it yielded a gummy mass, which showed no tendency to crystallise.

The whole of the solution (which had a pink colour) was evaporated somewhat on a water-bath, and then mixed with alcohol, when it grew turbid and deposited a crystalline salt in nodules. This crystalline salt dissolved with considerable difficulty on boiling it with water; the resulting solution, when allowed to evaporate in a desiccator, yielded crystals presenting the characteristic appearance of thiodiglycollate of barium, and which were proved by a determination of water and barium to consist of that substance—

·3899 grms. lost at 110° ·0907 grms. = 23·3 per cent.
·3899 gave ·2427 grms. sulphate of baryta = ·1427 grms. of barium = 36·6 „
Calculated for $S(CH_2—CO_2)_2Ba\,,5H_2O$ . . . . $\begin{cases} 24·0 \text{ „ water} \\ 36·5 \text{ „ barium.} \end{cases}$

It should be remarked that the thiodiglycollate was formed in comparatively small quantity.

The solution from which the thiodiglycollate had been separated was mixed with more alcohol, which, however, failed to precipitate any crystalline matter; it was then evaporated to dryness on a water bath, and the barium determined in the resulting gummy mass after it had been dried at 110° C.—

·3205 grms. gave ·2152 sulphate of baryta = ·12653 barium = 39·47 per cent.
·2882 „ ·1912 „ „ = ·1124 „ = 39·00 „

These numbers agree with the percentage of barium calculated for the anhydrous methyl-thioglycollate of barium—

$(CH_3—S—CH_2—CO_2)_2Ba$ requires 39·48 per cent. barium.

As no method of purifying this salt presented itself, it appeared desirable to saponify the ether with a base other than baryta, in order to obtain the corresponding salt for examination, and for that purpose caustic soda was employed, or rather a solution of sodium in alcohol. A quantity of the liquid product was boiled with this until the reaction ceased to be alkaline. The resulting solution contained a crystalline salt in suspension, which, no doubt, consisted of thiodiglycollate of sodium. It was filtered from this and evaporated to dryness on a water-bath, when it left a crystalline residue. This was treated with water and dissolved easily, leaving, however, a few oily drops, which probably consisted of unsaponified ether. The solution filtered from these was evaporated on a water bath to dryness, redissolved in alcohol, again evaporated to dryness, and the remaining salt dried at 110° C. till it ceased to lose weight.

The determination of sodium in the salt thus dried showed that it consisted of methyl-thioglycollate of sodium, $CH_3—S—CH_2—CO_2Na$.

·2025 gave ·1105 sulphate of sodium = ·0358 sodium = 17·7 per cent.
·2735 „ ·1510 „ „ = ·04891 „ = 17·5 „
$CH_3—S—CH_2—CO_2Na$ requires . . . . 17·8 „

Further proof of the presence of methyl-thioglycollate of ethyl in the product was afforded by the production of a crystalline compound when it was mixed with alcoholic corrosive sublimate solution.* The resulting compound was not, however, submitted to analysis.

The above experiments show that the action of iodacetic ethyl ether on sulphide of methyl is similar to the action of bromacetic acid on many hydrocarbon sulphides, that is to say, that the halogen and sulphur change places. The production of methyl-thioglycollate of ethyl as a far more abundant product than thiodiglycollate of ethyl shows, however, that the reaction occurs in two distinct stages, in the first of which only one of the methyl groups of sulphide of methyl is replaced by the group $CH_2—CO_2C_2H_5$, whilst in the second the remaining methyl group is similarly replaced. The action may be compared with that of iodacetic or bromacetic acid on ammonia, by which

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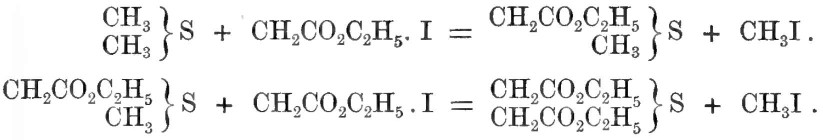
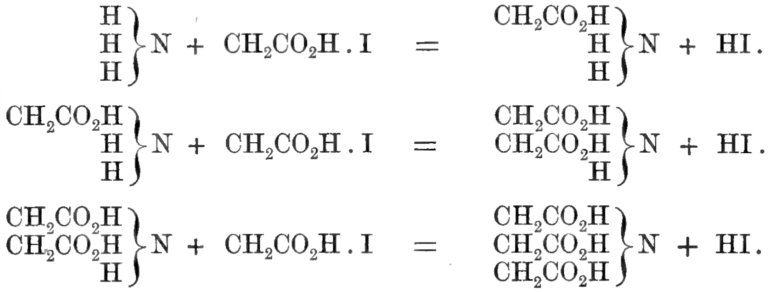
* J. Wislicenus, "Zeitsch. für Chem." 1865, p. 625, mentions a compound of thioglycollic ethyl ether and corrosive sublimate, to which he assigns the formula $C_4H_7SO_2.HgCl$. No doubt the compound obtained as above would have a similar composition.

glycocoll is formed as a first product, diglycol- and triglycol- amidic acids subsequently—



$$\begin{aligned}
&(1.)\quad \left.\begin{matrix}CH_3\\CH_3\end{matrix}\right\}S + CH_2CO_2C_2H_5.\,I = \left.\begin{matrix}CH_2CO_2C_2H_5\\CH_3\end{matrix}\right\}S + CH_3I\,.\\
&(2.)\quad \left.\begin{matrix}CH_2CO_2C_2H_5\\CH_3\end{matrix}\right\}S + CH_2CO_2C_2H_5\,.\,I = \left.\begin{matrix}CH_2CO_2C_2H_5\\CH_2CO_2C_2H_5\end{matrix}\right\}S + CH_3I\,.
\end{aligned}$$



$$\begin{aligned}
&(1.)\quad \left.\begin{matrix}H\\H\\H\end{matrix}\right\}N + CH_2CO_2H\,.\,I = \left.\begin{matrix}CH_2CO_2H\\H\\H\end{matrix}\right\}N + HI\,.\\
&(2.)\quad \left.\begin{matrix}CH_2CO_2H\\H\\H\end{matrix}\right\}N + CH_2CO_2H\,.\,I = \left.\begin{matrix}CH_2CO_2H\\CH_2CO_2H\\H\end{matrix}\right\}N + HI\,.\\
&(3.)\quad \left.\begin{matrix}CH_2CO_2H\\CH_2CO_2H\\H\end{matrix}\right\}N + CH_2CO_2H\,.\,I = \left.\begin{matrix}CH_2CO_2H\\CH_2CO_2H\\CH_2CO_2H\end{matrix}\right\}N + HI\,.
\end{aligned}$$

It is almost superfluous to remark that the iodide of trimethyl-sulphine, observed as a product of the reaction, owes its formation to the action of the iodide of methyl on the excess of sulphide of methyl taken; just as in the case of the action of ammonia on bromacetic or iodacetic acids, bromide or iodide of ammonium is produced by the action of the hydriodic or hydrobromic acid on the excess of ammonia taken.

The above reaction is one of many in which sulphide of methyl plays a similar part to that of a tertiary monamine or ammonia. The formation of the sulphine and thetine compounds are amongst these, as also the action of sulphide of methyl on iodide of acetyl, which gives rise to the formation of thiacetate and iodide of methyl.*

$$CH_3\text{—}CO\,.\,I + (CH_3)_2S = CH_3\text{—}CO\text{—}(SCH_3) + CH_3I\,.$$

This reaction being strictly comparable with that which occurs when ammonia acts on iodide of acetyl—

$$CH_3\text{—}CO\,.\,I + H_3N = CH_3\text{—}CO\text{—}(NH_2) + HI\,.$$

I may observe in conclusion that up to the present time I have not been able to satisfactorily explain the separation of a certain quantity of free iodine which always occurs when iodacetic ethyl ether and sulphide of methyl react on each other.*

Action of Bromacetic Ethyl Ether on Sulphide of Methyl.—Having ascertained that iodacetic ethyl ether forms no addition product with sulphide of methyl,

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* Cahours, "Comptes Rendus," lxxxi. (1875) 1164.

it seemed to be of importance to study the action of bromacetic ethyl ether on sulphide of methyl, to ascertain whether it would behave as the iodacetic ether or as bromacetic acid, and form an addition product.

Pure bromacetic ethyl ether was prepared by dissolving bromacetic acid in about twice the quantity of absolute alcohol required to form the ether, and then adding a quantity of oil of vitriol about equal in weight to the bromacetic acid taken. The mixture, after remaining for some time at rest, was thrown into a large excess of water, and the oily liquid which separated out washed with water and rectified. That portion which passed over from 159–162° C. was employed for experiment,—the boiling point of pure bromacetic ethyl ether being 159° C.

A preliminary experiment showed that a reaction occurred when the ether was mixed with sulphide of methyl, and that a solid product resulted.

25 grms. of the ether were mixed with 30 grms. of sulphide of methyl (equimolecular quantities of the two substances require for 25 grms. of the ether less than 10 grms. of sulphide of methyl, so that the latter was in large excess). The two substances mixed to a clear liquid, which, however, in a few seconds began to grow cloudy; oily drops gradually precipitated, and these increased in amount until more than a quarter of the whole was converted into a colourless heavy liquid, which remained as a distinct layer. After about an hour and a half an opaque colourless crystal appeared in this, and rapidly increased in size until in a few minutes the whole of the oily liquid was converted into a crystalline mass; and on allowing the mixture to remain undisturbed for a night the supernatant liquid had almost solidified from the presence of crystals.

These were detached from the flask in which the experiment had been made, washed six or seven times with sulphide of methyl, and then dried in vacuo.

When dry, they consisted of beautiful white scales,—very thin, and having a mother-of-pearl lustre,—which were exceedingly hygroscopic. They were very soluble in alcohol, and could not be readily recrystallised from it. Heated,

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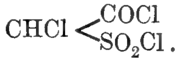
[*] Methyl-thioglycollic acid and its compounds have not, so far as I can ascertain, been previously prepared, though thioglycollic acid and the thioglycollates have been well studied.

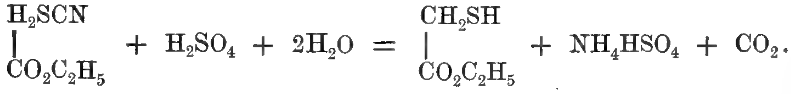
CARIUS ("Ann. der Chem. u. Pharm." cxxiv. 43) obtained the acid for the first time by the action of sulphydrate of potassium on chloracetic acid.

R. SIEMENS ("Berichte der Deutsch. Chem. Gesell." vi. 689) prepared the acid by the action of reducing agents on the body having the formula



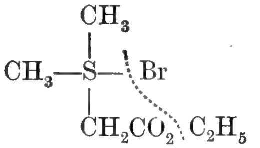
$$\mathrm{CHCl}<\begin{matrix}\mathrm{COCl}\\ \mathrm{SO_2Cl}\end{matrix}.$$

HEINTZ ("Ann. d. Chem. u. Pharm." cxxxvi. 223) prepared the ethyl ether by the action of boiling dilute sulphuric acid on the ethyl ether of sulphocyanate of glycolyl—



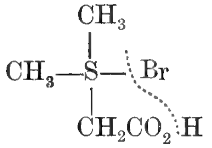
$$\begin{matrix}\mathrm{H_2SCN}\\ |\\ \mathrm{CO_2C_2H_5}\end{matrix} + \mathrm{H_2SO_4} + 2\mathrm{H_2O} = \begin{matrix}\mathrm{CH_2SH}\\ |\\ \mathrm{CO_2C_2H_5}\end{matrix} + \mathrm{NH_4HSO_4} + \mathrm{CO_2}.$$

they rapidly decomposed and gave off inflammable vapour. Owing to its deliquescence, the new substance was not analysed as such, but converted into platinum double salts, the analyses of which clearly prove it to consist of an addition product of sulphide of methyl and bromacetic ethyl ether, which may be called ethyl-bromate of dimethyl-thetine, just as the addition product of bromacetic acid and sulphide of methyl is called hydrobromate of dimethyl-thetine—



$$CH_3{-}\overset{\displaystyle CH_3}{\underset{\displaystyle CH_2CO_2C_2H_5}{S}}{-}Br$$

Ethyl-Bromate of Dimethyl-Thetine.



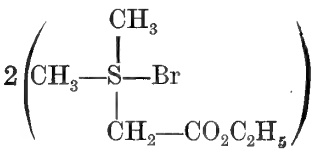
$$CH_3{-}\overset{\displaystyle CH_3}{\underset{\displaystyle CH_2CO_2H}{S}}{-}Br$$

Hydrobromate of Dimethyl-Thetine.

Chloro-Platinate of Ethyl-Bromate of Dimethyl-Thetine.—An aqueous solution of the ethyl-bromate yielded an abundant yellow crystalline precipitate with a solution of chloride of platinum. This was collected, washed with a little cold water, and recrystallised from hot water (in which it dissolved with difficulty). The recrystallised salt consisted of beautiful scales of a light orange colour.

A determination of platinum was made by calcining a weighed quantity of the salt—

·1450 grms. left ·0360 grms. = 24·8 per cent.

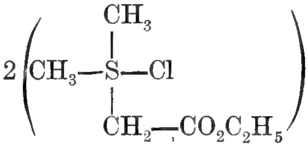


$$2\left(CH_3{-}\overset{\displaystyle CH_3}{\underset{\displaystyle CH_2{-}CO_2C_2H_5}{S}}{-}Br\right), PtCl_4 \text{ requires } 24·7 \quad „$$

A quantity of the ethyl-bromate of dimethyl-thetine was treated with excess of oxide of silver, and the solution filtered from the bromide of silver, which was at once produced; it was then mixed with chloride of platinum and concentrated on a water-bath; on cooling, a dark red salt separated out, containing water of crystallisation (which was partly lost in the desiccator). On recrystallisation from hot water its colour became lighter, and a platinum determination gave 28·02 per cent.

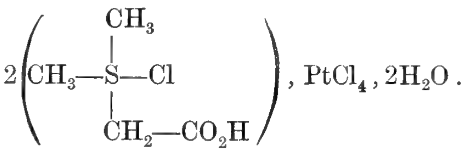
In another experiment similarly conducted, but in which the concentration lasted a longer time, a much lighter coloured salt was obtained, which closely resembled chloro-platinate of dimethyl-thetine in appearance, and which was found to contain 28·3 per cent. of platinum.

As the quantity of platinum calculated for the compound—



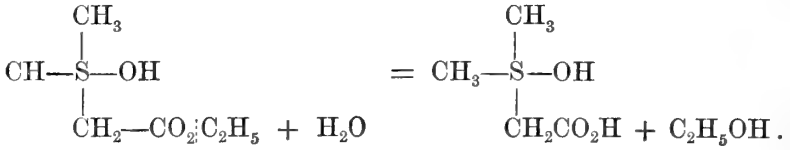
$$2\left(CH_3{-}\overset{\displaystyle CH_3}{\underset{\displaystyle CH_2{-}CO_2C_2H_5}{S}}{-}Cl\right), PtCl_4.$$

amounts to 27·8 per cent., whereas the chloro-platinate of dimethyl-thetine,—



$$2\left(\begin{array}{l} \quad\;\; CH_3 \\ \quad\;\;\; | \\ CH_3—S—Cl \\ \quad\;\;\; | \\ \quad\;\; CH_2—CO_2H \end{array}\right), PtCl_4, 2H_2O\,.$$

requires 28·6 per cent. of platinum, it is probable that the ethyl-hydrate of dimethyl-thetine produced by the action of oxide of silver on the ethyl-bromate is an unstable substance, which is readily resolved when heated with water into alcohol and the base dimethyl-thetine—



$$\begin{array}{l} \qquad\; CH_3 \\ \qquad\;\; | \\ CH—S—OH \\ \qquad\;\; | \\ \qquad\; CH_2—CO_2{:}C_2H_5 + H_2O \end{array} = \begin{array}{l} \qquad\quad CH_3 \\ \qquad\quad\; | \\ CH_3—S—OH \\ \qquad\quad\; | \\ \qquad\quad CH_2CO_2H + C_2H_5OH\,. \end{array}$$

The results of the above experiments show that the action of bromacetic ethyl ether on sulphide of methyl is of a totally different character from that of iodacetic ethyl ether, the former yielding an addition product with the greatest ease, while the latter yields no such a product. It appeared to be of some interest to complete the investigation by an experiment on the action of sulphide of methyl on chloracetic ether, and accordingly the experiment was made. A quantity of chloracetic ether was sealed up in a tube with about twice an equimolecular quantity of sulphide of methyl, the mixture heated in boiling water for a day or two, and then allowed to remain at rest for some weeks. When next examined the tube was found to contain abundance of colourless crystals, apparently blunt needles, arranged in radiating groups. Thinking that these probably consisted of chloride of trimethyl-sulphine, and that therefore the reaction had been of a kind similar to that which occurs when the iodacetic ether is employed, the tube was opened and some of the crystals (which were exceedingly deliquescent) removed, dissolved in water, and mixed with chloride of platinum solution. A light orange-coloured salt soon crystallised out in small plates. These were washed with cold water and dried. Two determinations of platinum gave 30·50 per cent. and 30·47 per cent., whereas the chloro-platinate of trimethyl-sulphine requires 34·9 per cent.

Curiously enough, the percentage of platinum required for the chloro-platinate of tri-*ethyl*-sulphine is 30·4 per cent., a number exactly agreeing with the amount yielded by the salt under examination.

It is almost inconceivable that chloride of tri-ethyl-sulphine should have been produced, and at present the nature of the reaction that occurs must remain in doubt, as the quantity of substance at my disposal was too small to admit of a further examination.

In conclusion, I take the opportunity of thanking my pupil Mr J. N. Collie for his assistance during this investigation.

XXII.—*A Class of Determinants.* By J. DOUGLAS HAMILTON DICKSON, M.A., Fellow and Tutor, Peterhouse, Cambridge.

Revised from a paper " A Continued Fraction" of date* 19*th February* 1877.

(Read 3d June 1878.)

(I.) It is proposed to consider determinants of the form

$$\begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_1, & a_2, & \ldots & a_r \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & b_1, & b_2, & \ldots & b_r \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & a_1, & a_2, & a_3, & \ldots & a_{r+1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & b_1, & b_2, & b_3, & \ldots & b_{r+1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_1, & a_2, & \ldots & & & & & & a_{r-2}, & a_{r-1}, & a_r, & \ldots & a_{2r-2} \\ b_1, & b_2, & \ldots & & & & & & b_{r-2}, & b_{r-1}, & b_r, & \ldots & b_{2r-2} \end{vmatrix}$$

and of the same form as this with the top horizontal and last vertical rows removed, in connection with the two series of quantities

$$\begin{array}{l} a_1, a_2, a_3, \ldots \\ b_1, b_2, b_3, \ldots \end{array}$$

(II.) From the two given sets of quantities

$$\begin{array}{l} a_1, a_2, a_3, \ldots \\ b_1, b_2, b_3, \ldots \end{array}$$

two new sets are deduced

$$\begin{array}{l} c_1, c_2, c_3, \ldots \\ d_1, d_2, d_3, \ldots \end{array}$$

by means of the auxiliary quantities

$$\begin{array}{l} \alpha_0, \alpha_2, \alpha_3, \ldots \\ \beta_0, \beta_2, \beta_3, \ldots \end{array}$$

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* I have to express my best thanks to Professor CAYLEY for his kind revision of this paper, and for his advice in accordance with which it was re-written and the forms of §§ II. and IV. entirely altered.

viz.,

$$\left.\begin{array}{l}\alpha_0 = \begin{vmatrix} a_1, & 1 \\ b_1, & 1 \end{vmatrix} = a_1 - b_1, \text{ and, } n = 2 \text{ or upwards, } \quad \alpha_n = \begin{vmatrix} a_1, & a_n \\ b_1, & b_n \end{vmatrix} \\[2ex] c_1 = \dfrac{\alpha_2}{\alpha_0} \quad , c_2 = \dfrac{\alpha_3}{\alpha_0}, \ldots\ldots\ldots\ldots \; c_n = \dfrac{\alpha_{n+1}}{\alpha_0} \end{array}\right\} \ldots \text{(A)}$$

$$\left.\begin{array}{l}\beta_0 = \begin{vmatrix} b_1, & 1 \\ c_1, & 1 \end{vmatrix} = b_1 - c_1, \text{ and, } n = 2 \text{ or upwards, } \quad \beta_n = \begin{vmatrix} b_1, & b_n \\ c_1, & c_n \end{vmatrix} \\[2ex] d_1 = \dfrac{\beta_2}{\beta_0} \quad , d_2 = \dfrac{\beta_3}{\beta_0}, \ldots\ldots\ldots\ldots \; d_n = \dfrac{\beta_{n+1}}{\beta_0} \end{array}\right\} \ldots \text{(B)}$$

Then in like manner from the sets (c, d) two new sets

$$\begin{array}{l} e_1, \; e_2, \; e_3, \; \ldots \\ f_1, \; f_2, \; f_3, \; \ldots \end{array}$$

by means of the auxiliary quantities

$$\begin{array}{l} \gamma_0, \; \gamma_2, \; \gamma_3, \; \ldots \\ \delta_0, \; \delta_2, \; \delta_3, \; \ldots \end{array}$$

viz.,

$$\left.\begin{array}{l}\gamma_0 = \begin{vmatrix} c_1, & 1 \\ d_1, & 1 \end{vmatrix} = c_1 - d_1, \text{ and, } n = 2 \text{ or upwards, } \quad \gamma_n = \begin{vmatrix} c_1, & c_n \\ d_1, & d_n \end{vmatrix} \\[2ex] e_1 = \dfrac{\gamma_2}{\gamma_0} \quad , e_2 = \dfrac{\gamma_3}{\gamma_0}, \ldots\ldots\ldots\ldots \; e_n = \dfrac{\gamma_{n+1}}{\gamma_0} \end{array}\right\} \ldots \text{(C)}$$

$$\left.\begin{array}{l}\delta_0 = \begin{vmatrix} d_1, & 1 \\ e_1, & 1 \end{vmatrix} = d_1 - e_1, \text{ and, } n = 2 \text{ or upwards, } \quad \delta_n = \begin{vmatrix} d_1, & d_n \\ e_1, & e_n \end{vmatrix} \\[2ex] f_1 = \dfrac{\delta_2}{\delta_0} \quad , f_2 = \dfrac{\delta_3}{\delta_0}, \ldots\ldots\ldots\ldots \; f_n = \dfrac{\delta_{n+1}}{\delta_0} \end{array}\right\} \ldots \text{(D)}$$

and so on indefinitely.

(III.) A few particular cases will present the question more clearly. We have

$$\alpha_n = \begin{vmatrix} a_1, & a_n \\ b_1, & b_n \end{vmatrix} \quad \ldots\ldots \quad (1)$$

Also

$$\beta_n = \begin{vmatrix} b_1, & b_n \\ c_1, & c_n \end{vmatrix}$$

or by equations (A),

$$\alpha_0\beta_n = \begin{vmatrix} b_1, & b_n \\ \alpha_2, & \alpha_{n+1} \end{vmatrix}$$

that is,

$$\alpha_0\beta_n = \begin{vmatrix} b_1 & , & b_n \\ \begin{vmatrix} a_1 , & a_2 \\ b_1 , & b_2 \end{vmatrix} & , & \begin{vmatrix} a_1 , & a_{n+1} \\ b_1 , & b_{n+1} \end{vmatrix} \end{vmatrix} = - \begin{vmatrix} \cdot & b_1 , & b_n \\ a_1 , & a_2 , & a_{n+1} \\ b_1 , & b_2 , & b_{n+1} \end{vmatrix} \quad . \quad . \quad (2)$$

Advancing in equations (2) each letter, *a*, *b*, α, β, to the next,

$$\beta_0\gamma_n = - \begin{vmatrix} \cdot & c_1 , & c_n \\ b_1 , & b_2 , & b_{n+1} \\ c_1 , & c_2 , & c_{n+1} \end{vmatrix}$$

which becomes, by one application of equations (A),

$$\alpha_0\beta_0\gamma_n = - \begin{vmatrix} \cdot & c_1 & , & c_n \\ b_1 , & b_2 & , & b_{n+1} \\ \begin{vmatrix} a_1 , & a_2 \\ b_1 , & b_2 \end{vmatrix} , & \begin{vmatrix} a_1 , & a_3 \\ b_1 , & b_3 \end{vmatrix} & , & \begin{vmatrix} a_1 , & a_{n+2} \\ b_1 , & b_{n+2} \end{vmatrix} \end{vmatrix} = - \begin{vmatrix} \cdot & \cdot & c_1 , & c_n \\ \cdot & b_1 , & b_2 , & b_{n+1} \\ a_1 , & a_2 , & a_3 , & a_{n+2} \\ b_1 , & b_2 , & b_3 , & b_{n+2} \end{vmatrix}$$

and by a second application

$$\alpha_0^2\beta_0\gamma_n = - \begin{vmatrix} \cdot & \cdot & \begin{vmatrix} a_1 , & a_2 \\ b_1 , & b_2 \end{vmatrix} , & \begin{vmatrix} a_1 , & a_{n+1} \\ b_1 , & b_{n+1} \end{vmatrix} \\ \cdot & b_1 , & b_2 , & b_{n+1} \\ a_1 , & a_2 , & a_3 , & a_{n+2} \\ b_1 , & b_2 , & b_3 , & b_{n+2} \end{vmatrix} = - \begin{vmatrix} \cdot & \cdot & a_1 , & a_2 , & a_{n+1} \\ \cdot & \cdot & b_1 , & b_2 , & b_{n+1} \\ \cdot & b_1 , & \cdot , & b_2 , & b_{n+1} \\ a_1 , & a_2 , & \cdot , & a_3 , & a_{n+2} \\ b_1 , & b_2 , & \cdot , & b_3 , & b_{n+2} \end{vmatrix}$$

or, subtracting the second top line from the third top line, and adding the second column from the left hand to the third column from the left hand,

$$\alpha_0^2\beta_0\gamma_n = - \begin{vmatrix} \cdot & \cdot & a_1 , & a_2 , & a_{n+1} \\ \cdot & \cdot & b_1 , & b_2 , & b_{n+1} \\ \cdot & b_1 & \cdot & \cdot & \cdot \\ a_1 , & a_2 , & a_2 , & a_3 , & a_{n+2} \\ b_1 , & b_2 , & b_2 , & b_3 , & b_{n+2} \end{vmatrix} = b_1 \begin{vmatrix} \cdot & a_1 , & a_2 , & a_{n+1} \\ \cdot & b_1 , & b_2 , & b_{n+1} \\ a_1 , & a_2 , & a_3 , & a_{n+2} \\ b_1 , & b_2 , & b_3 , & b_{n+2} \end{vmatrix} \quad . \quad . \quad (3)$$

In like manner the evaluation of δ_n may be deduced from γ_n ; thus

$$\beta_0^2\gamma_0\delta_n = c_1 \begin{vmatrix} \cdot & b_1 , & b_2 , & b_{n+1} \\ & c_1 , & c_2 , & c_{n+1} \\ b_1 , & b_2 , & b_3 , & b_{n+2} \\ c_1 , & c_2 , & c_3 , & c_{n+2} \end{vmatrix}$$

$$\therefore\ \alpha_0\beta_0^2\gamma_0\delta_n = c_1 \begin{vmatrix} \cdot & b_1 & b_2 & b_{n+1} \\ \cdot & c_1 & c_2 & c_{n+1} \\ b_1 & b_2 & b_3 & b_{n+2} \\ \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} & \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} & \begin{vmatrix} a_1 & a_4 \\ b_1 & b_4 \end{vmatrix} & \begin{vmatrix} a_1 & a_{n+3} \\ b_1 & b_{n+3} \end{vmatrix} \end{vmatrix} = -c_1 \begin{vmatrix} \cdot & \cdot & b_1 & b_2 & b_{n+1} \\ \cdot & \cdot & c_1 & c_2 & c_{n+1} \\ \cdot & b_1 & b_2 & b_3 & b_{n+2} \\ a_1 & a_2 & a_3 & a_4 & a_{n+3} \\ b_1 & b_2 & b_3 & b_4 & b_{n+3} \end{vmatrix}$$

Again

$$\alpha_0^2\beta_0^2\gamma_0\delta_n = -c_1 \begin{vmatrix} \cdot & \cdot & b_1 & b_2 & b_{n+1} \\ \cdot & \cdot & \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} & \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} & \begin{vmatrix} a_1 & a_{n+2} \\ b_1 & b_{n+2} \end{vmatrix} \\ \cdot & b_1 & b_2 & b_3 & b_{n+2} \\ a_1 & a_2 & a_3 & a_4 & a_{n+3} \\ b_2 & b_2 & b_3 & b_4 & b_{n+3} \end{vmatrix} = c_1 \begin{vmatrix} \cdot & \cdot & \cdot & b_1 & b_2 & b_{n+1} \\ \cdot & \cdot & a_1 & a_2 & a_3 & a_{n+2} \\ \cdot & \cdot & b_1 & b_2 & b_3 & b_{n+2} \\ \cdot & b_1 & \cdot & b_2 & b_3 & b_{n+2} \\ a_1 & a_2 & \cdot & a_3 & a_4 & a_{n+3} \\ b_1 & b_2 & \cdot & b_3 & b_4 & b_{n+3} \end{vmatrix}$$

which, by operations similar to those performed in evaluating γ_n, becomes

$$\alpha_0^2\beta_0^2\gamma_0\delta_n = c_1 \begin{vmatrix} \cdot & \cdot & \cdot & b_1 & b_2 & b_{n+1} \\ \cdot & \cdot & a_1 & a_2 & a_3 & a_{n+2} \\ \cdot & \cdot & b_1 & b_2 & b_3 & b_{n+2} \\ \cdot & b_1 & \cdot & \cdot & \cdot & \cdot \\ a_1 & a_2 & a_2 & a_3 & a_4 & a_{n+3} \\ b_1 & b_2 & b_2 & b_3 & b_4 & b_{n+3} \end{vmatrix} = b_1c_1 \begin{vmatrix} \cdot & \cdot & b_1 & b_2 & b_{n+1} \\ \cdot & a_1 & a_2 & a_3 & a_{n+2} \\ \cdot & b_1 & b_2 & b_3 & b_{n+2} \\ a_1 & a_2 & a_3 & a_4 & a_{n+3} \\ b_1 & b_2 & b_3 & b_4 & b_{n+3} \end{vmatrix} \quad . \quad . \quad (4)$$

In like manner from (4) we may write

$$\beta_0^2\gamma_0^2\delta_0\epsilon_n = c_1d_1 \begin{vmatrix} \cdot & \cdot & c_1 & c_2 & c_{n+1} \\ \cdot & b_1 & b_2 & b_3 & b_{n+2} \\ \cdot & c_1 & c_2 & c_3 & c_{n+2} \\ b_1 & b_2 & b_3 & b_4 & b_{n+3} \\ c_1 & c_2 & c_3 & c_4 & c_{n+3} \end{vmatrix}$$

Reducing the three lines containing c's at the same time, this becomes

$$\alpha_0^3\beta_0^2\gamma_0^2\delta_0\epsilon_n = c_1d_1 \begin{vmatrix} \cdot & \cdot & \cdot & \cdot & a_1 & a_2 & a_3 & a_{n+2} \\ \cdot & \cdot & \cdot & \cdot & b_1 & b_2 & b_3 & b_{n+2} \\ \cdot & \cdot & \cdot & b_1 & \cdot & b_2 & b_3 & b_{n+2} \\ \cdot & \cdot & a_1 & a_2 & \cdot & a_3 & a_4 & a_{n+3} \\ \cdot & \cdot & b_1 & b_2 & \cdot & b_3 & b_4 & b_{n+3} \\ \cdot & b_1 & \cdot & b_2 & \cdot & b_3 & b_4 & b_{n+3} \\ a_1 & a_2 & \cdot & a_3 & \cdot & a_4 & a_5 & a_{n+4} \\ b_1 & b_2 & \cdot & b_3 & \cdot & b_4 & b_5 & b_{n+4} \end{vmatrix}$$

or, as before,

$$\alpha_0^3\beta_0^2\gamma_0^2\delta_0\epsilon_n = c_1 d_1 \begin{vmatrix} \cdot & \cdot & \cdot & \cdot & a_1, & a_2, & a_3, & a_{n+2} \\ \cdot & \cdot & \cdot & \cdot & b_1, & b_2, & b_3, & b_{n+2} \\ \cdot & \cdot & \cdot & b_1, & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & a_1, & a_2, & a_2, & a_3, & a_4, & a_{n+3} \\ \cdot & \cdot & b_1, & b_2, & b_2, & b_3, & b_4, & b_{n+3} \\ \cdot & b_1, & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_1, & a_2, & a_2, & a_3, & a_3, & a_4, & a_5, & a_{n+4} \\ b_1, & b_2, & b_2, & b_3, & b_3, & b_4, & b_5, & b_{n+4} \end{vmatrix}$$

$$= b_1^2 c_1 d_1 \begin{vmatrix} \cdot & \cdot & a_1, & a_2, & a_3, & a_{n+2} \\ \cdot & \cdot & b_1, & b_2, & b_3, & b_{n+2} \\ \cdot & a_1, & a_2, & a_3, & a_4, & a_{n+3} \\ \cdot & b_1, & b_2, & b_3, & b_4, & b_{n+3} \\ a_1, & a_2, & a_3, & a_4, & a_5, & a_{n+4} \\ b_1, & b_2, & b_3, & b_4, & b_5, & b_{n+4} \end{vmatrix} \quad . \quad . \quad . \quad (5)$$

The method of reduction of determinants of the proposed forms is now apparent. I have, however, calculated ζ_n. Repeating the foregoing equations, (1) to (5), and adding the equation (6) for ζ_n, the series of the six equations is

$$\alpha_n = \begin{vmatrix} a_1, & a_n \\ b_1, & b_n \end{vmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\alpha_0\beta_n = - \begin{vmatrix} \cdot & b_1, & b_n \\ a_1, & a_2, & a_{n+1} \\ b_1, & b_2, & b_{n+1} \end{vmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\alpha_0^2\beta_0\gamma_n = b_1 \begin{vmatrix} \cdot & a_1, & a_2, & a_{n+1} \\ \cdot & b_1, & b_2, & b_{n+1} \\ a_1, & a_2, & a_3, & a_{n+2} \\ b_1, & b_2, & b_3, & b_{n+2} \end{vmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\alpha_0^2\beta_0^2\gamma_0\delta_n = b_1 c_1 \begin{vmatrix} \cdot & \cdot & b_1, & b_2, & b_{n+1} \\ \cdot & a_1, & a_2, & a_3, & a_{n+2} \\ \cdot & b_1, & b_2, & b_3, & b_{n+2} \\ a_1, & a_2, & a_3, & a_4, & a_{n+3} \\ b_1, & b_2, & b_3, & b_4, & b_{n+3} \end{vmatrix} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\alpha_0^3\beta_0^2\gamma_0^2\delta_0\epsilon_n = b_1^2 c_1 d_1 \begin{vmatrix} \cdot & \cdot & a_1, & a_2, & a_3, & a_{n+2} \\ \cdot & \cdot & b_1, & b_2, & b_3, & b_{n+2} \\ \cdot & a_1, & a_2, & a_3, & a_4, & a_{n+3} \\ \cdot & b_1, & b_2, & b_3, & b_4, & b_{n+3} \\ a_1, & a_2, & a_3, & a_4, & a_5, & a_{n+4} \\ b_1, & b_2, & b_3, & b_4, & b_5, & b_{n+4} \end{vmatrix} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$\alpha_0^3\beta_0^3\gamma_0^2\delta_0^2\epsilon_0\xi_n = - b_1^2 c_1^2 d_1 e_1 \begin{vmatrix} \cdot & \cdot & \cdot & b_1, & b_2, & b_3, & b_{n+2} \\ \cdot & \cdot & a_1, & a_2, & a_3, & a_4, & a_{n+3} \\ \cdot & \cdot & b_1, & b_2, & b_3, & b_4, & b_{n+3} \\ \cdot & a_1, & a_2, & a_3, & a_4, & a_5, & a_{n+4} \\ \cdot & b_1, & b_2, & b_3, & b_4, & b_5, & b_{n+4} \\ a_1, & a_2, & a_3, & a_4, & a_5, & a_6, & a_{n+5} \\ b_1, & b_2, & b_3, & b_4, & b_5, & b_6, & b_{n+5} \end{vmatrix} \quad . \quad . \quad . \quad . \quad (6)$$

where

$$\alpha_0 = a_1 - b_1\,, \quad \beta_0 = b_1 - c_1\,, \quad \gamma_0 = c_1 - d_1\,, \quad \&c.$$

From these six equations, (1) to (6), the determinants employed may be defined—as, for example, in (6)—by the symbol $\Delta_{7,\,n+2}$, 7 being the order of the determinant, and $n+2$ the suffix of the last constituent in its first line.

By working in like manner with the determinants $\Delta_{2r,\,n+r}$ and $\Delta_{2r+1,\,n+r}$, it can be shown that the law holds good generally.

(IV.) Writing $n = 2$ in the above equations, the determinants there written become of the forms of those proposed to be examined, and may be represented simply by Δ_m, where m is the order of the determinant.

Substituting for α_2, β_2, . . . their values from equations (A), (B), . . . the equations (1) to (6), with one more added for symmetry, become

$$\left.\begin{array}{rl} b_1 = & \Delta_1 \\ \alpha_0\, c_1 = & \Delta_2 \\ \alpha_0\, \beta_0\, d_1 = & -\,\Delta_3 \\ \alpha_0^2\, \beta_0\, \gamma_0\, e_1 = & \Delta_4 \,.\, b_1 \\ \alpha_0^2\, \beta_0^2\, \gamma_0\, \delta_0\, f_1 = & \Delta_5 \,.\, b_1\, c_1 \\ \alpha_0^3\, \beta_0^2\, \gamma_0^2\, \delta_0\, \epsilon_0\, g_1 = & \Delta_6 \,.\, b_1^2\, c_1\, d_1 \\ \alpha_0^3\, \beta_0^3\, \gamma_0^2\, \delta_0^2\, \epsilon_0\, \zeta_0\, h_1 = & -\,\Delta_7 \,.\, b_1^2\, c_1^2\, d_1\, e_1 \\ \&c. & \&c. \end{array}\right\} \quad . \quad . \quad . \quad (7)$$

where the sign depends only on the suffix of Δ, viz., where this is $\equiv 3$ (mod. 4) the sign is $-$, but otherwise $+$.

By division equations (7) give

$$\left.\begin{aligned}
\alpha_0 \frac{c_1}{b_1} &= \frac{\Delta_2}{\Delta_1} \\
\beta_0 \frac{d_1}{c_1} &= -\frac{\Delta_3}{\Delta_2} \\
\alpha_0 \gamma_0 \frac{e_1}{d_1} &= -\frac{\Delta_4}{\Delta_3} b_1 \\
\beta_0 \delta_0 \frac{f_1}{e_1} &= \frac{\Delta_5}{\Delta_4} c_1 \\
\alpha_0 \gamma_0 \epsilon_0 \frac{g_1}{f_1} &= \frac{\Delta_6}{\Delta_5} b_1 d_1 \\
\beta_0 \delta_0 \zeta_0 \frac{h_1}{g_1} &= -\frac{\Delta_7}{\Delta_6} c_1 e_1 \\
\&c. \qquad &\qquad \&c.
\end{aligned}\right\} \quad . \quad . \quad . \quad (8)$$

whence, if $\Delta_0 = 1$,

$$\left.\begin{aligned}
\beta_0 \frac{a_1 d_1}{b_1 c_1} &= -\frac{\Delta_0 \Delta_3}{\Delta_1 \Delta_2} a_1 \\
\gamma_0 \frac{b_1 e_1}{c_1 d_1} &= -\frac{\Delta_1 \Delta_4}{\Delta_2 \Delta_3} b_1 \\
\delta_0 \frac{c_1 f_1}{d_1 e_1} &= -\frac{\Delta_2 \Delta_5}{\Delta_3 \Delta_4} c_1 \\
\epsilon_0 \frac{d_1 g_1}{e_1 f_1} &= -\frac{\Delta_3 \Delta_6}{\Delta_4 \Delta_5} d_1 \\
\zeta_0 \frac{e_1 h_1}{f_1 g_1} &= -\frac{\Delta_4 \Delta_7}{\Delta_5 \Delta_6} e_1 \\
\&c. \qquad &\qquad \&c.
\end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

that is since $\beta_0 = b_1 - c_1$, $\gamma_0 = c_1 - d_1$, &c.

$$\left.\begin{aligned}
\frac{\Delta_0 \Delta_3}{\Delta_1 \Delta_2} &= \frac{d_1}{c_1}\left(\frac{c_1}{b_1} - 1\right) \\
\frac{\Delta_1 \Delta_4}{\Delta_2 \Delta_3} &= \frac{e_1}{d_1}\left(\frac{d_1}{c_1} - 1\right) \\
\&c. \qquad &\qquad \&c.
\end{aligned}\right\} \quad . \quad . \quad . \quad (10)$$

and so generally for $\frac{\Delta_m \quad \Delta_{m+3}}{\Delta_{m+1} \, \Delta_{m+2}}$.

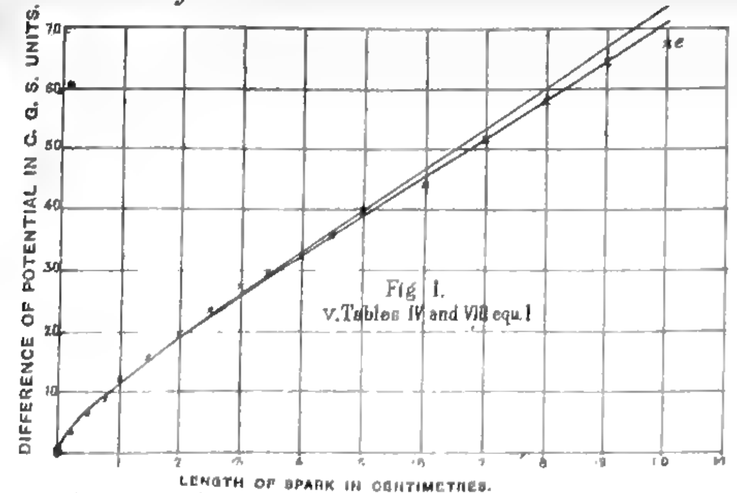


Fig 1.
v. Tables IV and VIII equ. 1

Dielectric, Air. Electrodes, Discs. Pressure of Atmosphere
Lower Curve is Curve of Observation. Upper Curve obtained by neglecting higher powers of S.

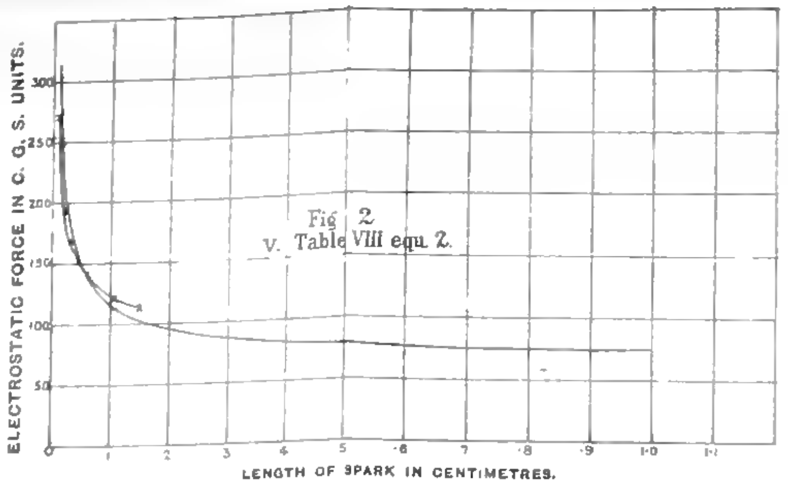


Fig 2.
v. Table VIII equ. 2.

Curve deduced from the Upper Curve of Fig. 1. Crosses mark values given by Sir William Thomson.

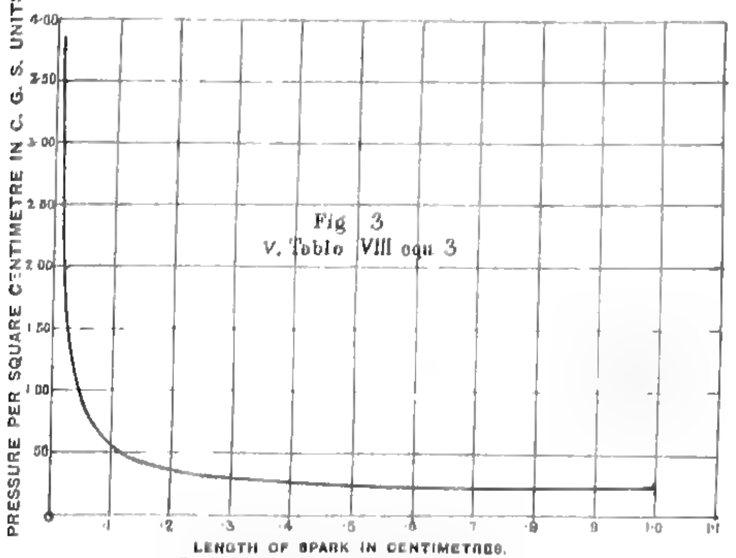


Fig 3
v. Table VIII equ. 3

Curve deduced from Upper Curve of Fig. 1

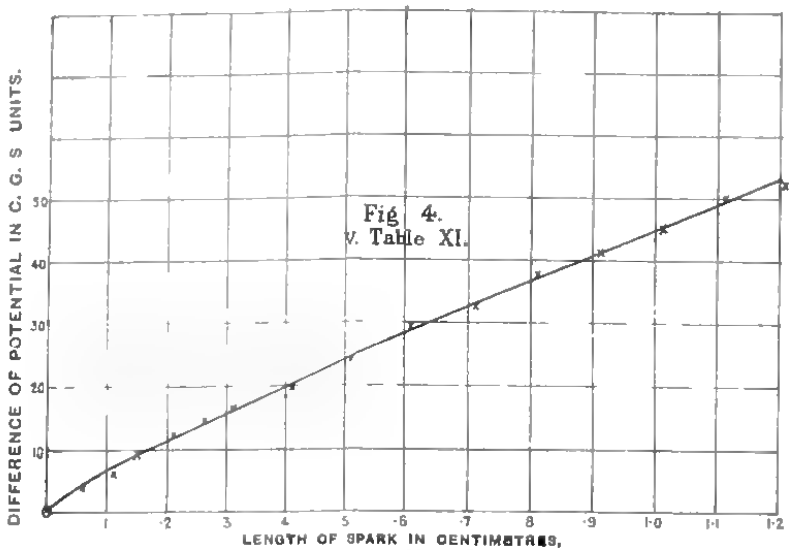


Fig 4.
v. Table XI.

Dielectric, Hydrogen. Electrodes, Discs. Pressure, Atmospheric.

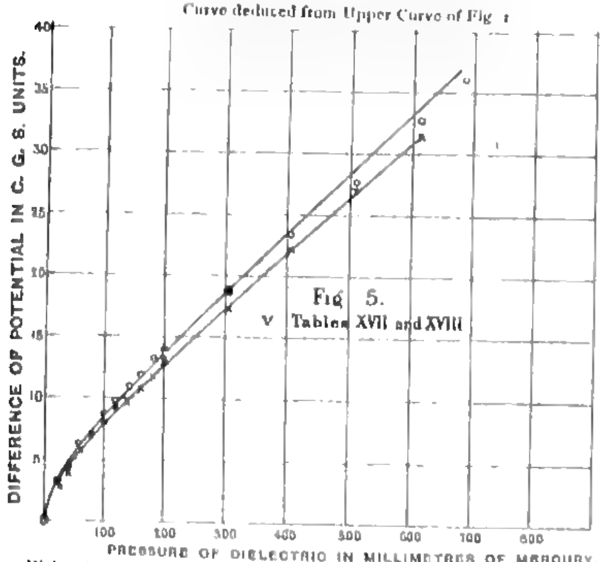


Fig 5.
v. Tables XVII and XVIII

Dielectric, Air. Electrodes, Discs. Length of Spark, ·5 centimetres.
Upper Curve with jars on. Lower with jars off.

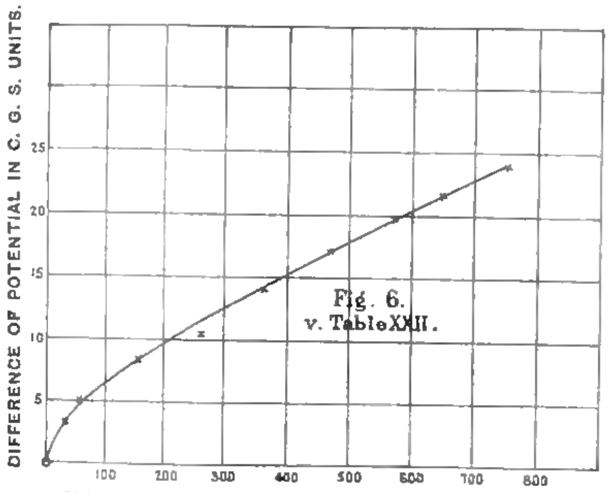


Fig. 6.
v. Table XXII.

Dielectric, Hydrogen. Electrodes, Discs. Length of Spark, ·5 centimetres.

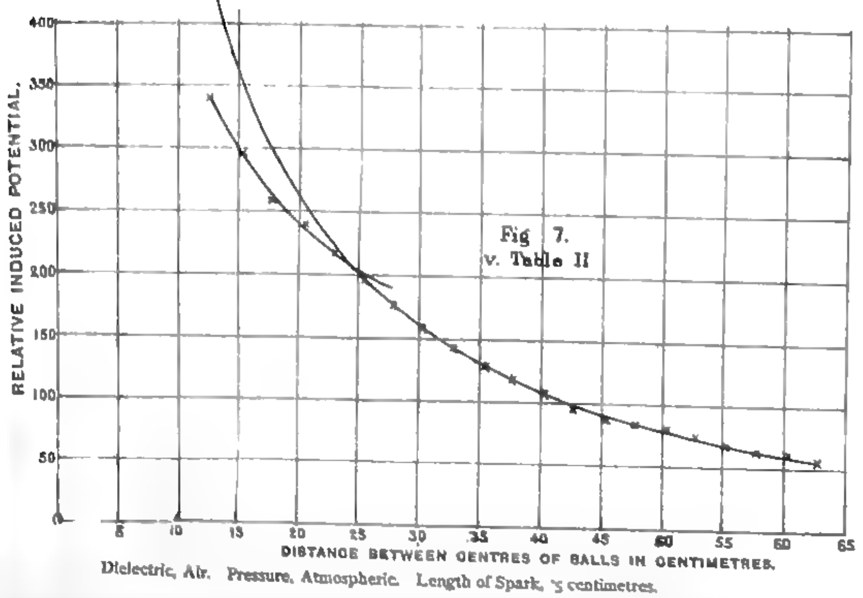


Fig 7.
v. Table II

Dielectric, Air. Pressure, Atmospheric. Length of Spark, ·5 centimetres.

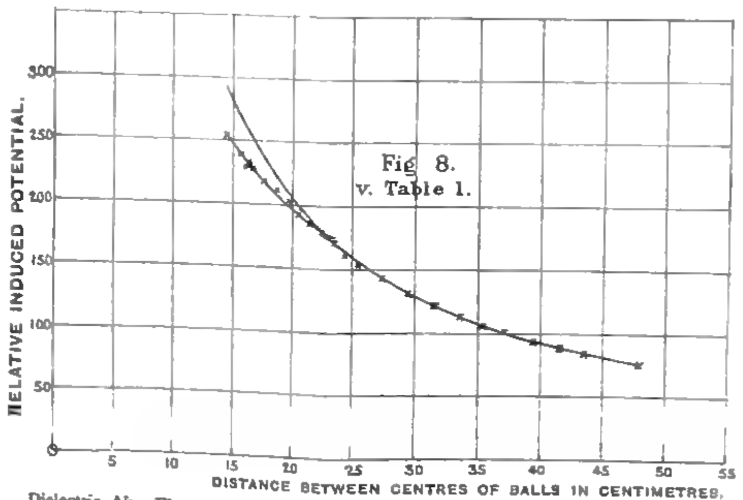


Fig 8.
v. Table 1.

Dielectric, Air. Electrodes, Balls. Pressure, 160 mm. Length of Spark, 6 centimetres.

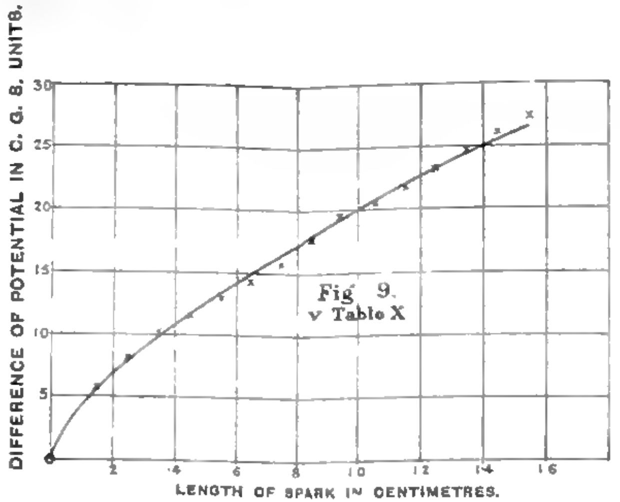


Fig 9.
v Table X

Dielectric, Air. Electrodes, Discs. Pressure, 180 mm.

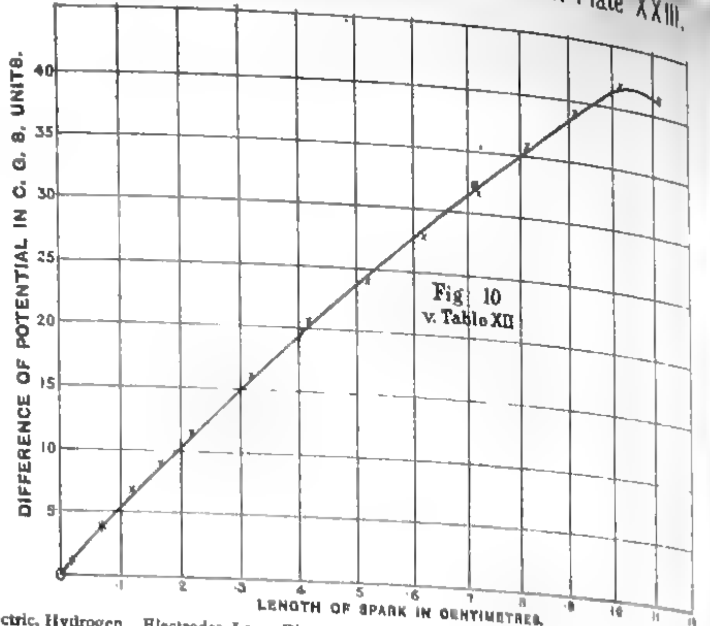


Fig 10
v. Table XII

Dielectric, Hydrogen. Electrodes, Large Discs. Pressure, Atmospheric. Discs heated before being put in

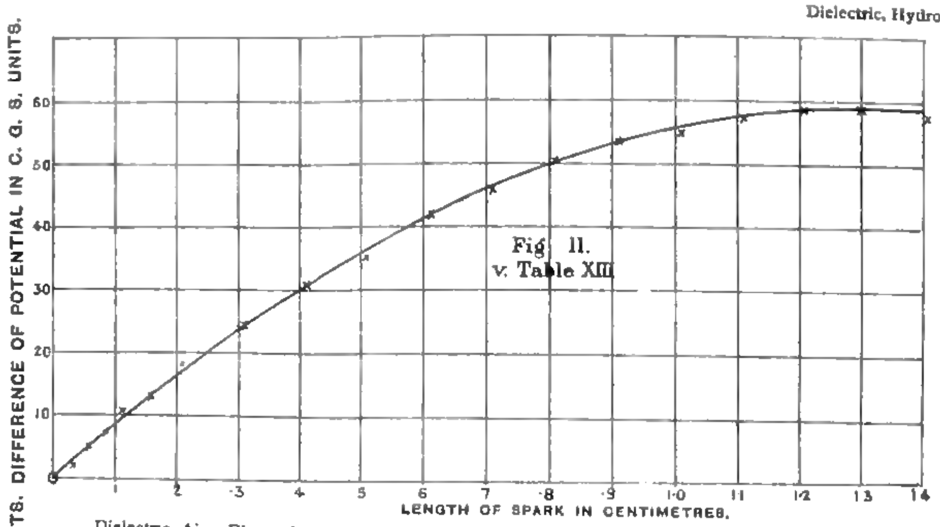


Fig 11.
v. Table XIII

Dielectric, Air. Electrodes, Discs. Pressure, Atmospheric. Discs and air inside receiver heated.

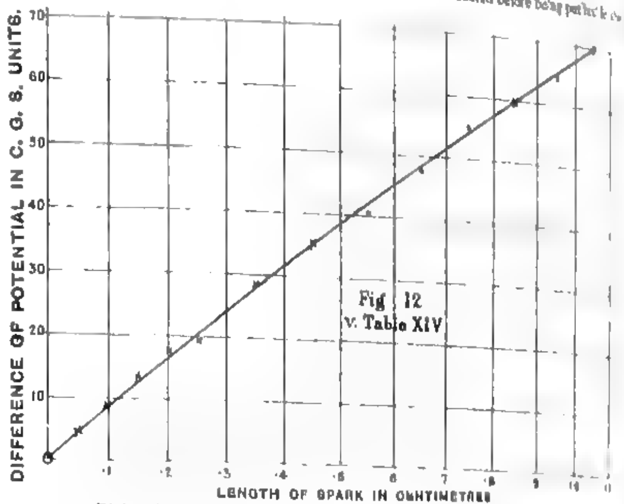


Fig 12
v. Table XIV

Dielectric, Air. Electrodes, Discs. Pressure, Atmospheric. Discs heated.

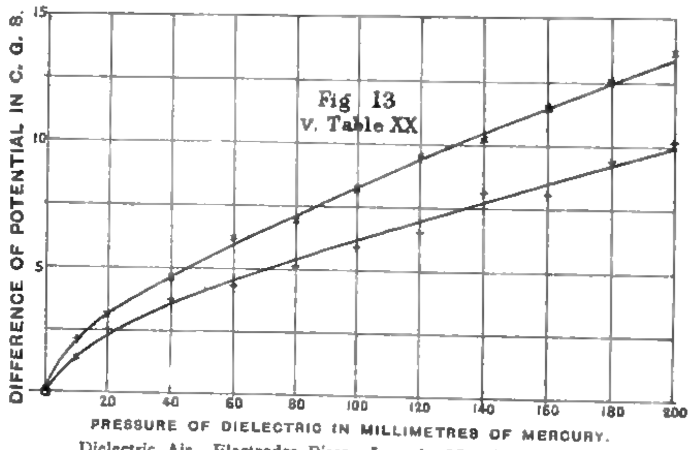


Fig 13
v. Table XX

Dielectric, Air. Electrodes, Discs. Length of Spark, ·5 centimetres
Upper Curve, Single Discharge. Lower Curve, Continued Discharge.

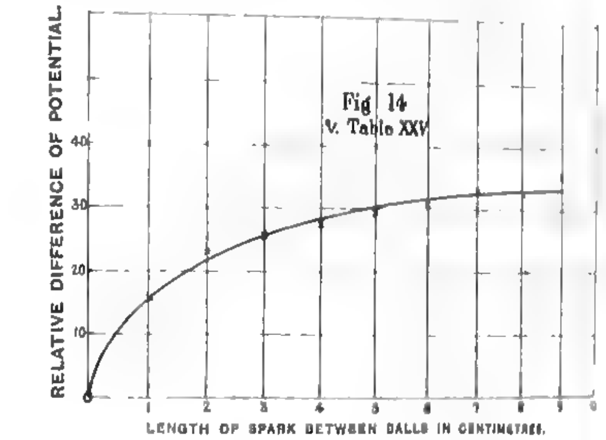


Fig 14
v. Table XXV

Dielectric, Air. Electrodes, Balls. Pressure, 40 mm.

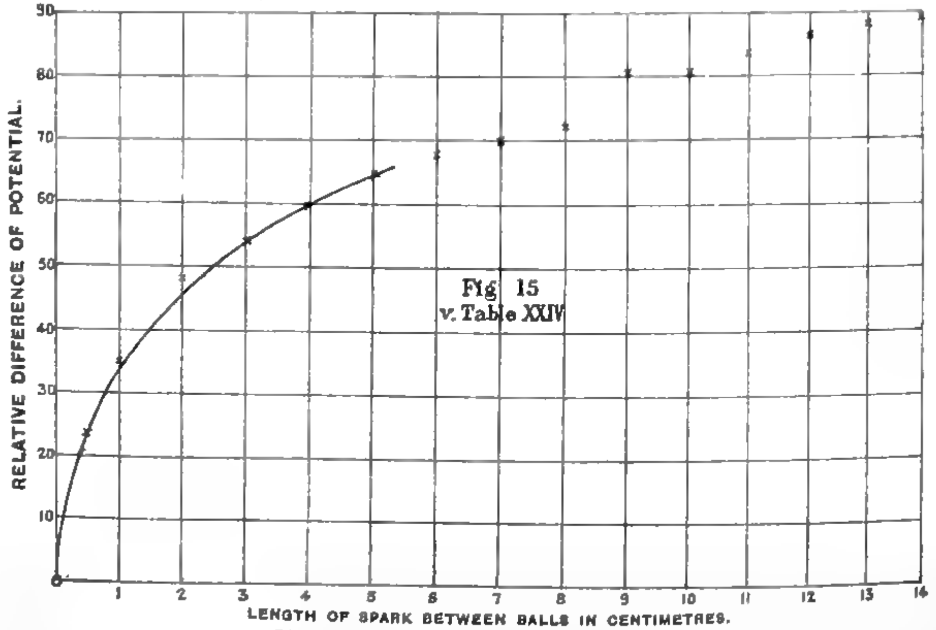


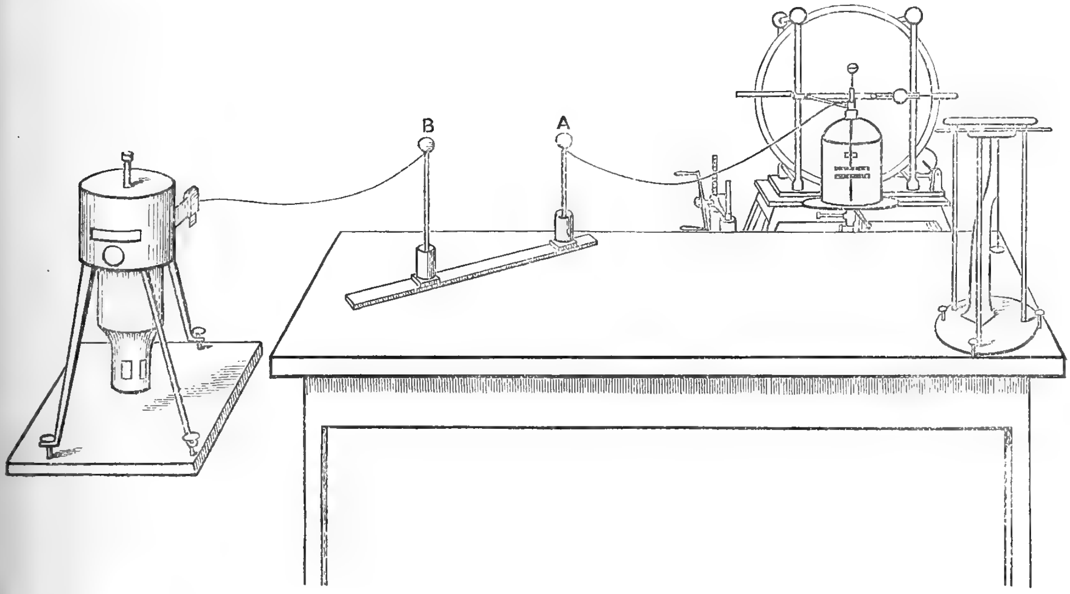
Fig 15
v. Table XXIV

Dielectric, Air. Electrodes, Balls. Pressure, 85 mm.

XXIII.—*On the Disruptive Discharge of Electricity: An Experimental Thesis for the Degree of Doctor of Science, Department A.* By Alexander Macfarlane, M.A., B.Sc. (Plate XXIII.)

(Read 18th February 1878.)

The experiments to which I shall refer were carried out in the physical laboratory of the University during the late summer session. I was ably assisted in conducting the experiments by three students of the laboratory,—Messrs H. A. Salvesen, G. M. Connor, and D. R. Stewart. The method which was used of measuring the difference of potential required to produce a disruptive discharge of electricity under given conditions, is that described in a paper communicated to the Royal Society of Edinburgh in 1876 in the names of Mr J. A. Paton, M.A., and myself,[*] and was suggested to me by Professor Tait as a means of attacking the experimental problems mentioned below.



The above sketch which I took of the apparatus *in situ* may facilitate the description of the method. The receiver of an air-pump, having a rod capable of being moved air-tight up and down through the neck, was attached to one of the conductors of a Holtz machine in such a manner that the conductor of the machine and the rod formed one conducting system. Projecting from the bottom of the receiver was a short metallic rod, forming one conductor

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[*] "Proc. R.S.E." vol. ix. p. 332.

with the metallic parts of the air-pump, and by means of a chain with the uninsulated conductor of the Holtz machine. Brass balls and discs of various sizes were made to order, capable of being screwed on to the ends of the rods. On the table, and at a distance of about six feet from the receiver, was a stand supporting two insulated brass balls, the one fixed, the other having one degree of freedom, viz., of moving in a straight line in the plane of the table. The fixed insulated ball A was made one conductor with the insulated conductor of the Holtz and the rod of the receiver, by means of a copper wire insulated with gutta percha, having one end stuck firmly into a hole in the collar of the receiver, and having the other fitted in between the glass stem and the hollow in the ball, by which it fitted on to the stem tightly. A thin wire similarly fitted in between the ball B and its insulating stem connected the ball with the insulated half ring of a divided ring reflecting electrometer.

Thus there were in all four conducting systems,—*First,* the interior surface of the room, with the uninsulated conductor of the Holtz, the lower electrode inside the receiver, the outside case and the uninsulated half of the divided ring of the electrometer. *Second,* the insulated conductor of the Holtz, with the rod of the receiver, the upper electrode, the covered wire, and fixed insulated ball. *Third,* the movable insulated ball, with the wire and insulated half of the divided ring of the electrometer. *Fourth,* the Leyden jar of the electrometer, with the vertical wire and horizontal needle. To keep these four systems of conductors insulated from one another was a condition essential to accuracy in the experiments. To satisfy this condition the two glass insulating stems A and B were thoroughly heated each morning before the fire, the upper part of the receiver was coated with shellac, while the lower part was coated in the inside with tinfoil, connected by a strip with the lower rod. The fourth system was generally charged before taking a series of observations by communicating to it a half-inch spark from the electrophorus. Its insulation was several times tested by taking two observations, distant by the lapse of twenty-four hours, of the reading at discharge under conditions constant, excepting as to the charge of the moving needle. I find that the readings on the 30th and 31st May for a spark ·5 centimetre long, and under given conditions, were 99 and 79·6 respectively; from which we may infer that four-fifths of the charge remained after a lapse of twenty-four hours.

When the potential of the second system was raised by driving the machine, the potential of the third was also raised; and this went on until a discharge took place between the electrodes inside the receiver, so that the deflection of the spot of light from zero was an indication of the difference of potential of the second and first systems when the discharge took place. By breaking the contact between the conductors of the Holtz machine before beginning to turn the wheel, and by turning the wheel slowly and steadily, we made the image of the wire move continuously, and to be at rest at the instant of discharge. After the discharge took place the image fell back and oscillated about zero or a point near it.

The force resisting the deflection of the mirror was the action of two external magnets upon several small magnets fixed to the back of the mirror. The two external magnets were at the beginning of the experiments properly placed on the top of the case, and kept by melted paraffin in the same positions throughout.

The distance between the electrodes, where the spark passed, was measured by resting a finely divided glass millimetre scale on the flat surface of the collar of the receiver, and adjusting the rod until a thin wire soldered to a ring forming the upper part of the rod coincided with a scratch on the scale. For some time we had no other means of inferring the pressure of the insulator inside the receiver than the indications of a gauge, the greatest range of which was 200 mms.; but finally we had a barometer tube in connection.

When the second system of conductors is charged to a positive potential, the potential of the third system is also positive, but necessarily less than that of the first. By increasing the distance between the balls A and B, the potential of the latter system may be reduced to any fraction of the potential of the former. Were this lessening not made, the potentials which formed the subject of the research would be too great to be measured by any electrometer. To ascertain minutely the law according to which the potential of the third system diminished when the distance of the ball B from the ball A was increased, I took the series of observations recorded in Tables I. and II., and plotted on figs. 7 and 8. The insulated balls used in the series of observations of 29th May were of unequal diameters—2·5 inches and 2 inches respectively; and they were supported by slightly dissimilar stems. They were replaced on the 12th June by the balls and stand represented in the sketch, and which were specially constructed for the purpose. The new balls are each of 2·25 inches diameter. The slide on which the ball B moves is graduated to half millimetres, so that its position during any series of observations can be accurately recorded. It was generally 42·75 centimetres, sometimes 32·75 centimetres.

In addition to the difference in the balls, the constants were very different, being in the one case the difference of potential required to pass a spark between two ball electrodes of one inch diameter through 6 centimetres of air at a pressure of 160 mm.; and in the other between two discs 4 inches diameter through ·5 centimetre of air at 740 mm. pressure. Yet the equations expressing the dependence of the potential of the third system upon the distance between the centres of the insulated balls, are very similar. In the case of the first curve, the equation $V = 4522{\cdot}6r^{-1} - 20{\cdot}24$, where V denotes the induced potential on the ball B, and r the distance of its centre from the centre of the ball A, satisfies all the observed values of r beyond 22·5 centimetres; while the equation

$$V = 2903{\cdot}5r^{-1} + 58{\cdot}073 - {\cdot}31807r,$$

got by multiplying the former function of r by $\cdot 642 + \frac{\cdot 11}{7} r$, satisfies all the observed values up to 22·5 centimetres; and in the case of the second curve, the equation

$$V = 6081\cdot 7 r^{-1} - 42\cdot 262$$

satisfies all the values of r beyond 24 centimetres; while the equation

$$V = 3188\cdot 4 r^{-1} + 98\cdot 32 - \cdot 83970 r,$$

got by multiplying the former function of r by $\cdot 52427 + \cdot 019822 r$, satisfies all the preceding observed values. The figures show these peculiarities well. As the curve is not discontinuous, the true function of r for V must coincide with the former function when r is large, and with the latter when r is small. The observations of 20th July, where a single observation was taken for each value of r, may be taken as showing the degree of precision finally arrived at in the experiments.

In the sketch there is represented on the corner of the table a long range absolute electrometer, which the Professor of Natural Philosophy procured on loan for me from Sir WILLIAM THOMSON, for the purpose of reducing the results to absolute measure. To provide for this reduction, I took each day four or five observations of the reading given when a spark passed between the two parallel metal discs at a distance of ·5 centimetre, the insulating air being at the pressure of the atmosphere. I compared the electrometers on five days by the following method:—I connected the insulated disc of the absolute electrometer with the insulated ball A, thus substituting the disc wire and ball for the second system of conductors. A charge was communicated to the substituted system sufficient to attract upwards the aluminium square of the balance; and the reading of the wire-image watched until from a slow escape of the charge along the insulating stems of the absolute electrometer, the attraction was just insufficient to keep the square in its sighted position. This method was found to give more consistent readings than when the insulating stems of the absolute electrometer were dried, and a coincidence obtained by moving the guard plate up or down. The observations on one of the days are given in Table XXVIII. The equation, by means of which the coefficient was deduced from the given data, is

$$V' - V = (D' - D) \sqrt{\frac{8\pi F}{A}} \cdot$$

The mean of eight independent determinations shows the potential required to pass a spark between the discs when at a distance of ·5 centimetre, and separated by air at atmospheric pressure to be 38·63 C.G.S. units.

The method above described has enabled me to investigate several of the laws of the disruptive discharge of electricity.

[*Added January* 1878.—In the tables appended we have entered almost all the observations made. The mode of treating the observations was as follows:—I plotted the arguments and entries of each series of observations on section paper, and drew the curve which best satisfied these points of observation. After comparing all the curves so obtained, which related to one question, I found an equation,

$$y' = f(x),$$

which satisfied a curve to a first approximation, and calculated out the values of y' for all the points of observation; then since

$$y = y'\phi(x),$$

where y denotes the most probable observed value,

$$\phi(x) = \frac{y}{y'},$$

$\phi(x)$ was obtained by plotting $\frac{y}{y'}$ on an enlarged scale, drawing a curve by the graphic method, and finding an equation for that curve.]

Measurement of the Difference of Potential required to pass a Spark through Air at the Atmospheric Pressure between Parallel Metal Plates at Different Distances.

This problem has been investigated for small distances by Sir WILLIAM THOMSON; and it was from a perusal of his paper on the subject, in "Papers on Electrostatics and Magnetism," and the discussion of these results in CLERK MAXWELL'S "Electricity," that I was led to take the above subject for an experimental inquiry. Tables III., IV., V., VI., VII., VIII., and figs. 1, 2, 3, give the observations and inferences on this problem. The large discs mentioned as forming the electrodes were each of brass, 4 inches diameter, well rounded at the edge, the one having its face flat, the other slightly convex. A spherometer, the side of the base of which is 2·5 inches, gave the height of the convex segment through its feet to be ·05 centimetre. The convex face belonged to the lower electrode. The diameter of the cylindrical part of the receiver used is 19 centimetres. The constants were the same for the five curves (Tables III. to VII.), excepting that the pressure and the temperature may have been slightly different. The maximum deflection could always be observed with precision, and the differences in the values for the same length of spark were probably due in great part to the spark not always passing exactly in the axis of the discs. The zero was always read after the second system of conductors had been completely discharged, which was always done after reading the maximum deflection. The values in the column headed Mean Difference of Potential are not always the mean of the values in the preceding column; but when not such, they are values got by concluding from independent evidence that greater weight ought to be attached to some of the entries than to the others. The entries to which greater weight has been attached are marked with a cross thus, ×. The 6th column gives the same quantity expressed in absolute measure, the 7th gives the value calculated from the equation at the top of the table, and the 8th gives the differences between the most probable observed value (column 6th) and the calculated value. The letter *e* denotes that the spark was observed to pass from the edge of the disc in a bent path, and not from the centre in a straight path. The series of readings was continued until the distance was reached at which the spark began to pass from the edge, but not further, as the spark then passes under new conditions. This distance was found to be 1 centimetre. In fig. 1 and in all the following figures the intersection of the cross marks the mean observed value, and the curve is drawn by means of the equation at the top of the table. The reading when the spark passed from the edge was invariably less than when central and straight.

The five equations agree well with one another. In the case of the first four the differences are negative at the beginning, but in the case of the fifth positive. The equation for the curve, Table VI., squared, is

$$V^2 = 3{\cdot}7142s^4 - 494{\cdot}68s^3 + 4348{\cdot}5s^2 + 889{\cdot}52s,$$

where V denotes the difference of potential of the discs, and s the length of the spark.

Here the terms involving s^4 and s^3 are small compared with the terms involving s^2 and s, so long as s is not greater than 1 centimetre, the distance at which the spark begins to pass from the edge. Their existence is therefore probably due to the want of infiniteness in the diameter of the discs compared with the greater lengths of spark. By neglecting these higher terms we get the following results:—

Table.	Function for V.	a.	b.
III.	$66{\cdot}687\sqrt{\{s^2+{\cdot}20393s\}}$	·10196	6·8000
IV.	$66{\cdot}510\sqrt{\{s^2+{\cdot}20305s\}}$	·10152	6·7523
V.	$67{\cdot}337\sqrt{\{s^2+{\cdot}20351s\}}$	·10175	6·8520
VI.	$65{\cdot}945\sqrt{\{s^2+{\cdot}20455s\}}$	·10227	6·7445
VII.	$68{\cdot}167\sqrt{\{s^2+{\cdot}21015s\}}$	·10507	7·1627
Mean,	$66{\cdot}940\sqrt{\{s^2+{\cdot}20503s\}}$	·10251	6·8623

Here a and b are the semi-axes of the hyperbola represented by the equation; a meaning the number of centimetres which must be supposed added to the length of spark to make the difference of potential proportional to the length of the spark, and b the amount of difference of potential due to such a distance. The values for a, being independent of the absolute value of the entries, afford a test of the accuracy of the experiments. The mean equation,

$$V=66{\cdot}940\sqrt{\{s^2+{\cdot}20503s\}},$$

is drawn on fig. 1, so that it may be compared with the equation which passes through the points of observation. Assuming, then, that the above mean equation is true when the discs are of infinite extent, the equation for the electrostatic force is

$$R=66{\cdot}940\sqrt{\{1+{\cdot}20503\tfrac{1}{s}\}},$$

where R denotes the electrostatic force, and s the length of the spark.

The values of R for several values of s are given in Table VIII., and the curve is drawn in fig. 2. Plotted along with it are several values for R, published by Sir WILLIAM THOMSON, p. 252 of "Papers." His measurements do not extend beyond 1·5 centimetre; the parts in common agree very well. From his observations, he concluded that the limiting value of R was something not much less than 130; the above equation gives 66·94.

According to the mathematical theory of electricity, if the conductors are in the form of two infinite parallel plates, then R is constant. The above function would give R constant were it not for the term involving s; in the above example $889{\cdot}52s$. It is probable, then, that there is a physical condition which the mathematical theorem in question assumes implicitly, but which is not fulfilled in the conditions of the experiment. Professor CLERK MAXWELL says with reference to this (p. 56 of "Electricity and Magnetism")—

"It is difficult to explain why a thin stratum of air should require a greater force to produce a disruptive discharge across it than a thicker stratum. Is it possible that the air very near to the surface of dense bodies is condensed, so as to become a better insulator? or does the potential of an electrified conductor differ from that of the air in contact with it by a quantity having a maximum value just before discharge, so that the observed difference of potential of the conductors is in every case greater than the difference of potentials on the two sides of the stratum of air by a constant quantity, equivalent to the addition of about ·005 of an inch to the thickness of the stratum?"

Several of the series of observations in the succeeding tables were made to decide between these two hypotheses; the results fully establish, in my judgment, the former. The quantity (·005 inch, that is, ·0127 centimetre) is considerably less than the quantity above denoted by a. But if the two curves for R, fig. 2, are similar, the equation for Sir WILLIAM THOMSON'S curve is

$$R=93{\cdot}322\sqrt{\{1+\frac{{\cdot}073718}{s}\}},$$

an equation satisfying well the given values, and according to it the value of a is ·037 centimetre.

The observations recorded in Table IX. were made with discs of much smaller diameter—1·5 inch. The value of a, which is ·28433 centimetre, is greater than the value obtained with the large discs. But observations with the small discs have not been made for as many lengths of spark as is desirable.

Table X. gives the results for the same conditions as in Tables III. to VII., excepting that the density of the air is much less. The equation obtained by neglecting the higher powers of s is

$$V = 18{\cdot}292\sqrt{\{s^2 + {\cdot}52322s\}},$$

which gives $a = {\cdot}26161$, and $b = 4{\cdot}7854$. Thus a is increased and b decreased. This leads us to infer that when the pressure is diminished the condensed stratum is also diminished—a conclusion which is supported by a peculiarity in Table XXIII., where, while the mean value for air before any exhaustion has been made is 152, the mean value after exhaustion thrice given is 142. a has always been found to be greater the smaller the pressure. The distance at which the spark was first observed to pass from the edge was 1·65 centimetre, compared with 1 centimetre for the ordinary pressure.

Table XI. (fig. 4) gives the results obtained when hydrogen was substituted for air. To effect this substitution I heated the brass discs thoroughly before the fire; then having screwed them on, I exhausted the air and let in hydrogen, repeating this process twice; but before taking any readings I left the hydrogen in for twenty-four hours. The curve through the readings is precisely similar to that for air, and the spark began to pass from the edge at very nearly the same distance. The equation obtained by neglecting the higher powers of s is

$$V = 43{\cdot}190\sqrt{\{s^2 + {\cdot}13690s\}},$$

giving a to be ·06845 centimetre. The ratio of the intercepts for hydrogen and for air is thus ·66, which is not very different from the ratio of their electric strengths[*]—·63 given in column 6th of Table XXIII.—and agrees exactly with the value given in column 7th. I made only one determination of the absolute value of the difference of potential required to give a spark through hydrogen under the standard conditions. The value obtained, 12·1, is very nearly half of that obtained from the value for air by means of the ratio of the electric strengths of the two gases. I have provisionally taken the latter value. The necessity for leaving the hydrogen in for a length of time before taking the readings becomes manifest when the above table is compared with Table XII., which gives the results when the observations were taken immediately after the gas was put in. The observations are plotted on fig. 10. The curve given by the equation

$$V = 53{\cdot}940s - 12{\cdot}9316s^2,$$

which represents a parabola, runs well through all the observed values. There

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[*] We have the high authority of Professor CLERK MAXWELL for using the term the electric strength of the gases to denote what has sometimes less properly been called their insulating power.

is then no term independent of s; and the coefficient of s, it may be observed, is somewhat greater than the $\frac{b}{a}$ of the equation

$$V=43{\cdot}190\sqrt{\{s^2+{\cdot}13690s\}}.$$

It thus appears that by heating the discs the cause of the existence of the term independent of s has been removed. Now, heating the discs removes condensed gas. The curvature involved in the parabola must be due to a decrease of temperature, as also to the increase of distance between the discs.

Table XIV. (fig. 12) contains the investigation for air under the same conditions. The results are precisely similar. The ratio of the coefficients of the equation for hydrogen is 4·17, while that of the coefficients of the equation for air is 4·45. The coefficient of s is somewhat greater than the $\frac{b}{a}$ of the equation

$$V=66{\cdot}940\sqrt{\{s^2+{\cdot}20503s\}}.$$

Their ratio is 1·30, while for hydrogen the ratio is 1·25. The investigation of Table XIII. (fig. 11) is the same as that of Table XIV., excepting that in heating the discs the air inside the receiver was also heated; whereas, in the case of Table XIV., only the discs were heated, a difference, in my opinion, sufficient to account for the difference in the magnitude of the coefficients. During this series of observations special notice was taken of the place where the spark passed. *c* denotes that it was at the centre, and *nc* that it was near the centre.

In Table XV. are observations undertaken to test whether a change in the capacity of the charged conductor has any effect upon the readings. The couple of small Leyden jars referred to are those usually hung on the conductors of the Holtz; they were always on when measurements were made, excepting in the case of Table XVIII. The capacity of the large jar when tested was found to be five times that of the couple of small jars. If the three last readings for the large jars are more correct than their predecessors, it would appear that the change of capacity has no effect upon the reading. But the difference of the curves of Tables XVII. and XVIII., if not otherwise explicable, supports the opposite conclusion.

Table XVI.—By "continued discharge," is meant a discharge which kept the spot of light at a fixed deflection. This deflection was always less than that for the corresponding single discharge. The image can be made to remain very steady with only a slight oscillation. The zero was more displaced than in the case of the single spark, owing probably to the greater amount of electrification introduced inside the electrometer. Between the fifth and sixth observations the apparatus was disturbed, which accounts for the larger differences. The observations of this table, taken together with those of Table XX., show that the curves for the continued discharge are similar to those for the single, but are of smaller magnitude.

Measurement of the Difference of Potential required to produce the same length of Spark at different pressures of the Dielectric.

In this independent variation we have not the complexity introduced by a change of distance between the electrodes. We accordingly have simpler equations; the simple hyperbola satisfies well all the points of observation.

The observations of Tables XVII. and XVIII. were taken in immediate succession. The latter give very accurately an hyperbola, the former require a small correction. I think that the latter curve was taken under more favourable auspices as regards the state of the observers; the readings in themselves were more difficult to take, on account of the slow velocity with which it was necessary to drive the machine owing to the small capacity of the conductor. The observations of Tables XIX. and XX. also give hyperbolas. The results are,—

Table.	Function for V.	*a*.	*b*.
XVII.	$\cdot04798\ \sqrt{\{p^2+205\cdot58p\}}$	102·79	4·6678
XVIII.	$\cdot044551\ \sqrt{\{p^2+200p\}}$	100	4·4551
XIX.	$\cdot046342\ \sqrt{\{p^2+199\cdot01p\}}$	99·51	4·6112
XX.	$\cdot046853\ \sqrt{\{p^2+207\cdot06p\}}$	103·53	4·8508
Mean	$\cdot045794\ \sqrt{\{p^2+202\cdot92p\}}$	101·46	4·6462

where a and b denote the semi-axes of the hyperbola, and p the pressure in millimetres of mercury. $\frac{b}{a}$ for XVIII., which is the curve obtained without Leyden jars, is smaller than for any other of the curves.

The observations for the continued discharge, Table XX., were taken each after the reading for the corresponding single discharge. By neglecting the higher terms of p, we get

$$V=\cdot035032\sqrt{\{p^2+205\cdot62p\}},$$

where V denotes the difference of potential of the discs, and p the pressure of the dielectric; which gives $a=102{\cdot}81$ compared with $103{\cdot}53$ for the single spark; and $b=3{\cdot}6016$ compared with $4{\cdot}8508$. The two curves are represented together on fig. 13.

Table XXI. contains an investigation of the same problem with this difference, that the length of spark was 1 centimetre, whereas in all the preceding series of observations it was ·5 centimetre. The equation is similar. The ratio of the $\frac{b}{a}$ of the ·5 centimetre curve to that of the 1 centimetre curve ought to be $\frac{38{\cdot}878}{70{\cdot}207}$, that is ·554. The mean equation and the equation of this table give ·568.

I regret that I did not take a series of observations in the reverse order, for it may be that the exhausting of the air before taking the observations took away the condensed air in great part. The curve for hydrogen instead of air (Table XXII. fig. 6) happened to be investigated in the reverse order. The mean points lie very accurately on an hyperbola, excepting that for 262, where there may have been an error in reading the number. The equation

$$V={\cdot}012\sqrt{\{p^2+600p\}}$$

gives $a=300$, and $b=3{\cdot}600$. The largeness of the a compared with that for air affords an additional ground for inferring that the order of procedure is not indifferent.

The Electric Strength of Different Gases.

Table XXIII.—The first values for air were taken before any exhaustion was made; and each of the observed differences of potential, with the exception of the second, the smallness of which is probably due to a mistake of 10 in reading, is decidedly greater than any afterwards observed for air. The less value of the subsequent readings for air must have been due to the exhaustions involved in the change of gas, which were always to 3 mm. or so. It does not seem due to any change of sensitiveness in the electrometer; for then there would have been a falling off in the third and fourth mean values. Hence 142 has been chosen, and not 152, as the proper reading for air, and the values given in column 6th are deduced from 142. Column 7th contains the mean of all other determinations made of the relative electric strength; and these means agree pretty closely with the values of column 6th. Column 8th contains FARADAY's values. (Section 1388 of "Experimental Researches in Electricity.") These numbers all agree in being less than the corresponding number in column 6th; but this is explained by the fact that FARADAY's number is the ratio of the thicknesses of the insulators when they are equally strong, not the ratio of the strengths of the insulators when they are equally thick. The one ratio is not equal to the other, unless the function connecting the difference of potential with the length of spark is that of the straight line; which was not the case under the conditions of his measurements.

Measurement of the Difference of Potential required to pass a Spark between two Spherical Balls at Different Distances.

Tables XXIV. to XXVII., and figs. 14 and 15.—The balls used as electrodes were each 1 inch in diameter, and when in use were screwed on to the rods. Observations were taken through a much greater range of distances than when the electrodes were discs, because no discontinuity was observed such as took place with the discs. I have not expressed the results in absolute measure in the tables, because the observations necessary for the immediate comparison were not taken, and also because there is need of another series of observations taken with the improvements in the apparatus introduced after these observations were made to decide definitely between various hypotheses which they suggest. In the case of Table XXIV., fig. 15, the equation was chosen to satisfy the observations between ·5 and 8, which it does very well; but it does not satisfy the rest of the observations. There certainly is a discontinuity in the observations, which was due to the commencement of an escape from the insulated wire. The four series of observations agree in giving the same kind of function for V. By making some assumptions the equations have been reduced to absolute measure, so that the relative magnitudes at least are correct.

Table.	Pressure.	Function for V.	Value of *s* for Maximum.
XXIV.	85	$10{\cdot}442s^{\frac{1}{2}} - {\cdot}36907s^{\frac{3}{2}}$	9·65
XXV.	40	$6{\cdot}8477s^{\frac{1}{2}} - {\cdot}26015s^{\frac{3}{2}}$	8·77
XXVI.	20	$4{\cdot}5069s^{\frac{1}{2}} - {\cdot}10043s^{\frac{3}{2}}$	14·96
XXVII.	30	$5{\cdot}7920s^{\frac{1}{2}} - {\cdot}17822s^{\frac{3}{2}}$	10·83

The values of *s* for a maximum coincide pretty closely with the values of *s*, at which the escape begins. This is seen well in fig. 15, where all the observed values up to the maximum lie well on the curve given by the function; one only is slightly out. These results confirm the conclusion that we inferred from our preliminary experiments of the summer of 1876, that the curve is to a first approximation a parabola.

In conclusion, our thanks are due to the Professor of Natural Philosophy for bearing the expense of what was new in the arrangement, and to the Professor of Chemistry for the loan of apparatus for the preparation of the gases.

TABLE I.—Induction Curve, 29th May 1877. Electrodes, balls. Pressure, 160 mm. Length of Spark, 6 centimetres.

Equ. I. $V = 4522{\cdot}6r^{-1} - 20{\cdot}24$. Equ. II. $V = 2903{\cdot}5r^{-1} + 58{\cdot}073 - {\cdot}31807r$.

Distance between Centres of Balls. r Centimetres.	Deflection. n.	Zero. n^1.	Observed Difference of Potential. $n^1 - n$.	Mean Difference of Potential. V^1.	Calculated Difference of Potential Equ. I. V.	Calculated Difference of Potential Equ. II. V.
14·5	230	484	254	253·7	291·66	253·70
	230	"	254			
	231	"	253			
15·5	248	"	236	238	271·54	240·47
	240	"	244			
	250	"	238			
16·5	254	"	230	229·3	253·86	228·79
	254	"	230			
	256	"	228			
17·5	270	"	214	219	238·19	218·42
	265	"	219 ×			
	265	"	219			
18·5	280	"	204	212	224·22	209·14
	270	"	214			
	267	"	217			
19·5	275	"	209	204	211·69	200·77
	280	"	204 ×			
	280	"	204			
20·5	290	"	194 ×	194	200·37	193·19
	285	"	199			
	284	"	200			
21·5	297	"	187 ×	187	190·11	186·28
	293	"	191			
	297	"	187			
22·5	306	"	178	178	180·76	179·96
	306	"	178			
	307	"	177			
23·5	310	"	174	172	172·21	174·15
	314	"	170			
	313	"	171			
24·5	318	"	166	161·3	164·36	...
	325	"	159			
	325	"	159			

TABLE I.—*continued.*

Distance between Centres of Balls. r Centimetres.	Deflection. n.	Zero. n^1.	Observed Difference of Potential. n^1-n.	Mean Difference of Potential. V^1.	Calculated Difference of Potential Equ. I. V.	Calculated Difference of Potential Equ. II. V.
25·5	334	484	150	155	157·11	...
	328	”	156			
	325	”	159			
27·5	335	”	149	144	144·22	...
	340	”	144 ×			
	340	”	144			
29·5	350	”	134	132	133·07	...
	352	”	132			
	354	”	130			
31·5	362	”	122	122·7	123·33	...
	360	”	124			
	362	”	122			
33·5	369	”	115	114	114·76	...
	370	”	114			
	372	”	112			
35·5	377	”	107	108	107·16	...
	375	”	109			
	376	”	108			
37·5	382	”	102 ×	102	100·36	...
	381	”	103			
	381	”	103			
39·5	392	”	92	93·3	94·25	...
	390	”	94			
	390	”	94			
41·5	399	”	85	88·7	88·74	...
	394	”	90			
	393	”	91			
43·5	399	”	85	84·7	83·73	...
	400	”	84			
	399	”	85			
48·0	410	”	74	74·7	73·98	...
	410	”	74			
	408	”	76			

TABLE II.—Induction Curve, 20th July 1877. Electrodes, large discs. Pressure, 740 mm. Length of Spark, ·5 centimetres.

Equ. I. $V=6081{\cdot}7r^{-1}-42{\cdot}262$. Equ. II. $V=3188{\cdot}4r^{-1}+98{\cdot}32-{\cdot}83970r$.

Distance between Centres of Balls in Centimetres. r.	Deflection. n.	Zero. n^1.	Observed Difference of Potential. n^1-n.	Calculated Difference of Potential Equ. I. V.	Calculated Difference of Potential Equ. II. V.
12·75	185	523	338	434·74	337·68
15·25	227	”	296	356·54	294·59
17·75	263	”	260	300·38	263·05
20·25	283	”	240	258·07	238·77

TABLE II.—*continued.*

Distance between Centres of Balls in Centimetres. *r*.	Deflection. *n*.	Zero. n^1.	Observed Difference of Potential. n^1-n.	Calculated Difference of Potential Equ. I. V.	Calculated Difference of Potential Equ. II. V.
22·75	305	523	218	224·45	219·00
25·25	328	526	198	199·15	203·39
27·75	347	523	176	176·90	189·92
30·25	363	"	160	158·79	...
32·75	380	"	143	143·44	...
35·25	393	"	130	130·27	...
37·75	405	525	120	118·85	...
40·25	417	"	108	108·84	...
42·75	428	"	97	100·00	...
45·25	436	"	89	92·14	...
47·75	440	"	85	82·21	...
50·25	444	"	81	78·77	...
52·75	450	"	75	73·03	...
55·25	456	"	69	67·82	...
57·75	462	"	63	63·05	...
60·25	464	524	60	58·68	...
62·75	470	"	54	54·66	...

TABLE III.—Distance Curve, 15th June 1877. Electrodes, large discs. Pressure, atmospheric.

Equation, $V=(67{\cdot}340-3{\cdot}2500s)\sqrt{\{s^2+{\cdot}2s\}}$.

Length of Spark in Centimetres. *s*.	Deflection. *n*.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Value in absolute measure. V.	Difference. V^1-V.
·05	473	500	27	27·3	7·479	7·512	−0·033
	475	"	27				
	476	504	28				
·1	463	505	42	42·3	11·589	11·608	−0·019
	463	"	42				
	462	"	43				
·2	436	"	69	68	18·630	18·862	−0·232
	435	"	70				
	440	"	65				
·3	408	"	97	93·3	25·561	25·702	−0·141
	415	507	92				
	418	509	91				
·4	388	"	121	120	32·876	32·352	+0·524
	390	510	120×				
	391	511	120				
·5	373	513	140	142·7	39·095	38·878	+0·217
	370	"	143				
	365	510	145				

TABLE III.—*continued.*

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Value in absolute measure. V.	Difference. V^1-V.
·6	350	515	165	164	44·931	45·304	−0·373
	354	514	160				
	350	517	167				
·7	330	„	187	189	51·780	51·640	+0·140
	330	523	193				
	330	516	186				
·8	310	522	212	211	57·807	57·904	−0·097
	312	523	211				
	315	525	210				
·9	300	523	223	227	62·192	64·091	−1·899
	308	525	217				
	303	530	227 ×				
1·0	283	533	250	249·3	68·300	70·207	−1·907
	286	536	250				
	290	538	248				

TABLE IV.—Distance Curve, 9th July 1877. Electrodes, large discs. Pressure, 750 mm. Temperature, 73° Fahrenheit.

Equation, $V=(67·015-2·5142s)\sqrt{\{s^2+·2s\}}$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Value in absolute measure. V.	Difference. V^1-V.
·025	502	512	10	10	3·75	5·02	−1·27
	502	„	10				
	502	„	10				
·05	495	„	17	18	6·75	7·48	−0·73
	493	„	19				
	494	„	18				
·075	488	„	24	24	9·00	9·60	−0·60
	488	„	24				
	488	„	24				
·1	479	„	33 ×	33	12·37	11·56	+0·81
	479	„	33				
	478	„	34				
·15	470	„	42	42	15·75	15·27	+0·48
	470	„	42				
	470	„	42				
·2	462	„	50	51·7	19·39	18·81	+0·58
	460	„	52				
	459	„	53				
·25	448	„	64	63	23·62	22·27	+1·35
	448	„	64				
	451	„	61				

TABLE IV.—*continued.*

Length of Spark in Centimetres. *s.*	Deflection. *n.*	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Value in absolute measure. V.	Difference. $V^1 - V$.
·3	436	512	76	74	27·75	25·66	+2·09
	438	”	74 ×				
	434	”	78				
·35	430	514	84	79	29·63	29·02	+0·61
	430	”	84				
	435	”	79 ×				
·4	428	”	86	86	32·25	32·34	−0·09
	430	516	86				
	430	”	86				
·45	422	”	94	95	35·62	35·63	−0·01
	421	”	95				
	421	”	95				
·5	410	”	106	105	39·38	38·90	+0·48
	412	518	106				
	415	”	103				
·6	408	”	110	118	44·25	45·38	−1·13
	400	”	118 ×				
	400	”	118				
·7	378	517	139	138·3	51·87	51·80	+0·07
	378	”	139				
	380	”	137				
·8	367	”	150	154·7	58·02	58·14	−0·12
	366	520	154				
	365	525	160				
·9	357	528	171	172	64·50	64·43	+0·07
	355	525	170				
	350	”	175				
1·0	*e* 332	510	178	179·3	67·25	70·66	−3·41
	e 330	515	185				
	e 340	”	175				

TABLE V.—Distance Curve, 14th June 1877. Electrodes, large discs. Pressure, atmospheric. Temperature, 74° Fahrenheit.
Equation, $V = \{67·925 - 2·9303s\} \sqrt{\{s^2 + ·2s\}}$.

Length of Spark in Centimetres. *s.*	Deflection. *n.*	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$,	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. $V^1 - V$.
·05	480	510	30	30	5·88	7·58	−1·70
	480	”	30				
	480	”	30				
·1	450	505	55 ×	55	10·78	11·71	−0·93
	452	”	53				
	452	”	53				
·2	415	”	90	90	17·65	19·05	−1·40
	415	”	90				
	415	”	90				

TABLE V.—*continued.*

Length of Spark in Centimetres. *s.*	Deflection. *n.*	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. V^1-V.
·3	377	510	133	134	26·27	25·97	+0·30
	390	512	122				
	380	515	135				
·4	350	522	172	167	32·74	32·70	+0·04
	337	503	166				
	347	510	163				
·5	313	512	199	199·3	39·08	39·33	−0·25
	313	512	199				
	316	516	200				
·6	288	520	232	234	45·88	45·84	+0·04
	288	523	235				
	290	525	235				
·7	257	527	270	269·3	52·81	52·29	+0·52
	260	530	270				
	265	533	268				
·8	255	540	285	296·5	58·14	58·66	−0·52
	245	545	300 ×				
	255	548	293 ×				
·9	233	552	319	323	63·34	64·96	−1·62
	240	557	317				
	235	568	333				
1·0	215	573	358	363	71·18	71·20	−0·02
	212	578	366				
	215	580	365				
1·1	*e* 186	”	394	394	77·26	77·37	−0·11

TABLE VI.—Distance Curve, 17th July 1877. Electrodes, large discs. Pressure, 737 mm.

Equation $V=\{66{\cdot}69-3{\cdot}7142s\}\sqrt{\{s^2+{\cdot}2s\}}$.

Length of Spark in Centimetres. *s.*	Deflection. *n.*	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. V^1-V.
·06	479	508	29	29	8·24	8·32	−0·08
	480	509	29				
·11	466	510	44	43·5	12·35	12·24	+0·11
	467	”	43				
·21	441	”	69	68	19·31	19·34	−0·03
	443	”	67				
·31	415	512	97	96	27·27	26·06	+1·21
	418	513	95				
·41	400	”	113	112·5	31·95	32·59	−0·64
	398	510	112				
·51	370	”	140	138	39·19	38·99	+0·20
	374	”	136				

TABLE VI.—*continued.*

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. V^1-V.
·61	352	510	158	156	44·31	45·29	−0·98
	358	512	154				
·71	327	"	185	183·5	52·12	51·49	+0·63
	330	"	182				
·81	310	"	202 ×	202	57·37	57·60	−0·23
	320	517	197				

TABLE VII.—Distance Curve, 13th June 1877. Electrodes, large discs. Pressure, atmospheric. Equation $V=(69{\cdot}875-8{\cdot}9375s)\sqrt{\{s^2+{\cdot}2s\}}$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Value in absolute measure. V.	Difference. V^1-V.
·1	460	505	45 ×	45	13·37	11·95	+1·42
	458	"	47				
	460	"	45				
·2	437	"	68	67	19·91	19·26	+0·65
	438	"	67 ×				
	438	"	67				
·3	415	"	90	89	26·45	26·02	+0·43
	418	"	87				
	414	"	91				
·4	403	"	102	105	31·20	32·48	−1·28
	400	"	105 ×				
	402	"	103				
·5	370	"	135	132	39·22	38·69	+0·35
	375	"	130				
	375	"	130				
·6	354	"	151	150	44·58	44·69	−0·11
	356	"	149				
	355	"	150				
·7	334	"	171	173	51·41	50·50	+0·91
	330	"	175				
	332	"	173				
·8	318	"	187	189	56·16	56·10	+0·06
	314	"	191				
	315	"	190				
·9	297	"	208	207·3	61·60	61·52	+0·08
	296	"	209				
	300	"	205				
1·0	280	"	225	226	67·16	66·75	+0·41
	280	"	225				
	276	"	229				
1·1	*e*268	"	237	239	71·02	71·80	−0·78
	*e*265	"	240				
	*e*265	"	240				

TABLE VIII.—Deduced Curves.

Difference of Potential Curve, $V = 66{\cdot}940\sqrt{\{s^2 + {\cdot}20503s\}}$.

Electrostatic Force Curve, $R = 66{\cdot}940\sqrt{\left\{1 + \frac{{\cdot}20503}{s}\right\}}$.

Pressure Curve, $p = {\cdot}18168\left\{1 + \frac{{\cdot}20503}{s}\right\}$.

Length of Spark in Centimetres. s.	Difference of Potential in C. G. S. units. V.	Electrostatic Force. $R = \frac{V}{s}$.	Pressure. $p = \frac{R^2}{8\pi \times 981{\cdot}4}$.
·025	5·076	203·05	1·3470
·05	7·559	151·18	·9267
·075	9·701	129·35	·6783
·1	11·691	116·91	·5542
·2	19·052	95·26	·3679
·3	26·056	86·85	·3058
·4	32·932	82·33	·2748
·5	39·745	79·49	·2562
·6	46·524	77·54	·2437
·7	53·281	76·11	·2349
·8	60·024	75·03	·2282
·9	66·756	74·17	·2231
1·0	73·484	73·48	·2189

TABLE IX.—Distance Curves, 4th June 1877. Electrodes, small discs. Pressure, atmospheric.

Equation, $V = 52{\cdot}840\sqrt{\{s^2 + {\cdot}56866s\}}$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential. V.	Difference. $V^1 - V$.
·25	468	498·5	30·5	28·5	24·74	23·91	+0·83
	470	”	28·5 ×				
	470	”	28·5				
	470	”	28·5				
·5	454	”	44·5	44·5	38·63	38·63	0·00
	454	”	44·5				
	454	”	44·5				
	454	”	44·5				
·75	436	”	62·5	60·75	52·74	52·56	+0·18
	438	”	60·5				
	437	”	61·5				
	440	”	58·5				
1·0	423	”	75·5	76·25	66·19	66·19	0·00
	423	”	75·5				
	422	”	76·5				
	421	”	77·5				
1·25	*e*410	”	88·5	89·2	77·43	79·67	−2·24
	*e*410	”	88·5				
	*e*408	”	90·5				

TABLE IX.—*continued.*

Electrodes, small discs. Pressure, 180 mm.

Equation, $V = 6{\cdot}0767\sqrt{\{s^2 + 15{\cdot}040s\}}$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential. V.	Difference. $V^1 - V$.
·25	482	498·5	16·5	14	12·15	11·88	+0·27
	485	"	13·5				
	485	"	13·5				
	486	"	12·5				
·5	476	"	22·5	19·5	16·93	16·94	−0·01
	480	"	18·5				
	479	"	19·5				
	480	"	18·5				
	480	"	18·5				
·75	475	"	23·5	25·1	21·79	20·91	+0·88
	472	"	26·5				
	472	"	26·5				
	473	"	25·5				
	475	"	23·5				
1·0	472	"	26·5	27·1	23·53	24·34	−0·81
	471	"	27·5				
	470	"	28·5				
	474	"	24·5				
	470	"	28·5				
1·25	462	"	36·5	33·3	28·91	27·42	+1·49
	466	"	32·5				
	465	"	33·5				
	466	"	32·5				
	467	"	31·5				
1·5	462	"	36·5	35·9	31·16	30·27	+0·89
	461	"	37·5				
	463	"	35·5				
	465	"	33·5				
	462	"	36·5				

TABLE X.—Distance Curve, 6th June 1877. Electrodes, large discs. Pressure, 180 mm.

Equation, $V=(19·724-3·0666s)\sqrt{\{s^2+·45s\}}$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. V^1-V.
·15	478	498	20	20	5·79	5·78	+0·01
	478	„	20				
	478	„	20				
·25	470	„	28 ×	28	8·11	7·93	+0·18
	469	„	29				
	470	„	28				
·35	463	„	35	35	10·13	9·87	+0·26
	463	„	35				
	463	„	35				
·45	457	„	41	40	11·58	11·67	−0·09
	458	„	40				
	459	„	39				
·55	453	„	45	45	13·03	13·38	−0·35
	452	„	46				
	454	„	44				
·65	447	„	51	49·7	14·39	14·99	−0·60
	448	„	50				
	450	„	48				
·75	444	„	54	54·3	15·72	16·53	−0·81
	444	„	54				
	443	„	55				
·85	438	„	60	61·3	17·75	17·99	−0·24
	436	„	62				
	436	„	62				
·95	430	„	68	67	19·40	19·39	+0·01
	430	„	68				
	433	„	65				
1·05	427	„	71	71	20·56	20·71	−0·15
	426	„	72				
	428	„	70				
1·15	423	„	75	75·7	21·92	21·97	−0·05
	422	„	76				
	422	„	76				
1·25	419	„	79	80	23·16	23·16	0·00
	417	„	81				
	418	„	80				
1·35	413	„	85	85·3	24·69	24·29	+0·40
	413	„	85				
	412	„	86				
1·45	408	„	90	90·3	26·14	25·36	+0·78
	409	„	89				
	406	„	92				
1·55	403	„	95	94·7	27·42	26·36	+1·06
	403	„	95				
	404	„	94				
1·65	e						

TABLE XI.—Hydrogen Distance Curve, 12th July 1877. Dielectric, Hydrogen. Electrodes, large discs. Pressure, 752 mm. Temperature, 70° Fahr.

Equation, $V=(43{\cdot}334-1{\cdot}042s)\sqrt{\{s^2+{\cdot}136s\}}$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. V^1-V.
·06	498 498	510 "	12 12	12	3·99	4·69	−0·70
·11	492 492	" "	18 18	18	5·99	7·11	−1·12
·16	482 482	" "	28 28	28	9·32	9·39	−0·07
·21	474 473	" "	36 × 37	36	11·98	11·62	+0·36
·26	468 469	511 512	43 43	43	14·31	13·82	+0·49
·31	463 462	511 510	48 48	48	15·98	15·99	−0·01
·41	453 452	511 "	58 59 ×	59	19·68	20·30	−0·62
·51	439 438	" "	72 73	72·5	24·12	24·57	−0·45
·61	423 423	" 512	88 × 89	88	29·28	28·80	+0·48
·71	414 413	" "	98 99	98·5	32·78	33·01	−0·23
·81	398 403 399	" " "	114 109 113	112	37·28	37·19	+0·09
·91	388 388	" "	124 124	124	41·26	41·35	−0·09
1·01	377 379 376	510 " 514	133 × 131 138 ×	136	45·26	45·49	−0·23
1·11	359 361 *e* 358	510 " 514	151 149 156	150	49·92	49·60	+0·32
1·21	356 363 343	" 512 "	158 149 169	158·7	52·82	53·69	−0·87

TABLE XII.—Hydrogen Distance Curve, discs having first been heated. 11th July 1877. Dielectric, Hydrogen. Electrodes, large discs. Pressure, atmospheric.

Equation, $V = 53·940s - 12·9316s^2$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. $V^1 - V$.
·02	506 505 505	510 " "	4 5 × 5	5	·95	1·07	−0·12
·07	492 490 491	" " "	18 20 × 19	20	3·80	3·71	+0·09
·12	475 473	" "	35 × 37	35	6·65	6·29	+0·36
·17	463 461	" "	47 × 49	47	8·93	8·80	+0·13
·22	450 448	" "	60 × 62	60	11·40	11·24	+0·16
·32	426 425	" "	84 85	84·5	16·06	15·94	+0·12
·42	403 402	" "	107 108	107·5	20·43	20·38	+0·05
·52	383 385	" "	127 × 125	127	24·13	24·55	−0·42
·62	361 361	" "	149 149	149	27·67	28·47	−0·80
·72	345 343	" "	165 167	166	31·55	32·13	−0·58
·82	323 322	" "	187 188	187·5	35·63	35·45	+0·18
·92	306 310	" "	204 × 200	204	38·77	38·68	+0·09
1·02	290 288	" "	220 222	221	41·99	41·57	+0·42
1·12	293 290	" "	217 × 220	217	41·52	41·03	+0·49

Table XIII.—Air Distance Curve, discs having first been heated, 13th July 1877. Dielectric, Air. Electrodes, large discs. Pressure, 742 mm. Temperature, 70° Fahr.

Equation, $V = 89{\cdot}240s - 33{\cdot}815s^2$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. $V^1 - V$.
·035	508	512	4 ×	4	2·06	3·08	−1·02
	507	510	3				
·06	500	"	10	10	5·15	5·23	−0·08
	500	"	10				
·085	495·5	"	14·5	14·5	7·47	7·34	+0·13
	495·5	"	14·5				
·11	489	"	21	21	10·82	9·41	+1·41
	489	"	21				
·16	483	509	26 ×	26	13·39	13·41	−0·02
	482	507	25				
	482	"	25				
·21	470	506	36	36	18·54	17·25	+1·29
	470	"	36				
·31	459	"	47	47·5	24·46	24·41	+0·05
	458	"	48				
·41	448	"	58	60	30·90	30·90	0·00
	446	508	62				
·51	439	"	69	69	35·54	36·71	−1·17
	440	509	69				
·61	428	"	81	81	41·72	41·86	−0·14
	428	"	81				
·71	421	"	88	88·7	45·69	46·31	−0·62
	419	"	90				
	nc 421	"	88				
·81	*nc* 412	"	97	98	50·48	50·10	+0·38
	c 410	"	99				
·91	*c* 405	"	104	103·3	53·21	53·21	0·00
	nc 404	"	105				
	c 408	"	101				
1·01	*c* 404	"	105	106·3	54·75	55·38	−0·63
	c 403	"	106				
	c 402	510	108				
1·11	*c* 400	512	112	111	57·17	57·40	−0·23
	nc 402	"	110				
1·21	*c* 393	"	119	113·5	58·46	58·40	+0·06
	c 396	511	115 ×				
	c 400	512	112 ×				
1·31	394	"	118	113·7	58·56	58·86	−0·30
	e 401	509	108				
	nc 397	"	115				
1·41	*c* 396	508	112	111·7	57·69	58·61	−0·92
	e 396	"	112				
	e 397	"	111				

TABLE XIV.—Distance Curve, discs having first been heated, 20th July 1877. Dielectric, Air. Electrodes, large discs. Pressure, 740 mm.

Equation, $V=87{\cdot}040s-19{\cdot}560s^2$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference V^1-V.
·05	510	523	13	11·5	4·50	4·30	+0·20
	513	„	10				
·1	500	„	23	23	8·99	8·51	+0·48
·15	488	„	35	35	13·69	12·62	+1·07
·2	477	„	46	46	17·99	16·63	+1·36
·25	473	„	50	50	19·56	20·54	−0·98
·35	450	„	73	73	28·56	28·07	+0·49
·45	433	„	90	90	35·21	35·21	0·00
·55	420	„	103	103	40·29	41·95	−1·66
·65	401	„	122	122	47·72	48·31	−0·59
·75	383	„	140	140	54·76	54·28	+0·48
·85	370	„	153	153	59·85	59·85	0·00
·95	358	„	165	165	64·54	65·03	−0·49
1·05	*e* 343	„	180	179	70·02	69·83	+0·19
	e 345	„	178				
·25	474	„	49	...	...	...	...
	473	„	50				
Taken after an interval of 45 minutes :—							
·25	468	523	55	...	...	...	...
	471·1	„	51·5				
·5	423	520	97	...	...	...	...
	423	„	97				

TABLE XV.—Change of Capacity of Charged Conductor, 4th July 1877. Dielectric, Air. Electrodes, large discs. Pressure, atmospheric. Length of spark, ·5 centimetres.

Capacity.	Deflection. n.	Zero. n^1.	Diff. of Potential. n^1-n.	Mean Diff. of Potential.
Large jar, . .	300	500	200	203
	295	496	201	
	290	490	200	
	293	486	193	
	296	505	209	
	297	„	208	
	296	„	209	

TABLE XV.—*continued.*

Capacity.	Deflection. n.	Zero. n^1.	Diff. of Potential. n^1-n.	Mean Diff. of Potential.
Couple of small jars,	298	505	207	207
	300	”	205	
	296	”	209	
	300	”	205	
	296	”	209	
Without any jars, .	315	510	295	209
	295	”	215	
	310	”	200	
	290	”	220	
	290	”	220	
	305	”	205	
	300	”	210	

TABLE XVI.—Distance Curve; Continued Discharge, 17th July 1877. Dielectric, Air. Electrodes, large discs. Pressure, 737 mm.

Equation, $V=\{46{\cdot}095-2{\cdot}567s\}\surd\{s^2+{\cdot}2s\}$.

Length of Spark in Centimetres. s.	Deflection n.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential. V.	Difference V^1-V.
·06	494	510	16	4·55	5·74	−1·19
·11	487	518	31	8·81	8·46	+0·35
·21	483	530	47	13·35	13·34	+0·01
·31	473	540	67	19·03	18·01	+1·02
·41	475	560	85	24·14	22·53	+1·61
·51	463	545	82	23·29	26·95	−2·66
·61	452	554	102	38·97	31·30	−2·33
·71	450	570	120	34·08	35·59	−1·51
·81	438	565	127	36·07	39·81	−3·74
·91	*e*	...	...	...	...	...
·51	438	520	82	...	...	...
	447	530	83			

TABLE XVII.—Pressure Curve, 3d July. First Curve. Jars on. Dielectric, Air. Electrodes, large discs. Length of spark, ·5 centimetre.

$$\text{Equation, } V=\{\cdot 048646-\cdot 0000033047p\}\sqrt{\{p^2+200p\}}.$$

Pressure in Millimetres of Mercury. p.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. $n^1—n$.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. V^1-V.
20	486	510	24	24	3·23	3·22	+0·01
	486	,,	24				
40	476	509	33	32·5	4·37	4·76	−0·39
	477	,,	32				
60	464	511	47	47	6·33	6·06	+0·27
	464	,,	47				
80	460	,,	51	52	7·00	7·26	−0·26
	459	,,	52 ×				
100	448	,,	63	63	8·48	8·39	+0·09
	448	,,	63				
120	440	513	73	72·5	9·76	9·49	+0·27
	440	512	72				
140	433	,,	79	80·5	10·84	10·56	+0·28
	430	,,	82				
160	425	,,	87	87	11·71	11·61	+0·10
	425	,,	87				
180	414	,,	98	98	13·19	12·64	+0·55
	414	,,	98				
200	408	,,	104	104	14·00	13·66	+0·34
	408	,,	104				
303	368	510	142	139	18·71	18·81	−0·10
	364	505	141				
	373	,,	132				
	366	506	140				
406	338	508	170 ×	175	23·56	23·84	−0·28
	325	,,	183				
	332	,,	176 ×				
	330	,,	178 ×				
509	304	,,	204	206	27·73	28·81	−1·08
	305	,,	203				
	298	,,	210				
	302	,,	206				
612	270	,,	238	242	32·57	33·73	−1·16
	265	,,	243				
	265	,,	243				
	266	,,	242				
685	242	,,	266	268	36·07	37·18	−1·11
	235	,,	273				
	240	,,	268				
	240	,,	268				

Table XVIII.—Pressure Curve, 3d July. Second Curve. Without jars on. Dielectric, Air. Electrodes, large discs. Length of spark, ·5 centimetre.

Equation, $V = \cdot 04455 \sqrt{\{p^2 + 200p\}}$.

Pressure in mm. of Mercury. p.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential. V.	Difference. $V^1 - V$.
25	492 492 491	512 " "	20 20 21	20·3	2·73	3·34	−0·61
40	482 484	" "	30 × 28	30	4·04	4·36	−0·32
60	472 470	" "	40 42	41	5·52	5·56	−0·04
80	460 460	" "	52 52	52	7·00	6·67	+0·33
100	454 453	" "	58 59	58·5	7·87	7·72	+0·15
120	443 443	" "	69 69	69	9·29	8·73	+0·56
140	440 440	513 "	73 73	73	9·83	9·72	+0·11
160	435 433	" "	78 80	79	10·63	10·69	−0·06
180	429 425	" "	84 88	86	11·58	11·65	−0·07
200	418 418	" "	95 95	95	12·79	12·60	+0·19
303	385 388	" "	128 × 125	128	17·23	17·39	−0·16
406	355 340 346 348	" " " "	158 173 167 165	165	22·21	22·10	+0·11
509	313 308	" "	200 × 205	200	26·92	26·73	+0·19
612	280 280	" "	233 233	233	31·36	31·41	−0·05

TABLE XIX.—Pressure Curve, 27th June 1877. Dielectric, Air. Electrodes large discs. Length of spark, ·5 centimetre.

$$\text{Equation, } V = \cdot 046342 \sqrt{\{p^2 + 199 \cdot 01 p\}}.$$

Pressure in mm. of Mercury. p.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. $V^1 - V$.
66	479	514	35	35	6·13	6·13	0·00
	479	"	35				
	479	"	35				
169	447	513	66	66	11·56	11·56	0·00
	447	"	66				
	447	"	66				
273	415	508	93	90	15·75	13·91	+1·84
	418	"	90 ×				
	414	510	96				
376	392	508	116	119	20·83	21·55	−0·72
	389	"	119 ×				
	393	510	117				
479	358	"	152 ×	152	26·60	26·41	+0·19
	360	515	155				
	356	"	159				
582	351	518	167	177	30·98	31·24	−0·26
	343	520	177 ×				
	350	523	173				
740	295	510	215 ×	215	37·63	38·63	−1·00
	310	517	207				
	318	525	207				

TABLE XX.—Pressure Curves, 29th June 1877. First Curve. Single Discharge. Dielectric, Air. Electrodes, large discs. Length of spark, ·5 centimetre.

$$\text{Equation, } V = \cdot 046853 \sqrt{\{p^2 + 207 \cdot 06 p\}}.$$

Pressure in mm. of Mercury. p.	Deflection. n.	Zero. n^1.	Difference of Potential. $n^1 - n$.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. $V^1 - V$.
20	478	495	17	20	3·13	3·16	−0·03
	475	"	20 ×				
	477	"	18				
60	490	530	40	39	6·11	5·93	+0·18
	490	529	39 ×				
	490	"	39 ×				

TABLE XX.—*continued.*

Pressure in mm. of Mercury. p.	Deflection. n.	Zero. n^1.	Difference of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential in abs. meas. V.	Difference. V^1-V.
100	465	515	50	52	8·15	8·20	−0·05
	452	510	58				
	461	"	49				
140	468	533	65	65	10·19	10·30	−0·11
	467	532	65				
	464	530	66				
180	430	510	80	80	12·54	12·29	+0·25
	430	"	80				
	428	508	80				
200	418	507	89	86·5	13·56	13·30	+0·26
	424	505	81				
	416	"	89				
40	483	510	27 ×	29·5	4·62	4·66	−0·04
	484	"	26				
	478	"	32 ×				
80	470	512	42	44	6·90	7·10	−0·20
	467	"	45				
	467	"	45				
120	458	517	59	60	9·40	9·27	+0·13
	455	516	61				
	455	"	61				
160	443	517	74	72	11·28	11·32	−0·04
	443	516	73				
	448	517	69				
10	502	511	9	11	1·72	2·18	−0·46
	500	510	10				
	498	509	11 ×				

Second Curve. Continued Discharge.

Equation, $V=(\cdot 03552-\cdot 000002413p)\sqrt{\{p^2+200p\}}$.

20	479	495	16	2·56	2·35	+0·21	
60	502	529	27	4·32	4·42	−0·10	
100	473	510	37	5·92	6·11	−0·19	
140	479	530	51	8·16	7·68	+0·48	
180	450	508	58	9·28	9·18	+0·10	
200	443	506	63	10·08	9·91	+0·17	
40	488	511	23	3·68	3·47	+0·21	
80	483	515	32	5·12	5·29	−0·17	
120	478	518	40	6·40	6·90	−0·50	
160	470	520	50	8·00	8·43	−0·43	
10	502	516	14	2·24	1·63	+0·61	

TABLE XXI.—Pressure Curve, 15th June 1877. Longer Spark. Dielectric, Air. Electrodes, large discs. Length of spark, 1 centimetre.

Equation, $V = {\cdot}080620 \sqrt{\{p^2 + 219{\cdot}84p\}}$.

Pressure in mm. of Mercury. p.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential. V.	Difference. $V^1 - V$.
20	485	505	20	20·4	5·59	5·58	+0·01
	486	„	19				
	484	506	22				
40	478	507	29	29·3	8·03	8·22	−0·19
	478	„	29				
	477	„	30				
60	470	„	37	38	10·41	10·45	−0·04
	468	506	38				
	468	„	38				
80	459	„	47	46	12·60	12·49	+0·11
	461	„	45				
	460	„	46				
100	455	508	53	53	14·52	14·42	+0·10
	453	507	54				
	455	„	52				
120	444	508	64	63	17·26	16·28	+0·98
	444	507	63				
	444	„	63				
140	435	„	72	71	19·45	18·10	+1·35
	435	„	72				
	436	„	71 ×				
160	423	„	84	83	22·74	19·87	+2·87
	424	„	83 ×				
	423	„	84				
180	418	508	90	84	23·01	21·63	+1·38
	422	509	87				
	425	„	84 ×				
200	408	500	92	89	24·38	23·36	+1·02
	408	502	94				
	415	504	89 ×				
740	268	513	245	248	67·94	67·94	0·00
	266	517	251				

TABLE XXII.—Pressure Curve, 12th July 1877. Dielectric, Hydrogen. Electrodes, large discs. Length of spark, ·5 centimetre. Temperature, 70° Fahr.

$$\text{Equation, } V = \cdot 024\sqrt{\{p^2 + 600p\}}.$$

Pressure in mm. of Mercury. p.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Mean Diff. of Potential in absolute measure. V^1.	Calculated Diff. of Potential. V.	Difference. V^1-V.
752	439	511	72	72·5	24·03	24·20	−0·07
	438	„	73				
648	445	510	65 ×	65	21·62	21·09	+0·53
	455	„	55				
572	450	„	60	59·5	19·81	19·65	+0·16
	452	„	58				
	450	„	60				
468	461	512	51	51	16·98	16·97	+0·01
	461	„	51				
364	470	„	42	42	13·98	14·22	−0·24
	471	„	41				
	470	„	42				
262	481	„	31	31	10·32	11·41	−1·09
	481	„	31				
158	485	„	27	25	8·32	8·31	+0·01
	488	„	24				
	487	„	25				
56	495	„	17	15	4·99	4·60	+0·39
	497	„	15 ×				
31	500	„	12	10	3·33	3·36	−0·03
	502	„	10 ×				

TABLE XXIII.—The Electric Strength of Different Gases, 5th July 1877. Electrodes, large discs. Pressure, 746 mm. Length of spark, ·5 centimetre.

Dielectric.	Deflection. n.	Zero. n^1.	Diff. of Potential. n^1-n.	Mean Diff. of Potential.	Electric Strength relative to that of Air.	Mean of all other Observations	Faraday's Values.
Air . . .	350	501	151	152	...	...	...
	360	„	141				
	348	„	153				
	344	„	157				
	347	502	155				
	352	„	150				
	350	„	152				
	345	„	157				

TABLE XXIII—*continued.*

Dielectric.	Deflection. n.	Zero. n^1.	Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential.	Electric Strength relative to that of Air.	Mean of all other Observations.	Faraday's Values.
CO_2, . .	370	502	132	135	·951	·952	·91
	363	,,	139				
	364	505	141				
	373	503	130				
	369	,,	134				
	363	505	142				
	374	,,	131				
	368	,,	137				
	364	507	143				
	380	,,	127				
	377	,,	130				
Air, . .	368	,,	139	142	1·000	...	...
	365	,,	142				
	370	,,	137				
	360	,,	147				
	360	,,	147				
	367	,,	140				
O_2, . .	370	505	135	132	·930	·938	·71
	372	,,	133				
	372	,,	133				
	370	,,	135				
	380	,,	125				
	373	,,	132				
	373	,,	132				
Air, . .	356	503	147	142	...	...	...
	370	,,	133				
	366	,,	137				
	358	,,	145				
	356	,,	147				
H_2, . .	417	505	88	90	·634	·664	·53
	420	510	90				
	420	,,	90				
	420	,,	90				
	420	511	91				
	420	512	92				
	423	,,	89				
Air, . .	375	510	135	141	...	...	...
	368	,,	142				
	368	,,	142				
	365	,,	145				
Coal Gas,	...	...	...	...	...	·935	·71

TABLE XXIV.—Long Distance Curve, 25th May 1877. Dielectric, Air. Electrodes, balls. Pressure, 85 mm. Temperature, 66° Fahr.

Equation, $V=34{\cdot}92s^{\frac{1}{2}}-1{\cdot}206s^{\frac{3}{2}}$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential. V^1.	Calculated Diff. of Potential. V.	Difference. V^1-V.
14	412	500	88	89·3	67·5	+21·8
	410	,,	90			
	410	,,	90			
13	413	,,	87	88·3	69·4	+18·9
	410	,,	90			
	412	,,	88			
12	413	,,	87	87	70·8	+16·2
	413	,,	87			
	413	,,	87			
11	420	,,	80	84	71·8	+12·2
	415	,,	85			
	414	,,	86			
10	419	,,	81	80·3	72·3	+8·0
	420	,,	80			
	420	,,	80			
9	420	,,	80	80·3	72·2	+8·1
	419	,,	81			
	420	,,	80			
8	428	,,	72	72	71·5	+0·5
	428	,,	72			
	428	,,	72			
7	434	,,	66	70	70·0	0·0
	430	,,	70 ×			
	430	,,	70 ×			
6	431	,,	69	67·7	67·8	−0·1
	433	,,	67			
	433	,,	67			
5	434	,,	66	64·7	64·6	+0·1
	435	,,	65			
	437	,,	63			
4	439	,,	61	60	60·2	−0·2
	441	,,	59			
	440	,,	60			
3	444	,,	56	54·3	54·2	+0·1
	447	,,	53			
	446	,,	54			
2	450	,,	50	48·3	46·0	+2·3
	453	,,	47			
	452	,,	48			
1	466	,,	34	34·7	33·7	+1·0
	463	,,	37			
	467	,,	33			
·5	472	,,	28	24·3	24·3	0·0
	480	,,	20			
	475	,,	25			

TABLE XXV.—Long Distance Curve, 22d May 1877. Dielectric, Air. Electrodes, balls. Pressure, 40 mm.

Equation, $V=16{\cdot}752s^{\frac{1}{2}}-{\cdot}6364s^{\frac{3}{2}}$.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. n^1-n.	Mean Diff. of Potential. V^1.	Calculated Diff. of Potential. V.	Difference. V^1-V.
1	480	496	16	16·3	16·1	+0·2
	479	"	17			
	480	"	16			
2	473	"	23	23	21·9	+1·1
	473	"	23			
	473	"	23			
3	470	"	26	25·7	25·7	0·0
	472	"	24			
	469	"	27			
4	470	"	26	27·3	28·4	−1·1
	468	"	28			
	468	"	28			
5	467	"	29	29·3	30·3	−1 0
	467	"	29			
	466	"	30			
6	465	"	31	30·7	31·7	−1·0
	466	"	30			
	465	"	31			
7	463	"	33	32·7	32·5	+0·2
	465	"	31			
	462	"	34			
8	463	"	33	32·7	33·0	−0·3
	463	"	33			
	464	"	32			
9	460	"	36	35·3	33·1	+2·2
	461	"	35			
	461	"	35			

TABLE XXVI.—Long Distance Curve, 23d May 1877. Dielectric, Air. Electrodes, balls. Pressure, 20 mm.

Equation, $V = 7{\cdot}4405 s^{\frac{1}{2}} - {\cdot}16580 s^{\frac{3}{2}}$.

Length of Spark. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential. V^1.	Calculated Diff. of Potential. V.	Difference. $V^1 - V$.	Temperature.
1	491	500	9 ×	9	7·3	+1·7	...
	488	"	12				
	491	"	9 ×				
2	488	"	12	10	10·1	−0·1	64·5° F.
	489	"	11				
	490	"	10 ×				
3	489	"	11	11·3	12·0	−0·7	...
	488	"	12				
	489	"	11				
4	488	"	12	12	13·6	−1·6	...
	488	"	12				
	488	"	12				
5	487	"	13	14	14·8	−0·8	...
	487	"	13				
	486	"	14 ×				
6	486	"	14	15	15·8	−0·8	66°
	488	503	15 ×				
	488	"	15				
7	486	"	17	16·7	16·6	+0·1	...
	487	"	16				
	486	"	17				
8	486	"	17	17	17·3	−0·3	...
	486	"	17				
	486	"	17				
9	485	"	18	17·5	17·8	−0·3	66·4°
	486	"	17				
	485·5	"	17·5				
10	483	502	19	18·7	18·3	+0·4	...
	483	"	19				
	484	"	18				
11	483	"	19	18·3	18·6	−0·3	...
	485	503	18				
	485	"	18				
12	484	"	19	19·3	18·9	+0·4	66·5°
	483	"	20				
	484	"	19				
13	481	500	19	19·3	19·1	+0·2	...
	483	503	20				
	485	504	19				
14	482	503	21	21	19·2	+1·8	67°
	483	"	20				
	482	504	22				
15	482	503	21	22	19·2	+2·8	...
	480	"	23				
	482	504	22				

TABLE XXVII.—Long Distance Curve, 17th May 1877. Dielectric, Air. Electrodes, small balls. Pressure, 30 mm.

Equation, $V = 13{\cdot}168 s^{\frac{1}{2}} - {\cdot}40519 s^{\frac{3}{2}}$.

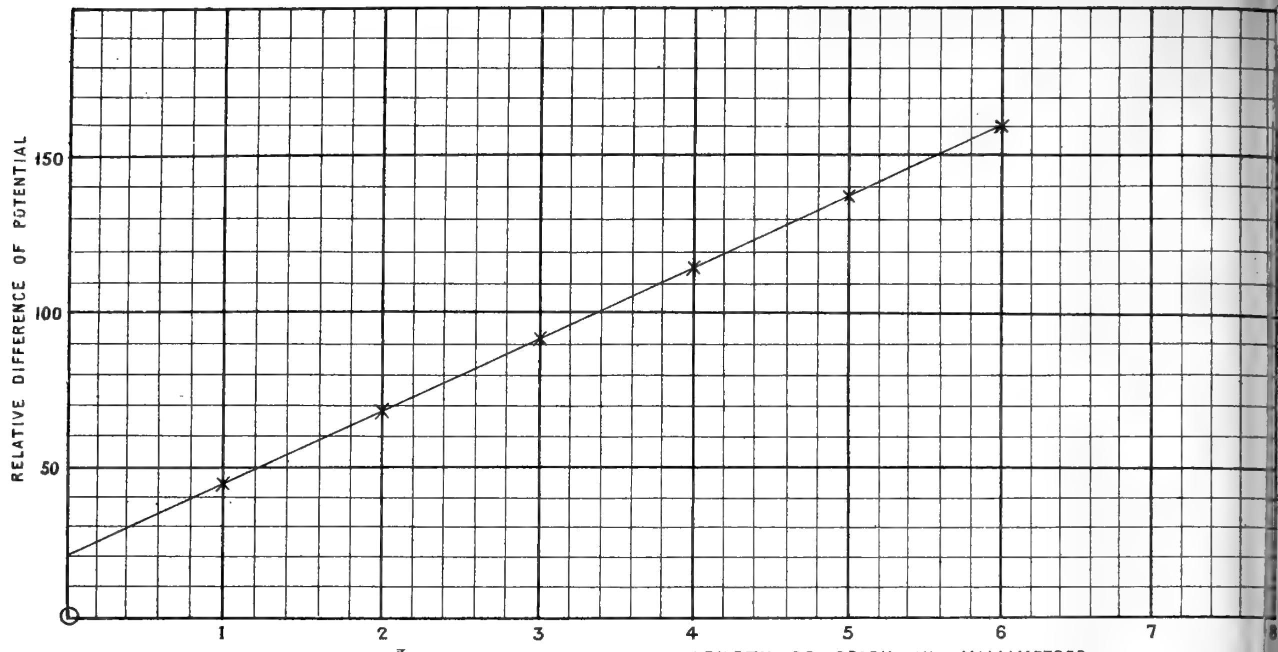
Length of Spark. s.	Deflection. n.	Zero. n^1.	Observed Diff. of Potential. $n^1 - n$.	Mean Diff. of Potential. V^1.	Calculated Diff. of Potential. V.	Difference. $V^1 - V$.
1	470	487	17	16	12·8	+3·4
	471	,,	16 ×			
	471	,,	16 ×			
2	468	,,	19	18	17·5	+0·5
	469	,,	18			
	470	,,	17			
2·5	467	,,	20	19·3	19·2	+0·1
	467	,,	20			
	469	,,	18			
5	465	,,	22	22·5	24·9	−2·4
	465	,,	22			
	464·5	,,	22·5 ×			
7·5	464	,,	23	25·5	27·7	−2·2
	462	,,	25 ×			
	461	,,	26 ×			
10	458	,,	29	28·7	28·8	−0·1
	458	,,	29			
	459	,,	28			
12·5	458	,,	29	28·3	28·6	−0·3
	459	,,	28			
	459	,,	28			
15	457	,,	30	29	27·5	+1·5
	460	,,	27			
	457	,,	30			
17·5	454	,,	33	33·3	25·4	+7·9
	454	,,	33			
	453	,,	34			

TABLE XXVIII.—Comparison of Electrometers, 10th July 1877.

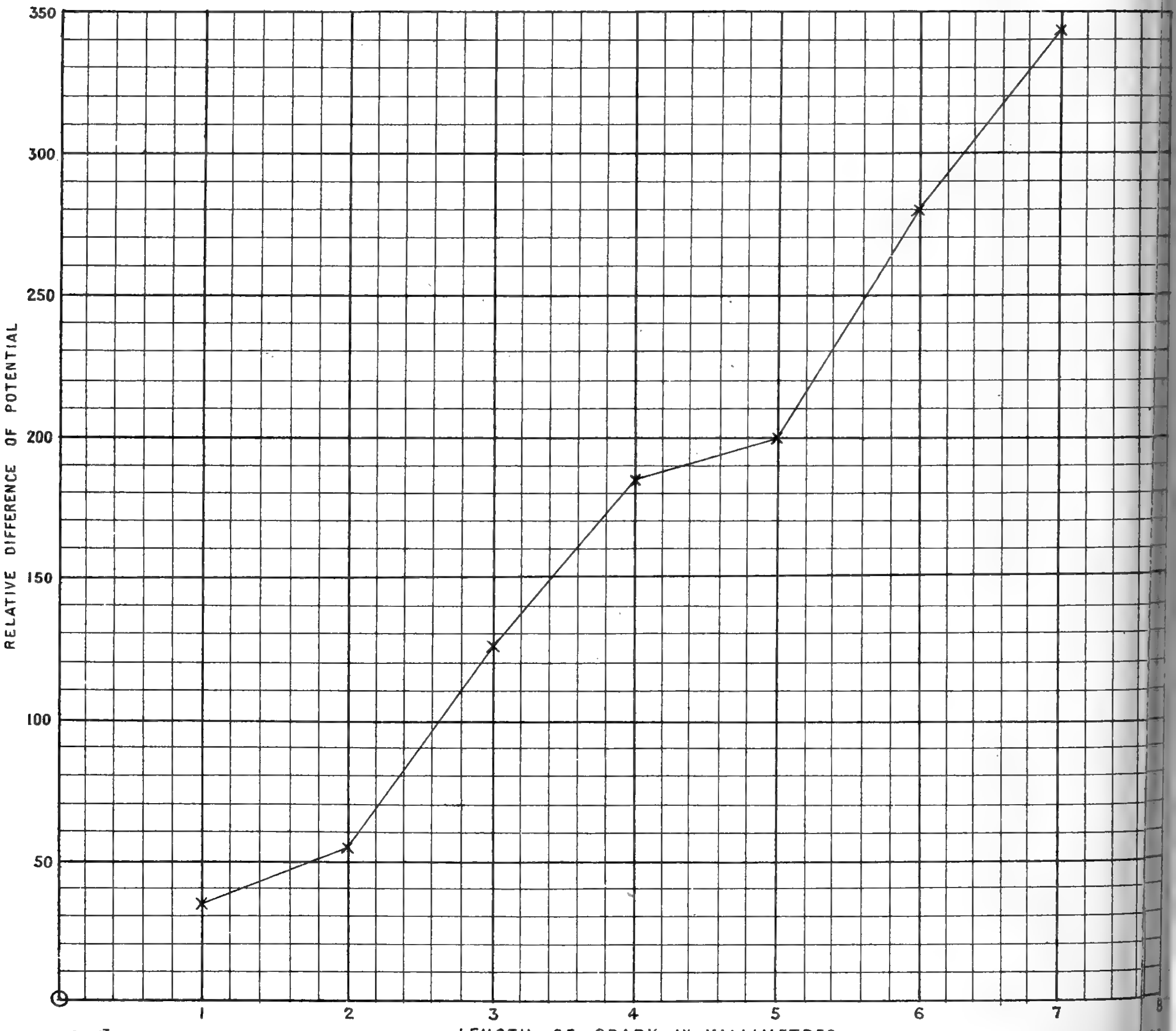
Mean of eight determinations, 38·63 C. G. S. units.

Distance between Plates of Absolute Electrometer in mm. D.	Deflection of Electrometer at moment of fall of Aluminium Square.	Zero.	Difference, giving Reading of Divided Ring Electrometer.	Mean Difference.	Value in absolute measure of Potential required for Standard Spark.
10	496	510	14	13	
	498	„	12		
	497	„	13		
	495	„	15		
	497	„	13		
	496	„	14		
	498	„	12		
	498	„	12		
7·5	502	„	8	8	47·70
	502	„	8		
	502	„	8		
	501	„	9		
	502	„	8		
	504	„	6		
	502	„	8		
8·75	500	„	10	10	39·75
	500	„	10		
	499·5	„	10·5		

ELECTRIC SPARK THROUGH TURPENTINE



Continuous discharge 25th Jan^y 1878



SINGLE DISCHARGE

25th Jan^y, 1878

XXIV.—*On the Discharge of Electricity through Oil of Turpentine.* By ALEXANDER MACFARLANE, M.A., B.Sc., and R. J. S. SIMPSON. (Plate XXIV.)

(Received 7th February 1878. Read 4th March 1878.)

At the beginning of this session we proceeded to apply the method of measuring large differences of potential described in former papers, to the investigation of the disruptive discharge through liquid dielectrics. We have now obtained some results for oil of turpentine.

A vessel capable of holding the liquid, and at the same time of forming an electrode, was constructed by fixing on a metal plate to one end of a hollow glass cylinder of diameter slightly less than that of the receiver of the air-pump. The vessel was placed on a metallic support, so as to be in conducting connection with the metal parts of the air-pump, and with the earth. The other electrode, which was either a circular disc, a spherical ball, or a conical point, was screwed on to the brass rod passing through the stuffing-box on the top of the receiver, and was thus capable of being adjusted to various heights in the turpentine. The bottom of the vessel is 16·7 centimetres, and the brass disc, which commonly formed the upper electrode, is 10 centimetres in diameter, and 4 millimetres thick.

The metal bottom of the vessel was first made of copper; but it was found that the turpentine, when charged, acted upon the copper very rapidly, forming a compound which it afterwards dissolved. Tinfoil pasted on the surface of the copper plate prevented the action taking place. To raise the surface of the lower electrode that it might be better seen, we put in a lead plate of 16·4 centimetres diameter and ·85 centimetres thick. When the liquid was charged, a lead compound was formed with great rapidity, and remained undissolved. The substance formed arranged itself in columns between the electrodes, in the direction of the lines of force. The charge made its way through the dielectric by these columns. We found that a tin plate was not acted on, but that it was always necessary to filter the liquid to get rid of any solid particles, as these, when present, never failed to arrange themselves in the lines of force; and when not of sufficient number to form a continuous chain, dissipated the charge by dancing between the plates. When a chain, so formed, was stretched between the electrodes, the index of the electrometer behaved as if a current were passing by means of the chain,—when the Holtz machine was turned at a uniform rate, the index remained steady; when it was turned faster, the index gave a greater reading; when slower, a less reading. Sometimes the current passing appeared to break the thread; when that happened, the electricity had an opportunity of beginning to pass in a different mode.

Three other modes of discharge were observed—by motion of the liquid; by a disruptive discharge; and by convection by means of gas-bubbles. When the disc was at the height of 4 or more millimetres above the plate, the surface of the oil of turpentine became agitated; while the behaviour of the index of the electrometer showed that the charge was being rapidly, but not instantaneously, lost. This agitation of the liquid ceased when any other kind of discharge began. When the Holtz machine was turned for some time with its conductors in contact, and the contact then suddenly broken, a circular ripple was first observed of diameter about equal to that of the disc, and then the agitation proceeded to extend over the surface. The existence of the agitation appeared to depend on the nearness of the upper electrode to the surface of the liquid (which was generally 4 centimetres deep), as well as on the amount of charge. The surface assumed a different form, according as the charge was positive or negative. When the charge was positive, the liquid rose up round the rod of the receiver; when negative, the liquid rose up the edge of the vessel, sometimes up to and over the rim—a height of about 5 centimetres. On one occasion, when the surface had been agitated for some time, and when, in consequence of diminished air-pressure, a considerable quantity of turpentine vapour must have been present, a red smoky flame was observed for an instant on the surface of the liquid. No luminous discharge was observed at the time.

When there was no chain of solid particles between the plates, it was generally possible to get a disruptive discharge, similar to that through air, even though the surface of the liquid was in great commotion. The spark was vertical and dazzling white; threadlike, sinuous, and sometimes forked, when the Leyden jars were off the conductors of the Holtz; but thicker and more direct when the jars were on. The flash gave a continuous spectrum, and sometimes appeared tinged with crimson, like the sparks through hydrogen, especially at the negative electrode. The sound accompanying the discharge was more intense than the sound accompanying the discharge through air under the same conditions. The index of the electrometer indicated that the discharge was complete. The disruptive discharge was accompanied by the formation of bubbles of gas. These were always attracted to the negative plate. When the electrification was neutralised, they of course adhered to the under surface of the disc; when the disc was electrified negatively, they adhered with still greater firmness; when positively, they were repelled so as either to remain suspended in the liquid, or to adhere to the lower electrode, according to the greater or less distances between the surfaces. The spark always passed from or to where the gas bubbles were; and bubbles seemed to be formed in the path of the spark. When the upper electrode was negative, it was easier to get a succession of sparks; and the number of gas bubbles formed was greater. This was probably due to the fact that in this case the bubbles adhered firmly to the surface. When the air pressure inside the receiver was diminished, the gas bubbles formed were larger and more numerous. At a pressure of half an atmosphere, with the upper plate positive, and at a distance of half a centimetre from the lower, the bubbles were observed to effect the discharge, by carrying the electricity with them to the negative electrode. It is possible to cause a shower of bubbles to descend from the upper to the lower surface. When they impinge on the lower plate, they emit a slight flash and corresponding sound. At very low pressures we found it impossible to get a spark.

When we substituted a brass ball of 8 mm. diameter for the upper disc, and charged it positively, the gas bubbles rolled outwards along the lower plate in straight lines from the centre, until they reached a point where their buoyancy lifted them up. There was a more rapid formation of gas bubbles when the ball was negative than when positive.

We also tried, for the upper electrode, a conical point 3 centimetres in height, by a diameter of 5 mm. at the base. When the extremity of the cone was 1 or 2 mm. from the lower electrode, the jars being off, the discharge passed in the form of an arc, which rotated in the direction of the hands of a watch, as we looked down upon it. At a distance of 2 centimetres the spark was sinuous and in the form of an arc.

It is probable that the gas liberated is due to the decomposition of the oil of turpentine, and not merely to the liberation of a gas loosely held; for the supply seemed inexhaustible, and after we had passed a great number of sparks, the liquid assumed a brownish tint, and a black deposit was found on the plates. We have not as yet got a sample of the oil of turpentine analysed.

We made several series of observations of the difference of potential required to produce a disruptive discharge at different distances between the disc and the plate. They all point to the same conclusion, and the following is representative :—

Relative Difference of Potential required to produce a Single Spark.
Dielectric, Oil of Turpentine. Electrodes, Disc and Plate. Pressure, 625 *mm.*

S. Distance between Disc and Plate.	Deflection. n.	Zero. n'.	$n' - n$.	V. Mean difference of Potential.
·1 centimetres,	448	490	42	35·5
	465	494	29	
·2 "	438	497	59	55·5
	448	500	52	
·3 "	335	490	155	127·5
	390	490	100	
·4 "	310	495	185	184
	315	498	183	
·5 "	280	500	220	198·5
	325	502	177	
·6 "	200	505	305	280
	250	505	255	
·7 "	165	510	345	342·5
	175	515	340	

We conclude that a straight line passing through the origin represents approximately the dependence of V upon S.

We have also obtained observations for a continued discharge. They are very regular in themselves, but their interpretation or correction is difficult. They indicate a straight line not passing through the origin, of much smaller inclination to the axis of S than the straight line for the single discharge.

Relative Difference of Potential required to produce a Continued Discharge.—
Conditions same as above.

S. Distance between Disc and Plate.	Deflection. n.	Zero. n'.	Difference. $n' - n$.
1 centimetres,	465	510	45
·2 "	470	535	65
·3 "	470	565	95
·4 "	470	588	118
·5 "	470	610	140
·6 "	465	625	160
·7 "	...	...	...

It is evident from the above table that the index before becoming steady fell back to the same reading always, and that the zero reading went on increasing. Though the ball in connection with the electrometer was touched, the image remained permanently at the place it took up when the contact of the conductors was made. The outside of the insulated wire was observed to be charged. The pressure was diminished to 625, to keep the receiver firmly in one position against the shaking of the Holtz machine.

The electric strength of oil of turpentine at 625 mm. pressure was found about double that of air at atmospheric pressure.

THE DISRUPTIVE DISCHARGE OF ELECTRICITY,

A. Macfarlane, D.Sc., and P. M. Playfair, M.A.

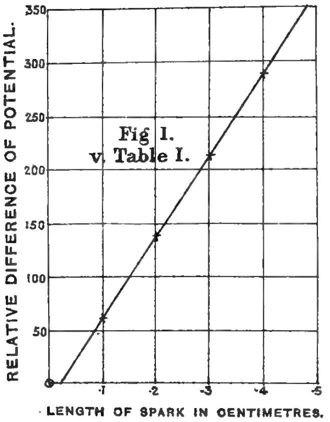


Fig 1.
v. Table I.

Dielectric, Paraffin Oil, Electrodes, Discs. Pressure of Atmosphere.

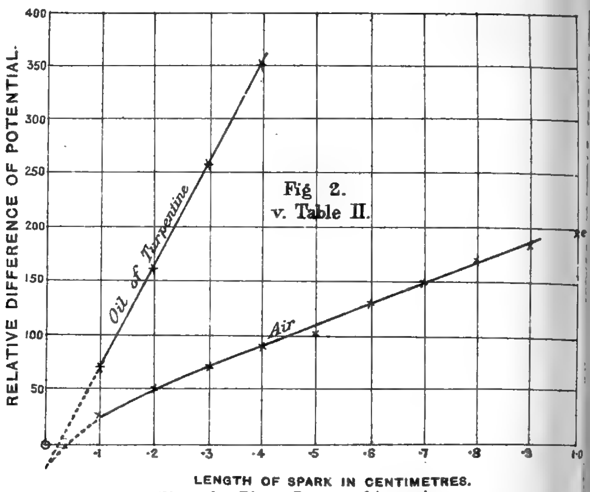


Fig 2.
v. Table II.

Electrodes, Discs. Pressure of Atmosphere.

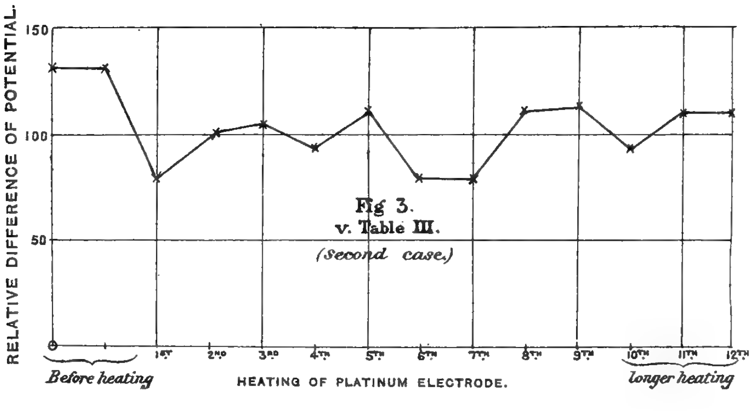


Fig 3.
v. Table III.
(second case.)

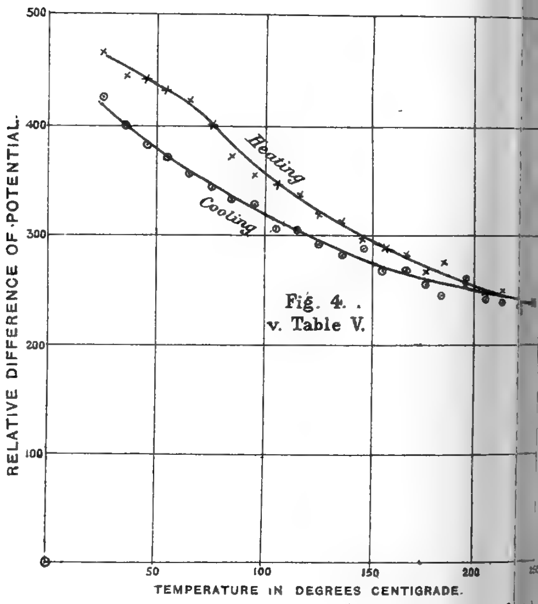


Fig. 4.
v. Table V.

Dielectric, Air. Electrodes, Discs. Length of Spark ·9 Centimetres. Pressure of

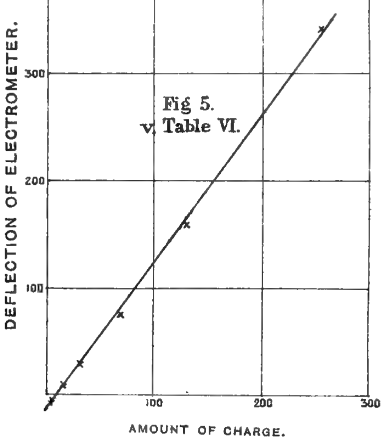


Fig 5.
v. Table VI.

XXV.—*On the Disruptive Discharge of Electricity.* By ALEXANDER MACFARLANE, D.Sc., and P. M. PLAYFAIR, M.A.

(Read 1st July 1878.)

During the months of May and June of this session we have endeavoured to investigate certain questions suggested by our experience of the discharge of electricity through the gases and through oil of turpentine,* for which purpose Dr MACFARLANE had received a grant from the Royal Society of London.

Discharge through Liquid Dielectrics.

When paraffin oil (the kind employed for illuminating purposes) was put into a glass vessel, in which were two brass plates arranged in the form of a condenser, and when the plates were charged by means of the Holtz machine, it exhibited the same phenomena as oil of turpentine. Gas bubbles were produced; they did not appear until after the passage of the spark. Once produced, they facilitated the passage of the spark through bringing the electrified surfaces virtually nearer to one another. Hence, when taking observations of the difference of potential required to pass a spark through layers of different thickness of the oil, it was always necessary to remove the bubbles generated by the passage of one spark before electrifying again. This was effected by bringing the discs into contact.

The axis of the bubble in the direction of the lines of force was observed to become elongated before discharge. When the charge was positive, the bubbles were attracted to one surface; and when negative, to the other. They were generally attracted to the positive surface, but sometimes to the negative. We have not been able to detect the condition on which this difference of behaviour depends. In the case of oil of turpentine, the attraction was always to the negative plate. The attraction was more marked when no jars were on the conductors of the Holtz machine. The gas liberated is a hydrocarbon, and there is a deposition of carbon simultaneously.

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* "On the Disruptive Discharge of Electricity," by ALEX. MACFARLANE, M.A., B.Sc., "Trans. R.S.E.," vol. xxviii. p. 633; and "On the Discharge of Electricity through Oil of Turpentine," by the same author and R. J. S. SIMPSON, "Trans. R.S.E.," vol. xxviii. p. 673.

Measurement of the Difference of Potential required to pass a Spark through a Liquid Dielectric at the Atmospheric Pressure between Parallel Metal Plates at Different Distances.

TABLE I.—Paraffin Oil, 14th June 1878.

Length of Spark in Centimetres. *s.*	Deflection. *n.*	Zero. *n′.*	Difference of Potential. *n′—n.*	Mean Difference of Potential.
·1	405 390	458 "	53 68	60·5
·2	315 325	456 458	141 133	137
·3	245 245	" "	213 213	213
·4	160 180	" "	298 278	288
·5	Escape.			

It was impossible to obtain the reading for ·5 centimetres; because the escape of electricity from the positive conductor caused the image to be driven off the scale before any discharge through the liquid took place. These observations when plotted (see diagram 1) lie very exactly on a straight line, which, however, has a small negative intercept on the axis of ordinates. From them we deduce—

$$V = 750\, s - 15,$$

where V denotes relative difference of potential, and s the length of the spark. Hence—

$$R = 750,$$

that is, the electrostatic force is constant. In absolute measure R = 364 C.G.S units.

On repeating our experiments with oil of turpentine, we got observations agreeing with the above and with our former results.

TABLE II.—Oil of Turpentine, 18th June 1878.

Length of Spark in Centimetres. s.	Deflection. n.	Zero. n'.	Difference of Potential. $n'-n$.	Mean Difference of Potential.
·1	383 390	458 "	75 68	71
·2	300 290	" "	158 168	163
·3	200 200	" "	258 258	258
·4	110 105	" "	348 353	350
·5	—40[*]	290		

Before putting in the liquid we took readings for sparks through the air, and we have plotted them along with the above in diagram 2. The curve for turpentine is precisely similar to that for paraffin. We deduce—

$$V = 922\,s-20,$$

where V denotes the relative difference of potential, and s the length of the spark. Hence—

$$R = 922,$$

that is, the electrostatic force is constant. In absolute measure R = 338 C.G.S. units.

Thus, in the case of a liquid, the electrostatic force is constant; while in the case of a gas it is variable; and this is, probably, because the liquid cannot be condensed on the surfaces of the electrodes, while the gas can.

Effect of Heating the Electrodes upon the Passage of the Electric Spark.

For the purpose of studying minutely the effect of heating the electrodes upon the passage of the spark, we constructed an apparatus suggested by Professor CLERK MAXWELL. Two pieces of thick platinum wire, $p\ p$ (fig. 1), were insulated in a brass plate b, by means of vulcanite plugs v, and placed so

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[*] Escape from the connections of the positive conductor of the Holtz began.

as to be in planes at right angles to one another and to the brass plate. The shortest distance between the wires is 4 mm. The figure represents the apparatus suspended by means of the upper wire from one conductor of the Holtz machine, and the lower wire connected with the positive conductor. Only the wire which was not at the time bearing the weight could be heated without introducing an alteration of the distance between the wires. The heating was effected by bringing the terminals of a battery of four Bunsen elements, arranged in multiple arc, into contact with the platinum wire, at a distance of an inch apart. The following series of observations is representative of all made :—

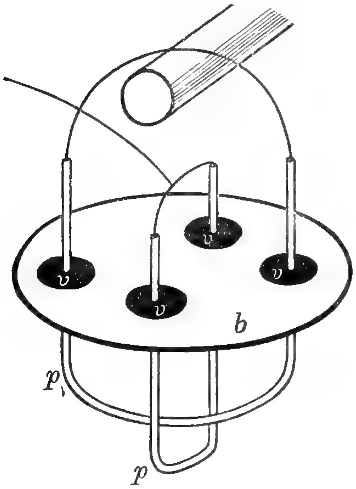


Fig. 1.

TABLE III.—Single Sparks between Platinum Wires, 21st June 1878.
Shortest distance between wires, 4 mm.

		Deflection. n.	Zero. n'.	Difference.	Mean.
Lower Wire, Negative and Heated	Before heating,	340	463	123	124·3
	„	338	462	124	
	„	335	461	126	
	Heated, . . .	335	460	125	122·3
	„	335	462	127	
	„	335	460	125	
	„	345	„	115	
	„	345	„	115	
	„	335	459	124	
	„	335	460	125	
Lower Wire, Positive and Heated	Before heating,	335	460	125	125
	„	335	„	125	
	Heated, . . .	385	„	75	93·3
	„	365	„	95	
	„	355	„	105	
	„	368	„	92	
	„	349	„	111	
	„	385	„	75	
	„	385	„	75	
	„	355	„	105	
	„	353	„	107	
allowed longer time to cool	„	370	„	90	100
	„	355	„	105	
	„	355	„	105	
Upper Wire, Positive and Heated	Before heating,	345	460	115	115
	„	„	„	„	
	„	„	„	„	

TABLE III.—*continued.*

		Deflection. n.	Zero. n'.	Difference.	Mean.
Upper Wire, Positive and Heated	Heated, . . .	370	460	90	92·5
	”	365	”	95	
	”	365	”	95	
	”	370	”	90	
Upper Wire, Negative and Heated	Before heating,	338	460	122	122
	”	”	”	”	
	Heated, . . .	375	”	85	84
	”	375	”	”	
	”	365	”	95	
	”	395	”	65	
	”	370	”	90	

The wire when heated was always made red-hot, and the charging was made as soon after the heating as possible. A greater time elapsed between heating and charging in the first case than in the three others, owing to a less convenient arrangement of the battery; which probably accounts for the comparatively small effect. The observations of case second are plotted on diagram 3. They appear to show that the effect of a heating had disappeared before the time of the next heating, for they indicate a line parallel to the axis of x.

When a continued spark was taken instead of a single, a similar diminution of the deflection was observed.

TABLE IV.—Continued Spark between Platinum Wires, 26th June 1878.
Shortest distance between wires, 4 mm. Positive wire operated on.

	Deflection. n.	Zero. n'.	Zero when Touched.	Difference of Potential. $n'-n$.	Mean Difference of Potential.
Before heating,	390	535	495	145	140
”	380	530	”	150	
”	405	540	500	135	
”	400	530	”	130	
Heated, . . .	390	525	”	135	133
”	390	530	495	140	
”	380	505	”	125	
Heated longer,	415	515	”	100	98
”	415	515	”	100	
”	415	510	”	95	

This diminution of the difference of potential required to effect the discharge must be due to a change at the surface of the wire; for the air between the wires cannot be so greatly rarified by the heating of the wire as to produce the effect.

Measurement of the Difference of Potential required to Pass a Spark through Air at Different Temperatures, the Pressure being Constant.

To investigate this question we constructed the vessel represented in fig. 2. A glass cylinder, *c*, fits into two brass plates, *p*, by means of grooves. The brass discs, *d*, which were those generally used in the experiments, were screwed on, the one to a brass rod rising from the lower plate, the other to a rod which moves inside a tube fixed to the upper plate. The upper plate contains an orifice for the purpose of allowing air to escape when the lower plate is heated, and also a hole for the insertion of a thermometer. The lower plate was put to earth, and the upper charged by being kept in contact with a projecting conductor of the Holtz. The heating was effected by means of a powerful Bunsen burner placed below the lower plate, and was carried on till the plate became red-hot.

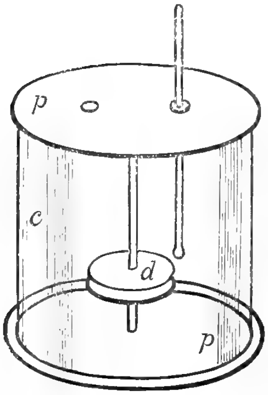


Fig. 2.

Table V.—Spark through Air at Different Temperatures, 6th June 1878. Electrodes, discs. Length of Spark, ·9 centimetres. Pressure, atmospheric.

	HEATING.			COOLING.		
Temperature in Degrees Centigrade. *t*.	Deflection. *n*.	Zero. *n'*.	Difference of Potential. *n'*−*n*.	Deflection. *n*.	Zero. *n'*.	Difference of Potential. *n'*−*n*.
25	−3 0	462 465	465 465 } 465	40 38 30	463 460 453	423 422 } 423 423

TABLE V.—*continued.*

Temperature in Degrees Centigrade. t.	HEATING.				COOLING.			
	Deflection. n.	Zero. n'.	Difference of Potential. $n'-n$.		Deflection. n.	Zero. n'.	Difference of Potential. $n'-n$.	
35	20	465	445	445	50	465	415	400
	20	,,	445		50	460	410	
					80	455	375	
45	55	485	430	442·5	60	452	392	386
	45	500	455		70	450	380	
55	55	485	430	430	84	460	376	372
	55	,,	430		92	,,	368	
65	45	468	423	420	93	455	362	357
	65	482	417		103	,,	352	
75	78	460	382	401	105	450	345	345
	35	455	420		105	,,	,,	
85	90	450	360	371	128	460	332	333
	78	460	382		125	,,	335	
95	105	453	348	352	115	450	,,	328
	100	457	357		123	445	322	
105	98	450	352	347	135	,,	310	303
	115	458	343		148	,,	297	
115	120	455	335	336	145	450	305	302
	120	458	338		150	,,	300	
125	135	450	315	319	165	455	290	290
	135	458	323		165	,,	,,	
135	125	440	315	311	180	470	,,	280
	145	453	308		195	465	270	
145	192	490	298	298 ×	188	475	287	286
	220	500	280		183	468	285	
155	170	450	280	290	205	460	255	266
	150	,,	300		180	456	276	
165	160	440	280	282	190	,,	266	265
	155	,,	285		192	,,	264	
175	195	455	260	267	198	455	257	256
	180	,,	275		200	,,	255	
185	175	450	275	276	228	460	232	246
	172	,,	278		200	,,	260	
195	200	,,	250	256	210	465	255	260
	188	,,	262		195	460	265	
205	210	455	245	248	212	455	243	243
	208	460	252		212	,,	,,	
215	210	462	252	251	208	,,	247	241
	230	480	250		220	,,	235	
225	208	460	252	252	210	460	250	237
	210	463	253		235	,,	225	
235	240	460	220	240 ×	220	450	232	231
	230	470	240		218	,,	230	
245	220	455	235	240				
	220	,,	,,					
	210	460	250					

We noted that after the conductors were fully discharged and the image had ceased to oscillate, it made an excursion in the negative direction, to the distance of ten divisions or so in the interval between the pairs of observations. These observations are plotted on diagram 4. The ordinates of the cooling curve are less than the corresponding ordinates of the heating curve—a result confirmed by two other series of observations. The falling off cannot be entirely due to a leaking of the charge of the electrometer; for readings taken after twenty-four hours showed that not more than one-third of the difference could be so accounted for. The bulb of the thermometer, being in the place indicated by fig. 2, must in each case have given the temperature of the air between the discs; hence the diminution was probably due to the temperature or other state of the discs.

Several series of observations were undertaken to determine whether the electrometer, by means of which all these observations were made, gives deflections which are strictly proportional to the inducing charge. A special method was required on account of the want of sensitiveness of the electrometer, and the considerable range of scale used. Acting upon Professor TAIT's suggestion, we put a charge upon the inducing ball of the pair on the stand, observed the deflection on the scale, then divided the charge by putting a similar and equal ball into contact with the ball on the stand, and then removing the auxiliary ball, read again, and so on. We obtained observations, all of which agree with the following:—

TABLE VI.—Calibration of Electrometer, 28th June 1878.

	Zero. n'.	Deflection. n.	Difference. $n'-n$.
Before division, . . .	505	165	340
After 1st division, . .	...	345	160
After 2d division, . .	...	430	75
After 3d division, . .	...	473	32
After 4th division, . .	...	493	12
After 5th division, . .	...	503	2
After 6th division, . .	...	509	−4

When the inducing ball was connected with the ground the zero was 515; when the electrometer was connected with the ground, 505.

When the entries of column fourth are plotted as ordinates to the numbers 256, 128, 64, 32, 16, 8, 4, as abscissæ (see diagram 5), they are seen to indicate a straight line, the intercept of which on the axis of ordinates is slightly negative, a peculiarity borne out by all our other series of observations. The readings indicate a straight line so exactly, that we may safely infer that the deflections of the image from zero are strictly proportional to the charge on the inducing balls.

We had arranged to complete the calibration of the electrometer on Thursday forenoon (27th June); but as the deflection on the scale, when the dividing ball was brought into contact, always fell rapidly in the negative direction, and went to a great distance beyond the proper zero, it was impossible to proceed. The indications were the same as when negative electricity is escaping into the surrounding air; and that such was the case then was evidenced by the severe thunderstorm which came on soon after.

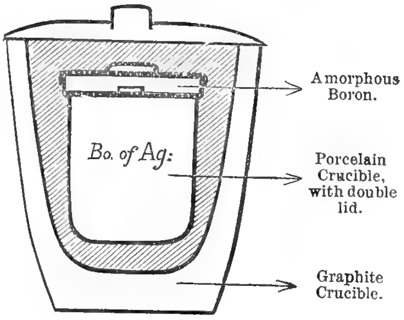
Our thanks are due to Professor TAIT, in whose laboratory these experiments were made, for many valuable suggestions.

XXVI.—*The Preparation and Properties of Pure Graphitoid and Adamantine Boron.* By R. M. MORRISON, D.Sc. (Edin.), and R. SYDNEY MARSDEN, B.Sc. (Edin.).

(Read 17th June 1878.)

In preparing a specimen of diamond boron by WÖHLER and DEVILLE'S method, the idea struck us of substituting silver, gold, or some other metal in place of the aluminium. We thought that by so doing we might be able to save considerable expense if the silver would dissolve the boron; because in the case of aluminium the byproducts are useless, whereas in the case of silver the byproducts are easily reconverted into metallic silver. On making an experiment with silver in place of aluminium, we found that it did dissolve boron, and that on dissolving the silver away with nitric acid a slate-grey powder was left behind. This powder, on being examined after purification, was found to be boron, and to contain nothing but boron; under the microscope, it presented a complex composition, some being transparent and some opaque,—the opaque portion consisting of hexagonal prisms exactly like those of graphite; the transparent portion being composed of plates and a few small tetrahedra. The quantity obtained was but small, so we repeated our experiment several times, at last obtaining sufficient to examine the properties of these crystals.

The yield of each experiment varied considerably, owing to variations in the manner in which each experiment was conducted. This point we will consider after discussing the method of preparation. The method which we employed in conducting these experiments is as follows:—The silver in small lumps and the amorphous boron were placed in alternate layers in a porcelain crucible, which was fitted with a double lid to keep it perfectly air-tight—the one fitting inside the crucible and the other outside—the intermediate space being filled with amorphous boron to prevent oxidation of the boron to be dissolved. Care was taken that the silver did not touch the sides of the crucible, and that the mass was well packed to prevent subsidence. This crucible was then placed inside a plumbago crucible, and the intermediate space was filled with pure sugar charcoal to burn any air which might be forced in by the blast. The



whole is then introduced into a gas furnace and heated to the temperature of boiling silver, and this temperature was kept up for five hours. During this time the silver melts and becomes quite saturated with the boron, which crystallises out on cooling. Special precautions were taken to allow of very gradual cooling, and by means of another furnace, heated to a bright red heat and which fitted exactly round the gas furnace, we found it possible to prolong the process of cooling over some six or seven hours, thus giving every facility for the formation of crystals.

On opening the plumbago crucible after the operation, care was taken not to break the porcelain crucible inside, which was then itself broken open. We were particularly careful not to get any of the crucible mixed up with the contents inside. These contents were found to be: 1st, a lump of metallic silver; and 2d, a slatish-grey powder.

The silver was well cleaned and dissolved in moderately strong nitric acid, and yielded from its interior a quantity of a steel-grey crystalline powder. This powder was then examined in order to ascertain its composition and properties. 1st, From its mode of preparation the only substances which could be present are boron and silver, and though improbable, owing to precautions taken, silica and alumina from the crucible. 2d, A portion was oxidised by means of fused nitre, and the mass dissolved in acid and tested for silver, but not even a trace was found. In a similar manner we have also proved silica and alumina to be absent; nor were we able to detect anything except boron. These crystals, therefore, must be pure boron, partly graphitic and partly adamantine. Thus boron is found to have three forms, viz.: amorphous, graphitoid, and adamantine; although the so-called graphitic boron of Wöhler and Deville has been found to be a boride of aluminium, and the adamantine to contain carbon and aluminium. The properties of this powder are as follows:—

1st, It is infusible at a white heat.

2d, It is not oxidised by air at a white heat, even superficially.

3d, It is completely oxidised, though slowly, by fused nitrate of potash.

4th, It is completely oxidised by nitric acid when treated with it for twenty-four hours at a temperature of 90°-100° C.

5th, It is completely volatilised when heated with a mixture of hydrofluoric and strong sulphuric acids.

6th, It does not alloy with platinum at a white heat.

7th, As a powder it is a non-conductor of electricity.

We have not yet had the opportunity of taking its specific heat.

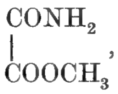
We have also tried a number of experiments, not only with silver, but also with the following metals: lead, tin, copper, antimony, bismuth, iron, magnesium, and zinc; and have obtained some very interesting results, but we have not as yet finished that research. The success of the boron experiments led us to try with carbon also, and the results have quite equalled our expectations. We are at present engaged with these experiments, and hope at some future time to lay the results before the Society.

XXVII.—*On a New General Method of Preparing the Primary Monamines, &c.* By R. Milner Morrison, D.Sc.

(Read 17th June 1878.)

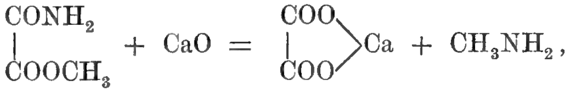
Having occasion at one time to require a quantity of methylamine, I tried a number of the best methods given in text-books and books of reference, but the smallness of the yield and the great trouble involved in the processes drew my attention to the fact that there was room for a good method of preparing the primary monamines. Hofman's method, viz., heating the iodides of the alcoholic radicals with ammonia, is of little use, as the trouble of separating the diamines and triamines formed at the same time is so great as to make the process practically useless if the primary monamines alone are required.

Keeping this fact before me, I tried for some years, in the intervals of more systematic work, to find the solution of this problem, and I think I have now found a process which, at the same time simple and inexpensive, gives a large yield of the primary monamines. This result has not been arrived at without several failures, and as I think that even negative results are often useful, I shall briefly refer to them. In the first place, together with Professor E. A. Letts, I tried to obtain methylamine from oxamate of methyl. As oxamate of methyl has the following constitutional formula, viz.—



$$\begin{array}{l} CONH_2 \\ | \\ COOCH_3 \end{array},$$

we expected that, if treated with lime, it would break up thus—

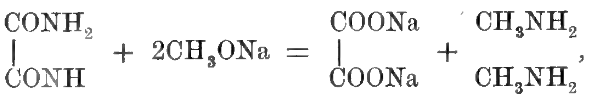


$$\begin{array}{l} CONH_2 \\ | \\ COOCH_3 \end{array} + CaO = \begin{array}{l} COO \\ | \\ COO \end{array}\!\!>Ca + CH_3NH_2,$$

Oxalate of Calcium. Methylamine.

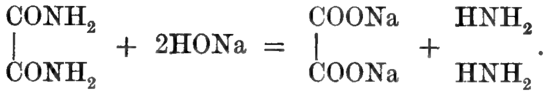
or rather the products of this reaction, conducted as it was at a high temperature, namely, carbonate of calcium, carbonic oxide, and methylamine. The reaction, however, does not go in this way, but in another and very curious way, which we need not at present consider.

Following out this idea, it struck me that if oxamide be treated with methylate of sodium, that methylamine would be the result. Thus—



$$\begin{array}{l} CONH_2 \\ | \\ CONH \end{array} + 2CH_3ONa = \begin{array}{l} COONa \\ | \\ COONa \end{array} + \begin{array}{l} CH_3NH_2 \\ \\ CH_3NH_2 \end{array},$$

as oxamide treated with caustic soda yields ammonia according to this reaction—



$$\begin{array}{l}CONH_2\\ |\\ CONH_2\end{array} + 2HONa = \begin{array}{l}COONa\\ |\\ COONa\end{array} + \begin{array}{l}HNH_2\\ \\ HNH_2\end{array}.$$

This reaction, however, though very pretty on paper, does not take place. I did not investigate the products of the reaction further than to assure myself that not a trace of methylamine was produced; but the reaction has been studied by some one else, who published his results about the same time that I worked on it.

Giving up all hope of arriving at what I wanted from this line of investigation, I next tried to obtain methylamine from the double sulphate of methyl and ammonium, a salt comparatively easily prepared. I first tried to obtain methylamine by acting on the double sulphate with caustic potash, expecting that the reaction would go in this manner- -

$$\begin{array}{l}CH_3\\ NH_4\end{array}SO_4 + 2KHO = \begin{array}{l}K\\ K\end{array}SO_4 + CH_3NH_2 + 2H_2O,$$

but only a very small trace of methylamine is thus produced. As the result of experiments conducted afterwards, I have found that though this is a very bad method for methylamine, yet, as we go higher in the series, a better yield is obtained; thus, if the double sulphate of ammonia and ethyl be employed, a small portion of ethylamine is produced, considerably larger than in the case of the corresponding methyl compound; and, to go higher still, the amyl sulphate of ammonium gives a proportionately very much larger yield. This method, however, is by no means satisfactory, as even under the most favourable circumstances but little of the substance is obtained.

The method, however, looked so promising that I was very unwilling to give it up, and therefore after some time tried a slight, but as it afterwards proved most important, variation. I employed, instead of the caustic potash or hydrate of potassium, unslaked lime, or oxide of calcium. I obtained in this manner a gas which, after being caught in hydrochloric acid, the liquid evaporated to dryness gave a chloride which was soluble to the extent of about one-half in absolute alcohol, and which, when heated with alkalies, gave off a gas which was inflammable. These being the qualitative tests for methylamine, I was led to repeat my experiment on a larger scale, the more so as the yield seemed to be larger. The weight of hydrochlorate produced in one operation I found to be about one-half of the weight of methyl-sulphate of ammonium taken and as nearly one-half of this was soluble in absolute alcohol, this represented 25 per cent. of hydrochlorate of methylamine of the substance taken; for chloride of ammonium, though not absolutely insoluble, in absolute alcohol is said to be totally so, if even a trace of hydrochlorate of methylamine be present.

Having prepared a quantity of hydrochlorate of methylamine in this way, I next tried to see if any dimethylamine or trimethylamine was formed at the same time. By the quantitative analysis of the hydrochlorate it is, however, proved that this is not the case, and this is confirmed by the analysis of the double chloride of the base and platinum.

In the first case the chlorine was estimated, and gave these results—

1st,	Per cent.	of chlorine	formed,	53·5
2d,	„	„	„	53·7
3d,	„	„	„	53·3

The calculated percentage of chlorine in hydrochlorate of methylamine is 52·7, that of ammonia is 66·35. This points to the conclusion that the substance was hydrochlorate of methylamine, containing a small quantity of ammonium chloride, but probably no hydrochlorate of di- or tri- methylamine, as in that case the percentage of chlorine would have been below instead of above the calculated amount; besides, it is hard to imagine how the reaction could possibly go in order to produce any di- or tri- methylamine. These inferences were borne out by the analyses of the platino-chlorides, as the numbers obtained closely approached the theoretical, always, however, tending to the assumption that some ammonium chloride was present.

Having thus obtained methylamine by this method, I tried if the other monamines could be prepared in the same manner. I have obtained ethylamine, propylamine, isobutylamine, and isoamylamine by this method; but, curiously enough, as we ascend the series the percentage of the double sulphate which is converted into hydrochlorate becomes smaller, so that the process is best adapted for the preparation of methylamine and ethylamine. This, however, is exactly the reverse of HOFMAN'S process, which in the case of methyl and ethyl gives not only methyl- and ethyl- amines, but also the di- and tri- amines; whereas, as we ascend the series a greater proportion of the monamines are produced, until, in fact, only the monamines are produced.

In conclusion, I may sum up the advantages of the process and its disadvantages.

It is simple, inexpensive, gives little trouble, and the yield is very good. The chief disadvantage is that the double sulphates of the alcohol radicals and ammonium are very deliquescent, and the presence of water decreases the quantity of amine obtained.

I append analyses of a number of the salts made by this method.

A portion of the crude hydrochlorate was exhausted with alcohol, filtered, and evaporated to dryness on the water-bath, and the purified salt so obtained treated with a small quantity of boiling absolute alcohol to get rid of the last traces of chloride of ammonium. This solution was precipitated with chloride of platinum, the precipitate washed with alcohol, dried in the water-bath, and then—at 120° C.—

·591 grms. taken, after ignition left ·2365 grms. platinum = 40· per cent.
·067 „ gave . . . ·0275 „ „ = 41·04 „

and at a different time—

1·0155 grms. left ·4355 grms. platinum = 42·6 per cent.
1·2236 „ „ ·528 „ „ = 42·7 „

but these were made from hydrochlorate not so thoroughly freed from chloride of ammonium. By calculation—

Ammonium chloro-platinate gives 44·18 per cent. Pt.
Methylamine „ „ 41·68 „ „
Dimethylamine „ „ 39·36 „ „
Trimethylamine „ „ 37·49 „ „

A portion of the twice purified hydrochlorate was taken for the estimation of the chlorine, which was done volumetrically. 1 c.c. of the silver solution = ·001 grms. chlorine—

·051 grms. required 27·5 c.c. = 53·9 per cent. Cl.
·08 „ 44· „ = 53·7 „ „
·11825 „ 63· „ = 53·3 „ „

Per cent. of chlorine by calculation in hydrochlorate of methylamine = 52·74; per cent. of chlorine in chloride of ammonium = 66·35.

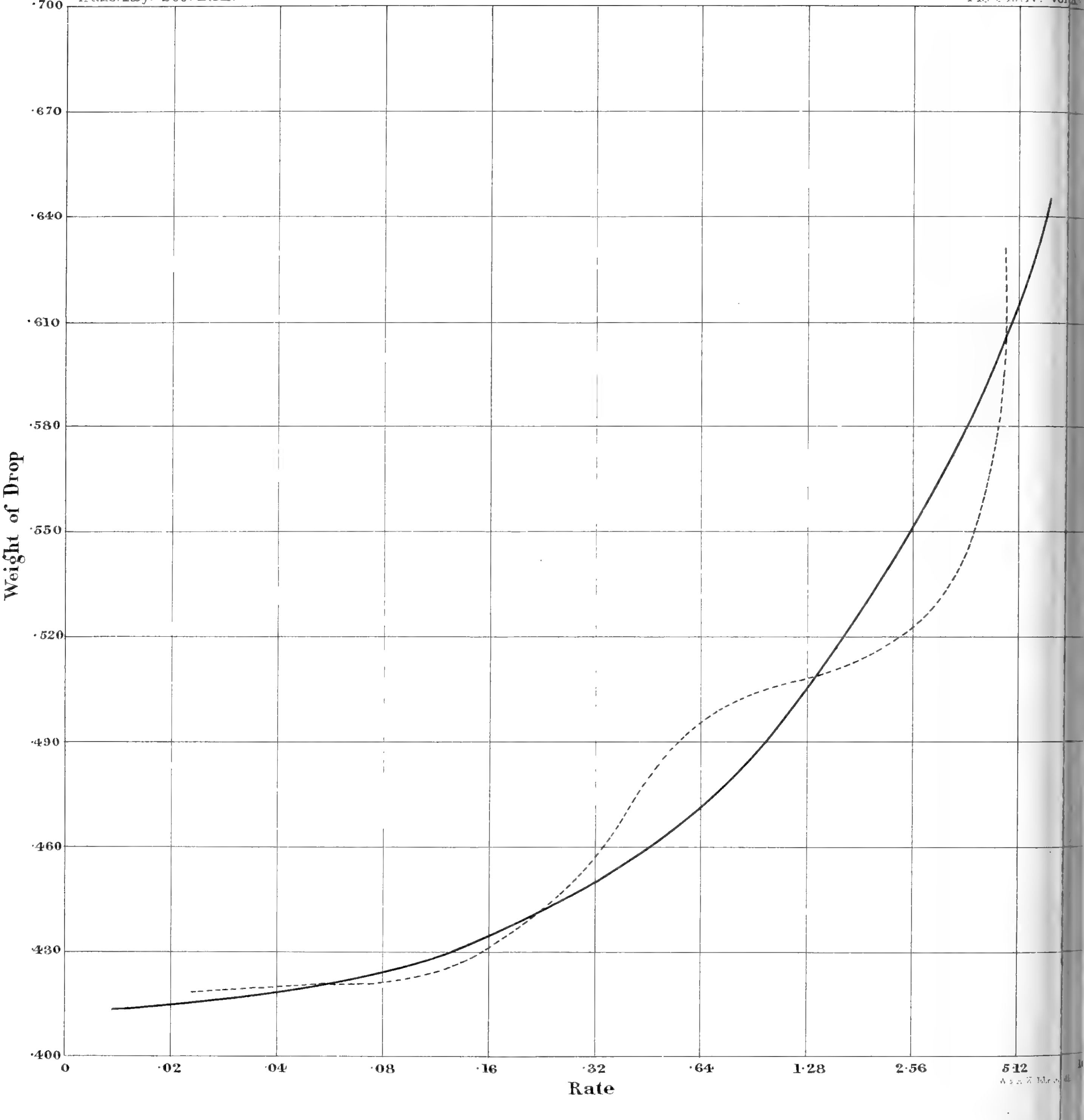
Of the hydrochlorate of ethylamine made by this process—

·114 grms. required 50·4 c.c. = 44·21 per cent. Cl.
·107 „ 47· „ = 43·83 „ „
·113 „ 50· „ = 44·2 „ „

By calculation, the percentage of chlorine in hydrochlorate of ethylamine = 43·55.

·1895 grms. of the ethylamine chloro-platinate gave ·077 grms. platinum = 40·7 per cent. Calculated percentage, 39·36.

·24 grms. of the isobutylamine salt gave ·087 grms. platinum = 36·2 per cent. Pt. Calculated percentage, 35·4.


·700
·670
·640
·610
·580
·550
·520
·490
·460
·430
·400
Weight of Drop
0
·02
·04
·08
·16
·32
·64
1·28
2·56
5·12
Rate

XXVIII.—*On a Method of Determining the Cohesion of Liquids.*

By J. B. Hannay, F.C.S. (Plate XXIV.*)

(Read 17th December 1877.)

While working at an investigation (about to be published), relating to the flow of liquid through capillary tubes, it was thought that the cohesion of a liquid might influence the rate of its flow, and a method was sought for, which would enable the cohesion of any liquid to be determined; the ordinary method of calculating the cohesion from observations of the flattening of a drop of the liquid resting upon a plane surface not being applicable to all liquids. The method which seemed most likely to yield results was that of observing the drops of a liquid so arranged that they fell from the end of a column of the same liquid, and accurately measuring the width of the neck of the drop at the instant of breaking, and the weight of the fluid which caused the rupture. It is easily seen that before this could be done a complete knowledge of the nature of drops would be required, and as the phenomenon of dropping has been made the subject of a long investigation by Professor Guthrie, an examination of his results was undertaken. They show that the drop increases in weight as the growth-time decreases, that is, the quicker the rate of flowing the larger the drop. Many experiments were made, which, agreeing with Professor Guthrie's numbers pretty closely, need not be quoted here. While, therefore, his experimental numbers may be taken as quite correct, they do not seem to prove the theory he has put forward in their explanation. The following is from his paper:—"The most prominent fact is that, on the whole, the drops undergo a continuous diminution in weight or size as G.t. increases. To such an extent is this the case, that the most rapidly falling drops of the above table are nearly twice as heavy as the most slowly falling ones. The cause of this is probably to be sought for in the circumstance that when the flowing to the solid is more slow, the latter is covered with a thinner film of liquid, so that, as the drop parts, the solid reclaims by adhesion more of the root of the drop than is the case when the adhesion of the solid to the liquid can satisfy itself from the thicker film which surrounds the drop in the case of a more rapid flow." It appeared that the correctness of this theory might be experimentally tested by the following method:—Suppose mercury dropping from a column of that liquid in a glass tube, of such dimensions that on being slightly inclined or shaken the mercury will run out, so that the glass acts only as a wall to retain the mercury as a column; this would fulfil the conditions of a liquid dropping from itself, and as there would be no solid to reclaim by adhesion a portion of the root of the drop, if Professor GUTHRIE's theory were correct we ought to have the same weight of drop whatever the rate. This experiment has been tried, and the result is that the increase is just as great as when dropping from a solid sphere, as in Professor GUTHRIE's experiments—a result which shows that the theory above given does not explain the phenomenon. The following theory is more consistent with facts:—There seem to be two causes for the increase in the size of the drop with the increase in the rate of dropping; the first being that, as the neck of the drop is a tube conveying liquid into the drop, the faster the flow the more liquid will run through that neck in a given time, and as there is always an interval between the beginning and the end of the breaking of the neck of the drop, the quicker the rate of flow the more liquid enters the drop during the act of breaking. The second is that the quicker the rate of flow the longer is the time of breaking, because the "stump" of the drop follows the old drop down before finally breaking, so that the quicker the rate of flow the more liquid enters the drop during the act of breaking, because the tube has a longer lifetime. Concisely stated, then, the quicker the rate the greater is the flow of liquid through the breaking neck, and the longer does that flow continue. It may not, perhaps, seem of much importance which theory is true, since the facts are the same; but in the investigation of cohesion the truth or falsity of either of these theories is the all-important consideration. It is easily seen that there must be a certain weight of drop which is sufficient to break the neck of the drop, and that this weight does not vary with the rate, but is always the same for the same width of neck, and for this reason may be called a "normal" drop. If Professor GUTHRIE's theory be followed to its legitimate conclusion, there is no such thing as a normal drop, as according to him a slow drop is an imperfect one, a part of which has been torn back by the attraction of the solid, so that we could only have by that theory a normal drop when the rate is infinitely quick, or, in other words, when we have a stream. It is apparent that by this theory the breaking weight of a drop cannot be arrived at; but, according to the theory above substituted, the drop is normal when its growth-time is infinitely long, that is, when there can be no flow into the drop through its neck when breaking. If we accept this theory, then we can, from experimental data, calculate the size of a normal drop, as we can determine the rate of decrease of weight with the decrease of rate, and reduce the rate to zero, when we shall have a normal drop. As it was necessary that the temperature should be constant, the following apparatus was planned for the work:—Fig. 1. The mercury was placed in the bulb A, whose contents were accurately ascertained, and allowed to run out at any required rate by the stop-cock; B, whose handle, C, travelled along a graduated scale D. The bore of the plug of the stop-cock was merely a narrow slit, instead of the usual round hole, so that the motion of the handle made only a slight increase or decrease on the flow. In this way the rate could be regulated to a nicety. If necessary, pressure could be applied through the tube E, while the tube F gave the lower part of the apparatus free access to the atmosphere. The tube from which the liquid drops was so adjusted that it just held the mercury as a column, so that it exerted no attractive influence upon the liquid. By means of this apparatus the dropping of liquids may be examined in various atmospheres, as by the introduction of another tube, like F, a current of any gas or vapour may be passed through the apparatus. It was found that with the above arrangement, if the drop fell from a sufficient height to be audible, the vibration of the apparatus interfered with the size of the drops, and when a high rate was used, if the droppings were not audible, the eye was quite incapable of counting correctly. The dotted curve in the plate shows the variation in the weight of the drop, owing to the vibration of the apparatus when the dropping of the mercury was audible. The explanation of this is, that when the drop is nearly normal, or about to break, should the swing be upwards it gains time, and consequently the drop is larger than it should be, and when the swing is downwards it breaks off the drop before it has attained its normal size. It was thus seen that for high rates of dropping an apparatus would be required in which the drop might fall outside the apparatus, so as to be audible without shaking, or that it might be counted automatically. The latter method was ultimately adopted, and the apparatus, of which the following is a sketch, shows the method used. It must be remembered, however, that this apparatus could only be used for ordinary temperatures, as the drop would

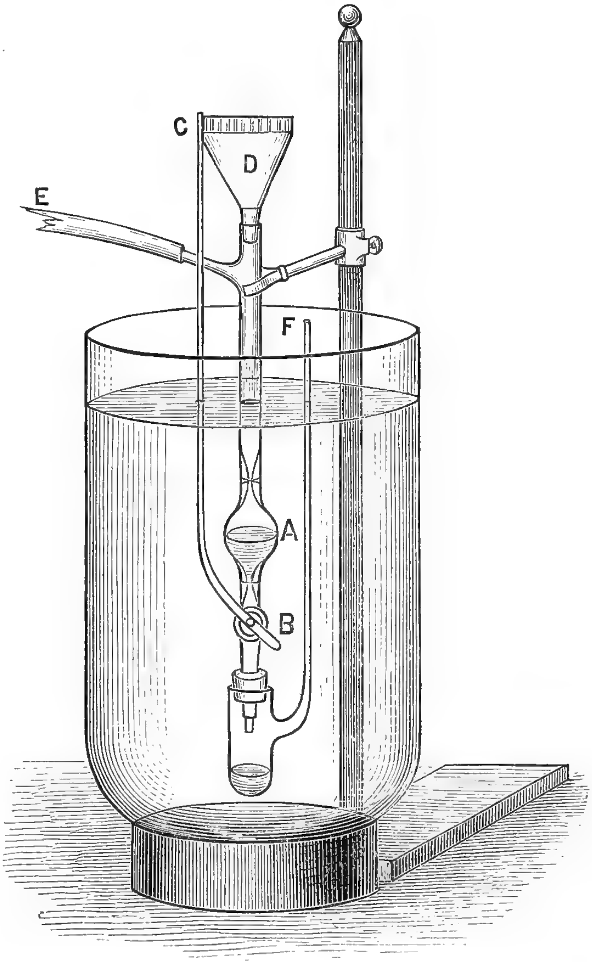


Fig. 1.

cool before breaking if high temperatures were used; so the apparatus in fig. 1 was used for temperature work, and that in fig. 2 for rate work. This apparatus has a pen at A, attached to which is a little tray, and both are held

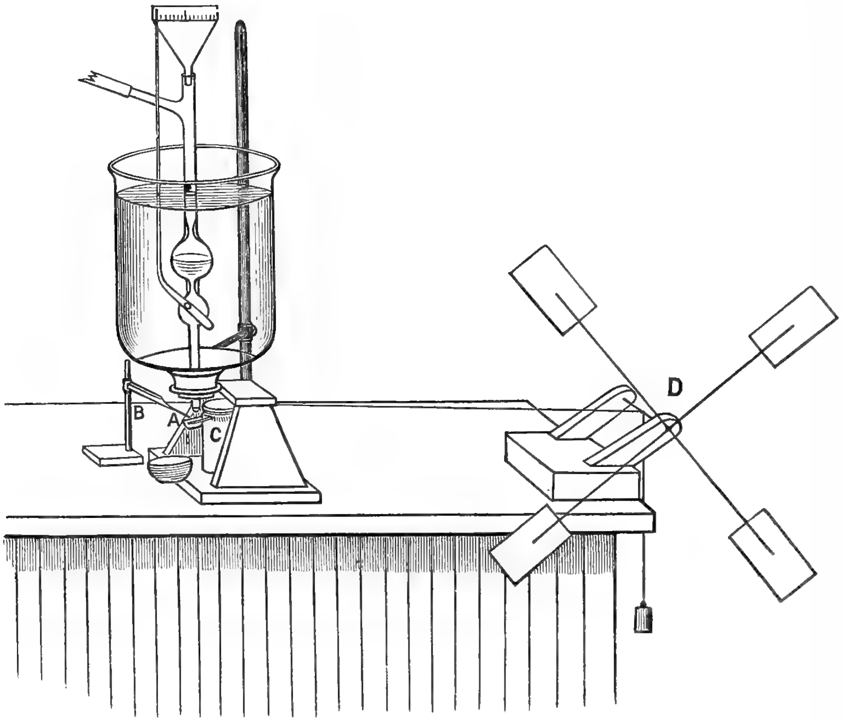


Fig. 2.

at the end of a spring B, which is twisted at right angles at the middle, so that it is both a vertical and horizontal spring. It is fixed, pressing against the cylinder C, and when the latter is rotated it draws a straight line round the paper coiled on the drum; but when a drop falls on the tray it bends the spring and makes a depression in the line, and by the number of depressions the number of drops is registered. Each drop, after striking the tray, falls into the little basin placed below. The rate of rotation is regulated by the fly D, which can be regulated to any resistance. With these two apparatus the investigation can be carried on with great ease and certainty, the only disturbing cause left being that when quick rates are used, the breaking of the neck communicates a vibratory motion to the forming drop, which slightly affects its size at different rates. The following table gives the numbers obtained while experimenting with mercury at the ordinary temperature :—

G.t.	Wt.	R.	G.t.	Wt.	R.
42·30″	·4136 grm.	·0096 grm.	0·49″	·4903 grm.	1·001 grm.
23·00	·4150 ,,	·0180 ,,	0·40	·5004 ,,	1·252 ,,
15·00	·4157 ,,	·0277 ,,	0·39	·5008 ,,	1·301 ,,
12·80	·4178 ,,	·0326 ,,	0·36	·5049 ,,	1·414 ,,
9 60	·4214 ,,	·0438 ,,	0·35	·5115 ,,	1·421 ,,
7·55	·4227 ,,	·0612 ,,	0·32	·5138 ,,	1·608 ,,
5·40	·4239 ,,	·0785 ,,	0·27	·5230 ,,	1·934 ,,
4·80	·4265 ,,	·0889 ,,	0·25	·5365 ,,	2·144 ,,
4·50	·4306 ,,	·0957 ,,	0·22	·5382 ,,	2·450 ,,
3·50	·4300 ,,	·1229 ,,	0·218	·5475 ,,	2·509 ,,
2·70	·4345 ,,	·1609 ,,	0·192	·5514 ,,	2·859 ,,
2·05	·4347 ,,	·2120 ,,	0·175	·5601 ,,	3·216 ,,
1·90	·4435 ,,	·2334 ,,	0·164	·5719 ,,	3·476 ,,
1·40	·4494 ,,	·3210 ,,	0·157	·5742 ,,	3·676 ,,
1·00	·4598 ,,	·4598 ,,	0·143	·5917 ,,	4·150 ,,
0·80	·4623 ,,	·5788 ,,	0·131	·6003 ,,	4·594 ,,
0·78	·4719 ,,	·6053 ,,	0·124	·6300 ,,	5·054 ,,
0·56	·4851 ,,	·8662 ,,	0·111	·6325 ,,	5·716 ,,
0·52	·4838 ,,	·9305 ,,	0·100	·6431 ,,	6·431 ,,

In this table Professor GUTHRIE's nomenclature is used, calling the time between the falling of the drops growth-time or G.t., calling w the weight of mercury the bulb holds at the temperature at which the experiment is made to the time required to empty the bulb, and n the number of drops. We have—

$$\text{G.t.} = \frac{t}{n},$$

$$\text{Wt.} = \frac{w}{n},$$

$$\text{R.} = \frac{w}{t}.$$

The subjoined curve illustrates graphically the above table, and the formula $\cdot 4130 + \cdot 0833r - \cdot 0074r^2$ is one which agrees pretty well with the results. It will be seen that the above table does not represent a very regular curve; but there are places where the increase in weight is less than what would be expected, so that no formula will exactly fit the curve. These numbers are indeed experimental irregularities, caused by the vibration of breaking being continued in the new drop, and affecting it as before explained, and they only occur where the rate of dropping and vibration are in some way in agreement. It must also be remembered that a drop changes in form after it is formed, so that this may in some way affect the results. When the rate is very slow, the first error disappears; and as the second must be inconsiderable, a minute examination of the rate of decrease at very long growth-times would be likely to yield a more correct formula, but the above serves to illustrate the method.

The neck of the drop was measured at the breaking-point, and the measurements were made as free from error as possible by having the growth-time very long, and following in the decreasing neck, till it was seen to give way. The average of a large number of readings gave 3·395 mm. as the diameter of the neck at 16° C. From rad. ^{2} × π, we find 9·0526 square millimetres as the area of the neck, and from $\frac{\text{weight}}{\text{area}}$, we find a breaking strain of 0·0456 grm. per square millimetre of liquid. Experiments made with varying temperature show the general result that the cohesion decreases with a rise of temperature faster than the density; but the difference is so slight, that a very minute investigation would require to be made before a formula could be given.

FIG 54. H. Distribution of Steam & Speed as for E; but connecting rod reduced to $3\frac{1}{2}$ times Stroke.

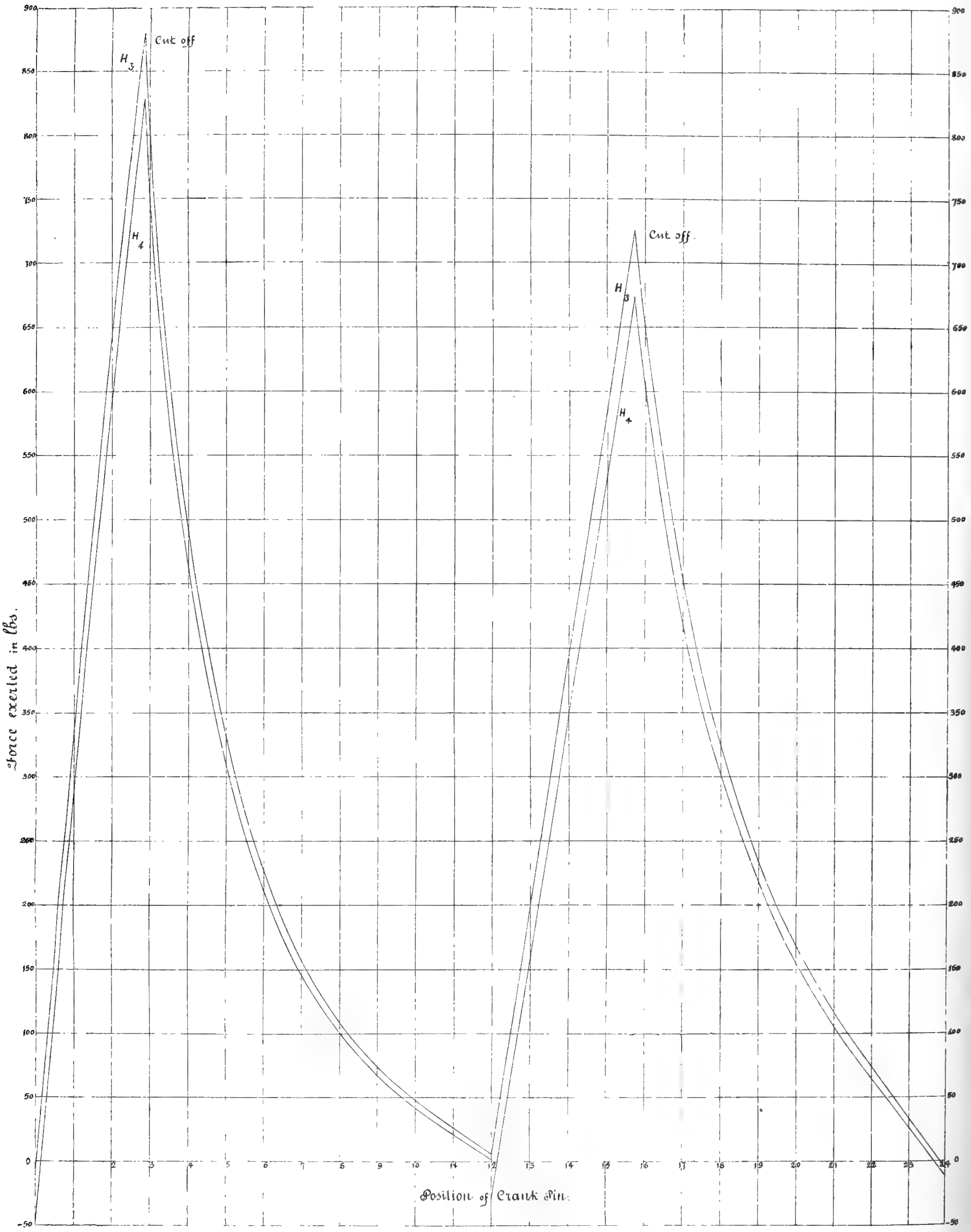


FIG 53. G. Speed infinitely slow – $p = 50$ lbs $r = 5\cdot93$. $p_3 = 3$ lbs.
$p_m - p_3 = 18\cdot905$ lbs per Square inch

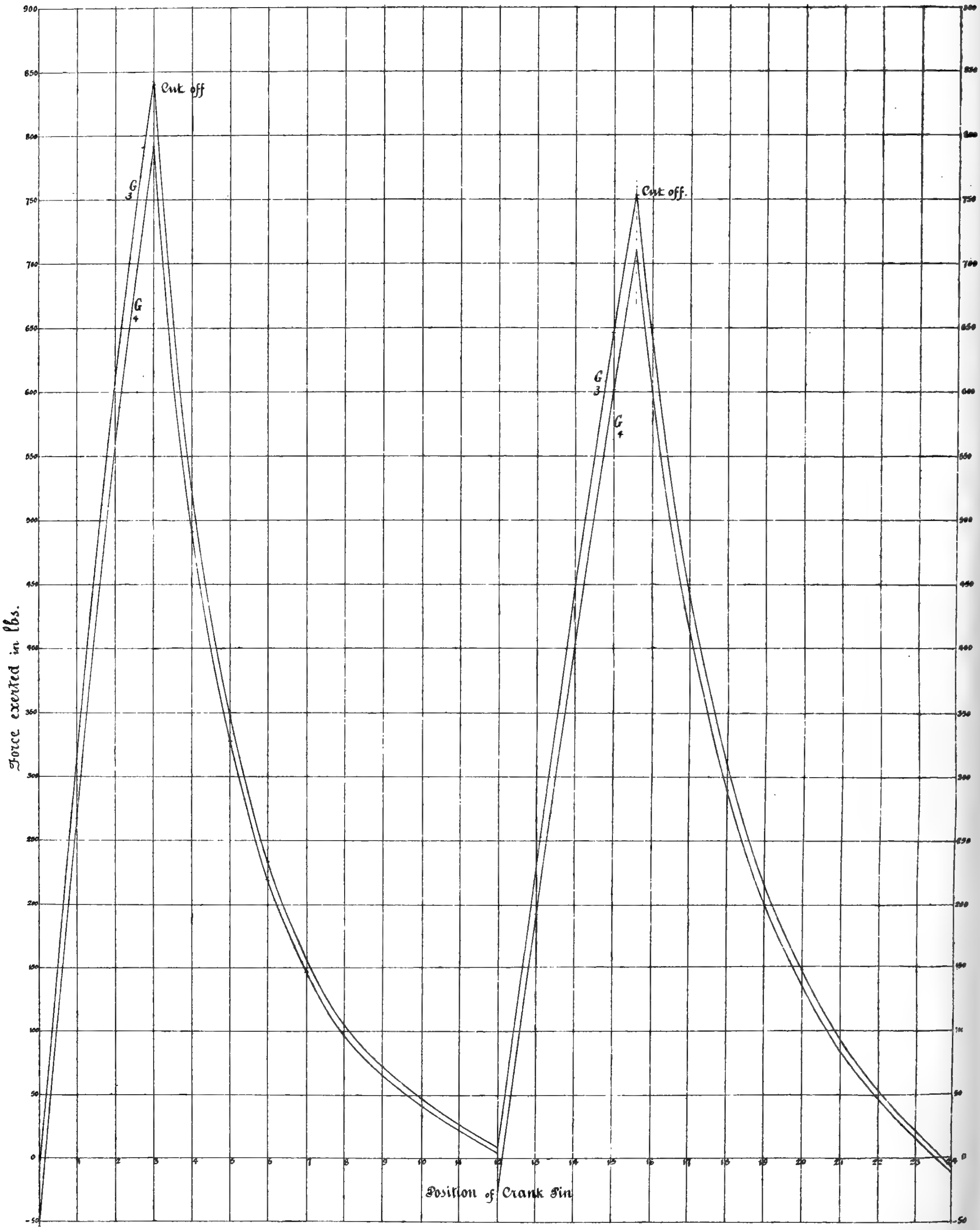


FIG 52. F. Speed 4 Rev. per Sec., $p_1 = 50$ lbs., $r = 5{\cdot}93$, $p_3 = 3$ lbs., giving $p_m - p_3$ 18·905 lbs. per sq. in.

F_1 same as E_1; F_2 same as E_2, F_3 loaded no Friction; F_4 loaded and Friction.

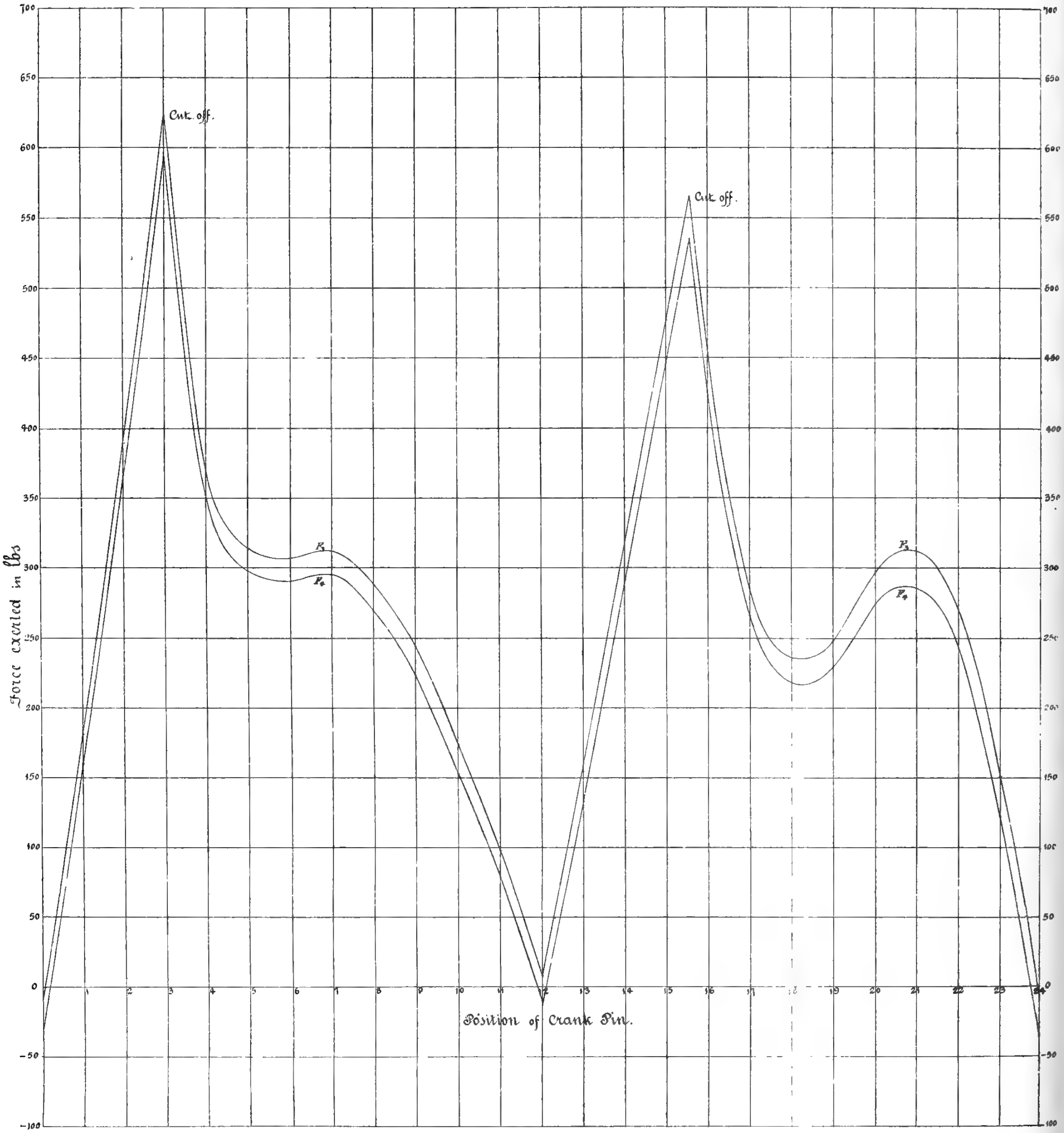


FIG 51. E. Speed 1 Rev. per Sec., p_i = 50 lbs., r = 5·93, p_3 = 3 lbs., giving $p_m - p_3$ = 18·905 lbs. per sq. in.
E_3 loaded no Friction, E_4 loaded and Friction.

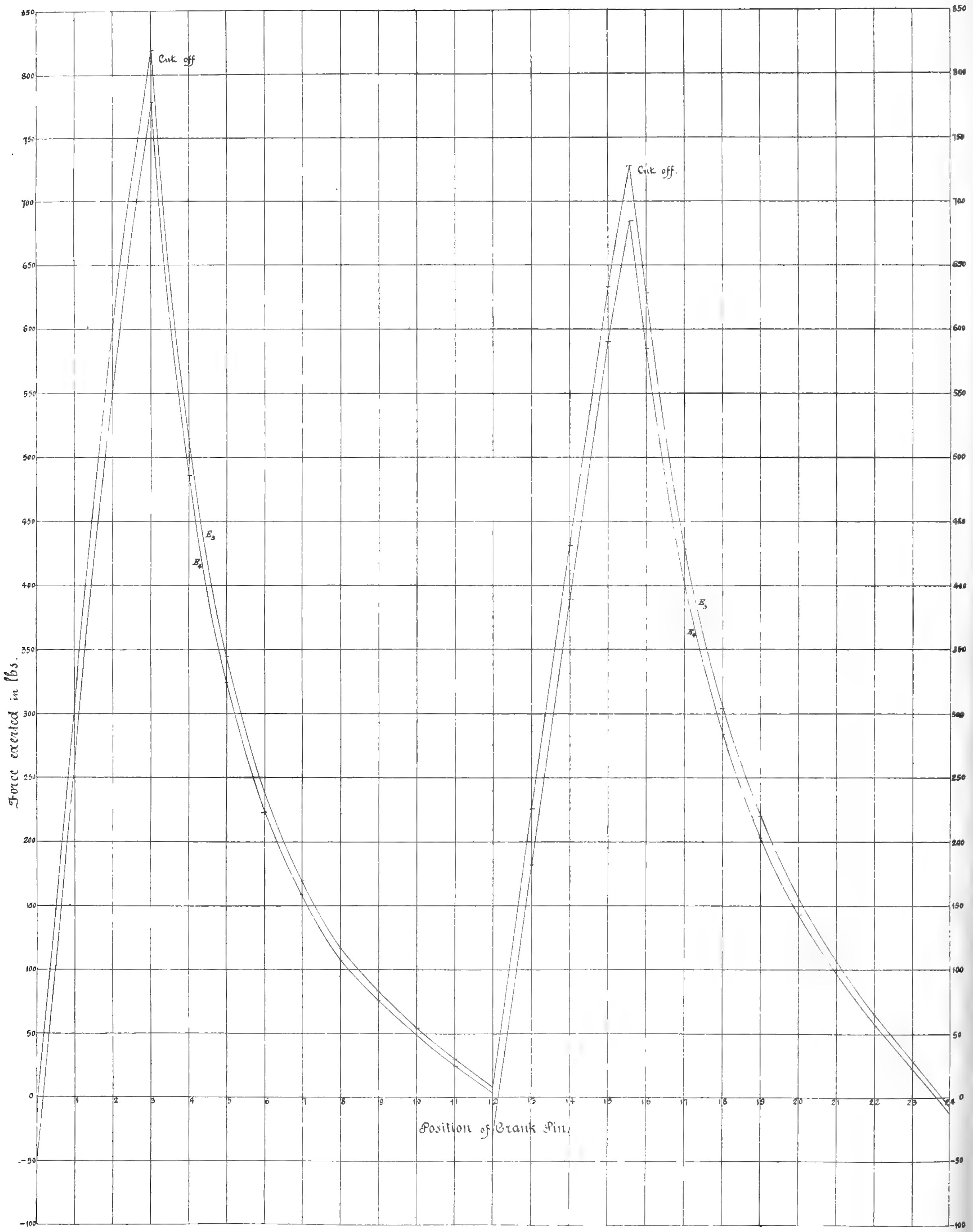


FIG 50. E Speed 1 Rev. per Sec., $p_i = 50$ lbs., $r = 5{\cdot}93$, $p_3 = 3$ lbs., giving $p_m - p_3 = 18{\cdot}905$ lbs. per sq. in.
E_1 unloaded, no Friction, E_2 unloaded and Friction.

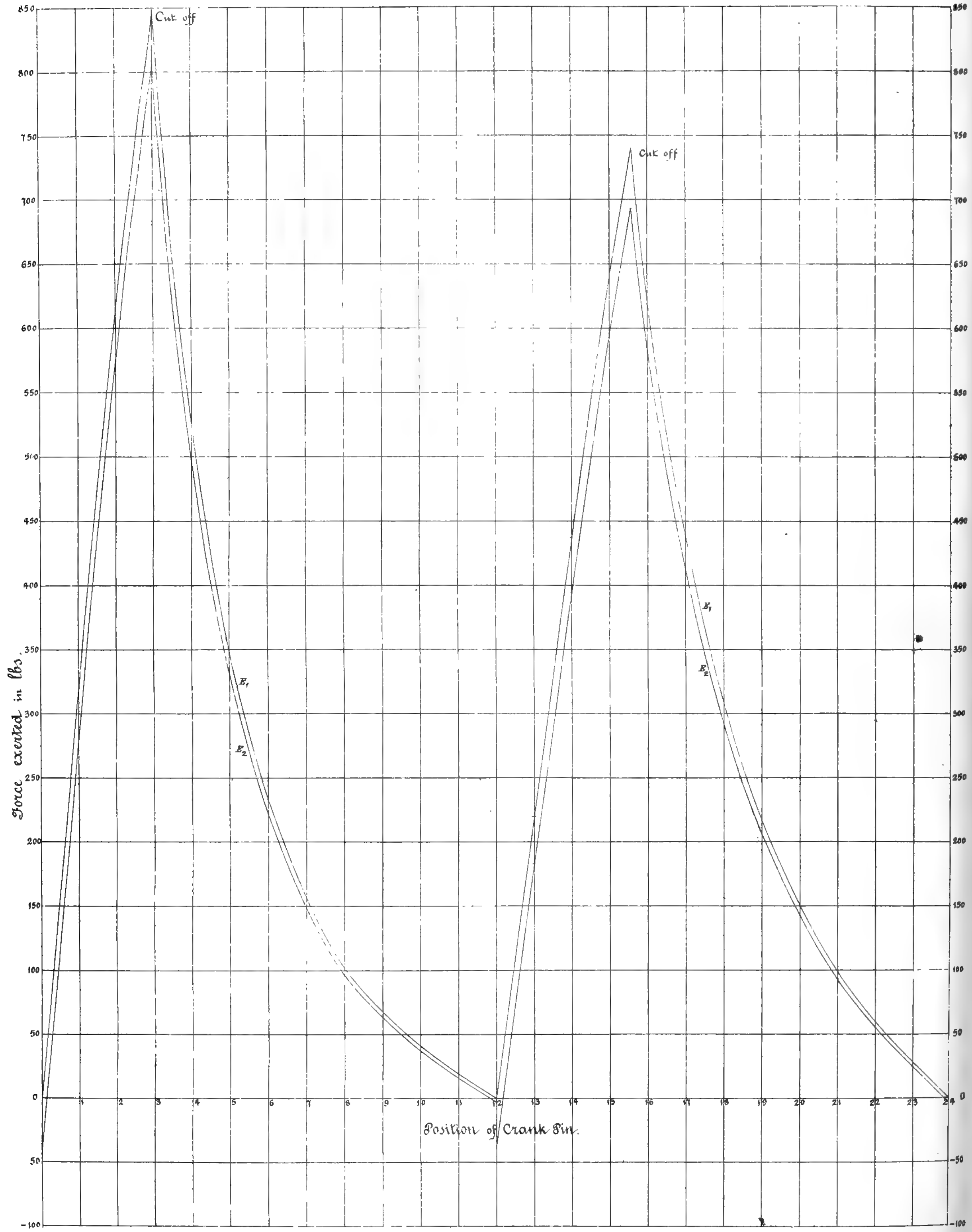


FIG 49. D. Speed 4 Rev. per Sec., $p_1 = 10$ lbs., $r = 12{\cdot}8$, $p_3 = 0{\cdot}516$, giving $p_m - p_3 = 2$ lbs. per sq. in.

D_1 same as C_1 D_2 same as C_2, D_3 Loaded, no Friction; D_4 Loaded, and Friction.

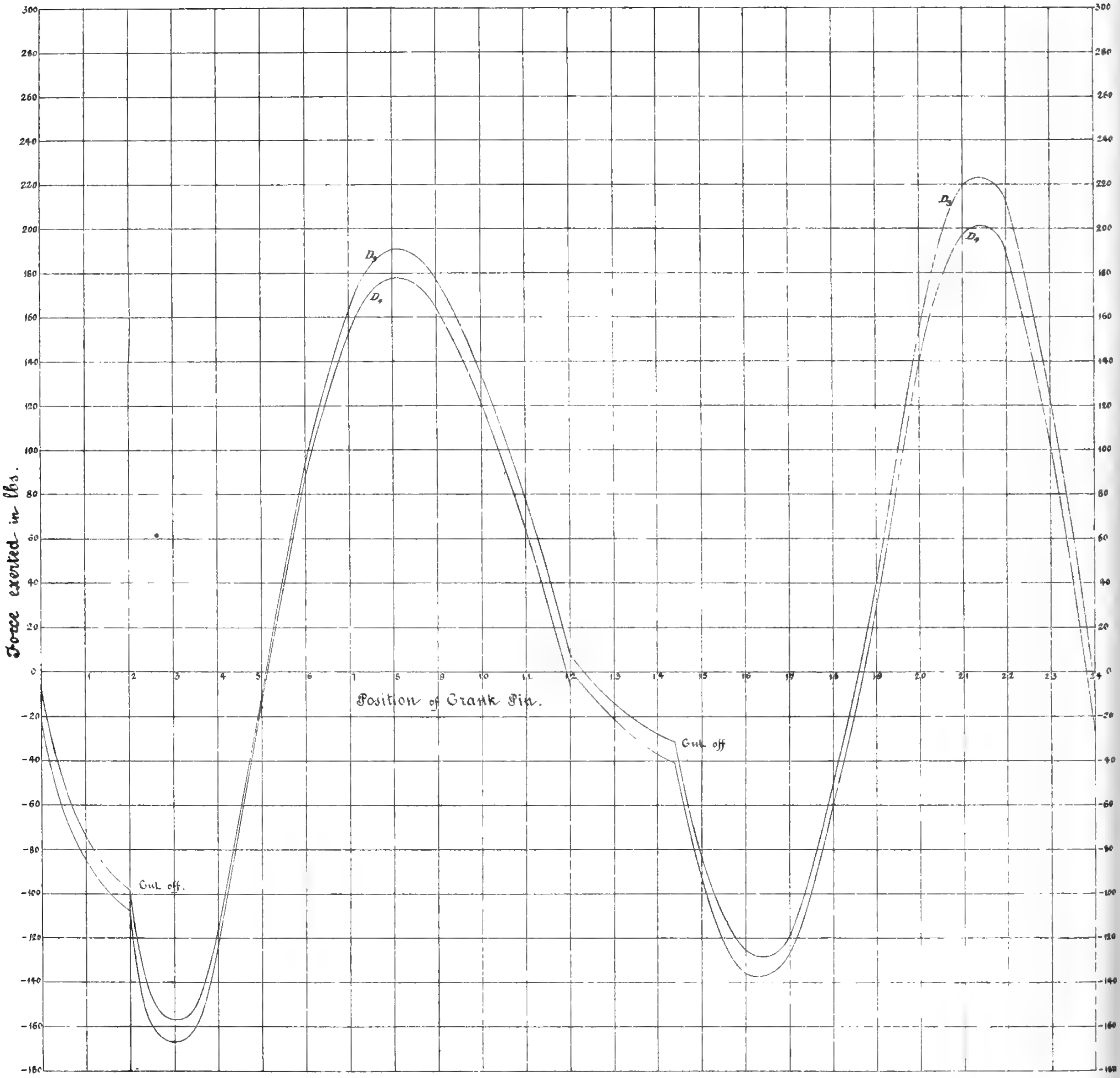


FIG 47. B. Speed 4 Rev. per Sec. Uniform effective pressure $p_m - p_3 = 2$ lbs. per sq. in.
B_1 same as A_1; B_2 same as A_2; B_3 loaded no Friction, B_4 loaded and Friction.

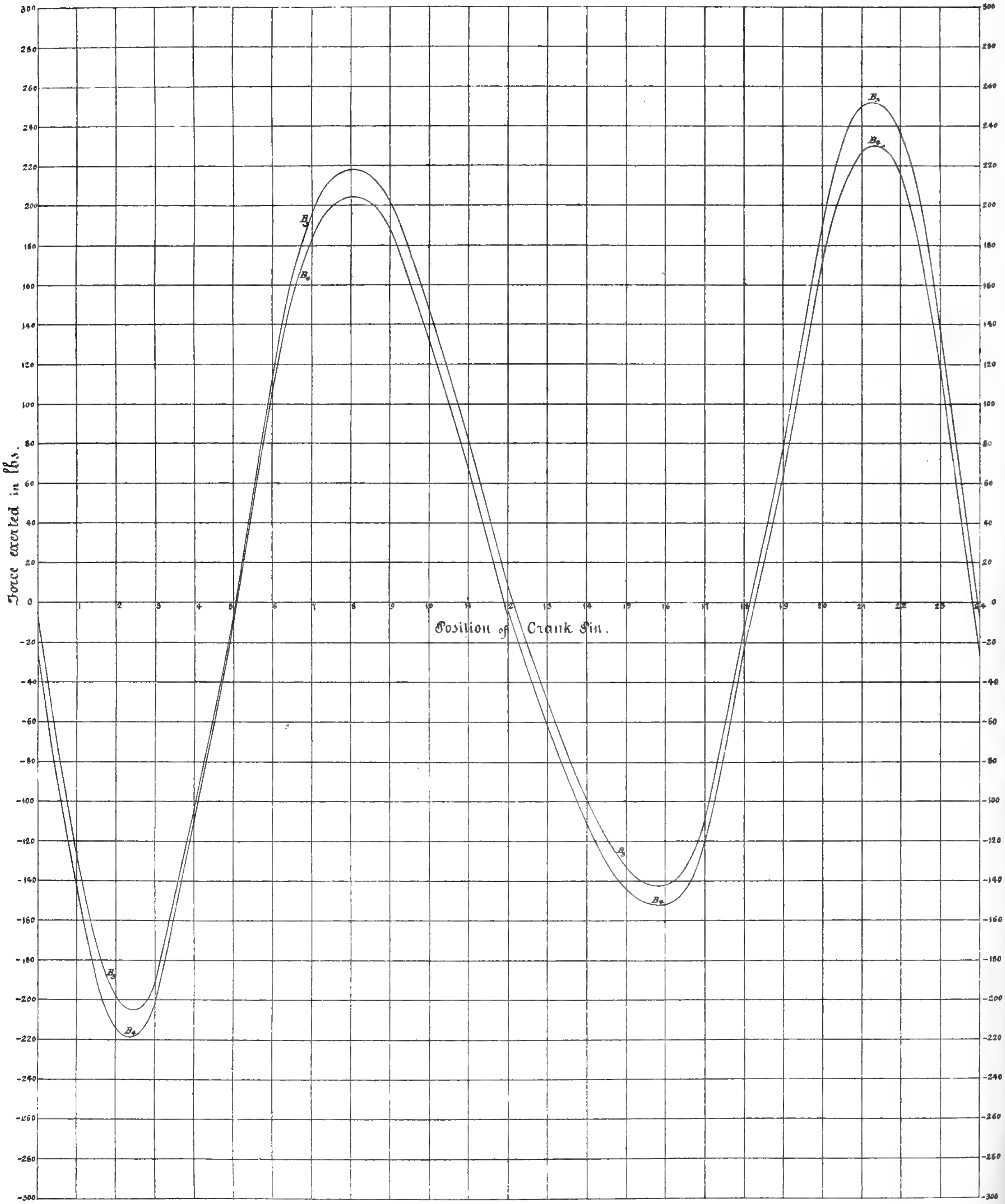
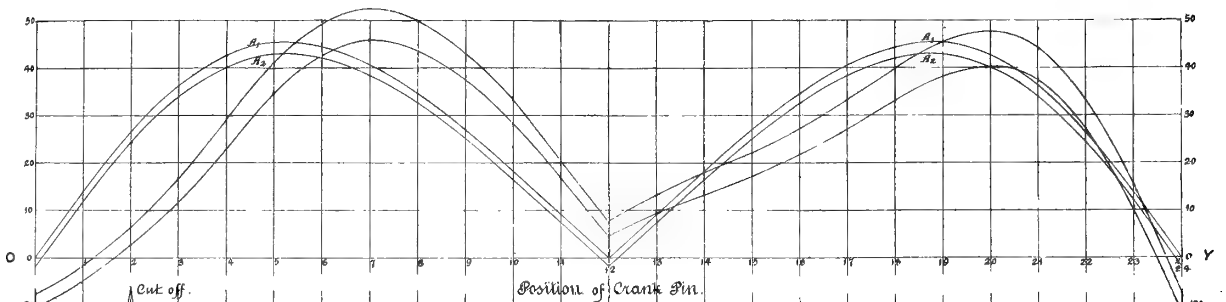


FIG 45. A. Speed 1 Rev. per Second, Uniform effective pressure $p_m - p_3 = 2$ lbs. per sq. in.

A_1 unloaded no Friction; A_2 unloaded and Friction; A_3 loaded no Friction; A_4 loaded and Friction.



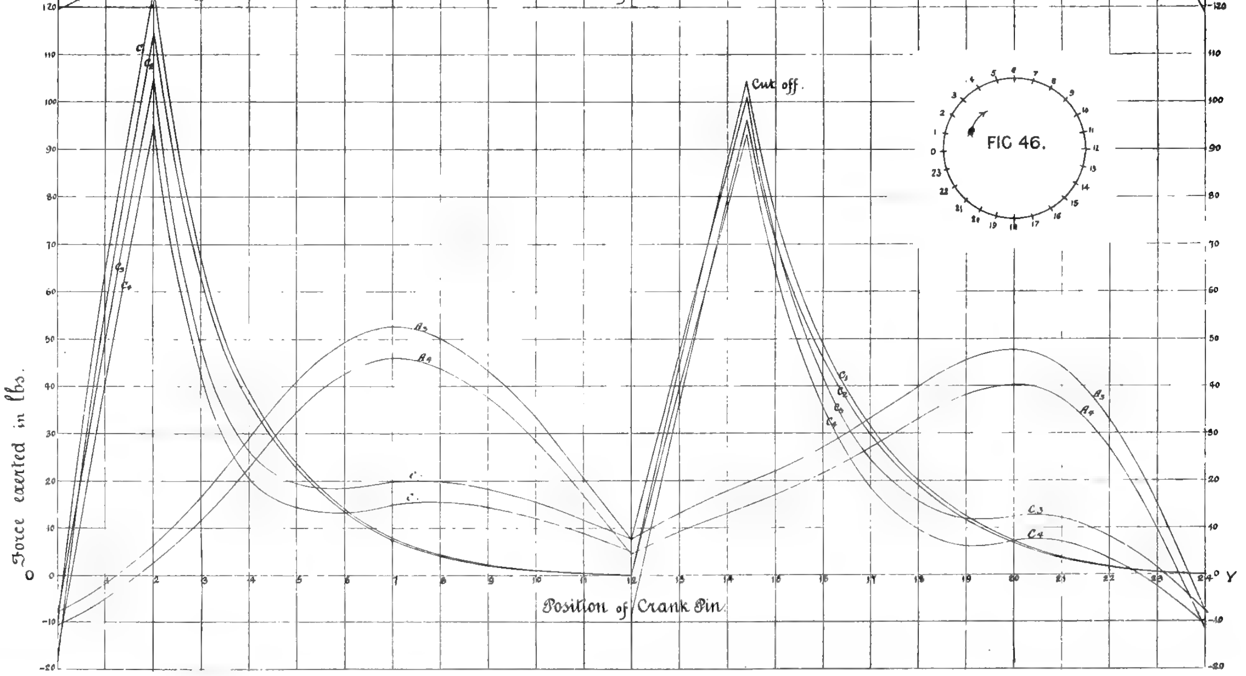


FIG 48. C. Speed 1 Rev. per Sec., $p_1 = 10$ lbs., $r = 12{\cdot}8$; $p_3 = 0{\cdot}516$, giving $p_m - p_3 = 2$ lbs. per sq. in.

C_1 unloaded no Friction, C_2 unloaded and Friction, C_3 loaded no Friction, C_4 loaded and Friction.

XXIX.—*On the Application of Graphic Methods to the Determination of the Efficiency of Machinery.* By Professor FLEEMING JENKIN. (Plates XXVI.–XXXIII.)

(Read 18th February 1878.)

PART II.—*The Horizontal Steam Engine.*

§ 31. In a previous paragraph (29) the loaded dynamic frame (fig. 38) for one position of a direct-acting horizontal steam-engine has been described, and the mode of drawing that frame explained, on the assumption that the loads L_a, L_b, L_c, L_d, are known: it was stated that these loads had been calculated for one particular engine. The present paper gives the varying effort which this engine is capable of exerting at each part of its stroke with given pressures (both constant and varying) in the cylinder, and given *constant* velocities of the crank-axle. The same calculations show the efficiency of the engine at each part of the stroke, and its total efficiency under the same circumstances. This problem has, it is believed, never hitherto been solved so as to take into account all the circumstances of mass, weight, and friction.

§ 32. No pains has been taken in the choice of the particular example. The object of the paper is not so much to draw general conclusions as to all steam-engines—which, indeed, differ too much in arrangement to make this feasible—as to show how, by a method of no great complexity, full information can be obtained as to any particular engine or class of engines.

The following are the particulars of the engine:—Stroke, 16 in.; diameter of cylinder, 8 in.; length of connecting-rod, 41 in.; centre of gravity of connecting-rod, 20 in. from the crank-pin; mass of connecting-rod, 34 lbs.; mass of piston and piston-rod, 46 lbs.; mass of balanced crank and that part of the fly-wheel which is borne by the main bearings, 200 lbs.; the mass of the frame fixed to the earth is indefinitely large; diameter of crank-shaft in bearings, $4\frac{1}{2}$ in.; diameter of crank-pin, $2{\cdot}4''$; diameter of pin at crosshead, $1{\cdot}8''$. The position of the resistance overcome (fig. 17, Part I.) is that of a tangent to an imaginary circle of 18 in. radius, concentric with the crank-shaft, the tangent being so inclined as to cut the centre line of the engine $19{\cdot}2''$ from the crank-shaft centre on the side towards the piston. This line may be regarded as the direction of the resistance exerted at the teeth of a spur-wheel. The position of this line exercises a very material influence on the results obtained, which must not, therefore, as was said before, be considered as applicable to engines generally. The coefficient of friction has been taken as $= \frac{1}{12}$.

§ 33. Eight distinct cases have been investigated. In these examples the action of the engine has been studied on four different assumptions—1st, neglecting weight, mass, and friction; 2d, neglecting weight and mass; 3d, neglecting friction, but taking mass and weight into account; 4d, taking mass, weight, and friction into account. A comparison of the results shows the influence of each element of the problem on the final result. These processes are obtained by what in the first part of the paper are called—1st, the dynamic frame; 2d, the dynamic frame with friction; 3d, the loaded dynamic frame; 4d, the loaded dynamic frame with friction. The examples will be called A, B, C, D, E, F, G, and H; the four assumptions will be indicated by the suffixes 1, 2, 3, 4, and it must be borne in mind that the suffix 4 corresponds to the complete solution, in which all the elements of the problem are taken into account. The results are graphically shown in a series of curves which will be called "Effort curves." These were drawn to a very large scale and have been reduced by a photographicprocess.

§ 34. Example A.—Speed of engine, 1 revolution per second; effective pressure on cylinder, 2 lbs. per square inch—uniform throughout the stroke. This example has been chosen to indicate the effect of running the engine with a very low pressure and no expansion. The vertical ordinates of curve A_1, fig. 45, Plate XXVI., indicate in pounds the total effort which the engine could exert along the assumed line of resistance, the total pressure on the cylinder being 100·5 lbs. The horizontal co-ordinates indicate the arc which the centre of the crank-pin has traversed, measured from the position in which the crank-pin is, at the end of its stroke, nearest to the cylinder. The co-ordinates of all the other curves have a similar signification. The ordinates for each curve have been calculated by separate figures for 24 evenly spaced positions of the crank, numbered as shown in fig. 46, when the piston and connecting rod lie to the left of the crank. The portion of the curve corresponding to the forward or front stroke, between the positions 0 and 12, will be called the front branch; the portion corresponding to the backward or back stroke, between 12 and 24, will be called the back branch. In A_1 the front and back branches are symmetrical relatively to ordinate 0 or 12. The relation of the ordinates of this curve to the total pressure 100·5 lbs. on the piston depends wholly on the relative velocity of the piston and a point on the circumference of a circle 3 feet diameter concentric with the crank-shaft. There are two dead points at 0 and 12, where, for an infinitely short arc, no resistance could be overcome.

In curve A_2 the front and back branches are sensibly symmetrical in the present example, although not rigidly so. Instead of a mere dead point we have now two dead arcs of different lengths, the longest being at the end of the front and beginning of the back strokes. These dead arcs together last for about 2·7 per cent. of each revolution. The negative ordinates throughout these dead arcs indicate that the engine, instead of driving, must be driven. The area enclosed by the curves above the line OY measures the work which the steam can do. The area enclosed below the line OY in A_2 measures the work which must be done (not by the steam) to pull the engine through the dead arcs. This work, in practical cases, is done by the fly-wheel, which, if large enough, might do the necessary work without allowing the speed to fluctuate sensibly—a condition assumed throughout the calculations.

The whole work done by the steam is $100{\cdot}5 \times 16 \times 12$, or 3216 inch lbs. The area of A_1 was found to be 3210 inch lbs., showing the error due to defective drawing to be very trifling; the area of A_2 (being the arithmetical difference between the positive and negative areas) is 2974 inch lbs. The efficiency, on assumption 2d, is therefore $\frac{2974}{3210} = 0{\cdot}927$.

In curve A_3 the front and back branches are no longer even approximately symmetrical to one another. There is no dead point at the end of the front stroke, and there is a long dead arc at the end of the back stroke. The maximum effect is greater in the front than in the back stroke. These effects are due to the weight and mass of the connecting-rod. A counterbalance might be so placed as to make the front and back branches resemble one another much more closely. The area of the curve A_3 (calculated as for A_2) is 3212 inch lbs., the total error due to imperfect draughtsmanship being only 4 inch lbs.

Curve A_4 resembles A_3 in its general outline, and when these are compared with A_1 and A_2 they show the effect of the weight and mass of the parts in increasing friction. The increase in the loss due to friction is not even approximately constant throughout the stroke, but is much greater when the crank is nearly at right angles to the centre line of the engine. The loss is greatest during the back stroke, producing inequality between the useful work done during the back and front strokes. The causes of each departure from symmetry in all these curves can be followed on the diagrams of the dynamic frames, but this paper would be unduly extended if these were all to be printed. Moreover, such directions have already been given for drawing these as will enable any one to investigate the cause of each effect now described or shown on the curves. The area of A_4 is 2602 inch lbs., so that the true efficiency of the engine running at this speed, and with this low pressure of steam, is $\frac{2602}{3212} = 0{\cdot}810$.

§ 35. Example B, fig. 47, Plate XXVII.—The effect of the resistance of the masses to acceleration as distinguished from the effect of the weights of the parts might have been exhibited by drawing effort curves—1st, on the assumption that although the parts resisted acceleration they had no weight; and, 2d, on the assumption that although the parts had weight they offered no resistance to acceleration, or were moving at an infinitely slow speed; then, comparing these curves with A_4 we should have seen the effect of each element of the problem. This, however, was thought unnecessary, because the effect of the resistance of the masses to acceleration is strikingly shown by the curves B_3 and B_4, being the effort curves of the same engine, with the same pressure in the cylinder, but running four times faster. When making one revolution per second, the resistances to acceleration are smaller than the weights of the elements in motion, as may be seen in Table I. of the Appendix, where these forces are given for each position. When the speed is increased to four revolutions per second, these forces are multiplied by sixteen, and their effect is then much greater than that of the weight of the parts and suffices completely to change the character of the effort curve. The negative portion of the curve lasts for nearly half the revolution of the crank, so that for nearly half of each revolution the fly-wheel would have to pull the engine round. This is true both for B_3 and B_4 for the curves without and with friction, and is simply due to the fact that during the first half of each stroke the reciprocating masses are being positively accelerated. The positive ordinates during the period when the engine is driving of course exceed those during which the engine is being driven; so that for curve B_3 the balance of positive area ought to be 3216 inch lbs. It actually is, on the drawing, 3256, the excess being due to small errors in drawing and computation. The area of B_4 is only 1744 inch lbs.

The efficiency, therefore, has sunk to 0·536; almost half the power of the engine is taken up in driving itself; the pressures on the joints caused by resistance to acceleration have at this high speed greatly increased the loss due to friction. The inequality between back and front strokes is also very great, the area of the front branch being 953 inch lbs., that of the back branch only 791. The loss due to friction sinks, however, almost to nothing at one point of the front stroke, very near the place where, in example A_4, it was a maximum. This may serve as a warning against hasty generalisations. In curve B_4 there are sensible sudden changes of efficiency at points 0 and 12, due to a sudden change in the position of the points where the elements bear on one another.

§ 36. Example C, fig. 48, Plate XXVI.—Example C was selected with the object of ascertaining how far the efficiency is affected by using the steam expansively instead of admitting it throughout the stroke. There is a very general idea that the sudden shock, as it is called, of admitting steam at a much higher pressure for a short time at the beginning of the stroke must diminish the efficiency of an engine. This is not so in the present example. An imaginary indicator diagram was selected for example C, drawn on the supposition that the steam was first admitted at a constant pressure of 10 lbs. per square inch; that r, the reciprocal of the fraction of the stroke during which steam enters at a constant pressure, was 12·8; that the steam expanded, according to an adiabatic curve, and was suddenly released at the end of the stroke, and that during the return stroke the back pressure was 0·516. These data give a total effective initial pressure of 476·7 lbs.; a total effective final pressure of 3·7 lbs.; and a mean effective intensity of pressure of 2 lbs. per square inch, as in examples A and B. The work done by the steam per stroke is, therefore, as before, 3212 inch lbs. The speed of the engine is taken, as for curve A, at one revolution per second. Curves C_3 and C_4 give the effects with and without friction; the area of C_3 is 3223 inch lbs., showing a very small error of execution; the area of C_4 is 2640; the ratio of these values gives 0·819 as the efficiency of the engine worked in this way. This efficiency is actually a little higher than that of curve A, which is repeated in this figure to allow it to be more readily compared with C. A similar result is to be observed in curve D, fig. 49, Plate XXVIII., constructed from the same data, but at the high speed of 4 revolutions per second. The area of D_3 is 3270 inch lbs., that of D_4 1965 inch lbs., giving an efficiency of 0·601—a value sensibly higher than that derived from curve B when the steam was admitted throughout the stroke. The errors due to imperfect drawing have generally the result of slightly increasing the effort, and the error in curve D_3 for this particular drawing (3270 over 3216) is nearly 1·7 per cent. The liability to error is much increased when, as here, a large portion of the area is negative. If this percentage were reckoned on the arithmetical sum of the areas, instead of on their difference, it would be insignificant. Notwithstanding this inevitable imperfection, there is every reason to expect that the errors in curves D_3 and D_4 resemble one another; and we have the less reason to suspect the accuracy of the conclusion, because we can see that since the tendency of resistance to acceleration during the beginning of each stroke is to diminish the effort, while that of a large initial pressure is to increase it, the two tendencies counteract one another without causing pressure on the main-bearings or crank-pin. Thus, in curves B_3 and B_4 we found that the loss due to friction at positions 5 and 18 was much reduced from this cause. In curves C_3 and C_4 we have this useful result of the mass of the moving parts without the reversal of the stresses which brings down the efficiency in B_3 and B_4. Similarly, in curves D_3 and D_4, the effect of the high initial pressure is to prevent the negative parts of the curve from falling so low as in example B. The extremely low efficiency of A_4, B_4, and C_4 illustrates the evil effect of using a large and heavy engine running with small mean pressures of steam. We not unfrequently see large engines ordered for factories, mines, or waterworks to allow for subsequent extension, or for large variations in the work required at different times. The above cases show how very serious the loss may be when an engine is habitually worked much below its power. The cases are no doubt extreme, but they show the tendency of the practice.

§ 37. Example E, figs. 50 and 51, Plates XXIX. and XXX.—The four cases hitherto analysed are not, strictly speaking, practical cases: they were chosen so as to bring into prominence the effects of friction and high speed, separately and combined. These effects are obviously most marked when the engine is running lightly loaded. We will now consider a more practical example. The curves E correspond to a speed of 1 revolution per second, and to an indicator diagram drawn as follows:—Initial intensity of pressure 50 lbs. per square inch, $r=5{\cdot}93$; back pressure, 3 lbs. per square inch; mean effective pressure, 18·905 lbs. per square inch; work done by steam, 30640 inch lbs. per revolution.

E_1 is the effort curve, neglecting mass, weight, and friction.

E_2 the effort curve, taking the friction into account which results from effect and resistance, but neglecting that due to weight and mass; the area of E_1 is 30590 inch lbs, the error being 50 inch lbs. The area of E_2 is 28400 inch lbs., making the efficiency 0·928 on this hypothesis.

E_3 is the effort curve, taking the effort mass and weight into account, but neglecting friction. The area is 30700, the error being 110 inch lbs. in the drawings.

E_4 is the effort curve, taking mass, weight, and friction into account—in other words, the true effort curve. Its area is 28030 inch lbs., giving an efficiency of 0·913, or a little less than that calculated for E_2. Now that the steam pressures are large, the loss due to the weight and mass of the parts is at this speed comparatively insignificant. The total inevitable loss due to friction in the machine alone, even excluding the accidental friction due to tightness of piston and glands, and the power required to work the valve gear, is nearly 9 per cent. of the whole indicated horse-power. This example shows the complete fallacy of the experimental method sometimes adopted with the object of testing the efficiency of an engine. The engine is run with no resistance, and the indicated H.P. observed. This is assumed approximately to represent the loss due to the engine itself when doing useful work against a large resistance; but from example A we see that the power required to overcome the friction in the engine when a small resistance was being overcome was only 3210–2974 inch lbs., or 236 inch lbs.; whereas in example E the power required is 1190 inch lbs., or nearly five times as much. This ratio would be somewhat diminished by the constant accidental resistances in glands, &c., and by the power required for valves and pumps. It must, however, always remain very large.

In E_1 and E_2 the discontinuity of the curves at positions 0 and 12 is very marked. There must also be a slight break near positions 6 and 18, due to the change in the bearing-points on the crosshead pin; but the frictional loss at this point is so insignificant that the break in the curve is not sensible. There is very little difference in general character between curves E_1 E_2 and E_3 E_4. Mass and weight play a very small part in the general result.

§ 38. Example F, fig. 52, Plate XXXI.—Example F is important and instructive, showing the result of running the engine at 4 revolutions per second instead of 1 revolution per second, but maintaining the same indicator diagram as in example E, the mean effective pressure being 18·905 lbs. as before. We might expect this high speed to diminish the efficiency, as in examples B and D, but this is not the case. Curves F_1 and F_2 are identical with E_1 and E_2 and are therefore omitted; curves F_3 and F_4 are, however, very different in character from E_3 and E_4. The resistance to acceleration causes the effort to be much more uniformly distributed. It greatly diminishes the maximum pressure, and largely increases the pressure during the second half of each stroke. The resistance to acceleration is not, however, so large relatively to the steam pressure as to produce negative ordinates, such as were found in cases B and D. The action of the reciprocating masses is, therefore, on the whole, beneficial, and we find the efficiency of the engine to be ·917, or somewhat higher than in example E. Much care has been taken in the computations to check this rather remarkable result, but there seems no reason to doubt its accuracy. The important conclusion to be drawn does not, however, depend on trifling differences—such as that between ·917 and ·913. It is that high speeds do not necessarily entail low efficiency, so that by judicious arrangement of the masses we can have small engines running rapidly with high expansion which, so far as frictional resistances are concerned, may be at least as efficient as the same engines running slowly, and necessarily more efficient than larger engines doing the same work.

§ 39. Examples G. and H, figs. 53 and 54, Plates XXXII. and XXXIII.—The unexpected results obtained rendered it desirable to investigate the case of infinitely slow speed, or that in which the effect of inertia was wholly disregarded. Curves G_3 and G_4 show the efforts with and without friction for the practical distribution of steam assumed in cases E and F. The total work done by the steam is 30770 inches, the useful work 20120, and the efficiency ·914—a result hardly differing sensibly from that obtained with the high speed of 4 revolutions per second.

§ 40. Example H.—Lastly, curves H_3 and H_4 show the efforts with and without friction when the engine is modified so as to make the connecting-rod only 28 in. long, or 3½ times the stroke. The weight of the new rod is taken as 28 lbs., and its centre of gravity 14″ from the crank end, and its radius of gyration about the cross-head 18·83 inches. The speed assumed was 1 revolution per second.* As calculated from the areas of the curves the whole work done by the steam is 30740 inch lbs., the useful work 27920 inch lbs., and the efficiency 0·91.

This result is of considerable practical value, showing that the efficiency of the engine is hardly diminished by shortening the connecting-rod to this extent.

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* The initial steam pressure, the ratio of expansion, and the back pressure were assumed to be the same as in example E.

§ 41. *General Conclusions.*—The most important conclusion to be drawn from the foregoing examples is, that the investigation of the efficiency of any given direct-acting engine is rendered comparatively easy by the new method. Next in importance may be ranked the warning not to judge hastily as to the result of any given modification in proportions or speed. The differences introduced by a change of speed are especially remarkable, and show how futile reasoning must be as to relative efforts and resistances as to stresses on the various parts of the engine, and the efficiency of any design when the inertia of the masses is left out of the calculation.

Further conclusions may be drawn as follows:—1st, that high speeds do not necessarily involve small efficiency; 2d, that a short connecting-rod is not very disadvantageous; 3d, that expansive working, even when carried to great lengths, does not necessarily involve a loss of efficiency.

Table I. gives an abstract of the numerical values obtained with this particular engine, but the reader must be warned against considering these results as generally applicable to other engines of different proportions.

TABLE I.—*Abstract of Results.*

	Example.	Pressure.			Speed.	Efficiency.
Engine described in § 34.	A_1 A_2 A_3 A_4	Low	Uniform	2 lbs. per square inch.	Slow, 1 rev. per sec.	· 27 ·81
	B_1 B_2 B_3 B_4		Uniform	2 lbs. per square inch.	Fast, 4 rev. ,,	·927 ·536
	C_1 C_2 C_3 C_4		Expansion	Cut off $\frac{1}{12\cdot8}$ mean effective = 2 lbs. per square inch.	Slow, 1 rev. ,,	·907 ·819
	D_1 D_2 D_3 D_4		Expansion	Cut off $\frac{1}{12\cdot8}$ mean effective = 2 lbs. per square inch.	Fast, 4 rev. ,,	·907 ·61
	E_1 E_2 E_3 E_4	Practical	Expansion	Cut off $\frac{1}{5\cdot93}$ mean effective pressure 18·905 lbs. per square inch.	Slow, 1 rev. ,,	·928 ·913
	F_1 F_2 F_3 F_4		Expansion		Fast, 4 rev. ,,	·928 ·917
	G G_2 G_3 G_4		Expansion		Infinitely slow, 0.	·928 ·914
Engine described in § 40.	H_3 H_4		Expansion		Slow, 1 rev. per sec.	·908

The necessary calculations and drawings for this paper have been made by my assistant Mr J. A. Ewing, to whom I am much indebted both for the accuracy with which the work has been done, and for the interest he has shown in adopting the novel method of investigation.

The appendix contains data which will allow the reader to verify the results arrived at without going through all the calculations.

This appendix has been drawn up by Mr Ewing.

APPENDIX TO PART II.

On the Application of Graphic Methods to the Determination of the Efficacy of Machinery.

To determine the forces required for the acceleration of the piston and connecting-rod in each position of the engine, we must know the acceleration of the piston, and the angular velocity and angular acceleration of the connecting-rod. If AC be the crank, and CB the connecting-rod, and if AB be called x, then $\frac{d^2x}{dt^2}$ is the acceleration of the piston; and if the angle CBA be called θ, $\frac{d\theta}{dt}$ is the angular velocity of the connecting-rod, and $\frac{d^2\theta}{dt^2}$ is its angular acceleration.

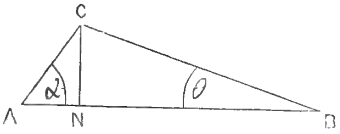

Fig. 55.

If the angle which the crank makes with AB be called a, $\frac{da}{dt}$ is the angular velocity of the crank.

Then, calling the crank radius r, and the length of the connecting-rod l, we have

$$\theta = \sin^{-1}\frac{\text{CN}}{\text{CB}} = \sin^{-1}\left(\frac{r\sin a}{l}\right)$$

$$\frac{d\theta}{dt} = \frac{r\cos a}{\sqrt{l^2 - r^2\sin^2 a}} \cdot \frac{da}{dt}$$

Differentiating again, and remembering that $\frac{d^2\alpha}{dt^2} = 0$, we obtain finally

$$\frac{d^2\theta}{dt^2} = -\frac{r \sin \alpha \, (l^2 - r^2)}{(l^2 - r^2 \sin^2 \alpha)^{\frac{3}{2}}} \left(\frac{d\alpha}{dt}\right)^2 .$$

Again

$$AB = AN + BN$$

$$x = r \cos \alpha + l \cos \theta$$

$$\frac{dx}{dt} = -r \sin \alpha \frac{d\alpha}{dt} - l \sin \theta \frac{d\theta}{dt} ,$$

and

$$\frac{d^2x}{dt^2} = -r \cos \alpha \left(\frac{d\alpha}{dt}\right)^2 - l \cos \theta \left(\frac{d\theta}{dt}\right)^2 - l \sin \theta \frac{d^2\theta}{dt^2} .$$

Substituting for $\frac{d\theta}{dt}$ and $\frac{d^2\theta}{dt^2}$ their values as determined above, and putting $r \sin \alpha$ for $l \sin \theta$, and $\sqrt{l^2 - r^2 \sin^2 \alpha}$ for $l \cos \theta$, we obtain finally

$$\frac{d^2x}{dt^2} = -r \left(\frac{d\alpha}{dt}\right)^2 \left\{ \cos \alpha + \frac{r l^2 \cos 2\alpha + r^3 \sin^4 \alpha}{(l^2 - r^2 \sin^2 \alpha)^{\frac{3}{2}}} \right\}$$

The forces required to accelerate the piston and the connecting-rod may now be calculated as follows :—

For the piston.—If M be the mass of the piston and piston-rod in lbs., the force in lbs. is

$$\frac{M}{g} \frac{d^2x}{dt^2} .$$

When this quantity is negative, the force acts towards the centre of the crank shaft.

For the connecting-rod.—The motion may be looked at as a translation of the whole rod in the direction of motion of the piston, combined with a rotation of the rod about the crosshead. Hence the force producing acceleration is the resultant of three components :—F_1, the force required for the linear acceleration in the direction of motion of the piston ; F_2, the force required for rotation about the crosshead at the angular velocity which the rod has at the instant under consideration. This acts towards the centre of rotation, and is equal and opposite to the so-called "centrifugal force ;" and, lastly, F_3, the force required to give the rod the angular acceleration which it has at the given instant.

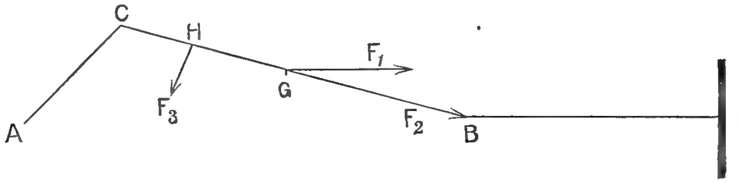

Fig. 56.

Let M′ be the mass of the connecting-rod in lbs., and G its centre of mass,

(*Note.*—In figs. 55 and 56 the engine has been represented as seen from behind, if the engine in the previous figures be considered as viewed from the front.)

distant l_0 from the crosshead B; also let k be its radius of gyration about B. Then the first component mentioned above, or F_1 is

$$\frac{M'}{g} \frac{d^2x}{dt^2}$$

and acts through G parallel to the path of the piston.

The second component, F_2, is

$$\frac{M'l_0}{g} \left(\frac{d\theta}{dt}\right)^2$$

and acts along the line of the rod towards B. The third component, F_3, is

$$\frac{M'l_0}{g} \left(\frac{d^2\theta}{dt^2}\right)$$

and acts at right angles to the rod through the centre of the percussion H, which is at a distance $\frac{k^2}{l_0}$ from B.

These three forces may be most conveniently compounded by shifting F_3 to G, and introducing a couple whose moment is $F_3 \cdot HG$, or

$$\frac{M'(k^2-l_0^2)}{g} \frac{d^2\theta}{dt^2}, \text{ or } \frac{I}{g} \frac{d^2\theta}{dt^2},$$

where I is the moment of inertia of the rod about G.

Forces equal and opposite to those components form the several parts of the whole resistance to acceleration, and when these are combined with the weight, the resultant is the whole load on the element. (See § 27, page 30, Trans. R.S.E., vol. xxviii.) This composition is effected graphically, and a single force is obtained acting through G, which has then to be shifted parallel to itself to such a distance as to give rise to the moment $\frac{I}{g} \frac{d^2\theta}{dt^2}$. This process gives a single force of determinate magnitude and position, as the load on the element in each position of the engine.

The following tables show the component parts of the acceleration and force in the cases which have been actually examined. In Table I. the connecting-rod is 41″ long, and its mass is 34 lbs.; l_0 is 20″, and $\frac{k^2}{l_0}$ is 34·08 inches. In Table II. the connecting-rod is 28″ long, and its mass is 28 lbs.; l_0 is 14″, and $\frac{k^2}{-}$ is 25·32 inches. In both cases the crank radius is 8″, the mass of the piston and piston-rod 46 lbs., and the speed is one revolution per second, whence

$$\frac{da}{dt} = 2\pi \cdot$$

The positions of the crank in column 1 are numbered thus:—Positions 0 and 24 are the same, and correspond to $\alpha=0$. Position 12 corresponds to $\alpha=180°$. The movement in fig. 56 is contrary to that of the hands of a watch, and the interval between two successive positions is 15°. Looked at from the other side, as in earlier figures of the engine, the movement would be in the direction of the hands of a watch.

TABLE I.

Connecting-rod, 41″. Crank, 8″. Mass of connecting-rod, 34 lbs. Mass of piston and piston-rod, 46 lbs. Speed, one revolution per second.

$l_0 = 21''$. I = 9576 (inches and lbs.)

Piston, mass = M.			Connecting Rod, mass = M′.					
Position of Crank.	$\frac{d^2x}{dt^2}$ feet and seconds.	$\frac{M}{g}\frac{d^2x}{dt^2}$ lbs.	$\frac{d\theta}{dt}$ radians and seconds.	$\frac{d^2\theta}{dt^2}$ radians and seconds.	F_1 or $\frac{M'}{g}\frac{d^2x}{dt^2}$ lbs.	F_2 or $\frac{M'l_0}{g}\left(\frac{d\theta}{dt}\right)^2$ lbs.	F_3 or $\frac{M'l_0}{g}\frac{d^2\theta}{dt^2}$ lbs.	Moment $\frac{I}{g}\frac{d^2\theta}{dt^2}$ lbs. and inches.
0	−31·45	−44·88	1·226	0	−33·21	2·78	0	0
1	−29·94	−42·77	1·186	−1·925	−31·61	2·60	− 3·56	− 47·7
2	−25·50	−36·46	1·067	−3·759	−26·93	2·10	− 6·95	− 93·2
3	−18·67	−26·66	0·876	−5·385	−19·71	1·42	− 9·95	−133·5
4	−10·60	−15·13	0·622	−6·702	−10·69	0·71	−12·38	−166·1
5	− 2·29	− 3·27	0·323	−7·557	− 2·42	0·19	−13·97	−187·3
6	5·24	7·48	0	−7·854	5·53	0	−14·51	−194·6
7	11·86	16·36	−0·323	−7·557	12·52	0·19	−13·97	−187·3
8	15·72	22·46	−0·622	−6·702	16·60	0·71	−12·38	−166·1
9	18·56	26·52	−0·876	−5·385	19·60	1·42	− 9·95	−133·5
10	20·14	28·76	−1·067	−3·759	21·27	2·10	− 6·95	− 93·2
11	20·91	29·87	−1·186	−1·925	22·08	2·60	− 3·56	− 47·7
12	21·19	30·27	−1·226	0	22·37	2·78	0	0
13	20·91	29·87	−1·186	1·925	22·08	2·60	3·56	47·7
14	20·14	28·76	−1·067	3·759	21·27	2·10	6·95	93·2
15	18·56	26·52	−0·876	5·385	19·60	1·42	9·95	133·5
16	15·72	22·46	−0·622	6·702	16·60	0·71	12·38	166·1
17	11·86	16·36	−0·323	7·557	12·52	0·19	13·97	187·3
18	5·24	7·48	0	7·854	5·53	0	14·51	194·6
19	− 2·29	− 3·27	0·323	7·557	− 2·42	0·19	13·97	187·3
20	−10·60	−15·13	0·622	6·702	−10·69	0·71	12·38	166·1
21	−18·67	−26·66	0·876	5·385	−19·71	1·42	9·95	133·5
22	−25·50	−36·46	1·067	3·759	−26·93	2·10	6·95	93·2
23	−29·94	−42·77	1·186	1·925	−31·61	2·60	3·56	47·7
24	−31·45	−44·88	1·226	0	−33·21	2·78	0	0

TABLE II.

Connecting-rod, 28″. Crank, 8″. Mass of connecting-rod, 28 lbs. Mass of piston and piston-rod = 46 lbs. Speed, one revolution per second.

$l_0 = 14''$.

Piston, mass = M.			Connecting Rod, mass = M′.					
Position of Crank.	$\frac{d^2x}{dt^2}$ feet and seconds.	$\frac{M}{g}\frac{d^2x}{dt^2}$ lbs.	$\frac{d\theta}{dt}$ radians and seconds.	$\frac{d^2\theta}{dt^2}$ radians and seconds.	F_1 or $\frac{M'}{g}\frac{d^2x}{dt^2}$ lbs.	F_2 or $\frac{M'l_0}{g}\left(\frac{d\theta}{dt}\right)^2$ lbs.	F_3 or $\frac{M'l_0}{g}\frac{d^2\theta}{dt^2}$ lbs.	Moment $\frac{I}{g}\frac{d^2\theta}{dt^2}$ lbs. and inches.
0	−33·76	−48·23	1·795	0	−29·36	3·27	0	0
1	−31·93	−45·61	1·739	− 2·703	−27·77	3·07	− 2·75	− 31·1
2	−26·70	−38·14	1·571	− 5·342	−23·22	2·50	− 5·42	− 61·3
3	−18·81	−26·87	1·296	− 7·797	−16·36	1·70	− 7·91	− 87·5
4	− 9·41	−13·44	0·926	− 9·863	− 8·18	0·87	−10·01	−113·3
5	− 0·08	− 0·11	0·483	−11·269	− 0·07	0·24	−11·43	−129·4
6	7·85	11·21	0	−11·770	6·83	0	−11·94	−135·2
7	13·54	19·34	−0·483	−11·269	11·77	0·24	−11·43	−129·4
8	16·91	24·16	−0·926	− 9·863	14·70	0·87	−10·01	−113·3
9	18·41	26·30	−1·296	− 7·797	16·01	1·70	− 7·91	− 87·5
10	18·88	26·97	−1·571	− 5·342	16·42	2·50	− 5·42	− 61·3
11	18·91	27·01	−1·739	− 2·703	16·44	3·07	− 2·75	− 31·1
12	18·80	26·86	−1·795	0	16·35	3·27	0	0
13	18·91	27·01	−1·739	2·703	16·44	3·07	2·75	31·1
14	18·88	26·97	−1·571	5·342	16·42	2·50	5·42	61·3
15	18·41	26·30	−1·296	7·797	16·01	1·70	7·91	87·5
16	16·91	24·16	−0·926	9·863	14·70	0·87	10·01	113·3
17	13·54	19·34	−0·483	11·269	11·77	0·24	11·43	129·4
18	7·85	11·21	0	11·770	6·83	0	11·94	135·2
19	− 0·08	− 0·11	0·483	11·269	− 0·07	0·24	11·43	129·4
20	− 9·41	−13·44	0·926	9·863	− 8·18	0·87	10·01	113·3
21	−18·81	−26·87	1·296	7·797	−16·36	1·70	7·91	87·5
22	−26·70	−38·14	1·571	5·342	−23·22	2·50	5·42	61·3
23	−31·93	−45·61	1·739	2·703	−27·77	3·07	2·75	31·1
24	−33·76	−48·23	1·795	0	−29·36	3·27	0	0

The engine of Table I. was also examined when running at a speed of four revolutions per second. This, of course, had the effect of multiplying $\frac{d\theta}{dt}$ by four, and $\frac{d^2\theta}{dt^2}$ and $\frac{d^2x}{dt^2}$ by sixteen. The Table for this set of circumstances may, therefore, be deduced from Table I. by multiplying column 4 by four, and columns 2, 3, 5, 6, 7, 8, and 9 by sixteen.

XXX.—*Thermal and Electric Conductivity.* By Professor TAIT.

(§§ 1–16, Read March 18, 1878—Revised (from a Shorthand Writer's extended Notes) December 4, 1878.)
(§§ 17–23, Read June 3, 1878.)

The following paper contains the results of an inquiry which has occupied me at intervals for somewhere about ten years. It was carried out in part at the expense of the British Association, and I have already reported results to that body in 1869 and 1871. But these provisional reports referred to very short ranges of temperature only, and the experiments were made with faulty thermometers, for which I had not the corrections which had been carefully determined by WELSH at Kew.

The inquiry arose from my desire to extend to other metals the very beautiful and original method which Principal FORBES devised, and which the state of his health prevented him from applying to any substance but iron. FORBES' experiments gave a result so very remarkable, and (as it seemed to me) so theoretically suggestive, that I wished to extend them to other pure metals, and also, in one or two cases at least, to alloys.

I believe that Principal FORBES had at least two reasons for undertaking his investigations :—(1.) When he commenced his inquiry, there was no really accurate or trustworthy determination of the absolute conductivity of any body whatever for heat. (2.) FORBES had himself, in 1833* and subsequent years, pointed out a very remarkable analogy between the conducting powers of metals for electricity and for heat, and had shown that these were almost precisely proportional to one another—that is to say, that the list of the average relative conductivities of different metals for electricity differed, from that of their relative conductivities with regard to heat, certainly not more than did the several electric lists furnished by different experimenters, and certainly less than the corresponding thermal lists. Hence it was natural to suppose that temperature might have a marked effect on thermal conductivity, as it was known to have such an effect on electric conductivity.

The great merit of FORBES' method † is, that it seeks the conductivity in terms of its definition, instead of seeking a value of the conductivity which will best satisfy the integral of FOURIER'S equation formed on the hypothesis of uniform conductivity, and of loss of heat from the surface of the bar in direct proportion to the temperature-excess above the surrounding air. Although

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* Proc. R. S. E., i. 5.

† Report B. A., 1852.

Forbes' paper has been printed in the Transactions of this Society,[*] I may make a few additional remarks on the methods he employed.

He used for the first part of the experiment, what he called the *statical experiment,* a bar of iron, 8 feet long by $1\frac{1}{4}$ inch square section. One end of this was raised to a high temperature by means of a pot containing melted solder, whose temperature was maintained nearly constant for eight or nine hours. The rest of the bar was exposed to the air of the laboratory, and of course parted with a portion (of the heat conducted to it) partly by radiation, partly by convection. It was found that after about eight hours a stationary distribution of temperature was attained, in which the gain of heat in any section of the bar by conduction was just neutralized by the surface loss. This temperature distribution was then accurately determined. In the second or *dynamical experiment,* a shorter bar, of exactly the same transverse dimensions, was employed; not, however, for the conduction of heat, but for the purpose of ascertaining at what rate its heat was lost by radiation and convection at different temperatures. For this purpose the bar was heated as uniformly as possible, once for all, and then allowed to cool in the air, its temperature being noted at measured intervals of time. The introduction of the experiments with the shorter bar was the main point of great importance in which Forbes improved the *experimental* part of the determination. And, as regards the subsequent calculations, it need only be said, to show the improvement he introduced, that had he followed Biot's mode of procedure he would probably have failed to discover that thermal conductivity (in some cases at least) depends on temperature. As I have already said, though Forbes' results were confined to iron, they were the first of any real value to the absolute measurement of thermal conductivity.

§ 1. Viewed in the light of the results attained, I do not now think so much as I was originally disposed to do of one of the chief reasons which led me to the present inquiry. But that does not in any way matter to my other chief reason; for, though an attractive hypothesis has been shown to be untenable, at all events without very considerable restrictions, some valuable and even curious measurements have been made. Forbes' results for iron have been, in all but one particular, closely reproduced by myself, but their most striking peculiarity, the falling off of conductivity by increase of temperature is, so far as I yet know, confined to the single metal which he experimented on. I had fancied that as the numerical results given by him seemed closely consistent with a conductivity varying inversely as the absolute temperature, such might be generally the case. Inquiring into possible physical reasons for this, I saw that if it were assumed that, in the steady linear propagation of heat, the amount of available

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[*] Trans. R. S. E., 1860-61, and 1864-5.

energy of the heat in three successive slices of a solid, of equal thickness, were always the lowest possible, consistent with the conditions of the experiment, FORBES' result would follow, and would give, in fact, an excellent instance of dissipation of energy.

§ 2. The subjects I set myself to inquire into were definitely these—

(1.) Whether in pure metals there is always a decrease of thermal conductivity with a rise of temperature. And for this purpose I chose the metals copper and lead, because we can easily and at small expense procure them in large quantity and in a state of great purity.

(2.) Whether different specimens of the same metal may not differ in thermal conductivity, at least as widely as they are known to do in electric conductivity; and for this purpose, in consequence of Sir W. THOMSON'S* remarkable observations on the electric conductivity of copper, I selected copper.

(3.) Whether an alloy, such as is chosen for resistance coils because its electric conductivity changes little with change of temperature, does not show a similar small change of thermal conductivity; for this purpose I chose the alloy, German silver, which is frequently used for such coils.

(4.) A fourth question, which I have not yet answered, was whether there may not be some conduction-peculiarity in a substance whose specific heat varies little with temperature. This was suggested to me by the theoretical notions above alluded to, and probably falls with them. For such a purpose there can be no doubt that the best substance is platinum, because its specific heat is known to alter very little, and Messrs JOHNSTON and MATTHEY were kind enough to offer to provide me with a bar of platinum of the same dimensions as FORBES' iron bar, at the comparatively small expense of working the material into the necessary form and working it down again. The value of the material of such a bar, it may be well to mention, would have been about £2000.

§ 3. The results I have hitherto published in the Reports of the British Association were, of course, strictly preliminary. For, besides the want of scale-errors for my thermometers, another great difficulty felt at the commencement of the experiments was that of maintaining a nearly constant temperature in the source of heat for the statical experiment. At the time I gave those provisional reports, I had operated only with temperatures not much higher than that of boiling water; through a range, in fact, barely sufficient to indicate with certainty a *change* of conductivity even in iron.

§ 4. Shortly after FORBES published the full result of his experiments on iron, another excellent and novel method, quite distinct in principle from his, was described by ÅNGSTRÖM.† Of that method I availed myself, with the help of

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* Proc. R. S., 1857 (June 15).

† Pogg. Ann., 1862. Phil. Mag., 1863, i.

the various bars and thermometers obtained for the present inquiry. In ÅNGSTRÖM's method it is so much more easy to calculate out the results, and derive the conductivity from the experiments, than in that of FORBES, that I have already —in 1872–73*—communicated to the Society the results obtained by this method, though I had years before made most of the experimental determinations required by FORBES' method, whose numerical consequences are only now produced. But my thermometers, though excellent for the use of FORBES' method, were not nearly delicate enough for the proper application of that of ÅNGSTRÖM. It requires, for its proper carrying out, the very accurate reading of small *changes* of temperature. Hence the results of 1872–73 can be looked upon as at best but very rough approximations. One great defect of ÅNGSTRÖM's method, as compared with that of FORBES, lies in the assumption (which forms part of its necessary basis) that the rate of surface loss is proportional directly to the excess of temperature over the surrounding air. Even for the moderate range of temperature employed in ÅNGSTRÖM's experiments,† this is not nearly correct. Hence, and for other reasons (for instance, his equations being formed as if k were constant), I do not accept his statement that the thermal conductivity of copper falls off as the temperature rises, as one which his method was competent to decide. Even with FORBES' much superior method, a range of at least 100° C. is absolutely necessary to settle such a point.

I have had several reasons for delay in publishing the results of these experiments. For the most part, the experiments themselves were made eight or nine years ago, but for the delay with regard to the calculations I am not wholly responsible. Since I obtained the assistance of Mr EVANS, however, there has been no unnecessary delay in the computations. Experimental difficulties of various kinds were, however, constantly cropping up. Besides the difficulty already alluded to, of maintaining a steady temperature of the source of heat, a very peculiar difficulty arose from the behaviour of the thermometers. These, after being exposed to high temperatures and cooled, showed a gradual rise of the zero points; and, in some of those which have been most frequently exposed to the highest temperatures, the zero point has risen as much as about five degrees. There were also very great difficulties about the heating of the short bar for the cooling experiment. Here my results were very different (at high temperatures) from those of FORBES. Again, the lead and copper, and sometimes (in extreme cases), even the iron and German silver, when highly heated, become oxidised, and the coating of oxide on the surface promotes radiation, if not also convection; and as the surface becomes oxidised to different degrees at different temperatures no one set of experiments with the short bar is strictly comparable with anything but one part of the long bar. That difficulty is not so much felt

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* Proc. R. S. E.

† Pogg. Ann., Band 118, 1863.

in the case of the iron, still it is felt to a certain extent in the case of all the metals tried. My results are all somewhat uncertain on this account. This uncertainty, and means of removing it, are discussed in § 13.

Another reason for the delay that has occurred in producing the results has been my endeavouring—to a certain extent fruitlessly—to give the results in terms of *absolute temperature*, by the help of air-thermometers. Much time has been spent on that work, yet, even with the assistance of Dr JOULE and others, I have not been able to get a really good set of determinations. The real difficulty lies in the fact that the holes cut in the bars for the insertion of the bulbs of thermometers are necessarily so small, that it is not possible to construct any efficient air-thermometer which can be made to take the place of the mercurial ones.

I have been assisted in the experimental part of the work by several of my Laboratory students; but most especially by my mechanical assistant, Mr T. LINDSAY, who has been throughout the inquiry as valuable to me as was his father to FORBES.

§ 5. The results now given are founded, some of them on experiments made before 1871, and some on experiments made last year. The calculations have all been carried out with care and accuracy by Mr EVANS (who used the processes described by FORBES), and their results have been verified by myself, partly by graphical methods, partly by various devices for interpolation, and in the majority of instances by calculation also.* But, as will be seen, I content myself at present with the statement of probable values only. I have only now arrived at nearly definite conclusions as to the best mode of

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* One of these interpolation methods is so easily applied, and (in consequence of the usual nature of the statical curves) gives results so fairly approximate, that it must be mentioned here as of great use if only in checking the results of the more complex calculations.

Let v_1, v_2, v_3, v_4, be the observed temperatures shown by the four thermometers, placed at intervals of three inches on the long bar. Let w be the number of degrees lost per minute by the thermometer in the short bar, when its temperature-excess above the air is nearly that of $\frac{1}{2}(v_2+v_3)$. Then the conductivity at the temperature $\frac{1}{2}(v_2+v_3)$, in terms of the units employed in § 15 below, is very approximately

$$\frac{w}{8(v_1-v_2-v_3+v_4)}.$$

[This formula assumes *third* differences of v_1 to vanish.] With a single bar of 20 inches, or so, in length (with four or more holes three inches apart), to be used alternately for the statical and for the dynamical experiment (in the former with its free end artificially cooled), I believe that very fair determinations of thermal conductivity may be made in a few hours by the use of the above formula. Had I known this ten years ago I should not have undertaken the repetition and extension of FORBES' experiments *under conditions exactly similar to his.* But, on the other hand, had I not undertaken this work, I should probably not have fallen upon this simple method.

I believe that it may be found applicable even to stout wires or rods, the temperatures being observed by a thermo-electric process. Thus these determinations may be made for very rare metals, and also for substances of very low conductivity. I hope, with the assistance of a party of my Laboratory students, to get a large number of metals examined by this method during next winter and summer sessions.

working, after having pushed to the extreme admissible limit every part of the process.

Before giving the results, it may be well to detail with some care the particulars in which my apparatus and modes of experimenting differ from those employed by FORBES.

§ 6. With regard to the bars employed—The iron bar experimented on was that last made for FORBES' experiments. My chief object in employing this bar was, of course, to ascertain how nearly I could reproduce FORBES' results; with the view of obtaining, as far as I had the means of doing so, a check upon my own work. A couple of copper bars were procured for me, at the instance of Mr WILLOUGHBY SMITH, from a firm largely engaged in furnishing copper cores for submarine cables. These were of the same dimensions as FORBES' iron bar but, while one (*Crown*) was made of copper of the highest electric conductivity, the other (*C*) was made of copper of the worst conductivity. The only difference in construction between these copper bars (as well as the other bars which I employed) and FORBES' iron bar, consisted in the necessary protection of the metal from the mercury which was employed to surround the bulbs of the thermometers when inserted in the holes. For this purpose it was necessary that the holes should be lined with iron; and, therefore, little cups like the heads of arrows are sunk into the copper, lead, and German silver bars. The thickness of the iron shell is so small that it is not sufficient to influence in the slightest measureable degree the progress of the heat along the bar. The copper was in the *hard* state. I propose, at some future time, when some of the desiderata after-mentioned are supplied, to have these bars *annealed* and repeat the measurement of their conductivity.

Along with the copper bars just described, I received some specimens of wire for electric testing. These were said to be made of the same materials. My experience of them has not been satisfactory, as different specimens from the same material show considerable differences in electric conductivity. I therefore defer the consideration of the electric conductivity of these materials till I have time to test for this purpose the long bars themselves.

The German silver bars, long and short, were cut from an exceedingly fine casting, procured for me by the late Mr BECKER. Its transverse section is of exactly the same dimensions as the others. The bars of lead were cast by Messrs MILNE, and are in all respects like the others, save that the bar for the statical experiments is not so long. It required special additional supports to prevent flexure.

The bar of gas-coke upon which some experiments have been made, was sawn from a block of coke obtained from Mr YOUNG of the Dalkeith gas-works. The bar is exactly of the same transverse section as the other bars employed, but though only a few inches in length, it was found sufficient. Even with the highest temperature applied at one end, after 10 hours exposure, there was scarcely any perceptible heating at the further end. The same bar served first for the statical experiment, and then was heated again for the cooling experiment.

§ 7. In procuring the thermometers, on whose accurate indications the whole value of the experimental work depends, I availed myself of the assistance of Dr BALFOUR STEWART, who was then director of the Observatory at Kew. Two sets of thermometers were made under his supervision, one set with long range and short degrees, the other with short range and long degrees, and all were tested by him. I had wished as far as possible to carry out FORBES' idea that it was better to use thermometers, even if they did not show the zero point, which even at high temperatures exposed only a small quantity of mercury in the stem, than to have a long column exposed to the air, with its temperature of course very different at different parts. Dr BALFOUR STEWART, however, told me that, so far as he knew, it was impossible to accurately graduate thermometers under these conditions; and he advised me to take the thermometers as he could make them and guarantee them, namely, mercurial ones, made of proper glass, carefully divided by graduating instruments at Kew. As this is a point of vital importance, I append in a foot-note an extract from Dr STEWART's letter.*

I have already spoken of the circumstance that when the bulbs of some of these thermometers had been heated several times to over 200° C., and especially when heated more than once to nearly 300° C., their indications began to be permanently altered in the way of increase; and in some of them which had been exposed in the holes or bores, closest to the source of heat, where they had been often raised to a temperature of 300° centigrade, it was found that the permanent alteration of zero was as much as 5 degrees. As it appeared that the probable nature of the distortion was a permanent shrinkage of the bulb, I calculated what should on that supposition be the behaviour of the instrument at different temperatures, and by comparing its indications step by step with those of another of the thermometers which had not been distorted by violent heating, I found the results of calculation verified. The altered instrument loses slightly

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* *Extract from a letter*, Dr STEWART *to* Prof. TAIT.

"KEW OBSERVATORY, 8*th December* 1868.

. . . . "We have come to the conclusion that each instrument ought to go down as low as the freezing point.

"It is possible, no doubt, starting with an instrument that includes the freezing point in order to determine the graduation constants, and afterwards taking out some mercury, to produce instruments that begin to register only at high temperatures. But there is an element of uncertainty introduced in taking out the mercury, which may not only cause a constant error, but an error of scale value. . . . .

(Signed) "B. STEWART."

to the other, so that at 300° C. it is little more than four degrees in advance instead of the five it had at zero.

But, after all, this change of error (for the altered instruments were used for the higher temperatures only) can be easily allowed for in correcting the readings for scale-errors; and it is very small in comparison with other inevitable errors of the determination. To mention only one of these, a very slight inexactitude in the position of the hole bored for one of the higher thermometers would involve a more serious error. And, in the mercury, or fusible metal, in each hole there is a most peculiar distribution of temperature, due to the fact that one side of the hole is very considerably hotter than the other.

§ 8. I have already mentioned the very great difficulty encountered in obtaining a properly uniform source of heat in the statical experiment. I tried various processes depending on boiling points, and all sorts of gas regulators, without success, until I got a very valuable suggestion from Dr Crum Brown. The principle is excessively simple, but in working it was found to be almost perfect. It necessitated none of the constant watching described by Forbes. All that was required was a reading of the whole set of thermometers every hour or half-hour.

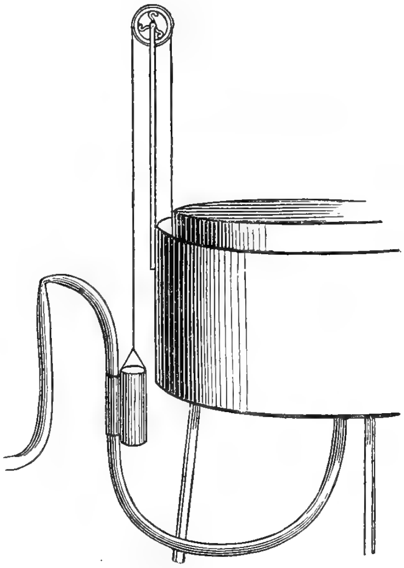
The following extract from my note-book tells its tale sufficiently:—

Gas lit at 6.25 A.M.

	12h. 25m. P.M.	1h. 11m.	1h. 51m.	2h. 58m.
Temperature at hole nearest source,	299	301	301·1	301

In fact, during the last three hours of the experiment referred to, the temperature, though about 300° C., varied by only about one-tenth of a degree. This was actually less than the change of temperature of the air of the room. Of course this, and a few others like it, are exceptional cases; but not possible, even as such, with any other arrangement I have tried. As a rule, a change of *at most* three degrees in the temperature shown by the thermometer nearest the source (and this change a very gradual one) was the utmost fluctuation during the last three hours in the great majority of the experiments. In the few cases in which there was a greater change, it was traced at once to the "burning-down" of one or more barrels of the six-barrelled Bunsen I employed. In such cases, the experiment was at once stopped, and the record crossed out.

Nothing more satisfactory could have been expected in a matter so very difficult as that of regulating the gas supply, when, as all know, in a town like Edinburgh, the pressure is sometimes varied arbitrarily by an amount almost equal to one-third or one-fourth of the whole; and where, especially towards dusk, there are very sudden changes, partly due to increased pressure in the gasometers, partly to the rapid lighting of many burners. The process employed by CRUM BROWN is to cut off, or increase, the supply of gas to a small gasholder by a sort of valve which acts almost instantaneously. The valve consists of an india-rubber tube, which is just on the point of being *nipped*—that is, being bent over so as almost completely to close it. A very slight motion of one end effects the difference between nipping and comparative openness, so that when this tube is appended to one of the weights of the gasholder, it maintains a perfectly regular pressure in the holder. In fact, it was not possible to observe, from half-hour to half-hour, any variation of level of the inverted vessel.



The theory of this application is that, where absolute regularity or steadiness cannot be had, the best substitute for it is extreme stability of equilibrium. There is, no doubt, a constant change going on, but any displacement produces such a disproportionately great force of restitution as practically to keep everything steady.

§ 9. Another source of great difficulty, which had been fully felt by FORBES, was the heating of the short bar. The method he finally adopted is perhaps not applicable, except to iron: at least when high temperatures are required. He plunged his iron bar bodily into a bath of melted fusible metal. The bar was wrapped in paper to prevent too sudden an abstraction of heat from the melted metal. I first tried to heat the bars by means of a sort of air-bath, but I found that in such a bath they all became oxidised before the temperature was sufficiently raised. I endeavoured to overcome this difficulty by putting successive covers on the bath, making it, in fact, almost air-tight, and passing a uniform current of dry carbonic acid gas through it.

These methods proved comparative failures, and the simple process ultimately adopted consisted in taking a brass gas-pipe, pierced along its upper side by a number of holes at equal intervals from one another. This burner was connected directly with the gasometer and produced a row of little jets. As these were of gradually diminishing intensity (in consequence of diminishing pressure), the tube was slightly inclined upwards from the gasholder. The bar (previously raised to a temperature of about 100°, by radiation from a fire, to prevent deposition of moisture from the flames) was placed over it in a horizontal position on a sort of rack, on which it was kept turning round and round, until it was very uniformly heated; being occasionally turned end for end. It was found that when the bar was not heated above 200° C., but little oxidation was produced during the time required for the heating. When it was necessary to raise the temperature higher, the nature of the effect on the surface was described by its colour, which was noted and compared with the effect found to be produced on different parts of the corresponding long bar by its more gradual heating. It would be very easy to burn a mixture of gas and air, and so to a great extent get rid of the possibility of smoking the surface, but practically it was found that no insuperable difficulties were introduced by taking the ordinary coal-gas. But, for a reason presently to be mentioned, the short bars had always to be raised to a temperature much higher than that at which the readings of the thermometers commenced. Thus all my results must necessarily be a little too large, as the cooling was in every case observed on a bar more oxidised than the portion of the long bar which had the same temperature.

§ 10. With reference to the estimation of the true temperature of the bulbs of the thermometers from the readings of a variably heated stem, the great difficulty experienced was one felt by FORBES also—one which he endeavoured to get rid of by detaching arbitrarily a column of mercury, and throwing it up into the little bulb at the top of the thermometer, thus working from an arbitrary zero. Dr BALFOUR STEWART told me it was almost impossible to get trustworthy results from the thermometer so treated, and I determined to take my chance of the insufficient heating of the column of mercury in the thermometer, which was not directly immersed in the mercury in the holes in the bar. I do not think very much error can be introduced by this, for the following reasons. If we calculate for a temperature of 250° C.—which is nearly the highest used in the greater number of the experiments—the utmost error that can be introduced in the indications of the thermometers used is somewhere about 10° C. That is to say, the highest temperatures were read at the most 10° less than they would have been if the whole thermometer had been exposed to the same temperature. This correction of 10° at 250° diminishes at lower temperatures, and increases at higher nearly as the square of the excess of temperature above the freezing point. But as the same thermometers, or exactly similar ones, were employed, under precisely* similar conditions, in the short bars as in the long ones, the *difference* between the corresponding errors in the two associated experiments must have been at most a fraction of a degree even at the higher temperatures. The numerical results, therefore, are stated in terms of the temperatures *so read*, and these involve (from this cause) an error in defect, of

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* Jan. 13, 1879. In spite of the contents of § 11*, now added, this is nearly true of my experiments, for the highest of the thermometer readings in the cooling bars were not used in the calculations.

somewhere about 10° at 250° C., and varying for other temperatures as above stated. I have preserved all the notes of experiments, as well as the thermometers, as it may ultimately be possible to get an air-thermometer which will enable me to reduce the determinations to a more accurate standard; but until that can be done it seems hopeless to expect to improve (in this particular) the method I have employed, however important might be the results.

§ 11. There is one respect, and one only, in which my results have been found to be not quite consistent with those of FORBES. This is in regard to the law of cooling of the short bar in terms of the temperature. FORBES, in fact, called special attention to this question, and he evidently felt considerable surprise at the result he obtained, for he tried it over and over again with the same conclusion. Although he pointed out that the initial uniformity of temperature of the heated bar would tend to produce the *appearance* of such a result, FORBES expressed himself as convinced that the curve representing the rate of cooling of the short bar in terms of the *temperature* begins to be straight about 150° C., and then bends over so as to become convex upwards. I have carried it considerably farther, in fact, up to estimated temperatures of at least 300° C., without finding the slightest trace of convexity. It is obviously essential that this discrepancy should be explained; and I think it depends on the fact that FORBES did not heat his short bar much above the temperature (200° C. or thereabout) at which the readings commenced. Under these circumstances the flow of heat from the interior of the bar is for some time retarded; in fact, till a state of things is arrived at in which the temperatures at different distances from the axis or from the ends of the bar cease to undergo a rapid *relative* change, the inserted thermometer does not indicate the true loss of heat by the bar. I think that this explanation is borne out by the fact that FORBES' results, with a bar of smaller section and length (in which the abnormal state is of shorter duration), agree more nearly with mine, so far at least as change of rate of cooling is concerned.

I easily reproduced FORBES' results by heating the bar only to the temperature at which the readings commenced. But to avoid this source of error I always, when it could be done, raised the temperature much above the point at which readings were to begin, so as, in fact, to read only when the normal state of cooling had been arrived at. In some of my experiments with iron the bar was heated to such an extent that mercury boiled furiously when put into the hole—and I had to employ fusible metal instead. In all cases I obtained results resembling those of FORBES during the first few minutes of cooling.

The following short table illustrates this difference, as well as the fact stated in § 9 that my numbers are *all* a little too high. The first column gives the temperature-excess over the air; the second contains the rate of cooling as given by FORBES; the third column contains results obtained (for the same temperatures) by a rough graphic method from my own numbers. The rates are in degrees C. per minute :—

Rates of cooling of Iron Bar.

			Ratio.
20°	0·275	0·29	1·06
50°	0·80	0·85	1·06
100°	1·84	1·95	1·06
160°	3·18	3·45	1·09
200°	3·78	4·60	1·22
260°	4·52	6·50	1·44

I have every reason to believe that FORBES' results, in this matter, for temperatures under 150° C. are more exact than mine, especially as his bar was not exposed to air during the heating. Thus it would appear that my numbers are, *throughout,* about 5 or 6 per cent. too high. The really vital difference between our results appears in the three last numbers in the column of ratios.

[§ 11.* *Added, January* 1879.]—I was so well satisfied with the explanation given above, as in character thoroughly consistent with the observations, that I did not work out its numerical consequences. While the paper was passing through the press, however, I tried to estimate the time required for the disappearance of the abnormal state, and arrived at conclusions which are not quite consistent with this mode of accounting for the difference between FORBES' results and my own. To make this statement intelligible, a short account of FOURIER's treatment of the problem is necessary.

The equation for the cooling of an infinitely long cylinder, in which the temperature depends only upon the distance from the axis, is (assuming conductivity constant)

$$k\left(\frac{d^2v}{dr^2}+\frac{1}{r}\frac{dv}{dr}\right)=\frac{dv}{dt}\,.$$

This linear equation FOURIER integrates by assuming as a particular integral

$$v=\epsilon^{-mt}u$$

where u is a function of r only. We thus have

$$\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}+\frac{m}{k}u=0\,,$$

The surface condition (assuming rate of surface-loss to be proportional to excess of temperature over air) is

$$k\left(\frac{du}{dr}\right)_0+hu_0=0\,.$$

From the first of these equations we have u in terms of m and r. The second gives an infinite number of real positive values of m, say m_1, m_2, &c., in ascending order of magnitude, in terms of r_0 (the radius of the cylinder), k, and h. Now h is easily found (approximately) from the rate of cooling, and k is known. Hence we determine the values of m, and have

$$v = \mathrm{A}_1 \epsilon^{-m_1 t} u_1 + \mathrm{A}_2 \epsilon^{-m_2 t} u_2 + \quad . \quad . \quad .$$

where the coefficients (A) are to be calculated so as to make v agree with the initial state when $t = 0$.

Without doing this, however, it is obvious that the proposed explanation given above depends for its validity on the supposition that m_2 is not enormously greater than m_1; for, if it be, the abnormal terms due to the original uniform heating will disappear with very great rapidity.

A rough calculation showed me that $\frac{m_2}{m_1}$ for the iron bar lies between 2000 and 3000. Hence the bar is barely out of the bath before these abnormal terms have become insensible. The effect due to the finite length of the bar is easily calculated by the help of FOURIER's method for a cube, which applies to a rectangular parallelepiped of any dimensions, symmetrically heated. It depends on the fact that the temperature at any point can be expressed as the product of three functions, each containing the time and *one* only of the cöordinates. I owe this hint to Professor CHRYSTAL.

Calling $2a$, $2b$, $2c$ the edges of the parallelepiped, this method leads to the following expression—

$$v = 64\, v_0 \Sigma\left(\frac{\sin na \cos nx}{2na + \sin 2na} \epsilon^{-kn^2 t}\right) . \Sigma\left(\frac{\sin n'b \cos n'y}{2n'b + \sin 2n'b} \epsilon^{-kn'^2 t}\right) . \Sigma\left(\frac{\sin n''c \cos n''z}{2n''c + \sin 2n''c} \epsilon^{-kn''^2 t}\right),$$

where the values of n, n', n'' are the roots of

$$na \tan na = \frac{ha}{k}, \quad n'b \tan n'b = \frac{hb}{k}, \quad n''c \tan n''c = \frac{hc}{k},$$

and v_0 is the initial uniform temperature.

With the data contained in the present paper, it is easy to obtain from the above the following values of $\frac{1}{v_0}\frac{dv}{dt}$ corresponding to a uniform initial temperature (v_0) of about 200° C., the bar being $1\frac{1}{4}$ inches square, by 20 inches in length, and only the slower vanishing terms being retained :—

Iron, . .	$-0{\cdot}0235\epsilon^{-0{\cdot}0235t}\,(1-0{\cdot}068\epsilon^{-0{\cdot}16t})$
Copper (Crown), .	$-0{\cdot}0262\epsilon^{-0{\cdot}0262t}\,(1-0{\cdot}06\epsilon^{-1{\cdot}08t})$.

Hence the rate of cooling is diminished initially as regards the longitudinal flux of heat by above 5 per cent. in both bars. [The omitted terms reduce this by one-fourth, *at first.*] In copper this is diminished to 1 per cent. (less than the errors of observation) in less than two minutes, so that it cannot be traced in any of the observations, as certainly two minutes must elapse after the heating before readings can commence. In iron the error is reduced to 2 per cent. after about six minutes; so that to this cause is due a part, but only a small part, of the difference between FORBES' results and mine. For the initial sluggishness of cooling is exhibited by copper as well as iron, so that there must be another and more effective cause besides longitudinal cooling.

I next tried (but without the least hope that it would help me) whether the discrepance might not be due to the fact that FOURIER assumes k to be constant. If we assume (for the range of temperature employed)

$$k = \frac{ak_0}{a + v}$$

which is not far from the truth, the equation is no longer linear, even for the infinitely long cylinder.[*] But I found that this would not account for the result to be explained, and that no substitution of a more accurate law of cooling than that adopted by FOURIER would remove the difficulty.

Thus I was driven to seek the main cause of the phenomenon in the thermometer, not in the bar, and I traced it to the fact that the mercury in the bulb is all but fully heated almost at once, but that the final adjustment in the bulb and stem takes place more gradually. No previous heating of the bulb will much help in such a case.

To test this explanation I heated the short iron bar, and immersed a thermometer bulb at once in one of the holes, reading it, as usual, every minute. After six minutes had elapsed, I inserted a second thermometer in a hole very near the first, and read it at half time between the continued readings of the first. After another period of six minutes a third thermometer was inserted close to the others. The result has fully verified the correctness of my conjecture. The following table, graphically calculated from the readings, explains itself. A refers to the first-mentioned thermometer, B to the second, C to the third. The thermometers were read as soon as they ceased to rise.

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[*] It is interesting, however, to know that it can be transformed into

$$ak_0\left(\frac{d^2\phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr}\right) = \epsilon^{\phi}\frac{d\phi}{dt}$$

which differs only by the factor ϵ^{ϕ} on the right from the equation for constant conductivity.

Rates of cooling of the same bar, simultaneously indicated by thermometers whose bulbs had been immersed for different periods.

Temperature-excess.	A	B	C
210° C.	5·15		
200°	4·98		
190°	4·75		
180°	4·42	4·10	
170°	4·06	3·89	
160°	3·70	3·61	3·28
150°	3·33	3·28	3·18
140°	2·96	2·91	2·88

NOTE.—For this experiment the bar, which was much discoloured, was not polished previous to heating; so that the numbers are necessarily larger than those in § 11 above. This does not affect the *relative* results.

In each of these columns the differences are obviously least at the top, and the corresponding points of inflection in the curves of cooling are obviously at temperatures which are the lower, the colder was the bar when the thermometer was inserted. Also, it will be observed that the thermometers arrive more quickly at the true temperature the lower it is—*i.e.*, the shorter the column of mercury in the stem. Another experiment gave analogous results with a copper bar. Thus the main difference between FORBES' results and mine is fully explained.

One result of this discussion is that in heating the short bars it is more important to prevent oxidation than to secure absolute uniformity of heating. Another is that the hypothesis of uniform temperature in the cross-sections of the long bar is practically very near the truth.

§ 12. In the treatment of the *Statical Curves* I have always used, as FORBES did, the formula

$$\log. v = A - \frac{Bx}{1 + Cx}.$$

It is easy to work with, and its results are usually accurate within the unavoidable errors of other parts of the determination.

Where, as with the iron and the German silver bars, the nature of the problem admitted it, I have constructed graphically each of two curves of statical distribution for the same metal (with the solder at very different temperatures), and, to the same abscissæ as the values of v, the calculated values of $\frac{dv}{dx}$. One of these drawings was on tracing paper, and was superposed upon the other, with the view not merely of detecting possible errors in the calculations, but also of testing how far the results might be trusted. On this point I have no remarks to offer further than this, that the values of $\frac{dv}{dx}$ for the lower temperatures, must, when they are small, as a rule be determined *graphically*.

When the highest temperature (observed) was over 300° C., it was impossible to reconcile it with the curve deduced from the above formula from the indications of the three succeeding thermometers. As this was obviously due to the rapid expansion of mercury near its boiling point, the irreconcilable observation (sometimes as much as 10° above the curve mentioned) was not taken into account.

§ 13. The *Curves of Cooling* were at first treated in the same way. But they had to be broken up into several sections, and it was not easy to decide (without great additional labour) how to obtain the most trustworthy value of the rate of cooling at a point common to two sections, from the more or less discordant values obtained from the separate formulæ for the sections.

I next tried to treat them by taking three points with abscissæ in arithmetical progression, and determining the common quantity to be subtracted from their ordinates, so that the intervening arc might be treated as *logarithmic*. [FORBES used the logarithmic curve, but he made it pass through three points without subtraction from their ordinates.]

This is a very good method so far as results go, and might be applied to all the different curves required for these experiments. But I found that, though the details which it involves are easy, even practised calculators were liable to get confused with their multiplicity.

Finally, for my own revision of the whole work, I adopted the following method. I constructed a curve, usually with 5^m, 10^m, and 20^m intervals for the abscissæ, whose ordinates were $\frac{1}{5}$th of the *changes* of temperature during the 5^m periods, or $\frac{1}{10}$th of the changes for the 10^m periods, &c. The scale for ordinates was usually much larger than that for abscissæ. The points so determined did not, of course, give a very smooth curve (especially where successive readings at intervals of 1^m or 2^m came to be within one or two tenths of a degree of a division on the scale), but it was very easy to draw a smooth curve so as to equalize the errors, and the ordinates of this curve are at once the desired values of rates of cooling. This process has proved exceedingly successful. It is very much less tedious, and much less liable to large error, than any other at all accurate one—and its results compare favourably with those obtained by the other methods above. I believe that this process, applied to the cooling of bars, especially if one be of platinum, will give good results as to change of specific heat with temperature.

I have already stated that as the short bars were always necessarily heated much above the temperatures at which their cooling was observed, my results are a little too large. The only really serious case is that of the copper bars. But for these the curve of cooling was observed *through the same range* for very different degrees of initial heating, and it was found that the only effect of oxidation was to increase all the ordinates through that range in a slowly increasing ratio, so that the assumed correction for oxidation was easily made, and probably pretty accurate. I cannot, however, feel certain that I have in all cases applied it rightly. It is not at all easy to pronounce on an equality of oxidation of two bars (so far as our present purpose is concerned) unless both be employed for the cooling experiment.

FORBES expressed an opinion (which I do not share) against electro-plating the bars to prevent oxidation. I intend to try this method; and also, if possible, the wrapping of the bar in thin sheet iron, so as to employ FORBES' bath of solder. I have made several experiments with bars *smoked.* The method promises well, except perhaps in the case of copper, but the calculations are not yet effected.

§ 14. The *Statical Curves of Cooling* were constructed exactly as described by FORBES. But there are two remarks of some importance to be made upon the mode of obtaining their areas.

In the first place, they are not even approximately logarithmic, except for small intervals. And even then the axis is not usually the asymptote. Their area between two ordinates is usually greater than that of a logarithmic with the same axis and passing through the two points.

Secondly,—It is a matter of great difficulty to determine what to allow for the portion, in theory infinitely long, but finite in area, which extends beyond the point of lowest observation of temperature on the long bar:—except in the case of the copper bars, where the temperature was kept at the further end *lower* than that of the surrounding air. The end of the bar was introduced into a large vessel of gutta-percha full of water, which was constantly renewed from below by means of a pipe connected with a large cistern. Thus the values of $\frac{dv}{dx}$ were never very small at any observed part of the bar.

The question here raised is a very important one. It is not at all probable that the thermal conductivity should, *in all the substances I have examined,* begin to change very much more rapidly below 50° C. than it had been changing during the whole range to that point from 200° C. or even from 300° C. Hence, when I found the conductivity to be well represented between these limits (in terms of the temperature) by a straight line, I have ignored (as almost certainly due to errors inseparable from the method employed) the somewhat marked and rapidly increasing curvature, which is indicated in many cases, for the lower 20° or 30° of observed temperatures. I justify this proceeding on the ground that (in addition to the fact that the areas, the smaller ones especially, are underrated by treating the curve as logarithmic) very slight differences in the quantity allowed for the infinitely prolonged area (a quantity whose value we can only guess at) make all the difference between a rapidly increasing curvature and a rapidly diminishing one (sometimes even with a point of contrary flexure), while barely affecting the run of the higher and much more extensive part of the curve. This remark does not require (as will be seen) to be applied to the case of iron, which appears to be a thoroughly exceptional one,—though manifest indications of it are to be seen in FORBES' diagram of the conductivities of the bar when naked and when covered with paper. Another cause may have some effect here. The excesses of temperature above that of the air are so small that an inevitable error of even 0°·1 may produce a serious effect on the calculated result.

If I have sufficient leisure, in the course of next session, I hope to settle this point by using a cold water bath applied near the *middle* of each of the iron, German silver, and lead bars, the source of heat being kept at as high a temperature as in the experiments already made. I now believe from experience that in measuring conductivity, at whatever temperature, things ought to be arranged so as to avoid any very slow flux of heat. And I also think that, especially for very good conductors, such as copper, the bars should be smoked.

§ 15. With these observations I submit the following values, by no means as final even so far as my own work is concerned but, as probably fair approximations to the truth. The units are the foot, minute, and degree centigrade, the unit of heat being that required to raise the temperature of a cubic foot of the substance by 1° C. [See the end of this section.]

IRON.

Temperature C.	Conductivity.	F.
0	0·0149	(0·0190)
50	0·0138	0·0131
100	0·0128	0·0115
150	0·0121	0·0107
200	0·0114	0·0100
250	0·0109	0·0094
300	*0·0105	0·0089
350	*0·0102	...

The two numbers marked with an asterisk are merely *probable* deductions from the curve representing the others. They are introduced to show the difference in character between my results and those of FORBES, due mainly to the difference in our estimates of the rate of cooling at high temperatures. The column headed F. is (graphically) interpolated from FORBES' table (*Trans. R.S.E.*, 1864, p. 102), which refers to the same bar under the same conditions. This table does not extend below 17° C., so that the number in brackets is to some extent conjectural. It is inserted to illustrate what I have said in § 14 above as to the rapid change of conductivity indicated when temperature excesses are small.

My numbers seem to point to a temperature, somewhere about red-heat, at which the thermal conductivity of iron is a minimum,—but this is quite uncertain.

COPPER.

	Crown.	C.
0	0·076	0·054
100	0·079	0·057
200	0·082	0·060
300	0·085	0·063

I have already (§ 13 above) stated that uncertainty must attach to all these determinations of conductivity of copper at high temperatures on account of the different amounts of oxidation of the short bars and of different parts of the long bars. The small increase of conductivity with rise of temperature, here shown, *may* depend upon too great a rate of cooling having been adopted for the hotter parts of the long bars.

GERMAN SILVER.

0	0·0088
100	0·0090
200	0·0092
300	0·0094

The several experiments on German silver, both statical and dynamical, did not show so satisfactory an agreement as those on the other bars. A set of mean values is therefore given.

LEAD.

0	0·0152
100	0·0160

The experiments on lead have not been conducted through a sufficient range of temperature to make the change here indicated certain.

The experiments on gas-coke proved a failure. The method is not adapted to substances of such low conductivity.

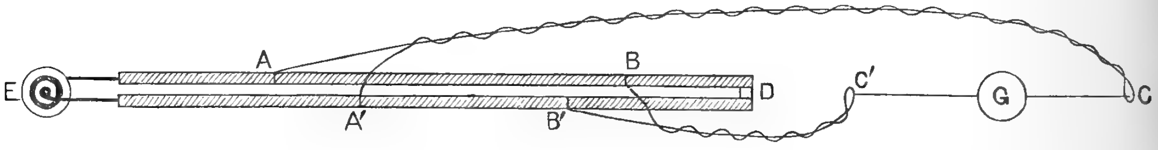
To convert these numbers to the usual unit of conductivity, they must be multiplied by the specific gravity and the specific heat of each substance: and also by the number of pounds in a cubic foot of water, if heat is to be measured in the usual thermal unit. The former constants I have as yet determined only roughly, and not for very great ranges of temperature. I need scarcely, therefore, add that in the calculations no heed has been taken of the change of specific heat with temperature. This would *increase* the values of k at higher temperatures, and thus reduce the change in conductivity in iron, and increase the small changes indicated for the other substances.

§ 16. As the above results, though the outcome of a very protracted investigation, are, for reasons already stated, only provisional, I do not think it necessary to print the details of the observations, graphical constructions, or calculations. Several points must be thoroughly cleared up before more definite statements can be made. Meanwhile the MSS. of the whole work is placed at the disposal of the Society.

§ 17. To determine the electric conductivity of the bars above described, I employed, in succession, three different methods. The results of these separate methods agreed with one another quite as well as did results by any one method made on different portions of the same bar. The German silver bar is the least uniform of the metallic bars, portions of it of 10 inches length at different parts varying through a range of as much as 5 per cent. in their conductivity. Slight defects in the casting, some of which are visible at the surface, of course easily account for this. I give the average value.

Neither the absolute nor the relative electric conducting powers of these bars were found to agree at all well with those of wires (said to be of the same material) which were furnished to me along with them. Hence some of my earlier statements to the *British Association* (especially with regard to copper) were inaccurate. The fortunate circumstance that I had no wire said to be of the same material as the FORBES iron bar, led me to test all the thick bars themselves for their electric conductivity.

§ 18. The first process I employed was that described by Sir W. THOMSON (Proc. R. S. 1861). The principle of the method will be easily seen from the following diagram.



The bars to be compared are placed parallel to one another, and connected by a small resistance D at one end, while the poles of a single cell E (sometimes short-circuited) are applied to the other ends for a period usually very short. Points A, A′, B, B′, are joined by resistances, *similarly divided* in C, C′; and these latter points are connected with the terminals of a sensitive galvanometer whose coil has a resistance, large in comparison with that of any other part of the arrangement.

Under these conditions, if i be the current in the battery, the current in the galvanometer coil is (to a sufficient approximation),

$$\frac{i}{g}\left\{\frac{aq-bp}{a+b}+\left(\frac{a}{a+b}-\frac{\alpha}{\alpha+\beta}\right)c\right\}$$

Here the resistances are AC $=a$, CA′ $=b$, BC′ $=\alpha$, C′B′ $=\beta$, AB $=p$, A′B′ $=q$, BDB′$=c$, galvanometer$=g$. If, for instance, $a=b$, $\alpha=\beta$, very exactly, and if we adjust B′ till there is no deflection, we have then

$$p=q,$$

i.e., AB and A′B′ have equal resistance. For accuracy by this method we must have, as THOMSON has pointed out, $\frac{a}{b}=\frac{\alpha}{\beta}$ very accurately, and c very small.

§ 19. The second and third methods which I employed require a differential galvanometer. This was very exactly adjusted, before the experiments, by putting the coils in multiple arc, and using the cell on them without a shunt. The exact balance was obtained by means of a box of resistance coils inserted in one or other of the branches. This being done, I connected one coil with A, B′, and the other with A′, B. Here the effect is approximately proportional to

$$i\left(\frac{q+c}{g}-e\frac{p+c}{g'}\right),$$

where g and g' are the resistances in the galvanometer coils, and e is the ratio of their deflecting forces on the needle when equal currents pass through them. The adjustment above described makes, very accurately,

$$g'=eg,$$

and the joint effect on the needle is therefore as

$$\frac{i}{g}(q-p).$$

Shifting B′ as before till there is no deflection, the resistances AB, A′B′ are equal.

§ 20. But I find by trial, that by far the most expeditious and simple method is to connect the coils of the differential galvanometer directly with A, B and A′, B′. Here the deflection is *accurately* proportional to

$$i\left\{\frac{q}{g+q}-e\frac{p}{g'+p}\right\}$$

so that the resistance c is not involved. I found, in fact, that I could, without sensible alteration of the balance, put for c (which, in addition to short portions of the thick bars, was usually a brightly polished cube of copper of the same section as the bars, and clamped very tightly between them), a short thin wire, which became red-hot when the current was allowed to pass for a few seconds. Nothing but absolutely *perfect* adjustment could have made this possible when using the other methods.

In my experiments the most unfavourable case gave

$$g > 30{,}000\, q,$$

so that q and p are practically equal when there is no deflection.

§ 21. I employed the bar C of inferior copper in all these comparative experiments. But the conductivity of the German silver bar is so much less that I could employ only 10 inches of it, as against 7 feet of the inferior copper. I therefore endeavoured, by experiments on short lengths of the two copper bars, to find approximately the correction required, in consequence mainly of the breadth of my contact pieces, very slightly, perhaps, in consequence of the great section of the bars. Here are the results in inches,—

C. A′B′.	Crown AB.	Ratio. Uncorrected.	Ratio. Corrected.	Mean of Corrected
49·66	85·7	1·726	1·732	1·729
48·47	*83·5	1·723	1·729	
41·47	71·25	1·718	1·725	
18·8	32·2	1·713	1·728	
16·65	*28·46	1·709	1·731	
9·25	15·7	1·697	1·729	

[*Note.*—In the experiments marked with an asterisk the arrangement was altered by shifting the crown bar to the other coil of the galvanometer. The agreement of these with the others is a good guarantee of the accuracy of the adjustments, and the goodness of the method is seen in the fact, that no observation deviates so much as $\frac{1}{4}$ per cent. from the mean. This is a striking verification of what was said above about the small effect of the holes bored in the bars, for the nippers were placed quite at random in the various experiments.]

The contact pieces were nippers of polished copper, 0·42 inch broad, which were easily slipped along the bars, and were tightened on them by screw clamps when the final adjustment was nearly arrived at.

It appears from the column of *corrected* ratios above, that it is only necessary to subtract 0·4 inch (the sum of the half breadths of the nippers, the wires being soldered to them symmetrically) from each of the measured distances to secure almost perfect uniformity. Thus I was led to see that the influence of the section of the copper bars is almost undiscoverable by such experiments.

§ 22. For the FORBES iron bar the following results were obtained (but with the correction 0·2 inch) :—

Fe.	C.	Ratio. Uncorrected.	Ratio. Corrected.
20	74·3	3·715	3·74
10	37·3	3·73	3·79
5	18·4	3·68	3·79

For German silver (mean of several experiments at different parts of the bar, with correction 0·2 inch),—

G. S.	C.	Ratio. Uncorrected.	Ratio. Corrected.
10	84·1	8·41	8·56

For lead (also with the correction 0·2 inch),—

Pb.	C.	Ratio. Uncorrected.	Ratio. Corrected.
14	93·7	6·69	6·77
10	66·9	6·69	6·80

These experiments were repeated for me by Mr D'Arcy Thompson, who used, as contact pieces, plates of copper pressed edgeways against the long bars in planes perpendicular to their axes. His results differ in no case from mine before the third significant figure.

§ 23. Taking the inferior copper as unit, both for thermal and for electric conductivity, we find the following table of conductivities at ordinary temperatures, with the rough results as to specific gravity and specific heat referred to in § 15 above:—

	Thermal.	Electric.
Copper (Crown), ...	1·41	1·729
„ C, ...	1·00	1·000
Forbes' Iron, ...	0·29	0·264
Lead, ...	0·12	0·149
German Silver, ...	0·14	0·117

The agreement of these numbers is by no means so close as is generally stated; but this is no longer remarkable, for it is well-known that the electric conductivity of all pure metals alters very much with the temperature, while we have seen that, as regards thermal conductivity, there is but slight change with either copper or lead, though there is a large change with iron. This accords with some results of my own on the electric conductivity of iron at high temperatures (Proc. R. S. E., 1872–3, p. 32), and with the results of the repetition of these experiments by a party of my laboratory students (Proc. R. S. E., 1875-6, p. 629).

The only alloy treated above, violates, as was to be expected, Forbes' rule for pure metals, for it seems to be superior to lead in thermal conductivity, while decidedly inferior to it as regards electric conductivity.

§ 24. The chief results of these papers may be thus briefly summarised:—

1. *The thermal conductivity of iron diminishes as its temperature is raised.*

This accords with the statement of Forbes, whose numbers for temperatures between 50° and 150° C. are probably very accurate.

2. *At temperatures above* 150° *C. the diminution of conductivity of iron is less rapid than that assigned by* Forbes. *The conductivity seems to reach a minimum somewhere about red-heat.*

3. *The thermal conductivity of copper and lead changes much less than that of iron with rise of temperature, and probably in the sense of increase instead of diminution. The same is true of German silver.*

4. *Electrically bad copper conducts heat worse than electrically good copper—but not in the same ratio.*

5. *The metals examined have the same order as conductors of heat and of electricity. The alloy violates this arrangement.*

Postscript.—As I have not given the experimental data for the first part of this paper, I may state here the peculiarity upon which the above deductions chiefly depend.

The law of cooling is nearly the same (to a constant factor) for iron and the two kinds of copper throughout the range of temperatures employed.

But the statical curve for iron differs considerably from that for copper. The ratios of the temperature-excesses at intervals of three inches along the long bars increase at higher temperatures in iron much faster than in copper. In fact, the inferior copper almost realises Lambert's result.

XXXI.—*On Thermodynamic Motivity.* By Sir W. THOMSON, F.R.S.

(Read April 3, 1876. Received for publication April 12, 1879.)

After having for many years felt with Professor TAIT the want of a word "to express the *Availability* for work of the heat in a given magazine, a term for that possession the waste of which is called *Dissipation*,"[*] I now suggest the word *Motivity* to supply this want.

In my paper on the "Restoration of Energy from an Unequally Heated Space," published in the *Philosophical Magazine* for January 1853, I gave the following expression for the amount of "mechanical energy" derivable from a body B, given with its different parts at different temperatures, by the equalisation of its temperature throughout to one common temperature[†] T, by means of perfect thermodynamic engines,—

$$W = J \iiint dx\, dy\, dz \int_T^t c\, d\, t \left(1 - \epsilon^{-\frac{1}{J}\int_T^t \mu\, d\, t}\right) \quad . \quad . \quad . \quad . \quad (1)\,;$$

where t denotes the temperature of any point x, y, z of the body; c the thermal capacity of the body's substance at that point and that temperature; J, JOULE's equivalent; and μ, CARNOT's function of the temperature t. Farther on in the same paper a simplification is introduced thus—

"Let the temperature of the body be measured according to an absolute

---

[*] Tait's "Thermodynamics," First Edition (1868), § 178.

[†] In the present article I suppose this temperature to be the given temperature of the medium in which B is placed; and thermodynamic engines to work with their recipient and rejectant organs respectively in connection with some part of B at temperature t, and the endless surrounding matter at temperature T. In the original paper this supposition is introduced subordinately at the conclusion. The chief purpose of the paper was the solution of a more difficult problem, that of finding the value of T,—a kind of average temperature of B to fulfil the condition that the quantities of heat rejected and taken in by organs of the thermodynamic engines at temperature T are equal. The burden of the problem was the evaluation of this thermodynamic average; and I failed to remark that when the value which the solution gave for T is substituted in the formula of the text, it reduces to $J\iiint dx\, dy\, dz \int_T^t c\, d\, t$, which was not very obvious from the analytical form of my solution, but which we immediately see must be the case by thinking of the physical meaning of the result; for, the sum of the excesses of the heats taken in above those rejected by all the engines must, by the first law of thermodynamics, be equal to the work gained by the supposed process. This important simplification was first given by Professor TAIT in his "Thermodynamics." It does not, however, affect the subordinate problem of the original paper, which is the main problem of this one.

scale, founded on the values of CARNOT's function, and expressed by the following equation :—

$$t=\frac{J}{\mu}-a\ ,$$

where a is a constant which may have any value, but ought to have for its value the reciprocal of the expansibility of air, in order that the system of measuring temperature here adopted may agree approximately with that of the air thermometer. Then we have

$$\epsilon^{-\frac{1}{J}\int_o^t \mu\,dt}=\frac{a}{t+a}\ , \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

It was only to obtain agreement with the zero of the ordinary centigrade scale of the air thermometer that the a was needed, and in the joint paper by JOULE and myself, published in the Transactions of the Royal Society (London) for June 1854, we agreed to drop it, and to define temperature simply as the reciprocal of CARNOT's function, with a constant co-efficient proper to the unit or degree of temperature adopted. Thus definitively, in equation (6) of § V. of that paper, we took $t=\frac{J}{\mu}$, and have used this expression ever since as the expression for temperature on the arbitrarily assumed thermodynamic scale. With it we have

$$\epsilon^{-\frac{1}{J}\int_T^t \mu\,dt}=\frac{T}{t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)\ ;$$

and by substitution (1) becomes

$$W=J\iiint dx\,dy\,dz\int_T^t c\,d\,t\left(1-\frac{T}{t}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Suppose now B to be surrounded by other matter all at a common temperature T. The work obtainable from the given distribution of temperature in B by means of perfect thermodynamic engines is expressed by the formula (4). If, then, there be no circumstances connected with the gravity, or elasticity, or capillary attraction, or electricity, or magnetism of B in virtue of which work can be obtained, that expressed by (4) is what I propose to call the whole Motivity of B in its actual circumstances. If, on the other hand, work is obtainable from B in virtue of some of these other causes, and if V denote its whole amount, then,

$$\mathfrak{M}=V+W \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

is what I call the whole Motivity of B in its actual circumstances, according to this more comprehensive supposition.

We may imagine the whole Motivity of B developed in an infinite variety of ways. The one which is obvious from the formula (5) is first to keep every part of B unmoved and to produce all the work producible by perfect thermodynamic engines equalising its temperature to T; and then keeping it rigorously at this temperature to take all the work that can be got from it elastically, cohesively, electrically, magnetically, and gravitationally, by letting it come to rest unstressed, diselectrified, demagnetised, and in the lowest position to which it can descend.

But instead of proceeding in this one definite way, any order of procedure whatever leading to the same final condition may be followed; and provided nothing is done which cannot be undone, that is to say, in the technical language of thermodynamics, provided all the operations be reversible, the same whole quantity of work will be obtained in passing from the same initial condition to the same final condition, whatever have been the order of procedure. Hence the Motivity is a function of the temperature, volume, figure, and proper independent variables for expressing the cohesive, the electric, and the magnetic condition of B, with the gravitational potential of B simply added (which when the force of gravity is sensibly constant and in parallel lines will be simply the product of the gravity of B into the height of the centre of gravity above its lowest position). So also is the *Energy* of a body B (as I first pointed out, for the case of B a fluid, in Part V. of my "Dynamical Theory of Heat," in the "Transactions of the Royal Society of Edinburgh" for December 15, 1851, entitled, "On the Quantities of Mechanical Energy contained in a Fluid in Different States as to Temperature and Density"). Consideration of the Energy and the Motivity, as two functions of all the independent variables specifying the condition of B completely in respect to temperature, elasticity, capillary attraction, electricity, and magnetism, leads in the simplest and most direct way to demonstrations of the theorems regarding the thermoelectric properties of matter which I gave in Part III. of "The Dynamical Theory of Heat" (March 1851); in Part VI. of "Dynamical Theory, Thermoelectric Currents" (May 1, 1854); in a paper in the Proceedings for 1858 of the Royal Society of London, entitled "On the Thermal Effect of Drawing out a Film of Liquid," and in a communication to the Royal Society of Edinburgh (Proc. R. S. E. 1869–70), "On the Equilibrium of Vapour at the Curved Surface of a Liquid;" and in my article on the "Thermoelastic and Thermomagnetic Properties of Matter," in the first number of the "Quarterly Journal of Mathematics" (April 1855); and in short articles in Nichol's "Cyclopædia," under the titles "Thermomagnetism," "Thermoelectricity," and "Pyroelectricity," put together and republished with additions in the *Philosophical Magazine* for January 1878, under the title "On the Thermoelastic, Thermomagnetic, and Pyroelectric Properties of Matter."

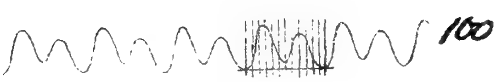
It would be beyond the scope of the present article to enter in detail into these applications, which were merely indicated in my communication to the Royal Society of Edinburgh of three years ago, as a very short and simple analytical method of setting forth the whole non-molecular theory of Thermodynamics.

Fig 12

"Attack" of ō one, voice 1.

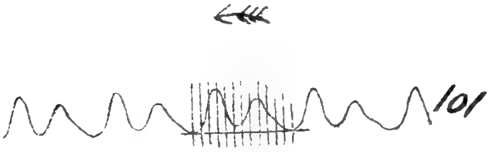
ō voice 1

Fig 13



Same utterance as No 8
but much later

Fig 14



Same utterance as No 10
but much later.

No! spoken by voice 5.

Fig 15

Fig 16

Fig 17

Fig 18

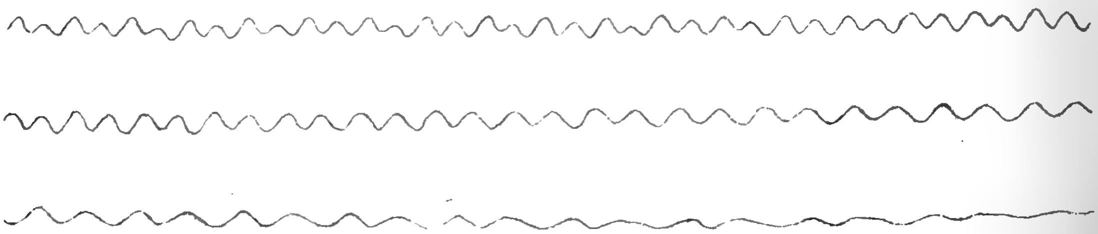


Fig 19

Fig 9

a° voice 5

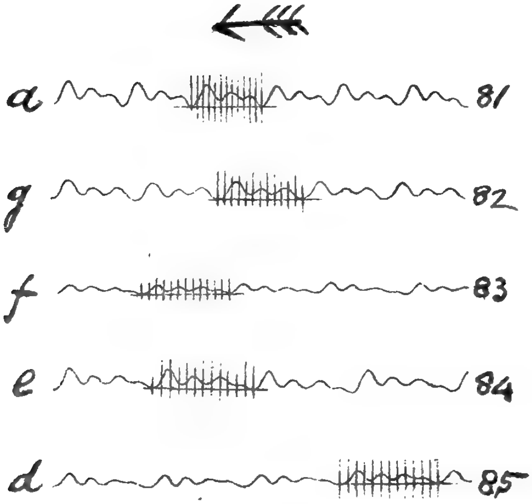


Fig 10

ā voice 5

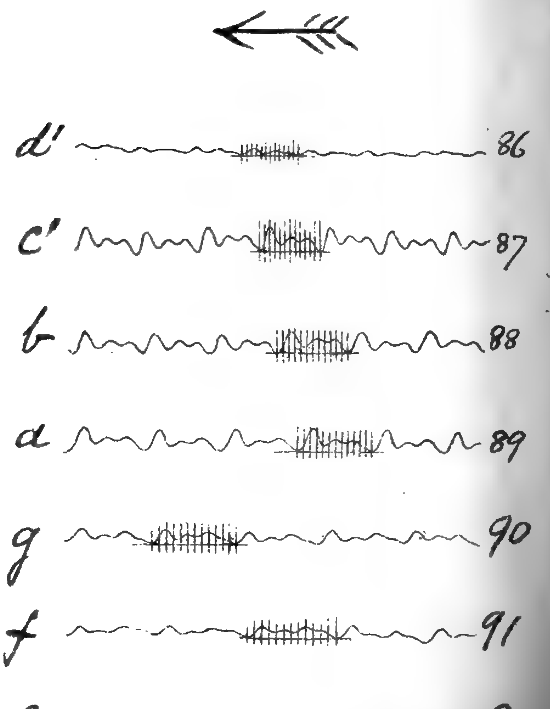


Fig 11.

ō, artificial
(Pitch by measurement)

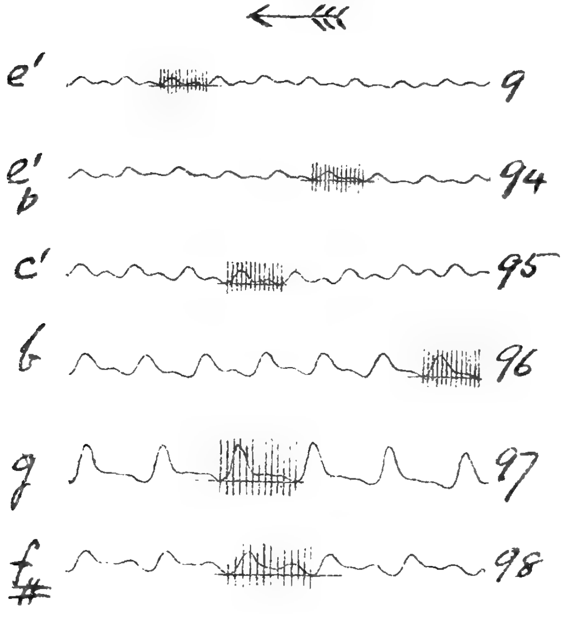


Fig 5. ō voice 6

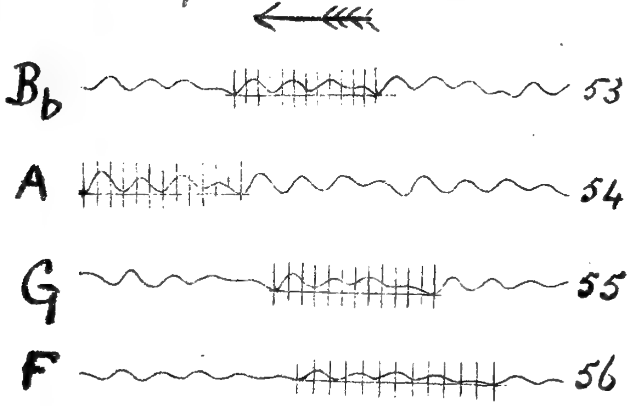


Fig 7

ū voice 5

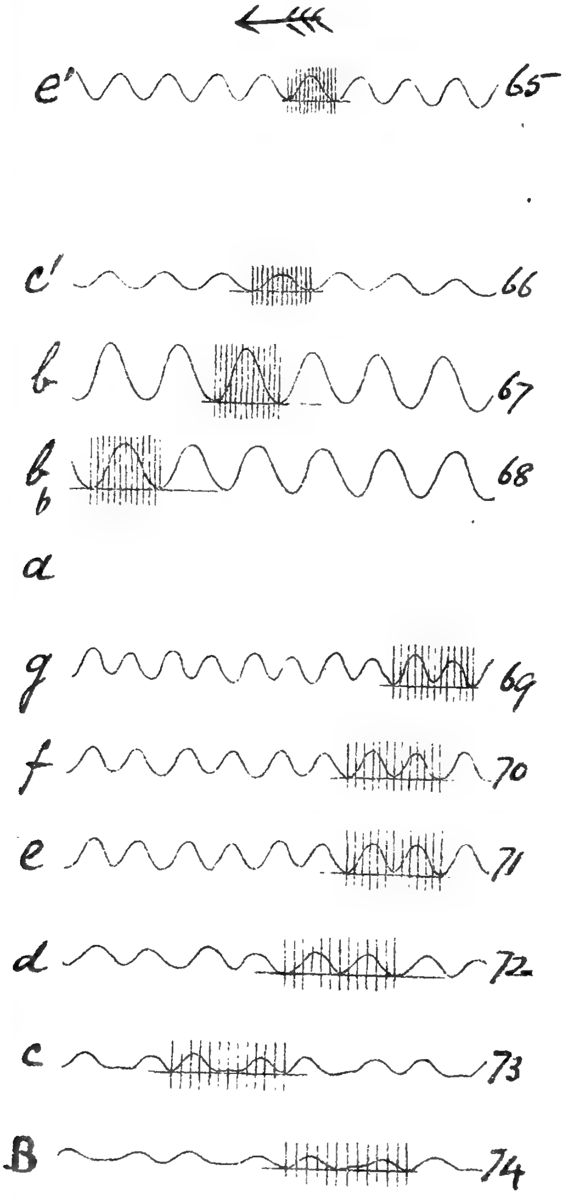


Fig 6 voice 2

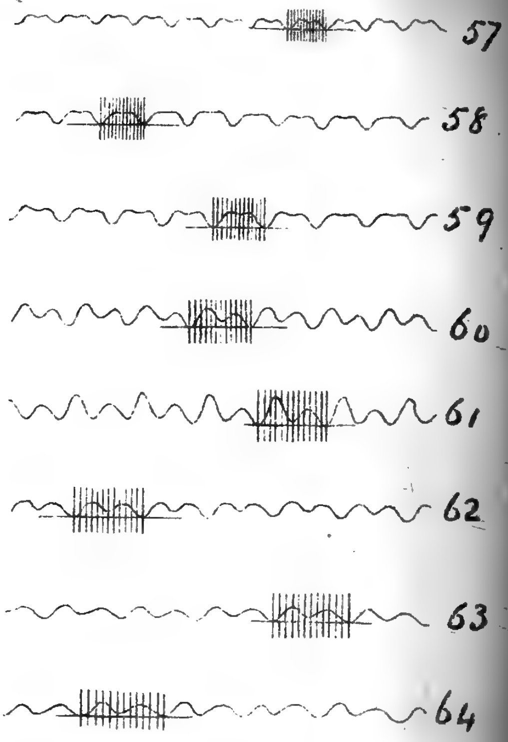


Fig 8

ū voice 1

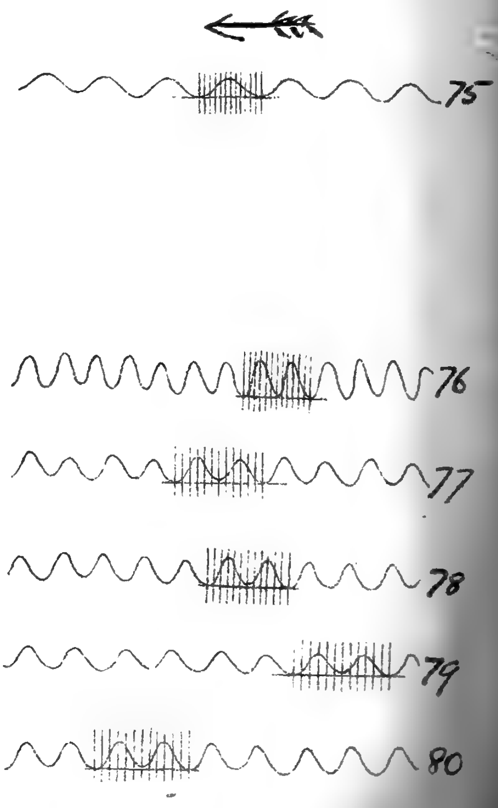


Fig 3. ō voice 3.

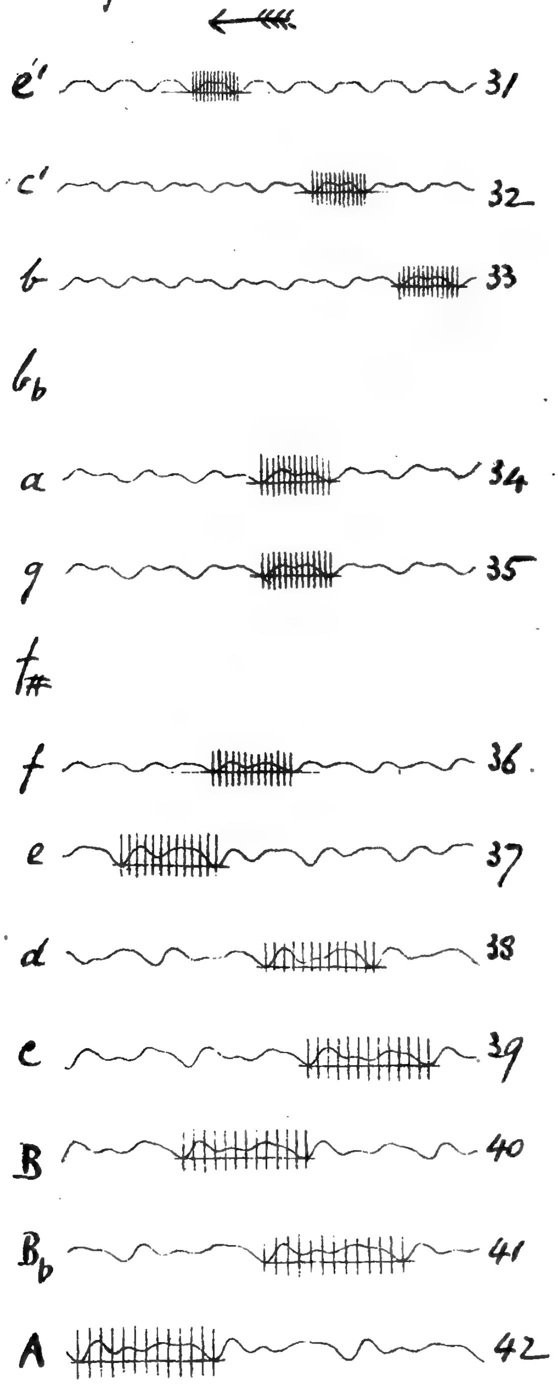


Fig 4. ō voice 4

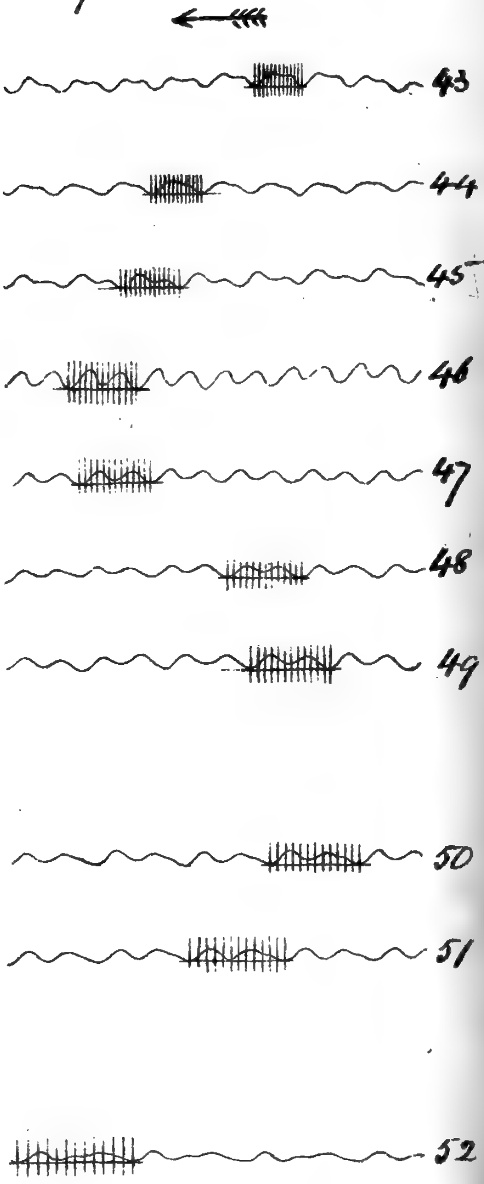


Fig 1. ō Voice 1

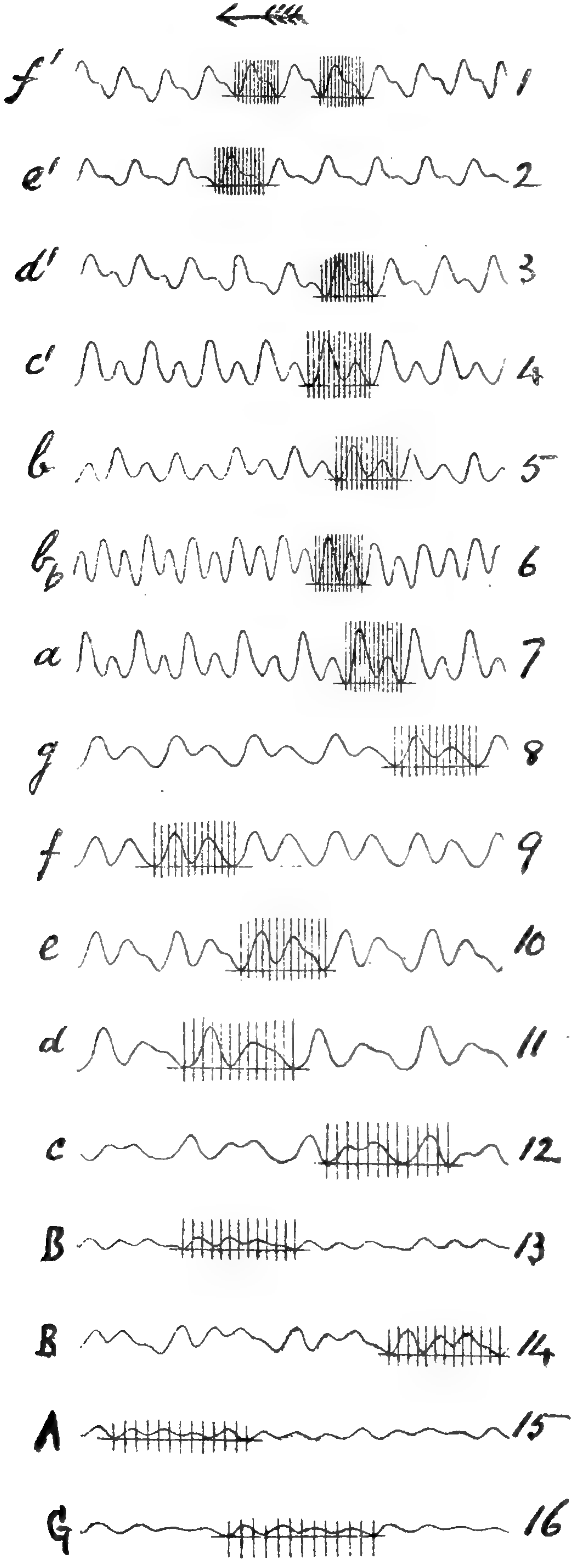
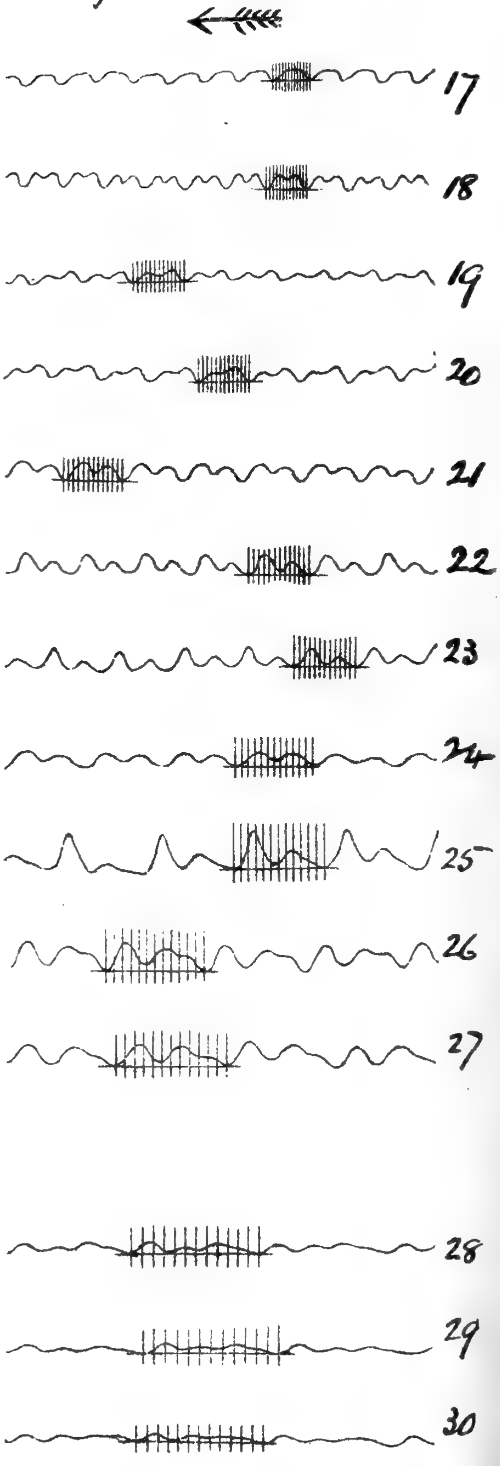
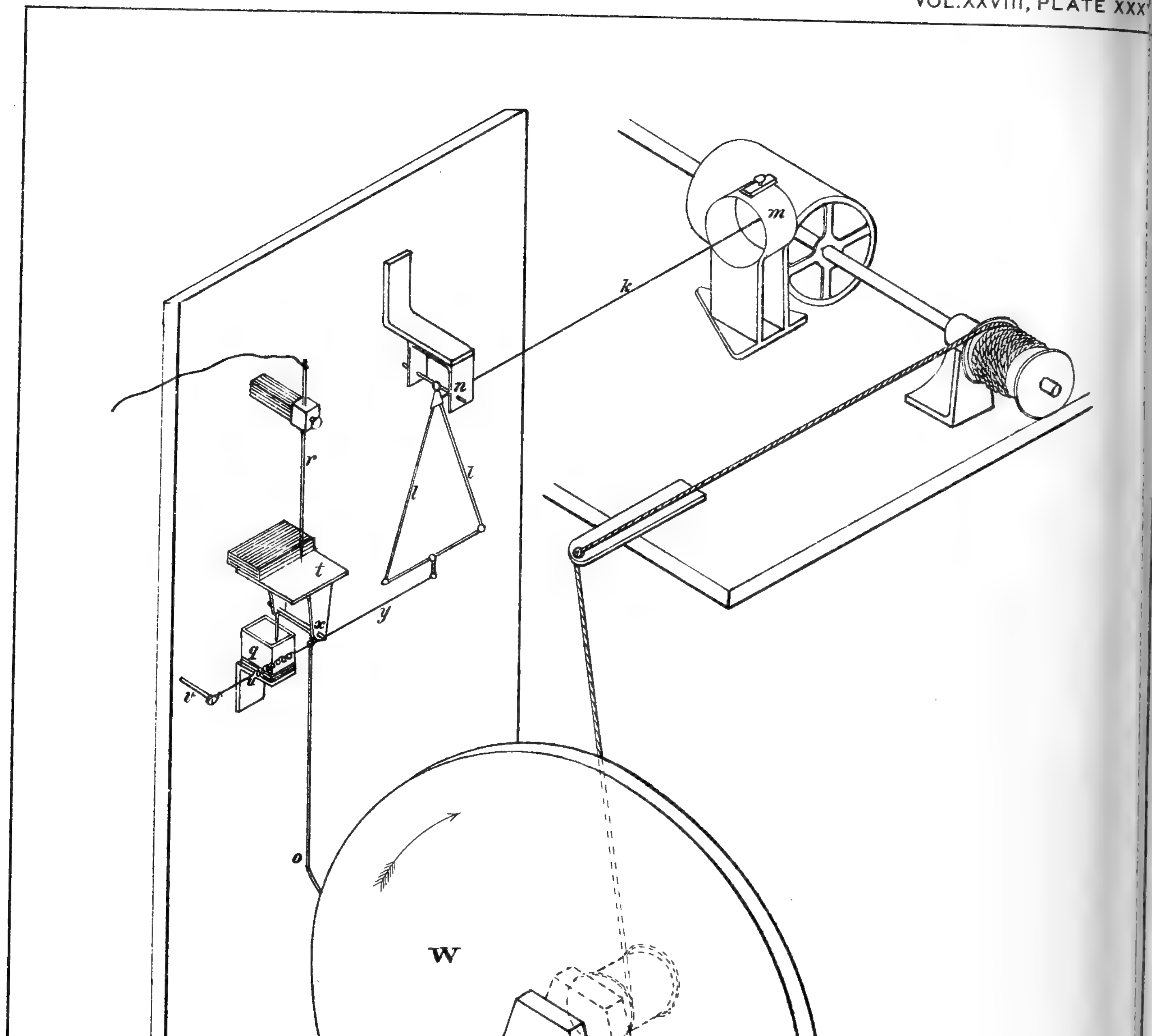


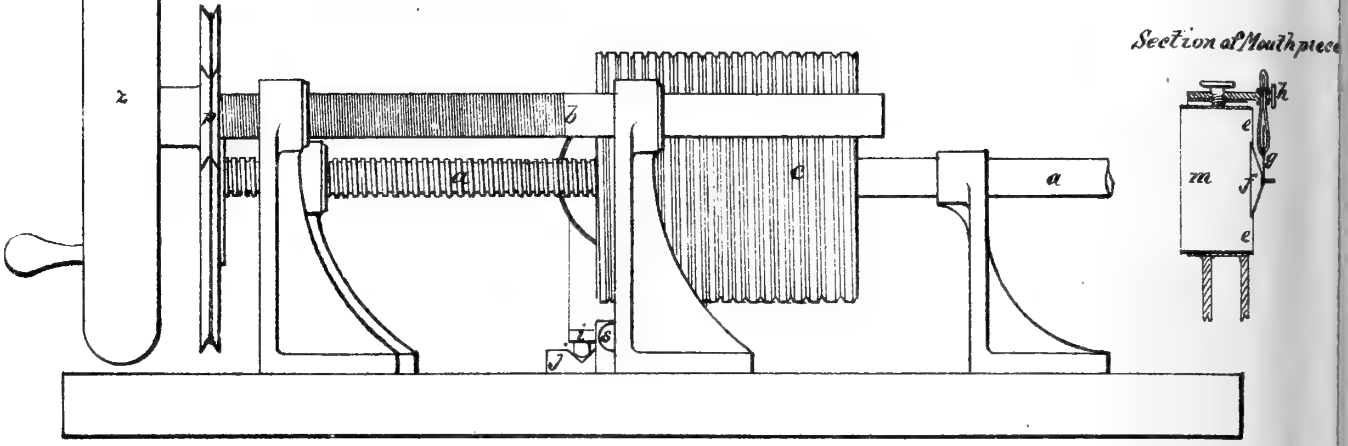
Fig 2 ō Voice 5





McFarlane & Erskine, Lithrs Ed

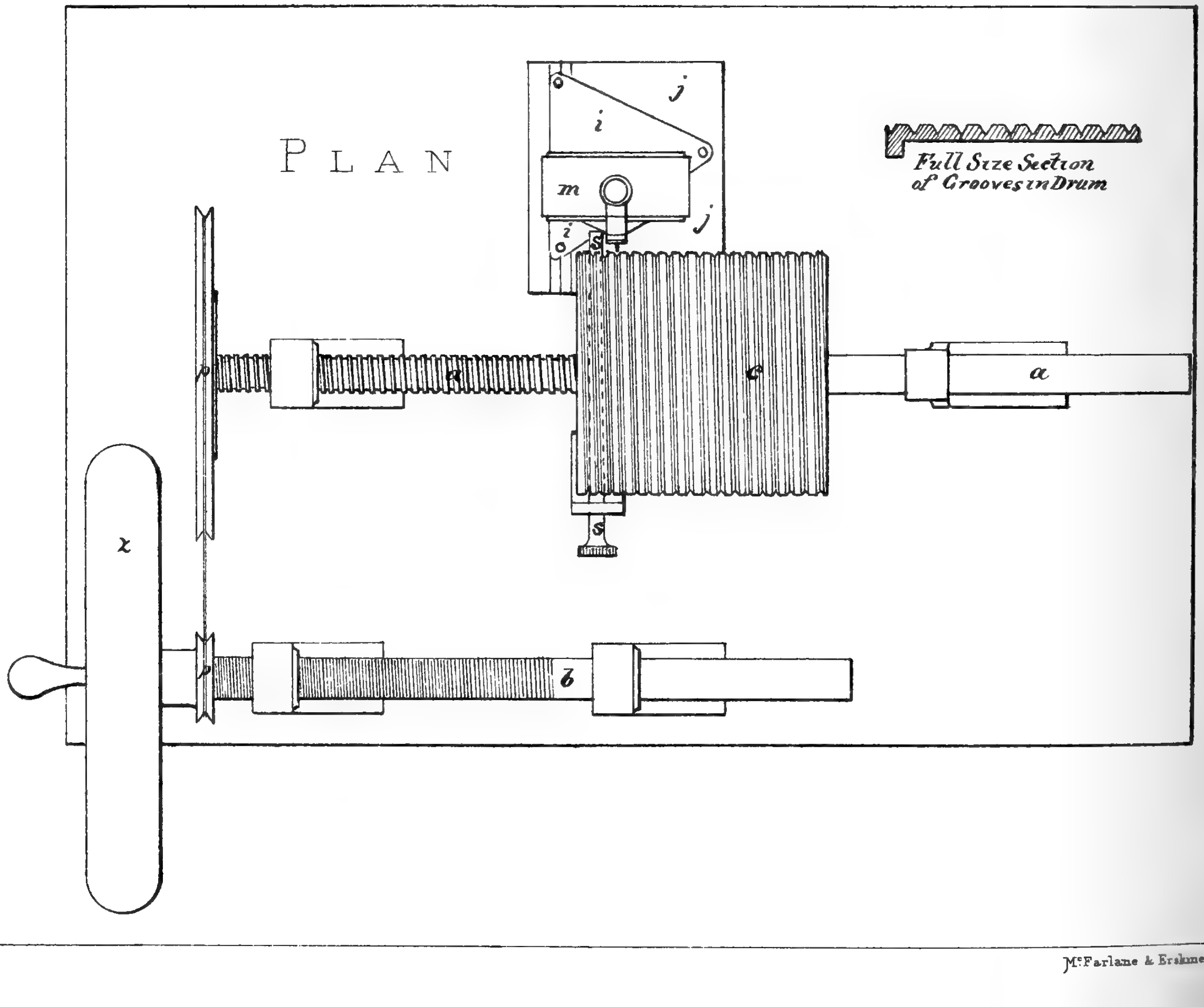
ELEVATION



Section of Mouthpiece

PLAN

Full Size Section of Grooves in Drum



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XXXII.—*On the Harmonic Analysis of certain Vowel Sounds.* By Professor FLEEMING JENKIN, F.R.SS.L. & E., and J. A. EWING, B.Sc., F.R.S.E. (Plates XXXIV. to XL.)

(Communicated June 3 and July 1, 1878. Now published, with additions down to July 19, 1878.)

The permanent record obtained with Mr T. A. EDISON's phonograph has afforded a new opportunity of investigating the nature of spoken sounds, and the method which this invention placed at our disposal appears to us to possess several important advantages.

This method consists in obtaining a magnified transcript on paper of the indentations impressed by spoken vowels on the tinfoil of the phonograph, and then subjecting the periodic wave-forms thus obtained to harmonic analysis.

The curves as drawn in ink on paper represent to a large scale the surface of a longitudinal section of the tinfoil made along the centre of the furrow impressed by the pricker of the phonograph. The forms impressed on the tinfoil depend essentially on the movement which a particle of air performs when the given sound is being uttered, and the harmonic constituents of each period of the continuous wave-form indicate the relative proportions in which the prime tone and its harmonics are present in the sound.

We thus obtain what may be called a harmonic analysis of the vowel sounds.

The curves obtained are precisely those which the phonautograph is intended to give, and our experiments confirm the accuracy of some of the records already obtained by DONDERS with that instrument. We consider, however, that our method has a double advantage over the older plan; for, in the first place, we can ascertain whether the curve produced on the tinfoil really does correspond accurately with the sound in question, by making this very curve reproduce the sound; we can thus assure ourselves that the proper periods of the disc and its connected parts have had no serious influence on the form of the curve; in the second place, the risk of this influence is much less with our arrangement than with the phonautograph, because the magnification of the curve which is necessary to allow of its examination by the eye, can with the phonographic record be performed at leisure, so that the motions of the marking points and multiplying levers can be reduced in speed until their inertia is without sensible influence on the curve finally registered. It is difficult to secure this condition with the phonautograph, especially upon high notes.

As compared with König's flames, the method now described has the great advantage of continuity. This advantage it also possesses over the direct analysis by means of resonators; and, moreover, it leaves nothing to the subjective appreciation of the observer.

The phonograph used in our experiments differed in some respects from any of which we have seen published accounts. A plan and side elevation of the instrument is shown in Plate XXXIV., the scale of which is one-third of the full size; *c* is the spirally-grooved cylinder on which the tinfoil which was to receive the impression was placed. It was supported by the axle *a*. On another axle *b*, was a heavy fly-wheel *z*. The two axles were connected by two pulleys *pp*, and a cord, marked by dotted lines in the figure. These pulleys were of such diameters that *b* made four revolutions for each revolution of *a*. Each axle had cut on it a screw working in a fixed nut used as one of the bearings. The screw on *a* was of the same pitch as the spiral groove on the cylinder *c*, and that on *b* was of one-fourth that pitch; so that as the fly-wheel revolved the pulleys which communicated motion from one axle to the other advanced together longitudinally, and remained always in the same plane with one another. By this arrangement the power of the fly-wheel to give uniformity of motion was much increased without giving too great a speed of revolution to the cylinder. The mouthpiece *m*, which is shown in section, and its mode of support, also deserve notice. The mouthpiece consisted of an inner and an outer brass tube; the outer tube was fixed to the stand, and the inner tube was clamped in its place by a screw passing through the top of the other. On the end of the inner tube the speaking disc was fastened—consisting of a ring *e*, of oil-silk and a central cone *f*, of stiff paper. At the apex of the cone was the pointer which indented the tinfoil. It was of hard steel, shaped like a slightly rounded chisel, and sharp in the horizontal plane. It was further supported and directed by a short piece of watch-spring *g*, which was rigidly secured to a projecting bracket *h*.

This form of speaking disc was adopted after experiments with a great number of different shapes and materials. By other appliances much louder sounds could be obtained from the phonograph, but these were defective in distinctness, while this arrangement combined a moderate degree of loudness with remarkable clearness and purity of utterance. The loudness of the articulation was increased by placing a wooden mouthpiece, such as is used in Prof. Graham Bell's telephone, at the other end of the tube *m*, but this was found to damage the purity of the sounds, no doubt by making the tube act more as a resonator, which unduly favoured certain tones. As a sort of gauge of the merit which our phonograph possessed as a means of recording and reproducing sound, we may mention that no difficulty was felt by hearers in making out sentences as repeated by it, which had been originally spoken in their absence.

The tube was secured to a triangular brass plate *i*, with three rounded feet, two of which stood in a V-groove in another brass plate *j*, fixed on the sole-plate of the instrument, while the third foot rested against a plane surface on a part of the plate *j*. These feet gave five points of support, and left the stand one degree of freedom of motion, namely, to slide along a horizontal line at right angles to the axis of the cylinder *c*. The sixth degree of constraint was given by the screw *s*, which abutted against the plate *i*, and which served to adjust the pressure of the pointer against the tinfoil. The stand holding the mouthpiece was kept pressing against the end of the screw *s* by a couple of india-rubber bands. This arrangement enabled the mouthpiece to be drawn back, or even completely removed with perfect facility, and then replaced at any time in absolutely the same position as before.

Our first aim was to secure well-defined curves of such dimensions as would allow them to be subjected to harmonic analysis, and to obtain these under such conditions as should leave no doubt that each curve did truly represent the sound spoken in, at least, its essential vowel quality, if not in all its minor characteristics.

Our method of obtaining these curves was as follows:—The sound to be examined was spoken or sung with the mouth of the speaker close to, but not touching, the end of the tube *m*, and while the sound was being uttered the fly-wheel was turned as steadily as possible by an operator who kept time with a metronome. The metronome gave rather more than three beats in a second, and the fly-wheel was turned so as to make one revolution for each beat. This gave a movement to the cylinder regular enough to enable the pitch of the sound, when not otherwise known, to be afterwards determined to within one semitone by measuring the records. Special care was taken to turn the phonograph regularly when the determination of the pitch was known to depend on a measurement of the marks. We employed two kinds of tinfoil; in the earlier experiments the weight of the foil was 268 grains per square foot, and in later experiments 314 grains per square foot. Our best results were obtained with the thicker tinfoil, the resistance of which seemed in no way to interfere with the accuracy of the curves; on the contrary, this resistance served to augment the directing force under which the disc vibrated, and allowed higher tones to be correctly registered.

The embossed or indented tinfoil record was transcribed so as to give a magnified curve drawn in ink on paper by means of the appliances shown in Plate XXXV. The pointer of the speaking disc was connected by a fibre *k*, of fine glass to a lever *l*, consisting (for the sake of lightness and rigidity) of a triangular framework of straws; this frame was pivoted in a fixed support at *n*. A silk thread *y*, joined the bottom end of *l* with a fine glass siphon, which formed a second lever pivoted at *x*, the upper end of which was bent round so as to dip into box *q*, holding ink, whilst the lower end was turned so as to point towards the periphery of a large wheel, which it approached but did not touch.

A long band of paper an inch wide was coiled round the circumference of W, and as this revolved past the siphon *o* it received the tracing in the form of a succession of fine spots of ink, which were deposited by the siphon as the result of a continuous electrification, which came as a discharge from the rod *r* to the plate *t*. The ink box *q* and the rod *r* were supported and insulated by vulcanite brackets, and the rod was connected by a wire to a small "mousemill" or inductive electrical machine. This method of registering the movements of a pointer was invented by Sir WILLIAM THOMSON, and is used in his telegraphic recorder. It has been used by us before in experiments on friction, and a full account of it will be found in the paper describing these experiments ("Phil. Trans.," vol. clxvii. p. 509). It has the immense advantage of doing away with all friction between the recording pencil and the paper on which the tracing is drawn.

The directive force needed to bring the siphon *o* quickly back to the vertical position after having been displaced was given by the spring *u*, which, although shown as a spiral in the drawing, actually consisted of a delicate straight thread of india-rubber with one end fastened to the siphon at the same point as the fibre *y*, while the other end was secured to the fixed support *v*. This spring was stretched sufficiently to make the system of levers very "dead-beat;" that is to say, their recovery after displacement was rapid and unaccompanied by any gradually dying away oscillation. All the fixed supports formed part of a very strong framework which projected from the iron table on which the phonograph stood, so that any shaking or movement of that table, such as might have been caused by a footfall in its neighbourhood, had no effect in altering the relative position of any of the parts. Glass was chosen for the connecting thread *k*, after many other substances had been rejected as unsuitable owing to their change of length under the varying stress caused by the greater or less extension of the spring *u*.

With this arrangement each displacement of the pointer of the phonograph through any very small distance produced a proportional displacement of the end of the siphon through about four hundred times that distance.

Hence when the phonograph containing the record of a spoken sound was placed in position as shown in the figure, and the pointer adjusted so as to press gently into the embossed furrow in the tinfoil, if the cylinder of the instrument were slowly turned the end of the siphon *o* would copy on a greatly enlarged scale the movements which had originally been made by the pointer and disc when under the influence of the voice. And if at the same time the electrical machine were set in action, and the wheel W turned so as to draw the paper ribbon past the siphon, an enlarged copy of the wave-forms of the spoken sounds would be transferred to the paper. To complete the apparatus it was only necessary to establish a mechanical connection between the wheel W and the cylinder *c* of the phonograph, so that the two might revolve with a constant velocity ratio. This was done very simply by putting a wooden drum *d* on the axle of the phonograph, another drum *w* on the axle of the wheel, and then connecting the two by a long string. The magnified ink curves were then obtained by putting the mousemill in action, and very slowly turning the wheel W, which pulled round with it the cylinder of the phonograph; the speed was always kept down sufficiently to prevent the inertia of the moving parts from affecting in the smallest degree the truth of the transcribed record.

The figures which are given (Plates XXXVI. to XL.) are exact copies of wave-forms traced out in this manner. The length of a complete period in the figures is about seven times its length on the tinfoil. Each figure gives only a few consecutive periods, chosen out of perhaps the two or three hundred which went to make up the complete utterance. These specimens are, however, in the case of sounds *sung* so as to preserve both pitch and vowel quality, really representative of the whole tracing. As a general rule, the tracings were remarkable for the constancy with which the same wave-form repeated itself over hundreds of periods; while, on the other hand, in a few cases there was a greater roughness and irregularity in the forms than could fairly be laid to the charge of the method of copying on the original state of the sheet of tinfoil. This irregularity may perhaps in some cases have been due to the presence of inharmonic constituents in the sounds, but at other times was certainly due to wavering in the voice. These cases of irregularity were rare, and on no occasion have we observed periodic variation of the wave with a recurrence to an old form, even in the most extended utterances.

The excursions of the end of the siphon were never so large as to make the obliquity of the levers a practical source of error.

After the record on the tinfoil had been used to produce the magnified curves, the phonograph was withdrawn from the multiplying apparatus, and the record was made to reproduce the sounds originally spoken. In no case was the transcribed curve accepted as satisfactory unless the tinfoil proved still able to give the original sound satisfactorily. This was done to guard against the acceptance of faulty curves, in which the forms impressed on the tinfoil had been partly obliterated by the process of transcribing. The adjustment of the pressure between the pointer and the tinfoil during that process, so that it should be neither so great as to obliterate the record nor so small as to prevent the pointer from going to the bottom of the indentations in the foil, was a matter of the greatest delicacy, and required constant attention. The precaution mentioned above of making the tinfoil record repeat the sounds after transcription served also as a continual check on the fidelity with which the phonograph itself was registering the spoken sounds.

After the curves had been drawn they were subjected to harmonic analysis to determine the amplitudes of their constituent partial tones. The curves may be regarded as giving a graphic representation of a functional relation between x and y where these are rectangular co-ordinates, the axis of x being parallel to the line joining successive maxima or minima. This function, being periodic, may by FOURIER's theorem be represented by the well-known expression—

$$y = A_0 + \Sigma_{n=\infty}^{n=1} \alpha \sin (nx + \beta),$$

where A_0 is a constant depending on the position chosen for the axis of x and the terms under the sign of summation are simple harmonic constituents corresponding to the partial tones of HELMHOLTZ: the prime being given when $n=1$, the second partial or octave of the prime when $n=2$, the third partial a twelfth of the prime when $n=3$, and so on. The successive values of α are the amplitudes of the several partials, and β their phase. The above expression may be written—

$$\begin{aligned} y = A_0 + A_1 \sin x + A_2 \sin 2x \ldots\ldots + A_n \sin nx + \ldots\ldots \\ + B_1 \cos x + B_2 \cos 2x \ldots\ldots + B_n \cos nx + \ldots\ldots \end{aligned}$$

where A and B are such that

$$\alpha_n = \sqrt{A_n^2 + B_n^2} \text{ and } \beta_n = \tan^{-1} \frac{B_n}{A_n}.$$

By drawing and measuring a number of values of y for a given periodic curve, a corresponding number of values of A and B can be evaluated. Our process of analysis was as follows :—A straight line was drawn as axis tangent to two successive *minimums* in the transcribed curve, so as to include one whole period. The length of the period was determined by comparison of several on each side of the one drawn, and a portion of the straight line equal to the period was set out and divided into twelve equal parts. From these divisions perpendicular lines were drawn cutting the curve. The length of these lines between the axis and the curve were measured in two-hundredths of an inch, by means of a scale graduated to twentieths (the decimals being estimated). The numbers obtained in this way formed twelve values of y for the one period chosen, and afforded the data for calculating the amplitude and phases of the first six constituent partial tones. Professor TAIT was kind enough to supply us with the solutions of the simultaneous equations for twelve value of y, having used them before in another investigation, and we have verified them independently. They are given below :—

$$A_0=\frac{1}{12}\left(y_1+y_2+y_3+y_4+y_5+y_6+y_7+y_8+y_9+y_{10}+y_{11}+y_{12}\right)$$

$$A_1=\frac{1}{12}\left\{2y_1+y_3-y_5-2y_7-y_9+y_{11}+\sqrt{3}(y_2-y_6-y_8+y_{12}).\right\}$$

$$A_2=\frac{1}{12}\left(2y_1+y_2-y_3-2y_4-y_5+y_6+2y_7+y_8-y_9-2y_{10}-y_{11}+y_{12}\right)$$

$$A_3=\frac{1}{6}\left(y_1-y_3+y_5-y_7+y_9-y_{11}\right)$$

$$A_4=\frac{1}{12}\left(2y_1-y_2-y_3+2y_4-y_5-y_6+2y_7-y_8-y_9+2y_{10}-y_{11}-y_{12}\right)$$

$$A_5=\frac{1}{12}\left\{2y_1+y_3-y_5-2y_7-y_9+y_{11}-\sqrt{3}(y_2-y_6-y_8+y_{12})\right\}$$

$$A_6=\frac{1}{12}\left(y_1-y_2+y_3-y_4+y_5-y_6+y_7-y_8+y_9-y_{10}+y_{11}-y_{12}\right)$$

$$B_1=\frac{1}{12}\left\{y_2+2y_4+y_6-y_8-2y_{10}-y_{12}+\sqrt{3}(y_3+y_5-y_9-y_{11})\right\}$$

$$B_2=\frac{1}{4\sqrt{3}}\left(y_2+y_3-y_5-y_6+y_8+y_9-y_{11}-y_{12}\right)$$

$$B_3=\frac{1}{6}\left(y_2-y_4+y_6-y_8+y_{10}-y_{12}\right)$$

$$B_4=\frac{1}{4\sqrt{3}}\left(y_2-y_3+y_5-y_6+y_8-y_9+y_{11}-y_{12}\right)$$

$$B_5=\frac{1}{12}\left\{y_2+2y_4+y_6-y_8-2y_{10}-y_{12}-\sqrt{3}(y_3+y_5-y_9-y_{11})\right\}$$

The process of measurement and calculation indicated above has been applied to more than a hundred curves. It extended only to the sixth partial tone, and, already laborious, would have been far more so if an attempt had been made to carry it further. To do so, however, seemed to be unnecessary; the results of the analysis showed that the curves were constituted essentially of low lying partials, and, independently of that, we knew beforehand that our phonograph was incapable of registering tones of very high pitch. A shrill whistle, however loud, made no impression on the tinfoil. No doubt, this accounts for the fact that the phonograph fails to reproduce certain vowels well, notably the sound *i* (as in machine), and the French or German *ü*. It shows, however, that those vowels which the phonograph does speak well have their essential characteristics determined by comparatively low partial tones; and our investigation has been confined to vowels of this class. Several of the curves, after analysis, were built up again graphically by synthesis of their constituent simple harmonic waves, and this process always gave an accurate reproduction of the original.

We have paid but little attention to the phases of the constituent tones. They did not appear to follow any simple rule. One and the same voice singing the same vowel at the same pitch on different occasions, although generally adhering to one wave-form, sometimes gave a changed phase relation of approximately the same constituents. On this account we have not taken the trouble to calculate the phase relation except in a few cases, but have confined our attention to the much more important peculiarity, the amplitude of the tones. It may be observed that the experiments have given thorough confirmation of HELMHOLTZ's discovery that vowel quality is not dependent on phase relation, so long as the constituent tones are unchanged.

To satisfy ourselves that the curves obtained were not affected to any practical extent by the particular form of instrument employed, we made numerous experiments with varied discs, springs, and mouthpieces, as well as with several different thicknesses of tinfoil, the result being to prove that the curves were so little affected by the special conditions of the apparatus, that they might be considered as giving a true and consistent record of the essential characteristics of the sound.

Having said so much by way of preface as to the mode of obtaining and analysing the curves, we now pass on to speak of the result. The experiments were chiefly directed to the two sounds *ō* and *ū* (the vowels in "oh!" and "food"). Several very different voices were employed. Voice No. 1 was a powerful baritone, with a very considerable range and good musical training. No. 2 was a high set and somewhat harsh voice of limited range, and without musical training. No 3 was a rich and well-trained bass voice of a man of eighty. Nos. 4 and 5 were somewhat alike, being voices of moderate range and power, and with some musical training; No. 5 was the stronger of the two. No. 6 was a powerful bass. Generally the vowel sounds were sung in tune with notes given by a piano, and when not otherwise described, it is to be understood that the pitch was determined in this way. In other cases, which will be named when they occur, the sounds were spoken or sung at random, and the pitch was determined afterwards by measuring the lengths of the curves.

Figure 1* (Plate XXXVI.) gives the wave-forms for a series of *ō*'s sung by voice No. 1. The pitch on which the vowel was sung is denoted by the letter alongside the curve. In naming pitches we have adopted the usual notation, in which c' has 256 complete vibrations per second, and c'' 512. A rise in the curve, as printed here, corresponds with a hollow in the tinfoil. The arrow pointing from right to left shows the direction in which the tinfoil passed under

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[*] These figures have been reproduced by a photo-lithographic process.

the vibrating pointer as the sounds were being uttered. The height of the waves is, as we have already said, about 400 times the depth of the marks on the tinfoil, and their length is about seven times that of those marks. The examples are all numbered separately by figures on the right hand side, for convenience of reference to the analysis which will be given later.

Figure 2, Plate XXXVI, gives a corresponding series of ō's sung by voice 5. They are placed so that the letters denominating the pitches serve both for figures 1 and 2. Figs. 3 and 4, Plate XXXVII., give ō's by voices 3 and 4.

In every case the specimens of the curves, which are given here, include the particular periods which were selected for analysis.

It will be seen that some of the curves in the upper parts of the scale bear considerable resemblance to that published by DONDERS, and obtained with the phonautograph. The general resemblance to the forms given by KÖNIG's flames is also obvious.

Table I. gives the results of the harmonic analysis of the set of ō's contained in figure 1. Tables II., III., and IV. correspond to the curves in figures 2, 3, and 4. The first diagonal line sloping upwards, and from left to right, gives the amplitude of the primes in the curves obtained by singing the vowel on different notes. The second diagonal line gives the amplitudes of the second partials; and so on up to the sixth partials, beyond which the analysis did not extend. The absolute pitch of each partial tone is given vertically under it by the letters printed along the bottom of the table. The six numbers in each horizontal row are the amplitudes of the six partials in one single utterance. The number placed opposite each horizontal row, on the left side of the table, is the reference number for the particular example, corresponding to the number on the right hand side of each curve in the figures. Thus, for example, No. 3 is ō sung on *d′*; the amplitude of its prime is 119, of the II. partial 76, of the III. partial 5, of the IV. partial 10, of the V. partial 3, and of the VI. partial 2.

[TABLE I.

TABLE I.—*Ō*, Voice 1.

	G	G♯	A	B♭	B	c	c♯	d	e♭	e	f	f♯	g	g♯	a	b♭	b	c'	c♯'	d'	e♭'	e'	f'	f♯'	g'	g♯'	a'	b♭'	b'	c''	c♯''	d''	e♭''	e''	f''	f♯''	g''	g♯''	a''	b♭''	b''	c'''	c♯'''	d'''	e♭'''	e'''	f'''	f♯'''	g'''	g♯'''	a'''	b♭'''	b'''	c''''
1																							121												71							7					1				5			4
2																						105												69							7					3				2			3	
3																				119												76							5					10				3			2			
4																		110												160							15					10				10			7					
5																	70												126							15					14				6			1						
6																75												185							13					8				11			1							
7															125												190							25					22				5			2								
8													69												103							27					6				2			2										
9											55												140							45					4				8			1												
10										72												131							73					7				10			4													
11								44												134							82					16				22			10															
12						18												95							61					33				3			0																	
13					25												15							28					31				6			5																		
14				37												58							61					47				11			0																			
15			15												15							18					29				6			3																				
16	13												0							15					40				8			4																						

TABLE II.—$\bar{O}$, VOICE 5.

	A	B♭	B	c	c♯	d	e♭	e	f	f♯	g	g♯	a	b♭	b	c'	c♯'	d'	e♭'	e'	f'	f♯'	g'	g♯'	a'	b♭'	b'	c''	c♯''	d''	e♭''	e''	f''	f♯''	g''	g♯''	a''	b♭''	b''	c'''	c♯'''	d'''	e♭'''	e'''	f'''	f♯'''	g'''	g♯'''	a'''	b♭'''	b'''	c''''
17																					53												19							6					2				3			1
18																				52												53							5					6				5			2	
19																		27												42							6					4				2			1			
20																47												41							5					3				3			2					
21															47												61							2					14				8			2						
22														48												70							13					7				1			3							
23													40												68							10					8				3			0								
24											32												50							3					6				1			2										
25									70												109							58					13				12			5												
26								41												88							64					13				5			2													
27						33												72							56					5				7			2															
28			6												38							23					25				6			3																		
29		18												26							15					15				2			2																			
30	15												8							22					21				4			0																				

The figures given for the amplitude of the partial tones are the actual amplitude calculated in $\frac{1}{200}$ths of an inch, from measurements of the transcribed curves shown above. They are therefore not merely relative, but afford an indication of the loudness of each utterance. This indication is, however, only a very rough one, for the amplitudes depend on many other conditions than the loudness of the tones; in particular, they are greatly affected by the closeness of the mouth to the vibrating disc, and no special care was taken to keep that distance constant.

Even in the absence of slight and unavoidable irregularities in the curves, it would have been almost impossible to have made the measurements with such accuracy as to determine with certainty the values in the unit place for the above numbers. The last figure of each group is not to be depended upon, and when an amplitude, such as 2 or 3, is given for any tone, that tone may very possibly have been entirely absent in the round ring, as it may very possibly have been present to double the extent indicated by the figures.

Table II. gives the analyses of the examples in fig. 2 of *ō* sung by voice 5.

The most cursory examination of the above curves and tables show that at different pitches the vowel sound *ō* is by no means composed of the same relative constituents, but, that as the pitch alters, changes take place which are fairly gradual and consistent throughout. Above *g*, indeed, the results might be described with little reference to absolute pitch. In that region of the scale the sound *ō* consists almost wholly of two partial tones—the first and the second, the proportion between which, however, depends partly on the pitch and partly on the quality of the voice. On the highest notes reached the proportion of prime to second with each voice is greater than it is a few tones lower down, but the proportion varies greatly with different voices, even at the same pitch. Below *g* the third partial begins to be prominent; the second, however, still remains very strong, and the prime moderately so. This state of things continue on *e* and *d*. Then the fourth partial appears, and is very strong on B, B♭, A, and even G (No. **16**); the prime, however, is now very weak; the second partial continues pretty strong, except on G, and the third is very strong. Thus we have, on going down the scale, the general result that one after another of the upper partial appears in succession, while first the prime and then the second partial become much weakened; there being always at least two partials strongly reinforced, and generally more than two.

Before, however, proceeding to examine these figures in detail, and to point out their bearing on existing theories of vowel sounds, we shall give a number of additional examples of *ō* sung by other voices. Fig. 3 gives the curves for a set of *ō*'s, sung by voice 3, and fig. 4 a similar set by voice 4.

Table III. and Table IV. contain the results of the analyses of the curves in figs. 3 and 4 respectively.

TABLE III.—Ō, Voice 3.

	A	B♭	B	c	c♯	d	e♭	e	f	f♯	g	g♯	a	b♭	b	c'	c♯'	d'	e♭'	e'	f'	f♯'	g'	g♯'	a'	b♭'
31																				53						
32																37										
33															36											
34													40												36	
35											46												29			
36									25												37					
37								40												67						
38						27												61							38	
39				19												48							33			
40			21												46							29				
41		28												41							25					36
42	26												41							35					39	

	b'	c''	c♯''	d''	e♭''	e''	f''	f♯''	g''	g♯''	a''	b♭''	b''	c'''	c♯'''	d'''	e♭'''	c'''	f'''	f♯'''	g'''	g♯'''	a'''	b♭'''	b'''
31						18							3					1				2			1
32		30							1					4				1			1				
33	31							2					4				1			1					
34						4					4				3			2							
35				2					2				2			1									
36		11					3				4			2											
37	35					5				5			2												
38				19				4			2														
39		18				12			4																
40	28				10			0																	
41				9			0																		
42			18			2																			

TABLE IV.—$\bar{O}$, VOICE 4.

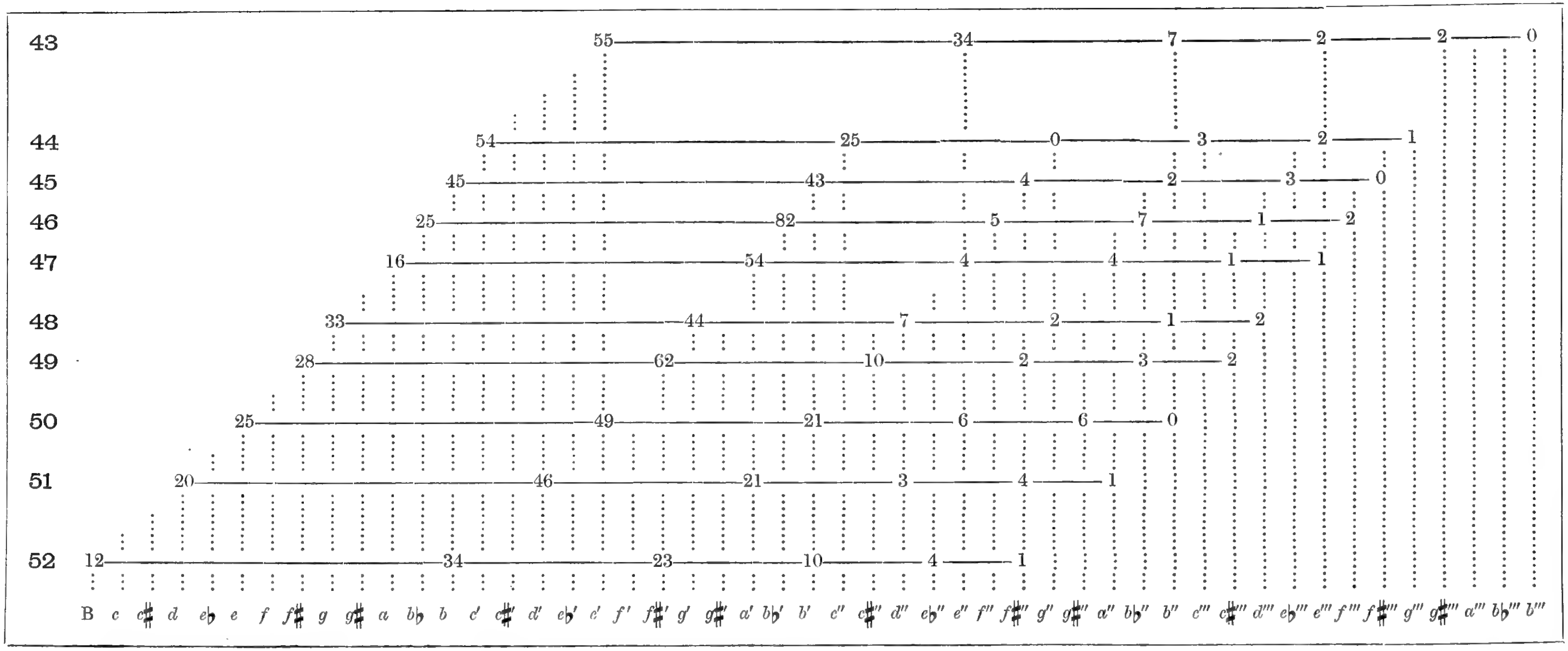


Fig. 5, PlateXXXVIII. and Table V., give the curves and analyses of four very low bass *ō*'s, sung by voice 6.

Fig. 6, Plate XXXVIII., gives a series of *ō*'s sung by voice 2. The pitch of each utterance was, in this case, determined by measurement of the curves.

Table VI. gives the analyses of the examples in fig. 6.

Table VII. has been drawn up to facilitate comparison of the results obtained for *ō* with different voices; and it brings out clearly the points of agreement and difference between them. It will be seen that from *g* upwards all the voices agree in forming *ō* essentially of two partial tones—the prime and the second; but the proportions in which these are present are far from constant. On *e*′, for example, the ratio of prime to second in voice 3 is 100 : 34, while in voice 5 it is 100 : 102. The second partial is conspicuously weak in voice 3 throughout this part of the scale. On *b* the difference between voice 1 and voice 5 is even greater than this. All the voices, however, are alike in one respect, that the proportion of the prime to the second partial is a *minimum* on the note *b*♭; that is to say, when the second partial falls on *b*♭′ it is specially strong. With voice 1 it is nearly two and a half times as strong as the prime, and with voice 4 it is actually more than three times as strong as the prime.

An examination of Tables I. to V. will show that the reinforcement of the partials, whether first, second, third, or fourth, reaches a maximum when that partial falls on *b*♭′—in other words, all tones thus falling on that pitch are specially conspicuous.

Below *g* the third partial comes in more or less rapidly; it is strong when the vowel is sung on *f*, *e*, and *d*. On *c* the fourth has become strong, the second and third remain so, but the prime has become conspicuously weaker. On B, B♭, and A, the fourth partial is very strong; with voice 6 on A, its amplitude is nearly nine times that of the prime. On G the second partial has become weak, and the fourth is still the most conspicuous. In the solitary example on F the fifth partial is immensely strong.

Thus we see that at the pitches ordinarily used in speech the vowel sound *ō* consists almost wholly of the two constituents—a prime and its octave—the ratio of whose amplitudes may vary widely. But when the range is extended so as to reach lower pitches, higher partials successively appear in such a way as to allow the highest strongly reinforced partial to remain in the neighbourhood of *b*♭′; and further, we observe that all the tones between the prime and the highest strongly reinforced partial continue prominent until the note sung descends to G and A. The second seems to disappear as the fifth comes in.

Generally, we may say, from an examination of the foregoing table, that there is a wide range of reinforcement, extending over about two octaves (from *f* or *g* to *f*″), within which all tones are more or less strongly reinforced, and that there is a specially strong reinforcement at the pitch *b*♭′.

TABLE V.—Ō, VOICE 6.

	F	F♯	G	G♯	A	B♭	B	c	c♯	d	e♭	e	f	f♯	g	g♯	a	b♭	b	c	c♯'	d'	e♭'	e'	f'	f♯'	g'	g♯'	a'	b♭'	b'	c''	c♯''	d''	e♭''	e''	f''
53						18												22							32					75				9			2
54					9												46							44					80				12			4	
55			34												30							8					45				9			7			
56	22												10							15					8				31			1					

TABLE V.—Ō, VOICE 2.

	f♯	g	g♯	a	b♭	b	c'	c♯'	d'	e♭'	e'	f'	f♯'	g'	g♯'	a'	b♭'	b'	c''	c♯''	d''	e♭''	e''	f''	f♯''	g''	g♯'	a'	b♭''	b''	c'''	c♯'''	d'''	e♭'''	e'''	f'''	f♯'''	g'''	g♯'''	a'''	b♭''	b'''	c''''	c♯''''
57													44												32							6					0				4			2
58											51												30							5					2				1			1		
59									66												40							4					0				3			2				
60						45												66							7					4				6			2							
61					49												104							18					6				4			2								
62				32												58							6					8				6			2									
63		23												51							14					3				2			2											
64	18												58							15					3				1			0												

TABLE VII.—Vowel Sound *Ō.*

Pitch.	Voice.	Amplitudes of the First Six Partial Tones.					
		I.	II.	III.	IV.	V.	VI.
f♯	2	44	32	6	0	4	2
f′	1	121	71	7	1	5	4
	5	53	19	6	2	3	1
e′	1	105	69	7	3	2	3
	2	51	30	5	2	1	1
	3	53	18	3	1	2	1
	4	55	34	7	2	2	0
	5	52	53	5	6	5	2
d′	1	119	76	5	1	3	2
	2	66	40	4	0	3	2
	5	27	42	6	4	2	1
c′	1	110	160	15	10	10	7
	3	37	30	1	4	1	1
	4	54	25	0	3	2	1
	5	47	41	5	3	3	2
b	1	70	126	15	14	6	1
	2	45	66	7	4	6	2
	3	36	31	2	4	1	0
	4	45	43	4	2	3	0
	5	47	61	2	14	8	2
b♭	1	75	185	13	8	11	1
	2	49	104	18	6	4	2
	4	25	82	5	7	1	2
	5	48	70	13	7	1	3
a	1	125	190	25	22	5	2
	2	32	58	6	8	6	2
	3	40	36	4	4	3	2
	4	16	54	4	4	1	1
	5	40	68	10	8	3	0
g	1	69	103	27	6	2	2
	2	23	51	14	3	2	2
	3	46	29	2	2	2	1
	4	33	44	7	2	1	2
	5	32	50	3	6	1	2
f♯	2	18	58	15	3	1	0
	4	28	62	10	2	3	2
f	1	55	140	45	4	8	1
	3	25	37	11	3	4	2
	5	70	109	58	13	12	5
e	1	72	131	73	7	10	4
	3	40	67	35	5	5	2
	4	25	49	21	6	6	0
	5	41	88	64	13	5	2
d	1	44	134	82	16	22	10
	3	27	61	38	19	4	2
	4	20	46	21	3	4	1
	5	33	72	56	5	7	2
c	1	18	95	61	33	3	0
	3	19	48	33	18	2	4
B	1	25	15	28	31	6	5
	3	21	46	29	28	10	0
	4	12	34	23	10	4	1
	5	6	38	23	25	6	3
B	1	37	58	61	47	11	0
	3	28	41	25	36	9	0
	5	18	26	15	15	2	2
	6	18	22	32	75	9	2
A	1	15	15	18	29	6	3
	3	26	41	35	39	18	2
	5	15	8	22	21	4	0
	6	9	46	44	80	12	4
G	1	13	0	15	40	8	4
	6	34	30	8	45	9	7
F	6	22	10	15	8	34	1

This summary, however, does not include one phenomenon, which is not so obvious as those already mentioned, and to which we are disposed to attach some importance. There is evidence that in certain cases the high partials, as they successively appear, do so somewhat abruptly, while, at the same time, the partial immediately lower, which had previously been the highest prominent partial, sometimes suffers a rather abrupt diminution in strength. A good instance of this is given by voice 6, Table V., where, in examples **53**, **54**, and **55** (or B♭, A, and G) the fourth partial is by far the most prominent, but in example **56**, which is only one tone lower, the fourth partial has sunk into comparative insignificance, and the fifth partial has become remarkably strong. The suddenness of this change is well brought out by Table VIII., which gives the amplitude of the fourth and fifth partials of the ō's sung by voice 6.

TABLE VIII.

Pitch of Prime.	B♭.	A.	G.	F.
Amplitude of IV.th Partial, . .	75	80	45	8
Amplitude of V.th Partial, . .	9	12	9	34

Other, though perhaps less striking instances of the same sort of action, are to be found in other parts of the table. Table IX. gives the amplitudes of the III.d partial, and Table X. those of the IV.th partial, in ō's sung by voices 1, 3, and 5, with the pitch of the partial at the top. It will be observed that there is a somewhat sudden rise at the places marked with a star, thus *. There is much less of this in voice 1 than in the other two.

TABLE IX. (THIRD PARTIALS.)

Pitch of Third Partial.	*c'''*	*b''*	*b♭'*	*a''*	*g♯''*	*g*	*f♯''*	*f''*	*e''*	*e♭''*	*d''*	*c♯''*	*c''*	*b'*	*b♭'*	*a'*	*g♯*	*g'*
Amplitude in Voice 1,	7	7	...	5	...	15	15	13	25	...	27	...	45	73	...	82	...	61
„ „ 3,	...	3	...	...	...	1	2	...	4	...	2	...	11*	35	...	38	...	33
„ „ 5,	6	5	...	6	...	5	2	13	10	...	3	...	*58	64	...	56	...	...

TABLE X. (Fourth Partials.)

Pitch of Third Partial.	*c'''*	*b''*	*b♭''*	*a''*	*g♯''*	*g''*	*f♯''*	*f''*	*e''*	*e♭''*	*d''*	*c♯''*	*c''*	*b'*	*b♭'*	*a'*	*g♯'*	*g,*
Amplitude in Voice 1,	10	14	8	22	...	6	...	4	7	...	16	...	33	31	47	29	...	40
" " 3,	4	4	...	4	...	2	...	3	5	...	19	...	18	*28·	36	39	...	...
" " 5,	3	14	7	8	...	6	...	13	13	...	5	...	...	*25	15	21	...	...

A very remarkable instance of an abrupt change in the constitution of a vowel sound is afforded by the letter *ū*, on which we have made a large number of observations which will now be described. Like *ō*, *ū* was well spoken by the phonograph. Figs. 7 and 8 give sets of curves for *ū*, sung by voices 5 and 1 respectively. In voice 5 the curves given for the sound *ū* above *a* are very approximately simple harmonic curves; almost the whole of the sound consists of the prime tones. This result agrees so far with that obtained with the phonautograph by Donders. But at *a* a sudden and remarkable change takes place. The period split up into two halves, each very nearly simply harmonic, and the analysis of this form shows that it consists of a feeble prime tone with an excessively strong second partial. This form is given on *g*, *f*, *e*, and *d*. On *c*, *d*, and B the third partial appears, but we cannot say that we have got any really good *ū*'s from any voice or notes below *c;* at these low pitches the vowel sound of this letter become very poor, the marks on the tinfoil were feeble, and the reproduction by the phonograph was little more than an inarticulate groan. For the sake of brevity, we will call the approximately simple harmonic form obtained for *ū* above *a* the *simple u*, and the form shown for the notes *e f*, *g f*, and *e*, the *duplex u*. At the first glance it is difficult to distinguish the duplex curve for *u* on *e* from the simple one for *u* on *e'*.

Table XI. gives the analyses of the *ū*'s in fig. 7. It will be observed that the note *a* is wanting in the examples given. It has been omitted because we had difficulty in obtaining a good *ū* back from the phonograph when the voice was sung on this note. The voice in question (No. 5) was very apt to give an *ō* quality to the sound, both as originally sung and as reproduced by the phonograph, and in that case the form of the curve obtained was at once recognisable as resembling *ō*. The attempts of this voice to sing *ū* at this pitch generally resulted in a vowel sound of very variable quality, and a correspondingly irregular form in the phonographic records. The voice seemed to have a difficulty in keeping the second partial sufficiently weak to give the simple form, or in making the second partial sufficiently strong to give the duplex form. But although no good specimen of voice 5 has been transcribed on this note,

TABLE XI.—$\bar{U}$, VOICE 5.

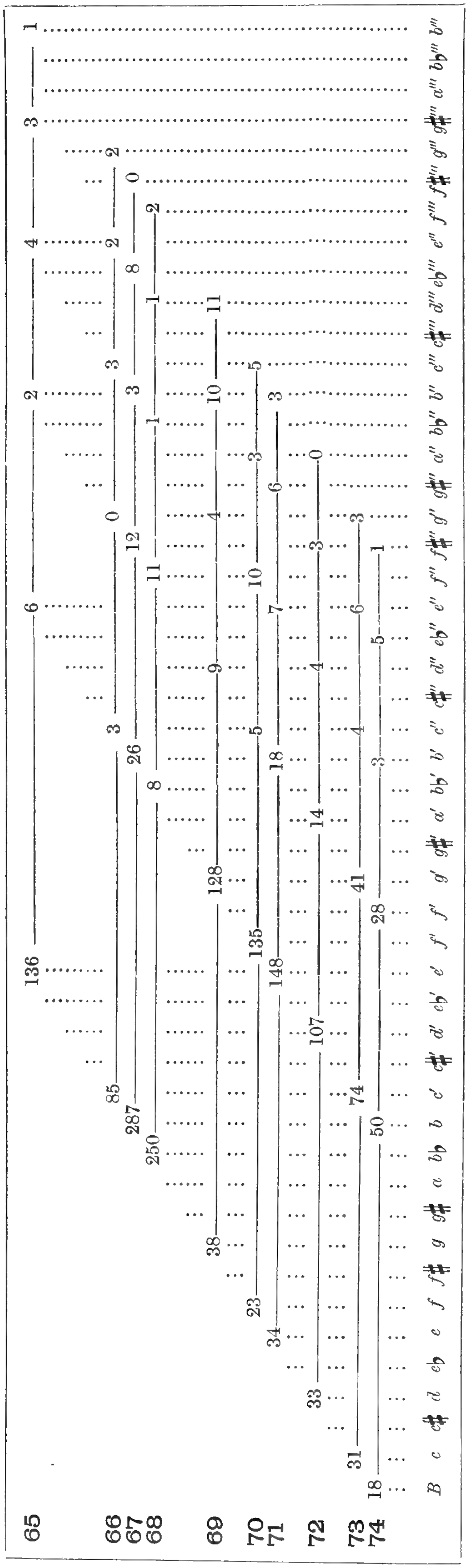
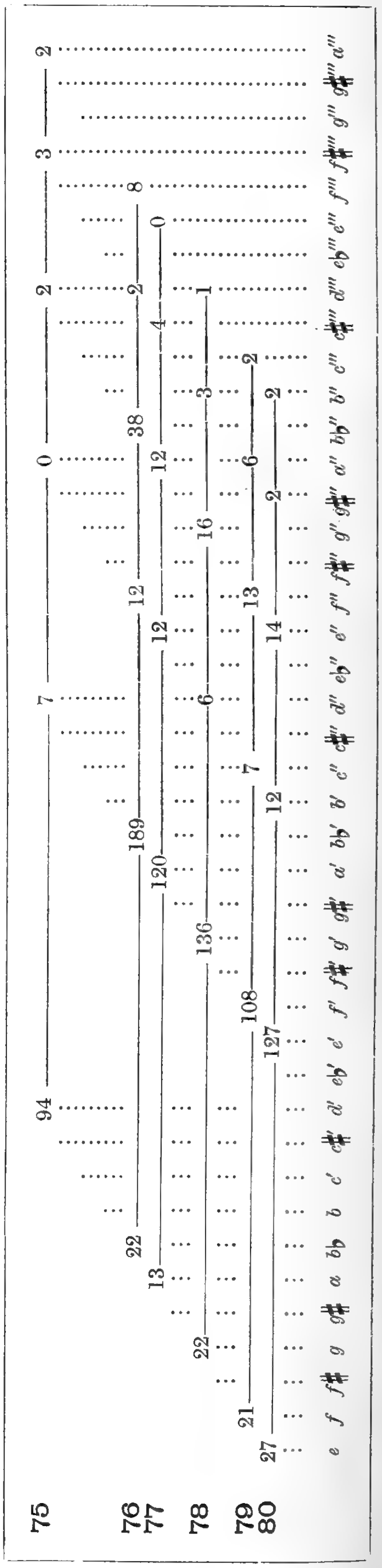


TABLE XII.—$\bar{U}$, VOICE 1.



we satisfied ourselves by frequent trials that in order to get a good *ū* the curve had either to be of the duplex or the simple type. The change required to be sudden. Anything between the two forms came out *ō*. And the same voice, in singing *ū* on this critical note, did actually sometimes produce the duplex form and sometimes the simple form. The two forms could be made to overlap.* We found the same thing true of other voices. Fig. 8 shows that voice 1 gave the duplex form as high as *b*♭. Another voice, whose range was extremely limited, gave the simple form on every note it could take, and carried it as low as *f*, which was this voice's lower limit.

Table XII. gives the analyses of the curves in fig. 8. It may be mentioned here that although comparatively few examples of *ū* curves have been transcribed and analysed, the simplicity of the curves was such that their form could often be observed fairly well by mere inspection of the tinfoil record; and experiments with several voices have given us confidence that the examples given above are really representative of the results to be obtained from this letter.

By way of further testing the vowel sound *ū* at the critical point where it changed from the simple to the duplex form in a given voice, attempts were made by voice 5 to *slur* both up and down past this pitch; that is to say, to go on pronouncing *ū*, and gradually raise or alter the pitch so as to pass the pitch *a*. In all these attempts, however, one or other of two things happened. Either the vowel quality as appreciated by the ear and the form of the curves as recognised by the eye changed to *ō* for an instant as the critical pitch was being passed, or else there was an interval of almost complete silence, and the curves sank to an insignificant size. This, as well as the evidence already given, goes to prove that an abrupt change must take place from the duplex to the simple form of *ū* if the vowel quality is to remain pure.

As some doubt may be felt whether this singular change in the wave-form of *ū* may not have been due to some peculiarity in the instrument, we may repeat that, being fully alive to this danger, we tried the experiment with changed mouthpieces, changed vibrating discs, and changed springs, but always with the result that on or about the pitch *a* certain voices made a sudden alteration in the constitution of their *ū*. Further, it must not be forgotten that another voice, using the same instrument, continued to give the simple form as low as *f*.

The following is a general summary of the results established by our experiments in the vowel sound *ū*.

(1.) The generic character of *ū* from *d* to *f*′ is given by the prominence of

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* The following note is taken from our diary:—"May 20. Voice No. 5 did duplex and single *ū*'s on *b*♭, *a*, and *g*. The duplex *ū* on *b*♭ was very *ō*-ish. The others were good. The single form on *a* was the best of all."

a single partial tone. Above *a* this partial is usually the prime; below *a* it is usually the octave of the prime.

(2.) In the lower or duplex form of *ū* the amplitude of the second partial is sometimes as much as nine times as great as that at the prime, but in pitches below *a* the sound will still be recognised as *ū*, even when the second partial is only from three to four times as large as the prime. The ratio of prime to second partial in this part of the scale is always much less for *ū* than it was for *ō*.

(3.) For pitches below *d* the experiments do not warrant very positive conclusions. The prominence of a single partial is less marked in this part of the scale, and at the same time the quality of the spoken vowel is exceedingly vague.

(4.) There does not appear to be any one pitch having the marked characteristics for *ū* which were possessed by the pitch *b*♭′ for *ō*.

(5.) There is a critical pitch in the neighbourhood of *a* or *b*♭ at which a sudden change takes place in the form and composition of the waves produced by certain voices when speaking or singing *ū*: at this point the pitch of the single prominent tone changes by a whole octave.

This change is not made on account of any requirement of the ear, but probably on account of the difficulty in adjusting the mouth-cavity so as to continue the simple form on the lower notes. This conclusion follows from the facts mentioned above, that some voices would on different occasions give sometimes the simple and sometimes the duplex forms on *a* and *b*♭ when singing what was at least generically the same vowel sound, and that another voice, also pronouncing the same vowel, gave the simple form as low as the note *f*.

The prominent tone in *ū* is generally found within the region *b*♭′ to *b*. What may be termed the average pitch of the constituents of *ū* is on notes near *d* and *e*, lower than that of the constituents of *ō*, owing to the absence of the third partial, but the average pitch for *ū* is higher than for *ō* on the notes *f* and *g*, and even on *b*♭ with voice 1: this arises from the comparative smallness of the prime. When the simple form is reached the average pitch of the constituents of *ū* is much lower than that of the constituents of *ō*. If, instead of considering the average pitch, we look at the pitch of the highest prominent constituent, we find that this pitch is the same for *ū* and *ō* when sung on *g* by voices 1 and 5, and also when sung on *a* and *b*♭ by voice 1. Before proceeding to consider these and the foregoing facts with reference to the theory of vowel sounds, we shall complete the account of our experiments.

A few observations were made of the curves given by the vowels *a*° (as in the word "awe") and *ā* (as in "father"). These vowels were fairly well spoken by the phonograph, though not quite so well as *ū* or *ō*. Fig. 9, Plate XXXIX.,

TABLE XIII.—Å, VOICE 5.

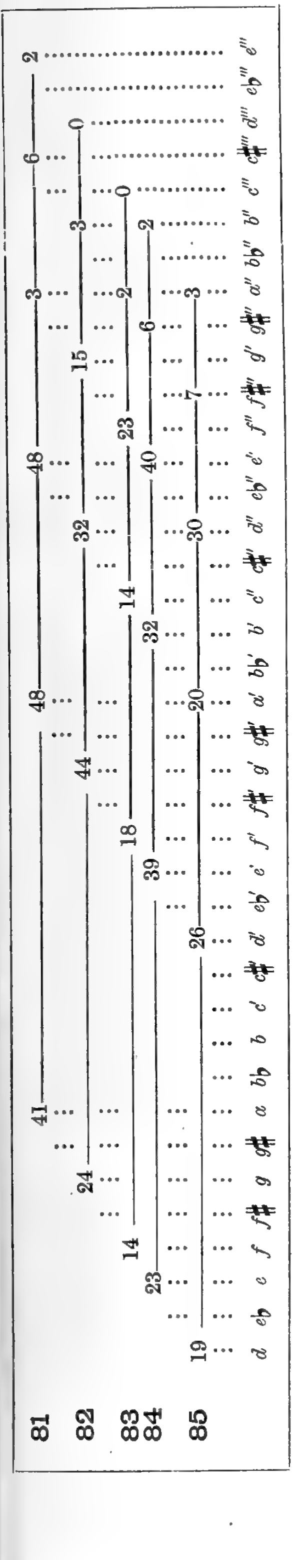


TABLE XIV.—Ā, VOICE 5.

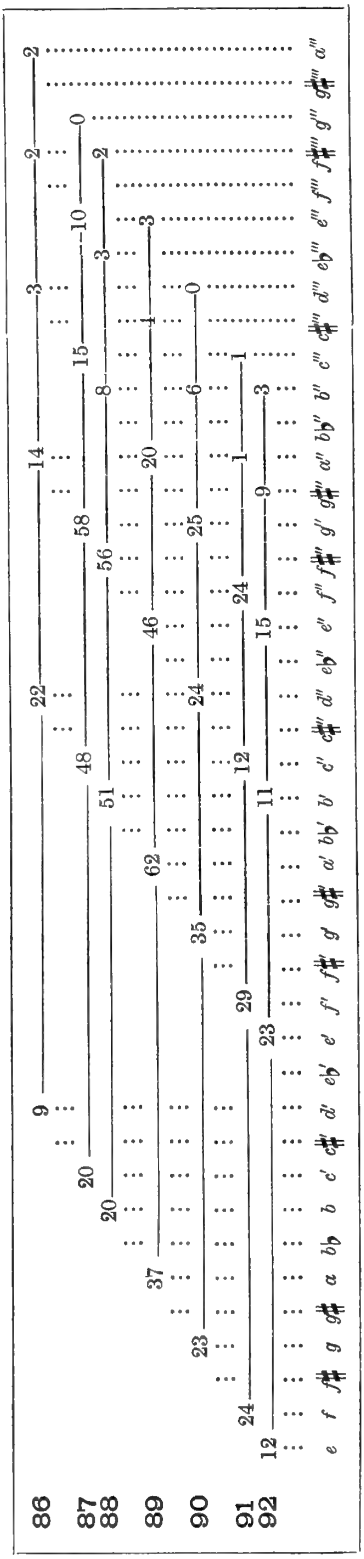
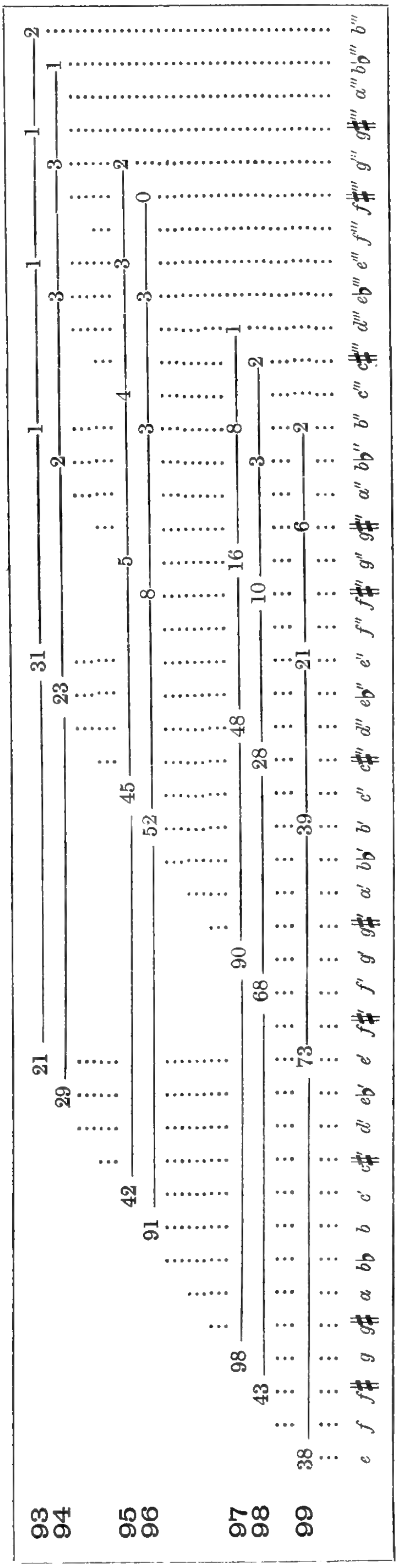


TABLE XV.—Ō, ARTIFICIAL.



gives a set of *a*°'s as sung by voice 5, and fig. 10 gives a set of *ā*'s by the same voice. The analyses of these are contained in Tables XIII. and XIV. respectively. The range over which good phonographic records were obtained from this voice was, it will be observed, extremely limited.

These tables show that where *ō* has only the first and second partials prominent, *a*° has three consecutive partials all more or less strong, and *ā* has four. On *e*, where *ō* was composed chiefly of three partials, *a*° has four and *ā* five. Generally, the average pitch of the reinforced group of partials is higher for *a*° than for *ō*, and higher still for *ā*; and the range over which the reinforcement extends in *a*° and *ā* is at least as great as it was in *ō*. The upper maximum of reinforcement for *ā* appears to be on or close to g'', that is, nine semitones higher than it was for *ō*. For *a*° it is probably about $e\flat''$ or e'', or from five to six semitones higher than for *ō*.

Experiments throwing additional light on the subject have been made by the help of an apparatus exhibited by Professor Crum Brown at the meeting of the Society held on June 3. It consisted of a resonating cavity or bottle of irregular form, made of gutta-percha, to which bagpipe reeds of various pitches were applied. There were several holes in the sides of the bottle, by closing or opening which its properties as a resonator could be altered. When the reed was blown the apparatus spoke or rather sung a vowel sound, whose quality depended on what holes were left open. It could be made to give the vowels *ō*, *a*°, *ā*, and *i*. In particular, it gave a remarkably good imitation of the vowel *ō* with one arrangement of the apertures in the sides. The form of the bottle was arrived at more or less tentatively. If the pitch of the reed was altered, the cavity remaining constant in every way, the same vowel continued to be given, at of course a changed pitch. Professor Crum Brown was kind enough to lend us this apparatus in order that we might investigate the *ō* which it spoke in the same way as we had investigated that vowel as produced by human voices. By holding this apparatus before the mouthpiece of the phonograph, and using reeds of various pitches, we have obtained curves for artificial *ō*'s ranging from e to e'. The pitch of the sound was determined in each case by measuring the record. These *ō*'s when reproduced by the phonograph came out very well—even better than the original sound, as the jarring noise of the reed was lost. Their vowel quality was recognisable without the smallest difficulty as *ō*, of perhaps a somewhat bright species. Fig. 11, Plate XXXIX., shows the curves got for the artificial *ō*'s, and Table XV. gives the results of their analysis. An inspection of it brings out several important facts.

In the first place, it will be observed that the artificial *ō*'s, which are known to have been produced by the resonance of a *constant* cavity, are marked, like the human *ō*'s, by a wide range of pitch throughout which the partial tones are more or less reinforced. We have a fourth partial distinctly reinforced in one case as high as *g*″ and a second partial as low as *e*′, and it is very probable that if the experiments had extended to lower notes we should have detected resonance at still lower pitches by the strengthening of the second partial. Without assuming that the primes below *e* are reinforced at all, we see that this cavity is capable of reinforcing more or less strongly all tones falling between *g*″ and *e*′ at least.

Further, it cannot be said that these analyses exhibit any specially strong resonance on or close to *b*♭′, as was observed in the human *ō*.

The artificial *ō*'s agree with those of the human voice in being composed almost wholly of the two first partials for notes above *b*. On *f* and *g* their third partials are more prominent than in the human *ō*'s, and they have in addition a moderate fourth partial; their second partials, however, are not so strong relatively to the prime. On *e* also there is a strong fourth partial.

The property possessed by this irregularly-shaped gutta-percha cavity of reinforcing tones over a wide range of pitch, was confirmed by another and quite independent experiment. A short tube was inserted into the neck of the bottle, in place of the reed, and the end of this tube was applied to the ear. The cavity was thus adapted to act as a resonator to sounds from outside. By striking in succession the keys of a pianoforte with this resonator applied to the ear we were able to observe the tones which were reinforced, by the peculiar bumming noise which they gave rise to in the bottle. On working down the scale, with the cavity arranged for the vowel sound *ō*, the first note at which resonance could be detected was *g*♯″. It became stronger on *g*″, stronger still on *f*♯, and excessively strong on *f*″. On *e*″ it was nearly equally strong; on *e*♭″ and *d*″ weaker, but still very strong. On *c*″ it again became very intense, and again fell off somewhat on lower notes. But even on *g*′ and *f*♯′ there was much more resonance than could be accounted for by the reinforcement of the second partial in the note struck. The presence of the upper harmonics in the sound given by the pianoforte wires prevented this method of observing from being suitable to pitches below *f*′. But the above experiment sufficed to show the cavity had at least two, and probably more, proper tones, so closely grouped as to have the general effect of enabling it to strengthen by resonance any tone whatever between certain wide limits of absolute pitch.

When the side apertures of the cavity were altered so as to suit it to the production of the vowel *a*°, the resonance rose in pitch; and this was still more the case when the cavity was arranged for the vowel *ā*. The highest proper tone (which for *ō* was on *f*″) rose to *c*‴, and there was perceptible reinforcement as high as *e*‴. In this case also the range of resonance was extensive, and was by no means confined to two or three tones.

We now pass to the more general conclusions which we conceive may be drawn from our experiments. In a letter which appeared in "Nature," No. 450, vol. xviii. p. 167, we have given a short account of what we believed to be the existing state of the theory of vowel sounds, and we will take advantage of that publication to make our reference in this place to previous writers on the subject as brief as possible. It is generally recognised that vowel qualities of tone are produced by the action of the oral cavities in reinforcing by resonance certain partial tones in the composite sound given by the vocal chords. The "constant cavity theory" of vowel sounds, which is taught by DONDERS, and which is frequently spoken of as the doctrine of HELMHOLTZ (although we have found no definite and rigid statement of it in his Tonempfindungen), also asserts that for a given vowel the resonance cavity of the mouth is unaltered at all pitches on which the vowel is sung, and that the cavities for different vowels are distinguished by their "proper" or "characteristic" tones, or, in other words, by their pitches of maximum resonances, which are nearly independent of age and sex, and depend solely on the vowels for pronouncing which the mouth has been arranged. Then, when the vowel is sung on any subtone of the pitch of maximum resonance of the cavity, the overtone corresponding to that pitch will be very strongly present. The theory does not give a definite answer to the question, what will happen when no partial of the note sung coincides even approximately with the pitch of the proper tone, or how the vowel is recognised in that case. The pitches of the characteristic tones for various vowels have been examined by noticing the pitch of the whispered vowel, and also by observing the resonance when tuning forks are held before the mouth, the mouth being set for a particular vowel; but they are far from identical, as determined by different observers. This, however, may be explained by saying that the quality of the vowels experimented on was not the same. HELMHOLTZ says that he has detected only one proper tone for the vowels *ū*, *ō*, a°, and *ā*. For *ō* he gives $b\flat'$, and for *ā* $b\flat''$, while a° occupies an intermediate position, variable with the quality of the vowel. For *ū* he says it is by no means easy to find the pitch of resonance by tuning forks: he gives f as the pitch for this vowel, adding, however, in a footnote that there appear to be great personal differences, so that small alterations of the pronunciation may drive the pitch up to f'.

HELMHOLTZ does not say (so far as we have seen) whether the mouth-cavities for these vowels, for which he has detected only one proper tone, differ phonetically in any other respect than in the pitch of that proper tone.

Passing now to our own results, it is clear that the quality of a vowel sound does not depend either on the absolute pitch of reinforcement of the constituent tones alone, or on the simple grouping of relative partials independently of pitch. Before the constituents of a vowel can be assigned, the pitch of the prime must be given; and, on the other hand, the pitch of the most strongly reinforced partial is not alone sufficient to allow us to name the vowel. To do this we must also know the relation of the constituent partials to one another.

The sound ū is produced mainly by the reinforcement of a single partial tone, generally lying in the region *a* to *a′*. The sound ō requires, at least, two strong partials. When there are only two they lie in the region between *g* and *f″*. Possibly the upper limit may extend even higher than *f″* with a tenor or a woman's voice. Other tones than the prime are reinforced in the sound ō over a region covering nearly two octaves, namely, from *f″* to *g*.

This great range over which reinforcement extends must be a distinguishing mark of ō as compared with ū, perhaps *the* distinguishing mark. We have seen that when ū and ō were sung on *b*♭ by voice 1, ō consisted of a strong first and second partial, ū of a weak first and strong second. The pitch of the most strongly reinforced tone in both was *b*♭′, the "characteristic tone" of ō. But, in point of fact, this was much more strongly reinforced in the ū than in the ō. The characteristic mark separating ō from ū cannot therefore have been the prominence of the tone *b*♭′: it must rather be sought for in the fact that for ū this tone alone was reinforced, whereas for ō the prime was reinforced to some extent also. Again, when voice 5 sang ō and ū on the same note, *b*♭, it gave a strong prime and second for ō, but a strong prime only for ū. Here, too, the distinction must lie in the extent over which the reinforcement acted, for the difference of average pitch of reinforcement was positively greater between the two ū's than between either of them and either of the ō's. The maximum resonance of the ū sung by voice 5 on *b*♭ was a whole octave above that of the ū sung by voice 1 at the same pitch; and, if we take the pitch of maximum resonance of the ō's as *b*♭, then the pitch of maximum resonance for ō and ū sung on *b*♭ by voice 1 was precisely the same. The argument is not in any way weakened by saying that the ū of one voice was not the same vowel as the ū of the other. Identically the same it cannot have been; nevertheless, on higher or lower notes, the two voices agreed as to the composition of ū, and generically the vowels were certainly the same. Speakers and hearers were unconscious of any generic change in the vowel sound ū when the pitch of maximum resonance of the oral cavity rose suddenly by a whole octave. The evidence appears conclusive that the prominence of a single partial tone due to the reinforcement given by a single proper tone of the oral cavity is insufficient to characterise a vowel.

It is equally clear that even in the case of the human voice, in singing or speaking, a given vowel does not simply produce a certain group of relative partial tones independently of absolute pitch. Possibly, indeed, the ear might recognise a single tone with a feeble accompaniment of upper partials as a sort of ū below the region within which the human voice produce the simple form of ū. Thus, HELMHOLTZ says that a *b*♭ fork when sounded alone gave a very dull ū, much duller than could be produced in speech; the sound became more like ū when the second and third partials were added feebly. This tone is an octave below the pitch where voice 1 ceased to give the simple form for ū. Similarly, it is possible that the group, consisting of a prime and its octave, might continue to give $\bar{o}$ when sounded below the limits within which the voice when singing $\bar{o}$ produces this simple group. Our own impression of the effect produced when the second of an $\bar{o}$ sung at a high pitch is made to speak in the phonograph at a lower speed, supports this view, but the vowel sound given in this way is proved by our curves to be different from the human $\bar{o}$, and we do not think that a mere subjective impression counts for much.

On the other hand, there is a decided resemblance, though scarcely an identity between the constituents of $\bar{o}$ at a low pitch and a° or $\bar{a}$ at higher pitches. The relative partials of $\bar{o}$ in the neighbourhood of *b*♭ resemble pretty closely those of $\bar{a}$ in the neighbourhood of *f* or *g*. Our experiments on a° and $\bar{a}$ are not sufficiently numerous to enable us to draw any very general conclusion as to these vowels, but they are sufficient, when taken in conjunction with the experiments on $\bar{o}$, to show that between certain vowels the main distinction must lie in the absolute pitch of the group of reinforced constituent tones.

We are thus brought back to our original statement, that in distinguishing vowels the ear is guided by two factors, one depending on the harmony or group of relative partials, and the other on the absolute pitch of the reinforced constituents.

It seems not a little singular that the ear should attach so distinct a unity to sounds made up of such very various groups of constituents, as we have obtained from different voices and at different pitches, as to recognise all these sounds as some one particular vowel.

We are forced to the conclusion already adopted by HELMHOLTZ and DONDERS, that the ear recognises the *kind of oral cavity* by which the reinforcement is produced; that although the sounds which come from the cavity differ so much that we are unable, when they are graphically represented and mathematically analysed, to detect any very prominent common feature, nevertheless by long practice the ear is able to distinguish between the different sorts of cavities which are made use of in pronouncing given vowels. Something of the same kind may, indeed, be observed in other sources of sound than the human voice. The characteristic by which we recognise a musical instrument is in many cases the peculiar nature of the resonance chamber which it possesses: and we continue without difficulty to recognise a substantial unity in the quality of sounds given by the instrument at different parts of the scale, where the action of the resonance chamber must be very different.

The question then remains, is the resonance cavity for a vowel sound constant at all pitches, and, if so, what properties does it possess which enable its modification of the sound given by the vocal chords to be recognised on a particular vowel? And further, if the cavity is not constant, but variable when the vowel is spoken at different pitches, what common feature is possessed by the modified forms of the cavity?

The vowel producing resonance cavities are clearly distinguished in virtue of two properties—first, the absolute pitch at which they produce a maximum reinforcement; and second, the area of pitch over which reinforcement acts. The latter property, when it is extensive, is very probably due to the existence of subordinate proper tones not far from each other in pitch. This property of the oral cavity has, we believe, been hitherto much neglected. In order completely to define the properties of the oral cavity, the relative reinforcing power at each pitch over the whole range of reinforcement ought to be determined; our experiments, however, give only a rough idea of this relative power throughout the range.

Professor Crum Brown's gutta-percha cavity, which produced the vowels *ō*, *a*°, *ā*, and *i* at very different pitches, proves conclusively that a *constant* cavity of irregular form and tolerably soft material is capable of producing any one of these vowels over a wide range of pitch. We have then to inquire whether the experiments show that these vowels (or more particularly the vowel *ō*, which has been most fully investigated), when produced by the human voice, are actually the result of a constant oral cavity; and further, whether the action of a constant cavity can explain the phenomena observed for the vowel *ū*, which was not one of those spoken by the bottle.

It does not appear possible that the reinforcement which we have observed for the sound *ū* can have been the effect of a constant oral cavity. There is, on the contrary, every evidence that, as the pitch on which the vowel was sung descended, the proper tone cavity was adjusted so as to bring its proper tone into unison, first with the prime tone, and then afterwards with the second partial. The proper tone of the cavity seems to have fallen note by note with the pitch of the vowel until it reached *ā*, or thereabout, when it suddenly rose an octave, and then again went on falling as before. This, at least, seems to be much the most natural view to take of the causes which produced the excessive prominence, first of the prime and afterwards of the second partial when *ū* was sung down the scale. It will be observed in Table XII. that in the duplex *ū*'s, when, according to this view, the maximum pitch of resonance of the cavity is in unison with the second partial, the fourth partials are appreciably strong, as might be expected to be the case.

We should then describe the *ū* cavity as an adjustable cavity, with a very limited range of resonance, whose effect is to reinforce strongly only one partial lying above the pitch *a*.

It is, indeed, possible that, so far as the small changes of pitch throughout the range of ordinary speech are concerned, even the *ū* cavity may be constant. We have no evidence either for or against such a view. But, when the range of pitch is extended as it is in song, the analyses show that the cavity can no longer be constant. We may add, as a mere conjecture, that one point of difference between vowel sounds uttered in speech and in song may conceivably be that in the former the cavity is constant, and in the latter it is tuned.

There is some evidence in the analyses that even the ō cavity is not quite constant. We have already drawn attention to the apparent abruptness with which some of the constituent partials come into, or disappear from, prominence, and this might be explained by supposing a certain adjustment or tuning of the cavity. The analyses in Table V. of ō's, sung by voice 6, appear to indicate that, in examples **53** and **54**, a strong upper proper tone of the cavity was in unison, or very nearly in unison, with the fourth partial of each. In example **55** it would appear from the proportion which the amplitude of the fourth partial bears to that of the fifth, that this proper tone was still much nearer to the fourth than the fifth partial; it seems to be following the fourth partial down the scale. But in example **56** it has apparently risen so as to be exactly, or very approximately, in unison with the fifth partial.

If we assume that the ō cavity is absolutely constant, we must describe it as a cavity reinforcing tones throughout nearly two octaves, or from *g* to *f″*, for we have found in some cases a strong second partial when the prime is on G and on *f′*. We are disposed to regard it as more probable, that in human voices the ō cavity is slightly tuned or modified according to the pitch on which the vowel is sung. We do not mean by this that the cavity adapts itself so as to be in unison with some particular partial, but only that when no partial falls on *b*♭′, the cavity will make its pitch of maximum resonance deviate a little from that pitch, so as to approach more nearly, and therefore reinforce more strongly, some partial tone lying near. And the same thing may occur with other proper tones of the cavity than the tone *b*♭′. If we regard the ō cavity as adjustable in this manner, we should describe it as a cavity capable of reinforcing tones over somewhat more than one octave. The range of adjustment need not exceed about six semitones, and the upper resonance would never deviate far from *b*♭′, on which pitch the strongest reinforcement can be given. Another way of putting the same view would be to say that the genuine character of ō is given by a cavity reinforcing tones over rather more than one octave, with an upper proper tone never far from *b*♭′, and that the human voice in singing may choose (to suit the pitch) an ō cavity, whose upper proper tone deviates slightly from *b*♭′, in order to bring some one strong proper tone of the cavity more nearly into unison with one of the partial tones of the note sung. Of course, this might be explained by saying that the voice chooses a different species of the generic vowel ō at different notes, so as to bring out as loud a resonance as possible. It is to be observed that the hypothesis of a tuned cavity is much strengthened by the evidence which exists for believing it to be tuned in the vowel ū, evidence which, it must be admitted, is much stronger than any which we have obtained for ō.

We should describe the *a*° and *a* cavities as differing from the *ō* cavity chiefly in having a higher pitch of resonance. Their range of reinforcement appear even more extensive than that of the *ō* cavity.

It is very satisfactory to find that the ō's given by the human voices which we have experimented with are marked by the strong resonance on $b_\flat'$, which HELMHOLTZ has noticed by quite different methods of observation. It tends to show that our *ō* was essentially the same vowel sound as his, and to give us confidence in the mode of experiment which we have adopted. This resonance is much less noticeable in voice 3 than in the other voices he has tried, and it does not appear to exist at all in the artificial *ō*'s given by Dr CRUM BROWN'S bottle. From this we may, perhaps, conclude that it is a peculiarity of the human voice rather than a really essential feature of an *ō* cavity.

Since the experiments were concluded, our attention has been drawn to a paper by FELIX AUERBACH* ("Pogg. Ann. Ergänzung," viii. 2), describing an investigation of vowel sounds conducted by him in the physical laboratory at Berlin, under the guidance of Professor HELMHOLTZ. He endeavoured to estimate, by means of resonators, the comparative strength of the several partial tones where given vowels were sung, and from the figures so obtained he has deduced what is in many respects a novel theory of vowel sounds. He conceives the number expressing the intensity of each tone to be capable of being split up into two factors, one of which is a function of the absolute pitch, and the other of the number of the partial. The second function is of course discontinuous; but the first, he states, may be expressed by an equation of the third order, giving a curve whose maximum occurs at a pitch which he calls the reduced characteristic tone, but with very extended perceptible reinforcement on both sides of that pitch. The expression, characteristic tone, appears to be used by AUERBACH in a sense very different from that of HELMHOLTZ. It will be observed that our results are so far in agreement with those of AUERBACH, that we recognise the relative and absolute factors as both entering into the composition of a vowel, but we cannot say that our figures agree at all with his estimates, or support any conclusions which he has drawn from them, except that the two elements of absolute pitch and relation between partials must both be taken into account. We have been unable to discover any constant multiplier resembling AUERBACH'S factors, and, indeed, the existence of any constant multipliers, such as he employs, is inconsistent with the great variation in proportion between the constituent tones which we observed with different voices and even the same voices.

In conclusion, we may notice one or two minor points brought out during

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* AUERBACH uses the word "factor" to denote a constant multiplier, which is not the sense in which the word has been employed on page 772.

the foregoing investigation. The magnified curves frequently showed well the variation of pitch during the "attack" of the voice upon a given note. The periods were almost always much too long at first, and gradually decreased in length until the proper pitch was reached, which sometimes did not happen until more than a dozen periods had been uttered. During this commencement the curves passed through various forms, which were recognisable as the forms corresponding to the same vowel at lower pitches. An instance of this is given in fig. 12, Plate XI., which is taken from the beginning of the utterance of the vowel ō, sung on *e* by voice 1, of which example **10**, fig. 1, has already been given as a specimen.

The vowel quality often deteriorated during a long-continued utterance, and this of course produced a corresponding change in the curves. The most trustworthy specimens were obtained as near to the beginning of the utterance as the gradual finding of the pitch just described would admit. Later on the sound was generally more musical and less vocal. Figs. 13 and 14 illustrate this change, which has often been attributed to the ear alone. Example **100** is taken from the same utterance as example **8**; and example **101** from the same utterance as example **10**; but **100** and **101** come much later in the respective utterances than the examples already given (fig. 1). The analyses of **100** and **101**, which should be compared with Nos. **8** and **10** in Table I., are as follows:—

Partial.	I.	II.	III.	IV.	V.	VI.
Example **100**	85	194	25	26	24	4
Example **101**	79	165	74	8	10	3

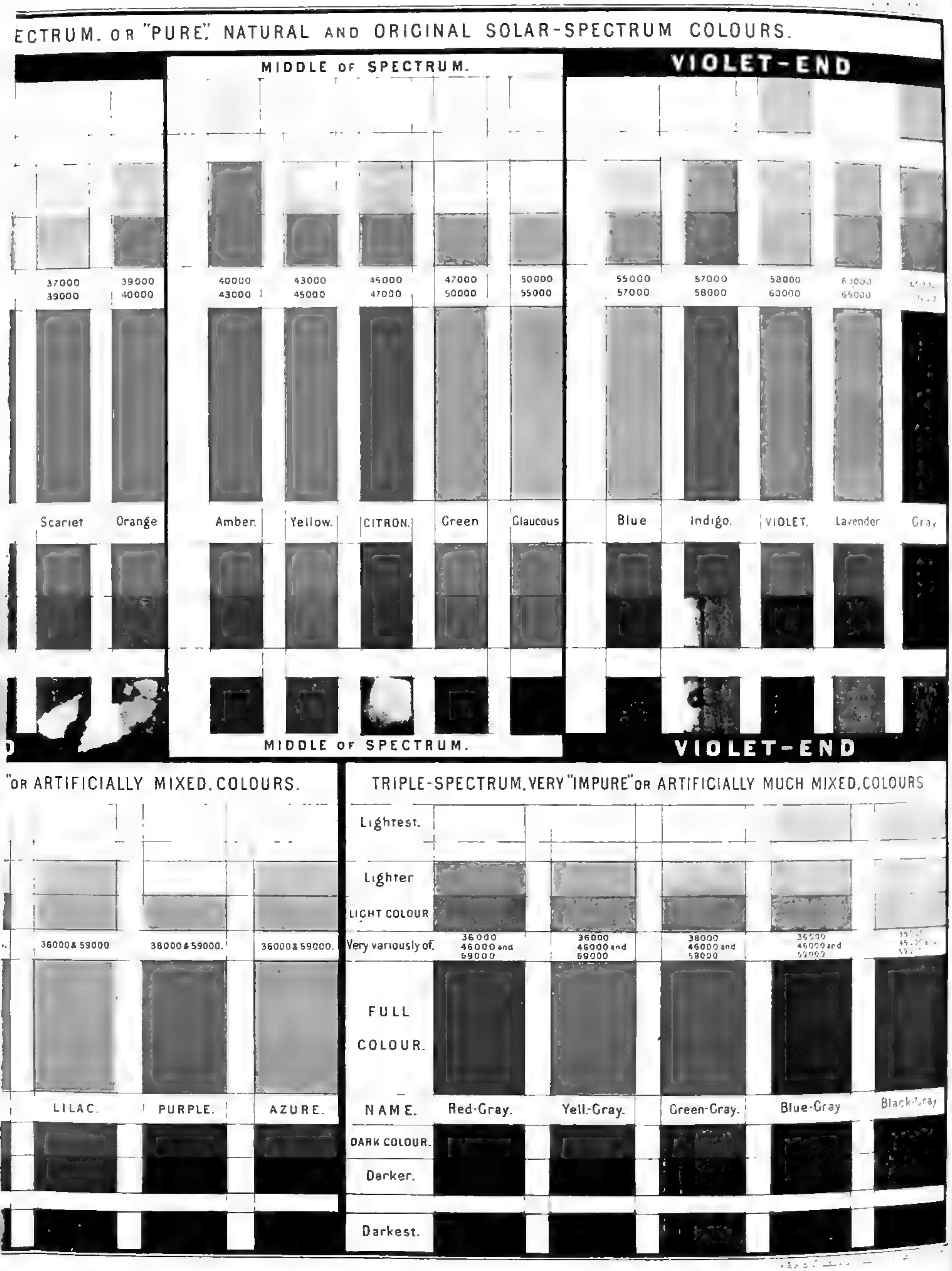
Finally, we give an example to show the modulation of pitch and vowel quality during the utterance of a spoken word. The word in question was the third of the series "No, no, no!" spoken by voice 5. Fig. 15, Plate XI., shows the commencement. What makes the *n* is not clear, but there seems to be a somewhat continued hum before the ō curve proper is reached. Fig. 16 gives a specimen a little later. Thirteen periods are left out between the end of fig. 15 and the beginning of fig. 16. Fig. 17 comes eight periods later than fig. 16; fig. 18 sixteen periods later than fig. 17; and fig. 19 thirteen periods later than fig. 18. From the beginning of fig. 19 to the end of the utterance there are no omissions. Fig. 19 shows the dropping of the pitch towards the close, and also the change in quality of the spoken vowel, making it approach ū. It will be observed that the change from ō towards ū occurs here by the strengthening of the second partial.

The apparatus which we have used to magnify the records in the tinfoil would suit very well to produce records on the tinfoil from any given periodic curve, by cutting a cam of the required form, and applying it to the rim of the wheel W (*vide* Plate XXXV.). By this means the phonograph might be made to speak artificial and vowel combinations of harmonic tones. We regret that want of time has prevented us from making this application of the apparatus.

In conclusion, we would draw attention to the excessive smallness of some of the prime tones in vowel sounds, as compared with the upper partials. Many instances of this occur in the tables. They show how feeble even the prime may be when not reinforced by oral resonance, and also how even an exceedingly weak prime is capable of determining the musical pitch of a compound sound.

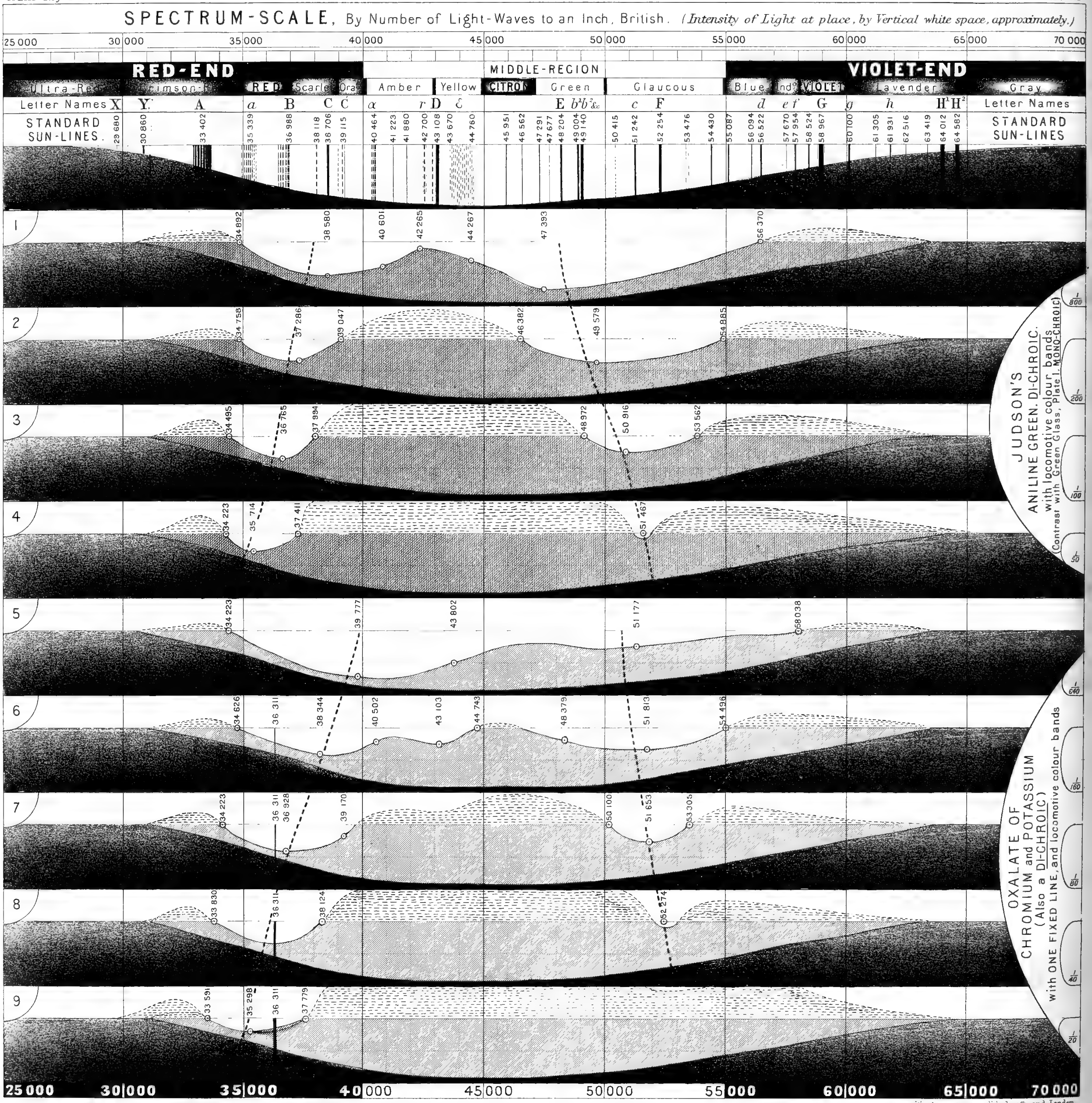
OPTICAL NOMENCLATURE OF COLOURS.

for Double-Star Observers and Others.



COLOURED FLUIDS.

(WATERY SOLUTIONS)



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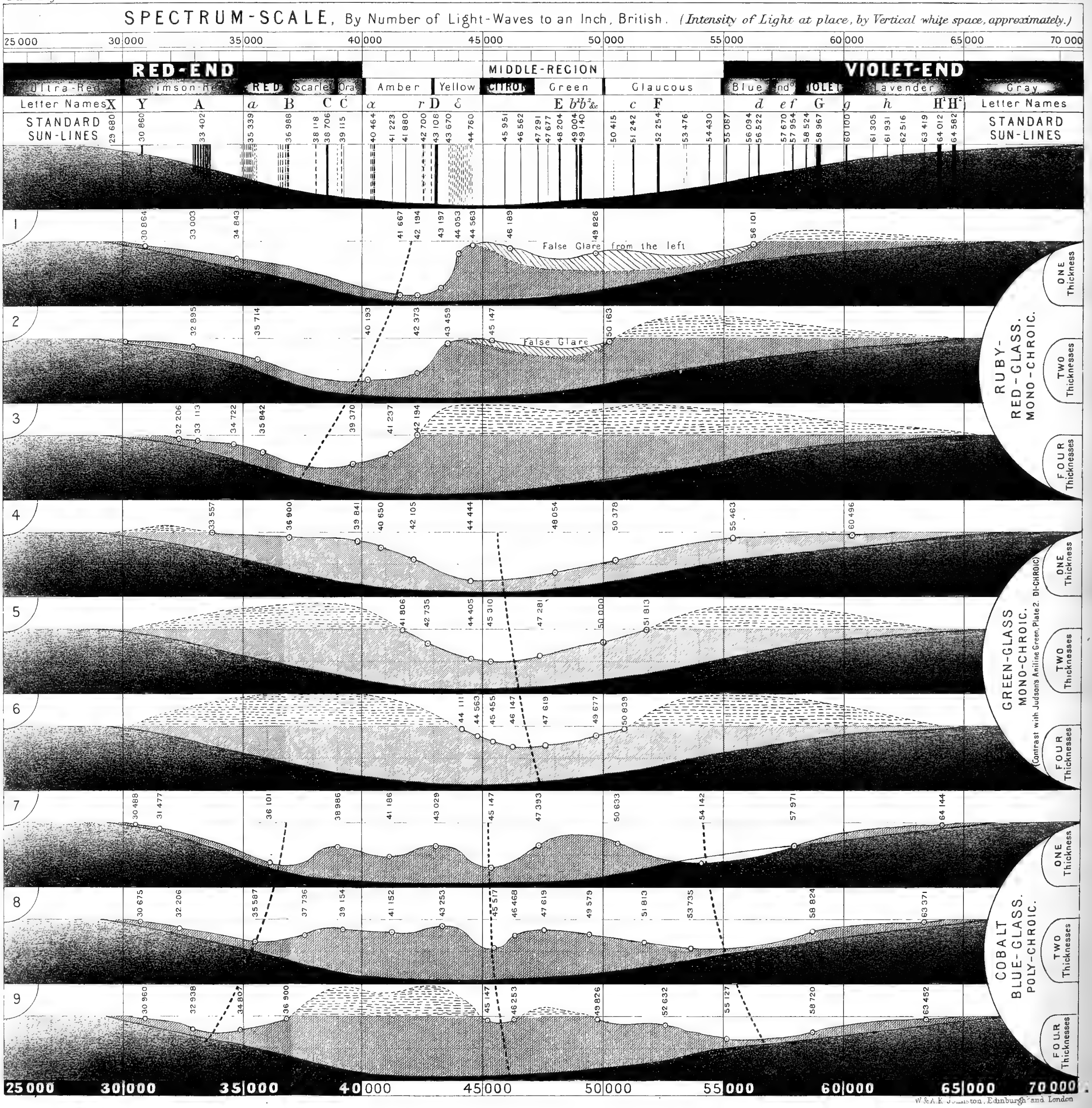
COLOURED GLASSES.



W.&A.K. Johnston, Edinburgh and London

XXXIII.—*Colour, in Practical Astronomy, Spectroscopically Examined.*

By Piazzi Smyth, Astronomer Royal for Scotland.

(Plates XLI.–XLIII.)

(Received May 24th, 1878.)

CONTENTS.

(The Plates are also mentioned on pp. 781, 782, and 789 to 791.)

The Apology.

Colours and colour sensations are much referred to still by double-star observers, who make use of a most extensive range of names, and differ widely from each other, in describing the many different tints which they claim to recognise by simple eye-observation in those interesting objects.

Material coloured shades, generally in glass, are also of daily use in both terrestrial and nautical astronomy, in decreasing or qualifying the intensity of sun's, moon's, and other rays; while special varieties of such media are still being inquired after for acting, if possible, with improvement of, rather than detriment to, the nicest definition of the optical instruments concerned.

And again coloured glasses have been latterly demanded, which should allow only the strictest mono-chromatic light to pass through them in one or another particular grade of the spectrum,—so as to allow, as one example only, the solar red-prominences to be plainly and broadly observed, without the usual spectroscopic draw-back of having to look at them through the narrowest of slits.

Inasmuch as fully successful results have hardly yet been obtained by the world in all these lines of inquiry; and as the best hitherto published data of experiment, so far as known to me, have not taken up several points of sensible importance to practical men,—I have employed a few weeks of enforced leisure in the beginning of this year, 1878,—in completing certain spectroscopic examinations of colours commenced in 1876, with the hope of supplying some of the indicated desiderata.

These new examinations were commenced under the form of observing "absorption-spectra;" a well known branch already of spectrum analysis, but applied now with some further precautions upon any, or all, of the signally colouring materials which I could easily obtain; observing each of them separately under several different thicknesses; and ultimately recording the features so made visible both in wave-number spectrum place; and, though roughly, in intensity and quantity as well.

The instrument used was my private "Aurora Spectroscope;" and from its series of several available powers of dispersion, I chose, as most suitable to the generally ill-defined visual phenomena concerned, that of only 1°·2 between A and H. It was given by a prism of very white flint glass constructed by M. Salleron of Paris, of large size but small refracting angle, viz., 30°. The objectives of both collimator and telescope were 2·3 inches in diameter, and the magnifying power of the latter, armed with Mr Hilger's bright-line-reference* eye-piece, carrying quartz lenses, was ten times linear. The micrometer measurements were made by moving the whole telescope angularly by a large tangent screw, the divisions of whose head were afterwards reduced to wave-numbers by reference to Angstrom's Normal Solar Spectrum.

More peculiarly still for the practical object in view, the coloured media were not placed immediately in front of the slit, but in an anterior arrangement of object-glasses which allowed 4 or 5 square inches of the medium, either in glass plates, or flat glass bottles, to be utilised. The slit itself, though rather wide, was yet only so moderately opened, that something could still be seen by its means, of the chief Fraunhofer lines in the day-light sky, if looked for. But the ultimate observations were always made at night, upon the foundation offered by the continuous spectrum of a bright coal-gas flame, as duly chronicled at the head of every list of numerical observations.

COLOURED GLASS SERIES. (*See also Plate* 1, *No.* 41 *of this Vol.*)

Coloured glasses were first examined, and were taken somewhat in spectrum order of colour, beginning with the ultra-red; so that they come

RED-LILAC,
RUBY-RED,
YELLOW-BROWN,
GREEN,
COBALT-BLUE, and
BLUE-LILAC.

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* This neat little contrivance, admirable up to a certain extent, was yet found to have a weakness in certain parts of the Spectrum, the nature, and subsequent complete cure of which will be found described in the "Postscript" to this paper, at pp. 802–804.

The observations made on the above varieties of glass, and on several thicknesses of each, followed by a few of the more immediate conclusions for practical use derivable from them, will be found,—

First, in the printed Appendix I., pp. 805–815;

Second, in graphical form in Plates, Nos. 1 to 3 MS., and Plate 1, engraved.

In these Plates, while the light which has been transmitted is indicated by a white wave depending from a level line, the light which has been stopped is shown by a differential dark area, between the white wave of transmitted light, and the outline (similarly depending) of the whole wave of light which had previously offered itself to be transmitted from the continuous spectrum of the bright reference flame employed.

This duplex wave method was adopted on considering that,—when light has been stopped out from any part of the length of an absorption spectrum, no conclusions can safely be formed therefrom, either as to the energy of the stopping power of the interposed and intercepting medium, or as to the accuracy of which the observation was capable—unless we know how much, or how little, light there originally was at that part to be stopped; the quantity moreover varying exceedingly in the course of any spectrum.

The particular projection adopted for all these *refraction* spectra is one which, while giving a scale of equal parts on the paper to "*Number* of Light waves in an Inch, British;" and in figures increasing most naturally in the same direction as the increase of refrangibility,—has the further advantage in practice of somewhat unlocking the contents of the closely compressed-up red-end of the spectrum, as given more or less by all prisms,—and yet not stretching that end out, nor oppositely compressing the violet end, to the needlessly extravagant degree for useful spectrum details, of all *diffraction* spectra.*

Our projection in fact represents nearly a mean between prismatic, and "grating," spectra; while its scale is as absolute and general as any other, and very much more agreeable to common sense and use than those which go against refrangibility, the very parent of any and all spectra by prisms. It is of course far from being absolutely new; though it has never beene mployed on a large scale; and has been too much lost sight of in recent years, when either arbitrary instrumental scales, or equal-part wave-length almost caricatures,

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* M. Angstrom's grand Normal Solar Spectrum Map is a type of diffraction projections. "Wave-lengths" form there a scale of equal parts along the whole of its length of 137 inches nearly. For one purpose of theory that wave-length scale may be convenient; but evidently Nature declines to be bound by it in other respects; for in the first 30 inches of it, there are only 98 solar lines; and in the last 30, so many as 522; so that they hardly have standing room.

An ingenious idea was proposed some years ago by Prof. Alexander S. Herschel, for employing both a nature-founded scale, and in equal parts on the paper for prismatic spectra almost as observed; or with all their exaggerated length of the violet end. The method consisted in employing, not wave-numbers per inch, but their squares; and it is nearly true for the average of prisms. But its numbers are impracticably large, and it does too little justice to diffraction spectra.

rather than representations, of prism-observed phenomena have monopolised attention among the greater spectroscopists.* (See also p. 842.)

COLOURED *FLUID* SERIES. (*See also Plate* 2, *No.* 42 *of this Vol.*)

The number of colours offered by commercial glass, being very limited, I next tried watery solutions of various well known chemicals, including a list of aniline dyes as prepared for the people by Mr JUDSON; and tried most of them at three, and some at six, different strengths.

In this manner there were examined altogether of

REDS, twelve; with notes on a few more;
YELLOWS, four;
GREENS, two;
BLUES, four; and
VIOLETS, five.

The individual observations are contained in their numerical printed form in Appendix II., pp. 816 to 842; and the first medium operated on, *potassic permanganate,* had several of its stages given by duplicate observations, to afford some idea of the amount of probable error in any single observation further on.

The graphical representation of all the more important of the above observations is given in MS. Plates 4 to 14 (reduced, for economy, to one *engraved* Plate, No. 2); in the order above indicated from Red to Violet and Lavender-Gray; the spectrum projection being the same as that already explained. (See also p. 842.)

FIRST RESULT, *from both glasses and fluids.*

The first general result to be drawn from all the observations is that,—the luminous bands left outstanding in these so-called absorption spectra of colouring materials are *moveable* in the spectrum with variation of the depth of the colour medium. For an increase of such depth invariably makes a red band move further to the red, and a blue band further to the blue, end of the Spectrum. There can therefore be no coloured glass ever depended on either to stop, or transmit any one mono-chromatic ray alone; for the said glass will act sometimes on one, and sometimes on another and neighbouring spectral ray, according to the temporary brilliance of the light, and depth of colouring matter.

This notable point was first ascertained by the varying micrometer measures

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* Further, we place the red-end, or natural beginning, of the spectrum to the left-hand because, although a recent reviewer of Mr Rand Capron's "Photographed Spectra" chides him for so doing; and says, "have not Kirchoff, and Huggins placed the red-end to the right;"—yet the opposite plan has been adopted by Fraunhofer, Brewster, Gladstone, Bunsen, Janssen, Roscoe, Watts, De Willengen *per* Secchi, Plucker, Hittorf, De Boisbaudran, and lastly Russell of Sydney in the "*Observatory*" for March 1879. While we may also claim the practice of the whole European family of nations in writing from left to right, never from right to left.—(*Added March* 8, 1879.)

for such bands of light; and was afterwards confirmed by the discovery (independent at least, though it may not be original) of a black line in the *Oxalate of Chromium and Potassium* spectrum. The said black-line, when first seen in a weak solution, was barely separated from the left-hand, or red, side of the red band of light. With an increase in the strength of the solution, the black-line was seen further in upon the band of light. With a further increase the line was central on the band of light. With still further increase it came towards the right-hand side; and with still further increase it was lost on that right-hand side.

Relative motion therefore of the band of light and the black-line was just as positively proved, as with the sun rising on the eastern horizon, passing through the spectator's visible breadth of the sky, and setting in the west. But, that it was the band of light that *really* moved, and not the line, was testified by the micrometer measures; for they showed one and the same invariable reading for the line; but progressively changing readings for the band.

Similar measures on three different strengths of *Didymium Nitrate* solution, showed that all the dark lines and bands in that substance's most remarkable spectrum were all of them fixed; just as fixed indeed as the Fraunhofer black-lines in the solar spectrum. Lines which FRAUNHOFER himself, with eminent propriety and truth called "fixed lines;" and which we now know are produced by absorption, some in the sun's, and some in the earth's, atmosphere.

To call therefore the luminous and colour-medium spectra we are now engaged upon, simply and as though distinctively "absorption spectra," does not give a sufficient idea of the chief peculiarity we have thus far found in them, viz., the undoubtedly locomotive, and not fixed, character of their luminous spectral bands; and from this very unexpected feature something further of interest may be educed by and by.

SECOND RESULT, *as to certain published laws.*

The next general result of our observations is singularly opposed to the chief asserted "laws" of the spectra presented by coloured transparent media, as taught in some, happily not all, of the elementary books on Spectroscopy in the present day.

Thus says one of these works,* otherwise much to be commended:—

"(1.) In all cases the *red* colour of any transmitted light, is due to the deficiency of transmitted rays belonging to the *blue* and *violet* end of the spectrum.

"(2.) Cobalt blue glass absorbs all the red and orange rays and most of the yellow and green, but transmits the blue rays and most of the indigo and blue-violet rays.

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* See p. 50, par. 3, of "The Spectroscope and its Work, 1877; by Richard A. Proctor, B.A., Cambridge; being one of the "Manuals of Elementary Science," prepared under the auspices of "The Committee of General Literature and Education, appointed by the Society for Promoting Christian Knowledge."

"(3.) In every case the colour of glass or crystal corresponds with the portion of the spectrum most freely transmitted."

Now each of these supposed laws of coloured glasses and spectral colours, is the subject of frequent contradiction by our observations; thus,—

1′. Red light is spectrally transmitted most red by media not generally esteemed red at all; but held rather to be characteristic of, to abound in, and transmit most freely, rays at the blue end of the spectrum.

2′. Cobalt-blue glass instead of absorbing as asserted all the red rays, transmits some of them most vividly. And this has been long known and extensively used in optical experiments for certain purposes by many scientists,—though somehow or other passed over in 1877 by the particular text-book quoted; and therefore requiring to be re-asserted very distinctly.

3′. In multitudes of cases the colour of some glass or crystal according to usual acceptation, instead of being the same as, is the very opposite of, that part of the spectrum most freely transmitted by them. Thus a certain dark blue-green dye, of which we shall have to describe some still more striking habitudes presently, transmits under certain conditions only one spectral ray of any kind of light, and that one is of a burning, fiery *red.*[*]

Of course there is an explanation of these apparent anomalies. But the anomalies are so numerous, and of such wide practical bearing, as to form in themselves a law of Nature, which cannot be safely neglected in any system of School Philosophy. And if, out of 29 colouring substances spectrally examined here, 12 of them have been accordant with the usual law, another 12 have been positively against it, and 5 have been of a more difficult character still.

PHYSICAL, AS AGAINST OPTICAL,
ARRANGEMENT OF COLOUR MEDIA.

Each of the three classes of spectrum-modifying substances just alluded to, contains examples of most various and diverse tints according to eye-observation. They cannot therefore, for their physical qualities, be efficiently classified by visible colour at all; but must be rated rather according to the spectral action itself just alluded to, and whose effects may be summarised thus:—

The first 12 substances have transmitted in the spectrum, for all depths of their respective tints, only *one* band of light, and may therefore be called

MONO-CHROIC.

The second 12 have transmitted, in all their deeper tints, *two* widely distinct spectrum bands, and are therefore to be called

DI-CHROIC.

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[*] This fluid, as being one of the aniline series of coal-tar dyes, must be of too modern discovery or manufacture to have come under the piercing examination of the late Sir David Brewster; though he had found other coloured solutions having in different, but lesser, degrees the property just mentioned; viz., that with increased thicknesses they transmitted not bluish, but red light only.

And the last 5 have transmitted three or more bands, so as to admit the name of

POLY-CHROIC.

When the media are very faint and thin, they will, indeed, all of them transmit something of every part and all the colours of the continuous spectrum of a gas light. But for our present inquiry, such action would still be termed MONO-CHROIC. Impure mono-chroic no doubt; but that, much rather than Poly-chroic, because the several colours which they do so transmit are all connected, one with the other, and form all together a single symmetrical great wave of light.

When however the media increase in depth of colouring matter, the transmitted light narrows in spectral place, and at length either makes one band only, as a decided and pure MONO-CHROIC; or two distinctly and actually separated-by-darkness bands, as a DI-CHROIC; or more of such discontinuous bands of spectral light, as a POLY-CHROIC.

With still greater depths of the media, one of the two bands of a DI-CHROIC sometimes gets an advantage over the other, until at length one of them is entirely lost; and the other alone remaining, the medium becomes a MONO-CHROIC again, but of a higher and purer order than at first. Of an otherwise most noteworthy order too, for the band left outstanding at the very last, is usually the one least characteristic of the colour of the fluid as generally known to the world: and the fact has been already remarked by some investigators long ago, to the extent of noting that some bluish substances did, when in increased thickness of layers, transmit *red* light only. (See also note to p. 784.)

Finally the POLY-CHROIC media proper, though approaching the phenomena of the *Di-chroics*, are never such clear and positive colours as either MONO-CHROICS or DI-CHROICS; and rather serve the purpose of shades for eventually extinguishing all light and colour.

Hence my first attempt at arranging colour-media according to these features of their spectral behaviour, runs thus:—

MONO-CHROICS.

Ammonia Chloride of Copper.
Ruby-red glass.
Judson's Marone red (Aniline series).
——— Magenta red.
——— Crimson red.
Port, Claret and other wines.
Iodine tincture.
Yellow-brown glass.
Bichromate of Potash (Potassium).

Tea-infusion.
Sulphate of Copper.
Green glass, stained green.

DI-CHROICS.

Permanganate of Potassium.
Oxalate of Chromium and Potash.
Cobalt-blue glass.
Litmus solution.
Judson's Peach-blossom dye.
——— Cambridge blue.
——— Oxford blue.
——— Violet.
——— Purple.
——— Mauve.
——— Green.
Green heavy flint glass, innately green from lead.

POLY-CHROICS.

Red-lilac glass.
Blue-lilac glass.
Judson's Lavender dye.
——— Plum-colour.
and perhaps Didymium Nitrate.

As the Di-chroic colours will chiefly occupy us in what is to come, we may as well tabulate here a further variety amongst them; viz., the tendency of some to become more red, and others blue, with increased thicknesses; or thus:—

RED-WARD Di-chroics.

Oxalate of Chrom. and Potass. slatey-blue.
Litmus blue.
Judson's Green.
——— Peach-blossom pink.
——— Violet.
——— Purple.
——— Ruby, and
——— Mauve.

BLUE-WARD Di-chroics.

Permanganate of Potassium pink.
Cobalt-blue glass.
Judson's Cambridge blue.
——— Oxford blue.

PRACTICAL APPLICATION IN SPECTROSCOPY AND OPTICS.

One of the most frequent utilisations of coloured glasses in solar spectroscopy is, to keep out of the field of view false glare from neighbouring and brighter parts of the spectrum. Wherefore arises the useful question,—What colour shall we adopt to transmit the utmost possible quantity, say, of the faint red end, and extinguish the utmost of every other part, of the spectrum?

Then comes the answer, strange evidently to the authors of some of the text-books, though well known to experienced observers,—"Use either blue or green, if dichroic, media; for they transmit red light, more red, or further to the red end of the spectrum, than any known red medium whatever." But the said blue or green media must be *very intensely* dichroic, and all the stained green *glasses* I have yet tried, have been exceedingly mono-chroic and therefore useless, even pernicious, in such an inquiry.

The quality demanded, or for transmitting red light, comes out numerically quantified from our observations, thus,—

OF THE REDNESS OF RED LIGHTS,

by its Wave-number Place in the Spectrum.

Colouring Material, and its depth.	Intensity of the light-band transmitted. Max. = 10.	Number of Waves at the place in a British Inch. Redness decreases as the number increases.	Class of Colour.		
			Mono-chroic.	Di-chroic.	Poly-chroic.
Two Cobalt-blue glasses,	4·1	35 587·	...	×	
$\frac{1}{200}$ of Cambridge Blue dye,	3·	36 166·	...	×	
$\frac{1}{100}$ of Judson's Green dye,	3	36 765	...	×	
$\frac{1}{80}$ Oxalate of Chromium and Potash; Blue,	4	36 928	...	×	
Two Blue-lilac glasses,	3·3	37 313	...	...	×
Claret wine, Purple-red, by day-light inspection,	3·	37 411	×		
$\frac{1}{600}$ Litmus Blue,	3·	37 779	...	×	
Lisbon Collares wine,	3·	37 879	×		
$\frac{1}{400}$ Judson's Violet dye,	3·	38 023	...	×	
$\frac{1}{200}$ Judson's Mauve-red,	3	38 226	...	×	
$\frac{1}{400}$ Judson's Purple,	3	38 285	...	×	
$\frac{1}{200}$ Judson's Peach-blossom,	4	38 343	...	×	
Leith supplied Port wine,	3	38 462	×		
$\frac{1}{50}$ Marone-red dye,	4	38 820	×		
Four Ruby-red glasses,	4·5	39 371	×		
$\frac{1}{200}$ Judson's Crimson,	4	39 417	×		
$\frac{1}{60}$ Judson's Ruby dye,	4	39 526	...	×	
$\frac{1}{400}$ Iodine solution, Brown in simple day-light,	4	39 526	×		
$\frac{1}{200}$ Magenta Red,	4	40 016	...	×	
$\frac{1}{8000}$ Permanganate of Potash, Pinkish,	4	40 016	×		
Lisbon Port wine, Brown-red,	4	40 650	...		×
Two Red-lilac glasses,	4·5	40 933	...		×

Whence we may easily see, that cobalt-blue glasses are pre-eminent for the exceeding redness of the red light they transmit; while ruby-red glasses, the very expression for red, for danger-signals, with the public, are low down in the scale, and their colour is found to verge more to the spectrum place of orange.

In the case also of the two *potassium* colours, it is instructive to observe, that the pink permanganate is low down or poor in the scale of red, and transmits little real red light; while the far bluer, almost slatey blue oxalate of chromium and potass is high up on the scale of red, and transmits a band of red light which is both splendidly and beautifully red, artistically as well as scientifically.

Somewhat similarly, though with *mutatis mutandis*, in the search for shades suitable to bring out the other faint end of the spectrum, and defend it from the brighter middle region, we may here arrange our variously coloured media in the order in which they were found to transmit light most nearly up towards the lavender-gray end; as thus

OF ULTRA-VIOLET, OR LAVENDER, AND GRAY, LIGHT, BY

Wave-number Place of the furthest light transmitted at that end of Spectrum.

Colouring Material, and its depth.	Intensity of the light at the place observed. Max. = 10·0.	Number of Waves at the place in a British Inch.	Class of Colour.		
			Mono-chroic.	Di-chroic.	Poly-chroic.
Two Blue-lilac glasses, ...	0·1	64 146·	...	...	×
Four Cobalt-blue glasses, ...	0·1	63 452·	...	×	
Two Red-lilac glasses, ...	0·1	63 452·	...	...	×
$\frac{1}{400}$ Judson's Purple dye, ...	0·1	62 267	...	×	
$\frac{1}{400}$ Judson's Violet, ...	0·1	61 920	...	×	
$\frac{1}{400}$ Judson's Ruby, ...	0·1	61 805	...	×	
$\frac{1}{30}$ Ammonia Chloride of Copper, . .	0·1	61 728	×		
$\frac{1}{4000}$ Permanganate of Potash, . . .	0·1	61 652	...	×	
$\frac{1}{200}$ Judson's Peach-blossom, . . .	0·1	61 577	...	×	
$\frac{1}{400}$ ——— Mauve dye, ...	0·1	61 237	...	×	
$\frac{1}{200}$ ——— Cambridge Blue, . . .	0·1	60 241	...	×	
$\frac{1}{50}$ ——— Oxford Blue, ...	0·1	60 241	...	×	
$\frac{1}{400}$ ——— Lavender dye, ...	0·1	59 916	...	...	×
$\frac{1}{2500}$ Litmus Blue solution, ...	0·1	59 453	...	×	
$\frac{1}{200}$ Judson's Green, ...	0·1	54 885	...	×	
$\frac{1}{80}$ Oxalate of Chromium and Potash, . .	0·1	53 305	...	×	
Four stained Green glasses, ...	0·1	50 839	×		

Where a position at the top of the column exhibits most favour by that medium to the more refrangible end of the spectrum.

Again for *Telescopic* utilisations, as for ensuring the finest definition to the Solar limb,—the most strictly monochroic, though dark, media may be selected from our Appendices, say Ruby-red glass, Judson's crimson dye, or ammonia-chloride of copper, blue; where the deep colour will ensure their acting as sun-shades as well. But, for the purpose of practically achromatising, a simple lens, with the least possible loss of light, bichromate of potash solution appears to excel every other tint tried as yet, being both a *monochroic*, and in nearly the most luminous part of the spectrum.

NOMENCLATURE OF COLOURS, OPTICALLY.

For Double-Star observers chiefly. (*See Plate* 3, *No.* 43 *of this Vol.*)

If all the spectral colours discerned in practice must, for theoretical purposes be reduced to a primary three only; then those three are, according to what has for some years past been generally held, and is now given by my renewed observations, taken under favourable circumstances, *not* red, yellow and blue; nor even red, green and violet, but rather

RED,
CITRON, *i.e.* yellow-green, and
VIOLET.

Red, therefore is characteristic of the red-end, or beginning of the spectrum; Citron, of the spectrum's middle and most luminous region; and Violet, of the spectrum's latter, closing, and more refrangible end.

But in practical colour-naming, it is both more convenient to start from a basis of a greater number of separate, if nature-presented, tints; and, over and above what may be done after an approximate fashion, by rude mixtures of the primary three,—there are, and with all exactness and certainty, very many more colours distinctly discernible in the simple solar spectrum as separate, peculiar, existences, when powerful practical methods have been correctly applied to show the colours on an extended scale, say 50 feet long, and in their utmost possible spectral purity, or freedom from admixture of one with the other, or with anything else.* Yet, after all of that increase in number of visible colours shall have been accomplished, a *single* solar spectrum, however rich in many, by no means gives us all, of Nature's colours as we see them day by day. Some, for instance of botanical colours,—and often the most exquisite of them all,—are only to be imitated spectrally by taking *two* spectra, and making their *opposite* ends overlap. While there are other tints still, which are only spectrally re-producible by three different spectra, or three portions, not naturally adjacent, of one spectrum, being artificially combined together, as in Professor CLERK MAXWELL's most ingenious and instructive "Colour-box."

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* See forward to pp. 791 and 792.

Hence in my own practical proceedings touching optical sensations only (as in naming double-star colours, which are moreover generally mixtures and impure)—I have found it expedient to arrange colours, however made up or attained to, in three divisions for eye-reference; thus—

(1.) SINGLE-SPECTRUM COLOURS.

Number.	Principal Divisions of the Spectrum.	Colours by name, chosen during actual spectroscopic observations of a high sun, at Lisbon, with a large dispersion power, a very narrow slit, no coloured glasses, yet no side reflections or false glare.	Fixed, black, Solar Lines within the same limits.	The more easily procured of Chemical Flame, and Electric-spark, bright Lines within the same limits.	Wave-number place per British Inch.
1	**RED-END** OF SPECTRUM.	Ultra-Red.	X.	...	25 000 30 000
2		Crimson-Red.	Y and A.	Potassium *a*.	30 000 35 000
3		RED.	*a* and B.	Lithium *a*.	35 000 37 000
4		Scarlet.	C.	Scarlet Hydrogen.	37 000 39 000
5		Orange.	...	Orange Oxygen.	39 000 40 000
6	**MIDDLE** OF SPECTRUM.	Amber.	*a* Band.	Orange-Amber Carbo-hydrogen.	40 000 43 000
7		Yellow.	D.	Sodium *a*.	43 000 45 000
8		CITRON.	Aurora line.	Citron Carbo-hydrogen.	45 000 47 000
9		Green.	E.	Thallium *a*, and Carbo-hydr., G. G.	47 000 50 000
10		Glaucous.	F.	Glaucous Hydrogen.	50 000 55 000
11	**VIOLET END** OF SPECTRUM.	Blue.	...	Cæsium *a* and *β*.	55 000 57 000
12		Indigo.	...	Indigo Nitrogen.	57 000 58 000
13		VIOLET.	G.	Violet Hydrogen.	58 000 60 000
14		Lavender.	H^1 and H^2.	Lavender Hydrogen	60 000 65 000
15		Gray.	...	...	65 000 70 000

(2.) DOUBLE-SPECTRUM COLOURS.

Number.	Name.	Spectral Region.
16	Amaranth.	Compound of 36 000 and 59 000 variously.
17	Rose.	ditto.
18	Lilac.	ditto.
19	Purple.	ditto.
20	Azure.	ditto.

(3.) TRIPLE-SPECTRUM COLOURS.

Number.	Name.	Spectral Region.
21	Red-gray.	Very compound; of 36 000, 46 000, and 59 000 variously.
22	Yellow-gray.	ditto.
23	Green-gray.	ditto.
24	Blue-gray.	ditto.
25	Black-gray.	ditto.

Throughout the series of 25 colours thus named, there should be one or two intermediate colour gradations easily imaginable between every pair. While of each of *them* again, as also of the previous ones, there are three or more degrees of lighter, or mixed with white; and three or more degrees of darker, or mixed with black (see Plate 3), easily realisable; and making in all about 400 practical tints for double-star observers to sharpen and define their colour senses upon; so long as they think it right, or expedient to confine themselves to eye-observations alone, touching star-colour, its causes and effects. (See p. 843.)

LIMITS OF NEWTON'S "*Nec variat lux fracta colorem.*"

To make violet, the artists tell us to mix red and blue; and to make lavender, add a little yellow to the previous mixture; and we *may* thereby obtain something which will pass muster with the eye alone, but not with the spectroscope, nor with photographic preparations. And inasmuch as blue is already lower in refrangibility than either violet or lavender, the addition of red or yellow or both can only make it lower than ever; nor is there any known pictorial pigment, so far as I know, which, being added to blue, can increase it in the scale of refrangibility.

Hence we fall back with all the more respect on NEWTON's earliest statement of colour in prism-scopy, as well representing a grand physical action in Nature of the most admirable, and otherwise inimitable, kind; when he laid down that, "every spectral colour has a certain refrangibility, and every degree of refrangibility a certain colour, peculiar, constant and unalterable."

Sir DAVID BREWSTER unfortunately imagined at one time, that he had upset that law, by having proved, with a mixture of coloured glasses superposed on prisms, that each of the then recognised seven colours of the spectrum existed more or less in *every* part of, and all along, the whole of the spectrum.

That total mistake (arising chiefly from the impurity of the colours of coloured-glasses), has been abundantly disproved by many persons since then; and it was indeed a necessary obstruction to be removed, before the modern spectrum analysis of MM. BUNSEN and KIRCHOFF, could be fairly established. But on the other hand, NEWTON's older law which had so much truth in it,—and for approximate purposes may be spoken of as true still, is nevertheless, I fear, by no means absolutely correct for minute, but measurable, quantities.

This conclusion arises first, from the peculiar *locomotive* character of colour bands in our absorption spectra described on pp. 782, 783: and it was proved again still more signally to me last summer when spectroscoping the high sun at noonday in Lisbon. For when the instrumental arrangements were peculiarly such that the solar spectrum colours (made pure by using the narrowest possible slit, and *no* coloured glass shades and yet eliminating side reflections), were so brilliant and decided that no one with tolerable eyes could refuse to see, not a mere three, but *at least* 15 different and distinct kinds of colour in a continued order along the chromatic scale (then 30 feet long, from red to lavender),—it was forced on my attention that any alteration of the brightness of the spectrum, caused either accidentally or purposely, would often shift the very colours themselves, by name, or by eye-appreciation as such, through one or two of the 15 colour-spaces, backwards or forwards as the case might be. At such times too, there was no alteration of refrangibility whatever; either according to instrumental construction, or actually observed steadiness of the Fraunhofer lines in the field of the measuring telescope: but there was undoubtedly a sort of small-range movement of the colour-scale as a whole, though happily only through the limits of a small epicycle as it were of adjustment attached to it.

Hence *colours*, even of the spectrum, however beautiful and though practically pure, are not amenable with full accuracy to the simple law of refraction of rays of light in a prism; and contain some unexplained phenomena still.*

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* This section has been written in February 1879.

VISION OF COLOURS, THROUGH COLOURED MEDIA.

So long as we confine ourselves, in such vision, to media of a MONO-CHROIC nature, very little, if any, unusual or particularly interesting, effect will be observed. A general glare of *red,* will no doubt appear in all the lights of a landscape, and especially of the sky, when we look through the ruby-red glass of railway signal-lamps ; or of *blue,* through solutions of ammonia chloride of copper ; or of *yellow,* through Bichromate of Potash. But neither the indigenous and previously existing red, nor the blue, nor the yellow substances of the earthly scene are themselves much discriminated, intensified, or beautified thereby.

The moment, however, that we employ DI-CHROIC media, and of any colour, some most startling changes may be witnessed.

Take for instance a weak, and therefore to the world at large, a slatey-blue solution of Oxalate of Chromium and Potash and look through it at the reds, whether of flowers or brick-fields in the landscape. In place of their being dulled, as they would infallibly be if mixed up with so much impure blue as colours on an artist's palette,—they now start forth brilliantly, gorgeously, red ; perhaps even three times more red than they ever were before !*

Or take a more unpromising medium still, a dark-blue and black-green (Judson's Aniline dye), and let it be as deep almost as writing ink. What should we expect to see through such an apparent hindrance to any vision at all, as that ?

The following is what I did see on the first occasion of trying the experiment, and you may judge of my surprise.

The chief feature already in the scene was a white cottage with a blue-slated roof, in a broad, sun-illumined field of bright green grass, with some red brick buildings in the distance. I then put the flat black bottle of Judson's most dark and blue-green fluid before my eye,—and lo ! the view was still of day-light brightness, there were still the red buildings in place, still the brilliant white cottage with its distinctly blue-slated roof,—but it was now standing in a field, not of green at all, but of glaring and evident *vermilion !*

I next looked down, through the dark medium as before, into a neighbour's garden just renovated for the approaching spring. There was a rockery of various coloured stones, leafless tree trunks of warm dark tints, and a brilliant gravel serpentine walk,—but it seemed to be bordered now, not with green

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* The Oxalate of Chromium and Potash was well examined by Sir David Brewster in his best days ; and he distinctly observed that though bluish in weak solutions, it allowed only red light to pass through its thicker solutions. Our recently observed effect depends upon that, but is accompanied by something further.

turf, but with scarlet cloth. *Couleur de rose,* do you suggest? No: it was *couleur de* soldier's coat; so excessively pronounced in red was the aspect at that moment of what I knew to be merely green grass: and yet the daisies in it, here and there, remained all of them exquisitely white.

Still again I looked through the black-green bottle at a plant of flourishing Palmetto in an earthenware pot on the table at my elbow. I scanned it very closely, while the sun shone upon it, over all its lights and shades, over all its leaves, stalks and stem almost microscopically, and made myself as sure as I could be by clear, strong eye-sight of anything whatever even of things nearest at hand,—that, though the earth in the pot was black, yet both plant and pot were absolutely of one and the same colour; and that colour was a signal species of red, of a more beautiful kind and even sublime degree of redness of red than I had ever met with in any sort or kind of gardening before!

The instant the dark bottle was removed, the plant was green, and the pot a dull brown-red. But how had their recent simultaneous and most superlative redness been brought about, through means too of an almost pitch-dark blue-green medium?

The reason thereof is duplex; but not very far to seek with the aid of spectroscopy. Judson's green, the medium concerned, is intensely di-chroic, and when very dark will transmit little, if any, part of the spectrum except the extreme red. Wherefore, if it meets with another di-chroic either green or brown, it allows none of that green or brown, however bright, to pass; and only such portions of their red spectral bands as correspond with its own red band.

But that band, as shown already in our Table of Reds, page 787, is far further up towards Nature's mysterious beginning and fountain-head of both the spectrum and red, than is any of the red of the, for railway signal-lamps most approved, ruby-red glass. Wherefore by so much, expressible too in spectrum number, is the red ray of di-chroic green vegetation, when seen through Judson's dark green-black dye, more magnificently, purely and perfectly red, than any red colour that human eye has yet seen without the discriminating assistance of the spectroscope.

But over and above the redness of any reds thus made manifest, there was such brilliance! Whence therefore came the light for that almost sun-shiny illumination, whose vigour lent such a positiveness to the transformation scene?

It comes in a manner out of the very darkness of the dark bottle itself; and by means of an interesting interference of the waves of light. In fact it reminds one closely, though with a variation, of the terrestrial circumstances under which the incandescent sodium vapour of the sun may be seen dark, in place of bright; viz., it must be looked at through another supply of also incandescent sodium vapour, but not so bright or hot as the first.

Similarly then, though *vice versâ,* the essence of success in making our colour experiment is, that the thing looked at, shall be brilliantly illuminated, while the thing looked through, shall be comparatively in the shade, as well as physically darker: for, if these conditions be well secured, the blue of the blue-green in the dark bottle looked through, so combines its undulations with those of the blue of the thing looked at, that they make something approaching to white light, in place of double blue-dark.

Hence if thus through the, in itself most dark,* bottle, we contemplate a piece of known cobalt-blue glass held before us in the sun-shine,—its blue colour is for the time entirely gone and nothing left behind except a faint residual pinkish tinge, in an otherwise transparent and white glass plate.

In the same manner, looking at another, and weaker bottle of Judson's green, but extra well illuminated,† its green is knocked out of it, and you might fancy the contents little but pure water.

CHEMICAL INDICATIONS, BY DI-CHROIC SPECTRAL MEDIA.

While green of grass is turned so positively into glaring red, there are other green substances which resist, and some which even change in the opposite direction,—under the influence of the di-chroic blue-green black bottle.

Thus while all plant leaves that I have tried, became more or less red,—litmus paper and litmus dyed cloth, grandly red; and two samples of green paper-hangings became decidedly red;—yet on the other hand,—

Blue of heaven,
Blue of slates,
Blue of sulphate of copper,
Blue-green of acetate of copper,
Certain green-painted Venetian blinds, and
Nine samples of green paper-hangings

refused to become red at all, and some of them verged towards glaucous.

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* One of Judson's $\frac{1}{4}$ ounce bottles of concentrated dye, mixed in 20 ounces of hot water and allowed to cool, gives nearly the requisite strength and darkness, in a flat bottle one inch thick internally. On trying solutions of various strength in an angular or prism-shaped bottle the dispersion was not found to be anomalous as with fuchsine, but normal as with glass, though with the addition of looking through the glass-prism with a flat film of Judson's green interposed, and thereby reducing the whole continuous spectrum of daylight to a red band and a green band, with a black band between them.

† As the parallel, though opposite, example in spectroscopy of the importance of the intense illumination of the thing looked at, than the thing looked through, to bring about the interference of waves and inversion of the simple appearance of the light of the nearer substance;—if we hold a spirit lamp burning salt between a spectroscope, and a dull distant object giving a weak day-light spectrum, no inversion takes place; and the salt lines of the lamp appear bright, utterly hiding thereby the dark, but only grey, D lines of the clouds.

But if we hold the same spirit lamp between the same spectroscope and the *Sun*, the bright salt lines of the poor little lamp are inverted instantly and add black, not white, to the already very dark Solar D lines.

Still more remarkably, an exquisite yellow-green muslin, in eye-colour like green of young lettuce leaves in a delicately prepared salad,—the most attractive thing therefore that muslin, among all green muslins, ever invented to induce young ladies into adopting it for a ball dress,—it not only refused to turn red at all, but went the opposite way and became of a blue green. I sent it therefore to my young friend Dr. PREVOST, now prosecuting chemistry in Oxford, requesting him to pronounce, if he could by some not too difficult chemical analysis, what the colouring material might be ?

Almost by return of post he answered, "Oh ! you need not mention my name, for the detection of the chief ingredient was so extremely easy. I merely proceeded in the usual elementary chemical manner, then applied MARSH's test, and 'arseniate of copper' was abundantly manifested."*

SUBJECTIVE CONDITION OF THE COLOUR-BLIND, SO-CALLED.

From the times of DALTON, and through the epochs of HERSCHEL, BREWSTER, and TROUGHTON, down to the late GEORGE WILSON, and the happily still existing Professors KELLAND and CLERK MAXWELL,—an immense deal of learned interest has always been expressed or taken in the peculiar optical condition under which some persons are born, and remain throughout life; and which has been called for shortness, though with eminent inaccuracy and even mischievous effect, "colour-blindness."

For some years past, the *laws* of such reputed incapacity for seeing colour, have been so exactly formulated in various scientific treatises, that there seemed little or nothing more remaining to be learnt. And yet I suspect, that the new instrument of investigation which the di-chroic spectrum principle puts into our hands, will require some of those laws to be re-written; and varieties of kind, as well as degree, to be recognised amongst different patients.

This new necessity however arises very excusably out of the almost impossible conditions of the problem as hitherto presented. You may, for instance, question a colour-blind person to any extent; but if *he* has never, from his birth, seen any difference between two colours, which to you are as opposite as any two can be,—how can he describe what he sees in terms of your impressions ? But if you are now enabled by the di-chroic medium to make, or un-

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* Shortly after the above was written I had the opportunity in Lisbon, with a bottle of this Judson's green to my eye, to see agaves, palm-trees, acacias, Indian corn, myrtle bushes, and oleanders, not green, but of every description of red from vermilion and cochineal, to deep Indian-red. Yet the green-sea water of the Tagus, absolutely refused to alter; though yellow-brown, unpainted oars, dipped into it, came up *blood-red* at every stroke !

In the Bay of Biscay, where the dull blue waves were running rather high under a cloudy sky, and brown sea-weed, like coils of hempen whale line, was floating about,—the bottle showed the waves of an ultra dark, sombre, almost threatening black-green, but the sea-weed of a magnificent coral red, almost luminous in the splendour and glory of its redness.

make yourself in a moment, "colour-blind," at least in its most generally acknowledged and leading characteristics,—your memory enables you most securely to describe how you saw things in one condition, as compared with your feelings in the other.

What then is the main outward observed fact, which we have to imitate, or realise on ourselves, in order to stand in the place of the orthodox "colour-blindness" both of literature, and of a great part of mankind ?

In the carefully prepared work of that most amiable and laborious of scientific enthusiasts, the late Dr GEORGE WILSON, —he has abundantly described the various objective features of "colour-blindness;" and considers the chief one to be, in one word, an inability to distinguish between *red* and *green;* these being actually and notoriously two distinct members of the only three esteemed primary colours: and separated further an immense spectral distance from each other, whenever the intervening secondary colours, formed by their mixtures, are allowed to appear. Blue and green are easily distinguished by most of such persons,—but red and green they can see *no* difference in.

As a symbol of that leading and most *bizarre* fact in "colour-blindness," the worthy Doctor had his book bound in *red* cloth, and armed with a bright *green* label. And I can also add from my own long experience of a late and most esteemed friend in another part of the world, that though he could distinguish smudgy, indistinct colours, including dull greens and impure reds, of almost any kind; and though he could recognise an immense variety of tints, including dark shades, in Nature; and could æsthetically enjoy their brighter beauties and also their more sombre chiaro-scuro effects and deeper shadows; and had moreover almost inimitable accuracy of vision for sharp defining and micrometric mensuration,—yet if, on a luminous sunny morning, a green leaf was of a very brilliant, pronounced *green,* and a red flower, as of the pomegranate, was of a very staring and declared *red,*—he could not then see, understand or believe that there was a particle of colour difference between them !

Now this being the very example most easy of artificial realisation with our Di-chroic dark bottle; and as difference of degree in that said bottle, realises also the next most extensive fact amongst the "colour-blind," viz., the inability to distinguish between *red* and *black,*—I presume that I have really got hold, at least in principle, of something like the general natural agency in the case of a large section of these individuals: viz., vision through a dark fluid in the eye, in some persons possibly verging to green, in others to blue, in others to brown, but in all of them necessarily di-chroic as to the spectrum.

Hence I can now, whenever there is occasion, take my place amongst a considerable number of the colour-blind, as one of themselves; and yet acquainted with colours in the very same manner as those persons who protest that *they* are not blind. Wherefore from such a position of double perceptive gifts, I now proceed to examine the received *laws* touching the colour-blind: chiefly as they have been well and succinctly collected by Professor EVERETT, in that useful book of reference, *his* edition of DESCHANEL'S *Natural Philosophy.*

The chief, of these long since postulated maxims,—(but wherein I am sorry to say, though colours are abundantly dealt with, the difference of physical constitution among matters held as showing to the eye one and the same colour,—is not at all, or barely, touched on)—are as follows,—

(1) "What is called colour-blindness has been found, in every case which has been carefully investigated, to consist in the absence of the elementary sensation corresponding to *red.*

(2) "The scarlet of the spectrum is thus visible to the colour-blind, not as scarlet, but as a deep, dark colour, probably a kind of dark *green.*

(3) "To such an eye, any colour can be matched by a mixture of yellow and blue, and such vision may therefore be called di-chroic.*

(4) "Objects which have the *same colour* to a tri-chroic (an ordinary) eye, have also the *same colour* to a di-chroic (colour-blind) eye."

To those most clearly stated laws, the following practical answers may now be rendered, as I am led to believe by the di-chroic medium, on the part of at least a large section of the colour-blind :—

(1′). In the case most characteristic of orthodox colour-blindness, viz., confusing between bright red and bright green,—it is not as laid down that the red is *not* seen, but that the green appears also as red; in fact just as red as the red.

(2′). As for the scarlet of the spectrum being said to be seen by such of the colour-blind only as a dark green or other dull colour,—it is seen by them more purely, gloriously, entirely red than by any other class of persons.

(3′). To say, that to any one with a colour-blind eye, which really sees red so grandly, as those looking through a strongly di-chroic fluid must do,—to say, I repeat, that any colour can be matched by a mixture of yellow and blue,—is simply a mistake. And such mistake has originated probably from *some* of the yellows shown to the parties concerned, being those which, to them, appeared more or less red. Thus I, looking through the colour-blinding, or rather di-chroicising medium, see in my colour box

Lemon yellow = faint rose pink set off in places with pale French gray. Naples yellow = orange red-lead. Cadmium yellow = vermilion red-lead. While Gilding, reminds me of the "red-gold" of rustic fairy-tales and song.

(4′). Objects which have the same colour to a tri-chroic, have the same to a di-chroic, eye,—says the law. Wherefore I look with my natural tri-chroic eye at Venetian green blinds, and at green grass, and see the two objects of the

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* Readers will please to note the different meaning here attached to this word, as compared to that in which we have ourselves been employing it spectrally, and must employ it again.

same colour, viz., green; but on converting my eye into the di-chroic kind by holding the blue-green black bottle before it, behold I see the two objects as different as possible from each other in colour, one being green and the other red.

Hence *this* reputed law requires decided correction or limitation; for an artificially di-chroicised eye certainly, and for many a naturally di-chroicised one probably: while experiment on the latter class, might well repay some advanced physiologist to institute; especially if he should make it on so extensive a scale as to eliminate accidents and varieties of human eyes, as to their internal coloured humours, whose tints, dear to the poet and the lover, are legion. Meanwhile, however, in certain cases, depending more on physics than optics, a di-chroicised natural eye (as in the above grass and venetian blind case) may perceive not a less, but a greater, variation of tints than a tri-chroic eye; and therefore has a sensible advantage in some exact questions.

"But," asks an æsthetic friend, "though it may be useful occasionally in mere mechanical science,—can it ever be desirable in the beautiful, the exalted Fine-Arts to see anything with an eye that makes green grass red?"

Yes it is! Especially when you find that such an eye has a whole gamut of choice and various reds. So that if it preserves some of its richest red for a carpet of standard green grass, it has orange red for yellow green; lakey red for blue-green; and purple red, madder-brown red, and bronzed red for the duller greens of diverse plants at sundry seasons of the year. While many of the most exquisite dove-coloured, and other balancing tints, are either left untouched, or so decidedly re-inforced, as to make a more effective picture for light and shade on the whole, than the ordinary view. Contemplate the cases however carefully, practically, both by anxiously painting them out from Nature* and also by reading up Art-authors,—and you may presently be gifted to see, that the new Di-chroic spectroscopy has at last most effectively solved for man, that ultra difficult Art-problem which has puzzled Tri-chroic-eyed painters for centuries; viz., how to dispose of, or allowably substitute, with pictorial mellowness, "that crude colour green."

The best Dutch and Flemish landscape painters, striving in their day after noble ideals, rather than mere realistic transcripts, eschewed green altogether; and employed, as *their* method of solving the problem (so far as they then dared), "warm brown" instead; aiming always for a grass lawn at the exquisitely melodious tint, the glorious colour, of "an old Cremona violin"; and

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* I made several coloured sketches this last summer in Lisbon, first as seen direct or simply by my natural tri-chroic eye, and then as seen by looking through the dark-green di-chroic bottle: and if the trees at an old mill door, did stand up in this latter case like gigantic red corals, there were magnificent shades of dove-coloured gray behind, and pale sea-green sky above them to satisfy the most fastidious eye as to abstract theory, and splendour of effect, though not as to ordinary terrestrial experience.

they claimed the introduction of so apparently opposite a tint as that species of brown, in place of green, as the only mode of preserving the *harmony* of their pictures. The Edinburgh National Gallery too, possesses a costly and, by real connoisseurs, highly appreciated landscape of Titian's (painted for an Emperor), with the trees and grass successfully rendered of very nearly the ancient violin hue. Some rash young Academicians of our present days, I am informed, presume to laugh at the long-shaped painting, and protest that any school-boy, of their teaching, could paint better than that! Greener, no doubt; but Titian's work, for deep reasons of his own, is most certainly without the slightest pretence or attempt to imitate green there, at least as ordinary men see it; while the sky is more glaucous than the raw blue which the lads, his criticizers, would probably have adopted.

Yet had the social and educated opinion of that age been sufficiently advanced in our modern dichroic spectroscopy, to have allowed the great colour artist to go just one step further; and use, in place of mere heated brown for green, that which Nature herself has long ago stamped as an exchangeable substitute for green in the cause of pictorial harmony; viz., some variety or other of that wondrous colouration, red,—there can be little doubt that Titian's work would have ranged through a greater number of octaves of tints and hues, and been a more ecstatic triumph of chromatic effect than it is; or more like one of Turner's masterpieces of Sunset, which goes on increasing in value from generation to generation, even in proportion as its rubric grandeur is worthily understood and intelligently admired. But when actually that is sometimes run down in very unscientific art coteries, innocent of the smallest acquaintance with instrumental spectroscopy and its colour-researching power, the real seat of the mischief may be, not that the stigmatised "colour-blind" persons have a di-chroicising fluid in their eyes, and which, in reality, enables them to see almost with a Titian or a Turner,—but that the general public has *not* anything of the kind; has in fact imperfect eyes; eyes admitting too much yellow; faded eyes without "visual purple," as poor, tame, flat and unprofitable to a true artist, as watery blood without healthy, colouring globules must be to any one whatever.

There is also another very notable artistical gain to the di-chroicised eye when contemplating Nature æsthetically, and with music in the soul,—in the facility which such an eye possesses for turning heavy, inky *blue*, but no shades of brown, into light. And this is a feature, the abstract principle and spectrum foundation of which, I am happy to acknowledge, is most clearly and independently, though involuntarily, set forth, in Professor Clerk Maxwell's extremely scientific paper in the Philosophical Transactions for 1860. For he there most expressly places, and pictures the brightest and *whitest* light of the whole

Spectrum to a "colour-blind" eye, as being near the place of the Fraunhofer line F, or the "blue," *i.e.* the blue-green or glaucous, Hydrogen locality.

Now if blue be seen when under some mixture with other colours which may give it a cold, dark, inky hue,—how all thorough artists do detest it! How many a natural scene has been condemned æsthetically for appearing to them, with their tri-chroic, uncoloured, eyes, lugubrious; weighed down by that heavy and most forbidding tint. Yet a di-chroic eye has seen the same identical region at the very same time, luminous, brilliant, glowing with red, and not without sufficient browns, purples, violets and even points both of black and white, to give force of character, variety of effect, and make its owner happy for the day.

In short, if there is any colour which may be on the whole, though it is certainly not in every particular, somewhat defective in the colour sensation of powerfully Di-chroicised (we can no longer call them colour-blind) individuals,—it is not red, but yellow; or that colour which, in painting, being added too largely to almost any mixtures of red and violet, makes them dirty, messy, odious at once. The reason too of so much freedom for di-chroite eyes, from the world's vulgarising leaven of overpowering yellow, is plain enough from our previous spectral examinations of colours. For the very essence of a di-chroic, as contrasted with a mono-chroic, medium is there shown to be, the dulling out of the yellow, or *middle,* region of the spectrum. So that whether eyes are tinged with visual purple, gray, green, coffee-brown, or anything else, so long as the medium is spectrally di-chroic, the yellow, and more especially the yellow-green or citron components in the scene *must* be shut out, to a greater or less degree.

But before that citron region, usually by far the brightest in our natural-eye spectrum, (indeed too bright, generally, for good vision of both the red and the violet simultaneously with it), is completely excluded, by increasing depths of di-chroicising media,—strange effects may be produced by its partially darkened tints mixing with others. Thus—take the particular red cloth cover of Dr GEORGE WILSON's symbolically bound volume. On placing it in sunshine, and looking at it through a dark (as $\frac{1}{80}$th to a $\frac{1}{90}$th)* solution of Judson's green dye,—the effect is, that it is seen of a *brighter* red.

But if we next look at it through a pale ($\frac{1}{300}$th) solution of the same dye, a something so weakly green that it produces little or no effect either on bright green grass or things in general,—what do we see?

A *black* book! Not very densely or darkly black, but nearer black than

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* From $\frac{1}{60}$ to $\frac{1}{90}$, according to the brightness of the sunlight at the time, is the strength of solution which has most effect in turning the greenest grass into the most brilliant scarlet. Stronger solutions as $\frac{1}{20}$ and $\frac{1}{40}$ both stop too much light, and make green grass only dully orange or buff.

gray. The reason of the seeming blackness being, apparently, that the depth of the blue tint in the pale bottle not being sufficient to reverse the waves and convert the blue of the lakey-red of Dr Wilson's book into light, the two blues add their depths of blue together, making them very dark blue indeed, to a mixture of some red, and much dark-greenish shade in the region of the half extinguished citron of the spectrum. A species of commingling which cannot fail, in a poor light, to produce something very like black.

Hence the subject of vision of colours through artificial Di-chroic media of different depths and different hues,—Judson's green being merely one among many and diverse-tinted colour-media which have more or less of the curious property,—seems to promise a wide range of phenomena most interesting to Science and possibly, to be, enriching to Art.

But precisely because the subject is so extensive, my own immediate duties require me now to take leave of its strange phenomena; and to trust that others better able and more professionally concerned, will continue the inquiry and extend the results.

POSTSCRIPT.

Colour-perfected Reference-line for Spectroscopic Mensuration.

All the accuracy of spectroscopic measurement, when performed as it generally is by looking into a telescope, depends, very much as in astronomical observations, on having good fiducial points of reference in the field of view.

These fiducial points must be sharp in themselves, in good focus, easily seen, and *without parallax*; while, for a bright field of observation, they may be black-lines or pointers; and for a dark field, bright ones. The latter are the more important in spectroscopy, because, while they may, on a certain construction, be usable in a bright field also, they alone have any efficiency in a dark field; and every spectrum has dark ends, however bright it may be in the middle.

With this view it was, that after trying several older and less perfect contrivances, I procured three years ago one of the widely praised bright-line references invented and manufactured by Mr Adam Hilger,[*] and employed it in the observations detailed at the beginning of this paper. Up to a certain extent it proved admirable; for the line of light (formed by a cut through black varnish on the face of a fox-wedge of glass dipping into the field of view and illumined by a lamp at its outer end) was sharp, easily seen even with the faintest illumination-light, and capable of being brought to best focus; while it was also fur-

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[*] Of 192 Tottenham Court Road, London.

nished with a wheel of coloured glasses, so as to allow the observer to make the reference-line of almost any colour he should please.

This wheel promised to be a charming addition, characteristic too for spectroscopy; and yet herein was ultimately found the weakness. For as the observer advanced from the red, to the violet, end of the spectrum, if there were any fine lines to measure there, they had more and more *parallax* (or the bright pointer line on them); until at last they seemed to roll from side to side with every movement of the eye at the eye-hole of the ocular, in a manner to defy all attempts at accuracy; and no change in the colour wheel could cure it.

After vainly trying decreased eye-holes, grayed surfaces for the wedge bearing the bright line, and every likely looking bit of blue, violet or lavender glass, from French, as well as English, makers, the following three steps were taken,—and were practically accomplished for me, I am happy to say by Mr HILGER himself, just as enthusiastically as if they had been to promote, rather than supplant, his own long approved bright-line arrangement.

The *first* step was, that to secure having a good black-line pointer for a bright field of view, a steel blade was prepared, cut to an angle of 45° at the top, and placed edgeways. It formed, with Mr HILGER's excellent workmanship in such extra-hard material, a finer, surer line than anything scratched through black varnish; and, when optical and mechanical adjustments were properly attended to, the extreme point of the blade, in the centre of the field of view, could be brought into perfect focus of both eye-piece and object-glass. It remained there too without any parallax through the whole of the brighter part of the Solar spectrum; or for as long as the pointer itself could still be seen, dark, on the darkening ends of the spectrum.

The *second* step was, that in order to turn at pleasure that dark, into a bright, pointer, the angle of the upper end of the steel, already well polished, was made to reflect to the eye the light of a lamp shining into the eye-piece from just above it. And it did reflect that light, but with the most absurd amount of parallax whenever the blue, violet or lavender regions of any spectrum were under observation. Thus if the steel, as a *black*-pointer, was placed in the middle of the violet-hydrogen's bright line, one could actually see the black point remain fixed there; while its lamp-illumined face, no matter what coloured glasses that illumining light was passed through, rolled from side to side.

The *third* step was, that in order to correct such a monstrous defect, the wheel of coloured glasses (with their chemical tints no doubt exceedingly vivid, but of *a different refrangibility* to the same eye-matched tints of the spectrum), was removed; and in its place was established a small spectroscope pivoting on the level of its slit, and so "lensed" as to throw an image of that slit, through its direct vision prism upon the steel point; while a side screw altered the angle of the little spectroscope, so as to bring one colour after another of its miniature spectrum of the illuminating lamp-flame, upon the said reflecting point.

In an instant the success of the proceeding was found practically perfect. For at any point of the spectrum of observation in the eye-piece, the observer, everything else being correct, could either make or unmake a fearful amount of parallax of the illuminated point accordingly as he illuminated it with a different, or the same, region of the illuminating spectrum. Either day-light, or lamp-light, and that last no more than a little flame of a sponge-lamp, burning only an ounce of rock oil through a whole day, will quite suffice for the illumination's origin.* While the number of spectral tints procurable thus for the point, one after the other, and by the easy plan of delicate turns to a fine screw,—proved almost a fascinating employment;—especially when the result completely meets the scientific object proposed, viz., the abolition of parallax for an illuminated reference-line in *any* part of a telescopic spectrum under examination.

Or, if there is one feature about the arrangement not yet fully perfect, it is merely that *green* light is not reflected very decidedly from steel. But that imperfection, Mr Hilger is at the present moment rectifying by making another pointer of the almost equally hard, far more anti-oxidisable, and much whiter metal, Iridium.† And so the trouble of three years has at last been successfully closed, by practically acknowledging,—that however a pigment, or a stained glass, may approach by eye-matching to a spectral tint, its *physical* action may be totally different, if it has any different prismatic refrangibility to the part of the spectrum being observed: and it may have something in that physical way very different indeed.

P. S.

Edinburgh, *Feb.* 21, 1879.

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* In my own spectroscope, this little flame being 12 inches from the illuminating arrangement has its image re-formed there by a common magnifying glass of 3 inches solar focus, placed half-way between.

† As this metal, or rather its alloy with Platinum, as Iridium-platinum, promises to fulfil a most important part in forming the "finder" and "reference-spectrum's" small mirror in front of a spectroscope's slit, as well as to serve for ruling diffraction "gratings" upon,—the following scrap of recent practical information about it, from Mr Hilger in his workshop, may perhaps usefully occupy this last corner of the page:—

"Iridium-platinum," he writes in March 1879, "is a very fine metal, but rather very dear. You may easily calculate what a small mirror of one or more inches would cost, as the ounce of it is charged thirty-five shillings!! The specific gravity is about the same as platina. A piece I bought the other day, $\frac{1}{8}$ inch thick by 1·5 inch square, rather full in size, came to £13. I almost fainted at such an awful price: still it was ordered, and I had to pay for it. So far as polish is concerned, it does get as white and bright as a silver mirror, and never tarnishes in the least. I have already polished several small mirrors of iridium-platinum, and it answers perfectly."

APPENDIX I.

COLOURED-GLASS SERIES OF SPECTRA, WITH PRISM 2;
Dispersion = 1°·2, from A to H of the Solar Spectrum.
Magnifying power of inspecting telescope = 10.

Order, Red to Violet.

December 1876, January 1878. Bar. = 29·7. Ther. = 61·0° generally. All the series taken on the foundation of the continuous spectrum of a union-jet gas (Coal-gas) burner viewed edgeways. Slit opened to $\frac{1}{10}$ space of D to E, and focussed on the D line.

COAL-GAS FLAME'S CONTINUOUS SPECTRUM.

	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
False glare from internal reflections . .	0·5	24·642	26 596	Ultra Red
True but faint black-red light . . .	0·3	24·965	30 864	Crimson Red
Strong red light seen at	3·0	25·145	34 305	Crimson Red
D slit, red side thereof	9·0	25·823	42 827	Amber
„ yellow side	9·3	25·873	43 384	Yellow
Maximum Light	**9·5**	**25·940**	**44 053**	Yellow
Green light ends, blue begins at . .	9·0	26·220	46 948	Citron
Blue Light ends, violet light begins . .	3·0	27·670	58 140	Violet
All light ends at	0·0	28·430	64 516	Lavender

ONE RED-LILAC GLASS INTERCEPTS.

First black-red light	0·1	24·913	30 581	Crimson Red
First decided red light	0·2	25·050	32 787	Crimson Red
First strong light	1·6	25·240	35 971	Red
Maximum Light	**6·7**	**25·770**	**42 194**	Amber
Red-Orange colour ends on first side of D line	6·5	25·823	42 827	Amber
Green yellow colour begins on second side of D	6·4	25·873	43 384	Yellow
Faint dark band in the green . . .	3·8	26·560	50 000	Glaucous
Darkest part of band	1·8	26·970	52 966	Glaucous
Dark band ends and violet light begins .	2·2	27·190	54 645	Glaucous
Violet light culminates	2·3	27·745	58 928	Violet
End of all light	0·0	28·280	63 291	Lavender

RED-LILAC GLASS—*continued.*

TWO RED-LILAC GLASSES INTERCEPT.

	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
First black-red light appears at	0·1	25·130	34 014	Crimson Red
First strong red light	0·7	25·226	36 062	Red
Maximum Light	**4·5**	**25·625**	**40 650**	Amber
Green shade begins	4·2	25·758	42 123	Amber
Bright green ends	2·0	26·040	45 208	Citron
Last faint green light, gray light afterwards	1·0	26·210	46 816	Citron
Middle of long darkest region . . .	0·2	26·800	51 733	Glaucous
First bluish violet	0·7	27·460	56 657	Blue
More decided violet	1·2	27·640	58 140	Violet
Maximum Violet	**1·4**	**27950**	**60 496**	Lavender
End of all light	0·0	28·300	63 452	Lavender

FOUR RED-LILAC GLASSES INTERCEPT.

	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
First faint black-red light at . . .	0·0	25·030	32 468	Crimson Red
First decided red light	0·4	25·170	34 722	Crimson Red
Maximum Light	**0·8**	**25·320**	**37 313**	Scarlet
Faint dark band	0·6	25·420	38 314	Scarlet
Second Maximum	**0·8**	**25·560**	**40 000**	Amber
All light ends	0·0	25·715	41 667	Amber

Six plates of this glass all but absolutely, and eight plates do absolutely, extinguish every trace of the lamp flame. This sort of glass, moreover, is more eminent as a darkener of all the spectrum than a transmitter of some portion in particular and a darkener of the rest. If, too, it does darken the yellow and green portions chiefly, it does so in a far more smooth and equable manner than cobalt-blue glass or blue-lilac glass. Hence Red-Lilac glass promises to be useful as sun-shades, and solar spectrum shades.

RUBY-RED GLASS.

COAL-GAS FLAME'S CONTINUOUS SPECTRUM (Repeated).

	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
False glare from internal reflections . .	0·5	24·642	26 596	Ultra Red
True but faint black-red light . . .	0·3	24·965	30 864	Crimson Red
Strong red light, seen at	3·0	25·145	34 305	Crimson Red
D slit, red side thereof	9·0	25·823	42 827	Amber
„ yellow side	9·3	25·873	43 384	Yellow
Maximum Light	**9·5**	**25·940**	**44 053**	Yellow
Green light ends, blue begins at . . .	9·0	26·220	46 948	Citron
Blue light ends, violet light begins . .	3·0	26·670	58 140	Violet
All light ends at	0·0	28·430	64 516	Lavender

RUBY-RED GLASS—*continued.*

ONE RUBY-RED GLASS INTERCEPTS.

	Intensity, Max. = 10.	*r.*	Wave-number in Brit. inch.	Colour Region by Wave-number.
False red glare at least as far	0·4	24·500	23 585	Ultra Red
First supposed true light	0·4	24·670	26 882	Ultra Red
More certain light	0·7	24·925	30 864	Crimson Red
Quite certain light	1·5	25·070	33 003	Crimson Red
Strong true red light	2·7	25·175	34 843	Crimson Red
Maximum Light	**8·2**	**25·710**	**41 667**	Amber
Yellow light begins	8·0	25·765	42 194	Amber
Green light begins	7·2	25·855	43 197	Yellow
Greenish black dark band	2·2	25·930	44 053	Yellow
Black reaches a maximum	0·7	25·975	44 563	Yellow
Black ends, and green band begins	0·8	26·155	46 189	Citron
Green band ends, and false violet, caused by red glare, begins	1·5	26·550	49 826	Green
End of all light	0·0	27·385	56 101	Blue
TWO RUBY-RED GLASSES INTERCEPT.				
False glare begins	0·1	24·550	24 814	Ultra Red
First true red light	0·1	24·803	28 986	Ultra Red
Decided red light	1·4	25·070	32 895	Crimson Red
Strong red light	4·2	25·220	35 714	Red
Maximum Light	**7·1**	**25·575**	**40 193**	Amber
Green-black begins	5·7	25·780	42 373	Amber
End of all true spectrum light	0·0	25·880	43 459	Yellow
Red false glare faintly begins	0·1	26·040	45 147	Citron
Red false faint glare ends	0·0	26·590	50 163	Glaucous
FOUR RUBY-RED GLASSES INTERCEPT.				
Red light begins at	0·1	25·015	32 206	Crimson Red
Red light decided at	0·3	25·080	33 113	Crimson Red
Very decided	1·2	25·170	34 722	Crimson Red
Full red light	2·4	25·230	35 842	Red
Maximum Light	**4·5**	**25·500**	**39 370**	Orange
Black shade begins	3·0	25·677	41 237	Amber
All light ends	0·0	25·770	42 194	Amber

Eight ruby-red glasses transmit a similar beam of red light, a little narrower, and more removed toward the ultra red, or at 37,820 wave-number.

These ruby-red glasses are therefore eminent for transmitting *one* colour, and stopping all others. And further, the colour they transmit with every increased thickness of the glass is *less* refrangible than before. Hence, while with one thickness, and even two, there is much orange and yellow light transmitted close up to D, there is only, with five thicknesses, a red band near the place of C. So very nearly indeed over it, that such glass may be usefully employed, perhaps, to facilitate solar prominence observations.

The plates of glass employed above were white glass, flashed only on the surface with the ruby material, and that often wavy and with striæ like horse hairs. Enquiry should be made if such glass can be had of pot-metal and good in optical quality. Two opticians *have* obligingly undertaken to furnish me with such sun-shades, but when their work arrived it was found not to be exactly the ruby-glass tint, nor so clear a colour.

YELLOW-BROWN GLASS.

COAL-GAS FLAME'S CONTINUOUS SPECTRUM (REPEATED).				
	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
False glare from internal reflections	0·5	24·642	26 596	Ultra Red
True but faint black-red light	0·3	24·965	30 864	Crimson Red
Strong red light seen at	3·0	25·145	34 305	Crimson Red
D slit, red side thereof	9·0	25·823	42 827	Amber
„ yellow side	9·3	25·873	43 384	Yellow
Maximum Light	**9·5**	**25·940**	**44 053**	Yellow
Green light ends, blue begins at	9·0	26·220	46 948	Citron
Blue light ends, violet light begins	3·0	27·670	58 140	Violet
All light ends at	0·0	28·430	64 516	Lavender
ONE YELLOW-BROWN (ANTI-PHOTOGRAPHIC) GLASS INTERCEPTS (t=63°).				
Red false glare in this region	0·2	24·500	23 585	Ultra Red
True red light begins	0·3	24·960	31 348	Crimson Red
Strong red light	1·2	25·090	33 422	Crimson Red
Stronger	2·1	25·170	34 722	Crimson Red
Very strong red light	4·0	25·280	36 697	Red
Chief Red light	7·2	25·730	41 806	Amber
Maximum Yellow Light	**7·4**	**25·830**	**42 974**	Amber
Green light begins	7·0	25·980	44 643	Yellow
Grass green	6·1	26·260	47 259	Green
Dark green	5·2	26·480	49 188	Green
Green-black and false violet shade	3·7	26·690	50 942	Glaucous
Blue-black colour	1·4	27·035	53 476	Glaucous
End of all true light, but not of false glare	0·5	27·390	56 101	Blue
Faint false glare ends	0·0	28·350	63 939	Lavender
TWO YELLOW-BROWN GLASSES INTERCEPT.				
False glare begins nearly	0·3	24·550	24 814	Ultra Red
First true red light	0·3	24·910	30 581	Crimson Red
Strong red light	1·0	25·040	32 680	Ultra Crimson Red
Stronger	2·2	25·198	35 248	Red
Very strong	3·4	25·270	36 364	Red
Red up to this, the Maximum Light	**6·4**	**25·820**	**42 735**	Amber
Yellow-green light begins	6·2	25·884	43 554	Yellow
Grass green	3·7	26·220	46 948	Citron
Black green	1·4	26·420	48 709	Green
End of all true light, but not of false glare	0·4	26·625	50 378	Glaucous
Ultra false glare ends	0·0	26·950	52 798	Glaucous

YELLOW-BROWN GLASS—*continued.*

FOUR YELLOW-BROWN GLASSES INTERCEPT.

	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
First red light	0·1	25·080	33 223	Crimson Red
Strong red light	0·6	25·200	35 248	Red
Very strong red light	2·0	25·330	37 313	Scarlet
Red up to this, the Maximum Light	**4·7**	**25·760**	**42 105**	Amber
Green begins	3·9	25·900	43 764	Yellow
Green black	1·6	26·100	45 767	Citron
End of all light	0·0	26·280	47 461	Green

Eight yellow-brown glasses transmit a barely visible red band, with maximum light still further towards the red end, or at 40,000 Wave N^{r}.

Hence this sort of glass transmits fairly its own band, which is on the C side of D; but it is a broader band, and fainter than the red band near C transmitted by ruby-red glass.

GREEN GLASS.

COAL-GAS FLAME'S CONTINUOUS SPECTRUM (Repeated).

	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
False glare from internal reflections	0·5	24·642	26 596	Ultra Red
True but faint black-red light	0·3	24·965	30 864	Crimson Red
Strong red light seen at	3·0	25·145	34 305	Crimson Red
D slit, red side thereof	9·0	25·823	42 827	Amber
„ yellow side	9·3	25·873	43 384	Yellow
Maximum Light	**9·5**	**25·940**	**44 053**	Yellow
Green light ends, blue begins at	9·0	26·220	46 948	Citron
Blue light ends, violet light begins	3·0	26·670	58 140	Violet
All light ends at	0·0	28·430	64 516	Lavender

ONE GREEN GLASS INTERCEPTS.

	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Faint gray false glare	0·1	25·100	33 557	Crimson Red
Strong gray glare	0·3	25·300	36 900	Red
First black-red true light	0·5	25·530	39 841	Orange
Strong black-red light	1·6	25·620	40 650	Amber
Red brown light	4·3	25·755	42 105	Amber
Maximum Light, grayish in colour	**7·2**	**25·960**	**44 444**	Yellow
Grass green light	6·0	26·350	48 054	Green
Blue shade	4·4	26·620	50 378	Glaucous
Blue black and faint violet	0·8	27·320	55 463	Blue
End of all light	0·0	27·950	60 496	Lavender

GREEN GLASS—*continued.*

TWO GREEN GLASSES INTERCEPT.

	Intensity, Max. = 10.	*r.*	Wave-number in Brit. inch.	Colour Region by Wave-number.
First brown red light	0·1	25·730	41 806	Amber
Strong brown red light	2·0	25·820	42 735	Amber
Brown red ends, gray begins . . .	4·7	25·960	44 405	Yellow
Maximum Light	**5·0**	**26·050**	**45 310**	Citron
Grass green colour	4·0	26·270	47 281	Green
Green black begins	1·6	26·570	50 000	Glaucous
End all	0·0	26·820	51 813	Glaucous
FOUR GREEN GLASSES INTERCEPT.				
First light seen	0·0	25·930	44 111	Yellow
Decided light	0·4	25·975	44 563	Yellow
End of shade	1·5	26·070	45 455	Citron
Maximum Light	**2·0**	**26·140**	**46 147**	Citron
Dark grass green begins	1·6	26·295	47 619	Green
Very dark	0·4	26·530	49 677	Green
End all	0·0	26·675	50 839	Glaucous

Eight green glasses show only the ghost of a band in or near 46,990 Wave N^{r}. Green glass is almost as much of an obscurer as transmitter; and what it transmits is a very broad band between, and also including, both D and E. This being the brightest part of the spectrum, green glass is not often wanted in Spectral Observations, to promote the transmission of that coloured light, and retard, or prevent, the transmission of all other colours.

COBALT-BLUE GLASS.

COAL-GAS FLAME'S CONTINUOUS SPECTRUM (Repeated).

	Intensity, Max. = 10.	*r.*	Wave-number in Brit. inch.	Colour Region by Wave-number.
False glare from internal reflections . .	0·5	24·642	26 596	Ultra Red
True but faint black-red light . . .	0·3	24·965	30 864	Crimson Red
Strong red light seen at	3·0	25·145	34 305	Crimson Red
D slit, red side thereof	9·0	25·823	42 827	Amber
„ yellow side	9·3	25·873	43 384	Yellow
Maximum Light	**9·5**	**25·940**	**44 053**	Yellow
Green light ends, blue begins at . .	9·0	26·220	46 948	Citron
Blue light ends, violet light begins . .	3·0	26·670	58 140	Indigo
All light ends at	0·0	28·430	64 516	Lavender

COBALT-BLUE GLASS—*continued.*

ONE COBALT-BLUE GLASS INTERCEPTS.

	Intensity, Max. = 10.	*r*.	Wave-number in Brit. inch.	Colour Region by Wave-number.
False glare begins, and the whole spectrum tends to bands	0·1	24·720	27 701	Ultra Red
First red true light begins	0·2	24·900	30 488	Crimson Red
Decided red light	0·6	24·970	31 447	Crimson Red
Maximum Red light band	**5·8**	**25·250**	**36 101**	Red
Middle of dark shade	4·2	25·460	38 986	Scarlet
Maximum *brown orange* light band	**5·2**	**25·670**	**41 186**	Amber
Middle of a darkish brown band	4·2	25·840	43 029	Yellow
Maximum of *green-yellow* light	**7·0**	**26·030**	**45 147**	Citron
Green shade begins	4·4	26·275	47 393	Green
Green ends and blue begins	4·0	26·650	50 633	Glaucous
Maximum blue light	**5·4**	**27·120**	**54 142**	Glaucous
Violet light begins	3·8	27·620	57 971	Indigo
End of all light	0·0	28·370	64 144	Lavender

The light transmitted is extravagant in length of spectrum, but distinguished by tendency to discontinuous bands chiefly in the red and yellow portions.

TWO COBALT-BLUE GLASSES INTERCEPT.

	Intensity, Max. = 10.	*r*.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins at	0·0	24·910	30 675	Crimson Red
Very decided light is seen at	0·6	25·010	32 206	Crimson Red
Maximum of Red Light	**4·1**	**25·220**	**35 587**	Red
End of red light	2·0	25·360	37 736	Scarlet
Middle of dark band	0·8	25·470	39 154	Orange
Maximum *Maroon* Light	**1·5**	**25·662**	**41 152**	Amber
Middle of black-brown shade band	0·6	25·860	43 253	Yellow
Maximum *Green-yellow* Light Band	**4·8**	**26·070**	**45 517**	Citron
Shade begins	2·5	26·170	46 468	Citron
Middle of shade	1·4	26·295	47 619	Green
End of shade	1·9	26·527	49 579	Green
Green light ends, blue begins	3·0	26·810	51 813	Glaucous
Maximum Blue Light	**4·0**	**27·080**	**53 735**	Glaucous
Violet light begins	1·7	27·720	58 824	Violet
End of all light	0·0	28·285	63 371	Lavender

COBALT-BLUE GLASS—*continued.*

FOUR COBALT-BLUE GLASSES INTERCEPT.

	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour-Region by Wave-number.
Light begins at	0·0	24·930	30 960	Crimson RED
Rises more decidedly at	1·0	25·060	32 938	Crimson RED
MAXIMUM RED LIGHT at	**2·2**	**25·175**	**34 807**	CRIMSON RED
Shade ends sharply	0·0	25·298	36 900	RED
After long dark space, faint green begins .	0·0	26·030	45 147	CITRON
Said faint green band ends	0·0	26·150	46 253	CITRON
Light begins again *Greenish-black* . .	0·0	26·545	49 826	Citron
Green-black shade ends	1·2	26·930	52 632	Glaucous
MAXIMUM BLUE LIGHT	**2·6**	**27·260**	**55 127**	BLUE
Violet light begins	1·5	27·720	58 720	VIOLET
End of all light	0·0	28·300	63 452	LAVENDER

Eight cobalt-blue glasses narrow and still farther remove apart the red and the blue bands of light, thus—

Red band culminates at 36,200 Wave N^{r}., or Between a and A, Solar.
Blue band culminates at 56,660 Wave N^{r}., or Between F and G, Solar.

BLUE-LILAC GLASS.

COAL-GAS FLAME'S CONTINUOUS SPECTRUM (REPEATED).

False glare from internal reflections . .	0·5	24·642	26 596	Ultra RED
True but faint black-red light . . .	0·3	24·965	30 864	CRIMSON RED
Strong red light, seen at	3·0	25·145	34 305	CRIMSON RED
D slit, red side thereof	9·0	25·823	42 827	AMBER
„ yellow side	9·3	25·873	43 384	YELLOW
MAXIMUM LIGHT	**9·5**	**25·940**	**44 053**	YELLOW
Green light ends, blue begins at . .	9·0	26·220	46 948	CITRON
Blue light ends, violet light begins . .	3·0	29·670	58 140	VIOLET
All light ends at	0·0	28·430	64 516	LAVENDER

ONE BLUE-LILAC GLASS INTERCEPTS.

False glare begins	0·1	24·730	27 933	Ultra RED
True black-red light begins	0·2	24·970	31 546	Crimson RED
Strong red light	1·7	25·170	34 722	Crimson RED
Full red light	3·3	25·270	36 364	RED
Red colour ends, MAXIMUM LIGHT, *Green-yellow*, begins	**7·4**	**25·900**	**43 764**	YELLOW
Grass green	5·0	26·430	48 733	GREEN
Blue begins	3·3	26·840	52 002	Glaucous
Violet begins	2·5	27·750	58 928	VIOLET
Violet colour, rich	1·0	28·230	62 854	LAVENDER
End of all light	0·0	28·397	64 392	LAVENDER

BLUE-LILAC GLASS—*continued.*

TWO BLUE-LILAC GLASSES INTERCEPT.

	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
First black-red light	0·1	24·960	31 447	Crimson Red
Strong red light	1·1	25·100	33 557	Crimson Red
Full red light	2·0	25·180	34 904	Crimson Red
Maximum Red Light	**3·3**	**25·330**	**37 313**	Scarlet
Middle of dark band	3·1	25·475	39 170	Orange
Maximum of Orange Light	**3·9**	**25·670**	**41 186**	Amber
Middle of a dark band	3·3	25·880	43 535	Yellow
Chief Maximum *Green-yellow*	**5·6**	**26·060**	**45 393**	Citron
Green edge of a very broad darkish band	3·3	26·180	46 555	Citron
Middle of said dark broad band	1·0	26·820	51 840	Glaucous
Violet edge of said dark band	1·7	27·220	54 765	Glaucous
Maximum of Violet Light	**2·3**	**27·910**	**60 168**	Lavender
End of all light	0·0	28·370	64 144	Lavender

FOUR BLUE-LILAC GLASSES INTERCEPT.

	Intensity, Max. = 10.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins at	0·0	24·975	31 646	Crimson Red
Maximum *Red light*	**2·6**	**25·260**	**36 311**	Red
Middle of dark band	1·7	25·480	39 262	Orange
Maximum *Orange light*	**2·7**	**25·660**	**41 085**	Amber
Middle of dark band	1·6	25·830	42 918	Amber
Maximum *Green light*	**2·9**	**26·015**	**44 984**	Yellow
Beginning of dark broad band	0·6	26·180	46 555	Citron
End of light	0·0	26·400	48 497	Green
Light begins again, violet in colour	0·1	27·580	57 637	Indigo
Maximum Violet Light	**1·3**	**28·060**	**61 387**	Lavender
End of all light	0·0	28·230	62 854	Lavender

Eight blue-lilac glasses show a mere ghost of a red band, with maximum in 34 360 Wave-number.

QUALITIES OF SIMPLE GLASSES.

From the above notes we may deduce for each kind of glass tried, as follows :—

Red-Lilac, is a general broad obscurer; though obscuring most over the green and blue next the yellow and violet; leaving the red near C, slightly in favour.

Wherefore this glass may be useful in dulling sun-light in the middle of the spectrum, and promoting examination of all the red region of it.

Blue-Lilac, obscures broadly the green and blue, and admits broadly, though faintly, the violet, but tends to produce narrow bands in the red and yellow. In greater thicknesses it admits a faint band of red near little *a* (Solar), and a faint, broadish band of violet between G and H ; in fact, farther towards the violet end than any other known glass.

Therefore this colour may be suitable for observations at the violet end of the spectrum.

QUALITIES OF SIMPLE GLASSES—*continued.*

Ruby-Red, is eminent, when thick, as a transmitter of one, but rather broad, band of light only ; transmitting that one too, easily, while stopping all others entirely, and the place of the band transmitted (when through five ordinary ruby-red glasses, and from a given intensity of light origin) is right on the place of the Solar C line.

Therefore this glass may perhaps be utilised in somewhat facilitating the observations of hydrogen manifestations over the surface of the Sun.

It is also a great corrector of want of achromaticity in telescopic images, but is not very agreeable to the eye.

Cobalt-Blue, is in many points the very opposite of Ruby-red, and yet transmits red, and even redder light, than the latter.

It is opposite to it in this, that it (cobalt-blue) transmits not one beam of light only, but patches of multitudes all along the spectrum ; and chiefly at either end of it : so that, when used for a sun-shade glass, it positively sets off at the worst any want of achromaticity of a telescopic image.

The colours which cobalt-blue most antagonises, and cuts up into narrow bands, are, 1st, orange-red, and then green-yellow.

The colours it most transmits are ultra red (between great A and little *a*, Solar), and then blue light (between F and G).

Therefore this glass is to be used for the faint ultra red of the spectrum ; no other sort of red-transmitting glass, showing red so far to the ultra red end. But its range of red is very limited, and it is even peculiarly inimical to the Solar C line and its neighbourhood.

Yellow-Brown, admits a rather broad beam of light on the red side of D, transmits that well, and stops others powerfully.

It is simply useful for anti-actinic purposes in photography.

Green Glass, does for the green side, what yellow-brown glass does for the red side, of the line D. It transmits rather a broad band of greenish light, transmits it well, and stops all others more and more powerfully, in proportion as removed from it in spectrum place. Hence green colour, even when slight, as in "heavy lead flint glass" for prisms, though transmitting green, of course, and even blue, *antagonises all the violet, lavender, and gray.* Hence for these parts of the spectrum, whether Solar, or chemical, whether to see the H lines of the Sun, or the violet lines of carbo-hydrogen flame, prisms should be selected of white flint glass only.

But the green of "heavy lead glass," differs remarkably touching *red* light from the green, (stained-green) glass plates above described, for it transmits much red light freely ; and hence prisms made of that heavy lead glass, with its very powerful dispersion qualities, may be employed to much advantage, in spite of their greenness, on the Red end of the spectrum.

COMBINATIONS OF COLOURED GLASSES.

No combinations of any of the above coloured glasses, transmitted light,—

(1.) Either so far towards and into the ultra red as *cobalt-blue glass ;* though that also transmits much green, blue, and violet light.

(2.) Or so nearly monochromatic, and easily or intensely, as well as so exactly over the C line, as *ruby-red glass ;* though its band is rather a broad one.

(3.) Or so far towards the violet as *blue-lilac ;* though that transmits red light as well.

COMBINATIONS OF COLOURED GLASSES—*continued.*

But—

(1′.) 2 Red + 2 Green, give a rather narrow band of *brown-orange,* useful for definition and pleasant to the eye as a sun-shade glass.

(2′.) 2 Yellow-Brown + 2 Green give a rather broad band not good for sun-shade or for correction of non-achromaticity of image, but good enough for ordinary vision; and as its centre is *nearly on the D line,* such illumination is pleasanter to the eye, and more like white light to it, than the yellow-brown alone, when used for anti-actinic purposes in ordinary muriatic, iodic and possibly some bromo-iodic, photography.

(3′.) 2 Blue + 2 Red give a rather broad band over *the place of the Solar B line.*

This combination affords a very safe, though not an agreeable, light for a photographer's dark room, when working with the most sensitive kinds of pure bromide of silver preparations. It is necessary too; for the Bromide, as Sir John Herschel showed nearly forty years ago, extends its action over "an extravagant length" of the Solar spectrum, as compared with the very limited range through which Iodide of silver is sensitive.

And—

(4′.) 4 Blue + 1 Red, where the 1 red cuts off the blue light of the blue, and does not much interfere with the ultra red of the blue, though dragging it somewhat back from the ultra red direction, as from A to *a,* Solar, gives a narrower band of peculiarly rich deep red.

This band, as to intensity of light, is weak; and it is far removed from the ordinary visual regions of the spectrum, but it must possess peculiar monochromatic, and for most chemicals anti-actinic, power; unless indeed the light be very strong, when there is always danger of the green and blue lights coming in also.

APPENDIX II.

COLOURED-FLUID SERIES OF ABSORPTION SPECTRA,

Observed with PRISM 2; DISPERSION = 1°·2, from A to H, Solar.
Magnifying power of inspecting telescope = 10.

(In Five Divisions:)

DIVISION I., RED MEDIA; DIV. II., YELLOW MEDIA; DIV. III., GREEN MEDIA; DIV. IV., BLUE MEDIA; DIV. V., VIOLET MEDIA.

DIVISION I.—RED MEDIA.

1. PERMANGANATE OF POTASH; OR POTASSIC PERMANGANATE.

Watery Solutions thereof; $\frac{1}{1000}$ to $\frac{1}{32000}$ (its colouring power being most intense), in flat glass bottles presenting films of 5 inches area, and 1 inch thick.

COAL-GAS LUMINOUS FLAME'S CONTINUOUS SPECTRUM,
as foundation for what follows.

With all this series of observations, the slit was used much narrower than with the coloured glass series; to the degree shown by the readings below for the breadth of the sodium line, and graphically exhibited in the Plates.

	Intensity, Ord. Max. = 10.	*r.*	Wave-number in Brit. inch.	Colour Region by Wave-number.
False red glare	0	24·86	29 087	Ultra Red
First true red light, ...	1	25·18	33 102	Crimson Red
Slit, by sodium light, 1st side, ...	11	26·01	43 015	YELLOW
„ „ 2d side, ...	11	26·03	43 203	YELLOW
MAXIMUM LIGHT, YELLOW	**12**	**26·03**	**43 403**	YELLOW
End of all light at violet	0	28·46	62 712	Lavender

1. POTASH PERMANGANATE, Watery Solutions—*continued.*

Solution = $\frac{1}{32000}$, pale pink in colour, when seen through 1 inch thickness by broad daylight.

	Intensity.	*r*.	Wave-number in Brit. inch.	Colour Region by Wave-number.
First reddish black light	0	25·33	35 026	Red
Decided red light	1	25·53	37 779	Scarlet
Maximum Light in Orange . . .	**8**	**25·84**	**41 374**	Amber
Ends in hazy edged dark band . . .	1	26·07	43 802	Yellow
Centre of said dark band	0	26·15	44 563	Yellow
That band ends in a fainter shade . .	2	26·21	45 147	Citron
Another dark band begins	1	26·27	45 746	Citron
Centre of said dark band	0	26·35	46 598	Citron
That band ends in a rather less dark shade	1	26·40	47 192	Green
Another dark band begins.	1	26·50	48 146	Green
Centre of said dark gray band . . .	0	26·58	48 828	Green
That band ends in a fainter shade . .	1	26·63	49 237	Green
Another darkish band begins . . .	1	26·72	50 025	Glaucous
Said greenish gray band ends . . .	2	26·84	51 099	Glaucous
A very faint dark greenish band begins .	2	26·93	51 894	Glaucous
Said band ends	3	27·06	53 079	Glaucous
Centre of a still fainter and more hazy band	3	27·18	53 996	Glaucous
Maximum Violet Light	**5**	**27·57**	**56 883**	Blue
End of all light	0	28·18	60 909	Lavender

Solution = $\frac{1}{16000}$, 1 inch thickness appears a bluish pink by daylight.

	Intensity.	1st Obser.	2d Obser.	Mean.	W. L.	Colour Region.
Red shade begins . . .	1	25·34	25·33	25·34	35 162	Red
Maximum Light, Orange .	**6**	**25·87**	**25·82**	**25·84**	**41 374**	Amber
Hazy beginning of absolute black	0	26·06	26·06	26·06	43 687	Yellow
Broad black band ends . .	0	27·06	27·08	27·07	53 135	Glaucous
Greenish lighter shade ends .	2	27·26	27·27	27·26	54 555	Glaucous
Maximum Violet Light . .	**4**	**27·63**	**27·67**	**27·65**	**57 504**	Indigo
Ends broadly and obscurely .	0	28·43	28·14	28·28	61 595	Lavender

1. POTASH PERMANGANATE, WATERY SOLUTIONS—*continued.*

Solution = $\frac{1}{8000}$, colour of 1 inch thickness by daylight, appears a darker bluish pink.

	Intensity.	1st Obser.	2d Obser.	Mean.	W. L.	Colour Region.
Red light begins	1	25·25	25·29	25·27	34 223	CRIMSON RED
MAXIMUM LIGHT, RED	**4**	**25·77**	**25·68**	**25·72**	**40 016**	AMBER
Hazy ending into black	0	26·02	26·03	26·02	43 309	YELLOW
Light begins again	0	27·32	27·30	27·31	54 885	GLAUCOUS
Greenish black shade ends	2	——	27·58	27·58	56 948	BLUE
MAXIMUM VIOLET LIGHT	**3**	**27·80**	**27·79**	**27·80**	**58 480**	VIOLET
End all	0	28·42	28·35	28·38	62 247	LAVENDER

Solution = $\frac{1}{4000}$, colour of 1 inch thickness by daylight, appears a violet pink.

Red light begins	1	25·28	25·30	25·29	34 495	CRIMSON RED
MAXIMUM LIGHT, RED	**4**	**25·63**	**25·62**	**25·62**	**38 820**	SCARLET
Hazy ending into black	0	25·97	25·99	25·98	42 882	AMBER
Light begins again	0	27·50	27·55	27·52	56 497	BLUE
MAXIMUM VIOLET LIGHT	**3**	**27·86**	**27·88**	**27·87**	**58 893**	VIOLET
Ends broadly and uncertainly	0	28·37	28·21	28·29	61 667	LAVENDER

Solution = $\frac{1}{2000}$, colour of 1 inch thickness, by daylight = violet.

Red light begins	0	25·26	25·33	25·30	34 722	CRIMSON RED
MAXIMUM LIGHT, RED	**3**	**25·40**	**25·41**	**25·40**	**36 010**	RED
Ends	0	25·56	25·52	25·54	37 879	RED SCARLET
Light begins again	0	27·72	27·80	27·76	58 241	VIOLET
MAXIMUM LIGHT, VIOLET	**2**	**27·97**	**27·97**	**27·97**	**59 524**	VIOLET
Ends	0	28·16	28·16	28·16	60 779	LAVENDER

Solution = $\frac{1}{1000}$, colour of 1 inch thickness by daylight = blue violet, and in depth = an ordinary sun-glass.

Red light begins	0	25·18			33 102	CRIMSON RED
MAXIMUM (RED) LIGHT	**2**	**25·30**			**34 626**	CRIMSON RED
Red light ends	0	25·44			36 603	RED
Violet light begins	0	27·76			58 241	VIOLET
MAXIMUM VIOLET LIGHT	**1·5**	**28·05**			**60 049**	LAVENDER
Violet light ends	0	28·25			61 380	LAVENDER

N.B.—Curious that the violet band becomes so much fainter than the red band with increased thickness of solution, when its colour to the eye, if thin, is red, and if thick is violet.

N.B.—Also, with every increased thickness, maximum of red moves more to red, and maximum of violet moves more to violet; so that at last, all the visible red band is beyond, or farther to the red, than any of the red light when thinner solution was employed.

2. JUDSON'S PEACH-BLOSSOM DYE; a blue Pink.

COAL-GAS LUMINOUS FLAME, as foundation.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
False red glare	0	24·86	29 087	Ultra Red
First true red light	1	25·18	33 102	Crimson Red
Slit, by sodium light, 1st side	11	26·01	43 015	Yellow
" " 2d side	11	26·03	43 203	Yellow
Maximum Light, Yellow	**12**	**26·03**	**43 403**	Yellow
End of all light at violet	0	28·46	62 712	Lavender
Solution pale, or $\frac{1}{3200}$.				
Red light begins	0	25·32	34 892	Crimson Red
Maximum Light, Orange	**9**	**25·77**	**40 601**	Amber
Ends in pale shade	4	25·98	42 882	Amber
Said pale shade ends	4	26·17	44 743	Yellow
Maximum Blue Light	**5**	**26·92**	**51 813**	Glaucous
End all	0	28·10	60 372	Lavender
Stronger solution, strength = $\frac{1}{800}$.				
Light begins	0	25·32	34 892	Crimson Red
Maximum Light, Red	**6**	**25·61**	**38 700**	Scarlet
Red light ends hazily in black	1	25·86	41 615	Amber
Black ends in green shade	2	26·77	50 454	Glaucous
Maximum Light, Violet	**4**	**27·60**	**57 110**	Indigo
End of all light	0	28·26	61 451	Lavender
Stronger solution, strength = $\frac{1}{200}$.				
Light begins	0	25·28	34 364	Crimson Red
Maximum Light, Red,	**4**	**25·58**	**38 344**	Scarlet
Ends	0	25·63	38 941	Scarlet
Light begins again	0	27·62	57 274	Indigo
Maximum Light, Violet	**2**	**27·92**	**59 207**	Violet
End all	0	28·28	61 595	Lavender
Stronger solution, strength = $\frac{1}{50}$. Only one band, the red one, visible.				
Light begins	0	25·27	34 223	Crimson Red
Maximum light, Red	**2**	**25·50**	**37 411**	Scarlet
End all	0	25·60	38 580	Scarlet

3. Judson's Dyes, MARONE RED.

COAL-GAS LUMINOUS FLAME, as foundation.	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
False red glare	0	24·86	29 087	Ultra Red
First true red light	1	25·18	33 102	Crimson Red
Slit, by sodium light, 1st side	11	26·01	43 015	Yellow
,, ,, 2d side	11	26·03	43 203	Yellow
Maximum Light, Yellow	**12**	**26·03**	**43 403**	Yellow
End of all light at violet	0	28·46	65 712	Lavender
Solution of strength = $\frac{1}{800}$. Admits only one broad, red to yellow, band.				
Light begins	0	25·34	35 162	Red
Maximum Light	**8**	**25·82**	**41 152**	Amber
Ends	0	26·13	44 366	Yellow
Solution of strength = $\frac{1}{200}$. Shows one red band only.				
Light begins	0	25·30	34 626	Crimson Red
Maximum Light	**6**	**25·72**	**40 016**	Amber
End all	0	25·95	42 571	Amber
Solution of strength = $\frac{1}{50}$.				
Light begins	0	25·33	35 026	Red
Maximum Light	**4**	**25·62**	**38 820**	Scarlet
End all	0	25·78	40 717	Amber

4. Judson's CARDINAL RED.

Like the above, but rather yellower.

5. Judson's Dyes—MAGENTA RED.

COAL-GAS LUMINOUS FLAME, as foundation.	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
False red glare	0	24·86	29 087	Ultra Red
First true red light	1	25·18	33 102	Crimson Red
Slit, by sodium light, 1st side	11	26·01	43 015	Yellow
,, ,, 2d side	11	26·03	43 203	Yellow
Maximum Light, Yellow	**12**	**26·03**	**43 403**	Yellow
End of all light at violet	0	28·46	62 712	Lavender

5. MAGENTA RED—*continued.*

Solution of strength = $\frac{1}{800}$. One strong red, and one weak violet band.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·35	35 298	RED
MAXIMUM RED	**6**	**25·80**	**40 933**	AMBER
Ends.	0	26·06	43 687	YELLOW
Begins again	0	27·77	58 309	VIOLET
FAINT VIOLET MAXIMUM	**1**	**28·10**	**60 372**	LAVENDER
End all	0	28·30	61 728	LAVENDER

Solution of strength = $\frac{1}{200}$.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·34	35 162	RED
MAXIMUM RED	**4**	**25·70**	**39 777**	ORANGE
End all	0	25·93	42 373	AMBER

Solution of strength = $\frac{1}{50}$. This red is most like the ruby-red glass, but still clearer.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·32	34 892	CRIMSON RED
MAXIMUM LIGHT, RED	**3**	**25·56**	**38 124**	SCARLET
End all	0	25·75	40 388	AMBER

6. Judson's PUCE OR PLUM COLOUR.

A not very red, but decidedly impure, red-purple, or red-lilac.

COAL-GAS LUMINOUS FLAME, as foundation.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
False red glare	0	24·86	29 087	Ultra RED
First true red light	1	25·18	33 102	CRIMSON RED
Slit, by sodium light, 1st side	11	26·01	43 015	Yellow
„ „ 2d side	11	26·03	43 203	Yellow
MAXIMUM LIGHT, YELLOW	**12**	**26·03**	**43 403**	Yellow
End of all light at violet	0	28·46	62 712	Lavender Gray

6. PUCE or PLUM COLOUR—*continued.*

Solution = $\frac{1}{800}$ strength.

	Intensity.	*r.*	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·40	36 010	Red
Maximum Light in yellow	**8**	**26·08**	**43 879**	Yellow
Middle of a dark band	2	26·37	46 816	Citron
Middle of a bright green band . . .	4	26·62	49 164	Green
Beginning of broad but dark blue band .	2	26·97	52 274	Glaucous
End all	0	28·20	61 039	Lavender

Solution = $\frac{1}{200}$ strength; shows one red-orange band only.

	Intensity.	*r.*	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·42	36 311	Red
Maximum	**4**	**25·80**	**40 933**	Amber
End all	0	26·10	44 072	Yellow

Solution = $\frac{1}{50}$ strength; shows nothing; proving the colour to be a better obscurer than transmitter.

7. Judson's CRIMSON, similar to MAGENTA generally.

COAL-GAS LUMINOUS FLAME as foundation.

	Intensity.	*r.*	Wave-number in Brit. inch.	Colour Region by Wave-number.
False red glare	0	24·86	29 087	Ultra Red
First true red light	1	25·18	33 102	Crimson Red
Slit, by sodium light, 1st side	11	26·01	43 015	Yellow
,, ,, 2d side	11	26·03	43 203	Yellow
Maximum Light, Yellow	**12**	**26·03**	**43 403**	Yellow
End of all light at violet	0	28·46	62 712	Lavender

Solution = $\frac{1}{800}$ strength. Shows only one red and orange band.

	Intensity.	*r.*	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·32	34 892	Crimson Red
Maximum Light	**7**	**25·82**	**41 152**	Amber
Light ends	0	26·09	43 975	Yellow

7. Judson's CRIMSON—*continued.*

Solution = $\frac{1}{200}$ strength. Shows one red band only.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·35	35 298	Red
Maximum Light	**4**	**25·67**	**39 417**	Orange
Light ends	0	25·92	42 265	Amber

Solution = $\frac{1}{50}$ strength.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·37	35 575	Red
Maximum Light	**2**	**25·55**	**37 994**	Scarlet
Light ends	0	25·68	39 526	Orange

8. Judson's RUBY.

COAL-GAS LUMINOUS FLAME as foundation.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
False red glare	0	24·86	29 087	Ultra Red
First true red light	1	25·18	33 102	Crimson Red
Slit, by sodium light, 1st side . .	11	26·01	43 015	Yellow
„ „ 2d side . .	11	26·03	43 203	Yellow
Maximum Light, Yellow	**12**	**26·03**	**43 403**	Yellow
End of all light at violet	0	28·46	62 712	Lavender

Solution = strength of $\frac{1}{800}$. Shows dark bands in citron and blue-green, but bright regions in the red and violet.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·35	35 298	Red
Maximum, Orange Light	**8**	**26·05**	**43 592**	Yellow
Beginning of dark band	3	26·22	45 249	Citron
Centre of dark band	1	26·32	46 275	Citron
End of dark band	3	26·47	47 824	Green
Centre of bright-green band	5	26·67	49 579	Green
Green ends, blue begins	4	27·00	52 549	Glaucous
End all	0	28·38	62 251	Lavender

8. Judson's RUBY—*continued.*

Solution $=\frac{1}{200}$ strength. Shows one bright red-orange band, and one do. violet band.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·32	34 892	Crimson Red
Maximum, Orange Light	**6**	**25·80**	**40 933**	Amber
Ends in black	0	26·12	44 267	Yellow
Begins again	0	27·58	56 948	Blue
Maximum, Violet	**2**	**27·87**	**58 893**	Violet
End all	0	28·31	61 805	Lavender
Solution $=\frac{1}{50}$ strength. Shows one red band only.				
Light begins	0	25·28	34 364	Crimson Red
Maximum Light	**4**	**25·68**	**39 526**	Orange
Ends	0	25·95	42 571	Amber

9. Judson's CLARET-RED.

Very like the above Ruby-Red.

10. Judson's PONCEAU, or POPPY-RED.

Also very like the above Ruby-Red, but with the red band rather more to the red, and the blue band fainter.

11. Judson's MAUVE-RED.

A strong colourer.

COAL-GAS LUMINOUS FLAME, as foundation.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
False red glare	0	24·86	29 087	Ultra Red
First true red light	1	25·18	33 102	Crimson Red
Slit, by sodium light, 1st side	11	26·01	43 015	Yellow
„ „ 2d side	11	26·03	43 203	Yellow
Maximum Light, Yellow	**12**	**26·03**	**43 403**	Yellow
End of all light at violet	0	28·46	62 712	LavenderGray

11. Judson's MAUVE-RED—*continued.*

Solution = $\frac{1}{3200}$ strength. Slightly dulls out the yellow, but leaves all the other colours nearly pure.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·32	34 892	Crimson Red
Maximum Red Light	**6**	**25·65**	**39 170**	Orange
Ends	0	25·89	41 964	Amber
Light begins again	0	26·94	52 002	Glaucous
Maximum Violet Light	**4**	**27·63**	**57 339**	Indigo
End all	0	28·43	62 559	Lavender

Solution = $\frac{1}{200}$ strength.

Light begins	0	25·32	34 892	Crimson Red
Maximum Red Light	**3**	**25·57**	**38 226**	Scarlet
Ends	0	25·72	40 016	Amber
Ghost of a violet band	1	28·05	60 049	Lavender

Solution = $\frac{1}{50}$ strength.

Light begins	0	25·30	34 626	Crimson Red
Maximum Red Light	**2**	**25·50**	**37 411**	Scarlet
Ends	0	25·58	38 344	Scarlet

12. CLARET WINE=RUBY RED.

Shows one narrow red band.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·26	34 095	Crimson Red
Maximum Light	**3**	**25·50**	**37 411**	Scarlet
End all	0	25·69	39 651	Orange

13. LISBON PORT-WINE = Reddish Brown.

Shows one band, from red to yellow, dully.

	Intensity.	*r*.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·39	35 868	Red
Maximum Light	**4**	**25·72**	**40 016**	Amber
End all	0	26·06	43 687	Yellow

14. PORT-WINE last from LEITH.

Shows one red band only; but that very red.

Light begins	0	25·31	34 758	Crimson Red
Maximum Light	**3**	**25·59**	**38 462**	Scarlet
End all	0	25·81	41 034	Amber

15. RED MARSALA.

Like the port, but paler.

Light begins	0	25·26	34 095	Crimson Red
Maximum Light	**4**	**25·70**	**39 777**	Orange
End all	0	26·09	43 975	Yellow

16. LISBON COLLARES.

Shows intense red band.

Light begins	0	25·29	34 495	Crimson Red
Maximum Light	**3**	**25·54**	**37 879**	Scarlet
End all	0	25·72	40 016	Amber

The above, when diluted with water, were still mono-chroic, though the band of each became broader, and included more or less citron and green, as well as red.

DIVISION II.—YELLOW MEDIA.

1. IODINE.

Tincture thereof diluted in water, March 9, 1878.

COAL-GAS LUMINOUS FLAME as foundation.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
False red glare	0	24·86	29 087	Ultra Red
First true red light	1	25·18	33 102	Crimson Red
Slit by sodium light, 1st side	11	26·01	43 015	Yellow
" " 2d side	11	26·03	43 203	Yellow
Maximum Light, Yellow	**12**	**26·03**	**43 403**	Yellow
End of all light at violet	0	28·46	62 712	Laven. Gray
Solution of $\frac{1}{800}$ strength, shows only one band, stretching from red to citron, dully.				
Light begins	0	25·31	34 758	Crimson Red
Maximum Light	**5**	**25·99**	**42 992**	Amber
End all	0	26·44	47 529	Green
Solution of $\frac{1}{400}$ strength.				
Light begins	0	25·30	34 626	Crimson Red
Maximum Light	**4**	**25·68**	**39 526**	Orange
End all	0	26·04	43 497	Yellow

2. BICHROMATE OF POTASH.—March 12, t = 660.

Solution in hot water; very like iodine, but brighter and yellower; and giving lighter and narrower band in spectrum.

Solution = $\frac{1}{48}$.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·30	34 626	Crimson Red
Maximum Light and salt line	**10**	**26·00**	**43 103**	Yellow
End all	0	26·38	46 948	Citron

2. BICHROMATE OF POTASH—*continued.*

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Solution = strength of $\frac{1}{24}$.				
Light begins	0	25·29	34 495	Crimson Red
Maximum Light	**8**	**25·96**	**42 680**	Amber
End all	0	26·28	45 851	Citron
Solution = strength of $\frac{1}{12}$, or close to saturation.				
Light begins	0	25·28	34 364	Crimson Red
Maximum Light	**6**	**25·94**	**42 481**	Amber
End all	0	26·23	45 331	Citron

This solution better suited than any other yet tried for achromatising simple objectives, one colour, its own yellow, excepted.

New and transparent orange bichromate solution for galvanic battery shows no black *lines*, only one broad band of light from red to yellow and citron.

Old, dark, greenish-brown bichromate solution, nearly used up in a battery, shows one reddish band, placed thus:—

r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
25·38	35 714	Red
25·50	37 411	Scarlet
25·67	39 417	Orange

and in this band is a hazy attempt at a dark line in—

r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
25·52	37 651	Scarlet

N.B.—This is not the clear sharp line seen in oxalate of chromium and potash.

3. GAMBOGE.

Primrose yellow in solution, but opaque.

4. TEA.

Tried in hot solution, gave one band in orange and citron, but dull; and when cold was nearly opaque.

DIVISION III.—GREEN MEDIA.

1. SULPHATE COPPER.

A faint colouring material, its $\frac{1}{5}$ giving a tint only as deep as $\frac{1}{160}$ of amm. sulph. copp.

It is an impure blue, destroying the red, no doubt, but dulling the violet and favouring the citron far too much.

COAL-GAS LUMINOUS FLAME as foundation.	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
False red glare	0	24·86	29 087	Ultra Red
First true red light	1	25·18	33 102	Crimson Red
Slit, by sodium light, 1st side	11	26·01	43 015	Yellow
„ „ 2d side	11	26·03	43 203	Yellow
Maximum Light, Yellow	**12**	**26·03**	**43 403**	Yellow
End of all light at violet	0	28·46	62 712	Lavender
Solution=$\frac{1}{20}$. Shortens the red, and dulls the orange slightly.				
Red light begins	0	25·58	38 344	Scarlet
Maximum Light, yellowish	**4**	**26·09**	**43 975**	Yellow
Green ends, blue begins	4	26·77	50 454	Glaucous
End all	0	28·12	60 518	Lavender
Solution=$\frac{1}{10}$. Shortens the red much, and dulls the orange and yellow.				
Red light begins	0	25·72	40 016	Amber
Maximum Light	**5**	**26·20**	**45 045**	Citron
Green ends, blue begins	3	26·77	50 454	Glaucous
End all	0	28·17	60 846	Lavender
Solution=$\frac{1}{5}$. Red reduced to merely a red-brown edging on left of green.				
Light brown-red begins	0	25·92	42 265	Amber
Maximum Light	**3**	**26·30**	**46 062**	Citron
Green ends, blue begins	2	26·85	51 177	Glaucous
End all	0	28·12	60 518	Lavender

This tint is so weak as to be next to useless as a colour; but should be preserved for comparison and contrast.

2. Judson's GREEN.

A decidedly blue green, generally, or in thin solutions; but in thick solutions looking red by transmitted light.

Solution $=\frac{1}{800}$ strength. Green is largely admitted, and next red. Yellow and violet are knocked out.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·32	34 892	Crimson Red
Maximum Red Light	**5**	**25·60**	**38 580**	Scarlet
Ends in Brown band	3	25·77	40 601	Amber
Middle of Brown band	2	25·92	42 265	Amber
End of Brown band	3	26·12	44 267	Yellow
Maximum Green Light	**6**	**26·42**	**47 393**	Green
End all	0	27·50	56 370	Blue

Solution $=\frac{1}{200}$ strength. Shows one true red band, and one green band only.

Light begins	0	25·31	34 758	Crimson Red
Maximum Red Light	**3**	**25·49**	**37 286**	Scarlet
Ends	0	25·64	39 047	Orange
Begins again	0	26·33	46 382	Citron
Maximum Green Light	**3**	**26·67**	**49 579**	Green
End all	0	27·31	54 885	Glaucous

Solution $=\frac{1}{100}$ strength. Shows one red band, and one green band clear and distinct. The bottle full, by transmitted light, makes gaslight look coppery; daylight greenish.

Light begins	0	25·29	34 495	Crimson Red
Maximum Red Light	**3**	**25·45**	**36 765**	Red
Ends	0	25·55	37 994	Scarlet
Begins again	0	26·60	48 972	Green
Maximum Green Light	**1**	**26·82**	**50 916**	Glaucous
End all	0	27·12	53 562	Glaucous

Solution $=\frac{1}{50}$ strength. Shows one red band only. Fluid ruby-red by transmitted gaslight, coppery by daylight.

Light begins	0	25·27	34 223	Crimson Red
Maximum Red Light	**2**	**25·38**	**35 714**	Red
Ends	0	25·50	37 411	Scarlet
Ghost of a green band	0	26·88	51 467	Glaucous

2. Judson's GREEN—*continued.*

After this, tried 1 to 4 of the usual green (yellow-green) glasses, but found them without any trace of a separate red band. Next tried Sol.$=10x$, in bottle on landscape, and, lo! the grass-greens were red; as red, or brighter red, than any of the red bricks and tiles near at hand! Stone colour and road colour remained much as before. Venetian shutters, *painted* green, were nearly black. The vermilion of grass came out best when Sun shone on it, and showed it, to the naked eye, *most* green and most distinct from red tile and red brick. Here seems at once a short cut to colour blindness; *viz.*, an eye lens or fluid as *di-chroic* as this blue-green!

DIVISION IV.—BLUE MEDIA.

1. COPPER AMMONIA-CHLORIDE, Water Solution.

Solution tried $\frac{1}{320}$, $\frac{1}{160}$, $\frac{1}{80}$, $\frac{1}{40}$, $\frac{1}{20}$, and $\frac{1}{10}$.

Solutions of below the strength of $\frac{1}{40}$, become milky, and may have to stand for a day or two to let some gray matter subside; that done, and the colours being noted by eye, by daylight transmitted through 1 inch thickness, $\frac{1}{320}$ has only the faintest tinge of blue; $\frac{1}{160}$=light blue, $\frac{1}{80}$ = blue, $\frac{1}{40}$ = intense blue, $\frac{1}{20}$ = slightly violet blue; and $\frac{1}{10}$ = also slightly violet blue.

COAL-GAS LUMINOUS FLAME, as foundation.

	Intensity.	*r*.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
First red glare	0	24 86	29 087	Ultra Red
First true red light	1	25·18	33 102	Crimson Red
Slit, by sodium light, 1st side	11	26·01	43 015	Yellow
„ „ 2d side	11	26·03	43 203	Yellow
Maximum Light, Yellow	**12**	**26·03**	**43 403**	Yellow
End of all light at violet	0	28·46	62 712	Lavender

Solution $\frac{1}{320}$. Though the colour is of the faintest, it has a most remarkable effect in dulling the over-light yellow region of the spectrum; and doing it broadly and smoothly, without any of the *bands* which cobalt-blue glass has.

First faint beginning	0	25·26	34 095	Crimson Red
Decided light	1	25·36	35 436	Red
Maximum Light (in Green)	**7**	**26·33**	**46 382**	Citron
Green ends, blue begins	5	26·90	51 653	Glaucous
Blue ends, violet begins	3	27·75	58 173	Violet
End all	0	28·42	62 500	Lavender

1. COPPER AMMONIA-CHLORIDE—*continued.*

Solution $\frac{1}{160}$. More decided dulling of red and orange, then of yellow and citron, but without perceptible band; and reduction of spectrum to nearly three colours only, red, green, a little blue, and then violet.

	Intensity.	*r*.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Red light begins	0	25·41	36 166	RED
Red ends in green	2	26·08	43 879	YELLOW
MAXIMUM LIGHT in GREEN	**4**	**26·59**	**48 900**	GREEN
Green ends in violet	3	27·02	52 715	GLAUCOUS
End all	0	28·30	61 728	LAVENDER

Solution = $\frac{1}{80}$. Red, orange, and yellow all gone; a portion of green, the blue and violet left.

Light begins	0	26·43	47 461	GREEN
Green ends in violet	3	27·11	53 476	GLAUCOUS
MAXIMUM LIGHT	**4**	**27·30**	**54 825**	GLAUCOUS
End all	0	28·30	61 728	LAVENDER

Solution = $\frac{1}{40}$. One violet-blue band only, with its anterior edge greenish.

Begins green-black	0	27·11	53 476	GLAUCOUS
MAXIMUM LIGHT	**3**	**27·50**	**56 370**	BLUE
End all	0	27·97	59 524	VIOLET

Solution = $\frac{1}{20}$. One violet band with first edge inclining to green-black.

Begins green black	0	27·33	55 036	BLUE
MAXIMUM LIGHT	**2**	**27·74**	**58 106**	VIOLET
End all	0	28·22	61 173	LAVENDER

Solution = $\frac{1}{10}$.

Begins	0	27·70	57 837	INDIGO
MAXIMUM	**1**	**27·90**	**59 067**	VIOLET
Ends	0	28·10	60 372	LAVENDER

2. Judson's Dyes, CAMBRIDGE BLUE.

Solution of strength = $\frac{1}{800}$.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·32	34 892	Crimson Red
Maximum Red Light	**6**	**25·60**	**38 580**	Scarlet
Ends in dark haze band	1	25·85	41 494	Amber
End of that haze band darkish . . .	2	26·15	44 563	Yellow
Maximum Green Light	**4**	**26·64**	**49 334**	Green
End all in violet	0	28·15	60 709	Lavender

Solution of strength = $\frac{1}{200}$. One narrow red band, and one broad blue band.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·30	34 626	Crimson Red
Maximum Red Light	**2**	**25·41**	**36 166**	Red
Ends	0	25·51	37 538	Scarlet
Light begins again broadly and hazily .	0	27·00	52 549	Glaucous
Maximum Blue Light	**2**	**27·55**	**56 722**	Blue
End all	0	28·08	60 248	Lavender

Solution of strength = $\frac{1}{50}$. Shows a faint blue band only.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	27·33	55 036	Blue
Maximum Light Blue	**1**	**27·59**	**57 013**	Indigo
End all	0	27·90	59 067	Violet

3. OXFORD BLUE.

Solution of strength = $\frac{1}{800}$. Brightens the red, dulls the yellow broadly, and admits green, blue, and violet.

Solution of strength = $\frac{1}{200}$.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·31	34 758	Crimson Red
Maximum Light Red	**3**	**25·49**	**37 286**	Scarlet
Ends	0	25·65	39 170	Orange
Light begins again	0	26·55	48 567	Green
Maximum Light Violet	**3**	**27·18**	**53 996**	Glaucous
End all	0	28·22	61 173	Lavender

3. OXFORD BLUE—*continued.*

Solution of strength = $\frac{1}{60}$. This red of the Oxford blue is farther to the red than that of the Cambridge blue.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·19	33 223	Crimson Red
Maximum Red Light	**1**	**25·31**	**34 758**	Crimson Red
Ends	0	25·43	36 456	Red
Light begins again	0	27·19	54 083	Glaucous
Maximum Violet	**2**	**27·58**	**56 948**	Blue
End all	0	28·08	60 248	Lavender

4. LITMUS.

(9th March 1878.)

A dark granulated solid, dissolving easily in water; blue in small depths, verging to red for greater, in transmitted light.

Solution $\frac{1}{10000}$; is a light blue liquid, brightening the red, and dulling yellow and violet of Spectrum.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·28	34 364	Crimson Red
Maximum Red Light	**7**	**25·62**	**38 820**	Scarlet
Beginning of faint dulness	4	25·81	41 034	Amber
Brightens	4	26·12	44 267	Yellow
Maximum Green light	**5**	**26·41**	**47 304**	Green
Maximum Blue	**3**	**27·15**	**53 763**	Glaucous
End all	0	28·04	59 988	Violet

Solution = $\frac{1}{2500}$. Shows one burning red broad band, next a black band over yellow and citron; then a bright green band, and finally a faint blue band.

Light begins	0	25·30	34 626	Crimson Red
Maximum Red Light	**5**	**25·58**	**38 344**	Scarlet
Light ends	0	25·75	40 388	Amber
Begins again	1	26·18	44 843	Yellow
Maximum Green	**4**	**26·60**	**48 972**	Green
Darker and blue	2	26·90	51 653	Glaucous
End all	0	27·96	59 453	Violet

4. LITMUS—*continued.*

Solution = $\frac{1}{600}$ Shows only one narrow red band.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·34	35 162	RED
MAXIMUM RED LIGHT	**3**	**25·53**	**37 779**	SCARLET
End all	0	25·61	38 700	SCARLET

Solution $\frac{1}{150}$, blocks all light out of spectrum.

Solution $\frac{1}{2500}$, does not redden green, but does redden Naples yellow, cadmium yellow, and other yellows.

Stronger solutions redden grass-greens, but not very brilliantly.

DIVISION V.—VIOLET MEDIA.

1. OXALATE OF CHROMIUM AND POTASSIUM.

A dark *lilac* looking powdery, lumpy matter, not easily dissolved in water; and having, by reflected light in solution, a dull inky blue-black colour. In the weaker solutions it shows by transmitted light a slatey-blue; but in stronger solutions a rich red colour. It is thus just the opposite of Permanganate of Potash, which is pink in weak solutions, and violet-blue in strong ones.

Solutions	$\frac{1}{640}$	$\frac{1}{320}$	$\frac{1}{160}$	$\frac{1}{80}$	$\frac{1}{40}$	$\frac{1}{20}$	
Appear	palest possible slate colour.	pale slate blue.	slate blue with pink tendency.	pinkish blue.	red-blue.	red.	By broadly transmitted daylight.

COAL-GAS LUMINOUS FLAME, as foundation.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
First red glare	0	24·86	29 087	Ultra RED
First true red light	1	25·18	33 102	CRIMSON RED
Slit, by sodium light, 1st side	11	26·01	43 015	Yellow
„ „ 2d side	11	26·03	43 203	Yellow
MAXIMUM LIGHT, YELLOW	**12**	**26·03**	**43 403**	YELLOW
End of all light at violet	0	28·46	62 712	Lavender

1. OXALATE OF CHROMIUM AND POTASSIUM—*continued.*

Solution $\frac{1}{640}$ tried; though so weak in colour to the eye, it notably dulls the green—cuts off the violet and the yellow, leaving the red pure and bright.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Red black light begins	0	25·27	34 223	Crimson Red
Maximum brightest Red light . . .	**6**	**25·70**	**39 777**	Orange
Red light ends, green begins . . .	5	26·07	43 802	Yellow
Green ends, blue begins	3	26·85	51 177	Glaucous
End all	0	27·73	58 038	Violet

Solution $\frac{1}{320}$. Though colour is so pale, yet the spectrum is very strongly banded.

Red light begins	0	25·26	34 095	Crimson Red
Maximum Red Light	**4**	**25·65**	**39 170**	Orange
Red light ends in dark haze . . .	2	26·02	43 309	Yellow
Middle of dark haze band	1	26·22	45 249	Citron
End of dark band	2	26·46	47 710	Green
Green light ends, and blue begins . .	3	26·85	51 177	Glaucous
End all	0	27·38	55 463	Blue

Solution = $\frac{1}{160}$. A fine line clearly seen in the red band: a red band and a green-blue band are now the only spectral lights.

Red light begins	0	25·30	34 626	Crimson Red
Fine black line!!!	1	25·42	36 311	Red
Maximum Red Light	**4**	**25·60**	**38 344**	Scarlet
Shady ending of red in black light . .	2	25·76	40 502	Amber
Sodium line, made in gas flame . . .	2	26·00	43 103	Yellow
Chief blackness of dark band . . .	0	26·17	44 743	Yellow
Shady ending of dark band . . .	1	26·53	48 379	Green
Maximum of green-blue light . . .	3	26·92	51 813	Glaucous
End all	0	27·25	54 496	Glaucous

Solution $\frac{1}{80}$. The line in the red is now very black, as well as neat and sharp, while the spectrum shows only as to light a red band and a fainter green-blue band.

Red light faintly begins	0	25·27	34 223	Crimson Red
Strong black spectral light . . .	3	25·42	36 311	Red
Maximum Red Light	**4**	**25·46**	**36 928**	Red
Red light ends in hazy black . . .	1	25·65	39 170	Orange
Light begins again hazily and greenly .	0	26·73	50 100	Glaucous
Maximum of such faint light . . .	**2**	**26·90**	**51 653**	Glaucous
End all	0	27·09	53 305	Glaucous

1. OXALATE OF CHROMIUM AND POTASSIUM—*continued.*

Solution $\frac{1}{40}$. The fixed, black line in the red band is now exceedingly intense, and is in the middle of light of said band.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins at	0	25·24	33 830	Crimson Red
The black, fixed line	3	25·42	36 311	Red
Maximum Light Region also . . .	**3**	**25·42**	**36 311**	Red
Red light ends, hazily	0	25·56	38 124	Scarlet
Ghost of a narrow green band . . .	0	26·97	52 274	Glaucous

Solution $\frac{1}{20}$. Fixed line very black, symptoms of other lines on right-hand side.

	Intensity.	r.	Wave-number in Brit. Inch.	Colour Region by Wave-number.
Light begins	0	25·22	33 591	Crimson Red
Maximum Red Light	**2**	**25·35**	**35 298**	Red
Black, fixed line	...	25·42	36 311	Red
Light line	0	25·53	37 779	Scarlet

The black, fixed line in the red, is the chief feature here of Oxalate of Chromium and Potash. It is a true spectrum analysis line, constant in spectrum place by the above measures. And it assists the measures again, in showing that luminous bands are totally different things; for they shift in place, both as tested by the measures and by this line, with the strength of the solution. This line, or lines, to be examined again with Prism 3 or 4. It was subsequently so examined and found to be a bundle of finer lines.

2. Judson's Dyes—VIOLET.

A very strong colour, staining all the glass vessels sadly. This colour is antipathic to yellow chiefly; knocking it out by a narrow dark band; even when the solution is very weak.

Solution of strength = $\frac{1}{1600}$: offers 1 narrow luminous red band and 1 blue and violet band, but knocks out orange, yellow, and citron.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·32	34 892	Crimson Red
Maximum Red Light	**4**	**25·62**	**38 820**	Scarlet
Ends	0	25·82	41 152	Amber
Begins again	0	26·77	50 454	Glaucous
Maximum light Violet	**3**	**27·58**	**56 948**	Blue
End all	0	28·25	61 380	Lavender

2. Judson's Dyes—VIOLET—*continued.*

Solution of strength = $\frac{1}{400}$: shows 1 red band and 1 violet; each pure, strong and distinct.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25.28	34 364	Crimson Red
Maximum Red Light	**3**	**25·55**	**37 994**	Scarlet
Ends	0	25·70	39 777	Orange
Begins again	0	27·28	54 705	Glaucous
Maximum Violet	**2**	**25·73**	**58 038**	Violet
End all	0	28·33	61 935	Lavender

Solution of strength = $\frac{1}{100}$.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·33	35 026	Red
Maximum Red Light	**2**	**25·47**	**37 064**	Scarlet
Ends	0	25·57	38 226	Scarlet
A separate faint band seen here . .	0	28·15	60 709	Lavender

Solution of strength = $\frac{1}{25}$. Its red band is remarkably red and pure.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·28	34 364	Crimson Red
Maximum Red Light	**2**	**25·39**	**35 868**	Red
End all	0	25·47	37 064	Scarlet

3. Judson's Dyes—LAVENDER.

A dark strong colour in itself by daylight; and in the spectrum, more of an obscurer of all rays, than a transmitter of some.

Solution of strength = $\frac{1}{800}$: produces broad dulling of the yellow, resulting in a long faintly, but nearly evenly, illuminated triple spectrum of red, green and blue.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·45	36 765	Red
Maximum Red Light	**4**	**25·83**	**41 271**	Amber
Fainter Maximum Green	3	26·46	47 710	Green
Still fainter Maximum Blue	2	27·53	56 561	Blue
End all	0	28·10	60 372	Lavender

3. Judson's Dyes—LAVENDER—*continued.*

Solution of strength = $\frac{1}{400}$: is eminent as a duller of yellow, orange, red and citron.

	Intensity.	*r.*	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·43	36 456	Red
Faint Maximum Red Light	**2**	**25·68**	**39 526**	Orange
Red light ends in a dark band	1	25·97	42 790	Amber
Green light begins after a dark band	1	26·33	46 382	Citron
Faint Green Maximum	2	26·67	49 579	Green
Green ends, blue begins	1	27·15	53 763	Glaucous
End all	0	28·03	59 923	Violet

Solution of strength = $\frac{1}{200}$, excludes all light except a ghost of violet band.

4. Judson's Dyes—PURPLE.

A rich, *i.e.* a lakey, blue. An intense colour.

Solution of $\frac{1}{3200}$ strength. Extinguishes yellow completely, citron almost, brightens the red, and the blue and violet.

	Intensity.	*r.*	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·35	35 298	Red
Maximum Red Light	**5**	**25·75**	**40 388**	Amber
Ends in a smoky band	1	25·92	42 265	Amber
End of that band	1	26·16	44 643	Yellow
Maximum Green light	3	26·75	50 276	Glaucous
Maximum Blue Light	3	27·39	55 525	Blue
End all	0	28·26	61 451	Lavender

Solution = $\frac{1}{800}$ strength. Shows one strong red band and one strong blue one.

Light begins	0	25·33	35 026	Red
Maximum Red Light	**4**	**25·63**	**38 941**	Scarlet
Ends	0	25·81	41 034	Amber
Begins again in green blue	0	26·97	52 274	Glaucous
Maximum Blue Light	**3**	**27·58**	**56 948**	Blue
End all	0	28·38	62 251	Lavender

4. Judson's Dyes—PURPLE—*continued.*

Solution $=\frac{1}{200}$ strength.

	Intensity.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	25·30	34 626	Crimson Red
Maximum Red Light	**2**	**25·52**	**37 651**	Scarlet
Light ends	0	25·67	39 417	Orange
Light begins again	0	27·46	56 054	Blue
Faint Maximum Blue	**1**	**28·00**	**59 701**	Violet
End all	0	28·33	61 935	Lavender
Solution $=\frac{1}{50}$ strength. Shows one red band only.				
Light begins	0	25·26	34 095	Crimson Red
Maximum Red Light	**1**	**25·43**	**36 456**	Red
End all	0	25·53	37 779	Scarlet

5. DIDYMIUM NITRATE.

Solution $=\frac{1}{20}$. Like water; nearly colourless, or with only a pale pinkish hue; yet makes many notable black lines and bands in spectrum.

	Intensity. Light Lines.	Intensity. Dark Lines.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Light begins	0	...	25·01	30 883	Crimson Red
Dark band 1st side	...	...	25·24	33 830	Crimson Red
2d side	...	3	25·30	34 626	Crimson Red
Light near this line	1	...			
Maximum Red Light	**5**	...	**25·85**	**41 494**	Amber
Salt line, from illuminating flame .	...	...	26·00	43 103	Yellow
Faint shade begins	...	1	26·00	43 103	Yellow
Stronger shade begins	...	3	26·06	43 687	Yellow
Central strong black line . . .	...	5	26·10	44 072	Yellow
End of that line group . . .	...	3	26·13	44 366	Yellow
Maximum Green Light . . .	**7**	...	**26·17**	**44 743**	Yellow
Dark haze line 1st edge . . .	...	...	26·55	48 567	Green
2d edge . . .	...	4	26·57	48 733	Green
Very faint haze band 1st edge .	...	...	26·65	49 407	Green
2d edge .	...	1	26·71	49 925	Green

5. DIDYMIUM NITRATE—*continued.*

	Intensity. Light Lines.	Intensity. Dark Lines.	r.	Wave-number in Brit. inch.	Colour Region by Wave-number.
Solution $=\frac{1}{20}$ strength—*continued.*					
Medium dark hazy line	...	2	27·04	52 910	GLAUCOUS
Very faint hazy line	...	1	27·13	53 619	GLAUCOUS
Medium dark hazy line band	...	3	27·22	54 259	GLAUCOUS
Broad band in violet { 1st edge	...	...	27·58	56 948	BLUE
2d edge	...	5	27·66	57 571	INDIGO
End all	0	0	28·21	61 102	LAVENDER
Solution $=\frac{1}{10}$.					
Light begins	0	...	24·94	30 030	CRIMSON RED
Black band { 1st edge	...	...	25·22	33 591	CRIMSON RED
2d edge	...	7	25·31	34 758	CRIMSON RED
Very faint line in faint light	1	1	25·51	37 538	SCARLET
MAXIMUM RED LIGHT	**5**	...	**25·92**	**42 265**	AMBER
Salt line	...	...	26·00	43 103	YELLOW
From and after which a slight shade	...	...			
Black band { 1st edge, hazy	...	8	26·05	43 592	YELLOW
2d edge	...	8	26·13	44 366	YELLOW
MAXIMUM GREEN LIGHT	**7**	...	**26·29**	**45 956**	CITRON
Dark hazy band { 1st edge	...	...	26·53	48 379	GREEN
2d edge	...	5	26·58	48 828	GREEN
Very faint but broad band { 1st edge	...	1	26·65	49 407	GREEN
2d edge	...	1	26·71	49 925	GREEN
Dark hazy line	...	3	27·04	52 910	GLAUCOUS
Very faint hazy line	...	1	27·13	53 619	GLAUCOUS
Dark, broader hazy line	...	4	27·21	54 201	GLAUCOUS
Grand black haze band { 1st edge	...	...	27·57	56 883	BLUE
2d edge	...	7	27·68	57 703	INDIGO
Faint haze line in violet	...	1	27·94	59 347	VIOLET
End all	0	...	28·15	60 709	LAVENDER
Solution $=\frac{1}{5}$.					
Light begins	0	...	24·99	30 637	CRIMSON RED
Black band { 1st edge	...	...	25·23	33 715	CRIMSON RED
2d edge	...	8	25·31	34 758	CRIMSON RED
Faint line in faint light	1	1	25·51	37 538	SCARLET
MAXIMUM RED LIGHT	**5**	...	**25·86**	**41 615**	AMBER
Salt line	...	...	26·00	43 103	YELLOW
Faint shade begins	...	1	26·00	43 103	YELLOW
Black band { 1st edge	...	...	26·04	43 668	YELLOW
2d edge	...	10	26·13	44 366	YELLOW
MAXIMUM GREEN LIGHT	**7**	...	**26·21**	**45 147**	CITRON

5. DIDYMIUM NITRATE—*continued.*

Solution$=\frac{1}{5}$ strength—*continued.*

	Intensity. Light Lines.	Intensity. Dark Lines.	*r.*	Wave-number in Brit. inch.	Colour Region by Wave-number.
Extremely faint line	...	1	26·46	47 710	GREEN
Black hazy band 1st edge	...	...	26·52	48 309	GREEN
2d edge	...	8	26·58	48 828	GREEN
Faint but broad band 1st edge	...	...	26·65	49 407	GREEN
2d edge	...	2	26·71	49 925	GREEN
MAXIMUM BLUE LIGHT	**4**	...	**26·82**	**50 916**	GLAUCOUS
Dark hazy line	...	5	27·04	52 910	GLAUCOUS
Faint hazy line	...	2	27·11	53 476	GLAUCOUS
Broader and darker line	...	6	27·21	54 201	GLAUCOUS
MAXIMUM INDIGO LIGHT	**3**	...	**27·42**	**55 741**	BLUE
Grand black haze band 1st edge	...	...	27·54	56 657	BLUE
2d edge	...	8	27·68	57 703	INDIGO
MAXIMUM VIOLET LIGHT	**2**	...	**27·81**	**58 548**	VIOLET
Faint hazy line	...	...	27·96	59 453	VIOLET
End all	0	...	28·21	61 087	LAVENDER

PLATE I., No. 41 OF THIS VOL. XXVIII.

This is a representation in black and white, of the effects of three degrees or depths of colour, of each of three diverse kinds of stained glass, viz., *ruby-red*, *rich green*, and *cobalt-blue*, in stopping out certain parts of the spectrum of a bright coal-gas flame.

The method of representation is symbolical, not imitation. In the latter case the spectrum-strip would have been of the same height from end to end; while bright bands and dark bands, all of equal height, would have differed from each other, solely in most refined degrees of shade; an ultra difficult pictorial result to produce and keep to, when the strictest economy in engraving is enforced on an author. The symbolical method has been therefore adopted, wherein degrees of light or shade are represented by amount of height above, or depth below, a certain horizontal base-line;—a method both easy to realise in any kind of multiplication of graphical work, and easy also of recognition by the student.

In either case, the spectrum necessarily begins and ends in darkness. The locomotion of colour-bands, with increased depth of the transparent colouring material (so much alluded to in the paper) is shown by the strong dotted lines connecting several spectra together. (See also p. 781.)

PLATE II., No. 42 OF THIS VOL. XXVIII.

Here, for economy's sake, only two of the many fluids examined have been dealt with. One of them, *Judson's green*, differs from the stained-green glass of Plate I., by being, though equally green, not a *mono-chroic*, but a *di-chroic;* and the parent on that account of many remarkable phenomena herein-before opened-up. The other, the bluish *Oxalate of Chromium and Potass*, is also *di-chroic*, but has this further feature of interest, that it shows a *fixed* black line, as well as locomotive bright colour bands. (See also p. 782.)

PLATE III., No. 43 of this Vol. XXVIII.

This Plate, of 25 very distinct and recoverable colours, with seven variations in degree of each, with which Messrs W. & A. K. Johnston, the Society's able engravers, have kindly taken a deal of trouble and much anxious interest, will, with their splendid chromatic printing, probably explain itself on inspection better than any mere words in this place can attempt to do: the principles on pp. 789 to 791 having been previously studied. P. S.

APPENDIX.

TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

1879.

CONTENTS.

LIST OF MEMBERS,

INCLUDING

COUNCIL, ALPHABETICAL LIST OF ORDINARY FELLOWS,

LIST OF HONORARY FELLOWS,

LIST OF ORDINARY FELLOWS, ELECTED FROM 1876 TO 1879, ACCORDING TO DATE OF ELECTION,

AND

LIST OF FELLOWS DECEASED, RESIGNED, AND CANCELLED.

THE COUNCIL

OF

THE ROYAL SOCIETY OF EDINBURGH,

ELECTED ON MONDAY, 25th NOVEMBER 1878.

PROFESSOR KELLAND, M.A., President.

HONORARY VICE-PRESIDENTS.

His Grace the DUKE of ARGYLL, K.T., D.C.L. Oxon., F.R.S.
Sir ROBERT CHRISTISON, Bart., M.D., F.R.C.P.E., D.C.L.
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OF

THE ORDINARY FELLOWS OF THE SOCIETY,

CORRECTED TO MAY 1879.

N.B.—*Those marked * are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. ,, ,, ,, Keith Medal.
N. ,, ,, ,, Neill Medal.
P. ,, ,, ,, contributed one or more Papers to the *Transactions*.

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1878		Campbell, John Archibald, M.D., Garland's Asylum, Carlisle	
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1874		* Chiene, John, M.D., F.R.C.S.E., 21 Ainslie Place	
1875		* Christie, John, 19 Buckingham Terrace	
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1872		* Constable, Archibald, 11 Thistle Street	
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1877		* Cunningham, George, Memb. Inst. Civil Engineers, 2 Ainslie Place	
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1848		Davidson, Henry, Muirhouse, Davidson's Mains	
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1879		* Denny, William, Bellfield, Dumbarton	90
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1867	P.	* Donaldson, James, M.A., LL.D., 20 Great King Street	
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1866		* Falshaw, Sir James, Bart., 14 Belgrave Crescent	
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1852		Fleming, Andrew, M.D., Deputy Surgeon-General, 3 Napier Road	
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1872		* Lee, Robert, Advocate, 26 Charlotte Square	
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1858		* Leslie, James, Memb. Inst. Civil Engineers, 2 Charlotte Square	
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1864		* Lindsay, William, Hermitage-Hill House, Leith	
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1871		* Logie, Cosmo Garden, M.D., Surgeon-Major, Royal Horse Guards, 47 Queensborough Gardens, Bayswater	
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1867		* M‘Candlish, John M., W.S., 4 Doune Terrace	
1866		* M‘Culloch, John, 11 Duke Street	
1871		* Macdonald, Angus, M.D., F.R.C.P.E., F.R.C.S.E., 29 Charlotte Square	
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1878		* M‘Gowan, George, 24 Seton Place	
1869		* MacGibbon, David, Architect, 89 George Street	
1879		* M‘Grigor, Alexander Bennett, LL.D., 19 Woodside Terrace, Glasgow	
1840		Mackenzie, John, New Club, Princes Street	215
1877		* Macfie, Robert A., Dreghorn Castle, Colinton	
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1872		* Maclagan, David, C.A., 9 Royal Circus	
1853		Maclagan, Major-General R., Royal Engineers, Lahore, Punjab	
1869		* Maclagan, R. Craig, M.D., 5 Coates Crescent	220
1870		* Macleod, George H. B., M.D., Professor of Surgery, in the University of Glasgow, 10 Woodside Crescent, Glasgow	
1876		* Macleod, Rev. Norman, 7 Royal Circus	
1872		* Macmillan, Rev. Hugh, LL.D., D.D., Seafield, Greenock	
1876		* Macmillan, John, M.A., 18 Duncan Street, Drummond Place	
1869	N P.	* M‘Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Murthly, Perthshire	225
1873	P.	* M‘Kendrick, John G., M.D., F.R.C.P.E., Professor of Physiology in University of Glasgow	
1864		* M‘Lagan, Peter, of Pumpherston, M.P., Linlithgow	
1869		* M‘Laren, John, Advocate, 46 Moray Place	
1866		* Macnair, John, 33 Moray Place	
1877		* Macnee, Sir Daniel, President of the Royal Scottish Academy, 6 Learmonth Terrace	230
1840	P.	M‘Neill, The Right Hon. Sir John, G.C.B., K.L.S., LL.D., Burnhead, Liberton	

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1864		* Marwick, James David, LLD., Town-Clerk, Glasgow	
1866		* Masson, David, LL.D., Professor of Rhetoric, 6 Minto Street	235
1856	K. P.	* Maxwell, James Clerk, M.A., F.R.S., Professor of Experimental Physics, Cavendish Laboratory, Cambridge; Glenlair, Dalbeattie	
1853		Mercer, Græme Reid, of Gorthie, Ceylon Civil Service	
1875		* Millar, C. H., of Blaircastle, 5 Palmerston Place	
1841		Miller, John, of Leithen, Memb. Inst. Civil Engineers, 2 Melville Crescent	
1869		* Miller, Oliver G., Panmure House, Forfarshire	240
1852		Miller, Thomas, A.M., LL.D., Rector, Perth Academy	
1833		Milne, Admiral Sir Alexander, Bart., G.C.B., Inveresk	
1878		* Milne, John, Mechanician, Trinity Grove, Edinburgh	
1875		* Milroy, John, C.E., 8 Salisbury Road	
1866		* Mitchell, Arthur, M.A., M.D., LL.D., Commissioner in Lunacy, 34 Drummond Place	245
1843		Mitchell, Joseph, Memb. Inst. Civil Engineers, Viewhill, Inverness	
1879		* Moinet, Francis W., M.D., F.R.C.P.E., 13 Alva Street.	
1865		* Moir, John J. A., M.D., F.R.C.P.E., 52 Castle Street	
1870		* Moncreiff, the Right Hon. Lord, Lord Justice-Clerk, 15 Great Stuart Street	
1871		* Moncrieff, Rev. William Scott, of Fossaway, Bishop-Wearmouth, Sunderland	250
1868		* Montgomery, Very Rev. Dean, 17 Atholl Crescent	
1866		* Morehead, Charles, M.D., F.R.C.P.L., 11 North Manor Place	
1879		* Morrison, J. B. Brown, of Finderlie and Murie, Perthshire	
1877	P.	* Morrison, Robert Milner, D.Sc., Senior Demonstrator of Chemistry in the University of Edinburgh, 13 Douglas Crescent	
1861	P.	* John Muir, D.C.L., LL.D., 10 Merchiston Avenue	255
1873		* Muir, M. M. Pattison, Caius College, Cambridge	
1874	P.	* Muir, Thomas, M.A., High School, Glasgow	
1870		* Munn, David, High School	
1857		Murray, John Ivor, 8 Huntriss Row, Scarborough	
1877		* Murray, John, Challenger Office, 32 Queen Street	260
1877		* Napier, John, Saughfield House, Glasgow	
1874		* Napier, James, F.R.S., Maryfield House, Bothwell	
1866		* Nelson, Thomas, St Leonard's, Dalkeith Road	
1870	P.	* Nicholson, Henry Alleyne, M.D., D.Sc., Professor of Civil and Natural History in the University of St Andrews	
1878		Norris, Richard, M.D., Professor of Physiology, Queen's College, Birmingham	265
1863		* Ormidale, Hon. Lord, 14 Moray Place	
1877		Panton, George A., 24 Bennetts Hill, Birmingham	
1837	P.	Parnell, Richard, M.D., 17 Merchiston Avenue	
1863		* Peddie, Alexander, M.D., F.R.C.P.E., 15 Rutland Street	
1868		* Peddie, John Dick, Architect, 33 Buckingham Terrace	270
1869		Pender, John, M.P., Manchester	
1873	P.	* Pettigrew, J. Bell, M.D., F.R.C.P.E., F.R.S., Professor of Medicine and Anatomy in the University of St Andrews.	

Date of Election.			
1849		Pirrie, William, M.D., Professor of Surgery, Marischal College, Aberdeen	
1859	P.	* Playfair, Right Hon. Lyon, C.B., M.P., LL.D., F.R.S., 68 Onslow Gardens, London	
1834	P.	Ponton, Mungo, W.S., Clifton, Bristol	275
1874		Powell, Baden Henry Baden-, Forest Department, India	
1877		Pole, Wm., F.R.S., Mus. Doc., Mem. Inst. Civ. Eng., 31 Parliament St., Westminster, S.W.	
1852		Powell, Eyre B., Director of Public Instruction, Madras	
1865		* Powrie, James, Reswallie, Forfar	
1875		Prevost, E. W., Ph.D., Agricultural College, Cirencester	280
1849		Primrose, Hon. B. F., C.B., 22 Moray Place	
1873		Pritchard, Andrew, 87 St Paul's Road, Highbury, London	
1868		* Raleigh, Samuel, C.A., Park House, Dick Place	
1869		Raven, Rev. Thomas Milville, M.A., The Vicarage, Crakehall, Bedale	
1865		* Redford, Rev. Francis, M.A., Rectory, Silloth	285
1836		Rhind, David, Architect, 19 Hill Street	
1875		* Richardson, Ralph, W.S., 16 Coates Crescent	
1872		Ricarde-Lever, Major F. Ignacio, Carlton Club, St James' Street, London	
1877		* Roberton, James, LL.D., Professor of Conveyancing in the University of Glasgow, 1 Park Terrace East, Glasgow	
1879		* Robertson, Major-General A. Cuningham, 86 Great King Street	290
1872		* Robertson, D. M. C. L. Argyll, M.D., 18 Charlotte Square	
1859		* Robertson, George, Memb. Inst. Civil Engineers, 47 Albany Street	
1860		* Robertson, William, M.D., F.R.C.P.E., 28 Albany Street	
1877	P.	* Robinson, George Carr, Demonstrator of Chemistry, Public Health Laboratory, University of Edinburgh, 9 Melville Terrace	
1876		* Rodger, J. F., S.S.C., 1 Royal Circus	295
1862	P.	* Ronalds, Edmund, LL.D., Bonnington House, Bonnington Road	
1852		Russell, Alexander James, C.S., 9 Shandwick Place	
1837	K. P.	Russell, John Scott, M.A., F.R.S., 5 Westminster Chambers, London	
1869	P.	* Rutherford, Wm., M.D., F.R.S., Professor of Institutes of Medicine in the University of Edinburgh, 14 Douglas Crescent	
1870		* Sanders, William R., M.D., F.R.C.P.E., Professor of General Pathology in the University of Edinburgh, 30 Charlotte Square	300
1863		* Sanderson, James, Surgeon-Major, 41 Manor Place	
1864		* Sandford, Rev. D. F., LL.D., 6 Rutland Square	
1849	P.	Sang, Edward, C.E., 6 Molendo Terrace	
1846		Schmitz, Leonard, LL.D., Belsize Park Gardens, London	
1875		Scott, Michael, Memb. Inst. Civil Engineers, 9 Great Queen St., Westminster, London	305
1864		* Sellar, W.Y., M.A., LL.D., Professor of Humanity, University of Edinburgh, 15 Buckingham Terrace	
1872		* Seton, George, M.A. Oxon., Advocate, 42 Greenhill Gardens	
1834		Sharpey, William, M.D., LL.D., F.R.C.S.E., F.R.S., Emeritus Professor of Anatomy, University College, London	
1872		* Sibbald, John, M.D., Commissioner in Lunacy, 3 St Margaret's Road, Whitehouse Loan	
1870		* Sime, James, M.A., Craigmount House, Dick Place	310
1871		* Simpson, A. R., M.D., F.R.C.P.E., Professor of Midwifery, 52 Queen Street	

Date of Election.			
1859	P.	* Skene, William F., LL.D., D.C.L., W.S., 27 Inverleith Row	
1876		* Skinner, William, W.S., Town-Clerk of Edinburgh, 35 George Square	
1868		* Smith, Adam Gillies, C.A., 5 Lennox Street	
1839		Smith, David, W.S. (Treasurer), 10 Eton Terrace	315
1871		* Smith, John, M.D., F.R.C.S.E., 11 Wemyss Place	
1863	P.	* Smith, John Alexander, M.D., F.R.C.P.E., 10 Palmerston Place	
1855		* Smith, R. M., 4 Bellevue Crescent	
1871		* Smith, Rev. W. Robertson, M.A., Professor of Hebrew, Free Church College, Aberdeen, 83 Crown Street, Aberdeen	
1846	K.P.	Smyth, Piazzi, Professor of Practical Astronomy, and Astronomer-Royal for Scotland, 15 Royal Terrace	320
1866		* Spence, James, F.R.C.S.E., Professor of Surgery, 21 Ainslie Place	
1874	P.	* Sprague, T. B., M.A., 29 Buckingham Terrace	
1850	P.	Stark, James, M.D., F.R.C.P.E., Huntfield, Biggar	
1844		Stevenson, David, Memb. Inst. Civil Engineers (Vice-President), 45 Melville Street	
1877		* Stevenson, James, 4 Woodside Crescent, Glasgow	325
1868		Stevenson, John J., Red House, Bayswater Hill, London, W.	
1848	P.	Stevenson, Thomas, Memb. Inst. Civ. Engineers, 17 Heriot Row	
1868		Stewart, Colonel J. H. M. Shaw, Royal Engineers, Madras	
1878		* Stewart, James R., M.A. Oxon., 10 Minto Street	
1866		* Stewart, T. Grainger, M.D., F.R.C.P.E., Prof. of Practice of Physic, 19 Charlotte Sq.	330
1873		* Stewart, Walter, 22 Torphichen Street	
1848		Stirling, Patrick J., LL.D., Kippendavie House, Dunblane	
1877		* Stirling, Wm., M.D., Sc.D., Professor of Institutes of Medicine in the University of Aberdeen	
1823		Stuart, Captain T. D., H.M.I.S.	
1870		* Swan, Patrick Don, Provost of Kirkcaldy	335
1848	P.	Swan, William, LL.D., Professor of Natural Philosophy, in the University of St Andrews	
1844		Swinton, A. Campbell, LL.D., of Kimmerghame, Dunse	
1875		* Syme, James, 10 Buckingham Terrace	
1872		Tait, the Rev. A., LL.D., Canon of Tuam, Moylough Rectory, Ballinasloe, Ireland	
1861	K.P.	* Tait, P. Guthrie, M.A., Professor of Natural Philosophy in the University of Edinburgh (Secretary), 38 George Square	340
1870		* Tatlock, Robert R., City Analyst's Office, 138 Bath Street, Glasgow	
1846		Taylor, Sir Alexander, M.D., Pau, France	
1872		* Teape, Rev. Charles R., M.A., Ph.D., 15 Findhorn Place	
1873		* Tennent, Robert, 21 Lynedoch Place	
1843		Thomson, Allen, M.D., F.R.C.S.E., F.R.S., Emeritus Professor of Anatomy in the University of Glasgow, 66 Palace Garden Terrace, London, W.	345
1870		* Thomson, Rev. Andrew, D.D., 63 Northumberland Street	
1875		* Thomson, James, LL.D., F.R.S., Professor of Engineering in the University of Glasgow, Oakfield House, University Avenue, Glasgow	
1863		* Thomson, Murray, M.D., Roorkee, East Indies	
1870		* Thomson, Spencer C., Actuary, 10 Chester Street	
1847	K.P.	Thomson, Sir William, LL.D., F.R.S. (Hon. Vice-President), Regius Professor of Natural Philosophy in University of Glasgow, Mem. of the Institute of France	350
1855		* Thomson, Sir Wyville C., LL.D., F.R.S. (Vice-President), Regius Professor of Natural History in the University of Edinburgh, Bonsyde, Linlithgow	

Date of Election.			
1870		* Thomson, William Burns, F.R.C.P.E., F.R.C.S.E., 1 Ramsay Gardens	
1849		Thomson, William Thomas, Actuary, 27 Royal Terrace	
1876		Thomson, William, Royal Institution, Manchester	
1878		Thorburn, Robert Macfie, Uddevalla, Sweden	355
1874	N.P.	* Traquair, R. H., M.D., Keeper of the Nat. Hist. Collections in the Museum of Science and Art, Edinburgh, and President of the Royal Physical Society, 8 Dean Park Crescent	
1874		* Tuke, J. Batty, M.D., 20 Charlotte Square	
1879		* Turnbull, John, of Abbey St Bathans, W.S., 49 George Square	
1867		* Turnbull, William, 14 Lansdowne Crescent	
1861	N.P.	* Turner, William, M.B., F.R.C.S.E., F.R.S., Professor of Anatomy in the University of Edinburgh, (SECRETARY), 6 Eton Terrace	360
1877		* Underhill, Charles E., B.A., M.B., F.R.C.P.E., F.R.C.S.E., 8 Coates Crescent	
1875		Vincent, Charles Wilson, Royal Institution, Albemarle Street, London	
1867		* Waddell, Peter, 5 Claremont Park, Leith	
1829		Walker, James, W.S., Tunbridge Wells	
1873		* Walker, Robert, M.A., University, Aberdeen	365
1864		* Wallace, William, Ph.D., Glasgow	
1870		* Watson, James, 45 Charlotte Square	
1866		* Watson, John K., 94 Blackford Road	
1873		* Watson, Morrison, M.D., Professor of Anatomy, Owens College, Manchester	
1866		* Watson, Patrick Heron, M.D., 16 Charlotte Square	370
1862	P.	Watson, Rev. Robert Boog, 3 Bruntsfield Place	
1877		Weldon, Walter, F.C.S., Rede Hall, Burstow, Surrey	
1873		Welsh, Major, Bengal Artillery	
1840		Welwood, Allan A. Maconochie, LL.D., of Meadowbank and Garvoch, Kirknewton, Edin.	
1876		White, Rev. Francis Le Grix, M.A., F.R. Hist. S., F.G.S., Leaming House, Ulleswater, Penrith, Cumberland	375
1868		Williams, W., Gayfield House	
1858		* Williamson, Thomas, M.D., F.R.C.S.E., 28 Charlotte Street, Leith	
1879		* Wilson, Andrew, Ph.D., Lecturer on Zoology and Comparative Anatomy in the Edinburgh Medical School, 118 Gilmore Place	
1877		* Wilson, Charles E., M.A., LL.D., H. M. Senior Inspector of Schools, 19 Palmerston Place	
1878		* Wilson, Rev. John, M.A., Bannockburn Academy	380
1875		Wilson, Daniel, LL.D., Professor, Toronto	
1834		Wilson, Isaac, M.D.	
1847		Wilson, John, Professor of Agriculture in the University of Edinburgh	
1863		* Wilson, J. G., M.D., F.R.C.S.E., 9 Woodside Crescent, Glasgow	
1873		Wilson, Robert, Engineer, Patricroft, Manchester	385
1870		Winzer, John, Assistant Surveyor, Civil Service, Ceylon	
1864		* Wood, Alexander, M.D., F.R.C.P.E., 12 Strathearn Place	
1864		* Wood, Andrew, M.D., F.R.C.S.E., 9 Darnaway Street	
1855		Wright, Thomas, M.D., Cheltenham	
1864		* Wyld, Robert S., LL.D., 19 Inverleith Row	390
1861		* Young, James, of Kelly and Durris, F.R.S., Wemyss Bay, by Greenock	
1863		* Young, John, M.D., Professor of Natural History in the University of Glasgow	392

LIST OF HONORARY FELLOWS

AT MAY 1879.

His Royal Highness the PRINCE OF WALES.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW X.)

Elected.		
1864	Robert Wilhelm Bunsen,	*Heidelberg.*
1867	Michel Eugène Chevreul,	*Paris.*
1858	James D. Dana,	*Newhaven, Connecticut.*
1877	Alphonse De Candolle,	*Geneva.*
1879	Franz Cornelius Donders,	*Utrecht.*
1855	Jean Baptiste Dumas,	*Paris.*
1877	Carl Gegenbaur,	*Heidelberg.*
1879	Asa Gray,	*Harvard College, Cambridge, United States.*
1864	Hermann Ludwig Ferdinand Helmholtz,	*Berlin.*
1879	Jules Janssen,	*Paris.*
1875	August Kekulé,	*Bonn.*
1868	Gustav Robert Kirchhoff,	*Berlin.*
1875	Herman Kolbe,	*Leipzig.*
1864	Albert Kölliker,	*Würzburg.*
1875	Ernst Eduard Kummer,	*Berlin.*
1845	Johann von Lamont,	*Munich.*
1864	Richard Lepsius,	*Berlin.*
1876	Ferdinand de Lesseps,	*Paris.*
1864	Rudolph Leuckart,	*Leipzig.*
1879	Johann Benedict Listing,	*Göttingen.*
1875	Joseph Liouville,	*Paris.*
1876	Carl Ludwig,	*Leipzig.*
1878	J. N. Madvig,	*Copenhagen.*
1855	Henry Milne-Edwards,	*Paris.*
1864	Theodore Mommsen,	*Berlin.*
1874	Louis Pasteur,	*Paris.*
1867	Benjamin Peirce,	*Cambridge, United States.*
1864	Karl Theodor von Siebold,	*Munich.*
1878	Otto Wilhelm Struve,	*Pulkowa, St Petersburg.*
1855	Bernard Studer,	*Berne.*
1874	Otto Torell,	*Lund.*
1868	Rudolph Virchow,	*Berlin.*
1874	Wilhelm Eduard Weber,	*Göttingen.*
1867	Friedrich Wöhler,	*Do.*

Total, 34.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW X.)

Elected.		
1849	John Couch Adams, LL.D., F.R.S., Mem. Inst. France,	*Cambridge.*
1835	Sir George Biddell Airy, K.C.B., M.A., D.C.L., LL.D., F.R.S.,	*Greenwich.*
1870	Thomas Andrews, M.D., LL.D., F.R.S.,	*Belfast (Queen's College).*
1866	Thomas Carlyle, LL.D., Ord. Boruss. "Pour le Mérite," Pres. Phil. Inst. Edin.,	*London.*
1865	Arthur Cayley, LL.D., F.R.S., Mem. Inst. France,	*Cambridge.*
1865	Charles Darwin, M.A., F.R.S., Mem. Inst. France,	*Down, Bromley, Kent.*
1874	John Anthony Froude, LL.D.,	*London.*
1876	Thomas Henry Huxley, D.C.L., LL.D., Sec. R.S., Mem. Inst. France,	*Do.*
1867	James Prescott Joule, LL.D., D.C.L., F.R.S., Mem. Inst. France,	12 *Wardle Road, Sale near Manchester.*
1849	William Lassell, LL.D., F.R.S.,	*Ray Lodge, Maidenhead.*
1845	Rev. Dr Humphrey Lloyd, D.D., F.R.S., M.R.I.A.,	*Dublin.*
1874	William Hallowes Miller, M.A., LL.D., D.C.L., F.R.S., Mem. Inst. France,	*Cambridge.*
1845	Richard Owen, C.B., M.D., LL.D., D.C.L., F.R.S., Mem. Inst. France,	*London.*
1876	Thomas Romney Robinson, D.D., LL.D., D.C.L., F.R.S., M.R.I.A.,	*Armagh.*
1865	General Sir Edward Sabine, R.A., K.C.B., LL.D., D.C.L., F.R.S., Mem. Inst. France.	*London.*
1876	Henry John Stephen Smith, M.A., LL.D., F.R.S., F.C.S.,	*Oxford.*
1878	Balfour Stewart, M.A., LL.D., F.R.S.,	*Manchester.*
1864	George Gabriel Stokes, M.A., LL.D., D.C.L., Sec. R.S.	*Cambridge.*
1874	James Joseph Sylvester, M.A., LL.D., F.R.S., Mem. Inst. France,	*Baltimore.*
1864	Alfred Tennyson, D.C.L., F.R.S., Poet Laureate,	*Freshwater, Isle of Wight.*

Total, 20.

LIST OF ORDINARY FELLOWS.

Elected from 1876 *to* 1879, *arranged according to the date of their Election.*

5th January 1877.

John Murray.
Walter Noel Hartley.
Walter Weldon.

5th March 1877.

George Broadrick.
John Napier.
James King.
George Cunningham.

2d April 1877.

William Jolly.
Robert Milner Morrison.
Charles E. Underhill.
Charles Edward Wilson.
George Carr Robinson.
John Gibson.

7th May 1877.

Robert A. Macfie.
William Stirling.

4th June 1877.

James Stevenson.
James Roberton.
George A. Panton.
Isaac Bayley Balfour.
Sir Daniel Macnee.
William Pole.

7th January 1878.

W. H. Allchin.
Richard Norris.
Daniel John Cunningham.
Alan Macdougall.
John Fredrick Bateman.
William King.
P. R. Scott Lang.

4th February 1878.

James Alfred Ewing.
John Wilson.
Robert Macfie Thorburn.
Andrew Peebles Aitken.
John Milne.

4th March 1878.

Charles Davidson Bell.
James Blyth.
R. K. Galloway.

1st April 1878.

The Lord Inverurie.

6th May 1878.

Alexander Macfarlane.
Samuel Drew.
George M'Gowan.
James Brunlees.
John Grahame Dalziel.

3d June 1878.

J. J. Kirk Duncanson.

6th January 1879.

J. B. Brown Morrison of Finderlie and Murie.
Andrew Wilson.
James Lambert Bailey.
Robert Cox.
John Hislop.
James Cossar Ewart.
George William Balfour.

3d February 1879.

A. Cunningham Robertson, Major-General.
William Denny.
Francis W. Moinet.

3d March 1879.

Thomas H. Cockburn Hood.
Thomas Gilray.
Alexander Bennet M'Grigor.
James Blaikie.

5th May 1879.

John Turnbull of Abbey St Bathans.

2d June 1879.

James Abernethy.

LIST OF FELLOWS DECEASED, RESIGNED, AND CANCELLED.

From November 1876 to June 1879.

HONORARY FELLOWS (BRITISH) DECEASED.

Sir Richard Griffiths.
William Henry Fox Talbot.

HONORARY FELLOWS (FOREIGN) DECEASED.

Claude Bernard.
Elias Magnus Fries.
Jean Joseph Leverrier.
John Lothrop Motley.
Victor Regnault.
Angelo Secchi.

ORDINARY FELLOWS DECEASED.

Dr James Allan, died in 1852. Death only intimated in 1878.
Dr James Warburton Begbie.
David Bryce.
Dr James Bryce.
Andrew Coventry.
Sir James Coxe.
Sir William Gibson-Craig, Bart.
James Cunningham, W.S.
Rev. D. T. K. Drummond.
G. Stirling Home Drummond.
Sir David Dundas, Bart.
Lewis D. B. Gordon.
Professor Robert Harkness.
Sir George Harvey.
William Keddie.
Professor Laycock.
Thomas Login.
Sir William Stirling Maxwell, Bart.
The Hon. Lord Neaves.
Martyn R. Roberts.
Alexander Russel.
Hugh Scott of Gala.
Dr Edward James Shearman.
James Thomson, died in 1870. Death only intimated in 1877.
Arthur, Marquis of Tweeddale.
Dr James Watson.

FELLOWS RESIGNED.

Dr Alexander Hunter.
Dr Thomas Smith Maccall.
Dr Thomas E. Thorpe.
Captain T. P. White.

ELECTIONS CANCELLED.

Ernest Bonar, whose name has been inserted by mistake in Lists subsequent to 1856.
Professor Arthur Gamgee, M.D.

LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH,

AS REVISED JANUARY 1873.

LAWS.

[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

Title.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows.

II.

The fees of Ordinary Fellows residing in Scotland.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.*

III.

Payment to cease after 25 years.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from farther payment.

IV.

Fees of Non-Resident Ordinary Fellows.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt from any farther payment. In the case of any Resident Fellow ceasing to reside

Case of Fellows becoming Non-Resident.

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* At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s., to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

A modification of this rule, in certain cases, was agreed to 3d January 1831.

in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall afterwards be printed in the circulars for three Ordinary Meetings of the Society, previous to the day of election, and shall lie upon the table during that time.

X.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall

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* "A. B., a gentleman well versed in Science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

XI.

Royal Personages.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X.

XII.

Recommendation of Honorary Fellows.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circulars for two ordinary meetings of the Society, previous to the day of election.

Mode of Election.

XIII.

Election of Ordinary Fellows.

The election of Ordinary Fellows shall only take place at the first Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote.

XIV.

Ordinary Meetings.

The Ordinary Meetings shall be held on the first and third Mondays of every month from November to June inclusively. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make.

XV.

The Transactions.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which

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* We hereby recommend ____________________
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

____

____

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To the President and Council of the Royal Society
of Edinburgh.

they shall deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

XVI.

How Published.

The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

XVII.

The Council.

There shall be elected annually, for conducting the publications and regulating the private business of the Society, a Council, consisting of a President; Six Vice-Presidents, two at least of whom shall be resident; Twelve Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

XVIII.

Retiring Councillors.

Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers.

An Extraordinary Meeting for the Election of Office-Bearers shall be held on the fourth Monday of November annually.

XX.

Special Meetings; how called.

Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties.

The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in November, he shall present the accounts for the preceding year, duly audited. At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

Auditor.

At the Extraordinary Meeting in November, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

XXIV.

Secretaries to Ordinary Meetings.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

XXV.

Curator of Museum and Library.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows.

XXVI.

Use of Museum and Library.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations, as the Council from time to time shall appoint.

XXVII.

Register Book.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

THE KEITH, BRISBANE, AND NEILL PRIZES.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The Keith Prize, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1879–80, for the "best communication on a scientific subject, communicated, in the first instance, to the Royal Society during the Sessions 1877–78 and 1878–79." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1879–80, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 1st June 1879.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the Paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented to the Society during the Sessions 1878–79 and 1879–80, whether they may have been given in with a view to the Prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr Patrick Neill of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate,

1. The Neill Prize, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1880–81.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented to the Society during the three years preceding the 1st May 1880,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, AND NEILL PRIZES, FROM 1827 TO 1879.

I. KEITH PRIZE.

1st Biennial Period, 1827–29.—Dr Brewster, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2d Biennial Period, 1829–31.—Dr Brewster, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3d Biennial Period, 1831–33.—Thomas Graham, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833–35.—Professor J. D. Forbes, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835–37.—John Scott Russell, Esq., for his Researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837–39.—Mr John Shaw, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839–41.—Not awarded.

8th Biennial Period, 1841–43.—Professor J. D. Forbes, for his Papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843–45.—Not awarded.

10th Biennial Period, 1845–47.—General Sir Thomas Brisbane, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847–49.—Not awarded.

12th Biennial Period, 1849–51.—Professor Kelland, for his papers "on General Differentiation, including his more recent communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851–53.—W. J. Macquorn Rankine, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853–55.—Dr Thomas Anderson, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855–57.—Professor Boole, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16th Biennial Period, 1857–59.—Not awarded.

17th Biennial Period, 1859–61.—John Allan Broun, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18th Biennial Period, 1861–63.—Professor William Thomson, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19th Biennial Period, 1863–65.—Principal Forbes, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society,

20th Biennial Period, 1865–67.—Professor C. Piazzi Smyth, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21st Biennial Period, 1867–69.—Professor P. G. Tait, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22d Biennial Period, 1869–71.—Professor Clerk Maxwell, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23d Biennial Period, 1871–73.—Professor P. G. Tait for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24th Biennial Period, 1873–75.—Professor Crum Brown, for his Researches "on the sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25th Biennial Period, 1875–77.—Professor M. Forster Heddle, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

II. MAKDOUGALL-BRISBANE PRIZE.

1st Biennial Period, 1859.—Sir Roderick Impey Murchison, on account of his Contributions to the Geology of Scotland.

2d Biennial Period, 1860–62.—William Seller, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3d Biennial Period, 1862–64.—John Denis Macdonald, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4th Biennial Period, 1864–66.—Not awarded.

5th Biennial Period, 1866–68.—Dr Alexander Crum Brown and Dr Thomas Richard Fraser, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6th Biennial Period, 1868–70.—Not awarded.

7th Biennial Period, 1870–72.—George James Allman, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8th Biennial Period, 1872–74.—Professor Lister, for his paper "on the Germ Theory of Putrefaction and the Fermentative Changes," communicated to the Society 7th April 1873.

9th Biennial Period, 1874–76.—Alexander Buchan, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

III. THE NEILL PRIZE.

1st Triennial Period, 1856–59.—Dr W. Lauder Lindsay, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2d Triennial Period, 1859–62.—Robert Kaye Greville, LL.D., for his Contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3d Triennial Period, 1862–65.—Andrew Crombie Ramsay, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and Memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4th Triennial Period, 1865–68.—Dr William Carmichael M'Intosh, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5th Triennial Period, 1868–71.—Professor William Turner, for his papers "on the great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6th Triennial Period, 1871–74.—Charles William Peach, for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7th Triennial Period, 1874–77.—Dr Ramsay H. Traquair, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

PROCEEDINGS

OF THE

STATUTORY GENERAL MEETINGS,

AND

LIST OF MEMBERS ELECTED AT THE ORDINARY MEETINGS.

FROM NOVEMBER 1876 TO NOVEMBER 1878.

STATUTORY MEETINGS.

NINETY-FOURTH SESSION.

Monday, 27th November 1876.

At a Statutory Meeting, Sir William Thomson, President, in the Chair, the Minutes of the Statutory Meeting of 22d November 1875 were read and confirmed.

The following Office-Bearers were elected for 1876–77 :—

Sir William Thomson, Knt., LL.D., President.
His Grace the Duke of Argyll,
Sir Robert Christison, Bart., M.D., } Honorary Vice-Presidents.
Professor Kelland,
Rev. W. Lindsay Alexander, D.D.,
David Stevenson, Esq., Mem. Inst. Civ. Eng.,
The Hon. Lord Neaves,
The Right Rev. Bishop Cotterill, D.D.,
Principal Sir Alexander Grant, Bart., } Vice-Presidents.
Dr John Hutton Balfour, General Secretary.
Professor Tait,
Professor Turner, } Secretaries to Ordinary Meetings.
David Smith, Esq., Treasurer.
Dr Maclagan, Curator of Library and Museum.

COUNCILLORS.

Dr Mitchell.
Dr Andrew Fleming, H.M.I.S.
Dr Charles Morehead.
Alexander Buchan, A.M.
Robert Wyld, Esq.
Dr Ramsay H. Traquair.
Thomas Harvey, LL.D.
Dr John M'Kendrick.
Dr J. Matthews Duncan.
Sir Wyville C. T. Thomson.
D. Milne Home, Esq., LL.D.
Professor Crum Brown.
James Bryce, Esq., LL.D.

The Treasurer laid on the table his Annual Report, certified by the Auditor.

George Auldjo Jamieson, Esq., was elected Auditor for the year 1875–76.

The Secretary reported as follows :—

Number of Ordinary Fellows at 5th November 1875	358
New Fellows Elected, 1875–76, ...	15
Total,	373
Deduct—Deceased, 9 ; resigned, 1, ...	= 10
Number of Ordinary Fellows at November 1876, ...	363
Add Honorary and Non-Resident Fellows, ...	56
Total Ordinary and Honorary Fellows at November 1876, . .	419
Honorary Fellows deceased during the year, ...	3

NINETY-FIFTH SESSION.

Monday, 26th November 1877.

At a Statutory Meeting, Sir William Thomson, President, in the Chair, the Minutes of the Statutory Meeting of 27th November 1876 were read and confirmed.

The following Office-Bearers were elected for 1877–78:—

Sir William Thomson, Knt., LL.D., President.	
His Grace the Duke of Argyll,	Honorary Vice-Presidents.
Sir Robert Christison, Bart., M.D.,	
Rev. W. Lindsay Alexander, D.D.,	Vice-Presidents.
David Stevenson, Esq., Mem. Inst. Civ. Eng.,	
The Right Rev. Bishop Cotterrill, D.D.,	
Principal Sir Alexander Grant, Bart.,	
David Milne Home, LL.D.,	
Sir C. Wyville Thomson, LL.D.,	
Dr John Hutton Balfour, General Secretary.	
Professor Tait,	Secretaries to Ordinary Meetings.
Professor Turner,	
David Smith, Esq., Treasurer.	
Dr Maclagan, Curator of Library and Museum.	

COUNCILLORS.

Dr Ramsay H. Traquair.
Thomas Harvey, LL.D.
Professor John T. M'Kendrick.
Professor Crum Brown.
Professor Kelland.
Professor Fleeming Jenkin.
Rev. R. Boog Watson.
Dr Hugh Cleghorn.
Professor T. R. Fraser.
Professor Rutherford.
Dr R. M. Ferguson.
Sheriff Hallard.

The Treasurer laid on the table his Annual Report, certified by the Auditor.

George Auldjo Jamieson was elected Auditor for the year 1877–78.

The Secretary reported as follows:—

Number of Ordinary Fellows at 5th November 1876,	363
New Fellows Elected, 1876–77,	21
Total,	384
Deduct—Deceased, 7; resigned, 2; cancelled, 2,	= 11
Number of Ordinary Fellows at November 1877,	373
Add Honorary Fellows,	55
Total Ordinary and Honorary Fellows at November 1877,	428
Honorary Fellows deceased,	3

NINETY-SIXTH SESSION.

Monday, 25th November 1878.

At a Statutory Meeting, Sir WILLIAM THOMSON, President, in the Chair, the Minutes of the Statutory Meeting of 26th November 1877 were read and confirmed.

The following Office-Bearers were elected for 1878–79:—

Professor KELLAND, M.A., President.	
His Grace the DUKE OF ARGYLL, Sir Robert CHRISTISON, Bart., M.D., Sir WILLIAM THOMSON, LL.D., having filled the office of President,	Honorary Vice-Presidents.
DAVID STEVENSON, Esq., Mem. Inst. Civ. Eng., The Right Rev. Bishop COTTERILL, D.D. Principal Sir ALEXANDER GRANT, Bart., LL.D. DAVID MILNE-HOME, Esq., LL.D. Sir WYVILLE C. T. THOMSON, LL.D. Professor DOUGLAS MACLAGAN, M.D.	Vice-Presidents.
Dr JOHN HUTTON BALFOUR, General Secretary.	
Professor TAIT, Professor TURNER,	Secretaries to Ordinary Meetings.
DAVID SMITH, Esq., Treasurer.	
ALEXANDER BUCHAN, Esq., Curator of Library and Museum.	

COUNCILLORS.

Professor FLEEMING JENKIN.
Rev. R. BOOG WATSON.
Dr HUGH CLEGHORN.
Professor T. R. FRASER.
Professor RUTHERFORD.
Dr R. M. FERGUSON.
Rev. W. LINDSAY ALEXANDER, D.D.
Dr THOMAS A. G. BALFOUR.
J. Y. BUCHANAN, Esq., M.A.
Rev. THOMAS BROWN.
ROBERT GRAY.
Dr WILLIAM ROBERTSON.

The TREASURER laid on the table his Annual Report, certified by the Auditor.

GEORGE AULDJO JAMIESON was elected Auditor for the year 1878–79.

The SECRETARY reported as follows:—

Number of Ordinary Fellows at 23d November 1877, . .	373
New Fellows Elected, 1877–78, ...	24
Total, .	397
Deduct—Deceased, 10; resigned, 2, ...	= 12
Number of Ordinary Fellows at November 1878, . .	385
Add Honorary Fellows, ...	53
Total Ordinary and Honorary Fellows at November 1878, .	438
Honorary Fellows deceased, ...	4

The following Public Institutions and Individuals are entitled to receive Copies of the Transactions and Proceedings of the Royal Society of Edinburgh:—

ENGLAND.

London, British Museum.
... Royal Society.
... Linnean Society.
... Royal Astronomical Society.
... Royal Asiatic Society.
... Society of Arts.
... Geological Society.
... Athenæum Club.
... Chemical Society.
... Institution of Civil Engineers.
... Royal Geographical Society.
... Royal Horticultural Society.
... Hydrographic Office, Admiralty.
... Royal Institution.
... Royal Society of Literature.
... Medico-Chirurgical Society.
... Museum of Economic Geology.
... Royal Observatory, Greenwich.
... Statistical Society.
... Royal College of Surgeons of England.
... United Service Institution.
... Zoological Society.

Cambridge Philosophical Society.
... University Library.
Historic Society of Lancashire and Cheshire.
Leeds Philosophical and Literary Society.
Manchester Literary and Philosophical Society.
Oxford, Bodleian Library.
Yorkshire Philosophical Society.

SCOTLAND.

Edinburgh, Advocates' Library.
... University Library.
... College of Physicians.
... Highland and Agricultural Society.
... Royal Medical Society.
... Royal Physical Society.
... Royal Scottish Society of Arts.
... Royal Botanic Garden.
Aberdeen, University Library.
Glasgow, University Library.
St Andrews, University Library.

IRELAND.

Library of Trinity College, Dublin.
Royal Irish Academy.

COLONIES, &c.

Calcutta, Asiatic Society.
Canada, Library of Geological Survey.
Melbourne, University Library.
Toronto, Literary and Historical Society.
Sydney, University Library.
New Zealand Institute.

CONTINENT OF EUROPE.

Amsterdam, Royal Institute of the Netherlands.
Basle, Natural History Society.
Berlin, Königliche Akademie der Wissenschaften.
... Physicalische Gessellschaft.
Berne, Society of Swiss Naturalists.
Bologna, Accademia delle Scienze dell' Istituto di Bologna.
Bordeaux, Societé des Sciences Physiques et Naturelles.
Brussels, Royal Academy of Sciences.
... The Royal Observatory.
... The Scientific Society of Brussels.
Buda, Literary Society of Hungary.
Christiania, University Library.
Copenhagen, Royal Academy of Sciences.
Dorpat, University Library.
Dresden, Die Leopoldinisch-Carolinische Akademie.
Erlangen, University Library.
Frankfort, Senckenbergische Naturforschende Gesellschaft.
Geneva, Société de Physique et d' Histoire Naturelle.
Giessen, University Library.
Göttingen, University Library.
Haarlem, Société Hollandaise des Sciences Exactes et Naturelles.

Haarlem, Musée Teyler.

Jena, Professor Schwalbe, Editor of Zeitschrift der Medicinisch-Physikalischen Gesellschaft.

Leipzig, Prof. Wiedemann, Royal Saxon Academy.

Lille, Société des Sciences.

Lisbon, Academia Real das Sciencias de Lisboa.

Lyons, Société d'Agriculture.

Madrid, Comision del Mapa Geologico de España.

Milan, Reale Istituto Lombardo di Scienze, Lettere, ed Arti.

Montpellier, Academie des Sciences et Lettres.

Moscow, Imperial Society of Naturalists.

... Société Impériale des Amis d'Histoire Naturelle, d'Anthropologie et d'Ethnographie.

Munich, Royal Academy of Sciences of Bavaria (2 copies).

Neufchatel, Société des Sciences Naturelles.

Naples, Dr Anton Dohrn, Palazzo Torlonia.

Palermo, The Society of Italian Spectroscopists.

... Signor Agostino Todaro, Giardino Botanico.

Paris, Académie des Sciences.

... L'Observatoire.

... École Polytechnique.

... Société de Géographie.

... Societé d'Agriculture.

... Société d'Encouragement pour l'Industrie Nationale.

... École des Mines.

... Dépôt de la Marine.

... Muséum du Jardin des Plantes.

... Société Géologique de France.

... Société Mathématique.

Rome, Accademia dei Lincei.

Rotterdam, Bataafsch Genootschap der Proefondervindelijke Wijsbegeerte.

St Petersburg, Académie Impériale des Sciences.

... Commission Impériale Archéologique.

St Petersburg, Pulkowa Observatory.

... Physikalisches Central-Observatorium.

Stockholm, Kongliga Svenska Vetenskaps-Academien.

Turin, Royal Academy of Sciences.

... M. Michelotti.

Upsala, Kongliga Vetenskaps-Societeten.

Venice, Reale Istituto Veneto di Scienze, Lettere ed Arti.

Vienna, Imperial Academy of Sciences.

... Novara Commission.

... Central Institute for Meteorology and Terrestrial Magnetism.

... Geologische Reichsanstalt.

... Zoologisch-Botanisches Verein.

Zurich University.

UNITED STATES OF AMERICA.

Boston, the Bowditch Library.

... Academy of Arts and Sciences.

... Society of Natural History.

Cambridge, Mass. U.S., Harvard University.

... Harvard College, Observatory.

New York, State Library.

Philadelphia, American Philosophical Society.

... Academy of Natural Sciences.

United States, Naval Observatory, Washington, D. C.

Washington, the Smithsonian Institution.

Yale College, Newhaven, Connecticut.

SOUTH AMERICA.

Buenos Ayres, Public Museum, per Dr Burmeister.

(*All the Honorary and Ordinary Fellows of the Society are entitled to the Transactions and Proceedings. See notice at foot of next page.*)

The following Institutions and Individuals receive the Proceedings only:—

ENGLAND.

London Meteorological Office, 116 Victoria Street.

London, Editor of "Nature."

London, The Meteorological Society, Westminster.

London Mathematical Society.

London Geologists' Association.

Cornwall Geological Society.

Liverpool Literary and Philosophical Society.

Newcastle Philosophical Society.

Oxford Ashmolean Society.

Scarborough Philosophical Society.

Whitby Philosophical Society.

SCOTLAND.

Botanical Society of Edinburgh.
Geological Society of Edinburgh.
Scottish Meteorological Society.
Geological Society of Glasgow.
Philosophical Society of Glasgow.

COLONIES.

Canada, Literary and Philosophical Society of Quebec.
Library of the Geological Survey, Canada.
China Branch of the Asiatic Society, Hongkong.
Literary Society of Madras.
Superintendent of Government Farms of Madras Presidency.
North China Branch of the Asiatic Society, Shanghae.
Royal Society of New South Wales.
Royal Society of Victoria.
Linnean Society of New South Wales.
University of Adelaide, South Australia.

CONTINENT OF EUROPE.

Abbé Moigno, Rédacteur des "Mondes," Paris.
Ém. Alglave, Directeur de la Revue des Cours Littéraires et Scientifiques, Paris.
Bordeaux, Société de la Géographie Commerciale.
Cassel, Verein für Naturkunde.
Catania, Accademia Gioenia di Scienze Naturali.
Chemnitz, Naturwissenschaftliche Gesellschaft.
Cherbourg, Société Nationale des Sciences Naturelles.
Ghent, University Library.
Gratz, Naturwissenschaftliches Verein für Steiermark.
Leipzig, Naturforschende Gesellschaft.
Milan, Società Crittogamologica Italiana.
Modena, Società dei Naturalisti.
Pisa, Nuovo Cimento.
Utrecht, Provinciaal Genootschap van Kunsten en Wetenschappen.

UNITED STATES.

California, Academy of Sciences.
Cincinnati Observatory.
Cincinnati, Society of Natural History.
Connecticut, Academy of Arts and Sciences.
Massachusetts, Peabody Academy of Science, Salem.
Wisconsin, Academy of Sciences, Arts and Letters.

NOTICE TO FELLOWS.

All Fellows of the Society who are not in Arrear in their Annual Contributions, are entitled to receive Copies of the Transactions and Proceedings of the Society, provided they apply for them within Five Years after Publication.

Fellows not resident in Edinburgh must apply for their Copies either personally, or by an authorised agent, at the Hall of the Society, within Five Years after publication.

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