TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XXXIII. PART I.—FOR THE SESSION 1885–86.

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[*Issued May 31, 1887.*]

TRANSACTIONS.

I.—*The Atomic Weight of Tungsten.* By JOHN WADDELL, B.A., D.Sc.

(Read 7th June 1886.)

From the results obtained by BERZELIUS in his experiments with tungsten, the number 189·28 is calculated as the atomic weight of that metal. Many later investigations have been made, in which uniformly a lower figure was arrived at, in the majority of cases very nearly 184, which has therefore been regarded as the atomic weight.

Some of the determinations were made with a special view to the support or overthrow of a theory. DUMAS, for instance, wished to discover whether the atomic weight of tungsten was exactly double that of molybdenum. In such cases it would be natural to suppose that special care would be taken to secure purity of materials. DUMAS obtained the numbers 96 and 184 as the atomic weights of molybdenum and tungsten, but he seems to have distrusted his results; for he remarks—"Is it necessary, however, to conclude from this discussion, that some simple ratios of the kind which one used to admit between molybdenum and tungsten cannot exist? I do not think so" (*Ann. Chim. Phys.*, [3] 55, 144).

In studying the literature of the subject, I felt that no security was afforded by the various experimenters of the purity of the compounds made use of by them. SCHEIBLER (*Jour. Prac. Chem.*, 83, 273), apparently considers a single recrystallisation of the sodium salt all that is necessary. This salt was used as a starting point, and though the further working up into barium metatungstate may have tended to purification, SCHEIBLER certainly does not prove that he has thus freed his tungsten from molybdenum and silicon. BERNOULLI (*Pogg. Ann.*, 111, 573) considered that he had obtained pure tungstic acid, though he gives no indication of an attempt to free from molybdenum. He prepared his tungstic acid from wolfram, which contained silica and niobic acid, from which the tungstic acid was separated by solution in ammonia. He found that the reduced metal, when ignited in a current of chlorine, was entirely changed into a volatile chloride. This test proves the absence of silica, which would have formed a residue, but has no bearing whatever upon the question, whether or no molybdenum were present; for molybdenum pentachloride, and tungsten hexachloride are produced under precisely the same conditions, and are very similar in appearance and characteristics.

Similar criticisms could be made on all the investigations to which I have referred. It appeared to be too readily assumed that recrystallisation would insure purity, no certainty being afforded either by a trustworthy mode of separation of all possible impurities, or by tests showing the absence of admixture.

In my work, I kept specially in view the elimination of silicon and molybdenum, at the same time using all precautions to free from other impurities as well.

The reason for paying special attention to silicon and molybdenum, was that these are very similar in many respects to tungsten, and are moreover very liable to exist in ores of that metal. The presence of both was proved in some scheelite with which preliminary experiments were made, and their elimination was found to be no very easy matter.

The material used as starting point in the subsequent investigation was commercial tungsten, a dark grey powder, containing 94–98 per cent. of the metal. The impurity was supposed by the manufacturers to be chiefly lower oxides of tungsten, and in particular molybdenum was said to be absent. The employment of metallic tungsten as starting point is not to be recommended, owing to the great difficulty of oxidation. The most convenient material, doubtless, is sodium tungstate, which can be obtained readily enough. In my work, however, some interesting facts with regard to oxidation were brought into prominence, for I tried several methods, the results of which are here given.

The first mode of oxidation tested was continued boiling with aqua regia, but at the end of a week a great deal of the metal was still unacted upon, and the method was considered unsatisfactory.

Another plan tried was mixing the metal with nitre, and throwing the mixture, small portions at a time, into a red hot crucible. Moderately good results were thus obtained; and the process would, I think, work fairly well on the large scale where proper furnaces and crucibles are available. On the small scale, the chief objection is that the crucible itself is attacked. A third method furnished good results, and considerable attention was devoted to the determination of the most favourable conditions. The process consisted in the ignition of a mixture of potassium chlorate, sugar, and metal. After an extended number of trials, the proportions found most satisfactory were 9 : 3 : 5.

This is the most expeditious method of oxidation, for 200 grammes of metal can readily be ignited in one operation, and two-thirds of the total quantity are oxidised in a few seconds. I found it most convenient to mould the materials into a cone, sufficient water having been added to make this operation possible. The cone having been placed on a large iron tray was lighted at the apex, so that the propagation of the combustion was downward. By this precaution, loss of material caused by spurting was in large measure avoided. The objection to this mode of oxidationis the great difficulty experienced in recovering tungstic acid from the great bulk of salt. If, however, appliances are at hand for evaporation, with strong hydrochloric acid on an oil bath and long continued drying at 120°, the process might be made fairly successful, otherwise the difficulties encountered in recovering the tungstic acid far outweigh the advantage gained by rapid oxidation.

The method which I found most convenient and satisfactory was the ignition of the metal in a porcelain tube, a current of air being passed over the red hot mass. The metal must form only a thin layer, else the oxidised material will obstruct the passage of air, for tungstic acid occupies nearly four times the volume of metallic tungsten. In my experiments, sulphurous fumes were produced in considerable quantity, evidently showing the presence of sulphide in the commercial tungsten. The vapours were absorbed by caustic soda solution, and in this solution traces of molybdenum were found. After about six hours' heating, the metal was, as a rule, to a great extent oxidised, and had acquired a green colour. If the operation be long continued, say for thirty hours, the compound produced possesses a canary-yellow colour, and is practically tungstic acid.

The green colour usually obtained was doubtless due to the presence of partially oxidised and totally unoxidised material. In ordinary cases complete oxidisation was not considered necessary, for in subsequent operations the small amount which had escaped the action of the hot air was easily eliminated. The greater part of the tungstate obtained was prepared from the impure tungstic acid just mentioned. The mode of treatment was as follows :—

The partially oxidised mass was fused in a platinum basin with one half of its weight of sodium carbonate. The fusion was complete in a few minutes. The fused mass after cooling was disintegrated with water, and the tungstate dissolved out, leaving the unoxidised metal as residue. It was found that the platinum basin was somewhat attacked, and subsequent examination proved the presence of lead in the tungsten residue. The solution of tungstate was boiled in a large silver basin with addition of ammonia carbonate, the latter being employed to separate out any silica or iron and aluminium hydrates possibly present. A very slight precipitate formed and was filtered off. The filtrate was again treated with ammonium carbonate, the process being repeated till no more precipitate was produced. The liquid was then evaporated to dryness, and thus a mixture of sodium tungstate and carbonate was obtained. For my purpose the presence of carbonate was no disadvantage, and I did not crystallise out the tungstate.

As I have already remarked, the method of purification hitherto almost universally adopted was recrystallisation. I employed the method of fractional precipitation as more likely to give a decided test. In the case of all the precipitations being alike, the probability of purity would nearly amount to a certainty, for it is extremely unlikely that the proportions of the impurities would be the same in each of the precipitates. In case of a difference appearing in the precipitations, it is natural to suppose that those most widely separated would exhibit the greatest dissimilarity, and that the middle fractionations would be similar and practically pure.

It was known that the great bulk of my salt was tungstate. Any impurities more readily precipitated than tungsten ought to be concentrated in the first fractionation, while anything less readily precipitated would be chiefly found in the last portion.

The question presented itself in what form it was best to precipitate the tungsten. From the soluble tungstate it was possible to throw down either an insoluble tungstate or tungstic acid. As I wished to determine the atomic weight of the metal by reduction of tungstic acid, it is evident that if the former method were employed it would be well to produce a tungstate which could be easily decomposed and changed to tungstic acid. Such a compound is mercurous tungstate, which loses its mercury on ignition. From some experiments tried with mercurous nitrate as precipitant, I decided that this method of fractionation was not so feasible as the precipitation of tungstic acid direct by means of hydrochloric acid.

Before proceeding with the fractionation, however, I freed the sodium tungstate so far as possible from molybdenum. The method employed was that recommended by Rose. Sufficient tartaric acid was added to a solution of the alkaline salt to prevent the precipitation of tungstic acid on acidification with hydrochloric acid. A stream of hydrogen sulphide was then passed through the solution, and an appreciable though small precipitate of molybdenum sulphide was thus obtained.

The filtered solution containing about 300 grammes of solid tungstate had a blue colour, owing to the presence of a small quantity of one of the lower oxides of tungsten. The liquid was decolorised by the passage of a current of air, and was then ready for fractional precipitation. It is to be noted that though sufficient tartaric acid had been added to the solution to prevent precipitation of tungstic acid by a small quantity of dilute hydrochloric acid, yet a considerable excess of the latter was capable of producing quite a precipitate. This fact is important, otherwise the method must have been greatly modified. It is further to be noted that the precipitation was gradual, hence there was ample opportunity for the liquid to be well mixed, and the precipitation was therefore not of a local character. The mode of procedure was the following:—

The liquid was boiled in a porcelain basin, and to it a measured quantity of pure hydrochloric acid was added. The boiling was then continued till the precipitate was formed in sufficient quantity, when the contents of the basin were removed to a large beaker and allowed to settle. The supernatant liquid was after some time decanted, and the precipitate washed once or twice by decantation. The precipitate was set aside for future use, the separate decantations were united and evaporated to the bulk of the original solution, and acid added as before. This process was repeated till eleven fractionations were obtained.

The first of these had a dark green colour, probably because the current of air had not thoroughly oxidised the liquid. The subsequent precipitates, as far as the seventh, were pale yellow; while the remaining fractionations were dirty green, and not so finely divided as those which preceded. These differences were, I think, caused by the fact that for the final precipitations the liquid required to be boiled down to small bulk, in order to obtain a reasonable quantity of tungstic acid. The tartaric acid under these circumstances probably exercises a reducing action, and in the small quantity of liquid the precipitate was in all likelihood aggregated by the continued boiling.

The third precipitate was purified, and used for estimation of the atomic weight. The precipitate was washed several times by decantation, and then repeatedly on a filter. It was then dissolved in pure ammonia, and after filtration reprecipitated by addition of pure hydrochloric acid. The solution in ammonia had a triple purpose. It insured the oxidation of the precipitate in case any lower oxide of tungsten were present. It separated any slight trace of impurity not soluble in ammonia. It aided the washing from sodium salts, for the solution and reprecipitation presented fresh surfaces to the action of the wash water. The washing (which was with water containing a little hydrochloric acid in order to prevent the precipitate running through the filter) was continued till the filtrate yielded no residue on evaporation, and a test portion of the precipitate gave only a slight indication of sodium by the spectroscope. The precipitate was afterwards dried and ignited in a current of air. The tungstic acid thus obtained had a beautiful pale canary-yellow colour, and was quite uniform in appearance. It was reduced in a porcelain tube by a current of pure hydrogen, the temperature being gradually raised from below dull redness to the highest obtainable by a strong blast in a Fletcher's furnace.

Assuming that tungstic acid has the composition expressed by the formula

WO_3, and that it is reduced to metallic tungsten by ignition in a stream of hydrogen, the weight of oxygen lost, compared with that of the tungsten left behind, gives all the data required for determining the atomic weight of the latter. As a mean of three experiments made with the precipitate described, I obtained the number 184·5. The tenth and eleventh precipitates were united, and treated in the same way as the third fractionation. It was difficult to wash, and the tungstic acid had a greenish tinge, and altogether did not appear so pure as what had been before obtained. During the reduction there was a slight volatilisation. The atomic weight, estimated in this sample, was 183·7. The seventh precipitate was subjected to similar treatment. Its appearance and behaviour were quite satisfactory, and the atomic weight calculated was 184. I made a number of other determinations which need not be described. Some of them were made with tungstic acid not freed from molybdenum by sulphuretted hydrogen. In a number of cases a slight volatilisation was observed, and in these the atomic weight estimated was low. So uniformly was the volatilisation noticed when the number obtained was below 184, that there is every reason to believe that where no volatilisation was observed none actually occurred.

The uniformity in result was greatly in favour of purity in the tungstic acid; but as silica is with difficulty precipitated from a silicate by means of hydrochloric acid, and as tungstic acid exhibits the same characteristics, I thought it advisable, if possible, to prove the absence of silica independently. This I did by Marignac's method of separation, which consists in fusing with hydrogen potassium sulphate. Tungstic acid forms a tungstate under these circumstances, while silica remains unaltered, and is left undissolved when the fused mass is treated with water. As no residue was left after solution, the absence of silica was established.

Subjoined is a table of the estimations described above:—

Fraction.	WO_3.	W left.	O_3 lost.	Atom weight.	Remarks.
III.	1·4006	1·1115	·2891	184·55	No volatilisation.
	·9900	·7855	·2045	184·37	”
	1·1479	·9110	·2369	184·59	”
VII.	·9894	·7847	·2047	184·00	”
X.	4·5639	3·6201	·9438	183·69	Slight volatilisation.

Of the above determinations, those denoted III. are the most trustworthy, from the fact that there are three concordant estimations. The mean of these is 184·5, which is the atomic weight of tungsten calculated from oxygen equal 16.

This reduced to 0=15·96 gives W=184·04.

I made a couple of determinations of the specific gravity of two specimens of metallic tungsten, the first A had not been specially freed from molybdenum, the second B was a portion of III. The determinations were made in a specific gravity bottle of 10 c.c. capacity. The bottle was weighed empty, and full of distilled water which had been boiled to expel air. The bottle was then dried and weighed again, then some tungsten was introduced, and another weighing was made. The metal was afterwards covered with water, and the bottle placed under an air-pump, in order to extract the air enclosed in the powder. The bottle was then filled with water, and a weighing again made, and as the water evaporated slowly, but perceptibly, weighings were taken when the meniscus in the capillary touched two fixed marks scratched on the stopper. How nearly the readings agreed, is shown in the table below.

Specimen.	Wt. tungsten.	Reading.	Weight water displaced.	Specific gravity.
A	1·0353	Upper mark	·0566	18·249
		Lower mark	·0566	18·249
B	·7187	Upper mark	·0382	18·772
		Lower mark	·0383	18·765

Most reliance should be placed upon the result obtained with B, because of the assured purity of the metal, and because a first trial would be liable to experimental errors. All such errors tend to give a low specific gravity.

It was not thought necessary to make another determination, because the number agrees so well with those obtained by the best authorities. The highest determination is that given by Roscoe, viz., 19·13; the majority of experimenters give figures lying between 18 and 19, while a number so low as 16·54 has been obtained. The density seems to depend upon the method of preparing the metal.

My work has been confirmatory of the commonly accepted atomic weight and specific gravity of tungsten; its chief value lies in the fact that the subject was attacked in a way so far as I know not hitherto attempted, and the corroborative evidence is therefore all the more trustworthy.

My thanks are due to Prof. Crum Brown and Dr Gibson for valuable suggestions and kindly assistance.

II.—*On Dew.* By Mr JOHN AITKEN.

(Read 21st December 1885.)

The immense amount that has been written on the subject of dew renders it extremely difficult for one to state anything regarding it which has not been previously expressed in some form. It has been examined over and over by minds of every type, and from every point of view; so that every possible explanation of the different phenomena seems to have been given, and so many passing thoughts recorded, that from the literary point of view the whole subject seems exhausted. As a necessary result, these different treatises are in many respects contradictory; and it would be quite impossible to construct anything like a consistent explanation and account of our subject, from the very voluminous writings of those who have treated it from the purely literary point of view, and whose ideas have been evolved from their inner consciousness, according to what seemed to them the fitness of things, and without questioning nature as to the truth of their conclusions. On the scientific side of the subject, however, the writings are not so voluminous, and additions to it are still required to enable us to determine which of the many conflicting opinions are correct.

In ancient times it was thought that the moon and stars had an important influence on dew, probably because there is most dew on those nights when these orbs shine brightly on the earth; thus confusing two things which have a common cause, and making one the effect of the other. ARISTOTLE placed the knowledge of this subject far in advance of his time. He defines dew to be humidity detached in minute particles from the clear chill atmosphere. The Romans, led by the writings of PLINY, returned again to the primitive idea that dew fell from the heavens. This idea retained its position during the course of the Middle Ages. Then began an endless variety of theories, such as, that the air is condensed into water by the cold, that the moon's rays caused it, and so on.

In the beginning of the eighteenth century clearer ideas began to be formed, and a reformation took place, in which, as in most reformations, the swing of the pendulum went to the extreme on the opposite side. Dew was no longer believed to descend from the heavens, for GERSTEN advanced the idea that it rose from the earth; and in this opinion he was followed by M. DU FAY and Professor MUSSCHENBROEK, the latter, however, afterwards made some observations which caused him to change his opinion. GERSTEN was led to think that dew rose from the ground, because he often found grass and low shrubs moist with it, while trees were dry. M. Du Fay followed up these observations with experiments, made by placing sheets of glass at different heights from the ground. He found that dew formed on the lowest pane first, and only appeared on the highest at a later hour; he also found that the lowest pane collected most moisture. Other observers gave somewhat different explanations of the phenomena connected with dew; but owing to a want of clearness, the subject did not advance much till the masterly *Essay on Dew* by Dr Wells made its appearance.

Dr Wells' experiments were so simple, and his interpretation of the different phenomena connected with dew so clear, that he has been justly considered the great master of this subject. In his Essay he struck a medium between the two previous theories as to the source of the moisture that forms dew. He did not think with the ancients that it fell from heaven, nor with Gersten that it rose from the earth, but that it was simply condensed out of the air in contact with the surfaces of bodies cooled by radiation below the dew-point of the air at the place. This opinion has, so far as I am aware, been generally received up to the present time.

Some experiments I have recently made on this subject have caused me to differ entirely from Dr Wells as to the source of the vapour that forms dew. As everything written by Dr Wells is, so to speak, stereotyped and final, there seems to be the greater reason that any of his conclusions that seem doubtful should be carefully criticised and fully investigated; I shall therefore give an account of the experiments that have caused me to differ from so great an authority.

Dr Wells thought that almost all the moisture deposited as dew at night was taken up by the air during the heat of the day; so that, according to his idea, vapour ascended from the earth during the day, and again descended and became condensed as dew on the surface of the earth at night. My observations have led me to the very opposite conclusion. All my experiments indicate that dew, on bodies near the surface of the earth, is almost entirely formed from the vapour rising at the time from the ground; at least this would appear to be the case generally in this climate, to which my experiments have been confined.

After Gersten gave his reasons for supposing that dew rose from the ground, and Du Fay extended the subject, Dr Wells combated their conclusions, and successfully showed that their experiments did not prove that vapour rose from the ground, and that all the phenomena adduced in favour of their theory could be equally well explained according to his own. With regard to Du Fay's reason for thinking that dew rises from the ground—namely, that it appears on bodies near the earth earlier than on those at a greater height—he says:* "But this fact readily admits of an explanation on other grounds, that have already been mentioned. 1. The lower air, on a

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**An Essay on Dew*, by William Charles Wells, p. 109.

clear and calm evening, is colder than the upper, and will, therefore, be sooner in a condition to deposit a part of its moisture. 2. It is less liable to agitation than the upper. 3. It contains more moisture than the upper, from receiving the last which has risen from the earth, in addition to what it had previously possessed in common with other parts of the atmosphere." Then he goes on to give reasons why vapour cannot be rising out of the ground, but adds, that some of it must be from this source, as bodies near the surface of the ground get dewed sooner than those higher up, though equally cold with them, but says, "the quantity from this cause can never be great," and proceeds to give his reasons, which are not altogether satisfactory, and need not be quoted here. He then sums up as follows:—"These considerations . . . . . warrant me to conclude that on nights favourable to the production of dew, only a very small part of what occurs is owing to vapour rising from the earth; though I am acquainted with no means of determining the proportion of this part to the whole."

I shall now proceed to detail the observations which have caused me to differ from the conclusion so distinctly set forth by Dr Wells in the above quotations. I need not say that all my experiments only confirm the conclusions of that observer as to the formation of dew—that is, as to the conditions most favourable for the deposition of moisture on the surfaces of bodies during dewy nights, while the earth is radiating heat into space. The point on which we differ is as to the source of the vapour that condenses on the radiating surfaces—a point which Dr Wells admits there were no facts to determine, his own opinion being formed by experiments that did not bear directly on the subject.

When I began to doubt the truth of the generally received opinion as to the source of the vapour, I found a difficulty in beginning my investigation, as it was not easy to arrange experiments to give a direct answer to the question. My intention at first was to test, by means of a delicate hygrometer, the humidity of the air at different heights from the ground and under different conditions. This plan had, however, soon to be abandoned, owing to the impossibility of making anything like accurate observations with any instruments at present in use.

For some time I have had in my possession a hair hygrometer constructed by Chevallier of Paris. This form of instrument is perhaps one of the best for the purpose; yet on making a few test experiments with it, for the special purpose under consideration, its indications were found to be nearly valueless. For instance, if the instrument was removed from saturated to drier air, and again replaced in the saturated, it was impossible to get the pointer back again to the same position on the scale; and as the amount of dryness it would be required to measure was a very small degree removed from saturation, the error in the indications might be greater than the actual amount of dryness.

Then again, all such hygrometers, as well as wet and dry bulb thermometers, will have their indications affected by radiation; they will surround themselves with an envelope of cooled air, as there is but little wind during the time the observations require to be made. Their indications would therefore be of little value, and investigation by means of them had to be abandoned.

What first caused me to doubt the present theory, and led me to suppose that dew is formed from vapour rising from the ground, was the result of some observations made in summer on the temperature of the soil at a small depth under the surface, and of the air over it, after sunset and at night. On all occasions in which these temperatures were taken, the ground a little below the surface was found to be warmer than the air over it. It is evident that, so long as these conditions exist, and provided the supply of heat is sufficient to keep the surface of the ground above the dew-point, there will be a tendency for vapour to rise and pass from the ground into the air, the moist air so formed will mingle with the air above it, and its moisture will be condensed, forming dew wherever it comes in contact with a surface cooled below its dew-point.

These considerations suggested another method of experimenting than by the use of hygrometers. If vapour is really rising from the ground during night, it seemed possible that it might be trapped on its passage to the air, and that this might be accomplished by placing over the soil something that would check the passage of the vapour, while it allowed the heat to escape. To carry out this idea, I placed over the soil shallow boxes or trays, made of tinplate and painted. These trays were 3 inches (76 mm.) deep, and more than a foot (305 mm.) square in area; they were placed in an inverted position over the soil to be tested.

The action of these trays will be somewhat as follows:—Supposing the roof of the small enclosure formed by the covering tray is not by the passing air or by radiation cooled below the temperature of the ground. Then evaporation will cease when the air between the tray and the ground is saturated, and no dew will collect on the inside of the enclosure. But if the tray is cooled below the temperature of the ground, vapour will condense on the inside, and more vapour will rise from the ground to supply its place, and this will go on so long as the ground is the warmer of the two. The effect of these trays will be very much the same as if there was no enclosure, and the air over the grass was nearly saturated, motionless, and of a lower temperature than the soil. But it is evident the trays will check the evaporation on most nights, on account of the slow circulation inside, and also on account of the air inside being always nearly saturated, which is not the case outside the enclosure, so that under most conditions it seems likely there will be less evaporation under the trays than outside them. This will be particularly the case on those nights when there is wind, and the air is not saturated, a condition which seems to be very frequent in our climate at ordinary elevations. We must remember that the air may not be saturated when dew is forming; and the dew-collecting surface requires to be cooled below the temperature of the air before it collects moisture.

In experimenting with these trays different kinds of ground were selected, and the trays placed over them after sunset, that is, after the earth had ceased to receive heat, and the heat-tide had begun to ebb. They were generally examined between 10 and 11 P.M., and again in the morning.

DEW ON GRASS.

Confining our attention to the trays placed over grass, the result of the experiments was that, on all occasions yet observed, there was—1. Always more moisture on the grass inside the trays than outside. 2. There was always a deposit of dew *inside* the trays. 3. There was often a deposit outside the trays, but the deposit on the outside was always less than on the inside, and sometimes there was no deposit outside when there was one inside.

Now I think these facts prove that far more vapour rises out of the ground during the night than condenses as dew on the grass. This excess is evidenced by the greater amount of moisture on the grass inside the trays than outside, and by the amount of dew condensed inside the box. Under the ordinary conditions found in nature, this excess is carried away by the wind and mixed up with the air, while some of it is deposited on bodies further away from the ground. It should be noticed that the inside of the tray was more heavily dewed than the outside. This shows there was a higher vapour tension inside than outside the enclosure, which proved that the vapour rising from the ground outside the tray had got mixed up with drier air, as it did not form so heavy a coating of dew as the inside air, even though it had the advantage of a slightly lower temperature than the inside, on account of it being the side of the metal from which the heat was radiating.

It may be as well to notice here some objections that may be made to this way of testing the point. It may be said, that though so much vapour does rise under these trays, yet if they were removed and the grass freely exposed, the vapour would not rise, and that the vapour rises because the tray keeps the ground under it warm. Observation certainly shows that the ground under the trays is kept slightly warmer than outside them. At night a thermometer is higher on the grass under the tray than on that outside, and next morning the ground at 3 inches below the surface is from 1 to 2 degrees warmer under the tray than outside its influence. This objection to the protecting influence of the trays has an appearance of reason about it; but if we examine the facts, I think it will be admitted that instead of being an argument against this method of experimenting it is rather a reason for it. We must remember the tray does not heat the ground; it does not add anything to its store of heat, and enable it to evaporate more moisture; it simply prevents so much of its store of heat escaping. Now heat escapes from the ground at night in two ways—first, by radiation, and second, by absorption—to supply the latent heat of evaporation. From the area covered by these trays radiation goes on much as at other places; the painted metal will radiate as much heat as the grass, but evaporation is checked, as there is but little circulation under the trays; and further, there is the heat recovered by the condensation inside the box. It would thus appear that the reduced evaporation and heat of condensation will be the principal causes of the higher temperature inside than outside; so that the trays, instead of increasing the evaporation, would rather seem to decrease it; and that the lower temperature outside is due to the greater evaporation there taking place, as both surfaces are exposed to the same loss by radiation.

There is an objection that might be made to the whole theory that dew is formed from vapour rising from the ground. It might be urged that it is impossible for the vapour to rise from the ground, and that these trays interfere with the conditions existing in nature. On a cold clear night, for instance, when the grass gets cooled before the dew-point, it might be said that it is quite impossible for the vapour to rise up through it, as it would be all trapped on its passage to the surface by contact with the cold blades, and that the trays placed over the grass prevent this condensation by stopping the radiation from the grass, and thus they allow the vapour to come up.

A little explanation will, however, show this objection to be groundless. On a dewy night no doubt the top of the grass is at a temperature below the dew point, and if we may take the temperature of a thermometer placed on the grass to be the same as that of the grass, which we may do without sensible error, if we then remove the thermometer and place it among the stems of the grass, the thermometer will rise; and if we place the bulb among the stems close to, but not in the ground, we shall find it to be very much warmer than at the surface. On dewy nights I have frequently found it as much as 10 to 12 degrees warmer. From this we see that the warm air diffusing upwards with its burden of vapour only meets with a very small amount of surface cooled below the dew-point, so that the greater part of the vapour is free to escape into the air.

Fairly considered, I think these trays more nearly represent natural conditions than might at first sight appear. Indeed, precisely similar results have been observed with natural conditions. If we examine plants with large blades, we shall often find, on dewy nights, that those leaves which are close to the ground have their under surfaces heavily dewed, while their upper surfaces are dry. The effect of the trays is very much the same as that of these large leaves on a perfectly calm night. The only difference is, the trays will lose more heat on account of their better conducting power, and more vapour will be condensed under them than under the bad conductor, while the temperature of the soil will be more nearly reduced to what it would have been if no large close surface prevented the free evaporation.

The experiments described were made in August and September, when the ground was very dry, owing to the unusually small rainfall during the previous months. On all occasions the inside of the tray was dewed, however dry the soil, and the inside was always more moist than the outside.

After these experiments were made, another method of testing the point under investigation suggested itself, and though, unfortunately, rather far on in the season for satisfactory work of this kind, I at once proceeded to carry it out, as it afforded a means of checking my previous experiments with the trays; but by this time October had arrived, and the conditions had very much changed. The temperature had fallen considerably, and the rainfall had greatly increased the humidity of the soil.

It is very evident that if vapour continues to rise from the ground during dewy nights, as well as during the day, the ground giving off vapour must lose weight. If this could be shown to be the case, it would prove in a more satisfactory manner than the previous experiments that vapour does rise from the ground during night, and that, therefore, dew on bodies near the surface of the ground is really formed from the vapour rising at the time, and not from the vapour that rose during the day.

In the first week of October experiments were begun to test this point, by weighing a small area of the surface of the ground, before and after dew had formed, to see whether the ground continued to give off vapour or not while dew was forming. For this purpose a number of shallow pans 6 inches (152 mm.) square and $\frac{1}{4}$ inch (6·3 mm.) deep were prepared. One of these pans was selected, and a piece of turf slightly smaller was cut from the lawn and placed in it. The pan with its turf was then carefully weighed with a balance sensitive enough to turn with $\frac{1}{4}$ grain; but in experiments of this kind, which must be done quickly, accuracy of only one grain was aimed at, lest the time required for more accurate weighing might cause loss of weight by evaporation. To prevent loss from this cause, the weighing was done in an open shed.

The turf was cut at sundown, and when dew began to form. The earth was removed from it till it weighed exactly 3500 grains (226·79 grammes). The pan with its turf was then rapidly restored to the lawn, and put in its place, where the turf had been cut out, and in as good contact with the ground as possible. The pan and turf were then brought back, the under side of the pan carefully cleaned and dried, and all weighed again to make sure nothing was lost in the manipulations; after which it was again restored to its place in the lawn, and left exposed while dew was forming. A few experiments were made in this way, in all of which the ground was found to lose weight. For instance, on the 7th October, the small turf freely exposed to the sky at 5.15 P.M., when weighed again at 6.30 P.M., was found to have lost 5½ grains (0·356 grammes), and by 10.15 P.M. it had lost 24 grains (1·555 grammes). Fuller particulars of these experiments will be given further on.

In making these experiments, the first thing done was to sink two thermometers in the ground, one to a depth of 3 inches (76 mm.), the other to a depth of 1 foot (305 mm.), and to place a third thermometer on the surface of the grass. Readings were taken when the experiment began, and again when the pans were removed for weighing. During the time the turfs were exposed, generally about 5 hours, the soil at 3 inches below the surface lost from 2 to 5 degrees, while at 12 inches the loss was small. No doubt part of the heat was lost by radiation, but in grass-land, where the surface of the soil is protected by a fairly good non-conductor, much of the heat will be spent in evaporating the moisture.

These experiments prove clearly that under the conditions then existing, the soil loses weight, and that vapour really rises from the ground even while dew is forming; therefore the dew then found on the grass must have been formed out of the vapour rising from the ground at the time. The dew on the grass was, in fact, so much of the rising vapour trapped by the cold grass. The blades of grass acted as a kind of condenser, and held back some of the vapour which would have escaped into the air.

It must not be supposed that these experiments in any way contradict the well-known observations of WELLS and others who have worked at the subject. It has long been the custom to expose different substances to radiation during the night, and to estimate the amount of dew on different nights by the *increase of weight* due to the moisture collected on them. It must be noticed that the conditions of the two sets of experiments are quite different. In those for estimating or measuring the amount of dew, the collecting body must not be in heat communication with the earth, an essential condition being that it shall receive no heat by conduction from surrounding bodies; whereas, in the experiments with the turf, the essential condition is that the body experimented on shall be in as good contact with the ground as possible. The result of these two conditions is, that in the former, the exposed surface loses heat by radiation into space, and soon gets cooled below the temperature of the air, and when cooled below the dew-point, dew collects upon it; while in the latter case the exposed surface is in good heat communication with the ground, and tends to keep hotter than the other surface; then being always moist it tends to give off vapour, which diffuses away from the hot ground and escapes into the air above, but in part is trapped by coming into contact with the cold grass.

The experiments were generally stopped at night. It would be of no use to let them go on till morning, unless one were in attendance at sunrise; for the early morning heat radiated from the sun and sky would cause an increased evaporation, and make the loss appear too great. On one occasion, however, when the morning was dull, weighings were made, and the soil was then found to have lost weight during the late night and morning.

The following simple observation is sufficient to convince us that, under the ordinary conditions of our climate, vapour is almost constantly escaping night and day from soil under grass. Go out any night, but it is best when terrestrial radiation is strong, place one thermometer on the grass, and push another under its surface, among the stems, but it need not be into the soil, and note the differance in temperature. As an example, I found, at 10.45 P.M. on the 10th October, this difference to be as much as 18·5 degrees. The thermometer on the surface of the grass was 24°, while the other, only about $1\frac{1}{2}$ inches underneath it, and not in the soil, was as high as 42°·5, the temperature of the air at the time being 32°·5. Of course, this difference varies, and is not always so great as on this occasion, when the sky was clear and the air still. An experiment of this kind causes us to doubt the value of the radiation observations made by comparing the readings of a thermometer placed on the grass with the temperature of the air in the screen; because the temperature of the thermometer on the grass varies greatly according to its position. If its bulb is supported near the tips of the stems, the temperature is much lower than when it is allowed to press the grass close to the ground, because in the latter position it receives a good deal of heat from the earth.

It might be objected that these experiments having been made late in the year, and when the soil was damp, they do not prove that evaporation would take place in summer when the soil was dry. Other considerations, however, lead us to suppose that this nightly evaporation does go on even after a continuance of dry weather, though I have no direct experiments to prove it, other than those made with the inverted trays. But I find that soil, after it has been kept for some time in a house, and when it looks dry and incapable of supporting vegetation, still gives off vapour, and saturates the air over it. This was shown by placing over some dry-looking soil a glass receiver, in which was hung the hair hygrometer. The instrument soon showed an incease of humidity inside the receiver, and after a time indicated saturation. To check the reading of the hygrometer, it was quickly removed and placed in saturated air, when it was not found to change its reading.

Now as soil, even when it appears dry, tends to give off vapour, and saturate the air in contact with it, it is evident that under most conditions of our climate the vapour tension at the surface of the ground, amongst the stems of the grass, must, owing to the higher temperature, be very much greater than at the tops of the blades; and as the air and vapour are warmer, they tend to rise and diffuse themselves, and so come into contact with the colder blades at the surface, where the moisture gets deposited as dew.

Having proved that, under the conditions existing during the experiments, the ground was giving off vapour during the night, I then proceeded to test the value of the observations previously described, and which were made by placing shallow trays over the grass, in order to see if those experiments were of any value. A small tray, similar to those used in the earlier experiments, was prepared. It was made to fit tightly into one of the shallow pans, in which, as before, was placed a small turf cut from the lawn. After the turf and its pan was weighed, the tray was placed over it, and the whole removed, and put in its place in the lawn. This was done at the same time as the other experiment previously described, in which the turf and pan, after being weighed, was freely exposed to radiation and evaporation. The result was that the tray was found to check the evaporation. The inside of the covering tray was dewed very much like another one placed over undisturbed grass. The turf covered by the tray lost only 6 grains (0·388 grammes) during the five hours, or about $\frac{1}{4}$ of the amount lost by the one freely exposed to the air. This shows that the trays check the evaporation; we may therefore conclude that the amount collected by them is less than would be given off by the exposed parts of the grass.

There seems to be reason for supposing that the amount lost per unit of area in these experiments, with the freely exposed turfs, is too low an estimate for the loss of the lawn at the parts where it was undisturbed, because the under sides of the pans were not in good contact with the ground beneath them. The experimental turf would not therefore be so warm as the rest of the ground, and its evaporation would therefore be less. Most of the heat was conveyed upwards towards the experimental turfs by the rising vapour, which condensed on the under sides of the pans on which the turfs rested, as they were always found to be dripping wet underneath when removed from the soil.

The question now comes to be, Does this evaporation take place from grass-land on all nights and in all weathers? So far as my observations at present go, evaporation is constantly going on, however strong the radiation. On all nights on which the inverted trays have been exposed, dew has collected on their inner surfaces. There is, however, an indirect way of testing this point which may be noticed here, as it is specially applicable to observations on grass land. As soil capable of supporting vegetation tends to saturate the air in contact with it, it will be admitted that so long as the soil is hotter than the air in contact with the grass, vapour will tend to diffuse upwards. Now I find by placing a minimum registering thermometer on the grass, and another on the top of the soil among the stems of the grass, that there is always a difference between the minimum *on* and *under* the grass, often amounting to a considerable number of degrees, this difference being greatest on nights when radiation is strongest, and least when windy and cloudy. It is only as the day advances that the temperature *on* the grass approaches that *under* it; this is caused by the upper thermometer being heated by solar radiation sooner than the lower; but as the air is by this time drier, there is no tendency for it to lose moisture by contact with the colder soil, though some of the dew condensed on the grass will, after it evaporates, diffuse downwards, and condense on the soil. It may therefore be safely concluded that, on almost all nights in this climate, vapour does rise from grass-covered land, and it is this vapour that we see as dew on the exposed surfaces of the grass.

Dew on Soil.

While the experiments previously described were being made on ground covered with grass, parallel ones were made on bare soil. The inverted trays placed over soil always showed a greater amount of condensed vapour inside them than those over grass. Sometimes there was a heavy deposit of dew inside, while there was none outside. This would be owing to the soil radiating directly to the trays, and to the amount of heat brought up and conveyed to the trays by the vapour. The temperature of the trays was thus in some cases kept above the dew-point of the air outside.

Experiments were also made by weighing a small area of the surface soil, to see if it also lost weight like the grass-land during dewy nights. One of the small pans was covered with a thin layer taken from the top of the soil. The pan and its soil was then weighed and put on the surface of the bare ground at the place where the soil had been taken out. It was left exposed the same time as the other trays with the turfs. On weighing, the soil was found to have lost 23 grains (1·490 grammes) in five hours, or nearly the same as the turf. Alongside this pan was placed another one of the same area, and with the same weight of soil, but covered with a small tray, to see whether the covering trays decreased the evaporation from soil as well as from grass. The result was the same as was found with the turf—a decrease in the evaporation. The protected soil lost only 8 grains (0·518 grammes).

The following are the details of a few of the experiments on grass-land and on bare soil made on different evenings, and show the temperatures and the loss of moisture per 0·25 square foot, or 0·023225 square metre, during the experiments :*—

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* Throughout this investigation I have adhered to the Fahrenheit scale, as it is the one generally used for meteorological purposes in this country, and because it possesses what appears to me practical advantages over the Centigrade scale. The degrees are of a more suitable size, and combine ease in reading with accuracy. This scale also avoids a fruitful source of error, experienced by many, when taking readings above and below zero.

OCTOBER 7, 1885. 5.30 P.M.

		Grass.	Soil.
Temperature of soil,	3 inches below surface,	47°·5	46°
" "	12 " "	47°·5	44°

AT 6.30 P.M.

Grass exposed,	lost	5½ grains,	or	0·356 grammes.
" under tray,	"	3 "		0·194 "
Bare soil exposed,	"	5½ "		0·356 "
" under tray,	"	2¾ "		0·178 "

AT 10.30 P.M.

		Grass.	Soil.
Temperature on surface,		36°·5	40°
" of soil,	3 inches below surface,	44°·5	42°·1
" "	12 " "	47°	44°

Grass exposed,	lost	24 grains,	or	1·555 grammes.
" under tray,	"	6 "		0·388 "
Soil exposed,	"	23 "		1·490 "
" under tray,	"	8 "		0·518 "

OCTOBER 12, 1885. 5.30 P.M.

		Grass.	Soil.
Temperature of soil,	3 inches below surface,	44°	45°
" "	12 " "	44°·5	43·5

AT 10.15 P.M.

		Grass.	Soil.
Temperature on surface,		31°·5	35°·5
" of soil,	3 inches below surface,	42°	40°·2
" "	12 " "	44°	43°·5

Grass exposed,	lost	30 grains,	or	1·944 grammes.
" under tray	"	6 "		0·388
Soil exposed,	"	22 "		1·425
" under tray,	"	6½ "		0·421

There was a little wind on this occasion, and very little dew formed. During the night the min. *on* the grass was 28°·5, under it 41°; and it was not till 10 o'clock next morning that the thermometer *on* the grass was as high as the one *under* it.

The following reading were taken about the 20th October, the exact date unfortunately is omitted in note-book :—

5.15 P.M.

Temperature of air—Dry bulb, 42·5; Wet bulb, 40.

		Grass.	Soil.
Temperature on surface,		39°	42°
" of soil,	3 inches below surface,	46°	48°
" "	13 " "	46°·3	46°·1

AT 10.40 P.M.

Temperature of air,	Dry bulb, 38°	Wet bulb, 36°	at 4′ 0″
" "	" 33½	" 33°	near ground.

		Grass.	Soil.
Temperature on surface,		31°	34°
" of soil,	3 inches below surface,	44°·2	43°
" "	12 " "	46°	46°

Grass exposed,	lost	9 grains,	or	0·583 grammes.
" under tray,	"	8 "		0·518 "
Soil exposed,	"	16½ "		1·069 "
" under tray,	"	9 "		0·583 "

NEXT MORNING AT 9 A.M.

		Grass.	Soil.
Temperature on surface,		39°·5	39°·5
" of soil,	3 inches under surface,	42°·5	40°·5
" "	12 " "	45°·5	45°·5

Grass exposed,	lost	19 grains,	or	1·231 grammes.
" under tray,	"	13 "		0·842 "
Soil exposed,	"	30 "		1·944 "
" under tray,	"	18 "		1·166 "

These figures cannot be supposed to represent anything definite, they only indicate a condition of matters which has not been previously observed. They show that evaporation in our climate is going on night as well as day during dry weather, but the extent to which it takes place cannot be gathered from these observations, as they are far too few for the purpose—too few alike with regard to seasons, humidities, and exposures; nor can the proportionate amount of evaporation from bare soil and from grass-land be arrived at from the weights given. These readings can only be considered true for the place and moisture at the time of the year when the experiments were made. For instance, the inverted trays over soil in my early experiments always indicated a larger evaporation from soil than from grass, while the later ones did not. But the early experiments were made over soil freely exposed to sunshine during the whole day, while the later ones were made at a place less freely exposed, on account of the situation where the first experiments were made being too far from the place of weighing. It is evident the amount of sunshine will be an important factor in this nightly evaporation, as it will greatly determine the amount of heat stored up during the day, and available for evaporation during the night.

I extremely regret the season was so far advanced before these experiments were begun, as most of the weather suitable for the purpose was past. I have, however, endeavoured to check my results as well as possible. Still I feel that what has been done is only preliminary. Similar experiments would require to be made during the whole year, to determine whether this evaporation is constantly going on or not in fair weather, and to determine its amount under different conditions. The varieties of soils, of humidities, and exposures are so great that an enormous number of experiments would require to be made to determine with any degree of accuracy the amount of evaporation that takes place from any large tract of land.

The temperatures of the soil and of the air during these experiments were not high, but we must remember they were taken in October. In summer we have to deal with much higher temperatures and greater vapour tensions, and therefore the possibilities of heavier dews. On the 18th August I find the temperature ½ inch under the surface of the soil at 4 P.M. was 82°, at 3 inches underneath it was 72°, the temperature of the air being 66°. At 9 P.M., at 3 inches deep, the temperature was 60° under grass and under bare soil. The temperature *on* the grass was 45°, while a thermometer placed *on* bare soil was 52°. Next morning the temperature at 3 inches under grass was 56°, and at the same depth under soil 52°. The soil at 3 inches down had thus lost 20 degrees during the night, and that nearer the surface would have lost a good deal more. Much of this loss would be spent in evaporating moisture. On this occasion it will be noticed that at night the difference between the temperature on the surface of the grass and on the bare soil was as much as 7 degrees, and this easily explains why the ground kept dry while the grass got wet.

So far as my limited observations go, evaporation is constantly going on from soil under grass, but on a few occasions it was doubtful whether the reverse process had not taken place, and vapour got condensed on the surface of bare soil. On one or two occasions in autumn, I observed soil which had been dry the previous day to be damp in the morning. The soil had evidently received an increase of moisture. But the question still remains, Whence this moisture? Came it from the air, or from the soil underneath? The latter seems the more probable source, as the higher temperature below would determine a movement of the moisture upwards by the vapour diffusing; and the surface soil being cold, the vapour would be trapped by it before it escaped into the air, in the same way as it is trapped by grass on grass land.

During summer it is difficult to trace the vapour condensed on the surface of the soil to its source, and to say definitely whether it came from the air or from the ground underneath. But on the morning of the 12th October I had an interesting opportunity of studying this question. During the night the radiation had been very powerful, the surface of the soil was greatly cooled, and a thin crust of frozen earth formed. After the sun had thawed the surface it was very wet. An examination of the soil before the sun had acted on it, showed that the vapour condensed near its surface had come from underneath. On lifting the small clods on the surface, it was observed that their under surfaces and sides, when close to each other, were all thickly covered with hoar-frost so thickly as to be nearly white, while the upper surfaces exposed to the passing air had but little deposited on them,—the interpretation of which seems to be, that the vapour rising from the hot soil underneath had got trapped in its passage through the cold clods. Its presence underneath and on the sides of the clods was an evidence that the moisture was on its passage from the ground, when it met with the cold surface which imprisoned it.

This hoar-frost on the sides and under the clods could not be due to vapour condensed from the passing air, because the upper surfaces of the clods had scarcely any deposited on them, and that in spite of the fact that the upper surfaces would be the colder, as they were those from which the radiation was taking place. It seems probable that even the vapour condensed on the upper surfaces of the clods was part of the vapour escaping from the soil, and was not taken from the passing air.

The occasions when the earth is most likely to receive vapour condensed upon it from the passing air, are not on clear nights when the radiation is strong, but rather when after strong radiation and cooling of the surface the weather changes, becomes cloudy, and a warm moist wind blows over the land. Occasions of this kind are seen most frequently after frosts, and undoubtedly much moisture is then condensed on the soil, but the moisture so condensed is not what we call dew.

Dew on Roads.

There is considerable difference among works on dew as to the absence of dew on roads, but almost all agree in stating that it is never formed on roads; and the presence of dew on grass, while none is visible on roads, is generally attributed to the greater radiating power of vegetation over that of the material of which our roads are composed. Now I find that this statement as to facts is wrong, and the explanation is also inaccurate. Dew really does form on roads in great abundance on dewy nights, and the material of the road is practically as good a radiator as the grass.

The reason why it is generally said that dew is not seen on roads is owing, not to the less radiating power of the stones, but to the fact that dew has not been looked for at the proper place. The blades of grass are practically non-conductors of heat, while stones conduct fairly well. The result of this is that we are not entitled to look for dew on the upper surfaces of stones, as on grass, but it must be sought for on their under sides, because the stones are good conductors, and the vapour tension under them is much higher than at their upper surfaces, owing to the higher temperature of the air laden with moisture rising from the ground. If we examine a gravel walk on a dewy evening, we shall find the under sides of the stones, especially those near the solid ground, to be dripping wet; and we may occasionally see isolated patches of stones wet on the upper surface, probably due to an openness in the ground at the place permitting a free escape of vapour.

Another reason why the upper surface of the gravel does not get wet, is that it is in good heat communication with the ground; the stones are thus kept warm; and as a good deal of the vapour rising from the ground is trapped by the under surfaces of the stones, the vapour which escapes these surfaces is not enough to saturate the air at the temperature of the exposed surfaces of the gravel. The following temperature, taken at 10 P.M. on the 25th September, will give an idea of the difference in temperature on the surface of grass and on gravel, and show why no dew is formed on the top of the stones while it collects on the grass. A thermometer placed on the surface of the gravel was 34°, while one placed near it, but on grass, was 30°, or 4° lower. At the surface of the soil under the grass the temperature was 40°, and it was almost exactly the same temperature at the bottom of the gravel which was $1\frac{1}{2}$ inches deep.

We see from the above that hot vapour, rising from the ground under grass, ascends till it comes into contact with the cold blades, and is condensed on their exposed surfaces; whereas on the gravel road the under sides of the stones are nearly as cold as their exposed surfaces, and much of the warm vapour gets condensed under them, while the vapour which escapes to the surfaces has its dew-point lowered by mixing with the surrounding air, and the upper surfaces of the stones being in good heat communication with the ground, are not cool enough to condense this vapour and form dew.

A simple manner of studying the formation of dew on roads is to take, say, two slates, and place one of them on the gravel and one on a hard part of the road. If these slates are examined on a dewy night, their under sides will be found to be dripping wet, though their upper surfaces and the road all round them are quite dry. This experiment also shows us that under most conditions of our climate vapour does rise from hard dry-looking roads on dewy nights.

In studying questions of this kind, and for showing the importance of the heat communicated by the earth to the radiating body, the following experiment may be useful. Place on the grass, soil, or road, a slate and a piece of iron, say an ordinary 7 lb. weight. Alongside of these place another slate and weight; but instead of the latter resting on the ground, elevate them a few inches on small wooden pegs driven into the earth. If we examine the surfaces of these bodies on dewy nights, the following will be the general result. While the grass all round is wet with dew, we shall find that the upper surfaces of the slate and the weight resting on the ground keep dry, and those of the elevated ones get wet like the grass. the reason for this is that the bodies on the ground as well as the elevated ones are constantly losing more heat by radiation than they receive by absorption; but those in contact with the ground have heat communicated to them by conduction and by the condensation of vapour on their under surfaces; their temperature is thus prevented from falling as low as that of the elevated bodies, which only receive heat from the passing air; the latter are thus cooled more by radiation than those on the ground. Bodies out of heat communication with the ground thus tend to cool more than those in contact with it; and while the former get cooled below the dew-point and collect dew, the latter keep warmer than the dew-point, and thus tend to keep dry, or if wetted to become dry again.

These considerations suggest a simple method of testing whether the surface of any particular part of the ground is giving off vapour or not. It is very evident that so long as the temperature of the surface of the soil is above the dew-point of the air, vapour will rise from the ground, and that if the surface is cooled by radiation below the dew-point, evaporation will cease, and vapour will condense upon it. In order to test this, all that is necessary is to place on the ground, and in good heat communication with it, some substance that is a good conductor, and shows dewing easily. A piece of metal covered with black varnish does well. It is painted black, not in order to radiate copiously, but because black shows any deposit of dew most quickly and easily. So long as this test surface keeps dry while in contact with the ground, the soil round it must be giving off vapour, because the temperature of its surface is higher than the dew-point. But if the temperature of the ground falls below the dew-point it will collect moisture, and this test surface will collect dew also, and will thus tell us that the surrounding soil is receiving moisture. In experiments such as these we are simply converting a small area of the earth's surface into a condescending hygroscope, and our test surface tells us whether the earth's surface at the place is cooled by radiation below the dew-point or not. So long as no dew forms on the test surface vapour is being given off.

These test surfaces must not be large, at most only two or three centimetres, because if large they would check the free passage of the vapour to the air, and so prevent the soil under them from cooling to the same amount as the surrounding ground; and further, it is difficult to get good contact with large surfaces, without which only a part of the test surface keeps clear, while the part not in contact gets dewed, even though the temperature of the surface of the ground is above the dew-point. This was confirmed by observations made on a frosty night. On lifting each test plate, it was observed that the soil was frozen to it under the clear parts, and no soil adhered under the parts that were dewed. In my experiments I have used small copper discs covered with black varnish, ordinary glass mirrors, and also small black mirrors, in order to get rid of the objection to ordinary silvered mirrors, namely, that they might not be good radiators. On no occasion up to the beginning of November have I yet seen dew on any of these at night, but it is difficult to say whether dew had not formed on them on some mornings, as the air was thick and misty, and the deposit then observed might have fallen as fine rain.

The changes in temperature of the surface of the soil due to radiation, give rise to a downward movement of heat during the day, and to an upward movement of it during the night. These heat changes will be accompanied by corresponding movements of the moisture in the soil. During day, after the surface is heated, the vapour tension being higher above than below, a downward movement of moisture will take place; and at night this process will be reversed, the tension of the vapour at a depth being greater than near the surface, the vapour rises and condenses in the colder soil. Part of the latent heat so liberated by the rising vapour is spent in radiation from the surface, part in evaporating moisture, and a little in heating the air cooled by contact with cold grass, &c.

We may conclude that, owing to the heat received during the day, and probably also to the internal heat of the earth, vapour continues to rise from the ground long after the sun has set, and in many conditions the vapour continues to rise the whole night; but under certain others it seems probable that the reverse will occasionally take place, and vapour condense on the ground. This is most likely to take place soonest on bare soil, especially on those parts of it that are in bad heat communication with the ground underneath. But over grass-land in most conditions of our climate, when dew is forming, the evaporation seldom seems to stop, but goes on night and day, on account of the surface of the soil being protected by the grass from losing its heat so quickly as the bare soil. The escaping vapour rises till it meets with some surface not in good heat communication with the ground, and which has been cooled by radiation, in the manner set forth by WELLS and others. These remarks refer to weather when dew is most abundant, as in spring, summer, and autumn, and do not apply to those conditions in which a warm vapour-laden air is brought over a cold ground.

DEW AND WIND.

It is well known that during windy nights no dew is formed. We previously knew that wind acts in two ways to prevent the formation of dew; to these two ways we must now add a third. Wind prevents the formation of dew—(1) by mixing the hot air above the surface of the ground with the air cooled near its surface, this tends to prevent the air being cooled to the dew-point; (2) the wind by its passage over the surface of radiating bodies prevents these surfaces being cooled much below the temperature of the air; the wind thus tends to prevent the air in contact with these surfaces being cooled below the dew-point; and (3) wind blowing over the surface of the ground rapidly carries away the vapour rising from the soil, and mixes it up with a large quantity of drier air. The wind thus tends to prevent an accumulation of damp air near the ground.

To illustrate this third effect of wind, let us use the observations made on the evening of October 12. The sky was clear, and there was a considerable amount of radiation, but a slight wind was blowing. The bare soil in the test-pan lost 22 grains and the corresponding turf lost 30 grains in about five hours. Almost no dew was formed on the grass, but trays placed over the bare soil and over grass had their inside surfaces covered with moisture, though not so heavily as was generally observed on dewy nights. The reason why so little dew formed on this occasion was, partly, that the wind prevented the temperature of the air near the ground falling as much as it would have done if it had been calm. In the screen the temperature only fell to 40°. On the grass, however, it fell to 31°·5, and on the soil to 35°·5; but a good deal depended on the exposure of the thermometer to the wind. From the above we see that, though wind was blowing, the thermometer on the grass fell a good deal below the temperature of the air, and showed a considerable amount of radiation. The wind apparently prevented the formation of dew on this occasion, principally by preventing an accumulation of moist air near the surface of the ground. The inverted trays showed that if the wind had fallen dew would have formed, because it formed in the still air under the trays. The deposit was not so heavy inside the trays on this occasion as was often seen in dewy nights, because the wind prevented the radiation cooling the top of the trays to the same extent as when it was calm.

DEW AND VEGETATION.

When I began to make observations on dew, one of the first things I did was to make a tour of the garden on a dewy night, and to examine the appearance of the plants. A very short survey was sufficient to show that something else was at work than radiation and condensation to produce the effects then seen. Let me briefly describe what I saw, and what at once struck me could not be explained by the ordinary laws of radiation and condensation. Certain kinds of plants were found to be covered with moisture, while others were dry. Many plants of the *Brassica* family were heavily covered with glistening drops; while beans, peas, &c., growing alongside them, were quite dry. Again, in clusters of plants of the same kind some were wet, while others were not; and not only so, but some branches were wet, while others on the same plant were dry. These differences were noticed to be quite irrespective either of their exposure to the sky, or to the probable humidity of the air surrounding them.

In illustration of this latter point, small clusters of dwarf French poppies may be mentioned. Most of the plants were quite dry, whilst others growing amongst them were dripping with moisture; and while some branches were dry, others on the same plant were studded with drops, and the general surface of the leaves in some cases wet. On examination of these plants next day, it was observed that those that were wet at night were all plants in vigorous growth, and the shoots that were dewed were those in which the vegetation seemed most active. It was also observed that it was always the same plants and branches that were dewed night after night during the short time the observations were made.

A closer examination of the leaves of broccoli plants showed better than any others that the moisture collected on them was not deposited in the manner we should expect if it had been deposited as an effect and according to the laws of radiation; nor was it deposited in accordance with the laws of condensation; indeed, every appearance was at variance with these laws. Examination showed that the moisture was collected in little drops placed at short distances apart, along the very edge of the leaf, while the rest of the leaf was often dry. Now, if the moisture had been condensed by cold produced by radiation, then it would have been most abundant on the upper surface of the leaf; but there would have been none on its windward edge. This is well seen when we expose a small glass plate on a dewy night; the windward edge is always dry, and the deposit is spread evenly over the rest of the plate up to the opposite margin, because the temperature of the air when it first strikes the plate is higher than the dew-point, and it has to travel over more or less of the surface of the glass before it is cooled enough to deposit its moisture. Again, if these drops on the edge of the leaf had been deposited according to the laws of condensation, then the moisture would have been deposited on the surface more in accordance with the distribution of temperature at the different points; the moisture would therefore have been more equally distributed, and not been in large isolated drops.

On further examining these plants, I placed the lantern behind the blade, and then observed that the position of the beautiful sparkling diamond-like drops that fringed its edge had a definite relation to the structure of the leaf; they were all placed at the points where the nearly colourless and semi-transparent veins of the leaf came to the outer edge, at once suggesting that these veins were the channels from which the drops had been expelled.

These isolated drops on the edges of the leaves were therefore evidently not dew, but an effect of the vitality of the plants. An examination of grass blades showed that they also tend to have large drops attached to them, while the rest of the blade is dry, and these drops were always found to be situated at certain definite points; they were always near the tips of the blades. *These large drops* seen on plants at night are therefore *not dew* at all, but are watery juices exuded by the plants.

Now this excretion of water by the leaves of growing plants is not a new discovery—it has been long well known. But what seems extremely curious is, that its relation to dew has never been recognised, at least so far as I am aware, and it must be admitted that it is one of considerable importance.

It is well known that plants transpire from their leaves an immense amount of moisture, which passes off in an invisible form. Prof. J. BOUSSINGAULT found that mint transpired 82 grammes of water per square metre in sunshine, and 36 grammes in shade; but if the roots of the plants were removed, they only transpired 16 and 15 grammes respectively. This simple experiment proves that the root sends into the stem of the plant a supply of water, that it acts as a kind of force-pump, and keeps up a pressure inside the tissues of the plant. This supply sent in by the root is in most conditions removed by means of transpiration from the surface of the leaves.

Now what will be the result if transpiration is checked, while the root continues to send forward supplies? It will evidently depend on two things—first, the pressure the root is capable of exerting before its action is stopped; and second, the freeness with which the water can escape from the leaves. If the root pressure is small, it will cease with the transpiration; but if it is great, the sap will be forced into the plant, and if nature has provided any outlets it will escape at these openings.

Dr J. W. MOOL* has given great attention to the subject, and has experimented on a number of plants. The method he employed in his researches was to place the leaves under the most favourable conditions for the excretion of drops, by diminishing the transpiration as far as possible, and by supplying them with water. He substituted for root pressure, a pressure produced by a column of mercury. Out of 60 plants experimented on by Dr MOOL, he found that the leaves of 29 excreted drops without being injected, 13 leaves became injected and excreted drops, and 18 became injected and did not excrete at all. He says that the excretion takes place by water-pores, and by ordinary stomata, while in some cases it occurs at surfaces possessing neither of these organs.

I have recently made a few experiments on this subject in its relation to dew. As, however, the season was far advanced before the experiments were begun, but little could be accomplished, for the activity of the plants was nearly over, and grass was almost the only plant possessing sufficient vitality for

* *Nature*, vol. xxii. p. 403.

experimenting. I however removed a branch of the poppy, which, during summer, had shown such a tendency to exude moisture, and connected it by means of an india-rubber tube with a head of water of about one metre. After placing a glass receiver over it, so as to check evaporation, it was left for two or three hours, when it was found to have excreted water freely—some parts of the leaves being quite wet, while drops had collected at other places.

The broccoli plants which had excited my interest in summer were also experimented with. A full-grown leaf was fitted into the apparatus, and the pressure applied. In a little over an hour it also exuded water, and soon got fringed with drops along its edge in exactly the same way that was observed on it in summer. Another leaf from the same plant, but much younger, being about one quarter grown, on being tested in the same way did not excrete at all, after the pressure had been applied for twenty-four hours. Here we have the same result as that noticed in summer—one leaf exudes, while another on the same plant does not.

If the water pressed into the leaf is coloured with aniline blue, the drops when they first appear are colourless, but before they grow to any size, the blue appears, showing that little water was held in the veins, but the whole leaf got coloured of a fine deep blue-green, like that seen when vegetation is very rank, showing that the injected liquid had penetrated through the whole leaf.

Most of my experiments on this subject were made with grass. I find that even in the middle of October, after having been severely frosted two or three times, which had probably reduced its vitality, it still exuded so abundantly that drops collected in air which was not saturated. A turf placed in a cellar, dry enough to keep glass quite free from dewy deposit, soon collected drops. These drops always appear near the tips of the blades; they are not exuded from every blade, and sometimes from only one on each stalk, but generally from more; and it is always from the blades that seem to have the greatest vitality, and are nearly, but not quite full grown. Sometimes it is the youngest blade that exudes, but if it is very small, it is the second youngest. As the blades grow old they cease to exude; but this seems to be due to some change in the blade at the point where it exuded, and not to a diminution of root pressure, as it exudes freely when the tip is cut off.

The question might be here raised, Are these drops really exuded by the plant? Are they not due to some condensing power possessed by the leaves, by the presence at these points of some substance possessing an affinity for water vapour, or some process by which they may extract moisture from the air? To get an answer to this question, I selected a small turf, placed over it a glass receiver, and left it till drops were excreted. Removing the receiver, a blade having a drop attached to it was selected. After being carefully dried, the tip of the blade was placed in a small glass receiver, so as to isolate it from the damp air of the larger receiver. This small covering glass measured about 10 mm. in diameter by about 15 mm. in height. Its open end was closed by means of a very thin plate of metal cemented to it. In the centre of this plate was pierced a small opening, of the same size and shape as the selected blade of grass. The tip of the blade was entered about 5 mm. into this small receiver, and to prevent moisture entering and coming in contact with the tip of the blade, an air-tight joint between the blade and the metal was made with india-rubber solution. The tip of the blade was thus isolated inside the small receiver in which the air was dry. The large glass receiver was then placed over the turf to prevent evaporation from the lower part of the blade, or the experiment was made in a room where the air was not very dry. After a time, generally some hours, the turf was examined. A drop was always found to have formed on the tip of the blade inside the small receiver, and this drop was, as nearly as could be judged, always as large as the drops formed in the moist air under the large receiver. It would thus appear that these drops are really exuded by the plant, and not extracted from the air.

These exuded drops seem to be almost entirely the result of root pressure, because if we cut off the roots, and place the stems in water, putting over all a glass receiver standing in water so as to saturate the air, and as a test that the conditions are favourable, placing a small turf alongside the cut grass under the receiver, we shall find that scarcely any drops make their appearance on the rootless stems, while those with roots have drops attached to them. Again, if we take one of these rootless stems, and attach it by means of the india-rubber tube to a head of water, it is found to exude drops at the tips of its blades in moist air in the same way as when it was attached to its roots.

These excreted drops are formed on grass on other than dewy nights. After rain, if there has been no wind, and the air near the ground becomes saturated, a rearrangement of the drops takes place. Some time after the rain has ceased, most of the blades will be found to be tipped with a drop at the same point as the exuded drop appeared at night—a position which no falling rain drop could keep. This tendency of plants to exude moisture explains why the grass is almost always wet during autumn. At that season evaporation is slow, and as the plants are constantly pouring in supplies to the drops, it takes a long time for the slow evaporation to overcome the wetting effect and dry up the grass.

The question as to what degree of humidity in the air is necessary before plants will exude drops, would seem to be greatly determined by the rate at which the supply is sent into the leaf. If the supply is greater than the evaporation from the whole surface of the leaf, the drop grows; but if the supply is less, it does not form, or if formed, it decreases in size. The rate of the supply will evidently depend on the kind of plant and the amount of its vital activity at the time. The formation of drops on plants that exude moisture will therefore depend on the rate of supply, the humidity of the air, and the velocity of the wind. It is not easy to get a satisfactory experimental answer to this question, on account of the soil near the grass tending to moisten the air over it. A small turf placed in an elevated position in the centre of a room has been observed to have drops on it, when there was a difference of more than one degree between the wet and the dry bulb thermometer hung alongside. As the drops are exuded at the tips of the blades, it is probable the air in contact with them was not much moistened by the small area of soil underneath.

These observations entirely do away with the explanation usually given of the tendency of grass to get wet early and heavily on dewy nights. It has generally been explained by saying that grass is a better radiator than most substances, and therefore cools more, and sooner, than other bodies. We now see that those drops that first make their appearance on grass are not drops of dew at all, and their appearance depends, not on the laws of dew, but on those of vegetation. Hence the varied distribution of moisture on plants and shrubs on dewy nights.

We have seen that much of the moisture that collects on plants at night does not form like dew on dead matter. Dead matter gets equally wet where equally exposed, and the moisture does not collect on it in isolated drops, as it does on plants. Those drops which appear on grass on clear nights are not dew, and they make their appearance on surfaces that are not cooled to the dew-point. If the radiation effect continues after these drops have been forming for some time, true dew makes its appearance, and now the plants get wet all over their exposed surfaces in the same manner as dead matter. This latter form of wetting or true dew is of rarer occurrence than we might at first imagine. On many nights on which grass gets wet, no true dew is deposited on it; and on all nights, when vegetation is active, the exuded drops always make their appearance before the true dew; so that when we walk in early evening over the wet lawn, it is not dew that we brush off the grass with our feet, but the sap exuded by the plant itself. The difference between these exuded drops and true dew can be detected at a glance. The moisture exuded by grass is always excreted at a point situated near the tip of the blade, and forms a drop of some size, which may form while the rest of the blade is dry, but true dew collects evenly all over the blade. The exuded liquid forms a large glistening diamond-like drop, whereas dew coats the blade with a fine pearly lustre.

I feel that the dissecting hand of science has here done an injury to our poetic feelings. Every poet who has sung of the beauties of nature has added his tribute to the sparkling dew-drop, and BALLANTINE in his widely-known song has taught a comforting lesson from the thought that "ilka blade o' grass keps its ain drap o' dew." No doubt the drop of dew to which the poets refer is the large sparkling diamond-like gem that tips the blades of grass, and which we now know is not dew at all. While, however, our interpretation of nature has changed, the teaching of the poet remains, and the sparkling dew-drop may still teach the same comforting lesson. We must, however, change our views regarding the source of the refreshing influence. We may no longer look upon it as showered down from without, but as welling up from within—no longer as taken by the chill hand of night and given to refresh and invigorate exhausted nature; we must rather look upon it as suggesting that we are provided with an internal vitality more than sufficient to restore our exhausted powers, after the heat and toil of the day are past.

RADIATION.

I have said in a previous part of this paper that the surface of bare soil and of roads will radiate at night as much heat as grass. It may be thought I have said this simply because we do not now require that grass should be the more powerful radiator to enable us to explain its greater wetness on dewy nights. Though it is not now necessary to suppose that grass is a powerful radiator, yet there is nothing in the above experiments to prove it either a good or a bad one. It therefore seemed desirable that some definite experiments be made on this point, and also to determine the radiating powers of different substances at night, as this is always an interesting and important point in questions connected with the deposition of dew; and the radiating power of grass, though not the principal cause of its wetness at night, might be still considered to play a subordinate part.

We have already a great number of experiments on the radiating powers of different substances. Unfortunately most of the accurate measurements of this kind are from laboratory experiments, and do not appear to bear very directly on our subject. FRANKLIN'S early experiments, made with different coloured cloths placed on snow, seem to have given our ideas an unfortunate bias on this subject. From observing the different depths to which cloths of different colours sunk in snow, when exposed to solar radiation, he came to the conclusion that the dark colours absorb most heat, and this conclusion seems for long to have influenced our ideas. If the heat radiated and absorbed by a surface was composed entirely of visible rays, then no doubt the colour of a body would be an index of its radiating and absorbing powers.

But as the eye gives us no information about the greater proportion of the radiant energy, its indications are of no value in determining the radiating and absorbing powers of different surfaces.

Experiment shows that different surfaces have different absorbing powers for different rays. Melloni, for instance, found that white lead absorbed only about half as much heat from a Locatelli lamp as lamp black did, while it absorbed as much as lamp black when the source of heat was copper at 100° C. It is evident from this, that we cannot take the result of experiments made in the laboratory, and apply them to surfaces exposed to the temperature of the sky on a clear night. It may be possible that the radiating and absorbing powers of different surfaces may bear the same proportion to each other when the temperature is 0°, and they radiate into space, as when their temperature is 100°, and they are exposed to surfaces at the ordinary temperature of the laboratory. This may be so, but till it is proved we cannot apply these laboratory experiments to the cooling effect of radiation at night.

Some experiments on the radiating power of different substances exposed to a clear sky were made by Daniell. He used for his purpose two similar parabolic reflectors. In the focus of each was placed the bulb of a thermometer. In experimenting he turned the reflectors to the sky, and coated the bulbs of the thermometers with the substances to be tested. Comparing garden mould with black wool, his measurements show, from the average of three readings given by him, that while the black wool fell 9° below the temperature of the air, the mould fell only 6°. The difference between the radiating powers of chalk and black wool, as given by him, was not quite so great. There seems to be an objection to this method of experimenting. The different surfaces here lose more heat by radiation into space than they receive. To supply this loss, they receive more heat by radiation from the reflector than they give, and they also receive heat from the surrounding air, conveyed to them by connection currents. Now in the experiment as arranged by Daniell, the two surfaces will not receive the same amount of heat from the latter source. The wool surface will not have such a free circulation of air over it as the other one; it will therefore not receive so much heat, and its temperature will thus tend to fall lower.

It appeared that something more might be done in this direction, and on consideration it was thought that the radiation thermometers, described by me in a previous paper, might be suitable for the purpose. It may be remembered that the principle on which these radiation thermometers is constructed is, that a large surface is more highly heated than a small one by radiation during the day, on account of the absorbed heat being more slowly taken away by the passing air from the former than from the latter; and for a similar reason a large surface is colder at night than a small one, as the small surface receives more heat, per unit of area, from the air than the large one. The absorbing and radiating surface of these instruments is a *large* flat area, painted black, and its temperature is taken by means of a thermometer, with its bulb placed under the centre of the radiating surface.*

The construction of these radiation instruments has been altered, and those used in this investigation were made of metal in place of wood, as described in the previous paper, the radiating surface being a thin plate of metal, 14 inches (355 mm.) square. A thin metal tube is fixed close to and parallel with the under surface of the plate. One end of the tube terminates at the centre of the plate, and the other at the edge. The thermometer is placed in this tube with its bulb under the centre of the plate, and to prevent heat escaping or being absorbed at the back, a considerable thickness of cotton wool is placed under it. The instrument is practically a shallow box, 14 inches square by 2 inches (51 mm.) deep, packed with cotton wool. One of the flat areas of the box is exposed to radiation, and its temperature is taken by means of a thermometer placed under its surface. In the following I shall refer to this instrument simply as the thermometer box.

One of the advantages of this form of instrument for solar radiation experiments is, that the readings given by different instruments agree with each other, at least this is the case so far as my experience goes; and it is well known that the vacuum radiation thermometers are unsatisfactory in this respect, no two almost ever reading alike. For instance, the vacuum radiation thermometers used at the Indian Stations, when compared with another of the same pattern as standard, were in some cases found to differ as much as 15°, though they were exact copies of each other, and similarly exposed.† I find that when the different instruments of the kind used by me are compared they agree very well when of the same size. It is of course necessary that they be of the same size—this results from the principle of their construction. It seems possible that we might make boxes of different sizes, and from them determine the law of variation for size; so that, knowing the size of the surface used in any particular set of observations, we could determine what temperature its readings corresponded to in another instrument of a different size, or all readings might be reduced to a standard size, say the temperature of a very large surface.

I may mention that the temperature given by an instrument of the size here described when placed in sunshine is a good deal above that indicated by a vacuum thermometer, which had been carefully prepared for me by Casella of London. Generally the readings were about 12 per cent. higher.

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* Thermometer Screens, *Proceedings of the Royal Society, Edinburgh*, No. 117, 1883–84.

† Report of the Meteorology of India, 1879, by H. F. Blanford, F.R.S.

One objection to these large-surface radiation thermometers is that they are more affected by wind than the vacuum ones. If it is a question of solar energy we are considering, this certainly is an objection, but if it is one of climate it will scarcely be so. I need not say that for questions of terrestrial radiation at night the vacuum thermometer is of no use.

In using these thermometer boxes for determining the radiating powers of different surfaces at night the following method was employed:—Two precisely similar boxes were prepared, and their upper surfaces painted black. They were placed in an elevated position in the open air, commanding a clear view of the sky all round. They were first exposed without anything on their surfaces, to see if their readings were exactly alike. In constructing them care was taken to put the same amount of cotton wool in each, in order that their non-conducting powers and heat capacities might be the same, so that both might take the same amount of heat to warm them, and both lose the same amount of heat at the back. On trial both instruments were found to read alike when similarly exposed.

As the sky radiation is a rather variable quantity, it would not do, on most nights, to leave one of these test surfaces bare, and use it as a standard with which to compare the other, over which we have put the substance to be tested, because the uncovered surface will follow the changes in the radiation more easily than the other, and will change more, and sooner, than the one covered with the substance to be tested, particularly if the substance is a bad conductor. The method generally adopted was to place both surfaces as nearly as possible under the same conditions. For instance, the first substances tested were black and white cloths of different materials; of each kind a black and a white was selected, each pair being as much alike as possible, of the same material, of the same weight, and of the same texture. A black one was placed over one thermometer box, and the corresponding white one over the other. After a time the readings were taken, and the position of the cloths reversed, the black being placed over the box where the white was, and *vice versa*, and readings again taken. Then if radiation remained constant one of the cloths was removed, and the other compared with the black surface.

The following table shows the results of some experiments made on the radiating power of black and white cloths tried in this way. The readings were taken on the evening of the 14th November. The sky on the occasion was quite cloudless. The air was very dry, and had scarcely any movement—an unusually favourable condition for conducting experiments of this kind. The radiating surfaces were placed at a height of about one metre from the ground, and a protected thermometer for taking the temperature of the air was placed alongside at the same height.

Air.	Substance.	Radiation.	Substance.	Radiation.
35°	No. 1, black	28°	No. 1, white	28°
35°	„ white	28°·5	„ black	28°·5
35°·5	” ”	28°·5	” ”	28°·5
35°·5	Paint black	28°·5	” ”	28°·5
36°	No. 1, black	28°·5	Paint black	28°·5
36°	No. 2, „	29°	No. 2, white	29°
35°	” ”	29°·2	” ”	29°·3
35°	„ white	29°	„ black	29°
35°·5	” ”	28°·7	” ”	28°·8
35°·5	” ”	28°	Paint „	28°
35°	No. 3, white	27°	No. 3, black	27°
35°	„ black	27°	„ white	27°
34°·5	Paint „	26°·5	” ”	26°·5
34°·5	” ”	26°·5	” ”	26°·5

In the above table, the first column shows the temperature of the air at the height of the radiating surfaces. In the second and fourth columns are the substances whose radiating powers are compared, No. 1 being black and white cotton cloths, No. 2 merino cloths, and No. 3 thick woollen cloths. In the third and fifth columns are the temperatures of the radiating surfaces. The following was the manner of conducting the experiments:—Take the first on the table. A black cotton cloth was spread over one thermometer box, and a white cotton one over the other; after a time, when the readings were taken, the temperature of the air was 35°, the black cloth 28°, and the white one 28°. The black cotton was now removed from its box, the white one put in its place, and the black one where the white one previously was. This was done to check any error from difference of exposure to wind or difference in thermometer boxes. After a time the readings were taken, and found to be—air 35°, white cotton 28·5, and black cotton 28·5. The radiation of the cloth was then compared with the radiation from the black paint on the surface of the radiation box. This was possible on this night, as there was no wind, and radiation was fairly constant.

In my first experiments with black and white cloths, they were found to be cooled to an unequal amount; but as the cloths used on this occasion were what first came to hand, and happened to be of unequal thickness and texture, and as there was wind blowing at the time, the heating effect of the passing air acted unequally on the different cloths, and prevented them from being cooled to the same amount; hence the necessity of using cloths of equal texture in experiments of the kind, especially when wind is blowing.

It will be observed that these experiments do not show any difference in the radiating powers of white and black cloths; nor do they show any difference in the radiating powers of cotton, wool, and paint. All radiate equally well, and have their surfaces cooled to the same amount when exposed to the same radiation. It will be noticed that the temperature of the radiating surfaces varied during the experiments, and was from 6 to 8 degrees below the temperature of the air.

These experiments make no claim to any great degree of accuracy; the conditions under which they are made make it difficult to get correct results, as the readings have to be taken with the aid of a lantern in the open air on cold nights, and as special thermometers had not been prepared for the radiation boxes, the thermometers used had to be partly withdrawn from the boxes before reading; there may therefore be a slight inaccuracy in the temperatures given. The error from this cause is not likely to be more than a quarter of a degree, and if there had been any great difference in the radiating powers of the surfaces, it would have shown on a scale of 6 to 8 degrees.

The following table gives the result of a comparison made between the radiating powers of grass and garden soil, on a calm evening when the air was dry. One of the thermometer boxes was sprinkled over with the soil, and over the other was put a layer of cut grass just sufficient to conceal all the black surface, and pressed down so as to make as flat a surface as possible:—

Temperature of air.	Temperature of grass.	Temperature of soil.
34°	25°·5	25°
35°	27°	26°·5
35°	27°	26°
35°	27°	26°·5
35°	27°	26°·5

From the above it will be seen that the garden soil was colder than the grass on this evening. When the grass was removed from the box and the soil compared with the black paint on the other box, the soil was found to be a little colder than the black paint, but not so much as it was colder than the grass. The reason for the soil being colder than the black paint would appear to be due to the evaporation taking place from its surface; the dew-point at the time was very low, and the top of the soil showed signs of drying. Compared with grass, this was not the reason for the difference, as the grass was slightly damp. The higher temperature of the grass would rather appear to be due to the nature of its surface. The passing air would communicate more heat to its irregular surface than it would to the more even one of the soil. Grass and soil were compared on other evenings on which the air was not so dry, and the exposed surfaces had vapour condensed on them; on these occasions the two surfaces radiated almost equally well.

This comparison of the radiating powers of grass and soil gives no support to the idea that the greater wetness of grass on dewy nights is owing to its greater radiating power. The radiating powers of the two surfaces seem to be practically the same; and if neither grass nor soil received heat from the ground, the soil would cool lowest, because the grass in its natural condition would get more heat from the passing air, on account of its surface being irregular and in small pieces, as we know that small surfaces receive from this cause much more heat, per unit of area, than larger ones, this being particularly the case when there is wind. From which we see that the smallness of the blades in grass is really an advantage, and prevents their surfaces being cooled by radiation so much as they would be if they were larger.

The number of substances tested for their radiating powers at night is not so great as was hoped for, on account of the rare occurrence of evenings on which work of this kind can be done in this climate; for not only must the sky be free from passing clouds, in order that the amount of radiation may be as constant as possible, but the air must also be very dry, in order that the dew-point may be lower than the temperature of the cold radiating surfaces. If vapour gets condensed on the radiating surfaces, the radiation from the film of ice, or water, will interfere with the results. In making the experiments, a large sheet of glass was generally exposed alongside of the radiation boxes, to show if vapour was being deposited on the radiating surfaces. But even with this precaution we cannot be sure we are testing the radiating powers of some substances experimented on, because some kinds of matter have an affinity for water, and condense vapour on their surfaces from unsaturated air. As an example of uncertain results, I may mention a comparison made between salt and sugar. These two substances have been found by other observers to radiate very unequally at 100° C.—sugar radiating twice as much as salt. When tested at night, they were found to radiate equally well; but as both substances have an affinity for water, their surfaces would have a film of moisture over them, which would increase the radiating power of the salt, and thus make the test of no value.

Among the few substances that have been found to radiate less heat at night than a black surface is sulphur. On the night of the 7th December, when the air was very dry and the glass plate kept undewed, the following readings were taken:—

Temperature of air.	Temperature of black surface.	Sulphur.
27°	21°	23°
26°	19°	21°·25

The sulphur was sifted over the one thermometer box and the other left bare.

It will be observed that the black surface radiated one half more than the sulphur. This experiment suggests that a sprinkling of sulphur might be used as a protection to delicate plants on frosty nights, but whether it would pay or not experience alone can determine.

Polished tin was also tested, a sheet of tin being placed over one box, and another sheet painted black put over the other, so as to make the conditions of both boxes similar. The amount radiated by the tin was small; when the temperature of the black surface fell 7°, the tin only fell about 1°, more or less, according to the perfection of the polish of its surface.

For meteorological purposes the following observations made on the radiating power of snow will be useful. I regret that owing to the absence of snow so far this winter, I have only had one opportunity of making observations on this substance. In the following table will be found the readings given by the thermometer boxes, one of which was left bare, and gave the radiation of black paint, while over the other was put a thin layer of snow. This was done on the forenoon of the 10th December, and readings were begun shortly after mid-day, and taken from time to time till evening:—

Hour.	Air.	Black.	Snow.	Difference.
12·30 P.M.	28°	24°	21°	−3°
1 ,,	28°	24°·5	22°	−2°·5
2 ,,	26°·8	22°	20°	−2°
5 ,,	23°	15°·5	16°	+0°·5
5·30 ,,	21°	15°	15°·5	+0°·5
6·30 ,,	21°	13°	13°·5	+0°·5
8 ,,	19°	9°	10°	+1°

In the above table, the first column gives the hour at which the temperatures were taken. In the second column are the temperatures of the air; in the third are the temperatures of the black radiating surface; in the fourth are the temperatures of snow surface; and in the fifth column are the differences between the temperature of the snow and the black surface at the hour the readings were taken. The day on which this comparison was made was fine, clear, calm, and frosty, with the sun shining brightly. The radiating surfaces had a clear view of the sky, but were protected from the direct rays of the sun.

It will be observed that while the sun was high the snow surface was very much colder than the other; while the black surface only fell 4° below the temperature of the air, the snow fell 7°. As the day advanced, and the sun sunk towards the horizon, this difference decreased to 2°·5 at 1 o'clock and to 2° at 2 o'clock; and at 5 o'clock, by which time the sun had set, the snow was a little warmer than the black surface, a condition in which it remained during the evening.

The reason for the snow being colder than the black surface during the day would seem to be, that both surfaces radiate and absorb "dark heat" about equally well, both surfaces therefore throw off about the same amount of heat; but while this is the case, their absorbing powers for the heat of the sun are very different, and though the sun was not shining directly on the surfaces, yet there is a considerable amount of its heat reflected to the surface of the earth from the atmosphere overhead. Now a black surface absorbs most of this reflected heat that falls upon it, while the snow absorbs very little. Hence, while both surfaces are radiating about the same amount of heat, the black surface is absorbing far more than the snow, and thus keeps warmer. As the sun sinks, the amount of its heat reflected by our atmosphere gets less and less, and the difference in the temperature on the two surfaces diminishes; and when at last the sun is quite under the horizon the temperature of the two surfaces becomes nearly equal. It will be, however, observed, that they never become quite the same, the snow being generally about half a degree warmer than the other. The whole of this difference is not, however, owing to difference in radiating powers; the snow will tend to give a slightly higher reading on account of its surface being rougher than that of the paint, thus causing it to receive more heat from the passing air than the black surface. And, further, from the conditions of the experiments, the readings being taken during a falling temperature, and the snow not being a good conductor of heat, the thermometer under it will take longer to fall than the one under the blackened metal. From these conditions it seems probable that there is not much difference between the radiating and absorbing powers of snow and black paint at night, while the difference is very considerable during the day.

It has been suggested to me that this difference in the radiating and absorbing powers of snow and black surface, such as soil, &c., will enable us to explain a difficulty long felt, regarding the hour at which the diurnal variation of temperature begins in countries covered with snow. Over those parts of the surface of our globe, where there is no snow, the temperature of the air begins to rise *before* sunrise, whereas in snow-clad regions this change does not take place till the sun is above the horizon—the explanation would appear to be that where the surface of the ground is dark it absorbs the heat of the sun, and warms the air whenever the rays begin to shine into the air overhead; but where covered with snow it is but little warmed by these early reflected rays, and it is not till the sun gets higher and shines on the surface of the earth that its effects begin to be felt.

GENERAL REMARKS.

We see as a result of these experiments, that in our climate at least, water vapour is almost constantly rising from the ground, and this takes place from fallow land, from grass land, and from roads, even on nights on which there is heavy dew. There seems to be but little doubt that the tide of vapour almost always flows outwards from the earth, and ebbs but rarely, save after it has been condensed to cloud and rain. The question as to whether any surface is in a condition to lose or gain moisture on a dewy night depends on its more or less perfect heat communication with the earth. Those surfaces, such as soil, rock, stones &c., which are in good heat communication with the earth, tend to keep warm, and to lose moisture; while those surfaces not in good communication with the earth, such as leaves of plants, roofs of sheds, &c., tend to lose their heat, and gain moisture. This is the reason why grass tends to collect true dew, while stones on the ground remain dry. Grass is a bad conductor, and forms a non-conducting layer over the ground, preventing the earth from losing its heat. The inside of this covering is hot by contact with the earth, while its outside is cooled by radiation; and as the grass is a bad conductor, its exposed surface gets cooled by radiation to a lower temperature than the better conducting soil and stones; hence the appearance of dew on it, while the earth is dry.

Since vapour is constantly rising from the earth on dewy nights, it follows that any measurements of dew we may make ought not to be added to the rainfall, as the water so collected is in no sense a measure of the moisture returned to the earth at night, nor is it even a proof that any water is then returned. The amount of dew measured is simply a somewhat rough indication of the amount of moisture received by plants and other bodies not in heat communication with the ground; while the ground itself does not receive any, but is rather giving off vapour.

Dew is most copious during clear weather, and these experiments show us that this condition of weather has a threefold action in the production of dew —first, cloudless skies are necessary at night, in order that radiation may be strong, and the surfaces of bodies cooled low enough to condense the vapour; second, clear skies are necessary in order that a copious evaporation may take place under a hot sun during the day; and third, the same conditions are necessary that the ground may be highly heated by the sun, and a large amount of heat stored up during the day to be spent in evaporating an abundant supply of vapour during the night.

What are known as radiation fogs are generally supposed to be due to cold air flowing down, at evening, from higher levels to lower and warmer ones, and the mixing of the airs resulting in a foggy condensation. The more probable explanation now seems to me to be, that they are caused by the uprising of the hot air and moisture from the ground, mixing with the colder air above the grass, much in the same way as a fog is produced over a river in sunny weather when the water is warm. So far as my observations go, these fogs generally form over flat damp fields, after hot sunny days, and they have been seen where there was no high ground from which cold air could flow in.

There almost seems reason for supposing that much of the moisture collected on grass, and which looks like true dew, may under many conditions be fine rain from fog formed in the manner above described. Because the hot air and vapour rising through the grass will tend to form fog, where it mixes with the cold air near the upper part of the grass, and if there is little wind this fog will settle on the blades. Under most conditions this fog will not form above the grass, and will not therefore be visible. It will often not form above the blades, because the hot moist air may there meet with too much dry air to supersaturate it, but there seems reason for supposing that it will be often formed amongst the stems of the grass.

During frosts we have excellent opportunities for studying the condensation of the vapour of our atmosphere, because it remains in the position where it is condensed and is easily seen, being neither absorbed by the ground nor dropped from the plants, &c., on which it may be deposited. I took the opportunity afforded by two nights of this kind for observing two opposite conditions of the air, and I shall here describe the effects of the radiation on the nights of the 14th and 15th November. During the afternoon the canopy of clouds that had hung over the earth for some days was gradually drawn aside, and moved away southwards. By 5 P.M. on the 14th the sky was cloudless. There was only a very slight movement of the air from the north, the radiation was strong, and the air dry. These conditions continued all night, and the minimum thermometer in the screen fell to 25°.

Next morning the ground and the grass were frozen. It was what is called a black frost. There was no hoar-frost on the trees, and what little there was on the grass was irregularly distributed. All the little hollows, of about a foot square in area and under, had a deposit of hoar-frost, while the higher parts of the grass had none. As there was no wind, and only a slow movement of the air, this peculiar distribution would not be caused by the heating effect of the passing air on the higher and more exposed blades, but was probably owing to its dryness. The small hollows being less freely exposed to the circulation, the air in them became more moistened from the vapour rising out of the ground than the air a little higher up, where it got mixed with a larger amount of the dry air. The test surface on the ground was quite dry at 9 P.M., and also next morning, showing that the ground had been giving off vapour all night.

In contrast with this, let us now look at the condition of matters on the following morning. During the whole of the 15th the air remained calm and frosty, and the cold intensified during the night. On the morning of the 16th the minimum thermometer indicated a temperature of 19°. On this occasion we had a hoar or white frost. Grass, fences, shrubs &c., were all white, and the trees even to their top branches. The air on this occasion had evidently got cooled to near its dew-point, and moisture had condensed on almost every exposed surface, causing nature to present a remarkable contrast to its appearance on the previous morning, though both mornings were frosty.

We shall now refer in more detail to some of the points most worth noticing on these mornings. It was observed that the distribution of the hoar-frost on the grass on the morning of the 16th was the reverse of what it was the previous morning, the high blades on this occasion having rather a thicker coating of hoar-frost than those lower down. The reason for this was that the higher blades were exposed to the passing air, which on this occasion was saturated; whereas those on the hollows had to depend for their supply on what rose from the ground, which could not be much under the conditions, as the bottom of the grass and the top of the soil were cooled below the freezing-point, and most of the rising vapour would be trapped before it reached the surface. We may also note here that the test surfaces on the soil were quite dry, and that the slate and iron weight resting on the grass were free from deposit; while the elevated slate and weight, the grass, and almost everything else had a coat of hoar-frost, showing that the ground kept hot enough to give off moisture. An examination of the bare soil also showed that at most parts of its surface vapour was being given off. Wherever the contact with the ground underneath was good, no hoar-frost formed, and it was deposited only on the small clods that were lying on the surface; of course, there was plenty of hoar-frost on the under sides of the large clods, but none on their upper surfaces.

The slates and weights resting on the grass were frequently examined while the frost lasted, which it did till the morning of the 19th. During all this time the radiation was great, and the temperature very low. The minimum on different nights was as low as 12°·5, 17°·5, and 19°. During all that time, though the ground received no heat direct from the sun, and but little in any way from above, yet the supply from beneath was sufficient to keep the temperature of the surface above the dew-point, and the slate and weight in contact with the ground remained black amidst the surrounding whiteness.

Another peculiarity of mornings such as this—which are of frequent occurrence during winter—is the deposition of moisture on trees. During my observations in summer I never saw shrubs dewed to a height of more than a few feet from the ground, while in winter tall trees frequently have vapour deposited on them to the top branches. But as my observations in warm weather are very limited on this point, and owing to a simple wetting not being so conspicuous as hoar-frost, it is possible trees may occasionally get wet in summer with dew without its being observed. It however seems probable that it will be of much more frequent occurrence in winter than in summer, owing to the much longer absence of the sun during winter nights.

Radiation from Snow.*

In a previous part of this paper reference has been made to the radiating and absorbing powers of snow. In the experiment detailed, comparative readings are given of the temperature of snow and of a black surface exposed in shade on a bright day. The temperatures taken under the conditions then existing showed the snow to be much colder than the black surface. This conclusion has since been confirmed by a number of readings with the radiation thermometer under different conditions of climate. Of these observations it will only be necessary to give those taken on two days. The following temperatures were taken on the 19th January. In these tables the contents of the columns are arranged as in previous one.

Hour.	Air.	Black.	Snow.	Difference.
10.0 A.M.	20°·2	16°·2	12°·0	−4°·2
2.0 P.M.	30°·0	30°·0	26°·0	−4°·0
2.30 „	30°·1	30°·2	26°·4	−3°·8
3.30 „	30°·2	30°·2	28°·0	−2°·2

In the morning the sky was clear, but by 2 P.M. it became overcast, with a thin uniform covering of clouds; and at 3.30, it was beginning to snow. The next readings were taken on the 5th February.

Hour.	Air.	Black.	Snow.	Difference.
10.0 A.M.	23°·0	28°·8	25°·0	−3°·8
12.0 P.M.	25°·5	31°·1	26°·1	−5°·0
2.0 „	30°·0	34°·8	30°·0	−4°·8
3.30 „	31°·0	34°·5	30°·5	−4°·0
5.15 „	31°·5	29°·0	29°·0	−0°·0

On this occasion also the sky was overcast. In these two tables exactly the same result is recorded, the black on both occasions being again much warmer than the snow. These two tables are given, as they were taken under quite different conditions. When the temperatures given in the first table

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* Read March 1, 1886.

were taken, the air was warmer than the radiating surfaces, while in the second the air was colder than the exposed surfaces, except towards evening. A number of other readings were taken under different conditions, but with the same results; the snow was always colder than the blacker surface during day, while other observations made at night show their radiating powers are then very similar.

Part of the cooling produced by the snow surface shown in the above tables would be due to evaporation from the snow. The amount due to this cause was not great, as the difference between the wet and dry bulb thermometers was small at the time. I regret, however, that the readings of these instruments have been lost, so I write from memory. It will, however, be observed that the difference in the radiating powers of the two surfaces continued during the whole time the readings were taken on the 19th January, though the weather changed to snow, and the air at that time would be nearly saturated. The readings given in the tables therefore give the total cooling effect of the snow, which is produced by two causes—radiation and evaporation.

This small absorbing power of snow for heat, reflected and radiated from the sky during the day, must have a most important effect on the atmosphere, causing its temperature to be much lower when the ground is covered with snow than when free from it. So that when a country becomes covered with snow —other things being equal—it will be accompanied by a depression of the mean temperature of the air; and, further, as cold tends to produce a stable condition of the atmosphere, not creating the current disturbances of heating, it would appear that once a country has become covered with snow there will be a tendency towards glacial conditions.

But this poor absorbing power of snow is not the only way in which it tends to produce a glacial climate. Snow, in addition to being a bad absorber of the heat of the sky, is also a very poor conductor of heat. In illustration of this, let me mention a few temperature observations made while the ground was covered with snow during January last. On the 18th of that month there was about 5½ inches (140 mm.) of snow on the ground; the night was clear, and radiation strong. At 8 P.M. the temperature of the surface of the snow was 3°, and a minimum thermometer also on the snow showed that it had been at 0° at an earlier hour. Taking a maximum thermometer, the index was brought down below the freezing-point, and the bulb plunged through the snow down to the grass. On examining it a short time afterwards the index was at 32°. In confirmation of this reading, it may be mentioned that on removing the snow the top of the soil was found to be unfrozen. These observations showed that there was a difference of about 30° between the temperature of the top and the bottom of the snow—that is, a distance of 5½ inches.

During the night the temperature of the air fell 5 degrees lower, so that the surface of the snow would be kept about zero for most of the night, yet next morning at 9.45 the bottom of the snow was still at 32°. The temperature of the surface of the snow had risen at this hour only to 8°·5. These observations were repeated on other occasions when the coating of snow was thinner; there was then less reduction in the temperature of the surface, but on all occasions when the snow was a few inches deep, the surface of the soil remained at 32°. As the ground was frozen when the snow fell, it would appear that the earth's heat slowly thawed it under the protection of the snow, and the temperature of 32°, which was below the surface when the soil was frozen, gradually rose to it, where it of course stopped, and the rising heat was spent in melting the snow.

The protection afforded by the bad conducting power of snow is evidenced in our climate, by the amount of vegetation that takes place underneath it, on those occasions when we have had snow on the ground for a length of time. After the snow is gone, many plants are found to have grown, and some advanced nearly to bloom under its protection. This same influence may, however, be seen at work in a more marked manner in spring on the slopes of the Alps and other lands covered with snow all winter. As the snow recedes, and the surface of the earth is gradually laid bare, the vegetation is found to be in an advanced state—many of the flowers, if not in bloom, are just ready to open.

This bad conducting power of snow, compared with soil and rock, will evidently tend to lower the temperature of the air over snow-clad lands, as a few inches of snow in our climate conserves the earth's heat, and prevents its surface being cooled below 32°. The surface of the snow thus gets very much colder than the bare surface of the earth would have been if no snow had been on it, and the air is thus cooled much more over a snow surface than over one of bare earth.

To get evidence of the statement that the surface of a country covered with snow is much colder than it would have been if free from snow, we have only to examine the surface of the ground under the two conditions. The surface of snow covering the ground receives so little heat from the earth that it gets cooled by radiation to such an amount that it is almost always during frost cooled below the dew-point, and is covered with a heavy deposit of hoar-frost. During the late snow I have frequently noticed a thickness of more than 12 mm. of beautiful ice crystals of this kind deposited on its surface. But while the snow is cooled so much, and has this deposit on it, the surface of the soil where it has been laid bare keeps quite free from this deposit, as it receives sufficient heat from below to keep its temperature above the dew-point. The air over snow-clad lands is thus always in contact with a highly-cooled surface; whereas when there is no snow, the air rests on a warmer one.

Taking then these two things together, the bad absorbing power of snow and its small conducting power for heat, we see that when snow has once got possession of a land, the tendency to glacial conditions is greatly increased; and if it were not for the disturbing effects of heat in warmer climates, it is hard to say how far glacial conditions might spread towards the equator.

Up to the time when the previous part of this paper was given in* I had been unableto find records of any meteorolgical station at which the condition of the ground with regard to snow had been recorded in addition to the usual temperature observations; even yet I have not succeeded in getting suitable records for our climate, and one cannot help regretting that observations of this kind are not more frequently kept in this country.

My conclusions, however, with regard to the effect of snow on climate, I now find are confirmed by the observations of Dr Woeikof, an abstract of whose work in this direction has just appeared in *Nature* of 18th February 1886 (vol. xxxiii. p. 379). Dr Woeikof has approached the subject from the observational side, and as his results bear directly on ours, I shall quote the following paragraph from the abstract referred to:—

"The year 1877 was a striking instance of how the absence of snow was accompanied by a far less notable lowering of temperature during the prevalence of anticyclones, than would have been the case had the soil been covered with snow. In 1877 there was no snow in Eastern Russia until Christmas, and in November and December the anticyclones occurred, accompanied by no wind, or only by feeble breezes. Quite bright weather lasted in December for more than ten days; and still, in the region which remained uncovered with snow, no great cold was experienced, as usually happens in such circumstances; the minima were 8° to 9° above their average values. The same conditions were noticed during the winters of 1879–80 and 1881–82 in West Europe, as shown by Dr Billwiller in the *Zeitschrift für Meteorologie* for 1882."

In the next paragraph, the abstract states that, in the opinion of Dr Woeikof, the higher temperature of November, as compared with March in South-East Russia, is due to the ground not being usually covered with snow in November.

These observations of Dr Woeikof confirm the conclusions arrived at in this paper from a consideration of the properties of snow. In the opinion of Dr Woeikof, the low temperature accompanying the snow is to be attributed to its bad conducting power. While we quite agree with the observer

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* Note added 26th February 1886.

that the bad conducting power of snow will be *a* cause of the lower temperature, yet we think that the observations recorded in this paper show that radiation plays a most important part in producing this result. It has been long well known that snow receives less heat from the direct rays of the sun than the surface of a dark body like soil or rock, and we now also know that, under any condition of sky yet tested, it absorbs but little of the sky radiation. From this we see that as the surface of the snow, under all conditions of sky, receives less radiant heat from without than the surface of the ground, the air in contact with it will be more cooled than the air over bare soil or rock.

In the radiation temperatures above recorded, the snow was generally about 4 degrees colder than the black surface during the day. We must not from this suppose that these 4 degrees are the greatest difference that could exist in the radiation temperatures of the two surfaces. These 4 degrees simply represent the difference that was maintained under the conditions when the tests were made. If the air circulation had been greater, the difference would have been less, and both would have been nearer the temperature of the air; and if the circulation had been less, the difference would have been greater. But while there may be a variation in the difference of the temperatures of the two surfaces, there will be but little difference in their respective cooling effects, as with a quick circulation a large amount of air will be cooled a little, while, when the circulation is slow, a little air will be cooled a great deal. Not only so, but on account of the temperature of space being so much colderthan the surface of the snow, the temperature of the snow will tend to sink about the same amount below the temperature of the air, even though the air be greatly cooled; so that, under the same conditions of radiation and atmospheric circulation, however much the air may have been cooled, the two surfaces will tend to maintain the same difference in temperature, and the snow will always tend to cool the air more than a black surface, when the surfaces are colder than the air, or to heat it less when the surfaces are warmer.

It is therefore evident that this radiation effect will be one of the causes tending to produce the lower temperature of the air while snow is on the ground, and it seems probable that the snow will not only reduce the minima temperatures, but also the maxima. It seems also probable that the bad absorbing power of snow will have a most important influence in retarding the approach of warm weather, as the earth under snow receives far less heat from the sun with the approach of spring than if the ground had been free from snow. This is, of course, altogether apart from the question of the retardation of the approach of warm weather by the sun's heat being spent in melting the snow, instead of warming the air.

SUMMARY.

Our principal conclusions may be summed up as follows:—*First,* From the experiments made with (*a*) the inverted trays, (*b*) by weighing small areas of turf, and (*c*) by observations of the temperatures on and under the grass during dewy nights, we have concluded that vapour is almost constantly rising from grass land, by night as well as day, in our climate. *Second,* From experiments made with (*a*) inverted trays, (*b*) by weighings of small areas of soil, and (*c*) by observations made with small test condensing surfaces, on dewy nights, we have concluded that, under most conditions of our climate, vapour rises from uncultivated areas of soil during night as well as day. It follows from these two conclusions that dew never "falls" on the earth; and for reasons given, it is only deposited on plants, and other bodies, not in good heat communication with the ground. *Third,* That the greater part of the dew condensed on bodies near the ground is formed of the vapour rising at the time from the earth, and very little of it from the vapour that rose during the day. *Fourth,* That dew forms copiously on roads, but owing to the stones being good conductors of heat, the vapour is deposited on the under sides of the stones, and not on the top as on grass. *Fifth,* Wind hinders the formation of dew by preventing an accumulation of damp air near the ground. *Sixth,* The "dew-drop" formed on grass and other plants is not dew at all, but is formed of the exuded sap of the plant. *Seventh,* Almost all substances, such as black and white cloths, garden mould and grass, radiate equally well at night. Among the few exceptions observed are polished metals and sulphur. *Eighth,* A covering of snow on the ground lowers the mean temperature of the air.

FURTHER REMARKS ON DEW.

APPENDIX.

(Read 19th July 1886.)

Since the preceding paper was written, a few opportunities have occurred for continuing the investigation under different conditions of climate, and some additions have also been made to strengthen the exudation theory of the "dew-drop" on grass and other plants, of which I shall here give a short account.

At the beginning of the paper, evidence is adduced to show that water vapour is almost constantly rising from the ground during night, as well as day, and it is there noted that the experiments were somewhat unsatisfactory on account of the rather damp condition of the soil at the time, the dry season being over before the investigation was made. In order to supplement these observations, a few others were made in the beginning of July of this year, when the soil was in about as dry a condition as it almost ever is in this climate.

An inverted tray was placed over bare soil which was so dry and powdery that it rose in dust when the edge of the tray was pressed into it. The tray was not placed on the ground till after 9 P.M.—that is, after the soil had lost a good deal of heat and moisture—yet in two hours the inside was wet, and in the morning it was covered with drops. While this experiment was going on, another tray was placed on the lawn at a place where the grass was burnt quite brown. The inside of this tray was also wetted, and to about the same amount as the one over the bare soil. It may be noted here, that the positions for making these tests were selected on account of their extreme dryness. The bare soil was light and open, while the lawn was extremely dry, having been laid down a few years ago with every precaution to ensure this condition; the under soil is dry and sandy, it was moreover well drained, the upper soil was removed, and a good depth of ashes put in its place, then a layer of sand, on which was put the turf with its inch or two of soil. A drier position could scarcely have been selected for the trial, and yet the result showed that in our climate bare soil and grass land, even when very dry, continue to give off vapour during dewy nights.

A few experiments were also made on this subject in March last at Hyères, in the south of France. In order that the test might be as severe as possible, I selected a spot where the soil seemed driest and most exposed to the sun and the wind; and as the soil was stony and lying on the hill-side, it could have no supply of water from below. An inverted tray placed over this arid ground collected a surprisingly great amount of moisture. The following are the notes of one of the tests made at Hyères on the 27th March last:—At 5.15 P.M. the sun had ceased to shine on the place selected for the experiment; at that hour the temperature of the soil was 73° F. at 3 inches below the surface, and 59° at 12 inches. At 5.45 the tray was put on the ground. Examined at 7.15 P.M. the outside was dry, but inside was quite wet, and by 10 P.M. the drops were so large they ran down the inside when the tray was placed vertically. This great amount of wetting was probably due, not only to the ground being highly heated during the day, but also to a considerable fall in temperature at night, by the cooling produced under the clear skies of that climate.

I much regret I have been unable to get any trustworthy information regarding the movement of the vapour near the surface of the ground in barren and desert countries. I have, however, received some information from travellers who have been in Australia and parts of South Africa, where rain does not fall for months at a time, and it goes to prove that even in these dry countries vapour rises from the ground at night, as they often found the under side of their waterproof bedding placed on the ground to be wet after camping out at night. One would scarcely have expected much moisture to be collected in this way on account of the warmth of the sleeper's body keeping up the temperature of the condensing surface. It therefore seems probable that the moisture will collect under those parts of the waterproof which are beyond the influence of the sleeper's body.

While the experiments above referred to on bare soil and grass land were being made on ground exceptionally dry in July, those with slates placed on the road were repeated. As in the experiments described in the first part of this paper, the slates placed on the hard dry part of the road and on the gravel got quite wet on their under sides at night, thus showing that, even under the exceptionally dry conditions existing at the time, vapour was still rising at night from the hard and arid roads.

In connection with the action of stones lying near the surface of the ground, we may here refer to a result observed by agriculturists, on which it seems to throw some light. It has been remarked that the removal of small stones from fields where the soil is light and open, often has a prejudicial effect on the crops. It must be admitted that accurate information on this point is not easily obtained, but the impression in some parts of the country certainly is, that the removal of small stones from fields is not to be recommended. Now the removal of stones may act prejudicially in different ways. In some cases the disintegration of the stones may add to the richness of the soil; their removal may therefore in some cases decrease the natural fertility of the field. No doubt the removal of the stones will permit of a more rapid evaporation from the soil during the day, but it does also seem probable that the peculiar action of stones, in trapping the moisture before it comes to the surface at night, will have a beneficial effect on light and dry soils. Part of the moisture is trapped by them before it can escape from the surface at night; and when the sun rises, the stones becoming warmer than the soil under them, the moisture leaves the stones and condenses in the soil underneath. In this way stones would seem to have a sort of conserving action on the moisture, and tend to check the prejudicial effects of continued dry weather.

In confirmation of this conserving action of bodies lying on the surface of the ground and improving its fertility, I would refer to a letter in *Nature*, vol. xxxiii. p. 583, by Lieut.-Col. A. T. FRASER. As this letter is so interesting, and bears directly on our subject, I may be allowed to quote from it here. At the beginning of his letter he says:—"Having had occasion to lay out a large quantity of iron hoes and picks, without handles, on the hard ground of an open inclosure in one of the driest districts of India (Bellary), where, in fact, these implements had been collected in the face of a scarcity, it was found, after they had lain a couple of months, that a thick, weedy, but luxuriant vegetation had sprung up, enough, though there was no rain, to almost hide the tools."

Lieut.-Col. FRASER also says he had previously noticed in the tropics a similar stimulating effect produced on vegetation when tools were left lying on the surface, but was unable to account for it till he read the abstract of the first part of this paper, when he at once recognised the manner in which the tools lying on the surface acted in hot and dry climates by checking the escape of vapour at night. No doubt much of the increased fertility observed would be due to the wetting the plants received by the vapour condensed on the under sides of the tools; still it will be admitted something must also be owing to the reduced evaporation during the day, produced by the metal tools checking the escape of the vapour.

This conservation of moisture by stones and other bodies lying on the surface of the ground, and its up and down movement during night and day, may be easily seen in the experiment with the slates. Examined at night the *road under the slates is as dry* as the exposed parts, while the under sides of the slates are wet; but if examined at a certain time in the morning, the under sides of the slates will be found to be dry, *while the road under them is wet.* But, if the slates are not examined till a later hour, both the slates and the road under them will be found to be dry, the moisture being driven deeper and condensed among the stones lower down.

Many people, after reading the conclusions arrived at in the first part of this paper, seem to have a difficulty in accepting the theory that vapour is constantly rising from the ground during night as well as day, and that the dew on grass is formed of this rising vapour, because it seems to them to contradict the teaching of Dr WELLS, and it also appears to be at variance with their experience. A little consideration will show that the results established by Dr WELLS are in no way affected by it. That investigator certainly did not think that much vapour rose from the ground at night, yet he was well aware that some might rise; his investigation was however principally confined to the condensation of the vapour after it is in the air, and he gave comparatively little attention to its source; whereas in this investigation WELLS' results are accepted with regard to the condensation, and an attempt is made to extend the subject by investigating some parts not worked out by him.

Others again have made a difficulty to the acceptance of this theory by extending the conclusions arrived at in this paper to other conditions than those for which they are true. They have assumed that if dew on grass and on bodies near the ground is formed of vapour rising at the time, then the dew found on bodies higher up in the air must also be formed of vapour rising at the time from the ground immediately underneath; and as this conclusion is opposed to experience, they seem inclined to dismiss the whole theory as unworthy of consideration. When we come to investigate what is taking place in nature we will see that this extension of the conclusion is by no means justified.

The reason why it is concluded that the dew on the grass, and on bodies close to the ground, is formed of vapour rising from the ground, is that at night the ground under the grass is always in a condition to give off vapour, and this rising vapour will tend to displace that which rose during the day; the stems and blades of grass will thus become surrounded by vapour that has risen during the night. A further reason is, that the ground under the grass is much warmer than the air over it, and the air in contact with the moist earth is nearly saturated; the tension of the vapour in the air rising through the stems of grass is thus higher than that in the air over them, and it is therefore in a more favourable condition for condensing and forming dew than the air higher up. This hot rising vapour will often indeed yield dew when the air over the grass can give none.

When, however, we come to consider what takes place higher up above the grass, or even at the tops of the blades, we meet with a much more complicated condition of matters, and we are now able to say very little about the source of the vapour condensed at these higher positions. Whenever we get above the protection of the grass, into the parts of the atmosphere exposed to air-currents, we can say very little as to the source of the vapour existing there, either as to the place where it changed to vapour, or the time when this change took place. No doubt some of the molecules in this upper air will have risen but recently from the ground, but some of them will certainly—if there is the slightest wind—be molecules that have risen during the day, and no doubt some of them will have ascended into the air many days previously; and while some will have but recently come from the ground immediately underneath, others will have travelled from lands and oceans far away. But while this may be so, it in no way affects the conclusion that vapour is almost constantly—night as well as day—given off by the ground, and that dew on grass, and on bodies close to the ground, is part of this rising vapour trapped by their cold surfaces.

Curiously enough, history here repeats itself. The theory that dew rises from the ground has before now been wrecked by the observation that dew forms on bodies placed high above the ground, and in situations where no vapour could have risen to them from beneath. Professor MUSSCHENBROEK rejected GERSTEN'S theory of rising dew after he found dew was deposited on bodies placed on the leaden roof of his observatory. He thought the dew formed under those conditions could not have risen from the ground, but must have fallen from the atmosphere. There must, therefore, evidently be a foundation somewhere for the rejection of the theory on the grounds stated, though it must be admitted it is difficult to find, as the statements are in no way opposed to each other. It will, therefore, be as well for us to consider here the cause of this appearance of opposition.

One cannot help thinking that a good deal of the difficulty experienced in reconciling the statement, that the dew formed on bodies near the surface of the earth is formed of vapour rising from the ground, with the fact that dew is found on bodies high up in the air is caused by a want of clearness in our ideas, or perhaps rather to a persistence of primitive ideas. Dew was in olden times often spoken of as falling from the heavens, and even yet we talk of falling dew. This expression is to a certain extent associated with the idea of falling rain—a process in which the moisture passes from place to place through the air, and falls on bodies exposed to it; and many seem to think that if dew comes out of the ground it should be found only on bodies exposed to the earth. That in fact rising dew is the converse of the old falling dew, whereas dew is only so much moisture taken by a cold surface from the store of vapour in the air. This explanation of the difficulty does seem somewhat absurd, but we all know that old habits of thought have a curious way of asserting themselves, and it seems the only way of explaining a difficulty so many have felt.

Let us picture an imaginary state of matters, in which all the conditions shall be as simple as possible. Suppose the vapour to be constantly rising from the ground, and that the air is absolutely still; and further, let us imagine that the vapour flows upwards through the air in a continuous stream, only varying in velocity at different hours. At 6 P.M. we will suppose the molecules of vapour that left the earth at 6 A.M. to have arrived at a height a. Let dew now begin to form, then the moisture condensed at 6 P.M., on bodies at the height a, will be vapour that rose from the ground at 6 A.M., and bodies at intermediate heights will have the vapour on them that rose at the intermediate hours. At 6 A.M. the following morning the vapour that rose at 6 on the previous evening will be at an elevation b, and the dew forming on all bodies lower than b at this hour will be vapour that rose during the night. So that in these ideal conditions, if we knew the rate of ascent of the vapour, we could tell the hour at which the vapour—condensed at any height—rose from the ground.

If this imaginary condition of matters was correct, then there would be some reason for expecting that dew would only be deposited on bodies placed over such areas as yield vapour. In nature, however, the conditions are much more complicated. The vapour does not flow upwards in a uniform stream, but is mixed with the air by eddies and wind currents, and carried to bodies far from where it rose; so that while we can say something about the source of the vapour condensed on bodies near the surface of the earth, yet the molecules in the higher air have no history we can interpret. While the vapour rising from the ground plays an important part in the phenomena of dew, as it is not only the source of that formed on bodies near the ground, but it also increases the amount deposited on bodies high up, yet the rising vapour is not essential to its formation, as dew may be deposited even though the country for many miles all round is dry and incapable of yielding any vapour. In such case the supply of vapour to form dew would depend on the evaporation of the dew, and on what was brought in by the winds.

THE "DEW-DROP."

The statement that the "dew-drops" formed on plants at night is not dew at all, but is formed of the exuded sap of the plant, has been rejected by some on account of its being contrary to all accepted ideas on the subject, while some who have accepted it, have given only an indifferent assent. It has therefore seemed desirable that further evidence be advanced in support of the statement, and also that some simpler methods be devised for studying the phenomena connected with the exudation of moisture by plants, so that those not accustomed to making difficult experiments may be able to demonstrate the point for themselves.

One of the simplest experiments of this kind is to cut a piece of turf, or, better still, lift a single grass plant with a clod of earth attached to it, which can generally be easily found in any garden. The leaves and stems of a single plant being separate and open, the phenomena are more easily observed on it than in the confused vegetation of a turf. Place the plant on a plate, and invert a tumbler or other vessel over it, so as to enclose the plant and rest it on the plate. This should be done when the soil is not too dry, otherwise water will require to be given to the plant. After it has been kept in moist air for about an hour drops will begin to exude, and the tip of nearly every blade will be found to be studded with a diamond-like drop.

In the above simple experiment there is nothing to tell us where the moisture came from to form the drops. It might be contended that it was condensed by the plant out of the moist air. It has, however, been shown in the first part of this paper, that when the tip of the blade is isolated from all supply of moist air, the drop at the end grows as quickly as the drops at the ends of the blades exposed to saturated air. This experiment is, however, a somewhat difficult one for any one not accustomed to work of this kind. The point may, however, be proved in a much simpler way. Take any exuding plant with a single stem, such as a broccoli or poppy, if it is growing in a pot so much the better, as it is more convenient both for making and seeing the results of the experiment. Prepare a circular disc of metal—say tin-plate—with a hole in its centre large enough for the stem of the plant to pass through, then cut the disc in two through the centre. Now place the disc on the pot with the stem of the plant passing through the hole, and join the two halves of the disc, either by soldering, or by cementing over the joint a strip of sheet india-rubber. A large glass receiver is now placed over the plant with its edges resting on the metal plate. In this way the plant is isolated in air from which it can extract no moisture, the metal plate preventing vapour rising from the soil underneath. If any drops now appear on the leaves, it is evident the air cannot be the source of the moisture.

When tested in this way, it will be found that any exuding plant will soon become studded with drops, and present exactly the appearance it would at night. The time the drops take to appear depends on certain conditions; amongst these are—1st, the kind of plant experimented on; 2nd, the state of vital activity in the plant at the time; and 3rd, the degree of dryness of the leaves at the time it is put under the receiver. If the leaves are very dry, it will take some time to fill the tissues of the plant with sap before the surplus begins to exude. A broccoli plant in fair health and condition may be expected to show drops in less than an hour. These drops are small at first, and gradually grow so large that they fall off by their own weight.

An experiment of this kind is so easily made by any one, that the interest and the information gained is ample reward for the little trouble taken in making it. Instead of using a metal plate as above described for isolating the plant from the damp soil, a simpler plan is to use a piece of sheet india-rubber, with a radial slit in it, for slipping it round the plant, the two edges being joined with india-rubber solution, the glass receiver being, as before, placed over the plant, and its edges resting on the rubber.

The evidence advanced in the first part of this paper in support of the statement that the "dew-drop" is exuded by the plant is—1st, that the drop at the tip of a blade of grass grows as quickly when isolated from all supply of moist air as when exposed to saturated air; 2nd, the blades of those plants which have drops attached to them on dewy nights exhibit no drops when separated from the root, even when supplied with water and placed in saturated air; and 3rd, when hydrostatic pressure is applied to the stalk of the leaf of any plant that has drops attached to it at night, it causes the leaf to exude at exactly the same points as the drops appeared on dewy nights. Although the above evidence is fairly conclusive, yet there is a point where it might be strengthened by the addition of a link to the chain. While it is shown that hydrostatic pressure will produce the same effects as are seen on plants at nights, yet no evidence is adduced to show that there is any internal pressure in those plants on which "dew-drops" have been observed. This omission has now been corrected, and some experiments made in connection with this point. The plants selected for experiment were of the same kinds as those which were observed to have "dew-drops" on them at night, and which showed exuded drops when subjected to hydrostatic pressure, such as broccoli, cauliflower, poppy, &c. For convenience, some plants were grown in flower pots, and the experiments made while they were still small.

For measuring the pressure inside the plants, U-shaped tubes half filled with mercury were used, and the pressure measured by the height to which the sap forced the mercury. These gauges did their work well enough, but they were somewhat slow in action, as it takes some time for sufficient liquid to be exuded to displace the mercury and force it up the tube. Where high pressures were required to be measured, the U tubes had to be abandoned, on account of their inconvenient height, as well as the length of time required for making an observation with them, and the pressures were measured by means of air-pressure gauges. These gauges were made of a short length of wide thermometer tube, having a bore of less than 1 mm. diameter, a short column of mercury being put in to form an index. The pressures were calculated from the volumes, and corrections made when necessary for temperature. The gauges were occasionally compared with a column of mercury to see that everything was correct. The pressures given cannot be considered correct to more than 10 mm. of mercury, but for the present purpose this degree of accuracy is sufficient, as the pressures are very indefinite, varying with so many conditions that anything like an exact figure cannot be looked for in experiments of this kind.

I shall now describe in detail an experiment made on a cauliflower, as it is similar to those made on other plants, of which it will only be necessary here to give the results. The pot containing the plant was placed on a sheet of metal, and a glass receiver got ready large enough to cover it entirely. The stalk of one of the blades was selected for making the connection between the pressure gauge and the plant. This stalk, while the leaf was still on it, was prepared for making a water-tight joint by filling up the longitudinal groove in its upper surface with beeswax, laid on with a slightly heated iron. The blade was now cut off, and the gauge attached by means of a short length of soft india-rubber tube; the stalk having been made round by means of the beeswax, a tight joint was easily made. With the exception of this one leaf cut off, all the others were left untouched. The receiver was now put over the plant to stop evaporation from the leaves, and everything left at rest. After a short time the pressure was seen beginning to rise in the gauge, and drops also began to show themselves all round the edges of the leaves. As time went on the drops increased in size, and the pressure went up to 290 mm. of mercury, at which point it stopped. This pressure can be considered correct only for the particular plant under the particular conditions. It simply meant that when the pressure rose to 290 mm., the whole of the supply of sap sent up by the root could find an exit by exudation. If the supply had been greater, or the exuding pores fewer or smaller, the pressure would no doubt have gone higher, and *vice versa*.

The plant in the above experiment was in the same condition as it would be on a dewy night. All evaporation from the leaves was stopped, and transpiration having ceased, the root continued to send up its supplies of sap, first filling the tissues of the plant, and then producing an internal pressure, which forced the sap to escape by the exuding pores. But what is the condition of the tissues during day when transpiration is going on? An answer to this was easily obtained by removing the receiver from the plant, and allowing evaporation to proceed from its leaves. The result was that the pressure inside the plant fell, and a negative pressure took its place; the mercury first fell in the U tube, and then rose on the other side. The mercury was drawn up in one case to a height of 140 mm., and in another plant to 180 mm., the height seeming to depend on the rate of evaporation, and the perfection and closeness of the tissues of the plant enabling it to stand a greater or less pressure before air forced its way inwards. We see from this that exuding plants during night, and at times when there is little evaporation, have an internal pressure tending to distend their tissues, and have a negative or external pressure during the day tending to press the tissues inwards. This internal pressure may help to explain the more rigid appearance of the leaves of plants at night; while the negative pressure or degree of vacuum produced inside the leaves by transpiration explains the manner in which water is taken up by cut flowers and branches of plants when their ends are placed in it, and it also explains something of the peculiar curving of leaves when withering.

I have said that the pressure above measured inside of the cauliflower plant would have been much greater if the exuding pores had been less in size or number—that, in fact, the pressure then measured was not the maximum root pressure. To test this point, the plant was now cut across near the bottom of the stem, within two or three centimetres of the root, so removing all the leaves with their exuding pores; and the pressure gauge was attached to the stem near the root. The gauge now rose very rapidly, and in a short time indicated a maximum pressure of 760 mm., the india-rubber connecting tube requiring to be strongly bandaged to prevent it bulging. It seems strange that the delicate tissues of a young plant should be able to produce and resist so great a pressure. We must however remember that this last registered pressure is one to which the plant is never subjected when under natural conditions, but even the 290 mm. measured when the plant was exuding freely does seem a great pressure to exist in plants.

A poppy tested in the same way showed an internal pressure of 175 mm. with its leaves all on, and exuding freely in saturated air. This lower pressure compared to the cauliflower, would seem to indicate that the exuding pores are larger or more numerous in the poppy than in the cauliflower, as the former are fully as wet as the latter on dewy nights. When the poppy was cut across, and the gauge attached to the main stem near the root, the pressure rose to as much as 1040 mm.

The pressure inside grass has not been easily measured, owing to the difficulty of making a tight joint with the gauge, and so delicate a structure as a grass stem. The highest pressure observed before the joint gave way was 160 mm. The measurement was taken with all the blades on, and exuding freely in saturated air. This pressure is just a little less than was found in the poppy when under similar conditions. No measurements have been made with all exudation stopped, on account of grass not growing in a form suitable for making a measurement of this kind.

The following are a few readings of the maximum root pressure given by different plants. These plants were small and still in the seed-bed, but too large for transplanting.

Cauliflower,	875	mm.
,,	920	,,
,,	1065	,,
Cabbage,	1310	,,

From the above figures it will be seen that the pressures given by the different cauliflower plants varied considerably; and that the cabbage experimented on was capable of exerting a root pressure equal to forcing its sap to a height of about 58 feet, thus showing an extraordinary reserve of energy.

It was shown in the first part of this paper that the leaves of these plants exuded when hydrostatic pressure was applied to their stalks, and we have now shown that there is abundance of pressure inside these plants at night to produce the exudation. Nothing like an attempt, however, has been made here to give either the exact pressure inside different kinds of plants or the pressure under different conditions. It has already been stated that the conditions affecting the pressure are much too varied for the figures to be settled by a few experiments; all that has been attempted is to show that in exuding plants, there is abundance of pressure to produce the results claimed.

In connection with this subject, it was interesting to notice the variation in the exudation of grass during the late continuance of dry weather. While the soil was damp exudation went on as usual, but when the ground got drier the exudation gradually got less and less, and at last it entirely ceased. Even when the grass was covered with an enclosure, and surrounded with saturated air, no exudation took place, and yet the grass was green and growing; it took some time after the grass was wetted before the activity was great enough to give rise to exudation. During this dry weather, while the grass had ceased to exude, it got moist at nights with the hot vapour rising from the ground, the lower parts, particularly where exposed to radiation, being wetter than the tops of the blades.

It is very difficult to get an idea of the number of plants that exude, so much depends on the vitality of the plant at the time, and on the amount of moisture in the ground, so that the same plant may emit drops at one time and not at another. We have seen that even so free an exuding plant as grass may cease to discharge; others cease with a less degree of dryness, and with a less decrease in vital activity. The number that exudes under favourable conditions is, however, much greater than we might at first imagine. In the south of France, in spring, a very great number of plants were observed to exude on dewy nights; even roses had their leaves fringed with drops, a condition in which I have never seen these plants in this country; but the activity of vegetable life in spring is very much greater in the south than with us.

This question of root pressure in plants is one of vast interest; so much still remains to be known about it. How is it that one plant must have the soil in which it grows full of water, while another requires it to be only damp? Another seems to be able to grow on nearly dry soil, whilst another still can by means of its air-roots extract moisture from air that is not saturated. What is the source of energy called into action by this latter class to enable it to condense the vapour in the air? Is it a chemical process? or a purely physical one, like the condensation of vapour by Professor TAIT's hygrometer when it is falling? or is it some unknown function of vitality? These questions, however, open up a field much too wide to be considered here.

28*th July* 1886.—After I had written the above paragraph, and as I supposed had closed the paper, it slowly dawned upon me that the surface of the leaves of all the different kinds of plants that have been observed to exude drops behaved themselves in a particular manner towards water. None of them seemed to be wetted by it. The glistening rain-drop on the grass shows that the blades of that plant are not wetted by water, the glistening being due to the reflection from the inside of the drop, where it rests on the blade, but does not touch it. But do all the other exuding plants repel water in the same manner? As it was raining while these thoughts passed through my mind, a visit to the garden was at once made, and the broccoli, poppy, and all the other exuding plants were examined. Every one of them was found to behave towards the rain-drops in the same manner as the grass. The rain-drops slipped off their surfaces "like water off a duck's back;" and where water collected in the hollows of the blades, the reflection from its internal surface showed it was not in contact with them.

The other plants—cultivated and uncultivated—in the garden were then examined, when most of them were found to be quite wet. The difference in their appearance from the exuding ones was very marked. At first sight the leaves of plants that got wet, like potatoes, beans, &c., looked almost as if they were dry, but in reality the water wetted them so perfectly all over, that it ran off, leaving only a thin and even film on their surfaces; whereas all the plants that exuded drops had their surfaces dry, save certain small areas where the natural surface of the blade had been destroyed. On thinking over the matter, it became evident that this property of leaves that exude drops at night ought to have been foreseen by me. The fact that the emitted moisture remains as a drop, shows that the surface of the leaf rejects water; if the leaf surface got wetted with water, the exuded liquid would have crept outwards from the exuding pore, and have wetted the leaf for some distance all around it. These exuded drops behave very much in the same manner as a drop of water attached to the end of a glass rod that is not very clean; the water does not wet the rod, but draws itself up into a drop. If the drop had been attached to a wooden rod or a piece of thread, or anything that was easily wetted, it would not have remained as a drop, but have spread itself all over the surface of the body.

When examining the plants in the garden during rain, in addition to those plants which I knew exuded drops at night, I noticed a number of others that rejected the rain drops, and kept their surfaces dry in the same manner as the exuding plants. Amongst these were Nasturtium, some of the Brassiceæ family not previously observed, and also some weeds. Now, it appeared that if the above reasoning is correct, these other dry-surfaced plants ought to exude drops, I therefore marked them, and on afterwards experimenting found that they also discharged drops like the others.

It almost looked at first sight as if this property of repelling water was a distinguishing characteristic of the leaves of all exuding plants; but on further considering the matter, the idea soon suggested itself that the other class of plants, the leaves of which got wetted with rain, might also exude moisture, as it was evident that if they did exude the discharge would be masked, for the moisture would not collect on them in drops, but spread itself over the leaves, and so become undistinguishable from dew. It therefore seemed desirable that other experiments be made on this class of plants, to see if any of them exuded moisture. It was evident that special precautions would be necessary to enable us to see the exuded moisture on leaves easily wetted, as it would not be so easily seen as the sparkling drop on water-repelling leaves.

For investigating this point, the most convenient plant I could find was a strong growing variety of everlasting flower (Helichrysum). This plant was one of those observed to have its leaves wet while it was raining, and no exuded drops were observed on it at night. The first thing determined was to see if there was any root pressure to cause exudation. The plant was cut across at the bottom of the stem, and the pressure gauge attached near the root. The root pressure was found to be 950 mm.; that is, this plant had as great an internal pressure as was found in the drop-exuding plants. In order to see whether it exuded when hydrostatic pressure was applied, the upper part of the plant, which was cut off for taking the root pressure, was removed to the laboratory, where it was connected by means of an india-rubber tube with a head of water of about 1·5 metres, and surrounded with saturated air. After a time drops appeared at the tips of most of the leaves, and also at some other points on them; but these drops were quite unlike those on grass, broccoli, and other water-repelling plants; they spread themselves on the leaves, and adhered to them, no reflection being given from the back of the flattened drop. It could, however, be easily seen, when the experiment was made in this way, that moisture is exuded from the plant, whereas at night no exuded moisture is perceptible. The reason for this is, that under the condition of the experiment, the exuded drop only spreads to a certain extent, and the outline of the wetted surface is defined, because the whole surface of the leaf is not wet; but at night the surface of the leaf is wet with dew, and the exuded drop spreads and thins away by imperceptible degrees into the dewed surface. This was illustrated in the above experiment by breathing on the leaf, so as to bring it into the same condition it is on dewy nights, the drop was then seen to spread rapidly outwards.

We see from the above that a plant may be exuding, and yet we may not be able to notice it. This is specially the case while dew is forming, that is under natural conditions; for dew is very generally forming while plants are exuding, and it is difficult to tell from an examination made at night whether any plant whose leaves have an affinity for water is exuding or not. It is therefore much better to test the plants under artificial conditions, by placing them in saturated air, but where no dew can be formed on their surfaces. This can be done by placing them at night under hand-glasses, and well protected from radiation, or even during the day under metal boxes, and well shaded. In this way a few plants, whose leaves got wet with rain, were tested, and all were found to exude if the evaporation from the leaves was stopped long enough, and time given for the tissues to get filled with sap. In all cases the exuded moisture adhered to the leaf and formed a wet patch. The plants tested were helichrysum, stocks, asters, mignonette, foxglove, celery, lettuce, turnips.

The plants were taken at hazard, and while some, such as mignonette and stocks, exuded little, the others discharged a good deal. The root pressure of a stock was measured, and found to be only about one-half that of the more freely exuding Helichrysum. The *root pressure* will, however, be only one factor in determining the amount exuded, as it is evident the *rate of supply* sent in by the root will be of as much importance; but no measurements of quantity have been made by me. It may be as well to note here, that though the few plants, taken at hazard, all showed powers of exuding, yet we must not therefore conclude that all plants have this property.

It is interesting to note the effects of these two ways in which the surface of leaves behave towards their exuded sap and water. Take the different kinds of turnips, for instance. The Swedish variety exudes freely, the liquid forming little drops fringing the leaves, while the moisture exuded by the other varieties spreads itself over the leaves. One result of this is, that after dewy nights the softer varieties dry sooner than the Swedish, because the exuded moisture, by spreading itself over the surface of the leaves, dries up much more quickly than the drops on the others. This seems to be the explanation of a fact frequently observed by sportsmen and others who have occasion to walk through turnip fields on autumn mornings, namely, that the softer varieties generally wet them much less than the swedes. Again, after rain the swedes take longer to dry than the others, because their surfaces do not get wet, but the water collects in drops, imperfectly attached to them, and also fills the hollows of their leaves; whereas the other kinds get wet, and the water runs off them, leaving only a thin film on their surface, which dries up much more quickly than the drops on the others. Further, when we walk through turnips immediately after rain, our feet brush the drops from the swedes in showers, which rapidly wet us, while the water adheres to and does not so easily leave the surfaces of the others.

This last part of the investigation takes us a step further, and shows us that not only is the dew-drop a result of the vitality of those plants on which it forms, but that much of the wetness spread over the leaves of others on dewy nights is produced by moisture exuded by the plants.

III.—*On the Foundations of the Kinetic Theory of Gases.* By Professor Tait.

(Revised May 14, 1886.)

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The attempt to account for the behaviour of gases by attributing their apparently continuous pressure to exceedingly numerous, but nearly infinitesimal, impacts on the containing vessel is probably very old. It certainly occurs, with some little development, in Hooke's tract of 1676, *Lectures de potentiâ restitutivâ, or of Spring;* and, somewhat more fully developed, in the *Hydrodynamica* of D. Bernoulli, 1738. Traces of it are to be found in the writings of Le Sage and Prévost some 80 or 90 years ago. It was recalled to notice in 1847 by Herapath in his *Mathematical Physics,* and applied, in 1848, by Joule to the calculation of the average speed of the particles in a mass of hydrogen at various temperatures. Joule expressly states[*] that his results are independent of the *number* of the particles, and of their directions of motion, as also of their mutual collisions.

In and after 1857 Clausius greatly improved the treatment of the problem by taking account not only of the mutual impacts of the particles but also of the rotations and internal vibrations which they communicate to one another, with the bearing of this on the values of the specific heats; at the same time introducing (though only to a limited extent) the statistical method. In this series of papers we find the first hint of the length of the mean free path of a particle, and the explanation of the comparative slowness of the process of diffusion of one gas into another. But throughout it is assumed, so far as the calculations

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[*] The paper is reprinted *Phil. Mag.* 1857, II. See especially p. 215.

are concerned, that the particles of a gas are all moving with equal speeds. Of the Virial, which CLAUSIUS introduced in 1870, we shall have to speak later.

In the *Philosophical Magazine* for 1860 CLERK-MAXWELL published his papers on the "Collisions of Elastic Spheres," which had been read to the British Association in the previous year. In this very remarkable investigation we have the first attempts at a numerical determination of the length of the mean free path. These are founded on the observed rate of diffusion of gases into one another; and on the viscosity of gases, which here first received a physical explanation. The statistical method is allowed free play, and consequently the law of distribution of speed among the impinging particles is investigated, whether these be all of one kind or a mixture of two or more kinds. One of his propositions (that relating to the ultimate partition of energy among two groups of colliding spheres), which is certainly fundamental, is proved in a manner open to very grave objections:—not only on account of the singular and unexpected ease with which the proof is arrived at, but also on account of the extraordinary rapidity with which (it seems to show) any forced deviation from its conclusions will be repaired by the natural operation of the collisions, especially if the mass of a particle be nearly the same in each system. As this proposition, in the extended form given to it by BOLTZMANN and others, seemed to render the kinetic theory incapable of explaining certain well-known experimental facts, I was induced to devote some time to a careful examination of MAXWELL's proof (mainly because it appears to me to be the only one which does not seem to evade rather than boldly encounter the real difficulties of the question[*]), with the view of improving it, or of disproving the theorem, as the case might be. Hence the present investigation, which has incidentally branched off into a study of other but closely connected questions. The variety of the traps and pit-falls which are met with even in the elements of this subject, into some of which I have occasionally fallen, and into which I think others also have fallen, is so great that I have purposely gone into very minute detail in order that no step taken, however slight, might have the chance of escaping criticism, or might have the appearance of an attempt to gloss over a real difficulty.

The greater part of the following investigation is concerned only with the most elementary parts of the kinetic theory of gases, where the particles are

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[*] Compare another investigation, also by CLERK-MAXWELL but based on BOLTZMANN's processes, which is given in *Nature*, viii. 537 (Oct. 23, 1873). Some remarks on this will be made at the end of the paper. Meanwhile it is sufficient to point out that this, like the (less elaborate) investigations of MEYER and WATSON, merely attempts to show that a certain state, once attained, is permanent. It gives no indication of the rate at which it would be restored if disturbed. As will be seen later, I think that this "rate" is an element of very great importance on account of the reasons for confidence (in the general results of the investigation) which it so strikingly furnishes.

regarded as hard smooth spheres whose coefficient of restitution is unity. The influence of external forces, such as gravity, is neglected; and so is that of internal (molecular) forces. The number of spheres is regarded as extremely great (say of the order 10^{20} per cubic inch): but the sum of their volumes is regarded as very small in comparison with the space through which they are free to move; as, for instance, of the order 10^{-3} or 10^{-4}. It will be seen that several of the fundamental assumptions, on which the whole investigation rests, are justified only by reference to numbers of such enormous magnitude, or such extreme minuteness, as the case may be. The walls of the containing vessel are supposed simply to *reverse* the normal velocity of every sphere impinging on them.

I. *One set of Equal Spheres.*

1. Very slight consideration is required to convince us that, unless we suppose the spheres to collide with one another, it would be impossible to apply any species of finite reasoning to the ascertaining of their distribution at each instant, or the distribution of velocity among those of them which are for the time in any particular region of the containing vessel. But, when the idea of mutual collisions is introduced, we have at once, in place of the hopelessly complex question of the behaviour of innumerable absolutely isolated individuals, the comparatively simple statistical question of the average behaviour of the various groups of a community. This distinction is forcibly impressed even on the non-mathematical, by the extraordinary steadiness with which the numbers of such totally unpredictable, though not uncommon, phenomena as suicides, twin or triple births, dead letters, &c., in any populous country, are maintained year after year.

On those who are acquainted with the higher developments of the mathematical *Theory of Probabilities* the impression is still more forcible. Every one, therefore, who considers the subject from either of these points of view, must come to the conclusion that continued collisions among our set of elastic spheres will, *provided they are all equal*, produce a state of things in which the percentage of the whole which have, at each moment, any distinctive property must (after *many* collisions) tend towards a definite numerical value; from which it will never afterwards markedly depart.

This principle is of the utmost value, when legitimately applied; but the present investigation was undertaken in the belief that, occasionally at least, its powers have been to some extent abused. This appears to me to have arisen from the difficulty of deciding, in any one case, what amount of completeness or generality is secured when the process of averaging is applied in successive steps from the commencement to the end of an investigation, instead of being reserved (as it ought to be) for a single comprehensive step at the very end.

Some of the immediate consequences of this principle are obvious without calculation: such as

(*a*) Even distribution, at any moment, of all the particles throughout the space in which they move.

(*b*) Even distribution of direction of motion among all particles having any one speed, and therefore among all the particles.

(*c*) Definite percentage of the whole for speed lying between definite limits.

These apply, not only to the whole group of particles but, to those in any portion of space sufficiently large to contain a very great number of particles.

(*d*) When there are two or more sets of mutually colliding spheres, *no one of which is overwhelmingly more numerous than another, nor in a hopeless minority as regards the sum of the others,* similar assertions may be made as to each set separately.

2. But calculation is required in order to determine the law of grouping as to speeds, in (*c*) above. It is quite clear that the spheres, even if they once had equal speed, could not possibly maintain such a state. [I except, of course, such merely artificial distributions as those in which the spheres are supposed to move in groups in various non-intersecting sets of parallel lines, and to have none but direct impacts. For such distributions are thoroughly unstable; the very slightest transverse impact, *on any one sphere,* would at once upset the arrangement.] For, when equal smooth spheres impinge, they *exchange* their velocities along the line of centres at impact, the other components being unchanged; so that, *only* when that line is equally inclined to their original directions of motion, do their speeds, if originally equal, remain equal after the completion of the impact. And, as an extreme case, when two spheres impinge so that the velocity of one is wholly in the line of centres at impact, and that of the other wholly perpendicular to it, the first is brought to rest and the second takes the whole kinetic energy of the pair. Still, whatever be the final distribution of speeds, it is obvious that it must be independent of any special system of axes which we may use for its computation. This consideration, taken along with (*b*) above, suffices to enable us to find this final distribution.

3. For we may imagine a space-diagram to be constructed, in which lines are laid off from an origin so as to represent the simultaneous velocities of all the spheres in a portion of space large enough to contain a very great number of them. Then (*b*) shows that these lines are to be drawn evenly in all directions in space, and (*c*) that their ends are evenly distributed throughout the space between any two nearly equal concentric spheres, whose centres are at the common origin. The density of distribution of the ends (*i.e.*, the number in unit volume of the space-diagram) is therefore a function of r, that is, of

$\sqrt{x^2+y^2+z^2}$. But the argument above shows, further, that this density must be expressible in the form

$$f(x)\,f(y)\,f(z)$$

whatever rectangular axes be chosen, passing through the origin. These joint conditions give only two admissible results: viz., either

$$f(x)=\mathrm{A}, \quad \text{or } f(x)=\mathrm{B}\varepsilon^{\mathrm{C}x^2}.$$

The first is incompatible with the physical problem, as it would make the percentage of the whole particles, which have one definite speed, increase *indefinitely* with that speed. The same consideration shows *à fortiori* that, in the second form of solution, *which is the only one left,* C must be negative. Hence the density of the distribution of "ends" already spoken of is

$$\mathrm{B}^3\varepsilon^{-hr^2}.$$

If n be the whole number of particles, *i.e.*, of "ends," we must obviously have

$$4\pi\mathrm{B}^3\int_0^\infty \varepsilon^{-hr^2}r^2dr=n.$$

The value of the integral is

$$\frac{1}{4}\sqrt{\frac{\pi}{h^3}};$$

so that the number of spheres whose speed is between r and $r+dr$ is

$$4\sqrt{\frac{h^3}{\pi}}n\varepsilon^{-hr^2}r^2dr \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This distribution will hereafter be spoken of as the "*special*" state.

The mean speed is therefore

$$4\sqrt{\frac{h^3}{\pi}}\int_0^\infty \varepsilon^{-hr^2}r^3dr=\frac{2}{\sqrt{\pi h}};$$

while the mean-square speed is

$$4\sqrt{\frac{h^3}{\pi}}\int_0^\infty \varepsilon^{-hr^2}r^4dr=\frac{3}{2h}.$$

This shows the meaning of the constant h. [Several of the results we have just arrived at find full confirmation in the investigations (regarding mixed systems) which follow, if we only put in these P for Q *passim*:—*i.e.*, pass back from the case of a mixture of spheres of two different groups to that of a single group.]

4. Meanwhile, we can trace the general nature of the process by which the "special" arrangement of speed expressed by (1) is brought about from any initial distribution of speed, however irregular. For impacts on the containing vessel do not alter r, but merely shift the particular "end" in question to a

different position on its spherical locus. Similarly, impact of equal particles does not alter the *distribution* of velocity along the line of centres, nor along any line perpendicular to it. But it does, in general, produce alterations in the distribution parallel to any line other than these.

Hence impacts, in all of which the line of centres is parallel to one common line, produce no change in the arrangement of velocity-components along that line, nor along any line at right angles to it. But there will be, in general, changes along every other line. It is these which lead gradually (though very rapidly) to the final result, in which the distribution of velocity-components is the same for all directions.

When this is arrived at, collisions will not, in the long run, tend to alter it. For then the uniformity of distribution of the spheres in space, and the symmetry of distribution of velocity among them, enable us (by the principle of averages) to dispense with the only limitation above imposed; viz., the parallelism of the lines of centres in the collisions considered.

5. In what precedes nothing whatever has been said as to the ratio of the diameter of one sphere to the average distance between two proximate spheres, except what is implied in the preliminary assumption that the sum of the volumes of the spheres is only a very small fraction of the space in which they are free to move. It is probable, though not (so far as I know) thoroughly proved, that if this fraction be exceedingly small the same results will ultimately obtain, but only after the lapse of a proportionately long time; while, if it be infinitely small, there will be no law, as there will be practically no collisions. On the other hand, if the fraction be a large one (*i.e.*, as in the case of a highly compressed gas), it seems possible that these results may be true, at first, only as a very brief *time-average* of the condition of the spheres in any region large enough to contain a great number:—that, in fact, the distribution of particles and speeds in such a region will be for some time subject to considerable but extremely rapid fluctuations. Reasons for these opinions will be seen in the next section of the paper. But it must also be noticed that when the particles fill the greater part of the space in which they move, *simultaneous* impacts of three or more will no longer be of rare occurrence; and thus a novel and difficult feature forces itself into the question.

Of course with infinitely hard spheres the probability of such multiple collisions would be infinitely small. It must be remembered, however, that the investigation is meant to apply to physical particles, and not to mere mathematical fictions; so that we must, in the case of a highly compressed gas, take account of the possibility of complex impacts, because the duration of an impact, though excessively short, is essentially finite.

II. *Mean Free Path among Equal Spheres.*

6. Consider a layer, of thickness δx, in which quiescent spheres of diameter s are evenly distributed, at the rate of n_1 per unit volume. If the spheres were opaque, such a layer would allow to pass only the fraction

$$1 - n_1 \pi s^2 \delta x / 4$$

of light falling perpendicularly on it. But if, instead of light, we have a group of spheres, also of diameter s, falling perpendicularly on the layer, the fraction of these which (whatever their common speed) pass without collision will obviously be only

$$1 - n_1 \pi s^2 \delta x ;$$

for two spheres must collide if the least distance between their centres is not greater than the sum of their radii. It is, of course, tacitly understood when we make such a statement that *the spheres in the very thin layer are so scattered that no one prevents another from doing its full duty in arresting those which attempt to pass.* Thus the fraction above written must be considered as differing very little from unity. In fact, if it differ much from unity, this consideration shows that the estimate of the number arrested will necessarily be exaggerated. Another consideration, which should also be taken into account is that, in consequence of the finite (though very small) diameter of the spheres, those whose centres are not in the layer, but within one diameter of it, act as if they were, in part, in the layer. But the corrections due to these considerations can be introduced at a later stage of the investigation.

7. If the spheres impinge obliquely on the layer, we must substitute for δx the thickness of the layer in the direction of their motion.

If the particles in the layer be all moving with a common velocity parallel to the layer, we must substitute for δx the thickness of the layer in the direction of the *relative* velocity.

If the particles in the layer be moving with a common velocity inclined at an angle $\frac{\pi}{2} - \theta$ to the plane of the layer, and the others impinge perpendicularly to the layer, the result will be the same as if the thickness of the layer were reduced in the ratio of $\sin\theta : 1$, and it were turned so as to make an angle θ with the direction of motion of the impinging particles.

8. Now suppose the particles in the layer to be moving with common speed v_1, but in directions uniformly distributed in space. Those whose directions of motion are inclined at angles between β and $\beta + d\beta$ to that of the impinging particles are, in number,

$$n_1 \sin\beta \, d\beta / 2 ;$$

and, by what has just been said, if v be the common speed of the impinging particles, the *virtual* thickness of the layer (so far as these particles are concerned) is

$$v_0 \delta x / v,$$

where

$$v_0 = \sqrt{v^2 + v_1^2 - 2vv_1 \cos\beta}$$

is the *relative* speed, a quantity to be treated as essentially positive.

Thus the fraction of the impinging particles which traverses this set without collision is

$$1 - n_1 \pi s^2 \delta x v_0 \sin\beta \, d\beta / 2v \, .$$

To find the fraction of the impinging particles which pass without collision through the layer, we must multiply together all such expressions (each, of course, infinitely nearly equal to unity) between the limits 0 and π of β. The logarithm of the product is

$$-\frac{n_1 \pi s^2 \delta x}{2v} \int_0^{\pi} \sqrt{v^2 + v_1^2 - 2vv_1 \cos\beta} \, . \sin\beta \, d\beta \, .$$

Making v_0 the variable instead of β, this becomes

$$-\frac{n_1 \pi s^2 \delta x}{2v^2 v_1} \int v_0^2 dv_0 \, .$$

If v be greater than v_1, the limits of integration are $v - v_1$, and $v + v_1$, and the expression becomes

$$-n_1 \pi s^2 \delta x \left(1 + \frac{v_1^2}{3v^2}\right) ;$$

but, if v be less than v_1, the limits are $v_1 - v$ and $v_1 + v$, and the value is

$$-n_1 \pi s^2 \delta x \left(\frac{v}{3v_1} + \frac{v_1}{v}\right) .$$

These give, as they should, the common value

$$-4 n_1 \pi s^2 \delta x / 3$$

when $v = v_1$.

9. Finally, suppose the particles in the layer to be in the "special" state. If there be n in unit volume, we have for the number whose speed is between the limits v_1 and $v_1 + dv_1$

$$n_1 = 4nv_1^2 dv_1 \sqrt{\frac{h^3}{\pi}} \epsilon^{-hv_1^2} \, .$$

Hence the logarithm of the fraction of the whole number of impinging particles, whose speed is v and which traverse the layer without collision, is

$$-4\pi n s^2 \sqrt{\frac{h^3}{\pi}} \delta x \left(\int_0^v \epsilon^{-hv_1^2} \left(v_1^2 + \frac{v_1^4}{3v^2}\right) dv_1 + \int_v^{\infty} \epsilon^{-hv_1^2} \left(\frac{vv_1}{3} + \frac{v_1^3}{v}\right) dv_1 \right)$$

The value of the factor in brackets is easily seen to be

$$-\frac{dV}{dh}+\frac{1}{3v^2}\frac{d^2V}{dh^2}+\left(\frac{2v}{3h}+\frac{1}{2h^2v}\right)\varepsilon^{-hv^2},$$

or

$$\frac{1}{4h^2v}\varepsilon^{-hv^2}+\left(\frac{1}{4h^2v^2}+\frac{1}{2h}\right)V,$$

where

$$V=\int_0^v \varepsilon^{-hv^2}dv,$$

and thus it may readily be tabulated by the help of tables of the error-function.

When v is very large, the ultimate value of the expression is

$$\frac{1}{4}\sqrt{\frac{\pi}{h^3}};$$

which shows that, in this case, the "special" state of the particles in the layer does not affect its permeability.

10. Write, for a moment,

$$-e\delta x$$

as the logarithm of the fraction of the particles with speed v which traverse the layer unchecked. Then it is clear that

$$\varepsilon^{-ex}$$

represents the fraction of the whole which penetrate unchecked to a distance x into a group in the "special" state. Hence the mean distance to which particles with speed v can penetrate without collision is

$$\frac{\int_0^\infty \varepsilon^{-ex}xdx}{\int_0^\infty \varepsilon^{-ex}dx}=\frac{1}{e}.$$

This is, of course, a function of v; and the remarks above show that it increases continuously with v to the maximum value (when v is infinite)

$$\frac{1}{n\pi s^2};$$

i.e., the mean path for a particle moving with infinite speed is the same as if the particles of the medium traversed had been at rest.

11. Hence, to find the *Mean Free Path* among a set of spheres all of which are in the special state, the natural course would appear to be to multiply the

average path for each speed by the probability of that speed, and take the sum of the products. Since the probability of speed v to $v+dv$ is

$$4\sqrt{\frac{h^3}{\pi}}\,\epsilon^{-hv^2}v^2dv$$

the above definition gives for the length of the mean free path,

$$4\sqrt{\frac{h^3}{\pi}}\int_0^\infty \epsilon^{-hv^2}v^2dv/e$$

or, by the expression for e above,

$$\frac{1}{n\pi s^2}\int_0^\infty \frac{\epsilon^{-hv^2}v^2dv}{\int_0^v \epsilon^{-hv_1^2}\left(v_1^2+\frac{v_1^4}{3v^2}\right)dv_1+\int_v^\infty \epsilon^{-hv_1^2}\left(\frac{vv_1}{3}+\frac{v_1^3}{v}\right)dv_1}.$$

This may without trouble (see § 9) be transformed into the simpler expression

$$\frac{1}{n\pi s^2}\int_0^\infty \frac{4x^4\epsilon^{-x^2}dx}{x\epsilon^{-x^2}+(2x^2+1)\int_0^x \epsilon^{-x^2}dx},$$

which admits of easy numerical approximation. The numerical work would be simplified by dividing above and below by ϵ^{-x^2}, but we prefer to keep the present form on account of its direct applicability to the case of mixed systems. And it is curious to note that $4\epsilon^{-x^2}$ is the third differential coefficient of the denominator.

The value of the definite integral (as will be shown by direct computation in an *Appendix* to the paper) is about

$$0{\cdot}677\,;$$

and this is the ratio in which the mean path is diminished in consequence of the motion of the particles of the medium. For it is obvious, from what precedes, that the mean path (at any speed) if the particles were quiescent would be

$$\frac{1}{n\pi s^2}.$$

[The factor by which the mean path is reduced in consequence of the "special" state is usually given, after CLERK-MAXWELL, as $1/\sqrt{2}$ or 0·707.

But this appears to be based on an erroneous definition. For if n_v be the fraction of the whole particles which have speed v, p_v their free path; we have taken the mean free path as

$$\Sigma(n_v p_v),$$

according to the usual definition of a "mean."

CLERK-MAXWELL, however, takes it as

$$\frac{\Sigma(n_v v)}{\Sigma(n_v v/p_v)},$$

i.e., the quotient of the average speed by the average number of collisions per particle per second. But those who adopt this divergence from the ordinary usage must, I think, face the question "Why not deviate in a different direction, and define the mean path as the product of the average speed into the average time of describing a free path?" This would give the expression

$$\Sigma(n_v v) \,.\, \Sigma(n_v p_v/v)\,.$$

The latter factor involves a definite integral which differs from that above solely by the factor $\sqrt{h/x}$ in the numerator, so that its numerical determination is easy from the calculations already made. It appears thus that the reducing factor would be about

$$\frac{2}{\sqrt{\pi}} \times 0{\cdot}650\,, \quad = 0{\cdot}734 \text{ nearly};$$

i.e., considerably more in excess of the above value than is that of CLERK-MAXWELL. Until this comparatively grave point is settled, it would be idle to discuss the small effect, on the length of the mean free path, of the diameters of the impinging spheres.]

III. *Number of Collisions per Particle per Second.*

12. Here again we may have a diversity of definitions, leading of course to different numerical results. Thus, with the notation of § 11, we may give the mean number of collisions per particle per second as

$$\Sigma(n_v v/p_v).$$

This is the definition given by CLERK-MAXWELL and adopted by MEYER; and *here* the usual definition of a "mean" is employed. The numerical value, by what precedes, is

$$16ns^2h^3\int_0^\infty \varepsilon^{-hv^2}v^3dv\left(\int_0^v \varepsilon^{-hv_1{}^2}\left(v_1{}^2+\frac{v_1{}^4}{3v^2}\right)dv_1+\int_v^\infty \varepsilon^{-hv_1{}^2}\left(\frac{vv_1}{3}+\frac{v_1{}^3}{v}\right)dv_1\right).$$

MEYER evaluates this by expanding in an infinite series, integrating, and summing. But this circuitous process is unnecessary; for it is obvious that the two parts of the expression must, *from their meaning*, be equal; while the second part is integrable directly.

13. On account of its bearing (though somewhat indirectly) upon the treat-

ment of other expressions which will presently occur, it may be well to note that a mere inversion of the order of integration, in either part of the above double integral, changes it into the other part.

Otherwise :—we may reduce the whole to an immediately integrable form by the use of polar co-ordinates; putting

$$v=r\cos\theta\,,\quad v_1=r\sin\theta\,,$$

and noting that the limits of r are 0 to ∞ in both parts, while those of θ are 0 to $\pi/4$ in the first part, and $\pi/4$ to $\pi/2$ in the second. [*This* transformation, however, is not well adapted to the integrals which follow, with reference to two sets of spheres, because h has not the same value in each set.]

14. Whatever method we adopt, the value of the expression is found to be

$$\sqrt{\frac{8\pi}{h}}\cdot ns^2=2\sqrt{\frac{2}{\pi h}}\pi ns^2\,;$$

and, as the mean speed is (§ 3)

$$\frac{2}{\sqrt{\pi h}}\,,$$

we obtain Clerk-Maxwell's value of the mean path, above referred to, viz.,

$$\frac{1}{n\pi s^2\sqrt{2}}\,.$$

But (in illustration of the remarks at the end of § 11) we might have defined the mean number of collisions per particle per second as

$$\frac{\Sigma(n_v v)}{\Sigma(n_v p_v)}\,,\ \text{or as}\ \frac{1}{\Sigma(n_v p_v/v)}\,;\ \&c., \&c.$$

The first, which expresses the ratio of the mean speed to the mean free path, gives

$$\frac{2}{\sqrt{\pi h}}\cdot\frac{\pi ns^2}{0{\cdot}677}\,;$$

and the second, which is the reciprocal of the mean value of the time of describing a free path, gives

$$\frac{1}{\sqrt{h}}\,\frac{\pi ns^2}{0{\cdot}650}\,.$$

The three values which we have adduced as examples bear to one another the reciprocals of the ratios of the above-mentioned determinations of the mean free path.

IV. *Clerk-Maxwell's Theorem.*

15. In the ardour of his research of 1859,* Maxwell here and there contented himself with very incomplete proofs (we can scarcely call them more than illustrations) of some of the most important of his results. This is specially the case with the investigation of the law of ultimate partition of energy in a mixture of smooth spherical particles of two different kinds. He obtained, in accordance with the so-called *Law of Avogadro,* the result that the average energy of translation is the same per particle in each system; and he extended this in a Corollary to a mixture of any number of different systems. This proposition, if true, is of fundamental importance. It was extended by Maxwell himself to the case of rigid particles of any form, where rotations perforce come in. And it appears that in such a case the whole energy is ultimately divided *equally* among the various degrees of freedom. It has since been extended by Boltzmann and others to cases in which the individual particles are no longer supposed to be rigid, but are regarded as complex systems having great numbers of degrees of freedom. And it is stated, as the result of a process which, from the number and variety of the assumptions made at almost every stage, is rather of the nature of playing with symbols than of reasoning by consecutive steps, that in such groups of systems the ultimate state will be a partition of the whole energy in equal shares among the classes of degrees of freedom which the individual particle-systems possess. This, if accepted as true, at once raises a formidable objection to the kinetic theory. For there can be no doubt that each individual particle of a gas has a very great number of degrees of freedom besides the six which it would have if it were rigid:—the examination of its spectrum while incandescent proves this at once. But if all these degrees of freedom are to share the whole energy (on the average) equally among them, the results of theory will no longer be consistent with our experimental knowledge of the two specific heats of a gas, and the relations between them.

16. Hence it is desirable that Clerk-Maxwell's proof of his fundamental Theorem should be critically examined, and improved where it may be found defective. If it be shown in this process that certain preliminary conditions are absolutely necessary to the proof even of Clerk-Maxwell's Theorem, and if these *cannot* be granted in the more general case treated by Boltzmann, it is clear that Boltzmann's Theorem must be abandoned.

17. The chief feature in respect of which Maxwell's investigation is to be commended is its courageous recognition of the difficulties of the question. In this respect it far transcends all other attempts which I have seen. Those

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* *Phil. Mag.*, 1860.

features, besides too great conciseness, in respect of which it seems objectionable, are :—

(*a*) He *assumes* that the transference of energy from one system to the other can be calculated from the results of a single impact between particles, one from each system, each having the average translational energy of its system.

Thus (so far as this step is concerned) the distribution of energy in each system may be any whatever.

(*b*) In this typical impact the velocities of the impinging spheres are taken as at right angles to one another, so that the relative speed may be that of mean square as between the particles of the two systems. The result obtained is fallacious, because in general the directions of motion after impact are found not to be at right angles to one another, as they would certainly be (on account of the perfect reversibility of the motions) were this really a typical impact.

(*c*) CLERK-MAXWELL proceeds as if every particle of one system impinged upon one of the other system at each stage of the process—*i.e.*, he calculates the transference of energy as if each pair of particles, one from each system, had simultaneously a typical impact. This neglect of the immensely greater number of particles which either had no impact, or impinged on others of their own group, makes the calculated rate of equalisation far too rapid.

(*d*) Attention is not called to the fact that impacts between particles are numerous in proportion to their *relative* speed, nor is this consideration introduced in the calculations.

(*e*) Throughout the investigation each step of the process of averaging is performed (as a rule) before the expressions are ripe for it.

18. In seeking for a proof of MAXWELL'S Theorem it seems to be absolutely essential to the application of the statistical method to premise :—

(A) That the particles of the two systems are thoroughly mixed.

(B) That in any region containing a very large number of particles, the particles of each kind separately acquire and maintain the error-law distribution of speeds—*i.e.*, each set will ultimately be in the "special" state. The disturbances of this arrangement produced in either system by impacts on members of the other are regarded as being promptly repaired by means of the internal collisions in the system itself. This is the sole task assigned to these internal collisions. We assume that they accomplish it, so we need not further allude to them.

[The warrant for these assumptions is to be sought as in § 4; and in the fact that only a small fraction of the whole particles are at any instant in collision; *i.e.*, that each particle advances, on the average, through a considerable multiple of its diameter before it encounters another.]

(C) That there is perfectly free access for collision between each pair of particles, whether of the same or of different systems; and that, in the mixture, the number of particles of one kind is not overwhelmingly greater than that of the other kind.

[This is one of the essential points which seem to be wholly ignored by BOLTZMANN and his commentators. There is no proof given by them that one system, while regulating by its internal collisions the distribution of energy among its own members, can also by impacts regulate the distribution of energy among the members of another system, when these are not free to collide with one another. In fact, if (to take an extreme case) the particles of one system were so small, in comparison with the average distance between any two contiguous ones, that they practically had no *mutual* collisions, they would behave towards the particles of another system much as LE SAGE supposed his ultra-mundane corpuscles to behave towards particles of gross matter. Thus they would merely alter the apparent amount of the molecular forces between the particles of a gas. And it is specially to be noted that this is a question of *effective diameters* merely, and not of masses :—so that those particles which are virtually free from the self-regulating power of mutual collisions, and therefore form a disturbing element, may be much more massive than the others.]

19. With these assumptions we may proceed as follows:—Let P and Q be the masses of particles from the two systems respectively; and when they impinge, let u, v be their velocity-components measured towards the same parts along the line of centres at impact. If these velocities become, after impact, u′, v′ respectively, we have at once

$$\mathrm{P}(\mathrm{u}'-\mathrm{u})=-\frac{2\mathrm{PQ}}{\mathrm{P}+\mathrm{Q}}(\mathrm{u}-\mathrm{v})=-\mathrm{Q}(\mathrm{v}'-\mathrm{v});$$

an immediate consequence of which is

$$\mathrm{P}(\mathrm{u}'^2-\mathrm{u}^2)=-\frac{4\mathrm{PQ}}{(\mathrm{P}+\mathrm{Q})^2}\left(\mathrm{Pu}^2-\mathrm{Qv}^2-(\mathrm{P}-\mathrm{Q})\mathrm{uv}\right)=-\mathrm{Q}(\mathrm{v}'^2-\mathrm{v}^2).$$

Hence, denoting by a bar the average value of a quantity, we see that transference of energy between the systems must cease when

$$\mathrm{P}\overline{\mathrm{u}^2}-\mathrm{Q}\overline{\mathrm{v}^2}-(\mathrm{P}-\mathrm{Q})\overline{\mathrm{uv}}=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (1),$$

and the question is reduced to finding these averages.

[I thought at first that $\overline{\mathrm{uv}}$ might be assumed to vanish, and that $\overline{\mathrm{u}^2}$ and $\overline{\mathrm{v}^2}$ might each be taken as one-third of the mean square speed in its system. This set of suppositions would lead to MAXWELL'S Theorem at once. But it is clear that, when two particles have each a *given* speed, they are more likely to collide when they are moving towards opposite parts than when towards the same parts. Hence $\overline{\mathrm{uv}}$ must be an essentially negative quantity, and therefore $\mathrm{P}\overline{\mathrm{u}^2}$ necessarily less than $\mathrm{Q}\overline{\mathrm{v}^2}$, if P be greater than Q. Thus it seemed as if the greater masses would have on the average less energy than the smaller. These are two of the pitfalls to which I have alluded. Another will be met with presently.]

20. But these first impressions are entirely dissipated when we proceed to calculate the average values. For it is found that if we write (1) in the form

$$P\overline{u^2-uv}-Q\overline{v^2-uv}=0,$$

the terms on the left are equal multiples of the average energy of a P and of a Q respectively. Thus MAXWELL'S Theorem is rigorously true, though in a most unexpected manner. There must surely be some extremely simple and direct mode of showing that $\overline{u^2-uv}$ is independent of the mean-square speed of the system of Qs. Meanwhile, in default of anything more simple, I give the investigation by which I arrived at the result just stated.

21. Suppose a particle to move, with constant speed v, among a system of other particles in the "special" state; the fraction of the whole of its encounters which takes place with particles, whose speed is from v_1 to v_1+dv_1 and whose directions of motion are inclined to its own at angles from β to $\beta+d\beta$, is (§ 8) proportional to

$$\varepsilon^{-kv_1^2}v_1^2dv_1v_0\sin\beta\, d\beta,$$

or as we may write it for brevity

$$\nu_1 v_0 \sin\beta\, d\beta.$$

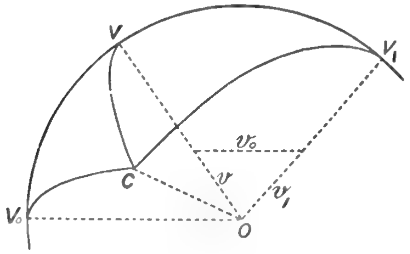
This is easily seen by remarking that, by § 8, while the particle advances through a space δx, it virtually passes through a layer of particles (such as those specified) of thickness $v_0\delta x/v$. Here (§ 3) $3/2k$ is the mean-square speed of the particles of the system.

Let the impinging particle belong to another group, also in the special state. Then the number of particles of that group which have speeds between v and $v+dv$ is proportional to

$$\varepsilon^{-hv^2}v^2dv=\nu,$$

as we will, for the present, write it.

Now let V, V_1, V_0, in the figure, be the projections of v, v_1, v_0 on the unit sphere whose centre is O; C that of the line of centres at impact. Then $VOV_1=\beta$. Let $V_0OV=\alpha$, $V_0OV_1=\alpha_1$, $V_0OC=\gamma$, and $VV_0C=\phi$. The limits of γ are 0 and $\pi/2$; those of ϕ are 0 and 2π. Also the chance that C lies within the spherical surface-element $\sin\gamma d\gamma d\phi$, is proportional to the area of the projection of that element on a plane perpendicular to the direction of v_0, *i.e.*, it is proportional to



$$\cos\gamma\sin\gamma\, d\gamma d\phi.$$

But by definition we have

$$\mathrm{u} = v \cos \mathrm{VOC} = v(\cos \alpha \cos \gamma + \sin \alpha \sin \gamma \cos \phi).$$
$$\mathrm{v} = v_1 \cos \mathrm{V_1OC} = v_1(\cos \alpha_1 \cos \gamma + \sin \alpha_1 \sin \gamma \cos \phi);$$

and by the Kinematics of the question, as shown by the dotted triangle in the figure, we have

$$v \cos \alpha - v_1 \cos \alpha_1 = v_0 ,$$
$$v \sin \alpha - v_1 \sin \alpha_1 = 0 .$$

Thus, as indeed is obvious from much simpler considerations,

$$\mathrm{u} - \mathrm{v} = v_0 \cos \gamma ,$$

so that

$$\overline{\mathrm{u}^2 - \mathrm{uv}} = \frac{\int vv_1 v_0 \sin \beta \, d\beta \, \mathrm{u}(\mathrm{u} - \mathrm{v}) \cos \gamma \sin \gamma \, d\gamma d\phi}{\int vv_1 v_0 \sin \beta \, d\beta \cos \gamma \sin \gamma \, d\gamma d\phi}$$

$$= \frac{\int vv_1 v_0 \sin \beta \, d\beta \, v \, (\cos \alpha \cos \gamma + \sin \alpha \sin \gamma \cos \phi) v_0 \cos^2 \gamma \sin \gamma \, d\gamma d\phi}{\int vv_1 v_0 \sin \beta \, d\beta \cos \gamma \sin \gamma \, d\gamma d\phi} ,$$

where each of the integrals is quintuple.

The term in $\cos \phi$ vanishes when we integrate with respect to ϕ :—and, when we further integrate with respect to γ, we have for the value of the expression

$$\frac{\frac{1}{2}\int vv_1 v_0 \sin \beta \, d\beta v v_0 \cos \alpha}{\int vv_1 v_0 \sin \beta \, d\beta} ,$$

where the integrals are triple.

Now

$$2 v v_0 \cos \alpha = v^2 + v_0^2 - v_1^2 ,$$

and

$$v v_1 \sin \beta \, d\beta = v_0 dv_0 ,$$

so that the expression becomes

$$\frac{\frac{1}{4}\int vv_1 \frac{v_0^2 dv_0}{vv_1} (v^2 + v_0^2 - v_1^2)}{\int vv \frac{v_0^2 dv_0}{vv_1}} .$$

It will be shown below (Part VI.), that we have, generally

$$\int vv_1 \frac{v_0^{2n} dv_0}{vv_1} = \frac{\mathrm{I}_{2n+1}}{2n+1} = \frac{\sqrt{\pi}}{4} n ! \frac{(h+k)^{\frac{2n-1}{2}}}{(hk)^{n+1}} ,$$

and that it is lawful to differentiate such expressions with regard to h or to k. Hence

$$\overline{\mathrm{u}^2 - \mathrm{uv}} = \frac{1}{4} \frac{\mathrm{I}_5/5 - \left(\frac{d}{dh} - \frac{d}{dk}\right) \mathrm{I}_3/3}{\mathrm{I}_3/3} = \frac{1}{h} .$$

Thus CLERK-MAXWELL's Theorem is proved.

22. The investigation of the separate values of the parts of this expression is a little more troublesome, as the numerators now involve second partial differential coefficients of I_1; but it is easy to see that we have

$$\overline{u^2}=\frac{1}{16}\,\frac{\left(\frac{d}{dh}-\frac{d}{dk}\right)^2 I_1-2\left(3\frac{d}{dh}-\frac{d}{dk}\right)I_3/3+I_5/5}{I_3/3}=\frac{h+2k}{2h(h+k)},$$

$$\overline{uv}=\frac{1}{16}\,\frac{\left(\frac{d}{dh}-\frac{d}{dk}\right)^2 I_1-2\left(\frac{d}{dh}+\frac{d}{dk}\right)I_3/3-3I_5/5}{I_3/3}=-\frac{1}{2(h+k)},$$

and, from these, the above result again follows.

[It is clear, from the investigation just given, that the expression for the value of $\overline{u^2-uv}$ would be the same (to a numerical factor *près*) *whatever* law we assumed for the probability of the line of centres having a definite position, and thus that MAXWELL's Theorem would be true, provided only that the law were a function of γ alone, and not of ϕ (*i.e.*, that the possible positions of the line of centres were symmetrically distributed round the direction of relative motion of the impinging particles). In my first non-approximate investigation (read to the Society on Jan. 18, and of which an Abstract appeared in *Nature*, Jan. 21, 1886) I had inadvertently assumed that the possible positions of C were equally distributed over the surface of the hemisphere of which V_0 is the pole, instead of over the surface of its diametral plane. The forms, however, of $\overline{u^2}$ and of $\overline{uv}$ separately, suffer more profound modifications when such assumptions are made.]

V. *Rate of Equalisation of Average Energy per particle in two Mixed Systems.*

23. To obtain an idea of the rate at which a mixture of two systems approaches the MAXWELL final condition, suppose the mixture to be complete, and the systems each in the special state, but the average energy per particle to be different in the two. As an exact solution is not sought, it will be sufficient to adopt, throughout, roughly approximate expressions for the various quantities involved. We shall choose such as lend themselves most readily to calculation.

It is easy to see, by making the requisite slight modifications in the formula of § 12, that, if m be the number of Ps and n that of Qs in unit volume, the number of collisions per second between a P and a Q is

$$2mns^2\sqrt{\frac{\pi(h+k)}{hk}},$$

where s now stands for the sum of the radii of a P and of a Q. For if, in the formula referred to, we put $(hk)^{3/2}$ for h^3, and also put k for h in the exponentials where the integration is with respect to v_1, it becomes

$$8ns^2(hk)^{\frac{3}{2}}\mathrm{I}_3/3\,,$$

according to the notation of § 21. This is the average number of impacts per second which a P has with Qs.

Hence, if ϖ be the *whole* energy of the Ps, ρ that of the Qs, per unit volume, the equations of § 19 become

$$\dot{\varpi}=-\frac{16}{3}\frac{\mathrm{PQ}}{(\mathrm{P}+\mathrm{Q})^2}s^2\sqrt{\frac{\pi(h+k)}{hk}}(n\varpi-m\rho)=-\dot{\rho}\,,$$

from which we obtain, on the supposition (approximate enough for our purpose) that we may treat $1/h+1/k$ as constant,

$$n\varpi-m\rho=\mathrm{C}\epsilon^{-t/\mathrm{T}}\,,$$

where

$$\frac{1}{\mathrm{T}}=\frac{16}{3}\frac{\mathrm{PQ}}{(\mathrm{P}+\mathrm{Q})^2}s^2(m+n)\sqrt{\frac{\pi(h+k)}{hk}}\,.$$

The quantity

$$n\varpi-m\rho=mn(\varpi/m-\rho/n)\,,$$

is mn times the difference of the average energies of a P and a Q, and (since $\epsilon^{4.6}=100$ nearly) we see that it is reduced to one per cent. of its amount in the time

$$t_1=4.6\mathrm{T}=\frac{13.8}{16s^2(m+n)}\frac{(\mathrm{P}+\mathrm{Q})^2}{\mathrm{PQ}}\sqrt{\frac{hk}{\pi(h+k)}}\text{ seconds.}$$

24. For a mixture, in equal volumes, of two gases in which the masses of the particles are not very different, say oxygen and nitrogen, we may assume as near enough for the purposes of a rough approximation

$$m=n=\frac{3}{2}\times 10^{20}\,,$$

whence $m+n$ (per cubic inch) is double of this,

$$\frac{3}{2h}=\frac{3}{2k}=(12\times 1600\text{ inch sec.})^2\,,$$

$$s=3\times 10^{-8}\text{ inch,}$$

so that

$$t_1=\frac{13.8\times 10^{16}\times 4}{16\times 9\times 3\times 10^{20}\times 12\times 1600}\sqrt{\frac{3}{4\pi}}=\frac{1}{3\times 10^9}\text{ seconds, nearly;}$$

and the difference has fallen to 1 per cent. of its original amount in this period, *i.e.*, after each P has had, on the average, about four collisions with Qs. This calculation has no pretensions to accuracy, but it is excessively useful as showing the nature of the warrant which we have for some of the necessary assumptions made above. For if the rapidity of equalisation of average energy in two systems is of this extreme order of magnitude, we are entitled to suppose that the restoration of the special state in any one system is a phenomenon taking place at a rate of at least the same if not a higher order of magnitude.

CLERK-MAXWELL's result as regards the present question is that, at every typical impact between a P and a Q, the difference of their energies is reduced in the ratio

$$\left(\frac{\mathrm{P}-\mathrm{Q}}{\mathrm{P}+\mathrm{Q}}\right)^2;$$

so that, if the masses were equal, the equalisation would be instantaneous.

VI. *On some Definite Integrals.*

25. It is clear that expressions of the forms

$$\int_0^\infty \varepsilon^{-hx^2}x^r dx \int_0^x \varepsilon^{-ky^2}y^s dy \quad \text{and} \quad \int_0^\infty \varepsilon^{-hx^2}x^r dx \int_x^\infty \varepsilon^{-ky^2}y^s dy\,,$$

where r and s are essentially positive integers, may lawfully be differentiated under the integral sign with regard to h or to k. In fact they, and their differential coefficients, which are of the same form, are all essentially finite.

As, in what immediately follows, we shall require to treat of the first of these forms only when r is odd and s even, and of the second only when r is even and s odd, it follows that their values can all be obtained by differentiation from one or other of the integrals

$$\int_0^\infty \varepsilon^{-hx^2}x dx \int_0^x \varepsilon^{-ky^2}dy = \frac{\sqrt{\pi}}{4h\sqrt{h+k}}\,,$$

and

$$\int_0^\infty \varepsilon^{-hx^2}dx \int_x^\infty \varepsilon^{-ky^2}y dy = \frac{\sqrt{\pi}}{4k\sqrt{h+k}}\,.$$

These values may be obtained at once by noticing that the second form is integrable directly; while, by merely inverting the *order* of integration, it becomes the first with h and k interchanged.

26. In §§ 21, 22 we had to deal with a number of integrals, all of one form, of which we take as a simple example

$$\mathrm{I}_3/3 = \int \nu\nu_1 \frac{v_0^{\,2}}{vv_1} dv_0$$

$$= \frac{1}{3}\int_0^\infty \varepsilon^{-hx^2}x dx\left(\int_0^x \varepsilon^{-ky^2}y dy\left(\overline{x+y}^3 - \overline{x-y}^3\right) + \int_x^\infty \varepsilon^{-ky^2}y dy\left(\overline{y+x}^3 - \overline{y-x}^3\right)\right).$$

From the remarks above it is clear that this can be expressed as

$$\frac{2}{3}\frac{\sqrt{\pi}}{4}\left\{\left(3\frac{d^2}{dhdk}+\frac{d^2}{dk^2}\right)\frac{1}{h\sqrt{h+k}}+\left(3\frac{d^2}{dkdh}+\frac{d^2}{dh^2}\right)\frac{1}{k\sqrt{h+k}}\right\}$$

$$=\frac{2}{3}\frac{\sqrt{\pi}}{4}\frac{3}{2}\left(\frac{k+3h}{h^2(h+k)^{\frac{5}{2}}}+\frac{3k+h}{k^2(h+k)^{\frac{5}{2}}}\right)$$

$$=\frac{\sqrt{\pi}}{4}\;\frac{(k^3+3k^2h)+(3kh^2+h^3)}{h^2k^2(h+k)^{\frac{5}{2}}}$$

$$=\frac{\sqrt{\pi}}{4}\;\frac{(h+k)^{\frac{1}{2}}}{(hk)^2}.$$

The peculiar feature here shown is the making up of the complete cube of $k + h$ in the numerator by the supply of the *first half* of its terms from the first part of the integral, and of the remainder from the second.* On trial I found that the same thing holds for I_5 and I_7, so that I was led to conjecture that, generally, as in § 21

$$\frac{\mathrm{I}_{2n+1}}{2n+1}=\frac{\sqrt{\pi}}{4}\,n\,!\;\frac{(h+k)^{\frac{2n-1}{2}}}{(hk)^{n+1}}.$$

After the preliminary work we have just given, it is easy to prove this as follows. We have always

$$\left((x+y)^{2n+1}-(x-y)^{2n+1}\right)\left((x+y)^2+(x-y)^2\right)=$$
$$(x+y)^{2n+3}-(x-y)^{2n+3}+\left(x^2-y^2\right)^2\left((x+y)^{2n-1}-(x-y)^{2n-1}\right).$$

Operate on this by

$$\int_0^\infty \varepsilon^{-hx^2}xdx\int_0^x \varepsilon^{-ky^2}ydy\;\left(\qquad\qquad\right),$$

and on the same expression, with x and y interchanged (when, of course, it remains true), by

$$\int_0^\infty \varepsilon^{-hx^2}xdx\int_x^\infty \varepsilon^{-ky^2}ydy\;\left(\qquad\qquad\right),$$

and add the results. This gives at once

$$-2\left(\frac{d}{dh}+\frac{d}{dk}\right)\mathrm{I}_{2n+1}=\mathrm{I}_{2n+3}+\left(\frac{d}{dh}-\frac{d}{dk}\right)^2\mathrm{I}_{2n-1}\,;$$

which is found on trial to be satisfied by the general value given above.

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* Prof. CAYLEY has called my attention, in connection with this, to the following expression from a Trinity (Cambridge) *Examination Paper* :—

$$\begin{aligned}(a+b)^{2n} &= (a+b)^n\ \ (a^n+b^n)\\ &+\ (a+b)^{n-1}\ (na^nb+nab^n)\\ &+\ (a+b)^{n-2}\left(\frac{n.n+1}{1.2}a^nb^2+\frac{n.n+1}{1.2}a^2b^n\right)\\ &\qquad\qquad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\\ &+(a+b)\frac{n.n+1\ .\ .\ .\ .\ 2n-2}{1\ .\ 2\ \ .\ .\ .\ .\ \ n-1}(a^nb^{n-1}+a^{n-1}b^n)\,.\end{aligned}$$

27. Partly as a matter of curiosity, but also because we shall require a case of it, it may be well to mention here that similar processes (in which it is no longer necessary to break the y integration into two parts) lead to the companion formula

$$\frac{\mathrm{I}_{2n}}{2n}=\int_0^\infty \varepsilon^{-hx^2}\,xdx\int_0^\infty \varepsilon^{-ky^2}\,ydy\left(\overline{x+y}^{2n}-\overline{x-y}^{2n}\right)/2n$$

$$=\frac{\pi}{4}\,\frac{1\,.\,3\,.\,5\ldots(2n-1)}{2^n}\,\frac{(h+k)^{n-1}}{(hk)^{\frac{2n+1}{2}}}$$

And we see, by **Wallis'** Theorem, that (when n is increased without limit) I_{2n} is ultimately the geometric mean between I_{2n-1} and I_{2n+1}.

VII. *Mean Path in a Mixture of two Systems.*

28. If we refer to § 10, we see that, instead of what was there written as $-e\delta x$, we must now write $-(e+e_1)\delta x$; where e_1, which is due to stoppage of a particle of the first system by particles of the second, differs from e in three respects only. Instead of the factor $4s^2$, which appears in e, we must now write $(s+s_1)^2$; where s_1 is the diameter of a particle of the second system. Instead of h and n we must write h_1 and n_1 respectively.

Hence the mean free path of a particle of the first system is

$$4\sqrt{\frac{h^3}{\pi}}\int_0^\infty \frac{v^2dv}{e+e_1}\varepsilon^{-hv^2}\,;$$

which, when the values of e and e_1 are introduced, and a simplification analogous to those in §§ 9, 11, is applied, becomes

$$\frac{1}{n\pi s^2}\int_0^\infty \frac{4\varepsilon^{-x^2}x^4dx}{x\varepsilon^{-x^2}+(1+2x^2)\int_0^x \varepsilon^{-x^2}dx+\frac{n_1h}{nh_1}\left(\frac{s+s_1}{2s}\right)^2\left(x_1\varepsilon^{-x_1^2}+(1+2x_1^2)\int_0^{x_1}\varepsilon^{-x^2}dx\right)}$$

in which

$$x_1=x\sqrt{\frac{h_1}{h}}\,.$$

Thus the values tabulated at the end of the paper for the case of a single system enable us to calculate the value of this expression also.

VIII. *Pressure in a System of Colliding Particles.*

29. There are many ways in which we may obtain, by very elementary processes, the pressure in a system of colliding particles.

(*a*) It is the rate at which momentum passes across a plane unit area; or the whole momentum which so passes per second. [It is to be noted that a loss of negative momentum by the matter at either side of the plane is to be treated as a gain of positive.]

In this, and the other investigations which follow, we deal with planes supposed perpendicular to the axis of x; or with a thin layer bounded by two such planes.

The average number of particles at every instant per square unit of a layer, whose thickness is δx, is $n\delta x$. Of these the fraction

$$\nu = 4\sqrt{\frac{h^3}{\pi}}\varepsilon^{-hv^2}v^2dv$$

have speeds from v to $v+dv$. And of these the fraction

$$\sin\beta\, d\beta/2$$

are moving in directions inclined from β to $\beta+d\beta$ to the axis of x. Each of them, therefore, remains in the layer for a time

$$\delta x/v\cos\beta,$$

and carries with it momentum

$$Pv\cos\beta$$

parallel to x. Now from $\beta=0$ to $\beta=\frac{\pi}{2}$ we have positive momentum passing towards x positive. From $\beta=\frac{\pi}{2}$ to $\beta=\pi$ we have an *equal* amount of negative momentum leaving x positive. Hence the whole momentum which passes per second through a plane unit perpendicular to x is

$$2\times\frac{1}{2}Pn\int_0^\infty \nu v^2\int_0^{\frac{\pi}{2}}\cos^2\beta\sin\beta d\beta=\frac{1}{3}Pn\overline{v^2}$$

where the bar indicates mean value. That is

$$\text{Pressure}=p=\frac{2}{3}\text{ (Kinetic Energy in Unit Volume).}$$

(*b*) Or we might proceed as follows, taking account of the position of each particle when it was last in collision.

Consider the particles whose speeds are from v to $v+dv$, and which are contained in a layer of thickness δx, at a distance x from the plane of yz. Each has (§ 10) on the average ev collisions per second. Thus, by the perfect reversibility of the motions, from each unit area of the layer there start, per second,

$$n\nu ev\delta x$$

such particles, which have just had a collision. These move in directions uniformly distributed in space; so that

$$\sin\beta\, d\beta/2$$

of them are moving in directions inclined β to $\beta+d\beta$ to the axis of x. Of these the fraction

$$\varepsilon^{-ex\sec\beta}$$

(where x is to be regarded as signless) reach the plane of yz, and each brings momentum

$$Pv\cos\beta$$

perpendicular to that plane. Hence the whole momentum which reaches unit area of the plane is

$$2\times\frac{1}{2}nP\int_0^\infty \nu v^2\int_0^{\frac{\pi}{2}}\cos\beta\sin\beta\,d\beta\int_0^\infty e\,dx\,\varepsilon^{-ex\sec\beta}$$

$$=nP\int_0^\infty \nu v^2\int_0^{\frac{\pi}{2}}\cos^2\beta\sin\beta\,d\beta\,,$$

the same expression as before.

(*c*) **Clausius'** method of the virial, as usually applied, also gives the same result.

30. But this result is approximate only, for a reason pointed out in § 6 above. To obtain a more exact result, let us take the virial expression itself. It is, in this case, if N be the number of particles in volume V,

$$\frac{1}{2}PN\overline{v^2}=\frac{3}{2}pV+\frac{1}{2}\Sigma(Rr)\,,$$

where R is the mutual action between two particles whose centres are r apart, and is positive when the action is a stress tending to bring them nearer to one another. Hence, omitting the last term, we have approximately

$$p=\frac{1}{3}\frac{N}{V}P\overline{v^2},$$

which we may employ for the purpose of interpreting the value of the term omitted.

[It is commonly stated (see, for instance **Clerk-Maxwell's** Lecture to the *Chemical Society*[*]) that, when the term $\frac{1}{2}\Sigma(Rr)$ is negative, the action between the particles is in the main repulsive:—"a repulsion so great that no attainable force can reduce the distance of the particles to zero." There are grave objections to the assumption of molecular repulsion; and therefore it is well to inquire whether the mere impacts, which must exist if the kinetic theory be true, are not of themselves sufficient to explain the experimental results which have been attributed to such repulsion. The experiments of **Regnault** on hydrogen first showed a deviation from **Boyle's** Law in the direction of less

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[*] *Chem. Soc. Jour.*, xiii. (1875), p. 493.

compression than that Law indicates. But ANDREWS showed that the same thing holds for all gases at temperatures and pressures over those corresponding to their critical points. And AMAGAT has experimentally proved that in gaseous hydrogen, which has not as yet been found to exhibit any traces of molecular attraction between its particles, the graphic representation of pV in terms of p (at least for pressures above an atmosphere, and for common temperatures) consists of a series of parallel straight lines. If this can be accounted for, without the assumption of molecular repulsion but simply by the impacts of the particles, a real difficulty will be overcome. And it is certain that, at least in dealing with hard colliding spheres if not in all cases, we have no right to extract from the virial, as the pressure term, that part only which depends upon impacts on the containing vessel; while leaving unextracted the part depending on the mutual impacts of the particles. The investigation which follows shows (so far as its assumptions remain valid when the particles are not widely scattered) that no pressure, however great, can bring a group of colliding spheres to a volume less than four times the sum of their volumes. If they were motionless they could be packed into a space exceeding the sum of their volumes in the ratio $6 : \pi\sqrt{2}$, or about $1{\cdot}35 : 1$, only.]

In the case of hard spheres we have obviously $r=s$; and, with the notation of § 19, remembering that $\mathrm{Q}=\mathrm{P}$, $k=h$, we have

$$\mathrm{R}=-\mathrm{P}(\mathrm{u}-\mathrm{v}).$$

Hence we must find, by the method of that section, the mean value of the latter expression. It is easily seen to be

$$-\mathrm{P}\frac{\int \nu\nu_1 v_0{}^2 \sin\beta\ d\beta \cos^2\gamma \sin\gamma d\gamma d\phi}{\int \nu\nu_1 v_0 \sin\beta\ d\beta \cos\gamma \sin\gamma d\gamma d\phi}=-\frac{2\mathrm{P}}{3}\frac{\int \nu\nu_1 v_0{}^3 dv_0/vv_1}{\int \nu\nu_1 v_0{}^2 dv_0/vv_1}$$

$$=-\frac{2\mathrm{P}}{3}\frac{\mathrm{I}_4/4}{\mathrm{I}_3/3}=-\mathrm{P}\sqrt{\frac{\pi}{2h}}.$$

But, § 14, the average number of collisions, per particle per second, is

$$2\sqrt{\frac{2}{\pi h}}\frac{\mathrm{N}}{\mathrm{V}}\pi s^2.$$

Hence, for any one particle, the sum of the values of R (distributed, on the average, uniformly over its surface) is, in one second,

$$\Sigma(\mathrm{R})=-\frac{2\mathrm{NP}}{h\mathrm{V}}\pi s^2=-\frac{4}{3}\frac{\mathrm{N}}{\mathrm{V}}\mathrm{P}\overline{v^2}\pi s^2=-p.4\pi s^2.$$

Thus it would appear that we may regard each particle as being subjected to the general pressure of the system; but as having its own diameter *doubled.*

It is treated, in fact, just as it would then be if all the others were reduced to massive points.

The value of the term in the virial is

$$\frac{1}{4}\,ns\Sigma(\mathrm{R})$$

because, though every particle suffers the above average number of collisions, it takes two particles to produce a collision. This is equal to

$$-np\pi s^3 = -6p \text{ (sum of volumes of spheres)};$$

so that the virial equation becomes

$$n\mathrm{P}\overline{v^2}/2 = \frac{3}{2}p\big(\mathrm{V} - 4 \text{ (sum of volumes of spheres)}\big),$$

which, in *form* at least, agrees exactly with AMAGAT'S* experimental results for hydrogen.

These results are closely represented at 18° C. by

$$p(\mathrm{V} - 2{\cdot}6) = 2731\,;$$

and at 100° C. by

$$p(\mathrm{V} - 2{\cdot}7) = 3518\,.$$

The quantity subtracted from the volume is sensibly the same at both temperatures. The right-hand members are nearly in proportion to the absolute temperatures. The pressure is measured in mètres of mercury. Hence the volume of the gas, at 18° C. and one atmosphere, is (to the unit employed)

$$2{\cdot}6 + 2731/0{\cdot}76 = 3596 \text{ nearly.}$$

Thus, by the above interpretation of AMAGAT'S results, we have at 18° C.

$$n\pi s^3 = 3{\cdot}9/3596\,.$$

CLERK-MAXWELL, in his *Bradford Lecture*,† ranks the various numerical data as to gases according to "the completeness of our knowledge of them." The mean free path appears in the second rank only, the numbers in which are regarded as rough approximations. In the third rank we have two quantities involved in the expression for the mean free path, viz., the absolute diameter of a particle, and the number of particles per unit volume (s and n of the preceding pages).

To determine the values of s and n separately, a second condition is required. It has usually been assumed, for this purpose, that the volume of a gas, "when reduced to the liquid form, is not much greater than the combined volume of the molecules." MAXWELL justifies this assumption by reference to the small compressibility of liquids.

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* *Annales de Chimie*, xxii. 1881.

† *Phil. Mag.*, 1873, ii. 453. See also *Nature*, viii. 298.

But, if the above argument be, even in part, admitted, we are not led to any such conclusion, and we can obtain ns^3 (as above) as a quantity of the second rank. We have already seen that ns^2 is inversely proportional to the mean free path, and is thus also of the second rank. From these data we may considerably improve our approximations to the values of n and of s.

Taking MAXWELL's estimate of the mean free path in hydrogen, we have (to an inch as unit of length)

$$\frac{0\cdot677}{\pi ns^2}=380.10^{-8}.$$

From these values of ns^2 and ns^3 we have, approximately, for 0° C., and 1 atmosphere,

$$n=16.10^{20}, \quad s=6.10^{-9}.$$

The values usually given are

$$n=3.10^{20}, \quad s=2\cdot3.10^{-8}.$$

It must be recollected that the above estimate rests on two assumptions, neither of which is more than an approximation, (*a*) that the particles of hydrogen behave like hard spheres, (*b*) that they exert no mutual molecular forces. If there were molecular attraction the value of ns^3 would be greater than that assumed above, while ns^2 would be unaltered. Thus the particles would be larger and less numerous than the estimate shows.

[Of course, after what has been said, it is easy to see that V should be diminished further by a quantity proportional to the surface of the containing vessel and to the radius of a sphere. But though this correction will become of constantly greater importance as the bulk occupied by a given quantity of gas is made smaller, it is probably too minute to be detected by experiment.]

IX. *Effect of External Potential.* (*Added June* 15, 1886.)

31. Another of MAXWELL's most remarkable contributions to the Kinetic Theory consists in the Theorem that a vertical column of gas, when it is in equilibrium under gravity, has the same temperature throughout. He states, however, that an erroneous argument on the subject, when it occurred to him in 1866, " nearly upset [his] belief in calculation."* He has given various investigations of the action of external forces on the distribution of colliding spheres, but

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* *Nature*, viii., May 29, 1873. MAXWELL's name does not occur in the Index to this volume, though he has made at least five contributions to it, most of which bear on the present subject:—viz. at pp. 85, 298, 361, 527, 537.

all of them are complex. The process of BOLTZMANN, alluded to in a foot-note to the introduction (*anté*, p. 66), and which CLERK-MAXWELL ultimately preferred to his own methods, involves a step of the following nature.

An expression, analogous to the f of § 3, but in which B and C are undetermined functions of the coordinates x, y, z, of a point, is formed for the number of particles per unit volume, at that point, whose component speeds, parallel to the axes, lie between given narrow limits. I do not at present undertake to discuss the validity or the sufficient generality of the process by which this expression is obtained, though the same process is (substantially) adopted by WATSON and others who have written on the subject. However obtained, the expression is correct. It can be established at once by reasoning such as that in §§ 2, 3, 4. To determine the forms of the aforesaid functions, however, a most peculiar method is adopted by BOLTZMANN and MAXWELL. The number of the particles per unit volume at x, y, z whose corresponding "ends" occupy unit volume at u, v, w in the velocity space-diagram (§ 3), is expressed in terms of these functions, and of $u^2+v^2+w^2$. The variation of the logarithm of this number of particles is then taken, on the assumption that

$$\delta x = u\delta t, \text{ \&c.,} \quad \delta u = -\frac{d\mathrm{U}}{dx}\delta t, \text{ \&c.,}$$

where U is the external potential; and it is equated to zero, *because the number of particles is unchangeable.* As this equation must hold good for all values of u, v, w, it furnishes sufficient conditions for the determination of B and C. The reasons for this remarkable procedure are not explained, but they seem to be as below. The particles are, as it were, followed in thought into the new positions which they would have reached, and the new speeds they would have acquired, in the interval δt, *had no two of them collided or had there been no others to collide with them.* But this is not stated, much less justified, and I cannot regard the argument (in the form in which it is given) as other than an exceedingly dangerous one; almost certain to mislead a student.

What seems to underlie the whole, though it is not enunciated, is a postulate of some such form as this :—

When a system of colliding particles has reached its final state, we may assume that (on the average) for every particle which enters, and undergoes collision in, a thin layer, another goes out from the other side of the layer precisely as the first would have done had it escaped collision.

32. If we make this assumption, which will probably be allowed, it is not difficult to obtain the results sought, without having recourse to a questionable

process of variation. For this purpose we must calculate the changes which take place in the momentum, and in the number of particles, in a layer; or, rather, we must inquire into the nature of the processes which, by balancing one another's effects, leave these quantities unchanged.

Recur to § 29, and suppose the particles to be subject to a potential, U, which depends on x only. Then the whole *momentum* passing per unit of time perpendicularly across unit surface of any plane parallel to yz is

$$\frac{1}{3}\mathrm{P}n\int_0^\infty \nu v^2 = \frac{\mathrm{P}n}{2h},$$

where n (the number of particles per cubic unit), and h (which involves the mean-square speed), are functions of x.

At a parallel plane, distant α from the first in the direction of x positive, the corresponding value is

$$\frac{1}{2}\mathrm{P}\left(1+\alpha\frac{d}{dx}\right)\frac{n}{h}.$$

But the difference must be sufficient to neutralise, in the layer between these planes, the momentum which is due to the external potential, *i.e.*,

$$-\mathrm{P}n\alpha\frac{d\mathrm{U}}{dx}.$$

Hence

$$\frac{1}{2}\mathrm{P}\alpha\frac{d}{dx}\frac{n}{h} = -\mathrm{P}n\alpha\frac{d\mathrm{U}}{dx}.$$

or

$$-2h\frac{d\mathrm{U}}{dx} = \frac{1}{n}\frac{dn}{dx} - \frac{1}{h}\frac{dh}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1).$$

Again, the *number* of particles which, in unit of time, leave the plane unit towards the side x positive is

$$\frac{1}{2}n\int_0^\infty \nu v\int_0^{\frac{\pi}{2}}\cos\beta\sin\beta\, d\beta = \frac{1}{4}n\int_0^\infty \nu v.$$

Hence those which leave the corresponding area at distance α are, in number,

$$\frac{1}{4}\left(1+\alpha\frac{d}{dx}\right)\left(n\int_0^\infty \nu v\right).$$

But, by our postulate of last section, they can also be numbered as

$$\frac{1}{4}n\int_\xi^\infty \nu v\,(1-\xi^2/v^2),$$

where

$$\xi^2 = 2\alpha\frac{d\mathrm{U}}{dx}.$$

This expression is obtained by noting that none of those leaving the first plane can pass the second plane unless they have

$$v^2 \cos^2\beta > 2a\frac{dU}{dx} \cdot$$

All of the integrals contained in these expressions are *exact*, and can therefore give no trouble. The two reckonings of the number of particles, when compared, give

$$-2h\frac{dU}{dx} = \frac{1}{n}\frac{dn}{dx} - \frac{1}{2h}\frac{dh}{dx} \quad . \quad . \quad . \quad . \quad (2).$$

From (1) and (2) together we find, first

$$\frac{dh}{dx} = 0,$$

which is the condition of uniform temperature; and again

$$n = n_0 \varepsilon^{-2h(U-U_0)},$$

which is the usual relation between density and potential.

[In obtaining (2) above it was assumed that, with sufficient accuracy,

$$\varepsilon^{-h\zeta^2} = 1 - h\zeta^2.$$

To justify this :—note that in oxygen, at ordinary temperatures and under gravity,

$$\frac{3}{2h} = 1550^2 \text{ in foot-second units,}$$

$$\frac{dU}{dx} = 32 \quad ,, \quad ,, \quad ,,$$

so that, even if $a = 1$ inch, we have approximately

$$h\zeta^2 = 2h\frac{a\,dU}{dx} = \frac{1}{300{,}000} \cdot]$$

It is easy to see that exactly similar reasoning may be applied when U is a function of x, y, z; so that we have, generally,

$$n = n_0 \varepsilon^{-2h(U-U_0)},$$

where h is an absolute constant. And it is obvious that similar results may be obtained for each separate set of spheres in a mixture, with the additional proviso from MAXWELL's Theorem (§§ 20, 21) that P/h has the same value in each of the sets.

APPENDIX.

The following little table has been calculated for the purposes of §§ 11, 28, by Mr J. Clark, Neil-Arnott Scholar in the University of Edinburgh, who used six-place logarithms:—

x	X_1	X_2	X_1/X_2	X_3	X_3/X_2
·1	·000099	·200665	·00049 +	·000990	·00493 +
·2	·001537	·405312	·00379 +	·007686	·01896 +
·3	·007420	·617838	·01198 +	·024676	·03994 –
·4	·021814	·841997	·02591 –	·054537	·06477 +
·5	·048675	1·081321	·04501 +	·097350	·09003 –
·6	·090418	1·339068	·06752 +	·150698	·11254 –
·7	·147091	1·618194	·09089	·210130	·12985 +
·8	·215978	1·921318	·11241 –	·269973	·14051 +
·9	·291870	2·250723	·12968 +	·324301	·14409 –
1·0	·367879	2·608351	·14104 –	·367879	·14104 –
1·1	·436590	2·995825	·14572 +	·396900	·13249 –
1·2	·491380	3·414479	·14388 +	·409409	·11990 +
1·3	·527004	3·865384	·13633 +	·405388	·10488 –
1·4	·541119	4·349386	·12441 +	·386514	·08887 –
1·5	·533581	4·867132	·10962 +	·355721	·07309 –
1·6	·506619	5·419114	·09348 –	·316637	·05843 –
1·7	·464174	6·005696	·07729 –	·273044	·04546 +
1·8	·409127	6·627149	·06203 +	·228404	·03447 –
1·9	·352543	7·283658	·04840 –	·185549	·02547 +
2·0	·293040	7·975359	·03674 +	·146520	·01837 +
2·1	·236390	8·702340	·02715 +	·112567	·01294 –
2·2	·185224	9·464667	·01956 –	·084193	·00889
2·3	·141065	10·262360	·01373 +	·061333	·00598 –
2·4	·104541	11·095474	·00941 +	·043559	·00393 –
2·5	·075390	11·964016	·00630 +	·030156	·00252 +
2·6	·052962	12·867980	·00411 –	·020370	·00158 +
2·7	·036242	13·807388	·00262 +	·013423	·00097 +
2·8	·024155	14·782249	·00162 +	·008627	·00058 +
2·9	·015700	15·792549	·00099 +	·005414	·00034 +
3·0	·009963	16·838302	·00057 +	·003321	·00019 +

Here $X_1 = x^4\epsilon^{-x^2}$ and $X_3 = x^3\epsilon^{-x^2}$, while $X_2 = x\epsilon^{-x^2} + (2x^2+1)\int_0^x \epsilon^{-x^2}dx$.

The sum of the numbers in the fourth column is 1·69268, so that the approximate value of the integral in § 11, which is 0·4 of this, is 0·67707.

The sum of the numbers in the sixth column is 1·62601, so that the value of the integral in [the addition to] § 11 is about 0·6504.

IV.—*The Eggs and Larvæ of Teleosteans.* By J. T. Cunningham, B.A. (Plates I.–VII.)

(Read 5th July 1886.)

The purpose of this memoir is (1) to make known a number of drawings and descriptions of the eggs, embryos, and larvæ of the species of Teleosteans which I have been able to study at the Scottish Marine Station; (2) to review as comprehensively as possible what is known at the present time concerning the structure of the embryos and larvæ of the species of Teleosteans, and to discover what features are common to each family or each order; (3) to discuss the changes which take place in the protoplasm and nucleus of the mature ovum immediately after it is shed, both when fertilised and when unfertilised. The ova of the following species were taken directly from the parent fish, and artificially fertilised. The necessary operations were carried out, in some cases by myself, on board fishing boats—usually steam trawlers from Granton. In many instances I did not myself go out in the boats, but the ova were obtained and brought to me at the laboratory by Alexander Turbyne, keeper of the station. But in every case there is no uncertainty as to the species of the fish from which the ova were taken; if there was any doubt, specimens of the parent fish were brought with the ova.

1. *Clupea harengus*, Linn. (Herring) (Pl. I. figs. 1–3).

The development of the herring has been described by Prof. C. Kupffer* in an elaborate memoir, which is illustrated by microscopic photographs. Among these, one figure of the hatched larva is given, but this is on too small a scale to exhibit the structure clearly. It is nearly two years since I studied the ova of the herring, and some of the drawings which I then made have been used to illustrate papers on particular problems in Teleostean development.† But, as far as I am aware, no good figures of the larva of the herring have been published, and I therefore think that the figures on Pl. I. will not be superfluous. Herring, as is well known, have two spawning seasons on the east coast of Britain—one in the spring, in February and March, and one in the

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* *Ueber Laichen und Entwicklung des Ostsee-Herings*, Berlin, 1878.

† "On the Significance of Kupffer's Vesicle," &c., *Quart. Jour. Micr. Sci.*, 1885; and "On Relations of Yolk to Gastrula," &c., *Ibid.*

autumn, in August and September. The eggs which I studied were obtained in August off the Longstone Lighthouse, Fearn Islands.

Embryonic Period and Temperature.—Hatching took place on the eighth and ninth days, the temperature varying from 11°·5 to 14°·5 C.

In the herring ovum the yolk consists of a number of nearly spherical translucent vitelline globules; there are no oil globules. The blastodisc is large in proportion to the yolk.

Diagnosis of Larva.—The length of the newly hatched larva is 5·2 to 5·3 mm., according to KUPFFER. The mouth is open, the body is wholly transparent except the eyes, which are of a deep black, and perfectly opaque; there are no red blood corpuscles; the notochord is unicolumnar; the anus is at a distance from the yolk sac, being 1 mm. from the end of the tail; the pectoral fin is present as a simicircular fold of membrane; the pelvic fin is not developed; compact chromatophores are present on the sides of the body and tail.

The larvæ of the herring I have taken occasionally, but not often, in the townet. Two were obtained at 5 fathoms depth west of Inchkeith, Oct. 7, 1885; 15 at a depth of 3 feet off St Abb's Head, Sept. 30, 1885; a few at 3 fathoms east of Inchkeith, May 14, 1885; and a few at 5 fathoms north-east of Inchkeith, April 15, 1885.

2. *Salmo levenensis* (Loch Leven Trout) (Pl. I. fig. 4).

This figure is taken from an alevin of the species obtained from Sir JAMES MAITLAND'S hatchery at Howietoun. The larva was three days old; the permanent anterior dorsal, caudal, and anal fins have begun to develop, but the median larval fold is present behind the anterior dorsal, behind the anal, and between the anus and the yolk. The pelvic fins have appeared; they are situated some distance in front of the anus, and they have no connection with the preanal larval fin, which extends between them up to the yolk sac.

3. *Osmerus eperlanus*, Lacép (Smelt or Sperling) (Pl. I. figs. 5, 6).

The mature egg of Osmerus, when first shed, is yellow in colour, and but slightly translucent; it is surrounded by a double zona radiata, the inner surface of which is, as in all Teleosteans, in immediate contact with the vitellus. When the eggs are allowed to fall on to stones or glass plates in water containing milt, they become attached and fertilised simultaneously. The attachment is effected in the following manner:—The outer zona radiata ruptures at the region of the ovum which is opposite the micropyle, and peals off the inner zona, becoming of course inverted in the process. Over a circular area surrounding the micropyle, the two layers of the zona remain firmly united. The outer surface of the external layer or zona externa is adhesive, and the ruptured edge becomes attached, so that the ovum swings in the water from the flexible suspensory membrane thus formed. I have elsewhere * described the separation of the two layers of the zona resulting in the formation of the suspensory membrane, but the relation of the united parts of the two layers to the micropyle is now described for the first time, and is shown in fig. 6 as it appears in optical section.

When fertilisation takes place, a large perivitelline space is formed by the elevation of the internal zona. Unfortunately, I was unable to obtain a sufficient number of healthy ova to study the development. Fig. 5 was taken from an ovum fertilised at Stirling on May 6; it was drawn on May 7, twenty-five hours after the egg was shed. It shows the character of the egg, and the relation of the blastodisc to the yolk; but the blastodisc was not segmented, and it is possible that the ovum was not really fertilised, the formation of the blastodisc occurring normally in ripe Teleostean ova without fertilisation. The ovum resembles somewhat that of the herring. It is a little more transparent than the herring ovum, and the structure of the yolk is different. In the egg of *Osmerus* there are a number of oil globules, varying much in size, while the yolk of the herring ovum has no oil globules. The diameter of the fertilised ovum is 1·3 mm.

4. *Pleuronectes platessa*, Linn. (Plaice) (Pl. II. figs. 1–3).

The eggs of the plaice were artificially fertilised on board a steam trawler outside the Isle of May, February 3, 1886. The egg is 1·95 mm. in diameter, and like the other eggs of the Pleuronectidæ which I have examined, has a perfectly homogeneous yolk. The perivitelline space is small. The larvæ were not actually hatched, but one taken from the ovum, when almost ready to hatch, is shown in fig. 3. Its length is 4·1 mm. The eye is faintly pigmented. There are three rows of yellow dendritic pigment cells down each side, and black dendritic cells in the head. The anus is open, and situated immediately behind the yolk sac. The notochord is multicolumnar; the pelvic fin not developed.

5. *Pleuronectes flesus*, Linn. (Common Flounder) (Pl. II. figs. 4–8).

Eggs of this species were obtained on March 30, 1886, in the Firth of Forth, in Aberlady Bay. It is the only species which has been found in abundance, and in the spawning condition, so far up the Firth. The egg is similar in all respects to that of the plaice except in size. It is 1·03 mm. in diameter. The newly hatched larva is transparent, and 3·01 mm. in length.

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* *Proc. Zool. Soc.*, London, 1886.

The anus is open; notochord multicolumnar;* small round pigment spots along the sides and head; pelvic fin not developed.

On March 5 of the current year I visited some fishing boats at Kincardine on the Forth. These boats were fishing with what are called bag-nets or stow-nets. A net of this kind is fashioned very much like a beam-trawl, and is fastened beneath the boat, so that its mouth faces the current of the tide; the fish are thus washed into the net. Among the fish taken on the occasion of my visit were a large number of *Pleuronectes flesus,* which are commonly called fresh-water flounders, or mud flounders. Nearly all of these fish had a number of small round white tumours on the fins and on the upper or dark side. The tumours are cutaneous, and have been described more than once (see M'Intosh, *Third Annual Report of Scottish Fishery Board*). The fishermen stated that these tumours were the eggs of the fish, that the mud flounder carried its eggs on its back. On another occasion a bottle was sent to me from Elie, said to contain flounder spawn; the contents when examined proved to be the greenish gelatinous egg-cases of some species of Chætopod, perhaps *Arenicola piscatorum,* and within the cases were the trochospheres, whose green colour was the cause of the colour of the cases.

5. *Pleuronectes limanda,* Linn. (Salt-water Flounder) (Pl. II. figs. 9–11; Pl. III. figs. 1–6).

Ripe specimens of this species were obtained by me in considerable numbers on board a steam-trawler six or seven miles east-north-east of the Isle of May, on May 21 of the current year. A number of the eggs were squeezed out, artificially fertilised, and conveyed to the Marine Station. Living specimens were also successfully carried to the aquarium, and upon eggs taken from these I was able to study the condition of the ripe eggs immediately on their escape from the oviduct, and the earliest processes of fertilisation and development.

The egg, after the formation of the perivitelline space, is ·84 mm. in diameter; the appearance, magnified 33 times at the close of simple segmentation, is shown in Pl. II. fig. 9.

Hatching took place on the third day; the temperature of the surface of the sea where the eggs were taken was 7°·5 C., and the temperature of the water containing the eggs varied from this to 10° C.

The newly hatched larva was 2·66 mm. in length; the structure closely similar to that of other species of the genus; notochord multicolumnar; mouth not open; small black pigment spots on sides of the body; anus close to the yolk, and not open.

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* The terms unicolumnar and multicolumnar applied to the notochord refer to the arrangement of the vacuoles, which are very conspicuous in newly hatched fish: in the herring and a few other cases these vacuoles are cubical, and form a single linear series; in other cases there are several series.

The figures given of the first processes of fertilisation and development will be considered in a subsequent section.

On Dec. 5, 1885, I trawled with a fine meshed shrimp trawl across the Drum Sands, which are situated between Queensferry Point and Cramond Island, and obtained a considerable number of young *Pleuronectes limanda*. These were about 2 inches long, and could be identified from the semicircular curve in the lateral line above the pectoral fin. Larger, nearly full-grown, specimens were also taken, and kept for some time in the aquarium, where they lived healthily.

In June of the current year, Mr Ramage, who is at present studying at the station, pointed out to me that the sands to the west of the laboratory were swarming with young flounders. These were about $\frac{1}{4}$ inch long, and had already reached the condition of the adult; they showed no trace of larval structures. But I was unable to identify these young fish, as the lateral line could not be clearly distinguished. It is of course probable that the young of many different species are present in such situations in the summer months. It is pretty certain that nearly all our valuable flat-fishes pass the early post-larval stages of their existence on littoral sand-flats. Mr George Brook informs me that large numbers of young flat-fish are destroyed by shrimpers in such situations. With regard to this particular locality, I have never seen any shrimping carried on in the neighbourhood.

6. *Pleuronectes cynoglossus*, Linn. (Witch) (Pl. III. figs. 7–9; Pl. IV., Pl. V.).

Of the developing eggs of this species I made a particularly careful study, with the intention of obtaining, if possible, greater certainty on the various points in dispute concerning the earliest changes that the mature ovum undergoes after being shed. A number of living specimens of the fish were trawled by the "Medusa," on 23rd and 24th June of the current year, at a place called Fairlie Patch, opposite the town of Fairlie, in the channel between the island of Cumbrae and the mainland. The fish were taken alive to the little laboratory known as the "Ark," which was originally a floating structure, but is now firmly established on the beach at the east side of Millport Bay. I have given a large number of figures, illustrating the successive stages in the development of this species. After the formation of the perivitelline space they are 1·155 mm. in diameter. The yolk is perfectly transparent, but the zona radiata is thicker than in most of the other species of the genus. The perivitelline space is very small. During the time the eggs were under observation the weather was very fine, and the laboratory being fully exposed to the sunshine, became in the middle of the day very hot. I had no means of regulating the temperature of the water containing the eggs, and on two occasions it rose to 20°·5 C.

This temperature was fatal to a large number of the developing eggs. The temperature of the water in which the eggs were first placed was 12°·5 C. With these great variations in temperature, hatching took place on the sixth day. The larva is not different from that of the other species of Pleuronectes; its length is 3·9 mm.; there is no pigment in the eye; a number of very minute pigment spots are scattered down the sides. The anus is not open, and the coalesced segmental ducts do not communicate with the rectum (see Pl. V. fig. 5). Pl. V. fig. 7, shows the condition of the larva a little more than forty-eight hours after hatching. The length is now increased to 5·9 mm.—a very rapid rate of growth. The median fin-fold is much wider. The eye is slightly pigmented, and pigment is largely developed in the skin of the body. The cutaneous chromatophores form five well-marked transverse stripes, arranged in longitudinal series along the sides, three of them on the tail, one in the region of the rectum, and one about the pectoral fin. No trace of the pelvic fin is to be seen. The operculum is present as a slight fold, and beneath it the first branchial cleft is widely open; behind this are four clefts indicated but not perforated. The mouth is also still wanting.

7. *Pleuronectes microcephalus* (Lemon Sole).

I have not obtained fertilised ova of this species, but I was able to ascertain from examination of unfertilised mature examples that there are no oil globules, and that the diameter measures 1·1 mm. The ripe females, from which the mature eggs came, were taken in the trawl east of May Island, May 22 of the current year.

8. *Gadus æglefinus*, Linn. (Haddock) (Pl. VI. fig. 1).

The ova of *Gadus morrhua*, *G. æglefinus*, and *G. merlangus*, in various stages of development, have been previously figured by me.* The larvæ, after hatching, were not described in the paper I refer to. The newly hatched cod has been correctly figured by John Ryder.† For the sake of comparison, I give a figure of the newly hatched larva of the haddock. The eye is pigmented, and there is a single row of dendritic chromatophores along each side ventrally. The anus is not open, nor the mouth; the pelvic fin is also wanting. In all respects, except in size, the larva of the haddock resembles that of the cod.

The following species of ova and larvæ were not obtained directly from the parent fish, but identified from other considerations.

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* "Relations of Yolk to Gastrula in Teleosteans," *Quart. Jour. Micr. Sci.*, 1885.

† Report of American Fish Commission for 1882, Washington, 1884.

9. *Cottus scorpius*, Linn. (Pl. VI. fig. 2).

The eggs ascribed to this species were brought in to the station on February 14 of the present year. They formed large masses of dark red colour, and were attached to the rocks between tide marks. The ova are but slightly translucent; the zona radiata is thick. The figure shows the appearance under a low power of the microscope. The yolk is homogeneous, except for the presence of scattered oil globules, irregular in number and size, and contains the pigment, which, yellowish-red as seen in each separate ovum, gives the whole mass a darker red colour. The diameter of the vitelline membrane is 2·03 mm., of the ovum 1·81. The identification is founded on some remarks of Professor M'Intosh, who observed the deposition of similar eggs in the aquarium of the Marine Laboratory at St Andrews (see *Third Annual Report Scottish Fishery Board*, 1885, App. F.).

Agassiz * has stated that the eggs of *Cottus grœnlandicus*, which is only a variety of *Cottus scorpius*, are pelagic. His conclusion rests apparently on the identification of the oldest stage of larvæ from a certain kind of pelagic eggs with the adult *Cottus*, and this mode of identification is of course not absolutely certain.

10. *Liparis Montagui*, Cuv. (Pl. VI. figs. 3, 4).

Small masses of adhesive eggs are frequently obtained attached to tufts of *Hydrallmannia falcata*, Hincks. I have obtained such specimens in the months of May and June, both from long lines laid outside the Isle of May and from the dredge in the upper parts of the Firth. By the fishermen the eggs in question are usually believed to come from the herring or the haddock, and even naturalists of some experience have confounded them with herring spawn, which also often adheres to specimens of *Hydrallmannia*. The mass from which fig. 3 was taken was attached to a piece of *Hydrallmannia* left by the tide on the beach near Cramond Island, and was obtained May 7, 1886. The longest diameter of the egg, including the vitelline membrane (zona radiata), was 1·27 mm., the transverse diameter of the yolk sac ·87 mm. The zona radiata is of considerable thickness, and shows a division into two layers. The yolk is homogeneous and transparent, and contains three or more oil globules of various sizes. The mass of eggs seen with the unaided eye was colourless and transparent. I have identified the ova as those of *Liparis Montagui*, from some remarks of Prof. M'Intosh in Report on the St Andrews Laboratory, in the *Third Annual Report of the Scottish Fishery Board*, but the identification is

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* *Proc. Amer. Acad. Arts and Sci.*, vol. xvii.; and *Memoirs of Mus. Comp. Zool.*, Harvard, vol. xiv. No. 1, pt. i.

not certain. M'INTOSH says that the eggs of *Liparis Montagui* are found in shallow water, attached to such zoophytes as Hydrallmannia and Sertularia, and also to red Algæ, and are of a pale straw colour. The eggs I have described were well advanced in development, so that the colour may have been present at an earlier stage, the colour of such eggs often disappearing as development proceeds. The eyes were considerably pigmented.

Fig. 4 is a sketch of a fish hatched from some eggs exactly similar to those above described, which were taken in the trawl between Inchkeith and Burntisland, April 29, 1884. The age of the young fish was two days after hatching. The eyes are deeply pigmented, the mouth completely developed, the pectoral fin is large, and covered with black pigment spots, and there is a row of similar spots along the ventral edge of the tail on each side. A small remnant of the yolk is still present, containing a single oil-globule.

11. *Cyclopterus lumpus*, Linn. (Lump-sucker) (Pl. VI. fig. 5).

To amateur naturalists on the coasts of Scotland the large masses of yellowish spawn of this fish, watched by the male parent, the "rawn and cock paidle," as they are called in the Scotch dialect, are a not unfamiliar sight. I regret to say I have not had an opportunity of personally observing the phenomenon in its natural state. But masses of the ova of *Cyclopterus* have been frequently brought into the station by boys; they are found attached to the rocks near the station, not far from low water mark. The colour of the eggs varies from red to pale yellow or nearly white. The yolk contains numerous oil globules of various sizes, arranged in a cluster at the ventral pole, but is otherwise homogeneous. The perivitelline space is small. The eggs are but slightly translucent. The diameter is 2·60 mm., inclusive of the vitelline membrane. The young *Cyclopterus*, when first hatched, is 4 mm. in length, but not so far advanced in development as the stage figured by AGASSIZ[*] of the same length. The anus is immediately behind the yolk sac, which forms such a contrast in size to the tail that the fish is tadpole-like in form. The body is quite opaque, and the blood red. The eyes are completely pigmented. The embryonic fin fold persists extending forwards dorsally a little beyond the anal region, but fin rays have appeared in the membrane. Both paired fins are well developed, the ventrals forming a median sucker, which differs only from that of the adult in exhibiting the fin rays in a more primitive condition. The skin contains numerous regularly distributed chromatophores.

The young *Cyclopterus*, both immediately after hatching and in later stages, occur very plentifully among the Algæ on the shore at Granton, and everywhere on the British coasts. They are also frequently taken in the tow-net at a

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[*] *Young Stages*, iii.

distance from the shore, but in this case are usually attached by the sucker to floating pieces of sea-weed. The eggs are deposited in January and February, and the young stages are to be found on the shore or in the tow-net throughout the summer. It was observed by Mr Jackson, in the Southport Aquarium, that the male parent, watching over the eggs, kept up a continual motion of his pectoral fins in close proximity to the eggs, and it appears that this is necessary to secure the sufficient oxygenation of the eggs, which are laid in such large masses that the central ones might easily in still water be asphyxiated. Young specimens of *Cyclopterus* were taken in the tow-net in the following localities:—Surface, 30 miles north-east of May Island, July 17, 1885; surface, near Inch Mickery, Aug. 26, 1885; surface, Firth of Forth, two occasions, 1884; surface, east of Craig Waugh, May 1884. I have never taken any large numbers either of these or any other fish larvæ in the tow-nets.

Species not identified.

A certain number of well-marked species have been obtained by tow-net collecting, which I have not yet been able to identify. There are two possible methods of identifying an unknown species of pelagic ovum. One is to compare it, or the larva hatched from it, with figures and descriptions of ova or larvæ already known; the other, to keep a number of specimens of the ovum in question alive until they hatch, and then to keep the larvæ till they attain the specific characters of the adult fish. Both of these methods are liable to error.

Species No. 12 (Pl. VII. fig. 2).

This form is easily distinguished by one conspicuous characteristic, namely, that the perivitelline space is very wide. The yolk is perfectly homogeneous and transparent. The diameter of the vitelline membrane is 2·1 mm., of the ovum 1·2 mm. The eggs were obtained in the latter end of March, both in 1885 and 1886, about 10 miles east of the Isle of May. Unfortunately, time could not be found to give sufficient attention to the form to isolate it and keep it alive till hatching took place. Thus the characters of the larva were not ascertained, and no egg at all similar has been taken directly from an adult fish.

Species No. 13 (Pl. VII. figs. 3, 4).

The eggs of this species were obtained in the tow-net, 16 miles beyond the Isle of May, on April 30, and off Gullane Ness, May 27, 1886. The diameter of the ovum, including the vitelline membrane, is ·84 mm. The perivitelline space is small; there is a single oil globule situated beneath the posterior end of the embryo. Some of the eggs were hatched, and fig. 4 shows the form of the larva immediately after hatching; the length is 2·1 mm.; the notochord, as seen in fig. 4*a*, is multicolumnar; there are black pigment spots on the body, but the eye is unpigmented; the pigment on the post-vitelline part of the body forms two black transverse bands. The intestine was, I believe, not open, but a solid extension of it extended to the ventral edge of the larval fin.

In a great many respects the present species agrees with *Motella mustela*, Linn., as described by GEORGE BROOK,* from eggs actually observed to be deposited by the parent. There are several minute points of difference. BROOK'S measurement of the ovum is ·655 to ·731 mm. in largest diameter, while he gives the length of the newly hatched larva as 2·25 mm.; thus the diameter of the ovum given by BROOK is slightly less than my measurement, while the length of the larva given by him is slightly greater than what I have stated. The position of the oil globule and rectum is also different in my figure from that in BROOK'S. But the points of agreement are more numerous and important than the points of difference; the arrangement of the pigment, for instance, is exactly the same in the two accounts. It is evident, therefore, that the species I have described is either *Motella mustela*, Linn., or some other of the four British species of *Motella*.

Pelagic eggs closely similar to the species here described, and to those of *Motella mustela* as described by BROOK, have been described by A. AGASSIZ and C. O. WHITMAN,† and referred with some uncertainty to *Motella argentea*, Rhein. Two other species of pelagic eggs have also been provisionally ascribed by those authors to the genus *Motella*.

Species No. 14 (Pl. VII. figs. 5, 6).

This form is well characterised; it possesses one feature which, as far as extant observations show, is present in no other pelagic ovum, namely, that the yolk is divided into a number of polyhedral masses. This egg is the most perfectly pellucid of all I have observed, and the planes of division in the yolk appear in optical section as extremely fine lines. The egg is slightly oval in shape, ·94 mm. by ·97 mm. in diameter. The newly hatched larva is 3·63 mm. in length, the notochord is unicolumnar, and the anus is separated from the yolk by two-thirds of the length of the post-vitelline part of the body, as in the herring; the larva is absolutely without pigment. The eggs were obtained in 1884 and 1886, in the latter end of May and during June. In each season they were taken within the Firth of Forth, between Gullane Ness and the island of Inchkeith. This form, from its conspicuous characteristics, has

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* *Linn. Soc. Jour.*, vol. xviii.

† "Pelagic Stages of Young Fishes," *Memoirs of Mus. Comp. Zool. Harv.*, vol. xiv., No. 1.

long been known, but all attempts to trace it with certainty into connection with a particular species of fish have hitherto failed. The eggs and larvæ were first described by A. AGASSIZ* in 1882, under the name *Osmerus mordax*, Gill, figures being given of the newly hatched larva and some older stages. AGASSIZ appears to have obtained his figures of the later stages from specimens taken in the tow-net, not from larvæ reared in captivity directly from the egg. He states that at first he supposed the larvæ to belong to some *Clupeoid* species, until he saw a paper by Mr H. J. RICE, on the development of *Osmerus*, when he became convinced that his specimens were really to be ascribed to *Osmerus mordax*. He points out that the oldest larva he figures has a striking resemblance to *Scombresox* and *Belone*. As a matter of fact this resemblance is not very exact, and as it is known that the eggs of all the *Scombresocidœ* are provided with filamentous processes of the vitelline membrane, it is certain that the ovum under consideration cannot belong to any member of that family. AGASSIZ also remarks that the resemblance of the development of *Osmerus* to that of the herring as given by SUNDEVALL† is very close. Now SUNDEVALL gives a figure of the larva of *Osmerus eperlanus*, which shows an oil globule in the yolk sac, and I have shown that the ovum of *O. eperlanus* is adhesive. Thus it is impossible that AGASSIZ' larva should be that of *Osmerus mordax*. Two species of the same genus could not differ so greatly in the structure of their ova and the conditions to which those ova are exposed, as do the pelagic ovum we have been considering, and the ovum of *Osmerus eperlanus*. Moreover, *Osmerus mordax* does not occur in the British seas. It is certain that the herring cannot be the parent of the ovum in question, in spite of the resemblance between the larva derived from it and the herring larva, for the ova of the herring are well known, and are not pelagic. This same pelagic ovum and larva have been described by V. HENSEN,‡ and that gentleman, courteously replying to inquiries of mine on the subject, said, in his opinion, the parent species was the sprat. But here we have the same difficulty as in the case of *Osmerus*. Can any species of *Clupea* have pelagic ova? No instance is yet known of a typically adhesive and a typically pelagic ovum occurring in the same genus. Nevertheless the segregation of the yolk in our pelagic ovum is not altogether incomparable with the condition of the yolk in the herring. It seems absolutely certain that the problematic ovum belongs to some physostomous fish, but hitherto no physostomous fish is known to have a pelagic ovum. It has struck me as possible that the parent we are seeking to discover is really the eel, *Anguilla vulgaris*. At all events the fertilised spawn of the eel has never been examined.

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* *Young Stages*, pt. iii.

† *Svensk. Vetensk. Akad.*, 1855.

‡ *Vierter Ber. Com. Unt. Deutsches Meere*, Berlin, 1883.

Species No. 15, Pl. VII. fig. 7.

These ova formed a cylindrical rope-like mass, and were brought up on a tow-net line from a depth of about 30 fathoms in the Gulf of Guinea. They were obtained by Mr John Rattray, on two occasions when he was on board the steamer "Buccaneer," a telegraph steamer placed at the disposal of Mr J. Y. Buchanan, for hydrographical investigations. The first occasion was on March 12 of the current year, in lat. 1° 17′ N., long. 13° 56·6′ W.; the second occasion was soon after, not far from the same locality. The depth of the ocean at the place was 2725 fathoms. The felted filaments in the rope-like mass were internal, the eggs external. Each ovum was 1·5 to 1·6 mm. in diameter. They are, as far as I am aware, the first Teleostean ova which have been found to have a group of filamentous processes at each of two opposite poles of the vitelline membrane. In these ova one group of processes is rudimentary and functionless, but nevertheless the system of processes is closely similar to that which occurs in *Myxine* (see my paper on "Reproductive Elements of *Myxine glutinosa*," *Quart. Jour. Micr. Sci.*, 1886). Gobiidæ, Blenniidæ, Pomacentridæ, Atherinidæ, Scombresocidæ are the only families known in which processes of the vitelline membrane occur, but it is impossible to say which of these families, if any, includes the parent of the ova described.

Identification of Ova.—The identification of the numerous pelagic ova which are taken in the tow-net at the mouth of the Firth, at different times of the year, cannot at present be carried out with complete certainty. If the eggs and larvæ of every species known to occur were adequately described and figured, the feat might be possible; but at present the identification of any egg taken from the sea must always be subject to a certain degree of scepticism. I have several times attempted to assign the eggs in a tow-net gathering each to its parent species, and have satisfied myself that I had separated the eggs of the Plaice, Cod, Haddock, *Pl. flesus,* and *Trigla gurnardus.* But there may be other species with closer resemblances to these than I am at present aware of.

General Comparative Review of the Structure of the Ova and Larvæ of Teleosteans.[*]

The method followed in the present section is to take the families of each Order successively, and inquire what is known concerning the characters of

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[*] The classification employed in the present section is—

I. Physostomi.
II. Physoclisti.
    1. Anacanthini.
    2. Acanthopterygii.
    3. Acanthopt. Pharyngognathi.
    4. Lophobranchii.
    5. Plectognathi.

the eggs and larvæ, then to ascertain what features are common to all the families of the order, and finally to compare the characters which belong to the several orders. We shall take the families as defined by GÜNTHER in the article "Ichthyology" of the *Encyclopædia Britannica.*

Fam. 1. SILURIDÆ.

The female of *Aspredo batrachus* attaches the eggs to the skin of her own ventral surface, and carries them about there until they are hatched. The male *Arius* carries the eggs about in his pharynx. The male *Callichthys* makes a nest.

An account of the breeding and development of *Amiurus albidus* (Lesueur), Gill, is given by JOHN A. RYDER in *Bull. U.S. Fish. Com.*, vol. iii. The ovum is adhesive, and $\frac{1}{6}$ inch in diameter after fertilisation; the vitellus was $\frac{1}{8}$ inch in diameter. The female deposited the whole of her eggs at one time in a tank, in one mass, which was 6 inches in length by 4 in width, by $\frac{3}{4}$ inch in thickness. The male watched over the mass with great assiduity till hatching occurred, and constantly fanned the eggs with his anal, ventral, and pectoral fins. The perivitelline space in the developing ovum was crowded with free refringent corpuscles, a fact not noted in any other Teleostean ovum. Hatching took place on sixth to eighth day. The intestine in the larva ends not very far behind the yolk sac.

Fam. 2. SCOPELIDÆ.

,, 3. CYPRINIDÆ.

The carps are all fresh-water fishes. The eggs are in most cases adhesive, and attached to aquatic plants. The zona radiata is double. *Carassius auratus,* L., the gold-fish, and the variety known as the telescope-fish, attach their eggs to water plants (M. VON. KOWALESWKI, *Zeit. f. wiss. Zool.,* Bd. xliii.).

The larvæ of *Cyprinus* (*Leuciscus*) *rutilus* and *C. idus* are figured by SUNDEVALL. These figures are curious. In the newly hatched larva they show the yolk apparently extending back to the anus; that is to say, although the anus is near the end of the tail, as in other physostomous larvæ, the yolk, instead of being ellipsoidal in shape, is elongated, and occupies, in addition to its usual space, the interval ordinarily taken up by the preanal median fin-fold. The latter structure is shown in a normal state of development in stages subsequent to the absorption of the yolk, and it is possible that the apparent anomaly in the earlier stages is due to want of definition in the drawings, as in SUNDEVALL's figures generally the limit between intestine and yolk sac is not clearly shown. The eggs were hatched in May. The newly hatched larva of *Leuciscus rutilus* is 6·5 mm. long, of *L. idus*, 7·3 mm.

Fam. 4. KNERIIDÆ.
„ 5. CHARACINIDÆ.
„ 6. CYPRINODONTIDÆ.

Many of the Cyprinodonts are viviparous. The males are always much smaller than the females. A. AGASSIZ has figured some late stages of one species, *Fundulus nigrofasciatus*, C. and V., but his youngest stage has the homocercal tail already complete, and does not allow one to judge of the characters of the newly hatched larva.

Fam. 7. HETEROPYGII.
„ 8. UMBRIDÆ.
„ 9. SCOMBRESOCIDÆ.

The position of this family is somewhat doubtful; it is placed by GÜNTHER among the Physostomi, although the air-bladder has no duct. By CLAUS (*Grundzüge der Zoologie*, 4th ed., 1882), the family is added to the Anacanthini. The peculiarities of the vitelline membrane in this family were first noticed and described by HAECKEL (Muller's *Archiv*, 1855). Prof. KÖLLIKER, in the *Verh. d. Physik u. Med. Ges. zu Würzburg*, 1858, corrected and added to HAECKEL's observations. A clear and satisfactory description of the membrane, with its filamentous processes, is given by JOHN A. RYDER, in his paper on the Development of *Belone longirostris* = *Belone truncata*, Günther (Lesueur) (*Bull. U. S. Fish Commission*, vol. i. 1881). From that paper we learn that the egg of Belone is much heavier than sea water, and sinks rapidly to the bottom when undisturbed; and also, that by means of the filaments, large numbers of the eggs spawned from the same female are fastened together, and the clusters usually become attached to foreign objects in the water, which objects may of course chance to be either fixed or in a state of free suspension. The vitellus is optically homogeneous, and the whole egg transparent, though, I infer, less so than pelagic ova. The larva, after hatching, is not figured, but as far as can be judged from figures of the embryo within the vitelline membrane, the anus is in immediate proximity to the yolk. This point cannot be definitely decided. The egg of *Belone truncata* is rather large, measuring, according to RYDER, $\frac{1}{7}$ inch diameter, or, as measured from the figure given by him, 3·49 mm.

The vitelline membrane is provided with filaments similar to those of Belone in the genera Scombresox, Hemirhamphus, and Exocoetus (flying fish). Arrhamphus' eggs have not been examined. The eggs of *Belone vulgaris*, Fleming, have been examined by Mr Francis Day, and a short description of the filaments is given in his *British Fishes.* The species occurs on the British coasts. It is not uncommon on the south coast, and, according to Parnell, enters the Firth of Forth in July. I have not obtained any specimens hitherto.

It is worthy of note, that if I am right in judging from Ryder's figure that the rectum in the larva of Belone is in contact with the yolk sac, this fact confirms the view of Claus, that the Scombresocidæ do not belong to the Physostomi.

Fam. 10. Esocidæ.

There is only one genus in this family, Esox, the pike. The eggs of *Esox lucius,* Linn., have frequently formed the subject of embryological investigation, and were part of the material on which was based the classical memoir of Lereboullet, "Recherches d'Embryologie Comparée sur le Brochet, l'Ecrevisse et la Perche" (*Ann. d. Sci. Nat.,* ser. iv. vol. i. 1854).

The eggs are small, and are deposited in February and March. They are adhesive and attached to aquatic plants in narrow creeks or ditches (Day).

The larvæ of the pike at different stages are described and figured by Sundevall (*Svenska Vet. Akad. Hand.,* 1855). The youngest stage figured is two days old. The anus is nearer to the end of the tail than to the yolk sac; the pectorals are developed, but not the ventrals; the eye is considerably pigmented, and chromatophores are scattered all over the body; the length is 10 mm.; the newly hatched larva is 9 mm. long. It is noteworthy that the pelvic fins have no relation in development to the ventral fin-fold; the latter persists, extending between the pelvic fins and in front of them long after they have begun to appear.

Fam. 11. Galaxiidæ.
,, 12. Mormyridæ.
,, 13. Sternoptychidæ.

Argyropelecus hemigymnus, Cocco, was dredged between the Shetland and Faroe Islands by the "Porcupine" in 1869. Most of the species are pelagic, some abyssal. The eggs are large (Day, *Brit. Fishes*).

Fam. 14. Stomiatidæ.
,, 15. Salmonidæ.

The ova of Salmo are large, heavy, and non-adhesive. In the newly hatched larva of this genus, or alevin as it is commonly called, the anus is at a distance from the yolk, a preanal embryonic fin separating the two, as in the herring. The notochord, however, is multicolumnar. As has already been mentioned in the case of Esox, the preanal fin-fold extends between and in front of the pelvic fins in the alevin of Salmo.

The larva of *Coregonus oxyrhynchus*, Nilss., at different stages is figured by SUNDEVALL (*loc. cit.*). The ova were deposited from 6th to 10th November, and hatched in the following February; they fall loose and separate to the bottom of the water; the diameter measures 3 mm. The newly hatched embryo is 11 mm. long; the anus is near the end of the tail, far removed from the yolk sac. The pelvic fins develop at the sides of the preanal fin long before the latter disappears, and the position of the pelvic fins is behind the anterior end of the preanal fin, where it meets the yolk-sac.

The ova of Thymallus (Grayling) are similar to those of Salmo, but smaller. The ova of *Osmerus* (*eperlanus* at all events) are adhesive, the external adhesive layer of the zona radiata peeling off from the inner, and forming a suspensory membrane. SUNDEVALL (*Svensk. Akad.*, 1855) gives figures and description of the newly hatched larva of *Osmerus eperlanus;* its length is 5·5 mm., the anus is near the end of the tail, there is a single oil globule in the yolk, the eye is slightly pigmented; the eggs were obtained May 2, hatched May 20, 1855. AGASSIZ and WHITMAN (*Pelagic Stages,* p. 38) remark that the development of the pelagic egg they believe to be *Osmerus mordax* closely resembles that of the herring as given by SUNDEVALL. They seem to have overlooked SUNDEVALL's figures of *Osmerus*. The presence of an oil-globule in the larva of the latter genus is sufficient to prevent its being confounded with the larvasupposed by the American authors to belong to *Osmerus mordax*.

Fam. 16. PERCOPSIDÆ.
,, 17. HAPLOCHITONIDÆ.
,, 18. GONORHYNCHIDÆ.
,, 19. HYODONTIDÆ.
Fam. 20. PANTODONTIDÆ.
,, 21. OSTEOGLOSSIDÆ.
,, 22. CLUPEIDÆ.

The ova of *Clupea harengus*, Linn., have been carefully studied. The ova are heavy and adhesive. The yolk is composed of a number of spherical or nearly spherical yolk spheres, with no oil-globules. The blastodisc is large, forming about one-fifth of the whole egg. The newly-hatched larva is pelagic and very transparent, the anus is far behind the yolk sac, the notochord unicolumnar, the eyes slightly pigmented, but no pigment in the rest of the body. The fertilised eggs and larvæ of *Clupea sprattus*, Linn., have never been observed. Eggs, apparently mature, were pressed by Mr DUNCAN MATTHEWS from a few specimens of the fish which had well-developed ovaries.

The ova were apparently adhesive, similar to those of the herring, but considerably smaller (Report on the Sprat Fishing of 1883–84, *Second Annual Report of the Scottish Fishery Board*, 1884).

The eggs of *Alosa sapidissima* have received much attention from the United States Fish Commission. They differ from those of the herring in not being adhesive; they are deposited in fresh or brackish water, and are but slightly heavier than the water itself, so that they remain in a state of suspension near the bottom. It is a curious fact that, although the artificial cultivation of shad ova has been practised on such a large scale in America, no memoir on the development of the fish has appeared in the publications of the U. S. Commission. I have not been able to find any figure of the ova at any stage of development, but Mr John A. Ryder, in a paper on the absorption of the yolk in embryo fishes (*Bulletin U. S. Fish Commission*, vol. ii., 1882), gives a figure of the anterior region of a larval *Alosa*, some days after hatching. All that can be drawn from this figure is that the notochord is multicolumnar.

Fam. 23. BATHYTHRISSIDÆ.
„ 24. CHIROCENTRIDÆ.
„ 25. ALEPOCEPHALIDÆ.
„ 26. NOTOPTERIDÆ.
„ 27. HALOSAURIDÆ.

Fam. 28. HOPLOPLEURIDÆ.
„ 29. GYMNOTIDÆ.
„ 30. SYMBRANCHIDÆ.
„ 31. MURAENIDÆ.

A great deal has been written about the reproductive organs of Anguilla and Conger. The fertilised ova have never been seen, but young eels about 2½ inches long are common enough in canals and rivers in spring. Specimens from the Forth and Clyde Canal were brought to me in April 1886. For an account of the investigations which have been made into the reproduction of the eel, see G. Brown Goode, *Bull. U. S. Fish. Commission*, vol. i., 1881.

From the above survey it is seen that no physostomous fish is known at present to have pelagic ova. In the newly-hatched larvæ, at present known, the anus is separated by a considerable interval from the yolk sac. In the Clupeidæ the notochord is unicolumnar, but this is not a feature common to the order, as that organ is multicolumnar in the newly-hatched Salmo. It seems pretty certain that the problematic ovum, which Agassiz and Whitman identified as belonging to *Osmerus mordax*, is derived from some physostomous fish. V. Hensen suggested, in a letter to me, that it was the ovum of the sprat, but without evidence this is improbable, and it is not supported by the account of the sprat's ovum given by Duncan Matthews. As no one has seen the embryo of the eel, it possibly belongs to Anguilla, but in that case one would expect to find the ova more plentiful.

ORDER II. ANACANTHINI.

Fam. 1. LYCODIDÆ.

I am not aware that the development of any species of this family has been studied.

Fam. 2. GADIDÆ.

Gadus.—A large number of the species of this genus have been studied—*Gadus morrhua, merlangus,* and *æglefinus* by myself, *G. morrhua* by JOHN A. RYDER. The eggs are, of course, closely similar except in size. The largest of the three species above mentioned are those of *G. æglefinus.* The eggs are pelagic, the yolk is optically homogeneous, and destitute of oil globules. In the newly hatched larva the anus is not open, the rectum is in immediate proximity to the yolk sac, the notochord is multicolumnar, the pelvic fins not developed, and the mouth not open. In the newly-hatched haddock the eyes are considerably pigmented; there are stellate chromatophores scattered over the sides of the trunk, and a single row of them along the ventral edge of each side of the tail.

Motella.—The development of *Motella mustela,* Linn., the five-bearded rockling, has been studied by GEORGE BROOK (*Jour. Linn. Soc.*, 1884, vol. xviii.). The eggs were deposited in his aquarium, under observation. The eggs are pelagic, and have usually one large oil globule, exceptionally more than one. (The buoyancy of the egg is in the paper attributed to the oil globule, an error which has been repeatedly made; there are many pelagic ova which have no oil globule.) The eggs are somewhat oval in shape and slightly variable in size. Length of longer axis, ·655 to ·731 mm.; of shorter, ·640 to ·716 mm. Hatching took place in 5½ to 6 days, at a temperature of 51° to 62° F. In the newly hatched larva the rectum is immediately behind yolk, but not open, and not extending to the edge of the fin-fold. The eyes are slightly pigmented, and there are two small patches of pigment on the tail. The anus was not open seven days after hatching; the mouth not open at hatching. Spawning took place in May and June.

Motella argentea, Rhein.—The young in various stages were identified and described by A. AGASSIZ, July 1882, in *Young Stages,* pt. iii. (*Proc. Amer. Acad. Arts and Sci.*, vol. xvii.). In the youngest stage, 4 mm. in length, the embryonic fin-fold is continuous, notochord multicolumnar (a point not ascertainable from BROOK's figures), pelvic fins palmate and large. In oldest stage, 3·4 cm. in length, two dorsal and one anal fin all distinct; pelvic fins very long and narrow. There is some uncertainty about the identity of the specimens; they may belong not to *Motella argentea,* but to some species of Onus.

Eggs taken by the tow-net at Newport, identified as belonging to *Motella argentea*, are described by AGASSIZ and WHITMAN in *Pelagic Stages of Young Fishes.* The identification is based on the character and distribution of the pigment in the larva hatched from the eggs, and is to some extent doubtful. The average size of the eggs is ·78 mm. There is a single oil globule (in one case two, which coalesced) which is large and colourless, and measures ·15 to ·16 mm. in diameter. The figure given of the newly-hatched larva agrees closely with BROOK's figure of *Motella mustela.* The embryonic period varied with the temperature from three to six days. The eggs were taken from May to July.

Motella cimbria, Nilsson (Linn.).—The four-bearded rockling. PARNELL's example, captured in June, had the ova almost mature. Three specimens were taken by me, in the trawl, off Fast Castle Point, Haddingtonshire, March 12, 1886. In these the reproductive organs were very small. The largest specimen was ·26 m. long.

A species allied to *Motella,* probably actually a species of that genus, is figured in *Pelagic Stages,* pl. xii. The ovum is ·70 mm. in diameter, and has a single oil-globule. The newly hatched larva agrees with that of *Motella mustela,* in that the rectum terminates, apparently blindly, immediately behind the yolk, and does not extend to the edge of the ventral fin-fold. The eggs were obtained in March and April. Another species allied to *Motella* is figured in *Pelagic Stages,* plate ii. figs. 1 to 3.

The ovum of *Merlucius* is mentioned and figured by KINGSLEY and CONN,* but the size is not stated. Like that of *Motella,* it has a single large oil globule at the vitelline pole.

The eggs and larvæ of *Lota vulgaris,* the burbot, have been described by CARL J. SUNDEVALL in *Svenska Vetensk. Akad. Hand.,* 1855. The species is entirely confined to fresh water, and is thus unique among the Gadidæ, all the rest of which are marine and produce pelagic ova. The ova of *Lota* are shed separate and loose at the bottom of the water; some ova are opaque, some transparent. According to SUNDEVALL, they are small, but measurements are not given. Figures of the newly hatched larva and somewhat later stages are given; the drawings are not quite adequate, but show some essential points. In the newly hatched larva there is a single oil globule in the yolk, and therefore probably in the ovum; the anus is close behind the yolk, but not in contact with it; the larva is 3 mm. in length. This larva bears a close resemblance to that of *Motella,* as figured by BROOK. There are two differences in *Lota;* the oil globule is not so far back, and the two transverse stripes of pigment in the tail of *Motella* are wanting. The pigment in an eight days old larva of *Lota*

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* *Memoirs of Boston Society of Natural History,* vol. iii. No. 6, 1883.

formed a series of spots along the dorsal edge of the side of the body and tail. This case is interesting, as showing how little modification is necessary to adapt the ova of two allied fishes to such apparently different environments as the surface of the sea and the bottom of a river or stream. The ova of the cod sink in fresh water, but they probably would not develop in that condition. The ova of *Trigla gurnardus* sank in the water of the Scottish Marine Station, but they invariably died in that condition after some days. The conditions in which the ova undergo development are not constant in a given family, but the structure of the ovum is more so, and the structure of the larva is always characteristic of families, and even to some extent of whole orders.

Fam. 3. OPHIDIIDÆ.

The eggs of *Fierasfer* (*acus* and *dentatus*) have been described by EMERY in his Naples Station monograph on the genus. The ovum has a single large oil globule; it is small, ·8 mm. in diameter. The ova when deposited are united together in masses, each mass containing many thousand eggs in a thick gelatinous envelope. The masses are pelagic, floating at the surface of the sea. In the newly hatched larva the anus is in immediate proximity to the yolk, which still contains its oil globule situated at the anterior end. A great deal of pigment along the sides of the trunk, and a single row of chromatophores on each side at the ventral edge of the tail. A little in front of the level of the anus a median dorsal papilla interrupts the continuity of the fin-fold. This papilla grows rapidly, and ultimately forms a long filament supported on a short upright stalk. The filament bears a number of leaf-like appendages, and is called the vexillum. No stages of embryonic development are figured, and in the figures of the larva the internal structure is not shown. The structure of the notochord cannot be seen. A very lucid and complete account is given of the ovarian development of the ovum. The vitelline nucleus is described, and shown to be merely the starting point of the development of the vitelline spheres, which by their coalescence form the yolk in the mature ovum. The oil globule similarly arises from the coalescence of a number of small ones. The differences in the structure of mature ova are thus explained, and no support is given to the ideas recently advanced concerning the origin of the yolk from follicular cells, or of the latter from the germinal vesicle.

Fam. 4. MACRURIDÆ.

Development not yet studied.

ANACANTHINI PLEURONECTOIDEI.

Fam. PLEURONECTIDÆ.

The development of a great number of species belonging to this family has been studied. In the preceding section of this memoir, ova of four species of Pleuronectes are described. Mention of the study of several species has been made by M'INTOSH. In Appendix F. of the *Third Annual Report of the Scottish Fishery Board,* he states that the ova of the cod, haddock, whiting, grey gurnard, common flounder, turbot, sole, lemon dab, common dab, and long rough dab had been examined in the Marine Laboratory at St Andrews. E. E. PRINCE describes the ova of *Pleuronectes platessa, P. flesus, P. limanda,* as well as those of *Gadus æglefinus, G. morrhua, G. merlangus,* and *Trigla gurnardus,* in *Ann. and Mag. Nat. Hist.*, May 1886, but gives no figures. The young of *Pleuronectes Americanus,* Walb., are described and figured by AGASSIZ in *Young Stages,* plate ii., from a stage at which the larva is 4 mm. in length. The eggs and newly hatched larva of this species are figured in *Pelagic Stages,* plate xvi. In the larva the rectum is, as far as can be judged from AGASSIZ's figure, a little distance behind the yolk, and the notochord seems to be unicolumnar; but on neither of these points is the figure very distinct. In all the species of Pleuronectes which I have figured the rectum is in contact with the yolk, and the notochord multicolumnar.

Pseudorhombus.—The eggs and larvæ of *Pseudorhombus oblongus,* Storer, the Sienna flounder, are figured by AGASSIZ in *Young Stages,* ii. plate ix. figs. 1–3, and in *Pelagic Stages,* plates xiv., xv., figs. 1–14. There is one oil globule, which in the newly hatched larva is at the posterior end of the yolk; at the same stage the rectum is in contact with the yolk. The character of the notochord is not shown in the figures. The egg of the transparent flounder, *Pseudorhombus oblongus,* Stein, has no oil globule, and no pigment on the yolk. The figures 1–4 on plate vi. of *Young Stages,* ii., given under the name of *P. melanogaster,* Stein, really belong to *Tautoga onitis.*

Rhombus maculatus, Mitch.—Some advanced larvæ of this species are figured in *Young Stages,* ii., but the eggs and newly hatched larva are not given.

Hippoglossoides limandoides.—I have been unable to obtain eggs of this species. Many specimens were obtained in the months of May and June, which were spent; occasionally a ripe male was obtained, but never a ripe female. It probably spawns in the neighbourhood of the Firth of Forth in April. M'INTOSH states that he obtained the ova of this species before 1st June 1884, but he does not describe them. (*Second Annual Report, S. F. B.*)

Arnoglossus megastoma, Donovan.—A specimen of this species, taken 16 miles E. by N. of May Island, April 30, 1886, in the trawl, was brought to the station. The ova were quite immature. THOMPSON at Belfast, according to DAY, ascertained that it spawned in October.

Plagusia.—The young, about 1 inch long, is figured by AGASSIZ in *Young Stages,* ii., but not the eggs or larvæ.

Thus all the Anacanthini, as far as at present known, except *Lota,* have pelagic ova, and in all the rectum at the time of hatching is in contact with the yolk. In *Gadus* and *Motella* the anus is not open, and does not extend to the margin of the ventral fin-fold.

ORDER III. ACANTHOPTERYGII.

DIV. I. PERCIFORMES.

Fam. 1. PERCIDÆ.

The young of *Labrax lineatus,* Bl. and Schn., are figured by A. AGASSIZ in *Young Stages,* iii. In the youngest stage figured, 3·5 mm. in length, the yolk sac is already absorbed. The larvæ were taken at the surface of the sea with the tow-net, but the eggs were not found. The ova of *Perca fluviatilis* are adhesive, and attached to fresh-water plants. An account of them is given by SUNDEVALL. Hatching occurred fourteen days after fertilisation. The newly hatched larva was 5 mm. long; there was a single oil globule in the yolk sac, and the anus was slightly separated from the latter. Spawning took place in May.

The ova of *Serranus cabrilla* are stated by HOFFMANN to be pelagic.

Fam. 2. SQUAMIPENNES.
,, 3. MULLIDÆ.
,, 4. SPARIDÆ.
Fam. 5. CIRRHITIDÆ.
,, 6. SCORPÆNIDÆ.

This family consists exclusively of marine fishes, and all the species that have hitherto been studied from the embryological point of view have pelagic ova.

An account of the ovum of *Scorpæna* is given in HOFFMANN'S memoir, published in the *Transactions of the Amsterdam Academy,* 1881. The species observed were *S. porcus* and *S. scrofa.* The ripe ovum before fertilisation consists of a perfectly homogeneous glassy yolk surrounded by a thin envelope of protoplasm, which has a faint reddish tinge, and is as usual principally accumulated at the micropylar pole. The ovum before fertilisation has a slightly oval form, ·95 mm. by ·84 mm. in diameter. After fertilisation the perivitelline space is very small. HOFFMANN deals only with fertilisation and segmentation, and gives no figures or descriptions of embryos or larvæ. His examination of the ova of *Scorpæna* was made at Naples. The ova of Scorpæna are deposited in masses, each mass consisting of a large number of ova enveloped by a slimy substance. HOFFMANN believes that the slimy substance is not a product of the egg-membrane, but probably the peculiarly modified connective tissue of the *theca folliculi.* This conclusion seems extremely unlikely, but no investigator has yet inquired into the origin of the gelatinous envelopes which contain the ova of *Scorpæna* or of *Fierasfer.* The newly hatched larva of *Scorpæna* is 2·07 mm. in total length, and the anus is almost in contact with the yolk sac, ·07 mm., according to HOFFMANN, being the distance between the two.

Hemitripterus americanus, C. and V. (*H. acadianus,* Storer.)—The ova and larvæ of this species are described by A. AGASSIZ in *Pelagic Stages.* The ovum is pelagic, and possesses a single oil globule, which in the early stages of development is at the pole of the yolk opposite the centre of the blastoderm. Diameter of ovum, 1·02 to 1·10 mm. The developing embryo is distinguished by the large number of brownish-yellow chromatophores which, interspersed with a few black ones, are present on the sides of the body of the embryo, and over the whole surface of the yolk. In the newly hatched larva the rectum is separated by a very slight interval from the yolk. The fin-fold is very wide, and in it are three pigment patches—two dorsal and one ventral. The anus is apparently not perforated. The structure of the notochord is not shown. The identification of these ova and embryos seems to be based on the characters of the older stages of the larvæ.

Fam. 7. NANDIDÆ.
„ 8. POLYCENTRIDÆ.
„ 9. TEUTHIDIDÆ.

Div. II. ACANTH. BERYCIFORMES.

Fam. 1. BERYCIDÆ.

Div. III. ACANTH. KURTIFORMES.

Fam. 1. KURTIDÆ.

Div. IV. ACANTH. POLYNEMIFORMES.

Fam. POLYNEMIDÆ.

Div. V. ACANTH. SCIÆNIFORMES.

Fam. 1. SCIÆNIDÆ.

Div. VI. ACANTH. XIPHIIFORMES.

Fam. 1. XIPHIIDÆ.

Div. VII. ACANTH. TRICHIURIFORMES.

Fam. 1. TRICHIURIDÆ.

Div. VIII. ACANTH. COTTO-SCOMBRIFORMES.

Fam. 1. ACRONURIDÆ.
„ 2. CARANGIDÆ.

Capros, according to DAY, was observed to shed pelagic ova by Mr DUNN at Megavissey, July 20, 1882.

Temnodon saltator, Linn. (*Pomatomus saltatrix,* Gill), is called the Blue-fish on the Atlantic coast of the United States. Pelagic ova, believed to belong to this species, are described by AGASSIZ in *Pelagic Stages,* and a long larva, 9 mm. in length, identified as Temnodon, is figured in *Young Stages,* pt. iii. pl. ii. The ova to which I have referred are, in one respect, unique among all the kinds of pelagic ova hitherto described. In AGASSIZ's own words, the egg exhibits a partial segmentation of the yolk—that is to say, at the stage when the embryonic ring has just been formed, there is a ring of definitely limited large cells round the edge of the blastoderm. After the blastoderm has enclosed the yolk, the large cells seem to form a complete envelope round the yolk beneath the blastoderm. To judge from the figures given by AGASSIZ and WHITMAN, I should have concluded that in this species of ovum the periblast, instead of being a syncytium, was divided into cells, and should have been ready to agree with the view expressed by those authors in their "Preliminary Notice" (*Proc. Amer. Acad. Arts and Sci.,* vol. xx.), namely, that the actual cleavage of the yolk in this instance was positive proof that the nucleated periblast in all cases, and the yolk, are "integrant portions of the ovum." But in *Pelagic Stages* it is stated that closer examination has shown that the large cells are situated beneath the periblast, and belong to the yolk; that they are not protoplasmic elements, but vitelline, although they have an epibolic growth, and extend round the unsegmented yolk as this becomes enclosed by the blastoderm and periblast. It is pointed out that the change in relations of these superficial yolk segments shows that a transposition occurs in the Teleostean ovum among the yolk elements closely analogous to the invaginatory movement of the yolk in holoblastic ova. The diameter of the ovum is ·70 to ·75 mm. The ova occur at Newport from the middle of June to middle of August. At the yolk pole there is a single large oil globule. The newly hatched larva is 2·15 mm. in diameter; the rectum is separated by a distance of ·275 mm. from the yolk sac; pigment is scanty; a series of black chromatophores along the dorsal edge of the tail, and a few brownish-yellow ones along the body and rectum. The structure of the notochord is not shown. The development of the young fish was traced till a stage at which it measured 9 mm. in length.

The presence of an oil globule, the externally segmented yolk, and the slight separation of the rectum from the yolk sac, are the diagnostic features in *Temnodon,* but how far these are characteristic of the family is not known.

Fam. 3. Cyttidæ.
,, 4. Stromateidæ.

This is a small family of marine fishes, containing only two genera. Figures of *Stromateus triacanthus,* Peck, from a length of 7 mm. upwards, are given by Agassiz in *Young Stages,* pt. iii. The notochord is apparently multicolumnar, but no other embryonic or larval characters are to be discovered from the figures. The species is called Butter-fish in America, and the young at the length of 10–20 mm. are in the habit of sheltering themselves beneath the umbrella of *Dactylometra,* one of the Scyphomedusæ.

Fam. 5. Coryphænidæ.
,, 6. Nomeidæ.
,, 7. Scombridæ.

According to Day, the eggs of *Scomber scomber*, the common mackerel, are shed in May and June, and in the Brighton Aquarium have been observed to be of the pelagic kind. The development of *Cybium maculatum,* the Spanish mackerel, has been described by John A. Ryder (*Bull. U.S. Fish Commission,* vol. i., 1881). The investigation was carried out in July 1880 at Mobjack Bay, Virginia, and in 1881 at Cherrystone Harbour, Va. The eggs hatched twenty-four hours after fertilisation, but the temperature to which they were exposed is not stated. Evidence was obtained that spawning naturally takes place at night. The ovum measures $\frac{1}{25}$ to $\frac{1}{20}$ inch in diameter, or, as measured from the figures given, ·856 to 1·06 mm. It is pelagic; there is a single large oil globule, otherwise the yolk is homogeneous; the perivitelline space is small. The newly hatched larva is 2·52 mm. long as measured from the figure; the notochord is multicolumnar; the anus immediately behind the yolk, and open; pigments spots are present on the body and round the oil globule, and also form one conspicuous transverse stripe in the middle of the tail. The oil globule in the hatched larva is situated on the ventral side of the yolk, a little posteriorly. The mouth opens twenty-one hours after hatching.

Fam. 8. TRACHINIDÆ.

The development of *Trachinus vipera* has been described by GEO. BROOK (*Lin. Soc. Jour.*, vol. xviii., 1884). The eggs were shed in that author's aquarium. Spawning takes place at night, and is continued through the months of May, June, and July. The ovum is pelagic, 1·32 mm. in diameter, and contains from 20 to 30 small oil globules. The oil globules are external to the vitellus, and contained in depressions of its surface. It is probable that this is often the case; it certainly is in *Trigla gurnardus*, but whether the oil globules are always external is doubtful. The perivitelline space is small. Hatching took place on ninth, tenth, and eleventh days, at a temperature of 54° to 60° Fahr. In the newly hatched larva the rectum is immediately behind the yolk sac, the notochord multicolumnar. The eyes are pigmented; black pigment cells are scattered over the body and the surface of the yolk sac, and aggregated in a transverse stripe at the middle of the tail. The ventral fins are well developed at the time of hatching. The length of the newly hatched larva is 3·5 mm. The yolk sac is absorbed, and the mouth well developed twenty-four hours after hatching.

Fam. 9. BATRACHIDÆ.

The young *Batrachus tau*, Lin., 2 mm. in length, has been figured by STORER (*Mem. Amer. Acad.*, v. pl. xix.). AGASSIZ figures a specimen 6 mm. in length in *Young Stages*, pt. iii., but this shows only traces of the larval characters. The anal is still continuous with the caudal fin, and the "ganoid" lobe of the tail is well marked.

Fam. 10. PEDICULATI.

The eggs of *Lophius piscatorius*, Lin., are described in *Young Stages*, pl. iii. The eggs are held together by gelatinous mucus in a single flat layer which floats horizontally in the sea, forming a large sheet 3 feet broad and 25 to 30 feet long. The spawn is shed on the American coast from June to

August. It has also been observed on the British coast, but I have not myself met with it. A more complete description, with better figures, is given by Agassiz and Whitman in *Pelagic Stages.* The egg is large, 1·75 mm. in diameter, and has a single immense oil globule ·4 mm. in diameter, of a transparent copper colour. Black chromatophores are developed very early, and are aggregated chiefly about the ventral side of the embryo, present in less abundance on the surface of the yolk sac, round the oil globule, and on the tail. In the newly hatched larva the yolk sac is globular, and very large in comparison with the body; the oil globule is ventral and posterior; the rectum is immediately behind the yolk; the eyes are deeply pigmented; the notochord multicolumnar; the pelvic fins not developed. The successive forms of the larva, which is up to a late stage pelagic, are described and figured in *Young Stages,* iii. Agassiz points out the resemblance, both in the character of the spawn and the structure and development of the larva, between *Lophius* and *Fierasfer,* comparing the long anterior dorsal spine in the former, which is a permanent organ, but develops at a very early stage, to the vexillum of *Fierasfer,* which is a temporary appendage disappearing completely in the adult. It seems probable that in the *Fierasfer* larva the vexillum is morphologically derived from a fin ray, as are the appendages in *Lophius.*

The male of *Antennarius,* another species of this family, a pelagic fish, makes, according to Günther, a nest, and guards the eggs deposited in it. We have thus in this family a series of steps in the transition from ordinary littoral adhesive ova to typical pelagic ova. The ova of *Antennarius* are probably adhesive, and are deposited in a pelagic nest. The ova of *Lophius* are also adhesive, but float as a detached mass unprotected by an apparatus formed from pelagic algæ. If the ova of *Lophius* were separate, instead of adhering together in a mass, they would be typical pelagic ova.

Fam. 11. COTTIDÆ.

The question of the ova of Cottus has been discussed in a previous section. The pelagic ova of *Trigla gurnardus* have been described.* In this family we have a greater difference between the ova of closely allied genera than in the preceding, for the eggs of *Cottus* are typical examples of littoral adhesive ova, while those of *Trigla* are typically pelagic. Sundevall (*loc. cit.*) gives an account of the development of *Cottus gobio* and *Cottus quadricornis.* The eggs of the former species are deposited in May. The larva twenty-four hours after hatching was 8 mm. long; there was a single oil globule in the yolk, and the rectum was in contact with the yolk sac. The larva of *Cottus quadricornis* is

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* "Yolk and Gastrula," J. T. Cunningham, *Quart. Jour. Micr. Sci.*, 1885.

very similar; its length three days after hatching was 11·5 mm. The eggs of both species are adhesive, and form masses sticking to objects on the shore.

Fam. 12. CATAPHRACTI.

The fertilised ova of *Agonus cataphractus* have never been described; but Prof. M'INTOSH, in *3rd Ann. Rep. S. F. B.*, says he found nearly mature ova in a specimen trawled near St Andrews on March 12. The ova had a pale salmon colour, were 1·3 mm. in diameter, and probably adhesive.

Fam. 13. PEGASIDÆ.

Div. IX. ACANTH. GOBIIFORMES.

Fam. 1. DISCOLALI.

The ova of *Cyclopterus lumpus* have been mentioned in the previous section. *Liparis* is the only other genus, and what is known of the spawn of *Liparis Montagui* has also been stated.

Fam. 2. GOBIIDÆ.

Gobius Ruthensparri is stated by DAY to have been bred in confinement by Mr ROBERTS of the Scarborough Museum. The ova were adhesive, and were deposited within the shell of a barnacle. The male watched over the mass of eggs, and fanned them with his fins.

HOFFMANN (*loc. cit.*, p. 19) gives a description and figure of the ovum of *Gobius minutus*. The ovum has a peculiar elongated pyriform shape, with a very large perivitelline space, and at the narrow end are a number of filaments, in the centre of which is the micropyle. The eggs are attached by the filaments.

The ova of *Callionymus lyra* are pelagic, and have been described by M'INTOSH, in *Ann. and Mag. Nat. Hist.*, Dec. 1885. On the 8th August a female specimen was obtained at St Andrews, from which ripe ova could be pressed out. The ova are pelagic, transparent and buoyant, small in size, being of about the same diameter as the ova of *Pleuronectes flesus*. The exterior surface of the vitelline membrane or zona radiata exhibits a reticulum of slightly elevated ridges, the meshes of the reticulum being hexagonal; from this characteristic the ova can be easily identified. At Millport, in June of the present year, I obtained a pelagic ovum from the tow-net which agreed exactly with Prof. M'INTOSH's description. M'INTOSH adds, that *Trophon*, hermit crabs, and bivalve mollusca were found in the stomach of *Callionymus*. He gives no figures or description of any embryonic or larval stages of the species.

There is here another example of a family in which some genera produce adhesive, others pelagic, ova.

Div. X. ACANTH. BLENNIIFORMES.

Fam. 1. CEPOLIDÆ.
„ 2. HETEROLEPIDOTIDÆ.
„ 3. BLENNIIDÆ.

The ova of *Anarrhichas lupus* have been discovered by Prof. M'INTOSH to be deposited in February; they are large, heavy, and non-adhesive, and the larvæ, when hatched, are well advanced in development (see *Nature*, June 17, 1886).

The ova of *Blennius galerita*, according to DAY, are adhesive, and attached to the under surface of stones.

Blennius pholis also deposits adhesive ova, which are attached to small caverns in the rocks of the sea-shore.

The ova of *Blennius* are stated by HOFFMANN to possess processes extending from the zona radiata.

The ova of *Centronotus gunnellus*, according to W. ANDERSON SMITH, are deposited from February to April. The ova are adhesive, and form a spherical mass about the size of a walnut; this ball is quite free, and both parents lie coiled round it.

In *Zoarces viviparus* the ova are retained during development within the cavity formed by the coalesced ovaries. Breeding takes place in the winter months, chiefly in December, January, and February. Specimens in which the young were ready to be born were obtained on the shore at Granton in February and March. The young at parturition are about 1½ inches long, and in all respects, except size, similar to the parents. I met with several specimens in which the young in the ovary had been killed by some cause or another, and when the cavity was cut into, their bodies were discovered in a shrunken state, but not decomposed.

Fam. 4. MASTACEMBELIDÆ.

Div. XI. ACANTHOPTERYGII MUGILIFORMES.

Fam. 1. SPHYRÆNIDÆ.
„ 2. ATHERINIDÆ.

Several stages of *Atherinichthys notata*, Günther (*Chirostoma notata*, Gill), are figured by AGASSIZ in *Young Stages*, pt. iii. In the youngest the embryonic fin-fold is still unaltered, but the yolk sac is absorbed, and the mouth open.

The ovum of this species is stated by RYDER to possess four filamentous processes connected with the vitelline membrane ("Development of *Belone longirostris*," *Bull. U.S. Fish Commission*, vol. i.). The threads or filaments are more completely described by RYDER in vol. ii. of the same bulletin, the fish being there called *Menidia*, which is a synonym. The threads are in length about eight times the diameter of the ovum, and when the latter is first emitted the threads lie coiled spirally round it. There can be little doubt that the four threads are merely the outer layer of the zona radiata in a specialised form, and are homologous with the suspensory membrane in *Osmerus*.

Fam. 3. MUGILIDÆ.

Div. XII. ACANTH. GASTROSTEIFORMES.

Fam. 1. GASTROSTEIDÆ.

The ova of *Gastrosteus* are adhesive, and deposited in nests made with water plants, and guarded by the male. *Spinachia vulgaris* makes nests of seaweeds, Fucus, &c., on the sea-shore; its ova are similar to those of *Gastrosteus*, but larger. It has been shown by Prof. KARL MÖBIUS of Kiel, that the white filaments, by which the nest of Spinachia is held together, are spun by the male fish, and that they are formed from a substance resembling mucin which is produced in the kidneys (see *Schr. Naturwiss. Vereins für Schleswig Holstein*, Bd. vi., 1885; translated in *Ann. and Mag. Nat. Hist.*, Aug. 1885: also E. E. PRINCE, *Ann. and Mag.*, Dec. 1885). The ova of Spinachia, according to PRINCE, are ·085 inch in diameter. A large mass of pale yellow oil globules are aggregated at the yolk pole. At temperature 41° to 51° Fahr., in June the ova hatched in twenty-five to forty days. No figures of the development are given by PRINCE.

Fam. 2. FISTULARIIDÆ.

Div. XIII. ACANTH. CENTRISCIFORMES.

Fam. 1. CENTRISCIDÆ.

Div. XIV. ACANTH. GOBIESOCIFORMES.

Fam. 1. GOBIESOCIDÆ.

Lepadogaster Decandolii.—Some observations on the development of this species are described by W. ANDERSON SMITH in *Proc. Roy. Phy. Soc. Edin.*, 1886. The ova are adhesive, attached to stones or shells, and watched over by both parents. Spawning takes place in June and July on the west coast of Scotland. The ovum has a single oil globule, and is hatched twenty-eight days after fertilisation. The ovum of *L. bimaculatus* are always found adhering to the inner surface of shells of *Pecten operculatus;* they are deposited likewise in June and July, and are guarded by at least one of the parents.

Div. XV. Acanth. Channiformes.

Fam. 1. Ophiocephalidæ.

These are fresh-water fishes of the Indian region. The male *Ophiocephalus* is stated by Günther to make a nest and guard the presumably adhesive ova.

Div. XVI. Acanth. Labyrinthibranchii.

Polyacanthus viridiauratus, Günther, the *Macropus viridi-auratus* of Lacépède, commonly called the Paradise-fish, is a native of the East Indian Archipelago, but is commonly kept in aquaria in Europe, and breeds freely in confinement. Some account of the ova is given by Dr Miecz. von Kowalewski in *Zeit. f. wiss. Zool.,* Bd. xliii. The perivitelline space is small; the yolk apparently broken up into small masses, and large oil globules are present; but the appearance of the living ovum is not described.

Order II. ACANTH. PHARYNGOGNATHI.

Fam. 1. Pomacentridæ.

The ovum of *Heliasis chromis* is described by Hoffmann (*loc. cit.,* p. 19). The name he uses seems to be slightly erroneous. It is the *Heliastes chromis* of Günther's British Museum catalogue. The ovum forms a somewhat long ellipsoid with blunt ends, and at one of the poles is a group of eight or nine long straight filaments attached at their basis to the vitelline membrane. The micropyle is situated in the centre of the group of filaments. Hoffmann remarks that the filaments in this ovum, and in *Belone, Blennius, Gobius,* represent the external zona in adhesive ova such as *Leuciscus* and *Perca,* in which the zona radiata is differentiated into two layers. On the other hand, in pelagic ova, such as those of *Scorpæna,* or in heavy non-adhesive ova, such as those of *Salmo,* no division into two layers can be discovered in the zona radiata. We may add to this comparison of Hoffmann's that it is probable from what Ryder observes concerning the development of the filaments in *Belone,* that these processes are actually formed by a splitting up and unequal development of the external zona; and thus there is no fundamental difference between the origin of the suspensory membrane in *Osmerus eperlanus* and the filamentous processes in *Belone, Atherinichthys, Heliastes,* &c. The yolk of Heliastes resembles in structure that of the herring, being composed of a number of ellipsoidal vitelline discs; but there is also present a large oil globule at the vitelline pole. The protoplasm in the mature unfertilised ovum forms as usual an envelope round the vitellus which is thickest beneath the micropyle, and thins away all round that point. The blastodisc and blastoderm during simple segmentation is large in proportion to the yolk. The perivitelline space is considerable. Hoffmann gives no figures of the embryonic or larval stages.

Fam. 2. Labridæ.

The development of a large number of the wrasses has been studied.

Tautoga onitis, Linn.—The pelagic ova of this species are described and figured in *Pelagic Stages*. The diameter of the ovum measures ·90 to ·95 mm. The yolk is homogeneous, and there is no oil globule; the perivitelline space is of moderate dimensions. The newly hatched larva is 3·05 mm. in length; the rectum is not in contact with the yolk sac, but at a distance of ·55 mm. from it (not nearly so far back as in *Clupea*). The anus is open, the notochord multicolumnar; the eye is scarcely pigmented, but there are small compact pigment spots along the dorsal region of the sides of both body and tail; the pectorals are scarcely developed, the ventrals not at all. The eggs of *Tautoga* were artificially fertilised, so that the identity of the ova and newly hatched larvæ is certain. But the authors point out that the ova of *Ctenolabrus*, *Ps. melanogaster*, and *Tautoga* are so similar, both in structure and size, that it is scarcely possible to distinguish them with certainty in the produce of the tow-net. The authors state that figs. 1, 2, 3, and probably fig. 4 (in my opinion fig. 4 also, certainly) in plate vi. of *Young Stages*, part ii., belong to *Tautoga*, and not to *Pseudorhombus melanogaster*. Thus the position of the rectum with respect to the yolk sac in the newly hatched larvæ is shown to be a constant family character; and there is no exception to the statement that in Pleuronectidæ the rectum at that stage is in contact with the yolk sac.

Ctenolabrus adspersus, Walb. (*C. cœruleus*, Storer).—The ova of this species have been described by Agassiz and Whitman in *Pelagic Stages*, and by Kingsley and Conn. The ovum is ·85 to ·92 mm. in diameter. Before fertilisation the peripheral layer of protoplasm is densely filled with refractive granules, which render the ovum opaque; but after fertilisation the granules disappear, and the egg becomes perfectly transparent. In the newly-hatched larva the rectum is separated by ·25 mm. from the yolk sac, the total length of the larva being 2·30 mm. There are black dendritic chromatophores along the sides of the body and tail. The time required for hatching varies from two to six days. The eggs are shed at Newport in the months of May and June.

The ova of *Julis vulgaris* are described by Hoffmann (*loc. cit.*, p. 43). The diameter of the ovum measures ·75 mm. The yolk is homogeneous, but contains an oil globule ·15 mm. in diameter. The ova are suspended separately in the water not united in masses. The dimensions of the newly-hatched larva are 1·77 mm. in total length, ·15 mm. from the yolk sac to the anus.

The ova of *Crenilabras,* of which genus HOFFMANN examined four species, are not pelagic, but adhesive. The zona radiata shows the division into two layers, which occurs in most adhesive ova. The diameter of the ovum is ·7 to ·75 mm. The yolk is not homogeneous, but contains a number of vitelline globules; there seem to be no oil globules. The newly-hatched larva is 3·6 mm. long, and the anus is ·6 mm. from the yolk sac.

Thus we see that considerable variations occur in the family of Labridæ in the character of the ova. Most of the genera produce pelagic ova, but the ova of *Crenilabrus* are adhesive. As in the Gadidæ, there is either a single oil globule in the yolk or none at all. Two characters seem constant throughout the family—(1) that the notochord is multicolumnar, (2) that the anus is at some little distance from the yolk sac, though not nearly so far back as on the Physostomi. The separation of rectum and yolk sac occurs also in the Carangidæ (*Temnodon*), and among the Physoclisti seems to be confined to these two families.

Fam. 3. EMBIOTOCIDÆ.

Fishes of the North Pacific, most abundant on the American coast. All viviparous.

Fam. 4. CHROMIDES.

ORDER V. LOPHOBRANCHII.

Fam. 1. SOLENOSTOMIDÆ.

According to GÜNTHER, the female bears the eggs attached to filaments developed on the ventral fins, the inner edges of which are united to the skin of the body.

Fam. 2. SYNGNATHIDÆ.

In *Siphonostoma typhle,* which is common on the British coasts, the ova are carried till the time of hatching by the male in a pouch formed by longitudinal folds of the skin behind the anus.

In *Syngnathus* there is a similar pouch in the male. According to RYDER, yolk contains numerous oil globules.

In *Nerophis* the ova are attached to the abdomen of the male by a viscid secretion in front of the anus. *N. lumbriciformis* and *N. aquoreus* are not uncommon on the east coast of Scotland, but I have not had an opportunity of examining the ova of either.

Some account of the development of *Hippocampus* is given by JOHN A. RYDER in *Bull. U.S. Fish. Commission,* Bd. 1. In the embryonic *Hippocampus* the fin-fold is wanting, in *Syngnathus* it is but slightly developed.

ORDER VI. PLECTOGNATHI.

Fam. 1. SCLERODERMI.
„ 2. GYMNODONTES.

The development of these has not been studied.

The Maturation and Fertilisation of the Teleostean Ovum.

In considering the subject of the phenomena which take place in the ripe Teleostean ovum immediately after its separation from the parent, two questions chiefly excited my curiosity, neither of which have I yet solved to my complete satisfaction. These questions refer to the account of the phenomena which has been given by Professor C. K. HOFFMANN.* The first is, Is there any foundation for HOFFMANN'S statement that the first segmentation spindle is directed radially, and divides into a superficial nucleus which belongs to the archiblast, and a deeper one which belongs to the periblast? The second is, Can we trace in the fish ovum the transformations of the nucleus which accompany the expulsion of the polar bodies, and compare these transformations with those which E. VAN BENEDEN † has described in *Ascaris megalocephala*.

The subject of the last question will be considered first. The investigation of the matter is one of considerable difficulty. It is necessary, in the first place, to have a plentiful supply of healthy living specimens of some species with pelagic ova; and in the second place, to have at command the most approved appliances and reagents for their microscopic examination. The first opportunity I had of making the attempt was in May of the present year, when I had a number of ripe *Pleuronectes limanda* alive in the aquarium of the Station.

In the ripe ovum of *P. limanda,* immediately on its escape from the ovary, the zona radiata is in immediate contact with the ovum. The condition of the ovum is shown in Plate III. fig. 1. There is no doubt that the eggs of all the species of *Pleuronectes* and *Gadus* are closely similar except in size, and RYDER is in error when he indicates a perivitelline space in his figure of the ripe newly shed ovum of the cod. There is a layer of protoplasm round the ovum in the neighbourhood of the micropyle, which thins out at the pole opposite the micropyle. In the living egg, within half an hour after it is shed, whether milt be added to the water in which it is contained or not, the expulsion of a transparent spherical polar body through the micropyle can be readily observed. Its appearance is shown in Plate II. fig. 10, taken from an unfertilised ovum, and Plate III. fig. 4, three hours after fertilisation. At this latter stage the perivitelline space has begun to appear; it develops first in a ring round the micropyle as a centre. The protoplasm, immediately after the ovum is shed, begins to collect at the micropylar pole of the ovum, and this process begins

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* See *Verhandelingen der konink. Akad. der Wetenschappen,* Th. xxi., Amsterdam, 1881.

† *La Maturation, Fécondation, etc., et la Division Cellulaire,* Paris et Gand, 1883.

the rhythm of segmentation. At the stage shown in Plate III. fig. 3, one and a half hours after fertilisation, the central part of the protaplasmic disc is much the thickest, forming a somewhat conical protuberance downwards into the yolk. The protuberance afterwards disappears, and at the end of three hours the blastodisc has the shape shown in Plate III. fig. 4, the lower surface being uniform. Then a second aggregation of the protoplasm begins, but this time towards two points, as shown in Plate III. fig. 5, producing the first division of the blastodisc. Thus the aggregation of the protoplasm towards the micropylar pole may be regarded as a contraction towards one central point, and the first division as due to a contraction towards each of two separate points.

The expulsion of a polar body through the micropyle I observed repeatedly in the ova of *Pleuronectes cynoglossus* studied at Millport last June. It took place in both fertilised and unfertilised ova. But I was unable to discover either in the living eggs, or in the fresh eggs treated with reagents on the slide, any nuclear spindle either before or during the expulsion of the polar body. In one or two instances I noticed a minute pyriform projection of protoplasm on the surface of the blastodisc after the latter had withdrawn itself from the vitelline membrane (Plate III. fig. 7) This might be either a second polar body, or simply the proximal part of the first drawn away with the receding blastodisc, from the inner end of the micropyle. AGASSIZ and WHITMAN, in *Pelagic Stages*, p. 19, mention the formation of two polar bodies in Teleostean ova, and undertake to describe them in a subsequent memoir.

My results, as far as they go, concerning the unfertilised ova, are in agreement with those of HOFFMANN. In the unfertilised ovum, as a rule, the expulsion of the polar globule through the micropyle and the concentration of the protoplasm take place just as in the fertilised ovum, with the exception that the latter process goes on much more slowly in the unfertilised ovum. I have never seen any traces of segmentation in the unfertilised ovum. The small protoplasmic body on the blastodisc seen two hours after fertilisation, and shown in Pl. III. fig. 7, was seen also at the same stage in the unfertilised ovum. The aggregation of the protoplasm in the fertilised ovum is finished about three hours after fertilisation, and the stage shown in Pl. III. fig. 4, is reached. At this stage the unfertilised ovum stops unchanged, being perfectly incapable of segmenting. Pl. III. fig. 9 shows an unfertilised ovum of *Pl. cynoglossus* six hours after shedding, at which time the fertilised ova were in the eight-cell stage, and the cells again dividing to form the sixteen-cell stage. The unfertilised ova were in the condition shown in Pl. III. fig. 9, twenty-four hours after being shed, and remained unchanged till they died. I was not able to determine with absolute certainty at what stage the spermatozoon entered the fertilised ovum. This occurs, as is evident from Pl. IV. fig. 2, during the first half hour, and I am inclined to believe that it takes place immediately the ripe ovum is exposed to the milt, so that the spermatozoon remains within the protoplasm while the polar globule or globules are being expelled.

To take up now the first of my two questions. The changes which occur in the blastodisc after the ovum has been exposed to the influence of milt are shown in the figures of the ova of *Pl. limanda* and *Pl. cynoglossus.* The separation of the vitelline membrane from the blastodisc occurs almost immediately after the ovum has been placed in sea water containing milt. The protoplasm aggregates at the micropylar pole, and half an hour after fertilisation it projects at the pole considerably into the yolk (Pl. III. figs. 3 and 8). At this stage, in ova treated with acetic acid and methyl green, I was able with a high power to see distinctly the male and female pronuclei in close proximity to one another (Pl. IV. fig. 2). I was not able to discover the spindle produced from the union of these two bodies. The first segmentation of the blastodisc takes place gradually by the aggregation of the protoplasm round two centres, as in Pl. III. fig. 5, and Pl. IV. fig. 1. The protoplasm towards each side of the blastodisc projects downwards into the yolk, so that there are now two of these projections instead of one, with a deep broad furrow in the under surface of the blastodisc between them. No furrow on the upper surface of the blastodisc is at first visible. After treatment of the ovum with acetic acid and methyl green at this stage, a nucleus can be made out very distinctly in the two halves of the blastodisc, and these are the only two nuclei in the ovum. The line joining the nuclei is a chord of the sphere of the ovum, and not a radius, as stated by HOFFMANN. The nuclei are best seen when the ovum is placed on the slide with the blastodisc downwards, so that the blastoderm is seen through the transparent yolk, the stained ova being mounted in glycerine. It was from an ovum in these conditions that Pl. IV. fig. 3, was taken, the ovum having been killed with acetic acid half an hour after fertilisation. The outline of the blastodisc on the surface of the ovum is not circular, but elliptical, and the plane of division passes through the short axis of the ellipse. This plane of division contains the principal axis of the ovum, by which I mean the axis passing through the centre of the blastodisc and the centre of the ovum. HOFFMANN states that the plane of the first division is perpendicular to the principal axis of the ovum (*loc. cit.*, p. 105). It seems to me possible that HOFFMANN may have been led into this error by the relative positions in which the two nuclei are seen when the ovum is in a certain position with respect to the axis of the microscope. To explain this I must refer to the diagrams shown in figs. 1 and 2. Fig. 1 represents a section of the ovum passing through the principal axis and perpendicular to the plane of the first division of the blastodisc. Now, if the axis of the microscope occupies, with respect to the ovum, the position *x y*, and the plane which is in focus, perpendicular of course to that axis, occupies the position shown by the line *a b*, then the appearance of the section of the ovum seen in this plane will be that represented in fig. 2. The two nuclei will be seen projected on to the focus-plane as at n^1n^2, fig. 2; while the under surface of that part of the blastodisc, which is nearer to the observer, will be projected on to the focus-plane as a curved line, apparently dividing the blastodisc into two portions, one internal and one external, each containing one of the nuclei. In Pl. III. fig. 8, the blastodisc is seen thus apparently divided before the first division has taken place, but the dividing line is nothing but the under surface of the near part of the blastoderm projected on to the focus-plane. A comparison between the diagram in fig. 2 and figs. 4 and 12, pl. iii. of HOFFMANN'S memoir, will show how completely his views are explained by my supposition. At the same time all his figures cannot be so explained. Figs. 2 and 4 of his pl. v. are in direct opposition to my results.

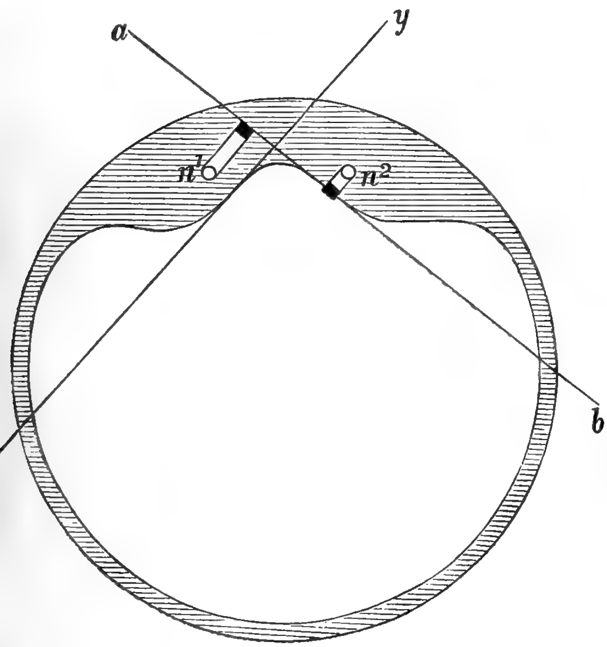

FIG. 1.

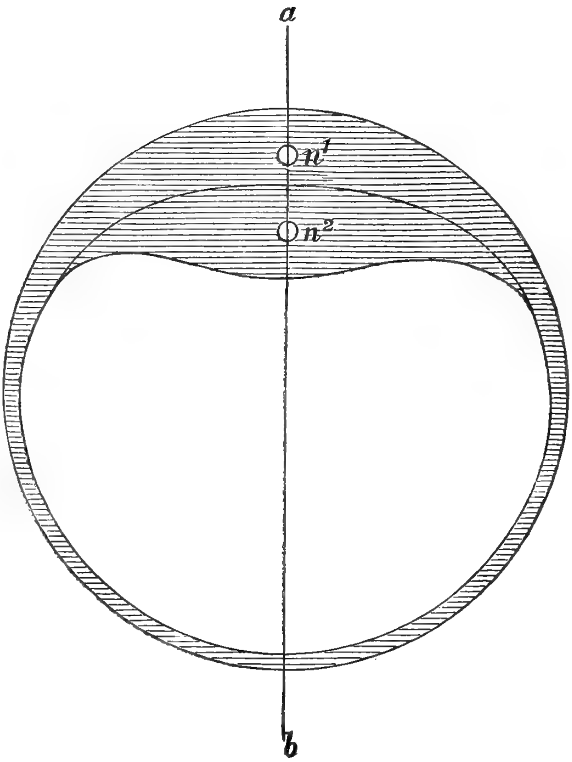

FIG. 2.

It is a remarkable fact that, as will be evident from a reference to Pl. IV. fig. 3, the nuclei of the two-cell stage are not at first in the thickest part of their respective cells. The centre of aggregation of the protoplasm lies nearer the edge of the blastoderm than the nucleus, and it would seem as if the protoplasm were active in the division and the nucleus passive, a hypothesis quite contrary to current conceptions. I have been unable to find any evidence of the existence of periblast up to the eight-cell stage. Pl. IV. fig. 5, shows an optical section of the four-cell stage, in which it is evident there is no separate subblastodermic layer.

Spawning Periods of some of the Fishes of the Firth of Forth.

	Jan.	Feb.	Mar.	April.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Clupea harengus, *L.*,	?	× ×	× ×					× ×	× ×	?	?	?
Osmerus eperlanus, *Lac.*,				× ×								
Gadus morrhua, *L.*,			× ×	× ×								
G. æglefinus, *L.*,			× ×	× ×								
G. merlangus, *L.*,			× ×	× ×								
Pleuronectes platessa, *L.*,		× ×										
Pl. flesus, *L.*,			× ×									
Pl. limanda, *L.*,					× ×							
Pl. cynoglossus, *L.*,						× ×						
Pl. microcephalus, *L.*,					× ×							
Trigla gurnardus, *L.*,				× ×	× ×	× ×	× ×					
Zoarces viviparus, *L.*,	× ×	× ×										
Spinachia vulgaris,						× ×						
Callionymus lyra,								× ×				

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DESCRIPTION OF PLATES.

Plate I.

Fig. 1. Egg of herring towards the close of period of simple segmentation, 11 hours after fertilisation. Aug. 26, 1884. Zeiss A, Oc 3.
Fig. 2. Anterior end of herring embryo, 6 days after fertilisation, 1 day before hatching. Sept. 1, 1884. Zeiss A, Oc 3.
Fig. 3. Herring larva nearly 24 hours after hatching. Aug. 22, 1884. Zeiss A, Oc 2.
Fig. 4. Outline of alevin of *Salmo levenensis*, 3 days after hatching. Magnified about 4 times.
Fig. 5. Ovum of *Osmerus eperlanus*, Lacép, 25½ hours after fertilisation. May 7, 1886. Mag. 33 times.
Fig. 6. Ovum of *Os. eperlanus*: optical section through suspensory membrane, internal zona, and micropyle. Zeiss CC, Oc 2.

Plate II.

Fig. 1. Embryo of *Pleuronectes platessa*, Linn., in ovo, 23 days 5 hours after fertilisation. Mag. 33 times. Feb. 26, 1886.
Fig. 2. Sculpturing of surface of vitelline membrane of ovum of *Pl. platessa*, L. Mag. 50 times.
Fig. 3. Larva of *Pl. platessa*, taken artificially from ovum on point of hatching. 27 days after fertilisation. Mag. 33 times.
Fig. 4. Ovum of *Pl. flesus*, 2 days 2½ hours after fertilisation. Mag. 33 times. The development of this egg was retarded: end of the period of simple segmentation.
Fig. 5. *Pl. flesus*, 22 hours after fertilisation, stage just before appearance of segmentation cavity. Mag. 33 times. March 31, 1886.
Fig. 6. *Pl. flesus*, 2 days 2½ hours. First appearance of segmentation cavity. Mag. 33 times.
Fig. 7. *Pl. flesus*, 2 days 22 hours. Mag. 33 times.
Fig. 8. *Pl. flesus*, newly hatched. 7 days. April 6, 1886. Pigment black, anus open. Mag. 33 times.
Fig. 9. Ovum of *Pleuronectes limanda*, L., 20¼ hours after fertilisation. Stage preceding formation of segmentation cavity. May 22, 1886. Mag. 33 times.
Fig. 10. Blastoderm of unfertilised ovum of *Pl. limanda*, showing expulsion of polar globule.
Fig. 11. Spermatozoon of *Pl. limanda*. Zeiss DD, Oc 4.

Plate III.

Fig. 1. Newly shed unimpregnated ovum of *Pl. limanda*, optical section showing micropyle and relations of protoplasmic layer. Zeiss A, Oc 3, Abbé's camera. Mag. 70 times.
Fig. 2. *Pl. limanda*, ½ hour after fertilisation. Zeiss A, Oc 3, camera. Mag. 70 times.
Fig. 3. *Pl. limanda*, 1½ hours after fertilisation. Mag. 70 times.
Fig. 4. *Pl. limanda*, 3 hours after fertilisation. Mag. 70 times.
Fig. 5. *Pl. limanda*, little more than 3 hours: process of first division. Mag. 70 times.
Fig. 6. *Pl. limanda*, larva newly hatched. May 28, 1886. Mag. 33 times.
Fig. 6*a*. Notochord of same. Zeiss CC, Oc 3.
Fig. 7. Blastodisc of *Pl. cynoglossus*, unfertilised 2 hours after shedding, shows what may be the second polar body. Mag. 70 times.

Fig. 8. Ovum of *Pl. cynoglossus*, 2 hours after fertilisation. Mag. 33 times.

Fig. 9. *Pl. cynoglossus*, unfertilised, 6 hours after shedding. Mag. 33 times.

PLATE IV.

Fig. 1. Blastoderm of *Pl. cynoglossus* in process of first division, 3½ hours after fertilisation. Zeiss CC, Oc 3.

Fig. 2. *Pl. cynoglossus*, ½ hour after fertilisation, ovum treated with acetic acid and methyl green, and examined entire in glycerine; optical section of blastodisc; shows male and female pronuclei. Zeiss CC, Oc 3, camera.

Fig. 3. Blastoderm of *Pl. cynoglossus*, immediately after first division: shows nucleus in each of the two cells. Acetic acid and methyl green. Zeiss Cc, Cc 2, without camera.

Fig. 4. *Pl. cynoglossus*, 4-cell stage, 4 hours. Mag. 33 times.

Fig. 5. Optical section of 4-cell blastoderm, acetic acid and methyl green: shows absence of periblast beneath blastoderm. Zeiss CC, Oc 2.

Fig. 6. *Pl. cynoglossus* 8-cell stage dividing; acetic acid only without glycerine, 6 hours after fertilisation. Mag. 33 times.

Fig. 7. *Pl. cynoglossus*, blastoderm commencing to spread, 24 hours. Mag. 33 times.

Fig. 8. *Pl. cynoglossus*, segmentation cavity, 1 day 5 hours. Mag. 33 times.

Fig. 9. *Pl. cynoglossus*, 1 day 8 hours. Mag. 33 times.

Fig. 10. *Pl. cynoglossus*, 1 day 23½ hours. Mag. 33 times.

Fig. 11. *Pl. cynoglossus*, 2 days 4 hours. Mag. 33 times.

Fig. 12. *Pl. cynoglossus*, 2 days 19 hours. Mag. 33 times.

PLATE V.

Fig. 1. *Pl. cynoglossus*, 2 days 19 hours. Mag. 33 times.

Fig. 2. *Pl. cynoglossus*, 3 days 1½ hour. Mag. 33 times.

Fig. 3. *Pl. cynoglossus*, 4 days. Mag. 33 times.

Fig. 4. Newly hatched larva of *Pl. cynoglossus*, 5 days. Mag. 33 times.

Fig. 4*a*. Notochord of *Pl. cynoglossus.*

Fig. 5. Condition of rectum, *r*, and coalesced ends of segmental ducts, *s.d.*, in newly-hatched larva. Zeiss CC, Oc 2.

Fig. 6. Condition of same parts, 30 hours after hatching.

Fig. 7. Larva of *Pl. cynoglossus*, 2 days after hatching.

PLATE VI.

Fig. 1. *Gadus æglefinus*, L., newly hatched. Mag. 33 times.

Fig. 2. Adhesive ovum from shore near station, probably *Cottus scorpini.* Mag. 33 times.

Fig. 3. Adhesive ovum attached to *Hydrallmannia falcata*, perhaps *Liparis Montagui.* Mag. 33 times.

Fig. 4. Young fish, 2 days after hatching. Hatched from adhesive ova taken in trawl April 29, 1884. *Liparis Montagui.*

Fig. 5. Ovum of *Cyclopterus lumpus.* Mag. 33 times.

PLATE VII.

Fig. 1. Newly hatched larva of *Cyclopterus lumpus*, ventral surface, from specimen preserved in spirit. Mag. 33 times.

Fig. 2. Pelagic ovum taken 10 miles E.S.E. of May Island, March 23, 1886. Mag. 33 times.

Fig. 3. Pelagic ovum taken in Firth of Forth, off Gullane Ness, May 27, 1886. Mag. 33 times.

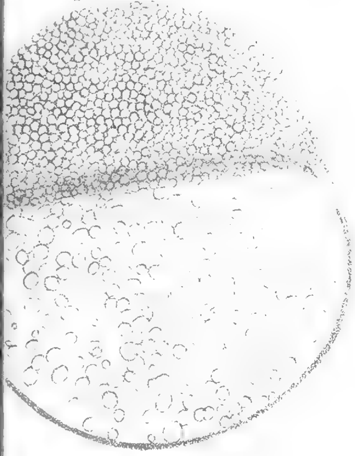
Fig. 4. Larva newly hatched from ovum shown in previous figure. Mag. 33 times.

Fig. 4*a*. Notochord of same. Zeiss CC, Oc 3.

Fig. 5. Pelagic ovum taken off Gullane Ness, May 27, 1886. Mag. 33 times.

Fig. 6. Larva newly hatched from same. Mag. 33 times.

Fig. 7. Teleostean ova from Gulf of Guinea. Mag. 18 times. Species unknown.



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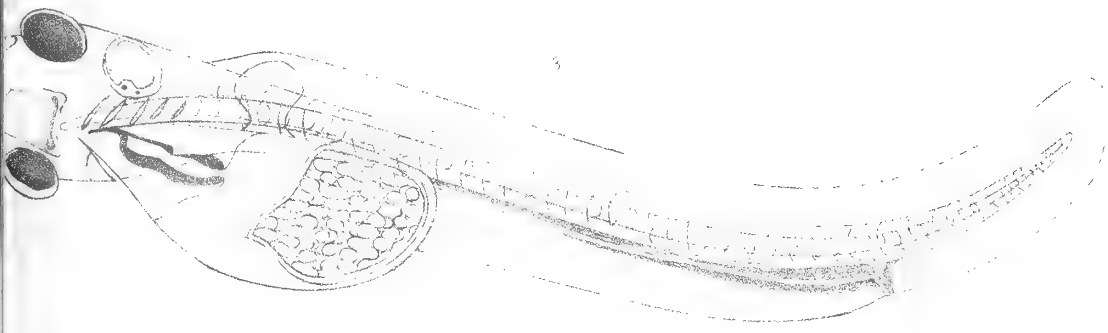

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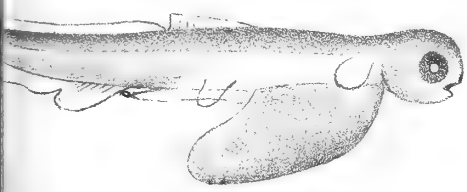
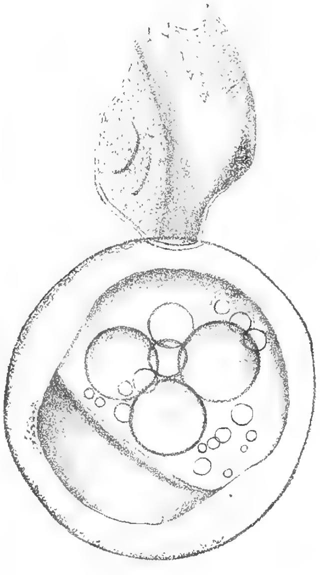
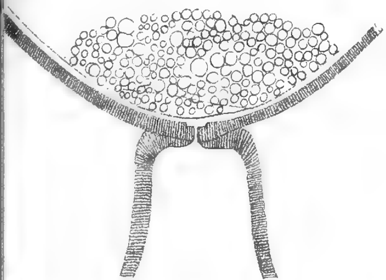
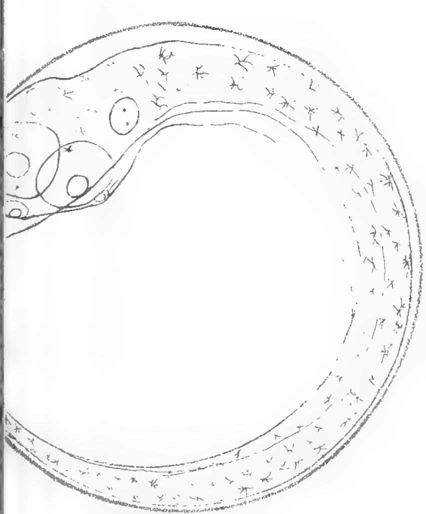






FIGS. 1-3, CLUPEA HARENGUS, L. FIG. 4, SALMO LEVENENSIS.
FIGS. 5, 6, OSMERUS EPERLANUS, LACÉP.

McLagan & Cumming imp.



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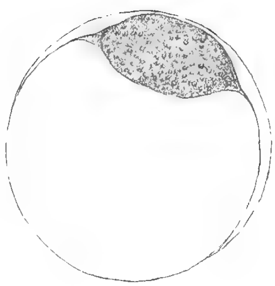

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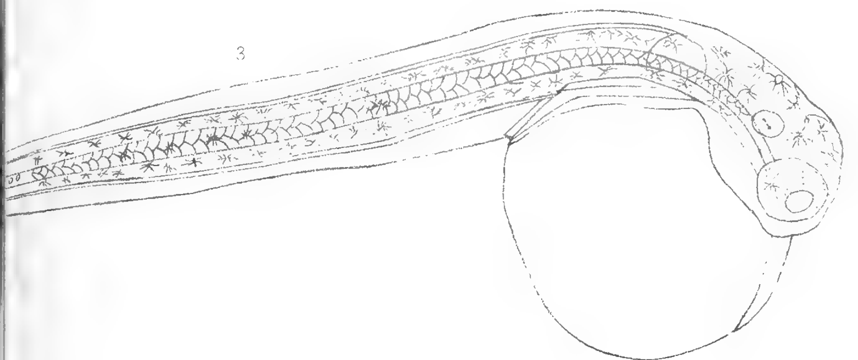

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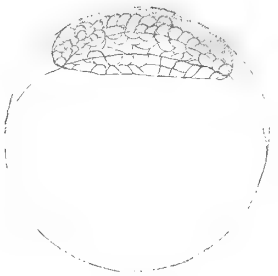

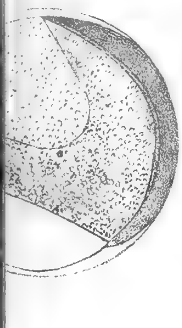

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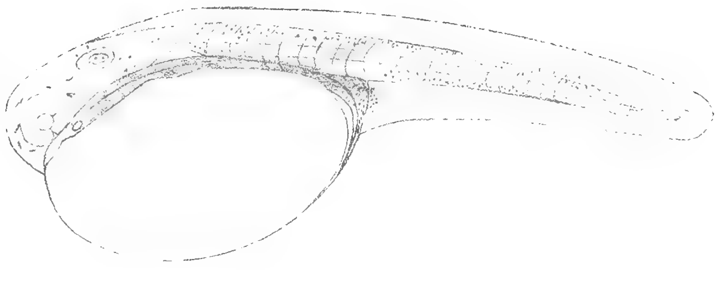

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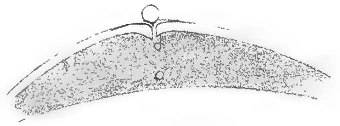

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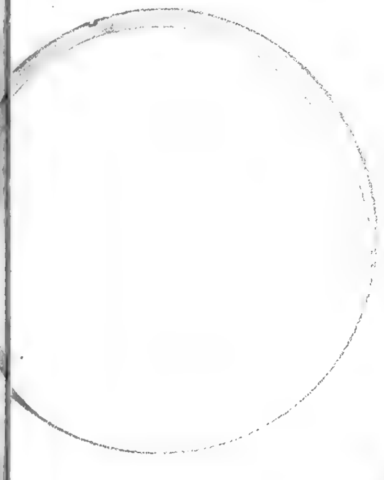
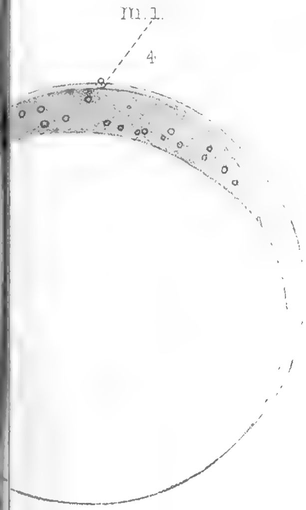
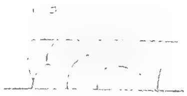
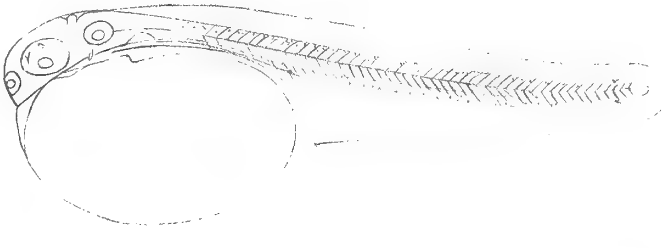
FIGS. 1-3, PLEURONECTES PLATESSA, L.   FIGS. 4-8, PL. FLESUS, L.
FIGS. 9-11, PL. LIMANDA, L.

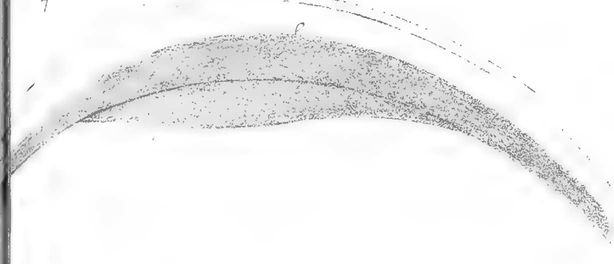
McLagan & Cumming imp.











FIGS. 1-6a, PLEURONECTES LIMANDA, L.   FIGS. 7-9, PL. CYNOGLOSSUS, L.

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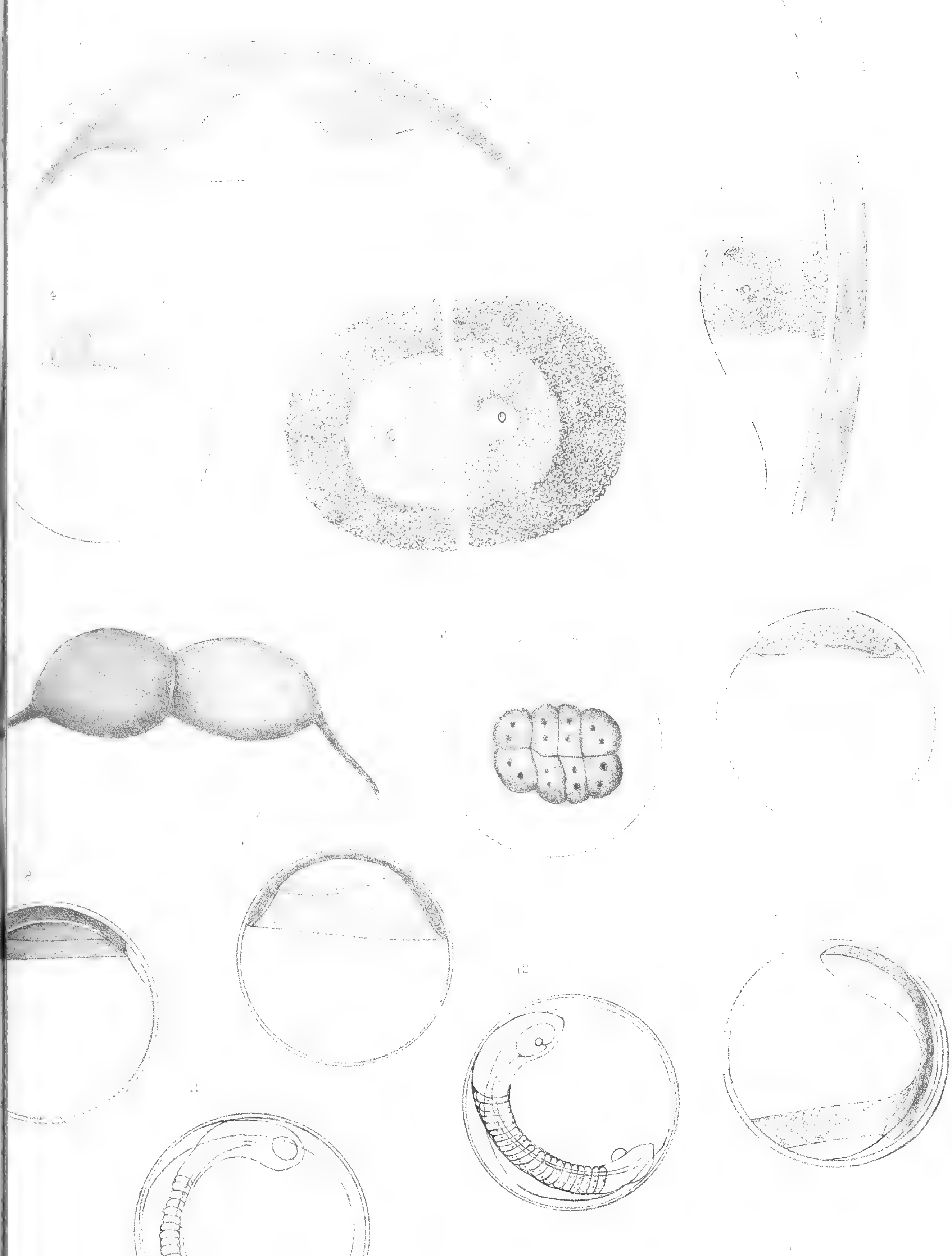
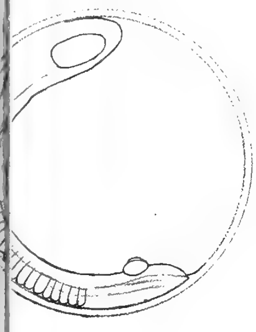


PLEURONECTES CYNOGLOSSUS, L.

M^{c}Lagan & Cumming



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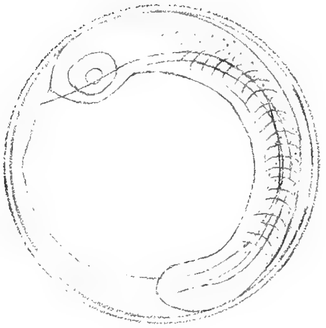

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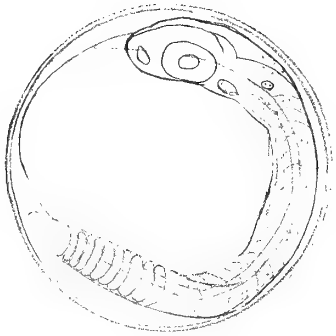

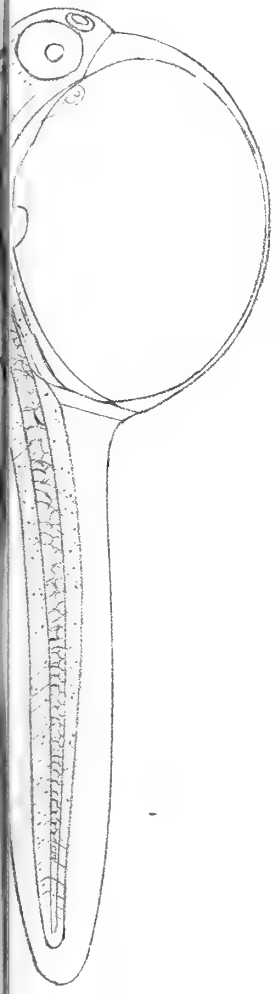

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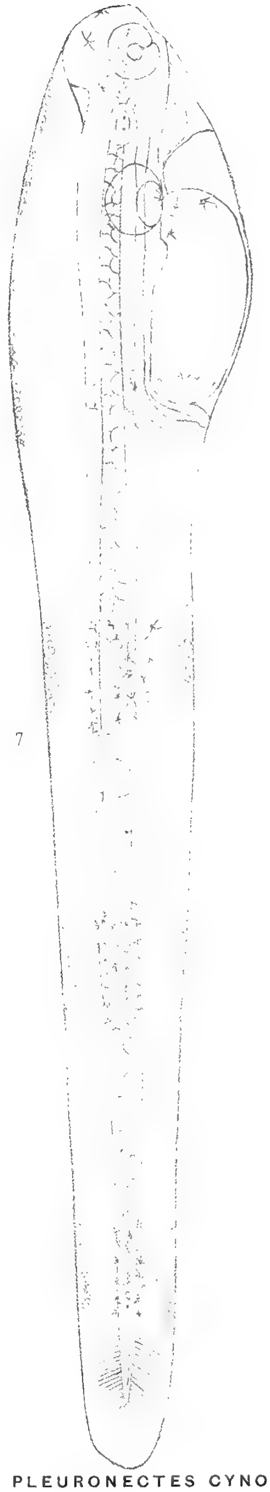

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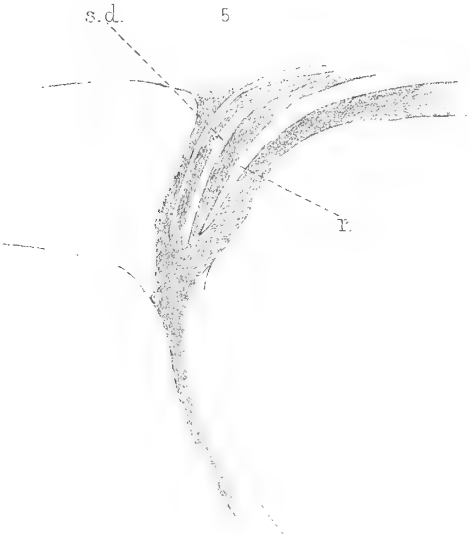

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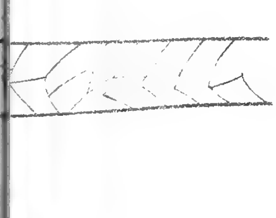

PLEURONECTES CYNOGLOSSUS, L.

M^{c}Lagan & Cumming imp

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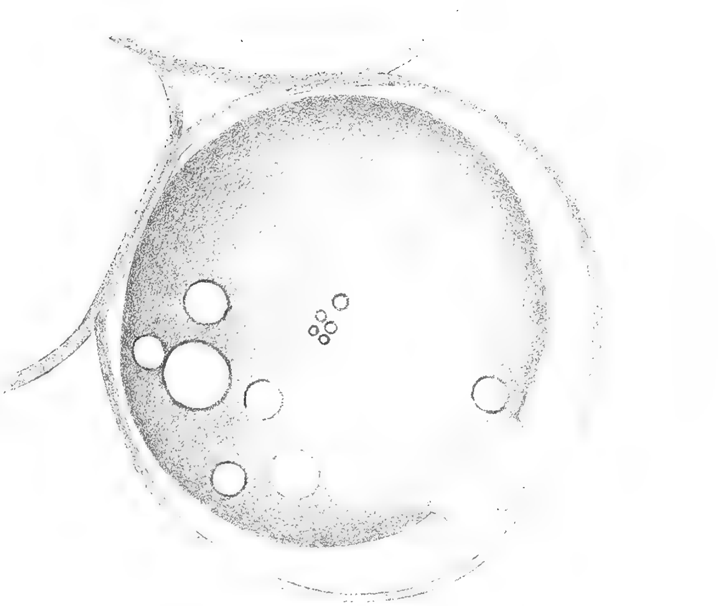

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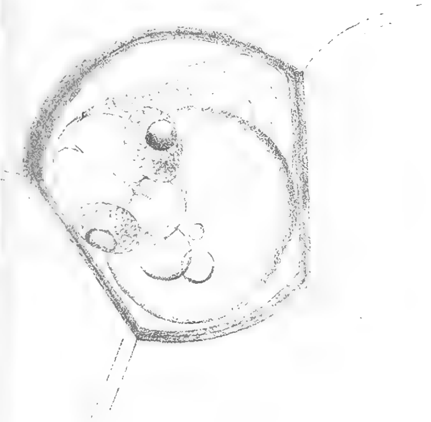

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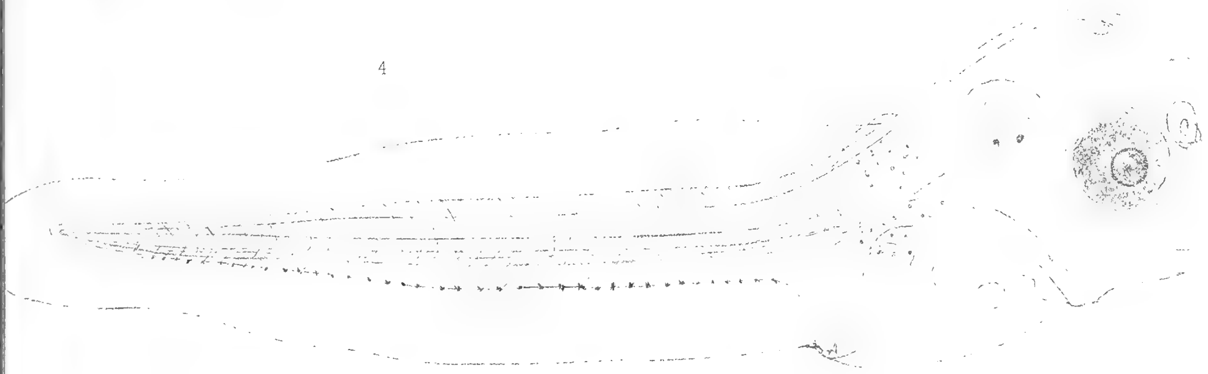
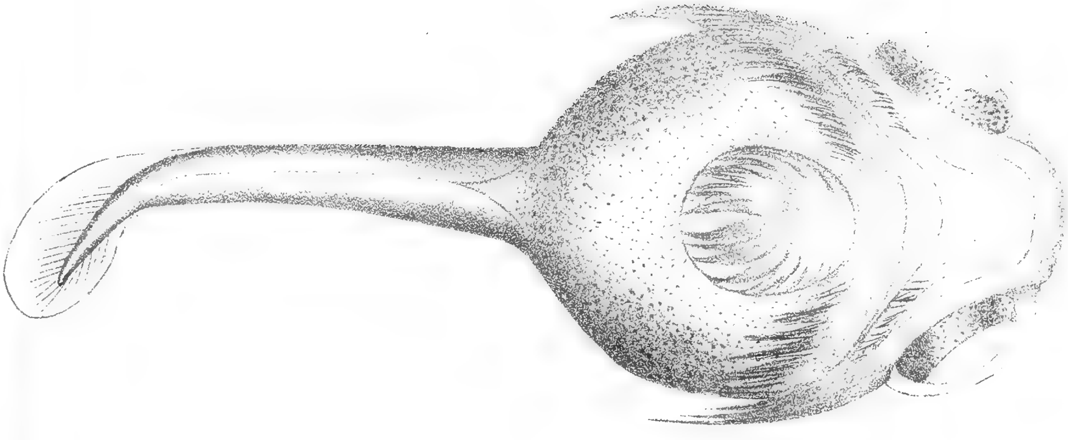


FIG. 1, GADUS AEGLEFINUS, L. FIG. 2, COTTUS SCORPIUS, L.?
FIGS. 3, 4, LIPARIS MONTAGUI, CUV. FIG. 5, CYCLOPTERUS LUMPUS, L.

M^{c}Lagan & Cumming imp.



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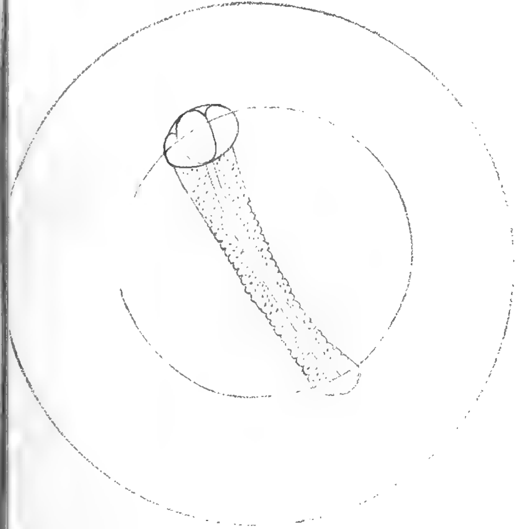

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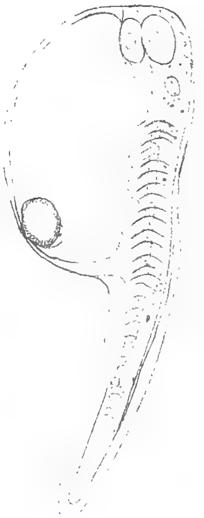

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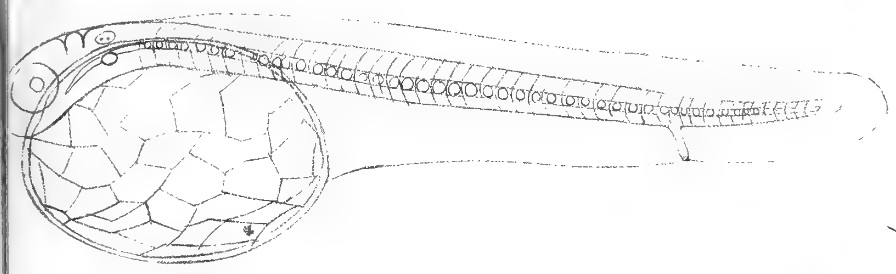

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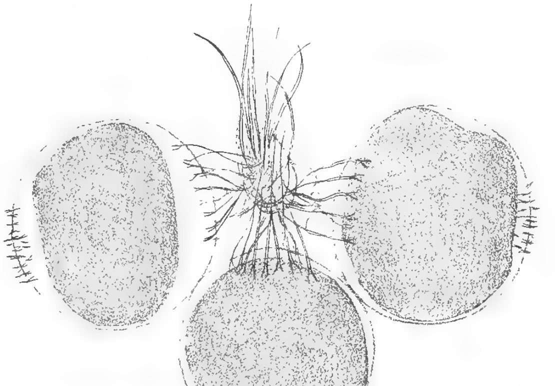

FIG. 1, CYCLOPTERUS LUMPUS, L. FIGS. 2-7, UNIDENTIFIED SPECIES.

Mᶜ Lagan & Cumming imp

V.—*On the Fructification of some Ferns from the Carboniferous Formation.* By Robert Kidston, F.R.S.E., F.G.S. (Plates VIII., IX.)

CONTENTS.

Calymmatotheca, Stur, emend.

Kidston, *Quart. Jour. Geol. Soc.*, vol. xl. p. 590.

In a Review * of Dr Stur's *Zur Morphologie und Systematik der Culm- und Carbonfarne*,† I pointed out that he appears to have included two types of fern fructifications in his genus *Calymmatotheca.*

The first type includes those forms originally placed in *Calymmatotheca* ‡ and consists of a number of exannulate sporangia arranged around a common point of attachment; the second type, which was first described in *Zur. Morph., u. Syst. d. Culm- u. Carbonfarne*,§ is there represented by *C. Avoldensis* and *C. Frenzli.* The fruit of these two species is apparently surrounded by an *involucre* or *indusium*, and it is only with this part of the fructification that we are at present acquainted. I also further indicated that in *Calymmatotheca*, as I proposed to restrict the genus, the fruiting portions of the frond are entirely deprived of foliage pinnules, whereas in the other type (*C. Avoldensis* and *C. Frenzli*) only a very slight modification takes place in the fertile portion of the fronds—the sporangia being borne on the ordinary foliage pinnules.

In a subsequent paper,‖ when describing the fruit of *Sphenopteris delicatula*, Sternb., I more fully explained the structure of such frutifications as those occurring in *C. Avoldensis* and *C. Frenzli*, and for these three species proposed the new genus *Zeilleria.*

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* *Geol. Mag.*, Dec. III. vol. i. p. 328, July 1884.

† *Sitzb. der k. Akad. der Wissensch.*, Band lxxxviii. 1 Abth., 1883.

‡ "Culm-Flora," Heft ii. p. 255, *Abhandl. d. k. k. geol. Reichsanst.*, vol. viii. Of the four figures of Calymmatothecous fruits given here by Stur, *C. Schimperi*, *C. minor*, *C. Haueri*, and *C. Stangeri*, my interpretation of the structure of these fruits is chiefly founded on *C. Stangeri*, as this seems the most perfectly preserved.

§ p. 799.

‖ *Quart. Jour. Geol. Soc.*, vol. xl. p. 590.

On the specimens of *Z. delicatula*, Sternb., sp., figured in the *Quart. Jour. Geol. Soc.*, vol. xl. pl. xxv., the development of the fruit can be traced. It at first consists of a globular indusium, which at maturity splits into four valves for the dissemination of the spores.

On the other hand, what I regard as the true interpretation of the fruit of *Calymmatotheca* is that which was first propounded by RENAULT * and more fully explained and illustrated by ZEILLER,† viz., that in *Calymmatotheca* the fruit consists of a number of *exannulate sporangia* arranged around a common point of attachment.

In the case of *C.* (*Sorocladus*) *asteroides*, Lesqx.,‡ there can be no doubt that the component parts of this star-like fructification are sporangia, and not thong-like segments of a split involucre.

In his last work, *Die Carbon-Flora der Schatzlarer Schichten*,§ Dr STUR freely criticises my remarks on his genus *Calymmatotheca* as employed by him in his *Zur Morph. u. Syst. d. Culm- u. Carbonfarne,* and still adheres to his original opinion that the portion of the fruit of *Calymmatotheca* with which we are acquainted, is the thong-like remains of a split indusium. He also mentions, in regard to his *C. Stangeri*, that he has observed in a few cases at the base of the beaker-like indusium, small convex elevations.

Notwithstanding this, I still think that Dr STUR is mistaken in his interpretation of the fruit of *Calymmatotheca*, and that in *C. Stangeri* the fruit consists of a number of sporangia arranged around a common axis, as in *C.* (*Sorocladus*) *asteroides*, Lesqx., *C.* (*Sphenopteris*) *bifida*, L. & H., and *C. affinis*, L. & H., sp., to be presently described. The small elevations at the base of the inner cavity of the indusium (?) of *C. Stangeri*, to which Dr STUR again refers in his *Carbon-Flora*,‖ have, I am afraid, no organic connection with the fruit, and are perhaps due to mineralisation or to the adhesion of some extraneous matter. I make this suggestion from an examination of the fruit of *C.* (*Sphenopteris*) *bifida*, L. & H., and *C.* (*Sphenopteris*) *affinis*, L. & H., which are similar in all external respects to STUR'S *Calymmatotheca;* and as in these cases the fruit is certainly not composed of thong-like segments of a split indusium, but of true exannulate sporangia, I am induced to believe that Dr STUR, through imperfect preservation of his specimens, is mistaken in their interpretation.

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* *Cours d. Botan. Foss.*, Troisième Année, p. 198, 1883.

† *Ann. d. Scienc. Nat.*, 6e sér. Bot., tome xvi. p. 182, pl. ix. figs. 10, 11.

‡ Lesquereux, *Rept. Geol. Survey of Illin.*, vol. iv. p. 406, pl. xiv. figs. 6, 7; *Coal Flora of Pennsyl.*, p. 328, pl. xlviii. fig. 9; see also Zeiller, *Ann. d. Scienc. Nat.*, *loc. cit.*, p. 182, pl. ix. figs. 10, 11.

§ *Abhandl. d. k. k. geol. Reichsanst.*, Band xi. Abth. 1, p. 239, Wien, 1885. It is to be regretted that Dr STUR here accuses M. ZEILLER of writing anonymously my Review of his *Carbon-Flora*, contributed to the *Geol. Mag.*, which communication M. ZEILLER had neither seen nor was aware of, till after its publication—especially as the paper more fully explaining my views on this subject was published in the *Quart. Jour. Geol. Soc.*, Aug. 1, 1884.

‖ *Loc. cit.*, p. 238.

In regard to *C. Haueri*, Stur,[*] the type has apparently been so imperfectly preserved that little of its structure can be discerned—the specimen is represented by little more than a mere carbonaceous stain on the matrix. It, however, shows the peculiar character of the sporangia (?) (segments of the indusium according to Dr STUR) being united in pairs by their basal portions, but whether the thong-like bodies are segments of a split indusium or sporangia cannot be satisfactorily settled from an examination of his figure. The union of the thong-like bodies in pairs is taken as an objection by Dr STUR to the sporangial explanation of these fruits, but as the union of sporangia is of frequent occurrence in the *Marattiaceæ,* their apparent partial union in *C. Haueri* does not militate against the view that in *Calymmatotheca* we are dealing with marattiaceous sporangia arranged in groups.

As to the affinities of these ferns, I have nothing to add to that mentioned in my former paper in the *Quart. Jour. Geol. Soc.*, where I said, "*Calymmatotheca* as here restricted (and as restricted in the present communication) is probably related to the *Marattiaceæ*, whereas *Zeilleria* appears to have affinities with the *Hymenophyllaceæ.*"[†] This is very different to that which Dr STUR gives as my views on the affinity of *Calymmatotheca,* where he states, "Dass *Calymmotheca* eine *Hymenophyllacee,* wie der ungenannte Autor meint, nicht sein könne, geht klar aus dem Fehlen des verlangerten oder fadenförmigen Receptaculums am Grunde der Kapsel hervor."[‡]

It was only the members of the genus *Zeilleria* that I thought might have *hymenophyllaceous* affinities, but as none of the specimens of this genus, which have come under my notice, have afforded any glimpse of the arrangement of the spores within the *indusium* (whether they were attached to a column or not), we can only throw out a suggestion as to the affinities of *Zeilleria,* a suggestion which must be corroborated or refuted as subsequent investigations decide. *Calymmatotheca,* however, as I have proposed to restrict it, possesses apparently an undoubted *marattiaceous* form of fruit.

The specimens to which reference was made when first treating of Dr STUR's genus *Calymmatotheca* were those on which the genus *Zeilleria* were founded,[§] hence Dr STUR is mistaken in assuming that my views were established on a *Hawlea.*

A further distinction that was pointed out between *Calymmatotheca* and *Zeilleria* is found in the fruiting portions of the fronds of *Calymmatotheca* being reduced to masses of fruit, unassociated with any ordinary foliage pinnules, whereas in *Zeilleria* the fruiting portion of the frond varies little from the ordinary barren condition, the fertile being mixed with the ordinary barren

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[*] *Culm-Flora,* Heft i. pl. i. fig. 2.

[†] *Quart. Jour. Geol. Soc.*, vol. xl. p. 591.

[‡] *Carbon-Flora,* p. 241.

[§] *Quart. Jour. Geol. Soc.*, vol. xl. p. 590.

pinnules. This condition is said by Dr STUR to occur in his *C. Schatzlariensis*,* but the figures of this species given on his pl. xxxviii. are very imperfect, and little can be learnt of the fruit from them; and from the meagre evidence afforded by the woodcut on p. 238, one would not like to say definitely whether this fern should be referred to *Calymmatotheca* or *Zeilleria*.†

Calymmatotheca bifida, L. & H., sp.

Pl. VIII. figs. 1, 2, 3, 4, 5, 6*a*; Pl. IX. figs. 16, 17.

Calymmatotheca bifida, Kidston, *Quart. Jour. Geol. Soc.*, vol. xl. p. 591 (foot-note).
Sphenopteris bifida, Lindley and Hutton, *Fossil Flora*, vol. i. pl. liii.
Sphenopteris bifida, Hibbert, *Trans. Roy. Soc. Edin.*, vol. xiii. p. 177, pl. vi. figs. 1, 2.
Sphenopteris bifida, Kidston, *Trans. Roy. Soc. Edin.*, vol. xxx. p. 537.
Sphenopteris bifida, Miller, *Testimony of the Rocks*, Edin., 1857, p. 466, fig. 129.
Trichomanites bifidus, Göppert, *Syst. fil. foss.*, p. 264, pl. xv. fig. 11.
Todea Lipoldi, Stur, *Culm Flora*, Heft. i. p. 71, pl. xi. fig. 8; Heft. ii. p. 291.
Todea Lipoldi, Schimper in Zittel, *Handbuch der Paleontologie*, Band ii. Heft. i. p. 107, fig. 75.
Sphenopteris frigida, Heer, *Foss. Flora Spitzbergens*,‡ p. 6, pl. i. figs. 1, 3–6.
(?) *Sphenopteris geniculata*, Heer, *Foss. Flora Spitzbergens*, p. 7, pl. i. figs. 8 and 10 (? 7 and 9).
Sphenopteris rutæfolia, Schmalhausen (not Gutbier), *Mém. de la Acad. Impér. d. Sciences de St Pétersbourg*, vii^e sér. vol. xxxi. No. 13, p. 4, pl. i. figs. 1–4 (? fig. 5), 1883.
Staphylopteris Peachii, Kidston (not Balfour), *Trans. Roy. Soc. Edin.*, vol. xxx. p. 539, pl. xxxi. fig. 6.
Sphenopteris (*Diplothmema*) *tracyana*, Lesquereux, *Coal Flora of Pennsyl.*, vol. iii. p. 766, pl. ci. fig. 2, 1884.

Description.—Frond divided into two symmetrical lanceolate portions by a dichotomy of the main axis. Pinnæ sub-opposite or alternate, linear; pinnules sub-opposite, or alternate and divided into 3–8 simple or bifid, narrow, linear, single-nerved segments. Fruiting pinnæ deprived of foliage pinnules, and borne on the main rachis in the neighbourhood of the bifurcation. Fruit consisting of about 16–20 linear sporangia arranged in a circle around a common axis, and situated at the extremities of the bifurcations of the fruiting pinnæ. Sporangia free in their upper portion, but united below.

Remarks.—The type specimen of this species, figured by LINDLEY and HUTTON, gives a very unsatisfactory idea of the true form of this fern, their example having evidently suffered so much from maceration before fossilisation took place, that the delicate limb of the pinnule has entirely decayed, the veins only remaining.

A much more characteristic figure than that given by the authors of the

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* *Die Carbon-Flora*, p. 265, pl. xxxviii. figs. 1, 2.

† It is unfortunate that many of the figures on the plates of Dr STUR's *Carbon-Flora* are so indistinct that it is quite impossible to discuss minute details of structure from them.

‡ *Kongl. Svenska Vetenskaps-Akademiens Handlingar*, Band xiv. No. 5, Stockholm, 1876.

Fossil Flora, is the small woodcut given by the late HUGH MILLER in the *Testimony of the Rocks.* This example is refigured on Pl. IX. fig. 16. Another figure, showing well the form of the pinnæ and pinnules of *C. bifida,* is given by STUR in his *Culm Flora,* under the name of *Todea Lipoldi.* The pinnules are divided into 3–7 very narrow linear segments, in each of which is a simple central vein. The limb of the pinnule is very narrow, forming only a slight border to the nerve.

On Pl. VIII. fig. 1, the specimen has so suffered from decay, that nothing of the pinnules now remains but the veins, the specimen being in fact reduced to the same state of imperfection as that of LINDLEY and HUTTON'S original type of the species. This example, however, is specially interesting, as it shows the position of the fruit on the frond, which is here seen to be situated in the neighbourhood of the bifurcation of the main axis. From the specimen drawn at fig. 2, it is shown that in the fruiting pinnæ the *synangia* are borne at the extremities of the little branches resulting from a third or fourth series of dichotomies.

On the surface of the *sporangia* are generally seen, in well-preserved examples, a few longitudinal fine ridges. At figs. 6*a* and 3*a*, two of the *synangia* are exhibited in profile. At fig. 4, and also in figs. 2 and 3, are represented flattened out groups of sporangia.

The sporangia individually are slightly fusiform, and united in their basal portion.*

The affinities of this fern seem to be undoubtedly with the *Marattiaceæ,* and in the genus *Kaulfussia* the *sporangia* are united to each other round a common point, an arrangement with which the fruit of *C. bifida* closely corresponds.

In *C. bifida* the *synangia* differ from those of *Kaulfussia* in the *sporangia* being free for a considerable portion of their length and in the position of the *synangia* on the fern. In *Kaulfussia* the *synangia* are scattered on the back of the frond. In the more essential structural characters of the fruit, however, the close analogy between the fruit of *C. bifida* and *Kaulfussia* is very striking.

Specimens showing the bifurcation of the rachis are not uncommon, and on the main axis below the bifurcation there are a few barren pinnæ, which are usually much smaller and less divided than those above the bifurcation. The pinnæ within the fork, formed by the dichotomy of the axis, are at the base of the two arms much shorter than those on the outer side of the fork, but they gradually increase in size as the arms of the fork separate from each other. The fronds never seem to have attained to large dimensions.

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* In one or two *sporangia* I think there can be detected a small elongated pore a little below the apex, but defer positively affirming its presence till I have more frequently seen its occurrence, and so convince myself that this appearance is not accidental.

The *sporangia* of *C. bifida* are more numerous, narrower, and slightly larger than those of *C. affinis*, L. & H., sp., to which I in error referred the first specimens of the fruit of *C. bifida.*[*]

Sph. frigida, Heer, and *Sph. geniculata,* Heer, seem to have been founded on imperfectly preserved fragments of *C. bifida,* L. & H., sp.

To *C. bifida* must also I think be referred *S. rutæfolia,* Schmalhausen (not Gutbeir) and *S. tracyana,* Lesquereux. Any one having only the original figure of LINDLEY and HUTTON to guide them in their identification of this species, may be very well excused for regarding *Todea Lipoldi, Sph. rutæfolia,* Schmalhausen, and *S. tracyana,* as specifically distinct from *C. bifida;* but from the examination of numerous specimens, many of which came from the original locality, I have no doubt that all the names included in the list of synonymy here given under *C. bifida* refer to this one species.

Description of Specimens.—Fig. 1. This specimen was collected by Mr JOHN JACKSON on the River Irthing, within a mile above Lampert, and communicated to me by Mr HUGH MILLER, F.G.S. The specimen is badly preserved, and reduced very much to the same condition as that of the original type figured by LINDLEY and HUTTON in their *Fossil Flora,* pl. liii., where the limb of the pinnules has entirely disappeared, leaving only the veins. Notwithstanding, however, the imperfect state of the example shown at fig. 1, it is interesting, as distinctly showing the position occupied by the fructification of this species, which is on the rachis below the dichotomy as well as on the base of the two arms of the fork.

Specimen from Lewis Burn, rather over 200 yards below Lewis Burn Colliery, N. Tynedale, Northumberland (fig. 2), in the Collection of the Geological Survey of England, collected by Mr J. RHODES. This example, which occurs in association with barren fragments of *C. bifida,* exhibits the characteristic dichotomisation of the fructifying branches of this species, and in fact of the genus *Calymmatotheca.* The rachis of the pinnæ here seems to undergo three series of dichotomies, at the extremities of the ultimate forks of which the *synangia* were borne.[†]

Specimen from Back Burn, opposite Cranecleuch New Houses, N. Tynedale, Northumberland, collected by Mr J. RHODES, in the Collection of the Geological Survey of England (fig. 3). The specimen figured lies on the corner of a small slab which contains a great many groups of the sporangia of this species. These groups seem to be much crushed and flattened out, and little of the intimate structure is discernible. Only slight traces of the stem which bore the synangia is preserved in this example. The

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[*] *Trans. Roy. Soc. Edin.*, vol. xxx. p. 539, pl. xxxi. fig. 6.

[†] See *Trans. Roy. Soc. Edin.*, vol. xxx. pl. xxxi. fig. 6.

synangium, lettered *a*, shows very clearly the union of the sporangia in their lower portions.

Specimen from Bateinghope Burn, Redesdale, Northumberland (figs. 4 and 5*a*). This small example shows four *synangia*, lettered respectively *a*, *b*, *c*, *d*, of which *a* and *b* are the two most perfect. Each *synangium* appears to contain from 18–20 *sporangia*. For about half their length the sporangia are free, but their basal portions are united. The individual sporangia, though now compressed, show a distinct rotundity, and have usually one or two well-marked longitudinal ridges. The sporangia must have originally possessed considerable substance, for in the fossil state they are frequently converted into a coaly material, which, from its brittle nature, when the stones containing the fossils are split, commonly causes the free portions of the sporangia to spring from the matrix, only leaving their impressions on the stone.

Specimen from Lewis Burn, over 200 yards below Lewis Burn Colliery, N. Tynedale, Northumberland (fig. 6), collected by Mr J. Rhodes, in the Collection of the Geological Survey of England. This small slab shows two different types of fern fructification lying side by side. That marked *a* is a *synangium* of *C. bifida*, but the other is evidently the remains of an *indusium* split into five segments. Unfortunately, very few fragments of this interesting fern fructification (6*b*) have been discovered ; but another, though imperfect example, shows the indusia attached to a rachis in a somewhat similar manner to those of *Sorocladus stellatus*, Lesqx.* It differs, however, from that species in the larger size of the indusium, and in the frond being apparently bipinnate, at least the small fragment showing these fruits attached to the rachis exhibits a bipinnate disposition of the indusia. I propose provisionally to designate this species as *Sorocladus antecedens*. The plant I here place in *Sorocladus* differs from *Zeilleria* in the fruiting portion being altogether destitute of ordinary foliage pinnules. The few fragments of this species which have been collected come from the same locality.

Specimen from Burdiehouse, Mid-Lothian, in the "Hugh Miller Collection," Museum of Science and Art, Edinburgh (Pl. IX. figs. 16 and 17). This example, which is the original of the small woodcut given by Hugh Miller in the *Testimony of the Rocks*, Edinburgh, 1857, p. 466, fig. 129, is reproduced here natural size. It shows one of the two main divisions of the frond, and bears about 22 pairs of opposite pinnæ. The pinnæ on the right of the figure are longer than those on the left, the latter having been situated within the fork of the frond.

The pinnæ are lanceolate, the longer ones bearing about 14 pairs of pinnules, which vary from simple to being divided into 8 linear, single-nerved, simple, or bifid segments, according to their position on the pinnæ. The

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* *Coal Flora of Pennsyl.*, p. 328, pl. xlviii. fig. 8.

pinnules towards the centre of the pinnæ are longest. The pinnules of the superior side of the pinnæ are longer than those on its inferior side. Figs. 17, *a*, *b*, *c*, show some of the pinnules slightly enlarged.

There is another example from Burdiehouse in the "Hugh Miller Collection," Museum of Science and Art, Edinburgh, which is interesting as showing very beautifully the bifurcation of the main axis, below which, as well as on the two arms of the fork, the frond bears barren pinnæ. It further shows that the pinnæ within the fork are shorter than those attached to the outer side of the fork, a character well shown in fig. 16.

Several specimens, also showing the bifurcation of the main axis, are contained in the Collection of the Geological Survey of Great Britain.

My thanks are due to Dr TRAQUAIR, keeper of the Natural History Department, Museum of Science and Art, Edinburgh, for permission to figure the specimen shown on Pl. IX. fig. 16.

Horizon.—Calciferous Sandstone Series.

Localities:—

Scotland.—Burdiehouse, near Edinburgh; Muir Burn, Kershope Burn, and Tweeden Burn, Liddesdale;* and River Esk, Glencartholm, Eskdale.*

England—Northumberland.—Shore section, Sandstone Quarry, a little south of Sea Houses; Bateinghope Burn, 1 mile from head of stream, Redesdale; east bank of Lewis Burn, Barney's Cut, a little more than $\frac{1}{4}$ mile south-west of Lewis Burn Bridge, North Tynedale; Lewis Burn, more than 200 yards below Lewis Burn Colliery, North Tynedale; Buck Burn, $\frac{3}{4}$ mile north-west of Willow Bog, Oakenshaw Burn, North Tynedale; Cranecleuch Burn, opposite Cranecleuch New Houses, Whickhope Burn, North Tynedale; foot of Sauchy Sike, Little Whickhope Burn, North Tynedale; Rigend Burn, Kielder, N. Tynedale. *Cumberland.*—River Irthing, $\frac{7}{8}$ mile north of Lampert (county boundary, Northumberland and Cumberland); foot of streamlet, $\frac{3}{4}$ mile south-west of Wileysike, River Irthing; River Irthing, 2 miles north-east of Waterhead; River Irthing, $\frac{3}{4}$ mile east of Waterhead; Bothrigg Burn, near its head, 1 mile east of the Flat, Bewcastle; stream between Oakshaw and Whintingstone, Clattering Ford, Bewcastle.†

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* Collected by Mr A. MACCONCHIE, Fossil Collector to the *Geol. Sur. of Scotland.*

† The specimens from Northumberland and Cumberland have been mostly collected by Mr J. RHODES, Fossil Collector to the *Geol. Sur. of England.*

Calymmatotheca affinis, L. & H., sp.

Plate IX. figs. 18–22.

Calymmatotheca affinis, Kidston, *Catalogue of Palæoz. Plants in Brit. Mus.*, p. 66, 1886.
Sphenopteris affinis, Lindley and Hutton, *Foss. Flora*, vol. i. pl. xlv.
Sphenopteris affinis, Hibbert, *Trans. Roy. Soc. Edin.*, vol. xiii. p. 178, pl. vi. fig. 4; Pl. v. *bis.*
Sphenopteris affinis, Peach, *Quart. Jour. Geol. Soc.*, vol. xxxiv. p. 131, pl. vii.
Sphenopteris affinis, Peach, *Trans. Bot. Soc. Edin.*, vol. xii. pp. 162 and 187.
Sphenopteris linearis, Brongniart (not Sternberg), *Hist. d. végét. foss.*, p. 175, pl. liv. fig. 1.
Sphenopteris linearis, Hibbert, *Trans. Roy. Soc. Edin.*, vol. xiii. p. 178, pl. vi. fig. 3.
Staphylopteris (?) *Peachii*, Peach, *Quart. Jour. Geol. Soc.*, vol. xxxiv. p. 131, pl. viii. figs. 1, 2, 3 (4 ?).
Sphenopteris frigida, Heer (*in part*) *Foss. Flora Spitzbergens*, pl. i. fig. 2.
Sphenopteris flexilis, Heer (*in part*), *Foss. Flora Spitzbergens*, p. 8, pl. i. figs. 11–27 (pl. ii. figs. 7–10 ?)

Description.—Frond divided into two symmetrical, lanceolate parts, tripinnate or decompound; primary pinnæ alternate, lanceolate; secondary and tertiary pinnæ alternate and broadly lanceolate; pinnules cuneate, entire or divided into 2–3 cuneate lobes. Veins numerous, radiating from the base of the pinnule, and dichotomising 2–3 times. Fructification consisting of 4–6 oblong exannulate sporangia borne at the extremities of the dichotomously divided fertile pinnæ, which are wholly deprived of foliage pinnules. Position of fertile pinnæ not yet observed, but probably holding the same position on the frond as those of *Calymmatotheca bifida.* Rachis smooth.

Remarks.—The plant figured by BRONGNIART as *Sphenopteris linearis* is evidently not STERNBERG'S fern of that name.* The specimen that has served as the type of STERNBERG'S *Sph. linearis* is so imperfect that, from any evidence afforded by the figure, it is very improbable it will ever be known what his fern really is.

On the other hand, the plant figured by BRONGNIART as *Sph. linearis* is the same as that earlier described by LINDLEY and HUTTON as *Sph. affinis.* The type figure of *Sph. affinis* is unfortunately not very characteristic of the species, and though small pinnuled forms occur, the pinnules are always more cuneate than shown in the figure given on plate xlv. (vol. i.) of the *Foss. Flora.* As far as this character is concerned, the figure given by BRONGNIART is more satisfactory, but perhaps the most characteristic figures are those given by HIBBERT.†

It may be added that the specimen figured by LINDLEY and HUTTON as *Sph. linearis*,‡ which is fortunately preserved in the Hutton Collection (Museum of Natural History, Newcastle-on-Tyne), is not the *Sph. linearis*, Bgt. (=*Sph.*

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* Sternberg, Vers. ii. p. 15, pl. xlii. fig. 4.

† *Trans. Roy. Soc. Edin.*, *loc cit.*

‡ *Foss. Flora*, vol. iii. pl. ccxxx.

affinis, L. & H.), but a fine specimen of the upper portion of *Sph. crassa,* L. & H. Their plate is not a satisfactory rendering of the fossil.*

C. affinis, in the dichotomisation of the main axis and the general distribution of the primary pinnæ on the two forks of the dichotomy, follows the same arrangement as that occurring in *C. bifida,* L. & H., sp.

Specimens of the fruit of *C. affinis* were first exhibited by the late Mr C. W. PEACH at the meeting of the Bot. Soc. Edin., May 1874.† These were subsequently named *Staphylopteris* (?) *Peachii* by the late Prof. BALFOUR. Later Mr C. W. PEACH found the *Staphylopteris* (?) *Peachii* united to *Sphenopteris affinis,*‡ but regarded it as a parasite. The structure, however, of *Staphylopteris* (?) *Peachii,* being that of a marattiaceous fructification, independently of the fact of a similar structure having been found organically attached to *C. bifida* in such a manner as to conclusively prove it is the fruit of that species, shows beyond all doubt that *Staphylopteris* (?) *Peachii* is the fruit of *Sphenopteris affinis,* and not a parasite.

Mr C. W. PEACH communicated a paper to the Geol. Soc. London on *Sph. affinis* and *Staphylopteris* (?) *Peachii,*§ in which he figured some small specimens of the latter fossil.||

In the same communication he describes and figures what he believed to be the true fruit of *Sph. affinis.*¶ The specimen from which this figure was taken was kindly shown me by its describer, but I could not distinguish other than some sand-grains or other inorganic matter adhering to the pinnules, which had been mistaken by my friend for fruit. I believe this view of the supposed fruit is that accepted by others who have seen the specimen.

A very good restoration of the complete frond of *C. affinis* is given by HUGH MILLER as a frontispiece to his *Testimony of the Rocks.*

The figures of *Sph. frigida* and *Sph. flexilis,* mentioned in the synonymy, appear to belong to this species.

Description of Specimens.—Fig. 18. From Burdiehouse, near Edinburgh. This specimen, which is preserved in a dark grey limestone, shows the general form of the fern. The pinnæ are lanceolate, the secondary pinnæ being somewhat more broadly lanceolate than the primary. The tertiary pinnæ bear 2–4 cuneate pinnules, which are either simple or compounded of 2–3 cuneate lobes. The veins are indistinctly preserved, the carbon of the plant being converted into a bright coal-like substance.

Fig. 19. From Harwood Burn, below Limefield House, near West Calder, Mid-Lothian, in the Collection of the Geological Survey of Scotland.

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* See Kidston, *Proc. Roy. Phys. Soc. Edin.,* vol. vii. p. 238.

† *Trans. Bot. Soc. Edin.,* vol. xii. p. 162.

‡ *Trans. Bot. Soc., loc. cit.,* p. 187.

§ *Quart. Jour. Geol. Soc.,* vol. xxxiv. p. 131.

|| Pl. viii. figs. 1–3 (4 ?).

¶ Pl. vii. fig. 2.

Fig. 19*b* shows one of the pinnæ of a large pinnuled form of *C. affinis.* This example, in the size of the pinnules, corresponds to BRONGNIART'S *Sphenopteris linearis.** The nervation is beautifully preserved, and is shown in the enlarged drawing, fig. 19*a.*

Fig. 20. From West Calder, collected by the late Mr C. W. PEACH. This sketch shows one of the largest fertile pinnæ of *C. affinis* with which I have met. The pinna is destitute of foliage pinnules, and ramifies by a series of dichotomies. At *a* are shown the ultimate branchlets, but the *synangia* have become detached from their parent stalks, a few of which, however, are seen at *a* and *b*. These consist of 4–5 sporangia.

Figs. 21–22 are copied from the plate which accompanies Mr C. W. PEACH'S paper in the *Quart. Jour. Geol. Soc.*, vol. xxxiv. p. 131. Fig. 21 shows the *synangia* attached to their supporting pedicels, and fig. 22 gives a single *synangium,* which is enlarged at fig. 22*b* to show the five exannulate sporangia of which it is composed.

Horizon.—Hitherto only found in the Calciferous Sandstone series, where in some localities it is plentiful.

Localities :—

Scotland—Berwickshire.—Bilsdean Creek, 1½ miles west of Cockburnspath (J. Bennie); Shore, west of Harbour, Cove, Cockburnspath (J. Bennie). *Fifeshire.*—Rocks above Kinghorn; Flisk Quarry, St Andrews; Grange Quarry, Burntisland; Kilmundy Limestone Quarry, Burntisland (J. Bennie); Kilmundy Sandstone Quarry, Burntisland (J. Bennie); Dodhead Quarry, Burntisland (J. Bennie); east side of the Binn near Burntisland (J. Bennie); Brosyhall Lime Quarry, east of Burntisland (J. Bennie); Binnend Shale Works, Burntisland (J. Bennie). *Haddingtonshire.*—Long Craigs Bay, 1½ miles west of Dunbar (J. Bennie). *Buteshire.*—Island of Arran (British Museum). *Dumfriesshire.*—Docken Beck, near Langholm (A. Macconochie); Glencartholm, Eskdale (A. Macconochie); Tinnis Burn, near Newcastleton, Liddesdale (A. Macconochie). *Linlithgowshire.*—Railway Cutting, Dalmeny Railway Station; Dalmeny Shore, halfway between Long Craig and Newhall Piers, Queensferry (J. Bennie). *Mid-Lothian.*—Raw Camps, Mid-Calder (*Fruit*); Straiton Oil Works, near Loanhead; Queen's Park, Edinburgh (Professor Ross); Craigleith Quarry, near Edinburgh; Lochend, Edinburgh; Addiewell; Water of Leith, below Redhall Mill Dam; Hailes Quarry, Kingsknowe, near Slateford (Professor D'Arcy Thomson), (*Fruit*), (T. Stock); Burdiehouse; Banks of the Almond, Cramond (R. F. B. Bishop); Suburban Railway Cutting, Edinburgh (J. Gaul); West Hermand, near West Calder (*Fruit*), (C. W. Peach); Harwood Burn, below Limefield House, near West Calder (J. Bennie); Currie (*Fruit*), (J. Bennie); Slateford; Shore at

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* *Loc. cit.*, pl. liv. fig. 1.

Wardie, near Granton (C. W. Peach); Inchkeith, Frith of Forth (J. Gaul).

England—Cumberland.—Bull Cleuch, Kirk Beck, Bewcastle (H. Miller). *Northumberland.*—Warksburn, North Tynedale (H. Miller).

Calymmatotheca asteroides, Lesqx., sp.

Calymmatotheca asteroides, Zeiller, *Ann. des Scienc. nat.*, 6ᵉ sér. Bot., vol. xvi. p. 182, pl. ix. figs. 10, 11.

Staphylopteris asteroides, Lesquereux, *Report Geol. Survey of Illin.*, vol. iv. p. 406, pl. xiv. figs. 6, 7; Schimper, *Traité d. paléont. végét.*, vol. iii. p. 512.

Sorocladus asteroides, Lesquereux, *Coal Flora of Pennsyl.*, p. 328, pl. xlviii. figs. 9, 9*b*.

Remarks.—Among many other specimens of fossil plants contained in the collection of the late William Henry Johnson, Dudley, I observed two small specimens of this species.

Unfortunately, they are not very well preserved, and do not add any additional information to the knowledge of the species. The general growth of the species is well shown in Lesquereux's figures, and their more minute structural details have been illustrated by Zeiller. The fructification consists of a number of elongated sporangia, usually six in number, arranged in a stellate manner around a common point of attachment. It is not yet known to which fern this fructification belongs, as the fertile portion shows no traces of the barren pinnules.

Horizon.—Middle Coal Measures.

Locality.—Coseley, near Dudley.

Zeilleria Avoldensis, Stur, sp.

Plate VIII. figs. 8–10.

Zeilleria Avoldensis, Kidston, *Quart. Jour. Geol. Soc.*, vol. xl. p. 591, 1884.

Calymmotheca Avoldensis, Stur, "Morph. u. Syst. d. Culm. u. Carbonfarne," *Sitzb. d. k. Akad. d. Wissensch.*, vol. lxxxviii. p. 171, fig. 37.

Die Carbon-Flora d. Schatzlarer Schichten, *Abhandl. d. k. k. geol. Reichsanst.*, vol. xi. Abth. i. p. 251, pl. xxxviii. fig. 1, text fig. 41 on p. 238.

Description.—Frond decompound (4–5 pinnate); primary pinnæ broadly lanceolate, secondary pinnæ lanceolate, and composed of about twenty pairs of tertiary pinnæ. The tertiary pinnæ are more or less lanceolate, but vary in outline according to their position on the frond. Pinnules attached to the rachis by their whole base and united among themselves, the free portion of the limb is ovate-triangular; medial nerve clearly defined, and giving off 2–4 simple lateral branchlets, all of which extend to the margin of the pinnule. Fruiting portion of the frond confined to the lower secondary pinnæ, where the fertile pinnules bear 1–3 pedicellate indusia at the extremities of the excurrent veins. In the earlier condition the indusia are oval, but at maturity split into four valves.

Remarks.—From the figure of this species given by Dr STUR in his *Carbon Flora*, the fronds of this fern must have attained to large dimensions. The form of the tertiary pinnæ varies much on the upper and lower parts of the fern; on the upper portion they are small, about 3 mm. long, 2 mm. broad, and more or less oval in outline; those towards the apex of the secondary pinnæ are more or less united among themselves. On the lower secondary pinnæ the tertiary pinnæ are 15–20 mm. or more long, and bear many pairs of alternate pinnules, which are usually united to each other for $\frac{1}{3}$ or $\frac{2}{3}$ of their length. The free portion is triangular, and has a well-defined central and usually two lateral veins, one given off from each side of the medial nerve. The fertile pinnules do not differ in form from the barren, except in the veins being produced to form little pedicels to which the oval indusia are attached (Pl. VIII. figs. 9, 10). Occasionally only the upper pinnules are fertile, but quite as frequently the lower, as well as the upper pinnules bear fruit.

The fruit of this fern, as pointed out by Dr STUR,* is composed of four valves. This four-cleft appearance, however, is only shown when the indusium has reached maturity and split for the dissemination of the spores; in the young state the indusia are oval as seen at Pl. VIII. fig. 9. As to the manner in which the spores are arranged within the indusium nothing is known.

Description of Specimen.—Pl. VIII. fig. 8. The example figured is the only British specimen of this species with which I have yet met. It shows a portion of a secondary pinnæ, bearing the remains of twelve tertiary pinnæ, none of which are very complete, but all are fertile, except the two upper pairs. The fertile pinnules bear in some cases three (fig. 10) and in others only one indusium (fig. 9).

At fig. 9 is exhibited the young, and at fig. 10 the more advanced condition of the indusia, where they have split into valves.

This specimen was in the collection of the late Mr HENRY JOHNSON, F.G.S., Dudley, from whom I received it for examination.

Horizon.—Middle Coal Measures.

Locality.—Corseley, near Dudley.

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* *Carbon Flora, &c.*, p. 254.

Neuropteris heterophylla, Brongniart.

Plate VIII. fig. 7.

Neuropteris heterophylla, Brongniart, "Classification des Végétaux Fossiles," Extract from *Mémoires du Muséum d' histoire naturelle*, tome viii. p. 33, pl. ii. fig. 6, 1822.
Hist. d. Végétaux Fossiles, p. 243, pl. lxxi. and pl. lxxii. fig. 2, 1828.
Neuropteris Loshii, Brongniart, *Hist. d. Végétaux Fossiles*, p. 242, pl. lxxii. fig. 1, and pl. lxxiii. 1828.

Several authors have described what they believe was the fructification of the genus *Neuropteris*, Brongniart, but in all these cases the supposed fruit was either a parasitic fungus, or the fern bearing the fruit described had been referred to the genus *Neuropteris* in error.

As early as 1826, HOFFMANN figured what he regarded as the fruit of his *Neuropteris ovata*.* This consisted of a single lanceolate pinnule, 3 cm. long and 1 cm. wide, whose basal extremity appears to me rather to lie under the stem which is supposed to have borne it than to be attached to it. The upper surface of this supposed fruiting pinnule shows an indistinct granulation. Its preservation is, however, so imperfect that it seems impossible to say that this supposed fruit belongs to *N. ovata*, or even to any other member of the genus *Neuropteris*.

The next supposed fruit of *Neuropteris* was figured by BRONGNIART in his *Hist. d. vêgét. foss.*, p. 239, plate lxv. figs. 3 and 3*a*, where certain linear swellings situated *between* the nerves are irregularly scattered over the upper surface of a pinnule of *Neuropteris flexuosa*. In a subsequent part of the same work, p. 326, BRONGNIART corrected this erroneous interpretation of these bodies, and refers them to parasitic fungi, a view which receives confirmation from the occurrence of similar organisms on ferns belonging to different recent genera.

Almost conclusive evidence against these bodies being the fruit of ferns is further afforded by their occupying the tissue of the pinnules *between* the veins, whereas the fruit of ferns is situated *on* some part of their nervation.

In 1880 FONTAINE and WHITE, in their *Permian and Upper Carboniferous Flora*, give a figure of *N. hirsuta*,† showing what they believed to be its fructification, but again this supposed fruit appears to be only another of those parasitic fungi, and one which seems very closely related to the species affecting the specimen of *N. flexuosa* described by BRONGNIART. The supposed *sori* figured by

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* "Uber die Pflanzenreste des Kohlengebirges von Ibbenbühren und vom Piesberge bei Osnabruck," in Keferstein's *Teuchland geognostisch-geologisch dargetsellt*, vol. iv. p. 158, pl. i. figs. 5–8, Weimar, 1826. His fig. 8 is the supposed fruiting pinnule.

† "Second Geol. Survey of Pennsylvania, Report of Progress P.P.," *The Permian or Upper Carboniferous Flora of West Virginia and S.-W. Pennsylvania*, p. 47, pl. viii. figs. 7, 8, Harrisburg, 1880.

Fontaine and White are also stated to lie *between* the veins, a circumstance which is fatal to the view that these bodies are the fructification of their fern.

Bunbury[*] had previously figured and described similar organisms on the pinnules of *N. Scheuchzeri* (*N. cordata,* Bunbury, not Brongniart),[†] and had rightly referred them to a disease of the parenchyma or a parasitic fungus.

In the Carboniferous formation, fossil parasitical fungi occur not only on various spcies of ferns, but on other plants also, and have been figured and described by various writers.[‡]

N. heterophylla, Brongt. (with which *N. Loshii,* Brongt., is now known to be synonymous), has also had its supposed fruit described by Gutbier in 1849,[§] but in this case, even if the bodies which were supposed by Gutbier to be the fruit of his fern really prove to be its fructification, we are still in ignorance of the fruit of *Neuropteris,* as Gutbier's fern does not belong to this genus, but to *Odontopteris.*[||]

The specimen now described exhibits very clearly the mode and character of the growth of the fruiting portion of *Neuropteris.* It was discovered by Mr T. Stock, by whom it was communicated to me for examination.

The fossil shows an axis *a* about 8 cm. long, which gives off apparently two pairs of lateral pinnæ, *b, c* and *d, e.* The terminal portion of the specimen ends in a number of dichotomous branchlets, the ultimate divisions being about 8 mm. long, and bearing the fruit at their summits. On the terminal part of the fossil there is no trace of the ordinary foliage pinnules. At *b* and *c* are shown what appears to be the remains of a pair of lateral pinnæ, each of which seems to have supported four fructifications. Associated with these pinnæ are the remains of a small number of ordinary barren pinnules. Of the two lower pinnæ, that marked *d* is very incomplete, and only shows some fragments of the ordinary barren pinnules; the corresponding opposite pinna is, however, more perfect, and shows three fructifications and a portion of a pedicel of a fourth. At the base of this pinnæ are preserved some remains of

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[*] *Quart. Jour. Geol. Soc.*, vol. iii. p. 424, pl. xxv. fig. 1*e* and 1*f.*

[†] See Zeiller, "Notes sur la Flore houillère des Asturies," *Mém. Soc. Géol. du Nord. Lille,* p. 6, 1882.

[‡] See Göppert, " Foss. Farrnkräuter," *Syst. fil. foss.*, p. 262, pl. xxxvi. fig. 4, *Excipulites Neesii;* Weiss, *Foss. Flora d. jüng. Slk. u. d. Rothl.*, p. 19; Schimper, *Traité d. paléont. végét.*, vol. i. p. 141, pl. i. fig. 19, and Explanation to pl. xxxii. figs. 6 7; Geinitz, *Vers. d. Steink. in Sachsen*, p. 2, pl. xxiii. fig. 13, *Excipulites Neesii*; pl. xxv. fig. 10, *Depazites Rabenhorsti;* Feistmantel, "Der Handendflötzzug," &c., *Archiv. d. Naturw. Landesdurchforschung von Böhmen,* iv. Band, No. 6 (Geol. Abth.), p. 62, pl. i. fig. 1, *Xylomides ellipticus;* Weiss, "Steinkohlen-Calamarien," *Abhandl. z. geol. specialkarte v. Preussen u. d. Thüringischen Staaten,* Band v. Heft. ii. p. 66, pl. i. fig. 2; Grand' Eury, *Flore carbon. du Départ. de la Loire et du centre de la France,* p. 10, *Excipulites punctatus* and *Hysterites cordaitis,* pl. i. fig. 7, &c.

[§] *Die Versteinerungen die Rothliegenden in Sachsen,* p. 12, pl. iv. figs. 2, 3.

[||] See Weiss, *Foss. Flora d. jüng. Stk. u. d. Rothl.*, p. 27 ; also for the fruit of *Odontopteris,* see Gand' Eury, *Flore carbon. du Départ. de la Loire,* p. iii. pl. xiii. fig. 4.

the ordinary barren pinnules. It would appear, therefore, that each of the lateral pinnæ supported four fruits, and on the terminal portion; though the remains of only eleven fructifications are seen, there were probably originally twelve. There may be combined in this part two lateral pinnæ and the apex of the frond, each bearing four fruits, but this cannot be clearly traced. At *f* is shown a small fragment of a pinna, drawn in the natural position it holds to the larger specimen. This shows the remains of two fructifications and portions of three barren pinnules, one of which is very perfect. The fortunate occurrence of barren pinnules associated with this fructification, conclusively identifies this interesting specimen with the genus *Neuropteris*, and further the barren pinnules *f** and *e** do not differ in any way from many shown on the figure of *N. heterophylla*, given by BRONGNIART in the *Hist. d. Végét. foss.*, plate lxxi. In the same beds from which this specimen was collected *N. heterophylla* is plentiful.

As to the affinities of this species, either with past or present existing genera of ferns, unfortunately this specimen does not afford sufficient data from which to form any opinion.

In the description of this specimen, I have therefore refrained from employing the terms *indusium* or *sporangium* to the little expansions at the extremities of the pedicels, as I cannot determine their true structure, though they are apparently composed of two or four segments.

Horizon.—Lower Coal Measures.

Locality.—Blairpoint, Dysart, Fife.

ALCICORNOPTERIS, n. gen., Kidston.

Generic Description.—Rachis ramifying by a series of dichotomies. Barren pinnæ composed of a foliaceous *Rhacophyllum*-like expansion. Fruiting portion consisting of much divided circinately convoluted pinnæ. Form and mode of attachment of sporangia to the fruiting pinnæ unknown.

Remarks.—This genus in the barren condition approaches closely to *Rhacophyllum*, and in its fruiting branches to SCHIMPER'S *Triphyllopteris collombi* and DAWSON'S *Cyclopteris acadica*.

Alcicornopteris convoluta, n. sp., Kidston.

Plate VIII. figs. 11–15.

Rhacophyllum Lactuca, Kidst. (not Sternb.), *Trans. Roy. Soc. Edin.*, vol. xxx. p. 540.

Description.—Rachis flattened with a central angular ridge, and dividing by a series of dichotomies. The primary (?) dichotomy forming an obtuse angle; those angles formed by subsequent dichotomies are a little more acute. The pinnæ of the barren fronds possess a broad foliaceous expansion cut into spirally bent lobes, in which the nerves are indicated by dichotomously dividing ridges. Fertile pinnæ dividing dichotomously and reduced to winged circinately convoluted rachis-like segments. The convolutions of the basal portion of the pinnæ overlap each other; their ultimate divisions are narrower, less prominently winged, and do not apparently overlap each other, or only do so to a limited extent.

Remarks.—Specimens of this fern have been in the Collection of the Geological Survey of Scotland for many years, that figured on Plate VIII. fig. 13, having been collected by the late Mr RICHARD GIBBS about twenty-five years ago. The portions of the species with which I first met were fragments of the scorpiod fruiting pinnæ. These, I thought, might perhaps belong to *Triphyllopteris Collombi,* Schimper,* or to *Cyclopteris* (*Aneimites*) *Acadica,* Dawson,† both of which species have very close affinities to each other, if not specifically identical.

Associated with the British examples, though careful examination was made, no barren pinnules were ever discovered that could be identified with either SCHIMPER'S or DAWSON'S ferns.

It was only towards the end of 1884 that my difficulties in the identification of this fern were removed by Mr JOHN RHODES, Fossil Collector to the Geological Survey of England, finding the specimens figured on Plate VIII. figs. 11, 12, which show the barren condition of this plant. I had previously seen a small fragment of the barren condition of *Alcicornopteris convoluta* from Docken Beck, Eskdale, collected by Mr A. MACCONOCHIE, one of the Fossil Collectors to the Geological Survey of Scotland, but had erroneously identified it as *Rhacophyllum Lactuca,*‡ to which small fragments have a great resemblance, so much is this the case, that with fragmentary specimens it is almost impossible to distinguish them. The fruiting portions of *A. convoluta* have apparently a more strongly winged rachis than occurs in *Cyclopteris Acadica,* or in *Triphyllopteris Collombi;* but here also small fragments of the ultimate segments of the fruiting portions of *Alcicornopteris convoluta* would be with difficulty distinguished from fragments of the fruiting portions of the two ferns already mentioned (see Plate VIII. fig. 15).

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* *Triphyllopteris Collombi,* Schimper-Zittel, *Handbuch der paléontologie,* ii. Band, 1 Lief, p. 114, fig. 84, 1879; *Traité d. paléont. végét.,* vol. i. p. 479, pl. cvii. fig. 13; "Les végét. foss. du terrain de trans. d. Vosges" (in *Le terrain de trans. d. Vosges,* by J. Koechlin-Schlumberger and W. Ph. Schimper, Strasburg, 1862), p. 339, pl. xxvii. figs. 8–11 (*Sphenopteris*).

† *Cyclopteris* (*Aneimites*) *Acadica,* Dawson, "Geological Survey of Canada," *Fossil Plants of the Lower Carboniferous and Millstone Grit Form. of Canada,* p. 26, pl. vii. figs. 53–63, 1873; *Acadian Geol.,* 2nd ed., p. 481, fig. 75, 1868; *Quart. Jour. Geol. Soc..,* vol. xxii. p. 153, pl. viii. fig. 32, 1865.

‡ *Trans. Roy. Soc. Edin.,* vol. xxx. p. 540.

On the other hand, when more perfect specimens are secured, the differences between the fruiting portions of *A. convoluta* and *C. Acadica* and *T. Collombi* are very well marked, and in the barren condition, the British species has no similarity with either SCHIMPER's or DAWSON's plants.

It may be questioned if this new plant should not be included in *Rhacophyllum*, with which its barren pinnæ have so great a resemblance, but against adopting this course is the fact that *Rhacophyllum* is essentially a Coal Measure genus, whereas *A. convoluta* has hitherto only been found in the Calciferous Sandstones, and then usually in the basement beds. Whatever view may be taken of the genus *Rhacophyllum*, whether as forming an individual genus or as a provisional one, only containing the accessory pinnules of other ferns, I am not in a position to decide; but in regard to *A. convoluta* there can remain no doubt as to its being an autonomous fern, and not a portion of another species.

In *Triphyllopteris Collombi* the fruit is borne at the extremity of the circinately bent segments, and it probably occupied a similar position on *A. convoluta*, but none of the specimens that have come under my notice have shown any traces of sporangia.

Description of Specimens.—Specimen from Horncliffe Dean, near the Mill, River Tweed, South of Horncliffe Village, Northumberland, collected by Mr J. RHODES (Plate VIII. fig. 11), in the Collection of the Geological Survey of England. This specimen shows a small portion of a barren frond. The rachis is very stout, and gives off apparently alternate pinnæ, possessing a midrib with a sharp angular ridge. This example is not well preserved, and does not show any perfect pinnæ or pinnules, but these were apparently cut into lobes, having curious spirally twisted segments, which at their point of separation formed an almost circular sinus giving the frond a curled appearance.

Specimen from River Tweed, 100 yards below Norham Castle, Northumberland, collected by Mr J. RHODES (Plate VIII. fig. 12), in the Collection of the Geological Survey of England. This specimen, though also fragmentary, is a very good example of the mode of ramification of *Alcicornopteris convoluta*. It exhibits a portion of a rachis 4 cm. long, and 5 mm. broad at the lower broken-over extremity, and 1 cm. wide immediately below the point where it bifurcates. The two arms of the first bifurcation go off from the parent rachis at almost right angles, and then again bifurcate. Both the upper forks of this second dichotomy are broken over, but the lower arm on the left forms a third series of dichotomies. On this is borne the barren pinnules. A portion of one of the corresponding forks of the right hand dichotomy of the third series is also present. Here again the pinnules are badly preserved, but show the same characteristics as fig. 11. The rachis appears to have been flat, and traversed by a prominent vascular system which appears as a triangular ridge. The helicoid nature of the frond is well shown on the frondose portion of this specimen. An imperfect fragment of a pinnule lies between the primary forks of the rachis, but its position there is accidental. The surface of the rachis is finely striated.

Specimen from Cove Shore, east of Cove Harbour, Berwickshire, collected by the late Mr R. Gibbs (Plate VIII. fig. 13), in the Collection of the Geological Survey of Scotland. This specimen, which is preserved in a hard micaceous sandstone containing many vegetable fragments, shows probably a primary dichotomy of the rachis. Before the pinnule segments are reached, there also appears here a threefold diohotomy of the axis, similar to that shown in fig. 12.

Except the main axis, the other portions of the specimen are indifferently preserved. The rachis is very distinctly striated longitudinally, and seems to have been originally flat, with a well-pronounced central angular ridge, probably representing the vascular system of the rachis. The portion of the rachis shown in this figure is so flat that it must be described as winged. In fact, the frondose expansion of the barren pinnæ seems to be only a further development of this wing.

Specimen from Long Craigs Bay, near Dunbar, Haddingtonshire, collected by Mr James Bennie (Plate VIII. fig. 14), in the Collection of the Geological Survey of Scotland. This example of a fruiting portion of the frond of *A. convoluta* from Long Craigs Bay is preserved in a fine-grained red shale. The main rachis and those springing from it are broadly winged, the vascular bundle appearing as an angular ridge running in the centre of the rachis. The lateral pinnæ, of which only the basal portions are preserved, are best seen to the right of the figure, and consist of a series of dichotomously divided helicoid segments. The segments of these pinnæ overlap each other, and produce an almost inextricable confusion of convolutions.

Specimen from River Tweed, about 100 yards below Norham Castle, Northumberland (Pl. VIII. fig. 15). This specimen probably exhibits the ultimate divisions of the fertile pinnæ, and a comparison of this figure with the fruiting examples *Triphyllopteris Collombi*, Schimper, figured in the *Handbuch der Paleontologie*, p. 144, fig. 87, and with the figures of *Cyclopteris Acadica* given by Dawson, in the *Fossil Plants of the Lower Carb. of Canada*, plate vii., will show the great similarity between certain portions of these three ferns. The rachis in this part of the pinnæ can scarcely be said to be winged, though distinctly flattened.

Horizon.—Calciferous Sandstone series.

Localities :—

Scotland—Berwickshire.—Cove Shore, ½ mile east of Cove Harbour, 1½ miles north-east of Cockburnspath. "In a hard bed of micaceous sandstone at the base of the Carbonificerous Rocks, or rather at the base of that portion of them which immediately overlies the red and yellow sandstones of Berwickshire"* (R. Gibbs, collector) Kimmerghame Quarry, near Duns; (A. Macconochie). *Haddingtonshire.*—Long Craigs Bay, east of Belhaven Bay,

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* "Memoirs of the Geol. Survey of Great Britain," *The Geology of Eastern Berwickshire*, p. 58, 1864.

1 mile west of Dunbar (J. Bennie). *Dumfriesshire.*—Docken Beck, 3 miles south of Langholm (A. Macconochie).

England—Northumberland.—River Tweed, 100 yards below Norham Castle; River Tweed, south of Horncliffe Village; Horncliffe Dean, near mill south of Horncliffe Village; River Coquet, ½ mile north-north-east of Holystone; Coomsdon Burn, ½ mile south-west from its junction with the River Rede; Hawk Burn, near Catcleugh, Redesdale; Spithope Burn, Redesdale; Crawley Dean (east of road), ⅓ mile south of Powburn, near Ingram. *Cumberland.*—Bull Cleugh, Kirk Beck, Bewcastle.

I am indebted to Dr A. Geikie, F.R.S., for permission to figure and describe the various specimens, mentioned in this communication, belonging to the Geological Surveys of England and Scotland.

EXPLANATION OF PLATE VIII.

Figs. 1–6*a*. *Calymmatotheca bifida*, L. & H., sp.
  Fig. 1. From River Irthing, near Lampert.
  Fig. 2. From Lewis Burn, near Lewis Burn Colliery, N. Tynedale, Northumberland.
  Fig. 3. From Back Burn, opposite Cranecleuch New Houses, N. Tynedale.
  Fig. 4. From Bateinghope Burn, Redesdale.
  Fig. 5. Synangium, lettered *a* on fig. 4; enlarged.
  Fig. 6*a*. From Lewis Burn, near Lewis Burn Colliery, Northumberland.
Fig. 6*b*. *Sorocladus antecedens*, Kidston.
Fig. 7. *Neuropteris heterophylla*, Brongt. From Blairpoint, Dysart, Fife.
Figs. 8–10. *Zeilleria Avoldensis*, Stur, sp. From Coseley, near Dudley.
Figs. 9–10. Pinnules; enlarged.
Figs. 11–15. *Alcicornopteris convoluta*, Kidston.
  Fig. 11. From Horncliffe Dean, River Tweed, Northumberland.
  Fig. 12. From Norham Castle, Northumberland.
  Fig. 13. From Cove Shore, east of Cove Harbour, Berwickshire.
  Fig. 14. From Long Craigs Bay, near Dunbar, Haddingtonshire.
  Fig. 15. From River Tweed, near Norham Castle, Northumberland.

EXPLANATION OF PLATE IX.

Figs. 16–17. *Calymmatotheca bifida*, L. & H., sp.
  Fig. 16. From Burdiehouse, near Edinburgh. Specimen in the "Hugh Miller Collection," Museum of Science and Art, Edinburgh (natural size).
  Fig. 17*abc*. Three Pinnules, enlarged.
Fig. 18. *Calymmatotheca affinis*, L. & H., sp. From Burdiehouse, Mid-Lothian.
Fig. 19. *Calymmatotheca affinis*, L. & H., sp. From Harwood Burn, below Limefield House, near West Calder. In the Collection of the Geological Survey of Scotland (fig. 19*a*, enlarged; 19*b*, natural size).
Fig. 20. *Calymmatotheca affinis*, L. & H., sp. From West Calder, Mid-Lothian. Collected by the late Mr C. W. PEACH.
Figs. 21, 22. *Calymmatotheca affinis*, L. & H. Copied from Peach, *Quart. Jour. Geol. Soc.*, vol. xxxiv. pl. viii. figs. 1*a* (= 21) and 3–3*a* (= 22*a*, *b*).

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McFarlane & Erskine, Lithrs Edinr

Fig. 1-6a C. BIFIDA, L.&H. sp. 6b S. ANTECEDENS, Kidston. 7. N. HETEROPHYLLA, Brongt.
8-10. Z. AVOLDENSIS, Stur. sp. 11-15. A. CONVOLUTA, Kidston.

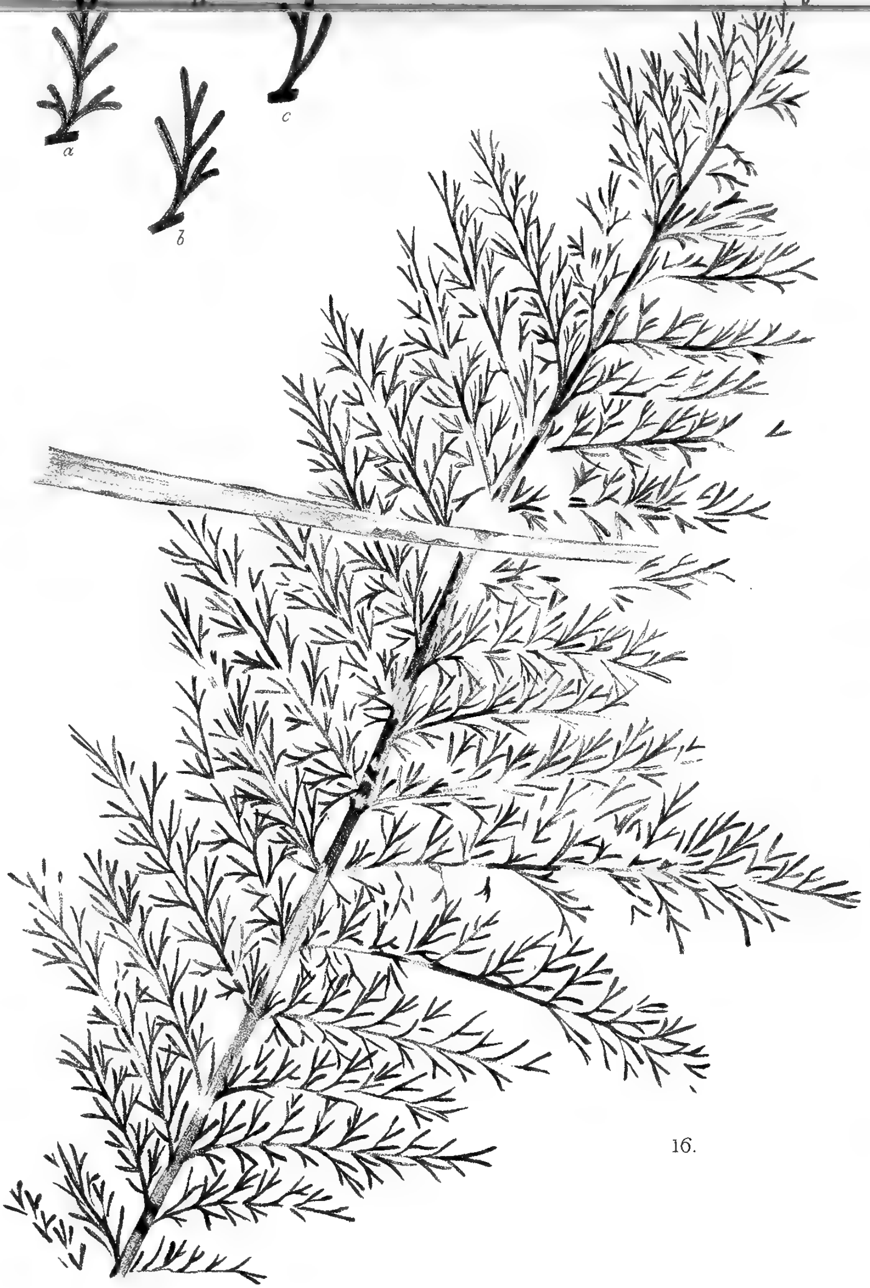

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Fig. 16-17. C. BIFIDA, L.&H. sp. 18-22. C. AFFINIS, L.&H. sp.

VI.—*On the Colours of Thin Plates.* By LORD RAYLEIGH. (Plate X.)

(Received 13th July 1886. Revised Aug. 1886.)

Introduction.

The first impression upon the mind of the reader of the above title will probably be, that the subject has long since been exhausted. The explanation of these colours, as due to interference, was one of the first triumphs of the wave-theory of light; and what YOUNG left undone was completed by POISSON, FRESNEL, ARAGO, and STOKES. And yet it would be hardly an exaggeration to say that the colours of thin plates have never been explained at all. The theory set forth so completely in our treatises tells us indeed how the composition of the light reflected depends upon the thickness of the plate, but what will be its colour cannot, in most cases, be foretold without information of an entirely different kind, dealing with the chromatic relations of the spectral colours themselves. This part of the subject belongs to Physiological Optics, as depending upon the special properties of the eye. The first attempt to deal with it is due to NEWTON, who invented the chromatic diagram, but his representation of the spectrum is arbitrary, and but a rough approximation to the truth. It is to MAXWELL that we owe the first systematic examination of the chromatic relations of the spectrum, and his results give the means of predicting the colour of any mixed light of known composition. Almost from the time of first reading MAXWELL'S splendid memoir, I have had the wish to undertake the task of calculating from his data the entire series of colours of thin plates, and of exhibiting them on NEWTON'S diagram. The results are here presented, and it is hoped may interest many who feel the fascination of the subject, and will be pleased to see a more complete theory of this celebrated series of colours.

The diagram (Plate X.) explains many things already known from observation, such as the poverty of the blue of the first order and of the green of the second order. For good blues we must look to the second and third orders, and for good greens to the third and fourth. The point in which the diagram disagrees most with descriptions by former observers, *e.g.*, HERSCHEL, relates to the precedence of the reds of the first and second orders. The first red has usually been considered inferior, but the reason appears to lie in its feeble luminosity, and consequent liability to suffer from contamination of white light. This and other questions are further discussed in the sequel.

The complementary colours, best obtained with the aid of polarised light, are also calculated and exhibited on a diagram.

§ 1. The calculation, according to YOUNG and POISSON, of the amount of light of given wave-length (λ) reflected from a thin plate is given in all treatises on physical optics. If D be the thickness, β the obliquity of the ray within the plate, $1 : e$, the ratio in which the amplitude is altered in one reflection, then for the intensity of light in the reflected system we have—

$$\frac{4e^2\sin^2(\pi V/\lambda)}{(1-e^2)^2+4e^2\sin^2(\pi V/\lambda)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which the intensity of the original light is taken to be unity, and V is written for $2\,D\cos\beta$. The colours exhibited in white light are to be found by combining the chromatic effects of all the rays of the spectrum.

When, as in NEWTON'S rings, the thickness of the plate varies from point to point, there is a series of colours determined by supposing D to vary in the above expression. This series is not absolutely independent of the material of which the plate is composed, even if we disregard the differences of brightness corresponding to the occurrence of e^2 in the numerator of our expression. On account of retarded propagation, the value of λ for a given ray is less in glass, for instance, than in air; and in consequence of dispersion there is no accurate proportionality, so that we cannot say absolutely that a definite thickness in glass corresponds to a definite, though different, thickness in air. Moreover, since e varies from one body to another, the denominator of (1) changes its value somewhat.

It is evidently impracticable to carry out calculations strictly applicable to all cases. If we take for λ the wave-length in air, we obtain results appropriate to the ordinary case of NEWTON'S rings; and in extending them to plates of other material, we in effect neglect the relatively small influence of dispersion.

Again, we may without much error neglect the variation of the denominator with wave-length, which amounts to supposing e^2 small, or that the two media do not differ much in refrangibility. In the case of glass and air the value of e^2 is about $\frac{1}{25}$. When $\sin^2(\pi V/\lambda)$ is small, it is of little consequence what the value of the denominator may be, and we may therefore identify it with $(1+e^2)^2$, taking instead of (1),

$$\frac{4e^2}{(1+e^2)^2}\sin^2\frac{\pi V}{\lambda} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is on this formula, strictly applicable only to a plate of air bounded by matter of small refrangibility, that the calculations and diagrams of this investigation are based.

§ 2. The colours of NEWTON'S scale are met with also in the light transmitted by a somewhat thin plate of doubly-refracting material, such as mica, the plane of analysis being perpendicular to that of primitive polarisation. To this case also our calculations are applicable, if we neglect the dispersion, and (as is usual) the light transmitted after two or more reflections at the surfaces of the plate.

If the analyser be turned through 90°, a new series of colours is exhibited complementary to the first series. The purity of the colours, as regards freedom from admixture with white, is greatest when the principal section of the crystal is inclined at 45° to the plane of polarisation, and it is in this case also that the colours of the first series attain their maximum brightness. If we represent the first series by $\sin^2(\pi V/\lambda)$, the second series in the case referred to will be represented by $\cos^2(\pi V/\lambda)$. It should be noticed that the colours of NEWTON'S rings seen by transmitted light are complementary to those seen by reflection; but the scale of colours is far more dilute than that obtainable as above with the aid of double refraction.

The colours of the first series are met with also in other optical experiments, *e.g.*, at the centre of the illuminated patch, when light issuing from a point passes through a small round aperture in an otherwise opaque screen.*

§ 3. In order to be able to calculate the colour of any given mixture of light, it is necessary to know the exact chromatic relations of the spectral rays themselves. This is precisely the question investigated by MAXWELL.† Selecting three rays as standards of reference, he expresses the colours of other rays in terms of them. The actual observations in all cases consisted in matches of two whites, one the original white which had not undergone prismatic analysis, the other a white compounded of three rays,—first of the three standard rays themselves, then of two standard rays in combination with a fourth ray which it was desired to express in terms of the standards. The auxiliary white was then eliminated.

The three points selected were at 24, 44, and 68 of the scale to which the spectrum was referred. "I chose these points, because they were well separated from each other on the scale, and because the colour of the spectrum at these points does not appear to the eye to vary very rapidly, either in hue or in brightness, in passing from one point to another. Hence, a small error of position will not make so serious an alteration of colour at these points, as if we had taken them at places of rapid variation; and we may regard the amount of the illumination produced by the light entering through the slits in these positions as sensibly proportional to the breadths of the slits.

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* Airy's *Tract on Optics*, § 79.

† "On the Theory of Compound Colours," *Phil. Trans.*, 1860.

"(24) corresponds to a bright scarlet about one-third of the distance from C to D; (44) is a green very near the line E; and (68) is a blue, about one-third of the distance from F to G."

A specimen observation is given:—

$$\text{"Oct. 18, J.}\quad 18{\cdot}5(24)+27(44)+37(68)=\text{W}\,.$$

This equation means that on the 18th of October the observer J (myself), made an observation in which the breadth of the slit X was 18·5, as measured by the wedge, while its centre was at the division (24) of the scale; that the breadths of Y and Z were 27 and 37, and their positions (44) and (68); and that the illumination produced by these slits was exactly equal, in my estimation as an observer, to the constant white W.

"The position of the slit X was then shifted from (24) to (28), and when the proper adjustments were made, I found a second colour-equation of this form—

$$\text{Oct. 18, J.}\quad 16(28)+21(44)+37(68)=\text{W}\,.$$

Subtracting one equation from the other, and remembering that the figures in brackets are merely symbols of position, not of magnitude, we find

$$16(28)=18{\cdot}5(24)+6(44)\,,$$

showing that (28) can be made up of (24) and (44), in the proportion of 18·5 to 6.

"In this way, by combining each colour with two standard colours, we may produce a white equal to the constant white. The red and yellow colours from (20) to (32) must be combined with green and blue, the greens from (36) to (52) with red and blue, and the blue from (56) to (80) with red and green."

The values employed in the present paper are those of MAXWELL's second observer K (whose vision in the region of the line F was more normal than his own)*, and are given in his table No. VI. For our purpose they require some extension, especially at the violet end. Thus the equivalents of (16), (84), (88), (92), (96), (100), are obtained by a graphical extrapolation from the curves given by MAXWELL. The adjoining table is deduced from his with some reduction, in order to exhibit the value, in terms of the three standards, of the illumination due to the unit width of slit in each case. It will be seen that the extrapolation at the upper end of the spectrum is necessary in order to make up anything like the full total of (68).

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* It is understood that K represents Mrs MAXWELL. In these matters a woman's observations are generally to be preferred to a man's, as less liable to irregularities of the kind described in *Nature*, Nov. 17, 1881.

TABLE I.

Scale.	Wave-length.	Colour.	(24)	(44)	(68)
16	2580	red	+·140	...	...
20	2450	red	·420	+·009	+·063
24	2328	scarlet	1·000	...	...
28	2240	orange	1·155	·360	−·006
32	2154	yellow	·846	·877	·005
36	2078	yellow-green	·484	1·246	·032
40	2013	green	+·127	1·206	−·008
44	1951	green	...	1·000	...
48	1879	bluish-green	−·063	·759	+·085
52	1846	blue-green	·055	·506	·282
56	1797	greenish-blue	·050	·340	·495
60	1755	blue	·047	·190	·753
64	1721	blue	−·033	·033	·905
68	1688	blue	...	...	1·000
72	1660	indigo	+·019	·006	·944
76	1630	indigo	·025	+·016	·693
80	1604	indigo	·005	−·028	·479
84	1580	...	...	...	·333
88	1560	...	...	...	·208
92	1540	...	...	...	·146
96	1520	...	...	...	·083
100	1500	...	...	...	·042
			+3·973	+6·520	+6·460

The colour produced by combining all the light which passed the prisms from (16) to (100) is the white of the apparatus. Its equivalent in terms of the standards is given by

$$W' = 3{\cdot}973(24) + 6{\cdot}520(44) + 6{\cdot}460(68).$$

It differs a little from the standard white of the original matches, *i.e.*,

$$W = 18{\cdot}6(24) + 31{\cdot}4(44) + 30{\cdot}5(68),$$

not only in consequence of omission of some extreme red and violet, but probably also on account of absorption by the prisms.

The colours of the spectrum were exhibited by MAXWELL in NEWTON'S manner, and are reproduced on our diagram (Plate X.), in which each colour is represented by the centre of gravity of three weights at the corners of an equilateral triangle, the magnitudes of the weights being taken proportional to the quantities of (24), (44), and (68) required to compound the colour, so that the corners themselves represent the standard colours.

The wave-lengths are given in FRAUENHOFER'S measure (in terms of the Paris inch).* The scale is such that for D, $\lambda = 2175$, and for F, $\lambda = 1794$.

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* 1 Paris inch = 2·7070 cm.

The fact that the spectrum colours lie, roughly speaking, upon two sides of the triangle (see Plate X.), indicates that all pure oranges and yellows can be made up by a mixture of pure red and pure green, and in like manner that all varieties of pure blue and blue-green can be compounded of pure violet and pure green. If, as there is reason to believe, the curve representing the spectrum is slightly rounded off at the green corner, this means that the *same* spectrum green is not available for both pure yellows and pure blues. The green lying most near the corner gives with red yellows, and with violet blues, which are somewhat less saturated than the corresponding colours of the spectrum.

TABLE II.

	$\mathrm{Sin}^2 \frac{\pi V}{\lambda}$		
	V=1846	V=3600	V=6800
16	·607	·896	·828
20	·490	·991	·420
24	·367	·980	·060
28	·275	·892	·013
32	·188	·737	·225
36	·118	·553	·572
40	·066	·379	·863
44	·028	·219	1·000
48	·003	·068	·865
52	·000	·024	·702
56	·007	·000	·391
60	·026	·026	·148
64	·052	·081	·023
68	·084	·164	·008
72	·119	·256	·089
76	·164	·373	·264
80	·209	·483	·467
84	·255	·589	·667
88	·297	·678	·817
92	·342	·762	·932
96	·389	·840	·994
100	·440	·904	·989

§ 4. The colours of thin plates are to be calculated in accordance with (2) from Table I., as white was calculated, but with introduction throughout of the factor $\sin^2(\pi V/\lambda)$. For each thickness of plate V is constant, but an integration over the spectrum is required. Table II. gives a specimen of the values of the factors, and may be considered to represent the brightness, at various points, of thespectrum that would be formed by analysing the light reflected. The three retardations given correspond to the reds of the first and second order, and to the green of the fourth order. In actual calculation these numbers would not occur (nor indeed those of Table I.), but would be represented by their logarithms.

From the necessity of determining a large number of points, the calculations ran to great length. They have not been performed throughout in duplicate, but have been so far re-examined as to exclude any error which could appreciably affect the diagram. In many cases neighbouring points verify one another to a sufficient degree of accuracy.

Table III.—*First Series.*

V.	(24).	(44).	(68).	V.	(24).	(44).	(68).
0	·77	1·65	2·28	5200	2·69	4·72	1·42
1006·5	3·82	6·46	5·87	5300	3·06	4·30	1·95
1300	3·75	5·07	2·79	5400	3·33	3·78	2·68
1500	3·01	3·20	·82	5600	3·51	2·75	4·15
1604	2·51	2·23	·27	5800	3·20	2·01	5·03
1688	2·04	1·49	·18	6000	2·53	1·77	4·93
1755	1·67	1·01	·27	6200	1·75	2·11	3·97
1846	1·20	·53	·69	6400	1·09	2·82	2·74
1951	·75	·26	1·63	6600	0·76	3·56	1·85
2013	·49	·26	2·25	6700	0·74	3·87	1·71
2154	·13	·67	3·81	6800	0·83	4·12	1·75
2328	·09	1·82	5·44	6900	1·00	4·26	1·99
2630	·99	4·44	5·87	7000	1·26	4·32	2·41
2927	2·59	5·95	3·37	7100	1·55	4·27	2·91
3100	3·29	5·77	1·71	7200	1·86	4·13	3·41
3300	3·78	4·68	·59	7400	2·45	3·69	4·17
3400	3·81	4·04	·58	7600	2·83	3·16	4·48
3500	3·74	3·17	·93	7800	2·93	2·76	4·02
3600	3·50	2·40	1·59	8000	2·72	2·56	3·24
3800	2·68	1·26	3·42	8200	2·35	2·62	2·45
4000	1·67	·93	5·08	8400	1·89	2·85	2·28
4200	·79	1·48	5·71	8600	1·52	3·17	2·52
4400	·29	2·68	5·02	8800	1·32	3·47	2·98
4600	·35	3·96	3·43	9000	1·34	3·65	3·69
4800	·91	4·91	1·86	9200	1·53	3·73	4·04
4900	1·33	5·13	1·31	9400	1·83	3·67	3·84
5000	1·79	5·17	1·03				

The final results, expressed as before in terms of the standards (24), (44), (68), are exhibited in Table III. In the first column are to be found the values of V (expressed in the same measure as λ). Thus, when V = 1688, the illumination vanishes at the point (68) on Maxwell's scale, for which λ = 1688. If the compound light reflected from a plate of this thickness were analysed by the prism, the centre of a dark band would be found at (68). Although the extinction is absolute at only one point, still the neighbouring region, which naturally contributes most of the colour-component (68), is very obscure, and thus the total of this component reaches only ·178, while the two other components are present in fair quantity. The resulting colour is a good orange.

As V increases, the dark band moves down the spectrum. When V = 1951, the centre of the band is at (44); thus nearly all the green is eliminated, and the colour is a rich purple. Again, when V = 2328, the centre of the band is at (24), the resulting colour is a rich blue. This band then moves out of the visible spectrum; but a new one presently makes its appearance, and begins to invade the spectrum from the violet end. When $V = 2 \times 1688$ or 3376, the ray (68) is again extinguished, and the colour is the yellow of the second order. For higher values of V, there may be two or more dark bands simultaneously, as appears in Table II., when V = 6800.

§ 5. Any sequence of colours may conveniently be represented on NEWTON'S diagram, in the manner adopted by MAXWELL for the particular sequence found in the spectrum. Such a curve would represent, for example, the colours of an absorbing medium, as the thickness traversed varies from nothing to infinity. In all such cases the curve starts from the point white, and ends at the point representative of that ray of the spectrum to which the medium is most transparent. For many coloured media the curve would not depart widely from a straight line ruled outwards from white to a point on one of the sides of the triangle. But when the medium is dichromatic, as for example a solution of chloride of chromium, the curve might start in one direction and ultimately come round to another. Thus in the case referred to the course of the curve from white would be towards the middle of the blue side of the triangle, then after a good progress in that direction it would bend round through yellow, and ultimately strike the triangle at a point near the red corner representative of the extreme visible rays at the lower end of the spectrum. The principal object of the present investigation was to exhibit in a similar manner upon NEWTON'S diagram the curve of the colours of thin plates. To find the point corresponding to the retardation 1688, we imagine weights proportional to the numbers 2·04, 1·49, ·18 to be situated at the three angular points of the triangle, and construct the centre of gravity of such weights. This point represents the colour due to retardation 1688.

§ 6. The diagram (Plate X.) embodies the results of Table III., so far as the *quality* of the effects is concerned. When the thickness, or retardation (V), is infinitely small, the amount of light reflected of course vanishes, but the *colour* approaches a limit, found by combining the constituents in quantities proportional to λ^{-2}, the limit of $\sin^2(\pi V/\lambda)$. This limiting blue of the first order would be the blue of the sky, according to the theory which attributes the light to reflection from thin plates of water in the form of bubbles. The blue of the sky is, however, really a much richer colour than this, and corresponds more nearly to that calculated on the supposition that the disturbance is due to spheres, or masses of other shape, small in all their dimensions relatively to the wave-lengths of light. According to this view, the colour is that found by taking the components of white light proportionally to λ^{-4}, instead of λ^{-2}.*

The curve, starting thus from a definite point, takes a nearly straight course in the direction of white (W′), which it passes a little upon the green side. The white of the first order on NEWTON's scale is thus somewhat greenish, as must obviously be the case when we consider that it arises when the maximum reflection is in the green or yellow portion of the spectrum, so that the red and blue must be relatively deficient; but the deviation from white is very small, and is not usually recognised. After leaving white the curve passes through the yellow, and approaches pretty close to the side of the triangle at a point representing the D-line in the orange.† The retardation is here 1688. The colour then reddens, but makes no approach to the spectrum reds lying near the corner of the triangle. Passing rapidly through the purple "transition-tint," it becomes bluer, until it attains the magnificent blue or violet of the second order, in the neighbourhood of $V = 2328$. At this.point there is a good approach to the corresponding spectrum colour, although the latter lies here a little outside the triangle. Leaving blue the colour rapidly deteriorates, becoming greener, but nowhere attaining a good green. The best yellow of the second order at 3400 is nearly as pure as the best of the first order, but inclines less to orange. The reds of the second order are even less pure than those of the first, but the inferiority diminishes as we approach the second transition-tint in the purple. The blue of the third order at 4200 is much inferior to the corresponding colour of the second order, but gradually acquires a superiority as it becomes greener near 4400. The blue-greens which follow, and the full greens from 4800 to 5000, are splendid colours, beyond comparison superior to the corresponding colours of the second order, but yet falling far short of the spectrum colours near (44). On the other hand, in the third order the yellows are not so pure as in the first and second orders, and there is even less approach to red, although a better show is made in the purple at 6000. In the transition from this purple to green, the blue falls short even of the blue of the first order, but the green at 6800 is very fine, sensibly equal to one of the greens of the third order. It will be remarked that in the fourth order greens there is little variety, the direction both on the outward and on the backward course being nearly in a line through white. On the return to white, which is

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* See several papers by the Author, published in the *Philosophical Magazine*, "On the Light from the Sky, its Polarisation and Colour," Feb. 1871, April 1871; "On the Scattering of Light by small Particles," June 1871; "On the Electro-Magnetic Theory of Light," August 1881, &c.

† The points 20, 24, 28, ... on the diagram, represent the spectrum colours as determined by MAXWELL.

very closely approached, a contrary curvature sets in, so that the earlier reds are more blue than the later. The curve then bends round on the yellow side of white, until it attains a rather feeble blue-green at 9000.

§ 7. It will be interesting to compare the diagram with descriptions by previous writers of NEWTON'S scale of colours. In his article on Light in the *Encyclopædia Metropolitana* (1830), Sir JOHN HERSCHEL says:—"The colours, whatever glasses be used, provided the incident light be white, always succeed each other in the same order; that is, beginning with the central black spot as follows:—

"First ring, or first order of colours,—*Black, very faint blue, brilliant white, yellow, orange, red.*

"Second ring, or second order,—*Dark purple or rather violet, violet, blue, green* (very imperfect, a yellow-green), *vivid yellow, crimson-red.*

"Third ring, or third order,—*Purple, blue, rich grass-green, fine yellow, pink, crimson.*

"Fourth ring, or fourth order,—*Green* (dull and bluish), *pale yellowish-pink, red.*

"Fifth ring, or fifth order,—*Pale bluish-green, white, pink.*

"Sixth ring, or sixth order,—*Pale blue-green, pale pink.*

"Seventh ring, or seventh order,—*Very pale bluish-green, very pale pink.*

"After these the colours become so pale that they can scarcely be distinguished from white.

"On these we may remark, that the green of the third order is the only one which is a pure and full colour, that of the second being hardly perceptible, and of the fourth comparatively dull and verging to an apple-green; the yellow of the second and third orders are both rich colours, but that of the second is especially rich and splendid; that of the first being a fiery tint passing into orange. The blue of the first order is so faint as to be scarce sensible, that of the second is rich and full, but that of the third much inferior; the red of the first order hardly deserves the name—it is a dull brick-colour; that of the second is rich and full, as is also that of the third; but they all verge to crimson, nor does any pure scarlet or prismatic red occur in the whole series."

HERSCHEL'S observations were made in the usual way with glass lenses,—a course convenient in respect of measurement of thicknesses, but incapable of doing justice to the colours, in consequence of the contamination with white light reflected at the upper surface of the upper plate and at the lower surface of the lower plate. The latter reflection should at any rate be got rid of by using a glass, either opaque, or blackened at the hind surface.

§ 8. For his description NEWTON used the soap-bubble, "because the Colours of these Bubbles were more extended and lively than those of the Air thin'd between two Glasses, and so more easy to be distinguished." He takes the colours in the reverse order, beginning with large retardations. I give his description as nearly as may be in his own words, but adapted to the more convenient notation followed by HERSCHEL :—

"The red of the fourth order was also dilute and dirty, but not so much as the former three; after that succeeded little or no yellow, but a copious green (fourth order), which at first inclined a little to yellow, and then became a pretty brisque and good willow-green, and afterwards changed to a bluish colour; but there succeeded neither blue nor violet.

"The red of the third order inclined very much to purple, and afterwards became more bright and brisque, but yet not very pure. This was succeeded with a very bright and intense yellow, which was but little in quantity and soon changed to green; but that green was copious and something more pure, deep and lively than the former green. After that followed an excellent blue of a bright sky colour (third order), and then a purple, which was less in quantity than the blue, and much inclined to red.

"The red of the second order was at first a very fair and lively scarlet, and soon after of a brighter colour, being very pure and brisque, and the best of all the reds. Then after a lively orange followed an intense bright and copious yellow, which was also the best of all the yellows; and this changed first to a greenish-yellow and then to a greenish-blue; but the green between the yellow and the blue was very little and dilute, seeming rather a greenish-white than a green. The blue which succeeded became very good, and of a very fair bright sky-colour, but yet something inferior to the former blue; and the violet was intense and deep, with little or no redness in it, and less in quantity than the blue.

"In the last red appeared a tincture of scarlet next to violet, which soon changed to a brighter colour, inclining to an orange; and the yellow which followed was at first pretty good and lively, but afterwards it grew more dilute, until by degrees it ended in perfect whiteness."*

§ 9. Some small discrepancies in the descriptions of NEWTON and HERSCHEL probably depend upon ambiguities in the use of colour names. In the rings of high order what NEWTON calls blue, HERSCHEL describes as bluish-green. Both observers remark upon the poverty of the green of the second order, but the diagram shows that it is superior to that of the fifth order. Neither NEWTON nor HERSCHEL seem to have done full justice to the green of the fourth order, which at its best rivals closely the corresponding colour of the third order. My own observations are in accordance with the teaching of the diagram, which shows, moreover, that as we depart from retardation 6800 the colour of the fourth order rapidly deteriorates by admixture with white, while the colours of the third order in the neighbourhood of 4800 retain their purity as they change in hue.

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* Newton's *Opticks*, 1704, book ii. p. 21.

One discrepancy between the diagram and the above descriptions will at once strike the reader. According to the diagram, the red and purple of the first order are superior to those which follow, whereas HERSCHEL says that the red of the first order hardly deserves the name. Judged by the standard of the spectrum red at (24), this criticism would apply to them all; but the question is as to the relative merits of the various reds. The explanation depends upon considerations of brightness, of which the curve takes no account. If we refer to Table III., we see that at 1846 the red component is 1·20, but that at the corresponding point for the red of the second order (between 3600 and 3800) it rises to about 3·0. The deficiency of brightness in the first order goes a long way by itself to explain the apparent inferiority, for dark red gives rather the impression of brown; but if there is the slightest admixture of white light, the comparison is still more unfair. It would be useless, for example, to take the colours from an air-plate between lenses. The feebly luminous red of the first order is then drowned in a relatively large proportion of white light, which tells much less upon the brighter, though less pure, red of the second order. This complication does not arise when soap-films are employed, and the red of the first order is evidently much improved; but the rapidity of transition at this part of the scale renders observation difficult. The best comparison that I have been able to make is with the aid of a beautiful mica combination kindly lent me by Rev. P. SLEEMAN. When this is examined in a dark room between crossed nicols, and lighted brilliantly from a part of the sky near the sun, the red of the first order is seen in great perfection, and I had no difficulty in believing it to be superior to that of the second order. It is not very easy to bring the rivals into juxtaposition under equal brightnesses; but there is, I think, no reason to doubt that the first order would come off victorious. The composition of the lights will be understood by reference to Table II.

§ 10. The only colours which can be said to make any approach to spectrum purity are the yellows of the first two orders, and the blue and green-blue of the second and third orders respectively. There is a corresponding difficulty in obtaining good greens by absorption. To do so it is necessary that the transmitted spectrum should terminate at two pretty well-marked points; in the case of red the difficulty is much less, all that is requisite being that the transmission should increase rapidly as the refrangibility of the light diminishes.

Besides the absolute brightness, there are two other circumstances which may influence the estimation of the colours of thin plates as normally presented. It is probable that in some cases the colours are much affected by contrast with their neighbours. To this cause we may attribute the difficulty in observing the transition between the reds and blue-greens of the fourth and higher orders. As the nearly neutral transition-tint is approached from either side, the effect upon the eye is improved by contrast, so as largely to compensate for the increasing poverty of the real colour. Much, again, depends upon the rapidity with which differences occur with varying retardation. When NEWTON speaks of the yellow of the second order as copious, he refers (I imagine) rather to the width of the band than to the brightness of the light. The diagram gives important information on this subject also. Compare, for example, in the first order, the change from 1500 to 1755, with that from 1755 to 1846 or 1951. The rapidity of the change in the latter interval is the foundation of the usefulness of the "transition-tint" in polarimetric work. If we wish to compare the rates of progress in different orders, we must distinguish according as we contemplate sensitiveness to small absolute, or to small relative, variations of retardation.

§ 11. The points of intersection of the curve are of interest, as corresponding to colours obtainable with two different thicknesses. The first that presents itself is the yellow, common to the first and second order. The table shows that the latter is the brighter. In the second and third orders the similar colours differ but little in brightness. One occurs in the blue and another in the greenish-yellow. Nor is there much difference of brightness between the otherwise nearly identical greens of the third and fourth orders. It follows that if observers are able to distinguish in all cases which order of colours they are dealing with, it must be by reference to a sequence, rather than by estimation of a single colour.

§ 12. With respect to the absolute retardations or thicknesses at which the various colours are formed, careful observations have been made by REINOLD and RÜCKER.* For comparison with their results I will take the green of the fourth order at 6800. In air at perpendicular incidence, this answers to a thickness of $3{\cdot}40 \times 10^{-5}$ Paris inches, or $9{\cdot}19 \times 10^{-5}$ cm. The numbers in their Table (p. 456), Column V., are

Green, ...	8·41
. . . .	8·93
Yellow-green, ...	9·64

so that the agreement is pretty good. I would remark in passing that the diagram does not recognise a yellow-green of this order; but the appearance of such may perhaps be explained by contrast.

§ 13. The series of colours complementary to those of Table III. are found by subtraction of the numbers there given from those representative of white, viz., 3·97, 6·52, 6·46, respectively. The resulting numbers are exhibited in Table IV., in which the first entry for zero retardation corresponds to the full white.†

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[*] "On the Electrical Resistance of Thin Liquid Films, with a Revision of Newton's Table of Colours," *Phil. Trans.*, 1881.

[†] In comparing with Table III., it should be remembered that the numbers there given under the head of $V = 0$ are relative only, the true values being infinitely small.

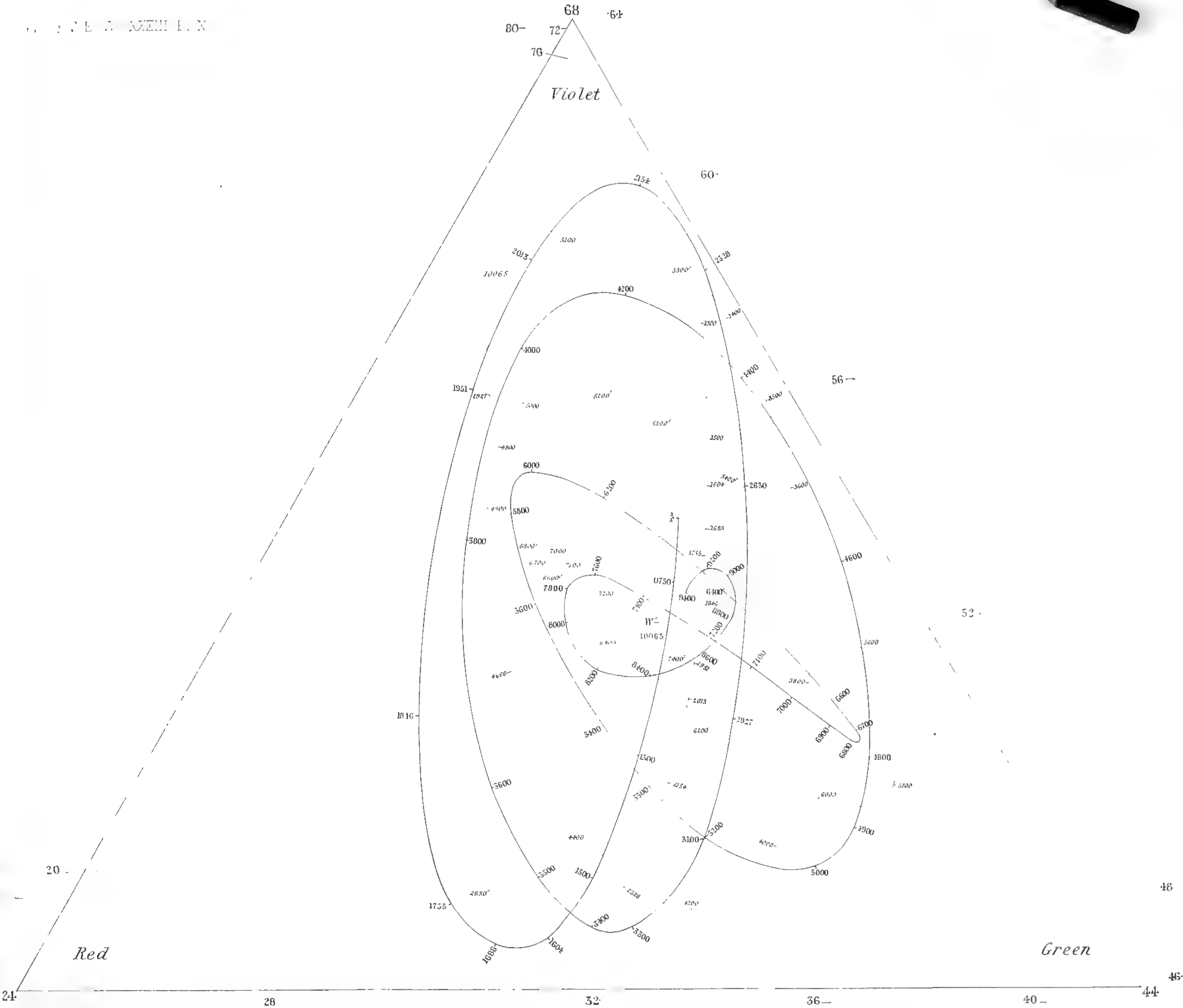
TABLE IV.—*Second (Complementary) Series.*

V.	(24).	(44).	(68).	V.	(24).	(44).	(68).
0	3·97	6·52	6·46	5200	1·28	1·80	5·04
1006·5	·15	·06	·59	5300	·91	2·22	4·51
1300	·22	1·45	3·67	5400	·64	2·74	3·78
1500	·96	3·32	5·64	5600	·46	3·77	2·31
1604	1·46	4·29	6·19	5800	·77	4·51	1·43
1688	1·93	5·03	6·28	6000	1·44	4·75	1·53
1755	2·30	5·51	6·29	6200	2·23	4·41	2·49
1846	2·77	5·99	5·77	6400	2·89	3·70	3·72
1951	3·22	6·26	4·83	6600	3·22	2·96	4·61
2013	3·48	6·26	4·21	6700	3·23	2·65	4·75
2154	3·84	5·85	2·65	6800	3·14	2·40	4·71
2328	3·88	4·70	1·02	6900	2·97	2·26	4·47
2630	2·98	2·08	·59	7000	2·72	2·20	4·05
2927	1·38	·57	3·09	7100	2·42	2·25	3·55
3100	·68	·75	4·75	7200	2·11	2·39	3·05
3300	·19	1·84	5·87	7400	1·52	2·83	2·29
3400	·16	2·48	5·88	7600	1·14	3·36	1·98
3500	·23	3·35	5·53	7800	1·04	3·76	2·44
3600	·47	4·12	4·87	8000	1·25	3·96	3·22
3800	1·29	5·26	3·04	8200	1·63	3·90	4·01
4000	2·30	5·59	1·38	8400	2·08	3·67	4·18
4200	3·19	5·04	·75	8600	2·45	3·35	3·94
4400	3·68	3·84	1·44	8800	2·65	3·05	3·48
4600	3·62	2·56	3·03	9000	2·63	2·87	2·77
4800	3·06	1·61	4·60	9200	2·44	2·79	2·42
4900	2·64	1·39	5·15	9400	2·15	2·85	2·62
5000	2·18	1·35	5·43				

The curve representative of this series of colours on NEWTON's diagram is given by the dotted line in the Plate, so far as the tabulated numbers permit. It starts from the point White, and passes rapidly through a whitish-yellow to a very dark red and purple at V = 1006·5. This part of the curve can not be drawn from the tabulated data,—a defect of no great consequence, for the quantity of light being so insignificant, its quality is of little interest. From V = 1300 onwards the curve is pretty well determined.

It will be seen that the two series of colours are of pretty much the same general character. The green at 5800 in the second series compares favourably with the greens of the third and fourth orders in the first series.

[illegible]



Archibald & Peck Engravers Edin.

VII.—*On the Electrical Properties of Hydrogenised Palladium.* By CARGILL G. KNOTT, D.Sc. (Edin.), F.R.S.E., Professor of Physics, Imperial University, Tokayo, Japan. (Plate XI.)

(Despatched to Royal Society of Edinburgh, May 25, 1886. Read 19th July 1886.)

In the following paper I desire to place on record the results of certain experiments which have lately engaged my attention. The facts established are, so far as I am aware, novel and in themselves interesting.

Many of the physical properties of hydrogenised palladium or hydrogenium have been carefully studied by GRAHAM, DEWAR, and others; but no one seems to have called attention to its thermoelectric peculiarities, or to have made a special study of its electrical resistance. These two inquiries form the subject of this paper. Throughout I shall use, for brevity's sake, the name Hydrogenium, which was applied by GRAHAM to the fully-saturated form. Here, however, it is applied generally to any alloy of the two substances, without any regard to a possible chemical compound of definite molecular constitution. The paper naturally divides itself into two sections—the first part relating to the electrical resistance, the second part to the thermoelectric properties.

ELECTRICAL RESISTANCE OF HYDROGENIUM.

The steady increase of the resistance of hydrogenium with the charge of hydrogen was noticed by DEWAR;[*] and further details were given by myself in a short paper published a few years ago.[†] There I obtained a resistance of 1·518 for fully-saturated hydrogenium, of which the originally pure palladium wire had a resistance of unity. In the present inquiry I have easily obtained a much greater increase of resistance—such as 1·634, 1·7775, and even as much as 1·83. Whether this may be a result of impurities being present in the acid which was used as the electrolyte, I cannot say. It may be noted, however, that the palladium wire itself was obtained in Paris, and was guaranteed to be very pure indeed.

The main purpose of the present investigation was to study the temperature characteristics of the resistance of hydrogen-charged palladium. Throughout each series of experiments the same palladium wire was used, the hydrogen

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[*] *Trans. Roy. Soc. Edin.*, vol. xxvii.

[†] *Proc. Roy. Soc. Edin.*, 1882–83.

being added in small successive doses. Not till the maximum saturation was reached was the wire subjected to any excessive heating, such as is generally supposed to be necessary to drive the hydrogen out. The temperature was regulated by means of an oil-bath, into which the wire, firmly bound to the ends of stout copper rods, dipped along with the thermometer which measured the temperature. The heating was applied gently by means of a spirit-lamp.

In the first series of experiments the temperature was raised gradually to about 300° C.; and in these experiments the loss of hydrogen was beautifully shown in the manner by which the resistance began to *decrease* at a temperature of about 260° C. A detailed description of two of the dozen experiments made will suffice, as all have almost exactly the same characteristics. I quote verbatim from my experimental book, just as the entries were made after the completion of the experiment.

Experiment made on January 27, 1886.

Resistance of hydrogenium at $10^\circ{\cdot}2$ C. $= 93{\cdot}6$ (10^{-2} ohm)
(the numbers observed are omitted).

Description of Results.—Up to temperature 175° C., the resistance of hydrogenium grows at a steady constant rate $= {\cdot}203$ per degree centigrade; or to that temperature

$$R = 91{\cdot}4 + {\cdot}203\,t$$

for this particular specimen; or, more generally, the resistance of a specimen of hydrogenium, whose resistance is 1 ohm at 0° C., is given by the formula

$$r = 1 + {\cdot}00222\,t.$$

The rate of increase then begins to increase slowly till 220° C. is reached.

From 220° to 260° the resistance remains practically steady, varying through a range of 1 in 140—that is, 0·7 per cent.

From 260° to 280° the resistance falls off very rapidly, attaining its maximum rate of decrease at 274°. The rate is then ·96 approximately.

At 280° the rate of decrease markedly diminishes, and seems to tend to evanescence till the highest temperature (295°) is reached.

From this temperature down again to the ordinary atmospheric temperature the resistance diminishes at a steady and almost constant rate, namely, 202 per degree centigrade; or

$$R = 63{\cdot}56 + {\cdot}202\,t$$

or, reduced as above,

$$r = 1 + {\cdot}00318\,t.$$

It is striking that the rates of change of the palladium in its two states are such that their *total* changes through a large range of temperature are the same.

Experiment of February 4, 1886.

Resistance of hydrogenium at 7° C. $=119{\cdot}3$ (10^{-2} ohm).

Heating Curve—

$$\text{From } 0° \text{ to } 140° \quad d\mathrm{R}/dt = {\cdot}194\,,$$

and

$$\mathrm{R} = 118 + {\cdot}194\,t,$$

or

$$r = 1 + {\cdot}00165\,t\,.$$

From 140° to 260°, the resistance grows to a maximum, such that $d\mathrm{R}/dt$ changes continuously, increasing to a maximum (·385) at temperature 225°±5°, and then diminishing to zero.

At 260° $d\mathrm{R}/dt$ changes sign and continues to increase numerically very rapidly, so that at 300° the resistance has fallen from 282·5 to 136. This means a very rapid rate of change at about 295°, approximately equal to 5.

The cooling curve gives, as average value,

$$d\mathrm{R}/dt = {\cdot}219$$

and

$$\mathrm{R} = 66{\cdot}5 + {\cdot}219\,t\,,$$

or

$$r = 1 + {\cdot}0033\,t\,.$$

These descriptions, with the accompanying curves (A) on Plate XI., are sufficient to bring out the peculiarities of the case. The various experiments, made with wires of different charge, gave very similar results. From these it appears that, up to a temperature of 150° C., hydrogenium of all degrees of charge behaves like palladium, except that the temperature-coefficient is generally, and always for the higher charges, smaller than for pure palladium. At higher temperatures the resistance seems to grow at a more rapid rate, and this peculiarity is more marked for the more highly charged metal. This is shown also in the fact that the difference between the lowest and highest resistances is greater for the more strongly hydrogenised wire. A little above 200°C., the hydrogen begins to escape, the first effect being that the resistance increases more and more slowly till it reaches a maximum. Diminution of resistance then sets in, sometimes with great rapidity, so that before 300° is reached the resistance has fallen nearly, if not quite, to what its value would be at that temperature for the original palladium. The rate at which this diminution sets in is of course a function of the time as well as of the temperature. I believe that, if the wire were kept at a steady temperature of 260°, or even lower, it would ultimately lose all its hydrogen. Observations on the change of resistance give, indeed, the most delicate means of studying the manner in which the hydrogen escapes, and would well repay a careful investigation.

These earlier experiments were made with a view to establish the broad features of the case. They suggested, however, various lines of further and more careful inquiry, of which one has just been mentioned. Another is obviously a following up of the remark made at the end of the description of the experiment of 27th January, which was indeed the very first of the series. The problem, expressed in its generality, is, What relation, if any, exists between the temperature-coefficient of resistance and the charge of hydrogen present? At first sight there is a tendency for the *total* change for a given rise of temperature to remain the same whatever charge of hydrogen is present. That is if we use throughout the whole series of experiments the same *wire* at different saturations, and draw curves of the resistance (as measured) in terms of the temperature, we shall obtain a family of curves which in their initial portions run parallel to each other, the lowest curve being that of pure palladium, the highest that of saturated hydrogenium. A closer study of the various experiments showed that this relation did not strictly hold; but before anything definite could be obtained, it was necessary to make a series of careful experiments with this special object in view. The results of these later experiments I shall now give, comparing them when possible with the results of the earlier series.

A palladium wire was taken of resistance ·927 ohms at 18° C. Its resistances at different temperatures up to 110° or so were carefully measured in an ordinary Wheatstone bridge. It was then charged with a small charge of hydrogen, and the same process of measurement of resistance gone through, and so on, with necessary additions of hydrogen, till the wire became saturated with the gas.

From the observations, the values of the resistance for each wire were interpolated so as to correspond to the temperatures 18°, 28°, 38°, &c. The subjoined table gives these interpolated values for six different hydrogeniums besides the pure palladium itself. In one experiment four terms only appear. This resulted from the breaking of the large glass beaker in which the wire was being heated:—

TABLE *showing Resistances in Ohms at various Temperatures of Palladium and Hydrogenium.*

Temperature.	Palladium.	I.	II.	III.	IV.	V.	VI.
18° C.	·927	·991	1·051	1·175	1·306	1·402	1·514
28°	·958	1·027	1·089	1·207	1·342	1·430	1·547
38°	·990	1·062	1·123	1·242	1·376	1·464	1·578
48°	1·022	1·098	1·157	1·273	1·410	1·497	1·611
58°	1·053	1·134	1·190	1·309	...	1·530	1·645
68°	1·084	1·171	1·225	1·346	...	1·563	1·677
78°	1·116	1·208	1·260	1·381	...	1·597	1·712
88°	1·147	1·244	1·296	1·418	...	1·632	1·746
98°	1·176	1·279	...	1·453	...	1·669	1·780
108°	1·206	...	...	1·491	...	1·700	1·813
118°	...	...	...	...	...	...	1·847

If a table of first differences is formed from these numbers, it will be found that all the hydrogeniums have the first difference nearly constant throughout, whereas in the palladium itself the first difference distinctly diminishes as the temperature rises.

The following table gives the mean of the successive differences for each specimen :—

Palladium.	I.	II.	III.	IV.	V.	VI.
·030	·036	·035	·035	·035	·033	·033

The gradual decrease in the values for the hydrogeniums as the charge of hydrogen increases is so regular, that it is difficult to regard it as accidental. Also the distinctly smaller value for the pure palladium is a significant fact; although it must be remembered that it is a mean of a series of steadily diminishing values, whose greatest value is nearly ·032. The others again are, as already pointed out, means of values which must be regarded as practically constant throughout the whole range.

The quantity, however, which should receive our closest attention is, as I have pointed out in my previous paper on the resistance of nickel at high temperatures, not $d\mathrm{R}/dt$, but $\mathrm{R}^{-1}d\mathrm{R}/dt$. It will be sufficient at present to form this quantity from the series of first differences, by dividing each by the mean of the resistances whose difference it is. The sanction for this simple mode is the "straight-linedness" of the numbers throughout. The following table shows these "logarithm-rates" arranged opposite the interpolated means of the temperatures of former table of resistances :—

TABLE *showing the Values at different Temperatures of the "Logarithm Rate"* ($R^{-1}dR/dt$) *for Palladium and Hydrogenium.*

(For convenience of tabulating, the numbers are multiplied by 10^4.)

Temperature.	Palladium.	I.	II.	III.	IV.	V.	VI.
23° C.	34	36	35	27	28	20	21
33°	33	33	31	28	25	23	20
43°	32	33	30	25	24	22	21
53°	30	33	28	28	...	22	21
63°	29	32	29	28	...	22	19
73°	30	31	28	25	...	21	21
83°	27	30	29	27	...	22	20
93°	25	27	...	25	...	22	19
103°	25	...	...	26	...	19	18
113°	...	...	...	...	...	...	18

If means are taken for the first three numbers in all the columns, and then means of the next four, a condensed table will be obtained, which may be regarded as giving fairly approximate values for the logarithm rates at temperatures 33° and 68° These are as follows:—

	Mean Values of $\frac{1}{R}\frac{dR}{dt}$ at	
	33°	68°
Palladium, . .	33	29
Hydrogenium I.	34	32
„ II.	32	28
„ III.	27	27
„ IV.	26	...
„ V.	22	22
„ VI.	21	20

These numbers bring out very clearly the fact that the first effect of adding hydrogen to palladium is to make the resistance of the wire somewhat more sensitive to changes of temperature, but that this greater sensitiveness soon disappears as more and more hydrogen is added. In the saturated condition, hydrogenium resembles other alloys in having a temperature-coefficient for change of resistance which is less than for pure metals.

This conclusion regarding the first effect of adding hydrogen is borne out by the results of the earlier series of experiments, which, though not having the same claims to accuracy, are now given for purposes of comparison. In the columns headed R'/R are tabulated the ratios of the hydrogenium wire resistances to the resistance of the wire in its pure palladium condition; and in the columns headed a'/a are tabulated the ratios of the corresponding temperature coefficients as given by the formula

$$R'_\theta = R'(1+a\theta),$$

where θ is the temperature measured in degrees of the centigrade scale—the zero of temperature reckoning being the temperature at which R′ is the resistance. In the earlier experiments this zero of reckoning is the centigrade zero; in the later experiments 33° and 68° C.

Comparison of Temperature Coefficients for different Specimens of Hydrogenium.

Earlier Series. 0° C.		Later Series. 33° C.		68° C.	
R′/R	α'/α	R′/R	α'/α	R′/R	α'/α
1·06	1·09	...	...	...	...
1·08	1·06	1·07	1·03	1·08	1·10
1·14	1·02	1·13	·97	1·13	·97
1·17	1·00	1·26	·82	1·24	·93
1·39	...	1·39	·79	...	...
1·47	·72	1·48	·67	1·44	·76
1·56	·76	1·6	·64	1·55	·69
1·78	·50	...	...	...	...

The most curious point established in these experiments seems to be that, to a fair approximation, the total change of resistance in a given palladium wire charged with hydrogen, due to a given change of temperature, is independent of the amount of hydrogen present. Thus, if we form the product of each pair of corresponding ratios in the table just given, we shall obtain for each column a series of values differing in no case from their mean by more than 6 or 8 per cent., with the single exception of the last pair in the first column. These means are respectively 1·11, 1·07, and 1·12. It is not unity, so that the pure palladium does not quite fall in with the hydrogeniums.

Another mode of expressing the fact here indicated is to say that, at any temperature below 150° C., the increase in resistance of a given palladium wire is a function simply of the amount of hydrogen taken in. This mode of regarding the phenomenon suggested the inquiry, Does the rate of in-take of hydrogen, or the total amount that can be absorbed, depend upon the temperature of the electrolyte? A direct experiment was tried by connecting two electrolytic cells in series with the source of current, and using, as negative electrodes in these cells, two equal portions of the same palladium wire. The liquid in the one cell was kept at a steady temperature of 90° C., while the other was at the ordinary temperature of the room, about 18° C. No difference, however, in the rates of charging, or in the final charge, was observed.

The peculiarity in the change of resistance above 150° C., and below the temperature at which loss of hydrogen sets in, is reserved for a further discussion; that is, if further experiments reveal anything new. The nature of the peculiarity is indicated in the curves, and has been already sufficiently touched upon.

The Thermoelectric Properties of Hydrogenium.

So far as I am aware, this subject has never been attacked by any experimenter. My first inquiry was, therefore, merely as to the existence of a thermoelectric current between pure palladium and hydrogenised palladium. I quite expected to find such a current, but was very much surprised at its magnitude. I cannot do better than quote the whole of the first experiment from my experimental book.

Resistance of palladium wire before hydrogenisation—

= ·64 ohms.

Resistance of same wire after hydrogenisation—

= ·99 ohms.

The hydrogenium was then bound to a palladium wire, put in circuit with a galvanometer, and the palladium-hydrogenium junction gradually heated in oil up to 300° C., and then allowed to cool. The galvanometer was then gauged by means of a standard Daniell, whose electromotive force was assumed to be 1·1 volts.

The temperatures, with the corresponding deflections, as given in the galvanometer scale, are as follows. The cold junction varied from 6°·2 to 8° C. throughout the experiment:—

Heating.		*Cooling.*	
Temperature.	Deflection.	Temperature.	Deflection.
38° C.	29·3	300° C.	218
67°	66	295°	191
96°·5	95·5	290°	185
121°	126	285°	179
150°	159	280°	171
182°	183	275°	167
200°	213	270°	161
220°	222	265°	157
235°	224	260°	149
240°	229	255°	145
245°	220	250°	138
250°	220	240°	130
255°	218	235°	125
260°	218	230°	122
265°	220	220°	111
270°	220	210°	101
280°	219	200°	94
285°	218	175°	80
290°	214	170°	75
295°	220	155°	71
300°	218	120°	56
...	...	90°	40

From these numbers the following facts were deduced:—

If e is the electromotive force expressed in volts, and t the temperature in degrees centigrade, it is found that—

Up to 200° C., temperature rising,

$$e = 1{\cdot}67 \times 10^{-5} \times 1{\cdot}11(t-8)\,.$$

From 200° to 300° C., e is practically steady.

From 300° to 200°, temperature falling,

$$e = 1{\cdot}67 \times 10^{-5}(-105{\cdot}5 + {\cdot}945t + {\cdot}000313t^2)\,.$$

From 200° to temperature of air,

$$e = 1{\cdot}67 \times 10^{-5} \times {\cdot}47(t-6)\,.$$

Hence we find, in C.G.S. units,

$$\begin{array}{lll}
(1) & \dfrac{de}{dt} = 1853 & \text{from } 0^\circ \text{ to } 200^\circ \text{ C. heating.} \\
(2) & \dfrac{de}{dt} = 785 & \text{from } 200^\circ \text{ to } 0^\circ \text{ C. cooling.} \\
(3) & \dfrac{de}{dt} = 0 & \text{from } 200^\circ \text{ to } 300^\circ \text{ C. heating.} \\
(4) & \dfrac{de}{dt} = 1578 + 1{\cdot}044t & \text{from } 300^\circ \text{ to } 200^\circ \text{ C. cooling.}
\end{array}$$

Finally, assuming the palladium to be the same as that investigated by TAIT, we find for the thermoelectric powers of hydrogenium, in conditions (1) and (2), referred to lead (as in EVERETT's *Units and Physical Constants**) the expressions—

$$(1)\quad p = 1128 - 3{\cdot}59t\,.$$
$$(2)\quad p = 160 - 3{\cdot}59t\,.$$

Hence, on the thermoelectric diagram the hydrogenium line is something like this. It begins near the iron line, runs parallel to the palladium line till 200° C. is reached, when it falls somewhat quickly to the palladium line, which it hugs up to 300° C. During cooling, it seems to start from the point it would have occupied had its course remained unchanged during the whole heating. From thence it runs at a less inclination than the palladium line until the temperature of 200° C. is reached, after which it remains parallel to the palladium line down to ordinary temperatures, and comes out a little below copper at 0° C.

Adopting TAIT's values as given by EVERETT for iron, copper, and

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* The signs are here changed so as to agree with TAIT's theory, which connects the inclinations of the thermoelectric lines with the THOMSON effects in the corresponding metals. I shall always speak of iron as lying *above* lead, and palladium as *below* lead, on the thermoelectric diagram.

palladium, we may indicate the peculiarities of the hydrogenium by means of the following table of thermoelectric powers (t in centigrade degrees):—

Iron,		$1734 - 4{\cdot}87t$
Copper,		$136 + {\cdot}95t$
Palladium,		$-625 - 3{\cdot}59t$
Hydrogenium	(0° − 200°),	$+1128 - 3{\cdot}59t$
〃	(200° − 300°),	$-625 - 3{\cdot}59t$
〃	(300° − 200°),	$+1578 - 2{\cdot}55t$
〃	(200° − 0°),	$+160 - 3{\cdot}59t$

It must be understood, of course, that these equations do not strictly hold at these temperatures which separate the one group from the other; at these points there are continuous although rapid transitions which baffle an arithmetic representation.

The peculiarities here indicated may be explained as due to the escape of the hydrogen during the heating and to its partial return during the cooling. Only the one extremity of the hydrogen-charged wire is heated, so that only from that portion will the hydrogen escape to any marked extent. Whether it escapes wholly out of the wire, or is partly driven into the contiguous colder portions, is not certain. The latter possibility is far from improbable, if the wire is undercharged to begin with. Whatever may be the case, however, it is obvious that at about 200° C. the portion of wire immersed in the hot oil begins to lose its occluded hydrogen. The thermoelectric system that now exists is of such a complicated nature that it would be difficult, perhaps impossible, to predict what should happen. There is a pure palladium wire joined in circuit with a hydrogenised palladium wire, whose charge as well as temperature varies continuously from the one extremity to the other. Supposing, as I shall establish later, that the thermoelectric position of hydrogenium is a function of the charge, we have to do with a chain of elements of continuously varying thermoelectric power. Adding to this the further complication that there is a time variation of both temperature and charge, we can scarcely expect to be able to prejudge the phenomenon. The mere fact of a *variation* of charge may —for ought we know to the contrary—bring into play an electromotive force of other than purely thermal origin. As the heating changes to cooling, the electromotive force which had remained so constant since 200° begins to fall away rapidly, but with diminishing rapidity as the temperature falls. There certainly seems to be an instability in the condition of the hydrogen during this cooling process, for just as the temperature below which hydrogenium was stable during the heating the thermoelectric properties recover their original ordinary characteristics. The final position of the hydrogenium line, after the cooling, shows that there has been a loss of hydrogen at the junction; but, as was proved by resistance measurement afterwards, a very small amount of hydrogen was lost to the wire as a whole—not more than 5 or 6 per cent.

The thermoelectric power of the palladium and hydrogenium couple, as shown by this first experiment, was so large as to suggest investigating the properties of the hydrogenium by coupling it with some other metal, such as platinum or copper. The use of the palladium itself is besides open to the possible objection that the passage of a current between it and hydrogenium may cause a transfer of hydrogen. There was, however, no positive evidence of such a possibility.

In the next experiments to be described a triple junction was formed of platinum, palladium, and hydrogenium. Rapidly alternating readings were taken of the platinum-palladium and platinum-hydrogenium electromotive forces, which were estimated in C.G.S. units. In this way the hydrogenium and palladium were directly compared. The following are the results of one of the experiments, the same wire being used as in the experiment already described. By the addition of a little more hydrogen the resistance was raised to 1·012 ohms.

Comparison of Electromotive Forces of Palladium (P*t*-P*d*) *and Platinum-Hydrogenium* (P*t*-H*d*) *at different Temperatures.*

The numbers are given in 10^{-4} of the C.G.S. units.

Heating.		*Cooling.*	
P*t*-P*d*.	P*t*-H*d*.	P*t*-P*d*.	P*t*-H*d*.
+ ·19	− ·56	122·5	102·3
2·06	− 3·36	120·1	98·5
5·57	− 6·82	117·6	94·8
9·51	− 10·86	108·1	90·3
11·9	− 10·7	99·4	84·5
12	− 9·33	93·7	79·6
18·5	− 7·4	89	74·2
23	+ ·9	75·6	62·5
27·5	+ 8·26	60·7	48·6
50	31·3	57·6	46
62·1	41·3	54·5	43·7
66·7	47·2	41·8	32·7
81·6	61·7	37·2	29·8
86·1	67·9	34·1	27
98·5	77	32·6	25·4
103·9	82	28·2	21·5
114·7	95·2	25·5	20
122·5	102·3	19·5	14·9
		17·4	13·5
		11·9	9·3
		8·3	6·1

In this experiment the triple junction was enclosed in a small porcelain tube and heated in a charcoal furnace. The temperature of 200° C. corresponds approximately to the value 16 in P*t*-P*d* column, and temperature 300° C. to the value 25. The curves marked B in Plate XI. bring out the peculiarities to the eye.

If we try to trace the hydrogenium line on the thermoelectric diagram from this curve, we shall get rather a curious result, which is shown roughly in the small diagram (C) in the upper left-hand corner of Plate XI. A glance at the electromotive force curve indicates, indeed, that the ratio of the thermoelectric powers of hydrogenium and palladium referred to platinum begins with a negative value greater than unity, passes through zero at about 150° C., becomes equal to positive unity at about 200° C., and attains a value *greater than unity* as the temperature passes through 300° C. This may be explained in two ways. It may be due to an electromotive force other than thermal, brought into existence by the variation of charge or a possible convection of hydrogen along the wire. Or, it may be due to the integral electromotive force of a row of hydrogenium elements of varying charge and temperature being a function of the space-rate of either variation. Ultimately the ratio of the thermoelectric powers becomes a little less than unity, and continues so until the temperature reaches its highest value. The cooling curve belongs to a diagram line which lies between palladium and platinum, dividing the space in the ratio of 1 to 4. The main features are identical with those of the first experiment, the differences in detail being the result of the much higher temperatures employed, in virtue of which more hydrogen is lost to the wire. In the small thermoelectric diagram the palladium line is drawn parallel to the platinum line, although they are really inclined at an angle which would make them meet at a point corresponding to −600° C. The irregularities due to the instability of the hydrogen appear in the thermoelectric experiments at a lower temperature than in the resistance experiments. This may well be due to the inequality of temperature distribution along the wire in the former case.

It now remained to make a study of the thermoelectric properties of hydrogeniums of intermediate charges. The object aimed at was to obtain careful determinations at temperatures below those at which arise the irregularities due to the instability of the hydrogen. I confined myself, therefore, to temperatures below 100° C. The arrangement and reduction of results, which present some novelties of operation, were as follows:—

A triple junction of platinum, pure palladium, and hydrogenium was immersed in oil, along with a centigrade thermometer, which, however, was used rather as an indicator than an accurate measurer. The platinum-palladium circuit was indeed the real thermometer; and in terms of its electromotive force the electromotive force of the platinum-hydrogenium circuit was obtained. The resistance of each specimen of hydrogenium was measured before and after the electromotive force measurements. The mean of these two resistance measurements, which were usually the same, and differed only in one case by as much as 1 per cent., may be regarded as a fair indicator of the amount of hydrogen present in the particular specimen. The same palladium wire was used throughout as the basis for the successive hydrogeniums.

The resistances were the same in both circuits, so that the galvanometer deflections were proportional to the electromotive forces. The circuits could be thrown on to the galvanometer in either direction and in rapid alternation. The sequence in which the readings were taken was as follows:—Let a, b, symbolise the readings of the two circuits, and let $+$ or $-$ be prefixed to indicate the direction in which the current flowed through the galvanometer. Then the order in which the observations were made was

$$+a-a-b+b-b-a+a-a-b+b-b \text{ \&c.}$$

From $+a$ to the next $+a$ represents a complete series, from which two corresponding readings may be obtained; and similarly from $+b$ to the next $+b$. An example, taken at random from the pages of the experimental book, will make the method clear:—

$+a$		$-a$	$-b$		$+b$
161		147		(344·5)	
	(308)		164·5		
					180
	(325)		164·5		
		155		(344·5)	
171					
		153·5		(362)	
	(324·5)		170		
					192

The zero line of the scale lay at the centre, and to this zero the zero position of the galvanometer spot of light was roughly adjusted. Hence the sum $161+147$ $(=308)$ is to a first approximation double the deflection due to the average current during the time taken to make the two readings; and similarly for all the other successive pairs of readings belonging to either circuit. The bracketed numbers show these sums, no two of which are of course simultaneous or correspond exactly to the same temperature. But let us take any four in chronological order, such that the first and last belong to one circuit, and the two intermediate ones to the other. Then we may assume, if the rate of change is slow, that the sums of these pairs are proportional to electromotive forces which correspond to the same temperature. In the example given, two pairs of corresponding numbers may be obtained by simply adding successive pairs of bracketed numbers of the same name, namely, 634, 689, and 650·5, 706·5. Every number so finally obtained depends on at least three readings. I shall give here a portion of one of the tables containing these reduced numbers, together with first differences, and the ratios of corresponding pairs of differences—which quantities were necessary for the final reductions.

Experiment of May 14, 1886.

Resistance of Hydrogenium = 1·4245 *Ohms.*

Temperature.	*Pt* – H*d*.	Differences.	*Pt* – P*d*.	Differences.	Ratio.
	63		164		
		23		49	·47
40°	86		213		
		23		50	·46
	109		263		
		19		51·5	·37
	128		314·5		
50°		25		61	·49
	153		365·5		
		21		55	·38
60°	174		420·5		

and so on.

The temperatures in the first column are not to be regarded as accurately determined. They merely indicate, to a sufficient approximation, the temperature of the moment. The ratios in the last column should give the ratios of the corresponding electromotive powers of hydrogenium and palladium referred to platinum. The mean of all for any one experiment will give a good value for this ratio at the mean temperature of 60° C., the cold junction being 20° C., and the highest temperature being 100° C. Instead of taking a single mean, however, I divided the experiment into two portions, and found the mean of all differences below 80° C. and the mean of all differences above 80° C. In this way I got two values for the ratio—one corresponding to temperature 50° C., and the other to temperature 90° C. These ratios, which are represented by the symbol *de′/de*, are given in the accompanying table, each specimen of hydrogenium being distinguished by its resistance estimated in terms of the resistance of the originally pure palladium wire; that is, by the ratio R′/R.

TABLE *comparing the Changes in Resistance and in Thermoelectric Power of Palladium Wire charged with different amounts of Hydrogen.*

R′/R	1	1·043	1·121	1·189	1·26	1·302	1·395	1·504	1·78
de′/de 50°	1	·88	·75	·77	·70	·63	·47	·44	– 1·58
de′/de 90°	1	·88	·76	·70	·65	·61	·53	·44	– 1·58
de′/de (mean)	1	·88	·755	·735	·675	·62	·50	·44	– 1·55

These numbers prove two things—

1. The greater the charge of hydrogen, the higher on the thermoelectric diagram is the hydrogenium line.
2. The displacement of the line for a given increment of charge is greater at the higher charges.

So striking is this second peculiarity that, whereas it requires the resistance of the wire to be increased by 50 per cent., so as to bring the line half way to the platinum line, it requires only 78 per cent. to make it move to the other side of the platinum line to a distance greater by 50 per cent. than the distance of the palladium line. This is indicated by the figures in the last column having a negative sign. The great displacement in the hydrogenium line at high charges, as proved by these experiments, is in agreement with the results of the earlier experiments.

It does not seem possible to draw any conclusion from the values given as to there being any change in the inclination of the hydrogenium line. The safest conclusion to draw would be that, on the whole, the ratio de'/de remains constant through the corresponding range of temperature—that is, through 40° C. If this were so, it would mean that the hydrogenium line is not parallel to the palladium line, since the latter is itself inclined to the platinum line. The thermoelectric powers of palladium to platinum at 50° C. and 90° C. are in the ratio of 70/75. If the hydrogenium line were parallel to the palladium line, the corresponding ratio for it would be smaller for all the cases studied except the last, for which it would be greater, being indeed greater than unity. There is certainly no indication of such a property in the tabulated numbers. The properties are indeed almost reversed.

Again, the constancy of the ratio de'/de for all temperatures (if surely established) would mean that the hydrogenium lines all pass through the point of intersection of the palladium and platinum—a very remarkable fact, should it be established by later experiments. We might then reason as to the existence of a *hydrogen* line passing through the same point. Certainly the constancy of this ratio is well marked in the last two cases, just the ones where, upon the very natural hypothesis of parallelism, it should be most distinctly variable. A closer study of the subject seems called for, and I hope before long to attack the problem in a somewhat different manner. It may be said that we can hardly expect to avoid irregularities due to a probable inequality in the distribution of the hydrogen charge. Such irregularities will obviously have more effect in thermoelectric measurements than in resistance measurements.

In connection with the subject of the thermoelectric properties of hydrogenium, the following will make an instructive lecture experiment. A palladium wire is hydrogenised throughout half its length, and its extremities attached to the terminals of a galvanometer. If, now, a flame is applied to the centre of this apparently single uniform wire, a large thermoelectric current is obtained, which grows to a maximum and then falls down to zero. This spurious neutral point is of course simply the result of hydrogen being lost to the heated portion. As the wire cools down again no such large current is obtained. The effect may be reproduced a number of times by following up with the flame the ever-shifting point of separation of the hydrogenium and the palladium.

SUMMARY.

The electrical resistance of hydrogenium increases with the temperature up to the point at which hydrogen begins to be given off. Thereafter the resistance begins to decrease till all the hydrogen has been driven away, after which increase sets in again as the now pure palladium wire is heated. The temperature-coefficient diminishes as the charge increases, and in such a manner that the total increase of resistance through a given range of temperature is nearly the same for the same palladium wire whatever the charge may be. In other words, at any given temperature below 150° C., the increase of resistance due to a given additional charge is the same. The changes of resistance between 200° C. and 300° C. afford a very delicate means of studying the manner in which the hydrogen escapes.

The thermoelectric current in a palladium-hydrogenium circuit flows from palladium to hydrogenium through the hot junction—that is, the hydrogenium line lies higher in the thermoelectric diagram than the palladium line. The higher the charge the higher the position. Saturated hydrogenium lies, for ordinary atmospheric temperatures, between copper and iron. Up to 150° C. the hydrogenium lines are straight lines, and nearly if not quite parallel to the palladium line. Above 200° C. rapid changes set in, the result no doubt of the loss of hydrogen at the junction. On cooling, the hydrogen seems to return partially to the extremity from which it has been driven. The effects at high temperatures are, however, complicated, because of the unequal distribution of hydrogen in the palladium.

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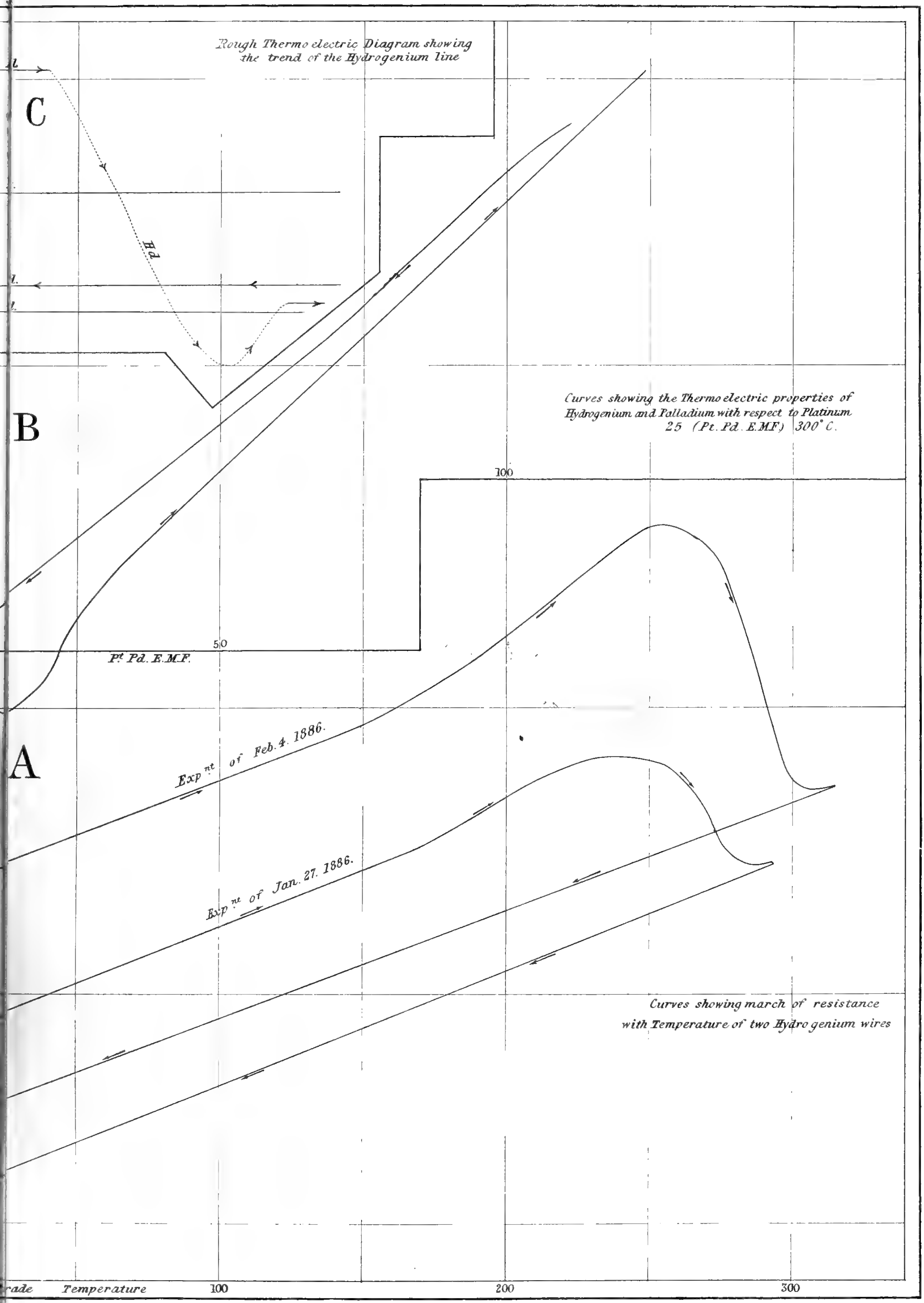

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VIII.—*The Electrical Resistance of Nickel at High Temperatures.* By CARGILL G. KNOTT, D.Sc. (Edin.), F.R.S.E., Professor of Physics, Imperial University, Tokayo, Japan. (Plate XII.)

(Read 5th July 1886.)

In the *Proceedings of the Royal Society of Edinburgh* for 1874–75 there is a short paper on the "Electrical Resistance of Iron at a High Temperature." It is the record of certain experiments made by three of us, then students in the Physical Laboratory of the University of Edinburgh; and its conclusion is that there is a peculiarity in the behaviour of iron as an electric conductor at the temperature of a dull red heat. At this temperature other physical peculiarities are known to exist, particularly as regards its thermal expansion, its thermal capacity, and its specific heat for electricity. The discovery of these striking properties we owe respectively to Dr GORE,[*] Professor BARRETT,[†] and Professor TAIT.[‡]

Professor TAIT'S discovery, that the THOMSON effect in iron changes sign at certain high temperatures, is in itself very striking; and, when taken in connection with other coexistent peculiarities, suggests various lines of inquiry. The most obvious is, perhaps, the question as to its occurrence in other metals. There is one other metal which rivals iron in thermoelectric eccentricity, namely, nickel. At a temperature of 200° centigrade, its THOMSON effect gradually changes sign from a considerable negative value to a large positive value, changing back again to nearly its original value at 300° C. If nickel thus agrees with iron in one exceptional feature, it may well be expected to agree in others. In short, Does nickel between the temperatures of 200° and 300° C. undergo exceptional changes in length? is there a phenomenon corresponding to BARRETT'S reglow? and has the electric resistance any unusual change at these temperatures? The first and third questions may be readily answered by experiment; the second, however, seems to offer almost insuperable difficulties as a subject of investigation. The following paper deals with the third of these inquiries.

I have thought it well to embody, along with the results for nickel, corresponding results for iron. The chief reason for this is, that in the experiments conducted in 1874 by Messrs SMITH and MACFARLANE and myself, only a

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[*] *Proc. Roy. Soc. Edin.*, 1869, and *Phil. Mag.*, 1869.

[†] *Phil. Mag.*, 1873.

[‡] *Trans. Roy. Soc. Edin.*, 1872–73.

general qualitative result was obtained. The method adopted was not one which lent itself to accurate quantitative determinations, and to obtain such is in itself an important quest. Moreover, the ultimate nature of the peculiarity can only be seen in its true light by a direct comparison of the individual characteristics as shown by iron and nickel. The great difficulty in making such a comparison arises from the high temperature at which the phenomenon shows itself in iron; and a further complication springs from the change in the metal, due to oxidation and tempering. In the case of nickel, however, the critical temperature is within reach of a mercurial thermometer, and the oxidation is insignificant. It is highly probable, then, that the results for nickel will be more definite and unmistakable than those for iron.*

In the experiments to be described, the resistances were measured by the simple form of Wheatstone bridge. The wires were generally tested in pairs, necessarily so in the measurements at very high temperatures. Four stout copper rods, 60 cm. long, ·7 square cm. cross-section, furnished with strong shoulder binding screws at the extremities, were fixed in a vertical position some little distance apart. Their lower extremities were joined in pairs by two wires, one of which was a specimen of nearly pure platinum, and the other the nickel, iron, or palladium wire which was being tested. The upper extremities of the rods were joined by stout copper wires to a commutator, which was in connection with a Wheatstone bridge resistance box of ordinary construction. The current was obtained from a gravity Daniell of high resistance; and the measurements were made by means of a dead-beat mirror astatic galvanometer constructed by ELLIOTT BROTHERS.

The earlier experiments and some of the later ones were carried out by Messrs HIRAYAMA and SANEYOSHI, two science students in the Imperial University, Tokayo, and were originally intended simply as an exercise in laboratory work. Two wires, one of nickel and the other of platinum, were coiled in long spirals, and fixed to the lower extremities in the manner already described. A vessel containing olive oil was then brought into position, so that the wires were wholly immersed. The temperature was gradually raised by means of a spirit-lamp, and the resistances measured at convenient intervals. The temperatures were given by a centigrade thermometer, whose bulb hung in the centre between the terminals of the wires. The oil was briskly stirred the whole time, so as to secure a practically uniform temperature throughout the mass. The thermometer itself was tested directly at freezing and boiling points, and the necessary corrections applied. The error being the same at both points, it was assumed to apply throughout the whole range of readings. Two specimens of nickel wire were studied, which we shall distinguish as the thick nickel and the thin nickel. Tables A and B give the observations for

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* See also *Proc. Roy. Soc. Edin.*, ix. 120, 1875–76. [P. G. T.]

these wires; tables C and D give the corresponding results for platinum and palladium. They are added simply for the sake of comparison.

The first column contains the corrected temperatures; the second the resistances corrected for the connections; and the third these resistances reduced so as to make the resistance at 0° C. equal to 100. The value of the resistance of the wire at 0° C. was in all cases calculated—in the case of platinum and palladium from the empirical parabolic formula which was found to agree well with the observations, and in the case of nickel by the method of successive differences from the first five or six numbers. This latter method was used because, for the nickel measurements, it was found quite impossible to obtain a suitable formula of ascending powers of temperature up even to the fourth. The reduced resistances of the third column for each wire are represented graphically in their relation to temperatures on Plate XII. (I.). The diameters and specific resistances of each wire are given at the head of each list of numbers.

The resistances throughout are measured with greater accuracy than the temperatures. In some cases, the temperature being steady, the resistance was adjusted by means of a large shunt set in an arc along with the units of the resistance box. In other cases—and this was latterly found the more convenient method—the resistance was fixed as near as the coils would allow, and the temperature of the wire slowly raised till the current through the galvanometer just vanished. Hence, although the resistances in table A appear only to the third significant figure, they are really certain to the fourth.

TABLE A.—*Thick Nickel.*

Diameter = ·05 cm. Specific resistance = 9697 (C.G.S.).

Temperature.	Resistance of Wire used.	Resistance reduced for comparison.
0° C.	·724 ohms	100
10°	·757	104·6
20°	·79	109·1
29°	·82	113·3
66°·4	·95	131·2
88°	1·03	142·3
98°·6	1·07	147·8
114°·6	1·14	157·5
122°·6	1·17	161·6
131°·6	1·21	167·1
142°·3	1·26	174·0
167°	1·37	189·2
187°	1·47	203·0
198°·5	1·53	211·3
215°	1·62	223·8
220°·3	1·65	227·9
233°	1·72	237·6
251°	1·83	252·8
255°	1·85	255·5
264°·8	1·91	263·8
267°·5	1·93	266·6

TABLE A.—*continued.*

Diameter = ·05 cm. Specific resistance = 9697 (C.G.S.).

Temperature.	Resistance of Wire used.	Resistance reduced for comparison.
278° C.	2·00 ohms	276·2
281°	2·02	279·0
293°	2·10	290·1
295°	2·11	291·4
300°	2·15	296·7
301°·4	2·16	298·4

TABLE B.—*Thin Nickel.*

Diameter = ·0154 cm. Specific resistance = 14500 (C.G.S.).

Temperature.	Resistance of Wire used.	Resistance reduced for comparison.
0° C.	1·914 ohms	100
9°	1·96	102·4
33°·4	2·09	109·2
56°·5	2·233	116·6
76°	2·38	124·3
99°	2·55	133·2
120°·4	2·716	141·9
143°·7	2·915	152·3
165°	3·091	161·5
182°·5	3·259	170·3
203°	3·457	180·6
226°	3·697	193·2
248°·8	3·96	206·9
266°	4·17	217·9
290°·4	4·48	234·1
305°	4·73	247·1

TABLE C.—*Platinum.*

Diameter = ·049 cm. Specific resistance = 16800 (C.G.S.).

Temperature.	Resistance of Wire used.	Resistance reduced for comparison.
0° C.	1·02 ohms	100
35°·5	1·11	108·8
58°	1·164	114·1
80°·5	1·208	118·4
99°·5	1·256	123·1
119°	1·30	127·5
141°·3	1·35	132·4
163°·8	1·40	137·3
186°·3	1·45	142·2
204°	1·484	145·5
221°·3	1·523	149·4
243°·1	1·58	154·9
261°	1·612	158·0
278°	1·642	161·0
295°·8	1·69	165·7

TABLE D.—*Palladium.*

Diameter = ·0396 cm. Specific resistance = 10680 (C.G.S.).

Temperature.	Resistance of Wire used.	Resistance reduced for comparison.
0° C.	1·595 ohms.	100
9°	1·64	102·8
31°	1·77	111·0
56°	1·914	120·0

TABLE D.—*continued.*

Diameter = ·0396 cm. Specific resistance = 10680 (C.G.S.).

Temperature.	Resistance of Wire used.	Resistance reduced for comparison.
76°·5 C.	2·02 ohms	126·7
99°	2·146	134·6
120°·4	2·265	142·0
144°	2·39	149·9
165°	2·508	157·3
184°·5	2·607	163·5
204°	2·72	170·5
226°	2·832	177·6
247°	2·94	184·4
268°·5	3·05	191·2
291°·5	3·163	198·3
305°	3·216	201·3

A glance at the columns of reduced resistances, or at the representative curves, shows at once one peculiarity of nickel as compared with the other metals, namely, the comparatively great increase of resistance throughout the measured range of temperature. The curves further show, by the manner of their curvature, that the rate of increase of resistance of a given nickel wire per degree centigrade *increases* as the temperature rises; whereas this rate of increase diminishes in the case of platinum and palladium. The same fact is readily shown from the numbers themselves by dividing the successive first differences of either column of resistances by the corresponding temperature differences.

The impossibility of representing the march of the nickel resistance by an empirical formula of ascending powers of the temperature has been already noticed. Some mode of formulating the results is, however, advisable, so as to make them numerically comparable with the results for platinum and palladium, which can be represented very approximately in the usual way. The following mode seems to be in many respects suitable:—

First, calculate by strict interpolation methods from five contiguous observations, the resistances corresponding to successive conveniently chosen temperatures, say, 0°, 50°, 100°, 150°, 200°, and 250°. Then tabulate, as in the subjoined table, the successive differences of these resistances. In the series of 2nd differences we recognise at once the impossibility of applying a parabolic equation to the results for nickel.

Table of Successive Differences of Thick Nickel Resistances.

Temperature.	Resistance.	1st Differences.	2nd Differences.
0°	·724		
		·168	
50°	·892		·016
		·184	
100°	1·076		·035
		·219	
150°	1·295		·024
		·243	
200°	1·538		·044
		·287	
250°	1·825		

Table of Successive Differences of Thin Nickel Resistances.

Temperature.	Resistance.	1st Differences.	2nd Differences.
0°	1·914		
		·275	
50°	2·189		·091
		·366	
100°	2·555		·045
		·411	
150°	2·966		·050
		·461	
200°	3·427		·084
		·545	
250°	3·972		

Although it is impossible to get a single parabolic equation to apply all through, we may calculate parabolic equations to apply to successive overlapping segments of a hundred degrees' range, taking as initial points the successive temperatures 0°, 50°, 100°, 150°. We thus obtain four equations, which will be found to agree closely with the observations. To compare these equations with those for other metals, such as platinum and palladium, would then be an easy matter.

It is to be remembered, however, that the usual method of representing observations by an empirical formula of ascending powers of the one variable has rarely any deep significance. What is of real importance in all such investigations is to know, first, what the value of a certain quantity is, and, second, how it varies under given conditions; and in many instances the latter is the main object of research. It is so in the present inquiry. It should be our object, then, to tabulate our results in such a manner that the rate of change of resistance per degree of temperature may be evident at a glance for all temperatures. The usual equation is of the form

$$R=R_0(1+\alpha t+\beta t^2),$$

from which we may almost at once calculate dR/dt for any temperature. What we wish, however, is not so much this quantity as the quantity $R^{-1}dR/dt$, which is the real rate of change of resistance.

In the following table this quantity is calculated for the four series of observations already given, so that the peculiarities of nickel may be readily indicated. The quantities are estimated for the temperatures 50°, 100°, 150°, 200°, since for these alone can be safely estimated the rates of change in the case of the nickel. The necessary calculation is most readily effected by means of the formula—

$$\frac{1}{R_t}\,\frac{dR_t}{dt}=\frac{1}{R_t}\left(\frac{\Delta_1}{\tau}+\frac{\Delta_2}{2\tau}\right),$$

where Δ_1 Δ_2 are the first and second differences in the series of resistances, corresponding to the temperatures $t-\tau$, t, $t+\tau$. The values for platinum and palladium are similarly estimated.

Temperature.	Values of $\frac{1}{R_t}\frac{dR_t}{dt}$ for			
	Thick Nickel.	Thin Nickel.	Platinum.	Palladium.
50° C.	·00395	·00293	·00218	·00302
100°	·00375	·00306	·00198	·00249
150°	·00357	·00294	·00170	·00225
200°	·00342	·00294	·00142	·00197

This table shows very distinctly the real nature of the difference between nickel and the other two metals; it is a difference only of degree. The quantity $R^{-1}dR/dt$ or $d \log R/dt$, we shall, for brevity's sake, call the logarithm rate, per unit rise of temperature being understood. It appears, then, that nickel differs from platinum or palladium, or most other metals, in the fact that its logarithm rate does not change so much with rise of temperature. In the case of the thin nickel, indeed, it is practically constant, so that the march of resistance with temperature could be very approximately represented by a simple logarithmic equation.

It may be noted that the logarithm rates for platinum and palladium are approximately inversely as the corresponding absolute temperatures. Hence we have

$$\frac{1}{R}\frac{dR}{dt}=\frac{k}{t}$$

For platinum, $k=\cdot 7$ roughly;
„ palladium, $k=\cdot 95$ „

Integrating and evaluating the constant by the condition

$$R=100 \text{ when } t=274,$$

we find, for platinum, the formula

$$R=1\cdot 97\times t^{0\cdot 7};$$

and, for palladium,

$$R=\cdot 483\times t^{0\cdot 95}.$$

These formulas will be found on trial to be in fair agreement with the numbers given in tables C and D.

We may also by integration of

$$\frac{1}{R}\frac{dR}{dt}=\cdot 003$$

obtain a formula for the thin nickel. Its form is

$$\text{Nap. log. } (R\times\cdot 0228)=\cdot 003\times t,$$

where t is, as before, the absolute temperature. This expression will likewise be found to suit the numbers given in the last column of table B.

There is no very obvious mode for obtaining a similar formula for the thick nickel.

It may be remarked that this mode of representing the temperature relations of resistance by a power of the absolute temperature—a power which may be fractional—includes as a special case the well-known statement that, for pure metals, the resistance is directly as the absolute temperature. For small ranges of temperature the equation

$$R = CT^k$$

may be easily thrown into the approximate form

$$R = R_0(1 + at + bt^2),$$

where T is absolute temperature, t centigrade, and the other quantities are constants. In this case a is to a first approximation equal to k times the reciprocal of 274.

We now pass to the discussion of the second series of experiments. In these the temperature was raised to a fairly bright red heat by means of a charcoal furnace. The four stout copper rods, with the attached wires which were to be tested, dipped into a porcelain vessel through suitable holes in the lid. The vessel itself stood inside a small charcoal furnace, and was heated by red charcoal dropped in around it. After reaching its highest temperature the charcoal and wires gradually cooled; and during this cooling the resistances of the two wires were measured in rapid alternation.

To obtain what might be regarded as simultaneous values of the resistances, means of successive pairs of readings for the one metal were interpolated. In every case the one wire was the same piece of platinum, whose indications served the purpose of a thermometer. In terms of its resistances, the resistances of the other wires could be expressed, graphically or otherwise. Many experiments were made with each kind of wire, and a vast number of observations accumulated. These I have not thought necessary to reproduce in the form in which they were obtained. The five curves of Plate XII. (II.), however, which tell their own tale clearly enough, are drawn from the observations, conveniently reduced, of the five best experiments. The reductions were the same as those made in the earlier series of experiments; that is, the resistance at 0° C. for each metal was reduced to 100, and the other resistances changed proportionally.

Although the numbers themselves are not reproduced, their essence is given in table E, which is really a comparative table of the resistances of certain wires at various temperatures from 0° C. to a fairly bright red heat. The series of platinum resistances, as shown in the first column, rises from 100 to 230 by successive additions of 10. From the reduced observations in the several experiments, the resistance of any metal corresponding to each one of the chosen platinum resistances can be readily calculated. Thus the number 401 in the fourth column means that a piece of thick nickel, whose resistance at 0° C. is 100, has a resistance of 401 at that temperature at which 190 is the resistance of a piece of platinum whose resistance at 0° C. is also 100. In short, the platinum column serves the purpose of a provisional temperature scale, in terms of which the resistances of the other metals are expressed. Under each column a row of differences is added. These bring out strongly the peculiarities which are disclosed by a glance at the curves of Plate XII. (II.). On the left of the platinum column a few numbers are given to indicate the temperature in degrees centigrade. Above the value 320° C., the estimation is only approximate, and is based on the assumption that platinum wire changes in resistance according to a parabolic function of the temperature. 700° C. may be regarded as a fair approximation to the highest temperature. Besides the nickels and palladium used in the former series of experiments, two specimens of iron were investigated, and are given for purposes of comparison.

TABLE E.—*Comparison of the Resistances of various Metals at different Temperatures.*

Temp. in ° C.	Platinum.		Palladium.		Thin Nickel.		Thick Nickel.		Iron (1).		Iron (2).	
	Resist.	Diff.	Resist.	Diff.	Resist.	Diff.	Resist.	Diff.	Resist.	Diff.	Resist.	Diff.
	230		303		350		476		713		718	
		10		14		12		19		80		75
580°	220		289		338		457		633		643	
		10		14		13		17		83		75
	210		275		(325)		440		550		568	
		10		12		13		22		65		54
	200		263		311		418		485		514	
		10		14		14		17		60		51
420°	190		(249)		297		401		425		463	
		10		15		15		23		54		58
	180		234		282		378		371		405	
		10		16		16		43		45		50
320°	170		218		266		335		326		355	
		10		17		31		52		42		50
270°	160		201		235		283		284		305	
		10		17		29		43		42		44
220°	150		184		206		240		242		261	
		10		17		30		34		28		42
180°	140		(167)		176		206		214		219	
		10		17		24		36		34		32
130°	130		150		152		170		180		187	
		10		17		19		29		32		32
84°	120		133		133		141		148		155	
		10		17		15		21		27		28
40°	110		(116)		(118)		(120)		121		127	
		10		16		18		20		21		27
0°	100		100		100		100		100		100	

First, we notice that palladium is very similar to platinum in the manner of its changings, tending, however, to diminish in rate of change as compared with the platinum at higher temperatures. Secondly, we see at a glance that the behaviour of the nickels is very peculiar. About a temperature of 180° or 200° C. the rate of growth of resistance of a given wire with temperature undergoes a marked increase, and experiences a more evident decrease at a temperature somewhat above 300° C. Throughout this range of temperature the comparatively great slope in the resistance curve is very striking.

Thirdly, there seems to be a similar increase in the rate of growth of resistance of iron wire, occurring at a temperature a little below 600° C. It was unfortunately impossible to attain a much higher temperature with the means at our disposal; but in the very highest readings obtained there was sometimes an indication of a decrease setting in, as in the case of nickel. In the curves as drawn the peculiarities of the iron are not very distinctly shown. It was thought better, however, to draw the curve to the same scale as the curves for the other metals than to proportion the co-ordinates to make it well-conditioned. The curves indicate at once the extremely great increase of resistance in iron as compared with other metals. This is in accordance with former experiments; and the results here obtained agree fairly well with Von WALTENHOFEN's results for steel (see WIEDEMANN's *Electricität,* vol. i. p. 525). The measurements made by other experimenters do not agree nearly so well—as a rule, a much smaller increase has been found.

In this paper, however, no special emphasis is laid on the results for iron, except that they cannot be represented by any ordinary empirical formula, such as C. W. SIEMENS has given. So far as they go, they bear out our result of twelve years ago, that the rate of growth in resistance of iron experiences a marked increase at a temperature of a dull red heat. This peculiarity has now been proved to exist in the case of nickel, occurring however at a much lower temperature. The further peculiarity, so distinct in the case of nickel—namely, the subsequent decrease in the rate of growth—probably exists also in the case of iron. Indeed, on Von WALTENHOFEN's authority, the continued increase of resistance of steel as the temperature rises from a red heat to a white heat tends to evanescence. This bears out the statement made above. Thus, it appears that iron and nickel agree in a certain peculiarity in the rise of their resistance with temperature. This peculiarity may be thus described. Within a certain range of temperature, the resistance of a given wire increases at a more rapid rate per temperature degree than at temperatures above or below this particular range. For nickel this range lies between 200° and 320° centigrade; for iron between a dull red and a bright red heat. Now, it is exactly within these ranges respectively that the thermoelectric peculiarities of nickel and iron occur. In no other metals have any similar peculiarities been observed. Hence we may regard it as an experimental truth that the interesting changes in the sign of the THOMSON effect in metals in which such changes do occur are accompanied by peculiar changes in the manner of growth of resistance with temperature.

In the case of the nickel, the simultaneousness of the two peculiarities was demonstrated by direct experiment. In effecting this direct comparison, we tried many modifications; but the essential characteristic of the experiment was to obtain, alternating with the resistance measurements, accurate determinations of the electromotive force of a nickel-palladium pair. In some cases the measurements of the platinum resistance were used as the temperature scale in which to express this electromotive force; in other cases the platinum wire was dispensed with, so far as resistance measurement was concerned, but was introduced as a third element in the thermoelectric junction, after the convenient manner invented by Professor TAIT. That is, the three wires—nickel, palladium, and platinum—were bound together as a triple junction, and the free extremities led off in such a way that the nickel-palladium circuit and palladium-platinum circuit could be thrown on to the galvanometer in rapid alternation. In this form of experiment the palladium-platinum circuit played the rôle of a thermometer. The platinum was very similar in its thermoelectric properties to the kind named "Soft Pt" in Professor TAIT's first approximation to a thermoelectric diagram. Its thermoelectric line was but slightly inclined to the palladium line, and the electromotive force of the palladium-platinum circuit increased at a somewhat quicker rate than the temperature as estimated in centigrade degrees.

In whatever way the temperature was virtually measured, whether by the resistance of platinum or the electromotive force of the palladium-platinum circuit, the experiment gave us the means of comparing directly the two peculiar effects of nickel. Two curves could be drawn, the one showing the march of the electromotive force of nickel-palladium with temperature, the other giving the same thing for nickel resistance. The resistance curve was similar in all respects to those already shown in Plate XII. (II.); the electromotive force curve reproduced with wonderful fidelity the old result of Professor TAIT. Beginning nearly straight at low temperatures, or if anything slightly concave upwards, it became, as the temperature approached 250° C., distinctly convex upwards. About 300° C. the neutral point was reached, and shortly after passing the vertex the curve became accurately straight, and continued so to the highest temperatures. In fact, it consisted practically of two straight portions, oppositely inclined to the line of temperatures, and connected by a parabolic arc with vertex at 300° C. This, of course, shows that the nickel line on the thermoelectric diagram, lying at low temperatures *below* the palladium line,* continues parallel thereto till the temperature reaches 200° C., after which it gradually bends up towards the palladium line. This it cuts through at the neutral point (300° C.), and almost immediately thereafter bends round again into parallelism with the palladium line. Now these two rapid bendings were found to occur just at the temperatures at which the peculiar bendings occurred in the resistance curve.

Similar experiments were tried on iron, with, however, doubtful results. This was certainly in the main due to the non-efficiency of the method of preparing and keeping a high temperature.

The main results of these experiments may be thus described :—

1. The rate of growth of the resistance of a given nickel wire with temperature is greater, on the average, than the corresponding quantity for platinum or palladium, and less than that for iron.
2. The "logarithm rate"—that is, the rate of change per unit rise of temperature of *unit* resistance at any temperature—falls off more slowly for nickel as the temperature rises to 200° C. than it does for platinum or palladium.
3. At about 200° C. the rate of resistance-growth for nickel increases markedly, and continues practically steady till about 320° C., when a sudden decrease occurs, and thereafter the resistance steadily increases at this diminished rate. In other words, between the limits of temperature specified, the slope of the resistance curve is much steeper than for any other temperature. The same peculiarity is probably possessed by iron between the temperatures of a dull red and a bright red heat.
4. The peculiarity occurs (in each case) between the limits of temperature within which the striking thermoelectric peculiarity discovered by Tait also occurs—a peculiarity which is quite unknown in the case of any other metal.
5. There is thus a strong presumption that the Thomson effect in metals has a close connection with the mutual relations of resistance and temperature; at any rate in metals in which the Thomson effect is proportional to the absolute temperature (according to Tait's theory), the "logarithm rate" of change of resistance seems to be very approximately inversely as the absolute temperature. In nickel and iron, in which the law of the Thomson effect is peculiar, such a simple relation between resistance and temperature does not hold.

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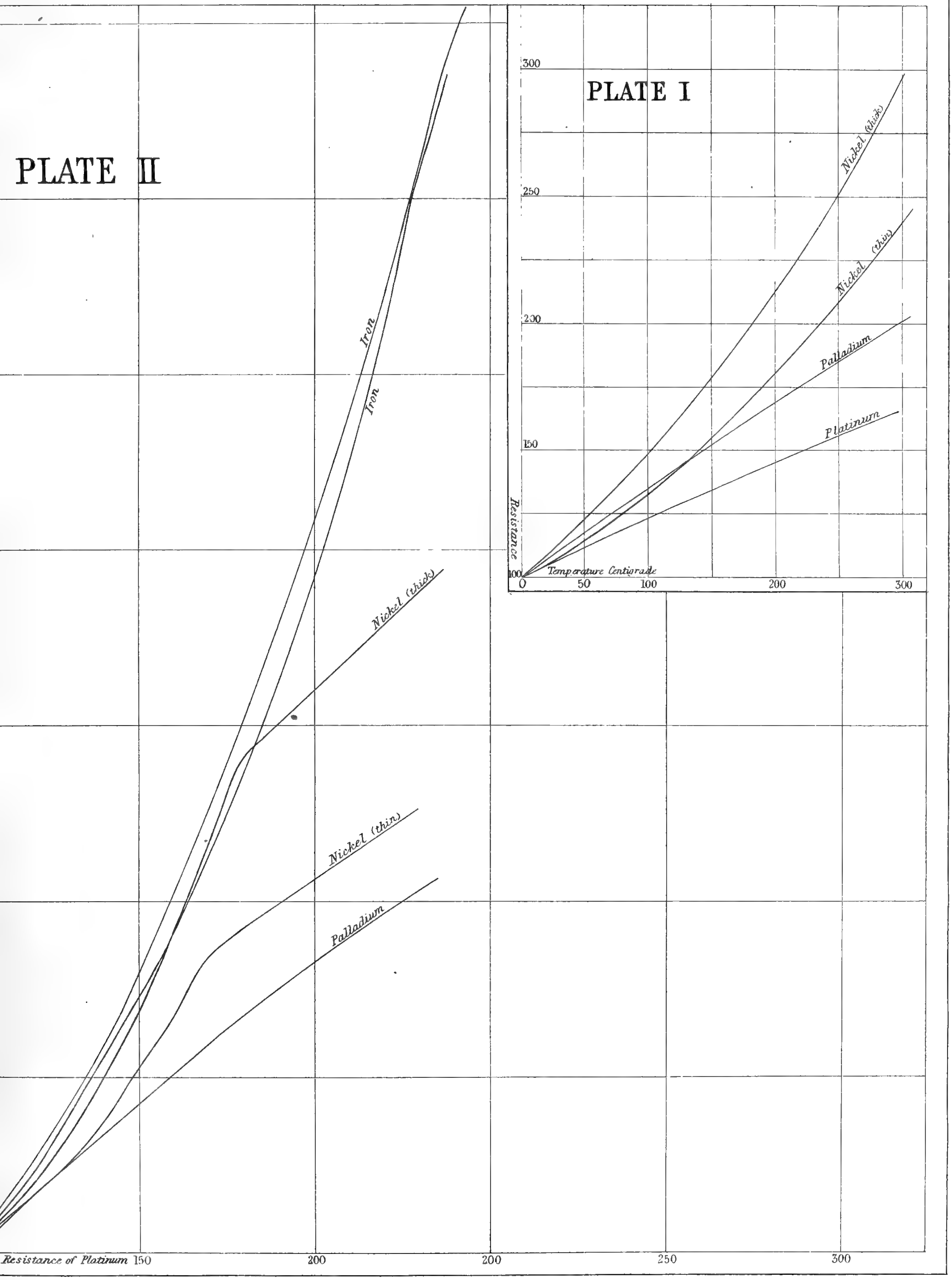
* It is to be regretted that certain writers still persist in turning the diagram, as it were, upside down, thus losing the advantage of Tait's improvement on Thomson's original form—an improvement which fits in so admirably with the *sign* of the Thomson effect.



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IX.—*The Formation of the Germinal Layers in Teleostei.* By GEORGE BROOK, F.L.S., Lecturer on Comparative Embryology in the University of Edinburgh. (Plates XIII.–XV.)

(Read 1st February 1886.)

Introduction.

A little more than a year ago, I was led to conclude that the primitive hypoblast in pelagic Teleostean ova was derived mainly from the unsegmented protoplasm forming the floor of the segmentation cavity. In this respect my results were mainly in harmony with the researches of LEREBOULLET (21), KUPFFER (18 and 19), KLEIN (16), Van BAMBEKE (3), and others, but opposed to the more recent investigations of HENNEGUY (10), HOFFMANN (14), KINGSLEY and CONN (15), RYDER (24), AGASSIZ and WHITMAN (1), and CUNNINGHAM (8.) At that time I adopted AGASSIZ and WHITMAN'S name of *periblast* for the subgerminal layer containing free nuclei, though I differed from those authors in my idea of the mode of origin of the nuclei. As, however, I regard the tissues derived from this layer as belonging chiefly to the parablastic group of HIS and WALDEYER, I propose to return for the present to the older name of *parablast.* I was not enabled to trace the origin of this layer in my earlier investigations, and thus failed to grasp its significance. More recently I have had an opportunity of studying the development of several other types, particularly the herring and cod. In the herring the germinal mound is not formed until after fertilisation. Partly owing to this circumstance, and partly also to the early period at which the assimilation of yolk commences, the eggs of the herring are particularly well suited for a study of the parablast question.

In order to understand the relation of the parablast to the yolk and to the embryo, it will be necessary to give a detailed account of the structure of the ripe unfertilised ovum, and of the early stages of development. In doing so, I have compared my results with the earlier investigations of BOECK, KUPFFER, and HOFFMANN.

The Ripe Unfertilised Ovum.

The ripe unfertilised ova of the herring vary considerably in size. This variation appears to be of a twofold nature. There is a certain variation found in the size of ova from a single female, or in those from a particular spawning district. There is also a variation in the *average size* of ova from one spawning district, as compared with those of another. KUPFFER (19) found that the ova of the Baltic herring have a diameter usually varying between ·92 and 1 mm., while exceptionally small ones may only measure ·85 mm. Boeck (2) gives the diameter of the herring ova on the Norwegian coast as 1·5 mm. Those which I have obtained from the Ballantrae herring averaged 1·17 mm. in diameter, and those of the Loch Fyne herring are about the same size. The egg is enclosed in an egg membrane, and outside the latter there is a viscous layer by which the ovum adheres to anything with which it comes in contact. It is by means of this substance that the eggs become attached to one another in the form of flattened cakes.

Viscous Layer.—If an isolated ovum is gently pressed from a ripe female and examined immediately under the microscope, it will be found that the adhesive material forms a comparatively even covering around the egg envelope. This is even better shown if the egg be pressed out into dilute osmic acid solution, or into spirit, and afterwards examined by means of sections. When thus hardened the viscous layer usually appears structureless, or presents faint and indistinct transverse streaks, such as would be shown in sections of a horny substance. Fig. 1 shows a section of the viscous layer and egg membrane in the unfertilised ovum. The relative thickness of the viscous layer varies very much. When it forms a comparatively even layer around the egg membrane it has usually about the same diameter as the membrane itself. When, however, before hardening, the viscous substance comes in contact with that of another egg, or with a foreign surface, a thickened welt is formed, which may be two or three times the thickness of the egg membrane. In such cases there is a corresponding thinning out of the layer in other parts of its circumference. Although usually structureless, I have once or twice seen well-marked transverse striations in the hardened viscous layer. Fig. 2 represents such an appearance, and is taken from a ripe egg which was pressed directly from the oviduct into osmic acid solution. The viscous layer has here about the average thickness, but is divided longitudinally into two strata. Each presents a well-marked transverse striation, which is more distinct than that in the inner portion of the egg membrane. The division of the viscous layer into two strata may possibly be due to shrinking, but it appears difficult to see how the transverse striations could be brought out by the same cause. They are very evenly distributed throughout.

This structure agrees with Hoffmann's (14) description of the appearance of the viscous layer in the unripe egg. According to this author, the *zona radiata in the nearly ripe egg* consists of two layers closely united together. The outer layer, which corresponds to the viscous layer, is perforated by very fine pore canals, and is sharply separated from the inner portion. In the *ripe egg*, placed directly in 1-10th per cent. osmic acid, the pore canals have almost entirely disappeared. If the ripe eggs are, however, first brought in contact with sea water, and afterwards fixed in osmic acid, the outer layer is seen to be quite separated from the inner one, and to constitute the viscous substance which serves for the attachment of the egg. This layer now frequently presents a laminated structure. From this description it will be seen that HOFFMANN regards the viscous layer as a metamorphosed part of the zona radiata. In this case it must be regarded as a secretion of the vitellus.

Egg Membrane.—KUPFFER has described two egg membranes, an outer *zona radiata* and an inner one, comparable with the true vitelline membrane of other forms. HOFFMANN has shown that in their origin these two portions of the egg membrane are very similar in structure, and that they are both to be regarded as portions of the *zona radiata;* there is no true *vitelline membrane* in the herring ovum. My investigations support HOFFMANN'S conclusions.

Yolk.—According to KUPFFER, the entire contents of the egg envelope are divisible into three parts—

1. A superficial layer, consisting of strongly refractive shining homogeneous globules ·008 to ·02 mm. in diameter, which he terms yolk granules (*Dotterkörner*).

2. Immediately below the thin superficial layer the yolk consists of larger and less refractive yolk spheres, which have a rounded or egg-shaped form. They vary in size from ·05 to ·08 mm., and constitute the greater portion of the yolk. These are the *Dotterkugeln* of KUPFFER.

3. There is also a scanty viscous mass of protoplasm, which is mixed up with the other two.

According to this observer, the fine granular layer of yellowish protoplasm, which later forms the blastoderm, is not present in the unfertilised ovum, nor is there a germinal disc.

As HOFFMANN has already pointed out, KUPFFER was mistaken in supposing that the *germinal protoplasm* does not already exist in the ripe unfertilised ovum. The reason why it is not visible in the living egg is twofold. In the first place, it does not form a distinct layer around the yolk, as is usually the case in Teleostean fish ova; and in the second, there is so slight a difference between the colour of the yolk spheres and that of the protoplasm, that in the living egg it is impossible to distinguish between them. A section of the unfertilised egg shows nevertheless that the germinal protoplasm exists in the herring as well as in other fish eggs. There is, however, this difference, that whereas in fishes generally the bulk of the germinal protoplasm is collected as a distinct layer on the surface of the yolk at the time the egg is ready for fertilisation, in the herring the protoplasm is distributed throughout the yolk. The egg contents consist in fact of a mass of delicate protoplasm, in which the yolk spheres are imbedded. This structure is well brought out in sections of ova stained in Mayer's carmine. I have obtained the best results by leaving the eggs in the staining solution from twenty-four to forty-eight hours. Subsequent treatment with acid alcohol extracts all the stain from the yolk, so that the protoplasm remains deeply stained with carmine, while the yolk retains its original pale yellow tint, which is, however, too weak to show in thin sections. Unless the acid alcohol is allowed to act for some hours the yolk is not entirely robbed of its stain.

Fig. 3 (Pl. XIII.) represents a section through the middle of an egg, prepared as I have described. The germinal protoplasm is seen to be finely granular under a high power, but the granulation is so delicate and regular that, with a low power, the protoplasm appears quite homogeneous. The large yolk spheres occupy the greater portion of the egg, and are comparatively evenly distributed. There is not so great a variation in the size of the yolk spheres at this as in later stages. Towards the margin, however, there is a diminution in size, leading up to the small yolk granules which lie immediately beneath the egg envelope. Kupffer says nothing as to the origin of these yolk granules, but Hoffmann has indicated their possible mode of origin. According to this observer, the unripe ovum contains little protoplasm, and a large number of the large yolk spheres about ·035 mm. in diameter. It is to be presumed that at this stage there are no *yolk granules,* although this is not distinctly stated. Later, in the ripe unfertilised ovum, the large yolk spheres are fewer in number, and there is more protoplasm. While most of the yolk spheres are comparatively homogeneous in structure, a few are seen to be filled with smaller spheres only ·002 mm. in diameter. These are often seen lying in clusters outside the larger spheres, and would appear to be set free by the rupture of the walls of the latter. Hoffmann does not, however, offer any theory on the subject, nor does he give any figures.

There is undoubtedly a greater proportion of protoplasm to food yolk in the ripe ovum of the herring than is to be found at an earlier stage. The small refractive yolk granules are also not present in the unripe ovum so far as my observations go, but I have not been able to trace their origin. I have never observed a collection of small spherical "granules" inside one of the yolk spherules, nor any collection of them which would lead one to infer the origin assigned to them by Hoffmann. Judging from the subsequent behaviour of the food yolk, I am not inclined to accept Hoffmann's view without further evidence. The yolk of Teleostean fish ova, and particularly of the herring, is simply a collection of food material which has no cellular value whatever, and does not undergo a segmentation comparable with that of amphibian ova. If Hoffmann's views are accepted, there must either be a segmentation in the yolk spheres themselves, or the small yolk granules are the result of a mechanical subdivision of the larger masses. The former proposition appears entirely unwarranted by the future behaviour of the yolk. A mass of yolk, which can form around itself a thin wall, subdivide its contained food material into a number of small spheres, and then by a rupture of its wall set the new products free, must surely possess more vital properties than are exhibited by the yolk of fish ova. Again, the appearance of the yolk granules is different from that of the yolk spherules. The former are dense and highly refractive, while the latter are only slightly refractive. The "yolk granules" disappear when the egg is fertilised. They do not appear to be incorporated within the protoplasm as so much food, but the dense droplets spread out into a thin film, which becomes indistinguishable amongst the germinal protoplasm. Such dense droplets are found in the ova of the Gadidæ and other fishes, and here play a similar part. They are probably oily in nature, and may be produced by the mechanical collection of the oily material contained in the yolk mass. Returning now to KUPFFER'S statements concerning the structure of the ripe unfertilised ovum, I hold that he is correct in his opinion that the germinal disc, in the strict sense of the word, does not yet exist in the unfertilised ovum. Undoubtedly he was mistaken in concluding that in the unfertilised ovum there is no division into formative and nutritive yolk. This has already been pointed out by HOFFMANN. This author, however, appears to regard the germinal protoplasm, which is distributed throughout the yolk in the ripe unfertilised ovum, as homologous with the germinal disc in other fish ova. This is not the case. The germinal protoplasm already existing in the unfertilised ovum does indeed, after fertilisation, become in great part used up in forming the germinal disc, but not entirely so. This protoplasm also increases in bulk considerably at the expense of the yolk before segmentation commences.

Unfortunately, I have not been enabled to make a thorough study of the development of the ovarian ovum of the herring. I have, however, examined sections of ovaries in various stages of development. The earliest stage which I have observed is one in which the primitive ovum is already surrounded by a follicle. The cell protoplasm contains very little food material, and the nucleus is very distinct. As the ovum increases in size yolk material becomes collected within the egg contents to such an extent as to hide the nucleus and its surrounding protoplasm, when only the whole egg is examined. As development goes on it can be made out from sections that the protoplasm increases in bulk at the expense of the yolk. It also spreads out amongst the yolk spheres. At an early stage the cell protoplasm is star-shaped, having a central somewhat thin area in which the nucleus is situated, and around this are a number of radiating protoplasmic processes, which are pushed in amongst the continually increasing bulk of yolk spheres. In this manner the protoplasm comes to form a network supporting the yolk spheres. At a comparatively early stage the germinal spot becomes obliterated, and the germinal vesicle also loses its primitively well-marked character. In the ripe unfertilised ovum I have not been able to recognise the germinal vesicle. The most approved methods of staining do not bring out any differentiation in the germinal protoplasm. The latest phase of the germinal vesicle which I have observed is represented in fig. 4.

It is here somewhat quadrangular, but irregular in outline, and from its margin fine threads of protoplasm are seen to penetrate the yolk mass. With a magnifying power of 700 diameters a beautiful reticular arrangement of the fibrillæ can be made out. There are distinct thickenings at the nodes, and besides the fibrillar reticulum there are a very large number of very small granules, which stain deeply with carmine. I have not been able to make out the process of degeneration which the germinal vesicle probably undergoes prior to the ovum becoming mature and ready for fertilisation.

The difference then between the ripe ovum of the herring and that of other Teleostean fishes appears to rest in the fact, that at the time the herring ovum is ready for impregnation the germinal protoplasm is not collected into a definite layer. Changes in the arrangement of the egg contents therefore occur in the herring ovum *after fertilisation*, which in the majority of fish ova have already been brought about without the aid of spermatozoa.

Kupffer also maintains that the relationship of the various parts of the ripe unfertilised ovum remains unaltered in sea water, *so long as the egg remains unfertilised;* in other words, that sea water, *per se*, has no influence on the ovum. On this point Boeck and others are at variance with Kupffer. The experiments which I have already described* were in part devised to test this point, and all tend to show that Kupffer's conclusion was justified. Others which were conducted more recently only tend to strengthen the same view. In the first place, the egg membrane does not separate from the yolk unless it has previously been penetrated by spermatozoa. Whether these enter at the micropyle only, or may force their way through the egg envelope at any part, I have not been able to decide; but the fact remains that unless an egg is submitted to the action of spermatozoa, the egg membrane does not separate to any appreciable degree from the yolk. In this connection reference may be made to my experimental researches of the fertilisation of herring ova (7). In one experiment, eggs which had remained *unchanged for twenty-four hours* in sea water were submitted to the action of spermatozoa. Within an hour the egg membrane had separated to a considerable extent from the yolk in a large number of the eggs. Although in most cases the eggs at this time retained only a feeble vitality, and the viscous covering of the egg membrane had become so hardened as to offer considerable obstruction to the entrance of spermatozoa, a breathing chamber was formed in the normal manner. It is evident, then, that the entrance of spermatozoa is necessary before this separation can take place. If the micropyle is an open canal, communicating freely with the surface of the yolk, there appears no reason why water should not enter the

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* Annual Report of the Fishery Board for Scotland, 1885.

egg through this channel. It would seem, however, on *a priori* grounds, that a micropyle can be of little service in aiding fertilisation in such an egg as that of the herring. Around the egg envelope at the time the egg leaves the oviduct there is a comparatively thick layer of a viscous substance which hardens in sea water. So long as it remains semifluid it collects at the lower pole, and if two or more eggs are in contact, the viscous substance around each forms a thick ring around the point of contact, and thus the whole mass is cemented together. It follows, therefore, that the moment an egg touches any object in its descent through the water it becomes attached to that object, and does not again change its position during development. It seems probable that the micropyle cannot have a fixed position in relation to the axis of the ovum, as is the case in pelagic ova. If this supposition is justified, it follows that the micropyle must frequently be covered by the thicker portions of the hardened viscous layer. In any case the opening of the micropyle must be filled with the viscous layer before this hardens, as KUPFFER has already pointed out. Thus spermatozoa would find as much difficulty in penetrating the ovum at the micropyle as at any other point. It has been proved by repeated experiment that herring ova may be fertilised from twelve to twenty-four hours after they have been placed in sea water, and by this time the viscous layer is so hard that the ova are not easily displaced from their point of attachment. In any case, therefore, spermatozoa are able to penetrate this hardened layer, which indeed offers more resistance than the egg membrane itself. It has been generally admitted that in fish ova the spermatozoa enter by the micropyle and by this only. The presence of a viscous covering of the egg envelope in such ova as that of the herring may modify the use of the micropyle. If the viscous covering of the eggs of the herring and allied forms is phylogenetically of recent origin, it may be that the changed conditions have rendered the micropyle useless. These remarks are, however, simply offered as suggestions, a thorough investigation of the whole subject being very desirable.

The behaviour of the germinal protoplasm during the time that a ripe unfertilised ovum remains in sea water cannot easily be observed in the living egg. In order to investigate this point, I have cut sections of a large number of eggs which have remained unfertilised in sea water for a time varying from one to forty-eight hours, and I have also examined sections of ripe ova taken from females which had been dead some hours. As I have previously described, the germinal protoplasm at first forms a comparatively even network between the yolk spheres. There is, it is true, a little more protoplasm at the surface of the yolk than towards the centre, but it must be remembered that the yolk spheres are smaller towards the surface, and there is thus more room for the protoplasm. This exact relationship is not long maintained in any case. There is, however, a difference in its behaviour in the fertilised and in the unfertilised ovum. The immediate effect of fertilisation is that a true germinal disc is formed, *of which the protoplasm already existing in the ripe unfertilised ovum forms only a part.* Such changes never take place without the stimulating aid of spermatozoa. Their exact nature will be described in due course; for the present it will be sufficient to describe the behaviour of the germinal protoplasm in the unfertilised egg. Simply stated, the germinal protoplasm acts as so much passive material so long as the egg remains unfertilised. It collects very slowly to the surface, it is true, but in a very different manner to that in which it would if fertilised. There is a *gradual* accumulation at the surface, and day by day the protoplasmic network is withdrawn more and more from the centre, *but it never collects into a disc-like prominence* in the unfertilised ovum. After a week's immersion in sea water an unfertilised ovum presents the appearance shown in fig. 5. It only differs from the recently-matured ovum in having more protoplasm around the circumference and less towards the centre of the egg. The protoplasmic filaments are never withdrawn to nearly the same extent in the unfertilised as in the fertilised egg. In the fertilised egg there are a number of branching protoplasmic filaments connecting the germinal disc with the yolk, but in the yolk pole the protoplasm is almost entirely withdrawn. In the unfertilised egg which has been some days in sea water this is not the case. The protoplasm forms a comparatively even layer at the surface of the yolk, and there is no division into animal and vegetative poles, the protoplasmic filaments not being withdrawn more at one part than at another.

It should be pointed out that this partial collection of the germinal protoplasm at the surface will take place whether the egg is placed in sea water or not. In order to conduct the experiment already referred to, ripe females were kept for a varying time in moist cloths. Sections of eggs kept under such conditions show that the germinal protoplasm begins to collect at the surface of the yolk soon after the egg is ripe, and that the amount of protoplasm found at the surface is, roughly speaking, proportional to the time which has been allowed to elapse before examination. Thus, then, sea water has nothing to do with causing the protoplasm to collect at the surface, nor, so far as I could make out, is this accomplished more rapidly in sea water than in the ovary itself.

Formation of the Germinal Mound.

The fact that at the time of impregnation the ovum of the herring exhibits nothing of the nature of a germinal disc, in the ordinary sense of the word, is a point of very great interest. In the history of the majority of fish ova, the influence of the sperm is not necessary for a separation of the germinal disc from the yolk, as this separation has already taken place before fertilisation. The case of the herring is, therefore, specially interesting from the fact that this accumulation of the germinal protoplasm at one pole can be watched under the microscope in the living egg. KUPFFER has made an elaborate series of investigations on the formation of the blastoderm in the herring ovum. He concludes that the germinal disc is formed by the combined influence of sea water and spermatozoa. KUPFFER'S conclusion is undoubtedly true to a certain extent, though not exactly in the manner in which he intended it. He was of opinion that the act of fertilisation brought about changes in the egg contents which resulted in the separation of the germinal protoplasm from the yolk, and the collection of this into a definite mound at the animal pole of the ovum. Although he mentions a certain small quantity of protoplasm mixed amongst the yolk spheres, he does not appear to have been aware that there is in the ripe unfertilised herring ovum a considerable collection of germinal protoplasm distributed throughout the yolk, the greater portion of which ultimately forms the germinal disc. Nevertheless, he was correct in the sense that the germinal protoplasm *is not collected into the form of a disc* until after fertilisation. On this account it appears to me that HOFFMANN is not entirely justified in his assertion that in the herring, as in other Teleostean fishes, the germinal layer exists before fertilisation. As we have seen, the germinal protoplasm which exists in the *unfertilised* egg must undergo a further development and growth before a germinal layer exists, which can be compared with that existing in most Teleostean fish ova before fertilisation. Van BAMBEKE (3) has called attention to the same point, and has shown that in *Tinca vulgaris*, as well as in the herring, the influence of spermatozoa is necessary before a *true* germinal disc is formed. If the germinal protoplasm existing in the unfertilised ovum simply *collected at the surface* after fertilisation, and then commenced to segment, it would be another matter. This protoplasm would then be directly comparable with the germinal disc or germinal layer of other fishes. If by the germinal layer is understood an amount of protoplasm which is distributed throughout the yolk, which, as a result of fertilisation, collects at the surface, *grows at the expense of the yolk,* and after a considerable increase in its bulk begins to segment, then there is a germinal layer in the ripe unfertilised ovum of the herring. But surely technical terms should have a definite and limited meaning, and it is impossible to regard the germinal layer of such eggs, for instance, as those of the *Gadidœ,* as equivalent with the protoplasm which, in the ripe unfertilised ovum of the herring, is distributed throughout the nutritive yolk.

Let us glance for a moment at the structure of a pelagic Teleostean ovum. The yolk is very transparent, and is not divided into a large number of yolk spheres, but consists only of one large vitelline sphere, which is comparatively homogeneous in structure. There may or may not be a special condensation of the oily contents into definite oil globules. From the fact that the yolk consists only of one large yolk sphere the outline is smooth, and there are no indentations of the surface of the vitelline mass as in the herring ovum. Outside the yolk the germinal protoplasm is collected into a distinct layer, usually of a pale yellow tint. This layer is always sharply marked off from the yolk at the time the ovum leaves the oviduct. As the egg floats in the water the germinal layer collects at the lower pole, leaving only a delicate film of protoplasm around the upper portion of the yolk sphere. Segmentation then commences if the egg has been fertilised. In the herring, however, the changes are somewhat different. At a period varying from fifteen to thirty minutes after the introduction of spermatozoa to sea water containing ripe eggs, the egg membrane begins to leave the yolk, and by the time an hour has elapsed the membrane has expanded to such an extent that a large cavity is formed all around the vitelline mass. This cavity is filled with water, which probably contains a portion of the egg contents in solution. The accompanying table shows the increase in the size of the egg after inception of water. The first two items are the measurements given by KUPFFER, and may be taken as a type of the eggs of the Baltic herring. The others are measurements of the eggs of herring taken from the Ballantrae Banks, off the Ayrshire coast.

	Diameter of unfertilised egg.	Diameter of yolk after 1 hour.	Diameter of egg capsule after 1 hour.	Diameter when breathing chamber is complete.			
				Yolk.		Egg capsule.	
				Greatest diameter.	Diameter at right angles.	Greatest diameter.	Diameter at right angles.
1	·92 mm.	...	...	·85 mm.	·82 mm.	1·2 mm.	(after 45 min.)
2	1·0 mm.	...	...	·97 mm.	·92 mm.	1·29 mm.	(,,)
3	1·194 mm.	1·147 mm.	1·486 mm.	1·298 mm.	1·204 mm.	1·768 mm. (after 5 hours)	1·65 mm. (after 5 hrs.)
4	average 1·1759 mm.	...	...	...	...	1·599 mm.	1·580 mm.
5						1·599 mm.	1·486 mm.
6						1·599 mm.	1·448 mm.
7						1·542 mm.	1·430 mm.

The eggs with which KUPFFER experimented were smaller than those from Ballantrae, but even taking this into account, it will be seen that the latter have proportionately a considerably greater diameter after the *breathing chamber* is formed. KUPFFER'S measurements were made forty-five minutes after fertilisation, and he does not state whether any further increase in the diameter of the egg capsule was noticed at a later period. In the measurements which are given for comparison, it will be seen that in example No. 3 the egg capsule measured 1·486 mm. one hour after fertilisation. At this stage both yolk and egg capsule were comparatively globular. At this stage the germinal protoplasm had collected to a considerable extent to the surface of the yolk, but was fairly evenly distributed around it. The later measurements are taken at a stage when the germinal protoplasm has collected into the form of a mound at the animal pole of the ovum. The *greatest diameter* therefore passes through the axis of the ovum at this stage. Whether the measurement 1·29 mm. represents the largest diameter which is reached by the eggs of the Baltic herring I cannot say, but it is probable that if there had been any further increase KUPFFER would have noted it. The early period at which this diameter was reached is probably to be attributed to the temperature at which the experiments were conducted. The temperature of the water during my experiments at the Rothesay Aquarium varied between 40° and 42° F.; whereas it is probable the temperature was considerably over 55° F. during KUPFFER'S experiments, as his embryos hatched out on the seventh day. Dr MEYER (22) has shown that young herring hatch out on the tenth or eleventh day at a temperature between 51°·8 and 53°·6 F.; whereas at 32° F. the earliest embryos do not hatch until the forty-seventh day. As the egg membrane leaves the yolk the surface of the latter can be more easily studied. It is then seen that the "yolk granules" on the surface are rapidly disappearing. KUPFFER says they are dissolved. However this may be, neighbouring "yolk granules" may be seen to run together and flatten out into a thin pellicle, which soon becomes indistinguishable, with the large yolk spheres for a background. They behave, in fact, very much like the small droplets of an oily nature which are found on the surface of the egg of the cod before fertilisation. Whether the "yolk granules" are really oil globules, or only yolk material richer in oil than the larger spheres, I cannot say, but their behaviour would seem to support the former supposition. KUPFFER next describes a series of *clear vacuoles* which arise at the surface of the yolk as transparent spots. These increase rapidly in size, and are pushed forward towards the centre of the yolk as a network of fine tubes. With the appearance of the clear vacuoles the germinal protoplasm begins to collect on the surface of the yolk. I have never been able to observe the vacuoles which KUPFFER describes, although I have searched for them repeatedly. With the act of fertilisation an activity is set up in the germinal protoplasm which causes it to collect rapidly on the surface of the yolk. Very early in this process the outer yolk spheres are a little wider apart than those towards the centre, and the protoplasm as it collects fills up the spaces between them. A little later the protoplasm is almost entirely withdrawn from the centre of the yolk, and there is then a thin layer of protoplasm on the surface, with a number of branching root-like processes extending some distance into the yolk. It is, I think, this collecting protoplasm which KUPFFER has mistaken for vacuoles and his series of coarse tubes. In sections of the egg about this stage which have been mounted unstained the germinal protoplasm is *very transparent*, whereas the yolk spheres are quite granular and of a yellowish tint. The appearance in the living egg is thus explained. The transparent portions between the yolk spheres are not vacuoles, and a system of tubes pushed down into the yolk, but the channels by which the germinal protoplasm (of which KUPFFER had no cognisance) makes its way to the surface. An hour after fertilisation (at 41° F.) a considerable quantity of the germinal protoplasm has collected at the surface of the yolk, and forms a distinct layer, varying from ·0188 to ·0564 mm. in thickness. At first this is quite clear and homogeneous, but soon fine granules make their appearance, and the whole layer becomes darker in tone.

From the moment that a layer of protoplasm has collected at the surface, an interesting series of phenomena is commenced, which is only terminated when the whole of the nutritive yolk has been consumed. The germinal protoplasm begins to grow at the expense of the yolk. Large masses of yolk are incorporated within the substance of the protoplasm and digested there. During this time the protoplasm is in constant motion, and flows slowly in thicker and thinner waves around the yolk. These phenomena are not new, but form part of a process which has frequently been described in connection with the *parablast.* They are, however, more marked and easily followed in the herring than in any other form with which I am acquainted. The process is one of *intracellular digestion,* and at a later stage probably forms an important mode in which the food yolk is used up in most meroblastic ova. The important point to be noted for the present is, that in the herring the yolk is partly consumed to form the germinal disc itself. KUPFFER supposed that nearly the whole of the germinal disc was formed in this manner, but it is probable that he had not studied *sections of the egg* in the earliest stages of development. He says, to begin with, that *the formative yolk appears as a continuous superficial layer.* It has already been seen that this is not so. When the greater portion of the germinal protoplasm has collected at the surface of the yolk, the *appearance* in an optical section of the living egg is no doubt as KUPFFER describes. Optical sections are, however, very misleading, and should only be used in *helping* to explain the appearance shown in actual section. A little experience may teach one how to interpret optical sections, but this experience can only be obtained from a study of actual sections of the egg.

Fig. 6 represents an optical section of the living egg of the herring an hour after fecundation. The germinal protoplasm is seen as a continuous superficial layer, which is considerably thicker at one side. The protoplasm is filled with fine and larger granules, and a number of small clear vesicles may also be made out under a moderately high power. The superficial layer of "yolk granules" has entirely disappeared, and the yolk now consists of a mass of large yolk spheres, which appear to be evenly distributed throughout. A careful examination of the peripheral row of yolk spheres shows that as yet there is little irregularity in their size and position. So much for the appearances presented by an optical section; let us now turn our attention to a stained section of the same stage. Such a section is represented in fig. 7. Remembering the structure of the ripe unfertilised ovum, it is easy to see now that the germinal disc *is in process of formation.* The protoplasm in collecting towards the surface has followed certain channels between the yolk spheres. There is still, however, a considerable portion of the protoplasm mixed amongst the yolk in the form of branching processes communicating with the surface layer. These processes vary in thickness, and also in their number and distribution in different parts of the circumference. Towards the germinal pole they are stronger, and penetrate further into the yolk than at any other point. In the yolk pole the filaments rarely penetrate beyond the second row of yolk spheres. The protoplasm is seen to be highly granular, and also to contain a number of small masses of yolk. It will thus be seen that the true relation of protoplasm to food yolk cannot be made out in optical section.

The appearance of granules in the protoplasm is the first sign of an active vegetative period in the history of the germinal protoplasm. In the section under consideration the digestive process has only just commenced, so that it may be better studied at a little later stage. Fig. 8 represents a section of an egg five hours after fertilisation, and at a time when the germinal protoplasm has almost entirely collected towards the germinal pole. Large masses of yolk are seen to be entangled in the protoplasmic processes, and throughout the germinal disc itself there are a number of yolk masses varying in size. At this stage great activity is manifested by the protoplasm immediately adjoining the yolk, and, as already stated, the whole mass of the germinal protoplasm has an undulating movement. In the living egg it frequently happens that there is a large temporary collection of the germinal protoplasm at the yolk pole. This has been termed the *Gegenhügel* by KUPFFER. Sometimes this accumulation is so large that it may easily be mistaken for the true germinal area.

At this stage KUPFFER describes an appearance in the living egg, which is intimately connected with the growth of the germinal disc. He says that when the germinal protoplasm forms a distinct layer on the surface of the yolk, the surface vacuoles disappear, and there remains only one or a pair of *large lacunæ* in the centre, which are often continued towards the flat base of the germinal disc by a stalk. These have much in common with the *latebra* of the hen's egg. According to KUPFFER, these *lacunæ* are not to be considered as distinct caverns, but as transition areas (*Schmelzungsheerde*) in which the globular yolk masses are transformed into a more clear and uniform mass. These, he maintains, are to be found till the end of development. RYDER suggests that the lacunæ, as described by KUPFFER, do not normally exist, but that they are produced by the hardening reagents used in preserving the ova. Such, however, is not the explanation. It is in the *living* egg that such an appearance is seen (see fig. 9) before any reagents have been used. There are no such lacunæ to be found in sections of hardened material. The appearance, I think, admits of a similar explanation to that which has been given for the "vacuoles," of which it is supposed to be the remnant. *The base of the germinal disc is not flat,* as was supposed by KUPFFER. There are proceeding from it a number of broad but tapering strands of protoplasm, which form the means of communication between the germinal disc and the yolk. In this region the mixture of yolk and protoplasm is more transparent than the more solid yolk mass, and thus in optical section appears as a cavity. The peculiar shape of these so-called lacunæ appears to be altered by hardening agents, as I have never in section met with such flask-shaped masses as are seen in the living egg. Later, as the protoplasmic filaments are withdrawn, the central mass of yolk spheres lose their rounded outline, and appear to fuse together; whereas the more peripheral ones retain their primitive form, being always more or less surrounded with protoplasm. Thus, again, in these later stages the central portion appears more transparent than the peripheral zone. KUPFFER'S interpretation also requires modification in another respect. One is led to conclude from his remarks that the yolk is directly transformed into protoplasm in the "transition area" already spoken of. This is not really the case. The yolk only becomes available for the use of the blastoderm after it has been assimilated and digested by the existing protoplasm. While the germinal disc is in progress of formation, its somewhat conical and ill-defined base is actively engaged in this process of assimilation. In the base of the disc large masses of food yolk may be seen entangled between the branching filaments, and these get smaller and smaller as they are pushed farther away from the active area.

It is important to note that at the time the upper portion of the disc is ready for segmentation the lower portion is still actively fulfilling a vegetative function.

It thus comes about that both KUPFFER and HOFFMANN were partly correct in their conceptions of the germinal disc, but, according to my view, neither of them were entirely so. It is by a union of both ideas that a true comprehension of the question is obtained.

The egg of the herring has now reached a stage when it is comparable with those of other fishes. Indeed, so nearly does it approach to WALDEYER'S (26) *ideal* of a meroblastic ovum that it might very well have served as his type. Before the first furrow appears the egg is made up as follows :—

1. Of a large collection of protoplasm in the germinal area in which segmentation subsequently commences.

2. Of a thin film of cortical protoplasm entirely surrounding the yolk, and which frequently presents a considerable dilatation at the yolk pole.
3. Of a number of filamentous protoplasmic processes, mainly confined to the base of the germinal area, which serve to keep up a communication between the latter and the more purely nutritive yolk.
4. Of the nutritive yolk itself, which constitutes the greater portion of the ovum.

Thus it will be seen that the egg of the herring fulfils all the conditions of WALDEYER'S typical meroblastic ovum. The rôle played by these constituent parts in the economy of the embryo is very marked. The part played by the cortical protoplasm, and the root-like filaments, is particularly well brought out in the herring. A discussion of the whole subject will, however, be deferred until we consider the origin and growth of the *parablast.* Shortly before the appearance of the first furrow the disc *as seen in optical section* has a diameter of ·84 mm., and is ·28 mm. in thickness. On account of the collection of the germinal protoplasm at one pole the egg loses its previously rounded outline, and has now a diameter of 1·60 mm. in its longer axis, and 1·48 mm. in a direction at right angles to this.

Segmentation.

The appearance and behaviour of the first segmentation nucleus in the fish ovum has not been satisfactorily explained. HOFFMANN (14), indeed, has figured in a very diagrammatic manner the appearance and position of this nucleus when it first divides; but so far as I am aware his observations have not been confirmed, nor have they received any support from the work of recent investigators. In the case of the herring I have used the most approved methods of fixing and staining the material, but have as yet failed to observe a nucleus of any kind *until after the third furrow has been formed.* The gradual disappearance of the germinal vesicle in the ovum as it approaches maturity has been already alluded to. Judging from analogy, a portion of the germinal vesicle must remain as the female pronucleus. Having failed to demonstrate this as a defined and recognisable mass, it appears necessary to assume that it is distributed throughout the germinal protoplasm. I am not prepared to prove this view, which undoubtedly is not in harmony with our information in other cases, but it receives considerable support from a knowledge of the behaviour of the germinal protoplasm during the early segmentation stages. At the time of the appearance of the first furrow I have not succeeded in demonstrating a nucleus either in the living egg or in prepared material. It has thus been impossible to test the statements of KUPFFER and HOFFMANN as to the direction of the first plane of cleavage *from actual observations of the nucleus,* or of any trace of karyokinetic figure. So far as the herring is concerned, other phases of the process may be followed, which give a clue not only to the *direction* of the early planes of cleavage, but also show in a most decided manner the true nature and mode of origin of the two primary germinal areas —the archiblast and parablast.

According to KUPFFER, the first furrow is *meridianal* in direction—that is, in the direction of the axis of the egg. The second is *equatorial*, and with the completion of this the germinal area is divided into archiblast and parablast. The third furrow is *meridianal*, but at right angles to the first, and after this the segmentation proceeds in the usual way.

HOFFMANN, on the other hand, maintains that in the fish ovum the first segmentation furrow takes an *equatorial* direction, so that instead of the first cleavage process resulting in the formation of two segmentation spheres, the germinal area is divided into *two layers* corresponding to the archiblast and parablast, each of which contains half of the original segmentation nucleus. According to this arrangement, the parablast is given an equal value with the archiblast at the outset, and yet HOFFMANN denies that it takes any part in the formation of the *tissues* of the embryo.

The majority of observers have not described an *equatorial* furrow in the Teleostean fish ovum until a considerably later stage, and it has been generally accepted that there is no furrow in the fish ovum which is equivalent to the first equatorial furrow (the third of the series) in the amphibian ovum.

AGASSIZ and WHITMAN (1) differ from other investigators in attributing the whole of the germinal protoplasm in the first instance to the archiblast, and derive the nuclei of the parablast from the archiblast as secondary products which are derived from its marginal cells.

From my own investigations, I conclude that in the herring, as in so many other forms, the first furrow which takes an equatorial direction is the *third of the series.* This furrow is therefore comparable with the third furrow in the amphibian ovum.

After the protoplasm in the germinal area has increased considerably in bulk at the expense of the yolk, there is usually a comparatively quiescent period. During this stage a part of the germinal protoplasm collects at the *yolk pole,* and there forms a small mound. As soon, however, as the first traces of a furrow are to be seen this mound gradually disappears, and the protoplasm of which it was formed slowly travels along the surface of the yolk to join in the approaching period of activity in the germinal area. There remains, however, a thin film of protoplasm around the yolk both in this and in later stages. This does not form a flat layer, but is seen in section to follow the outline of the yolk spheres, and frequently to fill in the spaces between them.

The surface of the germinal protoplasm, which has hitherto been much arched, commences to flatten towards the centre. As a result of this flattening, a vertical furrow is slowly pushed down towards the surface of the yolk, as seen in optical section, but stops short some distance above the latter. A study of stained sections of this stage shows several points which cannot be made out in the living egg. The upper portion of the germinal area contains very little food yolk, but between this and the main body of the yolk material there is a somewhat triangular area containing a larger quantity of yolk imbedded in the protoplasm, and from the base of which the protoplasmic processes are pushed down into the yolk. This area is actively engaged in assimilating food material, and is crowded with particles of yolk. At the time, therefore, that the upper and older portion of the germinal area is ready for segmentation, the material included in the lower portion adjoining the yolk is not so far advanced. It is one of the recognised laws of segmentation that the rapidity with which any part of an ovum segments varies, *ceteris paribus*, with the relative amount of protoplasm it contains. The protoplasm in this vegetative portion of the germinal area contains as yet too much undigested yolk for it to take part in the segmentation process. Thus the first furrow progresses more slowly as it approaches this area, and for the time being is arrested before it has reached the base of the germinal disc. Another point is also shown in sections of this stage, which will be considered more fully at a later period, but which should be mentioned here. The first furrow is not a continuous plane of cleavage in the first instance. There are in the line of cleavage a series of small vacuoles, which, as they become more elongated, run together, and so form a considerable portion of the furrow. Thus the first two segmentation spheres are in part separated by a process of vacuolation. The first two segmentation spheres become entirely separated from one another at their upper poles, the series of vacuoles aiding considerably in this process. The germinal area is thus divided into two portions, which are completely separated above, but which are united at the base, the first furrow not having completely penetrated to the base.

After the first furrow has been formed, a comparatively quiescent stage follows, during which a part of the germinal protoplasm again collects in a mound at the yolk pole. This is not an accidental occurrence, but has been already observed by KUPFFER, and I have had frequent opportunities of observing this process. During this quiescent period it is usual for a nucleus to make its appearance towards the centre of each segmentation sphere in pelagic fish ova. Such nuclei in the living ovum have the appearance of a more transparent area, usually distinctly marked off from the surrounding protoplasm. They usually disappear again before the next active stage begins, again to reappear during the following quiescent period. I have not observed such nuclei in the living egg of the herring at this stage, nor have I been able to make them out in stained preparations.

The formation of the second meridianal furrow is merely a repetition of what has been described for the first. The protoplasm at the yolk pole again joins that in the germinal area, and a second furrow is formed at right angles to the first. This second furrow, like the first, does not quite reach the base of the germinal protoplasm. When the first two furrows are completed, the protoplasm in the germinal area is imperfectly divided into four segments, which are not defined at the base. There is again a quiescent stage during which a small thickening of germinal protoplasm is again to be seen at the yolk pole.

The next furrow is equatorial in direction, and simply completes the contour of the four existing segmentation spheres. Before this can be formed, the protoplasm through which it will pass must have so far completed its process of assimilation as to allow segmentation to proceed. Shortly before the equatorial furrow commences, the protoplasm at the yolk pole again joins that in the germinal area, and the furrow is then formed very slowly, and is at first indistinct. It is situated towards the base of the germinal area, and with its completion there are formed four segmentation spheres, which are now isolated from the yolk. Below the furrow a small portion of the germinal protoplasm remains, part of which forms branching processes into the yolk immediately below the segmented portion, and the remainder becomes distributed around the yolk mass.

There is thus set up a division into two distinct layers, the archiblast and parablast. The archiblast is cut off from further direct communication with the yolk, and goes on segmenting. The parablast comprises that portion of the germinal protoplasm which is not included in the archiblast, and which remains as a connecting area between the latter and the yolk. For the time being it remains comparatively inactive, but later has a very important part to play.

The time occupied by all these changes is about 9½ hours, at a temperature of 41° to 44° F. There is a slight variation in the rapidity with which the various eggs develop, which becomes more marked as development proceeds. The following table shows the details of the process :—

Commencement of 1st furrow,	6¼ hours	after impregnation.
Completion „	6¾ hours	„
Commencement of 2nd furrow,	7¾ hours	„
Completion „	8⅓ hours	„
Commencement of 3rd furrow,	9⅓ hours	„
Completion „	9¾ hours	„

It will be noticed that in each case there is an hour's interval between the completion of one furrow and the commencement of the next. According to Kupffer's observations, the first furrow commences about two hours after impregnation. He does not, however, state the temperature at which the observations were made, but I gather from remarks in another portion of the paper that this was probably between 60°·8 and 64°·4 F.

The influence of temperature on the rate of development of fish ova is very great, and has been already studied by MEYER in the case of the herring. The sensibility of the ova of the herring to a change of temperature is almost as marked as that of pelagic fish ova. There can be no doubt, however, that under natural conditions the eggs of the herring are not so liable to sudden changes of temperature as are those which float at or near the surface of the sea. The range of temperature at which the eggs of the herring will develop normally is much wider than is the case for the eggs of the Salmonidæ.

On the completion of the four-cell stage—that is, after the formation of the third furrow—the segmented disc measured in one case ·9407 mm. in diameter, and ·3198 mm. in thickness; in another egg the measurements were ·9595 mm. and ·2822 mm. respectively. After the furrows are completed, the individual cells separate more or less from one another. This is accomplished in the following manner:—The adjacent cells begin to separate at the outer limit of the furrow between them, and at the same time a similar process commences at the inner extremity of the same furrow. Thus, at the point where the two furrows cross one another, there is a space formed, owing to the protoplasm receding somewhat from the former point of contact. In this respect the segmenting disc of the herring ovum presents a very different appearance from that which is seen in many other fish ova, for instance those of the Salmonidæ. Such an arrangement is of frequent occurrence amongst the Invertebrata, and is there connected with the formation of the segmentation cavity. In the herring the segmentation cavity arises at a much later stage, and has no connection with this structure, which, indeed, is here only a temporary one. As the cells become more completely separated, the central cavity is lost. This mode of separation of the cells in the early segmentation stages is not confined to the herring amongst fishes. A similar phase is found in the ova of *Perca, Leuciscus*, and other forms. In the species which show this partial separation of the early segmentation spheres, the cells at a later stage are always loosely aggregated together. On the other hand, in the group of which *Salmo* may be taken as the type, the cells are never so completely separated from one another, and there is almost an entire absence of those large spaces between the cells during the segmentation stage, which are of such common occurrence in the herring ovum. In this respect *Salmo* approaches more nearly to the Elasmobranch type, and the difference is probably connected with the distribution of food yolk.

Under normal conditions the separation of the four cells is never complete in the herring ovum. They present the appearance of four conical mounds united towards the base. I have seen similar effects in the eggs of the *Gadidæ,* but these, I think, have been caused by too great an elevation of temperature. *In such cases the separation may become complete,* when development is at once arrested, and the egg dies.

Returning now to the position held by KUPFFER, it will, I think, be seen that he must have been mistaken in the *order* of segmentation. It is only fair to state that his opinion, that the second furrow takes an equatorial direction, was founded on an observation of the process in the pike, and not in the herring. He concludes, however, that the early phases of segmentation are identical in both species. If the second furrow is *equatorial* in direction, why should the cortical protoplasm flow forwards from the yolk pole to the germinal area when the *third furrow* is about to be formed ? If there is an equatorial furrow already in existence, the upper portion of the germinal area which it defines must be cut off from communication with the lower part, and with the yolk. That this is not the case is shown by the fact that after the formation of each of the first three furrows, the cortical protoplasm diminishes in quantity. There is a further point. The cortical protoplasm flows from the yolk pole to the germinal area, and *becomes a part of the latter* during each of the first three segmentation stages. After this the remaining cortical protoplasm, which has very much decreased in bulk, does not again flow towards the germinal disc, *until the latter consists of a mulberry mass of cells, and awaits the co-operation of the parablast.* The cortical protoplasm continues to exist, and indeed to increase in bulk, but it is evident it can no longer take part in the segmentation of that portion of the germinal area which has been cut off from communication with it by the formation of an equatorial furrow. Nevertheless, it has a very important part to play.

The same arguments may be brought against the assertion of HOFFMANN, that the *first* furrow takes an equatorial direction in the fish ovum. HOFFMANN appears very certain about his interpretation, and gives a figure in which the nucleus is situated towards the base of the germinal disc, and in which the elongation of the nucleus during karyokinetic division takes a vertical direction. The plane of division is therefore at right angles to the nuclear axis, so that an equatorial furrow is formed.

It would not be just to deny the accuracy of such observations from a study of different material. HOFFMANN'S investigations were made on Julis, &c., and mine on the herring. It is possible that a difference in species may allow of a difference in the order of development. All I can say is, that in the herring neither the first nor the second furrow takes an equatorial direction, according to my own interpretation of the process. If either of them did, the whole plan of development would, to my mind, be changed.

Having concluded that the third furrow takes an equatorial direction in the herring ovum, it will be well to reflect on the significance of this fact. The main portion of the germinal protoplasm, which constitutes the archiblast, forms the *animal pole* of the egg, while the yolk, together with the residual protoplasm, is to be regarded as the *vegetative pole.* The animal pole at this stage consists of four segments or cells, while the whole of the vegetative pole *has the value of one cell.* The whole of the vegetative area may be compared to a gigantic fat cell in which the fat is replaced by food yolk. The function of the cortical protoplasm is to digest and absorb the food material as fresh nourishment is required by the growing organism. Having once grasped the significance of this point, the interpretation of future developmental phases does not present much difficulty.

The ovum of an amphibian is *holoblastic,* while that of the herring is *meroblastic,* yet this difference in the mechanical division of the ovum does not prevent a comparison of the two types. In an amphibian ovum, such as that of ***Rana,*** the majority of the germinal protoplasm has collected in the animal pole by the time that the first equatorial furrow is formed. The same is the case in the herring ovum. In ***Rana*** the vegetative pole consists at first of four large segments, which contain the greater portion of the yolk material, but which are also supplied with a considerable amount of protoplasm. The fact that at this stage the vegetative area consists of *four segments instead of one,* shows that the proportion of yolk to protoplasm is not so great as to entirely prevent the segmentation process from progressing. The proportion, indeed, is such that, in accordance with the law of segmentation, the division of the vegetative area is *slower,* and the resulting segments are *larger,* than is the case in the animal pole. It is also important to note, whilst making this comparison, that the protoplasm in the vegetative area of the amphibian ovum is not collected in any particular part, but that it is generally distributed throughout each segment. The same is the case with the yolk material. With further subdivision, therefore, each cell in the yolk pole consists partly of yolk and partly of protoplasm.

Thus each cell carries its food supply along with it. Such is not the case in the herring ovum, and it is this fact which constitutes the essential difference between the two types. The food yolk in the herring ovum does not segment. This absence of segmentation in the yolk of fishes arises from two causes—viz., the overwhelming preponderance of food yolk, and the absence of a sufficient quantity of protoplasm distributed through it. In the herring ovum the protoplasm in the yolk pole consists of a comparatively thin *cortical layer,* with a few branching processes pressing into the food material, which are almost entirely confined to that portion of the vegetative area on which the archiblast rests. *From the nature of this distribution,* the food supply cannot be assimilated in the same manner in the herring as it is in the amphibian ovum. In the herring the food yolk *must be digested* before it can be available as nourishment for the embryo. Had the separation of protoplasm from yolk been *complete* this end could not have been accomplished. It is therefore the function of the *residual protoplasm* in the yolk pole to make the food supply available for the use of the embryo. This is accomplished by a process which is essentially one of *intracellular digestion,* since the protoplasm in the yolk pole must be regarded as having the value of a cell, *and is directly derived from the germinal area.* The cortical protoplasm incorporates a portion of the yolk material within its substance, digests it, and thus adds to its bulk. A store of available material is thus laid up, which is utilised as occasion requires. It thus appears that the food yolk of the herring ovum is more nearly comparable *in its manner of assimilation* with the albuminous food supply of Lumbricus and some insects, than with the food yolk of the amphibian ovum. Nevertheless the yolk pole in the herring ovum has the same morphological value as that of the amphibian ovum, and it is mainly owing to the difference in distribution of the constituent parts that their subsequent behaviour is not identical. How far the tissues derived from each pole are identical in the two cases will be considered later.

Although I am firmly convinced that in the herring ovum the third furrow is an equatorial one, I am not at present prepared to assert that this is the case in all fish ova. Nevertheless, it appears to me probable that an equatorial furrow will ultimately be shown to exist in all Teleostean ova at an earlier stage than has generally been supposed.

To take the case of pelagic ova. In a paper on the development of *Trachinus vipera* (5) I have described the first furrow, which takes an equatorial direction, to be the fifth of the series, and to be formed in the four central cells of the 16-cell stage when the 32-cell stage is being produced. I am now, however, inclined to think that I have neglected to observe one furrow altogether—that, namely, which divides archiblast from parablast. At the time the ovum is fertilised the yolk consists of *one* large yolk sphere and not of a number of small ones, as is the case in the herring ovum. There is also no appreciable increase in the quantity of germinal protoplasm after the egg is fertilised. The germinal protoplasm in the ripe unfertilised ovum consists of an even layer entirely surrounding the yolk. As the circumference of the yolk sphere is quite smooth, the line of demarcation between protoplasm and yolk is well marked. After fertilisation the bulk of the germinal protoplasm sinks to the lower pole of the ovum to form the germinal disc. A thin film of protoplasm is, however, still left surrounding the yolk, and this gets thicker towards the germinal area. The first furrow appears as a longitudinal depression across the centre of the circular disc, and is pushed down towards the yolk. A little later each end of the furrow becomes forked. The forked extremities then grow round, so that the two on each side meet. Two cells are thus formed, whose outline is very distinct in the region of the original longitudinal furrow, but gets more and more indistinct towards the periphery of the disc. *The two cells thus formed do not enclose the whole of the protoplasm in the germinal area.* The protoplasm at this stage has a yellowish tinge, and in the living egg the faint yellow shade can be seen to extend outside the limits of the two segmentation spheres (see figs. 1–4, *loc. cit.*). By another vertical furrow at right angles to the first the germinal disc becomes divided into four cells. It will be remembered that in the herring the first equatorial furrow simply completes the base of the four existing segmentation spheres, and that with the completion of this furrow there is a division into archiblast and parablast. In pelagic fish ova, however, the germinal disc rests directly on the yolk sphere, there being no intermediate area which is actively engaged in assimilation at this stage. Thus the *base* of the early segmentation spheres is not easily recognised. Only in that portion of the germinal disc which rests on the cortical protoplasm, and not on the yolk, could any such furrow be made out. There is, however, reason to suppose that the base of the first two segmentation spheres is not defined, and that even the lateral line marking their periphery *does not reach the yolk* at this stage, from the fact that the two cells increase considerably in size after their outline has been defined. At the completion of the 4-cell stage the segmented disc is undoubtedly divided off from the cortical protoplasm, and there is a division into archiblast and parablast, as in the herring ovum. Whether the furrow (or equivalent of a furrow) which brought about this separation was formed before or after the second meridianal furrow, I cannot say definitely at present. Whichever be the case, it is clear that this furrow is homologous with the third furrow in the herring ovum, and also with the third in the amphibian ovum. In the case of the *Salmonidæ,* it appears to me also that the point which in reality corresponds with the first equatorial furrow in the amphibian ovum is reached at the 4-cell stage. When the base of these four cells comes to be defined there is still left a cortical layer of protoplasm, and a small quantity also mixed up with the yolk under the disc, as is the case in the herring ovum.

From what has been already said, it will be seen that I conceive the term *Archiblast* to be applicable to that portion of the ovum which is usually spoken of as the *germinal disc*—that portion, namely, which is included in the early segmentation stages. It is for this reason that I have preferred to use the term *germinal area* in describing all changes in the herring ovum *prior to the completion of the 4-cell stage.* The later stages of the segmentation process do not present any features of special interest. A stage in which the germinal disc consists of three rows of cells is shown in fig. 10. About 26 hours after fertilisation a *morula* mass of cells has been produced, which is represented in fig. 12. It will be seen from the figure that the cells of the archiblast have already become differentiated into two well-marked groups. The outer row of cells are elongated and flattened, forming an epithelioid layer, whereas the remainder are comparatively large round cells, which are only loosely aggregated together. During the time that segmentation has been progressing in the *archiblast* the cortical protoplasm has increased considerably in bulk. In sections it may be seen that this layer gradually forms a thickening at the yolk pole. When the *morula* stage is reached the cortical protoplasm leaves the yolk pole, and gradually collects around and under the segmented disc. Even at a considerably earlier stage the cortical protoplasm may be seen to be accumulating towards the disc (see fig. 13, Pl. XIV.); but after the morula stage is reached there is no longer any protoplasm to be observed at the yolk pole *in the living egg.* In stained sections there is always a thin film of protoplasm to be seen around the yolk spheres, and this is increased from time to time by the assimilation of more yolk.

The collection of unsegmented protoplasm will be spoken of as the *parablast.* As will be seen later, the part played by what I term *archiblast* and *parablast* in the herring ovum is not identical with that which has been described by HIS and others in other forms, but I conceive the same terms to be applicable.

The Part played by the Parablast.

Historical.—The mode of origin of the parablast, and the part which it plays in the economy of the embryo, has during the past few years been one of the most keenly contested problems in this branch of embryology. In 1868 HIS brought forward his well-known theory as to the development of the tissues in meroblastic ova (12). He held that in the chick the whole of the tissues of the future embryo were not derived from the three-layered blastoderm. That the blood and connective tissue series of structures arise independently of the segmented disc, and take their origin from the white yolk substance immediately underlying the blastoderm and outside the embryonic area. In their mode of origin the former set of tissues are known as *archiblastic,* and the latter as *parablastic.* According to HIS, the nuclei of the parablastic cells are derived from the white yolk spheres, which themselves have the value of cells. The segmented disc supplies the material for the three germinal layers, and the cells from the parablast find their way in between the cells of these layers. In the same year KUPFFER (18) described in Teleostean fish ova a layer of protoplasm outside the germinal disc, in which, at the close of segmentation, concentric rows of free nuclei make their appearance. His investigations were made on *Gasterosteus aculeatus,* and other forms, and the following short extracts from his paper show the position taken up at the time :—" Man sieht

. . . . . rings um den Rand des Keimhügels Kerne auftreten, die in ganz regelmässiger Weise angeordnet sind. Es sind wasserklare, runde Bläschen, ohne irgend welche Körnchen im Innern, die in concentrischen Kreislinien, auf das Centrum des Keimhügels bezogen, sich gruppiren." "Man sieht nämlich zwischen den bläschenartigen Kernen zarte Contouren auftreten, die genau an einander schliessende polygonale Felder umgrenzen, deren Mittelpuncte die Kerne einnehmen. Kurz es entsteht eine Lage eines regelmässigen, aus hexagonalen Zellen gebildeten Platten-epitheliums." ". . . . . Unter den sich furchenden Keime ein besonderes Blatt sich bilde, das nicht aus den Furchungszellen herzuleiten ist, denn die Zellen desselben entstehen frei in einer den Dotter bekleidenden dünnen Blastemschicht, indem als Erstes die Kerne derselben erscheinen." "Ob dieses Blatt wirklich zum Darmdrüsenblatt wird, muss dahingestellt bleiben, vielleicht ist es nur eine vorübergehende Bildung, was aber wohl unwahrscheinlich."

So far as I am aware, this is the first clear account of a nucleated zone of protoplasm outside the segmented disc in Teleostean fish ovum. It is true that at an earlier date LEREBOULLET (21) observed a similar layer of cells in the egg of the pike, which he concluded was transformed into the lowest germinal layer. He was, however, of opinion that the cells were derived from small yolk spheres (globules vitellins), and failed to recognise the independent formation of the nuclei, which by KUPFFER were ascribed to "free cell formation."

So far as the Teleostei are concerned, the existence of this layer has since been thoroughly established. The points on which more recent investigators differ are firstly—the source from which the nuclei in this layer are derived; and secondly, the ultimate fate of the cells derived from it.

Ten years ago KLEIN (16) made an important contribution to the subject, and reviewed the position taken up by earlier authors. This author concluded, from a study of the early stages of the trout, that besides the blastoderm proper, it is necessary to study closely the behaviour of the subgerminal and paragerminal substance, which "bears at all stages of development an important genetic relation to the blastoderm and the embryo." KLEIN calls the segmented portion of the blastoderm *archiblast,* and the unsegmented portion in connection with the yolk is the *parablast.* I have followed KLEIN's nomenclature in the present paper, although, as already pointed out, the terms are not used in the sense originally applied to them by HIS. The parablast layer was first correctly noted by OELLACHER (23), who described it as continuous with the germinal disc in early stages, but mistook it for a vitelline membrane. KLEIN restricts the term parablast to the thickened welt of protoplasm, having a somewhat triangular section which forms a rim round the segmented blastoderm in early stages. Van BAMBEKE (3) has described a similar layer in

Leuciscus, under the name of the "couche intermédiaire." Klein and Van Bambeke differ considerably in their accounts of the early shape and position of this layer, but it seems probable that both authors are correct so far as the species studied is concerned, and that the parablast varies in position in different species and at different stages of development. Klein thus describes the appearance of nuclei in the parablast:—"Searching carefully through the parablast with a moderately high power (Hartnack's No. 8) we detect numerous *isolated*, small, transparent bodies, *very faintly outlined*, so as to be rendered just perceptible; between these and *distinct* nuclei *all intermediate forms may be met with as regards general aspect, outline, and size. This obviously means new formation of nuclei.* It therefore stands to reason to assume that, inasmuch as at a period when nuclei may be seen to multiply by division, the formation of nuclei *de novo*, as it were, still takes place in the parablast, *the first nuclei of the parablast have also originated in the same manner*, i.e., *de novo*."

Klein is of opinion that the peripheral thickening of the archiblast is caused by an addition of cells from the subjacent parablast, and that a large part of the hypoblast is derived from this layer. The evidence of Van Bambeke also points to a similar conclusion.

Kingsley and Conn (15) describe an "intermediary layer" in the pelagic ova of several fishes, in which "free cell formation" takes place, but are inclined to regard the true hypoblast as derived, in the first instance, from an invagination of the ectodermal layer of the epiblast. Mr Kingsley, however, informs me that, since the publication of the paper referred to, he has been led to change his views on the subject.

Hoffmann (14) affirms that the parablast arises with the formation of the first furrow, *which takes an equatorial direction*, thus dividing the germinal disc into two layers, each of which contains half of the first segmentation nucleus. According to this author, nuclei are abundant in the parablast during later stages, but the layer is not destined to take part in the formation of the embryo. The parablast, according to Hoffmann's view, is rather to be regarded as a degenerate relic of the vegetative pole in holoblastic types, which, owing to the increase of yolk, is no longer able to fulfil its former functions.

According to Agassiz and Whitman (1), the nuclei found in the parablast (periblast) are derived from the margin of segmented blastoderm (archiblast), and these authors figure a sixteen-cell stage, showing the origin of the nucleated subgerminal layer. It would appear, however, from their observations, that the parablast is not concerned in formation of the hypoblast, but that this is formed by a process of true invagination.

Cunningham (8) supports the views of Agassiz and Whitman regarding the origin of the nuclei in the parablast, but is inclined to regard the hypoblast as resulting from an invagination of the outer, instead of the inner portion of the epiblast. Regarding the latter point, CUNNINGHAM may be said to stand almost alone in supporting HAECKEL'S view of the *formation* of the Teleostean gastrula, KINGSLEY, as already stated, having ceased to hold that view. The general question of the origin and significance of the parablastic layer in meroblastic ova has not received much attention in this country. BALFOUR (4 and 9), while accepting to a certain extent HIS'S view as to the development of free nuclei in the surface yolk of Elasmobranchs, and in the white yolk underlying the blastoderm of the chick, does not accept his terminology. According to BALFOUR, a number of cells are formed in the upper strata of the yolk, which unite with the cells of the blastoderm during the processes of invagination and differentiation of the germinal layers, but these are apparently of only secondary importance. If the so-called "germinal wall" of the chick embryo is a portion of the layer here termed parablast, and there seems no room for doubt on this point, BALFOUR certainly held that certain portions of *all* the germinal layers may chiefly or partly be produced from this layer. Speaking of the differentiation of the layers in the *area vasculosa* of the chick, he says :—"The mesoblast and hypoblast of the area opaca do not arise by simple extension of the corresponding layers of the area pellucida ; but the whole of the hypoblast of the area opaca, and a large portion of the mesoblast, and possibly even some of the epiblast, take their origin from the peculiar material which forms the germinal wall, and which is continuous with the hypoblast at the edge of the area opaca." In his latest contribution to the parablast question, HIS (13) withdraws from the position which he formerly held in respect to the cellular character of the white yolk spheres, and consequently of the derivation of nuclei and cells from them. According to his view, the relation of the parablast to the embryo may be shortly summarised in the following manner :—The segmented blastoderm (*archiblast*) gives rise to the three primary germinal layers, epiblast, mesoblast, and hypoblast, but these only give rise to *archiblastic* tissues.

The *epiblast* gives rise to the epidermis and the true glands derived from it, and to a part of the epithelium of the digestive tract, as well as to the nervous system.

The *hypoblast* forms the rest of the epithelium of the digestive tract, and the glands belonging thereto.

The *mesoblast* (or, more properly speaking, that portion of it which is derived from the archiblast) gives exclusively smooth and striped muscles, together with the epithelium of the urogenital tract. Mesoblast also gives rise to the primitive clothing of the cœlom; but, according to HIS, this is only transitory, and later another connective tissue covering takes the place of the primary one.

The whole of the blood and connective tissue, in its widest sense, are developed at a later period *outside the region of the segmented blastoderm*, and are therefore parablastic in their origin.

Thus the mesoblast in the ordinary sense (the middle germinal layer of Remak) is a compound, and not a simple layer, and the two portions may be spoken of as archiblastic and parablastic mesoblast. The parablastic mesoblast of His almost exactly corresponds with the mesenchyme of the brothers Hertwig (11).

Finally, then, according to His, the parablast has nothing to do with the formation of the primary germinal layers, but is utilised later to form that portion of the mesoblast which gives rise to the blood and connective tissue series.

More recently Waldeyer (26) has contributed a most important paper on the subject, which, I take it, goes to the root of the matter. Waldeyer calls attention to the structure of a typical meroblastic ovum, and to the relative distribution of protoplasm and yolk. The yolk is passive food material, which can only be utilised by the embryo after assimilation. Beneath the germinal disc there are a number of protoplasmic processes (Keimfortsätze) which press in amongst the passive food material; and there is also a thin cortical film of protoplasm around the yolk. Segmentation takes place in the germinal disc, but does not affect the protoplasmic processes or the cortical layer. Later nuclei appear in the protoplasm, which is as yet unsegmented, *and not in the yolk itself.* The cells thus produced give rise to parablastic tissues. Thus the parablast layer is derived from the original protoplasm of the ovum, and not from white yolk cells, and its nuclei are also derivatives of the first segmentation nucleus. Waldeyer distinguishes a *primary segmentation*, resulting in the formation of the archiblast, and a *secondary segmentation*, which frequently takes the form of budding, by which the parablastic tissues are derived. Waldeyer also points out that there is no essential difference between meroblastic and holoblastic eggs; but that throughout the animal kingdom a graduated series of modifications in the segmentation process are to be noticed, which are largely due to the varying quantity of passive food material contained within the ovum. It is also certain that the unequal distribution of the yolk is as important as its quantity in bringing about modifications in the segmentation process. According to Waldeyer's view, the formation of parablastic elements in holoblastic eggs is more easily explained than on His's view. During the segmentation process division takes place most rapidly in that part of the ovum containing least food yolk. At the base of the vegetative pole those cells are found which contain most yolk, and therefore segment more slowly. Those cells which are ready for tissue formation arrange themselves into the three primary layers, as in meroblastic ova, and constitute the archiblast. The cells not yet ready, and overladen with nutritive yolk, bud off later processes of protoplasm containing nuclei, which give rise to the parablastic elements of the embryo.

Waldeyer admits that parablast cells may take part in the formation of the hypoblast in some forms, as has been maintained by so many authors; but thinks its chief function in the higher vertebrates, at any rate, is to elaborate those cells which give rise to the blood and connective tissue elements.

Kollmann (17) maintains that the layer which gives rise to the blood and connective tissues represents a distinct advance on the triploblastic arrangement of invertebrates, and raises it to the rank of a primitive organ under the name of *acroblast*, and gives it an equal value with the other germinal layers. He points out that acroblast exists in Aves and Lacerta as a peripheral thickening between the epiblast and hypoblast before the mesoblast (the archiblastic mesoblast of Waldeyer) is formed. The cells in this thickening give rise to a series of amœboid wandering cells (poreutæ) by division, and these in their turn fill in the serous cavities between the other germinal layers, and form the blood and connective tissue.

My own Observations.—I will now describe the changes which take place in the parablast, as I have observed them in the herring and other forms.

In the preceding section I described the appearance of the parablast at the end of what I consider the *primary* segmentation stage in the herring. The parablast, which has increased very considerably in bulk at the expense of the yolk, leaves the periphery, and collects mainly under the archiblast. About twenty-six hours after fertilisation transverse sections of the egg present the appearance shown in fig. 12. The archiblast has become differentiated into two layers. The outer small and somewhat flattened cells, which stain deeply with carmine, constitute the epidermal layer of the epiblast. The cells more centrally situated are larger, more rounded, and do not stain so deeply. They are loosely aggregated together, and represent the nervous layer of the epiblast in other Teleostean types which I have examined. It must, however, be remembered that we have not yet arrived at the invagination stage, and the germinal layers will not be differentiated for some time. Beneath the archiblast the parablast appears as a thick layer of protoplasm *which is undergoing division into cells.* Clear vacuole-like spaces are recognisable at irregular intervals both in the peripheral parablast and in the part more centrally situated. Around these clearer spots the protoplasm is becoming divided off so as to form a number of cells. The lines of fainter colour in the parablast represent the planes of division. So far as I can make out, there is no karyokinetic figure during this process of cell formation in the parablast, and each nucleus arises independently of its neighbour, in a manner similar to that which I have described for *Trachinus*. The observations of Kupffer, Klein, and others, are very clear on this point; and I have frequently observed the full formation of nuclei both in the living egg and in the prepared material. The protoplasm in which these nuclei appear is, however, part of the original germinal layer of the ovum; and it thus appears probable that they are to be regarded as derivatives of the first segmentation nucleus. I thus regard the cells formed in the parablast as secondary segmentation products in the sense of Waldeyer, and must leave the question of nuclei open until we have more detailed information on the subject. If, indeed, the observations of Hoffmann should prove true for all Teleostean fishes, the question presents no further difficulty; but, as already stated, I have not been able to accept Hoffmann's views.

The cells thus formed in the parablast are next set free from their bed of unsegmented protoplasm, and join those derived from primary segmentation in the archiblast. About the same time the cells in the archiblast undergo division, and soon the cells derived from the parablast are no longer distinguishable from the archiblast cells. As already stated, fig. 12 represents a section of an ovum of the herring twenty-six hours after fertilisation. In fig. 14, which is from material preserved two hours later, the cells derived from the archiblast are easily distinguished from those which have been recently added from the parablast. The archiblast cells stain more deeply, and besides nuclei there are indications of an intracellular reticulum. Perhaps the most noticeable characteristic is that the archiblast cells are loosely collected together, and present in section a number of vacuoles between adjoining cells which have not yet become completely separated. The parablast cells, on the other hand, only stain faintly, show little structure, and are closely crowded together beneath the others. In fig. 16, which represents the appearance two hours later again, all difference between the two sets of cells is lost, and if it had not been for the two previous stages, one would not have known that the archiblast had received any addition of cellular elements from the parablast. The unsegmented portion of the parablast still remains as a somewhat thin film beneath the segmented blastoderm, and again increases rapidly in bulk by assimilation of food yolk. Four hours later than fig. 15 a number of cells are again in process of formation, which are destined to be included in the segmented blastoderm in a similar manner to the first batch. Fig. 17 represents a section of this stage. The basal portion of the segmented blastoderm is represented with its adjoining parablast. The two portions are not in contact at the periphery, but this is the result of a mechanical injury. An endeavour has been made to represent as nearly as possible the exact appearance and structure of each cell in the portion of the section represented. A gradual transition may be traced from the basal portion of the parablast, in which no nucleus is to be found, to the completely separated cells, which are already included in the morula. The first trace of cell formation is seen in a number of somewhat deeper stained patches of protoplasm, the colour getting more intense towards a centre, and gradually fading away from that point. A little later the cell contour becomes visible as a less deeply stained outline. In the next stage the nucleus appears as a less deeply stained portion in the centre of the cell. Its outline is almost circular, and a delicate more deeply stained reticulum may be observed in its interior. *The deepest staining is now around the periphery of the nucleus,* and the colour gradually becomes less intense towards the cell wall. Later the more deeply stained granules in the cell plasma arrange themselves in the form of a reticulum, which is ultimately connected with that of the nucleus. By this time the cells are indistinguishable from those already forming part of the morula. In some of the cells more transparent vacuole-like structures are found in the cell plasma, both in the parablast cells and in those of the morula, but are much more numerous in the latter. Possibly these structures may be connected with the nourishment of the cell, but this is not clear.

During the next few hours nearly the whole of the subgerminal parablast has been used up in budding off cells to join in the morula, which has in consequence increased very much in thickness, and presents a sharply curved upper surface. The subgerminal parablast in eggs forty-five hours after fertilisation consists of only a very thin film, as will be seen from fig. 18. The peripheral parablast, however, consists of a comparatively thick wedge-shaped mass, stretching from the base of the morula to the equator of the egg, and contains a considerable number of rows of nuclei. This portion of the parablast has been gradually increasing in importance while the changes which I have just described have been taking place in the subgerminal parablast, but has not taken part in these changes. It is this peripheral portion of the parablast which has usually received attention from various investigators, and in which the development of nuclei has been so often observed in the living egg. The nuclei appear in concentric rings around the base of the morula, the first ring being formed in the thickest portion of the layer, *i.e.,* the part immediately adjoining the morula. A faint cell outline can usually be observed in the protoplasm around each nucleus, but after a time this appears to become obliterated, though not so early as in *Trachinus.* In the living egg a single row of nuclei can be observed at a stage corresponding to fig. 12. At the stage shown in figure 14 there were three or four rows of cells in the peripheral parablast. In the living egg, of which fig. 18 is a section, the peripheral parablast has become cellular almost to the equator. The cells are arranged somewhat irregularly. In the portions more densely crowded with nuclei, the cell outline is polygonal, whereas where there are fewer nuclei the cell contour is more rounded. So far as I am aware, this stage represents the earliest one at which cells from the parablast have been described as taking part in the formation of the embryo in other Teleostean types. It corresponds to that of fig. 2 in my account of the pseudo-invagination in Trachinus (6). The extension of the morula over the yolk, the formation of the segmentation cavity, and the differentiation of the germinal layers has not yet commenced, though these changes immediately follow on this stage. So far as my investigations go, I am not aware of any case exactly comparable with this, though further investigations may probably reveal such. In all the Teleostean types with which I am acquainted, with the exception of the herring, the *primary* segmentation process results in the formation of a morula corresponding to that of fig. 18, and it is only in stages immediately following this that the elements resulting from secondary segmentation take part in the further differentiation of the blastoderm. In other words, in most Teleostean ova, the morula consists solely of archiblastic elements, and it is only after the formation of the segmentation cavity, and the commencement of invagination, that the parablastic elements come to be utilised. In the herring, as we have seen, at least two distinct batches of parablast cells are budded off, and unite with those of the archiblast before any trace of differentiation of the morula is to be found. The final morula in this case contains parablastic as well as archiblastic elements. The difference is important, and as I hope to show later, is connected with the early elaboration of the parablast, and probably also with the absence of a vitelline circulation in this type.

Shortly after the stage represented in fig. 18 the morula begins to spread out over the yolk, and the extension is accompanied by a thinning out of the central portion, which has hitherto been thickest. In this way a segmentation cavity is formed, which, however, never reaches so important a development in the herring as in some other Teleostean types. The parablast forms the floor of the segmentation cavity, while the cells of the morula form its roof and lateral boundaries. Fig. 21 (Pl. XV.) represents a longitudinal section in the axis of the embryo 68 hours after fertilisation. The roof of the segmentation cavity is formed of several rows of cells, the most external constituting the epidermal layer of the epiblast. Towards the periphery of the blastoderm there is a thickening forming the commencement of the blastodermic rim. There is no invagination of the epidermal layer of the epiblast, but the thickening may possibly be in part produced by an invagination of the nervous layer. I am, however, inclined to believe that the thickening, *in so far as it does not result from an addition or segregation of cells from the parablast*, is due to mechanical agencies. As the cells forming the roof of the segmentation cavity press towards the periphery, so as to aid the blastoderm in its extension over the yolk, they would naturally form a thickening on the under surface. The floor of the segmentation cavity is lined by a comparatively thick layer of parablast, in which a number of free nuclei are embedded. The parablast extends under the thickened peripheral portion of the blastoderm, and around its margin forms a thickened welt, in which a number of nuclei are also found.

Fig. 22 represents a portion of the section more highly magnified. The peripheral parablast is richly charged with nuclei, as also is that lining the floor of the segmentation cavity. These nuclei, before the formation of the peripheral thickening, were abundantly distributed *throughout* the parablast, as may be seen by a reference to fig. 20, which represents a slightly earlier stage. Now, however, the portion of the parablast on which the thickened rim rests is very thin, and is quite devoid of nuclei, whereas both in the peripheral parablast and in that portion in front of the thickening nuclei are still numerous. It is, therefore, impossible to avoid the conclusion that the nuclei, and a large portion of the protoplasm formerly situated in the region of the thickened rim have been used up in the formation of that thickening. The layer, which is pushed inwards from the thickening, constitutes the primitive hypoblast, so that I am brought back to my former observations on the development of Trachinus and Motella (6), and can only reiterate that this layer is mainly if not entirely formed by a segregation of cells from the parablast. A study of fig. 22 also brings out another important point. The cells of the morula rest directly on the parablast in the region of the blastodermic rim, whereas the two layers are separated more centrally by the segmentation cavity. The result is, that the primitive hypoblast is closely connected with the epiblast of the morula in the blastodermic rim, whereas the layer as it gradually fills in the segmentation cavity never adheres to the epiblast, but is always distinctly separated from it by a slight remnant of the cavity itself. It is in this manner that I would propose to get rid of one of the chief arguments against the parablastic origin of the primitive hypoblast. It has been argued by Henneguy (10) and others, that if the primitive hypoblast in Teleosteans was really formed from a different source than the archiblast, the two primary layers ought to be distinct throughout their entire length. In other words, that the separation of the primitive hyploblast and epiblast towards the centre of the embryonal shield, and their close union at the periphery, was a strong argument in favour of the origin of the primitive hypoblast as a true invagination of the archiblast. Hoffmann holds similar views. It will, however, be easily understood that this close union of the primitive layers at the periphery is equally the necessary result of the views which I advocate. The segmentation cavity does not extend across the whole diameter of the disc as was advocated by Haeckel, but the peripheral portion of the disc always rests on the parablast for about 1-6th of its diameter. It is exactly in this part, and in this part only, that the two layers are originally in close union.

I need not describe in detail the further advance of the primitive hypoblast, and the gradual obliteration of the segmentation cavity as the result. Throughout the process the parablast is very active in its elaboration of food yolk, and is continually supplying the new layer with more cells. Fig. 23 represents a *transverse* section of the body axis at a considerably later stage. It shows the primitive hypoblast in close union with the parablast, from which it is derived, but almost completely separated from the cells forming the roof of the segmentation cavity. Fig. 24, which is a more highly magnified view of a portion of this section, shows several important points. The intimate connection between yolk and parablast cannot fail to be noted, and the physiological function of the layer—the elaboration of fresh material for the embryo—is well brought out. The upper portion of the parablast contains a large number of free nuclei, which agree in every particular with the nuclei of the primitive hypoblast cells. Around some the protoplasm is seen to be constricted off to form cells. These are seen in all stages, from those completely embedded in unsegmented protoplasm to those adhering to the parablast only at one point.

Although at some points the epiblast and the primitive hypoblast are in contact owing to the rapid and prolific growth of the latter, it appears probable that the cells situated above the segmentation cavity give rise to the epiblast only. While the primitive hypoblast remains still undifferentiated, the cells of the epiblast collect, especially in the head region, forming a special thickening for the rudiment of the central nervous system. In the axis of the embryo the primitive hypoblast in the region of the head is pushed to each side by this enormous development, so that, in longitudinal section, the primitive hypoblast appears to cease in the posterior portion of the head swelling. Such a section is shown in fig. 25. The nuclei in the parablast are still prominent, and the two primitive layers are separated by a small space in the neck region, but are united towards the caudal extremity. Fig. 26 represents a portion of the section situated near the tail swelling, more highly magnified. The cells in the epiblast are closely packed together, whereas those in the primitive hypoblast are more loosely arranged. The two layers are not, however, distinctly separated, though a difference in the staining of the nuclei indicates the point of union of the two layers. The parablast still persists as a thin, unsegmented layer of protoplasm closely connected with the yolk, in which numerous free nuclei are embedded.

At a later stage in transverse section (fig. 30), the primitive hypoblast cells in the median line change their appearance considerably. A number of cells, forming a somewhat circular or slightly quadrate cord, lose their distinct outline, and seem to fuse together to form a more solid mass. This is the commencement of the notochord. The two lateral portions become separated from it as the lateral mesoblastic plates, and a single row of cells remains in connection with the parablast, which constitute the commencement of the permanent hypoblast. The lateral plates of mesoblast are completely separated from the epiblast, but above the notochord there is an arrangement of the cells which would lead one to suppose the two had been in close union. It thus appears that in the herring the three germinal layers are not completely differentiated until the notochord has made its appearance to separate the two lateral plates of mesoblast. Though I cannot speak with absolute certainty, there appears every probability that the primitive hypoblast gives rise to notochord, mesoblast, and permanent hypoblast, and that the morula mass of cells existing prior to the formation of the primitive hypoblast (the archiblast in other Teleostean types) persists as the epiblast. It is possible, however, but by no means sure, that some of its cells are included in the upper rows of mesoblast cells.

Theoretical Considerations.—Since my observations were completed and the bulk of the present paper written, I have received a paper by Dr Rückert (25) on the Formation of the Germinal Layers in Elasmobranchs, published about six months ago, in which the author has come to very similar conclusions to those here advocated. His observations were made chiefly on the eggs of *Torpedo*, and the following is a short summary of the results arrived at:—

1. The free nuclei in the yolk of meroblastic eggs (which Rückert terms merocytes) are segmentation products which have undergone a secondary modification under the influence of the food yolk. Their mode of origin is most clearly demonstrated by the peculiar segmentation of the richly deutoplasmic but still holoblastic eggs of many invertebrates (Insecta, Crustacea, Vermes). In these there is at first a total segmentation, but later the nuclei in the vegetative pole, together with the surrounding protoplasm, separate themselves from the deutoplasm, and while undergoing frequent division produce embryonal cells which are undistinguishable from those formed by regular segmentation. The deutoplasm in the vegetative pole becomes by this means passive food material, and the whole original holoblastic egg a meroblastic one.
2. In Elasmobranchs the merocytes are amœboid (rhizopodenartig) structures whose richly ramifying processes absorb and assimilate the surrounding yolk. They produce later a number of embryonal cells by endogenous cell formation or budding.
3. The embryonal cells produced from merocytes take part in the formation of all the germinal layers. In most animals they form the entoblast,

mesenchym, and the blood. In many Arthropods they likewise supply the entire ectoblast.

4. In Elasmobranchs they play only a subordinate part in the formation of the ectoblast, and must here probably be regarded as homologous with the vegetative pole of holoblastic eggs.
5. The merocytes arise with the first equatorial division of the segmenting ovum. The germinal disc which represents the animal pole goes on segmenting. The merocytes increase only to a trifling extent during this period. The blastula cavity appears between the morula (archiblast) and the superficial layer of yolk charged with merocytes—that is to say, between the animal and the vegetative poles of the egg. Its roof gives rise to the ectoblast; its floor supplies the ectoblast in the following manner:—The embryonal cells formed from the merocytes are pushed up into the blastula cavity from the yolk, and close its lumen. This process commences all around the periphery of the disc, but is later mainly confined to the posterior position (embryonal shield). The blood and mesenchym cells also arise from merocytes.

It will thus be seen that, according to RÜCKERT, the hypoblast is derived mainly from merocytes and not from a rearrangement of the "lower layer cells" of the primary segmentation, as has been maintained by previous authors. The merocytes of RÜCKERT undoubtedly corresponds with the free nuclei in the layer which I have termed parablast, and indeed in the Trout there is an approach to the structure described by RÜCKERT.

If these observations are correct, it will be necessary to modify considerably our ideas of a meroblastic egg. Although meroblastic ova are usually regarded as having been derived from holoblastic ones by the inclusion of an excess of passive food yolk, the two appear to be more closely related than has generally been admitted. We have been in the habit of regarding segmentation as only taking place in the germinal disc, and have usually derived the three primary germinal layers as a result of this segmentation process. The yolk is therefore in the main regarded as a passive food store, which is assimilated later through the digestive and circulatory systems. WALDEYER's idea of the interrelation of archiblast and parablast certainly modifies this view, and, I take it, is a step in the right direction. According to the views here advocated, a meroblastic egg produces the germinal layers from both animal and vegetative poles of the ovum, as is the case in holoblastic ova, but the means by which this end is attained is different in extreme cases. As RÜCKERT has, however, pointed out, there are a large number of types, particularly amongst the Arthropods and Mollusca, which, so to speak, bridge over the gap between the two extremes. The difficulty lies in the proper understanding of the vegetative pole in meroblastic ova. As already pointed out, the vegetative pole in a Teleostean such as the herring consists at first of only one cell, or if it contains more than one nucleus the protoplasm is at any rate unsegmented. The protoplasm is mainly peripheral, and surrounds a practically solid mass of passive yolk. There are at most a number of protoplasmic filaments pressing down amongst the yolk spherules which serve to bring the active and passive material into closer union. Too much stress cannot be laid on this fact, for to my mind it constitutes the main difference between meroblastic and holoblastic ova. A certain proportion of yolk to protoplasm may or may not prevent total segmentation, the result depending on their relative distribution in the ovum.

The development of the Decapod Crustacea shows that, to commence with, total segmentation may take place, and then that later a central unsegmented yolk mass may be formed, while the protoplasm and nuclei accumulate on the surface. This appears best explained by supposing that the protoplasm, when generally distributed throughout the yolk, was present in sufficient quantity to bring about total segmentation, but that, as it collects at the surface, a central mass of practically pure yolk is formed, which can no longer be assimilated in the same manner as that in a typical holoblastic egg such as that of *Amphioxus*. In this manner an ovum at first holoblastic becomes secondarily meroblastic. To return to our Teleostean ovum. The protoplasm in the vegetative pole increases rapidly in bulk by an assimilation of its enclosed food material, and thus is enabled to bud off cells which, had the *distribution* of yolk and protoplasm been otherwise, would have been produced by normal segmentation. Thus arises the distinction between primary and secondary segmentation, and the latter is seen to be only a modified form of the former. The first equatorial furrow, whenever it arises, divides the animal from the vegetative pole, and in meroblastic ova the segmentation in the vegetative pole, by becoming of the secondary type, accommodates itself to any relative proportion of yolk.

According to my observations, the separation of the animal from the vegetative pole in the herring occurs with the formation of the third furrow. It may be, however, that this is not the case in all other Teleosteans. I nevertheless regard the furrow or partial furrow which divides the peripheral protoplasm from that which undergoes primary segmentation in the germinal disc, as the equivalent of the first equatorial furrow in holoblastic types.

The archiblast in the herring, together with the cells derived from the parablast, prior to the formation of the segmentation-cavity, give rise to the epiblast. The vegetative pole then gives rise to the primitive hypoblast, which is in turn differentiated into the mesoblast and permanent hypoblast. I am not at present prepared to say whether this is the case in most of the Teleosteans. It appears, however, on *a priori* grounds, that at any rate in those forms which have a vitelline circulation the process may be modified. It may be that in such types the parablast is mainly concerned in the formation of the connective-tissue elements, and so cannot play so important a part in the formation of the germinal layers. In higher vertebrates (*e.g.*, the chick) the mesoblast in connection with the primitive streak is probably formed independently of the parablast, but it is generally admitted that the hypoblast is in part formed from that layer. So far as the chick is concerned, there appears little doubt that the blood and connective-tissue elements are derived from the parablast, and further investigation may show that in the trout and allied types there is a similar double origin of the mesoblast. One further point remains to be noted. The primitive hypoblast, as I have observed it in the herring, is precisely homologous with that of the Amphioxus, which, be it remembered, is a holoblastic type. In both cases the primitive hypoblast becomes differentiated into two lateral plates of mesoblast separated by the notochord, and what remains constitutes the permanent hypoblast.

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EXPLANATION OF PLATES.

Figures 5, 7, and 8 are taken from drawings by my friend Mr W. L. Calderwood; figures 6 and 9 were originally drawn from the living egg, and the colours are diagrammatic. All the other figures were drawn for me by Mr J. T. Thompson, M.B., C.M. I feel it only just to Mr Thompson to state that much of the delicacy of his original drawings has been unavoidably lost in their reproduction on stone. The figures are taken exclusively from ova of herring.

Plate XIII.

Fig 1.—Section of viscous layer and egg-membrane of a ripe unfertilised ovum. Externally is seen the homogeneous viscous layer, then follows the outer layer of zona radiata longitudinally striated, and most internally is the inner layer of zona radiata transversely striated. Gundlach, $\frac{1}{16}$, Oc. 3.

Fig. 2.—Section of abnormal egg-envelopes, showing viscous layer divided into two strata, and each exhibiting a well-marked transverse striation. Gundlach, $\frac{1}{16}$, Oc. 3.

Fig. 3.—Transverse section of an unfertilised ovum. The carmine-stained protoplasm is seen distributed throughout the yolk, and not collected into a definite layer. In its meshes lie the large unstained yolk-spheres, and they occupy the greater portion of the egg. Lying immediately beneath egg-membrane are seen smaller *yolk granules.* Swift, 1 inch.

Fig. 4.—Germinal vesicle of ovarian ovum. Latest stage observed. Shows the irregular, somewhat quadrangular, outline, the fine protoplasmic threads leaving its margin and going into yolk-mass, and the delicate nuclear reticulum. Gundlach, $\frac{1}{16}$.

Fig. 5.—Section of an unfertilised ovum after lying two days in sea-water. Differs only from recently-matured ovum in that the protoplasmic network has withdrawn a little from centre of egg, and there is rather more protoplasm at periphery, but *no disc-like prominence* is to be seen. Zeiss, A A, Oc. 3.

Fig. 6.—Optical section of a living egg, one hour after fertilisation. Germinal protoplasm is seen collecting at circumference as a continuous layer, considerably thicker at one side. *Yolk granules* are no longer visible. There is a large breathing-chamber. Zeiss, A A, Oc. 3.

Fig. 7.—Actual section of an ovum, one hour after fertilisation. Shows true relation of the protoplasm to the yolk. The protoplasm is collecting at surface, but there is still left a portion mixed amongst the yolk-spheres in the shape of branching processes, which towards germinal pole are stronger, and penetrate further into yolk. Zeiss, A A, Oc. 3.

Fig. 8.—Section of an egg, five hours after fertilisation. Germinal protoplasm has almost entirely collected at germinal pole. Large masses of yolk are entangled in the meshes of its processes, and a number of yolk-masses, varying in size, are also found in body of germinal mound itself. Zeiss, A A, Oc. 3.

Fig. 9.—Egg, nine hours after fertilisation; four-cell stage. Sketched from living egg, and coloured to harmonise with the other figures. Shows the separation of parablast from archiblast by the formation of an equatorial furrow. Lacunæ are indicated in the yolk, which are continued towards the base of disc by stalks. Swift, 1 inch.

Fig. 10.—Section of egg, twenty-one hours after fertilisation. Three rows of cells are seen in germinal disc. Cell reticulum is well brought out by differential carmine stain. Zeiss, A A.

Fig. 11.—Section of egg, twenty-four hours after fertilisation, showing morula shortly before addition of cells from parablast. The protoplasm of parablast is chiefly situated at yolk pole at this stage. Zeiss, A A.

Fig. 12.—Section of egg, twenty-six hours after fertilisation. Morula mass of cells (primary morula). An outer row of flattened epithelioid cells is differentiated = epidermal layer of epiblast of authors. Remaining mass consists of rounded cells loosely aggregated = nervous layer of epiblast of authors. Cortical protoplasm has left yolk pole to collect under and around segmented disc and nuclei, and outlines of cells can be made out in the subgerminal parablast. Primary segmentation has ended, and secondary segmentation has begun. Section is not quite through centre of egg. Zeiss, A. A.

Plate XIV.

Fig. 13.—An imperfect section of a stage shortly before that of fig. 12. Parablast is collecting towards disc. Zeiss, A A.

Fig. 14.—Egg, twenty-eight hours after fertilisation. Section through marginal part of morula (hæmatoxylin). Cells from archiblast easily distinguished from those which have been recently added from the parablast. Archiblast cells are stained more deeply, and there are indications of intracellular reticulum. Numerous vacuoles are seen between archiblast cells and bridges of protoplasm connecting the neighbouring cells together. Zeiss, D D.

Fig. 15.—Section of an egg, thirty hours after fertilisation. Unsegmented portion of parablast is seen as a thin film beneath the segmented disc. End of first budding-off process in parablast. Swift, 1 in.

Fig. 16.—Margin of morula of a stage about same as fig. 15, more highly magnified to compare with fig. 14. All distinction between the two kinds of cells is lost (hæmatoxylin). Zeiss, D. D.

Fig. 17.—Egg, thirty-four hours after fertilisation. Section represents basal portion of disc with the adjoining parablast; the two portions are not in contact at periphery, but this is the result of a mechanical injury. Cells are again to be seen in process of formation in the parablast. Differential carmine staining shows gradual transition from cells in process of formation at the base of the parablast, to those already included in the germinal disc. Swift, $\frac{1}{5}$.

Fig. 18.—Section of egg, forty-five hours after fertilisation. The final morula, containing in herring parablastic as well as archiblastic elements. Subgerminal portion of parablast is only a thin layer, peripheral portion, however, forms a thick wedge-shaped mass, extending from base of morula to equator of egg, and contains a considerable number of nuclei. Swift, 1 in.

Fig. 19.—Right corner of morula of fig. 18, more highly magnified and showing the nuclei in the peripheral parablast. Swift, $\frac{1}{5}$.

Fig. 20.—Section of egg, fifty-six hours after fertilisation, showing beginning formation of segmentation-cavity. Parablast forms its floor, and the cells of the morula form its roof and lateral boundaries. Nuclei are seen to be distributed *throughout* the parablast. Swift, 1 in.

Plate XV.

Fig. 21.—Longitudinal section of disc in axis of embryo, sixty-eight hours after fertilisation. Shows roof of segmentation-cavity formed of several rows of cells, its floor of parablast. Thickening at periphery forms commencement of blastodermic rim, and the layer pushed inwards from this thickening constitutes primitive hypoblast. Swift, 1 in.

Fig. 22.—More magnified view of a portion of fig. 21. Absence of nuclei in thinned out portion of parablast immediately below rim, but they are still numerous in peripheral parablast, and in parablast forming floor of segmentation-cavity. In region of blastodermic rim the cells of the morula rest directly on the parablast; but centrally the two layers are separated by

the segmentation-cavity. Hence the close union of the primitive layers at periphery of disc. Swift, $\frac{1}{5}$.

Fig. 23.—Transverse section through axis of embryo, 77½ hours after fertilisation. Shows primitive hypoblast in close union with parablast, from which it is derived, but almost completely separated from the cells forming roof of segmentation-cavity. Swift, 1 in.

Fig. 24.—More highly magnified portion of fig. 23. Shows the intimate connection between yolk and parablast. Upper part of parablast contains a number of free nuclei, agreeing in every particular with the nuclei of the primitive hypoblast cells, and around some of them protoplasm is constricted off to form cells. Zeiss, E.

Fig. 25—Longitudinal section, in axis of embryo of an egg, 92½ hours after fertilisation. Blastopore is near its closure. Primitive hypoblast appears to cease at posterior portion of head swelling, due to its being in this region pushed to each side by the enormous development of rudiment (keel) of central nervous system. Nuclei are still prominent in the parablast, and the two primitive layers are separated by a small space in neck region, but are united towards caudal extremity. Swift, 1 in.

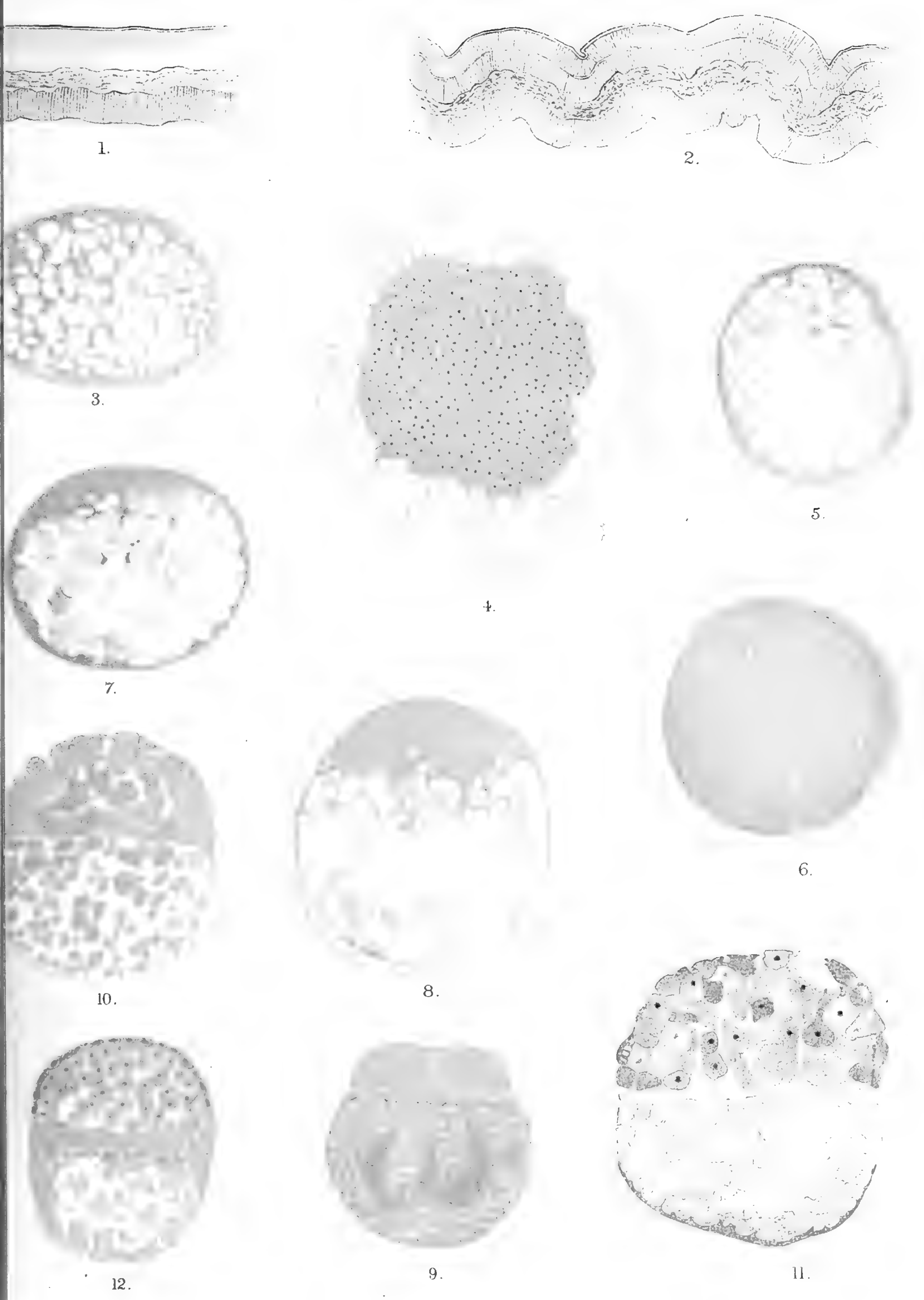
Fig. 26.—Shows a portion of fig. 25, in region of tail swelling, more highly magnified. Cells in epiblast are closely packed together, whereas those of primitive hypoblast are more loosely arranged. The two layers are, however, not distinctly separated, though a difference in the staining of the nuclei in the original preparation indicates point of union of the two layers. Parablast shows as a thin unsegmented layer of protoplasm, in which are embedded numerous nuclei. Zeiss, D. D.

Fig. 27.—Longitudinal section in axis of embryo, 101½ hours after fertilisation. Blastopore is just closed. Nervous epiblast is conspicuous in anterior region. Mesoblast is beginning to show division into somites, and hypoblast is distinctly separated in region of somites. Zeiss, A A.

Fig. 28.—More highly magnified portion of same embryo, as fig. 27, showing epiblast, mesoblast, and hypoblast, the last still in connection with the unsegmented protoplasm of the parablast. Zeiss, D D.

Fig. 29.—Transverse section through body axis of an embryo of same stage as figure 27. Upper part of section passes through the head region, and shows the thickened nervous epiblast (keel). Lower part passes through tail region; notochord not yet differentiated. Zeiss, A A.

Fig. 30.—Transverse section of an embryo, 112 hours after fertilisation. Shows separation of the notochord and the two lateral mesoblastic plates. A single row of cells remains in connection with the parablast, and constitutes the rudiment of the permanent hypoblast. Zeiss, A A.



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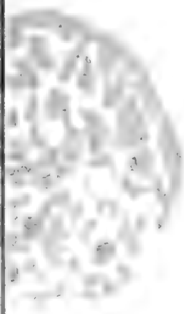
FORMATION OF THE GERMINAL LAYERS IN TELEOSTEI.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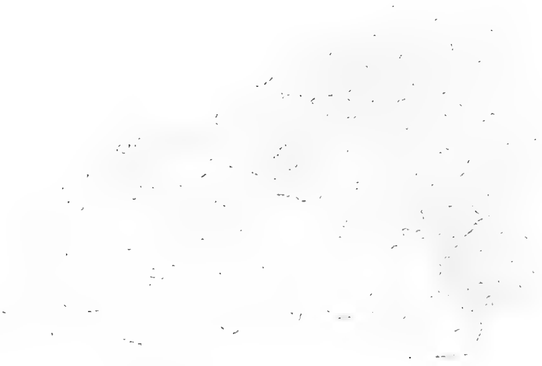
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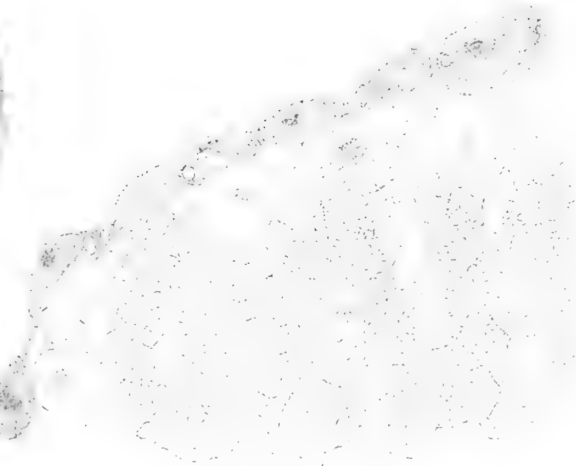
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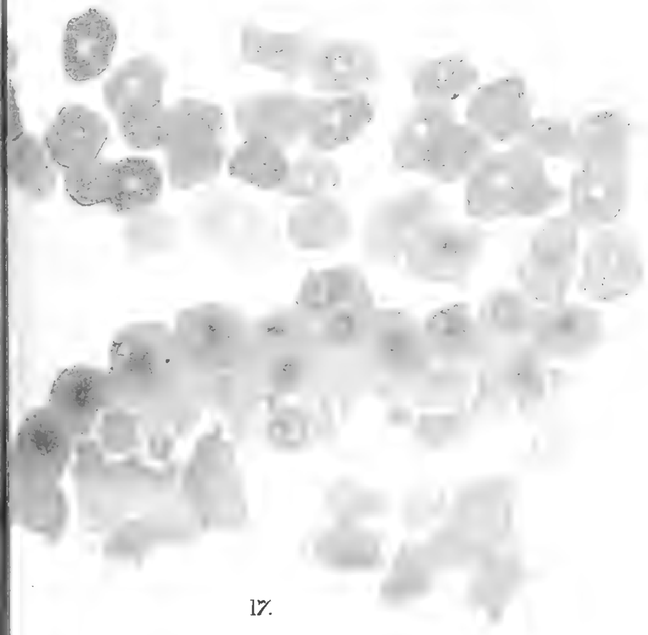
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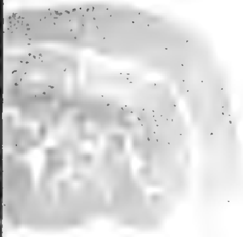
FORMATION OF THE GERMINAL LAYERS IN TELEOSTEI.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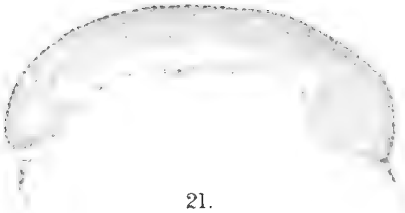
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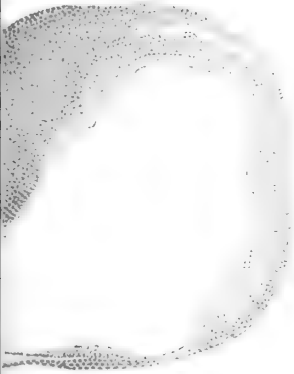
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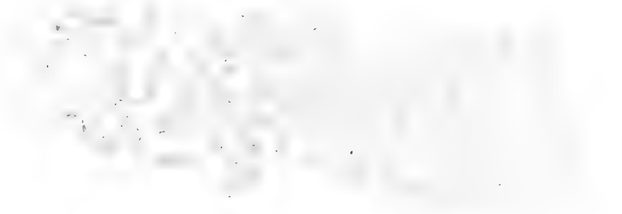
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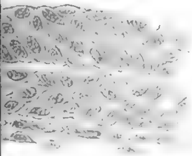
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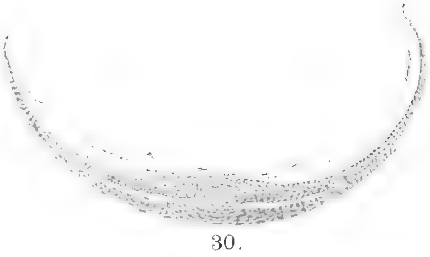
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FORMATION OF THE GERMINAL LAYERS IN TELEOSTEI

X.—*On the Structure of* Suberites domuncula, *Olivi* (*O. S.*), *together with a Note on peculiar Capsules found on the surface of Spongelia.* By J. Arthur Thomson. (Plates XVI., XVII.)

(Read 7th June 1886.)

Part I.

The sponge *Suberites domuncula* attracted the attention of the Italian naturalist Olivi* almost a hundred years ago, but was on account of its firm india-rubber like consistency erroneously regarded as an Alcyonium. Beyond the general diagnosis of Nardo† who erected the genus Suberites, the structure of the sponge has, I believe, remained virtually unknown. Not for this reason, however, but because the Monaxonia in general, of which Suberites is an example, are still for the most part but little known, Prof. F. E. Schulze of Berlin was good enough to ask me last winter to investigate the structure of this sponge. To him, therefore—perhaps the greatest living authority on the subject—I may be allowed to express my gratitude for the hospitable reception with which he welcomed a stranger to his laboratory, and for the constant interest and assistance with which he encouraged my work. That I have not succeeded in giving a perfect elucidation of the structure is largely due to the same cause which has kept it for so long almost wholly unknown. The crowded siliceous spicules, the compact consistence, the smallness of the ciliated chambers, make the histological analysis somewhat difficult. Such gaps as exist in my investigation I hope to be able to fill up by the study of related forms, and have with that end begun the study of *Suberites mana.*

Suberites domuncula is found covering the outside of a sea-snail shell, inhabited by a hermit crab. The change of position thus secured for the sponge is an obvious advantage of this commensalism, while the hermit-crab on the other hand is very effectively masked. Numerous polypes are also found embedded in the sponge. In none of the forms which I had the opportunity of examining was the hermit crab present, and in all the limy shell was to a greater or less extent eaten away, leaving in one instance only the apex. How the lime is precisely affected I was not able to discover. The cavity remaining after the shell has gone leaves a wide coiled canal, which will doubtless aid in the irrigation of the compact mass.

My material was obtained from the Berlin Aquarium, and after careful dehydration, was stained (generally with alum carmine or hæmatoxylin), and

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* Bronn's *Klassen und Ordnungen*, Vosmaer "Porifera," p. 33.

† *Ibid.*, p. 332.

sectioned in the usual fashion. I found it useful for general survey to make several large sections through the sponge.

The ectoderm of *Suberites* exhibits no marked peculiarities, but consists of a fine layer of small polygonal and apparently unequal cells, the contours of which were readily demonstrable by the silver nitrate or gold chloride method. With a lens the skin can be seen to be covered with fine pores, while numerous larger apertures are irregularly distributed over the surface. I was, however, unable to discover any oscular opening or openings. The larger apertures seemed, on closer examination, to be widened canals occupied by the abundant commensal polypes. Between the pores the points of the monact spicules projected slightly above the surface.

Not a little of the difficulty attending the investigation of *Suberites* is due to the very abundant occurrence of the large free siliceous needles. On a section through the whole sponge a radiate disposition can be recognised. They extend in large crowded brush-like bundles from the centre outwards, though considerable irregularity of arrangement is also observable. The brush-like bundles are best seen towards the surface between adjacent canals. The needles exhibit a simple unaxial form, running to a point at one end, and knobbed like a pin at the other (Plate XVI. fig. 4). I also observed a diact form, with double median nodes and with both extremities pointed, a modification readily derivable by fusion or doubling.

The Ciliated Chambers (fig. 4).—The disposition of the ciliated chambers, which is of course the principal point, is very difficult to determine owing to the number of spicules, the compactness of the sponge, the minuteness of the chambers themselves, and the adjacent relations of afferent and efferent canal systems. In sections of fortunate thickness and staining, the small chambers can be seen throughout the whole sponge, but more abundantly towards the periphery. They seem to have a somewhat more than hemispherical form, and exhibit on cross section as many as sixteen cells round the margin. These chambers are in direct communication with the finer branches of the ordinary canal system, which exhibits what is usually termed the fourth degree of complexity. An inner flattened nucleated epithelium could be detected as the lining of some of the canals. *The afferent and efferent canals lie side by side,* their parietal pores are adjacent, and beyond the distribution of the chambers and the direction of the increase in the diameter of the branch canals, I could detect no difference between them, and no special oscular or efferent regions.

The Connective Tissue.—The mesodermic connective tissue exhibits great variety of composition in different regions. The cells vary greatly in shape, from round and regular to polygonal and multipolar, or to long drawn out spindle-like forms. Fine connecting threads between adjacent cells could be readily recognised. In some cases, besides nucleus and nucleolus, the intracellular protoplasmic network could be distinctly seen. The great interest of the connective tissue, however, is its frequent modification into what may be termed *muscle-cells*. Disposed in compact strands, specially abundant in certain positions, these extended spindle-shaped cells certainly suggest a contractile function. A thick layer occurs just below the ectoderm; numerous well-defined, occasionally branching, strands run parallel to the surface somewhat further inwards; a similar compactness of disposition is very abundant in the region adjoining the gasteropod shell, and lastly these muscle-strands frequently occur round the larger canalicular passages. While the strands of closely-packed spindle-shaped cells are quite definitely distinguishable, evident transitions exist between the latter and ordinary connective tissue cells (figs. 5–8).

Reproductive Elements.—Embedded in the connective tissue matrix, I have observed the occurrence of developing sperms in the form of morula-like balls of minute cells, surrounded by an envelope of flattened connective tissue (fig. 11). These balls of cells correspond with the sperm-morulæ described by SCHULZE and POLEJÂEFF. As ova occur in the same specimen, *Suberites* appears to be hermaphrodite. The ova occur, frequently in extraordinary abundance, throughout the whole sponge. In some cases two nuclei were present in the ovum, or a nucleus with a stained aggregate at each pole,—probably the beginnings of division. The chromatic contents of the germinal vesicle exhibited a most varied appearance, which seems to me worthy of special notice. Not one nucleolus, but several apparent nucleoli, were very generally present; a smaller spherule often seemed to arise as a bud from the larger, or to lie adjacent to, though unconnected with it; but more frequently three, four, or five variously disposed nucleolid spheres of perfectly definite contour, and uniformly stained, occurred. They were sometimes of equal size, but oftener with one slightly larger than the others. An idea of the varied disposition can be best obtained by a glance at a few nuclei represented in fig. 10. My friend Dr HEIDER has observed a horse-shoe-shaped nucleolus in sponge-ova, which might of course in cross section explain some of the forms. A more complex form might obviously show several spheres on cross section, and that this is the explanation is suggested by the nuclear sections in fig. 10, several of which were cut through the same nucleus at different levels. It is also possible that we have here to deal with phenomena resulting, not from the complicated shape of the nucleolus, but from the occurrence of that multinucleolar condition which has, of late years, been repeatedly observed.

As *Suberites* is but ill-suited for minute histological study, which could in such a form at most result in the confirmation of what can be better observed in other types, I have contented myself with attempting to elucidate the general structure, and thus doing something towards increasing our knowledge of the almost unattacked *Monaxonia*.

PART II.

Appended Note on the Capsules found on the surface of Spongelia pallescens.

(Plate XVII.)

In the course of some studies on sponges, pursued in the laboratory of the Zoological Institute at Berlin, Professor F. E. SCHULZE directed my attention to certain peculiar club-shaped knobs which were formed on the surface of a Spongelia, and entrusted them to me for examination. The death of the Spongelia prevented me from tracing their further history, and I can therefore at present only note their interesting structure, in the hope that others who may have observed these or similar structures may be able to explain their import.

The knobs were of the size of a small pin's head, and were raised above the level of the already contracted sponge by stalks formed from projecting peaks of the horny skeletal framework. The shape of the knobs was oval or pear-shaped, and their contour was always perfectly defined.

Treatment with silver nitrate readily revealed the interesting fact that the knobs were surrounded by a well-defined ectoderm composed of cells of varying shape and size (fig. 11).

The contents of a teased-out knob seemed to consist of generally round cells (figs. 3, 6) of very varied size, and with distinct nuclei, while sections of stained knobs exhibited what appeared as a compact and intricate meshwork of fine filaments, the meshes of which were occupied by cells of varied size (figs. 4, 7). Towards the margins, and especially towards the base of the knob, the apparent network was looser, and there especially it could be seen that the structure was that of incipient tissue with undifferentiated cells, or of connective tissue in which the matrix was apparently wrinkled, or in some way modified round the cells, following their contours and producing the appearance of a very intricate filamentous network. The cells round the boundary, directly below the epithelium, were often very regular. Below these and at the base of knob, certain larger round cells (fig. 6) occurred, though by no means confined to these portions. Between these and the irregular cells of the meshwork intermediate forms were obvious. In some knobs the cells were almost all rounded (fig. 10), as if not compressed to the same extent. In others, as was especially well seen at the base, where the cells were less abundant, they exhibited the greatest variability of form (fig. 8).

As these knobs present perfect definiteness of structure, and are only in formal contact with the sponge, it seems possible that they may thus secure the persistence of the organism in unfavourable environment, which can of course only be verified by following their history. They might therefore be termed regenerative capsules. Somewhat similar aggregations or contractions were observed to occur in *Reniera*, but in these I was not able to make out any definite structure.

As a histological modification in response to a change in the environment, whether it be connected or not with securing the persistence of the endangered organism, the occurrence of these structures is perhaps worthy of record.

Explanation of Plates.

Plate XVI.—*Suberites.*

Fig. 1. External appearance, showing pores.
2. Large section, showing spicules, canals, and ova,
3. Ectodermic epithelium.
4. System of canals, showing ciliated chambers.
5–6. Muscle-cells.
7–8. Connective tissue cells of mesoderm.
9. Ova.
10. Germinal vesicles with nucleoli.
11. Sperm-morulæ.

Plate XVII.—Capsules of *Spongelia.*

Fig. 1. Capsules on surface of sponge.
2. Individual capsule on peak of sponge.
3. Contents of capsule squeezed out.
4. Longitudinal section.
5. Border of capsule.
6. Abundant round cells.
7. Longitudinal section.
8. Contained elements.
9. Cross section.
10. Longitudinal section with abundant round cells.
11. Ectoderm outlines.

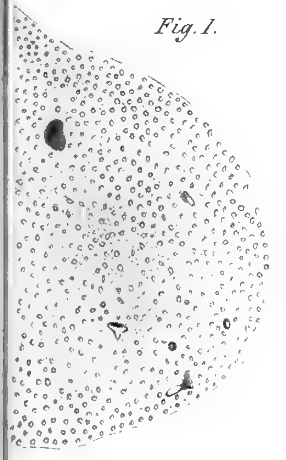


Fig. 1.

Fig. 3.

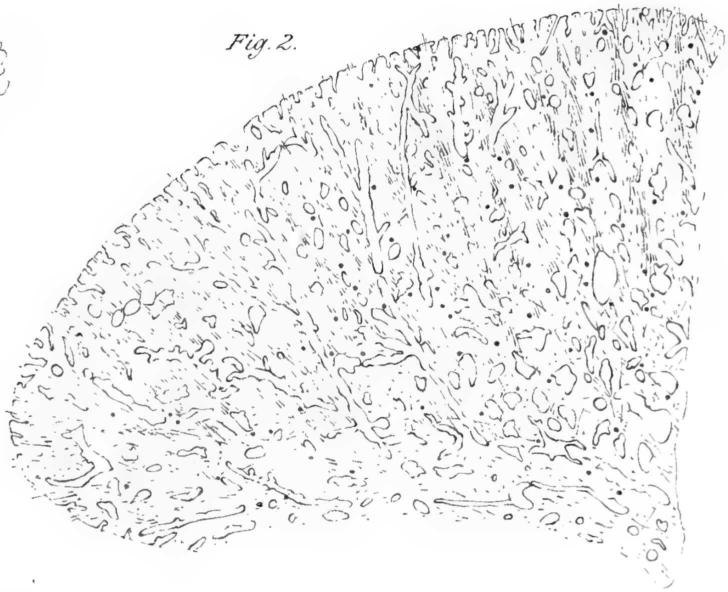
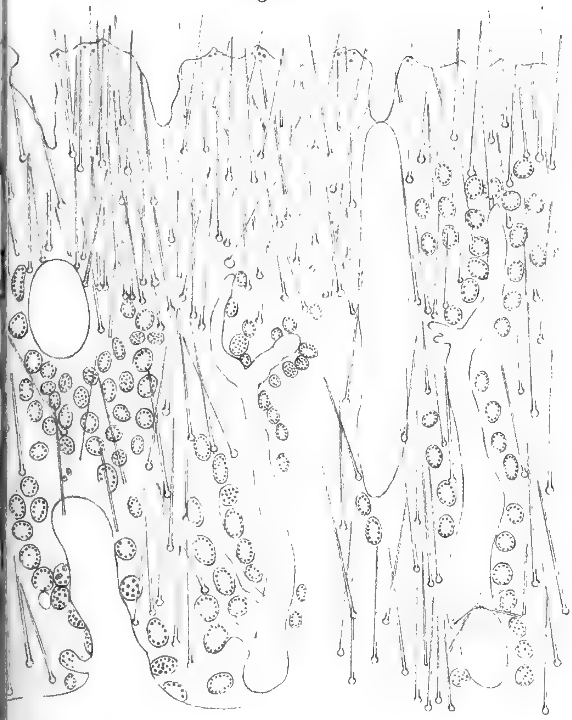


Fig. 2.

Fig. 4.



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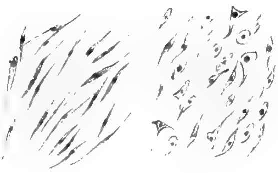


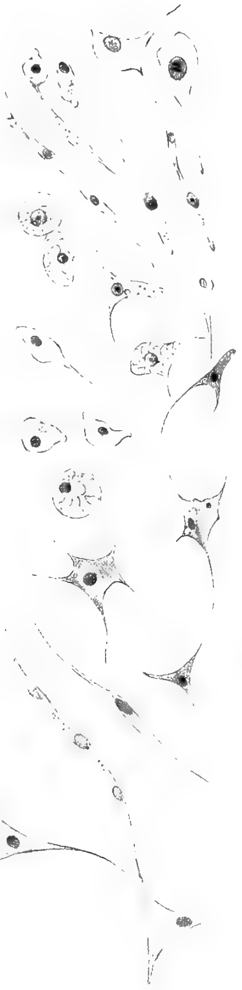


Fig. 8.

Fig. 9.

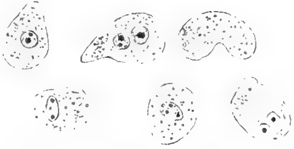


Fig. 5.

Fig. 6.

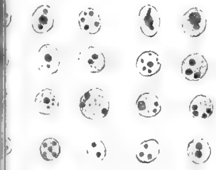
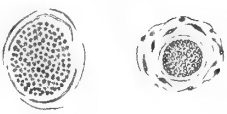


Fig. 10.

Fig. 11.



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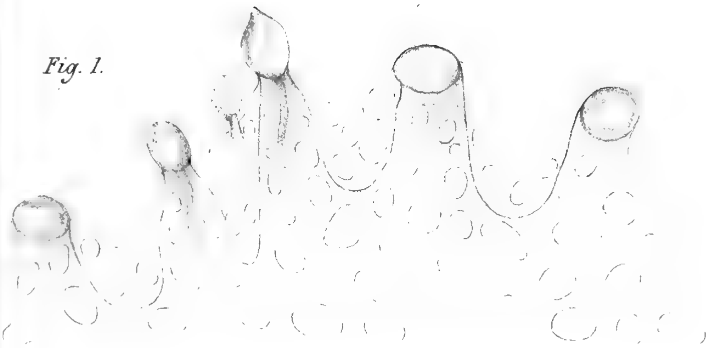

Fig. 2.

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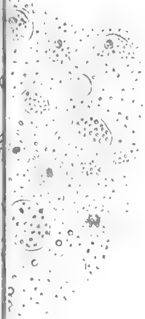

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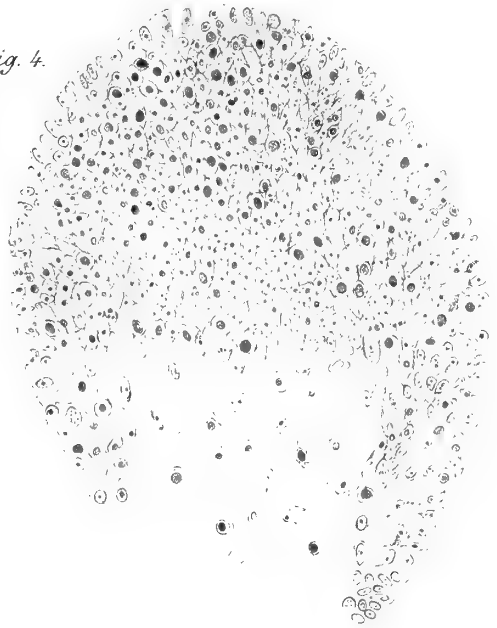
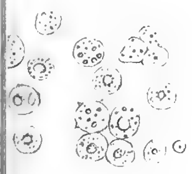


Fig. 5.

Fig. 7.

Fig. 6.



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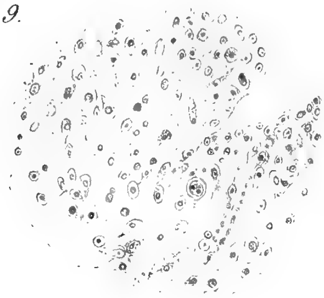

Fig. 8.

Fig. 10.

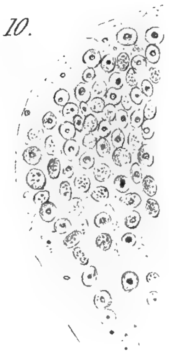

Fig. 11.

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XI.—*The Reproductive Organs of Bdellostoma, and a Teleostean Ovum from the West Coast of Africa.* By J. T. Cunningham, B.A.

(Read 5th July 1886.)

During a short visit I paid to Oxford in the month of June last I had the opportunity of examining, by the kind permission of Professor Moseley, a number of specimens of *Bdellostoma Forsteri,* which were some of a large number brought from the Cape by Mr Adam Sedgwick of Trinity College, Cambridge. This examination showed what, from the close affinity of the two forms, was naturally to be expected, namely, that the structure of the reproductive system and the development of the reproductive elements in Bdellostoma were very closely similar to the structure and development of the corresponding parts in Myxine. A short time ago I described before the Society some ovarian eggs of Myxine, obtained at the beginning of the present year, which were approaching maturity. In these eggs there were slight projections at the poles, and on the surface of the projecting parts a number of papillæ. The projections were caused by the growth of a number of threads from the vitelline membrane within the ovarian capsule, and the papillæ were the separate elevations produced by the threads. In one of the specimens of Bdellostoma which I examined at Oxford there were a number of ovarian eggs in an exactly similar condition. These eggs of Bdellostoma are of course much larger than those of Myxine; the eggs of the latter, in the condition I refer to, were 2·1 cm., those of the former are 3·5 cm. No one has seen the perfectly ripe eggs of Bdellostoma after their escape from the ovary, but the specimens I have described prove conclusively that the eggs of this species when shed are provided with a number of polar threads, which are processes of the vitelline membrane, exactly as in Myxine. I have not yet made a microscopic examination of the reproductive organs in Bdellostoma, but from what I could see by ordinary dissection, it is evident that all the peculiarities which exist in the reproductive system in Myxine occur also in Bdellostoma. A number of specimens possessed sexual organs, in the anterior part of which were minute ova, while the posterior part was evidently testicular tissue; and in one or two other specimens the whole organ seemed to be testicular. The small quantity of testicular tissue in a given specimen was also noticeable, as in Myxine. I found no specimens which showed indications of having recently discharged their eggs. I have ascertained from Mr Sedgwick that his specimens were collected in August and September, and this fact shows that the breeding period of Bdellostoma agrees with that of Myxine in falling within the coldest season of the year. *Myxine glutinosa*, in the North Sea, deposits its eggs in December, January, and February, and the two latter months agree in meteorological conditions with the months of August and September in the latitude of Cape Town.

The egg of Bdellostoma at the stage under consideration has a thicker and stronger vitelline membrane than the egg of Myxine. I found it impossible to strip off from preserved specimens of the latter the connective tissue and follicular epithelium without rupturing the vitelline membrane. In the eggs of Bdellostoma this could be accomplished with ease. The membrane, when exposed, was seen to be yellowish-brown in colour, and translucent. Round the micropylar end of the capsule formed by the membrane is seen a distinct thin line, forming a complete ring, and it is evident that the micropylar end forms an operculum which separates from the rest of the capsule along this line. STEENSTRUP has figured a detached operculum in the figure he gives of the ova of Myxine, but in the latter form I have not yet detected indications of the structure. There can be no doubt, from the appearance seen in the Bdellostoma ovum, that the escape of the embryo in the Myxinoids is effected by the removal of an operculum specially adapted for that purpose.

The Teleostean ova I have next to describe resemble in the character of the vitelline membrane the ova of the Myxinoids. Each ovum is spherical in shape, 1·5 to 1·6 mm. in diameter, and about one pole of the sphere is provided with a number of long thin flexible filaments springing from the vitelline membrane. Each filament commences at the attached base with a conical papilla, which is thicker than the filament itself. By the interlacing of the filaments a large number, many thousands, of eggs are connected together to form a cylindrical mass about an inch wide, and a foot or more in length. The felted filaments form a rope-like core to the cylinder, the eggs forming an external layer. Besides the long filaments, each egg shows a similar number of short filaments springing from the opposite pole. These are very slender, and only from 2 mm. to 1·5 cm. in length. In other respects they resemble the long filaments, of which they are evidently rudimentary representatives. They seem to have no function, being too small to afford any assistance in the process of attachment. It is probable, though I have not been yet able to demonstrate the fact, that the micropyle is situated in the centre of the region whence the long filaments arise. If this were so, the relations of the filaments and vitelline membrane in this Teleostean egg would be exactly similar to those which obtain in the ovum of the Myxinoids. And whatever be the position of the micropyle, it is interesting to note that the occurrence of a group of filamentous processes of the vitelline membrane at each of the two opposite poles of the ovum is not peculiar to the Myxinoids. It is as certain as an inference from the unfertilised ovum can be, that the segmentation of the egg of the Myxinoids is meroblastic, as in Teleosteans, and thus in two points the Myxinoid ovum agrees with the Teleostean, and differs from that of Petromyzon, while in respect of the mass of the yolk the Myxinoids agree more with Elasmobranchs.

I have not succeeded in identifying the species of fish to which belong the eggs above described. The eggs of several species are known to be provided with filamentous processes. In the Scombresocidæ the filaments are equal in length to the diameter of the ovum, and are uniformly distributed over the surface of the membrane. The filaments in this family were first described by Professor HAECKEL.* JOHN A. RYDER gives a very clear and complete account of them in the *Bulletin of the U. S. Fish Commission,* 1881, vol. i., as studied in *Belone longirostris.* In Chirostoma, one of the Atherinidæ, RYDER found there were only four filaments attached at one pole of the egg close together. In this latter case the filaments were during development closely wound round the vitelline membrane in one equator of the sphere, so that the method of their formation differs from that of the Myxinoid filaments, which are perpendicular to the surface of the membrane throughout their growth in the follicle.

Filamentous processes of the vitelline membrane occur also in the family Pomacentridæ; they have been described by HOFFMANN in *Heliastes chromis* of the Mediterranean (see *Konink. Akad. d. Vetensk. Amst.,* vol. xxi.). Here they occur at one end only of the ellipsoidal ovum. They occur also in Gobius and Blennius, but in neither of these cases are two sets of processes present, situated at opposite poles of the ovum. It is thus impossible to say whether the ova described in this paper belong to a fish of the family Scombresocidæ among the Physostomi, of the family Pomacentridæ, or coral-fishes among the Pharyngognathi, of the family Gobiidæ, Blennidæ or Atherinidæ, or to a species of some other family whose eggs are altogether unknown. The ova were obtained on two occasions, each time a single cylindrical "rope," by Mr JOHN RATTRAY, F.R.S.E., in the Gulf of Guinea. Mr RATTRAY was on board a steamer called the "Buccaneer" last winter, in the capacity of naturalist, having been invited to accompany Mr J. Y. BUCHANAN, who was carrying out some hydrographical investigations off the coast of Africa. The eggs were obtained in the following manner:—A small conical buoy was attached at the end of a rope, and along the rope were fastened two or three muslin tow-nets. The whole was then thrown overboard in such a way that the mouth of the tow-net faced whatever current was flowing. The eggs were found entangled on the line when the apparatus was

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* MULLER'S *Archiv,* 1855.

recovered. On the first occasion, March 12th of the current year, the position was lat. 1° 17′ N., long. 13° 56′·6 W. The depth at which the ova were caught by the line was 30 fathoms. The total depth of the ocean at the spot was 2725 fathoms. The other mass of eggs was taken in a similar way, not far off the locality just defined. Thus these eggs were in a pelagic condition, suspended in the water, and freely obeying the ocean current. Mr Rattray states they were very transparent.

P.S.—Since the above was written I have found that the meroblastic nature of the ovum of Myxine has been actually proved. Fertilised eggs, in which the blastoderm had already begun to spread over the yolk, were examined and described by W. Müller several years ago (*Jenaische Zeitschrift*, Bd. IX.). These eggs were from the collection of the Göteborg Museum, and were obtained at Lysekil in Bohuslän in 1854. W. Müller, however, did not give a correct account of the development of the vitelline membrane and polar threads.

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CONTENTS.

[*Issued April 13, 1888.*]

XII.—*On the Foundations of the Kinetic Theory of Gases.* II.

By Professor Tait.

(Read December 6, 1886, and January 7, 1887. Revised April 4, 1887.)

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[*Erratum* in Part I., *anté*, p. 65. For 1676, read 1678, as the date of Hooke's Pamphlet.]

In the present communication I have applied the results of my first paper to the question of the transference of momentum, of energy, and of matter, in a gas or gaseous mixture; still, however, on the hypothesis of hard spherical particles, exerting no mutual forces except those of impact. The conclusions of §§ 23, 24 of that paper form the indispensable preliminary to the majority of the following investigations. For, except in extreme cases, in which the causes tending to disturb the "special" state are at least nearly as rapid and persistent in their action as is the process of recovery, we are entitled to assume, from the result of § 24, that in every part of a gas or gaseous mixture a local special state is maintained. And it is to be observed that this may be accompanied by a common translatory motion of the particles (or of each separate class of particles) in that region; a motion which, at each instant, may vary continuously in rate and direction from region to region; and which, in any one region, may vary continuously with time. This is a sort of generalisation of the special state, and all that follows is based on the assumption that such is the most general kind of motion which the parts of the system can have, at least in any of the questions here treated. Of course this translational speed is not the same for all particles in any small part of the system. It is merely an average, which is maintained in the same roughly approximate manner as is the "special state," and can like it be assumed to hold with sufficient accuracy to be made the basis of calculation. The mere fact that a "steady" state, say of diffusion, can be realized experimentally is a sufficient warrant for this assumption; and there seems to be no reason for supposing that the irregularities of distribution of the translatory velocity among the particles of a group should be more serious for the higher than for the lower speeds, or *vice versâ*. For each particle is sometimes a quick, sometimes a slow, moving one:—and exchanges these states many thousand times per second. All that is really required by considerations of this kind is allowed for by our way of looking at the mean free paths for different speeds.

I may take this opportunity of answering an objection which has been raised in correspondence by Professor Newcomb, and by Messrs Watson and Burbury, to a passage in § 3 of the First Part of this paper.* The words objected to are put in Italics:—

"But *the argument above shows*, further, that this density must be expressible in the form

$$f(x)\,f(y)\,f(z)\,,$$

whatever rectangular axes be chosen, passing through the origin."

The statement itself is not objected to, but it is alleged that it does not follow from the premises assumed.

This part of my paper was introduced when I revised it for press, some months after it was read; the date of revision, not of reading, being put at the head. It was written mainly for the purpose of stringing together what had been a set of detached fragments, and was in consequence not so fully detailed as they were. I made some general statements as to the complete verification of these preliminary propositions which was to be obtained from the more complex investigations to which they led; thus showing that I attached comparatively little weight to such introductory matters. If necessary, a detailed proof can be given on the lines of § 21 of the paper. The "argument" in question, however, may be given as below. It is really involved in the italicised words of the following passage of § 1:—"in place of the hopeless question of the behaviour of innumerable absolutely isolated individuals, the comparatively simple statistical question of *the average behaviour of the various groups of a community.*"

Suppose two ideal planes, parallel to $x=0$, to move with common speed, x, through the gas. The portion of gas between them will consist of two quite distinct classes of particles:—the greatly more numerous class being mere

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* In the *Phil. Mag.*, for April 1887, the same objection is raised by Prof. Boltzmann; who has appended it to the English translation of his paper presently to be referred to. But he goes farther than the other objectors, and accuses me of reasoning in a circle.

fleeting occupants, the minority being (relatively) as it were permanent lodgers. These are those whose speed perpendicular to the planes is very nearly that of the planes themselves. The *individuals* of each class are perpetually changing, those of the majority with extraordinary rapidity compared with those of the minority; but each *class*, as such, forms a definite "*group* of the community." The method of averages obviously applies to each of these classes separately; and it shows that the minority will behave, so far as y and z motions are concerned, as if they *alone* had been enclosed between two *material* planes, and as if their lines of centres at impact were always parallel to these. The instant that this ceases to be true of any one of them, that one ceases to belong to the group;—and another takes its place. Their behaviour under these circumstances (though not their number) must obviously be independent of the speed of the planes. Hence the law of distribution of components in the velocity space-diagram must be of the form

$$f(x) \, . \, \mathrm{F}(y, z) \, ;$$

and symmetry at once gives the result above.

[(*Inserted March 5th,* 1887.) Another objection, but of a diametrically opposite character, raised by Mr BURBURY[*] and supported by Professor BOLTZMANN,[†] is to the effect that in my first paper I have unduly multiplied the number of preliminary assumptions necessary for the proof of MAXWELL'S Theorem concerning the distribution of energy in a *mixture* of two gases. In *form*, perhaps, I may inadvertently have done so, but certainly not in *substance.*

The assumptions which (in addition to that made at the commencement of the paper (§ 5) for provision against simultaneous impacts of three or more particles, which was introduced expressly for the purpose of making the results applicable to real gases, not merely to imaginary hard spheres,) I found it necessary to make, are (§ 18) as follows; briefly stated.

(A) That the particles of the two systems are thoroughly mixed.

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[*] The Foundations of the Kinetic Theory of Gases. *Phil. Mag.* 1886, I, p. 481.

[†] Über die zum theoretischen Beweise des Avogadro'-schen Gesetzes erforderlichen Voraussetzungen. *Sitzb. der kais. Akad.* XCIV, 1886, Oct. 7. In this article Prof. BOLTZMANN states that I have nowhere expressly pointed out that my results are applicable only to the case of hard spheres. I might plead that the article he refers to is a brief *Abstract* only of my paper; but it contains the following statements, which are surely explicit enough as to the object I had in view :—

"This is specially the case with his [MAXWELL'S] investigation of the law of ultimate partition of energy in a mixture of smooth spherical particles of two different kinds."

"It has since been extended by BOLTZMANN and others to cases in which the particles are no longer supposed to be hard smooth spheres."

"Hence it is desirable that MAXWELL'S proof of his fundamental Theorem should be critically examined." Then I proceed to examine it, *not* Professor BOLTZMANN'S extension of it. In my paper itself this limitation is most expressly insisted on.

(B) That the particles of each kind, separately, acquire and maintain the "special state."

(C) That there is free access for collision between each pair of particles, whether of the same or of different systems; and that the number of particles of one kind is not overwhelmingly greater than that of the other.

Of these, (A) and (B), though enunciated separately, are regarded as *consequences* of (C), which is thus my sole assumption for the proof of CLERK-MAXWELL'S Theorem. Professor BOLTZMANN states that the only necessary assumptions are:—that the particles of each kind be uniformly distributed in space, that they behave on the average alike in respect of all directions, and that (for any one particle?) the duration of an impact is short compared with the interval between two impacts. His words are as follows:—"Die einzigen Voraussetzungen sind, dass sowohl die Moleküle erster als auch die zweiter Gattung gleichförmig im ganzen Raume vertheilt sind, sich durchschnittlich nach allen Richtungen gleich verhalten und dass die Dauer eines Zusammenstosses kurz ist gegen die Zeit, welche zwischen zwei Zusammenstossen vergeht."

He farther states that neither system need have internal impacts; and that Mr BURBURY is correct in maintaining that a system of particles, which are so small that they practically do not collide with one another, will ultimately be thrown into the "special" state by the presence of a *single* particle with which they can collide.

Assuming the usual data as to the number of particles in a cubic inch of air, and the number of collisions per particle per second, it is easy to show (by the help of LAPLACE'S remarkable expression* for the value of $\Delta^n 0^m/n^m$ when m and n are very large numbers) that somewhere about 40,000 *years* must elapse before it would be so much as *even betting* that Mr BURBURY'S single particle (taken to have twice the diameter of a particle of air) had encountered, once at least, each of the 3.10^{20} very minute particles in a single cubic inch. He has not stated what is the average number of collisions necessary for each of the minute particles, before it can be knocked into its destined phase of the special state; but it must be at least considerable. Hence, even were the proposition true, æons would be required to bring about the result. As a result, it would be very interesting; but it would certainly be of no importance to the kinetic theory of gases in its practical applications.

I think it will be allowed that Professor BOLTZMANN'S assumptions, which (it is easy to see) practically beg the whole question, are themselves inadmissible

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* *Théorie Analytique des Probabilités. Livre* II, *chap.* ii, 4. [In using this formula, we must make sure that the ratio m/n is sufficiently large to justify the approximation on which it is founded. It is found to be so in the present case. At my request Professor CAYLEY has kindly investigated the correct formula for the case in which m and n are of the *same* order of large quantities. His paper will be found in *Proc. R. S. E.*, April 4, 1887.]

except as consequences of the mutual impacts of the particles in each of the two systems separately. Professor BOLTZMANN himself, indirectly and without any justification (such as I have at least attempted to give), *assumes* almost all that he objects to as redundant in my assumptions, with a good deal more besides. But he says nothing as to the *relative* numbers of the two kinds of particles. Thus I need not, as yet, take up the question of the validity of Professor BOLTZMANN'S method of investigation (though, as hinted in § 31 of my first paper, I intend eventually to do so); and this for the simple reason that, in the present case, I cannot admit his premises.

Mr BURBURY assumes the non-colliding particles to be in the "special state," and proceeds to prove that the single additional particle will not disturb it. But, supposing for a moment this to be true, it does *not* prove that the solitary particle would (even after the lapse of ages) reduce any non-colliding system, with positions at any instant, speeds, and lines of motion, distributed absolutely at random (for here there cannot be so much as plausible grounds for the introduction of Professor BOLTZMANN'S assumptions) to the "special state." If it could do so, the perfect reversibility of the motions, practically limited in this case to the reversal of the motion *of the single particle alone*, shows that the single particle would (for untold ages) continue to throw a system of non-colliding particles further and further *out* of the "special" state; thus expressly contradicting the previous proposition. In this consequence of reversal we see the reason for postulating a very great number of particles of *each* kind. If Mr BURBURY'S sole particle possessed the extraordinary powers attributed to it, it would (except under circumstances of the most exact adjustment) not only be capable of producing, but *would* produce, absolute confusion among non-colliding particles already in the special state. Considering what is said above, I do not yet see any reason to doubt that the assumption of collisions among the particles of each kind, separately, is quite as essential to a valid proof of MAXWELL'S Theorem as is that of collisions between any two particles, one from each system. I have not yet seen any attempt to *prove* that two sets of particles, which have no internal collisions, will by their mutual collisions tend to the state assumed by Professor BOLTZMANN. Nor can I see any ground for dispensing with my farther assumption that the number of particles of one kind must not be overwhelmingly greater than that of the other. A small minority of one kind must (on any admissible assumption) have an average energy which will fluctuate, sometimes quickly sometimes very slowly, within very wide and constantly varying limits.

DE MORGAN* made an extremely important remark, which is thoroughly applicable to many investigations connected with the present question. It is to the effect that "no *primary* considerations connected with the subject of

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* *Encyc. Metropolitana. Art.* Theory of Probabilities.

Probability can be, or ought to be, received if they depend upon the results of a complicated mathematical analysis." To this may be added the obvious remark, that the purely mathematical part of an investigation, however elegant and powerful it may be, is of no value whatever in physics unless it be based upon admissible assumptions. In many of the investigations, connected with the present subject, alike by British and by foreign authors, the above remark of De Morgan has certainly met with scant attention.]

In my first paper I spoke of the errors in the treatment of this subject which have been introduced by the taking of means before the expressions were ripe for such a process. In the present paper I have endeavoured throughout to keep this danger in view ; and I hope that the results now to be given will be found, even where they are most imperfect, at least more approximately accurate than those which have been obtained with the neglect of such precautions.

The nature of Clerk-Maxwell's earlier investigations on the Kinetic theory, in which this precaution is often neglected, still gives them a peculiar value ; as it is at once obvious, from the forms of some of his results, that he must have *thought them out* before endeavouring to obtain them, or even to express them, by analysis. One most notable example of this is to be seen in his *Lemma* (*Phil. Mag.* 1860, II. p. 23) to the effect that

$$\int_{-r}^{r} \pm \mathrm{U} x^m \, dx = \frac{2}{m+2} \frac{d}{dx}\left(\mathrm{U} r^{m+2}\right) ;$$

where U and r are functions of x, not vanishing with x, and varying but slightly between the limits $-r$ and r of x;—and where the signs in the integrand depend upon the character of m as an even or odd integer. This forms the starting point of his investigations in Diffusion and Conductivity. It is clear from the context why this curious proposition was introduced, but its accuracy, and even its exact meaning, seem doubtful.

In all the more important questions now to be treated, the mean free path of a particle plays a prominent part, and integrals involving the quantities e, or $e+e_1$ (as defined in §§ 9, 10, 28) occur throughout. We commence, therefore, with such a brief discussion of them as will serve to remove this merely numerical complication from the properly physical part of the reasoning.

X. *On the Definite Integrals*

$$\int_0^\infty \frac{\nu v^r}{e} \quad \text{and} \quad \int_0^\infty \frac{\nu v^r}{e_1 + z e_2} .$$

33. In the following investigations I employ, throughout, the definition of the mean free path for each speed as given in § 11. Thus all my results

necessarily differ, at least slightly, from those obtained by any other investigator.

By § 11 we see at once that

$$\int_0^\infty \frac{\nu v^r}{e} = \frac{1}{n\pi s^2}\int_0^\infty \frac{\varepsilon^{-hv^2} v^{r+2}dv}{\int_0^v \varepsilon^{-hv_1^2}(v_1^2+v_1^4/3v^2)dv_1 + \int_v^\infty \varepsilon^{-hv_1^2}(vv_1/3+v_1^3/v)dv_1},$$

$$= \frac{1}{n\pi s^2\sqrt{h^r}}\int_0^\infty \frac{4x^{r+4}\,\varepsilon^{-x^2}dx}{x\varepsilon^{-x^2}+(2x^2+1)\int_0^x \varepsilon^{-x^2}dx},$$

$$= \frac{\mathrm{C}_r}{n\pi s^2\sqrt{h^r}}, \text{ suppose.}$$

The finding of C_r is of course a matter of quadratures, as in the *Appendix* to the First Part of this paper, where the values calculated are, in this notation, C_{-1} and C_0; and Mr CLARK has again kindly assisted me by computing the values of C_1, C_3, C_5, which are those required when we are dealing with Viscosity and with Heat-Conduction in a single gas. The value of C_2 has also been found, with a view to the study of the general expression for C_r. These will be given in an *Appendix* to the present paper.

34. When we come to deal with Diffusion, except in the special case of equality of density in the gases, this numerical part of the work becomes extremely serious, even when the assumption of a "steady" state is permissible. As will be seen in § 28 of my first paper, we should have in general to deal with tables of double entry, for the expressions to be tabulated are of the form—

$$\int_0^\infty \frac{\nu v^r}{e_1+ze_2} = \frac{1}{n\pi s^2\sqrt{h^r}}\int_0^\infty \frac{4x^{r+4}\varepsilon^{-x^2}dx}{x\varepsilon^{-x^2}+(2x^2+1)\int_0^x \varepsilon^{-x^2}dx + z\left(x_1\varepsilon^{-x_1^2}+(2x_1^2+1)\int_0^{x_1}\varepsilon^{-x^2}dx\right)}$$

$$= {}_1\mathfrak{C}_r = \frac{{}_1\mathrm{C}_r}{n\pi s^2\sqrt{h^r}}, \text{ suppose.}$$

For the second gas the corresponding quantity will be written as ${}_2\mathfrak{C}_r$. Here

$$x_1 = x\sqrt{h_1/h},$$

and

$$z = \frac{n_1 h}{n h_1}\left(\frac{s+s_1}{2s}\right)^2;$$

so that they are numerical quantities, of which the first depends on the relative masses of particles of the two gases, while the second involves, in addition, not only their relative size but also their relative number. It is this last condition which introduces the real difficulty of the question, for we have to express the value of the integral as a function of z before we can proceed with the further details of the solution, and then the equation for Diffusion ceases to resemble that of FOURIER for Heat-Conduction.

The difficulty, however, disappears entirely when we confine ourselves to the study of the "steady state" (and is likewise much diminished in the study of a variable state) in the special case when the mass of a particle is the same in each of the two gaseous systems, whether the diameters be equal or no. For, in that case, we have $h_1 = h$ and $x_1 = x$, so that the factor $1/(1+z)$ can be taken outside the integral sign. Thus, instead of ${}_1\mathfrak{C}_r$, we have only to calculate C_r of the previous section.

XI. *Pressure in a Mixture of Two Sets of Spheres.*

35. Suppose there be n_1 spheres of diameter s_1 and mass P_1, and n_2 with s_2, P_2, per cubic unit. Let $s = (s_1 + s_2)/2$.

Then the average number of collisions of each P_1 with P_1s is, per second,

$$2\sqrt{\frac{2\pi}{h_1}} n_1 s_1^2 \,.$$

The impulse is, on the average (as in § 30),

$$-P_1\sqrt{\frac{\pi}{2h}} \,.$$

Similarly, each P_1 encounters, in each second (§ 23), the average number

$$2n_2\sqrt{\frac{\pi(h_1+h_2)}{h_1 h_2}} \,.\, s^2$$

of P_2s, and the average impact is

$$-\frac{P_1 P_2}{P_1+P_2}\sqrt{\frac{\pi(h_1+h_2)}{h_1 h_2}} \,.$$

Thus the average sum of impacts on a P_1 is, per second,

$$-2P_1\frac{\pi}{h_1} n_1 s_1^2, \text{ due to } P_1\text{s; and}$$

$$-2\frac{P_1 P_2}{P_1+P_2}\,\frac{h_1+h_2}{h_1 h_2}\pi n_2 s^2, \text{ due to } P_2\text{s}.$$

In the Virial expression $\frac{1}{2}\Sigma(Rr)$, {§ 30}, r must be taken as s_1 for the first of these portions, and as s for the second. Hence we have

$$\frac{1}{4}\Sigma(\mathrm{R}r) = -\frac{\pi}{2}\left\{\frac{\mathrm{P}_1}{h_1}n_1{}^2s_1{}^3 + 2\frac{\mathrm{P}_1\mathrm{P}_2(h_1+h_2)}{(\mathrm{P}_1+\mathrm{P}_2)h_1h_2}n_1n_2s^3 + \frac{\mathrm{P}_2}{h_2}n_2{}^2s_2{}^3\right\},$$

$$= -\frac{\pi}{n}p\{n_1{}^2s_1{}^3 + 2n_1n_2s^3 + n_2{}^2s_2{}^3\};$$

for

$$\frac{\mathrm{P}_1}{h_1} = \frac{\mathrm{P}_2}{h_2} = \frac{\mathrm{P}_1+\mathrm{P}_2}{h_1+h_2} = \frac{1}{n}\left(\frac{n_1\mathrm{P}_1}{h_1} + \frac{n_2\mathrm{P}_2}{h_2}\right) = \frac{2p}{n},$$

where

$$n = n_1 + n_2\,.$$

In the special case $s_1 = s_2 = s$, this becomes, as in § 30,

$$\frac{1}{4}\Sigma(\mathrm{R}r) = -\pi pns^3\,.$$

To obtain an idea as to how the "ultimate volume," spoken of in that section, is affected by the difference of size of the particles, suppose $n_1 = n_2$. The values of the above quantities are

$$-\frac{\pi n}{4}p\,\{s_1{}^3 + 2s^3 + s_2{}^3\} \text{ and } -\pi nps^3\,;$$

so that (as we might have expected) disparity of size, with the same mean of diameters, *increases* the quantity in question.

Thus, if

$$s_1 : s : s_2 :: 1 : 2 : 3\,,$$

the ratio of the expressions above is 11 : 8. The utmost value it can have (when s_1/s_2 is infinite, or is evanescent) is 5 : 2.

XII. *Viscosity.*

36. Suppose the motion of the gas, *as a whole,* to be of the nature of a simple shear; such that, relatively to the particles in the plane of yz, those in the plane x have a common speed

$$\mathrm{V} = \mathrm{B}x$$

parallel to y. V, even when x is (say) a few inches, is supposed small compared with the speed of mean square. We have to determine the amount of momentum parallel to y which passes, per second, across unit area of the plane of yz.

In the stratum between x and $x + \delta x$ there are, per second per unit surface, $nve v\delta x$ collisions discharging particles with speed v to $v + dv$ (distributed uniformly in all directions) combined, of course, with the speed of translation of the stratum. The number of these particles which cross the plane of yz at angles θ to $\theta + d\theta$ with the axis of x is

$$\epsilon^{-ex\sec\theta}\sin\theta d\theta/2\,.$$

[Strictly speaking, the exponent should have had an additional term of the order eBx^2/v; but this is insensible compared with that retained until x is a very large multiple of the mean free path. See the remarks in § 39 below.] Each takes with it (besides its normal contribution, which need not be considered) the abnormal momentum

$$PBx\,,$$

relatively to yz and parallel to y.

Hence the whole momentum so transferred from x positive is

$$\frac{PBn}{2}\int \nu v \int_0^{\frac{\pi}{2}} \sin\theta d\theta \int_0^{\infty} \varepsilon^{-ex\sec\theta} ex dx,$$

or

$$\frac{PBn}{2}\int_0^{\infty} \frac{\nu v}{e}\int_0^{\frac{\pi}{2}} \cos^2\theta \sin\theta d\theta = \frac{PBn}{6}\int_0^{\infty}\frac{\nu v}{e}.$$

Doubling this, to get the full differential effect across the plane of yz, it becomes (§ 33)

$$\frac{PBnC_1}{3\pi ns^2\sqrt{h}} = \frac{PBn\ 0{\cdot}838}{3\pi ns^2\sqrt{h}}\,.$$

The multiplier of B, *i.e.* of dV/dx, is the coefficient of Viscosity. Its numerical value, in terms of density and mean path, is

$$\frac{\rho\lambda}{\sqrt{h}}0{\cdot}412\,.$$

Clerk-Maxwell, in 1860, gave the value

$$\frac{\rho l}{\sqrt{h}}0{\cdot}376\,,$$

which (because $l = 707\lambda/677$, as in § 11) differs from this in the ratio 20 : 21. In this case the short cuts employed have obviously entailed little numerical error. Since $\rho\lambda$ is constant for any one gas, the Viscosity (as Maxwell pointed out) is independent of the density.

37. Both expressions are proportional to the square-root of the absolute temperature. We may see at once that, on the hypothesis we have adopted, such must be the case. For, if we suppose the speed of every sphere to be suddenly increased m fold, the operations will go on precisely as before, only m times faster. But the absolute temperature will be increased as $m^2 : 1$. Similar anticipations may be made in the cases of Diffusion and of Thermal Conductivity.

Maxwell was led by his experimental measures of Viscosity, which seemed to show* that it increases nearly in proportion to the first power of the absolute temperature, to discard the notion of hard spheres, and to introduce the hypothesis of particles repelling one another with force inversely as the fifth power of the distance. I have already stated that there are very grave objections to the introduction of *repulsion* into this subject, except of course in the form of elastic restitution. That the particles of a gas have *this* property is plain from their capability of vibrating, so that they must lose energy of translation by impact; and I intend, in the next instalment of this investigation, so far to modify the fundamental assumption hitherto made as to deduce the effects corresponding to a coefficient of restitution less than unity; and also to take account of molecular *attraction*, specially limited in its range to distances not much greater than the diameter of a sphere.

XIII. *Thermal Conductivity.*

38. We must content ourselves with the comparatively simple case of the steady flow of heat in one direction; say parallel to the axis of x. This will be assumed to be vertical, the temperature in the gas increasing upwards, so as to prevent convection currents. No attention need, otherwise, be paid to the effects of gravity.

Hence the following conditions must be satisfied :—

- (*a*) Each horizontal layer of the gas is in the special state, compounded with a definite translation *vertically*.
- (*b*) The pressure is constant throughout the gas.
- (*c*) There is, on the whole, no passage of gas across any horizontal plane.
- (*d*) Equal amounts of energy are, on the whole, transferred (in the same direction) across unit area of all such planes.

39. Let n be the number of particles per unit volume in the layer between x and $x+dx$; ν the fraction of them whose speed, relatively to the neighbours as a whole, lies between v and $v+dv$; α the speed of translation of the layer.

The *number* of particles which pass, per unit area per second, from x positive through the plane $x=0$, is the sum of those escaping, after collision, from all the layers for positive x, and not arrested on their way :—viz.,

$$\frac{1}{2}\int_0^\infty \int_0^{\frac{\pi}{2}} \int_0^a n\nu v e \epsilon^{-\sec\theta \int_0^x e dx} \sin\theta d\theta \frac{v\cos\theta-\alpha}{v\cos\theta} dx .$$

Here a, though in any ordinary case it need not be more than a very small fraction of an inch, is a quantity large compared with the mean free path of a

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* *Cf.*, however, Stokes, *Phil. Trans.*, 1886, vol. clxxvii. p. 786.

particle. Its value will be more exactly indicated when the reason for its introduction is pointed out.

The last factor of the integrand depends on the fact that the particles are emitted from *moving* layers:—involving the so-called DÖPPLER, properly the RÖMER, principle.

We neglect, however, as insensible the difference between the absorption due to *slowly* moving layers and that due to the same when stationary.

Because a, the range of x, is small we may write with sufficient approximation

$$n = n_0 + n_0'x, \text{ \&c., \&c.}$$

Introducing this notation, the expression above becomes

$$\frac{1}{2}\int_0^\infty \int_0^{\frac{\pi}{2}} \int_0^a n_0 \nu_0 v e_0 \left(1 + \left(\frac{n_0'}{n_0} + \frac{\nu_0'}{\nu_0} + \frac{e_0'}{e_0}\right) x + \ldots\right) \epsilon^{-\sec\theta \int_0^x e dx} \sin\theta \, d\theta \frac{v\cos\theta - \alpha}{v\cos\theta} dx \, .$$

Now, to the degree of approximation adopted,

$$\int_0^x e dx = e_0 x + e_0' x^2/2 \, .$$

The second term of this must always be very small in comparison with the first, even for an exceptionally long free path. But, if we were to make

$$x = 2e_0/e_0'$$

the second term would become *equal* to the first. Hence a, the upper limit of the x integration, must be made much smaller than this quantity. Thus we may write

$$\epsilon^{-\sec\theta \int_0^x e dx} = \epsilon^{-e_0 x \sec\theta}(1 - e_0' x^2 \sec\theta/2 + \ldots) \, .$$

We said, above, that

$$e_0 a = a \Big/ \frac{1}{e_0}$$

is a large number, say of the order 10^2. It appears then at once that terms in

$$\epsilon^{-e_0 a} = \epsilon^{-100} = 10^{-43} \text{ nearly}$$

may be neglected. Such terms occur at the upper limit in the integration with regard to x above, and what we have said shows, *first* why a had to be introduced, *second* why it disappears from the result.

Writing now only those factors of the above expression which are concerned in the integration with respect to x, we have

$$\int_0^a \left(1 + \left(\frac{n_0'}{n_0} + \frac{\nu_0'}{\nu_0} + \frac{e_0'}{e_0}\right) x + \ldots\right)\left(1 - e_0' x^2 \sec\theta/2 + \ldots\right) \epsilon^{-e_0 x \sec\theta} dx \, ,$$

or

$$\frac{1}{e_0}\left(\cos\theta + \frac{1}{e_0}\left(\frac{n_0'}{n_0} + \frac{\nu_0'}{\nu_0}\right)\cos^2\theta\right) .$$

The terms in e_0' are found to have cancelled one another, a result which greatly simplifies the investigation.

Had we complicated matters by introducing $\alpha_0+\alpha_0'x$ in place of α, the term in α_0' (which, if it exist at all, is at least very small) would have been divided on integration *twice* by e_0, a quantity whose value is, on the average, of the order $5 . 10^5$ (to an inch as unit of length).

The expression now becomes

$$\frac{1}{2}\int_0^\infty\int_0^{\frac{\pi}{2}} nv\left(1+\left(\frac{n'}{n}+\frac{v'}{v}\right)\frac{\cos\theta}{e}\right)\left(v\cos\theta-\alpha\right)\sin\theta d\theta\,.$$

We have omitted the zero suffixes, as no longer required; and, as the plane $x=0$ is arbitrary, the expression is quite general.

Omitting the product of the two small terms, and integrating with respect to θ, we have

$$\frac{1}{2}\int_0^\infty nv\left(v/2-\alpha+\left(\frac{n'}{n}+\frac{v'}{v}\right)v/3e\right).$$

The corresponding expression for the number of particles which pass through the plane from the negative side is, of course, to be obtained by simply changing the signs of the two last terms. Thus, by (c) of § 38, we have

$$\int_0^\infty nv\left(\alpha-\left(\frac{n'}{n}+\frac{v'}{v}\right)v/3e\right)=0\,,$$

or

$$\alpha=\int_0^\infty v\left(\frac{n'}{n}+\frac{v'}{v}\right)v/3e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

40. The *pressure* at the plane $x=0$, taken as the whole momentum (parallel to x) which crosses it per unit area per second, is to be found by introducing into our first integrand the additional factor

$$\mathrm{P}(v\cos\theta-\alpha)\,,$$

where P is the mass of a particle. There results

$$\frac{\mathrm{P}}{2}\int_0^\infty nv\left(v^2/3-v\alpha+\left(\frac{n'}{n}+\frac{v'}{v}\right)v^2/4e\right).$$

We must take the *sum* of this, and of the same with the signs of the two last terms changed; so that the pressure (which is constant throughout, by (b) of § 38) is

$$p=\frac{\mathrm{P}}{3}\int_0^\infty nvv^2=\frac{\mathrm{P}n}{2h} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

Thus n/h is constant throughout the gas.

[If a very small, thin, disc were placed in the gas, with its plane parallel to *yz, and the steady state not thereby altered,* the difference of pressures on its sides would be

$$nP\int_0^\infty \nu\left(2v\alpha-\left(\frac{n'}{n}+\frac{\nu'}{\nu}\right)v^2/2e\right)$$

or

$$p\frac{\lambda}{0\cdot677}\,\frac{h'}{h}\left(\frac{8}{3\sqrt{\pi}}\left(\frac{5}{2}C_1-C_3\right)-\frac{5}{2}C_2+C_4\right).$$

For ordinary pressures, and a temperature gradient 10° C. per inch, this is of the order 10^{-7} atmosphere only.]

41. For the *energy* which passes per second per unit of area across $x=0$, we must introduce into the first integrand of § 39 the additional factor

$$\frac{P}{2}\left(v^2-2v\alpha\cos\theta\right);$$

and the result of operations similar to those for the number of particles is

$$E=-\frac{P}{6}\int_0^\infty n\nu v^3\left(\left(\frac{n'}{n}+\frac{\nu'}{\nu}\right)\Big/e-5\alpha\Big/v\right)\quad.\quad.\quad.\quad.\quad.\quad(3.)$$

This expresses the excess of the energy passing from the negative to the positive side of $x=0$, over that passing from positive to negative; and, by (d) of § 38 must be constant.

42. To put (1) and (3) in a more convenient and more easily intelligible form, note that because

$$\nu=4\sqrt{\frac{h^3}{\pi}}\,\varepsilon^{-hv^2}v^2dv$$

we have

$$\frac{\nu'}{\nu}=\frac{3}{2}\frac{h'}{h}-h'v^2.$$

But, by (2),

$$\frac{n'}{n}=\frac{h'}{h}.$$

Thus, by (1),

$$\alpha=\frac{h'}{h}\int_0^\infty \nu v\left(\frac{5}{2}-hv^2\right)\Big/3e,$$

$$=\frac{h'}{3\sqrt{h^3}}\frac{1}{n\pi s^2}\left(\frac{5}{2}C_1-C_3\right),$$

$$=\frac{h'}{\sqrt{h^5}}\,\frac{P}{6p\pi s}\left(\frac{5}{2}C_1-C_3\right)\quad.\quad.\quad.\quad.\quad.\quad(1'.)$$

Similarly (3) becomes

$$E=\frac{h'}{\sqrt{h^5}}\,\frac{P}{6\pi s^2}\left(\frac{25}{4}C_1-5C_3+C_5\right)\quad.\quad.\quad.\quad.\quad.\quad(3'.)$$

43. The only variable factor $(h'/h^{\frac{5}{2}})$ in these expressions for α, and for E, is the same in both. Hence, as E does not vary with x, $h'/h^{\frac{5}{2}}$ is constant, and so also is α. Thus since, if τ be absolute temperature, we have

$$h\tau = \text{constant};$$

we find at once,

$$\tau^{\frac{3}{2}} = \text{A} + \text{B}x.$$

Thus the distribution of temperature, and therefore that of density, is determined when the terminal conditions are given. The formula just given agrees with the result first obtained by CLAUSIUS in an extremely elaborate investigation,* in which he showed that MAXWELL'S earliest theory of Heat-Conduction by gases is defective.

The general nature of the motion of the gas is now seen to be analogous to that of liquid mud when a scavenger tries to sweep it into a heap. The broom produces a translatory motion of the mud, which is counteracted by gravitation-sliding due to the surface gradient:—just as the displacement (by translation) of the whole gas, from hot to cold, is counteracted by the greater number of particles discharged (after collisions) from a colder and denser layer, than from an adjoining warmer and less dense layer.

44. The results of calculation of values of C_r given in the *Appendix* enable us to put the expressions (1′) and (3′) into the more convenient forms

$$\alpha = \frac{h'}{\sqrt{h^5}} \frac{\rho\lambda}{p} 0{\cdot}06 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1'')$$

$$\text{E} = \frac{h'}{\sqrt{h^5}} \rho\lambda \, 0{\cdot}45 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3'')$$

where it is to be remarked that the product $\rho\lambda$ is independent of the temperature of the gas.

The Conductivity, k, is defined by the equation

$$k\frac{d\tau}{dx} = -\text{E},$$

and thus its value is

$$k = \sqrt{\frac{\tau}{\tau_0^3}} \frac{\rho\lambda}{\sqrt{h_0^3}} 0{\cdot}45,$$

where τ_0, h_0 are simultaneous values of τ and h.

At 0° C. (*i.e.* $\tau = 274$) this is, for air, nearly 3.10^{-5} in thermal units on the pound-foot-minute-Centigrade system:—*i.e.* about 1/28,000 of the conductivity of iron, or 1/3600 of that of lead.† Of course, with our assumption

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* *Pogg. Ann.*, cxv, 1862; *Phil. Mag.*, 1862, I.

† *Trans. R. S. E.*, 1878, p. 717.

of hard spherical particles, we have not reckoned the part of the conducted energy which, in real gases, is due to rotation or to vibration of individual particles.

XIV. *Diffusion.*

45. The complete treatment of this subject presents difficulties of a very formidable kind, several of which will be apparent even in the comparatively simple case which is treated below. We take the case of a uniform vertical tube, of unit area in section, connecting two vessels originally filled with different gases, or (better) mixtures of the same two gases in different proportions, both, however, maintained at the same temperature; and we confine ourselves to the investigation of the motion when it can be treated as approximately steady. We neglect the effect of gravity (the denser gas or mixture being the lower), and we suppose the speeds of the group-motions to be very small in comparison with the speed of mean square in either gas. [In some of the investigations which follow, there are (small) parts of the diffusion-tube in which one of the gases is in a hopeless minority as regards the other. Though one of the initial postulates (*d* of § 1) is violated, I have not thought it necessary to suppress the calculations which are liable to this objection; for it is obvious that the conditions, under which alone it could arise, are unattainable in practice.]

Clerk-Maxwell's Theorem (§ 15), taken in connection with our preliminary assumption, shows that at every part of the tube the number of spheres per cubic unit, and their average energy, are the same. Hence, if n_1, n_2, be the numbers of the two kind of spheres, per cubic unit, at a section x of the tube

$$n_1+n_2=n=\text{constant}, \qquad (1.)$$

Also, if P_1, P_2, be the masses of the spheres in the two systems respectively, h_1 and h_2 the measures (§ 3) of their mean square speeds, we have

$$P_1/h_1=P_2/h_2=(n_1P_1/h_1+n_2P_2/h_2)/n=2p/n, \qquad (2.)$$

where p is the constant pressure.

Strictly speaking, the fact that there is a translational speed of each layer of particles must affect this expression, but only by terms of the first order of small quantities.

46. The number of particles of the P_1 kind which pass, on the whole, towards positive x through the section of the tube is (as in § 39)

$$n_1\alpha_1-n_1'\int_0^\infty \nu_1 v/3e_1\,;$$

where α_1 is the (common) translational speed of the P_1s, and $1/e_1$ the mean free path of a P_1 whose speed is v. We obtain this by remarking that, in the present problem, h_1 is regarded as constant, so that there is no term in ν_1'.

Hence, if G_1 be the mass of the first gas on the negative side of the section, divided by the area of the section, we have

$$\frac{dG_1}{dt} = -P_1(n_1\alpha_1 - n_1'{}_1\mathfrak{C}_1/3) \quad . \quad . \quad . \quad . \quad . \qquad (3.)$$

If G_2 be the corresponding mass of the second gas, we have (noting that, by (1), $n_1' + n_2' = 0$)

$$\frac{dG_2}{dt} = -P_2(n_2\alpha_2 + n_1'{}_2\mathfrak{C}_1/3) \quad . \quad . \quad . \quad . \quad . \qquad (4.)$$

From the definitions of the quantities G_1, G_2, we have also

$$\left.\begin{aligned} \frac{dG_1}{dx} &= P_1 n_1\,, \quad \frac{d^2G_1}{dx^2} = P_1 n_1'\,, \\ \frac{dG_2}{dx} &= P_2 n_2\,, \quad \frac{d^2G_2}{dx^2} = -P_2 n_1'\,. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (5.)$$

47. We have now to form the equations of motion for the layers of the two gases contained in the section of the tube between x and $x + \delta x$. The increase of momentum of the P_1 layer is due to the difference of pressures, behind and before, caused by P_1s; minus the resistance due to that portion of the impacts of some of the P_1s against P_2s in the section itself, which depends upon the relative speeds of the two systems, each as a whole. This is a small quantity of the order the whole pressure on the surfaces of the particles multiplied by the ratio of the speed of translation to that of mean square. The remaining portion (relatively very great) of the impacts in the section is employed, as we have seen, in maintaining or restoring the "special state" in each gas, as well as the MAXWELL condition of partition of energy between the two gases. If R be the resistance in question, the equations of motion are

$$\left.\begin{aligned} \frac{\partial}{\partial t}(P_1 n_1 \alpha_1 \delta x) &= -\frac{1}{2}\frac{d}{dx}\left(\frac{P_1 n_1}{h_1}\right)\delta x - R\delta x\,, \\ \frac{\partial}{\partial t}(P_2 n_2 \alpha_2 \delta x) &= -\frac{1}{2}\frac{d}{dx}\left(\frac{P_2 n_2}{h_2}\right)\delta x + R\delta x\,, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (6.)$$

where ∂ represents *total* differentiation.

48. To calculate the value of R, note that, in consequence of the assumed smallness of α_1, α_2, relatively to the speeds of mean square of the particles, the number of collisions of a P_1 with a P_2, and the circumstances of each, may be treated as practically the same as if α_1 and α_2 were each zero:—*except* in so far

that there will be, in the expression for the relative speed in the direction of the line of centres at impact, an additional term

$$(\alpha_1-\alpha_2)\cos\psi,$$

where ψ is the inclination of the line of centres to the axis of x. Thus to the impulse, whose expression is of the form

$$-\frac{2PQ}{P+Q}(u-v),$$

as in § 19 of the First Part of the paper, there must be added the term we seek, viz.,

$$-\frac{2P_1P_2}{P_1+P_2}(\alpha_1-\alpha_2)\cos\psi.$$

This must be resolved again parallel to x, for which we must multiply by $\cos\psi$. Also, as the line of centres may have with equal probability all directions, we must multiply further by $\sin\psi d\psi/2$, and integrate from 0 to π. The result will be the average transmission, per collision, per P_1, of *translatory* momentum of the layer parallel to x. Taking account of the number of impacts of a P_1 on a P_2, as in § 23, we obtain finally

$$R=\frac{4}{3}n_1n_2s^2\sqrt{\frac{\pi(h_1+h_2)}{h_1h_2}}\frac{P_1P_2}{P_1+P_2}(\alpha_1-\alpha_2) \quad . \quad . \quad . \quad (7.)$$

where s is the semi-sum of the diameters of a P_1 and a P_2.

49. To put this in a more convenient form, note that (2), in the notation of (5), gives us the relation

$$\frac{1}{h_1}\frac{dG_1}{dx}+\frac{1}{h_2}\frac{dG_2}{dx}=2p,$$

whence

$$G_1/h_1+G_2/h_2=2px. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8.)$$

We have not added an arbitrary constant, for no origin has been specified for x. Nor have we added an arbitrary function of t, because (as will be seen at once from (3)) this could only be necessary in cases where the left-hand members of (6) are quantities comparable with the other terms in these equations. They are, however, of the order of

$$\frac{d^2G_1}{dt^2},\ \frac{d^2G_1}{dxdt}\alpha_1,\ \&c.,$$

and cannot rise into importance except in the case of motions much more violent than those we are considering.

From (8) we obtain

$$\frac{dG_1}{dt}\Big/h_1+\frac{dG_2}{dt}\Big/h_2=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad (9.)$$

which signifies that equal *volumes* of the two gases pass, in the same time, in opposite directions through each section of the tube. This gives a general description of the nature of the cases to which our investigations apply.

But, by (3) and (4), we have for the value of

$$\mathrm{P}_1\mathrm{P}_2 n_1 n_2(\alpha_1-\alpha_2)$$

the expression

$$-\mathrm{P}_2 n_2\Big(\frac{d\mathrm{G}_1}{dt}-\frac{1}{3}\mathrm{P}_1 n_1{}'_1\mathfrak{C}_1\Big)+\mathrm{P}_1 n_1\Big(\frac{d\mathrm{G}_2}{dt}+\frac{1}{3}\mathrm{P}_2 n_1{}'_2\mathfrak{C}_1\Big);$$

or, by (9), (2), and (5)

$$-2ph_2\Big(\frac{d\mathrm{G}_1}{dt}-\frac{1}{3n}\,\frac{d^2\mathrm{G}_1}{dx^2}(n_{21}\mathfrak{C}_1+n_{12}\mathfrak{C}_1)\Big)\cdot$$

Substituting this for the corresponding factors of R in the first of equations (6), and neglecting the left-hand side, we have finally

$$0=-\frac{1}{2h_1}\,\frac{d^2\mathrm{G}_1}{dx^2}+\frac{8}{3}s^2\sqrt{\frac{\pi(h_1+h_2)}{h_1h_2}}\;\frac{ph_2}{\mathrm{P}_1+\mathrm{P}_2}\Big\{\frac{d\mathrm{G}_1}{dt}-\frac{1}{3n}\,\frac{d^2\mathrm{G}_1}{dx^2}(n_{21}\mathfrak{C}_1+n_{12}\mathfrak{C}_1)\Big\}$$

or

$$\frac{d\mathrm{G}_1}{dt}=\Big(\frac{3}{16s^2}\;\frac{\mathrm{P}_1+\mathrm{P}_2}{\sqrt{\pi(h_1+h_2)h_1h_2}}\cdot\frac{1}{p}+\frac{1}{3n}(n_{21}\mathfrak{C}_1+n_{12}\mathfrak{C}_1)\Big)\frac{d^2\mathrm{G}_1}{dx^2};$$

or, somewhat more elegantly,

$$\frac{d\mathrm{G}_1}{dt}=\Big(\frac{3}{8ns^2}\sqrt{\frac{h_1+h_2}{\pi h_1h_2}}+\frac{1}{3n}(n_{21}\mathfrak{C}_1+n_{12}\mathfrak{C}_1)\Big)\frac{d^2\mathrm{G}_1}{dx^2}\cdot \quad . \quad . \quad . \quad . \quad (10.)$$

50. This equation resembles that of FOURIER for the linear motion of heat; but, as already stated in § 34, the quantities $\mathfrak{C}_1$ which occur in it render it in general intractable. The first part of what is usually called the *diffusion-coefficient* (the multiplier of $d^2\mathrm{G}_1/dx^2$ above) is constant; but the second, as is obvious from (5) and (8), is, except in the special case to which we proceed, a function of $d\mathrm{G}_1/dx$; *i.e.* of the percentage composition of the gaseous mixture.

51. In the special case of equality, both of mass and of diameter, between the particles of the two systems, the diffusion-coefficient becomes

$$\mathrm{D}=\frac{3}{8ns^2}\sqrt{\frac{2}{\pi h}}+\frac{\mathrm{C}_1}{3n\pi s^2\sqrt{h}},$$

or

$$\mathrm{D}=\Big(\frac{3}{4}\sqrt{\frac{\pi}{2}}+\frac{\mathrm{C}_1}{3}\Big)\frac{\lambda}{0{\cdot}677\sqrt{h}}=\frac{\lambda}{\sqrt{h}}1{\cdot}8,$$

where λ is the mean free path in the system. Hence the diffusion-coefficient among equal particles is directly as the mean free path, and as the square root of the absolute temperature. FOURIER's solutions of (10) are of course applicable in this special case.

If we now suppose that our arrangement is a tube of length l and section S, connecting two infinite vessels filled with the two gases respectively; and, farther, assume that the diffusion has become steady, the equation (10) becomes

$$\frac{dG_1}{dt}=D\frac{d^2G_1}{dx^2},$$

where the left-hand member is constant. Also, it is clear that, since dG_1/dx must thus be a *linear* function of x, we have

$$\frac{dG_1}{dx}=Pn_1=Pn\left(1-\frac{x}{l}\right),$$

so that the mass of either gas which passes, per second, across any section of the tube is

$$SD\rho/l$$

where ρ is the common density of the two gases.

For comparison with the corresponding formulæ in the other cases treated below, we may now write our result as

$$l\frac{dG_1}{dt}=-\frac{P}{\pi s^2\sqrt{h}}1{\cdot}22.$$

Also, to justify our assumption as to the order of the translatory speed, we find by (3)

$$\alpha_1=\frac{1{\cdot}38\lambda}{(l-x)\sqrt{h}}.$$

Hence, except where $l-x$ is of the order of one thousandth of an inch or less, this is very small compared with $h^{-\frac{1}{2}}$. And it may safely be taken as impossible that n_1 can (experimentally) be kept at 0 at the section $x=l$.

If the vessels be of finite size, and if we suppose the contents of each to be always thoroughly mixed, we can approximate to the law of mixture as follows. On looking back at the last result, we see that for ρ we must now substitute the *difference* of densities of the first gas at the ends of the connecting tube. Let g_1, g_2 be the quantities of the two gases which originally filled the vessels respectively; and neglect, in comparison with them, the quantity of either gas which would fill the tube. Then, obviously,

$$\frac{dG_1}{dt}=-\frac{SD\rho}{l}\left(\frac{G_1}{g_1}-\frac{g_1-G_1}{g_2}\right),$$

whence

$$G_1=\frac{g_1g_2}{g_1+g_2}\left\{\frac{g_1}{g_2}+\epsilon^{-\frac{SD\rho}{l}\frac{g_1+g_2}{g_1g_2}t}\right\}.$$

This shows the steps by which the initial state $(g_1, 0)$ tends asymptotically to the final state $\left(\frac{g_1}{g_1+g_2}g_1, \frac{g_2}{g_1+g_2}g_1\right)$, in which the gases are completely mixed. When the vessels are equal this takes the simple form

$$G_1=\frac{g}{2}\left(1+\varepsilon^{-\frac{2SD\rho t}{gl}}\right).$$

52. In the case just treated there is no transmission of energy, so that the fundamental hypotheses are fully admissible. In general, however, it is not so. The result of § 41, properly modified to apply to the present question, shows that the energy which, on the whole, passes positively across the section x is, per unit area per second,

$$\frac{5}{4}\left(\frac{P_1 n_1 \alpha_1}{h_1}+\frac{P_2 n_2 \alpha_2}{h_2}\right)-\frac{1}{6}n_1'(P_1\,{}_1\mathfrak{C}_3-P_2\,{}_2\mathfrak{C}_3).$$

This, of course, in general differs from section to section, and thus a disturbance of temperature takes place. In such a case we can no longer assume that h_1 and h_2 are absolute constants; and thus terms in $\mathfrak{C}_5$ would come in; just as a term in C_5 appeared in the expression for energy conducted (§ 42). Thus, in order that our investigation may be admissible, the process must be conducted at constant temperature. This, in general, presupposes conditions external to the apparatus.

53. Though it appears hopeless to attempt a general solution of equation (10), we can obtain from it (at least approximately) the conditions for a steady state of motion such as must, we presume, finally set in between two infinite vessels filled with different gases at the same temperature and pressure. For the left-hand member is then an (unknown) constant, a second constant is introduced by integrating once with respect to x; and these, which determine the complete solution, are to be found at once by the terminal conditions

$$\frac{1}{P_1}\frac{dG_1}{dx}=n_1=\begin{cases} n \text{ for } x=0\,, \\ 0 \quad ,, \quad =l\,. \end{cases} \quad . \quad . \quad . \quad . \quad . \qquad (11.)$$

And, by a slight but obvious modification of the latter part of § 51 above, we can easily extend the process to the case in which the vessels are of finite size:—always, however, on the assumption that their contents may be regarded as promptly assuming a state of uniform mixture. The consideration of § 52, however, shows that the whole of the contents must be *kept* at constant temperature, in order that this result may be strictly applicable.

54. Recurring to the special case of § 51, let us now suppose that, while the masses of the particles remain equal, their diameters are different in the two gases. Thus, suppose $s_1>s_2$. Then it is clear that

$$s_1{}^2-s^2, \quad \text{and} \quad s^2-s^{\,2},$$

are both positive. In this case, infinite terminal vessels being supposed, (10) gives for the steady state

$$\mathrm{A}=\frac{\mathrm{P}}{\pi n\sqrt{h}}\left\{\frac{3}{4s^2}\sqrt{\frac{\pi}{2}}+\frac{\mathrm{C}_1}{3}\left(\frac{n_2}{n_1s_1^{\,2}+n_2s^2}+\frac{n_1}{n_1s^2+n_2s_2^{\,2}}\right)\right\}\frac{dn_1}{dx}\,; \quad . \quad . \quad (12.)$$

whose integral, between limits as in (11) above, is

$$-\mathrm{A}l=\frac{\mathrm{P}}{\pi n\sqrt{h}}\left\{\frac{3n}{4s^2}\sqrt{\frac{\pi}{2}}+\frac{\mathrm{C}_1n}{3}\left(\frac{1}{s^2-s_2^{\,2}}-\frac{1}{s_1^{\,2}-s^2}+\frac{2s_1^{\,2}}{(s_1^{\,2}-s^2)^2}\log\frac{s_1}{s}+\frac{2s_2^{\,2}}{(s^2-s_2^{\,2})^2}\log\frac{s_2}{s}\right)\right\}\,.$$

Here A is the rate of passage of the first gas, in *mass* per second per unit area of the section of the tube.

If now we put

$$s_1=s+\sigma,\ s_2=s-\sigma\,,$$

then, when σ is small compared with s, the multiplier of $\mathrm{C}_1n/3$ is

$$(1+\sigma^2/3s^2)/s^2\,,\ \text{nearly.}$$

When σ is nearly equal to s, *i.e.* one of the sets of particles exceedingly small compared with the other, it is nearly

$$1{\cdot}283/s^2\,.$$

Thus it appears that a difference in size, the mean of the diameters being unchanged, favours diffusion.

Suppose, for instance,

$$s_1\ :\ s\ :\ s_2\ ::\ 3\ :\ 2\ :\ 1\,,$$

and we have

$$\begin{aligned}\mathrm{A}&=-\frac{\mathrm{P}}{\pi ls^2\sqrt{h}}\left\{\frac{3}{4}\sqrt{\frac{\pi}{2}}+\frac{2\mathrm{C}_1}{3}\left(\frac{4}{15}+\frac{36}{25}\log\frac{3}{2}+\frac{4}{9}\log\frac{1}{2}\right)\right\}\,,\\ &=-\frac{\mathrm{P}}{\pi ls^2\sqrt{h}}\left\{\frac{3}{4}\sqrt{\frac{\pi}{2}}+\frac{\mathrm{C}_1}{3}1{\cdot}085\right\}=-\frac{\mathrm{P}}{\pi ls^2\sqrt{h}}1{\cdot}24\,,\\ &=-\frac{\rho\lambda}{l\sqrt{h}}\,1{\cdot}83\,.\end{aligned}$$

Compare this with the result for equal particles (§ 51), remembering that λ now stands for the mean free path of a particle of either gas in a space filled with the other :—and we see that (so long at least as the masses are equal) diffusion depends mainly upon the mean of the diameters, being but little affected by even a considerable disparity in size between the particles of the two gases. Thus it appears that the viscosity and (if the experimental part of the inquiry could be properly carried out) conductivity give us much more definite information as to the relative sizes of particles of different gases than we can obtain from the results of diffusion.

Equation (12) shows how the *gradient* of density of either gas varies, in the stationary state, with its percentage in the mixture. For the multiplier of $\frac{dn_1}{dx}$ is obviously a maximum when

$$\frac{1}{s^2+ys_1^2}+\frac{1}{s^2+s_2^2/y},$$

in which $y=n_1/n_2$, is so. This condition gives

$$n_1/n_2=y=s_2/s_1 .$$

Hence the gradient is *least* steep at the section in which the proportion of the two gases is inversely as the ratio of the diameters of their particles; and it increases either way from this section to the ends of the tube, at each of which it has the same (greatest) amount. This consideration will be of use to the full understanding of the more complex case (below) in which the masses, as well as the diameters, of the particles differ in the two gases.

55. Let us now suppose the mass per particle to be different in the two gases. The last terms of the right-hand side of (10), viz.,

$$\frac{1}{3n}(n_{21}\mathfrak{C}_1+n_{12}\mathfrak{C}_1)\frac{d^2\mathrm{G}_1}{dx^2},$$

may be written in the form

$$\frac{\mathrm{P}_1}{3\pi n}\frac{dn_1}{dx}\left\{\frac{(n-n_1)h_2}{\sqrt{h_1}}\int_0^\infty\frac{f(y)dy}{n_1h_2s_1^2\mathrm{F}(y)+(n-n_1)h_1s^2\mathrm{F}\left(y\sqrt{\frac{h_2}{h_1}}\right)}+\frac{n_1h_1}{\sqrt{h_2}}\int_0^\infty\frac{f(y)dy}{(n-n_1)h_1s_2^2\mathrm{F}(y)+n_1h_2s^2\mathrm{F}\left(y\sqrt{\frac{h_1}{h_2}}\right)}\right\}$$

where the meanings of f and F are as in § 34.

If we confine ourselves to the steady state, we may integrate (10) directly with respect to x, since $d\mathrm{G}_1/dt$ is constant. In thus operating on the part just written, the integration with regard to x (with the limiting conditions as in (11)) can be carried out under the sign of integration with respect to y:—and then the y integration can be effected by quadratures.

The *form* of the x integral is the same in each of the terms. For

$$\int_n^0\frac{(n-n_1)dn_1}{\mathrm{A}n_1+\mathrm{B}(n-n_1)}=\int_n^0\frac{n_1dn_1}{\mathrm{A}(n-n_1)+\mathrm{B}n_1}=\frac{n}{\mathrm{A}-\mathrm{B}}\left\{1+\frac{\mathrm{A}}{\mathrm{A}-\mathrm{B}}\log\frac{\mathrm{B}}{\mathrm{A}}\right\}.$$

This expression is necessarily negative, as A and B are always positive. When A and B are nearly equal, so that $\mathrm{B}=(1+e)\mathrm{A}$, its value is

$$-\frac{n}{\mathrm{A}}\left(\frac{1}{2}-\frac{e}{3}+\ldots\right),$$

so that, even when A and B are equal, there is no infinite term.

It is easy to see, from the forms of $F(y)$, and of its first two differential coefficients, that the equation

$$h_2 s_1^2 F(y) = h_1 s_2^2 F\left(y\sqrt{\frac{h}{h_1}}\right)$$

can hold for, at most, *one* finite positive value of y.

56. As a particular, and very instructive case, let us suppose

$$P_1 : P_2 :: h_1 : h_2 :: 16 : 1\,,$$

the case of oxygen and hydrogen.

(*a*) First, assume the diameters to be equal. Then the integral of (10), with limits as in (11), taken on the supposition that the flow is constant, is

$$l\frac{dG_1}{dt} = -\frac{P_1}{\pi s^2\sqrt{h_1}}\left\{\frac{3}{8}\sqrt{17\pi} - \frac{1}{3}\int_0^\infty dy\left(\frac{f(y) - 16f\left(\frac{y}{4}\right)}{F(y) - 16F\left(\frac{y}{4}\right)} + \frac{F(y)f(y) - 16^2F\left(\frac{y}{4}\right)f\left(\frac{y}{4}\right)}{\left(F(y) - 16F\left(\frac{y}{4}\right)\right)^2}\log\frac{16F\left(\frac{y}{4}\right)}{F(y)}\right)\right\}$$

As remarked above, the definite integral is essentially negative. For so is every expression of the form

$$\frac{a-b}{A-B} + \frac{Aa - Bb}{(A-B)^2}\log\frac{B}{A}$$

provided A, B, a, and b be all positive. When A and B are equal its value is

$$-\frac{1}{2A}(a+b).$$

I have made a rough attempt at evaluation of the integral, partly by calculation, partly by a graphic method. My result is, at best, an approximation, for the various instalments of the quadrature appear as the relatively small *differences* of two considerable quantities. Thus the three decimal places, to which, from want of leisure, I was obliged to confine myself, are not sufficient to give a very exact value. The graphical representations of my numbers were, however, so fairly smooth that there seems to be little risk of large error. The *full* curve in the sketch below shows (on a ten-fold scale) the values of the integrand (with their signs changed), as ordinates, to the values of y as abscissæ. The area is about $-2{\cdot}165$. Hence we have

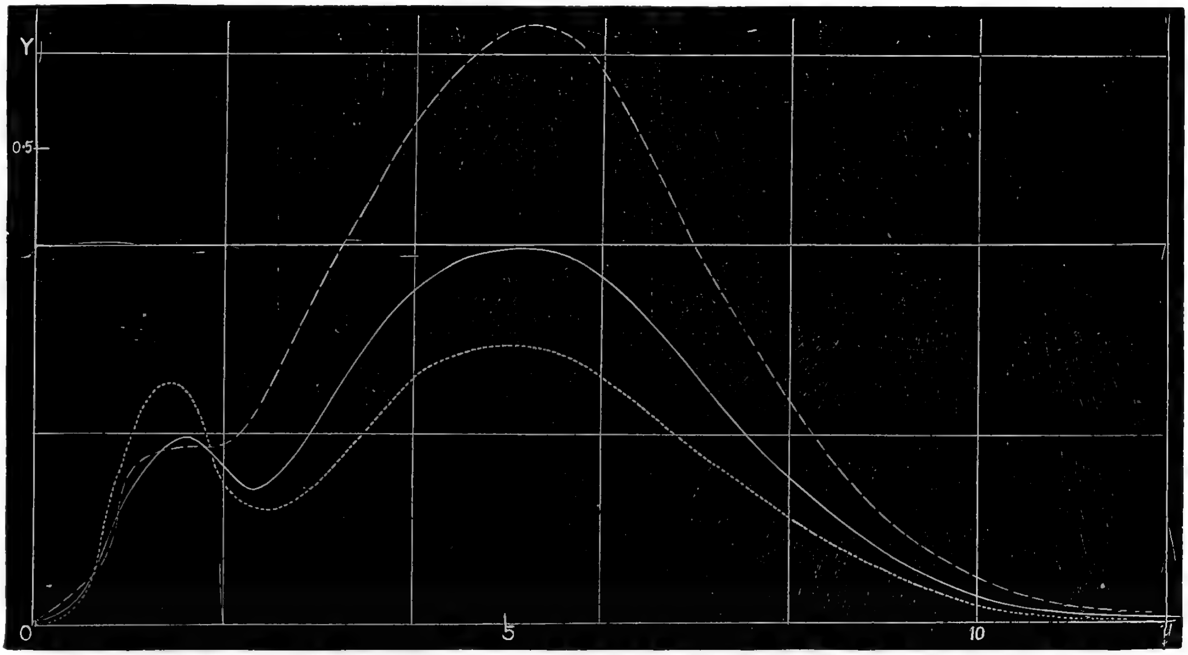
$$l\frac{dG_1}{dt} = -\frac{P_1}{\pi s^2\sqrt{h_1}}3{\cdot}463\,,$$

(*b*) Suppose next that the diameter of a P_1 is three times that of a P_2, but the semi-sum of the diameters is s as before. The definite integral takes the form

$$\int_0^\infty dy\left\{\frac{f(y)}{\frac{9}{4}F(y)-16F\left(\frac{y}{4}\right)}-\frac{16f\left(\frac{y}{4}\right)}{F(y)-4F\left(\frac{y}{4}\right)}+\frac{\frac{9}{4}F(y)f(y)}{\left(\frac{9}{4}F(y)-16F\left(\frac{y}{4}\right)\right)^2}\log\frac{64F\left(\frac{y}{4}\right)}{9F(y)}-\frac{64F\left(\frac{y}{4}\right)f\left(\frac{y}{4}\right)}{\left(F(y)-4F\left(\frac{y}{4}\right)\right)^2}\log\frac{4F\left(\frac{y}{4}\right)}{F(y)}\right\}$$

The corresponding curve is exhibited by the dashed line in the sketch, and its area is about $-3\cdot157$. Hence, in this case,

$$l\frac{dG_1}{dt}=-\frac{P_1}{\pi s^2\sqrt{h_1}}3\cdot793\,.$$



(*c*) Still keeping the sum of the semidiameters the same, let the diameter of a P_2 be three times that of a P_1. The integral is

$$\int_0^\infty dy\left\{\frac{f(y)}{\frac{1}{4}F(y)-16F\left(\frac{y}{4}\right)}-\frac{16f\left(\frac{y}{4}\right)}{F(y)-36F\left(\frac{y}{4}\right)}+\frac{\frac{1}{4}F(y)f(y)}{\left(\frac{1}{4}F(y)-16F\left(\frac{y}{4}\right)\right)^2}\log\frac{64F\left(\frac{y}{4}\right)}{F(y)}-\frac{576F\left(\frac{y}{4}\right)f\left(\frac{y}{4}\right)}{\left(F(y)-36F\left(\frac{y}{4}\right)\right)^2}\log\frac{36F\left(\frac{y}{4}\right)}{F(y)}\right\}.$$

The curve is the dotted line in the cut, and its area is about $-1\cdot713$. Hence we have

$$l\frac{dG_1}{dt}=-\frac{P_1}{\pi s^2\sqrt{h_1}}3\cdot312\,.$$

If we compare these values, obtained on such widely different assumptions as to the relative diameters of the particles, we see at once how exceedingly difficult would be the determination of diameters from observed results as to diffusion. (Compare § 54.)

But we see also how diffusion varies with the relative size of the particles, the sum of the diameters being constant. For the smaller, relatively, are the particles of smaller mass (those which have the greater mean-square speed) the more rapid is the diffusion.

And further, by comparison with the results of §§ 51, 54, we see how much more quickly a gas diffuses into another of different specific gravity than into another of the same specific gravity.

When the less massive particles are indefinitely small in comparison with the others, the diameter of these is s; and for their rate of diffusion we have

$$l\frac{dG}{dt} = -\frac{P_1}{\pi s^2 \sqrt{h_1}} 4 \cdot 26 .$$

When it is the more massive particles which are evanescent in size, the numerical factor seems to be about 3·48. Hence it would appear that, even in the case of masses so different, there is a *minimum* value of the diffusion-coefficient, which is reached before the more massive particles are infinitesimal compared with the others.

[At one time I thought of expressing the results of this section in a form similar to that adopted in the expression for D in § 51. It is easy to see that the quantity corresponding to λ would now be what may be called the mean free path of a *single* particle of one gas in a space filled with another. Its value would be easily calculated by the introduction of h_1 for h in the factor ν of the integral

$$\int_0 \frac{\nu}{e}$$

while keeping e in terms of h. This involves multiplication of each number in the fourth column of the *Appendix* to Part I. by the new factor $\varepsilon^{-(h_1-h)x^2} h_1^{\frac{3}{2}}/h^{\frac{3}{2}}$. But, on reflection, I do not see that much would be gained by this.]

APPENDIX.

The notation is the same as in the Appendix to Part I.

x	xX_1/X_2	x^2X_1/X_2	x^3X_1/X_2	x^5X_1/X_2
0·1	·000049	·000005	·000001	·000000
·2	·000758	·000152	·000030	·000001
·3	·003594	·001078	·000323	·000029
·4	·010364	·004146	·001658	·000265
·5	·022505	·011252	·005626	·001407
·6	·040512	·024307	·014584	·005250
·7	·063623	·044536	·031175	·015276
·8	·089928	·071942	·057554	·036834
·9	·116712	·105041	·094537	·076575
1·0	·141040	·141040	·141040	·141040
1·1	·160292	·176321	·193953	·234683
1·2	·172656	·207187	·248624	·358019
1·3	·177229	·230398	·299517	·506184
1·4	·174174	·243844	·341382	·669108
1·5	·164430	·246645	·369968	·832427
1·6	·149568	·239309	·382894	·980209
1·7	·131393	·223368	·379726	1·097407
1·8	·111654	·200977	·361758	1·172098
1·9	·091960	·174724	·331976	1·198432
2·0	·073480	·146960	·293920	1·175680
2·1	·057015	·119731	·251435	1·108829
2·2	·043032	·094670	·208274	1·008046
2·3	·031579	·072632	·167054	·883714
2·4	·022584	·054202	·130085	·749288
2·5	·015750	·039375	·098438	·615234
2·6	·010686	·027784	·072238	·488332
2·7	·007074	·019099	·051567	·375926
2·8	·004536	·012701	·035563	·278812
2·9	·002871	·008326	·024145	·203063
3·0	·001710	·005130	·015390	·138510
3·1	·001071	·003320	·010294	·098925
3·2	·000629	·002014	·006445	·065997
3·3	·000361	·001192	·003935	·042852
3·4	·000211	·000689	·002344	·027098
3·5	·000111	·000389	·001361	·016671
3·6	·000066	·000240	·000865	·010004
3·7	·000037	·000136	·000505	·005839
3·8			·000229	·003307
3·9			·000118	·001798
4·0			·000062	·000985
	2·095244	2·954862	4·630593	14·624154

Thus the values of C_1, C_2, C_3, and C_5 are respectively 0·838, 1·182, 1·852, and 5·849.

XIII.—*Tables for Facilitating the Computation of Differential Refraction in Position Angle and Distance.* By the Hon. LORD M'LAREN.

(Read 6th December 1886.)

The annexed tables are intended to facilitate the computation of the corrections for refraction which have to be applied to differential measures, such as are made with the micrometer or heliometer.

Differential measures are of two kinds:—(1) Direct measures of differences of right ascension and declination; and (2) measures of position angle and distance. In either case the observer only seeks to determine the relative positions of the objects under observation; and the correction for refraction consists in the applying to each reading a quantity representing the difference of the separate effects of refraction on the apparent places of the two stars, whose relative positions are to be determined. This might be effected by computing separately the displacement of each star caused by refraction, and taking the difference between these quantities for the required correction. But, in practice, the correction for refraction is obtained more easily and more accurately by differentiation.

When the measures to be corrected for refraction are direct measures of differences of right ascension and differences of declination, the quantities $\log \frac{d\,R}{d\,a}$ and $\log \frac{d\,R}{d\,\delta}$ may be tabulated for a given latitude, with the arguments, declination, and hour angle. The numerical values of these co-efficients for unit of arc (or 1″) are to be computed for all possible positions above the horizon; and then the correction is at once obtained by taking out $\log \frac{d\,R}{d\,a}$ and $\log \frac{d\,R}{d\,\delta}$ from the table and adding to each the logarithm of the number of seconds of arc in the corresponding measure. It is intended, in a subsequent paper, to submit a specimen of such a table prepared for the latitude of Edinburgh.

The correction for refraction in the case of observations of position angle and distance is a more troublesome matter; because the various readings of the position angles and distances for any pair of stars are not all taken at the same elevation above the horizon, and therefore each measure must be separately corrected for refraction before it can be combined with the others into a mean position angle or mean distance.

The analytical investigation of these corrections leads to the following expressions:—

If we call π' and π the true and apparent position angles; Δ' and Δ, the true and apparent distances of the two stars; ζ, the mean zenith distance of the field of view; η, the parallactic angle; and κ, the co-efficient of refraction, we have

$$\pi' = \pi - \kappa \tan^2 \zeta \sin (\pi - \eta) \cos (\pi - \eta).$$
$$\Delta' = \Delta + \kappa\, \Delta\, [1 + \tan^2 \zeta \cos^2 (\pi - \eta)].$$

In these expressions all the variable quantities are given directly by the readings, excepting $\tan^2 \zeta$ and η, the parallactic angle. Now, the last mentioned quantities are functions of the latitude, declination, and hour angle. They can therefore be tabulated for a given latitude, with the arguments, declination, and hour angle. The present tables give for the parallel 55° 56′ (which passes through Edinburgh), and also for 57° 30′ the quantities $\log \tan^2 \zeta$ and η for each ten minutes of hour angle, and for each interval of two degrees of declination from 40° north to 90°. The tables include the entire circumpolar region of the heavens visible from the respective latitudes, and one or other of them may be used for observations taken in any part of Scotland, without sensible error. Where great accuracy is desired, a table of differences applicable to the particular observatory may be obtained by interpolating between the two printed tables.

The computations for the two tables were made in the following manner:—Calling ϕ the latitude of the place of observation; Π, the polar distance corresponding to the interval of declination; and τ the hour angle—the quantities ζ and η are to be obtained by solving the spherical triangle, whose vertices are the pole, the zenith, and the star; whose sides are polar distance, zenith distance, and the co-latitude; and whose angles are hour angle, azimuth and parallactic angle.

To adapt the solution to logarithmic computation, the auxiliary angles M and N were computed for each 10 minutes of hour angle by the formulæ

$$\operatorname{Sin} M = \cos \phi \sin \tau.$$
$$\operatorname{Tan} N = \operatorname{cotan} \phi \cos \tau.$$

The resulting values of N and log cos M were tabulated, and the final computations were made by the formulæ

$$\operatorname{Cos} \zeta = \cos M \cos (\Pi - N).$$
$$\operatorname{Cos} \eta = \operatorname{cotan} \zeta \tan (\Pi - N).$$

The quantities $\log \tan^2 \zeta$ and η were directly computed for each alternate column of the tables. The intermediate columns were obtained by interpolation, checked by independent computation of a sufficient number of tabular places to ensure substantial accuracy in the last decimal place.

TABLE

CONTAINING

THE LOGARITHM OF TAN^2 ZENITH DISTANCE

AND THE

PARALLACTIC ANGLE

FOR

LATITUDE 55° 56′, AND DECLINATION 40° TO 90°.

H.	M.	40°	42°	44°	46°	48°	50°	52°	54°	H.	M.
0	0	8·9130	8·7840	8·6550	8·4725	8·2900	7·9968	7·7036	7·0560	24	0
0	10	8·9220	8·7942	8·6684	8·4961	8·3238	8·0544	7·7849	7·3145	23	50
0	20	8·9310	8·8064	8·6818	8·5197	8·3576	8·1119	7·8662	7·5730	23	40
0	30	8·9575	8·8405	8·7235	8·5751	8·4266	8·2260	8·0254	7·8225	23	30
0	40	8·9840	8·8746	8·7652	8·6304	8·4956	8·3401	8·1846	8·0720	23	20
0	50	9·0220	8·9201	8·8182	8·6989	8·5796	8·4493	8·3190	8·2258	23	10
1	0	9·0600	8·9656	8·8712	8·7674	8·6636	8·5585	8·4534	8·3796	23	0
1	10	9·1066	9·0194	8·9321	8·8393	8·7465	8·6571	8·5676	8·5021	22	50
1	20	9·1532	9·0731	8·9930	8·9112	8·8294	8·7556	8·6818	8·6246	22	40
1	30	9·2013	9·1275	9·0536	8·9791	8·9045	8·8385	8·7725	8·7203	22	30
1	40	9·2494	9·1818	9·1142	9·0469	8·9796	8·9214	8·8632	8·8160	22	20
1	50	9·3092	9·2361	9·1729	9·1113	9·0497	8·9959	8·9421	8·9016	22	10
2	0	9·3490	9·2903	9·2316	9·1757	9·1198	9·0704	9·0210	8·9872	22	0
2	10	9·3974	9·3419	9·2864	9·2337	9·1810	9·1348	9·0886	9·0560	21	50
2	20	9·4458	9·3935	9·3412	9·2917	9·2422	9·1992	9·1562	9·1248	21	40
2	30	9·4942	9·4439	9·3935	9·3464	9·2993	9·2581	9·2169	9·1859	21	30
2	40	9·5426	9·4942	9·4458	9·4011	9·3564	9·3170	9·2776	9·2470	21	20
2	50	9·5888	9·5421	9·4954	9·4522	9·4090	9·3714	9·3328	9·3025	21	10
3	0	9·6350	9·5900	9·5450	9·5033	9·4616	9·4248	9·3880	9·3580	21	0
3	10	9·6796	9·6359	9·5922	9·5513	9·5104	9·4744	9·4384	9·4081	20	50
3	20	9·7242	9·6818	9·6394	9·5993	9·5592	9·5240	9·4888	9·4581	20	40
3	30	9·7684	9·7267	9·6849	9·6455	9·6061	9·5709	9·5357	9·5051	20	30
3	40	9·8126	9·7715	9·7304	9·6917	9·6530	9·6178	9·5826	9·5520	20	20
3	50	9·8557	9·8109	9·7741	9·7356	9·6970	9·6618	9·6265	9·5957	20	10
4	0	9·8988	9·8583	9·8178	9·7794	9·7410	9·7057	9·6704	9·6394	20	0
4	10	9·9411	9·9005	9·8598	9·8215	9·7832	9·7478	9·7123	9·6808	19	50
4	20	9·9834	9·9426	9·9018	9·8636	9·8254	9·7898	9·7542	9·7222	19	40
4	30	0·0256	9·9845	9·9433	9·9049	9·8664	9·8306	9·7947	9·7621	19	30
4	40	0·0678	0·0263	9·9848	9·9461	9·9074	9·8713	9·8352	9·8019	19	20
4	50	0·1091	0·0673	0·0255	9·9862	9·9469	9·9103	9·8736	9·8397	19	10
5	0	0·1504	0·1083	0·0662	0·0263	9·9864	9·9492	9·9120	9·8775	19	0
5	10	0·1921	0·1492	0·1062	0·0656	0·0249	9·9871	9·9492	9·9139	18	50
5	20	0·2338	0·1900	0·1462	0·1048	0·0634	0·0249	9·9864	9·9502	18	40
5	30	0·2760	0·2310	0·1860	0·1439	0·1010	0·0621	0·0225	9·9854	18	30
5	40	0·3182	0·2720	0·2258	0·1829	0·1400	0·0993	0·0586	0·0206	18	20
5	50	0·3604	0·3129	0·2655	0·2214	0·1772	0·1357	0·0941	0·0549	18	10
6	0	0·4026	0·3539	0·3052	0·2598	0·2144	0·1720	0·1296	0·0893	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	40°	42°	44°	46°	48°	50°	52°	54°	H.	M.
0	0	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	24	0
0	10	5 0	5 48	6 35	8 25	10 15	13 45	17 15	27 6	23	50
0	20	10 0	11 35	13 10	16 50	20 30	27 30	34 30	54 12	23	40
0	30	14 40	16 51	19 6	23 13	27 19	35 22	43 22	61 6	23	30
0	40	19 20	22 11	25 2	29 35	34 8	43 11	52 14	68 0	23	20
0	50	22 59	26 3	29 7	33 58	38 49	47 25	56 0	69 36	23	10
1	0	26 38	29 55	33 12	38 21	43 30	51 38	59 46	71 12	23	0
1	10	29 47	33 7	36 27	41 31	46 35	54 8	61 41	71 46	22	50
1	20	32 56	36 19	39 42	44 41	49 40	56 38	63 36	72 19	22	40
1	30	35 11	38 34	41 56	46 40	51 24	57 52	64 20	72 12	22	30
1	40	37 26	40 48	44 10	48 39	53 8	59 6	65 4	72 5	22	20
1	50	39 11	42 27	45 43	50 0	54 16	59 47	65 17	71 51	22	10
2	0	40 56	44 6	47 16	51 20	55 24	60 27	65 30	71 36	22	0
2	10	42 9	45 12	48 14	52 4	55 54	60 38	65 22	71 0	21	50
2	20	43 22	46 17	49 12	52 48	56 24	60 48	65 14	70 23	21	40
2	30	44 15	47 2	49 48	53 13	56 37	60 44	64 52	69 40	21	30
2	40	45 8	47 46	50 24	53 37	56 50	60 40	64 30	68 57	21	20
2	50	45 40	48 11	50 41	53 44	56 47	60 23	63 59	68 11	21	10
3	0	46 12	48 35	50 58	53 51	56 44	60 6	63 28	67 25	21	0
3	10	46 29	48 46	51 3	53 46	56 29	59 40	62 51	66 31	20	50
3	20	46 46	48 57	51 8	53 41	56 14	59 14	62 14	65 37	20	40
3	30	46 53	48 58	51 2	53 27	55 52	58 41	61 30	64 45	20	30
3	40	47 0	48 58	50 56	53 13	55 30	58 8	60 46	63 52	20	20
3	50	46 56	48 49	50 41	52 51	55 0	57 30	59 59	62 52	20	10
4	0	46 52	48 39	50 26	52 28	54 30	56 51	59 12	61 52	20	0
4	10	46 40	48 21	50 2	51 58	53 54	56 8	58 21	60 52	19	50
4	20	46 28	48 3	49 38	51 28	53 18	55 24	57 30	59 53	19	40
4	30	46 10	47 40	49 9	50 49	52 38	54 37	56 36	58 52	19	30
4	40	45 52	47 16	48 40	50 9	51 58	53 50	55 42	57 50	19	20
4	50	45 24	46 45	48 5	49 34	51 12	52 59	54 45	56 47	19	10
5	0	44 56	46 13	47 30	48 58	50 26	52 7	53 48	55 43	19	0
5	10	44 26	45 39	46 52	48 16	49 39	51 15	52 50	54 39	18	50
5	20	43 56	45 5	46 14	47 33	48 52	50 23	51 52	53 34	18	40
5	30	43 21	44 26	45 30	46 45	48 0	49 25	50 50	52 27	18	30
5	40	42 46	43 46	44 46	45 57	47 8	48 28	49 48	51 19	18	20
5	50	42 6	43 3	44 0	45 7	46 13	47 29	48 45	50 11	18	10
6	0	41 26	42 20	43 14	44 16	45 18	46 30	47 42	49 3	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	40°	42°	44°	46°	48°	50°	52°	54°	H.	M.
6	0	0·4026	0·3539	0·3052	0·2598	0·2144	0·1720	0·1296	0·0893	18	0
6	10	0·4454	0·3956	0·3448	0·2981	0·2513	0·2076	0·1639	0·1226	17	50
6	20	0·4882	0·4363	0·3844	0·3363	0·2882	0·2432	0·1982	0·1559	17	40
6	30	0·5323	0·4789	0·4244	0·3747	0·3250	0·2737	0·2323	0·1887	17	30
6	40	0·5764	0·5204	0·4644	0·4131	0·3618	0·3141	0·2664	0·2215	17	20
6	50	0·6208	0·5626	0·5044	0·4511	0·3978	0·3484	0·2990	0·2528	17	10
7	0	0·6752	0·6048	0·5444	0·4891	0·4338	0·3827	0·3316	0·2840	17	0
7	10	0·7115	0·6483	0·5850	0·5276	0·4702	0·4173	0·3644	0·3102	16	50
7	20	0·7578	0·6917	0·6256	0·5661	0·5066	0·4519	0·3972	0·3364	16	40
7	30	0·8048	0·7356	0·6663	0·6043	0·5423	0·4857	0·4290	0·3716	16	30
7	40	0·8518	0·7794	0·7070	0·6425	0·5780	0·5194	0·4608	0·4068	16	20
7	50	0·9005	0·8243	0·7480	0·6807	0·6134	0·5527	0·4920	0·4362	16	10
8	0	0·9492	0·8691	0·7890	0·7189	0·6488	0·5860	0·5232	0·4655	16	0
8	10	0·9993	0·9148	0·8303	0·7570	0·6837	0·6186	0·5534	0·4939	15	50
8	20	1·0494	0·9605	0·8716	0·7951	0·7186	0·6511	0·5836	0·5222	15	40
8	30	1·1009	1·0069	0·9128	0·8329	0·7529	0·6828	0·6127	0·5494	15	30
8	40	1·1524	1·0532	0·9540	0·8706	0·7972	0·7145	0·6418	0·5766	15	20
8	50	1·2045	1·0995	0·9944	0·9078	0·8203	0·7449	0·6696	0·6023	15	10
9	0	1·2566	1·1457	1·0348	0·9441	0·8534	0·7754	0·6974	0·6279	15	0
9	10	1·3099	1·1927	1·0754	0·9804	0·8854	0·8046	0·7237	0·6523	14	50
9	20	1·3632	1·2396	1·1160	1·0167	0·9174	0·8337	0·7500	0·6766	14	40
9	30	1·4166	1·2857	1·1548	1·0513	0·9477	0·8611	0·7744	0·6989	14	30
9	40	1·4700	1·3318	1·1936	1·0858	0·9780	0·8884	0·7988	0·7211	14	20
9	50	1·5228	1·3765	1·2303	1·1181	1·0059	0·9136	0·8212	0·7417	14	10
10	0	1·5756	1·4213	1·2670	1·1504	1·0338	0·9387	0·8436	0·7622	14	0
10	10	1·6260	1·4636	1·3011	1·1800	1·0589	0·9611	0·8633	0·7799	13	50
10	20	1·6764	1·5058	1·3352	1·2096	1·0840	0·9835	0·8830	0·7977	13	40
10	30	1·7232	1·5441	1·3650	1·2355	1·1059	1·0030	0·9000	0·8131	13	30
10	40	1·7700	1·5824	1·3948	1·2613	1·1278	1·0224	0·9170	0·8285	13	20
10	50	1·8101	1·6149	1·4197	1·2826	1·1454	1·0379	0·9305	0·8406	13	10
11	0	1·8502	1·6474	1·4446	1·3038	1·1630	1·0535	0·9440	0·8526	13	0
11	10	1·8817	1·6726	1·4636	1·3199	1·1762	1·0652	0·9541	0·8618	12	50
11	20	1·9132	1·6979	1·4826	1·3360	1·1894	1·0768	0·9642	0·8710	12	40
11	30	1·9314	1·7123	1·4931	1·3450	1·1969	1·0832	0·9695	0·8761	12	30
11	40	1·9496	1·7266	1·5036	1·3540	1·2044	1·0896	0·9748	0·8812	12	20
11	50	1·9581	1·7333	1·5084	1·3580	1·2076	1·0925	0·9773	0·8831	12	10
12	0	1·9666	1·7399	1·5132	1·3620	1·2108	1·0953	0·9798	0·8849	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	40°	42°	44°	46°	48°	50°	52°	54°	H.	M.
6	0	41° 26′	42° 20′	43° 14′	44° 16′	45° 18′	46° 30′	47° 42′	49° 3′	18	0
6	10	40 41	41 33	42 25	43 23	44 21	45 24	46 26	47 48	17	50
6	20	39 56	40 46	41 36	42 30	43 24	44 17	45 10	46 33	17	40
6	30	39 9	39 55	40 40	41 32	42 23	43 18	44 13	45 28	17	30
6	40	38 22	39 3	39 44	40 33	41 22	42 19	43 16	44 20	17	20
6	50	37 30	38 9	38 47	39 33	40 18	41 12	42 5	43 6	17	10
7	0	36 38	37 14	37 50	38 32	39 14	40 4	40 54	41 52	17	0
7	10	35 44	36 18	36 51	37 31	38 10	38 57	39 43	40 38	16	50
7	20	34 50	35 21	35 52	36 29	37 6	37 49	38 32	39 23	16	40
7	30	33 49	34 17	34 45	35 20	35 54	36 36	37 18	38 5	16	30
7	40	32 48	33 13	33 38	34 10	34 42	35 23	36 4	36 47	16	20
7	50	31 47	32 11	32 35	33 2	33 29	34 9	34 49	35 30	16	10
8	0	30 46	31 9	31 32	31 54	32 16	32 55	33 34	34 13	16	0
8	10	29 42	30 9	30 25	30 48	31 10	31 44	32 16	32 55	15	50
8	20	28 38	29 8	29 18	29 41	30 4	30 33	31 2	31 36	15	40
8	30	27 31	27 54	28 6	28 28	28 49	29 16	29 43	30 15	15	30
8	40	26 23	26 39	26 54	27 14	27 34	27 59	28 24	28 54	15	20
8	50	25 13	25 26	25 39	25 59	26 18	26 42	27 5	27 32	15	10
9	0	24 2	24 13	24 24	24 43	25 2	25 24	25 46	26 10	15	0
9	10	22 28	23 15	23 12	23 28	23 43	24 3	24 23	24 47	14	50
9	20	21 53	22 17	22 0	22 12	22 24	22 42	23 0	23 23	14	40
9	30	20 29	20 46	20 42	20 54	21 5	21 21	21 37	21 57	14	30
9	40	19 3	19 14	19 24	19 35	19 46	20 0	20 14	20 30	14	20
9	50	17 47	17 56	18 4	18 15	18 26	18 39	18 51	19 7	14	10
10	0	16 30	16 37	16 44	16 55	17 6	17 17	17 28	17 44	14	0
10	10	15 8	15 18	15 27	15 34	15 41	15 51	16 1	16 15	13	50
10	20	13 46	13 58	14 10	14 13	14 16	14 25	14 34	14 45	13	40
10	30	12 26	12 28	12 30	12 42	12 53	12 58	13 11	13 21	13	30
10	40	11 6	10 58	10 50	11 10	11 30	11 30	11 48	11 56	13	20
10	50	9 40	9 40	9 40	9 52	10 3	10 7	10 19	10 26	13	10
11	0	8 14	8 22	8 30	8 33	8 36	8 43	8 50	8 55	13	0
11	10	6 52	7 0	7 8	7 6	7 3	7 15	7 26	7 32	12	50
11	20	5 30	5 38	5 46	5 38	5 30	5 46	6 2	6 9	12	40
11	30	4 10	4 14	4 18	4 17	4 15	4 23	4 31	4 35	12	30
11	40	2 50	2 50	2 50	2 55	3 0	3 0	3 0	3 0	12	20
11	50	1 25	1 25	1 25	1 28	1 30	1 30	1 30	1 30	12	10
12	0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

LOGARITHM OF TAN² Z FOR LAT. 55° 56′.

H.	M.	56°	58°	60°	62°	64°	66°	68°	70°	H.	M.
0	0	——	——	7·7420	8·0315	8·3210	8·4923	8·6636	8·7936	24	0
0	10	——	7·3788	7·7905	8·0598	8·3289	8·4978	8·6667	8·7952	23	50
0	20	7·3360	7·5880	7·8390	8·0880	8·3368	8·5033	8·6698	8·7967	23	40
0	30	7·6580	7·8240	7·9895	8·1904	8·3912	8·5439	8·6965	8·8164	23	30
0	40	7·9800	8·0600	8·1398	8·2927	8·4456	8·5844	8·7232	8·8360	23	20
0	50	8·1584	8·2095	8·2605	8·3836	8·5065	8·6307	8·7546	8·8606	23	10
1	0	8·3368	8·3590	8·3812	8·4745	8·5678	8·6769	8·7860	8·8851	23	0
1	10	8·4637	8·4772	8·4906	8·5640	8·6374	8·7310	8·8245	8·9161	22	50
1	20	8·5906	8·5953	8·6000	8·6535	8·7070	8·7850	8·8630	8·9471	22	40
1	30	8·6906	8·6891	8·6875	8·7299	8·7724	8·8394	8·9063	8·9811	22	30
1	40	8·7906	8·7828	8·7750	8·8064	8·8378	8·8937	8·9496	9·0150	22	20
1	50	8·8720	8·8625	8·8530	8·8761	8·8992	8·9465	8·9938	9·0517	22	10
2	0	8·9534	8·9422	8·9310	8·9458	8·9606	8·9993	9·0380	9·0883	22	0
2	10	9·0234	9·0103	8·9971	9·0065	9·0169	9·0483	9·0797	9·1235	21	50
2	20	9·0934	9·0783	9·0632	9·0672	9·0732	9·0973	9·1214	9·1587	21	40
2	30	9·1549	9·1382	9·1251	9·1224	9·1242	9·1429	9·1616	9·1938	21	30
2	40	9·2164	9·1981	9·1798	9·1775	9·1752	9·1885	9·2018	9·2288	21	20
2	50	9·2722	9·2545	9·2368	9·2296	9·2224	9·2317	9·2409	9·2634	21	10
3	0	9·3280	9·3109	9·2938	9·2817	9·2696	9·2748	9·2800	9·2980	21	0
3	10	9·3777	9·3581	9·3384	9·3260	9·3136	9·3154	9·3172	9·3313	20	50
3	20	9·4274	9·4052	9·3830	9·3703	9·3576	9·3560	9·3544	9·3645	20	40
3	30	9·4744	9·4513	9·4281	9·4135	9·3990	9·3972	9·3904	9·3971	20	30
3	40	9·5214	9·4973	9·4732	9·4568	9·4404	9·4384	9·4264	9·4297	20	20
3	50	9·5649	9·5399	9·5148	9·4969	9·4789	9·4721	9·4602	9·4604	20	10
4	0	9·6084	9·5824	9·5564	9·5369	9·5174	9·5057	9·4940	9·4911	20	0
4	10	9·6493	9·6223	9·5954	9·5746	9·5538	9·5401	9·5263	9·5209	19	50
4	20	9·6902	9·6623	9·6344	9·6123	9·5902	9·5744	9·5586	9·5506	19	40
4	30	9·7294	9·7008	9·6721	9·6485	9·6249	9·6025	9·5900	9·5796	19	30
4	40	9·7686	9·7392	9·7098	9·6847	9·6596	9·6405	9·6214	9·6086	19	20
4	50	9·8058	9·7755	9·7451	9·7189	9·6927	9·6716	9·6505	9·6357	19	10
5	0	9·8430	9·8117	9·7804	9·7531	9·7258	9·7027	9·6796	9·6628	19	0
5	10	9·8785	9·8462	9·8139	9·7854	9·7568	9·7325	9·7081	9·6891	18	50
5	20	9·9140	9·8807	9·8474	9·8176	9·7878	9·7622	9·7366	9·7154	18	40
5	30	9·9483	9·9141	9·8798	9·8489	9·8170	9·7907	9·7635	9·7407	18	30
5	40	9·9826	9·9474	9·9122	9·8801	9·8480	9·8192	9·7904	9·7660	18	20
5	50	0·0158	9·9797	9·9435	9·9101	9·8766	9·8464	9·8162	9·7901	18	10
6	0	0·0490	0·0119	9·9748	9·9400	9·9052	9·8736	9·8420	9·8141	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	56°	58°	60°	62°	64°	66°	68°	70°	H.	M.
0	0	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	24	0
0	10	90 0	137 2	158 40	167 30	168 22	172 50	174 32	174 58	23	50
0	20	88 56	118 24	147 42	155 0	162 16	165 40	169 4	169 55	23	40
0	30	87 46	111 38	135 29	144 42	152 59	157 14	161 29	163 29	23	30
0	40	86 36	104 51	123 6	134 24	143 42	148 48	153 54	157 3	23	20
0	50	85 25	101 11	116 56	127 35	137 13	142 49	148 25	151 56	23	10
1	0	84 14	97 30	110 46	120 45	130 44	136 50	142 56	146 49	23	0
1	10	83 8	94 50	106 32	115 56	125 21	131 39	137 57	142 5	22	50
1	20	82 2	92 10	102 18	111 7	119 58	126 28	132 58	137 20	22	40
1	30	80 56	90 5	99 10	107 29	115 45	122 10	128 34	133 10	22	30
1	40	79 50	88 0	96 10	103 51	111 32	117 51	124 10	128 59	22	20
1	50	78 46	86 14	93 41	100 52	108 3	114 10	120 17	125 9	22	10
2	0	77 42	84 27	91 12	97 53	104 34	110 29	116 24	121 19	22	0
2	10	76 38	82 52	89 6	95 23	101 39	107 21	113 3	117 55	21	50
2	20	75 34	81 17	87 0	92 52	98 44	104 13	109 42	114 31	21	40
2	30	74 29	79 49	85 8	90 39	96 10	101 26	106 41	111 24	21	30
2	40	73 24	78 20	83 16	88 26	93 36	98 38	103 40	108 17	21	20
2	50	72 23	76 59	81 35	86 26	91 14	96 6	100 55	105 25	21	10
3	0	71 12	75 38	79 54	84 26	88 58	93 34	98 10	102 33	21	0
3	10	70 11	74 26	78 15	82 34	87 52	91 16	95 40	99 56	20	50
3	20	69 0	72 48	76 36	80 41	84 46	88 58	93 10	97 18	20	40
3	30	67 54	71 30	75 3	78 55	82 47	86 48	90 49	94 49	20	30
3	40	66 48	70 11	73 30	77 9	80 48	84 37	88 28	92 19	20	20
3	50	65 40	68 51	72 0	75 28	78 56	82 36	86 16	89 59	20	10
4	0	64 32	67 31	70 30	73 47	77 4	80 34	84 4	87 39	20	0
4	10	63 24	66 14	69 4	72 11	75 17	78 38	81 58	85 26	19	50
4	20	62 16	64 57	67 38	70 34	73 30	76 41	79 52	83 12	19	40
4	30	61 7	63 39	66 11	68 59	71 46	74 49	77 50	81 4	19	30
4	40	59 58	62 21	64 44	67 23	70 2	72 56	75 50	78 55	19	20
4	50	58 48	61 3	63 19	65 51	68 23	71 9	73 54	76 48	19	10
5	0	57 38	59 46	61 54	64 19	66 44	69 21	71 58	74 41	19	0
5	10	56 27	58 29	60 30	62 48	65 5	67 36	70 7	72 47	18	50
5	20	55 16	57 11	59 6	61 16	63 26	65 51	68 16	70 53	18	40
5	30	54 3	55 53	57 42	59 46	61 50	64 8	66 26	68 52	18	30
5	40	52 50	54 34	56 18	58 16	60 14	62 25	64 36	66 51	18	20
5	50	51 37	53 16	54 55	56 47	58 39	60 14	62 48	65 2	18	10
6	0	50 24	51 58	53 32	55 18	57 4	59 2	61 0	63 13	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

LOGARITHM OF TAN²Z FOR LAT. 55° 56′.

H.	M.	56°	58°	60°	62°	64°	66°	68°	70°	H.	M.
6	0	0·0490	0·0119	9·9748	9·9400	9·9052	9·8736	9·8420	9·8141	18	0
6	10	0·0813	0·0428	0·0043	9·9684	9·9325	9·8995	9·8665	9·8369	17	50
6	20	0·1136	0·0737	0·0338	9·9968	9·9598	9·9254	9·8910	9·8598	17	40
6	30	0·1451	0·1039	0·0627	0·0245	9·9862	9·9505	9·9147	9·8821	17	30
6	40	0·1766	0·1341	0·0916	0·0521	0·0126	9·9755	9·9384	9·9043	17	20
6	50	0·2065	0·1629	0·1194	0·0786	0·0377	9·9993	9·9609	9·9254	17	10
7	0	0·2364	0·1918	0·1572	0·1050	0·0628	0·0231	9·9834	9·9464	17	0
7	10	0·2660	0·2199	0·1738	0·1304	0·0869	0·0459	0·0048	9·9663	16	50
7	20	0·2956	0·2480	0·2004	0·1557	0·1110	0·0686	0·0262	9·9862	16	40
7	30	0·3242	0·2752	0·2261	0·1799	0·1337	0·0902	0·0466	0·0052	16	30
7	40	0·3528	0·3023	0·2518	0·2041	0·1562	0·1117	0·0670	0·0242	16	20
7	50	0·3803	0·3284	0·2764	0·2274	0·1784	0·1323	0·0861	0·0422	16	10
8	0	0·4078	0·3544	0·3010	0·2507	0·2004	0·1528	0·1052	0·0599	16	0
8	10	0·4343	0·3793	0·3243	0·2727	0·2210	0·1721	0·1232	0·0765	15	50
8	20	0·4608	0·4042	0·3476	0·2946	0·2416	0·1914	0·1412	0·0931	15	40
8	30	0·4861	0·4279	0·3696	0·3153	0·2609	0·2094	0·1578	0·1086	15	30
8	40	0·5114	0·4515	0·3916	0·3359	0·2802	0·2273	0·1744	0·1240	15	20
8	50	0·5349	0·4736	0·4122	0·3551	0·2980	0·2439	0·1897	0·1382	15	10
9	0	0·5584	0·4956	0·4328	0·3743	0·3158	0·2604	0·2050	0·1524	15	0
9	10	0·5808	0·5163	0·4518	0·3920	0·3322	0·2754	0·2191	0·1655	14	50
9	20	0·6032	0·5370	0·4708	0·4097	0·3486	0·2909	0·2332	0·1785	14	40
9	30	0·6233	0·5557	0·4881	0·4257	0·3632	0·3046	0·2460	0·1903	14	30
9	40	0·6434	0·5744	0·5054	0·4416	0·3778	0·3183	0·2588	0·2020	14	20
9	50	0·6621	0·5910	0·5208	0·4559	0·3900	0·3304	0·2699	0·2121	14	10
10	0	0·6808	0·6085	0·5362	0·4701	0·4040	0·3425	0·2810	0·2222	14	0
10	10	0·6966	0·6229	0·5493	0·4823	0·4152	0·3528	0·2903	0·2309	13	50
10	20	0·7124	0·6374	0·5624	0·4944	0·4264	0·3630	0·2999	0·2395	13	40
10	30	0·7262	0·6499	0·5737	0·5048	0·4359	0·3719	0·3078	0·2469	13	30
10	40	0·7400	0·6625	0·5850	0·5152	0·4454	0·3807	0·3160	0·2543	13	20
10	50	0·7506	0·6722	0·5938	0·5232	0·4526	0·3873	0·3221	0·2600	13	10
11	0	0·7612	0·6819	0·6026	0·5312	0·4598	0·3940	0·3282	0·2657	13	0
11	10	0·7695	0·6894	0·6093	0·5374	0·4655	0·3992	0·3327	0·2699	12	50
11	20	0·7778	0·6969	0·6160	0·5436	0·4712	0·4044	0·3372	0·2740	12	40
11	30	0·7827	0·7011	0·6195	0·5466	0·4737	0·4068	0·3397	0·2760	12	30
11	40	0·7876	0·7053	0·6230	0·5496	0·4762	0·4092	0·3422	0·2783	12	20
11	50	0·7888	0·7067	0·6246	0·5512	0·4778	0·4105	0·3432	0·2793	12	10
12	0	0·7900	0·7081	0·6262	0·5528	0·4794	0·4118	0·3442	0·2803	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	56°	58°	60°	62°	64°	66°	68°	70°	H.	M.
6	0	50° 24′	51° 58′	53° 32′	55° 18′	57° 4′	59° 2′	61° 0′	63° 13′	18	0
6	10	49 10	50 39	52 7	53 48	55 29	57 22	59 15	61 22	17	50
6	20	47 56	49 19	50 42	52 18	53 54	55 42	57 30	59 31	17	40
6	30	46 40	47 59	49 17	50 49	52 20	54 3	55 45	57 42	17	30
6	40	45 24	46 38	47 52	49 19	50 46	52 23	54 0	55 52	17	20
6	50	44 7	45 17	46 26	47 49	49 12	50 45	52 18	54 3	17	10
7	0	42 50	43 55	45 0	46 19	47 38	49 7	50 36	52 14	17	0
7	10	41 32	42 34	43 36	44 50	46 4	47 29	48 53	50 28	16	50
7	20	40 14	41 13	42 12	43 21	44 30	45 50	47 10	48 42	16	40
7	30	38 52	39 49	40 46	41 50	42 53	44 11	45 29	46 56	16	30
7	40	37 30	38 25	39 20	40 18	41 16	42 32	43 48	45 10	16	20
7	50	36 11	37 12	37 53	38 50	39 46	40 56	42 6	43 25	16	10
8	0	34 52	35 59	36 26	37 21	38 16	39 20	40 24	41 39	16	0
8	10	33 31	34 25	34 59	35 51	36 42	37 43	38 43	39 54	15	50
8	20	32 10	32 51	33 32	34 20	35 8	36 5	37 2	38 8	15	40
8	30	30 47	31 25	32 3	32 49	33 34	34 27	35 20	36 23	15	30
8	40	29 24	29 59	30 34	31 17	32 0	32 49	33 38	34 38	15	20
8	50	27 59	28 33	29 7	29 47	30 27	31 13	31 58	32 54	15	10
9	0	26 34	27 7	27 40	28 17	28 54	29 36	30 18	31 10	15	0
9	10	25 10	25 40	26 10	26 45	27 19	27 58	28 37	29 27	14	50
9	20	23 46	24 13	24 40	25 12	25 44	26 20	26 56	27 43	14	40
9	30	22 16	22 43	23 10	23 39	24 7	24 42	25 17	26 1	14	30
9	40	20 46	21 13	21 40	22 5	22 30	23 4	23 38	24 18	14	20
9	50	19 23	19 46	20 9	20 33	20 56	21 27	21 57	22 33	14	10
10	0	18 0	18 19	18 38	19 0	19 22	19 49	20 16	20 48	14	0
10	10	16 28	16 46	17 3	17 24	17 44	17 39	18 33	19 3	13	50
10	20	14 56	15 12	15 28	15 47	16 6	16 28	16 50	17 18	13	40
10	30	13 30	13 43	13 55	14 13	14 30	14 52	15 14	15 36	13	30
10	40	12 4	12 13	12 22	12 38	12 54	13 16	13 38	13 54	13	20
10	50	10 32	10 41	10 49	11 2	11 14	11 32	11 50	12 8	13	10
11	0	9 0	9 8	9 16	9 25	9 34	9 48	10 2	10 21	13	0
11	10	7 38	7 40	7 42	7 52	8 2	7 54	7 46	8 21	12	50
11	20	6 16	6 12	6 8	6 19	6 30	6 0	5 30	6 20	12	40
11	30	4 38	4 40	4 42	4 48	4 53	4 38	4 23	4 52	12	30
11	40	3 0	3 8	3 16	3 16	3 16	3 16	3 16	3 23	12	20
11	50	1 30	1 34	1 38	1 38	1 38	1 38	1 38	1 42	12	10
12	0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	72°	74°	76°	78°	80°	82°	84°	86°	H.	M.
0	0	8·9212	9·0240	9·1268	9·2140	9·3012	9·3781	9·4550	9·5240	24	0
0	10	8·9226	9·0251	9·1276	9·2144	9·3012	9·3781	9·4550	9·5240	23	50
0	20	8·9240	9·0262	9·1284	9·2148	9·3012	9·3781	9·4550	9·5240	23	40
0	30	8·9362	9·0356	9·1350	9·2198	9·3046	9·3807	9·4568	9·5257	23	30
0	40	8·9488	9·0452	9·1416	9·2248	9·3080	9·3833	9·4586	9·5268	23	20
0	50	8·9665	9·0585	9·1504	9·2319	9·3134	9·3869	9·4604	9·5279	23	10
1	0	8·9842	9·0717	9·1592	9·2390	9·3188	9·3905	9·4622	9·5291	23	0
1	10	9·0077	9·0901	9·1724	9·2493	9·3261	9·3957	9·4653	9·5310	22	50
1	20	9·0312	9·1084	9·1856	9·2595	9·3334	9·4009	9·4684	9·5328	22	40
1	30	9·0558	9·1284	9·2010	9·2712	9·3413	9·4071	9·4729	9·5358	22	30
1	40	9·0804	9·1484	9·2164	9·2828	9·3492	9·4133	9·4774	9·5387	22	20
1	50	9·1095	9·1717	9·2339	9·2966	9·3592	9·4211	9·4829	9·5422	22	10
2	0	9·1386	9·1950	9·2514	9·3103	9·3692	9·4288	9·4884	9·5457	22	0
2	10	9·1673	9·2193	9·2712	9·3262	9·3812	9·4327	9·4942	9·5495	21	50
2	20	9·1960	9·2435	9·2910	9·3421	9·3932	9·4366	9·5000	9·5532	21	40
2	30	9·2259	9·2686	9·3112	9·3584	9·4056	9·4508	9·5059	9·5572	21	30
2	40	9·2558	9·2936	9·3314	9·3747	9·4180	9·4649	9·5118	9·5612	21	20
2	50	9·2859	9·3189	9·3520	9·3915	9·4310	9·4751	9·5191	9·5658	21	10
3	0	9·3160	9·3443	9·3726	9·4083	9·4440	9·4852	9·5264	9·5704	21	0
3	10	9·3453	9·3695	9·3938	9·4259	9·4580	9·4958	9·5336	9·5752	20	50
3	20	9·3746	9·3948	9·4150	9·4435	9·4720	9·5064	9·5408	9·5799	20	40
3	30	9·4038	9·4201	9·4364	9·4615	9·4866	9·5178	9·5489	9·5852	20	30
3	40	9·4330	9·4454	9·4578	9·4795	9·5012	9·5291	9·5570	9·5905	20	20
3	50	9·4606	9·4699	9·4793	9·4973	9·5152	9·5401	9·5649	9·5957	20	10
4	0	9·4882	9·4945	9·5008	9·5150	9·5292	9·5510	9·5728	9·6009	20	0
4	10	9·5154	9·5184	9·5214	9·5327	9·5439	9·5625	9·5811	9·6063	19	50
4	20	9·5426	9·5423	9·5420	9·5503	9·5586	9·5740	9·5894	9·6117	19	40
4	30	9·5692	9·5662	9·5631	9·5681	9·5731	9·5855	9·5979	9·6177	19	30
4	40	9·5958	9·5900	9·5842	9·5859	9·5876	9·5970	9·6064	9·6236	19	20
4	50	9·6209	9·6128	9·6042	9·6032	9·6022	9·6086	9·6149	9·6316	19	10
5	0	9·6460	9·6351	9·6242	9·6205	9·6168	9·6201	9·6234	9·6396	19	0
5	10	9·6701	9·6569	5·6438	9·6375	9·6311	9·6317	9·6322	9·6427	18	50
5	20	9·6942	9·6788	9·6634	9·6544	9·6454	9·6432	9·6410	9·6457	18	40
5	30	9·7179	9·7002	9·6824	9·6709	9·6594	9·6539	9·6493	9·6512	18	30
5	40	9·7416	9·7215	9·7014	9·6874	9·6734	9·6646	9·6576	9·6567	18	20
5	50	9·7639	9·7419	9·7199	9·7036	9·6872	9·6764	9·6665	9·6627	18	10
6	0	9·7862	9·7623	9·7384	9·7197	9·7010	9·6882	9·6754	9·6686	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	72°	74°	76°	78°	80°	82°	84°	86°	H.	M.
0	0	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	24	0
0	10	175 50	175 55	176 0	176 20	176 39	176 48	176 56	177 13	23	50
0	20	170 40	171 20	172 0	172 39	173 18	173 35	173 52	174 26	23	40
0	30	165 29	166 42	167 54	168 54	169 54	170 24	170 53	171 34	23	30
0	40	160 12	162 0	163 48	165 9	166 30	167 12	167 54	168 42	23	20
0	50	155 27	157 42	159 57	161 29	163 0	164 3	165 5	165 59	23	10
1	0	150 42	153 24	156 6	157 48	159 30	160 53	162 16	163 16	23	0
1	10	146 12	149 43	152 14	154 15	156 15	158 19	159 22	160 32	22	50
1	20	141 42	146 2	148 22	150 41	153 0	154 44	156 28	157 47	22	40
1	30	137 45	141 43	144 43	147 12	149 43	151 37	153 30	154 58	22	30
1	40	133 48	137 24	141 0	143 43	146 26	148 29	150 32	152 8	22	20
1	50	130 1	133 46	137 30	140 23	143 16	145 56	147 36	149 21	22	10
2	0	126 14	130 7	134 0	137 3	140 6	142 23	144 40	146 33	22	0
2	10	122 47	126 43	130 38	133 48	136 57	139 23	141 49	143 50	21	50
2	20	119 20	123 18	127 16	130 32	133 48	136 23	138 58	141 6	21	40
2	30	116 7	120 7	124 6	127 25	130 43	133 27	136 11	138 24	21	30
2	40	112 54	116 55	120 56	124 17	127 38	130 31	133 24	135 42	21	20
2	50	109 55	113 53	117 55	121 20	124 45	127 41	130 36	133 0	21	10
3	0	106 56	110 55	114 54	118 23	121 52	124 50	127 48	130 18	21	0
3	10	104 11	108 7	112 2	115 32	119 1	122 3	125 4	127 39	20	50
3	20	101 26	105 18	109 10	112 40	116 10	119 15	122 20	125 0	20	40
3	30	98 48	102 36	106 24	109 54	113 23	116 30	119 37	122 21	20	30
3	40	96 10	99 54	103 38	107 7	110 36	113 45	116 54	119 41	20	20
3	50	93 42	97 22	101 1	104 28	107 55	111 5	114 15	116 58	20	10
4	0	91 14	94 49	98 24	101 49	105 14	108 25	111 36	114 14	20	0
4	10	88 53	92 22	95 51	99 15	102 39	105 49	108 59	111 47	19	50
4	20	86 32	89 54	93 18	96 41	100 3	103 13	106 22	109 19	19	40
4	30	84 16	87 34	90 52	94 10	97 28	100 37	103 46	106 45	19	30
4	40	82 0	85 13	88 26	91 39	94 51	98 1	101 10	104 10	19	20
4	50	79 50	82 57	86 4	89 13	92 22	95 30	98 37	101 38	19	10
5	0	77 40	80 41	83 42	86 47	89 52	92 58	96 4	99 5	19	0
5	10	75 35	78 10	81 25	84 26	87 27	90 31	93 34	96 35	18	50
5	20	73 30	75 29	79 8	82 5	85 2	88 3	91 4	94 4	18	40
5	30	71 28	73 46	76 53	79 46	82 39	85 37	88 35	91 34	18	30
5	40	69 26	72 2	74 38	77 27	80 16	83 11	86 6	89 4	18	20
5	50	67 26	70 27	72 28	75 12	77 56	80 48	83 40	86 36	18	10
6	0	65 26	68 52	70 18	72 57	75 36	78 25	81 14	84 8	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	72°	74°	76°	78°	80°	82°	84°	86°	H.	M.
6	0	9·7862	9·7623	9·7384	9·7197	9·7010	9·6882	9·6754	9·6686	18	0
6	10	9·8074	9·7818	9·7562	9·7352	9·7142	9·6988	9·6833	9·6739	17	50
6	20	9·8286	9·8013	9·7740	9·7507	9·7274	9·7093	9·6912	9·6792	17	40
6	30	9·8494	9·8199	9·7905	9·7654	9·7403	9·7200	9·6997	9·6849	17	30
6	40	9·8702	9·8386	9·8070	9·7801	9·7532	9·7307	9·7082	9·6905	17	20
6	50	9·8898	9·8567	9·8235	9·7944	9·7652	9·7404	9·7155	9·6956	17	10
7	0	9·9094	9·8747	9·8400	9·8086	9·7772	9·7500	9·7228	9·7007	17	0
7	10	9·9278	9·8917	9·8556	9·8226	9·7895	9·7600	9·7305	9·7059	16	50
7	20	9·9462	9·9087	9·8712	9·8365	9·8018	9·7700	9·7382	9·7111	16	40
7	30	9·9638	9·9247	9·8855	9·8492	9·8128	9·7790	9·7452	9·7157	16	30
7	40	9·9814	9·9406	9·8998	9·8618	9·8238	9·7880	9·7522	9·7203	16	20
7	50	9·9980	9·9559	9·9137	9·8739	9·8342	9·7967	9·7592	9·7251	16	10
8	0	0·0146	9·9711	9·9276	9·8861	9·8446	9·8054	9·7662	9·7299	16	0
8	10	0·0298	9·9849	9·9403	9·8973	9·8543	9·8136	9·7728	9·7345	15	50
8	20	0·0450	9·9987	9·9530	9·9085	9·8640	9·8217	9·7794	9·7390	15	40
8	30	0·0593	0·0119	9·9647	9·9189	9·8732	9·8292	9·7851	8·7429	15	30
8	40	0·0736	0·0250	9·9764	9·9294	9·8824	9·8366	9·7908	9·7468	15	20
8	50	0·0867	0·0369	9·9872	9·9389	9·8906	9·8433	9·7960	9·7505	15	10
9	0	0·0998	0·0489	9·9980	9·9484	9·8988	9·8500	9·8012	9·7541	15	0
9	10	0·1118	0·0598	0·0078	9·9571	9·9064	9·8564	9·8064	9·7576	14	50
9	20	0·1238	0·0707	0·0176	9·9658	9·9140	9·8628	9·8116	9·7611	14	40
9	30	0·1345	0·0805	0·0264	9·9735	9·9206	9·8682	9·8157	9·7640	14	30
9	40	0·1452	0·0902	0·0352	9·9812	9·9272	9·8735	9·8198	9·7669	14	20
9	50	0·1543	0·0986	0·0429	9·9879	9·9330	9·8785	9·8230	9·7696	14	10
10	0	0·1634	0·1070	0·0506	9·9947	9·9388	9·8834	9·8280	9·7722	14	0
10	10	0·1714	0·1141	0·0567	0·0002	9·9436	9·8874	9·8311	9·7744	13	50
10	20	0·1794	0·1211	0·0628	0·0056	9·9484	9·8913	9·8342	9·7765	13	40
10	30	0·1860	0·1272	0·0683	0·0105	9·9527	9·8949	9·8371	9·7785	13	30
10	40	0·1926	0·1332	0·0738	0·0154	9·9570	9·8985	9·8400	9·7805	13	20
10	50	0·1979	0·1379	0·0779	0·0191	9·9603	9·9012	9·8420	9·7819	13	10
11	0	0·2032	0·1426	0·0820	0·0228	9·9636	9·9038	9·8440	9·7834	13	0
11	10	0·2070	0·1462	0·0853	0·0257	9·9661	9·9061	9·8461	9·7849	12	50
11	20	0·2108	0·1497	0·0886	0·0286	9·9686	9·9084	9·8474	9·7858	12	40
11	30	0·2126	0·1513	0·0899	0·0299	9·9698	9·9090	9·8479	9·7861	12	30
11	40	0·2144	0·1528	0·0912	0·0311	9·9710	9·9096	9·8484	9·7864	12	20
11	50	0·2154	0·1538	0·0922	0·0318	9·9713	9·9101	9·8490	9·7867	12	10
12	0	0·2164	0·1548	0·0932	0·0324	9·9716	9·9106	9·8496	9·7871	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	72°	74°	76°	78°	80°	82°	84°	86°	H.	M.
6	0	65° 26′	68° 52′	70° 18′	72° 57′	75° 36′	78° 25′	81° 14′	84° 8′	18	0
6	10	63 29	66 20	68 11	70 45	73 18	76 4	78 49	81 42	17	50
6	20	61 32	63 48	66 4	68 32	71 0	73 42	76 24	79 15	17	40
6	30	59 38	61 48	63 57	66 22	68 46	71 24	74 1	76 44	17	30
6	40	57 44	59 47	61 50	64 11	66 32	69 5	71 38	74 13	17	20
6	50	55 48	57 49	59 49	62 4	64 18	66 47	69 16	71 53	17	10
7	0	53 52	55 50	57 48	59 56	62 4	64 29	66 54	69 33	17	0
7	10	52 3	53 55	55 46	57 20	59 54	62 14	64 34	67 9	16	50
7	20	50 14	51 59	53 44	55 44	57 44	59 59	62 14	64 45	16	40
7	30	48 23	50 3	51 42	53 38	55 33	57 43	59 53	62 32	16	30
7	40	46 32	48 7	49 42	51 32	53 21	55 27	57 32	60 18	16	20
7	50	44 43	46 14	47 44	49 30	51 15	53 16	55 17	57 47	16	10
8	0	42 54	44 20	45 46	47 27	49 8	51 5	53 2	55 16	16	0
8	10	41 4	42 27	43 49	45 29	47 8	48 57	50 46	52 56	15	50
8	20	39 14	40 33	41 52	43 30	45 8	46 49	48 30	50 35	15	40
8	30	37 26	38 41	39 55	41 26	42 56	44 35	46 14	48 14	15	30
8	40	35 38	36 48	37 58	39 21	40 44	42 21	43 58	45 53	15	20
8	50	33 50	34 57	36 3	37 22	38 40	40 11	41 42	43 34	15	10
9	0	32 2	33 5	34 8	35 22	36 36	38 1	39 26	41 14	15	0
9	10	30 16	31 15	32 13	33 23	35 33	35 54	37 15	38 56	14	50
9	20	28 30	29 24	30 18	31 24	32 30	33 47	35 4	36 38	14	40
9	30	26 44	27 38	28 32	29 29	30 26	31 38	32 49	34 0	14	30
9	40	24 58	25 52	26 46	27 34	28 22	29 28	30 36	31 21	14	20
9	50	23 9	23 58	24 46	25 33	26 19	27 22	28 24	29 24	14	10
10	0	21 20	22 3	22 46	23 31	24 16	25 15	26 14	27 26	14	0
10	10	19 33	20 10	20 47	21 30	22 12	23 6	24 0	25 7	13	50
10	20	17 46	18 17	18 48	19 28	20 8	20 57	21 46	22 47	13	40
10	30	15 58	16 27	16 56	17 33	18 9	18 53	19 37	20 32	13	30
10	40	14 10	14 37	15 4	15 37	16 10	16 49	17 28	18 16	13	20
10	50	12 25	12 46	13 7	13 38	14 9	14 42	15 14	15 59	13	10
11	0	10 40	10 55	11 10	11 39	12 8	12 34	13 0	13 41	13	0
11	10	8 55	9 9	9 22	9 47	10 11	10 37	11 3	11 33	12	50
11	20	7 10	7 22	7 34	7 54	8 14	8 27	8 40	9 12	12	40
11	30	5 20	5 31	5 42	5 52	6 2	6 14	6 25	6 50	12	30
11	40	3 30	3 40	3 50	3 50	3 50	4 0	4 10	4 28	12	20
11	50	1 45	1 50	1 55	1 55	1 55	2 0	2 5	2 10	12	10
12	0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	Decl. 88° Log Tan²Z	Decl. 88° η	H.	M.
0	0	9·5938	180° 0′	24	0
0	10	9·5938	177 30	23	50
0	20	9·5940	175 0	23	40
0	30	9·5945	172 15	23	30
0	40	9·5950	169 30	23	20
0	50	9·5955	166 53	23	10
1	0	9·5960	164 16	23	0
1	10	9·5966	161 41	22	50
1	20	9·5972	159 6	22	40
1	30	9·5986	156 25	22	30
1	40	9·6000	153 44	22	20
1	50	9·6015	151 5	22	10
2	0	9·6030	148 26	22	0
2	10	9·6047	145 50	21	50
2	20	9·6064	143 14	21	40
2	30	9·6085	140 37	21	30
2	40	9·6106	138 0	21	20
2	50	9·6125	135 24	21	10
3	0	9·6144	132 48	21	0
3	10	9·6167	130 14	20	50
3	20	9·6190	127 40	20	40
3	30	9·6215	125 4	20	30
3	40	9·6240	122 28	20	20
3	50	9·6265	119 55	20	10
4	0	9·6290	117 22	20	0
4	10	9·6315	114 49	19	50
4	20	9·6340	112 16	19	40
4	30	9·6374	109 43	19	30
4	40	9·6408	107 10	19	20
4	50	9·6423	104 38	19	10
5	0	9·6438	102 6	19	0
5	10	9·6471	99 35	18	50
5	20	9·6504	97 4	18	40
5	30	9·6531	94 33	18	30
5	40	9·6558	92 2	18	20
5	50	9·6588	89 32	18	10
6	0	9·6618	87 2	18	0

H.	M.	Decl. 88° Log Tan²Z	Decl. 88° η	H.	M.
6	0	9·6618	87° 2′	18	0
6	10	9·6645	84 34	17	50
6	20	9·6672	82 6	17	40
6	30	9·6700	79 27	17	30
6	40	9·6728	76 48	17	20
6	50	9·6757	74 30	17	10
7	0	9·6786	72 12	17	0
7	10	9·6813	69 44	16	50
7	20	9·6840	67 16	16	40
7	30	9·6862	64 49	16	30
7	40	9·6884	62 22	16	20
7	50	9·6910	59 56	16	10
8	0	9·6936	57 30	16	0
8	10	9·6961	55 5	15	50
8	20	9·6986	52 40	15	40
8	30	9·7007	50 14	15	30
8	40	9·7028	47 48	15	20
8	50	9·7049	45 25	15	10
9	0	9·7070	43 2	15	0
9	10	9·7088	40 37	14	50
9	20	9·7106	38 12	14	40
9	30	9·7123	35 49	14	30
9	40	9·7140	33 26	14	20
9	50	9·7152	31 2	14	10
10	0	9·7164	28 38	14	0
10	10	9·7176	26 13	13	50
10	20	9·7188	23 48	13	40
10	30	9·7199	21 26	13	30
10	40	9·7210	19 4	13	20
10	50	9·7219	16 43	13	10
11	0	9·7228	14 22	13	0
11	10	9·7237	12 3	12	50
11	20	9·7242	9 44	12	40
11	30	9·7242	7 5	12	30
11	40	9·7244	4 24	12	20
11	50	9·7244	2 12	12	10
12	0	9·7246	0 0	12	0

TABLE

CONTAINING

THE LOGARITHM OF TAN^2 ZENITH DISTANCE

AND THE

PARALLACTIC ANGLE

FOR

LATITUDE 57° 30′, AND DECLINATION 40° TO 90°.

LOGARITHM OF TAN^2 Z FOR LAT. 57° 30′.

H.	M.	40°	42°	44°	46°	48°	50°	52°	54°	H.	M.
0	0	8·9979	8·8795	8·7611	8·6046	8·4480	8·2073	7·9667	7·4039	24	0
0	10	9·0023	8·8864	8·7705	8·6159	8·4612	8·2379	8·0145	7·5547	23	50
0	20	9·0136	8·8993	8·7849	8·6376	8·4904	8·2823	8·0743	7·7754	23	40
0	30	9·0324	8·9168	8·8012	8·6706	8·5400	8·3613	8·1826	7·9754	23	30
0	40	9·0570	8·9529	8·8488	8·7229	8·5969	8·4437	8·2916	8·1437	23	20
0	50	9·0879	8·9902	8·8924	8·7784	8·6644	8·5316	8·3987	8·2885	23	10
1	0	9·1225	9·0319	8·9393	8·8362	8·7330	8·6226	8·5121	8·4223	23	0
1	10	9·1607	9·0758	8·9909	8·8973	8·8042	8·7086	8·6129	8·5384	22	50
1	20	9·2016	9·1220	9·0424	8·9582	8·8740	8·7907	8·7073	8·6436	22	40
1	30	9·2444	9·1701	9·0958	9·0192	8·9425	8·8686	8·7945	8·7383	22	30
1	40	9·2881	9·2189	9·1496	9·0792	9·0087	8·9425	8·8763	8·8258	22	20
1	50	9·3329	9·2679	9·2029	9·1379	9·0728	9·0024	8·9520	8·9061	22	10
2	0	9·3775	9·3154	9·2552	9·1949	9·1345	9·0790	9·0234	8·9799	22	0
2	10	9·4227	9·3649	9·3069	9·2505	9·1939	9·1422	9·0903	9·0495	21	50
2	20	9·4676	9·4126	9·3576	9·3044	9·2511	9·2025	9·1538	9·1147	21	40
2	30	9·5123	9·4598	9·4071	9·3566	9·3059	9·2610	9·2159	9·1723	21	30
2	40	9·5566	9·5064	9·4558	9·4075	9·3592	9·3151	9·2709	9·2345	21	20
2	50	9·6008	9·5522	9·5036	9·4573	9·4109	9·3623	9·3247	9·2890	21	10
3	0	9·6445	9·5955	9·5505	9·5036	9·4608	9·4196	9·3784	9·3435	21	0
3	10	9·6871	9·6364	9·5953	9·5472	9·5087	9·4684	9·4279	9·3935	20	50
3	20	9·7298	9·6850	9·6402	9·5976	9·5550	9·5156	9·4761	9·4421	20	40
3	30	9·7718	9·7277	9·6840	9·6422	9·6004	9·5617	9·5229	9·4908	20	30
3	40	9·8138	9·7705	9·7271	9·6860	9·6448	9·6066	9·5683	9·5344	20	20
3	50	9·8551	9·8123	9·7694	9·7287	9·6879	9·6500	9·6120	9·5781	20	10
4	0	9·8962	9·8536	9·8111	9·7706	9·7301	9·6923	9·6546	9·6205	20	0
4	10	9·9369	9·8946	9·8522	9·8119	9·7716	9·7339	9·6961	9·6619	19	50
4	20	9·9778	9·9353	9·8929	9·8526	9·8123	9·7744	9·7367	9·7020	19	40
4	30	0·0184	9·9757	9·9331	9·8927	9·8522	9·8142	9·7762	9·7412	19	30
4	40	0·0587	0·0158	9·9728	9·9322	9·8915	9·8533	9·8150	9·7795	19	20
4	50	0·0988	0·0554	0·0122	9·9712	9·9301	9·8916	9·8529	9·8171	19	10
5	0	0·1388	0·0951	0·0513	0·0099	9·9684	9·9293	9·8902	9·8549	19	0
5	10	0·1788	0·1343	0·0898	0·0480	0·0062	9·9715	9·9267	9·8896	18	50
5	20	0·2190	0·1736	0·1282	0·0858	0·0434	0·0031	9·9628	9·9249	18	40
5	30	0·2593	0·2131	0·1669	0·1235	0·0801	0·0391	9·9981	9·9596	18	30
5	40	0·2994	0·2525	0·2055	0·1612	0·1169	0·0750	0·0332	9·9938	18	20
5	50	0·3398	0·2915	0·2431	0·1980	0·1529	0·1101	0·0673	0·0272	18	10
6	0	0·3805	0·3311	0·2817	0·2354	0·1890	0·1453	0·1016	0·0604	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	40°	42°	44°	46°	48°	50°	52°	54°	H.	M.
0	0	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	0° 0′	24	0
0	10	4 34	5 12	5 50	7 2	8 13	11 5	13 56	26 49	23	50
0	20	8 54	10 7	11 20	13 33	15 45	20 12	25 39	42 27	23	40
0	30	13 1	14 48	16 34	19 35	22 36	28 45	34 53	50 48	23	30
0	40	16 55	19 8	21 20	24 58	28 35	35 26	42 17	56 23	23	20
0	50	20 37	23 8	25 38	29 38	33 38	40 32	47 26	59 55	23	10
1	0	24 0	26 44	29 28	33 41	37 53	44 42	51 31	62 27	23	0
1	10	27 3	29 54	32 45	36 48	40 40	46 45	52 49	63 19	22	50
1	20	29 45	32 42	35 39	39 55	44 10	50 21	56 31	65 12	22	40
1	30	32 9	35 8	38 7	42 19	46 30	52 15	58 0	65 50	22	30
1	40	34 16	37 16	40 15	44 15	48 15	53 42	59 8	66 14	22	20
1	50	36 7	39 3	41 58	45 50	49 41	54 32	59 22	66 17	22	10
2	0	37 44	40 48	43 27	47 9	50 51	55 37	60 23	66 19	22	0
2	10	39 7	41 54	44 39	48 11	51 43	56 10	60 36	66 6	21	50
2	20	40 18	42 59	45 40	49 2	52 23	56 34	60 45	65 49	21	40
2	30	41 17	43 52	46 26	49 39	52 51	56 47	60 42	65 24	21	30
2	40	42 9	44 38	47 6	50 38	53 10	56 52	60 33	64 56	21	20
2	50	42 52	45 17	47 41	50 42	53 19	56 49	60 18	64 23	21	10
3	0	43 25	45 45	48 5	50 47	53 26	56 42	59 57	63 47	21	0
3	10	43 49	46 1	48 13	50 46	53 20	56 25	59 30	63 5	20	50
3	20	44 8	46 13	48 17	50 45	53 13	56 7	59 0	62 23	20	40
3	30	44 21	46 20	48 19	50 38	52 56	55 41	58 26	61 11	20	30
3	40	44 29	46 23	48 16	50 29	52 41	55 17	57 52	60 51	20	20
3	50	44 29	46 18	48 7	50 12	52 16	54 45	57 14	60 3	20	10
4	0	44 30	46 12	47 55	49 55	51 54	54 13	56 32	59 12	20	0
4	10	44 24	46 15	47 39	49 33	51 26	53 37	55 48	58 19	19	50
4	20	44 14	46 17	47 19	49 7	50 54	52 59	55 3	57 25	19	40
4	30	44 1	45 26	46 50	48 35	50 19	52 17	54 15	56 30	19	30
4	40	43 44	45 7	46 30	48 6	49 43	51 40	53 26	55 34	19	20
4	50	43 23	44 42	46 0	47 32	49 4	50 50	52 35	54 36	19	10
5	0	42 59	43 59	45 29	46 56	48 23	50 3	51 43	53 41	19	0
5	10	42 33	43 44	44 55	46 17	47 39	49 14	50 48	52 7	18	50
5	20	44 4	43 12	44 19	45 37	46 51	48 22	49 53	51 36	18	40
5	30	41 32	42 36	43 39	44 53	46 7	47 22	48 55	50 54	18	30
5	40	40 58	41 59	42 59	44 9	45 18	46 38	47 58	48 30	18	20
5	50	40 23	41 20	42 15	43 22	44 28	45 43	46 57	48 25	18	10
6	0	39 44	40 38	41 31	42 34	43 35	44 47	45 58	47 21	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	40°	42°	44°	46°	48°	50°	52°	54°	H.	M.
6	0	0·3805	0·3311	0·2817	0·2354	0·1890	0·1453	0·1016	0·0604	18	0
6	10	0·4217	0·3708	0·3200	0·2724	0·2249	0·1801	0·1354	0·0932	17	50
6	20	0·4627	0·4103	0·3579	0·3091	0·2603	0·2144	0·1685	0·1252	17	40
6	30	0·5039	0·4498	0·3957	0·3455	0·2952	0·2482	0·2011	0·1567	17	30
6	40	0·5458	0·4900	0·4340	0·3822	0·3304	0·2821	0·2337	0·1881	17	20
6	50	0·5879	0·5299	0·4720	0·4185	0·3651	0·3154	0·2655	0·2188	17	10
7	0	0·6305	0·5703	0·5101	0·4549	0·3998	0·3485	0·2973	0·2493	17	0
7	10	0·6729	0·6104	0·5478	0·4908	0·4338	0·3810	0·3283	0·2789	16	50
7	20	0·7163	0·6512	0·5861	0·5270	0·4680	0·4136	0·3592	0·3084	16	40
7	30	0·7603	0·6925	0·6246	0·5633	0·5020	0·4459	0·3899	0·3376	16	30
7	40	0·8045	0·7336	0·6628	0·5993	0·5359	0·4779	0·4200	0·3662	16	20
7	50	0·8489	0·7749	0·7008	0·6350	0·5691	0·5093	0·4494	0·3941	16	10
8	0	0·8942	0·8167	0·7392	0·6708	0·6024	0·5407	0·4789	0·4219	16	0
8	10	0·9393	0·8581	0·7770	0·7060	0·6349	0·5711	0·5072	0·4487	15	50
8	20	0·9855	0·9004	0·8153	0·7414	0·6676	0·6016	0·5357	0·4754	15	40
8	30	1·0312	0·9420	0·8529	0·7761	0·6993	0·6312	0·5630	0·5010	15	30
8	40	1·0784	0·9847	0·8910	0·8111	0·7313	0·6608	0·5904	0·5267	15	20
8	50	1·1250	1·0266	0·9283	0·8453	0·7622	0·6895	0·6167	0·5512	15	10
9	0	1·1722	1·0688	0·9655	0·8792	0·7928	0·7177	0·6426	0·5754	15	0
9	10	1·2184	1·1099	1·0014	0·9117	0·8221	0·7446	0·6671	0·5980	14	50
9	20	1·2648	1·1509	1·0370	0·9439	0·8507	0·7708	0·6909	0·6201	14	40
9	30	1·3112	1·1916	1·0721	0·9754	0·8787	0·7964	0·7141	0·6414	14	30
9	40	1·3574	1·2320	1·1067	1·0056	0·9059	0·8211	0·7364	0·6620	14	20
9	50	1·4020	1·2705	1·1391	1·0352	0·9313	0·8442	0·7571	0·6810	14	10
10	0	1·4472	1·3094	1·1716	1·0642	0·9568	0·8673	0·7778	0·6998	14	0
10	10	1·4898	1·3460	1·2022	1·0910	0·9798	0·8882	0·7966	0·7169	13	50
10	20	1·5314	1·3809	1·2304	1·1160	1·0016	0·9076	0·8136	0·7329	13	40
10	30	1·5690	1·4131	1·2572	1·1392	1·0212	0·9253	0·8294	0·7470	13	30
10	40	1·6064	1·4439	1·2814	1·1608	1·0402	0·9424	0·8446	0·7606	13	20
10	50	1·6386	1·4712	1·3038	1·1798	1·0558	0·9563	0·8568	0·7717	13	10
11	0	1·6702	1·4972	1·3242	1·1977	1·0712	0·9698	0·8684	0·7826	13	0
11	10	1·6952	1·5183	1·3414	1·2127	1·0840	0·9813	0·8786	0·7917	12	50
11	20	1·7192	1·5377	1·3562	1·2254	1·0946	0·9907	0·8868	0·7993	12	40
11	30	1·7362	1·5518	1·3674	1·2353	1·1032	0·9981	0·8930	0·8048	12	30
11	40	1·7506	1·5633	1·3760	1·2426	1·1092	1·0034	0·8976	0·8091	12	20
11	50	1·7572	1·5688	1·3804	1·2462	1·1120	1·0060	0·9000	0·8110	12	10
12	0	1·7612	1·5721	1·3830	1·2485	1·1140	1·0079	0·9018	0·8126	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	40°	42°	44°	46°	48°	50°	52°	54°	H.	M.
6	0	39° 44′	40° 38′	41° 31′	42° 34′	43° 35′	44° 47′	45° 58′	47° 21′	18	0
6	10	39 4	39 54	40 44	41 43	42 41	43 49	44 56	46 14	17	50
6	20	38 21	39 9	39 56	40 51	41 46	42 50	43 54	45 8	17	40
6	30	37 37	38 22	39 6	39 58	40 50	41 50	42 50	44 0	17	30
6	40	36 50	37 33	38 14	39 3	39 51	40 48	41 45	42 51	17	20
6	50	36 2	36 41	37 20	38 6	38 52	39 46	40 39	41 41	17	10
7	0	35 12	35 49	36 25	37 8	37 51	38 41	39 31	40 30	17	0
7	10	34 20	34 55	35 29	36 9	36 49	37 37	38 24	39 20	16	50
7	20	33 27	33 59	34 30	35 8	35 46	36 30	37 14	38 7	16	40
7	30	32 32	33 2	33 31	34 6	34 41	35 23	36 4	36 53	16	30
7	40	31 35	32 3	32 30	33 3	33 36	34 15	34 53	35 39	16	20
7	50	30 36	31 2	31 27	31 58	32 29	33 5	33 41	34 25	16	10
8	0	29 36	30 0	30 23	30 52	31 20	31 55	32 29	33 9	16	0
8	10	28 35	28 57	29 18	29 45	30 11	30 43	31 15	31 53	15	50
8	20	27 32	27 52	28 12	28 37	29 1	29 31	30 0	30 36	15	40
8	30	26 28	26 46	27 5	27 27	27 50	28 18	28 45	29 18	15	30
8	40	25 22	25 39	25 55	26 17	26 37	27 3	27 29	28 0	15	20
8	50	24 15	24 30	24 46	25 5	25 24	25 48	26 12	26 40	15	10
9	0	23 8	23 22	23 36	23 54	24 11	24 33	24 55	25 22	15	0
9	10	21 57	22 10	22 22	22 39	22 55	23 15	23 35	24 0	14	50
9	20	20 46	20 58	21 9	21 24	21 39	21 58	22 16	22 39	14	40
9	30	19 34	19 45	19 55	20 8	20 22	20 39	20 56	21 17	14	30
9	40	18 21	18 31	18 40	18 52	19 4	19 20	19 35	19 55	14	20
9	50	17 8	17 16	17 24	17 35	17 46	18 0	18 14	18 32	14	10
10	0	15 53	16 0	16 4	16 17	16 28	16 41	16 54	17 7	14	0
10	10	14 38	14 45	14 52	15 0	15 8	15 21	15 34	15 30	13	50
10	20	13 24	13 24	13 24	13 34	13 44	13 54	14 4	14 21	13	40
10	30	12 4	12 11	12 18	12 20	12 22	12 33	12 44	12 54	13	30
10	40	10 50	10 44	10 38	10 50	11 2	11 14	11 26	11 30	13	20
10	50	9 16	9 23	9 30	9 30	9 30	9 42	9 54	9 58	13	10
11	0	8 8	8 5	8 2	8 8	8 14	8 17	8 20	8 33	13	0
11	10	6 22	6 36	6 50	6 57	7 4	7 10	7 16	7 22	12	50
11	20	5 30	5 21	5 12	5 17	5 22	5 30	5 38	5 50	12	40
11	30	3 54	3 59	4 4	4 20	4 30	4 20	4 16	4 30	12	30
11	40	2 44	2 26	2 8	2 26	2 44	2 36	2 28	2 52	12	20
11	50	1 22	1 13	1 4	1 13	1 22	1 18	1 14	1 26	12	10
12	0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	56°	58°	60°	62°	64°	66°	68°	70°	H.	M.
0	0	6·8410	——	7·2762	7·6946	8·1129	8·3244	8·5357	8·6802	24	0
0	10	7·0948	——	7·4268	7·7791	8·1314	8·3363	8·5411	8·6842	23	50
0	20	7·4766	7·2220	7·5986	7·8831	8·1699	8·3624	8·5552	8·6945	23	40
0	30	7·7683	7·5400	7·8141	8·0247	8·2352	8·4075	8·5799	8·7121	23	30
0	40	7·9958	7·8580	8·0059	8·1563	8·3068	8·4586	8·6104	8·7346	23	20
0	50	8·1783	8·1731	8·1679	8·2787	8·3893	8·5185	8·6476	8·7577	23	10
1	0	8·3324	8·3222	8·3120	8·3911	8·4702	8·5796	8·6889	8·7944	23	0
1	10	8·4639	8·4495	8·4351	8·4932	8·5511	8·6427	8·7341	8·8294	22	50
1	20	8·5798	8·5630	8·5462	8·5876	8·6289	8·7047	8·7804	8·8656	22	40
1	30	8·6819	8·6633	8·6445	8·6740	8·7033	8·7658	8·8282	8·9037	22	30
1	40	8·7743	8·7549	8·7353	8·7554	8·7643	8·8249	8·8758	8·9420	22	20
1	50	8·8601	8·8386	8·8169	8·8246	8·8321	8·8777	8·9232	8·9811	22	10
2	0	8·9365	8·9146	8·8928	8·8992	8·9057	8·9381	8·9705	9·0205	22	0
2	10	9·0085	8·9858	8·9629	8·9634	8·9637	8·9904	9·0169	9·0547	21	50
2	20	9·0756	9·0524	9·0291	9·0267	9·0241	9·0432	9·0621	9·0982	21	40
2	30	9·1385	9·1145	9·0904	9·0847	9·0790	9·0875	9·1055	9·1362	21	30
2	40	9·1979	9·1735	9·1489	9·1363	9·1317	9·1363	9·1489	9·1737	21	20
2	50	9·2544	9·2295	9·2045	9·1935	9·1824	9·1808	9·1906	9·2046	21	10
3	0	9·3086	9·2829	9·2572	9·2439	9·2306	9·2315	9·2323	9·2473	21	0
3	10	9·3589	9·3328	9·3066	9·2914	9·2761	9·2732	9·2701	9·2816	20	50
3	20	9·4080	9·3811	9·3543	9·3372	9·3202	9·3148	9·3095	9·3164	20	40
3	30	9·4585	9·4294	9·4002	9·3815	9·3627	9·3548	9·3468	9·3501	20	30
3	40	9·5004	9·4724	9·4443	9·4243	9·4041	9·3936	9·3831	9·3832	20	20
3	50	9·5441	9·5155	9·4867	9·4653	9·4438	9·4310	9·4181	9·4154	20	10
4	0	9·5864	9·5572	9·5279	9·5051	9·4822	9·4674	9·4526	9·4470	20	0
4	10	9·6275	9·5970	9·5676	9·5436	9·5196	9·5029	9·4862	9·4779	19	50
4	20	9·6674	9·6368	9·6062	9·5810	9·5558	9·5372	9·5186	9·5079	19	40
4	30	9·7063	9·6745	9·6437	9·6174	9·5909	9·5704	9·5499	9·5373	19	30
4	40	9·7440	9·7121	9·6802	9·6527	9·6251	9·6032	9·5812	9·5660	19	20
4	50	9·7811	9·7485	9·7156	9·6871	9·6585	9·6349	9·6112	9·5940	19	10
5	0	9·8196	9·7849	9·7502	9·7204	9·6906	9·6657	9·6407	9·6214	19	0
5	10	9·8525	9·8185	9·7838	9·7524	9·7210	9·6952	9·6693	9·6481	18	50
5	20	9·8871	9·8520	9·8169	9·7851	9·7532	9·7253	9·6974	9·6744	18	40
5	30	9·9211	9·8848	9·8488	9·8160	9·7831	9·7538	9·7244	9·6997	18	30
5	40	9·9543	9·9175	9·8806	9·8466	9·8126	9·7820	9·7514	9·7250	18	20
5	50	9·9871	9·9490	9·9114	9·8762	9·8410	9·8091	9·7773	9·7492	18	10
6	0	0·0192	9·9804	9·9416	9·9054	9·8693	9·8383	9·8074	9·7755	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	56°	58°	60°	62°	64°	66°	68°	70°	H.	M.
0	0	0° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	24	0
0	10	39 41	109 28	152 20	160 25	168 30	170 32	172 34	173 35	23	50
0	20	59 15	98 28	131 39	144 23	157 6	161 14	165 23	167 20	23	40
0	30	66 42	94 46	120 12	133 52	147 31	152 58	158 25	161 13	23	30
0	40	70 29	91 4	111 19	124 58	138 36	145 14	151 52	155 23	23	20
0	50	72 23	89 7	105 51	117 59	130 7	138 23	145 40	149 49	23	10
1	0	73 23	87 10	101 14	113 1	124 47	132 20	139 54	144 31	23	0
1	10	73 48	85 40	97 48	108 54	120 0	127 17	134 34	139 31	22	50
1	20	73 52	84 8	94 48	104 40	114 33	122 4	129 36	134 50	22	40
1	30	73 40	82 49	92 23	101 25	110 26	117 44	125 3	130 25	22	30
1	40	73 19	81 30	89 58	98 18	106 38	113 43	120 49	126 16	22	20
1	50	72 51	80 20	88 3	95 40	103 19	110 7	116 55	122 7	22	10
2	0	72 15	79 7	85 59	93 5	100 10	106 42	113 15	118 40	22	0
2	10	71 36	77 54	84 12	90 48	97 24	103 37	109 51	115 12	21	50
2	20	70 52	76 41	82 29	88 38	94 46	100 41	106 37	111 54	21	40
2	30	70 6	75 13	80 20	86 20	92 19	97 58	103 37	108 45	21	30
2	40	69 18	74 18	79 17	84 40	90 1	95 23	100 46	105 45	21	20
2	50	68 28	73 7	77 46	82 50	87 53	92 58	98 3	102 53	21	10
3	0	67 36	71 57	76 17	81 0	85 44	90 35	95 26	100 7	21	0
3	10	66 40	70 45	74 50	79 17	83 44	88 20	92 57	97 27	20	50
3	20	65 45	69 35	73 24	77 36	81 47	86 11	90 35	94 58	20	40
3	30	64 55	68 27	71 59	75 77	79 55	84 8	88 20	92 28	20	30
3	40	63 50	67 14	70 37	74 21	78 5	82 53	86 4	90 26	20	20
3	50	62 51	66 3	69 15	72 47	76 19	80 7	83 55	87 51	20	10
4	0	61 51	64 52	67 53	71 14	74 35	78 12	81 49	85 36	20	0
4	10	60 50	63 41	66 32	69 43	72 53	76 20	79 46	83 24	19	50
4	20	59 46	62 29	65 11	68 12	71 12	74 30	77 47	81 16	19	40
4	30	58 45	61 18	63 51	66 42	69 33	72 42	75 50	79 11	19	30
4	40	57 41	60 6	62 31	65 14	67 56	70 56	73 55	77 8	19	20
4	50	56 37	58 24	61 11	63 46	66 20	69 11	72 2	75 7	19	10
5	0	55 38	57 15	59 51	62 18	64 45	67 28	70 11	73 9	19	0
5	10	54 25	56 29	58 32	60 51	63 10	65 46	68 22	71 13	18	50
5	20	53 18	55 15	57 12	59 25	61 37	64 5	66 33	69 17	18	40
5	30	52 11	54 2	55 52	57 58	60 4	62 27	64 46	67 23	18	30
5	40	51 2	52 48	54 33	56 32	58 31	60 48	63 1	65 31	18	20
5	50	49 53	51 33	53 13	55 7	57 0	59 9	61 17	63 44	18	10
6	0	48 43	50 18	51 52	53 40	55 28	57 36	59 43	61 55	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	56°	58°	60°	62°	64°	66°	68°	70°	H.	M.
6	0	0·0192	9·9804	9·9416	9·9054	9·8693	9·8383	9·8074	9·7755	18	0
6	10	0·0510	0·0112	9·9712	9·9340	9·8968	9·8623	9·8278	9·7966	17	50
6	20	0·0820	0·0411	0·0002	9·9618	9·9234	9·8877	9·8519	9·8193	17	40
6	30	0·1123	0·0714	0·0285	9·9890	9·9494	9·9125	9·8755	9·8413	17	30
6	40	0·1425	0·0995	0·0565	0·0158	9·9752	9·9369	9·8986	9·8631	17	20
6	50	0·1721	0·1279	0·0836	0·0418	0·0000	9·9605	9·9220	9·8841	17	10
7	0	0·2012	0·1558	0·1104	0·0674	0·0244	9·9837	9·9430	9·9057	17	0
7	10	0·2295	0·1829	0·1363	0·0921	0·0479	0·0060	9·9640	9·9244	16	50
7	20	0·2576	0·2098	0·1619	0·1165	0·0711	0·0280	9·9848	9·9438	16	40
7	30	0·2854	0·2362	0·1871	0·1405	0·0939	0·0496	0·0051	9·9629	16	30
7	40	0·3123	0·2620	0·2115	0·1637	0·1159	0·0703	0·0247	9·9812	16	20
7	50	0·3388	0·2870	0·2352	0·1862	0·1371	0·0903	0·0436	9·9988	16	10
8	0	0·3650	0·3118	0·2586	0·2083	0·1580	0·1138	0·0696	0·0199	16	0
8	10	0·3902	0·3356	0·2810	0·2295	0·1779	0·1288	0·0797	0·0325	15	50
8	20	0·4152	0·3592	0·3032	0·2504	0·1976	0·1473	0·0970	0·0487	15	40
8	30	0·4391	0·3817	0·3243	0·2703	0·2162	0·1648	0·1133	0·0639	15	30
8	40	0·4630	0·4041	0·3452	0·2900	0·2347	0·1822	0·1296	0·0790	15	20
8	50	0·4857	0·4254	0·3651	0·3087	0·2522	0·1985	0·1448	0·0932	15	10
9	0	0·5081	0·4464	0·3846	0·3269	0·2693	0·2145	0·1597	0·1071	15	0
9	10	0·5289	0·4658	0·4027	0·3438	0·2849	0·2291	0·1732	0·1196	14	50
9	20	0·5492	0·4846	0·4201	0·3601	0·3000	0·2432	0·1864	0·1327	14	40
9	30	0·5687	0·5028	0·4369	0·3757	0·3145	0·2568	0·1990	0·1434	14	30
9	40	0·5875	0·5203	0·4530	0·3907	0·3284	0·2696	0·2108	0·1544	14	20
9	50	0·6049	0·5363	0·4677	0·4044	0·3411	0·2814	0·2217	0·1644	14	10
10	0	0·6218	0·5533	0·4848	0·4191	0·3534	0·2929	0·2324	0·1745	14	0
10	10	0·6372	0·5663	0·4954	0·4299	0·3644	0·3031	0·2418	0·1830	13	50
10	20	0·6522	0·5797	0·5072	0·4411	0·3750	0·3130	0·2510	0·1914	13	40
10	30	0·6646	0·5914	0·5182	0·4510	0·3838	0·3211	0·2584	0·1982	13	30
10	40	0·6766	0·6026	0·5286	0·4607	0·3928	0·3293	0·2658	0·2052	13	20
10	50	0·6866	0·6117	0·5368	0·4684	0·4000	0·3358	0·2716	0·2105	13	10
11	0	0·6968	0·6208	0·5448	0·4758	0·4068	0·3421	0·2774	0·2159	13	0
11	10	0·7048	0·6282	0·5516	0·4820	0·4124	0·3473	0·2822	0·2204	12	50
11	20	0·7118	0·6345	0·5572	0·4870	0·4168	0·3517	0·2866	0·2241	12	40
11	30	0·7166	0·6389	0·5612	0·4907	0·4202	0·3547	0·2892	0·2268	12	30
11	40	0·7206	0·6426	0·5646	0·4940	0·4234	0·3576	0·2918	0·2292	12	20
11	50	0·7220	0·6441	0·5662	0·4954	0·4246	0·3587	0·2928	0·2301	12	10
12	0	0·7234	0·6452	0·5670	0·4962	0·4254	0·3594	0·2934	0·2306	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	56°	58°	60°	62°	64°	66°	68°	70°	H.	M.
6	0	48° 43′	50° 18′	51° 52′	53° 40′	55° 28′	57° 36′	59° 43′	61° 55′	18	0
6	10	47 32	49 2	50 32	52 14	53 56	55 53	57 49	60 1	17	50
6	20	46 21	47 16	49 11	50 48	52 25	54 16	56 7	58 13	17	40
6	30	45 9	46 30	47 50	49 22	50 54	52 40	54 26	56 26	17	30
6	40	43 56	45 12	46 28	47 56	49 23	51 4	52 44	54 39	17	20
6	50	42 43	43 55	45 7	46 30	47 53	49 28	51 4	52 53	17	10
7	0	41 29	42 37	43 44	45 3	46 22	47 53	49 23	51 8	17	0
7	10	40 15	41 19	42 23	43 38	44 52	46 18	47 44	49 24	16	50
7	20	38 59	39 59	41 0	42 10	43 21	44 43	46 4	47 39	16	40
7	30	37 42	38 39	39 36	40 43	41 50	43 7	44 25	45 55	16	30
7	40	36 25	37 19	38 13	39 16	40 19	41 33	42 46	44 12	16	20
7	50	35 8	35 59	36 49	37 49	38 48	39 58	41 7	42 29	16	10
8	0	33 49	34 37	35 25	36 21	37 17	38 40	40 3	41 3	16	0
8	10	32 30	33 16	34 0	34 53	35 46	36 48	37 50	39 3	15	50
8	20	31 11	31 53	32 35	33 25	34 14	35 13	36 11	37 20	15	40
8	30	29 51	30 31	31 10	31 56	32 43	33 38	34 33	35 38	15	30
8	40	28 30	29 7	29 43	30 27	31 11	32 1	32 50	33 53	15	20
8	50	27 9	27 28	28 17	28 58	29 38	30 27	31 15	32 13	15	10
9	0	25 48	26 20	26 52	27 30	28 8	28 53	29 38	30 33	15	0
9	10	24 24	24 54	25 24	26 0	26 35	27 17	27 59	28 50	14	50
9	20	23 1	23 29	23 56	24 29	25 2	25 42	26 21	27 8	14	40
9	30	21 38	22 3	22 28	22 59	23 29	24 6	24 43	25 26	14	30
9	40	20 14	20 37	21 0	21 28	21 56	22 30	23 3	23 44	14	20
9	50	18 49	19 11	19 32	19 58	20 23	20 54	21 25	22 2	14	10
10	0	17 20	17 57	18 34	18 41	18 48	19 17	19 46	20 21	14	0
10	10	15 56	16 15	16 34	16 53	17 12	17 40	18 8	18 38	13	50
10	20	14 38	14 48	14 58	15 20	15 42	16 6	16 30	16 56	13	40
10	30	13 4	13 19	13 34	13 49	14 4	14 27	14 50	15 13	13	30
10	40	11 34	11 51	12 8	12 21	12 36	12 54	13 12	13 34	13	20
10	50	10 2	10 17	10 32	10 47	11 2	11 14	11 26	11 47	13	10
11	0	8 46	8 53	9 0	9 15	9 30	9 37	9 44	10 4	13	0
11	10	7 28	7 34	7 40	7 51	8 2	8 8	8 14	8 32	12	50
11	20	6 2	6 2	6 2	6 5	6 8	6 26	6 44	6 47	12	40
11	30	4 46	4 41	4 36	4 36	4 36	4 45	4 54	5 8	12	30
11	40	3 16	3 8	3 0	3 8	3 16	3 22	3 28	3 35	12	20
11	50	1 38	1 34	1 30	1 34	1 38	1 41	1 44	1 48	12	10
12	0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	72°	74°	76°	78°	80°	82°	84°	86°	H.	M.
0	0	8·8246	8·9365	9·0485	9·1413	9·2342	9·3148	9·3954	9·4678	24	0
0	10	8·8272	8·9385	9·0497	9·1422	9·2346	9·3150	9·3954	9·4678	23	50
0	20	8·8338	8·9435	9·0531	9·1448	9·2364	9·3162	9·3962	9·4683	23	40
0	30	8·8443	8·9516	9·0587	9·1491	9·2393	9·3184	9·3974	9·4689	23	30
0	40	8·8588	8·9625	9·0663	9·1547	9·2431	9·3212	9·3992	9·4703	23	20
0	50	8·8792	8·9780	9·0768	9·1625	9·2482	9·3247	9·4012	9·4712	23	10
1	0	8·8998	8·9936	9·0874	9·1709	9·2543	9·3297	9·4042	9·4735	23	0
1	10	8·9245	9·0127	9·1007	9·1811	9·2614	9·3344	9·4074	9·4752	22	50
1	20	8·9508	9·0331	9·1154	9·1923	9·2692	9·3405	9·4118	9·4780	22	40
1	30	8·9791	9·0554	9·1315	9·2048	9·2780	9·3471	9·4162	9·4810	22	30
1	40	9·0083	9·0786	9·1489	9·2183	9·2876	9·3541	9·4206	9·4839	22	20
1	50	9·0390	9·1034	9·1678	9·2330	9·2981	9·3619	9·4256	9·4871	22	10
2	0	9·0604	9·1288	9·1872	9·2482	9·3092	9·3702	9·4312	9·4906	22	0
2	10	9·0924	9·1499	9·2073	9·2642	9·3209	9·3788	9·4366	9·4939	21	50
2	20	9·1343	9·1811	9·2280	9·2807	9·3334	9·3883	9·4434	9·4985	21	40
2	30	9·1663	9·2079	9·2494	9·2980	9·3465	9·3984	9·4502	9·5026	21	30
2	40	9·1983	9·2349	9·2712	9·3154	9·3595	9·4082	9·4568	9·5070	21	20
2	50	9·2300	9·2619	9·2937	9·3332	9·3725	9·4188	9·4634	9·5115	21	10
3	0	9·2622	9·2892	9·3163	9·3521	9·3880	9·4297	9·4704	9·5165	21	0
3	10	9·2929	9·3157	9·3384	9·3702	9·4019	9·4406	9·4780	9·5200	20	50
3	20	9·3234	9·3419	9·3605	9·3883	9·4161	9·4510	9·4858	9·5260	20	40
3	30	9·3534	9·3682	9·3827	9·4070	9·4311	9·4623	9·4934	9·5306	20	30
3	40	9·3834	9·3943	9·4051	9·4256	9·4461	9·4739	9·5018	9·5362	20	20
3	50	9·4125	9·4200	9·4274	9·4443	9·4611	9·4862	9·5112	9·5421	20	10
4	0	9·4414	9·4454	9·4494	9·4628	9·4761	9·4981	9·5200	9·5479	20	0
4	10	9·4696	9·4704	9·4712	9·4813	9·4913	9·5096	9·5278	9·5529	19	50
4	20	9·4971	9·4951	9·4930	9·4997	9·5065	9·5215	9·5366	9·5587	19	40
4	30	9·5242	9·5193	9·5144	9·5180	9·5216	9·5333	9·5450	9·5643	19	30
4	40	9·5508	9·5431	9·5355	9·5361	9·5367	9·5455	9·5542	9·5704	19	20
4	50	9·5768	9·5666	9·5564	9·5541	9·5517	9·5572	9·5631	9·5761	19	10
5	0	9·6020	9·5894	9·5768	9·5717	9·5666	9·5693	9·5720	9·5822	19	0
5	10	9·6268	9·6120	9·5971	9·5893	9·5813	9·5810	9·5804	9·5878	18	50
5	20	9·6514	9·6342	9·6170	9·6065	9·5960	9·5930	9·5900	9·5940	18	40
5	30	9·6750	9·6557	9·6363	9·6234	9·6104	9·6041	9·5978	9·5994	18	30
5	40	9·6985	9·6771	9·6556	9·6402	9·6247	9·6159	9·6072	9·6058	18	20
5	50	9·7212	9·6977	9·6743	9·6558	9·6374	9·6268	9·6162	9·6117	18	10
6	0	9·7435	9·7182	9·6929	9·6727	9·6526	9·6384	9·6242	9·6171	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	72°	74°	76°	78°	80°	82°	84°	86°	H.	M.
0	0	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	180° 0′	24	0
0	10	174 36	175 10	175 44	176 8	176 32	176 46	177 0	177 26	23	50
0	20	169 17	170 24	171 31	172 17	173 3	173 35	174 6	174 27	23	40
0	30	164 2	165 41	167 20	168 25	169 31	170 16	171 0	171 34	23	30
0	40	158 55	161 4	163 13	164 37	166 1	167 3	168 4	168 46	23	20
0	50	153 58	156 33	159 8	160 50	162 33	163 48	165 2	165 57	23	10
1	0	149 9	152 7	155 6	157 6	159 6	160 33	162 0	163 8	23	0
1	10	144 29	147 48	151 8	153 25	155 42	157 21	159 0	160 18	22	50
1	20	140 4	143 40	147 16	149 48	152 20	154 11	156 2	157 24	21	40
1	30	135 48	139 39	143 30	147 15	149 1	151 0	153 0	154 35	22	30
1	40	131 43	135 45	139 57	142 15	145 43	147 57	150 10	151 52	22	20
1	50	127 19	131 42	136 5	138 31	142 28	144 51	147 14	149 5	22	10
2	0	124 5	128 18	132 32	135 53	139 14	141 47	144 20	146 19	22	0
2	10	120 34	124 52	129 11	132 37	136 4	138 45	141 26	143 34	21	50
2	20	117 11	121 32	125 53	129 25	132 57	135 45	138 32	140 47	21	40
2	30	113 54	117 46	122 38	126 15	129 53	132 47	135 40	138 4	21	30
2	40	110 44	115 6	119 28	123 9	126 51	129 53	132 54	135 22	21	20
2	50	107 44	112 3	116 23	120 6	123 50	126 58	130 6	132 40	21	10
3	0	104 49	108 6	113 24	117 9	120 54	124 6	127 18	129 59	21	0
3	10	102 2	106 15	110 29	114 15	118 2	121 18	124 34	127 20	20	50
3	20	99 21	103 30	107 39	111 25	115 11	118 31	121 50	124 39	20	40
3	30	96 35	100 44	104 53	108 38	112 23	115 45	119 6	122 0	20	30
3	40	94 12	98 11	102 10	105 54	109 38	113 1	116 24	119 22	22	20
3	50	92 14	95 52	99 31	103 13	106 55	110 18	113 40	116 43	20	10
4	0	89 22	93 9	96 56	100 35	104 14	107 38	111 2	114 7	20	0
4	10	87 2	90 43	94 23	97 59	101 35	105 0	108 24	111 31	19	50
4	20	84 45	88 20	91 55	95 27	99 0	102 24	105 48	108 57	19	40
4	30	82 32	86 0	89 28	92 50	96 11	99 42	103 12	106 22	19	30
4	40	80 21	83 13	87 5	90 30	93 54	97 15	100 36	103 47	19	20
4	50	78 12	81 29	84 46	88 6	91 25	94 44	98 2	101 14	19	10
5	0	76 6	79 16	82 25	85 41	88 57	92 15	95 32	98 44	19	0
5	10	74 2	77 6	80 10	83 22	86 33	89 47	93 0	96 12	18	50
5	20	72 0	74 57	77 53	81 0	84 7	87 19	90 30	93 41	18	40
5	30	70 0	72 21	74 41	78 13	81 45	84 53	88 0	91 11	18	30
5	40	68 0	70 46	73 30	76 27	79 23	82 29	85 34	88 43	18	20
5	50	66 4	68 43	71 21	74 12	77 3	80 6	83 8	86 15	18	10
6	0	64 7	66 40	69 12	72 59	74 45	77 43	80 40	83 46	18	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

LOGARITHM OF TAN^2 Z FOR LAT. 57° 30′.

H.	M.	72°	74°	76°	78°	80°	82°	84°	86°	H.	M.
6	0	9·7435	9·7182	9·6929	9·6727	9·6526	9·6384	9·6242	9·6171	18	0
6	10	9·7654	9·7383	9·7111	9·6887	9·6663	9·6496	9·6328	9·6231	17	50
6	20	9·7866	9·7576	9·7286	9·7041	9·6796	9·6606	9·6416	9·6289	17	40
6	30	9·8072	9·7765	9·7458	9·7192	9·6926	9·6709	9·6492	9·6337	17	30
6	40	9·8275	9·7951	9·7627	9·7341	9·7056	9·6818	9·6580	9·6398	17	20
6	50	9·8471	9·8131	9·7790	9·7486	9·7181	9·6917	9·6652	9·6444	17	10
7	0	9·8663	9·8307	9·7951	9·7627	9·7304	9·7021	9·6738	9·6504	17	0
7	10	9·8847	9·8476	9·8104	9·7763	9·7422	9·7117	9·6812	9·6555	16	50
7	20	9·9029	9·8642	9·8256	9·7897	9·7538	9·7213	9·6888	9·6606	16	40
7	30	9·9206	9·8805	9·8404	9·8029	9·7653	9·7308	9·6964	9·6657	16	30
7	40	9·9377	9·8962	9·8547	9·8155	9·7763	9·7395	9·7028	9·6703	16	20
7	50	9·9540	9·9112	9·8684	9·8276	9·7868	9·7484	9·7100	9·6748	16	10
8	0	9·9702	9·9260	9·8818	9·8395	9·7972	9·7574	9·7176	9·6801	16	0
8	10	9·9854	9·9399	9·8945	9·8508	9·8070	9·7652	9·7234	9·6841	15	50
8	20	0·0013	9·9537	9·9070	9·8619	9·8167	9·7733	9·7298	9·6883	15	40
8	30	0·0144	9·9666	9·9187	9·8722	9·8258	9·7806	9·7354	9·6924	15	30
8	40	0·0284	9·9794	9·9304	9·8826	9·8349	9·7882	9·7416	9·6971	15	20
8	50	0·0415	9·9914	9·9413	9·8923	9·8433	9·7954	9·7464	9·7006	15	10
9	0	0·0544	0·0032	9·9520	9·9019	9·8518	9·8025	9·7532	9·7048	15	0
9	10	0·0659	0·0139	9·9619	9·9104	9·8590	9·8085	9·7580	9·7079	14	50
9	20	0·0771	0·0239	9·9707	9·9184	9·8661	9·8144	9·7626	9·7111	14	40
9	30	0·0877	0·0337	9·9796	9·9263	9·8730	9·8200	9·7670	9·7140	14	30
9	40	0·0979	0·0430	9·9880	9·9338	9·8795	9·8256	9·7716	9·7172	14	20
9	50	0·1072	0·0514	9·9956	9·9405	9·8854	9·8304	9·7754	9·7196	14	10
10	0	0·1166	0·0598	0·0030	9·9473	9·8916	9·8355	9·7794	9·7227	14	0
10	10	0·1242	0·0671	0·0100	9·9533	9·8966	9·8395	9·7824	9·7248	13	50
10	20	0·1318	0·0740	0·0162	9·9587	9·9012	9·8434	9·7856	9·7271	13	40
10	30	0·1380	0·0799	0·0212	9·9633	9·9054	9·8468	9·7882	9·7287	13	30
10	40	0·1446	0·0857	0·0268	9·9682	9·9096	9·8506	9·7916	9·7308	13	20
10	50	0·1494	0·0900	0·0306	9·9715	9·9124	9·8528	9·7932	9·7318	13	10
11	0	0·1544	0·0946	0·0348	9·9751	9·9154	9·8553	9·7952	9·7334	13	0
11	10	0·1586	0·0980	0·0374	9·9777	9·9180	9·8576	9·7972	9·7348	12	50
11	20	0·1616	0·1010	0·0404	9·9804	9·9204	9·8596	9·7988	9·7359	12	40
11	30	0·1644	0·1034	0·0424	9·9822	9·9220	9·8608	9·7994	9·7365	12	30
11	40	0·1666	0·1055	0·0444	9·9838	9·9232	9·8620	9·8008	9·7375	12	20
11	50	0·1674	0·1062	0·0450	9·9844	9·9238	9·8623	9·8008	9·7375	12	10
12	0	0·1678	0·1067	0·0456	9·9849	9·9242	9·8627	9·8012	9·7377	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	72°	74°	76°	78°	80°	82°	84°	86°	H.	M.
6	0	64° 7′	66° 40′	69° 12′	72° 59′	74° 45′	77° 43′	80° 40′	83° 46′	18	0
6	10	62 12	64 24	67 5	69 47	72 28	75 22	78 16	81 19	17	50
6	20	60 18	62 39	65 0	67 36	70 12	73 2	75 52	78 53	17	40
6	30	58 26	60 41	62 57	65 28	67 59	70 44	73 28	76 26	17	30
6	40	56 33	58 43	60 53	63 39	65 45	68 25	71 6	74 1	17	20
6	50	54 43	56 47	58 52	61 12	63 33	66 10	68 46	71 37	17	10
7	0	52 53	54 52	56 51	59 6	61 21	63 54	66 26	69 13	17	0
7	10	51 3	52 57	54 51	57 2	59 12	61 39	64 6	66 50	16	50
7	20	49 14	51 3	52 52	54 58	57 3	59 25	61 48	64 28	16	40
7	30	47 25	49 9	50 53	52 54	54 54	57 11	59 28	62 4	16	30
7	40	45 37	47 17	48 56	50 51	52 46	54 58	57 10	59 42	16	20
7	50	43 50	45 25	46 59	48 49	50 39	52 47	54 54	57 21	16	10
8	0	42 2	43 32	45 2	46 47	48 32	50 36	52 40	55 1	16	0
8	10	40 16	41 42	43 7	44 48	46 28	48 25	50 22	52 40	15	50
8	20	38 29	39 50	41 11	42 47	44 22	46 14	48 6	50 18	15	40
8	30	36 43	38 0	39 16	40 47	42 18	44 5	45 52	48 0	15	30
8	40	34 56	36 9	37 21	38 47	40 13	41 56	43 38	45 39	15	20
8	50	33 11	34 19	35 27	36 49	38 10	39 47	41 24	43 20	15	10
9	0	31 26	32 31	33 35	34 52	36 9	37 40	39 12	41 4	15	0
9	10	29 40	30 42	31 43	32 54	34 5	35 33	37 0	38 45	14	50
9	20	27 55	28 51	29 48	31 4	32 21	33 34	34 48	36 27	14	40
9	30	26 9	27 2	27 54	28 58	30 1	31 17	32 34	34 7	14	30
9	40	24 24	25 13	26 2	27 1	27 59	29 12	30 24	31 51	14	20
9	50	22 40	23 25	24 10	25 4	25 59	27 4	28 10	29 33	14	10
10	0	20 56	21 35	22 14	23 6	23 58	26 0	26 0	27 16	14	0
10	10	19 8	19 48	20 28	21 13	21 57	22 52	23 46	24 57	13	50
10	20	17 22	17 59	18 36	19 16	19 56	20 45	21 34	22 42	13	40
10	30	15 36	16 9	16 42	17 20	17 58	18 41	19 24	20 24	13	30
10	40	13 56	14 26	14 56	15 29	16 2	16 42	17 22	18 8	13	20
10	50	12 8	12 34	13 0	13 28	13 56	14 32	15 8	15 49	13	10
11	0	10 24	10 47	11 10	11 31	11 52	12 23	12 54	13 32	13	0
11	10	8 50	9 0	9 10	9 34	9 58	10 26	10 54	11 23	12	50
11	20	6 50	7 3	7 16	7 37	7 58	8 19	8 40	9 3	12	40
11	30	5 22	5 26	5 30	5 49	6 8	6 15	6 22	6 49	12	30
11	40	3 42	3 48	3 54	3 59	4 4	4 20	4 36	4 45	12	20
11	50	1 51	1 54	1 57	2 0	2 2	2 15	2 28	2 36	12	10
12	0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	12	0

Horizontal Argument, Declination.—Vertical Argument, Hour Angle.

H.	M.	Decl. 88° Log Tan ²Z	Decl. 88° η		H.	M.
0	0	9·5403	180°	0′	24	0
0	10	9·5403	177	52	23	50
0	20	9·5404	174	48	23	40
0	30	9·5404	172	8	23	30
0	40	9·5412	169	28	23	20
0	50	9·5416	166	52	23	10
1	0	9·5420	164	16	23	0
1	10	9·5430	161	36	22	50
1	20	9·5442	158	46	22	40
1	30	9·5458	156	10	22	30
1	40	9·5472	153	34	22	20
1	50	9·5486	151	2	22	10
2	0	9·5500	148	18	22	0
2	10	9·5512	145	42	21	50
2	20	9·5536	143	4	21	40
2	30	9·5550	140	28	21	30
2	40	9·5572	137	50	21	20
2	50	9·5596	135	14	21	10
3	0	9·5616	132	40	21	0
3	10	9·5632	130	6	20	50
3	20	9·5662	127	28	20	40
3	30	9·5678	124	54	20	30
3	40	9·5706	122	20	20	20
3	50	9·5730	119	46	20	10
4	0	9·5758	117	12	20	0
4	10	9·5780	114	38	19	50
4	20	9·5808	112	6	19	40
4	30	9·5836	109	32	19	30
4	40	9·5866	106	58	19	20
4	50	9·5896	104	26	19	10
5	0	9·5924	101	56	19	0
5	10	9·5952	99	24	18	50
5	20	9·5980	96	52	18	40
5	30	9·6010	94	22	18	30
5	40	9·6044	91	52	18	20
5	50	9·6072	89	22	18	10
6	0	9·6100	86	52	18	0

H.	M.	Decl. 88° Log Tan ²Z	Decl. 88° η		H.	M.
6	0	9·6100	86°	52′	18	0
6	10	9·6134	84	22	17	50
6	20	9·6162	81	54	17	40
6	30	9·6182	79	24	17	30
6	40	9·6216	76	56	17	20
6	50	9·6236	74	28	17	10
7	0	9·6270	72	0	17	0
7	10	9·6298	69	34	16	50
7	20	9·6324	67	8	16	40
7	30	9·6350	64	40	16	30
7	40	9·6378	62	14	16	20
7	50	9·6396	59	48	16	10
8	0	9·6426	57	22	16	0
8	10	9·6448	54	58	15	50
8	20	9·6468	52	30	15	40
8	30	9·6494	50	8	15	30
8	40	9·6526	47	40	15	20
8	50	9·6544	45	16	15	10
9	0	9·6564	42	56	15	0
9	10	9·6578	40	30	14	50
9	20	9·6596	38	6	14	40
9	30	9·6610	35	40	14	30
9	40	9·6628	33	18	14	20
9	50	9·6644	30	56	14	10
10	0	9·6660	28	32	14	0
10	10	9·6672	26	8	13	50
10	20	9·6686	23	46	13	40
10	30	9·6692	21	24	13	30
10	40	9·6700	18	54	13	20
10	50	9·6708	16	30	13	10
11	0	9·6716	14	10	13	0
11	10	9·6724	11	52	12	50
11	20	9·6730	9	26	12	40
11	30	9·6736	7	16	12	30
11	40	9·6742	4	54	12	20
11	50	9·6742	2	44	12	10
12	0	9·6742	0	0	12	0

XIV.—*On a Class of Alternating Functions.* By THOMAS MUIR, LL.D.

(Read 7th March 1887.)

A glance at the expression

$$\frac{(a-\alpha)(a-\beta)(a-\gamma)(a-\delta)}{(a-b)(a-c)(a-d)}+\frac{(b-\alpha)(b-\beta)(b-\gamma)(b-\delta)}{(b-a)(b-c)(b-d)}$$
$$+\frac{(c-\alpha)(c-\beta)(c-\gamma)(c-\delta)}{(c-a)(c-b)(c-d)}+\frac{(d-\alpha)(d-\beta)(d-\gamma)(d-\delta)}{(d-a)(d-b)(d-c)}$$

is sufficient to verify the fact that it is symmetric with respect to a, b, c, d, and also with respect to $\alpha, \beta, \gamma, \delta$. It is likewise, although not quite so evidently, an alternating function with respect to the interchange

$$\begin{pmatrix} a & b & c & d \\ \alpha & \beta & \gamma & \delta \end{pmatrix};$$

that is to say, if a and α be interchanged, and at the same time b and β, c and γ, d and δ, the function is not altered in magnitude, but merely changes sign. With a little trouble, indeed, the expression can be transformed into

$$(a+b+c+d)-(\alpha+\beta+\gamma+\delta),$$

or say

$$\Sigma a - \Sigma\alpha$$

This alternating function is only one of a large class to which it is proposed here to direct attention. It may be looked upon as in a certain sense the generator of the other members of the class, because they are derivable from it by prefixing to each of its component fractions a symmetric function of the three variables which occur only once in the corresponding denominator, *e.g.*, the symmetric function $bc+bd+cd$ prefixed as a factor to the first fraction, the like function $ac+ad+cd$ prefixed to the second fraction, and so on. The various kinds of symmetric functions which may be used in this way as prefixed factors are best expressed in the form

$$\begin{vmatrix} b^m & b^n & b^r \\ c^m & c^n & c^r \\ d^m & d^n & d^r \end{vmatrix} \div \begin{vmatrix} 1 & b & b^2 \\ 1 & c & c^2 \\ 1 & d & d^2 \end{vmatrix} \quad \text{or} \quad \frac{|b^m c^n d^r|}{|b^0 c^1 d^2|},$$

m, n, r, having any three values chosen from 0, 1, 2, 3, 4; for example, the above-mentioned instance

$$bc+bd+cd$$

is, in this form,

$$\frac{|b^0 c^2 d^3|}{|b^0 c^1 d^2|}.$$

The question then is—How can we by transformation set in evidence the fact that

$$\frac{|b^m c^n d^r|}{|b^0 c^1 d^2|} \cdot \frac{(a-\alpha)(a-\beta)(a-\gamma)(a-\delta)}{(a-b)(a-c)(a-d)} + \frac{|a^m c^n d^r|}{|a^0 c^1 d^2|} \cdot \frac{(b-\alpha)(b-\beta)(b-\gamma)(b-\delta)}{(b-a)(b-c)(b-d)}$$

$$+\frac{|a^m b^n d^r|}{|a^0 b^1 d^2|} \cdot \frac{(c-\alpha)(c-\beta)(c-\gamma)(c-\delta)}{(c-a)(c-b)(c-d)} + \frac{|a^m b^n c^r|}{|a^0 b^1 c^2|} \cdot \frac{(d-\alpha)(d-\beta)(d-\gamma)(d-\delta)}{(d-a)(d-b)(d-c)}$$

is an alternating function with respect to the interchange

$$\begin{pmatrix} a & b & c & d \\ \alpha & \beta & \gamma & \delta \end{pmatrix} ?$$

Since

$$|b^0 c^1 d^2| \;=\; \zeta^{\frac{1}{2}}(b,c,d) \;=\; (d-c)(d-b)(c-b) ,$$

the fractions evidently have the same denominator, viz.,

$$(d-c)(d-b)(d-a)(c-b)(c-a)(b-a) ,$$

or

$$\zeta^{\frac{1}{2}}(a\ b\ c\ d) ;$$

so that, if we expand the original numerators in descending powers of a, of b, &c., the expression becomes

$$\frac{1}{\zeta^{\frac{1}{2}}(a\,b\,c\,d)}\Big[-|b^m c^n d^r|\{a^4 - a^3\Sigma\alpha + a^2\Sigma\alpha\beta - a\Sigma\alpha\beta\gamma + \alpha\beta\gamma\delta\}$$
$$+|a^m c^n d^r|\{b^4 - b^3\Sigma\alpha + b^2\Sigma\alpha\beta - b\Sigma\alpha\beta\gamma + \alpha\beta\gamma\delta\}$$
$$-|a^m b^n d^r|\{c^4 - c^3\Sigma\alpha + c^2\Sigma\alpha\beta - c\Sigma\alpha\beta\gamma + \alpha\beta\gamma\delta\}$$
$$+|a^m b^n c^r|\{d^4 - d^3\Sigma\alpha + d^2\Sigma\alpha\beta - d\Sigma\alpha\beta\gamma + \alpha\beta\gamma\delta\}\Big] .$$

This, when the coefficients of $\Sigma\alpha$, $\Sigma\alpha\beta$, &c., are collected and condensed,

$$=\frac{1}{\zeta^{\frac{1}{2}}(a, b, c, d)}\Big[|a^m b^n c^r d^4| \;-\; |a^m b^n c^r d^3|\Sigma\alpha \;+\; |a^m b^n c^r d^2|\Sigma\alpha\beta$$
$$-\; |a^m b^n c^r d|\Sigma\alpha\beta\gamma \;+\; |a^m b^n c^r d^0|\Sigma\alpha\beta\gamma\delta \Big] ,$$

and no farther simplification is possible until the special values of m, n, r are given.

Taking in order the ten different sets of special values

$$0, 1, 2 ;\ 0, 1, 3 ;\ 0, 1, 4 ;\ 0, 2, 3 ;\ \ldots\ldots$$

and denoting the whole expression by $F_{m,n,r,}$ we see immediately that

$$F_{0,1,2} = \frac{1}{\zeta^{\frac{1}{2}}(a, b, c, d)}\Big[\ |a^0 b^1 c^2 d^4| \quad - |a^0 b^1 c^2 d^3|\Sigma\alpha \Big] ,$$

$$F_{0,1,3} = \frac{1}{\zeta^{\frac{1}{2}}(a, b, c, d)}\Big[\ |a^0 b^1 c^3 d^4| \quad + |a^0 b^1 c^3 d^2|\Sigma\alpha\beta \Big] ,$$

$$F_{0,1,4} = \frac{1}{\zeta^{\frac{1}{2}}(a, b, c, d)}\Big[-|a^0 b^1 c^4 d^3|\Sigma\alpha + |a^0 b^1 c^4 d^2|\Sigma\alpha\beta \Big] ,$$

$$
\begin{aligned}
\mathrm{F}_{0,2,3}&=\frac{1}{\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)}\Big[\quad |a^0b^2c^3d^4| \qquad\qquad -|a^0b^2c^3d^1|\,\Sigma\alpha\beta\gamma\Big]\,,\\
\mathrm{F}_{0,2,4}&=\frac{1}{\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)}\Big[-|a^0b^2c^4d^3|\,\Sigma\alpha \qquad -|a^0b^2c^4d^1|\,\Sigma\alpha\beta\gamma\Big]\,,\\
\mathrm{F}_{0,3,4}&=\frac{1}{\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)}\Big[\quad |a^0b^3c^4d^2|\,\Sigma\alpha\beta \quad -|a^0b^3c^4d^1|\,\Sigma\alpha\beta\gamma\Big]\,,\\
\mathrm{F}_{1,2,3}&=\frac{1}{\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)}\Big[\quad |a^1b^2c^3d^4| \qquad\qquad +|a^1b^2c^3d^0|\,\Sigma\alpha\beta\gamma\delta\Big]\,,\\
\mathrm{F}_{1,2,4}&=\frac{1}{\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)}\Big[-|a^1b^2c^4d^3|\,\Sigma\alpha \qquad +|a^1b^2c^4d^0|\,\Sigma\alpha\beta\gamma\delta\Big]\,,\\
\mathrm{F}_{1,3,4}&=\frac{1}{\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)}\Big[\quad |a^1b^3c^4d^2|\,\Sigma\alpha\beta \quad +|a^1b^3c^4d^0|\,\Sigma\alpha\beta\gamma\delta\Big]\,,\\
\mathrm{F}_{2,3,4}&=\frac{1}{\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)}\Big[-|a^2b^3c^4d^1|\,\Sigma\alpha\beta\gamma +|a^2b^3c^4d^0|\,\Sigma\alpha\beta\gamma\delta\Big]\,.
\end{aligned}
$$

Now, from the theory of alternants it is known that

$$
\begin{aligned}
|a^0b^1c^2d^3|&=\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)\,,\\
|a^0b^1c^2d^4|&=\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)\times\Sigma a\,,\\
|a^0b^1c^3d^4|&=\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)\times\Sigma ab\,,\\
|a^0b^2c^3d^4|&=\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)\times\Sigma abc\,,\\
|a^1b^2c^3d^4|&=\zeta^{\frac{1}{2}}(a,\,b,\,c,\,d)\times\Sigma abcd\,;
\end{aligned}
$$

and thus it follows that

$$
\begin{aligned}
\mathrm{F}_{0,1,2}&=\Sigma a-\Sigma\alpha\\
\mathrm{F}_{0,1,3}&=\Sigma ab-\Sigma\alpha\beta\,,\\
\mathrm{F}_{0,1,4}&=\Sigma ab\,.\,\Sigma a-\Sigma\alpha\beta\,.\,\Sigma a\,.\\
\mathrm{F}_{0,2,3}&=\Sigma abc-\Sigma\alpha\beta\gamma\,,\\
\mathrm{F}_{0,2,4}&=\Sigma abc\,.\,\Sigma a-\Sigma\alpha\beta\gamma\,.\,\Sigma a\,,\\
\mathrm{F}_{0,3,4}&=\Sigma abc\,.\,\Sigma\alpha\beta-\Sigma\alpha\beta\gamma\,.\,\Sigma ab\,,\\
\mathrm{F}_{1,2,3}&=\Sigma abcd-\Sigma\alpha\beta\gamma\delta\,,\\
\mathrm{F}_{1,2,4}&=\Sigma abcd\,.\,\Sigma a-\Sigma\alpha\beta\gamma\delta\,.\,\Sigma a\,,\\
\mathrm{F}_{1,3,4}&=\Sigma abcd\,.\,\Sigma\alpha\beta-\Sigma\alpha\beta\gamma\delta\,.\,\Sigma ab\,,\\
\mathrm{F}_{2,3,4}&=\Sigma abcd\,.\,\Sigma\alpha\beta\gamma-\Sigma\alpha\beta\gamma\delta\,.\,\Sigma abc\,,
\end{aligned}
$$

all the expressions on the right being manifestly alternating functions with respect to the interchange

$$\begin{pmatrix} a & b & c & d\\ \alpha & \beta & \gamma & \delta \end{pmatrix}.$$

These expressions are seen to be the ten determinants of the matrix

$$\left\|\begin{matrix} 1 & \Sigma\alpha & \Sigma\alpha\beta & \Sigma\alpha\beta\gamma & \Sigma\alpha\beta\gamma\delta\\ 1 & \Sigma a & \Sigma ab & \Sigma abc & \Sigma abcd \end{matrix}\right\|;$$

and consequently, if we represent this matrix by

$$\left\|\begin{matrix} \sigma_0 & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4\\ s_0 & s_1 & s_2 & s_3 & s_4 \end{matrix}\right\|,$$

the results take the form

$$\begin{aligned} \mathrm{F}_{0,1,2} &= |\, s_1\sigma_0 \,| \,, \\ \mathrm{F}_{0,1,3} &= |\, s_2\sigma_0 \,| \,, \\ &\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ \mathrm{F}_{2,3,4} &= |\, s_4\sigma_3 \,| \,, \end{aligned}$$

where the suffixes on the right are got by subtracting from 4 each of those omitted on the left.

With the help of this notation, also, we can combine all the results in one statement, viz.:—

If m, n, r *arranged in order of magnitude be any three of the values* 0, 1, 2, 3, 4, *and* u, v *arranged in order of magnitude be the remaining two, then*

$$\begin{aligned} &\frac{|\, b^m c^n d^r \,|}{|\, b^0 c^1 d^2 \,|} \cdot \frac{(a-\alpha)(a-\beta)(a-\gamma)(a-\delta)}{(a-b)(a-c)(a-d)} + \frac{|\, a^m c^n d^r \,|}{|\, a^0 c^1 d^2 \,|} \cdot \frac{(b-\alpha)(b-\beta)(b-\gamma)(b-\delta)}{(b-a)(b-c)(b-d)} \\ &+ \frac{|\, a^m b^n d^r \,|}{|\, a^0 b^1 d^2 \,|} \cdot \frac{(c-\alpha)(c-\beta)(c-\gamma)(c-\delta)}{(c-a)(c-b)(c-d)} + \frac{|\, a^m b^n c^r \,|}{|\, a^0 b^1 c^2 \,|} \cdot \frac{(d-\alpha)(d-\beta)(d-\gamma)(d-\delta)}{(d-a)(d-b)(d-c)} \\ &= |\, s_{4-u}\sigma_{4-v} \,| \,, \end{aligned}$$

where s *and* σ *are explained by the examples* $s_2 = \Sigma ab$, $\sigma_3 = \Sigma \alpha\beta\gamma$.

The case where $v=4$ has been given by Sylvester, being the subject of his unsolved problem No. 2810 in *Mathematics from Educational Times,* vol. xlv. p. 129.

Of course the foregoing results are not at all confined to two sets of four variables (a, b, c, d), $(\alpha, \beta, \gamma, \delta)$. Two sets of w variables have not been taken merely on account of inconvenience in writing. The typical term for the next case (two sets of *five* variables) is

$$\frac{|\, b^m c^n d^r e^s \,|}{|\, b^0 c^1 d^2 e^3 \,|} \cdot \frac{(a-\alpha)(a-\beta)(a-\gamma)(a-\delta)(a-\epsilon)}{(a-b)(a-c)(a-d)(a-e)}$$

where (m, n, r, s) is a set of four values taken from 0, 1, 2, 3, 4, 5.

XV.—*Expansion of Functions in terms of Linear, Cylindric, Spherical, and Allied Functions.* By P. Alexander, M.A. Communicated by Dr T. Muir.

(Read 20th December 1886.)

The expansion of $\phi(x)$ in terms of $G_0(x)$, $G_1(x)$, $G_2(x)$, &c., connected by a given law, being of great importance in mathematico-physical investigation, every method of effecting this expansion must have some interest for scientists.

I therefore proceed to propose what I think to be a new method, in the hope that it may prove to be useful.

Many special expansions of this nature have been effected by Fourier, Legendre, and others.

After I had developed my method, my attention was called to two papers on this subject showing methods of development of great generality. The titles of the papers are—König, J., "Ueber die Darstellung von Functionen u. s. w.," *Mathematische Annalen*, v. pp. 310–340, 1871; and Sonine, N., "Recherches sur les fonctions cylindriques," &c., *Mathematische Annalen*, xvi. pp. 1–80, 1879. König, assuming that

$$\phi(x_0+x)=F_0(x_0)\,.\,G_0(x)+F_1(x_0)\,.\,G_1(x)+F_2(x_0)\,.\,G_2(x)+\&c.$$

where G_0, G_1, G_2, &c., are an infinite series of functions of x, connected by some given law, and also subject to the condition that when x is nearly equal to c, each of them is capable of expansion in ascending integral powers of $(x-c)$, beginning in the case of $G_\rho(x)$ with $(x-c)^\rho$, proceeds to show that the coefficients $F_0(x_0)$, $F_1(x_0)$, &c., are to be deduced from the following—

$$\begin{aligned}
G_0(c)\,.\,F_0(x_0) &= \phi(x_0+c)\,,\\
G_0(c)\,.\,F_0'(x_0) &= F_0(x_0)\,.\,G_0'(c)+F_1(x_0)G_1'(c)\,,\\
G_0(c)\,.\,F_0''(x_0) &= F_0(x_0)\,.\,G_0''(c)+F_1(x_0)\,.\,G_1''(c)+F_2(x_0)\,.\,G_2''(c)\,,\\
\&c., & \qquad\qquad \&c.
\end{aligned}$$

Sonine shows that

$$S_0(a+x)=A_0(a)\,.\,S_0(x)-2\{A_1(a)S_1(x)-A_2(a)S_2(x)+\&c.\}$$

if the series is convergent, where $S_0(x)$ and $A_0(a)$ may be any functions whatever of x and a consistent with convergency, and $A_0(a)$, $A_1(a)$, $A_2(a)$, &c., and $S_0(x)$, $S_1(x)$, $S_2(x)$, &c., are connected by the following relations :—

$$\left.\begin{aligned} A_1(a) &= -\frac{d}{da}[A_0(a)], \\ A_{n+1}+2\frac{dA_n}{da}-A_{n-1} &= 0 , \end{aligned}\right\}$$

and

$$\left.\begin{aligned} S_1 &= -\frac{dS_0}{dx}, \\ S_{n+1}+2\frac{dS_n}{dx}-S_{n-1} &= 0 ; \end{aligned}\right\}$$

and hence

$$A_n=(-i)^n \cos n\Delta_1 . A_0 ,$$

and

$$S_n=(-i)^n \cos n\Delta . S_0 ,$$

where

$$i=\sqrt{-1}$$

and Δ_1 and Δ are operations defined by

$$i \cos \Delta_1=\frac{d}{da} ,$$

and

$$i \cos \Delta =\frac{d}{dx} .$$

KÖNIG'S method seems to be much more general than SONINE'S, as KÖNIG'S functions G_0, G_1, G_2, &c., may be connected by any law, while SONINE'S functions A_0, A_1, A_2, &c., are connected by one law only. But on the other hand, KÖNIG'S functions are limited by the condition that $G_\rho(x)$ must, when x nearly equals c, be capable of expansion in ascending integral powers of $(x-c)$ beginning with $(x-c)^\rho$, whereas SONINE'S functions are subject to no such condition.

Both methods give the expansion of $\phi(x)$ in terms of $J_0(x)$, $J_1(x)$, $J_2(x)$, &c., BESSEL'S functions. But neither of them give the expansion

$$\phi(x)=A_0J_n(k_0x)+A_1J_n(k_1x)+A_2J_n(k_2x)+\&c.,$$

where k_0, k_1, k_2, &c., are the roots of some equation of condition.

The most general method of expansion I have seen is that of expansion in normal co-ordinates employed by RAYLEIGH throughout his *Theory of Sound*, which is so satisfactory that had I become acquainted with it somewhat earlier, I would probably not have sought after the following method :—

The general problem is to determine A_0, A_1, A_2, &c., so that when possible

$$\phi(x)=A_0G_0(x)+A_1G_1(x)+A_2G_2(x)+\&c., \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where G_0, G_1, G_2, &c., are connected by some given law.

The solution of this in all its generality has not yet been obtained, but in most of the particular cases which have been solved the method seems to be to operate on (1) with an operator O_n such that

$$O_n . G_m = 0 , \qquad (2)$$

except $m = n$.
And, therefore,

$$O_n . \phi(x) = A_n O_n . G_n(x)$$

$$\therefore \quad A_n = \frac{O_n . \phi(x)}{O_n . G_n(x)} \qquad (3)$$

Following this lead, I have found an operator of this nature in the case where G_0, G_1, G_2, &c., are elementary solutions of the equation,

$$(\delta + g)G = 0 , \qquad (4)$$

when g has the values g_0, g_1, g_2, &c., derived from the condition

$$\{\sigma . G\}_{x=a} = 0 \qquad (5)$$

where δ and σ are operations which may have the forms

$$\delta = X_0 + X_1\left(\frac{d}{dx}\right) + X_2\left(\frac{d}{dx}\right)^2 + \text{\&c.}, \qquad (6)$$

$$\sigma = P_0 + P_1\left(\frac{d}{dx}\right) + P_2\left(\frac{d}{dx}\right)^2 + \text{\&c.}, \qquad (7)$$

where X_0, X_1, X_2, &c., and P_0, P_1, P_2, &c., may be either constants or functions of x.

The operator I have discovered for the solution of this problem is—

$$O_n = \{\sigma(\delta + g_n)^{-1}\}_{x=a} \qquad (8)$$

The proof is as follows :—

$$\sigma(\delta + g_n)^{-1} G_m = \sigma\{[g_n^{-1} - g_n^{-2}\delta + g_n^{-3}\delta^2 - \text{\&c.}]G_m + G_n\} . \qquad (9)$$

But from (4),

$$\therefore \left.\begin{aligned} \delta . G_m &= -g_m . G_m \\ \delta^2 . G_m &= -g_m \delta G_m = g_m^2 . G_m \\ \delta^3 . G_m &= g_m^2 \delta G_m = -g_m^3 G_m \\ &\text{\&c.} \qquad \text{\&c.} \end{aligned}\right\} \qquad (10)$$

$$\begin{aligned}\therefore\quad \sigma(\delta+g_n)^{-1}\mathrm{G}_m &= \sigma\{[g_n{}^{-1}+g_n{}^{-2}g_m+g_n{}^{-3}g_m{}^2+\&\mathrm{c.}]\mathrm{G}_m+\mathrm{G}_n\} \\ &= (g_n-g_m)^{-1}\sigma\mathrm{G}_m+\sigma\mathrm{G}_n \\ &= \frac{\sigma\mathrm{G}_m}{g_n-g_m}+\sigma\mathrm{G}_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)^*\end{aligned}$$

But from (5),

$$[\sigma\mathrm{G}_n]_{x=a}=0=[\sigma\mathrm{G}_m]_{x=a}$$

Hence (11) gives—

$$\left.\begin{aligned}\{\sigma(\delta+g_n)^{-1}\mathrm{G}_m\}_{x=a} &= \frac{0}{g_n-g_m}+0 \\ &= 0 \text{ if } m \text{ is not}=n \\ &= \frac{0}{0} \text{ if } m=n\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (12)$$

Hence from (3),

$$\begin{aligned}\mathrm{A}_n &= \frac{\mathrm{O}_n.\,\phi(x)}{\mathrm{O}_n.\mathrm{G}_n} \\ &= \left\{\frac{\sigma(\delta+g_n)^{-1}\phi(x)}{\sigma(\delta+g_n)^{-1}\mathrm{G}_n}\right\}_{x=a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)\end{aligned}$$

By the method of vanishing fractions,

$$\begin{aligned}\left\{(\sigma+g_n)^{-1}\mathrm{G}_n\right\}_{x=a} &= \left\{\frac{\sigma\,.\,\mathrm{G}_m}{g_n-g_m}\right\}_{(m=n,\, x=a)} \\ &= \left\{\frac{\dfrac{d}{dg_m}(\sigma\,.\,\mathrm{G}_m)}{\dfrac{d}{dg_m}(g^n-g_m)}\right\}_{(m=n,\, x=} \\ &= \left\{\frac{d}{dg_n}(\sigma\,.\,\mathrm{G}_n)\right\}_{x=}\end{aligned}$$

Hence (13) becomes

$$\mathrm{A}_n=-\left\{\frac{\sigma(\delta+g_n)^{-1}\phi(x)}{\dfrac{d}{dg_n}(\sigma\,.\,\mathrm{G}_n)}\right\}_{x=a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

which is the required solution.

As new results, especially when very general, are liable to suspicion, I proceed to test this by a particular example whose solution can be otherwise found.

Let

$$\delta=\left(\frac{d}{dx}\right)^2+\frac{\lambda}{x}\left(\frac{d}{dx}\right)-\frac{p^2+(\lambda-1)p}{x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

---

[*] In the proof of (11) it has been assumed that g_m is less than g_n. The same may be proved for g_m greater than g_n by expanding $(\delta+g_n)^{-1}$ in the reverse order.

and

$$\sigma=h+\frac{d}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

An elementary solution of (4) is in this case

$$\left.\begin{array}{l} G_n=\left(x\sqrt{g_n}\right)^{-\frac{\lambda-1}{2}} J_{p+\frac{\lambda-1}{2}}\left(x\sqrt{g_n}\right) \\ \text{or} \qquad \left(x\sqrt{g_n}\right)^{\frac{\lambda-1}{2}} K_{p+\frac{\lambda-1}{2}}\left(x\sqrt{g_n}\right) \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (17)^*$$

where J and K are BESSEL's functions of the first and second orders.

Supposing then that it is possible to expand $\phi(x)$ in terms of the first of these and operating on (1) with the operator

$$O_n=\int_0^a dx\, x^\lambda G_n$$

we have

$$\int_0^a \phi(x)x^\lambda G_n dx=A_0\int_0^a G_0G_nx^\lambda dx+A_1\int_0^a G_1G_nx^\lambda dx+\&c. \quad . \quad . \qquad (18)$$

But

$$\int_0^a G^mG_nx^\lambda dx=a^\lambda.\frac{\left\{G_m\frac{dG_n}{dx}-G_n\frac{dG_m}{dx}\right\}_{x=a}}{g_m-g_n} \quad . \quad . \quad . \quad . \qquad (19)$$

$$=-a^\lambda.\frac{\left\{G_n\left(h+\frac{d}{dx}\right)G_m\right\}_{x=a}}{g_m-g_n} \text{ from (5) and (16)}$$

$$\left.\begin{array}{ll} =-a^\lambda\left\{\frac{G_n\sigma\,.\,G_m}{g_m-g_n}\right\}_{x=a} & \text{from (16)} \\ \left.\begin{array}{l} =0 \text{ if } m \text{ is not } =n \\ =\frac{0}{0} \text{ if } m=n \end{array}\right\} & \text{from (5)} \end{array}\right\} \quad . \quad . \qquad (20)$$

By the method of vanishing fractions,

$$\int_0^a G_n^2x^\lambda dx=-a^\lambda\left\{\frac{G_n\sigma\,.\,G_m}{g_m-g_n}\right\}_{(m=n,\,x=a)}$$

$$=-a^\lambda\left\{G_n\frac{d}{dg_n}(\sigma G_n)\right\}_{x=a} \quad . \quad . \quad . \quad . \qquad (21)$$

---

* Or $G_n=(x\sqrt{g_n})^{-\frac{\lambda-1}{2}}J_{p+\frac{\lambda-1}{2}}(x\sqrt{g_n})+B_n(x\sqrt{g_n})^{\frac{\lambda-1}{2}}K_{p+\frac{\lambda-1}{2}}(x\sqrt{g_n})$.

Again,

$$x^\lambda\left[G_n\delta\phi-\phi\delta G_n\right]=x^\lambda\left\{G_n\left[\left(\frac{d}{dx}\right)^2+\frac{\lambda}{x}\left(\frac{d}{dx}\right)-\frac{p^2+(\lambda-1)p}{x^2}\right]\phi\right.$$

$$\left.-\phi\left[\left(\frac{d}{dx}\right)^2+\frac{\lambda}{x}\left(\frac{d}{dx}\right)-\frac{p^2+(\lambda-1)p}{x^2}\right]G_n\right\},$$

$$=x^\lambda\left\{G_n\left[\left(\frac{d}{dx}\right)^2+\frac{\lambda}{x}\left(\frac{d}{dx}\right)\right]\phi-\phi\left[\left(\frac{d}{dx}\right)^2+\frac{\lambda}{x}\left(\frac{d}{dx}\right)\right]G_n\right\},$$

$$=G_n\frac{d}{dx}\left[x^\lambda\frac{d\phi}{dx}\right]-\phi\frac{d}{dx}\left[x^\lambda\frac{dG_n}{dx}\right],$$

$$=\frac{d}{dx}\left\{x^\lambda\left[G_n\frac{d\phi}{dx}-\phi\frac{dG_n}{dx}\right]\right\}.$$

$$\therefore \qquad -x^\lambda\phi\delta G_n=\frac{d}{dx}\left\{x^\lambda\left[G_n\frac{d\phi}{dx}-\phi\frac{dG_n}{dx}\right]\right\}-x^\lambda G_n\delta\phi\,;$$

$$\therefore \qquad \int_0^a \phi G_n x^\lambda dx=-\frac{1}{g_n}\int_0^a x^\lambda\phi\delta G_n dx,\quad \text{from (4)}$$

$$=\left[\frac{x^\lambda\left(G_n\frac{d\phi}{dx}-\phi\frac{dG_n}{dx}\right)}{g_n}\right]_{x=a}-\frac{1}{g_n}\int_0^a x^\lambda G_n\delta\phi dx$$

or

$$\int_0^a \phi.G_n.x^\lambda dx=\frac{a^\lambda\left\{G_n\left[\frac{d\phi}{dx}+h\phi\right]\right\}_{x=a}}{g_n}-\frac{1}{g_n}\int_0^a \delta\phi.G_n x^\lambda dx$$

$$=\frac{1}{g_n}\left\{a^\lambda\left[G_n.\sigma\phi\right]_{x=a}-\int_0^a \delta\phi.G_n.x^\lambda dx\right\}.$$

Similarly,

$$\int_0^a \delta\phi.G_n.x^\lambda dx=\frac{1}{g_n}\left\{a^\lambda\left[G_n\sigma\delta\phi\right]_{x=a}-\int_0^a \delta^2\phi.G_n.x^\lambda dx\right\},$$

and so on.

Hence

$$\int_0^a \phi.G_n.x^\lambda dx=a^\lambda\left\{G_n.\sigma\left[\frac{\phi}{g_n}-\frac{\delta\phi}{g_n^2}+\frac{\delta^2\phi}{g_n^3}-\&c.\right]\right\}_{x=a}$$

$$=a^\lambda\{G_n\sigma(g_n+\delta)^{-1}\phi\}_{x=a} \qquad . \quad . \quad . \quad . \quad . \qquad (22)$$

But

$$\mathrm{A}_n = \frac{\mathrm{O}_n.\phi}{\mathrm{O}_n.\mathrm{G}_n}$$

$$= \frac{\int_0^a \phi.\mathrm{G}_n.x^\lambda dx}{\int_0^a \mathrm{G}_n^2 x^\lambda dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (23)*$$

Hence from (21) and (22) this becomes

$$\mathrm{A}_n = \frac{a^\lambda[\mathrm{G}_n \,.\, \sigma(\delta+g_n)^{-1}\phi]_{x=a}}{-a^\lambda[\mathrm{G}_n\frac{d}{dg_n}(\sigma\mathrm{G}_n)]_{x=a}}$$

$$= -\left\{\frac{\sigma(\delta+g_n)^{-1}\phi}{\frac{d}{dg_n}(\sigma\mathrm{G}_n)}\right\}_{x=a} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

which verifies (14) for this case.

If $\lambda = 0$ and $p = 0$ (FOURIER'S *Heat*, ch. vii. and viii.; RAYLEIGH'S *Sound*, § 135), then (23) or (24) will give an expansion of $\phi(x)$ in linear functions (trigonometric),

$$\phi(x) = \mathrm{A}_0 . \sqrt{\frac{2}{\pi}} \cos(x\sqrt{g_0}) + \mathrm{A}_1\sqrt{\frac{2}{\pi}} \cos(x\sqrt{g_1}) + \&c.$$

where g_0, g_1, &c., are the roots of

$$a\sqrt{g} \,.\, \tan(a\sqrt{g}) = ah \,.$$

If $\lambda = 1$ (FOURIER'S *Heat*, ch. vi.; RAYLEIGH'S *Sound*, § 201),

$$\phi(x) = \mathrm{A}_0 \,.\, \mathrm{J}_p(x\sqrt{g_0}) + \mathrm{A}_1 \,.\, \mathrm{J}_p(x\sqrt{g_1}) + \&c.,$$

where g_0, g_1, &c., are the roots of

$$\sqrt{g}\mathrm{J}'_p(a\sqrt{g}) + h\mathrm{J}_p(a\sqrt{g}) = 0 \,,$$

which gives an expansion of $\phi(x)$ in cylindric or BESSEL'S functions.

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* Since writing this I have proved that if $\delta = \mathrm{X}_2\left(\frac{d}{dx}\right) + \mathrm{X}_1\frac{d}{dx} + \mathrm{X}_0$

$$\mathrm{A}_n = \frac{\int_0^a \phi \,.\, \mathrm{G}_n \,.\, \frac{e^{\int\frac{\mathrm{X}_1}{\mathrm{X}_2}dx}}{\mathrm{X}_2} dx}{\int_0^a \mathrm{G}_n^2 . \frac{e^{\int\frac{\mathrm{X}_1}{\mathrm{X}_2}dx}}{\mathrm{X}_2} dx} \,;$$

but I find that Sturm and Liouville have anticipated me (Liouville's *Journal de Mathématiques*, vol. i., 1836).

If $\lambda=2$ (FOURIER'S *Heat*, ch. v.; RAYLEIGH'S *Sound*, ch. xvii.),

$$G_n=(x\sqrt{g_n})^{-\frac{1}{2}}J_{p+\frac{1}{2}}(x\sqrt{g_n}),$$

$$\therefore \quad \phi(x)=A_0(x\sqrt{g_0})^{-\frac{1}{2}}J_{p+\frac{1}{2}}(x\sqrt{g_0})+A_1(x\sqrt{g_1})^{-\frac{1}{2}}J_{p+\frac{1}{2}}(x\sqrt{g_1})+\&c.,$$

where g_0, g_1, g_2, &c., are the roots of

$$2a\sqrt{g}J'_{p+\frac{1}{2}}(a\sqrt{g})+(2ah-1)J_{p+\frac{1}{2}}(a\sqrt{g})=0,$$

which gives an expansion of $\phi(x)$ in terms of spherical functions (Kugelfunctionen).

XVI.—*On Cases of Instability in Open Structures.* By E. Sang, LL.D.

(Read February 7, 1887.)

In the course of some remarks on the scheme proposed for the Forth Bridge, which remarks are published in the eleventh volume of the *Transactions of the Royal Scottish Society of Arts,* I was led to enunciate, among other theorems, one of a somewhat unexpected character, to the effect that any symmetric structure built on a rectangular basis, having no redundant parts, and depending on longitudinal strain alone, is necessarily unstable. This theorem was established by arguments restricted to the single matter under consideration; it is one of an extensive class, and I now propose to discuss the subject from a general abstract point of view.

The whole subject is evolved in the working out of two inverse geometrical problems and their corresponding mechanical applications. The relative positions of a number of points being prescribed, we may have to secure these by linear connections; or, the lengths of these connections being given, we may seek to discover the relative positions of the points. And we may have to compute the strengths needed to enable these connections to resist strains applied at the various points.

The relative position of *two* points is determined by the length of the straight line joining them, and the material connection can only serve as the medium for the equipoise of equal and opposite strains applied at its two ends; it can offer no resistance to stresses directed obliquely to it. The opposing pressures may be directed inwardly so as to cause compression, or outwardly so as to cause distension; the former is an example of unstable, the latter an example of stable equilibrium.

The instability in the case of compression is familiarly exemplified by an attempt to balance a load on the top of a walking-stick, or by the buckling of a long, thin rod; stability can be obtained only by the use of something aside of the straight line. In the case of distension we have to observe that no member of a structure acts upon a contiguous member except by compression; we do not pull an object toward us, we always push it; each link of a chain pushes the other link; the pulling is internal to the links themselves. Every case of stretching necessarily implies at each end compression changed first into transverse strain and then into distension. This phenomenon, which, from habit, we regard as simple, is indeed a most complex one, whose intimate nature as yet surpasses our understanding. Hence it is that, beyond the abstract arrangement of the parts as represented by straight lines, there is the problem, far more difficult, requiring much more constructive skill, of contriving the manner of the junctions.

In general, the relative positions of *three* points, as A, B, C, are determined by the lengths of the three lines AB, BC, CA, joining them two and two. A pressure applied at the point A can be resisted by the linear members AB, AC only when its direction is in the same plane with them, and they must be enabled to offer resistance by pressures applied at B and at C, which again, if they be not in the direction BA, CA, must cause a stress on BC. Hence we have here a system of six pressures in equilibrium all having their directions in one plane.

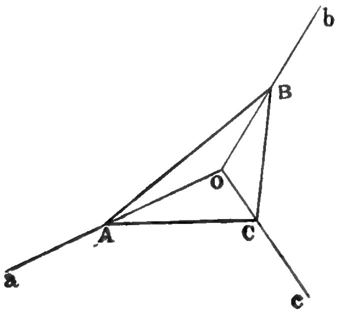

Fig. 1.

Let aA and bB be the directions of the pressures applied at A and at B, and let these be continued to meet in O; join also CO. According to the known law of equilibrium, the strains on AB and on AC are proportional to the sines of the angles OAC and BAO, which again are represented by the doubles of the expressions $\frac{AOC}{CA.AO}$ and $\frac{BOA}{OA.AB}$; wherefore, if we denote the strain on the member AB by the symbol $\overline{AB}$, we have the equality $\overline{AB}\cdot\frac{BOA}{AB}=\overline{CA}\cdot\frac{AOC}{CA}\cdot$ And on examining the equilibrium at B, we find also $\overline{AB}\cdot\frac{BOA}{AB}=\overline{CB}\cdot\frac{COB}{CB}$, so that the direction of the pressure applied at C must also pass through the same point O.

Again, on comparing the pressure applied at A with the strain on AB, we find

$$\overline{aA}:\overline{AB}::\frac{ABC}{CA.AB}\quad\frac{AOC}{CA.AO}$$

whence

$$\overline{aA}\cdot\frac{AOC}{AO}=\overline{AB}\cdot\frac{ABC}{AB},$$

and it follows that the six expressions

$$\overline{aA}\cdot\frac{BOA.AOC}{AO},\quad \overline{bB}\cdot\frac{COB.BOA}{BO},\quad \overline{cC}.\frac{AOC.COB}{CO},$$

$$\overline{AB}.\frac{ABC.BOA}{AB},\quad \overline{BC}\cdot\frac{ABC.COB}{BC},\quad \overline{CA}\cdot\frac{ABC.AOC}{CA},$$

are all of equal value.

On multiplying each of the first three by

$$\frac{AO.BO.CO}{BOA.AOC.COB}$$

we get the equalities

$$\overline{aA}\cdot\frac{\mathrm{BO.CO}}{\mathrm{COB}}=\overline{bB}\cdot\frac{\mathrm{CO.AO}}{\mathrm{AOC}}=\overline{cC}\cdot\frac{\mathrm{AO.BO}}{\mathrm{BOA}}$$

that is to say, the three external pressures applied at the points A, B, C balance each other just as if they had been applied directly to the point O.

When computing the internal strains caused by given external pressures, the area ABC occurs in every case as a division; if, then, the three points were in one straight line, that is, if the area ABC were zero, the internal strains would become infinitely great, unless the applied pressures were all in the same line with them. Here we have the first and very well known example of instability in construction.

If the point O be removed to a very great distance, the directions *a*A, *b*B, *c*C of the external pressures become parallel as in fig. 2. The intermediate pressure, in this figure *b*B, must be opposed to the direction of the others, its intensity being the sum of those at A and C.

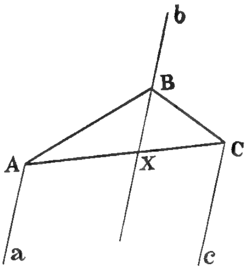

Fig. 2.

The relation of the strain on AC to the external pressure at B is then given by the formula

$$\overline{\mathrm{AC}}=\overline{b\mathrm{B}}\cdot\frac{\mathrm{AX.CX}}{\mathrm{AC.BX}};$$

so that if B were shifted along the line BX nearer to X, the strain on AC would be augmented in the inverse ratio of the new to the former BX; but the pressures $\overline{aA}$, $\overline{bB}$, $\overline{cC}$, would still remain proportional to the lines XC, CA, AX. Were B brought actually to X the strains would become infinite.

It is much to be regretted that, in lesson books on mechanics, the beginner is taught the properties of this impossible straight lever, without a hint of caution in regard to it. The strains on the arms, even that upon the fulcrum, are left out of view. In this way hazy notions are engendered; the load at A is said to balance that at C, although both be pressing in one direction.

The relative positions of four points, as A, B, C, D, fig. 3, are in general fixed by the lengths of the six lines AB, CD, AC, BD, AD, BC joining them two and two; these form the boundaries of a solid, called in Greek *tetrahedron*, which may get the English name *fournib*, shorter and quite as descriptive; the potters call it *crowfoot:* it is the simplest of flat-faced solids.

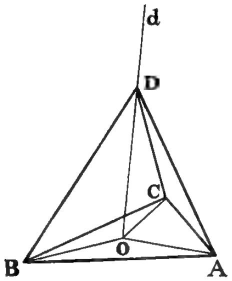

Fig. 3.

As in the triangle pressures applied at the corners can balance each other only when their directions meet in one point; so, reasoning by what is called *analogy*, we might infer that, of four pressures at the corners of a tetrahedron balancing each other, the directions must all tend to a single point. But this inference does not hold good; it may be that no two of these directions meet at all.

At each of the four points we have the equilibrium of four pressures, namely, the external pressure and the strains on the members meeting there. These strains can be computed when the direction and intensity of the applied pressure are known.

Thus let us continue the direction *d*D of a pressure applied at D until it meet the plane of ABC in some point O, and let AO, BO, CO be drawn. We have then the equalities

$$\frac{\overline{d\mathrm{D}}}{\mathrm{DO}\,.\,\mathrm{ABC}}=\frac{\overline{\mathrm{DA}}}{\mathrm{DA}\,.\,\mathrm{BOC}}=\frac{\overline{\mathrm{DB}}}{\mathrm{DB}\,.\,\mathrm{COA}}=\frac{\overline{\mathrm{DC}}}{\mathrm{DC}\,.\,\mathrm{AOB}}\,.$$

The points A, B, C and O remaining as they are, if D were brought nearer to O, the first of the above expressions would be augmented in inverse proportion to DO, and if D were brought actually to O, this term would become infinite, the strains $\overline{\mathrm{DA}}$, $\overline{\mathrm{DB}}$, $\overline{\mathrm{DC}}$ also infinite and the structure impossible.

When, in such an arrangement as fig. 3, the resistances at A, B, C are in a direction parallel to *d*D, their intensities are proportional to the opposite triangles, so that

$$\frac{\overline{d\mathrm{D}}}{\mathrm{ABC}}=\frac{\overline{a\mathrm{A}}}{\mathrm{BOC}}=\frac{\overline{b\mathrm{B}}}{\mathrm{COA}}=\frac{\overline{c\mathrm{C}}}{\mathrm{AOB}}\,,$$

and thus the distribution of the pressure among the ultimate resistances is independent of the distance DO.

In fig. 3 the point O is placed inside of the triangle ABC, and a pressure applied in the direction *d*DO causes compression in all the three members, DA, DB, DC. In fig. 4 O is placed outside of the line AC, and, with pressure in the direction *d*DO, the members DA, DC are compressed, while DB is distended.

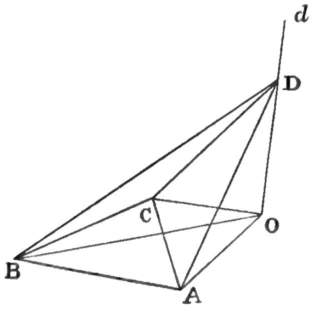

Fig. 4.

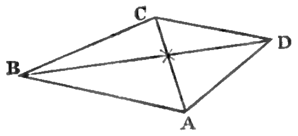

Fig. 5.

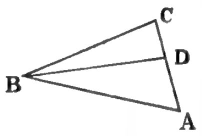

Fig. 6.

If here the point D were brought down to O, the structure would take the form of a plane tetragon ABCD, with its two diagonals AC and BD, as shown in fig. 5. Such a structure can offer no resistance to pressures inclined to its plane.

If the point D were on the straight line AC, as in fig. 6, it might seem that the five distances DA, AB, BC, CD, DB would suffice to secure the straightness of ABC; but on consideration we perceive that the two triangles ABD, CBD are merely hinged upon the common line DB.

In the cases of two, three, and four points, we have seen that the length of every line joining them in pairs is needed for fixing the relative positions; this rule does not hold for higher numbers. Thus, if a fifth point E be connected with three of the four corners of the tetrahedron ABCD, its relative position is determined, provided always that E be not in the plane of the three points with which it is joined; so that nine lines suffice for five points. The line joining E with the fourth point of the tetrahedron would be redundant.

In all such structures, three of the points, as A, B, C in fig. 7, must each have four concurring lines, and the remaining two, D and E, only three; and if no four of the five points be in one straight line, the system is self-rigid.

This rigidity will subsist although the two triple points D, E be in the same plane with any one pair of the quadruple ones, as in fig. 8, which is intended to show D, B, E, C as in one plane. The scheme then takes the appearance of a pyramid, having A for its apex, and the quadrangle DBEC for its base. Thus the flatness of a tetragon may be secured by connecting each of its corners with a fifth point not in the same plane.

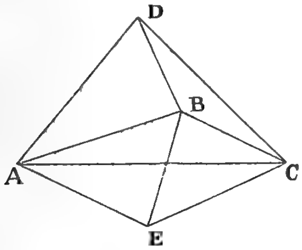

Fig. 7.

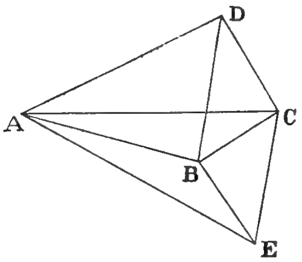

Fig. 8.

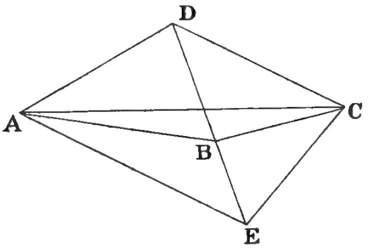

Fig. 9.

Moreover, the system still remains rigid although the points D and E be both in one plane with AB also. In this case DBE, the meeting of two planes, must be a straight line, as shown in fig. 9. Thus we see that, for the establishment of three points in a straight line, two auxiliary points must be introduced, with seven additional linear members.

We have now got a possible straight lever DBE. In order to examine the law of the balancing of pressures applied at B, D, E, we must trace out the strains on the various members, and their equilibriums at the five junctions, subject to the condition that there be no external pressure at A or at C. The result of this examination is, that the strains on the members are eliminated; that the directions of the applied pressures must all pass through one point;

and that their intensities must be proportional to the sines of the opposite angles,—this result being independent of the positions of the auxiliary points A and C.

Does it thence follow that we may omit those points altogether? Assuredly not; for our whole investigation proceeded on the ground that each member transmits from its one end to its other end the strain with which it is accredited.

Each additional point needs for its establishment three new linear members, so that in any self-rigid open structure, if n be the number of the points, there must be $3n-6$ linear connections; this formula failing only in the extreme case, $n=2$.

Hitherto we have been considering the self-rigidity of structures, and may now proceed to treat of the laws of stability in relation to the ground, taking first the case of a self-rigid structure to be kept firmly in position.

In every case the support must be derived from points in the ground, which points necessarily form by themselves a rigid structure, so that our problem assumes the general form of "how to connect one rigid structure with another." If f be the number of the points in the foundation, and n that of those in the supported structure, we have in all $f+n$ points in the compound, which must clearly be self-rigid. Hence the total number of linear members must essentially be $3f+3n-6$. But of these $3f-6$ are virtually included in the foundation, wherefore the number of the members above ground must in all possible cases be $3n$. Of these, however, $3n-6$ are already included in the supported structure, and thus we arrive at the important general law, "that the number of linear supports must be neither more nor less than six when the supported structure is self-rigid." This most elementary of the laws of support seems almost to be unknown, the enunciation of it takes even professional engineers by surprise.

All our portable direction-markers, our theodolites, alt-azimuths, levelling telescopes, have to be supported above the ground at a height convenient for the eye. It is essential that the stationary part of each be firmly held; yet in every case, with not one exception in the thousand, our geodetical instruments are set upon three slender legs, diverging almost from a point. In such an arrangement the steadiness in direction is derived exclusively from the stiffness of the legs, which, however, are very flexible. The well-known result is, that any strain in handling the instrument, even the pressure of a slight breeze, deranges the reading.

More than fifty years ago an instrument maker in London, Robinson by name, placed his beautifully made little alt-azimuths on a new kind of stand.

He connected three points in the stationary part of the instrument with three points in the ground by means of six straight rods inclined to each other. In this arrangement, the stability in every direction is derived from the resistance to longitudinal compression, the flexure of the rods having an infinitesimally small influence. It takes essentially the form of the octahedron or sixnib, as in fig. 10; a self-rigid structure having six connected points.

This beautiful Robinson stand keeps the alt-azimuth so firmly in position that, even during a heavy gale, the image of the moon may be seen to move without tremour across the cobwebs of the field-bar. Yet it has not been adopted by engineers and surveyors; the exigencies of the photographer, however, have determined his recourse to it.

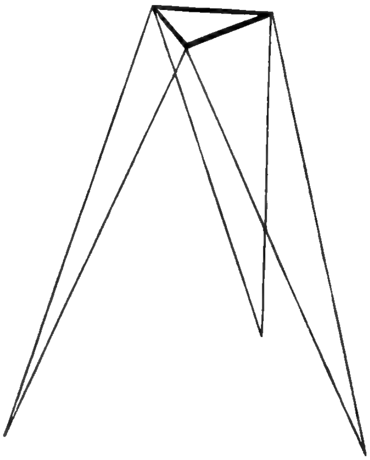
Fig. 10.

It is not essential that the supports meet two and two as in this arrangement; they may be quite detached, connecting six points in the supported structure with six points in the ground; but in all cases they must be so disposed that any dislocation whatever would imply a change in the length of some of them.

It might have sufficed merely to remark that this condition excludes the parallelism of the supports; but it is expedient to insist, seeing that, in the deplorable case of the Tay Bridge, the fabric was set upon two rows of upright columns. The opinion is still held that the effective base is equal to the whole breadth of such a structure, whereas the most casual examination may show that, no matter how broad the structure may be, its effective base is only that of a single column.

When the superstructure is not rigid in itself, or indeed whether it be so or not, the entire number of the members above the ground must be thrice that of the supported points. If we attempt to do with fewer the fabric must fall; if we place more we cause unnecessary internal strains. I hope in a subsequent paper to treat of redundancy, meantime our attention may be confined to structures having the proper number of parts.

If the supported points belong to one system they must be mutually connected, and, at the least, there must be as many of these connections as there are points, less one, wherefore the number of supports can never exceed twice the number of the points by more than one.

Out of the endless variety of cases we may select one class for examination, that in which the supported points are connected so as to form a polygon, not necessarily all in one plane. The number of the connections being already *n*, that of the supporting members must be $2n$, which may rest on $2n$ separate points in the ground, but which may be brought together in pairs or otherwise. If they be placed in pairs there are as many supporting as supported points.

Robinson's octahedral stand shows this arrangement when there are three supported points; we shall now take the case when four points are supported from four points in the ground, as in fig. 11, where the connected points A, B, C, D are shown as supported by the eight members AE, EB, BF, FC, CG, GD, DH, HA.

In general, that is when there is no regularity, such a structure contains all the elements of stability. The positions of the foundation points being known, if the lengths of the twelve members be prescribed, we shall have twelve equations of condition whereby to compute the twelve co-ordinates of the four points A, B, C, D. Or, viewing the matter from the mechanical side, external pressures applied at A, B, C, D may be resolved into their elements in three assumed directions, x, y, z, and so may the stresses on the various parts; there must be equilibrium at each of the points, in each of the three directions, and so again we have twelve equations whereby to compute twelve unknown quantities.

The algebraist at once perceives that the resulting divisor (or determinant as it is called) may happen to be zero, in which case the stress becomes infinite; that the dividend may be zero, showing that the particular member has no strain upon it; or even that both the dividend and the divisor may be zero at once, showing the structure to be indeterminate. But such investigations distract the attention from the objects under consideration to their mere representative symbols, and do not carry intellectual conviction along with them. Their true and highly important office is to determine accurately the various stresses, thereby enabling the constructor to apportion the strengths of the various parts.

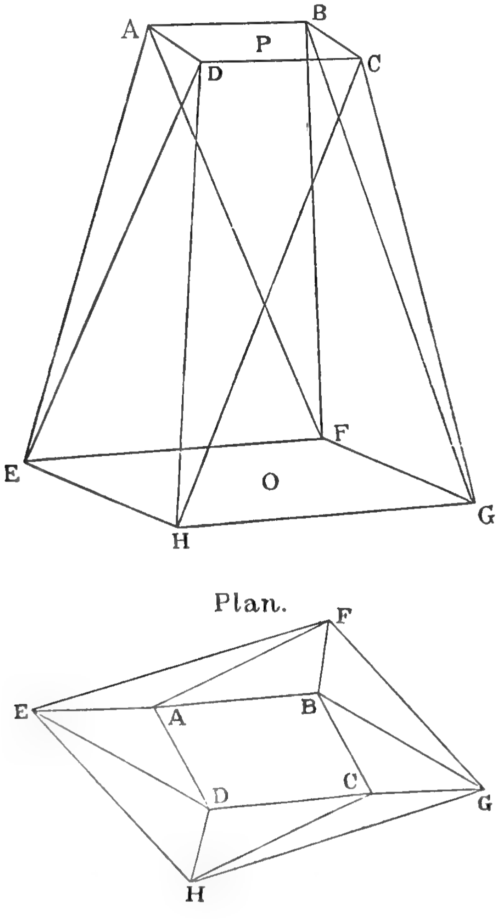

Fig. 11.

The determinateness, that is the stability, of a structure typified by fig. 11 ceases when we introduce symmetry or even semi-regularity. Let, for example, the figures EFGH, ABCD be rhomboids, having their middle points O and P in the same vertical line, in which case the opposite members are of equal lengths, AE to CG; EB to GD, and so on. Such a structure is clearly instable.

Since the lengths HA, AE are fixed, the point A must be on the circumference of a circle, having HE for its axis of rotation; and similarly for the points B, C, D. If now we suppose the point A to be pushed inwards, the members AB, AD will push the points D and B outwards, and, consequently, C will move inwards by exactly as much as A; the structure will adapt itself perfectly to its new position. In truth, we have here not twelve, we have only eleven data; for, if one of the connections, say CD, were removed, and the structure thus made obviously mobile, the distance CD would yet remain always equal to AB; that distance cannot be reckoned among the data.

So much for the geometrical mobility; let us examine the strains. Any horizontal pressure at A is decomposable into two,—one in the direction AB, the other in the direction AD. The stress $\overline{AB}$ transferred to the point B may again be decomposed into two; the one of these in the plane EBF parallel to EF is completely resisted at E and F by the stresses $\overline{EB}$, $\overline{EF}$, but the other, perpendicular to EF, meets with no resisting obstacle; it and the corresponding pressure at D may be counteracted by extraneous pressure there, or by a single pressure applied at C, equal to the pressure at A, and in the same direction with it. Thus the distortion of the fabric by an eastward pressure at A is prevented by a like pressure applied at C, not westward, but eastward also; in respect, however, to the strains on EB, BF, HD, DG, the effects of these counteracting pressures are cumulative.

These considerations would seem to warrant the conclusion that all structures of this class are necessarily unstable; however, before venturing to accept of this conclusion, it may be prudent for us to inquire whether the arguments on which it is founded be strong enough to bear such a weighty superstructure. Now the chief argument was that the longitudinal stress on AB, acting at B, tends to turn the triangle EBF on EF as an axis; but this tendency exists only so long as AB is out of the plane EBF, and ceases whenever AB comes to be in that plane; in other words, whenever AB is parallel to EF. Hence it follows that structures, represented in plan by fig. 12, having the two rhomboids ABCD and EFGH placed conformably, are rigid. The conclusion was not absolutely general.

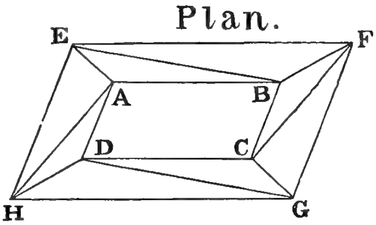

Fig. 12.

Though every rhomboid be not a rectangle, every rectangle is a rhomboid, and we might hastily thence conclude that these remarks concerning rhomboidal structures may be at once extended to rectangular ones. But we have just come from seeing an example in which peculiarity precludes generalisation, and thus it is expedient for us to examine specifically the case of rectangular structures.

The examination at once shows that the remarks made in regard to figs. 11 and 12 apply when the rhomboids pass into the form of rectangles; but the rectangle is symmetric, while the rhomboid is not so; the arrangement of the diagonals, as shown in these figures, is unsymmetric, and it thus remains for us to inquire into the laws of symmetry.

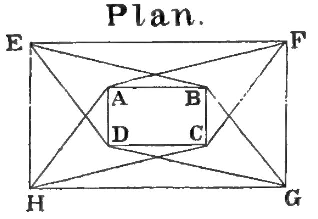

Fig. 13.

If, as shown in fig. 13, the rectangle ABCD be placed vertically over and conformably with EFGH, the arrangement is symmetric; it has already four out of the requisite twelve members, and the eight supports remain to be placed.

These may be inserted symmetrically as the eight diagonals, HA, AF, FC, CH; DE, EB, BG, GD, but then the structure becomes perfectly mobile; the points A and C move towards or from the central axis, while B and D move from or towards it.

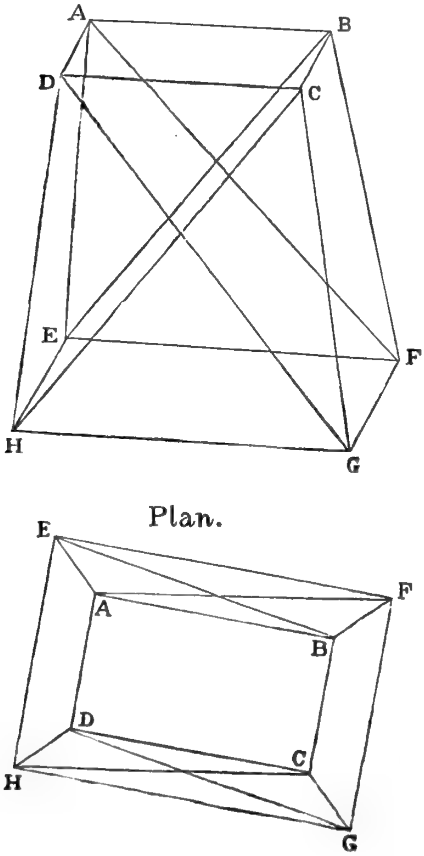

Fig. 14.

In any other symmetric arrangement the four corner parts, EA, FB, GC, HD, must appear, leaving four members yet to be distributed. If one of these be placed as the diagonal AF, symmetry requires also DG, BE, and CH, as shown in fig. 14.

The insecurity of this arrangement is obvious at a glance; no more need to have been said about it, but for its adoption in the scheme for the central towers of the proposed Forth Bridge. It presents two instances of that most vicious arrangement, the flattened tetrahedron; vicious because, while incapable of resisting any pressure not directed in its own plane, such a structure as EABF converts any twisting pressure into indefinitely exaggerated stress. It also presents two attempts to determine the shape of a quadrangle by the lengths of the four sides. Were the three open figures, EFBA, ABCD, DCGH, replaced by flat rigid plates, we should have, turning on the four parallel hinges, EF, AB, DC, HG, a most familiar example of instability.

Among open structures built on a rectangular base, instability is not confined to those with rhomboidal tops; for if, as in fig. 15, the triangle HAE be set up equal to GCF and EBF to HDC, so that AC may be parallel to EF and BD parallel to FG, the structure is movable. The diagonals AC and BD may be on one level and so cross each other, or the one may pass above the other at a distance on the plumb line OP. Since the three lines, AO, OP, PB, are mutually perpendicular—

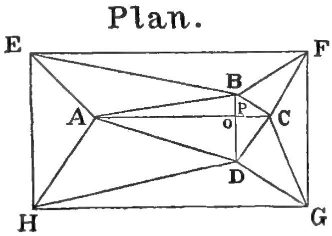

Fig. 15

$$AB^2=AO^2+OP^2+PB^2$$

and

$$CD^2=CO^2+OP^2+PD^2; \qquad \text{wherefore}$$

$$AB^2+CD^2=AO^2+OC^2+BP^2+PD^2+2.OP^2.$$

But the sum of BC^2 and AD^2 is equal exactly to the same quantity, and consequently

$$AB^2+CD^2=BC^2+DA^2,$$

so that one of these four is deducible from the remaining three; there are then only eleven data in this structure, instead of the twelve needed for rigidity. But it is to be observed that a dislocation must change the horizontality of AC and BD, so that the mutability may be only instantaneous, as in the case of maximum or minimum.

Passing now to the case of five supported points, we may remark that, by the introduction of a fifth point, a symmetric rigid structure may be built on a rectangular base.

Thus, if we place, as in fig. 16, the rhombus ABCD vertically over the rectangle EFGH, and complete the construction as in fig. 11, there results a symmetric structure, which, like all those of the same class, is changeable. On assuming, however, a point Z in the vertical axis of the system, and connecting it with each of the points A, B, C, D, we get a fabric both symmetric and rigid. The rigidity is confirmed thus:—If, supposing Z and its connections to be away, the points A and C be brought nearer, B and D would move apart; now, in virtue of the connections AZ, ZC, the shortening of AB would cause Z to rise, while, in virtue of the connections, BZ, ZD, the widening of BD would bring Z down; the opposition of these two tendencies keeps Z in its place.

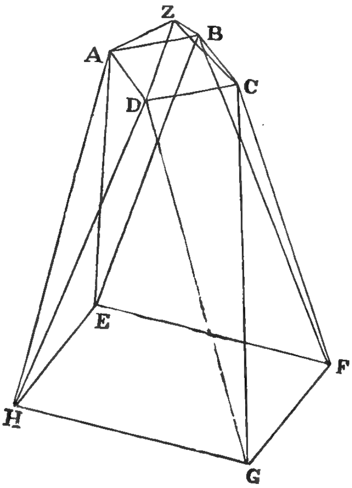

Fig. 16.

Here, in order to support five points, sixteen members are conjoined, and yet there does not seem to be any redundancy; moreover, the arrangement is quite symmetric. The equilibrium at each point gives rise to three equations of condition, and these fifteen equations cannot possibly serve to determine sixteen strains. But if we apply a pressure at any one of the five points, the fabric resists it, the various members are strained somehow, the law of equations notwithstanding. The explanation of this paradox may afford an instructive exercise to the student.

When the five points are arranged in the corners of a pentagon, each being carried by two supports, as shown in plan by fig. 17, the structure is rigid, provided the polygons be convex. Of this we easily convince ourselves by supposing one of the connections, say EA, to be removed, and by examining the motion of the link system thus left. The point A can move only in a circle, having KF for its axis; let A be moved inwards, the member AB will then cause the triangle FBG to turn outwards on FG as a hinge; BC will draw C inwards, CD will push D outwards, and lastly, DE will draw E inwards; wherefore the distance AE will be shortened, and the member AE can be replaced only when the structure is brought back to its former position.

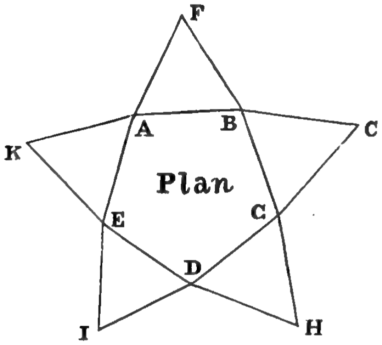

Fig. 17.

Following this line of argument one step further, we see that in the case of a hexagon the first and last points would move, the one outwards, the other inwards, and that so the distance might remain unchanged. When the hexagons are semi-regular or halvable, the distance remains absolutely unchanged, and the structure is indifferent as to position. This same remark applies to all polygons of an even number of sides.

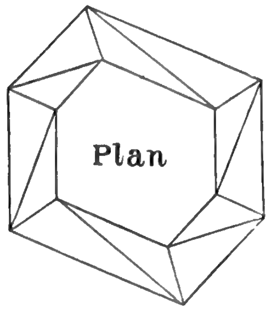

Fig. 18.

If, however, the upper and lower polygons be placed conformably, as in fig. 18, the structure is rigid, whether the number of supported points be even or odd.

These truths may be illustrated experimentally by preparing a few isosceles triangles as AFB, having perforations at A and B, through which an elastic string may be passed. On connecting a number of these, say seven, by a continuous thread, and spreading them out on a table, we form a flexible equal-sided heptagon, and when this is arranged regularly the points F are in the corners of a larger regular heptagon. If we now trace on a flat board a regular seven-sided figure, intermediate in size between these two, and secure the points F of the triangles in holes made at the corners, we shall have erected a structure analogous to that shown in fig. 17, and shall find it to be rigid.

If one of the triangles be removed, and the same process of construction followed with the remaining six, the resulting regular hexagonal structure is found to be instable.

Another reduction of the number brings us to the pentagonal structure, which again is stable; and still another removal gives the tetragonal instable fabric; and, lastly, when only three triangles are left, we have Robinson's octahedral stand.

The important distinction between the two cases of conformable or of unconformable polygons may be illustrated by preparing two pairs of triangles, one pair as EAB, GCD, of fig. 12, the other pair as FBC, HDA, and by connecting the sides, AB, BC, CD, DA, so as to form a flexible tetragon.

When the feet, E, F, G, H, are secured in the corners of a rhomboid or of a rectangle, the structure is rigid, if AB, BC be parallel to EF, FG; in all other cases it is instable.

These cases of instability in open structures have been elicited by means of the simplest considerations in Geometry and Statics; they lie indeed on the very surface of mechanical inquiry. They do not occur as isolated examples—they are arranged in extensive groups; and, being found in those classes of structures which may be called shapely, they stand out as warning beacons to those engaged in engineering pursuits.

XVII.—*On the Fossil Flora of the Radstock Series of the Somerset and Bristol Coal Field* (*Upper Coal Measures*). Part I. By Robert Kidston, F.R.S.E., F.G.S. (Plates XVIII.-XXVIII.)

(Read April 4, 1887.)

My attention for the last few years having been specially directed to the vertical distribution of the Carboniferous Fossil Flora, it is my intention to publish a series of papers dealing with this subject.

While carrying on these investigations it has been necessary, in addition to visiting public and private collections, to visit several of the coal fields for the purpose of collecting specimens, as in almost no case have the smaller and less attractive species been secured, and, as a rule, only what strikes the ordinary collector as being "a fine specimen" is preserved, to the exclusion of many less striking but often more valuable examples. Hence our public collections, and, with few exceptions, also our private collections, give a very imperfect idea of the richness of the flora of the Carboniferous Formation of Britain.

For the last four years I have annually paid a visit to the Radstock portion of the Somerset and Bristol Coal Field, with the object of collecting and examining the fossil flora of this area, from which were obtained several of the species described by Brongniart, and which is probably richer in fossil plants than any other coal field in Britain,—not only in the number of species it contains, but also in their excellent state of preservation.

A most important point in an investigation of this nature is to have the position of the beds from which the specimens have been derived accurately determined. In regard to the Radstock series of the Somerset and Bristol Coal Field, this qualification is amply fulfilled, making this area peculiarly suitable as a starting point for investigations in the vertical distribution of the British Carboniferous flora.

The geology of the Somerset and Bristol Coal Field, and especially the geology of the Radstock portion, has been fully worked out by several geologists, but it may not be out of place to introduce here a short geological sketch of this district, especially referring to the Radstock Series, from which all the specimens mentioned in the following list have been collected.

The Somerset and Bristol Coal Field extends from Cromhall in Gloucestershire to Frome in Somerset, and from Bath in the east to Nailsea and Clevedon on the west.

Its extreme length from north to south is 26 miles, and its width, from east to west (if the outlying Nailsea basin be included), is 24 miles. If the Nailsea basin be excluded, its width from Bath in the east to Bristol in the west is reduced to 12 miles.

The Coal Measures are mostly covered by Secondary rocks, Jurassic and Triassic, which are unconformable to the underlying Palæozoic strata.

The Carboniferous Formation lies in a trough surrounded at intervals on the north, west, and south by the Old Red Sandstone.

The general geological structure of this coal field will be most easily understood by referring to Plate XVIII., which gives a reduced sketch of a section prepared by Mr J. M'Murtrie, F.G.S., for the Royal Coal Commission in 1868.

This section shows the Secondary Formations lying unconformably on the upturned edges of the Palæozoic rocks. The centre of the basin is occupied by the Upper Division of the Coal Measures (Nos. in section 1, 2, 3). This rests on the Pennant Rock (No. 4), immediately beneath which is the Lower Division of the Coal Measures (5 and 6). Succeeding this is the Millstone Grit (7) and Carboniferous Limestone (8), which latter rests on the Old Red Sandstone (9).

It is necessary, however, to study in somewhat fuller detail the various *horizons* of the Coal Measures,—that is, all the Carboniferous rocks above the Millstone Grit. These, as already mentioned, resolve themselves into three great divisions, the *Upper* and *Lower Divisions* of the Upper Coal Measures, and the *Pennant Rock.*

The Upper Division of the Coal Measures, attaining a thickness of about 2200 feet, is separated into the *Upper* or *Radstock Series* (1), and the *Lower* or *Farrington Series* (3), between which are interposed a characteristic series of Red Shales (2).

The *Upper Division* (including the *Radstock* and *Farrington Series*) is separated from the *Lower Division* by the *Pennant Rock* (4), which attains a probable thickness of from 2500 to 3000 feet.

The *Lower Division*, of a thickness of about 2800 feet, is also divisible into two series, the upper of which is named the *New Rock Series,* and the lower the *Vobster Series.*

These two series are not so clearly separable from each other as those of the Upper Division are by the intervention of the Red Shales, being separated rather on account of the character of the veins and the nature of the strata than by the occurrence between them of any unproductive characteristic stratum of rock.

This coal field is traversed by many *faults,* some of which, especially the

Radstock great overlap fault, are well worthy of detailed study, but they do not fall in with the scope and object of these remarks.

Having now taken a general survey of the ground, let us return again to the *Upper Division* of the Coal Measures, which embraces the *Radstock* and *Farrington Series.*

The *Radstock* and *Farrington Series* occupy a basin, extending from Brislington in the north to Kilmersdon in the south, and form an oval tract whose length from north to south is about 12 miles, and whose width from east to west is about 5 miles.

There is another and smaller basin referable to the Radstock and Farrington Series, more particularly to the latter, which extends northwards from Pucklechurch for about 4 miles, with a width of about 2 miles.

From this portion I have not collected, nor have I seen many specimens from it, but this is most probably due to deficient collecting, and not to the absence of specimens.

I may mention here that the *Radstock* and *Farrington Series,* when viewed in their relation to the other coal fields of Great Britain, belong to their uppermost portion, and are the true *Upper Coal Measures,* altogether independently of their *local* position.

The coal of the *Upper* or *Radstock Series* is chiefly worked in the neighbourhood of Radstock, Writhlington, Midsomer-Norton, Camerton, Timsbury, and Paulton, and it is from the pits in the neighbourhood of these villages that most of the fossils referred to in this paper have been derived.

The *Radstock* or *Upper Series* of the *Upper Division* contains eight veins, viz.:—

The Withy Mills Seam,	1 ft. 4 in.
Great Vein,	2 „ 2 „
Top Little Vein,	1 „ 4 „
Middle Vein,	2 „ 2 „
Slyving Vein,	2 „ 4 „
Under Little Vein,	1 „ 2 „
Bull Vein,	2 „ 2 „
Nine-inch Vein,	1 „
Total,	13 ft. 8 in.

The total thickness of these veins is considerable, but in no case is the whole of it available at any one place.

In the majority of cases I have found it impossible to note the vein from which the various fossils came, but the whole of the *Radstock Series* are so intimately connected that the knowledge of the actual veins from which the fossils originate appears to be of little importance in the present instance.

The pits at which I have collected most are:—Braysdown Colliery, near

Radstock; this is one of the few collieries from which are worked the coals of both the *Radstock* and *Farrington Series.* Tyning and Ludlows Pits, Radstock, which are here treated as one, as they are connected and the débris of both is brought to the same rubbish tip near the Tyning Pit; in the localities given for the species, Tyning and Ludlows Pits are recorded as "Radstock." Middle Pit and Wellsway Pit, Radstock; Kilmersdon Pit and Lower Writhlington Pit, near Radstock; the Camerton Pits; and the Upper and Lower Conygre Pits, Timsbury.

The veins belonging to the *Radstock Series* worked at these collieries are:—

Braysdown Pit,	Great Vein, Top Little Vein, Middle Vein, Slyving Vein, Under Little Vein, and Bull Vein.
Ludlows and Tyning Pits,	Do.
Middle Pit,	Do.
Wellsway Pit,	Do.
Kilmersdon Pit,	Do.
Lower Writhlington Pit,	Do.
Camerton Pits,	Great Vein, Top Little Vein, Middle Vein, Slyving Vein, and Under Little Vein.
Upper Conygre Pit,	Do.
Lower Conygre Pit,	Do.

In addition to my own collecting, I have examined the specimens from the Radstock coal field in the Bath and Bristol Museums, and am indebted to the Rev. H. H. Winwood, F.G.S., for the use of specimens contained in the collection of the former, and to the Council of the Bristol Museum for a similar privilege; and I am further under obligation to Mr E. Wilson, Curator of the Bristol Museum, for giving every facility for examining the specimens under his charge.

I have also examined specimens from this coal field in the British Museum; University Museum, Oxford; Museum of the Geological Society of London; as well as some specimens from the same district in the Collection of the Geological Survey of England, in their Museum, Jermyn Street, London.

I am, however, principally indebted to Mr J. M'Murtrie, F.G.S., Radstock, for kindly placing at my disposal, for the purpose of examination and description, his fine collection of fossil plants from the Radstock Series. I am also further indebted to him for much information as to the geology of the neighbourhood, and from his various papers on the Geology of the Somerset and Bristol Coal Field the short geological sketch just given has been compiled.*

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* Those wishing fuller information on the geology of the Somerset and Bristol Coal Field will find it contained in the following papers and works:—Rev. Prof. W. Buckland and Rev. W. D. Cony-

SYNOPSIS OF SPECIES.

Fungi.

Excipulites, Göppert, 1836, *Die fossilen Farrnkrauter*, p. 262.

Excipulites callipteridis, Schimper, sp.

Excipula callipteridis, Schimper, *Traité d. paléont. végét.*, vol. i. p. 142, and Explanation to pl. xxxii. figs. 6, 7.
Excipula callipteridis, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 19.

Remarks.—On *Sphenopteris neuropteroides.* The minute fossils included here appear to be similar to those described by Schimper and Weiss as occurring on *Callipteris conferta,* Brongt. The central opening is not very well seen in our examples, which are situated on the limb of the pinnules between the veins.

It is interesting to find that both Lesquereux (*Coal Flora of Pennsyl.*, vol. i. p. 207, pl. xxxviii. fig. 2; *P. anceps* = *Sphen. neuropteroides*) and Zeiller (*Bull. soc. géol. de France*, 3e sér., vol. xii. p. 192) have noted the occurrence of these small fossils on the pinnules of *Sphen. neuropteroides.*

There seems little reason to doubt that *Excipulites* comprises a group of minute parasitic fungi.

Locality:—Radstock.

BEARE, "Observations on the South-Western Coal District of England," *Trans. Geol. Soc.*, ser. 2, vol. i. p. 210; G. C. Greenwell, "Notes on the Coal Field of East Somerset," *Trans. N. of Eng. Inst. of Mining Eng.*, vol. ii. p. 258; G. C. Greenwell, "On the Southern Portion of the Somerset Coal Fields," *Trans. S. Wales Inst. of Eng.*, vol. i. p. 147; H. Cossham, "On the Northern End of the Bristol Coal Field," *Trans. N. of Eng. Inst. of Mining Eng.*, vol. x. p. 97; G. C. Greenwell, "On the Somersetshire Sections of the Bristol Coal Field," *Trans. N. of Eng. Inst. Mining Eng.*, vol. x. p. 104; G. C. Greenwell and J. M'Murtrie, *The Radstock Portion of the Somerset Coal Field*, 1864, 8vo, Newcastle-on-Tyne; R. Etheridge, "On the Physical Structure of the Northern Part of the Bristol Coal Basin," *Proc. Cottsw. Nat. Field Club*, vol. iv. p. 28; C. Moore, "On Abnormal Conditions of Secondary Deposits where connected with the Somersetshire and South Wales Coal Basin, &c.," *Quart. Journ. Geol. Soc.*, vol. xxxiii. p. 449; J. M'Murtrie, "On the Carboniferous Strata of Somersetshire," *Proc. Bath Nat. Hist. and Antiq. Field Club*, vol. i. No. 2, p. 45; J. M'Murtrie, "The Faults and Contortions of the Somersetshire Coal Field," *ibid.*, vol. i. No. 3, p. 127; *Report of the Commissioners appointed to inquire into the several matters relating to Coal in the United Kingdom*, vol. i., 1871; John Anstie, *The Coal Fields of Gloucestershire and Somersetshire and their Resources*, 8vo, London, 1873; J. M'Murtrie, "The Geographical Position of the Carboniferous Formation in Somersetshire, &c.," *Proc. Bath Nat. Hist. and Antiq. Field Club*, vol. ii. p. 454; J. M'Murtrie, "Notes on the Physical Geology of the Carboniferous Strata of Somersetshire and associated Formations," *Somerset Arch. and Nat. Hist. Soc.*, 1875; J. M'Murtrie, "The Somersetshire Coal Fields, and the Method of working thin Seams in the Radstock District," *Proc. S. Wales Inst. of Eng.*, vol. xii. No. 5, p. 424; H. B. Woodward, *Memoirs of the Geological Survey, Geology of East Somerset and the Bristol Coal Fields*, 1876.

EQUISETACEÆ.

Calamites, Suckow, 1784, *Act. Acad. Theod. Palat.*, vol. v. p. 359.

Group I. **Calamitina** (emend.), Weiss, 1884, *Steinkohlen-Calamarien*, part ii. p. 59.*

Calamitina (Calamites) varians, var. **insignis,** Weiss.

Cal. (*Calamites*) *varians*, var. *insignis*, Weiss, *Steinkohlen-Calamarien*, part ii. p. 63, pl. i.; pl. xxviii. fig. 1.

Calamites varians, Germar, *Vers. v. Wettin u. Löbejun*, p. 49, pl. xx. figs. 1–3.

Remarks.—Rare.

Locality :—Camerton.

Group II.—**Eucalamites,** Weiss, 1884, *Steinkohlen-Calamarien*, part ii. p. 96.

Eucalamites (Calamites) (cruciatus) senarius, Weiss.

Calamites (*cruciatus*) *senarius*, Weiss, *Steinkohlen-Calamarien*, part ii. p. 114, pl. xiii. fig. 2.

Calamites approximatus, L. & H., *Fossil Flora*, vol. iii. pl. ccxvi.

Remarks.—The University Museum of Oxford possesses a fine specimen of this species from Camerton, which is the original of pl. ccxvi. of LINDLEY and HUTTON'S *Fossil Flora*. In the description of their plate (which is a reduced figure of the fossil), the authors say—"It agrees in a striking manner with the figures of ARTIS and ADOLPHE BRONGNIART, with the addition of a number of pits placed on the articulations, in a quincuncial manner, as in *Calamites cruciatus*. Hence it is probable that the latter proposed species will require to be reduced to *C. approximatus*."

The specimen referred to in the above quotation, and figured by the authors of the *Fossil Flora* on their plate ccxvi., belongs, however, to an entirely different group of *Calamites* from that in which *C. approximatus* is now placed.

In *Eucalamites*, which includes the Camerton plant, every node bears branches. In *Calamitina*, on the other hand, in which *C. approximatus*, Brongt., is enrolled, the branch-bearing nodes are separated by a greater or less number of nodes that do not produce branches.

The Camerton specimen of *Calamites* (*Eucalamites*) *senarius*, which is a compressed stem removed from the matrix, measures about 15 inches in length and about 3¾ inches in width at its lower extremity. It consists of sixteen perfect internodes and portions of two incomplete ones—one at each end of the fossil. On the circumference of each node are borne six branch scars. The internodes decrease slightly in length from below up, but in a somewhat irregular manner. Their exact measurement is—

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* *Abhandl. z. geol. special-karte v. Preussen u. Thuringischen Staaten*, Band v. part ii.

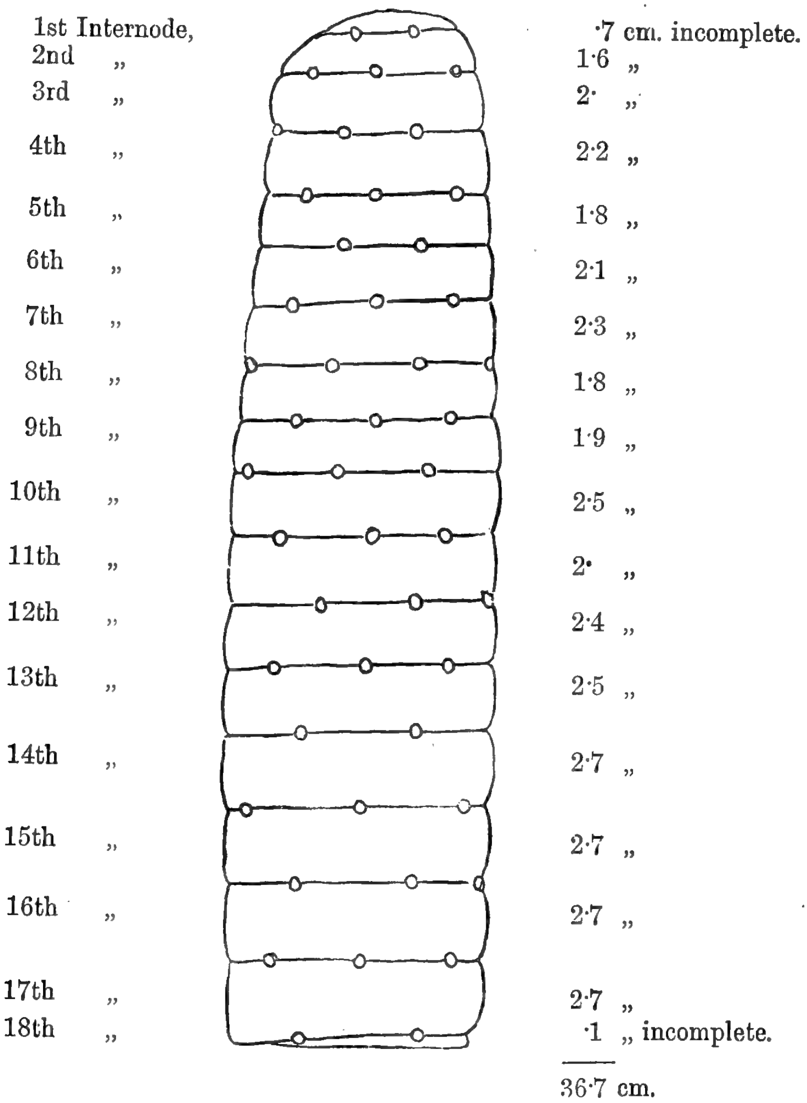

Fig 1. *Eucalamites cruciatus senarius,* Weiss ; Camerton (⅖ nat. size).

Locality :—Camerton.

Eucalamites (Calamites) ramosus, Artis.

Calamites ramosus, Artis, *Antedil. Phyt.,* pl. ii.
Calamites ramosus, Brongt., *Hist. d. végét. foss.,* p. 127, pl. xvii. figs. 5 6.
Calamites ramosus, Zeiller, *Végét. foss. du terr. houil.,* p. 15.
Cal. (*Eucalamites*) *ramosus,* Weiss, *Steinkohlen-Calamarien,* part ii. p. 98, pls. ii. fig. 3 ; v. figs. 1–2 ; vi. ; vii. figs. 1–2 ; viii. figs. 1, 2, 4 ; ix. figs. 1, 2 ; x. fig. 1 ; xx. figs. 1, 2.
Calamites nodosus, L. & H., *Foss. Flora,* vol. i. pls. xv., xvi.
Calamites nodosus, Lebour, *Illustrations of Foss. Plants,* p. 3, pl. ii. ; p. 7, pl. iii.
Calamites carinatus, Sternb., *Vers.,* i. fasc. 3, pp. 36 and 39, pl. xxxii. fig. 1 ; fasc. iv. p. xxvii.

As foliage :—

Annularia radiata, Brongt., *Prodrome,* p. 156.
Annularia radiata, Zeiller, *Végét. foss. du terr. houil.,* p. 24, pl. clx. fig. 1.
Asterophyllites radiatus, Brongt., *Class d. végét. foss.,* p. 35, pl. ii. fig. 7.
Asterophyllites foliosus, L. & H., *Foss. Flora,* vol. i. pl. xxv. fig. 1.

Asterophyllites foliosus, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 10, pl. xvi. figs. 1–3 (fig. 4?) (excl. pl. xv.).

Remarks.—Rare.

Locality :—Radstock.

Group III.—**Stylocalamites**, Weiss, 1884, *Steinkohlen-Calamarien*, part ii. p. 119.

Stylocalamites (Calamites) Suckowii, Brongt.

Calamites Suckowii, Brongt., *Hist. d. végét. foss.*, p. 124, pl. xiv. fig. 6; pl. xv. figs. 1–6; pl. xvi. figs. 2, 3, 4 (1?).

Calamites Suckowii, Feistmantel, *Vers. d. böhm. Kohlenab.*, p. 102, pl. ii. figs. 3, 4; pl. iii. figs. 1, 2; pl. iv. figs. 1, 2, pl. v.; pl. vi. fig. 1 (excl. as fruit *H. carinata*).

Calamites Suckowii, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 6, pl. xiii. figs. 1–6.

Calamites Suckowii, Roehl, *Foss. Flora d. Steink. Form. Westph.*, p. 9, pl. i. fig. 6; pl. ii. fig. 2.

Calamites Suckowii, Weiss, *Steinkohlen-Calamarien*, part i. p. 123, pl. xix. fig. 1 (1876); part ii. p. 129, pl. ii. fig. 1; pl. iii. figs. 2, 3; pl. iv. fig. 1; pl. xxvii. fig. 3 (1884).

Calamites Suckowii, Kidston, *Catal. Palæoz. Plants*, p. 24.

Calamites decoratus, Artis, *Antidel. Phyt.*, pl. xxv.

Calamites decoratus, Brongt. (in part), *Hist. d. végét. foss.*, p. 123, pl. xiv. figs. 1, 2 (excl. figs. 3, 4).

Remarks.—Not common.

I have observed two specimens of this species from Camerton,—one in the Bristol Museum, the other kindly given me by Mr G. West,—which show the peculiarity of a double row of tubercles, one row at each extremity of the ribs; thus each nodal line has a row of large tubercles immediately below it, and a row of smaller tubercles immediately above it. The same peculiarity is figured by Brongniart (*C. decoratus*, Brongniart (not Artis), *Hist. d. végét. foss.*, pl. xiv. figs. 3, 4) and Weiss (*C. ramosus*, *Steinkohlen-Calamarien*, part ii. p. 108, pl. ix. fig. 2). It is shown from the specimen given me by Mr West, which is a portion of the stem near its base, that the larger tubercles occupy the normal position, viz., the upper extremities of the ribs, and the smaller tubercles the lower extremity. Weiss's figure is therefore, as suspected by him, drawn in inverted position. My Camerton specimen also shows the further peculiarity, likewise mentioned by Weiss in the description of his example, that many of the ribs, instead of alternating at the nodes in a normal manner, are exactly opposite. This latter abnormality, however, in the case of a few ribs, is not a very uncommon phenomenon on stems of *Calamites.*

Localities :—Radstock; Camerton.

Stylocalamites (Calamites) cannæformis, Schloth.

Calamites cannæformis, Schlotheim, *Petrefactenkunde*, p. 398, pl. xx. fig. 1.

Calamites cannæformis, Brongt., *Hist. d. végét. foss.*, p. 131, pl. xxi.

Calamites cannæformis, Lindley and Hutton, *Foss. Flora*, vol. i. pl. lxxix.

Calamites cannæformis, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 115.

Calamites cannæformis, Zeiller, *Végét. foss. du terr. houil.*, p. 16.

Calamites pachyderma, Brongt., *Hist. d. végét. foss.*, p. 132, pl. xxii.

Remarks.—A few stems have been met with that should perhaps be referred to this species, which, however, appears to me to be a very ill-defined one, and in which are often placed large and badly-preserved stems that probably belong to *C. Suckowii* or some other species.

The type figure of *C. cannæformis* represents the terminal portion of a stem which does not seem to have been well preserved, hence there is difficulty in recognising the plant really meant by Schlotheim.

Localities:—Camerton; Welton Hill, Midsomer-Norton.

Stylocalamites (Calamites) Cistii, Brongt.

Calamites Cistii, Brongt., *Hist. d. végét. foss.*, p. 129, pl. xx.
Calamites Cistii, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 7, pl. xi. figs. 7, 8; pl. xii. figs. 4, 5; pl. xiii. fig. 7.
Calamites Cistii, Schimper, *Traité de paléont. végét.*, vol. i. p. 313.

Remarks.—More frequent than the foregoing species; not common, however.

Localities:—Radstock; Braysdown; Camerton; Wellsway; Lower Writhlington.

Calamocladus, Schimper, 1869, *Traité d. paléont. végét.*, vol. i. p. 323.

Calamocladus equisetiformis, Schloth., sp.

Calamocladus equisetiformis, Schimper, *Traité d. paléont. végét.*, vol. i. p. 324, pl. xxii. figs. 1–3.
Asterophyllites equisetiformis, Germar, *Vers. v. Wettin u. Löbejun*, p. 21, pl. viii.
Asterophyllites equisetiformis, Zeiller, *Végét. foss. du terr. houil.*, p. 19, pl. clix. fig. 3.
Hippurites longifolia, L. & H., *Fossil Flora*, vol. iii. pls. cxc., cxci.
Casuarinites equisetiformis, Schlotheim, *Flora d. Vorwelt*, p. 30, pl. i. figs. 1, 2; pl. ii. fig. 3.
Annularia calamitoides, Schimper, *Traité d. paléont. végét.*, vol. i. p. 349, pl. xxvi. fig. 1.

Remarks.—Not very frequent.

Localities:—Radstock; Braysdown; Wellsway; Upper Conygre Pit; Kilmersdon.

Annularia, Sternberg, 1820, *Vers. einer Geog. Botan. Darstellung d. Flora d. Vorwelt*, fasc. 2, p. 32.

Annularia stellata, Schloth., sp.

Annularia stellata, Zeiller, *Végét. foss. du terr. houil.*, p. 26, pl. clx. figs. 2, 3.
Casuarinites stellatus, Schloth., *Flora d. Vorwelt*, p. 32, pl. i. fig. 4.
Annularia longifolia, Brongt., *Prodrome*, p. 156.
Annularia longifolia, Germar, *Vers. d. Wettin u. Löbejun*, p. 25, pl. ix.
Annularia longifolia, Weiss, *Foss. Flora d. jüngst. Stk. u. Rothl.*, p. 30.
Asterophyllites longifolia, L. & H., *Foss. Flora*, vol. ii. pl. cxxiv.
Asterophyllites longifolia, Binney, *Palæontological Soc.*, 1868, p. 28, pl. vi. fig. 3.

As its fruit:—

Stachannularia tuberculata, Weiss, *Steinkohlen-Calamarien*, vol. i. p. 17, pl. i. figs. 2–4; pl. ii. figs. 1–3, 5 (left); pl. iii. figs. 3–10, 12.
Stachannularia tuberculata, Kidston, *Catal. Palæoz. Plants*, p. 56.
Buckmannia tuberculata, Sternberg, *Vers.*, i. fasc. 4, p. xxix. pl. xlv. fig. 2.

Remarks.—Frequent. STERZEL[*] notes the occurrence of specimens of *Annularia stellata* with *Stachannularia tuberculata* organically united, which proves what had previously been suspected, that this cone is the fruit of *Annularia stellata.*

The cones of this species are rare in the Radstock Coal Field.

Localities:—Radstock; Braysdown; Camerton; Upper Conygre; Lower Conygre; Paulton; Kilmersdon.

Annularia sphenophylloides, Zenker, sp.

Annularia sphenophylloides, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 11, pl. xviii. fig. 10.
Annularia sphenophylloides, Schimper, *Traité d. paléont. végét.*, vol. i. p. 347, pl. xvii. figs. 12, 13.
Annularia sphenophylloides, Sterzel, *Zeitsch. d. deut. geol. Gesell.*, vol. xxxiv. p. 685, pl. xxviii.
Annularia sphenophylloides, Weiss, *Foss. Flora d. jüngst. Stk. u. Rothl.*, p. 131.
Annularia sphenophylloides, Zeiller, *Végét. foss. du terr. houil.*, p. 25, pl. clx. fig. 4.
Annularia sphenophylloides, Kidston, *Catal. Palæoz. Plants*, p. 44.
Galium sphenophylloides, Zenker, *Neues Jahrb.*, 1833, p. 398, pl. v. fig. 6.
Annularia brevifolia, Brongt., *Prodrome*, p. 156.

As its fruit:—

Stachannularia calathifera, Weiss, *Steinkohlen-Calamarien*, vol. i. p. 27, pl. iii. fig. 11.

Remarks.—Frequent. Some large slabs were entirely covered with the leaves of this plant. STERZEL[†] has shown that the little cones, described by WEISS as *Stachannularia calathifera*, are the fruit of this species. The fruit is rare in the Radstock Coal Field, having only been found once at Radstock.

Localities:—Radstock; Upper Conygre; Lower Conygre; Camerton; Braysdown; Kilmersdon.

(?) RHIZOCARPEÆ.

Sphenophyllum, Brongniart, 1822, *Sur la Classification d. végét. foss.*, p. 34.

Sphenophyllum emarginatum, Brongniart.

Sphenophyllum emarginatum, Coemans and Kickx, *Bull. Akad. roy. Belgique*, 2^{e} sér. vol. xviii. p. 144, pl. i. fig. 2; pl. ii. figs. 1–3.
Sphenophyllum emarginatum, Geinitz (in part), *Vers. d. Steinkf. in Sachsen*, p. 12, pl. xx. figs. 1, 3, 4.
Sphenophyllum emarginatum, Schimper, *Traité d. paléont. végét.*, vol. i. p. 339, pl. xxv. figs. 15, 16.
Sphenophyllum Schlotheimii, L. & H. (not Brongt.), *Fossil Flora*, vol. i. pl. xxvii.

Remarks.—Frequent.

Localities:—Radstock; Braysdown; Camerton; Paulton; Upper Conygre; Lower Conygre.

FRUCTIFICATION.

Macrostachya, Schimper, 1869, *Traité d. paléont. végét.*, vol. i. p. 332.

Macrostachya infundibuliformis, Brongniart, sp.

Macrostachya infundibuliformis, Schimper, *Traité d. paléont. végét.*, vol. i. p. 133, pl. xxiii. figs. 15–17 (excl. figs. 13, 14).

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[*] *Zeitsch. d. deut. geol. Gesell.*, 1882, p. 685.

[†] *Loc. cit.*, p. 685.

Macrostachya infundibuliformis, Weiss, *Steinkohlen-Calamarien*, part i. p. 71, pl. vi. figs. 1–4; pl. xviii. figs. 1, 3, 4 (1876); part ii. p. 197 (1884).

Equisetum infundibuliforme, Brongt. (in part), *Hist. d. végét. foss.*, p. 119, pl. xii. figs. 14, 15 (excl. syn. and fig. 16).

Huttonia carinata, Germar, *Vers. v. Wettin u. Löbejun*, p. 90, pl. xxxii. figs. 1, 2.

Macrostachya carinata, Zeiller, *Végét. foss. du terr. houil.*, p. 23, pl. clix. fig. 4.

Equisetum, Brongt., *Class. d. végét. foss.*, p. 90, pl. iv. fig. 4.

Remarks.—Very rare; only two examples having been found.

Localities:—Radstock; Kilmersdon.

FILICACEÆ.

Sphenopteris, Brongniart, 1822, *Sur la Classification d. végét. foss.*, p. 33.

Sphenopteris tenuifolia, (Brongt. ?) Gutbier.

Plate XIX. fig. 2.

(?) *Sphenopteris tenuifolia*, Brongt., *Hist. d. végét. foss.*, p. 190, pl. xlviii. fig. 1.

Sphenopteris tenuifolia, Gutbier, *Abdrücke u. Vers. d. Zwickauer Schwarzkohl*, p. 39, pl. v. fig. 10; pl. x. fig. 9.

(?) *Cheilanthites tenuifolius*, Göppert, *Syst. fil. foss.*, p. 241.

Description.—Frond tripinnate; primary (?) and secondary pinnæ alternate, lanceolate; pinnules alternate, lanceolate; lower pinnules divided into numerous (as many as fourteen) segments; the lower segments are again divided into four to six simple or bifid lanceolate acute teeth; upper pinnules less divided, bearing simple, bifid or trifid acute lanceolate segments, into each of which extends a vein. Rachis of pinnæ thin. Fruit borne at the extremities of the secondary (?) pinnæ, and situated at the margin of the pinnule segments.

Remarks.—The specimen, of which a drawing is given natural size, shows two (?) primary pinnæ lying parallel to each other. As their parent rachis is not shown, their entire length cannot be estimated. The portions of the pinnæ preserved measure about 6 inches each.

This, the only example which I have seen, is beautifully preserved, and shows the most minute details of the pinnule cutting. The lower pinnules of the lower secondary pinnæ are much divided into broadly lanceolate segments, and the lower segments are again divided into a few simple and bifid acute lanceolate teeth, into each of which runs a vein. A careful drawing of one of these pinnules, magnified three times, is given on Plate XIX. fig. 2*b*. An upper pinnule, also enlarged three times, is shown at fig. 2*a*; the segments of this are, with one exception, bifid, but these details vary according to the position of the pinnule on the pinna, the uppermost pinnules being even less divided.

The fruit is borne on the upper secondary pinnæ, apparently at the margins of the pinnules. Its structure is not well shown, the fruit appearing as little indistinct groups at the extremities of the ultimate segmentation. Owing to the somewhat indistinct details of the fruit, I am led to believe it had not reached maturity, as the other parts of the specimen show their structure exquisitely.

I may note here that many of the Pecopteroids from this Coal Field are found in fruit, but always in an immature condition, and seldom show their structural details clearly, though in all other points the preservation of the fossils is very fine.

The mode of fructification of *Sphen. tenuifolia* appears to be similar to that of *Sphen. Gützoldi,* as figured by Gutbier.[*]

Stur regards as distinct species the *Sphen. tenuifolia,* Brongt., and the plant figured under that name by Gutbier, and has described a third species, *Sphenopteris* (*Calymmotheca*) *subtenuifolia.*[†]

Brongniart states in his description of his *Sphen. tenuifolia* that the specimen was preserved in a coarse-grained sandstone; hence he was not satisfied as to the thorough accuracy of his enlarged drawing of the pinnule. If, however, we compare the pinnule, as represented by Brongniart, with that on our Pl. XIX. fig. 2*a*, their similarity is very striking. On the other hand, Brongniart's plant has apparently a more rigid growth, and the main rachis is very thick for the size of the pinnæ; on our example the main rachis is unfortunately not shown.

With the fern from Zwickau, figured by Gutbier as *Sphen. tenuifolia,* our example agrees perfectly, his fig. 9, pl. x. being apparently identical with my fig. 2. The enlargement of the other example given by Gutbier in his pl. v. fig. 10*a*, does not seem to differ essentially from my fig. 2*a*, though the segments of the pinnules of his figure are shown to be a little stouter than in the Somerset plant.

Stur's *Sphen.* (*Calymmotheca*) *subtenuifolia* is very closely allied to *Sphen. tenuifolia,* if really distinct from it; but in the absence of enlarged details of the pinnule segmentation, a critical comparison can scarcely be made.

I have distinguished my example as *Sphenopteris tenuifolia,* Gutbier (? Brongt.), till the true relationship of these plants to each other is decided.

Locality:—Upper Conygre Pit.

Sphenopteris geniculata, Germar and Kaulfuss.
Plate XXI. fig. 1.

Sphenopteris geniculata, Germar and Kaulfuss, *Verhandl. d. K. Leop. Carol. Akad. d. Naturf.,* vol. xv. part ii. p. 224, pl. lxv. fig. 2, 1831.
Diplothmema geniculatum, Stur (in part), *Carbon-Flora,* p. 297, pl. xxxv. fig. 1.
Sphenopteris Kaulfussi, Schimper, *Traité d. paléont. végét.,* vol. i. p. 412.

Description.—Primary (?) pinnæ divided into two symmetrical portions; rachis flexuous, winged; secondary (?) pinnæ alternate, lanceolate; pinnules

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[*] *Vers. d. Rothliegenden in Sachsen,* p. 9, pl. ii. figs. 3, 4, 5.

[†] *Die Carbon-Flora der Schatzlarer Schichten,* p. 257, pl. xxxi. fig. 5, 1885.

alternate, the lower divided into numerous simple or bifid linear acute segments, into each of which runs a vein; upper pinnules less divided, and may consist of only one or two bifid segments.

Remarks.—The only British specimen of this rare species which I have seen is that figured on Pl. XXI. fig. 1. It shows very beautifully the peculiar dichotomisation of the pinnæ as developed on many *Sphenopterids.* The general outline of the pinnæ is more or less subrotund,—that of its two component parts oval. The stalk uniting this compound pinna to the rachis is not preserved, but was probably naked like that of the other members of this group of Sphenopteroids.

Two of the pinnules are enlarged at fig. 1*a.b.* Throughout their segmentation there can generally be traced a series of dichotomous divisions; this is exhibited in both enlargements, where each of the two larger segments dichotomises.

Both the primary (?) and secondary (?) rachis are distinctly winged and geniculate.

Sphenopteris geniculata appears to have been much confused with *Sphen. furcata,* Brongt., and, following GEINITZ, I united them in my Catalogue of Palæozoic Plants, but now regard the two species as essentially distinct.

Of the two figures of *Sphen. geniculata* given by STUR in his *Carbon-Flora,* that on his pl. xxxv. fig. 1, is evidently GERMAR and KAULFUSS' plant, but I think that on his pl. xxviii. fig. 1, is referable to *Sphen. furcata.* I believe also that the figure given by GEINITZ as *Sphen. geniculata* * (which he unites with *Sphen. furcata,* Brongt.) must be excluded from GERMAR and KAULFUSS' plant.

Locality:—Kilmersdon Pit.

Sphenopteris Grandini, Göppert, sp.

Sphenopteris Grandini, Schimper, *Traité d. paléont. végét.*, vol. i. p. 404.
Sphenopteris Grandini, Boulay, *Terr. houil. du nord de la France,* p. 27.
Hymenophyllites Grandini, Göppert, *Syst. fil. foss.*, p. 255, pl. xv. fig. 12.
Sphenopteris alata, Brongt., *Hist. d. végét. foss.*, p. 180, pl. xlviii. fig. 4.
Sphenopteris alata, Sauveur, *Végét. foss. de la Belgique,* pl. xvii. fig. 2.
Sphenopteris alata, Gutbier, *Vers. d. Zwick. Schwarzkohl,* p. 34, pl. v. figs. 16 and 17; pl. xi. fig. 1.

Remarks.—This species is rare in the Radstock Coal Field, but has been observed at several collieries.

STUR unites *Sphen. trichomanoides,* Brongt., with *Sphen. Grandini* † (*Sphen. alata,* Brongt.). On the other hand, it is suggested by BOULAY ‡ that *Sphen.*

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* *Vers. d. Steinkf. in Sachsen,* pl. xxiv. fig. 13.

† *Carbon-Flora d. Schatzlarer Schichten,* p. 304, 1885.

‡ *Loc. cit.*, p. 27.

trichomanoides is simply a pinna of *Sphen. furcata,* Brongt., and ZEILLER, though he includes *Sphen. trichomanoides* in his "Fougères du terrain houiller du nord de la France,* is inclined to accept BOULAY's suggestion.

The specimens from Radstock are similar to BRONGNIART's type figure.

Sphen. Grandini appears to me a very distinct plant, though, according to BOULAY, even it may be only a variety of *Sphen. furcata,* Brongt. This latter species has not yet been observed in the Radstock area, where *Sphen. Grandini,* though rare, is widely distributed.

The ferns figured by GEINITZ and LESQUEREUX as BRONGNIART'S plant belong to another species.†

Localities:—Radstock; Braysdown; Lower Conygre.

Sphenopteris macilenta, L. & H.

Sphenopteris macilenta, L. & H., *Fossil Flora,* vol. ii. pl. cli.
Sphenopteris macilenta, Geinitz, *Vers. d. Steinkf. in Sachsen,* p. 14, pl. xxiii. fig. 1.
Sphenopteris macilenta, Zeiller, *Bull. de la soc. géol. de France,* 3^e sér. vol. xii. p. 194.
Sphenopteris lobata, Gutbier, *Vers. d. Zwick. Schwarzkohl,* p. 44, pl. v. figs. 11, 13, 14, 15; pl. x. figs. 1–3.

Remarks.—Of very unfrequent occurrence.

Dr STUR is evidently in error in separating GEINITZ'S and GUTBIER'S figures from *Sphen. macilenta,* L. & H.,—a species in which the pinnule cutting varies much according to the position the pinnæ hold on the frond.‡

Localities:—Radstock; Braysdown; Camerton.

Sphenopteris Woodwardii, Kidston, n.s.

Plate XIX. fig. 1.

Description.—Frond tripinnate; rachis very stout; primary pinnæ subopposite, ascending; secondary pinnæ alternate or subopposite, lanceolate, ascending; pinnules alternate, oblong-lanceolate, pinnatifid, rarely divided into lobes; veins distinct, veinlets usually simple, occasionally bifid,—especially in those pinnules which are divided into lobes.

Remarks.—The specimen figured is the only example of this species with which I have met. It was collected on one of my earlier visits to Camerton, and though diligent search has since been made for additional specimens I have not yet succeeded in securing any.

The pinnules are most commonly merely pinnatifid, as shown at fig. 1*a*. The limb of the pinnules is of very delicate texture, but the veins are thick

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* *Bull. de la soc. géol. de France,* 3 sér., vol. xii. p. 194, 1883

† See Kidston, *Catal. Palæoz. Plants,* p. 78.

‡ *Carbon-Flora,* p. 375 (*Diplothmema macilentum*).

and prominent, being raised like threads on the surface of the pinnule. The veinlets are usually simple, but where the pinnules show a tendency to become lobed they are bifid, as seen in the lower lobe of fig. 1*b*. When the pinnules are divided into lobes, the vein in each lobe usually bifurcates, as shown in fig. 1*c*. These lobed pinnules are of somewhat irregular occurrence on the pinnæ. At the points marked *x* and *x'*, where examples of these lobed pinnules occur, that at *x* is situated on a basal secondary pinna, whereas that at *x'* is on a secondary pinna, placed well up a primary pinna. On the same primary pinna the majority of the pinnules, even on secondary pinnæ borne lower down on the rachis, are only pinnatifid, as seen at *z* and on the other pinnæ of the specimen.

The main rachis and those of the primary pinnæ are very stout, as compared with those of the secondary pinnæ; they are feebly striated, and bear slightly elevated points.

The only species to which the Camerton plant appears to have any resemblance are *Sphenopteris* (*Cheilantheites*) *grypophylla*, Göppert,[*] and *Sphen. bidentata*, Gutbier.[†]

From *Sphen. grypophylla*, *Sphen. Woodwardii* is easily distinguished by its lanceolate, upward-directed pinnæ, and the pinnatifid pinnules. In *Sphen. grypophylla* the pinnæ are long and linear, and spring from the rachis at almost right angles, and the pinnules are uniformly divided into bifid lobes. In addition, the whole general appearance of the two plants is characteristically distinct.

The type figure of *Sphen. bidentata*, Gutbier, is very fragmentary, and, except from the enlarged figure of the pinnule, a comparison of the species with any other is almost impossible. This enlarged pinnule shows a sharply bifid-toothed, spinous-like fern, which, both in the form of the pinnule and its segmentation, is essentially distinct from my plant.

I have great pleasure in naming this specimen after Dr HENRY WOODWARD, F.R.S., of the British Museum.

Locality:—Camerton.

Sphenopteris neuropteroides, Boulay, sp.

Sphenopteris neuropteroides, Zeiller, *Bull. de la soc. géol. de France*, 3ᵉ sér., vol. xii. p. 191, 1883.
Pecopteris neuropteroides, Boulay, *Le terr. houil. du nord de la France*, p. 32, pl. ii. figs. 6 and 6 *bis*, 1876.
Pseudopecopteris anceps, Lesqx., *Coal Flora of Pennsyl.*, vol. i. p. 207, pl. xxxviii. figs. 1–4, 1880.

Remarks.—Rare. I have compared a specimen of *Pseudopecopteris anceps*, Lesqx., from Pittston, communicated to Mr W. CASH, Halifax, by Mr R. D.

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[*] *Syst. fil. foss.*, p. 242, pl. xxxvi. figs. 1, 2. See also STUR, *Sphenopteris* (*Saccopteris*) *grypophylla*, *Carbon-Flora*, p. 176, pl. liii. figs. 3, 4, 5.

[†] Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 16, pl. xxiv. fig. 3.

LACOE, with the examples of *Sphen. neuropteroides* from Somerset, and find, as suspected by ZEILLER, that *Pseudopecopteris anceps* is identical with *Sphen. neuropteroides,* with which species it must therefore be united.

It is interesting to note that the British, as well as the American and French, specimens of this fern appear to be infested with a species of *Excipulites.*

Localities:—Radstock; Camerton; Withy; Clandown.

Sphenopteris cristata, Brongt., sp.

Sphenopteris cristata, Schimper, *Traité d. paléont. végét.*, vol. i. p. 397.
Sphenopteris cristata, Kidston, *Catal. Palæoz. Plants,* p. 74.
Pecopteris cristata, Brongt., *Hist. d. végét. foss.*, p. 356, pl. cxxv. figs. 4, 5.

Remarks.—The only specimen of this species with which I have met is that contained in the collection of the British Museum.

Locality:—Camerton.

Ptychocarpus, Weiss, 1869, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 94.

Description.—"Sori round or oval, divided by a longitudinal cleft into two oblong halves."

Remarks.—The fruit of the genus *Ptychocarpus* appears to consist of two sporangia lying side by side. The systematic position of the genus is near to *Asterocarpus* (= *Pecopteris*), but is separated from it by the sporangia being arranged in pairs, whereas in the *Asterocarpus-Pecopteroids* the fruit is composed of several stellately arranged sporangia. In the type of *Ptychocarpus* (*P. hexastichus*) the sporangia are surrounded by a narrow flat border, from which WEISS thinks that the two sporangia were covered by an indusium, which, springing from the centre of the medial line, extended over and beyond the sporangia.

He compares his genus *Ptychocarpus* to *Didymochlæna,* Desv., and the external resemblance of the *upper* surface of the fruiting pinnules of *Didymochlæna sinuosa,* as figured by BAUER,[*] to the species about to be described (*P. oblongus*) is very striking. In pointing out this external resemblance I do not at all infer any affinity between the recent and fossil genera.

Ptychocarpus oblongus, Kidston, n. sp.
Plate XX. fig. 2.

Description.—Frond tripinnate (?); pinnæ subopposite, lanceolate; pinnules subopposite, oblong, usually bearing four pairs of oblong lateral lobes and a terminal one. On each lobe is situated an oblong synangium (?), composed of

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[*] Bauer and Hooker, *Genera Filicum, or Illustrations of the Ferns and their other Allied Genera,* table viii. figs. 2, 3.

two sporangia (?). The main rachis and those of the pinnæ are stout, and pitted with scale-scars.

Remarks.—It is with considerable reservation that I place this species in the genus *Ptychocarpus,* Weiss.

The general appearance of my specimen exhibits a great similarity to Weiss' genus, but, on minute examination, I cannot positively affirm that the longitudinal cleft entirely divides the two supposed sporangia, as it does in *P. hexastichus,* Weiss (*loc. cit.,* pl. xi. fig. 2). The usual appearance of the fruit of the Camerton plant is shown at Plate XX. figs. 2*a* and *b*; at fig. 2*b* the cleft is seen to be much more prominent in the central part than at the two extremities, where it becomes indistinct. This may be merely caused by imperfect preservation, or even by the degree of maturity at which the specimen had arrived when embedded. This supposed bi-sporangial synangium is generally surrounded by a faint border, as shown at figs. 2*a* and *b,* which appears as a slight surrounding staining, but is clearly observable in many cases.

The pinnules seem to have been divided into lobes, on each of which was borne a (?) synangium. I am disposed to think that this surrounding border may not represent an indusium, but the margin of the limb of the pinnule on which the sporangia sat. The fruit seems to have had a firm consistency, and has still considerable elevation.

On collecting this fossil, my first impression was that each segment of the pinnules bore a *split exannulate sporangium,* but, as a result of further examination and comparison with the description of *Ptychocarpus,* Weiss, I have provisionally placed it in that genus.

The only example met with is that figured.

Locality:—Camerton.

Schizostachys, Grand' Eury, 1877, *Flore carbon du Département de la Loire, &c.,* p. 200.

Remarks.—This genus is characterised by its oblong, slightly curved, pedicellate sporangia, attached around a common point, or on the sides of a short pedicel. In the species described by Grand' Eury* the cellular tissue of the sporangia was still visible; some of these cells were especially prominent, and formed a band which encircled the sporangium. This band may perhaps represent an annulus. On the surface of the sporangia is a longitudinal line, to which these (?) annulus-forming cells seem to lie at a right angle.

Grand' Eury regarded his *Schizostachys frondosus* as the male inflorescence of *Noeggerathia;* Renault, on the other hand, places it among the ferns,† and this appears to be its true position.

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* *Schizostachys frondosus;* on his pl. xvii. fig. 3, named *Androstachys frondosus.*

† *Cours d. botan. foss.,* Troisième Année, 1883, p. 103.

Renault has further suggested that the fruit described by Grand' Eury as *Schizostachys* may perhaps be identical with that described by him as the fruit of *Zygopteris*.*

In the whole mode of its growth *Schizostachys ramosus*, Gr.' E., approaches so closely to *Schizopteris pinnata*, Gr.' E., and *S. cycadina*, Gr.' E., that it induces the supposition that it might be the fruit of one of these, or of a closely allied species.

As, however, the fruiting specimens show no barren foliage, they cannot be referred to these ferns with any degree of certainty.

Schizostachys sphenopteroides, Kidston, n. sp.

Plate XX. fig. 1.

Description.—Frond bipinnate; pinnæ subopposite, linear, lanceolate; pinnules subopposite, coriaceous, and composed of one sporangium on the upper pinnæ, and of two diverging sporangia on the lower pinnæ. Sporangia oblong, with a central line, from which extends a series of transverse bars. Rachis faintly striated.

Remarks.—A specimen is shown on Plate XX. fig. 1, natural size. The pinnules, which are quite destitute of any leafy expansion, are reduced to one or two sporangia, according to their position on the frond. The sporangia are oblong and straight, or very slightly curved. On their surface is a longitudinal line, from which the little transverse bars extend at right angles. The specimen is not sufficiently well preserved to exhibit the cellular structure of the sporangia, as in the case of that described by Grand' Eury, but it shows the transverse bars as drawn at fig. 1*a*. These bars extend over almost the whole of the exposed surface of the sporangia, but that they indicate the presence of an annulus I am unable to determine.

I apply the specific name of *sphenopteroides* to this species from its superficial resemblance to some of the members of that genus.

Locality:—Radstock.

Macrosphenopteris, Kidston, n. gen.

Description.—Pinnules very large, ovate, of delicate texture, provided with a central vein, from which spring numerous upward-directed dichotomous veinlets. Margin dentate or laciniate.

Remarks.—This genus is proposed for the *Adiantites Haidingeri*, Ettingshausen,† and the specimen about to be described.

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* *Loc. cit.*, p. 102.

† *Die Steinkohlenflora von Radnitz*, p. 34, pl. xix. fig. 3.

The genus *Adiantites,* Göppert, as originally employed by its author, had a very vague significance, and in it were placed ferns of very different character. The genus has been emended by SCHIMPER,[*] and as now defined, *Adiantites Haidingeri,* Ett., can no longer be included in it. It is therefore necessary to create a new genus for this fern and for the one I now describe as *Macrosphenopteris Lindsæoides.*

The remains of these ferns are treated as pinnules, as their general appearance points to this conclusion rather than to their being fronds.

Macrosphenopteris, in the delicate texture of the pinnules and the arrangement of the veins, shows affinities with *Sphenopteris,* hence the name (*Macrosphenopteris*) now proposed for it.

Macrosphenopteris Lindsæoides, Kidston, n. sp.

Plate XXVII. fig. 1.

Description.—Pinnules very large, of delicate texture, with a central vein, from which arise numerous ascending slightly curved dichotomous veinlets. Margin sinuous or dentate.

Remarks.—The specimen figured is the only example met with. It is unfortunately very imperfect, but the portion preserved shows that the pinnule must have been of large size. Its texture is very delicate, and the veins are distinct. At several parts of the pinnule the margin is thickened in a very peculiar manner; whether this is caused by a folding over of the margin or a thickening of the tissue at this part of the pinnule, cannot be determined. The appearance is not accidental, and is possibly connected with the fructification of the species; it has a strong superficial resemblance to the arrangement of the *indusia* of the genus *Lindsœa,* which has suggested the specific name of *Lindsœoides.*

Macrosphenopteris Lindsœoides, though closely related to *Macrosphenopteris Haidingeri,* Ett., sp., appears to have been a much larger species, with more distant nervation, and the margin not regularly dentate as in ETTINGSHAUSEN'S plant, where each of the veinlets seems to end in a small tooth.

The *Aphlebia pateriformis,* Germar,[†] may be allied to *Macrosphenopteris,* but "a distinct dichotomy of the longitudinal stripes (veins ?) is not recognisable" in GERMAR'S plant.

Locality :—Radstock.

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[*] *Traité d. paléont. végét.,* vol. i. p. 424 (*Adiantides*).

[†] *Vers. de. Steink. v. Wettin u. Löbejun,* fasc. 1, p. 5, pl. ii.

Neuropteris, Brongniart, 1822, *Sur la Classification des végétaux fossiles*, p. 33.

Neuropteris macrophylla, Brongt.

Plate XXI. fig. 2; Plate XXII. figs. 2, 3.

Neuropteris macrophylla, Brongt., *Hist. d. végét. foss.*, p. 235, pl. lxv. fig. 1.
Neuropteris macrophylla, Schimper, *Traité d. paléont. végét.*, vol. i. p. 434.
Neuropteris Clarksoni, Lesqx., in Roger's *Geol. of Pennsyl.*, vol. ii. p. 857, pl. vi. figs. 1–4.
Neuropteris Clarksoni, *Coal Flora of Pennsyl.*, p. 94, pl. ix. figs. 1–6.
Neuropteris Scheuchzeri, Kidston (not Hoffm.), *Catal. of Palæoz. Plants*, p. 95.
(?) *Osmunda*, Scheuchzer, *Herbarium diluvianum*, p. 48, pl. x. fig. 3, edition 1709.

Description.—Frond very large; pinnæ dividing by a series of dichotomies. Pinnules alternate, varying much in shape and size, triangular, lanceolate-acute, oblong-obtuse, and cyclopteroid. Midrib distinct, and extending to the apex; lateral veins numerous, arched, generally dichotomising four times, rarely five times, the last dichotomy being near the margin of the pinnule. Veins reaching the edge of the pinnule at an open angle. The cyclopteroid pinnules are situated on the rachis.

Remarks.—Neuropteris macrophylla was described by BRONGNIART from a specimen collected at Dunkerton, Somerset, which belonged to the Geological Society of London, and in whose collection it still remains. The species is frequent in the Radstock Coal Field, from which some large and fine specimens have been secured.

Having compared my specimens with the type, I am satisfied of their identity with it. This comparison was necessary as BRONGNIART'S representation of the nervation of his type is too coarse and distant. In fact, the nervation of *Neuropteris macrophylla* is much more like the nervation of *Neur. auriculata* as represented by BRONGNIART on his plate lvi. fig. A, than it is to the enlarged drawing that accompanies the original figure of the species. This led me to conclude that these two species were identical, but ZEILLER, to whom I sent specimens of *Neur. macrophylla,* kindly compared them with the type of *Neur. auriculata,* and informed me that *Neur. auriculata* has a much closer nervation than *Neur. macrophylla,* and the apex of the pinnules of *Neur. auriculata* is rounded. As the nervation forms a constant character for distinguishing the species of the genus *Neuropteris, Neur. auriculata* cannot be united with *Neur. macrophylla.* The form of the pinnules, at least in the present case, seems of little specific value. A figure is given on Plate XXII. fig. 3, of a specimen from Radstock in the Bath Museum. On the left side of the rachis the pinnules are oval, and very blunt, whereas on the right they are lanceolate. These differences are very clearly exhibited towards the apex of the fossil.

The posterior basal angle of the pinnules is usually more or less auricled. The various forms assumed by the pinnules of this species will be best appreciated by an examination of the three figures that accompany these notes. To Plate XXII. fig. 3, reference has already been made; at fig. 2 of the same plate a fragment of a pinna is given, which, in addition to showing lanceolate pinnules, exhibits the dichotomous ramification of the rachis. The pinnules situated at the angles formed by the dichotomies are of very irregular shape, being frequently triangular and irregular, sometimes even bifid at their apex, as if two pinnules had become confluent. The most interesting specimen I figure is that on Pl. XXI. fig. 2. On this example (which lies on the corner of a very large slab of *Neur. macrophylla*, which I received from Mr STEART, manager of the Braysdown Colliery), the gradual transition in the form of the pinnules from lanceolate to cyclopteroid can be followed. On some specimens in the collection of Mr J. M'MURTRIE, F.G.S., large cyclopteroid pinnules occur on a thick rachis, which may be the main rachis of the frond.

From *Neur. Scheuchzeri*, Hoffm., this species is easily distinguished by the absence of the small cyclopteroid pinnules at the base of the large terminal lobe, by its being destitute of the bristle-like hairs, and, above all, by its nervation. In *Neur. macrophylla* the ultimate dichotomy of the veins is much closer to the margin of the pinnule than the corresponding dichotomy in *Neur. Scheuchzeri*, and in the latter species the veins are closer.

From the figure of *Neur. Scheuchzeri* given by ZEILLER (*loc. cit.*, pl. xli. fig. 1), it appears that this species possesses a similar dichotomous ramification of the pinnæ to that which maintains in *Neur. macrophylla.*

Through the kindness of Mr C. CASH, F.G.S., Halifax, Yorkshire, I have been able to compare with my specimens of *Neur. macrophylla* a specimen of *Neur. Clarksoni*, Lesqx., from OLYPHANT, which was communicated to him by Mr R. D. LACOE, and find this last mentioned species is specifically identical with BRONGNIART'S plant.

I am inclined to refer the figure given by SCHEUCHZER (*loc. cit.*, pl. x. fig. 3) to *Neur. macrophylla*, Brongt., though several writers have placed it under *Neur. Scheuchzeri*, Hoffm. From the roughness of SCHEUCHZER'S figure, it is impossible definitely to refer it to either of these species.

While writing my Catalogue of Palæozoic Plants in the British Museum, believing that SCHEUCHZER'S figure should be referred to HOFFMANN'S *Neur. Scheuchzeri* (which, however, I treated as distinct from *Neur. cordata*, L. & H. = *Neur. hirsuta*, Lesqx.), I identified in error the specimens of *Neur. macrophylla* as *Neur. Scheuchzeri*, Hoffm.

The inaccurate drawing of the nervation of BRONGNIART'S type of *Neur. macrophylla* prevented me from identifying the specimens in the British Museum with his plant, to which, however, they really belong, and it was only on a subsequent examination of the specimens in the collection of the Geological Society of London that I detected the type of *Neur. macrophylla*, which enabled me to discover my former error.

Localities:—Dunkerton (Type); Wellsway; Radstock; Upper Conygre; Lower Conygre; Braysdown; Kilmersdon.

Neuropteris Scheuchzeri, Hoffmann.
Plate XXIII. figs. 1, 2.

Neuropteris Scheuchzeri, Hoffm., Keferstein's *Teuchland geognostisch-geologisch dargestellt.*, vol. iv. p. 156, pl. i.*b*, figs. 1–4, 1826.

Neuropteris Scheuchzeri, Zeiller, "Flore Houillère d. Asturies," p. 10 (*Mem. Soc. Géol. du Nord*, 1882).

Neuropteris Scheuchzeri, Zeiller, *Gites Minéraux de la France. Descrip. de la Flore Foss. Bassin houil. d. Valenciennes*, pl. xli. figs. 1–3, 1886.

Neuropteris angustifolia, Brongt., *Hist. d. végét. foss.*, p. 231, pl. lxiv. figs. 3–4.

Neuropteris acutifolia, Brongt., *Hist. d. végét. foss.*, p. 231, pl. lxiv. figs. 6, 7.

Neuropteris acutifolia, Ettingshausen, *Foss. Flora v. Radnitz*, p. 32, pl. xviii. fig. 5.

Neuropteris acutifolia, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 22, pl. xxvii. fig. 8.

Neuropteris acutifolia, Gutbier, *Vers. d. Zwick. Schwarzkohl*, p. 52, pl. vii. fig. 6.

Neuropteris cordata (not Brongniart), Bunbury, *Quart. Jour. Geol. Soc.*, vol. iii. p. 423, pl. xxi. fig. 1, *a–f.*

Neuropteris cordata, Göppert, *Foss. Flora d. perm. Form.*, p. 100, pl. xi. figs. 1, 2.

Neuropteris cordata, Lindley & Hutton, *Foss. Flora*, vol. i. pl. xli.

Neuropteris cordata, Kidston, *Catalogue of Palæozoic Plants*, p. 98.

Neuropteris hirsuta, Lesquereux, *Rep. Geol. Survey of Illin.*, vol. ii. p. 427, pl. xxxv. figs. 6–10.

Neuropteris hirsuta, Lesquereux, *Coal Flora of Pennsyl.*, p. 88, pl. viii. figs. 1, 4, 5, 7, 9, 12.

Dictyopteris cordata, Römer, *Palæontographica*, vol. ix. p. 30, pl. vi. fig. 4, 1862.

Dictyopteris Scheuchzeri, Römer, *Palæontographica*, vol. ix. p. 30, pl. ix. fig. 1, 1862.

Description.—Frond large, ultimate pinnæ alternate, lanceolate, and usually composed of a large terminal and two small cyclopteroid pinnules; medial vein extending to within a very short distance of the apex, lateral veins fine, close, numerous, usually divided four times,—the fourth dichotomy occurring about midway between the midrib and the margin, and reaching the edge of the pinnule at a wide angle. The surface of the pinnules bears irregularly scattered short bristle-like hairs. Large cyclopteroid pinnules are also present on the frond.

Remarks.—The specimen I figure, which is in the collection of the Bath Museum, shows a fragment of what must have been a very large frond. At the top of the specimen the pinnæ are reduced to a large pinnule with a basal lobe. The other pinnæ consist of a very large terminal pinnule, at the base of which, on each side of the rachis, is a small cyclopteroid pinnule. The terminal pinnule marked *x* is 8 cm. long, but the one immediately below it must have been larger.

Probably no species of the Carboniferous flora has been so much misunderstood and misidentified as *Neur. Scheuchzeri*, Hoffm. This arises from the imperfect figures and description of the type specimens, to which may be added the difficulty in obtaining the work in which the original description appears.

We are chiefly indebted to Zeiller for unravelling the synonymy of this species.

Neur. Scheuchzeri has, in the great majority of cases, been identified in error as *Neur. cordata*, Brongt.*

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* *Hist. d. végét. foss.*, p. 229, pl. lxiv. fig. 5.

Of *Neur. cordata*, BRONGNIART only figured a single pinnule, which in its form closely resembles the pinnules of *Neur. Scheuchzeri.* The type of *Neur. cordata* appears to be lost, but ZEILLER has discovered in the Museum at Paris many other specimens named *Neur. cordata* by BRONGNIART himself; these, however, embrace two species. Some of them are the true *Neur. cordata* as figured by BRONGNIART and others, are identical with the plants named *Neur. acutifolia,* Brongt., and *Neur. angustifolia,* Brongt. ZEILLER has very kindly sent me a specimen from the mines of Alais, Grand' Combe, of the plant he identifies as the true *Neur. cordata*, Brongt. With this before me there is no difficulty in recognising BRONGNIART'S *Neur. cordata* as essentially distinct from *Neur. Scheuchzeri,* with which, as will be presently seen, must be united *Neur. acutifolia,* Brongt., and *Neur. angustifolia*, Brongt. In *Neur. cordata* the veins are not nearly so close to each other as those of *Neur. Scheuchzeri,* and in addition to this the characteristic hairs of *Neur. Scheuchzeri* are absent from *Neur. cordata*;—even on specimens of *Neur. Scheuchzeri* where, through imperfect preservation, the hairs are not visible, the nervation is a sufficiently distinctive character by which to distinguish the two species.

The nervation of *Neur. Scheuchzeri,* enlarged three times, is shown on Plate XXIII. fig. 2. The hairs are omitted from this figure to avoid confusion. It will be observed from this drawing that the veins usually divide four times,—the first dichotomy being close to the midrib, the second and third dichotomy carry the veins to about midway between the central vein and the margin of the pinnule, and the arms of the fourth dichotomy extend from this point to the edge of the pinnule. A fifth dichotomy is but rarely observed, and equally rarely do its veins only divide three times throughout their course.

An enlarged drawing, to show the bristle-like hairs, is given at fig. 1*a*. These usually lie obliquely to the veins, imparting to the pinnule a *dictyopteroid* appearance, which has given rise to RÖMER'S *Dictyopteris cordata* and *Dictyopteris Scheuchzeri.*

On the original specimens of *Neur. Scheuchzeri* the presence of the hairs appears to have escaped HOFFMANN'S observation, or had been effaced through imperfect preservation, but, from his description of the plant and the accompanying figures, the identification of the specimens occurring in the Radstock Coal Field with HOFFMANN'S *Neur. Scheuchzeri* appears to be correct beyond doubt.

ZEILLER has examined the types of *Neur. acutifolia* and *Neur. angustifolia* which originated from "Camerton" and "near Bath," and "Wilkesbarre in Pennsylvania," and has observed on them the characteristic hairs, though BRONGNIART in his description of the two species does not indicate their existence. These two supposed species do not differ from each other except in the outline of the pinnules, which is not of sufficient value for specific distinction, and, further, it has been shown by ZEILLER that they are specifically identical with *Neur. Scheuchzeri,* Hoffm.

BUNBURY was the first to point out the presence of hairs on specimens of *Neur. Scheuchzeri* from Cape Breton, though he identified his plants as *Neur. cordata.*[*] He also mentions that he had observed these little hairs on specimens of *Neur. cordata,* L. & H. (not Brongniart), from Leebotwood (the locality from which LINDLEY and HUTTON's examples come), in the collection of the Geological Society of London, and this observation I am able to corroborate.

BUNBURY here also places *Neur. angustifolia,* Brongt., as a *variety* of *Neur. cordata,* and further suggests that *Neur. cordata, Neur. angustifolia, Neur. acutifolia,* and *Neur. Scheuchzeri* of BRONGNIART are all of them forms of *Neur. cordata.*

LESQUEREUX[†] at one time expressed a similar belief, but subsequently he treated *Neur. cordata, Neur. hirsuta,* and *Neur. angustifolia* as distinct, giving LINDLEY and HUTTON's figure as a reference under *Neur. cordata,* Brongt., but, as already stated, the specimens from Leebotwood belong to *Neur. Scheuchzeri,* and not to *Neur. cordata,* Brongt.[‡]

Neur. hirsuta, Lesq.,[§] agrees in every respect with *Neur. Scheuchzeri;* LESQUEREUX's species, however, was created before the true characters of *Neur. Scheuchzeri* were clearly understood; but it must now be reduced to a synonym of the latter-mentioned plant.

GÖPPERT gives a figure in his *Permian Flora* which he names *Neur. cordata.*[||] This does not show the small cyclopteroid pinnules that are generally present, but the form of the large pinnules and their nervation, as shown by his enlargement, agree entirely with *Neur. Scheuchzeri,* to which plant I believe his fern may belong.

ZEILLER, in his excellent remarks on *Neur. Scheuchzeri,* to which I am much indebted for a right understanding of this species, includes under *Neur. Scheuchzeri* the figure of a *Neuropteris* from England given by SCHEUCHZER in his *Herbarium Deluvianum,* pl. x. fig. 3 (edition 1709). It is impossible to speak definitely on the specific position of the fern figured by SCHEUCHZER, but I feel more inclined to identify it as *Neur. macrophylla,* Brongt., than *Neur. Scheuchzeri,* Hoffm.

While preparing the Catalogue of the Palæozoic Plants in the British Museum, with only figures and descriptions of these species before me, and all

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[*] *Quart. Jour. Geol. Soc.,* vol. iii. p. 424, 1847.

[†] In ROGER's *Geol. of Pennsyl.,* vol. ii. part 2, p. 857, 1858.

[‡] The nervation of LINDLEY and HUTTON's figures is very diagrammatic, and by no means represents the correct nervation of the plant they figure.

[§] *Coal Flora of Pennsyl.,* pp. 88, 89, 91, 1880.

[||] Pl. xi. fig. 1.

of them, with one exception, misidentifications, I now find that the plants I there placed under *Neur. cordata*, L. & H. (? Brongt.),[*] should be referred to *Neur. Scheuchzeri*, Hoffm., and the ferns I placed under *Neur. Scheuchzeri* must be referred to *Neur. macrophylla*, Brongt. Further remarks will be found on this subject under *Neur. macrophylla*, Brongt.

My thanks are due to the Rev. H. H. WINWOOD, F.G.S., for facilities given for figuring the fine specimen shown on Pl. XXIII. fig. 1.

Neur. Scheuchzeri occurs in several of the English coal fields.

Localities:—Braysdown; Radstock; Upper Conygre; Lower Conygre; Camerton; Wellsway.

Neuropteris flexuosa, Sternberg.

Neuropteris flexuosa, Sternb., *Vers.*, i. fasc. iv. p. xvi.
Neuropteris flexuosa, Brongt., *Hist. d. végét. foss.*, p. 239, pl. lxviii. fig. 2; pl. lxv. figs. 2, 3.
Neuropteris flexuosa, Schimper, *Traité d. paléont. végét.*, vol. i. p. 434, pl. xxx. figs. 12, 13.
Neuropteris flexuosa, Kidston, *Catal. Palæoz. Plants*, p. 93.
Osmunda gigantea, var. β, Sternb., *Vers.*, i. pp. 36 and 39, pl. xxxii. fig. 2.
Neuropteris plicata, Lesqx., *Coal Flora of Pennsyl.*, pl. x. figs. 1–4.

Remarks.—Frequent. The plants figured as *Neuropteris plicata* by LESQUEREUX,[†] seem clearly referable to *Neur. flexuosa*, Sternb., whatever may be the true value of STERNBERG's species.[‡]

Var. **rotundifolia.**

Neuropteris rotundifolia, Brongt., *Hist. d. végét. foss.*, p. 238, pl. lxx. fig. 1.
Neuropteris rotundifolia, Gutbier, *Vers. d. Zwick. Schwarzkohl*, p. 56, pl. vii. figs. 3, 4.

Remarks.—This form has been found at Camerton, but it passes into *Neur. flexuosa*, of which it can only be regarded as a varietal form.

Localities:—Radstock; Camerton; Upper Conygre; Lower Conygre.

Neuropteris ovata, Hoffmann.

Plate XXII. fig. 1.

Neuropteris ovata, Hoffmann, Keferstein's *Teuchland geognostisch-geologisch dargestellt*, vol. iv. p. 158, pl. 1*b*, figs. 5, 6, 7 (excl. fig. 8), 1826.

Description.—Frond much divided; rachis of primary pinnæ broad and finely striated; secondary pinnæ subopposite; rachis stout; pinnules alternate, oblong, apex rounded, superior basal angle rounded and sloping inwards, inferior angle produced as a rounded auricle. Veins fine, close, arched,

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[*] The true *Neuropteris cordata* has not yet, as far as I am aware, been discovered in Great Britain.

[†] *N. plicata*, Lesqx., *Coal Flora*, *loc. cit.*

[‡] *Vers.*, i. fasc. 4, p. xvi.; ii. p. 74, pl. xix. figs. 1–3.

usually divided four times, and meeting the margin of the pinnule at an acute angle. Midrib, strictly speaking, absent. Terminal lobe but little enlarged, broadly lanceolate, and generally confluent with uppermost pinnule or pinnules.

Remarks.—The specimens I identify as HOFFMANN'S plant agree in all respects with the figures and description given by him.

Neur. ovata has a great similarity in general appearance to *Neur. flexuosa*, Sternb., but is distinguished from it by constant and well-marked characters. Both species occur in the Radstock Coal Field; *Neur. flexuosa* is of frequent occurrence, but *Neur. ovata* is scarcely so common.

The terminal pinnule in *Neur. ovata* is never enlarged as in *Neur. flexuosa.* It is usually more or less broadly lanceolate, and at its basal extremity is connected with the uppermost pinnule or pinnules. The pinnules are auricled in a manner similar to those of *Neur. flexuosa,* but they do not overlap so much as in the latter-mentioned species. The veins are more arched than in *Neur. flexuosa,* and also appear to be more numerous.

A few of the upper pinnules are attached by their whole base to the rachis; the others are articulated by a short, almost imperceptible footstalk. A carefully enlarged drawing of a pinnule to show the nervation is given at fig. 1*a*. A true midrib can scarcely be said to be present. One or two veins, springing from the base of the pinnule, lie almost parallel, but, before reaching the apex, are lost in repeated dichotomies.

I have excluded from HOFFMANN'S reference his fig. 8, as there is really no evidence to show that this figure belongs to *Neur. ovata,* and much less that it should be regarded as the fruit of that species.*

HEER appears to have included under *Neur. flexuosa* more than one species of *Neuropteris.*† Some of his figures, I believe, should be referred to *Neur. ovata* (*cf.* pl. ii. fig. 2; pl. iii. fig. 2, &c.).

As neither the figure nor the description of the plant given by RÖMER as *Neur. ovata* agrees very well with HOFFMANN'S figures or description, I am doubtful of the correctness of RÖMER'S identification.‡

Neur. ovata is liable to be mistaken for a small form of *Neur. flexuosa,* but a comparison of well-preserved specimens of the two species will, I believe, at once show their specific individuality.

At fig. 1, Plate XXII., are given some pinnæ of *Neur. ovata,* drawn natural size.

Localities:—Upper Conygre; Camerton; Radstock; Wellsway.

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* See Kidston, *Trans. Roy. Soc. Edin.*, vol. xxxiii. pt. i. p. 150, 1887.

† *Flora foss. Helvetiæ*, p. 20, pls. ii. figs. 1–7; iii. 1–5; iv. 7–13; v. 2, 3.

‡ *Palæontographica*, vol. ix. p. 28, pl. vi. fig. 1.

Neuropteris rarinervis, Bunbury.

Neuropteris rarinervis, Bunbury, *Quart. Jour. Geol. Soc.*, vol. iii. p. 425, pl. xxii.
Neuropteris rarinervis, Lesqx., *Coal Flora of Pennsyl.*, p. 109, pl. xv. figs. 2–5.
Neuropteris rarinervis, Zeiller, *Bull. soc. géol. de France*, 3e sér., vol. xii. p. 197.
Neuropteris rarinervis, Kidston, *Catal. Palæoz. Plants*, p. 91.

Remarks.—Though occurring at most of the localities visited, this fern is nowhere plentiful in the Radstock Coal Field.

Localities :—Radstock; Camerton; Wellsway; Upper Conygre; Lower Conygre.

Neuropteris fimbriata, Lesqx.

Plate XXI. figs. 3–5.

Neuropteris fimbriata, Lesqx., *Geol. Report of Illin.*, vol. ii. p. 430; vol. iv. p. 384, pl. vi. fig. 4.
Neuropteris fimbriata, Lesqx., *Coal Flora of Pennsyl.*, vol. i. p. 81, pl. v. figs. 1–6.

Remarks.—A few isolated pinnules have been met with which may perhaps be referred to *Neur. fimbriata*, Lesqx. These are shown on Pl. XXI. figs. 3–5.

In the specimen given at fig. 5 the veins are a little finer than in those of figs. 3 and 4, and it may possibly be a small specimen of *Neuropteris* (*Cyclopteris*) *lacerata*, Heer.*

The other two figures, however, appear to agree more closely with LESQUEREUX'S *Neur. fimbriata*, but HEER'S and LESQUEREUX'S species approach very closely to each other, and the distinctive characters are not very prominent.

The fimbriation of the pinnules is a natural character, and not produced by an accidental flaying out of the tissue.

Localities :—Upper Conygre; Camerton; Wellsway.

Dictyopteris, Gutbier, 1835, *Abdrücke und Versteinerungen des Zwickauer Schwarzkohlengebirges*, p. 62.

Dictyopteris Münsteri, Eichwald, sp.

Plate XXI. fig. 6.

Odontopteris Münsteri, Eichwald, *Die Urwelt Russlands*, Heft i. p. 87, pl. iii. fig. 2, 1840.
Dictyopteris Münsteri, Schimper, *Traité d. paléont. végét.*, vol. i. p. 618, 1869.
Dictyopteris Münsteri, Zeiller, *Bull. soc. geol. de France*, 3e sér., p. 197, vol. xii.; *Études d. Gites Minéraux de la France; Bassin houil. d. Valenciennes, Descr. d. l. Flore Foss.*, pl. xlix. figs. 1–5, 1886.
Dictyopteris Hoffmanni, Römer, *Palæontographica*, vol. ix. p. 29, pl. vii. fig. 3, 1862.
Dictyopteris Hoffmanni, Schimper, *Traité d. paléont. végét.*, vol. i. p. 619.

Description.—Frond tripinnate, secondary pinnæ alternate, lanceolate; pinnules alternate, from shortly oval to oblong in form; upper pinnules united

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* *Flora foss. Helvetiæ*, p. 17, pl. vi. fig. 7.

to the rachis by their whole base, lower attached by a short footstalk, articulated. Anterior basal angle of pinnule rounded, posterior basal angle slightly auricled. Medial vein flexuous, and extending almost to the apex. Lateral veins dividing several times, anastomosing, and forming an irregular network. Meshes next the midrib longer than those further removed from it. Terminal pinnule lanceolate. The frond also bears large cyclopteroid pinnules.

Remarks.—I am indebted to Mr George West, Camerton, for the fine specimen of this species shown on Pl. XXI. fig. 6. From the inequality of the pinnæ on opposite sides of the rachis, the example is evidently only a pinna. At the apex are several large simple lanceolate pinnules; on the third highest pinna of those preserved, on each side of the terminal pinnule are a pair of almost semicircular pinnules (fig. 6*a* × 3) attached to the rachis by their whole base. On the lower pinnæ the pinnules are oblong, with occasionally slightly tapered apices.

The veins form a very loose and irregular network. Those next the flexuous midrib are elongated in the longer direction of the pinnule, *i.e.*, more or less parallel with the midrib. The meshes formed by the subsequent dichotomies of the veins are directed more upwards and outwards, and become smaller towards the margin of the pinnule. The reticulation is formed rather by a bending of the veins towards each other than by their actual union. The pinnules on the main figure are not so large as some shown at the part indicated by an *x*. Zeiller figures a cyclopteroid pinnule of this species (*loc. cit.*, fig. 4). These were probably borne on the main rachis as in *Neuropteris*.

A comparison of my example with Römer's *Dictyopteris Hoffmanni* leaves no doubt as to the identity of the two plants. But it seems equally clear that *D. Hoffmanni*, Römer, is only a more perfect specimen of *D. Münsteri*, Eich., sp., and this opinion has already been indicated by Zeiller.[*]

Through the kindness of Dr Weiss I have been enabled to compare a specimen of *D. Hoffmanni* from Piesberg (the original locality of this species) with the Camerton plant; their nervation is similar, but in the size of the pinnules the Piesberg example agrees more with the figures of *D. Münsteri*, as given by Zeiller, than the plant figured by me.

The presence of the large terminal pinnule does not seem to be a constant character, for on some of the specimens of *D. Münsteri*, figured by Zeiller, which agree exactly on their nervation with the specimen of *D. Hoffmanni* sent me by Dr Weiss, the terminal pinnules are comparatively small.

Locality:—Camerton.

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[*] *Bull. soc. geol. de France*, 3^{e} sér., vol. xii. p. 197.

Odontopteris, Brongniart, 1822, *Sur la classification des végétaux fossiles,* p. 34.

Odontopteris Lindleyana, Sternb.

Odontopteris Lindleyana, Sternb., *Vers.,* ii. p. 78.
Odontopteris obtusa, L. & H. (not Brongt.), *Fossil Flora,* vol. i. pl. xl.
(?) *Odontopteris heterophylla,* Lesquereux, *Geol. Rep. of Illin.,* vol. ii. p. 433, pl. xxxviii. figs. 2, 5.
(?) *Odontopteris heterophylla, Coal Flora of Pennsyl.,* vol. i. p. 129, pl. xxii. fig. 6.

Remarks.—The type of LINDLEY and HUTTON'S species is preserved in the University Museum, Oxford, but the figure given in the *Fossil Flora* is not a very correct representation of the specimen.

Odontopteris Lindleyana is very rare in the Radstock Coal Field, but occurs more plentifully in the shale over Pontydwaith Seam, Pochin Pit, near Tredegar, South Wales. The examination of the type and the additional specimens from Radstock and South Wales has shown that LESQUEREUX'S *Odontopteris heterophylla* is probably identical with the plant figured in error by LINDLEY and HUTTON as *Odontopteris obtusa.*

Localities :—Radstock; Braysdown.

Mariopteris, Zeiller, 1879, *Bull. soc. géol. de France,* 3e sér., vol. vii. p. 92.

Mariopteris nervosa, Brongt., sp.

Mariopteris nervosa, Zeiller, *Végét. foss. du. terr. houil.,* p. 69, pl. clxvii. figs. 1–4.
Pecopteris nervosa, Brongt., *Hist. d. végét. foss.,* p. 297, pl. xciv.; pl. xcv. figs. 1, 2.
Pecopteris nervosa, Lindley and Hutton, *Fossil Flora,* vol. ii. pl. xciv.
Alethopteris nervosa, Geinitz, *Vers. d. Steinkf. in Sachsen,* p. 30, pl. xxxiii. figs. 2, 3.
Pseudopecopteris nervosa, Lesqx., *Coal Flora of Pennsyl.,* vol. i. p. 197, pl. xxxvi. figs. 1–3.

Remarks.—Very rare.
Localities :—Radstock; Upper Conygre.

Mariopteris muricata, Schlotheim, sp.

Mariopteris muricata, Zeiller, *Végét. foss. du terr. houil.,* p. 71, pl. clxvii. fig. 5.
Pecopteris muricata, Brongt., *Hist. d. végét. foss.,* p. 352, pl. xcv. figs. 3, 4; pl. xcvii.
Alethopteris muricata, Roehl, *Foss. Flora d. Steink. Form. Westphalens,* p. 78, pl. xi. fig. 1.
Pseudopecopteris muricata, Lesqx., *Coal Flora in Pennsyl.,* p. 203, pl. xxxvii. fig. 2.
Filicites muricatus, Schlotheim, *Flora d. Vorwelt,* p. 54, pl. xii. figs. 21 and 23.

Remarks.—Very rare.
Locality :—Radstock.

EXPLANATION OF PLATES.

Plate XIX.

Fig. 1. *Sphenopteris Woodwardi*, Kidston, n. sp., Camerton (nat. size), p. 348.
Fig. 1*a.b.c.* *Sphenopteris Woodwardi*, pinnules enlarged, showing nervation.
Fig. 2. *Sphenopteris tenuifolia*, Gutbier (Brongt. ?), Upper Conygre Pit, Timsbury (nat. size), p. 345.
Fig. 2*a.b.* *Sphenopteris tenuifolia*, pinnules enlarged 3 times, showing nervation.
Fig. 3. *Sphenopteris species*, Old Mills Pit, Farrington, Guerney (nat. size). *Farrington Series*, p. 411.

Plate XX.

Fig. 1. *Schizostachys sphenopteroides*, Kidston, n. sp., Radstock (nat. size), p. 352.
Fig. 1*a.* *Schizostachys sphenopteroides*, sporangia enlarged 3 times.
Fig. 2. *Ptychocarpus oblongus*, Kidston, n. sp., Camerton (nat. size), p. 350.
Fig. 2*a.* *Ptychocarpus oblongus*, two pinnules enlarged 3 times.
Fig. 2*b.* *Ptychocarpus oblongus*, synanguim ? further enlarged.
Fig. 3. *Rhacophyllum spinosum*, Lesquereux, Radstock (nat. size). Specimen in the collection of Mr J. M'Murtrie, p. 389.

Plate XXI.

Fig. 1. *Sphenopteris geniculata*, Germar and Kaulfuss, Kilmersdon Pit (nat. size), p. 346.
Fig. 1*a.b.* *Sphenopteris geniculata*, pinnules enlarged, showing nervation.
Fig. 2. *Neuropteris macrophylla*, Brongt., Braysdown (nat. size), p. 354.
Fig. 3. *Neuropteris fimbriata*, Lesquereux, Wellsway Pit (nat. size), p. 361.
Fig. 4. *Neuropteris fimbriata*, Camerton (nat. size).
Fig. 5. *Neuropteris fimbriata* (?), Upper Conygre Pit, Timsbury (nat. size).
Fig. 6. *Dictyopteris Münsteri*, Eichwald, sp., Camerton (nat. size), p. 361.
Fig. 6*a.b.* *Dictyopteris Münsteri*, pinnules enlarged, showing nervation.

Plate XXII.

Fig. 1. *Neuropteris ovata*, Hoffmann, Camerton (nat. size), p. 359.
Fig. 1*a.* *Neuropteris ovata*, pinnule enlarged 2½ times, to show the nervation.
Fig. 2. *Neuropteris macrophylla*, Brongt., Radstock (nat. size), p. 354.
Fig. 2*a.* *Neuropteris macrophylla*, pinnule enlarged 2 times, to show the nervation.
Fig. 3. *Neuropteris macrophylla*, Radstock (nat. size). Specimen in the collection of the Bath Museum.
Fig. 3*a.* *Neuropteris macrophylla*, portion of pinnule enlarged, to show the nervation.

Plate XXIII.

Fig. 1. *Neuropteris Scheuchzeri*, Hoffmann, Radstock (nat. size). Specimen in the collection of the Bath Museum, p. 356.
Fig. 1*a.* *Neuropteris Scheuchzeri*, portion of pinnule enlarged, to show the hairs.
Fig. 2. *Neuropteris Scheuchzeri*, portion of pinnule enlarged 3 times, to show the nervation.
Fig. 3. *Trigonocarpus Noeggerathi*, Brongt. (nat. size). In the collection of Mr J. M'Murtrie, p. 403.
Fig. 4. *Rhabdocarpus multistriatus*, Presl, sp., Radstock (nat. size), p. 404.
Fig. 5. *Cardiocarpus Gutbiere*, Geinitz, Radstock (nat. size). In the collection of Mr J. M'Murtrie, p. 403.
Fig. 6. *Cardiocarpus*, Upper Conygre Pit, Timsbury (nat. size), p. 403.
Fig. 7. *Carpolithus ovoideus*, Göppert and Berger, Wellsway Pit (nat. size), p. 404.
Fig. 8. *Carpolithus ovoideus*, Camerton (nat. size).

PART II.

(Read 6th June 1887.)

Pecopteris, Brongniart.

Pecopteris, Brongt., *Sur la Classification des Végétaux Fossiles*, p. 33, 1822.
Cyatheites, Göppert, *Syst. fil. foss.*, p. 319, 1836.
Asterocarpus, Göppert, *Syst. fil. foss.*, p. 188, 1836.
Scolecopteris, Zenker, *Linnæa*, vol. xi. p. 509, 1837.
Hawlea, Corda, *Flora protogæa*, p. 90, 1845.

Remarks.—Several generic names, originating from some supposed likeness to recent genera, or from the arrangement of the sporangia, have been proposed by different authors for the ferns included here, and which were originally placed by Brongniart in his genus *Pecopteris.*

The name *Cyatheites* was given by Göppert to certain members of the genus on account of their supposed resemblance to some of the *Cyathea.* This supposed resemblance was dependent in great measure on imperfect preservation of the fruit.

The genus *Asterocarpus* of the same author was founded to comprise certain *Pecopterids*, in which the *exannulate sporangia* are arranged in a stellate manner ; the greater number of his *Cyatheites* are now known, from the structure of their fruit, to belong to his *Asterocarpus.*

In *Scolecopteris,* Zenker, the *exannulate sporangia* are also arranged in stellate groups, but the individual sporangia are produced upwards in a sharp point, thus differing from *Asterocarpus,* where the sporangia are short.

Hawlea, Corda, is most probably identical with *Asterocarpus.*

The upper surface of the pinnules of many species of *Pecopteris* is covered with short closely adpressed hairs. This villosity has been observed on many of the Radstock species, viz.,—*Pec. arborescens, Pec. arborescens,* var. *cyathea, Pec. oreopteridia, Pec. villosa,* and *Pec. Miltoni* (*Pec. abbreviata*), and I have also observed the same character on specimens of typical *Pec. Miltoni,* from Claycross, Derbyshire, and Ashton-under-Lyne, Lancashire, and on *Pec. polymorpha* from the Forest of Dean. On several of these species a villosity has previously been observed. It is probable that this villosity will be found to be much more common in the genus *Pecopteris* than generally supposed, as it is only observable on specimens in an exceptionally good state of preservation.

The fruit of many of the species of the genus has been observed and described.

Pecopteris arborescens, Schlotheim, sp.

Pecopteris arborescens, Brongt., *Hist. d. végét. foss.*, p. 310, pls. cii.; ciii.; figs. 2, 3.
Pecopteris arborescens, Germar, *Vers. v. Wettin u. Löbejun*, p. 97, pls. xxxiv, xxxv. (fig. 4?).
Pecopteris arborescens, Grand' Eury, *Flore Carbon du Départ. de la Loire*, p. 68, pl. viii. fig. 6.
Pecopteris arborescens, Zeiller, *Végét. foss. d. terr. houil.*, p. 81, pl. clxix. fig. 4.
Pecopteris arborescens, Kidston, *Catal. Palæoz. Plants*, pp. 113 and 253.
Cyatheites arborescens, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 24, pl. xxviii. figs. 7–11.
Cyatheites arborescens, Heer, *Flora foss. Helv.*, p. 27, pl. viii. figs. 1–4.
Cyathocarpus arborescens, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 84.
Filicites arborescens, Schlotheim, *Flora d. Vorwelt*, p. 41, pl. viii. figs. 13, 14.
Pecopteris platyrachis, Brongt., *Hist. d. végét. foss.*, p. 312, pl. cii. figs. 4, 5.
Pecopteris aspidioides, Brongt., *Hist. d. végét. foss.*, p. 311, pl. cxii. fig. 2.
Asplenites nodosus, Göpp., *Syst. fil. foss.*, p. 280, pl. xix. figs. 1–3.
Pecopteris cyatheoides, Schimper, *Traité d. paléont. végét.*, vol. i. p. 523, pl. xli. fig. 14.
Pecopteris cyathea, Brongt., *Hist. d. végét. foss.*, p. 307, pl. ci. figs. 1–3 (excl. fig. 4 = *P. Candolliana*).
Pecopteris cyathea, Grand' Eury, *Flore Carbon d. Départ. de la Loire*, p. 68, pl. viii. fig. 7.
Pecopteris cyathea, Zeiller, *Végét. foss. du terr. houil.*, p. 82, pl. clxix. figs. 5, 6.
Filicites cyatheus, Schlotheim, *Flora d. Vorwelt*, p. 38, pl. vii. fig. 11.
Aspidites decussatus, Göpp., *Syst. fil. foss.*, p. 369, pl. xxvi. figs. 1, 2.

Remarks.—In typical *Pec. arborescens* the veins are simple; in the form distinguished by SCHLOTHEIM as *Filicites cyatheus* the veins are sometimes simple, but usually once divided, and even occasionally divided three times. The pinnules are also more oblong than in typical *Pec. arborescens.*

The majority of botanists unite *Pec. cyathea* with *Pec. arborescens,* but, among recent writers, ZEILLER and GRAND' EURY keep them separate. The species is very common in the Radstock area, and occurs in a very fine state of preservation. After carefully examining many examples, though the specimens can generally be referred to their respective forms without much difficulty, they are so connected by intermediate conditions that I can only regard *Pec. arborescens* and *Pec. cyathea* as different states of one species.

The upper surface of the pinnules of some of the typical specimens of *Pec. arborescens* and the form *cyathea* is covered with short adpressed hairs, similar to those on the specimen of *Pec.* (*Scolecopteris*) *cyathea* figured by STUR.*

The *Asplenites nodosus,* Göpp., is only a somewhat imperfectly preserved fruiting specimen of *Pec. arborescens,* and his *Aspidites decussatus* is apparently the corresponding condition of SCHLOTHEIM'S *Filicites cyatheus.* Specimens agreeing with both these so-called species have been collected.

The form *cyathea* is of as frequent occurrence as typical *Pec. arborescens.*

Localities:—Radstock; Wellsway Pit, Braysdown; Kilmersdon Pit; Upper Conygre Pit; Camerton.

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* STUR, *Sitzb. d. k. Akad. d. Wissensch.*, vol. lxxxviii. Abth. i. p. 750, fig. 25, 1883.

Pecopteris Candolliana, Brongniart.

Pecopteris Candolliana, Brongt., *Hist. d. végét. foss.*, p. 305, pl. c. fig. 1.
Pecopteris Candolliana, Germar, *Vers. v. Wettin u. Löbejun*, p. 108, pl. xxxviii.
Pecopteris Candolliana, Grand' Eury, *Flore Carb. du Départ. de la Loire*, p. 69, pl. viii. fig. 8.
Pecopteris Candollei, Zeiller, *Végét. foss. du terr. houil.*, p. 84.
Cyathocarpus Candolleanus, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 85.
Pecopteris affinis, Brongt. (not Schlotheim), *Hist. d. végét. foss.*, p. 306, pl. c. figs. 2, 3.
Pecopteris cyathea, Brongt. (in part), *Hist. d. végét. foss.*, pl. ci. fig. 4.

Remarks.—Very rare.

Localities :—Radstock; Braysdown Colliery.

(?) Pecopteris asper, Brongniart.

Pecopteris asper, Brongt., *Hist. d. végét. foss.*, p. 339, pl. cxx. figs. 1–4.
Pecopteris asper, Zeiller, *Bull. soc. géol. d. France*, 3e sér., vol. xii. p. 202; *Flore foss. d. Bassin Houil. d. Valenciennes*, pl. xxix. figs. 1–3.

Remarks.—I refer to this species two small specimens collected at Timsbury, but, owing to their fragmentary nature, it is desirable that more perfect examples be examined before definitely recording the occurrence of this species.

Locality :—Upper Conygre Pit.

Pecopteris pennæformis, Brongniart.

Pecopteris pennæformis, Brongt., *Hist. d. végét. foss.*, p. 345, pl. cxviii. figs. 3, 4.
Pecopteris pennæformis, Brongt., *Class. d. végét. foss.*, p. 33, pl. ii. fig. 3.
Pecopteris pennæformis, Schimper, *Traité d. paléont. végét.*, vol. i. p. 505.
Pecopteris pennæformis, Zeiller, *Bull soc. géol. d. France*, 3e sér., vol. xii. p. 201, 1883; *Flore foss. d. Bassin houil. d. Valenciennes*, pl. xxx. figs. 1–4.
Pecopteris æqualis, Brongt., *Hist. d. végét. foss.*, p. 343, pl. cxviii. figs. 1, 2.

Remarks.—Extremely rare, only a single specimen having been met with.

Locality :—Camerton.

Pecopteris unita, Brongniart.
Pl. XXIV. figs. 2–9.

Pecopteris unita, Brongt., *Hist. d. végét. foss.*, p. 342, pl. cxvi. figs. 1–5.
Pecopteris unita, Kidston, *Catal. Palæoz. Plants*, p. 122 (excl. syn. *Pecopteris elegans*).
Pecopteris unita, Grand' Eury, *Flore Carb. du Départ. de la Loire*, p. 76, pl. viii. fig. 13.
Pecopteris unita, Lesqx., *Coal Flora of Pennsyl.*, p. 223, pl. xl. figs. 1–7.
Cyatheites unitus, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 25, pl. xxix. figs. 4, 5.
Cyathocarpus unitus, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 88, pl. xii. figs. 5, 6.
Goniopteris elliptica, Font. and White, *Perm. or Upper Carb. Flora*, p. 83, pl. xxx. fig. 1.

Pecopteris unita, *forma* emarginata, Göpp., sp.

Pecopteris longifolia, Brongt., *Hist. d. végét. foss.*, p. 273, pl. lxxxiii. fig. 2.
Pecopteris longifolia, Germar, *Vers. v. Wettin u. Löbejun*, p. 34, pl. xiii. figs. 2–4.
Stichopteris longifolia, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 97, pls. ix., x. figs. 7, 8.
Pecopteris emarginata, Bunbury, *Quart. Jour. Geol. Soc.*, vol. ii. p. 86, pl. vi., 1846.
Pecopteris emarginata, Lesqx., *Coal Flora of Pennsyl.*, p. 225, pl. xxxix. fig. 11.
Diplazites emarginatus, Göpp., *Syst. fil. foss.*, p. 274, pl. xvi. figs. 1, 2.

Remarks.—This species is common throughout the whole of the Radstock area, but usually occurs in a fragmentary condition, seldom more than isolated pinnæ being met with.

Many botanists regard *Pec. emarginata,* Göpp., sp. (= *Pec. longifolia,* Brongt., not Sternb.), as specifically distinct from *Pec. unita,* Brongt.; on the other hand, some regard them as only different portions of one species.

I have carefully collected specimens of the plants that have been referred respectively to *Pec. unita,* Brongt., and *Pec. emarginata,* Göpp., sp., and compared them with specimens of the latter species from Wettin, with which many of the Radstock examples agree, but have failed to discover any character by which *Pec. emarginata* can be separated specifically from *Pec. unita.* They seem to me so to pass into each other that their separation appears arbitrary, and not determined by any fixed character peculiar to either form.

A few specimens in fruit, identical with WEISS's figure of *Stichopteris emarginata* (=*Pec. emarginata,* Göpp., sp.), have also been met with.

For the satisfaction of those who may regard *Pec. emarginata* as a distinct species, its distribution is given separately under the distinction of *Pec. unita, forma emarginata.*

Description of Specimens Figured.

Pl. XXIV. fig. 3, *Pec. unita,* Brongt.; from New Mills Pit (*Farrington Series*).—This sketch shows the typical plant as figured by BRONGNIART in his *Hist. d. végét. foss.,* pl. cxvi. fig. 1. On the lower pinnæ the pinnules are separate to the base, but as the pinnæ approach the apex of the specimen (which from the inequality of the pinnæ on the two sides of the rachis is evidently a pinna and not the terminal portion of a frond), the pinnules become more or less united among themselves, till on the uppermost pinnæ the pinnules are so completely united that the pinnæ appear entire or only slightly dentate. At fig. 3*a* are given two pinnules, enlarged 2 times, from a lower pinnæ, to show the nervation.

The veinlets are sometimes almost straight, but usually curved upwards (as in fig. 5*a*), though occasionally curved outwards (as in fig. 3*a*).

Pl. XXIV. fig. 9, *Pec. unita,* Brongt.; Camerton.—This figure shows the pinnules united to each other for about two-thirds of their entire length. The specimen is a portion of a primary (?) pinna nearer its base than that just described (fig. 3), and corresponds to BRONGNIART's fig. 5, pl. cxvi. The veinlets are curved upwards,—the two contiguous basal veinlets coalescing and extending to the base of the notch that separates the free portions of neighbouring pinnules. This arrangement of the nervation—the union of the two basal contiguous veins and the formation of a veinless triangle at the base and between the contiguous pinnules—has induced some botanists to employ PRESL'S genus *Goniopteris* for this and some allied species.

Pl. XXIV. fig. 5, *Pec. unita*, Brongt.; Old Mills Pit (*Farrington Series*).—This small specimen differs from the last in the segments being slightly more elliptical, the specimen being in fact the *Goniopteris elliptica*, Fontaine and White.*

Pl. XXIV. fig. 4, *Pec. unita*, Brongt.; Camerton.—This would perhaps be regarded by some as *Pec. longifolia*, Brongt., but I believe it to be the uppermost entire pinnæ of *Pec. unita.* A like view is taken of a similar specimen figured by WEISS in his *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, pl. xii. fig. 5.

Probably the *Pec. lanceolata*, Lesqx., should also be referred to *Pec. unita* as its upper portion.†

Pl. XXIV. fig. 6, *Pec. unita, forma emarginata;* Camerton.—This specimen is clearly the *Pec. longifolia*, Brongt. (=*P. emarginata*, Göpp.).‡ Enlarged drawings of the nervation are given at fig. 6*a*. In comparing this specimen with that given at fig. 4, the differences are not greater than what occur in pinnæ situated on different parts of the same frond.

Figs. 4 and 6 are similar to the specimens GERMAR has figured as *Pec. longifolia* in his *Vers. d. Steink. v. Wettin u. Löbejun*, fasc. 3, pl. xiii. My fig. 4 corresponds to his fig. 2, and my fig. 6 to his figs. 3, 4.

Pl. XXIV. figs. 7, 8, *Pec. unita, forma emarginata.* Fig. 7 from Radstock; fig. 8 from Upper Conygre Pit, Timsbury.—These figures also represent the *Stichopteris longifolia*, Brongt., sp., of WEISS,§ which is evidently similar to *Pec. emarginatus*, Göpp., sp., as figured by BUNBURY.

Pl. XXIV. fig. 2, *Pec. unita, forma emarginata;* from Camerton.—This specimen would also be referred, by those who regard *Pec. unita* and *Pec. emarginata* or *longifolia* as distinct species, to the latter plant.

In regard to the various figures I have given in illustration of the different forms assumed by *Pec. unita*, if only characteristic specimens of *Pec. unita*, Brongt. (fig. 3), are dealt with on the one hand, or those characteristic of *Pec. longifolia* (figs. 2 and 6) on the other hand, one would probably be led to conclude that there were here two very distinct species. When, however, a large series of specimens is examined, these two supposed species are so intimately connected by intermediate forms that I have found myself unable definitely to say where *Pec. unita* ends and *Pec. emarginata* begins. I therefore class the latter as a form of *Pec. unita.*

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* *Perm. or Upper Carb. Flora*, p. 83, pl. xxx. fig. 1.

† *Pecopteris lanceolata*, Lesqx., *Coal Flora of Pennsyl.*, p. 227, pl. xxxix. figs. 9, 10=*Alethopteris lanceolata*, Lesqx., *Geol. Rep. of Illin.*, vol. iv. p. 398, pl. xiii. figs. 1–3.

‡ *Hist. d. végét. foss.*, pl. lxxxiii. fig. 2.

§ *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, pls. ix., x. figs. 7, 8.

|| *Quart. Jour. Geol. Soc.*, vol. ii. p. 86, pl. vi., 1846.

In my *Catalogue of Palæozoic Plants* I united *Pec. elegans* with *Pec. unita* in error.[*]

Localities:—

Pec. unita, Brongt.

Radstock; Braysdown Colliery; Upper Conygre Pit; Lower Conygre Pit; Kilmersdon Pit; Wellsway Pit; Camerton.

Pec. unita, Brongt., *forma emarginata*, Göpp., sp.

Wellsway Pit; Braysdown Colliery; Camerton; Radstock; Upper Conygre Pit.

Pecopteris villosa, Brongniart.

Pecopteris villosa, Brongt., *Hist. d. végét. foss.*, p. 316, pl. civ. fig. 3.

Remarks.—This species is included here merely as having been founded by Brongniart on a specimen from near Bath.

It is now well known that the outer surface of the pinnules of various species of *Pecopteris* possesses a villosity, and as this villosity often quite obscures the veins, it is occasionally extremely difficult, if not even impossible, to determine to which species such specimens should be referred. Zeiller suggests that *Pec. villosa* may perhaps belong to *Pec. abbreviata*, Brongt. (= *Pec. Miltoni*, Artis), but as the nervation on the type specimen is obliterated, it is quite impossible, in the absence of this most important character for the specific determination of the members of the genus *Pecopteris*, to decide to which species it should be referred.

Having observed a villosity on an undoubted specimen of *Pec. oreopteridia*, Schl., sp.,[†] and from the general outline of *Pec. villosa*, Brongt. agreeing so closely with that of *Pec. oreopteridia*, I am inclined to refer *Pec. villosa* to that species. The point, however, cannot be definitely settled from the meagre evidence before us.[‡]

Only one thing seems clear, that *Pec. villosa*, Brongt., most probably represents a condition of a species known under another name when the veins are not obscured by the villosity, and is not itself a plant possessing a true individuality.

The *Cyatheites villosus*, Geinitz,[§] is referable to *Pec. abbreviata*, Brongt. (= *Pec. Miltoni*, Artis).

Locality:—Near Bath (*Brongniart*).

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[*] See Zeiller, *Végét. foss. d. terr. houil.*, p. 93, pl. clxvi. figs. 5, 6.

[†] See Pl. XXVII. figs. 3, 4.

[‡] See Zeiller, "Notes sur la flore houillère des Asturies," *Mem. Soc. Géol. du Nord*, 1882, p. 12.

[§] *Vers. d. Steinkf. in Sachsen*, pl. xxix. figs. 6–8.

Pecopteris oreopteridia, Schlotheim, sp.

Pl. XXVII. figs. 3, 4; Pl. XXVIII. figs. 1, 2.

Pecopteris oreopteridia, Brongt., *Hist. d. végét. foss.*, p. 317, pl. civ. figs. 1, 2; pl. v. figs. 1, 2, 3.
Pecopteris oreopteridia, Renault, *Cours d. botan. foss.*, 1883, p. 110, pl. xviii. figs. 5, 5*bis*; pl. xix. figs. 7–12.
Pecopteris oreopteridia, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 66.
Pecopteris oreopteridia, Zeiller, *Bull. soc. géol. d. France*, 3e sér., vol. xiii. p. 138, pl. ix. figs. 1, 1*a*.
Cyatheites oreopteroides, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 25, pl. xxviii. fig. 14.
Filicites oreopteridius, Schlotheim, *Flora d. Vorwelt*, p. 36, pl. vi. fig. 9.

Remarks.—This species is common in the Radstock Series, and some very fine specimens have been collected. Of these, one which I received from Mr Job Moon, Camerton, deserves special note. This example shows three primary (?) pinnæ springing from a common rachis, of which only the central primary pinna is perfect. It measures 14½ inches in length, and its greatest diameter, which is towards the centre of the pinna, is 6 inches. The pinna to the right of this one is longer, but, not being perfect, I am unable to give its exact measurements. In outline the primary pinnæ are broadly lanceolate.

Of the three primary (?) pinnæ shown on the portion of the frond that has been preserved, the uppermost primary (?) pinna exhibits still, on the inferior side of its rachis, portions of 17 secondary (?) pinnæ. These are all barren.

The central primary (?) pinna bears about 36 pairs of alternate lanceolate secondary (?) pinnæ, of which the 6 or 7 lower pairs are barren, or only bear a few fruiting pinnules; the succeeding 6 or 7 pairs of pinnæ are soriferous, the fruit being borne on the central pinnules of the pinnæ. The remaining upper pinnæ are barren.

On the remaining and lowest of the three primary (?) pinnæ preserved on the specimen, the 15 lowest secondary (?) pinnæ are soriferous, the remaining upper pinnæ being barren.

Two secondary (?) pinnæ from the central primary (?) pinna are shown on Pl. XXVIII. figs. 1, 2. On fig. 1 the fruiting pinnules are seen to occupy the central part of the pinna; on fig. 2 only the third pair from the base are soriferous.

The most interesting point in connection with these soriferous pinnules is the occurrence on them of a dense villous covering. An enlarged drawing of such a pinnule is shown on Pl. XXVII. fig. 4. On a few of the barren pinnules a villosity can also be detected, but it is so slight that it cannot be compared in importance or prominence with that of the soriferous pinnules.

An enlarged barren pinnule (Pl. XXVII. fig. 3) shows from its nervation that this specimen is clearly referable to *Pec. oreopteridia*, Schl., sp.

Since detecting the presence of a villosity on the pinnules of *Pec. oreopteridia*, I have been led to suspect that perhaps *Pec. villosa,* Brongt., should be referred to this species.

Localities:—Radstock; Braysdown Colliery; Upper Conygre Pit; Camerton.

Pecopteris Cistii, Brongniart.

Pecopteris Cistii, Brongt., *Hist. d. végét. foss.,* p. 330, pl. cvi.

Remarks.—I only know this species from Brongniart's figures and description. Of the two specimens figured by him, one came from Dunkerton, near Bath, and the other from Wilkesbarre, Pennsylvania.

Dunkerton Pit is now closed, but I have carefully examined other localities where the same coals are worked in the hope of rediscovering this species, but have hitherto been unsuccessful. Lesquereux says in regard to *Pec. Cistii* —" Though I have seen many fragments referred to it, I have never been able to positively recognise in any the characters indicated by the author."* I can fully endorse this statement, for, though I have also seen specimens labelled "*Pec. Cistii,*" none of them possessed characters entirely in agreement with Brongniart's description, and were, I am afraid, only *Pec. oreopteridia.*

The type from Dunkerton, which belonged to the Museum of the University of Oxford, appears to have been lost or mislaid, for, when visiting that collection a short time ago, we were unable to discover it.

Locality:—Dunkerton (*Brongniart*).

Pecopteris Bucklandii, Brongniart.

Pecopteris Bucklandii, Brongt., *Hist. d. végét. foss.,* p. 319, pl. xcix. fig. 2.
Pecopteris Bucklandii, Grand' Eury, *Flore Carb. du Départ. de la Loire,* p. 75.
Pecopteris Bucklandii, Schimper, *Traité d. paléont. végét.,* vol. i. p. 504.
Pecopteris Bucklandii, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.,* p. 64.
Pecopteris pseudo-Bucklandii, Germar, *Vers. d. Steink. v. Wettin u. Löbejun,* p. 106, pl. xxxvii.

Remarks.—This species is very rare, but a few examples have been collected at Camerton—the original locality.

It is very doubtful if the fern figured as *Pec. Bucklandii* by Lindley and Hutton† really belongs to this species. If their figure is correct, I am inclined to think that it does not. Stur suggests that Lindley and Hutton's plant may perhaps be his *Hawlea Schaumberg-Lippeana,* but this is also very doubtful.‡

Locality:—Camerton.

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* *Coal Flora of Pennsyl.,* p. 244.

† *Fossil Flora,* vol. iii. pl. ccxxiii.

‡ *Carbon-Flora d. Schatzlarer-Schichten,* p. 120, pl. lvii. fig. 1; pl. lviii. figs. 1–4.

Pecopteris pteroides, Brongniart.

Pecopteris pteroides, Brongt., *Hist. d. végét. foss.*, p. 329, pl. xcix. fig. 1.
Pecopteris pteroides, Germar, *Vers. d. Steink. v. Wettin u. Löbejun*, p. 103, pl. xxxiv.
Pecopteris pteroides, Grand' Eury, *Flore Carb. du Départ. de la Loire*, p. 75.
Asterocarpus pteroides, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 91.

Remarks.—*Pec. pteroides* is extremely rare; the only specimen I have seen from the Radstock Series is one in the British Museum, labelled as coming from "near Bath."

The figures given by GEINITZ as *Pec.* (*Alethopteris*) *pteroides* do not appear to belong to this species.*

Locality:—"Near Bath."

Pecopteris crenulata, Brongniart.

Pecopteris crenulata, Brongt., *Hist., d. végét. foss.*, p. 300, pl. lxxxvii. fig. 1.
Pecopteris crenulata, Zeiller, *Bull. soc. géol. d. France*, 3e sér., vol. xii. p. 200, 1883; *Flore foss. du Bassin houil. de Valenciennes*, pl. xxv. figs. 1–4.

Remarks.—I refer to this species a single specimen collected at Camerton. It is, however, possible that some other examples which I have not yet been able satisfactorily to identify may belong to this species.

Locality:—Camerton.

Pecopteris polymorpha, Brongniart.

Pecopteris polymorpha, Brongt., *Hist. d. végét. foss.*, p. 331, pl. cxiii.
Pecopteris polymorpha, Grand' Eury, *Flore Carb. du Départ. de la Loire*, p. 74, pl. viii. figs. 10, 11.
Pecopteris polymorpha, Renault, *Cours d. botan. foss.*, p. 116, pl. xx. figs. 1–10, 1883.
Pecopteris polymorpha, Zeiller, *Végét. foss. d. terr. houil.*, p. 91, pl. clxix. figs. 1, 2, 3.
Pecopteris Miltoni, Brongt. (not Artis), in part, *Hist. d. végét. foss.*, pl. cxiv. figs. 2, 7 (other figs.?).

Remarks.—Not common. A good fruiting example was collected at Radstock.

I have detected the presence of short adpressed hairs on the upper surface of the pinnules of a specimen of this species from Trafalgar Colliery, near Drybrook, Forest of Dean. This villosity is so copious that on some of the pinnules the nervation is almost completely obscured.

Localities:—Radstock; Braysdown Colliery; Camerton.

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* *Vers. de Steinkf. in Sachsen*, pl. xxxii. figs. 1–5.

Pecopteris Miltoni, Artis, sp.

Pecopteris Miltoni, Germar, *Vers. d. Steink. v. Wettin u. Löbejun*, p. 63, pl. xxvii. (excl. syn. *Pec. polymorpha*, and *P. Miltoni*, Brongt., not Artis).

Pecopteris Miltoni, Sterzel, *Die Flora d. Rothl. im nordwest. Sachsen*, p. 6, pl. i. (xxi.) figs. 1–7 (in Dames & Kayser's *Paléont. Abhandl.*, Band iii. Heft ii. p. 240 (excl. syn. *Pec. polymorpha*).

Cyatheites Miltoni, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 27, pl. xxx. fig. 5 (fig. 6 ?), var. *abbreviata*, pl. xxx. figs. 7–8 ; pl. xxxi. figs. 1 (2, 3 ?), 4 (refs. in part).

Filicites Miltoni, Artis, *Antedil. Phytol.*, pl. xiv.

Pecopteris crenata, Sternberg, *Vers.*, i. p. xx. pl. x. fig. 7 ; ii. p. 154.

Hawlea pulcherrima, Corda, *Flora protogæa*, p. 90, pl. lvii. figs. 7, 8.

Hawlea Miltoni, Stur, *Carbon-Flora*, p. 108, pl. lix. and pl. lx. (excl. figs. 3–4., syn. in part).

(?) *Goniopteris brevifolia*, Schimper, *Traité d. paléont. végét.*, vol. i. p. 546.

Pecopteris abbreviata, Brongt., *Hist. d. végét. foss.*, p. 337, pl. cxv. figs. 1–4.

Pecopteris abbreviata, L. & H., *Fossil Flora*, pl. clxxxiv.

Pecopteris abbreviata, Zeiller, "Notes sur la flore houillère des Asturies," p. 12, (*Mem. Géol. Soc. du Nord*, 1882) ; *Flore foss. du Bassin houill. de Valenciennes*, pl. xxiv. figs. 1–4, 1886.

Cyatheites villosus, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 25, pl. xxix. figs. 6-8.*

Remarks.—A great difference of opinion exists among botanists in regard to the specific value of *Pec. Miltoni*, Artis, sp., *Pec. abbreviata*, Brongt., and *Pec. polymorpha*, Brongt.

For a few years I have carefully collected those species and visited several of the British Coal Fields where they occur, with the special object of satisfying myself as to the true relations of *Pec. Miltoni*, *Pec. abbreviata*, and *Pec. polymorpha* to each other.

Several authors have united them under one name.† In the barren condition, the discrimination of the species is often difficult. In *Pec. polymorpha* the nervation is closer, the divisions of the veinlets more numerous and straighter than in *Pec. Miltoni* and *Pec. abbreviata*, where the nervation has often a slight flexuosity. *Pec. abbreviata*, as will be seen, I regard as identical with *Pec. Miltoni.*

Even in the barren condition, I believe, *Pec. polymorpha* can be safely separated from all other species, if the specimens are well preserved and at all typical. Its fruit, however, at once establishes its individuality, and clearly separates it from *Pec. Miltoni.*

Pec. Miltoni was described by ARTIS from El-se-car Colliery, near Milton Furnace, Yorkshire, in 1825 :—*Pec. abbreviata* by BRONGNIART from mines near

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* I am very doubtful if the following species referred by STUR to *Pec. Miltoni* should be so included :—*Asplenites heterophyllus*, Göpp.; *Aspl. crispus*, Göpp. ; *Baluntites Martii*, Göpp. The following, also included by the same author, appear to me to have no connection with *Pec. Miltoni* :—*Adiantites giganteus*, Göpp. ; *Cyclopteris obliqua*, L. & H. ; *Schizopteris lactuca*, Roehl. (*Foss. Flora Westph.*, pl. xviii.) ; and *Cyclop. oblata*, L. & H.

† Geinitz, *loc. cit.*; Germar, *loc. cit.*; Sterzel, *loc. cit.*

Bath (Radstock Coal Field), and from the mines of Anzin, near Valenciennes, Départment du Nord.

I may mention here that the plants figured by BRONGNIART as *Pec. Miltoni, Hist. d. végét. foss.*, pl. cxiv., with perhaps the exception of his fig. 8, probably do not belong to this species, but to his own *Pec. polymorpha.** BRONGNIART'S figs. 2 and 7 evidently belong to *Pec. polymorpha*; his figs. 1, 3, 4, 5, and 6 most probably also belong to the same fern, but on these figures I express no definite opinion. His fig. 8 has been raised to specific rank by SCHIMPER, under the name of *Goniopteris brevifolia,*† but I think it is referable to *Pec. Miltoni.* I have collected at Radstock specimens which I cannot distinguish from it. BRONGNIART'S figure does not give much data from which to form any satisfactory opinion.

The type of *Pec. Miltoni* has disappeared, but while visiting some museums and private collections in Lancashire, Yorkshire, and Derbyshire, where coals are worked on or about the same horizon as those from which *Pec. Miltoni* was derived, I met with a number of undoubted specimens of it; but I am specially indebted to Mr GEORGE WILD, Bardsley Colliery, Ashton-under-Lyne, and to Dr PEGLER, Stonebroom, Derbyshire, for facilities for examining specimens of this species.

In my several visits to Radstock I also collected many fine specimens of the plant described as *Pec. abbreviata* by BRONGNIART. These specimens I have compared with the figures and descriptions of *Pec. Miltoni* and *Pec. abbreviata,* and have also compared the specimens from different localities with each other, as well as with some from the Coal Field of Valenciennes, kindly sent me by M. ZEILLER, but have failed to discover any character by which they can be separated.

It is admitted by all, including those who regard *Pec. Miltoni* and *Pec. abbreviata* as distinct species, that *Pec. abbreviata* at all events is very polymorphic, and those who are most intimate with this fern are most cognizant of this fact.

ZEILLER has carefully entered into this subject in his *Notes sur la flore houillère des Asturies* in his remarks on *Pec. abbreviata.* He describes the little hairs on the upper surface of the pinnules of this species, whose presence, often entirely obscuring the nervation, has led to its being identified as *Pec. villosa,* and as an instance he cites the identification of *Pec. abbreviata* as *Pec. villosa* by GEINITZ.‡

Whether the *Pec. villosa,* Brongt., can be referred to *Pec. abbreviata* or not, must in the meantime from want of evidence remain an open question.

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* *See* ZEILLER, *Mem. Soc. Géol. du Nord, loc. cit.*

† *Traité d. paléont. végét.*, vol. i. p. 546.

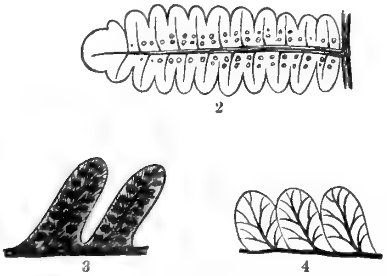
‡ That this so-called *Pec. villosa* is in reality the *Pec. Miltoni* (= *Pec. abbreviata,* Brongt.) will, I think, be admitted by all who have studied the subject.

Referring to the specific value of *Pec. Miltoni*, Artis, sp., and *Pec. abbreviata*, Brongt., ZEILLER says :—" In regard to the question whether *Pec. Miltoni* and *Pec. abbreviata* are not identical, and of which *Pec. Miltoni* is the older name, having been founded in 1825, though not arriving at a wholly sure conclusion, I incline meanwhile towards the negative.

" The general form of the pinnules indicated by ARTIS appears very analogous to those of *Pec. abbreviata*, but the nervation is not figured, which renders a comparison almost impossible, this character being almost the only one by which one is able satisfactorily to support it; further, the figure and diagnoses given by the author indicate the sori to be marginal, or almost marginal, while I have already mentioned that the groups of capsules of *Pec. abbreviata* cover all the inferior surface of the pinnule, and are by no means marginal. The figures given by GEINITZ, under the name of *Cyatheites Miltoni*,* show likewise the fructification almost marginal (pl. xxx. figs. 6, 6*a*, and also *Cyatheites Miltoni* var. *abbreviatus*, figs. 8, 8*a*, and 8*b*). This character of the disposition of the *sori* appears to me sufficiently important to compel one to regard *Pec. abbreviata* as decidedly distinct from *Pec. Miltoni.* When, as to its union with *Pec. polymorpha*, proposed by various authors, it is hardly necessary to mention that the characters of the fructification separate absolutely these two species, *Pec. abbreviata* having the short capsules of *Asterotheca*, and *Pec. polymorpha* the long sharp capsules of *Scolecopteris.* They belong further to different horizons" (" *niveaux différents* ").†

I must first refer to *Pec. Miltoni* as being of older date than *Pec. abbreviata.*

As is frequently the case with *Pec. abbreviata*, in *Pec. Miltoni* the nervation is seldom shown on account of the dense villosity with which the upper surface of the pinnules is covered. This villosity is in all respects identical to that occurring on the pinnules of specimens which have been distinguished as *Pec. abbreviata*, Brongt., from the Radstock Series. At text fig. 4 is given an enlarged drawing of three pinnules of *Pec. Miltoni*, Artis, sp., from Claycross, to show the nervation. This is absolutely identical in all respects with the enlargement of *Pec. abbreviata*, given by BRONGNIART on his pl. cxv. fig. 3*a*. There can be no doubt that the plant occurring at Claycross, Derbyshire (Middle Coal Measures), is the true *Pec. Miltoni*, as its growth, segmentation, and the



Pecopteris Miltoni, Artis, sp.
Fig. 2. From Bardsley Colliery, Ashton-under-Lyne, Lancashire.
Figs. 3, 4. From Claycross, Derbyshire (Middle Coal Measures).
Figures enlarged two diameters.

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* *Vers. d. Steinkf. in Sachsen*, p. 27, pl. xxx. figs. 5–8; pl. xxxi. figs. 1–4.

† ZEILLER, *Notes sur la flore houillère des Asturies*, p. 13. I may remark in passing, that in England *Pec. polymorpha* and *Pec. Miltoni* (including *Pec. abbreviata*) occur on the same horizon.

general character of many of the specimens is identical with the figure given by ARTIS.

Pec. Miltoni, Artis, sp. (whatever view may be taken of the relationship of *Pec. abbreviata* to it), is very polymorphic in the form and size of the pinnules. ARTIS has only figured one condition of his plant, a condition which probably corresponds to BRONGNIART'S *Pec. abbreviata*, fig. 1.

BRONGNIART gave several figures of his species, and these have been well supplemented by ZEILLER.* Forms corresponding to the figures of these authors occur among the Yorkshire, Lancashire, and Derbyshire specimens of *Pec. Miltoni;* in the barren condition, neither from the form of the pinnules nor their nervation can I discover any fixed character by which *Pec. abbreviata*, Brongt., can be separated from *Pec. Miltoni,* Artis, sp.

The only remaining point of comparison is the fructification. On this ARTIS says:—"*Fructifications surrounding the leaflets near, but not entirely on the margin.*" And again—"*The fructifications seated on the back of the leaves are not so closely seated on the margin as is expressed in the plate.*"

Many of the specimens of *Pec. Miltoni* which I have examined, from the counties already mentioned, are in fruit, though none have been in a condition to exhibit its minute structure, such as the number of sporangia that compose the sori, or the shape of the individual sporangia. These specimens, however, clearly indicate the position of the sori, which appear as little circular dots, as shown in the woodcut, figs. 2 and 3.

Fig. 2 is a pinnæ from a Lancashire example; fig. 3 from a Derbyshire plant. On both, the position of the fruit, as clearly indicated, is not marginal. In fact ARTIS, in referring to his own figure, clearly states that the fruit is not so closely seated on the margin as is expressed on his plate. Now, if his figure be carefully examined, it will be seen that on many of the pinnules the fruit holds almost a central position between the margin and the midrib, and certainly if it is not so near the margin as represented, it cannot be other than situated almost midway between the midrib and the margin, and then the sori will cover the whole of the under surface of the pinnule, as figured by ZEILLER.† In the pinnæ, where the pinnules are united throughout the greater part of their length, the fruit forms a single or double row along the midrib of the pinnæ; or, in other words, the pinnules only bear one or two groups of sori, situated at their base, as seen in woodcut, fig. 2.

Fig. 3 shows two pinnules densely clothed on their upper surface with short hairs,‡ which quite obliterate the veins. The fruit is seen as circular dots holding a similar position to those figured on *Pec. abbreviata* by ZEILLER. From

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* *Flore foss. du Bassin houil. d. Valenciennes,* pl. xxiv.

† *Loc. cit.*, pl. xxiv. figs. 3, 4, 4*a*, 4*b*.

‡ These are also present on most of the pinnæ of the specimen from which fig. 1 was taken.

the examination of several such specimens I am led to conclude that the fruit of *Pec. Miltoni*, Artis, sp., and *Pec. abbreviata*, Brongt., is essentially the same, and therefore there exists no specific difference between *Pec. Miltoni* and *Pec. abbreviata*, hence BRONGNIART'S species must be united with *Pec. Miltoni*, Artis, sp.

REMARKS ON SOME FIGURES OF THE SPECIES.

Pec. Miltoni, Germar, *Vers. v. Wettin u. Löbejun*, Heft 6, pl. xxvii. 1849.

The figures given here are very characteristic of *Pec. Miltoni*, Artis, sp. The nervation agrees entirely with that of the specimens from Lancashire, Yorkshire, and Derbyshire. His fig. 2 corresponds to ARTIS'S type.

I have received from Dr WEISS a specimen of *Pec. Miltoni* from Wettin, which confirms this opinion.

GEINITZ, *Vers. d. Steinkf. in Sachsen*, 1855.
Cyatheites Miltoni, pl. xxx. figs. 5, 6; pl. xxxi. figs. 1–4.
Cyatheites Miltoni, var. *abbreviatus*, pl. xxx. figs. 7, 8.

Of these, fig. 5 (pl. xxx.) is characteristic of ARTIS'S species, but I have never seen the fruit as represented at fig. 6. Fig. 8 (pl. xxx.) probably corresponds to my woodcut, fig. 2. Pl. xxxi. fig. 4, gives a fair idea of the position of the fruit. Fig. 3 probably does not belong to *Pec. Miltoni*; it is not, at all events, a characteristic figure of the species.

It must be noticed that in all the fruiting examples given by GEINITZ the *sori are immature*, and only represented by circular swellings. Were the sori more fully developed, they would occupy a much larger area of the surface of the pinnule.

Hawlea Miltoni, Stur, *Carbon-Flora*, pl. xlix. fig. 1, 1885.

Though no enlarged details of this specimen are given, the fossil as represented in the plate is absolutely identical with typical *Pec. Miltoni*. All the other figures in this plate are too indistinct to admit of any criticism. Pl. xl. figs. 3, 4, I exclude from *Pec. Miltoni*.

Pec. Miltoni, Sterzel, *Flora d. Rothliegenden im nordw. Sachsen*, pl. i. figs. 1–7.*

The figures given here, though small, possess the characters of *Pec. Miltoni*.

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* In Dames and Kayser, *Palæontologische Abhandl.*, vol. iii. Heft. 4, p. 237, Berlin, 1886.

Hawlea pulcherrima, Corda, *Flora protogæa*, pl. lvii. figs. 7, 8, 1845.

I have already stated my belief that *H. pulcherrima* is a fruiting specimen of *Pec. Miltoni*, Artis, sp.[*] STUR gives as the difference between the fruit of *H. pulcherrima* and *Pec. Miltoni*, that the former has shorter and broader pinnules (*Tertiarabschnitte*) and shorter and broader sporangia than *Pec. Miltoni*, where the pinnules and sporangia are narrower and longer. The fern being so polymorphic, and the form of the pinnules so greatly depending on their position on the frond, render this difference in form as a specific character in the case under discussion quite valueless. In regard to the other supposed distinguishing character—the form of the sporangia,—the slight differences pointed out by STUR are far too slight to be of real value; they are even scarcely distinguishable in his text figure which illustrates this point,[‡] and entirely disappear when *H. pulcherrima* is compared with ZEILLER's figures of the fruit of *Pec.* (*abbreviata*) *Miltoni.*[§]

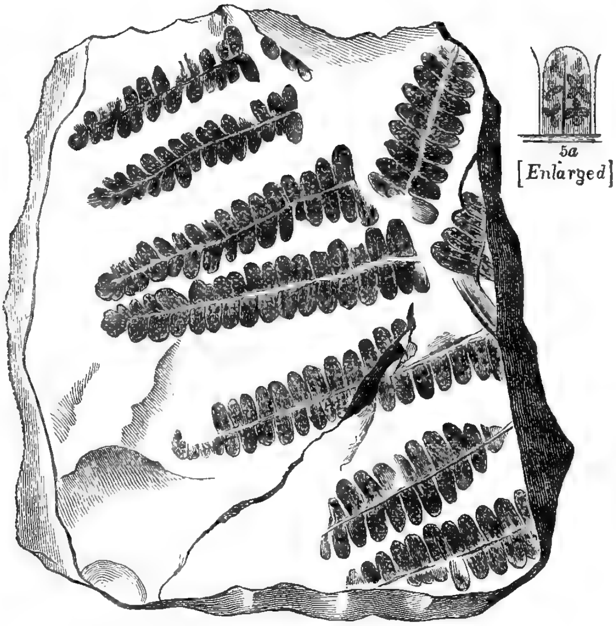

Fig. 5. (Nat. size.)
Pec. Miltoni, Artis, sp. (= *H. pulcherrima*, Corda), Forest of Wyre, Worcestershire.
In the collection of the British Museum.[†]

I give a text figure (5) of a specimen from the Forest of Wyre, Worcestershire, in the collection of the British Museum, which appears to me to be at

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[*] *Catalogue of Palæoz. Plants*, p. 121. In the remarks here appended to *Pec. Miltoni* I have inadvertently referred to *H. pulcherrima* as *Hawlea* "*elegans.*" I have also (p. 256) referred *A. aquilina*, Geinitz, *Vers. d. Steinkf. in Sachsen*, pl. xxxi. figs. 5–7, to *Pec. Miltoni*. I still think it probable that his figures 6, 7 are referable to this species, but am more doubtful about his figure 5. They are therefore omitted meantime from the synonymy of *Pec. Miltoni*, Artis, sp.

[†] I am indebted to Dr Woodwart, F.R.S., for permission to describe this specimen.

[‡] *Loc. cit.*, p. 106, fig. 17.

[§] *Loc. cit.*, pl. xxiv. figs. 3, 4.

once the *H. pulcherrima* of CORDA and the fruit of *Pec. Miltoni.* The fossil occurs as a dark brown impression of the lower surface of the frond on a yellow-brown matrix. On the left of the rachis, of which only a small portion is shown, are the remains of 7 pinnæ. The 3 lower ones are imperfect, but the 4 upper show their complete length. The pinnules at the base of the pinnæ are contiguous and oblong, with blunt apices; the upper pinnules are confluent. The whole of the surface of the pinnules is thickly covered with stellate groups of sporangia, hence the veins are only seen at a few points of the specimen, and even then indistinctly. The *sporangia* are oval, and arranged in stellate groups of 3–6 sporangia, though 4 is the common number in each little star. The central point of attachment of the groups of sporangia is situated midway between the central vein and the margin of the pinnule, which latter bears a greater or less number of sori. In this example the pinnules near the apex of the pinnæ bear very few sori; those about the centre bear three on each side of the midrib, whilst the lowest pinnules bear 5 or 6 sori on each side of the main nerve. The little stellate groups of sporangia measure in their greatest diameter 1 mm. to 1·5 mm., the individual sporangia ranging from ·5 to ·75 mm. This specimen was associated with barren examples of *Pecopteris Miltoni,* Artis, sp.

Localities :—Radstock ; Braysdown Colliery ; Camerton ; Welton ; Wellsway Pit ; Lower Conygre Pit.

Pecopteris Lamuriana, Heer.

Pecopteris Lamuriana, Heer, *Urwelt der Schweiz,* p. 13, fig. 12.
Pecopteris Lamuriana, Zeiller, *Bull. soc. géol. d. France,* 3e sér., vol. xiii. p. 139.
Alethopteris Lamuriana, Heer, *Flora foss. Helv.,* p. 32, pl. xii. figs. 6, 7.

Remarks.—This species is very rare, only two specimens having been collected.

Localities :—Radstock ; Braysdown Colliery.

Pecopteris pinnatifida, Gutbier, sp.

Neuropteris pinnatifida, Gutbier, *Vers. d. Zwick. Schwarzkohl,* p. 61, pl. viii. figs. 1–3.
Neuropteris pinnatifida, Gutbier, *Vers. d. Zechst. u. Rothl.,* p. 13, pl. v. figs. 1–4.
Pecopteris pinnatifida, Schimper, *Traité d. paléont. végét.,* vol. i. p. 507.
Pecopteris Geinitzii, Gutbier, *Vers. d. Zechst. u. Rothl.,* p. 16, pl. ii. fig. 10; pl. ix. figs. 1–3; pl. xi. figs. 5, 6.
? *Pecopteris fruticosa,* Gutbier, *Vers. d. Zechst. u. Rothl.,* p. 16, pl. v. figs. 8. 9.
Asterocarpus pinnatifidus, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.,* p. 93.

Remarks.—Of this fern four specimens have been collected,—three at Radstock and one at the Upper Conygre Pit, Timsbury. The figures given by GUTBIER of this species are very rough, but with the plant as figured in his

Vers. d. Zwick. Schwarzkohl, pl. viii. figs. 1–3, and in his *Vers. d. Zechst. u. Rothl.*, pl. v. figs. 1, 2, the Somerset examples agree entirely.

The nervation of this species is described by GUTBIER as "nerves strong, once bifurcated, in pairs, or fascicled (*fiderig*) according to the pinnulation." *

In the almost entire pinnules of one of our specimens, one of the arms of the dichotomously divided lateral veins usually divides again.

Pec. Geinitzii has been united with *Pec. pinnatifida* by WEISS and SCHIMPER, and in this I have followed them. Such a course, however, makes it difficult to reconcile the two figures given by GUTBIER of the fruit of his *Pec. pinnatifida* † with those he gives of the fruit of his *Pec. Geinitzii,* which is *Asterocarpous!*

Localities:—Radstock; Upper Conygre Pit.

Corynepteris, Baily.

Corynepteris, Baily, *Geol. Survey of Ireland,* Explan. to accompany Sheet 142, p. 16, 1860.
Grand' Eurya, Zeiller, *Ann. d. sc. nat. Bot.*, 6e sér., vol. xvi. p. 203, 1883.

Corynepteris erosa, Gutbier, sp.

Pecopteris erosa, Gutbier, *Gaea von Sachsen,* p. 81.
Pecopteris erosa, Lesqx., *Coal Flora of Pennsyl.*, vol. i. p. 255, pl. xliv. figs. 1 and 3.
Alethopteris erosa, Geinitz, *Vers. d. Steinkf. in. Sachsen,* p. 29, pl. xxxii. figs. 7–9.
Grand' Eurya erosa, Zeiller, *Ann. d. sc. nat. Bot.*, vol. xvii. p. 9, 1884.

Remarks.—Very rare. Only one specimen of this species has come under my notice from the Radstock Series.

From the structure of the fruit of this fern, ZEILLER places it in his genus *Grand' Eurya,* but from an examination of the type of *Corynepteris,* Baily, I am led to conclude that ZEILLER'S *Grand' Eurya* is identical with BAILY'S *Corynepteris,* and the latter, being the older generic name, is here adopted.

Locality:—Camerton.

Dactylotheca, Zeiller, *Ann. d. sc. nat. Bot.*, 6e sér., vol. xvi. p. 184, 1883.

Dactylotheca plumosa, Artis, sp.

Filicites plumosa, Artis, *Antedil. Phyt.*, p. 17, pl. xvii.
Pecopteris plumosa, Brongt., *Hist. de végét. foss.*, p. 348, pls. cxxi. cxxii.
Pecopteris delicatula, Brongt., *Hist. d. végét. foss.*, p. 349, pl. cxvi. fig. 6.
Aspidites Silesiacus, Göpp., *Syst. fil. foss.*, p. 346, pls. xxvii. and xxxix. fig. 1.
Senftenbergia crenata, Stur (not L. & H.) (in part), *Carbon-Flora,* p. 72, pl. xlv. fig. 1 (?).
Senftenbergia plumosa, Stur (in part), *Carbon-Flora,* p. 91, pl. li. fig. 1 (figs. indistinct, excl. syn. *Pec. pennæformis,* Brongt.).
Dactylotheca plumosa, Kidston, *Catal. Palæoz. Plants,* p. 128 (excl. syn. *Pec. acuta*).
Pecopteris dentata, L. & H., *Fossil Flora,* vol. ii. pl. cliv.

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* *Vers. d. Zwick. Schwarzkohl,* p. 61.

† *Vers. d. Zechst. u. Rothl.*, pl. v. figs. 3, 4.

Dactylotheca plumosa, var. dentata, Brongt., sp.

Pecopteris dentata, Brongt., *Hist. d. végét. foss.*, p. 346, pls. cxxiii. cxxiv.
Pecopteris dentata, Zeiller, *Végét. foss. du terr. houil.*, p. 86, pl. clxviii. figs. 3, 4.
Cyatheites dentatus, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 26, pl. xxv. fig. 11 (in part); pl. xxix. figs. 10–12; pl. xxx. figs. 1–3 (4?).
Dactylotheca dentata, Zeiller, *Ann. d. sc. nat. Bot.*, 6e sér., vol. xvi. p. 184, pl. ix. figs. 12–15; *Bull. soc. géol. d. France*, 3e sér., vol. xii. p. 201.
Senftenbergia plumosa, Stur, *Carbon-Flora*, pl. li. fig. 2 (3?).

Remarks.—Two distinct forms of this fern occur in Britain, which have usually been regarded as distinct species. In the above synonymy I have attempted to separate them.

This species was first described by ARTIS in 1825, under the name of *Filicites plumosus*, and what I regard as a variety of ARTIS's plant was described by BRONGNIART as *Pec. dentata* in 1828.

After examining numerous specimens of *Dactylotheca* (*Pec.*) *plumosa* and *Dactylotheca* (*Pec.*) *dentata*, I have been led to believe that *Pec. dentata* can only be regarded as a well-marked variety of *Pec. plumosa*, a view that has been indicated by GEINITZ.*

The two forms are, however, so well marked that a varietal name is demanded in the case of one of them; hence *Pec. dentata*, Brongt., being of later date than *Dacty. plumosa*, Artis, sp., BRONGNIART'S form must be distinguished as the variety, and is therefore here designated *Dacty. plumosa*, var. *dentata*.

Dactylotheca plumosa, var. *dentata*, Brongt., sp., is extremely common in the Radstock Series, and some very fine examples have been collected. The typical form, *Dactylotheca plumosa*, Artis, sp., is very rare, having only been met with at Timsbury.

Dactylotheca plumosa, Artis, sp., is more plentiful in the Middle than in the Upper Coal Measures, where the variety *dentata* is rare.

Dr STUR seems to have confused *Dacty.* (*Senftenbergia*) *plumosa* with several other species.† He unites *Aspidites Silesiacus*, Göpp.,‡ with *Sphen. crenata*, L. & H. Now *Aspidites Silesiacus*, Gopp. (specimens of which have been kindly sent me by Prof. WEISS), is identical with *Dactylotheca plumosa*, Artis, sp., with which I have been able to compare it. On the other hand, *Sphenopteris crenata*, L. & H.,§ I believe to be an entirely different species. Again STUR, on his plate li., gives three figures of *Dactylotheca* (*Senftenbergia*) *plumosa;* one of these appears to be *Dacty. plumosa* (fig. 1), the other two probably are referable to the var. *dentata*. STUR further unites with his *Dactylotheca* (*Senftenbergia*)

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* *Vers. d. Steinkf. in Sachsen*, p. 26.

† *Carbon-Flora*.

‡ *Syst. fil. foss.*, p. 346, pls. xxvii. and xxxix. fig. 1.

§ *Lindley and Hutton*, vol. i. pl. xxxix.; vol. ii. pls. c. ci.

plumosa the *Pec. pennæformis*, Brongt., a species which, when the authentic plant is seen, is specifically distinguishable by many well-marked characters.

In my *Catalogue of Palæozoic Plants* I united *Pec. acuta*, Brongt., with *Dactylotheca plumosa* (including var. *dentata*), Artis, sp. I have since had a specimen of *Pec. acuta*, Brongt., from Prof. WEISS, and now find my uniting them was an error. The uniformly simple nervation of *Pec. acuta*, as well as the acute falcate form of its pinnules, easily separate it from *Dacty. plumosa*, var. *dentata*. These differences are, however, better seen on actual specimens than in BRONGNIART'S figure.

Aspidites Glockeri, Göpp.,[*] was also regarded by me as a variety of *Dacty. plumosa*, Artis, sp., but perhaps it is not safe positively to refer GÖPPERT'S plant to ARTIS'S species without a comparison of specimens.

The fruit of *Dacty. plumosa*, var. *dentata*, has been described by ZEILLER.[†] It consists of broadly-lanceolate *marattiaceous* exannulate sporangia situated on the arms of the secondary nerves. The sporangia are not united to one another, and appear to open by a longitudinal cleft.

From characters derived from the structure of this fruit, ZEILLER has founded the genus *Dactylotheca* for *Pecopteris dentata*, Brongt., which is adopted here.

Dactylotheca plumosa, Artis, sp.

Locality :—Upper Conygre Pit.

Dacty. plumosa, var. *dentata*, Brongt., sp.

Localities :—Radstock; Braysdown Colliery; Wellsway Pit; Upper Conygre Pit; Lower Conygre Pit; Camerton.

Dicksoniites, Sterzel, *Botanisches Centralblatt*, Band xiii. Nos. 8, 9, 1883.

Dicksoniites Pluckenetii, Schlotheim, sp.

Dicksoniites Pluckenetii, Sterzel, *Botan. Centralblatt*, Band xiii. Nos. 8, 9, pl. vi. ; *Zeitsch. d. deut. geol. Gesell.*, 1886, p. 773, pl. xxi.
Pecopteris Pluckenetii, Brongt., *Hist. d. végét. foss.*, p. 355, pl. cvii. figs. 1–3.
Pecopteris Pluckenetii, Germar, *Vers. d. Steinkf. v. Wettin u. Löbejun*, p. 41, pl. xvi.
Pecopteris Pluckenetii, Zeiller, *Végét. foss. d. terr. houil.*, p. 90, pl. clxviii. figs. 1, 2.
Alethopteris Pluckenetii, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 30, pl. xxxiii. figs. 4, 5.
Cyatheites Pluckenetii, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 67.
Filicites Pluckenetii, Schlotheim, *Flora d. Vorwelt*, p. 52, pl. x. fig. 19.
Pecopteris bifurcata, Sternb., *Vers.*, i. p. xix. pl. lix. fig. 2; ii. p. 151.

Remarks.—Though this species has been noted from several localities, it is by no means common. All the examples collected are referable to the typical form.

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[*] *Syst. fil. foss.*, p. 375, pl. xxix. figs. 1–4.

[†] *Ann. d. sc. nat. Bot.*, 6^{e} sér., vol. xvi. p. 184, pl. ix. figs. 12–15.

The fruit has been figured and described by STERZEL,* and I am indebted to Mr E. WILSON, Clifton, for a small fruiting specimen which he collected at Welton. This example shows the upper surface of the frond. The *sori* are scattered on the pinnules in an irregular manner, and appear to have been placed in little pockets, which produce on the upper surface of the pinnules a corresponding small circular wart-like elevation,—such as is seen in some recent genera of ferns.

Localities:—Radstock; Kilmersdon Colliery; Upper Conygre Pit; Welton.

Alethopteris, Sternberg, *Vers. einer geol. botan. Darstellung d. Flora d. Vorwelt*, i. fasc. iv. p. xxi., 1820.

Alethopteris lonchitica, Schlotheim, sp.

Alethopteris lonchitica, Schimper, *Traité d. paléont. végét.*, vol. i. p. 554.
Alethopteris lonchitidis, Sternb., *Vers.*, i. fasc. iv. p. xxi.; *Vers.*, ii. p. 142.
Pecopteris lonchitica, Brongt., *Hist. d. végét. foss.*, p. 275, pl. lxxxiv.
Pecopteris heterophylla, L. & H., *Fossil Flora*, vol. i. pl. xxxviii.
Pecopteris Mantelli, Brongt., *Hist. d. végét. foss.*, p. 278, pl. lxxxiii.
Pecopteris Mantelli, L. & H., *Fossil Flora*, vol. ii. pl. cxlv.
Alethopteris Mantelli, Zeiller, *Végét. foss. du terr. houil.*, p. 74, pl. clxiii. figs. 3, 4.
Pecopteris urophylla, Brongt., *Hist. d. végét. foss.*, p. 290, pl. lxxxvi.
Filicites lonchiticus, Schlotheim, *Flora d. Vorwelt*, p. 55, pl. xi. fig. 22.

Remarks.—On only two occasions have I collected *Aleth. lonchitica* in the Radstock Series. This species is extremely common in the Lower Coal Measures, less plentiful in the Middle Coal Measures, where *Aleth. Serlii* begins to appear, and extremely rare in the Upper Coal Measures, where *Aleth. Serlii* becomes very abundant.

Aleth. lonchitica, Schl., sp., is very variable in the form and size of the pinnules, and there appears to be no definite line of demarcation between the typical form and *Aleth. Mantelli*, *Aleth. heterophylla*, and *Aleth. urophylla*. *Aleth. heterophylla* and *Aleth. urophylla* appear to represent the same form of the plant.

In *Aleth. Mantelli* the pinnules are longer and narrower. On the lower corner of the slab containing the type specimen of *Aleth. heterophylla* figured by LINDLEY & HUTTON occurs a fragment of *Aleth. Mantelli;* and on the upper corner of the type specimen of *Aleth. Mantelli*, Brongt., occurs a specimen of *Aleth. heterophylla!*

After examining many specimens of the species, I have been led to the conclusion that the plants mentioned in the synonymy given here are only different forms of one fern—possibly in some cases even different portions of the same frond; and to the synonyms mentioned above I believe others might be added.

Localities:—Radstock; Braysdown Colliery.

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* *Botanisches Centralblatt* and *Zeitsch. d. deut. geol. Gesell.*

Alethopteris Serlii, Brongniart, sp.

Alethopteris Serlii, Sternb., *Vers.*, ii. p. 144.
Alethopteris Serlii, Lesqx., *Coal Flora of Pennsyl.*, vol. i. p. 176, pl. xxix. figs. 1–5.
Alethopteris Serlii, Zeiller, *Végét. foss. d. terr. houil.*, p. 75, pl. clxiii. figs. 1, 2.
Pecopteris Serlii, Brongt., *Hist. d. végét. foss.*, p. 292, pl. lxxxv.
Pecopteris Serlii, L. & H., *Fossil Flora*, vol. iii. pl. ccii.

Remarks.—This fern is extremely abundant—of so frequent occurrence that one can scarcely split a slab without finding some fragments of it.

Brongniart notices two varieties—

(*a*) *Europœa*—with obtuse pinnules.
(*b*) *Americana*—with acute pinnules.

The first variety he received from near Bath and Dunkerton, and the latter from Wilkesbarre, Pennsylvania.

Both of these forms are equally common in the Radstock Series, but do not form real *varieties*, for on a specimen from Radstock some of the pinnæ are var. *Europœa* and others var. *Americana;* and further, even on the same pinna I have observed that the pinnules on one side of the rachis were acute, and on the other shorter and obtuse.

Localities:—Radstock; Braysdown Colliery; Huish Pit; Wellsway Pit; Upper Conygre Pit; Lower Conygre Pit; Camerton; Dunkerton.

Alethopteris Grandini, Brongniart, sp.

Alethopteris Grandini, Grand' Eury, *Flore Carbon. du Départ. d. la Loire*, p. 107.
Alethopteris Grandini, Zeiller, *Flore foss. du Bassin houil. de Valenciennes*, pl. xxxviii. figs. 1, 2.
Pecopteris Grandini, Brongt., *Hist. d. végét. foss.*, p. 286, pl. xci. figs. 1–4.

Remarks.—This species is very rare, only four or five specimens having come under my notice.

Localities:—Radstock; Braysdown Colliery; Wellsway Pit; Upper Conygre Pit.

Alethopteris aquilina, Schlotheim, sp.

Alethopteris aquilina, Schimper, *Traité d. paléont. végét.*, vol. i. p. 556, pl. xxx. figs. 8–10.
Pecopteris aquilina, Brongt., *Hist. d. végét. foss.*, p. 284, pl. xc.
Filicites aquilina, Schlotheim, *Flora d. Vorwelt*, p. 38, pl. iv. fig. 7; pl. v. fig. 8.

Remarks.—This species is rare in the Radstock Series, but the specimens appear to me to be identical with Schlotheim's plant. It is true that this author does not give details of the nervation, but dealing with his figures and descriptions, the evidence seems to be in favour of Brongniart being correct in identifying the plant he figures on pl. xc. as Schlotheim's species.

Dr Stur holds a different opinion, and separates the plant figured by Brongniart from *Aleth. aquilina* as founded by Schlotheim, and for Brong-

NIART's plant creates a new species, which he places in the genus *Danæites* under the name of *D. sarepontanus*.* I am, however, of opinion that Dr STUR is in error in separating BRONGNIART's figure from *Aleth. aquilina*, Schloth., sp.

It is doubtful if the plant figured by GEINITZ as *Aleth. aquilina* belongs to SCHLOTHEIM's fern.†

One specimen from Radstock, 10½ inches long, shows portions of fifteen pinnæ on the left side of the rachis and eleven on the right, the most perfect of which is 5½ inches long.

Localities:—Radstock; Wellsway Pit; and Braysdown Colliery.

Alethopteris obliqua, Brongniart, sp.

Alethopteris obliqua, Schimper, *Traité d. paléont. végét.*, vol. i. p. 557.
Pecopteris obliqua, Brongt., *Hist. d. végét. foss.*, p. 320, pl. xcvi. figs. 1–4.

Remarks.—I have received from Mr GEORGE WEST, Camerton, a small specimen of a fern showing two pinnæ which agree entirely with BRONGNIART's figs. 3, 4, the originals of which came from Oldham.

Alethopteris Davreuxi, Brongniart, sp.
Plate XXIV. fig. 1.

Alethopteris Davreuxi, Göpp., *Syst. fil. foss.*, p. 295 (excl. syn.).
Alethopteris Davreuxi, Zeiller, *Bull. soc. géol. d. France*, 3e sér., vol. xii. p. 199; *Flore foss. d. Bassin houil. d. Valenciennes*, pl. xxxii. fig. 1.
Pecopteris Davreuxi, Brongt., *Hist. d. végét. foss.*, p. 279, pl. lxxxviii.

Description.—Frond large, heterophyllous; primary and secondary pinnæ alternate; pinnules alternate. Secondary pinnæ on upper part of frond entire or more or less lobed, and united by their basal portions; secondary pinnæ on basal part of lower primary pinnæ, free and divided into many pairs of alternate pinnules. Pinnules oval or subrotund, united among themselves, the lowest inferior pinnule occupying the angle formed by the union of the rachis of the secondary and primary pinnæ. Terminal lobe oblong. The central vein in the pinnules is prominent in its lower part, and gives off numerous upward-directed once-divided veinlets. A few simple or dichotomous veins also enter the pinnules direct from the rachis. In the entire and lobed pinnæ the veins are fascicled and divided several times.

Remarks.—The specimen figured on Plate XXIV. fig. 1, has been kindly lent me for description by the Council of the Bristol Museum. It shows the remains of six primary pinnæ (numbered i.–vi.) on the left of the rachis and two on the right (numbered vii., viii.). The position of the rachis which has borne these pinnæ, and of which very little remains, is indicated by an arrow. The primary pinnæ i.–ii. bear quite entire secondary pinnæ; the lateral veins in these are slightly fascicled. On the primary pinnæ iii., the lower secondary pinnæ

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* *Carbon-Flora*, p. 223, pl. lxi. fig. 2.

† *Vers. d. Steinkf. in Sachsen*, p. 27, pl. xxxi. figs. 5–7.

become sinuous and lobed; in iv. this is more marked, a few of the lower secondary pinnæ bearing the ultimate pinnulation of the species.

The fasciculate arrangement of the veins is specially observable in those pinnæ with dentate or slightly lobed margins, but even in the upper entire pinnæ the veins show the same tendency. Pinna v. represents the most completely divided condition of the fern.

The pinnules are ovate, the pair at the base of the pinnæ being usually subrotund. The pinnules are very blunt, and united for about one-third of their length. The central vein is strongly marked, lying in a little furrow, and divides into several branches at its apex. From its sides are given off numerous once-divided veinlets. A few veins also enter the pinnules direct from the rachis to which they are attached. These are either simple or divided (see fig. 1*a*).

The species is very rare in the Radstock Series, only two specimens having come under my notice; both of these are, however, fine. One is in the collection of the Bristol Museum, and the other is in the collection of the Bath Museum.

Localities:—Radstock; Camerton.

Spiropteris, Schimper, *Traité d. paléont. végét.*, vol. i. p. 688, 1869.

Spiropteris, sp.

Remarks.—Under this name I place a specimen of *Pecopteris* in circinate vernation. It is impossible to determine the species to which this example belongs.

Locality:—Braysdown.

Rhacophyllum, Schimper, *Traité d. paléont. végét.*, vol. i. p. 684, 1869.

Rhacophyllum crispum, Gutbier, sp.

Fucoides crispus, Gutbier, *Vers. d. Zwick. Schwarzkohl*, p. 13, pl. i. fig. 11; pl. vi. fig. 18.
Aphlebia crispa, Zeiller, *Végét. foss. d. terr. houil.*, p. 95.
Rhacophyllum Lactuca, Schimper, *Traité d. paléont. végét.*, vol. i. p. 684, pl. xlvi. fig. 1; pl. xlvii. fig. 1 (2?); vol. iii. p. 524.
Schizopteris Lactuca, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 19, pl. xxvi. fig. 1.
Schizopteris Lactuca, Germar, *Vers. d. Steink. v. Wettin u. Löbejun*, p. 45, pls. xviii., xix.
Fucoides linearis, Gutbier, *Vers. d. Zwick. Schwarzkohl*, p. 13, pl. i. figs. 10, 12.

Remarks.—Not common.

Localities:—Radstock; Braysdown Colliery; Camerton.

Rhacophyllum filiciforme, Gutbier, sp.

Rhacophyllum filiciforme, Schimper, *Traité d. paléont. végét.*, vol. i. p. 685, pl. xlviii. figs. 3–6.
Fucoides filiciformis, Gutbier, *Vers. d. Zwick. Schwarzkohl*, p. 11, pl. i. figs. 3, 6, 7, 8, 13 (excl. syn.).
Fucoides crenatus, Gutbier, *Vers. d. Zwick. Schwarzkohl*, p. 14, pl. i. fig. 14.
Schizopteris Gutbieriana, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 19, pl. xxv. figs. 11–14.

Remarks.—Not common.

Localities :—Radstock ; Camerton.

Rhacophyllum Goldenbergii, Weiss.
Plate XXVII. fig. 2.

Rhacophyllum Goldenbergii, Weiss in Schimper, *Traité d. paléont. végét.*, vol. i. p. 686, pl. xlvi. fig. 2.

Description.—Frond membranous, pinnate (?) ; pinnæ lanceolate, pinnatifid ; teeth of pinnæ directed upwards, simple or bifid, lanceolate acute. Midrib distinct, and giving off a veinlet to each tooth. Main rachis broadly winged, in which can be traced, for a considerable distance, the downward course of the midribs that enter the pinnæ.

Remarks.—This species is extremely rare in the Radstock Series, only two specimens having been observed, both of which are in the collection of Mr J. M'Murtrie, F.G.S. The example figured, however, is from Pucklechurch,* and was presented to the Bristol Museum by Mr J. C. Blackmore.

The frond appears to have been of very delicate membranous character, in which the veins are distinctly seen.

In one of the specimens of this species in Mr M'Murtrie's collection, the midrib of the pinnæ appears thicker than in that figured, but the condition of preservation in the two specimens is different. That figured shows the upper surface of the frond, the carbon remaining as a thin brown film, whereas in the other examples an impression of the under surface of the frond is exhibited, from which the carbonaceous matter is entirely removed.

The British specimens agree very well with the figure of *Rhacophyllum Goldenbergii* given by Schimper, but the Saarbrück example does not seem to have been so well preserved as that from Pucklechurch. The teeth of the pinnatifid pinnæ appear to be simple, with the exception of the basal tooth on the superior margin, which is bifid.

As far as I am aware, this plant has hitherto only been recorded from Saarbrück.

Locality :—Radstock.

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* This specimen is probably from the neighbourhood of Pucklechurch—not actually from the village of that name, and therefore most likely from a bed on the horizon of the Farrington Series, *i.e.*, the coal-producing series immediately below the Radstock Series.

Rhacophyllum spinosum, Lesquereux.

Plate XX. fig. 3.

Rhacophyllum spinosum, Lesqx., *Coal Flora of Pennsyl.*, p. 320, pl. lviii. figs. 4, 5.

Description.—Frond tripinnate, pinnæ alternate, diverging from the broad flat membranous rachis at an acute angle; pinnules alternate, lanceolate, of delicate texture, with 2–3 short spine-like teeth, and a prominent lanceolate terminal lobe. In the centre of the flattened membranous rachis of the pinnæ is a nerve, from which simple veins are given off to each pinnule, in which again a single veinlet seems to extend into each tooth.

Remarks.—The above description varies somewhat from that given by Lesquereux in his *Coal Flora,* p. 320. It is there stated—"The veins are clearly seen in parallel fascicles on the rachis, and may be followed into the lateral pinnæ, where they disappear, probably there dividing into very thin branches, and passing into the lobes." "The rachis is distinctly dotted."

In comparing the outline and general form of the Radstock example with Lesquereux's figures, especially with his figure 5, the identity of the two ferns is complete, the only differences between them being the simple veins in the Radstock specimen, and the absence of the dots on the rachis. In the Radstock example, unfortunately, the main rachis is almost entirely removed, and only a faint indication of it is given on the matrix. The scales, however, might have been shown on the rachis of the pinnæ, but there is no indication of their presence,—but this is a character of secondary importance, as their presence or absence depends so much on the part of the frond under examination, and the state of preservation of the fossil.

In regard to the discrepancy in the nervation, the specimen I figure shows clearly that a single vein penetrates the centre of the broad membranous rachis of the pinnæ, which gives off to each pinnule a similar single vein; I could further distinguish the presence of a single veinlet in the teeth of some of the pinnules.

I am, therefore, led to conclude that what has been regarded as veins by Lesquereux are only external striæ, and this view is strengthened from the examination of the last species (*R. Goldenbergii*), which belongs to the same group of *Rhacophylli,* and where the veins are of a similar simple nature as those occurring in the Radstock specimen of *Rhacophyllum spinosum,* Lesqx.

The only example of this fern that I have seen is that figured on Plate XX. fig. 3; it is in the collection of Mr J. M'Murtrie, F.G.S., who has kindly submitted it to me for examination and description.

Locality:—Radstock.

Megaphyton, Artis, *Antediluvian Phytology*, p. 20, 1825.

Megaphyton frondosum, Artis.
Plate XXVI. fig. 4.

Megaphyton frondosum, Artis, *Antedil. Phyt.*, pl. xx.
Megaphyton frondosum, Kidston, *Catal. Palæoz. Plants*, p. 143.
Megaphyton approximatum, L. & H., *Fossil Flora*, vol. ii. pl. cxvi.
Megaphyton distans, L. & H., *Fossil Flora*, vol. ii. pl. cxvii.

Remarks.—Some notes on this fern stem will be found in my *Catalogue of Palæozoic Plants,* where the difficulties in the limitation of this species are indicated.

Locality :—Radstock.

Megaphyton elongatum, Kidston, n. sp.
Plate XXVI. fig. 1.

Description.—Frond scars arranged in two opposite vertical rows, distant; those of one row alternative with those of the opposite corresponding vertical row. Frond scars elongated, rounded at their upper extremity, and gradually merging into the stem below. Vascular cicatrice oval, situated towards the apex of the scar. Stem striated, and bearing numerous cicatricules of aerial rootlets.

Remarks.—This species is the most frequently occurring fern stem in the Radstock Series, but in mentioning this it must not be inferred that the fossil is plentiful. One specimen from Radstock, removed from its matrix, is 3 feet 11 inches in length, and at one extremity is 5 inches across, and at the other extremity 3¾ inches; the distance between the top of one scar and the top of that immediately succeeding it varies from 7 to 8 inches in this example. On another specimen the summits of the scars are about 10 inches apart.

The example figured, of which the sketch is reduced one half, is in the collection of the Bristol Museum. It does not show well the *Megaphyton* character of the stem, but was selected for figuring on account of the frond scar being more clearly defined on this specimen than on any of the compressed stems that show the two opposite vertical rows of scars.

The specimen I identified as *Caulopteris Cistii,* from Radstock, in the collection of the British Museum, is probably a fragment of a stem of this species.

Localities :—Radstock ; Middle Pit ; Camerton.

Caulopteris, Lindley & Hutton, *Fossil Flora*, vol. i. p. 121, 1832.

Description.—Stems of arborescent ferns, bearing distant or contiguous, circular or oval, smooth sars, arranged quincuncially, containing :—Type 1. An inner circular or oval closed ring, more or less following the contour of the frond scar; within this is a second oval scar, open at its upper aspect, the free ends being bent inwards, and forming a "horse-shoe scar." Type 2. A closed inner circular or oval line, within which is a variously bent transverse scar. Stems usually bearing numerous aerial rootlet cicatrices, placed on the stem between the frond scars.

Scars on stems deprived of their outer envelope, oval or elongate-elliptical, upper and lower extremities rounded or pointed, and generally confluent, occasionally showing within the scar traces of an inner oval cicatrice; whole stem striate.

Remarks.—Several explanations have been given of the structure of the scars of *Caulopteris*, some maintaining that the large inner circle of the frond is closed; others, that its superior margin is open, and that the two free ends bend inwards, and thus form the characteristic "horse-shoe scar." Both of these descriptions of the structure of the scars appear to be correct.

ZEILLER was the first to demonstrate the closed nature of the inner circle of the scar of *Caulopteris*,* inside of which he detected a bent transverse band, which latter he regarded as the true vascular cicatrice (text fig. 6). GRAND' EURY, however, states in his *Flore carbonifère du Département de la Loire et du Centre de la France*,† that, though in the Upper Coal Measures of the centre of France the *Caulopteris* with "horse-shoe"-shaped inner scars are very rare, they do exist, and on such a specimen he founds his *Caulopteris protopteroides*, remarking, at the same time, that this form of *Caulopteris* was not so plentiful in France as he had seen it in England. GRAND' EURY‡ was further able to show, from the examination of a siliceous specimen of *Caulopteris*, that the inner closed circle, as well as the transverse central scar, is a vascular tract.

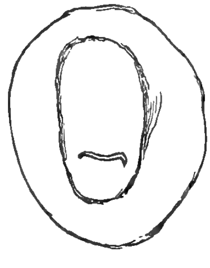

Fig. 6.

Caulopteris peltigera, Brongt., sp., showing inner closed vascular tract, within which is contained the transverse vascular cicatrice (copied from ZEILLER, half nat. size).

The genus has in England, as far as I am aware, only been found in the Upper Coal Measures, and on the majority of the specimens the scar is of the "horse-shoe" form.

Another very interesting point has been brought out by ZEILLER in his paper to which reference has already been made, that those fossils, for which CORDA proposed the genus *Ptychopteris* § (of which *Caulopteris macrodiscus* is the type), only represent the inner portion of *Caulopteris*, corresponding probably to the sheath of *sclerenchyma* that surrounds the central ligneous cylinder.|| The figure ZEILLER gives of *Caulopteris patria* shows both these conditions on the same specimen.¶

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* "Note sur quelques troncs de Fougères fossiles," *Bull. soc. géol. d. France*, 3e sér., vol. iii. p. 574, 1874–75.

† p. 84. ‡ *Loc. cit.*, p. 84. § *Flora protogœa*, p. 76.

|| *Bull. soc. géol. d. France*, 3e sér., vol. xii. p. 203, 1883.

¶ *Bull. soc. géol. d. France*, 3e sér., vol. iii. pl. xvii. fig. 4. This is named on the plate *Caulopteris peltigera*, Brongt., sp. (?), but has since been described as a new species by GRAND' EURY under the name of *Caulopteris patria*. See also ZEILLER, *Végét. foss. d. terr. houil.*, p. 100; and GRAND' EURY, *Flore carbon. du Départ. de la Loire*, p. 87.

Those fern stems, therefore, of the type of *Caulopteris macrodiscus* lose all true specific value, but as at present it is usually impossible to refer them to their parent species, I have mentioned them under *C. macrodiscus,* merely with the object of noting the occurrence of such fossils.

On Plate XXVI. fig. 2, is shown a specimen, natural size, which exhibits the formation of a *Caulopteris macrodiscus* fern stem from one of *C. peltigera* type. There is shown here part of the outer envelope still adhering to the stem, which when removed, exhibits beneath it the striated surface of *macrodiscus.*

Caulopteris primæva, Lindley & Hutton.

Caulopteris primæva, L. & H., *Foss. Flora,* vol. i. pl. xlii.
Caulopteris primæva, Schimper, *Traité d. paléont. végét.,* vol. i. p. 707.
Sigillaria Lindleyi, Brongt., *Hist. d. végét. foss.,* p. 419, pl. cxl. fig. 1.

Remarks.—This species is very rare. As far as I am aware, only two specimens of it have been discovered. One of these is that figured by LINDLEY and HUTTON, from Radstock, and the other is from Camerton in the collection of the Bristol Museum.

Localities:—Radstock; Camerton.

Caulopteris anglica, Kidston, n. sp.

Plate XXVI. fig. 3.

(?) *Caulopteris peltigera,* Geinitz, *Vers. d. Steinkf. in Sachsen,* pl. xxxiv. fig. 3.

Description.—Frond cicatrices almost contiguous, oval, slightly narrowed above. Vascular cicatrices nearly equally distant from all parts of the circumference; double, outer closed, inner "horse-shoe" shaped.

Remarks.—All the specimens of *Caulopteris* (with the exception of *C. primæva* and *C. macrodiscus*) from the Radstock area, which were in a sufficiently good state of preservation for specific determination appear to belong to this species.

On Pl. XXVI. fig. 3, is given, half natural size, an example from Radstock in the collection of the Bath Museum. The scars are oval, and slightly narrowed at the upper extremity. Some of them show a second inner circle, outside of and surrounding the "horse-shoe" scar. This is well seen in the middle scar in the left-hand row. The same character I have observed in other specimens. It is quite probable, then, that the outermost of the two inner circles correspond to the closed circle of ZEILLER, and the "horse-shoe" scar to the inner curved transverse scar, only much more developed.

On Plate XXVI. fig. 5, is shown an isolated scar with a very distinct "horse-shoe," but with no trace of the surrounding closed circle. The scar is quite smooth, but the absence of the surrounding closed circle makes it improbable that this small specimen belongs to *Caulopteris anglica.*

The frond scars often touch laterally, but are occasionally separate.

Caulopteris anglica differs from *Caulopteris protopteroides*, Grand' Eury,* in the closed vascular circle surrounding, and not being within, the "horse shoe" as in GRAND' EURY's species, and further I have not observed any trace of the small inner transverse vascular cicatrice, such as is described as occurring in *Caulopteris protopteroides.*

The specimen figured by GEINITZ as *Caulopteris peltigera* is probably referable to *Caulopteris anglica.*

Localities :—Radstock ; Camerton.

Caulopteris macrodiscus, Brongniart, sp.

Plate XXV. figs. 1, 2.

Caulopteris macrodiscus, Feistmantel, *Der Hangendflötzzug*, p. 85, pl. iv.†
Sigillaria macrodiscus, Brongt., *Hist. d. végét. foss.*, p. 418, pl. cxxxix
Caulopteris Phillipsii, L. & H., *Fossil Flora*, vol. ii. pl. cxl.
Ptychopteris macrodiscus, Corda, *Flora protogæa*, p. 76.
Ptychopteris macrodiscus, Germar, *Vers. d. Steink. v. Wettin u. Löbejun*, p. 115, pl. xl. fig. 1.

Remarks.—Perhaps the fossils placed here may be stems of *Caulopteris anglica,* deprived of their cortical envelope. They must not in any case be regarded as a distinct species.

Two of these stems are shown on Plate XXV. figs. 1, 2. The specimen illustrated at fig. 1 shows an inner oval cicatrice on the frond scars. Fig. 2 represents the more common appearance of the fossil.

Caulopteris Phillipsii, L. & H., does not appear to differ in any way from *Caulopteris macrodiscus,* Brongt.

Localities :—Radstock ; Camerton (type of *Caulopteris Phillipsii,* L. & H.).

Caulopteris, sp.

Remarks.—I place here a few specimens which, on account of their imperfect preservation, cannot be specifically identified, but may be distinct from those already given.

Localities :—Radstock ; Camerton.

LYCOPODIACEÆ.

Lepidodendron, Sternberg.

Vers. eines geol. botan. Darstellung d. Flora d. Vorwelt, i. fasc. i. p. 25 ; fasc. v. p. 10, 1820.

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* *Flore carbon. d. Départ. de la Loire*, p. 85.

† In *Archiv d. naturw. Landesdurchforschung v. Böhmen,* iv. Band Nr. 6, Geolog. Abth., Prag, 1881.

Lepidodendron aculeatum, Sternberg.

Lepidodendron aculeatum, Sternb., *Vers.*, i. fasc. i. pp. 20, 23, pl. vi. fig. 2; pl. viii. fig. 1 *b*; fasc. ii. p. 25, pl. xiv. figs. 1–4; fasc. iv. p. x.
Lepidodendron aculeatum, Schimper, *Traité d. paléont. végét.*, vol. ii. p. 20, pl. lix. fig. 3.
Lepidodendron aculeatum, Kidston, *Catal. Palæoz. Plants*, p. 153.
Lepidodendron aculeatum, Zeiller, *Flore foss. d. Bassin houil. d. Valenciennes*, pl. lxv.

Remarks.—Frequent.

A specimen in the roof of one of the workings of Braysdown Colliery, of which a piece was shown me by the manager, Mr STEART, was 40 feet long, but as both its upper and lower extremities were embedded, its complete length could not be ascertained. At some of the other collieries, fragments of very large stems were also seen.

Localities:—Radstock; Braysdown Colliery; Wellsway Pit; Camerton.

Lepidodendron Worthenii, Lesquereux.

Lepidodendron Worthenii, Lesqx., *Coal Flora of Pennsyl.*, vol. ii. p. 388, pl. lxiv. figs. 8, 9.
Lepidodendron Worthenii, Zeiller, *Flore foss. d. Bassin houil. d. Valenciennes*, pl. lxxi.

Remarks.—This species is comparatively common in the Radstock Series.

Localities:—Radstock; Wellsway Pit; Braysdown Colliery; Camerton.

Lepidodendron lanceolatum, Lesquereux.

Plate XXVII. fig. 5; Plate XXVIII. figs. 3, 4.

Lepidodendron lanceolatum, Lesqx., *Coal Flora of Pennsyl.*, p. 369, pl. lxiii. figs. 3–5.

Description.—Scars lanceolate or broadly fusiform, smooth, keeled; vascular cicatrice small, situated in upper part of scar; cicatricules indistinct. Leaves lanceolate, acute, slightly curved.

Remarks.—None of the specimens of this species with which I have met give a clear view of the vascular cicatrice, nor does it appear to have been well seen in LESQUEREUX's specimen. His enlarged fig. 5 has been probably taken from an imperfectly preserved example, and seems to represent a leaf-scar, from which the leaf has been forcibly torn.

What I regard as the cone of this species is shown at Pl. XXVII. fig. 5.

Localities:—Radstock; Camerton; Upper Conygre Pit; Braysdown.

Lepidodendron rhombicum, Sternberg, sp.

Lepidodendron rhombicum, Schimper, *Traité d. paléont. végét.*, vol. ii. p. 37.
Bergeria rhombica, Presl, in Sternb., *Vers.*, ii. p. 184, pl. lxviii. fig. 18.
Bergeria angulata, Presl, in Sternb., *Vers.*, ii. p. 184, p. lxviii. fig. 17.
Bergeria quadrata, Presl, in Sternb., *Vers.*, ii. p. 184, pl. lxviii. fig. 19.

Remarks.—On several occasions, at Timsbury, I met with specimens of

Lepidodendra which agree in the form of their leaf-scars with PRESL'S *B. rhombica, angulata,* and *quadrata.* On one example, about 7 inches long, the lower leaf-scars are *B. quadrata;* about half-way up the stem they assume the shape of the leaf-scars of *B. angulata,* and towards the top of the specimen the scars assume the ordinary Lepidodendroid form. I do not regard *L. rhombicum* as a true species, but think it probably includes only peculiar conditions of the leaf-scars of several species of Lepidodendra. The bolsters never seem to show any clear traces of the vascular cicatrice or cicatricules.

Locality:—Upper Conygre Pit.

Lepidophloios, Sternberg, *Vers. eines geol. botan. Darstellung d. Flora d. Vorwelt,* vol. i. fasc. vi. p. 13, 1825.

Lepidophloios, sp.

Remarks.—This genus is very rare. A small specimen of a *Lepidophloios* (sp.) was collected at Camerton, and a Halonian condition (*Halonia regularis*) of the same genus at Radstock.

Localities:—Radstock; Camerton.

Lepidophyllum, Brongniart, *Prod. d'une hist. d. végét. foss.,* p. 87, 1828.

Lepidophyllum majus, Brongniart.

Lepidophyllum maius, Brongt., *Prod.,* p. 87.
Lepidophyllum majus, Feistmantel, *Vers. d. Böhm. Kohlenab.,* Abth. ii. p. 41, pl. xiii. figs. 1–4.
Lepidophyllum trinerve, L. & H., *Fossil Flora,* vol. ii. pl. clii.
Lepidophyllum binerve, Lebour, *Illustrations of Fossil Plants,* p. 103, pl. lii.
Lepidophyllum lanceolatum, Lebour (not Brongt.), *Illustrations of Fossil Plants,* p. 105, pl. liii.

Remarks.—Not common. Probably the foliage of *Lepidophloios.*

Localities:—Radstock; Camerton; Upper Conygre Pit.

Lepidophyllum, sp.

Plate XXVII. figs. 7*a*, 7*b*.

Remarks.—Allied to *Lepidophyllum brevifolium,* Lesqx.*

Localities:—Radstock; Braysdown Colliery.

Lepidostrobus, Brongniart, *Prod. d'une hist. d. végét. foss.,* p. 87, 1828.

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* *Coal Flora of Pennsyl.,* pl. lxix. fig. 33.

Lepidostrobus spinosus, Kidston, n. sp.

Lepidostrobus, Brongniart, *Hist. d. végét. foss.*, vol. ii. pl. xxii. figs. 2, 3, 8.

Remarks.—A single specimen of a cone, similar to that figured by BRONGNIART (*loc. cit.*), has been collected. Though this fossil is rare in the Radstock Series, it is of frequent occurrence in some of the British Coal Fields.

Locality:—Braysdown Colliery.

Lepidostrobus, sp.

Remarks.—There is placed here a cone, the exposed portion of whose bracts is rhomboidal. It appears to be a well-marked species, but is only represented by a single specimen.

Locality:—Radstock.

Sigillaria, Brongniart, *Sur la classification d. végét. foss.*, p. 9, 1822.

Sigillaria major, L. & H., sp.

Ulodendron majus, L. & H., *Fossil Flora*, vol. i. pl. v. (excl. ref.).
Ulodendron minus, L. & H., *Fossil Flora*, vol. i. pl. vi. (excl. ref.).
Lepidodendron discophorum, König, *Icones fossilium sectiles*, pl. xvi. fig. 194.
Sigillaria discophora, Kidston, *Catal. Palæoz. Plants*, p. 174 (excl. syn. *Sig. Preuiana* and *Bothrodendron punctatum*); *Annals and Mag. Nat. Hist.*, 5th ser., vol. xvi. p. 251, pl. iv. fig. 5; pl. v. fig. 8; pl. vii. figs. 12, 13 (excl. syn. *Sig. Preuiana* and *Bothrodendron punctatum*).

Remarks.—Very rare. The leaf-scars in the specimen placed here are not distinctly preserved, but from what is discoverable of their form they seem to agree with those of *Sigillaria* (*Ulodendron*) *major*, especially with that form described as *U. minor* by LINDLEY and HUTTON.

As I find LINDLEY and HUTTON's name for this plant has priority over *Lepidodendron discophorum*, König, the former author's specific designation is now adopted.*

Locality:—Radstock.

Sigillaria Serlii, Brongniart.

Sigillaria Serlii, Brongt., *Hist. d. végét. foss.*, p. 433, pl. clviii. fig. 9.
Sigillaria Serlii, Carruthers, *Geol. Mag.*, new series, Dec. ii. vol. x. p. 49, 1883.

Remarks.—I only know this plant as occurring in the Radstock Series from the figure given by Mr CARRUTHERS in the *Geol. Mag.*

Long grass-like leaves, the *Cyperites bicarinata*, L. & H.,† similar to those shown in Mr CARRUTHERS's figure, are common throughout the whole of the

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* I hope presently to publish my reasons for still retaining these Ulodendroid-Lycopods in Sigillaria, a position which some botanists are not inclined to accord them.—*May* 1887.

† *Foss. Flora*, vol. i. pl. xliii. figs. 1, 2.

Radstock area, but are evidently the foliage of more than one species of Sigillaria.

I am strongly inclined to regard *Sig. Serlii* as only a young condition of *Sig. Brardii*, Brongt.

Locality:—Radstock.

Sigillaria M'Murtrieii, Kidston.

Sigillaria M'Murtrieii, Kidston, *Ann. and Mag. Nat. Hist.*, 5th ser., vol. xv. p. 357, pl. xi. figs. 3–5, 1885; *Proc. Roy. Phys. Soc. Edin.*, vol. viii. p. 415, pl. xxi. figs. 3–5.

Remarks.—Rare.

Localities:—Radstock; Braysdown Colliery.

Sigillaria monostigma, Lesquereux.

Sigillaria monostigma, Lesqx., *Rept. Geol. Survey of Illin.*, vol. ii. p. 449, pl. xlii. figs. 1–5, 1866; *Coal Flora of Pennsyl.*, vol. ii. p. 468, pl. lxxiii. figs. 3–6, 1880; vol. iii. p. 793, 1884.
Sigillaria monostigma, Schimper, *Traité d. paléont. végét.*, vol. ii. p. 101.
Asolanus camptotænia, Wood, *Trans. Amer. Phil. Soc.*, vol. xiii. p. 342, pl. ix. fig. 3, 1866.
Sigillaria camptotænia, Zeiller, *Flore foss. du Bassin houil. d. Valenciennes*, pl. lxxxviii. figs. 4–6.

Remarks.—Rare. WEISS * is of opinion that *Sig. monostigma* is not specifically distinct from *Sig. rimosa*, Gold.; † on the other hand, SCHIMPER thinks them different.

In the meantime I prefer to regard *Sigillaria monostigma* as specifically distinct from *Sigillaria rimosa*, Gold., on account of the difference in the structure of the cicatricules.

Localities:—Braysdown Colliery; Lower Writhlington Pit.

Sigillaria tessellata (Steinhauer), Brongniart.

Sigillaria tessellata, Brongt., *Hist. d. végét. foss.*, p. 436, pl. clvi. fig. 1; pl. clxii. figs. 1–4.
Sigillaria tessellata, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 44, pl. v. figs. 6–8.
Sigillaria tessellata, Zeiller, *Végét. foss. du terr. houil.*, p. 132, pl. clxxiii. fig. 2; *Flore foss. d. Bassin houil. d. Valenciennes*, pl. lxxxv., pl. lxxxvi. figs. 1–6.
Favularia tessellata, L. & H., *Fossil Flora*, vol. i. pls. lxxiii.–lxxv.
Phytolithus tessellatus, Steinhauer, *Trans. Amer. Phil. Soc.*, vol. i. p. 295, pl. vii. fig. 2.
(?) *Sigillaria elegans*, Brongt., *Hist. d. végét. foss.*, p. 438, pl. cxlvi. fig. 1; (? clv.) and clviii. fig. 1.
(?) *Favularia elegans*, Sternb., *Vers.*, i. p. 14, pl. lii. fig. 4.
(?) *Sigillaria Knorrii*, Brongt., *Hist. d. végét. foss.*, p. 444, pl. clvi. figs. 2, 3; pl. clxii. fig. 6.
(?) *Calamosyrinx Zwickaviensis*, Petzholdt, *Neues Jahrb.*, 1842, p. 181, pl. v.
(?) *Sigillaria sexangula*, Sauveur, *Végét. foss. de la Belgique*, pl. liii. fig. 1.

Remarks.—This is the most common *Sigillaria* in the Radstock Series. The specific limitation of this extremely variable species is a very difficult and still unsettled point. All the specimens which I have seen from Somerset are

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* WEISS, *Foss Flora d. jüngst. Stk. u. d. Rothl.*, p. 160.

† GOLDENBERG, *Flora Saræpontana fossilis*, Heft ii. p. 22, pl. vi. figs. 1–4.

referable to typical *Sig. tessellata,* especially as figured by ZEILLER (*loc. cit.*, pl. clxxiii. fig. 2).

It seems probable that *Sig. elegans* and *Sig. Knorrii* are at the most only varieties of *Sig. tessellata,* and SCHIMPER * would not only unite these with *Sig. tessellata,* but many other so-called species, and in the Catalogue of Palæozoic Plants in the British Museum, I follow him in placing a number of species under *Sig. tessellata.* Some botanists, however, regard many of these plants as specifically distinct from *Sig. tessellata.* ZEILLER places considerable value for specific distinction on the disposition of the cone scars. According to this author, in *Sig. tessellata* they are placed in the bottom of the furrows separating the ribs, and are of rounded or quadrangular contour, placed in vertical parallel rows, and encircle the stem in a band of about equal width. In *Sig. elegans,* on the other hand, the scars are said to be larger, sometimes assuming a quadrangular or polygonal contour, and occur on the ribs as well as in the furrows that separate the ribs, and form a band of scars in vertical series, but the series are not so long as in *Sig. tessellata.*†

Examples of *Sig. tessellata* showing verticels of cone scars are frequently found at the localities here mentioned for the species.

Bifurcating branches of *Sig. tessellata* are rare, but lately I have seen two specimens—one from Camerton in the collection of Mr GEORGE WEST, and the other from Bardsley, Ashton-under-Lyne, Lancashire, in the collection of Mr GEORGE WILD.

Localities:—Radstock; Braysdown Colliery; Camerton.

Sigillaria lævigata, Brongniart.

Plate XXVIII. fig. 5.

Sigillaria lævigata, Brongt., *Hist. d. végét. foss.*, p. 471, pl. cxliii.
Sigillaria lævigata, Kidston, *Catal. of Palæoz. Plants,* p. 193 (excl. syn. *S. ovata*).
Sigillaria lævigata, Zeiller, *Végét. foss. du terr. houil.*, p. 125; *Flore foss. d. Bassin houil. de Valenciennes,* pl. lxxviii. figs. 1–4.

Description.—Stem furrowed, furrows straight, ribs smooth, leaf-scars more or less distant, oval, narrowed upwards, higher than broad, or height about equal to width, lower and upper margins rounded, lateral angles distinct, from which extend downwards two raised lines that reach the scar immediately below or vanish a little above it. Vascular cicatricules situated above the centre of the scar,—lateral linear, central transversely oval. A short distance above the leaf-scar is usually placed a small cicatricule.

Decorticated stems, striated, lateral vascular cicatricules linear-reniform, increasing much with age, and becoming confluent.

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* *Traité d. paléont. végét.*, vol. ii. p. 81.

† ZEILLER, *Ann. d. sc. nat. Bot.*, 6ᵉ sér., vol. xix. p. 275.

Leaves long, grass-like.

Remarks.—Not common. The width of the ribs varies much according to the age of the specimen.

On some old stems, which, however, may belong to *Sig. reniformis* (as these two species cannot be clearly distinguished when decorticated), the distance apart of the vertical rows of leaf-scars was as much as 4¾ inches. The example from which this measurement was taken might be referred to *Sig. catenulata* or *Sig. alternans.*

Localities :—Radstock ; Braysdown Colliery.

Sigillaria reniformis, Brongniart.

Sigillaria reniformis, Brongt., *Hist. d. végét. foss.*, p. 470, pl. cxlii.
Sigillaria reniformis, Lesqx., *Coal Flora of Pennsyl.*, p. 501, pl. lxx. figs. 5–9.
Sigillaria reniformis, Sauveur, *Végét. foss. de la Belgique,* pl. l. fig. 1.
Sigillaria reniformis, Zeiller, *Flore foss. du Bassin houil. de Valenciennes,* pl. lxxxiv. figs. 4–6.

Remarks.—Rare. I have observed at several localities large decorticated stems with long geminate cicatricules (=*Sig. alternans*). It is, however, unsatisfactory to refer all these specimens to *Sig. reniformis,* as *Sig. lævigata* and probably other species have similar large cicatricules on decorticated stems of old individuals.

Locality :—Radstock.

var. Radstockensis, Kidston.

Plate XXVII. fig. 6.

Description.—Stem furrowed, furrows straight, wide, longitudinally striated ; leaf-scars close and placed on slightly raised square platforms; scars much broader than long, hexagonal, upper margin emarginate, lower margin rounded, lateral angles prominent ; cicatricules situated above the centre of scar, the two lateral oval, the central double, transverse. Two short straight lines extend from the lateral angles, and two from the rounded angles of the lower boundary line of the scar, which divide the lower half of the platform on which the leaf-scar is situated into one equal and two unequal compartments. About midway between the leaf-scars is a transverse furrow. Decorticated stem striated.

Remarks.—The only specimen I have seen of this variety of *Sig. reniformis* is that figured, and is in the collection of Mr J. M'Murtrie, Radstock. It differs from the type in the presence of the slightly raised square platforms on which the scars sit, and in the arrangement of the lines that extend from the leaf-scars. On the decorticated stem the cicatricules are oblong-geminate, as in *Sig. reniformis.*

Locality :—Radstock.

Sigillaria alternans, Sternberg, sp.

Sigillaria alternans, L. & H., *Fossil Flora*, vol. i. pl. lvi.
Sigillaria alternans, Feistmantel, *Vers. d. böhm. Kohlenab*, Abth. iii. p. 23, pl. v. fig. 3; pl. vi. figs. 1–3; pl. vii. figs. 1, 2.
Syringodendron alternans, Sternb., *Vers.*, i. fasc. 4, p. xxiv. pl. lviii. fig. 2.
Sigillaria catenulata, L. & H., *Fossil Flora*, vol. i. pl. lviii.

Remarks.—Sigillaria alternans must not be regarded as a species, but only as a decorticated condition of *Sig. reniformis, Sig. lævigata*, and possibly other species.

Localities:—Radstock; Braysdown Colliery.

Sigillaria notata, Steinhauer, sp.

Phytolithus notatus, Steinhauer, *Trans. Amer. Phil. Soc.*, vol. i. p. 294, pl. vii. fig. 3, 1818.

Remarks.—The type of this species was described by Steinhauer from Dunkerton. I have not, however, seen additional specimens from the Radstock Series.

Locality:—Dunkerton.

Sporangia.

Plate XXVII. fig. 9.

Remarks.—One of these sporangia is shown on Pl. XXVII. fig. 9. Several names have been applied to these organisms. That figured here is perhaps similar to LESQUEREUX'S *Carpolithes perpussillus.*[*]

While examining Scotch carboniferous shales for spores with Mr JAMES BENNIE, great quantities of these fossils were met with in some of the shales. They varied much in size and form, and we believe them to be sporangia.

Locality:—Upper Conygre Pit. On slab with *Lepidodendron lanceolatum*, Lesqx.

Sporangia (?).

Plate XXVII. fig. 8.

Description.—More or less circular bodies about 5 mm. in diameter, with a small circular depression at the base, about 1 mm. in diameter, from which diverge four curved lines. The sides bear two slightly depressed areas.

Remarks.—These fossils are probably Lycopod sporangia. The small circular depression in all likelihood marks the point of attachment to the bract, and the slight lateral depressions may have been caused by the mutual pressure of similar bodies placed close together.

Locality:—Camerton.

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[*] *Coal Flora of Pennsyl.*, vol. iii. p. 825, pl. cxi. fig. 23; ZEILLER, *Flore foss. du Bassin houil. de Valenciennes*, pl. xciv. fig. 18.

Lycopod macrospores.

Remarks.—I have received from Mr Henry E. Hippisley a specimen of spore coal from the "Slyving Vein" of the Radstock Coal Seams almost entirely composed of *macrospores* bearing the characteristic triradiate ridge, embedded in a matrix, chiefly composed of *microspores.* These *macrospores* are probably similar fossils to those described as *Trigonocarpus sporites* by Weiss.*

Stigmaria, Brongniart, *Sur la classification d. végét. foss.*, p. 9, 1822.

Stigmaria ficoides, Sternberg, sp.

Stigmaria ficoides, Brongt., *Class. d. végét. foss.*, p. 9, pl. i. fig. 7.
Stigmaria ficoides, L. & H., *Fossil Flora,* vol. i. pls. xxxi.–xxxvi.
Variolaria ficoides, Sternb., *Vers.*, i. fasc. i. pp. 22 and 24, pl. xii. figs. 1–3.

Remarks.—Frequent.

Localities :—Radstock; Camerton; Lower Conygre Pit; Kilmersdon.

var. minor, Geinitz.

Stigmaria ficoides, var. *minor,* Geinitz, *Vers. d. Steinkf. in Sachsen,* p. 49, pl. iv. fig. 6; pl. x. fig. 1.
Stigmaria ficoides, var. *minor,* Zeiller, *Végét. foss. du terr. houil.*, p. 141.

Remarks.—Only one specimen has been met with.

Locality :—Braysdown Colliery.

Stigmaria anglica, Sternberg, sp.

Plate XXVIII. figs. 9, 10.

Lepidodendron anglicum, Sternb., *Vers.*, i. fasc. iv. p. xi. pl. xxix. fig. 4.
Aspidiaria anglica, Sternb., *Vers.*, ii. p. 181, pl. lxviii. fig. 11.
Stigmaria reticulata, Brongt., *Prod.*, p. 87.

Description.—Cicatrice circular or subrhomboidal, enclosed in contiguous rhomboidal fields; vascular cicatricule oval, and situated towards the upper part of the cicatrice.

Remarks.—This peculiar Lepidodendron-like *Stigmaria* was described by Sternberg from Paulton, and placed by him in Lepidodendron. Subsequently Brongniart removed it from this genus and placed it in *Stigmaria,* to which genus the plant really belongs, notwithstanding its Lepidodendroid appearance. Brongniart, however, substituted the specific name of *reticulata* for the older one of *anglica,* but the original name for the species is here reinstalled.

I have examined a good many specimens of this fossil, mostly from

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* Weiss, *Foss. Flora d. jüngst Stk. u. d. Rothl.*, p. 204, pl. xviii. figs. 22, 23. See also Bennie and Kidston, *Proc. Roy. Phys. Soc. Edin.*, vol. ix. p. 82, "On the Occurrence of Spores in the Carboniferous Formation of Scotland."

Camerton, and portions of some of these are represented on Plate XXVIII. figs. 9, 9*a*, 10, 10*a*, 10*b*. The appearance of the species varies considerably with age. Fig. 10 shows the youngest condition of the plant with which I have met; fig. 10*a* is from the same specimen, but a little further up the root. Figs. 9 and 9*a* show parts of another example; these illustrate portions of the same specimen slightly separated laterally, but at the same height on the root. In many cases the lateral lines of the rhomboidal meshes are much more conspicuous than the upper and lower oblique boundary lines; in other cases the two oblique lines are so feebly indicated that the fossil assumes the appearance of a ribbed Sigillarian. This condition is slightly indicated in fig. 9*a*.

At fig. 10*b* a single enlarged scar is given to show the true Stigmarian nature of the vascular cicatrice.

This fossil attained considerable size. The largest roots that show their complete width measure 5½ inches across, but some of the broken fragments are evidently parts of larger specimens. One of these large specimens, belonging to Mr George West, Camerton, shows a bifurcation of the root into two almost equal forks.

Localities:—Paulton (type); Radstock; Camerton.

Cordaites, Unger, *Genera et species Plantarum Fossilium*, p. 277, 1850.

Cordaites angulosostriatus, Grand' Eury.

Cordaites angulosostriatus, Grand' Eury, *Flore carbon. du Départ. de la Loire*, p. 218, pl. xix.
Cordaites angulosostriatus, Zeiller, *Végét. foss. du terr. houil.*, p. 144, pl. clxxv. figs. 2, 3.
Cordaites angulosostriatus, Renault, *Cours. d. botan. foss.*, 1881, p. 90, pl. xii. fig. 3.

Remarks.—This species is generally distributed throughout the whole of the Radstock Series, and in some localities is very plentiful.

Localities:—Radstock; Upper Conygre Pit; Camerton.

Poacordaites, Grand' Eury, *Flore carbonifèr du Départ. de la Loire*, p. 222, 1877.

Poacordaites microstachys, Goldenberg, sp.

Poacordaites microstachys, Zeiller, *Végét. foss. du terr. houil.*, p. 146, pl. clxxv. fig. 1.
Cordaites microstachys, Goldenberg, in Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 194 (woodcut figs. 1–3, p. 195).
Poacordaites linearis, Grand' Eury, *Flore Carbon. du Départ. de la Loire*, p. 225, pls. xxiii. and xxiv. figs. 1–3.

Remarks.—Very rare.

Localities:—Radstock; Braysdown Colliery.

Cardiocarpus, Brongniart, *Prodrome d'une hist. d. végétaux fossiles*, p. 87, 1828.

Cardiocarpus Gutbieri, Geinitz.
Plate XXIII. fig. 5.

Cardiocarpus Gutbieri, Geinitz, *Vers. d. Steinkf. in Sachsen*, p. 39, pl. xxi. figs. 23–25.

Remarks.—The specimen is smaller than the type, but is probably referable to *Cardiocarpus Gutbieri.*

Locality :—Radstock.

Cardiocarpus fluitans, Dawson.
Plate XXIII. fig. 6.

Cardiocarpum fluitans, Dawson, *Quart. Jour. Geol. Soc.*, vol. xxii. p. 165, pl. xii. fig. 74.
Samaropsis fluitans, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 209, pl. xviii. figs. 24–30.
Samaropsis fluitans, Zeiller, *Flore foss. du Bassin. houil. de Valenciennes*, pl. xciv. fig. 7.

Remarks.—Several specimens of these little seeds have been met with, but only at one locality.

Locality:—Upper Conygre Pit.

Trigonocarpus, Brongniart, *Prodrome d'une hist. d. végét. foss.*, p. 137, 1828.

Trigonocarpus Noeggerathi, Sternberg, sp.
Plate XXIII. fig. 3.

Trigonocarpus Noeggerathi, Brongt., *Prod.*, p. 137.
Trigonocarpus Noeggerathi, Göpp. & Berger, *De fruct. et semin.*, pp. 15 and 18, pl. i. figs. 1, 2 (excl. ref. L. & H.).
Trigonocarpus Noeggerathi, Zeiller, *Flore foss. du Bassin. houil. de Valenciennes*, pl. xciv. figs. 8–11.
Trigonocarpus Noeggerathi, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 204 (excl. ref. L. & H.).
Palmacites Noeggerathi, Sternb., *Vers.*, i. fasc. 4, p. xxxv. pl. lv. figs. 6, 7.
Trigonocarpum areolatum, Göpp. & Berger, *De fruct. et semin.*, p. 19, pl. i. figs. 3, 4.

Remarks.—All the figures given by LINDLEY and HUTTON, in their *Fossil Flora*, as *Trigonocarpum Noeggerathi*, are referable to the fruit figured without name by PARKINSON in his *Organic Remains*, vol. i. pl. vii. figs. 6–8, to which BRONGNIART, in his *Prodrome*, applied the name of *Trigonocarpum Parkinsoni.*

Localities :—Radstock ; Camerton ; Lower Conygre Pit.

(?) **Trigonocarpus Dawesii**, Lindley & Hutton.

Trigonocarpum Dawesii, L. & H., *Fossil Flora*, vol. ii. pl. ccxxi.

Remarks.—The specimen included here is in the collection of the Bristol

Museum. It is slightly larger than LINDLEY and HUTTON's figure, but I think may be referred to their species.

Locality :—Camerton.

Rhabdocarpus, Göppert & Berger, *Fructibus et seminibus*, p. 20, 1848.

Rhabdocarpus multistriatus, Presl, sp.

Plate XXIII. fig. 4.

Rhabdocarpus multistriatus, Lesqx., *Coal Flora of Pennsyl.*, p. 578, pl. lxxxv. figs. 22, 23.
Rhabdocarpus multistriatus, Kidston, *Catal. of Palæoz. Plants*, p. 213.
Carpolithes multistriatus, Sternb., *Vers.*, ii. pl. xxxix. fig. 1, 2.

Description.—Seed 3·5 cm. long and 2·1 cm. broad; exposed surface, bearing about eight longitudinal ridges, on and between which run fine parallel striæ.

Remarks.—In size this specimen is somewhat smaller than that figured in STERNBERG; otherwise there is no apparent difference. The presence of the fine longitudinal striæ on the Radstock specimen may be due to better preservation.

The relationship of several described species of *Rhabdocarpi* to *Rhabdocarpus multistriatus* is difficult to determine. These are referred to in my *Catalogue of Palæozoic Plants.*

Locality :—Radstock.

Carpolithus, Sternberg, *Vers. einer geol.-botan. Darstellung d. Flora d. Vorwelt*, vol. ii. p. 208.

Carpolithus ovoideus, Göppert & Berger, sp.

Plate XXIII. figs. 7, 8.

Carpolithus ovoideus, Grand' Eury, *Flore carbon. du Départ. de la Loire*, p. 239.
Rhabdocarpus (?) *ovoideus*, Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 206, pl. xvii. fig. 4; pl. xviii. figs. 10–14, 18–21.
Rhabdocarpus ovoideus, Göpp. & Berger, *De fruct. et semin.*, p. 22, pl. i. fig. 17.
Rhabdocarpus Germarianus, Göpp., *Foss. Flora d. Perm. Form.*, p. 270, pl. lxiv. fig. 14.
Carpolithes membranaceus, Göpp. & Berger, *De fruct. et semin.*, p. 25, pl. ii. figs. 19 and 20.
Carpolithes ellipticus, Sternb., *Vers.*, i. fasc. 4, p. xl. pl. vii. fig. 1.

Remarks.—Rare.

Localities :—Wellsway Pit; Camerton.

GENERAL REMARKS.

The genus *Calamites*, though represented by several species, is not of very frequent occurrence. *Calamocladus* is also rare. *Sphenophyllum*, as far as at present known, is only represented by one species. Though seven species of *Sphenopteris* have been met with, none of them are of frequent occurrence. *Neuropteris* is represented by six species, of which *N. macrophylla* and *N. Scheuchzeri* are the most common, the former being especially plentiful. *Dictyopteris* is very rare, only a single specimen having been seen. *Odontopteris* is not much more common. No less than fifteen species of *Pecopteris* have been observed; of these *P. arborescens, P. Miltoni, P. oreopteridia,* and *P. unita* are the most plentiful, the two first-mentioned species being especially common. *Dactylotheca* is also of frequent occurrence. Of the six species of *Alethopteris,* only *A. Serlii* is common, but that is extremely plentiful. None of the species of *Rhacophyllum* are common. *Megaphyton* and *Caulopteris* are also rare. Lepidodendra are not very common, but *L. Wortheni* is the species most frequently met with. The genus *Lepidophloios* is very rare. The *Sigillariæ* are represented by eight species, the commonest being *S. tessellata. Cordaites,* though only represented by one species, is in some localities very plentiful. *Poacordaites* is, on the other hand, very rare. The genera *Cardiocarpus, Trigonocarpus, Rhabdocarpus,* and *Carpolithus* are of somewhat rare occurrence.

For the purpose of comparing the fossil plants from the Radstock Series with those of other coal fields, I append a table, in the first column of which are given the plants of the Radstock Series; Column II. shows the Radstock species that occur in the Coal Measures of France; and Column III. those common to the Zwickau Coal Beds.* Columns IV. and V. give the fossil plants common to both the Radstock and the Saarbrück and Ottweiler Series respectively.†

It must be borne in mind that the following Tables only show wherein the Floras of these respective areas *agree* with that of the Radstock Series, and do not take any note of the plants occurring in the various horizons taken for comparison which do not occur in the Radstock Series. If this point were taken into consideration, it would be found that each series has species more or less peculiar to itself.

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* Compiled from Geinitz, *Foss. Flora d. Steinkf. in Sachsen.*

† Weiss, *Foss. Flora d. jüngst. Stk. u. d. Rothl.*, p. 237.

[TABLE.

TABLE *comparing the Flora of the* Radstock Series *with that of other Coal Fields.*

Number.	Name.	Page.				Saarbrück Series.			Ottweiler Series.		
			I.	II.	III.	IV.			V.		
			Radstock Series.	Coal Measures of France.*	Zwickau Coal Field.	Lower.	Middle.	Upper.	Lower.	Middle.	Upper.
1	Excipulites callipteris, *Schimp. sp.*	339	×								
2	Calamitina varians, *var.* insignis, *W.*	340	×			× †	× †	× †	× †	× †	× †
3	Eucalamites cruciatus, *var.* senarius, *W.*	340	×	*a*		× ‡	× ‡				
4	ramosus, *Artis sp.*	341	×	*d*							
5	Stylocalamites Suckowii, *Brongt. sp.*	342	×	*d*	×	×	×	×	×	×	×
6	cannæformis, *Schl. sp.*	342	×	*a*	×	×	×				
7	Cistii, *Brongt. sp.*	343	×	*d*	×	×	×	×			
8	Calamocladus equisetiformis, *Schl. sp.*	343	×	*b*		×	×	×	×	×	×
9	Annularia stellata, *Schl. sp.*	343	×	*b*	×	×	×		×	×	×
10	sphenophylloides, *Zenker sp.*	344	×	*b*	×		×		×	×	×
11	Sphenophyllum emarginatum, *Brongt.*	344	×	*b*	×	×	×		×	×	×
12	Macrostachya infundibuliformis, *Brgt. sp.*	345	×	*a*	×	×	×				×
13	Sphenopteris tenuifolia, *Gutbier* (*? Brongt.*)	345	×		×						
14	geniculata, *Germ. & Kaul.*	346	×		?						
15	Grandini, *Göpp. sp.*	347	×		×						
16	macilenta, *L. & H.*	348	×		×						
17	Woodwardi, *Kidston*	348	×								
18	neuropteroides, *Boulay sp.*	349	×	*c*							
19	cristata, *Brongt.*	350	×	*a*	×						
20	Ptychocarpus oblongus, *Kidston*	350	×								
21	Schizostachys sphenopteroides, *Kidston*	352	×								
22	Macrosphenopteris Lindsæoides, *Kidston*	353	×								
23	Neuropteris macrophylla, *Brongt.*	354	×								
24	Scheuchzeri, *Hoffm.*	356	×	*c*	×						
25	flexuosa, *Sternb.*	359	×	*c*	×						
26	ovata, *Hoffm.*	359	×								
27	rarinervis, *Bunbury*	361	×	*c*							
28	fimbriata, *Lesqx.*	361	×								
29	Dictyopteris Munsteri, *Erchw. sp.*	361	×	*c*							
30	Odontopteris Lindleyana, *Sternb.*	363	×								
31	Mariopteris muricata, *Schl. sp.*	363	×	*c*							
32	nervosa, *Brongt. sp.*	363	×	*c*	×						
33	Pecopteris arborescens, *Schl. sp.*	366	×	*a*	×		×		×		×
34	Candolliana, *Brongt.*	367	×	*a*					×		×
35	asper, *Brongt. sp.* (*?*)	367	×	*c*							
36	pennæformis, *Brongt.*	367	×	*c*							
37	unita, *Brongt.*	367	×	*a*	×	×	×		×		×
38	villosa, *Brongt.*	370	×								
39	oreopteridia, *Schl. sp.*	371	×	*a*			×		×		×
40	Cistii, *Brongt.*	372	×								
41	Bucklandii, *Brongt.*	372	×	*a*			×				
42	pteroides, *Brongt.*	373	×	*a*	×	×	×		×	×	×
43	crenulata, *Brongt.*	373	×	*c*							
44	polymorpha, *Brongt.*	373	×	*a*							
45	Miltoni, *Artis sp.*	374	×	*c*		×	×	×	×	×	×
46	Lamuriana, *Heer*	380	×	*a*							
47	pinnatifida, *Gutbier sp.*	380	×								
48	Corynepteris erosa, *Gutbier sp.*	381	×		×						

* The letters in this column show the Horizons in which the plants occur in the Coal Measures of France, as under :—

a, Superior (= Upper Superior) } Superior or Upper Coal Measures.
b, Lower Superior, }
c, Middle Coal Measures.
d, Species common to *a*, *b*, *c*.

† *C. varians.* ‡ *C. cruciatus.*

Number.	Name.	Page.				Saarbrück Series.			Ottweiler Series.		
			I.	II.	III.	IV.			V.		
			Radstock Series.	Coal Measures of France.	Zwickau Coal Field.	Lower.	Middle.	Upper.	Lower.	Middle.	Upper.
49	Dactylotheca plumosa, *Artis sp.* *var.* dentata, *Brongt. sp.*	381	×	*c* *d*	×	×	×	×	×		×
50	Dicksoniites Pluckenetii, *Schl. sp.*	383	×	*a*	×	×	×		×		×
51	Alethopteris lonchitica, *Schl. sp.*	384	×	*c*							
52	Serlii, *Brongt. sp.*	385	×	*c*		×	×	×		?	
53	Grandini, *Brongt. sp.*	385	×	*a c*							
54	aquilina, *Schl. sp.*	385	×	*a*	×	×	×		×		×
55	obliqua, *Brongt. sp.*	386	×	*c*							
56	Davreuxi, *Brongt. sp.*	386	×	*c*							
57	Spiropteris, *sp.*	387	×								
58	Rhacophyllum crispum, *Gutbier sp.*	387	×	*d*	×	×	×				×
59	filiciforme, *Gutbier sp.*	388	×		×						
60	Goldenbergii, *Weiss*	388	×								
61	spinosum, *Lesqx.*	389	×								
62	Megaphyton frondosum, *Artis*	390	×	*c*	×						
63	elongatum, *Kidston*	390	×								
64	Caulopteris primæva, *L. & H.*	392	×								
65	anglica, *Kidston*	392	×								
66	macrodiscus, *Brongt. sp.*	393	×	*a*	×						
67	Lepidodendron aculeatum, *Sternb.*	394	×	*d*	×						
68	Worthenii, *Lesqx.*	394	×	*c*							
69	lanceolatum, *Lesqx.*	394	×								
70	rhombicum, *Presl*	394	×								
71	Lepidophloios, *sp.* (Halonia)	395	×	*b*							
72	Lepidophyllum majus, *Brongt.*	395	×		×	×	×				
73	*sp.*	395	×								
74	Lepidostrobus spinosus, *Kidston*	396	×								
75	*sp.*	396	×								
76	Sigillaria (Ulodendron) major, *L. & H. sp.*	396	×	*c*							
77	Serlii, *Brongt.*	396	×								
78	M'Murtrieii, *Kidston*	397	×								
79	monostigma, *Lesqx.*	397	×	*b*							
80	tessellata, *Brongt.*	397	×	*b*							
81	lævigata, *Brongt.*	398	×	*c*							
82	reniformis, *Brongt.*	399	×	*c*		×	×				?
83	alternans, *Sternb. sp.*	400	×			×	×		×		×
84	notata, *Steinhauer sp.*	400	×								
85	Sporangia	400	×								
86	„ (?)	400	×								
87	Macrospores (Lycopod) (= Trigonocarpus sporites, *W.*)	401	×			×	×				
88	Stigmaria ficoides, *Sternb. sp.*	401	×	*d*	×	×	×	×	×	×	×
89	„ „ *var.* minor, *Geinitz*	401	×								
90	anglica, *Sternb. sp.*	401	×								
91	Cordaites angulosostriatus, *Grand' Eury*	402	×	*a*							
92	Poacordaites microstachys, *Gold. sp.*	402	×				×				
93	Cardiocarpus Gutbieri, *Geinitz*	403	×		×						
94	fluitans, *Dawson*	403	×	*b*			×				×
95	Trigonocarpus Noeggerathi, *Sternb. sp.*	403	×	*c*		×	×				×
96	(?) Dawsii, *L. & H.*	403	×								
97	Rhabdocarpus multistriatus, *Presl sp.*	404	×								
98	Carpolithes ovoideus, *Göpp. & Berger*	404	×			×	×	×			

A summary of the results brought out by these columns shows that, of the

98 species occurring in the Radstock Series—
55 are common to the Coal Measures of France (excluding the *Houiller inférieur*).*
30 ,, ,, Zwickau Coal Field.
24 ,, ,, Lower Saarbrück Series.
30 ,, ,, Middle ,, ,,
9 ,, ,, Upper ,, ,,
17 ,, ,, Lower Ottweiler Series.
10 ,, ,, Middle ,, ,,
22 ,, ,, Upper ,, ,,

The Radstock Series belong to the uppermost beds of the British Coal Measures, with which perhaps the Zwickau Beds are homotaxial. Although the middle division of the Saarbrück Beds contains as many as thirty species in common with the Radstock Series, the floras of the Saarbrück and Ottweiler Series, taken as a whole, indicate a somewhat higher horizon than that of the Radstock Beds.

A characteristic Permian conifer, *Walchia piniformis*, Schl., sp., has been observed in the Middle Saarbrück Series, and it also probably occurs in the Ottweiler Beds. No evidence of this genus has been found in the Radstock Series.

It is, however, interesting to observe that in the Upper Ottweiler Series (which overlie the Saarbrück Series) no fewer than twenty species occur that are also found in the Radstock Series. Beds of the same age as the Radstock Series appear to be absent from France. Their position is evidently between the Upper and Middle Coal Measures of the French Coal Fields. For the comparative list of the French Superior and Middle Coal Measure plants (Column II.) I am indebted to M. Zeiller, who has spared no trouble in providing me with the desired information. To elucidate this point more fully, I give some extracts from his letters on this interesting subject.

"Your list (Column I.) seems to me to indicate exactly the passage from the Middle Coal Measures to the Upper Coal Measures, something like the highest zone of the Saarbrück Beds of Weiss, or the base of the Coal Field of Saxony. You will see in my text of the Coal Field of Valenciennes (when it appears, which will not be so soon as I would wish †) what I say on this classification.

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* It may be well to note here the probable equivalents of the French and British Carboniferous Rocks :—

France.	*Britain.*	
Upper Coal Measures (Houiller Supérieur and Supérieur inférieur)	Absent.	
Absent	Radstock Series (= Upper Coal Measures of Britain).	
Middle Coal Measures (Houiller Moyen) .	Middle Coal Measures. Lower Coal Measures.	
Inferior Coal Measures (Houiller inférieur) .	Lower Carboniferous	Carboniferous Limestone Series. Calciferous Sandstone Series.

† The Atlas of Plates only has been issued.

"The species in your list that I have not found in our *Bassin du Nord,* but which occur in our *Bassins du Centre,* I have marked '*Superior*'; others, which are found both in the *Bassin de Valenciennes,* but especially in its highest part, and at the base of our *Bassins de la Loire* and *Alais,* as *Sphenophyllum emarginatum* and *Sig. monostigma,* or even extending into the highest beds, as *Asterophyllites equisetiformis, Annularia stellata, Ann. sphenophylloides,* and *Alethopteris Grandini,* I have marked '*Inferior Superior.*' Others again, that I have only known in the *Bassin de Valenciennes* and not in our *Bassins du Centre,* I have marked '*Inferior,*' among these last a certain number occur in the north of France, towards the top of the Basin, that is to say, in the oil or gas-coal beds at the *Pas de Calais,* which the Abbé BOULAY has already shown, and which I show in my turn, to be the highest part of the *Bassin de Valenciennes.* These are *Sphen. neuropteroides, Neur. Scheuchzeri, Neur. rarinervis, Dictyopteris Munsteri, Pecopteris crenulata, Aleth. Serlii, Ulodendron majus,* and *Sigillaria reniformis.* What I have not marked as to *horizon* are those which I have not seen in France, but *Sphen. Grandini* (=*Sphen. alata,* Brongt.) is from Geislautern, that is to say, from the summit of the middle zone or from the base of the highest zone of the *Saarbrück Beds*; *Corynepteris erosa* is from Saxony. These two indications agree with what I have said already.

"On the other hand, I remark the absence of several species which with us are abundant in the *Upper Coal Measures,* particularly *Sphenophyllum oblongifolium, Sphenophyllum angustifolium, Sphenophyllum longifolium* (to which my *Sphenophyllum Thoni* should, I now believe, belong), *Neuropteris cordata, Odontopteris Reichiana, Dictyopteris Brongniarti, Callipteridium pteridium* (= *Pec. ovata,* Brongt. = *Call. mirabile,* Rost., sp.), *Pec. arguta* (= *Fil. fœminœformis,* Schl.), *Sigillaria Brardii,* and *Sigillaria spinulosa.*

"*From this, Radstock represents, in my opinion, a horizon* (niveau) *that we have not in France, but corresponds to the interval between the end of the Coal Deposits in the north of France and the beginning of the Coal Fields of the centre.*"

In a later letter M. ZEILLER further says—"The mixture they (the Radstock Series) present of Upper Coal Measure species with Lower Coal species is unquestionable, but I do not at all mean to say by that that they are equal both to all or part of our Lower Coal (or more exactly our Middle Coal) and of our Upper Coal; in my opinion they are *intermediate* between the two, and are situated immediately above our highest beds of the north of France and below our lowest beds of St Etienne, Gard, &c.; perhaps their base is equivalent to the summit of the first and their summit the equivalent of the base of the latter, but on the whole I think their position is between the two."

APPENDIX.

The Fossil Flora of the *Farrington, New Rock,* and *Vobster Series* has not been nearly so fully worked out as that of the *Radstock Series,* but still sufficient has been done to make a record of the species known to occur in these horizons of considerable value.

In none of these series are fossil plants so plentiful as in the Radstock Series, hence the difficulty of working out their flora,—in fact, it can only be done satisfactorily by those residing in the neighbourhood, who can take advantage of collecting when shafts are being sunk or new roads being driven underground, or when in any other mining operations good fossiliferous shales are met with. Some of the localities at the time of my visit appeared to be very barren, while at other times I know they yielded a very good harvest.

Next to the *Radstock Series,* the *Farrington Series* is that from which I have collected most, but the time devoted to it has been small compared to that given to the examination of the *Radstock Series.* I may remark in passing, *that palæontologically the Farrington Series cannot be separated from the Radstock Series, of which, in fact, they seem to form a part.*

The records from the *New Rock* and *Vobster Series* are chiefly obtained from the study of specimens in the museums already referred to; but I am also much indebted for information regarding the flora of these series to Mr E. Wethered, F.G.S., Cheltenham, and to Mr S. Jordan, Clifton, who kindly gave me every facility for examining the fossil plants in their collections.

The *Pennant Rock* seldom yields well-preserved examples, owing to the coarse-grained nature of the rock. Any specimens seen were usually coarse casts of *Calamites* or Lepidodendra.

The *Red Shales* which separate the *Radstock* and *Farrington Series* are also very barren, but the plants observed were similar to those of the two series just mentioned.

The flora of these various horizons are treated of in descending series :—

I. Farrington Series.

(Upper Coal Measures.)

Eucalamites (*Calamites*) *ramosus,* Artis.
Loc.—Parkfield.

Stylocalamites (*Calamites*) *Suckowii*, Brongt.
Loc.—Farrington Pit, Farrington-Gurney.

Annularia stellata, Schl., sp.
Loc.—Old Mills Pit, Farrington-Gurney; Farrington Pit; Parkfield.

Annularia sphenophylloides, Zenker.
Loc.—Old Mills Pit.

Sphenophyllum emarginatum, Brongt.
Loc.—Old Mills Pit; Parkfield; Farrington Pit.

Sphenopteris macilenta, L. & H. (?)
Loc.—Old Mills Pit.

Sphenopteris neuropteroides, Boulay, sp.
Loc.—Old Mills Pit.

Sphenopteris, sp. Pl. XIX. fig. 3 shows a small fragment of *Sphenopteris* having considerable similarity to the fruiting specimens of *Sphenopteris* (*Diplothmema*) *Zeilleri*, Stur,* figured by ZEILLER, but is too fragmentary for a satisfactory identification. On the same slab are the indistinct remains of another specimen, which is apparently specifically distinct from that just mentioned.

Loc.—Old Mills Pit.

Neuropteris macrophylla, Brongt.
Loc.—"Top Vein," Parkfield; Old Mills Pit; Farrington Pit.

Neuropteris Scheuchzeri, Hoffm.
Loc.—Foxcote, near Radstock; Middle Pit, Radstock; Farrington Pit; Old Mills Pit.

Neuropteris flexuosa, Brongt.
Loc.—Old Mills Pit.

Neuropteris ovata, Hoffm.
Loc.—"Hollybush Vein," Parkfield; Old Mills Pit.

Neuropteris rarinervis, Bunbury.
Loc.—Old Mills Pit; Foxcote.

Mariopteris nervosa, Brongt., sp.
Loc.—Old Mills Pit; Farrington Pit.

Pecopteris arborescens, Schl., sp.
Loc.—Old Mills Pit; Pucklechurch, near Mangotsfield; Parkfield.

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* = *Diplothmema acutilobum*, Zeiller, *Ann. d. sci. nat. Bot.*, 6^{e} sér., vol. xvi. pl. xi. fig. 2; see also *Flore foss. d. Bassin houil. d. Valenciennes*, pl. xv. fig. 5.

Pecopteris unita, Brongt.
Loc.—Farrington Pit; Old Mills Pit.

Pecopteris oreopteridia, Schl., sp.
Loc.—Parkfield.

Pecopteris pteroides, Brongt.
Loc.—Parkfield.

Pecopteris Miltoni, Artis, sp.
Loc.—Farrington Pit.

Dicksoniites Pluckenetii, Schl., sp.
Loc.—Farrington Pit; Foxcote.

Alethopteris lonchitica, Schl., sp.
Loc.—Shale over "Top Vein," Parkfield; Middle Pit, Radstock.

Alethopteris Serlii, Brongt., sp.
Loc.—Old Mills Pit; Farrington Pit; Middle Pit, Radstock.

Alethopteris Grandini, Brongt., sp.
Loc.—Old Mills Pit.

Rhacophyllum crispum, Gutbier, sp.
Loc.—Parkfield; (?) Old Mills Pit.

Rhacophyllum Goldenbergi, Weiss.
Pl. XXVII. fig. 2.
Loc.—Pucklechurch, near Mangotsfield.

Rhacophyllum, sp.
Loc.—Farrington Pit.

Caulopteris macrodiscus, Brongt.
Pl. XXV. fig. 1.
Loc.—Coal Pit Heath, near Bristol.

Caulopteris, sp.
Loc.—Old Mills Pit.

Lepidodendron Worthenii, Lesqx.
Loc.—Old Mills Pit.

Lepidostrobus.
Loc.—Old Mills Pit.

Sigillaria monostigma, Lesqx.
Loc.—Old Mills Pit.

Sigillaria M'Murtrieii, Kidston.
Loc.—Farrington Pit.

Sigillaria tessellata, Brongt.
Loc.—Foxcote, near Radstock.

Sigillaria reniformis, Brongt.
Loc.—Coal Pit Heath

Sigillaria principis, Weiss.
Pl. XXVIII. figs. 6–8.

Sigillaria principis, Weiss, Zeiller, *Flore foss. du Bassin houil. de Valenciennes*, pl. lxxix. figs. 1, 2, 1886.

Description.—Stem furrowed, furrows straight, ribs smooth; leaf-scars more or less distant, oval or suborbicular, lateral angles distinct, vascular cicatricules situated slightly above the centre of the scar, the two lateral lunate, the central punctiform. Leaf-scar surmounted by a small cicatrice. Decorticated stem striate.

Remarks.—The specimens from Farrington-Gurney agree with fig. 2 of ZEILLER'S plate, where the scars are orbicular, and not so oval as in his fig. 1. In the Somerset examples there is no trace of the downward running lines that proceed from the lateral angles. In some cases the scars are slightly emarginate, in others they are simply rounded at the top. The central portion of the ribs is flattened in my fossils, but this may be due to pressure. Only two specimens were met with, and, from the position in which they were found, they probably belong to the same individual.

Loc.—Old Mills Pit, Farrington-Gurney.

Sigillaria Voltzii, Brongt.
Loc.—Coal Pit Heath.

Sigillaria elongata, var. *minor*, Brongt.
Loc.—Parkfield.

Stigmaria ficoides, Sternb., sp.
Loc.—Parkfield; Old Mills Pit; Foxcote, near Radstock.

Cordaites angulosostriatus, Grand' Eury.
Loc.—Old Mills Pit; Foxcote.

Cardiocarpus fluitans, Dawson.
Loc.—Old Mills Pit.

Trigonocarpus Noeggerathi, Sternb., sp.
Loc.—Old Mills Pit.

Carpolithes ovoideus, Göpp. & Berger.
Loc.—Old Mills Pit.

II. PENNANT ROCK.

Stylocalamites (*Calamites*) *Suckowii*, Brongt.
Loc.—Crewshole, near Bristol.

Stylocalamites (*Calamites*) *cannæformis*, Schl., sp. (?).
Loc.—Crewshole (perhaps only *C. Suckowii*).

Calamitina (*Calamites*) *approximatus*, Brongt.
Loc.—Downend, near Mangotsfield; Crewshole; Fish Ponds, near Bristol.

" *Ulodendron*."
Loc.—Downend, near Mangotsfield.

" *Halonia*."
Loc.—Fish Ponds, near Bristol.

Stigmaria ficoides, Sternb., sp.
Loc.—Near Bristol.

III. NEW ROCK SERIES.

Stylocalamites (*Calamites*) *Suckowii*, Brongt.
Loc.—" Thoroughfare Seam," Kingswood, near Bristol.

Sphenopteris trifoliolata, Artis, sp.
Loc.—Above " Toad Vein," Deep Pit, Kingswood.

Mariopteris nervosa, Brongt.
Loc.—Deep Pit, Kingswood.

Pecopteris arborescens, Schl., sp.
Loc.—Golden Valley.

Pecopteris Miltoni, Artis, sp.
Loc.—" Top Vein," Warmley, near Bristol; Golden Valley.

Lepidostrobus, sp.
Loc.—Kingswood.

" *Ulodendron* " (*Sig. major*, L. & H., sp. (?)).
Loc.—" Little Toad Vein," Speedwell Colliery, Kingswood.

Sigillaria monostigma, Lesqx.
Loc.—Warmley, near Bristol.

Sigillaria tessellata, Brongt.
Loc.—Kingswood.

Sigillaria mamillaris, Brongt.
Loc.—Warmley, near Bristol.

Sigillaria mamillaris, var. *abbreviata* (Brongt.), Weiss.
Foss. Flora d. jüngst. Stk. u. d. Rothl., p. 165.
Loc.—"Great Toad Vein," Kingswood.

Sigillaria scutellata, Brongt.
Loc.—Kingswood.

Sigillaria rugosa, Brongt.
Loc.—"Great Toad Vein," Kingswood.

Sigillaria Schlotheimii, Brongt.
Loc.—2 feet above "Toad Vein," Kingswood.

Stigmaria ficoides, Sternb., sp.
Loc.—Kingswood; Bedminster, near Bristol.

Cordaites, sp.
Loc.—Deep Pit, Kingswood.

IV. VOBSTER SERIES.

Calamitina (*Calamites*) *varians,* Sternb.
Loc.—Edford Colliery, near Radstock.

Stylocalamites (*Calamites*) *cannæformis* (?), Schl., sp. (or *C. Suckowii,* Brongt.)
Loc.—Edford Colliery.

Sphenophyllum emarginatum, Brongt.
Loc.—Ashton Pits, near Bristol.

Pecopteris oreopteridia, Schl., sp.
Loc.—Ashton Pits, near Bristol.

Lepidodendron aculeatum, Sternb.
Loc.—Edford Colliery.

Lepidodendron rimosum, Sternb.
Loc.—Bed between Stone Rag and Main Seam, Edford.

Sigillaria Sillimani, Brongt.
Loc.—Ashton Pits, near Bristol.

Sigillaria mamillaris, Brongt.
Loc.—Ashton Pits, near Bristol.

EXPLANATION OF PLATES.

Plate XXIV.

Fig. 1. *Alethopteris Davreuxi*, Brongt., sp.; from Camerton, nat. size. Specimen in the collection of the Bristol Museum. 1*a*, two pinnules × 2, p. 386.
Fig. 2. *Pecopteris unita, forma emarginata*, Göpp., sp.; Camerton, p. 367.
Fig. 3. *Pecopteris unita*, Brongt.; New Mills Pit, Farrington-Gurney (*Farrington Series*). 3*a*, two pinnules × 2, showing the nervation.
Fig. 4. *Pecopteris unita, forma emarginata*, Göpp., sp.; Camerton. 4*a*, portion × 2, showing the nervation.
Fig. 5. *Pecopteris unita*, Brongt., *forma elliptica* (*G. elliptica*, Font. and White); Old Mills Pit, Farrington-Gurney (*Farrington Series*), nat. size. 5*a*, pinnule × 2, showing nervation.
Fig. 6. *Pecopteris unita, forma emarginata*, Göpp., sp.; Camerton. 6*a*, portion × 2, showing nervation.
Fig. 7. *Pecopteris unita, forma emarginata*, Göpp., sp.; Radstock, nat. size, showing fructification (= *Stichopteris longifolia* (Brongt.), Weiss).
Fig. 8. *Pecopteris unita, forma emarginata*, Göpp., sp.; Upper Conygre Pit, Timsbury, nat. size, showing fructification (= *Stichopteris longifolia* (Brongt.), Weiss).
Fig. 9. *Pecopteris unita*, Brongt.; Camerton, nat. size. 9*a*, pinnule × 2, showing nervation.

Plate XXV.

Fig. 1. *Caulopteris macrodiscus*, Brongt., sp.; Coal Pit, Heath, near Bristol (*Farrington Series*), half nat. size. Specimen in the collection of the Bristol Museum, p. 393.
Fig. 2. *Caulopteris macrodiscus*, Brongt., sp.; Radstock, half nat. size. Specimen in the collection of Mr J. M'Murtrie.
Fig. 3. *Megaphyton frondosum*, Artis; Radstock half nat. size. In the collection of Mr J. M'Murtrie, p. 390.

Plate XXVI.

Fig. 1. *Megaphyton elongatum*, Kidston, half nat. size. Specimen in the collection of the Bristol Museum, p. 390.
Fig. 2. *Caulopteris*, sp., Radstock, nat. size, showing the epidermal layer removed in part, and exhibiting the *Caulopteris macrodiscus* condition. In the collection of Mr J. M'Murtrie.
Fig. 3. *Caulopteris anglica*, Kidston; Radstock, half nat. size. Specimen in the collection of the Bath Museum, p. 392.
Fig. 4. *Megaphyton frondosum*, Artis; portion of the outer surface of the stem of specimen figured on Pl. XXV. fig. 3, nat. size, showing rootlets, p. 390.
Fig. 5. *Caulopteris*, sp., Radstock, nat. size, isolated scar, showing "horse-shoe" cicatrice.

Plate XXVII.

Fig. 1. *Macrosphenopteris Lindsæoides*, Kidston, n. sp.; Radstock, nat. size, p. 353.
Fig. 2. *Rhacophyllum Goldenbergii*, Weiss; Pucklechurch, nat. size. Specimen in the collection of the Bristol Museum, p. 388.
Figs. 3, 4. *Pecopteris oreopteridia*, Schloth., sp.; Camerton. Fig. 3, barren pinnule × 3, showing nervation. Fig. 4, fruiting pinnule × 3, showing the villose upper surface, p. 371.
Fig. 5. *Lepidodendron lanceolatum*, Lesqx.; Radstock, Cone, nat. size, p. 394.

Fig. 6. *Sigillaria reniformis*, Brongt., var. *Radstockensis*, Kidston; Radstock, nat. size. 6*a*, scar × 1½ In the collection of Mr J. M'Murtrie, p. 399.

Fig. 7. *Lepidophyllum*, sp. *a*, Radstock; *b*, Braysdown Colliery, nat. size, p. 395.

Fig. 8. *Sporangium*, Camerton, nat. size. 8*a*, × 2, p. 400.

Fig. 9. *Sporangium*, Upper Conygre Pit, Timsbury, nat. size. 9*a*, × 2, p. 400.

PLATE XXVIII.

Figs. 1, 2. *Pecopteris oreopteridia*, Schloth., sp.; Camerton. Two pinnæ from different parts of the same specimen, nat. size, p. 371.

Fig. 3. *Lepidodendron lanceolatum*, Lesqx.; Camerton, nat. size, stem showing leaf-scars, p. 394.

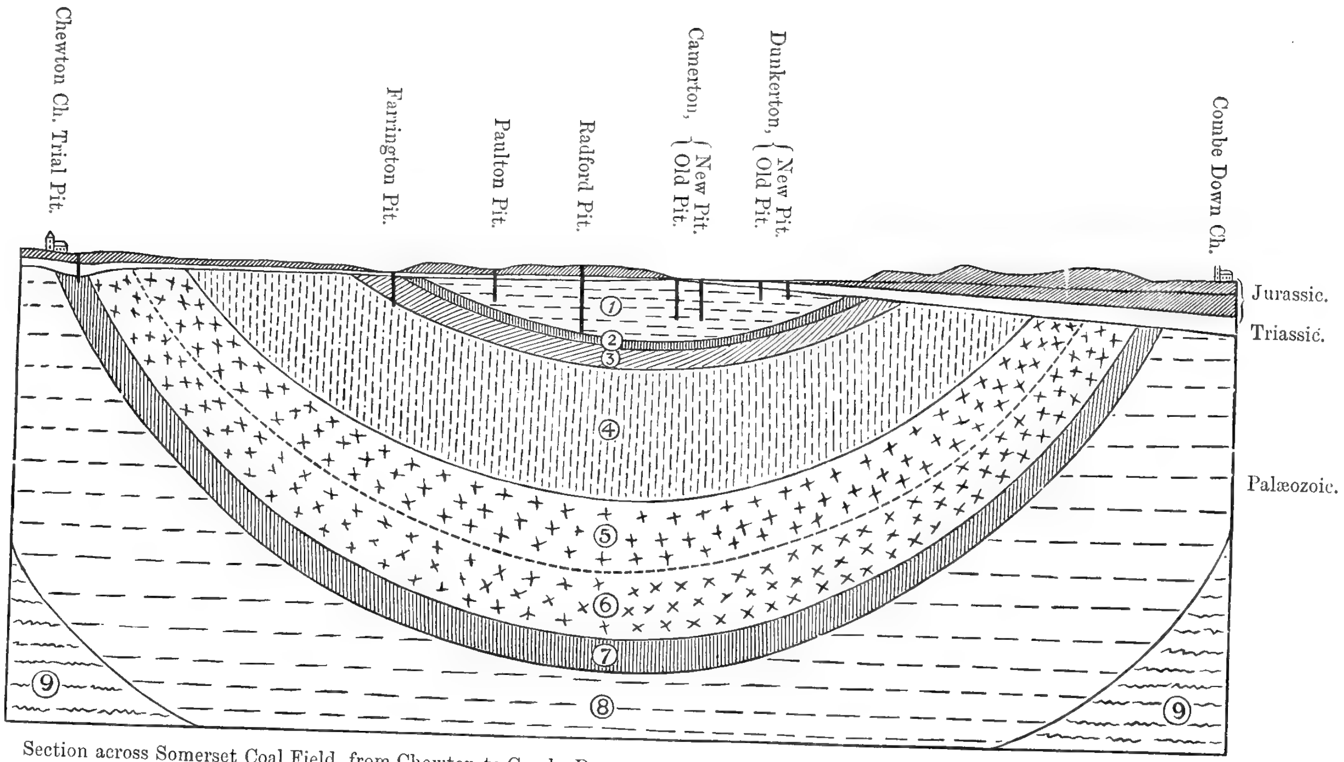
Fig. 4. *Lepidodendron lanceolatum*, Lesqx.; Camerton, nat. size, small branches showing foliage.

Fig. 5. *Sigillaria lævigata*, Brongt.; Radstock, nat. size. 5*a*, leaf-scar × 2, p. 398.

Figs. 6, 7. *Sigillaria principis*, Weiss; Old Mills Pit, Farrington-Gurney (*Farrington Series*), nat. size. 6*a*, leaf-scar × 2, p. 413.

Fig. 8. *Sigillaria principis*, Weiss, same locality as last. 8*a*, leaf-scar × 2, p. 413.

Figs. 9, 9*a*, 10, 10*a*. *Stigmaria anglica*, Sternb., sp.; Camerton, nat. size. 10*b*, scar × 2, p. 401.



Section across Somerset Coal Field, from Chewton to Combe Down, near Bath. (Reduced from section prepared by J. M'Murtrie, Esq., F.G.S., for the Royal Coal Commission, 1868, and published with his Paper read before the Somersetshire Archæological and Natural History Society, at Frome, 11th August 1875.)

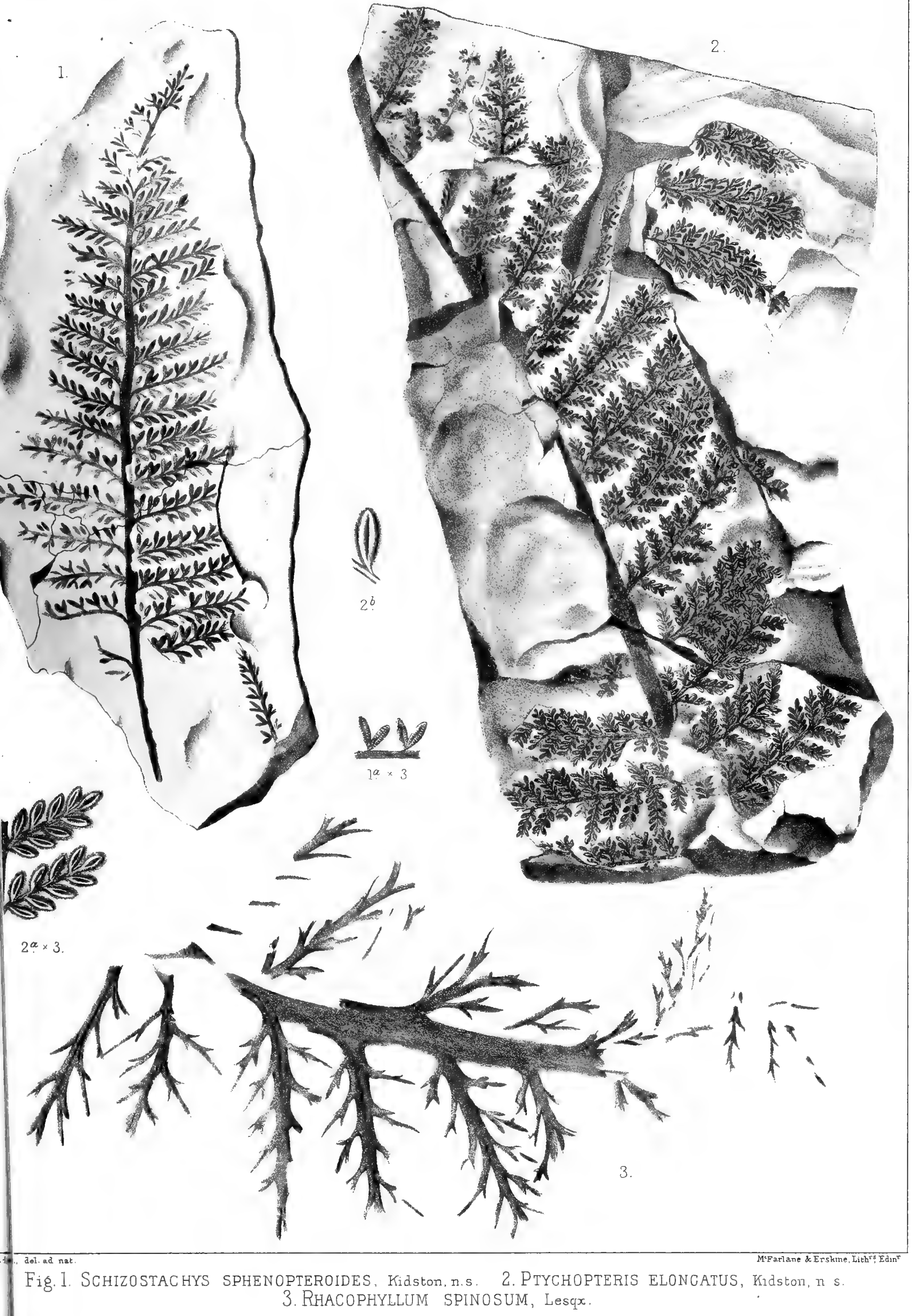
Coal Measures, . . . 1. 2. 3. } Upper Division, . . { 1. Radstock Series. 2. Red Shales. 3. Farrington Series.
4. Pennant Rock.
5. 6. } Lower Division, . . { 5. New Rock Series. 6. Vobster Series.
Millstone Grit, . . . 7.
Mountain Limestone, . . . 8.
Old Red Sandstone, . . . 9.

Robt Kidston, del ad nat

McFarlane & Erskine, Lithrs Edinr

Fig. 1. SPHENOPTERIS WOODWARDI, Kidston. n. s. 2. SPHENOPTERIS TENUIFOLIA, Gutbier, (Brongt.?) 3 a.b. SPHENOPTERIS, sp.

Fig. 1w ANN. SPHENOPHYLLOIDES, Zenker, sp. Fig 1y ANN. STELLATA, Schloth, sp.



del. ad nat.
McFarlane & Erskine, Lithrs Edinr

Fig. 1. SCHIZOSTACHYS SPHENOPTEROIDES, Kidston, n.s. 2. PTYCHOPTERIS ELONGATUS, Kidston, n.s.
3. RHACOPHYLLUM SPINOSUM, Lesqx.

idston del. ad nat. McFarlane & Erskine, Lith^{rs} Edin^{r}

Fig. 1. SPHENOPTERIS GENICULATA, Germar & Kaulfuss. 2. NEUROPTERIS MACROPHYLLA, Brongt.
Fig. 3-5. NEUROPTERIS FIMBRIATA, Lesquereux, (?) 6. DICTYOPTERIS MUNSTERI, Eichwald, sp.

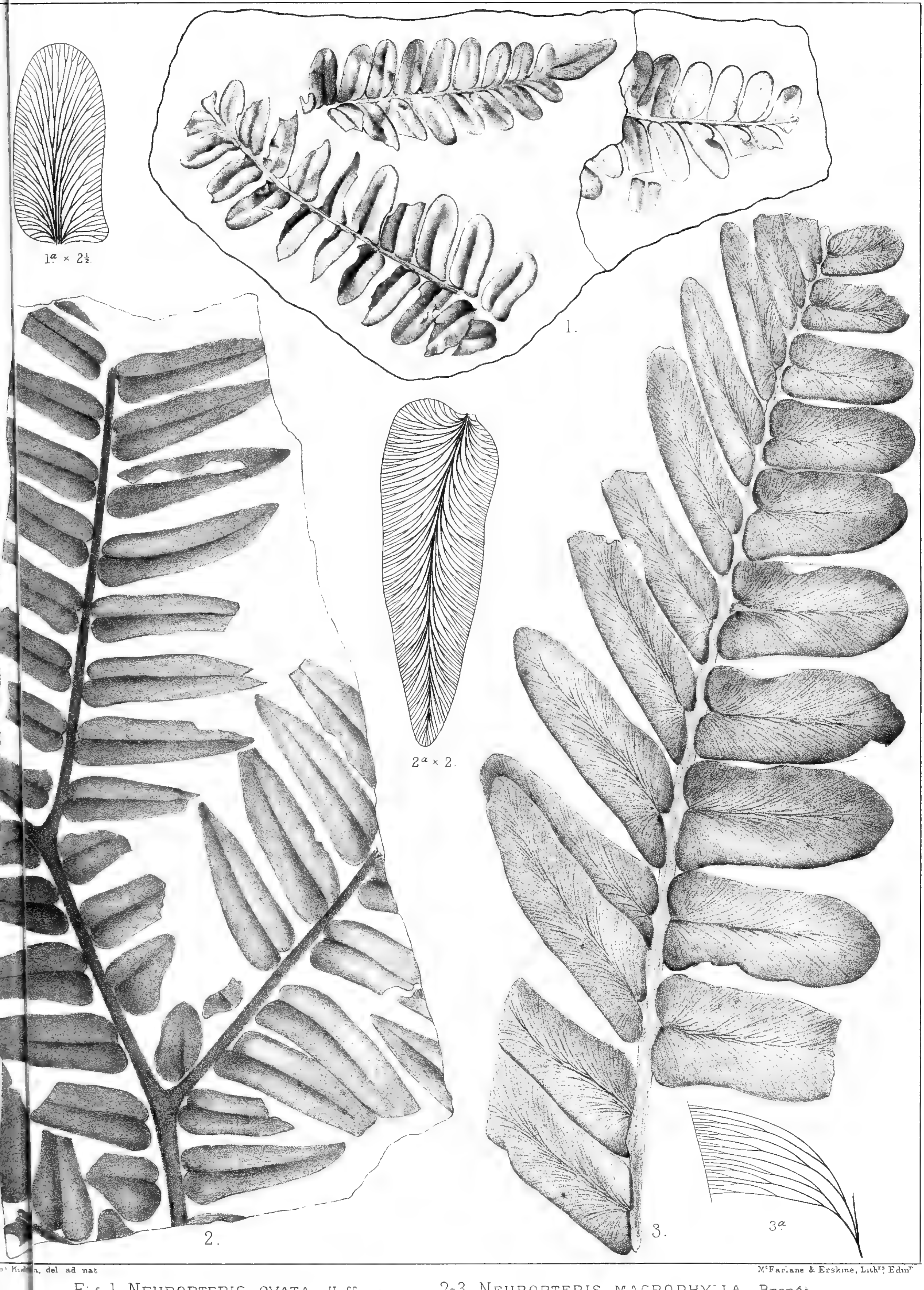


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Fig. 1, NEUROPTERIS OVATA, Hoffmann. 2-3, NEUROPTERIS MACROPHYLLA, Brongt.

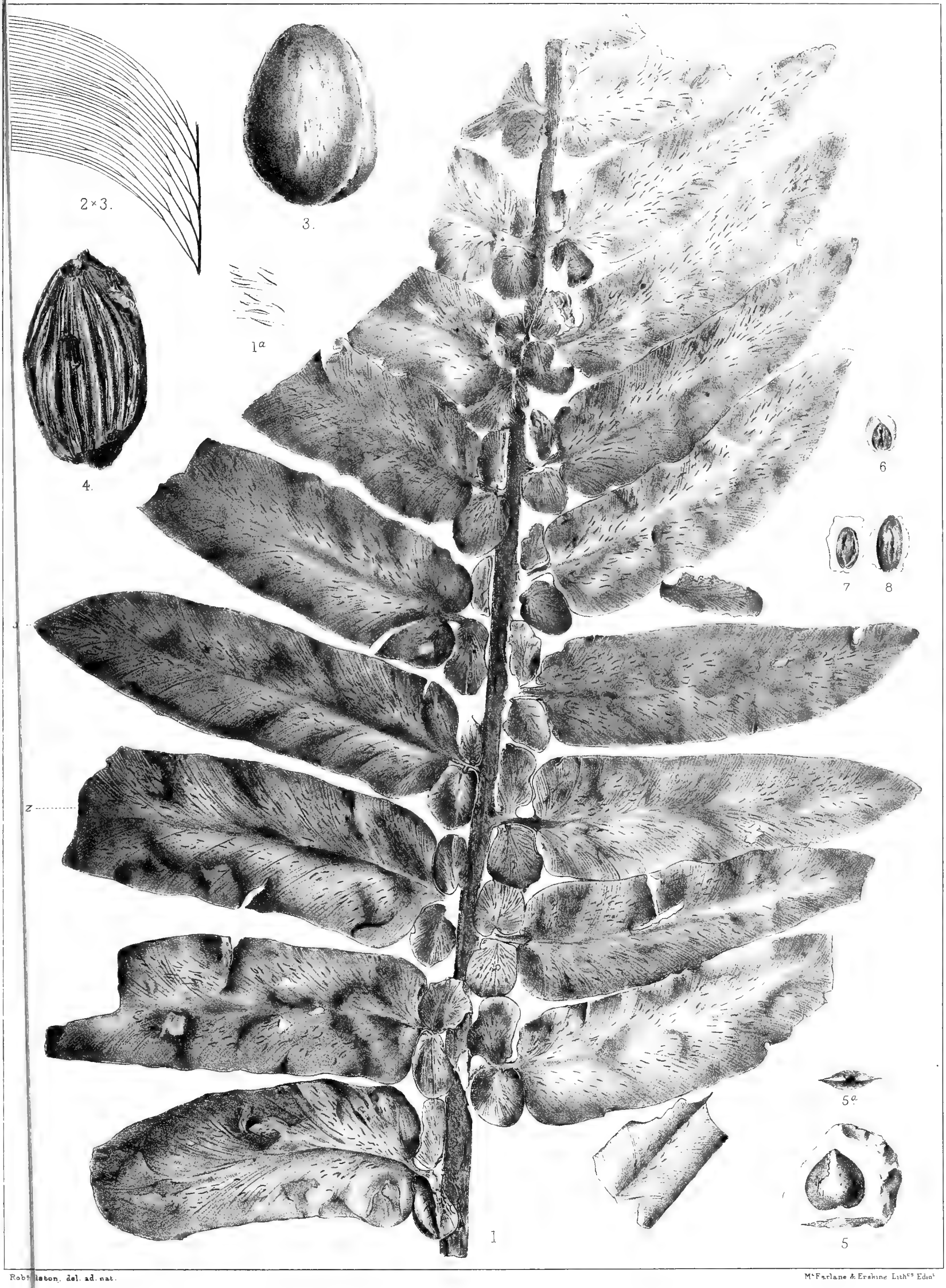

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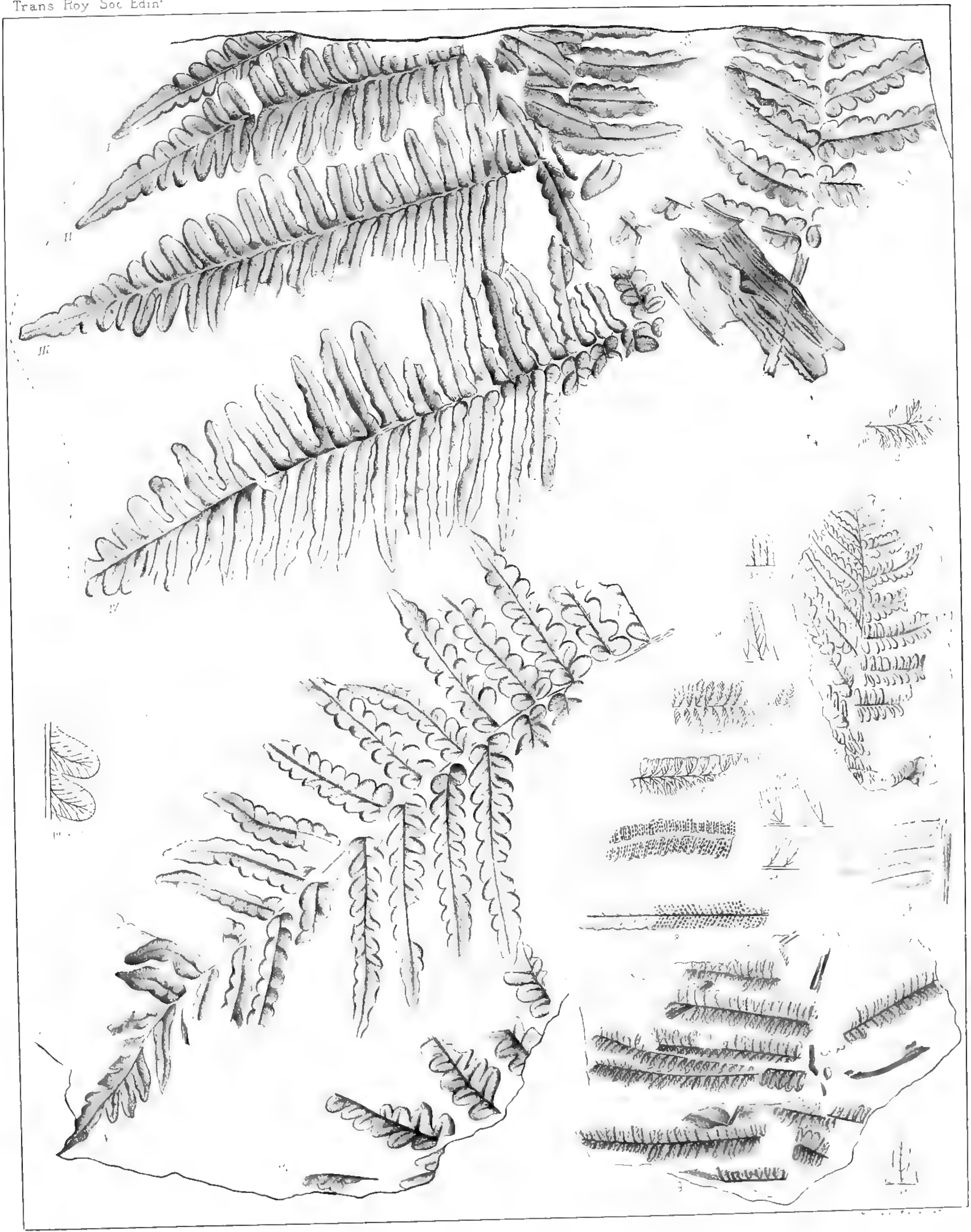
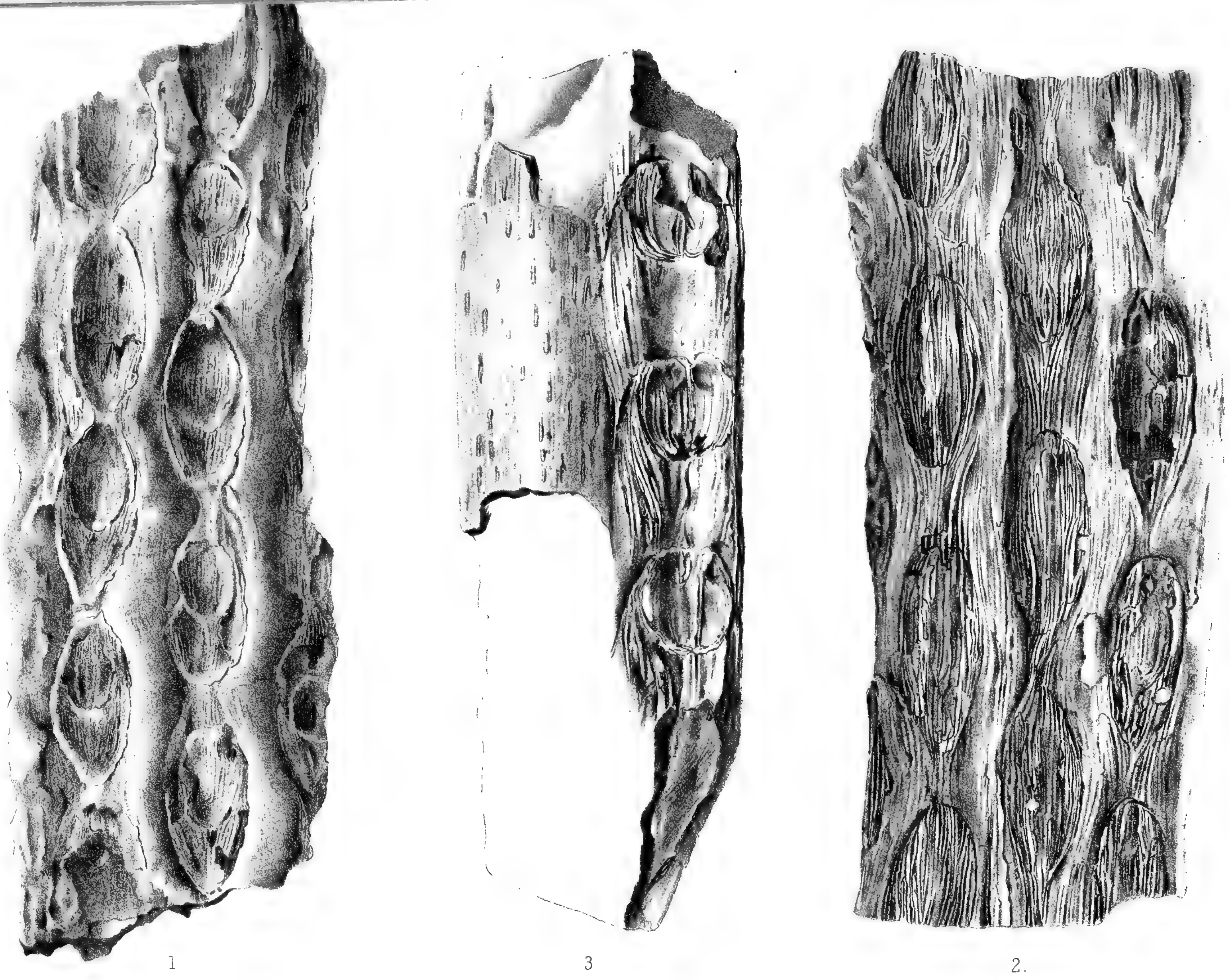
Fig. 1-2. NEUROPTERIS SCHEUCHZERI, Hoffm. 3. TRIGONOCARPUS NOEGGERATHI, Brongt.
Fig. 4. RHABDOCARPUS MULTISTRIATUS, Presl sp. 5. CARDIOCARPUS GUTBIERI, Geinitz.
Fig. 6. CARDIOCARPUS. 7-8. CARPOLITHUSOVOIDEUS, Göppert & Berger.



Robt Kidston del

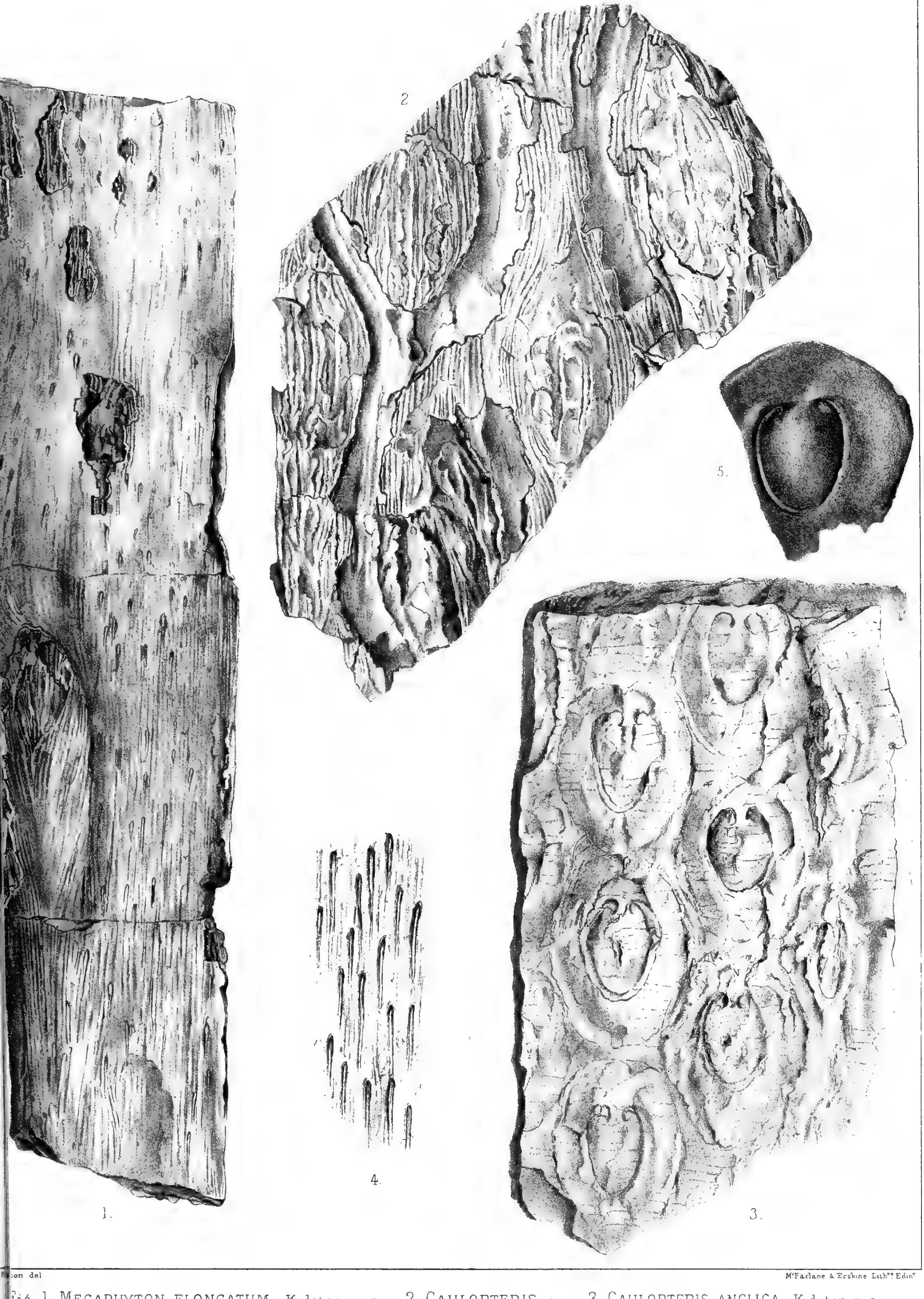
Fig 1. ALETHOPTERIS DAVREUXI Brongt sp   Fig 2-9. PECOPTERIS UNITA Brong.



Robt Kidston. del.

McFarlane & Erskine, Lithrs Edinr

Fig. 1-2. CAULOPTERIS MACRODISCUS, Brongt, sp. 3. MEGAPHYTON FRONDOSUM, Artis.



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ig 1. MEGAPHYTON ELONGATUM, Kidston n.s. 2. CAULOPTERIS, sp 3. CAULOPTERIS ANGLICA, Kidston, n.s.
Fig. 4. MEGAPHYTON FRONDOSUM, Artis. 5. CAULOPTERIS, sp.

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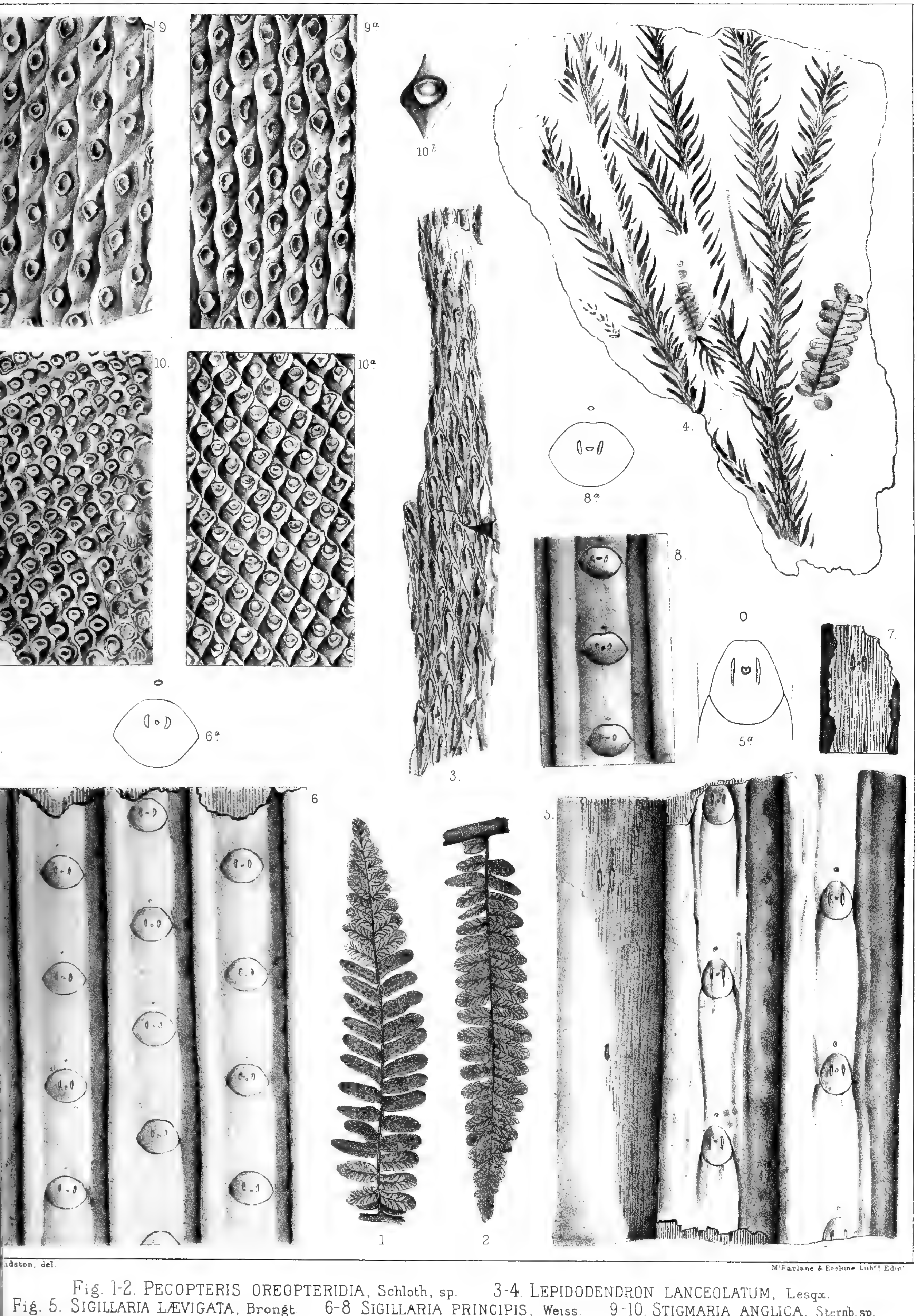
McFarlane & Erskine, Lith^{rs} Edin^{r}

Fig. 1. MACROSPHENOPTERIS LINDSÆOIDES, Kidston, n. s. 2. RHACOPHYLLUM GOLDENBERGII, Weiss.
Fig. 3-4. PECOPTERIS OREOPTERIDIA, Schloth. sp. 5. LEPIDODENDRON LANCEOLATUM, Lesqx. (Cone)
6. SIGILLARIA RENIFORMIS, Brongt. var. RADSTOCKENSIS, Kidston. 7. LEPIDOPHYLLUM, sp. 8-9, SPORANGIA.



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M'Farlane & Erskine Lith^rs Edin^r

Fig. 1-2. PECOPTERIS OREOPTERIDIA, Schloth, sp. 3-4. LEPIDODENDRON LANCEOLATUM, Lesqx.
Fig. 5. SIGILLARIA LÆVIGATA, Brongt. 6-8 SIGILLARIA PRINCIPIS, Weiss. 9-10. STIGMARIA ANGLICA, Sternb. sp.

XVIII.—*A Diatomaceous Deposit from North Tolsta, Lewis.* By John Rattray, M.A., B.Sc., of H.M. "Challenger" Commission, Edinburgh. (Plate XXIX.)

(Read May 2, 1887.)

The sample of the remarkable deposit of Diatomite, upon which the subjoined observations have been made, was forwarded to me some time ago by William Morrison, Esq., of the Academy, Dingwall, through my friend Mr John Gunn. With the exception of a brief notice in the newspapers soon after the discovery of the deposit by Mr Morrison a few months ago, and a paper entitled "On some New Localities for the Mineral Diatomite, with Notes on the Chemical Composition of the Specimens exhibited," in the *Mineralogical Magazine and Journal of the Mineralogical Society,* vol. vii. No. 32, pp. 30–34, July 1886, by W. Ivison Macadam, F.C.S., F.I.C., Professor of Chemistry, New Veterinary College, Edinburgh, no statistics have been published in connection with the deposit, nor has any account hitherto been given of the *Diatomaceæ* which play so prominent a part in its composition.

Mr Morrison* has made the following communication to me with respect to the deposit itself, and to the general features of the district in which it is found:—"The sample of Diatomite sent is from the bottom of a drained† fresh-water loch (Loch Osabhat), North Tolsta, within the civil parish of Stornoway. The area of deposit is about 1½ acres, and is covered with a skin of peaty soil of about 3 feet thick. I found an average depth of 7½ feet for the *pure* deposit over this area. The Diatomite is found pure to the rocky bottom of the loch. I examined sections of it in the trenches for draining cut through it, and saw no appearance of underlying deposit. The trenches, of course, did not go to the bottom of the deposit in the middle or deepest part, but, so far as I can judge, the Diatomite was pure and unmixed to the very *bed-rock* of the loch. In mass and *in situ* the deposit, when cut into, is bluish and fœtid, but when dry both smell and colour disappear. The rocks of the district are of the coarsely granular gneiss of the island, alternating with argillaceous schists. The rock around the loch is gneiss, much kaolinised. North of the conglomerate junction at Gress and on to North Tolsta the rocks are argillaceous schists of a yellowish-green colour. I was struck by the large number of jaspery pieces of gneiss I met on the road to the loch, and quite as much struck by the extreme kaolinised appearance of the rocks in the immediate neighbourhood

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* Letter, dated April 20, 1887.

† From Professor Macadam's paper it appears that this loch was drained in 1874.

of the loch itself. In the middle of the drained area is a 'crannoge,' the only fertile spot within the area, built on piles driven into the deposit of Diatomite, and overlaid by flat stones, over which lies the soil of the 'crannoge.' A causeway on piles connected this ancient lake-dwelling with the shore. Fragments of unglazed pottery and deer's horns were found when trenching around this little island. It may be of collateral interest to mention that directly south from Tolsta, across the Broad Bay, at Garrabost, is a deposit of black boulder plastic clay, now and for years back worked into bricks, at the 100-feet level. The deposit of boulder clay has in it fragments of recent and boreal shells. The average height of Loch Osabhat above the sea is 300 feet, the shore is 200 yards distant from the loch, and the coast is precipitous."

The colour of the dried material is light drab, and its texture is open and porous. Save at the margins, where mineral particles of considerable size—from 3 to 5 mm. in diameter—are to be found, but few minerals occur throughout its mass, the average size of such as do occur being in the specimen examined from 0·1 to 0·15 mm., or even somewhat greater, in diameter.

The results of an analysis* of the deposit have shown it to contain 13·875 per cent. of organic matter, consisting of chlorophyll, cellulose, &c., and 86·125 per cent. of inorganic constituents, the analysis calculated free from moisture, giving 13·874 per cent. of organic and 86·125 of inorganic components. The inorganic portion has yielded 1·326 parts of ferric oxide, 0·753 parts of aluminic oxide, and 94·495 parts of silica, mostly in the form of Diatoms.

The weight of a cubic foot of the Diatomite is said to be 54 lbs.

The following general features may be pointed out in connection with this deposit:—

(1) As indicated above, its frustules are entirely of fresh-water origin.

(2) As in external appearance, so in microscopic character, it exhibits a great degree of uniformity throughout (α) with respect to number of genera represented, which are few, and (β) with respect to their ratio to one another. Thus the preponderance of *Naviculæ, Epithemiæ, Eunotiæ,* and *Surirellæ* is to be contrasted with the much less frequent presence of *Cymbellæ, Encyonemæ, Synedræ, Fragilariæ, Tabellariæ, Cocconeidæ,* and *Cyclotella.*

(3) Among the heavier material *Naviculæ* and *Surirellæ* predominate, whilst among the lightest minute forms of *Navicula, Cymbellæ, Fragilariæ,* and *Cyclotella* are most common.

(4) The rare occurrence of some species, even of large size, is remarkable, *e.g., Gomphonema geminatum,* Ag., var. *bipunctata,* nov., and contrasts curiously with the great abundance of other species of the same genus, *e.g., Gomphonema acuminatum,* Ehrenb. Among the rarer species the following may also be noted:—*Encyonema ventricosum* (Kütz.), Grun.; *Navicula mesolepta,* Kütz.;

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* Macadam, *op. cit.*, pp. 33, 34.

Navicula macilenta, Grun.; *Navicula tenella*, Breb.; *Navicula inæquistriata*, n. sp.; *Navicula interrupta* (W. Sm.); *Epithemia gibba*, Kütz., var. *rectimarginata*, nov.; *Epithemia hyndmanii*, W. Sm.; *Eunotia major* (W. Sm.), Raben., var. *semelconstricta*, nov.; *Eunotia gracilis* (Ehrenb.), Rab., var. *semelmonticulata*, nov.; *Synedra ulna*, Ehrenb., var. *tolstensis*, nov.; *Tabellaria fenestrata*, Kütz.; *Tabellaria flocculosa*, Kütz.

(5) Although not characterised by many new species, several interesting and novel varieties occur, and various curious points of structure in connection with forms already well known are clearly brought out. Among the new forms the curious converse relationship in external form between *Eunotia major*, var. *semelconstricta* and *Eunotia gracilis*, var. *semelmonticulata*, is noteworthy.

(6) As shown by the measurements given below, the scarcely appreciable change in the average fineness of punctation in some species (*e.g.*, *Gomphonema acuminatum* and *Cyclotella antiqua*—marginal circlet) notwithstanding the size of the specimen is somewhat remarkable, although in the latter case it is often associated with irregularity of sculpturing in other parts of the valves.

Tribe I. RAPHIDIEÆ.

Fam. 1. CYMBELLEÆ.

1. *Cymbella helvetica*, Kütz.

Frequent. General form corresponding to that figured by W. SMITH (*Synop. Brit. Diat.*, vol. i. pl. ii. fig. 24); but (1) the extremities are less acute, as shown by A. SCHMIDT (*Atlas d. Diat.*, pl. x. fig. 18); and (2) the striation of the valve is not so arcuate. The striation differs from that on the frustule figured under this name by Van HEURCK (*Synop. d. Diat. d. Belg.*, pl. ii. fig. 15), as no hyaline area is found around the central nodule.

Length, 0·0600 mm.; greatest breadth at median inflation, 0·0137 mm.; striæ, 10 in 0·01 mm.

2. *Encyonema ventricosum* (Kütz.), Grun.

Rare. Some specimens agree very closely in form with that figured by W. SMITH (*op. cit.*, vol. ii. pl. lv. fig. 346*a*) as *Encyonema cæspitosum*, Kütz., but the valval striation is disposed in a different manner. Instead of the lines forming arches, with their convexities directed away from the median transverse line of the frustule, the concavities are so disposed in the present case. The specimens agree most closely with that figured by A. SCHMIDT (*Atlas*, pl. x. fig. 59).

Length, 0·0325 mm.; greatest breadth, 0·0125 mm.; distance of central nodule from convex side, 0·0075 mm.

Fam. 2. NAVICULEÆ.

3. *Navicula mesolepta*, Kütz.

The specimens observed agree almost in every detail with SCHMIDT'S figure (*op. cit.*, pl. xlv. fig. 70), but not with the form which he figures as *Navicula* (*Pinnularia*) *biceps*, Greg., with which he supposes the former to have the closest affinities, if it be not indeed identical. The figures of the forms given by KÜTZING (*Die kieselschaligen Bacillarien oder Diatomeen*, pl. xxviii. fig. 73, and pl. xxx. fig. 34) differ in the more undulated character of the margins and the more distinct appearance of the median raphe.

Length, 0·0625 mm.; greatest breadth, 0·0100 mm.; diameter of terminal knobs, 0·0075 mm.; striæ very faint, 14 to 15 in 0·01 mm.

4. *Navicula obtusa*, W. Sm., var. *lata*, nov. (Pl. XXIX. figs. 1*a*, 1*b*).

Valve elliptical; extremities slightly protruded, very obtuse; marginal flexure between terminal and median areas slight. Striation very faint, running for the most part in a parallel transverse direction, but towards the apices becoming very slightly oblique. Marginal zone of striæ leave a subelliptical central hyaline area, near the outer boundary of which a band of still more delicate striæ occur in the shape of an arch of large curvature. Hyaline areas around the raphe well marked, especially as the centre is approached, but becoming much more constricted towards the extremities, so that the striæ almost abut against the raphe. Terminal hyaline spaces subrotund, distinct. Median raphe not very well defined, with very minute central and terminal nodules. Connecting zone (Pl. XXIX. fig. 1*b*) broad, with parallel edges, median depression of central area of valve well seen from the zonal aspect.

Length, 0·0425 mm.; greatest breadth, 0·0187 mm.; length of protruded ends, 0·0050 mm.; distance between the two central extremities of the median raphe, 0·0050 mm.; diameter of terminal hyaline areola, 0·0043 mm. Striæ 12 in 0·01 mm. around the margin; 15 to 16 mm. in the arched bands on the central hyaline space.

Not uncommon.

5. *Navicula gibba*, A. S., *Atlas d. Diat.*, pl. xlv. figs. 50, 51.
=*Pinnularia gibba*, Ehrenb.

Several specimens. Typical. The costæ on the central side of the terminal nodule extend close up to the raphe for a short distance, as partly but somewhat imperfectly indicated in SCHMIDT'S and SMITH'S figures.

Length, 0·1125 mm.; greatest breadth at centre of valve, 0·0125 mm.; breadth of swollen extremities, 0·0105 mm.; breadth in intermediate area, 0·0100 mm. Striæ, 10 in 0·01 mm.

6. *Navicula macilenta*, Grun., var. *elliptica*, nov. (Pl. XXIX. fig. 4*a*).
=*Pinnularia macilenta* (Grun.) Ehrenb., var. *elliptica*.

A few specimens of this variety occur. They correspond in most of their characters with the forms figured by A. SCHMIDT (*op. cit.*, pl. xliii. figs. 7–9), but especially with his fig. 8 as regards the ornamentation of the valve, and with his fig. 7 as regards the disposition of the raphe. The following divergences may, however, be indicated :—(1) The costæ are much more obliquely disposed on the periphery of the central inflation. (2) The terminal hyaline areolæ are not subrotund but elliptical, the major axis of this ellipse being directed along the longitudinal axis of the raphe. (3) The raphe is faintly marked, but runs straight throughout its length, its terminal and central extremities being very small. The outlines of the central and circumraphideal hyaline areas agree with those of the specimen figured by A. SCHMIDT (fig. 8).

7. *Navicula acuta* (W. Sm.).
=*Pinnularia acuta*, W. Sm.
=*Pinnularia radiosa*, Kütz., var. *acuta*, Van Heurck (*op. cit.*, pl. vii. fig 19).

Fairly common. Typical. Length, 0·0900 mm.; greatest breadth, 0·0100 mm.

8. *Navicula gigas*, A. S., *Atlas d. Diat.*, pl. xlii. fig. 1.
=*Pinnularia gigas*, Ehrenb.

Common. The specimens agree in almost all points with the frustules figured by SCHMIDT. (1) The halo running longitudinally on each side of the valve across the transverse costæ is, however, more clavate in outline at its extremities, showing a slight structural modification in the flexure of the costæ. (2) The terminal hyaline areolæ are even more rounded, and upon these the flexed extremities of the raphe present no thickening, but become uniformly attenuated to very sharp acicular points. (3) The central nodule on one side of, and between, the proximal extremities of the raphe is not round in outline, but slightly reniform or flattened towards the margins of the frustule, but rounded at its ends. (4) The transverse diameter of the costæ is somewhat less.

Length, 0·2625 mm.; greatest breadth, 0·0450 mm.; diameter of terminal areolæ, 0·0075 mm.; diameter of transversely directed costæ, 0·0020 to 0·0025 mm.

9. *Navicula major* (W. Sm.).

=*Pinnularia major*, W. Sm.

Common. The following are the peculiarities of some of the specimens from the deposit:—(1) The central hyaline area does not form a more or less elliptical space around the central nodule, there being merely a slight expansion, which is sometimes confined to one side only (SCHMIDT, *op. cit.*, pl. xlii. fig. 17). (2) The terminal hyaline areola is not subrotund, but is, at both extremities of some of the specimens, of sharply reniform outline, arising from the elongation into it of one of the subterminal costæ. One side of the areola is larger than the other. (3) On the proximal side of the terminal areolæ the costæ extend almost up to the median raphe. (4) The costal halo is arched over the central nodule, but less markedly so than figured by Van HEURCK (*op. cit.*, pl. v. fig. 3). (5) The direction of the subterminal costæ is more oblique than is figured either by Van HEURCK or W. SMITH.

Length, 0·1225 mm.; greatest breadth, 0·0180 mm.; distance between central ends of raphe, 0·0025 mm.; transverse diameter of terminal reniform nodules, 0·0050 mm.

10. *Navicula viridis* (Kütz., W. Sm.).

=*Pinnularia viridis*, W. Sm.

Common, and of various sizes. The peculiarities of some of the specimens are:—(1) The well-defined subrotund hyaline terminal areolæ. (2) The disposition of the costæ, which run obliquely outwards and towards the central transverse axis at the extremities, but obliquely outwards and away from that axis towards the centre. (3) Two or three of the costæ on the proximal side of the terminal areolæ extend close up to the median raphe. (4) The central nodule is but faintly defined, sometimes the inner ends of the costæ in the region bounding the clear central space form an almost straight line, whilst their shorter length on the opposite side is often not very marked. (5) The general outline of some of the specimens is much less rounded than that of those figured by A. SCHMIDT (*op. cit.*, pl. xlii. figs. 19–21). They present, however, certain affinities to *Navicula alternans*, Schum., both with respect to ornamentation and to the character of the terminal areolæ, though the want of symmetry between the two sides of the valve is very much less marked.

Length, 0·1625 mm.; greatest breadth, 0·0235 mm.; diameter of terminal areola, 0·0040 mm.; striæ, 6 in 0·01 mm.

11. *Navicula oblonga* (W. Sm.), var. *subparallela*, nov. (Pl. XXIX. fig. 2).

Margins of valve almost straight, and tapering but very slightly to the extremities. Apices bluntly rounded. Terminal areolæ ovate, and placed very near the extremities; central hyaline space elliptical. Hyaline areas bordering the raphe lentelliptical. Costæ disposed as in the type in the central region, but altogether different at the extremities, being directed very obliquely outwards and towards the central transverse axis of the valve for a space of 0·0125 mm.

Length, 0·0900 mm.; greatest breadth, 0·0100 mm.; diameter of terminal areolæ, 0·0028 mm. Striæ 10 to 12 in 0·01 mm.

This Diatom agrees in form with the *Navicula oblonga* figured by A. Schmidt (*op. cit.*, pl. xlvii. fig. 67), but the details of its ornamentation are altogether different, especially with respect to the terminal costæ and the form of the median hyaline area.

12. *Navicula tenella*, Breb.

A few specimens occur in the deposit which agree essentially with the forms figured by Van Heurck (*op. cit.*, pl. vii. figs. 21, 22). The striation is straight and radiating, the median raphe is well defined, and the central clear space an ellipse of great excentricity, but the extremities are somewhat more obtuse, and the somewhat larger terminal areolæ are situated just beneath the margin.

Length, 0·0500 mm.; greatest breadth, 0·0100 mm.; striæ, 8 to 10 in 0·01 mm.

13. *Navicula cardinalis* (Ehrenb.).

= *Pinnularia cardinalis*, Ehrenb.

Several specimens. Some of the forms may be distinguished from that figured by W. Smith (*op. cit.*, pl. xix. fig. 166) (1) by the somewhat greater swelling at the centre of the frustule; (2) by the occurrence of hyaline areas around the median raphe, which are either bounded by straight lines externally, or are only very slightly elliptical; (3) by the presence of a less regularly contoured hyaline central area; and (4) by the more rounded appearance of the terminal areolæ.

Length, 0·1225 mm.; length of central hyaline area along the longitudinal axis, 0·0150 mm.; greatest breadth at middle, 0·0175 mm.; diameter of terminal areola, 0·0050 mm.; costæ, 8 in 0·01 mm.

14. *Navicula cardinalis* (Ehrenb.), var. *subconstricta*, nov. (Pl. XXIX. fig. 3).

Several specimens of small size occur which cannot be assigned to the typical species on account of the following peculiarities:—(1) The apices are separated from the main body of the frustule by the presence of a shallow but distinctly defined constriction (*a*, *a*). (2) The terminal hyaline spaces are almost round, and are placed immediately within the margin of the valve. (3) The transverse and obliquely-disposed costæ are straight or subarcuate in outline, but present no undulations along their margins. (4) Several of the costæ on the proximal side of the terminal spaces are longer than those adjoining them, and extend close up to the median raphe. (5) The transverse central hyaline space is infundibulate towards both margins, and is slightly broader on one side of the valve than on the other. (6) The costæ on one half of the valve are somewhat larger than on the opposite side, so that a slight asymmetry results. The wider side of the transverse central hyaline space is placed on that side of the valve upon which the length of the costæ is greatest.

Length, 0·0825 mm.; greatest breadth, 0·0125 mm.; transverse diameter of terminal space, 0·0028 mm.; width of transverse central hyaline space, 0·0050 mm. (narrower side), and 0·0062 mm. (wider side); striæ, 12 in 0·01 mm.

15. *Navicula inaequistriata*, n. sp. (Pl. XXIX. fig. 4).

Valve faintly elliptical, the margins in the median area bulging but very slightly; apices obtusely rounded. Raphe excentric throughout, but passing very near to one side towards its distal end, which is flexed abruptly towards the centre; its distal flexure is much more pronounced than its proximal, whilst its middle part is slightly arched in the opposite direction. Median hyaline area inequilateral, its outline on the side next the wider half of the valve being much more convex than on the opposite side. Terminal hyaline areas subrotund or elliptical, and sharply circumscribed. Striation well defined, radiating around the central hyaline space, becoming transverse about halfway between the centre and the apices, and again becoming oblique towards the extremities. Striæ straight or but very faintly subarcuate.

Length, 0·0800 mm.; greatest breadth, 0·0100 mm.; greatest transverse diameter of central hyaline space, 0·0035 mm.; diameter of terminal areolæ, 0·0020 mm.; striæ, 10 in 0·01 mm.

16. *Navicula interrupta* (W. Sm.).

= *Pinnularia interrupta*, W. Sm.

Very rare. The character of the ornamentation agrees essentially with that of the frustule figured by W. Smith (*op. cit.*, vol. i. pl. xix. fig. 184), the central hyaline space being somewhat infundibulate at both sides, but it differs from Schmidt's specimen (*Atlas*, pl. xlv. fig. 72) in the entire absence of a diamond-shaped median hyaline area. The outline of the central part of the valve is somewhat more convex than is indicated by Smith, or than the corresponding part of the *Navicula hilseana*, Janisch, which is very closely allied to it, but less so than that of *Navicula brauniana*, Grun., to which it also bears great external similarity. The appearance of the capitate terminal knob distinguishes it from *Navicula appendiculata* var. *irrorata*, Grun., although the character of the central hyaline region is very much the same in both.

Length, 0·0600 mm.; greatest breadth, 0·0105 mm.; transverse diameter of terminal nodules, 0·0060 mm.

The specimen observed was extremely hyaline, the valval sculpturing being only clearly recognised after considerable difficulty.

17. *Stauroneis phœnicenteron*, Ehrenb.

Specimens sometimes somewhat damaged. Striation exceedingly delicate. Length, 0·1525 mm.; greatest breadth, 0·0250 mm.; breadth of transverse stauros, 0·0040 mm.

18. *Stauroneis anceps*, Ehrenb.

Rare. General form of valve typical. Stauros not hour-glass shaped, but with parallel margins, and running outwards to the margins of the valves. Length, 0·0800 mm.; greatest breadth, 0·0175 mm.; transverse diameter of terminal knob, 0·0050 mm.; transverse diameter of stauros, 0·0025 mm. Striation exceedingly delicate.

Fam. 3. GOMPHONEMEÆ.

19. *Gomphonema geminatum*, Ag., var. *bipunctata*, nov. (Pl. XXIX. fig. 5).

Valves inflated at the centre and at both extremities, the outlines of all the inflations being less acute than in the typical species. This frustule differs in the following respects from that figured by W. Smith:*—(1) If the wider extremity be called *distal*, and the narrower, to which the stipes should be

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* *Synopsis British Diatomaceæ*, vol. i. pl. xxxvii. fig. 235, 1853.

attached, *proximal*, then the distal inflation passes much less abruptly into the distal constriction. (2) The outline of this latter forms a much smaller curve, being very similar to that figured by KÜTZING in his *Die kieselschaligen Bacillarien oder Diatomeen*, pl. xiii. fig. 2*a*. (3) The margins of the median inflation are also less curved, so that there is in the valve a far easier transition from the apical to the median, and again similarly from the latter to the proximal or basal region. (4) At the side of the central nodule, instead of five elliptico-cuneate unilateral cellules, there are here but two, which are far less prominent, on account of their smaller size,—the area occupied by both, together with the intervening space, being 0·0025 mm. (5) These cellules are not placed in the middle of the circumnodular hyaline area, but are somewhat nearer its distal than its proximal extremity. (6) This area, formed by the abbreviation of the limiting striæ is much larger, and its outline is by no means regular, inasmuch as some of the individual striæ pass a good deal further inwards than others. The outlines of the slightly-arched clear spaces surrounding those parts of the median raphe between the central and terminal nodules are, however, quite regular, as figured by W. SMITH and RALFS. (7) The terminal nodules of the raphe are very much less pronounced at the distal and proximal ends of the frustule. (8) The hyaline space at the proximal extremity has an entirely distinct character. It is not bounded by straight moniliform striæ, which pass from the distal side of the proximal nodule obliquely outwards to the widest part of the proximal inflation, but the inner ends of the terminal striæ, which run obliquely outwards and downwards to a narrower part of this inflation, nearer the base of the frustule, form an almost regular curve, convex on the side next the nodule on both sides of the valve. (9) The striation varies in one important feature. Around the upper lateral margin of the distal inflation the moniliform lines are not straight, but arcuate, the concavity here directed towards the apex of the frustules. This arcuation is continued all round from the straight stria, which runs outwards to the widest part of the distal inflation, to the apex, attaining its maximum of curvature at about two-thirds of this distance from the apex. On the distal constriction and the proximal half of the apical inflation the striæ are again slightly, though very distinctly, arcuate, the concavity of the curves being directed basally. (10) To the right of the distal lentelliptical clear space, and towards its lower part, is a hyaline sub-hexagonal areola, extending almost from the median clear space over about two-thirds of this half of the valve. At first sight this seems to be the result of local erosion, but, since it is sharply defined, and as there is no indication of any such action elsewhere, the ornamentation being remarkably clear throughout, there is a strong probability that it is a characteristic of the valve itself.

Length of distal inflation,	0·0175 mm.	
„ median inflation,	0·0600 „	
„ proximal inflation,	0·0250 „	
		Entire frustule = 0·1025 mm.
Breadth of distal inflation,	0·0250 mm.	
„ „ constriction,	0·0200 „	
„ median inflation,	0·0300 „	
„ proximal constriction,	0·0150 „	
„ „ inflation,	0·0200 „	

Number of striæ in 0·01 mm., 6; the diameter of the puncta composing the striæ does not exceed 0·0008 mm.

Although this form cannot be assigned to the typical species, its peculiarities are not of so divergent a character as to merit more than varietal distinction.

Rare.

20. *Gomphonema acuminatum*, Ehrenb. (Pl. XXIX. fig. 5*a*).

A considerable amount of divergence is found in the external form and sculpturing of the numerous figures that have been given by EHRENBERG (*Mikrogeologie*, pl. xv. figs. 87*a*, *b*); KÜTZING (*Die kieselschaligen Bacillarien oder Diatomeen*, pl. xiii. fig. 3); W. SMITH (*op. cit.*, vol. i. pl. xxviii. fig. 238); Van HEURCK (*Synopsis des Diatomées de Belgique*, pl. xxiii. fig. 16), &c., of this diatom. The peculiarities noticeable in specimens from the present deposit are the following:—(1) The cuneate valve is provided with three distinct marginal inflations, the proximal, though least marked, beginning in the immediate vicinity of the narrow extremity. In the case of the great distal inflation the gradient is much greater towards the apical crest than on its proximal side. In no case has asymmetry, such as is figured by KÜTZING and RALFS, been observed. Although, as will be seen from the subjoined measurements of a number of frustules, the ratio between the length of the three regions of the valves varies somewhat, the distal is usually about one-half of the length of the median. (2) The median bluntly conical apical crest has in most of the specimens a height of about 0·0025 mm., and it has been noted that, although the size of the frustule may vary very considerably, the height of this crest does not change, although its transverse diameter at the base slightly increases. Many of the specimens in the deposit correspond most closely to that figured by Van HEURCK, but in most of them the median and proximal inflations are somewhat more pronounced than in that form, and the ratio between their length and breadth do not agree, the Lewis specimens being relatively broader. (3) The arrangement of the transverse striæ in some of the specimens is somewhat different from that shown by W. SMITH and Van HEURCK, and agrees more closely with that shown in RALFS' and KÜTZING'S figures. In the distal inflation the striæ are in straight and not in arched lines, and at the greatest transverse diameter of this part one runs at right angles to the tangent at its surface, those adjoining it radiating on both sides. At the narrowest part of the constriction, between the distal and median inflations, another single stria on each side of the median raphe stands at right angles to the median longitudinal axis of the frustule. On both sides of the central stria of the median inflation the striæ are again disposed in a radiating manner, as indicated by W. SMITH, Van HEURCK, &c., but this radiating condition does not in all cases extend to the proximal extremity without interruption, the striation of the proximal inflation repeating exactly, though in miniature, the arrangement found in the median. (4) The striæ extend from the outer margins well up to the central raphe, which is by no means so pronounced in size as is indicated by any of the figures given by Van HEURCK. The striæ on the valves are often somewhat indistinct. (5) The terminal proximal unornamented areola is very minute, not exceeding 0·0012 mm. in diameter in the direction of the longitudinal axis.

TABLE *giving the Lengths and Breadths of Entire Frustules, and of parts of the same, with the average number of Striæ in each case. All the Measurements are given in Millimetres :—*

No.	Length of entire frustule.	Length of distal inflation.	Length of median inflation.	Length of proximal inflation.	Breadth of distal inflation.	Breadth of median inflation.	Breadth of proximal inflation.	Breadth of distal constriction.	Breadth of proximal constriction.	Striæ in 0·01 mm.
1	0·0650	0·0150	0·0275	0·0200	0·0125	0·0112	0·0068	0·0075	0·0050	10
2	0·0575	0·0112	0·0250	0·0188	0·0125	0·0100	0·0068	0·0075	0·0050	10
3	0·0725	0·0175	0·0300	0·0225	0·0150	0·0125	0·0075	0·0100	0·0068	10
4	0·0600	0·0125	0·0262	0·0188	0·0125	0·0125	0·0062	0·0075	0·0055	10
5	0·0850	0·0162	0·0400	0·0263	0·0137	0·0125	0·0062	0·0075	0·0050	8·5
6	0·0675	0·0137	0·0300	0·0213	0·0125	0·0106	0·0055	0·0056	0·0050	10
7	0·0625	0·0137	0·0275	0·0188	0·0125	0·0118	0·0056	0·0062	0·0050	10
8	0·0625	0·0125	0·0275	0·0200	0·0125	0·0100	0·0056	0·0062	0·0050	10
9*	0·0750	0·0162	0·0325	0·0238	0·0137	0·0100	0·0062	0·0068	0·0050	8
10*	0·0725	0·0150	0·0325	0·0225	0·0131	0·0112	0·0062	0·0062	0·0050	8
11	0·0650	0·0125	0·0275	0·0225	0·0125	0·0105	0·0055	0·0062	0·0050	10
12	0·0700	0·0150	0·0300	0·0225	0·0125	0·0110	0·0055	0·0062	0·0048	10
13	0·0575	0·0125	0·0250	0·0175	0·0112	0·0100	0·0055	0·0056	0·0050	10
14	0·0650	0·0150	0·0275	0·0200	0·0125	0·0125	0·0056	0·0075	0·0050	10
15	0·0650	0·0125	0·0275	0·0225	0·0125	0·0112	0·0056	0·0070	0·0050	9
Average Approx.	0·0658	0·0138	0·0285	0·0209	0·0127	0·0112	0·0059	0·0069	0·0051	9

The length of the apical crest in all cases was = 0·0025 mm., and this added to the lengths of the three inflations, each reckoned from the narrowest part of the constrictions, makes up the entire length of the frustule.

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[*] In taking the averages, Nos. 9 and 10, which are varietal forms, have not been taken into account.

With respect to the above specimens, of which the measurements have been given, the following remarks may be made :—

No. 1. The peculiarities of this form have been described above, and are represented in Pl. XXIX. fig. 5*a*.

No. 2. Striæ wider in distal and median inflations than in intervening constrictions. Those in the inflations arcuate and radiate in opposite directions, as in *Gomphonema acuminatum*, var. *intermedia*, Grun. (Van HEURCK, *op. cit.*, pl. xxiii. fig. 21), but the external configuration of the distal inflation showed that the frustule belonged, not to the variety, but to the species itself.

No. 3. Striæ as in No. 2. From the outline of the distal inflation, which is here much less acute than in the type, though less rounded than in GRUNOW'S var. *intermedia*, it may be inferred that this was a transitional form between the two, and one which may very probably have its closest affinity with that variety.

No. 4. Striæ somewhat more distant than in the previous forms, especially at the middle of the inflations, but presenting no arcuation, or only the very faintest trace of this, at the distal end, and agreeing more in this respect with the figure given by W. SMITH than with that of Van HEURCK. Differing from *Gomphonema acuminatum*, var. *coronata* (= *Gomphonema coronatum*, Ehrenb.), (1) by the less marked distal constriction, and (2) by the character of the distal striation.

No. 5. Terminal crest, 0·0025 mm. long; its breadth at the base, 0·0030 mm. Although this frustule is one of the largest observed, its crest has not increased in height, but only slightly in breadth.

No. 6. Striation in distal inflation similar to that of the corresponding area of *Gomphonema acuminatum*, Ehrenb., var. *intermedia*, Grun., but the general form shows that the specimen belonged to the type.

No. 7. Striation of distal inflation similar to that of *Gomphonema acuminatum*, Ehrenb., figured by Van HEURCK.

No. 8. Similar to No. 6.

No. 9. This is the *Gomphonema acuminatum*, var. *coronata* of Van HEURCK.

No. 10. Same as No. 9. The number of striæ varies considerably in the different parts of the frustules, being fewest in the median inflation and greatest towards the proximal ends.

No. 11. Typical specimen: distal striæ but very faintly radiate. Proximal inflation distinct and proximal extremity more obtuse than in many forms.

No. 12. Distal constriction more marked that the type figured by Van HEURCK; distal margin of apical inflation more transverse, with straight striæ.

Nos. 13–15. Typical specimens as figured by Van HEURCK.

21. *Gomphonema acuminatum*, var. *coronata*, Van Heurck.
See *supra*, Rare.

22. *Gomphonema acuminatum*, var. *laticeps?* Van Heurck, *op. cit.*, pl. xxiii. fig. 17.

Although approaching this variety, the specimen from the deposit differs from it in the following respects :—(1) The distal constriction is much more pronounced. (2) The median inflation does not pass directly into the proximal region of the frustule, but is definitely limited by a second proximal constriction, which is well marked. (3) There is a distinct proximal inflation. (4) Whilst the striation is arcuate and radiating in the distal area, it is radiating, but straight, in the median, and similar, though much less pronounced, in the proximal. The striation of the median region does not, therefore, pass uninterruptedly into that of the proximal, but is almost as clearly separated from it by an area at the proximal constriction, in which the striation is perpendicular to the longitudinal axis, as the striation of the median inflation is from that of the distal by a similar arrangement. The median nodule is prominent, but the raphe is only faintly marked throughout the remainder of its course.

Length of distal inflation,	0·0100 mm.	entire frustule = 0·0412.
„ median „	0·0137 „	
„ proximal „	0·0150 „	
„ apical crest,	0·0025 „	
Breadth of distal inflation,	0·0100 „	
„ „ constriction,	0·0043 „	
„ median inflation,	0·0075 „	
„ proximal constriction,	0·0030 „	
„ „ inflation,	0·0043 „	

Striæ in 0·01 mm. = 10.

Rare.

Fam. 4. COCCONEIDEÆ.

23. *Achnanthidium flexellum* (Kütz., de Breb.).

= *Cocconeis thwaitesii*, W. Sm.

Not uncommon. All the specimens essentially agree with the type figured by W. SMITH (*op. cit.*, vol. i. pl. iii. fig. 33), the only circumstance of importance, by which some forms in the deposits may be distinguished, being the absence of the somewhat prominent circles of marginal puncta, or the very indistinct character of these.

Average length, 0·0425 mm.; breadth, 0·0175 mm.; breadth of terminal obtuse extremities, 0·0075 to 0·0085 mm.

24. *Cocconeis placentula*, Ehrenb.

Not uncommon. Valval ornamentation very faint, but raphe and central nodule well marked. Length, 0·0275 mm.; greatest breadth, 0·0175 mm.

Tribe II. PSEUDORAPHIDIEÆ.

Fam. 5. FRAGILARIEÆ.

25. *Epithemia gibba*, Kütz.

Many of the specimens in the preparations consist of a single valve, rupture of the connecting zone having taken place. The only notable circumstance in which the valves differ from that figured by W. SMITH (*op. cit.*, vol. i. pl. i. figs. 13*b*., 13*c*.), is in the character of the costæ. These are less markedly arched throughout; in the central portion they are almost quite straight, but slight arcuation appears towards the extremities, so disposed that the convexity of the arches is directed away from the centre.

Length, 0·1625 mm.; breadth of a single valve (zonal aspect) 0·0100 mm.; breadth of entire frustule, 0·0200 mm.; breadth of narrowest part, 0·0162 mm.; breadth at inflated extremities, 0·0175 mm. Striæ, 8 in 0·01 mm.

26. *Epithemia gibba*, Kütz., var. *rectimarginata*, nov. (Pl. XXIX. fig. 6).

Valval view slightly plano-convex, one of the margins of the valve next connecting zone straight. Dorsal inflation considerable, median, from its apex the dorsal margin slopes outwards in a slightly concave line. Apices very bluntly rounded, much inflexed on the side of the connecting zone. Costæ simple, irregular, straight or slightly arcuate at different parts of the valve.

Length, 0·1500 mm.; breadth of a single valve at median inflation, 0·0100 mm.; breadth half way between this inflation and the extremity, 0·0065 mm.; striæ, 6 in 0·01 mm.

This is a rare variety, and the valve figured was found isolated. It is at once distinguished by its bluntly rounded extremities and by the irregularity of its costæ. A slight arcuation is found in these, especially on one side of the frustule, near its centre, but they speedily become straight. Want of uniformity, either in distance between the costæ or in their direction, occurs at various places, and these irregularities, though well marked, are not found at corresponding points on both sides of the central transverse line. There is no intercostal punctation, such as is figured in the varieties *parallela* and *ventricosum* of GRUNOW.

27. *Epithemia argus* (Ehrenb.), Kütz., var. *amphicephala*, Grun.

Specimens presenting a very considerable amount of affinity to this variety of GRUNOW are occasionally found. They may be distinguished, however, from the latter by the following characters:—(1) The convex margin of the valve is much less flattened, and there is no slight inflation on any part of the concave side. (2) The constrictions between the extremities and the median portion of the valve are less deep, so that the transition from one part of the valve to another is more easy. (3) The transverse costæ are irregular in distribution.

From *Epithemia alpestris*, W. Sm. (*op. cit.*, vol. i. pl. i. figs. 7, 7*a*, 7*b*), the frustule figured (Pl. XXIX. fig. 7) differs (1) by the presence of fine puncta, disposed in several lines and running across the valve in the intercostal areas; (2) the ventral margin is much more straight, and the recurvature of the extremities is far less marked; (3) the disposition of the costæ is much more irregular.

In some specimens, with a more concave ventral side, there is, on both sides of the middle line, a very slight tumidity, extending over a considerable space, and taking away from the curvature considerably. On the entire surface of the valve from sixteen to eighteen costæ occur.

Length of an average specimen, 0·0625 mm.; greatest breadth, 0·0100 mm.; length of recurved obtuse extremities, 0·0050 mm.

Not uncommon, and approaching somewhat the *Epithemia intermedia* of Hilse.

28. *Epithemia hyndmanii*, W. Sm.

Few specimens. These differ from that figured by W. Smith in the following respects:—(1) The obtusely-rounded extremities are slightly recurved, but much more than is represented by Van Heurck (*op. cit.*, pl. xxxi. figs. 3, 4); (2) the outline of the valve on the concave side is uniform, unlike Van Heurck's specimen; (3) the costation is peculiar, inasmuch as the ribs on the two sides of that which runs partially across the valve in the median transverse line are not symmetrically disposed, but are much more flexed—appearing almost subgeniculate—on one side than on the other.

Length, 0·1150 mm.; greatest breadth, 0·0200 mm.; average distance between the costæ, 0·0025 to 0·0028 mm. Intercostal puncta very faint, 10 to 12 in 0·01 mm.

29. *Epithemia sorex*, Kütz.

Not uncommon. Typical, but more akin to forms figured by Van Heurck (*op. cit.*, pl. xxxii. figs. 6–10) and Kützing than to those figured by W. Smith (*op. cit.*, vol. i. pl. i. figs. 9*a*, 9*b*). The two costæ bounding the subtriangular central dorsal area are often perfectly straight throughout their length, and it is not without interest to note that in some valves all the costæ are straight.

Length, 0·0425 mm.; greatest breadth, 0·0125 mm.; average distance between the valval costæ, 0·0018 to 0·025 mm.

30. *Epithemia proboscidea,* Kütz.

Common. Average length, 0·0350 mm.; greatest breadth, 0·0225 mm.; breadth of valves (zonal aspect), 0·0075 mm.; and of connecting zone, 0·0085 mm.

31. *Epithemia rupestris,* W. Sm.

Less common than the preceding. The specimens agree in general form with SMITH's type, but the transverse costæ are very distinct both in size and appearance, being very much narrower and perfectly straight, instead of broad and arcuate.

Length, 0·0500 mm.; greatest breadth, 0·0050 mm.; average distance between costæ, 0·0025 mm. to 0·0030 mm.

32. *Eunotia tetraodon,* Ehrenb.

Very abundant, and typical; ornamentation perfectly preserved. Length (valval aspect), 0·0475 mm.; greatest breadth, 0·0200 mm.; height of median dorsal ridges above median sulcus, 0·0050 mm.; striæ on middle of valve, 8 to 10 in 0·01 mm.

33. *Eunotia diodon,* Ehrenb.

Several frustules occur, but this species is by no means so abundant as the preceding one. Length (valval aspect), 0·0500 mm.; greatest breadth, from concave side to apex of one of the opposite dorsal inflations, 0·0125 mm.; breadth between dorsal ridges, 0·0100 mm.; distance between apices of ridges, 0·0125 mm. (*i.e.* = breadth of valve across one of the ridges); striæ in middle of valve 10 in 0·01 mm.

Although it is common for this species to have the transverse diameter at the inflations considerably greater than that of the extremities, forms occur in the deposit in which these diameters are nearly equal. The following are the dimensions of one such specimen examined:—Length, 0·0825 mm.; breadth of terminal areas, 0·0100 mm.; breadth at dorsal valval inflations, 0·0095 mm.

34. *Eunotia major* (W. Sm.), Raben.
= *Himantidium majus,* W. Sm.
= *Eunotia biceps* and *monodon,* Ehrenb.

Not unfrequent. The specimens exactly correspond to that figured by Van HEURCK (*op. cit.,* pl. xxxiv. fig. 14), as far as the arrangement of striæ in slightly radiating lines is concerned, but the general outline of some of the frustules shows a very slight subcentral inflation on the concave margin. Length, 0·1000 mm.; breadth, 0·0125 mm.; striæ, 8 in 0·01 mm. Specimens of smaller size not unfrequent.

35. *Eunotia major* (W. Sm.), Raben., var. *semelconstricta,* nov. (Pl. XXIX. fig. 8.)

Rare. This curious form is closely akin in general outline to the *Eunotia pectinalis,* var. *biconstricta* of GRUNOW (Van HEURCK, *op. cit.*, pl. xxxiii. fig. 19). It agrees essentially with the typical form of *Eunotia major* in its striation. The great ventral constriction is somewhat nearer one extremity of the frustule than the other, and its two sides are quite unsymmetrical, that next the extremity towards which it is placed being considerably more convex than the opposite side. There is sometimes a second slight constriction on the concave side, where the inflation bounding the greater constriction passes into the all but straight "ventral" line, which proceeds from it to the extremity.

Length, 0·1050 mm.; greatest breadth, 0·0125 mm.; breadth at great constriction, 0·0100 mm. Striæ, 9 in 0·01 mm.

The drawing in the plate represents a form in which the constriction was only of moderate depth. In some specimens observed it is at once more marked and bounded by more convex margins.

36. *Eunotia pectinalis* (Kütz.), Raben., var. *undulata,* Ralfs.
=*Himantidium undulatum,* W. Sm.

The undulations on the convex side are five in number, and of these the median is often much more pronounced than in the form figured by Van HEURCK (*op. cit.*, pl. xxxiii. fig. 17). Moreover, the inflation on the concave side is often much more sharp, whilst the transverse striation is relatively more dense in the areas intervening between the central portion and the terminal parts. Specimens also occur in which the dorsal undulations are not more than three in number, and of these the central is the more pronounced. In such forms there is the same median "ventral" inflation opposite the median dorsal as in others with the usual five undulations.

Length of specimen, 0·0650 mm.; breadth at central inflation, 0·0075 mm.; breadth at narrower parts adjoining the centre, 0·0050 mm. Not very abundant.

37. *Eunotia gracilis* (Ehrenb.), Raben., nec. W. Sm.
=*Himantidium gracile,* Ehrenb.

Not unfrequent. The specimens agree essentially with that figured by Van HEURCK (*op. cit.*, pl. xxxiii. figs. 1, 2), the valves being regularly lunate, with gently recurved extremities, and the striæ faintly radiating. Average length, 0·1000 mm.; greatest breadth at the middle, 0·0075 mm.; at the slight constriction at the origin of the recurved extremity the transverse diameter is 0·0060 mm. Striæ 10 in 0·01 mm.

38. *Eunotia gracilis* (Ehrenb.), Raben., var. *semelmonticulata,* nov. (Pl. XXIX. fig. 9.)

Rare. Valve lunately curved dorsally, with a median dorsal depression opposite to the ventral inflation. Ventrally there is a characteristic large subcentral protuberance with uniformly convex margin, but situated a little towards one end of the frustule, thereby rendering it asymmetrical. Extremities obtuse, slightly recurved. Striation uniform throughout, slightly radiating, and at the extremities faintly arcuate, the convexities of the curves being directed towards the centre.

Length, 0·1325 mm.; greatest breadth, 0·0100 mm.; striæ, 12 in 0·01 mm.

39. *Synedra ulna*, Ehrenb., var. *tolstensis*, nov. (Pl. XXIX. fig. 12).

Very rare. The specimen figured was broken across transversely at a distance of 0·1625 mm. from the extremity. Professor Van Heurck has figured several varieties with knobbed extremities, but none of these agree with the present form with respect to the form of the apices. The slightly diverging character of the margins show very close affinities to the specimen figured by Van Heurck on his pl. xxxviii. fig. 3 (*op. cit.*), and with this, too, the transverse striation of the valve fully agrees, but the terminal knob is much less prominent, its transverse diameter being 0·0045 mm, and the striation in the knob is not oblique, but parallel, to that found in other parts of the valve. The transverse diameter of the constriction separating the terminal knob from the remainder of the valve is 0·0030 mm. The appearance of the terminal region of the frustule somewhat resembles that of *Synedra ulna*, var. *danica*, Kütz., but in the present case the extremity is more rounded, and the constriction bounding it of a more definite character. No central hyaline space occurs in the fragmentary frustule observed. Striæ, 15 in 0·01 mm.

40. *Fragilaria virescens,* Ralfs, var. *exigua,* Grun.

Not common. The specimens observed agree with those figured by Van Heurck (*op. cit.*, pl. xlv. fig. 3). Length, 0·0225 mm.; breadth (valval aspect), 0·0035 mm. Striation very inconspicuous.

Fam. 6. TABELLARIEÆ.

41. *Tabellaria fenestrata,* Kütz.

Rare, and sometimes in fragments. Typical. Length, 0·0600 mm.; breadth of terminal inflation, 0·0500 mm.; breadth of median inflation, 0·0055 mm.; breadth of intermediate area, 0·0030 mm.

42. *Tabellaria flocculosa,* Kütz.

Rare. Typical. Length, 0·0300 mm.; greatest breadth (α) at central inflation, 0·0050 mm.; (β) at terminal inflations, 0·0035 mm.; breadth of intervening portion, 0·0030 mm. Striation very faint.

Fam. 7. SURIRELLEÆ.

43. *Surirella nobilis,* W. Sm.

Very common, and of variable dimensions. The outline is ovate, the margins being more or less rounded and sometimes flattened. The arrangement of the alæ and canaliculi is in many cases essentially the same as in SMITH's figures (*op. cit.*, vol. i. pl. viii. fig. 63), but in several frustules variations are found around the wider extremities of the valves. The canaliculi, namely, are not bounded by straight lines, but become very arcuate, and for a space occupied by three of these give the valve a scale-like appearance (Pl. XXIX. fig. 10). The median reniform hyaline terminal areola is separated by a small lenticular hyaline space from the margin of the valve, and its centrally directed convex half is divided uniformly by a well-marked, straight, and tapering line, which represents the apex of the median lentelliptical area, the outlines of which are defined in an exceedingly faint manner. A somewhat similar arrangement in the structure at the wider extremity of the valves is met with in *Surirella saxonica,* Auersw., *S. robusta,* Ehrenb. (=*S. nobilis,* W. Sm., according to Grunow), *S. bifrons,* Kütz., *S. tenera,* Greg. (=*S. diaphana,* Bleisch), *S. valida,* A. S., *S. rattrayi,* A. S., but in all of these the reniform shape is replaced by a more rounded or even by a bluntly triangular outline, the small lenticular marginal space being either much reduced and rounded, as in some varieties of *S. tenera,* Greg., or concavo-convex, the concavity being directed towards the margin, as in *S. valida,* A. S. The infundibulate form of the canaliculi is well pronounced, though much less marked than in *S. valida,* A. S., but not unlike that in many forms of *S. bifrons,* Kütz.

Some perfect specimens of large dimensions have the outlines of the valves not convex throughout its entire extent, as normally occurs, but very slightly concave in the central area. One such specimen examined showed the following dimensions :—Length, 0·2700 mm.; breadth at the widest part, 0·0725 mm.; transverse diameter at the concave area, 0·0550 mm. Generally the flattening of the convexity of the margins advances as the frustules increase in length. This becomes clear from a comparison of the undernoted measurements of frustules occurring in the deposit. It is further to be noted that, notwithstanding this curious relationship, the relative and absolute sizes of the canaliculi do not alter at all, or only within very narrow limits, and in the middle zone, having a slightly larger diameter in larger forms, the average distance between the outer extremity of one and that of its immediate neighbour being 0·0125 mm. in large specimens belonging to group A (see *infra*), and 0·0155 mm. in those belonging to group B.

No.		Length of Specimen.	Breadth of Specimen.	Ratio.
A. Larger forms,	1	0·3050 mm.	0·0725 mm.	4·206 : 1
	2	0·3500 „	0·0800 „	4·375 : 1
	3	0·3000 „	0·0750 „	4· : 1
	4	0·3125 „	0·0700 „	4·464 : 1
	5	0·3050 „	0·0700 „	4·357 : 1
B. Smaller forms,	6	0·1750 „	0·0675 „	2·592 : 1
	7	0·1775 „	0·0650 „	2·73 : 1
	8	0·1850 „	0·0700 „	2·642 : 1
	9	0·1800 „	0·0750 „	2·4 : 1
	10	0·1775 „	0·0725 „	2·448 : 1

In acid preparations detached rings of this species are very common. In some specimens observed the alæ assumed a somewhat panduriform shape, and were perfectly symmetrical on both sides of the valves, but careful examination with high powers showed that the appearance was due to the rupture of the alæ from the valve, and to their subsequent inflexion towards one another upon the surface,

44. *Surirella inaequisculpta*, n. sp. (Pl. XXIX. fig. 11).

Valve linear, and slightly constricted in the middle. Extremities dissimilar, both conical, but one much more acute at the apex than the other. Alæ parallel to the margin throughout. Canaliculi very variable in size and disposition throughout the valve; towards the two extremities narrower, oblique, and arcuate, the concavities being directed towards the apices, in the median zone larger, transverse, mostly very faintly defined. Central area on valve much reduced, linear, very indefinitely marked out. Clear areola at more obtuse extremity small, subtriangular, with rounded sides and very blunt angles, that at acute extremity still more minute, subrotund. No clear space intervenes between this areola and the margin.

Length, 0·2075 mm.; greatest transverse diameters, 0·0425 mm. and 0·0400 mm., the smaller towards the more acute extremity; transverse diameter of median constriction, 0·0375 mm.; distance between canaliculi in middle zone, 0·0062 to 0·0075 mm.; diameter of apical hyaline areola at the more obtuse extremity, 0·0075 mm., that at the less obtuse end, 0·0050 mm.

Rare.

Among the linear subpanduriform species of *Surirella* this frustule approaches *Surirella arcta*, A. S. (*op. cit.*, pl. xxiii. fig. 23), in outline more closely than any other, but it may be distinguished from it in several important respects :—(1) the two apical cones are not similar; (2) the terminal areolæ are not identical at both ends; (3) the canaliculi are much more indistinct, and at the extremities they are much finer and more arcuate. From *Surirella gracilis*, Grun., it is distinguished (1) by its more marked median constriction; (2) by a more pronounced absence of uniformity in the size of the canaliculi at various parts; and (3) by the somewhat more arcuate appearance of the extremities. The closely allied *Surirella angusta*, Kütz., may also be distinguished by the character of the extremities and canaliculi; whilst *Surirella linearis*, W. Sm., though also having close affinities, differs markedly in the relatively much larger size of the valval ornamentation and the distinct character of the terminal areolæ.

Tribe III. CRYPTORAPHIDIEÆ.

Fam. 8. Melosireæ.

45. *Cyclotella antiqua*, W. Sm.

Common, typical, and of various sizes. The following table gives the diameter, number of cuneate cellules, and of marginal puncta in the valve in several of the specimens observed :—

No.	Diameter of Disc.	Cellules on Valve.	Marginal Puncta.
1	0·0200 mm.	9	10 in 0·01 mm.
2	0·0250 „	10	10 „
3	0·0175 „	8	10 „
4	0·0245 „	10	10 „
5	0·0175 „	9	8 „
6	0·0185 „	9	8 „
7	0·0200 „	10	10 „
8	0·0185 „	9	10 „
9	0·0200 „	10	10 „
10	0·0150 „	8	10 „
11	0·0225 „	7	8 „

The following circumstances are noteworthy:—(1) The variability in the number, size, and regularity of the valval cellules: in many cases those on one side of the same valve are larger than those on the other. In some large valves (No. 11) these cellules are larger and less abundant than in others (Nos. 2, 4) of very nearly the same size. (2) The number of marginal puncta do not vary greatly in their distance apart, whatever the size of the disc may be.

EXPLANATION OF PLATE.

Figs. 1*a*, 1*b*.	*Navicula obtusa*, W. Sm., var. *lata*, nov.
Fig. 2.	*Navicula oblonga*, W. Sm., var. *subparallela*, nov.
Fig. 3.	*Navicula cardinalis* (Ehrenb.), var. *subconstricta*, nov.
Fig. 4.	*Navicula inaequistriata*, n. sp.
Fig. 4*a*.	*Navicula macilenta*, Grun., var. *elliptica*, nov.
Fig. 5.	*Gomphonema geminatum*, Ag., var. *bipunctata*, nov.
Fig. 5*a*.	*Gomphonema acuminatum*, Ehrenb.
Fig. 6.	*Epithemia gibba*, Kütz., var. *rectimarginata*, nov.
Fig. 7.	*Epithemia argus* (Ehrenb.), Kütz., var. *amphicephala* (?), Grun.
Fig. 8.	*Eunotia major* (W. Sm.), Raben., var. *semelconstricta*, nov.
Fig. 9.	*Eunotia gracilis* (Ehrenb.), Raben., var. *semelmonticulata*, nov.
Fig. 10.	*Surirella nobilis*, W. Sm.
Fig. 11.	*Surirella inaequisculpta*, n. sp.
Fig. 12.	*Synedra ulna*, Ehrenb., var. *tolstensis*, nov.

All the figures are × 600.

Fig 1a

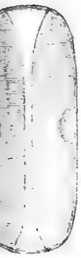


Fig 1b.

Fig 2.

Fig 3

Fig 4

Fig. 4a.

Fig 6

Fig. 11.

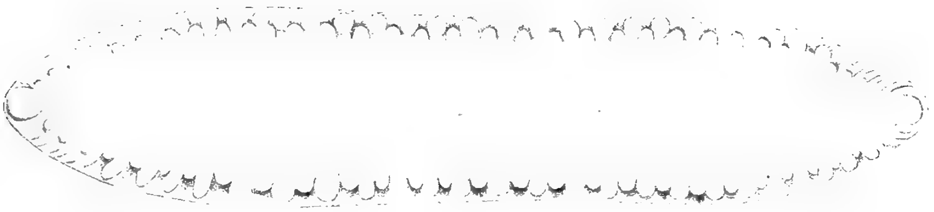


Fig. 5a.

Fig 5

Fig 10

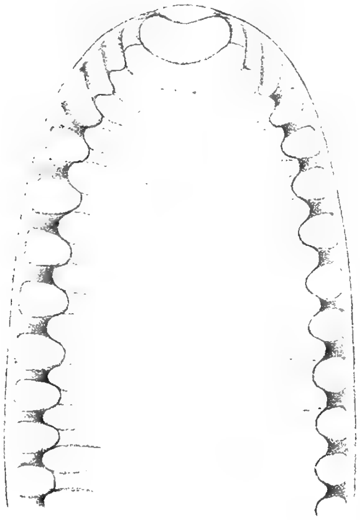
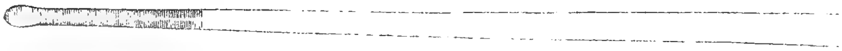


Fig. 8

Fig. 9.

Fig. 7.

Fig. 12.



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F Huth Lith^{r} Edin^{r}

XIX.—*On the Minute Structure of the Eye in certain Cymothoidæ.* By FRANK E. BEDDARD, M.A., F.R.S.E., F.Z.S., Prosector to the Zoological Society, and Lecturer on Biology at Guy's Hospital. (Plate XXX.)

(Read 15th July 1887.)

The structure of the eye in Isopods has been less studied than in any other group of Arthropods; the only modern descriptions of the minute anatomy of the eye in these crustaceans known to me are by GRENACHER* of *Porcellio*, by BULLAR† of *Cymothoa*, by BELLONCI‡ of *Sphæroma*, by myself§ of the genus *Serolis*. I have lately had the opportunity, while engaged upon my Report upon the "Challenger" Isopoda, of investigating the eye in several species of *Æga* and allied genera. The structure of the eye in these Isopods differs very materially from the descriptions given by GRENACHER and BULLAR, but agrees very closely with the structure of the eye in *Serolis*.

In all the types which I have studied there is a very close agreement in the structure of the different parts of the eye.

To each element of the eye (*ommatidium*) there is a cuticular lens. The *vitreous body* is clear and of a yellowish colour; it is plainly divided into two halves in correspondence with its origin from two cells. In no particular does the vitreous body depart from the characteristic structure exhibited in *Serolis*, *Sphæroma*, and other Isopods.

Underneath the *vitrella* is placed the *retinula*. This consists of seven distinct cells, which are deeply pigmented; the retinula cells commonly project upwards nearly as far as the cornea, and thus form a sheath for the vitreous body, as is the case, for example, in *Porcellio*; the portion of the retinula cells which borders upon the vitreous body is flattened out and of a greater width than the portion which lies below (fig. 16).

In the possession of seven retinula cells, the Cymothoidæ differ from the Serolidæ, where there are only four. BULLAR has figured seven retinula cells in *Cymothoa*, and this number may probably, therefore, be regarded as typical of the family. *Porcellio* and *Ligia* agree in the number of cells in the retinula.

Each retinula cell bears on its inner side, as in other Isopods and Crustacea

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* *Sehorgan d. Arthropoden*, Göttingen.

† *Phil. Trans.*, 1878.

‡ *Atti. r. Acad. Lincei.*, 1879.

§ Report on the Isopoda collected during the voyage of H.M.S. "Challenger," Zool., *Chall. Exp.*, Pt. xxiii.

generally, a chitinous structure known as the rhabdomere. BULLAR* has not figured or described more than a clear point at the summit of each cell of the retinula in *Cymothoa*, which, however, he regards as the equivalent of the rhabdomere (Sehstäbchen). In all the forms which I investigated the rhabdomere of each retinula cell was very largely developed, rather more so in some cases than in others. It always presented a remarkably striated appearance (see figs. 5, 6), clear lines alternating with dark lines; the appearance, in fact, suggested that it was composed of a number of separately formed rods or plates, between which a certain amount of pigment had been deposited; its structure is unlike that of most other Isopods and Crustacea generally, where each rhabdomere is a simple plate applied to the edge of the retinula cell. The rhabdomere of *Hyperia galba*† seems, however, to resemble it in certain points.

At the upper extremity the seven rhabdomeres are in contact, although even here they retain for the most part their independence, *i.e.*, they do not become fused. This is illustrated in fig. 15, which represents a section through the upper part of the retinula and rhabdom. Lower down the several rhabdomeres diverge from each other; each is somewhat conical in form, and projects downwards from its secreting retinula cell (see figs. 5, 12). The rhabdomeres do not, however, project freely into an interspace left between the retinula cells. They are closely enveloped by two large round cells, which fit into the interspace between the retinula cells.

These two cells are each furnished (figs. 5, 8, 9, *h*) with a distinct granular nucleus, in the centre of which is a spherical nucleolus; the boundary between the two cells was in no case very distinct. They are quite homogeneous and transparent; the outer surface of the cells was occasionally spotted with pigment granules, which may or may not be deposited on the outermost layer of the cells themselves. In transverse (fig. 7) and longitudinal (fig. 5) sections these cells are seen completely to envelop the rhabdom, which is plunged into their interior. In the species illustrated in figs. 5, 7, 8, 9, these cells were very conspicuous in teased preparations both depigmented and undepigmented; in the species illustrated in fig. 14 they were not conspicuous in such preparations, owing to the larger size of the rhabdom; in sections of both, however, they were perfectly obvious.

In my account of the eye of *Serolis*‡ I have described and figured a pair of precisely similar cells, showing an exactly corresponding relation to the rhabdom of the cells of the retinula.

In this structural feature the Cymothoidæ show therefore a close resemblance to the Serolidæ, and both differ in a corresponding degree from all other

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* *Loc. cit.*, p. 513, pl. 46, fig 13.

† CARRIÈRE, *loc. cit.*, p. 161, fig. 124, *rh.*

‡ *Loc. cit.*, p. 20 *et seq.* pl. ix.

Isopoda, and indeed from other Crustacea whose eyes are built upon the same plan, and of which we have any adequate knowledge. So far, therefore, the facts described in the present paper are a further confirmation of the justice of MILNE-EDWARDS' view respecting the affinities of the Serolidæ. I would remark that other cases are known where the structure of the eye has been found to indicate affinities,—notably Professors LANKESTER and BOURNE'S results in comparing the eyes of *Limulus* and *Scorpio.*[*] At the time when I wrote my account of the structure of the eye in *Serolis,* I was unable to do more than recall the facts, and dwell upon the differences from the eyes of other Arthropods.

Since that time an important paper has appeared upon the eyes of Molluscs and Arthropods, which has suggested to me an explanation of the apparently anomalous structure of the eye in the Serolidæ[†] and Cymothoidæ.

The author of this paper, Dr PATTEN, is entirely at variance with GRENACHER as to the morphology of the Arthropod compound eye.

GRENACHER regards the ommateum as composed of two layers of cells; the outer layer, which is grouped into pairs or groups of four, secretes the cuticular lenses, as well as the crystalline cone or vitreous body. LANKESTER and BOURNE have appropriately termed each group a *vitrella,* to correspond with the term *retinula,* to be referred to immediately. The inner layer of cells is again grouped into fours or sevens, &c., each group being termed a *retinula;* each cell of the retinula secretes a hard, chitinous body, the rhabdomere; the rhabdomeres usually (but not always) unite to form a centrally-placed rhabdom. The retinula cells are prolonged into nerve filaments. Pigment cells are also commonly found isolating the adjacent vitrellæ and retinulæ.

PATTEN agrees with GRENACHER in regarding the ommateum as composed of two layers of cells, but these layers do not correspond with those of GRENACHER.

The outermost layer consists of a flattened epidermis, but little modified, which is the matrix of the cuticular facets. Below this come four clear cells, termed retinophoræ; these secrete and enclose an axial, chitinous structure, which corresponds superiorly to the crystalline lens and inferiorly to the rhabdom of GRENACHER; the retinophoræ are surrounded and isolated by a variable number of pigment cells, among which the retinulæ of GRENACHER form one series. The nerve passes up the axial rod and branches into a plexus at its upper extremity.

If Dr GRENACHER'S conclusions are to be accepted, the retinula cells and the rhabdom (striated spindle) in the Decapod eye correspond absolutely to the retinula cells and rhabdom in the Isopoda; the chief difference between the two

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[*] *Quart. Jour. Micr. Sci.,* vol. xxiii. (1883).

[†] *Mitth. a. d. Zool. Stat. zu Neapel,* Bd. vi. (1886), p. 542.

types is, according to GRENACHER's figures, that in the Isopoda the vitreous body is short, more or less oval in form, and is made up of two closely opposed halves in correspondence with the *two* cells which secrete it; the vitreous body is also separated by a considerable interval from the chitinous product of the retinula cells—the rhabdom. In the Decapoda, on the other hand, the "vitrella" is composed of four cells, and the vitreous body is consequently separable into four portions, each of which is a product of one of the four cells. The vitreous body (or crystalline cone) is immensely elongated and conical in form, and is in actual contact with, or at most separated by an extremely minute interval from, the upper extremity of the rhabdom; the latter is, like the corresponding structure in the Isopod eye, formed as a secretion from a number of retinula cells. One of the main points concerning which Dr PATTEN is at issue with Dr GRENACHER relates to the formation of the crystalline cone and the rhabdom in the Decapod eye. GRENACHER, as already stated, regards this structure as being made up of two different parts; the upper crystalline cone is secreted by the four cells of the "vitrella;" the lower dilated portion, known as the striated spindle or rhabdom, is the product of the numerous cells of the retinula.

PATTEN, on the contrary, brings forward reasons for believing that there is no distinction between these two parts of the axial crystalline rod, and considers that they form a continuous structure, both being formed by the cells of the vitrella, which he terms retinophoral cells; the retinula cells of GRENACHER, on this hypothesis, sink to the level of mere pigment cells which surround the axial retinophoræ.

In a recent review of PATTEN's work this conclusion is considered to be probably correct, but to require some further confirmation.*

Dr PATTEN unfortunately did not direct his attention to the unravelling of the structure of the Isopod eye, and this is to be regretted, as in his opinion the eye in that group is in a primitive condition.

One of the strongest reasons for accepting GRENACHER's theory of the Arthropod eye is its extreme simplicity and the exact correspondence which it enables one to demonstrate between the structural features of different Crustacean eyes. It is quite easy, for example, by an inspection of his figures to see the essential similarity, masked only by some difference in detail, between each element of the eye of *Porcellio* and the rather more complicated elements of the Decapod eye.

If we are to accept Dr PATTEN's views it is quite impossible to compare, except in the most general way, the eye of an Isopod with that of a Decapod. Dr PATTEN claims as a merit of his own views, "that they lead to the reduction of the essential parts of all visual organs to one structural plan, which can be followed through the whole animal kingdom from the lowest to the highest."

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* *Quart. Jour. Micr. Sci.*, Oct. 1886.

It would be an additional merit if they served to render clear the modifications of the eye in a single phylum—the Crustacea—where it can hardly be doubted that the eyes are genetically connected.

Dr Patten states (p. 677) that "the presence of the corneal facets in certain higher forms only of insects and crustacea indicates that they are of late origin; moreover, the presence of a thick corneal hypodermis and the absence of corneal facets in such animals as *Branchipus*, the Isopods, Amphipods, and many insects show this condition to be a primitive one." Turning to the account of the structure of the eye in the amphipoda given in Bronn's "Thierreichs" (p. 343), I find that this very character, viz., the presence or absence of corneal facets, is made use of as a mark of distinction between the two groups of the Amphipoda and Isopoda—"Die structur der Amphipoden-Augen anlangend, so weichen sie von denjenigen der Isopoden dadurch sehr wesentlich ab, dass der sie bekleidende Theil des Kopf-Integumentes in keine nähere Beziehung zu den lichtbrechenden Medien tritt, dass mit anderen Worten also Cornea-Bildungen vollständig fehlen." It had not occurred to me, even before referring to the literature* of the subject, that in figuring the corneal facets in the Cymothoidæ, I was adding a new fact to our knowledge of Isopod anatomy. Dr Patten has also announced the discovery of the cuticular hypodermis, which, in all the Arthropod eyes examined by him, was found to intervene between the cuticle and the cells which secrete the vitreous bodies; it is true that these structures have been for the most part overlooked, but I would refer Dr Patten to fig. 2 of plate xlii. of the same volume, where they are distinctly figured in an Amphipod (*Phronima sedentaria*).† I have not succeeded in finding them in any of the species of Cymothoidæ which I have examined, but have not the least doubt that Dr Patten is right in supposing that they exist in all compound Crustacean eyes.

Whatever may be the value of Dr Patten's statements respecting the continuity of the crystalline cones and the rhabdom in the Decapod eye, there can be no question that at least in many Isopoda these structures are perfectly discontinuous. Grenacher's figure of *Porcellio* shows this fact, and my own figures illustrating the present memoir are in complete accord with Grenacher's. I have not been able to trace any continuity whatever between the rhabdom and the vitreous body, and the interval between these two structures is (see figs. 1, 2, &c.) a considerable one. Moreover, the cells which secrete the crystalline cones are bounded by a very distinct outer layer, which is as obvious at the lower end as at the upper end of these cells (see fig. 1). Hence it is clear that in these Isopods, at any rate, the cells of the vitrella are not prolonged

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* *Cf.* also Bullar's paper quoted above (pl. 46, fig. 12).

† Mentioned also in Balfour's *Comparative Embryology*, vol. ii. p. 396; see also Carrière, *Sehorgan der Thiere*, 1885, p. 158, where they are figured and described in *Gammarus pulex*.

downwards to encircle the rhabdom. This being the case, what cells do form the rhabdom? It appears that GRENACHER has already solved this problem, and that his retinula cells give rise to the rhabdom. In my preparations of the eyes of the Serolidæ and Cymothoidæ I can find no reason for doubting that the rhabdom is a product of the retinula cells; the absolute continuity between the retinula cells and the rhabdom is illustrated in fig. 12, and the same figure shows that the division of the rhabdom into seven radially-disposed rhabdomeres corresponds with the number (seven) of the retinula cells. In view of these facts, it is hard to believe that the rhabdom is anything else than the chitinous secretion of the retinula cells.

So far my results are confirmatory of those of GRENACHER and at variance with those of PATTEN. But it must always be borne in mind that PATTEN has not investigated the Isopod eye.

The nature of the remarkable hyaline cells present in the Cymothoidæ and in the Serolidæ now remains to be considered.

The large size of these cells plainly indicates that they are of some importance in the eye.

In seven of my figures of the eye of *Serolis* I have indicated a delicate filament (r') passing* out from between the two hyaline cells, running back towards the membrane bounding the ommateum; I have since re-examined my sections and teased preparations of the eye of *Serolis cornuta* and *S. schythei*, and find that this structure is nearly invariably present; I have succeeded in finding it in so many cases that I am inclined to believe it is in reality always present. I have traced the filament through the hyaline cells, at the upper end of which it becomes frayed out into a spindle-shaped bundle of fibrils; the exact mode of termination of these I have not been able to ascertain; they seemed to pass up into the axis of the rhabdom. In the paper referred to I have asserted their continuity with the rhabdom; a renewed examination of my preparation enables me, on the whole, to confirm this statement; they always appeared to end in the way described. I have re-drawn isolated elements of the eye of *Serolis cornuta* and *S. schythei*, in order to illustrate the above-described facts (figs. 1, 2, 3.). Fig. 3 illustrates an element of the eye of *S. cornuta;* figs. 1, 2 an isolated element of the eye of *S. schythei*; in this latter species there are two hyaline cells present, and the filaments passing down from the lower extremity of the rhabdom can be easily seen by careful focussing to pass through the substance of these cells. In *S. cornuta* also there are clearly two hyaline cells present.

The bundle of delicate fibrils which appear to start from the rhabdom unite a little way from the rhabdom, but still in the interior of the hyaline cell or cells, into a single thread, which certainly reaches as far as the membrane

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* *Loc. cit.*, pl. ix. figs. 3, 4, 5.

bounding the ommateum; in sections I have recognised these fibrils in the interior of the hyaline cells, but have not been able to trace them beyond the membrane into the tissue of the ganglion; indeed, in sections, I never succeeded in finding the fibril in the space between the hyaline cells and the membrane bounding the ommateum; the extension of the fibre, as far at least as this membrane, was perfectly obvious in teased preparations, both depigmented and undepigmented. In the Cymothoidæ I did not observe the fibrils in the interior of the hyaline cells, but in transverse sections of the prolongations of the retinula cells beyond the membrane of the ommateum there was very frequently an axial body (fig. 13 *a.*), which may very possibly be this same fibre.

The nature of this axial fibre would be hard to determine merely from a study of the eye of the adult crustacean. Fortunately, however, I have been able to examine the eyes of some very young examples of *Serolis schythei* taken from the brood pouch of the mother; I had not completely studied these specimens before preparing my memoir on *Serolis*. In some of these the hyaline cells appeared to be fully developed, and the ommatidia only differed from those of the adult in their smaller size. The vitreous body as completely filled the interior of the vitreous cells as in the adult, showing that the increase in size of the vitreous body advances *pari passu* with the growth in size of the vitreous cells. In other specimens, indistinguishable in point of size, I was unable to detect the hyaline cells. In the depigmented teased preparations (see fig. 19) the retinula cells appeared to be close together, and in the axial space was a delicate fibre, appearing to branch superiorly into a number of fibrils; this bundle of fibrils lies between the upper ends of the retinula cells and has a conical form; the interspaces are filled with granular matter. The whole structure appears to be the rhabdom; in my figure of an ommatidium of *Serolis* (*loc. cit.*, pl. ix. fig. 5), I have evidently sketched a rhabdom of a young specimen, and erroneously associated it with the ommatidium of an adult. Outside the rhabdom, in the immature eye, are thickenings of the margin of the retinula cells (figs. 17, 18); these have a very different appearance from the enclosed rhabdom. They ultimately become the structure illustrated in figs. 2, 3*r*, which I have regarded as the rhabdom.*

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* GRENACHER has not noted the presence of special pigmented cells within the ommateum of Isopods. In *Serolis* there are two or three rows of such cells surrounding each element of the eye; the cells themselves are not easily to be made out, but their nuclei are particularly distinct in teased preparations, when the pigment has been dissolved away by nitric acid. I am not certain as to the exact number of these nuclei in each element; the action of the nitric acid is such that the cells, of which these are the nuclei, are not merely depigmented, but are entirely dissolved away. I could never discover any traces of the cell protoplasm in such preparations; although these nuclei often appear on a superficial inspection to lie in the retinula cells, carefulo cu ssing shows that this is not the case; moreover, the nuclei themselves are smaller than, and in other respects different from, the nuclei of the retinal cells.

Now the rhabdom of the young eye is so closely similar in every particular to the peculiar axial structure (figs. 1, 2, 3r') of the adult eye, that it seems impossible to come to any other conclusion than that they are identical.

Although the retinula cells are easily freed from the axial bundle of fibrils, the latter are not left in such preparation entirely denuded; they are, in fact, seen to be surrounded by a variable amount of granular matter, and near to the lower end of the conical body one or two nuclei were occasionally to be seen. These nuclei, as illustrated in the figure, differ in their characters from the nuclei of the retinula cells. Sometimes only one was to be seen, but in this case it invariably lay to one side of the axial cone, indicating that the other nucleus had been present, but had been lost in the process of teasing up the material; when both nuclei were present they were seen to be symmetrically disposed towards one another, and to lie on either side of the extremity of the axial cone.

In transverse sections of the young eye (fig. 18) these nuclei are distinctly visible, and it is possible that the cells to which they belong, which are evidently the hyaline cells of the adult eye, take some share in the formation of the rhabdom.

In any case the rhabdom of the young ommatidium is evidently principally formed by the retinula cells, for in transverse sections (figs. 17, 18) it is seen to be four-sided, in correspondence with the four retinula cells.

The structure which I have termed the rhabdom in the adult eye (r, figs. 1, 2, 3, &c.) appears to be therefore a new structure, differing somewhat in its character from the rhabdom of the young eye, which, however, persists although very inconspicuous.

It will not escape the attention of those who have read Dr PATTEN's paper, that these hyaline cells bear not a little resemblance to his "*retinophoræ.*" One of the principal positions which he takes up with respect to the invertebrate eye is, that the cells which are immediately concerned in the function of vision are *colourless cells,* and contain a network of nerves. These colourless cells are grouped in various numbers, and each group is separated from a neighbouring group by circles of pigment cells, which he terms retinula, among which the retinula of GRENACHER form only one series.

In identifying a clear colourless cell or cells in each retinula, my results agree with those of PATTEN. With regard to the Arthropod eye, however, the agreement between my results and those of PATTEN end here. The retinophoræ, according to that author, form the crystalline cone and the rhabdom, which is merely a continuation of it. The hyaline cells of *Serolis*, &c., are plainly distinct from the cells which form the crystalline cone, and they do not form the rhabdom. At present, therefore, I am unable to compare the eye of *Serolis* and the Cymothoidæ with the eye of the Decapoda and Insects as interpreted by PATTEN;

unless it be supposed that the crystalline cones of *Serolis*, &c., are not homologous with the crystalline cones of the Decapod, but with the hypodermal layer of the latter, which on this hypothesis must be assumed to be absent in *Serolis.* This, however, is a purely gratuitous assumption, and does not seem likely. On the other hand, my results do harmonise with those of GRENACHER, as regards the distinction between the crystalline cone and the rhabdom.

As Dr PATTEN has pointed out, the retina of the Molluscan eye contains clear cells and pigmented cells, and he has shown reasons for believing that the former are mainly connected with the process of vision. The eye of Annelids is also similar in this respect. In *Nereis*, for example, the retinal layer contains clear cells (termed by CARRIÈRE* "secret-zellen") and pigmented cells.

I have no facts to bring forward which would allow of the hypothesis that the hyaline cells of the Serolidæ and Cymothoidæ are visual cells in the sense of being connected with nerve fibres.

The main facts in the present paper and the conclusions to which they lead are as follows:—

The Serolidæ and Cymothoidæ possess eyes which differ in certain important particulars from the compound eyes of all other Crustaceans, as at present understood.

The points of difference concern the retinulæ. Each retinula consists, in the first place, of four (*Serolis*) or seven (Cymothoidæ) elongated cells resembling those of other Isopoda; each of these cells secretes a chitinous body, the rhabdomere. In *Cymothoa* (BULLAR) the individual rhabdomeres retain their distinctness. In other Cymothoidæ and in the Serolidæ the rhabdomeres become fused to form an axially placed rhabdom, which has often a complicated form, and in which a large quantity of pigment is deposited. The Serolidæ (not the deep-sea species) and many Cymothoidæ possess a pair of large hyaline nucleated cells, surrounded by the other retinula cells. In the axis of these, and enclosed by them (in the Serolidæ), is a delicate fibre, passing back as far as the ommatial membrane, and expanding anteriorly into a conical body, which appears to penetrate into the axis of the rhabdom.

In young specimens of *Serolis schythei* the future hyaline cells are small and granular, and enclose the extremity of this axial cone and fibres, which may be partly a product of their activity, though chiefly formed by the other retinula cells. Each retinula therefore consists of two central clear cells (corresponding in number to the cells of each vitrella), surrounded by four or seven pigmented cells.

The pigmented retinula cells are connected with transversely striate fibres, which pass into the ganglion, and are generally regarded as nerve fibres. The

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* Figured by CARRIÈRE, *loc. cit.*, p. 31, figs. 26, 26A.

hyaline cells do not end in a nervous filament, unless the axial cuticular rod, which is hollow, encloses a nerve fibre. The specialisation of the retinula into clear and pigmented cells recalls the eye of certain Annelids and Molluscs. The eye is "diplostichous," the upper row of cells forming the vitrella, and the lower row the retinula. To this extent, therefore, my results harmonise with those of Grenacher rather than those of Patten.

EXPLANATION OF PLATE XXX.

Lettering.—*v.* Vitrella.
ret. Retinula.
r. Rhabdom.
r'. Filiform extremity of the rhabdom.
h. Hyaline cells.
n. Nuclei of the same.

N.B. Cuticular structures, *i.e.*, corneal lens, vitreous body, rhabdom, are coloured yellow.

Fig. 1. Isolated ommatidium of *Serolis schythei.*
Fig. 2. Hyaline cells and adjacent structures of the same more highly magnified.
Fig. 3. Hyaline cells and rhabdom of *S. cornuta.*
Fig. 4. Transverse section through ommatidium of *Cirolana* at different levels.
Fig. 5. A single ommatidium of same species. *A*, nucleus of hyaline cell highly magnified.
Fig. 6. An isolated rhabdomere of same.
Fig. 7. Transverse section through retinula of same.
Figs. 8, 9. Isolated retinula of same; all the pigmented retinula cells are not shown.
Fig. 10. Nucleus of hyaline cell.
Fig. 11. Two nerve rods, continuous anteriorly with pigmented retinula cells, showing their transverse striation.
Fig. 12. Transverse section of retinula of *Æga*, sp.
Fig. 13. Transverse section of prolongations of retinula cells below ommateal membrane.
Fig. 14. Isolated retinula of same.
Fig. 15. Transverse section of retinula of same at its upper extremity.
Fig. 16. Isolated retinula cell of same.
Figs. 17, 18. Transverse section through retinula of young *Serolis schythei.*
Fig. 19. Isolated ommatidia of same.
Fig. 20. Retinula cell of *S. cornuta*, showing two nuclei.

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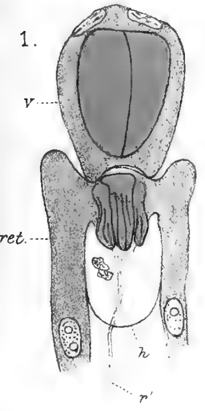

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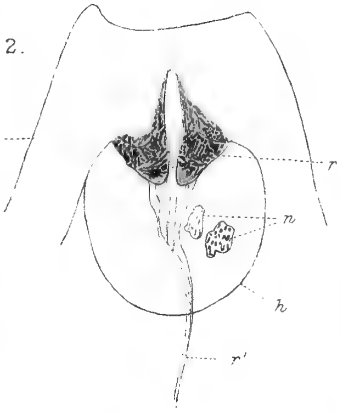

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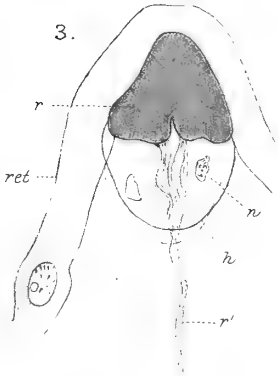

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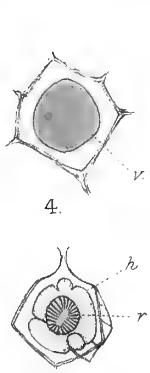

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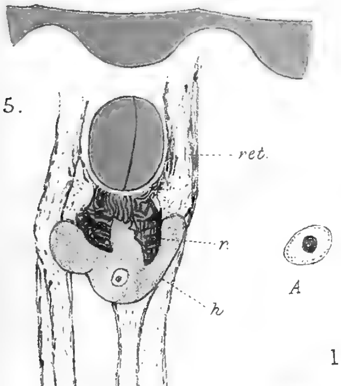

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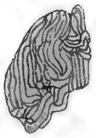

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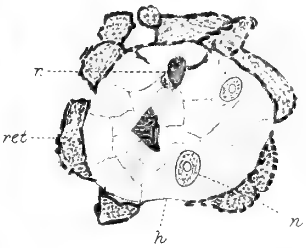

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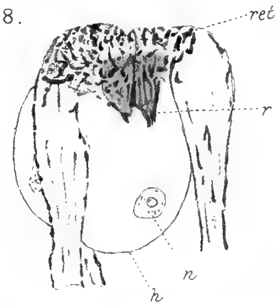

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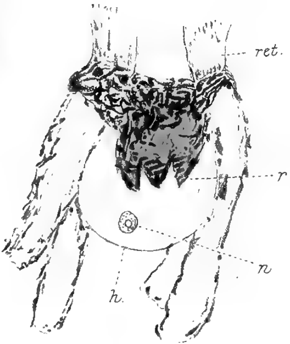

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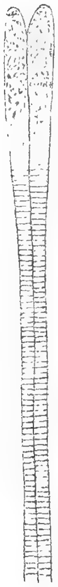

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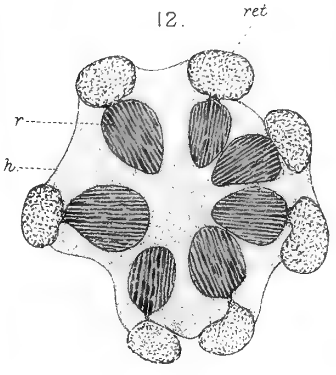

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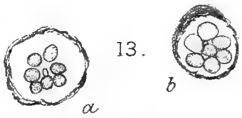

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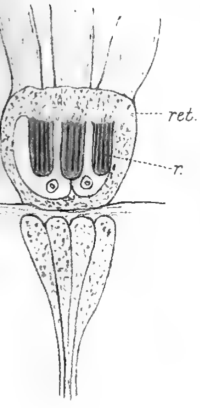

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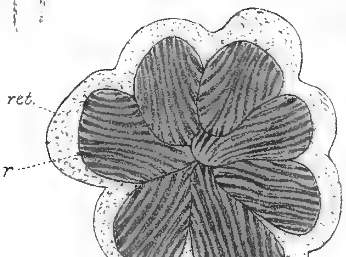

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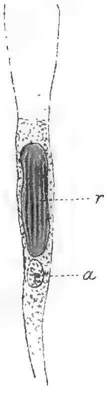

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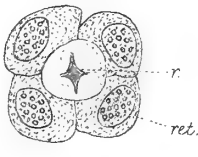

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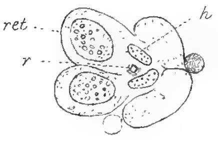

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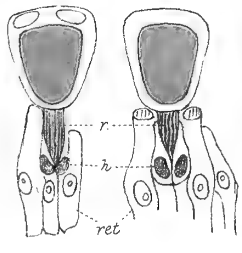

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XX.—*Report on the* Pennatulida *dredged by H.M.S. “Porcupine.”*

By A. Milnes Marshall, M.D., D.Sc., M.A., F.R.S., Beyer Professor of Zoology in the Owens College; and G. H. Fowler, B.A., Ph.D., Berkeley Fellow of the Owens College, Manchester. Communicated by John Murray, Esq. (Plates XXXI., XXXII.)

(Read 4th July 1887.)

Introduction.

The Pennatulida obtained on the cruises of the “Porcupine” during the summers of 1869 and 1870 comprise representatives of seven genera and nine species, of which one genus (*Deutocaulon*), and one variety (*candida*) of *Pennatula phosphorea*, are new to science.

The following table exhibits the distribution and number of specimens of the various forms :—

Station.	Locality.	Date.	Depth in fathoms.	Name.	Number of specimens.
1.	51° 51′ N.–11° 50′ W.	1869	110	Svava glacialis	1
”	” ”	”	”	Deutocaulon hystricis	4
24.	56° 26′ N.–14° 28′ W.	1869	109	Pennatula phosphorea (?)	4
”	” ”	”	”	Deutocaulon hystricis	1
36.	48° 50′ N.–11° 9′ W.	1869	725	Protoptilum Carpenteri	2
47.	59° 34 N.– 7° 18′ W.	1869	542	Kophobelemnon stelliferum	5
52.	60° 25′ N.– 8° 10′ W.	1869	384	Pennatula phosphorea	1
54.	59° 56′ N.– 6° 27′ W.	1869	363	Funiculina quadrangularis	1
57.	60° 14′ N.– 6° 17′ W.	1869	632	Kophobelemnon stelliferum	2
61.	62° 1′ N.– 5° 19′ W.	1869	114	Pennatula phosphorea	3
90.	59° 41′ N.– 7° 34′ E.	1869	458	Kophobelemnon stelliferum	4
	Loch Scavaig, Skye.	1869	45–60	Pennatula phosphorea	1
6.	48° 26′ N.– 9° 44′ W.	1870	358	Pennatula phosphorea	2
31.	35° 56′ N.– 7° 6′ W.	1870	477	Pennatula phosphorea (?)	2
	Tangier Bay.	1870	35	Pteroides griseum	8
	”	”	”	Pennatula rubra	2

Works on the subject, to which reference is made, are indicated by figures in heavy type throughout, and a list of them is appended.

For the terms “polyp” and “zooid,” long admitted to be unsatisfactory, and indicating the completely and incompletely developed members of the colony respectively, we have preferred to use “autozooid” and “siphonozooid,” terms first suggested by Professor H. N. Moseley (**1**, p. 118).

CLASSIFICATION.

The classification given by KÖLLIKER (**2**, p. 33), and condensed below, exhibits the systematic relations of the forms obtained by the "Porcupine." While this classification is the most recent and the best yet arranged, we cannot regard it as entirely satisfactory, for it is founded without due regard to morphological distinctions, and on characters which are rather accidental than essential. By it closely allied forms, *e.g.*, *Protocaulidæ* and *Virgularidæ*, appear to be far too widely separated. To rearrange the *Pennatulida*, however, were premature, till new and intermediate forms have been recorded, and a more thorough investigation of the anatomy and life-history of the group effected.

Order PENNATULIDA.

Section I. PENNATULEÆ: autozooids fused together to form leaves.
  Subsection i. *Penniformes:* leaves well developed.
    Family 1. Pteroididæ: *Pteroides griseum.*
      2. Pennatulidæ: *Pennatula phosphorea.*
        Pennatula rubra.

  Subsection ii. *Virgularieæ:* leaves small.
    Family 1. Virgularidæ: *Svava glacialis.*
      2. Stylatulidæ.

Section II. SPICATÆ: no leaves, autozooids sessile.
  Subsection i. *Funiculineæ:* autozooids in distinct rows.
    Family 1. Funiculinidæ: *Funiculina quadrangularis.*
      2. Stachyptilidæ.
      3. Anthoptilidæ.

  Subsection ii. *Junciformes:* autozooids in single series, or indistinct rows.
    Family 1. Kophobelemnonidæ: *Kophobelemnon stelliferum.*
      2. Umbellulidæ.
      3. Protocaulidæ: *Deutocaulon hystricis.*
      4. Protoptilidæ: *Protoptilum Carpenteri.*

Section III. RENILLEÆ.

Section IV. VERETILLEÆ.

DESCRIPTION OF SPECIES.

Order PENNATULIDA.

Section I. PENNATULEÆ.

Subsection i. *Penniformes.*

Family 1. Pteroididæ.

Genus *Pteroides* (Herkl.).

Pteroides griseum (Köll.).

Of this form eight specimens were obtained from Tangier Bay, all imperfect, and evidently broken by the dredge: of these, six have been broken off sharp at the middle or lower part of the rachis, one consists of the rachis entire with about an inch of the calcareous axis of the stalk, and one of the entire stalk with only the lower part of the rachis still attached.

Out of this species KÖLLIKER has erected two varieties, *brevispinosum* and *longispinosum*, both of which are represented among our specimens; but no sharp line can be drawn between them, the one variety passing into the other by slight gradations. Our specimens present no feature worthy of remark, beyond the fact that the feather is in all cases slightly longer than wide; whereas CARUS gives as a part of the definition, "latior quam longior." The longest specimen reaches only to 7·5 cm.

One specimen, consisting of the rachis only, does not agree exactly with any of KÖLLIKER'S descriptions. The softness of the rachis, the greater distance between the pairs of leaves, and the shape of the leaves, resembling more a sickle than a fan, give it a very different appearance to that of the more ordinary form; but *P. griseum* is so variable that we have hesitated to found a new species for the reception of this specimen. The dimensions are as follows:—Length of rachis (incomplete), 3·6 cm.; length of leaf, 1·5 cm.; breadth of feather, 2·8 cm.; breadth of leaf, 6 mm.; number of pairs of leaves, 15; number of rays (Hauptstrahlen), 7; distance of leaves apart, ·5 cm. The rays are prominent up to the top of the rachis; the siphonozooid plate short and broad.

KÖLLIKER (3), pp. 66–71, pl. iii. figs. 21–23; CARUS (4), p. 63.

Family 2. Pennatulidæ.

Genus *Pennatula* (L.).

Pennatula phosphorea (L.), var. *aculeata* (Köll.).

Two specimens of this variety were dredged at Station 6, 1870, in length 12·8 cm. and 9·5 cm. The first, which is entire, bears 7–9 autozooids on each leaf, and is slightly (naturally) distorted; the other, which has lost the extreme end of the stalk, and in which the leaves are formed of 6–7 autozooids, is, except for the smaller size of the siphonozooids, closely similar to the specimens described and figured by MARSHALL (5), p. 123, pl. xxi., xxii. Neither present any special features.

Pennatula phosphorea (L.), var. *lancifolia*, subvar. *variegata* (Köll.).

Two complete specimens, of length 9·7 cm. and 9·5 cm., were obtained in 1869 from Station 52 and from Loch Scavaig respectively. The rachis and stalk are very thin-walled and fleshy. The upper 2·5 cm. of the larger specimen are bent sharply downwards, not, apparently, as the result of accident, as the specimen could not be straightened without damage.

KÖLL. (3), p. 130, pl. viii. fig. 70; ix. fig. 73.

Pennatula rubra (Ell.).

Two fragments, in length 2 cm. and 1·1 cm. respectively, and consisting of the upper part of the rachis only, were obtained, with *Pteroides griseum*, from Tangier Bay. The larger specimen has thirteen pairs of leaves, formed of 32–40 autozooids. The general appearance is more compact than in KÖLLIKER'S figure.

KÖLL. (3), p. 135, pl. ix. figs. 74, 75; CARUS (4), p. 63.

Pennatula phosphorea, var. *candida*, n., Pl. XXXI. figs. 1, 2; and Pl. XXXII. fig. 3.

Of this new variety three fragments, all incomplete, were dredged at Station 61 in 1869. Two of these consist of the upper part of the rachis—the one much crushed, the other, of which the dimensions are given below, sufficiently well preserved to be figured. The third is a fragment from the middle of the rachis.

The colony is white throughout, the spicules, which are everywhere present, being entirely colourless. In many respects the specimens might have formed simply a white subvariety of var. *aculeata*, with which they have many characters in common. The shape of the leaves, however, and their diminutive basis, point to its being an intermediate form between the varieties *aculeata* and *angustifolia*, and as such we have introduced a new name for it.

On the dorsal surface is a deep median groove, free from siphonozooids; a similar but much shallower one was figured by MARSHALL (5), pl. xxi. fig. 5, in *P. phosphorea*, var. *aculeata*.

The autozooids, 7–9 in number, agree in appearance with those of other species of Pennatula, and are arranged in a single series. They possess very long tentacles, in which, as throughout the whole colony, colourless spicules are present. The margin of the calyx is notched into eight well-developed and sharp teeth.

The leaves are slender and flat, slightly twisted, and much resemble those of *P. phosphorea,* var. *aculeata.*

The siphonozooids are, on the dorsal surface of the rachis, arranged on each side of the median groove, and extend between the bases of the leaves on to the ventral surface; here they are packed very closely, and cover the whole of this face, leaving no median stripe bare, such as occurs in some forms. The ventral siphonozooids are much larger than those on the dorsal surface, especially near the bases of the leaves. All have a well-marked calyx, beset with marginal teeth.

Reproductive organs of the male sex were detected in a fully-formed leaf from one specimen.

Length of specimen (incomplete), ...	1·7 c.m.
Diameter of rachis, ...	2 mm.
Number of leaves, ...	12
Number of autozooids in a well-grown leaf, ...	7–9
Length of longest leaf, ...	4·2 cm.
Base of leaf, ...	1·0–15 cm.

Pennatula phosphorea? Pl. XXXII. figs. 4–7.

Two small specimens, probably of *Pennatula phosphorea,* were dredged at Station 31, 1870. The leaves are long, slender, pointed markedly upwards, and distinctly alternating. The siphonozooids are comparatively large, and one of unusual size is placed at the termination of the rachis on the ventral side. Both specimens are entire.

	A.	B.
Length of specimen, ...	3·1 cm.	4·1 cm.
„ stem, ...	1·6 cm.	1·9 cm.
„ rachis, ...	1·5 cm.	2·2 cm.
Number of leaves, ...	14	17
Length „	2 to 8 mm.	2 to 11 mm.
Number of autozooids per leaf, . .	1 to 5	

Four very small specimens, probably also of *Pennatula phosphorea,* were taken at Station 24, 1869. In all cases a terminal autozooid of considerable size is placed at the end of the rachis, and just at its base on the ventral surface an unusually large siphonozooid. The specimens are of a uniform white tint, the spicules being entirely devoid of colour; and it is interesting to note that in the two young specimens last described some of the spicules are coloured, others colourless, the resulting tint being of a lightish pink, much fainter than that of the ordinary *P. phosphorea.* The spicules are numerous everywhere except at the very bottom of the stalk.

Specimen A. (Pl. XXXII. figs. 4, 5).—The terminal autozooid is entirely normal in respect of tentacles, marginal teeth, &c.: it is 2·2 mm. long, and ·4 mm. wide at the middle of its length. The leaves are in pairs, the left member being a little in advance of the right.

The first pair of leaves consists, on the left, of a single autozooid 3·5 mm. long, the top of which is almost on a level with that of the terminal one; on the right the leaf is formed of two autozooids, the main one of which is rather larger than that last mentioned, and bears on its dorsal surface a second, which is ·3 mm. wide, and free for a length of ·58 mm. At the base of this leaf, as in many others in these specimens, is a slight swelling, probably the commencement of a third autozooid.

Of the second pair of leaves, that on the left, in length 4·8 mm., is formed of three autozooids; that on the right, also of three autozooids, is the largest of all on the colony, measuring 5 mm. in length.

Below these are three pairs of leaves, diminishing in size and in the number of the autozooids of which they are composed, the outermost, *i.e.*, most ventral, autozooid being always the largest on the leaf. Below these are two pairs of papillæ, the rudiments of future leaves.

The terminal siphonozooid is placed ventrally to the base of the terminal autozooid, and is in all the specimens much larger than any other. The rest are arranged in a series of about twenty pairs down the ventral surface of the rachis, extending as far as the fifth pair of leaves. Externally they appear as simple fan-shaped spicular sheaths.

Specimen B.—The terminal autozooid is much shorter than either of the two which compose the first pair of leaves, and is placed much more dorsally than is the case in the other three specimens. The right side has suffered considerable damage. There are in all five pairs of leaves.

Specimen C.—The leaves are placed rather opposite than alternately to each other, and the general appearance distinctly suggests deformity. There is a terminal autozooid bent sharply to the left. There are four pairs of leaves, the largest consisting of four autozooids; below these four pairs are slight papillæ.

Specimen D. (Pl. XXXII. figs. 6, 7).—The terminal autozooid is one of the largest on the colony, measuring 3·3 mm. in length, and ·75 mm. in diameter.

Of the first pair of leaves, each of which consists of two autozooids, that on the left is 3·9 mm. in length, and projects ·66 mm. beyond the terminal autozooid, the smaller autozooid being 1·9 mm. in length, and fused for the greater part with the main one; that on the right is closely similar. The leaves are in close contact at their bases, and are slightly displaced in the figure, as in the natural condition they almost concealed the terminal autozooid.

The calcareous axis extends quite to the lower end of the stalk, where it is hooked: it measures ·15 mm. in diameter at its widest part.

The following table gives the dimensions of the leaves in this specimen, the proportions of which may be taken as typical for all the four :—

Pair 1.	Left leaf,	2	autozooids,	length	3·9, 1·9 mm.
	Right „	2	„	„	4·0, 2·5 mm.
2.	Left „	3	„	„	4·3, 3·2, 2 mm.
	Right „	3	„	„	4·5, 3·3, 2 mm.
3.	Left „	3	autozooids,	length of leaf,	4·2 mm.
	Right „	3	„	„	4·0 „
4.	Left „	3	„	„	3·0 „
	Right „	3	„	„	2·4 „
5.	Left „	3	„	„	1·9 „
	Right „	3	„	„	1·9 „

Pairs 6–9 have the form of obliquely placed ridges, in which no truly formed autozooids are discernible.

The general dimensions of the four specimens are as follows :—

	A.	B.	C.	D.
Length of specimen,	20 mm.	21 mm.	19 mm.	23 mm.
„ stalk,	9 „	10 „	9 „	11 „
„ rachis,	11 „	11 „	10 „	12 „
„ longest leaf,	5 „	3·5 „	...	4·3 „
Diameter of rachis,	·6 „	1·1 „	1·1 „	·85 „
Number of pairs of leaves,	5	5	4	5

These four specimens are, we believe, the youngest that have yet been described: the presence in each of them of a large terminal autozooid, with an unusually large siphonozooid at its base, is of very considerable interest, and affords strong reason for holding that the mode of formation of the colony in *Pennatula* is similar to that described by Wilson for *Renilla* (*Phil. Trans.*, part iii. 1885).

Subsection ii. *Virgularieæ.*

Family 1. Virgularidæ.

Genus *Svava* (Kor. and Dan.).

Svava glacialis, var. *alba* (Kor. and Dan.).

One fragment of this interesting form, 6·5 cm. in length, was taken in 1869 at Station 1, comprising the rachis cut off just at its junction with the stalk, and incomplete above. The autozooids, three or four of which go to form a leaf (except at the highest part of the rachis, where there are two only to a leaf), are for the most part united by their bases only; in the very lowest leaves alone are they fused for the greater part of their length. In the centre of the rachis there are about six pairs of leaves in a centimetre. The autozooids appear to arise as transverse or slightly oblique ridges, divided into three or four from the first. In the lowest leaves the dorsal polyp is, as usual, slightly smaller than the other two; and in this region the shallow dorsal furrow is most distinct.

The siphonozooids, figured by Koren and Danielssen as a more or less transverse row at the base of each pair of leaves, in our specimen form a row in shape like a V, the apex of which is mid-dorsal, and lies about half-way between two pairs of leaves, each arm of the V being generally composed of four siphonozooids.

Koren and Danielssen (6), p. 5, pl. i. figs. 8–11.

Section II. SPICATÆ.

Subsection i. *Funiculineæ.*

Family 1. Funiculinidæ.

Genus *Funiculina* (Lam.).

Funiculina quadrangularis (Pall.).

The rachis of one young specimen was taken at Station 54 in 1869, noticeable only for the very great abundance of spicules in the autozooids.

Köll. (3), p. 370.

Subsection ii. JUNCIFORMES.

Family 1. Kophobelemnonidæ.

Genus *Kophobelemnon* (Asbj.).

Kophobelemnon stelliferum (Müll.).

This form was obtained more abundantly than any other during the cruises of the "Porcupine," but none of the specimens present any unusual features.

i. Station 57, 1869.—Two young specimens, both entire, were obtained here; the one 7·2 cm., the other 8·8 cm. in length. Each bears nine autozooids.

ii. Station 90, 1869.—Four specimens were dredged here. Two of these are perfect and very small, measuring only 4·5 cm. and 5·4 cm. in total length; they bear three and six autozooids respectively. The third specimen, the upper part of the rachis only, measuring 2·9 cm., bears nine autozooids, distinguished for their somewhat unusual length and slenderness. The fourth is a fragment of a very unusually large specimen, and consists of 6 cm. of the upper part of the rachis. The autozooids, twenty-one in number, as in the last specimen, appear unusually long and slender: while the cell is but 3 mm. in diameter, its length is 2 cm. to 2·5 cm., and that of the tentacles 1 to 5 cm. How much of this apparent slenderness is due to the comparative amount of retraction of the autozooids is, of course, doubtful, since, in spite of the great abundance of spicules, the autozooids are highly retractile.

iii. Station 47, 1869.—Five specimens, in length 4 to 6 cm., distinctly belonging to the var. *durum* of Kölliker, were obtained here.

Köll. (3), p. 304, pl. xxi. figs. 179–181.

Family 3. Protocaulidæ.

Genus *Deutocaulon* (Nob.).

Deutocaulon hystricis (sp. n.), Pl. XXXII. figs. 8, 9.

This new genus was dredged at Stations 1 and 24 in 1869. It is of intermediate complexity between Protocaulon, which may be regarded as the simplest and most primitive Pennatulid, and such forms as Cladiscus and Svava. To the former it approaches nearly, in the fact that the autozooids arise singly; to the latter, in the formation of incomplete leaves by partial union of two or three autozooids.

To discriminate exactly between generic and specific characters, when but one species has been obtained, is naturally not practicable; but the following may be regarded as definitions approximately correct:—

Deutocaulon, genus novum. "Pennatulida ex familiâ Protocaulidarum, quorum autozooidea, singulatim orta, pennæ laterales fiunt: calyx nullus: axis cylindratus."

Deutocaulon hystricis, species nova. "Rachis erecta, gracillima, cuius utraque facies levis est, et sine sulco medio ullo: stipitis pars inferior tumida, infima unca. In infima rachidis parte autozooidea lateralia, singulatim alternatimque posita; in media rachide, pennæ, primum alternæ, tum adversæ, ex duobus—tribus polypis fiunt. Spicula nulla. Siphonozooidea (?)."

All of the specimens, four in number, are very badly preserved, so that it is difficult to determine their characters.

The sarcosome is very thin throughout. No distinct cell nor any calycal processes were observable in the retracted autozooids; but at the tops of the radial chambers were collections of non-calcareous particles, very bright by reflected light. Reproductive organs were nowhere recognised in the specimens.

This form supplies a link, as was stated above, between the simpler more complicated forms of the Virgularian type. In Protocaulon, according to Kölliker (2), p. 26, pl. vii. fig. 23, the autozooids arise singly, and continue single throughout; in Deutocaulon they arise singly, but higher on the rachis are found rudimentary leaves formed of 2 to 3 polyps, generally fused by their bases only: in Cladiscus, figured by Koren and Danielssen (6), p. 101, pl. ix. figs. 13–15, the autozooids are from the first in more or less developed leaves, consisting each of three polyps, fused only by their bases; and in the more common forms, such as Virgularia itself, they arise in leaves, more completely developed by a more extensive fusion of more numerous autozooids. This new species, therefore, together with the family Protocaulidæ, to which it must be referred (characterised by the origin of the autozooids singly), should in a rearrangement of the Pennatulida be brought much closer to the Virgularidæ than is at present the case.

It is possible that our specimens are identical with the provisional species, *Virgularia gracillima* of Kölliker (2), p. 10. As, however, his specimen was imperfect below, it is not possible to decide the point with certainty. Should the two species, however, prove to be identical, a new genus will be necessary for their reception.

The dimensions, &c., of the specimens are as follows:—

	A. Both ends imperfect.	B. Upper end imperfect.	C. Both ends imperfect.	D. Upper end imperfect.
Total length,	3·5 cm.	4·8 cm.	3·0 cm.	3·6 cm.
Length of rachis,	3·5 cm.	2·3 cm.	2·5 cm.	1·0 cm.
Length of stalk,	none.	2·5 cm.	·5 cm.	2·5 cm.
Diameter of axis,	·3 mm.	·2 mm.	·2 mm.	·25 mm.
Diameter of rachis,	·35 mm.	·23 mm.	·27 mm.	·3 mm.
Diameter of stalk (greatest),	...	·27 mm.	...	·65 mm.
Diameter of stalk (least)	...	...	·37 mm.	·35 mm.
Number of polyps per leaf	3	2	2	...
Distance apart of pairs of leaves,	1·6 mm.	...	1·2 mm.	1 mm.

Of these, the specimens marked C and D are figured (Pl. XXXII. figs. 8, 9). Those marked A, B, C are from Station 1; D from Station 24.

Family 4. Protoptilidæ.

Genus *Protoptilum* (Köll.).

Protoptilum Carpenteri (Köll.).

Two fragments of the rachis of this species were obtained at Station 36, 1869, the one 22 mm. in length, the other 38 mm., but with the sarcosome stripped from the axis at both ends. Both agree well with the type specimen figured and described by Kölliker, but are not identical with it, though it was also dredged by the "Porcupine." In the one specimen the autozooids are distinctly arranged in groups of three each, the median dorsal one projecting more outwards than, and being slightly anterior to, the two lateral. In the other, they are placed more closely together, and no regular arrangement is recognisable.

With the exception of a narrow median ventral stripe, which is of a yellowish tint, the whole colony is of a brilliant red, the colour being due to the spicules, which on the ventral stripe are almost completely absent. Generally two prominent marginal teeth are placed on the abaxial edge of the calyx, and a varying number of lesser ones is also present.

Köll. (3), p. 274, pl. xxiv. figs. 223, 224.

Geographical Distribution.

With regard to the horizontal distribution, it is worthy of remark that Deutocaulon Hystricis adds another to the simple forms already obtained in the North Atlantic and the North Sea (Lygomorpha, Svava, Cladiscus), but that the genus to which it most closely approximates, viz., Protocaulon, has been obtained from Japan only. The latter observation is of especial interest, because not only in the facies of the terrestrial fauna, but also in the fishes, marine shells (Gwyn Jeffreys, *Jour. Linn. Soc. Zool.*, xii. pp. 100–109, 1876), and Ophiurids (Hoyle, *Proc. Roy. Soc. Edin.*, xii. p. 717), a close relation between the North Atlantic and Japanese faunas has been traced. The distribution of *Svava glacialis* is by our specimens extended several degrees N. and W.

The vertical distribution of Deutocaulon, dredged only just below the 100 fathom line, and next to Protocaulon the simplest Pennatulid recorded, is a direct contradiction to the assertion of Kölliker (2) that the simplest Pennatulida inhabit only great depths.

NOTE.

A species described as new by Professor MARSHALL (5), p. 129, under the name of *Virgularia tuberculata*, appears on close comparison to be identical with the *Cladiscus gracilis* of KOREN and DANIELSSEN (7), p. 101, pl. ix. figs. 13–15. The latter name naturally has priority, and *V. tuberculata* ceases to exist as a separate species. At the same time we consider the generic characters as given by the Norwegian authors too slight to justify a separation of the form from Virgularia.

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(5) Marshall, A. M., Report on the *Pennatulida* dredged by H.M.S. "Triton," *Trans. Roy. Soc. Edin.*, vol. xxxiii. pt. i. p. 119.
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(7) Koren and Danielssen, *Fauna Littoralis Norvegiæ*, pt. iii.

EXPLANATION OF PLATES.

PLATE XXXI.

Figs. 1, 2. *Pennatula phosphorea*, var. *candida*, n. Fig. 1 is from the dorsal; fig. 2 from the ventral surface. × 5.

PLATE XXXII.

Fig. 3. *Pennatula phosphorea*, var. *candida*, n. A single leaf from a different specimen to that figured on the previous Plate. × 7.

Figs. 4, 5. *Pennatula phosphorea* (?), young. Specimen A of the description (see p. 458). × 6.

Figs. 6, 7. *Pennatula phosphorea* (?), young. Specimen D of the description (see p. 458). × 6.

Figs. 8, 9. *Deutocaulon hystricis*, gen. sp. n. × 7

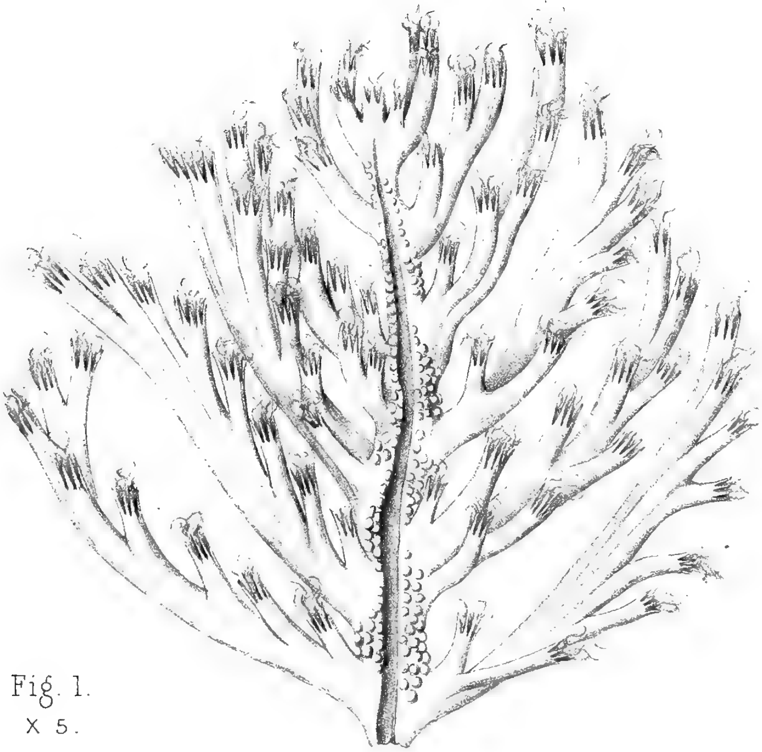


Fig. 1.
X 5.

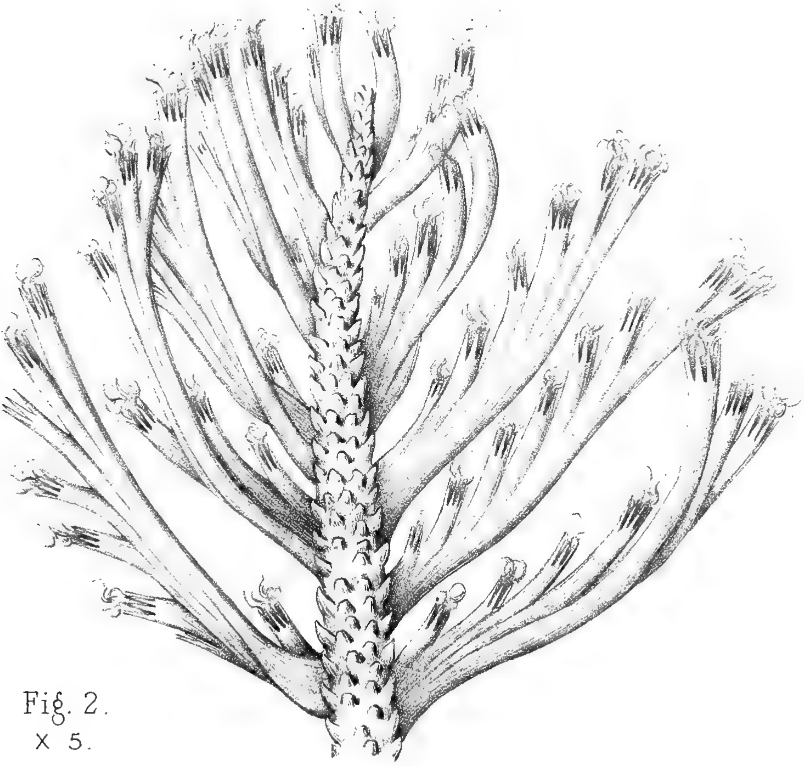


Fig. 2.
X 5.

.M. del.

F. Huth, Lithr Edinr.

Fig. 4.
× 6.

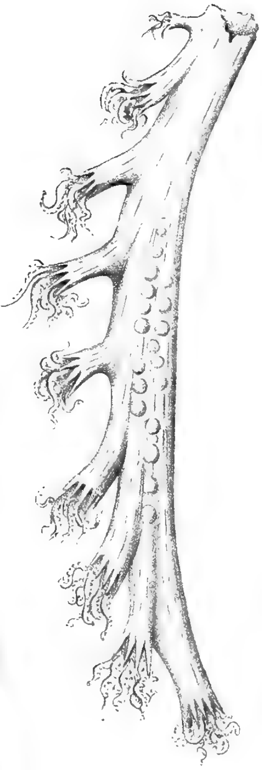


Fig. 3.
× 7.

Fig. 5.
× 6.

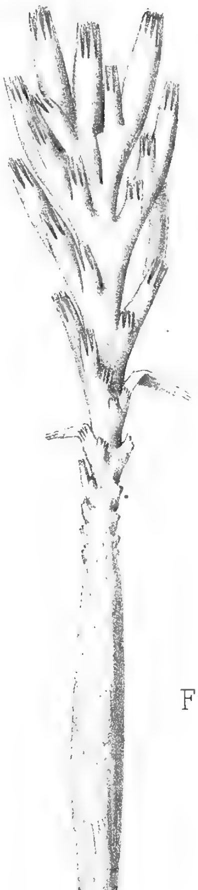


Fig. 6.
× 6.

Fig. 7.
× 6.

Fig 9.
× 7.

Fig. 8.
× 7.

M.M. & G.H.F. del.   F. Huth, Lith^r. Edin^r.

XXI.—*On the Determination of the Curve, on one of the coordinate planes, which forms the Outer Limit of the Positions of the point of contact of an Ellipsoid which always touches the three planes of reference.* By G. PLARR, Docteur ès-Sciences. Communicated by Professor TAIT.

(Read July 18, 1887.)

INDEX TO CONTENTS.

The question is the following:—We consider the solid angle formed by three planes at right angles to each other, and into the space of this single octant we introduce a given ellipsoid, and cause its surface to be tangent to each of the three sides of the solid angle. The position of the points of tangence will of course be variable in each plane according to the orientation given to the axes of the ellipsoid, but it is evident that on each of the planes the positions of the point of contact will be unable to outpass certain limits so long as the ellipsoid fulfils the condition of remaining tangent simultaneously to the three planes: these limiting positions of the point of contact in one, as for example, of the planes, will form a certain curve, and the proposed question will be: the determination of that curve, *the limiting curve* as we shall call it in the sequel.

§ I.

Let α, β, γ designate the unit vectors in the direction respectively of the three principal axes of the ellipsoid; let a, b, c designate the lengths of these axes; then if ω represents the central vector of a point on the surface of the ellipsoid, and if we put

$$\phi\omega = \frac{\alpha S\alpha\omega}{a^2} + \frac{\beta S\beta\omega}{b^2} + \frac{\gamma S\gamma\omega}{c^2},$$

or briefly

$$\phi\omega = \Sigma\frac{\alpha S\alpha\omega}{a^2},$$

the equation of the ellipsoid will be

$$S\omega\phi\omega = 1;$$

and if ω_0 designates the central vector of a point outside the surface, the equation

$$S\omega_0\phi\omega = 1$$

will represent that of the tangent plane passing through the extremity of ω.

Let O designate the apex of the solid trirectangular angle, and i, j, k the unit vectors in the direction of the three edges. We may assume that O and i, j, k remain fixed in position and direction. From this it follows that the centre O_1 of the ellipsoid and the direction of the trirectangular axes α, β, γ will be the variables of the question.

We designate by θ the vector OO_1 of the centre of the ellipsoid in one of its variable positions, and by

$$\rho, \sigma, \tau,$$

the vectors of the points of contact of the ellipsoid with the three planes respectively

$$(j, k), (k, i), (i, j)$$

briefly designated, these vectors having their origin in O. From their definition they satisfy the conditions

$$S\rho i = 0, \quad S\sigma j = 0, \quad S\tau k = 0.$$

We call, on the other hand,

$$\rho_1,\ \sigma_1,\ \tau_1$$

the central vectors of the points of contact, so that we have

$$\rho=\theta+\rho_1,$$
$$\sigma=\theta+\sigma_1.$$
$$\tau=\theta+\tau_1.$$

§ II.

At the extremity of ρ_1 the tangent plane is by hypothesis the plane (j, k) itself, so that the normal to the plane is parallel to i. At the same time, the normal has the direction of $\phi\rho_1$, thus we may put

$$\phi\rho = i\mathrm{N},$$

N being a scalar to be determined.

This equation gives

$$\rho_1=\mathrm{N}\phi^{-1}i,$$

where ϕ^{-1} is the inverse of ϕ, and is defined by

$$\phi^{-1}\omega=\alpha a^2\mathrm{S}\alpha\omega+\beta b^2\mathrm{S}\beta\omega+jc^2\mathrm{S}j\omega,$$

or briefly by

$$\phi^{-1}\omega=\Sigma\alpha a^2\mathrm{S}\alpha\omega.$$

Having

$$\mathrm{S}\rho_1\phi\rho_1=1,$$

ρ_1 being a central vector of a point on the surface, we get by the two vector equations:

$$1=\mathrm{N}^2\mathrm{S}i\phi^{-1}i.$$

This determines N ambiguously, but we have the equation of the tangent plane to give it its sign: namely, the tangent plane at the extremity of ρ_1 passes through the point O; we may, therefore, for the central vector ω_0 put

$$\omega_0=(-\theta).$$

Thus the equation of the tangent plane

$$\mathrm{S}\omega_0\phi\rho_1=1,$$

owing to $\phi\rho_1=i\mathrm{N}$, becomes

$$-\mathrm{NS}\theta i=1.$$

This gives

$$\mathrm{N}=\frac{-1}{\mathrm{S}\theta i}=\frac{1}{\sqrt{\mathrm{S}i\phi^{-1}i}}.$$

If we put

$$\theta=iu+jv+kw,$$

it will be necessary to assume that the values of u, v, w will continually be *positive* in consequence of our admission that the ellipsoid be contained in the one octant of which the edges are in the directions $+i,\ +j,\ +k$.

We have thus

$$N=\frac{1}{u}=\frac{1}{\sqrt{Si\phi^{-1}i}}.$$

A consideration of what takes place in the two other tangent planes will give corresponding values of v and of w.

We thus have the three expressions

$$u^2=Si\phi^{-1}i$$
$$v^2=Sj\phi^{-1}j$$
$$w^2=Sk\phi^{-1}k$$

with the restriction that the determination of the square roots of these expressions be the positive one exclusively.

By the value of N and of analogous values for the cases of σ_1 and τ_1, we have now

$$\begin{cases} \rho_1=\dfrac{1}{u}\phi^{-1}i\,, \\ \sigma_1=\dfrac{1}{v}\phi^{-1}j\,, \\ \tau_1=\dfrac{1}{w}\phi^{-1}k\,. \end{cases}$$

These values are to be introduced into

$$\rho=\theta+\rho_1\,,\ \sigma=\ \&c.$$

We may at once introduce also the following notations :—

$$x_1=Sj\phi^{-1}k\,,\quad x_0=Sj\phi k\,,$$
$$y_1=Sk\phi^{-1}i\,,\quad y_0=Sk\phi i\,,$$
$$z_1=Si\phi^{-1}j\,,\quad z_0=Si\phi j\,.$$

§ III.

Let us now calculate the variations of the above vectors when the directions of α, β, γ vary. It will be sufficient for our purpose to consider the variation of ρ only.

We represent the unit vectors α, β, γ by

$$\begin{cases} \alpha=piq \\ \beta=pjq \\ \gamma=pkq\,, \end{cases}$$

where $p(\ )q$ is to express the operator of a conical rotation, in which $pq=1$, and $\mathrm{T}p=\mathrm{T}q=1$.

For any position of the point of tangence at the extremity of ρ (generally comprised within the inside of the limiting curve), we represent the variations of the quantities by the characteristic δ, reserving the sign d of the differential to the increase of ρ when the extremity of this vector moves on the limiting curve.

In this latter case we assume (having $\mathrm{S}\rho i=0$)

$$\rho=jy+kz\,;$$

consequently

$$d\rho=jdy+kdz$$

represents the element of the limiting curve.

In the general case p changes into $p+\delta p$; putting

$$\epsilon=2\delta p\,.\times q$$

we get

$$\delta\alpha=\mathrm{V}\epsilon\alpha\,,\quad \delta\beta=\mathrm{V}\epsilon\beta\,,\quad \delta\gamma=\mathrm{V}\epsilon\gamma$$

Applying these to $\delta(\phi^{-1}\omega)$ we get

$$\delta(\phi^{-1}\omega)=\Sigma\mathrm{V}\epsilon\alpha\,.\,\alpha^2\mathrm{S}\alpha\omega+\Sigma\alpha\alpha^2\mathrm{S}\omega\mathrm{V}\epsilon\alpha$$

namely

$$\delta(\phi^{-1}\omega)=\mathrm{V}(\epsilon\phi^{-1}\omega)+\phi^{-1}(\mathrm{V}\omega\epsilon)$$

This written for $\omega=i,\ j,\ k$, gives first

$$\begin{aligned}2u\delta u&=\mathrm{S}i\delta(\phi^{-1}i)\\&=\mathrm{S}i[\mathrm{V}\epsilon\phi^{-1}i+\phi^{-1}\mathrm{V}i\epsilon]\\&=-2\mathrm{S}\,.\,\epsilon\mathrm{V}i\phi^{-1}i\end{aligned}$$

Hence (and *mutatis mutandis*):

$$\delta u=-\frac{1}{u}\mathrm{S}\,.\,\epsilon i\phi^{-1}i$$

$$\delta v=-\frac{1}{v}\mathrm{S}\epsilon j\phi^{-1}j\,,$$

$$\delta w=-\frac{1}{w}\mathrm{S}\epsilon k\phi^{-1}k\,.$$

With these expressions we get

$$\delta\theta=-\Sigma\frac{i}{u}\mathrm{S}\epsilon i\phi^{-1}i\,,$$

$$\delta\rho_1=\frac{1}{u}(\mathrm{V}\epsilon\phi^{-1}i+\phi^{-1}\mathrm{V}i\epsilon)+\frac{\phi^{-1}i}{u^3}\mathrm{S}\,.\,\epsilon i\phi^{-1}i\,,$$

and the second members evidently represent each a linear vector function of ϵ.

We designate them respectively by $\psi_0\epsilon$ and by $\psi_1\epsilon$, so that

$$\delta\theta = \psi_0\epsilon$$

$$\delta\rho_1 = \psi_1\epsilon\,.$$

And putting

$$\psi_0 + \psi_1 = \psi$$

we have now

$$\delta\rho = \psi\epsilon\,.$$

Generally having, for any vector ω,

$$\psi_0\omega = -\Sigma\frac{i}{u}\mathrm{S}\omega i\phi^{-1}i$$

$$\psi_1\omega = \frac{1}{u}\mathrm{V}(\omega\phi^{-1}i + \phi^{-1}\mathrm{V}i\omega) + \frac{\phi^{-1}i}{u^3}\mathrm{S}\omega i\phi^{-1}i\,,$$

the conjugates $\psi_0'\omega$ and $\psi_1'\omega$ will be

$$\psi_0'\omega = -\Sigma\frac{1}{u}\mathrm{V}i\phi^{-1}i\mathrm{S}i\omega\,,$$

$$\psi_1'\omega = -\frac{1}{u}(\mathrm{V}\omega\phi^{-1}i + \mathrm{V}i\phi^{-1}\omega) + \frac{1}{u^3}\mathrm{V}i\phi^{-1}i\mathrm{S}\omega\phi^{-1}i$$

with

$$\psi\omega = \psi_0\omega + \psi_1\omega\,,$$

$$\psi'\omega = \psi_0'\omega + \psi_1'\omega\,.$$

§ IV.

The condition which must be fulfilled when the extremity of ρ is to move on the limiting curve may be stated as follows:—

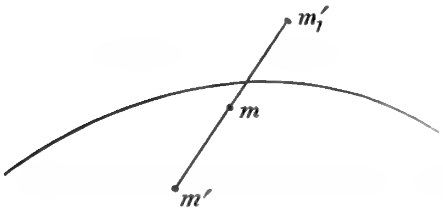
Generally the extremity of ρ is comprised within the inside of the curve in question. In that case two different axes of infinitesimal rotation, say ϵ and ϵ_1, will produce variations $\delta\rho$ differing generally from each other in direction and length. We may assume generally, also, that if ϵ and ϵ_1 be opposed to each other so that

$$\mathrm{U}\epsilon + \mathrm{U}\epsilon_1 = 0\,,$$

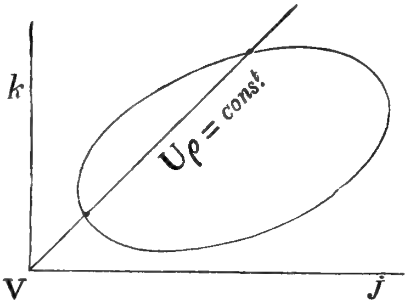
we will also have

$$\psi(\mathrm{U}\epsilon) + \psi(\mathrm{U}\epsilon_1) = 0\,.$$

But when the extremity of ρ is infinitesimally near the limiting curve, then we cannot any more admit this, because if ϵ should bring the point m (extremity of ρ) to the point m' *inside* the curve, then the rotation $\epsilon_1 = (-\epsilon)$ would have to bring the point m to m_1' *outside*, a circumstance which cannot take place, as it is against the very definition of the curve.



We may try the supposition that for a point ρ on the limiting curve the value of $\psi\epsilon$ will vanish for any direction of ϵ, without even excepting that direction of ϵ which might render $\psi\epsilon$ parallel to the element $d\rho$ of the curve.



If, for example, we put the question to determine the maximum of $T\rho$ under the condition

$$U\rho = \text{constant},$$

then we get the two equations

$$S\rho\delta\rho = 0 \text{ and } V\rho\delta\rho = 0,$$

which result into

$$\delta\rho = \psi\epsilon = 0.$$

Another supposition to be tried will be to assume that for a point on the limiting curve the direction of $\psi\epsilon$ will be parallel to $d\rho$ for any direction of ϵ, so that in that case the possibly not vanishing $\psi\epsilon$ will be invariable as to ϵ.

These two suppositions expressed respectively by the condition

$$\psi\epsilon = 0,$$

and by the condition

$$V(d\rho\psi\epsilon) = 0,$$

in each case for any direction of ϵ, are not excluding each other, and we will be able to show that the consequences of the second of the conditions involve the consequences of the first of them.

We may remark that the first of the conditions, namely,

$$\psi\epsilon = 0, \text{ for any } \epsilon,$$

will also present itself when we look upon the limiting curve as being the general envelope of all the possible curves which the extremity of ρ may describe under various limitations.

We may further remark that the condition

$$V . d\rho\psi\epsilon = 0,$$

namely

$$V(d\rho\delta\rho) = 0,$$

expresses the answer to the problem of finding the curve comprising the *area-maxima* of all the areas comprised by curves described by the extremity of ρ.

In order to conceive how the extremity of ρ may be caused to describe certain definite curves (both in the case of an envelope and in the case of an area to be considered), we must reflect that the expression of ρ depends on the *three* scalar elements of the versor p, and that we may conceive a certain relation between the three to be established, so that only two of them may be considered as independent. But here there arises the theoretical difficulty of our inability of guessing at the true nature of the relations to be established so long as the true nature of the limiting curve is ignored. This our inability of establishing the proper limitations between the three scalar elements of p will force us into the fear that, instead of an outermost limiting curve, we will deduce a possibly self-intersecting curve with many branches, but certainly all evidently contained in the outermost limiting curve.

§ V.

From the equation $\mathrm{V}d\rho\psi\epsilon=0$ we draw the three scalar equations—

$$\begin{aligned} \mathrm{S}id\rho\psi\epsilon &= \mathrm{S}\,.\,\epsilon\psi'\mathrm{V}id\rho=0\,, \\ \mathrm{S}jd\rho\psi\epsilon &= \mathrm{S}\,.\,\epsilon\psi'\mathrm{V}jd\rho=0\,, \\ \mathrm{S}kd\rho\psi\epsilon &= \mathrm{S}\,.\,\epsilon\psi'\mathrm{V}kd\rho=0\,. \end{aligned}$$

As we have assumed

$$d\rho=jdy+kdz\,,$$

we get

$$\begin{aligned} \mathrm{V}jd\rho &= idz\,, \\ \mathrm{V}kd\rho &= -idy\,. \end{aligned}$$

The two last of the scalar equations will give

$$\mathrm{S}\epsilon\psi'i=0\,.$$

But we easily find that $\psi'i$ is identically zero, because substituting i for ω in the expressions of $\psi_0'\omega$, $\psi_1'\omega$ we get

$$\psi_0'i=-\frac{\mathrm{V}i\phi^{-1}i}{u}i^2=+\frac{\mathrm{V}i\phi^{-1}i}{u}$$

and

$$\psi_1'i=\frac{1}{u}[-2\mathrm{V}i\phi^{-1}i]+\frac{\mathrm{V}i\phi^{-1}i}{u^3}\mathrm{S}i\phi^{-1}i\,,$$

which owing to $u^2=\mathrm{S}i\phi^{-1}i$ become

$$\psi_1'i=-\frac{1}{u}\mathrm{V}i\phi^{-1}i\,,$$

so that $\psi_0'i+\psi_1'i=\psi'i=0$ identically.

There remains the first scalar equation

$$\mathrm{S}\epsilon\psi'\mathrm{V}id\rho=0\,,$$

or

$$\mathrm{S}\epsilon[dy\psi'k-dz\psi'j]=0\,.$$

As this equation is to be satisfied for any direction of ϵ we must have

$$\psi'\mathrm{V}id\rho=0\,,$$

or, under another form:

$$dy\psi'k-dz\psi'j=0\,.$$

From this second form of the equation we deduce

$$\mathrm{V}.\psi'j\psi'k=0.$$

This parallelism of $\psi'j$ and $\psi'k$ is also the consequence of the condition

$$\psi\epsilon=0\,,\ \text{for any}\ \epsilon\,,$$

because treating this last equation successively by $\mathrm{S}.i$, $\mathrm{S}.j$, $\mathrm{S}.k$ it gives first

$$\mathrm{S}.i\psi\epsilon=\mathrm{S}.\epsilon\psi'i=0\,,$$

which is identically satisfied, because $\psi'i$ is identically zero.

Then the two other equations,

$$\mathrm{S}j\psi\epsilon=\mathrm{S}.\epsilon\psi'j=0\,.$$
$$\mathrm{S}k\psi\epsilon=\mathrm{S}.\epsilon\psi'k=0$$

have to be satisfied for any direction of ϵ. This must be effected by the least number of assumptions, lest we get too many conditions between the three scalar elements of p. The only way to reduce the number of conditions is to assume the parallelism between $\psi'j$ and $\psi'k$ again.

§ VI.

We cannot refrain from sketching briefly a method of development of the equations

$$\psi'\mathrm{V}id\rho=0\,,$$

and

$$\mathrm{V}.\psi'j\psi'k=0\,,$$

which we have abandoned owing to the complication arising from the rationalising the expressions containing the radicals u, v, w together.

If we develop namely $\psi'j$ and $\psi'k$ into their components parallel to i, j, k, putting (for reasons easily to be accounted for)

$$\psi'j=-i\mathrm{P}_1-j\mathrm{Q}-k\mathrm{R}_2\,,$$
$$\psi'k=+i\mathrm{P}_2+j\mathrm{R}_1+k\mathrm{Q}\,,$$

where with the notations

$$\begin{cases} x_1=\mathrm{S}j\phi^{-1}k \\ y_1=\mathrm{S}k\phi^{-1}i \\ z_1=\mathrm{S}i\phi^{-1}j \end{cases}$$

we have

$$\mathrm{P}_1=\frac{x_1}{v}-\frac{y_1}{u}\,,\ \mathrm{P}_2=\frac{x_1}{w}-\frac{z_1}{u}\,;$$

$$\mathrm{Q}=\frac{1}{u^3}(u^2x_1-y_1z_1)\,,$$

$$\begin{cases} \mathrm{R}_1=\dfrac{y_1{}^2}{u^3}-\dfrac{y_1}{w}+u-\dfrac{w^2}{u}\,, \\ \mathrm{R}_2=\dfrac{z_1{}^2}{u^3}-\dfrac{z_1}{v}+u-\dfrac{v^2}{u}\,, \end{cases}$$

then the equation $\psi' \mathrm{V} i d\rho = 0$ decomposes itself into the three

$$\begin{aligned} 0 &= \mathrm{P}_2 dy + \mathrm{P}_1 dz , \\ 0 &= \mathrm{R}_1 dy + \mathrm{Q} dz , \\ 0 &= \mathrm{Q} dy + \mathrm{R}_2 dz . \end{aligned}$$

The elimination of $\frac{dz}{dy}$ gives the equations

$$\begin{aligned} \mathrm{P}_1\mathrm{Q} - \mathrm{P}_2\mathrm{R}_2 &= 0 , \\ \mathrm{P}_2\mathrm{Q} - \mathrm{P}_1\mathrm{R}_1 &= 0 , \end{aligned}$$

and their consequence

$$\mathrm{R}_1\mathrm{R}_2 - \mathrm{Q}^2 = 0 .$$

It is evident that each of these equations contains the three radicals u, v, w all *three* at the same time, and their transformation into rational expressions depending on the nine coefficients

$$\begin{aligned} &\mathrm{S}\alpha i , \quad \mathrm{S}\alpha j , \quad \mathrm{S}\alpha k , \\ &\mathrm{S}\beta i , \quad \mathrm{S}\beta j , \quad \&c., \\ &\mathrm{S}\gamma i, \ \&c., \end{aligned}$$

will lead to hopeless complications from a practical point of view. Of course the expressions of Q, R_1, R_2 may be slightly simplified by the use of the direct function ϕ instead of ϕ^{-1}, but the fact remains that powers of uneven degree of u, v, w will affect together always each of the equations.

We will only add the remark that *theoretically* the two scalar equations above, and the two scalar equations to be derived from

$$\rho = \theta + \frac{\phi^{-1} i}{u} ,$$

will constitute four equations from which the three elements of the versor p may be supposed to be eliminated, so that the problem is theoretically definite.

We may also remark that when $\psi'(\mathrm{V} i d\rho) = 0$ is satisfied, that is when the point of tangence is situated on the limiting curve, then we have for the primitive function

$$\psi(\mathrm{V} i d\rho) = -d\rho(\mathrm{R}_1 + \mathrm{R}_2) .$$

As $\mathrm{V}(i d\rho)$ is directed normally to the curve, the equation shows that a rotation ϵ parallel to the normal produces a displacement of the extremity of ρ in the direction of the tangent.

From the above equation we draw by the application of the operator ψ:

$$\psi^2(\mathrm{V} i d\rho) = -\psi d\rho(\mathrm{R}_1 + \mathrm{R}_2) .$$

Now as $\psi' i = 0$ we have also $\psi(\kappa) = 0$, where

$$\kappa = \mathrm{V}(\psi' j \psi' k) \text{ generally.}$$

The cubic relative to ψ becomes a quadric, namely,

$$-\mathrm{M}_1\omega+\psi^2\omega+\kappa\mathrm{S}i\omega=0,^*$$

where

$$\mathrm{M}_1=-\mathrm{S}i\psi'j\psi'k(=\mathrm{R}_1\mathrm{R}_2-\mathrm{Q}^2).$$
$$\mathrm{M}_2=-\mathrm{S}j\psi'j-\mathrm{S}k\psi'k=+\mathrm{Q}-\mathrm{Q}=0\,.$$

The quadric can also be put under the form

$$\psi^2\omega=\mathrm{V}\left[i\mathrm{V}(\kappa\omega)\right].$$

Putting $\omega \parallel \mathrm{V}id\rho$ we get

$$\begin{aligned}\psi^2(\mathrm{V}id\rho)&=\mathrm{V}.\left[i\mathrm{V}(\kappa\mathrm{V}id\rho)\right]\\&=\mathrm{V}.\,i(d\rho\mathrm{S}\kappa i-i\mathrm{S}\kappa d\rho)\\&=\mathrm{V}id\rho(-\mathrm{M}_1)\,.\end{aligned}$$

Hence by the above:

$$\psi(d\rho)=\mathrm{V}id\rho\times\frac{\mathrm{M}_1}{\mathrm{R}_1+\mathrm{R}_2},$$

and

$$\mathrm{M}_1=0\ \text{by}\ \psi'(\mathrm{V}id\rho)=0\,.$$

This shows that when the point of tangence is on the limiting curve we have

$$\psi(d\rho)=0\,,$$

but this remark is unessential as to the sequel.

§ VII.

Let us examine $\psi\omega$ more particularly. First we see that

$$\begin{aligned}\psi_0'\theta&=-\Sigma\mathrm{V}i\phi^{-1}i\frac{\mathrm{S}i\theta}{u}\\&=\Sigma\mathrm{V}i\phi^{-1}i\,,\end{aligned}$$

owing to $\theta=iu+jv+kw$. As ϕ and ϕ^{-1} are self-conjugate-vector functions, it follows that

$$\Sigma\mathrm{V}i\phi^{-1}i=0\,;$$

thus we have

$$\psi_0'\theta=0\,.$$

This equation at once gives us the certainty that the primitive function $\psi_0\omega$ will vanish also when ω takes a particular direction. (We have shown this necessity in a paper published in the *Proc. Roy. Soc. Edin.* of the year 1881–82, p. 342.)

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* *Proc. Roy. Soc. Edin.*, 1882–83, p. 342.

Let us call ζ the vector, for which we have

$$\psi_0\zeta=0\,.$$

This equation gives the three scalar equations:

$$\mathrm{S}.\zeta i\phi^{-1}i=0\,,$$
$$\mathrm{S}.\zeta j\phi^{-1}j=0\,,$$
$$\mathrm{S}.\zeta k\phi^{-1}k=0\,.$$

In developing $\mathrm{V}.\,i\phi^{-1}i=\mathrm{V}.jk\phi^{-1}i$, &c., by the formula

$$\mathrm{V}(\mathrm{V}\lambda\mu\,.\,\nu)=\lambda\mathrm{S}\mu\nu-\mu\mathrm{S}\lambda\nu\,,$$

we get easily, employing the notations $x_1=\mathrm{S}j\phi\ {}^1k$, $y_1=\mathrm{S}k\phi^{-1}i$, $z_1=\mathrm{S}i\phi^{-1}j$:

$$0=y_1\mathrm{S}\zeta i-z_1\mathrm{S}\zeta k\,,$$
$$0=z_1\mathrm{S}\zeta k-x_1\mathrm{S}\zeta i\,;$$

hence

$$x_1\mathrm{S}\zeta i=y_1\mathrm{S}\zeta j=z_1\mathrm{S}\zeta k=\text{ a scalar }=-\mathrm{N}\,;$$

hence

$$\zeta=\mathrm{N}\left(\frac{i}{x_1}+\frac{j}{y_1}+\frac{k}{z_1}\right)\cdot$$

We assume $\mathrm{N}=x_1\,y_1\,z_1$, introducing thus a factor for the sake of which some precautions are to be taken when two of the three x_1, y_1, z_1 should vanish at the same time. We have thus

$$\zeta=iy_1z_1+jz_1x_1+kx_1y_1$$
$$=\Sigma iy_1z_1\,.$$

By $\psi_0\zeta=0$, and having generally

$$\psi_0+\psi_1=\psi\,,$$

we get

$$\psi\zeta=\psi_1\zeta\,.$$

This shows that when ϵ takes the direction of ζ, then the extremity of θ, namely, the centre of the ellipsoid, remains unmoved, whereas the extremities of ρ_1, σ_1, τ_1 displace themselves by the rotation round the instantaneous axis directed parallel to ζ.

We have in this particular case

$$d\rho=\psi_1\zeta dt=\psi\zeta dt\,,$$

where dt has a convenient scalar value.

§ VIII.

Let us now calculate $\psi_1\zeta$ explicitly. The expression of $\psi_1\omega$ gives

$$\psi_1\zeta=\frac{1}{u}\left[\mathrm{V}\zeta\phi^{-1}i+\phi^{-1}\mathrm{V}i\zeta+\frac{\phi^{-1}i}{u^2}\mathrm{S}\zeta i\phi^{-1}i\right].$$

Now ζ may be written

$$\zeta=iy_1z_1+x_1(jz_1+ky_1).$$

Having

$$\phi^{-1}i=-i\mathrm{S}i\phi^{-1}i-j\mathrm{S}j\phi^{-1}i-k\mathrm{S}k\phi^{-1}i=-iu^2-jz_1-ky_1,$$

we get

$$\zeta=i(y_1z_1-x_1u^2)-x_1\phi^{-1}i.$$

The coefficient of i, namely,

$$\begin{aligned}y_1z_1-x_1u^2&=\mathrm{S}k\phi^{-1}i\mathrm{S}i\phi^{-1}j-\mathrm{S}k\phi^{-1}j\mathrm{S}i\phi^{-1}i\\&=\mathrm{S.V}ki\mathrm{V}\phi^{-1}j\phi^{-1}i,\\&=-\mathrm{S}.j\mathrm{V}\phi^{-1}i\phi^{-1}j\\&=-\frac{1}{m}\mathrm{S}j\phi k,\end{aligned}$$

where we apply the formulas

$$m\phi^{-1}\mathrm{V}\lambda\mu=\mathrm{V}.\phi\lambda\phi\mu$$

$$m\phi^{-1}\mathrm{V}.\phi^{-1}\lambda'\phi^{-1}\mu'=\mathrm{V}.\lambda'\mu',$$

hence

$$\mathrm{V}.\phi^{-1}\lambda'\phi^{-1}\mu'=\frac{1}{m}\phi\mathrm{V}\lambda'\mu',$$

and introduce

$$\lambda'=i,\ \mu'=j.$$

Thus we get

$$\zeta=-\frac{i\mathrm{S}j\phi k}{m}-x_1\phi^{-1}i,$$

$$=-\frac{ix_0}{m}-x_1\phi^{-1}i.$$

This gives at once

$$\mathrm{S}.\zeta i\phi^{-1}i=0.$$

Then

$$\mathrm{V}.\zeta\phi^{-1}i=-\mathrm{V}.i\phi^{-1}i.\frac{\mathrm{S}j\phi k}{m};$$

and

$$\phi^{-1}\mathrm{V}i\zeta=-\phi^{-1}(\mathrm{V}i\phi^{-1}i).x_1.$$

Applying to the second member

$$m\phi^{-1}\mathrm{V}\lambda\mu=\mathrm{V}.\phi\lambda\phi\mu$$

for

$$\lambda=i,\ \mu=\phi^{-1}i,$$

we get

$$\phi^{-1}\mathrm{V}i\zeta=-\frac{x_1}{m}\mathrm{V}\phi i.i=\frac{x_1}{m}\mathrm{V}i\phi i.$$

These values give (as $x_1 = \mathrm{S}j\phi^{-1}k$):

$$\psi_1\zeta = \frac{1}{u}\left[-\mathrm{V}i\phi^{-1}i\frac{\mathrm{S}j\phi k}{m} + \frac{\mathrm{V}i\phi i\mathrm{S}j\phi^{-1}k}{m}\right].$$

Hence

$$mu\psi_1\zeta = \mathrm{V}.i\phi i\mathrm{S}j\phi^{-1}k - \mathrm{V}.i\phi^{-1}i\mathrm{S}j\phi k.$$

Developing $\mathrm{V}i\phi i$ and $\mathrm{V}i\phi^{-1}i$ into their components parallel to j, k:

$$\mathrm{V}i\phi i = \mathrm{V}.jk\phi i = j\mathrm{S}k\phi i - k\mathrm{S}j\phi i,$$
$$\mathrm{V}i\phi^{-1}i = \mathrm{V}.jk\phi^{-1}i = j\mathrm{S}k\phi^{-1}i - k\mathrm{S}j\phi^{-1}i,$$

we get

$$\begin{aligned} mu\psi_1\zeta &= j[\mathrm{S}k\phi i\mathrm{S}j\phi^{-1}k - \mathrm{S}k\phi^{-1}i\mathrm{S}j\phi k] \\ &\quad + k[-\mathrm{S}j\phi i\mathrm{S}j\phi^{-1}k + \mathrm{S}j\phi^{-1}i\mathrm{S}j\phi k]. \end{aligned}$$

Now ϕ and ϕ^{-1} being self-conjugate, we replace $\mathrm{S}k\phi i$, $\mathrm{S}k\phi^{-1}i$, respectively by

$$\mathrm{S}i\phi k,\ \mathrm{S}i\phi^{-1}k,$$

in the term affected by j, and

$$\mathrm{S}j\phi i,\ \mathrm{S}j\phi^{-1}i,\ \mathrm{S}j\phi^{-1}k,\ \mathrm{S}j\phi k$$

by

$$\mathrm{S}i\phi j,\ \mathrm{S}i\phi^{-1}j,\ \mathrm{S}k\phi^{-1}j,\ \mathrm{S}k\phi j,$$

in the term affected by k. This gives

$$\begin{aligned} mu\psi_1\zeta &= j[\mathrm{S}i\phi k\mathrm{S}j\phi^{-1}k - \mathrm{S}i\phi^{-1}k\mathrm{S}j\phi k] \\ &\quad + k[-\mathrm{S}i\phi j\mathrm{S}k\phi^{-1}j + \mathrm{S}i\phi^{-1}j\mathrm{S}k\phi j]. \end{aligned}$$

The coefficients of j, k are respectively

$$\mathrm{S}.\mathrm{V}ij\mathrm{V}.\phi^{-1}k\phi k = \mathrm{S}.k\phi^{-1}k.\phi k$$

and

$$\mathrm{S}.\mathrm{V}ki\mathrm{V}\phi^{-1}j\phi j = \mathrm{S}.j\phi^{-1}j.\phi j.$$

So that, with a change of sign ($\mathrm{S}k\phi^{-1}k.\phi k = -\mathrm{S}k\phi k\phi^{-1}k$, &c.), and introducing the notation

$$\mathrm{W}_1 = \mathrm{S}i\phi i\phi^{-1}i,$$
$$\mathrm{W}_2 = \mathrm{S}.j\phi j\phi^{-1}j,$$
$$\mathrm{W}_3 = \mathrm{S}.k\phi k\phi^{-1}k,$$

we have

$$mu\psi_1(\zeta) = -j\mathrm{W}_3 - k\mathrm{W}_2.$$

Let us further consider for any vector λ:

$$\begin{aligned} \mathrm{V}.\phi\lambda\phi^{-1}\lambda &= \mathrm{V}.\left(\Sigma\frac{\alpha\mathrm{S}\alpha\lambda}{a^2}\right)(\Sigma\alpha a^2\mathrm{S}\alpha\lambda) \\ &= \alpha\beta\mathrm{S}\alpha\lambda\mathrm{S}\beta\lambda\left(\frac{b^2}{a^2} - \frac{a^2}{b^2}\right) \\ &\quad + \gamma\alpha\mathrm{S}\gamma\lambda\mathrm{S}\alpha\lambda\left(\frac{a^2}{c^2} - \frac{c^2}{a^2}\right) \\ &\quad + \beta\gamma\mathrm{S}\beta\lambda\mathrm{S}\gamma\lambda\left(\frac{c^2}{b^2} - \frac{b^2}{c^2}\right). \end{aligned}$$

Hence, as $\alpha\beta=\gamma$, $\gamma\alpha=\beta$, $\beta\gamma=\alpha$, we get generally

$$\mathrm{S}\lambda\phi\lambda\phi^{-1}\lambda=\mathrm{S}\alpha\lambda\mathrm{S}\beta\lambda\mathrm{S}\gamma\lambda\times\Lambda,$$

where

$$\Lambda=\left(\frac{b^2}{a^2}-\frac{a^2}{b^2}\right)+\left(\frac{a^2}{c^2}-\frac{}{a^2}\right)+\left(\frac{c}{b^2}-\frac{b^2}{c^2}\right).$$

It is evident that, if we add and subtract unity, the second member will represent the development of

$$\left(\frac{c^2}{a^2}-\frac{b^2}{a^2}\right)\left(\frac{a^2}{b^2}-\frac{c}{b^2}\right)\left(\frac{b^2}{c^2}-\frac{a^2}{c^2}\right)$$

$$=\left(\frac{-1}{a^2b^2c^2}\right)(a^2-b^2)(c^2-a^2)(b^2-c^2)=\Lambda.$$

We have thus

$$\mathrm{W}_1=\mathrm{S}\alpha i\mathrm{S}\beta i\mathrm{S}\gamma i\,.\,\Lambda$$

$$\mathrm{W}_2=\mathrm{S}\alpha j\mathrm{S}\beta j\mathrm{S}\gamma j\,.\,\Lambda$$

$$\mathrm{W}_3=\mathrm{S}\alpha k\mathrm{S}\beta k\mathrm{S}\gamma k\,.\,\Lambda.$$

Introducing the notation

$$\alpha=ia_1+ja_2+ka_3$$

$$\beta=ib_1+jb_2+kb_3$$

$$\gamma=ic_1+jc_2+kc_3,$$

we finally get

$$\mathrm{W}_1=-a_1b_1c_1.\Lambda,$$

$$\mathrm{W}_2=-a_2b_2c_2.\Lambda,$$

$$\mathrm{W}_3=-a_3b_3c_3.\Lambda.$$

$$mu\psi_1\zeta=[ja_3b_3c_3+ka_2b_2c_2]\times\Lambda.$$

We put generally for $h=1, 2, 3$:

$$\mathrm{W}_h'=a_hb_hc_h,$$

then

$$\mathrm{W}_h=-\mathrm{W}_h'\Lambda$$

and

$$mu\psi_1\zeta=-(j\mathrm{W}_3'+k\mathrm{W}_3')\Lambda.$$

§ IX.

If we treat the fundamental equation

$$\psi'(\mathrm{V}id\rho)=0$$

by S . ζ, we get

$$\mathrm{S}\,.\,\zeta\psi'\mathrm{V}id\rho)=\mathrm{S}\,.\,\mathrm{V}id\rho\psi\zeta=0;$$

and, as

$$\psi\zeta=\psi_1\zeta=-\frac{1}{mu}(j\mathrm{W}_3+k\mathrm{W}_2),$$

we get

$$0=\mathrm{W}_3\mathrm{S}\,.\,j\mathrm{V}id\rho+\mathrm{W}_2\mathrm{S}\,.\,k\mathrm{V}id\rho,$$

namely,

$$0 = W_3(-Skd\rho) + W_2Sjd\rho ,$$

and for $d\rho = jdy + kdz$ the result becomes

$$W_3dz - W_2dy = 0 .$$

We have, therefore, for the element of the limiting curve,

$$\begin{aligned} d\rho &= (jW_3 + kW_2)dt' \\ &= -mu\psi_1(\zeta)dt' . \end{aligned}$$

Of course, there are two more conditions to be satisfied in order that the extremity of ρ be actually on the curve.

We may remark that ζ may also be put under a second and a third form, namely, the first being

$$\zeta = -\frac{ix_0}{m} - x_1\phi^{-1}i ,$$

we have also

$$\zeta = -\frac{jy_0}{m} - y_1\phi^{-1}j ,$$

$$\zeta = -\frac{kz_0}{m} - z_1\phi^{-1}k ,$$

where

$$x_0 = Sj\phi k ,\ y_0 = Sk\phi i ,\ z_0 = Si\phi j .$$

Hence if we put by analogy

$$\delta\sigma = (\psi_0 + \psi_2)\epsilon ,\ \delta\tau = (\psi_0 + \psi_3)\epsilon$$

we get, it is true,

$$-mu\psi_2\zeta = (kW_1 + iW_3) ,$$

$$-mu\psi_3\zeta = (iW_1 + jW_1) ,$$

but these two last expressions are only exceptionally representing $d\sigma$, $d\tau$ respectively, namely, they do so only when $Vjd\sigma$, or $Vkd\tau$, satisfy an equation analogous to $\psi' Vid\rho = 0$.

We put

$$\zeta' = jW_3' + kW_2'$$

and

$$\zeta_0 = jW_3 + kW_2 = -mu\psi_1\zeta ,$$

then we have

$$\zeta_0 = -\zeta\Lambda ,$$

and also

$$d\rho = \zeta' dt'' .$$

If we put

$$\zeta = x_0\phi^{-1}i - x_1\phi i ,$$

we have

$$\zeta_0 = Vi\zeta_1 .$$

Further, owing to $d\rho = \zeta_0 dt'$, the fundamental equation

$$\psi' \mathrm{V} i d\rho = 0$$

becomes

$$\psi' \mathrm{V} i \zeta_0 = 0 \, .$$

Now

$$\begin{aligned} \mathrm{V} i \zeta_0 &= \mathrm{V}(i \mathrm{V} i \zeta_1) \\ &= \zeta_1 i^2 - i \mathrm{S} i \zeta_1 \end{aligned}$$

as $\psi' i =$ identically zero, the equation becomes now simply

$$\psi'(\zeta_1) = 0 \, .$$

As we have

$$\zeta' = j \mathrm{W}_3' + k \mathrm{W}_2' \, ,$$

it follows that

$$\mathrm{V} i \zeta' = k \mathrm{W}_3' - j \mathrm{W}'_2 \, ,$$

and the fundamental equation also takes the form

$$0 = \mathrm{W}_3' \psi' k - \mathrm{W}_2' \psi' j \, .$$

If we replace $\psi' k$ and $(-\psi' j)$ by their expressions (already stated in § VI.),

$$\begin{aligned} \psi' k &= i \mathrm{P}_2 + j \mathrm{R}_1 + k \mathrm{Q} \, , \\ -\psi' j &= i \mathrm{P}_1 + j \mathrm{Q} + k \mathrm{R}_2 \, , \end{aligned}$$

the equation will decompose itself into three vector components parallel respectively to *i*, *j*, *k*, which have to be annulled separately, so that we get

$$\begin{aligned} 0 &= \mathrm{W}_3' \mathrm{P}_2 + \mathrm{W}_2' \mathrm{P}_1 \, , \\ 0 &= \mathrm{W}_3' \mathrm{R}_1 + \mathrm{W}_2' \mathrm{Q} \, , \\ 0 &= \mathrm{W}_3' \mathrm{Q} + \mathrm{W}_2 \mathrm{R}_2 . \end{aligned}$$

These reproduce the equations of the same form stated in § VI., in which *dy* and *dz* are replaced proportionally by W_3' and by W_2'. These three equations are equivalent to only *two distinct ones*, because as, for example, the necessary relation

$$\mathrm{R}_1 \mathrm{R}_2 - \mathrm{Q}^2 = 0$$

will help to transform the third of the equations into the second one,

$$\mathrm{W}_3' \mathrm{Q} + \mathrm{W}_2' \mathrm{R}_2 = \frac{\mathrm{Q}}{\mathrm{R}_1} (\mathrm{W}_3' \mathrm{R}_1 + \mathrm{W}_2' \mathrm{Q}) = 0 \, .$$

For this reason the equation

$$\psi'(\zeta_1) = 0$$

can yield but *two distinct* scalar equations. We will get them by treating $\psi'(\zeta_1) = 0$ by $\mathrm{S}.i(\ )$ and by $\mathrm{S}.\phi i(\ )$.

Curiously enough, the scalars

$$S.i\psi'\zeta_1 \text{ and } S.(\phi^{-1}i)\psi'\zeta_1$$

give the same result, at factors "près" which cannot generally be annulled; this happens owing to the fact that

$$x_0\psi i+mx_1\psi(\phi^{-1}i)=\frac{\zeta_0}{u} \;;$$

but we shall establish this relation further on.

§ X.

Let us treat first the case

$$S.i\psi'(\zeta_1)=S.\zeta_1\psi i=0 \,.$$

We have, by § III. :

$$\begin{aligned}\psi i&=\psi_0 i+\psi_1 i\\ \psi_0 i&=-\frac{j}{v}Sij\phi^{-1}j-\frac{k}{w}Sik\phi^{-1}k\\ &=\left(-\frac{j}{v}+\frac{k}{w}\right)x_1\\ \psi_1 i&=\frac{1}{u}Vi\phi^{-1}i=\frac{1}{u}Vjk\phi^{-1}i\\ &=\frac{1}{u}(jy_1-kz_1)\,;\end{aligned}$$

hence

$$\psi i=j\left(\frac{y_1}{u}-\frac{x_1}{v}\right)+k\left(\frac{x_1}{w}-\frac{z_1}{u}\right),$$

and as

$$\zeta_1=x_0\phi^{-1}i-x_1\phi i \,,$$

we deduce

$$\begin{aligned}S\zeta_1\psi i=x_0&\left[z_1\left(\frac{y_1}{u}-\frac{x_1}{v}\right)+y_1\left(\frac{x_1}{w}-\frac{z_1}{u}\right)\right]\\ -x_1&\left[z_0\left(\frac{y_1}{u}-\frac{x_1}{v}\right)+y_0\left(\frac{x_1}{w}-\frac{z_1}{u}\right)\right]\\ =&\frac{x_1}{u}[y_0z_1-z_0y_1]\\ +&\frac{x_1}{v}[-x_0z_1+x_1z_0]\\ +&\frac{x_1}{w}[x_0y_1-x_1y_0]\,.\end{aligned}$$

If now we consider the expressions of W_1, W_2, W_3, we get

$$\begin{aligned}W_1&=SVjkV\phi i\phi^{-1}i=z_1y_0-z_0y_1\,,\\ W_2&=SVkiV\phi j\phi^{-1}j=x_1z_0-x_0z_1\,,\\ W_3&=SVijV\phi k\phi^{-1}k=y_1x_0-y_0x_1\,.\end{aligned}$$

This gives

$$S\zeta_1\psi i=x_1\left(\frac{W_1}{u}+\frac{W_2}{v}+\frac{W_3}{w}\right).$$

As x_1 vanishes only for particular directions of the system α, β, γ, we must assume

$$\frac{\mathrm{W}_1}{u}+\frac{\mathrm{W}_2}{v}+\frac{\mathrm{W}_3}{w}=0\,,$$

or briefly

$$\Sigma\frac{\mathrm{W}_1}{u}=0\,.$$

Let us form also

$$\mathrm{S}\,.\,\phi^{-1}i\psi'\zeta_1=\mathrm{S}\,.\,\zeta_1\psi(\phi^{-1}i)\,.$$

We have

$$\psi(\phi^{-1}i)=\psi_0(\phi^{-1}i)+\psi_1(\phi^{-1}i)\,.$$

First,

$$\psi_0(\phi^{-1}i)=-\frac{j}{v}\mathrm{S}\phi^{-1}i\,.\,j\phi^{-1}j-\frac{k}{w}\mathrm{S}\phi^{-1}i\,.\,k\phi^{-1}k\,.$$

Applying

$$\mathrm{V}.\,\phi^{-1}\lambda\phi^{-1}\mu=\frac{1}{m}\phi\mathrm{V}\lambda\mu$$

and

$$\mathrm{S}j\phi^{-1}j\phi^{-1}i=-\frac{1}{m}\mathrm{S}j\phi k,\ \&c.,$$

we get

$$\psi_0(\phi^{-1}i)=\frac{x_0}{m}\left(\frac{j}{v}-\frac{k}{w}\right)\cdot$$

Secondly,

$$\psi(\phi^{-1}i)=\frac{1}{u}[\mathrm{V}\phi^{-1}i\phi^{-1}i+\phi^{-1}(\mathrm{V}i\phi^{-1}i)]+\frac{\phi^{-1}i}{u^3}\mathrm{S}\phi^{-1}i\,.\,i\phi^{-1}i$$

$$=\frac{1}{u}\phi^{-1}(\mathrm{V}i\phi^{-1}i)=\frac{1}{um}\mathrm{V}(\phi i\,.\,i)$$

$$=\frac{1}{um}[-j\mathrm{S}k\phi i+k\mathrm{S}j\phi i]=\frac{1}{um}[-jy_0+kz_0]\,.$$

Hence

$$m\psi(\phi^{-1}i)=j\left(\frac{x_0}{v}-\frac{y_0}{u}\right)+k\left(\frac{-x_0}{w}+\frac{z_0}{u}\right)\cdot$$

Having

$$\zeta_1=x_0\phi^{-1}i-x_1\phi i\,,$$

we get

$$\begin{aligned}m\mathrm{S}\zeta_1\psi\phi^{-1}i=x_0&\left\{z_1\left(\frac{x_0}{v}-\frac{y_0}{u}\right)+y_1\left(\frac{-x_0}{w}+\frac{z_0}{u}\right)\right\}\\ -x_1&\left\{z_0\left(\frac{x_0}{v}-\frac{y_0}{u}\right)+y_0\left(\frac{-x_0}{w}+\frac{z_0}{u}\right)\right\}\\ =\frac{x_0}{u}(y_1z_0-y_0z_1)&\qquad=\ \frac{x_0}{u}(-\mathrm{W}_1)\\ +\frac{x_0}{v}(x_0z_1-x_1z_0)&\qquad+\ \frac{x_0}{v}(-\mathrm{W}_2)\\ +\frac{x_0}{w}(x_1y_0-x_0y_1)&\qquad+\ \frac{x_0}{w}(-\mathrm{W}_3).\end{aligned}$$

Hence

$$m\mathrm{S}\xi_1\psi(\phi^{-1}i)=-x_0\Sigma\frac{\mathrm{W}_1}{u}\cdot$$

By the juxtaposition of ψi and $\psi(\phi^{-1}i)$,

$$\psi i = j\left(\frac{y_1}{u} - \frac{x_1}{v}\right) + k\left(\frac{x_1}{w} - \frac{z}{u}\right)$$

$$m\psi(\phi^{-1}i) = j\left(\frac{x_0}{v} - \frac{y_0}{u}\right) + k\left(-\frac{x_1}{w} + \frac{z_0}{u}\right).$$

we remark that we have

$$\begin{aligned} x_0\psi i + mx_1\psi(\phi^{-1}i) &= j\left[x_0\left(\frac{y_1}{u} - \frac{x_1}{v}\right) + x_1\left(\frac{x_0}{v} - \frac{y_0}{u}\right)\right] \\ &\quad + k\left[x_0\left(x_1 - \frac{z_1}{u}\right) + x_1\left(-\frac{x_1}{\omega} + \frac{z_0}{u}\right)\right] \\ &= \left[\frac{j}{u}(x_0y_1 - x_1y_0 +)\frac{k}{u}(x_1z_0 - x_0z_1)\right] \\ &= \frac{1}{u}(jW_3 + kW_2) \\ &= \frac{1}{u}\zeta_0 . \end{aligned}$$

This accounts for the fact that

$$S\zeta_1\zeta_0 = uS\zeta_1\left[x_0\psi i + mx_1\psi(\phi^{-1}i)\right]$$

becomes identically zero, because ζ_1 and ζ_0 are perpendicular to one another by the definition of ζ_1. Hence

$$S\zeta_1\psi(\phi^{-1}i) = -\frac{x_0}{x_1m}S\zeta_1\psi i = 0 .$$

It follows that for any line ω directed in the plane of i and of $\phi^{-1}i$, namely in the plane normal to the ellipsoid at the extremity of ρ_1 and containing the vector ρ_1, the results of

$$S.\omega\psi'(\zeta_1) = 0$$

will solely give the equation

$$\Sigma\frac{W_1}{u} = 0 .$$

§ XI.

Let us now form the result

$$S.\phi i\psi'\zeta_1 = S.\zeta_1\psi(\phi i) = 0 .$$

As ϕi differs generally in direction from i and from $\phi^{-1}i$ we may expect an equation distinct from the preceding one.

We have

$$\begin{aligned} \psi_0(\phi i) = &-\frac{i}{u}S\phi i.i\phi^{-1}i \\ &-\frac{j}{v}S\phi i.j\phi^{-1}j \\ &-\frac{k}{w}S\phi i.k\phi^{-1}k . \end{aligned}$$

$$\psi_1(\phi i) = \frac{1}{u} \mathrm{V}(\phi i \phi^{-1} i) + \frac{1}{u} \phi^{-1} \mathrm{V} i \phi i$$

$$+ \frac{1}{u^3} \phi^{-1} i \mathrm{S} \phi i . i \phi^{-1} i .$$

Treating first $\psi_0(\phi i)$ by $\mathrm{S}.\zeta_1$, the coefficient of $\mathrm{S}\zeta_1 i$ will be

$$= -\frac{1}{u} \mathrm{S}\phi i . i \phi^{-1} i = +\frac{\mathrm{W}}{u} .$$

Then, remarking that we have

$$\zeta_1 = x_0 \phi^{-1} i - x_1 \phi i ,$$

$$\mathrm{S}\zeta_1 j = x_0 z_1 - x_1 z_0 = -\mathrm{W}_2 ,$$

$$\mathrm{S}\zeta_1 k = x_0 y_1 - x_1 y_0 = +\mathrm{W}_3 ,$$

we get

$$\mathrm{S}\zeta_1 \psi_0 \phi i = \mathrm{S}\zeta_1 i \cdot \frac{\mathrm{W}_1}{u} + \frac{\mathrm{W}_2}{v} \mathrm{S}(\phi i . j \phi^{-1} j)$$

$$- \frac{\mathrm{W}_3}{w} \mathrm{S}.(\phi i . k \phi^{-1} k) .$$

We will at once eliminate $\frac{\mathrm{W}_3}{w}$ by $\Sigma \frac{\mathrm{W}_1}{u} = 0$,

taking

$$-\frac{\mathrm{W}_3}{w} = \frac{\mathrm{W}_1}{u} + \frac{\mathrm{W}_2}{v} .$$

Thus

$$\mathrm{S}\zeta_1 \psi (\phi i) = \frac{\mathrm{W}_1}{u} [\mathrm{S}\zeta_1 i + \mathrm{S} k \phi^{-1} k \phi i]$$

$$+ \frac{\mathrm{W}_2}{v} [\mathrm{S} j \phi^{-1} j \phi i + \mathrm{S} k \phi^{-1} k \phi i] .$$

As ϕ^{-1} is self-conjugate we have for the coefficient of $\frac{\mathrm{W}_2}{v}$

$$\mathrm{S}(j \phi^{-1} j + k \phi^{-1} k) \phi i = -\mathrm{S} i \phi^{-1} i . \phi i = +\mathrm{W}_1 .$$

Also

$$\mathrm{S}\zeta_1 i + \mathrm{S} k \phi^{-1} k \phi i = \mathrm{S} i (x_0 \phi^{-1} i - x_1 \phi i)$$

$$+ \mathrm{S}.\mathrm{V} i j \mathrm{V} \phi^{-1} k \phi i$$

$$= x_0 \mathrm{S} i \phi^{-1} i - x_1 \mathrm{S} i \phi i + \mathrm{S} i \phi i . x_1 - \mathrm{S} j \phi i \mathrm{S} i \phi^{-1} k$$

$$= \mathrm{S} k \phi j \mathrm{S} i \phi^{-1} i - \mathrm{S} i \phi j \mathrm{S} k \phi^{-1} i$$

$$= \mathrm{S}.\mathrm{V} k i \mathrm{V} \phi^{-1} i \phi j$$

$$= -\mathrm{S} j \phi j \phi^{-1} i$$

We have thus, first,

$$\mathrm{S}\zeta_1 \psi_0 (\phi i) = -\frac{\mathrm{W}_1}{u} \mathrm{S} j \phi j \phi^{-1} i$$

$$+ \mathrm{W}_1 \frac{\mathrm{W}_2}{v} .$$

Secondly, in $S.\zeta_1\psi_1\phi i$ the first term of $\psi_1\phi i$ disappears, because $\zeta_1 = x_0\phi^{-1}i - x_1\phi i$. We get

$$S\zeta_1\psi_1(\phi i) = \frac{1}{u}S.\phi^{-1}\zeta_1\left[V i\phi i - \frac{iW_1}{u^2}\right].$$

Taking the ϕ^{-1} of the expression of ζ_1

$$S\zeta_1\psi_1(\phi i) = \frac{1}{u^3}S(x_0\phi^{-2}i - x_1 i)(V.i\phi i u^2 - iW_1)$$

$$= \frac{1}{u^3}[x_0[u^2 Si\phi i\phi^{-2}i - W_1 Si\phi^{-2}i] + x_1 W_1 i^2].$$

In the coefficient of x_0 we have

$$Si\phi i\phi^{-2}i = \frac{1}{m}Si\phi i[m_1\phi^{-1}i - m_2 i + \phi i]$$

$$= \frac{m_1}{m}W_1,$$

$$Si\phi^{-2}i = \frac{1}{m}Si[m_1\phi^{-1}i - m_2 i + \phi i]$$

$$= \frac{m_1 u^2}{m} + \frac{m_2}{m} + \frac{Si\phi i}{m}.$$

This gives

$$S\zeta_1\psi_1(\phi i) =$$

$$= \frac{1}{u^3}\left\{ x_0\left[u^2\frac{m_1}{m}W_1 - W_1\left(\frac{m_1 u^2}{m} + \frac{m_2}{m} + \frac{Si\phi i}{m}\right)\right] - x_1 W_1 \right\}$$

$$= -\frac{W_1}{u^3}\left\{\frac{x_0}{m}(m_2 + Si\phi i) + x_1\right\}.$$

Summing together $S\zeta_1(\psi_0 + \psi_1)\phi i$ we have the factor W_1 affecting all the terms, and the equation

$$S\zeta_1\psi(\phi i) = 0,$$

in which for abbreviating we put

$$\frac{x_0}{m}(m_2 + Si\phi i) + x_1 = X$$

becomes

$$\frac{W_1}{u^3}\left[\frac{W_2 u^3}{v} - u^2 Sj\phi j\phi^{-1}i - X\right] = 0.$$

If, instead of eliminating $\frac{W_3}{w}$, we had eliminated $\frac{W_2}{v}$ from $S\zeta_1\psi_0(\phi i)$, we would have got

$$\frac{W_1}{u^3}\left[-\frac{W_3}{w}u^3 - W_1 u^2 - u^2 Sj\phi j\phi^{-1}i - X\right] = 0.$$

But as we have

$$-[W_1 + Sj\phi j\phi^{-1}i] = -[S.(Vi\phi i + Vj\phi j)\phi^{-1}$$

$$= +S.k\phi k\phi^{-1}i,$$

the last but one equation becomes

$$\frac{W_1}{u^3}\left[-\frac{W_3}{w}u^3+u^2Sk\phi k\phi^{-1}i-X\right]=0.$$

As the factor $W_1=-a_1b_1c_1\Lambda$ vanishes only for particular directions of α, β, γ, the general solution will be either

$$\frac{W_2}{v}u^3=Y=u^2Sj\phi j\phi^{-1}i+X,$$

or

$$\frac{W_3}{w}u^3=Z=u^2Sk\phi k\phi^{-1}i-X.$$

But the equation $\Sigma\frac{W_1}{u}=0$ is a consequence of these two, because

$$\begin{aligned}(Y+Z)&=u^2S(Vj\phi j+Vk\phi k)\phi^{-1}i\\&=-u^2Si\phi i\phi^{-1}=-W_1u^2;\end{aligned}$$

we may therefore look upon the expressions of $\frac{W_2u^3}{v}$ and $\frac{W_3u^3}{w}$ as constituting the two equations sought for.

§ XII.

If we express the quantities Y and Z in function of the nine coefficients a_1, b_1, c_1, a_2, b_2, &c., we will find that the expressions contain the same constant factor Λ which we have found to affect the quantities W_1, W_2, W_3.

First as to X we have

$$x_1=Sj\phi^{-1}k=\Sigma a^2a_2a_3$$

$$x_0=Sj\phi k=\Sigma\frac{a_2a_3}{a^2}$$

$$Si\phi i=\Sigma\frac{a_1}{a^2}$$

$$m_2=-\Sigma\frac{1}{a^2},\quad m=\frac{-1}{a^2b^2c^2}.$$

$$\begin{aligned}X&=x_1+\frac{x_0}{m}(m_2+Si\phi i)\\&=\Sigma a_2a_3\left\{a^2+\frac{a^2b^2c^2}{a^2}\times\Sigma\left(\frac{1}{a^2}-\frac{a_1^2}{a^2}\right)\right\}\\&=\Sigma a_2a_3\left\{a^2+b^2c^2\left[\frac{1-a_1^2}{a^2}+\frac{1-b_1^2}{b^2}+\frac{1-c_1^2}{c^2}\right]\right\}\\&=\Sigma a_2a_3\left\{a^2+\frac{b^2c^2}{a^2}(1-a_1^2)+c^2(1-b_1^2)+b^2(1-c_1^2)\right\}\\&=\Sigma a_2a_3\left\{a^2+b^2+c^2+\frac{b^2c^2}{a^2}(1-a_1^2)+(-c^2b_1^2-b^2c_1^2)\right\}.\end{aligned}$$

As $\Sigma a_2 a_3 = 0$, the term in $a^2+b^2+c^2$ disappears, and as $a_1{}^2+b_1{}^2+c_1{}^2=1$, we get

$$X = \Sigma a_2 a_3\left[b_1{}^2\left(\frac{b^2c^2}{a^2}-c^2\right)+c_1{}^2\left(\frac{b^2c^2}{a^2}-b^2\right)\right]$$

$$X = \Sigma a_2 a_3 b_1{}^2 \times c^2\left(\frac{b^2}{a^2}-1\right)+\Sigma a_2 a_3 c_1{}^2 \times b^2\left(\frac{c^2}{a^2}-1\right).$$

The first term may be written

$$= \Sigma a_2 a_3 b^2 c^2\left(\frac{1}{a^2}-\frac{1}{b^2}\right)b_1{}^2\,;$$

the second term

$$= \Sigma a_2 a_3 b^2 c^2\left(\frac{1}{a^2}-\frac{1}{c^2}\right)c_1{}^2\,.$$

By circular permutation of a, b, c, a_1, &c., we move the letters one step forward in the Σ of the first term (a into b, b into c, &c.), and one step backward in the Σ of the second term (a into c, b into a, &c.), the terms become respectively,

$$\Sigma b_2 b_3 c^2 a^2\left(\frac{1}{b^2}-\frac{1}{c^2}\right)c_1{}^2\,,$$

and

$$\Sigma c_2 c_3 a^2 b^2\left(\frac{1}{c^2}-\frac{1}{b^2}\right)b_1{}^2\,.$$

Thus we get:

$$X = \Sigma a^2\left(\frac{1}{b^2}-\frac{1}{c^2}\right)[b_2 b_3 c_1{}^2 c^2 - c_2 c_3 b_1{}^2 b^2]\,.$$

Thus far as to X. For the other term in $Y = u^2 Sj\phi j\phi^{-1}i + X$ we have

$$V.\phi j\phi^{-1}i = \left(\frac{\alpha a_2}{a^2}+\frac{\beta b_2}{b^2}+\frac{\gamma c_2}{c^2}\right)(\alpha a^2 a_1 + \beta b^2 b_1 + \gamma c^2 c_1)\,.$$

Applying $\beta\gamma = \alpha$, $\gamma\beta = -\alpha$, &c., we get

$$\begin{aligned} V\phi j\phi^{-1}i = {}& \alpha\left(\frac{c^2}{b^2}b_2 c_1 - \frac{b^2}{c^2}c_2 b_1\right) \\ & +\beta\left(\frac{a^2}{c^2}c_2 a_1 - \frac{c^2}{a^2}a_2 c_1\right) \\ & +\gamma\left(\frac{b^2}{a^2}a_2 b \; - \frac{a^2}{b^2}b_2 a_1\right). \end{aligned}$$

Treating by $S.j$, and applying $S\alpha j = -a_2$, $S\beta j = -b_2$, $S\gamma j = -c_2$,

$$\begin{aligned} Sj\phi j\phi^{-1}i = {}& -\frac{c^2}{b^2}a_2 b_2 c_1 + \frac{b^2}{c^2}c_2 a_2 b_1 \\ & -\frac{a^2}{c^2}b_2 c_2 a_1 + \frac{c^2}{a^2}a_2 b_2 c_1 \\ & -\frac{b^2}{a^2}c_2 a_2 b_1 + \frac{a^2}{b^2}b_2 c_2 a_1 \\ = {}& \Sigma b_2 c_2 a_1\left(\frac{a^2}{b^2}-\frac{a^2}{c^2}\right) \\ = {}& \Sigma a^2\left(\frac{1}{b^2}-\frac{1}{c^2}\right)\cdot a_1 b_2 c_2\,. \end{aligned}$$

We have now for $\mathrm{Y}=u^2\mathrm{S}j\phi j\phi^{-1}i+\mathrm{X}$

$$\mathrm{Y}=\Sigma a^2\left(\frac{1}{b^2}-\frac{1}{c^2}\right)[u^2a_1b_2c_2+c^2c_1{}^2b_2b_3-b^2b_1{}^2c_2c_3].$$

We have

$$u^2=a^2a_1{}^2+b^2b_1{}^2+c^2c_1{}^2 .$$

Substituting this into the [ ] we get

$$\begin{aligned}&a^2a_1{}^3b_2c_2\\&+b^2b_1{}^2a_1b_2c_2-b^2b_1{}^2c_2c_3\\&+c^2c_1{}^2a_1b_2c_2+c^2c_1{}^2b_2b_3 .\end{aligned}$$

The terms in the second line are

$$=b^2b_1{}^2c_2(a_1b_2-c_3) ,$$

and as

$$-c_3=\mathrm{S}\gamma k=\mathrm{SV}a\beta\mathrm{V}ij=\mathrm{S}aj\mathrm{S}\beta i-\mathrm{S}ai\mathrm{S}\beta j=a_2b_1-a_1b_2$$

we get

$$a_1b_2-c_3=a_2b_1 ,$$

so that the terms in b^2 become

$$b^2b_1{}^3c_2a_2 .$$

The terms of the third line are

$$c^2c_1{}^2b_2(a_1c_2+b_3),$$

and as

$$\begin{aligned}b_3&=-\mathrm{S}\beta k=\mathrm{S}a\gamma ij\\&=\mathrm{S}aj\mathrm{S}\gamma i-\mathrm{S}ai\mathrm{S}\gamma j\\&=a_2c_1-a_1c_2 ,\end{aligned}$$

hence

$$a_1c_2+b_3=a_2c_1 ,$$

the terms in c^2 become

$$c^2c_1{}^3b_2a_2 .$$

We have now

$$\mathrm{Y}=\Sigma a^2\left(\frac{1}{b^2}-\frac{1}{c^2}\right)(a^2a_1{}^3b_2c_2+b^2b_1{}^3c_2a_2+c^2c_1{}^3a_2b_2)$$

And if we put

$$\mathrm{Y}'=\Sigma a^2a_1{}^3b_2c_2 ,$$

and consider that by § VIII. we have

$$\Sigma a^2\left(\frac{1}{b^2}-\frac{1}{c^2}\right)=-\Lambda ,$$

namely, there we have

$$(+\Lambda)=\frac{-a^2}{b^2}+\frac{a^2}{c^2}-\frac{b^2}{c^2}+\frac{b^2}{a^2}-\frac{c^2}{a^2}+\frac{c^2}{b^2} ,$$

we have thus

$$\mathrm{Y}=-\mathrm{Y}'\Lambda .$$

With the notation already indicated in § VIII. as to W_h, the equation

$$\frac{u^3\mathrm{W}_2}{v}-\mathrm{Y}=0$$

becomes now

$$\left(Y' - \frac{u^3 W_2'}{v}\right)\Lambda = 0 .$$

The equation $\frac{u^3 W_3}{w} - Z = 0$ may be easily transformed by the help of the preceding one.

We have, namely, by § XI.,

$$Y + Z = -u^2 W_1 .$$

This gives

$$\begin{aligned} Z &= (u^2 W_1' + Y')\Lambda \\ Z &= \Sigma(a^2 a_1{}^2 a_1 b_1 c_1 + a^2 a_1{}^3 b_2 c_2)\Lambda \\ &= \Sigma a^2 a_1{}^3 (b_1 c_1 + b_2 c_2)\Lambda \\ &= -(\Sigma a^2 a_1{}^3 b_3 c_3)\Lambda . \end{aligned}$$

We put

$$Z' = \Sigma a^2 a_1{}^3 b_3 c_3 ;$$

then the equation in question becomes

$$\left[Z' - \frac{u^3 W_3'}{w}\right]\Lambda = 0 .$$

Generally we treat the case in which a^2, b^2, c^2 are different from one another. The equations are therefore

$$Y' = \frac{u^3}{v} W_2' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (I.)$$

$$Z' = \frac{u^3}{w} W_3' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (II.)$$

In order to render these equations rational in respect to S*ai*, S*aj*, &c., we require only *one* squaring of both members in each equation. This is not the case with the equation $\Sigma \frac{W_1}{u} = 0$.

We may put the two equations under the form

$$\begin{aligned} \frac{u^3}{v} &= \Sigma \frac{a^2 a_1{}^3}{a_2} \\ \frac{u^3}{w} &= \Sigma \frac{a^2 a_1{}^3}{a_3} , \end{aligned}$$

and remark as a curiosity the relation drawn, as, for example, from the first

$$\sqrt{\frac{(\Sigma a^2 a_1{}^2)^3}{(\Sigma a^2 a_2{}^2)}} = \Sigma\left[\frac{(a a_1)^3}{(a a_2)}\right] ,$$

so that the second member expresses a certain mean value of the terms appearing in the first member.

§ XIII.

We are now able to eliminate v and w from the expression of ρ. We have, namely,

$$\rho=\theta+\frac{\phi^{-1}i}{u}=jy+kz\,.$$

We have also

$$\phi^{-1}i=-i\mathrm{S}i\phi^{-1}i-j\mathrm{S}j\phi^{-1}i-k\mathrm{S}k\phi^{-1}i;$$

hence with

$$\theta=iu+jv+kw,$$

we get

$$\rho=iu+jv+kw-iu-j\frac{z_1}{u}-k\frac{y_1}{u}=jy+kz\,.$$

We deduce

$$y=v-\frac{z_1}{u}\,,$$

$$z=w-\frac{y_1}{u}\,.$$

Multiplying respectively by

$$u\mathrm{Y}',\ \ u\mathrm{Z}',$$

and applying (I.), (II.), we get

$$yu\mathrm{Y}'=uv\mathrm{Y}'-z_1\mathrm{Y}'=u^4\mathrm{W}_2{}'-z_1\mathrm{Y}'$$

$$zu\mathrm{Z}'=uw\mathrm{Z}'-y_1\mathrm{Z}'=u^4\mathrm{W}_3{}'-y_1\mathrm{Z}'.$$

Having

$$z_1=\Sigma a^2a_1a_2,\ \ y_1=\Sigma a^2a_3a_1,$$

and

$$u^2=\Sigma a^2a_1{}^2,$$

we may write

$$yu\mathrm{Y}'=\Sigma a^2a_1[u^2\mathrm{W}_2{}'a_1-\mathrm{Y}'a_2]$$

$$zu\mathrm{Z}'\ =\Sigma' a^2a_1[u^2\mathrm{W}_3{}'a_1-\mathrm{Z}'a_3]\,.$$

Of the factors between [ ], the first is

$$a_2\begin{bmatrix}a^2a_1{}^3b_2c_2+b^2a_1b_1{}^2b_2c_2+c^2a_1c_1{}^2b_2c_2\\ -a^2a_1{}^3b_2c_2-b^2b_1{}^3c_2a_2-c^2c_1{}^3a_2b_2\end{bmatrix}$$

$$=a_2[b^2b_1{}^2c_2(a_1b_2-b_1a_2)+c^2c_1{}^2b_2(a_1c_2-a_2c_1)],$$

and the corresponding factor in $zu\mathrm{Z}'$ is the same with the index 2 changed into 3,

$$=a_3[b^2b_1{}^2c_3(a_1b_3-b_1a_3)+c^2c_1{}^2b_3(a_1c_3-a_3c_1)].$$

But

$$a_1b_2-b_1a_2=\mathrm{S}\alpha i\mathrm{S}\beta j-\mathrm{S}\beta i\mathrm{S}\alpha j=\mathrm{S}\alpha\beta ji=-\mathrm{S}\gamma k=c_3;\ \ \text{\&c., \&c.}$$

The factors are respectively

$$a_2[b^2b_1{}^2c_2c_3-c^2c_1{}^2b_2b_3]$$

and

$$a_3[-b^2b_1{}^2c_3c_2+c^2c_1{}^2b_3b_2].$$

This gives

$$yu\mathrm{Y}'=\Sigma a^2a_1a_2[b^2b_1{}^2c_2c_3-c^2c_1{}^2b_2b_3]$$

$$zu\mathrm{Z}'\ =\Sigma a^2a_1a_3[-b^2b_1{}^2c_2c_3+c^2c_1{}^2b_2b_3].$$

If in the second terms we permute the letters one step forward in the series a, b, c, a, &c., the terms become respectively

$$\begin{aligned} &-\Sigma a^2b^2b_1b_2{a_1}^2c_2c_3 \\ &+\Sigma a^2b^2b_1b_3{a_1}^2c_2c_3 \,. \end{aligned}$$

Joining them to the first terms we have

$$\begin{aligned} yu\mathrm{Y}' &= \Sigma a^2b^2a_1b_1c_2c_3[b_1a_2-a_1b_2] \\ zu\mathrm{Z}' &= \Sigma a^2b^2a_1b_1c_2c_3(-b_1a_3+a_1b_3]. \end{aligned}$$

And as

$$\begin{aligned} b_1a_2-a_1b_2 &= \mathrm{S}\beta a j i = \mathrm{S}\gamma k = -c_3 ; \\ -b_1a_3+a_1b_3 &= \mathrm{S}a\beta k i = \mathrm{S}\gamma j = -c_2 , \end{aligned}$$

we get

$$\begin{aligned} yu\mathrm{Y}' &= -\Sigma a^2b^2a_1b_1c_2{c_3}^2 = -\mathrm{Y}_1 \,, \\ zu\mathrm{Z}' &= -\Sigma a^2b^2a_1b_1{c_2}^2c_3 = -\mathrm{Z}_1 \,. \end{aligned}$$

We may remark that the second members may be put under the form

$$\begin{aligned} &+\Sigma a^2b^2[a_2b_2+a_3b_3]c_2{c_3}^2 \quad \text{(in the case of the first equation)} \\ &= a^2b^2c^2\left[\mathrm{W}_2{}'\Sigma\frac{{c_3}^2}{c^2}+\mathrm{W}_3{}'\Sigma\frac{c_2c_3}{c^2}\right] \end{aligned}$$

so that

$$yu\mathrm{Y}' = a^2b^2c^2[\mathrm{W}_2{}'\mathrm{S}k\phi k+\mathrm{W}_3{}'\mathrm{S}j\phi k].$$

Likewise in the case of the second equation

$$zu\mathrm{Z}' = a^2b^2c^2\Sigma[\mathrm{W}_2{}'\mathrm{S}k\phi j+\mathrm{W}_3{}'\mathrm{S}j\phi j].$$

Hence dividing the first by $\mathrm{W}_2{}'$, the second by $\mathrm{W}_3{}'$, and remarking that

$$\frac{\mathrm{Y}'}{\mathrm{W}_2{}'} = \frac{u^3}{v} \,, \quad \frac{\mathrm{Z}'}{\mathrm{W}_3{}'} = \frac{u^3}{w} \,,$$

we get

$$\begin{aligned} y &= \frac{v}{u^4}a^2b^2c^2\left[\mathrm{S}k\phi k+\frac{\mathrm{W}_3{}'}{\mathrm{W}_2{}'}\mathrm{S}j\phi k\right] \\ z &= \frac{w}{u^4}a^2b^2c^2\left[\mathrm{S}j\phi j+\frac{\mathrm{W}_2{}'}{\mathrm{W}_3{}'}\mathrm{S}k\phi j\right] . \end{aligned}$$

As

$$\frac{dz}{dy} = \frac{\mathrm{W}_2{}'}{\mathrm{W}_3{}'}$$

we may also write these equations under the form

$$\begin{aligned} y\mathrm{W}_2{}'dt^{\mathrm{iv}} &= v\mathrm{S}.d\rho\phi k \,, \\ z\mathrm{W}_3{}'dt^{\mathrm{iv}} &= w\mathrm{S}.d\rho\phi j \end{aligned}$$

dt^{iv} being a convenient infinitesimal scalar.

§ XIV.

To express the nine coefficients a_1, b_1, c_1, a_2, &c., which determine the position of the system α, β, γ, we adopt for α the angle A which α forms with i and the angle B which the projection of α on the plane (j, k) forms with j. Thus we have

$$\alpha = i \cos \mathrm{A} + j \cos \mathrm{B} \sin \mathrm{A} + k \sin \mathrm{B} \sin \mathrm{A}\,.$$

For abbreviation's sake, we put

$$\cos \mathrm{A} = a_0,\ \sin \mathrm{A} = a'\,;$$
$$\cos \mathrm{B} = b_0,\ \sin \mathrm{B} = b'\,.$$

Then comparing

$$\alpha = ia_1 + ja_2 + ka_3\,,$$
$$\alpha = ia_0 + ja'b_0 + ka'b'\,,$$

we deduce

$$a_1 = a_0\,,\ a_2 = a'b_0\,,\ a_3 = a'b'\,.$$

Calling β_0 and γ_0 the directions forming with α a three-rectangular system in such a position that β_0 be coplanar with α, and i, we have then

$$\beta_0 = \gamma_0 \alpha\,,$$

and γ_0 being perpendicular to i and to α we have

$$\mathrm{S}\gamma_0 i = 0\,,\ \mathrm{S}\gamma_0 \alpha = 0\,,$$

hence

$$n\gamma_0 = \mathrm{V}i\alpha$$

with

$$\gamma_0^2 = -1\,.$$

Thus

$$-n^2 = \mathrm{V}^2 i\alpha = \mathrm{S}^2 i\alpha - i^2\,.\,\alpha^2\,,$$

hence

$$-n^2 = a_0^2 - 1 = -a'^2\,.$$

We put

$$n = a'\ (\text{not} = -a').$$

Thence

$$a'\gamma_0 = \mathrm{V}i\alpha = ka'b_0 - ja'b'\,,$$
$$\gamma_0 = -jb' + kb_0\,.$$

This gives

$$\beta_0 = (-jb' + kb_0)[ia_0 + a'(jb_0 + kb')]\,,$$

hence

$$\beta_0 = kb'a_0 - ib'^2a' + jb_0a_0 - ia'b_0^2\,,$$

namely,

$$\beta_0 = -ia' + jb_0a_0 + kb'a_0\,.$$

We now turn the system α, β_0, γ_0 round α as axis and to the amount of an angle C, and we shall have

$$\beta = p'\beta_0 q'\,,\ \gamma = p'\gamma_0 q',$$

where

$$p' = \cos \tfrac{1}{2}\mathrm{C} + \alpha \sin \tfrac{1}{2}\mathrm{C}$$
$$q' = \cos \tfrac{1}{2}\mathrm{C} - \alpha \sin \tfrac{1}{2}\mathrm{C}\,.$$

This gives, leaving $\gamma_0\alpha$ in the place of β_0,

$$\beta=(\cos\tfrac{1}{2}C+\alpha\sin\tfrac{1}{2}C)\gamma_0\alpha(\cos\tfrac{1}{2}C-\alpha\sin\tfrac{1}{2}C)\,,$$
$$\gamma=(\cos\tfrac{1}{2}C+\alpha\sin\tfrac{1}{2}C)\gamma_0(\cos\tfrac{1}{2}C-\alpha\sin\tfrac{1}{2}C)\,.$$

Developing

$$\beta=\cos^2\tfrac{1}{2}C.\gamma_0\alpha-\sin^2\tfrac{1}{2}C(\alpha\gamma_0\alpha^2)$$
$$+\sin\tfrac{1}{2}C\cos\tfrac{1}{2}C(\alpha\gamma_0\alpha-\gamma_0\alpha^2)\,,$$
$$\gamma=\cos^2\tfrac{1}{2}C.\gamma_0-\sin^2\tfrac{1}{2}C(\alpha\gamma_0\alpha)$$
$$+\sin\tfrac{1}{2}C\cos\tfrac{1}{2}C(\alpha\gamma_0-\gamma_0\alpha)\,.$$

Having $S\gamma_0\alpha=0$ we deduce

$$\alpha\gamma_0\alpha^2=-\alpha\gamma_0=\gamma_0\alpha$$
$$\alpha\gamma_0\alpha=2\alpha S\gamma_0\alpha-\gamma_0\alpha^2=\gamma_0\,.$$

Hence

$$\beta=\gamma_0\alpha(\cos^2\tfrac{1}{2}C-\sin^2\tfrac{1}{2}C)+2\gamma_0\sin\tfrac{1}{2}C\cos\tfrac{1}{2}C\,.$$
$$\gamma=\gamma_0(\cos^2\tfrac{1}{2}C-\sin^2\tfrac{1}{2}C)+2\alpha\gamma_0\sin\tfrac{1}{2}C\cos\tfrac{1}{2}C\,.$$

Replacing

$$\beta_0 \text{ for } \gamma_0\alpha$$
$$-\beta_0 \text{ for } \alpha\gamma_0$$

we have

$$\beta=\beta_0\cos C+\gamma_0\sin C$$
$$\gamma=\gamma_0\cos C-\beta_0\sin C\,;$$

or putting also

$$\cos C=c_0\,,\ \sin C=c'\,,$$

and replacing for β_0, γ_0 their expressions, we get

$$\beta=(-ia'+jb_0a_0+kb'a_0)c_0$$
$$+(-jb'+kb_0)c',$$
$$\gamma=(-jb'+kb_0)c_0$$
$$+(ia'-ja_0b_0-ka_0b')c'\,.$$

We have thus

$$\beta=-ia'c_0+j(a_0b_0c_0-b'c')+k(a_0b'c_0+b_0c')$$
$$\gamma=ia'c'+j(-a_0b_0c'-b'c_0)+k(-a_0b'c'+b_0c_0)\,.$$

This gives us the table of values, including those derived from α:

$$\begin{array}{lll}
a_1=a_0\,, & a_2=a'b_0\,, & a_3=a'b' \\
b_1=-a'c_0\,, & b_2=a_0b_0c_0-b'c'\,, & b_3=a_0b'c_0+b_0c' \\
c_1=a'c'\,, & c_2=-a_0b_0c'-b'c_0\,, & c^3=-a_0b'c'+b_0c_0\,.
\end{array}$$

§ XV.

We will now express the quantities u^2, v^2, w^2, W_1', W_2', W_3', Y', Z', Y_1, Z_1 in function of the three angles A, B, C, and their dependents a_0, a', b_0, b', c_0, c'.

If we examine the values of a_2, b_2, c_2, and compare them to those of a_3, b_3, c_3, we see that to pass from an expression in which a_2, b_2, c_2 enter alone (exclusive of a_3, b_3, c_3) to the similar expression in a_3, b_3, c_3 we have only to change

$$b_0 \text{ into } b', \text{ and } b' \text{ into } (-b_0).$$

First

$$u^2=\Sigma a^2a_1{}^2=a^2a_0{}^2+a'^2(b^2c_0{}^2+c^2c'^2)$$

which is independent of b_0, b'. Then

$$\begin{aligned} v^2&=\Sigma a^2a_2{}^2\\ v^2&=a^2a'^2b_0{}^2+b^2(a_0b_0c_0-b'c')^2\\ &\quad+c^2(-a_0b_0c'-b'c_0)^2, \end{aligned}$$

namely,

$$\begin{aligned} v^2&=b_0{}^2[a^2a'^2+b^2a_0{}^2c_0{}^2+c^2a_0{}^2c'^2]\\ &\quad+2b_0b'[-b^2a_0c_0c'+c^2a_0c_0c']\\ &\quad+b'^2[b^2c'^2+c^2c_0{}^2]. \end{aligned}$$

Introducing

$$\begin{aligned} \cos 2\mathrm{B}&=\mathrm{B}_0\\ \sin 2\mathrm{B}&=\mathrm{B}', \end{aligned}$$

we have

$$\begin{aligned} 2b_0{}^2&=1+\mathrm{B}_0\\ 2b'^2&=1-\mathrm{B}_0. \end{aligned}$$

By analogy we introduce also

$$\cos 2\mathrm{C}=\mathrm{C}_0,\ \sin 2\mathrm{C}=\mathrm{C}',$$

but we will leave $c_0{}^2$, c'^2 as they are, putting only $2c_0c'=\mathrm{C}'$. Thus by multiplying both members of the expression of v^2 by 2,

$$\begin{aligned} 2v^2&=(1+\mathrm{B}_0)[a^2a'^2+a_0{}^2(b^2c_0{}^2+c^2c'^2)]\\ &\quad+(1-\mathrm{B}_0)[b^2c'^2+c^2c_0{}^2]\\ &\quad-\mathrm{B}'(b^2-c^2)a_0\mathrm{C}'. \end{aligned}$$

$$\begin{aligned} 2v^2&=[a^2a'^2+a_0{}^2(b^2c_0{}^2+c^2c'^2)+(b^2c'^2+c^2c_0{}^2)]\\ &\quad+\mathrm{B}_0[a^2a'^2+a_0{}^2(b^2c_0{}^2+c^2c'^2)-b^2c'^2-c^2c_0{}^2]\\ &\quad-\mathrm{B}'(b^2-c^2)a_0\mathrm{C}'. \end{aligned}$$

By the above-mentioned change of b_0 into b', b' into $-b_0$ we get

$$\begin{aligned} \mathrm{B}_0&=b_0{}^2-b'^2 \text{ changed into } b'^2-b_0{}^2=-\mathrm{B}_0,\\ \mathrm{B}'&=2b_0b' \text{ changed into } -2b'b_0=-\mathrm{B}'. \end{aligned}$$

Putting

$$\begin{aligned} &[a^2a'^2+a_0{}^2(b^2c_0{}^2+c^2c'^2)+b^2c'^2+c^2c_0{}^2]=x_0\\ &\mathrm{B}_0[a^2a'^2+a_0{}^2(b^2c_0{}^2+c^2c'^2)-(b^2c'^2+c^2c_0{}^2)]-\mathrm{B}'(b^2-c^2)a_0\mathrm{C}'=x', \end{aligned}$$

we have at once

$$\begin{aligned} 2v^2&=(x_0+x')\\ 2w^2&=(x_0-x'). \end{aligned}$$

We remark that by the value of u^2 and by

$$a_0^2+a'^2=1,\ c_0^2+c'^2=1,$$

we have

$$u^2+x_0=a^2a_0^2+a'^2(b^2c_0^2+c^2c'^2)$$
$$+a^2a'^2+a_0^2(b^2c_0^2+c^2c'^2)+b^2c'^2+c^2c_0^2,$$
$$u^2+x_0=a^2+b^2+c^2=l_1.$$

Then

$$W_2=a_2b_2c_2$$
$$W_2=a'b_0(a_0b_0c_0-b'c')(-a_0b_0c'-b'c_0)$$
$$=a'b_0[-a_0^2b_0^2c_0c'+b'^2c_0c'+a_0b_0b'(c'^2-c_0^2)]$$
$$4W_2=a'b_0[-a_0^2(1+B_0)C'+(1-B_0)C'-2a_0B'C_0]$$
$$=a'b_0[a'^2C'+B_0C'(-1-a_0^2)-2B'a_0C_0].$$

If we put

$$W'=B_0C'(1+a_0^2)+2B'a^0C_0,$$

and by analogy,

$$W_0=a'^2C',$$

we have

$$4W_2'=a'b_0(W_0-W')$$
$$4W_3=a'b'(W_0+W').$$

As to $W_1'=a_1\ b_1\ c_1$ it is independent of B_0, B', namely, we get

$$4W_1'=-2a_0a'^2C'.$$

For the calcul of $Y'Z'$, we may prepare the following values. First, we have already

$$4b_2c_2=a'^2C'-W',$$
$$4b_3c_3=a'^2C'+W'$$

Secondly,

$$c_2a_2=-(a_0b_0c'+b'c_0)a'b_0$$
$$=-a'(b_0^2a_0c'+b_0b'c_0)$$
$$4c_2a_2=-2a'[(1+B_0)a_0c'+B'c_0]$$
$$4c_2a_2=(-2a')[a_0c'+(B_0c'a_0+B'c_0)]$$
$$4c_3a_3=(-2a')[a_0c'-(B_0c'a_0+B'c_0)].$$

Thirdly,

$$a_2b_2=a'b_0(a_0b_0c_0-b'c')$$
$$=\tfrac{1}{2}a'[a_0c_0(1+B_0)-B'c'].$$
$$4a_2b_2=(2a')[a_0c_0+(B_0a_0c_0-B'c')]$$
$$4a_3b_3=(2a')[a_0c_0-(B_0a_0c_0-B'c')].$$

Substituting into

$$Y'=a^2a_1^3b_2c_2+b^2b_1^3c_2a_2+c^2c_1^3a_2b_2$$

we get

$$4Y'=a^2a_0^3.[a'^2C'-W']$$
$$+b^2(-a\ \ c_0^3)(-2a')[a_0c'+B_0c'a_0+B'c_0]$$
$$+c^2(a'^3c'^3)(2a')[a_0c_0+B_0a_0c_0-B'c'].$$

Grouping the terms independent of B_0, B' separately from those which are dependent on B_0, B', and putting

$$4Y' = X_0 + X'$$
$$4Z' = X_0 - X' ,$$

we have (remarking $2c_0c' = C'$),

$$\begin{aligned} X_0 &= a^2a_0^3a'^2C' \\ &\quad + b^2a'^4a_0C'c_0^2 \\ &\quad + c^2a'^4a_0C'c'^2 \\ X_0 &= a'^2a_0C'[a^2a_0^2 + a'^2(b^2c_0^2 + c^2c'^2)] \\ X_0 &= -2W_1'u^2 , \end{aligned}$$

and

$$\begin{aligned} X' = &- a^2a_0^3W' \\ &+ b^2a'^4c_0^2[B_0a_0C + 2B'c_0^2] \\ &+ c^2a'^4c'^2[B_0a_0C' - 2B'c'^2] . \end{aligned}$$

Replacing

$$W' = B_0C'(1 + a_0^2) + 2B'a_0C_0$$

we get

$$\begin{aligned} X' = B_0a_0C'[&-a^2a_0^2(1 + a_0^2) + a'^4(b^2c_0^2 + c^2c'^2)] \\ &+ 2B'[-a^2a_0^4C_0 + a'^4(b^2c_0^4 - c^2c'^4)] . \end{aligned}$$

Putting

$$b^2c_0^2 + c^2c'^2 = d$$
$$b^2c_0^4 - c^2c'^4 = e ,$$

we have

$$\begin{aligned} X_0 &= a_0C'[a^2a_0^2a'^2 + a'^4d] \\ X' &= B_0a_0C'[-a^2a_0^2(1 + a_0^2) + a'^4d] \\ &\quad + 2B'[-a^2a_0^4C_0 + a'^4e] . \end{aligned}$$

§ XVI.

The equations (I.) (II.) of § XII. put under the form

$$vY' - u^3W_2' = 0$$
$$wZ' - u^3W_3' = 0$$

when rendered rational as to u, v, w, and when multiplied by 2^5, are of course

$$\text{(I.)}' \qquad (2v^2) \times (2^4Y'^2) - u^6 . 2^5W_2'^2 = 0$$
$$\text{(II.)}' \qquad (2w^2) \times (2^4Z'^2) - u^6 . 2^5W_3'^2 = 0 .$$

The first becomes

$$(x_0 + x')(X_0 + X')^2 - u^6 2a'^2b_0^2(W_0 - W')^2 = 0 .$$

We group the terms so as to put those of even order in B_0, B' together, and those of uneven order together, replacing $2b_0^2$ by $1 + B_0$.

Thus we have for the first equation (I′) :

$$x_0(\mathrm{X}_0{}^2+\mathrm{X}'^2)+2x'\mathrm{X}_0\mathrm{X}'-u^6a'^2[\mathrm{W}_0{}^2+\mathrm{W}'^2-2\mathrm{B}_0\mathrm{W}'\mathrm{W}_0]$$
$$+x'(\mathrm{X}_0{}^2+\mathrm{X}'^2)+2x_0\mathrm{X}_0\mathrm{X}'-u^6a'^2[\mathrm{B}_0(\mathrm{W}_0{}^2+\mathrm{W}'^2)-2\mathrm{W}_0\mathrm{W}']=0\,.$$

The second equation (II′) will contain the same terms as the first, with this difference, that the terms in the second line will have changed signs because of the change of sign of x', X′ and W′, and of B_0.

Thus the terms of the first line will be = zero separately, and the terms of the second line will be = zero separately also.

The terms of the first line will be obtained by the sum

$$(v^2\mathrm{Y}'^2-u^6\mathrm{W}_2{}'^2)+(w^2\mathrm{Z}'^2-u^6\mathrm{W}_3{}'^2)=0\,. \quad . \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

the terms in the second line in question will be obtained by subtraction

$$(v^2\mathrm{Y}'^2-u^6\mathrm{W}_2{}'^2)-(w^2\mathrm{Z} \;-u^6\mathrm{W}_3{}'^2)=0\,. \quad . \quad . \quad . \quad . \quad . \quad \text{(IV.)}$$

By the application of

$$\mathrm{B}_0{}^2+\mathrm{B}'^2=1$$

we may transform the terms of order zero in (III.) or the terms of the first order in (IV.) in order to render the equations homogeneous as to B_0, B′. The equations will then be respectively of the forms

$$\mathrm{G}_0\mathrm{B}_0{}^2+\mathrm{G}_1\mathrm{B}_0\mathrm{B}'+\mathrm{G}_2\mathrm{B}'^2=0\,,$$
$$\mathrm{H}_0\mathrm{B}_0{}^3+\mathrm{H}_1\mathrm{B}_0{}^2\mathrm{B}'+\mathrm{H}_2\mathrm{B}_0\mathrm{B}'^2+\mathrm{H}_3\mathrm{B}'^3=0\,.$$

More explicitly it can easily be shown that these equations are of the forms

$$\mathrm{G}_0{}'(\mathrm{B}_0a_0\mathrm{C}')^2+\mathrm{G}_1{}'(\mathrm{B}_0a_0\mathrm{C}')\mathrm{B}'+\mathrm{G}_2\mathrm{B}'^2=0$$
$$\mathrm{H}_0{}'\mathrm{B}_0{}^3(a_0\mathrm{C}')^2+\mathrm{H}_1{}'\mathrm{B}_0{}^2\mathrm{B}'(a_0\mathrm{C}')+\mathrm{H}_2\mathrm{B}_0\mathrm{B}'^2+\mathrm{H}_3{}'\mathrm{B}'^3(a_0\mathrm{C}')^2=0.$$

As to their degree the terms are complete rational functions of the tenth degree in (III.), and twelfth in (IV.), in respect to both a_0', a', and c_0, c'.

If we look on the whole question from a theoretical point of view, we may say that the question is now solved, because the elimination of $\frac{\mathrm{B}_0}{\mathrm{B}'}$ from the two last equations will give us a relation between the two angles A and C, so that one of them, A as for example, being looked upon as an independent variable, will determine the other angle C in this hypothesis, and consequently B, and mediately y and z will depend upon A. Of course this theoretical result, when put to the practice, will lead to inextricably complicated multiple solutions, owing to the high degree of the resultant of the elimination of $\frac{\mathrm{B}_0}{\mathrm{B}'}$.

For the present we have contented ourselves with the treatment of several particular cases, namely,

(a) when $B_0^2 = +1$, $B' = 0$,

that is, when the angle B has its extremity at the end of any of the four quadrants;

(b) when $B_0 = 0$, $B'^2 = +1$,

when the extremity of B lies in the bisecting lines of the four quadrants;

(c) when the angles A and C answer to the values

$$3 \cos^2 A = 1, \quad C'^2 = +1, \quad C_0 = 0,$$

in which case the relation (IV.) decomposes itself into the three linear factors

$$(b^2 + c^2) B_0 + (b^2 - c^2) B' a_0 C',$$
$$a^2 B_0 + (a^2 + 2c^2) B' a_0 C',$$
$$a^2 B_0 - (a^2 + 2b^2) B' a_0 C',$$

and the equation (III.) becomes an identity.

XXII.—*On the Partition of Energy between the Translatory and Rotational Motions of a Set of Non-Homogeneous Elastic Spheres.* By Professor W. BURNSIDE.

(Read July 18, 1887.)

At the suggestion of Prof. TAIT, an attempt has been made in this paper to apply the method used by him in § 21 of his paper on "The Foundations of the Kinetic Theory of Gases" to a case of the question of the distribution of energy in a system of non-homogeneous impinging spheres.

The problem may be stated as follows:—Given a very great number of smooth elastic spheres, equal and like in all respects, whose centres of figure and centres of inertia do not coincide, and the sum of whose volumes is but a small fraction of the space in which they move, it is required to find the ultimate distribution of energy among the various degrees of freedom when by collisions the system has attained a "special state."

The received result for the general problem, of which this is a comparatively simple case, is that the energy is distributed equally among the various degrees of freedom. MAXWELL's original proof of this (*Phil. Mag.*, 1860, ii. 37) is hardly more than a statement; while the reasoning given by WATSON, following BOLTZMANN, is on account of its vagueness difficult either to criticise or to verify. The result arrived at here is submitted with great diffidence, owing to its being directly opposed to the foregoing, but it is hoped that the nature of the reasoning is such that each step may be followed and accepted or rejected without doubt. As far as possible the notation of Prof. TAIT's paper is adhered to.

To specify the nature of the spheres, A, B, C are taken as the principal moments of inertia at the centre of inertia; c as the distance of the centre of figure from the centre of inertia; and α, β, γ as the direction-cosines of the line joining these two points with respect to the principal axes.

In the special state of the system it is assumed—

(i.) That the distribution of the linear velocities of the spheres follows the same law as for a system of homogeneous spheres, viz., that the number of spheres whose speeds lie between v and $v+dv$ per unit volume is

$$4\sqrt{\frac{h^3}{\pi}}n \, . \, \epsilon^{-hv^2}v^2dv\,,$$

and that the velocities are equally distributed as regards direction.

(ii.) That the number of spheres per unit volume whose angular velocities about the A-, B-, and C-axes at the centre of inertia lie between ω_1 and $\omega_1+d\omega_1$, ω_2 and $\omega_2+d\omega_2$, and ω_3 and $\omega_3+d\omega_3$ respectively is

$$\sqrt{\frac{k_1k_2k_3}{\pi^3}}n \,.\, \epsilon^{-k_1\omega^2_1-k_2\omega^2_2-k_3\omega^2_3}d\omega_1 d\omega_2 d\omega_3 \,.$$

[This assumption is made by analogy from the form for the speeds, and can only be justified by results.]

(iii.) That for any sphere all directions of the velocity of the centre of inertia with regard to the principal axes are equally likely.

(iv.) That the distance c between the centre of figure and centre of inertia of a sphere is very small compared with the radius.

The last assumption is made for the following reason:—The "opacity," or power of intercepting impinging particles, of a layer of such spheres as are being considered will depend both on the linear and angular velocities of the spheres, and the probability of a collision between different spheres will no longer be proportional to their relative speed, but to some function of their linear and angular velocities, which even if it could be expressed analytically would almost certainly be of a most intractable form. If, however, the distance c is assumed to be very small in comparison with the radius, the probabilities of a collision between different spheres, and the mean free path, will be sensibly independent of the angular velocities, and hence the same as for a system of homogeneous spheres, while there will still be an interchange between the energies of translation and rotation at each collision.

Let u, u' be the velocities of the centre of inertia of a sphere in the line of centres before and after an impact:—

ω_1, ω_2, ω_3, ω'_1, ω'_2, ω'_3 the angular velocities about the principal axes at the centre of inertia before and after an impact:—

l, m, n the direction-cosines of the line of centres with respect to the principal axes at an impact:—

and let large and small letters be used to distinguish the values of these quantities for the two impinging spheres.

Write also for brevity,

$$\begin{array}{ll} \mathrm{N}\beta-\mathrm{M}\gamma=\mathrm{P}\,, & n\beta-m\gamma=p\,, \\ \mathrm{L}\gamma-\mathrm{N}\alpha=\mathrm{Q}\,, & l\gamma-n\alpha=q\,, \\ \mathrm{M}\alpha-\mathrm{L}\beta=\mathrm{R}\,, & m\alpha-l\beta=r\,. \end{array}$$

The spheres are said to be elastic in the sense that the energy of a pair of colliding spheres is unaltered by the impact; and MAXWELL shows, in the paper already referred to, that in the impact of two such spheres the relative velocity of the points of contact in the direction of the line of centres is simply reversed after impact.

Hence the dynamical equations may be written (the mass of a sphere being taken as unity)

$$\begin{aligned} &\mathrm{U}'+u'=\mathrm{U}+u \\ &\mathrm{U}'-u'-c[\mathrm{P}\Omega'_1+p\omega'_1+\mathrm{Q}\Omega'_2+q\omega'_2+\mathrm{R}\Omega'_3+r\omega'_3] \\ &=u-\mathrm{U}+c[\mathrm{P}\Omega_1+p\omega_1+\mathrm{Q}\Omega_2+q\omega_2+\mathrm{R}\Omega_3+r\omega_3] \end{aligned}$$

$$\begin{aligned} \mathrm{A}(\Omega'_1-\Omega_1) &= c\mathrm{P}(\mathrm{U}-\mathrm{U}') \\ \mathrm{B}(\Omega'_2-\Omega_2) &= c\mathrm{Q}(\mathrm{U}-\mathrm{U}') \\ \mathrm{C}(\Omega'_3-\Omega_3) &= c\mathrm{R}(\mathrm{U}-\mathrm{U}') \end{aligned}$$

$$\begin{aligned} \mathrm{A}(\omega'_1-\omega_1) &= cp(u'-u) \\ \mathrm{B}(\omega'_2-\omega_2) &= cq(u'-u) \\ \mathrm{C}(\omega'_3-\omega_3) &= cr(u'-u)\,. \end{aligned}$$

The elimination of the angular velocities after impact from these equations leads to the following equations for U′ and u':—

$$\begin{aligned} &\mathrm{U}'(1+c^2\mathrm{K})-u'(1+c^2k)=u(1-c^2k)-\mathrm{U}(1-c^2\mathrm{K})+2c\varpi \\ &\mathrm{U}' \qquad\qquad +u' \qquad\qquad =u \qquad\qquad +\mathrm{U}\,, \end{aligned}$$

where again for brevity

$$\begin{gathered} \frac{\mathrm{P}^2}{\mathrm{A}}+\frac{\mathrm{Q}^2}{\mathrm{B}}+\frac{\mathrm{R}^2}{\mathrm{C}}=\mathrm{K} \\ \frac{p^2}{\mathrm{A}}+\frac{q^2}{\mathrm{B}}+\frac{r^2}{\mathrm{C}}=k \\ p\omega_1+q\omega_2+r\omega_3+\mathrm{P}\Omega_1+\mathrm{Q}\Omega_2+\mathrm{R}\Omega_3=\varpi\,. \end{gathered}$$

Hence the increase of energy of translation due to the impact (=T say)

$$\begin{aligned} &= \tfrac{1}{2}\ (\mathrm{U}'^2+u'^2-\mathrm{U}^2-u^2) \\ &=2c\,.\frac{\big(u-\mathrm{U}+c\varpi\big)\big(2\varpi-c(k+\mathrm{K})(u-\mathrm{U})\big)}{\big(2+c^2(k+\mathrm{K})\big)^2}\,. \end{aligned}$$

In the special state of the system the mean value of this quantity, as also of the increases in the energies of rotation about the three principal axes, must vanish; and the three independent results so obtained should give k_1, k_2, k_3 in terms of h.

In determining the mean value of any quantity connected with a collision of the spheres here considered, the fourth assumption made above permits the integrations involving the speeds and directions of motion of the colliding spheres and the direction of the line of centres to be performed just as in § 21 of Prof. Tait's paper above referred to. The additional integrations to be performed in this case will be obtained as follows:—Suppose lines drawn from the centre of the unit sphere (in the figure of the paragraph referred to) parallel to the A- and B-axes of each sphere meet the unit sphere in a, A, b, B; that ds, dS are elements of the surface of the sphere, surrounding a, A; and that ψ, Ψ are the angles made by ab, AB with arcs joining a, A to some fixed point. Then, to satisfy the third assumption,

$$\iint \ldots\ldots \, ds\, d\mathrm{S}\, d\psi\, d\Psi$$

must be taken between the limits 0 and 2π for ψ and Ψ, and over the whole sphere for each of the surface-integrals.

Finally, as regards the magnitudes of the angular velocities, the integrations are

$$\iint \ldots\ldots \epsilon^{-k_1(\omega^2{}_1+\Omega^2{}_1)-k_2(\omega^2{}_2+\Omega^2{}_2)-k_3(\omega^2{}_3+\Omega^2{}_3)} d\omega_1 d\Omega_1 d\omega_2 d\Omega_2 d\omega_3 d\Omega_3 \,,$$

and the limits for ω_1, Ω_1, &c., are $\pm\infty$. It might appear that, having taken all possible positions for the principal axes, the limits for ω_1, Ω_1, &c., should be 0 and ∞; but a little consideration will make it clear that these latter limits would be correct only for bodies symmetrical with regard to three mutually rectangular planes through the centre of inertia.

In the process of finding $\overline{\mathrm{T}}$ let the integrations with respect to ω_1, Ω_1, &c., be first performed. The only part of T which will contribute anything after integration is

$$2c^2\frac{2(p^2\omega^2{}_1+q^2\omega^2{}_2+r^2\omega^2{}_3+\mathrm{P}^2\Omega^2{}_1+\mathrm{Q}^2\Omega^2{}_2+\mathrm{R}^2\Omega^2{}_3)-(k+\mathrm{K})(u-\mathrm{U})^2}{\left[2+c^2(k+\mathrm{K})\right]^2}\,.$$

Performing on this expression the integrations indicated, and dividing by the corresponding part of the denominator of $\overline{\mathrm{T}}$, the result is

$$2c^2\frac{(p^2+\mathrm{P}^2)\left(\frac{1}{k_1}-\frac{(u-\mathrm{U})^2}{\mathrm{A}}\right)+(q^2+\mathrm{Q}^2)\left(\frac{1}{k_2}-\frac{(u-\mathrm{U})^2}{\mathrm{B}}\right)+(r^2+\mathrm{R}^2)\left(\frac{1}{k_3}-\frac{(u-\mathrm{U})^2}{\mathrm{C}}\right)}{\left[2+c^2(k+\mathrm{K})\right]^2}\,.$$

The value, at the same step in the process, of the average increase in ω_1-energy at a collision is similarly found to be

$$2c^2\frac{(p^2+\mathrm{P}^2)\left(\left(\frac{(u-\mathrm{U})^2}{\mathrm{A}}-\frac{1}{k_1}\right)+\frac{1}{2}c^2\left\{(q^2+\mathrm{Q}^2)\left(\frac{1}{\mathrm{A}k_2}-\frac{1}{\mathrm{B}k_1}\right)+(r^2+\mathrm{R}^2)\left(\frac{1}{\mathrm{A}k_3}-\frac{1}{\mathrm{C}k_1}\right)\right\}\right)}{\left[2+c^2(k+\mathrm{K})\right]^2}\,,$$

and from this, by an interchange of symbols, the values of the corresponding quantities for the other two rotations may be written down.

In the four expressions so far obtained, write

$$\frac{\mathrm{A}}{k_1}=\frac{\mathrm{B}}{k_2}=\frac{\mathrm{C}}{k_3}=\lambda\,;$$

they become respectively

$$2c^2 \frac{k+K}{[2+c^2(k+K)]}(\lambda-(u-U)^2)$$

$$-2c^2 \cdot \frac{\frac{p^2+P^2}{A}}{[2+c^2(k+K)]^2}(\lambda-(u-U)^2)$$

&c., &c.

In completing the process of averaging on these new expressions, the integrations

$$\iint \ldots\ldots ds dS d\psi d\Psi$$

affect only the first factors of each of them, and a consideration of the meaning of the factors shows that in each case the integral is a function of the constant quantities A, B, C, α, β, γ and c.

[Neglecting terms in c^4 as compared with those in c^2, the approximate values of the averages of these factors are

$$\frac{c^2}{3}\left(\frac{\beta^2+\gamma^2}{A}+\frac{\gamma^2+\alpha^2}{B}+\frac{\alpha^2+\beta^2}{C}\right),$$

$$-\frac{c^2}{3}\cdot\frac{\beta^2+\gamma^2}{A}, \quad -\frac{c^3}{3}\cdot\frac{\gamma^2+\alpha^2}{B}, \quad -\frac{c^2}{3}\cdot\frac{\alpha^2+\beta^2}{C}.\Big]$$

Hence the three equations

$$\frac{A}{k_1}=\frac{B}{k_2}=\frac{C}{k_3}=\overline{(u-U)^2}$$

are *a* solution, and therefore must be *the* solution, of the problem of the "special state."

The quantity $\overline{(u-U)^2}$ finally is affected only by the integrations of the § 21 of Prof. TAIT'S paper already referred to; and indeed its value may be written down at once from the result of that article. For

$$\overline{(u-U)^2}=\overline{u^2-uU}+\overline{U^2-Uu}=\frac{2}{h}\cdot$$

The required result then is

$$\frac{A}{k_1}=\frac{B}{k_2}=\frac{C}{k_3}=\frac{2}{h};$$

or, in words:—

The average energies of rotation of a sphere about each of the three principal axes are equal, and the whole average energy of rotation of a sphere is twice the average energy of translation.

The forms for the average changes in the rotation-energies at a collision indicate that, if at any time before the special state is attained the three rotation energies are equal, they will generally tend to become unequal again; and therefore the problem of determining at what rate the system tends to reach the special state would be intractable even if it were legitimate to suppose the second assumption to hold throughout.

If specially constituted spheres, however, are taken in which

$$\frac{\beta^2+\gamma^2}{A}=\frac{\gamma^2+\alpha^2}{B}=\frac{\alpha^2+\beta^2}{C}=\frac{2}{A+B+C},$$

an attempt may be taken to determine the rate in question, for the forms of the average changes in energy at a collision then show that if the equations

$$\frac{A}{k_1}=\frac{B}{k_2}=\frac{C}{k_3}$$

hold at any one instant, they will always hold.

Suppose, then, that in this case x, y are the whole energies of translation and rotation per unit volume; so that

$$x=\frac{3n}{4h} \qquad y=\frac{3nA}{4k_1}.$$

By § 14 of Prof. TAIT's paper in conjunction with the forms found above

$$\dot{x}=-\dot{y}=\sqrt{\frac{2\pi}{h}}n^2s^2\frac{2c^2}{A+B+C}\left(\frac{A}{k_1}-\frac{2}{h}\right)$$

$$=\sqrt{\frac{8\pi n x}{3}}\,\frac{8c^2s^2}{A+B+C}\,\frac{y-2x}{3}.$$

If 3E be the whole energy per unit volume, and if

$$\sqrt{\frac{8\pi n}{3}}\cdot\frac{8c^2s^2}{A+B+C}=\frac{1}{T\sqrt{E}},$$

$$\dot{x}=\frac{1}{T}\sqrt{\frac{x}{E}}\left(E-x\right);$$

the complete solution of which is

$$\frac{2\sqrt{E}}{\sqrt{x}-\sqrt{E}}+1=\text{constant}\ \epsilon^{\frac{t}{T}},$$

on the supposition that the energy of translation is originally greater than one-third of the total energy.

The ratio of the quantity T found here with that in § 23 of Prof. TAIT's paper, supposing there the numbers and masses of the two sets equal, is

$$\frac{6c^2}{A+B+C}.$$

Hence it would seem that if c is of the order $s\times 10^{-3}$, and therefore this fraction comparable with 10^{-6}, the rate at which the special state would be attained is still extremely rapid.

Note.—With regard to the form

$$\sqrt{\frac{k_1k_2k_3}{\pi^3}}\,n\epsilon^{-k_1\omega_1^2-k_2\omega_2^2-k_3\omega_3^2}d\omega_1 d\omega_2 d\omega_3$$

for the number of particles per unit volume with angular velocities between given limits, the fact that ω_1, ω_2, ω_3 are periodic functions while the particle is moving freely, suggests at first sight a difficulty. The period for each particle is a function of its energy of rotation and of its angular momentum. Suppose the particles whose energy of rotation lies between given close limits divided into sets, the angular momentum in each set also lying between given close limits. Then (compare KIRCHHOFF'S *Vorlesungen*, p. 64)

$$\omega_1 = p\,\mathrm{cn}(\lambda t+\mu)\,,\quad \omega_2 = q\,\mathrm{sn}(\lambda t+\mu)\,,\quad \omega_3 = r\,\mathrm{dn}(\lambda t+\mu)\,,$$

where p, q, r, λ and the modulus of the elliptic functions are constants for any one set.

The individuals of each set are distinguished by the values of μ. If then the circumstances are such that μ may be regarded as a uniformly varying quantity between limits separated by a period for each set, the number of particles corresponding to the product $d\omega_1\, d\omega_2\, d\omega_3$ will be independent of the time.

XXIII.—*A Contribution to our Knowledge of the Physical Properties of Methyl-Alcohol.* By W. Dittmar, F.R.SS. Lond. & Edin., and Charles A. Fawsitt. (Plate XXXIII.)

(Read May 2, 1887.)

Since its discovery by Dumas and Péligot in 1834, methyl-alcohol has been the subject of a great many researches, and as a result we have long had a perfectly certain knowledge of its atomic composition, and a very accurate knowledge of a great many of its reactions. Yet the physical properties of the *substance* CH_4O have not yet been determined with a satisfactory degree of precision. At this we need not wonder. For the study of the transmutations of a species a very impure specimen may suffice, and a series of such studies may leave no doubt about the correct atomic formula of the species in question, and consequently also, if it is a volatile substance, about its perfect gas density. But no other physical properties can be determined otherwise than by direct experiments on a pure specimen. And pure methyl-alcohol is very difficult to obtain. In whatever reasonable sense we may take the word "pure," as attached to the name of a chemical preparation, "pure" methyl-alcohol must be admitted to have been little more than a chemical fiction until Wöhler in 1852 discovered his well-known (oxalate) process for its extraction from wood-spirit.

As a consequence of this discovery, the properties of "methyl-alcohol" suffered a remarkable change; what had before been known as a more or less unpleasantly smelling liquid, which boils at about 60° C., and turns brown on treatment with caustic alkalies, assumed the form of an almost inodorous liquid, boiling at or near 66° C., and behaving to caustic alkali pretty much as pure ethyl-alcohol does. And it has since exhibited a fair degree of constancy in its properties in the hands of numerous observers, although the wood-spirits which these have used for their preparations must have been of very different kinds. This tends to show that Wöhler's alcohol (if carefully prepared) is at least a fairly close approximation to the ideal substance, and this impression is confirmed by a research of Krämer's, who prepared methyl-alcohol from purified formate of methyl, and found it to boil at very nearly 66° C.

The successive application of the oxalate and of the formate process would probably yield a very pure preparation, because the former tends to eliminate the more volatile, the latter the less volatile of the impurities; and we very much regret now that this idea did not suggest itself to our minds in the preparative stage of our work. We thought only of the WÖHLER process; how we applied it we shall now proceed to explain.

The raw material which we started with was a particular fraction of wood-spirit, which had been collected for us in the course of an *industrial* distillation. 50 c.c. of this spirit, when subjected to distillation in a fractionating flask, gave the following results:—

Boiling-point at a given stage of the distillation $= t°$.

Total volume of distillate obtained at that stage $= v$ c.c.

$t =$	67°	69°	70°	72°	75°
$v =$	0	20	30	40	50 c.c.

Two analyses by the iodide-method gave 71·8 per cent. of real CH_4O; by KRÄMER and GRODZKI's iodoform test we found 5·6 per cent. of acetone.

Before applying the oxalate process we thought we had better subject our alcohol to some kind of preliminary purification. We accordingly tried to remove the acetonic compounds by means of alkaline bi-sulphite; but no modification of the method led to satisfactory results. We then gave a trial to the now almost forgotten method of KANE (fixation of the CH_4O by means of a large excess of fused chloride of calcium, removal of what remains volatile by distillation, regeneration of the alcohol from the $CaCl_2$-compound by distillation with water). In this case we worked quantitatively, and determined the acetone in the several fractions as iodoform. The result was highly discouraging; a considerable proportion of the methyl-alcohol escaped combination with chloride of calcium, and the part fixed by the reagent included a considerable proportion of acetone.

After this second failure we decided upon confining ourselves to the use of caustic soda as a preliminary purifier; this agent was sure to decompose at least the acetate of methyl; and, if used dry and in quantity, could be expected to destroy at least part of the acetone. In order to ascertain how far the power of the reagent goes in the latter sense, we heated 100 c.c. of the crude spirit with 150 grammes of powdered caustic soda over a water-bath at the "wrong end" of a condenser for some hours, and next distilled over what could be volatilised by immersion of the flask in a boiling water-bath. We obtained 8 c.c. of a distillate smelling strongly of ammonias, and containing 4·6 per cent. of its weight of acetone. The residue was then decomposed with water, and distilled over a naked flame. It furnished a distillate which when tested with iodine and caustic soda *gave no iodoform.* We did not stop to ascertain what had become of the acetone, but at once applied the process to larger quantities of the spirit, with this modification, however, that we used less proportions of caustic soda in order not to lose too much of the CH_4O, which could not be expected to survive the process in its entirety. Not caring to lose time by a systematic elaboration of the process, we at once carried it out with quantities of 3–4 litres of crude spirit, and in each case adjusted the proportion of caustic soda according to our judgment. A so-called Papin digestor, as sold for the boiling of meat under pressure, served as a retort, and worked well. The best results, on the whole, we obtained in an operation in which 3·5 litres of crude spirit were worked up with 1400 grammes of powdered caustic soda. The crude product, when distilled out of a boiling water-bath, yielded 1·5 litres of a highly acetonic distillate, which was put aside. The residue, when distilled with 1·75 litres of water over a naked flame, yielded 1·9 litres of a strong methyl-alcohol, free from acetone and free of unpleasant odour. From their specific gravity, and assuming them to be aqueous methyl-alcohol, the 1·9 litres of distillate contained 1396 grammes of real CH_4O as against the 2354 of CH_4O which the 3·5 litres of spirit contained according to the iodide test. In other cases the yield was considerably less, no doubt through partial oxidation of the methyl-alcohol into formate or oxalate,

$$CH_4O + NaOH = CHNaO_2 + 2H_2$$

and

$$2CHNaO_2 = C_2Na_2O_4 + H_2 .$$

We, of course, do not recommend the process for manufacturing purposes; but it did good service to us by placing us in possession of some 6 litres of very strong and almost acetone-free methyl-alcohol. From it we prepared crystallised oxalate of methyl (or rather a mixture of this ester and methyl-oxalic acid) by a method which Mr Alexander Watt many years ago worked out in Dr Crum Brown's laboratory, and which we will describe shortly because we believe it is little known to chemists generally.

Purified alcohol, ...	400 c.c.
Oil of vitriol, ...	200 c.c.
Oxalic acid crystals, ...	500 grammes.

The oxalic acid is mixed with the vitriol, the spirit is then added, and the whole cautiously heated over a water-bath until the oxalic acid is dissolved. The liquid, on cooling and standing over night, deposits an abundant crop of methylic crystals, which are collected and squeezed out in a powerful press.

For the regeneration of the alcohol the crystals were heated with water in a flask connected more directly with an inverted condenser, kept at 70° C.; what remains uncondensed there passes down a condenser, and is collected in a receiver. The resulting distillate is sufficiently strong to be fit for immediate treatment with carbonate of potash, which latter was applied repeatedly as long as it acted visibly.

The resulting product was dehydrated further, first by distillation over quicklime, and then by distillation over baryta-lime, a *quasi* apology for baryta, of which reagent we had not a sufficient stock at our disposal. Nitrate of baryta is heated in a platinum crucible until it ceases to decrepitate. It is then ground up finely, and mixed with its own weight of perfectly anhydrous quicklime (as obtained by the dehydration of slaked lime at a red heat), the mixture put back into the crucible, and then kept for two to three hours at a red heat, conveniently within a muffle; to be converted into a mixture of BaO and CaO, of which 100 parts require, by calculation, 24·6 parts of water for conversion of the two oxides into monohydrates—4·3 for the baryta, 20·3 for the lime. The platinum crucible remains unattacked, and the product is easily reduced to a fine powder. Our general method for these dehydrations was as follows:—The alcohol to be operated upon is analysed approximately by determining its specific gravity; it is then mixed with only a little more than the calculated quantity of the respective dehydrator, and next "tortured" with it under an inverted condenser. The mixture is then distilled from out of a water-bath, in which the flask is entirely immersed, and the distillation continued as long as anything comes over. The distillate is again operated upon in the same way until two successive distillates, obtained at the baryta-lime stage, exhibit the same specific gravity at the same temperature. At the later stages of the dehydration the receiver is connected with a "vitriol tower" to keep out atmospheric water, and all unnecessary transvasations are avoided, the flask intended for distillation serving as the receiver in the preceding one. For the determination of the specific gravities we generally used a Westphal balance; in the later stages we determined the difference of specific gravity between two successive distillates by means of the differential method, which one of us described in the *Chemical News*, vol. xliv. p. 5, some years ago. The idea which guided us was to effect a complete dehydration with the least possible loss of alcohol as alcoholate of baryta or lime.

We felt quite sure that a methyl-alcohol which suffered no diminution of specific gravity on renewed distillation over baryta-lime really is anhydrous; yet some doubts about this arose in our minds at a later stage, and we distilled a presumed to be anhydrous alcohol over dehydrated sulphate of copper. The result was that the specific gravity suffered a measurable diminution. Possibly the alcohol then operated upon had attracted a little moisture from the air (notwithstanding the care with which it had been protected) since its final treatment with baryta-lime. We did not inquire into this point, but made it a rule henceforth not to accept an alcohol as anhydrous unless it had been made constant in specified gravity by means of, ultimately, dehydrated sulphate of copper.

As the result of a long and tedious series of dehydrations, we came into possession of a few litres of baryta-lime proof methyl-alcohol, and the question was to prove its freedom from *organic* impurities. As a first step towards this end, we determined the vapour density of our alcohol by means of an apparatus of our own invention, the description of which we prefer to reserve for an appendix to this memoir; suffice it to point to Plate XXXIII. fig. 1, and to state that it is constructed on the Gay-Lussac principle, in such a manner as to avoid the uncertainties in the variable density of the suspended mercury column, and that the vapour volume in it is measured under very nearly the prevailing atmospheric pressure at about 100° C. Three determinations made with from 90 to 100 milligrammes of alcohol gave the values 16·17, 16·27, 16·22; hydrogen = 1; *i.e.*, a little more than the theoretical number 16·00.*

We could not help noticing that our three numbers are a little above the value demanded by CH_4O; yet as the excesses lie almost within the limits of unavoidable errors, we preferred to accept our numbers as showing that our preparation was, at the worst, a fair approximation to the ideal substance. We now regret that we did not endeavour to attain a higher degree of precision in our vapour-density determinations by a suitable modification of our apparatus, and adopt the *exact* vapour density at a high enough temperature as *the* final test of purity. At the time it appeared to us better to try, and, if possible, prove, the purity of our alcohol by a series of chemico-physical tests, which all agreed in this, that the given alcohol was subjected to some *chemical* process of fractionation, and the two fractions were compared with each other, and the mother substance, in regard to some exactly measurable physical property.

To give a better idea of what we actually did, let us quote a few special cases.

I. 300 c.c. of a certain alcohol (I.) were "tortured" with 90 grammes of dry caustic potash, and the resulting mass was distilled by means of a water-bath as long as anything came over. The distillate, amounting to 120 c.c., was put aside as "A."

From the residue of alcoholate and hydrate of potash the alcohol was regenerated by distillation with water as fraction "B," which, according to its volume and specific gravity, contained about 142 grammes of absolute alcohol. Each of the two fractions was dehydrated completely by repeated distillation over baryta-lime until its specific gravity became constant. The final specific gravities, determined by the Westphal balance, and reduced to 0° C. by the same formula, were—that of A = 0·8138, that of B = 0·8142, *i.e.*, the two fractions were practically identical.

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* Krämer and Grodzki (*Berichte der Deutschen Chem. Ges.*, 1876, p. 1928) determined the vapour densities of synthetically prepared mixtures of methyl-alcohol with acetone or dimethyl-acetal, and arrived at the curious result that the densities of the mixtures differed from the calculated numbers. I have recalculated the numbers from their own data, and arrived at values which agree quite closely with those demanded by theory, *i.e.*, the assumption that the several vapours mix without contraction or expansion. Anybody who cares can easily satisfy himself that I am right. I am glad to avail myself of this opportunity for disinterring a piece of meritorious work which got lost by an unfortunate *lapsus calami* in the construction of a formula.—W. D.

Even absolute identity of specific gravity of course does not *prove* chemical identity. What goes a great deal further, as was shown by REGNAULT, is equality of vapour tension at a *series* of temperatures; and as a proof of chemical purity independence of the vapour tension, at a given temperature, of the volume of vapour produced from a given weight of substance. This method we proposed to ourselves to chiefly rely on, and we accordingly employed it pretty extensively. The apparatus we used will be fully described in a later section; meanwhile Plate XXXIII. fig. 2 may be referred to as giving a sufficient idea of its construction and of the way it is used.

To test a given alcohol, we either charged one limb of the apparatus with only one or two drops, the other with some 2 c.c. of the preparation, established a convenient temperature (by means of a water-bath) and external pressure, and took the difference of level between the mercury menisci in the two communicating tubes; or else we subjected the given alcohol to some kind of chemical fractionation, and compared nearly equal volumes of the two fractions in regard to their tensions at a selection of temperatures.

Being anxious to avoid everything that might disturb the proximate composition of a specimen, we at first expressly refrained from boiling off the absorbed air from the samples to be shut up in the tensiometer; but we soon found that we thus introduced an error which is in general far greater than we had anticipated. After having recognised this error of judgment, we made an attempt at correction for the absorbed air by measuring the tension of a given specimen at two (or more) widely different volumes. Our vapour *density* apparatus (suitably modified) lent itself well for this purpose. From 1·9–2 c.c. of a certain alcohol, which we supposed to be *very* pure, were introduced into the tube over mercury; a fixed-upon temperature was established by means of a water-bath, and the vapour tension then determined at three different volumes.

We did not succeed in maintaining absolute equality of temperature at the three different volumes, but had sufficient data from previous experiments for determining the necessary coefficient $\Delta p/\Delta t$ for reducing the several observations to a standard temperature. Two such series of experiments on the same specimen gave the following results:—

First Series: $t=16°\cdot 0$ C.

	Volume of Vapour in c.c.	Observed Tension in mm. of Mercury.
(1)	16·8	97·05
(2)	66·8	83·10
(3)	129·0	81·18

Second Series: $t=11°\cdot 0$ C.

	Volume of Vapour in c.c.	Observed Tension in mm. of Mercury.
(1)	8·5	95·74
(2)	83·2	62·50
(3)	129·9	60·08

From either of the two series it should be possible theoretically to calculate the quantities of air retained by the liquid; but a little reflection, based on the requisite formulæ, shows that the experimental data do not afford the necessary degree of precision for this purpose.

Assuming that at any of the three volumes all the air is in the vapour, and taking x as representing its volume in c.c. at t°, and 1 mm.'s pressure, we have,

First Series (16°).

By combining (1) and (3) . . . . $x=306$
" (2) and (3) . . . . $x=266$
Mean, . . $x=286$

From this mean and by equation $p=\frac{x}{v}+p_0$, where p stands for the total pressure as observed, and p_0 for the partial pressure of the alcohol vapour, we have

From	(1)	(2)	(3)
$p_0=$	80·0	78·8	79·0 .

Second Series (11°).

By combining (2) and (3) we have

$$p_0=55{\cdot}79 \text{ and } x=557{\cdot}2 \text{ c.c.}$$

From (1) and (3) we have

$$p_0=57{\cdot}60 \text{ and } x=322{\cdot}2 \text{ c.c.}$$

From (1) and (2) we have

$$p_0=58{\cdot}73 \text{ and } x=300{\cdot}4 \text{ c.c.}$$

From these results we clearly saw that a sufficient exactitude for p_0 could not be obtained in this manner, and we therefore fell back upon the old method of "boiling out" the specimens to be operated upon in their respective tubes before shutting them up. From a large mass of notes we extract the following:—

Two alcohols, A and B, were mixed to produce about 300 c.c. of "D." The specific gravity found was such that, according to our *present* alcoholometric tables, the percentage of absolute alcohol was 98·66.* Four distillations over anhydrous sulphate of copper brought down the specific gravity so that it now corresponded to 98·81. This alcohol was distilled by itself, and the distillate collected in two approximately equal fractions. Their specific gravities, reduced to per cents., corresponded to $\overset{\text{I.}}{99{\cdot}02}$ $\overset{\text{II.}}{98{\cdot}90}$; as we *now* see; at the time we took them both as representing absolute alcohol.

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[*] We propose, for the convenience of the reader, to quote the results in this manner; at the time we had, of course, to go by the specific gravity as a mere index of strength.

The two fractions were mixed, again dehydrated by anhydrous sulphate of copper and distilled, the first few drops of distillate being rejected. Two drops of what followed immediately were shut up in the right limb of the tensiometer; the distillation was then carried on to near the end, and two drops of the very last runnings collected in the left limb of the tensiometer. A comparison of the tensions gave the following results:—

$t = 30°$	$50°$	$65°$ C.
$\Delta p = 1{\cdot}3$	$1{\cdot}5$	$3{\cdot}0$ mm.

in favour of the earlier runnings.

The alcohol "D" was now tested, so to say, against itself; the left limb of the tensiometer was charged with only two drops, the right limb with some 2 c.c. of the alcohol; both were boiled before being shut up. A comparison of the tensions at 3 temperatures gave the following results:—

$t = 30°$	$50°$	$65°$ C.
$\Delta p = 6{\cdot}1$	$7{\cdot}1$	$7{\cdot}9$ mm.

in favour of the larger specimen.

These results were rather discouraging, because they seemed to show that the alcohol was impure. To show the relevancy of the Δp's, we add the corresponding Δt's, which are

$$\Delta t = 0°{\cdot}7 \qquad 0°{\cdot}4 \qquad 0°{\cdot}3\,.$$

It however still remains to be proved that the tension of even pure methyl-alcohol is *absolutely* independent of the ratio of the mass of the vapour to the mass of the liquid which it is in contact with. Methyl-alcohol, in reference to its conversion from liquid into vapour, exhibits anomalies. It "*bumps*" badly when distilled out of a glass vessel, *i.e.*, it may be heated considerably above its boiling point before it actually boils. According to our experience, it sometimes exhibits a similar anomaly in experiments for determining its vapour tension by the statical method.

The *air-free* alcohol, when shut up over mercury under a pressure which is considerably less than its maximum tension at the temperature of the bath, may form no vapour at all until the apparatus is being shaken, when a sudden formation of vapour sets in with explosive violence.

We are here referring to a series of experiments in which the two limbs of the tensiometer were charged with nearly equal quantities of the same alcohol; but only one of the two samples was deprived of its air before being shut up (in the left limb). The temperature of the bath being kept rigorously constant (reading of a sensitive but *un*corrected thermometer 51°·7), the tensions of the two samples were determined at a variety of volumes. The heights of the mercury columns were left *un*corrected for temperature, as we aimed only at comparisons. The following table gives the results of *one* of a number of series of observations; v stands for the volume of vapour in the right limb (where the air had *not* been removed); the vapour volume in the left was not much different from v in any case:—

v in c.c.	Vapour Tension.		
	Unboiled. mm.	Boiled. mm.	
11·5	412·6	409·7	409·9 ± 0·3
9·8	412·8	409·5	
8·1	413·0	409·9	
6·3	413·0	410·1	
4·7	412·8	409·8	
3·1	413·3	410·2	
1·1	416·2	411·8	
Max.—Min.	3·6	2·3	

Other similar series of determinations with the identical samples gave substantially the same results. There would be no use in troubling the reader with any further account of our tensiometric and specific gravity tests; we prefer to give our general conclusion, which was that our methyl-alcohol, although of a high order of purity, was not sufficiently pure to do justice to even our (home-made) apparatus for measuring the tension of a *given* vapour. But what could we do towards the further purification of our substance? The only course we could think of was to determine the tension-curve of the alcohol as it stood up to about its boiling-point; to then subject it to some kind of chemical purification (say conversion into formate, purification of the same, and regeneration of the alcohol from the purified ester), to determine the tension-curve of the purified alcohol, and compare it with that of the original preparation. Our tensiometer would have enabled us to determine corresponding *differences* of tension with a very high degree of certainty.

Supposing these differences to exceed the limits of observational errors, the second alcohol must be purified again, say by the oxalate method; and so on until two successive tension curves coincide practically. The mean curve could confidently be adopted as *the* tension-curve, and the last alcohol as sufficiently pure for any physical determination.

This, indeed, had been our programme from the first; but at the end of our pioneering experiments our available time had very nearly been exhausted. We accordingly decided upon just accepting our alcohol as the best apology for the ideal substance which we were able, under the circumstances, to produce, and using it for what we were forced to let stand as our final determinations.

Our apology for publishing these in the following section is the conviction that the numbers, though not what we would wish them to be, are probably better approximations to the truth than those given in the present handbooks of chemistry.

The Tension-Curve of Methyl-Alcohol.

In regard to it our programme from the first was limited in the sense that we did not intend to go beyond about 760 mm. as our maximum value.

The apparatus which we used is substantially the same as that which one of us employed many years ago for comparing the vapour tensions of the two fatty esters $C_3H_6O_2$.* Figure 2 on Plate XXXIII. shows the form which it assumed on the present occasion. The part which receives the liquids to be operated upon consists of a glass U-tube A, with a vertical tube soldered in between the two limbs. The two side tubes have an inner diameter of about 1 centimetre; the section of the middle tube is about equal to those of the two side tubes taken together. The side tubes are contracted somewhere near the upper end, and a well-ground glass stopper is fitted into the neck of the cup at the top. The exit end of the central tube communicates with a large bottle, and through it with a syphon-barometer B. In the latter the close limb is so long that the vacuum can be expanded into very much more than the customary volume. This long limb terminates in a funnel-shaped cup, the neck of the funnel being provided with a well-ground-in stopper. A mercury reservoir R, connected with a short side-branch from the open limb by means of a long piece of capillary india-rubber tubing, enables one to bring the two mercury-menisci into convenient positions. A small air-pump constructed so that it may serve for exhaustion or compression, and communicating more immediately with the bottle, serves to establish the required degree of attenuation in the latter. The three limbs of the W-tube and those of the manometer bear etched-in millimetre scales. The close limb of the manometer, from the top down to the lowest occurring position of the meniscus, is calibrated so that the unavoidable residuum of air in that limb can be determined by ascertaining the height of mercury supported by the atmosphere at two widely different vacuum volumes. Before charging the W-tube, a true zero plane for the three scales must be found by charging the apparatus with mercury up to a little beyond the three nominal zero-marks, making the limbs exactly plumb, and reading the three menisci in reference to their respective scales. Supposing the zero in the middle tube to be taken as the standard, this gives the corrections for the mercury columns in the two side tubes. By means of the well-known artifices, it is easy to fill the bends of the tensiometer with *air*-free mercury. More mercury is then run into the middle

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* *Chem. Soc. Jour.*, [2], vi. 477; *Annal. d. Chem. u. Pharm.*, Suppl., vi. 313; *Jahresb. f.* 1868, p. 500.

tube through a long-necked stop-cock funnel until the metal comes almost close to the necks of the two cups. Supposing now two specimens of methyl-alcohol to be operated upon, a small quantity of one is introduced into, say, the left limb, boiled there to expel its air, the stopper inserted, a little mercury poured into the cup, a little of the respective alcohol added, and the cup closed by means of a small glass cap fixed on by means of good india-rubber tubing, so that the two glass rims touch each other inside. The india-rubber is well wired on both sides. In a similar manner the right limb is charged with the other specimen, and the greater part of the mercury of the middle tube syphoned out. The charged tensiometer is suspended in an exactly vertical position within a large square water-bath, the front and back of which consist of plate glass, and connected air-tight with the bottle, and thus indirectly with the manometer. A standard thermometer, from GEISSLER of Bonn, suspended in the water-bath, gave the temperatures. The higher temperatures were established by means of a properly adjusted mixture of hot and cold water, and maintained by means of steam sent into the bath through two block-tin pipes which pervaded the bath in its entire height. By properly regulating the current of steam, and perpetual agitation of the water in the bath, we soon learned to keep even the highest temperatures constant to within ±0°·1 C. Immediately before and immediately after each series of experiments the zero correction of the thermometer was ascertained, to be allowed for in the ultimate record. To test the thermometer for the exactitude of its calibration, we constructed an air-thermometer—pretty much on the Jolly principle—and determined the true temperature-values for a large number of the marks on the Geissler standard by several series of experiments. As we had no real cathetometer at our disposal, we were not able to bring down the *uncertainty* of the temperature values, as determined by the air-thermometer, to less than 0°·1 to 0°·2 C., but within these limits the Geissler instrument (as corrected for the zero displacement) proved correct. We subsequently procured a large thermometer from Mr CASELLA (London), which was made by him out of a long capillary tube which we had calibrated most thoroughly by means of RUDBERG's method. This standard thermometer, by its calibration table, gives temperatures correctly to within about 0°·02 C.; but, unfortunately, before we had a chance of using it for standardising the Geissler instrument, the scale of the latter (which is on a separate glass strip enclosed with the thermometer stem within a glass jacket) became loose, and we could not manage to refix it in absolutely its original position.

The millimetre scales on all our instruments we made ourselves by means of an excellent screw-engine from BIANCHI of Paris, which Professor TAIT kindly lent to us.

In the numerous preliminary tension determinations, which we referred to in an earlier part of this paper, we always worked with two samples of alcohol at the same time, and these determinations included a complete rehearsal of the determination of the entire tension-curve. But in the final series we preferred to charge only one limb to reduce the number of readings by one. The actual routine of the work hardly requires to be described. After having established the desired temperature, we read first the three tensiometer limbs, then those of the manometer, and, lastly, the thermometer a second time, by means of a horizontal telescope. The temperature was in all cases rigorously constant, as far as one could read. To eliminate part of the error arising from unavoidable variations of temperature during a series of readings, we found it an improvement to close the open end of the manometer, and thus fix its mercury-menisci in their positions, immediately after reading the limbs in the tensiometer. As a rule, we commenced with the lowest temperature to proceed step by step to the highest, and then retraced our steps, so that each series consisted of an ascending and a descending section.

The alcohol used for the final tension determinations was specially dehydrated (by means of $CuSO_4$), and a sufficiency kept in a sealed-up tube until the tensiometer was ready for its reception.

The height of the several mercurial columns was reduced to 0° C., but *not* reduced to any standard latitude, for an obvious reason.

A preliminary survey of the results showed that they fell in approximately with the equation

$$\log p = a + bt\,.$$

We accordingly for a first approximation adopted this function, determined the constants a and b graphically, and from them calculated the values log p, corresponding to the several observational t's. For a second approximation we laid down the t's as abscissæ, and the corresponding values,

$$\text{“}\Delta y\text{”} = (\log p \text{ as observed}) - (\log p \text{ as calculated}),$$

as ordinates in a system of rectangular coordinates, when Δy appeared to be a function of t, according to an equation of the form $\Delta y = \alpha + \beta t + \gamma t^2$. We then calculated the constants α, β, and γ from three measured ordinates, and thus established an equation,

$$\log p = a' + b't + ct^2 + \delta(\log p)\,,$$

where $\delta(\log p)$ stands for the residual correction needed to establish equality between the two sides of the equation. These residuals $\delta(\log p)$, when represented graphically in function of t, suggested an equation of the 4th or 5th degree; but on looking more critically into the matter, we found that this final curve registered observational errors rather than anything else, and that a small constant correction applied to a' did full justice to our results. The final formula adopted was

$$\log p = 1{\cdot}4731 + 0{\cdot}02649t - 0{\cdot}0000742t^2.$$

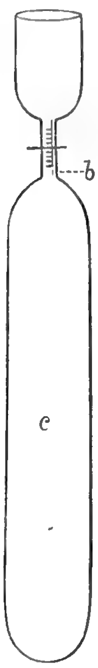
The following table gives the results of our final series of determinations; *i.e.*, the logg. of the observed p's, contrasted with the values calculated by means of the formula. The column "Exp.—Calc." gives the correction to be applied to the calculated logarithm of p to obtain log (p as found). The last column, under Δp, gives the corresponding difference between the two values p themselves:—

Temp. C.	Logarithm of		Exp.—Calc.	Δp in mm. of Mercury.
	Calculated Tension.	Observed Tension.		
4°·15	1·5817	1·5761	−0·0056	0·6
9°·15	1·7092	1·7097	+0·0005	0·07
9°·95	1·7293	1·7318	+0·0025	0·4
14°·15	1·8330	1·8373	+0·0043	0·7
19°·15	1·9531	1·9542	+0·0011	0·2
24°·15	2·0695	2·0738	+0·0043	1·2
29°·15	2·1821	2·1837	+0·0016	0·6
34°·15	2·2911	2·2940	+0·0029	1·3
39°·15	2·3963	2·3947	−0·0016	1·0
44°·15	2·4978	2·4988	+0·0010	0·7
49°·15	2·5956	2·5936	−0·0020	2·0
54°·15	2·6898	2·6895	−0·0003	0·3
59°·15	2·7802	2·7792	−0·0010	1·5
64°·15	2·8669	2·8682	+0·0013	2·4
65°·15	2·8837	2·8847	+0·0010	1·8

By means of the interpolation formula, given above, the table on the following page was calculated.

The Specific Gravity of Anhydrous Methyl-Alcohol.

For these determinations we used small cylindrical bottles of the form represented in the figure on page 522. The body c holds about 20 c.c.; the stem b bears a millimetre scale; 1 mm. corresponds to very nearly 0·01 c.c. To ascertain the capacity at any mark and any temperature which it might be convenient to use, we first determined the capacities for water of 14°·7 (the "15" of a certain delicate thermometer) up to 0, 5, 10, 15, 20 mm. We then determined the capacities for water up to 0 mm. (directly or indirectly) at exactly 0° (in melting ice), and at temperatures near 15°, 20°, 30°, 35°, 40°, 45°,



50°, 60°, 65°. To check the results we also determined the capacities for mercury at a similar series of temperatures. From the sum total of our experiments we calculated a formula $C_0 = C + kt$ for the capacity at $t°$ in grammes of water of the density corresponding to 4° C. (*i.e.*, the capacity in c.cs.), adopting such values for C_0 and k as fell in best with our best determinations.

In all these determinations, as also in those of the capacities for alcohol, the temperature 0° was established by means of a bath of melting ice, higher temperatures by means of a water-bath. The alcohol operated upon was protected against atmospheric moisture by means of a dried well-fitting cork inserted in the funnel-shaped tap of the bottle. The exact tare of the bottle was taken without the cork; but in the weighing the cork (compensated for by a piece of metal placed in the other pan) was left on until the still remaining difference of weight could be determined (after the removal of the cork and of its tare) by three consecutive readings of the oscillating needle. The results are given in the table on next page:—t gives the

Vapour Tension of Methyl-Alcohol in Millimetres of Mercury of 0° C.

Temp.	Tension.	Diff.	Temp.	Tension.	Diff.	Temp.	Tension.	Diff.
0°	29·7	...	25°	122·7	+6·2	50°	409·4	+17·7
1°	31·6	+1·9	26°	129·3	6·6	51°	427·7	18·3
2°	33·6	2·0	27°	136·2	6·9	52°	446·6	18·9
3°	35·6	2·0	28°	143·4	7·2	53°	466·3	19·7
4°	37·8	2·2	29°	151·0	7·6	54°	486·6	20·3
5°	40·2	2·4	30°	158·9	7·9	55°	507·7	21·1
6°	42·6	2·4	31°	167·1	8·2	56°	529·5	21·8
7°	45·2	2·6	32°	175·7	8·6	57°	552·0	22·5
8°	47·9	2·7	33°	184·7	9·0	58°	575·3	23·3
9°	50·8	2·9	34°	194·1	9·4	59°	599·4	24·1
10°	53·8	3·0	35°	203·9	9·8	60°	624·3	24·9
11°	57·0	3·2	36°	214·1	10·2	61°	650·0	25·7
12°	60·3	3·3	37°	224·7	10·6	62°	676·5	26·5
13°	63·8	3·5	38°	235·8	11·1	63°	703·8	27·3
14°	67·5	3·7	39°	247·4	11·6	64°	732·0	28·2
15°	71·4	3·9	40°	259·4	12·0	65°	761·1	29·1
16°	75·5	4·1	41°	271·9	12·5	66°	791·1	30·0
17°	79·8	4·3	42°	285·0	13·1	67°	822·0	30·9
18°	84·3	4·5	43°	298·5	13·5			
19°	89·0	4·7	44°	312·6	14·1	64°·96	760·0	
20°	94·0	5·0	45°	327·3	14·7			
21°	99·2	5·2	46°	342·5	15·2			
22°	104·7	5·5	47°	358·3	15·8			
23°	110·4	5·7	48°	374·7	16·4			
24°	116·5	6·1	49°	391·7	17·0			

temperature (corrected); *h* the level of the alcohol in the stem of the bottle; "Cap" the corresponding weight of alcohol in grammes; $_4S_t$ the weight of 1 c.c. of alcohol of t^0, in grammes, reduced to the vacuum. In these reductions the density of the air was assumed to be at the constant value of 1·2 grammes per litre, which is quite permissible in a case like the present:—

Bottle No. I.				Bottle No. II.		
t.	*h.*	Cap.	$_4S_t$.	*h.*	Cap.	$_4S_t$.
0°	20·0	16·3670	0·810 24	20·0	16·3845	0·810 11
4°·7	20·0	16·2770	0·805 69	20·0	16·2940	0·805 53
9°·7	7·0	16·0955	0·801 20	8·0	16·1155	0·801 19
14°·7	19·9	16·0985	0·796 66	20·0	16·1195	0·796 69
19°·7	20·0	16·0088	0·792 09	20·0	16·0250	0·791 94
29°·7	20·0	15·8217	0·782 64	20·0	15·8370	0·782 41
39°·7	20·0	15·6305	0·772 93	19·8	15·6455	0·772 82
49°·7	20·0	15·4420	0·763 41	20·0	15·4580	0·763 28
59°·7	20·0	15·2380	0·753 12	18·0	15·2420	0·753 13
64°·7	1·0	15·0150	0·748 25	−2·5	14·9910	0·747 85

The two bottles were immersed in the same bath.

A preliminary survey of the results showed that, in accordance with MENDELEJEFF'S proposition, $\frac{\Delta S}{\Delta t}$ is very nearly constant, so that an equation $S_0 - S_t = at + bt^2$ was sure to do sufficient justice to the observed relations. By a proper combination of observations we arrived ultimately at the interpolation formula,

$$S_0 - S_t = 90{\cdot}53t + 0{\cdot}085057t^2,$$

which, as the following comparisons show, sums up the results satisfactorily:—

	Specific Gravity at $t^\circ = {_4S_t}$.	
t.	Calculated.	Observed.
0°,	·810 18	·810 18
4°·7,	·805 91	·805 61
9°·7,	·801 32	·801 20
14°·7,	·796 69	·796 67
19°·7,	·792 02	·792 02
29°·7,	·782 54	·782 52
39°·7,	·772 90	·772 87
49°·7,	·763 09	·763 34
59°·7,	·753 10	·753 12
64°·7,	·748 05	·748 05

The alcohol used for these experiments had *not* been rendered air-free by boiling, but we of course took good care, especially in the determinations at higher temperatures, to make sure that there were no air-bells at the sides of the bottle when the level in the stem was read off. To form an idea of the error introduced by allowing the dissolved air to remain, we made a determination at 14°·7 with air-free alcohol. The alcohol was boiled in the specific-gravity bottle under an inverted (dry) condenser, then corked up while still hot, brought to 14°·7, &c. The specific gravity of the air-free alcohol was found equal to 0·79683, *i.e.*, by 0·00016 higher than that of the original preparation. The difference barely emerged from the limit of unavoidable errors, and we thought that we should probably risk more than we could possibly gain if we adopted the more refined method as *the* method.

Being in possession of a sufficient quantity of the kind of highly purified alcohol which had served for the above determinations, we thought we ought to supply to the chemical community what has hitherto been felt as a desideratum, namely, a set of tables giving the specified gravity of any aqueous methyl-alcohol as a function of its percentage and temperature. We accordingly prepared, by exact gravimetric synthesis, a series of aqueous methyl-alcohols, containing as nearly as possible 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95 per cent. by weight of absolute alcohol, and then determined their specific gravities in each case at 0°, 9°·7, and 19°·7 C. The *modus operandi* was exactly the same as that for the anhydrous alcohol. All the weighings were reduced to the vacuum, and water of 4° adopted as the standard substance at all temperatures. The numbers given as specific gravities accordingly may be read as giving each the weight of 1 c.c. in grammes. All determinations were made in duplicate—one with bottle No. I., another with bottle No. II. In the following table of results, "p" gives the percentage of real CH_4O; the three columns D give 100,000 times the weight of 1 c.c., *i.e.*, the value ${}_4S_t$ for t=0°, 9°·7, and 19°·7 respectively as a mean of two determinations. "Δ" is the deviation of the two determinations from the mean :—

Specific Gravities ${}_4S_t$ *of Aqueous Methyl-Alcohols.*

p	At 0°.		At 9°·7.		At 19°·7.	
	D	Δ	D	Δ	D	Δ
95·062	823 82	4·5	815 30	20·5	806 30	6·5
89·990	837 46	10·5	829 00	8·0	820 44	8·0
79·959	863 54	4·5	855 48	2·5	847 24	10·0
70·063	886 78	6·0	879 45	4·5	871 58	6 5
60·020	908 95	5·0	901 95	3·5	894 75	6·0
50·022	928 62	0·5	922 30	0·5	915 73	3·5
40·028	945 85	5·0	940 45	7·0	934 67	0·5
30·023	960 39	3·5	956 11	7·5	951 58	0·0
20·032	972 46	2·5	970 00	6·5	966 67	3·0
10·018	984 22	4·0	983 42	0·5	981 54	2·0
5·008	991 41	6·0	991 18	3·5	989 61	1·5

With these data before us we began by formulating the relation for the several kinds of aqueous alcohol operated upon, between specific gravity on the one hand, and temperature between 0° and 20° as limit values on the other. With the stronger alcohols the simple formula $S_0 - S_t = at$ afforded quite a satisfactory degree of approximation; the actual function, it is true, proved obviously *non*-linear, but, when the formula $S_0 - S_t = at + bt^2$ was substituted, b assumed, in general, so very small a value that, *with its own a*, the simpler function was practically as correct as the more complex one. Thus we found for absolute alcohol:—

At	0°	9°·7	19°·7
$_4S_t =$	81018	80120	79202
$S_0 - S_t =$	0	898	1816
$\frac{S_0 - S_t}{t} =$	...	92·58	92·17

whence

$$\frac{S_0 - S_t}{t} = 92{\cdot}98 - 0{\cdot}041t;$$

or, if we adopt the linear function

$$\frac{S_0 - S_t}{t} = 92{\cdot}37 \pm 0{\cdot}20,$$

which latter function gives, for 20°, $S_0 - S_{20} = 1847{\cdot}4 \pm 4{\cdot}0$, *i.e.*, an uncertainty of $\pm 0{\cdot}00004$ only.

Our *general* formula for $S_t = f(t)$, however, gives calculated values for 0°, 10°, and 20°, which when used as a basis for the calculation of the a, of the linear function, assigns to it the value 91·75 ± 0·43. We found subsequently that this value falls in better with the general relation between a and percentage than the value 92·37.

For the *aqueous* alcohols we found the following values for the constant of the *linear* function:—

Percentage of CH_4O.	Value of a.
100	91·75 ± 0·43
95	89·40 ± 0·48
90	86·81 ± 0·41
80	82·86 ± 0·12
70	76·35 ± 0·79
60	72·17 ± 0·09
50	65·26 ± 0·11
40	56·20 ± 0·53
30	44·39 ± 0·27
20	27·30 ± 2·04
10	8·25 for 9°·7 and 13·60 for 19°·7

From 20 per cent., or rather somewhere between 20 and 30 per cent. downwards, the value a as we see becomes *in*constant. Retaining the second term in the formula $S_0 - S_t = at + bt^2$, we found the following values for *this* a and for b :—

Percentage of CH_4O.	*a.*	*b.*
100 to 80	...	Some small value
70	74·03	+0·158
60	72·43	−0·018
50	64·94	+0·022
40	54·63	+0·107
30	43·59	+0·055
20	21·30	+0·408
10	3·06	+0·535
5	−4·05	+0·672
0*	−5·65	+0·685

Seeing that the equation $S_0 - S_t = at$ affords a sufficient approximation for all alcohols from 30 per cent. to 100 per cent., we tried to calculate an interpolation formula for the relation between the per-unitage p of CH_4O and the value of a which should cover the whole of this interval, but found that an equation of the second degree did not establish sufficient agreement between experiment and calculation. We ultimately divided the interval into sections as follows :—

I. From $p = 1$ to $p = 0·6$ (*i.e.*, from 100 per cent. to 60 per cent.). Adopted formula

$$a = a_0 + bp + cp^2 ;$$

the constants were calculated from the six experimental values by the method of the least squares, and found to be

$$\begin{aligned} a_0 &= 35·018 ; \log a_0 = 1·544\ 29 \\ b &= 68·379 ; \log b = 1·834\ 92 \\ c &= -11·718 ; \log c = 1·068\ 84 \end{aligned}$$

Contrasting the calculated with the experimentally determined values of a, we have

p.	*a.* By Formula.	*a.* By Direct Det.
1·	91·68	91·75
0·95	89·40	89·40
0·90	87·07	86·81
0·80	82·22	82·86
0·70	77·14	76·35
0·60	71·83	72·17

---

* By Rosetti's Table for the specific gravities of water. Our constants give +4°·1 as the temperature of maximum density.

II. From $p=0{\cdot}6$ to $p=0{\cdot}30$. Again adopting the function $a_0+bt+ct^2$, and calculating by the method of the least squares (which in this case, it is true, is almost out of court), we found—

$$\begin{aligned} a_0 &= -\ \ 6{\cdot}388\,; & \log a_0 &= 0{\cdot}805\ 37 \\ b &= +207{\cdot}53\ ; & \log b &= 2{\cdot}317\ 08 \\ c &= -127{\cdot}88\ ; & \log c &= 2{\cdot}106\ 82 \end{aligned}$$

which gives the following values :—

p.	a_0. By Formula.	a. By Direct Det.
0·6	72·09	72·17
0·5	65·41	65·26
0·4	56·16	56·20
0·3	44·36	44·39

III. For alcohols of less than 30 per cent. we adopted the function

$$S_0-S_t=at+bt^2,$$

and accordingly had to establish the relations between a and p and between b and p. No doubt the best mode of procedure would have been to bring the equation into some form like

$$S_0-S_t=(a_0+xp+yp^2)t+(\beta_0+zp+wp^2)t^2,$$

and to calculate the constants directly from all the experimental data by means of the method of the least squares; but we shrank from the very troublesome calculations which this would have involved, and satisfied ourselves with establishing the relations $a=f(p)$ and $b=\phi(p)$ by separate graphic interpolations.

The results were as follows :—

p	Const. a.		Const. b.	
	Curve.	Exp.	Curve.	Exp.
0	− 6·0	− 5·65*	+0·705	+0·685*
0·05	− 2·2	− 4·05	+0·648	+0·672
0·10	+ 3·3	+ 3·06	+0·581	+0·535
0·20	+20·0	+21·30	+0·398	+0·408
0·30	+44·0	+43·59	+0·060	+0·055

The values a and b for $p=0{\cdot}01$, $0{\cdot}02$ . . . . . . up to $0{\cdot}30$ were read from their curves and tabulated (see the alcohometric table below).

The agreement between experiment and calculation is not as perfect as we

---

* Rosetti's Table.

should have wished; but we could not see our way towards doing better, and consequently left the subject on one side to proceed to calculate interpolation formulæ for

The Relation between the Per-Unitage p *of Methyl-Alcohol and the Specific Gravity* $_4S_0$ *at* 0°.

As the specific gravity for $p=0$ must be that of water at 0°, which we will call W_0, we at once adopted the difference $_4W_0 - {}_4S_0$ as our dependent variable, but found it convenient to take $_4W_4=1000$, and adopted "y" as a symbol for the value which the difference then assumes; while "x" was substituted as a handier symbol for the per-unitage of CH_4O.

A preliminary graphic interpolation showed that there is a change of curvature somewhere about $x=0{\cdot}20$ (corresponding to 20 per cent.), showing that if a parabolic formula worked at all it must at least be of the third degree. Warned by Mendelejeff's experience with ethyl-alcohol, we never attempted to obtain *one* formula for the whole curve, but at once decided upon dividing it into sections. As the part from $x=0{\cdot}25$ upwards exhibited no change of curvature, we tried a variety of functions, including the general equation of the second degree ($Ay^2+Bxy+Cx^2$, &c.), for summing up the relation $y=f(x)$ for $x=0{\cdot}3$ to $1{\cdot}0$ in one formula, but arrived at no satisfactory result. After a deal of pioneering, we ultimately came to divide the curve into the following sections:—

I. From $x=0$ to $x=0{\cdot}4$.
II. " $x=0{\cdot}3$ to $x=0{\cdot}7$.
III. " $x=0{\cdot}6$ to $x=1{\cdot}0$.

First Interval.

We began by bringing the formula $y=ax+bx^2+cx^3$ into the form

$$a+bx+cx^2-\frac{y}{x}=0\,,$$

and then proceeded to calculate the constants a, b, c by means of the method of the least squares. This, as we now see, was an error of judgment, because it is the error in y and not that in $y \div x$ which must be brought to its minimum value; yet the result was satisfactory all the same. We found

$a=+185{\cdot}079$; $\log a=2{\cdot}267\ 357$.
$b=-348{\cdot}682$; $\log b=2{\cdot}542\ 429$.
$c=+559{\cdot}542$; $\log c=2{\cdot}747\ 833$.

affording a sufficient approximation, as seen by the following comparison. "Δy" means the uncertainty in the y as determined experimentally (see p. 524).

x.	y calculated.	y by exp.	Δy.
0·05008	8·465	8·46	±0·060
0·10019	15·609	15·65	0·040
0·20033	27·582	27·42	0·025
0·30025	39·281	39·49	0·035
0·40029	54·104	54·02	0·050

As a glance at the two columns "y" shows, the error of the formula amounts to $-0·209$ in the case of $x=0·3$; the other errors, arithmetically, range from 0·005 to 0·162.

Second Interval.

$x=0·3$ to $0·7$.

By proceeding exactly as in the case of the first interval, we found

$a=+129·871$; $\log a=2·113\ 512$.
$b=-\ 26·5315$; $\log b=1·423\ 762$.
$c=+102·71$; $\log c=2·011\ 629$.

Having found before that a general formula of the third degree for the whole interval from $x=0·3$ to $1·0$ gave fair, though not quite sufficient approximations, we made only two comparisons between y calculated and y found.

x.	y calculated.	y found.	Δy.
0·30	39·332	39·49	0·035
0·70	113·021	113·09	0·060

Third Interval.

$x=0·6$ to $1·0$.

Again, operating as before, we found

$a=+\ 89·001$; $\log a=1·949\ 395$.
$b=+109·012$; $\log b=2·037\ 476$.
$c=-\ \ 8·2891$; $\log c=0·918\ 51$.

We made only one comparison this time:

For $x=1$; $y=a+b+c=189·724$.
By experiment, $y=189·69$.

After having made sure of perfect arithmetical correctness all round, we next calculated by means of the formulæ the values ${}_4S_0$ for all the experimental values x, using each formula for the full range which it had been calculated for. We thus found that the best agreement between experiment and calculation was obtained by using formula II. for the interval $x=0{\cdot}4$ to $x=0{\cdot}6$, and formula I. and III. for their entire intended ranges. But the actual utilisation of the formulæ in this manner would have produced unpleasant, though by no means alarming, discontinuities in the final table at $x=40$ and $x=60$; we therefore ultimately decided upon exhausting each formula, so as to obtain duplicate values for the y's corresponding to $x=0{\cdot}3$ to $0{\cdot}4$, and to $x=0{\cdot}6$ to $0{\cdot}7$, and for each such x took the mean of the two competing values.

This is the history of the entries for ${}_4S_0$ in the following tables.

From these values ${}_4S_0$ and the values $a=(S_0-S_t)\div t$ as calculated by means of the interpolation formula, pp. 526 and 527, we calculated the specific gravities ${}_4S_{15{\cdot}56}$ for 60° F. for all the percentages from 31 to 100.

The corresponding values ${}_4S_{15{\cdot}56}$ for the alcohols from 30 per cent. downwards might have been calculated similarly from the values a and b in formula $S_0-S_t=at+bt^2$ as obtained graphically from the directly calculated values (see p. 527), but we had no perfect faith in these interpolations, and therefore preferred to calculate the specific gravity at 60° F. of each of the alcohols experimented on from the results obtained at 0°, 14°·7, and 19°·7, and, from the set of values ${}_4S_{15{\cdot}56}$ thus obtained, to calculate the coefficients of a formula $y=ax+bx^2+cx^3$ by the method of the least squares.

In this case $y={}_4W_{15{\cdot}56}-{}_4S_{15{\cdot}56}$, where the first symbol stands for the specific gravity of water at 60° F.; water at 4° being taken as = 1000. Calculating on the basis of $a+bx+cx^2-\frac{y}{x}=0$, we found

$$a=+180{\cdot}522;\ \log a=2{\cdot}256\ 530.$$
$$b=-191{\cdot}450;\ \log b=2{\cdot}282\ 055.$$
$$c=+318{\cdot}220;\ \log c=2{\cdot}502\ 727.$$

To test the equation, we calculated the y's for 20 per cent. and 40 per cent., and found

	Value of y by	
x.	Formula.	Experiment.
0·20033	31·039	30·92
0·40029	61·995	61·98

The values given in the following table for ${}_4S_{15{\cdot}56}$ up to 30 per cent., are calculated from this formula; those calculated in the same way for 31, 32, . . . 40 per cent. agreed very well with those deduced from the values ${}_4S_0$ and the equation $S_{15{\cdot}56}=S_0-15{\cdot}56\,a$; but we preferred to let the latter stand uncorrected. As some of our readers may consider this an error of judgment, we give the two values in the following table:—

Percentage of CH_4O.	Value of $_4S_{15\cdot56}$ calculated From $_4S_0$ and linear equations.	 From general equation for $_4S_{15\cdot56}$.
30	953 67	953 55
31	952 11	952 03
32	950 53	950 48
33	948 94	948 91
34	947 32	947 32
35	945 67	945 70
36	943 99	944 05
37	942 28	942 37
38	940 55	940 66
39	938 77	938 91
40	936 97	937 13

TABLE *giving the Specific Gravities of Aqueous Methyl-Alcohol at* 0° *and* 15°·56 C.; *Water of* 4° = 100,000.

I. From 0 to 30 per cent. of CH_4O. $_4S_t = {_4S_0} - (at + bt^2)$.

Percentage of CH_4O.	Specific Gravity at 0°.	Difference.	*a.* (Pos. or Neg.).	*b.* (All Pos.).	Specific Gravity at 15°·56.	Difference.
0	999 87	...	−6·0	+0·705	999 07	...
1	998 06	−181	5·4	·694	997 29	178
2	996 31	175	4·8	·681	995 54	175
3	994 62	169	3·9	·670	993 82	172
4	992 99	163	3·0	·659	992 14	168
5	991 42	157	2·2	·648	990 48	166
6	989 90	152	1·2	·634	988 93	155
7	988 43	147	−0·2	·621	987 26	167
8	987 01	142	+0·9	·609	985 69	157
9	985 63	138	2·1	·596	984 14	155
			All Positive.			
10	984 29	134	+3·3	0·581	982 62	152
11	982 99	130	4·8	·569	981 11	151
12	981 71	128	6·2	·552	979 62	149
13	980 48	123	7·8	·536	978 14	148
14	979 26	122	9·5	·519	976 68	146
15	978 06	120	11·0	·500	975 23	145
16	976 89	117	12·5	·480	973 79	144
17	975 73	116	14·5	·461	972 35	144
18	974 59	114	16·2	·440	970 93	142
19	973 46	113	18·3	·420	969 50	143
20	972 33	113	20·0	·398	968 08	142
21	971 20	113	22·2	·373	966 66	142
22	970 07	113	24·3	·350	965 24	142
23	968 94	113	26·4	·321	963 81	143
24	967 80	114	29·0	·291	962 38	143
25	966 65	115	31·3	·261	960 93	145
26	965 49	116	33·8	·230	959 49	144
27	964 30	119	36·0	·191	958 02	147
28	963 10	120	38·8	·151	956 55	147
29	961 87	123	41·1	·106	955 06	149
30	960 57	130	44·0	·063	953 55	151

II. From 30 to 100 per cent. $_4S_0 - _4S_t = at$.

Percentage.	Specific Gravity at 0°.	Difference.	a.	Specific Gravity at 15°·56.	Difference.
30	960 57	− 130	+44·36	953 67	− ...
31	959 21	136	45·66	952 11	156
32	957 83	138	46·93	950 53	158
33	956 43	140	48·17	948 94	159
34	955 00	143	49·39	947 32	162
35	953 54	146	50·58	945 67	165
36	952 04	150	51·75	943 99	168
37	950 51	153	52·89	942 28	171
38	948 95	156	54·01	940 55	173
39	947 34	161	55·10	938 77	178
40	945 71	163	56·16	936 97	180
41	944 00	171	57·20	935 10	187
42	942 39	161	58·22	933 35	175
43	940 76	163	59·20	931 55	180
44	939 11	165	60·17	929 75	180
45	937 44	167	61·10	927 93	182
46	935 75	169	62·01	926 10	183
47	934 03	172	62·90	924 24	186
48	932 29	174	63·76	922 37	187
49	930 52	177	64·60	920 47	190
50	928 73	179	65·41	918 55	192
51	926 91	182	66·19	916 61	194
52	925 07	184	66·95	914 65	196
53	923 20	187	67·68	912 67	198
54	921 30	190	68·39	910 66	201
55	919 38	192	69·07	908 63	203
56	917 42	196	69·72	906 57	206
57	915 44	198	70·35	904 50	207
58	913 43	201	70·96	902 39	211
59	911 39	204	71·54	900 26	213
60	909 17	222	71·96	897 98	228
61	907 06	211	72·37	895 80	218
62	904 92	214	72·91	893 58	222
63	902 76	216	73·45	891 33	225
64	900 56	220	73·98	889 05	228
65	898 35	221	74·51	886 76	229
66	896 11	224	75·05	884 43	233
67	893 84	227	75·57	882 08	235
68	891 54	230	76·10	879 70	238
69	889 22	232	76·62	877 14	256
70	886 87	235	77·14	874 87	227
71	884 70	230	77·66	872 62	225
72	882 37	233	78·18	870 21	241

II. From 30 to 100 per cent.—*continued.*

Percentage.	Specific Gravity at 0°.	Difference.	*a*.	Specific Gravity at 15°·56.	Difference.
73	880 03	234	78·69	867 79	242
74	877 67	236	79·20	865 35	244
75	875 30	237	79·71	862 90	245
76	872 90	240	80·22	860 42	248
77	870 49	241	80·72	857 93	249
78	868 06	243	81·23	855 42	251
79	865 61	245	81·73	852 90	252
80	863 14	247	82·22	850 35	255
81	860 66	248	82·72	847 79	256
82	858 16	250	83·21	845 21	258
83	855 64	252	83·70	842 62	259
84	853 10	254	84·19	840 01	261
85	850 55	255	84·67	837 38	263
86	847 98	257	85·16	834 73	265
87	845 39	259	85·64	832 07	266
88	842 78	261	86·12	829 38	269
89	840 15	263	86·59	826 68	270
90	837 51	264	87·07	823 96	272
91	834 85	266	87·54	821 23	273
92	832 18	267	88·01	818 49	274
93	829 48	270	88·48	815 72	277
94	826 77	271	88·94	812 93	279
95	824 04	273	89·40	810 13	280
96	821 29	275	89·86	807 31	282
97	818 53	276	90·32	804 48	283
98	815 76	277	90·78	801 64	284
99	812 95	281	91·23	798 76	288
100	810 15	280	91·68	795 89	287

APPENDIX.

The vapour density apparatus referred to in the context is constructed on the same principle as the measurer in the set of apparatus for gas analysis which was introduced by one of us some years ago (see "*Challenger*" *Memoirs, Physics and Chemistry,* vol. i. p. 143). As shown by Plate XXXIII. fig. 1, it consists of a Gay-Lussac burette-like combination of a wide with a narrow glass tube, which, by means of a long tube of capillary india-rubber slipped over the lower end *a*, communicates with a movable mercury reservoir *b*. The main tube near its upper end is contracted so as to produce a neck which is provided with a very well ground-in glass stopper. The same tube bears a millimetre scale; the gas volumes corresponding to the several points are determined by gravimetric calibration with mercury. The apparatus when used is fixed within a glass jacket, in which the required temperature is produced and maintained by means of a continuous current of steam or other vapour.

To make a determination, the apparatus is filled completely with mercury, and all air-bells are carefully removed. A known weight of the liquid to be operated upon is sealed up in a small bulb which must be almost absolutely full, so that, when heated to the respective temperature, it bursts by internal liquid pressure. The bulb is introduced through the neck, held down by the stopper, while some mercury is made to rise over it into the funnel, and the stopper then inserted. A few centimetres of mercury within the funnel make an absolutely gas-tight joint. The mercury reservoir is in general so placed that the pressure within the apparatus is less than that of the atmosphere; at the very end, however, it is so adjusted that the two menisci, in the narrow and in the wide tube, are in the same horizontal plane. The pressure of the vapour is then equal to $B+p$, where B is the height of the barometer, and p the small excess of capillary depression in the narrow as compared with the wide tube. The correction p is easily determined by filling the apparatus partly with mercury, and reading the difference of level between the two menisci when both tubes are open to the atmosphere. It amounts to less than one millimetre.

The special advantage claimed for the apparatus is that there is no need of any correction for the temperature of the mercury within the apparatus, which, with the ordinary constructions, is so uncertain.

When the apparatus was used for the special tension determinations reported on on pages 514 and 515 of this Memoir, the upper end of the narrow side tube communicated with the air-space of the mercury reservoir, and through it with that of the shorter limb of the barometer, and with the air-pump, as shown by the figure.

The same artifice might be resorted to for determining vapour densities at low pressures; but we have had no occasion to make such determinations.

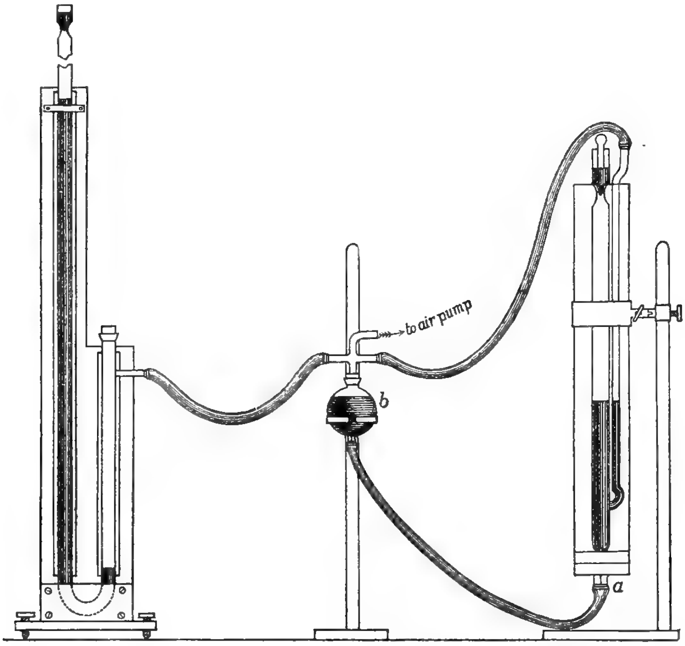


Fig. 1.

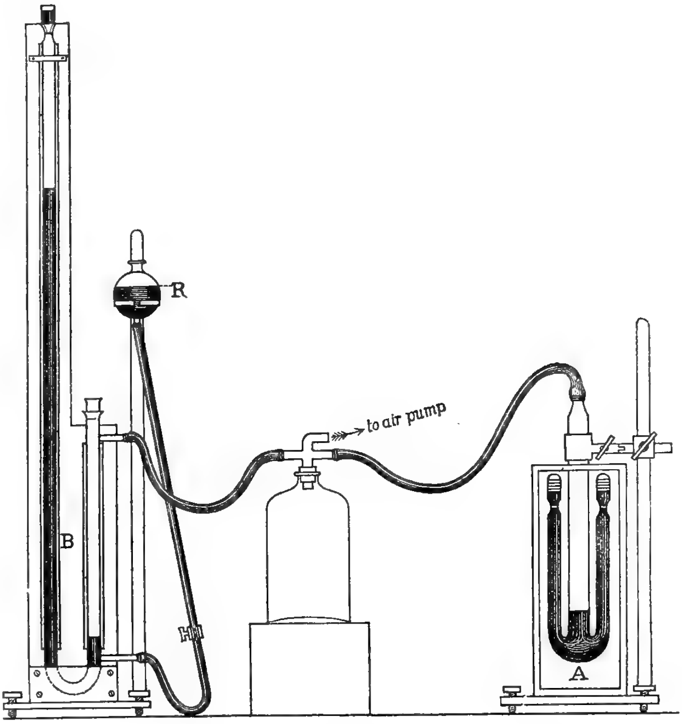


Fig. 2.

XXIV.—*On the Thermal Conductivity of Iron, Copper, and German Silver.* By A. CRICHTON MITCHELL, Esq. Communicated, with an Introduction, by Professor TAIT. (Plates XXXIV., XXXV.)

(Read 4th July 1887. Revised 24th October 1887.)

[INTRODUCTION.

Shortly after I read to the Society my paper on "Thermal and Electric Conductivity" (*Trans. R. S. E.*, 1878), in which I stated that the results were "by no means final, even so far as my own work is concerned," I was requested by Sir WYVILLE THOMSON to undertake the examination of the "Pressure Errors of the 'Challenger' Thermometers." This investigation led to another on the "Compression of Sea-Water," and allied subjects, which is not yet finished. Meanwhile, though I had prepared everything for my promised repetition of the experiments on Thermal Conductivity, the bars formerly used having been nickelised, &c., I found that it would be impossible for me to carry out the investigation. I therefore asked Mr MITCHELL, who, as Neil-Arnott Scholar, had already done good and careful work on Thermal Conductivity in my Laboratory, to repeat the experiments under the altered conditions. I put at his disposal all the apparatus which was employed in the former research. The Government Grant Committee allowed a sum for the payment of a computer to reduce the results, and the observations were at once commenced. The results are now laid before the Society, and are probably as good as the method and the thermometers employed can furnish.

As regards the method, one grand defect is the uncertainty as to the relative amounts of surface loss of heat in the two parts of the experiment. The nickelising has, to a very great extent at least, removed the part of this uncertainty which was due to oxidation of the bars; but there remains another part, not at all easy to reckon and allow for, which depends on the fact that each thermometer in the long bar is maintained for hours in a nearly constant state of graduated temperature throughout its stem, while the corresponding state of that in the short bar not only varies rapidly as the cooling proceeds, but probably always materially differs from it. No attempt has been made to correct the results so far as this cause of error (which is probably of no great importance) is concerned. It is clear that its effect will be to make the rate of cooling a little too small at the lower, as compared with the higher, temperatures.

Another defect, which indeed FORBES pointed out, is due to the very small temperature-gradients towards the colder end of the long bar. Mr MITCHELL has carried out my suggestion of an artificial cooling of the middle of the bar, and it is highly interesting to compare together the results he has obtained with and without this cooling.

ANGSTRÖM expressly stated (*Pogg. Ann.*, cxviii. 1863) that no account need be taken of the change of specific heat with temperature. In my paper above referred to, I said that it appears that, in iron especially, this change produces a very considerable effect on the estimated values of the conductivity. In default of better data, Mr MITCHELL has used those given (after NICOL and others) in a short paper in *Proc. R. S. E.*, p. 126, 1880–81, and in my *Heat*, § 246. The importance of this correction is shown by the comparison of the results obtained from it with those obtained when it is not applied. Mr MITCHELL'S experimental results are given in such a form that any subsequent improvement in these data can be taken advantage of without further experiment, and with very little trouble in the matter of calculation. The fact that the various short bars were exactly similar in surface in his experiments has enabled him to make a rough test of the accuracy of these data.

In the short paper above referred to, I showed that the consideration of the rise of specific heat with temperature would destroy if not overcome the apparent fall of conductivity of iron at higher temperatures. But I had not then the means of properly applying the correction without repeating about one-half of the laborious calculations incident to FORBES' method. Mr MITCHELL has in his calculations taken account of this consideration: and it must be regarded as one of the chief features of his paper that he has thus shown that iron does not form an exception to the law that ordinary metals *improve* in thermal conductivity as their temperature is raised.

As I am responsible for the methods employed by Mr MITCHELL in the experiments and calculations, though not for the calculations themselves, I must state here the directions given and the grounds for them, at least in so far as they introduce processes differing (to any considerable extent) from those used by FORBES or by myself.

1. As to the empirical formula (B) for the statical curve, in the special case of the iron bar when there was no artificial cooling.

This I obtained by plotting the logarithms of the temperature excesses as ordinates, the abscissæ being distances along the bar. The curve so obtained was nearly straight at the lower temperatures, and became rapidly more curved at higher temperatures. I therefore treated it as a branch of a hyperbola, and found its asymptote. Thus the *form* of the empirical expression was suggested at once.

2. The allowance for the unequal heating of the stems of the thermometers was obtained thus :—

Let v be the observed temperature (not the temperature excess), w the true temperature, and in accordance with § 10 of my paper $e=10/250^2=0{\cdot}00016$. From the result of Mr MITCHELL's comparison of the two thermometers, one partially, the other wholly, immersed in a paraffin bath, I have been confirmed in my assumption of an error of 10° at 250° C. Then we have

$$w=v+ev^2.$$

Thus, for the true temperature-gradient in the statical experiment,

$$\frac{dw}{dx}=(1+2ev)\frac{dv}{dx}.$$

Similarly, for the true rate of cooling, we have

$$\frac{dw}{dt}=(1+2ev)\frac{dv}{dt}.$$

The quantities on the right-hand sides are given by the experiments, or deduced directly by graphical methods or calculation.

For the statical curve of cooling it is easy to see in this way that each instalment of area must be multiplied by

$$1+2e\frac{v_1+v_2}{2},$$

where v_1 and v_2 are the limiting temperatures of the instalment.

It is clear that this correction increases the gradient at any point of the bar in a greater ratio than that in which it increases the total area of the corresponding part of the curve which expresses the flux of heat; so that its effect must be to diminish the estimates of conductivity at higher, more than at lower, temperatures.

3. I was much surprised at the first results obtained by Mr MITCHELL for the rates of cooling at high temperatures. At my instance he has repeated this part of the experiment in a form similar to that which I had employed, and certainly less likely to entail error, and the data thus obtained have been incorporated in the paper, in so far as they relate to the specified tables. [The remaining small difference between our results may be due to an over-estimate in my 6 p.c. reduction for oxidation.]

4. There still remains a possible source of error, due to the thermometers themselves:—Kew Standards though they be. This arises from the way in which the 200° C. and 300° C. points were determined at Kew. The

tubes having been carefully calibrated before filling, the standard points 0° C. and 100° C. were directly determined in the usual manner. But the positions of 200° C. and 300° C. were determined by taking successive portions of the tube whose volume (cold) corresponded to that of the portion (also cold) from 0° C. to 100° C. I have not the means of making allowance for this defect, which will probably mar all experiments of the kind until suitable air-thermometers are employed.

5. The fact that the conductivity deduced from experiments on the iron bar, when its full length is employed, differ so considerably from those obtained when it is artificially cooled in the middle, appears to be intimately connected with a remark made in my paper (§ 14) that "in measuring conductivity, at whatever temperature, things ought to be arranged so as to avoid any slow flux of heat." It seems that, even after the lapse of eight hours, the steady state of temperature has *not* been reached in the colder parts of the long iron bar.

6. As the numerical data, concerning specific gravity and specific heat, which Mr MITCHELL has (in default of better) been obliged to employ, are only rough estimates, I asked him to test them by finding the ratios of the rates of cooling of copper and iron at various common temperatures. The surface material was the same in the two bars, and their dimensions equal, so that the *amount* of heat lost in a given (short) time must have been the same for each at the same temperature. The ratio of the rates of cooling should therefore be constant for all temperatures if, and only if, the rate of change of specific heat with temperature be the same for each of the two materials. The result does not seem to favour the accuracy of the assumed data, but the process employed is not by any means an accurate one.

7. As my determinations of the relative electric conductivities of the bars had been verified by Mr D'ARCY THOMPSON, there is no necessity for their repetition. But, using them, with Mr MITCHELL'S results for thermal conductivity, my comparative table (*Trans. R. S. E.*, 1878, p. 739) should be altered (subject, of course, to correction for improved values of specific gravity and specific heat) to something like the following :—

	Thermal.	Electric.
Copper (Crown), ...	1·5	1·729
" (C.), ...	1·0	1·000
Forbes' Iron, ...	0·23	0·264
Lead, ...	0·12	0·149
German Silver, ...	0·13	0·117

P. G. TAIT.]

I. Experimental Methods.

As already stated, this work was intended to be a repetition of Prof. Tait's experiments upon the same bars, and accordingly every care was taken that it should be conducted under conditions as nearly as possible similar; except, of course, the important feature of the nickel films. For a full account of the original experimental details, reference may be made to the papers of Forbes (*Trans. R. S. E.*, 1860–61 and 1864–65); and for subsequent improvements, to that of Prof. Tait (*Trans. R. S. E.*, 1878). To what is there given concerning the experimental means employed little can be added. The set of thermometers, the position and use of each, the maintenance of regularity in gas pressure, and consequently in the steadiness of temperature during the last few hours of the long bar experiment—all were exactly the same as in the preceding work; and any slight novelty in this part of the work was more by way of addition to, than change in, the method. This was essential in order that a thorough comparison might be made between the two series of results.

The four metallic bars experimented upon were, respectively, wrought iron (that originally employed by Forbes), copper of superior electric conductivity (copper, crown), copper of inferior electric conductivity (copper, C.), and German silver. These, together with the short bar corresponding to each, were all plated with nickel. The plating, although comparatively light, was of good quality. Before beginning work on one of the bars, it was polished with fine rouge and paraffin oil, and afterwards thoroughly dried; while during the progress of the experiments on the long bars these were kept free from dust, &c., or anything likely to interfere with the normal amount of radiation. It was found that, when the distances along the bars, especially between the closer holes, were measured by a dividing engine, some small temperature correction would require to be made for the (slight) inexactness of the position of the holes. In no case was this correction greater than one-third of a degree C., and this, since it was only required at the highest temperatures, was well within the limits of other inevitable and undetermined errors.

The thermometers used in this inquiry were, as already stated, those used by Prof. Tait in his experiments. They were made under the direction of Dr Balfour Stewart, and afterwards carefully corrected at Kew. A fair idea of their quality *before* they had been used for any experimental work of this nature may be obtained from the following table, which gives the corrections according to the Kew certificates :—

Kew Standard, No.	Range Centig.	Correction at			
		0° C.	100° C.	200° C.	350° C.
*431	0° – 100°	0·0	–0·1	...	...
432	0° – 100°	0·0	–0·1	...	...
433	0° – 100°	0·0	0·0	...	...
434	0° – 100°	0·0	0·0	...	...
435	0° – 100°	0·0	+0·1	...	...
436	0° – 200°	0·0	0·0	0·0	...
437	0° – 200°	0·0	+0·2	+0·1	...
438	0° – 200°	0·0	+0·1	+0·1	...
439	0° – 200°	0·0	0·0	0·0	...
440	0° – 200°	0·0	+0·2	+0·1	...
441	0° – 350°	0·0	+0·15	+0·1	...
*442	0° – 350°	0·0	+0·15	+0·25	...
443	0° – 350°	0·0	+0·1	+0·25	...
444	0° – 350°	0·0	+0·25	...	+0·3
*445	0° – 350°	–0·1	–0·1	...	+0·1
446	0° – 350°	0·0	+0·2	...	+0·3
447	0° – 350°	0·0	+0·15	...	+0·1
*449	0° – 350°	0·0	+0·1	...	+0·1
*450	0° – 350°	0·0	+0·05	...	+0·05

* Not used in the experiments.

But after several exposures to high temperatures, sometimes nearly 300°, the thermometers began to show an error, and that one of increase, in the indications. The amount of this error in each instrument was found by comparing the reading taken when the long bar had cooled down overnight after the previous day's heating, with the simultaneous reading of a thermometer in the short bar standing in the vicinity of the long bar. This latter thermometer was only used for temperatures which did not exceed that of the air, and had never been subjected to any such treatment as had those it was employed to correct. The difference of the two readings named was taken as the error of the first, and was applied in the reduction of the experiments. In the case of the thermometers used in the hottest hole in the bars, this error amounted to about seven degrees at ordinary temperatures.

A series of corrections for the different thermometers was thus obtained, and in order to test whether the readings obtained from one day's work, and after the application of these corrections, were consistent with those of another day, the thermometers in the first three holes were interchanged among each other, and also with the thermometer which was always used to indicate the temperature in the cooling experiments on the short bar. It was found that the different sets of readings when so tested were consistent with each other.

This, however, is a source of less error than that which arises from the circumstance that the temperature (especially at any point near the hot end) of the long bar during an experiment is given by a thermometer with an unequally heated stem. In connection with this, Prof. TAIT calculated that for a temperature of 250° C. "the utmost error that can be introduced in the indications of the thermometers used is somewhere about 10° C. That is to say, the highest temperatures were read, at the most, 10° *lower* than they would have been if the whole thermometer had been exposed to the same temperature. This correction of 10° at 250° diminishes at lower temperatures and increases at higher, nearly as the square of the excess of the temperature above the freezing point" (*Trans. R. S. E.*, 1878).

Wishing, if possible, to obtain some experimental verification of this computation, I had two thermometers constructed, with stem, bulb, capacity, bore, and length of degree similar to each other and to those used in determining the temperature towards the hotter end of the long bars. They were tested in the following manner. One was placed vertically, with *bulb* only immersed, in a bath of melted paraffin wax; the other, placed horizontally, was wholly immersed in the same bath. The paraffin was maintained at a steady temperature throughout. After their indications had become stationary, the thermometers were read, and the excess of the reading of the wholly immersed, over that of the partially immersed, thermometer noted. From several experiments of this kind, it was calculated that the error in question amounts, at 250° C. to 9°·5, an exceedingly close verification of the previously calculated estimate.

In the reduction of the experimental readings, and in the deduction of conductivity this error has not been taken into account, chiefly because the thermometer used in the experiment on the cooling of the short bar was almost exactly the same in construction as those used in the long bar, and that therefore the difference in this respect between the two experiments must be small. In *Appendix* II., however, I have endeavoured to correct for this source of error.

Statical Experiment.—The bar to be experimented upon had one end inserted into an iron crucible containing melted solder, which is heated by a powerful Bunsen burner. Heat conducted along the bar raises the temperature of each portion of it, and this is carried on until the flow of heat across each section has become steady, a state indicated by the steadiness of the thermometric indications along the whole length of the bar. This condition was attained after (about) 7 hours in the case of iron, 6 in German silver, and after 5 hours in copper. This steadiness of state is then maintained for some time, generally at least an hour, after which the distribution of temperature along the bar, as well as the temperature of the unheated short bar in the vicinity, is carefully determined.

Initially, therefore, the success of this part of the experimental work depends upon the maintenance of this particular distribution of temperature, after it has been reached. This means that (besides such minor details as draughts of air, and a steady or a slowly changing temperature in the laboratory) great regularity in gas pressure has to be ensured.

The gas regulator, devised by Prof. Crum Brown, was employed in this inquiry also. Its working is as effective as its construction is simple; and its automatic action, hour after hour, relieves the experimenter of what would otherwise be the necessity of constantly noting and regulating the temperature of the melted solder in the crucible.

The following extract from my note-book shows sufficiently how perfect are the results obtained by employing it:—

German Silver.

July 9, 1886.
Gas lit 6·50 a.m.

Hour p.m.	2.30	2.45	3.0	3.15	3.30	3.45	4.0	4.15	4.30
Temp. in hole nearest hot end,	270·85	270·9	270·7	270·6	270·7	270·6	270·7	270·7	270·75

This, showing as it does only an extreme variation in the temperature excess of ·09 per cent., bears out what has been said. The room in which the experiments were conducted was lighted from the north alone, the south windows and the doors were kept closed, and the air temperature throughout the day did not change so rapidly as to cause any uncertainty in the results.

A slight modification of the experiment on the long bar was tried, enabling the method to give results with more certainty. Where, as in iron, the flux of heat across any section at the lower temperatures (*i.e.*, below 20°) is small, a small error in the determination of the temperatures makes a comparatively large difference in the estimate of conductivity. In order to avoid this, a suggestion of Prof. Tait's (first given in *Trans. R. S. E.*, 1878, p. 734) was adopted. While the bar was heated in the ordinary manner, a cold water bath was placed halfway up the bar towards the heated end, and through it a stream of water was continually passed from below. This had the effect of "steepening" the temperature-gradient, and thus allowing the measurement or calculation of the tangent to be made with greater exactness. This process is almost necessary in the case of metals the change in whose conductivity with increased temperature is so small that the ordinary experiment of Forbes is insufficient to detect its sign and amount.

An idea of the effect of this midway cooling on the distribution of temperature along the bar may be obtained from the following table, in which are compared the temperature excesses at the various holes in the bar, both in the ordinary experiment and in that with the cooling bath.

IRON.

Distance from origin in inches.*	Temperature Excess °C.	
	11/6/86. Bar *not* cooled.	13/4/87. Bar cooled 46 in. from origin.
0	229·3	229·35
3	162·5	163·3
6	118·45	119·85
9	87·6	89·5
15	50·35	51·0
21	29·8	29·95
27	18·25	17·65
33	11·4	9·6
45	4·6	−1·05 †

* The hole in the bar nearest the heated end is taken as origin.

† The − sign indicates that the actual temperature of the bar at this point was *below* that of the short bar standing in the vicinity.

Cooling Experiment.—The chief difficulty hitherto in this part of the experimental method has been the oxidation of the short bar when heated to a high temperature. This introduces an uncertainty into the final results owing to the necessity of applying a value of the "rate of cooling" of the short bar at a particular temperature, to a portion of the long bar whose surface at that temperature is generally in a different state from that of the short bar at the same temperature. And although a correction may be applied to the ordinates in the curve of cooling in order to remove this, its application must in many cases be a matter of doubt.

In this work, however, this uncertainty has been almost entirely removed by the nickel-plating of the bars; the difficulty arising from an oxidised surface never even presented itself, for at the end of each experiment the surface of the short bar on cooling had retained its original brightness, and had, except in a slight manner to be yet noted, in no way been affected. So that the condition which is necessarily involved in the deduction of the conductivity, viz., that the surface of the long bar at any particular temperature should be the same as, or, at least, strictly comparable with, that of the short bar when at the same temperature, was very fully realised.

Each of the short bars experimented upon was so placed, on bearings, in a rack, that it was possible to rotate it, on its long axis, while it was being heated. The temperature of the bar was raised by placing beneath it a row of 50 very small Bunsen burners. The heating was proceeded with as cautiously as possible, and generally (where it finally reached about 270°) occupied about 2 hours. This, so far, guaranteed an equable distribution of temperature throughout the bar. But, for a reason to be shortly stated, the results thus obtained were not used.

To this method of heating there is an objection. There is always a *slight* smoking of the Bunsen burners, and this, together with moisture deposited at the commencement of the heating, dulls the bright plated surface of the bar to some extent. At the instance of Prof. TAIT, who was surprised to find the rates of cooling greater with the nickelised than with the plain bars, these experiments were repeated in a form calculated to avoid any deposition of moisture, and to avoid the smoking as far as possible. The bars were heated to over 100° C. before a clear fire, and then *as quickly as possible* raised to a high temperature over the row of Bunsen burners. The results of *this* set of experiments have been used in the calculations. In *Appendix* I. a comparison between the results of the two methods is given.

The temperature of the bar during cooling was observed by means of a particular thermometer, which was used almost exclusively (with an exception already stated) for this purpose. As might be expected, the zero of this instrument rose after repeated exposures to such high temperatures as it was employed to indicate. But this error was carefully estimated. As already detailed, the short bar during the progress of the statical experiment was placed near the long bar, while its temperature (practically that of the surrounding air) was recorded by a thoroughly trustworthy thermometer. The thermometer used in the cooling experiment was simultaneously placed in one of the holes in the short bar, and its readings compared with those of the other thermometer in the bar. A continuous process of correction was thus established; and by always placing the thermometer in this position when not used for the cooling experiment, the gradual change which the amount of the error underwent as the experimental work proceeded was carefully noted.

Prof. TAIT, in the paper already referred to, pointed out the importance of raising the short bar to a temperature considerably higher than that *actually* required for the observation of the rate of cooling at any particular temperature. The exact reason for this is, that although the mercury in the bulb of the thermometer is heated almost at once, the column of mercury in the stem is not so heated for some little time after, and that the cooling of the two regularly together also does not ensue for some time. For example, in order to obtain a determination of the rate of cooling at 200° C. (as accurately as the method will allow), it is necessary to raise the bar to a temperature of 250° or 260°, insert the thermometer as quickly as possible, and allow the bar to cool down through the temperature required. For a full experimental proof, see *Trans. R. S. E.*, 1878, pp. 730, 731. This affords a complete explanation of the curious result FORBES arrived at, viz., that the curve showing rate of cooling in terms of temperature excess exhibited a point of flexure about 150° C.

By adopting this precaution I have verified Prof. TAIT'S results, and although in some cases the temperature excess was as much as 280°, no such result as that obtained by FORBES was even indicated.

In addition to the usual cooling experiments, a series was conducted at temperatures as high as it was possible to safely observe with a mercurial thermometer, generally about 330° C. These experiments were of great use in accurately estimating the rates of cooling which are necessary for the higher portions of the statical temperature curve. They were not made until all the normal experimental work was over, as they involved some risk of the safety of the thermometers employed.

II. DEDUCTION OF CONDUCTIVITY.

Statical Curve.—Although the manner in which this curve was constructed was similar to that in previous work, it may yet be well to describe it in detail.

From among all the statical experiments on the metal bar for which the curve was required, that one was chosen which, in its approach to, and maintenance of, the final thermal distribution, appeared to be the most steady, and when considered in all respects, the most successful. This was termed the *Standard Experiment.* The readings taken during the steady state of the bar were then examined, and one special set of readings, apparently the most trustworthy, was selected. The appropriate corrections having been applied, the true temperature excess at each hole in the bar was obtained by subtracting from the corrected reading the reading taken simultaneously of the temperature of the short bar. The series of numbers thus obtained were then laid down as points on a curve, in which the abscissæ represented distance along the experimental bar, and the ordinates denoted temperature excesses. The other experiments on the same bar were then taken in the order of their apparent merits, regarded in the same way as in selecting the standard experiment. The temperature excesses were plotted separately for each experiment, on a sheet of tracing paper superposed upon a blank part of the sheet of divided paper on which had been marked the points obtained from the standard experiment. The axes on both sheets were coincident. After each experiment had been so represented on the tracing paper, it was carefully drawn along, keeping the horizontal axes still coincident, over the sheet beneath, until the points marked upon both lay in one smooth curve. Each point on the superposed tracing paper was then transferred to that, immediately covered by it, on the sheet below. In this manner a series of points was obtained, through which a smooth curve was drawn, exhibiting the relation existing between temperature excess and position along the bar—what FORBES termed the statical curve of temperature.

It may be remarked that once the standard experiment, along with, say, two other decidedly trustworthy experiments, is chosen, and the three represented by a curve in the manner already described, we have therein furnished a test of the correctness of the others; indicated by the extent to which they agree with the first. By this means, the calculator is enabled to set aside as less trustworthy, certain of the experiments. The only discrepance, which falls to be reported in this connection, between the several experiments on the same bar, lies in the uncertainty connected with the representation by the curve of the temperature excess at the cool end of the bar. Here it was somewhat difficult to trace the curve accurately, and to determine the point on the bar where, for all practical purposes, at least, the temperature excess disappears. An error of a tenth of a degree is quite sufficient to cause this doubt. This circumstance argues strongly in favour of the adoption of that modified form, already detailed, of the long bar experiment, viz., where the bar is cooled midway by the application of a cold water bath; for in this case there is no dubiety whatever as to the point where the horizontal axis is crossed by the curve of temperature-excess.

Regarding the equation representing the curve, two formulæ were employed:—

$$\log v = \log A - \frac{bx}{1+cx} \quad \ldots\ldots\ldots\ldots \quad (A)$$

$$\log v = \log A + \frac{b}{c+x} - ex \quad \ldots\ldots\ldots\ldots \quad (B)$$

where v = temperature excess,

x = distance along bar, reckoned from any arbitrary origin; and where

A, b, c, and e are constants.

The formula (A), involving three constants, originally employed by REGNAULT (*Mém. Ac. Sci.*, vol. xxi.) to represent the relation between the temperature and pressure of saturated water-vapour, was that used by FORBES and also by Prof. TAIT, to represent the curve of temperature-excess in iron. But the differences between the *calculated* and *observed* values of v, both as shown by the tables in FORBES' last paper, and also when used by myself, were such as to lead to the construction, for use in this work, of the empirical formula (B). It was constructed by Prof. TAIT to suit as closely as possible, by four disposable constants, the curve plotted from the logarithms of the temperature excesses obtained in one of my earlier experiments on FORBES' iron bar. For the iron curve, it has been found to work very well, and certainly much better than its predecessor. But in the case of both copper bars and German silver, as well as the iron bar cooled midway, the equation (A) has been found to be more applicable, and has accordingly been used. In some cases it was found better to take the curve in two or more sections, employing for each the proper corresponding values of the constants.

A conception of the relative value, for this purpose, of the two formulæ may be obtained from the two tables subjoined, showing the degree of approximation of the value of v, the temperature excess, as calculated by the formula, to the value as given by the observational curve. The first, headed (F), is extracted from FORBES' paper already quoted; the second, headed (M), is the record of one of my own experiments, along with the values of v calculated from formula (B); while the third table, also headed (M), shows in how far the same experimental results are approximated to by formula (A). These tables refer, of course, to iron.

I. (F).

x, in feet.	v by experimental curve.	v by formula $v=\log A-\frac{bx}{1+cx}$	Difference.
	°C.	°C.	
0	...	272·7	...
0·25	190·5	191·0	+0·5
0·5	134·7	135·9	+1·2
0·75	97·3	98·23	+0·93
1·0	72·0	72·0	0·0
1·25	53·6	...	...
1·5	40·8	40·21	−0·59
2·0	24·2	23·58	−0·67
2·5	14·8	...	...
3·0	9·33	9·08	−0·25
4·0	4·0	4·0	0·0
5·0	1·8	1·96	+0·16
6·0	0·9	...	...
8·0	0·28	0·36	0·08

II. (M).

x, in feet.	v by experimental curve.	v by formula $\log v=\log A+\frac{b}{c+x}-ex$	Difference.
0·0	247·2	246·6	−0·6
0·25	172·1	173·8	+1·7
0·5	125·25	125·5	+0·25
0·75	92·3	92·2	−0·1
1·25	52·0	52·0	0·0
1·75	30·35	30·35	0·0
2·25	18·2	18·2	0·0
2·75	11·15	11·1	−0·05
3·75	4·3	4·3	0·0
4·75	1·85	1·7	−0·15
5·75	0·7	0·7	0·0

III. (M).

x, in feet.	v by experimental curve.	v by formula $\log v = \log A - \frac{bx}{1+cx}$	Difference.
0·0	247·2	245·4	−1·8
0·25	172·1	172·1	0·0
0·5	125·25	125·45	+0·2
0·75	92·3	92·3	0·0
1·25	52·0	52·0	0·0
1·75	30·35	30·6	+0·25
2·25	18·2	18·72	+0·52
2·75	11·15	11·65	+0·5
3·75	4·3	4·35	+0·05
4·75	1·85	2·53	+0·68
5·75	0·7	1·34	+0·64

The next table shows how far the formula (A), on the other hand, suits the case of the iron bar cooled midway.

x, in feet.	v by experimental curve.	v by formula $\log v = \log A - \frac{bx}{1+cx}$.	Difference.
0·0	206·35	206·35	0·0
0·25	150·85	150·75	−0·1
0·5	111·4	111·2	−0·2
0·75	83·45	82·85	−0·6
1·25	47·7	47·25	−0·45
1·75	28·15	27·9	−0·25
2·25	16·5	16·95	+0·45

The agreement shown here, though not so close as that shown in II. above, was much closer than that which the use of formula (B) gave.

LAMBERT (*Pyrometrie,* Berlin, 1779) showed, from experiment, that the curve of stationary temperature along a bar could be represented by an equation of the form

$$v = A\epsilon^{-px}.$$

whence

$$\log v = \log A - px.$$

So that the curve representing, at each point of the bar, the logarithms of the temperature excesses should be a straight line.

This result was shown to hold for the case of copper (C) by Prof. TAIT (*Trans. R. S. E.*, 1878).

Diagram 2 shows the degree of approximation to it in this work. With the exception of that of German silver, the curves for the other bars did not exhibit the result in question to the same extent.

In connection with the consideration of the statical curve, it will be well to compare in this respect the results of this work with those of Principal FORBES and Prof. TAIT. The curves for iron, according to these three different sets of experiments, are shown in Diagram 1. From the disposition of these curves one to another, it will be seen that (1), that given by FORBES (marked F), shows a greater rate of change of temperature along the bar than either of the other two; (2) that my own experiments, *i.e.*, on the nickel-plated bar (marked M), show a slower rate of change than those of Prof. TAIT (marked T); (3) that all three agree with comparative closeness at the lower temperatures.

The first of these remarks is confirmed when the values of dv/dx at different temperature excesses are plotted. The values which FORBES used are then seen to be all greater than those used in the deduction of conductivity in this work; on the average they are about 10 per cent. larger. No details of this kind (*i.e.*, values of tangents used, &c.) are printed in Prof. TAIT's paper, otherwise they would have formed an interesting comparison.

In the calculation of the tangents the values obtained by differentiation of the formula representing the curve have mainly been relied upon, except at the lower temperatures, where graphical measurements have been made and used. But some allowance must be made for the discrepance between the calculated and observed values of v. It was effected in this way. By the sign and amount of the difference between these two values, it can be determined whether at any given point the curve (by calculation), in bending away (below or above it) from the observational curve, makes the calculated values of dv/dx too small or too great. By taking other values of the constants, a value of dv/dx may be obtained with a probable error, at any given point, of opposite sign to that obtained by the first value of the constants. Thus it is known between what two limits lies the accurate value of the tangent at the given point.

Excepting the remark made that the value of the tangents graphically measured agreed satisfactorily with the value calculated from the formula, FORBES seems to have used *alone*, and without any such modification as is mentioned above, the values of dv/dx as they are given directly by one particular set of values of the constants,—viz., that set which appeared to give the best agreement with the observed temperature excess along the bar. Treating the curve in sections, as he did, partly avoids the difficulty; still, it cannot be doubted that without any allowance of the kind mentioned, the calculated values of the tangent to the curve, constrained as it is to pass through three points (or four, as the case may be), cannot be received and adopted without involving the possibility of occasional large error.

Some assistance may be obtained from the graphic representation of the value of dv/dx for different values of v. It sometimes gives a means of casting out untrustworthy estimates, and at all events prevents the entrance into the calculations of any great error. Diagram 3 illustrates its use in the case of the iron bar.

Curves of Cooling.—In the reduction of the observations, the same methods were employed as in previous work. The deduction of the rates of cooling was performed mainly by constructing a curve whose ordinates represented successive differences of temperature of the cooling bar (or $\frac{1}{2}$, $\frac{1}{3}$, &c. difference, where the interval of observation was 2, 3, &c. times the time-unit), and whose abscissæ represented the mean temperature excesses during the interval. It is not accurate to say a curve was thus constructed; in reality, there was thus obtained (as is shown in Diagram 4) a number of irregularly disposed points, through which was drawn a smooth curve, the ordinates of which were the rates of cooling at the temperature excess given by the respective abscissæ. In order to check the results derived in this manner, the following method of "grouped" readings was adopted. Six readings of the cooling bar were selected, taken at equal intervals. Correcting for thermometric errors, and subtracting the air temperature, we obtain six successive temperature excesses. Divide their sum by six, and there results the mean temperature excess during the five time-intervals which elapse between the reading of the first and last. Next take the five successive differences of temperature observed by the six readings; divide their sum by five, and we get the mean rate of cooling at the mean temperature excess during the interval. We thus have obtained, by taking many such groups of readings, a series of estimates of the rate of cooling at different excesses of temperature. When so plotted, the points nearly all lie on a smooth curve which may easily be drawn through them.

It was usual to go over each cooling experiment twice in this manner. All the readings were grouped into sets of six, and the results deduced. Going over them a second time, the readings were again placed in groups, so that one group in this second reduction overlapped equally two groups in the first; thus giving a second series of intermediate points on the curve.

In Diagram 5 is shown the curve thus obtained, in the case of one of the cooling experiments on iron at the higher temperatures. In order to show how satisfactory is this method, the following comparison, made between the ordinates (*i.e.*, rates of cooling) of this curve, obtained from "grouped" readings, and those of the curve obtained from "single" readings, may be considered :—

Temperature Excess.	Rate of Cooling.	
	From "Grouped" Readings.	From "Single" Readings.
°C.		
170	3·52	3·52
180	3·87	3·87
190	4·20	4·20
200	4·56	4·56
210	4·92	4·93
220	5·28	5·28
230	5·70	5·69
240	6·10	6·13
250	6·55	6·60
260	7·06	7·10
270	7·54	7·70

The above table is given to illustrate the results of the method of reduction; for a particular reason (connected with the circumstances under which the above experiment was conducted) the above values of the rate of cooling were not finally employed in the deduction of conductivity.

This process of grouping readings was found valuable in the deduction of the rate of cooling at the highest temperatures employed; since the irregular falling of the mercurial column almost necessitated some equalisation of this kind. This irregular descent of the mercury column is observed in the very best thermometers, and is aggravated in the present case by the (necessary) smallness of the bore, and also perhaps by the rapidity of the cooling at the higher temperatures.

Forbes' peculiar result as to the rate of cooling at different temperature excesses has already been alluded to, as also has its explanation. A comparison of results for the case of iron in the same way as has already been given for the statical temperature curve, may now be given. Diagram 6 shows the three final results, as they were given by Principal Forbes (marked F), by Prof. Tait (marked T, and reduced, according to his statement, by 6 per cent.), and lastly, the results of experiments made in connection with this work (marked M); the last, however, having been modified as explained in *Appendix* I.

The last two mentioned results clearly disprove the result of Forbes' experiments, viz., that at or about 150° C. the curve should exhibit a point of contrary flexure.

The relation of this to the final results will be noted later.

The following table gives the ordinates of the three different curves beyond 150° of temperature excess, up to which they practically agree. The heading of the columns indicates the set of experiments to which they belong, as explained above:—

Temperature Excess.	Rate of Cooling.		
°C.	F.	T.	M.
150	2·99	3·02	3·02
160	3·19	3·27	3·28
170	3·36	3·53	3·56
180	3·50	3·80	3·815
190	3·64	4·08	4·11
200	3·76	4·37	4·4
210	3·89	4·66	4·7
220	4·01	4·96	5·03
230	4·14	5·26	5·37
240	4·28	5·56	5·74
250	4·40	5·87	6·12
260	4·52	6·2	6·55
270	...	...	6·93

The area of the "statical curve of cooling" was arrived at by treating the curve in sections, and estimating what amount had to be subtracted from or added to the ordinates of each section in order to make it a logarithmic. The areas were checked by the use of a quadrature formula. When it was required to correct the areas for the change in specific heat with increase of temperature, the arithmetical mean of the two temperature excesses on the statical curve, and which included the particular section, was taken; multiplied by the increase per degree and the uncorrected area multiplied by this product with unity added to it.* By plotting a curve, as in Diagram 7, whose ordinates represent the area of the statical curve of cooling beyond different values of the temperature excess, represented by the abscissæ, a useful means of checking results arrived at by the methods mentioned is supplied.

In the case of the two copper bars, the cool end was maintained at a steady temperature by means of a cold water bath. In estimating the area of the statical curve of cooling some allowance must be made for the heat so carried off from the end of the bar. This quantity which has to be added is estimated as follows. Let T_1, T_2, T_3, &c., be the tangents at three successive increasing values of x, viz., x_1, x_2, x_3, &c., and A_1, A_2, A_3, &c., the area of the statical curve of cooling beyond the same values of x. Approximately, $k=\frac{A_1-A_2}{T_1-T_2}$. When the quantity on the right hand side of this equation is multiplied by

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* The following numbers were used in correcting for change of specific heat as representing the rise per degree Centigrade :—

Iron,	·0014
Copper (Crown),	·00088
Copper (C.),	·0009
German Silver,	·0009

They are based on the results of experiments made in 1868, by the late Mr J. W. Nicol in Prof. Tait's Laboratory; the materials used, with the unfortunate exception of the iron, being portions of the bars tested for conductivity.

T_2, we obtain an estimate of the number of units of heat flowing past the section represented by the next value of x, viz., x_2. From the area of the statical curve of cooling we derive another estimate, generally less. The difference between the two is the quantity of heat carried off by the cooling bath at the end of the bar. It is to be noted that (as the error, if any, in the values of the tangents, is usually such as to make them alternately too large and too small) where the estimates of this quantity vary, a process of averages is, within certain limits, permissible.

The following table gives the rate of surface loss, or cooling, for the four metals experimented upon. Those for each metal are the mean of several cooling experiments :—

Rates of Cooling.

Temperature Excess. °C.	Iron.	Copper (Crown).	Copper (C.).	German Silver.
5	...	·047	...	...
10	·11	·116	·13	·145
20	·26	·28	·28	·32
30	·43	·465	·46	·51
40	·61	·66	·64	·73
50	·807	·85	·83	·94
60	1·0	1·04	1·03	1·17
70	1·19	1·25	1·235	1·37
80	1·405	1·466	1·45	1·62
90	1·605	1·7	1·68	1·88
100	1·83	1·927	1·91	2·14
110	2·04	2·16	2·15	2·41
120	2·28	2·39	2·39	2·7
130	2·51	2·63	2·63	2·97
140	2·77	2·87	2·88	3·25
150	3·02	3·14	3·15	3·54
160	3·28	3·38	3·43	3·84
170	3·56	3·665	3·72	4·14
180	3·815	3·98	4·04	4·46
190	4·11	4·31	4·4	4·79
200	4·4	4·66	4·75	5·11
210	4·7	4·98	5·11	5·43
220	5·03	5·33	5·465	5·77
230	5·37	5·57	5·84	6·11
240	5·74	5·89	6·2	6·42
250	6·12	6·22	6·57	6·78
260	6·55	6·58	6·95	7·15
270	6·93	6·92	...	...

The following table shows the result of dividing the rate of cooling in copper (Crown), by that of iron at the same temperature. The column of ratios show that the specific heat of copper rises *more quickly* than what has been allowed for in the final deduction of conductivity. This, of course, by increasing the areas, would increase the conductivity at the higher temperatures.

The third column in the table shows the quotient resulting from the division of each of the ratios by the first.

Temperature Excess.	Rate of Cooling.		Ratio of these Rates.	$\frac{\text{Ratios}}{1{\cdot}054.}$
	Copper (Crown).	Iron.		
10	·116	·11	1·054	1·0
20	·28	·26	1·077	1·021
40	·66	·61	1·082	1·026
60	1·04	1·0	1·04	·987
80	1·466	1·405	1·043	·989
100	1·927	1·83	1·053	·999
120	2·39	2·28	1·048	·994
140	2·87	2·77	1·036	·983
160	3·38	3·28	1·03	·979
180	3·98	3·815	1·044	·989
200	4·66	4·4	1·059	1·005
220	5·33	5·03	1·059	1·005
240	5·89	5·74	1·026	·971

III. Final Results.

The units in terms of which the conductivity is given are those of foot, pound, minute; while the thermometric scale is, of course, Centigrade.

Iron.

(Ordinary Form of Experiment.)

x, in feet.	v.	$-\frac{dv}{dx}$.	Area of Statical Curve of Cooling to the Succeeding Value of x.	Area of Statical Curve of Cooling beyond x.	Area corrected for change of Spec. Heat.
0·0	247·2	30·1	13·52	43·669	53·049
0·25	172·1	19·54	8·703	30·149	34·598
0·5	125·25	13·22	5·96	21·446	23·885
0·75	92·3	9·2	7·2	15·486	16·877
1·25	52·0	4·79	3·74	8·286	8·785
1·75	30·35	2·92	1·92	5·0	5·38
2·25	18·2	1·52	1·056	2·626	2·708
2·75	11·15	1·0	·949	1·57	1·612
3·75	4·3	...	·381	...	...
4·75	1·85	...	·172	...	...
5·75	0·7	...	* ·062	...	...

Hence, thermometric conductivity of Iron.

(1) From areas uncorrected for sp. ht. change.

0°.	100°.	200°.	300°.
·0125	·01156	·01062	·00968

(2) From areas corrected for sp. ht. change.

·0129	·01266	·01243	·0122

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* The last number in each of these columns gives the whole area of that part of the curve beyond the corresponding value of x.

Iron.

(Bar cooled midway.)

x, in feet.	v.	$-\frac{dv}{dx}$.	Area of Statical Curve of Cooling to Succeeding Value of x.	Area of Statical Curve of Cooling beyond x.	Area Corrected for Change in Specific Heat.
0·0	206·35	21·83	11·17	37·557	44·084
0·25	150·85	15·51	7·55	26·384	29·95
0·5	111·4	11·0	5·27	18·834	20·896
0·75	83·45	7·9	6·49	13·564	14·825
1·25	47·7	4·27	3·304	7·074	7·638
1·75	28·15	2·3	1·81	3·77	4·112
2·25	16·5	1·3	·96	1·96	2·192
2·75	9·0	·62	·3	1·0	1·2

Hence, thermometric conductivity of Iron.

(1) From areas uncorrected for sp. ht. change.

0°.	100°.	200°.	300°.
·0112	·0116	·0120	·0124

(2) From areas corrected for sp. ht. change.

·0118	·0129	·0140	·0151

Copper (Crown).

x, in feet.	v.	$-\frac{dv}{dx}$.	Area of Statical Curve of Cooling to Succeeding Value of x.	Area of Statical Curve of Cooling beyond x.	Area Corrected for Change in Specific Heat.
0·0	238·25	10·58	16·64	114·74	130·151
0·25	211·65	9·12	13·811	98·102	109·91
0·5	185·95	7·85	11·064	84·29	93·42
0·75	162·6	6·83	17·877	73·22	80·47
1·25	126·8	5·2	13·327	55·35	60·003
1·75	99·65	3·94	10·032	42·02	45·123
2·25	78·85	3·02	7·505	31·99	34·123
2·75	62·7	2·3	10·359	24·48	26·021
3·75	39·65	...	5·976	...	...
4·75	24·8	...	3·167	...	...
5·75	13·45	...	1·375	...	...
6·75	6·4	...	·309	...	...

Hence, thermometric conductivity of Copper (Crown).

(1) From areas uncorrected for change in sp. ht.

0°.	100°.	200°.	300°.
·0728	·0736	·0744	·0752

(2) From areas corrected for change in sp. ht.

·0745	·0785	·0825	·0865

Copper (C.).

x, in feet.	v.	$-\frac{dv}{dx}$.	Area of Statical Curve of Cooling to Succeeding Value of x.	Area of Statical Curve of Cooling beyond x.	Area Corrected for Change in Specific Heat.
0·0	238·15	12·21	16·66	98·199	111·357
0·25	202·95	10·27	13·065	81·539	91·121
0·5	175·6	8·8	10·488	68·474	75·619
0·75	149·95	7·48	15·354	57·986	63·519
1·25	111·6	5·46	12·93	42·632	46·119
1·75	83·95	3·9	7·767	29·702	31·844
2·25	62·55	2·9	5·53	21·935	23·439
2·75	48·1	2·17	7·09	16·405	17·545
3·75	27·9	1·26	3·71	9·315	10·398
4·75	16·15	0·66	2·02	5·605	6·555
5·75	8·6	...	·85	...	...
6·75	3·0	...	·135	...	...

Hence, thermometric conductivity of Copper (C.).

(1) From areas uncorrected for sp. ht. change.

0°.	100°.	200°.	300°.
·0512	·053	·0548	·0566

(2) From areas corrected for sp. ht. change.

0°.	100°.	200°.	300°.
·0515	·056	·0605	·065

German Silver.

x, in feet.	v.	$-\frac{dv}{dx}$.	Area of Statical Curve of Cooling to Succeeding Value of x.	Area of Statical Curve of Cooling beyond x.	Area Corrected for Change of Specific Heat.
0·0	243·95	31·425	15·792	42·944	48·998
0·25	169·7	21·17	9·927	27·152	29·863
0·5	116·15	14·35	6·228	17·225	18·487
0·75	80·9	9·79	5·595	10·997	11·597
1·25	39·95	4·63	2·974	5·402	5·591
1·75	20·0	2·24	1·309	2·428	2·484
2·25	10·45	1·128	·624	1·119	1·134
2·75	5·45	0·606	·307	·495	0·495
3·75	1·55	0·16	·144	·188	0·188
4·75	0·5	...	·036	...	...
5·75	0·15	...	·008	...	...

Hence, thermometric conductivity of German Silver.

(1) From areas uncorrected for change in sp. ht.

0°.	100°.	200°.	300°.
·0068	·0079	·0090	·0101

(2) From areas corrected for change in sp. ht.

0°.	100°.	200°.	300°.
·0069	·0084	·0099	·0114

It only now remains to compare briefly the results given above with previous work upon the same bars. The particular values of the rates of cooling, and also of the tangents to the curve of stationary temperature excess, which FORBES used, have been already alluded to. Herein, it would seem, lies the explanation of his result that the thermometric conductivity of iron decreases as temperature increases. Employing, as he did, values of dv/dx, 10 per cent. *larger* than those found in this work, and also values of the rate of cooling which are now known to be much too *small* at any temperature excess beyond 150° C., these two factors are quite sufficient to account for the large decrease in conductivity which he stated for iron.

These remarks are fully confirmed by the results of Prof. TAIT, which are intermediate between those of FORBES and those given by me in the tables above. In Diagram 1 it is shown that the rates of change of temperature excess at different points along the long bar were, for the curve representing the experiments in question, intermediate between those of FORBES and my own. Again, in regard to the rates of cooling, the same may be said, as has already been referred to in connection with Diagram 6. This explains why Prof. TAIT's results for iron (viz., that its thermal conductivity is as nearly as possible constant), should lie between those of FORBES (already stated) and those given in this paper, where the conductivity of iron is shown to increase; for, because of reasons already given, the results derived from the experiments on the iron bar cooled midway, are certainly more trustworthy than those of the ordinary form of the experiment.

With respect to the copper and German silver bars, the results for these differ mainly from those of Prof. TAIT in regard to the rate at which the conductivity rises. In both samples of copper the rise is less, but in German silver more than in the previous work on these bars. But as to the absolute value of the conductivity there is no great difference between the two series of estimates.

In conclusion, I have to tender thanks to Prof. TAIT for the interest he has shown throughout in this inquiry, and for much valuable assistance and guidance both in the experimental part of the work and in the more laborious work of reduction. I have also to thank my friend Mr ROBERT DICKINSON for the care he has bestowed upon the accurate execution of the diagrams published along with this paper.

APPENDIX I.

In the determination of the rate of cooling, when the bar experimented upon was heated to about 100° C. by radiation from an open fire, previous to its being heated rapidly over the small Bunsen burners, it was found that the rates of cooling of the bar so treated were, for temperature excesses above 150° C., in a constantly decreasing ratio to those deduced from the experiments in which the bar had been subjected to a prolonged heating, during which the smoking action of the burners so far altered the brightness of the surface as to produce the effect mentioned. In the final deduction of conductivity these lower rates of cooling have been used. The following table shows, in the case of iron and German silver, the difference between the two series of estimates of the rate of cooling. In both copper bars the difference was noticeable only at the very highest temperatures used in the cooling experiments, and as, besides being comparatively small, such difference was at a temperature excess not reached in the long bar experiment, its effect on the area of the "statical curve of cooling" was almost inappreciable.

It must be remarked, however, that even with the larger differences between the two series of rates of cooling in the cases of iron and German silver, the two corresponding estimates of the conductivity differ only by about 1 per cent.

Temperature Excess.	Iron.			German Silver.		
	Lower.	Higher.	Ratio.	Lower.	Higher.	Ratio.
150	3·02	3·02	1·0	3·5	3·54	·988
160	3·28	3·28	1·0	3·79	3·84	·987
170	3·56	3·57	·997	4·14	4·17	·992
180	3·815	3·88	·983	4·46	4·51	·989
190	4·11	4·2	·978	4·79	4·92	·973
200	4·4	4·52	·973	5·11	5·28	·967
210	4·7	4·88	·963	5·48	5·66	·968
220	5·03	5·28	·952	5·77	6·05	·954
230	5·37	5·60	·958	6·11	6·4	·954
240	5·74	6·13	·936	6·42	6·79	·945
250	6·12	6·59	·928	6·78	7·15	·948
260	6·55	7·09	·923	7·15	7·55	·947
270	6·93	7·59	·913	...	...	...

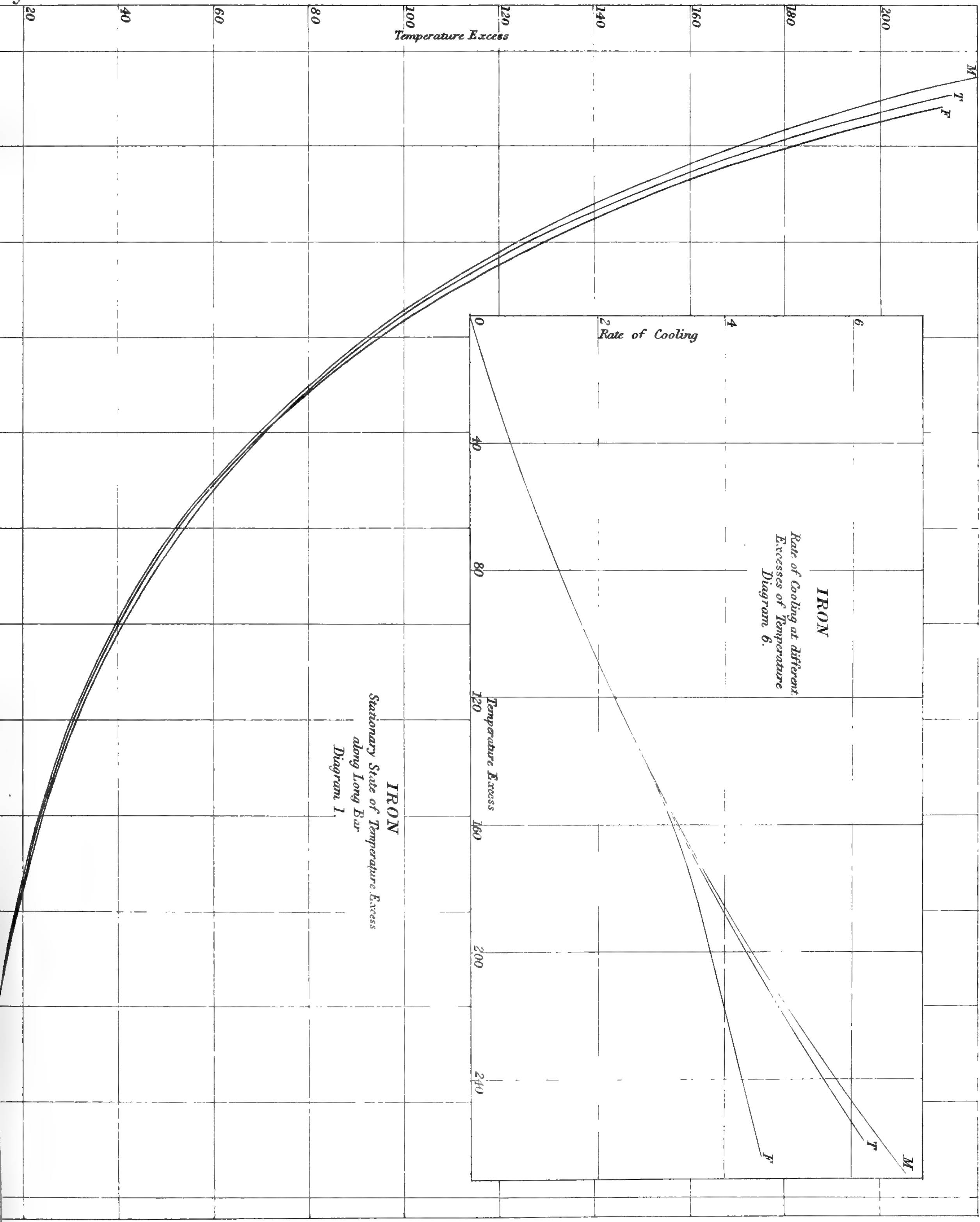
APPENDIX II.

In the text of this paper it is stated that the error due to the circumstance of the thermometric readings being those of a variably heated thermometer is not taken into account in the final results given above. While the paper was being printed, I applied to the tangents and areas the corrections given by

Prof. Tait in the introduction to this paper. The results of the application of these corrections are given in the following table along with the results given already. The numbers in brackets indicate the conductivity as deduced from data uncorrected for the error in question, those without brackets indicate that from corrected data.

It will be observed from a comparison of the corresponding numbers, that the effect of applying this correction is to make the conductivity more nearly constant at different temperatures.

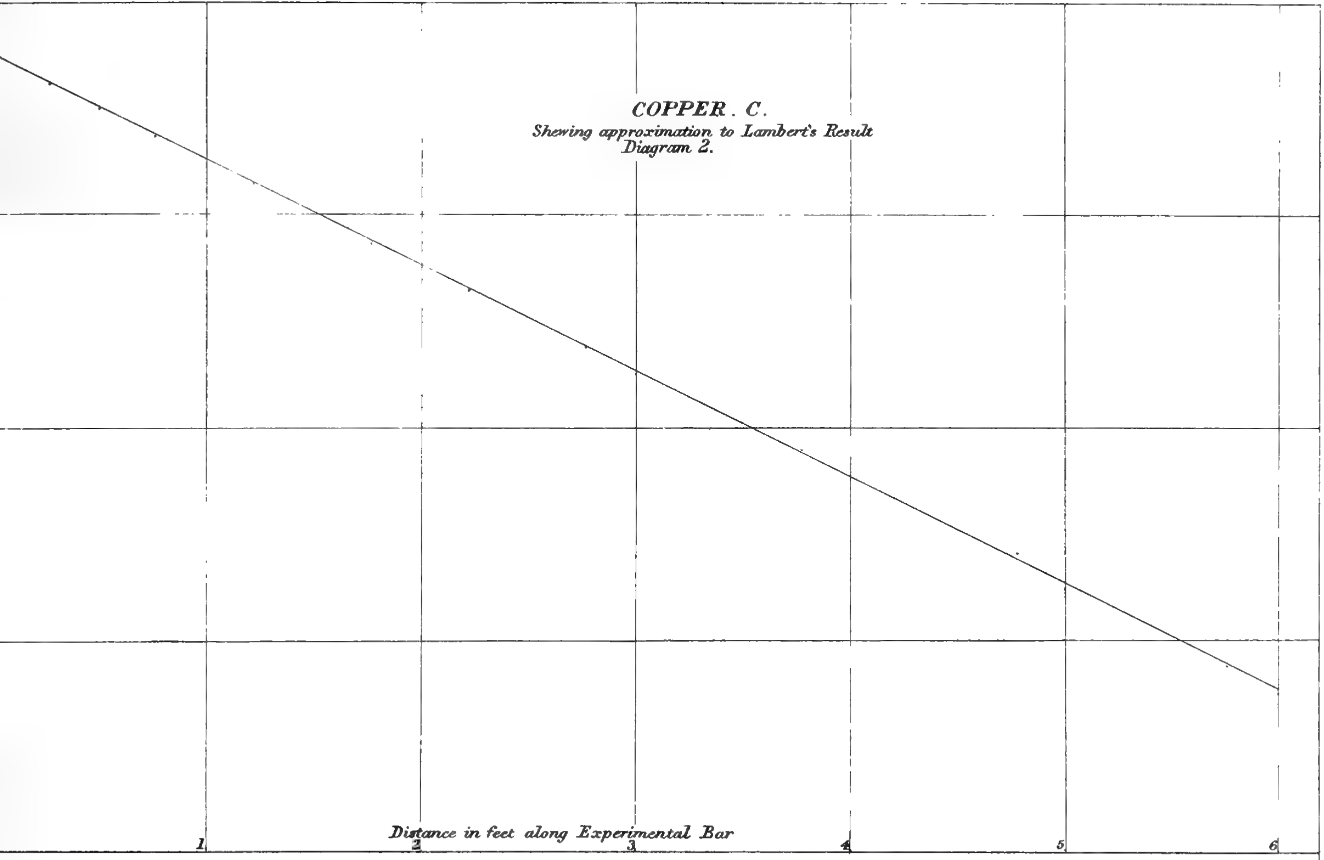
Temperature, C., . .		0°.	100°.	200°.	300°.
Iron (Ordinary).	Not corrected for specific heat change.	(·0125) ·0127	(·01156) ·0115	(·01062) ·0103	(·00968) ·0091
	Corrected for specific heat change.	(·0129) ·0131	(·01266) ·0123	(·01243) ·0115	(·0122) ·0107
Iron (Cooled Bar).	Not corrected for specific heat change.	(·0112) ·01154	(·0116) ·01162	(·0120) ·01170	(·0124) ·01178
	Corrected for specific heat change.	(·0118) ·0119	(·0129) ·01274	(·0140) ·01358	(·0151) ·01442
Copper (Crown).	Not corrected for specific heat change.	(·0728) ·0705	(·0736) ·0710	(·0744) ·0715	(·0752) ·0720
	Corrected for specific heat change.	(·0745) ·0776	(·0785) ·0792	(·0825) ·0808	(·0865) ·0824
Copper (C.).	Not corrected for specific heat change.	(·0512) ·0509	(·0530) ·052	(·0548) ·0531	(·0566) ·0542
	Corrected for specific heat change.	(·0515) ·0520	(·0560) ·0554	(·0605) ·0588	(·0650) ·0622
German Silver.	Not corrected for specific heat change.	(·0068) ·0070	(·0079) ·0078	(·0090) ·0086	(·0101) ·0094
	Corrected for specific heat change.	(·0069) ·0068	(·0084) ·0082	(·0099) ·0096	(·0114) ·0110

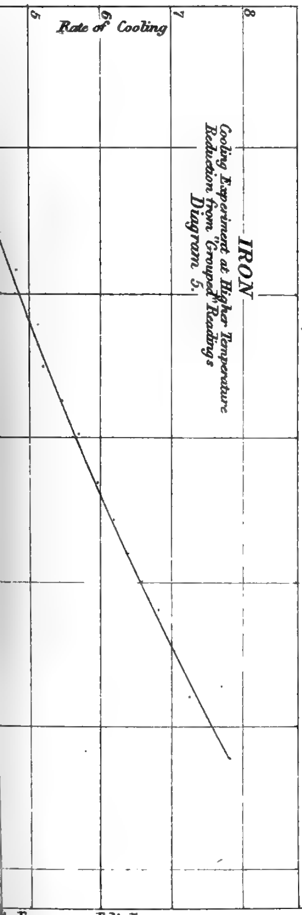


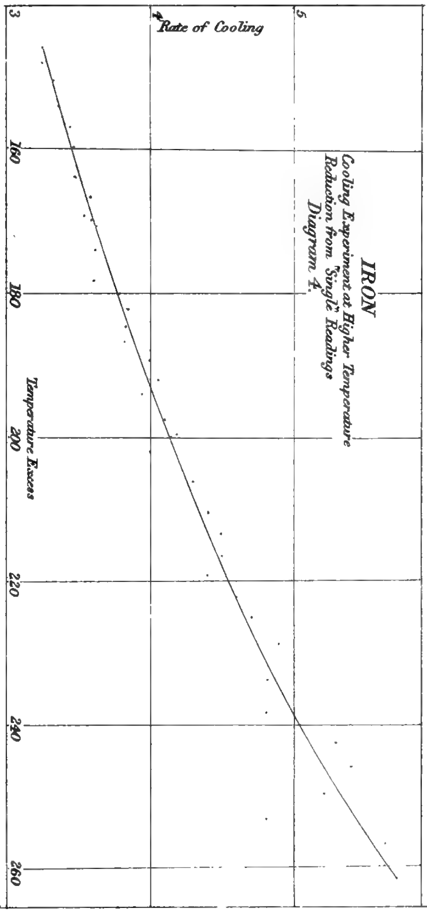
IRON
Stationary State of Temperature Excess
along Long Bar
Diagram 1.

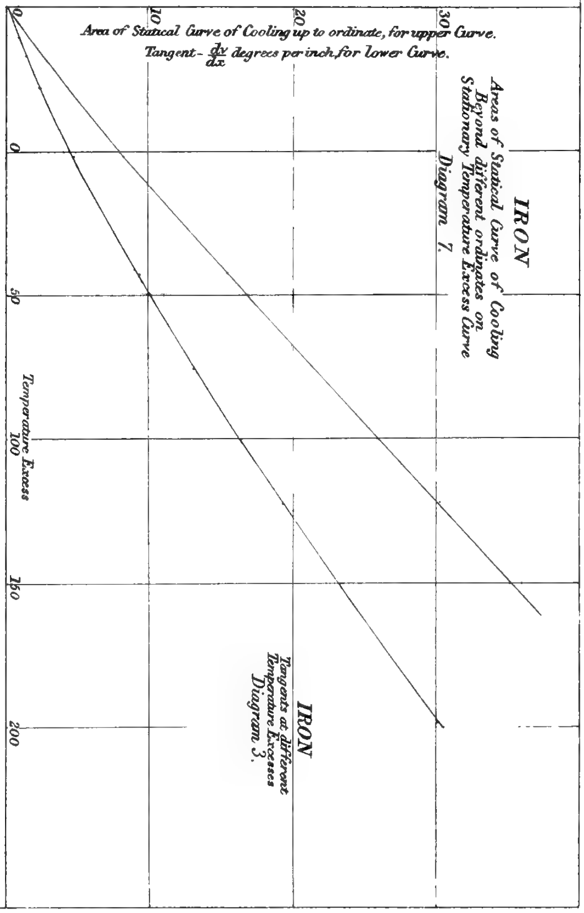
IRON
Rate of Cooling at different
Excesses of Temperature
Diagram 6.

Peck Engravers Edin^{r}


COPPER . C.
Shewing approximation to Lambert's Result
Diagram 2.


IRON
Cooling Experiment at Higher Temperature
Reduction from "Grouped" Readings
Diagram 5.


IRON
Cooling Experiment at Higher Temperature
Reduction from "Single" Readings
Diagram 4.


IRON
Areas of Statical Curve of Cooling
Beyond different ordinates on
Stationary Temperature Excess Curve
Diagram 7.

IRON
Tangents at different
Temperature Excesses
Diagram 3.

XXV.—*Critical Experiments on the Chloroplatinate Method for the Determination of Potassium, Rubidium, and Ammonium; and a Redetermination of the Atomic Weight of Platinum.* By W. DITTMAR and JOHN M'ARTHUR.

(Read 18th July 1887.)

The analytical methods referred to in our heading are infected with numerous sources of error, amongst which until lately the uncertainty of our knowledge of the combining constant "Pt" played a prominent part. This uncertainty, it is true, has been removed to a great extent by SEUBERT'S investigation "Ueber das Atomgewicht des Platins," *Liebig's Annalen* (for 1881), vol. ccvii. p. 1 *et seqq.* The value 194·8 * for Pt, which he ultimately adopted, falls in well with his analyses of the chloroplatinates of potassium and ammonium, and there can be no doubt that his chloroplatinates were close approximations to the ideal substances. Hence his atomic weight 194·8 must be admitted to be nearer the truth than the value 198 of ANDREWS, which, until lately, was so generally employed by chemists in the calculation of their analyses. But it does not follow that in, for instance, the determination of chloride of potassium "as metallic platinum," the factor $2KCl : Pt = 0·7657$, calculated from SEUBERT'S Pt, gives a more correct result than even the factor 0·75252, which follows from $K = 39 : Cl = 35·5$, and $Pt = 198$. The experience of practical analysts might almost be said to point the other way. A forcible illustration is afforded by FINKENER'S test-analyses in support of his own form of the chloroplatinate method for the determination of potassium. FINKENER separates out the potassium as chloroplatinate (in admixture with the sulphates of the foreign metals present), and weighs it as platinum. In reducing his results, he used ANDREWS' value, $Pt = 198$ (or one not far removed from it), and obtained close approximations to his syntheses. Had he calculated with SEUBERT'S number, 194·8, his results, if reduced to K_2O, would have been *too high by nearly* 2 *per cent.* All this, of course, goes no hair's-breadth towards invalidating SEUBERT'S result; it only shows that those analytical factors which are, by theory, equal to $K_2 : Pt$; $K_2 : PtCl_6K_2$, &c., must be determined *directly* by standard experiments; and separately for the several methods. This is what we have attempted (in a limited sense) to do. But a purely empirical determination of these factors by even the most exact experiments would have been of little use. If, for

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* Let us state at once that in this memoir all atomic weights are referred to $O = 16$. For the atomic weights of potassium, ammonium, and chlorine we adopt STAS' numbers, as calculated by LOTHAR MEYER and SEUBERT; $K = 39·136$; $NH_4 = 18·056$; $Cl = 35·455$.

instance, the *analyst's* factor for $K_2 : Pt$ differs from the ratio $2 \times 39{\cdot}136 : 194{\cdot}8$, the causes of this difference must be ascertained, and this necessarily involves an inquiry into the true value for "Pt"; STAS' numbers for KCl, &c., can, of course, be taken for granted. We accordingly did take up this inquiry, and in this sense our investigation joins on to SEUBERT'S, which, of course, was welcome to us, as affording a most valuable basis for this branch of our work.

Before proceeding to detail our experiments, let us state that, unlike our predecessors, we did not merely analyse our chloroplatinates, but in their productions used exactly known quantities of standard solutions of chloroplatinic acid and (for instance) chloride of potassium; so that the weights of platinum and alkaline chloride contained in the precipitated chloroplatinate could be ascertained by determining the small quantities of these which passed into the mother-liquor, and deducting them from the weights of platinum and chloride of potassium employed in the precipitation. In the preparation and application of these standard solutions (and of standard solutions generally) we used a system which was introduced some years ago by one of us in connection with a series of analyses of ocean water,* and which consists in this, that the solutions are standardised both by volume and by weight. The volume-titre serves only for calculating the number of cubic centimetres of solution containing a predetermined quantity of reagent: the exact weight of the latter is calculated from the weight of the measured off quantum and the weight-titre.

The necessary special reagents were prepared in the following manner:—

Chloroplatinic Acid.

For the preparation of this reagent, we utilised two supplies of what had been sold to us as "pure platinum." One of these came from the St Petersburg Works, through the kind agency of a St Petersburg gentleman, who happened to be in Glasgow at the time; the other, from Messrs Johnson, Matthey, & Co. of London. The St Petersburg metal, when examined, turned out to be contaminated with iridium so largely that we presume a mistake must have been made in the transmission or execution of our order. To purify the metal we dissolved it in aqua regia, as far as possible,† and subjected the crude chloroplatinic acid thus produced to the process of SCHNEIDER, which is so fully described by SEUBERT in his memoir. The pure chloroplatinate of ammonium obtained was reduced to metal by ignition, the platinum washed with hydrochloric acid, then with water, dissolved in aqua

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* "Challenger Reports," Physics and Chemistry, vol. i., published by order of Her Majesty's Stationery Office, &c. Adam and Charles Black (and others).

† A good deal of black iridium remained.

regia, and eliminated from the solution by means of pure hydrogen in the wet way. The metallic platinum was then filtered off, was washed successively with hot water, hot hydrochloric acid, and again with water, dried, and ignited in a porcelain crucible. When 23·338 grms. of this metal were dissolved in dilute aqua regia, with the view of preparing a standard solution, a black, sooty-looking residue remained, which was removed by filtration, ignited, and weighed. It amounted to 25·7 mgs. The solution was freed from nitric acid by repeated evaporation on a water-bath with hydrochloric acid, the ultimate residue dissolved in water, diluted to 471 c.c. in a tared bottle, and the solution weighed. It weighed 509·969 grammes, and, taking the part soluble in aqua regia as pure platinum, contained 23·3164 grms. of this metal. For roughly quantitative work, it was assumed to contain 49·5 mgs. of platinum $= \frac{1}{4} \times 198$ mgs. per c.c.

Strictly speaking, we had no right to assume that all the metal contained in this solution was pure platinum; but as the solution exhibited *a normal colour, and gave light yellow precipitates with chlorides of potassium and ammonium*, we thought it might safely be used as pure chloroplatinic acid of the above titre, the more so as the atomic weight of iridium is only a little less than that of platinum, and a small admixture of the former metal in a chloroplatinate could not appreciably affect the percentage of metal in it. We accordingly did use this solution in some of our experiments.

Matthey's Platinum.—To examine this metal for impurities, 10 grms. of it were dissolved in hydrochloric acid, with the aid of the least sufficiency of nitric. The solution was very light in colour, and *no* visible residue remained. The solution was transferred to a large platinum basin, mixed with a large excess of caustic soda,* boiled for 20 minutes, the hypochlorite destroyed by alcohol, and the resulting liquid acidified strongly with hydrochloric acid. No permanent precipitate was produced. The solution was precipitated with sal-ammoniac, the chloroplatinate (which was light yellow) filtered off, and the filtrate concentrated to a small volume, and allowed to stand to recover a second instalment of chloroplatinate. The filtrate from this was evaporated to dryness, the sal-ammoniac burned off, the residue washed with water, ignited, and weighed. It weighed 123 mgs. On treatment with aqua regia, a white residue (silica ?) remained, which weighed 32·6 mgs. after ignition. Hence metal in solution = 90·4 mgs. This solution was mixed with a weighed quantity of a standard solution of chloride of potassium, containing 64·79 mgs. of KCl (corresponding to about 0·95 time 90·4 mgs. of platinum), the mixture evaporated to a very small volume, and mixed, after cooling, with absolute alcohol, to precipitate the "chloroplatinate" produced. The precipitate, after

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* Specially made from pure (Trommsdorff's) carbonate-crystals, in a nickel basin.

drying at 110° C., weighed 217·9 mgs.; 193·5 mgs. of it, when reduced in hydrogen (dry-way), yielded 77·8 mgs. of metal, or 87·61 mgs. per 217·9 mgs. of precipitate. The filtrate from the "chloroplatinate" contained 2·4 mgs. of "platinum," and 0·49 mg. of chloride of potassium (determined and weighed as $PtCl_6K_2$). Hence chloride of potassium in the precipitate = 64·79 (weight of KCl used) − 0·49 = 64·3 mgs. Platinum (1) by synthesis, = 88·0; (2) by analysis, = 87·61; hence platinum per 2KCl by (1) = 204·2; by (2) = 203·3. These numbers are certainly higher than the true atomic weight of platinum, but, considering the small scale on which the analysis was made, and that all the impurities of the 10 grms. of original metal were concentrated in that small remnant of 90·4 mgs. of metal, the *original* metal, we thought, could be accepted as sufficiently pure. To prepare a standard solution, 49·5327 grms. of Matthey's metal (sponge) were ignited in a porcelain crucible, which brought down the weight to 49·5302 grms. These were dissolved in a Berlin basin, under a funnel, in aqua regia, made from specially prepared acids, the nitric acid expelled by repeated evaporation with hydrochloric acid, the ultimate residue dissolved in water, diluted to 1 litre, and weighed. A small quantity of an insoluble residue separated out on dissolving the chloroplatinic acid crystals in water; it was filtered off, ignited, and weighed, and its weight, 3·4 mgs., deducted from the original weight of platinum taken, as a correction. The solution was preserved as containing very nearly 49·5 mgs. of metal per c.c., and exactly 0·45930 grm. per 10 grammes of solution.

The vast majority of our quantative syntheses and analyses were made with this or some other standard solution derived from *Matthey's metal.* Wherever St Petersburg metal was used, this is specially stated.

Chloroplatinic acid, as prepared by means of the aqua regia process, is liable to be contaminated with the nitroso-compound $PtCl_6(NO)_2$. After having made a considerable number of syntheses and analyses of alkaline chloroplatinates, we found that one of our solutions at least was thus contaminated. It is difficult to see how a small admixture of the nitroso-body could affect a synthesis of *potassium* or *rubidium* chloroplatinate, yet its presence is at best no improvement; hence, after that discovery, we tried to make platinum solution by means of hydrochloric acid and chlorine gas, *i.e.*, the method which SEUBERT used once (for his *Darstellung* III., p. 16 of his memoir), to return to the old process. We soon saw how it came that SEUBERT tried the process only once; the solution of the metal takes place with an exasperating degree of sluggishness.

In the following form, however, we found the method to work satisfactorily. The platinum, preferably in the spongy form, is introduced into a light-coloured glass-stoppered bottle of, say, 2 litres' capacity; a sufficiency of fuming hydrochloric acid is poured on it, and the bottle filled with chlorine gas (purified by washing with water, and filtration through asbestos), the stopper put on, and the whole allowed to stand.

After about twelve hours the chlorine is mostly absorbed. The bottle is then refilled with chlorine, again allowed to stand, &c., until the metal is all, or mostly, dissolved. The solution is decanted off, again chlorinated to make sure of the absence of platinous salt, evaporated on a water-bath to expel the surplus chlorine and hydrochloric acid, and diluted to the proper volume. Of course, if it is meant to be used as an exact standard solution, the remnant of undissolved metal is collected, washed, ignited, weighed, and allowed for. With a "*chlorine Kipp*" (see DITTMAR'S *Exercises in Quantitative Analysis*, p. 137) at hand, the method is not so tedious as it appears at first sight. We have long come to adopt it even for ordinary laboratory purposes. When used in connection with work here reported on, it is referred to as the "*chlorine process.*"

Chloride of Potassium.

A few of our earlier experiments were made with a chloride obtained from recrystallised chlorate, but in the majority of cases we prepared our chloride of potassium from recrystallised *per*chlorate. The perchlorate is heated in a platinum basin until the bulk of the oxygen is removed. The rest is then expelled by fusing the residue in a platinum crucible, and keeping it at a dull-red heat until every trace of gas-bells has disappeared. The fused salt is poured into a platinum basin, allowed to cool, broken up, and used in this condition. The neutrality of the fused salt was made sure of by dissolving 2 grms. in water, and titrating the "alkali" with very dilute standard solutions of hydrochloric acid and caustic potash, in the presence of aurine as an indicator. The "alkali" was found equivalent to $\frac{1}{220} \times \frac{1}{2}K_2O$ mgs. = 0·2 mg., or rather *nil.*

Pure Sal-Ammoniac.

See the section on chloroplatinate of ammonium, page 627.

Rubidium Chloride.

See the section on its chloroplatinate, page 618.

Standard Solution of Nitrate of Silver.

As a rule, we prepared this reagent from pure nitrate, in which the water had been determined immediately before use. A quantity, containing $n \times 17$ grms. of dry nitrate, was dissolved to n litres. For the standardisation of the solution a standard solution of chloride of potassium was made synthetically by dissolving $\frac{1}{10}$ KCl grms. in water, diluting to 1 litre, and *weighing* the solution produced before the last mixing. 50 (or 100) c.c. of this solution were *weighed* into a tared phial, 51 (or 101) c.c. of silver solution added, the mixture reweighed to obtain the weight of the silver solution added, then shaken, and the chloride of silver allowed to settle in the dark. The supernatant liquor was decanted off, iron-alum added, and the exact quantity of dissolved silver determined by to-and-fro titration with decimal (*i.e.* centi-normal) solutions of sulphocyanate of ammonium and nitrate of silver. *These* solutions were standardised only by volume, and measured off in burettes. From the result and the specific gravity of the stronger silver solution, it was easy to calculate the exact number of *grammes* of the latter which precipitate $\frac{1}{10}$ KCl = 74·59 grms. of chloride of potassium. This number was recorded as the "weight-equivalent" of the reagent, and utilised in the calculation. In a perfectly similar manner we proceeded in our determination of chlorine by titration. The given solution of chloride was diluted to a known weight. A small fraction, weighed out, served for a preliminary titration of the chlorine in which the reagents* were merely measured by volume. In a larger aliquot part then the exact weight of the chlorine was determined by a method of gravimetric titration closely analogous to the one explained for the standardisation of the silver solution. This particular modification of VOLHARD's method having been worked out by one of us (long ago) for the determination of the chlorine in sea-water (see "Challenger" Memoirs, Physics and Chemistry, vol. i. in DITTMAR's Report on the Composition of Ocean Water), it will be referred to as the *Challenger method.*

In most of our analyses we determined the chlorine by means of this titrimetric method; sometimes we checked the results by gravimetric determinations in other aliquot parts of the respective solutions. In a few cases, when we had only little material at our disposal, we combined VOLHARD's method with the ordinary gravimetric method. The chlorine was precipitated by means of a known sufficient *weight* of the standard silver solution, and the chloride of silver collected and weighed. In the filtrate and wash-waters the still unprecipitated silver was then determined by VOLHARD's method, with volumetrically adjusted decimal (centi-normal) solutions. As each such duplicate analysis was preceded by a preliminary titration of the chlorine, we had no difficulty in so adjusting the weight of silver solution to be added, that the excess of silver left unprecipitated could be conveniently determined in the way explained.

After this introduction, we will proceed next to report on a series of experiments for determining the composition of chloroplatinate of potassium as produced under conditions similar to those prevailing in certain forms of the corresponding analytical method.

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* The silver solution and a volumetric deci-normal solution of sulphocyanate.

I. EXPERIMENTS ON THE COMPOSITION OF CHLOROPLATINATE OF POTASSIUM.

(*First Series.*)

The general *modus operandi* was as follows:—A known weight of chloride of potassium (weighed out in the form of a gravimetrically standarised solution) was mixed with, in general, a slight or moderate excess of standard chloroplatinic acid solution, whose weight was determined and recorded likewise. After production of the chloroplatinate (in some way or other), the mother-liquor was decanted and filtered off, the chloroplatinate washed (as a rule first with small instalments of water, then with alcohol), dried at a certain temperature, and weighed. The mother-liquor, after removal of the alcohol from the alcoholic portion, was diluted with water, placed in an Erlenmeyer flask, and the platinum reduced out by purified hydrogen in the wet way at 80 to 90° C. After complete reduction, the hydrogen was displaced by carbonic acid (to avoid explosions), the platinum filtered off, ignited, and weighed. The filtrate and wash-waters were diluted to a certain weight, and aliquot weighed parts used for the repeated determination of the fixed chlorine, *i.e.*, the chlorine present as KCl. For this purpose the respective liquid was evaporated to dryness, the residue dried further at 130°, then moistened with water, the solution re-evaporated, and the residue heated again to 130°, so that the free hydrochloric acid could be assumed to be away, which point, however, was always made sure of by the application of a thread of delicate litmus-paper to the last solution. In the chloride of potassium thus isolated, the chlorine was determined; generally by means of the "Challenger" method. In the later experiments a very small quantity of fixed chlorine present in the chloroplatinic acid used was determined in a large quantity of the respective reagent, and allowed for. By deducting the weight of platinum and chloride of potassium found in the mother-liquor from the weights originally taken, we obtained the weights that had passed into the precipitate.

In addition thereto, the precipitated chloroplatinate, as a rule at least, was analysed, more or less completely. Our method of analysis, in the earlier experiments, was to reduce the chloroplatinate in a boat standing within a combustion-tube, in hydrogen gas. To avoid loss the out-going gas was made to bubble through a little water contained in a bulbed U-tube. The contents of the U-tube were evaporated to dryness, the residue united with the aqueous extract of the ignited mass and the washings of the combustion-tube, the whole evaporated to dryness, the residue re-dissolved in water, and the small quantity of platinum, which invariably separated out (even from perfectly clear liquors) collected and weighed. In one case the chloride of potassium thus obtained was weighed directly; as a rule, we relied for it on the synthetical data. The platinum was washed, ignited in a small porcelain crucible, and weighed.

We soon, however, came to discard this method, and to reduce our chloroplatinates in the *wet way.* The chloroplatinate, after having been dried at a certain temperature and weighed, was placed in an Erlenmeyer flask, along with a sufficient quantity of water to dissolve the chloroplatinate in the heat, the platinum reduced out in the wet way, filtered off, and weighed,* and aliquot parts of the filtrate used for the determination of (in general) the fixed *and* the total chlorine.

In the following reports on the several experiments, the symbol A stands for the weight of chloride of potassium used in the synthesis; P for the approximate weight of platinum used, as chloroplatinic acid, per 2KCl = 149·18 parts of chloride of potassium; M for the exact weight of chloroplatinate which, according to analysis or synthesis, contains 2KCl parts of chloride of potassium.

Experiment I. (St Petersburg Metal).

A = 0·74 grm. (about); P = 201·6. Platinum solution poured into that of the chloride of potassium. Total volume of mixture, 40 c.c. The whole was evaporated to 2–3 c.c., and, as no excess of platinum was visible, another 1 c.c. of platinum solution was added (and its water evaporated away); hence, finally, P = 209·3. The residue was washed with absolute alcohol (which became distinctly yellow); the chloroplatinate dried at 110°, weighed, and analysed in the dry way.

The results, referred to 2KCl parts, were as follows (in the analysis of the precipitate only the platinum was determined):—

	Platinum.	Chloroplatinate, M.
Synthesis, ...	196·26	491·07
Analysis of precipitate, ...	196·31	

Hence Pt = 196·29; and thence, by calculation, $PtCl_6K_2 = 487{\cdot}29 = M - 3{\cdot}78$; hence water, &c., in the chloroplatinate = 0·700 per cent.

Experiment II. (Matthey's Metal).

C = 1·85495; P = 218·4. Platinum poured into potassium salt. Evaporation to about 5 c.c., then addition of absolute alcohol, as in Experiment I. Precipitate washed with absolute alcohol, and dried, first at 110°, then at 130°. Reduction by hydrogen effected in the dry way. Combined chloride of

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* Sometimes a film of platinum adheres firmly to the sides of the flask. This is easily recovered by dissolving it in *aqua regia*, evaporating to dryness in a porcelain crucible, and igniting the residue.

potassium determined only synthetically; the platinum both ways. Results, referred to 2KCl parts, were—

	Platinum.	Chloroplatinate, dried at 110°.	130°.
Synthesis, . . .	195·71	488·43 (= M′)	488·28 (=M″).
Analysis, . . .	195·63		

Hence, by calculation, P=195·67, and $PtCl_6K_2$=486·67. Hence water, &c.
in M′, =1·76 parts, or 0·360 per cent.
in M″, =1·61 „ 0·330 „

Experiment III. (Matthey's Metal).

A=1·85556; P=206·4 parts. The two reagent solutions were evaporated separately to about 30 c.c. each, and then mixed; the platinum being poured into the potassium salt. Mixture evaporated to 5 c.c., and allowed to stand over night. The precipitate washed by decanting filtration with three instalments of water, each=3 c.c.; 15 c.c. of absolute alcohol were then added, and the precipitate washed with absolute alcohol. The precipitate was dried at 130°, first for 5 hours, and then for other 24 hours; and in part of the final residue the *water* was determined *directly* (see page 578). The combined chloride of potassium was determined only by synthesis; the platinum both ways. Found per 2KCl parts,

$$\left.\begin{array}{lcl}\text{Platinum, by synthesis,} & = & 195{\cdot}75\\ \text{Platinum, by analysis,} & = & 195{\cdot}56\end{array}\right\}195{\cdot}66.$$

and for the weight of the chloroplatinate:—

Dried at 130° for 5 hours.	29 hours.	Anhydrous, by water determination.
M′=487·78	M″=487·29	M‴=486·63

From Pt=195·66; by calculation, $PtCl_6K_2$=486·66; hence water, &c.,

1·12	0·63	−0·03
or 0·230	0·129	*nil* per cent.

Experiment IV. (Matthey's Metal).

This experiment was wrought side by side of Experiment III., on the same scale, and in the same way, except that the chloride of potassium was poured into the platinum, and that the excess used of the latter was a little greater. The two chloroplatinates were dried, side by side, in the same chamber, only while, after 29 hours at 130°, No. III. was taken out to be analysed, No. IV. was dried for an additional 12 hours at 150°. The water in the finally dried substance was determined *directly* (see page 578). During the heating process involved in the determination of the water, the salt decrepitated so much, that a reduction of the residue in hydrogen could not have been effected with quantitative precision. Hence we relied entirely on the synthesis for both the fixed chlorine and the platinum.

Platinum taken, per 2 KCl parts of chloride of potassium, = 214·3 parts.

Found per 2KCl parts.

Platinum = 195·79, and chloroplatinate

	Dried at 130° for 5 hours.	29 hours.	29 hours, and at 150° for 12 hours.	Anhydrous Salt.
M =	489·67	488·67	488·07	487·05

Now, for Pt = 195·79 : $PtCl_6K_2$ = 486·79.

Hence for water, &c.,

In M parts	2·88	1·88	1·28	0·26
	or 0·588	0·385	0·262	0·0534 per cent.

Experiment V. (Matthey's Metal).

A = 3·7822 ; P = 195·0.

The chloride of potassium was dissolved in 20 c.c. of water, and the platinum solution (107·6 c.c.) added, cold. Mixture allowed to stand over night, liquor decanted off through a small filter, precipitate washed with small successive instalments of water, until the small excess of either reagent, which could be presumed to be present, was sure to be washed away (wash-water used, = 4 × 5 = 20 c.c.). The small portion of the precipitate which had gone on the filter was then washed with a little absolute alcohol, but *these* washings were collected by themselves. The aqueous filtrate and washings (some 130 c.c.) were evaporated to dryness over a water-bath in a platinum basin ; the residue, along with the small quantity of chloroplatinate from the filter dissolved in boiling water (about 150 c.c.), and the solution cooled down finally in ice, to cause as much as possible of the chloroplatinate to crystallise out. The crystals "I." obtained were allowed to settle, the liquor decanted off through the original filter, and the filtrate evaporated down to 20 c.c. The solution was cooled down, and allowed to stand over night, when another smaller crop of crystals "II." came out. The mother-liquor was decanted off through the filter, and the three portions of recovered chloroplatinate washed systematically with small instalments of cold water, each equal to about 2 c.c. Each 2 c.c. of water went first on crystals "I.", then on crystals "II.", and lastly on and through the filter. Four such washings were carried out. The last mother-liquor and these washings were analysed as usual to find the weights of chloride of potassium and platinum to be deducted from the quantities originally taken, in order to find how much of each was contained in the three precipitates of chloroplatinate, namely, the bulk of the original precipitate (it, after drying at 130°, weighed 9·6244 grms.), and the two crops of chloroplatinate recovered from filter and mother-liquor. The weights of these latter were not determined, but from the total combined platinum in the three portions we can calculate the total weight of chloroplatinate synthetically analysed. The total platinum obtained amounted to 4·88266 grms., whence, if we go by Experiment "III.", and the "M" for 5 hours' drying at 130° given there as corresponding to 195·66 of platinum, we have 12·1724 grms. for the total chloroplatinate, whence, by difference, 2·5480 grms. for the two minor portions conjointly.

Found, synthetically, per 2KCl parts, platinum = 196·64. *Exact* data for "M" absent.

Note.—The aqueous filtrate and washings contained 60·4 mgs. of platinum, which need 46·2 mgs. of chloride of potassium to be made into chloroplatinate; instead of these 46·2, we found 77·9 mgs.

Experiment V*a.* (Matthey's Metal).

A = 4·5385 grms. weighed directly, and dissolved in 100 c.c. of water. P = 203·4 parts, calculated from the volume of platinum solution used, which was 125 c.c.

The chloride of potassium was poured into the platinum solution, and the mixture allowed to stand over night. Next morning the mother-liquor was decanted off through a small filter, and the precipitate washed three times, each time with 10 c.c. of water; 20 c.c. of absolute alcohol were then poured on the precipitate in the basin, the small portion from the filter washed in with absolute alcohol, and the whole allowed to stand for ¼ hour, to effect at least a partial dehydration. The precipitate was then washed with absolute alcohol, and next kept over oil of vitriol for half a day. It was then dried at 100° for a short time, transferred to a flat platinum crucible, and dried for 4 hours at 110°, when its weight was found to be 12·1063 grms. This precipitate was put into a Geissler tube. A portion (A), amounting to 2·9398 grms., was however weighed out at once for analysis, and another (B) (2·9255 grms.), at the same time, for the determination of the water volatile at 150°, which, as we may state at once, amounted to 11·5 mgs., or 0·393 per cent. (after 23 hours' drying). The reduction of "A" was conducted in the *wet* way with 200 c.c. of water, the platinum filtered off, and weighed; (it weighed 1·1768 grms.). Aliquot portions of the filtrate served for the determination of the fixed and total chlorine. The former was determined once; (found 0·42449 grms. per total solution); the latter twice; found 1·27143, and 1·27122, mean = 1·27133 grms. adopted. Hence we had, by calculation, per 2KCl parts—

Platinum.	Loose Chlorine.	M_{110}.	M_{150}.
196·58	$1{\cdot}99496 \times Cl_2$	491·087	489·166

"M_{110}" stands for chloroplatinate dried for 4 hours at 110° C.;

"M_{150}" for the same, dried in addition for 23 hours at 150° C.

The number for the loose chlorine suggests the presence of $0·00504 \times (O = 16$ parts of oxygen) in the PtX_4 part of the salt. Admitting this, we have, if $Pt = 196·58$, for the composition of M parts of this kind of chloroplatinate—

Platinum,	196·58
2KCl,	149·18
$1·99496 \times Cl_2$,	141·46
$0·00504 \times O$,	0·08
	487·30

Hence water, &c. in.

	M_{110}.	and	M_{150}.
Per M parts,	3·79		1·87
Per cent.,	0·771		0·381

The oxychloride oxygen amounts to very little, supposing it to be present at all. Taking the two determinations of the total chlorine as so many determinations of three times the fixed chlorine, and combining them with the direct determination of the latter, we had:—

Mean value of fixed chlorine = 0·42401 ; and for 2KCl parts

Platinum,	196·80
2KCl,	149·18
4Cl,	141·82
$PtCl_6K_2$,	487·80

and for the values M,

$$M_{110} = 491·64 ; \; M_{150} = 489·72.$$

and for water, etc., in these,

Per M parts,	3·84	1·92

The results remain substantially the same.

Experiment VI. (Matthey's Metal).

At the time when this experiment was planned, we had arrived at the conviction that the atomic weight of platinum is very nearly, if not exactly, equal to 196, and the object of the experiment was to test this number in the most direct manner possible, namely, as follows:—

0·7548 grm. of chloride of potassium was dissolved in 20 c.c. of water, and the solution transferred to a tared glass-stoppered bottle of about 120 c.c. capacity. 20·5 c.c. of standard platinum solution were then measured out, weighed exactly, and added to the chloride of potassium. From the weight and the known titre of the solution, it followed that the weight of platinum added amounted to 0·99086 grm. or to 195·9 per 2KCl. The idea was to let the chloroplatinate separate out as far as possible, to draw off some of the clear liquor, and, by its analysis for platinum and fixed chlorine, prove the absence of excess, or determine such small excess as there might be, of either reagent. But very little chloroplatinate did settle out. We therefore added enough of absolute alcohol to nearly fill the bottle, shook up the contents, and allowed to stand, stopper on, over night. Next morning the whole was weighed, as much as possible of the clear liquor drawn off, weighed, (in a stoppered phial), and analysed for platinum, total chlorine, and fixed chlorine. For this purpose, the solution (after addition of water but without removing its alcohol), was kept at a temperature near to, but below, its boiling-point, within a conical flask, through which a very slow current of hydrogen was constantly passing. The out-going hydrogen was made to bubble through a quantity of water contained in a bulbed U-tube, to catch any hydrochloric acid that might go off. Let us at once state that no chlorine was found in this wash-water after the experiment. The platinum was filtered off and weighed; the filtrate diluted to a known weight, and aliquot parts used for the determination of the fixed and total chlorine. As there was not enough of material for repeating either determination, the chlorine, in each case, was precipitated with a known weight of standard nitrate of silver, the precipitated chloride of silver weighed, and the excess of silver in the filtrate determined by Volhard's method, to check the gravimetric determination. From the weight of platinum obtained, the corresponding weight of chloroplatinate of potassium was calculated, the resulting number multiplied by the ratio of bottle contents to liquor analysed, and the result deducted from the total chloroplatinate (solid or dissolved) calculated from the total platinum used, to obtain a first approximation to the weight of solid chloroplatinate in the precipitation bottle, at the time when the liquor was withdrawn. The result, when subtracted from the weight of the total contents, gave an approximation to the weight of the total mother-liquor. From this weight a second approximation to the dissolved chloroplatinate was obtained, and thus a second approximation to the weight of the solid chloroplatinate, and consequently also to that of the mother-liquor. This second result served for the reduction of the chlorines formed in the drawn off part to the total liquor.

We might have stated before that the platinum solution, before being used, was analysed to find the quantities of platinum, total chlorine, and fixed chlorine present in the weight of reagent employed. To enable the reader to form his own opinion on the probable uncertainty in the final results, we quote the following numbers:—

Found, for the total liquor drawn off—

	Total Chlorine.	Fixed Chlorine.
Gravimetrically, ...	0·38263	0·02676
Titrimetrically, ...	0·38240	0·02675
Means (adopted), ...	0·38251	0·02675

Calculated weight of total mother-liquor = 114·272 grms.;
Weight of drawn-off liquor analysed, = 99·718. Hence,

Grms. of	Chloride of Potassium.	Loose Chlorine.	Platinum.
In reagents,	0·75518	1·06538	0·99086
In mother-liquor,	0·06453	0·40768	0·07907
In precipitate,	0·69065	0·65770	0·91179
Ratios,	149·18 :	142·063 :	196·95

The loose chlorine equals 4·00685 × Cl, which, considering the complexity of the method, may be accepted as a sufficient approximation to 4·0000.

The value Pt, calculated from the loose chlorine, comes to 196·61; calculated from the total chlorine, to 196·72. This last number probably is the most exact of the three; but any are considerably above 196.

The following experiment was made with the view of finding a *lower limit* for the value of the constant:—

Experiment VII.

9·3125 grms. of chloride of potassium were dissolved in 50 c.c. of water, and mixed with a measured volume of standard platinum solution, containing 2·45 grms. of metal, *i.e.*, only about one-fifth of what the chloride of potassium would demand for its conversion into chloroplatinate. Total volume of mixture = 100 c.c. about. After a night's standing a large precipitate of chloroplatinate had settled out; it was washed with small successive instalments of water, until the excess of chloride of potassium, by calculation, was reduced to about 10 mgs. The washing was then continued with a cold-saturated solution of chloroplatinate of potassium. About 60 c.c. of this solution were used in all; after it followed two washings with 50 per cent., and at last a few with absolute, alcohol. The precipitate was dried, first for a night over vitriol, and then at 100° until constant within 4 to 5 mgs. It was then divided into two approximately equal parts, and both analysed by reduction with hydrogen in the wet way. That combination of the gravimetric with the titrimetric *modus*, which had served in Experiment VI., was used again.

Analysis of Part.	A.	B.
Substance analysed,	3·1626	2·6760 grms.
Platinum obtained,	1·2601	1·0672 „
Total chlorine (grav.),	1·36772	1·15775 „
„ (titr.),	1·36730	1·15750 „
„ (mean),	1·36751	1·15763 „
Fixed chlorine (grav.),	0·45921	0·38894 „
„ (titr.),	0·45894	0·38855 „
„ (mean),	0·45908	0·38875 „

Referring in both cases to 2KCl parts, we have—

	Platinum.	Total Chlorine.	Substance = M.
A,	194·64	211·227	488·50
B,	194·66	211·158	488·12
Means,	194·65	211·192	488·31

Adopting the mean, it would follow that Pt = 194·65, and that this Pt is combined with 211·192 = 5·95663 × Cl, supplemented by 0·04337 × (OH or ½O). But we have no excuse for presuming that oxychloride-oxygen is present; on the other hand, we have good reason for suspecting the presence of surplus chloride of potassium. Referring the mean results to 4 × Cl parts of *loose* chlorine, we have

Platinum,	196·78
1·01097 × K_2Cl_2,	150·82
4 × Cl of loose Cl,	141·82
Total,	489·42
And M, =	493·67
Water (?)	4·25

This, we believe, is the correct mode of interpreting the analysis.

It does not follow that 196·78 is the true atomic weight of platinum.

Experiment VIII. (Matthey's Metal).

All the experiments reported on so far were made with platinum solutions, made by means of the ordinary aqua regia process. We have reason to suspect that these solutions contained small quantities of the nitroso-compound $PtCl_6(NO)_2$, which *may* have had an evil effect, although it is not quite easy to see how this can have been the case. However this may be, the following experiments (VIII. and IX.) are free of this flaw, because they were carried out with platinum solution made by the *chlorine process* (see page 565). Their most important feature, however, is that a very considerable excess of chloroplatinic acid was used in the preparation of the chloroplatinates.

In Experiment VIII. 3·045 grms. of chloride of potassium were dissolved in 40 c.c. of water, and precipitated with 120 c.c. of a platinum solution containing 6·0 grms. of platinum, so that the platinum added per 2KCl parts amounted to 294·0 parts. The potassium salt was poured into the platinum solution, the mixture allowed to stand over night, the precipitate washed by decanting filtration, first with 5 c.c. of water (*i.e.*, chloroplatinic acid solution), and then with absolute alcohol; the aqueous and alcoholic washings were preserved separately. The precipitate was dried at 100° to 105° in a watch-glass for 4½ hours, and then for another hour in a Geissler tube at the same temperature. The precipitate acquired a constant weight in a remarkably short time. After having been dried, it was divided into three parts, A, B, C.

A = 3·3511 grms. was devoted to a direct determination of the water. Water obtained = 30·8 mgs. = 0·919 per cent.

B (= 2·8375) and C (2·8884 grms.) were dissolved separately in hot water and reduced by hydrogen, for the determination of the platinum, the fixed chlorine, and the total chlorine.

The results were as follows:—

Analysis of "B."—Substance = 2·8375; platinum therefrom = 1·1312. *Total* chlorine [in fraction analysed, by titration = 0·308683; gravimetrically = 0·308791: mean = 0·308737; whence, for the whole], 1·22855 grms. *Fixed* chlorine [by titration = 0·306219; gravimetrically = 0·306615; whence, for the whole], 0·409294 grms.

Hence per 2KCl parts:—

Platinum.	Total Chlorine.	Substance analysed. Dried at 100° C.	Anhydrous.
195·98	212·85 = 6·0033 Cl.	491·60	487·08

Sum of components = 486·98, which leaves no room for oxychloride oxygen.

Analysis of "C."—Substance = 2·8884; platinum = 1·1518. *Total* chlorine [calculated from mean of 0·314118 and 0·314255], 1·25006. *Fixed* chlorine [calculated from mean of 0·312018 and 0·312574], 0·417213.

Hence per 2KCl parts:—

Platinum.	Total Chlorine.	Substance. Dried at 100° C.	Anhydrous.
195·76	212·46 = 5·9924 × Cl.	490·92	486·40

Sum of components = 486·76, or adding in (0·0076 × ½O = 0·061) 486·82. But the small chlorine-deficit had better be viewed as an observational error, like the excess in the analysis of "B." The filtrate from the chloroplatinate (A + B + C), which still contained some 2 grms. of platinum, was utilised for

Experiment IX.

thus. It was mixed with 1 grm. of chloride of potassium, *i.e.*, two-thirds of what the 2 grms. of platinum required for their conversion into chloroplatinate and the mixture wrought in pretty much the way prescribed by Mr Tatlock in his form of the chloroplatinate process for determining potassium; *i.e.*, it was evaporated to near dryness on a water-bath, some added water evaporated over the residue to eliminate the free hydrochloric acid as fully as possible, and the residue, after cooling, digested in the cold with 10 c.c. of water, *i.e.*, virtually 10 c.c. of a 6 to 7 per cent.* platinum solution, for an hour. The liquor was then decanted off through a filter, and the precipitate washed, first twice with small added volumes of 5 per cent.* platinum solution, and at last with 95 per cent. (by weight) alcohol. It was dried at 100° in a Geissler tube (the weight became constant very soon) and weighed. It weighed 4·0487 grms. It was divided into three parts, A, B, and C.

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* Meaning a solution containing so many centigrammes of metallic platinum per c.c.

"A" was used for a direct determination of the water. 1·3573 grms. of substance gave 1·1 mgs., or 0·081 per cent. "B" and "C" were analysed by wet-way reduction. Found for

	"B."	"C."	Mean.
Substance,	1·0990	1·5917	
Platinum obtained,	0·4416	0·6403	
Substance per 1 grm. of platinum,	2·4887	2·4859	2·4873

The agreement being satisfactory, the filtrates from the platinum were mixed, and used for the determination of the fixed, and of the total, chlorine.

Results.—Substance = 2·6912 (by direct weighing preceding the dividing into "B" and "C"). *Total* chlorine [calculated from the mean of 0·23554 and 0·23582] = 1·17466. *Fixed* chlorine [calculated from the mean of 0·23462 and 0·23501] = 0·39223.

Hence per 2KCl parts:—

Platinum.	Total Chlorine.	Substance. Dried at 100°. M′.	Substance. Anhydrous. M″.
195·59	212·362 = 5·9896 × Cl.	486·53	486·14

Neglecting the small chlorine-deficit, we have for the sum of components 486·59, which, singularly, agrees better with M′ than with M″. Perhaps the 1·1 mgs. of water found in "A" was the result of observational errors,—water absorbed after weighing of the substance, or water out of the joints, &c.

In the preparation of the two chloroplatinates VIII. and IX. we were very much struck by the promptitude with which they acquired a constant weight in the drying chamber; all our previous chloroplatinates used to continue losing weight for hours and hours, and hardly ever really exhibited absolute constancy of weight.

The most remarkable feature in these chloroplatinates, however, is, that although produced in the midst of a very large excess of chloroplatinic acid, they contained rather less platinum per 2KCl parts than we had found in the chloroplatinates previously produced in the presence of small excesses, or even negative excesses, of the platinic reagent. Yet it does not follow that even those quasi-exceptional chloroplatinates (of Experiments VIII. and IX.) were free of surplus platinum. At the time when Experiments VIII. and IX. were planned, this question had already been expiscated experimentally to some extent by special experiments on chloroplatinates V. and V*a.*, which were done at the time when the analysis of V. and V*a.* were carried out, but which we prefer, in this memoir, to treat of separately in the following section.

II. ON THE COMPOSITION OF CHLOROPLATINATE OF POTASSIUM.

Second Series of Experiments.

Chloroplatinate of potassium, as has been long known, is liable to contain water, and indeed in most cases does contain water, so intimately combined with the rest that it cannot be completely expelled at even 150°. That this water should all be present as such, as a mere enclosure within the crystals, is difficult to believe; it is more likely to be present, at least partly, in the form of hydroxyl, functioning as part of the loose chlorine in the ideal substance. From certain observations of SEUBERT'S, indeed, it appears that whenever chloroplatinate of potassium is recrystallised from hot water, part of the chlorine passes into solution, and is replaced, of course, by its equivalent of hydroxyl or oxygen. That this exchange should take place only in hot, and not at all in cold, solutions, is by no means probable. Any chloroplatinate is liable to be thus contaminated, and as long as its purity is not proved, and quite apart from any free chloride of potassium, or surplus platinum in this form or that, which may adhere to it, must be looked upon as a (mixed) oxychloride of the general formula $PtCl_{(6-2y)}O_y . K_2 + xH_2O$, where y of course is a fractional number, and x may be greater than y, because the salt may contain combined water in addition to the water present as hydroxyl. To be able to make a direct and complete analysis of a "chloroplatinate," we must have methods for the direct determination of the water and of the oxychloride-oxygen.

The determination of the water presents no difficulty; it indeed is so easy that we wonder that SEUBERT did not effect it with his chloroplatinate of potassium, and thus remove the cloud of uncertainty which hangs over those of his calculations of the atomic weight of platinum which are based on the ratio of platinum to non-platinum in the chloroplatinate. All that is required is, from a known weight of substance, contained in a porcelain boat standing in a combustion-tube, to expel what goes off at a dull red heat, to remove the liberated chlorine by passing the volatile products through a spiral of red-hot sheet silver, placed in the exit-end of the combustion-tube, to collect the thus purified water in a tared chloride of calcium tube, and weigh it. The water determinations referred to in the above reports were carried out in this manner.* To avoid the uncertainties arising from the use of cork joints, the exit end of the combustion-tube was drawn out to quill size, and the entrance end of the chloride of calcium tube joined on by means of a piece of india-rubber, in such a manner that only a narrow line of the latter was exposed to the out-going vapours. This joint, during the whole of the operation, was

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* The water determination in Experiment III. forms an exception, in this sense, that the substance was heated in a current of *nitrogen*, and the products filtered through a stopper of *copper* gauze (instead of metallic silver).

kept at 100° to 105° by means of a small chamber made of asbestos pasteboard, which enclosed it; and the first step in each analysis was to make this joint anhydrous by passing a current of chloride of calcium dry air through the (heated) tube, until the chloride of calcium tube attached to its end ceased to gain weight, taking care to have the silver spiral at a red heat during an earlier stage of this preliminary operation. The expulsion of the water of the substance was effected in a slow current of chloride of calcium dry air.

The direct determination of the oxychloride-oxygen is a far more difficult problem, which we did not succeed in solving. To keep other chemists from wasting their time, we will shortly sketch out the two methods which we tried.

First Method.—A known weight of the chloroplatinate (contained in a boat standing within a combustion-tube, which has a spiral of red-hot sheet silver near its exit end) is heated to redness in a current of *carbonic oxide*, with the view of determining the water as such, and any surplus oxygen as carbonic acid. The former is collected in chloride of calcium, the latter in an evacuated flask containing a measured volume of standard baryta-water to be determined titrimetrically in the way which one of us employed successfully in the analysis of sea-water (see "Challenger Memoir"; also Dittmar's *Exercises in Quantitative Analysis*, section on Sea-water Analysis, pages 227–230). To test the method, we applied it to a known weight of pure fused chloride of silver. The carbonic oxide was prepared from oxalic acid, and kept in a Pisani gas-holder over strongly alkaline pyrogallate of soda. Certain other precautions which were taken are too obvious to be stated; suffice it to say that we obtained 16 mgs. of carbonic acid from a substance containing *no* oxygen.*

Second Method.—A known weight of the substance is heated in a current of pure hydrogen to obtain all the oxygen (present as water, or in other forms) as water, which is collected in a U-tube charged with pumice and oil of vitriol. By avoiding all cork-joints we had no difficulty in collecting our water in that U-tube, without any hydrochloric acid condensing along with it. To obtain air-free hydrogen, we employed a Kipp apparatus, charged with zinc and boiled out dilute sulphuric acid, and so communicating with a supplementary "Kipp," discharging hydrogen, as to maintain an atmosphere of hydrogen in the upper bulb of the other. The gas obtained was passed through a series of U-tubes charged with sulphuric acid and pumice, and from the last U-tube direct into a combustion-tube containing red-hot copper-gauze (and drawn out at both ends to avoid cork joints), and thence again through a U-tube containing sulphuric acid and pumice; from this last tube the gas entered the combustion-tube to do duty. This method, like the first, was rehearsed with pure chloride of silver, and the result was that the chloride of silver yielded a very

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* No doubt some $COCl_2$ had been produced from the CO and AgCl, and not *all* been decomposed by the silver spiral, although we operated at the highest temperature permissible in a Bohemian glass tube.

appreciable proportion of oxygen! (2·03 grms. gave 15·6 mgs.). Where this oxygen came from we are unable to say; perhaps it came out of the glass of the combustion-tube. If SCHÖNBEIN were still alive he would perhaps say that the oxygen came out of the chloride of silver; we need not add that this is not our explanation of the phenomenon.

While we are compiling this memoir, it strikes us that the oxygen in a chloroplatinate might perhaps be determined by heating it, behind red-hot silver, in a combustion-tube connected with, and previously evacuated by means of, a Sprengel pump. The chlorine should be retained as AgCl, and the oxygen go off as such (with the water), so that it could be collected, determined, and identified by the methods of gas analysis. Unfortunately the idea did not present itself to our minds at the time, and we had to rely on an obvious indirect method for determining the oxygen of a chloroplatinate. This is what we did in the following experiments.

Supplement to Experiment V. *of First Series.*

The principal chloroplatinate produced in this experiment,* while being dried at 130°, became slightly discoloured from some unexplained cause. Hence, instead of analysing it according to our general plan, we wrought it as follows :—

The dried precipitate was dissolved in water in a large platinum basin, the solution filtered, and the filtrate allowed to cool with occasional agitation. The crystalline deposit was washed twice, each time with 10 c.c. of water. It was then recrystallised from hot water. The crystals obtained were dried at 120° C., weighed, dissolved in water, and reduced in hydrogen in the wet-way, to determine the platinum and the two chlorines.

The results were as follows :—

	Fixed Chlorine.	Loose Chlorine.	Platinum.	Substance.
Absolute weights, .	0·52125	1·02916	1·4374	3·5704 grms.
Relative „ .	$Cl_2=70·91$	140·005 $=3·9488 \times Cl$	195·54	485·71 „

The chlorine-deficit is too great to be explained by observational errors; the salt must be assumed to have contained 0·0512 equivalents of oxygen, instead of chlorine, per molecule.

Admitting this, we have, for the composition of 485·71 parts,

Platinum, ...	195·54
Chloride of potassium, ...	149·18
Other chlorine, ...	140·00
Oxygen (as hydroxyl), ...	0·41
Water (in the hydroxyl), ...	0·46
Total, . . .	485·59

instead of 485·71, which is very satisfactory.

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* We mean the bulk of the original precipitate; *i.e.*, that part of it which did *not* pass on the filter, and which, as stated on page 570, amounted to 9·6244 grms. after drying at 130° C.

The two mother-liquors derived from the recrystallisations were analysed (by wet-way reduction in hydrogen, &c.), so as to determine the absolute quantities of platinum, fixed chlorine, and loose chlorine contained in them.

The first-mother liquor gave :—

	Platinum.	Fixed Chlorine.	Loose Chlorine.
Absolute weights, .	0·7914	0·28214	0·59644
Relative " .	196·67	1·9772 × Cl	4·1798 × Cl
" " .	194·00	1·951 × Cl	4·124 × Cl

The number 196·67 for Pt was chosen at the time, because it resulted from the synthesis of the original chloroplatinate *; we now utilise it as an upper limit, while 194 is used as a lower limit, for the unknown true "Pt." Either mode of calculation leads to the conclusion that the liquor contained chloroplatinic acid and (really free) hydrochloric acid besides $PtCl_6H_2$.

These analyses consequently *prove*, what SEUBERT only surmised, namely, that a chloroplatinate recrystallised from hot water contains oxygen in place of part of the chlorine of its $PtCl_4$, and that the mother-liquor contains hydrochloric acid.

Before inquiring into the origin of the chloroplatinic acid, let us give the results of the analysis of the *second mother-liquor*. It contained—

	Platinum.	Fixed Chlorine.	Loose Chlorine.
Absolute weights, .	0·5249	0·19199	0·36097
Relative " .	196·67	2·0286 × Cl	3·8140 × Cl
" " .	194·00	2·0014	3·7629 × Cl

Either mode of calculation brings out oxychloride-oxygen, and with Pt = 196·67 we obtain, moreover (for so much platinum), 0·0286 × KCl of surplus chloride of potassium. But the silver solution used for the determination of the fixed chlorine (in the respective fraction of the filtrate from the platinum), amounted to only 27·3 grms.; and either of the two numbers for the fixed chlorine (per Pt parts of platinum) must be considered uncertain by about ± 0·002 of its value (because the weight of the platinum cannot be presumed to be free of error). Correcting it down by 0·002 of its amount, and reducing to Pt = 195·5 (which number we ultimately came to adopt as the most probable value, *vide infra*), we have—

Platinum.	Fixed Chlorine.	Loose Chlorine.
195·50	2·0128 × Cl	3·7920 × Cl

It really would appear that a little chloride of potassium has been eliminated in the second recrystallisation; but it is not permissible to draw such a weighty conclusion from a single analysis made on such a small scale. We prefer to look upon the fraction 0·0128 as resulting from observational errors.

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* And a small slip in a calculation; the correctly calculated value is 196·64.

Supplement to Experiment V*a. of First Series.*

Of the 12 grms. of chloroplatinate produced in this experiment, the greater part was used for the following experiments :—

6·2356 grms. of chloroplatinate, dried at 110°, was dissolved in about 250 c.c. of boiling water in a platinum basin, and the solution allowed to stand (for about 40 hours), when a large crop of crystals was found to have separated out. These were washed thrice with small instalments of water and put aside as "Crystals C." The mother-liquor was distilled down, in a flask provided with a ground-in alembic and attached to a Liebig's condenser, to about 25 c.c. The distillate contained a small quantity of hydrochloric acid, which was determined gravimetrically by nitrate of silver; its chlorine amounted to 0·87 mg.

The contents of the flask, on cooling, deposited a crop of crystals which were collected and put aside as "E," the mother-liquor being labelled "D." Each of the products was analysed *in toto* (by reduction by hydrogen in the wet-way) so as to obtain the absolute weights of platinum, &c.

Crystals "C."

Substance *un*weighed. Platinum 1·9329. *Total chlorine** 2·07648 and 2·07725; mean, 2·07686. *Fixed chlorine* 0·46467 and 0·46450; mean 0·464585 per fraction analysed. For the whole 0·701157.

Hence per 2KCl parts :—

Platinum.	Loose Chlorine.	Oxygen (calculated).
195·48	139·129	
	or 3·9241 × Cl	0·0759 × ½ O

About 2 per cent. of the loose chlorine is replaced by oxygen.

Crystals "E."

	Fixed Chlorine.	Loose Chlorine.	Platinum.	Oxygen.
Absolute weights,	0·15260	0·29754	0·4195	
Relative „	70·91	138·26	194·93	
		=3·8996 × Cl		0·1004 × ½ O

Mother-liquor "D."

	Platinum.	Fixed Chlorine.	Loose Chlorine.†
Absolute weights,	0·1441	0·05044	0·11940
Relative „	195·00‡	68·253	161·58
		or 1·9250 × Cl	or 4·5574 × Cl

Hence, for the composition of the solution (if we take Pt = 195, as we

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* The AgCl in the titrations had a tinge of pink indicating platinum.

† Including the 0·87 mg. from the distillate.

‡ Instead of 194·93, as found for "E."

did at the time; any other value which one could reasonably substitute would lead to, essentially, the same conclusion)—

$$PtCl_4\left\{\begin{matrix}1{\cdot}925\ KCl\\0{\cdot}075\ HCl\end{matrix}\right\}+0{\cdot}482\ HCl.$$

To check our work, we added up the several instalments of platinum, &c. found in "C," "E," and "D," and contrasted them with the quantities found in the original chloroplatinate. We had for

	Platinum.	Fixed Chlorine.	Loose Chlorine.
"C"+"E"+"D," .	2·4965	0·90419	1·79265 grms.
Original precipitate, .	2·4961	0·90038	1·79623 „

Considering the complexity of the operations involved in obtaining the upper set of numbers, the agreement is very satisfactory. From our analysis of chloroplatinates V. and V*a*., and of their derivatives, we see that in both cases the original chloroplatinate, by being recrystallised, lost chlorine and platinum, with formation of free hydrochloric and chloroplatinic acids, which passed into solution, and the most plausible explanation of the result is to assume that the original chloroplatinates were mixtures of the constitution

$$PtCl_6K_2+x(Pt(OH)_6H_2 \text{ or } PtO_3H_2).$$

and that part of the hydrochloric acid formed in the substitution of oxygen or hydroxyl for $PtCl_4$-chlorine served to dissolve away the $xPtO_3H_2$ of oxyplatinate of hydrogen as chloroplatinic acid.

Now, in the original chloroplatinate of Experiment V. the weight of platinum "Pt" per 2KCl parts was (by synthesis) =196·67; while in the twice recrystallised salt the corresponding quantity was = 195·54.

In the case of chloroplatinate V*a*. we had "Pt"=196·58, and for its derivatives—

Salts		"C."	and	"E."
"Pt."	=	195·48		194·93

The analysis "E" was carried out on a relatively small scale. The mean of the other two values is 195·51; and, supposing our theory to be correct, the true atomic weight of platinum should be either equal to or less than 195·51. Now the degree of completeness with which the surplus platinum is eliminated by recrystallisation should be the higher the purer the chloroplatinate started with, and of all our chloroplatinates those produced in Experiments VIII. and IX. by means of a large excess of chloroplatinic acid apparently come nearest to the ideal substance; hence we thought the best thing we could do would be to prepare a large supply of such chloroplatinate, to analyse it, and then to see what value would come out for the weight of platinum per 2KCl parts, after recrystallisation from water and hydrochloric acid respectively, which latter solvent we hoped would eliminate the surplus platinum more completely, and resubstitute chlorine for the oxychloride oxygen. This programme was carried out in the following experiments:—

Experiment X.

In each of two parallel experiments, I. and II.,* a known weight (about 3·8 grms.) of pure chloride of potassium was dissolved in 50 c.c. of water, and the solution mixed with 150 c.c. of chloroplatinic acid solution, made from Matthey's metal by means of the chlorine process. This reagent contained 0·05 grm. of metal per c.c.; hence the excess of platinum used amounted to about 2·5 grms. The mixture was evaporated down on a water-bath, as far as possible, on stirring, the residue mixed with a little water, and re-evaporated. After cooling, 25 c.c. of water were added to produce a "ten per cent" platinum solution, and the mixture allowed to stand cold, *in the case of "I." for an hour, in the case of "II." for some* 12 *hours,* with occasional stirring. The precipitate was then thrown on a filter, the basin rinsed with 3 c.c. of the 5 per cent. platinum solution, and the precipitate, which was now all on the filter, washed with other 3 c.c. of the same reagent. After the liquor had drained off, the precipitate was washed exhaustively with alcohol of 95 per cent. (by weight). It was then dried in the filter at 100°, the bulk transferred to a tared glass-stoppered cylinder, and in it dried exhaustively at 100° C. The small remnant on the filter was dissolved off with hot water, evaporated in a tared crucible, and weighed by itself.

The aqueous and alcoholic washings (after removal of the alcohol by distillation from the latter) were reduced with hydrogen in the wet-way, the platinum was filtered off, and the filtrate evaporated to dryness to recover the potassium which had escaped precipitation. This potassium was determined as chloroplatinate (Fresenius' modus).

The results, so far, were as follows:—

	I	II.
Chloride of potassium taken, ...	3·8062	3·8061 = A.
„ left unprecipitated, .	0·0319	0·0302
„ in the chloroplatinate,	3·7743	3·7759 = A_0.
Chloroplatinate obtained, ...	12·3807	12·3691 = C.
Hence A_0 : C,	0·30485	0·30525

which numbers seem to show that chloroplatinate "II." having been lixiviated with chloroplatinic acid for 12 hours (instead of for one hour, like "I."), was purer than No. "I." Each of the two chloroplatinates was divided into three parts: one (A) for the determination of the platinum and total chlorine, another (B) for the determination of the water by the direct method, and a third (C) for

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* This second experiment was carried out for us by Mr JAMES ROBSON.

recrystallisation work. *The water-determinations* (carried out with 2·2 and 2·4 grms. of substance in I. and II. respectively) gave the following results:—

Found in	I.	II.
Per cents. of water,	0·353	0·269

The analyses were made with about 2·4 and 2·7 grms. of substance, dried at 100°. Their results, when combined with the synthetical results for the fixed chlorine, and *reduced to* $Cl_2 = 70·91$ parts of the latter, were as follows:—

	Total Chlorine.	Platinum.	Substance Analysed. M.
I.	213·08=6·0099 × Cl	196·01	489·35
II.	212·84=6·0032 × Cl	195·82	488·69

Neglecting the small excesses of chlorine found, we have for

	I.	II.
Platinum,	196·01	195·82
2KCl,	149·18	149·18
4 × Cl of loose chlorine,	141·82	141·82
Water (as above),	1·72	1·31
Total,	488·73	488·13
M.,	=489·35	488·69
Unaccounted for,	0·62	0·56

These deficits might be allowed to pass as the *cumulative* effects of observational errors; but possibly they may be owing to an element of uncertainty in the above numbers for the quantities of water. The samples "B," after having been weighed out for the determination of their water, were kept within their weighing-tubes,* over oil of vitriol, for a considerable time, before these determinations came to be carried out. On reweighing the *weighing tubes and contents*, they were found to have lost ("I".) 1·3 and ("II.") 2·1 mgs. Assuming these losses to have been suffered merely by the tubes and boats, the percentages of water come out as above reported. Assuming them to have been suffered by the chloroplatinates, their percentages of water rise to ("I.") = 0·412, and ("II.") = 0·356. The corresponding weights per M parts of chloroplatinate then are—

	I.	II.
Water,	2·01	1·74
Total components,	489·02	488·56
Deficits *now*,	0·33	0·13

which is more satisfactory.

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* Made each out of two lipless test-tubes sliding over one another.

Recrystallisations.

The parts "C," amounting conjointly to about 15·0 grms., were dissolved separately, each in 600 c.c. of boiling water, the solutions cooled down, finally in ice, the mother-liquors decanted and filtered off, and the crystals redissolved and recrystallised as before. The two twice-crystallised products were then united, dissolved in hot water, and the solution filtered, to make sure of the absence of every trace of dust in the final product. The filtered solution was evaporated over a water-bath, until crystals had abundantly separated out in the heat, then cooled down, finally in ice, the crystals collected, washed, first with a little cold water, and then exhaustively with absolute alcohol; the latter washings being collected by themselves. The crystals were dried over oil of vitriol; they amounted to 4·8 grms., showing that some 10 grms. of salt were contained in the several mother-liquors. The mother-liquors from the first and second crystallisations, when evaporated over a water-bath, gave off towards the end vapours of hydrochloric acid, showing that abundance of chlorine must have been eliminated, and its place taken by oxygen or hydroxyl. The whole of the liquors (minus a small quantity which had been used for these and other tests) were evaporated to dryness over a water-bath, and the residue washed with 91 per cent. (by weight) alcohol. The alcoholic liquors amounted to about 35 c.c.; they exhibited a faint yellow colour, indicating presence of chloroplatinic acid. 2·5 mgs. of platinum as chloroplatinic acid, when added to the same volume of alcohol, produced the same intensity of colour; hence (it would appear) very little platinum had been eliminated by the recrystallisations as chloroplatinic acid, or to speak more correctly, the $PtCl_6H_2$ and the KCl in the (acid) mother-liquor were almost equally balanced, representing just so much $PtCl_6K_2$ dissolved in hydrochloric acid.

The chloroplatinate thus recovered was dried, recrystallised from water, the crystals mixed with the above 4·8 grms. of recrystallised salt, and the two again recrystallised conjointly. The crystals were washed, first with water, then exhaustively with 91 per cent. alcohol, dried over a water-bath, and preserved in a glass-stoppered bottle as "*Recrystallised precipitate.*" Our mode of procedure may appear irrational to some of our readers, and so it is, in a sense. Our original programme was, starting from the chloroplatinate of Experiment X. to recrystallise it, and analyse both the crystals and the mother-liquor; then to recrystallise the crystals, and analyse the second mother-liquor; and so on, until the platinum per 2KCl would become constant, but we had not sufficient material for carrying out this scheme.

The aqueous mother-liquors obtained were mixed with 10 c.c. of 20 per cent. hydrochloric acid, evaporated over a water-bath to about 70 c.c., the liquor cooled down in ice, the crystals collected, washed first with 4 per cent.

hydrochloric acid, then with water, and lastly and exhaustively with the 91 per cent. alcohol. They were dried, transferred to a glass-stoppered tube, and kept as "*Precipitate recrystallised from hydrochloric acid.*" In order to see whether the chloroplatinic acid and the chloride of potassium were still balanced against each other in the hydrochloric mother-liquor, this liquor was evaporated down on a water-bath to about 10 c.c., and mixed, after cooling, with 71 c.c. of 91 per cent. alcohol, to bring down the chloroplatinate (and chloride of potassium, if present). The precipitate, amounting to about 0·66 grm., after having been washed with 91 per cent. alcohol, was extracted, very cautiously, with small successive instalments of ice-cold water. The aqueous washings were evaporated to dryness, the residues again taken up in a little water, filtered, and again evaporated to dryness. This last residue weighed 1·3 mgs., and consisted partly of free chloride of potassium; it gave a precipitate with added chloroplatinic acid. Possibly the alcoholic liquors may have contained more free chloride of potassium, but we unfortunately forgot to examine them.

XI. *Analysis of the Chloroplatinate recrystallised from Water.*

This preparation was divided into two parts (amounting to about 2·8 and 3·4 grms.), and then analysed separately.

Found per 2KCl parts—

	"Chloroplatinate."	Platinum.	Loose Chlorine.
I.	484·62	195·50	138·55 = 3·9077 × Cl
II.	484·91	195·69	138·66 = 3·91077 × Cl
Mean,	484·77	195·60	138·60 = 3·9092 × Cl

Substituting 0·0908 × 17 of hydroxyl for the chlorine deficit, we have (Mean of I. and II.)—

Platinum,	195·60
Chloride of potassium,	149·18
Loose Chlorine,	138·60
Hydroxyl,	1·54
	484·92
M.,	484·77
Excess,	0·15

XII. *Analysis of the Chloroplatinate recrystallised from Hydrochloric Acid*
(*by Mr Robson*).

This preparation was also analysed twice (substance used = 1·9379 and 2·3525 grms.); but unfortunately, the determination of the *fixed* chlorine in the second analysis miscarried. Reducing to unit weight of chloroplatinate analysed we had—

	Platinum.	Total Chlorine.	Fixed Chlorine.
I., ...	0·40203	0·43639	0·14587
II., ...	0·40179	0·43662	lost
Mean,*	0·40191	0·43650	(0·14587)

* Or reducing to 2KCl parts,

Chloroplatinate.	Platinum.	Loose Chlorine.
486·10	195·37	141·28=3·9847×Cl

The *water* was determined directly in 3·2310 grms.: found 5·9 mgs., or 0·1826 per cent., or 0·888 parts per M parts of chloroplatinate. Substituting 0·01534×17 of hydroxyl for the chlorine deficit, we have—

Platinum, ...	195·37	
K_2Cl_2, ...	149·18	
3·98466×Cl, . . .	141·28	
$0{\cdot}01534\times\frac{1}{2}O$, . . .	0·12	
$0{\cdot}01534\times\frac{1}{2}H_2O$, . .	0·14	
Other water, by analysis, .	0·75	0·888−0·138= 0·750
Total, . .	486·84	
M., . =	486·10	
Excess of analysis, =	0·74	

which is *not* too much.

It is surprising to see that the action of the hydrochloric acid did not resubstitute Cl for all the (OH) of the recrystallised salt.

The following table summarises what we found regarding the quantity of platinum (“Pt”) present, per 2KCl parts, in our several chloroplatinates:—

1. *Chloroplatinates prepared by Simple Precipitation; Platinum moderately (if at all) in excess.*

Experiments.	P.*	Volume of mixture of Reagents per 1 grm. of KCl used.	Pt.	
I.	209	Mixture evaporated to small volume, then alcohol added,	196·28	Mean Pt= 196·23
II.	218		195·67	
III.	206	32 c.c.	195·67	
IV	214	32 „	195·79	
V.	195	34 „	196·64	
V*a*.	203	50 „	196·58	
VI.	196	(Precipitated by alcohol.)	196·95	

* “P” stands for weight of platinum used as chloroplatinic acid, per 2KCl parts.

2. *Chloroplatinate prepared by precipitating Chloroplatinic Acid with a large excess of Chloride of Potassium.*

Experiment.	P.	Volume of mixture of Reagents per 1 grm. of KCl used.	Pt.
VII.	40	53·7 c.c.* (See context.)	194·65 (196·78 per 4Cl parts of loose chlorine).

3. *Chloroplatinate made by precipitating Chloride of Potassium with a large excess of Chloroplatinic Acid.*

Experiment.	P.	Volume of mixture of Reagents per 1 grm. of KCl used.	Pt.
VIII.	294	160 c.c.	195·87

4. *Chloroplatinates made from Chloride of Potassium, by evaporating with a large excess of Chloroplatinic Acid, washing with first* 5 *per cent. Platinum Solution, and lastly with Alcohol.*

Experiments.	P.	Volume of mixture of Reagents per 1 grm. of KCl used.	Pt.	
IX.	294	(Evaporation),	195·59	Mean = 195·81
X. (I.)	294	„	196·01	
X. (II.)	294	„	195·82	

5. *Recrystallised Chloroplatinates.*

Supplementary Experiment to V.,	195·54	Mean = 195·50
„ „ to V*a*.,	195·48	
Experiment XI., (Recrystallisation of X. from water),	195·60	
„ XII., (Recrystallisation from dilute hydrochloric acid),	195·37	

From our recrystallisation experiments (if our theory be correct), it follows that the true atomic weight of platinum (apart from observational errors) lies at, or is less than, 195·50.

This number, 195·50, then, affords an upper limit to the unknown number Pt. SEUBERT's experiments enable us to find a lower limit.

SEUBERT, in his Memoir, gives the results of two analyses of one, and of six of another, preparation of chloroplatinate of potassium. The chloride of potassium and the platinum were determined in each case, the loose chlorine only three times. His mean results, referred to 100 parts of substance analysed, were as follows:—(Mean relative uncertainty means the mean deviation from the mean, measured by the mean itself; or the uncertainty per unit of the quantity determined).

	Per cents.	Mean relative uncertainty.
Platinum,	40·110	± 0·0004
Chloride of potassium,	30·685	± 0·0010
Loose chlorine,	29·144	± 0·0017
Error, water, &c.,	0·061	
	100·000	

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* Per 1 grm. chloride of potassium as calculated from the platinum, assuming Pt to correspond to 2KCl.

Reducing to 2KCl parts, we have

2KCl.	Platinum.	Loose Chlorine.	Chloroplatinate as analysed.
149·18	195·00 ± ·27	141·688 or 3·9963 × Cl ± ·011 Cl	486·17

But SEUBERT'S chloroplatinate was prepared by precipitating a (rather dilute, ice-cold) solution of chloroplatinic acid, with, in two cases, 1·33 times, in the other six cases, twice, the calculated weight of chloride of potassium. His precipitate, therefore, in all probability, contained loosely combined (surplus) chloride of potassium.

Side by side with the potassium salt, SEUBERT prepared ammonium chloroplatinate by a closely similar process; the latter he recrystallised to remove "*niedergerissenen salmiak*"; the former he accepted as normal, although it also suffered a loss of alkyl chloride on recrystallisation; but in this case, it appears, SEUBERT assumes that the eliminated KCl came out of the chloroplatinate itself. For this, we submit, he had no excuse. Our view of the matter is that *both* his chloroplatinates contained *niedergerissenes* alkyl-chloride. In the case of his chloroplatinate of potassium, a very little surplus chloride of potassium was sufficient to make his value Pt by half a unit too low. In order to see by how much we would have to correct down his proportion of KCl in his chloroplatinate, let us refer his numbers to Pt = 195·50; they then read as follows:—

Chloride of Potassium.	Platinum.	Loose Chlorine.
149·562 ± 0·21	(195·50)	(4·0065 ± 0·0084) Cl

for which we may substitute, without correcting by more than the mean errors

149·35 = 1·00114 × K_2Cl_2	(195·50)	4·000 × Cl

Now SEUBERT'S chloroplatinate of potassium, from the way in which it was prepared, was bound to contain some "*niedergerissenes*" chloride of potassium; that its proportion should have amounted to less than 0·00114 of the chloride of potassium of the real chloroplatinate in his precipitate, is not at all likely; hence we are justified in concluding that his analyses of chloroplatinate of potassium fall in better with our Pt = 195·5, than with his own 194·8.

A *critique* of his analysis of the *ammonium* salt leads to a similar result. The results of these analyses may be summarised as follows:—Found for "Pt" (referred to O = 16).

1. By determining the weight-ratio of platinum to non-platinum, in a salt precipitated from sal-ammoniac solution, by an *excess* of chloroplatinic acid (Darstellung I.), ...	195·18
2. By determining the same ratio in a salt obtained with an excess of sal-ammoniac (and *not* recrystallised) (Darstellung II.), ...	194·53

3. By determining the same ratio in a salt prepared from salt of Darstellung II. by recrystallisation, ... 195·16
4. By determining the same ratio in a salt prepared from chloroplatinic acid by addition of about 2·5 times the calculated weight of sal-ammoniac, washing, and *recrystallising* the precipitate (Darstellung IV.), ... 195·50
5. By determining the weight-ratio of platinum to *total chlorine* in three preparations, namely:—
   - (*a*) One of *Darstellung II. not* recrystallised.
   - (*b*) One of *Darstellung III.* (precipitation of chloroplatinic acid, prepared by means of chlorine gas and hydrochloric acid, with a somewhat considerable excess of sal-ammoniac).
   - (*c*) One of *Darstellung IV.*

The values Pt (calculated by us from his numbers for platinum and total chlorine) were as follows:—

Salt, ...	*a*,	*b*,	*c*,
Pt, . . . =	195·55	195·78	196·10

Now, all these chloroplatinates were liable to contain water. Those which were *not* recrystallised,—excepting (1),—were almost bound to contain free sal-ammoniac; those which were, probably contained OH instead of part of their Cl. In the case of determinations (5), (*a*), (*b*), (*c*), however, the presence of water does not tell upon the results; hence, if we had only these three analyses to go by, we should say, results (*a*) and (*b*) are probably too low, because (free sal-ammoniac) chlorine was determined as chloroplatinate-chlorine; (*c*) is probably too high, because the salt contained oxygen in place of part of its chlorine, and we should take the mean of the mean of (*a*) and (*b*), and of (*c*) as the most probable value, and put down Pt = 195·89 ± 0·22.

Of result (1) it is difficult to say whether it is more likely to be too high or too low, because it may have contained surplus platinum; we must accept the 195·18 as it stands.

Result (2) is sure to be too low, because it must be presumed to have contained both water and surplus sal-ammoniac. Hence the value 194·53 is *less* than the true Pt.

Preparations (3) and (4) were probably free of surplus sal-ammoniac or platinum; but they may have contained water, which would make the resulting Pt too low; and they probably contained hydroxyl in place of their chlorine, which would tend the opposite way. But result (3) is derived from only one analysis. Hence (as (2) is out of court), the most reasonable mode of utilising the determinations (1) to (4), is to take the mean of (1) and (4); or rather, as (1) included six and (4) included nine analyses, to take $\text{Pt} = (6 \times 195{\cdot}18 + 9 \times 195{\cdot}50) \div 15 = 195{\cdot}37$, and assuming this to have, say, 10 times the "weight" of the result deduced from analyses (5), we have, finally, Pt = 195·42, which number falls in well enough with SEUBERT's analyses of chloroplatinate of potassium, and may be adopted as being virtually his final result. Combining it with our own number 195·50, we have Pt = 195·46, or rather 195·5 (because the uncertainity on either side is more than ± 0·1), as being *at present* the most probable value of the constant.

The true number, we mean a number ranking in probable precision with, say, MARIGNAC's number for chlorine, will, we hope, be determined one day, but if so, it must be derived from other experiments than analyses of chloroplatinates, which are clearly unfit for the purpose.

If, instead of searching for the true atomic weight, we want the *quasi* "constant," which tells us how much platinum is associated with 2KCl parts of chloride of potassium, in a chloroplatinate produced in the ordinary methods of *analysis*, even our value is too low by about half a unit. So at least we must conclude from our experiments on FINKENER's and on TATLOCK's form of the chloroplatinate process for the determination of potassium.

These experiments are detailed in the next following section.

III. FINKENER'S AND TATLOCK'S METHODS OF POTASH DETERMINATION.

Finkener's Method.

This method is not so widely known in this country as it ought to be, we therefore begin by shortly explaining it. Assuming, for the sake of greater definiteness, that the substance to be analysed is a mixture of chlorides and sulphates of potassium, sodium, and magnesium, it is dissolved in water, and the solution mixed with a quantity of sulphuric acid sure to be sufficient for converting all the foreign oxides into sulphates; a quantity of platinum solution, a little more than equivalent to the potassium to be determined is now added, and, if necessary, so much water that the chloroplatinate precipitate produced is dissolved in the next operation, which is to heat the mixture on a boiling water bath. The solution produced is evaporated on a water-bath to the consistence (after cooling) of a magma. This is allowed to cool, mixed with ether-alcohol,* and allowed to stand, well covered, until the precipitate has settled completely. The precipitate,—a mixture of chloroplatinate of potassium and the sulphates of the foreign metals,—is washed with ether-alcohol, to be worked up in one or other of the following two ways:—(A) The precipitate is heated in hydrogen gas to dull redness, so as to reduce the chloroplatinate to Pt + 2KCl, the product treated with water (then with hydrochloric acid, if necessary, and again with water); the residual platinum ignited and weighed, to be calculated into potassium. (B) The precipitate is lixiviated, as quickly as possible, with concentrated (cold) solution of sal-ammoniac, until the filtrate is

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* "Ether alcohol," in connection with FINKENER's method, always means 1 volume of anhydrous ether and 2 volumes of absolute alcohol.

free of sulphate; the residue (chloroplatinate of potassium plus sal-ammoniac) is dried, ignited in hydrogen, the platinum collected as before, and weighed. In this case, of course we have the option of determining the chloride of potassium extracted by water from the ignited residue, either quite directly by evaporation, &c., or indirectly by determining its chlorine.

When one of us, some years ago, was commissioned by the Challenger Authorities to carry out exact analyses of a large number of samples of ocean-water, he inquired critically into the several known methods which might have been used for the determination of the small proportion of potassium present in sea-water salts, and found that FINKENER'S was the only one which afforded a fair approximation to the truth. To render his results susceptible of subsequent correction, he brought the FINKENER method for his purpose into a definite form, regarding which it may suffice here to state the main points as follows:—*

100 c.c. of sea-water are weighed, evaporated to near dryness, and the salts made into normal sulphates; these are dissolved in water, any sulphate of lime, &c., is filtered off, and in the filtrate, the potassium is determined by FINKENER'S method (form A) by means of a quantity of chloroplatinic acid containing 200 mgs. of platinum, *i.e.*, about twice the calculated quantity.

A number of analyses of synthetically prepared mixtures showed that the platinum-weight, when multiplied by $K_2O \div Pt = 94 \div 198 = 0{\cdot}4747$, gave results about 1 per cent. short of the potash used (as chloride of potassium). When the factor, calculated from SEUBERT'S atomic weight for platinum ($Pt = 194{\cdot}8$) and STAS' value for K_2O was used (*i.e.*, the factor 0·48386)† the results were about by 0·01 of their value too high. We may state, in passing, that it was this observation which gave the start to the experiments reported above as Nos. I. to VII. in the First Series.

After the "Challenger" analyses had been reported, we again determined, for our own satisfaction, what degree of exactitude would have been attained if we had separated out the potassium and sodium as chlorides (free of calcium and magnesium), and then applied what we are in the habit of calling "*Fresenius' Method*," because it is the one recommended for mixtures of the two alkyl-chlorides in his handbook of analysis.

Leaving the errors involved in the elimination of the lime and magnesia on one side, we prepared a mixture containing very nearly the weights of chloride of potassium and sodium present in 100 c.c. of ocean-water (that of the former, of course, adjusted exactly), and analysed it by means of the following method:—

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* For details, see "Challenger Memoirs," Physics and Chemistry, vol. i. (appendix), p. 233 *et seqq.*; see also body of Memoir, pp. 12 *et seqq.*

† $\frac{\text{New factor}}{\text{Old factor}} = 1{\cdot}0192.$

The mixture is dissolved in a little water, mixed with more of chloroplatinic acid than necessary for the conversion of both metals into chloroplatinates, and evaporated to a magma on a water-bath. The residue, after cooling, is digested in 30 c.c. of alcohol of 80 per cent. by volume, the liquid decanted off through a small filter, and the precipitate washed with the same alcohol, until the last runnings give only an opalescence with nitrate of silver. The washed precipitate is dried on the filter, dissolved off with boiling water, the solution evaporated in a tared crucible to dryness, dried exhaustively at, first 105° C., then at 130°, and weighed. From the weights, the potassium is calculated (as K_2O) in order to see what the exact, but unreasoning, application of the method would lead to.

But the precipitate can neither be presumed to contain the whole of the potassium, nor to be pure; hence—

Firstly, the crude precipitate is reduced in hydrogen, the platinum weighed, and from the aqueous solution of the chloride of potassium, the latter recovered by a renewed application of FRESENIUS' method.

Secondly, the filtrate from the original chloroplatinate (which contains the sodium and excess of chloroplatinic acid), is freed from its alcohol, by evaporation to dryness, the residue reduced in hydrogen, the alkalies are extracted with water, and made into normal sulphates. In these, the potassium which escaped precipitation is recovered by FINKENER'S method (sal-ammoniac form). The platinum obtained in the reduction of this small quantity of chloroplatinate of potassium is weighed, and the chloride of potassium reconverted into chloroplatinate by FRESENIUS' process, to be weighed as such.

The pure chloride of potassium used for preparing the mixtures was made from recrystallised chlorate.* For the preparation of potash-free chloride of sodium, we used two methods, of which the following gave the best results:—Ordinary "pure" sulphate of soda is dissolved in water, and the solution saturated with hydrochloric acid gas, to precipitate about one-half of the alkali metal as chloride, which is collected on a funnel stopped up with a round glass bead fitting pretty closely into the neck of the funnel, and washed with fuming hydrochloric acid. The salt is then dried, redissolved in water, and reprecipitated by hydrochloric acid gas. The dried product contained a mere trace of sulphate, which was neglected. To test the salt for potassium, a large quantity of it was made into sulphate, and 23 grms. of this subjected to the FINKENER process, sal-ammoniac form. The potassium extracted was determined in the way just described, and its chloroplatinate identified by microscopic inspection. It amounted to 0·38 mg. of K_2O calculated from the platinum, and to 0·43 mg. calculated from the chloroplatinate. From the mean, 0·40 mg., we

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* We subsequently came to prefer the *per*chlorate for obvious reasons.

calculated that 100 grms. of the original chloride of sodium contained 2·1 mgs. of potassium calculated as K_2O. This potassium was allowed for in the test experiments.

Two such experiments were made, and both are reported in the "Challenger Memoir," along with some further details which are here omitted for brevity's sake. We satisfy ourselves with quoting one of the two reports.

Chloride of sodium used, ...	2·9 grms.	
Potassium present, calculated as K_2O in mgs. by	Old Atomic Weights.*	New Atomic Weights.†
Chloride of potassium taken, ...	50·04	50·12
„ in the chloride of sodium, .	0·06	0·06
	50·10	50·18

Analysis.—Potassium found, calculated as K_2O *in milligrammes.*

	Old Atomic Weights.*	New Atomic Weights.†
I. In the crude chloroplatinate by calculation, .	47·90	48·35
Ia. By calculation from the metallic Pt obtained therefrom, ...	47·62	48·53
II. In the pure chloroplatinate, ...	47·06	47·50
IIa. Calculating from the metallic Pt obtained, . .	46·62	47·51
III. In filtrate from crude chloroplatinate, . .	2·64	2·68
IV. In filtrate from pure chloroplatinate, ...	0·85	0·87
Sum of II., III., and IV., ...	50·55	51·05
Excess over synthesis, ...	0·45	0·87
Sum of IIa., III., and IV., ...	50·11	51·06
Excess over synthesis, ...	0·01	0·88
From the numbers given under I. and the synthesis we see that FRESENIUS' method, if used as it stands, would have given a loss of potash amounting to . . .	2·20	1·83
Or, per 100 of K_2O to be determined, to . . .	4·4	3·7

Note.—The chloroplatinates were weighed after being dried at 130°. The weights, after drying at 105°, were only about 0·001 more per unit weight of precipitate.

For the analysis of salt mixtures poor in potassium FINKENER's is the only method that works at all; for the analysis of potassiferous substances generally, it of course only competes with other methods, but over any of these it offers the great advantage of being in a high degree independent of the nature of the foreign bases present. It must be admitted, however, that the FINKENER method, in the form in which it came out of the inventor's hands, owes what there is in it of precision, to some extent, to compensation of errors. FINKENER's own test-analyses prove this: they gave very exact results, *because* FINKENER, in reducing his platinum-weights to potash, used the old atomic

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* K = 39; Cl = 35·5; Pt = 198.

† K = 39·13; Cl = 35·454; Pt = 194·8.

weight of platinum (Pt = 198); if he had used SEUBERT'S value, which no doubt comes nearer to the truth, his results would have been by about 2 per cent. of their value higher (in calculating for K_2O).

We thought it worth while to try and ascertain the several errors involved in the method, with the view of either evading them, or finding a formula for their correction.

There is, however, one other form of the platinum process which, although of less general applicability, as far as it goes, would appear to be as independent of the nature of the impurities in the substance analysed as FINKENER'S. We refer to Mr TATLOCK'S method, which, with anything that falls within the denomination of "potash salt" (pure or impure) is well known to give, to say the least of it, fair results.

We accordingly decided to test both these methods, as far as they compete side by side with each other. Passing over a deal of pioneering work, which, instructive as it was to ourselves, would probably not interest the majority of our readers, we will begin with our experiments on FINKENER'S method, and in the first instance detail certain experiments made for determining its value for the determination of relatively minute quantities of potassium salts, diffused throughout a substance consisting chiefly of sulphates and chlorides of sodium and magnesium. Let us state at once that we always worked with mixtures of this kind, because other acid-tests (than SO_4 or Cl_2) are of rare occurrence, and other bases than MgO and Na_2O are easily removed by analytical processes. From what we are going to say, every chemist will easily see to what extent the method is more widely applicable,—as a matter of probability at least.

Blank Experiments with Unmixed Sulphate of Soda.

The sulphate was made from the chloride of sodium which had served for the experiments detailed above, pages 594 and 595. It contained potassium =1·93 mgs. of chloride in 71·08 grms.

Experiment I.—1·5 grms. of the sulphate of soda (anhydrous) were dissolved in 30 c.c. of water, the solution mixed with chloroplatinic acid, containing 50 mgs. of metallic platinum, and evaporated to a magma. To it, alcohol (1 vol.) was added, and then ($\frac{1}{2}$ vol. of) ether, to dissolve out the chloroplatinic acid, which was washed away as far as possible with ether-alcohol. The remaining salt was almost, but not quite white. This was no more than we had expected, having previously found that the mixture obtained in FINKENER'S process is liable to contain an excess of platinum. To remove this admixture, the salt, after removal of the ether and alcohol by drying at a gentle heat, was redissolved in water, and "re-Finkenerised" without platinum; *i.e.*, again evaporated to a magma, and the latter washed with ether-alcohol. The platinum thus extracted was determined, and found to amount to 0·8 mg. The *recrystallised** sulphate, when heated in hydrogen gas under a funnel, and (the product) treated with water, left 1·2 mgs. of platinum, corresponding to 0·91 mg. of chloride of potassium. The 0·04 mg. (which had been present in the sulphate used) are not deducted.

Experiment II.—A repetition of I., except that 100 mgs. of H_2SO_4 in the form of standard acid, were added in the recrystallisation, with the view of decomposing the chloroplatinate of sodium suspected to be present. Platinum obtained from the recrystallised salt, = 0·5 mg. = 0·34 mg. of chloride of potassium (corrected for the potassium in the sulphate of soda used).

Experiment III.—Three grammes of sulphate of soda used, and the sulphate obtained recrystallised *twice*, *without* addition of sulphuric acid. Residual platinum, = 0·4 mg. = 0·30 mg. of chloride of potassium, or 0·22 mg. after deducting the 0·08 mg. really present.

Experiment IV.—A repetition of Experiment III., except that 200 mgs. of H_2SO_4 were added in each recrystallisation. Residual platinum exactly the same; *i.e.*, a quantity indicating 0·22 mg. of adventitious chloride of potassium.

Experiments III. and IV. were made side by side of each other.

Experiment V.—To study the effect of the added sulphuric acid on chloroplatinate of potassium, 0·2006 grm. of pure chloroplatinate of potassium was dissolved in 40 c.c. of boiling water, containing 25 mgs. of H_2SO_4, the mixture evaporated to a magma, and Finkenerised. The resulting salt was "recrystallised" again with 25 mgs. of H_2SO_4. Both mother-liquors were yellow. They contained, that from the first precipitation 0·9 mg., that from the second 0·6 mg. of platinum, equal to 0·68 and 0·46 mg. of chloride of potassium respectively.

Seeing that sulphuric acid, under the circumstances, decomposes chloroplatinate of potassium appreciably, we tried, in

Experiment VI., the effect of sulphate of *lithium* on chloroplatinate of potassium, because, supposing it to prove inert towards chloroplatinate of potassium, it would have afforded an admirable reagent for the elimination of foreign chloroplatinates from a FINKENER residue. 0·5 grm. of chloroplatinate of potassium when Finkenerised with addition of 0·1 grm. of pure sulphate of lithia, yielded a filtrate containing 3·8 mgs. of platinum, = 2·9 mgs. of chloride of potassium. On "recrystallising" with 50 mgs. of the lithia salt, the liquor contained 4·0 mgs. of platinum, equal to 3·0 mgs. of chloride of potassium.

This shows that sulphate of lithia is *not* available for the purpose aimed at.

Experiments with Sulphate of Soda, containing added Potassium.

I. A quantity of sulphate of soda equivalent to 8·73 grms. of chloride (NaCl), and a known weight of a standard solution of chloride of potassium

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* We adopted this word for designating the operation described.

containing 235·5 mgs. of KCl, were dissolved in hot water, and *Finkenerised* with a quantity of platinum solution containing 600 mgs. of platinum, *i.e.* about twice the quantity demanded by the potassium present. The magma obtained on evaporation was mixed with 50 c.c. of absolute alcohol, and allowed to stand for three quarters of an hour; 25 c.c. of ether were then added, the precipitate well pounded up with a pestle, and the whole allowed to stand under a small bell-jar for two hours. The mother-liquor was then decanted off through a filter, and the precipitate washed with ether-alcohol (2 volumes of alcohol and 1 volume of ether), until the last runnings gave only an opalescence with nitrate of silver. The alcohol-ether filtrate was kept as "I."

From the precipitate the sulphates were extracted by lixiviation with (140 grammes of) saturated sal-ammoniac solution. The last runnings gave only a slight turbidity with chloride of barium.

The solution obtained was put aside as "II."

The remaining chloroplatinate was dissolved in hot water and in the solution, reduced by hydrogen. The platinum obtained was filtered off and weighed; it amounted to 311·0 mgs. The filtrate from the metallic platinum was evaporated to dryness, the residue ignited to drive off the ammonia-salt, and the resulting crude chloride of potassium weighed; it amounted to 241·5 mgs. This salt was dissolved in water, and its potassium eliminated and weighed as chloroplatinate, by a method which was essentially TATLOCK'S, (extraction of foreign matter from the evaporated residue by 5 per cent.* chloride of platinum solution, and washing finally with strong alcohol). From the alcoholic washings the alcohol was distilled off, the residue united with the aqueous washings and filtrates, and from the mixture the platinum eliminated by hydrogen in the wet way. The solution was weighed and fractionated for the determination of the sulphuric acid, and of the total bases as sulphates.

The following is a summary of the results, stated in milligrammes :—

Chloride of potassium, used as such,	235·52
„ „ in the sulphate of soda used,	0·29
Total,	235·81

Chloride of Potassium obtained.

A. (Apparently) in the crude chloroplatinate, as calculated from the Pt (factor $=\cdot76117$,	236·72
Excess found,	0·29
B. In the alcohol-ether filtrate and washings; determined as $PtCl_6K_2$. Weight calculated from this precipitate = 0·61 mg.; from the platinum eliminated from it by $H_2 = 0\cdot84$ mg.; mean,	0·73

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* Meaning a solution containing 5 centigrms. of platinum metal per c.c. The phrases "5 per cent.," "10 per cent.," &c., platinum solution must be read in this sense.

C. From the sal-ammoniac liquor ("II."); determined in ignited residue as $PtCl_6K_2$, and as Pt eliminated from the same. Weight calculated from the $PtCl_6K_2$ was 1·22; from the Pt it was 1·60 mgs.; mean, 1·41; corrected for the KCl contained in the sal-ammoniac used,* . . . . 1·13

D. *The crude chloride of potassium,* extracted from the crude chloroplatinate, weighed 241·5 mgs. According to the analysis of the filtrate from the $PtCl_6K_2$ obtained from it, it contained 6·57 mgs. of Na_2SO_4; deducting this we have—

F. Pure chloride of potassium, ... 234·93

The pure chloroplatinate of potassium obtained from the crude chloride of potassium was weighed; its platinum extracted by H_2 in the wet-way, and weighed likewise.

D_0. The KCl, calculated

$$\left.\begin{array}{llll}\text{From the} & PtCl_6K_2, & \text{was} & 231{\cdot}48\\ \quad\text{,,} & Pt & \text{,,} & 231{\cdot}85\end{array}\right\}\text{mean, ... } 231{\cdot}67$$

This D_0 then was the weight of chloride of potassium which the FINKENER process had actually separated out; the close correctness of the number "A" was owing to the presence of foreign chloroplatinates in what was nominally $PtCl_6K_2$.

E. In the analysis of the crude chloride of potassium, by means of the platinum process, the little potassium which escaped precipitation was determined as $PtCl_6K_2$, and calculated as KCl; found equal to . . . . . 3·46

To check the work, let us add up the various instalments of chloride of potassium found.

$D_0+B+C+E$, ...		237·0
	Excess found, ...	1·2
$F+B+C$, ...		236·8
	Excess found, ...	1·0

II. *A Second Experiment* differed from the ("I.") just detailed in the following points:—(To the same quantity of sulphate of soda as had been used in "I.") only 23·86 mgs. of chloride of potassium were added, and the mixed solution Finkenerised with *five* times the calculated quantity of platinum solution. The salt mixture ($x Na_2SO_4 + PtCl_6K_2$), after a final wash with pure ether, was allowed to dry, and "recrystallised" without any addition (of H_2SO_4, &c.). In the treatment of the "recrystallised" salt mixture ($x Na_2SO_4 + PtCl_6K_2$) with sal-ammoniac, an *exhaustive* extraction of the sulphate was not insisted upon, for not decomposing too much of the chloroplatinate of potassium. The impure residue obtained after treatment with 90 grms. of sal-ammoniac solution was dissolved in hot water, the platinum reduced out by H_2 and weighed. The sal-ammoniac liquor was preserved. The results, so far, were as follows (in milligrammes):—

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* Determined by a blank carried out on a large scale.

Chloride of potassium used, including 0·29 mg., from The Na_2SO_4,	24·15
Found.	
(Apparently) in the chloroplatinate-mixture, as calculated from the platinum extracted,	19·18
In the second ether-alcohol filtrates, 5·8 mgs.	
Of these 5·8 mgs. of platinum, however, 0·8 was present as $PtCl_6K_2$, hence the recrystallisation process, besides removing 5·8 – 0·8 = 5·0 mgs. of platinum present as chloroplatinate, removed also 0·8 mg. of platinum which ought to have remained.	
Hence, if recrystallisation had not been resorted to, the chloride of potassium found would have been	23·59

Errors in the actual and imagined determinations = – 4·97 and – 0·56 respectively.

Experiment II*a*.—The residue ($y Na_2SO_4 + PtCl_6K_2 + z NH_4Cl$) obtained in the inexhaustive treatment with sal-ammoniac was, as stated, reduced with hydrogen in the wet-way, to weigh the platinum present in it. The filtrate from the platinum was evaporated to dryness, the residue made into sulphates, and thus the original potassium—apart from what had gone into the two ether-alcohol filtrates, and into the sal-ammoniac extract—recovered, as part of a mixture of alkali-sulphates. This mixture was dissolved, Finkenerised with only a small excess of platinum (50 mgs. of Pt instead of about 32 as calculated), and *no* "recrystallisation" effected. The washed chloroplatinate plus sulphate was treated exhaustively with sal-ammoniac (25·3 grms. solution); the sal-ammoniac liquors being put aside. The chloroplatinate ($+ x NH_4Cl$) was dissolved in water, and the platinum reduced out; it weighed 24·2 mgs., equal by calculation to 18·42 mgs. of chloride of potassium.

The chloride of potassium was recovered from the filtrate: it weighed 18·4 mgs., including, however, 0·2 mg. of insoluble matter, hence corrected weight = 18·2 mgs. It was dissolved in water, the SO_3 precipitated by nitrate of baryta (precipitate calculated as $Na_2SO_4 = 0·29$ mg.), and the chlorine of the filtrate determined by nitrate of silver. Chloride of potassium calculated from the weight of the chloride of silver = 18·11 mgs.

To be able to check our work, we determined the potassium in the two ether-alcohol filtrates of Experiment II., and in the two sal-ammoniac liquors, obtained in Experiments II. and II*a*. respectively.

Summary of Results.

		Mgs.
Chloride of potassium taken,		24·15
KCl present in the chloroplatinate of II*a*.; calculated from the Pt (24·2 mgs.) is,		18·42
The crude KCl from the same, corrected by deducting SO_3 as Na_2SO_4,	17·91	18·01
Calculated from the chlorine of the crude chloride of potassium,	18·11	
A_0 = mean of 18·42 and 18·11,		18·27

B. KCl obtained from first ether-alcohol filtrates of Experiment II.,		0·73
B′. KCl obtained from second ether-alcohol filtrates of Experiment II.,		0·58
C. KCl from the two sal-ammoniac liquors, worked conjointly:—		
Calculated from $PtCl_6K_2$, . 4·81 } mean = 5·03, deducting 0·23 present in		
„ Pt, . . 5·21 } sal-ammoniac,		4·80
X. Ether-alcohol filtrates of Experiment II*a.* were not worked up; they contained, let us say, the mean of B and B′,	=	0·65
Total chloride of potassium found = $A_0+B+B'+C+X$,	=	25·03
Excess of KCl found,	=	0·88

The *relative* exactitude in Experiments II. and II*a.*, it is true, is not very high; but the *absolute* precision of the results is high, considering that the substance analysed was, virtually, a mixture of the two chlorides, NaCl and KCl, which contained only 0·2766 per cent. of the latter. Hence the Finkener process, whatever it might be otherwise, is invaluable as a means for the determination of small quantities of potassium which escaped the meshes of other analytical methods, and in this sense, amongst others, we have used it largely.

Experiments on synthetically prepared Mixtures containing relatively large quantities of Potassium.

We could not detail all our experiments of this kind without filling a great many pages. We prefer to give the conclusions which we drew from a considerable experience concerning the Finkener process, and then pass on to reporting, mainly, on our final series, which was carried out by what we at the end came to recognise as the best form of the process for general purposes. The conclusions referred to are these,—

1. In the analysis of a mixture of chlorides and sulphates of the bases K_2O, Na_2O, MgO for K_2O, it is not necessary to begin by converting the bases into normal sulphates;* it suffices to add a sufficient quantity of sulphuric acid, equivalent, by calculation, to the chlorine present. In the test analyses to be reported on, we added measured volumes of standard sulphuric acid calculated from our knowledge of the weight of chlorine present. In actual practice, a preliminary determination of the latter by, say, Mohr's method, would give the necessary guidance.

2. In the treatment of the residue (of chloroplatinate of potassium plus sulphates) it is expedient to add, first a sufficiency of absolute alcohol, say 10 c.c., to allow to stand for some time, and then to add the necessary (5 c.c. of) ether, and allow to stand longer, but under a small bell-jar on a glass plate. For a long time it was our rule to let the alcohol act for half an hour, and the alcohol and ether for other two hours; but we subsequently found that five

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* Finkener does not say it is; but for a time it was *our* method.

minutes for the alcohol and then twenty-five minutes for the alcohol and ether suffice. In two test analyses (of a "95 per cent." salt, *vide infra*) we used this shorter mode, and found that the errors in the chloride of potassium amounted to only +0·09 and +0·94 mgr., for about 753 mgs. to be determined. The mixture is filtered, and the precipitate washed with ether-alcohol. To prepare it for "recrystallisation," it is washed finally with plain ether, and allowed to dry.

3. The "*recrystallisation*" of the mixture ($PtCl_6K_2 + xR_2SO_4$) obtained in the FINKENER process is necessary, in general, for the removal of chloroplatinates, but addition of sulphuric acid in this subsidiary operation does no good.

4. The alcohol and the ether must be absolutely free of ammonia; we always distilled them with a little phosphoric acid before use.

5. With the generality of substances, the sal-ammoniac form of the process offers no advantages over the straight-forward determination of the platinum in the "recrystallised" precipitate (of sulphates and $PtCl_6K_2$).

6. The determination of the platinum is best effected by reduction with hydrogen in the *wet* way.

In our final series, the FINKENER and the TATLOCK processes were worked side by side of one another, in this sense, that for every mixture analysed by means of one of the processes, a substantially identical mixture was analysed by the other. Yet we prefer, meanwhile, to detail our test experiments on the FINKENER process first, and by themselves.

For the preparation of the mixtures to be analysed, we used the following materials:—

(1) *Standard solutions of chloride of potassium* made from perchlorate. The solutions were standardised synthetically by weight, and, immediately after their preparation, quantities containing the desired amounts of salt were *weighed* out into so many bottles, which were marked, and kept ready for use.

(2) *Chloride of sodium solution,* prepared from potassium-free salt, and standardised volumetrically. 1 c.c. contained 8·194 mgs. of dry salt.

(3) *A sulphate of magnesia solution,* prepared from *pure* (alkali-free) magnesia (MgO)* by solution in a very slight excess of standard sulphuric acid, and diluting to a definite volume. 1 c.c. contained 1·539 mgs. of MgO.

(4) *A standard solution of sulphuric acid,* made from distilled acid. 1 c.c. = 47·87 mgs. H_2SO_4.

For the preparation of a mixture for analysis, one of the portions of chloride of potassium solution (see (1)), was mixed with measured volumes of solutions (2) and (3), and in general (4). The resulting solution was then Finkenerised

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* For mode of preparation, see "Challenger Memoir," p. 16.

with a measured volume of a standard solution of chloroplatinic acid prepared from chemically pure metal by (in most cases) the *chlorine* process (see page 564).

Our final experiments were made in sets of, in general, four analyses of the same kind of mixture. Before passing to these sets, we will shortly report on a single experiment, made with the view of seeing how the FINKENER process works with relatively pure potassium salts.

Analysis of a Mixture of Sodium and Potassium Chlorides, containing 98·4 *per cent of* KCl.

The solution analysed contained 7·1 mgs. of Na_2SO_4, = 5·9 mgs. of NaCl, and exactly 0·3701 grms. of KCl. It was mixed with sulphuric acid equal to 49 mgs. of H_2SO_4 (to give the sodium a better chance of separating out as sulphate), and then Finkenerised with 11 c.c. of a platinum solution, of which 10 c.c. would have sufficed by calculation.* Salt-mixture $(xR_2SO_4+PtCl_6K_2)$ "recrystallised." Platinum obtained from the final chloroplatinate (by reduction in the wet way) = 0·4855 grm. = 0·3696 grm. of KCl : error, = −0·5 mg.

The *second* ether-alcohol filtrate was worked up for potassium. It amounted to 0·3 mg. of KCl. Assuming the *first* ether alcohol filtrate (which was not analysed) to have contained the same quantity, we have—

Total KCl recovered, = 369·6 + 0·6 = 0·3702, instead of 0·3701 grm. The weight of platinum eliminated by "recrystallisation" was 3·1 mgs. Hence, if this operation had been omitted, we should have had 0·4855 + 0·0031 = 0·4886 grm. of platinum, equal to 0·3719 of KCl; *i.e.*, a positive error = 1·8 mgs.

We now pass to the *series* of trials referred to.†

I. *Set of Experiments.*

Subject:—a "95 per cent." salt.‡ Chloride of potassium operated upon in each analysis, about 0·75 grm. Sulphuric acid required to replace every Cl of the mixture by $\frac{1}{2}SO_4$, = 10·9 c.c.; actually added, = 12·0 c.c. The whole was evaporated to dryness, and the residue ignited to expel the chlorine. Platinum solution required by calculation, = 20 c.c., we added 21·5 c.c., *i.e.*, a very small excess, which, as we *now* know, was a mistake. The mixture of $PtCl_6K_2$ and

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* See last line of this page.

† The designations of the following sets of experiments do not in general indicate the order in which they were carried out.

‡ Meaning a mixture containing 95 parts of chloride of potassium in 100 of total anhydrous salts.

sulphates obtained was "*recrystallised*," the crystals obtained dissolved, the platinum reduced out by hydrogen and weighed.*

Results in Grammes.

Experiment.	1.	2.	3.	4.	5.
I. KCl taken, ...	·75750	·75984	·75791	·76042	·75982
II. Platinum obtained, .	·9959	·9952	·9917	·9945	·9991
III. KCl if $=0·76117 \times$ II., .	·75805	·75752	·75485	·75698	·76048
Error, *i.e.*, III.–I., .	$+·55$	$-2·32$	$-3·06$	$-3·44$	$-0·66$ mgs.

The first alcohol and ether washings from all the five analyses were worked up for potassium by FINKENER's method, sal-ammoniac form. Ultimately the KCl was determined in FRESENIUS's way, as $PtCl_6K_2$. Found for the five analyses, 1·37 mgs. of KCl.

The second alcohol and ether washings contained 39·1 mgs. of platinum, equal to 29·76 of chloride of potassium, or 5·95 mgs. per analysis. So much more would have been found (than quantities III.) if the recrystallisation had been omitted. But a determination of the potassium (as Pt) showed that 0·26 mg. of KCl per analysis was present in the ether-alcohol liquor.

Viewing the five analyses as one, we have—

A. Total chloride of potassium taken, ...	3·7955 grms.
a. KCl lost in the ether-alcohol liquors, ...	0·0027 „
$A_0 = A - a$, ...	3·7928 „
p. Platinum obtained, ...	4·9764 „

$A : p = 0·76270$

$A_0 : p = 0·76216$

$(p : A_0) \times (149·18 = K_2Cl_2)$ . . = *quasi* Pt = 195·73 „

$(p : A) \times$ „ . . . . = 195·60 „

II. *Set of Experiments.*

This set was carried out before we had come to adopt the recrystallisation modus for purifying the FINKENER product.

It seemed to us at the time that the most exact method of potassium determination would be to eliminate the bulk of the potassium as chloroplatinate by precipitation from purely aqueous solutions, and to utilise the FINKENER process only for the recovery of the unprecipitated remnant.

The substance worked upon was the 95 per cent. salt used in the I. Set. A known weight of chloride of potassium (amounting to about 0·76 grm.) was dissolved with the necessary impurities, and the solution next evaporated to about 5 c.c. About 1·5 times the calculated minimum of platinum solution was now added to produce some 35 c.c. of mixture, which was allowed to stand over night. The mother-liquor was then decanted off through a small filter,

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* In only one case, No. (4), did the filtrate contain a trace of platinum. It was recovered by H_2S, the PtS_2 made into Pt, and its weight (0·4 mg.) added on.

the precipitate washed with small instalments of water until pure by calculation (18 c.c. of water were used in all), then with dilute, and finally with absolute alcohol. The precipitate was dried at 110° and weighed. It was then dissolved in water, the platinum reduced out by hydrogen, and weighed likewise.

The alcoholic washings were evaporated to dryness, the aqueous liquors added, and after addition of enough of standard sulphuric acid for replacing the Cl of the chlorides present by $\frac{1}{2}SO_4$, the whole evaporated, Finkenerised, and the sulphates removed by sal-ammoniac solution. The resulting chloroplatinate was dissolved in hot water, reduced by hydrogen, the platinum filtered off, and the filtrate next evaporated to dryness, and the residue ignited, to drive off the ammonia-salt. In the ignited salt, the potassium was determined by means of the FRESENIUS' form of the chloroplatinate process. In some cases the chloroplatinate obtained was reduced in hydrogen (wet way), and the platinum weighed.

The factors used were —

For reducing $PtCl_6K_2$ to K_2Cl_2, ... 0·30435
„ Pt „ . . . . 0·76117

In Experiments II. to V. chloroplatinic acid made by the chlorine process was used; in Experiment I. a reagent made with aqua regia.

Results.

Experiment.	1.	2.	3.	4.	5.	
Chloride of potassium used, ...	·75380	·75308	·75497	·75467	·75390	grm.
I. Bulk of chloroplatinate, . .	2·3579	2·3776	2·3824	2·3528	2·3569	„
II. K_2PtCl_6 from liquors, . . .	·1138	·0887	·0926	·1202	·1190	„
III. Total $PtCl_6K_2$, ...	2·4717	2·4663	2·4750	2·4730	2·4759	„
IV. KCl corresponding, ...	·75226	·75062	·75326	·75266	·75354	„
V. Platinum from I., ...	·9456	·9529	·9549	·9430	·9423	„
VI. „ „ II., ...	·0454	...	...	·0487	·0484	„
VII. =V.+VI., ...	·9910	...	...	·9917	·9907	„
VIII. KCl corresponding to V., . .	·71976	·72532	·72684	·71778	·71725	„
IX. „ „ VI. (or II.),	·03456	(·02700)	(·02818)	·03707	·03684	„
X. =VIII.+IX., ...	·75432	(·75232)	(·75502)	·75485	·75409	„
Error in IV. (in mgs.), . .	−1·54	−2·46	−1·71	−2·01	−0·36	mg.
„ X. „ . . .	+0·52	−0·76	+0·05	+0·18	+0·19	„
SO_3 found in I. „ . . .	...	0·51	0·75	...	...	„

Total chloroplatinate produced, *i.e.* (I.+II.), ... 12·3619 grms.
Total chloride of potassium employed, ... 3·7704 „
Hence chloride of potassium per unit of chloroplatinate dried at 110° C., ... 0·30500 „
Total platinum obtained in experiments 1, 4, and 5, ... =2·9734 =*p*.
Total chloride of potassium for experiments 1, 4, and 5, ... =2·2624 =A.
Hence $A : p = 0·76087$
and $(p : A) \times K_2Cl_2 = 196·06$.

III. *Set of Experiments.*

Subject, the same "95 per cent." salt as was used in Sets I. and II. Method the same as that used in Set I.; except that the salts were made into sulphates only virtually, by addition of the calculated amount of standard sulphuric acid. Platinum added on Finkenerising, = 25 c.c. or 1·25 times the calculated minimum.

Product ($PtCl_6K_2$ and $x\ R_2SO_4$) "*recrystallised.*"

Results.

Experiment.	1.	2.	3.	4.
I. Chloride of potassium taken,	·75330	·75253	·75327	·75426 grm.
II. Platinum obtained, . .	·9900	·9880	·9902	·9907 „
III. KCl found if = ·76117 II., .	·75356	·75204	·75371	·75409 „
Error = III. − I., . .	+0·26	−0·49	+0·44	−0·17 mg.

The ether-alcohol liquors not analysed.

Uniting the four analyses into one, we have—

Chloride of potassium taken, ... = A = 3·0134 grms.
Platinum obtained, ... = p = 3·9589 „
Hence A : p. = 0·76116.
(p : A) × K_2Cl_2 = "Pt" = 195·99.

IV. *Set of Experiments.*

The solutions analysed were such as to represent very nearly a salt consisting of 82 per cent. of chloride of potassium, 15 of chloride of sodium, and 3 of sulphate of magnesia. The quantity of chloride of potassium used was about 0·656 grm. per analysis.

The solution to be analysed was mixed with the quantity of standard sulphuric acid equivalent to the chlorides present, and 1·25 times the calculated volume of platinum solution, and Finkenerised; the resulting product being "recrystallised." All else as in III.

Results.

Experiment.	1.	2.	3.	4.
I. Chloride of potassium taken,	·65390	·65294	·65032	·65046 grm.
II. Platinum obtained, . . .	·8597	·8553	·8543	·8547 „
III. KCl, corresponding = II. × 0·76117, .	·65438	·65103	·65027	·65057
Error, III. − I., . . .	+0·48	−1·91	−0·05	+0·11 mg.

Uniting the four analyses into one, we have—

Total chloride of potassium used, ... $=2·6076$ grms. $=$ A.
„ platinum obtained, ... $=3·4240$ „ $=p$.
Hence A : $p=0·76157$
and $(p : \text{A})\times \text{K}_2\text{Cl}_2=195·89$.

V. *Set of Experiments.*

Salt analysed consisted of 33·3 per cent. of chloride of potassium, 33·3 of chloride of sodium, and 33·3 of sulphate of magnesium.

Method exactly as in IV. Set.

Results.

Experiment.	1.	2.	3.	4.
I. Chloride of potassium taken,	·26603	·26606	·26490	·26490 grm.
II. Platinum obtained, . .	·3501	·3500	·3484	·3491 „
III. KCl, corresponding, *i.e.*, 0·76117 × II., . .	·26649	·26641	·26519	·26572 „
Error, III. – I., . .	+0·46	+0·35	+0·29	+0·82 mg.

A = Total chloride of potassium used, $=1·0619$ grms.
$p=$ „ platinum obtained, $=1·3976$ „
Hence A : $p=0·75980$
and $(p : \text{A})\times \text{K}_2\text{Cl}_2=196·34$.

Tatlock's Method.

This method was invented expressly for the assaying of commercial potash-salts, *i.e.*, of salt mixtures similar in constitution to those which we employed for our test-analyses by FINKENER's method. Mr TATLOCK's method, according to his own description,* is as follows:—Assuming the substance to be analysed to have been converted into a standard solution, a quantity equal to ("10 *grains*" =) 0·6 to 0·7 grm. of dry salt is measured off, to be analysed as follows:—*For every one gramme of salt* the solution is diluted to about 40 c.c.; it is then acidified with a few drops of hydrochloric acid, and mixed with 50 c.c. of a "5 per cent." chloride of platinum solution, meaning a solution contain-5 centgrms. of metal per c.c. The mixture is evaporated to near dryness over a water-bath, and the residue re-evaporated with addition of a little water, to more fully eliminate the free hydrochloric acid. [Observe that this large proportion of platinum is prescribed for *all* kinds of salts, rich or poor. Now 1 grm. each of the anhydrous salts, NaCl, MgCl_2, MgSO_4, demands only 1·69, 2·08, 1·65 grms. of platinum, assuming Pt to be equal to 198; hence Mr TATLOCK's intention apparently is to have sufficient chloroplatinic acid present for converting all the metals into chloroplatinates, and, in addition thereto, some

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* As communicated by him to a Committee of the British Association, and published by them in a Report presented to the Meeting at Glasgow, in 1876.

0·8 grm. (we calculate from the NaCl number) as surplus chloroplatinic acid.] Some 5 c.c. more of the choride of platinum solution are mixed with the residue (which, in the case of NaCl, will produce some 6 c.c. of a 17 per cent. solution); the whole is stirred well, and set aside in a cool place for at least an hour, with occasional stirring. The precipitate is then thrown on a very small filter, the basin rinsed out with about 15 drops of the platinum solution, and the precipitate on the filter washed with 16–24 drops more. The basin, and filter and contents, are then washed with the smallest possible quantity of alcohol of 95 per cent. (by weight or volume?; we used 95 per cent. by weight), and dried at 100° C. The precipitate is transferred as far as possible to a tared capsule, and further dried until it assumes a distinct orange colour. The filter with the remnant of precipitate adhering to it is incinerated, and the residue calculated as $Pt+K_2Cl_2$. The weight of the chloroplatinate of potassium, multiplied by 0·3056, gives the weight of chloride of potassium to be determined. [The factor is calculated from $Cl=35·457$; $K=39·137$; $Pt=197·19$.]

Whenever in the following we state that an analysis was executed according to "*Tatlock's directions*," these directions were followed closely as above given, except that we allowed ourselves to recover the small quantity of chloroplatinate sticking to the filter, by dissolving it off in hot water, and evaporating the solution to dryness in the tared crucible intended to receive the main quantity, and that we continued the drying process at 100° until the weight became constant.

In the case of a substance rich in sulphates, TATLOCK recommends to add a quantity of pure chloride of sodium. This rule, however, is obviously based on the misapprehension that "platinum solution" is one of $PtCl_4$* while it really is one of $PtCl_6H_2$. Yet the chloride of sodium may do good by substituting acid sulphate of soda for the H_2SO_4 liberated, and besides, by displacing some of the HCl in the surplus $PtCl_6H_2$.

Of the various sources of error involved in TATLOCK'S method, the most obvious is the appreciable solubility of chloroplatinate of potassium in water, and aqueous liquids generally. We therefore, at an early stage of our investigation, determined the

Solubility of the Chloroplatinate

in the following reagents :—Five small flasks were tared, each charged with 0·2 grm. of chloroplatinate of potassium, and a convenient volume of the respective

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* We used to be under this erroneous impression ourselves until some three years ago, when we analysed a carefully prepared platinum solution (which had been specially freed from extra hydrochloric acid) for chlorine and platinum. It contained very nearly 6 × Cl for 1 × Pt, which, by the way, is in accordance with an old analysis of "chloride of platinum," quoted in GMELINS'S handbook as having been made by VAUQUELIN.

solvent, the flasks stopped up, and allowed to stand with occasional agitation. Whenever the precipitate threatened to dissolve completely, an additional weighed instalment of chloroplatinate was added. After six days the contents of the flasks were weighed, filtered, and the filtrates analysed for the dissolved chloroplatinate. The temperature during those six days varied from about 13° in the mornings, to about 16°·5 in the evenings. The results were as follows :—

100 parts by weight of solvent dissolve *q* parts of $PtCl_6K_2$.

	Solvent.	*q*.
A.	Water,	0·628
B.	Hydrochloric acid of 5 per cent.,	0·662
C.	"5 per cent." chloride of platinum solution,	0·233
D.	Solution of chloride of platinum, containing 0·05 grm. of added (real) HCl, and 0·05 grm. of platinum, per c.c.,	0·168
E.	Sulphuric acid, containing $\frac{1}{2}SO_3=40$ grms. per litre,	0·900

Methods of Analysis: A. and B.—Evaporation to dryness, and weighing of residue, dried at 150° C., as $PtCl_6K_2$.

C. and D.—Evaporation to dryness on a water-bath, treatment with absolute alcohol, and weighing of the washed precipitate after drying at 150° as $PtCl_6K_2$.

E.—The liquid almost neutralised with pure (potassium-free) caustic soda, the platinum reduced out by hydrogen, and weighed.

We regret now not to have determined the action of stronger solutions of chloroplatinic acid, because Mr TATLOCK virtually begins by washing his chloroplatinate with a "17 per cent." solution of the reagent.

In now passing to our analyses, we begin with a series in which we deliberately departed from certain of Mr TATLOCK'S rules, in order to bring the errors into greater prominence, and also on the chance of being able to rectify these by suitable modifications of the process. Let us at once confess that our success in the latter direction amounted to very little, if anything.

Preliminary Trials.

Experiment I.—The solution analysed contained exactly 0·3471 grm. of chloride of potassium, and about 100 mgs. Na_2SO_4, and 150 mgs. of $MgSO_4$; it consequently represented a salt of "58 per cent." Evaporated down with platinum solution equal to 515 mgs. of platinum, or 50 mgs. more than demanded by the potassium. Residual magma washed five times, each time with 0·5 c.c. *of water*, then exhaustively with absolute alcohol. The chloroplatinate dried at 150°, and weighed; then dissolved in water, the platinum reduced out, and weighed likewise.

The aqueous washings were evaporated with 1 c.c. of normal sulphuric acid (49 mgs. of H_2SO_4) to a magma and Finkenerised; no recrystallisation; sal-ammoniac form applied. Resulting chloroplatinate dissolved, platinum reduced out and weighed.

Alcoholic Washings.—After removal of the alcohol by distillation, the platinum was removed by hydrogen, the filtrate evaporated to dryness, residual salts made into neutral sulphates and Finkenerised; sal-ammoniac process.

The results were as follows (mgs.) :—

I.	Chloride of potassium taken,	347·11 = A.
II.	First chloroplatinate precipitate,	1026·5 = C.
III.	KCl corresponding (= II. × 0·30553),*	313·63
IV.	Platinum from II.,	411·3 = *p*.
V.	KCl corresponding (IV. × 0·76117),	313·07
VI.	KCl in aqueous filtrates,	33·11
VII.	„ alcoholic washings, weighed as $PtCl_6K_2$,	1·12
I. – (VII. + VI.),		312·88 = A_0.

$$A_0 : C = 0{\cdot}3048 : A_0 : p = 0{\cdot}7607 ; \frac{(K_2Cl_2) \times p}{A_0} = 196{\cdot}11.$$

Conclusion.—It is quite possible, by operating as described, *i.e.*, without wasting so much platinum as TATLOCK does, to obtain a chloroplatinate fit for the balance; but the chloroplatinate includes only about 90 per cent. of the chloride of potassium.

Experiment II.—The solution analysed represented 1·04 grms. of a "67 per cent." salt, including 0·24 grm. of Na_2SO_4, 0·20 of NaCl, and 0·10 of $MgCl_2$. Platinum used, 2·6 grms., or 2·49 per grm. of salt analysed. TATLOCK's directions followed, except that the washing with 5 per cent. platinum solution was *continued* until the last runnings contained *only a trace* of SO_4R_2. Washing completed with 95 per cent. alcohol. Precipitate weighed, after drying at 100°, and after further drying at 150°. $A = 0{\cdot}7018$ grm. Chloroplatinate obtained, dried at 100° = $C' = 2{\cdot}2745$: same dried at 150° = $C'' = 2{\cdot}2737$. Platinum from C, by wet-way reduction, = 0·9132 grm. = *p*. Potassium in filtrates collected by FINKENER's (sal-ammoniac) process, and weighed as $PtCl_6K_2$. – KCl thus found = 0·00819.

$C' \times 0{\cdot}30435$ * $= 0{\cdot}69224$; $p \times 0{\cdot}76117 = 0{\cdot}69510$.

Mean = 0·69367 ; loss = 0·00813, = 1·16 per cent. of the KCl taken. KCl in precipitate by synthesis = 0·69361 = A_0.

$A_0 : C' = 0{\cdot}30500$; $A_0 : p = 0{\cdot}75954$.

$$\frac{p \times K_2Cl_2}{A_0} = 196{\cdot}41.$$

Conclusion.—It will *not* do to "improve" upon TATLOCK's method by washing with platinum solution, until the SO_3 is proved to be away.

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* These factors were calculated from results for "M" in the first series of potassium experiments. We did not consider it necessary to recalculate the analyses with our present factors.

Further Trials.

These were carried out with solutions representing 95 per cent., 82 per cent., 33·3 per cent. salts, which were prepared from the same materials as those used for the corresponding trials with FINKENER'S method. In the case of each kind of salt, indeed, the two methods were worked side by side of each other, so as to give no advantage to either.

General Method.—A solution representing 0·648 grm. of a "95 per cent." salt mixed with a few drops of hydrochloric acid, and 32·4 c.c. of 5 per cent. chloride of platinum; mixture evaporated to a magma, mixed with 2 to 3 c.c. of water, and evaporated again. After cooling, 3·25 c.c. of platinum solution added, and allowed to stand for an hour. So far, all the four analyses conducted in the same way.

In (1) *and* (2).—Washing with chloroplatinic acid continued until the impurities by calculation were reduced to about 0·06 mg. and the SO_3 to 0·012 (but a direct test with $BaCl_2$ showed that there must have been more). The washing then completed with 95 per cent. alcohol.

In (3).—After decanting off the mother-liquor, the precipitate was washed once, with 1 c.c. of platinum solution. The precipitate was then dissolved in hot water, and the solution, after addition of 2 c.c. of platinum solution, re-evaporated as far as possible on a water-bath. 2 c.c. of water were then added, and the whole allowed to stand for an hour. The precipitate was then filtered off, washed with two successive cubic centimetres of platinum solution (when as a matter of calculation, the impurities should have been reduced to 0·25 mg.; the SO_3 to 0·06 mg., yet a drop tested with $BaCl_2$ gave a precipitate), and lastly with 95 per cent. alcohol.

In (4), TATLOCK'S directions were strictly obeyed.

Chloroplatinates dried at 100° to 105°, weighed, and reduced with hydrogen, (wet way), to determine their platinum.

The results are stated in the following table (in grammes):—

Experiment,	1.	2.	3.	4.
I. Chloride of potassium taken,	·61075	·61033	·60992	·60917
II. Chloroplatinate obtained,	1·9721	1·9696	1·9755	1·9860
III. Platinum from "II.",	·7927	·7918	·7947	·7964
IV. II. × 0·30435,	·60021	·59945	·60124	·60444
Error = IV. minus I.; mgs.,	−10·54	−10·88	−8·68	−4·73
V. III. × 0·76117,	·60338	·60269	·60490	·60620
Error = V. minus I.; mgs.,	−7·37	−7·64	−5·02	−2·97

Filtrates—from (1), (2), and (3),* freed from alcohol by distillation: their potassium recovered by FINKENER'S (sal-ammoniac) process, and determined

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* Those of (4) were lost by a disaster in the laboratory.

as $PtCl_6K_2$. The KCl recovered amounted to 22·55 mgs. or 7·52 mgs. per analysis.

Uniting analyses (1), (2), and (3) into *one*, using the same symbols as before, we have—

$$A_0 : C = \cdot 30563 ; \; A_0 : p = \cdot 76011 ; \; (p : A_0) \; K_2Cl_2 = 196\cdot 26.$$

For experiment (4) : by our analysis A : C = 0·30673.

The Committee's (Tatlock's) factor is 0·30560.

Final Experiments.

These were all carried out *strictly* according to Tatlock's directions.

I. *Series.*

In it, a 95 per cent. salt, virtually the same as that used for Set I. of the Finkener analyses, was used. Chloride of potassium taken per analysis, ·6104 to ·6124 grm. The chloroplatinates dried at 100°, and their weights multiplied by the Committee's factor, 0·30560, to find the chloride of potassium. The *errors* were, in

Analysis,	(1)	(2)	(3)	(4)
	−1·60	−1·60	+0·91	−1·06 mgs.

The platinum of each precipitate was determined as usual; calculating from the weight of the platinum, by multiplying with 0·76117, the errors were—

(1)	(2)	(3)	(4)
−2·66	−3·43	−0·30	−1·58 mgs.

The volume of the platinic washings = 4·6 c.c. ; that of the alcoholic, about 19 c.c. per analysis.

Total KCl recovered from united washings (by Finkener's sal-ammoniac process), determined as $PtCl_6K_2$ = 11·11 mgs., or 2·78 mgs. per analysis.

Uniting the four analyses into *one*, we have—

$$A = 2\cdot 44447 \text{ grms.} \qquad A_0 = 2\cdot 43336 \qquad C = 7\cdot 9880$$

Hence KCl by Committee's factor, = 2·44112, = "T."

$T - A_0$ = +7·76 mgs. ; $T - A$ = −3·35 mgs., showing that the smallness of the latter difference is owing to impurities in the chloroplatinate.

Calculating constants from the slumped analysis, we have—

$$A \; : C = 0\cdot 30602 \text{ (Committee's factor} = 0\cdot 30560).$$
$$A_0 : C = 0\cdot 30463 \qquad A_0 : p = 0\cdot 76019 \qquad (p : A_0) \; K_2Cl_2 = 196\cdot 24$$

Let us note down before passing on, that the KCl found in the filtrates amounted to 11·11 ÷ 18·6, = 0·597 mg. per c.c. of aqueous platinic washings.

II. *Series: with* 82 *per cent. Salt.*

Comprising again 4 analyses, each made with about 0·52 grm. of chloride of potassium.

Taking "T" as symbol for the weight of chloride of potassium found in an analysis, from the chloroplatinate by the Committee's factor, and A as designating the corresponding weight of chloride of potassium taken, we had:—

Analysis,	(1)	(2)	(3)	(4)
T−A in mgs.,	−1·49	−0·90	−1·21	−1·22
Volume of platinic filtrates,	4·8	4·8	4·8	5·0 c.c.

These, united with the alcoholic washings, contained in all 9·71 mgs. of KCl, or 2·43 per analysis, or 0·500 mg. per c.c. of aqueous *platinic* filtrates.

Uniting the four analyses into *one*, we had—

A=2·10646; A_0=2·09675; C=6·8771;
T=2·10164; T−A= −4·82 mgs.; T−A_0=+4·89 mgs.
A : C=0·30630; A_0 : C=0·30489.
From (1), (3) *and* (4). Total platinum=*p*=2·0689; (platinum from (2) lost).
A_0=1·57274.
A_0 : *p*=0·76018; (*p* : A_0) K_2Cl_2=196·24.

III. *Series:* 33·3 *per cent. Salt.*

Four analyses, each with about 0·215 grm. of chloride of potassium.

Analysis,	(1)	(2)	(3)	(4)
T−A (in mgs.),	−0·66	−0·48	−0·88	−0·44
Volume of platinic filtrates,	5·8	5·9	5·3	5·3 c.c.

From *all* the filtrates, including alcoholic, KCl recovered as usual, and found, = 9·10 mgs., or 2·28 mgs. per analysis, or 0·408 mg. per c.c. of aqueous platinic filtrates.

Uniting the four analyses into *one*, we had—

A=0·86132; A_0=0·85222; C=2·8104; *p*=1·1259;
T=0·85886; T−A= −2·46 mgs.; T−A_0= +6·64 mgs.
A : C=0·30648; A_0 : C=0·30324; A_0 : *p* =0·75692; (*p* : A_0) K_2Cl_2=197·09.

The precipitate obviously was impure, and included foreign chloroplatinates.

Before passing on, let us summarise the principal result of our test analyses by the two methods.

A. *Finkener's.*

Each set of analyses united into one. Column I. refers to the respective page of this memoir; Column II. specifies the set of analyses referred to; Column III. the percentage of KCl in the salt analysed.

Page.	Set.	Per cents. of KCl in Salt.	A : p.	(p : A) K_2Cl_2.
604	I.	95	(·76270)	195·60
605	II.	95	(·76087)	196·06
606	III.	95	·76116	195·99
606	IV.	82	·76157	195·89
607	V.	33·3	·75980	196·34
	Means,	. .	·76084	195·98
	Calculated from III., IV., & V.			All.

(p : A_0)K_2Cl_2=195·73 (Set I.).

Mean (p : A_0) K_2Cl_2 may perhaps be put down at 195·98+0·13=196·11.

Set I. Salts made into normal sulphates; platinum used = 1·075 times the calculated minimum.

Set II. A combination of TATLOCK'S and FINKENER'S methods.

Set III. Only sulphuric acid added; no evaporation: platinum used, =1·25 times the calculated minimum.

Only sets III., IV., and V. correspond to *our present* method: hence the mean factor A : p was calculated from only these 3 sets.

We have recalculated the 12 analyses of sets III., IV., and V., with the new factor 0·76084, and found the "errors" of the recalculated numbers.

Set,	III.				IV.				V.			
A, about	0·75				0·65				0·266 gm.			
Analysis,	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Error,	−·07	−·82	+·11	−·50	+·20	−2·19	−·33	−·17	+·34	+·23	+·18	+·70 mg.

Mean error = ± 0·49 mg., or, excluding No. 2 in IV., it is ± 0·33.

B. *Tatlock's.*

In the following table, the first column refers to the respective page of this memoir; the third gives the percentage of KCl in the set of salts analysed; the figures in the second are reference marks.

Page.		Per cent. of KCl in Salt.	A : C.	A_0 : C.	(C : A_0)K_2Cl_2.	A_0 : p.	(p : A_0)K_2Cl_2.
612	(1)	95	·30673 *	·30563	488·11	·76011	196·26
612	(2)	95	·30602	·30463	489·71	·76019	196·24
613	(3)	82	·30630	·30489	489·29	·76018	196·24
613	(4)	33·3	·30648	(·30324)	491·96	(·75692)	(197·09)
	Means,	.	·30627	·30505	489·04	·76016	196·25
	Calculated from		(2), (3), & (4)	(1), (2), & (3)	(1), (2), & (3)	(1), (2), & (3)	(1), (2), & (3)

Recalculating the 12 analyses we made (by TATLOCK'S exact method) with the factor 0·30627, we arrive at the following errors for the individual results:—

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* From the one analysis of this set in which Tatlock's directions were strictly obeyed.

	(2)				(3)				(4)			
Salt of,	95 per cent.				82 per cent.				33·3 per cent.			
A about,	·610				·520				·215 grm.			
Analysis,	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
Error,	−·26	−·26	+2·25	+·28	−·34	+·25	−·06	−·07	−·19	−·01	−·41	+·03 mg.

Mean error, = ± 0·37, or excluding No (3) under (2), = ± 0·20 mg. We see that the degree of precision afforded by the two methods is about the same, and is amply sufficient for all practical purposes. But it still remains to be seen how far the Tatlock method is available for the analysis of salts which are relatively poor in potash.

Analyses made with the view of deciding this question will be submitted presently; but we prefer to interpolate a duplicate analysis, by both methods, of an imitation of the double salt $MgK_2S_2O_8 + 6H_2O$.

0·5370 grm. of chloride of potassium was converted into neutral sulphate (obtained 0·6273 grm. equivalent, by calculation, to 0·5368 of KCl); this was dissolved in water, and diluted to a known weight. Two portions of the solution were weighed out, each mixed with the calculated weights of a standard solution of $MgSO_4$, prepared from pure oxide by solution in dilute sulphuric acid, and analysed, one by the Tatlock method, the other by our form of the Finkener process.

	Found by the method of	Finkener.	Tatlock.
I.	Chloride of potassium used, ...	·29771	·23929 grm.
II.	Chloroplatinate obtained, ...	?	·7782 „
III.	II. × ·30627 (see page 614) = ...	...	·23834 „
IV.	Platinum from II., ...	·3913	·3119 „
V.	IV. × 0·76084 (see page 614) = ...	·29772	·23731 „
	"Excess" of KCl found by III., ...	...	− 0·95 mg.
	„ „ V., ...	+0·01	− 1·98 „

The Tatlock method, as we see, gave a deficit of about 1 mg. of KCl; but this, after all, is only $\frac{1}{240}$th of the quantity to be determined, which suffices for all purposes; or, in other words, the Tatlock method works well enough even with unmixed sulphates.

The substance analysed contained the equivalent of about 50·6 per cent. of KCl.

We now pass to

A Set of Three Analyses of a "10 *per cent.*" *Salt.*

A standard solution was prepared, which represented a mixture containing (about) 79 of NaCl, 6 of Na_2SO_4, and 5 of $MgSO_4$ in 90 parts. The 10 per cent. of KCl were weighed out specially for each analysis, as a standard solution.

Analysis (1)—Solution Finkenerised with twice the calculated weight of platinum, and enough (by calculation) of standard sulphuric acid to displace the chlorine of the chlorides. Mixture ($xR_2SO_4 + PtCl_6K_2$) "recrystallised," platinum reduced out in the wet-way, and weighed.

Analysis (2)—Solution Finkenerised exactly as in (1); but recrystallisation omitted. From mixture $xR_2SO_4 + PtCl_6K_2$, the sulphates extracted by sal-ammoniac; the residual chloroplatinate dissolved, the platinum reduced out, and weighed. The KCl contained in the filtrate beside NH_4Cl recovered by evaporation and ignition, and weighed; then dissolved in water, wrought with $PtCl_6H_2$ (in FRESENIUS' way), and the chloroplatinate weighed.

In Analysis (3), TATLOCK's method was applied in all strictness, except that the chloroplatinate received an extra washing with 6 drops of platinum solution.

Analysis (1)—*Finkener's Method; Recrystallisation.*

A = 77·22 mgs.; p = 102·2; $p \times 0·76084 = 77·76$;
Error = + 0·54 mg., or 0·7 per cent of A.

Analysis (2)—*Finkener's Method; Sal-Ammoniac Form.*

A = 162·44 mgs.; p = 217·4; $p \times 0·76084 = 165·41$;

Excess over A = 2·97 mgs. Crude KCl from filtrate (from p) = 166·9; Excess over A = 4·5 mgs. Chloroplatinate from the crude chloride of potassium = 527·6 mgs.; whence by multiplication with 0·30627 = 161·59 mgs. of chloride of potassium; deficit against A = 0·85 mg., or 0·52 per cent. Platinum out of the last chloroplatinate = p' = 212·0; $p' \times 0·7608 = 161·30$, which is less than A by 1·14 mgs., or 0·70 per cent.

Analysis (3)—*Tatlock's Method.*

A = 65·44 mgs.; C = 209·2; C × 0·30627 = 64·07 T; T − A = −1·37, or 2·1 per cent of A. Platinum from C = 84·0 = p. $p \times 0·7608 = 63·91 = T'$. $T' - A = -1·53$ mgs., or 2·3 per cent. of A.

Analysis (2) did not do justice to its method, through unobserved causes, it is true. Yet the error in (1) or (2) did not rise beyond 0·7 per cent. of the small quantity to be determined. The TATLOCK method loses 2 per cent. of the chloride of potassium to be determined; *i.e.*, it would report 9·8 instead of 10 per cent. We believe the line of the applicability of TATLOCK's process must be drawn at about the "10 per cent." salt.

Three Analyses of synthetically prepared Sea-Water Salts.

(Average ocean-water salts contain 2·11 per cent. of potassium calculated as KCl.)

A kind of potash-free sea-water was made from pure materials: chloride of sodium, magnesia, and standard solutions of sulphuric and hydrochloric acids. For each analysis a volume corresponding to so and so much average ocean-water was measured out, and the exact weight of a standard solution of chloride of potassium added. The methods were exactly the same as those used in the preceding set.

Analysis (1)—*Finkener's Method;* "*Recrystallisation.*"

Chloride of potassium used as such, ...	=148·69 mgs.	
„ „ present in the chloride of sodium used, .	= 0·19 „	
Total chloride of potassium, ...	=148·88 „	=A.

p = 198·3; $p \times 0·76084$, = 150·87; excess over A = 1·99 mgs., or 1·3 per cent. of A:—partly through compensation of errors.

The ether-alcohol washings, when worked up for potassium as usual, gave,

$$\left.\begin{array}{l}\text{The first washings } PtCl_6K_2 = 0·36 \text{ mg.} \\ \text{The second } \quad „ \quad\quad „ \quad = 0·61 \text{ „}\end{array}\right\}\text{of KCl.}$$

This analysis was made by means of our *present* form of the FINKENER process; the foreign bases were converted into sulphates only virtually, *i.e.*, by adding the calculated volume of standard sulphuric acid, &c.

In the "Challenger" analyses, the first step always was to actually convert all the bases into neutral sulphates, which probably ensures greater constancy in the results.

Analysis (2)—*Finkener's Method; Sal-Ammoniac Form.*

Total chloride of potassium operated upon (including that of the NaCl), = A = 148·59 mgs.

Platinum from the chloroplatinate ($+xNH_4Cl$), = 202·1 = p. $p \times 0·76084$, = 153·77; excess over A = 5·18 mgs., or 3·3 per cent of A. Crude KCl (from filtrate from platinum), = 146·0. Chloroplatinate from the same = 0·4642 grm. = 142·17 = "a" mgs. of KCl; this is less than A by 6·42 mgs., or 4·3 per cent. of A.

KCl recovered (as ultimately $PtCl_6K_2$) from sal-ammoniac liquors, =6·54 mgs. = Δ; from ether-alcohol washings = 0·46 mg. = δ. By addition, $a + \Delta = 148·71$ mgs. $(a + \Delta) - A = +0·12$ mg., or 0·08 per cent. of A.

Here again the sal-ammoniac process failed to do justice to itself; in many similar cases we obtained better results, in the sense that far less potassium passed into the sal-ammoniac. The method, unfortunately, is somewhat capricious; the chloroplatinate does not always stand the sal-ammoniac treatment equally well. To determine small quantities of potassium correctly, the sal-ammoniac liquors *must* be worked up; and in no case dares the crude chloride of potassium be accepted as *pure KCl.* Yet the sal-ammoniac form of the FINKENER method is invaluable, being the only method for extracting a *small* proportion of potassium from a mass of, *e.g.*, soda-salts; only it must be wrought with great circumspection, and in its *original form* be only used as a mode of *extracting* the potassium, not as a method for its determination.

Analysis (3)—*Tatlock's Method.*

Chloride of potassium used = A = 14·22 mgs.

(We could not have used as much as in (1) or (2) without wasting an unreasonable amount of platinum solution.) C = 41·0; C × 0·30627 = 12·56 = T. T − A = −1·66 mgs., or 11·7 per cent. of A.

Platinum from C = 16·9 mgs. = 12·86 mgs. of KCl. Chloride of potassium recovered from the washings, and weighed, ultimately as $PtCl_6K_2$ = 2·92 mgs.; hence A_0 = 11·30. T − A_0 = + 1·26 mgs., or 11·1 per cent. of A_0.

With salt-mixtures like sea-water salts the TATLOCK method obviously loses its applicability. Nor was it ever intended for such mixtures.

IV. EXPERIMENTS ON CHLOROPLATINATE OF RUBIDIUM.

These experiments were planned at a very early stage of our investigation. They were suggested by the obvious consideration that for the synthetical determination of the weight-ratio Pt : 2Cl between the platinum and fixed chlorine in chloroplatinates, chloride of rubidium should be better adapted than the potassium-salt, because, while itself soluble in alcohol, its chloroplatinate is less soluble in water than chloroplatinate of potassium. For a similar reason, chloride of cæsium should be preferable to chloride of rubidium; but we shrank from the great expense which would have been involved in procuring the necessary supply of the rarer of the two rare alkalies.

Our raw material for the preparation of chloride of rubidum was a supply of "*rubidium alum*" from Trommsdorff in Erfurt. The alum was dissolved in hot water, and its rubidium precipitated by addition of chloroplatinic acid, the precipitate washed, reduced in the dry way with hydrogen, and the chloride of rubidium extracted with water. As it turned out to contain a very appreciable quantity of sulphate, it was redissolved, reconverted into chloroplatinate, and recovered from the latter by means of hydrogen. The salt thus obtained was contaminated with sulphuric acid, and two or three repetitions of the cycle of operations failed to eliminate this impurity quite completely. Going by the aspect of the chloride of barium precipitate, the last precipitation, indeed, seemed to have effected no improvement; we therefore evaporated the whole of our (last) chloride of rubidium to dryness, and thus obtained about 32 grms.

of a salt, which, apart from that trace of sulphuric acid, seemed to be very pure. In its aqueous solution, sulphuretted hydrogen produced no change; sulphide of ammonium had no immediate effect, and, on long standing, only an almost invisible precipitate settled out. 0·5 grm. of the salt, when dissolved and mixed with iodide of potassium, gave no precipitate (absence of thallium). The spectrum-apparatus revealed no trace of potassium.

In order to, at the same time, determine the sulphuric acid, and rehearse a method for its removal, 5 grms. of the salt were dissolved, chloride of barium added, the precipitate allowed to settle, and weighed. It amounted to 6·5 mgs., indicating 0·045 per cent. of SO_3 in the preparation.

From the (concentrated) filtrate, the rubidium was precipitated by addition of a slight excess of chloroplatinic acid, the chloroplatinate allowed to settle, and washed by decanting filtration, first with water, then with 50 per cent., and lastly with absolute, alcohol. The weight of the chloroplatinate, after a preliminary drying at 120°, amounted to 11·187 grms.

1·0094 grms. of this chloroplatinate were dissolved in hot water, in an Erlenmeyer flask, the platinum was reduced out by hydrogen, and filtered off, and the filtrate evaporated to dryness over a water-bath. The residue was dissolved in 50 c.c. of water, and separate portions examined. 10 c.c. when mixed with sulphide of ammonium, gave a mere *trace* of (FeS and Al_2O_3?); 10 c.c. when Nesslerised, gave 0·08 mg. of NH_3, corresponding to 0·4 mg. per 1 grm. of salt; 10 c.c. when mixed with chloride of barium, and other 10 c.c. when mixed with sulphuric acid, gave *both* slight clouds of sulphate of baryta. From the last two tests, it was clear that the application of chloride of barium to our stock of salt would have done little good, and we accordingly decided upon using the preparation as it was.

A portion of it was dehydrated in a platinum crucible without fusion, then fused very cautiously, and poured out into a platinum basin. 18·1321 grms. of such salt were dissolved in water to 300 c.c., and thus converted into 313·522 grms. of a standard solution, of which every 1 grm. contained 57·834 mgs. of salt. Three titrimetric determinations of the chlorine gave the following results:—

Analysis,	1.	2.	3.
Approximate weight of solution taken, .	10·4	20·8	20·8 grms.
Chlorine per gramme of solution, . . .	16·8253	16·8200	16·8226 mgs.

Mean = 16·8226, corresponding to 29·087 per cent. of chlorine in the original salt.

Taking Rb = 85·4, and Cs = 133·0, the precentages of chlorine in the chlorides of the two metals are—

In	RbCl.	CsCl.
Equal to, ...	29·337	21·047

Hence our "chloride of rubidium" contained

Real chloride of rubidium, ...	96·985
Chloride of cæsium, ...	3·015
	100·000

These 3 per cent. of cæsium chloride, although inconvenient, did not unfit the salt for our purpose; all we had to do was to base our calculations of "Pt" not upon the weight of alkyl-chloride present in the respective chloroplatinate, but upon the weight of the chlorine in that chloride.

The reason why we took such pains in standardising our rubidium solution, of course was, that we intended to rely chiefly on the synthetical data of our experiments; but unfortunately, these became almost valueless through a change in the strength of the rubidium solution, which was observed only after the greater part of the work had been completed; yet they were, and still are, of great use to ourselves as affording checks for the respective analyses. Only the latter are reported on in the following paragraphs:—

To begin with a case where no quantitative synthesis was attempted, let us give the results of an analysis of the chloroplatinate of rubidium referred to above, as having been obtained incidentally from about 5 grms. of salt.

3·2495 grms. of this preparation (weighed after a preliminary drying at 120° C.) were placed in a "Geissler tube," and dried systematically, first at 120°, then at 130°, and lastly at 150° C. The weights recorded were as follows:—

After 2 hours at 120°, ...	3·2457
" " 130°, ...	3·2432
" 9 " 150°, ...	3·2342

Even then the weight was not constant; yet the drying process was stopped, and 3·1970 grms. of the salt transferred to an Erlenmeyer flask, to be reduced with hydrogen in the wet way, and analysed in the way we had before applied to many specimens of the potassium salt.

The reduction set in very readily, but was very slow in coming to an end. It took in all about six days for its completion, although every morning the precipitate was broken up with a glass rod, to bring the hidden chloroplatinate to the surface. When at last the reduction seemed to be completed, the platinum was filtered off, and the filtrate divided gravimetrically for the determination of the fixed and of the total chlorine. Suspecting that the platinum might include some undecomposed chloroplatinate, the greater part was removed from the filter, and next dehydrated at a dull-red heat. It weighed 1·0454 grms.; after subsequent strong ignition this weight was 1·0452 grms. and after a succeeding strong ignition in hydrogen it was 1·0449 grms. Hence it appears that the proportion of undecomposed chloroplatinate in the "platinum" was at the worst extremely small. The total weight of platinum obtained amounted to 1·0923 grms.

For the isolation of the fixed chlorine, the respective portion of the solution was evaporated to dryness, the residue dried at 130° for 2 hours, then dissolved in water, the solution re-evaporated, and the residue again dried at 130° for 1½ hours. The solution of the thus dried salt was absolutely neutral to litmus.

The determinations of the chlorine were effected by the "Challenger" method as usual.

Summary of Results.

Found per 2Cl=70·91 parts of *fixed chlorine.*

RCl by direct weighing.*	Platinum.	Loose Chlorine.	Substance.
244·33	203·39	144·71 =4·0816 × Cl.	595·29

These results at the time surprised us very much; but we have no difficulty now (after our later experience with the potassium salt) in explaining them. Part of the 203·39 parts of platinum must be assumed to be present as PtX_6H_2; the X_6 including the 0·0816 × Cl of chlorine, besides the necessary amount of oxygen or hydroxyl. Taking Pt as 195·5 as it follows from our potassium experiments, we have for the composition of 595·29 of the chloroplatinate :—

2RCl,	244·33	
1·04036 × Pt,	203·39	
Loose chlorine,	144·71	
Hydroxyl,	2·73	= 0·1606 × OH.
Hydrogen,	0·08	
	595·24	

which agrees very well with the above 595·29 of substance analysed; only the *closeness* of the agreement is probably accidental, as the direct determination of the "RCl" was made only on a very small scale.*

We now pass to those synthetical experiments with standardised solutions, of which, unfortunately, only the analytical parts are worth publishing.

Experiment I.

40 c.c. of the rubidium solution were evaporated to 15 c.c., poured into 40·4 c.c. of standard platinum solution (1 c.c. = 49·5 mgs. of metal), and the

---

* A weighed portion of the filtrate from the platinum evaporated to dryness, and made neutral, as above explained, and weighed. *Actual* weight = ·2213 grm. for the fraction analysed. The salt was analysed, and found to contain 29·09 per cent. of chlorine, *i.e.*, almost exactly as much as the original chloride of rubidium.

mixture was allowed to stand over night. Next morning the liquor was decanted through a small filter, the precipitate washed four times with water (10 c.c. each time), and then twice with absolute alcohol. Absolute weight of platinum used = 1·9976 grms.; platinum per Cl_2 = 70·91 parts of fixed chlorine = "P" = 201·8 (nearly, the exact data need not be reproduced *here*). Total chloroplatinate produced = 5·78 grms. It was dried at 130° C. for 5 hours, and then divided into two parts, A and B. A served for a determination of the water by the direct method, described page 578; water found = 0·862 per cent. B was reduced with hydrogen in the *wet* way (which took about 10 days), and analysed as usual. Found, per Cl_2 parts of fixed chlorine: alkyl chloride, by calculation from the percentage of chlorine in the original chloride of rubidium, = 243·79; platinum = 201·08; loose chlorine = 4·0776 × Cl; chloroplatinate (dried at 130°), = M = 593·81; hence, assuming that Pt = 195·5, and that only so much of the platinum is there as $PtCl_6R_2$, we have for M parts—

Platinum,	201·08
R_2Cl_2,	243·79
Other chlorine,	144·57
Hydroxyl,	1·59
Hydrogen,	0·06
Other water,	4·28
	595·37
M.,	=593·81
Excess over M.,	= 1·56

Experiment II.

Conducted pretty much like Experiment I. Platinum used = 1·48 grms.; P = 299. Chloroplatinate dried at 130° C. Water *not* determined. As the reduction again progressed very slowly, the determination of the *total* chlorine was omitted. Found per Cl_2 = 70·91 parts of fixed chlorine, M = 583·60; platinum = 196·21 only, although such a large excess of platinum solution had been employed for the production of the chloroplatinate! Just as in the case of the chloroplatinate of potassium, the presence of a large excess of chloroplatinic acid seems to prevent precipitation of surplus platinum as $Pt(OH)_6H_2$; the basic salt, which would otherwise have clung to the precipitate, passes into solution. Assuming that the loose chlorine in this case was double of the fixed, we have, for the composition of M parts of chloroplatinate—

R_2Cl_2,	243·79 (calculated).
Cl_4,	141·82
Platinum,	196·21
Water (?),	1·79
	583·61

Experiment III.

This experiment was carried on side by side of the preceding one, from which it differed in this, that the rubidium was kept in excess over the platinum solution, and that the latter was poured into the former. Platinum used = 0·987 grm ; P = 132·6. Chloroplatinate dried at 130° C., but constancy of weight not insisted on. Determination of total chlorine and of water again omitted. Found per Cl_2 = 70·91 parts of chlorine ; M = 602·43 ; platinum = 204·94 ; *i.e.*, higher than ever before. [The synthesis gave 204·8 or 205·7 according to whether one or other of two limit values were adopted for the uncertain titre of the rubidium solution.]

Experiment IV.

Here again an excess of rubidium was used, and the chloroplatinic acid poured into the RbCl. Precipitate treated, and dried at 130° C., as before. For its analysis, however, it was reduced by hydrogen in the *dry* way. Platinum used = 0·991 grm. P = 133·8. Found per Cl_2 parts of fixed chlorine, M = 611·08 ; platinum = 206·89.

Experiment V.

A parallel experiment to IV., from which it differed only in this that an *excess of platinum solution* was started with, and the rubidium chloride poured into it. Platinum used = 1·4861 grms.; P = 301. Found, per Cl_2 parts of fixed chlorine, platinum = 198·19 ; M not determined exactly. Here again, as in Experiment II., an excess of platinum used in the preparation of the chloroplatinate prevented, *to some extent,* precipitation of surplus platinum.

Experiment VI.

In this experiment a very large excess of chloride of rubidium was employed; and the reduction of the chloroplatinate effected in the *wet* way. To make sure of no chloroplatinate escaping reduction, it was divided into two parts (each equal to about 1 grm.), each part dissolved *completely* in hot water, and then submitted to the action of the hydrogen. To avoid loss of loose chlorine, the hydrogen entered from the Kipp's apparatus, and passed out from the flasks through U-tubes, the bends of which were closed with a layer of water. The contents of these protection tubes, however, when tested after the experiment with nitrate of silver, were found to contain no trace of hydrochloric acid.

Platinum used = 0·6917 grm.; P = 92·9. The chloroplatinate this time was dried only at 100°. Found per Cl_2 parts of fixed chlorine:—$M = 592·56$; platinum $= 201·95$; loose chlorine $= 4·0224 \times Cl$. Hence, assuming $Pt = 195·5$, and that the surplus platinum $(201·95 - 195·50 = 6·45)$ is present as PtX_6H_2; we have, for the composition of M parts—

Platinum,	201·95
2RCl, by calculation,	243·79
$4·0224 \times Cl$,	142·61
Hydroxyl,	2·99
Hydrogen,	0·07
Other water,	1·15
	592·56

Experiment VII.

This experiment was carried out pretty much in the same way as Experiment VI. in the potassium series, and its object, like that of the latter, was to ascertain the composition of the unwashed chloroplatinate precipitate. 20 c.c. of the rubidium solution were weighed out, and poured into a tared bottle of 120 c c.'s capacity; a known (predetermined) weight of platinum solution was then added, the bottle filled up to near the shoulder with water (total volume = 200 c.c. about), the whole mixed, and allowed to stand over night. Platinum used = 0·969 grm.; P = 195·2.

After determination of the exact weight of the whole, as much as possible of the clear liquor was sucked off into a tared Erlenmeyer flask, weighed, and analysed by reduction with hydrogen. To be able to determine the loose chlorine of the precipitate, the chloroplatinic acid solution was analysed immediately before the experiment, by reducing a known weight with hydrogen, weighing the platinum, and determining the chlorine in the filtrate; the small quantity of fixed chlorine in that solution being determined by itself, and allowed for. The uncertainty in the titre of the rubidium solution of course affected this experiment as well as it did the (unpublished) synthetical data of Experiments I. to VI.; yet this uncertainty is not sufficient to invalidate the principal result, which was that (for every Cl_2 parts of fixed chlorine) the platinum amounted to 205·48 parts, ± say one unit; but even 204·5 would be a very high result. The loose chlorine amounted to $4·151 \times Cl$; a large excess over 4.

Experiment VIII.

From the above experiments we concluded that our "chloroplatinates of rubidium" were not even normally constituted alkyl-chloroplatinates, *i.e.*, mixtures of the composition $PtCl_6R_2$; but included hydrogen instead of part of the R_2, and hydroxyl or oxygen instead of part of the Cl_6; that, in short they were mixtures of the general composition $PtCl_{(6-x)}(OH)_x + (R_{2-y}H_y)$, and we deemed it worth while to try and reduce the y to *nil* by using chloroplatinate of *sodium* as a precipitant and the "x," by heating the precipitate in a current of dry hydrochloric acid.

In accordance therewith (St Petersburg) platinum solution equal to 1·100 grms. of metal was evaporated with 0·7616 grm. of chloride of sodium, *i.e.* 1·15 Na_2Cl_2 for 195 of platinum, to dryness on a water-bath, the residue redissolved in 20 c.c. of water, and added to the chloride of rubidium solution to produce a mixture containing about 0·68 grm. of surplus chloride of rubidium by calculation. The mixture was allowed to stand over night, and the precipitate then washed by decanting filtration, first with water, then with 50 per cent., and lastly with absolute, alcohol. The greater part of the dried precipitate was transferred to a large porcelain boat, and in it, within a combustion tube drawn out at the exit end, kept in a current of air, dried with oil of vitriol, at 120° C., until its weight became constant, which took about six hours in all. The weight of the dried chloroplatinate was 2·7791 grms. The salt was then replaced in the combustion tube, a U-tube filled with glass beads moistened with vitriol attached to the exit end by means of a short india-rubber joint (see section on determination of water, page 578), and this *joint* next kept at 120°, in a current of dry air, until the U-tube ceased to gain weight; the U-tube consisted entirely of glass. The air was displaced by dry carbonic acid, the latter by hydrochloric acid, and this gas allowed to act at 120° C. for 3½ hours. The hydrochloric acid was then displaced by carbonic acid at 50°, and the latter by dry air, at the same temperature. The boat had gained 0·5 mg., the sulphuric acid U-tube 6·4 mgs. The contents of the latter, when diluted largely with water, and tested with nitrate of silver, gave a mere opalescence of chloride of silver. Hence the 6·4 mgs. of a gain were water, and, assuming that the substance lost water, and hydroxyl with formation of water ($HO + HCl = H_2O + Cl$), the water expelled amounted to 3·0 mgs., and the hydroxyl replaced by chlorine to 3·21 mgs. and the chlorine taken up in place of the latter to 6·7 mgs. The chloroplatinate was kept over solid caustic soda for a night, to remove any adhering HCl, but it suffered no loss of weight. For its analysis, it was dissolved *completely* in hot water, reduced by hydrogen, &c. As there was not sufficient material for duplicate analyses, the total and the fixed chlorine were each determined twice in the same quantity of solution, viz., gravimetrically, and again by determining the weight of unprecipitated silver titrimetrically. The results reduced to 2Cl parts of fixed chlorine were as follows:—

M = 581·37; platinum = 194·35; loose chlorine = 139·81, = 3·9434 × Cl. Hence we have for the composition of M parts—

Platinum,	194·35
2RCl, by calculation,	243·79
Loose chlorine,	139·81
Hydroxyl (0·0566 × OH),	0·96
	578·91
M found,	= 581·37
Water (?),	= 2·46

The 2RCl, however, may have included NaCl, which would make the deficit still greater, or it may have contained relatively more chloride of cæsium than the original "chloride of rubidium," which would have the opposite effect. Taking the experiment as it stands, it would appear that Pt = 194·35, *i.e.*, a little less than SEUBERT'S value; but it would be absurd to draw this conclusion from this one isolated experiment. A more plausible hypothesis is that the chloroplatinate contained surplus alkyl-chloride, which of course depresses the value Pt as calculated from the fixed chlorine. Assuming that the chloroplatinate contained its full complement of chlorine, and reducing the results to 6 × Cl parts of total chlorine, we have M = 586·90; and for the composition of those M parts—

Platinum,	196·20
1·00952 × R_2Cl_2,	246·11
4 × Cl,	141·82
	584·13
M found,	= 586·90
Deficit,	= 2·77

If we reduce to Pt = 195·50, we have M = 584·82; and for the composition of M parts,

Platinum,	195·50
1·00593 × R_2Cl_2,	245·23
3·9668 × Cl,	140·64
0·0332 × OH =	0·56
	581·93
M found,	= 584·82
Deficit,	= 2·89

That the chloroplatinate analysed contained 0·00593 × R_2Cl_2 of free alkyl chloride is no improbable assumption.

None of our rubidium experiments afford the data for calculating even a limit value for the atomic weight of platinum; yet they are interesting, as showing that the tendency of chloroplatinate of potassium to carry down surplus platinum, in the form of chiefly hydroxide $Pt(OH)_6H_2$, when produced by precipitation of chloroplatinic acid with alkyl chloride, is greatly intensified in the rubidium salt; and in its case exhibits itself strongly even in the presence of a large excess of rubidium chloride.

Our last Experiment, VIII., may perhaps be referred to as showing that this irregularity can be avoided by using chloroplatinate of *sodium* as a precipitant, and (may we add ?) avoiding an excess of rubidium salt.

When we were engaged in these experiments we had not yet discovered the method of recrystallisation for the removal of surplus platinum, but an experiment made incidentally with one of our chloroplatinates, viz., the one whose analysis is quoted on page 621), proves that *this* chloroplatinate did contain surplus platinum. 2 grammes of it were boiled with 100 c.c. of water, the residue (*a*) filtered off hot, the filtrate, which was strongly acid, cooled down, the precipitate formed by the cooling process filtered off, and the filtrate evaporated to dryness on a water-bath. The residue was treated with 8 c.c. of water, and the solution filtered. It gave no precipitate on addition of chloride of platinum, but a strong precipitate on addition of chloride of rubidium solution. The first residue (*a*) when boiled with water, furnished a solution, which when evaporated, and (the residue) treated with 8 c.c. of water, gave a filtrate which produced slight precipitates with both reagents.

V. Experiments on Chloroplatinate of Ammonium.

For the preparation of the necessary supply of pure sal-ammoniac, we started from a kind of commercial *liquor ammoniæ,* of 0·88 sp. gr., which had been sold to us as having been prepared from volcanic sal-ammoniac. 300 c.c. of this liquor were mixed with 65 c.c. of specially prepared hydrochloric acid of 20 per cent., so as to neutralise about $\frac{1}{10}$th of the volatile alkali, and the mixture heated, to drive out the still uncombined ammonia, which was passed into 360 c.c. of the hydrochloric acid. The resulting liquid was alkaline. It was evaporated in a Berlin basin over a water-bath, until a considerable quantity of sal-ammoniac had crystallised out, the residue allowed to cool, the magma of crystals collected on a funnel connected with a Bunsen pump, and washed with small instalments of 10 per cent. ammonia solution, made from the *volcanic liquor*, by distilling off ammonia from it, and passing it into water. The salt was then transferred to a shallow basin, and kept under a bell-jar in an atmosphere of (dilute) ammonia, over caustic soda sticks, which latter were renewed from time to time. After about a fortnight, the sal-ammoniac was transferred to another bell-jar, and under it, kept over oil of vitriol for 24 hours, to remove the ammonia adhering to the crystals. After this last operation, the sal-ammoniac was assumed to be pure, and was bottled up for the experiments.

To make sure of its purity, however, a known weight of the salt (about 1·6 grms.), was dissolved in water to a known weight, and the chlorine determined in two aliquot parts, both gravimetrically (with a known weight of silver used as nitrate) and titrimetrically, *i.e.*, by determining the silver left unprecipitated, by means of our form of VOLHARD's method.

Analysis,	I.	II.	
(1) Sal-ammoniac used,	·268735	·537277	grm.
(2) Chlorine calculated from the weight of the AgCl obtained,	·178059	·356143	„
(3) From the weight of the silver precipitated,	·178028	·355885	„
(4) Mean of (2) and (3),	·178044	·356014	„
(5) Substance analysed per 35·456 parts of chlorine,	53·515	53·507	
Mean,	53·511		
STAS' value is,	53·506		

This sal-ammonic was used in the following experiments.

I. *Series.*

The general mode of operating was as follows :—

A known weight of the salt (about 0·54 grm.) weighed out as a standard solution, was mixed with a slight excess of chloroplatinic acid, the mixture evaporated to almost dryness over a water-bath, the residue treated with a mixture of equal volumes of absolute alcohol and absolute ether (both specially distilled with a little syrupy phosphoric acid, to eliminate any ammonia), and washed with such ether-alcohol until free of soluble chlorine. Only a small portion of the precipitate was allowed to get on the filter. To recover this small portion, it was allowed to dry in the air, then dried further at 100°, and what could not be removed mechanically, dissolved off with hot water; the solution was evaporated to dryness over a water-bath, the bulk of the precipitate added, and the whole kept in a drying-chamber at 100° to 110° until constant in weight. The chloroplatinate was then transferred to an Erlenmeyer flask, with some 300 c.c. of water, the platinum reduced out with hydrogen, filtered off, and weighed. The filtrate was diluted to a known weight, and a small portion utilised for a preliminary determination of the chlorine. A larger portion then served for the exact determination of the chlorine, by means of that combination of the gravimetric and VOLHARD's method, which had served for the analysis of the sal-ammoniac.

The ether-alcohol washings were collected and utilised for the determination of the small amount of ammonia which had escaped precipitation. These determinations, however, proved valueless, because we found out (only when all the work had been done) that the chloroplatinic acid used was contaminated appreciably with the nitroso-compound, $PtCl_6(NO)_2$.

We therefore here satisfy ourselves with stating that the weights of unprecipitated sal-ammoniac *appeared* to amount to from 0·6 to 0·9 mg. per 0·54 grm. of sal-ammoniac operated upon.

Five experiments were made in this manner. The results are given in the following table, in which P stands for the weight of platinum used per $2NH_4Cl$ parts of sal-ammoniac; A signifies the weight of sal-ammoniac used; C, that of the chloroplatinate obtained; *p*, that of the platinum obtained from the chloroplatinate; Pt′ the weight of platinum present per $6 \times Cl$ parts of total chlorine.

Experiment,	I.	II.	III.	IV.	V.
P,	224 very nearly throughout.				
A : C,	·23873	·23970	(·24139)	·23948	·23984
A : *p*,	·54464	·54658	lost	·54616	·54743
Pt′,	196·90	196·59	...	196·73	196·48
p : C,	·43833	·43854	...	·43847	·43812

Mean Values contrasted with those calculated from Seubert's Pt = 194·8.

		Mean.	Calculated from Pt = 194·8.
A : C (omitting III.),	=	·23944	·24121
A : *p*,	=	·54620	·54934
Pt′,	=	196·68	194·8
p : C,	=	·43837	·43909

As the values Pt′, and more still those of *p* : C, agree fairly; while those for A : C and A : *p* vary to an unpleasant extent, we thought that a variable part of the ammonia might have been lost in the evaporations,—the presence of nitroso-compound in the reagent had not been noticed yet,—and therefore tried the following experiment:—The same quantities of standard sal-ammoniac and chloroplatinic acid solutions as has been used in Experiment V. were mixed in a retort connected with a Liebig's condenser, and distilled down in a current of air, while immersed in a steam-bath, and thus kept near 100° C. This somewhat tedious operation was continued until only about 1 c.c. of liquid was left in the retort. In the distillate, after neutralisation with ammonia-free caustic soda, the ammonia was determined by NESSLER'S colorimetric method. It amounted to only 0·01 mg. This tends to show that under the circumstances chloroplatinate of ammonium is not liable to dissociation (into $2NH_3$ and $PtCl_6H_2$); but then the ammonia, which otherwise would have escaped as such, may have been destroyed by the nitroso-compound.

II. *Series.*

The chloroplatinates obtained in the above experiments were obviously not of a constant composition, reducible to the general formula $PtCl_6(NH_4)_2$. Thinking that the irregularities were perhaps caused by the process of *evaporation*, we prepared two quantities of chloroplatinate of ammonium by mere precipitation in the cold. In one case (I.) the chloroplatinic acid was in excess, and the sal-ammoniac poured into it; in the other (II.), the sal-ammoniac solution was in excess, and the chloroplatinic acid poured into it. In either case, 3·00 grms. of platinum (as $PtCl_6H_2$) were operated upon, and the two solutions, before being mixed, attenuated by addition of quantities of water so adjusted that the total volume on both sides was 150 c.c. In both cases the mixture was allowed to stand over night. Next morning the clear liquid was decanted through a small filter; the precipitate was then washed by decanting filtration, first with small instalments of water until the soluble chlorine was reduced to a fraction of milligramme (by calculation), then with absolute alcohol until all the soluble chlorine was proved to be away (by testing with nitrate of silver). The precipitates were dried at 105° to 110° until constant in weight. Each of the two chloroplatinates was divided into two approximately equal parts, and each part analysed by itself; the platinum was reduced out, filtered off, and weighed; the filtrate divided into two aliquot parts, and in each the chlorine determined gravimetrically. VOLHARD's method was not used in this case, as our stock of standard silver had become exhausted. All duplicate determinations made agreed very well with each other. In the following table we give the values, $C : p$; Pt' = weight of platinum per $6 \times Cl$ parts of total chlorine ; Pt'' = weight of platinum per $Cl_6(NH_4)_2$ = 248·84 parts of non-platinum in the precipitate, also the approximate weight P of platinum employed per $2NH_4Cl$ parts of sal-ammoniac.

Experiment,	I.		II.		Seubert's Values.
Part,	A.	B.	A.	B.	
$p : C$,	·43838	·43837	·43836	·43834	·43909
Pt',	197·04	196·97	197·13	197·14	194·8
Pt'',	194·19	194·23	194·22	194·20	
P,	200		192		

The most remarkable feature in these results is that the values $p : C$, and P', were the same in Experiment I. as in Experiment II. Clearly neither the mean of the values Pt' nor that of the values Pt'' can pretend to be a close approximation to the true Pt. A reasonable hypothesis to account for the difference between Pt' and Pt'' is to assume that the chloroplatinate was a mixture of the composition $PtCl_6(NH_4)_2 + xPt(OH)_6H_2 + yH_2O$. By calculating the four analyses as *one*, we found per $6 \times Cl$ parts of chlorine—

Chloroplatinate, ... = M = 449·53
Platinum, ... = Pt' = 197·06

Assuming $6 \times Cl$ to be associated with $2NH_4Cl$ = 107·01, we have for the composition of the precipitate—

Platinum,	= 197·07
$2NH_4Cl$,	= 107·01
$4 \times Cl$,	= 141·82
Unaccounted for,	= 3·63
M,	= 449·53

What value must we assign to Pt, so that the $(OH)_6H_2$, combined with the surplus platinum (197·07 − Pt) equals these 3·63 parts ?

An easy calculation shows that we must take Pt = 190·4, which, of course, is quite inadmissible, but part of these 3·63 parts may be water.

III. *Series.*

The Experiments reported on under I. and II. were made a good long time ago. Quite lately, when we were engaged in summing up results generally, it struck us that they had better be supplemented by new experiments made with a chloroplatinic acid prepared from metal by means of hydrochloric acid and chlorine gas, and we accordingly did so. The following three syntheses and analyses were made with the same kind of sal-ammoniac as had served for Series I. and II., and the general *modus operandi* was the same as in Series I. The ether-alcohol washings were subjected to distillation to remove the ether-alcohol, the residue mixed with water, the platinum reduced out by hydrogen, and the mixture produced distilled with ammonia-free caustic soda, so as to concentrate the ammonia into a small quantity of hydrochloric solution. This solution was mixed with chloroplatinic acid, evaporated to dryness, and the residue washed with ether-alcohol, &c., to recover the ammonia as chloroplatinate. The weight of sal-ammoniac operated upon in each experiment was about 0·814 grm., the weight of platinum used per $2NH_4Cl$ was 227 parts, or about 1·15 times the quantity demanded by Pt = 197. The chloroplatinate precipitates were dried at 100° C. until constant in weight.

The results are given in the following tables :—

Experiment,	(1)	(2)	(3)*
Sal-ammoniac taken,	·81359	·81388	·81352 = A.
Sal-ammoniac in ether-alcohol filtrate,	·00083	·00083	·00160
„ in precipitate,	·81276	·81305	·81192 = A_0.
Chloroplatinate obtained,	3·4179	3·4139	3·4063 = C.
Platinum in C,	1·4925	1·4940	1·4883 = *p*.
Total chlorine,	1·61684	1·61865	1·61525
A : C,	·23804	·23840	·23883
A_0 : C,	·23780	·23816	·23836
A : *p*,	·54512	·54476	·54661
A_0 : *p*,	·54456	·54421	·54553
p : C,	·43667	·43762	·43693
Platinum per $2NH_4Cl$ (from A_0),	196·51	196·64	196·16 Pt_0
„ per 6 × Cl,	196·15	196·35	196·01 Pt′
Sal-ammoniac (A_0) per 6 × Cl,	106·82	106·85	106·93
From (1) and (2), chloroplatinate per $2NH_4Cl$,	449·68		
Ditto ; total chlorine,	213·08 = 6·0099 × Cl.		

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* This experiment was carried out for us by Mr James Robson.

Hence, for the composition of the precipitate according to experiments (1) and (2)—

Platinum,		196·58
$2NH_4Cl$,		107·01
$4·01 \times Cl$,		142·17
Total,		445·76
$(C : A_0) \times 107·012$,	=	449·68
Unaccounted for,	=	3·92

Probably part of the 196·58 parts of platinum is combined with the $0·01 \times Cl$ and hydroxyl into $Pt(Cl \text{ or } OH)_6H_2$, which would account for *part* of the 3·92 of undetermined components.

These experiments again are compatible with any value for the atomic weight of platinum, from 196·58 down to about 190.

Yet our experiments are of some value, because they show that if we weigh sal-ammoniac as chloroplatinate of ammonium, or as "platinum," we must not reduce our weighings to sal-ammoniac (or ammonia, or nitrogen) by means of factors calculated from the atomic weights concerned, but employ empirically determined factors.

According to our experiments, these factors are.

Series,	I.	III.	Theoretically if Pt=194·8.
A : C,	·23944	·23842	·24121
A : p,	·54620	·54550	·54934

Means of I. and III.

A : C,	=	·23893
A : p,	=	·54585

We are painfully aware that our individual determinations of these factors do not agree with one another as closely as we should wish. Yet we believe, when used as such in an analysis, they will afford a very fair approximation to the truth.

Summary of Results.

(1) The value Pt = 194·8 (O = 16), which SEUBERT deduced from his analyses of chloroplatinates, is too low; his own analyses, if properly interpreted, show that the true value is by a considerable fraction of a unit higher.

(2) From our own experiments on chloroplatinate of potassium, it appears that the true "Pt," though perhaps a shade below, lies close to, 195·5. This value falls in perfectly well with SEUBERT's analyses; hence it, *at present*, is the most probable value.

(3) Taking "Pt" as the number which must be substituted for Pt in the

calculation of the ratios $2KCl : Pt$; $2KCl : PtCl_6K_2$; $2NH_4Cl : Pt$, in order to obtain the correct factors for the reduction of analytically obtained chloroplatinate, or chloroplatinate-platinum, to chloride of potassium or ammonium, even our number 195·5 is too low; 196 affords in general a closer approximation. But "Pt," taken in this sense, is no constant at all. Those factors must be determined directly by standard experiments. The results of our own standard experiments are given, and contrasted with theoretically calculated ratios, in the following table. The entries "Ta" refer to TATLOCK'S method; the entries "F," to our own modification of FINKENER'S process as we used it for the analysis of the "95, 82, 33·3 per cent." salt-mixtures; the entries "N" to the customary platinum method for the determination of ammonia.

Method.	Symbols.	Theoretical Factors. Values calculated for Pt=194·8.	Pt=195·5.	Pt=196.	D. & M'A.'s Empirical Factors.	Notes.
Ta.	$2KCl : PtCl_6K_2$, .	·30707	·30665	·30633	·30627	(1)
Ta.	$2KCl : Pt$, . .	·76571	·76307	·76112	·76016	(2)
F.	$2KCl : Pt$, . .	·76571	·76307	·76112	·76084	
N.	$2(NH_4)Cl : PtCl_6(NH_4)_2$,	·24123	·24084	·24057	·2389	(3)
N.	$2(NH_4)Cl : Pt$, . .	·54934	·54737	·54598	·5459	(4)

Notes.

(1) Refers to the chloride of potassium in the substance analysed.

(2) Refers to the chloride of potassium contained in the chloroplatinate precipitate.

(3) and (4) both refer to the sal-ammoniac to be determined, not to that contained in the chloroplatinate precipitate.

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,, Part 2.	1 5 0	1 1 0
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TRANSACTIONS

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CONTENTS.

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XXVI.—*The* Polychæta Sedentaria *of the Firth of Forth.* By J. T. CUNNINGHAM, B.A., Fellow of University College, Oxford, Superintendent of the Granton Marine Laboratory; and G. A. RAMAGE, Vans Dunlop Scholar in Edinburgh University. (Plates XXXVI.-XLVII.)

(Read 15th July 1887.)

The studies of which the results are here set forth were carried on at the Granton Marine Laboratory of the Scottish Meteorological Society, in the years 1886 and 1887. Our memoir is by no means a monograph, although our original aim was to investigate every species taxonomically, anatomically, and embryologically. Much further study would have been necessary to carry out this aim at all completely, but in August 1887, both of us, for different reasons, had to abandon our work at Granton and leave Scotland. We have thought it better to publish the notes and drawings we had made, because they will probably be of service to British naturalists interested in the Polychæta, no extensive work on these forms having appeared in English since Johnston's *Catalogue of Non-Parasitical Worms in the British Museum*, which was published 1865, and which is now a very inadequate guide to the study of the subject. The greater part of the work of collecting, and much the larger part of the drawings, were done by Mr RAMAGE. A discussion of some anatomical points has been published separately by Mr CUNNINGHAM ("Some Points in the Anatomy of Polychæta," *Quart. Jour. Micr. Sci.*, 1887).

Fam. SPIONIDÆ, Sars, 1861.

Ariciæ naidinæ.—A. S. Oersted, Zur Classification der Annulaten, Arch. f. Naturges., x., 1844.

Spionidæ.—M. Sars, Christ. Vid. Selsk. Forh., 1861, p. 61.

OERSTED's family Ariciæ included Aricia, Scoloplos, Aonis, Leucodorum, Nerine, Spio, Disoma, Sphærodorum, Cirratulus, Dodecaceria, Ophelina, Ophelia, and Eumenia. JOHNSTON, in his Ariciadæ, included the Cirratulidæ and Spionidæ, but separated the Opheliaceæ and Eumenia. SARS defined the Spionidæ by the characters common to Nerine, Spio, Leucodore, Spione, and Disoma, exclusive of any other genus; and MALMGREN accepts this definition, but splits up the genus Nerine into two, Nerine and Scolecolepis.

Characters of the Family.—A large number of usually short somites, all but the first and last provided with well-developed parapodia, and some or all of them with cirriform ciliated branchiæ. These arise near the base of the notopodium, and are bent towards the middle line of the dorsal surface. The lamella of the notopodium is usually continued along the branchia. The neuropodium and notopodium each consist of a tubercle and short broad lamella. In the anterior parapodia setæ all simple and needle-shaped; in the middle of the body shorter uncini, bidentate at the apex and sheathed, appear in the neuropodium: farther back uncini occur, together with the needle-shaped bristles in both neuropodium and notopodium. The buccal somite bears no parapodia, but possesses two long cirriform tentacles, grooved and ciliated: these are probably homodynamous with the branchiæ. The præoral lobe is rudimentary, and continuous with a ridge on the dorsal surface of the buccal somite; this ridge ends in a conical projection, and bears the eyes, usually 4 in number. The anal segment variable; sometimes provided with a funnel-shaped collar round the anus, sometimes with a number of short processes. The nervous system is situated in the epidermis, and not very distinctly defined; between the cords is a very large median neural canal.

Genus *Nerine*, Johnston.

Nerine coniocephala, Johnst., Mag. Zool. and Bot., ii., 1838.

This genus was first defined by Johnston in 1838, but the definition given by G. O. Sars in 1861 is much more exact. Malmgren, in his *Annulata Polychæta,* 1867, split up the genus into two, one containing the species *Nerine coniocephala,* Johnst., the other Scolecolepis, containing *Nerine vulgaris* of Johnston, with two other species of Nerine described by Sars. The definition of Nerine in this more restricted sense has never been given; it may be drawn up as follows:—

Segments very short, notopodial lamella coalesced with branchia along the whole length of the latter in the anterior part of the body; distinct in the middle of the body; in the posterior part the branchiæ are absent, and the body-walls very thin. Cephalic lobe as a small ridge on dorsum of first somite projecting into a rounded tubercle anteriorly, and having a minute occipital tubercle at its posterior end. Eye spots 4 on cephalic lobe. Anus dorsal in aspect with a sub-anal lobe. Uncini unidentate, sheath small. Ventral neural canal single and large.

Nerine coniocephala, Johnston.

Nerine coniocephala, Johnst., Cat. Brit. Mus., p. 201, plate xvii. f. 9–13.
N. foliosa, Sars, Christiania Vid. Selsk. Forh., 1861, p. 61; Malmgren, Annulata Polychæta, 1867; M'Intosh, Fauna of St Andrews.

Sars places a note of interrogation after the name *coniocephala,* Johnston, in his list of synonyms. Johnston, in his Catalogue, affixes the same mark to

Nerine foliosa, Sars. MALMGREN also expresses a doubt as to the identity of the *coniocephala* of Johnston with the *folosia* of Sars. We can find no reason for this uncertainty, as the description and figures of JOHNSTON obviously agree with the description given by SARS of *folosia.*

Specific Characters.—The colour in the living animal is yellowish, the branchiæ being red, from the presence of pseudhæmal vessels, and the posterior part of the body having a green colour, due to the intestine seen through the body-walls.

A broad lamina extending along the whole length of the branchial filament in the anterior somites; posteriorly the lamina extends a less distance along the branchia, leaving an increasing distal portion free. The size is large, full grown specimens being over 6 inches long and $\frac{1}{4}$ inch broad; it is much larger than the following species.

Habits.—This worm is found at Granton burrowing in a stiff grey underclay, containing carbonised plant stems: it occurs in the littoral zone, and probably beyond it, at a depth of about 1 foot in the clay. It is also sometimes found under stones resting on muddy sand. When uncovered and placed in water, it is unable to crawl, but simply writhes and contorts itself, sometimes with great violence, and often breaks itself into pieces.

Anatomy.— The nephridia are not present in the anterior part of the body, where the notopodial lamina extends along the branchia. In the posterior segments the nephridium is lateral in position, on a level with the dorsal end of the neuropodial fascicle of setæ, and anterior to these. It consists of a spherical vesicle with internal and external ducts, both short. It is ciliated internally throughout. The internal duct passes through the mesentery in front of it, and opens into the cœlom by a funnel-shaped opening. We have been unable to find an actual aperture to the exterior, but the walls of the external duct become continuous with the epidermic cell layer. This duct is short, and passes horizontally forward from the vesicle, coming into relation with the epidermis at a point situated within the constriction of body-wall which separates adjacent somites. The ovary is a cellular mass attached to the dorsal side of the nephridial vesicle. In the centre of the ovary is a pseudhæmal vessel, and dorsally the cellular mass is produced into a flat band.

Nerine cirratulus, Claparède (Delle Chiaje).

Lambricus cirratulus, Delle Chiaje, Mém. su. gli. anim. s. Vert., iv. 196.
? *Nereis foliata,* Dalyell, Powers of the Creator, vol. ii. p. 155.
Nerine cirratulus, Clap., Chét. du Golfe de Naples, 1868.

Specific Characters.—Ocelli 4, two on each side on the cephalic ridge immediately in front of the base of the tentacles, forming a slightly curved transverse line, concave forwards. First somite bears neuropodial and notopodial setæ, but no branchiæ. Branchiæ bent over the back in all the following somites; lamina extending along the outer edge of the branchia, little more than half-way in anterior somites, a less distance posteriorly. Length, 7–12 cm. Colour in life distinctly green; branchiæ red, from the blood within them. Segments longer than in *N. coniocephala.*

This species has not before been recorded as occurring on the British coasts. After careful comparison, we are obliged to conclude that our specimens are not specifically distinct from CLARAPÈDE'S *N. cirratulus,* though there are minute points of difference. It is not probable that these are to be attributed to inaccuracy on the part of CLARAPÈDE: it is more likely that the specimens from localities so far apart as the Firth of Forth and the Bay of Naples present differences which are too slight to separate the species into two. In CLARAPÈDE'S figure of the parapodium and branchia, the lamina of the latter extends nearly to the lip, and has at its own extremity a rounded outline. In the anterior somites of our specimens the lamina does not extend so far along the branchia, and has distally a pointed corner (fig. 2 A). In the posterior somites of our form the lamina is of still smaller extent, and is rounded as in CLARAPÈDE'S figure. That author describes the ova very accurately; their structure is peculiar in two respects. They have a well-developed vitelline membrane, which is covered externally with a reticulation of hexagonal meshes. The shape is that of a slightly flattened ellipsoid, and round the longest circumference is a single series of vacuoles in the external region of the vitellus. The meaning of these vacuoles is unknown. CLAPARÈDE observed that when, after the ova had escaped into the water, the vitellus contracted, the vacuoles were ejected from it as vesicles into the perivitelline space. We have seen the formation of the perivitelline space, but did not observe the ejection of the vesicles, but this phenomenon was observed by one of us in the ovum of *N. coniocephala* (figs. 1 D, 1 E), where the vesicles form a double series.

The early development of this species is very peculiar in certain points. The ova are plagic, and we recognised them early in February amongst the product of the tow-net worked close to the shore. In the vessels of the Laboratory the ova sank to the bottom, which shows that they are heavier than the water, and only kept in suspension by the agitation due to the tides and waves. The segmentation we have not studied in detail: but from the appearance of the single stage observed it seems that the segmentation is complete and unequal, and that a gastrula is produced by epibole, as in other Chætopoda. At a later stage two rings of long S-shaped cilia appear, one at the broader anterior end of the body, the other at the posterior end. The vitelline membrane persists at this stage, and for long afterwards forming a cuticle for the larva, and retaining its characteristic honeycomb-like reticulum of projecting ridges. The bands of cilia project through this vitelline membrane (fig. 2 H). By proper focussing a bundle of cilia can be seen passing through one of the usually hexagonal meshes of the reticulum. The cilia doubtless pierce the membrane during their growth. In front of the anterior band of cilia the greater part of the surface of the larva is separated by a considerable space from the vitelline membrane, but a median anterior projection, shaped like a truncated cone, extends forwards to the membrane in the direction of the longer axis, and from this projection, that is from the apex of the præoral lobe, a bundle of long stiff sensory hairs projects through the vitelline membrane: these hairs are quite motionless, and are probably cuticular, not protoplasmic like cilia. Between the anterior and posterior rings of cilia is another space between the surface of the embryo and the vitelline membrane. At a later stage the embryo elongates, and the sides of the embryo come into contact with the vitelline membrane, especially at two points on each side, from which two pairs of fascicles of long setæ grow out. These setæ are directed backwards, and the anterior are much the longer. These bristles represent two pairs of parapodia (fig. 2 C). By the elongation of the embryo the vitelline membrane is brought into closer contact with the embryo, so that the spaces described above are more or less completely eliminated. The cilia have been described as forming rings, but we are not certain that the rings are complete: the anterior band is continuous across the dorsal side, but is formed of a number of tufts, not of a regular series. Apparently the mouth and anus are not yet formed, but a central cavity is seen in optical sections in the hypoblast. Two eye spots of dark pigment are present on the dorsal side in front of the anterior ciliated band. The larva is thus a telotroch. Figs. 2 I, 2 J, represent a later stage of the larva.

Anatomy.—The nephridia and ovaries in this species resemble those of the preceding, but the efferent duct of the nephridium is longer, and its external aperture more dorsal in position.

Genus *Scolecolepis* (Blainville), Malmgren.

Malmgren (*Ann. Polych.*, p. 199) takes this name for a genus, in which he includes the *Nerine vulgaris* of Johnston and Sars, and *N. currata* and *N. oxycephala* of Sars. The name was used by Blainville in 1828 (in *Dict. Sci. Nat.*, t. 57, we presume) for the *Lumbricus squamatus* of Müller's *Zool. Dan.*, which is possibly identical with *Nerine vulgaris*, Johnston. We use the name as applied by Malmgren. That author gives no definition of the characters of the genus, and we have not supplied the defect. We can only point out that the chief peculiarity is that the cephalic lobe is somewhat T-shaped, having anteriorly on each side a slight transverse process, instead of ending in a point, as in Nerine. In other respects the genus resembles Nerine.

Scolecolepis vulgaris (Johnston), Malmgren.

Nerine vulgaris, Johnston, Cat. Brit. Mus.
Scolecolepis vulgaris, Malmgren, *loc. cit.;* M'Intosh, Fauna of St Andrews.

Common at Granton under stones near low-water mark, among blackened rotting sea-weed. The animal is long, and much more slender than Nerine. The external characters are represented in Pl. XXXVII. figs. 3, 3 A, &c.

Genus *Spio*, Oersted.

The name was first used by FABRICIUS, but defined in its present sense by Oersted (*Arch. f. Naturges.*, x. 1, p. 106).

The chief characters are—Body minute filiform, cephalic lobe notched at anterior extremity; occipital tentacles very long, and thick in proportion to the body; constrictions between somites deep; notopodia with setaceous bristles only; uncini in the neuropodia bidentate; branchial cirri continuous with laminæ; anus provided with four short processes.

Spio seticornis, Fabr.

Nereis seticornis, Fabricius, Fn. Grönl., p. 306.
Spio seticornis, Fabr., Schr. Naturf. Freunde, Berlin, vi. p. 260.
Spio seticornis, Oersted, Arch. f. Naturges., x. 1.

Specific Characters.—Branchial cirri on the 3rd segment (2nd setigerous) well developed, distinct from notopodial lamella; 2nd, 4th, and following somites without branchial cirri; these occur again on the 12th or 13th, and several following somites, disappearing again towards the posterior extremity; 8th and following neuropodia bearing only uncinate setæ. Tubes of sand, long, tough, and flexible (*vide* figs. 4, 4 A, &c.).

Habits.—We have always found the tubes of this species in the middle and upper part of the littoral zone, in clefts of rocks and under stones. The tubes are very abundant, but from their length and the fragile nature of the worm, it is difficult to extract the latter from its dwelling without injury.

Anatomy.—The nerve cords are wide apart, and there are no neural canals. The epidermis on the ventral surface of each somite is greatly thickened and glandular, containing a large number of mucous cells, which stain deeply. Ova are seen in transverse sections of the middle part of the body nearly filling the body-cavity.

Genus *Leucodore*, Johnston.

Leucodore ciliata, Johnst., Mag. Zool. and Bot., ii. p. 57.
Polydora, Bosc, Histoire Nat. des Vers, Paris, An. x.

Branchial cirri confined to middle of the body, absent at either extremity; tentacles shorter than in Spio; first four somites behind the buccal, bearing acicular setæ only; in 5th somite the dorsal setæ are much elongated uncini, only the points projecting beyond the skin; branchial cirri present on segments following the 5th, first being very small; an infundibuliform membrane, incomplete dorsally, surrounds the anus.

Leucodore ciliata, Johnston.

Leucodore ciliata, Johnston, *loc. cit.;* Malmgren, Ann. Polych.
Leucodore ciliatus, Johnston, Cat. Brit. Mus., p. 205.
Polydora ciliata, M'Intosh, Fauna, St Andrews, p. 127.

Specific Characters.—Those of the genus.

Habits.—This worm inhabits soft mud tubes which are not very firmly constructed, and which fill up narrow chinks and clefts in rocks. In some oysters which were kept in the summer of 1886 in floating cages in Granton Quarry, and which became coated with sediment, large numbers of this worm were found between the projecting edges of the shell laminæ, the end of the tubes often projecting some distance. It is stated by HUXLEY that *Leucodore ciliatus* bores holes in the oyster shells. We have not found this to be the case; the tubes of the worm did not actually pierce the shell in any case, and we have not observed them on oysters newly dredged from the middle of the Firth.

Anatomy.—The nerve cords are rather wide apart and there is no neural canal.

CLAPARÈDE (*Chétopodes du Golfe de Naples*, 1868) describes in, *Leucodore Agassizii*, a glandular sac beneath the base of each branchia in somites posterior to the 5th (the modified one). He also describes the nephridium as arising from an internal aperture beneath this sac, turning then in the next somite towards the lateral border of the body, then reflected on itself, and passing to open to the exterior near the dorsal median line. In *Leucodore ciliatus* we have made out by compression both the glandular sac and the nephridium.

The nephridium is easily distinguished in the living worm after compression by the black granular matter contained in its cells. This matter is doubtless composed of urinary concretions. The glandular sac is derived from the epidermis.

Fam. MAGELONIDÆ.

Magelona papillicornis, Fritz Müller.

Mœa mirabilis, Johnst., Cat. Brit. Mus.; M'Intosh, Fauna of St Andrews.

Magelona papillicernis, Fr. Müller, Anneliden-fauna der Insel Santa Catherina, Arch. f. Naturges., 1858; M'Intosh, Z. f. w. Z., Bd. 31.

This peculiar and interesting species is common at Granton, and is got in abundance by digging in sand in the littoral zone. We have formed a special family for it, as it cannot be admitted into the *Spionidæ*, with which it is most nearly allied, or into any other family. It has a richly corpusculated blood, contained in special vessels, a large flat præoral lobe, and two long occipital tentacles furnished with long movable papillæ. These tentacles bear no cilia. There is an anterior thoracic region, which is much more muscular than the rest of the body, flattened, and somewhat reduced in diameter (*vide* Pl. XXXVIII. figs. 6, 6 A, &c.).

Fam. ARICIIDÆ, Malmgren.

The principal distinction between this family and the *Spionidæ* is the absence in the former of the occipital tentacles, or cirri of the buccal somite. The family is equivalent to the *Ariciæ veræ* of Oersted (*Arch. f Naturges.*, x., 1844), with the exclusion of Aonis, Grube, which is a synonym of Nerine. The family includes the genera *Aricia*, Savigny, *Scoloples* (Blainville), Oersted, and *Theodisca* (Fr. Müller), Claparède.

Scoloplos armiger (Müll.), Blainville.

Lumbricus armiger, Müller, Zool. Dan., i. p. 22.

Scoloplos armiger, Oersted, Ann. Dan. Consp., p. 37; Malmgren, Ann. Polych.; M'Intosh, Fauna of St Andrews.

Abundant, burrowing in sand in company with Magelona. The anterior somites bear chætæ only, without tubercles. Farther back there are both long tubercles and simple branchiæ, and the parapodia are approximated together on the contracted dorsal surface. The anal segment bears a pair of long filaments (figs. 7, 7 A, &c.).

Theodisca mamillata, Claparède.

Got off Laminarian roots at the Birnie Rocks, near Granton Quarry.

Specific Characters.—One pair eyes. Cephalic lobe, obtusely rounded. Branchiæ dorsal, commencing on 5th setigerous somite. Notopodium, consisting of a single cylindrical process, in front of the base of which is a fascicle of long capillary bristles. Neuropodium, a mammiform projection with a nipple-like process at the end; on the anterior surface exteriorly are a row of capillary setæ, proximally a large number of short thick setæ bifid at the apex; the latter are present only in the first 10–15 somites, not in the others. Among the dorsal bristles in the middle part of the body are some 2-pronged at the end (Pl. XXXVIII. fig. 8).

Fam. CIRRATULIDÆ, Vict. Carus.

Body round, parapodia near the ventral surface, the notopodia separated by a wide dorsal region. Parapodia themselves not prominent, with simple chætæ and acicula. Tentacular filaments in a transverse row on the dorsal surface near the anterior end. A pair of branchial filaments on several of the somites. Head long and conical.

Cirratulus cirratus, Malmgren (O. F. Müll.).

Cirratulus borealis, Lam., Anim. s. Vert., v. p. 302; Oersted, Annul. Dan. Consp., p. 43; Grœnl. Annul. Dorsibr., p. 54; Grube, Fam. Annel., p. 67; Johnston, Brit. Mus. Cat., 1867, p. 210.

Lumbricus cirratus, Müll., Zool. Dan. Prodr., p. 214.

Cirratulus cirratus, Malmgren, Ann. Polychæta, p. 205; M'Intosh, Fauna of St Andrews.

Transverse series of tentacular filaments on 1st somite behind the buccal; this is also the 1st setigerous somite. Buccal somite with two constrictions. Branchial filaments on the body few and scattered, and their origin separate from the base of the notopodium.

There is no doubt that the species we have here is identical with JOHNSTON'S *C. borealis.* It is equally certain that it is the same as the *C. borealis*, Lam., of OERSTED; and as both JOHNSTON and MALMGREN affirm that OERSTED'S name designates the same worm as *Lumbricus cirratus*, Müller, we cannot understand why MALMGREN (*Annulata Polychæta*) expresses a doubt as to the identity of JOHNSTON'S *C. borealis* with his own *C. cirratus*, Müller.

Habits.—Common on the shores of Granton Quarry and on the shore in the neighbourhood. Found usually extended beneath stones which are partially buried in rather soft muddy ground.

There has been considerable uncertainty among authorities as to the character of the portion of the body in this worm which is anterior to the conspicuous transverse series of filaments. In front of the segment which bears the latter there are two constrictions, and in front of the most anterior of these projects the blunt-pointed præoral lobe, which bears a pair of transverse rows of eyes. JOHNSTON counts the part between the eyes and the 1st annulation as the 1st segment; thus, according to him, the thicker tentacular filaments in transverse series arise from the 4th segment. OERSTED takes in JOHNSTON'S 1st segment as part of the præoral lobe, and states that the branchial filaments arise on the 3rd segment. We have come to the conclusion that there is nothing in front of the anterior transverse filaments except buccal somite and præoral lobe, the latter bearing the eyes. The buccal somite is longer than in most cases, equal in length to three of the succeeding somites. Longitudinal sections show that there are no mesenteries corresponding to the superficial annulations, and the ventral nerve cord does not extend in front of the 1st setigerous somite.

The real distinction between the dorsal tactile filaments of the anterior end and the lateral branchial filaments is, that the former are provided with a groove similar to that which occurs in the occipital tentacles of Nerine, &c.

Claparède's account of these filamentous appendages, which are so characteristic of this genus, and which give it its name, is not quite correct. There are two kinds of these filaments,—the tentacular, which are confined to the 1st setigerous somite in this species, to the 4th and 5th setigerous in the following, and the branchial, which are present on the sides of a great number of the somites. There is never more than one pair of branchial filaments to a somite in the present species; the attachment of the branchia is same distance dorsal of the notopodial bristles. The branchia is a thin flexible cylinder, composed of epidermis, muscular wall, and cavity. In the cavity run two longitudinal vessels continuous with each other at the distal extremity of the filament; a closely set series of transverse small capillary vessels connect these two main vessels; these run in the muscular layer close beneath the epidermis (see fig. 9 c). The tentacular filaments appear in the living animal opaque white, while the branchiæ are of a brilliant red: in spirit specimens the tentacular filaments are still white, while the branchial are dark, sometimes quite black, owing to the pigment, which is abundant in the epidermis of the latter. The tentacle also consists of epidermis, muscular layer, and cavity. In the cavity runs a *single* longitudinal pseudhæmal vessel, which ends cæcally at the distal extremity of the filament, and gives off no branches. On the ventral surface of the filament is a single deep groove running the whole length of the filament, and lined with a coating of extremely short cilia. When sections of the tentacle are made it is seen that the formation of the groove is entirely due to two parallel thickenings of the epidermis; no other structure in the tentacle takes any share in its formation. The number of the tentacles is considerable. In a large specimen we counted 10 pairs, and there is always attached to the tentacle-bearing somite a single pair of branchiæ in addition. These branchiæ are ventral in origin to the transverse series of tentacles. The number of the tentacles, however, is not constant, but increases with the age and size of the individual; in specimens 4 cm. long only 5 pairs are present. Claperède attributes two grooves to the branchiæ, and none to the tentacles, but his account of the vascular supply is correct. He does not discuss the relations of the groove to the epidermis.

In both the branchiæ and the tentacles there are nerves. In either case these are seen in a prepared section as a pair of fibrillar strands situated in the deeper region of the epidermis, and running longitudinally. In the branchia the two nerves are at opposite sides of the filament; in the tentacles they are placed one on each side of the groove (Pl. XXXIX. figs. 9 D, 9 F).

The nerve cords in the body are not separated from the epidermis, though they are defined with considerable distinctness. The fibrillar portion of the cords does not stain in borax carmine. There is no differentiation of ganglia, the nerve cells being distributed uniformly along the ventral side of the cords.

The dorsal vessel in this and all other species of Cirratulus has a very peculiar structure, which has been described by CLAPARÈDE.* By examining the living animal under a low power, the dorsal vessel can be seen to contain a bright red liquid, and to be bordered by three black granular-looking cords, one dorsal and two lateral. The vessel is much constricted where it passes through the mesenteries, and much enlarged in the body of the somite. It also has a very tortuous course when the animal is in an average state of contraction. CLAPARÈDE studied the vessel in *Andouinia filigera* (= *C. tentaculatus*), and states that the wall is formed by a muscular tunic formed of annular fibres, within which the "cordons bruns" are placed. He was unable to decide whether these cords were between the muscular tunic and the vessel's own wall, or within the latter. Our sections show clearly the relations of the cords, and also their minute structure, on which CLAPARÈDE says nothing. It should have been mentioned that the dorsal cord is often interrupted, and occasionally coalesces with one of the lateral. The cords are in the interior of the vessel, and although they are usually in contact with the inner surface of the wall, they are not continuous with it: the pseudhæmal fluid is frequently seen in the sections between one of the cords and the wall of the vessel. The structure of the cords is very peculiar: it has a glandular character, but it is difficult to decide whether the organs have a glandular function, as they have neither lumen nor efferent duct, nor have we been able to trace any connection between them and any other organ of the body. The cords are composed of elongated cells placed perpendicularly to the interior of the cord, which is enclosed by an extremely thin basement membrane. In our sections the cells are not stained. They contain a number of minute rounded granules similar to those which are seen in the secreting cells of nephridia in several Chætopoda, and also somewhat similar to granules in the secretory cells of the epithelium of the intestine. These granules are in greatest abundance towards the external parts of the cells.

The dorsal vessel gives off on each side a large vessel in the 3rd somite, the branch passes backwards and forms a lateral vessel, from which the

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* *Chétopodes du Golfe de Naples*, Genève, 1868.

afferent vessels to the branchiæ pass off. The dorsal vessel is continued to the anterior end as a thin trunk. The glandular cords end at the point where the lateral vessels are given off. We have not yet ascertained if there are communications in each somite, between the dorsal vessel and the lateral.

Reproduction.—We found a number of specimens swollen with eggs, at the end of March 1887, under stones, on the banks of Granton Quarry. These commenced to deposit their eggs when placed in a basin of sea water. The eggs were fastened together by transparent mucus, forming a soft mass without any definite shape, adhering to the stones and mud among which the animals were placed. On examination, the manner in which the ova escaped was ascertained without much difficulty. A female individual was isolated in a small glass vessel, and the surface of its body cleaned from eggs and dirt: in a short time a little group of eggs appeared on the side of each somite, and on examination with a lens, an egg could be seen half escaped, in several somites, a little ventral to the neuropodial setæ. When the animal was placed under the microscope, and viewed with a low power, and with reflected light on a dark background, a small round aperture was seen in the position just defined where the egg was seen escaping, that is ventral to the neuropodial setæ, and the eggs were seen escaping slowly one by one from this aperture. The process could be watched for a long time. The eggs as they escaped were enveloped in mucus excreted from the skin, and so formed little groups close to the aperture whence they escaped. A large individual, which though containing eggs, was not quite ripe, was next examined, and the genital apertures, although in this case contracted, could be seen. It was evident that these genital apertures were not formed temporarily by rupture, in order to let out the eggs, but definite permanent openings present throughout life. Some of the females were very large, the largest measuring 9 cm. in length by 5 cm. in diameter near the head. The males were much smaller, the largest being 5·5 cm. in length. The males had genital apertures exactly similar to those of the females. Subsequent preparation of series of sections from these specimens showed that the pores from which the genital product escaped were the external apertures of simple nephridia, present throughout nearly the whole length of the body. For a full account of the nephridial system, see a paper by one of us, *Quart. Jour. Micr. Sci.*, 1887.

C. tentaculatus, Fleming (Montagu).

Terebella tentaculata, Montagu, in Linn. Trans., ix. p. 110.
Cirratulus Lamarckii, Audouin and M. Edw., Litt. de la France, ii. p. 271.
Cirratulus tentaculatus, Johnston, Cat. Brit. Mus., p. 209.
Andouinia filigera, Clap., Chét. du Golfe de Naples.

Several long coiled tentacular filaments in transverse series, arising from dorsum of 5th and 6th setigerous somites; lateral filaments on some somites in front of these, annulations on the buccal somite. Lateral branchial filaments, arising immediately above the base of the notopodia. Larger than *C. borealis*, length 10–13 cm. and upwards, diameter 5 mm.

Habits.—Found in similar localities to those described for *C. cirratus*, but nearer low-water mark; it also burrows more deeply than the latter species.

As JOHNSTON remarks, it is somewhat difficult to ascertain to which somite the tentacular filaments belong. Eyes are not visible in preserved specimens, but faint pigmented lines are to be seen on the cephalic lobe in the living animal. The region in front of the 1st chætiferous somite has the same characters as in *C. cirratus*. The origin of the lateral filaments is a constant and marked character of this species. The colour is dark red.

Chœtozone, Malmgren.

Chœtozone, Malmgren, Ann. Polych., p. 206.

This genus was established by MALMGREN, in 1867, for the following single species. The distinguishing character is the arrangement of the chætae of the posterior somites, in an extended linear transverse series, so that they almost encircle the body.

Chœtozone setosa, Malmgren.

Chœtozone setosa, Malmgren, *loc. cit.*

Specific Characters.—Smaller than *Cirratulus cirratus;* only two tentacles which are much thicker than the branchial filaments, and arise immediately in front of the first pair of parapodia; tentacles with same characters as those of Cirratulus. Eyes wanting; anterior end of cephalic lobe acuminate; 6 annuli in front of 1st pair of parapodia. Branchial filaments arising close to base of notopodium, slightly behind it. Anterior pair of nephridia opening at base of 1st pair of parapodia, extending back through 2 or 3 somites, and visible to the unaided eye, on account of their brown granular appearance (Pl. XXXIX. fig. 11).

Anatomy.—There are 3 glandular cords in the dorsal vessel, as in Cirratulus, which are quite black, and very conspicuous in the living animal. The lateral branchial filaments occur on some of the somites throughout the body, and are not limited to a certain region, as in Dodecaceria. The chætæ are all long and setaceous.

Dodecaceria, Oersted.

Dodecaceria, Oersted, Annul. Dan. Consp., p. 44.

A genus also containing only a single species, the following:—

Dodecaceria concharum, Oersted.

Terebella ostreæ, Dalzell, Powers of the Creator, ii. p. 209.

Dodecaceria concharum, Oersted, Annul. Dan. Consp., p. 44; Johnston, Brit. Mus. Cat., 1865; M'Intosh, Fauna of St Andrews.

As only one species of this genus is known, a separate diagnosis of genus and species is unnecessary.

The form is allied to Chætozone, from which it differs principally in two points—(1) the absence of the peculiar arrangement of setæ in the posterior part of the body which characterises Chætozone; (2) the fact that there are only a few pairs of branchial cirri, confined to the anterior somites. Like Chætozone, the species possesses a single pair of cephalic cirri, which are thicker than the somatic, and usually twisted spirally. Anterior end of cephalic lobe cylindrical, obtuse, without the annulations seen in Cirratulus. Eyes none; mouth almost terminal. Posterior part of body considerably thicker than anterior. Colour dull olive-green (Pl. XXXIX. fig. 12).

Habits.—Not uncommon among roots of Laminaria. According to JOHNSTON, it lives in burrows drilled in the shell of *Cyprina islandica*, but we have not observed it in this condition.

Fam. THELETHUSIDÆ.

No cephalic tentacles: parapodia not prominent; arborescent branchiæ on several successive somites. Size large.

Genus *Arenicola*, Lamarck.

Arenicala piscatorum, Lam., Syst. d. Anim. s. Vert., 1802, p. 324.

Cephalic lobe rudimentary; tentacles and eyes absent; proboscis small and soft. Arborescent branchiæ commencing some distance behind anterior end; surface of somites marked with transverse ridges and furrows, and shallower longitudinal furrows. Burrows in mud or sand, which it devours.

Arenicola marina, Malmgren (Linn.).

Lumbricus marinus, Linnæus, S. N. xii. 1, 2, p. 1077.

Arenicola piscatorum, Lamarck, 1802, Syst. d. Anim. s. Vert., p. 234; Johnston, Cat. Brit. Mus.; Cuvier, Regne Anim. éd. accomp. de Planches, Annel., Tab. 8.

Arenicola marina, Malmgren, Annulata Polychæta, &c.; M'Intosh, Fauna of St Andrews.

Specific Characters.—Branchiæ on 7th to 19th chætiferous somites inclusive; caudal region of considerable length, without chætæ or branchiæ. Length, 8 inches to 1 foot, diameter about $\frac{1}{2}$ inch.

Habits.—Very common at Granton in flats of somewhat muddy sand on shore, and beyond low-water mark. The sides of its burrows are stained with the yellowish-green exudation from its body. The intestine is always full of the sand and mud in which it lives, and from which it derives its entire nourishment. This sand is ejected in a cylindrical rod from its anus, and this forms a spiral coil on the surface of the shore: near the "cast" is usually a wide hole, from which the head is protruded when the tide is up. The worm is usually found at least a spade's depth from the surface when the tide is out.

Anatomy.—The 1st somite is destitute of bristles; the following 6 bear each a dorsal fascicle of hair-like bristles and a ventral torus uncinigerus, but no branchiæ. The following 13 bear both fascicle and torus, and in addition a pair of plumose branchiæ; the rest of the body, which is variable in length, is thinner than the rest, and has neither fascicle, torus, nor branchiæ, but is cylindrical, and covered uniformly with papillæ. This part forms the caudal appendage: the number of somites in it we have not determined.

On dissection, it is found that the body-cavity is a large and well-defined space in the anterior and branchial regions, but almost obliterated in the caudal appendage. Transverse mesenteries are only present in the anterior 4 somites, and here they are incomplete; in the rest of the body they are absent. There is a septum between the buccal and the 1st chætiferous somite, none between the 1st and 2nd chætiferous; one behind the 2nd, and one behind the 3rd chætiferous.

Between two successive parapodia are seen externally 5 constrictions, of which the 5th is the deepest. Between the 5th and the parapodia is a prominent ring ending in a sharp line. The mesenteries are attached to the body-wall at the 2nd constriction. There are 6 pairs of nephridia, visible in spirit specimens as brown glandular masses attached to the body-wall; they belong to somites 5–10. CLAPARÈDE, in his description of *Arenicola Grubii*, describes only 5 pairs of nephridia, belonging to the 4th to the 8th setegerous somites; that is, somites 5–9. He does not mention any difference in the case of *A. marina*. In the latter the first pair of nephridia is smaller than the succeeding, and is to that extent rudimentary. The internal opening of the 1st segmental organ is on the anterior face of the mesentery, between somites 4 and 5.

The nephridium is a wide thin-walled tube, showing on dissection a black colour, which is due to the concretions in the glandular cells lining the cavity. The peripheral end of the tube opens to the exterior by a pore visible in the fresh animal to the naked eye, and situated immediately behind the dorsal extremity of the torus uncinigerus. Anterior to the pore in a living uninjured animal the black tube of the nephridium is seen through the semitransparent skin, and is very conspicuous. The peripheral end of the tube, which is posterior in position, is somewhat dilated, and lighter in colour than the rest. Anterior to this the tube, which is placed longitudinally, narrows; but at the anterior end, which lies near the posterior extremity of the one in front, the tube is of considerable width. Across the internal side of the anterior portion, that is on the side turned towards the body-cavity, lies a membranous appendage, whose edge, which is turned towards the median ventral line of the animal, is seen to be furnished with a conspicuous vascular fringe of a bright red colour. The anterior extremity of the fringe is seen to be connected with a transverse blood-vessel passing from the body-wall immediately behind the dorsal fascicle of setæ, and proceeding to the ventral vessel. From this transverse blood-vessel the whole organ probably receives its blood supply. The nephridium is confined beneath the bands of oblique muscles, which pass from the ventral nerve cord to the lateral body-wall. When these are cut through, the whole nephridium can be removed without difficulty; and examination of the membranous appendage shows that it is double, consisting of two membranes, between which is a long slit-like aperture leading into a membranous funnel which opens into the interior of the nephridial tube. The vascular fringe forms, in the original position of the parts, the dorsal border of this internal or cœlomic aperture, while the ventral border is entire and non-vascular. Both edges of the aperture, including the processes of the fringe, are covered with short cilia, while inside the aperture are seen very long cilia which extend throughout the tube.

In 1868, when CLAPARÈDE'S *Chétopodes du Golfe de Naples* was published, the gonads of Arenicola were unknown. CLAPARÈDE says that most authors had assigned the nephridia to the generative system, sometimes under the name of ovaries, sometimes under that of testes. QUATREFAGES (*Histoire Nat. des Annelés*) called them genital organs. GRUBE had assured himself that the ovaries were not to be sought in these organs, for he had seen the ova formed on the wall of the vascular cæca of the perivisceral cavity; he was inclined to regard the nephridia as testicles. CLAPARÈDE says this is impossible, because Arenicola has the sexes separate; he was of opinion that GRUBE was right as to the formation of the ova, and he gives a figure of one of the cæcal pseudhæmal vessels with a layer of cells surrounding it. He then leaves the question of the genital organs, and proceeds to give a not very accurate description of the nephridia. We were for some time at a loss as to where the genital products really took origin. In February we found two or three specimens which had a few ova in the body-cavity; in these cases, and in other specimens where mature ova were not present, loose cellular masses were often found in the neighbourhood of the nephridia, and against the posterior wall of the septum between somites 4 and 5. After considerable search, these masses were traced to a cord of cellular tissue attached to the membranous funnel of each nephridium. In most specimens the cells of this cord were so small and so undifferentiated that it was not possible at first to be certain it was a gonad. But in specimens which contained but few ova in the cœlom, it was obvious enough that this mass of cells was the female gonad, some of the cells in it being distinctly recognisable as ova (fig. 13 D). The ova in the body-cavity were of various sizes (fig 13 E), and it was evident that they went through the greater part of their development after being detached from the gonad. Male elements were in other specimens found freely suspended in the liquid of the cœlom, but only in small quantity; they were in the form of sperm polyplasts, or bundles of spermatozoa (fig. 13 F). The gonad in this case showed, instead of ova and germinal cells, systems of small cells, 6 or 8 in each system, evidently derived from the segmentation of germinal cells, and destined to fall off into the cœlom, and there form sperm-polyplasts, and ultimately spermatozoa.

None of the worms examined in February and March were sexually mature, and it was afterwards found that *Arenicola marina* does not shed its genital elements till August and September. MAX SCHULTZE was, we think, mistaken in ascribing the ova and embryos whose development he describes (*Entw. von Arenicola, etc.*, Halle, 1856) to this species. As shown in the following section, the cocoons he gathered were most probably these of *Scoloplos armiger*. It is yet uncertain whether the present species forms cocoons at all, or sheds its ova separately in the mud.

Eggs and Larvæ belonging, according to Max Schultze, to Arenicola.

At the beginning of February, we found large numbers of gelatinous cocoons on the surface of the sands near the Station at low tide. From MAX SCHULTZE's description (*loc. cit.*) we concluded at first that these were the spawn of *Arenicola marina*. That author discovered the cocoons he described as belonging to Arenicola on the flat shores of the island of Neuwerk, not far from Cuxhaven. In most points his account suits the spawn we obtained, but his cocoons had a pink colour, due to the ova within. We have not observed any colour in our specimens; the jelly was transparent, and the ova and embryos opaque white. The colour may not be a constant attribute.

The cocoons or gelatinous masses are about 2 cm. long and 1 cm. broad. They are pear-shaped, and the narrower end is produced into a long cylindrical stalk, about 3 cm. in length, which contains no ova. The stalk is usually imbedded in the sand.

It was very easy to keep these ova alive, and hatch them in captivity. They were simply placed in a shallow dish with sea water, and a little sand at the bottom. The larvæ lived several days after hatching. Pl. XL. figs. 14 A to 14 F, show a series of stages in the development of these larvæ. There are three transverse bands of cilia, which persist until the setæ begin to appear, and there is also, as shown in fig. 14 D, a ciliated longitudinal line or groove in the median line of the ventral surface. The first transverse band is præoral, the second postoral, and the third perianal. There is a well-developed præoral lobe and a single pair of black ocelli. In these points the larva is more like the adult *Scoleplos armiger* than the adult Arenicola; but the character which seemed to us to indicate more definitely the former species as the adult form is the pair of anal cirri which appear in the oldest stage of the larva figured (comp. Pl. XL. fig. 14 F, and Pl. XXXVIII. fig. 7 B). The first somite to be defined is the 1st postoral, and the others are constricted off in succession from the growing posterior end. Dorsal cirri appear on the two most posterior somites in the oldest stage figured.

Fam. HERMELLIDÆ.

Sabellaria, Lamarck.

Sabellaria spinulosa, R. Leuckart.

Sabellaria lumbricalis, Johnston, Cat. Brit. Mus.

Sabellaria spinulosa, R. Leuckart, Arch. für. Naturges., xv. 1; Malmgren, Ann. Polych., etc.; M'Intosh, Fauna of St Andrews.

We have identified our specimens with this species chiefly on account of the form of the paleæ of the outer row in the operculum. MALMGREN'S figure of the entire animal of this species far less resembles our specimens than his figure of *S. alveolata*, L. In the latter figure the branchiæ are long, as in our specimens (Pl. XLI. fig. 17 C); in the other figure they are much shorter. The position of the chætæ in the 1st and following few somites, and the breadth of the notopodia in the posterior somites, are other points in which MALMGREN'S figure of *S. alveolata* agrees with our specimens of *S. spinulosa*. It is difficult to avoid the conclusion he has by mistake interchanged the figures of the entire animal in the two species.

Habits.—The flat mud banks in the estuary of the Forth in the neighbourhood of Charleston contain myriads of this species, the tubes of which render these banks hard and firm. The dredge worked over these banks brings up large masses of these tubes, and scarcely anything else. We have also found specimens encrusting rocks, stones, and shells in the coralline zone near the Granton Laboratory, and inhabiting tubes of sand adhering to oyster, Pecten, and other shells dredged in the Firth.

Anatomy.—The morphology of Sabellaria offers a difficult problem, namely, that of deciding on the true significance of the cephalic processes on which the operculum is carried. Many zoologists have called these processes kopflappen, or cephalic lobes. It seems impossible that these lobes should be really cephalic, for there is no other case among the Chætopoda in which chætæ occur on the true cephalic lobes, or prostomium. It seems probable, from the way in which the anterior parapodia are bent forwards, specialised in form, and increased in size, for the protection of the head in other cases, *e.g.*, in Trophonia, Ampharete, and Pectinaria (which form a progressive series), it seems probable from these examples that the operculum in Sabellaria, with its three rows of paleæ, is derived from the most anterior parapodia. And the examination of a series of longitudinal sections of the anterior end of Sabellaria shows evidence which supports this hypothesis. The cerebral ganglia are found to be placed immediately above the anterior end of the œsophagus, and in contact with the integument of the ventral surface of the base of the opercular peduncles. It follows then that the whole substance of the peduncles belongs to the region of the body behind the cerebral ganglia, that is, to the postoral somites, and not to the præoral lobe. If it were otherwise, the cerebral ganglia would of course be situated on the dorsal side of the peduncles. The question next arises, Do the paleæ of the operculum represent one pair of parapodia, or how many pairs? At present there seems to be no evidence on which to decide this question.

The bases of the peduncles are united dorsally, but terminally they are independent, and between them there is a conical process projecting from the tissue which unites their bases. Each peduncle is flat towards the median plane, or even slightly concave; externally it is convex. On the ventral side each bears a large number of thin flexible ciliated tentacles: these are arranged in a series of short transverse rows, each row being continuous at the base. There are 7 or 8 of these rows.

At the base of the opercular processes, within the cavity on their ventral side, and at the dorso-lateral corners of the mouth, arise a single pair of much thicker tentacles (Pl. XLI. fig. 17 A). These, which are also ciliated, are true præstomial tentacles similar to those of Nerine. In the parts around the mouth the præstomium, the buccal somite, and the ventral part of the opercular segment, must, according to our views, all be present, but they cannot be defined. Behind the opercular peduncles occurs the 1st normal somite, which bears the first pair of dorsal branchiæ, and two chætiferous lobes, one of which is notopodium, the other neuropodium. Each of these lobes runs out to a conical process anteriorly, and the ventral pair are in close relation to the mouth. The chætæ are quite simple. In the next three somites, the notopodial setæ are specially modified, the shape is shown in fig. 17 C 4, and it is seen that they approximate to the form of the outer opercular paleæ. These three somites, together with the preceding one, may be considered as forming the prothoracic region. The succeeding somites forming the thoracic region, all bear branchiæ, which diminish in size towards the posterior end. The notopodia are elongated fin-like processes, bearing uncini; the neuropodial setæ are simple, and very long. The abdomen or tail, which is bent up ventrally, bears neither branchiæ nor parapodia.

Ammotrypane aulogastra, Rathke.

Ammotrypane aulogastra, Rathke, Nov. Act. Nat. Cur., xx. 1, tab. x. f. 1–3.
Ophelina acuminata, Oersted, Consp. Ann. Dan., p. 45; Arch. f. Naturgesch., x. 1, p. 111.
Ophelia acuminata, Grube, Fam. Ann.; Johnston, Cat. Brit. Mus., 1865.
Ammotrypane aulogastra, Malmgren, Annulata Polych.; Of. k. vetensk. Akad. Förhand., 1865 ; M'Intosh, Fauna of St Andrews, 1875.

The specific characters as given by JOHNSTON are—Body fusiform ; snout tipped with a small globule; branchial cirri to all the segments; anal extremity spoon-shaped, with two small fusiform appendages in front of the vent.

This is not quite correct: the 1st setigerous somite bears no branchia ; and the spear-shaped anal extremity is provided with cirri round its posterior edge; and in addition to the two small fusiform appendages in front of the vent, a long cirrus arises between these. This median cirrus usually lies in the hollow of the spoon-shaped appendage, and is therefore not easily seen (figs. 15 to 15 C).

A species of Opalina commonly occurs in the intestine of this species; it is of an elongated cylindrical shape, and towards one end repeated transverse fissions take place, the resulting segments remaining for some time connected so as to form a linear series (Pl. XLII. fig. 15 D).

Genus *Ophelia*, Savigny.

Generic Character.—Longitudinal constriction limited to posterior two-thirds of the body.

Ophelia limacina, Sars (Rathke).

Ammotrypane liniacina, H. Rathke, Nov. Act. Nat. Cur., xx. 1 ; Johnston, Cat. Brit. Mus.; Quatrefages, Ann. ii.
Ophelia bicornis, Oersted, Grönl. Ann. Dors.
Ophelia limacina, Sars, Nyt. Mag., vii. ; Malmgren, Ann. Polychæta, *loc. cit.;* M'Intosh, Fauna of St Andrews.

The worm is thicker than the preceding species ; the notopodial cirri are shorter and thicker, and confined to the posterior part of the body. The longitudinal constriction begins behind the 8th somite (7th chætiferous). The posterior extremity has a circlet of small papillæ.

Found between tide marks at Granton.

Eumenia crassa, Oersted.

Polyphysia crassa, Quatrefages, Annelès, ii.

Eumenia crassa, Oersted, Ann. Dan. Consp. Arch. f. Naturges., x. 1; Sars, Nyt. Mag., vii. 12 (description); Johnston, Cat. Brit. Mus.; Malmgren, Ann. Polychæt.; M'Intosh, Fauna of St Andrews.

The body is cylindrical, the thickest part near the anterior end. The first six chætiferous somites bear branchiæ in the form of a thick cluster of papillæ sprouting from a central stem, and situated in front of the notopodium (Pl. XLII. fig. 18 A). Each division of the parapodium consists of a pointed lamella towards the outside of the parapodium, and a fan of simple acicular chætæ towards the centre of the same. Such parapodia are present on all the somites except the buccal and anal. The anterior end is conical, terminating in two small divergent points; the anal segment is without appendages.

Habits.—Dredged in the Firth.

Lipobranchius, n. g.

Lipobranchius Jeffreysii (M'Intosh).

Eumenia Jeffreysii, W. C. M'Intosh, On the Structure of British Nemerteans, and some new British Annelids, Trans. Roy. Soc. Edin., 1868.

This worm we have not examined in detail: it evidently belongs to the family Scalibregmidæ, being similar in many respects to Eumenia, in which genus M'Intosh placed it. We have formed a new genus for it on account of the entire absence of branchiæ or branchial cirri. It is fusiform, both ends being conical, and it is not easy at first to distinguish which end is anterior and which posterior. The worm, in fact, looks like a maggot. Parapodia are present on all the somites except the buccal and anal, and are all similar in structure. Each consists of two similar mammillate processes bearing a few small chætæ at the apex. The ventral body-wall is much more muscular than the dorsal, and in spirit the former is contracted so that the body is curved (Pl. XLII. fig. 19). On each somite there are about three transverse corrugated ridges, but there are no definite constrictions between adjacent somites, which are only indicated by the parapodia.

Large numbers of specimens of this form were dredged in several places in the Clyde by the "Medusa." The worm is found only on muddy bottoms, and lies enclosed in a thick tube formed from the mud hardened by an excretion from the skin.

M'Intosh says the chætæ are of two kinds—one kind simple, the other bifurcated into two large branches. He also says there are two short thick tentacles on the head, and several elongated processes round the anus. We have not seen these, but our specimens were contracted, and we have not dissected them. Our specimens were identified by Prof. M'INTOSH himself, and our only object here is to record the occurrence of this curious form in the Clyde sea-area, as the specimens sent to Prof. M'INTOSH were taken by Mr JEFFREYS among the Hebrides in 1866, and off Shetland in 1867. We have never found the species in the Firth of Forth, but have nevertheless taken this opportunity of mentioning its occurrence in the Clyde.

Fam. AMMOCHARIDÆ.

Genus *Owenia,* Delle Chiaje.

Owenia filiformis, Delle Chiaje ; Claparède, Chæt. Naples.

Commonly got by dredging, inhabiting very thin flexible sand tubes, from which it is very difficult to detach it entire.

In small specimens the mouth is surrounded by a funnel-shaped lip quite entire except for a ventral notch. It is only in the larger specimens that the branched processes are seen. It seems from this that they are merely outgrowths from the periphery of the mouth, and have not really any connection with the branchiæ of the Serpulidæ, as suggested by CLAPARÈDE.

Fam. AMPHICTENIDÆ.

Pectinaria belgica, Lamarck (Pallas).

Nereis cylindraria belgica, Pallas, Misc. Zool., p. 122.

Pectinaria belgica, Lam., Amm. s. Vert., p. 350 ; Johnston, Cat. Brit. Mus.; Malmgren, Nordiska Hafs-Annulater Svensk. Akad. Forh., 1865, p. 356 ; M'Intosh, Fauna, St Andrews.

Mr HARVEY GIBSON (" Notes on some of the Polychæta," *First Report on the Fauna of Liverpool Bay,* Lond., 1886) has, by carefully neglecting the distinguishing differences between this species and *Amphitrite auricoma,* Müller, attempted to prove that the two forms are identical. He points out that in the original figures of PALLAS of *Nereis cylindraria,* variety *belgica,* " the stiff golden comb shows one continuous and uniform series of teeth, not two series, as in *P. auricoma;*" and that figures by subsequent authors, *e.g.,* M'INTOSH and MALMGREN, show the two combs in *Pectinaria belgica* with perfect distinctness. Moreover, certain references in PALLAS's text imply that his species had two distinct combs. Mr HARVEY GIBSON concludes—" Either PALLAS's draughtsman has made an error in most of the figures of *P. belgica,* and failed to represent the comb with sufficient accuracy, hence leading MÜLLER into error when comparing his form with that of PALLAS, or PALLAS's figures are correct (although his references in the text are wrong), and his species is distinct from that of MÜLLER (for the condition of the comb appears to be the only important difference between the two). Looking at the inaccuracy of the drawings as compared with var. *capensis* in PALLAS's work, and taking into account the indistinctly double series of teeth shown in figs. 5, 8, and 9 of var. *belgica,* I think that probably the former view is the most likely to be the correct one. In that case *P. auricoma* of MÜLLER disappears, and becomes *P. belgica* of PALLAS." How a zoologist, after actually referring to the description and figures given by MÜLLER of *Amphitrite auricoma*, and to the description and figures of both species given by MALMGREN in the *Nordiska Hafs-Annulater*, could suppose the condition of the comb to be the only important difference between the two species, is perfectly incomprehensible. The two distinguishing features of *Amphitrite auricoma* given by MÜLLER are (1) the curvature of the tube, (2) the serration of the margin of the flattened area behind the palmulæ. MALMGREN mentions both these characters and figures them, and he examined specimens of both species; only MALMGREN made the two characters generic instead of specific, and calls MÜLLER's species *Amphictene auricoma.* We can state with certainty that in our specimens the tube is perfectly straight, and the margin of the area behind the palmulæ perfectly entire. The presence of two separate combs is a constant character throughout the whole family Amphictenidæ.

Habits.—We obtained this species at low tide at Granton, on the surface of sandy flats. The tubes are often half buried, with the thin end projecting straight or obliquely from the sand. We have also taken it with the dredge.

MALMGREN states there are 17 pairs of fascicles of capillary setæ, and 13 pairs of uncinigerous pinnulæ, beginning at the 4th setigerous somite; but we find the last two pairs of parapodial projections before the "scapha," or abdominal region, are destitute of both capillary setæ and uncini; there are thus only 15 pairs of fascicles of setæ, and 11 pairs of uncinigerous pinnulæ, the latter commencing at the 4th setigerous somite. The first three pairs of fascicles of setæ are smaller than those following. In front of the 1st setigerous somite are two pairs of branchiæ. The pair of combs, which at first sight seem to belong to the buccal somite, probably really belong morphologically to the 1st branchial, of which somite they represent the notopodial setæ. The semicircular membrane between the tentacles and the palmulæ belongs to the buccal somite, while the tentacles themselves belong to buccal somite and præoral lobe. That this is the real interpretation of the combs is proved by comparison with Ampharete; in that genus the branchial somite, which follows the buccal, bears ventral to the branchia a dorsal fascicle of setæ, specialised into a palmula, and there can be little doubt that it is simply the great development of this palmula of the 1st post-buccal somite which has produced the condition seen in Pectinaria, and in the other genera of the Amphictenidæ.

The "scapha" represents the reduced abdominal region, and commences at the 21st somite, counting the buccal as the 1st. It consists of 5 or 6 somites; it is bent down towards the ventral side; its first somite bears two series of spines, representing the notopodia; the dorsal surface of the scapha is concave, and its lateral margins are thin and crenulated; it terminates in a spatulate membrane overhanging the anus. The scapha fills up the lumen of the tube, which the animal inhabits, posteriorly, as the palmulæ, and the flat surface behind them occlude the tube anteriorly.

Anatomy.—(See Pl. XLII. fig. 20 E).—In Pectinaria the arrangement of the dorsal blood-vessel is similar to that seen in Amphitrite; the intestine close behind the mouth is bent into a long S-shaped loop; the pharynx is narrow, then opens into a thick-walled smooth portion, extending nearly to the posterior end of the thorax, this turns forward as a thin-walled yellow portion, which reaches forward to the pharynx, and then turns back again, as a very thin-walled transparent portion full of sand. The dorsal blood-vessel on the first of these portions is ventral, and is formed by blood-sinuses on the gut communicating with a ventral sinus; the latter at the commencement of the narrow pharynx forms a circumintestinal ring, which opens into the dorsal "heart," as in Amphitrite. The heart contains a cellular "cardiac body."

There are three pairs of nephridia in Pectinaria, of which the first pair are the largest; all the organs are of the usual type, each consisting of a tube bent upon itself, and provided with a nephrostome, and an opening to the exterior. There is a transverse septum separating the buccal somite from the following. The nephrostome of the 1st nephridium is on the anterior side of this septum; the nephridia are brown or black in colour, and this most exterior one reaches dorsalwards above the intestine. The anterior nephridium opens to the exterior, some little space ventral of the origin of the 1st branchia. Between the nephridial opening and the root of the branchia is the opening of a peculiar glandular organ, whose function we have been unable to ascertain. On dissection of a fresh specimen, this gland is seen as a milk-white opaque cylindrical body, about $\frac{1}{8}$th inch long (Pl. XLII. fig. 20 E, *w*), free everywhere, except at the point where it is continuous at its external aperture with the body-wall. The efferent duct of this gland is lined by a high columnar epithelium, of which the component cells are solid and columnar; throughout the rest of the gland, though there is a layer of long solid nucleated cells near the basement membrane, these are covered by other layers of large vacuolated cells, whose walls form a network almost, but not quite obliterating the lumen of the gland. The wall of the gland is well supplied with pseudhæmal vessels. The 2nd branchiferous (*i.e.*, 3rd) somite and the following are unprovided with nephridia; but the latter, *i.e.*, the 4th somite, contains a nephrostome belonging to the nephridium of the 5th somite; the 6th somite is also provided with a nephridium, whose nephrostome is in the 5th somite. The nephrostomata are simple elongated funnels, with their apertures directed forwards; they are not provided with such a series of digitate processes as are seen in Lanice and Arenicola. The gonads are of the usual type, masses of indifferent cells attached to the exterior of the nephrostomata, on the mediad side. The reproductive cells become detached at a very early stage, and pass through the rest of their development in a detached condition in the cœlom. It is certain that the spermatozoa reach the exterior by passing through the nephridia. In a series of sections of a ripe male, we saw the nephrostomata and various parts of the nephridial tubes quite distended with spermatozoa. Between the two posterior nephrostomata and the body-wall, pass membranes which are rudiments of transverse septa. There is also a rudiment of a septum between somites 2 and 3 (*i.e.*, 1st and 2nd branchiferous). The ventral epidermic glandular tissue, so conspicuous and extensive in Terebellidæ, is restricted in Pectinaria to the first two somites.

Fam. AMPHARETIDÆ.

Ampharete gracilis, Malmgren.

Ampharete gracilis, Malmgren, Nordiska Hafs-Annulater, p. 365.

Dredged near Inchkeith, 29th October 1886; depth about 9 fathoms.

Specific Diagnosis.—Abdominal segments, 13. Palmula composed of 12 to 14 setæ, very slender, attenuated at extremity. Branchiæ filiform, attenuated at end, long, unequal; anterior longer than the posterior. Uncini 5–6 dentate. Anal somite crenulated at apex, without cirri.

Length of our specimen, 30 mm.

Analysing the somites here, as usual, we have cephalic—præoral lobe and buccal somite. Branchial—3, the 1st bearing palmulæ = modified notopodial fascicle, and 4 branchiæ on each side, the other two bearing only notopodial fascicles. Thoracic—12, each bearing notopodial fascicle borne on tubercle, and transverse uncinigerous torus, with 5–6 dentate uncini. Abdominal—13 with uncinigerous torus = neuropodium only, + anal segment without cirri (*vide* Pl. XLII. figs. 21 to 21 c).

Genus *Melinna*.

M. SARS, in describing *Sabellides cristata* (*Fauna Littoralis Norvegiœ*, ii. p. 19), says that it perhaps deserves to be placed in a separate genus, but decides provisionally to retain it under Sabellides. MALMGREN made the separation suggested by SARS, and instituted the genus Melinna (*Nordiska Hafs-Annulater*, p. 371), of which he gives the following diagnosis (in Latin): —Præstomium smooth, without elevated frontal part, with anterior margin transverse. Buccal segment produced into a ventral lip equal in length to the præstomium. Palmulæ wanting. Branchiæ filiform, 4 on each side. A single small spine in form of a hook, curved backwards, on each side behind the insertion of the branchiæ. The three anterior setigerous somites coalesced, forming as it were a sheath, free in front, aduate behind, inferiorly and laterally surrounding the oral and branchiferous region. 4th setigerous somite furnished dorsally with a membranous crest, which is equally denticulated on its anterior edge. Fascicles of capillary setæ present in 18 segments, the 3 anterior without a tubercle, 15 following furnished with a subcylindrical tubercle. Uncinigerous pinnulæ commencing from the 4th setigerous segment, and present up to the end of the body. A minute subconical papilla above the uncinigerous pinnula in the segments of the posterior part of the body, which are destitute of capillary setæ. Capillary setæ slightly curved, winged (limbatæ). Uncini pectiniform, subtriangular, with rounded angles, about 4 teeth.

Melinna cristata (Sars), Malmgren.

Sabellides cristata, Sars, Fauna Littoralis Norvegiæ, ii. p. 19.

Melinna cristata, Malmgren, Nordiska Hafs-Annulater, Svensk. Akad. Forh., 1885; M'Intosh, Fauna of St Andrews.

Dredged north of Inchkeith; also on Middle Bank opposite Granton, 6 fathoms, August 1886. M'INTOSH at St Andrews records it merely as frequent in the stomachs of cod.

Examination of this species has shown that neither the description of SARS nor that of MALMGREN is rigidly correct. From its external characters the body may be divided into 5 regions—(1) the cephalic, including the buccal somite; (2) the branchial; (3) the thoracic; (4) the abdominal.

The cephalic region is composed of the præstomium and buccal somite. The præstomium is but slightly developed; from it spring a number of filiform tentacles which can be withdrawn into the mouth, a condition they usually retain in spirit specimens.

The branchial region consists of 4 somites, not 3, as MALMGREN and SARS believed; these somites form a collar. A ridge projects from the anterior part of the 2nd somite ventrally, and is continued laterally on each side as far as the posterior boundary of the 4th somite. Between the lateral ridges is a deep dorsal depression, from the bottom of which the branchiæ, 4 on each side, arise. The branchiæ are filiform, but thicker than the tentacles; they belong to the 2nd and 3rd somites, or to one of these. Posteriorly the dorsal depression is bounded by a denticulated transverse ridge which projects from the dorsum of the 5th setigerous somite. The 4th setigerous somite bears, immediately ventral to the edge of the ridge, a fascicle of long notopodial setæ, and a single series of neuropodial short setæ. Between the notopodium and the neuropodium is a short space. On the 3rd setigerous somite there are similar setæ, but there is no interval between the notopodium and neuropodium. In the first two setigerous somites the notopodial and neuropodial setæ form a single series, and the former only differ from the latter by being slightly thinner and longer, and in colour; the neuropodial setæ are coloured brownish-yellow, the notopodial are colourless. MALMGREN states that the pinnulæ uncinigeræ are absent in the three anterior somites; but, as we have shown above, their homologues are present. SARS also believed the ventral branch of the feet to be entirely wanting in the first three somites. The neuropodial setæ of the branchial region are not uncini, they are simple short bristles cylindrical in shape, and of uniform thickness nearly to the end, terminating in a short transparent slender point. The end of the thick part is strongly pigmented, having a brownish-yellow colour.

The strong curved hook behind the origin of the branchiæ is probably a chæta of the 2nd notopodium specialised.

The thoracic region consists of 14 somites, each of which is provided with a notopodium and a neuropodium. The notopodium consists of a small fascicle of capillary setæ, each having a terminal winged blade (fig. 22 D). The setæ are of two lengths, some projecting far from the body, others much shorter, only the blade emerging. The fascicle is borne by a small tubercle flattened antero-posteriorly. The neuropodium consists of a single series of 4-toothed uncini, borne on a transverse "torus."

The abdomen includes the rest of the body, and comprises about 42 somites. This region is distinguished by the entire absence of the notopodial setæ, though the notopodial tubercle is recognisable in the first few somites. The uncini, borne on a torus which diminishes in size towards the posterior end, are present in all the somites.

Fam. TEREBELLIDÆ.

Subf. AMPHITRITINAE.

Amphitrite Johnstoni (Malmgren).

Amphitrite figulus, Dalyell, Powers of the Creator; M'Intosh, Fauna of St Andrews.

Terebella nebulosa, Johnston, Brit. Mus. Cat.

Amphitrite Johnstoni, Malmgren, Nordiska Hafs-Annulater, p. 377.

With regard to JOHNSTON'S synonym, it is to be remarked that his description applies to the species here in question, but according to MALMGREN some of the British Museum specimens catalogued by JOHNSTON as of this species really belong to *Thelepus circinnata*.

In this species there are 90 to 100 somites. The prostomium bears a large number of long simple filamentous tentacles. The prostomium has two prominent margins, one anterior and the other posterior, and on the surface between these the tentacles arise. On each side the prostomium runs out into a lappet or lobe, which increases the tentaculiferous surface. Next follows the buccal somite, which is well developed, but not otherwise remarkable. Then follow three somites, each bearing a pair of branchiæ. The branchiæ consists of a number of branches arising from a short, rather thick stem, which is directed forwards, and bears the branches on its posterior surface, and each branch immediately divides into two or more filaments, which are somewhat long, and are spirally curled, especially when contracted; the whole forms a large bushy plume or brush. The first notopodial fascicle of chætæ is borne on the 3rd branchiferous somite. There are 24 pairs of notopodial fascicles; behind the 26th post-buccal somite they are absent: in one specimen, however, a single fascicle was present on one side on the 27th post-buccal somite. The chætæ have a peculiar point or blade: there is a well-developed pair of lateral wings which extend to a short distance from the end, the rest of the blade being slightly curved and minutely denticulated on the concave side. The series of tori uncinigeri begins at the 4th post-buccal somite, and extends throughout the body, the posterior ones becoming shorter and much more prominent. The uncinus is short, and provided with tendons: it has 3 or 4 minute teeth on the outer edge of a single large one. On the ventral surface of each of the first 14 post-buccal somites, there is a single median "scutum ventrale," of which the last is small and rudimentary: the others occupy the whole ventral region of the somite between the ventral ends of the neuropodial tori (Pl. XLIII. fig. 23, 23 A, 23 B).

Habits.—Common between tide marks among Laminarian roots and under stones: it lives in mud, and does not form a very perfect tube; the mud in its immediate vicinity is merely glued together by the secretion of the worm's body to form a case. The worm reaches a considerable size, some of our specimens being over 15 cm. long and 12 mm. broad in the anterior part. MALMGREN also gives these as the maximum dimensions. Specially fine specimens are abundant on the shore at Joppa.

Anatomy.—There are 15 to 17 pairs of nephridia in the first 15 to 17 post-buccal somites. As in Trophonia, there are median vertical vessels passing from the ventral vessel to the intestine. In the anterior portion of the body the dorsal vessel is represented only by a perienteric sinus, which is most developed on the ventral side of the intestine, but at the extreme anterior end this sinus opens into a free dorsal contractile vessel forming a heart. This heart contains a cellular cardiac body, as in Trophonia (*vide* Pl. XLIII. fig. 23 C).

Amphitrite cirrata, Müller.

Terebella cirrata, Montagu, in Linn Trans., xii. p. 342; Johnston, Cat. Brit. Mus.; Leuckart, Archiv f. Naturges., i. 1849.

Amphitrite cirrata, Müller, Prodr. Zool. Dan.; Malmgren, Nordiska Hafs-Annulater, p. 375.

Two or three specimens obtained in the dredge on one occasion, on the Röst, north of Inchkeith, October 29, 1886.

Specific Diagnosis.—Chætiferous tubercles in 17 segments. Prostomium behind the tentacles without lateral lobes. Branchia with very short stem, from which spring a number of elongated filaments nearly equal in length, and spirally curled. A minute conical papilla in six segments, the 3rd chætiferous to the 9th. Ventral scutes, 10 in number, rectangular in shape; tube of mud or clay (Pl. XLIII. fig. 24).

Terebella Danielsseni, Malmgren.

T. Danielsseni, Mgn., Nordiska Hafs-Annulater; Stockholm Forhandlingar, 1865.

Specific Characters.—Segments short; branchiæ flabelliform, decreasing much in size posteriorly, branching dichotomously from a short stem; ultimate twigs very short. Two secondary teeth on the uncinus, one distinct the other minute; the manubrium of the uncinus with an entire curved outline at the corner beneath the teeth (*vide* Pl. XLIII. fig. 25).

Middle Bank, opposite Granton, 6 fathoms, August 11, 1886; only one specimen dredged.

Genus *Lanice*, Malmgren, 1865.

The points which distinguish this genus from Terebella are not very marked. Both have 17 pairs of notopodial fascicles; both have 3 pairs of arborescent branchiæ: the principal difference is the shape of the buccal somite, which in Lanice is produced anteriorly so as to form a large under lip, which is absent in Terebella. Eye spots are present in Terebella, absent in Lanice. In Lanice the 2nd post-buccal somite has a large vertical lobe on each side, and the glandular scuta ventralia form a continuous area, of a bright red colour in the living animal, instead of forming a metameric series.

Lanice conchilega, Mgn. (Pallas).

Nereis conchilega, Pallas, Misc. Zool., p. 131.
Terebella conchilega, Savigny, Syst. Annel.; Johnston, Brit. Mus. Cat.
Terebella littoralis seu arenaria, The Sand Mason, Dalyell, Powers of the Creator.
Lanice conchilega, Malmgren, Nord. Hafs-Annulater, p. 380; M'Intosh, Fauna of St Andrews.

Only one species of the genus is known, and it would be better to dismiss the genus and call the species *Terebella conchilega*. JOHNSTON, in his description of *T. conchilega*, does not mention the fringes at the mouth of the tube; but his original is *Nereis conchilega*, Pallas, which is undoubtedly this species, as the following quotation from the *Miscellanea Zoologica* will show:—"Subtus tænia prominens plana, pulchre rubra ad caput rotundato initio incipit, ultraque mediam corporis longitudinem producta, angustatur, tandemque evanescit. Caput animalis squamis quatuor planiusculis carnosis munitum est; quarum binæ majores contiguæ semiovatæ; exteriorque ad latera utrinque una cui setæ dorsales primi paris respondent." Johnston's *T. littoralis* is simply Dalyell's *T. littoralis seu arenaria;* and although the latter counted only 16 fascicles of chætæ, he mentions a "broad, taper, smooth, velvet bright carmine stripe along the belly between a transverse row of ellipses," which proves that his Sand Mason is our *Lanice conchilega*. Dalyell also mentions the fringe of branched filaments, made of particles of sand, round the mouth of the tube which the worm inhabits: he says this fringe occurs at both ends of the tube, but this must be a mistake.

Habits.—The tube is made of particles of sand, and at Granton we have always found it buried vertically in sand, with only an inch or so protruding above the surface. The tube is very long, and as the animal is always at the deep end of it, careful digging is required to extract it uninjured. The filaments of the fringe are hollow, and when the head of the worm is protruded the tentacles are partially contained in them, and so protected. The projecting part of the tube, with its tuft of tubules, has a very characteristic appearance on a sandy shore, and there are probably few sandy shores on the coast of Europe where these tufts are not to be seen. The worm also occurs to some distance beyond low-water mark (*vide* Pl. XLIII. fig. 26).

For an account of the nephridia, which in this species form a continuous tube on each side by coalescence, see a paper "On some Points in the Anatomy of Polychæta," by J. T. Cunningham, *Quart. Jour. Micr. Sci.*, 1887.

Scione maculata (Dalyell).

Terebella maculata, Johnston, Cat. Brit. Mus., p. 240; Dalyell, Pow. Creator, ii. p. 203.

We have placed this species in Malmgren's genus Scione, because it has 16 pairs of setigerous tubercles, and only one pair of branchiæ. Our specimens agree closely with Malmgren's *Scione lobata*, but that in ours there are numerous ocelli behind the tentacles, while Malmgren's generic diagnosis includes the words "oculi nulli." The species agrees with *lobata*, in having a lateral semicircular lobe projecting on each side from the 3rd somite, the one immediately behind the branchiferous. These lobes are evidently the "pair of short, obtuse, pellucid stumps not distinguished by obvious peculiarities," mentioned by Dalyell. We have no doubt that our specimens are of the species *Terebella maculata* as described by Dalyell, and it is extremely probable that that species is really a synonym of *Scione lobata*, Malmgren. The chætæ are limbate, with an entire point; the uncinus has two secondary teeth above the principal one. The tentacula are few in number, and in the living animal spotted. Round the anus are 6 or 7 conical papillæ (Pl. XLIV. fig. 27).

Habits.—We dredged several specimens, Nov. 11, 1886, on the Röst, north of Inchkeith, in 7 or 8 fathoms; bottom, shells, stones, and black muddy sand.

Genus *Thelepus*, Malmgren (R. Leuckart, 1849).

The two distinguishing features of this genus, as defined by Malmgren, are the presence of fascicles of capillary setæ throughout the whole length of the body, and the presence of numerous filiform branchiæ, arising separately in a transverse series on each side, in two of the anterior somites.

The genus was named by R. Leuckart (*Arch. f. Naturges.*, xv. 1), who founded it on a fragment containing only a few of the posterior segments of a single specimen. Leuckart believed his specimen to form both a new species and a new genus. Malmgren retained the generic name, but identified the characters of the species with those given by previous authors, under other names.

Thelepus circinnata, Malmgren (Fabr.).

Amphitrite circinnata, Fabr., Fauna Grœnl., p. 286.

Terebella conchilega, Dalyell, Pow. Creator.

Thelepus circinnata, Mgn., Nordiska Hafs-Annulater.

Thelepus circinnatus, M'Intosh, Fauna of St Andrews; Leslie and Herdman, Inv. Fauna of the Firth of Forth.

Venusia punctata, Johnston, Cat. Brit. Mus.

Specific Diagnosis.—That of the genus.

Description.—The filiform branchiæ are borne on the first two somites behind the buccal; the first of these has no chætæ of any kind; fascicles of capillary chætæ are present on all the somites, from the 2nd branchiferous to the end of the body. Uncinigerous tori are present from the 3rd chætiferous somite to the end of the body. The uncini have 2 teeth. Behind the series of tentacles, on a narrow transverse band, bordered by two ridges, are numerous ocelli. The dorsal surface of the animal is uniformly marked with clear oily-looking spots. Glandular scuta ventralia, one on each of the first 16 somites. The tube is slightly sinuous, flexible. It is formed of a thin transparent membrane resembling mica, which is covered externally with pieces of shell, small stones, fragments of Polyzoa, &c.

Habits.—We took it in the dredge off Anstruther, and obtained it also from fishermen's lines, worked 50 miles E. by S. from the May Island; depth about 30 fathoms. It is rather a deep-water form, not found between tide marks, though JOHNSTON says it is very common in the coralline region. According to MALMGREN, it is common on the arctic and northern shores of Europe, and extends as far south as the Mediterranean, ranging from 3 to 250 fathoms in depth.

Subf. POLYCIRRINÆ.

No branchiæ.

Ereutho Smitti, Malmgren.

Ereutho Smitti, Mlmgn., Nordiska Hafs-Annulater, p. 391.

Polycirrus Smitti, M'Intosh, Fauna of St Andrews.

We have identified this form from MALMGREN's diagnosis; GRUBE's genus Polycirrus has fasciculi of capillary setæ on 40 of the anterior segments; the genus Ereutho, as defined by MALMGREN, on only 13. MALMGREN gives the following analysis of the Subf. Polycirridea:—

Uncini nulli. Fasciculi setarum	in 6 segmentis,	Lysilla, Malm.
Uncini hamati. Fasciculi setarum capillarium,	in 13 segmentis,	Ereutho, Malm.
	in 19 to 22 segmentis,	Leucariste, Malm.
	in 40 seg. vel ultra,	Polycirrus, Gr.
Uncini elongati, sublineares aciculiformes. Fasciculi setarum in 10 segmentis,		Amæa, Malm.

Habits.—Not uncommon, frequently found in holes among roots of Laminaria, also under stones in the littoral zone, dredged occasionally on the middle bank, in 5 to 7 fathoms. It lives in holes in mud, sand, or debris, and does not form a separate tube for itself.

The prostomium forms a considerable tongue-shaped projection, which bears along its dorsal edge the numerous simple flexible tentacula: the buccal somite is not conspicuous. There is a single ventral scuta behind the mouth, corresponding to the buccal and 1st setigerous somite: behind this are 8 pairs of scutes belonging to somites 2 to 9.

One of the most peculiar features of the genus is the entire absence of uncini or uncinigerous tori, that is of neuropodial elements, in the 13 somites which bear the capillary setæ. Behind the 13th somite, small uncinigerous tori only are present: the uncini are small, uniserial, with two small teeth on the dorsal end.

This worm is of yellowish-white colour when alive, and is somewhat transparent. It writhes when placed in clean water into complicated contortions. The anterior region is swollen, the rest of the body cylindrical.

Subf. CANEPHORIDEA.

One branchia divided distally into 4 comb-like processes.

Terebellides Strœmi, Sars.

Terebellides Strœmi, Sars., Beskriv. og Jakttag., p. 48; Malmgren, Nordiska Hafs-Annulater, p. 396; M'Intosh, Fauna of St Andrews.

Dredged 1 mile from Isle of May; N.W. of Inchkeith, 13 to 16 fathoms, on muddy ground. We have never found the species on the shore. It is evidently restricted to muddy ground at depths of over 10 fathoms. M'INTOSH found large specimens in stomachs of cod and haddock. MALMGREN gives as its distribution Spitzbergen, Greenland, Iceland, Scandinavia, Britain, Baltic Sea, and, according to GRUBE, the Adriatic Sea, on muddy ground, 10 to 250 fathoms depth, everywhere rather abundant.

This form is peculiar among the Terebellidæ; it is the only species of the genus, and MALMGREN places the genus by itself in a separate subfamily, the Canephoridea.

The prostomium is membranous and flexible, the edge being sinuously plicate, and bearing the numerous short thin tentacles at its edge dorsally. The ventral part of the buccal somite is enlarged, and the dorsal contracted, so that the mouth is thrown to the dorsal side, and the end of the body appears truncated. The ventral part of the buccal somite carries a transverse membranous projection or crest. There is one single branchia attached to the 1st setigerous somite; the structure of this is unlike that of any other branchiæ in the family. It consists of a peduncle transversely flattened, which bears four pectiniform processes, in two pairs, the anterior pair being much larger than the posterior, and overlapping them. Each process is formed by a stem running backwards and bearing on its upper side a series of thin laminæ semicircular in shape. The laminæ are placed transversely to the axis of the stem and to the longitudinal axis of the worm, and are so closely crowded together that the branchia seems, on a cursory examination, solid. The stem of the posterior pectinate process is of a conspicuous opaque white colour. There are notopodial fascicles of capillary setæ in 18 somites, commencing with the somite behind the buccal. These setæ are almost straight, with a very long attenuated extremity, winged. From the 6th setigerous somite to the 18th the neuropodial element is an uncinigerous torus, bearing uncini in a single series: these uncini have a long handle or manubrium inserted in a socket in the torus. Behind the 18th somite the notopodial element is wanting; the neuropodial is a flat pinnula provided with uncini of the typical kind, *i.e.*, pectiniform and short, with three or four teeth (*vide* Pl. XLIV. fig. 29).

Anatomy.—There is but one pair of nephridia, situated in the first chætiferous somite.

Larva belonging to the Terebellidæ.

In fig. 30 is shown the appearance under the microscope of a pelagic larva obtained by the tow-net near shore at Granton, and believed by us to belong to some species of the family Terebellidæ. This larva was contained in a transparent tube, evidently secreted by itself; it is not an uncommon thing for the larvæ of tubicolous forms, even when leading a pelagic existence, to be provided with tubes. The intestine in this particular specimen is slightly convoluted, a point in which the larva differs from any adult Terebellid. Larval tentacles are growing from the prostomium, and there are a pair of otocysts behind the anterior end, probably closely connected with the œsophageal nerve commissures. Anteriorly the somites are provided with fascicles of acicular chætæ; but on the posterior half there are curious thin knobbed processes, a larval form of parapodia, which we have not further investigated. The whole larva was very transparent.

Fam. SABELLIDÆ=SABELLACEA, Malmgren.

The Sabellidæ were separated from the Serpulidæ by MALMGREN: the distinguishing characters are as follows:—Body almost cylindrical, or but slightly depressed, straight, pointed at the posterior end, consisting of two parts—an interior of few (5 to 12) somites, in which the tori uncinigeri are ventral, and the fascicles of capillary chætæ dorsal, and the rest of the body in which the tori uncinigeri are dorsal and the fascicles of capillary chætæ ventral. In the posterior part there is a ventral longitudinal sulcus, sometimes continued on to the dorsum of the anterior part. The first segment has a collar. The branchiæ are long straight filaments, in two groups, those of each coadnate at the base; each filament being provided interiorly with a double series of short thin processes; exteriorly the branchial filaments are usually naked, but sometimes provided with eye-spots, or short spatulate processes. Tube of the animal straight, cylindrical, membranous, usually coated with black mud, sometimes with grains of sand or other particles.

Sabella, Lin.

Ventrical sulcus not continued on to the dorsum of the thoracic region. Collar narrow, divided, with widely separated projecting corners dorsally, and reflected ventral lobes. Fascicles of capillary chætæ commencing on the collar somite, tori uncinigeri on the somite next to the collar. Uncini of the anterior region in double series, of two forms, one avicular the other cuspidate (Pl. XLIV. fig. 31 D). Branchiæ connected only at the base, dorsal appendages none, ocular spots sometimes present. Two tentacles, shorter than, and underneath the branchiæ.

Sabella pavonia, Sav.

Sabella penicillus, Lin., Syst. Nat., xii. p. 1269 (excl. syn. Rond., Ellis, et Syst. Nat., x.).

Amphitrite penicillus, Lam., Hist. Nat. Anim. s. Vert., 1818, v. (excl. syn. Fabr. et Brug.).

Sabella pavonia, Savigny, Syst. des Annél. Desc. de l'Egypte, 2nd. edit., 1826, xxi.; Malmgren, Nordiska Hafs-Annulater; M'Intosh, Fauna of St Andrews.

Specific Characters.—Size large, 12 to 15 inches in length. Segments of thoracic region 8 to 12. (In our figure 31 A, Pl. XLIV., the number shown, 15, is incorrect.) Branchiæ 35 to 45 on each side, long, thin flexible; no ocular spots; white, with minute purple spots or bands at equal distances. Two grooved tentacles, ¼th or somewhat more than ¼th the length of the branchiæ.

Habits.—Tube always covered densely with fine black mud, and usually enclosed in a colony of *Alcyonium digitatum*, which has usually a smooth surface without digitate processes. We obtained it not very frequently from haddock lines worked in about 30 fathoms, off the mouth of the Firth of Forth, the hooks catching easily in the coating of Alcyonium. The coating of Alcyonium shows that the tube projects far from the surface of the sea bottom.

Figs. 31 B, 31 C, show longer and shorter capillary chætæ from the thoracic region; figs. 31 D, the two forms of uncini.

Laonome Kröyeri, Malmgren.

Laonome Kröyeri, Malmgren, Nordiska Hafs-Annulater, p. 400.

One specimen got in dredge on Middle Bank, October 29, 1886.

Genus *Chone*, Kröyer.

Described by MALMGREN as follows:—Body somewhat round, sublinear, pointed posteriorly, with the abdominal furrow very conspicuous, and continued on to the dorsum of the anterior part of the body; anus terminal; no ventral cleft; somites divided into two by a transverse furrow. Collar adpressed to the branchiæ, divided into two parts dorsally, entire, without ventral or lateral incisions. Anterior part of the body composed of eight somites. No ventral scutes. Setigerous tubercles beginning from the collar somite, with setæ of two forms in the anterior part of the body, the longer form bordered, the apex much attenuated and slightly curved; the shorter form, subspatulate, the apex shortly mucronate, unequally bordered on both sides, one margin having a much broader border than the other; in the posterior part of the body all the capillary setæ of the same form, with a narrow border, and the apex long and tapered. Uncinigerous tori beginning from the second setigerous somite with a single row of uncini, in the anterior part of the body, beaked, with a somewhat elongated manubrium, and the vertex of the rostrum subserrulate, but in the posterior part of the body they are short and avicular. Branchiæ forming a semicircle on each side, connected by membrane for more than half their length ; their apices free, bordered on each side, with no dorsal appendages nor eye spots. Tentacular cirri round, filiform, unequal, many on each side.

Chone infundibuliformis, Kröyer.

Tubularia penicillus, Fabricius, Fauna Grœnl., p. 438.
Chone infundibuliformis, Kröyer, Om. Sabellerne, Danske Vid. Selsk. Forh., 1856, p. 33 ; Malmgren, Nordiska Hafs-Annulater, p. 404.

On the Röst and North Channel, north of Inchkeith, 1st November 1886 ; also west of Oxcars, November 1884.

The specific characters as given by MALMGREN are as follows :—Body of 50 to 80 somites, the breadth equal to a twelfth or eighth part of the length. Collar moderately produced, of equal height on both sides ; anterior margin entire, running down into a vertical sinus, and divided by a deep dorsal furrow, linear and not gaping. Branchiæ having their free apex of moderate length, and entirely enclosed in a leaf-like bordering membrane. Colour in spirit entirely white ; branchiæ in most living animals intensely purple, especially towards the apex, either all of one colour or spotted with yellowish-white ; in the smaller specimens, the branchiæ paler red, spotted with white or pale yellow. It inhabits a tube of yellowish membrane, covered with a coat of sand externally, fixed to stones, or often to Ascidians. Length 15 to 18 mm., breadth 1·7 to 6·5 mm. ; length of branchiæ may reach 27 mm. (Pl. XLIV. fig. 32).

Genus *Amphicora*, Ehrenberg (1836).

Fabricia, Blainville (1828), Leuckart, Claparède.
Othonia, Johnston (1834), Gosse.

The characters of this genus are as follows :—Body of 13 somites, the first nine of which belong to the thorax, and the last four to the abdomen. All are setigerous except the first and last. In the thorax the notopodial setæ are capillary, and the neuropodial setæ uncinate. In the abdomen the arrangement is reversed, the notopodial setæ being uncinate, and the neuropodial setæ capillary. Branchiæ with three branches in each tuft. Eye spots two, towards the sides of the first somite. A large labium or under lip bounds the mouth ventrally. Last somite closely united with that which precedes it, with a sort of post-anal lobe projecting backwards dorsally to the anus, and bearing two eye spots. Capillary setæ limbed on one side, curved, and much drawn out at the tip. Uncini with a long curved manubrium, and several teeth on the vertex.

Amphicora Fabricia (Müll.).

Tubularia Fabricia, Prodr., Z. Danicæ, n. 3066.

Othonia Fabricia, Johnston, Cat. Brit. Mus.

Amphicora sabella, Ehrenberg, Mitth. Ges. Nat. Freunde in Berlin, 1866, p. 2 (Heligoland).

Amphicora Fabricia, Malmgren, Annulata Polychæta, p. 225; M'Intosh, Fauna of St Andrews.

Fabricia quadripunctata, Claparède, Etudes Anat. Annel. Turbel. etc. in Mém. Soc. Phys. et Hist. Nat. Geneve, xvi. 1.

Occurs abundantly on the roots of Fuci growing on shale rocks at Granton. Colour reddish, length three or four lines. Inhabits a slender tube of mud two or three times the length of its body. When a few of these tubes are teased up in a watch-glass the minute worms come crawling out, wriggling along very actively, and moving usually tail first. This is the more curious, when we observe the post-anal lobe and eye spots described on the last somite.

Fam. ERIOGRAPHIDEA, Malmgren.

Genus *Myxicola* (Koch), Grube.

Arippasa, Johnston, Cat. Brit. Mus.

Eriographis, Grube, Fam. d. Annel.

Is described by MALMGREN as follows :—Body round, somewhat thick, giving off a quantity of mucus; somewhat attenuated towards each end, more so posteriorly than anteriorly; anus terminal; collar none. First somite produced inferiorly into a very short triangular acuminate process, pointing forward. Ventral furrow not conspicuous, but continued into the dorsum of the anterior part of the body. Anterior part of the body composed of eight somites, with slender capillary setæ with a narrow limb, and a few elongated uncini, with sub-rostrate apex beneath the minute fascicles of setæ. Posterior part of body of numerous subannular somites, with fascicles of setæ disappearing and very small uncini very shortly hooked, with a much elongated tooth on the vertex, very numerous, and forming a transverse series from the dorsum round the ventral surface, both above and below the fascicle of setæ. Branchiæ forming a semicircle on each side, almost completely connected by membrane; rays filiform, disposed in two rows internally. Eyes none. Two very short broad compressed rounded tentacles surrounding the mouth on each side.

Myxicola Steenstrupi, Kröyer.

Myxicola Steenstrupi, Kröyer, Dansk. Vid. Selsk. Forh., 1856, p. 17.
Myxicola Sarsi, Kröyer, *loc. cit.*, p. 9; Sars, Christiania Vid. Selsk. Forh., 1861, p. 130.
Myxicola Steenstrupi, Malmgren, Nordiska Hafs-Annulater, p. 409.
Arippasa infundibulum, Johnston, Cat. Brit. Mus.

This species is not mentioned by M'Intosh in his Fauna of St Andrews. We have dredged it on the Middle Bank on shelly ground on several occasions.

Malmgren describes the specific characters as follows:—"Body of 45 to 70 somites, in the posterior part more or less distinctly biannulate, short in middle of body, length equalling a fifth to a seventh part of the breadth. Branchiæ with from 15 to 22 branches on each side, equalling or slightly surpassing a third part of the length of the body, with apex naked, involuted, tapered, edged by membrane, almost equalling a fourth part of the length of the branchiæ; radii of moderate length, very thin, tapered, nearly flexible." Colour dull greenish, branchiæ tinged with purple, especially towards the tips. Always found enveloped in transparent mucous.

Fam. SERPULIDÆ, Malmgren.

We have not minutely studied either the literature or the animals in this family, but we believe the following account to be correct.

Gen. *Serpula*, Lin., Philippi.

Serpula vermicularis, Linn.

Serpula vermicularis, Linn., S. N., xii. p. 1266; Malmgren, Annulata Polychæta, p. 228; M'Intosh, Fauna of St Andrews.

Common in deep water.

Genus *Filigrana* (Oken).

Branchiæ with eight branches, two of which are expanded at their extremity, and obliquely truncated, so as to form opercula. Tubes calcareous, very slender, and adhering together in masses.

Filigrana implexa, Berkeley.

Filograna implexa, Berkeley, Zool. Journ., v. p. 427; Sars, Faun. Litt. Norv., i. p. 86; Malmgren, Annulata Polych.; M'Intosh, Fauna of St Andrews.
Protula Dysteri, Huxley, Anatomy of Invertebrates, 1871.

A large mass of the tubes of this species was obtained from Newhaven fishermen; it was brought up on haddock lines, some distance outside the May. We dredged a number of masses of the tubes, with the worm living within them, on the Röst, November 1, 1886.

Johnston mentions no specimens of Huxley's *Protula Dysteri*, the description of which he quotes, and there is nothing in that description to differentiate the worm from *Filograna implexa*, of which Johnston gives the characters from his own examination of specimens from Devonshire. Our identification of the two names is made on the authority of Professor M'Intosh. Our figure of the larva was taken from some which were found among living specimens of the worm (Pl. XLV. fig. 35).

Genus *Pomatocerus* (Phil.).

Branchiæ with many branches in each tuft; opercular tentacle thick, with two processes near the upper end. Tube entirely adherent, white, with a distinct keel on its upper surface, and a spine over the aperture.

Pomatocerus triqueter, L. Mörch.

Serpula triquetra, Sars, Reise, 1849; Danielssen, Reise, 1857.
Pomatocerus triqueter, Mörch, Revisio critica Serpulidarum Natur. Tidskr., 3 R. 1 B., 1863; Malmgren, Ann. Polych.; M'Intosh, Fauna of St Andrews.
Vermilia triquetra?, Philippi, in Ann. and Mag. Nat. Hist., xiv. 156, pl. iii. f. F.; Grube, Fam. Annel., 92.
Serpula conica, Johnston, Cat. of Worms in British Museum.

Very abundant everywhere, attached to rocks, stones, and shells.

Colour deep rich blue on the thorax, abdomen red in the females, and whitish in the males when sexually mature; branchiæ barred, and spotted with deep brown, blue, orange, white, and sometimes red. The distribution of the colour is subject to considerable variation. Capillary setæ in thorax curved, limbed, and finely pointed; those of the abdomen with the limb immensely expanded on one side, and serrated; uncini with ten points, and the manubrium all but obsolete (Pl. XLV. fig. 36).

Spirorbis borealis, Mörch.

Serpula spirorbis, Linn, S. N., xii. p. 1265; Müller, Prodr. Z. D., p. 236.
Spirorbis borealis, Mörch, Naturh. Tidskr., 3 R. 1 B., 1863; Malmgren, Annulata Polychæta; M'Intosh, Fauna of St Andrews.

Common on Fuci on the shore. Very common on *Fucus serratus* at Dunbar. We found it there breeding freely at the beginning of June. The ova form a cylindrical cord, consisting of two or more linear series, and lying beneath the animal in the tube, not in the operculum. The embryos were well advanced in development before hatching (Pls. XLV. and XLVI. fig. 37).

Spirorbis lucidus (Mont.), Mörch.

Serpula porrecta and *spirillum*, Fabr., F. Gr., p. 378.
Serpula spirillum, Müller, Prodr. Z. D., p. 236.
Spirorbis lucidus, Mörch, Malmgren and M'Intosh, *loc. cit.*

Common on hydroids dredged in the Firth (Pl. XLV. fig. 38).

Fam. CHLORHÆMIDÆ.

Trophonia plumosa, Johnston, (Müll.).

Amphitrite plumosa, Müller, Prodr. Z. D. n. 2621, p. 216.
Trophonia plumosa, Johnston, Cat. Brit. Mus.; Malmgren, Annulata Polychæta; M'Intosh, Fauna of St Andrews.

Common in the mud among Laminarian roots, also under stones in the Laminarian zone.

The pseudhæmal system consists of a dorsal and a ventral longitudinal vessel, united by a pair of lateral commissures for each somite. The posterior portion of the intestine receives its blood supply by vessels passing vertically upwards from the ventral longitudinal trunk. The long backward loop is supplied by two trunks arising one behind the other from the ventral longitudinal vessel, in about the 9th and 10th somites; from these the fluid finds its way into a series of lacunar spaces in the intestinal wall, through which it passes forward to flow into a large heart placed over the œsophagus. The heart is somewhat pyriform, being thicker at its posterior end, where it receives the blood from the intestine, and tapering anteriorly, where it divides into two trunks just behind the cerebral ganglion. These give off vessels to supply the branchiæ. The dorsal longitudinal vessel opens into the heart about the middle of its length.

The above account differs very much from that given by HORST (*Zool. Anz.*, viii. p. 12), who from what he saw in transverse sections of Brada, concluded that in Chlorhæmidæ the only representative of a free dorsal vessel was the heart of the anterior region, the dorsal vessel of other annelids being represented by the blood-sinus in the wall of the intestine. HORST, therefore, asserted that the Chlorhæmidæ possessed the same relations in the pseudhæmal vessel as the Enchytræidæ according to the researches of VEJDOVSKY. The view which seems to us the true one to take of the condition of things found in Trophonia is as follows:—Behind the loop of the intestine there is a series of vessels passing from the intestinal walls to the dorsal trunk. The dorsal vessel is to be regarded as continued by that part of the heart which is in front of the point where the dorsal vessel joins it. The part behind this represents (enormously enlarged) simply the vessels connecting the looped part of the intestine with the dorsal vessel. It is probable that the loop of the intestine corresponds to just two somites, for there are two vessels passing from the ventral vessel to the loop: the posterior part of the heart corresponds to two corresponding dorsal communicating vessels.

The meaning of the glandular body contained in the heart is difficult to surmise. HORST considers it to have been originally derived from the intestinal epithelium. We have found that in the adult Trophonia the cardiac body is entirely separate from the intestine, although at the posterior end of the heart the glandular body and the intestinal body-wall are in extremely close contact. The continuity of the blood-sinuses in the intestinal wall with those of the heart is obvious enough in sections. The minute structure of the cardiac body is closely similar to that of a nephridium. The whole organ or gland consists of a number of tubes having for the most part a longitudinal direction; some of these tubes have a lumen, the walls being lined with transparent nucleated cells containing granules, while in other tubes similar cells completely fill the cavity. (See Cunningham, "Some Points in the Anatomy of Polychæta," *Quart. Jour. Micr. Sci.*, 1887.)

There is only a single pair of nephridia placed longitudinally in the head. They extend posteriorly as far as the 6th to 8th somite. In colour they are yellowish green or white to the naked eye. They open anteriorly on the invaginated first somite at the side of the tentacles. We have not been able to discover any cœlomic opening or nephrostome.

When a piece of the nephridium is taken in the fresh condition and slightly teased in a drop of sea water, the whole structure of the glandular epithelium is beautifully seen. Scattered all over the field of the microscope are seen globular gelatinous-looking bodies, each with a slight globular concretion at its centre. In places a number of these gelatinous globules are seen united together: there are also spherical cells with delicate walls, each containing 10 or 12 of the globules. Again, there are a number of smaller cells, each containing several of the black concretions of smaller or larger size, and each cell bearing a single large cilium or flagellum, vibrating either very rapidly or more slowly. There is usually only one cilium to a cell, but occasionally a cell is seen bearing two. These ciliatedcells vary much in size, and the smaller ones are not spherical, but have an irregular outline. It is evident that these smallest cells, which are quite transparent, are almost entirely composed of unaltered protoplasm; even the smallest usually contain a single minute concretion. In the ciliated cells of moderate size it is seen that the substance containing the concretions is modified protoplasm, and it forms a separate spherical mass surrounded by a delicate layer of protoplasm which is continuous with the cilium. In the oldest and largest cells both cilium and protoplasm have disappeared, and nothing is left but a thin-walled vesicle containing a group of gelatinous globules, each with its concretionary nodule at the centre. These vesicles easily break up, and thus the gelatinous globules are scattered over the slide. The process of secretion then here consists in the successive evolution of minute protoplasmic ciliated cells into the large non-ciliated vesicles carrying concretions; the vesicles drop off in succession from the epithelium into the gland cavity, and so reach the exterior. A similar mode of secretion can be made out in the nephridia of all Chætopods, and also of Mollusca.

The ovaries in the female and testes in the male have similar positions and relations. There are in each sex four gonads in two pairs, lying on either side of the intestine. Each organ is a much elongated thick band or mass. In the natural condition all four organs are rolled up with the loop of the intestine into a smooth cylindrical mass, enclosed by a membrane. When the organs are disentangled the elongated form of the gonads is seen, and it becomes evident that they are free everywhere except at the anterior end, which is attached to one of the metameric transverse vessels of the body-wall. A single blood-vessel given off from the lateral vessel is seen running the whole length of the organ, and supplying it with capillaries. No efferent vessel was to be seen. The somites to which the gonads thus belong are the 7th and 8th setigerous. The ovaries have a bright green colour; the testes are pale yellow. The male specimens, as far as our observations go, are larger than the female. We have no clue as to the manner in which the generative products escape (Pl. XLVI. fig. 39).

Genus *Flabelligera,* Sars, 1829.

The distinguishing characters of the genus Flabelligera are the presence of a thick mucous sheath surrounding the body, and containing long, knobbed epidermic processes, and the large number (*circ.* 40) of branchial filaments on each side of the head. The usual single pair of ciliated tentacles are present, ventral to the branchiæ; these are much thicker than the branchiæ. The bristles of the 1st setigerous somite only are directed anteriorly, and being long and numerous, form a kind of palisade round the head region, which is completely retractile. In the rest of the somites the dorsal fascicle contains only 4 or 5 bristles, which terminate in slender points: a number of the epidermic projecting cells, of a special form, surround the dorsal bristles, climbing up them, as CLAPARÈDE says, like a climbing plant up a tree, and terminating at the same level as the bristles.

Flabelligera affinis, Sars.

Flabelligera affinis, Sars, 1829, Bidrag til Söedyrenes Naturalh., i. p. 31; Beskriv. og. Jakt., p. 47.
Siphonostomum vaginiferum, R. Leuckart, Arch. Naturg., xv. 1, p. 164.
Siphonostoma uncinata, Johnston, Cat. Brit. Mus., p. 223.
Flabelligera affinis, Malmgren, Annulata Polychæta, p. 193; M'Intosh, Fauna of St Andrews.

Dredged on Middle Bank in August 1886. Also found occasionally at Granton in the Laminarian zone under stones, at low water, spring tides.

It seems in the highest degree probable that the *S. diplochaites* described by CLARAPÈDE (*Chet. de Naples*, p. 369) is the same species as the present one; that the Mediterranean and northern forms are all of one species. That this is so is shown by comparing the synonymy given by MALMGREN (*Ann. Polyc.*) and CLAPARÈDE: both give *Chlorœma Edwardsi*, Dujardin, as synonyms. MALMGREN gives *Siphonostoma uncinata*, Johnston, as a synonym: JOHNSTON and CLAPARÈDE both give *S. Edwardsi*, Grube (*Fam. Annel.*), as a synonym.

In Siphonostomum the arrangement of the pseudhæmal system is closely similar to that described in Trophonia, but there are some slight differences. The dorsal vessel is not so large, and is confined to the anterior part of the body. At the extreme posterior part of the body no distinct dorsal vessel is present; the transverse lateral vessels of the integument form a network. But the anterior part of the vessel, as in Trophonia, receives the lateral vessels of the integument, and the vessel joins the heart a short distance in front of its posterior end. Into this vessel, near its junction with the heart, open several median vessels, bringing blood from the lateral vessels of the integument in front of the junction.

In *Flabelligera affinis* there are 6 pairs of gonads, instead of 2 as in Trophonia. They have the same relations as in the latter genus; each is supplied by a central vessel from the ventral longitudinal trunk, and the longer posterior gonads are bound up together with the loop of the intestine into a cylindrical mass, by means of a thin confining membrane. As in Trophonia, the ovaries are green, the testes white.

The same bending of the intestine occurs as in Trophonia; the blood in the sinus round the posteriorly directed limb of the loop passes backwards, and the successive waves can be seen passing backwards in the living animal, through the transparent epidermis. The heart has similar structure and relations to those described in Trophonia. CLAPARÈDE mentions the tubular excretory glands at the side of the pharynx, without recognising their homology with nephridia. They have the same structure as those of Trophonia (Pl. XLVI. fig. 40).

Fam. MALDANIDÆ.

Genus *Nicomache.*

This genus was first defined by MALMGREN, in his *Nordiska Hafs-Annulater,* as follows:—Body subcylindrical, posteriorly somewhat attenuated, of 26 segments, 22 of which are setigerous, while the two very short præanal somites are naked. Cephalic lobe coalesced with the naked buccal somite, oval, convex, inclined, and without a projecting border. *Superior setæ* capillary; some thicker, smooth, bordered, with the apex long and attenuated; others thinner, shorter, not bordered, smooth inferiorly, superiorly beset with very minute spines, adpressed in two series. *Inferior setæ:* in the three anterior setigerous somites, there is only one kind of seta, conical in form; in the rest, many minute uncini in a single series, with a tridentate beak at the vertex, with a fascicle of hairs bent over it beneath the beak. Anal segment infundubiliform, the margin surrounded by short cirri. Anus terminal in the bottom of the funnel.

Nicomache lumbricalis, Malmgren, Fabr.

Sabella lumbricalis, Fabricius, Fauna Grönl., p. 374, n. 369.

Clymene lumbricalis, Sars, Fauna littor. Norvegiæ, ii. p. 16, Tab. ii. f. 23–26; Christiania Vid. Selsk. Forh., 1861, p. 92.

Clymene borealis, Johnston, Dalyell, Pow. Creat., ii. 255, pl. xxxv. f. 5.

Commonly found among Laminarian roots, also occasionally under stones.

Specific Characters the same as those of the genus. Colour pinkish. The dorsal surface of the first few somites is abundantly spotted with red and white (Pl. XLVII. fig. 41).

Genus *Axiothea,* Malmgren.

First defined by MALMGREN, in his *Nordiska Hafs-Annulater,* as follows:—Body subcylindrical, of 24 somites, 18 of which are setigerous, and the 4 præanal naked. Cephalic lobe coalesced with the naked first somite, inclined with a projecting border, and anteriorly projecting into a short process. Superior setæ capillary; some longer, smooth and bordered; others shorter and thinner, finely pinnate on each side towards the tip. Inferior setæ: a single row of uncini numerous in all the setigerous somites, but in the three anterior fewer than in those following; vertex of the rostrum 4-dentate, with sometimes a 5th tooth, very small, disappearing or obsolete; a fascicle of hairs arising beneath the rostrum and bent over it. Last somite infundibuliform, with the margin ciliated. Anus terminal in the bottom of the funnel.

Axiothea catenata, Malmgren.

Got in great numbers inhabiting fine tubes buried in the sand, with only their upper ends protruding. The tubes often have a branch in the lower part of their course, and extend down to a depth of 6 or 8 inches.' The upper end is quite plain and open. The worms lie in their tubes with either their head or their tail uppermost indifferently, so that they can evidently turn in them. The locality whence our specimens were got was the flat sands for two or three hundred yards to the west of the Birnie Rocks, where the upper ends of the tubes form a sort of miniature forest all over the surface. Length 3 or 4 inches when fully extended, but when contracted, it is much less. Colour pinkish, paler towards the anterior end, with broad bands of deep red surrounding the body at intervals (Pl. XLVII. fig. 42).

Fam. CAPITELLIDÆ (HALELMINTHIDÆ, Mgn.).

Genus *Capitella*, Blainville.

Capitella Fabricii, Blan., Dict. des Sci. Nat., tome lvii., 1828.

Body very extensile, somites long and numerous: no branchiæ, parapodia rudimentary, represented by slight dorsal and ventral tubercles on the middle and posterior region of the body. Setæ of two kinds, subulate and uncinate, also genital setæ: subulate setæ confined to the anterior 6 or 8 somites, uncini to the rest. Male genital opening single, dorsal, between 8th and 9th somites; female, between 7th and 8th. Ocelli, 2 or more small lateral on præoral lobe. Two pits immediately behind eyes from which two ciliated knobbed processes can be protruded. Head conical and pointed.

Capitella capitata, Van Beneden, (Fabricius).

Lumbricus capitalus, Fabricius, Fn. Grönl., p. 279.

Valla ciliata, Johnston, Cat. Brit. Mus.

Capitella capitala, Van Beneden, Bullet Acad. Roy. Belgique, 2 ser., iii., 1857; Malmgren, Annulata Polychæt.; M'Intosh, Fauna of St Andrews.

Habits, &c.—Very common under stones in the littoral zone. The two ciliated retractile processes not seen unless the animal is watched with a low power for some time in the living condition.

Specific Characters.—Two minute eye-specks, just in front of the pits, whence the ciliated processes proceed.

As was pointed out by CLAPARÈDE (*Annélides Chéteopodes du Golfe de Naples*), the male genital aperture is single, situated between the 8th and 9th somites, and the notopodial setæ of these somites are modified for copulation. These setæ are long, thick, and bluntly pointed; there are 4 fascicles of them,—the 4 notopodial fascicles of the two segments, which have been shifted towards one another and towards the median dorsal line. In one male specimen, we investigated by examination, under the microscope, after compression, the distribution of the two kinds of setæ, with the following result:—In the neuropodia of the 8th and 9th somites, sheathed uncini only present. In the 6th somite, the notopodium has one uncinus on the dorsal side, the other setæ being all subulate; in the neuropodium all the setæ are subulate. In the first 5 somites there are only subulate setæ. Behind the 9th somite, sheathed uncini only present in both neuropodia and notopodia.

There seems to be still some doubt as to the process of copulation and reproduction in Capitella. VAN BENEDEN has regarded a pouch in connection with the genital opening in the male as testicle, but CLAPARÈDE doubts this interpretation. According to CLAPARÈDE, there are a pair of ovaries in the females in each segment, except a few of the most anterior. It is probable that the semen is somehow retained by the female near the genital opening, and the ova fertilised as they are expelled.

There are several segmental organs in each of the segments behind the genital opening, except the most anterior. These have been described by EISIG in "Die segmental organe der Capitelliden," *Mitth. Zool. Stat. Neapel.*, Bd. i., 1879. He says that, previously to his paper, of the generative apparatus of *Capitella capitata* only the greifhaken of the ♂, discovered by V. BENEDEN, and the pores of the female discovered by CLAPARÈDE, were known (V. BENEDEN, *Bull. Acad. de Belg.*, 1857, iii., Nos. 9 and 10; CLAPARÈDE, *Annélides Chétopodes du Golfe de Naples*, p. 274).

CLAPARÈDE, *loc. cit.*, says it is easy to find the sexual pores of the females, which are in the form of transverse clefts, on the ventral surface between the 7th and 8th somites, a little internal to the line of the "external" fascicles of bristles.

CLAPARÈDE was apparently aware that the copulatory setæ were dorsal. He speaks of these setæ as the *internal* fascicles of bristles transformed; and in speaking of the *ventral* female pores, he says they are internal to the external bristles. EISIG found that the pair of ventral pores described by CLAPARÈDE occurred in both sexes, and were in both the apertures of internally ciliated tubes; these tubes, if homologous with segmental organs, belong to the 8th somite; the 7th and 9th somites have nephridia in the larva; the 8th, none except the tubes referred to. Eisig found these tubes full of spermatozoa in both sexes, and believes that in the ♂ they act as penis, in the female as vulva plus receptaculum seminis.

As can easily be seen in a specimen of 8 mm. length, mounted whole in Canada balsam, the ganglion of the 9th setigerous somite is continuous ventrally with the epidermis; dorsally it receives the anterior nerve cord, and in front of this point both cord and ganglia are internal to the longitudinal layer of muscles; posteriorly both cord and ganglia are external to the circular layer of muscles, and in contact with the epidermis.

There are no pseudhæmal vessels; the body-cavity is crowded with red nucleated corpuscles, which are present in great numbers (Pl. XLVII. fig. 43).

Notomastus latericius (Sars).

Notomastus latericius, Sars, Fauna littoralis Norvegiæ, pl. ii., 1856.

Capitella rubicunda, Keferstein, Z. f. w. Z., Bd. xii.; Claparède, Beob. über. Anat. etc. an den Küste von Normandie, Leipzig, 1863.

Notomastus latericius, Malmgren, Ann. Polychæta; Of. Kongl. Vet. Akad. Förhandlingar, 1867.

The genus Notomastus was established by M. Sars, with the following diagnosis:—Anterior part of the body composed of 12 somites, each divided by a transverse constriction into 2 rings: 1st somite without, the the other 11 with, on each side, two fascicles of capillary setæ; no parapodial processes. Posterior part of the body longer and thinner; on each side two slight parapodial tubercles transversely elongated, and bearing uncini in a transverse series.

It is extremely characteristic of this genus, that, as described by Sars in his account of the animal, in the first 8 to 13 segments of the posterior or uncinate region, the two notopodial ridges coalesce into a single prominence in the dorsal median line.

Grube placed his genus Dasybranchus in the family Telethusidæ, with Arenicola. Sars thought Notomastus also belonged to the same family. But *Notomastus latericius* has no pseudhæmal vessels, and has numerous red corpuscles in the cœlomic fluid, in which respects its affinity to Capitella is clearly shown. There can be little doubt that the *Capitella rubicunda* of Keferstein and Claparède is identical with the *N. latericius* of Sars, and the discovery of the corpusculated cœlomic fluid in Notomastus belongs therefore to Claparède.

We refrain from giving a specific diagnosis, because the species of the genera have not been sufficiently compared. The colour is red.

Habitat.—Several specimens were dredged in November 1886, in 6 to 12 fathoms, in mud N.W. and W. of Inchkeith. They were found by sifting the mud, and were always broken; no complete specimen being obtained. SARS found his specimens on the N.W. shores of Norway, some in the Laminarian zone, some at 50 to 60 fathoms, some at 20 to 30 fathoms. CLAPARÈDE found it in the littoral zone on the coast of Normandy.

Anatomy.—The ventral nerve cord lies internal to both circular and longitudinal layers of muscles, throughout the body: the cord is not in immediate contact with any other organs, but is suspended in the cœlom by strands of muscular and connective tissue, which pass from the sides of the sheath surrounding the cord into certain strands connected with the circular muscles. There are no differentiated ganglia; the cord is of uniform width throughout its length, except where slight constrictions mark the boundary between contiguous somites. The sheath surrounding the cord is fibrous, and contains small muclei. A neural canal runs along the dorsal median line of the cord. On the dorsal side there are no neural cells, but these form a continuous layer at the sides and ventrally, the layer being thickest laterally. The internal core of the cord is composed of minute fibrils.

The intestine is very narrow, both in the anterior and posterior regions of the body, but especially in the latter.

The arrangement of the muscles is noticeable. The longitudinal layer is very strongly developed (Pl. XLVII. fig. 44).

DESCRIPTION OF PLATES.

PLATE XXXVI.

Fig. 1. *Nerine coniocephala*, anterior end. 1 A, parapodium from anterior region, with branchia and produced dorsal lamina; 1 B, parapodium from middle region, with branchial cirrus; 1 C, uncinus; 1 D, ripe unfertilised ovum; 1 E, ovum, with four segments.

Fig. 2. *Nerine cirratulus*, anterior end with proboscis everted. 2 A, parapodium and branchial cirrus from anterior region; 2 B, parapodium and branchia from middle region; 2 C, uncinus; 2 D, anterior end, with proboscis retracted; 2 E, posterior end, dorsal surface; 2 F, longitudinal vertical section, showing nephridia and ovaries; 2 G, larva of Nerine, with 2 somites provided with provisional chætæ.

PLATE XXXVII.

Fig. 2 H. Trochosphere of Nerine, dorsal surface, from life. E, Zeiss, Oc. 2, Camera, Feb. 21, 1887; 2 I, advanced larva of Nerine, dorsal surface; 2 J, same, ventral surface.

Fig. 3. *Scolecolepis vulgaris*, anterior end. 3 A, posterior end; 3 B, parapodium from anterior region, and branchia; 3 C, parapodium and branchia from middle region.

Fig. 4. *Spio seticornis*, anterior end. 4 A, posterior end; 4 B, uncinus.

Fig. 5. *Leucodore ciliatus*, anterior end. 5 A, posterior end; 5 B, modified dorsal chætæ of 5th chætiferous somite; 5 C, uncinus.

PLATE XXXVIII.

Fig. 6. *Magelona papillicornis*, anterior end, dorsal surface. 6 A, same, lateral surface; 6 B, posterior end, dorsal surface; 6 C, parapodium; 6 D, uncinus.

Fig. 7. *Scoloplos armiger*, anterior end, dorsal surface. 7 A, same, lateral surface; 7 B, posterior end, dorsal surface; 7 C, transverse section, middle region; 7 D, 7 E, 7 F, 7 G, chætæ.

Fig. 8. *Theodisca mammillata*, anterior end, lateral surface. For remaining figs. of this species see Pl. XL.

Fig. 9. *Cirratulus cirratus*, anterior end, dorsal surface. 9 A, same, ventral surface; 9 B, same, lateral surface.

Fig. 10. *Cirratulus tentaculatus*, anterior end, dorsal surface. 10 A, same, lateral surface.

PLATE XXXIX.

Fig. 9 C. Branchia of *C. cirratus*, optical longitudinal section, fresh—*bl*, *bl*, the two main blood-vessels; *tr*, *tr*, transverse connecting small vessels. 9 D, tranverse section of branchia—*bl*, *bl*, as before; *n*, *n*, nerves; *c*, *c*, cœlomic cavity. 9 E, tentacle, optical longitudinal section, fresh—*bl*, single blood-vessel; *g*, longitudinal groove. 9 F, transverse section of tentacle—*bl*, blood-vessel; *c*, *c*, cœlomic cavity; *g*, longitudinal groove; *n*, *n*, nerves. 9 G, anterior end showing anterior pair of nephridia; 9 H, nephridium.

Fig. 10 B. Chætæ of *Cirratulus tentaculatus*.

Fig. 11. *Chætozone setosa*.

Fig. 12. *Dodecaceria concharum*, anterior end.

Fig. 13. *Arenicola marina*. 13 B, 13 C, chæta and uncinus.

PLATE XL.

Fig 13 D. Portion of immature ovary of *Arenicola marina*, fresh, Zeiss DD, Oc. 2; 13 E, immature ova from body-cavity, February, fresh, Zeiss DD, Oc. 2; 13 F, sperm polyplast from body-cavity, same time, same power, fresh.

Fig. 14 A. Embryo probably of *Scoloplos armiger*, early stage, taken from pear-shaped gelatinous cocoon, Feb. 3, 1887, Zeiss CC, Oc. 3; 14 B, later stage, Feb. 4th; 14 C, same stage as previous figure, optical section; 14 D, more advanced, some days after hatching; 14 E, later stage, chætæ commencing to appear; 14 F, latest stage observed.

Fig. 8 A. *Theodisca mammillata*, anterior end, dorsal surface. 8 B, posterior end of same species; 8 C, capillary chæta; 8 D, 8 E, uncini of same.

PLATE XLI.

Fig. 15. *Ammotrypane aulogastra*, anterior end, dorsal surface. 15 A, anterior end, lateral surface—*a*, soft proboscis; *b*, cephalic organ. 15 B, posterior end, lateral surface; 15 C, parapodium and branchia as seen in a transverse section.

Fig. 16. *Ophelia limacina*, entire specimen, lateral surface.

Fig. 17. *Sabellaria spinulosa*, entire specimen, lateral surface. 17 A, anterior end, ventral surface, highly magnified; 17 B, anterior end, lateral surface; 17 C, paleæ of the operculum; 17 D, chæta of neuropodium, and uncinus of the notopodium.

PLATE XLII.

Fig. 15 D. Opalina from the intestine of *Ammotrypane aulogastra*.

Fig. 18. *Eumenia crassa*, lateral surface. 18 A, anterior parapodium, with branchia; 18 B, posterior parapodium of same.

Fig. 19. *Lepobranchus Jeffreysi*, lateral view.

Fig. 20. *Pectinaria Belgica*, lateral surface. 20 A, anterior end, dorsal surface; 20 B, anterior end, ventral surface; 20 C, capillary chæta; 20 D, uncinus. 20 E, specimen dissected from ventral surface—*w. l*, white gland; *ne*, nephridia.

Fig. 21. *Ampharete gracilis*, lateral view. 21 A, head, anterior surface; 21 B, capillary chæta; 21 C, uncinus.

Fig. 22. *Melinna cristata*, lateral view. 22 A, anterior end, dorsal surface, magnified; 22 B, anterior end, ventral surface; 22 C, dorsal hook, magnified; 22 D, capillary chæta; 22 E, uncinus; 22 F, chæta.

PLATE XLIII.

Fig. 23. *Amphitrite Johnstoni*, lateral view. 23 A, capillary chæta; 23 B, uncinus. 23 C, specimen opened from dorsal surface—*ht*, heart; *ne*, nephridia; *int*, intestine. 23 D, nephridium separated, magnified.

Fig. 24. *Amphitrite cirrata,* lateral view. 24 A, capillary chæta; 24 B, uncinus.
Fig. 25. *Terebella Danielsseni.* 25 A, capillary chæta; 25 B, uncinus.
Fig. 26. *Lanice conchilega.* 26 A, uncinus.

PLATE XLIV.

Fig. 27. *Scione maculata.* 27 A, posterior end, magnified; 27 B, chæta; 27 C, uncinus.
Fig. 28. *Ereutho Smitti,* ventral surface. 28 A, chæta; 28 B, uncinus.
Fig. 29. *Terebellides Strœmi.* 29 A, chætæ; 29 B, uncinus.
Fig. 30. Larva of a species of the Terebellidæ.
Fig. 31. *Sabella penicillus.* 31 A, ventral view of anterior end; 31 B, thoracic chæta; 31 C, abdominal chæta; 31 D, uncini.
Fig. 32. *Chone infundibuliformis.* 32 A, dorsal surface, anterior end.

PLATE XLV.

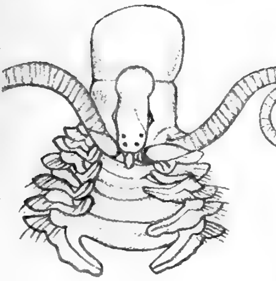
Fig. 32 B, 32 C, thoracic chætæ. 32 D, thoracic uncinus; 32 E, abdominal seta; 32 F, abdominal uncinus.
Fig. 33. *Amphicora Fabricia.* 33 A, chæta and uncinus.
Fig. 34. *Myxicola Steenstrupi.* 34 A, chæta and uncinus.
Fig. 35. *Filigrana implexa,* ventral surface. 35 A, larva.
Fig. 36. *Pomatocerus triqueter.* 36 A, abdominal chæta; 36 B, thoracic chæta; 36 C, uncinus.
Fig. 37. *Spirorbis borealis.* 37 B, uncinus.
Fig. 38. *Spirorbis lucidus.* 38 A, chæta of 1st thoracic somite; 38 B, chæta of 2nd thoracic somite; 38 C, chæta of abdomen.

PLATE XLVI.

Fig. 37 C, Part of a string of ova of *Spirorbis borealis,* from the shell; June 15, 1887; Zeiss, CC. Oc. 2.
Fig. 39. *Trophonia plumosa,* anterior end, with the buccal somite fully protruded, as it often is when the animal is killed with spirit—*a,* branchiæ; *b,* tentacles; *c,* mouth. 38 A, dissection of *Trophonia plumosa*—*a,* inverted buccal somite; *b,* retractor muscle; *d,* nephridia; *e, e,* ovaries; *f,* heart; *g,* dorsal pseudhæmal vessel; *h,* ventral psuedhæmal vessel; *i,* intestine. 38 B, from the epithelium of the nephridium, Zeiss, CC. Oc. 3.
Fig. 40. *Flabelligera affinis.* *a,* branchiæ; *b,* tentacles; *c, c,* nephridia; *d,* heart; *g,* intestine; *h,* setæ of notopodium; *i,* uncini of notopodium; *j,* glandular papillæ. 40 A, glandular papillæ, × 400; 40 B, uncinus of notopodium.

PLATE XLVII.

Fig. 41. *Nicomache lumbricalis,* anterior end, lateral view. 41 A, posterior end, lateral view; 41 B, uncinus.
Fig. 42. *Axiothea catenata,* anterior end, lateral view, proboscis protruded. 42 A, anterior end, ventral surface; 42 B, anterior end, dorsal surface; 42 C, posterior end; 42 D, chæta; 42 E, chæta; 42 F, uncinus.
Fig. 43. *Capitella capitata,* anterior end. 43 A, chæta; 43 B, uncinus. 43 C, copulatory armature—*a,* pouch. 43 D, corpuscles of body-cavity, after treatment with acetic acid, Zeiss E, Oc. 3.
Fig. 44. *Notomastus latericius,* anterior end, dorsal surface, proboscis everted. 44 A, same, lateral surface; 44 B, uncinus.



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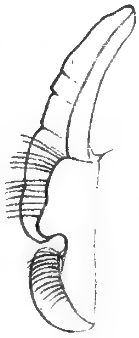

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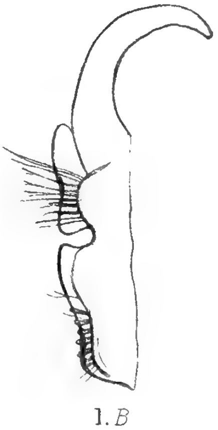

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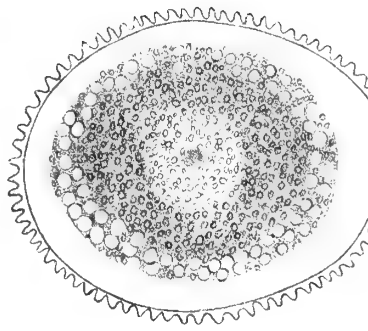

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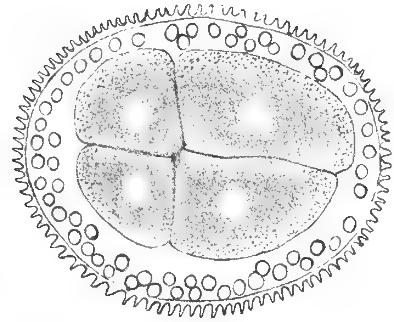

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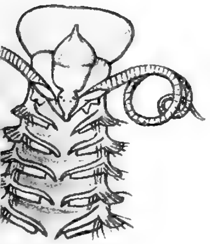

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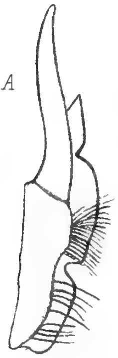

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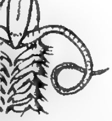

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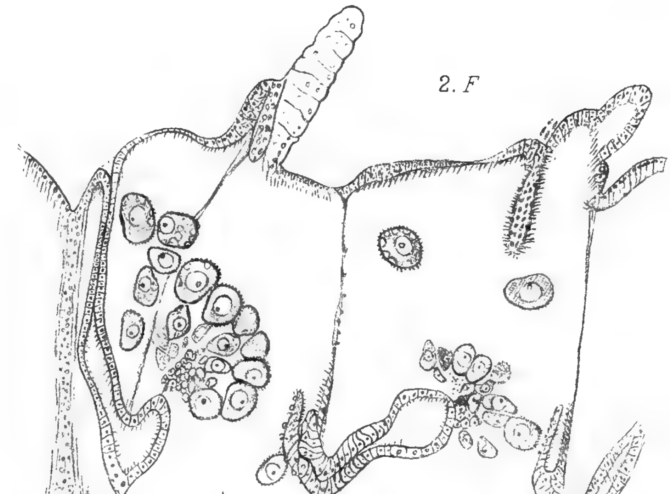

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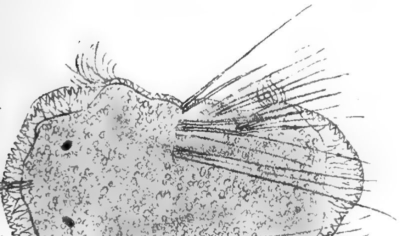

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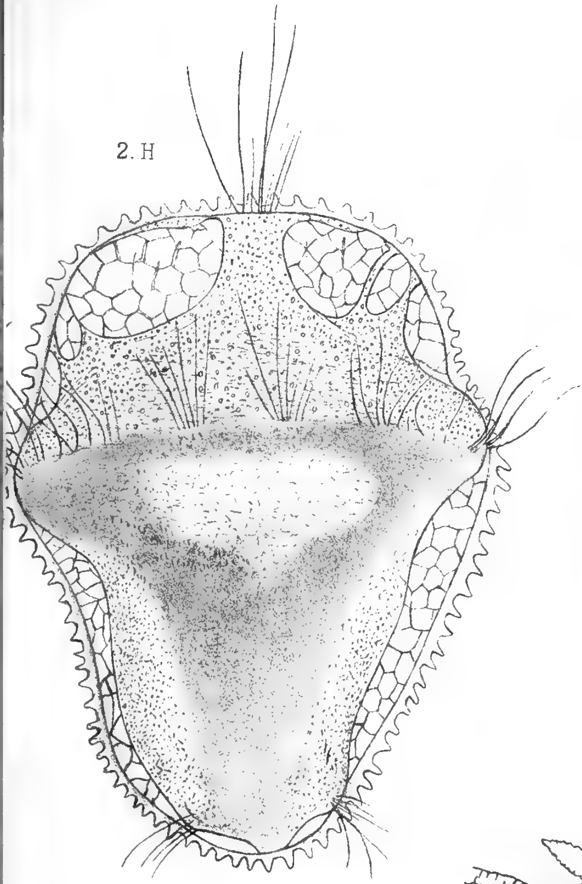
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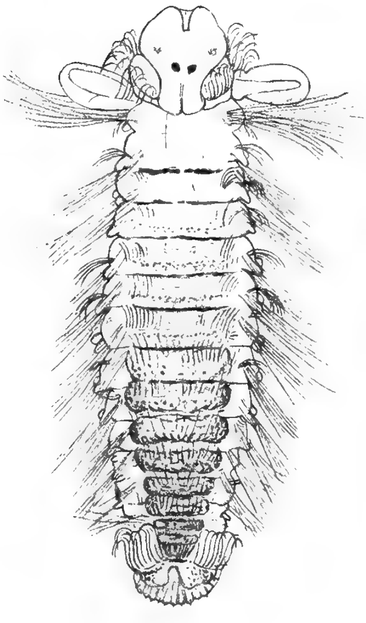
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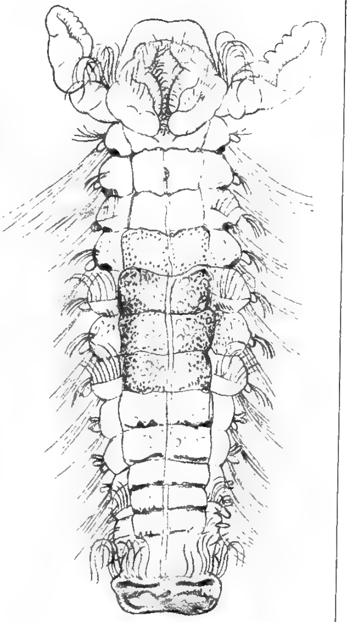
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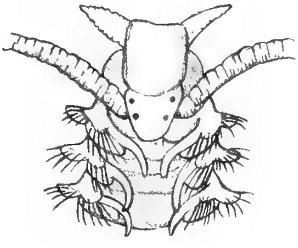
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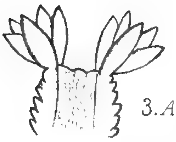
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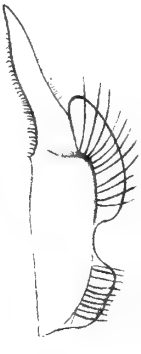
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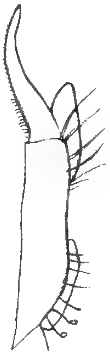
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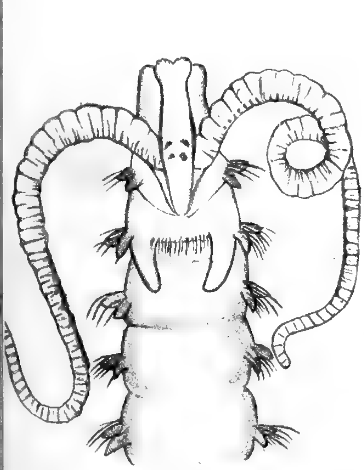
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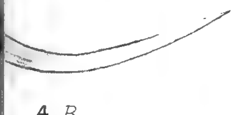
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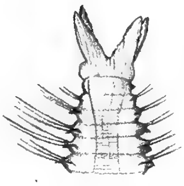
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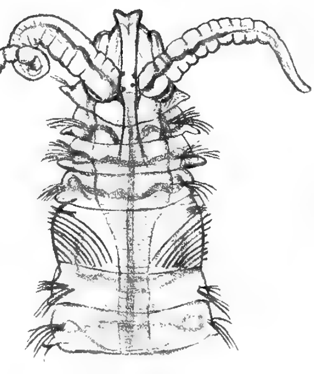
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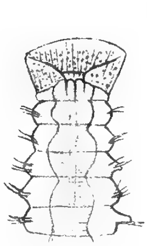
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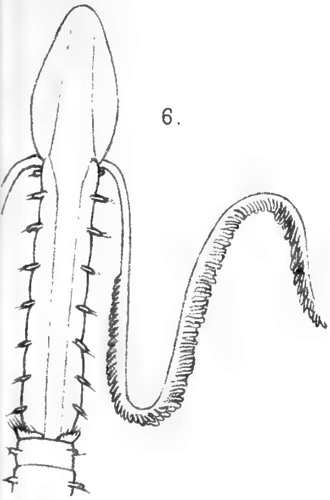
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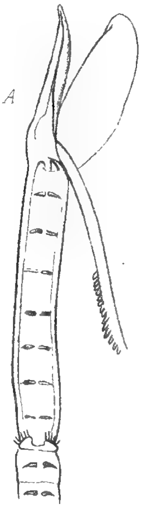
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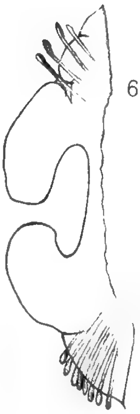
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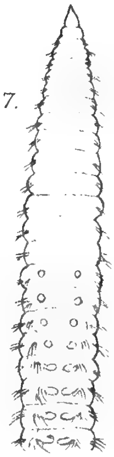
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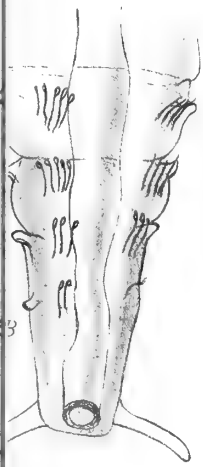
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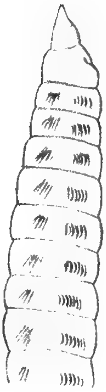
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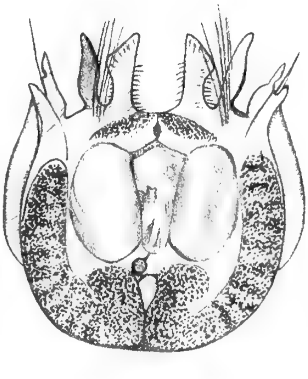
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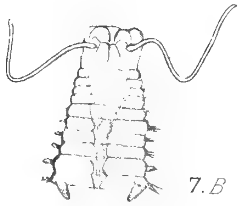
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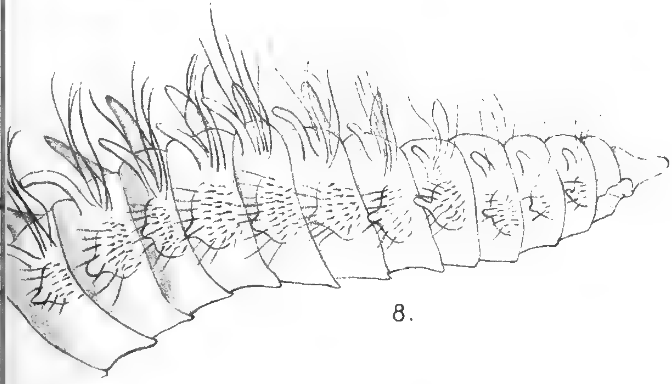
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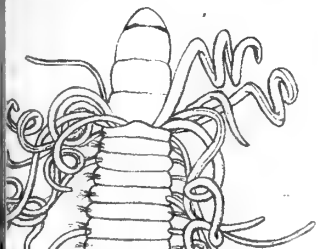
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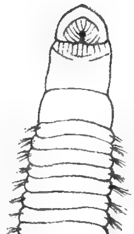
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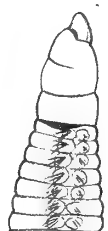
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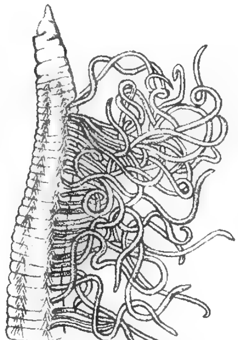
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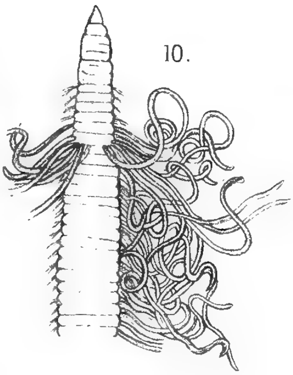
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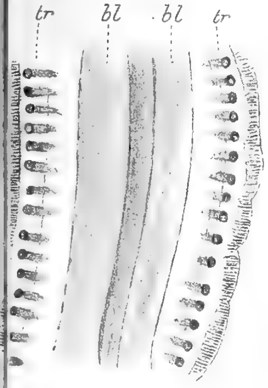

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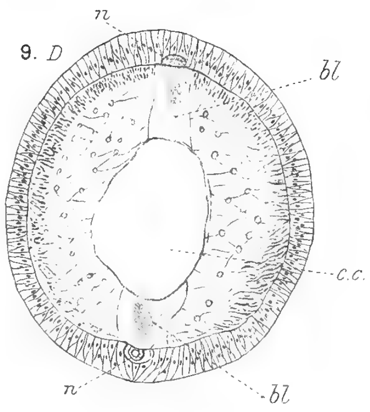

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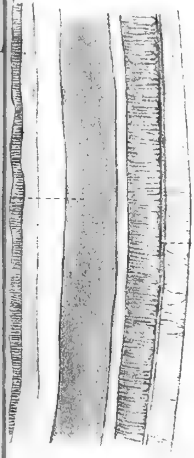

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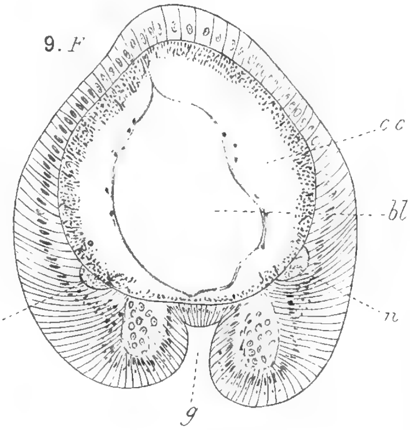

13. B.

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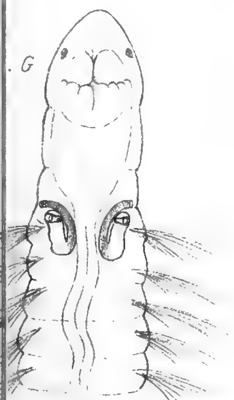

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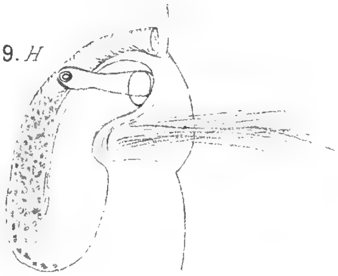
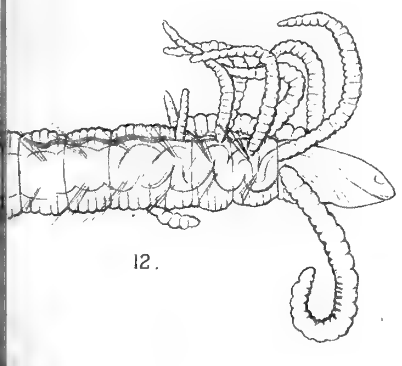


13. C.



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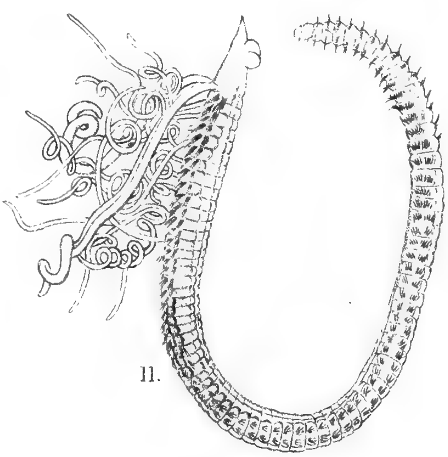
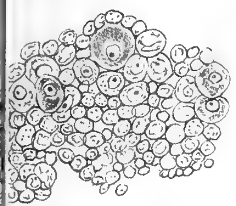
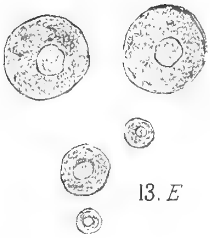


11.

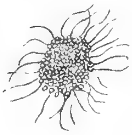
McFarlane & Erskine, Lithrs. Edinr.



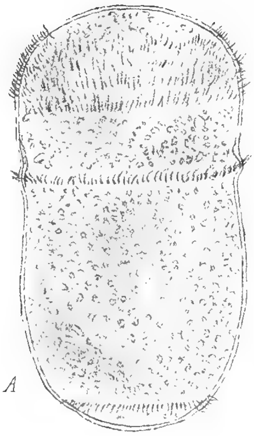
13. *D*



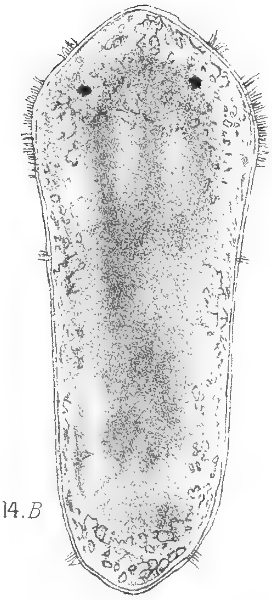
13. *E*



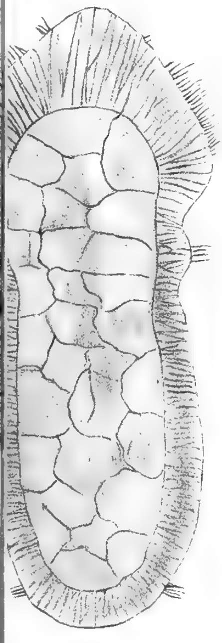
13. *F*



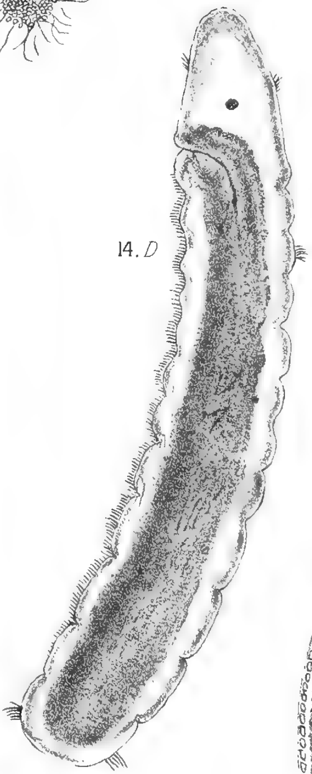
14. *A*



14. *B*

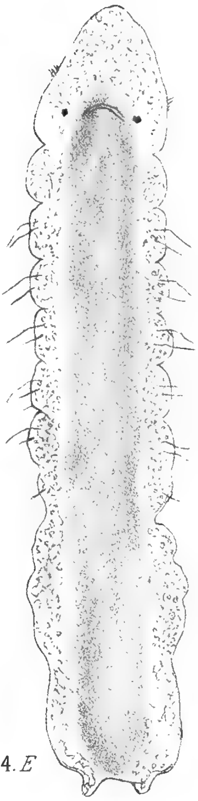


14. *C*

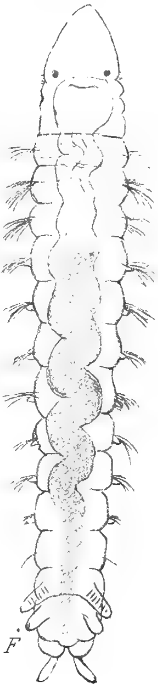


14. *D*

8. *C*



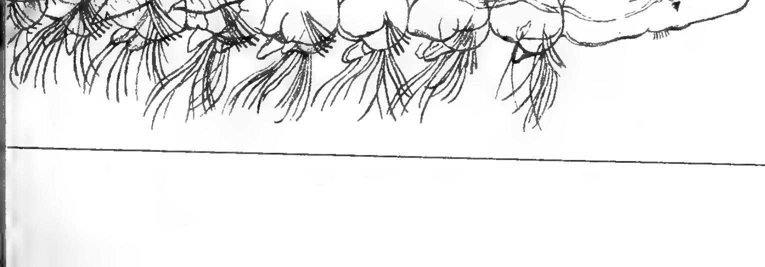
14. *E*



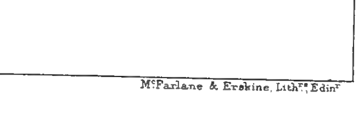
14. *F*

8. *D*

8. *E*



8. *A*



8. *B*

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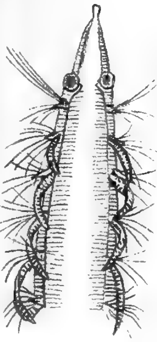

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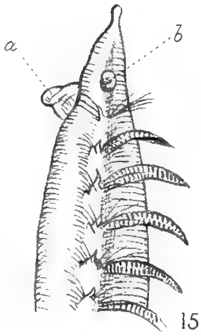
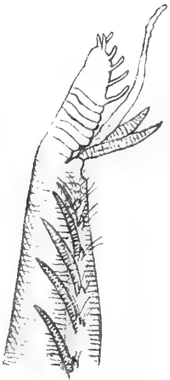
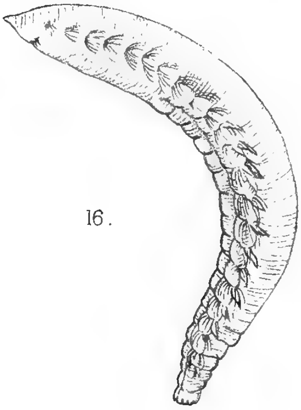


15. A



15. B



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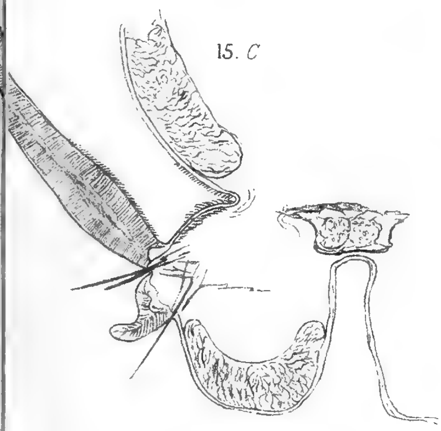
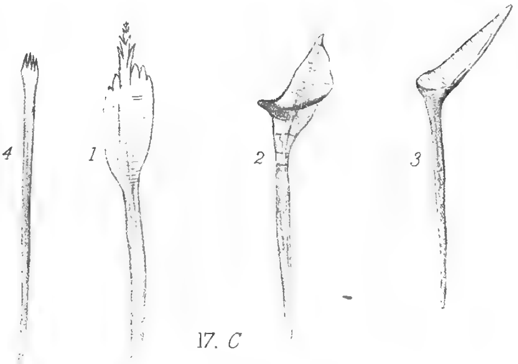
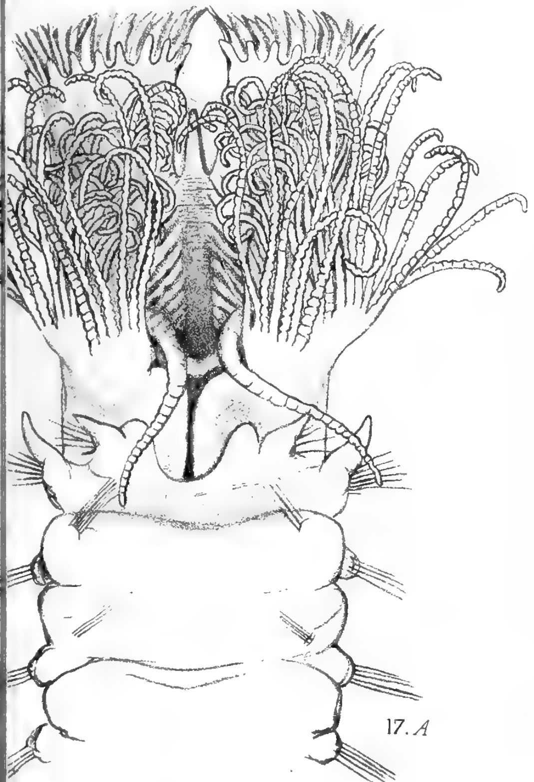


15. C

17.

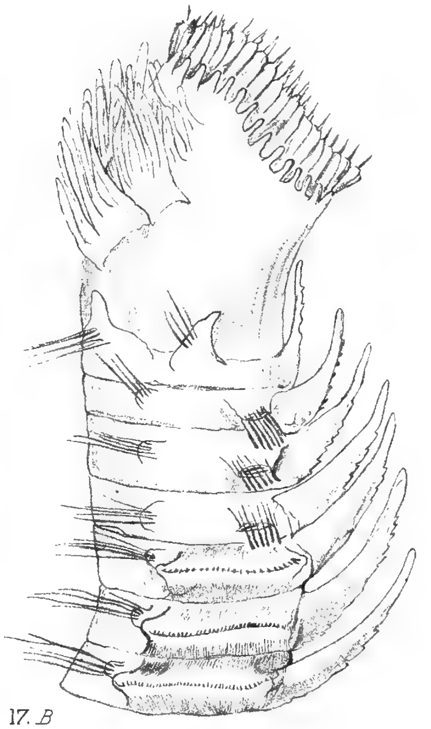


17. C



17. A

17. D



17. B

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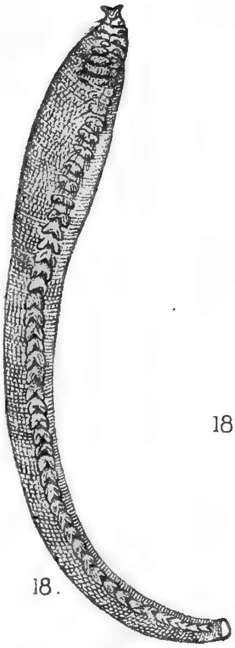
15. *D*



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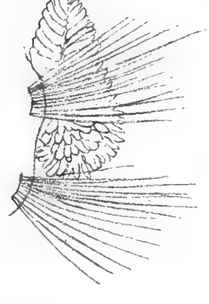
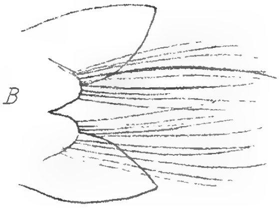
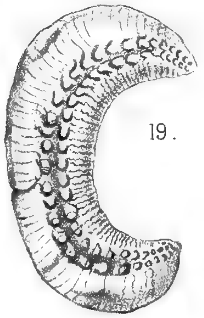


18. *A*



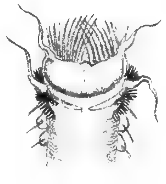
18. *B*



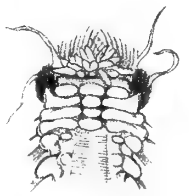
19.

20.

20. *C*

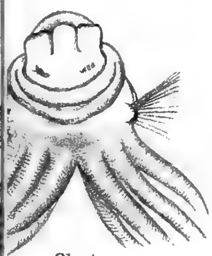


20. *A*

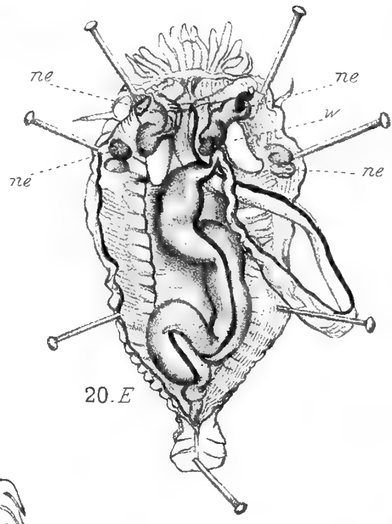


20. *B*

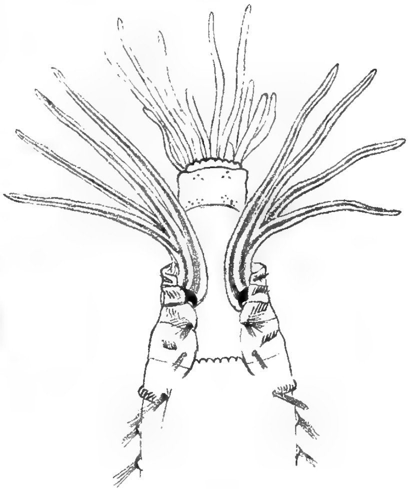
20. *D*



21. *A*

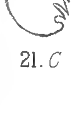


20. *E*



22. *A*

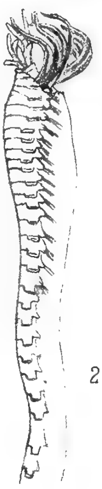
21. *B*



21. *C*

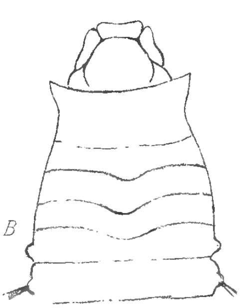
22. *C*

22. *D*



22.

22. *F*



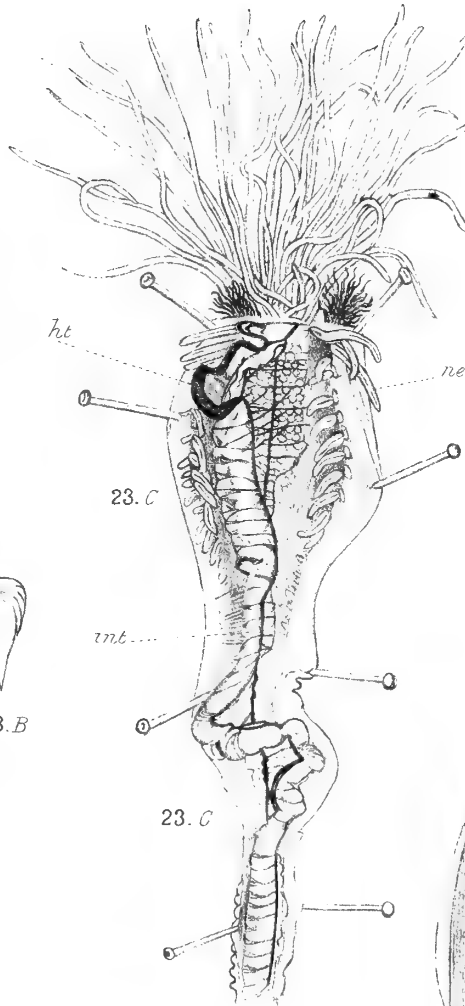
22. *B*

22. *E*

23.

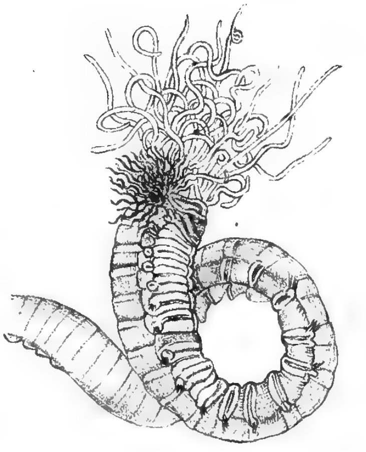
23. A

23. B



23. C

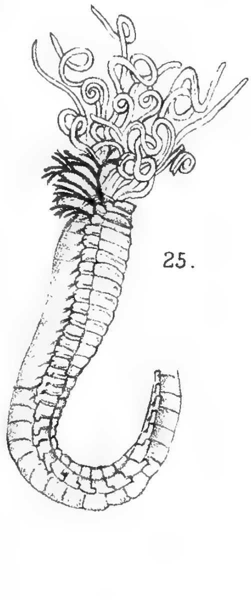
23. C



24.

24. A

24. B



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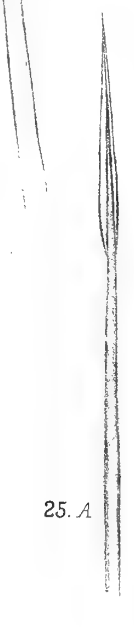
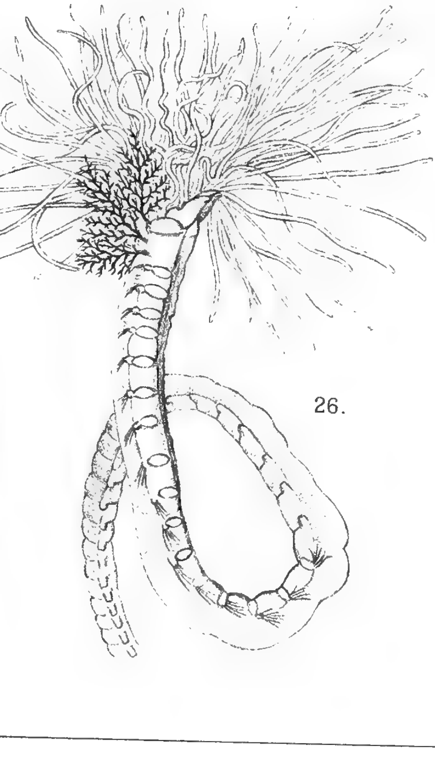


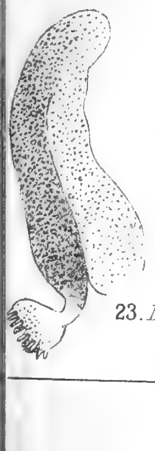
25. A

25. B



26.

26. A



23. D

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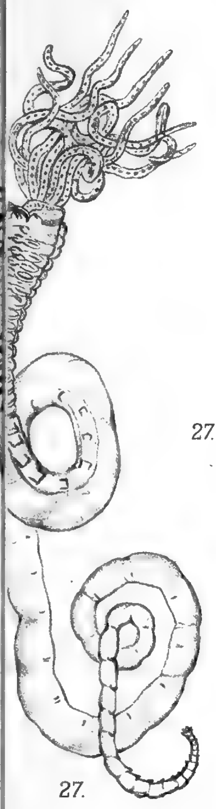
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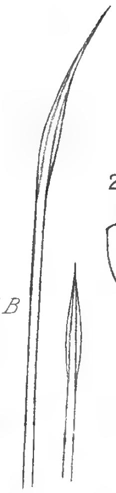
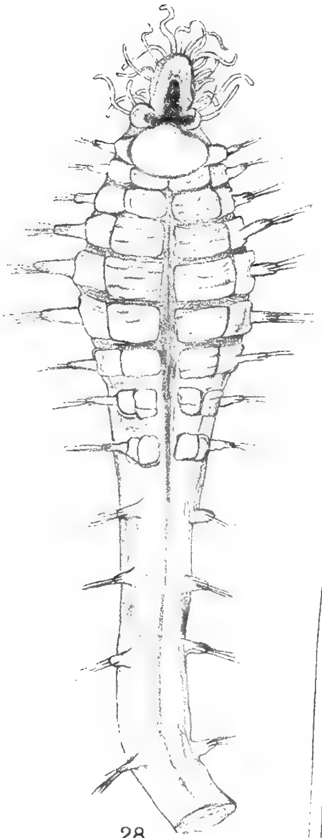
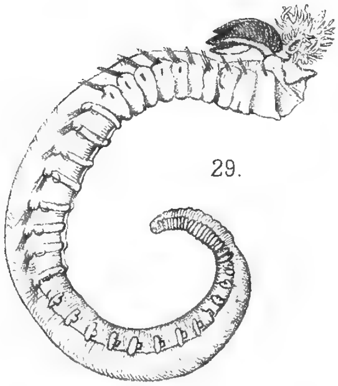

27.B


27.C

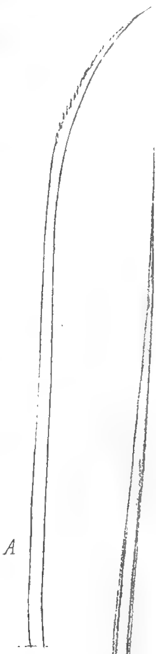

27.A


28.

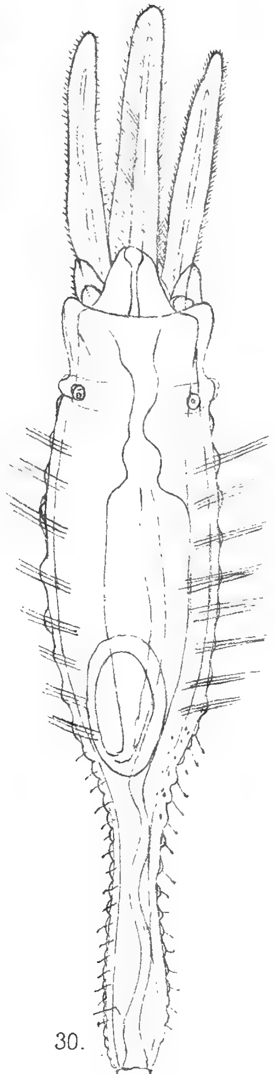

28.B


29.


29.B


28.A


29.A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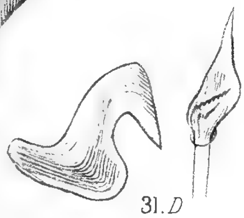
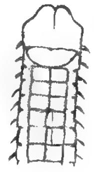
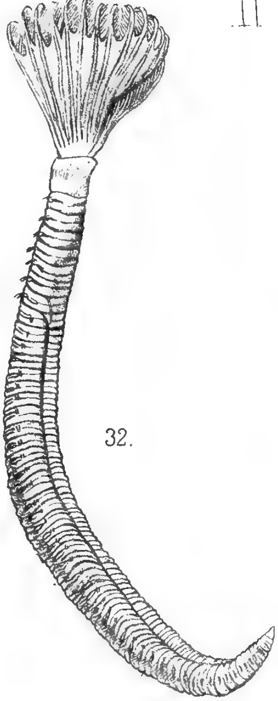
1.


31.D


31.A


31.B


31.C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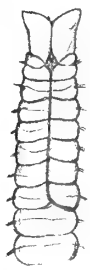

32.A

McFarlane & Erskine, Lithrs Edinr.

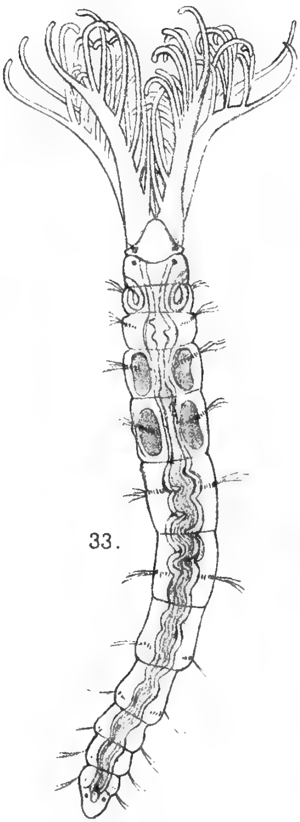

B


32. C

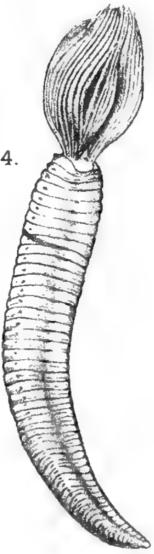

32. D


32. E


32. F


33.


33. A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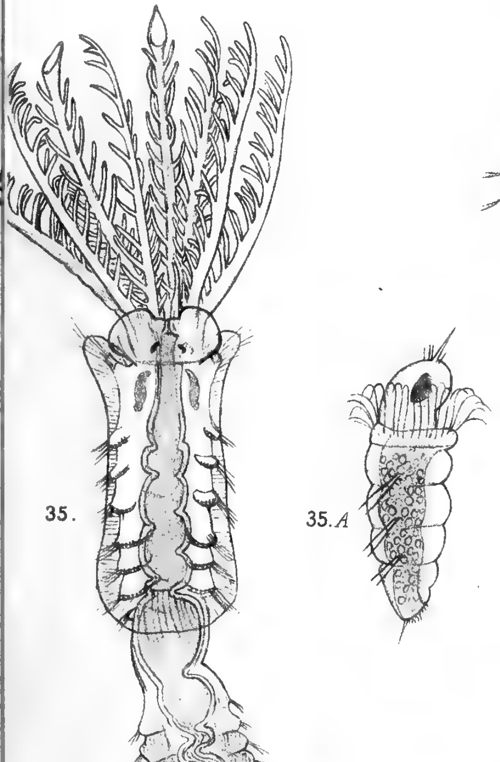
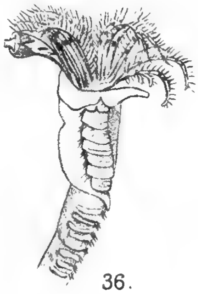
34. A

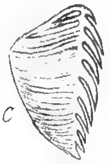

35.

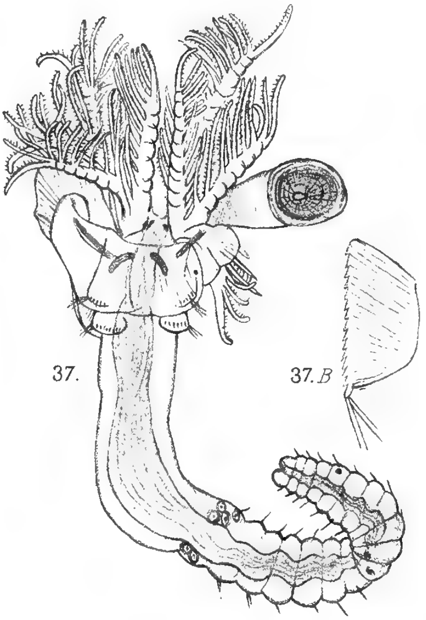
35. A


36.


36. A


36. B


36. C


37.

37. B


38.


38. A


38. B


38. C

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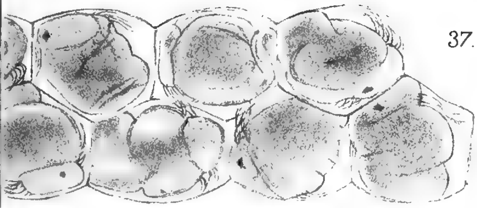
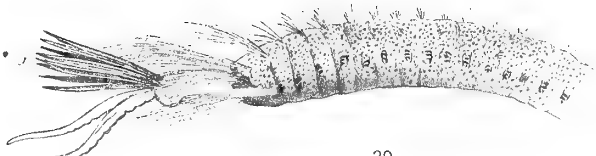
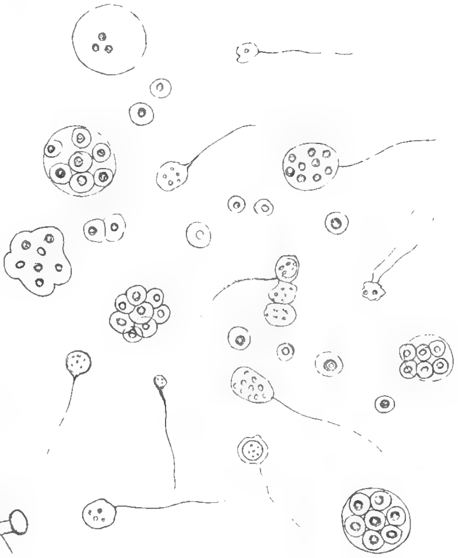


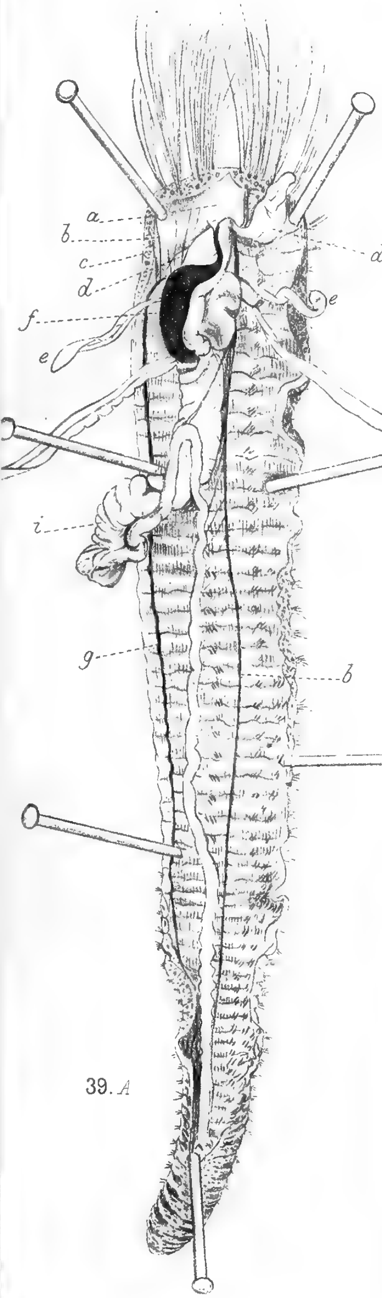
37. *C*



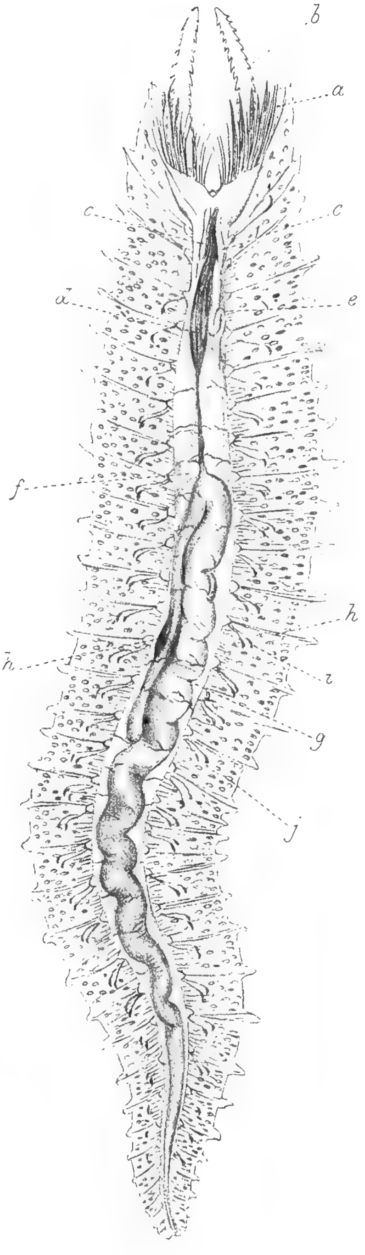
39.



39. *B*



39. *A*



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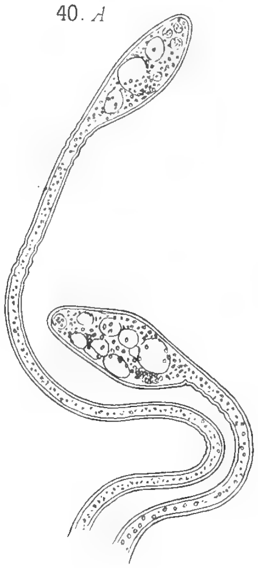
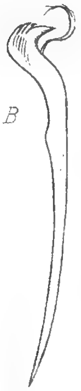


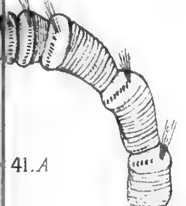
40. *A*

40. *B*

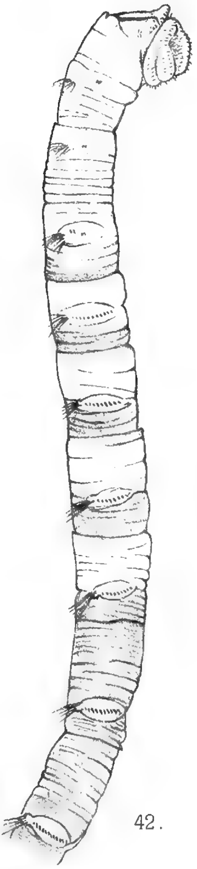
41.



41. *B*



41. *A*



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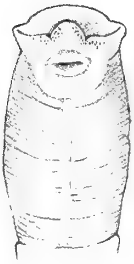
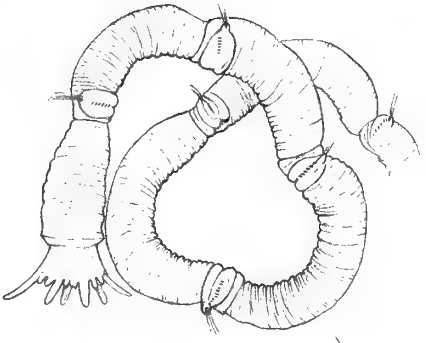
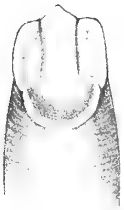


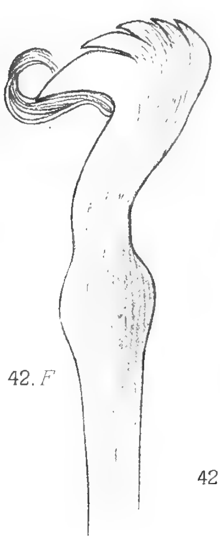
42. *A*



42. *C*



42. *B*

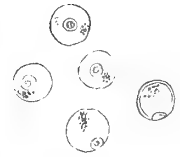


42. *F*

42. *D*

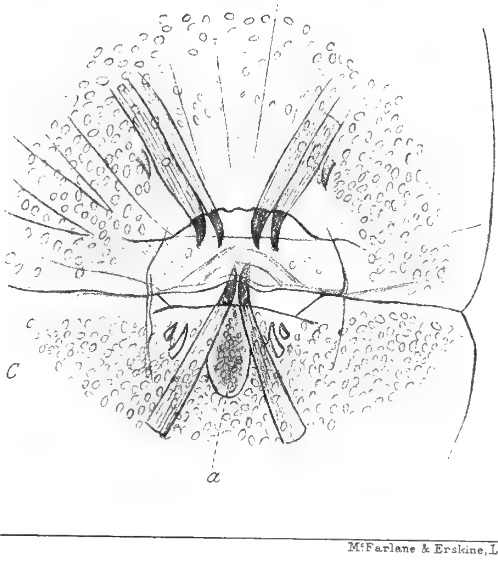
42. *E*

43. *A*



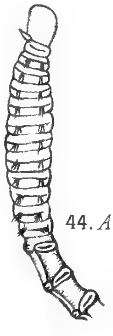
43. *D*

43. *B*

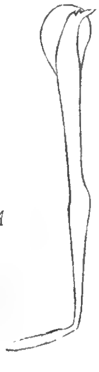


43. *C*

44.



44. *A*



44. *B*

M^{c}Farlane & Erskine, Lith^{rs} Edin^r

APPENDIX.

TRANSACTIONS

OF THE

ROYAL SOCIETY OF EDINBURGH.

CONTENTS.

LIST OF MEMBERS.

COUNCIL.

ALPHABETICAL LIST OF ORDINARY FELLOWS,

AND LIST OF HONORARY FELLOWS.

At January 1888.

THE COUNCIL
OF
THE ROYAL SOCIETY OF EDINBURGH,
NOVEMBER 1887.

PRESIDENT.

Sir WILLIAM THOMSON, LL.D., D.C.L., F.R.S., Foreign Associate of the Institute of France, Regius Professor of Natural Philosophy in the University of Glasgow.

HONORARY VICE-PRESIDENTS, HAVING FILLED THE OFFICE OF PRESIDENT.

His Grace the DUKE of ARGYLL, K.G., K.T., D.C.L. Oxon., F.R.S., F.G.S.
The Right Hon. Lord MONCREIFF, LL.D., Lord Justice-Clerk.

VICE-PRESIDENTS.

JOHN MURRAY, Ph.D., Director of the Challenger Expedition Commission.
D. MILNE HOME of Milne-Graden, LL.D.
Sir DOUGLAS MACLAGAN, M.D., President of the Royal College of Physicians, Edin., F.R.C.S.E., and Professor of Medical Jurisprudence in the University of Edinburgh.
The Hon. Lord MACLAREN, LL.D. Edin. and Glas., F.R.A.S., one of the Senators of the College of Justice.
The Rev. Professor FLINT, D.D., Corresponding Member of the Institute of France.
GEORGE CHRYSTAL, M.A., LL.D., Professor of Mathematics in the University of Edinburgh.

GENERAL SECRETARY.

P. GUTHRIE TAIT, M.A., Professor of Natural Philosophy in the University of Edinburgh.

SECRETARIES TO ORDINARY MEETINGS.

Sir WILLIAM TURNER, M.B., F.R.C.S.E., F.R.S., Professor of Anatomy in the University of Edinburgh.
ALEXANDER CRUM BROWN, M.D., D.Sc., F.R.C.P.E., F.R.S., Professor of Chemistry in the University of Edinburgh.

TREASURER.

ADAM GILLIES SMITH, Esq., C.A.

CURATOR OF LIBRARY AND MUSEUM.

ALEXANDER BUCHAN, Esq., M.A., LL.D., Secretary to the Scottish Meteorological Society.

COUNCILLORS.

S. H. BUTCHER, M.A., LL.D., Professor of Greek in the University of Edinburgh.
JOHN G. M‘KENDRICK, M.D., F.R.C.P.E., F.R.S., Professor of the Institutes of Medicine in the University of Glasgow.
THOMAS MUIR, M.A., LL.D., Mathematical Master in the High School of Glasgow.
WILLIAM CARMICHAEL M‘INTOSH, M.D., LL.D., F.R.S., F.L.S., Professor of Natural History in the University of St Andrews.
Sir ARTHUR MITCHELL, K.C.B., M.A., M.D., LL.D., Commissioner in Lunacy.
STAIR A. AGNEW, Esq., C.B., M.A., Advocate, Registrar-General.
A. FORBES IRVINE, Esq. of Drum, LL.D.
ROBERT M. FERGUSON, Esq., Ph.D.
J. BATTY TUKE, M.D., F.R.C.P.E.
FREDERICK O. BOWER, M.A., F.L.S., Regius Professor of Botany in the University of Glasgow.
GERMAN SIMS WOODHEAD, M.D., F.R.C.P.E.
ROBERT COX, Esq. of Gorgie, M.A.

ALPHABETICAL LIST

OF

THE ORDINARY FELLOWS OF THE SOCIETY,

CORRECTED TO JANUARY 1888.

N.B.—*Those marked * are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. ,, ,, ,, Keith Medal.
N. ,, ,, ,, Neill Medal.
V. J. ,, ,, ,, the Victoria Jubilee Prize.
P. ,, ,, ,, contributed one or more Papers to the TRANSACTIONS.

Date of Election.			
1879		Abernethy, Jas., Memb. Inst. C.E., Prince of Wales Terrace, Kensington	
1871		* Agnew, Stair A., C.B., M.A., Advocate, Registrar-General, 22 Buckingham Terrace	
1881		Aitchison, James Edward Tierney, C.I.E., M.D., F.R.S., F.L.S., Brigade-Surgeon, Secretary to the Surgeon-General, H.M.F. Bengal, and Naturalist with the Afghan Delimitation Commission, H.M. Bengal Army, North Bank, Simla, Punjab, India, 55 Parliament Street, London, S.W.	
1878		* Aitken, Andrew Peebles, M.A., Sc.D., F.I.C., 18 Dublin Street	
1875	K.P.	* Aitken, John, Darroch, Falkirk	5
1878		Allchin, W. H., M.B. (Lond.), F.R.C.P., Physician to the Westminster Hospital, 5 Chandos Street, Cavendish Square, London	
1856	B. P.	Allman, George J., M.D., F.R.S., M.R.I.A., F.L.S., Emeritus Professor of Natural History, University of Edinburgh, Ardmore, Parkstone, Dorset	
1886		* Anderson, Arthur, M.D., C.B., Ex-Inspector-General of Hospitals, Pitlochry	
1874		Anderson, John, M.D., LL.D., F.R.S., Superintendent of the Indian Museum, and Professor of Comparative Anatomy in the Medical College, Calcutta, 71 Harrington Gardens, Lond.	
1883		* Anderson, Robert Rowand, LL.D., 19 St Andrew Square	10
1883	P.	Andrews, Thomas, F.R.S., F.C.S., Memb. Inst. C.E., Ravencrag, Wortley, near Sheffield	
1881		Anglin, A. Hallam, M.A., LL.D., M.R.I.A., Professor of Mathematics, Queen's College, Cork, Brighton Villas, Western Road, Cork	
1867		* Annandale, Thomas, M.D., F.R.C.S.E., Professor of Clinical Surgery in the University of Edinburgh, 34 Charlotte Square	
1883		Archibald, John, M.B., C.M., Lynton House, Brixton Rise, London	
1886		* Armstrong, George Frederick, Professor of Engineering in the University of Edinburgh	15
1849		Argyll, His Grace the Duke of, K.T., D.C.L., F.R.S. (HON. VICE-PRES.), Inveraray Castle	
1887		* Ashdown, Herbert H., M.B., 49 Upper Bedford Place, Russell Square, London	
1885		* Baildon, H. Bellyse, B.A., Duncliffe, Murrayfield, Edinburgh	
1879		* Bailey, James Lambert, Royal Bank of Scotland, Ardrossan	
1875		* Bain, Sir James, 3 Park Terrace, Glasgow	20
1843		Balfour, Colonel David, of Balfour and Trenabie, Balfour Castle, Kirkwall	
1879		* Balfour, George W., M.D., LL.D., F.R.C.P.E., 7 Walker Street	
1877	P.	* Balfour, I. Bayley, Sc.D., M.D., C.M., F.R.S., Sherardian Prof. of Botany in the Univ. of Oxford	

Date of Election			
1870		* Balfour, Thomas A. G., M.D., F.R.C.P.E., 51 George Square	
1886		* Barclay, A. J. G., M.A., 5 Ethel Terrace	25
1872		* Barclay, George, M.A., 17 Coates Crescent	
1883		* Barclay, G. W. W., M.A., 40 Princes Street	
1887		Barlow, W. H., Memb. Inst. C.E., High Combe, Old Charlton, Kent	
1882		Barnes, Henry, M.D., 6 Portland Square, Carlisle	
1874		Barrett, William F., M.R.I.A., Professor of Physics, Royal College of Science, Dublin	30
1887		* Bartholomew, J. G., 32 Royal Terrace	
1878		Bateman, John Frederic La Trobe, Memb. Inst. C.E., F.R.S., F.G.S., F.R.G.S., 18 Abingdon Street, Westminster	
1857		Batten, Edmund Chisholm, of Aigas, M.A., 16 Pelham Crescent, South Kensington, London	
1880		* Bayly, General John, C.B., R.E., 58 Palmerston Place	
1882	P.	Beddard, Frank E., M.A. Oxon., Prosector to the Zoological Society of London, Zoological Society's Gardens, Regent's Park, London	35
1887		* Begg, Ferdinand Faithful, 6 Draper's Gardens, London	
1886		* Bell, A. Beatson, Chairman of Prison Commissioners, 130 George Street	
1874		* Bell, Joseph, M.D., F.R.C.S.E., 2 Melville Crescent	
1876		* Belcombe, Rev. F. E., 14 Merchiston Avenue	
1887		* Bernard, J. Mackay, 25 Chester Street	40
1875		Bernstein, Ludwik, M.D., Lismore, New South Wales	
1881		* Berry, Walter, Danish Consul-General, 11 Atholl Crescent	
1880		* Birch, De Burgh, M.D., Professor of Physiology, University College, Leeds, 16 De Grey Terrace, Leeds	
1884		* Black, Rev. John S., 6 Oxford Terrace	
1850		Blackburn, Hugh, M.A., LL.D., Emeritus Professor of Mathematics in the University of Glasgow, Roshven, Ardgour	45
1863	P.	Blackie, John S., Emeritus Prof. of Greek in the University of Edin., 9 Douglas Crescent	
1862		Blaikie, The Rev. W. Garden, M.A., D.D., LL.D., Professor of Apologetics and Pastoral Theology, New College, Edinburgh, 9 Palmerston Road	
1878	P.	* Blyth, James, M.A., Professor of Natural Philosophy in Anderson's College, Glasgow	
1884		Bond, Francis T., M.D., B.A., M.R.C.S., 1 Beaufort Buildings, Spa, Gloucester	
1872		* Bottomley, J. Thomson, M.A., Lecturer on Nat. Philosophy in the Univ. of Glasgow	50
1869		* Bow, Robert Henry, C.E., 7 South Gray Street	
1886		* Bower, Frederick O., M.A., F.L.S., Regius Professor of Botany in the University of Glasgow, 45 Kerrsland Terrace, Hillhead, Glasgow	
1884		Bowman, Frederick Hungerford, D.Sc., F.R.A.S., F.C.S., F.L.S., F.G.S., West Mount, Halifax, Yorkshire	
1871		* Boyd, Sir Thomas J., Chairman of the Scottish Fishery Board, 41 Moray Place	
1873		* Boyd, William, M.A., Peterhead	55
1886		* Bramwell, Byrom, M.D., F.R.C.P.E., 23 Drumsheugh Gardens	
1886		Brittle, John Richard, Memb. Inst. C.E., Vanbrugh Hill, Blackheath, Kent	
1877		Broadrick, George, Memb. Inst. C.E., The Hall Cross, Doncaster	
1887		* Brown, A. B., C.E., 19 Douglas Crescent	
1864	K. B. P.	* Brown, Alex. Crum, M.D., D.Sc., F.R.C.P.E., F.R.S. (SECRETARY), Professor of Chemistry in the University of Edinburgh, 8 Belgrave Crescent	60
1881		* Brown, J. A. Harvie, of Quarter, Dunipace House, Larbert, Stirlingshire	
1883		* Brown, J. Graham, M.D., C.M., F.R.C.P.E., 16 Ainslie Place	

Date of Election.			
1885		* Brown, J. Macdonald, M.B., F.R.C.S.E., 6 Atholl Place	
1861	P.	Brown, Rev. Thomas, 16 Carlton Street	
1870		Browne, Sir Jas. Crichton, M.D., LL.D., 7 Cumberland Ter., Regent's Park, Lond.	65
1883		* Bruce, Alexander, M.A., M.B., M.R.C.P.E., 16 Alva Street	
1878		Brunlees, Sir James, Memb. Inst. C.E., 5 Victoria Street, Westminster	
1867		* Bryce, A. H., D.C.L., LL.D., 42 Moray Place	
1869	B. P.	* Buchan, Alexander, M.A., LL.D., Secretary to the Scottish Meteorological Society (CURATOR OF LIBRARY), 72 Northumberland Street	
1870	P.	* Buchanan, John Young, M.A., F.R.S., 10 Moray Place	70
1882		* Buchanan, T. Ryburn, M.A., M.P. for the City of Edinburgh, 10 Moray Place	
1887		* Buist, J.B., M.D., F.R.C.P.E., 1 Clifton Terrace	
1887		* Burnet, John James, Architect, 1 Granby Place, Hillhead, Glasgow	
1887		* Burton, Cosmo Innes, B.Sc., F.C.S., 6 Montpellier, Viewforth, Edinburgh	
1883		* Butcher, S. H., M.A., LL.D., Professor of Greek in the University of Edinburgh, 27 Palmerston Place	75
1887		* Cadell, H. M., B.Sc., H.M. Geological Survey, 13 Douglas Crescent	
1869		* Calderwood, Rev. H., LL.D., Professor of Moral Philosophy in the University of Edinburgh, Napier Road, Merchiston	
1879		* Calderwood, John, F.I.C., Belmont Works, Battersea, London	
1878		Campbell, John Archibald, M.D., Garland's Asylum, Carlisle	
1887		* Capstick, J. W., Lecturer in Mathematics and Physics, University College, Dundee	80
1874		Carrington, Benjamin, M.D., Eccles, Lancashire	
1882		* Cay, W. Dyce, Memb. Inst. C.E., 107A Princes Street	
1876		* Cazenove, The Rev. John Gibson, M.A., D.D., 22 Alva Street, Chancellor of St Mary's Cathedral	
1885		* Chambers, Robert, 10 Claremont Crescent	
1866		* Chalmers, David, Redhall, Slateford	85
1874		* Chiene, John, M.D., F.R.C.S.E., Professor of Surgery in the University of Edinburgh, 26 Charlotte Square	
1875		* Christie, John, 19 Buckingham Terrace	
1872		Christie, Thomas B., M.D., F.R.C.P.E., Royal India Asylum, Ealing, London	
1880	K. P.	* Chrystal, George, M.A., LL.D., Prof. of Mathematics in the Univ. of Edin., 5 Belgrave Crescent	
1875		* Clark, Robert, 7 Learmonth Terrace	90
1886	P.	* Clark, The Right Hon. Sir Thomas, Bart., Lord Provost of Edinburgh, 11 Melville Crescent	
1863		Cleghorn, Hugh F. C., of Stravithie, M.D., LL.D., F.L.S., St Andrews, United Service Club, 14 Queen Street	
1875		* Clouston, T. S., M.D., F.R.C.P.E., Tipperlin House, Morningside	
1882		* Coats, Sir Peter, of Auchendrane, President of the Glasgow and West of Scotland Horticultural Society, Auchendrane, Ayr	
1887		* Cockburn, John, 6 Atholl Crescent	95
1887		* Coleman, Joseph James, Ardarroch, Bearsden, Glasgow	
1886		Connan, Daniel M., M.A., Education Department, Cape of Good Hope	
1872		* Constable, Archibald, 11 Thistle Street	
1863		Cowan, Charles, of Westerlea, Murrayfield	
1879		* Cox, Robert, of Gorgie, M.A., 34 Drumsheugh Gardens	100

Date of Election.			
1875		* Craig, William, M.D., F.R.C.S.E., 7 Bruntsfield Place	
1886		* Croom, John Halliday, M.D., 25 Charlotte Square	
1887		* Crawford, William Caldwell, Lockharton Gardens, Slateford, Edinburgh	
1887		* Cumming, A. S., M.D., 18 Ainslie Place	
1878		* Cunningham, Daniel John, M.D., Professor of Anatomy in Trinity College, 69 Harcourt Street, Dublin	105
1886		* Cunningham, David, Memb. Inst. C.E., Harbour Chambers, Dock Street, Dundee	
1877		* Cunningham, George Miller, 2 Ainslie Place	
1884	P.	* Cunningham, J. T., B.A., Marine Biological Laboratory, Plymouth	
1871		* Cunynghame, R. J. Blair, M.D., 6 Walker Street	
1841	P.	Dalmahoy, James, 9 Forres Street	110
1878		* Dalziel, John Grahame, 2 Melville Terrace, Pollokshields, Glasgow	
1885		* Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, 3 Great King Street	
1867		* Davidson, David, Somerset Lodge, Wimbledon Common, Wimbledon	
1848		Davidson, Henry, Muirhouse, Davidson's Mains	
1884		Davy, Richard, M.B., F.R.C.S., Surgeon to the Westminster Hospital, 33 Welbeck Street, Cavendish Square, London	115
1870		* Day, St John Vincent, C.E., 115 St Vincent Street, Glasgow, and 12 Rothesay Place, Edin.	
1876		* Denny, Peter, Memb. Inst. C.E., Dumbarton	
1869	P.	* Dewar, James, M.A., F.R.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London	
1869	P.	* Dickson, Alexander, M.D., Professor of Botany in the University of Edinburgh, 11 Royal Circus	
1884		* Dickson, Charles Scott, Advocate, 59 Northumberland Street	120
1876	P.	* Dickson, J. D. Hamilton, M.A., Fellow and Tutor, St Peter's College, Cambridge	
1869		* Dickson, William, 38 York Place	
1863	P.	Dittmar, W., LL.D., F.R.S., Professor of Chemistry, Anderson's College, Glasgow	
1885		Dixon, J. M., M.A., Professor of English Literature in the University of Tokio, Japan	
1881		* Dobbin, Leonard, Ph.D., 16 Kilmaurs Road	125
1867	P.	* Donaldson, J., M.A., LL.D., Principal of the United College of St Salvador and St Leonard, St Andrews	
1882		* Dott, D. B., Memb. Pharm. Soc., 7 Victoria Terrace, Musselburgh	
1866		* Douglas, David, 22 Drummond Place	
1878		Drew, Samuel, M.D., D.Sc., Chapelton, near Sheffield	
1880		* Drummond, Henry, F.G.S., Prof. of Nat. History in the Free Church College, Glasgow	130
1860		Dudgeon, Patrick, of Cargen, Dumfries	
1863	P.	Duncan, J. Matthews, M.A., M.D., F.R.C.P.E., LL.D., F.R.S., 71 Brook Street, London	
1870		* Duncan, John, M.D., F.R.C.P.E., F.R.C.S.E., 8 Ainslie Place	
1876		* Duncan, James, of Benmore, Kilmun, 9 Mincing Lane, London	
1878		* Duncanson, J. J. Kirk, M.D., F.R.C.P.E., 22 Drumsheugh Gardens	135
1859		Duns, Rev. Professor, D.D., New College, Edinburgh, 14 Greenhill Place	
1874		* Durham, William, Seaforth House, Portobello	
1869		* Elder, George, Knock Castle, Wemyss Bay, Greenock	
1885		* Elgar, Francis, LL.D., The Admiralty, London	
1875		Elliot, Daniel G., New York	140

Date of Election.			
1880		* Elliot, T. Armstrong, M.A., 6 Sanderson Road, Newcastle-on-Tyne	
1855		Etheridge, Robert, F.R.S., Assistant-Keeper of the Geological Department at the British Museum of Natural History, 14 Carlyle Square, Chelsea, London	
1884		* Evans, William, F.F.A., 18A Morningside Park, Edinburgh	
1863	P.	Everett, J. D., M.A., D.C.L., F.R.S., Professor of Natural Philosophy, Queen's College, Belfast	
1879		* Ewart, James Cossar, M.D., F.R.C.S.E., Professor of Natural History, University of Edinburgh, 3 Great Stuart Street	145
1878	P.	* Ewing, James Alfred, B.Sc., F.R.S., Professor of Engineering and Drawing in University College, Dundee	
1875		Fairley, Thomas, Lecturer on Chemistry, 8 Newton Grove, Leeds	
1866		* Falshaw, Sir James, Bart., Assoc. Inst. C.E., 14 Belgrave Crescent	
1859		Fayrer, Sir Joseph, K.C.S.I., M.D., F.R.C.P.L., F.R.C.S.L. and E., LL.D., F.R.S., Honorary Physician to the Queen, 53 Wimpole Street, London	
1883		* Felkin, Robert W., M.D., F.R.G.S., Fellow of the Anthropological Society of Berlin, 20 Alva Street, Edinburgh	150
1868		* Ferguson, Robert M., Ph.D., 12 Moray Place	
1874		* Ferguson, William, of Kinmundy, F.L.S., F.G.S., 21 Manor Place, Edinburgh, and Kinmundy House, Mintlaw	
1886		Field, C. Leopold, F.C.S., Upper Marsh, Lambeth, London	
1852		Fleming, Andrew, M.D., Deputy Surgeon-General, 3 Napier Road	
1876		* Fleming, J. S., 16 Grosvenor Crescent	155
1880		* Flint, Robert, D.D., Corresponding Member of the Institute of France, Professor of Divinity in the University of Edinburgh (VICE-PRESIDENT), Johnstone Lodge, 54 Craigmillar Park	
1872		* Forbes, G., Professor, M.A., F.R.S., F.R.A.S., M.S.T.E. and E., 34 Great George Street, Westminster	
1859		Forlong, Major-Gen. J. G., F.R.G.S., R.A.S., Assoc. C.E., &c., 11 Douglas Crescent	
1828		Foster, John, Liverpool	
1887		Fowler, Sir John, Memb. Inst. C.E., Thornwood Lodge, Kensington, London	160
1858		Fraser, A. Campbell, M.A., LL.D., D.C.L., Professor of Logic and Metaphysics in the University of Edinburgh, 20 Chester Street	
1867	B. P.	* Fraser, Thomas R., M.D., F.R.C.P.E., F.R.S., Professor of Materia Medica in the University of Edinburgh, 13 Drumsheugh Gardens	
1885		* Fraser, A. Y., M.A., Secretary to the Mathematical Society of Edinburgh, 4 Mayfield Road	
1867		Gayner, Charles, M.D., Oxford	
1880	P.	* Geddes, Patrick, Assistant to the Professor of Botany in the University of Edinburgh, and Lecturer on Zoology, 6 James' Court, Lawnmarket	165
1861	B. P.	Geikie, Archibald, LL.D., F.R.S., F.G.S., Director of the Geological Surveys of Great Britain, and Head of the Geological Museum, 28 Jermyn Street, London	
1871	B. P.	* Geikie, James, LL.D., F.R.S., F.G.S., Professor of Geology in the University of Edinburgh, 31 Merchiston Avenue	
1881		* Gibson, G. A., D.Sc., M.D., F.R.C.P.E., 17 Alva Street	
1877		* Gibson, John, Ph.D., 29 Greenhill Gardens	
1885	P.	* Gibson, R. J. Harvey, M.A., Lecturer on Botany, Victoria University, 98 Coltart Road, Prince's Park, Liverpool	170
1887		* Gilmour, William, 10 Elm Row	
1879		* Gilray, Thomas, M.A., Professor of English Language and Literature and Modern History in University College, Dundee	

Date of Election.			
1880		* Gilruth, George Ritchie, Surgeon, 48 Northumberland Street	
1850		Gosset, Major-General W. D., R.E., 70 Edith Road, West Kensington, London	
1867		* Graham, Andrew, M.D., R.N., Army and Navy Club, 36 Pall Mall, London	175
1880		* Graham, James, 195 Bath Street, Glasgow	
1851		Grant, The Rev. James, D.D., D.C.L., 15 Palmerston Place	
1883		* Gray, Andrew, M.A., Professor of Physics in University College, Bangor, North Wales	
1880	P.	Gray, Thomas, B.Sc., 17 Broomhill Avenue, Partick	
1886		* Greenfield, W. S., M.D., Professor of General Pathology in the University of Edinburgh, 7 Heriot Row	180
1872		* Grieve, David, 19 Abercorn Terrace, Portobello	
1884		* Grieve, John, M.A., M.D., F.L.S., 212 St Vincent Street, Glasgow	
1886		* Griffiths, Arthur Bower, Ph.D., Principal and Lecturer on Chemistry in the School of Science of the City and County of Lincoln, 15 Broadgate, Lincoln	
1883		Gunning, R. H., M.D., LL.D., 12 Addison Crescent, Kensington	
1886		* Haddington, The Right Hon. the Earl of, Tyninghame House, Haddington	185
1867		* Hallen, James H. B., F.R.C.S.E., F.R.P.S.E., Inspecting Veterinary Surgeon in H.M. Indian Army, Pebworth, near Stratford-on-Avon	
1881	P.	* Hamilton, D. J., M.B., F.R.C.S.E., Professor of Pathological Anatomy in the University of Aberdeen, 1A Albyn Place, Aberdeen	
1876	P.	* Hannay, J. Ballantyne, Cove Castle, Loch Long	
1886		* Hare, Arthur W., M.B., F.R.C.E., Professor of Surgery, Owens College, Victoria University, 3 Adelphi Terrace, Salford, Manchester	
1869		Hartley, Sir Charles A., K.C.M.G., Memb. Inst. C.E., 26 Pall Mall, London	190
1877		Hartley, Walter Noel, F.R.S., Professor of Chemistry, Royal College of Science for Ireland, Dublin	
1870		* Harvey, Thomas, M.A., LL.D., Rector of the Edinburgh Academy, 32 George Square	
1880	P.	* Haycraft, J. Berry, M.B., B.Sc., Lecturer on Physiology in the University of Edinburgh	
1875		Hawkshaw, Sir John, Memb. Inst. C.E., F.R.S., F.G.S., 33 Great George St., Westminster	
1870		Heathfield, W. E., F.C.S., 1 Powis Grove, Brighton	195
1862		Hector, Sir James, C.M.G., M.D., F.R.S., Director of the Geological Survey, Wellington, New Zealand	
1876	K. P.	* Heddle, M. Forster, M.D., Emeritus Professor of Chemistry in the University of St Andrews	
1884		* Henderson, John, jun., 4 Crown Terrace, Dowanhill, Glasgow	
1881	N. P.	* Herdman, W. A., D.Sc., Professor of Natural History in University College, Liverpool	
1871		Higgins, Charles Hayes, LL.D., Alfred House, Birkenhead	200
1859		Hills, John, Lieut.-Colonel, Bombay Engineers, C.B., United Service Club, London	
1879		Hislop, John, Secretary to the Department of Education, Wellington, New Zealand	
1885		Hodgkinson, W. R., Ph.D., F.I.C., F.C.S., Professor of Chemistry and Physics at the Royal Military Academy and Royal Artillery College, Woolwich, 75 Vanbrugh Park, Blackheath, London	
1828	P.	Home, David Milne, of Milne-Graden, LL.D., F.G.S. (VICE-PRESIDENT), 10 York Place	
1879		* Hood, Thomas H. Cockburn, F.G.S., Walton Hall, Kelso	205
1881	P.	* Horne, John, F.G.S., Geological Survey of Scotland, 41 Southside Road, Inverness	
1883	P.	* Hoyle, William Evans, M.A., M.R.C.S., Office of Challenger Commission, 32 Queen Street	
1886		Hunt, Rev. H. G. Bonavia, Mus. D. Dublin, Mus. B. Oxon., F.L.S., La Belle Sauvage, London	
1872		* Hunter, Major Charles, Pläs Cöch, Llanfair, Anglesea, and 17 St George's Sq., London	
1887		* Hunter, James, F.R.C.S.E., F.R.A.S., 20 Craigmillar Park	210

Date of Election.			
1887		* Hunter, William, M.D., 8 West Maitland Street	
1864		* Hutchison, Robert (Carlowrie Castle), and 29 Chester Street	
1855		Inglis, Right Hon. John, LL.D., D.C.L., Lord Justice-General of Scotland, and Chancellor of the University of Edinburgh, 30 Abercromby Place	
1882		* Inglis, J. W., Memb. Inst. C.E., Myrtle Bank, Trinity	
1874		* Irvine, Alex. Forbes, of Drum, LL.D., Advocate, Sheriff of Argyll (VICE-PRESIDENT), 25 Castle Terrace	215
1886		* Irvine, Robert, Royston, Granton, Edinburgh	
1875		Jack, William, M.A., Professor of Mathematics in the University of Glasgow	
1882		* Jamieson, A., Prof., Memb. Inst. C.E., Principal, the Glasgow and West of Scotland Technical College, Glasgow	
1860		Jamieson, George A., 24 St Andrew Square	
1880		Japp, A. H., LL.D., The Limes, Elmstead, near Colchester	220
1865		* Jenner, Charles, Easter Duddingston Lodge	
1869		Johnston, John Wilson, M.D., Surgeon-Major, 11 Windsor Street	
1867		* Johnston, T. B., F.R.G.S., Geographer to the Queen, 9 Claremont Crescent	
1874		Jones, Francis, Lecturer on Chemistry, Monton Place, Manchester	
1877		* Jolly, William, H.M. Inspector of Schools, F.G.S., Ardgowan, Pollokshields	225
1866		* Keiller, Alexander, M.D., F.R.C.P.E., LL.D., 21 Queen Street	
1886	P.	* Kidston, Robert, F.G.S., 24 Victoria Place, Stirling	
1877		* King, The Right Hon. Sir James, of Campsie, LL.D., Lord Provost of Glasgow, 12 Claremont Terrace, Glasgow	
1880		* King, W. F., Lonend, Russell Place, Trinity	
1883		* Kinnear, The Hon. Lord, one of the Senators of the College of Justice, 2 Moray Place	230
1878		* Kintore, The R. H. the Earl of, M.A. Cantab., Keith Hall, Inglismaldie Castle, Laurencekirk	
1875		* Kirkwood, Anderson, LL.D., 7 Melville Terrace, Stirling	
1880	P.	* Knott, C. G., D.Sc., Prof. of Natural Philosophy in the Imperial University of Tokio, Japan	
1875		* L'Amy, John Ramsay, of Dunkenny, Forfarshire, 107 Cromwell Road, London	
1886		* Laing, Rev. George, 17 Buckingham Terrace	235
1878		* Lang, P. R. Scott, M.A., B.Sc., Professor of Mathematics in the University of St Andrews	
1885		* Laurie, A. P., B.A., B.Sc., Nairne Lodge, Duddingstone, Edinburgh	
1870		* Laurie, Simon S., M.A., Professor of Education in the University of Edinburgh, Nairne Lodge, Duddingstone	
1881		* Lawson, Robert, M.D., Deputy-Commissioner in Lunacy, 24 Mayfield Terrace	
1872		* Lee, Alexander H., C.E., Blairhoyle, Stirling	240
1872		* Lee, The Hon. Lord, one of the Senators of the College of Justice, Duddingstone House, Edinburgh	
1882		* Leslie, Alexander, Memb. Inst. C.E., 12 Greenhill Terrace	
1883		* Leslie, George, M.B., C.M., Old Manse, Falkirk	
1863		Leslie, Hon. G. Waldegrave, Leslie House, Leslie	
1858		Leslie, James, Memb. Inst. C.E., 2 Charlotte Square	245
1874	P.	* Letts, E. A., Ph.D., F.I.C., F.C.S., Professor of Chemistry, Queen's College, Belfast	
1870	B. P.	* Lister, Sir Joseph, Bart., M.D., F.R.C.S.L., F.R.C.S.E., LL.D., D.C.L., F.R.S., Professor of Clinical Surgery, King's College, Surgeon Extraordinary to the Queen, 12 Park Crescent, Portland Place, London	

Date of Election.			
1882		* Livingston, Josiah, 4 Minto Street	
1861		Lorimer, James, M.A., Advocate, Professor of Public Law in the University of Edinburgh, 1 Bruntsfield Crescent	
1884		* Low, George M., Actuary, 19 Learmonth Terrace	250
1849		Lowe, W. H., M.D., F.R.C.P.E., Woodcote, Inner Park, Wimbledon	
1886		Lyster, George Fosbery, Memb. Inst. C.E., Gisburn House, Liverpool	
1855		Macadam, Stevenson, Ph.D., Lecturer on Chemistry, Surgeons' Hall, Edinburgh, 11 East Brighton Crescent, Portobello	
1887		M'Aldowie, Alexander M., M.D., Brook Street, Stoke-on-Trent	
1885		* M'Bride, Charles, M.D., Wigtown	255
1883		* M'Bride, P., M.D., F.R.C.P.E., 16 Chester Street	
1867		* M'Candlish, John M., W.S., 27 Drumsheugh Gardens	
1886		* Macdonald, The Right Hon. J. H. A., C.B., Q.C., M.P., LL.D., M.S.T.E. and E., Lord Advocate of Scotland, 15 Abercromby Place	
1886		* Macdonald, William J., M.A., 6 Lockharton Terrace	
1847		Macdonald, W. Macdonald, of St Martin's, Perth	260
1878		* MacDougall, Alan, Memb. I.C.E., Mail Building, 52 King Street West, Toronto, Canada	
1878	P.	Macfarlane, Alex., M.A., D.Sc., LL.D., Professor of Physics in the University of the State of Texas, Austin, Texas	
1885	P.	* Macfarlane, J. M., D.Sc., 15 Scotland Street	
1877		* Macfie, Robert A., Dreghorn Castle, Colinton	
1878		* M'Gowan, George, F.I.C., Ph.D., University College of North Wales, Bangor	265
1886		* MacGregor, Rev. J., D.D., 11 Cumin Place, Grange	
1880	P.	MacGregor, J. Gordon, M.A., D.Sc., Professor of Physics in Dalhousie College, Halifax, Nova Scotia	
1879		* M'Grigor, Alexander Bennett, LL.D., 19 Woodside Terrace, Glasgow	
1869	N. P.	* M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Professor of Natural History in the University of St Andrews, 2 Abbotsford Crescent, St Andrews	
1882		* Mackay, John Sturgeon, M.A., LL.D., Mathematical Master in the Edinburgh Academy, 69 Northumberland Street	270
1873	P.	* M'Kendrick, John G., M.D., F.R.C.P.E., F.R.S., Professor of the Institutes of Medicine in the University of Glasgow	
1840		Mackenzie, John, New Club, Princes Street	
1843	P.	Maclagan, Sir Douglas, M.D., President of the Royal College of Physicians, Edinburgh, and F.R.C.S.E., Professor of Medical Jurisprudence in the University of Edinburgh (VICE-PRESIDENT), 28 Heriot Row	
1853		Maclagan, General R., Royal Engineers, 4 West Cromwell Road, S. Kensington, London, S.W.	
1869		* Maclagan, R. Craig, M.D., 5 Coates Crescent	275
1864		* M'Lagan, Peter, of Pumpherston, M.P., Clifton Hall, Ratho	
1869		* M'Laren, The Hon. Lord, LL.D. Edin. and Glasg., F.R.A.S., one of the Senators of the College of Justice (VICE-PRESIDENT), 46 Moray Place	
1870		* Macleod, Sir George H. B., M.D., F.R.C.S.E., Regius Prof. of Surgery in the University of Glasgow, and Surgeon in Ordinary to the Queen in Scotland, 10 Woodside Crescent, Glasgow	
1876		* Macleod, Rev. Norman, D.D., 7 Royal Circus	
1883		* Macleod, W. Bowman, L.D.S., 16 George Square	280
1872		* Macmillan, Rev. Hugh, D.D., LL.D., Seafield, Greenock	
1876		* Macmillan, John, M.A., B.Sc., Mathematical Master, Perth Academy	

Date of Election.			
1884		* Macpherson, Rev. J. Gordon, M.A., D.Sc., Ruthven Manse, Meigle	
1883		* M'Roberts, George, F.C.S., Ardeer, Stevenston, Ayrshire	
1858		Malcolm, R. B., M.D., F.R.C.P.E., 126 George Street	285
1880	P.	Marsden, R. Sydney, M.B., C.M., D.Sc., F.I.C., F.C.S., Pembroke House, King Street, Stockton-on-Trent	
1882	P.	Marshall, D. H., M.A., Professor of Physics in Queen's University and College, Kingston, Ontario, Canada	
1869		Marshall, Henry, M.D., Clifton, Bristol	
1864		* Marwick, Sir James David, LL.D., Town-Clerk, Glasgow	
1866		* Masson, David, LL.D., Professor of Rhetoric and English Literature in the University of Edinburgh, 58 Great King Street	290
1885	P.	* Masson, Orme, D.Sc., Professor of Chemistry in the University of Melbourne	
1883		* Matthews, James Duncan, Springhill, Aberdeen	
1885		* Mill, Hugh Robt., D.Sc., F.C.S., Scot. Marine Station, Granton, 3 Glenorchy Terrace, Edin.	
1886		* Miller, Hugh, H.M. Geological Survey Office, George IV. Bridge	
1852		Miller, Thomas, M.A., LL.D., Emeritus Rector of Perth Academy, Inchbank House, Perth	295
1885		* Miller, William, S.S.C., 59 George Square	
1833		Milne, Admiral Sir Alexander, Bart., G.C.B., Inveresk	
1886		* Milne, William, M.A., B.Sc., Mathematical and Science Teacher, High School, Glasgow	
1866		* Mitchell, Sir Arthur, K.C.B., M.A., M.D., LL.D., Commissioner in Lunacy, 34 Drummond Pl.	
1865		* Moir, John J. A., M.D., F.R.C.P.E., 52 Castle Street	300
1870		* Moncreiff, The Right Hon. Lord, of Tullibole, Lord Justice-Clerk, LL.D. (Honorary Vice-President), 15 Great Stuart Street	
1871		* Moncrieff, Rev. Canon William Scott, of Fossaway, Christ's Church Vicarage, Bishop-Wearmouth, Sunderland	
1868		* Montgomery, Very Rev. Dean, M.A., D.D., 17 Atholl Crescent	
1887		Moos, Nanabhay A. F., L.C.E., B.Sc., Assistant Professor of Engineering, College of Science, Bombay	
1887		More, Alexander Goodman, M.R.I.A., F.L.S., 77 Leinster Road, Dublin	305
1877	P.	* Morrison, Robert Milner, D.Sc., F.I.C.	
1873		* Muir, M. M. Pattison, Prælector on Chemistry, Caius College, Cambridge	
1874	K.P.	* Muir, Thomas, M.A., LL.D., Mathematical Master, High School, Glasgow, Beechcroft, Bothwell, Glasgow	
1877		Mukhopâdhyay, Âsûtosh, M.A., F.R.A.S., Examiner in Mathematics in the University of Calcutta, Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta	
1870		* Munn, David, M.A., 2 Ramsay Gardens	310
1857		Murray, John Ivor, M.D., F.R.C.S.E., M.R.C.P.E., 24 Huntriss Row, Scarborough	
1877	N. P.	* Murray, John, LL.D., Ph.D., Director of the Challenger Expedition Commission, 32 Queen Street, and United Service Club (Vice-President)	
1887		Muter, John, M.A., F.C.S., Winchester House, 397 Kennington Road, London	
1884		Mylne, R. W., C.E., F.R.S., 7 Whitehall Place, London	
1877		* Napier, John C., Audley Mansions, Grosvenor Square, London	315
1887		* Nasmyth, T. Goodall, M.B., C.M., D.Sc., Foulford, Cowdenbeath, Fife	
1883		* Nelson, Thomas, St Leonard's, Dalkeith Road	
1884		* Newcombe, Henry, F.R.C.S.E., 5 Dalrymple Crescent, Edinburgh	

Date of Election.			
1866		* Nicholson, J. Shield, Professor of Political Economy in the University of Edinburgh, Eden Lodge, Eden Lane, Newbattle Terrace	
1880	P.	* Nicol, W. W. J., M.A., D.Sc., Lecturer on Chemistry, Mason College, Birmingham	320
1878		Norris, Richard, M.D., Professor of Physiology, Queen's College, Birmingham	
1886		Oliver, James, M.D., C.M., M.R.C.P., Assistant Physician, Hospital for Women, 18 Gordon Square, London	
1884		* Omond, Robert Traill, Superintendent of Ben Nevis Observatory, Fort-William, Inverness	
1877		Panton, George A., 95 Colmore Row, Birmingham	
1886		* Paton, D. Noel, M.D., B.Sc., 4 Walker Street	325
1881	N.P.	Peach, B. N., F.G.S., Acting Palæontologist of the Geol. Sury. of Scot., 13 Dalrymple Cres.	
1863		Peddie, Alexander, M.D., F.R.C.P.E., 15 Rutland Street	
1887		* Peddie, William, Assistant to the Professor of Natural Philosophy, Edinburgh University	
1886		* Peebles, D. Bruce, Tay House, Bonnington, Edinburgh	
1869		Pender, John, 18 Arlington Street, Piccadilly, London	330
1883		Phillips, Charles D. F., M.D., 10 Henrietta Street, Cavendish Square, London, W.	
1859	P.	Playfair, The Right Hon. Sir Lyon, K.C.B., M.P., LL.D., F.R.S., 68 Onslow Gardens, London	
1877		Pole, William, Memb. Inst. C.E., Mus. Doc., F.R.S., 31 Parliament Street, Westminster	
1886		* Pollock, Charles Frederick, M.D., F.R.C.S.E., 1 Buckingham Terrace, Hillhead, Glasgow	
1874		Powell, Baden Henry Baden-, Forest Department, India	335
1852		Powell, Eyre B., C.S.I., M.A., 28 Park Road, Haverstock Hill, Hampstead, London	
1880		* Prentice, Charles, C.A., Actuary, 8 St Bernard's Crescent	
1875		Prevost, E. W., Ph.D., The Poplars, Shuttington, Tamworth	
1849		Primrose, Hon. B. F., C.B., 22 Moray Place	
1882		* Pryde, David, M.A., LL.D., Head Master of the Ladies' College, 10 Fettes Row, Edinb.	340
1885		* Pullar, J. F., Rosebank, Perth	
1880		* Pullar, Robert, Tayside, Perth	
1884		Ramsay, E. Peirson, M.R.I.A., F.L.S., C.M.Z.S., F.R.G.S., F.G.S., Fel. of the Imperial and Royal Zool. and Bot. Soc. of Vienna, Curator of Australian Museum, Sydney, N.S.W.	
1882		* Rattray, James Clerk, M.D., 61 Grange Loan	
1885	P.	* Rattray, John, M.A., B.Sc., Natural History Department, British Museum, London	345
1869		Raven, Rev. Thomas Milville, M.A., The Vicarage, Crakehall, Bedale	
1883		* Readman, J. B., 9 Moray Place	
1875		* Richardson, Ralph, W.S., 10 Magdala Place	
1872		Ricarde-Seaver, Major F. Ignacio, Conservative Club, St James' Street, London and 2 Rue Lafitte, Boulevard des Italiens, Paris	
1883		* Ritchie, R. Peel, M.D., F.R.C.P.E., 1 Melville Crescent	350
1877		* Roberton, James, LL.D., Professor of Conveyancing in the University of Glasgow, 1 Park Terrace East, Glasgow	
1880		Roberts, D. Lloyd, M.D., F.R.C.P.L., 23 St John Street, Manchester	
1872		* Robertson, D. M. C. L. Argyll, M.D., F.R.C.S.E., Surgeon Oculist to the Queen for Scotland, and President of the Royal College of Surgeons, 18 Charlotte Square	
1859		Robertson, George, Memb. Inst. C.E., Athenæum Club, Pall Mall, London	
1886		* Robertson, J. P. B., Q.C., M.P., 19 Drumsheugh Gardens	355
1877	P.	* Robinson, George Carr, F.I.C., Lecturer on Chemistry in the Royal Institution, Hull	
1881		* Rogerson, John Johnston, B.A., LL.B., Merchiston Castle Academy	

Date of Election.			
1862	P.	Ronalds, Edmund, LL.D., Bonnington House, Bonnington Road	
1881		Rosebery, The Right Hon. the Earl of, LL.D., Dalmeny	
1880		Rowland, L. L., M.A., M.D., President of the Oregon State Medical Society, and Professor of Physiology and Microscopy in Williamette University, Salem, Oregon	360
1880		* Russell, J. A., M.A., B.Sc., M.B., F.R.C.P.E., Woodville, Canaan Lane	
1869	P.	* Rutherford, Wm., M.D., F.R.C.P.E., F.R.S., Professor of the Institutes of Medicine in the University of Edinburgh, 14 Douglas Crescent	
1863		Sanderson, James, Deputy Inspector-General of Hospitals, F.R.C.S.E., 8 Manor Place	
1864		Sandford, The Right Rev. D. F., LL.D., Bishop of Tasmania	
1849	B. P.	Sang, Edward, C.E., LL.D., Sec. to the Royal Scottish Society of Arts, 31 Mayfield Road	365
1846		Schmitz, Leonard, LL.D., 81 Linden Gardens, London	
1887		* Schulze, Adolf P., 2 Doune Gardens, Kelvinside, Glasgow	
1885		Scott, Alexander, M.A., D.Sc., 4 North Bailey, Durham	
1880		Scott, J. H., M.B., C.M., M.R.C.S., Prof. of Anatomy in the Univ. of Otago, N. Z.	
1875		Scott, Michael, Memb. Inst. C.E., 35 Dudley Road, Tunbridge Wells	370
1864		* Sellar, W. Y., M.A., LL.D., Professor of Humanity in the University of Edinburgh, 15 Buckingham Terrace	
1872		* Seton, George, M.A., Advocate, 42 Greenhill Gardens	
1887		* Sexton, A. H., F.C.S., Professor of Chemistry, College of Science and Arts, Glasgow	
1872		* Sibbald, John, M.D., Commissioner in Lunacy, 3 St Margaret's Road, Whitehouse Loan	
1870		* Sime, James, M.A., South Park, Fountainhall Road	375
1871		* Simpson, A. R., M.D., F.R.C.P.E., Professor of Midwifery in the University of Edinburgh, 52 Queen Street	
1859	P.	Skene, Wm. F., W.S., LL.D., D.C.L., Historiographer-Royal for Scotland, 27 Inverleith Row	
1876		* Skinner, William, W.S., Town-Clerk of Edinburgh, 35 George Square	
1868		* Smith, Adam Gillies, C.A. (TREASURER), 64 Princes Street	
1882	P.	Smith, C. Michie, B.Sc., Professor of Physical Science, Christian College, Madras, India	380
1885		* Smith, George, F.C.S., Polmont Station	
1883		Smith, James Greig, M.A., M.B., 16 Victoria Square, Clifton	
1871		* Smith, John, M.D., LL.D., F.R.C.S.E., President of the Medico-Chirurgical Society, 11 Wemyss Place	
1855		Smith, Robert Mackay, 4 Bellevue Crescent	
1886		* Smith, Major-General Sir R. Murdoch, K.C.M.G., R.E., Director of Museum of Science and Art, Edinburgh	385
1871	P.	* Smith, Rev. W. Robertson, M.A., LL.D., Librarian to the University of Cambridge	
1880		Smith, William Robert, M.D., D.Sc., 74 Great Russell Street, Bloomsbury Square, London	
1846	K.B. P.	Smyth, Piazzi, Professor of Practical Astronomy in the University of Edinburgh, and Astronomer-Royal for Scotland, 15 Royal Terrace	
1880		Sollas, W. J., M.A., D.Sc., late Fellow of St John's College, Cambridge, and Professor of Geology and Mineralogy in the University of Dublin, 4 Clyde Road, Dublin	
1882		* Sorley, James, F.F.A., C.A., 2 Dean Park Crescent	390
1874	P.	* Sprague, T. B., M.A., 29 Buckingham Terrace	
1850	P.	Stark, James, M.D., F.R.C.P.E., of Huntfield, Underwood, Bridge of Allan	
1885		* Steggall, J. E. A., Prof. of Mathematics and Natural Philosophy in University Coll., Dundee	
1886		* Stevenson, C. A., B.Sc., C.E., 45 Melville Street	
1884		* Stevenson, David Alan, B.Sc., C.E., 45 Melville Street	395
1877		* Stevenson, James, F.R.G.S., 4 Woodside Crescent, Glasgow	

Date of Election.			
1868		Stevenson, John J., 18 Queen's Road, Bayswater, London	
1868		Stewart, Major J. H. M. Shaw, Royal Engineers, 61 Lancaster Gate, London, W.	
1878		* Stewart, James R., M.A., 10 Salisbury Road	
1866		* Stewart, T. Grainger, M.D., F.R.C.P.E., Professor of the Practice of Physic in the University of Edinburgh, 19 Charlotte Square	400
1873		* Stewart, Walter, 22 Torphichen Street	
1848		Stirling, Patrick J., LL.D., Kippendavie House, Dunblane	
1877		* Stirling, William, D.Sc., M.D., Brackenbury Professor of Physiology and Histology in Owens College and Victoria University, Manchester	
1823		Stuart, Captain T. D., H.M.I.S.	
1870		* Swan, Patrick Don, Provost of Kirkcaldy	405
1848	P.	Swan, Wm., LL.D., Emeritus Professor of Natural Philosophy in the University of St Andrews, President of the Royal Scottish Society of Arts, Ardchapel, Helensburgh	
1844		Swinton, A. Campbell, of Kimmerghame, LL.D., Duns	
1875		Syme, James, 9 Drumsheugh Gardens	
1885		* Symington, Johnson, M.D., F.R.C.S.E., 2 Greenhill Park	
1872		Tait, the Very Rev. A., D.D., LL.D., Provost of Tuam, Moylough Rectory, County Galway, Ireland	410
1861	K.P.	Tait, P. Guthrie, M.A., Professor of Natural Philosophy in the University of Edinburgh (GENERAL SECRETARY), 38 George Square	
1870		* Tatlock, Robert R., City Analyst's Office, 138 Bath Street, Glasgow	
1872		* Teape, Rev. Charles R., M.A., Ph.D., 15 Findhorn Place	
1873		* Tennent, Robert, 23 Buckingham Terrace	
1885		* Thompson, D'Arcy W., Professor of Natural History in University College, Dundee	415
1884		* Thoms, George Hunter, of Aberlemno, Advocate, Sheriff of the Counties of Orkney and Zetland, 13 Charlotte Square	
1870		Thomson, Rev. Andrew, D.D., 63 Northumberland Street	
1887		* Thomson, Andrew, M.A., D.Sc., Assistant to the Professor of Chemistry in the University College, Dundee, 1 Blackness Crescent, Dundee	
1875	P.	* Thomson, James, LL.D., F.R.S., Professor of Engineering in the University of Glasgow, 2 Florentine Gardens, Hillhead, Glasgow	
1887	P.	* Thomson, J. Arthur, M.A., Lect. on Zoology, School of Medicine, Edin., 10 Kilmaurs Rd.	420
1880		Thomson, John Millar, King's College, London	
1863		Thomson, Murray, M.D., Professor of Chemistry, Thomason College, Roorkee, India	
1870		* Thomson, Spencer C., Actuary, 10 Eglinton Crescent	
1847	K. P. V.J.	Thomson, Sir William, LL.D., D.C.L., F.R.S. (PRESIDENT), Regius Professor of Natural Philosophy in the University of Glasgow, Foreign Associate of Institute of France, and Member of the Prussian Order *Pour le Mérite*	
1882		Thomson, Willliam, M.A., B.Sc., Professor of Mathematics, Victoria College, Stellenbosch, Cape Colony	425
1870		* Thomson, Wm. Burns, F.R.C.P.E., F.R.C.S.E., 110 Newington Green Road, London, N.	
1876		Thomson, William, Royal Institution, Manchester	
1878		Thorburn, Robert Macfie, Uddevalla, Sweden	
1874	N.P.	* Traquair, R. H., M.D., F.R.S., F.G.S., Keeper of the Natural History Collections in the Museum of Science and Art, Edinburgh, 8 Dean Park Crescent	
1874		Tuke, J. Batty, M.D., F.R.C.P.E., 20 Charlotte Square	430
1879		Turnbull, John, of Abbey St Bathans, W.S., 49 George Square	

Date of Election.			
1861	N.P.	Turner, Sir William, M.B., F.R.C.S.E., F.R.S., Professor of Anatomy in the University of Edinburgh, and President of the Royal Physical Society (SECRETARY), 6 Eton Terrace	
1877		* Underhill, Charles E., B.A., M.B., F.R.C.P.E., F.R.C.S.E., 8 Coates Crescent	
1875		Vincent, Charles Wilson, Royal Institution, Albemarle Street, London	
1867		* Waddell, Peter, 5 Claremont Park, Leith	435
1873		* Walker, Robert, M.A., University, Aberdeen	
1886		* Wallace, Robert, Professor of Agriculture and Rural Economy in the University of Edinburgh	
1864		* Wallace, William, Ph.D., City Analyst's Office, 138 Bath Street, Glasgow	
1883		* Watson, Charles, Redhall, Slateford	
1870		* Watson, James, C.A., 45 Charlotte Square	440
1866		* Watson, John K., 14 Blackford Road	
1866		* Watson, Patrick Heron, M.D., F.R.C.P.E., F.R.C.S.E., LL.D., 16 Charlotte Square	
1862	P.	Watson, Rev. Robert Boog, B.A., Free Church Manse, Cardross, Dumbartonshire	
1887		* Webster, H. A., Librarian to the Univ. of Edinb., 7 Duddingstone Park, Portobello	
1873		Welsh, David, Major-General, R.A., 1 Barton Terrace, Dawlish	445
1840		Welwood, Allan A. Maconochie, LL.D., of Meadowbank and Garvoch, Kirknewton	
1882		* Wenley, James A., Treasurer of the Bank of Scotland, 5 Drumsheugh Gardens	
1887		* White, Arthur Silva, Secretary to the Scottish Geographical Society, 22 Duke Street	
1881		Whitehead, Walter, F.R.C.S.E., 202 Oxford Road, Manchester	
1883		Wickham, R. H. B., M.D., F.R.C.S.E., Medical Superintendent, City and County Lunatic Asylum, Newcastle-on-Tyne	450
1887		* Wieland, G. B., Whitehill, Rosewell, Mid-Lothian	
1879		* Will, John Charles Ogilvie, M.D., 305 Union Street, Aberdeen	
1868		* Williams, W., Principal and Professor of Veterinary Medicine and Surgery, New Veterinary College, Leith Walk	
1879		* Wilson, Andrew, Ph.D., Lecturer on Zoology and Comparative Anatomy in the Edinburgh Medical School, 118 Gilmore Place	
1877		* Wilson, Charles E., M.A., LL.D., H.M. Senior Inspector of Schools for Scotland, 19 Palmerston Place	455
1878		* Wilson, Rev. John, M.A., 27 Buccleuch Place	
1875		Wilson, Daniel, LL.D., President of the University of Toronto, and Professor of English Literature in that University	
1882		Wilson, George, M.A., M.D., 23 Claremont Road, Leamington	
1834		Wilson, Isaac, M.D.	
1847		Wilson, John, LL.D., Emeritus Professor of Agriculture in the University of Edinburgh, Sandfield, Tunbridge Wells	460
1870		Winzer, John, Chief Surveyor, Civil Service, Ceylon, 7 Dryden Place, Newington	
1880		* Wise, Thos. Alex., M.D., F.R.C.P.E., F.R.A.S., Thornton, the Beulah, Upper Norwood	
1886		* Woodhead, German Sims, M.D., 6 Marchhall Crescent	
1884		Woods, G. A., M.R.C.S., Carlton House, 57 Houghton Street, Southport	
1864		* Wyld, Robert S., LL.D., 19 Inverleith Row	465
1887		* Yeo, John S., Carrington House, Fettes College	
1882		* Young, Andrew, 22 Elm Row	
1882		* Young, Frank W., F.C.S., Lecturer on Natural Science, High School, Dundee, Woodmuir Park, West Newport, Fife	
1882		* Young, Thomas Graham, Durris, Aberdeenshire	469

LIST OF HONORARY FELLOWS

AT JANUARY 1888.

His Royal Highness The PRINCE OF WALES.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW X.).

Elected.		
1884	Pierre J. van Beneden,	*Louvain.*
1864	Robert Wilhelm Bunsen,	*Heidelberg.*
1867	Michel Eugène Chevreul,	*Paris.*
1888	Rudolph Julius Emmanuel Clausius,	*Bonn.*
1877	Alphonse de Candolle,	*Geneva.*
1883	Luigi Cremona,	*Rome.*
1858	James D. Dana,	*New Haven, Conn.*
1879	Franz Cornelius Donders,	*Utrecht.*
1877	Carl Gegenbaur,	*Heidelberg.*
1879	Asa Gray,	*Harvard University.*
1888	Ernest Haeckel,	*Jena.*
1883	Julius Hann,	*Vienna.*
1884	Charles Hermite,	*Paris.*
1864	Hermann Ludwig Ferdinand von Helmholtz,	*Berlin.*
1879	Jules Janssen,	*Paris.*
1875	August Kekulé,	*Bonn.*
1864	Albert Kölliker,	*Würzburg.*
1875	Ernst Eduard Kummer,	*Berlin.*
1876	Ferdinand de Lesseps,	*Paris.*
1864	Rudolph Leuckart,	*Leipzig.*
1881	Sven Lovén,	*Stockholm.*
1876	Carl Ludwig,	*Leipzig.*
1878	J. N. Madvig,	*Copenhagen.*
1888	Demetrius Ivanovich Mendeléef,	*St Petersburg.*
1886	Alphonse Milne-Edwards,	*Paris.*
1864	Theodore Mommsen,	*Berlin.*
1881	Simon Newcomb,	*Washington.*
1886	H. A. Newton,	*Yale College.*
1874	Louis Pasteur,	*Paris.*
1886	L'Abbé Renard,	*Louvain.*
1881	Johannes Iapetus Smith Steenstrup,	*Copenhagen.*
1878	Otto Wilhelm Struve,	*Pulkowa.*
1886	Tobias Robert Thalen,	*Upsala.*
1874	Otto Torell,	*Lund.*
1868	Rudolph Virchow,	*Berlin.*
1874	Wilhelm Eduard Weber,	*Göttingen.*

Total, 36.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW X.).

Elected.		
1849	John Couch Adams, LL.D., F.R.S., Corresp. Mem. Inst. France,	*Cambridge.*
1835	Sir George Biddell Airy, K.C.B., M.A., LL.D., D.C.L., F.R.S., Foreign Associate Inst. France,	*Greenwich.*
1865	Arthur Cayley, LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. France,	*Cambridge.*
1884	Edward Frankland, D.C.L., LL.D., F.R.S.,	*London.*
1874	John Anthony Froude, LL.D.,	*London.*
1881	The Hon. Justice Sir William Robert Grove, M.A., LL.D., D.C.L., F.R.S.,	*London.*
1883	Sir Joseph Dalton Hooker, K.C.S.I., M.D., LL.D., D.C.L., F.R.S., F.G.S., Corresp. Mem. Inst. France,	*London.*
1884	William Huggins, LL.D., D.C.L., F.R.S.,	*London.*
1876	Thomas Henry Huxley, LL.D., D.C.L., F.R.S., F.L.S., F.Z.S., F.G.S., Corresp. Mem. Inst. France,	*London.*
1867	James Prescott Joule, LL.D., D.C.L., F.R.S., Corresp. Mem. Inst. France,	*Manchester.*
1845	Sir Richard Owen, K.C.B., M.D., LL.D., D.C.L., F.R.S., Foreign Associate Inst. France,	*London.*
1886	The Lord Rayleigh, D.C.L., LL.D., Sec. R.S.,	*London.*
1881	The Rev. George Salmon, D.D., LL.D., D.C.L., F.R.S., Foreign Associate of the Institute of France,	*Dublin.*
1884	J. S. Burdon Sanderson, M.D., LL.D., F.R.S.,	*Oxford.*
1864	George Gabriel Stokes, M.A., LL.D., D.C.L., Pres. R.S., Corresp. Mem. Inst. France,	*Cambridge.*
1874	James Joseph Sylvester, M.A., LL.D., F.R.S., Corresp. Mem. Inst. France,	*Oxford.*
1864	The Right Hon. Lord Tennyson, D.C.L., LL.D., F.R.S., Poet Laureate,	*Isle of Wight.*
1883	Alexander William Williamson, LL.D., F.R.S., V.P.C.S., Corresp. Mem. Inst. France,	*London.*
1883	Colonel Henry Yule, C.B., LL.D., Member of the Council of India,	*London.*

Total, 19.

ORDINARY FELLOWS ELECTED

During Session 1886–87,

Arranged according to the Date of their Election.

6th December 1886.

Âsûtosh Mukhopâdhyay, M.A., F.R.A.S.
Joseph James Coleman.
John James Burnet.

3rd January 1887.

Nanabhay A. F. Moos, L.C.E., B.Sc.

7th February 1887.

J. B. Buist, M.D.
A. B. Brown, C.E.
Ferdinand Faithful Begg.
J. Arthur Thomson, M.A.
Andrew Thomson, M.A., D.Sc.
William Caldwell Crawford.
J. G. Bartholomew.
William Hunter, M.D.
Thomas Goodall Nasmyth, D.Sc.
Sir John Fowler, M. Inst. C.E.
W. H. Barlow, M. Inst. C.E.
John Muter, M.A., F.C.S.

7th March 1887.

Arthur Silva White.
William Peddie, D.Sc
H. M. Cadell, B.Sc.
G. B. Wieland.
A. H. Sexton, F.C.S.

4th April 1887.

J. Mackay Bernard.
Herbert H. Ashdown, M.D.
William Gilmour.
James Hunter, F.R.A.S.
Alexander M. M'Aldowie, M.D.
Alexander Goodman More, M.R.I.A.

2nd May 1887.

H. A. Webster.
John S. Yeo.
John Cockburn.
A. S. Cumming, M.D.
J. W. Capstick.

6th June 1887.

Cosmo Innes Burton, B.Sc., F.C.S.
Adolf P. Schulzt.

FELLOWS DECEASED OR RESIGNED

During Session 1886–87.

ORDINARY FELLOWS DECEASED.

William Brown, F.R.C.S.E.
William Denny, C.E.
Alexander Gibson, Advocate.
Lord Gifford.
Robert Gray.
D. Rutherford Haldane, M.D.
James Pringle, Provost of Leith.
Alexander James Russell, C.S.
Thomas Stevenson, Hon. Vice-President.
Rev. Francis Le Grix White.

RESIGNED.

James Tait Black, Esq.
Donald Crawford, Esq., M.P.
J. B. Brown Morrison, Esq.

FOREIGN HONORARY FELLOWS DECEASED.

Session 1886–87.

Gustav Robert Kirchhoff.
Bernard Studer.
Hermann Kolbe.

LAWS

OF THE

ROYAL SOCIETY OF EDINBURGH.

AS REVISED 20TH FEBRUARY 1882.

LAWS.

[By the Charter of the Society (printed in the *Transactions*, Vol. VI. p. 5), the Laws cannot be altered, except at a Meeting held one month after that at which the Motion for alteration shall have been proposed.]

I.

THE ROYAL SOCIETY OF EDINBURGH shall consist of Ordinary and Honorary Fellows.

Title.

II.

Every Ordinary Fellow, within three months after his election, shall pay Two Guineas as the fee of admission, and Three Guineas as his contribution for the Session in which he has been elected; and annually at the commencement of every Session, Three Guineas into the hands of the Treasurer. This annual contribution shall continue for ten years after his admission, and it shall be limited to Two Guineas for fifteen years thereafter.*

The fees of Ordinary Fellows residing in Scotland.

III.

All Fellows who shall have paid Twenty-five years' annual contribution shall be exempted from further payment.

Payment to cease after 25 years.

IV.

The fees of admission of an Ordinary Non-Resident Fellow shall be £26, 5s., payable on his admission; and in case of any Non-Resident Fellow coming to reside at any time in Scotland, he shall, during each year of his residence, pay the usual annual contribution of £3, 3s., payable by each Resident Fellow; but after payment of such annual contribution for eight years, he shall be exempt

Fees of Non-Resident Ordinary Fellows.

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* A modification of this rule, in certain cases, was agreed to at a Meeting of the Society held on the 3rd January 1831.

At the Meeting of the Society, on the 5th January 1857, when the reduction of the Contributions from £3, 3s. to £2, 2s., from the 11th to the 25th year of membership, was adopted, it was resolved that the existing Members shall share in this reduction, so far as regards their future annual Contributions.

Case of Fellows becoming Non-Resident.

from any further payment. In the case of any Resident Fellow ceasing to reside in Scotland, and wishing to continue a Fellow of the Society, it shall be in the power of the Council to determine on what terms, in the circumstances of each case, the privilege of remaining a Fellow of the Society shall be continued to such Fellow while out of Scotland.

V.

Defaulters.

Members failing to pay their contributions for three successive years (due application having been made to them by the Treasurer) shall be reported to the Council, and, if they see fit, shall be declared from that period to be no longer Fellows, and the legal means for recovering such arrears shall be employed.

VI.

Privileges of Ordinary Fellows.

None but Ordinary Fellows shall bear any office in the Society, or vote in the choice of Fellows or Office-Bearers, or interfere in the patrimonial interests of the Society.

VII.

Numbers Unlimited.

The number of Ordinary Fellows shall be unlimited.

VIII.

Fellows entitled to Transactions.

The Ordinary Fellows, upon producing an order from the TREASURER, shall be entitled to receive from the Publisher, gratis, the Parts of the Society's Transactions which shall be published subsequent to their admission.

IX.

Mode of Recommending Ordinary Fellows.

Candidates for admission as Ordinary Fellows shall make an application in writing, and shall produce along with it a certificate of recommendation to the purport below,* signed by at least *four* Ordinary Fellows, two of whom shall certify their recommendation from personal knowledge. This recommendation shall be delivered to the Secretary, and by him laid before the Council, and shall afterwards be printed in the circulars for three Ordinary Meetings of the Society, previous to the day of election, and shall lie upon the table during that time.

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* "A. B., a gentleman well versed in Science (*or Polite Literature, as the case may be*), being "to our knowledge desirous of becoming a Fellow of the Royal Society of Edinburgh, we hereby "recommend him as deserving of that honour, and as likely to prove a useful and valuable Member."

X.

Honorary Fellows, British and Foreign.

Honorary Fellows shall not be subject to any contribution. This class shall consist of persons eminently distinguished for science or literature. Its number shall not exceed Fifty-six, of whom Twenty may be British subjects, and Thirty-six may be subjects of foreign states.

XI.

Royal Personages.

Personages of Royal Blood may be elected Honorary Fellows, without regard to the limitation of numbers specified in Law X.

XII.

Recommendation of Honorary Fellows.

Honorary Fellows may be proposed by the Council, or by a recommendation (in the form given below*) subscribed by three Ordinary Fellows; and in case the Council shall decline to bring this recommendation before the Society, it shall be competent for the proposers to bring the same before a General Meeting. The election shall be by ballot, after the proposal has been communicated *viva voce* from the Chair at one meeting, and printed in the circulars for two ordinary meetings of the Society, previous to the day of election.

Mode of Election.

XIII.

Election of Ordinary Fellows.

The election of Ordinary Fellows shall only take place at the first Ordinary Meeting of each month during the Session. The election shall be by ballot, and shall be determined by a majority of at least two-thirds of the votes, provided Twenty-four Fellows be present and vote.

XIV.

Ordinary Meetings.

The Ordinary Meetings shall be held on the first and third Mondays of every month from December to July inclusively; excepting when there are five Mondays in January, in which case the Meetings for that month shall be held on its third and fifth Mondays. Regular Minutes shall be kept of the proceedings, and the Secretaries shall do the duty alternately, or according to such agreement as they may find it convenient to make.

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* We hereby recommend______________________________
for the distinction of being made an Honorary Fellow of this Society, declaring that each of us from our own knowledge of his services to (*Literature or Science, as the case may be*) believe him to be worthy of that honour.

(To be signed by three Ordinary Fellows.)

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____
____

To the President and Council of the Royal Society
of Edinburgh.

XV.

The Transactions.

The Society shall from time to time publish its Transactions and Proceedings. For this purpose the Council shall select and arrange the papers which they shall deem it expedient to publish in the *Transactions* of the Society, and shall superintend the printing of the same.

The Council shall have power to regulate the private business of the Society. At any Meeting of the Council the Chairman shall have a casting as well as a deliberative vote.

XVI.

How Published.

The Transactions shall be published in parts or *Fasciculi* at the close of each Session, and the expense shall be defrayed by the Society.

XVII.

The Council.

That there shall be formed a Council, consisting—First, of such gentlemen as may have filled the office of President; and Secondly, of the following to be annually elected, viz.:—a President, Six Vice-Presidents (two at least of whom shall be resident), Twelve Ordinary Fellows as Councillors, a General Secretary, Two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library.

XVIII.

Retiring Councillors.

Four Councillors shall go out annually, to be taken according to the order in which they stand on the list of the Council.

XIX.

Election of Office-Bearers.

An Extraordinary Meeting for the Election of Office-Bearers shall be held on the fourth Monday of November annually.

XX.

Special Meetings; how called.

Special Meetings of the Society may be called by the Secretary, by direction of the Council; or on a requisition signed by six or more Ordinary Fellows. Notice of not less than two days must be given of such Meetings.

XXI.

Treasurer's Duties.

The Treasurer shall receive and disburse the money belonging to the Society, granting the necessary receipts, and collecting the money when due.

He shall keep regular accounts of all the cash received and expended, which shall be made up and balanced annually; and at the Extraordinary Meeting in November, he shall present the accounts for the preceding year, duly audited.

At this Meeting, the Treasurer shall also lay before the Council a list of all arrears due above two years, and the Council shall thereupon give such directions as they may deem necessary for recovery thereof.

XXII.

Auditor.

At the Extraordinary Meeting in November, a professional accountant shall be chosen to audit the Treasurer's accounts for that year, and to give the necessary discharge of his intromissions.

XXIII.

General Secretary's Duties.

The General Secretary shall keep Minutes of the Extraordinary Meetings of the Society, and of the Meetings of the Council, in two distinct books. He shall, under the direction of the Council, conduct the correspondence of the Society, and superintend its publications. For these purposes he shall, when necessary, employ a clerk, to be paid by the Society.

XXIV.

Secretaries to Ordinary Meetings.

The Secretaries to the Ordinary Meetings shall keep a regular Minute-book, in which a full account of the proceedings of these Meetings shall be entered; they shall specify all the Donations received, and furnish a list of them, and of the Donors' names, to the Curator of the Library and Museum; they shall likewise furnish the Treasurer with notes of all admissions of Ordinary Fellows. They shall assist the General Secretary in superintending the publications, and in his absence shall take his duty.

XXV.

Curator of Museum and Library.

The Curator of the Museum and Library shall have the custody and charge of all the Books, Manuscripts, objects of Natural History, Scientific Productions, and other articles of a similar description belonging to the Society; he shall take an account of these when received, and keep a regular catalogue of the whole, which shall lie in the Hall, for the inspection of the Fellows.

XXVI.

Use of Museum and Library.

All Articles of the above description shall be open to the inspection of the Fellows at the Hall of the Society, at such times and under such regulations, as the Council from time to time shall appoint.

XXVII.

Register Book.

A Register shall be kept, in which the names of the Fellows shall be enrolled at their admission, with the date.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND VICTORIA JUBILEE PRIZES.

The above Prizes will be awarded by the Council in the following manner :—

I. KEITH PRIZE.

The Keith Prize, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1887–88 for the "best communication on a scientific subject, communicated, in the first instance, to the Royal Society during the Sessions 1885–86 and 1886–87." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded at the commencement of the Session 1887–88, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 1st June 1888.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented to the Society during the Sessions 1886–87, 1887–88, whether they may have been given in with a view to the prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr Patrick Neill of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate,

1. The Neill Prize, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1888–89.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented to the Society during the three years preceding the 1st May 1888,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

IV. VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. Gunning, is to be awarded triennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize is open to men of science resident in or connected with Scotland.

The first award shall be in the year 1887, and shall consist of a sum of money. In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND VICTORIA JUBILEE PRIZES, FROM 1827 TO 1887.

I. KEITH PRIZE.

1st Biennial Period, 1827–29.—Dr Brewster, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2nd Biennial Period, 1829–31.—Dr Brewster, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3rd Biennial Period, 1831–33.—Thomas Graham, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4th Biennial Period, 1833–35.—Professor J. D. Forbes, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5th Biennial Period, 1835–37.—John Scott Russell, Esq., for his Researches "on Hydrodynamics," published in the Transactions of the Society.

6th Biennial Period, 1837–39.—Mr John Shaw, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7th Biennial Period, 1839–41.—Not awarded.

8th Biennial Period, 1841–43.—Professor James David Forbes, for his papers "on Glaciers," published in the Proceedings of the Society.

9th Biennial Period, 1843–45.—Not awarded.

10th Biennial Period, 1845–47.—General Sir Thomas Brisbane, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11th Biennial Period, 1847–49.—Not awarded.

12th Biennial Period, 1849–51.—Professor Kelland, for his papers "on General Differentiation, including his more recent communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13th Biennial Period, 1851–53.—W. J. Macquorn Rankine, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14th Biennial Period, 1853–55.—Dr Thomas Anderson, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15th Biennial Period, 1855–57.—Professor Boole, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16th Biennial Period, 1857–59.—Not awarded.

17th Biennial Period, 1859–61.—John Allan Broun, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18th Biennial Period, 1861–63.—Professor William Thomson, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19th Biennial Period, 1863–65.—Principal Forbes, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society,

20th Biennial Period, 1865–67.—Professor C. Piazzi Smyth, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21st Biennial Period, 1867–69.—Professor P. G. Tait, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22nd Biennial Period, 1869–71.—Professor Clerk Maxwell, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23rd Biennial Period, 1871–73.—Professor P. G. Tait, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24th Biennial Period, 1873–75.—Professor Crum Brown, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25th Biennial Period, 1875–77.—Professor M. Forster Heddle, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26th Biennial Period, 1877–79.—Professor H. C. Fleeming Jenkin, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II. having appeared in the volume for 1877–78.

27th Biennial Period, 1879–81.—Professor George Chrystal, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28th Biennial Period, 1881–83 —Thomas Muir, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29th Biennial Period, 1883–85.—John Aitken, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

II. MAKDOUGALL-BRISBANE PRIZE.

1st Biennial Period, 1859.—Sir Roderick Impey Murchison, on account of his Contributions to the Geology of Scotland.

2nd Biennial Period, 1860–62.—William Seller, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3rd Biennial Period, 1862–64.—John Denis Macdonald, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4th Biennial Period, 1864–66.—Not awarded.

5th Biennial Period, 1866–68.—Dr Alexander Crum Brown and Dr Thomas Richard Fraser, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6th Biennial Period, 1868–70.—Not awarded.

7th Biennial Period, 1870–72.—George James Allman, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8th Biennial Period, 1872–74.—Professor Lister, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9th Biennial Period, 1874–76.—Alexander Buchan, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10th Biennial Period, 1876–78.—Professor Archibald Geikie, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11th Biennial Period, 1878–80.—Professor Piazzi Smyth, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12th Biennial Period, 1880–82.—Professor James Geikie, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faröes," published in the Transactions of the Society.

13th Biennial Period, 1882–84.—Edward Sang, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculation of Logarithms both of Numbers and Trigonometrical Ratios, —the former communication being published in the Proceedings of the Society.

III. THE NEILL PRIZE.

1st Triennial Period, 1856–59.—Dr W. Lauder Lindsay, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2nd Triennial Period, 1859–62.—Robert Kaye Greville, LL.D., for his Contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3rd Triennial Period, 1862–65.—Andrew Crombie Ramsay, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and Memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geographical Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4th Triennial Period, 1865–68.—Dr William Carmichael M'Intosh, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5th Triennial Period, 1868–71.—Professor William Turner, for his papers "on the great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6th Triennial Period, 1871–74.—Charles William Peach, for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7th Triennial Period, 1874–77.—Dr Ramsay H. Traquair, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8th Triennial Period, 1877–80.—John Murray, for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9th Triennial Period, 1880–83.—Professor Herdman, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10th Triennial Period, 1883–86.—B. N. Peach, Esq., for his Contributions to the Geology and Palaeontology of Scotland, published in the Transactions of the Society.

IV. VICTORIA JUBILEE PRIZE.

1st Triennial Period, 1887–90.—Sir William Thomson, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.

PROCEEDINGS

OF THE

STATUTORY GENERAL MEETING,

22ND NOVEMBER 1886.

STATUTORY MEETING.

HUNDRED AND FOURTH SESSION.

Monday, 22nd November 1886.

At a General Statutory Meeting,

Mr Gray in the Chair.

The Secretary read apologies for absence from the Earl of Haddington, Sir Douglas Maclagan, and Professor M'Intosh.

The Minutes of last General Statutory Meeting of 23rd November 1885 were read, approved, and signed.

The Secretary intimated that Voting Papers as to the Hour of Meeting had been issued to the Fellows of the Society in the following form, with the result now declared:—

I.	Hour of Meeting to remain as hitherto,	110
II.	Hour to be changed from 8 P.M. to 4 P.M.,	50
III.	One of the Meetings in each Month to be held at 4 P.M., the other at 8 P.M.,	83

It was stated on behalf of the Council that occasional extra Meetings, to be held at 4 P.M., would be introduced when a sufficient number of Communications happened to be in the Secretary's hands.

The Chairman named Mr Andrew Young and Professor Duns as Scrutineers of the Balloting Lists. They reported that the following Council was unanimously elected:—

Sir William Thomson, LL.D., F.R.S., President.	
A. Forbes Irvine, Esq. of Drum,	Vice-Presidents.
John Murray, Ph.D.,	
David Milne Home, LL.D.,	
Professor Sir Douglas Maclagan,	
The Hon. Lord Maclaren,	
The Rev. Professor Flint, D.D.,	
Professor Tait, M.A., General Secretary.	
Professor Sir Wm. Turner, F.R.S.,	Secretaries to Ordinary Meetings.
Professor Crum Brown, F.R.S.,	
Adam Gillies Smith, Esq., C.A., Treasurer.	
Alexander Buchan, Esq., M.A., Curator of Library and Museum.	

COUNCILLORS.

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Professor Dickson.
Professor Shield Nicholson.
T. B. Sprague, Esq., M.A.
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Professor M'Intosh.
Robert Gray, Esq.
Arthur Mitchell, Esq., C.B.
Stair A. Agnew, Esq., C.B., M.A.
Robert M. Ferguson, Esq., Ph.D.

The Treasurer's Accounts for the past Session, with the Auditor's Report thereon, were approved.

On the motion of Professor Crum Brown, the Auditor was reappointed.

Mr Forbes Irvine proposed a vote of thanks to the Chairman, which was unanimously agreed to.

William Thomson, *Pres.*

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